# The Doppler effect on indirect detection of decaying dark matter

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**Introduction:** For a large field of view instrument [1]:

$$\mathcal{F} = \frac{\Gamma}{4\pi\, m_s} \int_\Omega \int_0^\infty d\Omega\, ds\, \rho[r(s,\Omega)]\,. \tag{1}$$

We can rewrite Eqn. 1 as

$$\frac{d^2\mathcal{F}}{d\Omega\, dE} = \frac{\Gamma}{4\pi\, m_s} \int_0^\infty ds\, \rho[r(s,\Omega)]\, \frac{dN(E)}{dE}. \tag{2}$$

Similar to the previous paper, we can write

$$\frac{d\tilde{N}(E, r[s,\Omega])}{dE} = \int dE'\, \frac{dN(E')}{dE'}\, G(E-E', \sigma_{E'})\,, \tag{3}$$

where the convolution function $G(E, \sigma_E)$ takes the form of a Gaussian with an width of $\sigma_E = (E/c)\sigma_{v_{\rm LOS}}$. We assume that $\sigma_{v_{\rm LOS}} \approx \sigma_{v_r}(r[s,\Omega])$.

I will now show the derivation of these formulae. Let us assume that the velocity distribution is $f(v)$ and the differential spectrum is $dN/dE = \delta(E-E_0)$. The effect of including this velocity distribution is that it takes the mono energetic spectrum to $\dfrac{d\tilde{N}}{dE} = \delta\left(E - E_0(1 \pm \dfrac{v_0}{c})\right)$. From this we can intuitively derive the following formula which is valid for a general $f(v)$:

$$\frac{d\tilde{N}}{dE} = \int f(v)\, \frac{dN}{dE'}\, G(E, E')\, dv\, dE' \tag{4}$$

where $G(E, E')$ is the convolution function. To estimate a functional form of $G(E, E')$, we can use the test case $f(v) = \delta(v - v_0)$, and $dN/dE = \delta(E' - E_0)$ to determine $G(E, E') = \delta(E - E'(1 \pm v/c))$.

Let us now consider $f(v) = \dfrac{1}{\sqrt{2\pi}\sigma_v}\, e^{v^2/2\,\sigma_v^2}$, so that

$$\frac{d\tilde{N}}{dE} = \int \delta(E' - E_0)\, \frac{1}{\sqrt{2\pi}\sigma}\, e^{v^2/2\,\sigma^2} \times \delta(E - E'(1 \pm v/c))\, dv\, dE'. \tag{5}$$

We have $\delta(E - E'(1 \pm v/c)) = \dfrac{c}{E'}\, \delta\left(v + c - \dfrac{E}{E'} c\right)$ . We can do the integrals to find

$$\frac{d\tilde{N}}{dE} = \frac{1}{\sqrt{2\pi}}\, \frac{c}{\sigma_v E_0} \exp\left(\frac{-(E-E_0)^2}{2E_0^2\, \sigma_v^2/c^2}\right) \tag{6}$$

which we can compare with a regular Gaussian to derive $\sigma_E = (E/c)\sigma_v$.

**Methods:** We use a suite of Milky Way zoom-in simulations run by [2] using the L-GADGET cosmology code (a descendant of GADGET-2, [3]) to study the Doppler-shifted line emission due to sterile neutrino decay.

The decay signal spectral intensity is traditionally defined in terms of a line integral along the viewing direction $\psi$:

$$\frac{dI(\psi, E)}{dE} = \frac{\Gamma}{4\pi m_\chi} \frac{dN(E)}{dE} \int_\phi \rho_\chi(r[s,\phi])\, ds \tag{7}$$

where the integral term is the well-known "J-factor," which captures the enhancement of the signal due to substructure.

The main insight of [4] is that for a detector with sufficient spectral resolution, the decay spectrum $\frac{dN(E)}{dE}$ can no longer be considered to be independent of position. This is due to Doppler shifting induced by the Sun's motion around the galactic center as well as broadening due to the position-dependent velocity dispersion $\sigma(r)$. Qualitatively speaking, the observed spectrum is given by the rest-frame decay spectrum $\frac{dN(E)}{dE}$ broadened by the dark matter velocity dispersion $\sigma(r)$, shifted by the Sun's velocity relative to the dark halo $\delta E_{MW} = \frac{E v_{MW}}{c}$, and integrated along the line of sight (LOS):

$$\frac{dI(\psi, E)}{dE} = \frac{\Gamma}{4\pi m_\chi} \int_\phi \rho_\chi(r[s,\phi])\, \frac{d\tilde{N}(E - \delta E_{MW}, r[s,\phi])}{dE}\, ds \tag{8}$$

Here, $\frac{d\tilde{N}}{dE}$ is the rest-frame spectrum broadened by the local (position-dependent) velocity dispersion. [4] model this as a Gaussian convolution with a width dependent on an analytic prescription for $\sigma(r)$.

Rather than attempting to analytically integrate (8), we construct the full spectral intensity seen by the detector directly from the N-body particles, incorporating Doppler shift and velocity dispersion in a straightforward and natural way. This is similar in spirit to both the "sightline" method employed by [5] and the velocity distribution function sampling of [6], both of whom eschew analytic prescriptions in favor of operating directly on the information available in the simulation data.

We do this by approximating the LOS integral using a thin cone, then sampling all simulation particles lying inside the cone. This sampling cone subtends the solid angle $\Omega_s$ and is understood to lie along $\psi$, the viewing direction as before. Replacing the integral with a sum over all particles $p$ in the sampling cone, we obtain the following expression:

$$\frac{dI(\psi, E)}{dE} = \frac{\Gamma}{4\pi m_\chi} \sum_{p \in \Omega_s} \frac{1}{r_p^2} \frac{dN[E(1 - v_p/c)]}{dE} \tag{9}$$

where $r_p$ is the scalar distance to particle $p$ and $v_p$ is the velocity projected along the line of sight. Intuitively, we are "stacking the spectra" from the individual simulation particles, with weights reflecting the $r^{-2}$ dependence in the observed flux. One can see that by considering the LOS velocity of each particle independently, we automatically capture the spectral convolution introduced by the bulk velocity dispersion. In the special case where $\frac{dN(E)}{dE}$ is a line, computing the observed spectrum is then as simple as building a $r^{-2}$-weighted histogram of the LOS velocities for all particles in the sampling cone.

**Counting statistics and differentiating line centroids:** Given a telescope with effective area $A_{eff}$ and field of view (FOV) $\Omega_{FOV}$, the number of photons entering the detector during an exposure of length $t$ is

$$N_\gamma = \frac{\Gamma t A_{eff}}{4\pi} \frac{m_s}{m_\chi} \frac{\Omega_{FOV}}{\Omega_s} \sum_{p \in \Omega_s} \frac{1}{r_p^2} \tag{10}$$

**Conclusions:**

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