Submitted by: Ranji Raj

Assignment 29 (Robustness & Reliability in MOP)

Consider the following bi-objective minimization problem with 2 decision variables and 1 inequality constraint:

$$\begin{aligned} min \quad & F(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x})) = (|\sin(\frac{\pi}{2} \cdot x_1)|, \sqrt{x_1} + \frac{e^{x_2 - 1}}{3}) \\ subject\ to \quad & g_1(\vec{x}) = 8 \cdot x_1 + 2 \cdot x_2 - {x_1}^2 \geq 18 \end{aligned}$$

And the solutions $x_A = (2,2)$ and $x_B = (5,1)$. A Latin hypercube strategy is used for sampling, but instead of randomly selecting the solution inside each box, the upper right solution of the box is selected (e.g. for a box whose corners are $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$, the solution $(1,1)$ is selected). The neighborhood has a size of $H = 4$ and $\delta_1 = \delta_2 = 1$. Please answer the following questions:

- What are the mean effective fitness values of x_A and x_B? What can you say about the robutsness of the solutions according to these mean effective fitness values?

For x_A, let the 4 neighbours be (2,1), (1, 2), (2, 3), (3, 2) &
For x_B, let the 4 neighbours be (6,1), (5, 2), (4,1), (5,0)

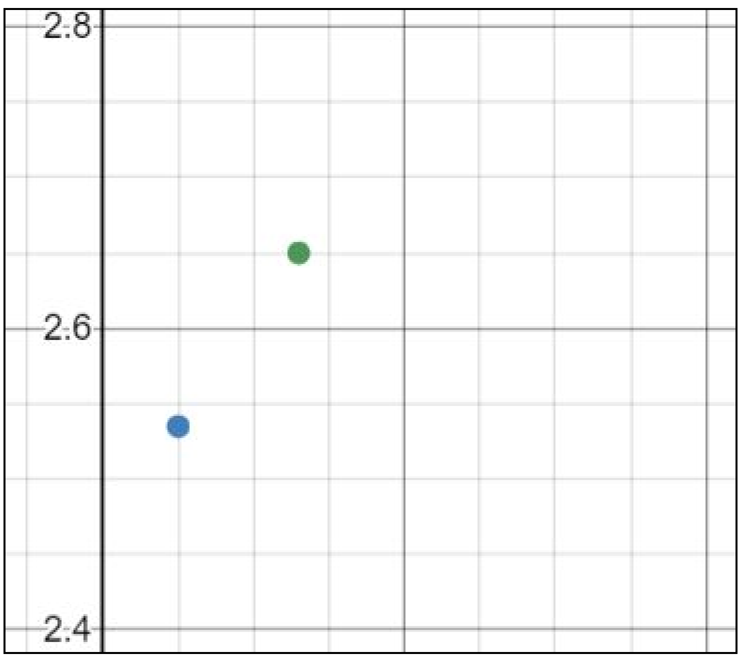
Computing the fitness values & constraints,

Neighbour, x_A,	Fitness, $F(\vec{x}_A) = (f_1(\vec{x}_A), f_2(\vec{x}_A))$	Constraint, $g_1(\vec{x}_A)$
(2,1)	(0.05,1.74)	$14 \ngeq 18$
(1,2)	(0.02,1.90)	$11 \ngeq 18$
(2,3)	(0.05,3.87)	$18 \geq 18$
(3,2)	(0.08, 2.63)	$19 \geq 18$

$f(x_A^{eff})$ = (0.05, 2.535)

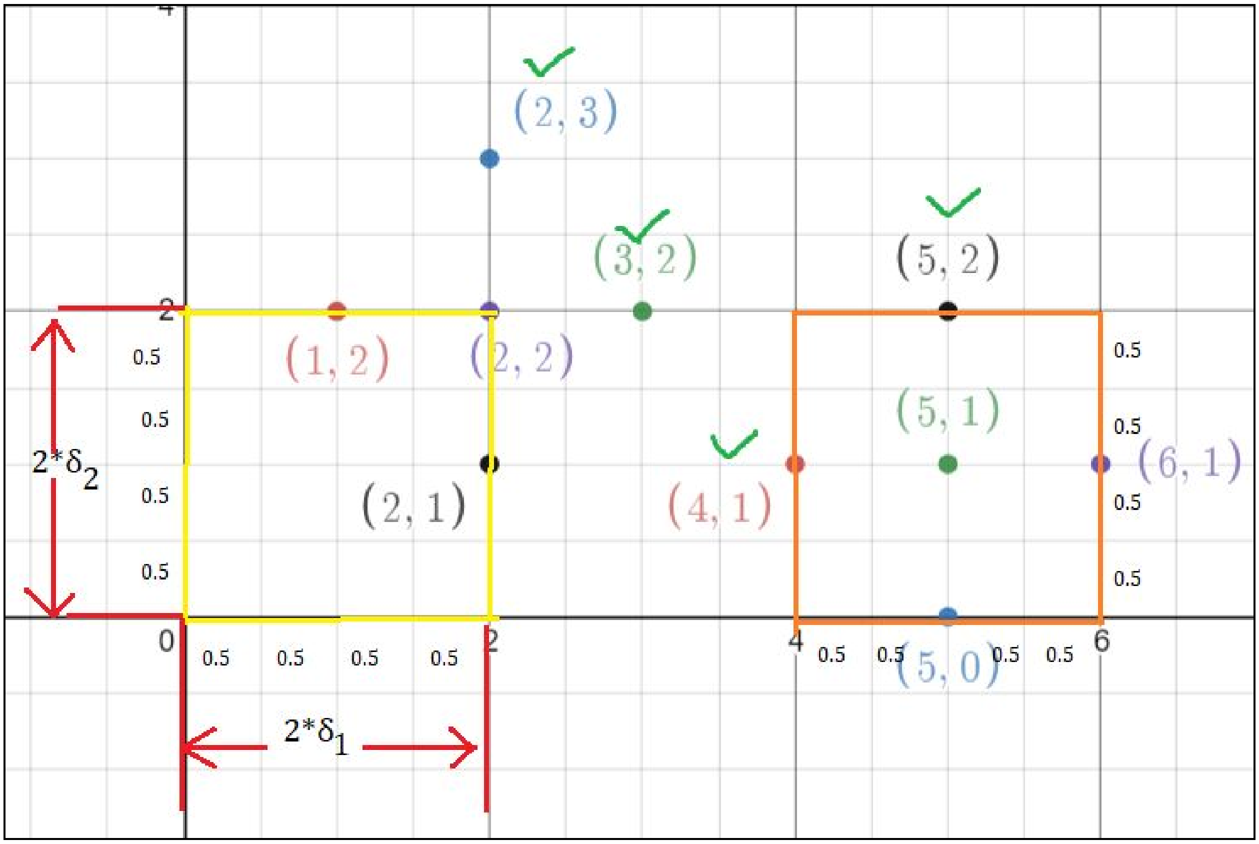
Neighbour, x_B,	Fitness, $F(\vec{x}_B) = \left(f_1(\vec{x}_B), f_2(\vec{x}_B)\right)$	Constraint, $g_1(\vec{x}_B)$
(6,1)	(0.16,2.78)	$14 \ngeq 18$
(5,2)	(0.13,3.14)	$19 \geq 18$
(4,1)	(0.10,2.33)	$18 \geq 18$
(5,0)	(0.13, 2.35)	$15 \ngeq 18$

$f\left(x_B^{eff}\right)$ = (0.13, 2.65)



x_A more robust than x_B

- Using the same Latin hypercube strategy, what is the reliability value of x_A and x_B?



For 2 (*r*) cases, out of 4 (N) in each of the solutions satisfies the constraints.

So the estimate of reliability by $\frac{r}{N}$ for x_A & x_B we get $\frac{2}{4}$ = 0.5 for both solutions.

- What should be the value of the required probability R so both solutions x_A and x_B are reliable? What value R would make both of them unreliable?

Since $\frac{r}{N} \geq R$ holds for probability a value of **R = 0.5** makes both solutions x_A & x_B as reliable.

Any value of **R >0.5** violates $\frac{r}{N} \geq R$, thus making x_A & x_B unreliable.