Strategic Sensing in Cognitive Radio Networks

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Under Supervision of

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Introduction

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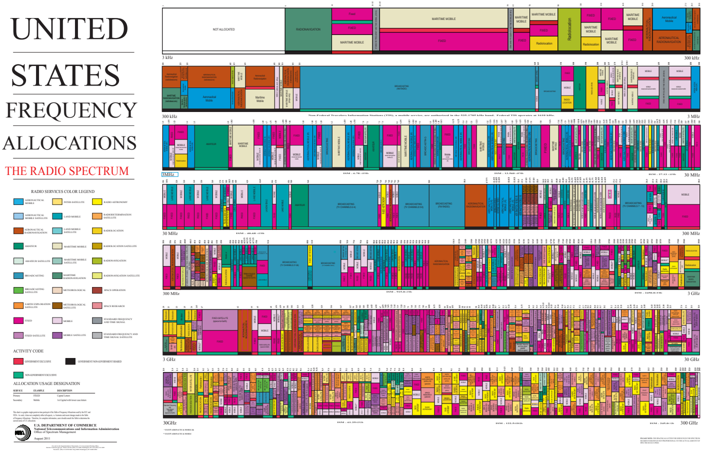

Figure: US frequency allocations of the radio spectrum

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An optional solution:

Cognitive Radio (CR) - A dynamically configured transceiver.

- First published by Mitola, J. & Maguire, G.Q., Jr. (1999).
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  - Chanel-state estimation and predictive modeling.
  - **Transmit-power control and dynamic spectrum management.**

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Two identical servers, S_Q and S_L, and a single queue.

Customers choose between two options: *Sense* S_L or *Join* S_Q.

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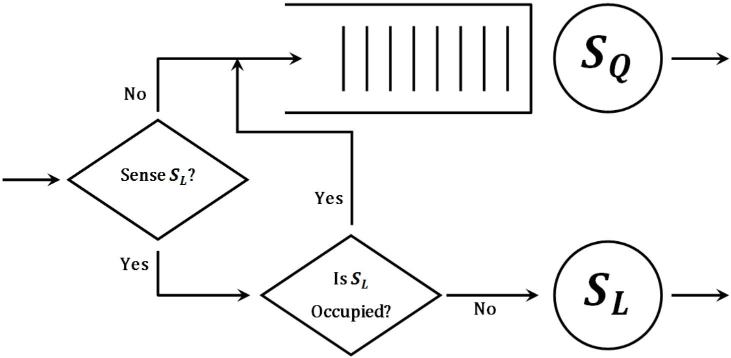

Figure: Customers' flow chart of the system

System-Model

Formulation:

- Identical rational individualistic customers
- Arrival rate $\sim \mathrm{Poisson}(\Lambda)$
- Service duration $\sim \mathrm{Exp}(\mu)$
- p - sensing probability
- $(X(t), Y(t))$ - the state at time t, where $X(t) \in \{0, 1, 2, \ldots\}$ and $Y(t) \in \{0, 1\}$
- $P_{i,j}$ - the stationary probability of state (i, j)

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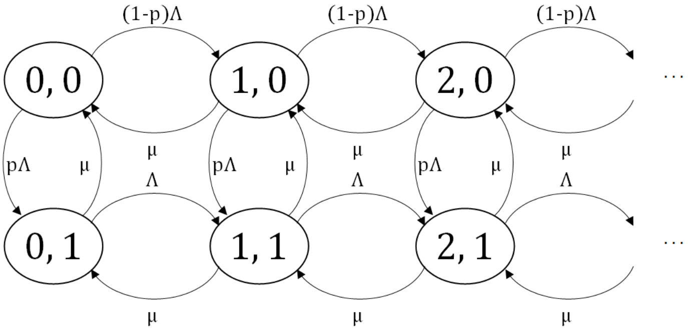

Figure: The Markov chain describing the transitions between states in the system

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This is a particular case of the *Heterogeneous Arrivals & Service* model of Yechiali, U. & Naor, P. (1971).

S_L is an independent M/M/1/1

The stationary probabilities are found solving the set of equations:

$$\begin{cases} \Lambda P_{0,0} - \mu P_{1,0} - \mu P_{0,1} = 0 \, , \\ (\mu + \Lambda) P_{0,1} - p \Lambda P_{0,0} - \mu P_{1,1} = 0 \, . \end{cases} \tag{1}$$

and $\forall n \in \{1, 2, \dots\}$:

$$\begin{cases} (\mu + \Lambda) P_{n,0} - (1-p) \Lambda P_{n-1,0} - \mu P_{n+1,0} - \mu P_{n,1} = 0 \, , \\ (2\mu + \Lambda) P_{n,1} - p \Lambda P_{n,0} - \Lambda P_{n-1,1} - \mu P_{n+1,1} = 0 \, . \end{cases} \tag{2}$$

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Defining $\rho := {}^{\Lambda}/_{\mu}$, after some development we achieve the following results:

$$\Pr(Y = 1) = \sum_{i=0}^{\infty} P_{i,1} = \frac{p\rho}{1 + p\rho} ; \tag{3}$$

$$\Pr(Y = 0) = 1 - \Pr(Y = 1) = \sum_{i=0}^{\infty} P_{i,0} = \frac{1}{1 + p\rho} ; \tag{4}$$

$$\begin{aligned} \hat{\rho}(p, \rho) :=& \frac{1}{\mu} \left[(1 - p)\Lambda + \Pr(Y = 1) \cdot p\Lambda \right] \\ =& \rho - \frac{1}{1 + p\rho} p\rho = 1 - \left(P_{0,0} + P_{0,1} \right) . \end{aligned} \tag{5}$$

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System-Utilization

Proposition

For each $\rho \in (0, \varphi)$, there exists a lower bound for p, denoted $\underline{p}$, such that the system is stable iff $p \in (\underline{p}, 1]$.

Proof.

For stability we demand $\hat{\rho}(p, \rho) < 1$.
Using (5) and isolating p in the inequality we get:

$$p > \frac{\rho - 1}{\rho(2 - \rho)} =: \underline{p}$$

By the monotonicity of $\hat{\rho}(p, \rho)$, we assume $p = 1$ concluding that

$$\hat{\rho}(1, \rho) = \frac{\rho^2}{1 + \rho} < 1 \quad \Leftrightarrow \quad \rho < \frac{1 + \sqrt{5}}{2} = \varphi$$

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Equilibrium Strategy

We assume $\rho < \varphi$ and define:

- $c_w > 0$ - the waiting cost per unit time.
- $c_s > 0$ - the cost of sensing.

We denote $\gamma := c_w/\mu c_s$ and write down the costs:

$$\begin{cases} C_N(p) = \frac{c_w}{\mu}\mathrm{E}\left[L(p)\right] \ ; \\ C_S(p) = c_s + \Pr(Y=1)\cdot\frac{c_w}{\mu}\mathrm{E}\left[L(p) \mid Y=1\right] \ ; \end{cases} \tag{6}$$

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Equilibrium Strategy

We assume $\rho < \varphi$ and define:

- $c_w > 0$ - the waiting cost per unit time.
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Proposition

For every $\rho \in (0, \varphi)$, and for every value $\gamma > 0$, a unique equilibrium strategy $p_e \in [0, 1]$ exists.

This result arises in spite of the fact that the function $C_S(p)$ is not necessarily monotone:

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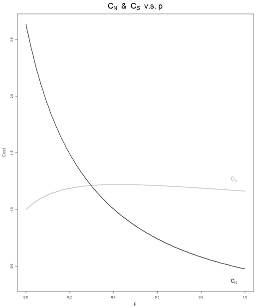

Figure: $\gamma = 1$ and $\rho = 0.725$

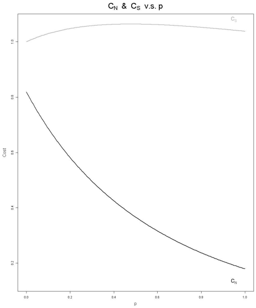

Figure: $\gamma = 1$ and $\rho = 0.45$

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In equilibrium, it must hold that $C_S(p) = C_N(p)$, or equivalently:

$$\gamma \mathrm{E}\left[L(p) \mid Y = 0\right] = \frac{1}{\Pr(Y = 0)} = 1 + p\rho \,. \qquad (8)$$

Proposition

The function $\mathrm{E}\left[L(p) \mid Y = 0\right]$ *is continuous and monotone non-increasing in* p.

To prove this we use the Sample Path Analysis technique, comparing two systems under the same sequence of events.

This property together with (8) immediately indicate that p_e is unique.

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Proof.

Define:

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  - $T_{i+1} - T_i \sim \exp(\Lambda)$;
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Assume, w.l.o.g that

- $p < p'$
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We shall show:

$$\mathrm{E}[X' \mid Y' = 0] \leq \mathrm{E}[X \mid Y = 0] .$$

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Proof (Cont.)

We apply the following modifications:

(i) If $Y(T_i) = 1$ (or $Y'(T_i) = 1$), customer i preempts the one in service, and the preempted customer is routed to S_Q (or S'_Q).

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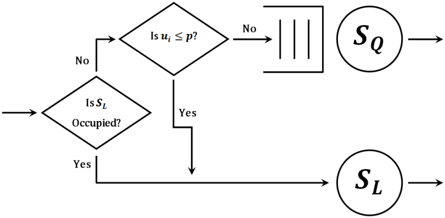

Figure: Customers' flow chart of the modified system

Equilibrium Strategy

Proof (Cont.)

(a) If there is a customer in S_L (or S_L'), it must be the last customer.

(b) Customers i begins service at T_i.

(c) Joining $S_L \Rightarrow$ Joining S_L'

$$\forall t \in [0, \infty) : \{Y(t) = 1\} \Rightarrow \{Y'(t) = 1\} ;$$

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Equilibrium Strategy

Proof (Cont.)

(a) If there is a customer in S_L (or S'_L), it must be the last customer.

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Proof (Cont.)

From (d) + (e),

$$\forall t \in [0,\infty) : X(t) \geq X'(t)\,; \quad \text{or,} \quad X \succcurlyeq X'\,; \tag{9}$$

In fact,

$$\mathrm{E}[X' \mid Y' = 0] \leq \mathrm{E}[X \mid Y' = 0]\,,$$

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Note that for some $\lambda_1, \lambda_2 \in [0,1]$ s.t. $\lambda_1 + \lambda_2 = 1$:

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In the paper we show explicitly that

$$\mathrm{E}_{Y=0}[X] \leq \mathrm{E}_{Y=0}\left[X \,\middle|\, \begin{array}{l}\text{busy period}\\ \text{has begun}\end{array}\right] = \mathrm{E}_{Y=0}[X \mid Y' = 1]\,,$$

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to complete the proof of the proposition. □

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Equilibrium Strategy

Proposition

The pure strategy $p = 0$ is an equilibrium strategy (in other words $p_e = 0$) iff:

$$\rho \leq \frac{1}{1+\gamma} .$$

Proof.

This is immediate, as $p = 0$ is the M/M/1 regular case and

$$\mathrm{E}[L(0) \mid Y = 0] = \mathrm{E}[L(0)] = \frac{\rho}{1-\rho} .$$

Substituting this in (8) and isolating ρ we get the desired result. $\square$

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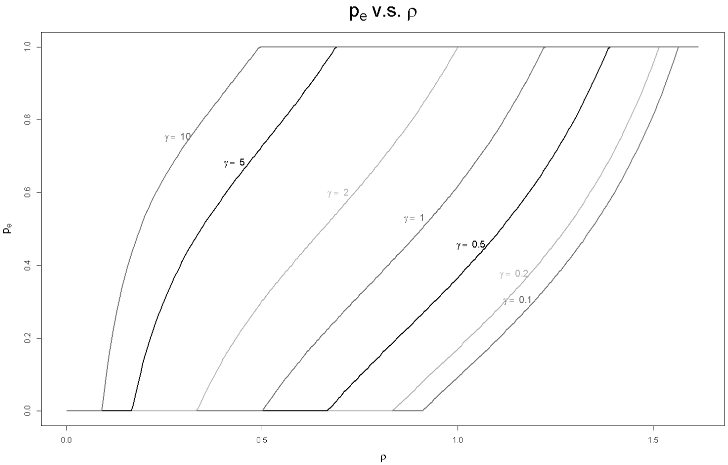

Figure: The equilibrium strategy p_e as a function of ρ for a various values of γ

Social Optimization

The social objective function, $C(p)$, is defined as

$$C(p) := (1-p)C_N(p) + pC_S(p)\,. \tag{11}$$

Denote p^* the socially optimal strategy. Accordingly,

$$p^* := \arg\min_{p\in[0,1]} C(p)\,. \tag{12}$$

and the values can be computed numerically.

For some values of ρ and γ, we spotted, counter intuitively that $p_e < p^*$.

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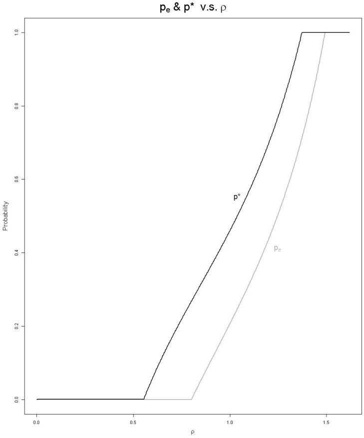

Figure: $\gamma = 0.25$

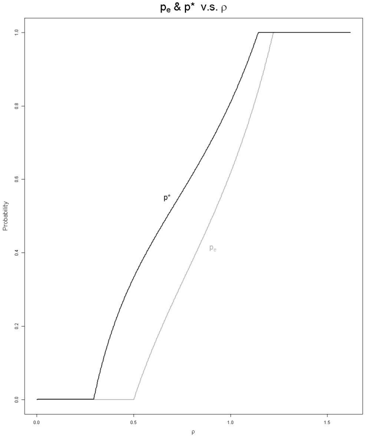

Figure: $\gamma = 1$

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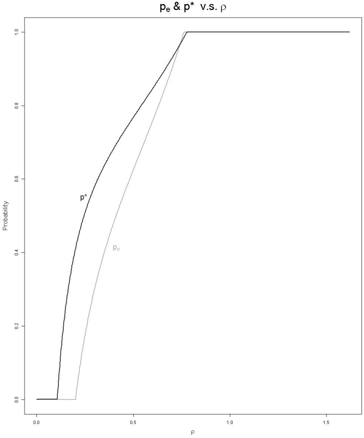

Figure: $\gamma = 4$

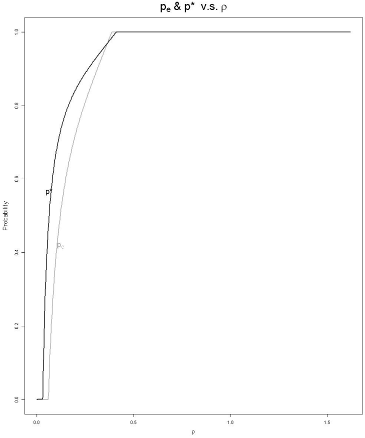

Figure: $\gamma = 16$

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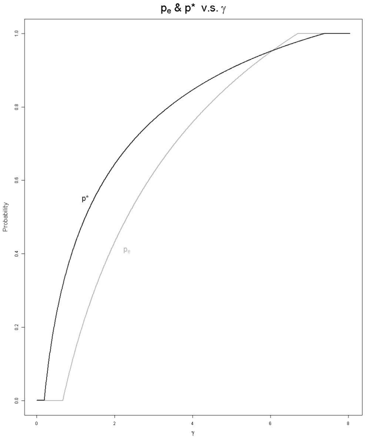

Figure: $\rho = 0.6$

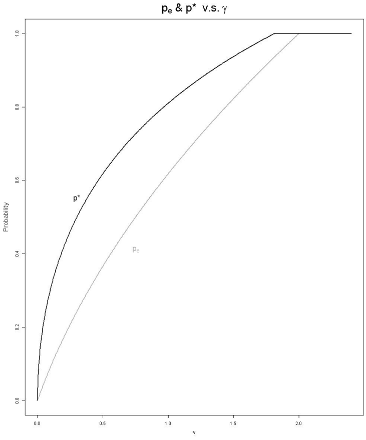

Figure: $\rho = 1$

Thank you for listening!