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Assignment 33 (GD & IGD)

Take a look at the Inverted Generational Distance (IGD) and Generational Distance (GD) performance indicators from the lecture and answer the following questions.

- Please explain how each of these metrics works, and what are the requirements for computing them.

❖ Generational Distance (GD)

- GD is a **convergence** metric.
- Given: PO solution set (*P*), set of solutions (*S*).
- Measures the average distances of solutions in **S** to **P.**
- MOEA which has **smaller** GD value has **good** convergence.

$$\boxed{GD(P,S) = \frac{\left(\sum_{i=1}^{|S|} d_i^{\,q}\right)^{\frac{1}{q}}}{|S|}}$$

Where d_i = euclidean distance in **objective space** =

$\min_{k\in|P|}\sqrt{\sum_{j=1}^{m}\left(f^{\,i}_{j}-f^{\,*}_{j}{}^{(i)}\right)^{2}}$ & i ∈ S.

❖ Inverted Generational Distance (IGD)

- IGD is a **diversity** metric.
- Given: PO solution set (*P*), set of solutions (*S*).
- Measures the average distances of solutions in **P** to **S.**
- MOEA which has **smaller** IGD value has **good** diversity.

$$\boxed{IGD(P,\ S) = \frac{\left(\sum_{i=1}^{|P|} d_i^{\,q}\right)^{\frac{1}{q}}}{|P|}}$$

Where d_i = euclidean distance in **objective space** =

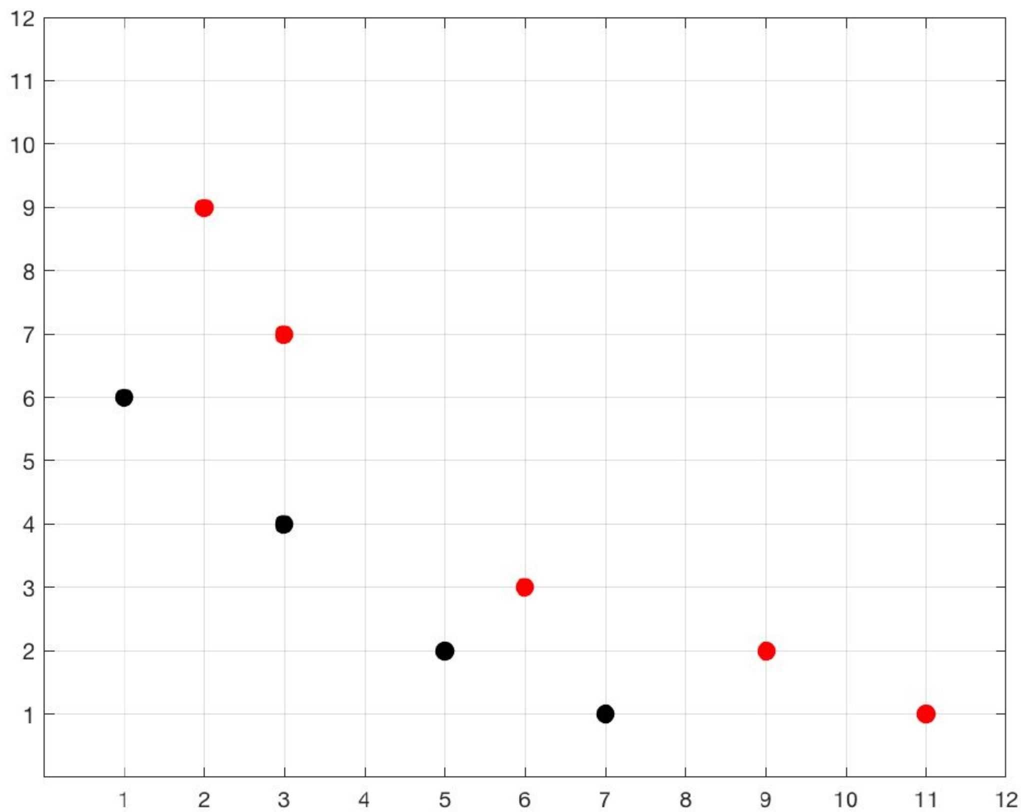
$\min_{k\in|S|}\sqrt{\sum_{j=1}^{m}\left(f_{j}^{\,i}-f_{j}^{\,*(i)}\right)^{2}}$ & i ∈ P.

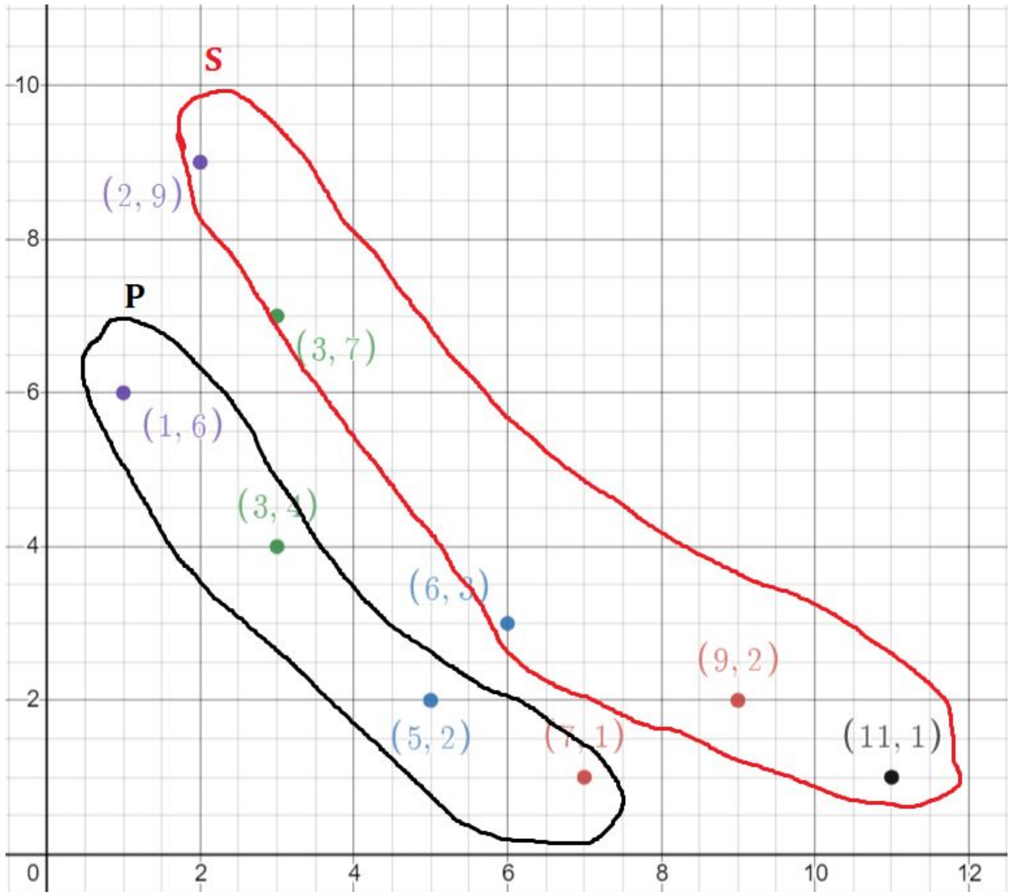
- Please describe the difference between the IGD and the IGDX performance indicators.

A variant of IGD when distances to the **search spaces** (decision variable space) are measured by means of Euclidean distances is called **IGDX**.

$$d_i = \min_{k \in |S|}\left(\sqrt{\sum_{j=1}^{n}\left(x^i{}_j - x_j{}^{*(i)}\right)^2}\right)$$

- In the following, you see a set of solutions produced by an algorithm (red points) and a sample of the Pareto-optimal solutions (black points) in a multi-objective optimization problem. Both objectives should be minimized. Compute the GD and IGD values of these solution sets, using $q = 1$.





Solutions	(2, 9) ↓	(3, 7) ↓	(6, 3) ↓	(9, 2) ↓	(11, 1) ↓
(1, 6) →	3.16	2.23	5.83	8.94	11.18
(3, 4) →	5.09	3.00	3.16	6.32	8.54
(5, 2) →	7.16	5.38	1.414	4.00	6.08
(7, 1) →	9.43	7.21	2.23	2.23	4.00

$$GD(P, S) = \frac{\sum_{i=1}^{5} d_i}{5} = \frac{3.16 + 2.23 + 1.414 + 2.23 + 4}{5} = \mathbf{2.608}$$

$$IGD(P, S) = \frac{\sum_{i=1}^{4} d_i}{4} = \frac{2.23 + 3 + 1.414 + 2.23}{4} = \mathbf{2.21}$$