Phys210: Mathematical Methods in Physics II

Homework 4

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Policies

- Please adhere to the *academic integrity* rules: see my explanations here for further details!
- For the overall grading scheme or any other course-related details, see the syllabus.
- Non-graded question(s) (if any) are for your own practice!
- Unless stated otherwise, you are expected to show your derivation of the results.
- The homework is due April 19$^{\text{th}}$ 2024, 23:59 TSI.

(1) Problem One *(8 points)*

(1.1) (a)

Consider the higher order function f of the type

$$f :: \mathbb{N} \to \left(\mathbb{R}^7 \to \mathbb{R}\right) \tag{1.1}$$

where the argument of the domain of f shall be written as a subscript. In this notation, define

$$f_1 = (x_1, \ldots, x_7) \to x_3^2 x_5 \tag{1.2a}$$

$$f_2 = (x_1, \ldots, x_7) \to \frac{x_2^2 x_5}{1 + x_6} \tag{1.2b}$$

$$f_3 = (x_1, \ldots, x_7) \to \frac{x_7^2 x_1}{1 + x_4} \cos(x_1) \tag{1.2c}$$

Compute $\mathsf{grad}(f_1)$, $\mathsf{grad}(f_2)$, and $\mathsf{grad}(f_3)$.

(1.2) (b)

Let us define the following higher order function

$$g :: \mathbb{R} \to \left(\mathbb{R}^2 \to \mathbb{R}\right) \tag{1.3a}$$

$$g = \alpha \to \left((x, y) \to (x^\alpha + y^{-\alpha})\frac{\partial}{\partial x} - (x^{-\alpha} + y^\alpha)\frac{\partial}{\partial y}\right) \tag{1.3b}$$

Compute $\mathsf{div}(g(-1))$, $\mathsf{div}(g(0))$, and $\mathsf{div}(g(1))$.

(1.3) (c)

Generalize the higher order function defined in part **(b)** as follows:

$$h :: \mathbb{R} \to \left(\mathbb{R}^3 \to \mathbb{R}\right) \tag{1.4a}$$

$$h = \alpha \to \left((x, y, z) \to (x^\alpha + y^{-\alpha})\frac{\partial}{\partial x} - (x^{-\alpha} + y^\alpha)\frac{\partial}{\partial y}\right) \tag{1.4b}$$

Compute $\mathsf{curl}(h(-1))$, $\mathsf{curl}(h(0))$, and $\mathsf{curl}(h(1))$.

(2) Problem Two *(not graded)*

Mathematica can be used to compute vector differentiation; for instance,

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Grad[Cos[x + y], {x, y,z}]
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computes the grad(k) for the function $k = (x, y, z) \to \cos(x + y)$.

Use Mathematica to solve the first problem.