We consider the classical algorithm in Clawpack with the minmod TVD limiter and solve the p-system:

$$\epsilon_t - u_x = 0, \tag{1a}$$

$$(\rho(x)u)_t - \sigma_x(\epsilon, x) = 0, \tag{1b}$$

where  $\sigma(\epsilon, x) = \exp(K(x)\epsilon) - 1$ ,

$$\rho(x) = \frac{\rho_A + \rho_B}{2} + \frac{\rho_A - \rho_B}{2} \sin(2\pi x),$$

$$K(x) = \frac{K_A + K_B}{2} + \frac{KA - KB}{2} \sin(2\pi x).$$

The domain is given by  $\Omega = [0, 400]$ .

## Generation of stegotons

To generate a stegoton we start with a zero initial condition and consider the following left boundary condition:

$$\epsilon(0,t) = 0,$$

$$u(0,t) = \begin{cases} -0.1 \left[ 1 + \cos(t_0 \pi) \right], & \text{if } |t_0| \le 1, \\ 0, & \text{otherwise,} \end{cases}$$

where  $t_0 = \frac{t-2.5}{2.5}$ . Note that once t > 5, the left boundary is zero. To produce the stegotons in the paper, we proceed as follows:

- Generation of the main stegoton. Run create\_stegotons/run\_psystem.py, which solves (1) up to a final time t = 400. The resolution is given by  $\Delta x = 1/1024$ .
- Isolation of the stegoton. Inside cut\_stegoton, create a folder called \_output and copy create\_stegoton/\_output/\*0400\* to the newly created folder. Finally, run cut\_stegoton/cut.py. The isolated stegoton will be placed in cut\_stegoton/\_output\_cut\_steg. The stegoton is isolated based on the stress. We locate the peak of the wave and move to the left and right until the solution is smaller than 10<sup>-12</sup>.
- Refinement of the isolated stegoton. Inside refine\_cut\_stegoton, create a folder called \_output\_cut\_steg and copy all the files inside cut\_stegoton/\_output\_cut\_steg to the newly created folder. Run refine\_cut\_stegoton/refine\_steg.py. To refine the stegoton, we start with the isolated stegoton with  $\Delta x = 1/1024$  and perform a high-order polynomial reconstruction. Using the reconstruction, we obtain the cell averages of the solution for different refinements.

After following these steps, the data (in h5 files) for the multiple refinements will be placed in refine\_cut\_stegoton/.