On the Performance of Cognitive Access with Periodic Spectrum Sensing

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ABSTRACT

The problem of cognitive access of parallel channels occupied by primary users is considered. The transmissions of primary users are modeled as independent continuous-time Markov processes. A secondary cognitive user employs a slotted transmission format and a periodic sensing strategy such that it decides if and where to transmit according to its sensing outcomes. The objective of the cognitive user is to maximize its throughput while satisfying collision constrains imposed by the primary users. Three access policies are analyzed. The optimal access policy is obtained based on a formulation of constrained Markov decision processes. A simple suboptimal memoryless policy is obtained by the use of instantaneous sensing outcome. As an upper bound of the optimal policy, a memoryless policy assuming the access of full channel states is also considered. For a symmetric system, we show that the simple memoryless policy is asymptotically optimal when the collision constraint is tight by proving that the performances of all three policies converge to the same value.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication; C.4 [**PERFORMANCE OF SYSTEMS**]: Performance attributes

General Terms

Performance

Keywords

Dynamic spectrum access, Constrained Markov decision processes, Resource allocation.

1. INTRODUCTION

We consider a hierarchical overlay cognitive network with N parallel communication channels shared by primary and secondary users [1]. The Primary Users (PUs) communicate through dedicated channels, oblivious to the presence of Secondary Users (SUs). On the other hand, a secondary user (SU) transmits opportunistically by first sensing a candidate channel. Based on the sensing outcome, the SU will decide if and where to transmit. The SU must also obey certain interference constraints so that its transmission will not interfere the communication of PUs beyond the acceptable levels. The first such cognitive coexistence scheme was proposed in [7].

When the occupancy behavior of PUs can be modeled statistically, the access policy of an SU can then be designed based on a certain optimization criterion. A natural objective for an SU is to maximize its throughput subject to collision constraints. For example, the network designer may want to assure the PUs that, whenever they transmit, the probability of colliding with an opportunistic SU is below a threshold, say, less than 1%. With such a guarantee, the PU may be willing to allow cognitive transmissions if it is compensated accordingly. Given the permission from PUs, an SU will try to capitalize transmission opportunities in the network.

The challenge to design an access protocol for a SU is twofold. First, the policy needs to choose a channel to sense according to its perspective on where transmission opportunities may lie. Second, once the sensing outcome is obtained, it needs to decide whether and how to transmit. For example, if the sensing outcome indicates that the channel is idle, it may choose to transmit for a period of time. If during its transmission, the PU assigned to that channel is idle, the transmission of the SU is successful[1], and the successfully decoded bits contribute to the throughput of the SU. Otherwise, the transmission of the PU collides with that of the

[1] We ignore the transmission failure caused by noise.

SU when the PU starts to transmit before the SU finishes. When the sensing outcome indicates that the PU is using the channel, the SU of course should not transmit on this channel, but it may decide to transmit immediately on a different channel (without sensing). Such an access scheme, first considered in [3], allows a fuller exploitation of transmission opportunities but with the additional implementation complexity.

1.1 Summary of Results and Context

In this paper, we assume that the transmissions of PUs are modeled as independent continuous-time Markov processes. A secondary cognitive user employs a slotted transmission format and a periodic sensing strategy such that it decides if and where to transmit according to its sensing outcomes. The objective of the cognitive user is to maximize its throughput while keeping the collision rate with the PUs below a given level.

We analyze the performance of three extreme access schemes: the optimal access with periodic sensing a suboptimal memoryless access using periodic sensing, and the optimal access with full observation Beyond the numerical evidence shown in [3], we provide a rigorous proof for the result that the policy assuming full observations serves as an upper bound for the optimal policy with period sensing although this relation sounds quite reasonable intuitively.

We also present in this paper an analysis of the gaps among these performance figures specifically when given the symmetric system which indicates the identical parameters and collision constraints. In particular, we prove that, when the collision constraint is tight, the optimal access policy performs identical to the policy that assumes the full observation of all channels. This implies that restricting sensing to periodic form carries no penalty.

Our results in someway are restrictive and narrow. For instance, we only considered the case when collision constraints are tightened. The reader will notice that the collision constraints for which no sensing is optimal are rather tight. The conditions presented in this paper is only sufficient and the practical significance of such scenarios is not clear. Also, we have assumed that sensing results are "perfect." In practice, sensing errors are inevitable. This and a number of other relevant issues are not addressed in this paper; they are subjects of further investigations.

1.2 Related Work

We will restrict ourselves to the problem of cognitive access in a hierarchical network of primary and secondary users. In this context, we refer to a recent survey by Zhao and Sadler [1]. We highlight here some related hierarchical access schemes and summarize the main contributions of this work. Related work in a broader context can be found in [4].

The joint design of sensing and access policy that maximizes the throughput of a cognitive SU subject to collision constraints is difficult in general, and it becomes tractable only when certain structures are imposed on the primary and secondary users. In [7, 8], Zhao *et al.* consider the case when all users follow a slotted transmission structure: when a PU has packets to transmit, it will do so at the beginning of the slot. When a cognitive SU needs to transmit, it will sense a channel and transmit only if the channel is idle. Such a slotted structure eliminates the possibility collision when sensing is perfect. The problem then becomes one of choosing the best channel to sense based on past experiences. If the transmissions of the PUs behave according to a Markov chain, the optimal policy can be formulated as a Partially Observable Markov Decision Process (POMDP). The POMDP can be solved by a linear program, but it suffers from the "curse of dimension" as the number of variables grows exponentially. However, under certain conditions, Zhao, Krishnamurthy, and Liu showed that the optimal policy is in fact myopic with a simple counting structure [12]. See also [13].

If the network does not impose the slotted structure (*e.g.* wireless LAN networks), the SU cannot assume collision-free transmissions even if the sensing result indicates the channel is free. The first work addressing this is [3] where the authors impose a slotted structure on the SUs while assuming that the PUs follow a continuous-time Markov process. Under such a model, the optimal policy becomes a constrained POMDP[2] for which no computationally tractable solutions exist. The authors of [3] impose the restriction that the SU can only sense the channel sequentially in a round robin fashion. Such a periodic sensing (PS) structure simplifies the problem from a constrained POMDP to a Constrained Markov Decision Process (CMDP) with a *finite* number of states. The optimal *periodic sensing* (PS) policy can again be obtained by a linear program. All three policies mentioned in Section 1.1 has also been proposed and compared numerically in [3]. Other related work assuming un-slotted PUs can be found in [14, 15, 16]. A formulation based on OFDMA physical layer can be found in [17].

2. SYSTEM MODEL

We follow the setting and notations in [3]. Assume that there are N parallel channels (indexed from 0 to $N-1$) available for transmissions by the PUs and an SU. Consider a hierarchical access scheme in which the PUs access the channels according to a certain protocol and the SU tries to access one of the N channels opportunistically.

We assume that the SU employs a slotted protocol. As illustrated in Fig. 1, in each slot, the SU i) senses one of the N channels at the beginning of the slot, ii) uses the sensing result to decide if and in which channel to transmit. For the periodic sensing access, the SU will sense the channel sequentially and make access decisions based on past sensing results.

The occupancy of each channel by a PU is assumed to evolve independently according to a homogeneous continuous-time Markov chain with idle ($X_i = 0$) and busy state ($X_i = 1$), respectively. The holding times are exponentially distributed with parameters λ_i^{-1} for the idle and μ_i^{-1} for the busy states, respectively. The state transition rate matrix (Q-matrix) under the continuous-time Markov process is given by

$$Q_i \triangleq \begin{pmatrix} -\lambda_i & \lambda_i \\ \mu_i & -\mu_i \end{pmatrix} \quad i = 0, 1, \ldots, N-1.$$

The stationary distribution of the ith PU's process can then be determined as

$$v_i(0) = \frac{\lambda_i}{\lambda_i + \mu_i}, \quad v_i(1) = \frac{\mu_i}{\lambda_i + \mu_i}. \tag{1}$$

[2]This can also be viewed as a constrained Markov decision process with uncountable state space.

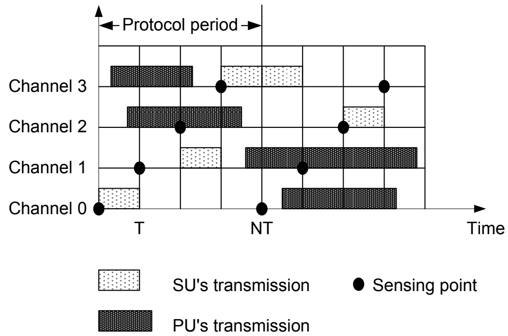

Figure 1: Sensing and Transmission Structure

3. SENSING SCHEME

Here we present a periodic sensing scheme and its induced Markov Chain model. The periodic sensing scheme, first introduced in [3], decouples sensing and access. The sensing and transmission events of the SU in PS protocol can be described as follows and illustrated in Fig. 1. The SU senses the channel in an increasing order at the beginning of each slot, starting from the smallest index (say, channel 0). Based on this and most recent N sensing results, the SU takes an action of either transmitting in one of the N channels or not transmitting at all.

Formally, define the k-th slot as an interval $I_k \triangleq [kT, (k+1)T]$ where T is the slot size. Assuming that sensing is perfect, we put together the sensing results of the most recent N slots to form an N-dimensional vector random process $Z(k) = [Z_0(k), \cdots, Z_{N-1}(k)]^T \in \{0,1\}^N$ defined by

$$Z_i(k) = \begin{cases} X_i(kT) & \text{if } i = k \bmod N \\ Z_i(k-1) & \text{otherwise} \end{cases} \qquad (2)$$

where $i = 0, 1, \cdots, N-1$ with $k = N, N+1, \cdots$ as its discrete-time index, and 'mod' denotes the modulus operation.

As shown in [3], the process $Z(k)$ is irreducible and periodic with period N. For each $q = 0, 1, \cdots, N-1$, the process $Z(pN+q), p = 1, 2, \cdots$, has the stationary distribution

$$f_q(z) = \prod_{i=0}^{N-1} (1_{[z_i=0]} v_i(0) + 1_{[z_i=1]} v_i(1)) \qquad (3)$$

where $1_{[\cdot]}$ denotes the indicator function.

4. OPTIMAL ACCESS

We present here the optimal access policy with periodic sensing based on the Constrained Markov Decision Process (CMDP) framework first introduced in [3]. A stationary random access policy π of the SU based on the periodic sensing process $Z(k)$ is a mapping from the value $Z(k) \in \{0,1\}^N$ to the probability distribution of the action taken in slot I_k by the SU defined on the set $\mathcal{A} = \{-1, 0, \cdots N-1\}$. The meaning of an action value $A_k \geq 0$ means a transmission in the A_k-th channel whereas $A_k = -1$ means no transmission in any channel.

We aim to maximize the throughput of the secondary system while abiding by constraints on the level of interference. Mathematically, we can formulate it as maximizing the average number of successful transmissions (of the SU)

$$J(\pi) = \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} E_\pi(r(Z(k), A_k, k)) \qquad (4)$$

where the expectation is taken over the probability distribution induced by a policy π. Here the immediate reward $r(Z, A_k, k)$, denoting the SU's successful transmission probability, can be analytically evaluated by

$$r(Z, A_k, k) = \begin{cases} g(Z, k \bmod N, A_k), & A_k \geq 0 \\ 0 & A_k = -1 \end{cases} \qquad (5)$$

where

$$g(Z, q, i) \triangleq \exp(-\lambda_i T)[\exp(Q_i \tau(i,q) T)]_{[z_i,0]}. \qquad (6)$$

Here $\tau(i,q) \triangleq (N+q-i) \bmod N$ is the "age" (in terms of number of slots) of the sensing result, and $[\exp(Q_i\tau(i,q)T)]_{[z_i,0]}$ is the transition probability of chain X_q from state z_q to 0.

At the same time, the constraints on interference to individual PU is defined as the asymptotic ratio of collision and successful transmission slots of the PU:

$$C_i(\pi) = \lim_{K\to\infty} \frac{\sum_{k=1}^{K} E[c(Z(k), i, k) \Pr(A_k = i|\pi, Z(k), k)]}{E(B_i(K))} \qquad (7)$$

where $B_i(k)$ is the total number of slots occupied by the PU in channel i up to time KT, and $\Pr(A_k = i|\pi, Z(k), k)$ represents the probability that channel i is chosen by policy π for the SU to transmit, given sensing result $Z(k)$. The cost $c(Z, A_k, k)$ which denotes the immediate collision probability is defined as

$$c(Z, A_k, k) = \begin{cases} 1 - r(Z, A_k, k) & A_k \geq 0 \\ 0 & A_k = -1 \end{cases}.$$

The optimal stationary random access policy achieving highest throughput can be found by solving the following CMDP problem:

$$\max_\pi J(\pi) \;\text{ subject to } C_i(\pi) \leq \gamma_i, \quad i = 0, \ldots, N-1, \qquad (8)$$

where $0 \leq \gamma_i \leq 1$ are given constants representing tolerable collision level in channel i.

5. THREE COGNITIVE ACCESS POLICIES

We give a brief description of three cognitive access policies. The first is a memoryless access (MA) policy that uses instantaneous sensing outcome and transmits with a fixed probability. This policy is suboptimal and its performance serves as a lower bound. The second is the optimal policy based on periodic channel sensing (PS). Finally, we present the case when the cognitive radio has full observation (FO) of all channel states. Given the Markovian channels evolution, this policy is the best possible policy among all slotted transmission systems (with or without periodic sensing). The performance of FO policy serves as an upper bound. The MA, PS and FO policies are the same as presented in [3].

5.1 Memoryless Access (MA) Policy: π^{MA}

If the access policy has no memory, the transmission will only depend on the current sensing outcome. Specifically, if in the kth slot, the SU senses a busy channel $q = k \bmod N$, no transmission is made. Otherwise it will transmit in the sensed channel q with probability β_q^{MA} given in [3],

$$\beta_q^{\mathrm{MA}} = \min(\frac{\gamma_q N(1-v_q(0)\exp(-\lambda_q T))}{1-\exp(-\lambda_q T)}, 1). \qquad (9)$$

which is decided such that collision constraints are satisfied while maximizing the throughput for the SU when channel q is sensed idle, and the collision constraint of channel q is γ_q. The throughput of this policy is:

$$J(\pi^{\mathrm{MA}}) = \frac{1}{N}\sum_{q=0}^{N-1} v_q(0)\beta_q^{\mathrm{MA}} \exp(-\lambda_q T). \qquad (10)$$

where $v_q(0) = \mu_q/(\lambda_q + \mu_q)$ is the stationary probability for channel q to be idle.

5.2 Optimal Access Policy with Periodic Sensing (PS): π^{PS}

In general, giving the state of the induced Markov chain $Z(k) = z$ in slot k, we need a strategy to choose one channel to transmit to maximize the throughput while satisfying the collision constraint. Such a policy can be obtained using the constrained Markov decision process (CMDP) formulation presented earlier.

Let the probability that we choose action $i \geq 0$ based on z and q be denoted by $\beta_{q,i}^{\mathrm{PS}}(z)$, and no transmission takes place with probability $\beta_{q,-1}^{\mathrm{PS}}(z) = 1 - \sum_{i=0}^{N-1}\beta_{q,i}^{\mathrm{PS}}(z)$. Given the set of collision constraints γ_i, the solution to CMDP (8) is given by the following linear program:

$$J(\pi_{\mathrm{opt}}^{\mathrm{PS}}) = \max_{\beta^{\mathrm{PS}}} \frac{1}{N}\sum_{q=0}^{N-1}\sum_{z\in\{0,1\}^N} f_q(z)\sum_{i=0}^{N-1} g^{\mathrm{PS}}(z,q,i)\beta_{q,i}^{\mathrm{PS}}(z) \qquad (11)$$

subject to

$$\sum_{q=0}^{N-1}\sum_{z\in\{0,1\}^N} \frac{f_q(Z)(1-g^{\mathrm{PS}}(z,q,i))\beta_{q,i}^{\mathrm{PS}}(z)}{N(1-v_i(0)\exp(-\lambda_i T))} \leq \gamma_i, \qquad \forall i \qquad (12)$$

$$\sum_{i=0}^{N-1}\beta_{q,i}^{\mathrm{PS}}(z) \leq 1, \forall q, z, \qquad \beta_{q,i}^{\mathrm{PS}}(z) \in [0,1], \ \forall q, z, i \qquad (13)$$

where $\beta^{\mathrm{PS}} = (\beta_{q,i}^{\mathrm{PS}}(z), q, i, z)$.

5.3 Optimal Cognitive Access with Full Observation (FO): π^{FO}

Full observation means that all the channels can be sensed simultaneously at the beginning of every time slot. Thus the access decision will be made based on the information $X(k) = x$ in time slot k, *i.e.*, the SU chooses the action $i \geq 0$ based on the observation $x = (x_i)$ with the probability $\beta_i^{\mathrm{FO}}(x)$. The optimal FO policy is a one step policy, and the immediate reward is given by

$$g^{\mathrm{FO}}(x,i) = \begin{cases} \exp(-\lambda_i T) & x_i = 0 \\ 0 & x_i = 1. \end{cases} \qquad (14)$$

Then the optimal access policy $\pi_{\mathrm{opt}}^{\mathrm{FO}}$ can also be solved through a linear programming problem as follows:

$$J(\pi_{\mathrm{opt}}^{\mathrm{FO}}) = \max_{\beta^{\mathrm{FO}}} \sum_{x\in\{0,1\}^N} f(x)\sum_{i=0}^{N-1} g^{\mathrm{FO}}(x,i)\beta_i^{\mathrm{FO}}(x) \qquad (15)$$

subject to

$$\sum_{x\in\{0,1\}^N} \frac{f(x)(1-g^{\mathrm{FO}}(x,i))\beta_i^{\mathrm{FO}}(X)}{(1-v_i(0)\exp(-\lambda_i T))} \leq \gamma_i, \qquad \forall i \qquad (16)$$

$$\sum_{i=0}^{N-1}\beta_i^{\mathrm{FO}}(x) \leq 1, \forall x, \qquad \beta_i^{\mathrm{FO}}(x) \in [0,1], \ \forall x, i \qquad (17)$$

where $\beta^{\mathrm{FO}} = (\beta_i^{\mathrm{FO}}(X), i, X)$.

6. THE OPTIMALITY OF MEMORYLESS ACCESS

When only one channel can be sensed at a time, the PS is the optimal policy. It cannot be easily discussed in an analytical way while we can still give its lower and upper bound of throughput analytically through the two heuristic policies MA and FO accordingly to estimate its performance.

Throughout this paper, we denote $J(\pi^{\mathrm{MA}})$, $J(\pi_{\mathrm{opt}}^{\mathrm{PS}})$ and $J(\pi_{\mathrm{opt}}^{\mathrm{FO}})$ as the optimal throughputs of MA, PS and FO, respectively.

PROPOSITION 1.

$$J(\pi^{\mathrm{MA}}) \leq J(\pi_{\mathrm{opt}}^{\mathrm{PS}}) \leq J(\pi_{\mathrm{opt}}^{\mathrm{FO}}). \qquad (18)$$

PROOF. The first inequality should be obvious as π^{MA} is on instance of the access policy that uses past N sensing outcomes and $\pi_{\mathrm{opt}}^{\mathrm{PS}}$ is optimal among such policies.

The second inequality, though intuitive, is not that obvious. We will just give a brief proof due to space limitation. For any PS solution $\beta_{q,i}^{\mathrm{PS}}(z)$, we can find a particular FO solution $\beta_i^{\mathrm{FO}}(x)$ accordingly:

$$\beta_i^{\mathrm{FO}}(x) = \frac{1}{N}\sum_{q=0}^{N-1}\sum_{z\in\{0,1\}^N} \Pr_q(z|x)\beta_{q,i}^{\mathrm{PS}}(z). \qquad (19)$$

so that when applying this solution, the FO throughput is equivalent to the according PS throughput and the FO constraints (16) and (17) are also satisfied which is not difficult get. It means that for the optimal PS policy $\pi_{\mathrm{opt}}^{\mathrm{PS}}$, there always exists an according available FO policy π^{FO} having the same throughput. Therefore,

$$J(\pi_{\mathrm{opt}}^{\mathrm{PS}}) = J(\pi^{\mathrm{FO}}) \leq J(\pi_{\mathrm{opt}}^{\mathrm{FO}}) \qquad (20)$$

and the second inequality exists. □

PROPOSITION 2. *The throughput of the MA policy is*

$$J(\pi^{\mathrm{MA}}) = \frac{1}{N}\sum_{q=0}^{N-1}[1_{\gamma_q\in(\gamma_q^{\mathrm{MA}},1]} v_q(0)\exp(-\lambda_q T) + 1_{\gamma_q\in[0,\gamma_q^{\mathrm{MA}}]}\frac{\gamma_q v_q(0) N\exp(-\lambda_q T)(1-v_q(0)\exp(-\lambda_q T))}{(1-\exp(-\lambda_q T))}] \qquad (21)$$

where $\gamma_q^{\mathrm{MA}} = \frac{(1-\exp(-\lambda_q T))}{N(1-v_q(0)\exp(-\lambda_q T))}$.

PROOF. From the MA optimal policy structure (9), we get the γ_q^{MA} through making $\frac{\gamma_q N(1-v_q(0)\exp(-\lambda_q T))}{1-\exp(-\lambda_q T)} = 1$, the policy is thus:

$$\beta_q^{\mathrm{MA}} = \begin{cases} \frac{\gamma_q N(1-v_q(0)\exp(-\lambda_q T))}{(1-\exp(-\lambda_q T))} & \gamma_q \in [0, \gamma_q^{\mathrm{MA}}] \\ 1 & \gamma_q \in (\gamma_q^{\mathrm{MA}}, 1]. \end{cases} \qquad (22)$$

and then the throughput can be got from (10) without much difficulty. □

Referring to the identical channels, it means that all the N channels have the identical parameters, e.g. $\lambda_i = \lambda, v_i(0) = v(0),\ \forall i$. Then we have the following proposition.

PROPOSITION 3. *Given the identical channels and the uniform collision constraint* $\gamma_i = \gamma, \forall i$,

$$J(\pi^{\mathrm{MA}}) = \begin{cases} \frac{\gamma v(0) N \exp(-\lambda T)(1-v(0)\exp(-\lambda T))}{(1-\exp(-\lambda T))} & \gamma \in [0, \gamma^{\mathrm{MA}}] \\ v(0)\exp(-\lambda T) & \gamma \in (\gamma^{\mathrm{MA}}, 1] \end{cases} \tag{23}$$

where

$$\gamma^{\mathrm{MA}} = \frac{(1-\exp(-\lambda T))}{N(1-v(0)\exp(-\lambda T))}. \tag{24}$$

PROOF. This proposition is easy to get when just applying the identical channel parameters $\lambda_i = \lambda, v_i(0) = v(0),\ \forall i$ and the uniform collision constraint $\gamma_i = \gamma, \forall i$ in the conclusion of the Proposition 2. $\square$

THEOREM 1. *Given identical channels and uniform collision constraint* γ,

$$J(\pi^{\mathrm{FO}}_{opt}) = \begin{cases} \frac{\gamma N(1-v(0)\exp(-\lambda T))\exp(-\lambda T)}{(1-\exp(-\lambda T))} & \gamma \in [0, \gamma^{\mathrm{FO}}] \\ (1-v(1)^N)\exp(-\lambda T) & \gamma \in (\gamma^{\mathrm{FO}}, 1]. \end{cases} \tag{25}$$

PROOF. In the FO problem, only idle channel can bring a positive reward when the SU accesses. When the channel is busy, accessing can only cost collision but no rewards. Thus it is not difficult to see that the following FO problem has the equivalent maximum throughput to the original FO problem:

$$J(\pi^{\mathrm{FO}}_{\mathrm{opt}}) = \exp(-\lambda T) \max_{\beta^{\mathrm{FO}}} \sum_{Z\in S} \sum_{i=0}^{N-1} f(x) 1_{(x_i=0)} \cdot \beta_i^{\mathrm{FO}}(x) \tag{26}$$

subject to

$$\sum_{x\in\{0,1\}^N} \frac{f(x)1_{(x_i=0)}\cdot(1-\exp(-\lambda T))\beta_i^{\mathrm{FO}}(x)}{(1-v(0)\exp(-\lambda T))} \le \gamma \qquad \forall i, \tag{27}$$

$$\sum_{i=0}^{N-1} \beta_i^{\mathrm{FO}}(x) \le 1 \qquad \forall x. \tag{28}$$

Define

$$W(\gamma) = \frac{\gamma N(1-v(0)\exp(-\lambda T))\exp(-\lambda T)}{(1-\exp(-\lambda T))}.$$

When summing up all the N inequations in (27), and multiply $\exp(-\lambda T)$ we have

$$\sum_{x\in S}\sum_{i=0}^{N-1} f(x)1_{(z_i=0)}\cdot\beta_i^{\mathrm{FO}}(x)\exp(-\lambda T) \le W(\gamma). \tag{29}$$

Note that the left side in (29) is exactly the objective in (26), which means $W(\gamma)$ supplies an upper bound of the FO objective $J(\pi^{\mathrm{FO}}_{\mathrm{opt}})$. Thus we try to find a particular solution to achieve the upper bound $W(\gamma)$. To achieve this upper bound is equivalent to make the equal mark in (29) satisfied, which is also equivalent to that all the N equality relations in (27) are satisfied simultaneously. First, we will give this solution, when $\gamma \in [\gamma^{(k-1)}, \gamma^{(k)}],\ 1 \le k \le N$:

$$\beta_i(x) = \begin{cases} 1/m & x_i = 0; x \text{ contains m } '0'\text{s}, 0 < \text{m} < \text{k} \\ \Delta\beta & x_i = 0; x \text{ contains k } '0'\text{s} \\ 0 & \text{others} \end{cases} \tag{30}$$

where

$$\Delta\beta = \frac{\left(\frac{\gamma(1-v(0)\exp(-\lambda T))}{1-\exp(-\lambda T)} - \sum_{j=1}^{k-1}\frac{C_N^j}{N} v(0)^j v(1)^{N-j}\right)}{\left(\frac{kC_N^k}{N} v(0)^k v(1)^{N-k}\right)} \tag{31}$$

and

$$\gamma^{(k)} = \begin{cases} 0 & k = 0, \\ \frac{\sum_{j=1}^k \frac{C_N^j}{N} v^j(0) v^{N-j}(1)(1-\exp(-\lambda T))}{1-v(0)\exp(-\lambda T)} & 0 < k \le N. \end{cases} \tag{32}$$

In (30), "x contains m '0's" means that there are m idle observations in the observation vector x, e.g. $x = (1,1,0,1,\cdots,1)^T$ means "x contains 1 '0'". For simplicity, we also define

$$\gamma^{\mathrm{FO}} \triangleq \gamma^{(N)} = \frac{(1-v(1)^N)(1-\exp(-\lambda T))}{N(1-v(0)\exp(-\lambda T))}. \tag{33}$$

When given the collision constraint $\gamma \in (\gamma^{(k-1)}, \gamma^{(k)}]$, the solution is determined according to (30)-(32). Then it is not difficult to testify that all the N equalities in (27) can be satisfied simultaneously. Therefore the upper bound of the FO object $W(\gamma)$ is the optimal FO throughput:

$$J(\pi^{\mathrm{FO}}_{\mathrm{opt}}(\gamma)) = \frac{\gamma N(1-v(0)\exp(-\lambda T))\exp(-\lambda T)}{1-\exp(-\lambda T)}, \quad \gamma \in (0, \gamma^{\mathrm{FO}}]\ . \tag{34}$$

Therefore the maximum achievable throughput is obtained as follows when $\gamma = \gamma^{\mathrm{FO}}$:

$$\begin{aligned} & J(\pi^{\mathrm{FO}}_{\mathrm{opt}}(\gamma^{\mathrm{FO}})) \\ = \ & \exp(-\lambda T) \sum_{x\in\{0,1\}^N} f(x)\left(\sum_{i=1}^N 1_{(x_i=0)}\beta_i^{\mathrm{FO}}(x)\right) \\ = \ & \exp(-\lambda T)(1-v(1)^N). \end{aligned} \tag{35}$$

When $\gamma \in (\gamma^{\mathrm{FO}}, 1]$, given $\gamma' < \gamma''$ in it, we can conclude that $J(\pi^{\mathrm{FO}}_{\mathrm{opt}}(\gamma')) \le J(\pi^{\mathrm{FO}}_{\mathrm{opt}}(\gamma''))$. Considering the extreme case $\gamma = 1$, the constraint (16) can be ignored, because (16) are satisfied for all the available solutions in (17), and then

$$J(\pi^{\mathrm{FO}}_{\mathrm{opt}}(\gamma = 1)) = \exp(-\lambda T)(1-v(1)^N) = J(\pi^{\mathrm{FO}}_{\mathrm{opt}}(\gamma^{\mathrm{FO}})).$$

So when $\gamma \in (\gamma^{\mathrm{FO}}, 1]$, we have
$J(\pi^{\mathrm{FO}}_{\mathrm{opt}}(\gamma^{\mathrm{FO}})) \le J(\pi^{\mathrm{FO}}_{\mathrm{opt}}(\gamma)) \le J(\pi^{\mathrm{FO}}_{\mathrm{opt}}(\gamma = 1)) = J(\pi^{\mathrm{FO}}_{\mathrm{opt}}(\gamma^{\mathrm{FO}}))$, which means when $\gamma \in (\gamma^{\mathrm{FO}}, 1]$, $J(\pi^{FO}_{opt}(\gamma))$ can be given as

$$J(\pi^{FO}_{opt}(\gamma)) = \exp(-\lambda T)(1-v(1)^N). \tag{36}$$

$\square$

From Proposition 2 and Proposition 3, we can get the analytical lower bound of the optimal PS performance. When given identical channels and uniform collision constraints, Theorem 1 supplies an analytical upper bound of the optimal PS performance. So given identical channels and uniform collision constraints, we can get the performance gap between the upper and lower bound:

$$J(\pi^{\mathrm{FO}}_{\mathrm{opt}}) - J(\pi^{\mathrm{MA}}) = \begin{cases} \frac{\gamma(1-v(0))N\exp(-\lambda T)(1-v(0)\exp(-\lambda T))}{(1-\exp(-\lambda T))} & \gamma \in [0, \gamma^{\mathrm{MA}}] \\ \left(\frac{\gamma N(1-v(0)\exp(-\lambda T))}{(1-\exp(-\lambda T))} - v(0)\right)\exp(-\lambda T) & \gamma \in (\gamma^{\mathrm{MA}}, \gamma^{\mathrm{FO}}] \\ (v(1)-v(1)^N)\exp(-\lambda T) & \gamma \in (\gamma^{\mathrm{FO}}, 1]. \end{cases} \tag{37}$$

We can see that

$$\lim_{\gamma\to 0}(J(\pi^{FO}_{opt}) - J(\pi^{MA})) = 0, \tag{38}$$

which means that when the collision constraint is very tight, the MA policy performs sufficiently close to the FO policy and thus to the PS policy, and we can approximately consider the MA policy is the optimal policy. It makes sense that when N is large, the linear program in PS is difficult to solve while MA policy can be easy to get, and lie in its simplicity when the strategy needs to adjust frequently in response to frequent changes in parameters of the PU.

7. NUMERICAL SIMULATION

In our experiments, the choices of the parameters are motivated from experiments conducted in [11]. The idle times can be approximated by an exponential distribution with parameter $\lambda^{-1} = 4.2$ms, and we also assume $\mu^{-1} = 1$ms for the channel's busy period. We focus on the case $N = 6$ and consider the tendency of throughput variation as we loosen the constraints. By assuming a slot size $T = 0.25$ms, we obtain the throughput characteristics. By checking the numerical results (curves are omitted due to space limit), as expected, the optimal throughputs of MA, PS and FO are in a non-decreasing order. When the constraint is tight, viz.,$\gamma \in$[0,0.033], PS and FO perform the same and grow linearly with the increasing value of γ while MA gives a suboptimal performance. When γ becomes bigger, viz., $\gamma \in$[0.034,0.05], there is a loss in the throughput of the optimal PS compared with the optimal FO, which accords to the intuitive analysis. When γ is big enough, the optimal throughout of the FO keeps a maximum constant value. The MA performance will not increase when $\gamma > 0.04$ and it shows that it is indeed a suboptimal policy. We also do the numerical computation to validate the correctness of the analytical expressions. When substituting the parameters in the expressions (24), (23), (33) and (35), we have $\gamma^{\rm MA} = 0.0403$, $J(\pi^{\rm MA}(\gamma^{\rm MA})) = 0.7610$, $\gamma^{\rm FO} = 0.0403$ and $J(\pi^{\rm FO}_{\rm opt}(\gamma^{\rm FO})) = 0.9422$. These are all matching our numerical results.

8. ACKNOWLEDGMENTS

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