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Digital Assignment - I

Q1) $p_1 = t^3 + 2t^2 + 2t - 1$
$p_2 = 2t^3 + t^2 + 3t + 4$
$p_3 = t^3 + 2t^2 + t - 7$

So $W = \text{span}(P_1, P_2, P_3)$

We will first check whether P_1, P_2, P_3 are linearly independent or not.

$\Rightarrow a p_1 + b p_2 + c p_3 = 0$
for some $a, b, c \in R$

$\Rightarrow a(t^3 + 2t^2 + 2t - 1) + b(2t^3 + t^2 + 3t + 4) + c(t^3 + 2t^2 + t - 7) = 0$
$\Rightarrow t^3(a + 2b + c) + t^2(2a + b + 2c) + t(2a + 3b + c) + (-a + 4b - 7c) = 0$

$$\Rightarrow \begin{aligned} a + 2b + c &= 0 \\ 2a + b + 2c &= 0 \\ 2a + 3b + c &= 0 \\ -a + 4b - 7c &= 0 \end{aligned}$$

In matrix form $Ax = 0$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 3 & 1 \\ -1 & 4 & -7 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Reducing Matrix A into rref form:-

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$\Rightarrow R_3 \leftarrow R_2 - R_1$; $R_3 \leftarrow R_3 - 2R_1$

$$A \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & -1 & -1 \\ -1 & 4 & -7 \end{bmatrix}$$

$\Rightarrow R_4 \leftarrow R_4 + R_1$; $R_2 \leftarrow R_2/3$

$$A \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 6 & 6 \end{bmatrix}$$

$\Rightarrow R_1 \leftarrow R_1 - 2R_2$; $R_3 \leftarrow R_3 + R_2$; $R_4 \leftarrow R_4 - 6R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -6 \end{bmatrix}$$

$\Rightarrow R_3 \leftarrow (-R_3)$

$$A \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -6 \end{bmatrix}$$

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$\Rightarrow R_1 \leftarrow R_1 - R_3 \; ; \; R_4 \leftarrow R_4 + 6R_3$

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence now our equation becomes:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow a = b = c = 0$

$\Rightarrow p_1, p_2, p_3$ are linearly independent & we know p_1, p_2 & p_3 spans W. So p_1, p_2 & p_3 are basis for W. Dimension of $W = 3$

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Q2] $V = \begin{cases} V_1 = (0,1,3,-2) \\ V_2 = (5,1,-2,1) \\ V_3 = (10,3,-1,0) \end{cases}$ $W = \begin{cases} W_1 = (1,-1,2,1) \\ W_2 = (0,2,-2,-2) \\ W_3 = (1,1,0,-1) \end{cases}$

$\dim(V) = 3, \quad \dim(W) = 3$

$V + W = \text{span}(V_1, V_2, V_3, W_1, W_2, W_3)$

Hence maximal set of ~~~~ above will be its basis.

$Q = [v_1 \; v_2 \; v_3 \; w_1 \; w_2 \; w_3]$

$$Q = \begin{bmatrix} 0 & 5 & 10 & 1 & 0 & 1 \\ 1 & 1 & 3 & -1 & 2 & 1 \\ 3 & -2 & -1 & 2 & -2 & 0 \\ -2 & 1 & 0 & 1 & -2 & -1 \end{bmatrix}$$

Finding rref of Q:

$\Rightarrow R_1 \leftrightarrow R_2$

$$Q \sim \begin{bmatrix} 1 & 1 & 3 & -1 & 2 & 1 \\ 0 & 5 & 10 & 1 & 0 & 1 \\ 3 & -2 & -1 & 2 & -2 & 0 \\ -2 & 1 & 0 & 1 & -2 & -1 \end{bmatrix}$$

$\Rightarrow R_3 \leftarrow R_3 - 3R_1 \; ; \; R_4 \leftarrow R_4 + 2R_1$

$$Q \sim \begin{bmatrix} 1 & 1 & 3 & -1 & 2 & 1 \\ 0 & 5 & 10 & 1 & 0 & 1 \\ 0 & -5 & -10 & 5 & -8 & -3 \\ 0 & 3 & 6 & -1 & 2 & 1 \end{bmatrix}$$

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$\Rightarrow R_2 \leftarrow R_2/5$

$$Q \sim \begin{bmatrix} 1 & 1 & 3 & -1 & 2 & 1 \\ 0 & 1 & 2 & 1/5 & 0 & 1/5 \\ 0 & -5 & -10 & 5 & -8 & -3 \\ 0 & 3 & 6 & -1 & 2 & 1 \end{bmatrix}$$

$\Rightarrow R_3 \leftarrow R_3 + 5R_2 \; ; \; R_4 \leftarrow R_4 - 3R_2$

$$Q \sim \begin{bmatrix} 1 & 1 & 3 & -1 & 2 & 1 \\ 0 & 1 & 2 & 1/5 & 0 & 1/5 \\ 0 & 0 & 0 & 6 & -8 & -2 \\ 0 & 0 & 0 & -8/5 & 2 & 2/5 \end{bmatrix}$$

$\Rightarrow R_3 \leftarrow R_3/6$

$$Q \sim \begin{bmatrix} 1 & 1 & 3 & -1 & 2 & 1 \\ 0 & 1 & 2 & 1/5 & 0 & 1/5 \\ 0 & 0 & 0 & 1 & -4/3 & -1/3 \\ 0 & 0 & 0 & -8/5 & 2 & 2/5 \end{bmatrix}$$

$\Rightarrow R_4 \leftarrow R_4 + (8/5) R_3$

$$Q \sim \begin{bmatrix} 1 & 1 & 3 & -1 & 2 & 1 \\ 0 & 1 & 2 & 1/5 & 0 & 1/5 \\ 0 & 0 & 0 & 1 & -4/3 & -1/3 \\ 0 & 0 & 0 & 0 & -2/15 & -2/15 \end{bmatrix}$$

$\Rightarrow R_4 \leftarrow -15/2 \, (R_4)$

$$Q \sim \begin{bmatrix} 1 & 1 & 3 & -1 & 2 & 1 \\ 0 & 1 & 2 & 1/5 & 0 & 1/5 \\ 0 & 0 & 0 & 1 & -4/3 & -1/3 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

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$\Rightarrow R_3 \leftarrow R_3 + 4/3(R_4)$; $R_1 \leftarrow R_1 - 2R_4$

$$Q \sim \begin{bmatrix} 1 & 1 & 3 & -1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$\Rightarrow R_2 \leftarrow R_2 - (1/5)R_3$; $R_1 \leftarrow R_1 + R_3$

$$Q \sim \begin{bmatrix} 1 & 1 & 3 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$\Rightarrow R_1 \leftarrow R_1 - R_2$

$$Q \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} = U$$

The column's which contain leading 1's will form the basis for the column space of Q. ~~Thus $\{v_1, v_2, v_3, w_1\}$~~
Thus $\{v_1, v_2, w_1, w_2\}$ is the maximal linearly independent set.

Hence

Basis of ~~[illegible]~~ $(V+W) = \left\{ \begin{pmatrix} 0 \\ 1 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -2 \\ 2 \end{pmatrix} \right\}$

$\therefore \underline{\mathrm{Dim}(V+W) = 4}$

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For finding basis of $(V \cap W)$, we need to find the null space of Q.

Hence equating

$$QX = 0 \Rightarrow UX = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $x_1 + x_3 = 0$; $x_2 + 2x_3 = 0$; $x_4 + x_6 = 0$; $x_5 + x_6 =$

As x_3 & x_6 are free variables, let $x_3 = t$ & $x_6 = s$

Hence ~~$x_1 = -x_3$; $x_2 =$~~

$x_1 = -t$; $x_2 = -2t$, $x_3 = t$,

$x_4 = -s$; $x_5 = -s$; $x_6 = s$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = s \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \\ 6 \end{bmatrix} = s\, n_s + t\, n_t$$

Hence null space of Q = span(n_s, n_t), which are linearly independent.

Hence $\text{Dim}(V \cap W) = 2$

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~~Ans~~ Now we know,

$Dim(V \cup W) = 4$,

$Dim(V) = 3$, $Dim(W) = 3$

$Dim(V \cap W) = 2$

& $Dim(V) + Dim(W) - Dim(V \cap W) = 4$

$= Dim(V \cup W)$

Hence Proved

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Q3] Given $T: P_1 \to P_2$ & $T(p(x)) = \int_0^x p(x)\,dx$

Basis of P_2 $(\alpha) = \{1+x^2, 2+x, x+x^2\}$
Basis of P_1 $(\beta) = \{1+x, 1\}$

To find : $[T]_\beta^\alpha$

Finding $[T]_\beta$

$T(\beta_1) = \int_0^x (1+x)\,dx = x^2/2 + x$

$T(\beta_2) = \int_0^x 1\,dx = x$

To find $[T]_\beta^\alpha$, we need find each $T(\beta)$ wrt to α.

Hence $T(\beta_1) = a\alpha_1 + b\alpha_2 + c\alpha_2$

$\Rightarrow x^2/2 + x = a(x^2+1) + b(x+2) + c(x^2+x)$

$\Rightarrow a+c = 1/2$
$b+c = 1$
$a+2b = 0$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}$$

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Augmenting the matrix:

$$\left[\begin{array}{cccc} 1 & 0 & 1 & 1/2 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 \end{array}\right]$$

rref form: $\left[\begin{array}{cccc} 1 & 0 & 0 & -1/3 \\ 0 & 1 & 0 & 1/6 \\ 0 & 0 & 1 & 5/6 \end{array}\right]$

Hence $a = -1/3$, $b = 1/6$, $c = 5/6$

$\|^{ly}$ for $T(\beta_2) = a\alpha_1 + b\alpha_2 + c\alpha_2$

$\Rightarrow x = a(x^2+1) + b(x+2) + c(x^2+x)$

$\Rightarrow$ $a + c = 0$

$b + c = 1$

$a + 2b = 0$

$$\Rightarrow \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 \end{array}\right]$$

rref form: $\left[\begin{array}{cccc} 1 & 0 & 0 & -2/3 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & 2/3 \end{array}\right]$

Hence $a = -2/3$, $b = 1/3$, $c = 2/3$

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Hence $[T]_B^{\alpha} = \begin{bmatrix} -1/2 & -2/3 \\ 1/6 & 1/3 \\ 5/6 & 2/3 \end{bmatrix}$