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LARRY ABBOT

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NEURONAL LEARNING IN THE TEMPORAL DOMAIN

PRL 105, 218102 (2010) PHYSICAL REVIEW LETTERS week ending 19 NOVEMBER 2010

Theory of Spike Timing-Based Neural Classifiers

Ran Rubin,[1,2] Rémi Monasson,[2,3] and Haim Sompolinsky[1,4,5]
[1]*Racah Institute of Physics, Hebrew University, 91904 Jerusalem, Israel*
[2]*Laboratoire de Physique Théorique de l'ENS, CNRS, Université Paris 6, 24 rue Lhomond, 75005 Paris, France*
[3]*Simons Center for Systems Biology, Institute for Advanced Study, Einstein Drive, Princeton, New Jersey 08540, USA*

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Neuron
Article
CellPress

Learning Precisely Timed Spikes

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Temporal Support Vectors for Dynamical Systems and Spiking Neurons

Abstract

E-I BALANCE IS REQUIRED FOR HIGH-CAPACITY, NOISE-ROBUST NEURONAL COMPUTATION

RAN RUBIN

with

LARRY ABBOTT & HAIM SOMPOLINSKY

Flatiron Institute, Nov. 2017

A NEW BALANCE

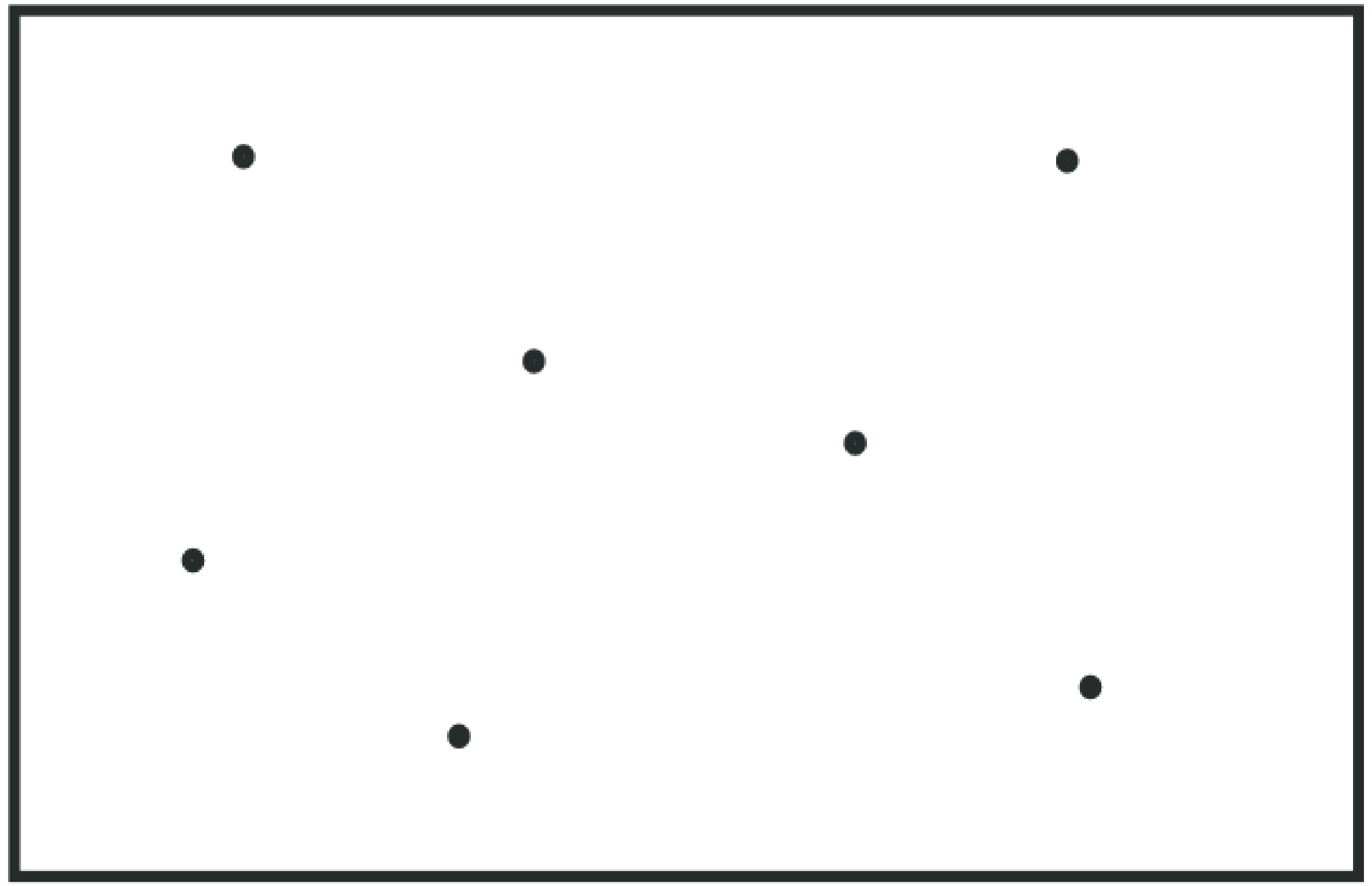
ASSOCIATIVE MEMORY NETWORKS

ASSOCIATIVE MEMORY NETWORKS

Network State

ASSOCIATIVE MEMORY NETWORKS

- Memory States - Attractors



Network State

ASSOCIATIVE MEMORY NETWORKS

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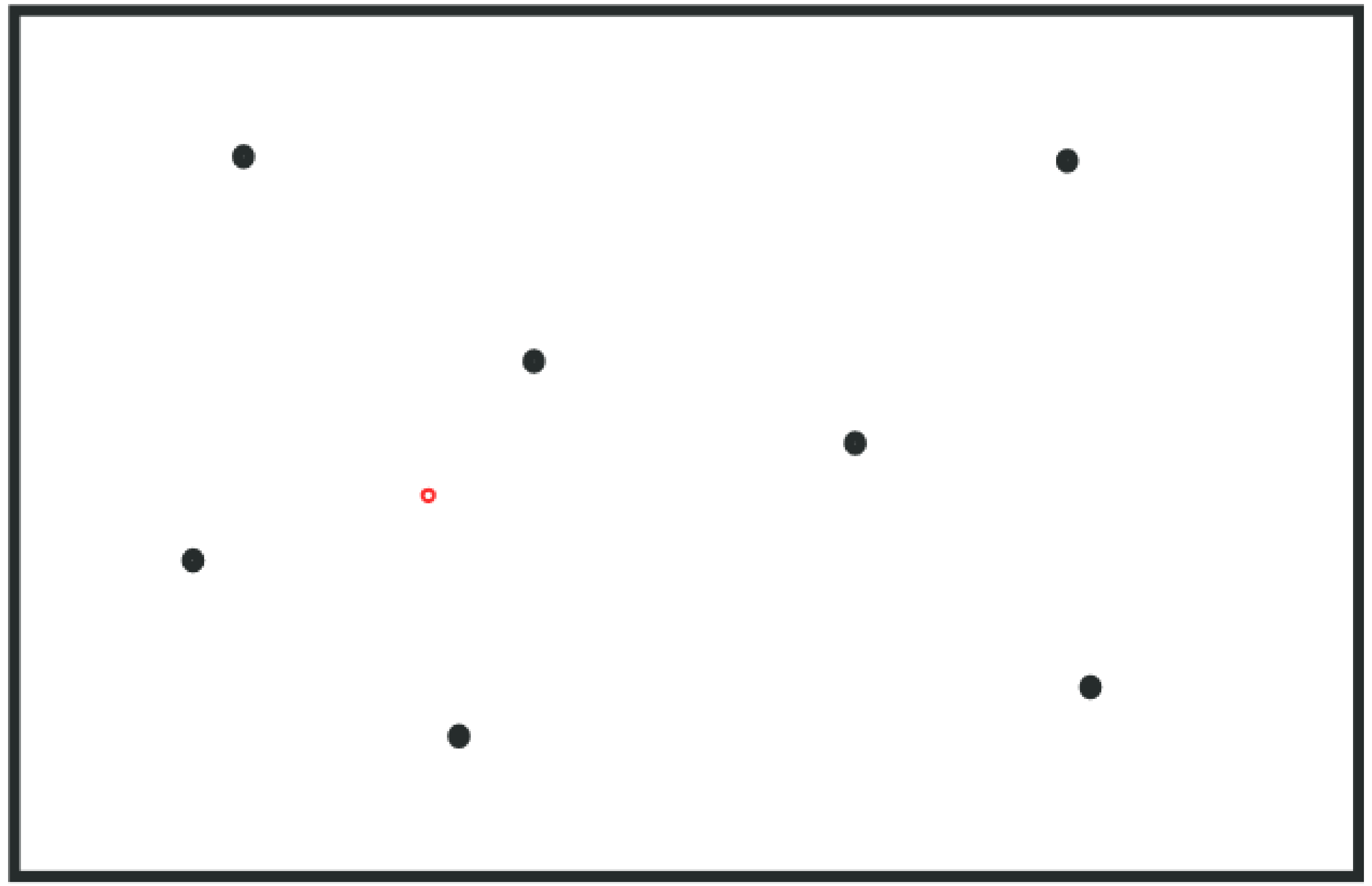

Network State

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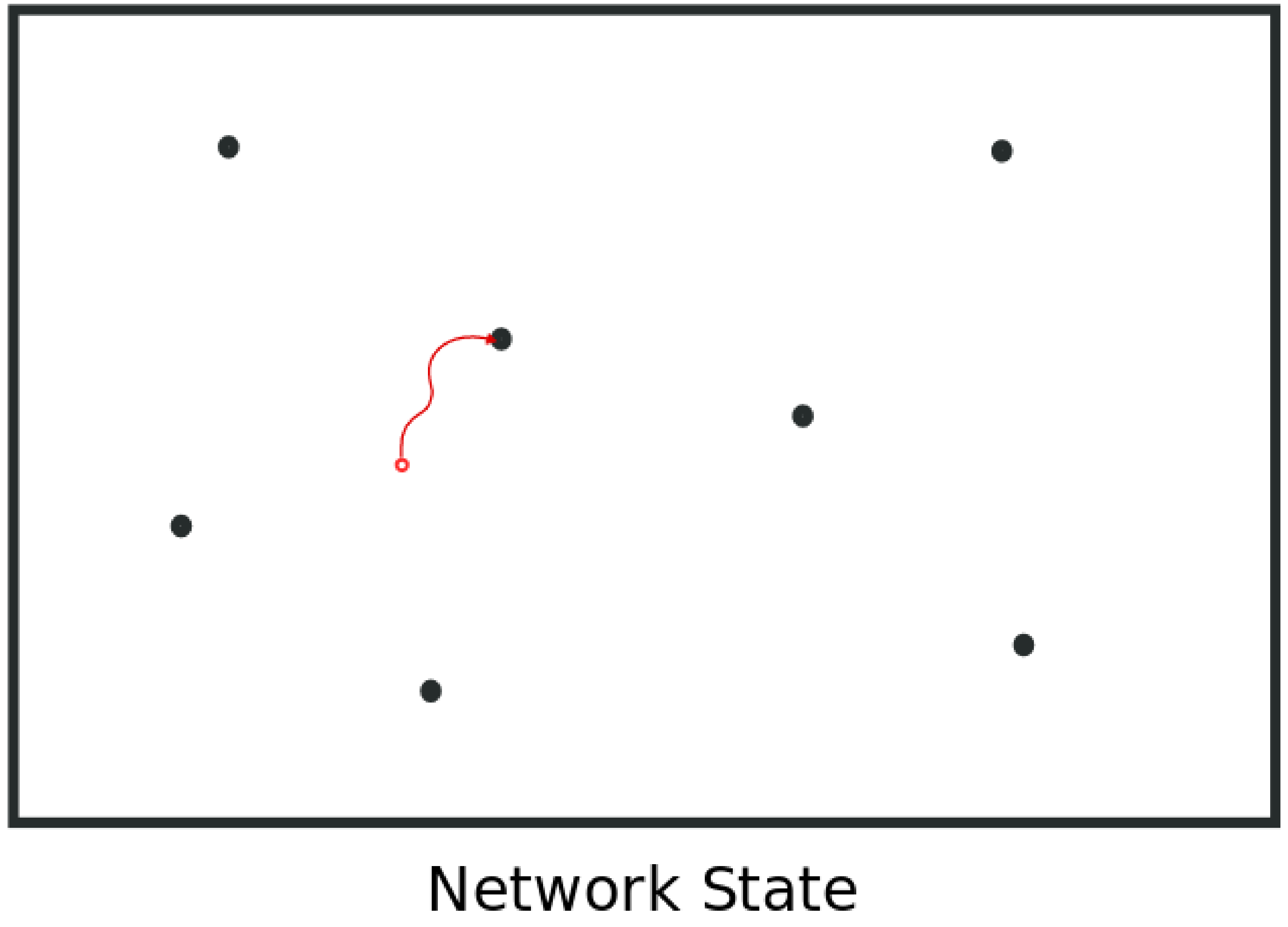
- Memory States - Attractors



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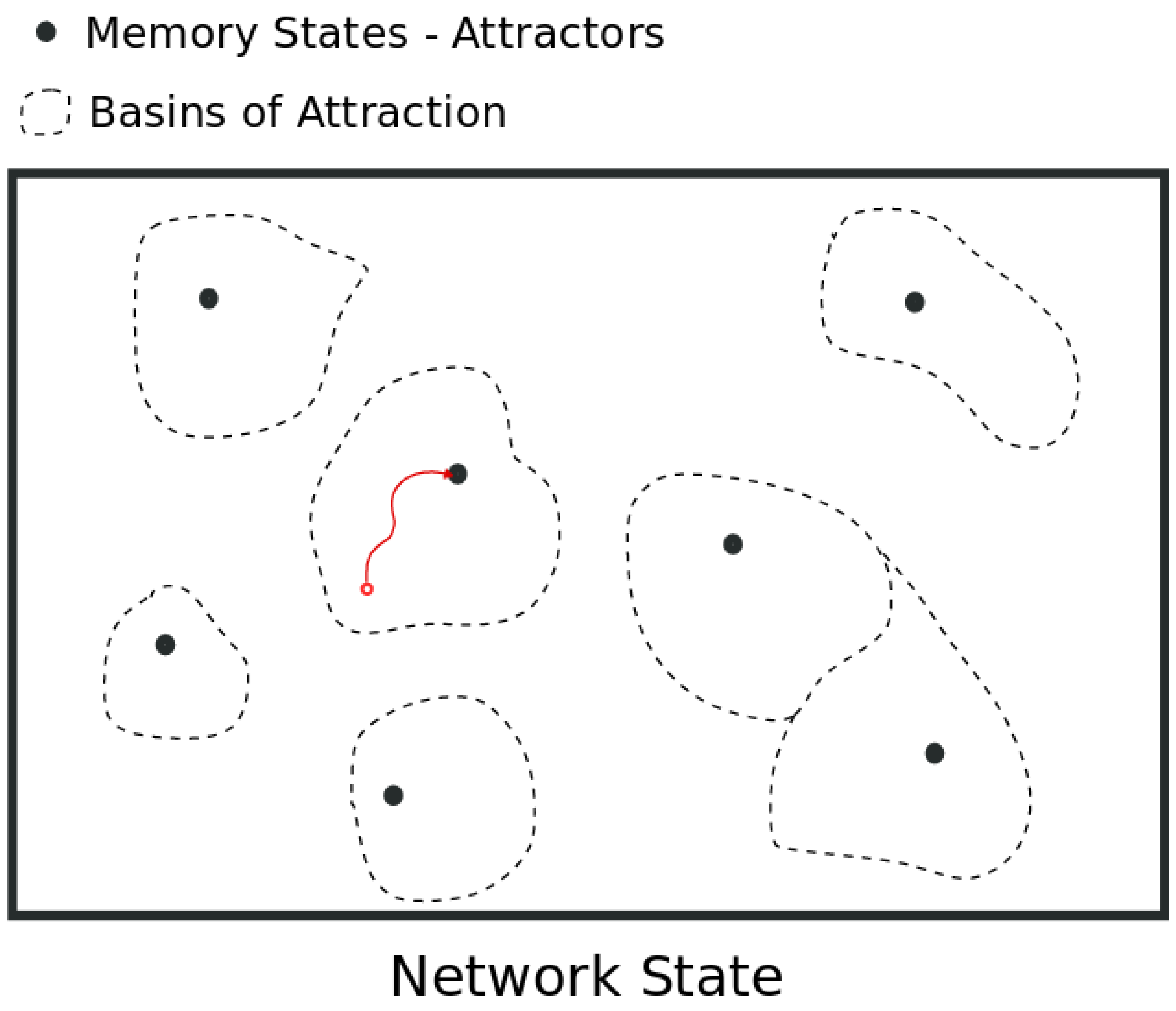
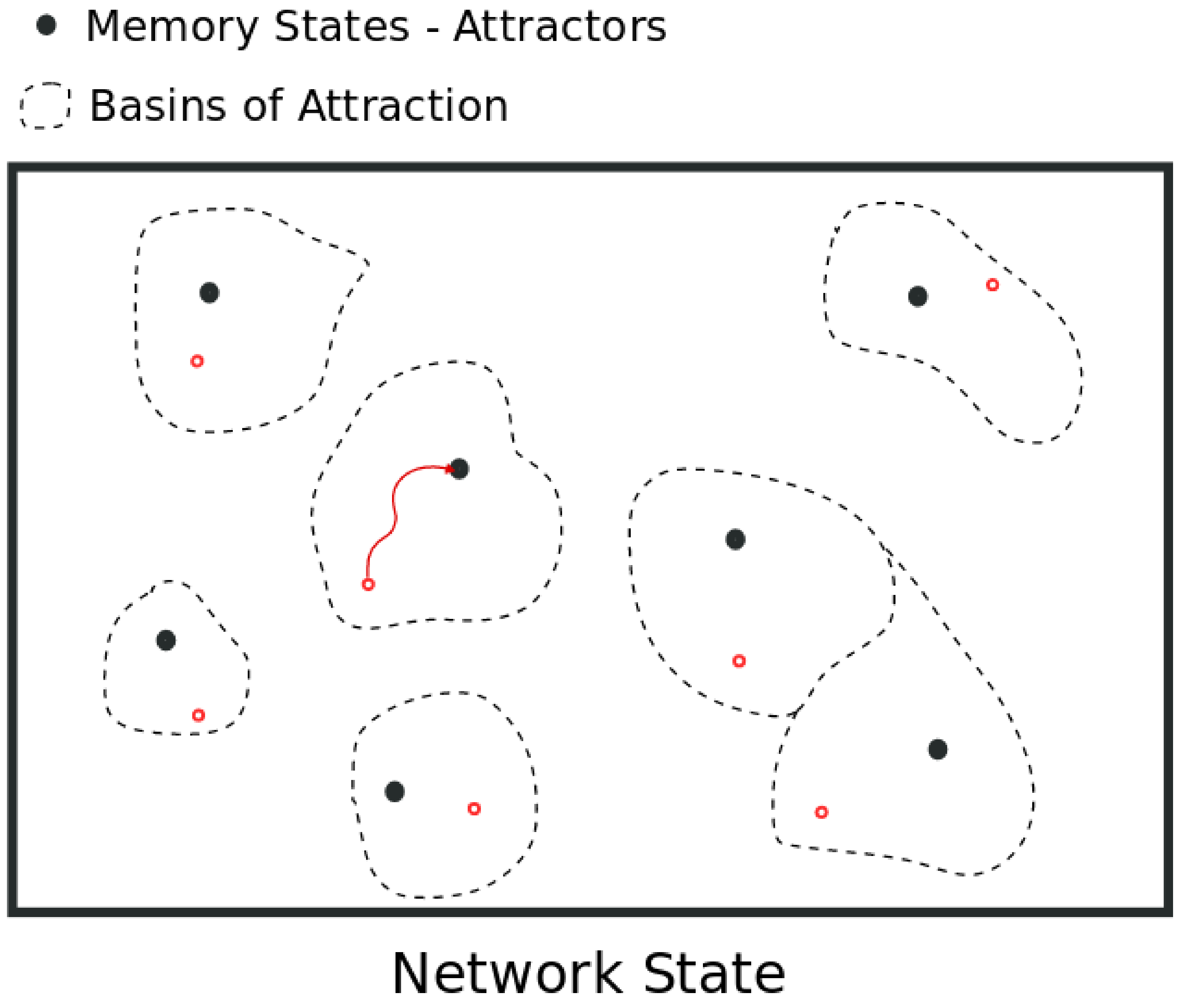


ASSOCIATIVE MEMORY NETWORKS



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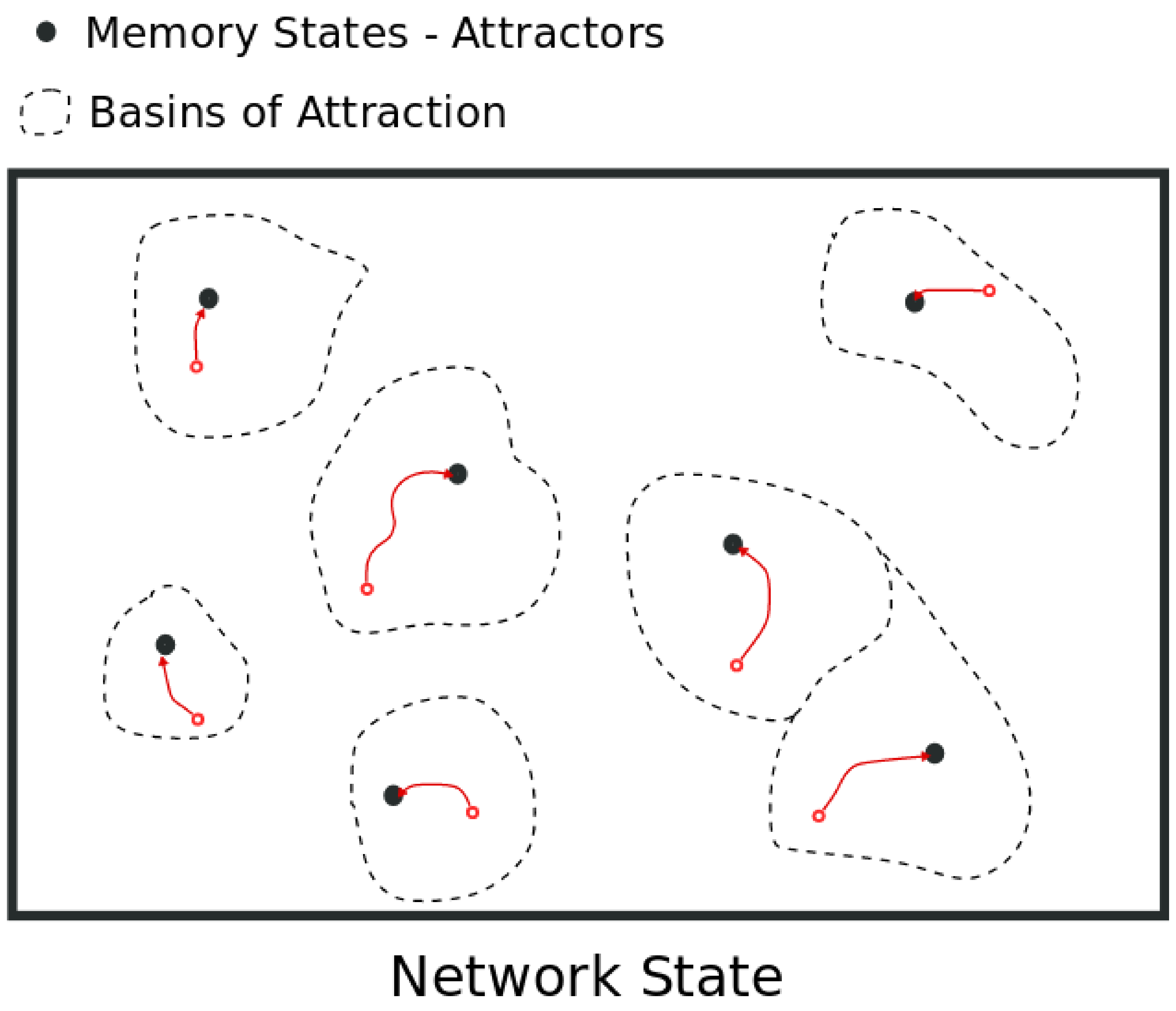
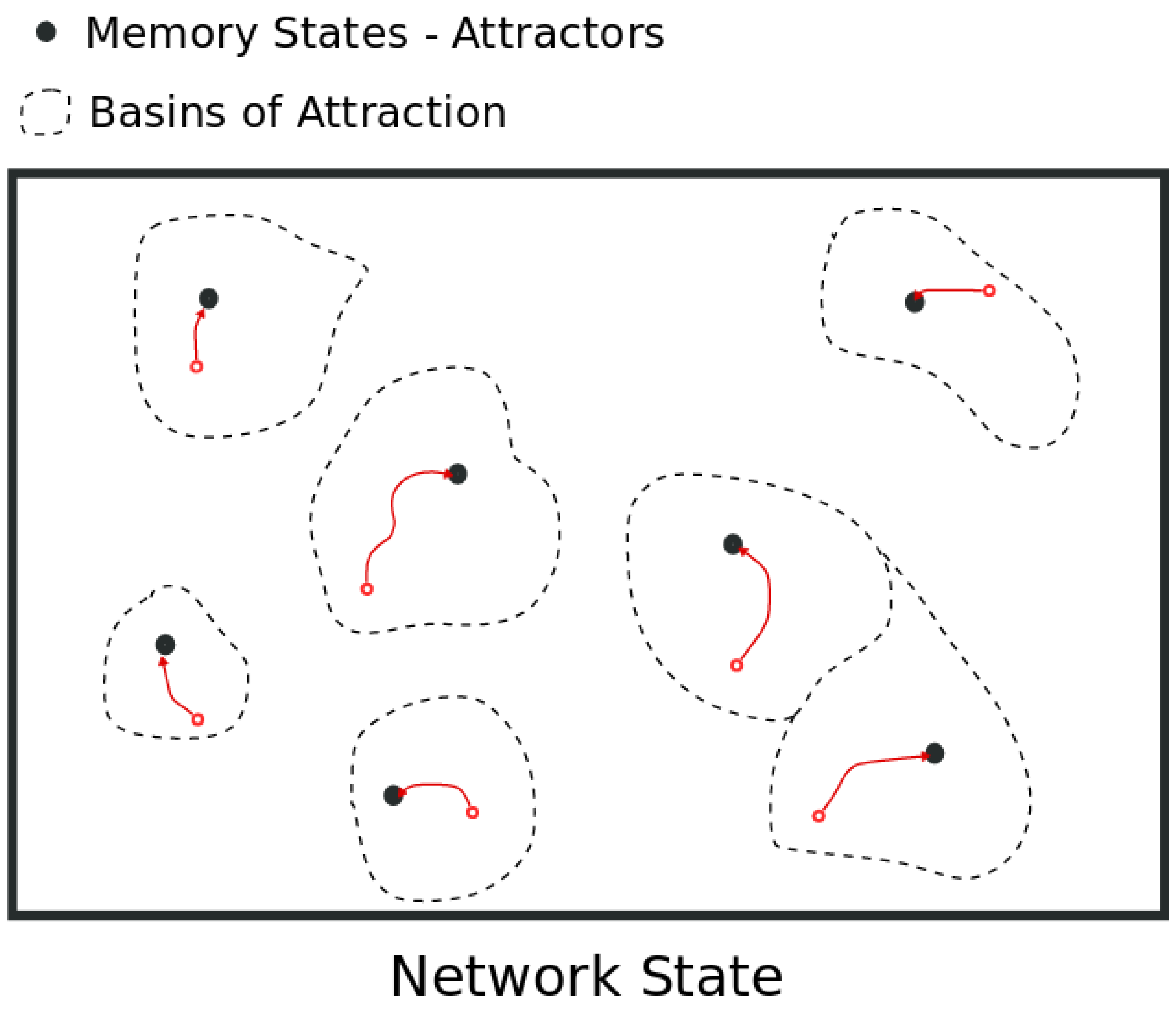


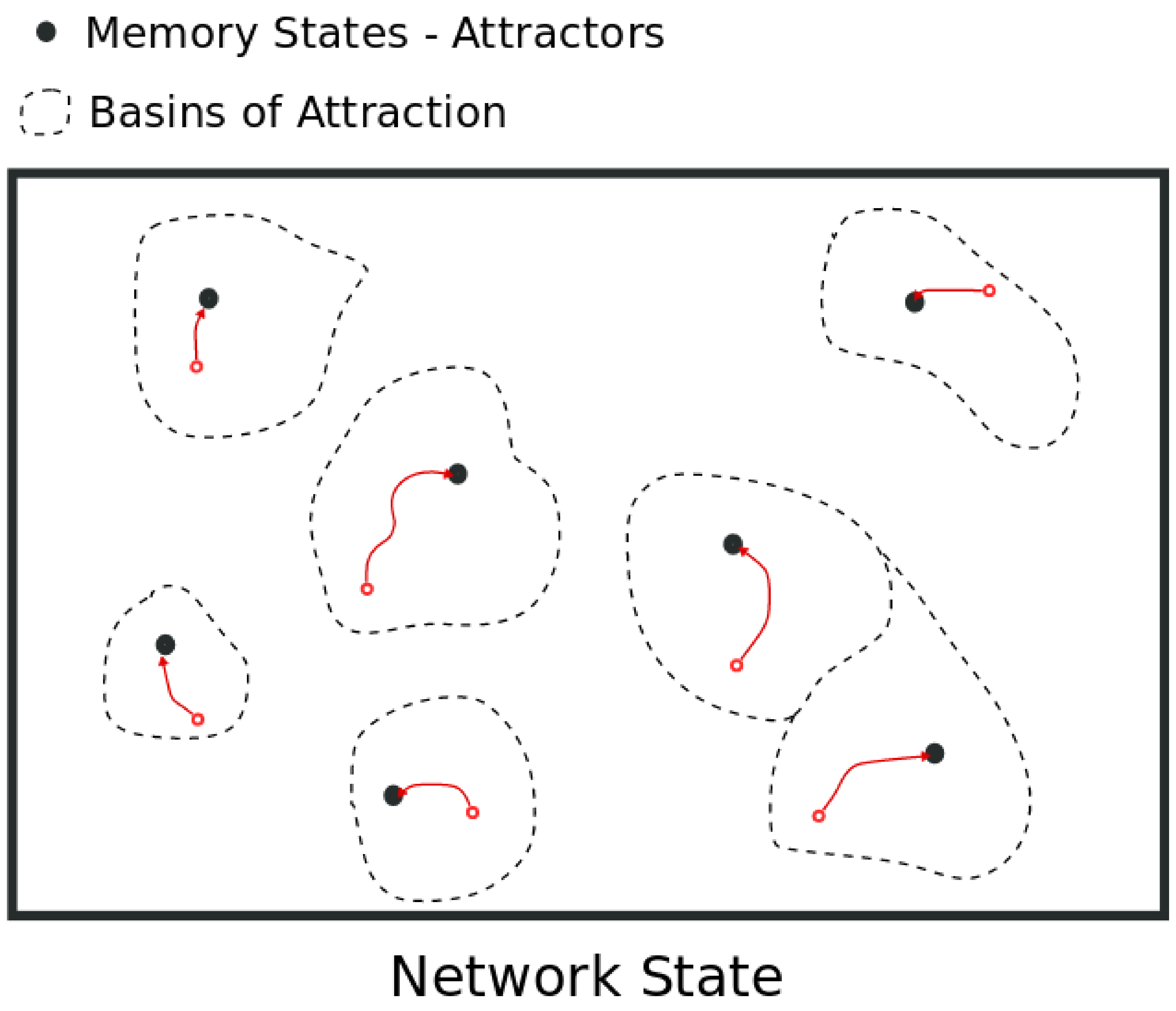
ASSOCIATIVE MEMORY NETWORKS



Analysis:

- Capacity

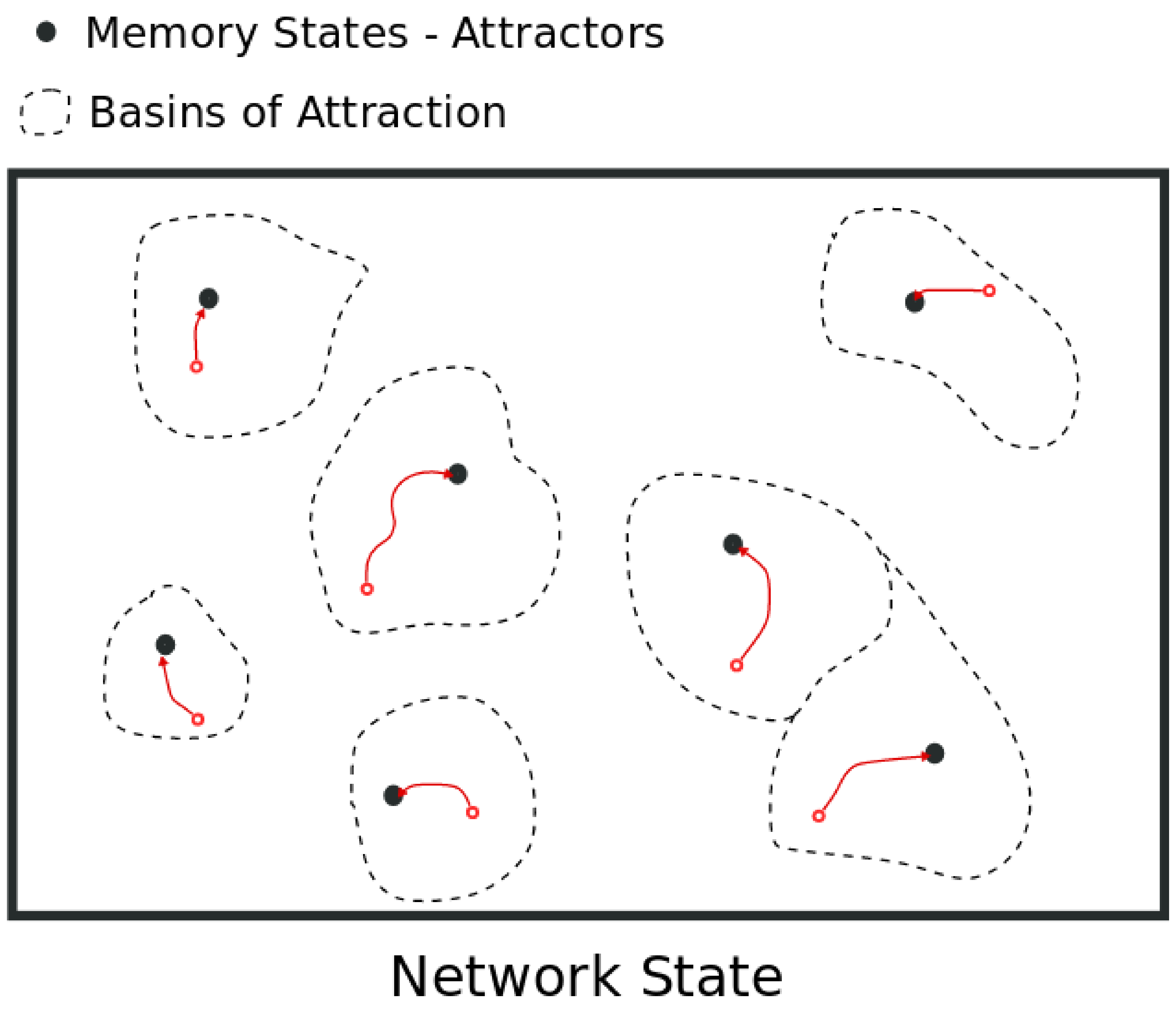
ASSOCIATIVE MEMORY NETWORKS



Analysis:

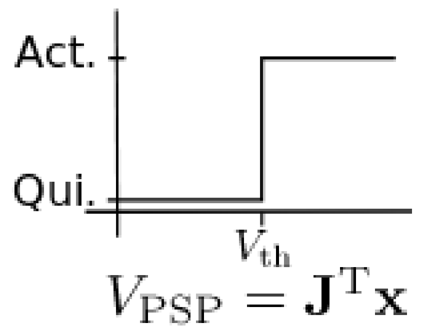
- Capacity
- Size of Basins

ASSOCIATIVE MEMORY NETWORKS

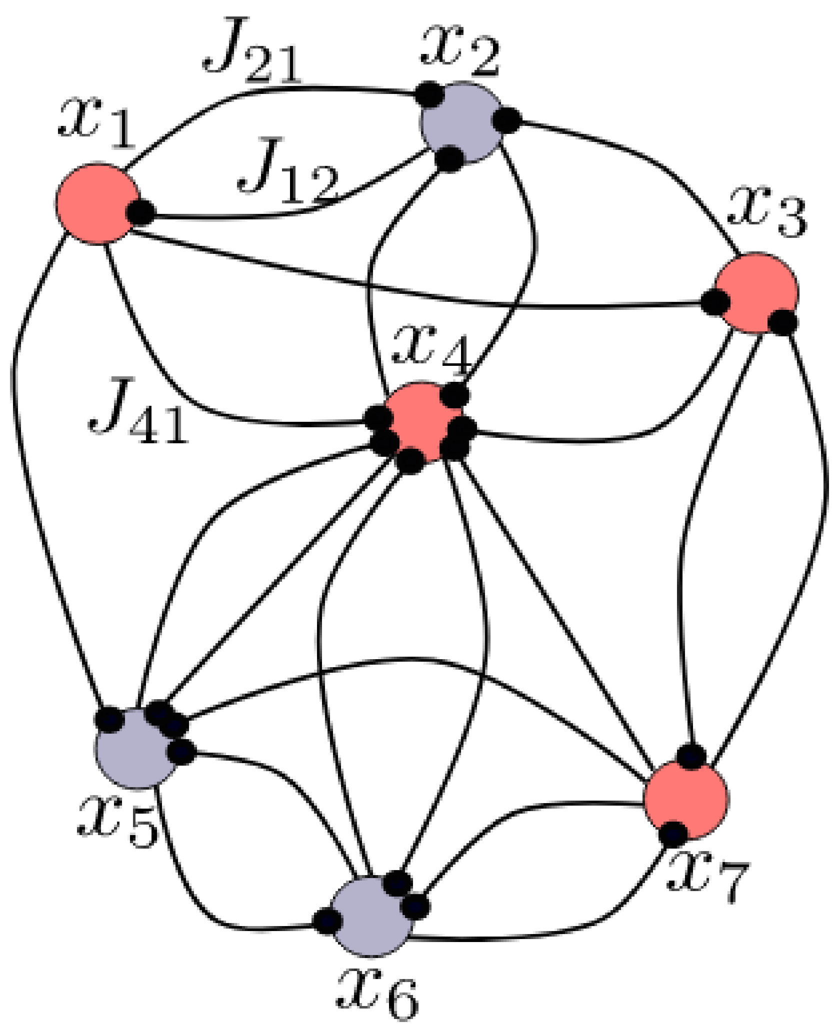


Analysis:

- Capacity
- Size of Basins
- Stability/Robustness to Noise

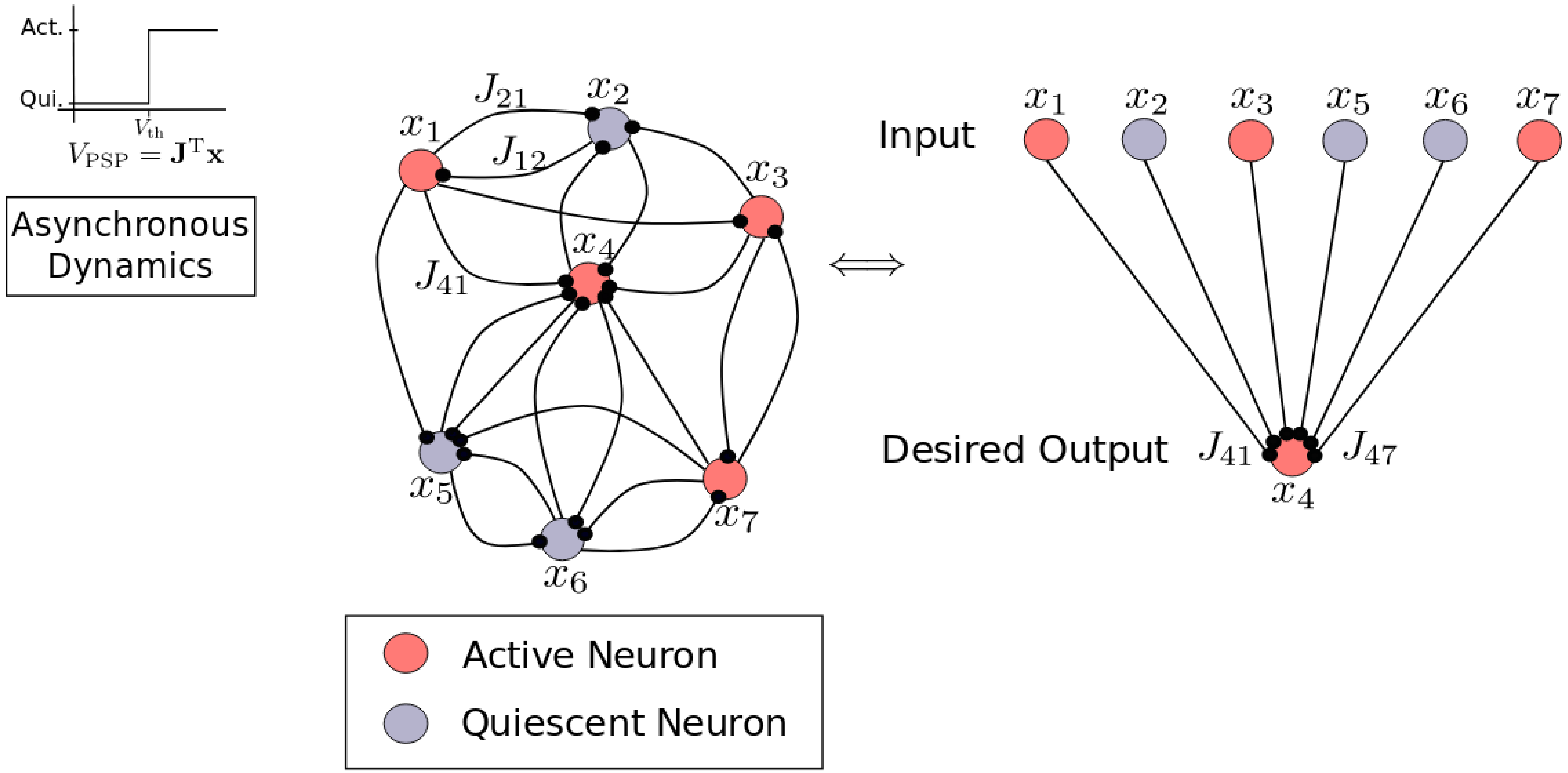


Asynchronous Dynamics

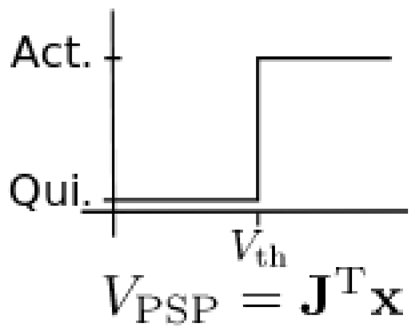


Active Neuron

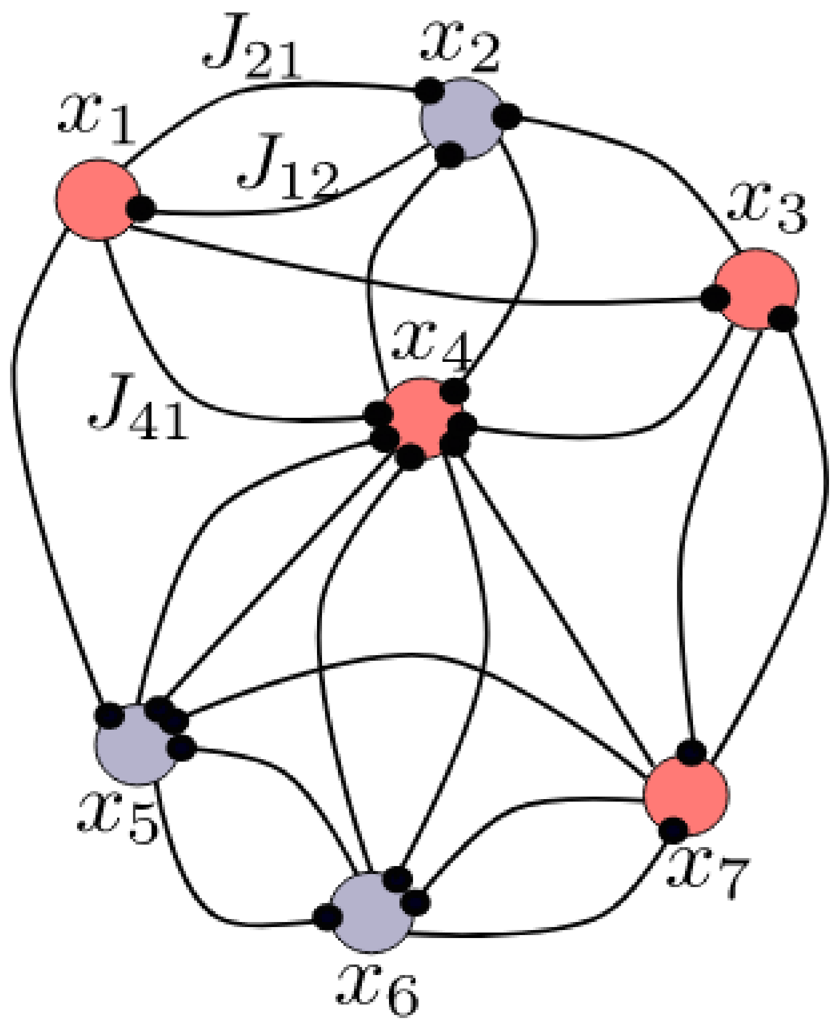
Quiescent Neuron



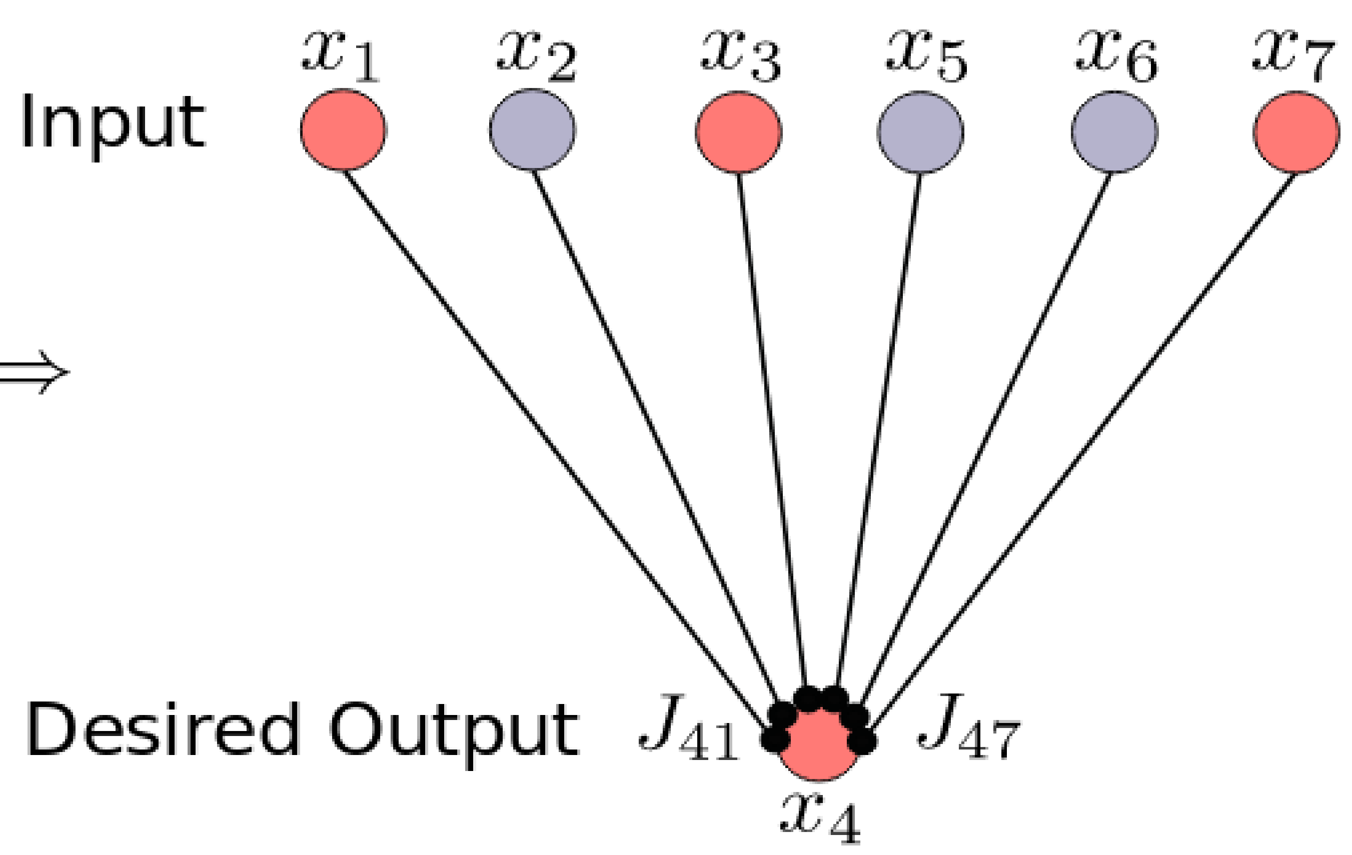
Act.
Qui.
V_{th}
$V_{\mathrm{PSP}} = \mathbf{J}^{\mathrm{T}}\mathbf{x}$
Asynchronous Dynamics
J_{21}
J_{12}
J_{41}
x_1
x_2
x_3
x_4
x_5
x_6
x_7
Active Neuron
Quiescent Neuron
$\Longleftrightarrow$
Input
x_1 x_2 x_3 x_5 x_6 x_7
Desired Output
J_{41}
J_{47}
x_4



Asynchronous Dynamics

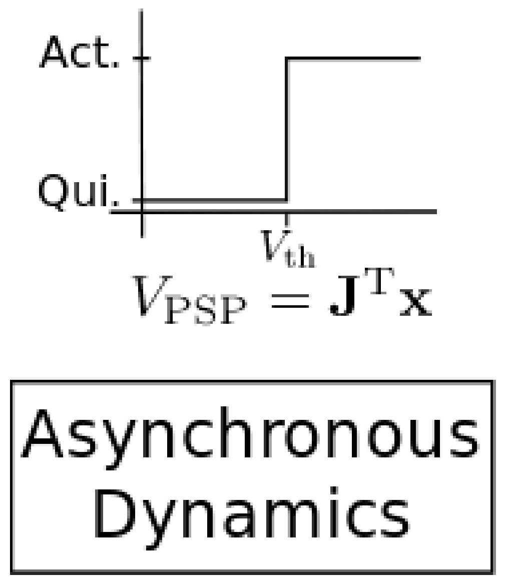


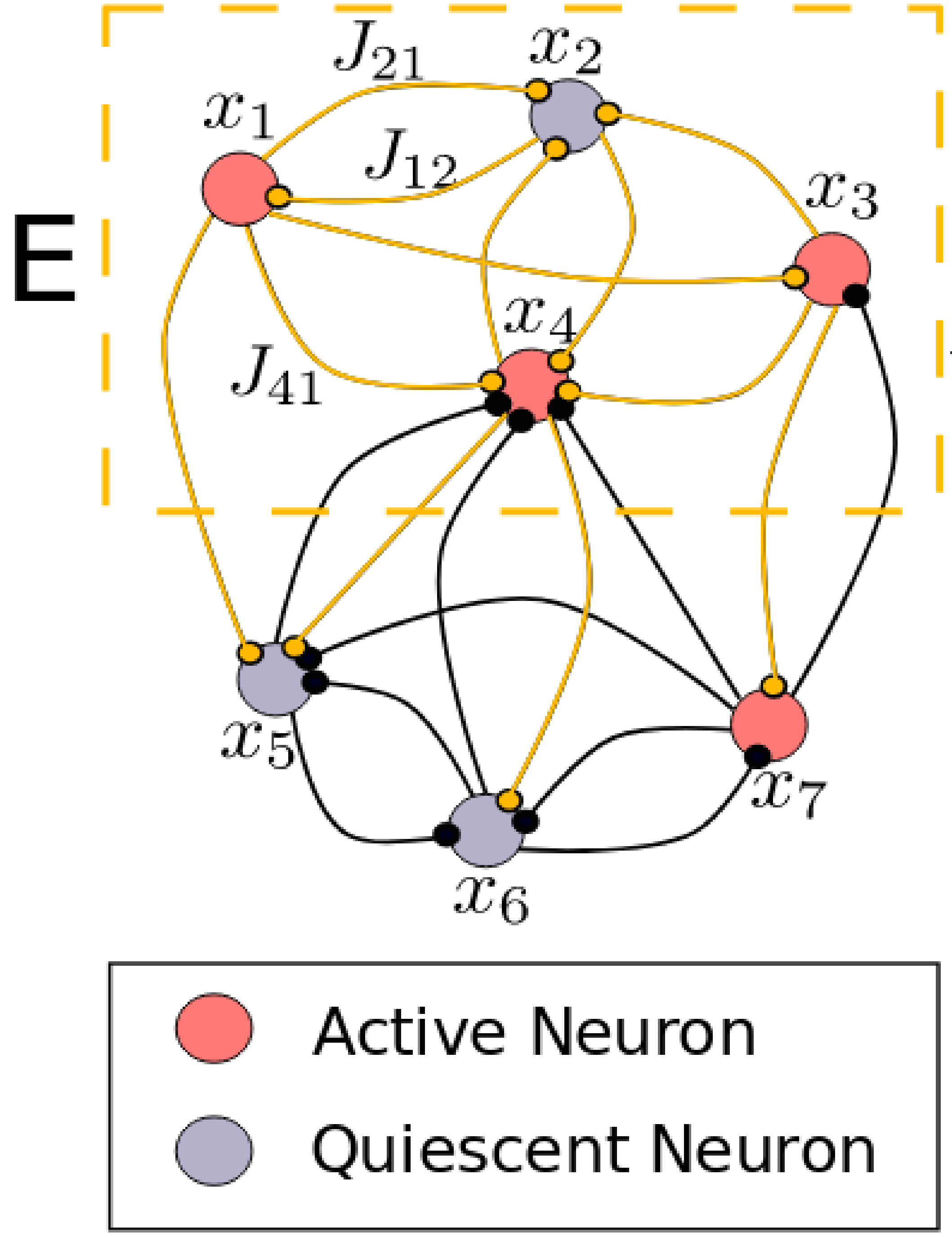
$\Longleftrightarrow$



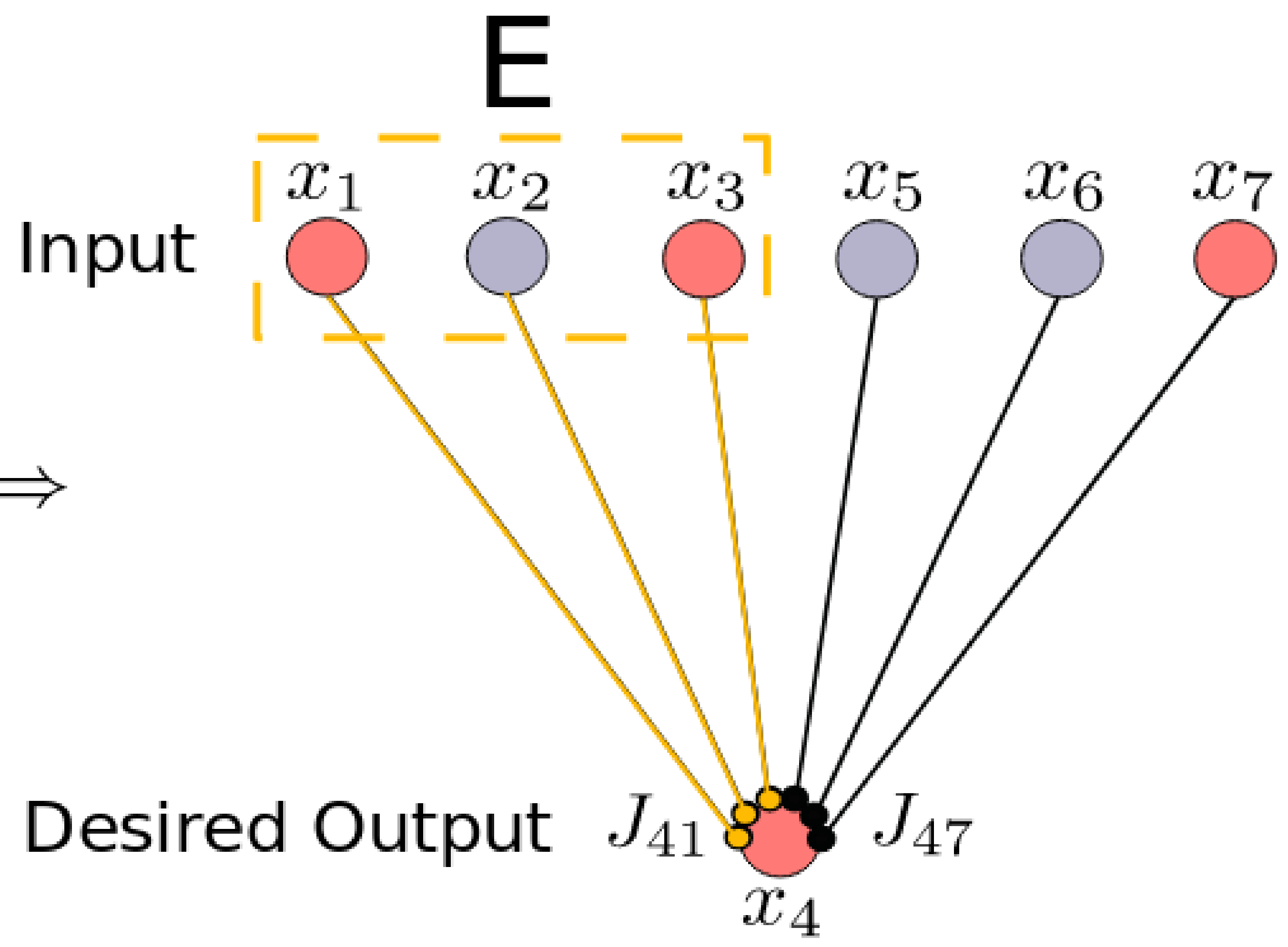
Active Neuron
Quiescent Neuron

Capacity: $\alpha_c = 2$

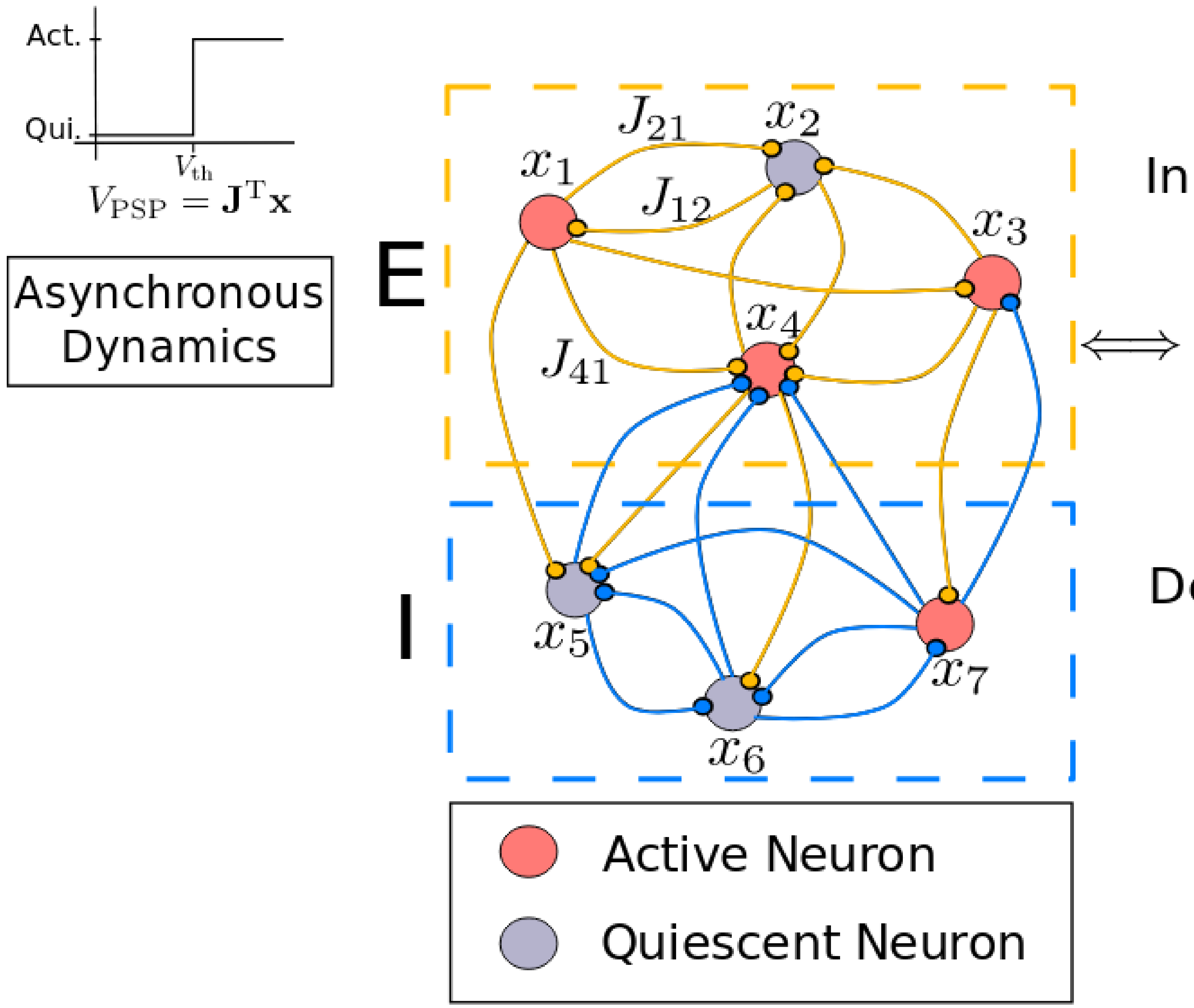


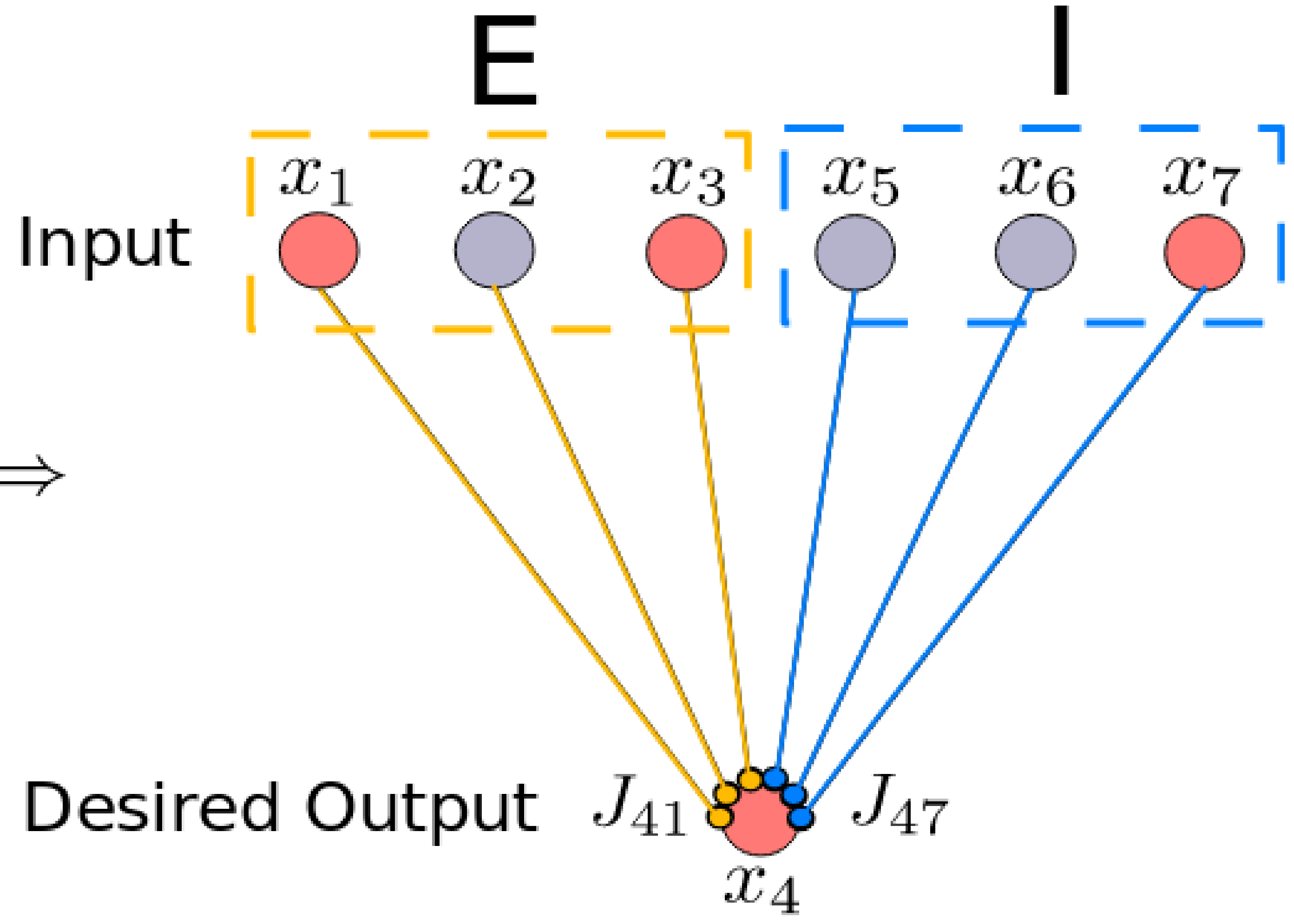


$\Longleftrightarrow$



$J_{ij} \geq 0 \text{ for } j \in \mathrm{E}$

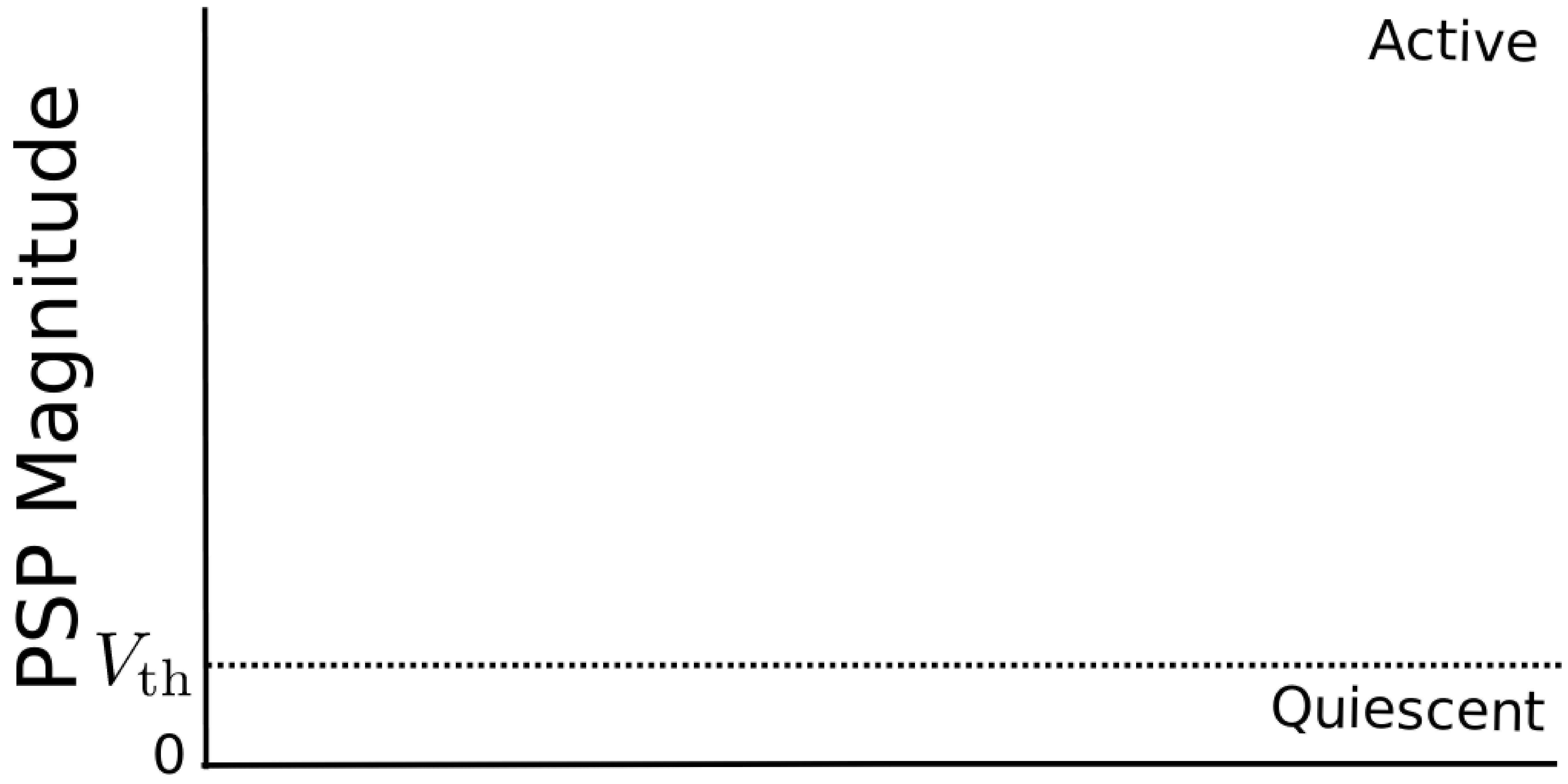




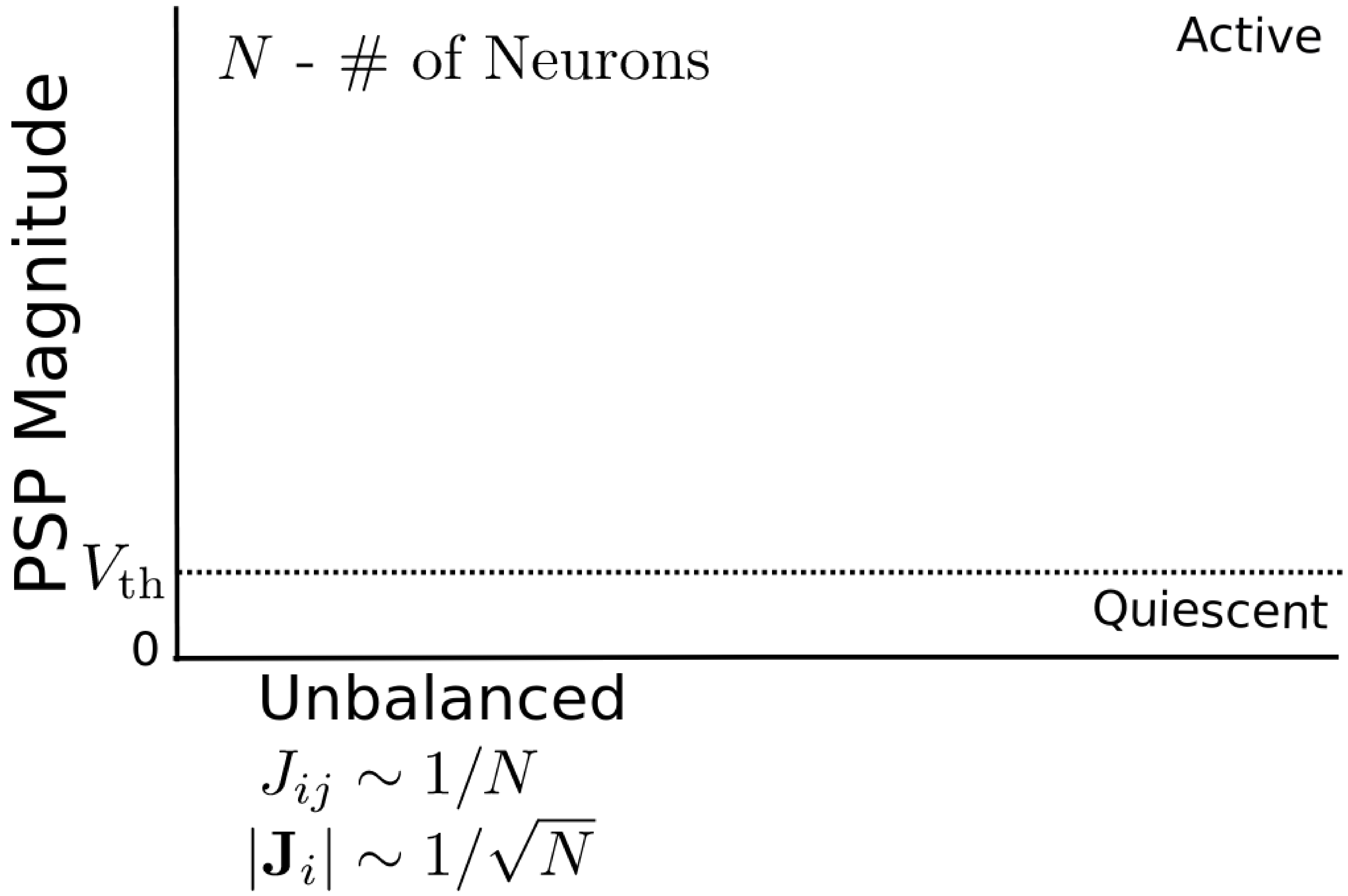
$$J_{ij} \geq 0 \text{ for } j \in \mathrm{E}$$

$$J_{ij} \leq 0 \text{ for } j \in \mathrm{I}$$

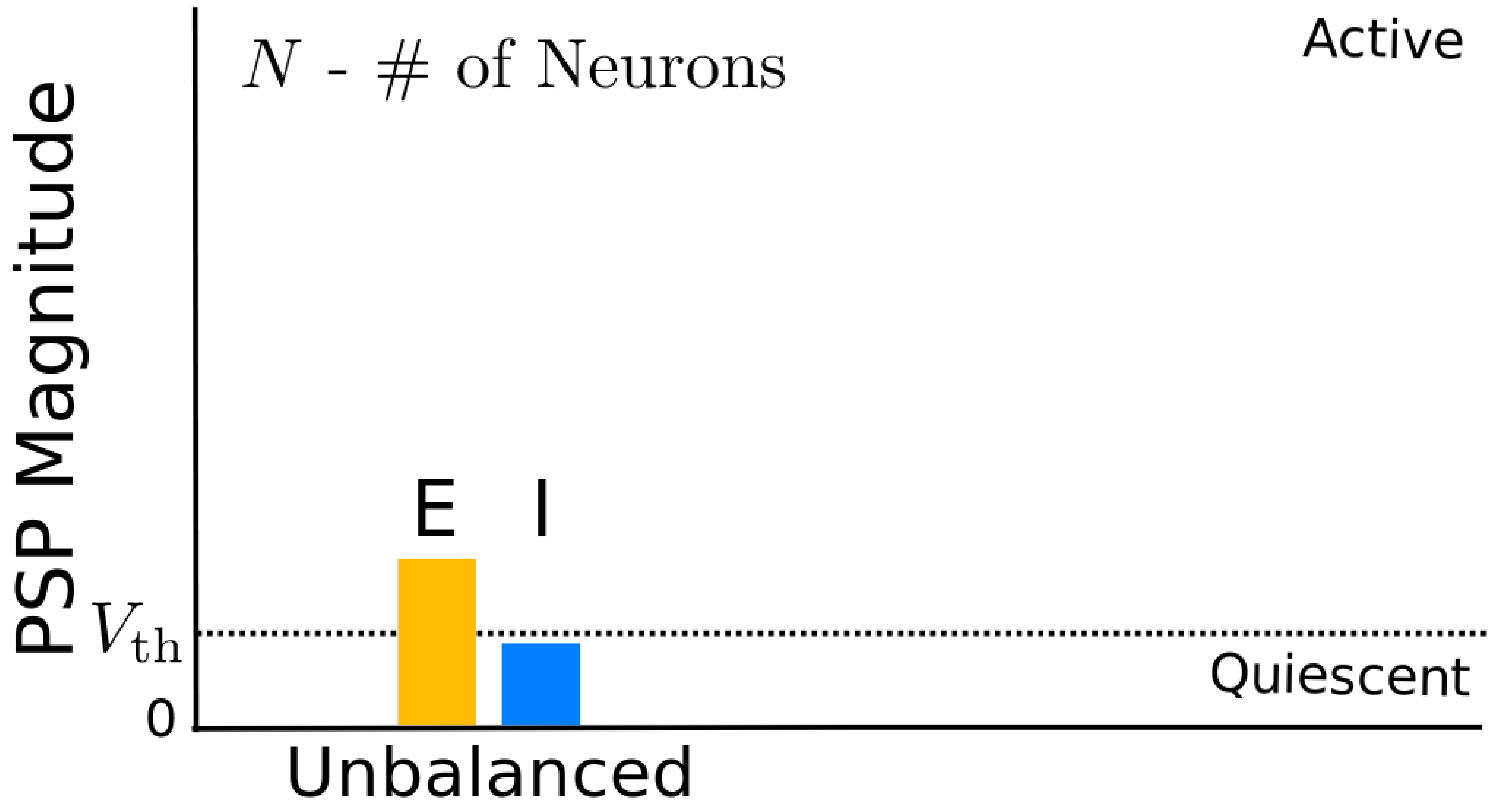
BALANCED VS. UNBALANCED WEIGHTS



BALANCED VS. UNBALANCED WEIGHTS



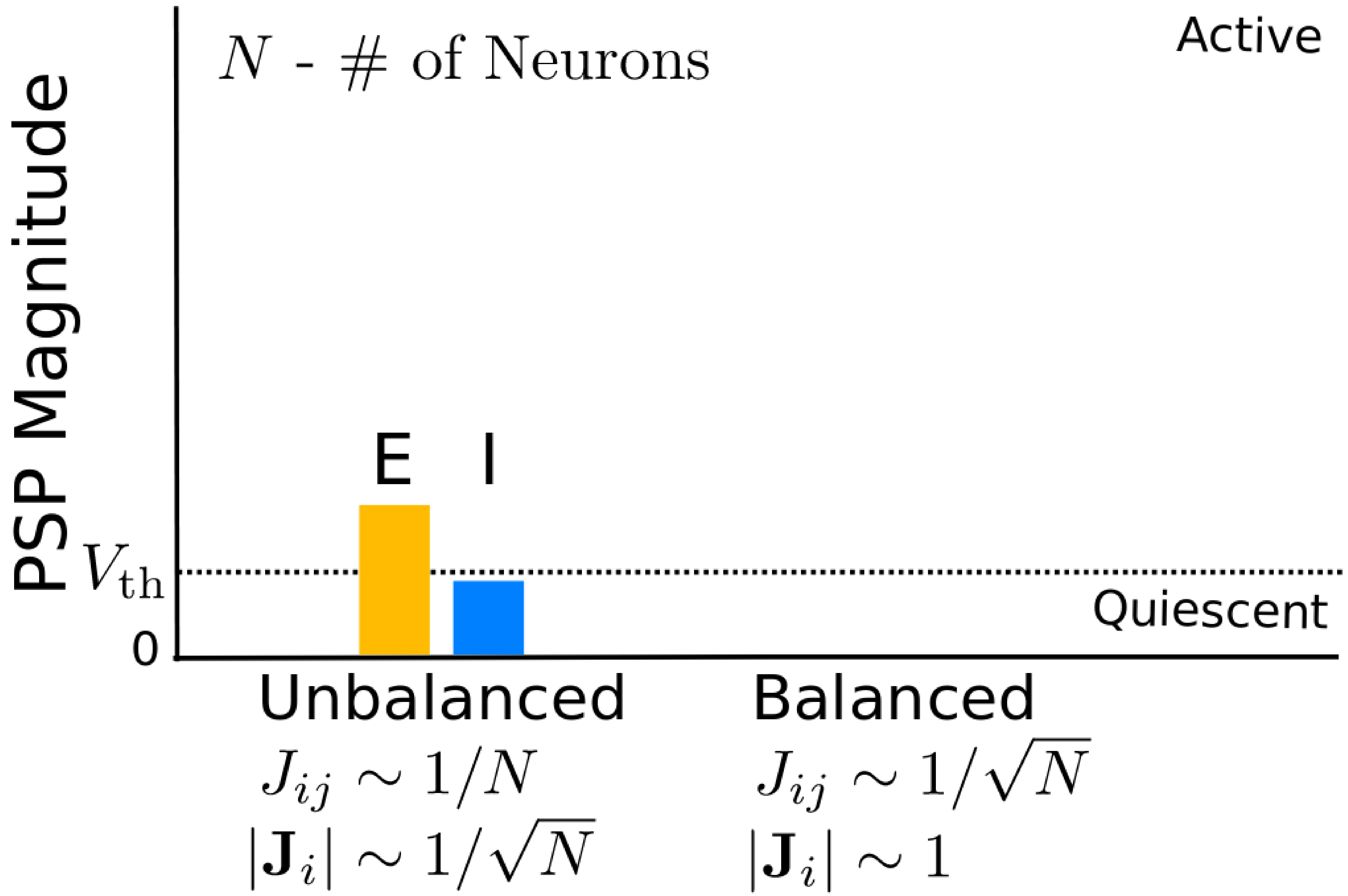
BALANCED VS. UNBALANCED WEIGHTS



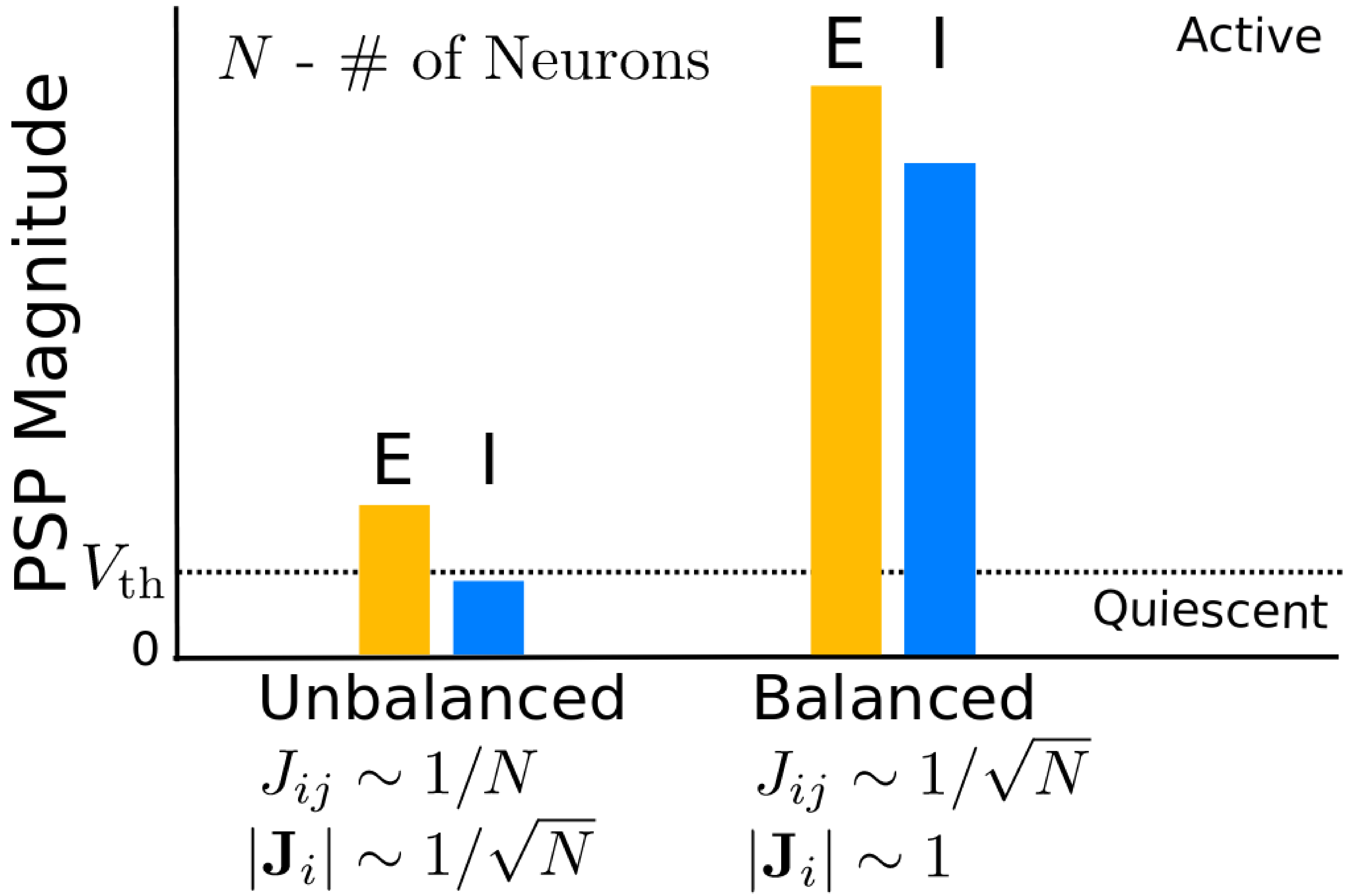
$$J_{ij} \sim 1/N$$

$$|\mathbf{J}_i| \sim 1/\sqrt{N}$$

BALANCED VS. UNBALANCED WEIGHTS



BALANCED VS. UNBALANCED WEIGHTS



ROBUSTNESS TO NOISE

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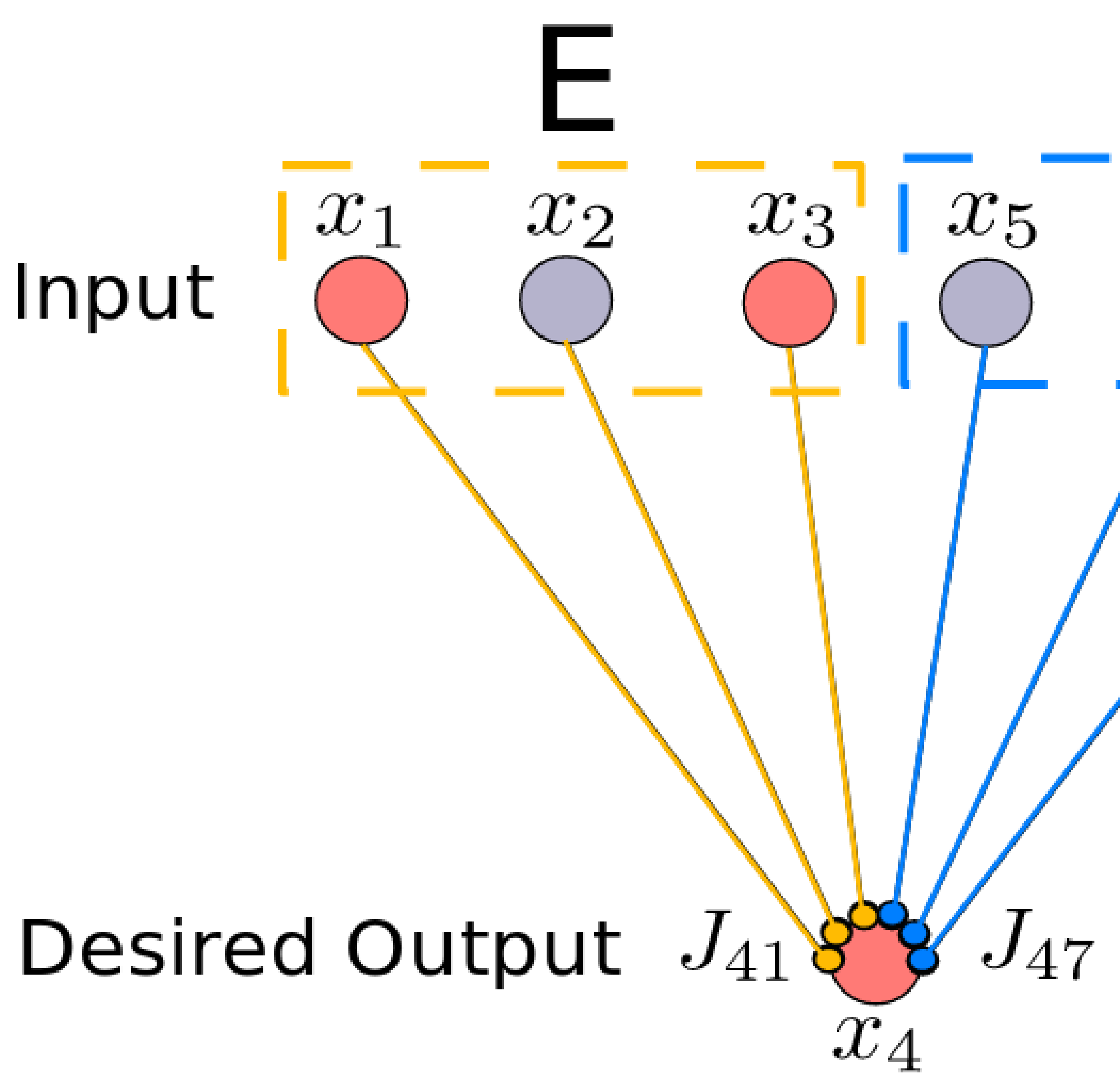

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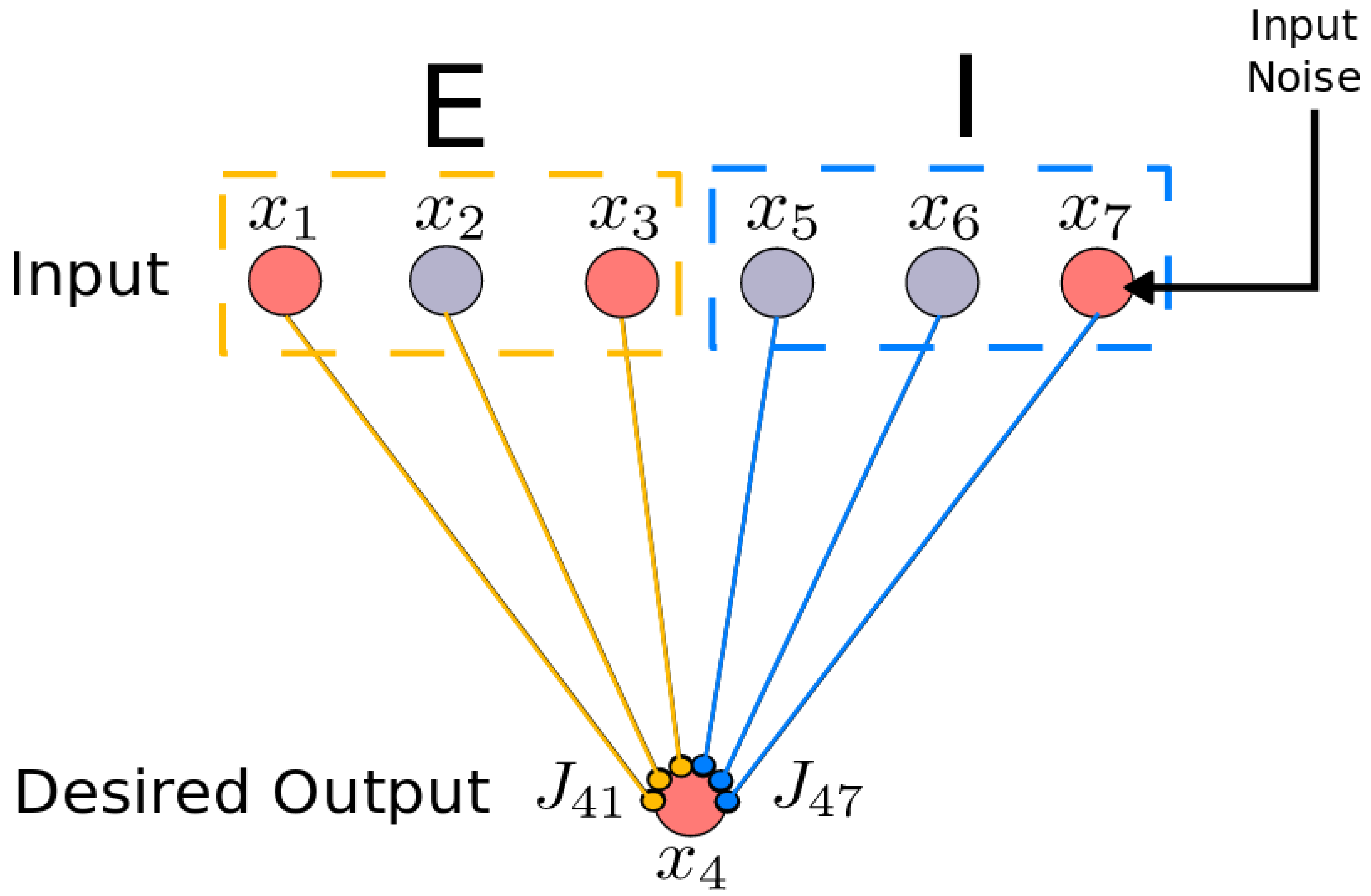
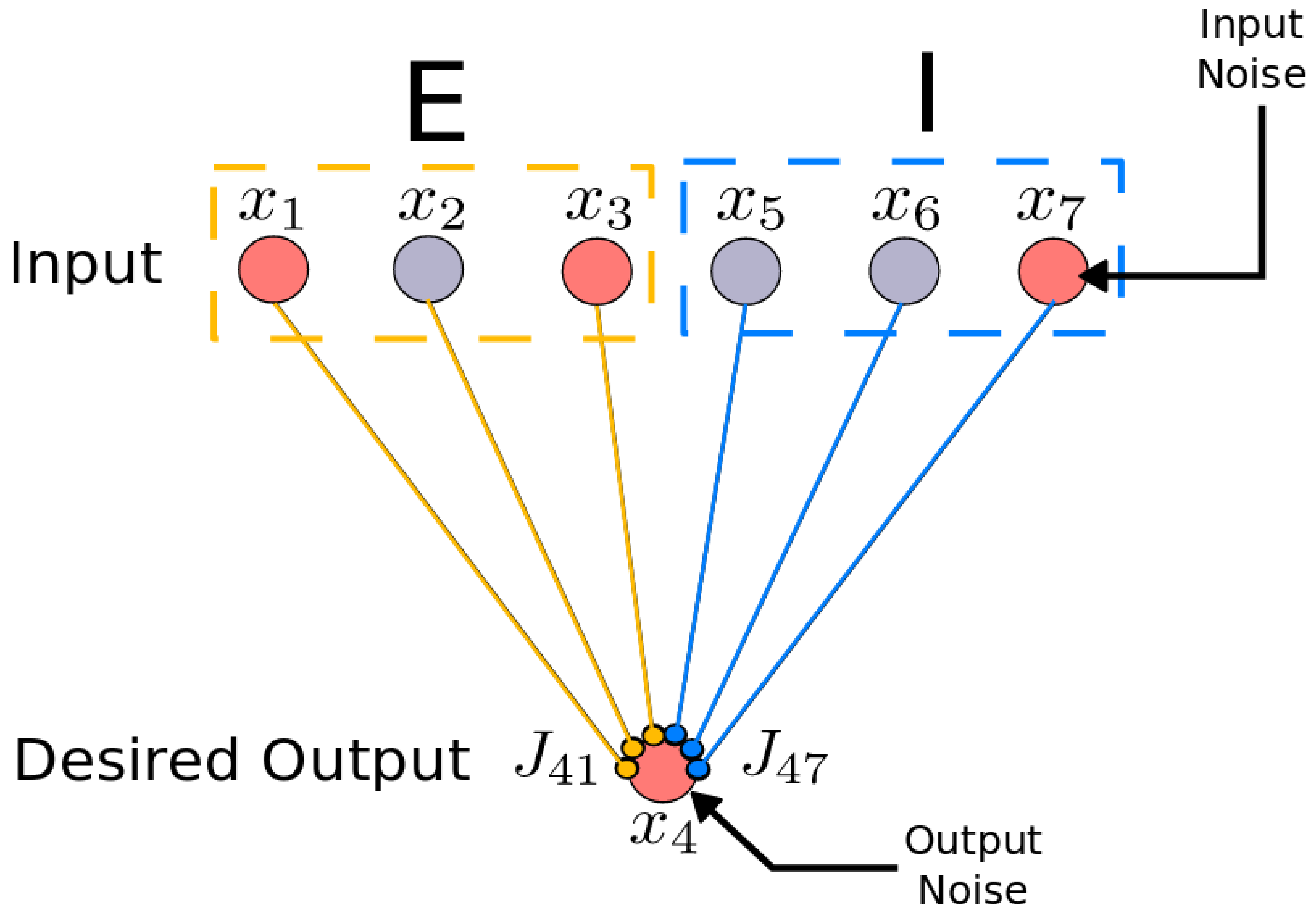
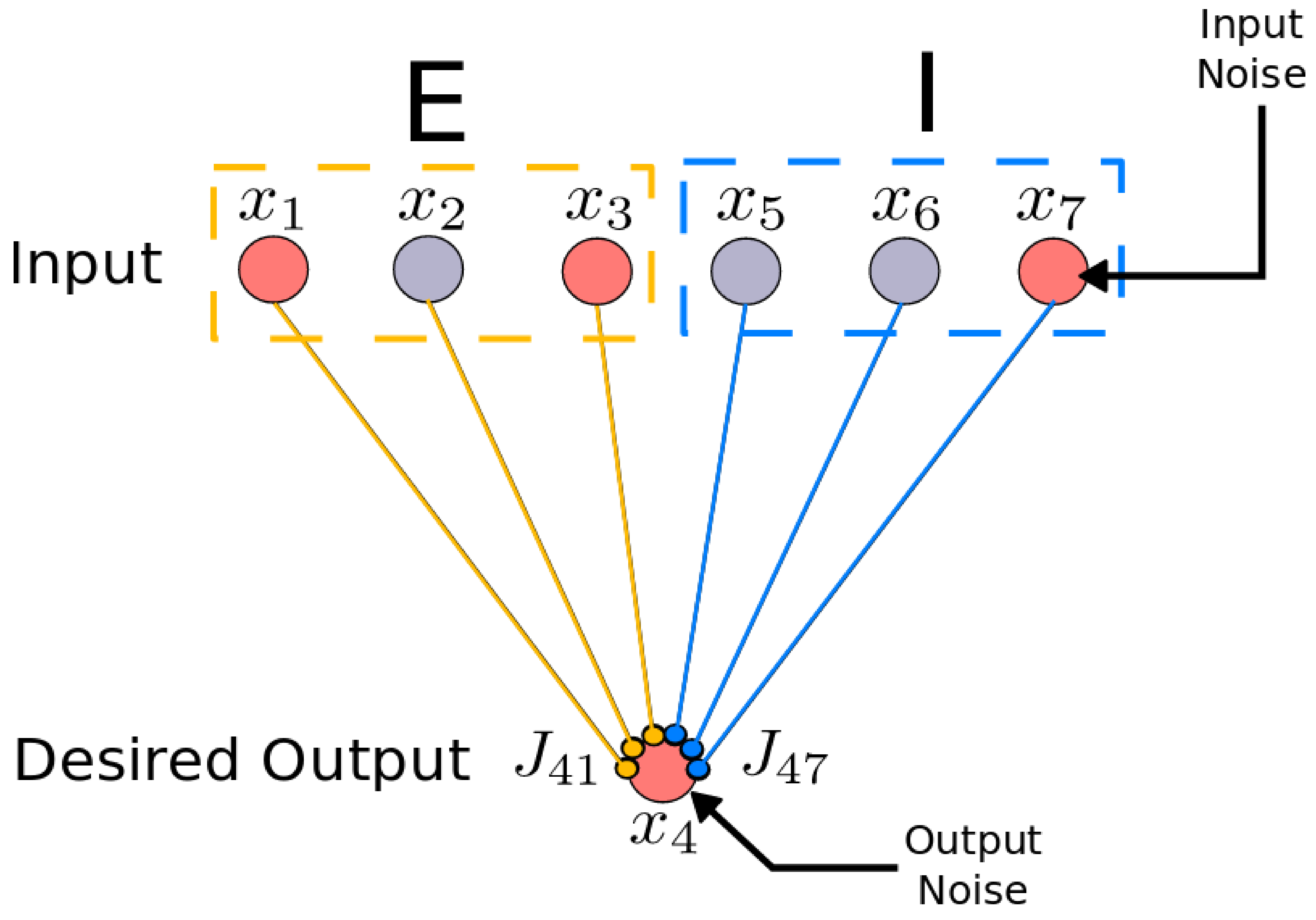


ROBUSTNESS TO NOISE

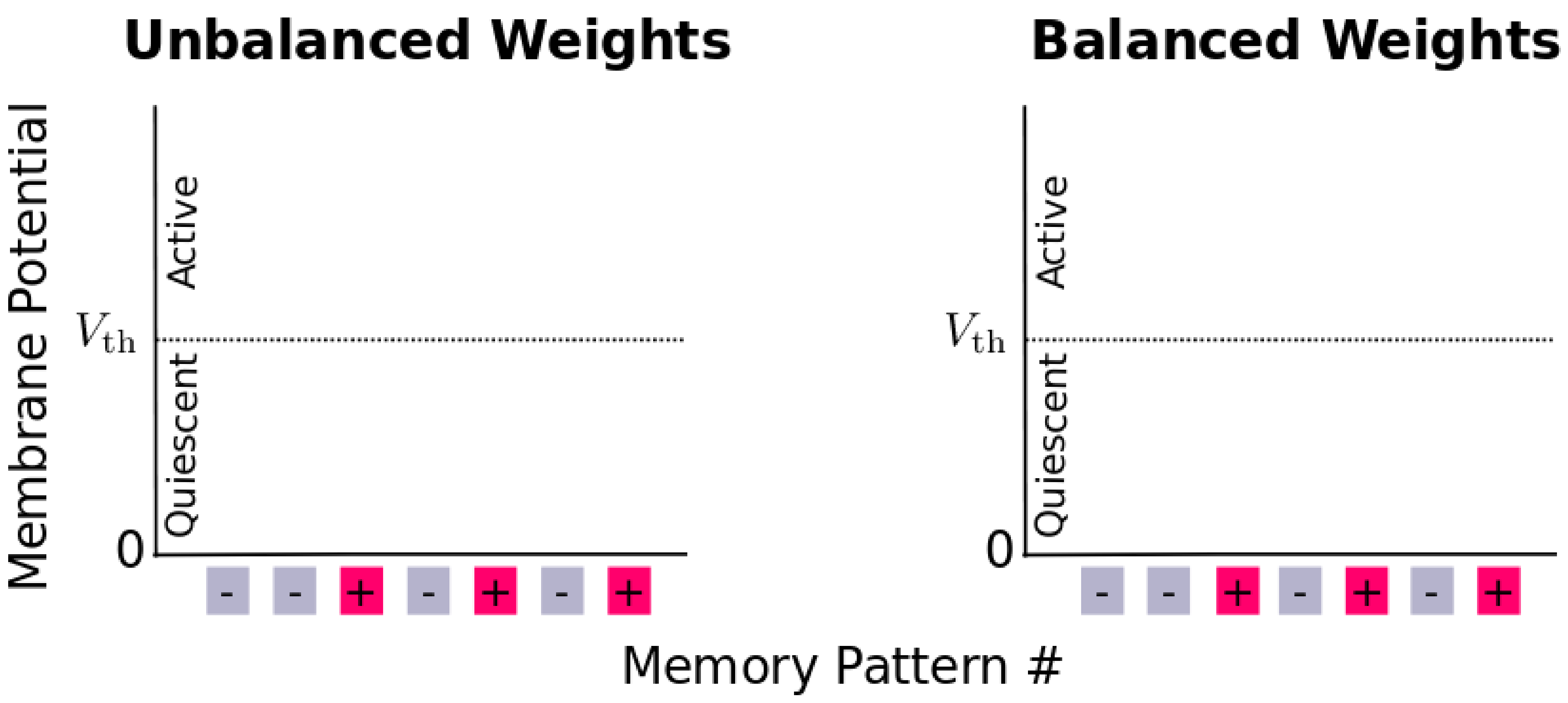


ROBUSTNESS TO NOISE

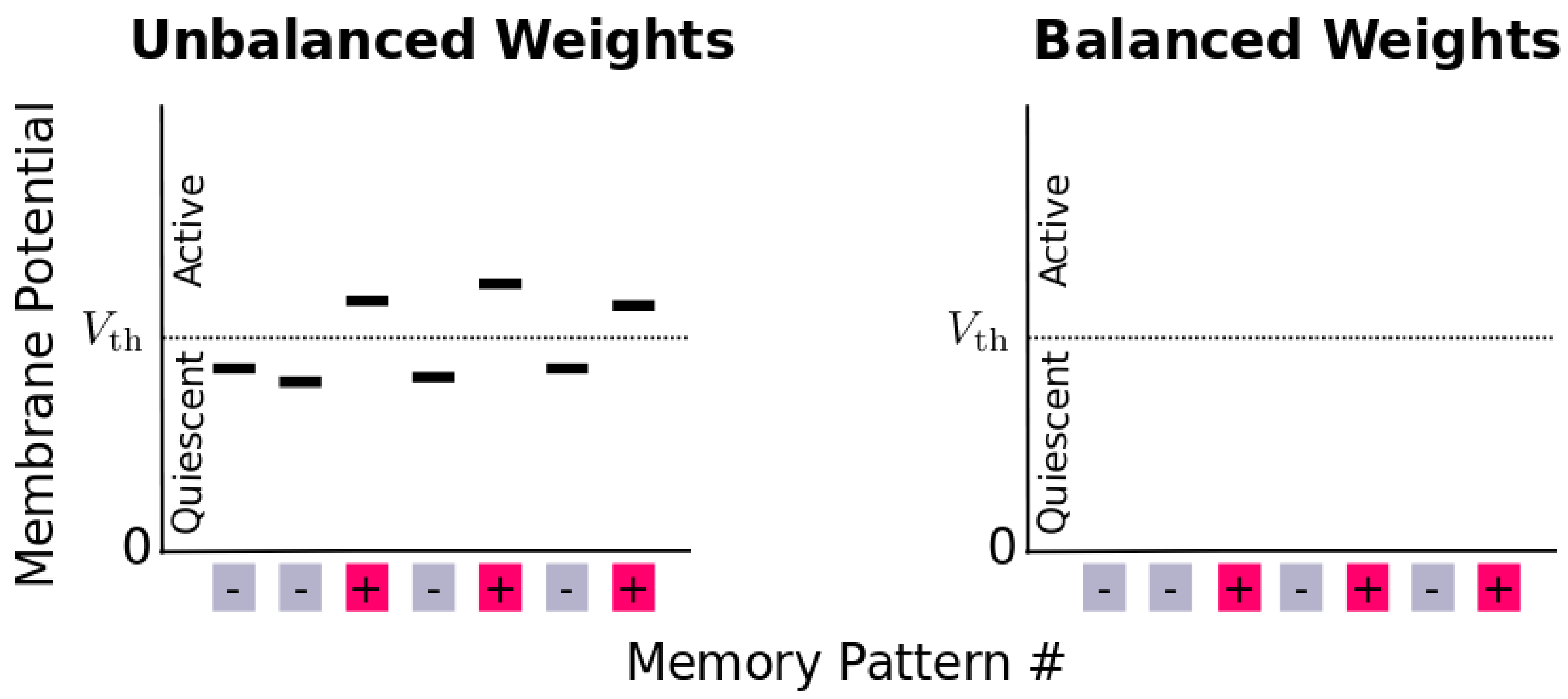


$$V_{\text{PSP}} = \sum_{j=1}^{N} J_{ij} \left(x_i + n_i^{\text{in}}\right) + n^{\text{out}}$$

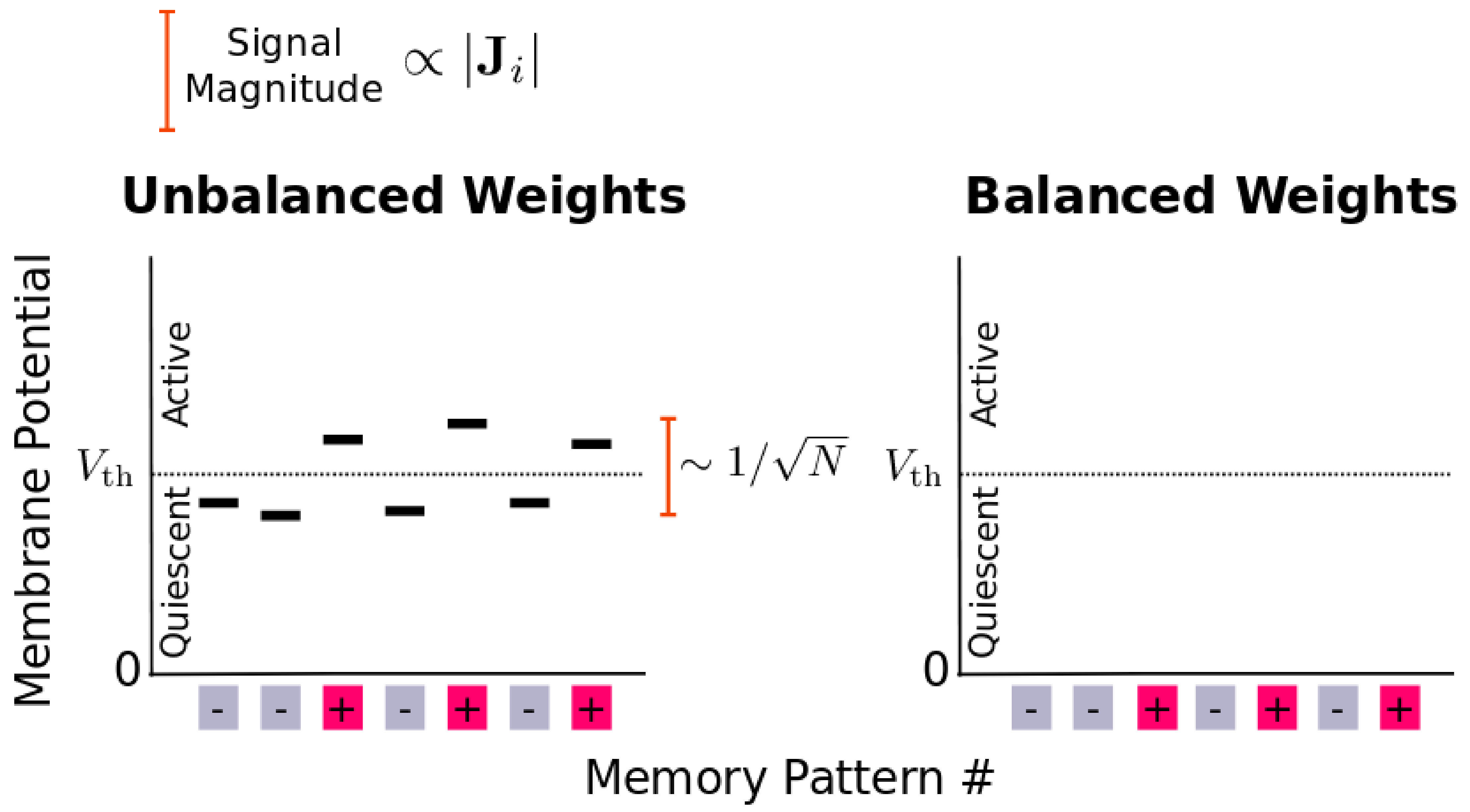
SIGNAL TO NOISE



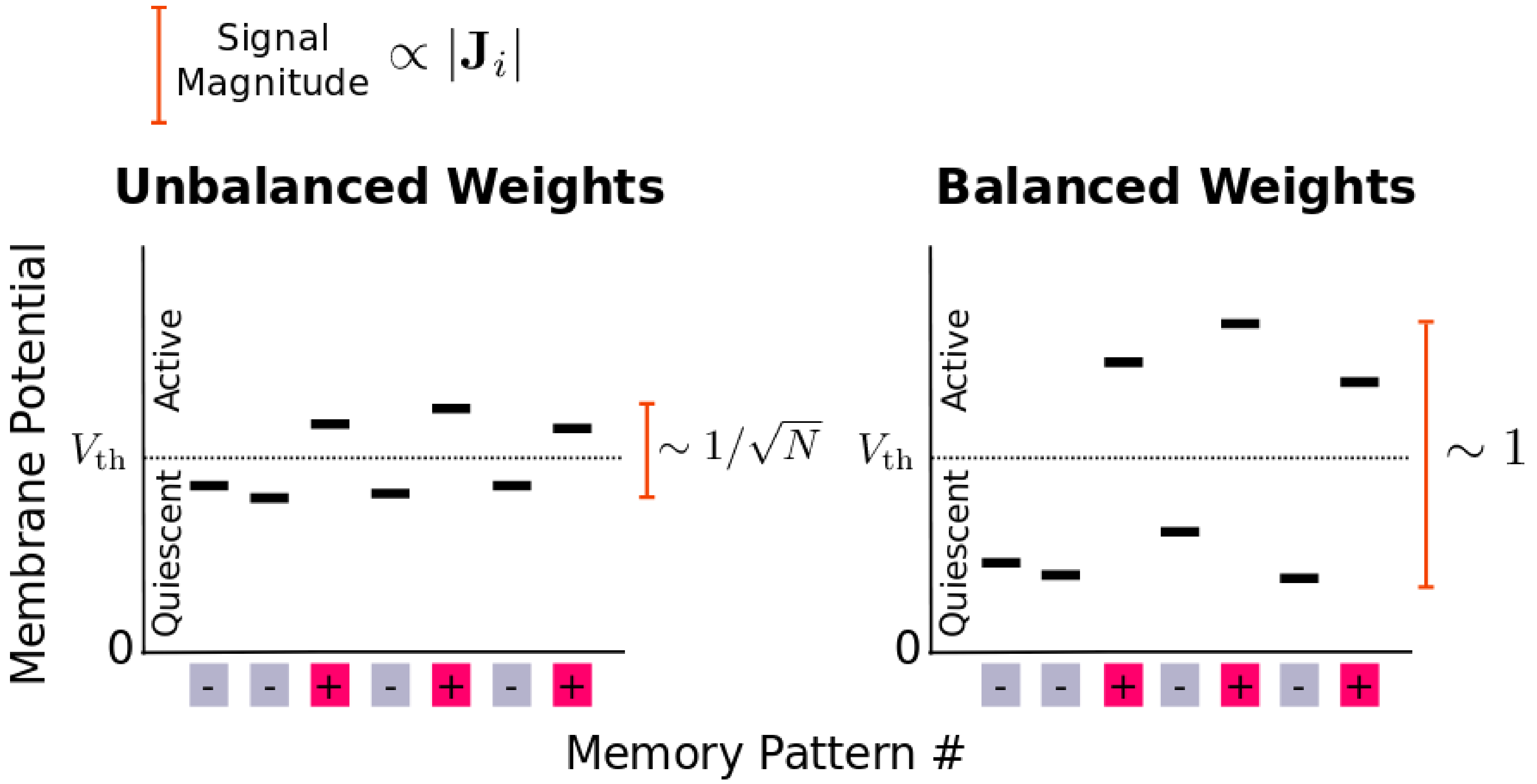
SIGNAL TO NOISE



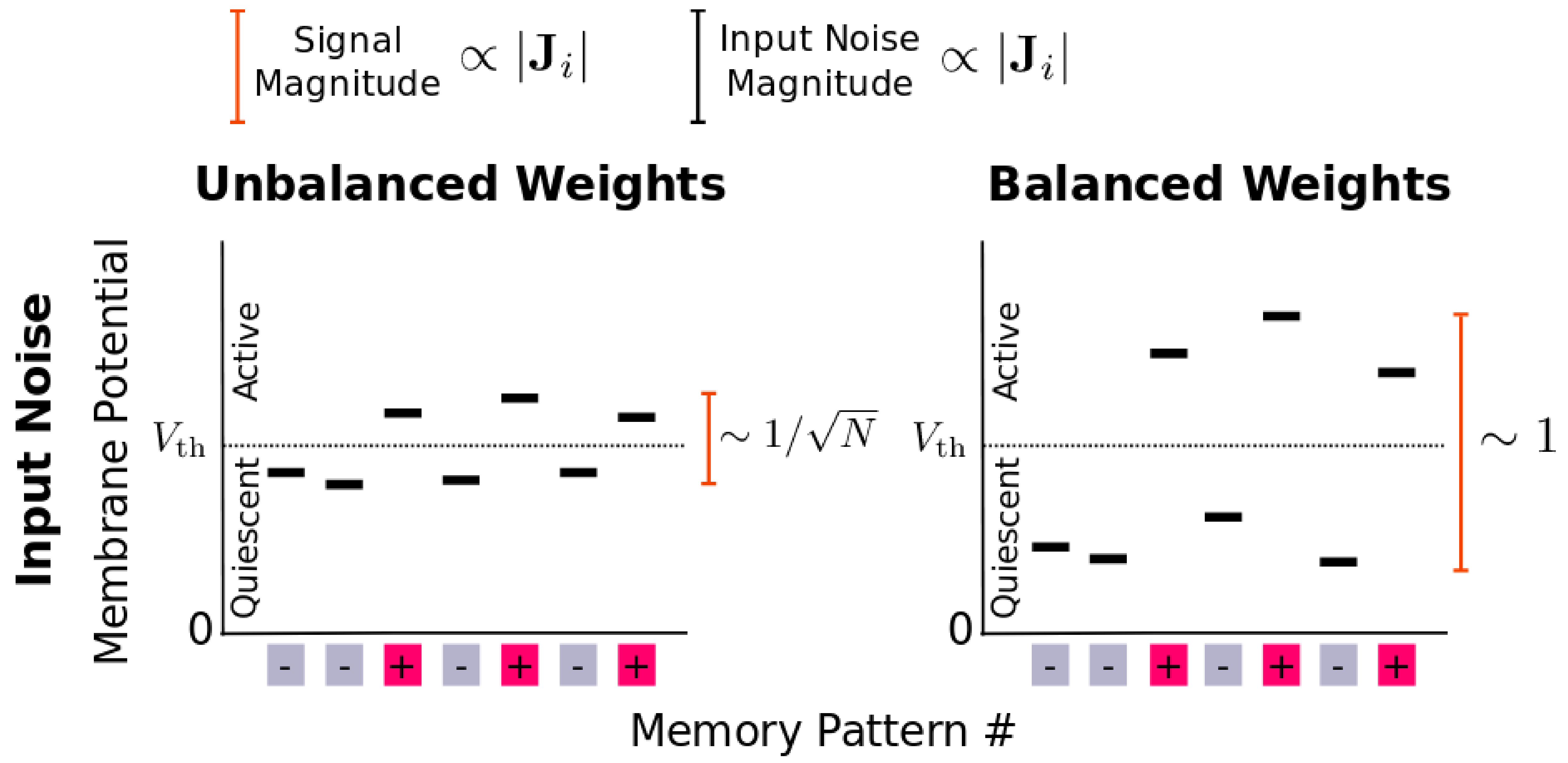
SIGNAL TO NOISE



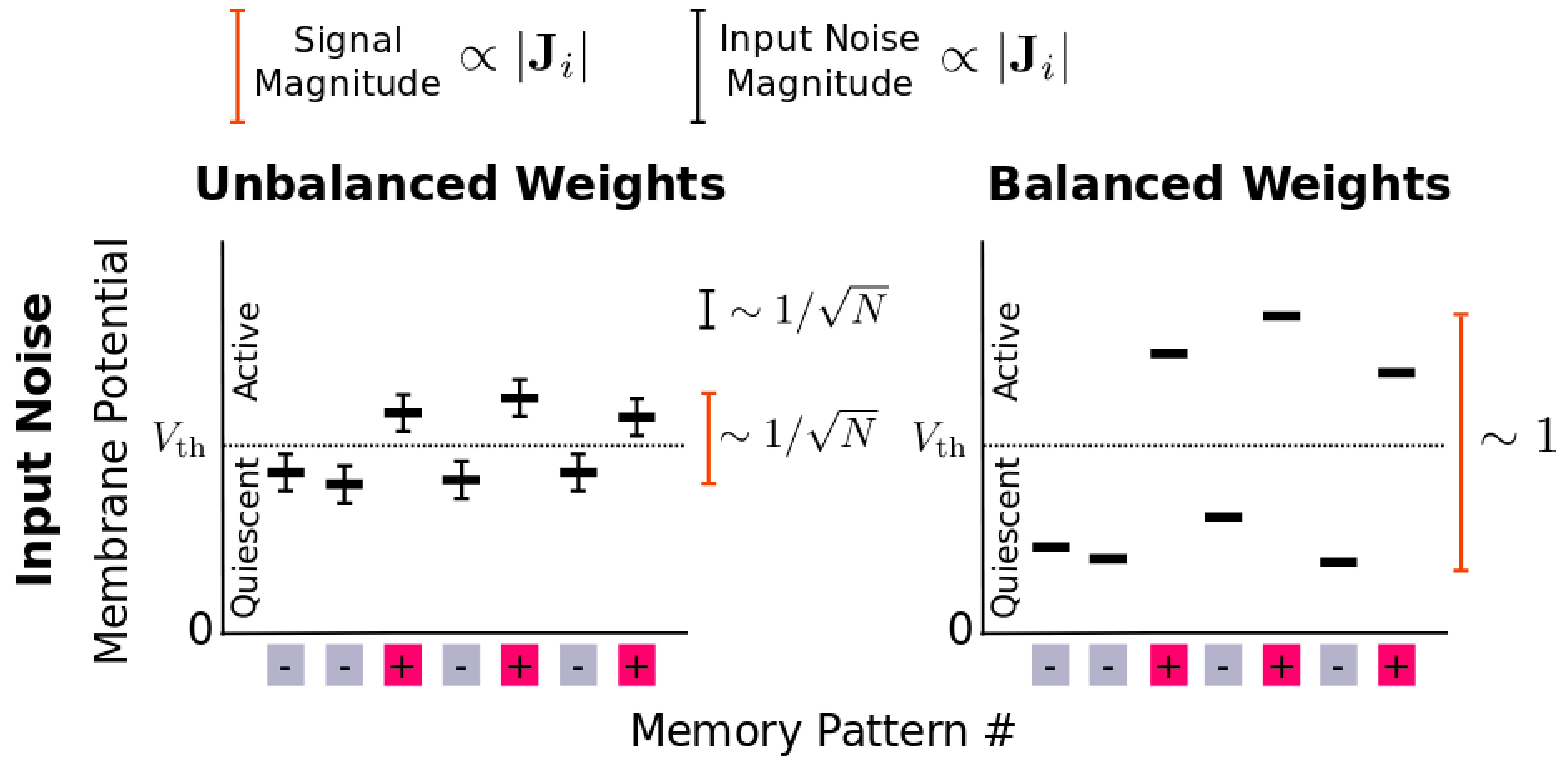
SIGNAL TO NOISE



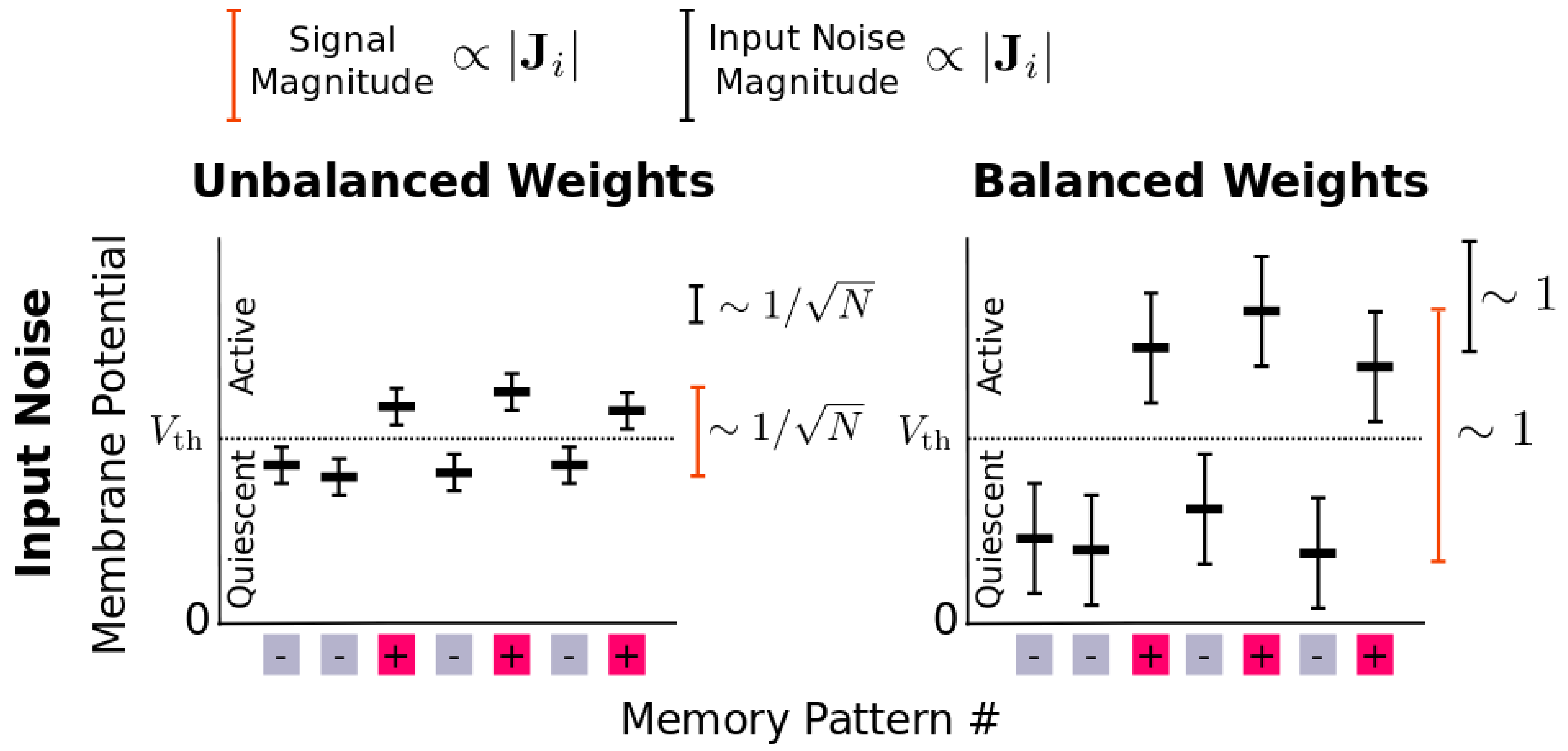
SIGNAL TO NOISE



SIGNAL TO NOISE

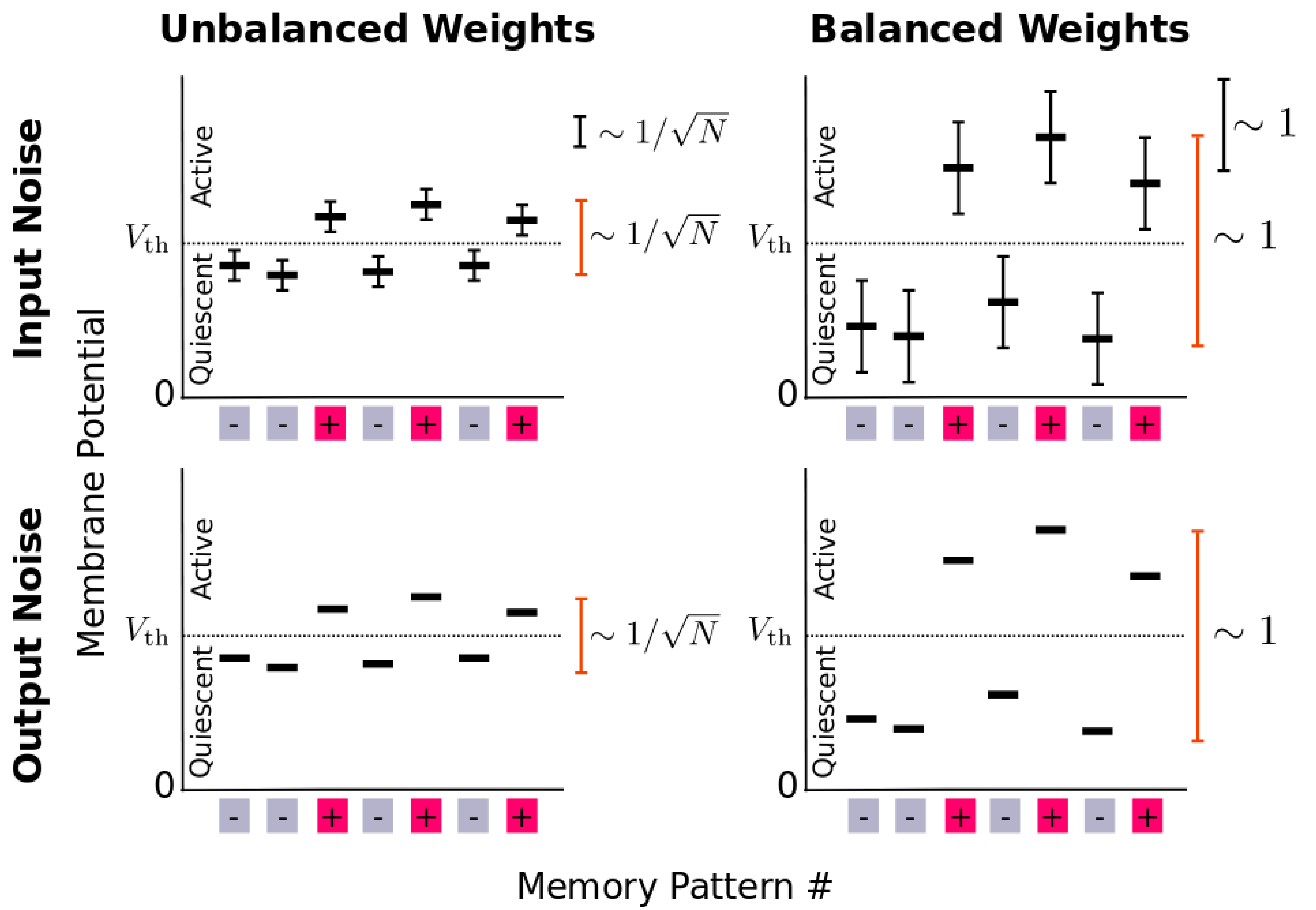


SIGNAL TO NOISE

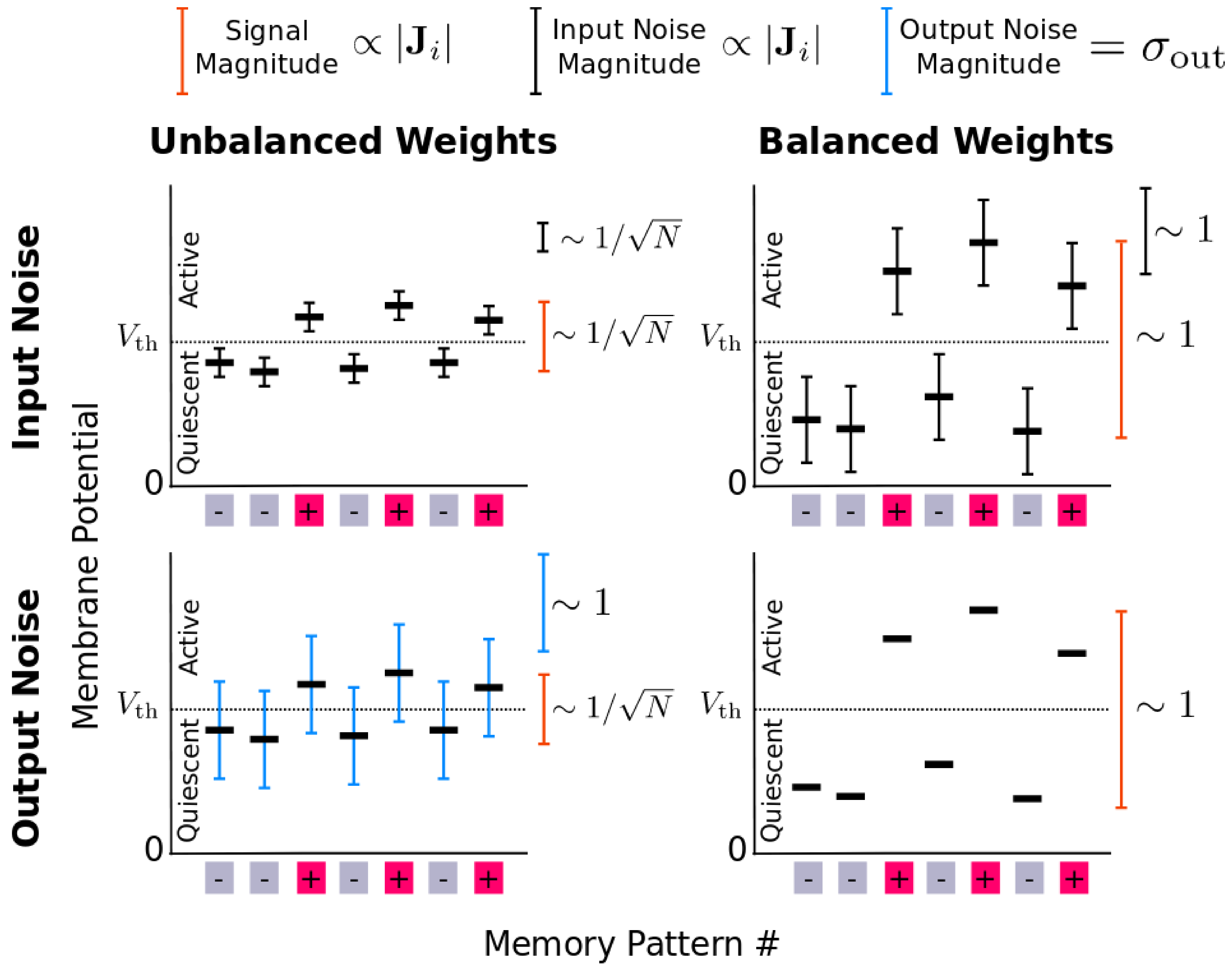


SIGNAL TO NOISE

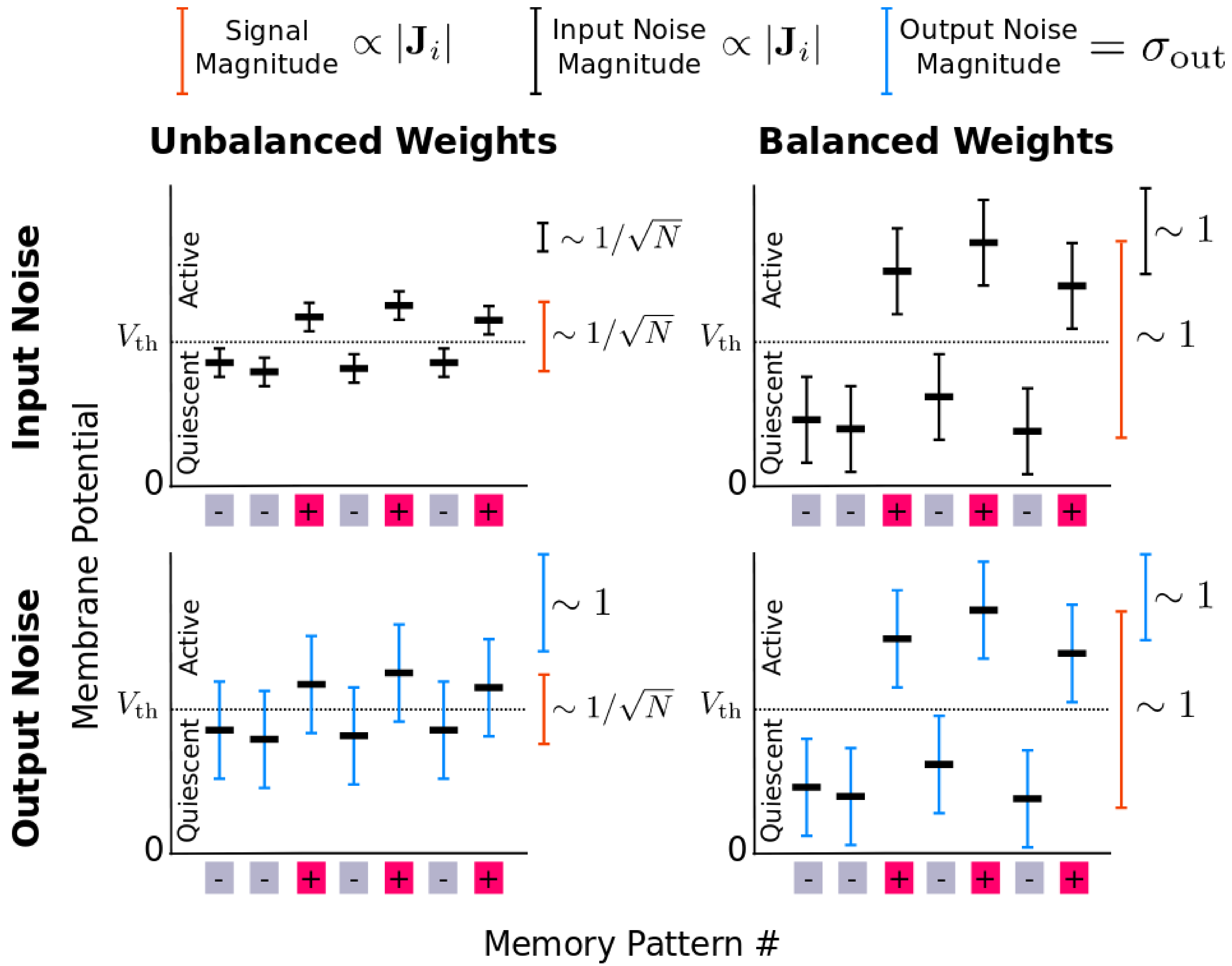
Signal Magnitude $\propto |\mathbf{J}_i|$ — Input Noise Magnitude $\propto |\mathbf{J}_i|$ — Output Noise Magnitude $= \sigma_{\text{out}}$



SIGNAL TO NOISE



SIGNAL TO NOISE



ONLY BALANCED SYNAPTIC WEIGHTS CAN PRODUCE NETWORKS THAT ARE ROBUST TO BOTH INPUT AND OUTPUT NOISE.

COMPARE TWO ASSOCIATIVE MEMORY NETWORKS:

COMPARE TWO ASSOCIATIVE MEMORY NETWORKS:

Balanced:

Each $\mathbf{J}_i$ is balanced.

COMPARE TWO ASSOCIATIVE MEMORY NETWORKS:

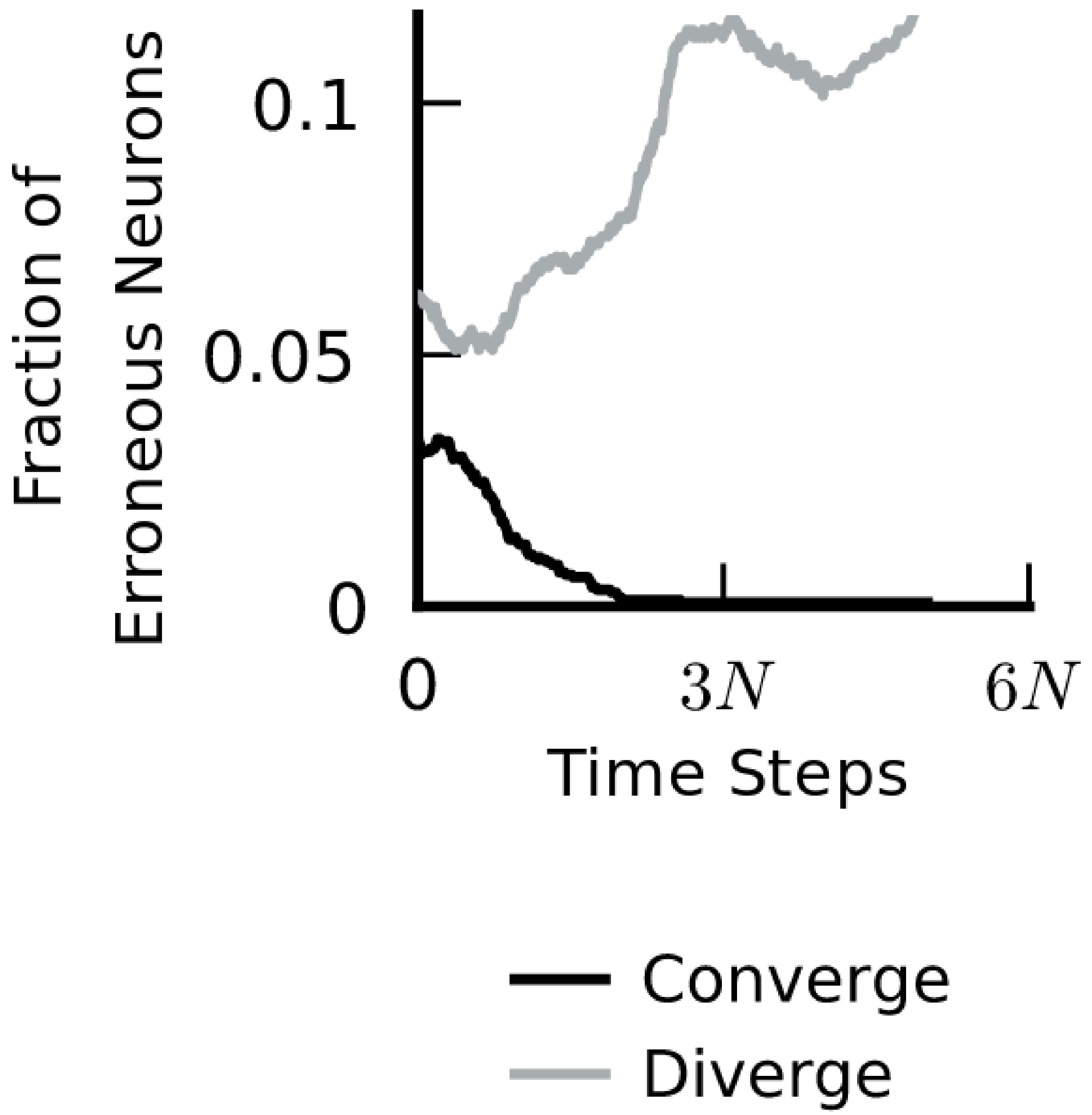
Balanced:

Each $\mathbf{J}_i$ is balanced.

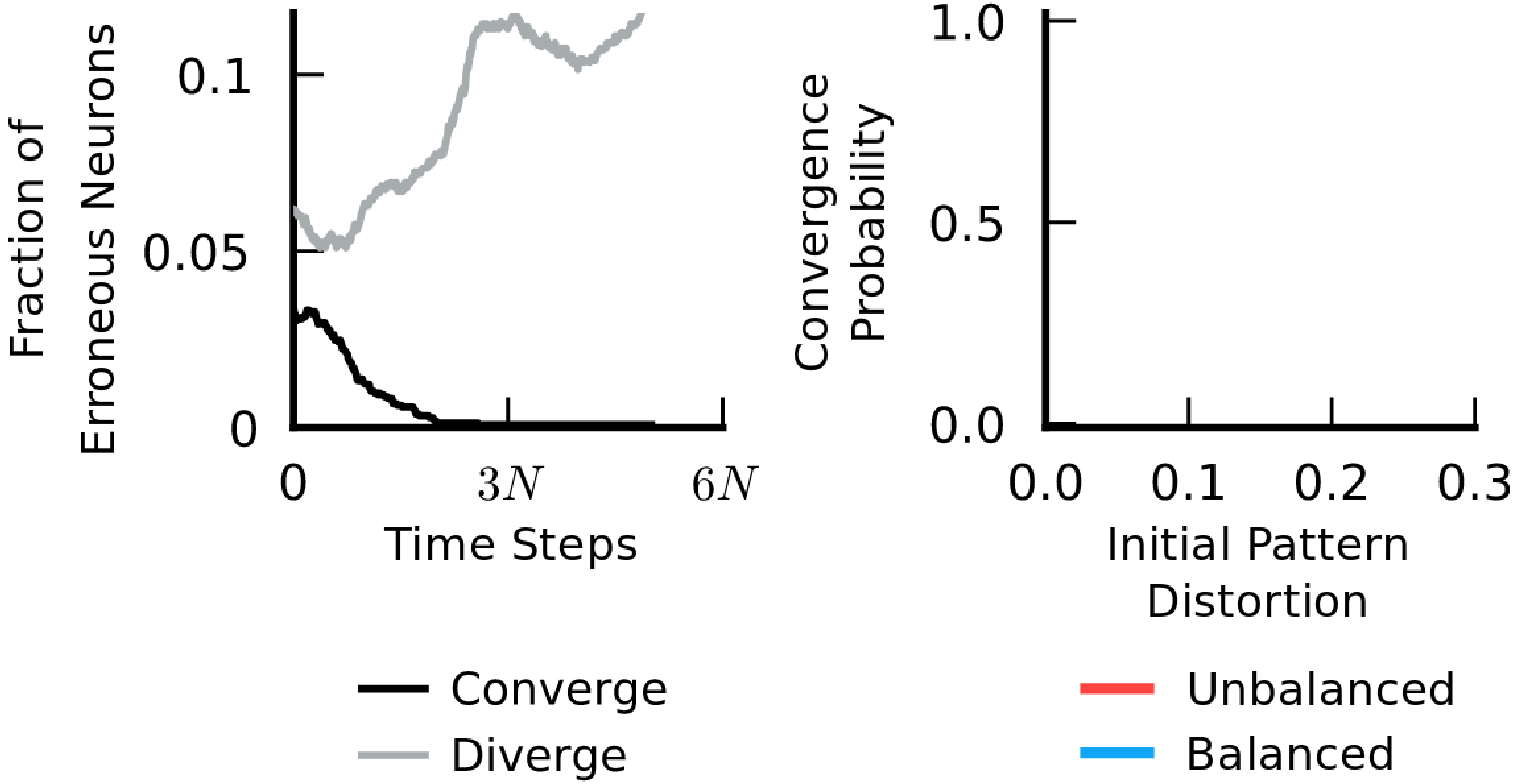
Unbalanced:

Each $\mathbf{J}_i$ is unbalanced.

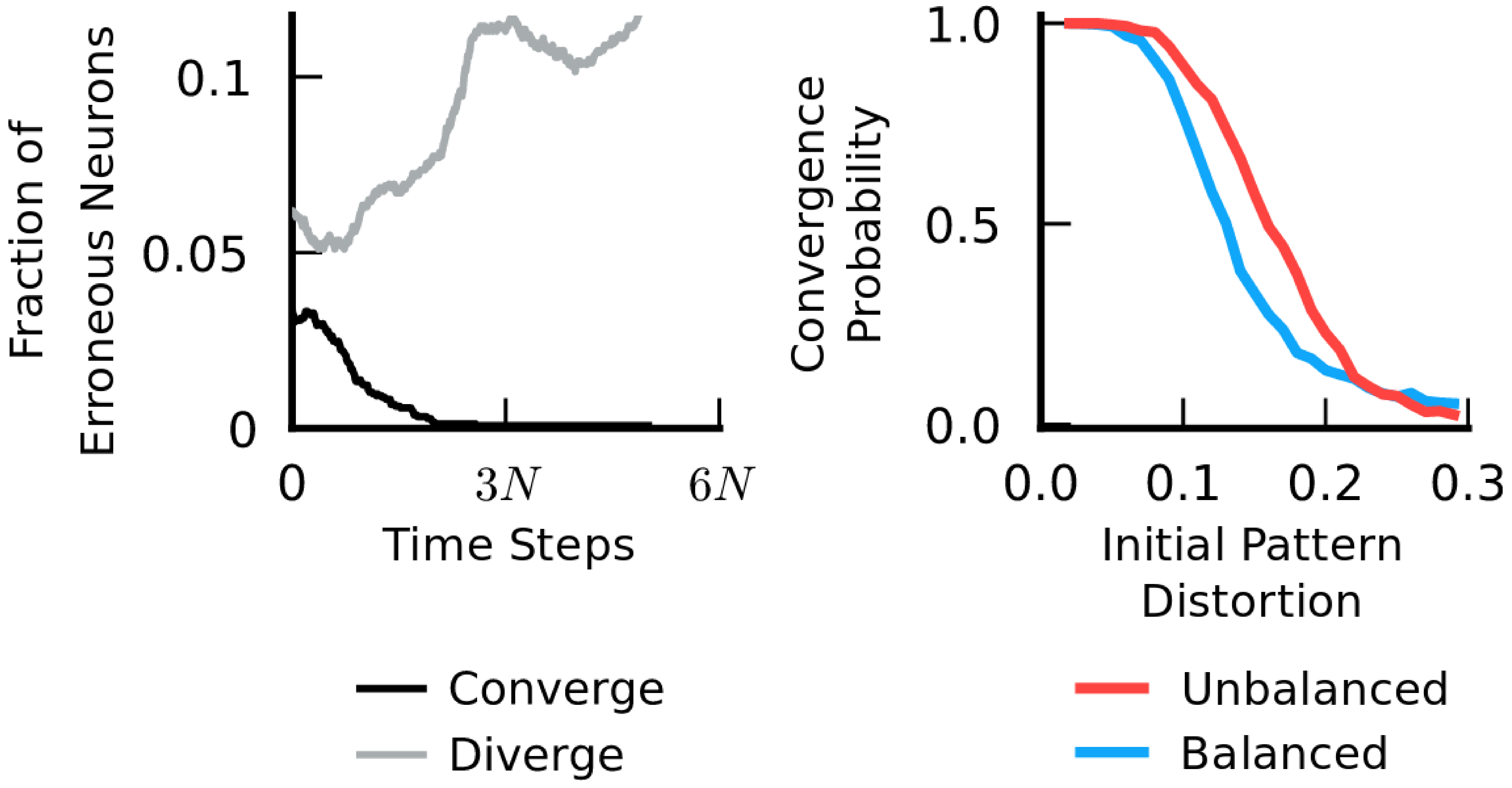
BASINS OF ATTRACTION



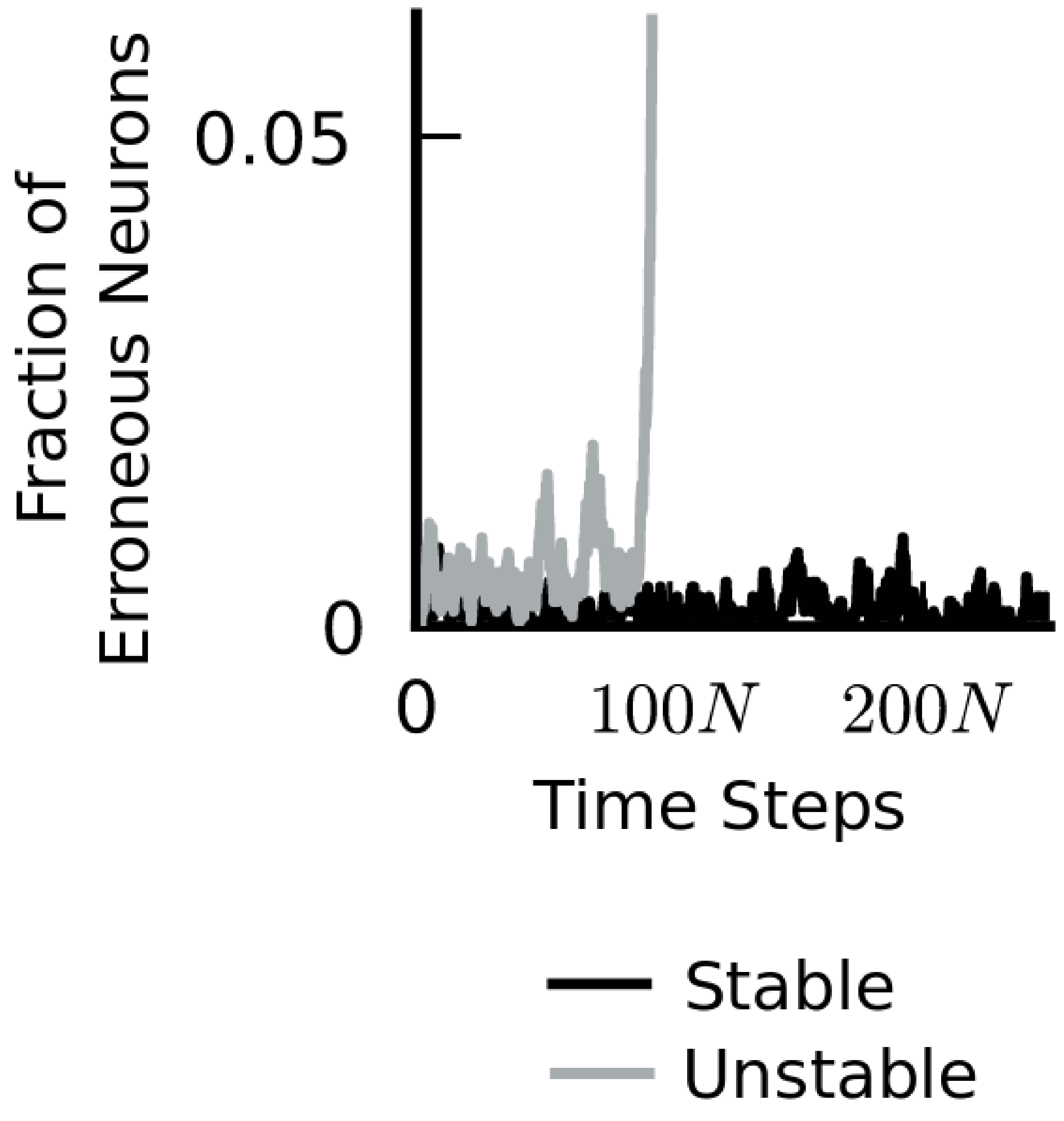
BASINS OF ATTRACTION



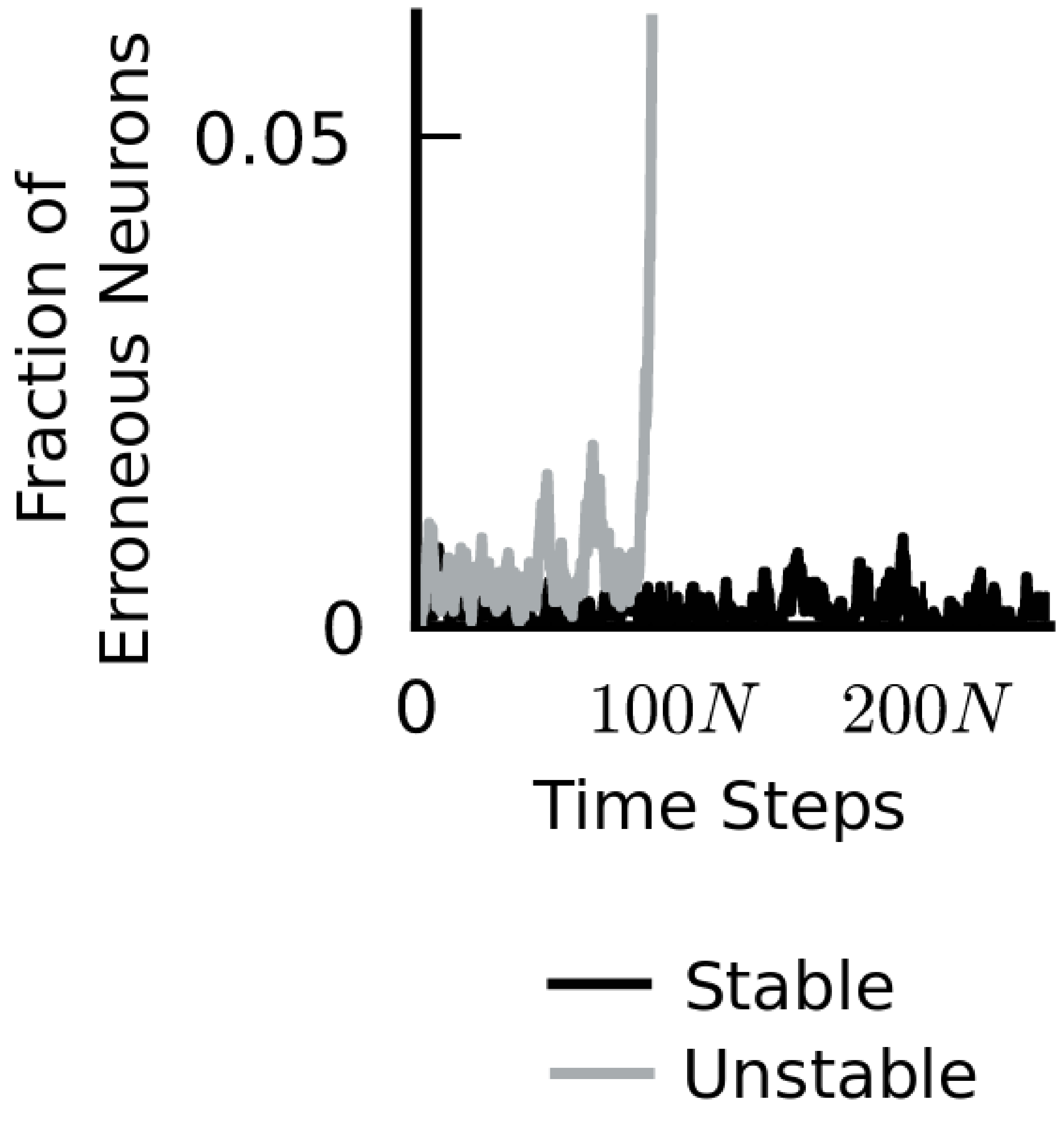
BASINS OF ATTRACTION

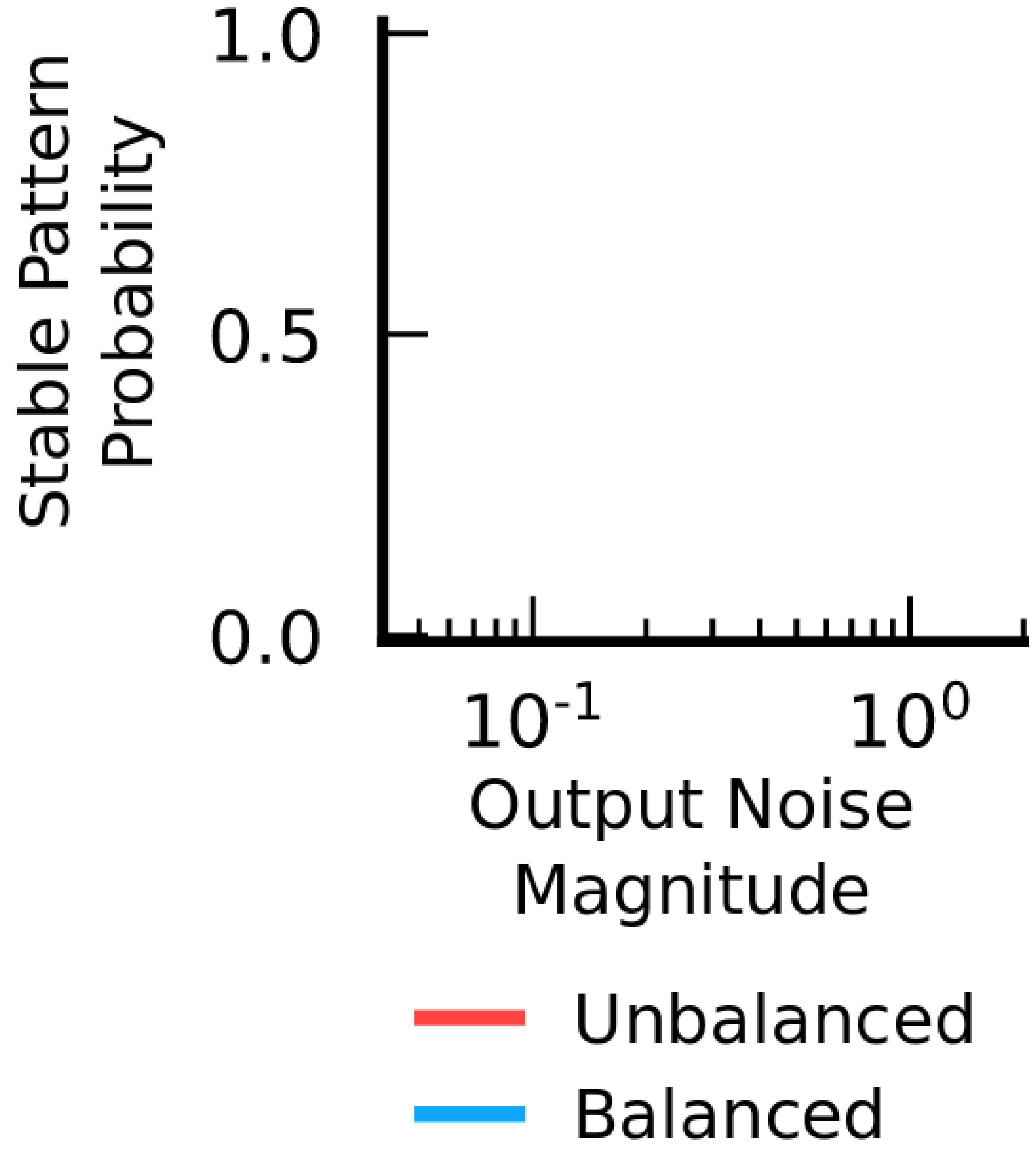


STOCHASTIC DYNAMICS

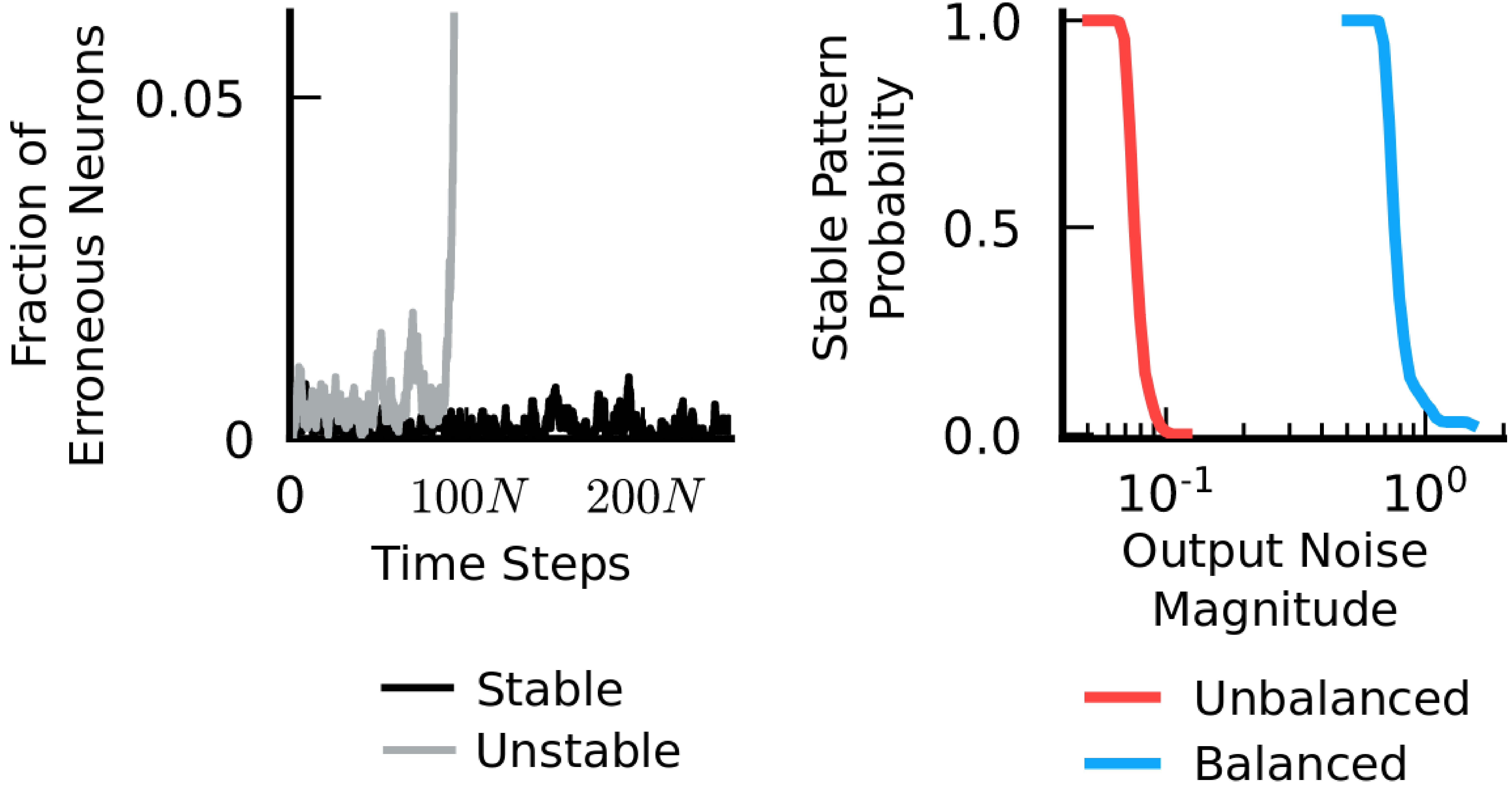


STOCHASTIC DYNAMICS





STOCHASTIC DYNAMICS



CAPACITY AND ROBUSTNESS FOR RANDOM MEMORY PATTERNS

CAPACITY AND ROBUSTNESS FOR RANDOM MEMORY PATTERNS

- P random memory states, $\{\boldsymbol{x}^{\mu}\}_{\mu=1}^{P}, x_i \in \{0, 1\}$ i.d.

CAPACITY AND ROBUSTNESS FOR RANDOM MEMORY PATTERNS

- P random memory states, $\{\boldsymbol{x}^{\mu}\}_{\mu=1}^{P}$, $x_i \in \{0, 1\}$ i.d.
- Pattern statistics:
  - $\mathrm{mean}\,(x_i)$ given by $\bar{x}_{\mathrm{exc}}$ or $\bar{x}_{\mathrm{inh}}$
  - $\mathrm{std}\,(x_i)$ given by σ_{exc} or σ_{inh}

GARDNER THEORY

$$V_{\mathrm{G}} = \int \mathcal{D}\left(\boldsymbol{J}\right) \prod_{\mu=1}^{P} \Theta\left[(2x_i^{\mu} - 1)\Big(\boldsymbol{J}^{\mathrm{T}} \boldsymbol{x}^{\mu} - V_{\mathrm{th}}\Big) - K\right]$$

GARDNER THEORY

$$V_{\mathrm{G}} = \int \underbrace{\mathcal{D}\left(\boldsymbol{J}\right)} \prod_{\mu=1}^{P} \Theta\left[(2x_i^{\mu} - 1)\left(\boldsymbol{J}^{\mathrm{T}}\boldsymbol{x}^{\mu} - V_{\mathrm{th}}\right) - K\right]$$

Sign
constraints
+
$|\boldsymbol{J}| \leq \Gamma$
$\Gamma \sim \mathcal{O}\left(1\right)$

GARDNER THEORY

$$V_{\mathrm{G}} = \int \underbrace{\mathcal{D}\left(\boldsymbol{J}\right)}_{\substack{\text{Sign} \\ \text{constraints} \\ + \\ |\boldsymbol{J}| \leq \Gamma \\ \Gamma \sim \mathcal{O}(1)}} \prod_{\mu=1}^{P} \Theta\left[(2x_i^{\mu} - 1)\left(\underbrace{\boldsymbol{J}^{\mathrm{T}} \boldsymbol{x}^{\mu}}_{\substack{\text{Membrane} \\ \text{potential}}} - V_{\mathrm{th}}\right) - K\right]$$

GARDNER THEORY

$$V_{\mathrm{G}} = \int \underbrace{\mathcal{D}\left(\boldsymbol{J}\right)}_{\substack{\text{Sign}\\ \text{constraints}\\ +\\ |\boldsymbol{J}| \leq \Gamma\\ \Gamma \sim \mathcal{O}(1)}} \prod_{\mu=1}^{P} \Theta\left[\underbrace{(2x_i^{\mu} - 1)}_{\substack{\text{Desired}\\ \text{state}}}\left(\underbrace{\boldsymbol{J}^{\mathrm{T}}\boldsymbol{x}^{\mu}}_{\substack{\text{Membrane}\\ \text{potential}}} - V_{\mathrm{th}}\right) - K\right]$$

GARDNER THEORY

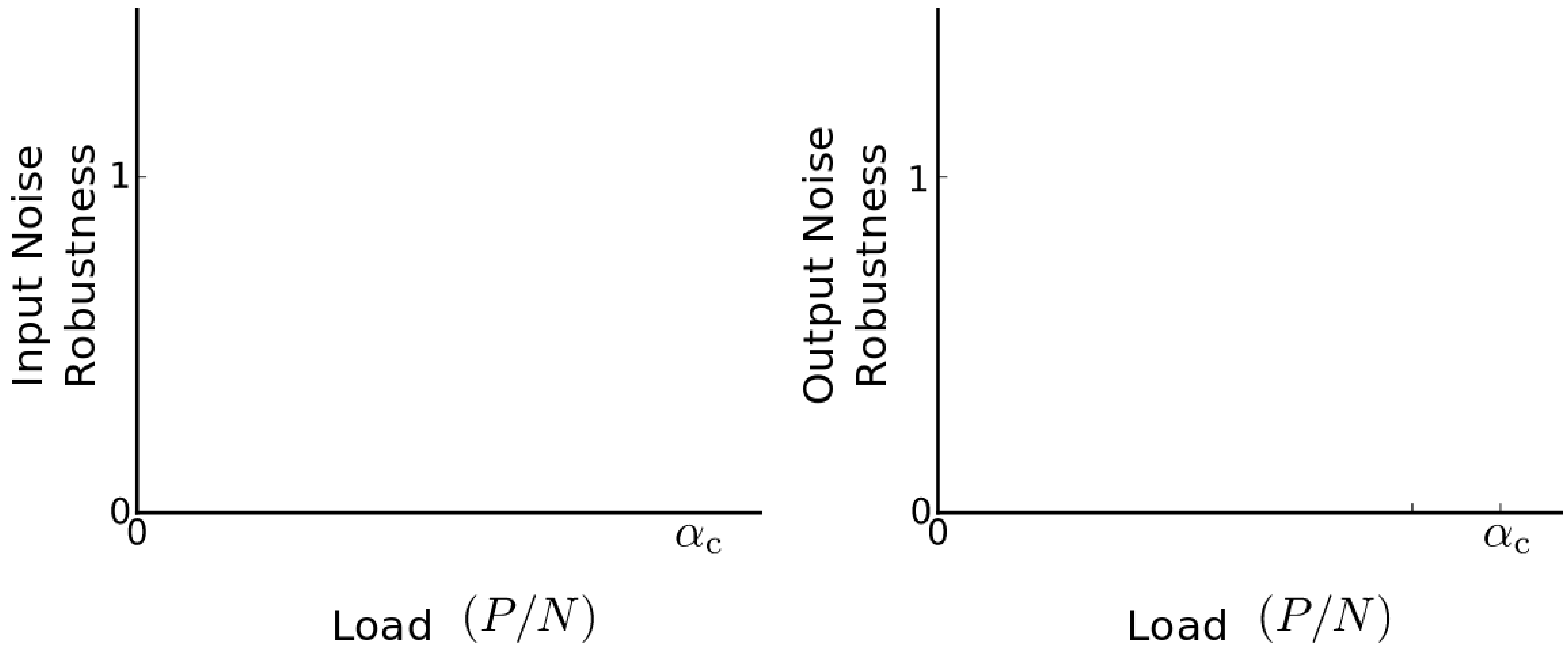
$$V_{\mathrm{G}} = \int \underbrace{\mathcal{D}\left(\boldsymbol{J}\right)}_{\substack{\text{Sign constraints} \\ + \\ |\boldsymbol{J}| \leq \Gamma \\ \Gamma \sim \mathcal{O}(1)}} \prod_{\mu=1}^{P} \Theta\left[\underbrace{(2x_i^{\mu} - 1)}_{\text{Desired state}}\Big(\underbrace{\boldsymbol{J}^{\mathrm{T}}\boldsymbol{x}^{\mu}}_{\text{Membrane potential}} - V_{\mathrm{th}}\Big) - \underbrace{K}_{\substack{\text{Output noise robustness:} \\ K = \kappa_{\mathrm{out}} \\ \text{Input noise robustness:} \\ K = |\boldsymbol{J}|\,\kappa_{\mathrm{in}}}}\right]$$

GARDNER THEORY

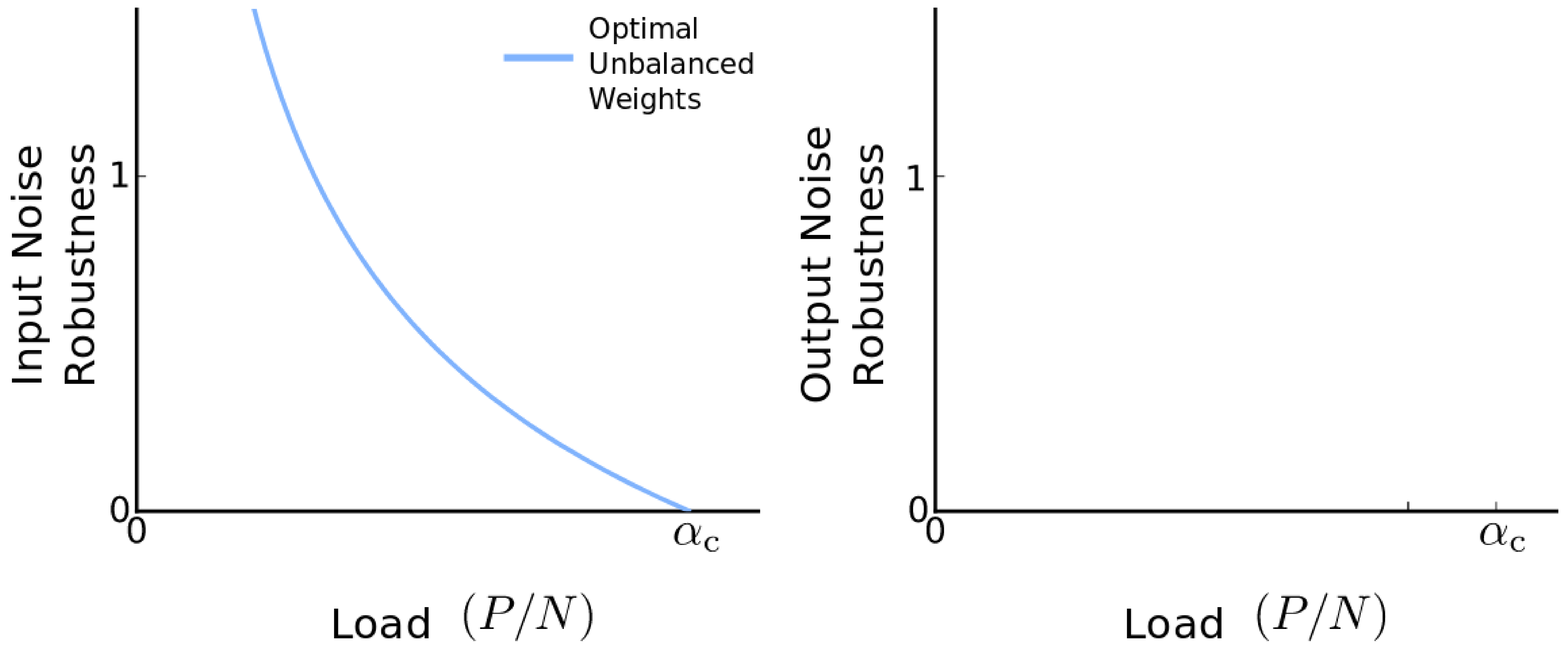
$$V_{\mathrm{G}} = \int \underbrace{\mathcal{D}\left(\boldsymbol{J}\right)}_{\substack{\text{Sign}\\\text{constraints}\\+\\|\boldsymbol{J}| \leq \Gamma\\\Gamma \sim \mathcal{O}(1)}} \prod_{\mu=1}^{P} \Theta\left[\left(\underbrace{2x_i^{\mu}}_{\substack{\text{Desired}\\\text{state}}} - 1\right)\left(\underbrace{\boldsymbol{J}^{\mathrm{T}}\boldsymbol{x}^{\mu}}_{\substack{\text{Membrane}\\\text{potential}}} - V_{\mathrm{th}}\right) - \underbrace{K}_{\substack{\text{Output noise}\\\text{robustness:}\\K = \kappa_{\mathrm{out}}\\\text{Input noise}\\\text{robustness:}\\K = |\boldsymbol{J}|\,\kappa_{\mathrm{in}}}}\right]$$

Calculate $\langle \ln V_{\mathrm{G}} \rangle$ for given P and κ_{in} or κ_{out}.

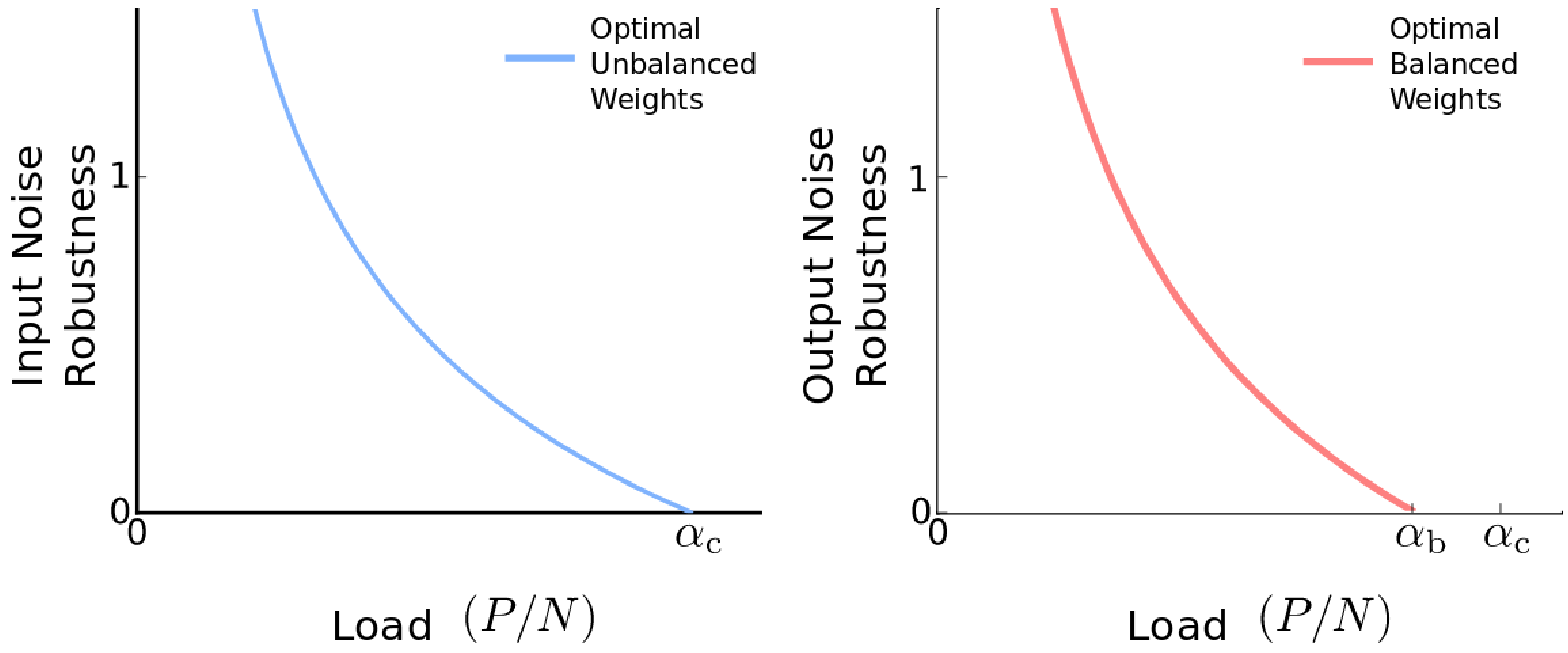
SINGLE NEURON ROBUSTNESS



SINGLE NEURON ROBUSTNESS



SINGLE NEURON ROBUSTNESS



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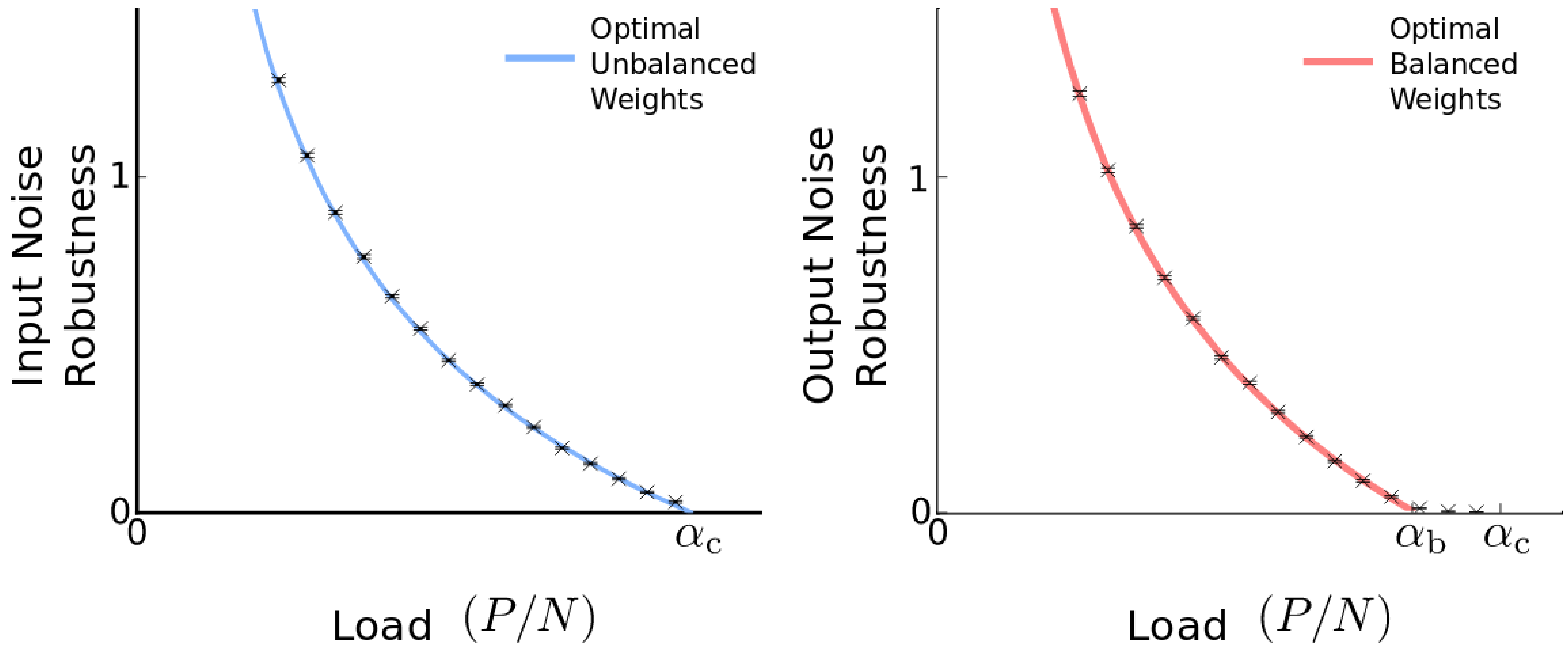

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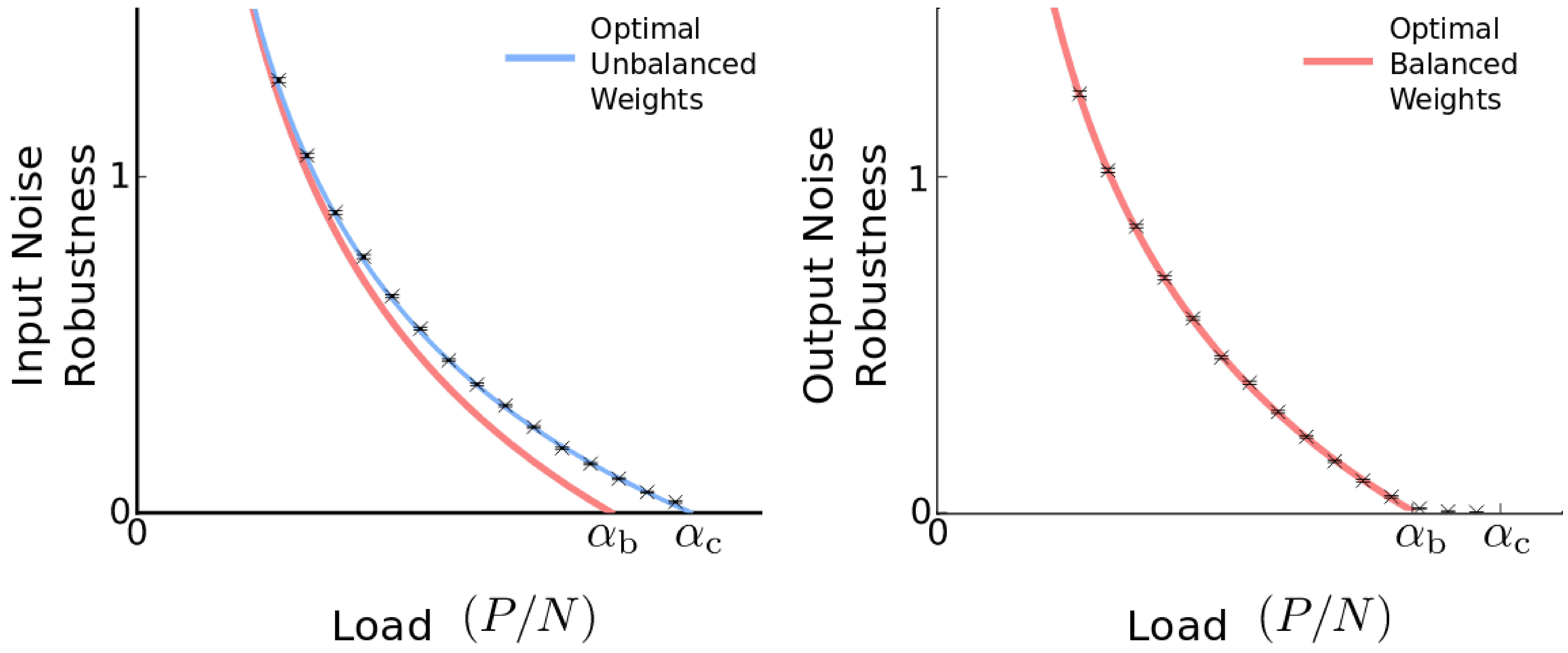

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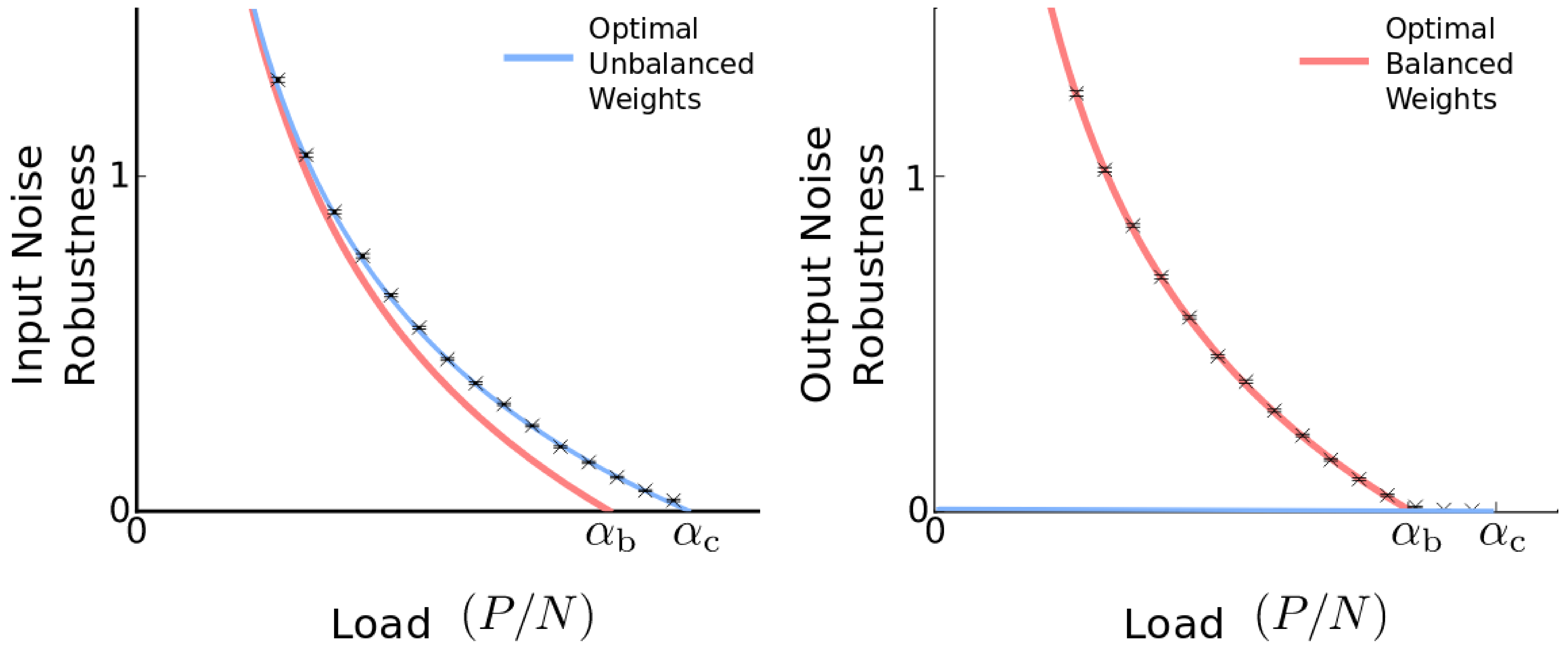
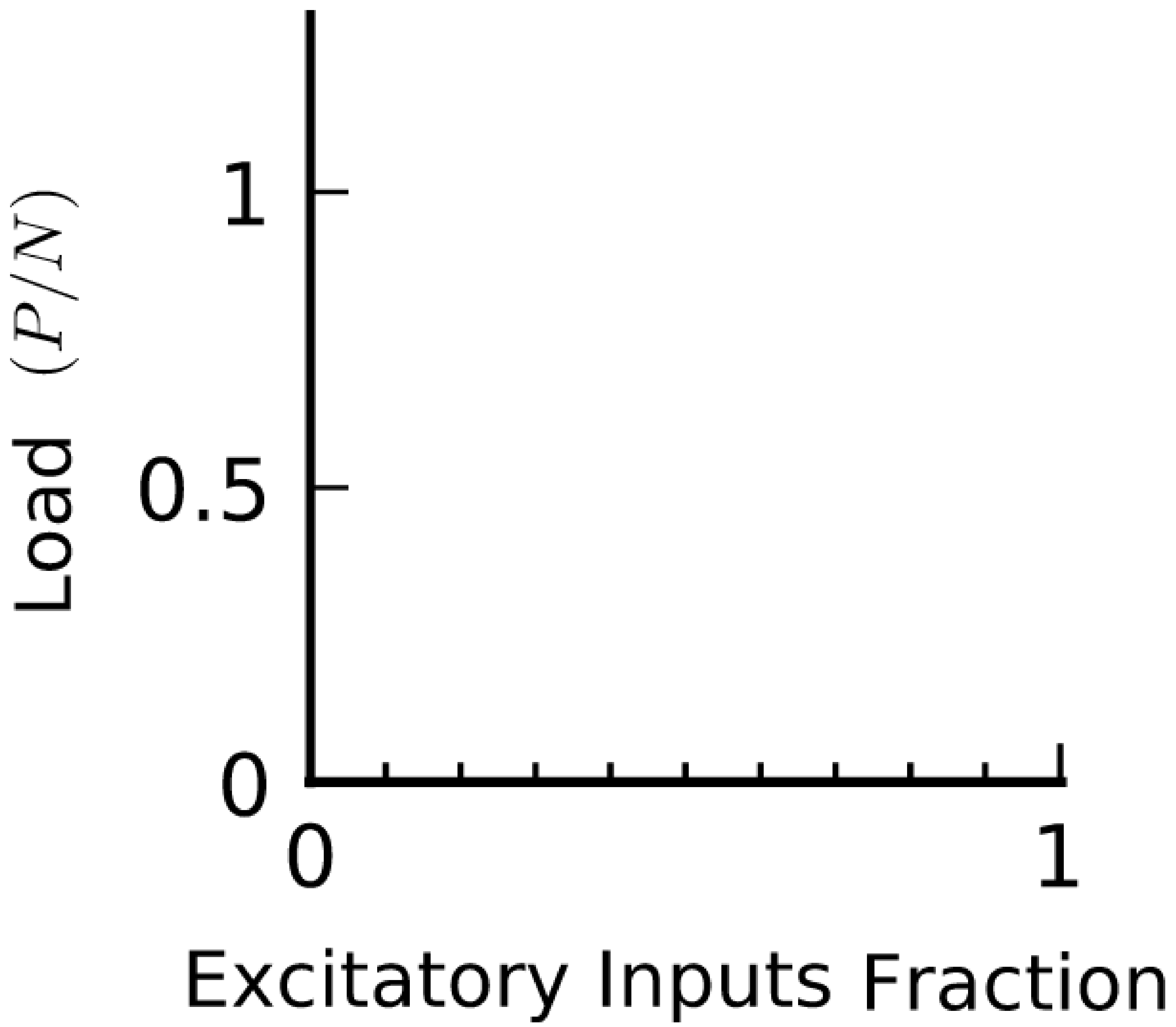


SINGLE NEURON CAPACITY



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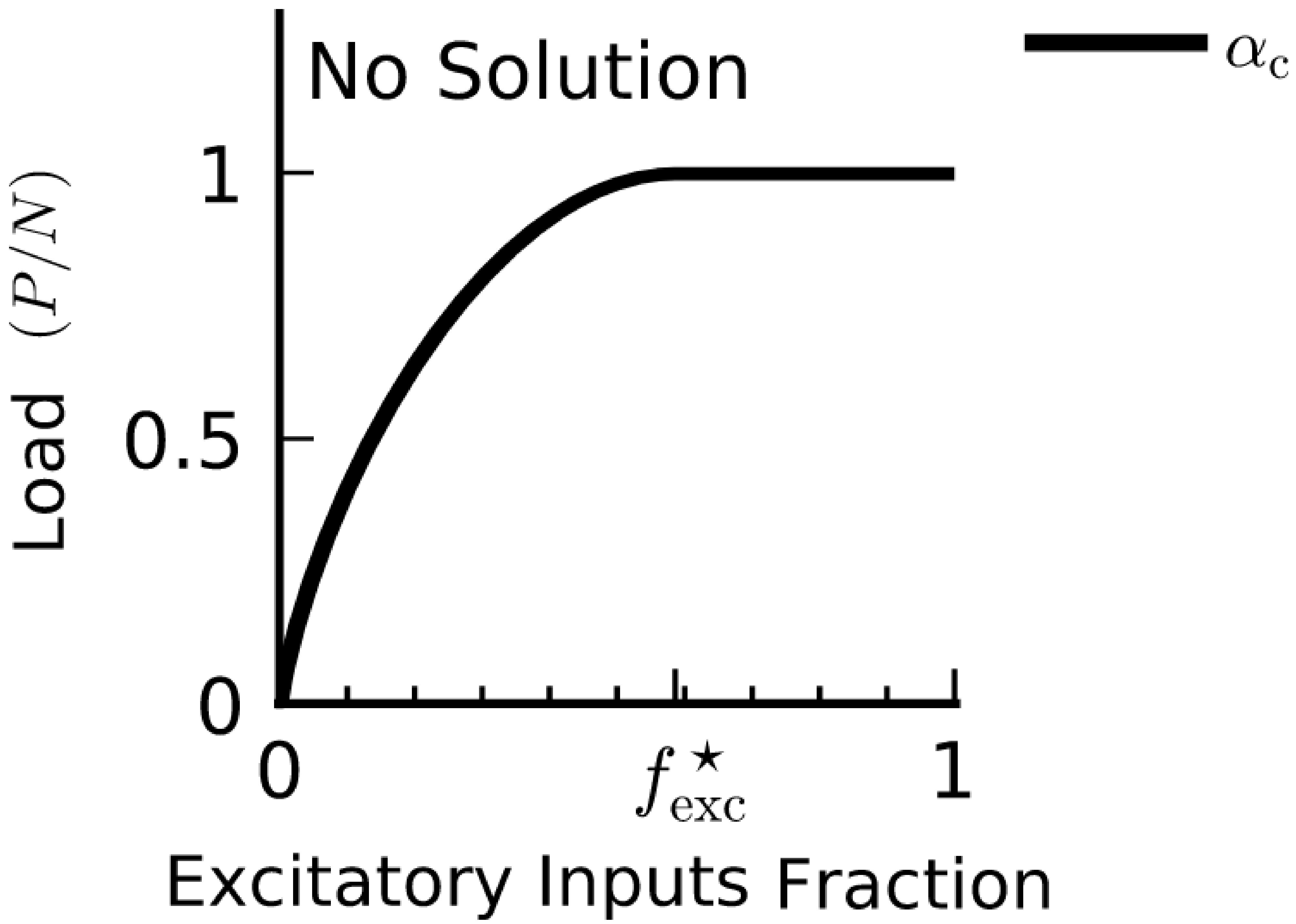

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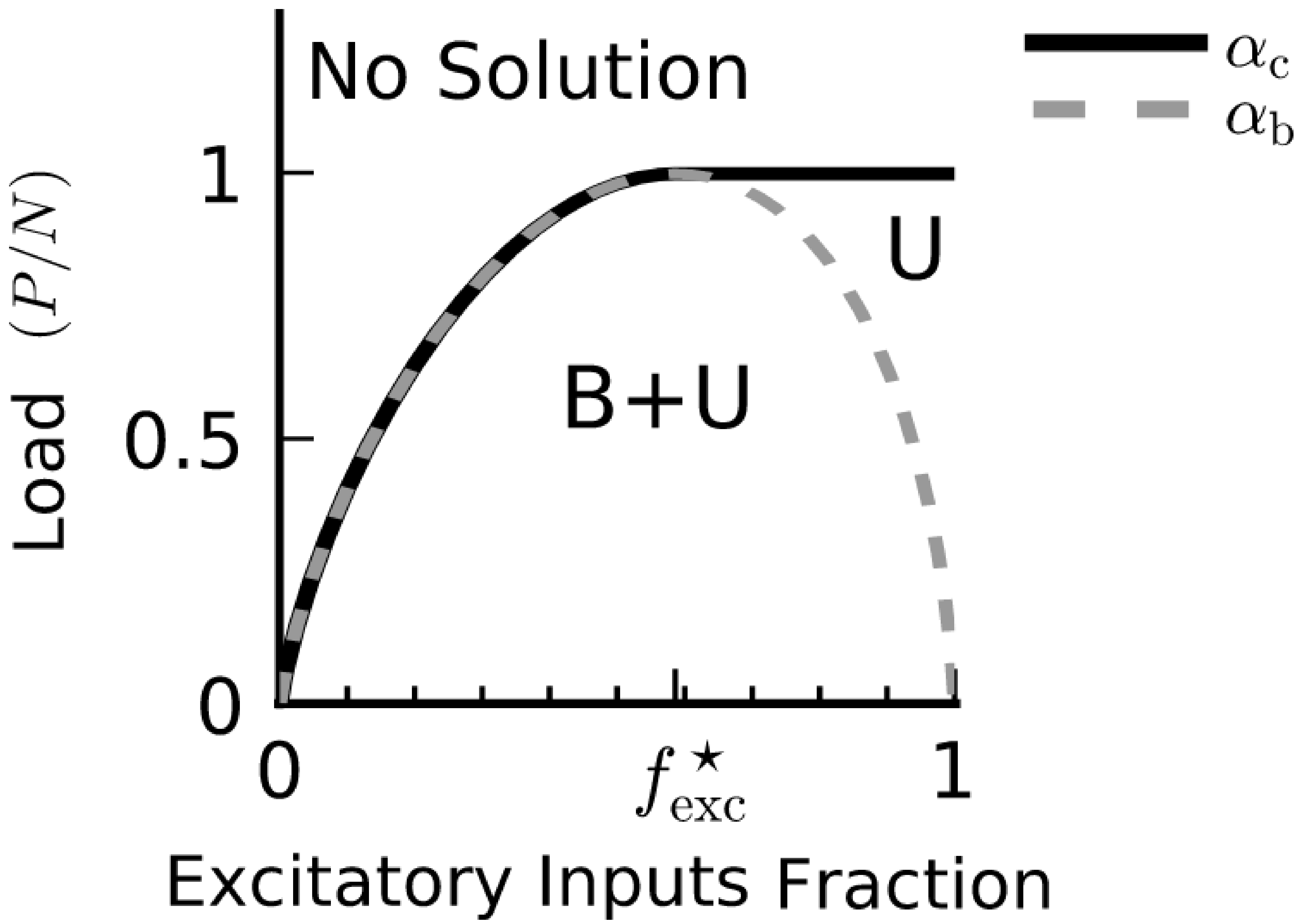

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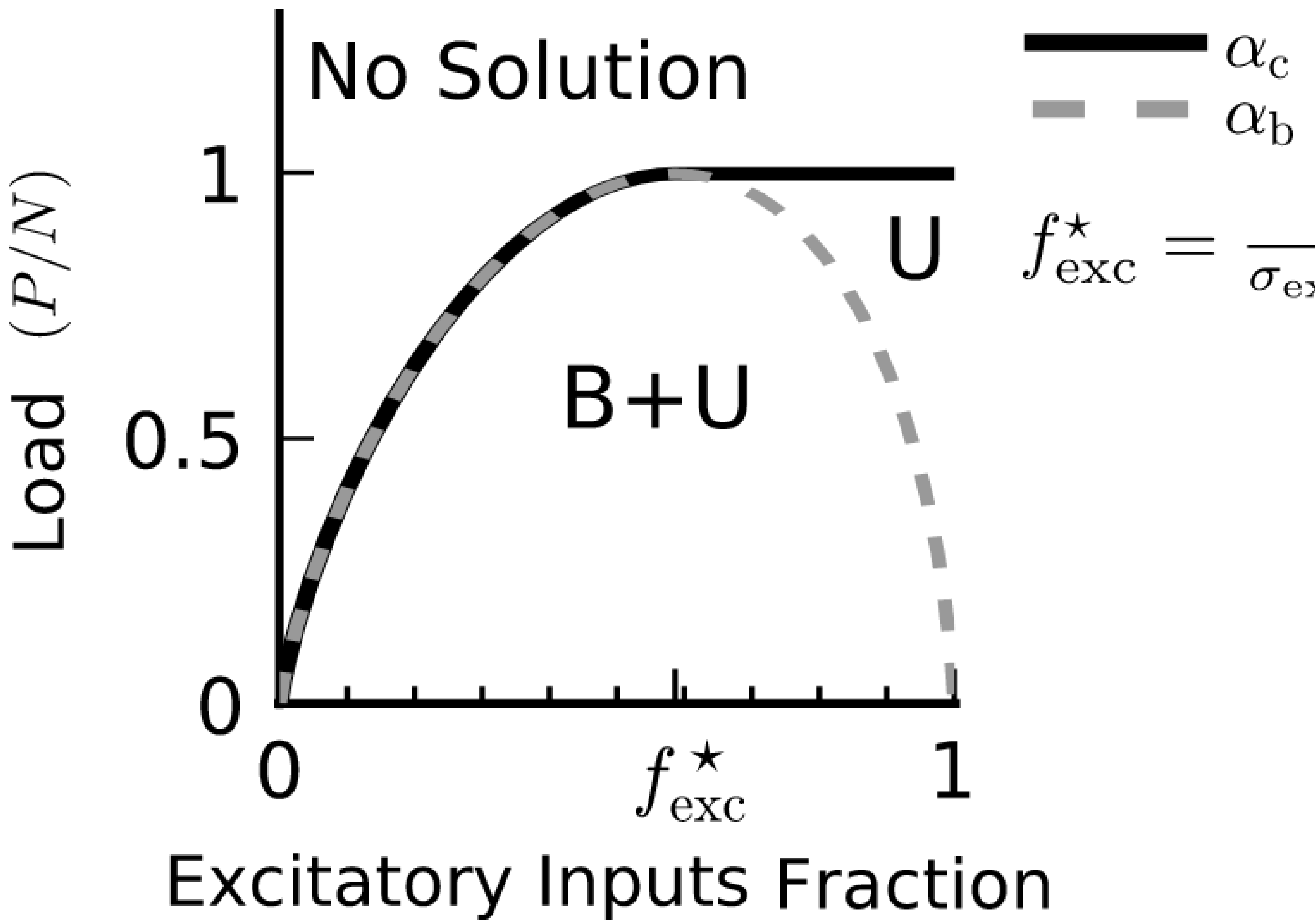
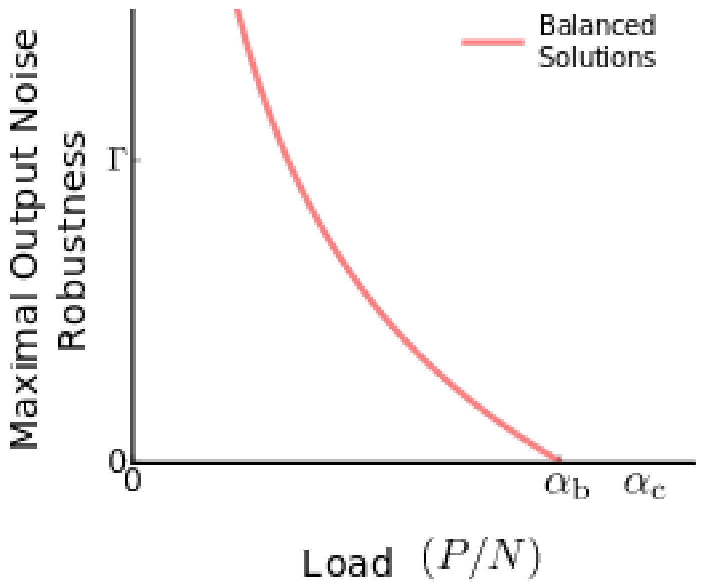
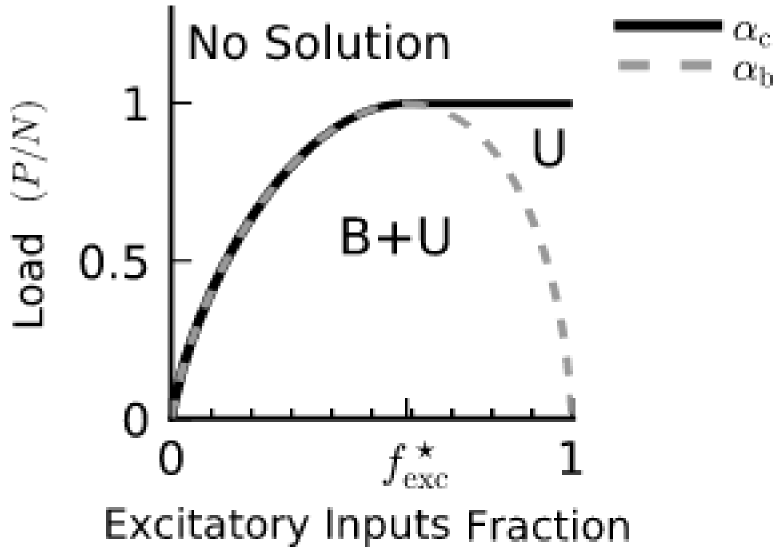


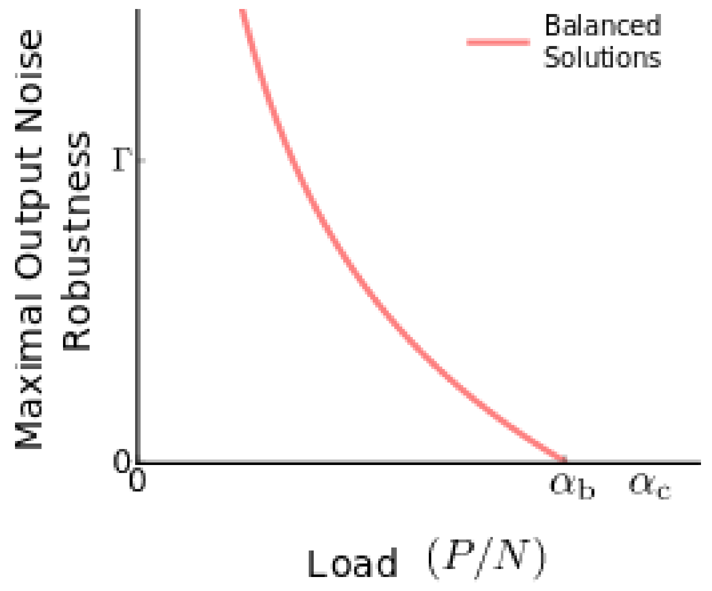
$$f^\star_{\mathrm{exc}} = \frac{\sigma_{\mathrm{exc}}/\bar{x}_{\mathrm{exc}}}{\sigma_{\mathrm{exc}}/\bar{x}_{\mathrm{exc}}+\sigma_{\mathrm{inh}}/\bar{x}_{\mathrm{inh}}}$$

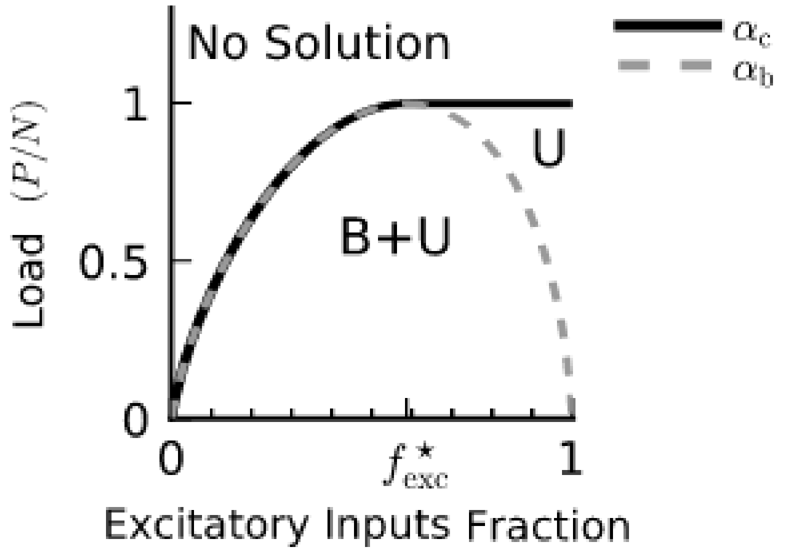
NETWORK CAPACITY

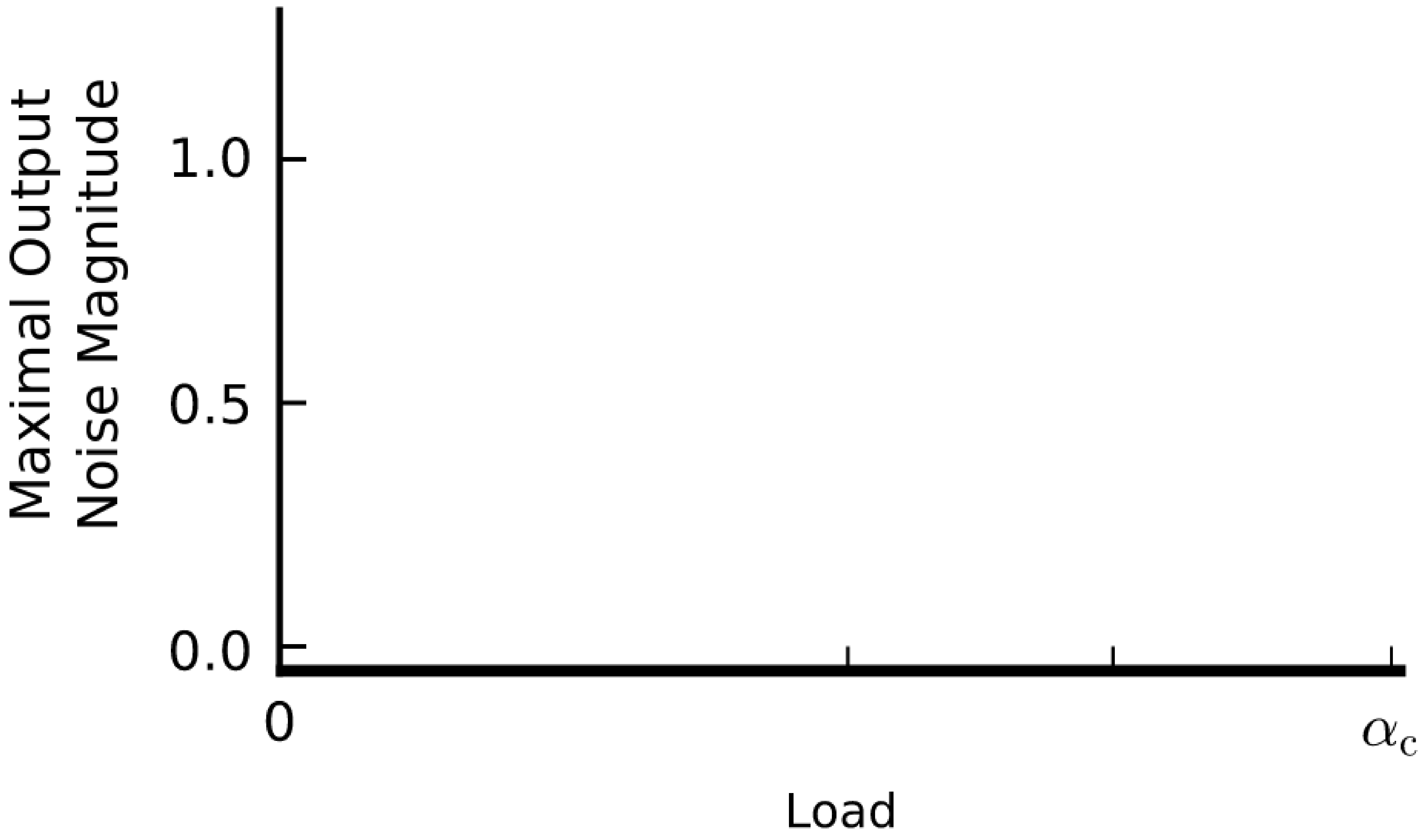




NETWORK CAPACITY







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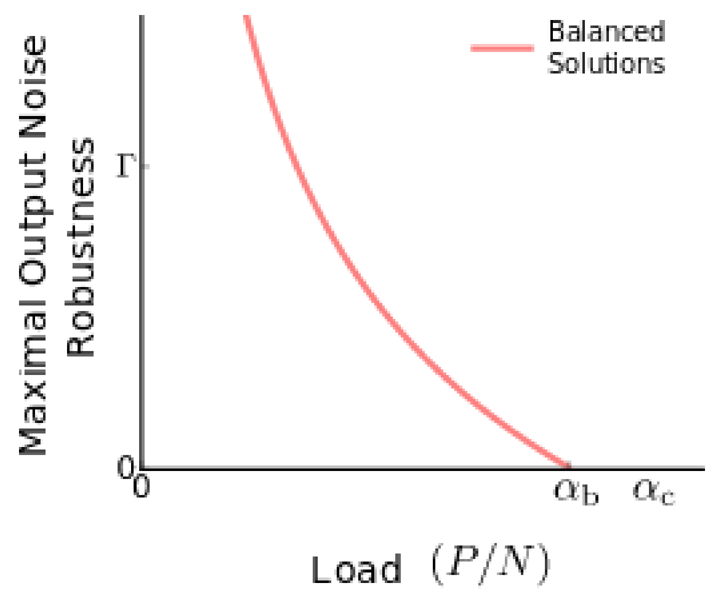
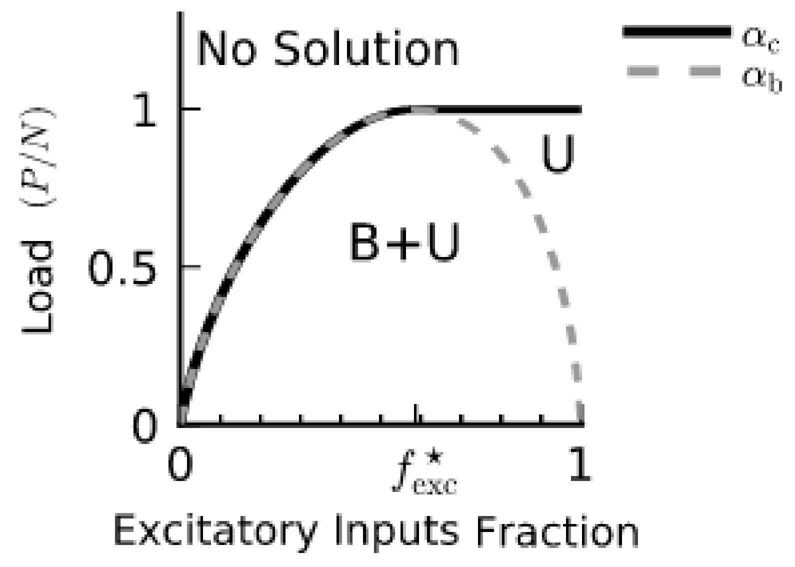
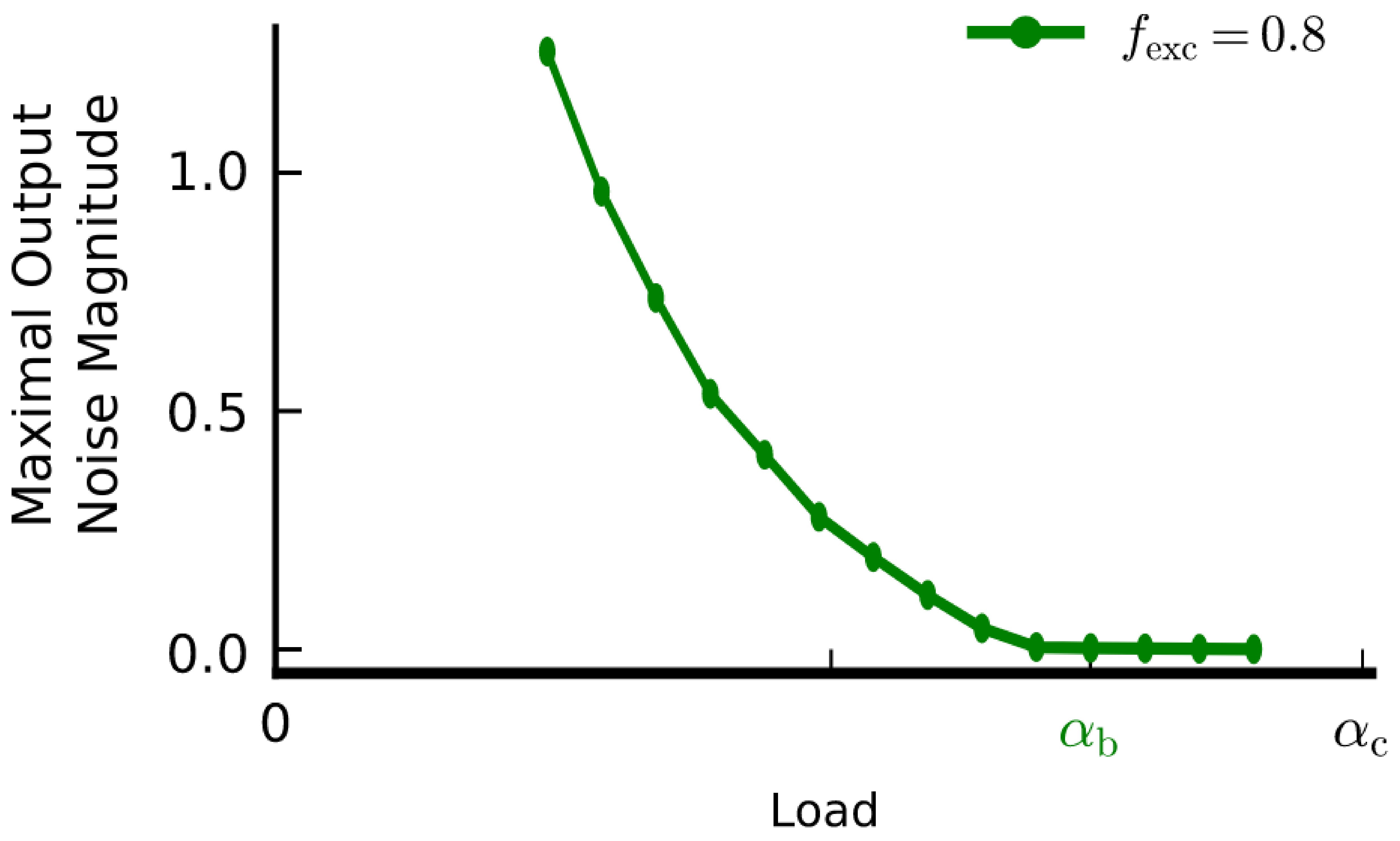
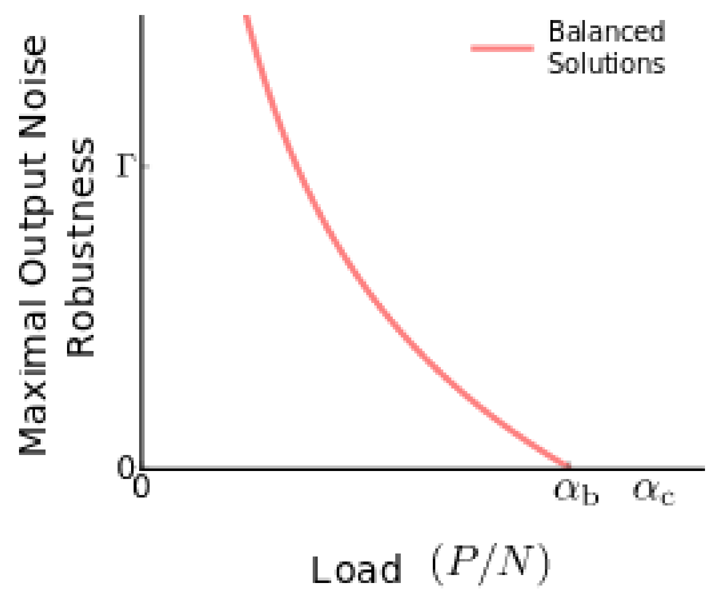
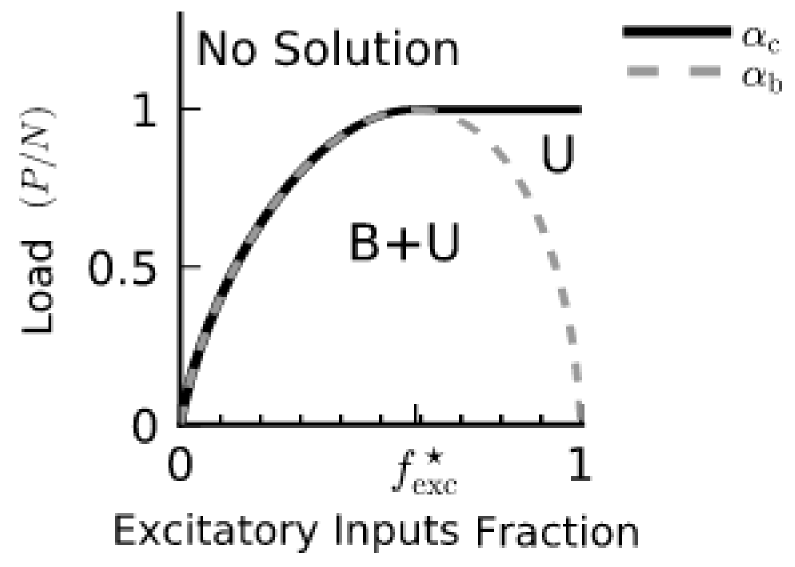


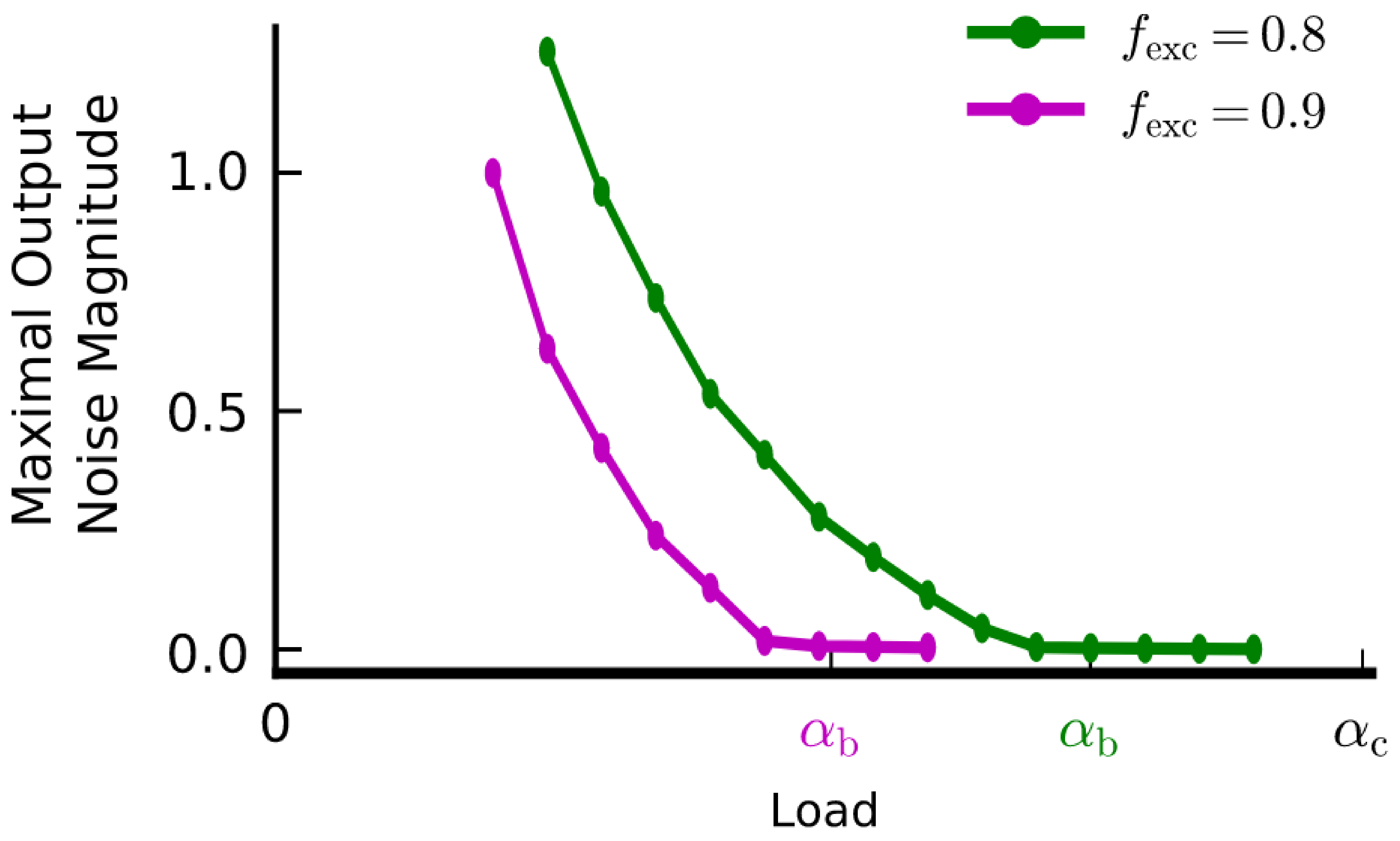




NETWORK CAPACITY







ROLE OF INHIBITION IN ASSOCIATIVE MEMORY NETWORKS

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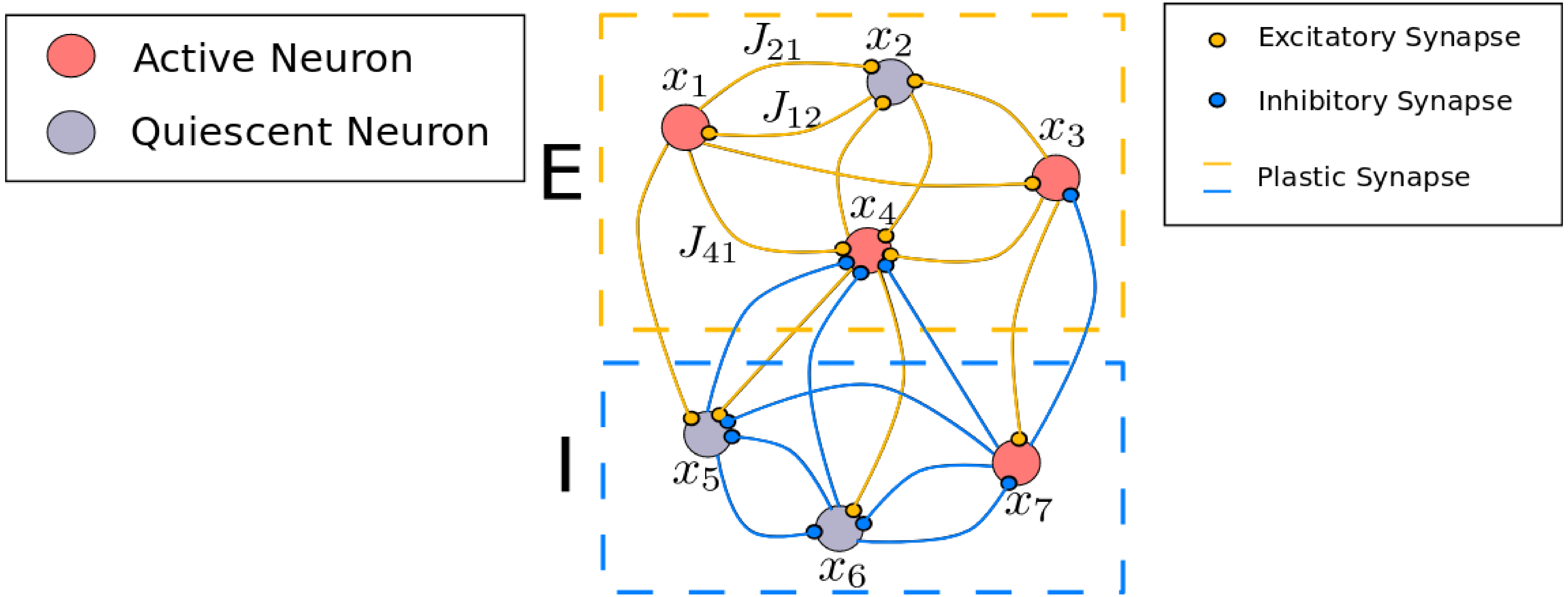
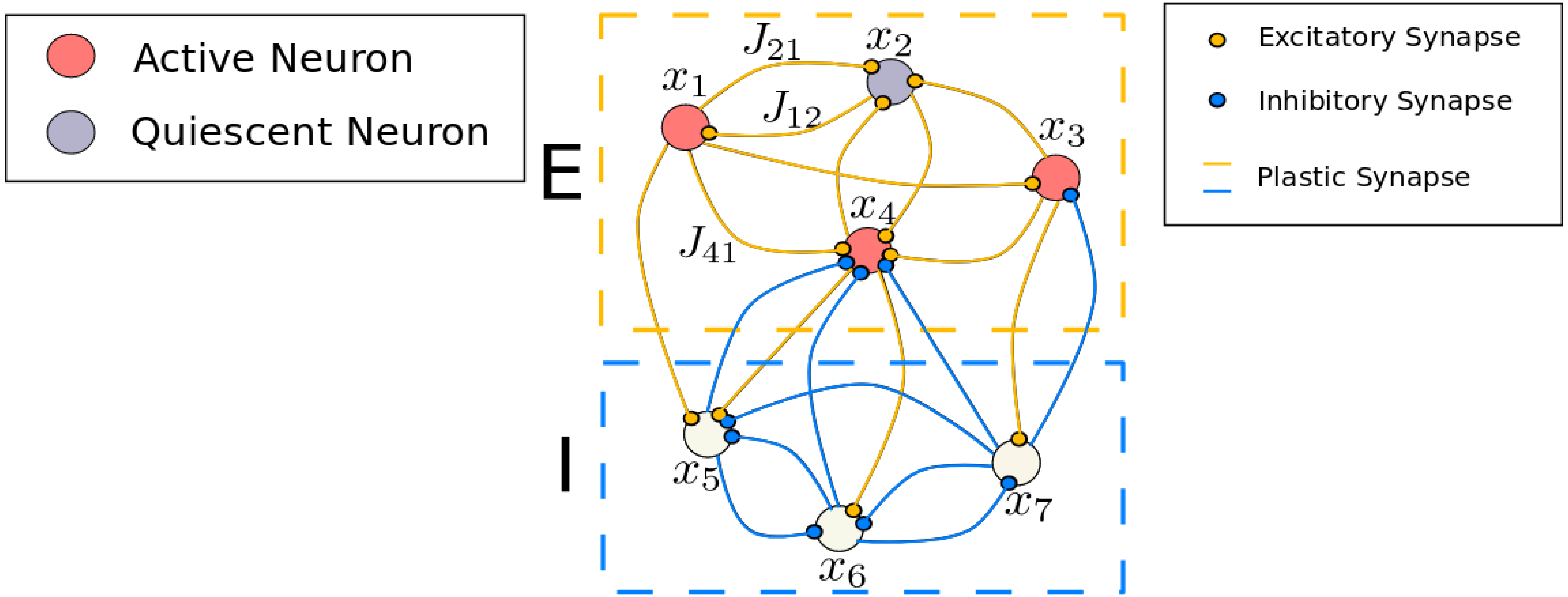
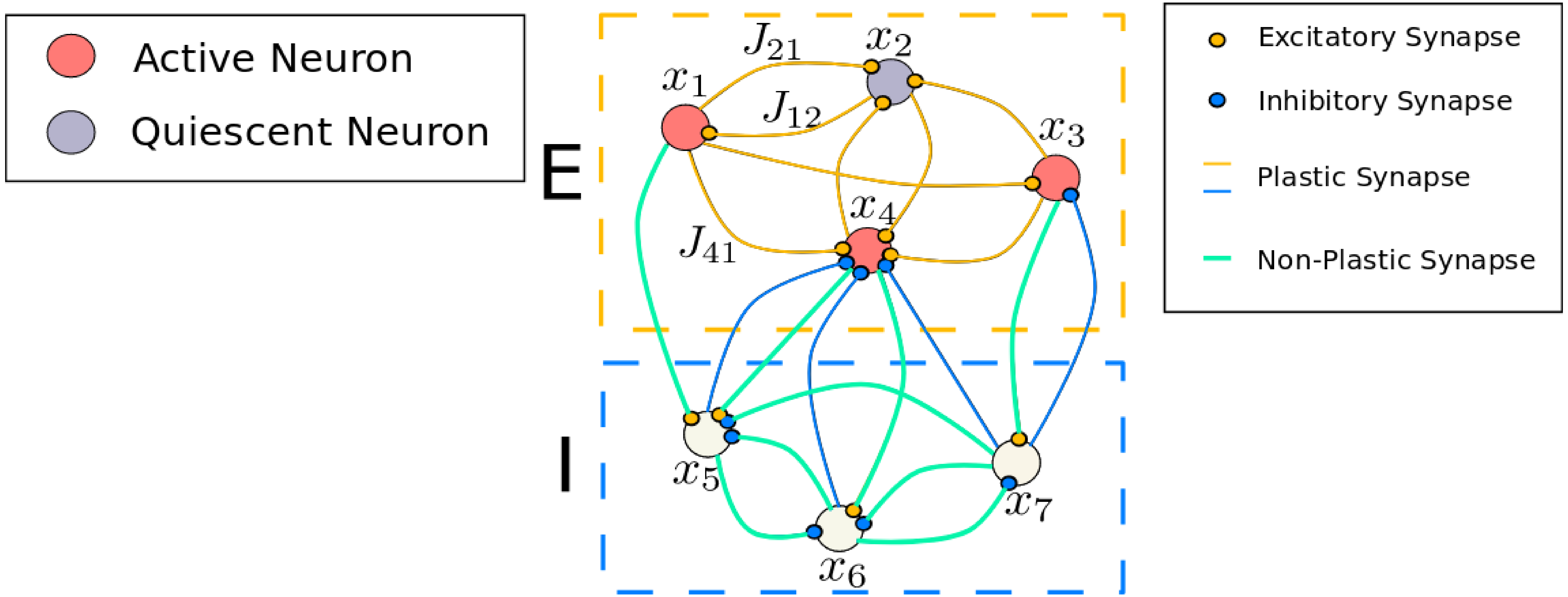


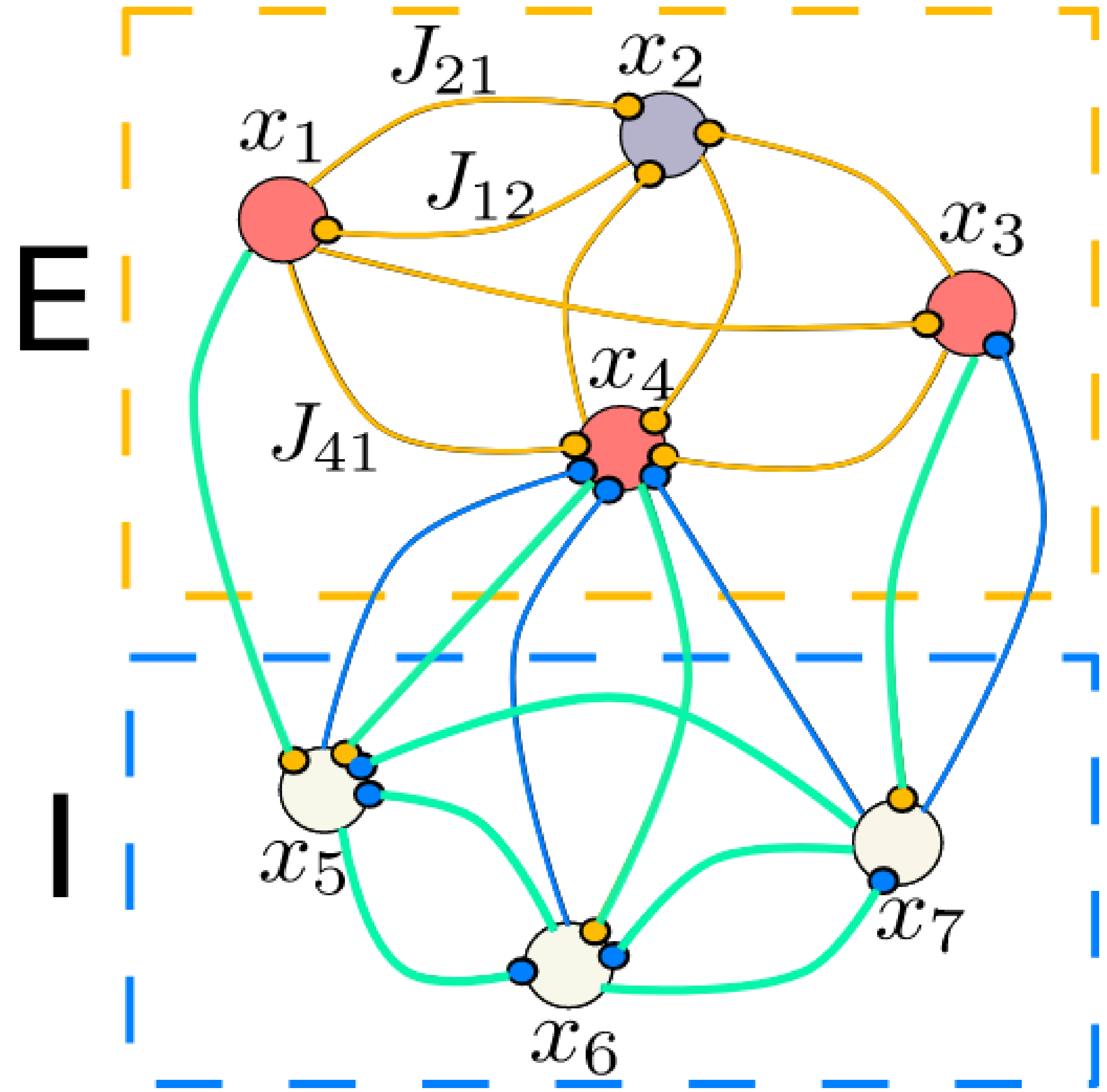
ROLE OF INHIBITION IN ASSOCIATIVE MEMORY NETWORKS



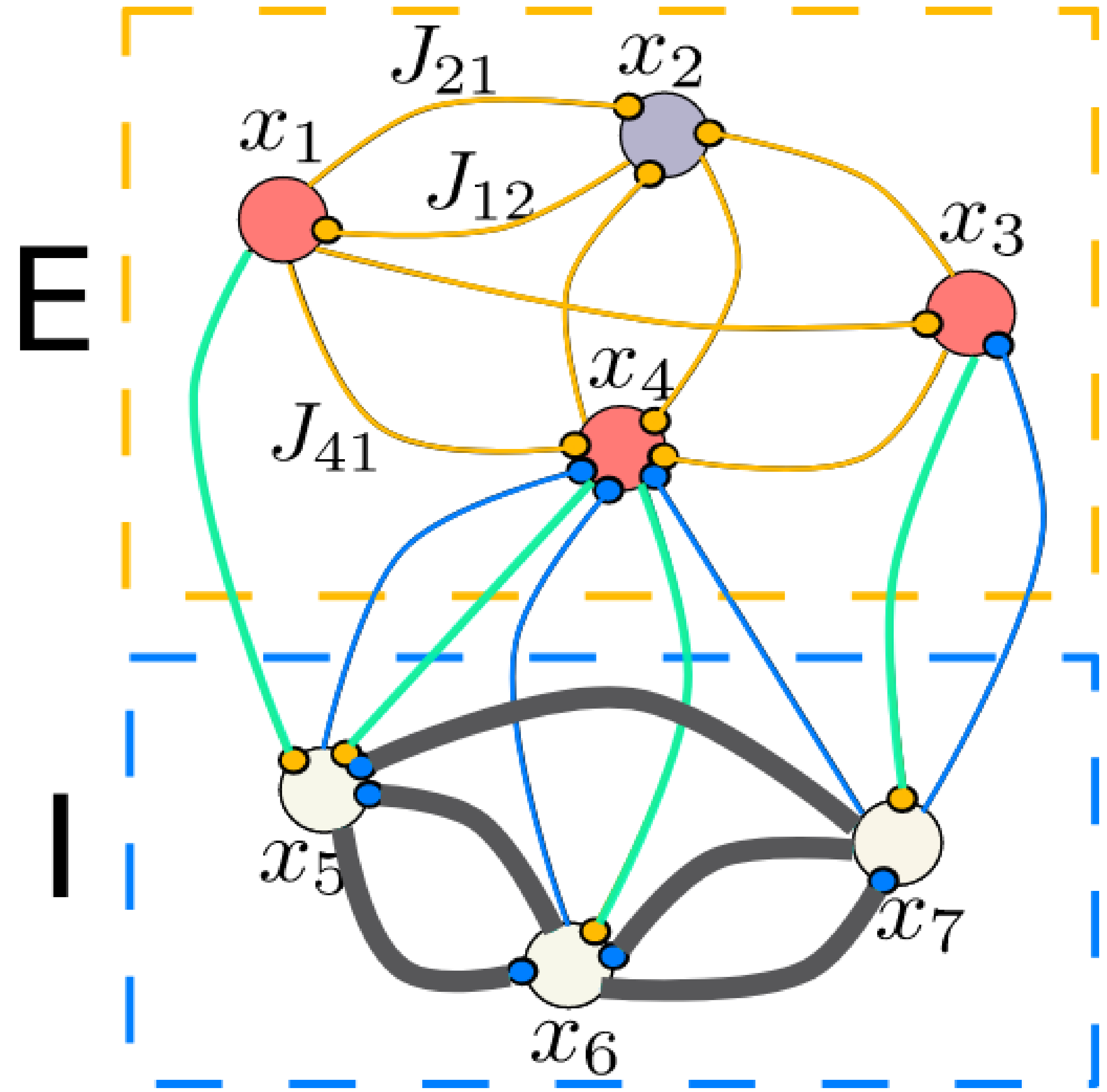
ROLE OF INHIBITION IN ASSOCIATIVE MEMORY NETWORKS



NETWORKS WITH RANDOMLY CONNECTED INHIBITORY POPULATION

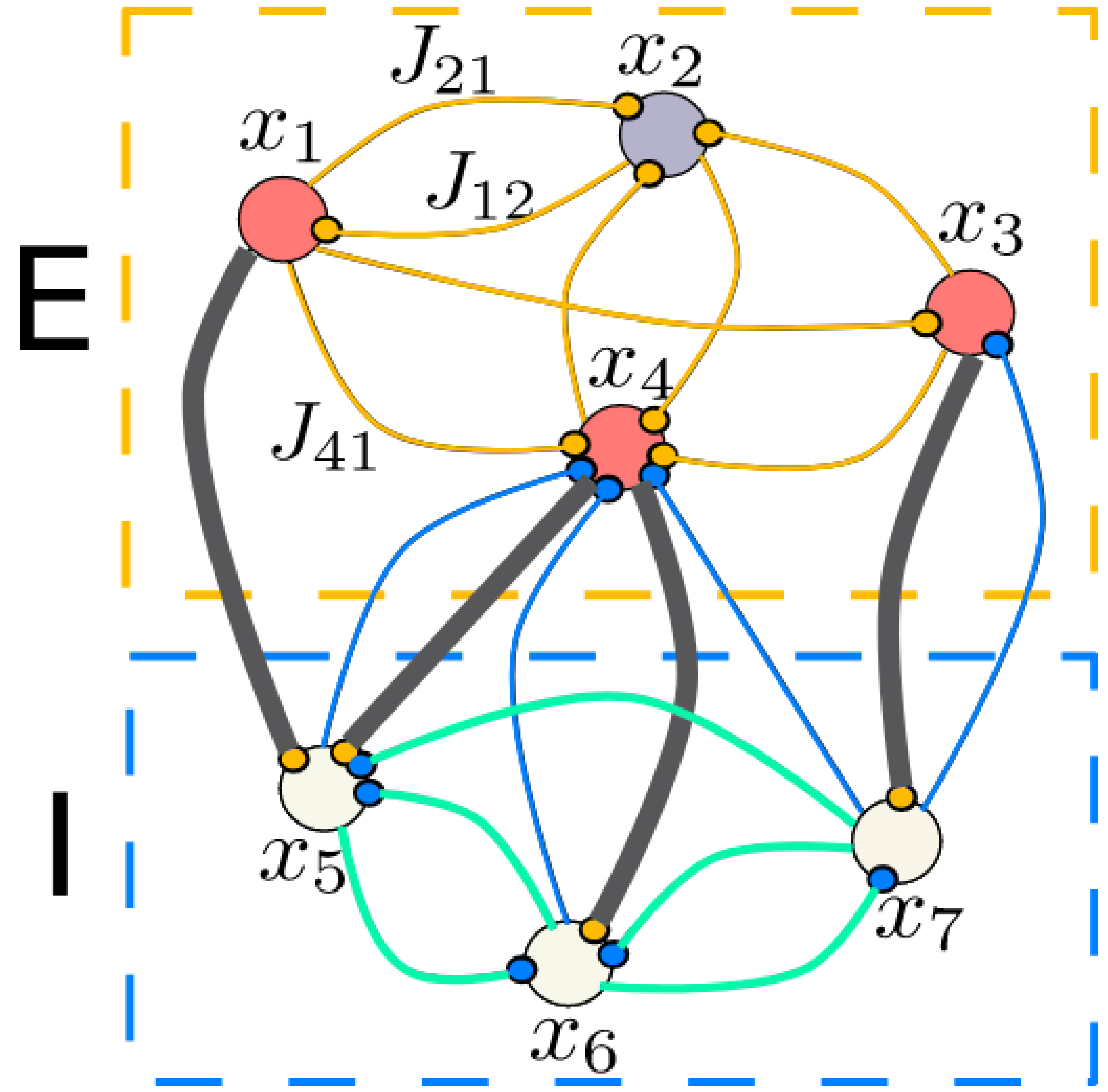


NETWORKS WITH RANDOMLY CONNECTED INHIBITORY POPULATION



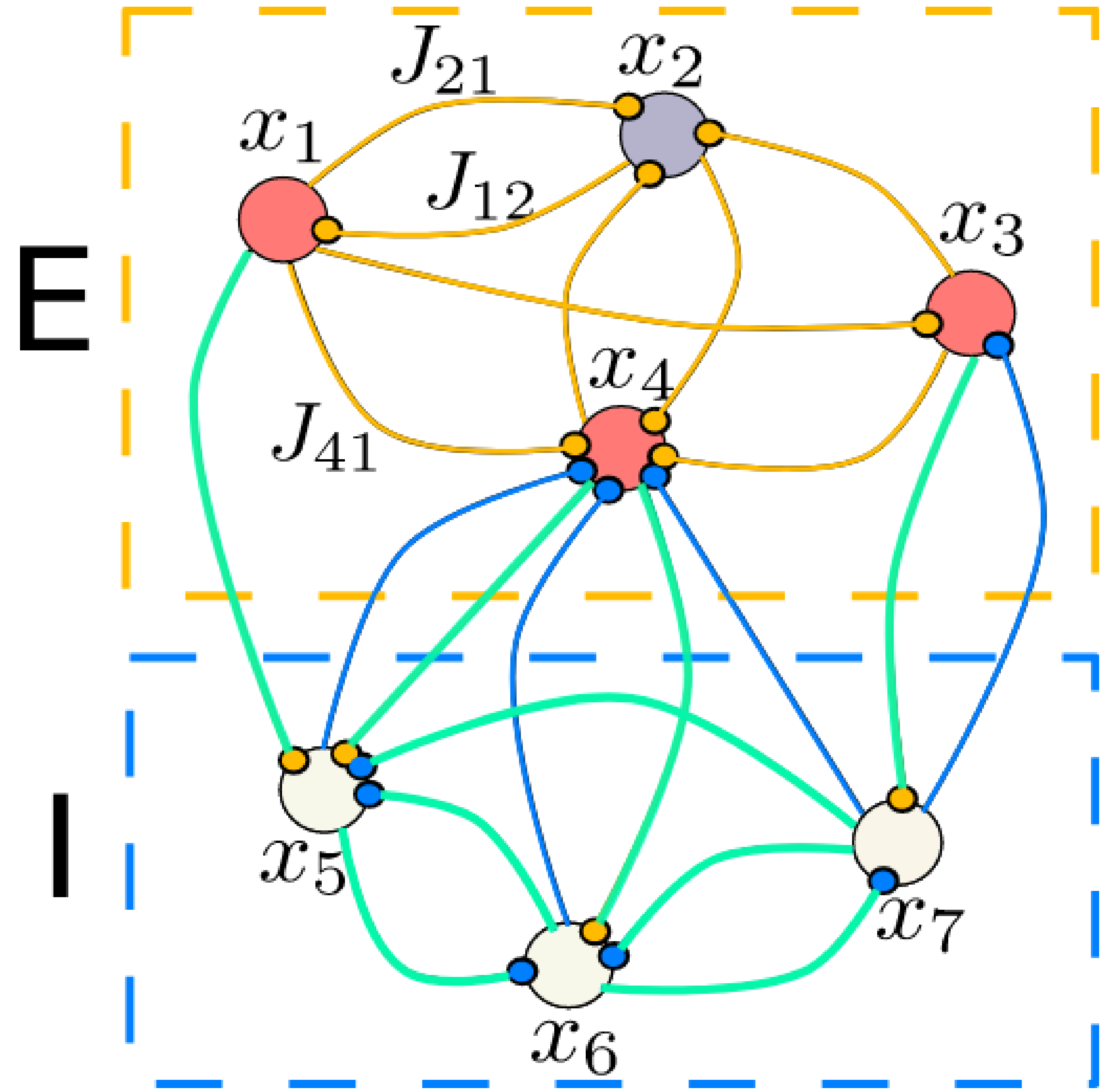
- Random I to I connections with $\langle J_{ij} \rangle = \bar{J}^{\mathrm{II}} \sim \frac{1}{\sqrt{N}}$.

NETWORKS WITH RANDOMLY CONNECTED INHIBITORY POPULATION

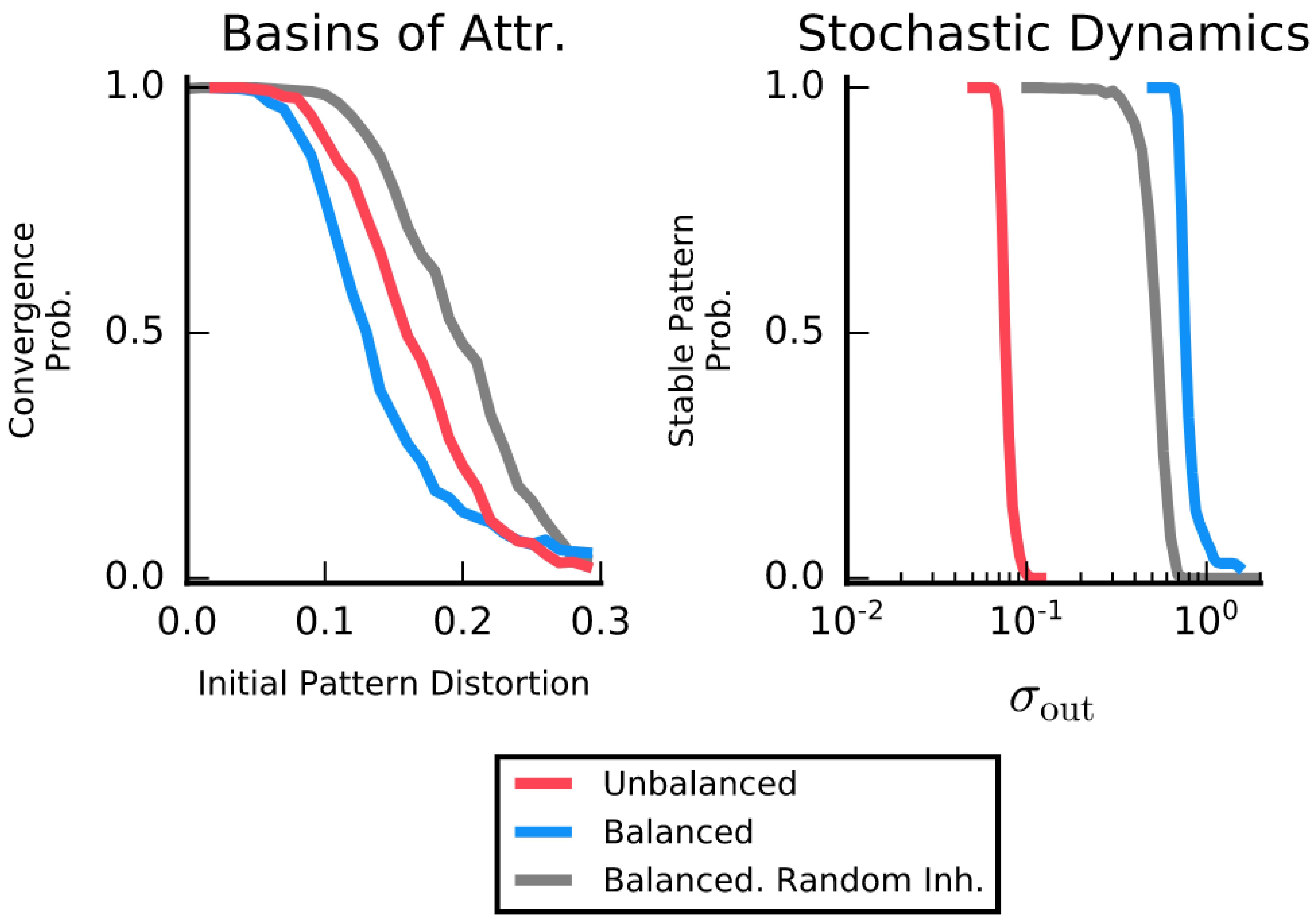


- Random I to I connections with $\langle J_{ij} \rangle = \bar{J}^{\mathrm{II}} \sim \frac{1}{\sqrt{N}}$.
- Random E to I connections s.t. mean I activity is m_{I} .

NETWORKS WITH RANDOMLY CONNECTED INHIBITORY POPULATION

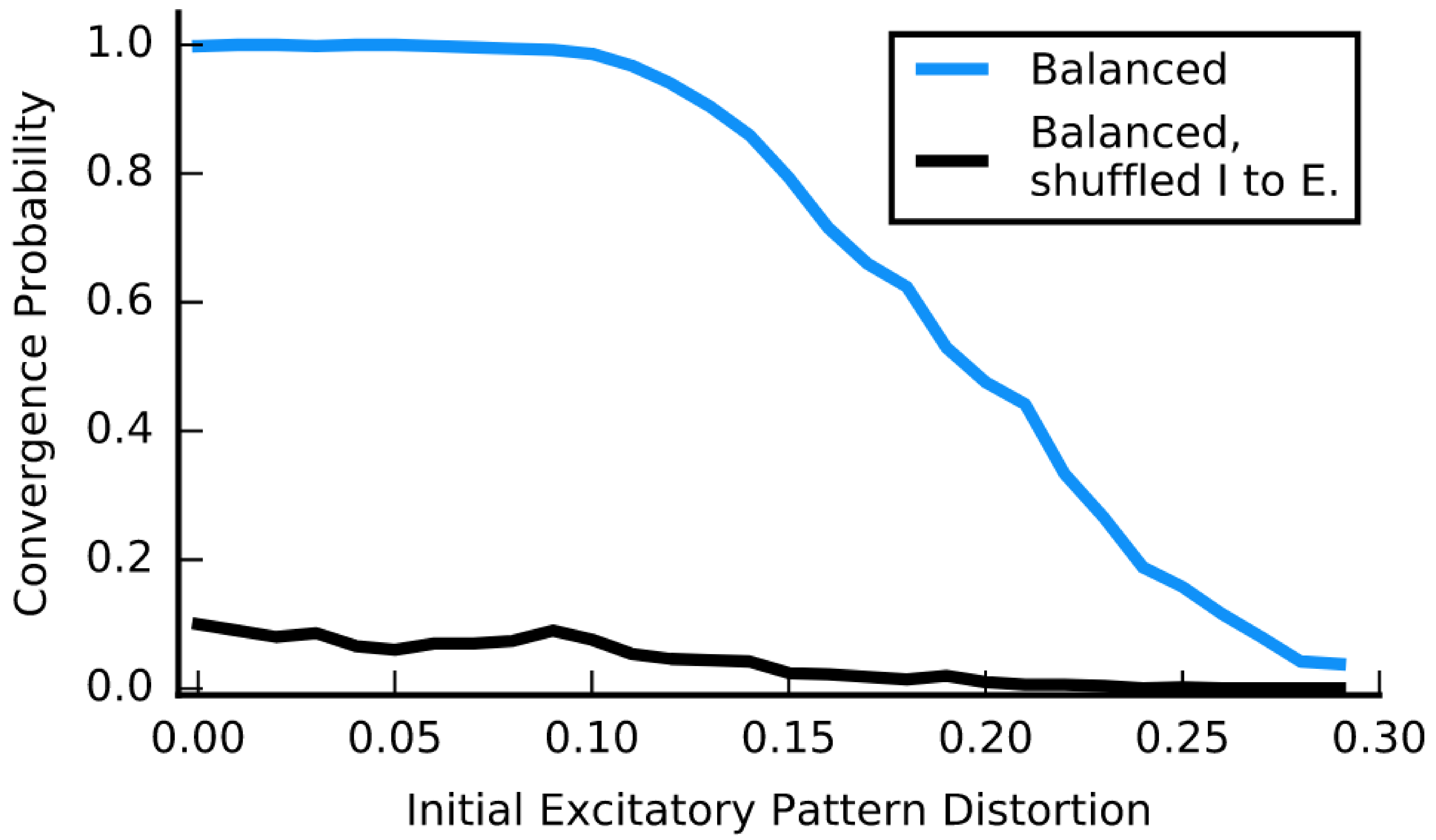


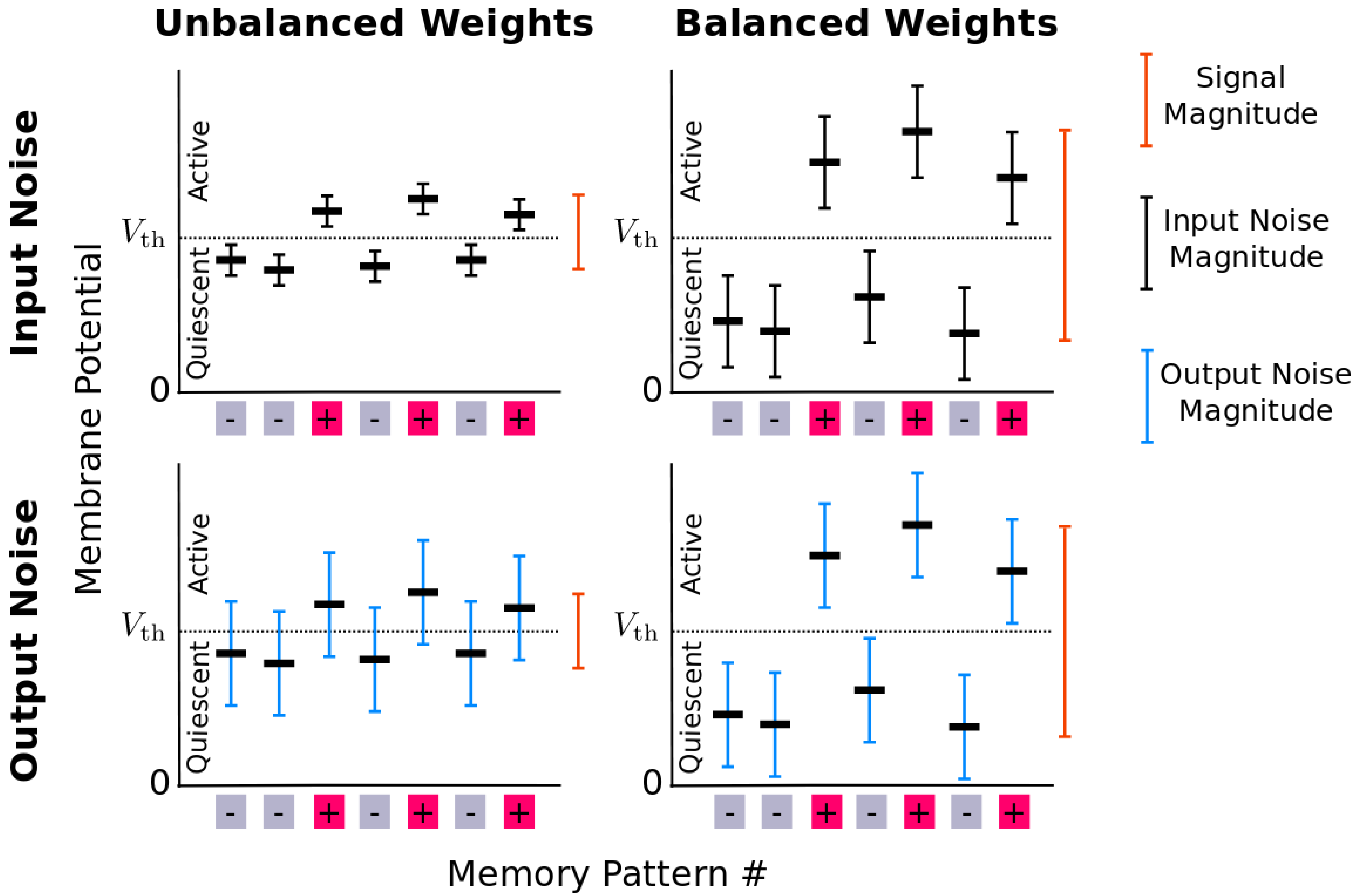
- Random I to I connections with $\left\langle J_{ij} \right\rangle = \bar{J}^{\mathrm{II}} \sim \frac{1}{\sqrt{N}}$.
- Random E to I connections s.t. mean I activity is m_{I}.
- I network in async. state.


Basins of Attr.
Stochastic Dynamics
Convergence Prob.
Initial Pattern Distortion
1.0
0.5
0.0
0.0
0.1
0.2
0.3
Stable Pattern Prob.
10^{-2}
10^{-1}
10^{0}
σ_{out}
Unbalanced
Balanced
Balanced. Random Inh.

ROLE OF INHIBITION

ROLE OF INHIBITION





Unbalanced Weights
Balanced Weights
Input Noise
Output Noise
Membrane Potential
Active
Quiescent
V_{th}
0
- - + - + - +
Memory Pattern #
Signal Magnitude
Input Noise Magnitude
Output Noise Magnitude

Successful
Neuronal
Learning

+

Output
Noise

Successful
Neuronal
Learning

$+$

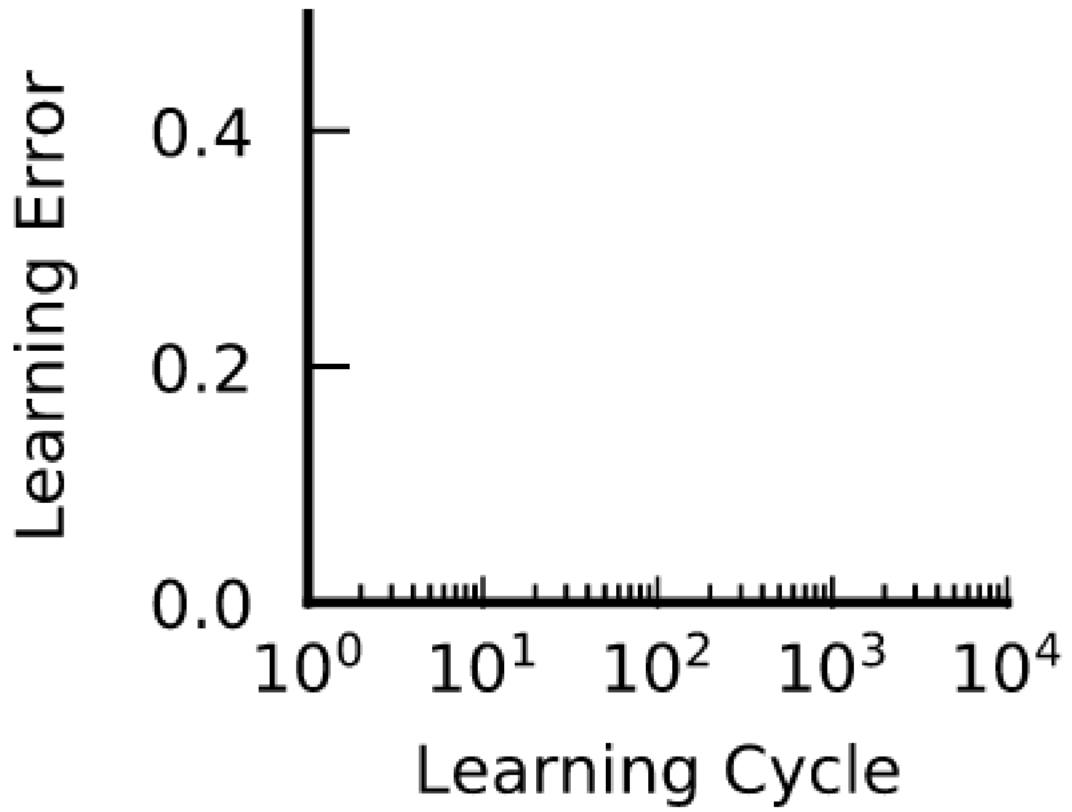
Output
Noise

$\Longrightarrow$

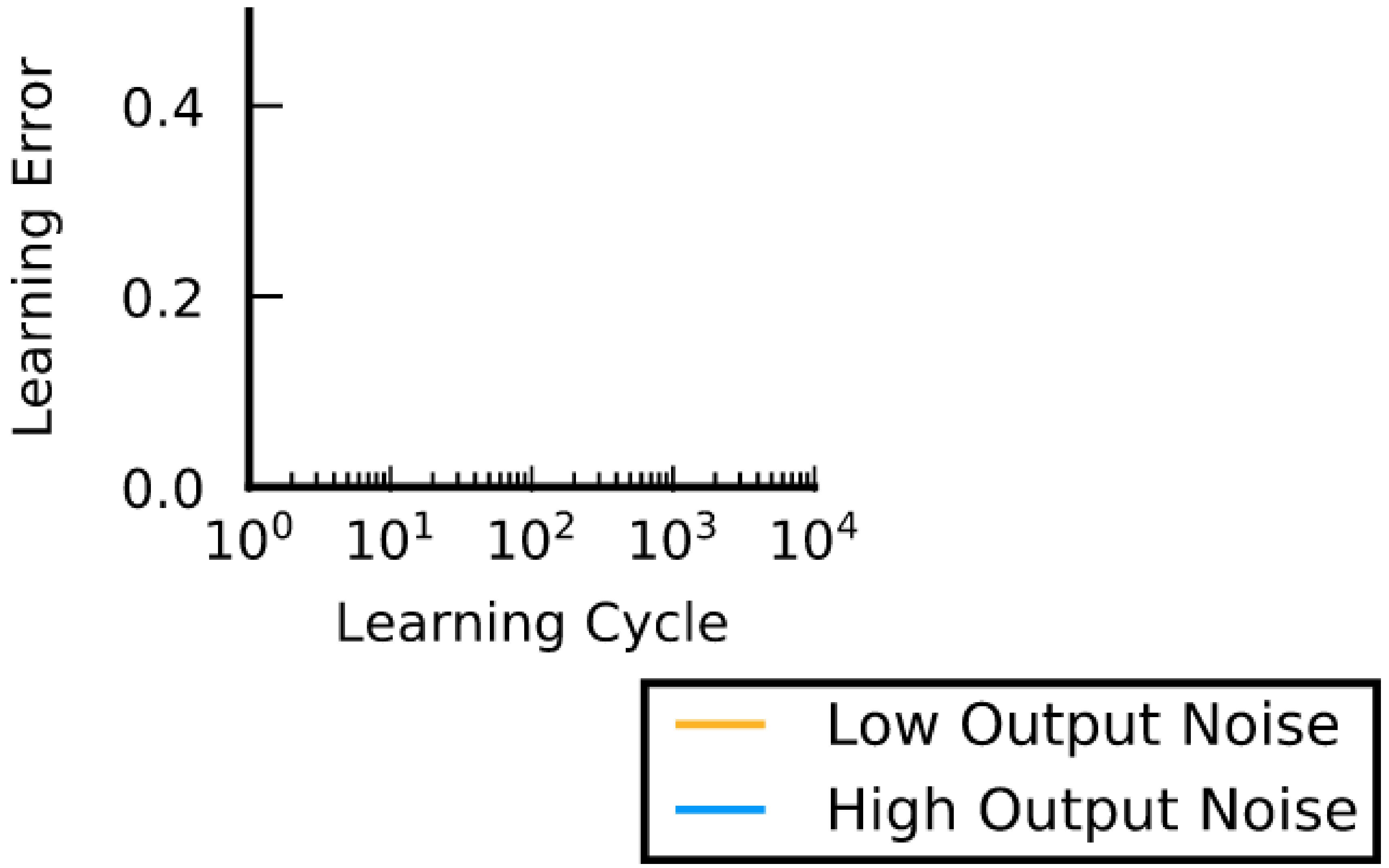
Balanced
Synaptic
Weights

EMERGENCE OF E-I BALANCE

EMERGENCE OF E-I BALANCE



EMERGENCE OF E-I BALANCE



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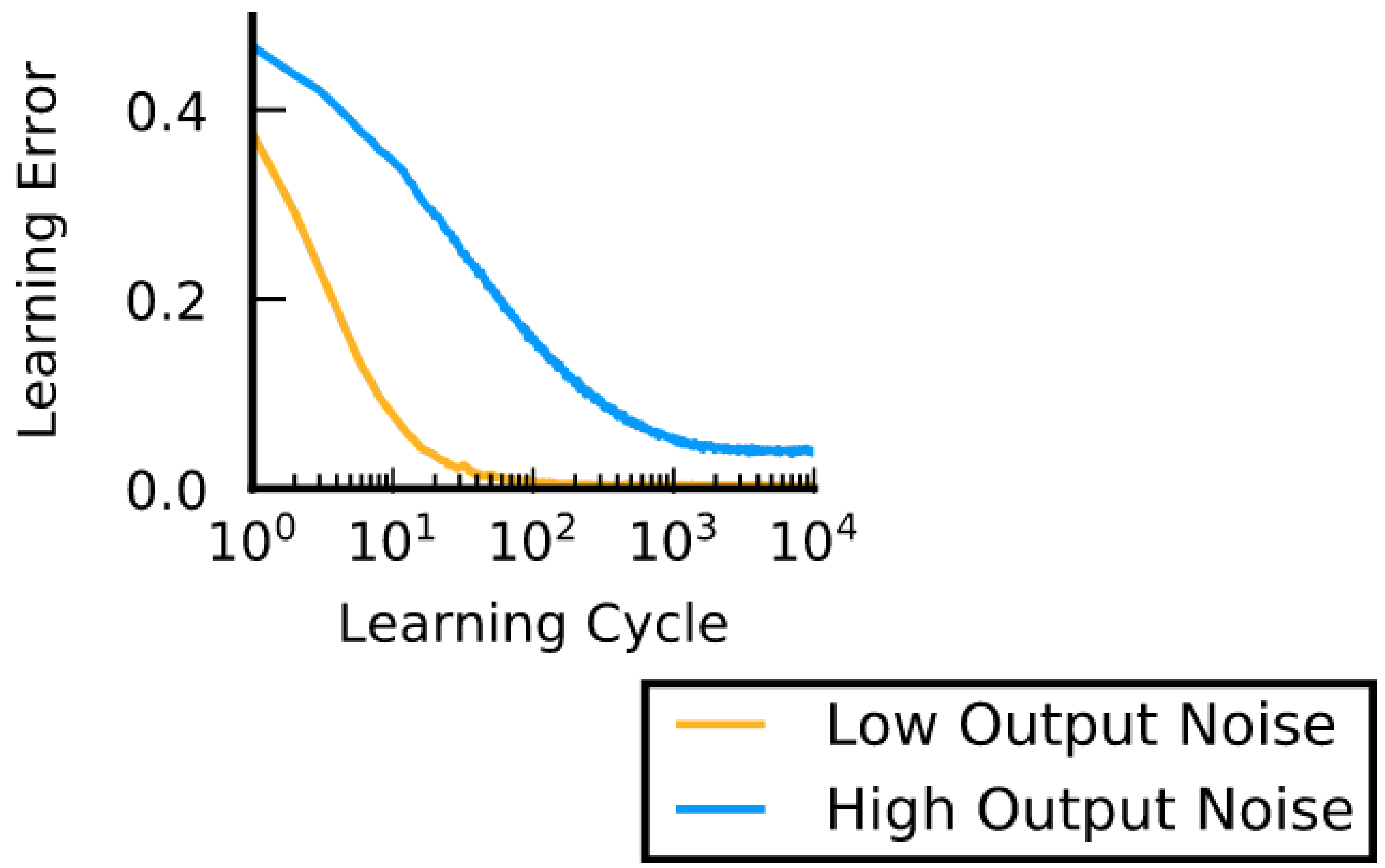

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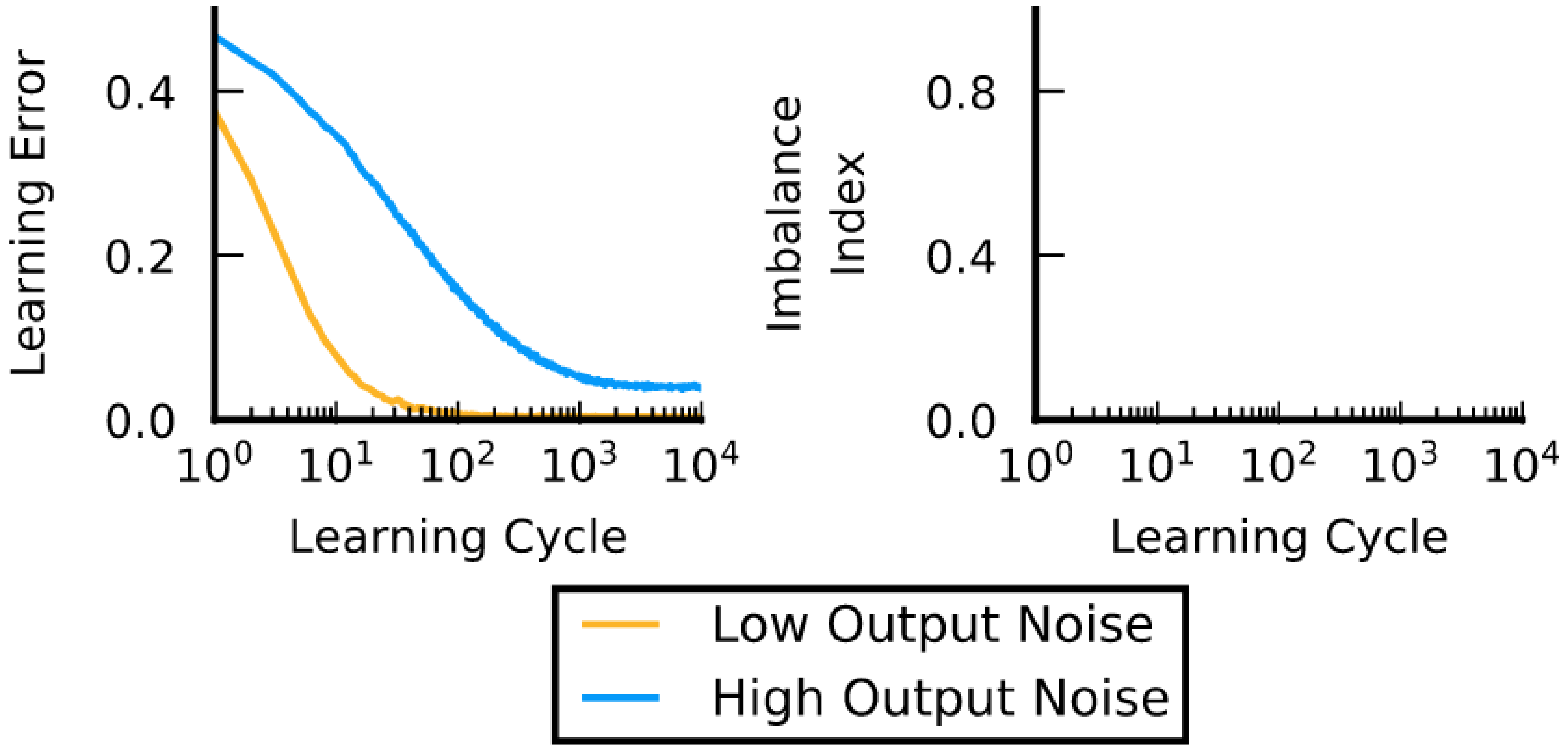
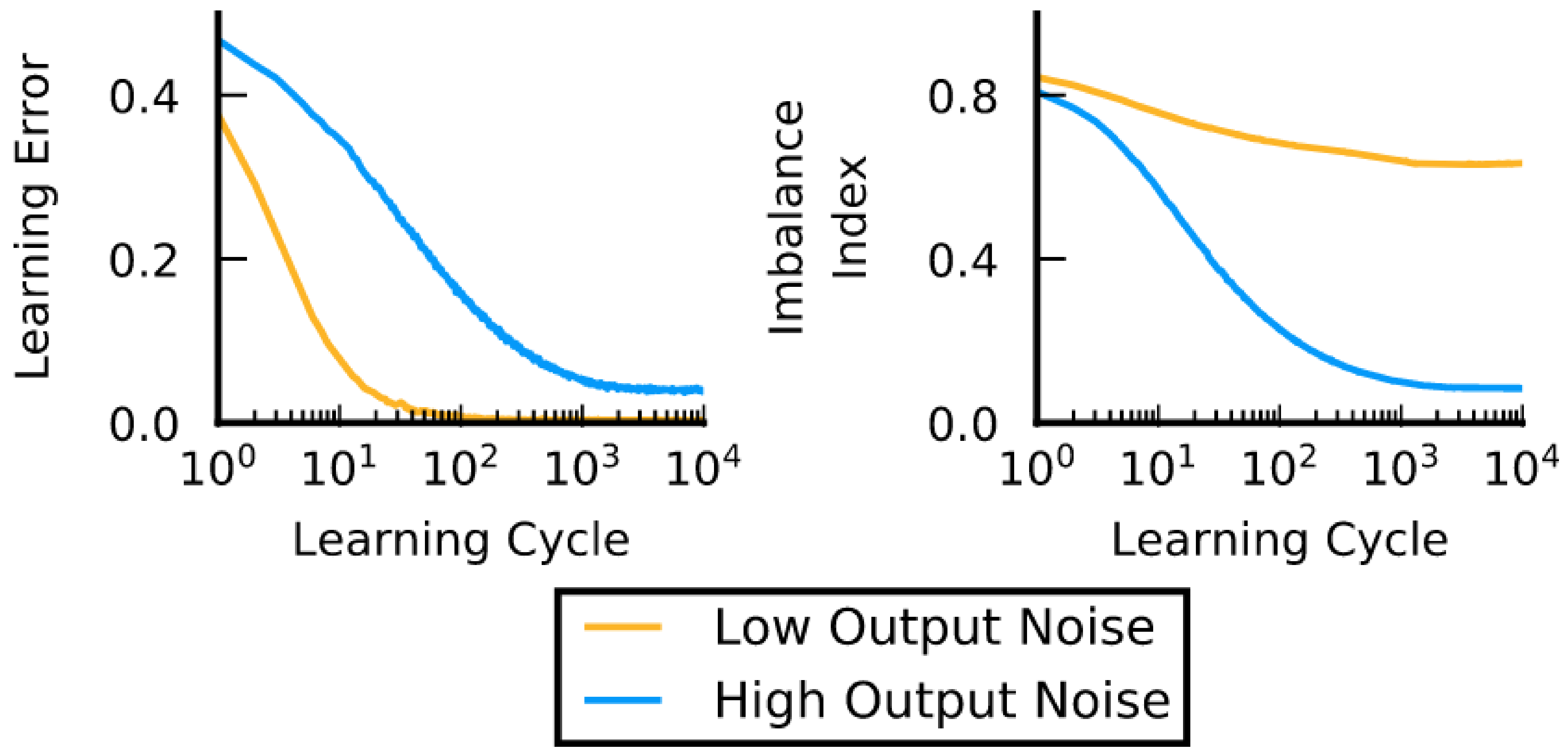


EMERGENCE OF E-I BALANCE



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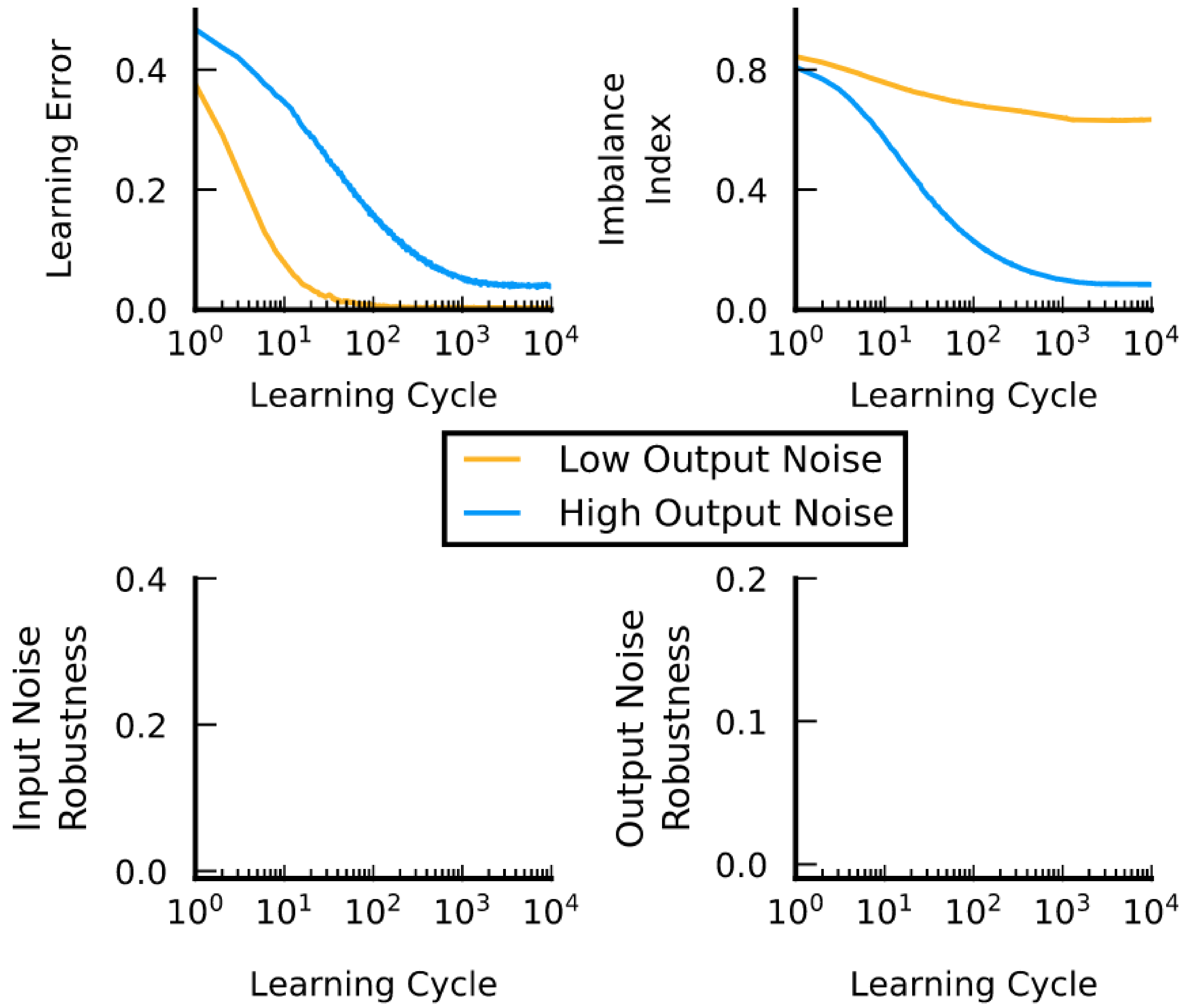

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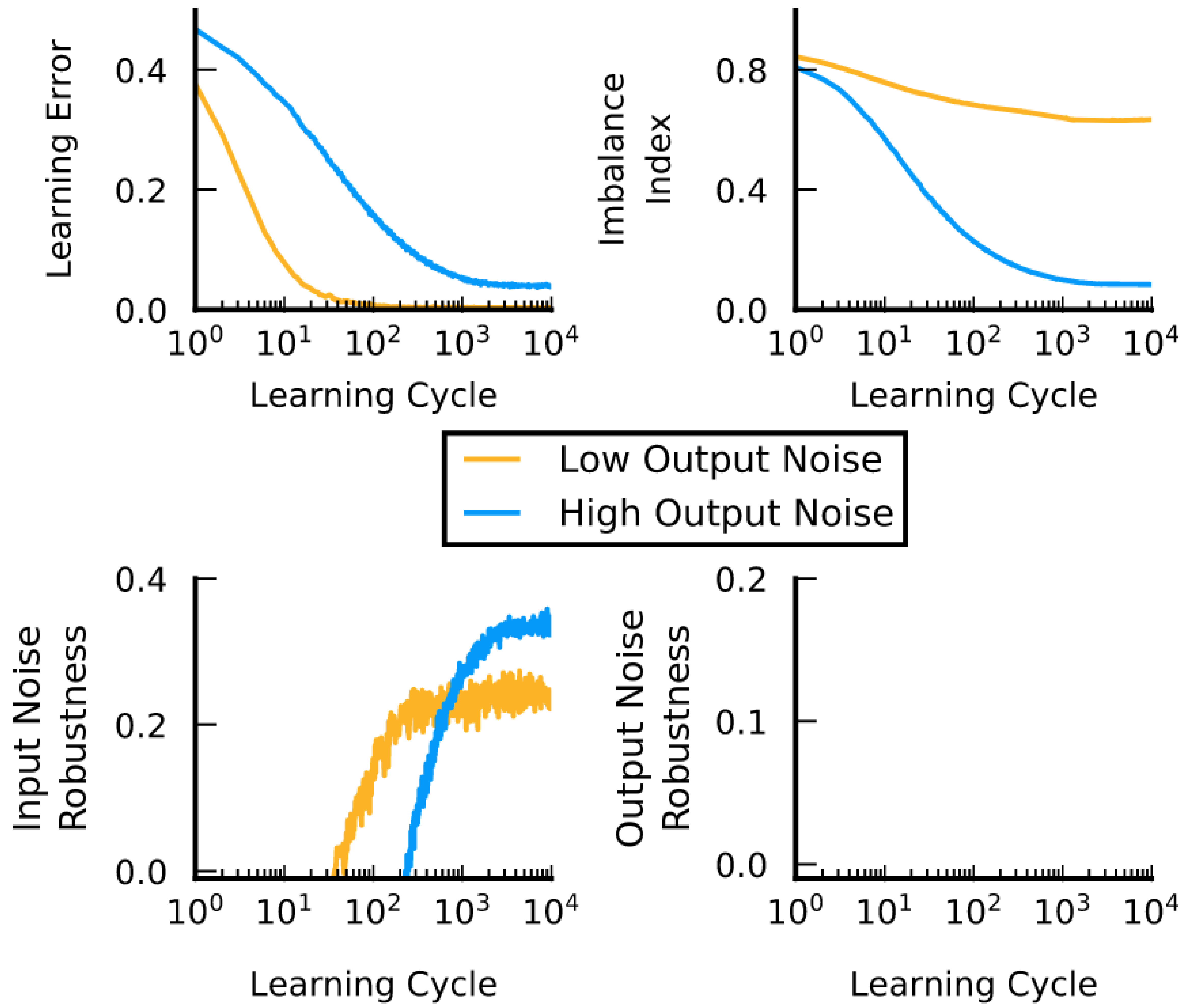
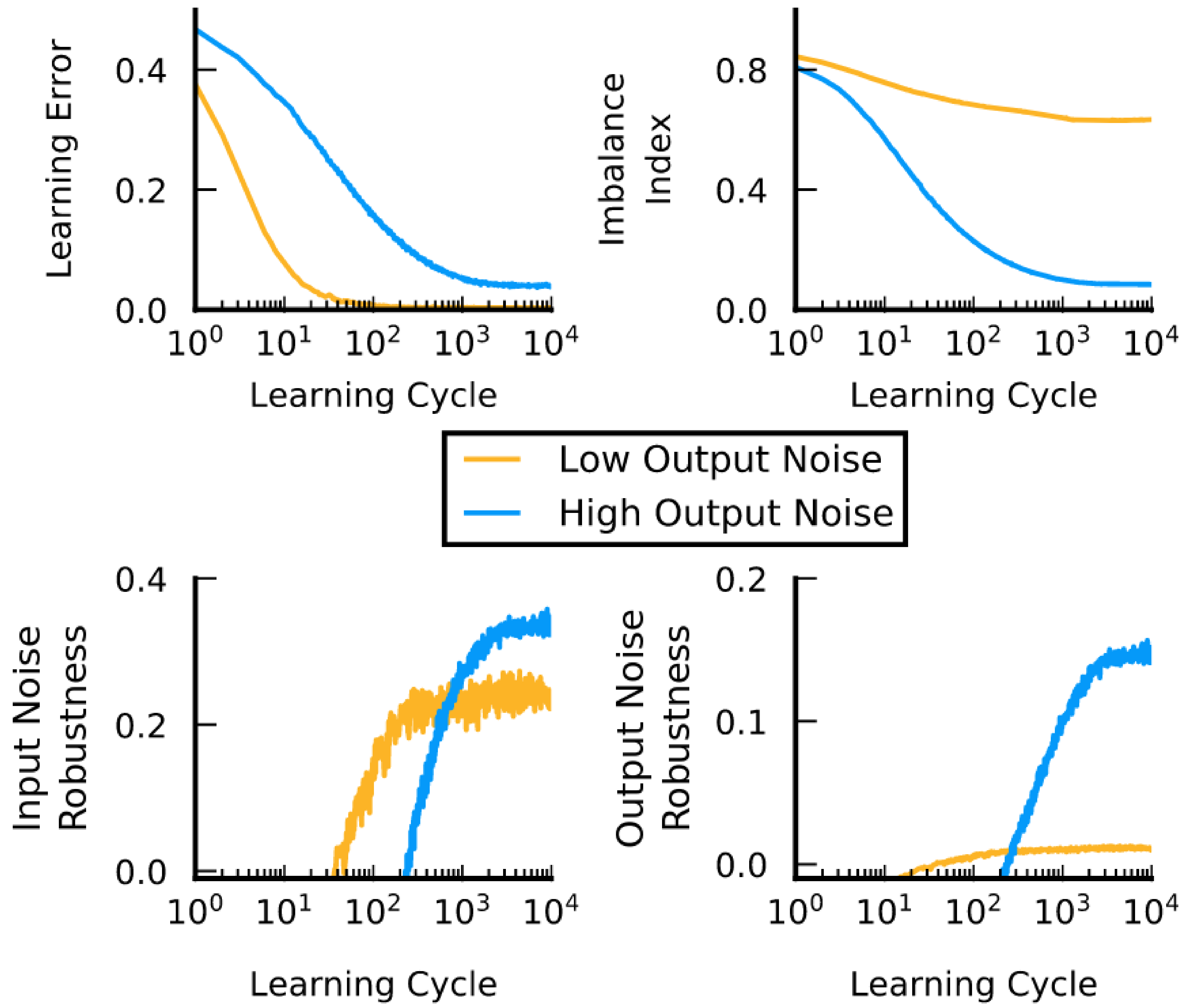


EMERGENCE OF E-I BALANCE



NEW BALANCE

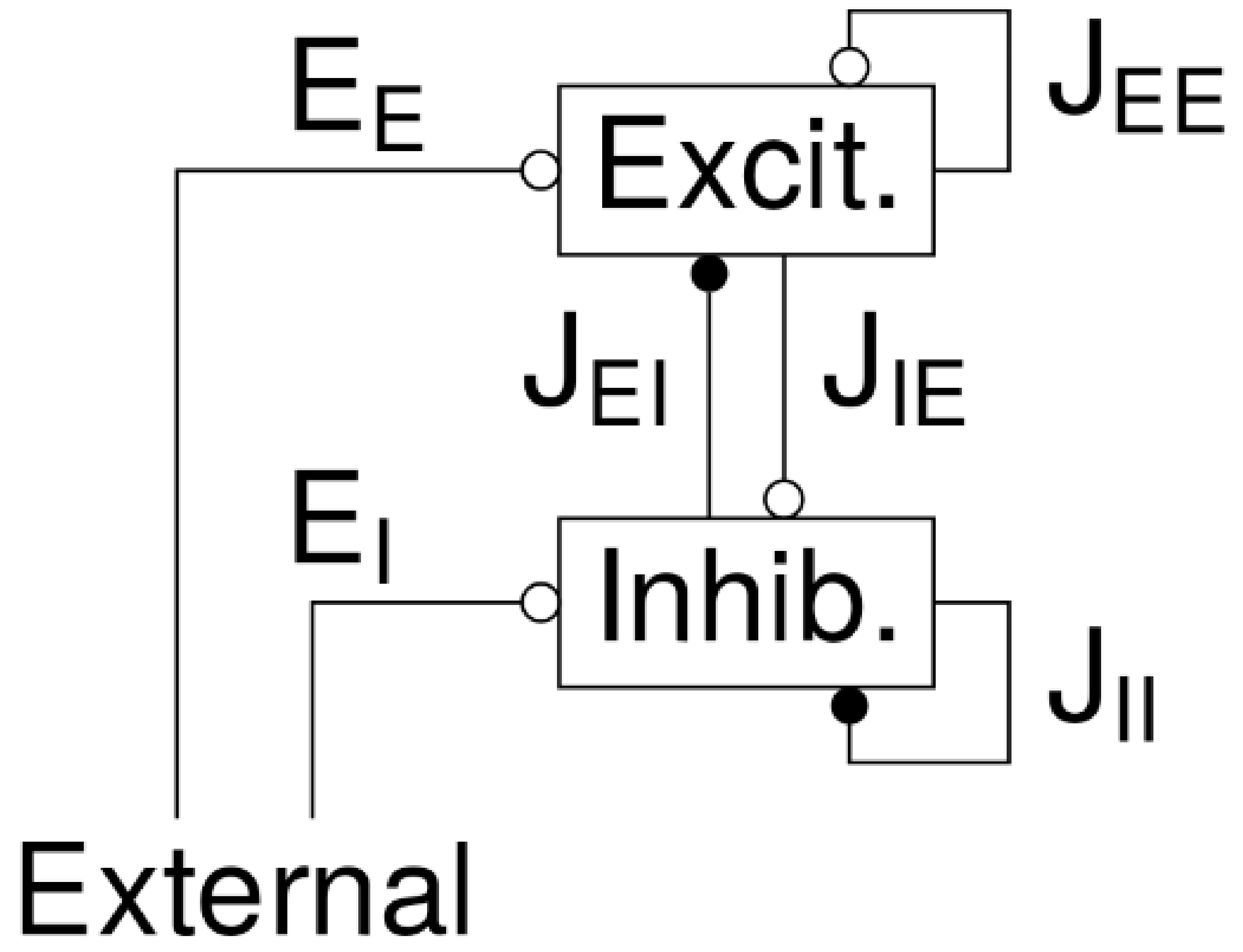
NEW BALANCE

OLD BALANCE

CLASSICAL BALANCED NETWORKS

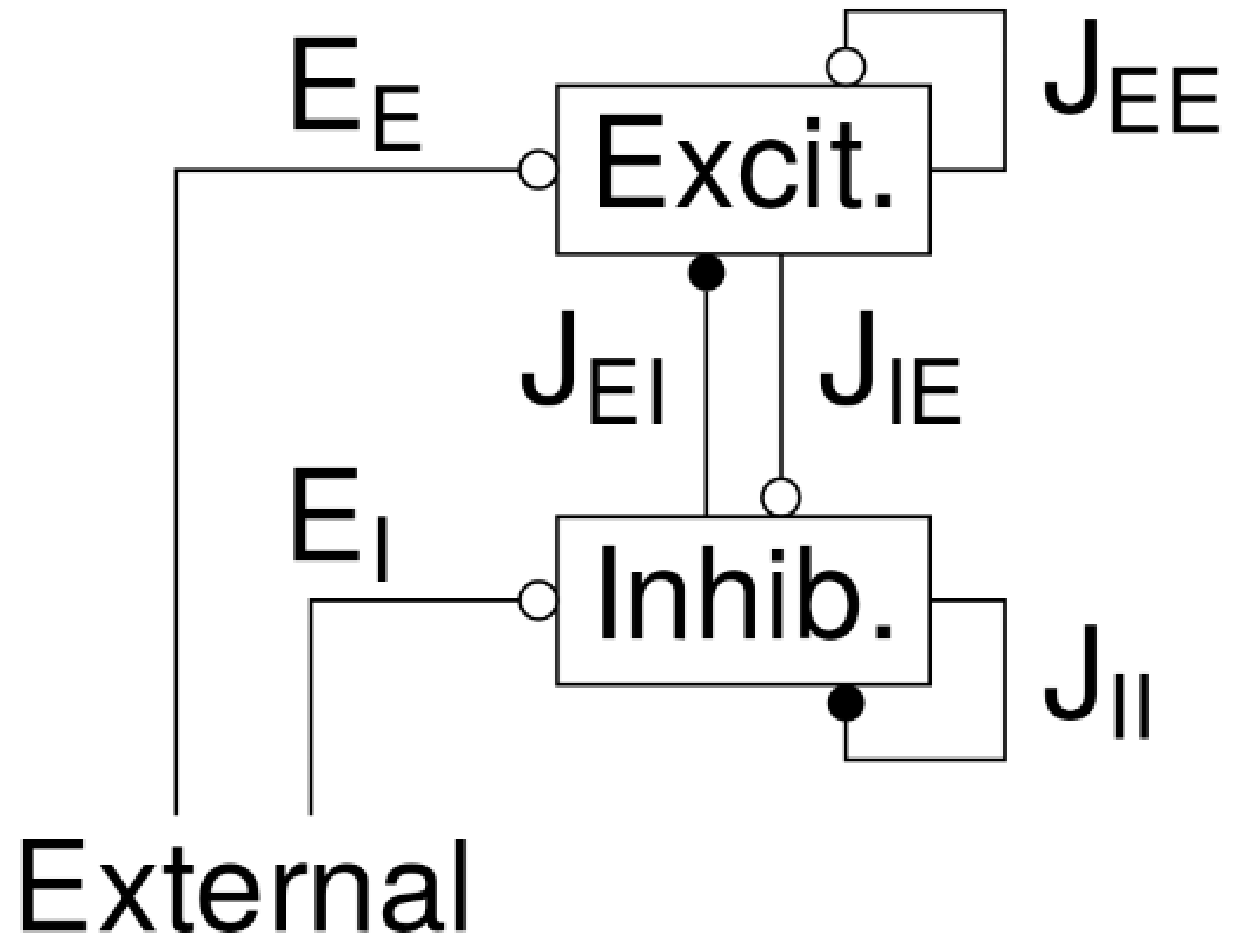
van Vreeswijk and Sompolinsky, Science, 1996

CLASSICAL BALANCED NETWORKS



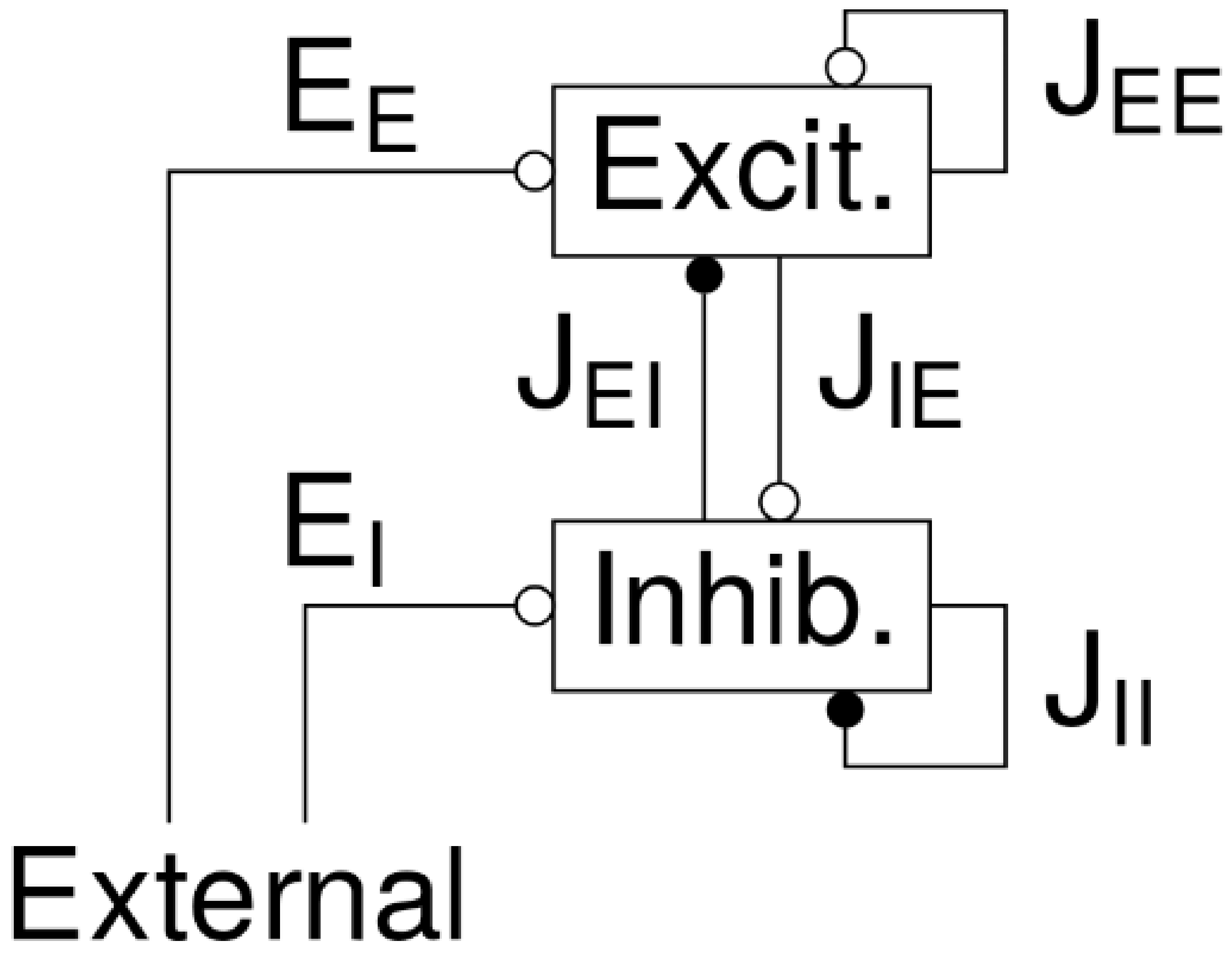
van Vreeswijk and Sompolinsky, Science, 1996

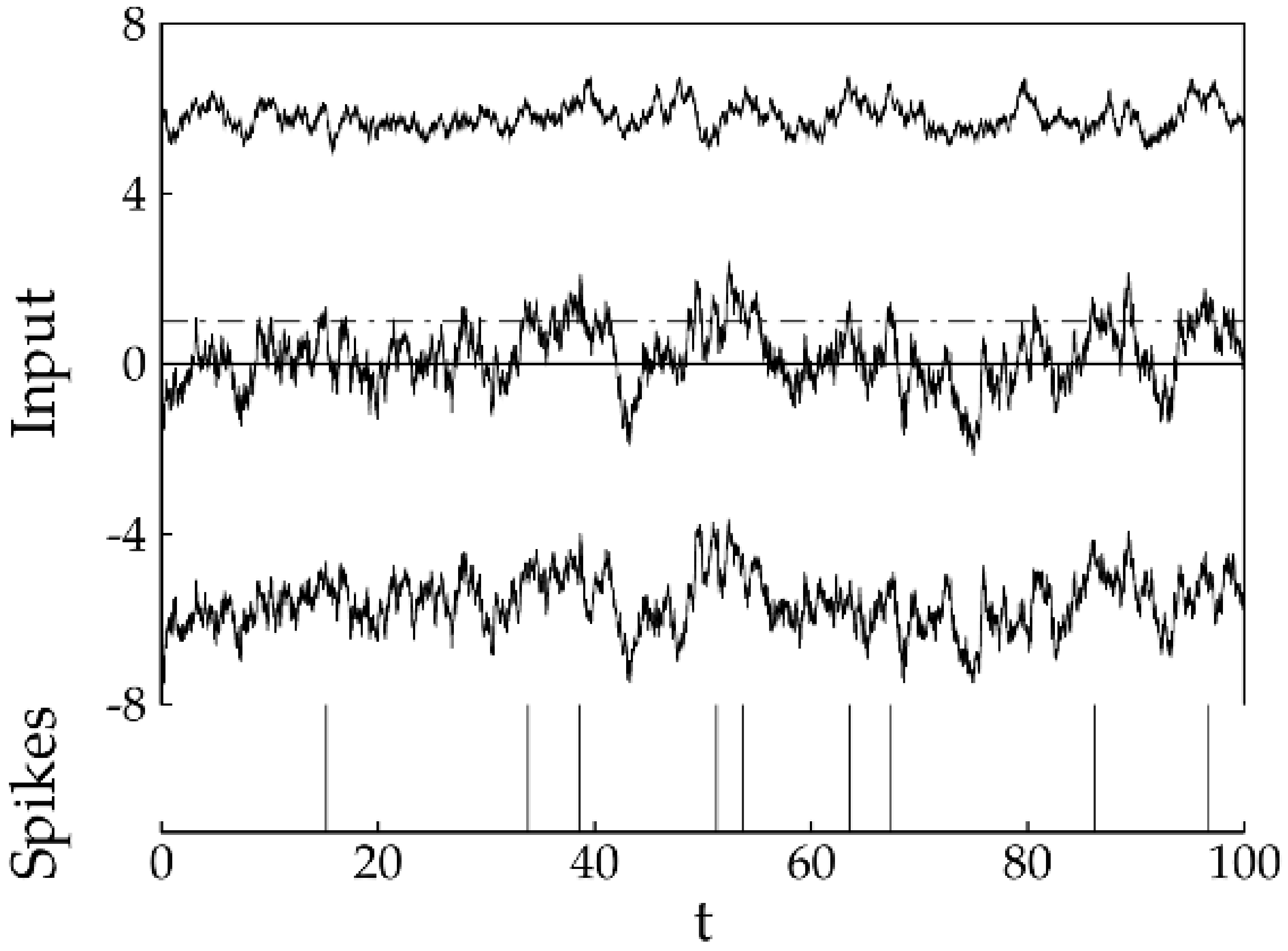
CLASSICAL BALANCED NETWORKS



van Vreeswijk and Sompolinsky, Science, 1996

CLASSICAL BALANCED NETWORKS





van Vreeswijk and Sompolinsky, Science, 1996

NEW BALANCE

OLD BALANCE