

- Determinant of a matrix $A = [a_{11}]_{1 \times 1}$ is given by

$$|A| = a_{11} \quad (1)$$

- Determinant of a matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad (2)$$

is given by

$$|A| = \left| \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \right| \quad (3)$$

$$= a_{11}a_{22} - a_{12}a_{21} \quad (4)$$

- Determinant of a 3×3 matrix is obtained by expanding along a row or a column.
- For any square matrix A , determinant satisfies the following properties:

- $|A^T| = |A|$
- Interchanging any two rows or columns changes the sign of determinant
- If any two rows or columns are identical or proportional, then $|A| = 0$
- If each element of a row or column is multiplied by k , then determinant is multiplied by k

- Area of a triangle with vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2} \left| \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \right| \quad (5)$$

- Minor of an element a_{ij} is obtained by deleting the i^{th} row and j^{th} column.

7. Cofactor of a_{ij} is defined as

$$A_{ij} = (-1)^{i+j} M_{ij} \quad (6)$$

8. Adjoint of a square matrix is the transpose of the matrix of cofactors.

9. A square matrix is singular if $|A| = 0$ and non-singular if $|A| \neq 0$.

10. Inverse of a non-singular matrix A is given by

$$A^{-1} = \frac{1}{|A|} (\text{adj } A) \quad (7)$$

11. For matrix equation $AX = B$:

- If $|A| \neq 0$, unique solution exists
- If $|A| = 0$ and $(\text{adj } A)B \neq 0$, no solution exists
- If $|A| = 0$ and $(\text{adj } A)B = 0$, solution may or may not exist