Nuclear Astrophysics: Neuton Star Physics

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Introduction to Particle Physics

Introduction:
Particle physics explores the fundamental building blocks of the universe and the forces that govern their interactions. It aims to understand the structure and behavior of matter at the smallest scales, uncovering insights into the origins and fundamental laws of the cosmos.

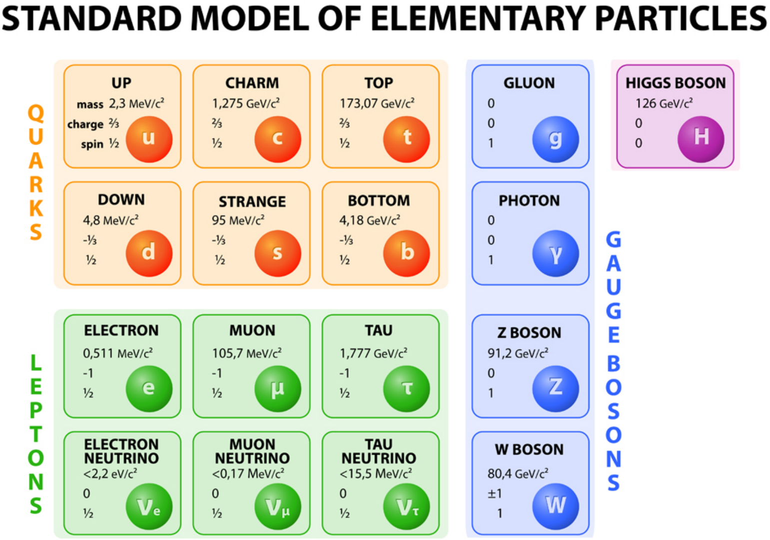
Particles and Forces:
Fundamental Particles:

- **Leptons:** Light particles like electrons and neutrinos.
- **Quarks:** Form protons and neutrons; six types (up, down, charm, strange, top, bottom).
- **Bosons:** Force carriers, including photons, gluons, W/Z bosons, and the Higgs boson.

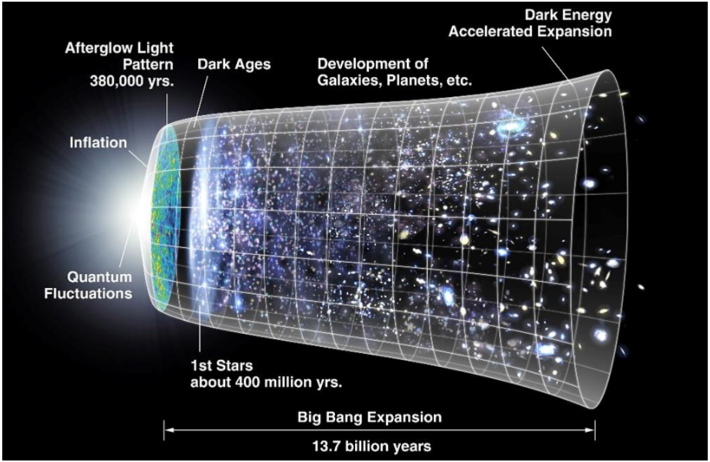
Fundamental Forces:

- **Electromagnetic:** Governs charged particle interactions.
- **Strong Nuclear:** Binds quarks and atomic nuclei.
- **Weak Nuclear:** Enables radioactive decay and nuclear fusion.
- **Gravitational:** Influences massive objects but is negligible at particle scales.

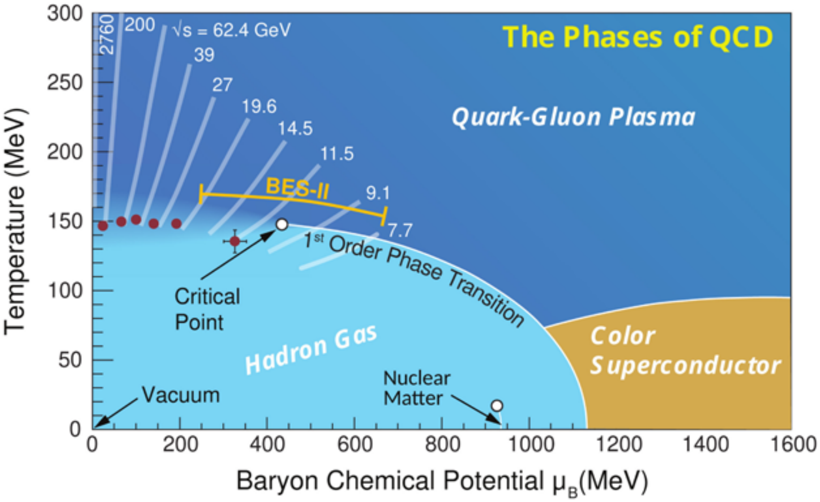
The Standard Model



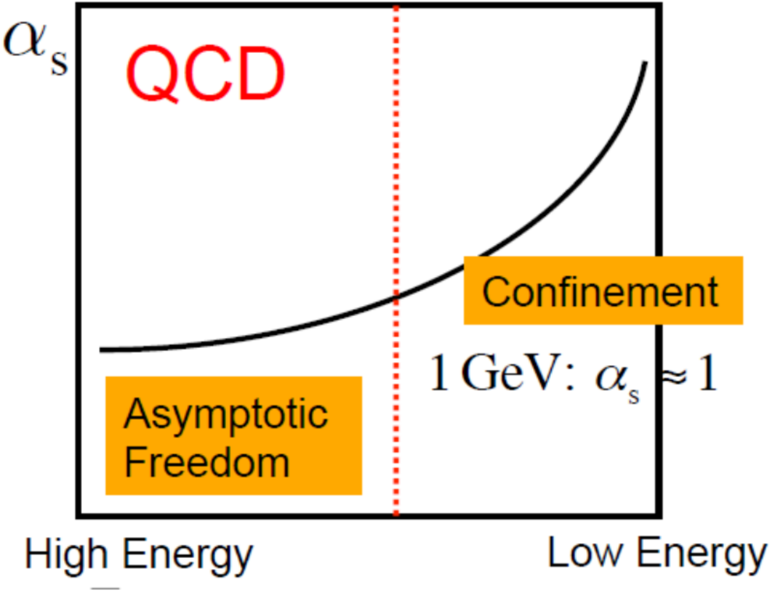
Big Bang Theory



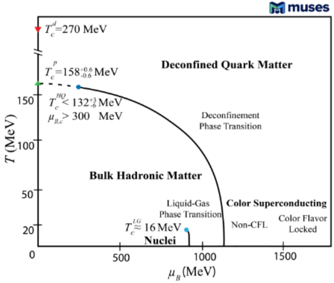
QCD Phase Diagram



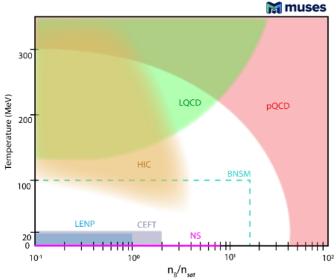
Confinement and Asymptotic Freedom



Models of the QCD Phase Diagram



R.K., et al., Phys. Rev. D 109, 074008 (2024)



R.K., et al., Living Rev. Rel. 27, 3 (2024)

Introduction to the Walecka Model

The Walecka Model, also known as the Relativistic Mean Field (RMF) theory, is a theoretical framework used to describe the properties of nuclear matter. It models the interactions between nucleons (protons and neutrons) through the exchange of scalar and vector mesons.

Scalar Meson (σ) Term:

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}\left(\partial^{\mu}\sigma\partial_{\mu}\sigma - m_{\sigma}^{2}\sigma^{2}\right)$$

Vector Meson (ω^{μ}) Term:

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_{\omega}^{2}\omega^{\mu}\omega_{\mu}$$

where $\omega_{\mu\nu} = \partial_{\mu}\omega_{\nu} - \partial_{\nu}\omega_{\mu}$

Saturation Properties of Nuclear Matter

Empirical nuclear matter exhibits a saturation point at a density $\rho_0 \approx 0.16\,\mathrm{fm}^{-3}$, where the energy per nucleon is minimized (approximately $-16\,\mathrm{MeV}$). This reflects a balance between attractive and repulsive interactions in nuclear forces.

A reliable nuclear model should reproduce the following properties:

- Saturation density ρ_0
- Binding energy per nucleon at saturation
- Compressibility modulus K
- Symmetry energy S
- Slope parameter L

Lagrangian of the Model

The total Lagrangian of the Walecka model combines contributions from the fermionic, scalar, and vector fields, along with interaction terms. It forms the foundation for deriving the equations of motion in mean-field theory.

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{int}} \\
&= \bar{\psi} i (i\gamma^{\mu}\partial\mu - m_N)\psi_i + g_{\sigma}\bar{\psi}_i \sigma \psi_i - g_{\omega}\bar{\psi} i \gamma^{\mu}\omega\mu\psi_i \\
&\quad + \frac{1}{2}(\partial^{\mu}\sigma\partial_{\mu}\sigma - m_{\sigma}^2\sigma^2) - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega^{\mu}\omega_{\mu}
\end{aligned}
$$

Mean-Field Approximation

In uniform nuclear matter, the mean-field approximation simplifies the equations of motion by treating meson fields as classical constants. Only time-like components of the vector field are retained.

Assumptions:

$$\sigma(x) \to \bar{\sigma}, \quad \omega^{\mu}(x) \to \delta^{\mu 0}\bar{\omega}_0$$

Nucleon Field Equation:

$$\left[i\gamma^{\mu}\partial_{\mu} - m_N + g_{\sigma}^{i}\bar{\sigma} - g_{\omega}^{i}\gamma^{0}\bar{\omega}_0\right]\psi_i = 0$$

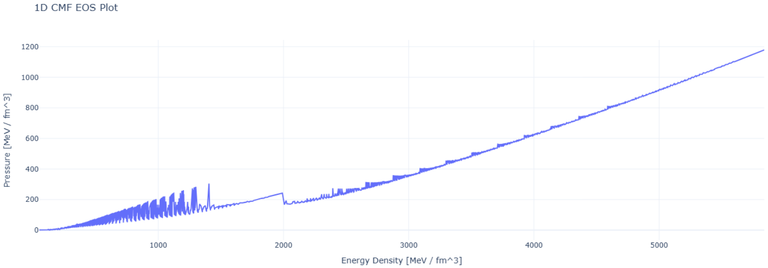
$$M_i^* = m_N - g_{\sigma}^{i}\bar{\sigma}$$

$$\left[i\gamma^{\mu}\partial_{\mu} - M_i^* - g_{\omega}^{i}\gamma^{0}\bar{\omega}_0\right]\psi_i = 0$$

Meson Field Equations:

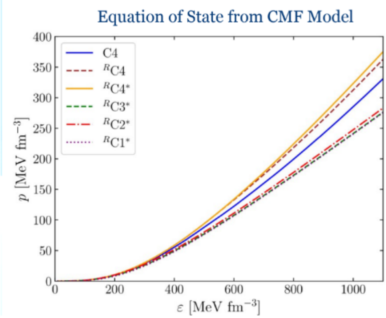
$$\bar{\sigma} = \frac{g_{\sigma}^{i}}{m_{\sigma}^{2}} n_{s_i}, \quad \bar{\omega}0 = \frac{g\omega^{i}}{m_{\omega}^{2}} n_{B_i}$$

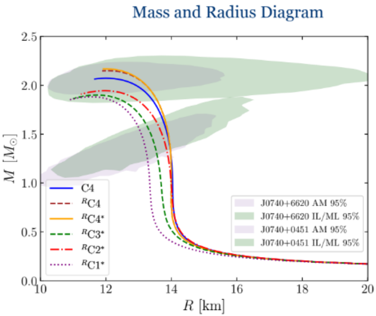
EoS in Neutron Star Matter from the CMF Model



Equation of State curve generated using the CMF model

EoS: Pressure vs Energy Density & Mass-Radius Curve of a Neutron Star





Conclusion

Through the study of effective models like the Walecka model and exploration of the QCD phase diagram, we gain valuable insight into the behavior of matter at high densities and temperatures. The application of the CMF model to neutron star matter enables the calculation of realistic equations of state, consistent with astrophysical observations. These approaches form a crucial link between nuclear theory and the physics of compact astrophysical objects.