# Optimal opportunistic sensing in cognitive radio networks

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**Abstract**

The authors are interested in evaluating the performance of a Cognitive Radio (CR) network composed of Secondary Users (SUs) and Primary Users (PUs). We look for optimizing the decision process of the secondary mobiles when they have to choose between licensed or unlicensed channels. In fact, the system is composed of several channels where only one unlicensed channel is shared between all the secondary mobiles, when they decide to use this particular channel. As the secondary mobiles are equipped with cognitive radios, they are able to sense the licensed channels and use one of them if it is free. We consider first the global system and look for the optimal proportion of secondary mobiles that sense the licensed channels in order to optimize an average performance of the system. Second, we assume that each secondary mobile decides to sense or not the licensed channels and, we are interested in an equilibrium situation as the SUs are in competition. After showing the existence and the uniqueness of equilibrium, we evaluate the performance of this equilibrium by looking at the price of the anarchy of the system.

## I. Introduction

A big new challenge in the networking community is how to put 'cognition' into networks. The term cognition is described as the faculty for a mobile or a network to adapt its communication parameters (transmission power for mobiles or frequency for a base station) to perturbations of its environment. A radio system having this capability is called a cognitive radio. This new field of research has started with the work of Mitola [1] and the faculty of new frequency channels usage. In wireless networks, in contrast to wired networks, the capacity is limited to the radio spectrum. Since the FCC has proposed, in November 2002, to open the use of many bands that has already been assigned but not sufficiently utilized, CR based wireless network architectures have been proposed in order to allow SUs to access licensed channels. Indeed, the FCC report reveals that the electromagnetic spectrum has gaps, i.e. frequency bands that has been assigned to licensed users, at a particular time and specific geographic location, are not being utilized. Note that locating unused frequencies, accounting for the energy spent in sensing the licensed channels, represent a big challenge for SUs. Moreover, the proposed CR architectures do not guarantee some QoS levels for SUs, which is mainly impacted by the PUs' activity and the interaction between SUs.

The operation model described in [2] introduces a new set of theoretical problems involving game theory, queuing theory and decision theory. Specifically, we focus, in this paper, on SUs having the faculty to sense the licensed bands and access them if idle or to access a dedicated spectrum. We are interested in designing an optimal spectrum sensing and access strategy for unlicensed users. In the first part of this paper, we consider a slotted communications for PUs and SUs. Indeed, we consider that the system is perfectly synchronized, and we assume that PUs and SUs have the same slot duration and moreover, we ignore the sensing errors, i.e. the false alarm and missing probability of sensing are zero. Thus, if the SU senses a licensed channel as idle, it still idle during the whole time slot. Most of previous works in the OSA area for CR networks have already taken these assumptions (see [3] and [4]). In the second part of this paper, we consider a more realistic scenario in which PUs operate in a non-slotted mode. Due to the agreement between the SP and PUs, the number of licensed channels should be higher than the number of PUs transmitting simultaneously. We further assume that PUs are able to determine whether there is a free channel or not. As the PUs have the highest priority to access their own licensed channels, if all the licensed channels are occupied, a new PU preempts a SU that is using a licensed channel. The rejected SU abort the transmission and try to transmit the whole packet at the next time slot. As the access to licensed channels is opportunistic, successful SUs' transmissions are highly dependent of the presence of PUs, and therefore the dedicated channel represents a guarantee of a lower bound of QoS for SUs.

Lots of recent works deal with CR technologies and their performances. The survey paper [5] gives some interesting problems for evaluating the performance of CR systems. Specifically, S. Haykin describes that mathematical tools and game theory, can be applied to analyze complex wireless systems. In [6], the authors have considered an energy efficient spectrum access policy. Each SU senses the spectrum and selects subcarriers taking into account data rate requirements and maximum power limit. This work is close to ours as the authors have studied the problem by considering a non-cooperative behavior among SUs and moreover, they have considered energy efficient allocation scheme. Note that the authors have considered that each SU that has traffic to transmit systematically senses the spectrum and locates the available subcarrier set. Thus, the authors have decoupled the sensing and the access decision and the

OSA problem is resumed to a decision about which channel to access from the set of available subcarriers. However, in our model, we consider that SUs may decide to access the dedicated channel without sensing the licensed spectrum.

The authors of [7] have proposed an OSA algorithm for SUs composed of two parts: first, a SU decides if the licensed channel is idle or not. Second, it determines whether this channel is a good opportunity or not based on channel sensing statistics. However, they have not considered the impact of multiple SUs. In fact, they have focused on the model of one SU accessing opportunistically a channel licensed for a PU.

Unlike most of previous works in the DSA area, we study decision-making methods and the corresponding equilibrium analysis using the queuing theory. Jagannathan et al. have considered in [8] a model similar to ours, where SUs choose to either acquire dedicated spectrum or to use spectrum holes. They have considered a pricing model and have studied the interaction between SUs as a non-cooperative game. There are several differences between their work and ours. First, they have considered that SUs sense systematically the licensed spectrum and make the decision about transmitting over the licensed channels or through the dedicated spectrum after the sensing outcome, however we consider that SUs choose the transmission medium before sensing in order to economize the energy spent for sensing when accessing to the dedicated bands. Secondly, they have considered that there is a centralized component that schedule SUs trying to access the licensed channels, however we consider that SUs are in competition, and collisions occur when several SUs access the same licensed channels. Moreover, the authors have not considered the energy spent for sensing the licensed channels.

The remainder of this paper is as follows. In the next section, we present the system model that will be studied in this paper. Section III focus on the model where PUs' transmissions are slotted. In Section IV we present the non-slotted model for PUs, and we consider that PUs may preempt a SU in service over the licensed channels. Before concluding the paper, we present some numerical illustrations for both the slotted and the non-slotted models.

## II. The system model

In this paper, we consider a system composed of $K + 1$ channels, where PUs are licensed to use $K$ channels and one dedicated channel is shared between all the SUs. The PUs (resp. SUs) arrive following a Poisson process with rate $\lambda_p$ (resp. $\lambda_s$). Note that each SU decides whether to sense the licensed channels or not. If it senses the spectrum and finds one free channel, it uses this channel for transmission. If it senses all the licensed channels as occupied, then the SU joins the dedicated channel reserved for all

TABLE I

THE SYSTEM PARAMETERS' DESCRIPTION

| Parameter | Description |
|---|---|
| $\lambda_p$ | arrival rate of PUs |
| $\lambda_s$ | arrival rate of SUs |
| $\mu_p$ | service rate in a licensed channel |
| $\mu_s$ | service rate for a SU in the dedicated channel |
| $K$ | the number of channels allocated for the PUs |
| $p$ | probability of sensing the licensed channels |
| $\alpha$ | the cost of sensing one channel for a SU |
| $\rho(p)$ | $\frac{(\lambda_p+p\lambda_s)}{\mu_p}$ |

the SUs. We denote by $p$ the probability that a SU senses the licensed channels. This probability may be considered as the proportion of SUs that chooses to sense the spectrum. This repartition of SUs can be set by a central controller, or determined individually by SUs in a decentralized manner. Moreover, we consider that SUs are operating via a limited battery and therefore have to be energy efficient. We assume that there is a cost $\alpha$ for sensing one licensed channel and moreover, if a SU decides to sense, it senses all the $K$ licensed channels. Many works, such as [9] and [8], have considered that SUs sense all the licensed channels. Some other works have considered periodic sensing (see [10]), whereas authors of [11] have considered that the SU selects and senses randomly one licensed channel. The service rates are denoted by $\mu_p$ (resp. $\mu_s$) for the licensed channels (resp. the dedicated channel), and are supposed to have an exponential distribution. The system model is depicted in Figure 1 and is composed of two sub-systems. The first one, namely subsystem $S_1$, represents the secondary subsystem, and the primary subsystem, denoted by $S_2$, is licensed for PUs and open for SUs' opportunistic access.

We give in the following some intuitions about the optimal OSA strategy for SUs in our model. Because of the cost of sensing, when the blocking probability in the primary subsystem $S_2$ increases, the SU has less incentive to sense the spectrum. In fact, if the SU does not find a free licensed channel, it uses the dedicated channel and pays also the sensing cost. However, if it decides to use the dedicated channel without sensing, it does not pay the sensing cost but transmits the packet with higher delay than using the licensed channels. Moreover, more there are SUs in the subsystem $S_1$, higher is the transmission delay for all the SUs using that subsystem. Therefore, there is a tradeoff for SUs whether to sense or not the licensed channels. We summarize, in Table I, the parameters of the model.

Obviously, SUs have to deal with two following performance metrics: the packets' delays and the energy spent for transmission. In fact, if the SU senses the licensed channels and finds one free channel, it transmits the held packet with a lower delay than transmitting over the dedicated channel. However, it spends energy for sensing the licensed channels. We define the main global metric of the system, which is the average total cost $U_S$ as a composition of the two following parts: the average sojourn time of a SU inside the system and the cost of sensing:

- The average sojourn time, denoted by $T_S$, depends on several parameters: the arrival rates of PUs and SUs, the service rates, the number of licensed channels and the sensing probability.
- The sensing cost $c_s$ depends on the number of licensed channels, and on the probability of sensing. We assume that this cost is linear with the number of licensed channels, i.e. $c_s(p, K) = \alpha K p$. In fact, the cost of sensing represents the energy spent in sensing. Note that SUs are supposed to sense all the licensed channels.

The average total cost, denoted by $U_S$, for a SU that chooses to sense the licensed channels with probability $p$ is given by:

$$U_S(p, K) \quad = \quad T_S(p, K) + c_s(p, K) = T_S(p, K) + \alpha p K. \tag{1}$$

The average sojourn time $T_S$ of a SU inside the system depends on the decision taken by the SU: to use the licensed channels or the dedicated one. We denote by $T_{S_1}$ (resp. $T_{S_2}$) the sojourn time if the SU that decides to transmit over the dedicated channel (resp. licensed channels). We assume that the sensing period is negligible compared to the sojourn time in both subsystems. Thus, the average sojourn time $T_S$ is expressed by:

$$T_S(p, K) = (1 - p) T_{S_1}(p, K) + p T_{S_2}(p, K). \tag{2}$$

## III. The slotted model

In this section, we consider that SUs and PUs evolve in a slotted model, and that they have the same time slots' durations. Moreover, we consider a perfect sensing, i.e. the false alarm and the missing probability equal $0$. The secondary subsystem $S_1$ is composed of the two following types of users (see Figure 1):

- SUs that have not sensed the licensed channels,
- SUs that have sensed all the licensed channels as occupied (they was blocked) and then, went back to the dedicated channel.

As the dedicated channel is equally shared between all the SUs, the subsystem $S_2$ can be modeled using an M/M/1 queue with a PS policy.

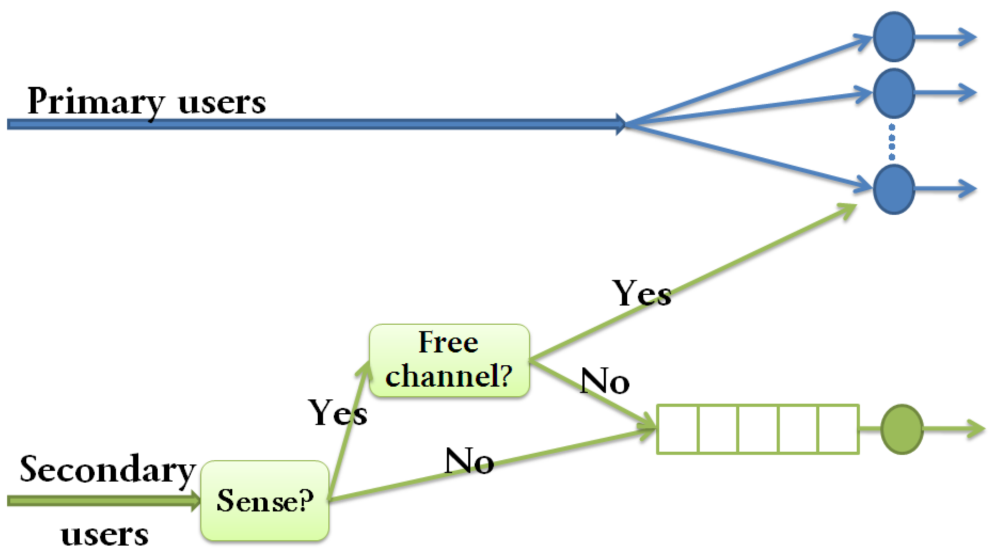


Fig. 1. The OSA model for CR networks

The second system, namely $S_2$, is composed of the two following types of users:

- PUs,
- SUs that have sensed the licensed channels and have found, at least, one free channel.

The subsystem $S_2$ can be modeled using an M/M/c/K queue, known as the Erlang-B model, with arrival rate $\lambda_p + p\lambda_s$. The blocking probability, which is the probability that any mobile finds all channels occupied, is given by the following Erlang-B formula:

$$\Pi(p, K) = \frac{\frac{\rho(p)^K}{K!}}{\sum_{n=0}^{K}(\frac{\rho(p)^n}{n!})}, \tag{3}$$

where $\rho(p) = \frac{(\lambda_p + p\lambda_s)}{\mu_p}$. This blocking probability depends not only on the number of licensed channels $K$, but also on the probability $p$ of sensing. In fact, this probability increases the input rate in the subsystem $S_2$ increases, and the blocking probability $\Pi(p, k)$ increases.

In the next section, we focus on the optimal sensing probability or the optimal proportion of SUs that sense licensed channels, which minimizes two important metrics: the average sojourn time and the average total cost.

### A. *Optimization of global performances*

*1) Average sojourn time:* We focus, in this section, on the average sojourn time in the system. The arrival rate in the dedicated channel (subsystem $S_1$) is composed of SUs that have not sensed the licensed channels, and SUs that have sensed and have not found free licensed channels. Then the arrival rate of SUs for that dedicated channel is $\lambda_s(1-p)+\lambda_s p\Pi(p,K)$. We assume that the maximum arrival rate, that is $\lambda_s$, which corresponds to the case where all SUs do not decide to sense, is lower than the service rate $\mu_S$. Then, we have a sufficient stability condition for the M/M/1 queue with a PS policy, which models the subsystem $S_1$. Note that the dedicated channel is shared between all the SUs, and thus more there is SUs transmitting over the dedicated channel, and higher is the sojourn time in the system (higher is the transmission delay). This assumption may be achieved by using an admission control mechanism by the SP for SUs, in order to guarantee some QoS requirements.

The average sojourn time $T_{S_1}$ for a SU, depending on the probability $p$ that SUs sense the licensed channels and the number of licensed channels, is expressed as follows:

$$T_{S_1}(p,K) = \frac{1}{\mu_s - \lambda_s(1-p(1-\Pi(p,K)))}. \tag{4}$$

If a SU decides to sense the licensed channels, its average sojourn time depends on the arrival rate of the PUs $\lambda_p$, and the proportion of SUs $p\lambda_s$ that have decided to sense the licensed channels. A SU that has not found a free channel, go back to the dedicated channel (see figure 1). Then, we can determine explicitly the average sojourn time $T_{S_2}$ for a SU that decide to sense the licensed channels:

$$T_{S_2}(p,K) = \frac{1-\Pi(p,K)}{\mu_p} + \Pi(p,K)T_{S_1}(p,K) = \frac{1-\Pi(p,K)}{\mu_p} + \frac{\Pi(p,K)}{\mu_s - \lambda_s(1-p(1-\Pi(p,K))}. \tag{5}$$

Henceforth, the average sojourn time of a SU in the system is given by the following expression:

$$T_S(p,K) \quad = \quad \frac{1-p}{\mu_s - \lambda_s(1-p(1-\Pi(p,K))} + \frac{p(1-\Pi(p,k))}{\mu_p} + \frac{p\Pi(p,K)}{\mu_s - \lambda_s(1-p(1-\Pi(p,K))}. \tag{6}$$

For notation convenience, let us consider the following function: $X(p,K) = p(1-\Pi(p,K))$. By replacing the function $X(p,K)$ in the expression of the average sojourn time, we obtain the following simpler expression of the average sejourn time:

$$T_S(X(p,K)) = \frac{1-X(p,K)}{\mu_s - \lambda_s + \lambda_s X(p,K)} + \frac{X(p,K)}{\mu_p}. \tag{7}$$

Our aim is to minimize the average sojourn time. Before, we prove, in the following lemma, that the function $X(p,K)$ is increasing with the probability of sensing.

*Lemma 1:* The function $X(p,K) = p(1-\Pi(p,K))$ is increasing with the sensing probability $p$.

*Proof:* See the Appendix A.

Given this result, we are able to determine the probability $\widetilde{p}$ that minimizes the average sojourn time $T_S(p, K)$.

*Proposition 1:* Consider a fixed number of licensed channels $K$. Then, the average sojourn time $T_S(p, K)$ is minimized when the sensing probability is:

$$\widetilde{p} = \begin{cases} 0 & \text{if} \quad \mu_s > \lambda_s + \sqrt{\mu_s \mu_p}; \\ 1 & \text{if} \quad \frac{\lambda_s - \mu_s + \sqrt{\mu_s \mu_p}}{\lambda_s} > 1 - \Pi(1, K); \\ p' & \text{otherwise.} \end{cases}$$

where $p'$ is the unique solution of:

$$\frac{\lambda_s - \mu_s + \sqrt{\mu_s \mu_p}}{\lambda_s} = p(1 - \Pi(p, K)). \tag{8}$$

*Proof:* Consider the average sojourn time $T_S(p, K)$ depending on the sensing probability $p$ and the number of licensed channels $K$. First, we look for the derivative of this function with respect to the sensing probability $p$:

$$\frac{\partial T_S}{\partial p}(p, K) = \frac{\partial T_S}{\partial X(p, K)}(X(p, K)) \times \frac{\partial X}{\partial p}(p, K).$$

The derivative of the average sojourn time equals 0 if and only if $\frac{\partial T_S}{\partial X(p,K)}(X(p, K)) = 0$, as we have already proved, in Lemma 1, that $\frac{\partial X}{\partial p}(p, K)$ is strictly positive.

From Equation (7), the derivative of the average sojourn time with respect to the function $X(p, K)$ is expressed as follows:

$$\frac{\partial T_S}{\partial X(p, K)}(X(p, K)) = \frac{-\mu_s \mu_p + (\mu_s - \lambda_s + \lambda_s X(p, K))^2}{\mu_p(\mu_s - \lambda_s + \lambda_s X(p, K))^2}.$$

Therefore, $\frac{\partial T_S}{\partial p}(p, K) = 0$ is equivalent to:

$$AX(p, K)^2 + BX(p, K) + C = 0,$$

where $A = \lambda_s^2, \quad B = 2\lambda_s(\mu_s - \lambda_s)$ and $C = (\mu_s - \lambda_s)^2 - \mu_s \mu_p$. Thus, the solutions of $\frac{\partial T_S}{\partial p}(p, K) = 0$ are expressed by:

$$X_1 = \frac{\lambda_s - \mu_s - \sqrt{\mu_s \mu_p}}{\lambda_s} \quad \text{and} \quad X_2 = \frac{\lambda_s - \mu_s + \sqrt{\mu_s \mu_p}}{\lambda_s}.$$

As the arrival rate for SUs $\lambda_s$ is positive, the derivative of the average sojourn time with respect to the function $X(p, K)$ is decreasing between $X1(p, K)$ and $X_2(p, K)$. Moreover, we have assumed the following sufficient stability condition of the $M/M/1$ queue that models the subsystem $S_1$: *the arrival rate of SUs verifies* $\lambda_s < \mu_s$.

Therefore, the first solution $X_1(p,K) < 0$. Note that this solution is not interesting for our problem, as $X(p,K)$ is always positive. Thus, we focus on the second solution, $X(p,K) = X_2(p,K)$.

Moreover $X(p,K)$ is increasing with p. Therefore, the function $T_S(x)$ is increasing for $x \in [0, X_2(p,K)[$ and decreasing for $x \in [X_2(p,K), 1]$. The function $X(p,K)$ equals 0 for a null sensing probability, and $X(1,K) = 1 - \Pi(1,K)$ when all SUs sense the licensed channels. Then, depending on the value $X_2$, we have the following values of sensing probability that minimize the average sojourn time:

- If $X_2 \leq X(0,K)$, then the function $T_S(p,K)$ is strictly increasing with the sensing probability over $[0,1]$ and the solution is $\widetilde{p} = 0$.
- If $X_2 \geq X(1,K)$, then the function $T_S(p,K)$ is strictly decreasing with the sensing probability over $[0,1]$ and the solution is $\widetilde{p} = 1$.
- Otherwise, if $X(0,K) < X_2 < X(1,K)$ then the function $T_S(p,K)$ has a unique minimum when $p = p'$, where $p'$ is the solution of:
$$\frac{\partial X}{\partial p}(p',K) = X_2 \Rightarrow p'(1 - \Pi(p',K)) = \frac{\lambda_s - \mu_s + \sqrt{\mu_s \mu_p}}{\lambda_s}.$$

■

Therefore, for a given number of licensed channels $K$, the minimum sojourn time $T_S(\widetilde{p},K)$ for a SU is expressed as follows:

$$T_S(\widetilde{p},K) = \begin{cases} \frac{1}{\mu_s - \lambda_s} & \text{if} \quad \mu_s > \lambda_s + \sqrt{\mu_s \mu_p}; \\ \frac{1-\Pi(1,K)}{\mu_p} + \frac{\Pi(1,K)}{\mu_s - \lambda_s \Pi(1,K)} & \text{if} \quad \frac{\lambda_s - \mu_s + \sqrt{\mu_s \mu_p}}{\lambda_s} > 1 - \Pi(1,K); \\ \frac{\mu_s - \sqrt{\mu_s \mu_p}}{\lambda_s \sqrt{\mu_s \mu_p}} + \frac{\lambda_s - \mu_s + \sqrt{\mu_s \mu_p}}{\mu_s \lambda_s} & \text{otherwise.} \end{cases}$$

We focus, in the next section, on minimizing the average cost function $U_S$. The sensing probability $\widetilde{p}$ minimizes the average sojourn time of SUs in the system if they do not care about the energy spent for sensing. Intuitively, SUs have less incentive to sense the licensed channels if there is a cost for sensing. We prove in the following section that this intuition is true.

*2) Average cost:* In order to avoid the interference with PUs, the SUs have to sense the licensed channels before accessing them, and pay a cost for sensing. Note that SUs spend energy for sensing the spectrum. In fact, we model by the sensing cost the energy spent for sensing the licensed channels. The average cost function $U_S(p,K)$ for a SU that senses the licensed channels with a probability $p$, is expressed as follows:

$$U_S(p,K) \quad = \quad T_S(p,K) + \alpha p K = \frac{1 - p(1 - \Pi(p,K))}{\mu_s - \lambda_s + \lambda_s p(1 - \Pi(p,K))} + \frac{p(1 - \Pi(p,K)}{\mu_p} + \alpha K p. \tag{9}$$

We denote by $\Pi'(p,K)$ the derivative of the blocking probability with respect to the sensing probability $p$. The following proposition states the sensing probability that minimizes the average cost function.

*Proposition 2:* For all values of $\alpha$ and $K$, the average cost function $U_S(p,K)$, defined in Equation (9), is minimized when the sensing probability is equal to:

$$p = min(1, max(p_0, 0)) := p^*,$$

where $p_0$ is the solution of the following equation:

$$1 - \Pi(p,K) - p\Pi'(p,K) = -\alpha K \left(1 + \frac{\mu_s \mu_p}{(\mu_s - \lambda_s + \lambda_s p(1 - \Pi(p,K)))^2 - \mu_s \mu_p}\right). \qquad (10)$$

*Proof:* By replacing the function $X(p,K)$ in Equation (9), the average cost function can be rewritten as follows:

$$U_S(p,K) = \frac{1 - X(p,K)}{\mu_s - \lambda_s + \lambda_s X(p,K)} + \frac{X(p,K)}{\mu_p} + \alpha p K.$$

After some algebra, the derivative of the average cost function, with respect to the sensing probability $p$, can be expressed by:

$$\frac{\partial U_S}{\partial p}(p,K) = \frac{\frac{\partial X}{\partial p}(p,K)((\mu_s - \lambda_s + \lambda_s X(p,K))^2 - \mu_s\mu_p) + \alpha K \mu_p(\mu_s - \lambda_s + \lambda_s X(p,K))^2}{\mu_p(\mu_s - \lambda_s + \lambda_s X(p,K))^2}.$$

Note that $\frac{\partial X(p,K)}{\partial p} = 1 - \Pi(p,K) - p\Pi'(p,K)$. Thus, the derivative of the cost function $U_S(p,K)$ equals 0 if and only if:

$$-\alpha K (1 + \frac{\mu_s \mu p}{(\mu_s - \lambda_s + \lambda_s X(p))^2 - \mu_s \mu p}) = 1 - \Pi(p,K) - p\Pi'(p,K).$$

We have already proved that the derivative of $X(p,K)$ with respect to $p$ is always positive. Therefore, the derivative of the average cost function with respect to the sensing probability equals 0 if and only if $p = min(1, max(p_0, 0)) := p^*$, where $p_0$ is the solution of Equation 10. ■

We have found the optimal repartition $p^*$ of SUs that senses the licensed channels in order to minimize the average cost function. Intuitively, the probability of sensing the licensed channels decreases with the sensing cost, as SUs have more incentive to use the dedicated channel due to the cost of sensing . The following proposition prove that this intuition is true and states the following relation between $p^*$ and $\widetilde{p}$.

*Proposition 3:* For all values of $\alpha$ and $K$, the proportion of SUs $p^*$ that senses the licensed channels, when SUs are energy-efficient, is less than the proportion $\widetilde{p}$ of SUs that senses the licensed channels without the energy constraint, i.e. $p^* \leq \widetilde{p}$.

*Proof:* We prove this proposition by contradiction. Assume that there exists $\alpha_0 > 0$ and $K_0$ such that $\widetilde{p} < p^*$. As $p^*$ minimizes the average cost function, we have:

$$U_S(p^*, K) \leq U_S(p, K).$$

Therefore, the average cost when SUs sense with a probability of $p^*$ is lower than the one when SUs sense with a probability $\widetilde{p}$:

$$\begin{aligned} T_S(p^*, K) + \alpha p^* K &\leq T_S(\widetilde{p}, K) + \alpha \widetilde{p} K, \\ \alpha K(p^* - \widetilde{p}) &\leq T_S(\widetilde{p}, K) - T_S(p^*, K). \end{aligned}$$

Note that $\widetilde{p}$ minimizes the average sojourn time $T_S(p, K)$, i.e.:

$$T_S(\widetilde{p}, K) - T_S(p^*, K) \leq 0.$$

Moreover, we have have assumed that $p^* - \widetilde{p} > 0$, which leads to a contradiction with $\alpha K(p^* - \widetilde{p}) \leq T_S(\widetilde{p}, K) - T_S(p^*, K)$. In fact, $\alpha K(p^* - \widetilde{p})$ is positive and $T_S(\widetilde{p}, K) - T_S(p^*, K)$ is negative. ■

This result is somehow intuitive. In fact, when the sensing cost increases, the SUs have less incentive to sense the licensed channels and prefer to use the dedicated one.

The main drawback of the optimal sensing probability $p^*$, the solution of the global optimization, is that it needs a central controller, in order to develop an optimal OSA mechanism. Indeed, the SP has to design the network such that a proportion $p^*$ of SUs senses the licensed channels. In practice, it would be difficult to control and to design such centralized control. To overcome this hurdle, we look in the next section for a distributed mechanism, based on individual decisions of SUs about the OSA.

### B. *Individual opportunistic sensing policy*

The main characteristic of the next generation networks is the transition from well-structured networks to infrastructure-less networks, and from centralized management to decentralized networks. Recently, several researches have focused on self-adaptive networks and autonomous devices. In this section, we consider a distributed system where SUs decide individually whether to sense or not the licensed channels. In fact, SUs tries to minimize, solely, its average cost function. Specifically, this system can be modeled by a non-cooperative game with an infinite number of players (as we do not restrict neither the time horizon of the system nor the number of SUs). Note that Game theory principle may be applied for resource allocation problems in a decentralized manner for wireless communications (see the survey paper [12] for some examples). Thus, we consider a game theoretical approach in order to design a decentralized OSA mechanism for SUs that decide individually whether to access the dedicated channel or to sense the licensed channels.

*1) Game model:* We consider that each SU decides on its probability $p$ to sense or not the licensed channels. It looks for minimizing its average cost function $U(p, p', K)$, which depends on its probability $p$, and the probability $p'$ of all the other SUs. The individual average cost function $U(p, p', K)$ is expressed as follows:

$$U(p, p', K) \quad = \quad (1 - p)T_{S_1}(p', K) + pT_{S_2}(p', K) + \alpha pK. \tag{11}$$

Note that the contribution to the cost by any individual SU is zero as we are not limited to a fixed number of SUs. Then, the equilibrium of this game is a Wardrop equilibrium, which has been first studied in the context of road traffic since the 1950s in [13]. For clarification purposes, we denote by $U_S(p, K) = U(p, p, K)$. Let us define the equilibrium for our non-cooperative game as a strategy, which minimizes the cost function $U$ against others using the NE strategy, defined in the following theorem.

*Theorem 1:* The sensing probability $p^E$ is a NE policy for the OSA problem between SUs if and only if:

$$p^E = \arg\min_p U(p, p^E, K), \ \forall p \in [0, 1].$$

*2) Equilibrium:* The following proposition proves the existence and the uniqueness of a NE strategy for our non-cooperative game between SUs.

*Proposition 4:* For all values of $\alpha$ and $K$, the NE policy for the OSA problem between SUs exists and is unique. Moreover, the sensing probability at the NE, $p^E$, is expressed as follows:

$$p^E = \begin{cases} 0 & \text{if} \quad \frac{1}{\mu_s - \lambda_s} < \frac{\alpha K}{1 - \Pi(0,K)} + \frac{1}{\mu_p}; \\ 1 & \text{if} \quad \frac{1}{\mu_s - \lambda_s \Pi(1,K)} > \frac{\alpha K}{1 - \Pi(1,K)} + \frac{1}{\mu_p}; \\ p' & \text{otherwise,} \end{cases}$$

where $p'$ is the solution of the following equation:

$$\frac{1}{\mu_s - \lambda_s(1 - p(1 - \Pi(p, K)))} = \frac{\alpha K}{1 - \Pi(p, K)} + \frac{1}{\mu_p}. \tag{12}$$

*Proof:* From Equation (11), the first argument derivative of the average cost function is expressed as follows:

$$\frac{\partial U}{\partial p}(p, p') = T_{S_2}(p', K) - T_{S_1}(p', K) + \alpha K.$$

The probability $p^E$ is a NE strategy for the OSA problem if and only if the first argument derivative of the average cost function equals 0, i.e.:

$$\alpha K + T_{S_2}(p^E, K) = T_{S_1}(p^E, K).$$

This equation characterizes a NE strategy for SUs. After some algebra, this expression may be expressed as follows:

$$(T_{S_1}(p^E, K) - \frac{1}{\mu_p})(1 - \Pi(p^E, K)) = \alpha K,$$

Then, the necessary and sufficient condition for the existence of a NE strategy for the OSA problem between SUs is:

$$T_{S_1}(p^E, K) = \frac{\alpha K}{1 - \Pi(p^E, K)} + \frac{1}{\mu_p}.$$

Note that the left-hand side function is continuous and decreasing with $p$ (see Equation (12)), whereas the right-hand side function is continuous and increasing in $p$. Therefore, this equation has a unique solution inside $]0, 1[$ if and only if:

$$\frac{1}{\mu_s - \lambda_s} > \frac{\alpha K}{1 - \Pi(0, K)} + \frac{1}{\mu_p},$$

and

$$\frac{1}{\mu_s - \lambda_s \Pi(1, K)} < \frac{\alpha K}{1 - \Pi(1, K)} + \frac{1}{\mu_p}.$$

Consider that $\frac{1}{\mu_s - \lambda_s \Pi(1,k)} > \frac{\alpha K}{1 - \Pi(1,k)} + \frac{1}{\mu_p}$, then the average cost function function is strictly decreasing, for $p \in [0, 1]$. Therefore, the sensing probability at the NE is $p^E = 1$.

Moreover, if $\frac{1}{\mu_s - \lambda_s} < \frac{\alpha K}{1 - \Pi(0,k)} + \frac{1}{\mu_p}$, then the average cost function is strictly increasing, for $p \in [0, 1]$. Therefore, the sensing probability at the NE is $p^E = 0$.

Otherwise, the sensing probability at the NE is the unique solution of Equation (12). Finally, a NE strategy for the OSA problem between SUs exists and is unique. ■

We have proved that the NE policy exists and is unique. Moreover, the average cost of a SU at the NE is expressed by:

$$U_S(p^E, K) = \begin{cases} \frac{1}{\mu_s - \lambda_s} & \text{if } \frac{1}{\mu_s - \lambda_s} < \frac{\alpha K}{1 - \Pi(0,K)} + \frac{1}{\mu_p}; \\ \frac{1 - \Pi(1,K)}{\mu_p} + \frac{\Pi(1,K)}{\mu_s - \lambda_s \Pi(1,K)} + \alpha K & \text{if } \frac{1}{\mu_s - \lambda_s \Pi(1,K)} > \frac{\alpha K}{1 - \Pi(1,K)} + \frac{1}{\mu_p}; \\ \frac{\alpha K}{1 - \Pi(p',K)} + \frac{1}{\mu_p} & \text{otherwise,} \end{cases}$$

where $p'$ is the solution of of Equation (12).

Given the existence and the uniqueness of a NE strategy for the OSA problem between SUs, the following proposition compares the sensing probability at the NE and the optimal sensing probability.

*Proposition 5:* For all values of $\alpha$ and $K$, the optimal sensing probability $p^*$ is higher than the sensing probability at the NE $p^E$, i.e. $p^E \leq p^*$.

*Proof:* See Appendix A. ■

This result is somehow intuitive. In fact, there is a lack of performances due to the selfishness of SUs in the decentralized system. Combining this result with the result of Proposition 3, we obtain the following relation:

*Lemma 2:* For all values of $\alpha$ and $K$, we have the following relation between the sensing probability that minimizes the average sojourn time, the optimal sensing probability, and the sensing probability at the NE:

$$\forall \alpha, K, \quad p^E \leq p^* \leq \widetilde{p}. \tag{13}$$

In fact, SUs sense less the licensed channels when they are energy-efficient, which is somehow intuitive. Moreover, SUs have less incentive to sense the licensed channels in a self-adaptive context than in a centralized network. Furthermore, the following proposition gives us a higher bound of the average total cost at the NE.

*Proposition 6:* For all values of $\alpha$ and $K$, we have the following higher bound of the average cost function when using a NE policy:

$$U_S(p^E, K) \leq \frac{1}{\mu_s - \lambda_s}.$$

*Proof:* Consider that $\frac{1}{\mu_s - \lambda_s} < \frac{\alpha K}{1-\Pi(0,K)} + \frac{1}{\mu_p}$. Therefore, the average cost function is expressed as follows:

$$U_S(p^E(\alpha, K)) = \frac{1}{\mu_s - \lambda_s}.$$

Second, Consider that $\frac{1}{\mu_s - \lambda_s \Pi(1,K)} > \frac{\alpha K}{1-\Pi(1,K)} + \frac{1}{\mu_p}$. Thus, the average cost function verifies:

$$U_S(p^E, K) = \frac{1-\Pi(1,K)}{\mu_p} + \frac{\Pi(1,K)}{\mu_s - \lambda_s \Pi(1,K)} + \alpha K \leq \frac{1}{\mu_s - \lambda_s \Pi(1,K)} \leq \frac{1}{\mu_s - \lambda_s}.$$

Otherwise, the average cost function can be bounded as follows:

$$U_S(p^E, K) = \frac{\alpha K}{1 - \Pi(p', K)} + \frac{1}{\mu_p} = \frac{1}{\mu_s - \lambda_s(1 - p'(1 - \Pi(p', K)))} \leq \frac{1}{\mu_s - \lambda_s}.$$

Finally, the higher bound of the average cost function is:

$$U_S(p^E, K) \leq \frac{1}{\mu_s - \lambda_s}.$$

■

It is well-known that the utility of the global optimization is higher than the utility when using NE strategies. Giving the existence and the uniqueness of the NE strategy for SUs, we focus in the next section on the lack of performance (utility) induced by the competition between SUs. In order to measure this gap of performance, we introduce the metric of the PoA.

### C. Price of anarchy

Koutsoupias and Papadimitriou [14] have introduced the concept of *Price of Anarchy*, which captures the deterioration of the performance of a decentralized system, due to the selfishness of its agents. This metric is well studied in routing games [15], where the PoA describes the worst-possible ratio between the total latency of a NE strategy and the latency of an optimal routing of the traffic. This metric describes the gap of performance in terms of individual utility between an optimal centralized system and a totally decentralized system.

The PoA is expressed as the ratio between the optimal utility (obtained with a centralized system) and the utility at the NE (obtained with a decentralized system when using a NE policy). In our context, we define the PoA as follows:

$$PoA = \frac{U_S(p^*, K)}{U_S(p^E, K)} \leq 1. \tag{14}$$

Our aim is to determine an expression of the minimal value of the PoA or to bound it, in order to measure the worst performance of the decentralized system. The following proposition gives us the worst-case lack of performance when upgrading from centralized networks to self-adaptive networks

*Proposition 7:* For all values of $\alpha$ and $K$, we have the following lower bound of the PoA:

$$PoA(\alpha, K) \geq 2(\frac{\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s - \lambda_s)}}{\lambda_s}) := \underline{PoA}.$$

*Proof:* See Appendix A.

This closed-form of the lower bound of the PoA is very interesting as it does not depend neither on the sensing cost $\alpha$ nor on the number of licensed channels $K$. Therefore, the SP may tune the service rate of the dedicated channel, $\mu_s$, and the arrival rate of SUs, $\lambda_s$, by using some admission control for example, in order to minimize the gap between the NE and the global optimization's performance.

In the following section, we present some numerical illustrations that validate our theoretical findings.

### D. Numerical Illustrations

This section presents the performance analysis of the proposed OSA mechanism. For this end, we have performed extensive Matlab simulations with different configurations of the system. Furthermore, two performance metrics have been considered: the sensing cost and the capacity of the system (number of licensed channels). We fix the arrival rate for PUs (reps. SUs) at 0.6 (reps. 0.8). Moreover, we have considered different service rates for the licensed channels ($\mu_p = 0.8$) and the dedicated channel ($\mu_s = 1.1$). Under these setting the PoA is analytically evaluated to $PoA = 0.7524$ from Proposition 7.

We focus, first, on the case of one licensed channel, and we set the sensing cost to $0.1$. Figure 2 illustrate the average total cost depending on the probability of sensing of SUs. We observe that the average total cost is minimized when the SUs sense the licensed channels with a probability $p = 1$, i.e. all SUs sense the licensed channels. In fact, since the sensing cost is relatively low ($c_s = 0.1$), all the SUs have incentive to sense the licensed channels.

Secondly, we consider multiple licensed channels and we set $K$ to 10. As we have already assumed that the sensing cost is linear with the number of licensed channels, choosing to sense the licensed channel become costly for the SUs with the increase of the number of licensed channels ($c_s = 1$). We plot in Figure 2 the average total cost, with $K = 10$ licensed channels, and we observe that the SUs have less incentive to sense the licensed channels compared to the first scenario ($K = 1$). In fact, the average total cost is minimal when SUs sense the licensed channels with a probability of $0.427$.

*1) Sensing cost:* We evaluate, in the present section, the impact of the sensing cost parameter $\alpha$ on the performance of the proposed OSA mechanism, given a fixed number of licensed channels ($K = 10$). Mobile devices equipped with a CR have usually a limited battery and thus have to be energy efficient. The main challenge of designing an energy-aware CR is to determine the appropriate sensing strategy, as SUs spend energy for sensing the licensed channels.

We plot in Figure 3 the optimal probability of sensing for SUs $p^*$ and the sensing probability of SUs at the NE policy $p^E$. We remark that both probabilities are decreasing with the sensing cost $\alpha$. This result is intuitive, as increasing the sensing cost decreases the incentive of SUs to sense the licensed channels. Furthermore, this observation validates the analytical result obtained in Proposition 5. In fact, the optimal sensing probability $p^*$ (obtained from the global optimization of the centralized system) is always higher than $p^E$ (the sensing probability obtained at the NE).

It is straightforward that the non-cooperative behavior of SUs induces a worst performance compared to the centralized system. We focus on the gap of performance induced when migrating from centralized to decentralized networks. We illustrate the PoA, defined by Equation (14), in Figure 4. We observe that the minimum of the PoA equals $0.7559$. Note that theoretically, the PoA is higher than $0.7524$. Thus, the performances obtained by simulations are slightly better than the lower bound obtained analytically from Proposition 7. Given this result, we are able to design a decentralized OSA mechanisms for energy-efficient SUs in self-adaptive CR networks, which is at worst 75% far from the optimal.

Note that the energy spent for sensing the licensed channels depends not only on the cost of sensing $\alpha$, but also on the number of licensed channels $K$, as the sensing cost $c_s$ is assumed to be linear with $K$. We evaluate, in the following section, the impact of the capacity on the performance of the proposed

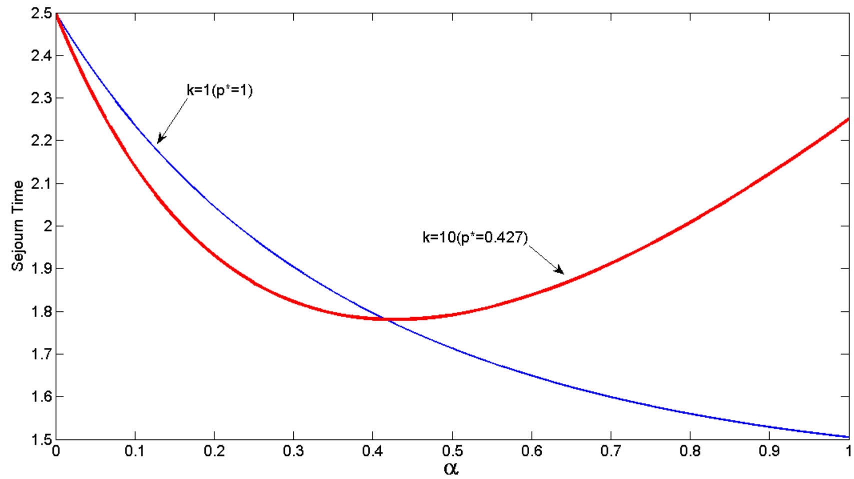


Fig. 2. The average total cost $U_S(p)$ for $\alpha = 0.1$, with one licensed channel, $K = 1$, and ten licensed channels, $K = 10$.

OSA mechanism.

*2) Capacity:* In this section, we are interested in the impact of the number of licensed channels on the proposed OSA mechanism. We fix the sensing cost $\alpha$ at $0.3$, and we vary the number of licensed channel from $1$ to $20$. An interesting analysis of [16] shows that the average number of available licensed channels in TV white-bands is about $15$. Note that under these settings, the blocking probability decreases with the number of licensed channels whereas the sensing cost increases.

Figure 11 depicts the impact of the number of licensed channels on both the optimal sensing probability and the sensing probability at the NE for SUs. We observe that both $p^*$ and $p^E$ are decreasing, and that $p^*$ is always higher than $p^E$. This result has already been proved analytically in Proposition 5. We plot in Figure 5 the average total cost with the number of licensed channels. We remark that the average cost is minimal for $K = 2$. Note that increasing the capacity of the system increases the opportunities in the primary subsystem $S_1$, but also increases the sensing cost $c_s$.

Similarly to the sensing cost analysis, we measure the gap of performance between the global system and the decentralized system through the PoA. Figure 6 illustrates the PoA depending on the number of licensed channels $K$. The worst-case performance gap is $0.7619$ obtained with $4$ licensed channels. This result is slightly higher than the analytical result of the Proposition 7, which says that the lower bound of the PoA equals $0.7524$.

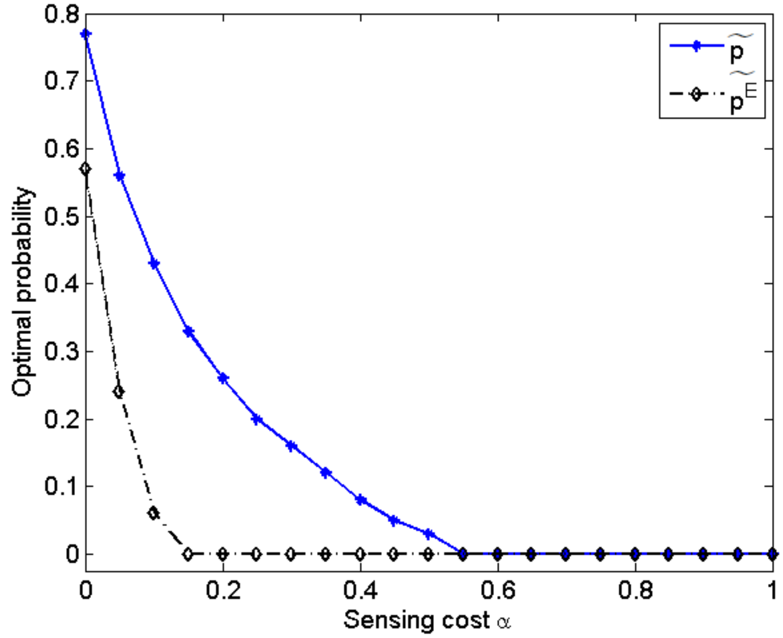


Fig. 3. The optimal probability of sensing depending on the sensing cost $\alpha$.

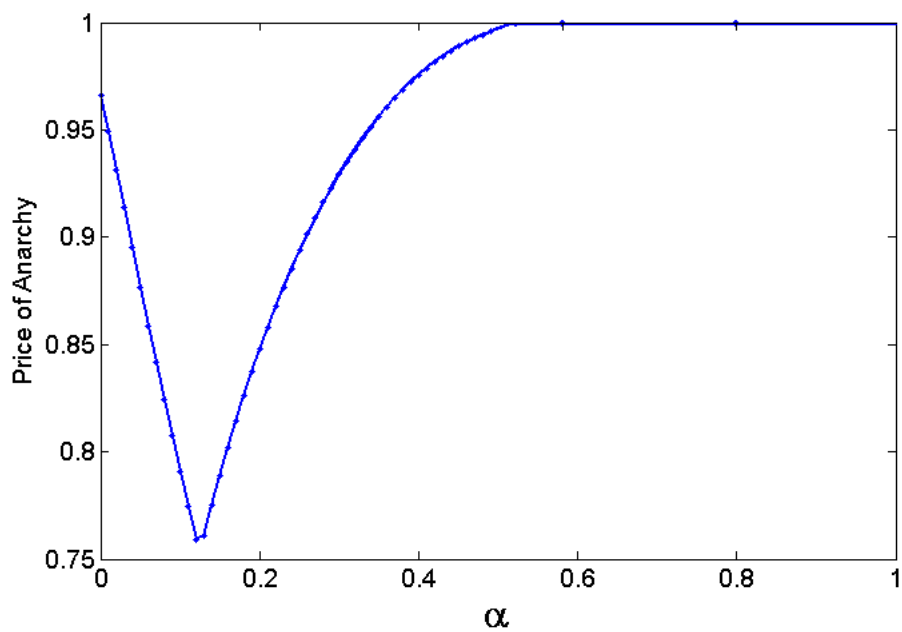


Fig. 4. The price of anarchy depending on the sensing cost $\alpha$.

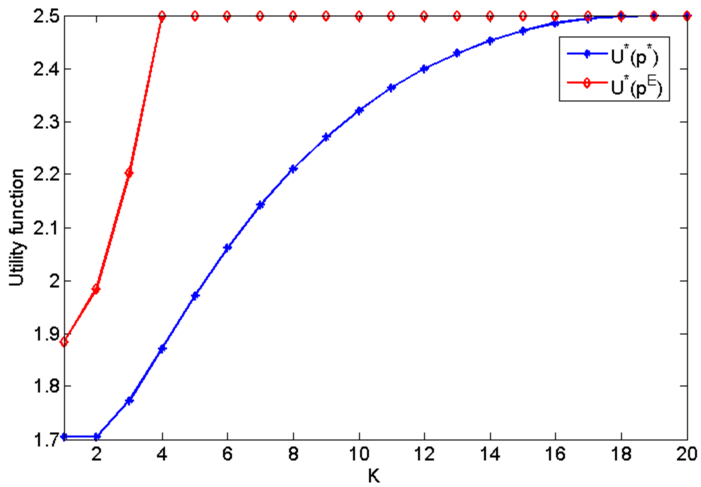


Fig. 5. The average total cost depending on the number of licensed channels in both the centralized and the decentralized system for the slotted model.

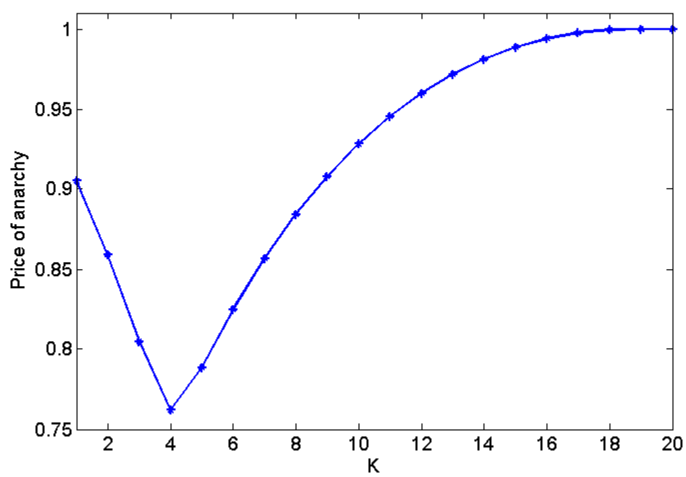


Fig. 6. The price of anarchy depending on the number of licensed channels in the slotted model.

### *E. Summary*

In this section, we have defined an optimal OSA policy for SUs. Moreover, we have proposed a decentralized policy for self-interested SUs and we have evaluated the gap of performance between both approaches through the PoA metric. Nonetheless, we have taken the assumptions that PUs operate in a slotted model, and that they are perfectly synchronized with SUs. We release theses assumptions in the next section by considering that the PUs evolve in a non-slotted regime, and that they may preempt a SU using the channel at their arrival. Releasing these assumptions significantly complicates the problem, as SUs have to face the reject form the licensed channels by PUs, as well as the competition with each other.

## IV. The non-slotted model

In the present section, we release some assumptions that has been taken in order to simplify the study of the system. Indeed, we consider a more realistic model, in which PUs evolve in a non-slotted mode and moreover, have the highest priority on their own channels. Thus, if a PU does not find a free licensed channel, it rejects one SU and start transmission. We assume that due to the agreement between the SP and PUs, the former ensure that there is always a licensed channel that is not used by PUs when a new PU arrives to the system, and that the PU can find such channel. Moreover, we consider that SUs can detect that a PU is present and free immediately the channel. We further assume that if the SU is rejected, it gets no reward, as it has to retransmit the whole packet at the next time slot. When there are several SUs using licensed channels, a PU chooses randomly one SU to reject. We model, in the following section, the reject probability of SU in the primary subsystem $S_1$.

### A. Reject probability

We denote by $X_p(t)$ (resp. $X_s(t)$) the number of PUs (resp. SUs) using the licensed channels at the time slot $t$, where $X_p(t)+X_s(t) \leq K$. Specifically, the subsystem of licensed channels can be modeled using a bi-dimensional Markov process, $Y(t) = \{X_p(t), X_s(t)\}$. The probability that a SU will be rejected when using a licensed channel is denoted by $P_r(p,K)$. This probability depends on the proportion $p$ of SUs that senses the licensed channels, and the number of licensed channels. Note that each SU that joins the system with a Poisson process observes the system in its stationary regime, according to the PASTA property.

We denote by $P_0(n,m)$ the probability that a SU will be rejected, when it joins a licensed channel and the primary subsystem has already $n$ PUs and $m$ SUs. Note that we have necessary $n+m<K$, and the reject probability is therefore expressed as follows:

$$P_r(p,K) \quad = \sum_{n,m/n+m=0}^{n+m=K-1} P_0(n,m)\pi(n,m), \tag{15}$$

where $\pi(n,m)$ is the stationary probability of the Markov process $Y(t)$ described in Figure 7. The stationary probabilities $\pi(n,m)$ can be computed using standard tools of Markov theory. Let us focus on the reject probabilities $P_0(n,m)$, it is possible to express the relation between probabilities $P_0(n,m)$ as a linear system. Note that for all states $(X_p(t), X_s(t)) = (n,m)$, such that $n+m = K-1$, $P_0(n,m)$ is expressed as follows:

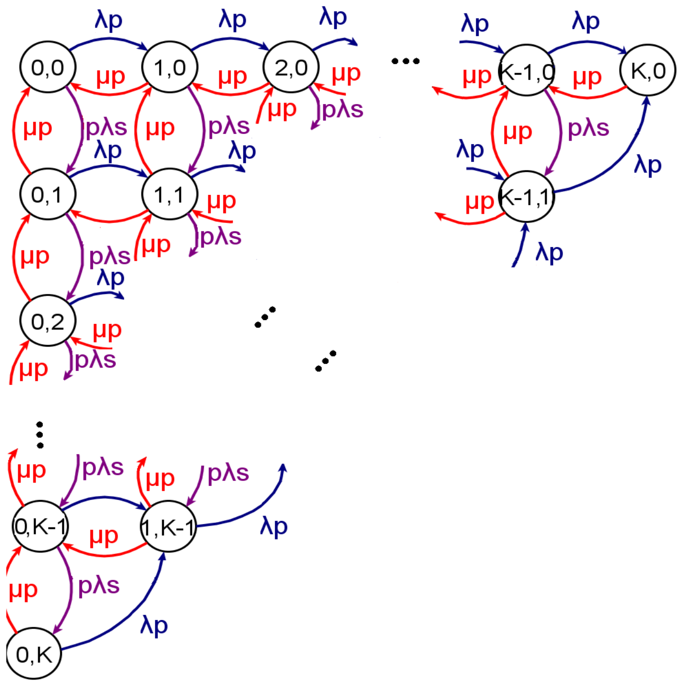


Fig. 7. The bi-dimensional Markov chain of Y(t).

$$P_0(n,m) = \begin{cases} \frac{1}{K}\frac{\lambda_p}{\lambda_p+\mu_p} + \frac{K-1}{K}\frac{\lambda_p}{\lambda_p+\mu_p}P_0(1,K-2) + \frac{\mu_p}{\lambda_p+\mu_p}P_0(0,K-2) & \text{if} \quad n=0, \\ \frac{\lambda_p}{\lambda_p+2\mu_p} + \frac{\mu_p}{\lambda_p+2\mu_p}P_0(K-2,0) & \text{if} \quad m=0, \\ \frac{1}{m+1}\frac{\lambda_p}{\lambda_p+2\mu_p} + \frac{m}{m+1}\frac{\lambda_p}{\lambda_p+2\mu_p}P_0(n+1,m-1) & \\ +\frac{\mu_p}{\lambda_p+2\mu_p}(P_0(n-1,m)+P_0(n,m-1)) & \text{otherwise.} \end{cases}$$

Otherwise, for $n+m<K-1$, the probability $P_0(n,m)$ is expressed as follows:

$$P_0(n,m)=\begin{cases} \frac{p\lambda_s}{p\lambda_s+\lambda_p+\mu_p}P_0(n,m+1)+\frac{\lambda_p}{p\lambda_s+\lambda_p+\mu_p}P_0(n+1,m) \\ \qquad +\frac{\mu_p}{p\lambda_s+\lambda_p+\mu_p}P_0(n,m-1) & \text{if} \quad n=0, \\ \frac{p\lambda_s}{p\lambda_s+\lambda_p+2\mu_p}P_0(n,m+1)+\frac{\lambda_p}{p\lambda_s+\lambda_p+2\mu_p}P_0(n+1,m) \\ \qquad +\frac{\mu_p}{p\lambda_s+\lambda_p+2\mu_p}P_0(n-1,m) & \text{if} \quad m=0, \\ \frac{p\lambda_s}{p\lambda_s+\lambda_p+2\mu_p}P_0(n,m+1)+\frac{\lambda_p}{p\lambda_s+\lambda_p+2\mu_p}P_0(n+1,m) \\ \qquad +\frac{\mu_p}{p\lambda_s+\lambda_p+2\mu_p}(P_0(n-1,m)+P_0(n,m-1)) & \text{otherwise.} \end{cases}$$

We assume that the reject probability $P_r(p,k)$ is increasing with the sensing probability $p$. This assumption is somehow realistic. Indeed, the greater is the number of SUs that choose to sense, the higher is the probability to be rejected by PUs. In the following section, we study the impact of the reject probability on the average cost function, and we determine the optimal OSA policies for SUs.

### B. Average cost

The average sojourn time $T^r_{S_1}$ for a SU that chooses to join the dedicated channel without sensing licensed channels is given by:

$$T^r_{S_1}(p,K)=\frac{1}{\mu_s-\lambda_s(1-p(1-\Pi(p,K))(1-P_r(p,K)))}. \tag{16}$$

This average sojourn time $T^r_{S_1}$ depends on the proportion $p$ of SUs that sense licensed channels and the number of licensed channels $K$. Note that $T^r_{S_1}$ is also impacted by the probability $P_r(p,K)$ that a SU could be rejected during its service in a licensed channel. Moreover, the average sojourn time $T^r_{S_2}$ of a SU that chooses to sense licensed channels is expressed by:

$$T^r_{S_2}(p,K)=\frac{\Pi(p,K)+(1-\Pi(p,K))P_r(p,K)}{\mu_s-\lambda_s(1-p(1-\Pi(p,K))(1-P_r(p,K)))}+\frac{(1-\Pi(p,K))(1-P_r(p,K))}{\mu_p}. \tag{17}$$

Henceforth, the average sojourn time of a SU in the non-slotted model is expressed as follows:

$$\begin{aligned} T^r_S(p,K) &= \frac{1-p(1-\Pi(p,K)-(1-\Pi(p,K))P_r(p,K))}{\mu_s-\lambda_s(1-p(1-\Pi(p,K))(1-P_r(p,K)))} \\ &+ \frac{p(1-\Pi(p,K))(1-P_r(p,K))}{\mu_p}. \end{aligned} \tag{18}$$

Therefore, the average cost function, that each SU aims to minimize, is expressed as:

$$\begin{aligned} U^r_S(p,K) &= \frac{1-p(1-\Pi(p,K))(1-P_r(p,K))}{\mu_s-\lambda_s(1-p(1-\Pi(p,K))(1-P_r(p,K)))} \\ &+ \frac{p(1-\Pi(p,K))(1-P_r(p,K))}{\mu_p}+\alpha pK. \end{aligned} \tag{19}$$

For notation convenience, we define $Y(p,K) = p(1-\Pi(p,K))(1-P_r(p,K))$. By substituting $Y(p,K)$ in the expression of the average cost function, we obtain the following expression of $U_S^r(p,K)$:

$$U_S^r(p,K) = \frac{1 - Y(p,K)}{\mu_s - \lambda_s(1 - Y(p,K))} + \frac{Y(p,K)}{\mu_p} + \alpha p K.$$

The first intuition one can make is that releasing the assumption that PUs evolve in a slotted model induces a loss of performance.

Let us denote by $p_r^*$ the optimal sensing probability of a SU in the non-slotted model. The following proposition gives us a relation between the average cost obtained with the slotted system and the average cost obtained with the non-slotted model.

*Proposition 8:* For all values of $\alpha$ and $K$, the optimal value of the average cost function is higher in the non-slotted model than in the slotted one:

$$U_S(p^*,K) \leq U_S^r(p_r^*,K).$$

*Proof:* Suppose, first, that $(\mu_s - \lambda_s(1 - Y(p,K)))^2 > \mu_s\mu_p$. Then, it follows that $(\mu_s - \lambda_s(1 - X(p,K)))^2 > \mu_s\mu_p$ and thus, we deduce from Proposition 2 that $p^* = 0$. Therefore, the average cost function is expressed by:

$$U_S(p^*,K) = \frac{1}{\mu_s - \lambda_s}.$$

Let us derive, in the following, the average cost function with respect to the reject probability:

$$\frac{\partial U_S^r}{\partial P_r}(P_r) = \mu_s p(1 - \Pi(p,K))\left(\frac{1}{(\mu_s - \lambda_s(1 - p(1 - \Pi(p,K))(1 - p_r)))^2} - \frac{1}{\mu_s\mu_p}\right) \leq 0.$$

We remark that $U_S^r(P_r)$ is decreasing with $P_r$. Thus, we have the following lower bound of the average cost function:

$$U_S^r(P_r) \geq U_S^r(1) = \frac{1}{\mu_s - \lambda_s} + \alpha p_r^* K \geq \frac{1}{\mu_s - \lambda_s},$$

which leads to:

$$U_S(p^*,K) \leq U_S^r(p_r^*,K).$$

Second, suppose that $(\mu_s - \lambda_s(1 - Y(p,K)))^2 \leq \mu_s\mu_p$. Therefore, $U_S^r(P_r)$ is increasing with $P_r$. Then, we obtain analogously that $U_S^r(P_r) \geq U_S^r(0)$.

Finally, the average cost function in the non-slotted model is higher than the average cost function in the slotted one, i.e. $U_S(p^*,K) \leq U_S^r(p_r^*,K)$. ■

This result is somehow intuitive as the reject of a SU introduces a lack of performances to the system. We focus, in the next section, on the study of the non-slotted self-adaptive CR network model.

### C. Individual optimization

We consider a distributed system in which each SU decides individually whether to sense or not the licensed channels. In fact, each SU decides on its probability $p$ of sensing the licensed channels. Note that a SU aims to minimize its average cost function $U_r(p,p',K)$, which depends on its probability $p$ and the probability $p'$ of the other SUs. Thus, the average cost function is expressed as follows:

$$U_r(p,p',K) \quad = \quad (1-p)T^r_{S_1}(p',K) + pT^r_{S_2}(p',K) + \alpha pK. \tag{20}$$

We prove, in the following proposition, that the non-cooperative OSA for SUs has a unique NE.

*Proposition 9:* For all values of $\alpha$ and $K$, the NE strategy for the OSA problem exists and is unique. Moreover the sensing probability at the NE $p_r^E$ is given by:

$$p_r^E(\alpha,K) = \begin{cases} 0 & \text{if} \quad \frac{1}{\mu_s-\lambda_s} < \frac{\alpha K}{(1-\Pi(0,K))(1-P_r(0,K))} + \frac{1}{\mu_p}, \\ 1 & \text{if} \quad \frac{1}{\mu_s-\lambda_s\Pi(1,K)} > \frac{\alpha K}{(1-\Pi(1,K))(1-P_r(1,K))} + \frac{1}{\mu_p}, \\ p'_r & \text{otherwise}, \end{cases} \tag{21}$$

where $p'_r$ is the solution of the following equation:

$$\frac{1}{\mu_s - \lambda_s(1-p(1-\Pi(p,K))(1-P_r(p,K)))} \quad = \quad \frac{\alpha K}{(1-\Pi(p,K))(1-P_r(p,K))} + \frac{1}{\mu_p}. \tag{22}$$

*Proof:* Let us focus on the average cost function $U_r$. The first partial derivative of the average cost function is:

$$\frac{\partial U_r}{\partial p}(p,p') = T^r_{S_2}(p',K) - T^r_{S_1}(p',K) + \alpha K.$$

Therefore, the probability $p'_r$ is a NE for SUs if and only if it satisfies the following equation:

$$\alpha K + T^r_{S_2}(p'_r,K) = T^r_{S_1}(p'_r,K).$$

This equation characterizes a NE for the OSA problem between SUs, and can be rewritten as follows:

$$(T^r_{S_1}(p'_r,K) - \frac{1}{\mu_p})((1-\Pi(p_r^E,K))(1-P_r(p'_r,K))) = \alpha K,$$

which leads to the following expression of the average sejourn time:

$$T^r_{S_1}(p'_r,K) = \frac{\alpha K}{(1-\Pi(p'_r,K))(1-P_r(p'_r,K))} + \frac{1}{\mu_p}.$$

From Equation (16), the average sojourn time $T^r_{S_1}$ for a SU that chooses to join the dedicated channel without sensing licensed channels is given by:

$$T^r_{S_1}(p'_r,K) = \frac{1}{\mu_s - \lambda_s(1-p'_r(1-\Pi(p'_r,K))(1-P_r(p'_r,K)))}. \tag{23}$$

Note that the left hand side function of Equation (22) is continuous and decreasing with $p'_r$. However, the right-hand side function is continuous and increasing with $p'_r$. Thus, the solution of this equation exists and is unique if and only if:

$$\frac{1}{\mu_s - \lambda_s} > \frac{\alpha K}{(1-\Pi(0,K))(1-P_r(0,K))} + \frac{1}{\mu_p}$$

and

$$\frac{1}{\mu_s - \lambda_s \Pi(1,K)} < \frac{\alpha K}{(1-\Pi(1,K))(1-P_r(1,K))} + \frac{1}{\mu_p}.$$

Firstly, let us consider that $\frac{1}{\mu_s-\lambda_s} < \frac{\alpha K}{(1-\Pi(0,K))(1-P_r(0,K))} + \frac{1}{\mu_p}$. Then, the utility function is strictly increasing and then the sensing probability at the NE is $p_r^E = 0$.

Second, if $\frac{1}{\mu_s-\lambda_s\Pi(1,K)} > \frac{\alpha K}{(1-\Pi(1,K))(1-P_r(1,K))} + \frac{1}{\mu_p}$, then the utility function is strictly deceasing and the sensing probability at the NE is $p_r^E = 1$.

Finally, the NE for our OSA problem exists and is unique. Moreover, the sensing probability at the NE is expressed by Equation (21). ■

For notation convenience, we denote for all $p$ and $K$, $U_S^r(p,K)$ by $U_r(p,p,K)$. The average total cost of a SU at the NE is given by:

$$U_S^r(p^E,K) = \begin{cases} \frac{1}{\mu_s-\lambda_s} & \text{if } \frac{1}{\mu_s-\lambda_s} < \frac{\alpha K}{(1-\Pi(0,K))(1-P_r(0,K))} + \frac{1}{\mu_p}, \\ \frac{1-(1-\Pi(1,K))(1-P_r(1,K))}{\mu_s-\lambda_s(1-(1-\Pi(1,K))(1-P_r(1,K)))} & \\ +\frac{(1-\Pi(1,K))(1-P_r(1,K))}{\mu_p} + \alpha K & \text{if } \frac{1}{\mu_s-\lambda_s\Pi(1,K)} > \frac{\alpha K}{(1-\Pi(1,K))(1-P_r(1,K))} + \frac{1}{\mu_p}, \\ \frac{\alpha K}{(1-\Pi(p'_r,K))(1-P_r(p'_r,K))} + \frac{1}{\mu_p} & \text{otherwise.} \end{cases} \quad (24)$$

Furthermore, the following proposition gives us a higher bound of the average total cost at the NE.

*Proposition 10:* For all values of $\alpha$ and $K$, we have the following higher bound of the average cost function at the NE:

$$U_S^r(p_r^E,K) \leq \frac{1}{\mu_s - \lambda_s}.$$

*Proof:* Firstly, consider that $\frac{1}{\mu_s-\lambda_s} < \frac{\alpha K}{(1-\Pi(0,K))(1-P_r(0,K))} + \frac{1}{\mu_p}$. Therefore, the average cost function is expressed, according to Equation (24), as follows:

$$U_S^r(p_r^E,K) = \frac{1}{\mu_s - \lambda_s}.$$

Second, we consider that:

$$\frac{1}{\mu_s - \lambda_s(1-(1-\Pi(1,K))(1-P_r(1,K)))} > \frac{\alpha K}{(1-\Pi(1,K))(1-P_r(1,K))} + \frac{1}{\mu_p}.$$

Then, the average cost function can be expressed as follows:

$$\begin{aligned} U_S^r(p_r^E, K) &= \frac{\Pi(1,K)(1-P_r(1,K))}{\mu_s - \lambda_s(1-(1-\Pi(1,K))(1-P_r(1,K)))} \\ &\quad + \frac{(1-\Pi(1,K))(1-P_r(1,K))}{\mu_p} + \alpha K \\ &\leq \frac{1}{\mu_s - \lambda_s(1-(1-\Pi(1,K))(1-P_r(1,K)))} \\ &\leq \frac{1}{\mu_s - \lambda_s}. \end{aligned}$$

The inequalities come from $0 \leq (1-\Pi(1,K))(1-P_r(1,K)) \leq 1$. Otherwise, we have the following higher bound of the average cost function:

$$\begin{aligned} U_S^r(p_r^E(\alpha, K)) &= \frac{\alpha K}{(1-\Pi(p_r^E, K))(1-P_r(p_r^E, K))} + \frac{1}{\mu_p} \\ &= \frac{1}{\mu_s - \lambda_s(1-p_r^E(1-\Pi(p_r^E, K))(1-P_r(p_r^E, K)))} \\ &\leq \frac{1}{\mu_s - \lambda_s}. \end{aligned}$$

Finally, we have, for all values of $\alpha$ and $K$, the following higher bound of the average cost function at the NE:

$$U_S^r(p_r^E, K) \leq \frac{1}{\mu_s - \lambda_s}.$$

■

Given the existence and the uniqueness of the NE for the proposed OSA mechanism in the non-slotted model, we study, in the next section, the gap of performance between the average cost at the NE and the average cost of the centralized system.

### *D. Price of anarchy*

The PoA models the lack of performance between the utility at the NE and the optimal utility. Specifically, the PoA is defined by the following ratio:

$$PoA_r(\alpha, K) = \frac{U_s^r(p_r^*, K)}{U_s^r(p_r^E, K)} \leq 1. \tag{25}$$

Let us focus on the expression of PoA. Similarly to the slotted model, our aim is to determine a lower value of the PoA or to bound it, in order to define the worst-possible lack of performance of the decentralized system. The following proposition gives us a lower bound of the price of anarchy, called $\underline{PoA_r}$.

*Proposition 11:* For all values of $\alpha$ and $K$, we have the following lower bound of the PoA:

$$PoA_r(\alpha, K) \geq 2(\frac{\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s - \lambda_s)}}{\lambda_s}) := \underline{PoA_r}.$$

*Proof:* See Appendix A. ∎

This closed-form lower bound of the PoA is interesting, as it depends neither on the sensing cost $\alpha$, nor on the number of licensed channel $k$. Thus, the SP has only to tune $\mu_s$ and $\lambda_s$ in order to maximize the performance of the decentralized system.

In the following section, we present some numerical illustrations that validate our theoretical findings.

### *E. Numerical illustrations*

This section presents the performance analysis of the proposed OSA mechanism. For this end, we have performed extensive Matlab simulations with different configurations of the system. Furthermore, two performance metrics have been considered: the sensing cost and the capacity of the system. We consider the same values of the system model parameters defined in Section III-D. Moreover, we assume that PUs may preempt SUs in service. We first focus on the sensing cost parameter $\alpha$. Then, we study the impact of the capacity on the OSA mechanism.

*1) Sensing cost:* We evaluate, in this section, the impact of the sensing cost $\alpha$ on the performance of the proposed OSA mechanism. Figure 8 illustrates the average cost function in both the slotted PUs transmissions and the non-slotted model. We observe that the average cost of SUs is always higher in the non-slotted model than in the slotted one, which validates the results of Proposition 8.

We observe, in Figure 9, that the optimal probability of sensing the licensed channels is decreasing with $\alpha$ in both models. However, we remark that the optimal probability of sensing in the non-slotted model $p_r^*$ is more sensitive to the sensing cost $\alpha$ than the optimal probability of sensing in the slotted model $p^*$. In fact, in the non-slotted model, the reject probability decreases the benefit of sensing in term of utility.

Let us focus on the lack of performance induced by the non-cooperative behavior of SUs in the decentralized model. We have obtained from Proposition 11 a lower bound of the price of anarchy $\underline{PoA_r} = 75.24\%$. This result is lower that the minimum value of the PoA obtained from Figure 10, which is $0.8289$.

The number of licensed channels has a major influence on the behavior of SUs and impacts not only the average sojourn time, but also the energy consumption, as the sensing cost grow linearly with the

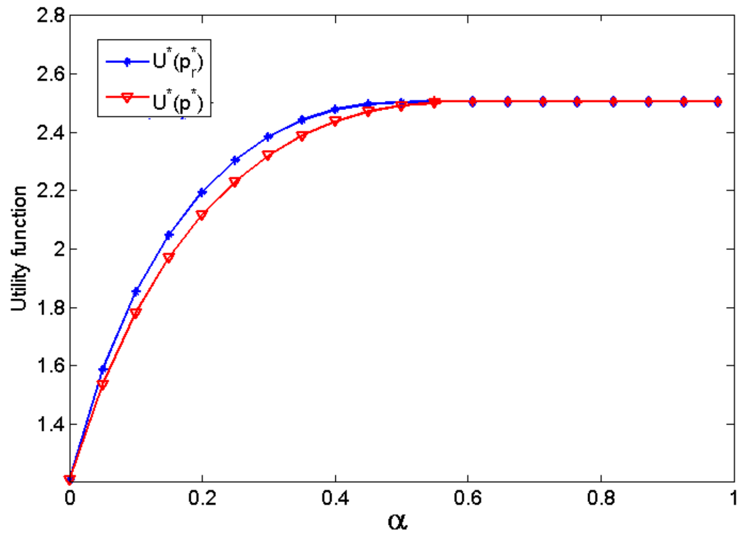


Fig. 8. The global optimum depending on the sensing cost $\alpha$ in both the slotted and the non-slotted models.

number of licensed channels. We depict, in the next section, the impact of the capacity on the performance of the proposed OSA policy.

*2) Capacity:* In the present section, we are interested in the impact of the number of licensed channels on the performance of the proposed OSA mechanism for SUs. We set the sensing cost $\alpha$ to $0.3$ and we vary the number of licensed channel from $1$ to $20$. Note that under these settings, the blocking probability decreases with the number of licensed channels whereas the sensing cost increases.

Firstly, we observe, in Figure 12, that both the optimal sensing probability $p_r^*$ and the sensing probability at the NE $p_r^E$ are decreasing with number of licensed channel $K$. Moreover, we remark that the sensing probability at the NE is lower or equal than the optimal sensing probability. In fact, the non-slotted model is more sensitive to the number of licensed channels than the slotted model. Second, we obtain from Figure 13 that the non-slotted model induces a higher average cost for SUs compared to the slotted model. Finally, we conclude with the analysis of the price of anarchy depending on the number of licensed channels $K$. In Figure 14, we observe that the minimal value of the price of anarchy is $0.8672$, which is not so far from the lower bound given by Proposition 11, which is $75.24\%$.

Both the sensing cost and the capacity of the system (number of licensed channels) are important factors in the performance of CR users. The SP may tune the system parameters in order to optimize the QoS for its SUs without the need for a centralized controller.

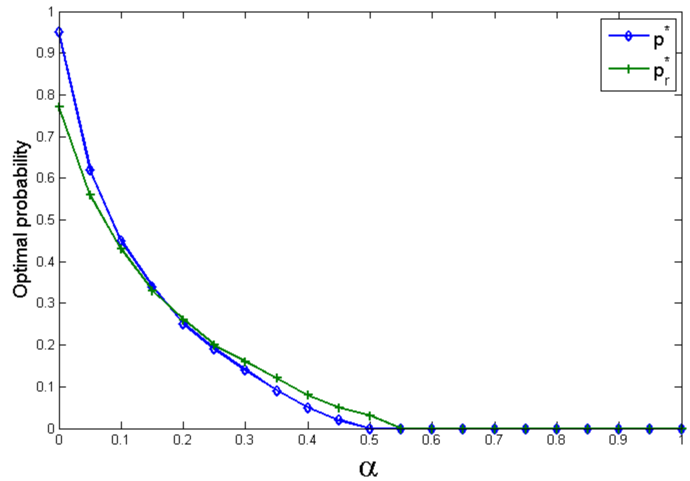


Fig. 9. The optimal probability of sensing depending on the sensing cost $\alpha$.

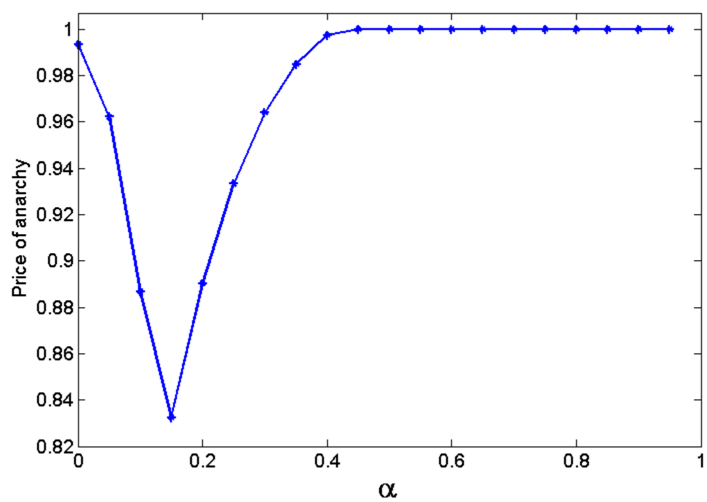


Fig. 10. The price of anarchy depending on $\alpha$.

## V. Conclusion

In this paper, we have proposed global and individual OSA mechanisms for SUs. We have studied the performance of these approaches and we have evaluated the gap of performance between them using the well-studied metric: the PoA. Simulation results have validated our theoretical finding. We have also considered both the slotted and the non-slotted model for PUs. In perspectives, we would like to propose

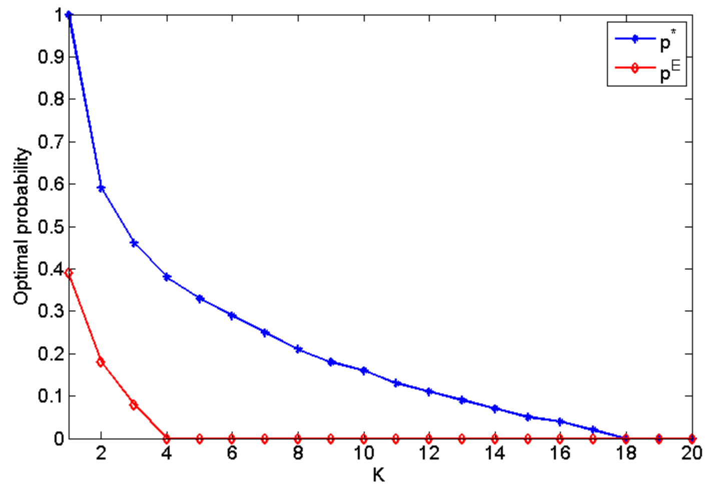


Fig. 11. The probability of sensing depending on the number of licensed channels in both the centralized and the decentralized systems.

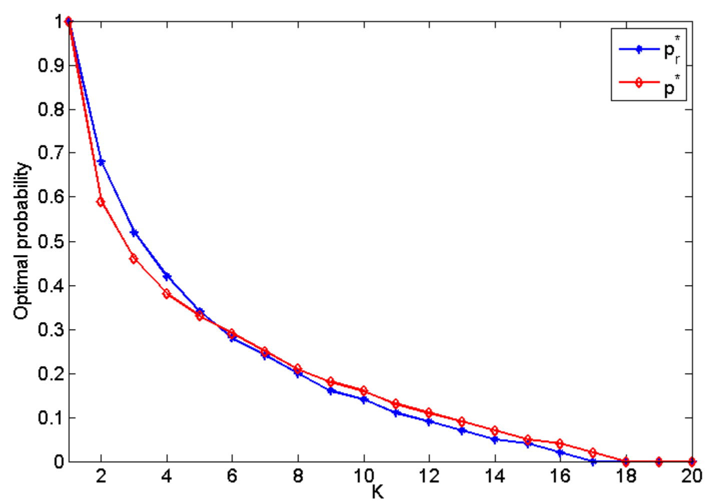


Fig. 12. The probability of sensing depending on the number of licensed channels in non-slotted model.

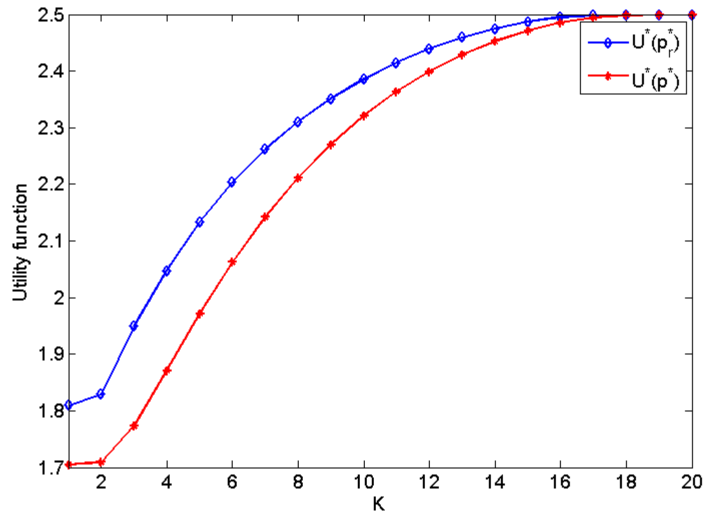


Fig. 13. The average total cost with the number of licensed channels in both the slotted and the non-slotted models.

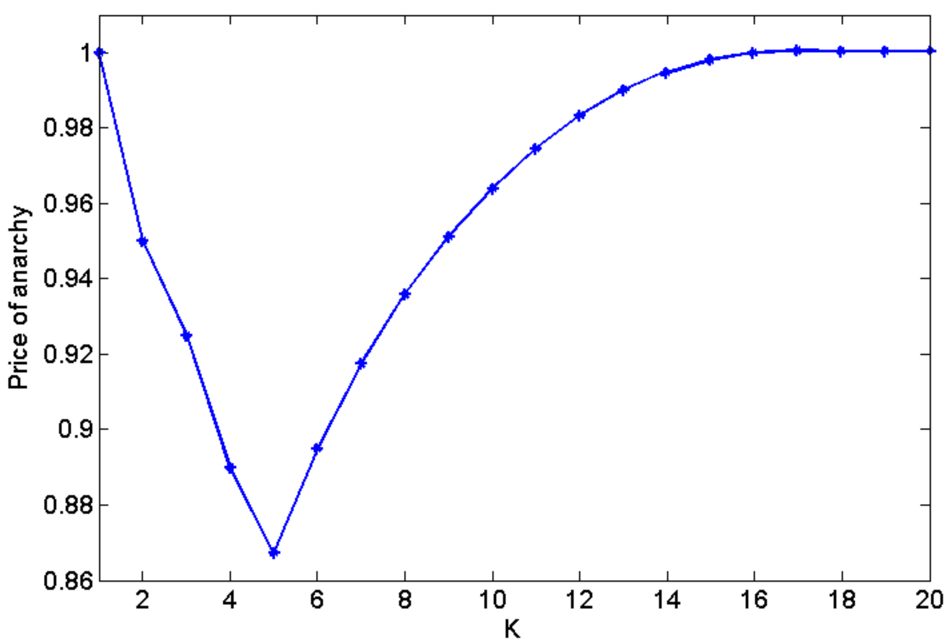


Fig. 14. The price of anarchy with the number of licensed channels $K$ in the non-slotted model.

smart sensing algorithms such that SUs will not have to sense all the licensed channels but only few of them. Those algorithms could be based on a MDP that considers on the number of licensed channels already used. In order to guarantee the QoS for PUs, we will introduce the constraint that the blocking probability for a PU must be lower than a threshold $p_{max}$.

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## APPENDIX

Consider the function $X(p, K) = p(1 - \Pi(p, K))$, and denote by $\Pi'(p, K)$ the derivative of $\Pi(p, K)$ with respect to the sensing probability $p$. Then, the partial derivative of the function $X(p, K)$ with respect to the sensing probability $p$ is:

$$\frac{\partial X}{\partial p}(p,K) = 1-\Pi(p,K)-p\Pi'(p,K) \tag{26}$$

$$= 1-\frac{\frac{\rho^K}{K!}}{\sum_{n=0}^{K}\frac{\rho^n}{n!}}-p\frac{\lambda_s}{\mu_p}\frac{\frac{\rho^{K-1}}{(K-1)!}\sum_{n=0}^{K}\frac{\rho^n}{n!}-\frac{\rho^K}{(K)!}\sum_{n=0}^{K-1}\frac{\rho^n}{n!}}{(\sum_{n=0}^{K}\frac{\rho^n}{n!})^2}. \tag{27}$$

Let us denote by $\gamma = p\frac{\lambda_s}{\mu_p}$. Then, the partial derivative of $X(p,K)$ with respect to the sensing probability $p$ can be rewritten as follows:

$$\frac{\partial X}{\partial p}(p,K) = \frac{\sum_{n=0}^{K-1}\frac{\rho^n}{n!}\sum_{n=0}^{K}\frac{\rho^n}{n!}+\gamma\frac{\rho^K}{K!}\sum_{n=0}^{K-1}\frac{\rho^n}{n!}-\gamma\frac{\rho^{K-1}}{(K-1)!}\sum_{n=0}^{K}\frac{\rho^n}{n!}}{(\sum_{n=0}^{K}\frac{\rho^n}{n!})^2}.$$

Consider, first, that $\rho < K$. It follows that $\gamma < \rho < K$, and then, $\gamma\frac{\rho^{K-1}}{(K-1)!}$ is lower than $K\frac{\rho^{K-1}}{(K-1)!}$. Therefore, we obtain the following expression of the partial derivative of $X(p,K)$ with respect to the sensing probability:

$$\frac{\partial X}{\partial p}(p,K) > \frac{\sum_{n=0}^{K-1}\frac{\rho^n}{n!}\sum_{n=0}^{K}\frac{\rho^n}{n!}+\gamma\frac{\rho^K}{K!}\sum_{n=0}^{K-1}\frac{\rho^n}{n!}-\sum_{n=0}^{K-1}\frac{\rho^{K-1}}{(K-1)!}\sum_{n=0}^{K}\frac{\rho^n}{n!}}{(\sum_{n=0}^{K}\frac{\rho^n}{n!})^2}.$$

As we have already supposed that $\rho < K$, there exists $0 < j < K$ such that $j \le \rho < j+1$. Specifically, under the above mentioned assumptions, the partial derivative of the function $X$ with respect to the sensing probability, $\frac{\partial X}{\partial p}(p,K)$ is higher than the following expression:

$$\frac{\sum_{n=0}^{j}\frac{\rho^{K-1}}{(K-1)!}\sum_{n=0}^{K}\frac{\rho^n}{n!}+\sum_{n=j+1}^{K-1}\frac{\rho^n}{n!}\sum_{n=0}^{K}\frac{\rho^n}{n!}+\gamma\frac{\rho^K}{K!}\sum_{n=0}^{K-1}\frac{\rho^n}{n!}}{(\sum_{n=0}^{K}\frac{\rho^n}{n!})^2}\frac{-\sum_{n=0}^{j}\frac{\rho^{K-1}}{(K-1)!}\sum_{n=0}^{K}\frac{\rho^n}{n!}-\sum_{n=j+1}^{K-1}\frac{\rho^n}{n!}\sum_{n=0}^{K}\frac{\rho^n}{n!}}{(\sum_{n=0}^{K}\frac{\rho^n}{n!})^2}$$

which leads to:

$$\Rightarrow X'(p) > \frac{\gamma\frac{\rho^K}{k!}\sum_{n=0}^{K-1}\frac{\rho^n}{n!}}{(\sum_{n=0}^{K}\frac{\rho^n}{n!})^2} > 0.$$

Second, consider that $\rho \ge k$. Then, the derivative of $X(p,k)$ with respect to the sensing probability $p$ can be expressed as follows:

$$\frac{\partial X}{\partial p}(p,K) = \frac{\sum_{n=0}^{K-1}\frac{\rho^n}{n!}\sum_{n=0}^{K}\frac{\rho^n}{n!}+\gamma\frac{(\rho-K)}{K}\frac{\rho^{K-1}}{(K-1)!}\sum_{n=0}^{K-1}\frac{\rho^n}{n!}-\gamma\frac{\rho^{K-1}}{(K-1)!}\frac{\rho^{K-1}}{(K-1)!}}{(\sum_{n=0}^{K}\frac{\rho^n}{n!})^2}.$$

Let us define $A$ and $B$ as follows:

$$A = \frac{\gamma\frac{(\rho-K)}{K}\frac{\rho^{K-1}}{(K-1)!}\sum_{n=0}^{K-1}\frac{\rho^n}{n!}}{(\sum_{n=0}^{K}\frac{\rho^n}{n!})^2} > 0, \text{ and}$$

$$B = \frac{\sum_{n=0}^{K-1}\frac{\rho^n}{n!}\sum_{n=0}^{K}\frac{\rho^n}{n!}-\gamma\frac{\rho^{K-1}}{(K-1)!}\frac{\rho^K}{(K)!}}{(\sum_{n=0}^{K}\frac{\rho^n}{n!})^2}.$$

Note that $\frac{\partial X}{\partial p}(p,k) = A + B$. It is clear that the first component of $\frac{\partial X}{\partial p}(p,k)$, $A$, is positive. The stability condition of the $M/M/1$ queue modeling the subsystem $S_1$ is:

$$(1-p)\lambda_s + p\lambda_s \Pi(p,K) < \lambda_s < \mu_s.$$

Therefore, a sufficient stability condition, for the $M/M/1$ queue, is $\gamma\Pi(p,K) < 1$. Note that $B$ can be rewritten as follows:

$$B = \frac{\sum_{n=0}^{K-1} \frac{\rho^n}{n!} \sum_{n=0}^{K} \frac{\rho^n}{n!} - \frac{\rho^{K-1}}{(K-1)!} \sum_{n=0}^{K} \frac{\rho^n}{n!} \gamma \frac{\frac{\rho^{K-1}}{(K-1)!}}{\sum_{n=0}^{K} \frac{\rho^n}{n!}}}{(\sum_{n=0}^{k} \frac{\rho^n}{n!})^2}.$$

which leads to:

$$B \geq \frac{\sum_{n=0}^{K-1} \frac{\rho^n}{n!} \sum_{n=0}^{K} \frac{\rho^n}{n!} - \frac{\rho^{K-1}}{(K-1)!} \sum_{n=0}^{K} \frac{\rho^n}{n!}}{(\sum_{n=0}^{k} \frac{\rho^n}{n!})^2} > 0.$$

Finally, as both components $A$ and $B$ of the partial derivative of $X(p,K)$ with respect to the sensing probability are positive, we obtain that $\frac{\partial X}{\partial p}(p,K)$ is always positive, and thus the function $X(p,K)$ is increasing with the sensing probability $p$. ■

We prove this proposition by contradiction. Assume that there exists a sensing cost $\alpha_0 > 0$ and a number of licensed channels $K_0$ such that $p^E > p^*$. As $p^*$ minimizes the average cost function, we have:

$$T_S(p^*, K) + \alpha p^* K \leq T_S(p^E, K) + \alpha p^E K.$$

However, $p^E$ is the sensing probability at the NE. Therefore, we have the following inequality:

$$T_S(p^*, p^E) + \alpha p^* K \geq T_S(p^E, K) + \alpha p^E K,$$

After some algebra, the last relation is equivalent to:

$$\frac{1}{\mu_p} + (p^*(1-\Pi(p^*,K)) - 1)(\frac{1}{\mu_p} - T_{S_1}(p^*,K)) \leq \frac{1}{\mu_p} + (p^*(1-\Pi(p^E,K)) - 1)(\frac{1}{\mu_p} - T_{S_1}(p^E,K)),$$

Note that $p^*(1-\Pi(p^E,K))$ is decreasing with $p^E$ for a given value of $p^*$. Moreover, $(\frac{1}{\mu_p} - T_{S_1}(p^E,K))$ is decreasing with $p^E$ (see Equation 4). This leads to a contradiction with the assumption that $p^E > p^*$, and therefore the left hand side of this inequality is decreasing with the probability $p^E$.

Finally, for all $\alpha$ and all $K$, the optimal sensing probability is higher than the sensing probability at the NE, i.e. $p^E \leq p^*$. ■

The price of anarchy is expressed by the following ratio:

$$PoA(\alpha, K) = \frac{U_S(p^*(\alpha,K))}{U_S(p_r^E(\alpha,K))}.$$

Suppose, first, that $\frac{1}{\mu_s-\lambda_s} < \frac{\alpha K}{1-\Pi(0,K)} + \frac{1}{\mu_p}$. Then, the price of anarchy can be expressed as follows:

$$PoA(\alpha, K) = \frac{\frac{p^*(1-\Pi(p^*,K))}{\mu_p} + \alpha p^* K \frac{1-\Pi(p^*,K)}{1-\Pi(p^*,K)} + \frac{1-p^*(1-\Pi(p^*,K))}{\mu_s-\lambda_s(1-p^*(1-\Pi(p^*,K)))}}{\frac{1}{\mu_s-\lambda_s}}.$$

As the blocking probability $\Pi(p, K)$ is increasing with $p$, the price of anarchy is higher than the following lower bound:

$$\begin{aligned} PoA(\alpha, K) &\geq \frac{\frac{p^*(1-\Pi(p^*,K))}{\mu_p} + \alpha p^* K \frac{1-\Pi(p^*,K)}{1-\Pi(0,K)} + \frac{1-p^*(1-\Pi(p^*,K))}{\mu_s-\lambda_s(1-p^*(1-\Pi(p^*,K)))}}{\frac{1}{\mu_s-\lambda_s}}, \\ &= p^*(1-\Pi(p^*,K)) \frac{\frac{1}{\mu_p} + \frac{\alpha K}{1-\Pi(0,K)}}{\frac{1}{\mu_s-\lambda_s}} + \frac{\frac{1-p^*(1-\Pi(p^*,K))}{\mu_s-\lambda_s(1-p^*(1-\Pi(p^*,K)))}}{\frac{1}{\mu_s-\lambda_s}}. \end{aligned}$$

Let us denote $X(p^*, K) = p^*(1 - \Pi(p^*, K))$. Therefore, the lower bound of the price of anarchy can be rewritten as:

$$PoA(\alpha, K) > X(p^*, K) + \frac{\frac{1-X(p^*,K)}{\mu_s-\lambda_s(1-X(p^*,K))}}{\frac{1}{\mu_s-\lambda_s}}.$$

Second, suppose that $\frac{1}{\mu_s-\lambda_s\Pi(1,K)} > \frac{\alpha K}{1-\Pi(1,K)} + \frac{1}{\mu_p}$. It follows that $1-\Pi(1,K) < \frac{\lambda_s-\mu_s+\sqrt{\mu_s\mu_p}}{\lambda_s}$. From Proposition 1, we have that $\widetilde{p}(K) = 1$ and then the price of anarchy is maximal:

$$PoA(\alpha, K) \geq \frac{T_S(\widetilde{p})}{U_S(p_r^E)} = 1.$$

Otherwise, the price of anarchy is expressed as follows:

$$PoA(\alpha, K) = \frac{\frac{p^*(1-\pi(p^*,K))}{\mu_p} + \alpha p^* K \frac{1-\pi(p^*,K)}{1-\pi(p^*,K)} + \frac{1-p^*(1-\pi(p^*,K))}{\mu_s-\lambda_s(1-p^*(1-\pi(p^*,K)))}}{\frac{\alpha K}{1-\Pi(p^E,K)} + \frac{1}{\mu_p}}.$$

Moreover, Proposition 5 says that for all values of sensing cost $\alpha$ and number of licensed channels $K$, the sensing probability when using a NE policy $p^E(\alpha, K)$ is lower than the optimal sensing policy $p^*(\alpha, K)$. As the blocking probably $\Pi(p, k)$ is increasing with the sensing probability, we obtain that:

$$\Pi(p_r^E(\alpha, K), K) \leq \Pi(p^*(\alpha, K)).$$

Thus, the price of anarchy is bounded by the following expression:

$$\begin{aligned} PoA(\alpha, K) &\geq \frac{p^*(1-\pi(p^*,K))(\frac{1}{\mu_p} + \frac{\alpha K}{1-\pi(p^E,K)}) + \frac{1-p^*(1-\pi(p^*,K))}{\mu_s-\lambda_s(1-p^*(1-\pi(p^*,K)))}}{\frac{\alpha K}{1-\Pi(p^E,K)} + \frac{1}{\mu_p}} \\ &= X(p^*, K) + \frac{\frac{1-X(p^*,K)}{\mu_s-\lambda_s(1-X(p^*,K))}}{\frac{\alpha K}{1-\Pi(p^E,K)} + \frac{1}{\mu_p}}. \end{aligned}$$

We have proved in Proposition 6 that $U_S(p^E(\alpha, K)) \leq \frac{1}{\mu_s-\lambda_s}$, then the lower bound of the price of anarchy can be expressed as follows:

$$PoA(\alpha, K) \geq X(p^*, K) + \frac{\frac{1-X(p^*,K)}{\mu_s-\lambda_s(1-X(p^*,K))}}{\frac{1}{\mu_s-\lambda_s}}.$$

For all the three previous cases, we may consider the following lower bound of the price of anarchy:

$$\forall \alpha, K, \quad PoA(\alpha, K) \geq X(p^*) + \frac{\frac{1-X(p^*)}{\mu_s-\lambda_s(1-X(p^*))}}{\frac{1}{\mu_s-\lambda_s}}.$$

Our aim is to minimize this lower bound of the price of anarchy. After some algebra, the lower bound of the price of anarchy may be expressed as follows:

$$PoA(\alpha, K) \geq X(p^*, K) + \frac{(\mu_s-\lambda_s)(1-X(p^*, K))}{\mu_s-\lambda_s(1-X(p^*, K))} = \frac{\mu_s-\lambda_s(1-X^2(p^*, K))}{\mu_s-\lambda_s(1-X(p^*, K))}.$$

Let us define the following function $F(X) = \frac{\mu_s-\lambda_s(1-X^2(p^*,K))}{\mu_s-\lambda_s(1-X(p^*,K))}$. The derivative of the function $F$ with respect to $X(p, k)$ is:

$$\frac{\partial F}{\partial X(p^*, K)}(X(p^*, K)) = \frac{\lambda_s^2 X^2(p^*, K) + (2\mu_s\lambda_s - 2\lambda_s^2)X(p^*, K) + \lambda_s^2 - \lambda_s\mu_s}{(\mu_s-\lambda_s(1-X(p^*, K)))^2}.$$

Therefore, $F'(X(p^*, K)) = 0$ when $X(p^*, K) = \frac{\lambda_s-\mu_s\pm\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}$. Note that we have $F(0) = 1$. Then, $F(X(p^*, K))$ is decreasing between $\frac{\lambda_s-\mu_s-\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}$ and $\frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}$. Finally, the lower bound of the price of anarchy is minimized when:

$$X(p^*, K) = \frac{\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}$$

and its minimal value is:

$$F(X(p^*, K)) = \frac{\mu_s - \lambda_s(1-(\frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s})^2)}{\mu_s - \lambda_s(1-\frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s})}.$$

Finally, for all values of $\alpha$ and $K$, we obtain the lower bound of the price of anarchy:

$$PoA(\alpha, K) \geq 2(\frac{\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}).$$

■

The price of anarchy, in the non-slotted model, is expressed by the following ratio:

$$PoA_r(\alpha, K) = \frac{U_S(p^*(\alpha, K))}{U_S(p_r^E(\alpha, K))}.$$

Suppose, first, that $\frac{1}{\mu_s-\lambda_s} < \frac{\alpha K}{(1-\Pi(0,K))(1-P_r(0,K))} + \frac{1}{\mu_p}$. Then, the price of anarchy can be expressed as follows:

$$\begin{aligned} PoA_r(\alpha, K) &= \frac{\frac{p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K))}{\mu_p} + \alpha p^* K \frac{(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K))}{1-\Pi(p^*,K)}}{\frac{1}{\mu_s-\lambda_s}} \\ &+ \frac{\frac{1-p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K)))}{\mu_s-\lambda_s(1-p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K)))}}{\frac{1}{\mu_s-\lambda_s}}. \end{aligned}$$

As the blocking probability $\Pi(p, K)$ is increasing with $p$, the price of anarchy is higher than the following lower bound:

$$\begin{aligned} PoA_r(\alpha, K) &\geq \frac{\frac{p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K))}{\mu_p} + \alpha p^* K \frac{(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K))}{(1-\Pi(0,K))(1-P_r(0,K))}}{\frac{1}{\mu_s-\lambda_s}} \\ &+ \frac{\frac{1-p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K))}{\mu_s-\lambda_s(1-p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K)))}}{\frac{1}{\mu_s-\lambda_s}} \\ &= p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K)) \frac{\frac{1}{\mu_p} + \frac{\alpha K}{(1-\Pi(0,K))(1-P_r(0,K))}}{\frac{1}{\mu_s-\lambda_s}} \\ &+ \frac{\frac{1-p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K)))}{\mu_s-\lambda_s(1-p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K)))}}{\frac{1}{\mu_s-\lambda_s}}. \end{aligned}$$

Let us denote $Y(p^*, K) = p_r^*(1 - \Pi(p_r^*, K))(1 - P_r(p_r^*, K))$. The lower bound of the price of anarchy can be rewritten as:

$$PoA_r(\alpha, K) > Y(p^*, K) + \frac{\frac{1-Y(p^*,K)}{\mu_s-\lambda_s(1-Y(p^*,K))}}{\frac{1}{\mu_s-\lambda_s}}.$$

Second, suppose that $\frac{1}{\mu_s-\lambda_s\Pi(1,K)} > \frac{\alpha K}{(1-\Pi(1,K))(1-P_r(1,K))} + \frac{1}{\mu_p}$. It follows that $1 - \Pi(1, K) < \frac{\lambda_s-\mu_s+\sqrt{\mu_s\mu_p}}{\lambda_s}$.

From Proposition 1, we have that $\widetilde{p}(K) = 1$ and then the price of anarchy is maximal:

$$PoA_r(\alpha, K) \geq \frac{T_S(\widetilde{p})}{U_S(p_r^E)} = 1.$$

Otherwise, the price of anarchy is expressed as follows:

$$\begin{aligned} PoA_r(\alpha, K) &= \frac{\frac{p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K)))}{\mu_p} + \alpha p^* K \frac{(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K))}{(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K))}}{\frac{\alpha K}{1-\Pi(p_r^E,K)} + \frac{1}{\mu_p}} \\ &+ \frac{\frac{1-p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K))}{\mu_s-\lambda_s(1-p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K)))}}{\frac{\alpha K}{1-\Pi(p_r^E,K)} + \frac{1}{\mu_p}} \end{aligned}$$

Moreover, Proposition 5 says that for all values of sensing cost $\alpha$ and number of licensed channels $K$, the sensing probability when using a NE policy $p_r^E(\alpha, K)$ is lower than the optimal sensing policy $p_r^*(\alpha, K)$. As the blocking probably $\Pi(p, k)$ is increasing with the sensing probability, we obtain that:

$$\Pi(p_r^E(\alpha, K), K) \leq \Pi(p^*(\alpha, K)).$$

Thus, the price of anarchy is bounded by the following expression:

$$\begin{aligned}
PoA_r(\alpha,K) &\geq \frac{p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K))(\frac{1}{\mu_p}+\frac{\alpha K}{(1-\pi(p_r^E,K))(1-P_r(p_r^E.k))})}{\frac{\alpha K}{(1-\pi(p_r^E,K))(1-P_r(p_r^E.k))}+\frac{1}{\mu_p}} \\
&+ \frac{\frac{1-p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K))}{\mu_s-\lambda_s(1-p_r^*(1-\Pi(p_r^*,K))(1-P_r(p_r^*,K)))}}{\frac{\alpha K}{(1-\pi(p_r^E,K))(1-P_r(p_r^E.k))}+\frac{1}{\mu_p}} \qquad (28) \\
&\geq Y(p^*) + \frac{\frac{1-Y(p^*)}{\mu_s-\lambda_s(1-Y(p^*))}}{\frac{\alpha K}{(1-\pi(p_r^E,K))(1-P_r(p_r^E.k))}+\frac{1}{\mu_p}}.
\end{aligned}$$

We have proved in Proposition 6 that $U_S(p_r^E(\alpha,K)) \leq \frac{1}{\mu_s-\lambda_s}$, then the lower bound of the price of anarchy can be expressed as follows:

$$PoA_r(\alpha,K) \geq Y(p^*,K) + \frac{\frac{1-Y(p^*,K)}{\mu_s-\lambda_s(1-Y(p^*,K))}}{\frac{1}{\mu_s-\lambda_s}}.$$

For all the three previous cases, we may consider the following lower bound of the price of anarchy:

$$\forall \alpha, K, \quad PoA_r(\alpha,K) \geq Y(p^*,K) + \frac{\frac{1-Y(p^*,K)}{\mu_s-\lambda_s(1-Y(p^*,K))}}{\frac{1}{\mu_s-\lambda_s}}.$$

Our aim is to minimize this lower bound of the price of anarchy. After some algebra, the lower bound of the price of anarchy may be expressed as follows:

$$PoA_r(\alpha,K) \geq Y(p^*,K) + \frac{(\mu_s-\lambda_s)(1-Y(p^*,K))}{\mu_s-\lambda_s(1-Y(p^*,K))} = \frac{\mu_s-\lambda_s(1-Y^2(p^*,K))}{\mu_s-\lambda_s(1-Y(p^*,K))}.$$

Let us define the following function $F(Y(p^*,K)) = \frac{\mu_s-\lambda_s(1-Y^2(p^*,K))}{\mu_s-\lambda_s(1-Y(p^*,K))}$. The derivative of the function $F$ with respect to $Y(p,K)$ is:

$$F'(Y) = \frac{\lambda_s^2 Y^2(p^*,K) + (2\mu_s\lambda_s - 2\lambda_s^2)Y(p^*,K) + \lambda_s^2 - \lambda_s\mu_s}{(\mu_s-\lambda_s(1-Y(p^*,K)))^2}.$$

Therefore, $F'(Y)=0$ when $Y(p^*,K) = \frac{\lambda_s-\mu_s\pm\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}$. Note that we have $F(0)=1$. Then, $F(Y)$ is decreasing between $Y(p^*,K) = \frac{\lambda_s-\mu_s-\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}$ and $Y(p^*,K)\frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}$. Furthermore, the lower bound of the price of anarchy is minimized when:

$$Y(p^*,K) = \frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s}$$

and its minimal value is:

$$F(Y(p^*,K)) = \frac{\mu_s-\lambda_s(1-(\frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s})^2)}{\mu_s-\lambda_s(1-\frac{\lambda_s-\mu_s+\sqrt{\mu_s(\mu_s-\lambda_s)}}{\lambda_s})}.$$

Finally, for all values of $\alpha$ and $K$, we obtain the following expression of the lower bound of the price of anarchy for the non-slotted model:

$$PoA_r(\alpha, K) \geq 2(\frac{\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s - \lambda_s)}}{\lambda_s}).$$

■