

## Assignment 31 (Constraint Handling in MOP)

Consider the following bi-objective minimization problem with 2 decision variables and 1 inequality constraint:

$$\begin{aligned} \min \quad & F(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x})) = (x_1^2 + x_2^2, |x_1 - x_2 - 2 \cdot x_1 \cdot x_2 + 2|) \\ \text{subject to} \quad & g_1(\vec{x}) = |x_1 + x_2| > 2 \end{aligned}$$

And a population consisting of the following eight solutions:

$$\begin{array}{llll} x^{(a)} = (0, 0) & x^{(b)} = (1, 1) & x^{(c)} = (2, 2) & x^{(d)} = (-3, 0) \\ x^{(e)} = (1, 2) & x^{(f)} = (-1, 0) & x^{(g)} = (3, -2) & x^{(h)} = (1, 3) \end{array}$$

Take a look at the constraint handling techniques in MOPs and answer the following questions.

- What advantages and disadvantages do you find between using penalty function for each objective and constrain-domination?

### **Penalized objective function**

#### Advantages:

- Using a **penalty function** convert a constrained problem to an unconstrained one.
- The new fitness value is computed based on value of **fitness function & penalty term**.
- The amount of constraint violation is indicated by the **penalty term**.

#### Disadvantages:

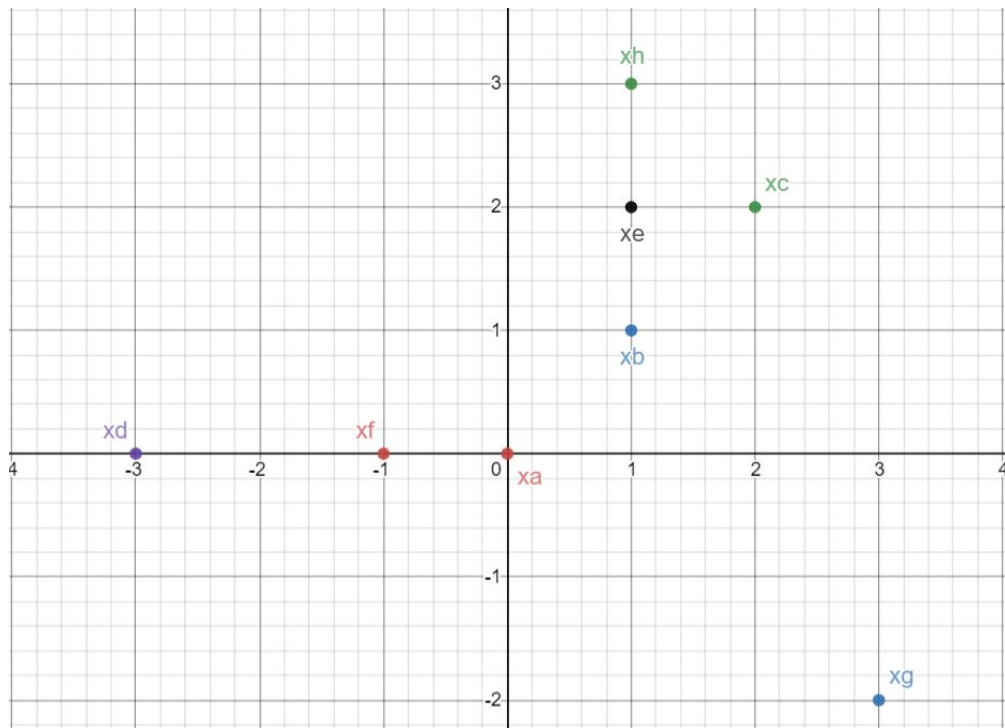
- Penalty term sometimes **distorts** the objective function. (static penalty)
- Small R-value  $\Rightarrow$  lesser distortion  $\Rightarrow$  optimum may not be found.
- Large R-value  $\Rightarrow$  distortion creates artificial local minima  $\Rightarrow$  near true optimum.

### **Constrain-domination**

#### Advantages:

- No conversion involved instead, measure the constraint violation for the solutions.

- Which solutions of the population are constrain-dominated by solution  $x^{(d)}$ ?



Solutions, $(x^*)$	Fitness, $F(\vec{x})$	Constraint, $g_1(\vec{x})$
$x^{(a)}$	(0, 2)	$0 \not> 2$
$x^{(b)}$	(2, 0)	$2 \not> 2$
$x^{(c)}$	(8, 6)	$4 > 2$
$x^{(d)}$	(9, 1)	$3 > 2$
$x^{(e)}$	(5, 3)	$3 > 2$
$x^{(f)}$	(1, 1)	$1 \not> 2$
$x^{(g)}$	(13, 19)	$1 \not> 2$
$x^{(h)}$	(10, 6)	$4 > 2$

$x^{(d)}$  **constraint dominates**  $x^{(a)}$ ,  $x^{(b)}$ ,  $x^{(f)}$ ,  $x^{(g)}$ ,  $x^{(h)}$

- Which solutions are constrain-dominated by solution  $x^{(b)}$ ?

$x^{(b)}$  **constraint dominates** none. It has a **larger** constraint violation value than any other solutions.

- Which solutions are non-dominated according to constrain-domination?

$$x^{(c)}, x^{(d)}, x^{(e)}$$

- Which solutions are the most reliable?

$$x^{(c)}, x^{(d)}, x^{(e)}, x^{(h)}$$