Assignment 3

Ran Liu RLIU361@GATECH.EDU
903515184

Problem 1

question 1

Indices: $i = 1, 2, ..., n$ for different machines. $j = 1, 2, ..., m$ for different cities.

Data: b_j is the maximum kilograms of machinery that city j can store up. w_i is the weighs of machine i in unit kilograms. $u_{i,j}$ is the utility gained from city j with machine i.

Variables: $x_{i,j}$ is the number of machine i that city j can obtain, it is an integer with zero and one as its possible values.

Objective: maximize $\sum_{j=1}^{m} \sum_{i=1}^{n} x_{i,j} u_{i,j}$

Constraints: $\sum_{j=1}^{m} x_{i,j} \leq 1$ for every i. $\sum_{i=1}^{n} x_{i,j} w_i \leq b_j$ for every j.

Bounds: $x_{i,j} = 0$ or $x_{i,j} = 1$.

question 2

Result as shown in Figure 1.

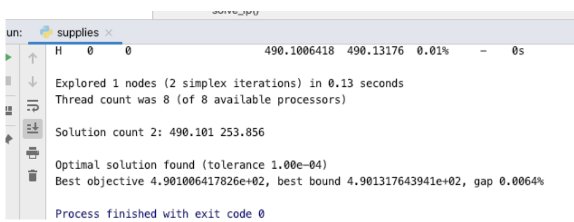

Figure 1

Problem 2

question 1

Indices: $i = 1, 2, ..., n$ are the dimensions of vector in R^n space. $j = 1, 2, ..., m$ are the numbers/ids of constraints.

Data: $A_{j,i}$ is the constraint space, where $A_{j,} = a_j$ and b_j together form the jth constraint. c_i is the cost vector in space R^n. λ is another constraint number.

Variables: $x_i \in R^n$ is the vector. d_j is an integer indicating whether if the jth constraint is satisfied.

Objective: minimize$\sum_{i=1}^{n} c_i x_i$

Constraints: $\sum_{i=1}^{n} A_{j,i} x_i \geq \lambda$ for every j. $\sum_{i=1}^{n} A_{j,i} x_i + (b_j - \lambda) d_j \geq b_j$ for every j. $\sum_{j=1}^{m} d_j \leq m - k$.

Nonnegativity: $d_j = 0$ or $d_j = 1$.

question 2

Result as shown in Figure 2.

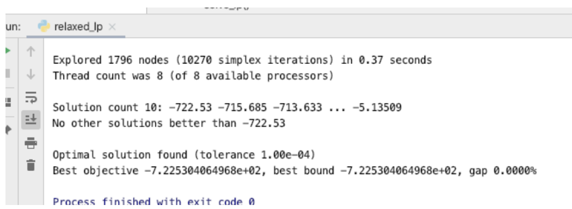

Figure 2

Problem 3

question 1

Indices: $t = 1, 2, ..., T$ for different time periods.

Data: c_t is the fixed setup cost if we decide to perform a production run during time period t. p_t is the production cost for each unit if we decide to perform a production run during time period t. s_t is the storage cost for each unit if we have product on hand at the end of time period t. b_t is the penalty of unfulfilled demand for each unit if we deliver less than the demand in time period t. D_t is the demand in time period t.

Variables: x_t is how many units of product to produce in time period t. e_t is how many units of product to store in time period t. f_t is a binary indicator of whether if the production is performed in time period t. g_t is how many units of product unfulfilled in time period t. M is a large number.

Objective: minimize $\sum_{t=1}^{T}(f_t c_t + x_t p_t + e_t s_t + g_t b_t)$

Constraints: $x_t \leq f_t M$ for every t. Explanation: if there is production, $x_t \geq 0$, thus $f_t = 1$. Otherwise, if there is no production, $x_t = 0$, thus $f_t \leq 1$, through optimization $f_t = 0$ eventually.

$e_0 = x_0 + g_0 - D_0$ and $e_{t+1} = e_t + x_{t+1} + g_{t+1} - D_{t+1}$ for $t \leq T - 1$.

$g_0 \geq D_0 - x_t$, $g_t \geq D_t - e_{t-1} - x_t$ for every t, and $g_T = 0$.

Bounds: x_t, e_t, f_t, g_t all have lower bounds of 0, and are all integers. f_t has upper bound of 1. M has a lower bound of 1.

question 2

Result as shown in Figure 3.

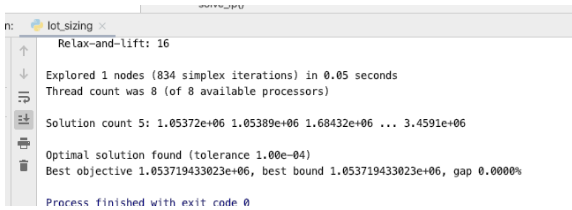

Figure 3

Problem 4

question 1

Indices: $i = 1, 2, ..., m$ are for different machines. $j = 1, 2, ..., n$ are for different jobs. $k = 1, 2, ..., m$ are for different stages. $l = 1, 2, ..., n$ are for different orders/sequences for each machine.

Data: $I_{j,k}$ is the index of machine needed by job j at k stage. $t_{i,j}$ is the time for machine i to complete job j at its specific stage when stage k requires machine i.

Variables: $x_{i,l}$ is the starting time of the lth job on machine i. $y_{j,i,l}$ is the binary indicator of whether if job j is executed at the lth order of machine i. In other words, y is eight $l * i$ matrices with only four 1 indicating when the machine i is processing the job j. e_i is the ending time of machine i. e^{MAX} is the maximum of e_i.

Objective: minimize e^{MAX}.

Constraints:

- $e_i = \sum_{j=1}^{n}(x_{i,n} + t_{i,j})y_{j,i,n}$ for every i. This means that the ending time of machine i equals the sum of the starting time of machine i of its last job and the time of the last job, which is controlled by a binary integer $y_{j,i,n}$. Again, the meaning of $y_{j,i,n}$ is that whether if job j is performed on machine i in the $l = n$th order.
- $e^{MAX} = \max_i e_i$.
- $\sum_{i=1}^{m}\sum_{l=1}^{n} y_{j,i,l} = 4$ for every job j.
- $\sum_{l=1}^{n} y_{j,i,l} = 1$ for every machine i in every job j.
- $\sum_{j=1}^{n} y_{j,i,l} = 1$ for every machine i in every order l. This means that, in the lth order of machine i, there can be only one job working.
- The above three constraints make $y_{j,i,l}$ a sparse tensor of shape $n \times m \times n$.
- $\sum_{j=1}^{n} y_{j,i,l}(x_{i,l} + t_{i,j}) \leq \sum_{j=1}^{n} y_{j,i,l+1}x_{i,l+1}$ for l in range(n-1) for every i. This means that in every machine, the ending time of step l must be eariler than the starting time of step l+1.
- $\sum_{l=1}^{n}(x_{I[j,k],l} + t_{I[j,k],j})y_{j,I[j,k],l} \leq \sum_{l=1}^{n} x_{I[j,k+1],l}y_{j,I[j,k+1],l}$ for every stages k in range(m-1) for every job j.

Bounds: $y_{j,i,l} = 0$ or $y_{j,i,l} = 1$ since they are binary indicators. $x_{i,l} \geq 0$, $e_i \geq 0$, and $e^{MAX} \geq 0$.

question 2

Result as shown in Figure 4.

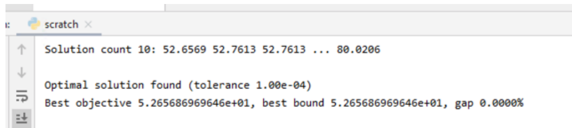

Figure 4