Assignment #5 (A Prior Methods)

DM - Decision Maker PO - Pareto Optimal

Apriori	Interactive	Posteriori
Prerequisite Requires Well-Known knowledge about the value function prior to problem solution.	Does not require any prior knowledge about any value function.	Requires Mathematical programming methods & EA knowledge.
Role of DM: DM is actively involved & must indicate the assumption before the optimization process.	DM is involved in the search process & decides upon which parts of PO front are considered with the optimization Program.	DM will be provided a set of PO solutions & the most suitable one will be selected which provides good diversity & good convergence.
Methods - Weighting method - ∈ constraint method - Lexicographic Ordering	- Pairwise Comparison - Reference point method - Classification of objectives	- EMO - NSGA-II

Weighting Method. (working principle) 5-1

- uses scalarization function:
- Converts a MOP to SOP.

i.e; Minimize $(f_1(\vec{x}), f_2(\vec{x}), \cdots, f_m(\vec{x}))$

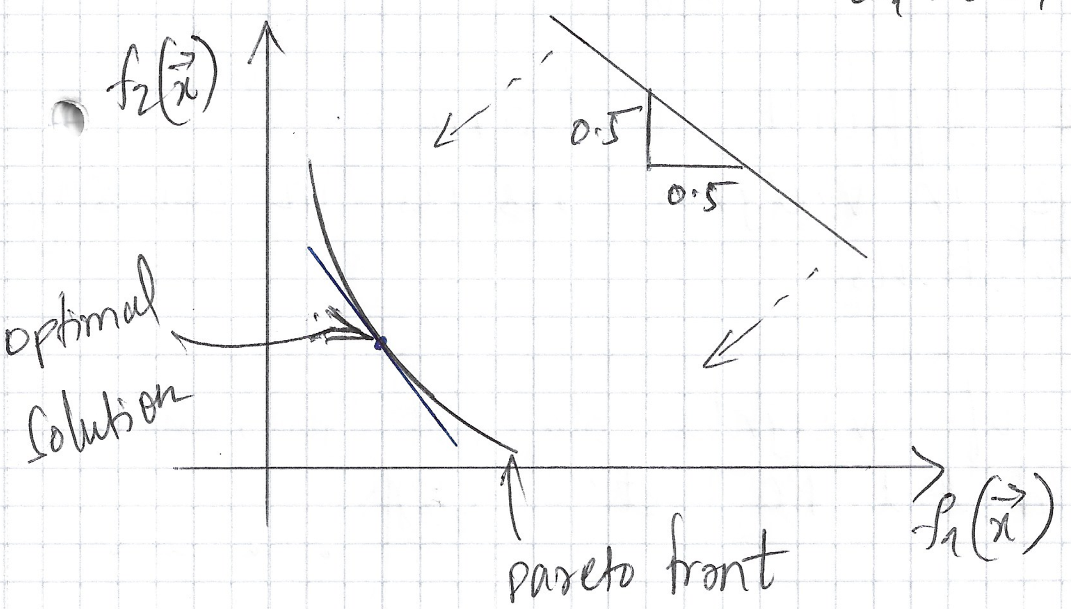
to:

$$\text{Minimize}\left(w_1 f_1(\vec{x}) + w_2 f_2(\vec{x}) + \cdots + w_m f_m(\vec{x})\right)$$

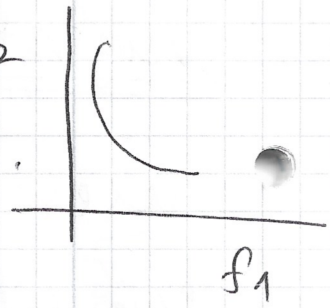
where $\sum_{i=1}^{m} w_i = 1$

- It finds one optimal solution out of many.
- uses weights indicating the slope (position of solution.

$\omega_1 = 0.5,\ w_2 = 0.5$

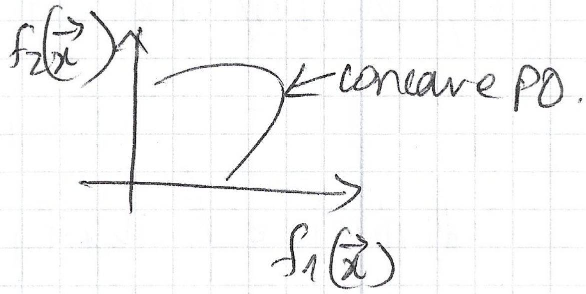


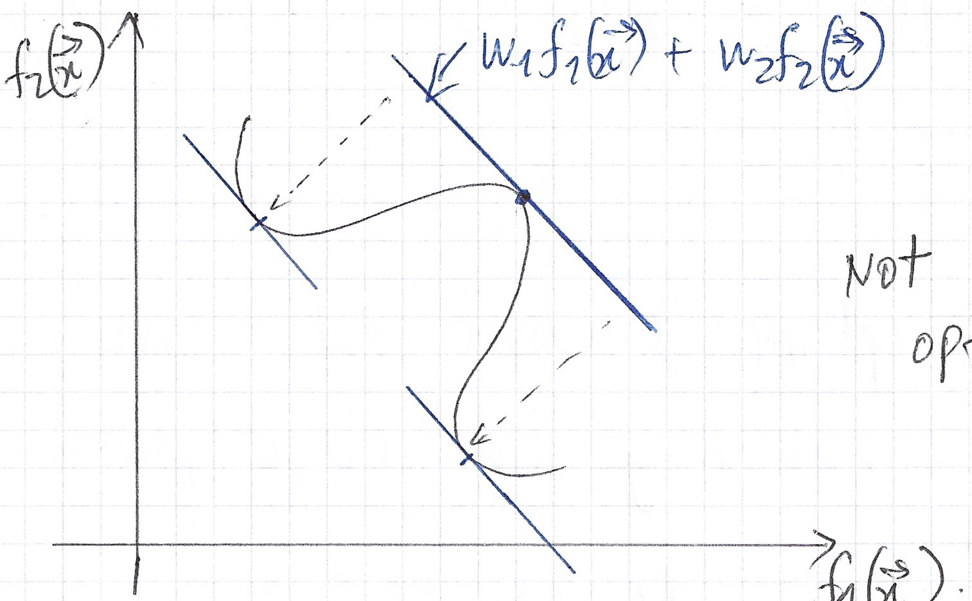
Advantages: For convex problems. one optimal solution is guaranteed.



Drawbacks:

① Non convex problems -

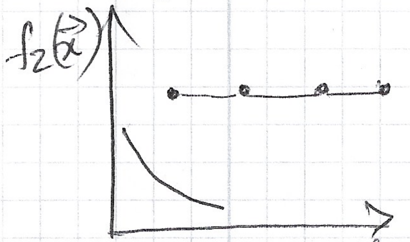




Not able to provide optimal solution.

② Weights are difficult to learn.
Small change in weights results in choosing different solutions.

③. Distribution of weights do not guarantee. evenly distributed PO set.



④ Preference of weights
(Some preferences won't be considered as Strongly as DM preferences)

ex: Selecting a location for long term stay.

Location	Rent (€)	Uni dist (km)	Sup dist (Km)	Result
A	230	5	5	2.8
B	185	6	7	2.2
C	190	7	7	2.7
wts.	0.8	0.1	0.1	

Solution set: { a (2,6), b (2,4), c (3.5, 3.5), d (4,2), e (8,1) }.

Case #1 $(w_1, w_2) := (0.5, 0.5)$ $\sum_{i=1}^{2} w_i = 1$

$w_1 f_1(a) + w_2 f_2(a)$	$0.5 \times 2 + 0.5 \times 6$	4
$w_1 f_1(b) + w_2 f_2(b)$	$0.5 \times 2 + 0.5 \times 4$	3
$w_1 f_1(c) + w_2 f_2(c)$	$0.5 \times 3.5 + 0.5 \times 3.5$	3.5
$w_1 f_1(d) + w_2 f_2(d)$	$0.5 \times 4 + 0.5 \times 2$	3
$w_1 f_1(e) + w_2 f_2(e)$	$0.5 \times 8 + 0.5 \times 1$	4.5

min(4, 3, 3.5, 3, 4.5) two optimal solutions b & d.

Case #2 $(w_1, w_2) := (0.1, 0.9)$ $\sum_{i=1}^{2} w_i = 1$

$w_1 f_1(a) + w_2 f_2(a)$	$0.1 \times 2 + 0.9 \times 6$	5.6
$w_1 f_1(b) + w_2 f_2(b)$	$0.1 \times 2 + 0.9 \times 4$	3.8
$w_1 f_1(c) + w_2 f_2(c)$	$0.1 \times 3.5 + 0.9 \times 3.5$	3.5
$w_1 f_1(d) + w_2 f_2(d)$	$0.1 \times 4 + 0.9 \times 2$	2.2
$w_1 f_1(e) + w_2 f_2(e)$	$0.1 \times 8 + 0.9 \times 1$	1.7

$\min(5.6, 3.8, 3.5, 2.2, 1.7) \Rightarrow$ one optimal solution 'e'.

5.2 ε- Constraint Method (working principle)

Convert a MOP to SOP by keeping one objectives as optimization function & rest as constraints.

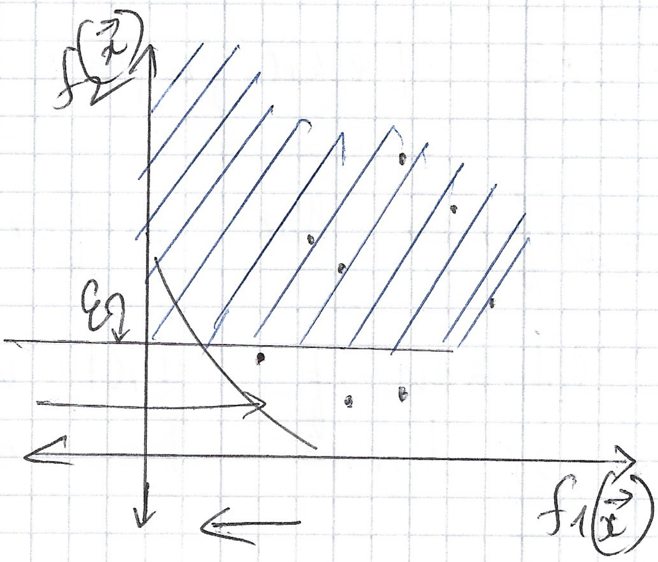
MOP: $\text{Min}(f_1(\vec{x}), f_2(\vec{x}), \ldots, f_m(\vec{x}))$ s.t $\vec{x} \in S$

$\Downarrow$

$\min(f_1(\vec{x})) \mid f_2(\vec{x}) \leq \varepsilon_2$

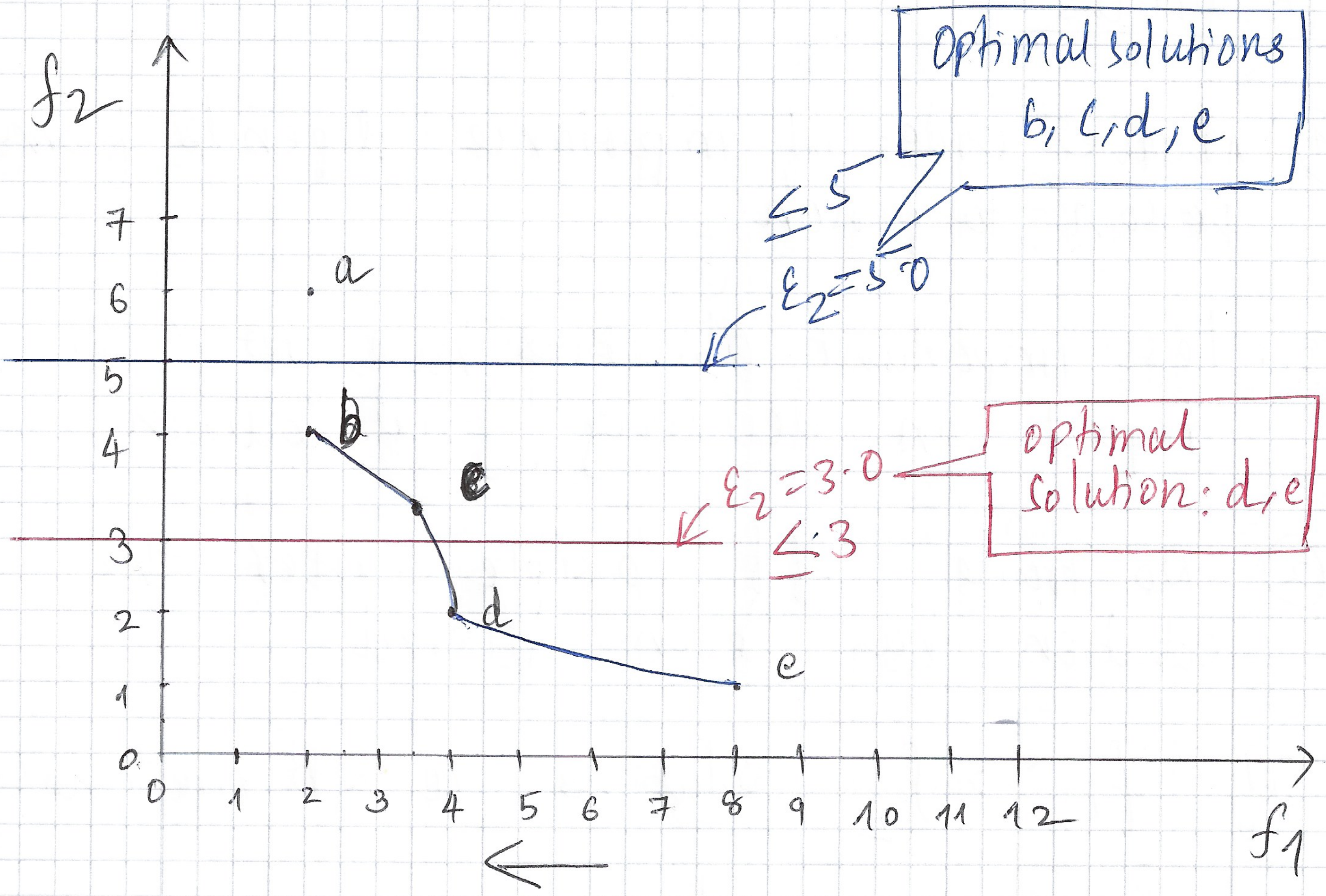
Advantage:

- PO solution can be found easily.



Drawback:

- Pareto front should be determined in advance without which putting the ϵ (hyperparameter) may be difficult.



5.3 Lexicographic Ordering (Working Principle)

- Defines the preferences among objectives.
- Solution of first objective is influenced by or drives the remaining objectives.

Advantages-

1. Solution is PO
2. Sucessive decision making is possible.

Drawbacks-

1. Absolute order of importance depends from person-to-person.
2. less important objectives have very less impact on deciding final solution.
3. No Trade off possible after the solution has been filtered

If $f_1 >> f_2$ Solutions a & b are chosen

If $f_2 >> f_1$ solutions e is chosen.