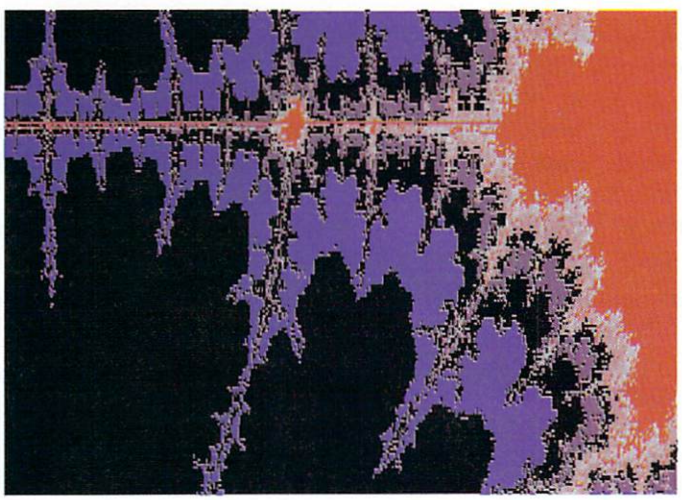
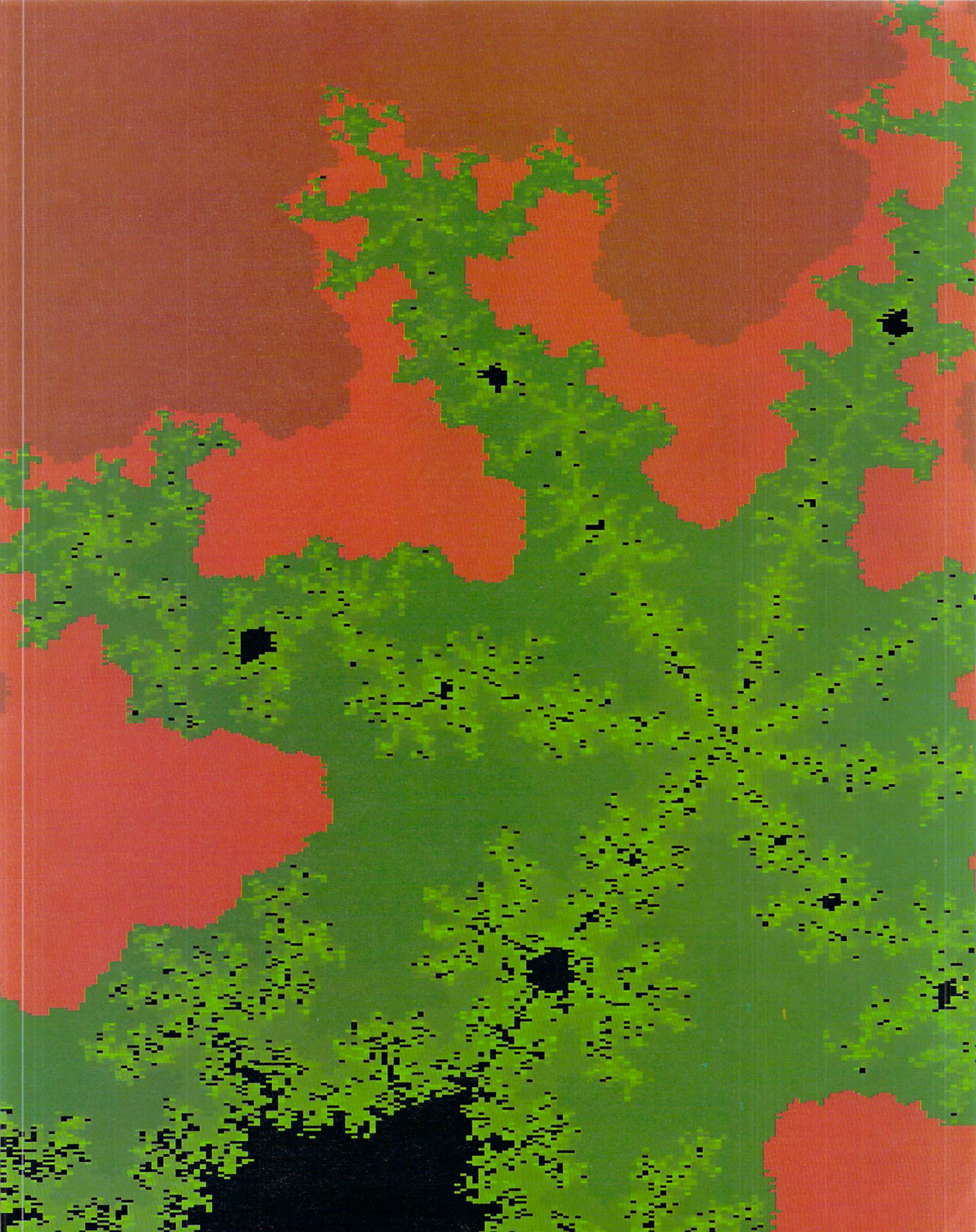
Digital Canvas

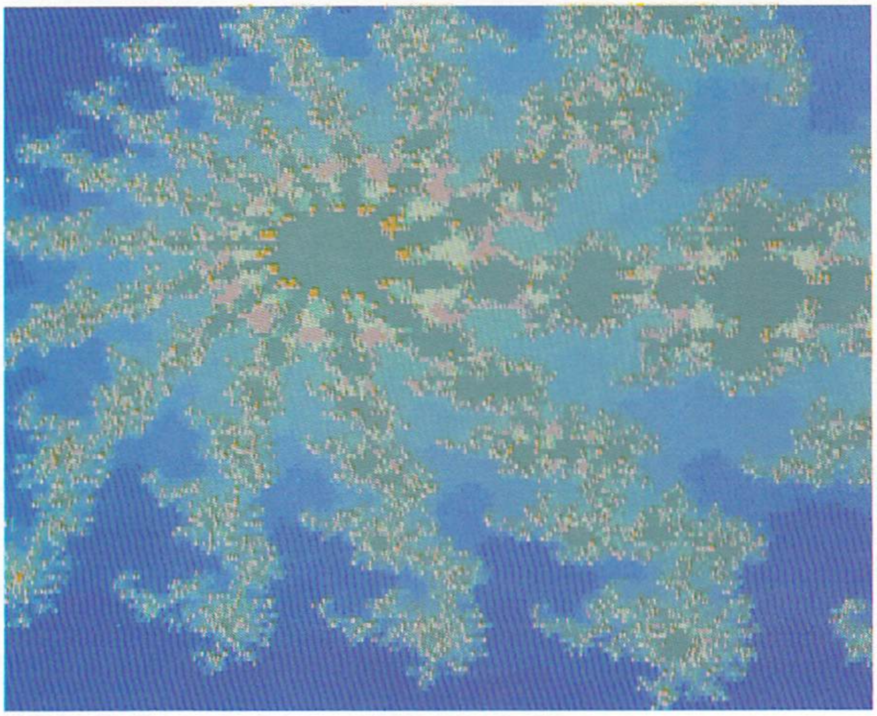


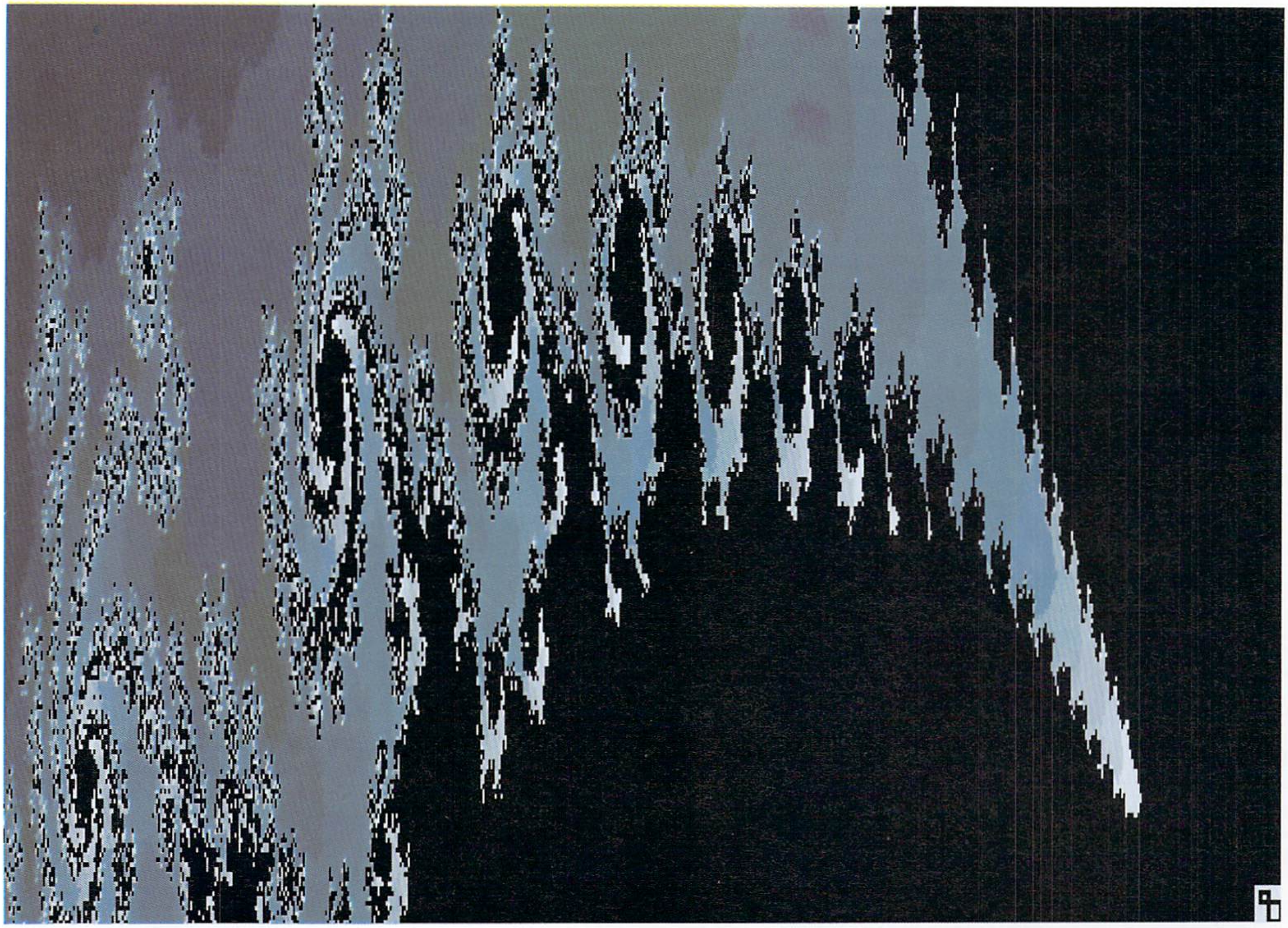
This month's Digital Canvas features mathematically-generated graphics called Fractals, created from the Mandelbrot Set. Mathematician Steve Bonner, who produced these images, feels they are ample "evidence that math and art are inherently linked, and ought never be separated." Leonardo would have agreed.

Steve Bonner, a long-time resident of Rockville, Maryland, received his undergraduate degree from the University of Maryland, and his Master of Arts in mathematics from The American University in Washington, DC. A serious student of analytic number theory, Steve recently was led to the so-called Mandlebrot Set through investigations in complex analysis. It is his conviction that if "the student of mathematics cannot either literally or figuratively picture what it is he's studying, then all attempts at understanding are futile."

Steve has pursued his interests with the Amiga, and submitted these images with remarks about his satisfaction that he no longer needs a mainframe or elaborate graphics hardware to study two- and three-dimensional objects with accuracy. We hope these pictures give you a feel for the intricate and complex beauty that can be visualized through the language of mathematics, whether or not you ever really learned to speak it.■







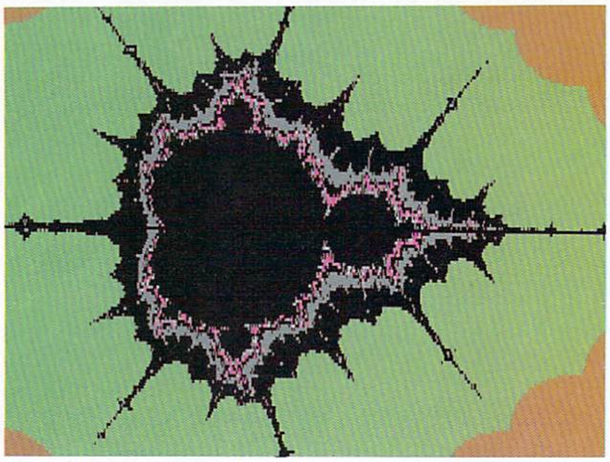
Fractals and Mandelbrots

By Bob Racko

Fractals get their name from their observed characteristics. Fractals occupy a fractional-dimension, for instance, somewhere between 1d (a line) and 2d (a plane). To qualify as a fractal, you have to nearly fill up all the space between one dimension and another.

To pull this off with a line requires a curve in the line (or fold) that follows a very crooked path and doubles back and forth without crossing over itself. The ratio of area covered this way versus the length of the line gives the "fraction" of the fractal.

Fractals also have another observable characteristic—no matter how much you enlarge them, there is always more detail available. For this reason, fractals are chosen to model such things as shorelines, mountain ranges and even noise. A single fractal generator can generate an infinite variety of self-similar forms that hold up well under enlargement.



Sometimes this fractal characteristic of enlargement gives an almost exact replica of some outer structure. A curved line that contains a number of subcurve copies has a special name—a "Peano curve."

The Mandelbrot Set

The theory behind the Mandelbrot set comes from the discovery that certain polynomials in the complex plane develop symmetric fractal behavior when iterated over a finite range.

That's a mouthful. In plain English, this means that you can make a pretty picture showing patterns of infinite sub-symmetry if you take a simple formula like this:

$$Z = Z * Z + C$$

where Z is a complex number, initially 0, and C is an initial real-x, imaginary-y point, and map out the border of all points where Z refuses to grow larger than a certain size. The border is a fractal pattern.

Programming Issues

Among the various Mandelbrot plotting programs, a simple standard holds in order to help the "explorers" communicate interesting border areas on the complex plane to one another. Areas of interest are given by their southwest corner and side. The southwest corner is typically the lower-right corner of a square area and the "side" is the length of a side of that square.

Another term bantered about by Mandelbrotonians is called the gap. The gap for a given picture tells the distance between two adjacent pixels (assuming everybody uses raster graphics). The gap is computed by dividing the side factor by the number of pixels available in one direction.

Many of the programs developed so far plot every point over a given (x,y) area and set an arbitrary limit to the number of iterations on a given point. Values that come out larger than some maximum length (usually two) before the iteration limit is reached are assigned a color based upon the number of iterations completed. The method for selecting a given point can be random if the random number generator can be guaranteed to deliver all possible values (the linear-congruential ones do).

Other, more complicated programs find a point that is "in the set" (i.e., runs out of iterations or fails to grow enough to make a difference) and another point, next to it, which is "not in the set" (i.e., gets too big) and then check points to either side to trace the outline of the boundary.

All these programs take quite a lot of time to run because they try many points on the plane. They have to—the Mandelbrot set border is more than a line; it covers an area in fractal form, zig-zags and all. Naturally, there is a frenzy of programming activity devoted to reducing the time required to either plot all those points or deliver a reasonable (but grainy) picture for further zooming. This currently can vary from 12 minutes to 12 hours depending on implementation features such as language used (Basic, Basic with Asm routines or C language) or plot algorithm (Straight Forward For Loop, Random Points, Curve Trace) and the ratio of in-set versus not-in-set points.■

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