# Mathematical Methods in Physics I

## Homework I

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# Problem 1

*Solution.* (a) The definition of `sqr2` is

$$\begin{aligned} \texttt{sqr2} &:: (\mathbb{C} \to \mathbb{C}) \to (\mathbb{C} \to \mathbb{C}) \\ \texttt{sqr2} &= (x \mapsto f(x)) \mapsto (x \mapsto g(x)) \end{aligned}$$

This high-order function takes a function $f$ as an input and outputs a function $g$ such that $g(x) = 2f(x)$.

(b) The derivative of $\cos x$ is $-\sin x$. Hence, the type and definition of the high-order function acting on cos are

$$\begin{aligned} \left(\frac{d}{dx} + \mathcal{I}\right) \cdot \cos &:: (\mathbb{R} \to \mathbb{R}) \to (\mathbb{R} \to \mathbb{R}) \\ \left(\frac{d}{dx} + \mathcal{I}\right) \cdot \cos &= (x \mapsto \cos x) \mapsto (x \mapsto -\sin x + \cos x) \end{aligned}$$

(c) The type of the operator $\mathcal{C}$, when acting on real variables and functions, is

$$\mathcal{C} :: \big[(\mathbb{R} \to \mathbb{R}) \to (\mathbb{R} \to \mathbb{R})\big] \to \big[(\mathbb{R} \to \mathbb{R}) \to (\mathbb{R} \to \mathbb{R})\big]$$

(d) The action of $\exp\left(\frac{d}{dx}\right)$ on $x^4$ is

$$\begin{aligned} \exp\left(\frac{d}{dx}\right) \cdot x^4 &= x^4 + \frac{1}{1!}\left(4x^3\right) + \frac{1}{2!}\left(12x^2\right) + \frac{1}{3!}\left(24x\right) + \frac{1}{4!}\left(24\right) + 0 + \dots \\ &= x^4 + 4x^3 + 6x^2 + 4x + 1 \end{aligned}$$

After the $n$th derivative of $x^4$, all subsequent terms of the Taylor series expansion become zero, as the derivative of a constant is zero. Consequently, we are left with an expression containing a finite number of terms. Applying the binomial formula $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$, we obtain

$$\exp\left(\frac{d}{dx}\right) \cdot x^4 = (x+1)^4$$

which is the result of shifting the argument. ∎