



## **CBE 750 Course Project**

# **Optimization Model for Operating Battery Systems in Multi-Level Electricity Markets**

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## Problem Statement

- Aim: To operate a battery system in a multi-level electricity market to maximize the daily revenues
- Battery systems can be integrated in a power grid to provide energy and regulation services
- Two components of revenues considered:
  - Energy sale: Battery provides net power to the grid and the grid pays the battery for the amount of energy sold
  - Providing regulation capacity or “band”: Grid pays the battery for providing a “band” around mean power within which grid can regulate net power output from battery
- Energy sale is organized in a three level market while regulation market has two levels
- Net battery power can be negative when the battery is drawing power from grid to recharge
- Problem is formulated as a deterministic linear program
- A comparison of ~~monthly~~ yearly revenues will be made by solving the optimization problem with two approaches:
  - Rolling horizon of 1 day
  - Solving complete optimization problem for 1 ~~month~~ year

# Battery Operation Model



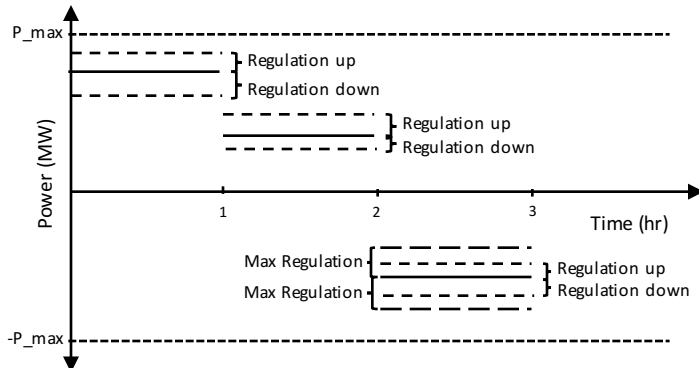
$$\pi(t)$$

$$P(t) \leq 1 \text{ MW}$$



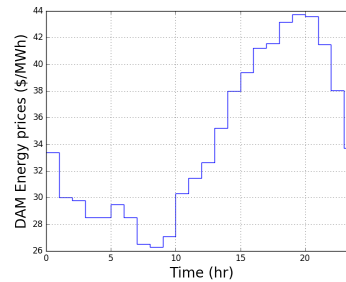
$$E(t) \leq 0.5 \text{ MWh}$$

Example for day-ahead market:

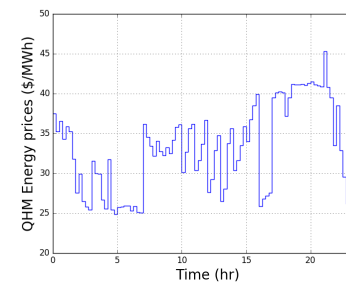


## Energy Prices:

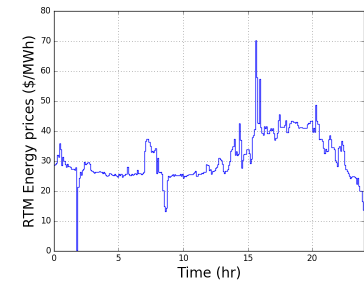
Day-ahead market  
(Time scale: 1 hour)



Quarter-hourly market  
(Time scale: 15 min)

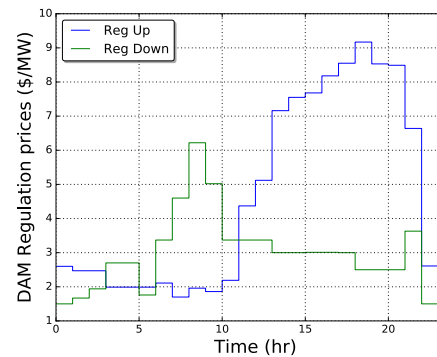


Real-time market  
(Time scale: 5 min)

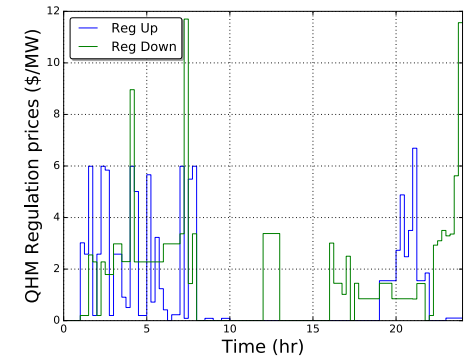


## Regulation Prices:

Day-ahead market  
(Time scale: 1 hour)



Quarter-hourly market  
(Time scale: 15 min)



# Optimization Model for one day: Constraints and Objective Function

$$R := \{1, \dots, n_r\}, Q := \{1, \dots, n_q\}, D := \{1, \dots, n_d\}$$

$$\text{where } n_r = 3, n_q = 4, n_d = 24$$

$$i \in R, j \in Q, k \in D$$

$$\Delta t_r = 5 \text{ min}, \Delta t_q = 15 \text{ min}, \Delta t_d = 60 \text{ min}$$

Energy balance equations for the battery at each subinterval:

$$E_{i,j,k} = E_{i-1,j,k} - \eta(P_{i,j,k} + P_{j,k} + P_k)\Delta t_r$$

$$E_{1,j,k} = E_{n_r,j-1,k} - \eta(P_{1,j,k} + P_{j,k} + P_k)\Delta t_r$$

$$E_{1,1,k} = E_{n_r,n_q,k-1} - \eta(P_{1,1,k} + P_{1,k} + P_k)\Delta t_r$$

$$E_{1,1,1} = E_0 - \eta(P_{1,1,1} + P_{1,1} + P_1)\Delta t_r$$

Energy regulation at each subinterval:

$$P_{i,j,k} + P_{j,k} + P_k + R_{j,k}^U + R_k^U \leq P_{max}$$

$$P_{j,k} + P_k + R_{j,k}^U + R_k^U \leq P_{max}$$

$$P_k + R_k^U \leq P_{max}$$

$$P_{i,j,k} + P_{j,k} + P_k - R_{j,k}^D - R_k^D \geq -P_{max}$$

$$P_{j,k} + P_k - R_{j,k}^D - R_k^D \geq -P_{max}$$

$$P_k + R_k^D \geq -P_{max}$$

Bounds on variables:

$$0 \leq E_{i,j,k} \leq E_{max}$$

$$-P_{max} \leq P_{i,j,k} \leq P_{max}$$

$$-P_{max} \leq P_{j,k} \leq P_{max}$$

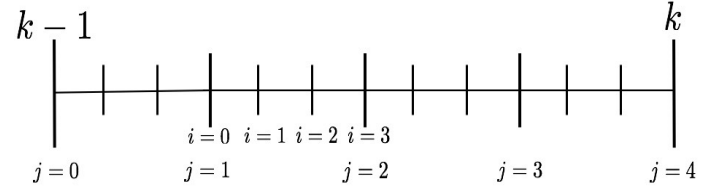
$$-P_{max} \leq P_k \leq P_{max}$$

$$0 \leq R_{j,k}^U \leq R_{max}$$

$$0 \leq R_k^U \leq R_{max}$$

$$0 \leq R_{j,k}^D \leq R_{max}$$

$$0 \leq R_k^D \leq R_{max}$$



Revenue from energy market:

$$Rev_E = \sum_{i \in R} \sum_{j \in Q} \sum_{k \in D} \Delta t_r (\pi_{i,j,k}^E P_{i,j,k}) + \sum_{j \in Q} \sum_{k \in D} \Delta t_q (\pi_{j,k}^E P_{j,k}) + \sum_{k \in D} \Delta t_d (\pi_k^E P_k)$$

Revenue from regulation market:

$$Rev_R = \sum_{j \in Q} \sum_{k \in D} \Delta t_q (\pi_{j,k}^U R_{j,k}^U) + \sum_{k \in D} \Delta t_d (\pi_k^U R_k^U) + \sum_{j \in Q} \sum_{k \in D} \Delta t_q (\pi_{j,k}^D R_{j,k}^D) + \sum_{k \in D} \Delta t_d (\pi_k^D R_k^D)$$

Objective:

$$Max Rev_E + Rev_R$$

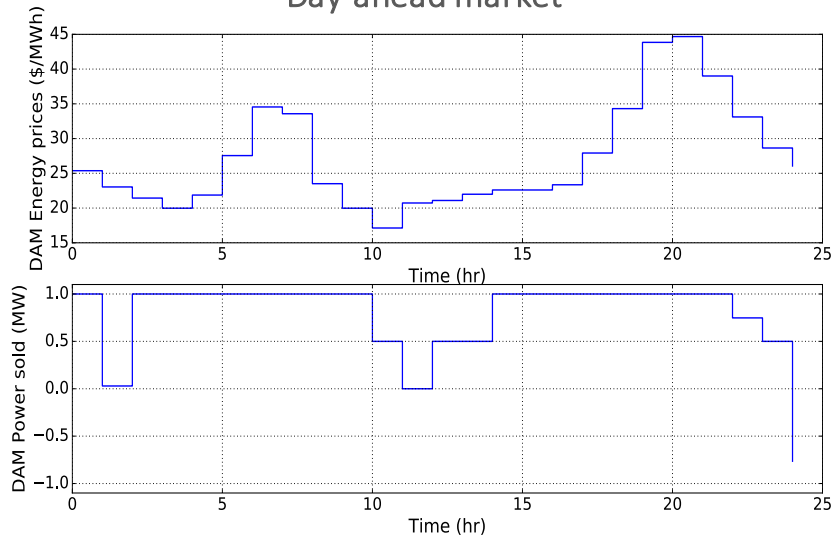
## Results: Solving Complete Optimization Problem for 1 year

- Approach:
  - A single optimization problem formulated to maximize revenues of the whole year
- Problem size:
  - 911041 linear constraints
  - 1103761 variables
- Computation time (Solved in Julia using Gurobi solver)
  - ~ **20 seconds**
- Total annual revenues: **\$293,125**
  - Revenues from energy sale: \$201,281
  - Revenues from regulation: \$91,844

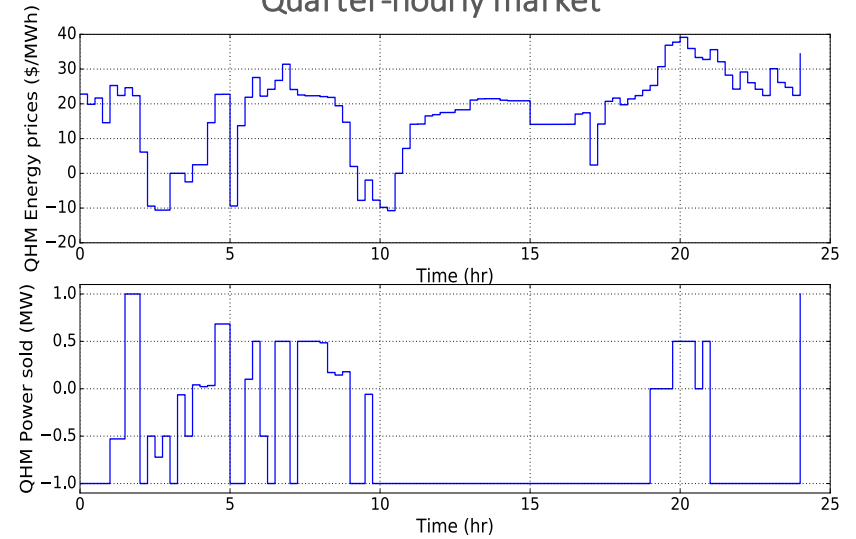
# Results: Simultaneously Solving Complete Optimization Problem for 1 year

## Energy trading profile in a day:

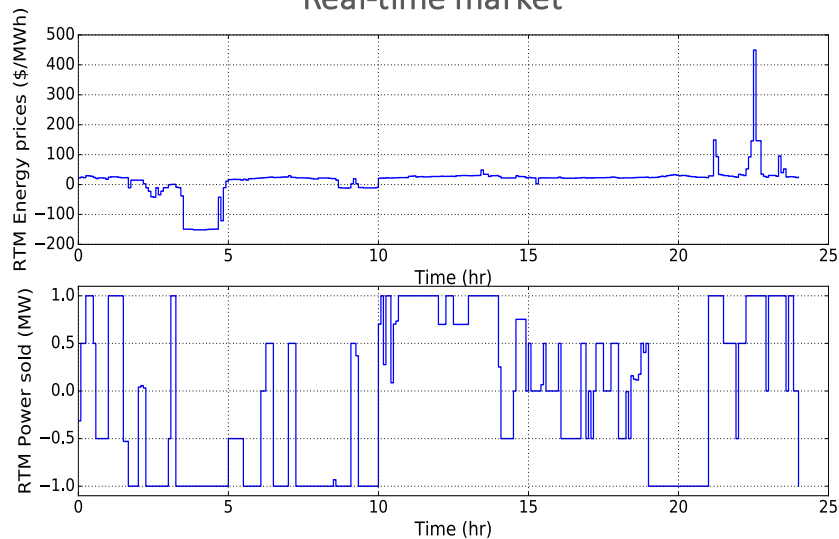
### Day-ahead market



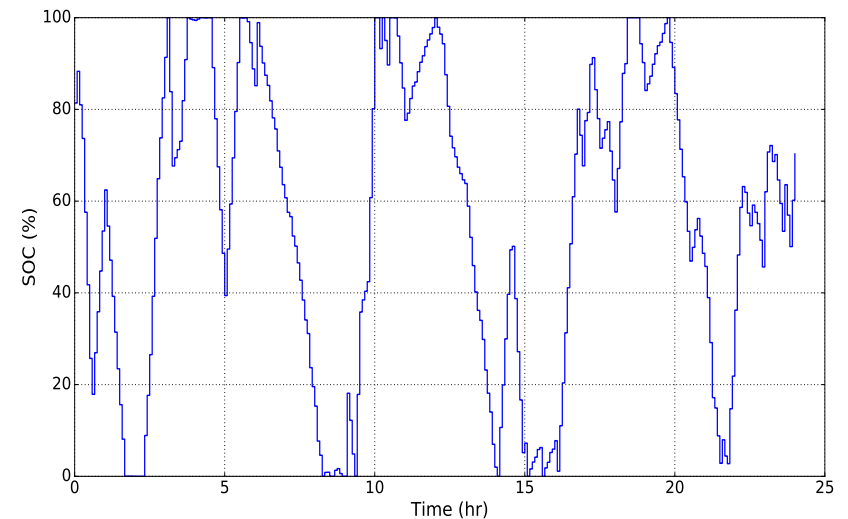
### Quarter-hourly market



### Real-time market

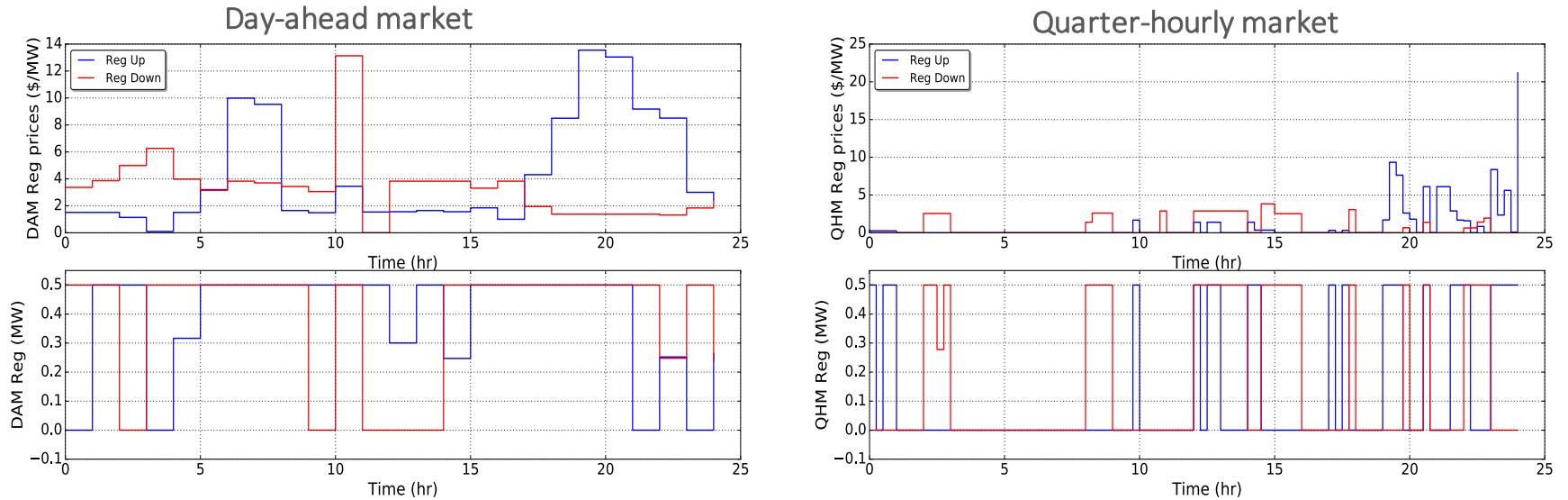


### State of charge of battery

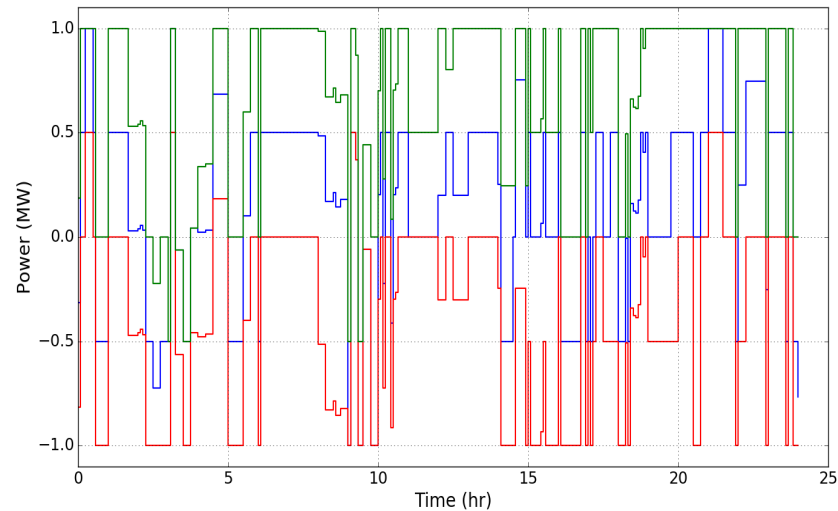


# Results: Simultaneously Solving Complete Optimization Problem for 1 year

## Regulation trading profile in a day:



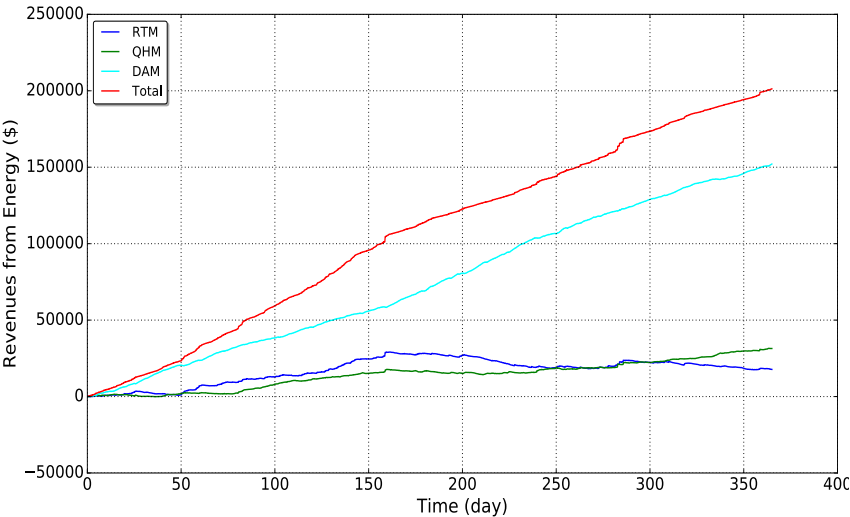
## Power and regulation band



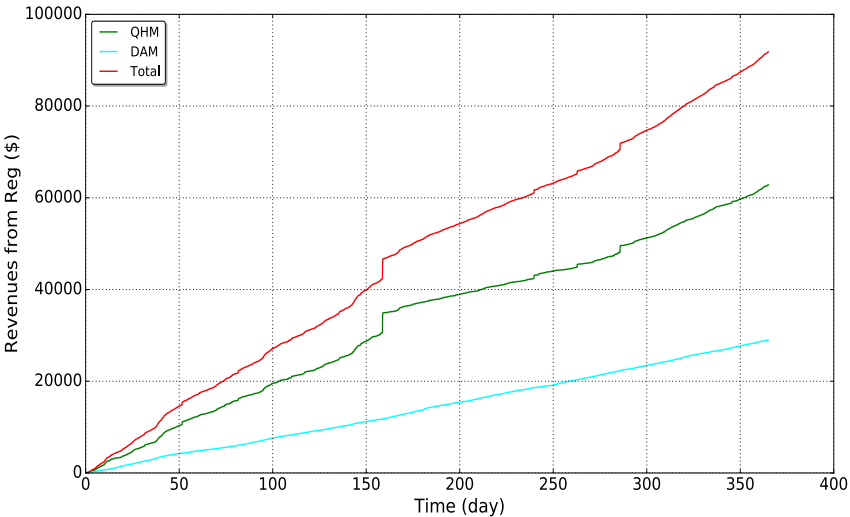
# Results: Solving Complete Optimization Problem for 1 year

## Cumulative revenues in a year:

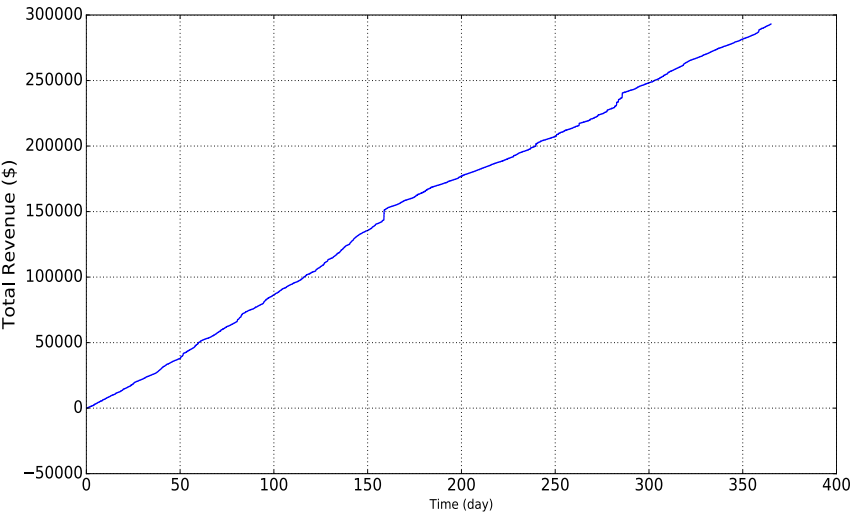
Revenues from energy sale



Revenues from regulation



Total revenues





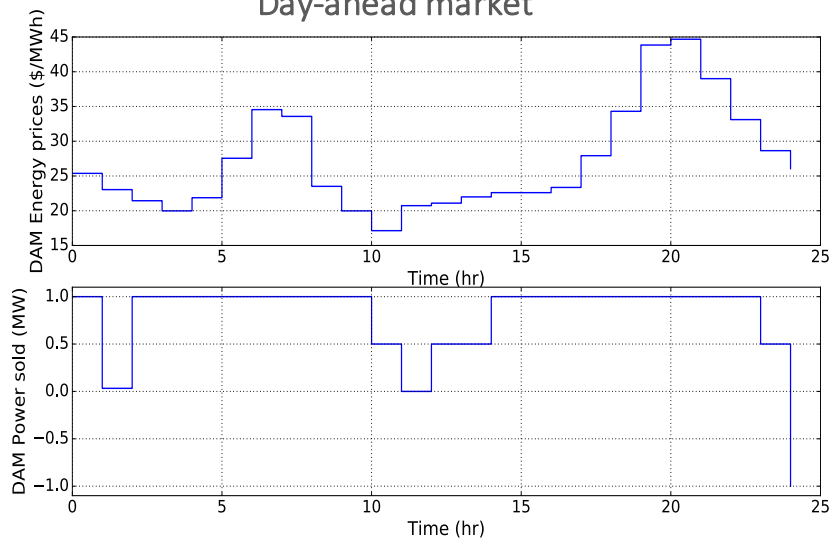
## Results: Solving with Rolling Horizon of 1 Day

- Approach:
  - An optimization problem formulated to maximize revenues of each day sequentially
  - Takes the solution of the end of previous day as the initial point for the present day
- Problem size for each day:
  - 2497 linear constraints
  - 3025 variables
- Computation time (Solved in Julia using Gurobi solver)
  - 0.01-0.02 seconds for each day
  - 5-10 seconds for whole year
- Total annual revenues: **\$296,985**
  - Revenues from energy sale: \$205,835
  - Revenues from regulation: \$91,150

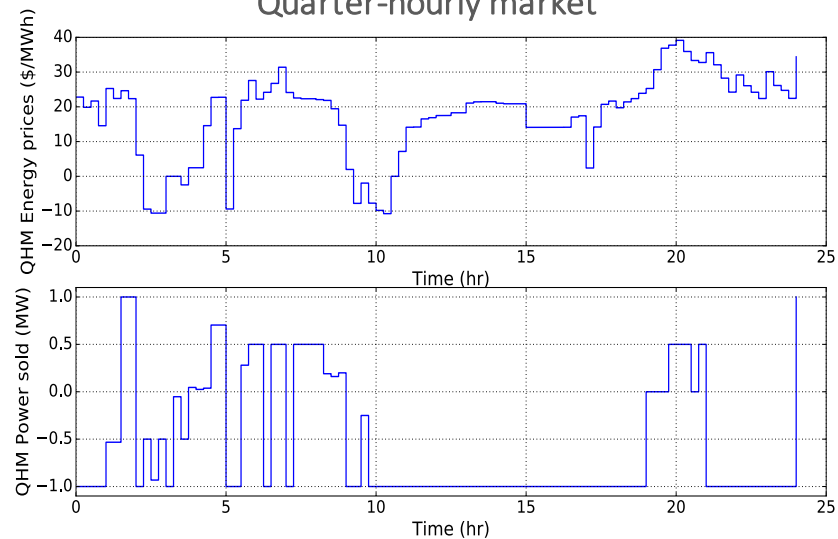
# Results: Solving with Rolling Horizon of 1 Day

## Energy trading profile in a day:

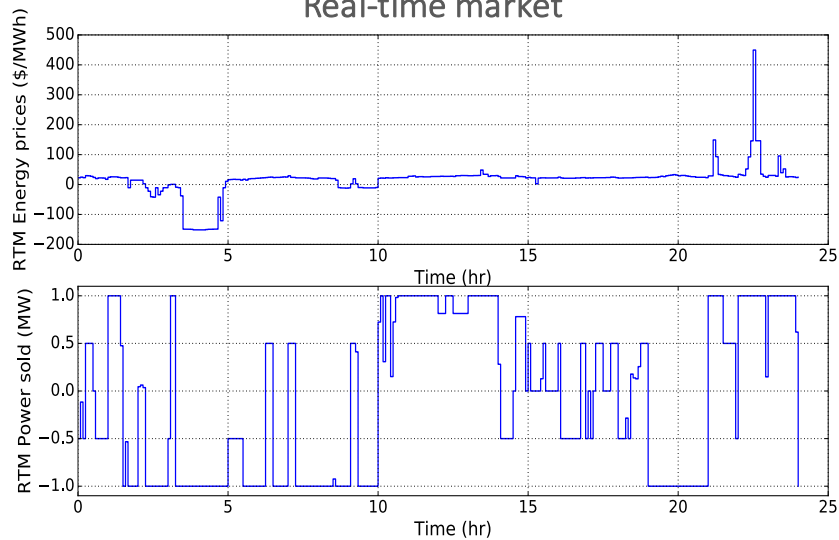
### Day-ahead market



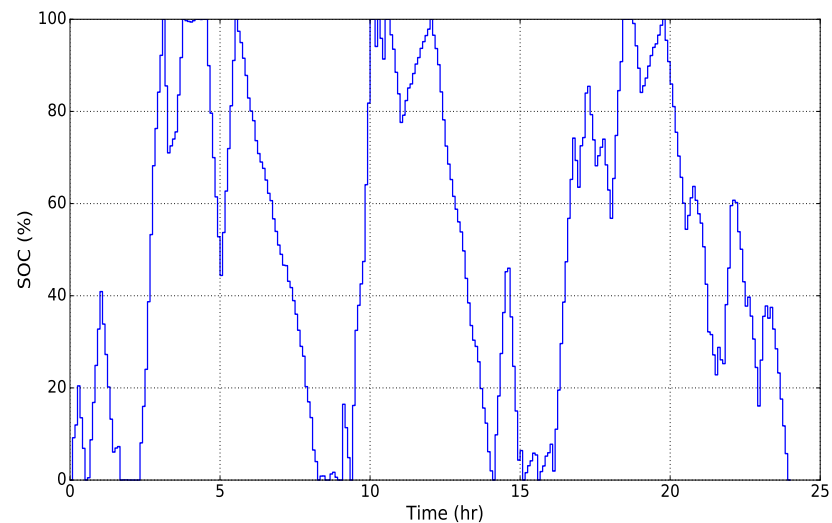
### Quarter-hourly market



### Real-time market



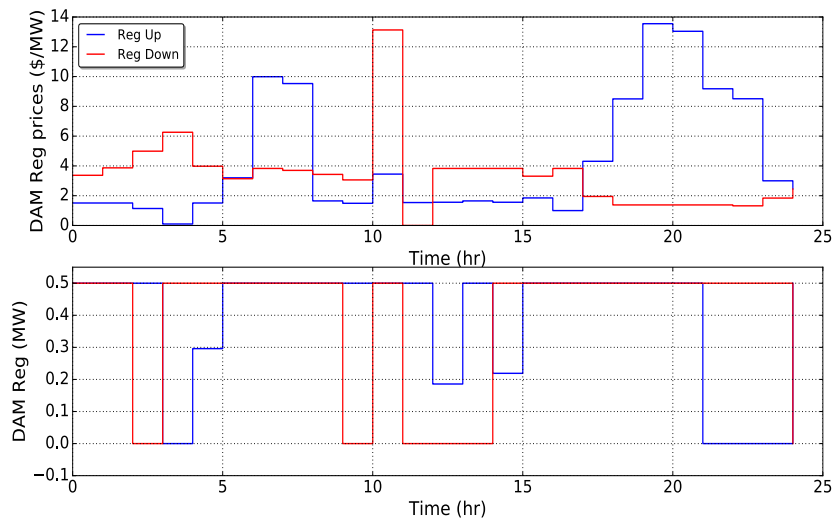
### State of charge of battery



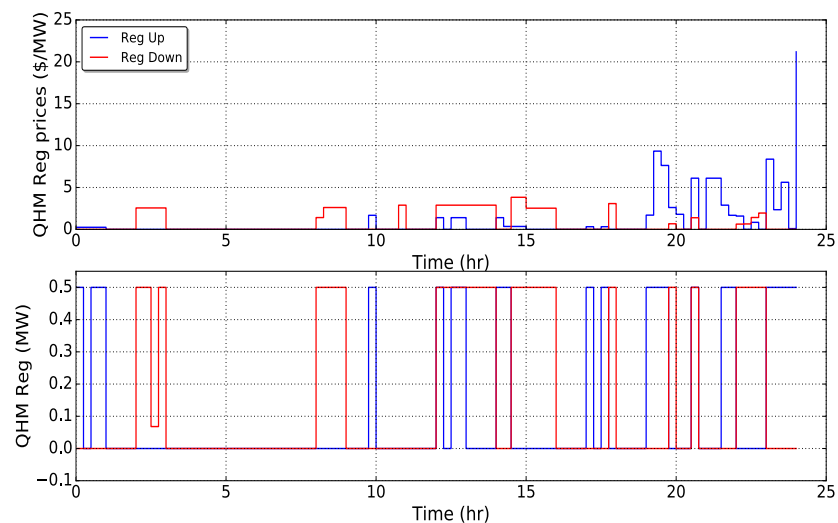
# Results: Solving with Rolling Horizon of 1 Day

## Regulation trading profile in a day:

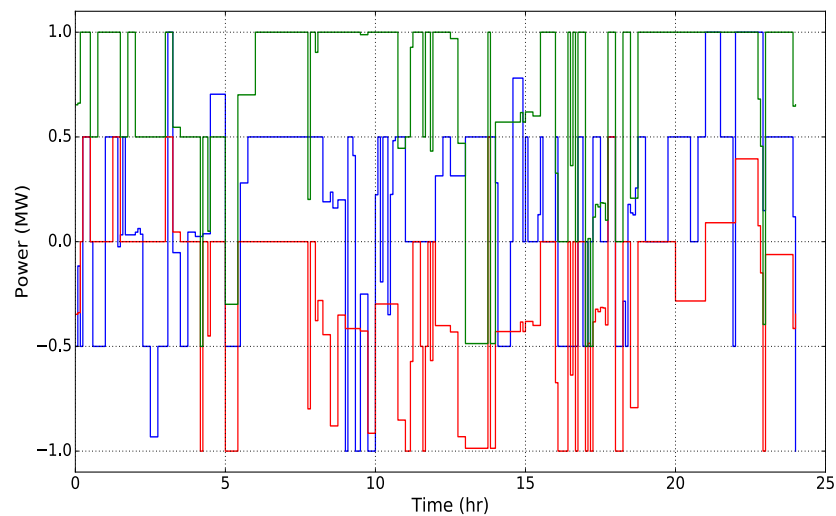
Day-ahead market



Quarter-hourly market



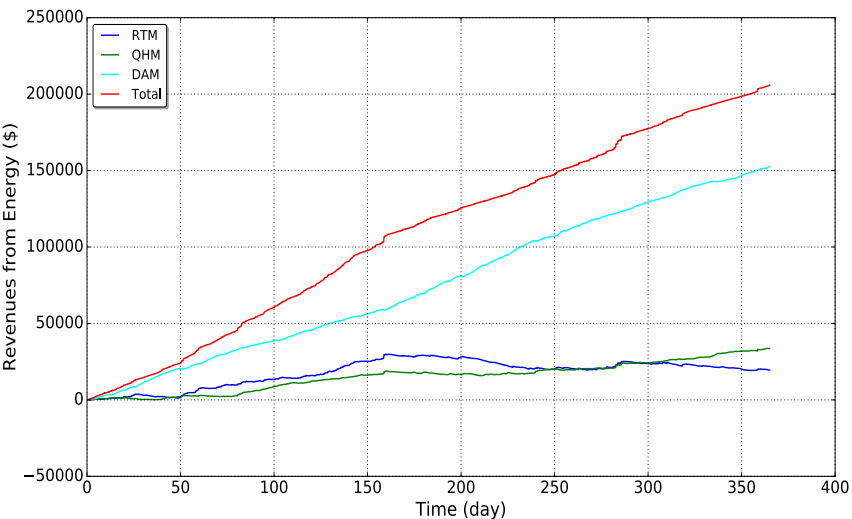
Power and regulation band



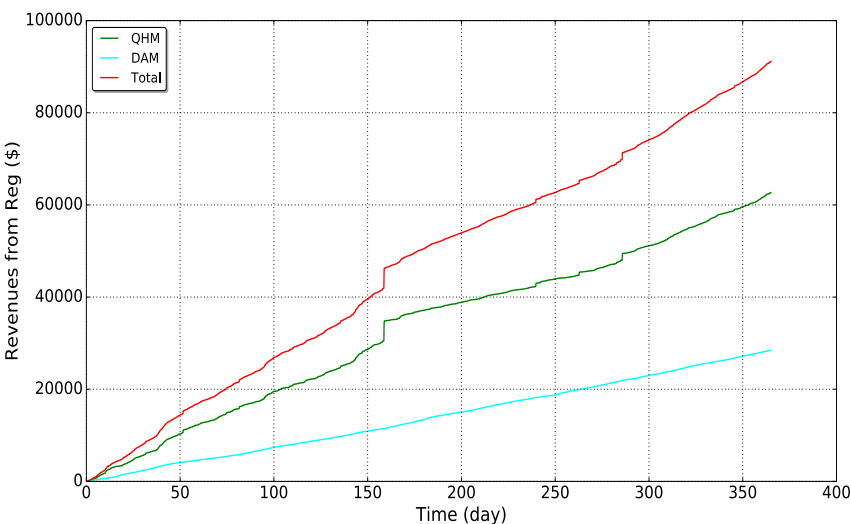
# Results: Solving with Rolling Horizon of 1 Day

## Cumulative revenues in a year:

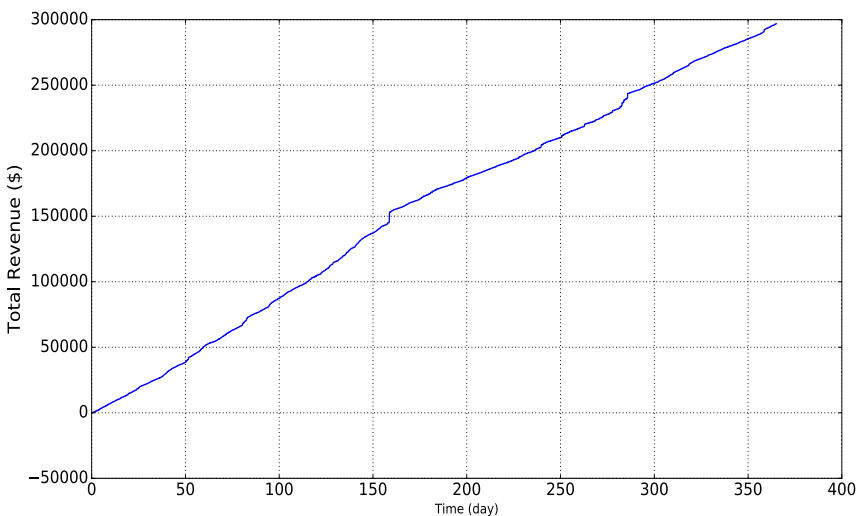
Revenues from energy sale



Revenues from regulation



Total revenues



## Comparison

### Simultaneously solving

- Problem size:
  - 911041 linear constraints
  - 1103761 variables
- Computation time
  - ~ 20 seconds
- Total annual revenues: **\$293,125**
  - Revenues from energy sale: \$201,281
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### Rolling horizon

- Problem size for each day:
  - 2497 linear constraints
  - 3025 variables
- Computation time
  - 0.01-0.02 seconds for each day
  - ~ 5-10 seconds for whole year
- Total annual revenues: **\$296,985**
  - Revenues from energy sale: \$205,835
  - Revenues from regulation: \$91,150

## Conclusions

- Rolling horizon approach is much computationally faster than the simultaneous approach
- Although trends of the solutions are similar, rolling horizon approach results in slightly higher annual revenues than the simultaneous approach
- Expect the rolling horizon approach to perform much better than the simultaneous approach to solve stochastic model for this problem
- A real-time optimal controller can be designed to operate the battery in an optimal way with rolling horizon approach

Thank you!