Cooperative Spectrum Sensing in Cognitive Radio, Part I: Two User Networks

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Abstract— **In cognitive radio networks, cognitive (unlicensed) users need to continuously monitor spectrum for the presence of primary (licensed) users. In this paper, we illustrate the benefits of cooperation in cognitive radio. We show that by allowing the cognitive users operating in the same band to cooperate we can reduce the detection time and thus increase the overall agility. We first consider a two-user cognitive radio network and show how the inherent asymmetry in the network can be exploited to increase the agility. We show that our cooperation scheme increases the agility of the cognitive users by as much as 35%. We then extend our cooperation scheme to multicarrier networks with two users per carrier and analyze asymptotic agility gain.**

In Part II of our paper [1], we investigate multiuser single carrier networks. We develop a decentralized cooperation protocol which ensures agility gain for arbitrarily large cognitive network population.

Index Terms— **Cognitive radio, cooperation, agility gain, detection time.**

I. Introduction

RECENTLY there has been tremendous interest in the field of *software defined radio* (SDR) and its relatively newer version *cognitive radio* (CR). SDR, which has been introduced in [2], achieves significant improvements over services offered by current wireless networks. With SDR, the software embedded in a cellular phone, for example, would define the parameters under which the phone should operate in real-time as its user moves from place to place. Today's cellular phone parameters, by contrast, are fixed in terms of frequency band and protocol. CR is even smarter than SDR. CR is designed to sense the changes in its surroundings. Thus it learns from its environment and performs functions that best serve its users. This is a very crucial feature of CR networks which allow users to operate in licensed bands without a license.

Though the above technologies promise tremendous gain, practical systems are yet to be developed that allow multiple users to share the spectrum in a smart way. In [3], a possible RF implementation architecture is discussed for front-end CR networks. Since the cognitive (unlicensed) users are utilizing the licensed band, they must detect the presence of licensed (primary) users in a very short time and must vacate the band for the primary users. One may expect this to be trivial but as shown in [4], there are fundamental limits to the detection capabilities of CR networks. In essence, at a very high level, CR networks essentially perform two operations: 1) Dynamic access of unused spectrum (also referred to as *Dynamic Spectrum Access* (DSA)). 2) Transmission on the available spectrum until the licensed user is detected (also referred to as spectrum sensing).

The problem of DSA for CR networks is extensively treated in [5]–[9]. In [5], a partially observable Markov chain formulation is used to derive optimal allocation strategies that are aimed to maximize the overall throughput. In [6] [7] utility-based selection procedures are described that allocate spectrum based on certain cost functions that are to be minimized. In [9] [8], a game-theoretic approach is adapted for decentralized channel assignment. In particular, the inherent tradeoff between information overhead and network throughput in CR networks is studied in detail in [8].

The problem of spectrum sensing has been discussed in [10]–[13]. In [10], the RF receivers of sensor node detectors are allowed to exploit the local oscillator leakage power to sense and locate the primary users. The detectors then convey the channel usage pattern to the cognitive radios. In [12], a neural network approach is proposed for cyclic spectral analysis to detect signals in unknown bands. In [11], power- and frequency-based sensing techniques are proposed for primary user detection in CR networks employing OFDM technology. This is challenging in itself, since the CRs have to locate contiguous bands that are left unused by the primary users. In [13], a collaborative spectrum sensing approach is proposed to detect the primary user. It is shown that information exchange between cognitive radios enhance the probability of detection of the primary user.

In this paper, we propose cooperative spectrum sensing techniques for cognitive radio networks. Cooperative networks achieve diversity gain by allowing the users to cooperate [14] [15]. In [15], a possible implementation of a cooperative protocol in a CDMA system is discussed. Cooperative schemes with orthogonal transmission in a TDMA system have been recently proposed in [16] and [17]. Broadly speaking, cooperative protocols are of two kinds [16]: 1) *Amplify-and-forward* (AF) and 2) *Decode-and-forward* (DF). It has been shown in [16] that two user single hop networks in which one of the user acts as a relay for the other, result in lower outage probabilities. In particular, it is shown that the (AF) protocol [16], in which the relay transmits the signal obtained from the transmitter without any processing, achieves full diversity. In this paper, we study the effect of the AF

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cooperation protocol on the spectrum sensing capabilities of cognitive radio network.

A short version of the results in this paper can be found in [18] and [19]. Our work is different from [13] in the sense that [13] do not consider cooperative techniques like AF and DF protocols. In [13], hard decision about the presence of the primary user from each cognitive user is pooled together and a majority logic is used to determine the presence of the primary user. In contrast, we exploit the spatial diversity inherent in a multiuser scenario to achieve better performance of cognitive networks.

The organization of the paper is as follows: In Section II, we formulate the detection problem for a simple two user cognitive network and describe the detector we use for detecting the presence of the primary user. We show that the inherent asymmetry of the network can lead to faster detection with cooperation. We then study the effect of power constraint on the performance of cooperative detection schemes and describe important properties of such networks. In Section III, we analyze agility gain for a two user network and show improvement in agility. In Section IV, we consider multicarrier networks with utmost two users per carrier and study cooperation schemes employing varying degrees of cooperation. Finally, in Section V, we present our conclusion.

II. Detection Problem

In this section, we describe the channel model that will be used throughout the paper, formulate the primary user detection problem, and propose a practical cooperation scheme to improve the agility of a simple two-user cognitive network.

A. Channel Model

In this paper, we assume that all channels experience Rayleigh fading. Moreover, channels corresponding to different cognitive users are assumed to be independent. If a signal x is sent, the received signal y is given by

$$y = fx + w,$$

where the fading coefficient f and the additive noise w are modelled as independent complex Gaussian random variables. Unless otherwise mentioned, the noise in this paper is assumed to be of zero mean and unit variance. Throughout the paper, we assume that there is a centralized controller (capable of both receiving and sending) with which all the cognitive users communicate. We also assume that each user has access to its channel state information. This is facilitated by allowing pilot symbols to be transmitted at regular intervals.

B. Constrained Cooperation Scheme

An important requirement of a cognitive radio architecture is to detect the presence of primary users as quickly as possible. For this reason cognitive users should continuously sense the spectrum. Consider a network with two cognitive radio users U_1 and U_2 operating in a fixed TDMA mode for sending data to some common receiver as shown in Figure 1. Suppose that a primary (licensed) user starts using the band. Then the two cognitive users need to vacate the band as soon

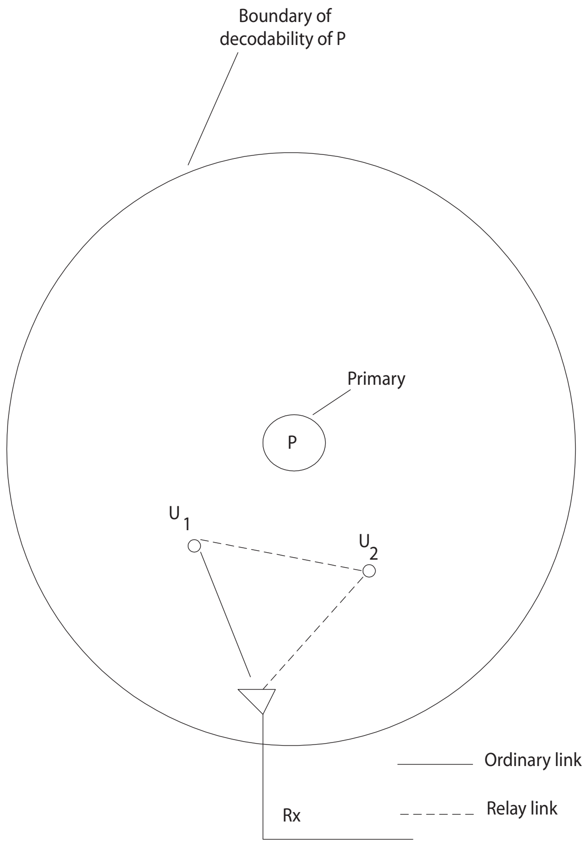

Fig. 1. Cooperation in cognitive network.

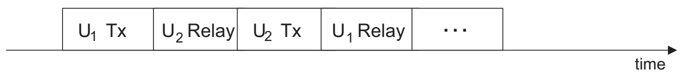

Fig. 2. Relay protocol used.

as possible to make way for the primary user. However, the detection time becomes significant if one of the users, say U_1, is far away from the primary user and the signal received from the primary user is so weak that the cognitive user U_1 takes a long time to sense its presence. We show that cooperation between the cognitive users can reduce the detection time of the "weaker" user thereby improving the "agility" of the overall network. We shall define these terms more precisely in Section III.

Throughout the paper, we allow the cognitive users, U_1 and U_2, to cooperate, with U_2 acting as a relay for U_1. Figure 1 describes a scenario where two cognitive users U_1 and U_2 are engaged in transmitting data to a common receiver in a particular frequency band. Slotted transmission is used wherein U_1 and U_2 transmit in successive slots following the AF protocol [16] as shown in Figure 2. Accordingly in time slot T_1, U_1 transmits and U_2 listens. In time slot T_2, U_2 relays the information received in the previous slot. Unknown to both these users, there is a primary user who has higher priority in occupying the band. It is crucial that presence of this primary user be detected as soon as possible. In time slot T_1, the signal received by U_2 from U_1 is given by,

$$y_2 = \theta h_{p2} + a h_{12} + w_2, \tag{1}$$

where h_{pi} denotes the instantaneous channel gain between the primary user and U_i, h_{12} denotes the instantaneous channel gain between U_1 and U_2, and w_2 denotes the additive Gaussian noise. We assume that h_{p2}, h_{12}, and w_2 are zero-mean complex Gaussian random variables which are pairwise independent. Also, we assume that the channels are reciprocal, i.e., $h_{12} = h_{21}$. In (1), a denotes the signal sent from U_1 and θ denotes the primary user indicator; $\theta = 1$ implies presence of the primary user and $\theta = 0$ implies its absence. If the transmit power constraint of U_1 is P then,

$$E\{|ah_{12}|^2\} = PG_{12},$$

where $G_{12} = E\{|h_{12}|^2\}$ refers to the channel gain between the users U_1 and U_2. Since h_{p2}, h_{12}, and w_2 are assumed independent, we have from (1) that

$$E\{|y_2|^2\} = \theta^2 P_2 + PG_{12} + 1,$$

where $P_i = E\{|h_{pi}|^2\}$ refers to the received signal power at U_i from the primary user. In time slot T_2, the relay user, U_2, relays the message from U_1 to a common cognitive receiver. The relay user has a maximum power constraint $\tilde{P}$. Hence it measures the average received signal power [20] and scales it appropriately so that its power constraint $\tilde{P}$ is satisfied. In time slot T_2, when U_2 is relaying the message of U_1 to the receiver, U_1 also listens to its own message. The signal received by U_1 from U_2 is given by

$$\begin{aligned} y_1 &= \sqrt{\beta_1} y_2 h_{12} + \theta h_{p1} + w_1 \\ &= \sqrt{\beta_1} h_{12}(\theta h_{p2} + a h_{12} + w_2) + \theta h_{p1} + w_1, \quad (2) \end{aligned}$$

where h_{p1} is the instantaneous channel gain between the primary user and U_1, w_1 is additive Gaussian noise, and β_1 is a scaling factor [16] used by U_2 to relay the information to the common receiver. In fact, β_1 is given by [16] [20]

$$\beta_1 = \frac{\tilde{P}}{E\{|y_2|^2\}} = \frac{\tilde{P}}{\theta^2 P_2 + PG_{12} + 1}.$$

After the message component is cancelled, the user U_1 is left with the signal

$$Y = \theta H + W, \quad (3)$$

where $H = h_{p1} + \sqrt{\beta_1} h_{12} h_{p2}$ and $W = w_1 + \sqrt{\beta_1} h_{12} w_2$. The detection problem can be now stated as:
Given the observation

$$Y = \theta H + W, \quad (4)$$

the detector decides on

$$\mathcal{H}_1 : \theta = 1$$

or

$$\mathcal{H}_0 : \theta = 0.$$

This is a very standard detection problem for which there are many choices of detector available in the literature. We describe the detector used in this paper below.

C. Energy Detector

In this paper, we utilize the *energy detector* (ED) [21] to show advantage of the proposed cooperation scheme. The reasons for choosing ED are twofold: (1) We want to show the effect of cooperation in cognitive networks. Hence the choice of detector is not critical. (2) We model the signal as a random variable with known power. Hence ED is optimal [21]. It can be shown that given $h_{12} = h_{21}$, the random variables H and W in (4) are complex Gaussian distributed with zero-mean and variances,

$$\sigma_H^2 = P_1 + \beta P_2 h \quad (5)$$

and

$$\sigma_W^2 = 1 + \beta h, \quad (6)$$

respectively, where

$$h = \frac{|h_{12}|^2}{E\{|h_{12}|^2\}} = \frac{|h_{12}|^2}{G_{12}}$$

and

$$\beta = \frac{\tilde{P} G_{12}}{\theta^2 P_2 + PG_{12} + 1}. \quad (7)$$

Here we assume that U_1 has access to the channel state h_{12}. This is facilitated by allowing pilot symbols to be transmitted at regular intervals. Since h_{12} is complex Gaussian, it is easily seen that h has the *probability density function* (pdf) given by

$$f(h) = \begin{cases} e^{-h} & h > 0 \\ 0 & h \leq 0. \end{cases}$$

The ED forms the statistics

$$T(Y) = |Y|^2$$

and compares with a threshold λ which is determined by a prespecified probability of false alarm α.

Define

$$\varphi(t; a, b) = \int_0^{\infty} e^{-h - \frac{t}{a+bh}} \, dh \quad (8)$$

for positive t, a, and b. Let $F_i(t)$ denote the *cumulative density function* (cdf) of the random variable $T(Y)$ under hypothesis $\mathcal{H}_i, i = 0, 1$. Since Y given h is complex Gaussian, it is obvious that $T(Y)$ given h is exponential. Also, from (6), we get

$$E\{T(Y)|\mathcal{H}_0, h\} = E\{|W|^2 | h, \theta = 0\} = 1 + \frac{PG_{12}}{\tilde{P}G_{12} + 1} h.$$

For $\mathcal{H}_0$ ($\theta = 0$),

$$\begin{aligned} F_0(t) &= P(T(Y) > t | \mathcal{H}_0) \\ &= \int_0^{\infty} P(T(Y) > t | \mathcal{H}_0, h) f(h) \, dh \\ &= \varphi\left(t; 1, \frac{PG_{12}}{\tilde{P}G_{12} + 1}\right). \end{aligned}$$

Similarly, it can be shown that

$$F_1(t) = \varphi\left(t; P_1 + 1, \beta(P_2 + 1)\right),$$

where β is given by (7). For a given probability of false alarm α, we need to find the threshold λ such that

$$\varphi\left(\lambda; 1, \frac{PG_{12}}{\tilde{P}G_{12} + 1}\right) = \alpha. \quad (9)$$

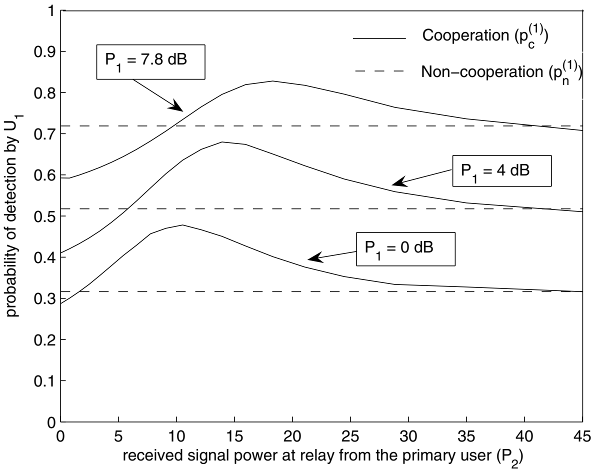

Fig. 3. Cooperation improves probability of detection (Constrained scheme, $\alpha = 0.1$).

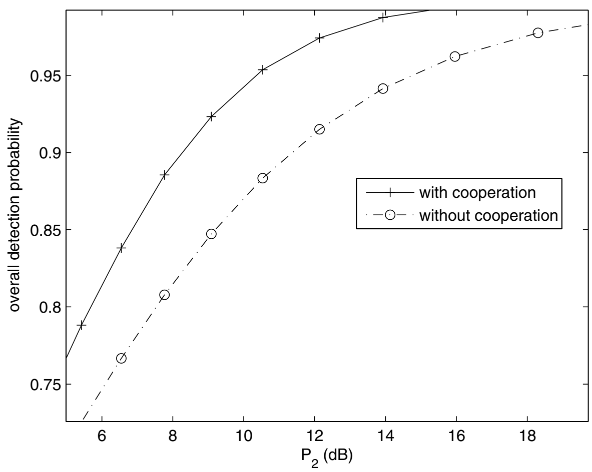

Fig. 4. Overall detection probability (Constrained scheme), $P_1 = 0$ dB, $\alpha = 0.1$.

In fact, λ can be uniquely determined since φ in (8) is strictly decreasing in t. Consequently, the probability of detection by U_1, with cooperation from U_2, is found to be

$$p_c^{(1)} = \varphi\left(\lambda; P_1 + 1, \beta(P_2 + 1)\right). \tag{10}$$

When there is no cooperation between U_1 and U_2, β_1 in (2) is zero. In this case, let $p_n^{(1)}$ and $p_n^{(2)}$ denote the respective detection probabilities. Working along similar lines, for the system model of our paper, it can be shown that

$$p_n^{(1)} = \alpha^{\frac{1}{P_1+1}} \tag{11}$$

and

$$p_n^{(2)} = \alpha^{\frac{1}{P_2+1}}. \tag{12}$$

Setting $\alpha = 0.1$, in Figure 3, we have plotted $p_n^{(1)}$ and $p_c^{(1)}$ as a function of P_2 for $P = \tilde{P} = 0$ dB [1]. We have plotted $p_n^{(1)}$ and $p_c^{(1)}$ for three different values of P_1: $P_1 = 0$ dB, 4 dB, and 7.8 dB. For each value of P_1, we note that the constrained cooperation scheme is beneficial ($p_c^{(1)} > p_n^{(1)}$) for a certain range of P_2. Also, the maximum achievable probability gain is dependent on the received signal power of the cognitive user U_1 from the primary user.

We now look at the overall detection probability of CR networks; i.e., the probability that the primary user is detected by U_1 or U_2. Such information is necessary to prove that the considered parameter setup is reasonable in the investigated scenario with licensed users. When the two users detect the primary user independently, it can be easily shown that the overall detection probability of the two user cognitive radio network is given by

$$p_n^{(1)} + p_n^{(2)} - p_n^{(1)} p_n^{(2)}$$

where $p_n^{(1)}$ and $p_n^{(2)}$ are given by (11) and (12), respectively. Similarly, when the users employ the cooperation scheme described above, the overall detection probability is given by

$$p_c^{(1)} + p_n^{(2)} - p_c^{(1)} p_n^{(2)}$$

where $p_c^{(1)}$ is given by (10). In Figure 4, we have evaluated and plotted the above expressions as a function of P_2 for $P_1 = 0$ dB assuming $\alpha = 0.1$. As can be seen from the figure, cooperation between cognitive users improves the overall probability of detection reasonably. It is important to note that even a reasonable increase in probability of detection is quite important since the cognitive radios are designed to continuously monitor spectrum and detect the presence of the primary users.

[1]When we mean $P = 0$ dB, we actually mean the SNR $\frac{P}{\sigma_w^2}$, where σ_w^2 refers to the additive Gaussian noise power. However, we assume $\sigma_w^2 = 1$ as already mentioned in the beginning of the paper.

D. Unconstrained Cooperation Scheme

In this section, we modify a crucial assumption we had imposed on the relay user in the previous section. As before, let h_{12} denote the instantaneous channel gain between U_1 and U_2 and $E\{|h_{12}|^2\} = G_{12}$ denote the average channel gain (see Figure 1). It is obvious from (2) that though the average channel gain between the users is G_{12}, the channel gain as seen by U_1 would be $\beta_1 G_{12}$ where β_1 is the scaling factor used to satisfy power constraint. In this section, we allow the relay user to adjust the scaling factor β_1 in (2) in such a way that the average channel gain as seen U_1 is always a constant irrespective of the channel gain between U_2 and the primary user. Note that this indirectly implies that there is no constraint on the transmit power of the relay user U_2. We choose,

$$\beta_1 = \frac{1}{E\{|h_{12}|^2\}} = \frac{1}{G_{12}}$$

to maintain the average channel gain as seen by U_1 at 0 dB. This is chosen purely for convenience. Any other constant value of β_1 would yield similar results. The detection problem for this unconstrained cooperation scheme is identical to that of the constrained scheme and is given by (4). We specifically discuss the unconstrained cooperation scheme because we wish to study the effect of channel gain between the cognitive users on the performance of cooperative detection schemes.

Working along similar lines as in Section II.C, we can show that the probability of detection of the primary user by U_1, with U_2 acting as the relay, is given by

$$p_c^{(1)} = \varphi\left(\lambda; P_1 + 1, P_2 + 1\right), \tag{13}$$

where λ is uniquely determined by

$$\alpha = \varphi\left(\lambda; 1, 1\right). \tag{14}$$

If $p_c^{(2)}$ denotes the probability of detection for U_2 with cooperation from U_1, it can be shown that

$$p_c^{(2)} = \varphi(\lambda; P_2 + 1, P_1 + 1),$$

where λ satisfies (14). As before, let the detection probabilities of U_1 and U_2 without any cooperation be given by (11) and (12), respectively. We have proved the following proposition in Appendix I:

Proposition 1:

(a) For all $P_2 > P_1$, $p_n^{(2)} > p_c^{(2)}$ and
(b) There exists P_2^* s.t. for all $P_2 > P_2^*$, $p_c^{(1)} > p_n^{(1)}$. ■

The message from Proposition 1 is clear. The first part of the proposition states that since $p_n^{(2)} > p_c^{(2)}$, we allow only U_2 to help U_1 and not vice-versa, since U_1 receives lesser signal power from the primary user than U_2. The second part of the proposition states that for U_2 to be helpful as a relay to U_1, it should receive sufficiently large signal power from the primary user than U_1. It is important to note that the constrained cooperation scheme discussed in Section II.C behaves differently in this aspect from the unconstrained cooperation scheme. In the constrained cooperation scheme, even if the relay user U_2 receives sufficiently large signal power from the primary user, we do not gain much in terms of agility if the channel gain between U_1 and U_2 is very weak. In contrast, in the unconstrained cooperation scheme, the channel gain between the users is immaterial in determining the performance of the network. This is because, U_1 always "sees" an effective constant channel gain irrespective of the position of U_2.

Compared to the non-cooperative case, the average SNR is increased by cooperation provided $P_2 > P_1$. From (5) and (6), the instantaneous SNR is given by

$$E\{\gamma_c|h\} = \frac{P_1 + P_2 h}{1 + h}.$$

Hence the average SNR at U_1 in case of cooperation is given by

$$\begin{aligned}\overline{\gamma}_c &= \int_0^\infty E\{\gamma_c|h\} f(h)\, dh \\ &= \int_0^\infty \frac{P_1 + P_2 h}{1 + h} e^{-h}\, dh = P_1 \int_0^\infty \frac{1 + \frac{P_2}{P_1} h}{1 + h} e^{-h}\, dh.\end{aligned}$$

For the non-cooperative case the average SNR at U_1 is given by

$$\overline{\gamma}_{nc} = P_1.$$

Thus the SNR gain is,

$$\overline{\gamma} = \frac{\overline{\gamma}_c}{\overline{\gamma}_{nc}} = \int_0^\infty \frac{1 + \frac{P_2}{P_1} h}{1 + h} e^{-h}\, dh,$$

which is greater than one if and only if $P_2 > P_1$. It has been shown in [22] [23] that for the detection of a Gaussian signal in Gaussian noise it requires on an average $O(\frac{1}{SNR^2})$ samples. Thus higher the SNR, lower the number of samples needed to detect the primary user.

III. Agility of the Two User Cognitive Radio Network

So far, we have devoted ourselves to improvement in detection probabilities through cooperation. The final goal, however, is to reduce the overall detection time. In general, detection time and detection probability do not follow a strictly inverse relationship for complicated networks. To show the effect of cooperation on the overall detection time, we shall define two types of protocols employing different degrees of cooperation. We assume that there is a central controller (capable of both receiving and sending) with which all the cognitive users communicate.

1) *Non Cooperative (NC) Protocol*: All the users detect the primary user independently. However the first user to detect the presence of the primary user informs the other users through the central controller.

2) *Totally Cooperative (TC) Protocol*: This employs the cooperation scheme described in Section II. Thus two users operating in the same carrier, if placed sufficiently near to each other, cooperate to find the presence of the primary user. The first user to detect the presence of the primary user informs the others through the central controller.

As before, we assume that there are two users U_1 and U_2 cooperating to find the presence of the primary user, with U_2 acting as a (potential) relay for U_1. Let τ_n be the number of slots taken by user U_1 to detect the presence of the primary user under the NC protocol. This detection time τ_n can be modelled as a geometric random variable, i.e.,

$$\Pr\{\tau_n = k\} = (1 - p_n^{(1)})^{k-1} p_n^{(1)},$$

where $p_n^{(1)}$ denotes the probability of detection by user U_1 in a single slot under the NC protocol and is given by (11). Let T_n and T_c denote the detection time taken by the two user network under the NC and TC protocol, respectively. We have proved the following proposition in Appendix II:

Proposition 2:

$$T_n = \frac{2 - \frac{p_n^{(1)} + p_n^{(2)}}{2}}{p_n^{(1)} + p_n^{(2)} - p_n^{(1)} p_n^{(2)}},$$

and

$$T_c = \frac{2 - \frac{p_c^{(1)} + p_n^{(2)}}{2}}{p_c^{(1)} + p_n^{(2)} - p_c^{(1)} p_n^{(2)}}. \blacksquare$$

We define the agility gain of the TC protocol over the NC protocol when there are two users as

$$\mu_{n/c}(2) \triangleq \frac{T_n}{T_c}.$$

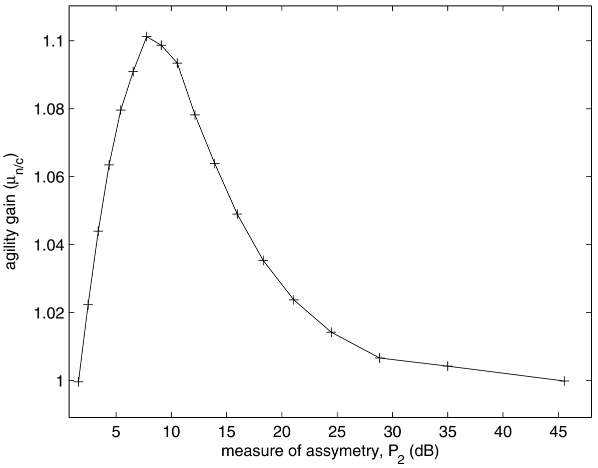

Fig. 5. Agility gain in two user networks under the constrained scheme, $P_1 = 0$ dB, $\alpha = 0.1$.

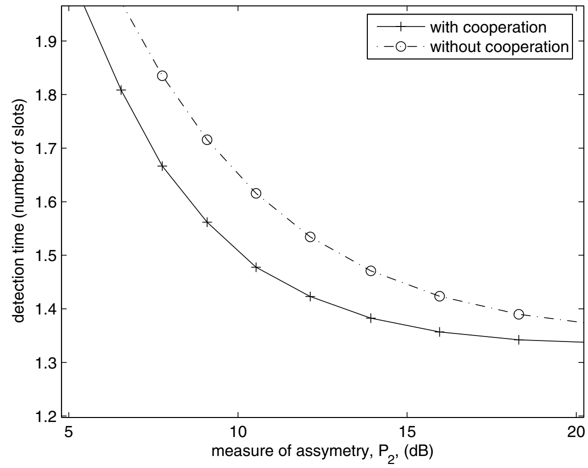

Fig. 6. Detection time (number of slots) (Constrained scheme), $P_1 = 0$ dB, $\alpha = 0.1$.

Note that the agility gain is a function of P_1 and P_2. We will discuss the agility under the constrained and the unconstrained schemes separately.

A. Agility under the constrained scheme

In Figure 5, we have plotted the agility gain $\mu_{n/c}$ under the constrained cooperation scheme as a function of the network asymmetry P_2 for $\alpha = 0.1$ and $P_1 = 0$ dB. We find that for this scenario, the maximum agility gain is about 11%. This increase in agility is useful in the long term since the cognitive radios need to continuously monitor spectrum for the presence of the primary user. Further, for completeness sake, in Figure 6 we also plot the actual time taken in terms of number of slots to detect the primary user. This gives the measure of the time saved in detecting the primary user through cooperation. As can be seen from the figure, the received signal power from the primary user at the relay user is important in determining the actual savings either in terms of agility or detection time.

We now consider agility under the unconstrained scheme where the relay user has sufficiently large enough power to transmit the received message. We find that the effect of increasing the relay transmit power results in agility gain of as much as 35%.

B. Agility under the Unconstrained scheme

In the unconstrained cooperation scheme, the gain between the users U_1 and U_2 is kept fixed irrespective of the value of P_1 or P_2. We then study how the performance of the system varies with the false alarm probability α. Setting $P_1 = 0$ dB, in Figure 7, we have plotted the agility gain $\mu_{n/c}(2)$ of a two user asymmetric network as a function of the asymmetry P_2 for different values of false alarm probability α. As the network becomes more and more asymmetric, agility gain increases. From the figure, we note that for $\alpha = 0.1$ as much as 35% increase in agility is achievable. Hence if one of the cognitive users receives a strong signal the primary user, then we are guaranteed fast detection even if the other user receives a weak signal.

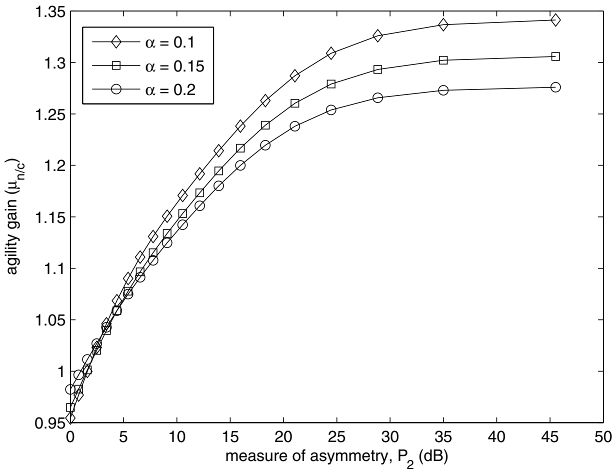

Fig. 7. Agility gain in two user networks under the unconstrained scheme.

We would like to know what is the maximum achievable agility gain as the relay user U_2 is taken arbitrarily close to the primary user. The following proposition, proved in Appendix III, indicates the maximum achievable agility gain by a two-user cognitive network under the unconstrained cooperation scheme:

Proposition 3: Let two users U_1 and U_2 operate under the unconstrained cooperation scheme with received signal power due to the primary user at user U_1 chosen to be $P_1 = 0$ dB. As the received signal power at U_2 due to the primary user $P_2 \to \infty$, we have

$$\mu_{n/c}^{\infty}(2) = \lim_{P_2 \to \infty} \mu_{n/c}(2) = \frac{3 - \sqrt{\alpha}}{2}. \blacksquare$$

Since $\alpha < 1$, we find that $\mu_{n/c}^{\infty}$ is greater than one. This implies asymptotic agility gain through cooperation.

IV. Multicarrier Cognitive Network

In this section we assume that there are $2m$ cognitive users. The total bandwidth B is equally divided into m sub-bands each of bandwidth $\Delta_m = \frac{B}{m}$. There are two cognitive users working on each sub-band following the cooperation protocol described in Section II. It must be noted that the primary user, if present, uses the whole bandwidth B. If P_1 denotes the signal power received from the primary user at cognitive user U_1 when it uses the whole band, the signal power received at U_1 now is scaled by a factor of m, and is given by $P_1' = \frac{P_1}{m}$. The noise power at U_1 is similarly scaled by a factor of m. For example, suppose that users U_1 and U_2 form a link in a particular sub-band with U_2 acting as a relay for U_1 under the unconstrained cooperation scheme described in Section II.D. The detection problem for that sub-band is identical to (4) except that

$$\sigma_H^2 = \frac{1}{m}(P_1 + P_2 h)$$

and

$$\sigma_W^2 = \frac{1}{m}(1 + h)$$

where, as before, h denotes the scaled channel gain between the pair of users. Thus the detection problem for multicarrier networks is similar to that of single carrier networks except for a scaling factor of m. Hence for a given probability of false alarm α, we need to find the threshold λ_m such that

$$\varphi\left(\lambda_m; \frac{1}{m}, \frac{1}{m}\right) = \alpha,$$

where $\varphi(.)$ is given by (8). It can be easily shown that $\lambda_m = \frac{\lambda}{m}$ where λ is given by (14). Therefore, the probability of detection by U_1 with cooperation from U_2 is found to be

$$p_c^{(1)} = \varphi\left(\lambda_m; \frac{P_1+1}{m}, \frac{P_2+1}{m}\right) = \varphi(\lambda; P_1+1, P_2+1).$$

It follows that the detection probability for a particular user is the same and independent of the bandwidth the user is occupying.

It can be seen that the probability that the cognitive users in a sub-band detect the primary user under the TC and the NC protocol are given, respectively, by

$$p = p_c^{(1)} + p_n^{(2)} - p_c^{(1)} p_n^{(2)}$$

and

$$p' = p_n^{(1)} + p_n^{(2)} - p_n^{(1)} p_n^{(2)}.$$

Since U_2 acts as a relay for U_1 and not vice-versa, it is obvious from the discussion in Section III that $p > p'$. Note that there are m sub-bands occupying the band. For $k = 1, 2, ..., m$, let p_k and $p_k' (< p_k)$ denote the probability of detection of the k^{th} sub-band to detect the presence of the primary user with and without cooperation, respectively. Here we assume that users in one sub-band cannot cooperate with the users in other sub-bands. Cooperation is allowed only within a sub-band. As before we shall consider TC and NC protocols separately. We have proved the following proposition in Appendix IV:

Proposition 4: The average number of slots taken for detection under the TC and NC protocol are given, respectively, by

$$T_c(m) = \frac{1}{1 - \prod_{k=1}^{m}(1 - p_k)}$$

and

$$T_n(m) = \frac{1}{1 - \prod_{k=1}^{m}(1 - p_k')}. \blacksquare$$

We now wish to know whether there is any gain in agility due to cooperation. Defining the agility gain to be

$$\mu_{n/c}(m) = \frac{T_n(m)}{T_c(m)},$$

we have proved the following in Appendix V:

Proposition 5: If there exists K with $1 \le K \le m$ and $p_K > p_K'$, then

$$\mu_{n/c}(m) > 1.$$

As $m \to \infty$,

$$\lim_{m\to\infty} \mu_{n/c}(m) = 1. \blacksquare$$

Thus, even if there is one sub-band which has a high detection probability, it follows that the overall detection time for the network is reduced considerably. In essence, for all finite cognitive user population we have agility gain, though, asymptotically, there is little difference between NC and TC protocols. Note also that this setup is different from the one we are about to discuss in Part II of our paper [1], where the m users are allowed to operate in a single carrier. Thus the results in this paper are very different from the results in [1].

V. Conclusion

In this paper, we have shown the benefits of cooperation in increasing the agility of cognitive radio networks. We have first considered a simple two user cooperative cognitive network and showed improvement in agility by exploiting the inherent asymmetry. We have analyzed two schemes employing different degrees of cooperation: (1) *non-cooperative* (NC), where each user detects the primary user independently, but the first user to detect the primary user informs the other cognitive users through a central controller and (2) *totally cooperative* (TC), where the users follow the AF cooperation protocol to reduce the detection time. We have showed that cooperation between cognitive nodes increases the overall agility of the network. We have also studied the effect of power constraint on cooperation schemes and some important properties of such networks. Furthermore, we have extended our cooperation scheme to multicarrier networks with utmost two users per carrier and have derived expressions for agility gain.

In Part II of our paper [1], we extend our cooperation scheme to multiuser single carrier networks. We consider a practical scenario where the cognitive users are randomly distributed. We use the results from this paper, to develop a decentralized cooperation protocol which ensures agility gain for arbitrarily large cognitive population.

Appendix I: Proof of Proposition 1

To prove Proposition 1, we need the following fact.

Fact 1: Let the function $\varphi(.;.,.)$ be as defined in (8). For any $t > 0$ and $\beta > 0$,

$$\lim_{\alpha\to\infty} \varphi(t; \alpha, \beta) = \lim_{\alpha\to\infty} \varphi(t; \beta, \alpha) = 1.$$

Proof: From (8) it follows that

$$\varphi(t;\alpha,\beta) = \int_0^\infty e^{-h} e^{-\frac{t}{\alpha+\beta h}}\, dh \le \int_0^\infty e^{-h}\, dh = 1.$$

Using the fact that $1 - e^{-x} \le x$ for all $x \ge 0$ we get that

$$\begin{aligned} 0 &\le 1 - \varphi(t;\alpha,\beta) \\ &= \int_0^\infty e^{-h}(1 - e^{-\frac{t}{\alpha+\beta h}})\, dh \\ &\le \int_0^\infty e^{-h} \frac{t}{\alpha+\beta h}\, dh \\ &= \int_0^1 e^{-h} \frac{t}{\alpha+\beta h}\, dh + \int_1^\infty e^{-h} \frac{t}{\alpha+\beta h}\, dh \\ &\le \int_0^1 \frac{t}{\alpha+\beta h}\, dh + \int_1^\infty e^{-h} \frac{t}{\alpha+\beta}\, dh \\ &= t\frac{\ln(1+\frac{\beta}{\alpha})}{\beta} + \frac{te^{-1}}{\alpha+\beta} \to 0, \end{aligned}$$

either as $\alpha \to \infty$ or $\beta \to \infty$. ■

Proof (of Proposition 1): We prove (a) first. From (13) we have that

$$p_c^{(2)} = \varphi(\lambda; P_2+1, P_1+1) = \int_0^\infty e^{-h} e^{-\frac{\lambda}{a+bh}}\, dh$$

and

$$p_n^{(2)} = \alpha^{\frac{1}{a}},$$

where $a = P_2 + 1$, $b = P_1 + 1$, and λ satisfies

$$\alpha = \int_0^\infty e^{-h} e^{-\frac{\lambda}{1+h}}\, dh.$$

It should be noted that since $P_2 > P_1$,

$$\int_0^\infty e^{-h} e^{-\frac{\lambda}{1+\frac{b}{a}h}}\, dh \le \int_0^\infty e^{-h} e^{-\frac{\lambda}{1+h}}\, dh = \alpha. \tag{I.1}$$

Let a' be such that $\frac{1}{a} + \frac{1}{a'} = 1$. We then have

$$\begin{aligned} p_c^{(2)} &= \int_0^\infty e^{-h} e^{-\frac{\lambda}{a+bh}}\, dh \\ &= \int_0^\infty \left(e^{-\frac{h}{a}} e^{-\frac{\lambda}{a+bh}}\right)(e^{-\frac{h}{a'}})\, dh && \text{(I.2)} \\ &\le \left(\int_0^\infty e^{-h} e^{-\frac{a\lambda}{a+bh}}\, dh\right)^{\frac{1}{a}} \left(\int_0^\infty e^{-h}\, dh\right)^{\frac{1}{a'}} && \text{(I.3)} \\ &\le \alpha^{\frac{1}{a}} = p_n^{(2)}, && \text{(I.4)} \end{aligned}$$

where (I.3) follows from (I.2) and Hölder's inequality ([24], pp. 128) and (I.4) follows from (I.3) and (I.1).

To prove (b), we note from Fact 1 that

$$p_c^{(1)} = \varphi(\lambda; P_1+1, P_2+1) \to 1$$

as $P_2 \to \infty$. Thus $\exists\ P_2^* > 0$ such that $p_c^{(1)} > p_n^{(1)}$ if $P_2 > P_2^*$. ■

Appendix II: Proof of Proposition 2

To derive the detection time under TC protocol, we shall assume that U_1 transmits first followed by U_2 and so on. Since the primary user location is known to both the cognitive users, U_2 acts as a relay for U_1 and not vice-versa. Let $p_c^{(1)}, p_n^{(1)}$, and $p_n^{(2)}$ be given by (10), (11), and (12) respectively. Under the TC protocol, it follows that the individual detection probabilities of U_1 and U_2 are, respectively, $\tilde{p} = p_c^{(1)} = 1 - \tilde{q}$ and $p' = p_n^{(2)} = 1 - q'$. It can be seen that the time for detection is given by

$$\begin{aligned} T_1 &= \tilde{p} + 2\tilde{q}p' + 3\tilde{q}q'\tilde{p} + 4\tilde{q}q'\tilde{q}p' + ...2n\tilde{q}^n {q'}^{n-1} p' + ... \\ &= \tilde{p}(1 + 3\tilde{q}q' + 5\tilde{q}^2{q'}^2 + ... + (2n+1)\tilde{q}^n {q'}^n + ...) \\ &+ 2\tilde{q}p'(1 + 2\tilde{q}q' + 4\tilde{q}^2{q'}^2 + ... + 2n\tilde{q}^n{q'}^n + ...). \\ &= \frac{2-\tilde{p}}{\tilde{p} + p' - \tilde{p}p'}. \end{aligned}$$

If on the other hand, U_2 is allowed to transmit first, the average detection time T_2 is given by

$$T_2 = \frac{2-p'}{\tilde{p} + p' - \tilde{p}p'}.$$

Hence the overall average detection time is given by

$$T_c = \frac{T_1 + T_2}{2} = \frac{2 - \frac{\tilde{p}+p'}{2}}{\tilde{p} + p' - \tilde{p}p'}.$$

The NC protocol is identical to the TC protocol except that here neither U_1 nor U_2 act as a relay for one another. Thus the probabilities of detection of U_1 and U_2 are given, respectively, by $p = p_n^{(1)}$ and $p' = p_n^{(2)}$. Following a similar derivation as in the case of TC protocol, we get

$$T_n = \frac{2 - \frac{p+p'}{2}}{p + p' - pp'}. \ ■$$

Appendix III: Proof of Proposition 3

From Fact 1, Appendix I, and (10) it follows that $p_c^{(1)} \to 1$ as $P_2 \to \infty$. From (12) it follows that $p_n^{(2)} \to 1$, as $P_2 \to \infty$. Thus, from Proposition 2, we get that $T_n \to \frac{3-\sqrt{\alpha}}{2}$ and $T_c \to 1$, as $P_2 \to \infty$. Therefore $\mu_{n/c} = \frac{T_n}{T_c} \to \frac{3-\sqrt{\alpha}}{2}$, as $P_2 \to \infty$. ■

Appendix IV: Proof of Proposition 4

If X is a geometric random variable with

$$\Pr\{X = k\} = (1-p)^{k-1}p,$$

we shall say $X \sim \mathcal{G}(p)$.

Let the random variable X_k denote the time taken for detection for the k^{th} sub-band for $1 \leq k \leq m$ under the TC protocol. In the TC protocol, once the primary user is detected by one sub-band, all the other sub-bands are informed through the common base-station. Hence the average time taken for detection is given by

$$T_c(m) = E\{\min(X_1, X_2, ...X_m)\}.$$

To derive the average detection time, we need the following fact.

Fact 2: Let $X_i \sim \mathcal{G}(p_i)$ for $i = 1, 2, ...n$ and define

$$X = \min(X_1, X_2, ...X_n).$$

Then, if $X_1, X_2, ...X_n$ are i.i.d., $E(X) = \frac{1}{1-\prod_{i=1}^{n}(1-p_i)}$. ■

As before, let p_k and p'_k be the detection probability of the primary user by k^{th} sub-band under the TC and NC protocol, respectively. Since $X_k \sim \mathcal{G}(p_k)$ in case of TC protocol, it follows from Fact 2 that $T_c(m) = \frac{1}{1-\prod_{k=1}^{m}(1-p_k)}$. Similarly, it can be shown that $T_p(m) = \frac{1}{1-\prod_{k=1}^{m}(1-p'_k)}$. ■

APPENDIX V: PROOF OF PROPOSITION 5

Since for all k, $p_k \geq p'_k$ and $p_K > p'_K$, we get that

$$q_c = \prod_{k=1}^{m}(1-p_k) < \prod_{k=1}^{m}(1-p'_k) = q_{nc}.$$

Thus $T_c(m) = \frac{1}{1-q_c} > \frac{1}{1-q_{nc}} = T_n(m)$ which implies that $\mu_{p/c}(m) > 1$. To prove the second part of the proposition, we proceed the following way: Consider a particular sub-band indexed by k with two users U_1 and U_2 occupying the band. Suppose U_1 and U_2 detect the primary user without any help from each other, then from (11) the probability of detection for U_j for $j = 1, 2$ satisfies $p_c^{(j)} = \alpha^{\frac{1}{P_j+1}} \geq \alpha$, where P_j is the received signal power at user U_j from the primary user. Since the probability of detection of the primary user by the sub-band under the NC protocol is $p'_k = 1-(1-p_n^{(1)})(1-p_n^{(2)})$, it follows that $1-p'_k \leq (1-\alpha)^2$. Thus, $q_{nc} = \prod_{k=1}^{m}(1-p'_k) \leq (1-\alpha)^{2m} \to 0$, as $m \to \infty$. If on the other hand, U_1 detects the presence of the primary user with help from U_2, then from (10) the detection probability of U_1 is given by,

$$p_c^{(1)} = \varphi(\lambda; P_1 + 1, P_2 + 1)$$

where λ satisfies (9). Thus

$$p_c^{(1)} = \int_0^\infty e^{-h} e^{-\frac{\lambda}{P_1+1+(P_2+1)h}} dh \geq \int_0^\infty e^{-h} e^{-\frac{\lambda}{1+h}} dh = \alpha.$$

Note that in this case U_2 detects the primary user by itself. If p_k denotes the probability of detection by the k^{th} sub-band under the TC protocol, we get $1-p_k = (1-p_n^{(2)})(1-p_c^{(1)}) \leq (1-\alpha)^2$ for all $1 \leq k \leq m$. Thus $q_c = \prod_{k=1}^{n}(1-p_k) \leq (1-\alpha)^{2m} \to 0$ as $m \to \infty$ from which we get that $\mu_{n/c}(m) = \frac{1-q_c}{1-q_{nc}} \to 1$ as $m \to \infty$. ■

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