www.ietdl.org

Published in IET Communications
Received on 29th June 2010
Revised on 20th December 2010
doi: 10.1049/iet-com.2010.0537

In Special Issue: Cognitive Communications

ISSN 1751-8628

Optimal opportunistic sensing in cognitive radio networks

O. Habachi Y. Hayel

CERI/LIA, Université d'Avignon et des Pays de Vaucluse, France
E-mail: oussama.habachi@univ-avignon.fr

Abstract: The authors are interested in evaluating the performance of a cognitive radio network composed of secondary and primary mobiles, and look for optimising the decision process of the secondary mobiles when they have to choose between licensed or unlicensed channels. In fact, the system is composed of several channels where only one unlicensed channel is shared between all the secondary mobiles, when they decide to use this particular channel. As the secondary mobiles are equipped with cognitive radios, they are able to sense the licensed channels and use one of them if it is free. The authors consider first the global system and look for the optimal proportion of secondary mobiles that sense the licensed channels in order to optimise an average performance of the system. Second, the authors assume that each secondary mobile decides to sense or not the licensed channels and, are interested in an equilibrium situation as the secondary mobiles are in competition. After showing the existence and the uniqueness of equilibrium, the performance of this equilibrium is evaluated by looking at the price of the anarchy of the system.

1 Introduction

A big new challenge in the networking community is how to put 'cognition' into networks. The term cognition is described as the faculty for a mobile or a network to adapt its communication parameters (transmission power for mobiles or frequency for a base station) to perturbations of its environment. A radio system having this capability is called a cognitive radio. This new field of research has started with the work of Mitola [1] and the faculty of new frequency channels usage. In wireless networks, in contrast to wired networks, the capacity is limited to the radio spectrum. In order to provide better services with higher quality of service (QoS), in November 2002 the Federal Communication Commission (FCC) proposed to open the use of many bands that are under-used. Indeed, the FCC report reveals that the electromagnetic spectrum has gaps, band of frequencies assigned to a primary user (PU), that at a particular time and specific geographic location is not being utilised by that user. In a cognitive network, it has been defined two types of users or mobiles: primary and secondary. On the one hand, the primary mobiles have made an agreement with the service provider and their traffic has some priority. On the other hand, when the secondary users (SUs) do not have access to the channels state database, they are able to sense licensed channels in order to use one of them if it is not occupied as the secondary mobiles are equipped with a cognitive radio. In this paper, we are interested in designing an optimal sensing strategy for the secondary mobiles. We consider, as PUs are licensed, that they can preempt secondary mobile. This means that, if all licensed channels are occupied, a new primary mobile can preempt a secondary mobile that is using a licensed channel. We evaluate the loss of performance for a secondary mobile in this case.

1.1 Related works

Lots of recent articles deal with cognitive radio technologies and their performances. The survey paper [2] gives lots of very interesting problems for evaluating the performance of cognitive radio systems. Specifically, the author describes how mathematical tools and specifically game theory can be applied to analyse these complex wireless systems. In [3], the authors consider an energy-efficient spectrum access policy. Each SU will sense the spectrum and select subcarriers taking into account data rate requirements and maximum power limit. This work is close to ours as the authors study the problem by considering a non-cooperative behaviour among new users and moreover, they consider energy-efficient allocation scheme. Game theory seems an ideal mathematical tool for evaluating performances of cognitive radio systems. It has already been employed in [4] for studying optimal power control and channel allocation schemes considering a hierarchical network with leaders (PU) and followers (SUs). Another game model has been proposed in [5] for the performance of carrier-sense multiple access. The authors focus on how to control the carrier-sense threshold for improving network performance in a non-cooperative setting. This work is close to ours but it is not in a cognitive radio context.

A more related work is proposed in [6]. The authors study decentralised medium access control (MAC) protocols such that SUs search for spectrum opportunities without a central controller. They consider a partially observable Markov decision process (MDP) and propose an analytical framework based on this tool. There are several differences with our work. First, they consider only licensed channels and SUs look for opportunities inside those channels. In our context, SUs can decide not to sense, and share a common unlicensed channel. Second, the authors do not consider the competition between the SUs. They look for optimal sensing and channel selection schemes that maximise the expected total number of bits delivered over a finite number of slots. Their optimal and sub-optimal policies depend on belief vector based on all past decisions and observations of channel's state. A related model is proposed in [7]. Indeed, the authors propose an optimal periodic sensing mechanism for secondary mobile. The periodic sensing decouples sensing and access. The optimal policy is based on constrained MDP and each secondary mobile maximises the average number of successful transmissions. Finally, the authors propose three cognitive access policies: memoryless, periodic sensing and full observation.

In [8], the authors propose an algorithm for SUs in two steps: first, any secondary mobile decides whether or not a channel is idle; second, he determines whether this channel is a good opportunity or not based on channel sensing statistics. Some other papers, far from our study but in the same cognitive radio context, on opportunistic sensing and MAC protocols are described in [9–12].

1.2 Contributions of the paper

The contributions of our paper deal with several points. First, our model is based on a queuing system which gives interesting closed-form results. Second, we analyse an important problem which has never been studied in the literature on cognitive radio networks, which is the non-cooperative game between the SUs in order to access to the licensed channels. Using this framework, we are able to determine the optimal sensing policy for a SU, in order to minimise his energy cost, and also to evaluate the gap of performance when the SUs decide selfishly about their sensing policy or if there is a central controller that determines the best sensing policy for any SU. This non-cooperative game approach is a first step in order to construct efficient decentralised algorithm based on reinforcement learning, for example [13].

1.3 Organisation of the paper

The paper is organised as follows: In the next section, we introduce the analytical model based on queuing tools. Then, in Section 3, we consider an optimal individual policy where each secondary mobile decides to sense or not the licensed channels. We compare the performance of the decentralised system, in which each secondary mobile determines his sensing policy, to the centralised problem by looking at the price of anarchy (PoA) in Section 4. Before concluding the paper and giving some perspective, we give some numerical illustrations of our results in Section 5.

2 System model and sensing policy

The system is composed of PUs and SUs. The PUs use K licensed channels (see Fig. 1). The SUs have the possibility to access to one of the licensed channels by using the cognition rules in order to benefit from the high QoS of licensed channels. We assume that the SUs do not have access to the channels state and then, if a SU wants to use a licensed channel, he has to sense before. Moreover, a SU can access to unlicensed bands (which is modelled as only one channel here) then without sensing the licensed channels. A SU is a mobile having the faculty to adapt his transmission parameters and to support different communication standards (e.g. 3G, WiMAX, WiFi, …). The possibility of having a channel reserved for secondary mobile has been used in [14]. In fact, authors present a model in which SUs have the possibility to access to both licensed and unlicensed bands. Therefore the SUs can access to the K licensed channels or decide to use the unlicensed bands modelled by a common channel. We assume that primary (resp. secondary) users arrive following a Poisson process with a rate λ_p (resp. λ_s). The service rates is μ_p (resp. μ_s) for the licensed channels (resp. common unlicensed channel). Those assumptions on Markov processes for arrivals and service time are also used in several works like in [6, 7].

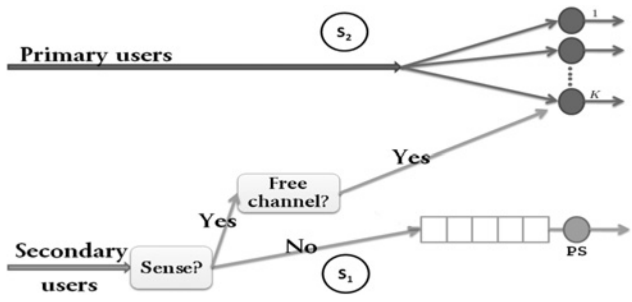

Fig. 1 *Model of opportunistic sensing for secondary mobiles*

Subsystem S_1 is an $M/M/1$ queue and S_2 is an $M/M/K/K$ queue

The system is depicted in Fig. 1 and is composed of two sub-systems: licensed and unlicensed bands. The first one, namely system S_1, represents the unlicensed band used by SUs, which do not sense the licensed channels. The capacity of this channel is equally shared among all the SUs, which does not sense licensed channels. Then, the subsystem S_1 modelling the SUs who do not choose to sense the licensed channels can be modelled using an $M/M/1$ queue with a Poisson arrival rate. These SUs are served following a process sharing (PS) policy. In order to guarantee the stability of this queue, we take the following assumption:

Assumption 1: We assume that the arrival rate λ_s of SUs is strictly lower than the service rate μ_s.

The second system with the licensed channels, namely S_2, is composed of the two following types of mobiles: (i) the primary mobiles that have found one free channel and (ii) the secondary mobiles that have sensed the licensed channels and have found a free channel.

Each SU decides to sense k licensed channels with k from 0 to K. Those k channels are randomly selected following a uniform distribution between the K licensed channels. For all $k \in \{0, \dots, K\}$, we denote by P_k the probability that a SU decides to sense k licensed channels. We write P as the vector $(P_0, \dots, P_K)$. We denote by α the cost of sensing one channel for a secondary mobile. Then the total cost of sensing depends on how many licensed channels that SU decides to sense.

After deciding on which channel (licensed or unlicensed) he wants to transmit, any SU gets a cost for his transmission which is linear with his expected sojourn time. Indeed, more time a secondary will spend using a channel,

doi: 10.1049/iet-com.2010.0537

more energy he will use. We denote by β the energy cost for one unit of time spent in the system. Then any SU wants to determine its optimal sensing policy (how many licensed channels to sense) in order to minimise an average cost function which is a linear combination between the average delay and the sensing cost.

The arrival rate in the common secondary channel (subsystem S_1) is composed of secondary mobiles that do not sense the licensed channels. Then the arrival rate of secondary mobile for that particular channel is $\lambda_s P_0$. We have assumed that the maximum arrival rate λ_s is lower than the service rate μ_s. Then, we have a sufficient stability condition of this $M/M/1$ queue. The average sojourn time (delay) $T_{S_1}(P)$ for a SU that chooses to join the common secondary channel, that is not to sense any licensed channel, is given by

$$T_{S_1}(P) = \frac{1}{\mu_s - \lambda_s P_0} \tag{1}$$

2.1 Blocking probability

For all $k \geq 1$, any SU that senses k channels and does not find any free channel, is blocked. Let B_k be the probability that a SU sensing k channels is blocked. We have

$$B_k(P) = \sum_{i=1}^{K} \pi_i(P, k) \frac{C_i^k}{C_K^k}$$

where π_i is the stationary probability that there are i users in the subsystem S_2 (PASTA principle). As the SUs arrive to the system with a Poisson process, we can use the PASTA principle [15] which says that any SU will see the system in stationary regime.

2.2 Preemptive mechanism

When a PU chooses a channel, a SU using this licensed channel will be rejected. This preemptive rule is one of the basic assumptions in a cognitive radio network with primary and SUs [16].

We denote by $X_p(t)$ (resp. $X_s(t)$) the number of PUs (resp. SUs) using the licensed channels at time t with $X_p(t) + X_s(t) \leq K$. The subsystem of licensed channel can be modelled using a bi-dimensional Markov process, $Y(t) = \{X_p(t), X_s(t)\}$. The probability that a SU will be rejected using a licensed channel is denoted by $P_r(P)$. We denote by $P_0(n, m)$ the probability that a SU will be rejected when he joins a licensed channel at the primary system has already n PUs and m SUs. Note that we have necessary $n + m < K$ and the reject probability is

$$P_r(P) = \sum_{n,m/n+m=0}^{n+m=K-1} P_0(n, m)\pi(n, m) \tag{2}$$

where $\pi(n, m)$ is the stationary probability of the Markov process $Y(t)$ described in Fig. 2.

The stationary probabilities $\pi(n, m)$ can be computed using standard tools of Markov theory. It is possible to express the relation between the probabilities $P_0(n, m)$ has a linear system. Indeed, we have for all states $(X_p(t), X_s(t)) = (n, m)$ such that $n + m = K - 1$ (see equation at the bottom of the page)

Otherwise for $n + m < K - 1$ (see equation at the bottom of the page)

Then, for any secondary mobile that decides to sense k

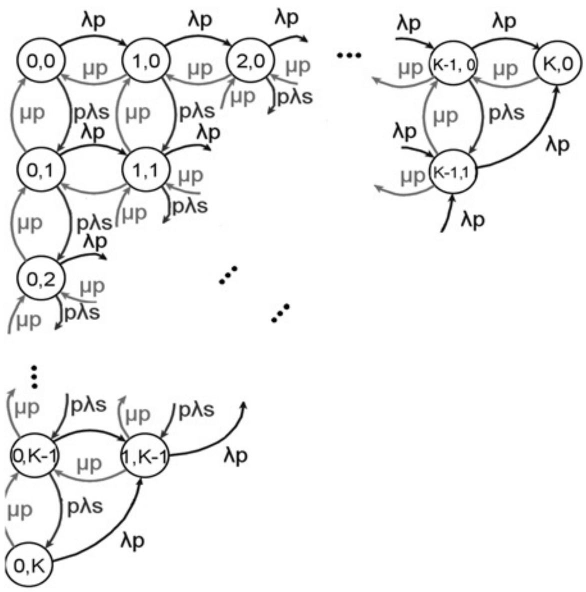

Fig. 2 *Markov chain of Y(t)*

$$P_0(n, m) = \begin{cases} \dfrac{1}{K}\dfrac{\lambda_p}{\lambda_p + \mu_p} + \dfrac{K-1}{K}\dfrac{\lambda_p}{\lambda_p + \mu_p} P_0(1, K-2) + \dfrac{\mu_p}{\lambda_p + \mu_p} P_0(0, K-2) & \text{if } n = 0, \\ \dfrac{\lambda_p}{\lambda_p + 2\mu_p} + \dfrac{\mu_p}{\lambda_p + 2\mu_p} P_0(K-2, 0) & \text{if } m = 0, \\ \dfrac{1}{m+1}\dfrac{\lambda_p}{\lambda_p + 2\mu_p} + \dfrac{m}{m+1}\dfrac{\lambda_p}{\lambda_p + 2\mu_p} P_0(n+1, m-1) + \dfrac{\mu_p}{\lambda_p + 2\mu_p}(P_0(n-1, m) + P_0(n, m-1)) & \text{otherwise} \end{cases}$$

$$P_0(n, m) = \begin{cases} \dfrac{p\lambda_s}{p\lambda_s + \lambda_p + \mu_p} P_0(n, m+1) + \dfrac{\lambda_p}{p\lambda_s + \lambda_p + \mu_p} P_0(n+1, m) + \dfrac{\mu_p}{p\lambda_s + \lambda_p + \mu_p} P_0(n, m-1) & \text{if } n = 0, \\ \dfrac{p\lambda_s}{p\lambda_s + \lambda_p + 2\mu_p} P_0(n, m+1) + \dfrac{\lambda_p}{p\lambda_s + \lambda_p + 2\mu_p} P_0(n+1, m) + \dfrac{\mu_p}{p\lambda_s + \lambda_p + 2\mu_p} P_0(n-1, m) & \text{if } m = 0, \\ \dfrac{p\lambda_s}{p\lambda_s + \lambda_p + 2\mu_p} P_0(n, m+1) + \dfrac{\lambda_p}{p\lambda_s + \lambda_p + 2\mu_p} P_0(n+1, m) + \dfrac{\mu_p}{p\lambda_s + \lambda_p + 2\mu_p} \\ \quad (P_0(n-1, m) + P_0(n, m-1)) & \text{otherwise} \end{cases}$$

licensed channels, the average sojourn time $T_{S_2}(P,k)$ is

$$T_{S_2}(P,k)=\frac{(1-B_k(P))(1-P_r(P))}{\mu_p} \quad (3)$$

A SU spends energy on transmitting and on sensing channels. The average cost $\bar{U}_S$ of a SU is determined by the summation of the cost of the average sojourn time inside the system and the cost of sensing. This average cost depends on the probability vector P and is expressed by

$$\bar{U}_S(P)=\beta\left(P_0\frac{1}{\mu_s-P_0\lambda_s}+\sum_{i=1}^{K}P_i\frac{(1-B_i(P))(1-P_r(P))}{\mu_p}\right)+\alpha\sum_{i=1}^{K}iP_i \quad (4)$$

It is straightforward that SU wants to minimise his average cost function and, without loss of generality, we can consider $\beta=1$.

2.3 Two-action policy

In order to simplify the analysis, we consider the particular case of the two following strategies: (i) to sense all the K licensed channels ($k=K$), and (ii) not to sense licensed channels ($k=0$). We denote by p the probability of sensing (all the K licensed channels), that is, the probability vector is such that $P=(1-p, 0, \ldots, 0, p)$.

The subsystem S_2 is composed of N channels having a service rate of μ_p. Transmissions on one of these can be modelled using an $M/M/K/K$ queue, known as the Erlang-B model [17], with a Poisson arrival rate of $\lambda_p+p\lambda_s$.

Therefore the blocking probability, which is the probability that any user finds all the channels occupied, is given by the following Erlang-B formula

$$B_K(p)=\frac{\rho(p)^K}{K!}\left(\sum_{n=0}^{K}\left(\frac{\rho(p)^n}{n!}\right)\right)^{-1}$$

with $\rho(p)=((\lambda_p+p\lambda_s)/\mu_p)$. This blocking probability depends on the number of licensed channels K, but also on the probability p. This probability is equal to the proportion of SUs that senses. If this proportion increases, the input rate in the subsystem S_2 increases and then the blocking probability $B_K(p)$ will also increase. Thus, there is a tradeoff for secondary mobiles to sense or not.

Finally, the average user cost function defined in (4) can be simplified to the following expression

$$\bar{U}_S(p)=\frac{1-p}{\mu_s-\lambda_s(1-p)}+\frac{p(1-B_K(p))(1-P_r(p))}{\mu_p}+\alpha pK \quad (5)$$

We define as the optimal sensing policy, the probability p^* which minimises the average cost function $\bar{U}_S(p)$, that is

$$p^*=\arg\min_p \bar{U}_S(p)$$

We are now interested in a decentralised point of view, where each SU, individually, decides to sense or not. In this particular decentralised context, we look for existence of an equilibrium situation and, if it exists, for its uniqueness.

3 Equilibrium sensing policy

We consider the distributed system in which each secondary mobile decides to sense or not the licensed channels selfishly. Each SU decides on his probability to sense p. This decision is based on minimising his average total cost $U(p,\ p')$ which depends on his probability p and the probability p' of the other secondary mobiles that sense

$$U(p,p')=(1-p)T_{S_1}(p')+pT_{S_2}(p')+\alpha pK \quad (6)$$

We have the following relation for all p: $U(p,p)=\bar{U}_S(p)$. Let us define an equilibrium for our non-cooperative game as a strategy that minimises the average cost function U against others using the equilibrium strategy.

Definition 1: The individual decision p^E is an equilibrium if and only if

$$p^E=\arg\min_p U(p,p^E)$$

Our game is also a population game as the number of players is not finite and can grow to infinity. In this type of game, an equilibrium strategy is computed looking at an individual optimal strategy against a population profile [18]. We now prove that our non-cooperative game between secondary mobiles has a unique equilibrium.

Proposition 1: For all K and α, the equilibrium $p^E(\alpha,\ K)$ exists and is unique. The equilibrium is given by (see equation at the bottom of the page)

with p' solution of the following equation

$$\frac{1}{\mu_s-\lambda_s(1-p)}=\alpha K+\frac{(1-B_K(p))(1-P_r(p))}{\mu_p} \quad (7)$$

Proof: From (6), the first argument derivative of the cost function is

$$\frac{\partial U}{\partial p}(p,p')=T_{S_2}(p')-T_{S_1}(p')+\alpha K$$

The probability p^E is an equilibrium if and only if it satisfies

$$p^E(\alpha,K)=\begin{cases}0, & \text{if } \dfrac{1}{\mu_s-\lambda_s}<\alpha K+\dfrac{(1-B_K(0))(1-P_r(0))}{\mu_p} \text{ and } \dfrac{1}{\mu_s}<\alpha K+\dfrac{(1-B_K(1))(1-P_r(1))}{\mu_p}\\ 1, & \text{if } \dfrac{1}{\mu_s-\lambda_s}>\alpha K+\dfrac{(1-B_K(0))(1-P_r(0))}{\mu_p} \text{ and } \dfrac{1}{\mu_s}>\alpha K+\dfrac{(1-B_K(1))(1-P_r(1))}{\mu_p}\\ p', & \text{otherwise}\end{cases}$$

doi: 10.1049/iet-com.2010.0537

the following equation

$$\alpha K + T_{S_2}(p^E) = T_{S_1}(p^E)$$

This equality is equivalent to

$$T_{s_1}(p^E) = \alpha K + \frac{(1 - B_K(p^E))(1 - P_r(p^E))}{\mu_p}$$

which leads to

$$\frac{1}{\mu_s - \lambda_s(1 - p^E)} = \alpha K + \frac{(1 - B_K(p^E))(1 - P_r(p^E))}{\mu_p}$$

The left-hand side function is continuous and strictly decreasing with p (see (7)) and the right-hand side function is also continuous and strictly decreasing in p. Then this equation has a unique solution inside [0, 1] if and only if

$$\frac{1}{\mu_s - \lambda_s} > \alpha K + \frac{(1 - B_K(0))(1 - P_r(0))}{\mu_p} \quad \text{and}$$
$$\frac{1}{\mu_s} < \alpha K + \frac{(1 - B_K(1))(1 - P_r(1))}{\mu_p}$$

or

$$\frac{1}{\mu_s - \lambda_s} < \alpha K + \frac{(1 - B_K(0))(1 - P_r(0))}{\mu_p} \quad \text{and}$$
$$\frac{1}{\mu_s} > \alpha K + \frac{(1 - B_K(1))(1 - P_r(1))}{\mu_p}$$

Moreover, if $(1/\mu_s) > \alpha K + [\{(1 - B_K(1))(1 - P_r(1))\}/\mu_p]$ and $\{1/(\mu_s - \lambda_s)\} > \alpha K + [\{(1 - B_K(0))(1 - P_r(0))\}/\mu_p]$ then the cost function is strictly decreasing and the equilibrium is $p^E = 1$. On the other hand, if $(1/\mu_s) < \alpha K + [\{(1 - B_K(1))(1 - P_r(1))\}/\mu_p]$ and $\{1/(\mu_s - \lambda_s)\} < \alpha K + [\{(1 - B_K(0))(1 - P_r(0))\}/\mu_p]$, the cost is strictly increasing and the equilibrium is $p^E = 0$. □

We proved that the equilibrium exists and is unique. Then, the average total cost of a secondary mobile at the equilibrium is given by (see equation at the bottom of the page)

We are now able to compare this equilibrium sensing policy with the optimal sensing policy without knowing in closest form the latter. Note that this particularity is not usually true in a non-cooperative game. In order to do this, we take the following assumption:

Assumption 2: We assume that the reject probability $P_r(p)$ is increasing with the sensing probability p. Indeed, if more secondary mobile choose to sense licensed channels, then a secondary mobile has a higher probability to be rejected by PUs.

Proposition 2: For all α and K, the optimal sensing probability p^* is higher than the equilibrium sensing probability p^E, that is, $p^E(\alpha, K) \le p^*(\alpha, K)$.

Proof: We assume that there exists $\alpha_0 > 0$ and K_0 such that $p^E(\alpha_0, K_0) > p^*(\alpha_0, K_0)$. For the rest of the proof and for simplicity, we denote $p^*(\alpha_0, K_0)$ by p^* and $p^E(\alpha_0, K_0)$ by p^E. Moreover, we have

$$T_S(p^*) + \alpha p^* K \le T_S(p^E) + \alpha p^E K$$

But, as p^E is an equilibrium, we have

$$T_S(p^*, p^E) + \alpha p^* K \ge T_S(p^E) + \alpha p^E K$$

From these two relations, we obtain $T_S(p^*, p^*) \le T_S(p^*, p^E)$. Thus, using the Assumption 2, we have

$$\begin{aligned}(1 - p^*)T_{S_1}(p^*) + \frac{p^*(1 - B_K(p^*))(1 - P_r(p^*))}{\mu_p} \\ \le (1 - p^*)T_{S_1}(p^E) + \frac{p^*(1 - B_K(p^E))(1 - P_r(p^*))}{\mu_p}\end{aligned}$$

and then

$$\begin{aligned}(1 - p^*)(T_{S_1}(p^*) - T_{S_1}(p^E)) \le \frac{p^*(1 - P_r(p^*))}{\mu_p}(B_K(p^*) \\ - B_K(p^E))\end{aligned}$$

But, it is clear that T_{S_1} is decreasing with p and B_K is increasing with p, then for $p^E > p^*$, the left-hand side is positive and right-hand one is negative which gives a contradiction. Thus, for all α and K, we have $p^E \le p^*$. □

Then, the proportion of SUs that senses the licensed channels at the equilibrium is lower than the optimal proportion that senses. This lack of performances is due to the competition between SUs. Finally, we look for the gap of performance in terms of individual cost, induced by the decentralised decision process. This gap can be measured using the PoA framework.

4 Price of anarchy

The PoA gives the lack of performance between the total cost of the worst Nash equilibrium and of an optimal routing of the traffic. In this model, the PoA is expressed as the ratio between the optimal cost of a centralised system and the cost at the equilibrium in a model with preemptive rules.

$$\bar{U}_S(p^E) = \begin{cases} \dfrac{1}{\mu_s - \lambda_s}, & \text{if } \dfrac{1}{\mu_s - \lambda_s} < \alpha K + \dfrac{(1 - B_K(0))(1 - P_r(0))}{\mu_p} \text{ and } \dfrac{1}{\mu_s} < \alpha K + \dfrac{(1 - B_K(1))(1 - P_r(1))}{\mu_p} \\ \dfrac{(1 - B_K(1))(1 - P_r(1))}{\mu_p} + \alpha K, & \text{if } \dfrac{1}{\mu_s - \lambda_s} > \alpha K + \dfrac{(1 - B_K(0))(1 - P_r(0))}{\mu_p} \text{ and } \dfrac{1}{\mu_s} > \alpha K + \dfrac{(1 - B_K(1))(1 - P_r(1))}{\mu_p} \\ \alpha K + \dfrac{(1 - B_K(p'))(1 - P_r(p'))}{\mu_p}, & \text{otherwise} \end{cases}$$

Then, it is computed as follows

$$\mathrm{PoA}(\alpha, K) = \frac{\bar{U}_s(p^*(\alpha, K))}{\bar{U}_s(p^E(\alpha, K))} \leq 1 \tag{8}$$

Proposition 3: For all K and α, we have

$$\bar{U}_S(p^E(\alpha, K)) \leq \frac{1}{\mu_s - \lambda_s}$$

Proof: First, if $(1/\mu_s) < \alpha K + [\{(1 - B_K(1))(1 - P_r(p^E))\}/\mu_p]$ and $\{1/(\mu_s - \lambda_s)\} < \alpha K + [\{(1 - B_K(0))(1 - P_r(0))\}/\mu_p]$ then $\bar{U}_S(p^E(\alpha, K)) = \{1/(\mu_s - \lambda_s)\}$.

Second, if $(1/\mu_s) > \alpha K + [\{(1 - B_K(1))(1 - P_r(1))\}/\mu_p]$ and $\{1/(\mu_s - \lambda_s)\} > \alpha K + [\{(1 - B_K(0))(1 - P_r(0))\}/\mu_p]$ then

$$\bar{U}_S(p^E(\alpha, K)) = \frac{(1 - B_K(1))(1 - P_r(1))}{\mu_p} + \alpha K \leq \frac{1}{\mu_s} \leq \frac{1}{\mu_s - \lambda_s}$$

Otherwise

$$\bar{U}_S(p^E(\alpha, K)) = \alpha K + \frac{(1 - B_K(p^E))(1 - P_r(p^E))}{\mu_p} = \frac{1}{\mu_s - \lambda_s(1 - p^E)} \leq \frac{1}{\mu_s - \lambda_s}$$

Finally, we have for all α and K: $\bar{U}_S(p^E(\alpha, K)) \leq \{1/(\mu_s - \lambda_s)\}$. □

We try now to find the lower bound of the PoA, in order to define the worst-possible lack of performance of the decentralised system. The aim is to determine a minimum expression of the PoA or to bound it, in order to measure the worst performance of the decentralised system compared to the centralised one. We have the following proposition which gives a lower bound of the PoA. This bound is very interesting because it does not depend on the sensing cost nor on the number of licensed channels.

Proposition 4: For any α and any number of licensed channels K, we have

$$\mathrm{PoA}(\alpha, K) \geq 2\left(\frac{\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s - \lambda_s)}}{\lambda_s}\right) := \underline{\mathrm{PoA}}$$

Proof: The $\mathrm{PoA}(\alpha, K)$ is expressed by the following ratio

$$\mathrm{PoA}(\alpha, K) = \frac{\bar{U}_S(p^*(\alpha, K))}{\bar{U}_S(p^E(\alpha, K))}$$

First, if $(1/\mu_s) > \alpha K + [\{(1 - B_K(1))(1 - P_r(1))\}/\mu_p]$ and $\{1/(\mu_s - \lambda_s)\} > \alpha K + [\{(1 - B_K(0))(1 - P_r(0))\}/\mu_p]$, then we have $p^E = 1$. As we said earlier in Proposition 2, $p^* \geq p^E$, then $p^* = 1$ and we have

$$\mathrm{PoA}(\alpha, K) = 1$$

Otherwise, let us focus on the gap between the cost function at the equilibrium and the optimal cost function. We have for all p^*, α and K

$$\bar{U}_S(p^E) - \bar{U}_S(p^*) = -\frac{p^*(1 - B_K(p^*))(1 - P_r(p^*))}{\mu_p} - \alpha K p^* + \frac{p^*\mu_s - \lambda_s p^E(1 - p^*)}{(\mu_s - \lambda_s(1 - p^*))(\mu_s - \lambda_s(1 - p^E))}$$

It is clear that the difference between the cost function at the equilibrium and the optimal cost function is maximal when $p^E = 0$. Then the PoA is minimal when $\bar{U}_S(p^E) - \bar{U}_S(p^*)$ is maximised.

Then PoA is minimised when $p^E = 0$; we focus on the analysis of the PoA in this particular case.

Let us suppose that when $p^E = 0$, we have $[\{(1 - B_K(p^*))(1 - P_r(p^*))\}/\mu_p] + \alpha K < \{1/(\mu_s - \lambda_s)\}$. Then, we have for all p^*, α and K

$$U(p^*, p^*) < \frac{p^*}{\mu_s - \lambda_s} + \frac{1 - p^*}{\mu_s - \lambda_s(1 - p^*)}$$

Consequently, we obtain

$$U(p^*, 0) < \frac{p^*}{\mu_s - \lambda_s} + \frac{1 - p^*}{\mu_s - \lambda_s} = \frac{1}{\mu_s - \lambda_s}$$

Here, we have a contradiction. In fact $U(0, 0) = \{1/(\mu_s - \lambda_s)\} > U(p^*, 0)$, and if $p^E = 0$ is an equilibrium, then $U(0, 0) < U(p', 0)$ for all p'. Finally, we have when $p^E = 0$, $[\{(1 - B_K(p^*))(1 - P_r(p^*))\}/\mu_p] + \alpha K \geq \{1/(\mu_s - \lambda_s)\}$.

Moreover, when $p^E = 0$, we have the following expression of the PoA (see equation at the bottom of the page)

$$\mathrm{PoA}(\alpha, K) = \frac{\{(p^*(1 - B_K(p^*))(1 - P_r(p^*)))/\mu_p\} + \alpha p^* K + \{(1 - p^*)/(\mu_s - \lambda_s(1 - p^*))\}}{1/(\mu_s - \lambda_s)}$$

Thus, combining previous results, the PoA is bounded by

$$\mathrm{PoA}(\alpha, K) \geq p^* + \frac{\{(1 - p^*)/(\mu_s - \lambda_s(1 - p^*))\}}{1/(\mu_s - \lambda_s)}$$

We look for the minimum of this lower bound. Then, after some manipulations we obtain

$$\mathrm{PoA}(\alpha, K) \geq p^* + \frac{(\mu_s - \lambda_s)(1 - p^*)}{\mu_s - \lambda_s(1 - p^*)} = \frac{\mu_s - \lambda_s(1 - (p^*)^2)}{\mu_s - \lambda_s(1 - p^*)}$$

We denote the following function $F(X) = \{(\mu_s - \lambda_s(1 - X^2))/(\mu_s - \lambda_s(1 - X))\}$. We have $F'(X) = \{(\lambda_s^2 X^2 + (2\mu_s\lambda_s - 2\lambda_s^2) X + \lambda_s^2 - \lambda_s\mu_s)/((\mu_s - \lambda_s(1 - X))^2)\}$.

Then $F'(X) = 0$ when $X = \{(\lambda_s - \mu_s \pm \sqrt{\mu_s(\mu_s - \lambda_s)})/\lambda_s\}$; and moreover we have $F(0) = 1$. Then, the bound is minimised when $X = \{(\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s - \lambda_s)})/\lambda_s\}$ and

doi: 10.1049/iet-com.2010.0537

its minimum is

$$F(X) = \frac{\mu_s - \lambda_s(1 - (\{\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s - \lambda_s)}\}/\lambda_s)^2)}{\mu_s - \lambda_s(1 - \{(\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s - \lambda_s)})/\lambda_s\})}$$

Finally, for all α and K, we obtain the lower bound of the PoA

$$\mathrm{PoA}(\alpha, K) \geq 2\left(\frac{\lambda_s - \mu_s + \sqrt{\mu_s(\mu_s - \lambda_s)}}{\lambda_s}\right)$$

□

5 Numerical illustrations

In this section, we present some numerical results with different configurations of the system. We consider the following values: $\lambda_s = 0.7$, $\lambda_p = 0.6$, $\mu_p = 0.8$ and $\mu_s = 1.1$. When the number of licensed channels is small $K = 1$, this optimal probability is $p^* = 1$ and that means that every secondary mobile is sensing the licensed channels at the optimum point. When the number of licensed channels becomes bigger, for example $K = 10$, we obtain $p^* = 0.427$ (Fig. 3). Indeed, the cost of sensing becomes so large, as each secondary mobile senses all the licensed channels, the proportion of secondary mobile that are sensing the licensed channels at the optimum is decreasing.

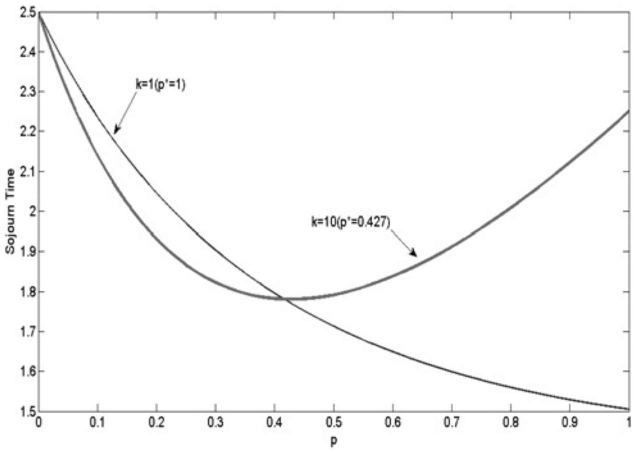

Fig. 3 *Average total cost $U_S(p)$ with $\alpha = 0.1$ and $K = 1$ or $K = 10$*

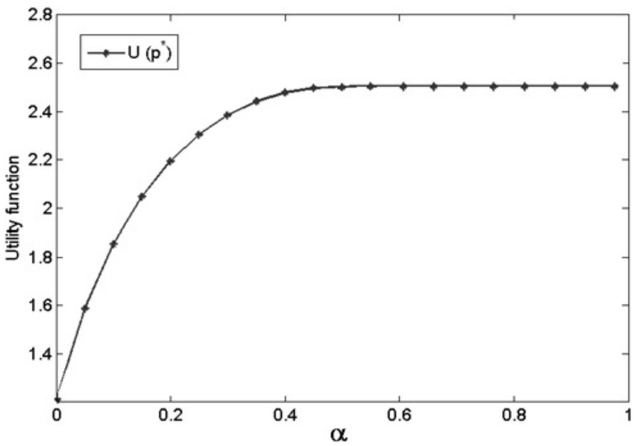

Fig. 4 *Global optimum depending on the sensing cost α*

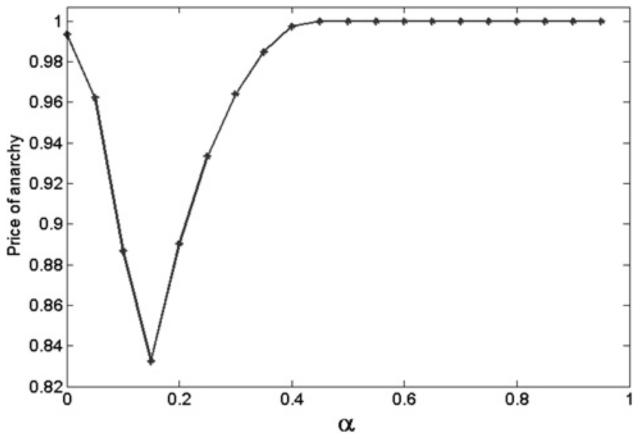

Fig. 5 *PoA depending on α*

5.1 Sensing cost

We analyse the impact of the sensing cost parameter α on the system performance with fixed number of licensed channels $K = 10$. We can validate the results of Proposition 3 with Fig. 4. We have obtained from Proposition 4 a lower bound of the PoA which is $\underline{\mathrm{PoA}} = 75.24\%$. This result is close to the minimum of the PoA that is 82.89% (Fig. 5). Then, we have obtained a good lower bound of the worst performance of the decentralised system (Fig. 6).

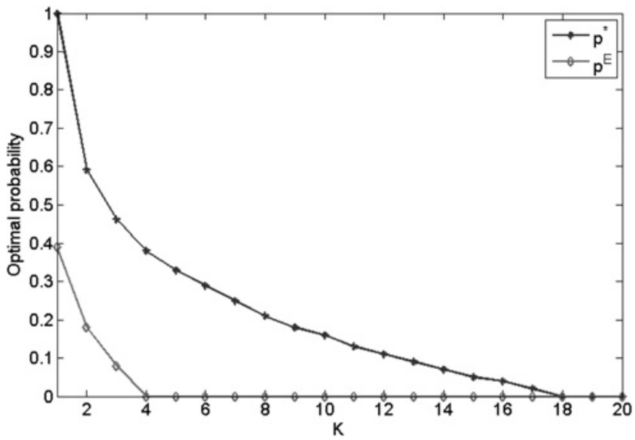

Fig. 6 *Optimal probability of sensing depending on the number of licensed channels*

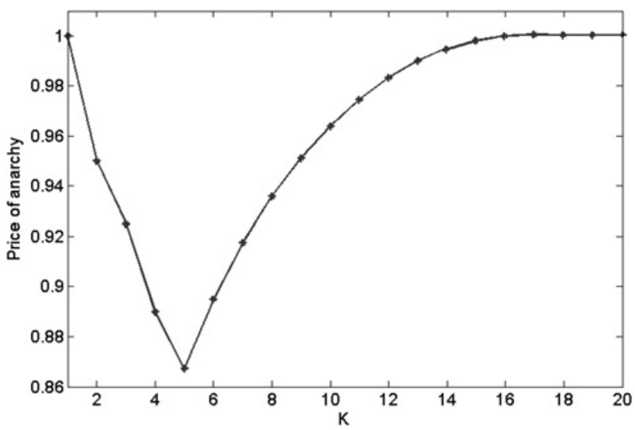

Fig. 7 *PoA with K*

doi: 10.1049/iet-com.2010.0537

5.2 Capacity

We are interested in the impact of the number of licensed channels on the performance of the opportunistic sensing mechanism of secondary mobiles. We consider the sensing cost $\alpha = 0.3$. By increasing the number of licensed channels, the blocking probability of secondary mobile will be smaller but the cost of sensing, which is linear with the number of licensed channels will increase. In Fig. 7, we observe that the minimal value of the PoA is 0.8672 which is not so far from the lower bound given in Proposition 4 which is 75.24%.

6 Conclusions and perspectives

In this paper we have defined an optimal sensing policy for opportunistic secondary mobile for those who have access to licensed channels. We have evaluated this optimal individual policy and compared it with the global optimum looking at the PoA of the system. In perspectives, we would like to propose smart sensing algorithms such that secondary mobiles will not have to sense all the licensed channels but only few of them. Those algorithms could be based on a MDP that considers on the number of licensed channels already used. In order to guarantee the QoS for PUs, we will introduce the constraint that the blocking probability for a PU must be lower than a threshold $p_{\max}$.

7 References

1 Mitola, J.: 'Cognitive radio: an integrated agent architecture for software defined radio'. PhD thesis, Royal Inst. Technol. (KTH), Stockholm, Sweden, 2000

2 Haykin, S.: 'Cognitive radio: brain-empowered wireless communications', *IEEE JSAC*, 2005, **23**, (2), pp. 201–220

3 Gao, S., Qian, L., Vaman, D.: 'Distributed energy efficient spectrum access in cognitive radio wireless ad hoc networks', *IEEE Trans. Wirel. Commun.*, 2009, **8**, (10), pp. 5202–5213

4 Bloem, M., Alpcan, T., Basar, T.: 'A Stackelberg game for power control and channel allocation in cognitive radio networks'. Proc. ValueTools, 2007

5 Park, K., Hou, J., Basar, T., Kim, H.: 'Noncooperative carrier sense game in wireless networks', *IEEE Trans. Wirel. Commun.*, 2009, **8**, (10), pp. 5280–5289

6 Zhao, Q., Tong, L., Swami, A., Chen, Y.: 'Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: a POMDP framework', *IEEE J. Sel. Areas Commun.*, 2007, **25**, (3), pp. 589–600

7 Li, X., Zhao, Q., Guan, X., Tong, L.: 'On the performance of cognitive access with periodic spectrum sensing'. Proc. ACM Workshop on Cognitive Radio Networks, 2009

8 Liu, X., Shankar, S.: 'Sensing-based opportunistic channel access', *Mobile Netw. Appl.*, 2006, **11**, (11), pp. 577–591

9 Zheng, H., Peng, C.: 'Collaboration and fairness in opportunistic spectrum access'. Proc. IEEE Int. Conf. on Communication (ICC), 2005

10 Wang, W., Liu, X.: 'List-coloring based channel allocation for opem-spectrum wireless networks'. Proc. IEEE VTC, 2005

11 Su, H., Zhang, X.: 'Cognitive radio based multi-channel MAC protocols for wireless ad hoc networks'. Proc. Globecom, 2007

12 Huang, S., Liu, X., Ding, Z.: 'Opportunistic spectrum access in cognitive radio networks'. Proc. IEEE Infocom, 2008

13 Sastry, P., Phansalkar, V., Thathachar, A.: 'Decentralized learning of Nash equilibria in multi-person stochastic games with incomplete information', *IEEE Trans. Syst. Man Cybern.*, 1994, **24**, (5), pp. 769–777

14 Akyildiz, I.F., Lee, W.-Y., Vuran, M.C., Mohanty, S.: 'NeXt generation dynamic spectrum access cognitive radio wireless networks: a survey', *Comput. Netw.*, 2006, **50**, (13), pp. 2127–2159

15 Wolff, R.: 'Poisson arrivals see time averages', *Oper. Res.*, 1982, **30**, (2), pp. 223–231

16 Cavdar, D., Yilmaz, H.B., Tugcu, T., Alagoz, F.: 'Resource planning in cognitive radio networks'. Proc. 6th Int. Conf. Symp. on Wireless Communication Systems, 2009

17 Senthilkumar, L., Sankaranarayanan, V.: 'Provisioning Erlang-B model based flow admission control for packet networks', *J. Inf. Sci. Eng.*, 2008, **24**, (5), pp. 1537–1550

18 Wiecek, P., Altman, E., Hayel, Y.: 'Stochastic state dependent population games in wireless communication', *IEEE Trans. Autom. Control*, 2011, **56**, (3), pp. 492–505

19 Wardrop, J.G.: 'Some theoretical aspects of road traffic research'. Proc. Inst. Civil Eng., Part 2, 1952, pp. 325–378

IET Commun., 2012, Vol. 6, Iss. 8, pp. 797–804
doi: 10.1049/iet-com.2010.0537