

- Determinant of a matrix  $A = [a_{11}]_{1 \times 1}$  is given by

$$|A| = a_{11}$$

- Determinant of a matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

is given by

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

- Determinant of a  $3 \times 3$  matrix is obtained by expanding along a row or a column.
- For any square matrix  $A$ , determinant satisfies the following properties:
  - $|A^T| = |A|$
  - Interchanging any two rows or columns changes the sign of determinant
  - If any two rows or columns are identical or proportional, then  $|A| = 0$
  - If each element of a row or column is multiplied by  $k$ , then determinant is multiplied by  $k$
- Area of a triangle with vertices  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is given by

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

- Minor of an element  $a_{ij}$  is obtained by deleting the  $i^{th}$  row and  $j^{th}$  column.

- Cofactor of  $a_{ij}$  is defined as

$$A_{ij} = (-1)^{i+j} M_{ij}$$

- Adjoint of a square matrix is the transpose of the matrix of cofactors.
- A square matrix is singular if  $|A| = 0$  and non-singular if  $|A| \neq 0$ .
- Inverse of a non-singular matrix  $A$  is given by

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

- For matrix equation  $AX = B$ :
  - If  $|A| \neq 0$ , unique solution exists
  - If  $|A| = 0$  and  $(\text{adj } A)B \neq 0$ , no solution exists
  - If  $|A| = 0$  and  $(\text{adj } A)B = 0$ , solution may or may not exist