Assignment #7 (Cone Domination)

7.1. Let $\vec{x}_1, \vec{x}_2 \in S$

Then $\vec{x}_1$ is said to Cone-dominate $\vec{x}_2$ under fulfillment of two conditions:

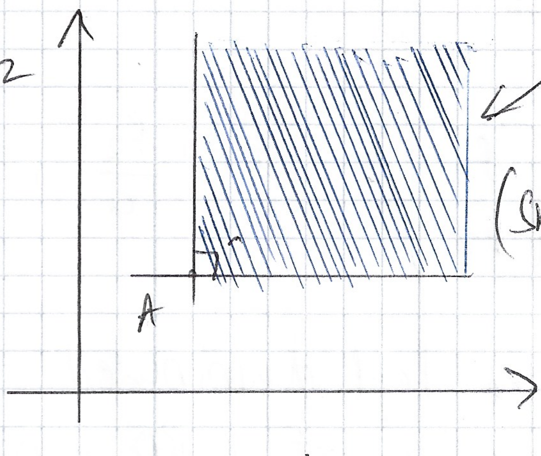
(1). $\Omega_i(\vec{x}_1) \leq \Omega_i(\vec{x}_2) \quad \forall\ i = 1, \dots, m$

(2) $\Omega_j(\vec{x}_1) < \Omega_j(\vec{x}_2)$ atleast one j.

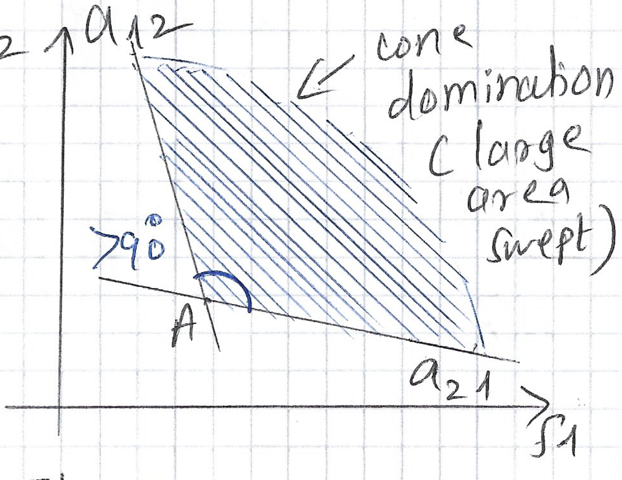
where $\Omega_i(\vec{x}) = f_1(\vec{x}) + \sum_{j=1, j \neq i}^{m} a_{ij} f_j(\vec{x});\ \forall i, \dots, m$

for a 2-objective problem with f_1 & f_2:

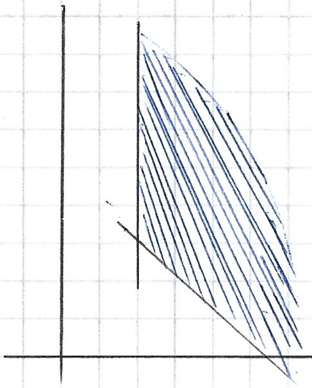
$$\begin{bmatrix} \Omega_1 & \Omega_2 \end{bmatrix}_{1\times2} = \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}_{2\times2} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}_{2\times1}$$



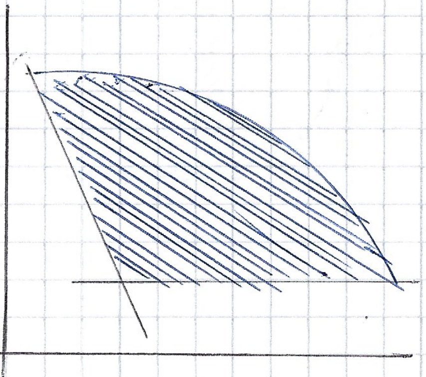
normal domination (Small area swept)



$$\begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 1/a \\ 0 & 1 \end{bmatrix}$$



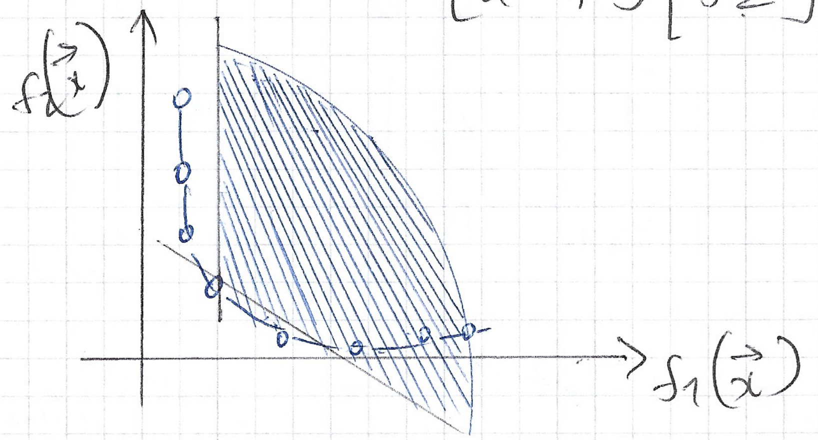
Cone angle: Symmetry transformation.

$$\vec{\Omega} = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1m} \\ a_{21} & 1 & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & 1 \end{bmatrix}$$

$$a_{ij} = \tan\left(\frac{\phi - 90}{2}\right), \forall i, j, i \neq j$$

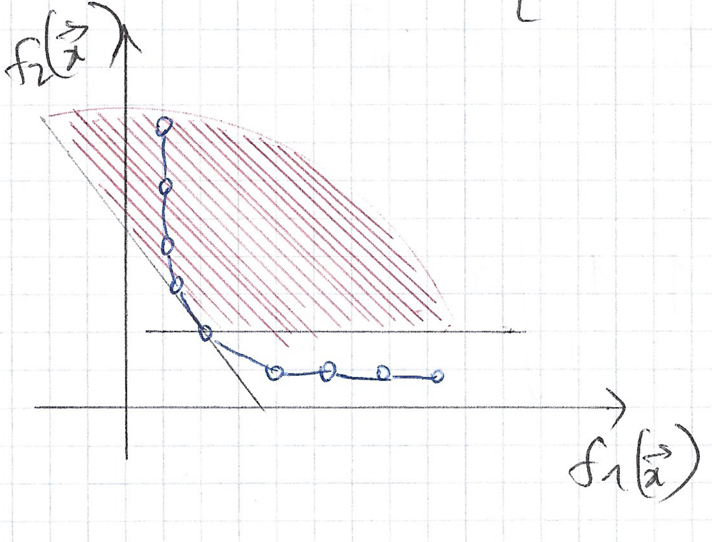
Results:

C1: $[\Omega_1, \Omega_2] = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = [f_1, af_1 + f_2]$



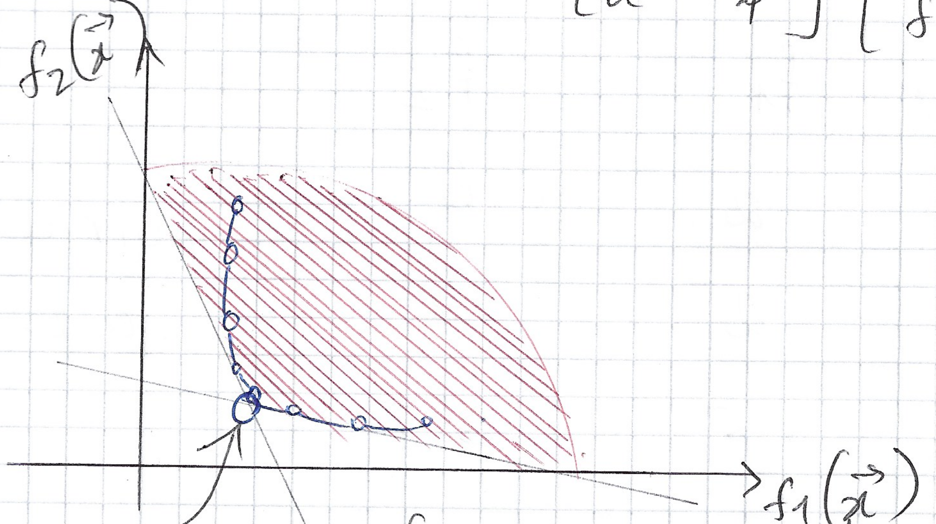
upper left of PO found
Lower right dominated

C2: $[\Omega_1, \Omega_2] = \begin{bmatrix} 1 & 1/a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = [f_1 + \frac{f_2}{a}, f_2]$



upper left dominated
lower right of PO found

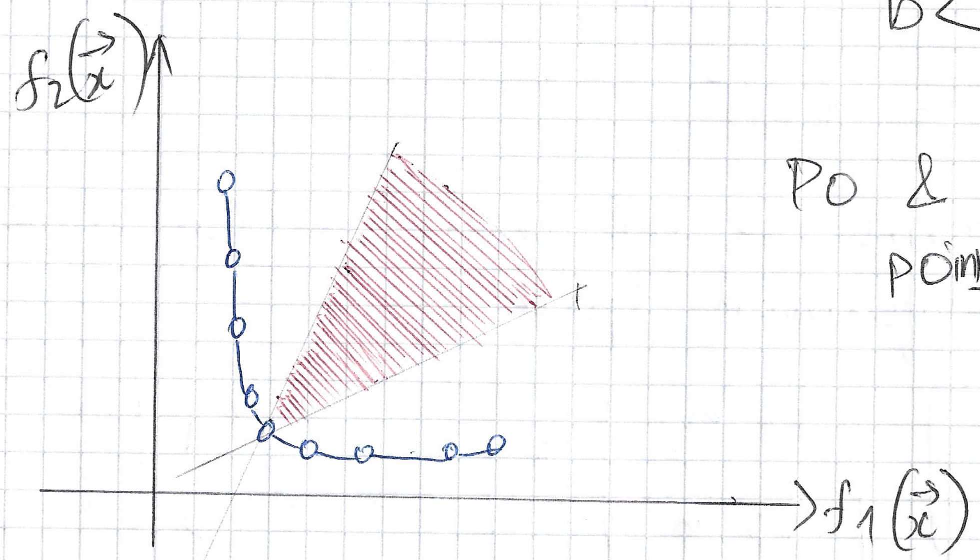
C3: $[\Omega_1, \Omega_2] = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = [f_1 + a f_2, \; a f_1 + f_2]$



Knee point found, rest dominated.

C4: $[\Omega_1, \Omega_2] = \begin{bmatrix} 1 & b \\ b & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = [f_1 + b f_2, \; b f_1 + f_2]$

$b < a$.



PO & other dominated points is part of PO.

7.2 From solution space we have,

Solution pt.	f_1	f_2
a	2	6
b	2	4
c	3.5	3.5
d	4	2
e	8	1

original space.

Matrix, $\begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix}$; $0.3 = \tan\left(\frac{\phi - 90}{2}\right)$.

Cone angle $= 123^\circ$

we transform the original solution space to new transformed space as.

$$\begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix} \begin{bmatrix} f_1(a) \\ f_2(a) \end{bmatrix} = \begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$\Omega_1 = 2 + 1.8 = 3.8$; $\Omega_2 = 0.6 + 6 = 6.6$

Similarly we find the transformations of other points,

solution pt.	Ω_1	Ω_2
a	3.8.	6.6
b	3.2	4.6
c	4.55.	4.55
d	4.6	3.2
e	8.3	3.4

Transformed space.

a vs. b

$\Omega_1(a) \not\leq \Omega_1(b)$ & $\Omega_2(a) \not\leq \Omega_2(b)$

$\Rightarrow$ a does not cone-dominates b.

a vs. c.

$\Omega_1(a) \leq \Omega_1(c)$ & $\Omega_2(a) \not\leq \Omega_2(c)$.

$\Rightarrow$ a does not cone-dominates ~~b~~ c

a vs. d.

$\Omega_1(a) \leq \Omega_1(d)$ & $\Omega_2(a) \not\leq \Omega_2(d)$.

$\Rightarrow$ a does not cone-dominates d.

a vs. e

$\Omega_1(a) \leq \Omega_1(e)$ & $\Omega_2(a) \not\leq \Omega_2(e)$.

$\Rightarrow$ a does not cone-dominates e.

b vs. a

$\Omega_1(b) \leq \Omega_1(a)$ & $\Omega_2(b) < \Omega_2(a)$

$\Rightarrow$ b CONE-DOMINATES a. $\boxed{\text{DOMINATES}}$

b vs. c

$\Omega_1(b) \leq \Omega_1(c)$ & $\Omega_2(b) \nless \Omega_2(c)$

$\Rightarrow$ b does not cone-dominates c.

b vs. d

$\Omega_1(b) \leq \Omega_1(d)$ & $\Omega_2(b) \nless \Omega_2(d)$

$\Rightarrow$ b does not cone-dominates d.

b vs. e

$\Omega_1(b) \leq \Omega_1(e)$ & $\Omega_2(b) \nless \Omega_2(e)$

$\Rightarrow$ b does not cone-dominates e.

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c vs. a

$\Omega_1(c) \nleq \Omega_1(a)$ & $\Omega_2(c) < \Omega_2(a)$

$\Rightarrow$ c does not cone-dominates a.

__c vs. b__

- $\Omega_1(c) \nleq \Omega_1(b)$ & $\Omega_2(c) < \Omega_2(b)$.

$\Rightarrow$ c does not cone-dominates b.

__c vs. d__

$\Omega_1(c) \leq \Omega_1(d)$ & $\Omega_2(c) \nless \Omega_2(d)$

$\Rightarrow$ c does not cone-dominates d.

- __c vs. e__

$\Omega_1(c) \leq \Omega_1(e)$ & $\Omega_2(c) \nless \Omega_2(e)$

$\Rightarrow$ c does not cone-dominates e.

__d vs. a__

$\Omega_1(d) \nleq \Omega_1(a)$ & $\Omega_2(d) < \Omega_2(a)$.

$\Rightarrow$ d does not cone-dominates a.

__d vs. b__

$\Omega_1(d) \nleq \Omega_1(b)$ & $\Omega_2(d) < \Omega_2(b)$.

$\Rightarrow$ d does not cone-dominates b.

__d vs. c__

$\Omega_1(d) \nleq \Omega_1(c)$ & $\Omega_2(d) < \Omega_2(c)$.

$\Rightarrow$ d does not cone-dominates c.

__d vs. e__

$\Omega_1(d) \leq \Omega_1(e)$ & $\Omega_2(d) < \Omega_2(e)$ $\boxed{\text{DOMINATES}}$

e vs. a

$\Omega_1(e) \nleq \Omega_1(a)$ & $\Omega_2(e) < \Omega_2(a)$.

$\Rightarrow$ e does not cone-dominates a.

e vs. b

$\Omega_1(e) \nleq \Omega_1(b)$ & $\Omega_2(e) < \Omega_2(b)$.

$\Rightarrow$ e does not cone-dominates b.

e vs. c

$\Omega_1(e) \nleq \Omega_1(c)$ & $\Omega_2(e) < \Omega_2(c)$.

$\Rightarrow$ e does not cone-dominates c

e vs. d

$\Omega_1(e) \nleq \Omega_1(d)$ & $\Omega_2(e) \nless \Omega_2(d)$.

$\Rightarrow$ e does not cone-dominates d.

NON-CONE DOMINATED POINTS: b, c, d

We obtain ~~same set of~~ points in case of PARETO-DOMINANCE. (b, c, d, e)

7.3. Matrix, $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$ cone angle $= 143^\circ$.

- Transformation from original solution space to transformed space as,

$$\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} f_1(a) \\ f_2(a) \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$\therefore \Omega_1 = 2 + 3 = 5 \quad \Omega_2 = 1 + 6 = 7.$

- Similarly for other points,

Solution. pt.	Ω_1	Ω_2.
a	5	7
b	4	5
c	5.25	5.25
d	5	4
e	8.5	5

a vs. b

$\Omega_1(a) \nleq \Omega_1(b)$ & $\Omega_2(a) \nleq \Omega_2(b)$

$\Rightarrow$ a does not cone-dominates b.

- a vs. c

$\Omega_1(a) \leq \Omega_1(c)$ & $\Omega_2(a) \nleq \Omega_2(c)$

$\Rightarrow$ a does not CD. c.

a vs. d

$\Omega_1(a) \leq \Omega_1(d)$ & $\Omega_2(a) \nless \Omega_2(d)$.

$\Rightarrow$ a does not CD d.

a vs. e

$\Omega_1(a) \leq \Omega_1(e)$ & $\Omega_2(a) \nless \Omega_2(e)$.

$\Rightarrow$ a does not CD e.

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b vs. a

$\Omega_1(b) \leq \Omega_1(a)$ & $\Omega_2(b) < \Omega_2(a)$.

$\Rightarrow$ b cone-dominates a. DOMINATES

b vs. c

$\Omega_1(b) \leq \Omega_1(c)$ & $\Omega_2(b) < \Omega_2(c)$.

$\Rightarrow$ b cone-dominates c DOMINATES

b vs. d

$\Omega_1(b) \leq \Omega_1(d)$ & $\Omega_2(b) \nless \Omega_2(d)$.

$\Rightarrow$ b does not CD. d.

b vs. e

$\Omega_1(b) \leq \Omega_1(e)$ & $\Omega_2(b) \nless \Omega_2(e)$.

$\Rightarrow$ b does not CD e.

c vs. a

- $\Omega_1(c) \nleq \Omega_1(a)$ & $\Omega_2(c) < \Omega_2(a)$.

$\Rightarrow$. c does not CD a.

c vs. b

$\Omega_1(c) \nleq \Omega_1(b)$ & $\Omega_2(c) \nless \Omega_2(b)$.

$\Rightarrow$. c does not CD b.

- c vs. d

$\Omega_1(c) \nleq \Omega_1(d)$ & $\Omega_2(c) \nless \Omega_2(d)$.

$\Rightarrow$. c does not CD d.

c vs. e

$\Omega_1(c) \leq \Omega_1(e)$ & $\Omega_2(c) \nless \Omega_2(e)$.

$\Rightarrow$. c does not CD. e.

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d vs. a

$\Omega_1(d) \leq \Omega_1(a)$ & $\Omega_2(d) < \Omega_2(a)$.

$\Rightarrow$. d CD a. $\boxed{\text{DOMINATES}}$

d vs. b

$\Omega_1(d) \nleq \Omega_1(b)$ & $\Omega_2(d) < \Omega_2(b)$.

$\Rightarrow$. d does not CD b.

$\underline{\underline{d \text{ vs. } c}}$

$\Omega_1(d) \leq \Omega_1(c)$ & $\Omega_2(d) < \Omega_2(c)$.

$\Rightarrow d$ CD c $\boxed{\text{DOMINATES}}$

$\underline{\underline{d \text{ vs. } e}}$

$\Omega_1(d) \leq \Omega_1(e)$ & $\Omega_2(d) < \Omega_2(e)$

$\Rightarrow d$ CD e $\boxed{\text{DOMINATES}}$

$\underline{\underline{e \text{ vs. } a}}$

$\Omega_1(e) \nleq \Omega_1(a)$ & $\Omega_2(e) < \Omega_2(a)$

$\Rightarrow$ e does not CD a.

$\underline{\underline{e \text{ vs. } b}}$

$\Omega_1(e) \nleq \Omega_1(b)$ & $\Omega_2(e) \nless \Omega_2(b)$.

$\Rightarrow$ e does not CD b.

$\underline{\underline{e \text{ vs. } c}}$

$\Omega_1(e) \nleq \Omega_1(c)$ & $\Omega_2(e) < \Omega_2(c)$

$\Rightarrow$ e does not CD c

$\underline{\underline{e \text{ vs. } d}}$

$\Omega_1(e) \nleq \Omega_1(d)$ & $\Omega_2(e) \nless \Omega_2(d)$

$\Rightarrow$ e does not CD d.

$\boxed{\text{NON-CONEDOMINATED POINTS: } b, d}$

7.4. Symmetric angle, $\varphi = 160°$

Cone angle defined as,

$$a_{ij} = \tan\left(\frac{\varphi - 90}{2}\right), \forall i, j, \; i \neq j$$

Put $\varphi = 160°$

$$a_{ij} = \tan\left(\frac{160 - 90}{2}\right) = \tan(35) = \underline{\underline{0.70}}$$

Required matrix, $\vec{\Omega}$ $\begin{bmatrix} 1 & 0.7 \\ 0.7 & 1 \end{bmatrix}$