Analysis of Multi-Scale Energy Markets using Stochatic Optimization Techniques

ISyE 719 Course Project

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Abstract

Electricity markets operate at multiple timescales (from hours to milliseconds) to ensure that supply and demands are matched in real time. These markets involve uncertainties because future electricity prices and demands are unknown at the time of decision-making, e.g. energy sale and purchase commitments by generators. We use stochastic programming techniques to study flexibility and economic opportunities provided by a battery in these markets, namely day-ahead (1-hour timescale) and real-time (5-minute timescale) markets. In this work we consider uncertainty only in electricity loads and determine optimal participation strategies using methods like receding horizon scheme, dual dynamic programming and *fullproblem (scenario sampling)*. We also determine bounds on expected policy costs by using perfect information and two-stage approximation (with restriction on states). We compare the costs of participation in exclusively in day-ahead market and both day-ahead and real-time markets. Our results show that market participation only in day-ahead energy markets can *significantly* reduce economic flexibility as compared to participating in both levels of markets.

1   Introduction

A diverse set of energy systems such as generators, batteries, wind turbines and flywheels can participate in electricity markets. This participation is governed by rules set by ISOs (Independent System Operators) such as California ISO, PJM (Pennsylvania-New Jersey-Maryland) Interconnection and Midcontinent ISO, under whose jurisdiction the market falls. The markets are structured at multiple time levels, namely day ahead (hourly market commitments) and real time markets (commitments ranging from minutes to seconds). In day-ahead markets the electricity is traded in intervals of 1 hour with the prices being constant in each interval of 1 hour and varying with intervals. Real time markets, on the other hand can have varying time scales depending on the ISO operating it. The real-lime market is a spot market in which utilities can buy power to meet the last few increments of demand not covered in their day ahead schedules. It is also the market that secures energy reserves, held ready and available for ISO use if needed, and the energy needed to regulate transmission line

stability [1].The frequency of energy price variation is different for day-ahead and real time markets (Figure 1). The real time market is more volatile, and at times the prices can even go negative here. This provides an opportunity to the building-battery system to buy electricity at negative price and meet its load demands. The fast dynamics of battery helps it to capture these spikes in prices and maximize its revenue potential.

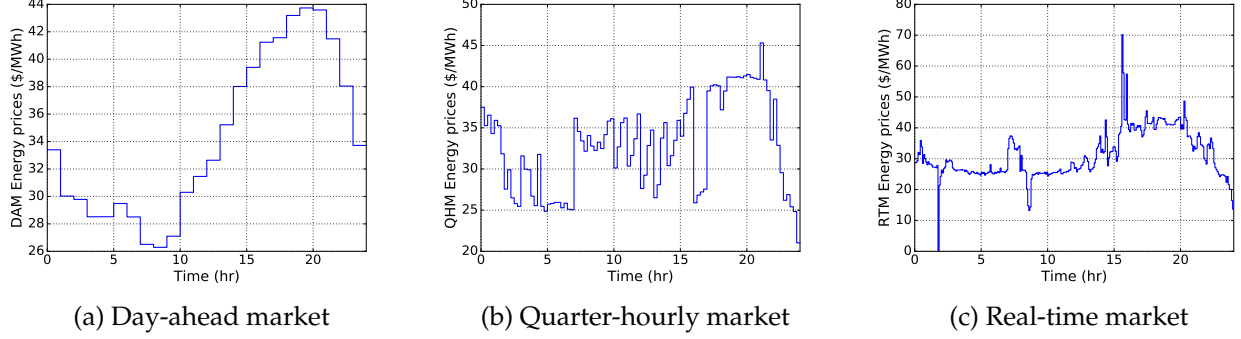(a) Day-ahead market   (b) Quarter-hourly market   (c) Real-time market

Figure 1: Energy prices for one day in the markets in California

Apart from price variations, the introduction of more renewable power sources in the grid such as wind and solar energy, there are greater variations and uncertainties in the net load as well. Since these sources depend on intermittent conditions such as weather, they introduce slow dynamics to the grid. Thus systems with faster dynamic responses such as battery and building systems are becoming increasingly important to balance these fluctuations and provide dynamic flexibility to the power grid. Also, factors such as transmission losses, generation cost and congestion affect the value of products (energy, regulation, spinning , non-spinning reserves) offered at different timescales. These fluctuations being inherently uncertain, determining the optimal participation policy requires analysis using stochastic optimization techniques.

2 Problem Definition and Decision-Making Setting

We consider a rechargeable Li-ion battery with a building that is inter-connected to the power grid for providing electricity services. Electricity services imply that batteries can provide power to or draw power from the grid. In our current setting, we do not consider participation in regulation or other ancillary services. The operator of the power grid (ISO) compensates the battery-owners for the electricity services provided. The goal for the battery-owners is to maximize the revenues generated by providing services to the grid and at the same time meeting the load demands from building. We consider battery and building as one system (building-battery system) and any unmet load demand from the building is penalized with the corresponding electricity price in real time market.

We generate a compact scenario tree consisting of 12 sub-intervals between any two hours. One scenario represents a load profile in these 12 subintervals. The participation in market begins at time t = 0 hrs (Figure 2) and decisions can be made latest till 1.5 hrs before any hour (according to

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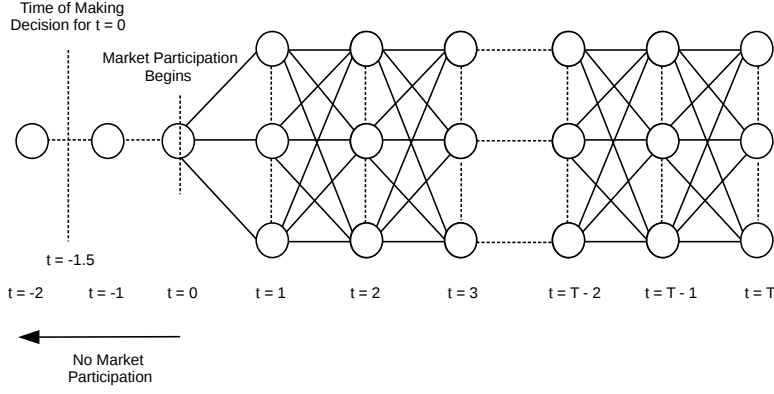[1]http://www.caiso.com/market/Pages/MarketProcesses.aspx

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Figure 2: Scenario tree at the beginning of market participation

market rules set by ISO). Thus our decision making starts at -1.5 hrs (as shown in Figure 2) and our participation begins at t = 0 hrs.

We divide an annual dataset for electricity loads (available over 5-minute intervals) into 52 subsets of weekly load profiles, i.e 2016 intervals of 5-minute each. This division of data helps in capturing the different load profiles over the weekdays and the weekend. We then use these 52 weekly profiles to generate sample scenarios for our computational experiments (Section 4). Since the loads are correlated between time intervals, we formulate a multivariate normal distribution for weekly loads using a mean vector and a covariance matrix. The mean vector (of size 2016 x 1) is the mean of the 52 datasets, but the covariance matrix (of size 2016 x 2016) calculated directly from the 52 datasets cannot be used, since its rank can at most be 52. So we use the Ledoit-Wolf Covariance Estimator [1] (to tackle rank deficiency) for estimating a full rank covariance matrix. We generate 50 samples of load profiles (Figure 3) for a week using the mean and covariance matrix obtained.
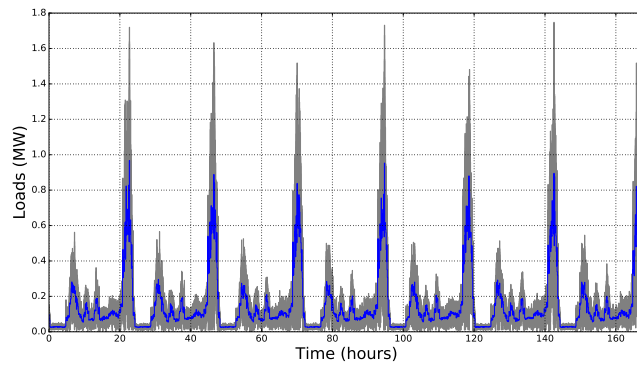

Figure 3: Load scenarios for a week

In this work we assume uncertainty only in the load demands and consider the day-ahead market participation to be first stage decision while real time market decision are treated as recourse. The goals of this work include to :

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- Investigate the economic potential of participating in various markets within a stochastic programming setting i.e. compare expected revenue from different markets and explore the benefits of participating in both real time and day-ahead markets

- Employ different multi-stage stochastic optimization solution methods and compare their performance. The methods compared are:

  - Sample average approximation of extensive form
  - Upper and lower bound estimations using perfect information and two-stage approximation (restriction)
  - Stochastic dual dynamic programming (SDDP)
  - Receding horizon

3 Optimization Model for Building-Battery System

3.1 Deterministic Setting

First, we formulate an optimization problem to minimize the cost (or maximize revenue) for the building-battery system operating in the electricity market in a deterministic setting assuming no uncertainty in any of the market and demand quantities. In this formulation, the building-battery system participates in the market at two timescales, hourly and 5-minute intervals. Since the price and load signals are discrete, real-time prices and loads available at 5-minute intervals and day-ahead prices at hourly intervals, we make the zero-order hold assumption, i.e. these time-varying signals are constant within an interval of corresponding duration of their timescale. We also assume that the energy stored in the battery before beginning of market participation is fixed at E_0. The building-battery system is illustrated with a schematic diagram in Figure 5.

   With these assumptions, we develop the optimization problem formulation for the system is described below.

3.1.1 Sets used in the model

$$\mathcal{T_R} := \{1, .., n_{\text{rtm}}\}$$
$$\mathcal{T_D} := \{1, .., n_{\text{dam}}\}$$

Here, $\mathcal{T_R}$ is the set of time indices corresponding to each 5-minute subintervals in the real-time market within each hour, so $n_{rtm} = 12$. $\mathcal{T_D}$ is the set of time indices corresponding to the hourly intervals of the day-ahead market. n_d can be chosen appropriately for any number of hours we plan to schedule the market participation policy for the building-battery system.

3.1.2 Time discretization used in the model

A time instance for a day-ahead market variable in the model is represented by only the index k ($k \in \mathcal{T_D}$) which corresponds to the interval between hours $k-1$ and k. On the other hand, a time

instance for a real-time variable in the model is represented by index (i, k), where $i \in \mathcal{T}_{\mathcal{R}}$ and $k \in \mathcal{T}_{\mathcal{D}}$. The indices i and k in a time instance (i, k) for a real-time variable correspond to the end of the respective 5-minute interval. For example, the time instance $(5, 7)$ corresponds to the end of the interval between 4th and 5th 5-minute interval within the 7th hour.

So, if a variable or time-varying parameter is indexed by only one index k it implies that it varies at hourly intervals and belongs to the day-ahead market, whereas a variable or time-varying parameter indexed by (i, k) is a real-time variable.

The time discretization and zero-order hold have been illustrated for the battery power in real-time and day-ahead markets in Figure 4, in which the red
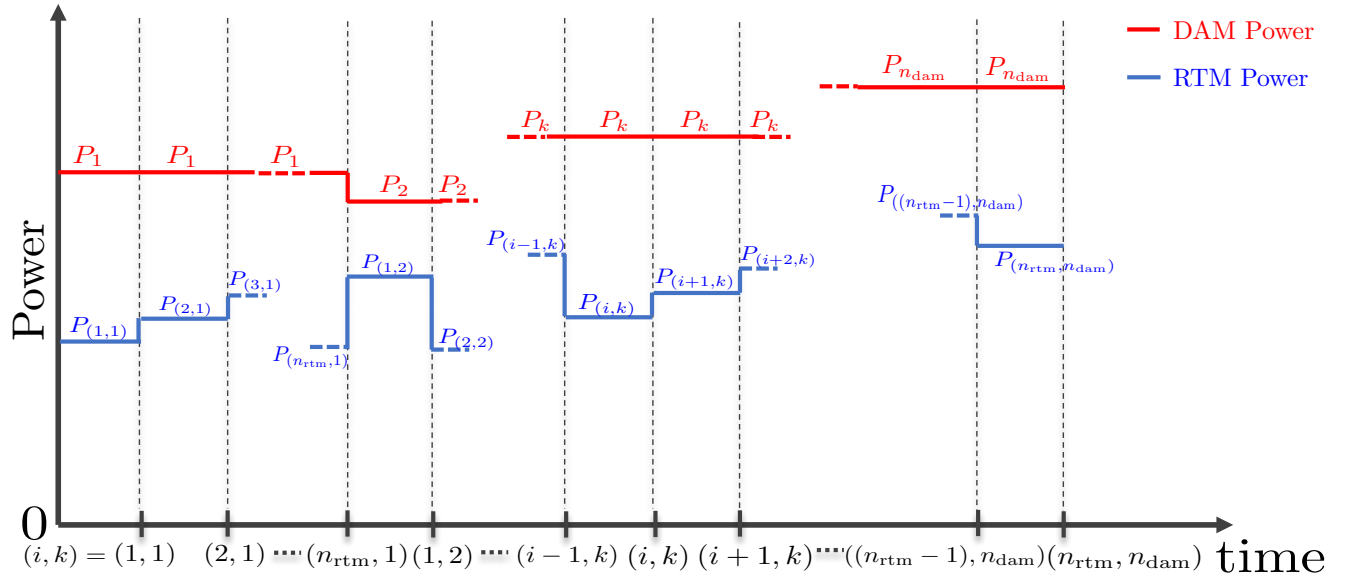

Figure 4: Time Discretization and Zero-Order Hold for Variables

3.1.3 Parameters in the model:

- **Market parameters:**

  - $\pi_k, \forall k \in \mathcal{T}_{\mathcal{D}}$: Electricity price in day-ahead market in the interval between hour $k - 1$ to k

  - $\pi_{i,k}, \forall i \in \mathcal{T}_{\mathcal{R}}, k \in \mathcal{T}_{\mathcal{D}}$: Electricity price in real-time market in the 5-minute interval between $(i - 1, k)$ and (i, k) within k^{th} hour

  - Δt: Time interval (in unit of hours) in the real-time market (which is 5 minutes)

- **Battery parameters:**

  - $E_{max} = 0.5$ MWh: Energy storage capacity of the battery

  - $P_{max} = 0.5$ MW: Maximum discharging or charging rate of the battery

  - $E_0 = E_{max}$: Energy stored in the battery before beginning of market participation

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- **Building loads:**

  - $L_{i,k}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}$: Building loads in the 5-minute interval between $(i-1, k)$ and (i, k) within k^{th} hour

3.1.4 Variables in the model:

- **Decision variables:**

  - $P_k, \forall k \in \mathcal{T}_\mathcal{D}$: Power committed by battery in day-ahead market in the interval between hour $k-1$ to k
  - $P_{i,k}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}$: Power committed by battery in real-time market in the 5-minute interval between $(i-1, k)$ and (i, k) within k^{th} hour
  - $L_{i,k}^{sup}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}$: Power supplied by battery to the building in the 5-minute interval between $(i-1, k)$ and (i, k) within k^{th} hour

- **State variables:**

  - $E_{i,k}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}$: Energy stored in the battery at time (i, k) (at the end of ith 5-minute interval within k^{th} hour)
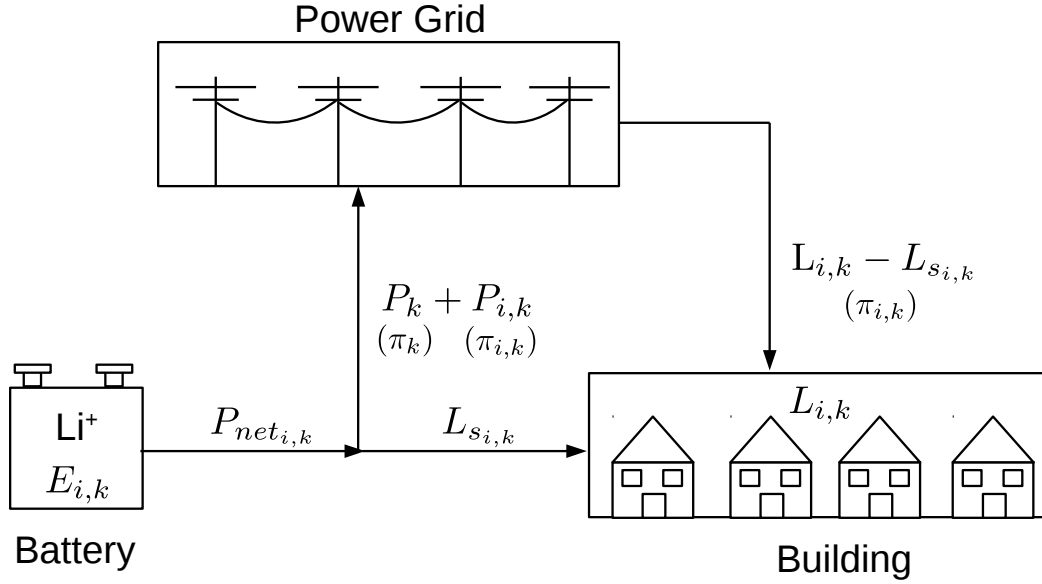

Figure 5: Illustration of Battery-Building System Interconnected with Power Grid

3.1.5 Constraints and objective function

- Net discharge (power) from the battery at any time has to be less than the maximum discharge capacity of the battery, P_{max}.

$$P_{net_{i,k}} = P_k + P_{i,k} + L_{i,k}^{sup}, \quad \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D} \tag{1}$$

- Balance on the energy stored in the battery at each time

$$E_{i,k} = E_{i-1,k} - P_{net_{i,k}}\Delta t_r, \quad \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D} \tag{2}$$

In Equation (2), $E_{0,k} = E_0$ for $k \in \{1\}$ and $E_{0,k} = E_{n_{rtm},k-1}$ for $k \in \mathcal{T}_\mathcal{R} \setminus \{1\}$.

- Bounds on variables:

$$0 \le E_{i,j,k} \le E_{max}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D} \tag{3a}$$

$$-P_{max} \le P_{i,k} \le P_{max}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D} \tag{3b}$$

$$-P_{max} \le P_k \quad \le P_{max}, \forall k \in \mathcal{T}_\mathcal{D} \tag{3c}$$

$$-P_{max} \le P_{net_{i,k}} \le P_{max}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D} \tag{3d}$$

$$0 \le L_{i,k}^{sup} \le L_{i,k}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D} \tag{3e}$$

- Objective Function:

$$\min \quad -\sum_{i \in \mathcal{T}_\mathcal{R}} \sum_{k \in \mathcal{T}_\mathcal{R}} \pi_{i,k} P_{i,k} \Delta t_r - \sum_{k \in \mathcal{T}_\mathcal{D}} \pi_k P_k \Delta t_r n_{rtm} + \sum_{i \in \mathcal{T}_\mathcal{R}} \sum_{k \in \mathcal{T}_\mathcal{R}} \pi_{i,k}(L_{i,k} - L_{i,k}^{sup})\Delta t_r \tag{4}$$

3.2 Optimization Model Under Uncertainty

Now, we consider uncertainty in the building loads and extend the optimization formulation in deterministic setting (Section 3.1) to formulate an extensive form of stochastic optimization problem to minimize the expected cost (or maximize expected revenue) for the building-battery system. In the formulation for the optimization problem under uncertainty, we consider all the the market participating conditions to be the same as in the deterministic setting except for the uncertainty in the loads. We use the same sets of time indices, time-discretization approach and variable/parameter notations as developed in Section 3.1 with the exception that uncertainty has to be added to all real-time variables because of the uncertain loads. We represent this uncertainty with a sample space Ξ and an outcome from this space is denoted by ξ. As described in Section 2, we consider that when a scenario $\xi \in \Xi$ realizes at an hour k the loads at all real-time subintervals between hour $k-1$ and k become known and the scenario-tree grows at every hour. So, the uncertain load at every time is denoted as $L_{i,k}(\xi)$. Similarly, we denote the resulting uncertainty in all real-time variables, namely the energy transacted by the battery with real-time market ($P_{i,k}(\xi)$), the energy supplied by the battery to the building ($L_{i,k}^{sup}(\xi)$) and the energy stored in the battery at every time ($E_{i,k}(\xi)$). The day-ahead energy participation by the battery is a first-stage decision which needs to be made before the realization of uncertainty. Furthermore, for the extensive form of the stochastic programming model, we assume that the uncertainty can be represented by a set of N possible scenarios $\mathcal{S}$ and all possible realizations of uncertainty lies in this set. So, $s \in \mathcal{S}$ represents a possible scenario of the realization of uncertainty. Also, we make the assumption that the scenarios are equiprobable. We use these assumptions to build the extensive form of the stochastic program to minimize the expected cost (maximize expected revenue) for the system and is described below.

3.2.1 Variables in the extensive form model

- **Decision variables:**

  - $P_{k,s}, \forall k \in \mathcal{T}_\mathcal{D}, \forall s \in \mathcal{S}$: Power committed by battery in day-ahead market in the interval between hour $k-1$ to k in scenario s

  - $P_{i,k,s}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}, \forall s \in \mathcal{S}$: Power committed by battery in real-time market in the 5-minute interval between $(i-1, k)$ and (i, k) within k^{th} hour in scenario s

  - $L_{i,k,s}^{sup}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}, \forall s \in \mathcal{S}$: Power supplied by battery to the building in the 5-minute interval between $(i-1, k)$ and (i, k) within k^{th} hour in scenario s

- **State variables:**

  - $E_{i,k,s}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}, \forall s \in \mathcal{S}$: Energy stored in the battery at time (i, k) (at the end of ith 5-minute interval within k^{th} hour) in scenario s

3.2.2 Constraints and objective function

- Net discharge (power) from the battery at any time has to be less than the maximum discharge capacity of the battery, P_{max}.

$$P_{net_{i,k,s}} = P_{k,s} + P_{i,k,s} + L_{i,k,s}^{sup}, \quad \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}, s \in \mathcal{S} \tag{5}$$

- Balance on the energy stored in the battery at each time

$$E_{i,k,s} = E_{i-1,k,s} - P_{net_{i,k,s}} \Delta t_r, \quad \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}, s \in \mathcal{S} \tag{6}$$

In equation (6), $E_{0,k,s} = E_0$ for $k \in \{1\}$ and $E_{0,k,s} = E_{n_{rtm},k-1,s}$ for $k \in \mathcal{T}_\mathcal{R} \setminus \{1\}$.

- Bounds on variables:

$$0 \leq E_{i,j,k,s} \leq E_{max}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D} \tag{7a}$$

$$-P_{max} \leq P_{i,k,s} \leq P_{max}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}, s \in \mathcal{S} \tag{7b}$$

$$-P_{max} \leq P_{k,s} \leq P_{max}, \forall k \in \mathcal{T}_\mathcal{D}, s \in \mathcal{S} \tag{7c}$$

$$-P_{max} \leq P_{net_{i,k,s}} \leq P_{max}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}, s \in \mathcal{S} \tag{7d}$$

$$0 \leq L_{i,k,s}^{sup} \leq L_{i,k,s}, \forall i \in \mathcal{T}_\mathcal{R}, k \in \mathcal{T}_\mathcal{D}, s \in \mathcal{S} \tag{7e}$$

- Nonanticipativity constraints for first-stage decisions ($P_{k,s}$):

$$P_{k,s} = \frac{1}{N} \sum_{s \in \mathcal{S}} P_{k,s}, \quad \forall k \in \mathcal{T}_\mathcal{D}, s \in \mathcal{S} \tag{8}$$

- Objective Function:

$$\min \quad \frac{1}{N} \sum_{s \in \mathcal{S}} \left[-\sum_{i \in \mathcal{T}_\mathcal{R}} \sum_{k \in \mathcal{T}_\mathcal{R}} \pi_{i,k} P_{i,k,s} \Delta t_r - \sum_{k \in \mathcal{T}_\mathcal{D}} \pi_k P_{k,s} \Delta t_r n_{rtm} + \sum_{i \in \mathcal{T}_\mathcal{R}} \sum_{k \in \mathcal{T}_\mathcal{R}} \pi_{i,k} (L_{i,k,s} - L_{i,k,s}^{sup}) \Delta t_r \right] \tag{9}$$

4 Computational Experiments

4.1 Economic Potential of Different Timescales of Electricity Markets

In this case study, we sample 50 scenario paths (out of the 50^{168} possibilities) for a week's time period. We then construct a full model for for this week and use real price signals from 01/01/2015 to 01/07/2015 in California ISO [2]. For every 1 hour interval in the model, we sample a scenario, and use the load data corresponding to that scenario for the 12 intervals (5 minutes each) in that hour. This helps to reduce the size of scenario tree (from $50^{168 \times 12}$ to 50^{168}). We solve this model (with 50 sampled paths) 100 times to get a confidence interval on the expected revenue. We then compare this expected revenue (obtained by participating in both real time and day-ahead market) to the cases when the battery participates only in either real time or day-ahead market alone. Highest revenue is

Table 1: Revenue Breakup from Participation in Different Markets

Market Participated	Total Revenue ($)	Unmet Load Cost ($)	DAM Revenue ($)	RTM Revenue ($)
DAM + RTM	711.08	-8.75	793.12	-90.79
DAM	-921.73	77.00	-844.73	N/A
RTM	-31.32	-9.10	N/A	-40.42
None	-10,293.23	10,293.23	N/A	N/A

earned when the battery participates in both the real time and day-ahead markets (Table 1). Whereas, if it participates in just day-ahead or real time market alone, the revenue is much less or even negative (results in loss). Consider the case when it participates in only day ahead market. It is evident from Figure 10 and 11 that the battery needs to charge itself (buy energy from the grid) in order to meet the building load demands. The price of energy is on average higher in day-ahead market than that in real time markets. Thus with no participation in real time market in does not have the flexibility to buy energy from the real-time market at lower prices and sell to day-ahead market (at a higher price) to make profits. The building loads need to be met in real-time (at every 5-minute interval) while the battery has the opportunity to buy or sell energy from the grid only at every hour. Thus the battery is forced to store enough energy to meet the building requirement for an hour by buying energy in hourly intervals from the day-ahead market. Whenever it has slightly excess energy available it tries to sell that energy to the day-ahead market to minimize the overall cost.

On the other hand, when it participates in both day-ahead and real-time markets, it has the flexibility to buy from the real-time market and sell to the day-ahead market while saving sufficient energy to meet the building requirement in real-time. This is possible because the real-time electricity prices are slightly lower than the day-ahead market prices on average. The real-time prices also fall below the day-ahead market prices very often (Figures 7a and 7b) and because of its fast dynamics, the battery can capture these moments by buying from real-time market and selling in day-ahead market to make profit. Thus market participation in both time scales is important for maximizing

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[2]http://oasis.caiso.com/mrioasis/logon.do

the economic potential. The typical problem in a stochastic setting (considering only 50 samples of load scenarios) for 1 week planning consists of 520900 variables, 722500 linear constraints, and takes Gurobi 35-40 seconds to solve it. If the number of scenarios is increased, the problem size would grow exponentially. While on increasing the planning period the problem size increases linearly. Thus for problems involving simultaneous participation in both real time and day-ahead market, we use advanced stochastic techniques for solving. We also compare their performance with respect to solution time and the expected revenue obtained.
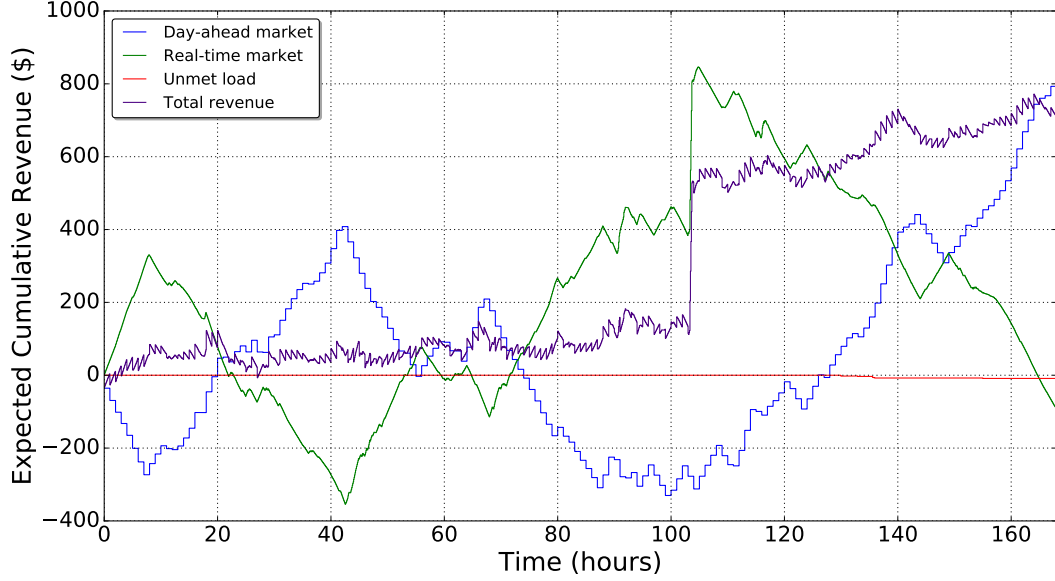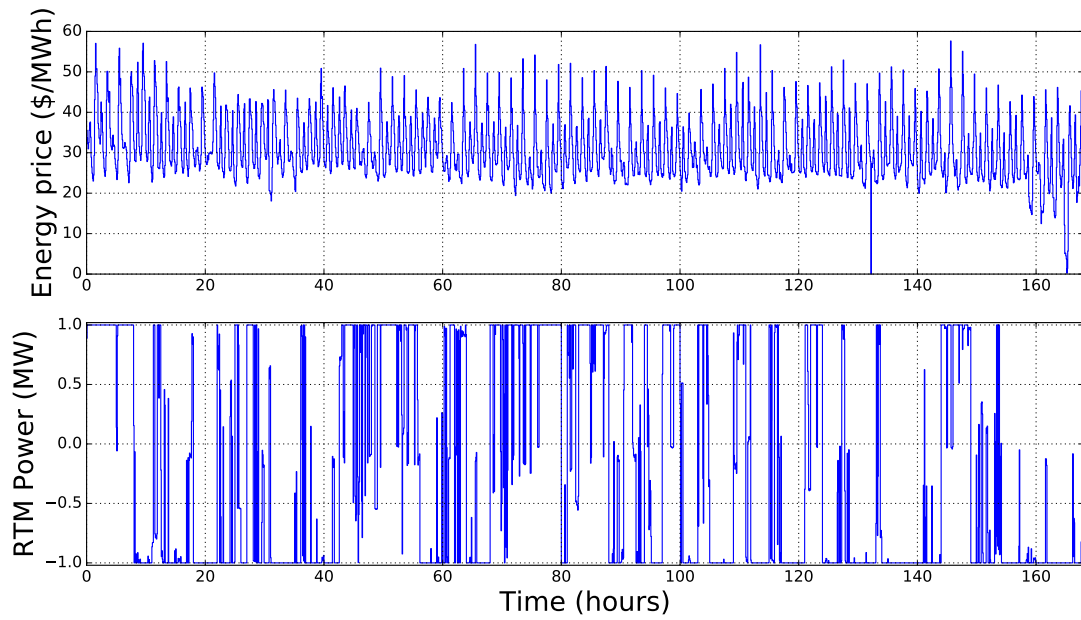

Figure 6: Trajectory of cumulative revenues when battery participates in both day-ahead and real-time markets
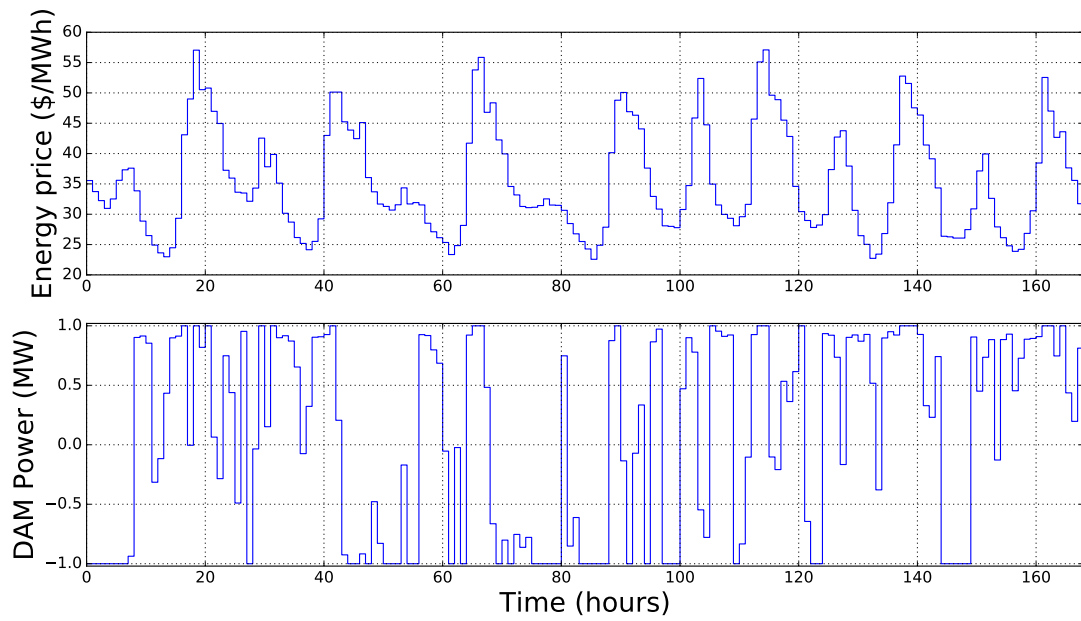
(a) Day-ahead market

(b) Real-time market

Figure 7: Energy participation policy when battery participates in both day-ahead and real-time markets
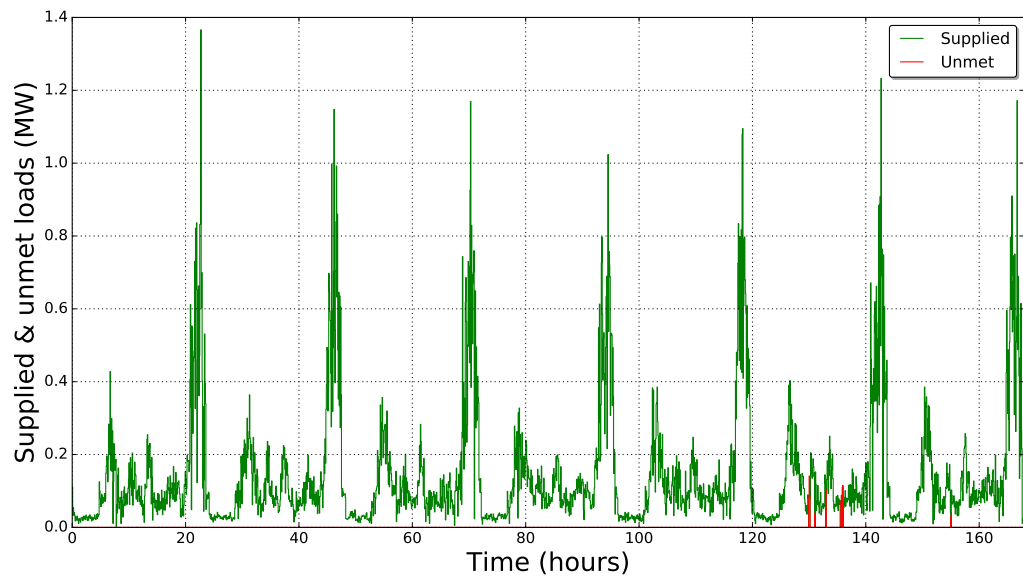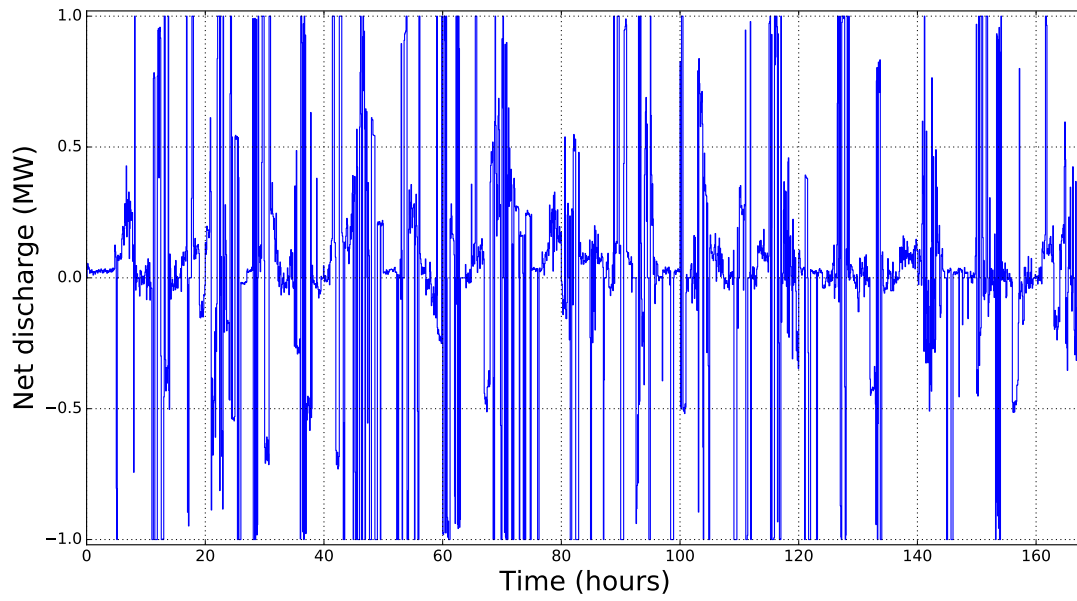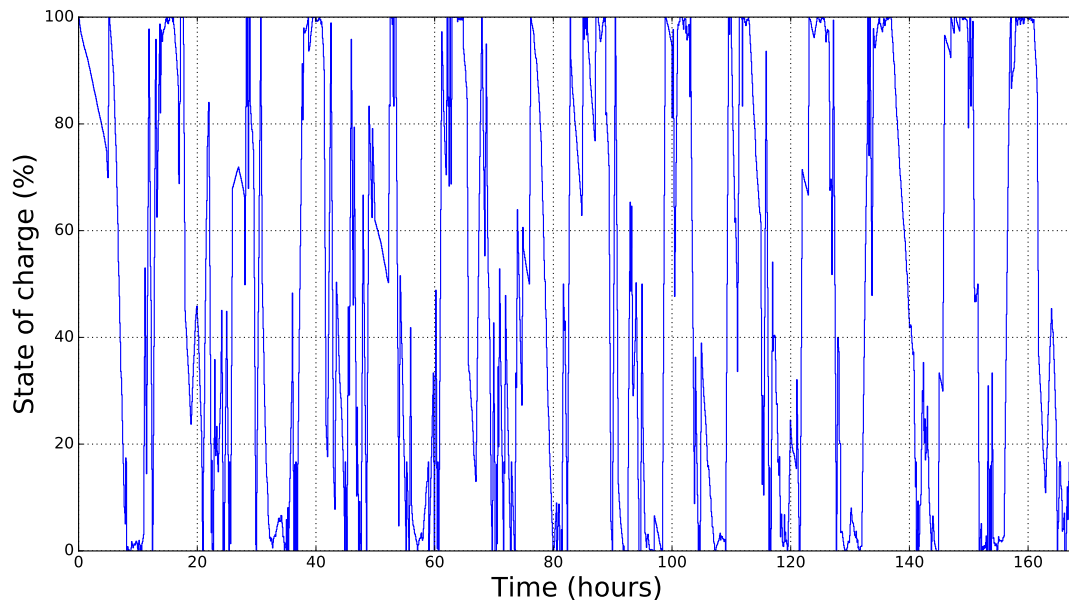
Figure 8: Power supplied to building and unmet load when battery participates in both day-ahead and real-time markets

(a) Net battery discharge

(b) State of charge trajectory

Figure 9: Operating condition of battery when participating in both day-ahead and real-time markets
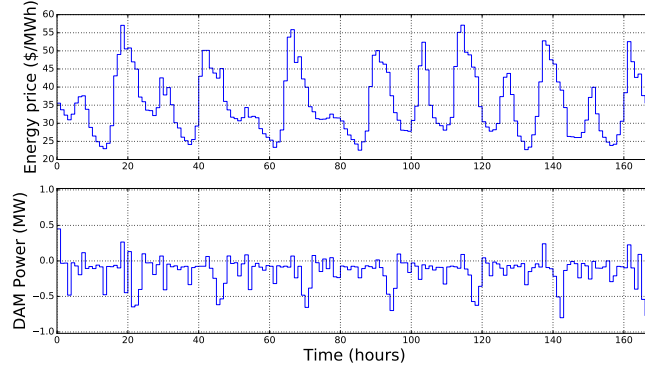
Figure 10: Energy participation policy when battery participates only in day-ahead market.
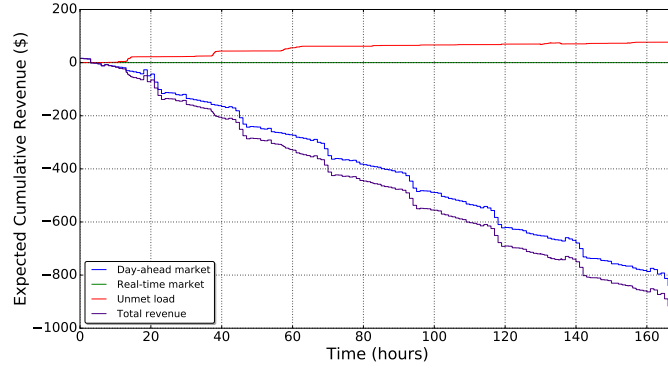

Figure 11: Trajectory of cumulative revenue when battery participates only in day-ahead market

4.2 Sample Average Approximation for the Full Extensive Form

we solve for 100 different sets of 50 sample paths from the scenario tree and obtain the expected value and confidence interval over it. Expected objective over 100 runs for stochastic fullproblem = -712.9901530720865 Confidence interval on objective over 100 runs = (-713.3345792874304,-712.6457268567426)

4.2.1 Upper Bound from Mean Value Problem

total cost: -697.61

4.2.2 Estimating Bounds with Perfect Information

expected cost: -714.18 better than -711

4.2.3 Estimating Bounds with Two-Stage Approximation: Restriction

expected cost: -698.92

For a set of 50 sample scenarios for load profiles for a week's planning period, Figure 12 shows the distribution of the revenues obtained in different scenarios. It is evident from Figure 12 that the policy from the perfect information solution always gives the highest revenues in all scenarios and the policy from the extensive form of stochastic program with the same sample scenarios also achieves much better revenues as compared to the cases with two-stage approximation with restriction on state variables and the mean-value approximation of the same sample scenarios.
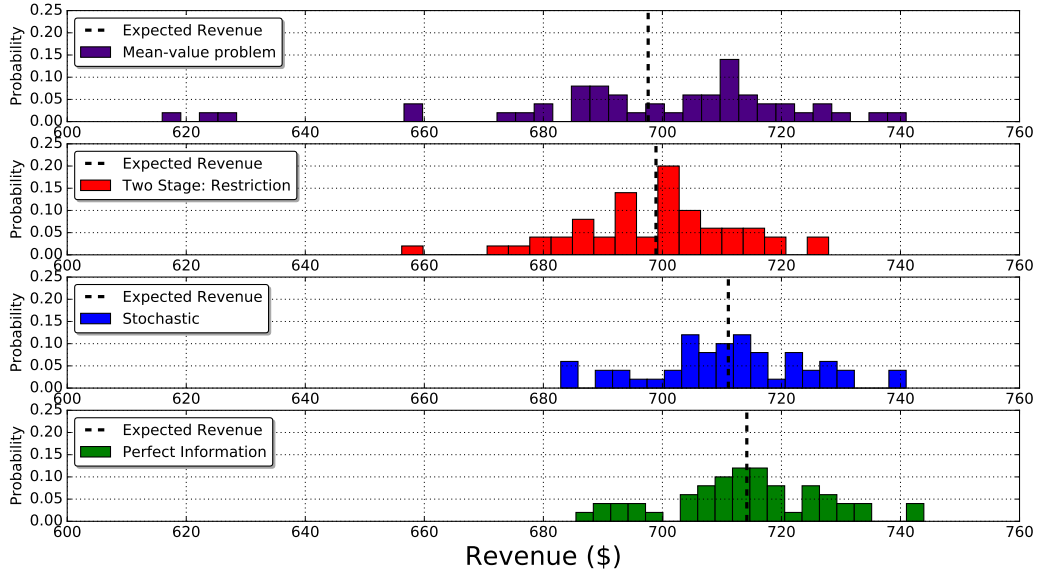

Figure 12: Distribution of the revenues obained in a set of 50 sample scenarios

4.3 Stochastic Dual Dynamic Programming

final lower bound : -554.88 final upper bound : -554.79 confidence interval: [-554.57, -555.02] time : 825.8 sec

Our state variable $E_{i,k}$ (energy-level of battery at any time) captures the history of load demands realized and the decisions made in the previous stages; thus providing stage-wise independent scenario realizations. The stochastic dual dynamic programming (SDDP) is applicable in this case.

The stochastic problem at each node is for making the energy commitment decisions from that hour to the next hour i.e we formulate a problem for decisions in future (where we have not yet observed any realizations). This is because we only have samples of scenarios for that future hour and not the perfect information. To get a lower bound, we have to the solve the problem at root node at which the realized scenario for random data has to be known. In our case we consider the root node problem to be the stochastic program between t = -1 hr and 0 hr. Since we are not participating

before t = 0 hr, we know that the random data (load in our case) is 0 between t = -1 hr and t = 0 hr. Hence the stochastic program at our root node is minimizing the θ_1 variable (capturing the future costs), such that $\theta_1 \geq \underline{\theta}$. The solution to this gives a starting lower bound. We give a lower-limit $\underline{\theta}$ for θ_1 because our cost is negative since we are maximizing revenue (or minimizing negative revenue). In case of root node, just θ_1 is being minimized, which becomes unbounded. Thus we need to provide initial lower bound for θ_1. $\underline{\theta}$ is found by assuming energy-market participation with a full capacity of battery in both markets for the full planning period i.e. for negative electricity prices, battery participates in full negative capacity (buying power) and for positive prices it sells power at full capacity. Thus at root node, θ_1 will be the maximum revenue that is achievable without any constraints on battery storage capacity.

In our multi-scale problem, the state variables ($E_{i,k}$) are defined at 12 sub-intervals (of 5 minutes each) within a stage. In standard (one timescale) SDDP algorithm, we take dual of the constraint linking the state variable at the current stage to the state variable at the previous stage. This does not work in case of multi-scale problems since the state variables vary at a time scale of 5 minutes within a stage, while the stages are realized at every one hour. The value of the state variable that is carried forward to the next stage is only the state at the last sub-interval of the current stage i.e the state variables are linked within each sub-intervals and not directly between the stages. We therefore use the sum of duals of the constraints inter-linking these state variables at all sub-intervals within the stage; this sum in effect links the final value of the state variables between consecutive stages. With this modified approach we implement our version of the SDDP algorithm as follows:

1. Generate a compact scenario tree with 50 nodes at every stage (168)

   Forward Pass:

2. Solve the root node problem and get θ_1. Set the current lower bound to θ_1.

3. Sample a load profile for stage j.

4. Formulate an optimization problem as described in Section 3.1, with $n_{\text{dam}} = j$, $E_{0,j} = E_{n_{\text{rtm}},j-1}$ and using the loads sampled in Step 3. Add a θ_{j+1} (capturing future costs) variable (bounded by $\underline{\theta}$) to the objective function.

5. Solve the optimization problem formulated in Step 4, get $E_{n_{\text{rtm}},j}$ Set $j = j + 1$.

6. *If* j ¡= 168, go to Step 3 *else* go to Step 7 and set j = 168.

   Backward Pass: *for s = 1,2,....,N*

   - Formulate a scenario sub-problem for scenario s, as described in Section 3.1, with $n_{\text{dam}} = j$, $E_{0,j} = E_{n_{\text{rtm}},j-1}$ (obtained from forward pass) and using the loads in scenario s.
   - Solve the scenario subproblem and get the duals corresponding to each of the n_{rtm} constraints linking the state variables at each subintervals.
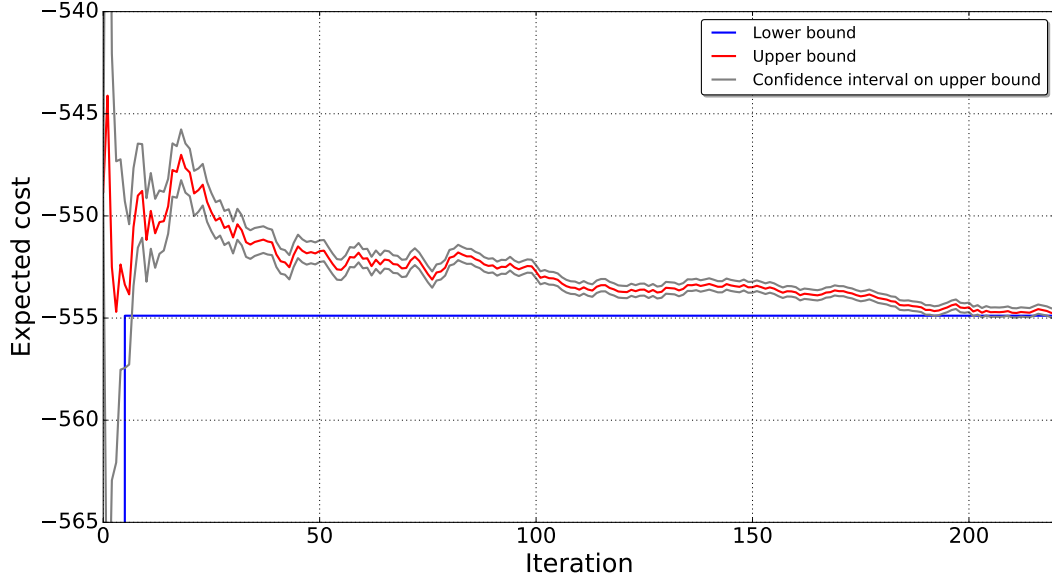
   end for

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Figure 13: Trajectory of the lowerand upper bounds obtained at every iteration of stochastic dual dynamic programming

4.4 Receding Horizon Scheme

In practical situations, we may not have the sample scenarios for load profiles available for full week or a longer planning period that we want to schedule our market participation policy for. In situations when we have the profiles for load scenarios are available for less time periods, a receding horizon scheme is useful and can be implemented to minimize the cost looking at only that much ahead in future for which we have the sample load scenarios. The receding horizon scheme is expected to do worse than the case when we have sample scenarios for load profiles available for the full planning period, but it still outperforms a policy obtained using mean-value counterpart in the same situation (when we have information available for less period of time in future).

We assume that at every hour τ, we have a set of samples for the scenarios for load profiles available only for the next T hours while we want to plan for a period T_P. At τ, we can solve a stochastic program $\mathcal{P}_\tau$, where $\mathcal{P}_\tau$ is the extensive form of the stochastic program described in Section 3.2 with $n_{\mathrm{dam}} = T$. To solve this stochastic program, it requires the inputs $L_{i,k,s}$, $\pi_{i,k}$ and π_k $\forall i \in \mathcal{T}_\mathcal{R}, k \in \{\tau + 1, ..., \tau + T\}, s \in \mathcal{S}$ and the initial energy level of the battery at that hour τ, E_τ. For compact representation, let $L_\tau(\xi_\tau)$ denote the set of load profiles in all scenarios for future T hours and π_τ be the set of prices in day-ahead and real-time markets for hour $\tau + 1$ to $\tau + T$. We can now solve the T-hour horizon stoachastic program $\mathcal{P}_\tau (E_\tau, \pi_\tau, L_\tau(\xi_\tau))$.

The receding horizon scheme which we implement on the building-battery system is described below.

1. Consider $\tau = 0$.

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2. With E_τ, $L_\tau(\xi_\tau)$ and π_τ, solve the stochastic program $\mathcal{P}_\tau\left(E_\tau, \pi_\tau, L_\tau(\xi_\tau)\right)$ over a T-hour horizon.

3. By solving $\mathcal{P}_\tau\left(E_\tau, \pi_\tau, L_\tau(\xi_\tau)\right)$, obtain a profile for net battery discharge, P_{net}, between hours τ and $\tau + 1$ corresponding to each scenario of load profile in the interval.

4. Using a realized scenario for load (random) between hours τ and $\tau + 1$, evolve battery storage level to $E_{\tau+1}$.

5. Set $\tau = \tau + 1$.

6. If $\tau = T_P$, then stop, else go to step 2.

We sample 50 paths upto next 24 hours horizon

5   Conclusions

6   Future Work

6.1   Price Uncertainty

6.2   Three Layer Markets

References

[1] Olivier Ledoit and Michael Wolf. A well-conditioned estimator for large-dimensional covariance matrices. *Journal of multivariate analysis*, 88(2):365–411, 2004.