# ENGO 623 Inertial Surveying and INS/ GPS Integration

## Project 2: Stochastic Modeling of Inertial Sensor Erros

Project report

Submitted by

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## 1 ABSTRACT

Residual random errors remain in inertial sensor data even after calibrating them. This is mainly because of the noise associated with inertial sensors. Although these noise are random, they are correlated in nature, which enables us to model them stochastically. This stochastic model can be included in INS error models for various applications. In this project different error modeling schemes are analyzed. In section 2, the applicability of modeling Gauss-Markov (GM) process with the autocorrelation sequence (ACS) is verified. Alternative to GM process, Autoregressive (AR) stochastic modeling is investigated. In section 3, the significance of de-noising is discussed in brief. Estimation of AR coefficients using Yule-Walker method, Covariance method, and Burg's method is demonstrated. In section 4, the results of de-noising, coefficient determination using various methods are discussed and compared. Finally, conclusion is provided in section 5.

# 2 INTRODUCTION

Post calibrated data of a stationary IMU (simply referred as sensor output in this report) contain the residual sensor errors which are not white in nature; rather they form a set of colored noise which has a distinct ACS sequence (Noureldin. A, Spring 2013). This residual error can be modeled with the help of a shaping filter which minimizes the lest square error between inertial sensor output and shaping filter output. Most of the inertial systems model the residual errors as a first order GM process, which is parameterized by their large correlation time and variance. It is defined by the following first order differential equation (Sameh et al, 2004).

$$\dot{b}(t)= -\beta\, b(t) + \sqrt{2\beta\sigma^2}\, w(t) \qquad (1)$$

Where, b(t) is one type of sensor error, residual bias, β is the correlation time and σ is the standard deviation of the noise.

## 2.1 Gauss-Markov models

Equation (1) has a typical autocorrelation sequence, which will be discussed following. ACS of first order GM process (GM-1) is defined as follows.

$$R_{xx}(t)=\sigma^2 e^{-\beta|t|} \qquad (2)$$

Similarly, second and third order GM process has ACS defined as (Gelb, A. 1974),

$$R_{xx}(t)=\sigma^2 e^{-\beta|t|}\ (1+\beta\backslash t\backslash) \text{ for GM-2,} \qquad (3)$$

$$R_{xx}(t)=\sigma^2 e^{-\beta|t|}\ (1+\beta\backslash t\backslash+(1/3)\beta^2\backslash t\backslash^2) \text{ for GM-3,} \qquad (4)$$

As said earlier if inertial sensors errors to be modeled stochastically with first order GM-1 process, their autocorrelation sequence can be verified against that of GM-1. Figure 1 shows the ACS of first order, second order and third order GM process defined by equations (2-4) with correlation time ~ 175 minutes. It also shows the ACS for Xbow gyro's Z data. It can be verified that ACS of the sensor does not match with that of GM-1, GM-2 or GM-3. Similar observations are found for other sensors as well. Refer Appendix Figure 13 and Figure 14 respectively for auto-correlation sequence plot of Novatel and Xbow data. Hence stochastic modeling of sensor data with GM process cannot be accurate due to improper autocorrelation sequence determination. Hence it has to be modeled with techniques such as autoregressive (AR), moving average (MA), autoregressive moving average (ARMA). This project is limited to analysis with AR models.

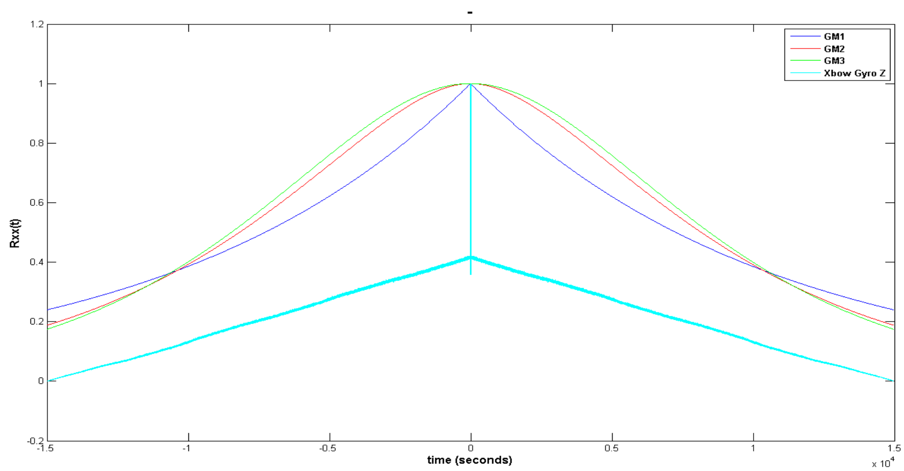



**Figure 1 GM model compared with sensor data, Xbow Gyro Z**

## 2.2 AR model:

The shaping filter of AR process has the transfer function (TF) *G(z)* expressed as,

$$G(z)=\frac{1}{1+\sum_{i=1}^{N} aiZ^{-1}} \qquad (5)$$

Then, AR process is described by, $u(n)=-\sum_{i=1}^{N} aiZ^{-1}+\alpha(n)$ (6)

here, $u(n)$ is sensor output, $a_i$ is the i$^{th}$ coefficient of the TF, N is the order of the filter, $\alpha(n)$ is the white innovation sequence. (Noureldin. A, Spring 2013).

The estimation has the following AR model in finite impulse response (FIR) filter form.

$$\hat{u}(n)=-\sum_{i=1}^{N} aiZ^{-1} \qquad (7)$$

Where, $\hat{u}(n)$ is the estimated sensor output. The coefficients of the model are tuned such that the estimation error is minimum in least square sense.

$$e(n) = u(n) - \hat{u}(n) \tag{8}$$

This project discusses three methods of estimating the coefficients they are: Yule-Walker method (YWM), Co-variance method (CM), and Burg's method (BM).

# 3 METHOD

In this section, the procedure followed in stochastic modeling of the inertial sensors is discussed. Residual sensors generally contain high frequency noise components. AR process needs higher order modeling to combat for the noise effect, which increases the complexity. Other way is to de-noise the signal before beginning the stochastic modeling. There are various techniques for de-noising the signal. Following two are investigated in this analysis; moving average and wave-let de-noising.

## 3.1 De-noising

### 3.1.1 Moving average (MA)

A moving average smoothening process is explored, which uses the MATLAB function 'smooth' in the analysis. A simple moving average filter with variable span, from 0.5 s to 10 s is explored (MATLAB 2012a). Figure 2 shows the result of de-noising performed on Xbow Gyro Z data. Standard deviation (*std*) of the sensor data reduced as the span is increased. Although moving average is simple, it needs a large time span to de-noise the data with *std* being less than a desirable limit. Hence an alternative approach, wave-let de-noising is explored.

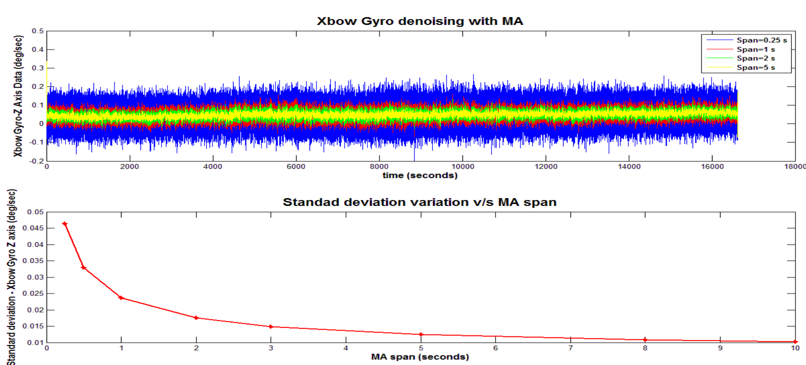



**Figure 2 Moving average de-noising: Xbow Gyro Z**

### 3.1.2 Wavelet de-noising (WDEN)

Wavelet de-noising is preferred for INS sensors before beginning the stochastic modeling. In this analysis, ‘db3’ wavelet function with $5^{th}$ level of decomposition is used with Stein’s Unbiased Risk mode of threshold selection, soft thresholding, and rescaling using level dependent estimation (MATLAB R2012a). Figure 3 shows the de-noising effect on Xbow Gyro data. Figure 4 shows the ACSs of Xbow gyro data de-noised with db3, db5 and db8.

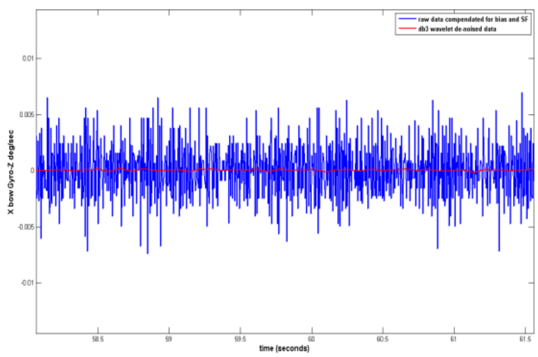



**Figure 3 Wavelet de-noising Xbow gyro data**

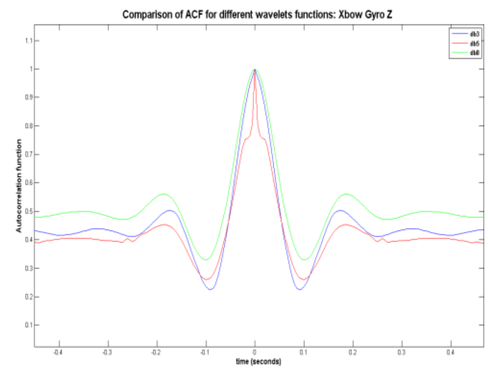



**Figure 4 Different wavelet compared - ACS Gyro data**

## 3.2 RMSE Computation

Once the coefficients of the AR model are obtained, an estimate of *u(n)* is computed (Equation-7). The estimated error (Equation-8) is verified for its root mean square value (RMSE). RMSE for different order of the AR model is verified. A best minimal order for AR process is the one which has least RMSE. Generally for most of the inertial sensors, RMSE will slowly decrease for orders beyond 3 or 4. It is better to a minimal best order so that Kalman-filter error state matrix will be computationally simpler.

## 3.3 Verification

The estimated data $\hat{u}(n)$ is compared with its ACS against the ACS of sensor de-noised data. A perfect matching is illustrated for one example in the next section.

The procedure for AR stochastic modeling of the strap-down inertial sensor data discussed in above sections is represented as a flow chat in Figure 5.

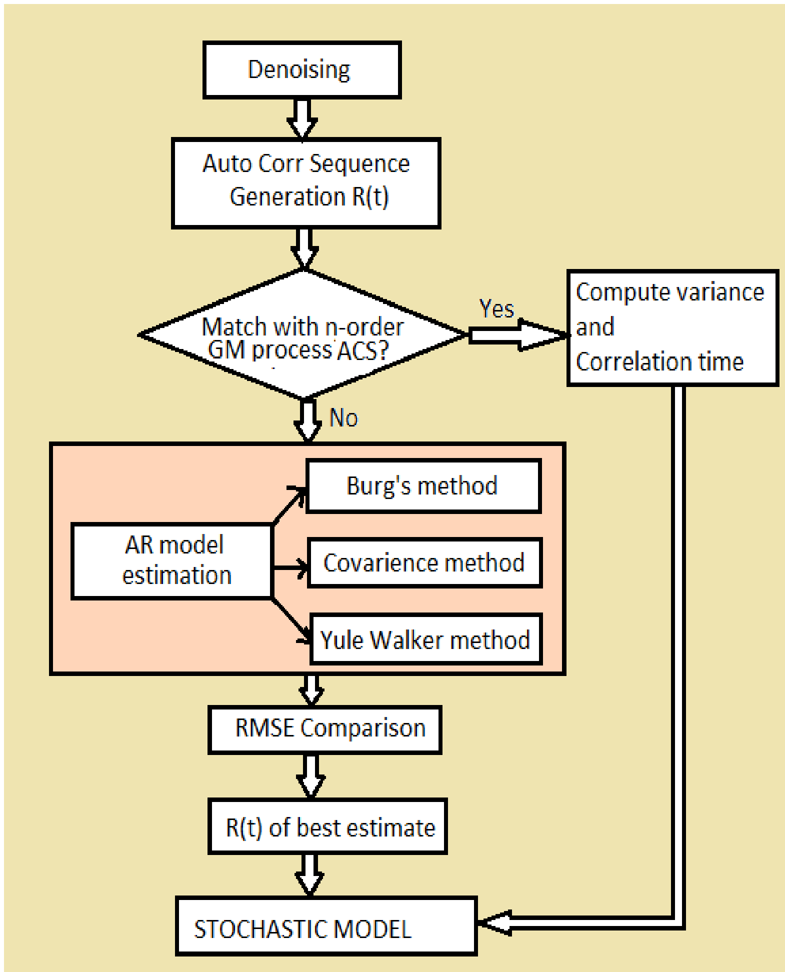


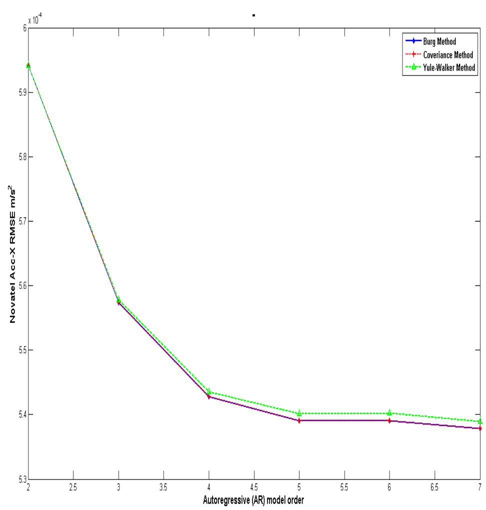

**Figure 5 Flowchart: stochastic modeling of the sensor data**

# 4 RESULTS & DISCUSSIONS

## 4.1 Autoregressive (AR) model estimation:

In this project MATLAB functions 'arburg', 'arcov', and 'aryule' are used to estimate the coefficients of N-order AR model, where N is the order of the model. In the analysis of the result, only Novatel accelerometer's (tactical grade) X axis data and Xbow gyro's (MEMS grade) Z axis data are chosen since similar observations were found in other axis sensors of respective grades, tactical or MEMS. The sample data mentioned above are first denoised with wavelet function db3 with five-level decomposition. Then it is passed through each of the estimator programs and verified for different order of the estimator.

Figure 6 and Figure 7 show the root mean square error (RMSE) of estimator data for three algorithms for Novatel accelerometer and Xbow gyroscope respectively. X axis is the order of the estimator. These figures show that RMSE decreases with increase in the order of the estimator. It is an expected result as many previous samples will be used to predict the present sample when the order of the filter is increased. For lower orders, RMSE is quite high, which if incorporated, results in poor stochastic modeling. However, for any order above 3 or 4, the RMSE remains approximately at the same level. Hence, in order to have a better stochastic model for each axis of the Xbow sensor, a AR model of order 3 or higher can be realized. It is also important to note that increasing the model order will result in complex error model structure, which is the input to the Kalman filter. Hence, generally 3$^{rd}$ order model is much preferred to reduce the computation complexity. It is also evident from the figures that both Burg's method and covariance method perform better than



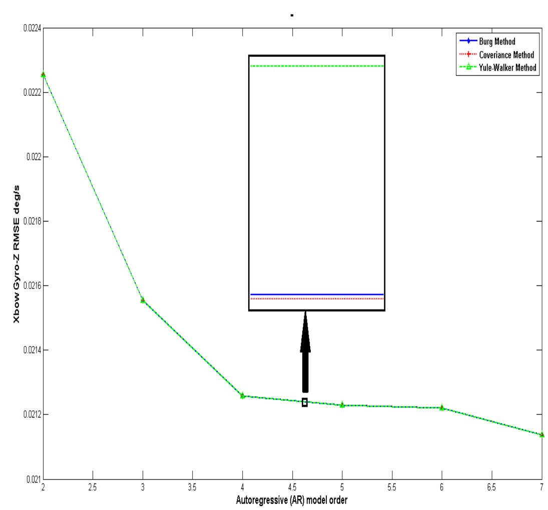




**Figure 6 RMSE v/s order of the estimator Novatel Acc** **Figure 7 RMSE v/s order of the estimator Xbow Gyro**

the Yule-Walker method. The poor performance of Yule-walker is expected for shorter data as it does windowing operation on the data (J Makhoul, 1975). In this experiment, 250 second data is used to examine all the three algorithms. For the Xbow data estimation, Yule-Walker method

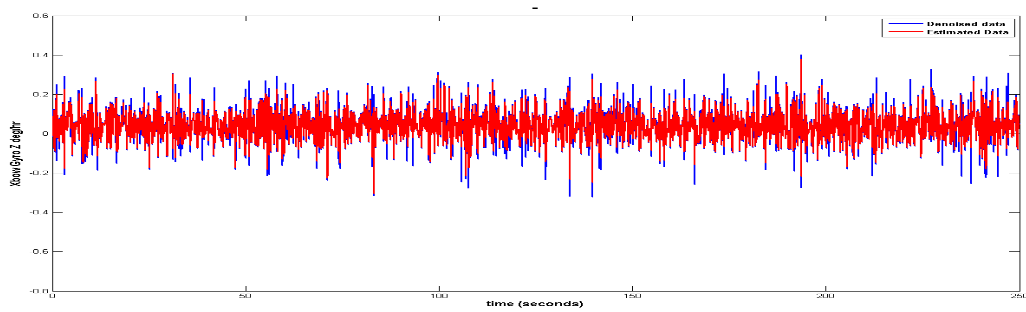



**Figure 8 Comparing estimated output from Burgs method and denoised data input for Xbow gyro**

showed almost similar performance as Burg or Covariance method, however, Burg and Covariance methods are a little better than Yule-Walker, as it is evident in Figure 7. It should be noted that computation of model coefficients in Yule-Walker method involves taking the matrix inverse, hence it takes relatively large computation load compared to other two methods. Covariance method for the given data performed similarly to the Burg's method. But for short data duration, Burg's method gave less RMSE when compared to Covariance method. In general, Covariance and Yule-walker method do not guarantee stability. Above mentioned reasons allowed us to choose estimation of coefficients using Burg's method because of it increased stability. Figure 9 shows the required time profile for 3 estimation methods. This is generated for a data record of length 10 seconds using MATLAB tic-toc events. As illustrated in the graph, Yule-Walker estimator needs significant time as compared to other to methods.

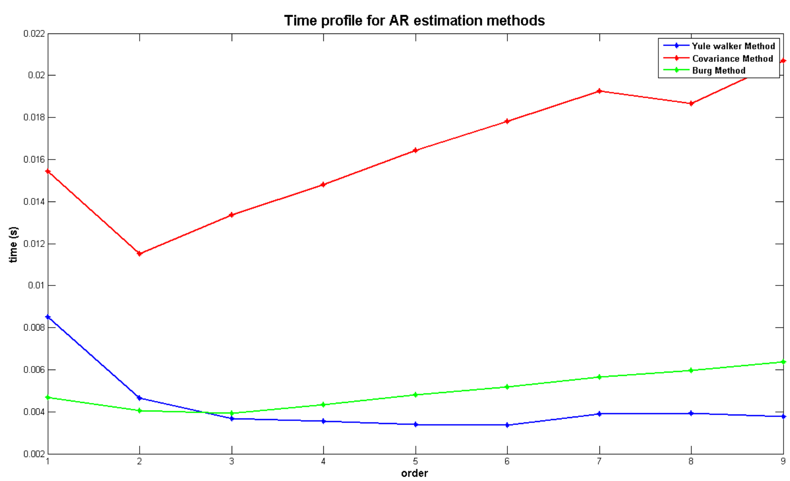



**Figure 9 Time profile for 3 estimators**

Estimated data is generated as per the equation (7). Figure 10 shows the estimated Xbow gyro data which was generated with 2nd order Burg's method, overlapped on de-noised data. Figure 6 shows the Novatel accelerometer data estimated in the same manner. Both results show that estimated output is matching with that of de-noised data.

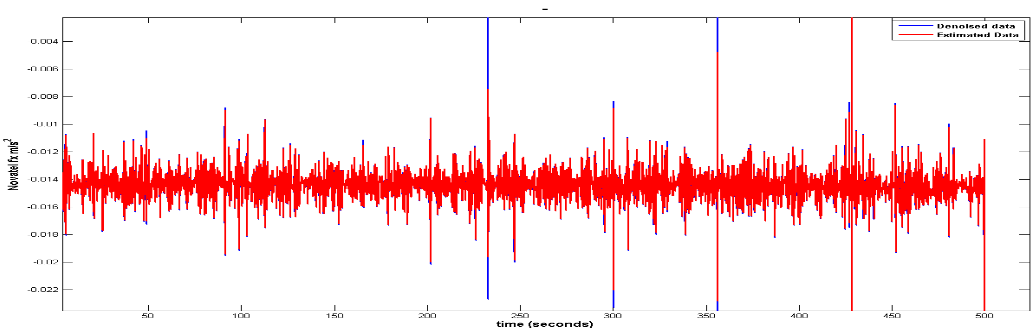



**Figure 10 Comparing estimated output from Burgs method and denoised data input for Novatel accelerometer**

## 4.2 Verification:

Estimated data $\hat{u}(n)$ can be verified with time domain analysis as discussed in previous plots. It can also be verified by comparing the ARF of $\hat{u}(n)$ and $u(n)$. Figure 12 shows the ACS of sensor data overlapped on GM models of order 1, 2 and 3. As discussed earlier, GM models could not properly model the sensor noise. Figure 13 shows zoomed-in view around $R_{xx}(0)$. Estimated data ACS overlaps exactly with that of de-noised data.

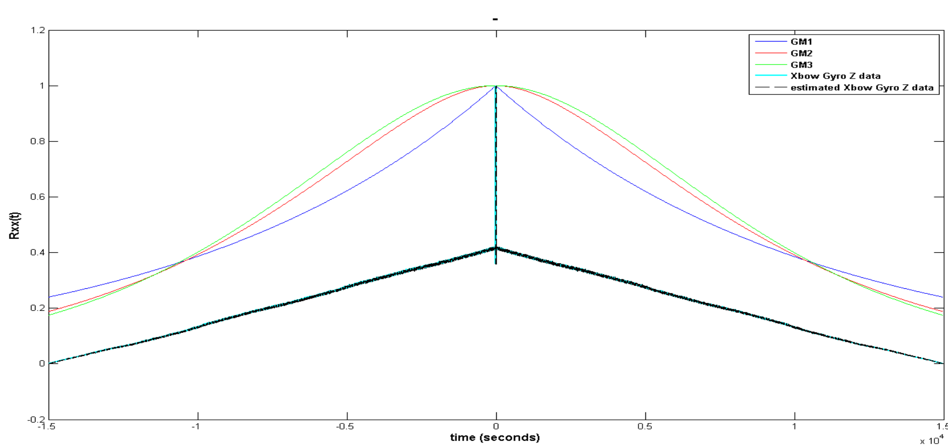



**Figure 11 ACS of Guass-Markov processes compared with sensor data**

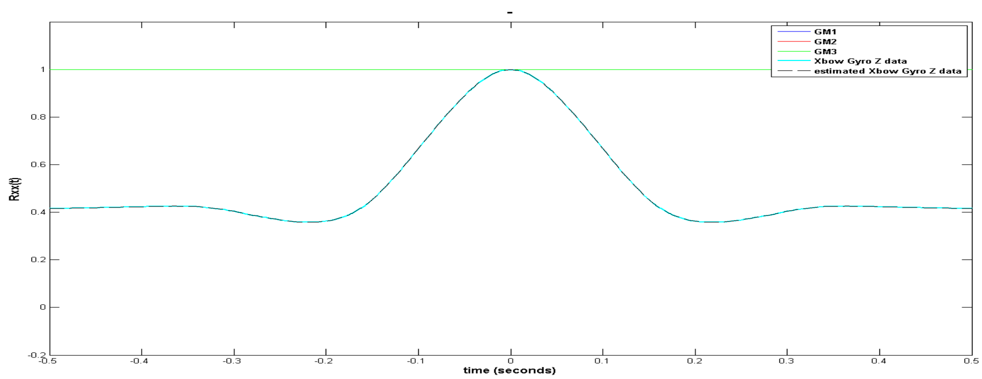



**Figure 12 ACS zoomed to compare estimated data and de-noised data**

# 5 CONCLUSIONS

Stochastic modeling of inertial sensors with 1st order Gauss-Markov, or, even 2nd and 3rd order is not accurate as the autocorrelation sequences of two processes do not match. Higher order AR models are explored with AR process. De-noising of sensor data is necessary to have a better estimate of the coefficients and to avoid having a higher order model. Wavelet de-noising with db5 wavelet is demonstrated. AR model parameters are estimated using three techniques, Yule-walker method, Covariance method and Burg's method of different orders. Burg's method out performs other two methods as it yields a least RMSE with short data. Also for a given short data Burg method generates a better estimation with less time as compared to the Covariance method.

# 6 REFERENCES

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## APPENDIX

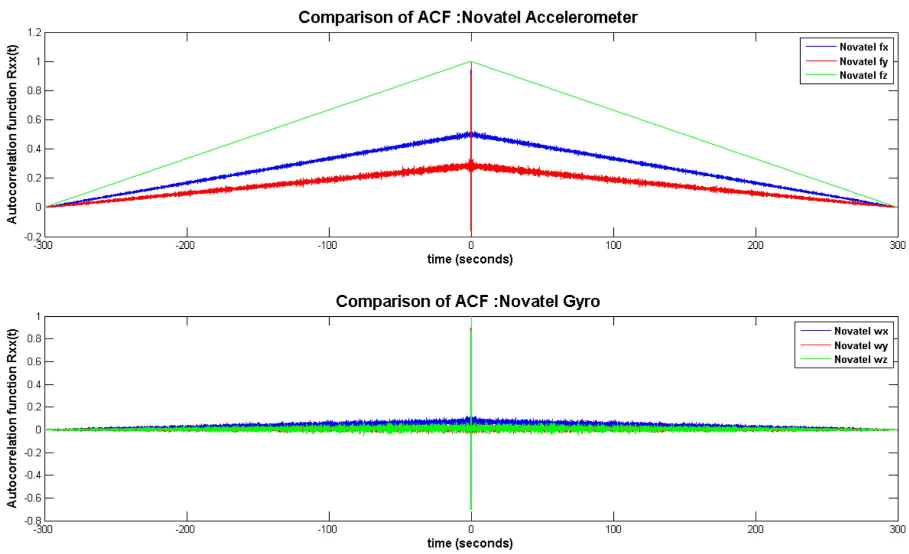


**Figure 13 Novatel Auto-correlation plot**

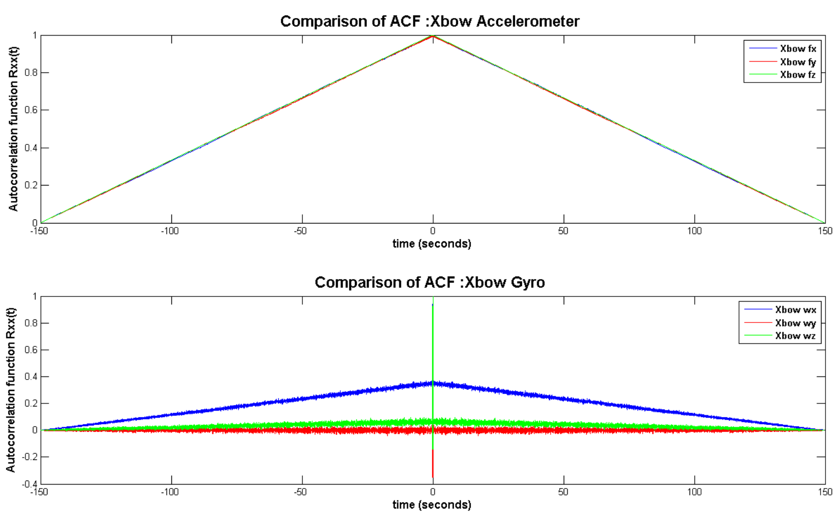


**Figure 14 Xbow Auto-correlation plot**