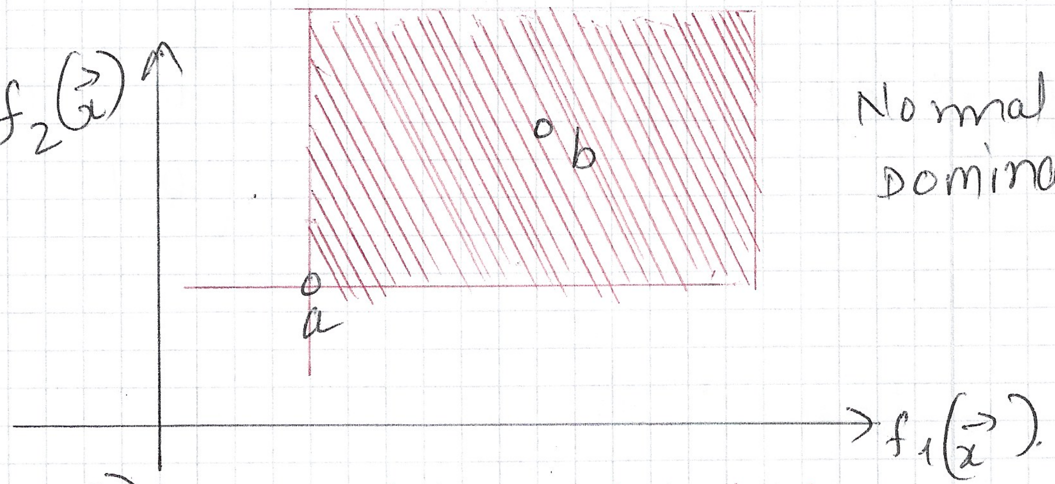
Assignment #8 (ϵ - Domination) 8-1

Let $\vec{x}_1, \vec{x}_2 \in S$.

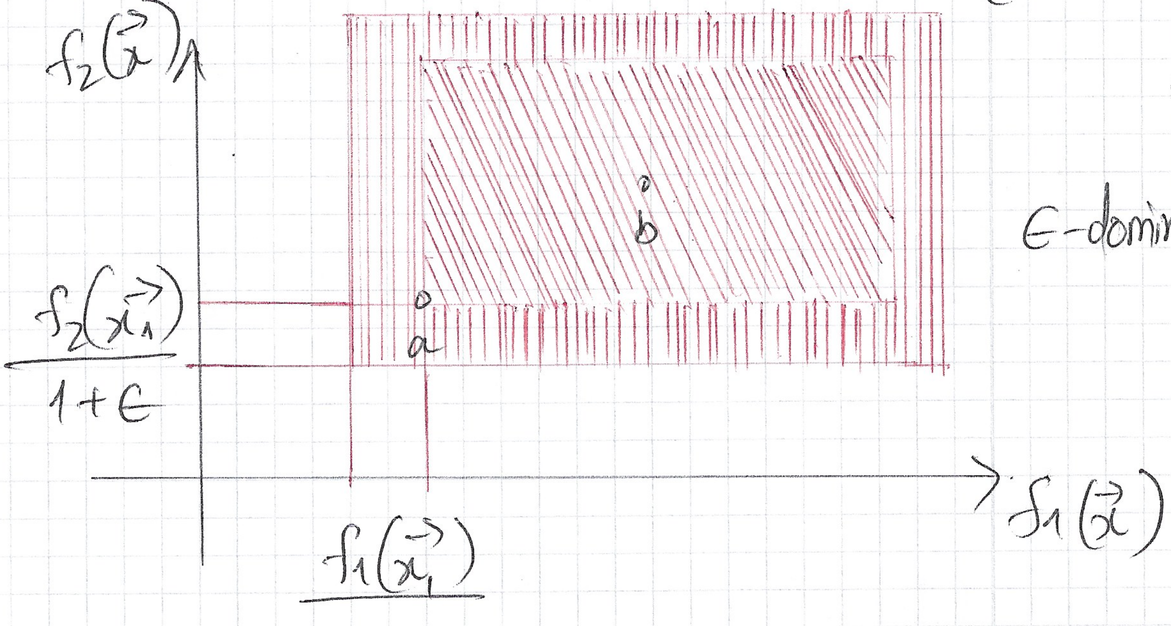
Then $\vec{x}_1$ ϵ-DOMINATES $\vec{x}_2$ under fulfilment of two conditions:

① $\frac{1}{1+\epsilon} f_i(\vec{x}_1) \leq f_i(\vec{x}_2) \quad \forall i = 1, \dots, m$

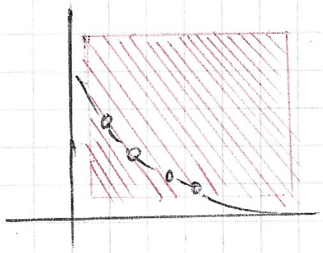
② $\frac{1}{1+\epsilon} f_j(\vec{x}_1) < f_j(\vec{x}_2)$ for at least one j



Normal Domination



ϵ-domination



8.2 $\epsilon = 0.5$

- From solution space we have,

Solution pt.	f_1	f_2
a	2	6
b	2	4
c	3.5	3.5
d	4	2
e	8	1

converting to ϵ - value,

$$\frac{1}{1+0.5} f_1(a) = \frac{2}{3}(2) = \underline{\underline{1.33}}.$$

$$\frac{1}{1+0.5} f_2(a) = \frac{2}{3}(6) = 4$$

Similarly,

Solution pt.	$\epsilon(f_1)$	$\epsilon(f_2)$
a	1.33	4
b	1.33	2.667
c	2.33	2.33
d	2.667	1.33
e	5.33	0.667

a vs. b

$$\frac{1}{1+\epsilon} f_1(a) \leq f_1(b) \quad \& \quad \frac{1}{1+\epsilon} f_2(a) \nleq f_2(b)$$

$\Rightarrow$ a does not ϵ-dominates b.

a vs. c

$$\frac{1}{1+\epsilon} f_1(a) \leq f_1(c) \quad \& \quad \frac{1}{1+\epsilon} f_2(a) \nless f_2(c)$$

$\Rightarrow$ a does not ϵ-dominates c.

a vs. d

$$\frac{1}{1+\epsilon} f_1(a) \leq f_1(d) \quad \& \quad \frac{1}{1+\epsilon} f_2(a) \nless f_2(d)$$

$\Rightarrow$ a does not ϵ-dominates d.

a vs. e

$$\frac{1}{1+\epsilon} f_1(a) \leq f_1(e) \quad \& \quad \frac{1}{1+\epsilon} f_2(a) < f_2(e)$$

$\Rightarrow$ a does not ϵ-dominates e.

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b vs. a

$$\frac{1}{1+\epsilon} f_1(b) \leq f_1(a) \quad \& \quad \frac{1}{1+\epsilon} f_2(b) < f_2(a)$$

$\Rightarrow$ b ϵ-dominates a $\boxed{\text{DOMINATES}}$

b vs. c

$$\frac{1}{1+\epsilon} f_1(b) \leq f_1(c) \quad \& \quad \frac{1}{1+\epsilon} f_2(b) < f_2(c)$$

$\Rightarrow$ b ϵ-dominates c $\boxed{\text{DOMINATES}}$

b vs. d

$$\frac{1}{1+\epsilon} f_1(b) \leq f_1(d) \quad \& \quad \frac{1}{1+\epsilon} f_2(b) < f_2(d).$$

$\Rightarrow$ b ϵ-dominates d. $\boxed{\text{DOMINATES}}$

b vs. e

$$\frac{1}{1+\epsilon} f_1(b) \leq f_1(e) \quad \& \quad \frac{1}{1+\epsilon} f_2(b) \nless f_2(e).$$

$\Rightarrow$ b does not ϵ-dominates e

c vs. a

$$\frac{1}{1+\epsilon} f_1(c) \nleq f_1(a) \quad \& \quad \frac{1}{1+\epsilon} f_2(c) < f_2(a).$$

$\Rightarrow$ c does not ϵ-dominates a.

c vs. b

$$\frac{1}{1+\epsilon} f_1(c) \nleq f_1(b) \quad \& \quad \frac{1}{1+\epsilon} f_2(c) < f_2(b).$$

$\Rightarrow$ c does not ϵ-dominates b.

c vs. d

$\frac{1}{1+\epsilon} f_1(c) \leq f_1(d)$ & $\frac{1}{1+\epsilon} f_2(c) < f_2(d)$

$\Rightarrow$ c ϵ-dominates d. DOMINATES

c vs. e

$\frac{1}{1+\epsilon} f_1(c) \leq f_1(e)$ & $\frac{1}{1+\epsilon} f_2(c) \nless f_2(e)$

$\Rightarrow$ c does not ϵ-dominates e.

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d vs. a

$\frac{1}{1+\epsilon} f_1(d) \nleq f_1(a)$ & $\frac{1}{1+\epsilon} f_2(d) < f_2(a)$

$\Rightarrow$ d does not ϵ-dominates a.

d vs. b

$\frac{1}{1+\epsilon} f_1(d) \nleq f_1(b)$ & $\frac{1}{1+\epsilon} f_2(d) < f_2(b)$.

$\Rightarrow$ d does not ϵ-dominates b.

d vs. c

$\frac{1}{1+\epsilon} f_1(d) \leq f_1(c)$ & $\frac{1}{1+\epsilon} f_2(d) < f_2(c)$.

$\Rightarrow$ d ϵ-dominates c. DOMINATES d vs. e

- $\frac{1}{1+\epsilon} f_1(d) \leq f_1(e)$ & $\frac{1}{1+\epsilon} f_2(d) \nleq f_2(e)$.

$\Rightarrow$ d does not ϵ-dominates e.

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e vs. a

$\frac{1}{1+\epsilon} f_1(e) \nleq f_1(a)$ & $\frac{1}{1+\epsilon} f_2(e) < f_2(a)$

$\Rightarrow$ e does not ϵ-dominates a.

e vs. b.

$\frac{1}{1+\epsilon} f_1(e) \nleq f_1(b)$ & $\frac{1}{1+\epsilon} f_2(e) < f_2(b)$.

$\Rightarrow$ e does not ϵ-dominates b.

e vs. c.

$\frac{1}{1+\epsilon} f_1(e) \nleq f_1(c)$ & $\frac{1}{1+\epsilon} f_2(e) < f_2(c)$.

$\Rightarrow$ e does not ϵ-dominates c.

e vs. d.

$\frac{1}{1+\epsilon} f_1(e) \nleq f_1(d)$ & $\frac{1}{1+\epsilon} f_2(e) < f_2(d)$.

$\Rightarrow$ e does not ϵ-dominates d.

$\boxed{\text{NON} - \epsilon - \text{DOMINATED POINTS: } b, e}$

8-3. $\epsilon = 1$

ϵ-value table,

Solution pt.	$\epsilon(f_1)$	ϵf_2
a	1	3
b	1	2
~~d~~c	1.75	1.75
d	2	1
e	4	0.5

a vs. b.

$\frac{1}{1+\epsilon} f_1(a) \leq f_1(b)$ & $\frac{1}{1+\epsilon} f_2(a) < f_2(b)$

$\Rightarrow$ a ϵdominates b. $\boxed{\text{DOMINATES}}$

a vs. c.

$\frac{1}{1+\epsilon} f_1(a) \leq f_1(c)$ & $\frac{1}{1+\epsilon} f_2(a) < f_2(c)$.

$\Rightarrow$ a ϵ-dominates c $\boxed{\text{DOMINATES}}$

a vs. d

$\frac{1}{1+\epsilon} f_1(a) \leq f_1(d)$ & $\frac{1}{1+\epsilon} f_2(a) \nless f_2(d)$

$\Rightarrow$ a does not ϵ-dominates d.

a vs. e

$\frac{1}{1+\epsilon} f_1(a) \leq f_1(e)$ & $\frac{1}{1+\epsilon} f_2(a) \nless f_2(e)$

$\Rightarrow$ a does not ϵ-dominates e

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b vs. a

$\frac{1}{1+\epsilon} f_1(b) \leq f_1(a)$ & $\frac{1}{1+\epsilon} f_2(b) < f_2(a)$

$\Rightarrow$ b ϵ-dominates a $\boxed{\text{DOMINATES}}$

b vs. c

$\frac{1}{1+\epsilon} f_1(b) \leq f_1(c)$ & $\frac{1}{1+\epsilon} f_2(b) < f_2(c)$

$\Rightarrow$ b ϵ-dominates c. $\boxed{\text{DOMINATES}}$

b vs. d

$\frac{1}{1+\epsilon} f_1(b) \leq f_1(d)$ & $\frac{1}{1+\epsilon} f_2(b) \nless f_2(d)$

$\Rightarrow$ b does not ϵ-dominates d.

b vs. e

$\frac{1}{1+\epsilon} f_1(b) \leq f_1(e)$ & $\frac{1}{1+\epsilon} f_2(b) \nless f_2(\cancel{d}e)$

$\Rightarrow$ b does not ϵ-dominates e c vs. a

$$\frac{1}{1+\epsilon} f_1(c) \leq f_1(a) \;\&\; \frac{1}{1+\epsilon} f_2(c) < f_2(a)$$

$\Rightarrow$ c ϵ-dominates a. $\boxed{\text{DOMINATES}}$

c vs. b

$$\frac{1}{1+\epsilon} f_1(c) \leq f_1(b) \;\&\; \frac{1}{1+\epsilon} f_2(c) < f_2(b)$$

$\Rightarrow$ c ϵ-dominates b. $\boxed{\text{DOMINATES}}$

c vs. d

$$\frac{1}{1+\epsilon} f_1(c) \leq f_1(d) \;\&\; \frac{1}{1+\epsilon} f_2(c) < f_2(d)$$

$\Rightarrow$ c ϵ-dominates d $\boxed{\text{DOMINATES}}$

c vs. e

$$\frac{1}{1+\epsilon} f_1(c) \leq f_1(e) \;\&\; \frac{1}{1+\epsilon} f_2(c) \nless f_2(e)$$

$\Rightarrow$ c does not ϵ-dominates e

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e vs. a

$$\frac{1}{1+\epsilon} f_1(e) \nleq f_1(a) \;\&\; \frac{1}{1+\epsilon} f_2(e) < f_2(a)$$

$\Rightarrow$ e does not ϵ-dominates a.

e vs. b

- $\frac{1}{1+\epsilon} f_1(e) \nleq f_1(b)$ & $\frac{1}{1+\epsilon} f_2(e) < f_2(b)$

$\Rightarrow$ e does not ϵ-dominates b.

e vs. c

$\frac{1}{1+\epsilon} f_1(e) \nleq f_1(c)$ & $\frac{1}{1+\epsilon} f_2(e) < f_2(c)$

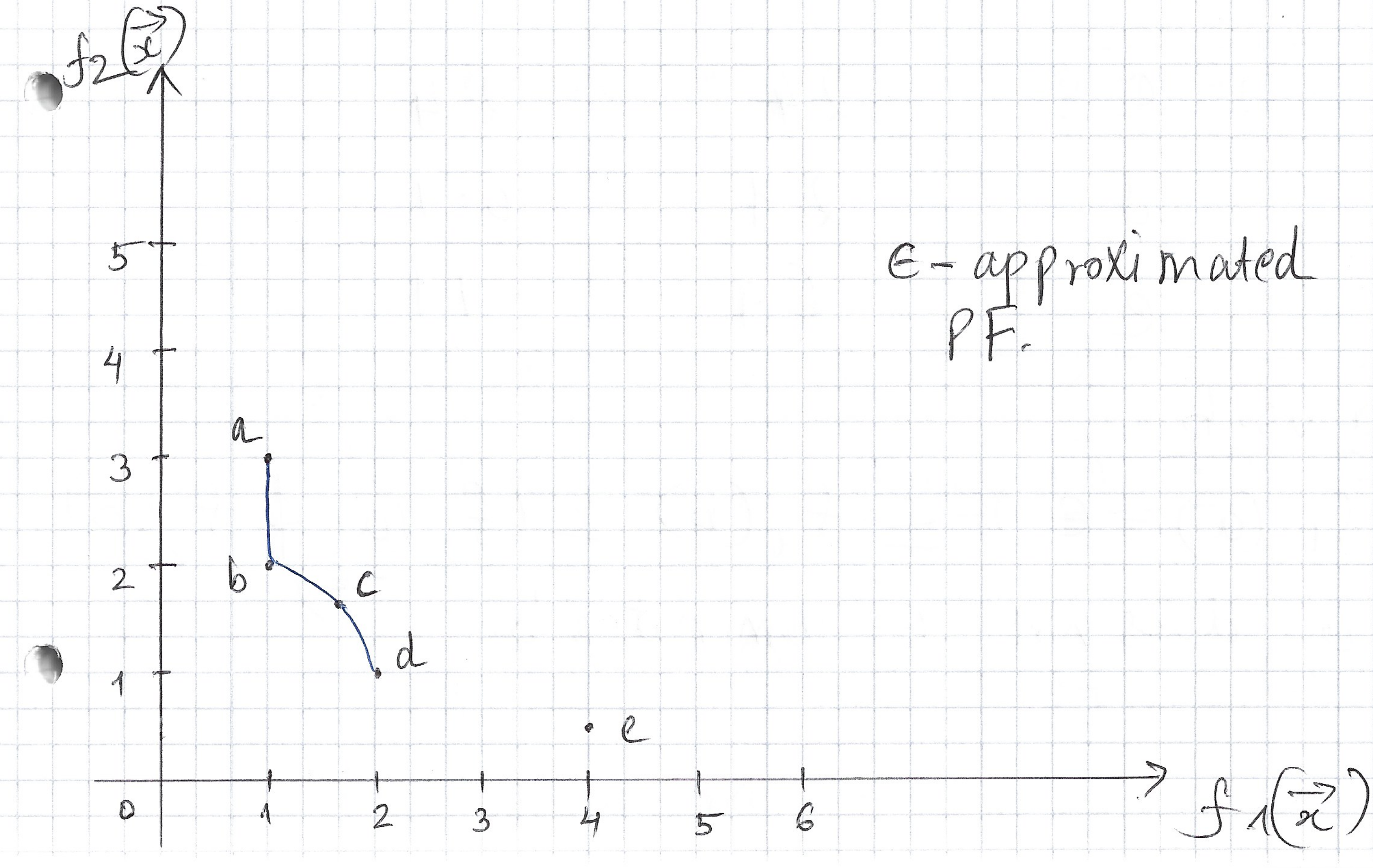
$\Rightarrow$ e does not ϵ-dominates. c.

e vs. d

$\frac{1}{1+\epsilon} f_1(e) \leq f_1(d)$ & $\frac{1}{1+\epsilon} f_2(e) < f_2(d)$.

$\Rightarrow$ e does not ϵ-dominates d.

$\boxed{\text{NON-}\epsilon\text{-DOMINATED POINT(S): } e}$



8-4. using Simplified definition;

Let $\vec{x}_1, \vec{x}_2 \in S$, then $\vec{x}_1$ ϵ-dominates $\vec{x}_2$ only under two conditions:

① $(f_i(\vec{x}_1) - \epsilon) \leq f_i(\vec{x}_2) \quad \forall i = 1, \dots m$

② $(f_j(\vec{x}_1) - \epsilon) < f_j(\vec{x}_2)$ atleast one j.

using $\epsilon = 1.1$, we calculate ϵ-value table as,

pt. solution	$\epsilon(f1)$	$\epsilon(f2)$
a	0.9	4.9
b	0.9	2.9
c	2.4	2.4
d	2.9	0.9
e	6.9	-1.1

a vs. b

$(f_1(a) - \epsilon) \leq f_1(b)$ & $(f_2(a) - \epsilon) \nless f_2(b)$

$\Rightarrow$ a does not ϵ- dominates b.

$\underline{a \text{ vs. } c}$

- $(f_1(a) - \epsilon) \leq f_1(c)$ & $(f_2(a) - \epsilon) \nless f_2(c)$.

$\Rightarrow$ a does not ϵ- dominates c

$\underline{a \text{ vs. } d}$

$(f_1(a) - \epsilon) \leq f_1(d)$ & $(f_2(a) - \epsilon) \nless f_2(d)$

$\Rightarrow$ a does not ϵ- dominates d

- $\underline{a \text{ vs. } e}$

$(f_1(a) - \epsilon) \leq f_1(e)$ & $(f_2(a) - \epsilon) \nless f_2(e)$.

$\Rightarrow$ a does not ϵ- dominates e.

$\underline{b \text{ vs. } a}$

$(f_1(b) - \epsilon) \leq f_1(a)$ & $(f_2(b) - \epsilon) < f_2(a)$

- $\Rightarrow$ b ϵ -dominates a $\boxed{\text{DOMINATES}}$

$\underline{b \text{ vs. } c}$

$(f_1(b) - \epsilon) \leq f_1(c)$ & $(f_2(b) - \epsilon) < f_2(c)$.

$\Rightarrow$ b ϵ- dominates c $\boxed{\text{DOMINATES}}$

$\underline{b \text{ vs. } d}$

- $(f_1(b) - \epsilon) \leq f_1(d)$ & $(f_2(b) - \epsilon) \nless f_2(d)$

$\Rightarrow$ b does not ϵ-dominates d b vs. e

$(f_1(b) - \epsilon) \leq f_1(e)$ & $(f_2(b) - \epsilon) \nleq f_2(e)$.

$\Rightarrow$ b does not ϵ-dominates e.

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c vs. a

$(f_1(c) - \epsilon) \nleq f_1(a)$ & $(f_2(c) - \epsilon) < f_2(a)$.

$\Rightarrow$ c does not ϵ-dominates a

c vs. b

$(f_1(c) - \epsilon) \nleq f_1(b)$ & $(f_2(c) - \epsilon) < f_2(b)$.

$\Rightarrow$ c does not ϵ-dominates b.

c vs. d

$(f_1(c) - \epsilon) \leq f_1(d)$ & $(f_2(c) - \epsilon) \nleq f_2(d)$.

$\Rightarrow$ c does not ϵ-dominates d.

c vs. e

$(f_1(c) - \epsilon) \leq f_1(e)$ & $(f_2(c) - \epsilon) \nleq f_2(e)$.

$\Rightarrow$ c does not ϵ-dominates e.

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d vs. a

$(f_1(d) - \epsilon) \nleq f_1(a)$ & $(f_2(d) - \epsilon) < f_2(a)$

$\Rightarrow$ d does not ϵ-dominates a.

d vs. b

- $(f_1(d)-\epsilon) \nleq f_1(b)$ & $(f_2(d)-\epsilon) < f_2(b)$.

$\Rightarrow$ d does not ϵ-dominates b.

d vs. c

$(f_1(d)-\epsilon) \leq f_1(c)$ & $(f_2(d)-\epsilon) < f_2(c)$.

$\Rightarrow$ d ϵ-dominates c. DOMINATES

- d vs. e

$(f_1(d)-\epsilon) \leq f_1(e)$ & $(f_2(d)-\epsilon) < f_2(e)$.

$\Rightarrow$ d ϵ-dominates e DOMINATES

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e vs. a

$(f_1(e)-\epsilon) \nleq f_1(a)$ & $(f_2(e)-\epsilon) < f_2(a)$.

$\Rightarrow$ e does not ϵ-dominates a

e vs. b

$(f_1(e)-\epsilon) \nleq f_1(b)$ & $(f_2(e)-\epsilon) < f_2(b)$.

$\Rightarrow$ e does not ϵ-dominates b

e vs. c

$(f_1(e)-\epsilon) \nleq f_1(c)$ & $(f_2(e)-\epsilon) < f_2(c)$.

$\Rightarrow$ e does not ϵ-dominates c.

e vs. d

$$(f_1(e) - \epsilon) \nleq f_1(d). \quad \& \quad (f_2(e) - \epsilon) < f_2(d)$$

$\Rightarrow$ e does not ϵ-dominates d.

$$\boxed{\begin{array}{l}\text{SO, IN ORDER TO ASSURE ONLY POINTS} \\ \text{b \& d are NON - } \epsilon\text{-DOMINATED} \\ \epsilon \text{ value of } 1.1 \text{ must be USED.}\end{array}}$$