Phys210: Mathematical Methods in Physics II

Homework 5

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Policies

- Please adhere to the *academic integrity* rules: see my explanations here for further details!
- For the overall grading scheme or any other course-related details, see the syllabus.
- Non-graded question(s) (if any) are for your own practice!
- Unless stated otherwise, you are expected to show your derivation of the results.
- The homework is due May 3rd 2024, 23:59 TSI.

(1) Problem One *(4 points)*

Consider the higher order function $\mathcal{I}$

$$\mathcal{I} :: \mathbb{R} \to (\mathbb{R} \to \mathbb{R}) \tag{1.1a}$$

$$\mathcal{I} = a \to \left(b \to \oint_{\partial\Omega} (L(a,b,x,y)dx + M(a,b,x,y)dy) \right) \tag{1.1b}$$

where the integration is over the boundary of an $2d$ region Ω (hence denoted $\partial\Omega$). Here, the functions L and M are defined as

$$L, M :: (\mathbb{R}, \mathbb{R}, \mathbb{R}, \mathbb{R}) \to \mathbb{R} \tag{1.2a}$$

$$L = (a,b,x,y) \to \cos(xy)\left(1 + x^a y^b e^{xy}\right) \tag{1.2b}$$

$$M = (a,b,x,y) \to \cos(xy)\left(1 + x^b y^a e^{xy}\right) \tag{1.2c}$$

For a particular value of a in terms of b (say $a = f(b)$), the value $\mathcal{I}(a)(b) :: \mathbb{R}$ is *independent* of the value b: find this function f.

Hint 1: You do not need to solve any integrals for this question; simply use the relevant *integral theorem*!

Hint 2: The question equivalently asks what is the value of a in terms of b such that $\oint_{\partial\Omega} \cos(xy)\left(\left(1 + x^a y^b e^{xy}\right)dx + \left(1 + x^b y^a e^{xy}\right)dy\right)$ is b–independent.

(2) Problem Two *(4 points)*

Consider the line integral $\mathcal{J}$ over a path γ given as

$$\begin{aligned}\mathcal{J} = \int_\gamma \Big[&-\left(((a-2)yz+1)\sin\left(x+y^2\right)\right)dx + 5z\cos\left(x+y^2\right)dy \\ &-2y(b+5yz)\sin\left(x+y^2\right)dy + cy\cos\left(x+y^2\right)dz \Big]\end{aligned} \tag{2.1}$$

(2.1) (a)

Find the values a, b, c for which this integration is *path-independent*.

(2.2) (b)

Compute the integration for the case it is path independent: take the initial and final points of the path as $(x_i, y_i, z_i) = (0, 0, 0)$ and $(x_f, y_f, z_f) = (\pi, 1/10, -2)$.

(3) Problem Three *(not graded)*

Mathematica can be utilized to solve these questions efficiently; for instance,

```
With[{
        L = Function[{x, y}, Cos[x  y] (1 + x^a y^b Exp[x  y])],
        M = Function[{x, y}, Cos[x  y] (1 + x^b y^a Exp[x  y])]
},
        FullSimplify[D[M[x, y], x] − D[L[x, y], y]]
]
```

does the most of the computing for the first question, whereas comparison with

```
Grad[Cos[a  x + b  y^2] (c + d  y  z), {x, y, z}]
```

should be sufficient to deduce a, b, c in the second problem.