Submitted by: Ranji Raj

Assignment 20 (SPEA - II)

Take a look at the concept and components of Strength Pareto Evolutionary Algortihm 2 (SPEA2) and answer the following questions.

- Describe how SPEA2 works. What additional components are necessary to convert the general EA into SPEA2?

Source: Lecture slides

- We find the non-dominated solutions from the initial population *P(t)* and stores in an externally second, continuously updated population called ***Archive (A)***.
- Compute the ***fitness using the ranking*** process taking into consideration how many of the solutions are dominated or dominates by an individual.
- When the solutions end up in a very crowded area in the front then a ***nearest neighbour density estimation (k-NN)*** procedure is used to guide the further search process.
- And finally, an ***archive truncation*** method is performed to preserve the boundary solutions. If the final solutions are smaller than the capacity then it is left empty else truncation is carried out.
- The archive can keep certain solutions ***forever*** which could even survive to the last archive.

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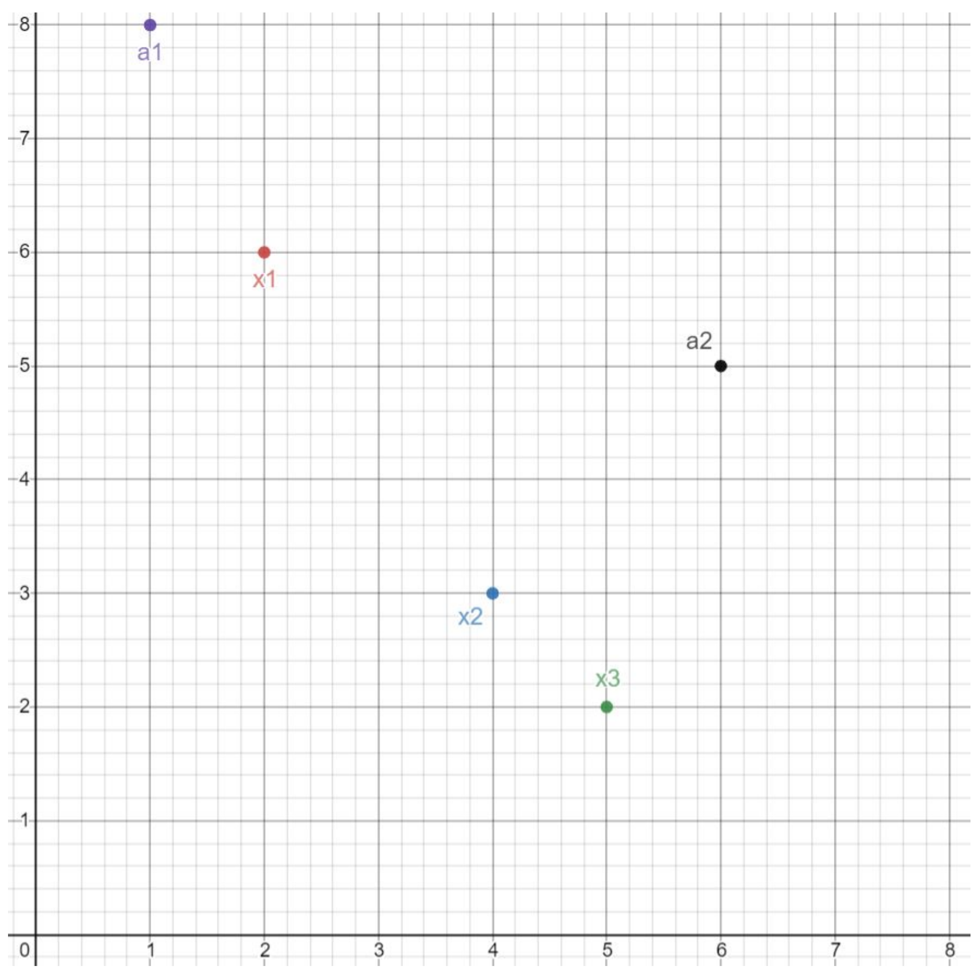
- For a 2-objective minimization problem, during the execution of SPEA2 and after the evaluation of solutions, the individuals in the population and archive are the following:

Solution	$f_1(x)$	$f_2(x)$
Population		
x_1	2	6
x_2	4	3
x_3	5	2
Archive		
a_1	1	8
a_2	6	5

Given that the archive size is $N_A = 3$, and using a $k = 2$ for the k-th nearest neighbour method, compute the fitness value for each indiviual and update the Archive.



Objective space with individuals in population and archive

We calculate the Strength and Raw values of each solution,

$S(x_1)$	0	$R(x_1)$	0
$S(x_2)$	1	$R(x_2)$	0
$S(x_3)$	1	$R(x_3)$	0
$S(a_1)$	0	$R(a_1)$	0
$S(a_2)$	0	$R(a_2)$	2

For, k = 2, we calculate the (euclidean) distances for each solution to other solutions in the objective space and apply 2-NN,

Sorted σ^2 - table for x_1

$\sigma^1_{x1\rightarrow a1}$	2.23
$\sigma^2_{x1\rightarrow x2}$	3.6
$\sigma_{x1\rightarrow a2}$	4.12
$\sigma_{x1\rightarrow x3}$	5

Sorted σ^2 - table for x_2

$\sigma^1_{x2\rightarrow x3}$	1.414
$\sigma^2_{x2\rightarrow a2}$	2.82
$\sigma_{x2\rightarrow x1}$	3.6
$\sigma_{x2\rightarrow a1}$	5.83

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Sorted σ^2 - table for x_3

$\sigma^1_{x3 \to x2}$	1.414
$\sigma^2_{x3 \to a2}$	3.16
$\sigma_{x3 \to x1}$	5
$\sigma_{x3 \to a1}$	7.21

Sorted σ^2 - table for a_1

$\sigma^1_{a1 \to x1}$	2.23
$\sigma^2_{a1 \to a2}$	5.83
$\sigma_{a1 \to x2}$	5.83
$\sigma_{a1 \to x3}$	7.21

Sorted σ^2 - table for a_2

$\sigma^1_{a2 \to x2}$	2.82
$\sigma^2_{a2 \to x3}$	3.16
$\sigma_{a2 \to x1}$	4.12
$\sigma_{a2 \to a1}$	5.83

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Calculate densities *D,* for each solution by D(i) = $\frac{1}{\sigma_i^2 + 2}$ where, 0 < D(i) < 1 and their fitness values,

$D(x_1)$	0.12	$F(x_1)$	0.12
$D(x_2)$	0.16	$F(x_2)$	0.16
$D(x_3)$	0.15	$F(x_3)$	0.15
$D(a_1)$	0.09	$F(a_1)$	0.09
$D(a_2)$	0.12	$F(a_2)$	2.12

We have 4 non-dominated (fitness < 1) solutions but, archive size N_A = 3 so we do truncation.

$\sigma_{x1\rightarrow a1 = 2.23} < \sigma_{x1\rightarrow x2 = 3.6} < \sigma_{x1\rightarrow x3 = 5}$ So we eliminate a_1 and update our archive with individuals x_1,x_2, x_3 .