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Assignment 26 (MOPSO)

We are using a Multi-Objective Particle Swarm Optimization (MOPSO) method to solve a 2-objective minimization problem. The algorithm is currently in timestep t_2. The population consists of four particles $\vec{x}_1$, $\vec{x}_2$, $\vec{x}_3$ and $\vec{x}_4$. In the following table you find the positions, velocities and fitness values of the particles at the timesteps t_0, t_1 and t_2.

Particle	$\vec{x}_1$	$\vec{x}_2$	$\vec{x}_3$	$\vec{x}_4$
$\vec{v}_i(t_0)$	$(0.0, 0.0)$	$(0.0, 0.0)$	$(0.0, 0.0)$	$(0.0, 0.0)$
$\vec{x}_i(t_0)$	$(2.0, 2.0)$	$(-3.0, 2.0)$	$(5.0, -1.0)$	$(-1.0, -2.0)$
$f(\vec{x}_i, t_0)$	$(5.0, 5.0)$	$(3.0, 5.0)$	$(7.0, 8.0)$	$(2.0, 7.0)$
$\vec{v}_i(t_1)$	$(-1.0, -1.0)$	$(1.0, -1.0)$	$(-2.0, 0.0)$	$(0.0, 1.0)$
$\vec{x}_i(t_1)$	$(1.0, 1.0)$	$(-2.0, 1.0)$	$(3.0, -1.0)$	$(-1.0, -1.0)$
$f(\vec{x}_i, t_1)$	$(6.0, 2.0)$	$(4.0, 5.0)$	$(5.0, 2.0)$	$(1.0, 6.0)$
$\vec{v}_i(t_2)$	$(1.0, -1.0)$	$(1.0, 2.0)$	$(-1.0, -2.0)$	$(-1.0, 0.0)$
$\vec{x}_i(t_2)$	$(2.0, 0.0)$	$(-1.0, 3.0)$	$(2.0, -3.0)$	$(-2.0, -1.0)$
$f(\vec{x}_i, t_2)$	$(8.0, 1.0)$	$(5.0, 4.0)$	$(6.0, 3.0)$	$(2.0, 4.0)$

The archive $A(t_1)$ at the end of timestamp t_1 consists of the following solutions:

$\vec{x}$	$\vec{f}(\vec{x})$
$(-1.0, -1.0)$	$(1.0, 6.0)$
$(-3.0, 2.0)$	$(3.0, 5.0)$
$(3.0, -1.0)$	$(5.0, 2.0)$

- Update the archive $A(t_2)$ with the new solutions of timestep t_2. The size of the archive is unlimited.

	$\vec{f}(\vec{x})$
(-1.0, -1.0)	(1.0, 6.0)
(-3.0, 2.0)	(3.0, 5.0)
(3.0, -1.0)	(5.0, 2.0)
(-1.0, 3.0)	(5.0, 4.0)
(-2.0, -1.0)	(2.0, 4.0)

- The update of the personal best (cognitive component) is done by finding the best solution through a lexicographical ordering in which the first objective is the most important one, i.e. $f_1 \gg f_2$, for odd solutions $(\vec{x}_1, \vec{x}_3)$; while even solutions $(\vec{x}_2, \vec{x}_4)$ give more importance to the second objective. Determine the personal best $P_b(\vec{x}_2, t_2)$ for particle $\vec{x}_2$.

	$\vec{f}(\vec{x})$
(-1.0, 3.0)	(5.0, 4.0)

- The leader selection (social component) is done using the sigma method. The swarm uses a fully connected neighborhood. Select the leader $P_g(\vec{x}_2, t_2)$ for particle $\vec{x}_2$.

σ_j for $\vec{x}_2$ @ t_0 = - 0.47 ; σ_j for $\vec{x}_2$ @ t_2 = 0.21

σ_i for $\vec{x}_2$ @ t_1 = - 0.21

	$\vec{f}(\vec{x})$
(-3.0, 2.0)	(3.0, 5.0)

- Calculate the updated velocity $(\vec{v}_2(t_3))$ and position $(\vec{x}_2(t_3))$ for the next iteration of the MOPSO for particle x_2. Use $w = 0.5$, $\phi_1 = (1, 0.5)$, $\phi_2 = (0.5, 1)$, $c_1 = 0.3$ and $c_2 = 0.2$.

For particle $\vec{x}_2$, the **first velocity** component,
0.5(1) + (0.3)(1)(3+1) + (0.2)(0.5)(5+1) = 2.3

Then the **first position** component,
2.3 - 1 = 1.3

For particle $\vec{x}_2$, the **second velocity** component,
0.5(2) + (0.3)(0.5)(5-3) + (0.2)(1)(4-3) = 1.5

Then the **second position** component,
1.5 + 3 = 4.5

This gives $\vec{v}_2(t_3)$ = **(2.3, 1.5)** and $\vec{x}_2(t_3)$ = **(1.3, 4.5)** for particle $\vec{x}_2$