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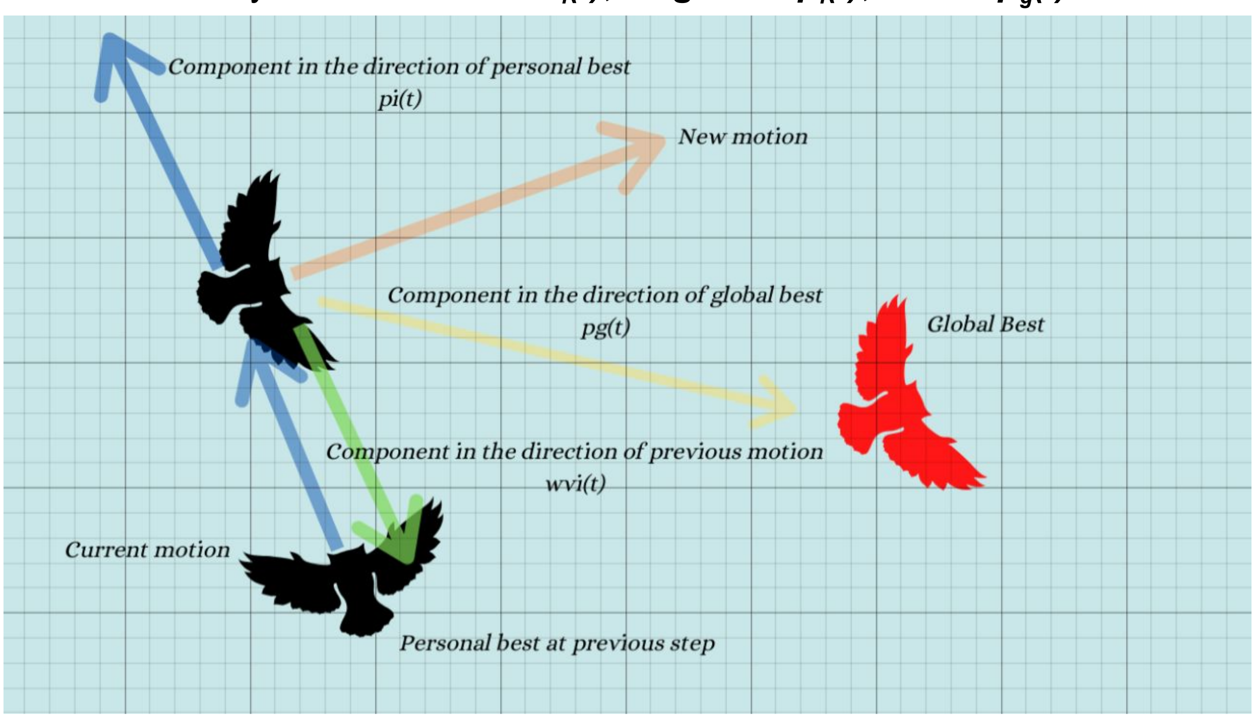
Assignment 19 (PSO)

We are using a Particle Swarm Optimization (PSO) method to solve a minimization problem. The algorithm started at time t_0 and is currently in timestep t_3. The population consists of three particles $\vec{x}_1$, $\vec{x}_2$ and $\vec{x}_3$. In the following table you find the positions, velocities and fitness values of the particles at the timesteps t_0, t_1, t_2 und t_3.

Particle	$\vec{x}_1$	$\vec{x}_2$	$\vec{x}_3$
$\vec{v}_i(t_0)$	$(0.00, 0.00)$	$(0.00, 0.00)$	$(0.00, 0.00)$
$\vec{x}_i(t_0)$	$(2.00, 2.00)$	$(-5.00, -2.00)$	$(3.00, -5.00)$
$f(\vec{x}_i, t_0)$	8.00	29.00	34.00
$\vec{v}_i(t_1)$	$(0.00, 0.00)$	$(2.10, 1.20)$	$(-0.30, 2.10)$
$\vec{x}_i(t_1)$	$(2.00, 2.00)$	$(-2.90, -0.80)$	$(2.70, -2.90)$
$f(\vec{x}_i, t_1)$	8.00	9.05	15.70
$\vec{v}_i(t_2)$	$(0.00, 0.00)$	$(2.31, 1.32)$	$(-0.33, 2.31)$
$\vec{x}_i(t_2)$	$(2.00, 2.00)$	$(-0.59, 0.52)$	$(2.37, -0.59)$
$f(\vec{x}_i, t_2)$	8.00	0.62	5.96
$\vec{v}_i(t_3)$	$(-0.78, -0.44)$	$(0.92, 0.53)$	$(-1.02, 1.26)$
$\vec{x}_i(t_3)$	$(1.22, 1.56)$	$(0.33, 1.05)$	$(1.35, 0.67)$
$f(\vec{x}_i, t_3)$	3.92	1.21	2.27

- Explain briefly how PSO works.

- For each particle *i,* located at $x_i(t) \in \mathbb{R}^n$ in the search space the position is updated at each iteration.
- Defined by : **Momentum** $\boldsymbol{w^*v_i(t)}$**, Cognitive** $\boldsymbol{p_i(t)}$**, Social** $\boldsymbol{p_g(t)}$



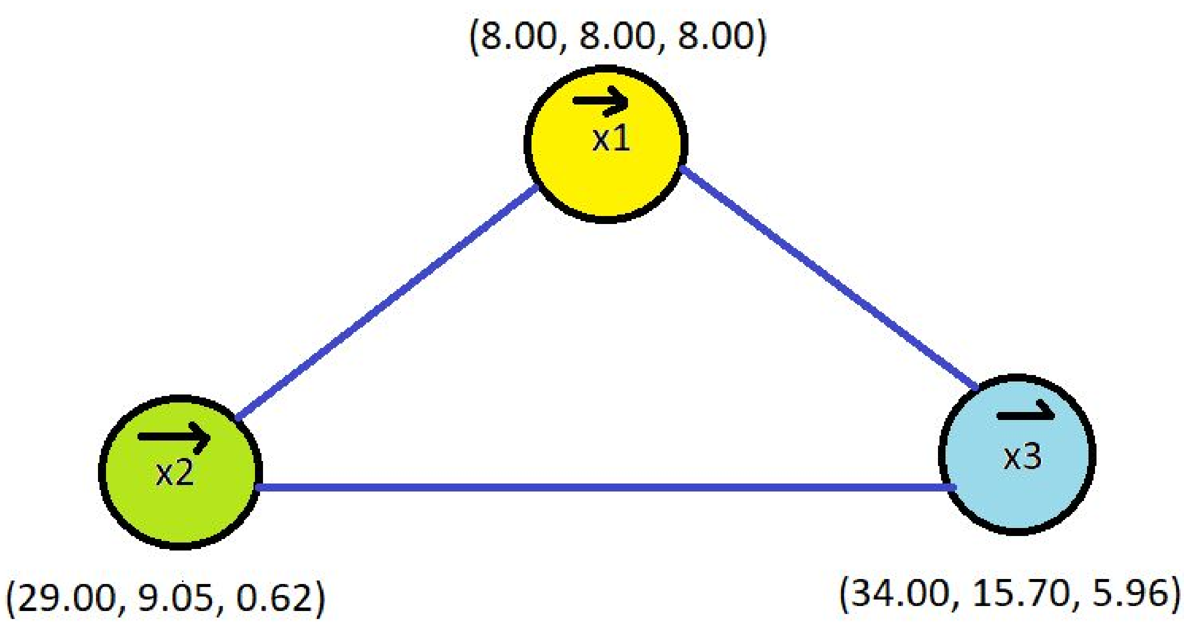
- The position is updated by, $x_i(t+1) = x_i(t) + v_i(t+1)$
- The velocity is updated by, $v_i(t+1) = w*v_i(t) + c_1\Phi_1(P_g(t)-x_i(t)) + c_2\Phi_2(P_i(t)-x_i(t))$

w = inertia weight
$\Phi_1 = \Phi_2$ = Learning coefficients
$c_1 = c_2$ = attraction rates

- Works for continuous optimization problems.

- For each particle i, determine in t_3 its previous best position $\vec{P}_i(t_3)$ and the previous best position in its neighborhood $\vec{P}_g(t_3)$. The PSO uses a fully connected neighborhood topology.

We have a **minimization** problem and also the PSO uses a **fully connected** neighbourhood topology in which all members are connected to one another.



For particle $\vec{x}_1$ previous best position in t_3, $\vec{P}_i(t_3)$ = 8.00 at all fitness
For particle $\vec{x}_2$ previous best position in t_3, $\vec{P}_i(t_3)$ = 0.62 at $f(\vec{x}_i, t_2)$
For particle $\vec{x}_3$ previous best position in t_3, $\vec{P}_i(t_3)$ = 5.96 at $f(\vec{x}_i, t_2)$

For particle $\vec{x}_1$ previous best neighbourhood position in t_3, $\vec{P}_g(t_3)$ = 0.62 at $f(\vec{x}_i, t_2)$ with the neighbour $\vec{x}_2$

For particle $\vec{x}_2$ previous best neighbourhood position in t_3, $\vec{P}_g(t_3)$ = 5.96 at $f(\vec{x}_i, t_2)$ with the neighbour $\vec{x}_3$

For particle $\vec{x}_3$ previous best neighbourhood position in t_3, $\vec{P}_g(t_3)$ = 0.62 at $f(\vec{x}_i, t_2)$ with the neighbour $\vec{x}_2$

- Calculate the updated velocities ($\vec{v}_i(t_4)$) and positions ($\vec{x}_i(t_4)$) for the next iteration of the PSO. Use $w = 0.4$, $\phi_1 = 0.3$, $\phi_2 = 0.3$ and $c_1 = c_2 = 1$.

For particle $\vec{x}_1$, the **first velocity** component,
0.4(-0.78) + (1)(0.3)(1.21 - 1.22) + (1)(0.3)(3.92 - 1.22) = 0.495

Then the **first time** component,
0.495 + 1.22 = 1.715

For particle $\vec{x}_1$, the **second velocity** component,
0.4(-0.44) + (1)(0.3)(1.21 - 1.56) + (1)(0.3)(3.92 - 1.56) = 0.427

Then the **second time** component,
0.427 + 1.56 = 1.987

This gives $\vec{v}_i(t_4)$ = **(0.495, 0.427)** and $\vec{x}_i(t_4)$ = **(1.715, 1.987)** for particle $\vec{x}_1$

For particle $\vec{x}_2$, the **first velocity** component,
0.4(0.92) + (1)(0.3)(2.27 - 0.33) + (1)(0.3)(0.62 - 0.33) = 1.037

Then the **first time** component,

1.037 + 0.33 = 1.367

For particle $\vec{x}_2$, the **second velocity** component,
0.4(0.53) + (1)(0.3)(2.27 - 1.05) + (1)(0.3)(0.62 - 1.05) = 0.449

Then the **second time** component,
0.449 + 1.05 = 1.499

This gives $\vec{v}_i(t_4)$ = **(1.037, 0.449)** and $\vec{x}_i(t_4)$ = **(1.367, 1.499)** for particle $\vec{x}_2$

For particle $\vec{x}_3$, the **first velocity** component,
0.4(-1.02) + (1)(0.3)(1.21 - 1.35) + (1)(0.3)(2.27 - 1.35) = -0.174

Then the **first time** component,
-0.174 + 1.35 = 1.176

For particle $\vec{x}_3$, the **second velocity** component,
0.4(1.26) + (1)(0.3)(1.21 - 0.67) + (1)(0.3)(2.27 - 0.67) = 1.146

Then the **second time** component,
1.146 + 0.67 = 1.816

This gives $\vec{v}_i(t_4)$ = **(-0.174, 1.146)** and $\vec{x}_i(t_4)$ = **(1.176, 1.816)** for particle $\vec{x}_3$

Particle	$\vec{x}_1$	$\vec{x}_2$	$\vec{x}_3$
$\vec{v}_i(t_4)$	(0.495, 0.427)	(1.037, 0.449)	(-0.174, 1.146)
$\vec{x}_i(t_4)$	(1.715, 1.987)	(1.367, 1.499)	(1.176, 1.816)