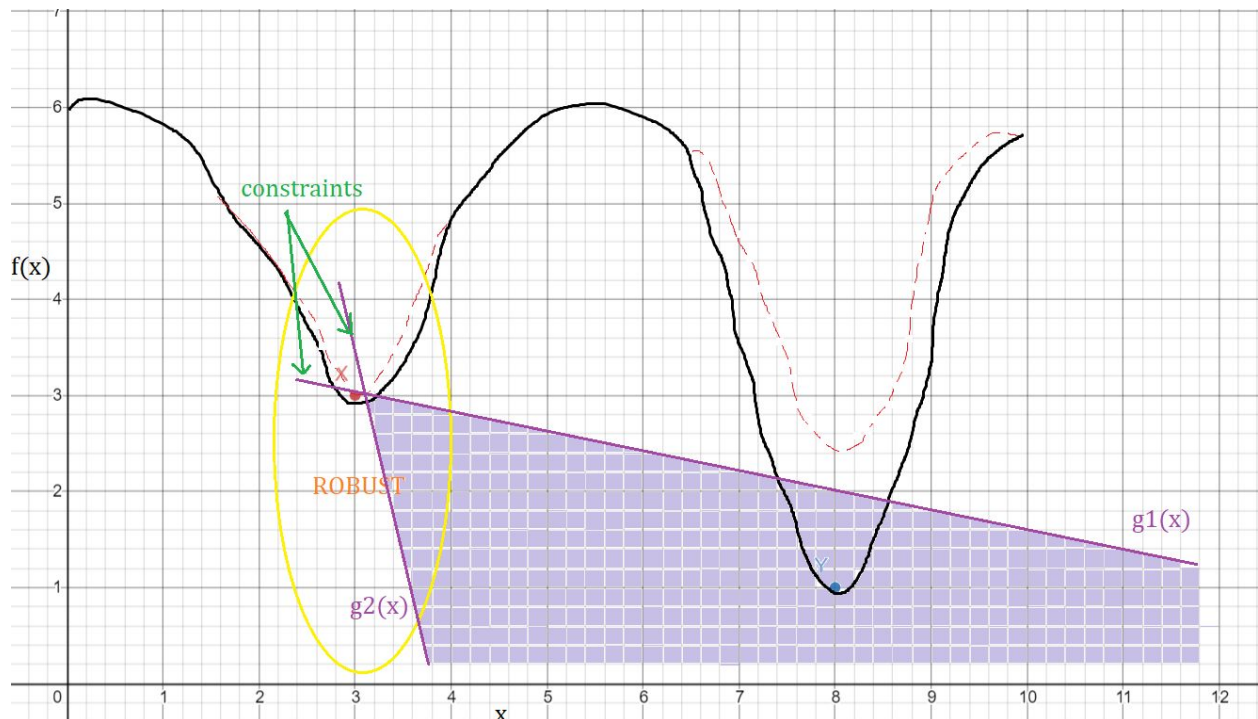


Assignment 28 (Robustness II)

Please answer the following questions related to Robustness.

- Explain the difference between robustness and reliability. Show a graphic example with at least one solution that is robust but not reliable, and another solution that is reliable but not robust.
1. **Robust designs** are designs where the variation in performance of the function wrt variation in relevant variables is **minimal**.
 2. **Reliable designs** are designs at which the chance of system failure is **low**.



- What is the difference between robustness in single- and in multi-objective problems? Explain what changes are needed in type I and type II robustness for multi-objective problems.
1. In MOP, we should consider the robustness for all m -objectives (as preferred by DM).
 2. The combined effect of variations in all m -objectives has to be used as a measure of sensitivity to variable perturbation.

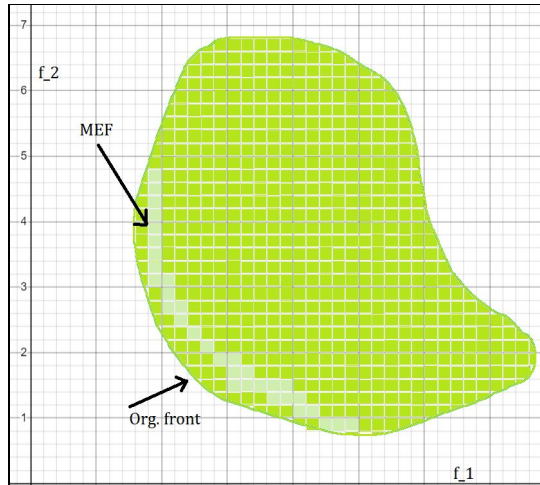
For any solution to be **Type I - MO Robust solution** if it is the global Pareto-optimal solution of form:

$$\text{Min } F^{\text{eff}}(x) = (f_1^{\text{eff}}(x), f_2^{\text{eff}}(x), \dots, f_m^{\text{eff}}(x)) \text{ subj to } x \in S$$

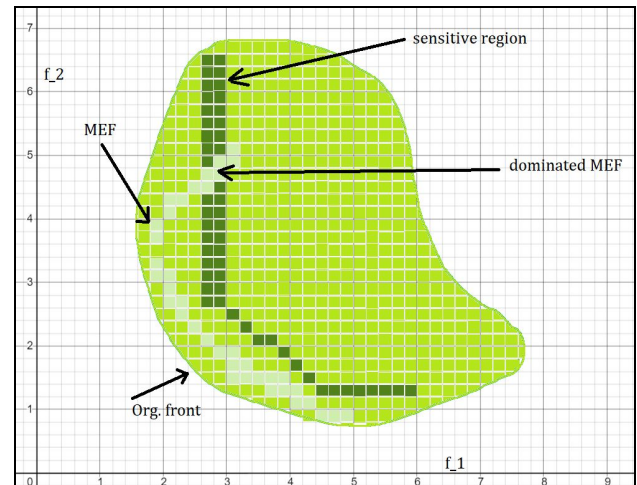
$$\text{(where, } f_j^{\text{eff}}(x) = \frac{1}{|B_\delta(x)|} \int_{y \in B_\delta(x)} f_j(y) dy; |B_\delta(x)| \Rightarrow \text{vol. of the } \delta\text{-neighbourhood).}$$

The obtained PF for this problem is called **Mean Effective Front** \Rightarrow cat. in 4 cases.

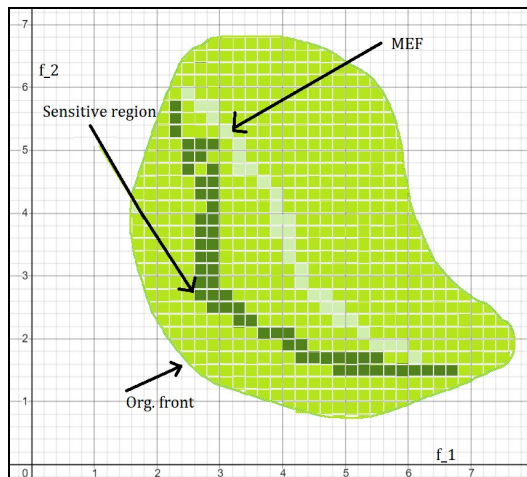
C1: Entire PF + MEF robust.



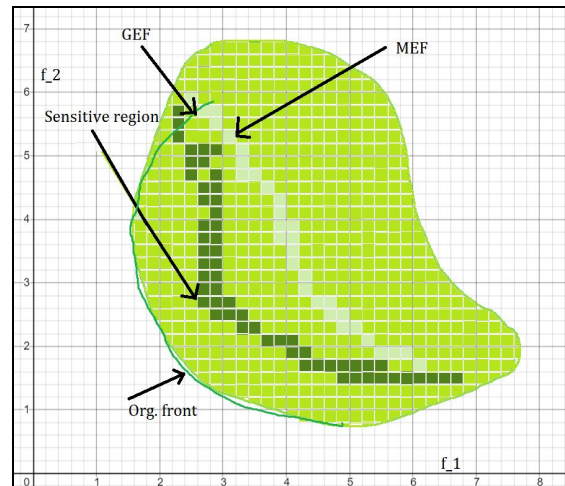
C2: Part of MEF robust



C3: MEF non-robust.



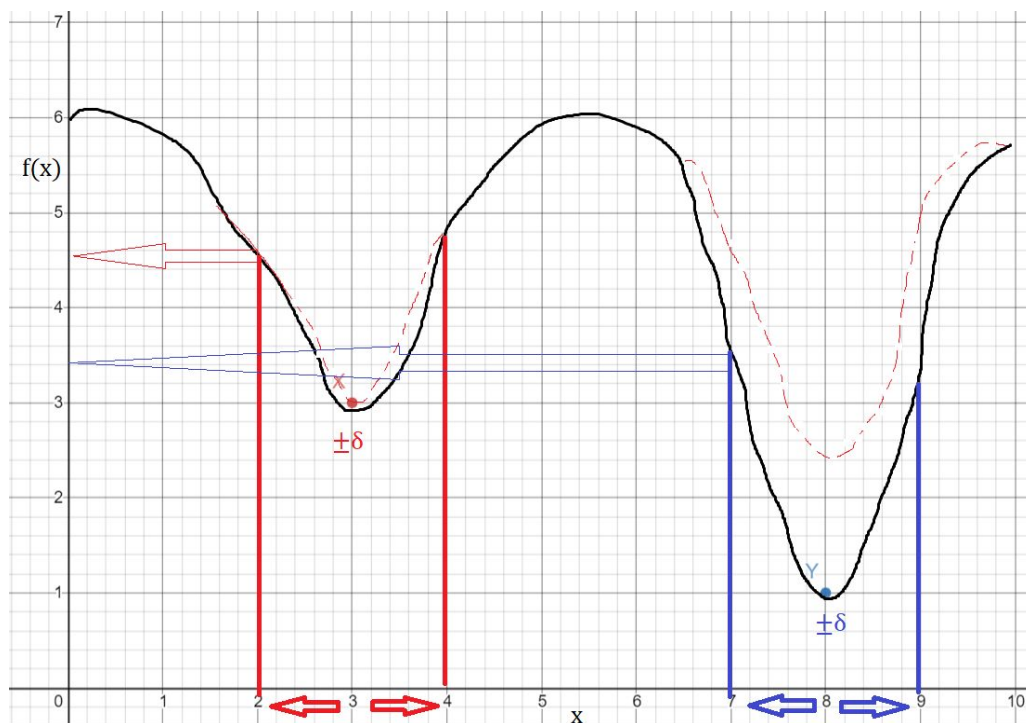
C4: Part of GEF non-robust.



For any solution to be **Type II - MO Robust solution** if it is the (global) feasible Pareto-optimal solution of form:

$$\begin{aligned} \text{Min } F(x) &= (f_1(x), \dots, f_m(x)) \\ \text{subj to } \frac{\|F^P(x) - F(x)\|}{\|F(x)\|} &\leq \eta \text{ \& } x \in S \end{aligned}$$

- In type I robustness, what is the influence of the parameter δ ?
1. In the context of type I robustness, the **hyperparameter** δ represents the **neighbour** of a particular solution by going $\pm\delta$ around a solution i.e. $x+\delta$ or $x-\delta$. ($\delta = 0 \Rightarrow$ no sampling).
 2. The volume or size of the neighbourhood (set of neighbours) is defined by $B_\delta(x)$.
 3. The choice of δ is very critical as allows to provide **acceptable** solutions in an optimization problem.



- Discuss how the parameter η influences type II robustness.

1. η is a user-defined threshold used in the context of type II robustness

$$\frac{\|f^{eff}(x) - f(x)\|}{\|f(x)\|} \leq \eta \quad \& \quad f^{eff}(x) = f^P(x)$$

2. If the η value is increased \Rightarrow accept every solution.
3. If the η value is decreased $\Rightarrow f^{eff}(x) - f(x)$ is indifferent.