Relations and Functions

Relation is a subset of cartesian product.

Eg: $A = \{Red, Blue\}$

$B = \{Bag, Shirt, jeans\}$

$A \times B = \{(Red, Bag)(Red, shirt)(Red, jeans)$
$(Blue, Bag)(Blue, shirt)(Blue, jeans)\}$

Cartesian product $\longrightarrow$ is the set of all ordered pairs.

$A = \{1, 2\}$

$B = \{a, b, c\}$

$A \times B = \{(1,a)(1,b)(1,c)(2,a)(2,b)(2,c)\}$

$B \times A = \{(a,1)(a,2)(b,1)(b,2)(c,1)(c,2)\}$

$A \times B \neq B \times A$

$R = \{(x,y) : x \text{ is Red colour}, x \in A, y \in B\}$

$R = \{(Red, Bag), (Red, Shirt)(Red, jeans)\}$

Types of Relation :-

1. Empty Relation
2. Universal Relation.

Empty Relation:

A relation R in a Set A is called empty relation, if no element of A is related to any element of A.

$$(i.e) \quad R = \phi \subset (A \times A)$$

$A \times B = \{ (Red, Bag) \ (Red, shirt) \ (Red, jeans)$
$(Blue, Bag) \ (Blue, shirt) \ (Blue, jeans) \}$

$R = \phi = \{ \}$ eg: Green colour.

(or)

Relation has no elements.

Universal Relation:

If the Relation has all the elements. It is a Universal relational.

R = Colours in the 1^{st} place.

$R = A \times B = \{ (Red, Bag) \ (Red, shirt) \ (Red, jean)$
$(Blue, Bag) \ (Blue, shirt) \ (Blue, jean) \}$

A relation R in a set A is called Universal relation, if each element of A is related to every element of A. (ie) $R = A \times A$.

$$A = \{1, 2, 3\}$$

$$A \times A = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$$

R = all Natural Numbers in 1st place

Equivalence Relation ;

1. Reflexive
2. Symmetric
3. transitive

A Relation R in a Set A is called an equivalence relation if R is Reflexive, Symmetric and transitive.

(i) Reflexive :-

A relation R is a non-empty set A is called Reflexive if $\underset{\text{(for all)}}{\forall}\ a \underset{\text{(belongs)}}{\in} A$, $(a, a) \in R$ (ie) $a \underset{\text{(Related)}}{R} a$

Eg : $A \times A$ $\quad A = \{1, 2, 3\}$

$$A \times A = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$$

$$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$$

$R_2 = \{(1,1)\}$ X

$R_3 = \{(1,1), (2,2) (3,3) (2,1) (3,1)\}$ ✓

$R_4 = \{\ \} = \phi$ X

$R_5 = \{(3,3) (2,2) (2,1) (2,3)\}$ X

(ii) Symmetric :- (Identital)

A Relation R in a set of A is called Symmetric if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$ for all $a_1, a_2 \in A$.

Example : $A = \{1, 2, 3, 4\}$

$R_1 = \{(1,1) (2,2) (2,1) (1,2)\}$ ✓

$R_2 = \{(1,2)\}$ X

$R_3 = \{(1,1)\}$ ✓

$R_4 = \{\ \}$ ✓

$R_5 = \{(1,3) (4,2) (3,1) (2,4)\}$ ✓

$R_6 = \{(1,4) (4,1) (3,2)\}$ X

(iii) Transitive :-

A Relation R in a set A is called transitive if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R$ for any $a_1, a_2, a_3 \in A$.

Example : $A = \{1, 2, 3\}$ doubles

$R_1 = \{(1,1)(2,2)(3,3)\}$ ✓

$R_2 = \{(1,1)(2,2)(2,3)\}$ ✓

$R_3 = \{(2,3)(3,1)(2,1)\}$ ✓

$R_4 = \{\ \}$ ✓

$R_5 = \{(1,2)\}$ ✓

$R_6 = \{(1,2)(2,2)\}$ ✓

Ex : 1.1

1. Determine whether each of the following Relations are reflexive, Symmetric and transitive.

(i) Relation R in the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$

$A = \{1, 2, 3, 4, 5, \ldots \ldots 14\}$

(subset)

$R \subset A \times A$

$R = \{ (x, y) : 3x - y = 0 \}$

$3x - y = 0$

$3x = y$

$\boxed{y = 3x}$

$x = 1, \quad y = 3$

$x = 2, \quad y = 6 \qquad x, y \in A$

$x = 3, \quad y = 9$

$x = 4, \quad y = 12$

$x = 5, \quad y = 15 \quad \times$

$R = \{ (1,3) (2,6) (3,9) (4,12) \}$

Reflexive : $\forall a \in A, (a,a) \in R$

$(1,1) \notin R, \quad (2,2) \notin R \; \ldots \; (14,14) \notin R$

$\therefore$ It is not Reflexive.

Symmetric : for any $a, b \in A$ if $(a,b) \in R$

$\Rightarrow (b,a) \in R$,

$(1,3) \in R$ but $(3,1) \notin R$

R is not Symmetric

Transitive :- for any $(a,b) \in R$ and $(b,c) \in R$
$\Rightarrow (a,c) \in R$.

$(1,3) \in R$ and $(3,9) \in R$ but $(1,9) \notin R$

$\therefore$ R is not transitive.

(iii) Relation R in the Set N of Natural numbers defined as, $N = \{1, 2, 3 \ldots\ldots \infty\}$

$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$

x & y are natural Numbers.

x as 1, 2, 3

$y = x + 5$

$x = 1, \quad y = 6 \qquad R = \{(1,6)\ (2,7)\ (3,8)\}$

$x = 2, \quad y = 7$

$x = 3, \quad y = 8$

Reflexive :- Since $(1,1) \notin R$

R is not Reflexive.

Symmetric :- R is not symmetric.

Transitive :- $(1,6) \in R$, $(2,7) \in R$, $(3,8) \in R$

$\therefore$ R is not transitive.

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$R = \{(x, y) : y \text{ is divisible by } x\}$

(Should be divide perfectly).

eg: $2/1 = 2$ ✓

So, y/x $4/2 = 2$ ✓

$2/6 = 1/3$ ✗

Should be bigger.

$R = \{(1,1)(1,2)(1,3)$
$(1,4)(1,5)(1,6)$
$(2,2)(3,3)(2,4)$
$(4,4)(5,5)(2,6)$
$(3,6)(6,6)\}$

Reflexive :- It is reflexive

Symmetric :- $(1,2) \in R$ but $(2,1) \notin R$

It is not Symmetric.

Transitive :- $(1,2) \in R$, $(2,4) \in R$, $(1,4) \in R$

It is transitive.

(iv) Relation R in the Set Z of all integers defined as

$R = \{(x, y) : x - y \text{ is an integer}\}$

$Z = \{-\infty, \ldots -4, -3, -2, -1, 0, 1, 2, 3 \ldots +\infty\}$

$R = \{(\text{any pair})\}$

Reflexive :- $R = \{(1,1)(2,2) \ldots\}$

It is Reflexive

Symmetric :- It is Symmetric.

Transitive :- Everything belongs to R.

This is transitive.

$\therefore$ It is equivalence Relation.

(V) Relation R in the Set A of Human beings in a town at a particular time given by

(a) $R = \{(x,y)\}$: x and y work at the Same place$\}$

$A = \{$ human beings $\}$

Reflexive :- x and x are Same persons, they work at same place.

R is Reflexive.

Symmetric :- x and y work at Same place

y and x " " "

(ie) $(x,y) \in R$, $(y,x) \in R$

R is Symmetric

Transitive :- x and y work at Same place

y and z " " "

then x and z " " "

It is transitive.

$(x,y) \in R$, $(y,z) \in R$, $(x,z) \in R$.

(b) $R = \{(x, y) : x$ and y live in the same locality$\}$

Reflexive : x and x are the same person.

So $(x, x) \in R$

R is reflective

Symmetric : x & y live in the same locality

y and x " " "

$(x, y) \in R$, $(y, x) \in R$

$\therefore$ R is symmetric

Transitive : x & y live in same locality

y & z " " "

then x and z also " "

$(x, y) \in R$, $(y, z) \in R \Rightarrow (x, z) \in R$

R is transitive.

It is ~~an~~ Equivalence Relation.

(c) $R = \{(x, y) : x$ is exactly 7cm taller than $y\}$

Reflexive :- x and x are the same persons

he cannot be taller than himself.

$(x, x) \notin R$

$\therefore$ R is not reflexive.

Symmetric :- x is exactly 7cm taller than y

than y is not taller than x

$(x, y) \in R$, $(y, x) \notin R$

$\therefore R$ is not Symmetric.

Transitive :- x is exactly 14cm taller than z.

(if x is exactly 7cm taller than y and y is exactly taller than z)

$(x, y) \in R$, $(y, z) \in R$ $\therefore$ $(x, z) \notin R$

$\therefore$ R is not transitive.

(d) $R = \{(x, y) : x \text{ is wife of } y\}$

Reflexive :- x and x are Same persons

She cannot be wife for herself

$(x, x) \notin R$

$\therefore$ R is not Reflexive

Symmetric :- If x is wife of y,

y cannot be wife of x

$(x, y) \notin$, $(y, x) \notin R$

$\therefore$ R is not symmetric.

Transitive :- If x is wife of y

then y cannot be wife of anybody else

$(x, y) \in R \quad (y, z) \notin R$

$\therefore$ R is not transitive.

2. Show that $R = \{(a,b) : a \leq b^2\}$, where R is the Set of Real Number. Check whether it is neither reflexive nor Symmetric nor transitive.

$$R = \{(a,b) : a \leq b^2\}$$

Reflexive :- $(a, a) \in R \ \forall \ a \in A$

$$a \leq a^2 \Rightarrow 2 \leq (2)^2 \quad -2 \leq (-2)^2$$

$$2 \leq 4 \quad -2 \leq 4$$

eg: $\frac{1}{3}$ $a^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$\cancel{\frac{1}{3} \leq \frac{1}{9}}$ (This not true)

(0.33) (0.111)

$\therefore$ R is not Reflexive.

Symmetric : $(a, b) \in R$, $(b, a) \in R$; for all $a, b \in A$

eg: $a = \frac{1}{3}$, $b = 1$

$$a \leq b^2$$

$$\frac{1}{3} \leq (1)^2$$

$$\frac{1}{3} \leq 1 \quad \checkmark \ (\text{This is true})$$

$$b \le a^2$$

$$1 \le \left(\frac{1}{3}\right)^2$$

$$1 \le \frac{1}{9} \quad \text{(This is not true)}$$

$\therefore$ R is not symmetric.

Transitive :- $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$

$a = 2$, $b = -2$, $c = -1$

$a \le b^2$ $\quad$ $b \le c^2$ $\quad$ $a \le c^2$

$2 \le (-2)^2$ $\quad$ $(-2) \le (-1)^2$ $\quad$ $2 \le (-1)^2$

$2 \le 4$ ✓ $\quad$ $-2 \le 1$ ✓ $\quad$ $2 \le 1^2$ ✗ (Not true)

$\therefore$ R is not transitive.

3. Check whether the Relation R defined in the set $\{1,2,3,4,5,6\}$ as $R = \{(a,b) : b = a+1\}$ is reflexive, symmetric or transitive.

$$A = \{1,2,3,4,5,6\}$$

$$R = \{(a,b) : b = a+1\}$$

$$= \{(1,2)\,(2,3)\,(3,4)\,(4,5)\,(5,6)\}$$

Reflexive :- $(1,1)\,(2,2)\,\ldots\,(6,6)$

There is no value in R

$\therefore$ R is not Reflexive.

Symmetric :- $(1,2) \in R$, $(2,1) \notin R$

It is not Symmetric.

Transitive :- $(1,2) \in R$ $(2,3) \in R \Rightarrow (1,3) \notin R$

It is not transitive.

So the given relation is not Reflexive, Symmetric and transitive.

4. Show that the Relation R in R defined as,
$R = \{(a,b) : a \leq b\}$, is reflexive, and transitive but not Symmetric.

Reflexive :- $(a, a) \in R$ $a \leq a$

eg $2 \leq 2$

$\therefore$ R is Reflexive.

Symmetric :- $(1,2) \in R$, $(2,1) \in R$

$1 \leq 2$ ✓ $2 \leq 1$ ✗

$\therefore$ R is not symmetric

Transitive :- $(a,b) \in R$, $(b,c) \in R$, $(a,c) \in R$

$a \leq b$ $b \leq c$ $\therefore a \leq c$

Therefore R is Transitive.

5. Check whether the Relation R in R defined by $R = \{(a, b) : a \le b^3\}$ is reflective, Symmetric or transitive.

$$R = \{(a,b) : a \le b^3\}$$

Reflexive :- $a \le a^3$

$2 \le (2)^3 \qquad \frac{1}{3} \le \left(\frac{1}{3}\right)^3$

$2 \le 8 \qquad \frac{1}{3} \le \frac{1}{27}$ ✗

$\therefore$ It is not Reflexive.

Symmetric :- $1 \le 3^3 \Rightarrow 1 \le 27$

$3 \le 1^3 \Rightarrow 3 \le 1$ ✗

$\therefore$ It is not Symmetric.

Transitive :- $10 \le 3^3 \Rightarrow 10 \le 27 \quad (a,b)$

$3 \le 2^8 \Rightarrow 3 \le 8 \quad (b,c)$

$10 \le 2^3 \Rightarrow 10 \le 8$ ✗

$\therefore$ It is not Transitive.

6. Show that the Relation R in the set $\{1, 2, 3\}$ given by $R = \{(1,2)(2,1)\}$ is symmetric but neither reflexive nor transitive.

set $A = \{1, 2, 3\}$

Reflexive : $(a, a) \in R$

$(1,1)\ (2,2)\ (3,3) \notin R$

R is not Reflexive.

Symmetric : $(1,2) \in R$ & $(2,1) \in R$

$\therefore R$ is Symmetric.

transitive : $(1,2) \in R$, $(2,1) \in R \Rightarrow (1,1) \notin R$

R is not transitive.

7. Show that the Relation R in the Set A of all the books in library of a college, given by $R = \{(x,y) :$ x and y have Same no. of pages$\}$ is an equivalence Relation.

$R = \{(x,y) : x$ & y have Same no. of pages$\}$

Reflexive : $(a,a) \in R$

$(x,x) \in R$ x & x have Same no of pages.

R is Reflexive.

Symmetric : $(a,b) \in R \Rightarrow (b,a) \in R$

$(x,y) \in R$ x and y have Same no of pages

y and x "

$\therefore R$ is Symmetric.

Transitive :- $(a,b) \in R$, $(b,c) \in R$, $(a,c) \in R$

$(x, y) \in R$, $(y, z) \in R$

x & y have same no. of pages

y and z " " "

then x & z " " ,

$\therefore$ R is transitive.

8. $R = \{ (a,b) : |a-b| \text{ is even} \}$

Reflexive : $(a,a) \in R$

$|a-a| = |0| = 0$

eg: $|2-2| = |0| = 0$

$(a,a) \in R$ $|a-a|$ is even.

$\therefore$ R is reflexive.

Symmetric :- $|a-b| = |b-a|$

If $|a-b|$ is even then $|b-a|$ is also even.

if $(a,b) \in R$, then $(b,a) \in R$

$\therefore$ R is symmetric.

Transitive :- $|a-b|$ is even, then $(a-b)$ is even

$|b-c|$ " " $(b-c)$ " "

$\therefore |a-c|$ is also even

$\therefore$ R is transitive $\therefore$ Equivalence relation.

* In $\{1, 3, 5\}$ → all elements are odd.

So, difference between any 2 odd numbers is always even.

Hence all elements of $\{1, 3, 5\}$ are related to each other.

* In $\{2, 4\}$ → all elements are even

So, difference " " 2 even "

" " " .

Hence all elements of $\{2, 4\}$ are related to each other.

So, difference of odd number and even number is always odd.

If difference not even,

Modulus of diff is also not even.

Hence $\{1, 3, 5\}$ are not related to $\{2, 4\}$

9. $A = \{x \in Z : 0 \leq x \leq 12\}$

(i) $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$

(ii) $R = \{(a, b) : a = b\}$

$A = \{0, 1, 2, 3, \ldots \ldots 12\}$

Reflexive : $(a, a) \in R$

$|a-a| = 0$ is a multiple of 4.
$(4 \times 0 = 0)$

$R = \{0, 4, 8, 12\}$

$\therefore R$ is Reflexive.

Symmetric : $|a-b| =$ multiple of 4

$|b-a| =$ " "

$\therefore R$ is Symmetric

transitive : $|a-b|$ is a multiple of 4

$|b-c|$ is a " "

$a - \cancel{b} + \cancel{b} - c = a-c$ is a multiple of 4

$|a-c|$ is a " "

$(a,c) \in R$ $\therefore R$ is transitive.

So the given relation is Reflexive, Transitive and Symmetric. Hence it is a Equivalence Relation.

$|a-b|$ is a multiple of 4

$|1-1| = 0$ $\therefore$ " "

$|1-2| = |-1| = 1 \rightarrow$ not multiple

$|1-3|$, $|1-4|$ $\nearrow$ $|1-9| = |-8| = 8$

$|1-5| = |-4| = 4$ Multiple of 4

The set of elements related to 1 is $\{1, 5, 9\}$

(iii) $R = \{(a,b) : a = b\}$

Reflexive : $(a,a) \in R$

$a = a$ $\therefore$ So $(a,a) \in R$

R is Reflexive.

Symmetric : If $(a,b) \in R$, $(b,a) \in R$

If $a = b$ $\therefore$ then $b = a$ (It is true)

(eg) $1 = 2$ $2 = 1$

R is Symmetric.

Transitive : $a = b$ & $b = c$, $a = b = c$

$\therefore$ R is transitive. $(a,c) \in R$.

Hence R is reflexive, Symmetric & transitive.

It is a equivalence Relation.

$R = \{(a,b) : a = b\}$

all elements related to 1 $\Rightarrow$ $a = 1$

If $a = 1$ then $b = 1$

$(1,1)$ satisfies the Relation.

Hence the set of elements related to 1 is $\{1\}$

10. Given an example of a Relation which is

(i) Symmetric but neither reflexive nor transitive

Let $A = \{1, 2, 3\}$

$R = \{(2,1)\ (1,2)\}$

Reflexive : $(1,1)\ (2,2)\ (3,3) \notin R$.

R is not Reflexive.

Symmetric : $(1,2) \in R \Rightarrow (2,1) \in R$

$\therefore$ R is Symmetric.

Transitive : $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$

$(2,1) \in R$, no 'c' value

R is not transitive.

(ii) Transitive But neither reflexive nor Symmetric.

$R = \{(a,b) : a < b\}$

Reflexive : $(a, a) \in R$

$a < a$ which not possible.

$\therefore$ R is not reflexive.

Symmetric : $(a,b) \in R$, $(b,a) \in R$.

$(a,b) \in R \Rightarrow a < b$

$(b,a) \in R \Rightarrow b < a$ (Which is not possible)

$\therefore$ R is not Symmetric.

Transitive : $(a,b) \in R \Rightarrow a < b$

$(b,c) \in R \Rightarrow b < c$

$a < b < c \Rightarrow (a < c) \in R$

R is transitive.

Hence Relation R is transitive but not reflexive and Symmetric.

(iii) Reflexive and Symmetric but not transitive

Let $A = \{4, 6, 8\}$

$R = \{(4,4)\,(6,6)\,(8,8), (4,6)\,(6,4)\,(6,8)\,(8,6)\}$

Reflexive : $(a,a) \in R$

$(4,4)\,(6,6)\,(8,8) \in R$

$\therefore$ R is Reflexive.

Symmetric : $(a,b) \in R \Rightarrow (b,a) \in R.$

$(4,6) \in R \Rightarrow (6,4) \in R$

$\therefore$ R is Symmetric.

Transitive : $(a,b) \in R,\ (b,c) \in R \Rightarrow (a,c) \in R$

$(4,6) \in R,\ (6,8) \in R$ no 'c'.

$(4,8) \notin R$

$\therefore$ R is not transitive.

(iv) Reflexive and transitive but not Symmetric.

$$A = \{1, 2, 3\}$$

$$R = \{(1,1)(2,2)(3,3)(1,2)(2,3)(1,3)\}$$

Reflexive : $(a,a) \in R$

$(1,1), (2,2), (3,3) \in R$

$\therefore$ R is reflexive.

Symmetric : $(a,b) \in R \Rightarrow (b,a) \in R$

$(1,2) \in R, (2,1) \notin R$

R is not Symmetric

Transitive : $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$

$(1,2) \in R, (2,3) \in R \Rightarrow (1,3) \in R$

$\therefore$ R is transitive.

(v) Symmetric and transitive but not reflexive.

$$A = \{1, 2, 3\}$$

$$R = \{(1,2)(2,1)(1,3)(3,1)(2,3)(3,2)\}$$

Reflexive : $(a,a) \in R$

$(1,1)(2,2)(3,3) \notin R$

R is not Reflexive.

Symmetric : $(a,b) \in R \Rightarrow (b,a) \in R$

$(1,2) \in R$, $(2,1) \in R$

$(1,3) \in R$, $(3,1) \in R$

$\therefore$ R is Symmetric.

Transitive : $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$

$\underset{a\ b}{(1,2)} \in R$ $\quad$ $\underset{b\ c}{(2,3)} \in R$ $\quad$ $\underset{(a,c)}{(1,3)} \in R$

$(2,1) \in R$ $\quad$ $(1,3) \in R$ $\quad$ $(2,3) \in R$

$\therefore$ R is transitive

Hence R is Symmetric & transitive but not reflexive.

11. $R = \{(P,Q) : OP = OQ\}$

Reflexive :- $(a,a) \in R$

$(P,P) \in R$ $\quad$ $OP = OP$ It is true

$\therefore$ R is reflexive.

Symmetric : $(a,b) \in R \Rightarrow (b,a) \in R$

$(P,Q) \in R$ $\quad$ $OP = OQ$

$(Q,P) \in R$ $\quad$ $OQ = OP$ (It is true)

R is symmetric.

Transitive : $(a,b) \in R \quad (b,c) \in R \Rightarrow (a,c) \in R$

$(P,Q) \in R \qquad OP = OQ$

$(Q,R_1) \in R \qquad OQ = OR_1$

$OP = OQ = OR_1 \Rightarrow OP = OR_1$

$\therefore$ R is transitive.

Since R is Reflexive, Symmetric and Transitive

Hence Equivalence Relation.

Circle is set of all points which are at equal distance from a fixed point.

P

fixed point is the Origin.

$\therefore$ The set of point Related to P form a Circle.

12. $R = \{(T_1, T_2) : T_1 \sim T_2\}$

Reflexive : $(a,a) \in R$

$(T_1, T_1) \in R$

$\therefore$ T_1 is Similar to itself.

Symmetric : $(a,b) \in R \Rightarrow (b,a) \in R$

$(T_1, T_2) \in R \Rightarrow T_1 \sim T_2$

$(T_2, T_1) \in R \Rightarrow T_2 \sim T_1$

$\therefore$ R is symmetric.

Transitive: $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$

$(T_1, T_2) \in R$ $\quad T_1 \sim T_2$

$(T_2, T_3) \in R$ $\quad T_2 \sim T_3$

$T_1 \sim T_2 \sim T_3$

So, R is transitive.

$T_1 \rightarrow 3, 4, 5$

$T_2 \rightarrow 5, 12, 13$ $\quad \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$

$T_3 \rightarrow 6, 8, 10$ $\quad T_1$ and T_3 are Similar

(i.e) T_1 & T_2 are related.

13. $R = \{(P_1, P_2) : P_1$ and P_2 have Same no of Sides$\}$

Reflexive: $(a,a) \in R$

$(P_1, P_1) \in R$.

P_1 and $P_1 \longrightarrow$ Same polygons

P_1 and P_1 will same no of Sides (This is true)

$\therefore$ R is Reflexive.

Symmetric: $(a,b) \in R \Rightarrow (b,a) \in R$

P_1 and $P_2 \rightarrow$ Same no. of Sides.

P_2 and $P_1 \rightarrow$ " "

$(P_1, P_2) \in R \Rightarrow (P_2, P_1) \in R$

$\therefore$ R is Symmetric

Transitive :- $(a,b) \in R,\ (b,c) \in R \Rightarrow (a,c) \in R$

P_1 and $P_2 \rightarrow$ Same no. of Sides.

$(P_1, P_2) \in R$

$(P_2, P_3) \in R \Rightarrow$ Same no. of Sides

$P_2 = P_3$

$P_1 = P_2 = P_3 \Rightarrow P_1 = P_3 \quad (P_1, P_3) \in R$

$\therefore$ R is transitive.

Relation R is a Equivalence Relation.

All triangles are related to the triangles having Sides 3, 4, 5.

14. $R = \{ (L_1, L_2) : L_1 \parallel L_2 \}$

Reflexive :- $(a, a) \in R$

$(L_1, L_1) \in R \quad L_1 \parallel L_2$

R. is Reflexive

Symmetric : $(a,b) \in R \Rightarrow (b,a) \in R$

$(L_1, L_2) \in R \quad L_1 \parallel L_2$

$L_2 \parallel L_1$

$\therefore$ R is Symmetric.

Transitive :- $(a,b) \in R$ $(b,c) \in R \Rightarrow (a,c) \in R$

$(L_1, L_2) \in R$ $L_1 \parallel L_2$ — ①

$(L_2, L_3) \in R$ $L_2 \parallel L_3$ — ②

from ① & ②

$L_1 \parallel L_2 \parallel L_3$

$L_1 \parallel L_3$ $(L_1, L_3) \in R$

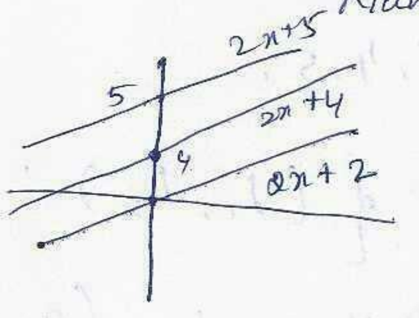
$\therefore$ R is transitive

Set of all lines related to the line $y = 2x + 4$

$y = mx + c$

$y = 2x + 4$

$m = 2$, $c = k$ (R) any real Number.



15. Reflexive :- $(a,a) \in R$

$\{1, 2, 3, 4\} \Rightarrow \{(1,1)\ (2,2)\ (3,3)\ (4,4)\}$

R is Reflexive.

Symmetric : $(a,b) \in R \Rightarrow (b,a) \in R$

$(1,2) \in R$, R is not Symmetric.

$(2,1) \notin R$

16. $R = \{(a,b) : a = b-2 ; \underline{b > 6}\}$. choose Correct answer.

(a) $(2,4) \in R$ (b) $(3,8) \in R$ (c) $(6,8) \in R$

(d) $(8,7) \in R$

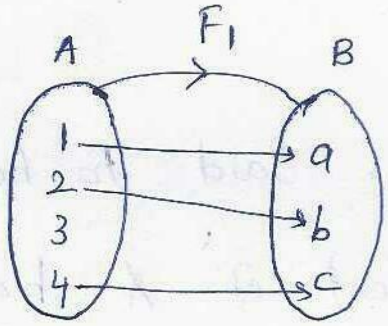
$b = 8-2 = 6$ $a = b-2$

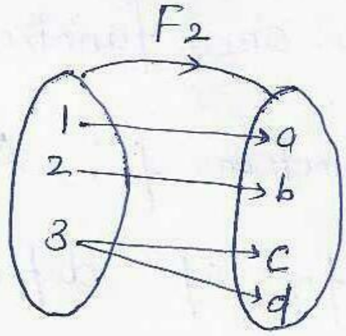
$7-2 = 5$ $= 8-2$

$= 6$.

Ex : 1.2.

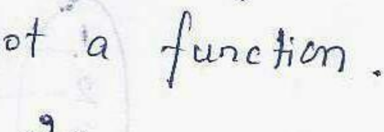
A relation R from A to B is said to be a function if all the elements of A has one and only one image in B.

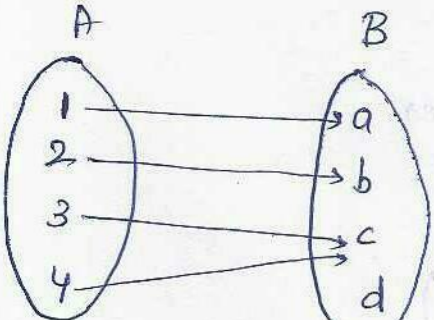




This is not a function since 3 has no image.

3 has 2 images, F_2 is not a function.





F_3 is a function as all the elements of A has one and only one image.

Domain - The Set of all first elements in the Ordered pair of A.

Co-domain - The Set of all elements of the Second set in B.

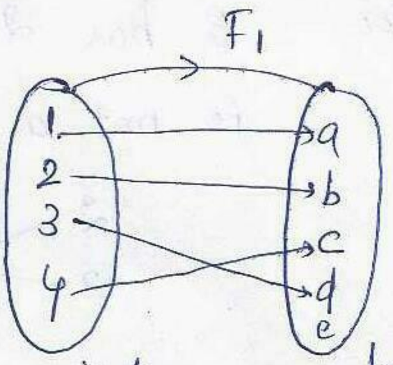
Range - The Set of Second elements in the Ordered pair in B.

(Only mapped) $\rightarrow \{a, b, c\}$

Types of functions :

1. One-to-One function :

A function $f : A \leftrightarrow B$ is Said to be One to One fn. if different element of A has different images.

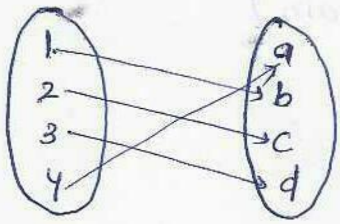


left over iruka Koodathy

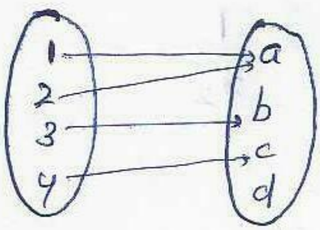
left over irukalam.

2. Onto - function : (Surjective)

A function $f : x \rightarrow y$ is Said to be Onto function if every element of y has Pre-image in X (i.e) for every $y \in Y$ those exist a pre image $x \in X$ such that $f(x) = y$.



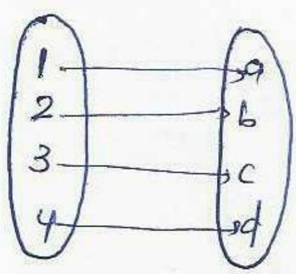
No left over, for every elements of set B, has pre-image in set A.



Since element d has no pre image, it is not onto function.

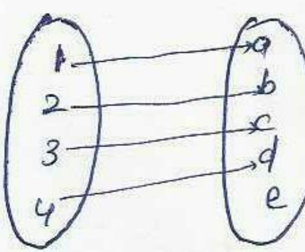
3. One-One and Onto (or bijective):

A function $f : x \to y$ is said to be bijective, if f is both One-One and Onto.



function is One-One and Onto

∴ It is bijective.



Function is One-One but not Onto.

It is not bijective.

1. One-One $R_* = R - \{0\}$

$R = \{0, \pm 1, \pm 2, \pm 3 \ldots \frac{1}{2}, \frac{1}{3}$

$f(x) = \frac{1}{x}$

$x_1, x_2 \in A$ (domain)

$f(x_1) = f(x_2)$

$x_1 = x_2$

$x_1, x_2 \in R_* \therefore f(x_1) = f(x_2)$

$\frac{1}{x_1} = \frac{1}{x_2}$

$x_2 = x_1$

$x_1 = x_2 \therefore$ It is one-one fn.

$f : A \to B$

Onto fn :-

$f(x) = y$

$y = 1/x$

$x = 1/y$

$f(x) = y$

$f(1/y) = \frac{1}{1/y} = y$

It is Onto fn.

$f : N \longrightarrow R_* \hookrightarrow R - \{0\}$

$\downarrow$

Natural no.

One-One

$x_1, x_2 \in N$

$f(x_1) = f(x_2)$

$\frac{1}{x_1} = \frac{1}{x_2}$

$x_1 = x_2$ $\therefore$ It is one-one

On to function

$f(x) = \frac{1}{x}$

$2 \in R_x$, 2 is not the Reciprocal of any natural number.

$f(x) = 2$

$\frac{1}{x} = 2$

$\frac{1}{2} = x \notin N$

$\therefore$ It is not an Onto fn.

2. Check injectivity (One-One) and Surjectivity (Onto)

(i) $f : N \to N$ given by $f(x) = x^2$

domain co-domain

One-One

$x_1, x_2 \in N$ (Domain) $f(x) = x^2$

$f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$$x_1^2 - x_2^2 = 0$$

$$(x_1 - x_2)(x_1 + x_2) = 0$$

$$x_1 = x_2 \qquad x_1 = -x_2 \ \times$$

$\therefore$ It is One to One fn.

Onto function;

$$f(x) = x^2$$

$$x = 1, 2, 3 \ldots \in N \ (\text{domain})$$

$$\text{Range} = \{ f(x) = x^2 ; x \in N \}$$

$$= \{ 1^2, 2^2, 3^2, 4^2 \ldots \}$$

$$\text{Range} = \{ 1, 4, 9, 16 \ldots \}$$

Co-domain $\Rightarrow$ Set of all Natural nos.

Co-domain $\neq$ Range

$\therefore$ It is not onto function.

It is One to One but Not Onto function.

(ii) $f : Z \to Z$ given by $f(x) = x^2$

$\downarrow$ domain $\quad \downarrow$ co-domain.

One-one, $\quad x_1, x_2 \in Z \qquad f(x) = x^2$

$$f(x_1) = f(x_2)$$

$$x_1^2 = x_2^2$$

$x_1^2 - x_2^2$ $\qquad$ $x_1^2 = x_2^2$

$(x_1 + x_2)(x_1 - x_2) = 0$ $\qquad$ $x_1 = \pm\sqrt{x_2^2}$

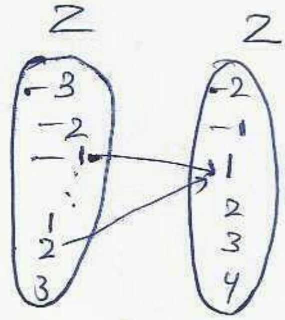
$x_1 = x_2$, $x_1 = -x_2$ $\qquad$ $x_1 = \pm x_2$

$x_1 = \pm x_2$

Eg: $f(x) = x^2$

$x = -1$ $\quad f(-1) = (-1)^2 = 1$

$x = 1$ $\quad f(1) = 1^2 = 1$



The image 1 has 2.
pre-image 1 and -1. $f(-1) \neq f(1)$

$\therefore$ It is not one-one.

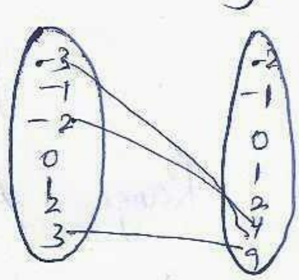
Onto function :- $\qquad f(x) = x^2$

$x = 0, \pm 1, \pm 2, \pm 3 \ldots \in Z$ (domain)

Range $\{ f(x) = x^2, x \in z \} = \{ 0, 1, 4, 9 \ldots \}$

Co-domain $= \{ 0, \pm 1, \pm 2, \pm 3 \ldots \}$

$\therefore$ 2, -2 etc belong to Co-domain
Z but don't belong to Range.



$\therefore$ f is not Surjective $\qquad (-2)^2 = 4$

$\therefore$ f is neither injective nor Surjective.

(iii) Given $f: R \to R$ given by $f(x) = x^2$

$$\underset{\text{domain}}{R} \to \underset{\text{Co-domain}}{R}$$

One-One :- $x_1, x_2 \in R$

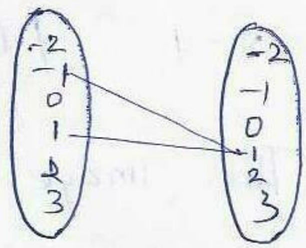
$f(x_1) = f(x_2)$ $\qquad f(x) = x^2$

$x_1^2 = x_2^2$

$x_1 = \pm\sqrt{x_2^2}$

$x_1 = \pm x_2$

$x_1 = x_2, \quad x_1 = -x_2$



$x = -1 \qquad f(-1) = (-1)^2 = 1$

$f(1) = 1^2 = 1$

$f(-1) \neq f(1)$

hence it not one-one fn

Onto function :- $f(x) = x^2$

Range $f(x) = x^2$, $x \in R$ is the set of all +ve real no's including 0

$x = \sqrt{2}$

$f(x) = (\sqrt{2})^2 = 2$

Range $\neq$ co domain.

Hence it is not onto function.

$\therefore$ fn is neither One-One (nor) Onto function.

(iv) Given :- $f : N \to N$ given by $f(x) = x^3$

domain Co-domain

One-One fn : $x_1, x_2 \in N$

$$f(x_1) = f(x_2)$$

$$x_1^3 = x_2^3$$

$$x_1^3 - x_2^3 = (x_1 - x_2)\underbrace{(x_1^2 + x_1x_2 + x_2^2)}_{\text{not possible}}$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$\therefore$ It is one-one function.

Onto function :-

$$f(x) = x^3$$

$x = 1, 2, 3, 4, \ldots \in N$ (domain)

Range set $\{f(x) = x^3, x \in N\} = \{1^3, 2^3, 3^3, 4^3 \ldots\}$

$= \{1, 8, 27, 64 \ldots\}$

2, 3, 4, 5, 6, 7 ..... it doesn't have pre image.

$\therefore$ fn is not onto.

$\therefore$ The fn is one-one but not onto.

(v) $f: Z \to Z$ given by $f(x) = x^3$

$$x_1, x_2 \in Z$$

$$f(x_1) = f(x_2)$$

$$x_1^3 = x_2^3$$

$$x_1 = x_2$$

$\therefore$ It is one-one function.

Onto function :

$x = 0, 1, -1, 2, -2 \ldots \in Z$ (domain)

Range $= \{ f(x) = x^3, x \in Z \}$

$= \{0, 1, -1, 8, -8 \ldots\} \neq$ codomain

$x = 1, \quad f(1) = 1^3$

$x = -1, \quad f(-1) = -1$

$(2, -2, \pm 3 \ldots)$ belongs to Co-domain but doesnt belong to Range.

Codomain $\neq$ Range

$\therefore$ It is not onto.

3. $[x] \to$ greatest integer less than equal to x

$$[1.2] \to 1$$

$$[1.9] = 1$$

$$[2] \to 2$$

$$f \to \underset{\text{domain}}{R} \longrightarrow \underset{\text{co-domain}}{R}$$

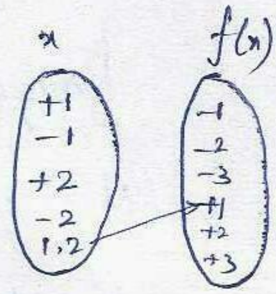
One-One :- $f(x) = [x]$

$$f(1.2) = [1.2] = 1$$

$$f(1.9) = [1.9] = 1$$

$$f(2.1) = [2.1] = 2$$

$$f(2.5) = [2.5] = 2$$



Different elements 1.2, 1.9, 2.1, 2.5 have the same image 1 and 2

$\therefore$ f is not one-one function.

Onto :- $f(x) = [x]$

Range Set $\to$ Set of integers.

Range $\neq$ Co.domain

It is not Onto function

4. $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

One-One function :- $|-1| = 1$ $\quad |-2| = 2$

$|1| = 1$ $\quad |2| = 2$

different elements $-1, 1$ have Same image as 1, So it is not one-one.

Onto :- Co-domain $\rightarrow$ Real no's

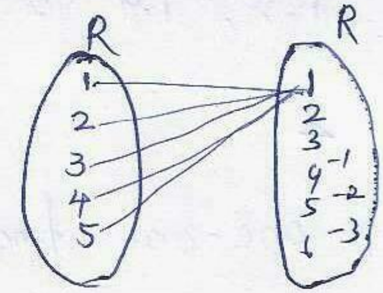
Range $\rightarrow$ Real no's of +ve values.

Range $\neq$ Co-domain.

$\therefore$ It is not Onto function.

5. $f(x) = \begin{cases} 1 & ; \text{ if } x > 0 \\ 0 & ; \text{ if } x = 0 \\ -1 & ; \text{ if } x < 0 \end{cases}$

One-One



diff elements $1, 2, 3, 4 \cdots$ have the Same image -1

$\therefore$ It is not One-One.

Onto :- Co-domain $\rightarrow$ Set of Real no's

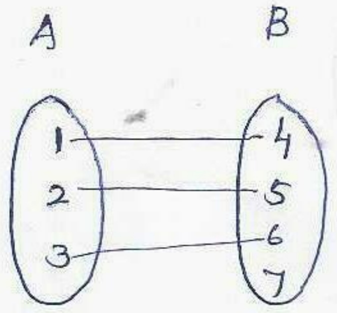
Range $\rightarrow \{1, 0, -1\}$

Co-domain $\neq$ Range

$\therefore$ It is not onto function.

6. $A = \{1, 2, 3\}$. $B = \{4, 5, 6, 7\}$

$$f = \{(1,4), (2,5), (3,6)\}$$



Since every element of A has a Unique image, hence f is one-one.

7. (i) $f : R \rightarrow R$ defined by $f(x) = 3 - 4x$

$$f(x) = 3 - 4x$$

One-one, $x_1, x_2 \in R$ (domain)

$$f(x_1) = f(x_2)$$

$$\cancel{3} - 4x_1 = \cancel{3} - 4x_2$$

$$\cancel{-4}x_1 = \cancel{-4}x_2$$

$$x_1 = x_2$$

$\therefore$ It is one-one function.

Onto function : $f(x) = 3 - 4x$

$$f(x) = y, \quad y \in R$$

$$3 - 4x = y$$

$$-4x = y - 3$$

$$x = y - 3/-4$$

$$f(x) = f\left(-\frac{(y-3)}{4}\right)$$

$$= 3 - 4x$$

$$= 3 - \cancel{4}\left(\frac{y-3}{-\cancel{4}}\right)$$

$$= \cancel{3} + y - \cancel{3}$$

$$f(x) = y$$

Thus for every $y \in R$, there exist $x \in R$

$$f(x) = y$$

$\therefore$ function is onto.

(ii) $f: R \longrightarrow R$ defined by $f(x) = 1 + x^2$

$$f(x) = 1 + x^2$$

One-one,

$$f(x_1) = f(x_2)$$

$$\cancel{1} + x_1^2 = \cancel{1} + x_2^2$$

$$x_1^2 = x_2^2$$

$$x_1^2 - x_2^2 = 0$$

$$(x_1 + x_2)(x_1 - x_2) = 0$$

$$x_1 = -x_2, \quad x_1 = x_2$$

$$f(-1) = 1 + (-1)^2 = 2$$

$$f(1) = 1 + 1^2 = 2$$

The pre image -1 and $+1$ have same image 2.

Hence it is not One-one fn.

Onto : $f(x) = 1 + x^2$

$$f(x) = y, \quad y \in R$$

$$1 + x^2 = y$$

$$x^2 = y - 1$$

$$x = \pm\sqrt{y-1}$$

$\therefore$ y is real no, it can be Negative also.

$$y = -2$$

$$x = \pm\sqrt{-2-1} = \pm\sqrt{-3} \text{ which is not}$$

real no. x is not real no.

Co-domain $\neq$ Real No.

Range = +ve value of Real no.

$\therefore$ f is not onto-function.

8. $f : A \times B \rightarrow B \times A \qquad f(a,b) = (b,a)$

one-one, $f(x_1) = f(x_2)$

$$x_1 = (a,b), \quad x_2 = (c,d)$$

$$f(x_1) = f(x_2)$$

$$f(a,b) = f(c,d)$$

$$(b,a) = (d,c)$$

$$b = d \quad a = c$$

$$x_1 = a, \ b = c, \ d = x_2$$

$$x_1 = x_2$$

$\therefore$ It is one-one function

Onto :-

$$f : A \times B \rightarrow B \times A$$

$$f(a,b) = (b-a)$$

$$f(x) = (b,a)$$

$$y = (b,a)$$

$(b,a) \in B \times A$ (co-domain)

$b \in B, \ a \in A$

for every $(b,a) \in B \times A$

there exist $(a,b) \in A \times B$

Such that $f(x) = y$

It is possible for all $a \in A$ and $b \in B$

$\therefore$ f is onto function.

9. $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ n/2, & \text{if } n \text{ is even} \end{cases}$ for all $n \in N$

One-one.

$$f(1) = \frac{1+1}{2} = \frac{2}{2} = 1$$

$$f(2) = \frac{2}{2} = 1$$

$$f(1) = f(2) \quad \text{but} \quad 1 \neq 2$$

Both $f(1)$ and $f(2)$ have same image 1.

$\therefore f$ is not one-one fn.

Onto :- $f(x) = y \qquad y \in N$

$$y = \frac{n+1}{2} \qquad y = n/2$$

$$2y = n+1 \qquad 2y = n$$

$$2y - 1 = n \qquad y =$$

for all natural no of y x is odd. (n) — for all natural no of y, n is even

Range = Co-domain.

$\therefore f$ is Onto function.

10. One-One fn (i) : $f(x) = \dfrac{x-2}{x-3}$

$$f(x_1) = f(x_2)$$

$$x_1 = x_2$$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$(x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$$

$x_1 = x_2$ It is one-one fn.

Onto function :-

$$f(x) = \frac{x-2}{x-3} \qquad f(x) = y$$

$$y = \frac{(x-2)}{x-3}$$

$$y(x-3) = x-2$$

$$xy - 3y = x - 2$$

$$xy - x = -2 + 3y$$

$$x(y-1) = 3y - 2$$

$$x = \frac{3y-2}{y-1} \qquad y \in R - \{1\}$$

$$x \in R - \{3\}$$

$$y = f(x)$$

$$f(x) = f\left(\frac{3y-2}{y-1}\right)$$

$$f(x) = \frac{x-2}{x-3}$$

$$\frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = \frac{\frac{3y-2-2y+1}{y-1}}{\frac{3y-2-3y+1}{y-1}}$$

$$= \frac{3y-2y}{1} = \frac{y}{1}$$

$$f(x) = y$$

Thus for every $y \in B$, there exist $x \in A$ such that $f(x) = y$.

Hence it is onto function.

11. $f(x) = x^4$

$f(-1) = (-1)^4 = 1$

$f(1) = 1^4 = 1$ $\therefore$ It is not one-one fn.

$f(-2) = 16$ The image 1 has 2 - pre images.

$f(2) = 16$ -1 & 1.

Onto :- $f(x) = y$

$y = x^4$

$x = \sqrt[4]{y} = y^{1/4}$

$\therefore$ It is not on-to function.

12. One - One.

$$f(x_1) = f(x_2)$$

$$f(x) = 3x$$

$$\cancel{3}x_1 = \cancel{3}x_2$$

$$x_1 = x_2$$

$\therefore$ This is one - one fn.

Onto function,

$$f(x) = y$$

$$3x = y$$

$$x = y/3$$

$$f(x) = f\left(y/3\right) = 3x$$

$$= \cancel{3}\left(y/\cancel{3}\right)$$

$$= y$$

$$f(x) = y$$

This is Onto function..

Miscellaneous.

1. Show that the function $f : R \rightarrow \{x \in R : -K x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$ $x \in R$ in One to One and Onto function.

$$f : R \rightarrow \{x \in R \;\; -1 < x < 1\}$$

$$f(x) = \frac{x}{1+|x|}$$

$$|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{x}{1+x}, & x > 0 \\ \dfrac{x}{1-x}, & x < 0 \end{cases}$$

One-one function,

for $x \geq 0$

$$f(x_1) = f(x_2)$$

$$\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$$

$$x_1(1+x_2) = x_2(1+x_1)$$

$$x_1 + x_2 \cdot x_1 = x_2 + x_2 \cdot x_1$$

$$x_1 = x_2$$

Case - II, for $x < 0$

$$f(x_1) = f(x_2)$$

$$\frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$$

$$x_1(1-x_2) = x_2(1-x_1)$$

$$x_1 - x_1 \cdot x_2 = x_2 - x_2 \cdot x_1$$

$$x_1 = x_2$$

Case - III $\quad x_1 \geq 0, \quad x_2 < 0$

if $f(x_1) = f(x_2)$

$$\frac{x_1}{1+x_1} = \frac{x_2}{1-x_2}$$

$$x_1(1-x_2) = x_2(1+x_1)$$

$$x_1 - x_1 \cdot x_2 = x_2 + x_1 x_2$$

$$x_1 - x_2 = x_1 x_2 + x_1 x_2$$

$$x_1 - x_2 = 2 x_1 \cdot x_2$$

$$(+ve) - (-ve) = 2(+ve)(-ve)$$

$$+ve = (-ve)(2)$$

It is not possible.

It is one to one function.

$f(x_1) \neq f(x_2)$, when x_1 and x_2 are not equal

Onto function :

for $x \geq 0$

$$f(x) = y$$

$$f(x) = \frac{x}{1+x}$$

$$y = \frac{x}{1+x}$$

$$y(1+x) = x$$

$$y + xy = x$$

$$y = x - xy$$

$$y = x(1-y)$$

$$\frac{y}{1-y} = x$$

Case II for $x < 0$

$$f(x) = \frac{x}{1-x}$$

$$y(1-x) = x$$

$$y = x(1+y)$$

$$\frac{y}{1+y} = x$$

$y \in \{ x \in R : -1 x < 1 \}$

i.e. Value of y is from -1 to 1.

$$-1 < y < 1$$

If $y = 1$

$x = \frac{y}{1-y}$ will not be defined.

If $y = 1$

$x = \frac{y}{1+y}$ will not be defined

So, x is defined for all values of y and $x \in R$.

$\therefore$ f is onto function.

hence f is one-one and onto.

2. Show that the function $f: R \to R$ given by $f(x) = x^3$ is injective.

$$f \to R \to R$$

$f(x) = x^3$

Injective :- Let x_1 & $x_2 \in R$

$$f(x_1) = f(x_2)$$

$$x_1^3 = x_2^3$$

$$x_1 = x_2$$

Since if $f(x_1) = f(x_2)$, then $x_1 = x_2$

$\therefore$ It is One-one fnc.

3. Let $X = \{1, 2, 3\}$

$P(X)$ = Set of all Subsets of X

$\{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}\ \{2,3\}, \{1,3\}\ \{1,2,3\}\}$

$A\ R\ B$ means $A \subset B$

Relation is

$R = \{(A, B) : A$ & B are sets, $A \subset B\}$

Reflexive : every set is a subset of itself

$A \subset A \therefore (A, A) \in R$

$\therefore$ R is Reflexive.

Symmetric : If $(A, B) \in R$, then $(B, A) \in R$

If $(A, B) \in R$

$\Rightarrow A \subset B$ But $B \subset A$ is not true

$A = \{1\}, \quad B = \{1, 2\}$

$A \subset B$, $B \subset A$ [2 is not in A]

↳ It isnot possible.

$\therefore$ R is not Symmetric.

transitive :

Since $(A, B) \in R$ and $(B, C) \in R$

$\Rightarrow$ If $(A \subset B)$ and $B \subset C$

then $A \subset C$

$(A, C) \in R$

eg: $A = \{1, 2\}$ $B = \{1, 2, 3\}$ $C = \{1, 2, 3, 4\}$

$A \subset B$, $B \subset C$

$\Rightarrow A \subset C$ (all elements of A are in C)

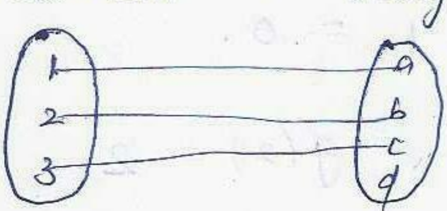
$\Rightarrow A R C$

$\therefore$ R is transitive.

But R is not an equivalence Relation as R is not Symmetric.

4. Find the no. of all onto functions from the Set $\{1, 2, 3 \dots n\}$ to itself.

Co-domain = Range (Onto)



Domain = $\{1, 2, 3\}$

Co-domain = $\{a, b, c, d\}$

Range = $\{a, b, c\}$

according to Question,

1 has 3 choices $\rightarrow$ n choice

2 " 2 choices $\rightarrow$ $(n-1)$ choice

3 " 1 choices $\rightarrow$ $(n-2)$ choice

Total No. of all onto functions

$= (n)(n-1)(n-2) \ldots\ldots 1 = n!$

5. Function f and g are equal if $f(a) = g(a)$ for all $a \in A$.

$f(x) = x^2 - x, \; x \in A$

$f(-1) = (-1)^2 - (-1) = 2$

$f(0) = 0$

$f(1) = 0$

$f(2) = 2$

$g(x) = 2\left|x - \frac{1}{2}\right| - 1, \; x \in A$

$g(-1) = 2\left|-1 - \frac{1}{2}\right| - 1$

$= 2\left|\frac{-2-1}{2}\right| - 1$

$= 2\left|-\frac{3}{2}\right| - 1$

$2 \times 3/2 - 1 = 2.$

$g(0) = 2\left|0 - 1/2\right| - 1$

$= 2\left|-1/2\right| - 1 = 0.$

$g(1) = 0$

$g(2) = 2$

$f(-1) = g(-1) = 2$

$f(0) = g(0) = 0$

$f(1) = g(1) = 0$

$f(2) = g(2) = 2$

$\therefore$ f an g are equal.

6. $A \times A = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)$
$(3,1)\ (3,2)\ (3,3)\}$

$R_1 = \{(1,1)(2,2)\ (3,3)\ (1,2)\ (2,1)\ (1,3)\ (3,1\}$

$(2,1)\ (1,3)$ $\qquad$ $(3,1)\ \ (1,2)$

$a, b\ \ b, c$ $\qquad$ $(3,2)$

$(a, c) \Rightarrow (2,3)$

$R_2 = \{(1,1)(2,2)\ (3,3)\ (1,2)\ (1,3)\ (2,3)$
$(2,1)\ \ (3,1)\ \ (3,2)\}$

Ans : $\underline{1}$.

7. Let $A = \{1,2,3\}$. Then Number of equivalence relation containing $(1,2)$ is,

(a) 1 $\qquad$ (b) 2 $\qquad$ (c) 3 $\qquad$ (d) 4

R, S, T

$A \times A = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (1,3)\ (2,1)\ (2,3)$
$(3,1)\ \ (3,2)\}$

$R_1 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,1)\}$

$R_2 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,1)$

$(1,3)\ (3,1)\ (2,3)\ (3,2)\}$

$(3,3)\ (3,2)$ $(1,2)\ (2,3)$ $(2,1)\ (1,3)$

$(3,2)$ $(1,3)$ $(2,3)$

Ans : 2.