

International Mathematical Olympiad (IMO) Problems

IIIT Bangalore

IMO 2016 – Day 1 (Monday, July 11, 2016)

1. Triangle BCF has a right angle at B. Let A be the point on line CF such that $FA = FB$ and F lies between A and C. Point D is chosen such that $DA = DC$ and AD is the bisector of $\angle DAB$. Point E is chosen such that $EA = ED$ and AD is the bisector of $\angle EAC$. Let M be the midpoint of CF. Let X be the point such that AMXE is a parallelogram where $AM \parallel EX$ and $AE \parallel MX$. Prove that the lines BD, FX, and ME are concurrent.
2. Find all positive integers n for which each cell of an $n \times n$ table can be filled with one of the letters I, M, O such that row, column, and diagonal conditions are satisfied.
3. Let $P = A_1A_2 \dots A_n$ be a convex polygon with integer-coordinate vertices on a circle. Let S be its area. If the squares of side lengths are divisible by an odd integer v , prove that $2S$ is divisible by v .

IMO 2016 – Day 2 (Tuesday, July 12, 2016)

1. A set of positive integers is fragrant if every element shares a prime factor with another. Let $P(n) = n^2 + n + 1$. Find the smallest b such that $\{P(a+1), \dots, P(a+b)\}$ is fragrant.
2. After erasing exactly k linear factors from both sides of $(x-1)(x-2) \dots (x-2016)$, find the least k such that the equation has no real solutions.
3. Frogs jump along intersecting segments.
 - (a) Prove it is possible if n is odd.
 - (b) Prove it is impossible if n is even.

IMO 2017 – Day 1 (Tuesday, July 18, 2017)

1. For $a_0 > 1$, define a sequence by a given rule. Determine all values of a_0 for which the sequence attains the same value infinitely often.
2. Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfying $f(f(x)f(y)) + f(x+y) = f(xy)$.
3. Hunter and invisible rabbit pursuit problem with bounded tracking error.

IMO 2017 – Day 2 (Wednesday, July 19, 2017)

1. Geometry problem involving tangents and a cyclic configuration.
2. Soccer players standing arrangement problem.
3. Polynomial identity on primitive lattice points.

IMO 2018 – Day 1 (Monday, July 9, 2018)

1. In a circle configuration, prove that lines DE and FG are parallel or coincide.
2. Find all integers $n \geq 3$ satisfying given recursive relations.
3. Anti-Pascal triangle existence problem with 2018 rows.

IMO 2018 – Day 2 (Tuesday, July 10, 2018)

1. A site is any point (x, y) in the plane such that x and y are both positive integers less than or equal to 20.

Initially, each of the 400 sites is unoccupied. Amy and Ben take turns placing stones, with Amy going first. On her turn, Amy places a new red stone on an unoccupied site such that the distance between any two sites occupied by red stones is not equal to $\sqrt{5}$. On his turn, Ben places a new blue stone on an unoccupied site. (A site occupied by blue stones is allowed to be at any distance from any other occupied site.) They stop as soon as one player cannot place a stone.

Find the greatest K such that Amy can ensure that she places at least K red stones, no matter how Ben places his blue stones.

2. Let a_1, a_2, \dots be an infinite sequence of positive integers. Suppose that there is an integer $N > 1$ such that, for each $n \geq N$, the number

$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1}$$

is an integer. Prove that there is a positive integer M such that $a_n = a_{n+1}$ for all $n \geq M$.

3. A convex quadrilateral $ABCD$ satisfies

$$AB \cdot CD = BC \cdot DA.$$

Point X lies inside $ABCD$ such that

$$\angle XAB = \angle XCD \quad \text{and} \quad \angle XBC = \angle XDA.$$

Prove that

$$\angle BXA + \angle D XC = 180^\circ.$$