Assignment 4

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Problem 1

question 1

Indices: $i = 1, 2, ..., n$ for different bonds. $j = 1, 2, ..., p$ for different sectors.

Data: B is the total budget, its unit: dollars. f_i is the unit price of bond i, its unit: dollars. g_i is the fixed purchasing fee of bond i, its unit: dollars. r_i is the annual yield rate of bond i. s_i is the associated sector of bond i. q_i is the quality of bond i. c_j is the subset of all bonds that are inside sector j. l is the subset of all bonds that have quality C.

Variables: x_i is how many units of bond i are purchased, its unit: 100 units of bonds. y_i is a binary indicator of whether if bond i is purchased. h_j is the contribution of sector j in terms of value.

Objective: maximize $\sum_{i=1}^{n} 100x_i r_i$

Constraints: $x_i \le My_i$ for every i, where $M = \frac{B}{100 \min\limits_i f_i}$ is a sufficiently big number (constant). $\sum_{i=1}^{n}(g_i y_i + 100x_i f_i) \le B$. $h_j = \sum_{i \in c_j} 100x_i f_i$ for every j. $0.5ph_j \le \sum_{i=1}^{n} 100f_i x_i$ for every j. $\sum_{i \in l} 100f_i x_i \le 0.1 \sum_{i=1}^{n} 100f_i x_i$.

Bounds: x_i is integer with lower bound 0. y_i is integer with lower bound 0 and upper bound 1. h_j has lower bound 0.

question 2

Result as shown in Figure 1.

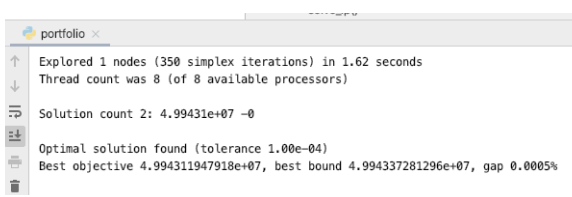


Figure 1

Problem 2

question 1

Indices: $i = 1, 2, ..., n$ are for different cities used for accepting products. $j = 1, 2, ..., n$ are for different cities used for giving products.

Data: b_i is the supply (positive) or demand (negative) for a certain product in city i. $f_{i,j}$ is the fixed building cost of establishing a link between cities i and j. $c_{i,j}$ is the transportation cost of shipping one unit of product through an established link between cities i and j.

Variables: $x_{i,j}$ is how many units of product to transport from city j to city i. $y_{i,j}$ is a binary indicator showing whether if there is a link established between city i and city j.

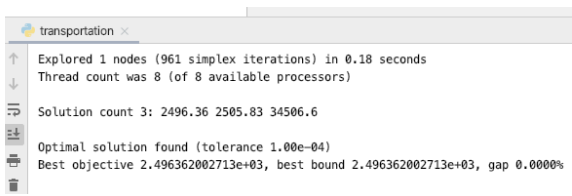
Objective: minimize$\sum_{i=1}^{n}\sum_{j=1}^{n}(x_{i,j}c_{i,j} + y_{i,j}f_{i,j})$

Constraints: $x_{i,j} \leq y_{i,j}M$ for every i and j, where $M = \sum_{i=1}^{n} \mid b_i \mid$ is a sufficiently big number. $b_i + \sum_{j=1}^{n} x_{i,j} - \sum_{j=1}^{n} x_{j,i} \geq 0$ for every city i.

Bounds: $y_{i,j} = 0$ or $y_{i,j} = 1$ for every $y_{i,j}$. $x_{i,j} \geq 0$.

question 2

Result as shown in Figure 2.



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transportation
Explored 1 nodes (961 simplex iterations) in 0.18 seconds
Thread count was 8 (of 8 available processors)

Solution count 3: 2496.36 2505.83 34506.6

Optimal solution found (tolerance 1.00e-04)
Best objective 2.496362002713e+03, best bound 2.496362002713e+03, gap 0.0000%
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Figure 2

Problem 3

question 1

Indices: $i = 1, 2, ..., m$ for different rows, and $j = 1, 2, ..., n$ for different columns.

Data: $h_{i,j}$ is the value of position (i,j) inside matrix $h \in R^{m\times n}$. c_j is the value of position j inside vector $c \in R^n$.

Variables: F_j is a binary indicator of whether if column j is inside the selected subset F. $x_{i,j}$ is a matrix of size (m,n), which is also a binary indicator of whether if element in position (i,j) is the maximum.

Objective: maximize $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{i,j}h_{i,j} - \sum_{j=1}^{n} c_jF_j$

Constraints: $x_{i,j} \leq F_j$ for every i and j. $\sum_{j=1}^{n} x_{i,j} = 1$ for every row i.

Bounds: $F_j = 0$ or $F_j = 1$. $x_{i,j} = 0$ or $x_{i,j} = 1$.

question 2

Result as shown in Figure 3.

Problem 4

question 1

Result as shown in Figure 4.

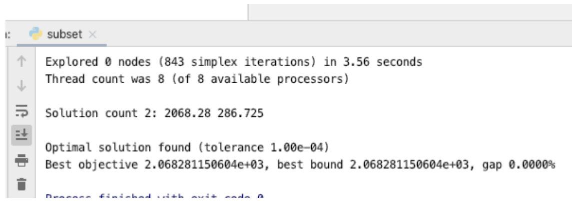

Figure 3

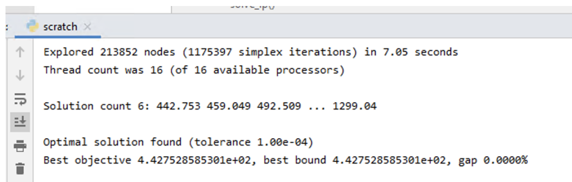

Figure 4

question 2

Result as shown in Figure 5.

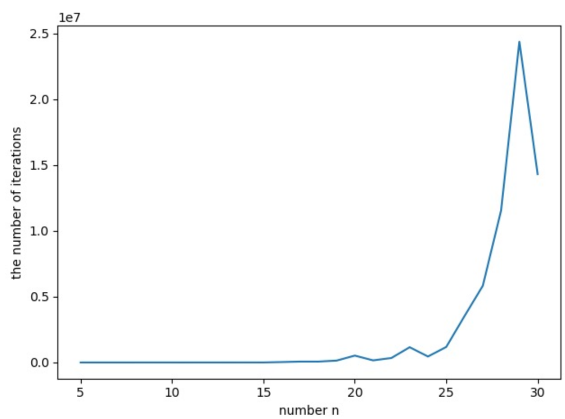

Figure 5

Observation: the total number of iterations is almost flat when the number n is small, which means that the increase of iterations is not significant at the beginning. When n is bigger than a specific number, the number of iterations grows exponentially or even quicker, which shows that the increase of iterations will become significant when n is big enough. Also, there are some fluctuations inside the plot when increasing the number n. This fluctuation may be caused by the periodic symmetry of data that is randomly generated from our id.

question 3

Result as shown in Figure 6.

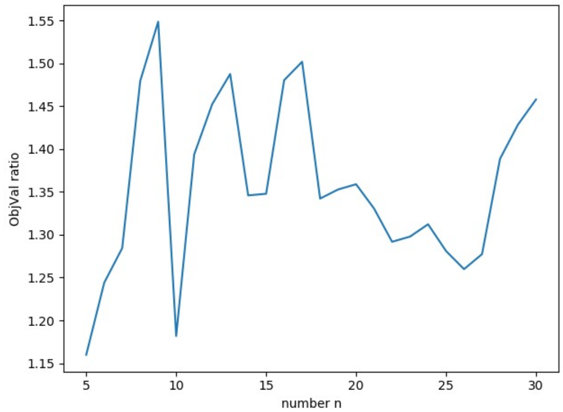

Figure 6

Observation: the ratio of the optimal objective values is small and remains roughly the same when increasing the number n (from 1.55 to 1.15). There is no significant increase or decrease of that ratio. There are some fluctuations when increasing the number n. Again, this fluctuation may be caused by the periodic symmetry of data that is randomly generated from our id.