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Assignment 30 (Constraint Handling)

Take a look at the constraint handling techniques from the lecture and answer the following questions.

- What is constraint handling and why is it needed in optimization algorithms?

- Constraint handling the process of adding constraints (equality, inequality, box constraints) to an optimization problem such that it performs the task in a more constrained manner.

- Two variations of constraint handling:
  - For SOP

$$\min f(x)$$
$$\text{subj to } h_j(x) = 0;\ j = 1 \ldots J$$
$$g_k(x) \geq 0;\ k = 1 \ldots K$$
$$x^L \leq x \leq x^U$$

  - For MOP

$$\min F(x) = \left(f_1(x), \ldots, f_m(x)\right)$$
$$\text{subj to } h_j(x) = 0;\ j = 1 \ldots J$$
$$g_k(x) \geq 0;\ k = 1 \ldots K$$
$$x^L \leq x \leq x^U$$

Relevance:

- Real-world optimization problems and decision making almost always have constraints.
- Engineering design problems: stress constraints, material properties.

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- Consider the following single-objective problem with 2 decision variables and 2 inequality constraints.

$$\begin{aligned} min \quad & f(\vec{x}) = 3 \cdot x_1 + 4 \cdot x_2 \\ subject\ to \quad & g_1(\vec{x}) = 0.4 \cdot x_1 - 0.6 \cdot x_2 \geq 0 \\ & g_2(\vec{x}) = -0.1 \cdot x_1 + 0.8 \cdot x_2 - 0.1 \geq 0 \end{aligned}$$

For this problem, we decided to use the static penalty method with one fixed value of $R = 3$ as shown on slide EMO-6-35. For the three solutions $\vec{x}^{(a)} = (2, 4)$, $\vec{x}^{(b)} = (1, 5)$ and $\vec{x}^{(c)} = (7, 2)$, please compute the fitness of the solutions, their constraint violations for each of the two constraints and the penalized objective function values.

Fitness values, constraint violations & penalized objective function values,

$f(\vec{x}^{(a)})$ = 3(2) + 4(4) = **22**

$g_1(\vec{x}^{(a)})$ = 0.4(2) - 0.6(4) = **1.6** ≥ 0

$g_2(\vec{x}^{(a)})$ = - 0.1(2) + 0.8(4) - 0.1 = **2.9** ≥ 0

$F(\vec{x}^{(a)})$ = 22 + 3(1.6 + 2.9) = **35.5**

$f(\vec{x}^{(b)})$ = 3(1) + 4(5) = **23**

$g_1(\vec{x}^{(b)})$ = 0.4(1) - 0.6(5) = **2.6** ≥ 0

$g_2(\vec{x}^{(b)})$ = - 0.1(1) + 0.8(5) - 0.1= **3.8** ≥ 0

$F(\vec{x}^{(b)})$ = 23 + 3(2.6 +3.8) = **42.2**

$f(\vec{x}^{(c)})$ = 3(7) + 4(2) = **29**

$g_1(\vec{x}^{(c)})$ = 0.4(7) - 0.6(2) = **1.6** ≥ 0

$g_2(\vec{x}^{(c)})$ = - 0.1(7) + 0.8(2) - 0.1 = **0.8** ≥ 0

$F(\vec{x}^{(c)})$ = 29 + 3(1.6 + 0.8) = **36.2**