Unit 2: Scan Conversion Algorithms:

2.1 Scan Converting a Point and a straight line: DDA Line Algorithm, Bresenham's Line Algorithm

DDA Algorithm (Digital Differential Analyzer)

- It's a line drawing algorithm based on obtaining successive pixel value. i.e. Δx and Δy.
- Basis of DDA method is to take unit step along one of the coordinate.

Eg: if we have $dx = 11$ and $dy = 6$ then we would take unit steps along x-coordinate and compute the steps m along y

$$\text{Slope } (m) = \frac{dy}{dx}$$

if ($m <= 1.0$) then
    let X_step = 1
    $\{dx = 1, dy = 0 \text{ or } 1\}$
else
    ($m > 1.0$)
    let Y_step = 1
    $\{dy = 1, \text{Y\_step} = 0 \text{ or } 1\}$

Consider a line with positive slope:
If slope $(m) \leq 1$ then we sample at unit x interval ($\Delta x = 1$) and compute each successive y values as:

$$\Delta x = 1$$

$$m = \frac{\Delta y}{\Delta x} \quad \therefore \Delta y = m$$

$$= \frac{y_{k+1} - y_k}{1} \rightarrow y_{k+1} - y_k = m$$

$$\therefore y_{k+1} = y_k + m$$

For a line with positive slope if $m > 1$, then we sample at unit y-interval and calculate each successive x-values as: $\Delta y = 1$

$$\Delta x = \frac{\Delta y}{m} = \frac{1}{m}$$

$$m = \frac{dy}{dx}$$

$$= \frac{1}{x_{k+1} - x_k}$$

$$\therefore x_{k+1} = x_k + \frac{1}{m}$$

Algorithm for DDA method of line drawing:

1. Input two end points (x_0, y_0) as left end point and (x_n, y_n) as right end point.
2. Calculate the horizontal deflection $dx = x_n - x_0$ and verticle deflection $dy = y_n - y_0$
3. Test if (absolute (dx) > absolute (dy))

   Steps = absolute (dx)

   else

   Steps = absolute (dy)
4. Calculate $\Delta x = \frac{dx}{steps}$ and ~~[illegible]~~ $\Delta y = \frac{dy}{steps}$
5. Start with (x_0, y_0) to determine offset needed at each step to generate the next pixel position along the line steps times and plot (x_0, y_0) as (x, y)
6. DO

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        for (k=1 to steps)
            {   x = x + del x
                y = y + del y
            }
End do
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Q. Using a DDA algorithm digitize, a line from point P(3,6) to (12,13).

Solution

$$(x_0, y_0) = (3,6)$$
$$(x_n, y_n) = (12,13)$$

$dx = x_n - x_0 = 12 - 3 = 9$

$dy = y_n - y_0 = 13 - 6 = 7$

Here $dx > dy$

$\therefore$ steps $= 9$

$$\Delta x = \frac{dx}{\text{steps}} = \frac{9}{9} = 1$$

$$\Delta y = \frac{dy}{\text{steps}} = \frac{7}{9} = 0.78$$

$$m = \frac{\Delta y}{\Delta x} = \frac{0.78}{1} = 0.78 \text{ i.e. } < 1$$

x_k	y_k	x_{k+1} $(x+\Delta x)$	y_{k+1} $(y+\Delta y)$	Rounded
3	6	4	6.78	(4,7)
4	6.78	5	7.56	(5,8)
5	7.56	6	8.34	(6,8)
6	8.34	7	9.12	(7,9)
7	9.12	8	9.90	(8,10)
8	9.90	9	10.68	(9,11)
9	10.68	10	11.46	(10,11)
10	11.46	11	12.24	(11,12)
11	12.24	12	13.02	(12,13)

Q. plot the line between the points (16,18) to (10,10).

Solution:

$(x_0, y_0) = (16, 18)$

$(x_n, y_n) = (10, 10)$

$dx = x_n - x_0 = |10 - 16| = 6$

$dy = y_n - y_0 = |10 - 18| = 8$

$dy > dx$

$\therefore$ steps $= 8$

Now, $\Delta x = \frac{dx}{steps} = -\frac{6}{8} = -0.75$

$\Delta y = \frac{dy}{steps} = -\frac{8}{8} = -1$

x_k	y_k	x_{k+1} $(x+\Delta x)$	y_{k+1} $(y+\Delta y)$	Rounded
16	18	15.25	17	(15, 17)
15.25	17	14.5	16	(15, 16)
14.5	16	13.75	15	(14, 15)
13.75	15	13	14	(13, 14)
13	14	12.25	13	(12, 13)
12.25	13	11.5	12	(12, 12)
11.5	12	10.75	11	(11, 11)
10.75	11	10	10	(10, 10)

Advantages of DDA algorithm

1. Faster than the direct use of line equation and it does not do any floating point multiplication.

Disadvantages of DDA Algorithm

1. Cummulative error due to precesion in the floating point representation which may cause calculated point to drift away from their actual position.

Bresenham's line Drawing algorithm

- It is an improvement upon DDA algorithm to use integer arithmetic only.
- It is an incremental scan conversion algorithm.
- Advantage is, it uses only integer calculation.
- Basic idea is to find the closest pixel position to the line path.
- Here we test the sign of an integer parameter 'P_k' whose value is proportional to the difference between the separation of the pixel position from the actual line $y = mx + c$ i.e.

$$P_k \propto (d_1 - d_2)$$

Where,

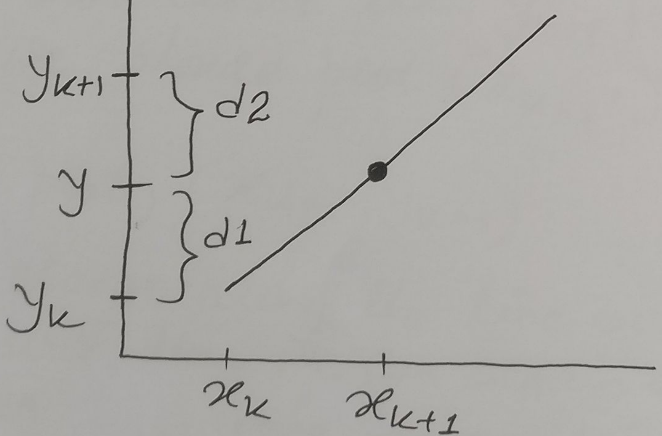
P_k = integer value

$d1, d2$ = Seperation of two pixel.

For slope $m < 1$

Let us consider a line with position slope less than one ($m<1$) starting from (x_1, y_1) and processing left to right, we take sample at unit x-direction (interval) in each successive column (x-position) and plot the pixel whose scan ~~line~~ line y-value is closest to the line path. Suppose x_k and y_k has just plotted, then two coordinate pixels to be plotted are (x_{k+1}, y_k) or (x_{k+1}, y_{k+1}), if $d1$ and $d2$ are the vertical seperation of pixel from the line then the value of y at x_{k+1} or at sampling position (x_{k+1}) is

$$y = mx + c \quad \text{——— } (1)$$



Then $d1 = y - y_k = (mx_{k+1} + C) - y_k$

and $d2 = y_{k+1} - y = (y_k + 1) - (mx_{k+1} - C)$

Now, the difference between two seperation is $d1 - d2$

$$= (mx_k + c) - y_k - (y_k + 1 - x_{k+1} m - c)$$

$$= 2mx_{k+1} - 2y_k + 2c - 1 \text{ ——————— } A$$

Now multiplying Δx on both sides we get,

$$\Delta x (d1 - d2) = 2 \cdot \frac{\Delta y}{\cancel{\Delta x}} \cdot \cancel{\Delta x} \cdot x_{k+1} - 2\Delta x y_k + 2\Delta x c - \Delta x$$

or, $P_k = \Delta x (d1 - d2) = 2\Delta y x_{k+1} - 2y_k \Delta x + 2c\Delta x - \Delta x$

or, $P_k = 2\Delta y (x_k + 1) - 2y_k \cdot \Delta x + 2c\Delta x - \Delta x \quad \left[\because x_{k+1} = x_k + 1\right]$

or, $P_k = 2\Delta y x_k + 2\Delta y - 2y_k \cdot \Delta x + 2c\Delta x - \Delta x$

$$\therefore P_k = 2\Delta y x_k - 2y_k \Delta x + B \text{ ——————— } (ii)$$

where,

$B = 2\Delta y + 2c \cdot \Delta x - \Delta x$ is constant term and is independent of any pixel position

If $P_k < 0$ then $d1 - d2 < 0 \Rightarrow d1 < d2$

so, y_k is more more closer than y_{k+1} from the line path and we should plot (x_{k+1}, y_k)

Else, we should plot (x_{k+1}, y_{k+1})

Now coordinate change along the line occur is unique unit interval, then we can obtain successive decision parameter P_{k+1} using increment integer calculation so, decision parameter for next step $(k+1)$ is:

$$P_{k+1} = 2\Delta y x_{k+1} - 2y_{k+1} \Delta x + B \text{ ——————— } (iii)$$

Now Subtracting eq^n(ii) from eq^n(iii)

$$P_{k+1} - P_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

or, $$P_{k+1} = P_k + 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k) \quad \text{——(iv)}$$

If $P_k < 0$ then, we plot lower pixel

$$x_{k+1} = x_k + 1$$
$$y_{k+1} = y_k$$

Then, $$\boxed{P_{k+1} = P_k + 2\Delta y}$$

Else $P_k \geq 0$ then we plot upper pixel

$$x_{k+1} = x_k + 1$$
$$y_{k+1} = y_k + 1$$

Then, $$\boxed{P_{k+1} = P_k + 2\Delta y - 2\Delta x}$$

For initial decision Parameter

To start from initial point, we need initial point with its initial decision parameter (P_0) which can be derived from equation — A

$$d1 - d2 = 2m x_{k+1} - 2y_k + 2C - 1$$

or, $d1 - d2 = 2m(x_k + 1) - 2y_k + 2C - 1$

or, $d1 - d2 = 2m x_k + 2m - 2y_k + 2C - 1$

or, $d1 - d2 = 2(m x_k - y_k + C) + 2m - 1$

or, $d1 - d2 = 2(m x_k + C - y_k) + 2m - 1$

or, $d1 - d2 = 2(y_k - y_k) + 2m - 1$ $\left[\because y_k = m x_k + C\right]$

or, $d1 - d2 = 2m - 1$

or, $d1 - d2 = 2 \cdot \frac{\Delta y}{\Delta x} - 1$

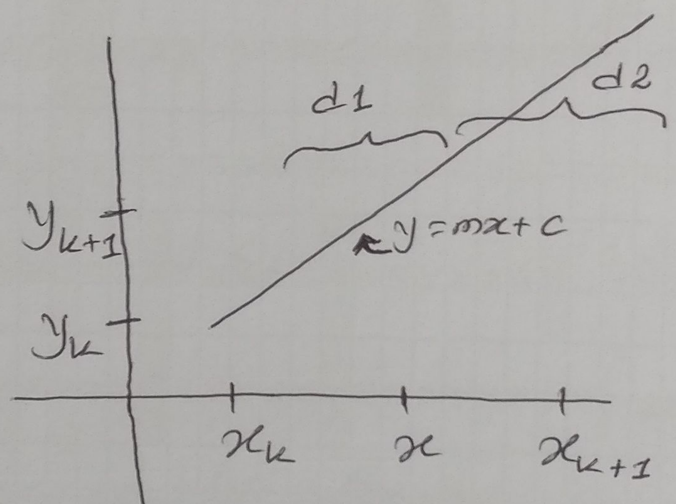
Or, $d1 - d2 = \frac{2\Delta y - \Delta x}{\Delta x}$

Or, $\Delta x(d1 - d2) = 2\Delta y - \Delta x$

$\therefore$ $\boxed{P_0 = 2\Delta y - \Delta x}$

For slope ($|m| > 1$)

Suppose x_k and y_k are the initial coordinate of the line which has been plotted in figure. Then two candidate pixels to be plotted are (x_k, y_{k+1}) or (x_{k+1}, y_{k+1})



If d1 and d2 are horizontal seperation of pixels from the line then the value of x at y_{k+1} or at sampling position y_{k+1} is

$$y = mx + c$$

Or, $x = \frac{y - c}{m}$

Or, $x = \frac{y_{k+1}}{m} - \frac{c}{m}$ ——— (i)

Then

$$d1 = x - x_k$$

$$d1 = \frac{y_{k+1}}{m} - \frac{c}{m} - x_k$$

and $d2 = x_{k+1} - x$

Or, $d2 = x_{k+1} - \frac{y_k+1}{m} + \frac{c}{m}$

$$d1 - d2 = \frac{y_k+1}{m} - \frac{c}{m} - x_k - x_{k+1} + \frac{y_k+1}{m} - \frac{c}{m}$$

Or, $d1 - d2 = \frac{2.(y_k+1)}{m} - \frac{2c}{m} - 2x_k - 1$ ——— (ii)

Multiplying Δy on both sides,

$\Delta y(d1-d2) = 2\Delta x y_k + 2\Delta x - 2c\Delta x - 2\Delta y x_k - \Delta y$

Let us define a decision $P_k = \Delta y(d1-d2)$

$P_k = 2\Delta x y_k - 2\Delta y x_k + B$ ——— (iii)

Where, $B = 2\Delta x - 2c\Delta x - \Delta y$

If $P_k < 0$ then $d1 - d2 < 0 \Rightarrow d1 < d2$

So,

x_k is more closer than x_{k+1} from the line and we should plot (x_k, y_{k+1})

else we should plot (x_{k+1}, y_{k+1})

Now,

Coordinate change along the line occur, at unique unit interval. Then we can obtain successive decision parameter using increment integer i.e.

$P_{k+1} = 2\Delta x . y_{k+1} - 2.\Delta y x_{k+1} + B$ ——— (iv)

we get,

$P_{k+1} - P_k = 2\Delta x y_{k+1} - 2\Delta x y_k - 2\Delta y x_{k+1} + 2\Delta y x_k$

$\therefore P_{k+1} = P_k + 2\Delta x (y_{k+1} - y_k) - 2\Delta y (x_{k+1} - x_k)$

If $P_k \geq 0$ then $y_{k+1} = y_k + 1$

$x_{k+1} = x_k + 1$

$$\therefore \boxed{P_{k+1} = P_k + 2\Delta x - 2\Delta y}$$

for initial decision parameter from eqn (ii)

$$d1 - d2 = \frac{2(y_k + 1)}{m} - \frac{2c}{m} - 2x_k - 1$$

or, $d1 - d2 = \frac{2y_k}{m} + \frac{2}{m} - \frac{2c}{m} - 2x_k - 1$

or, $d1 - d2 = \frac{2}{m} + 2\left[\frac{y_k}{m} - \frac{c}{m} - x_k\right] - 1$

or, $d1 - d2 = \frac{2}{m} - 1 \quad \left[\because x_k = \frac{y_k}{m} - \frac{c}{m}\right]$

or, $\Delta y (d1 - d2) = 2\Delta x - \Delta y$

or, $\therefore \boxed{P_0 = 2\Delta x - \Delta y}$

Bresenham's Line Algorithm

1. Input end points (x_1, y_1) and (x_2, y_2)
2. Compute

$$\Delta x = |x_2 - x_1|$$
$$\Delta y = |y_2 - y_1|$$

3. if $(x_2 > x_1)$

$\quad\quad lx = 1$

$\quad$ else

$\quad\quad lx = -1$

4. if $(y_2 > y_1)$

$\quad\quad ly = 1$

$\quad$ else

$\quad\quad ly = -1$

5. plot (x_1, y_1)
6. if $(\Delta x > \Delta y)$ $\{$ i.e. $|m| < 1 \}$

6.1 $P_0 = 2 \cdot \Delta y - \Delta x$

6.2 for $k = 0$ to Δx

if $P_k < 0$

$$x_1 = x_1 + lx$$
$$P_{k+1} = P_k + 2\Delta y$$

else

$$x_1 = x_1 + lx$$
$$y_1 = y_1 + ly$$
$$P_{k+1} = P_k + 2\Delta y - 2\Delta x$$

else $\{ |m| \geq 1 \}$

6.1 $P_0 = 2\Delta x - \Delta y$

6.2 for $k=0$ to Δy

if $P_k < 0$

$y_1 = y_1 + ly$

$P_{k+1} = P_k + 2\Delta x$

else

$x_1 = x_1 + lx$

$y_1 = y_1 + ly$

$P_{k+1} = P_k + 2\Delta x - 2\Delta y$

Q. Digitize the end points (3,10) and (6,2) using Bresenham's line drawing algorithm.

<u>Solution</u>

$(x_1, y_1) = (3, 10)$

$(x_2, y_2) = (6, 2)$

$\Delta x = |x_2 - x_1| = |6-3| = |3| = 3$

$\Delta y = |y_2 - y_1| = |2-10| = |-8| = 8$

$6 > 3$ i.e. $x_2 > x_1 \Rightarrow lx = 1$

$10 > 2$ i.e. $y_1 > y_2 \Rightarrow ly = -1$

and $\Delta y > \Delta x$ i.e. $|m| \geq 1$

K	Po	X_{k+1}, Y_{k+1}	P_{k+1}
0	-2	$(3, 10-1=9)$	$-2+2\cdot3=4$
1	4	$(3+1=4, 9-1=8)$	$4+2\times3-2*8=-6$
2	-6	$(4, 8-1=7)$	$-6+2*3=0$
3	0	$(4+1=5, 7-1=6)$	$0+2*3-2*8=-10$
4	-10	$(5, 6-1=5)$	$-10+2*3=-4$
5	-4	$(5, 5-1=4)$	$-4+2*3=2$
6	2	$(5+1=6, 4-1=3)$	$2+2*3-2*8=-8$
7	-8	$(6, 3-1=2)$ $=(6,2)$	———

Q. Digitize the end points (16,18) and (10,10) using Bresenham's line drawing algorithm

Solution

$(x_1, y_1) = (16, 18)$ $\quad x_1 > x_2 \Rightarrow l_x = -1$

$(x_2, y_2) = (10, 10)$ $\quad y_1 > y_2 \Rightarrow l_y = -1$

$\Delta x = |x_2 - x_1| = |10 - 16| = 6$

$\Delta y = |y_2 - y_1| = |10 - 18| = 8$

$P_0 = 2\Delta x - \Delta y = 2*6-8 = 4$

K	P_K	X_{K+1}, Y_{K+1}	P_{K+1}
0	4	(16-1=15, 18-1=17)	4+2*6-2*8 = 0
1	0	(15-1=14, 17-1=16)	0+12-16 = -4
2	-4	(14, 16-1=15)	-4+12 = 8
3	8	(13, 15-1=14)	8+12-16 = 4
4	4	(13-1=12, 14-1=13)	4+12-16 = 0
5	0	(12-1=11, 13-1=12)	0+12-16 = -4
6	-4	(11, 12-1=11)	-4+2*6 = 8
7	8	(11-1=10, 11-1=10)	———