Assignment 27 (Robustness I)

In the lecture we have learned about robustness of solutions in optimization. Please answer the following questions.

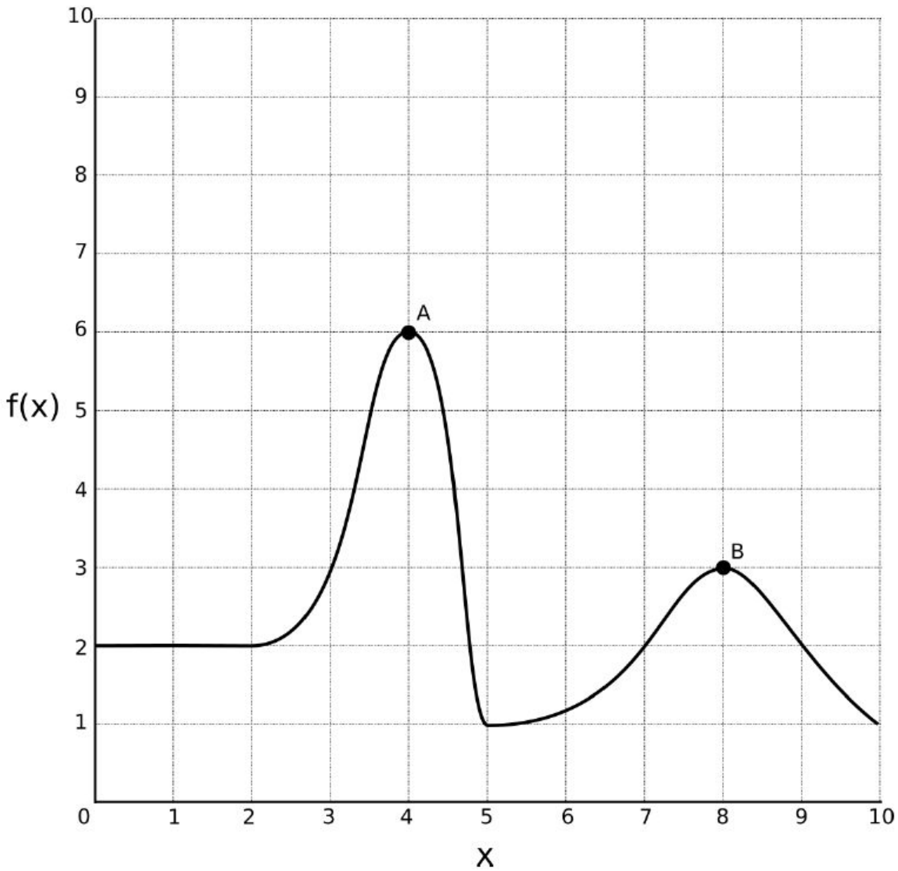
- What is robustness in the context of optimization? Why is robustness an important factor in single- and multi-objective optimization?

- Due to the uncertainties present in the real-world systems, one must **search for solutions that well performs** under all possible scenarios.
- Importance: **less** sensitive to perturbations.

- What is the difference between type I and type II robustness from the lecture? Which additional element is necessary in an optimization algorithm to be able to use type II robustness?

- **Robust I** - global minimizer of the mean effective function wrt δ - neighbourhood.
- **Robust II** - Measure the difference between the function value $f(x)$ & the effective function $f^P(x)$ (perturbed function).
- The additional element for type II usability: A user-defined threshold, **η**

- In the following we show a plot of the fitness function in a single-objective maximization problem. For the two solutions A (at $x_A = 4$) and B (at $x_B = 8$), compute their respective mean effective fitness values $f^{eff}(x_A)$ and $f^{eff}(x_B)$. The neighborhood has a size of 3 and consists of the samples at $x + \delta$ for $\delta \in \{-1, 0, 1\}$.



Neighbours and function values of x_A & x_B:

4+0 = 4	$f(x)$ = 6	8+0 = 4	$f(x)$ = 3
4-1 = 3	$f(x)$ = 2.9	8-1 = 7	$f(x)$ = 2
4+1 = 5	$f(x)$ = 1	8+1 = 9	$f(x)$ = 2
	$f^{eff}(x_A)$ = 3.3		$f^{eff}(x_B)$ = 2.33

- Which of the two solutions A and B is more robust according to type I robustness?

Since we are given a single-objective **maximization** problem comparing the mean effective fitness values we observe solution **A** as more robust.

- Which of the two solutions A and B is more robust according to type II robustness, using a value of $\eta = 0.3$? Use f^{eff} as the perturbed function.

Using the definition of Type II robustness and $f^{P}(x) = f^{eff}(x)$,

$$\frac{\left\| f^{eff}(x_A) - f(x_A) \right\|}{\left\| f(x_A) \right\|} = \frac{\sqrt{(3.3 - 6)^2}}{\sqrt{6^2}} = 0.45 \not\leq 0.3$$

$$\frac{\left\| f^{eff}(x_B) - f(x_B) \right\|}{\left\| f(x_B) \right\|} = \frac{\sqrt{(2.33 - 3)^2}}{\sqrt{3^2}} = 0.22 < 0.3$$

Hence, solution **B** is more robust.