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Strategic Sensing in Cognitive Radio Networks

Thesis Submitted Towards the Degree of Master of Science in Operations Research

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Abstract

We study a noncooperative multi-player game problem of individual rational users sending data-packets ("customers" in the terminology of queueing theory) in a Cognitive Radio Network (CRN) with the opportunity of spectrum sensing. The system is composed of two channels (or "servers"): One unlicensed channel shared freely among all users, and one in which the transmission entails the cost of sensing, and can be denied if the channel is already occupied. It is the users' prerogative to decide whether to use the shared channel or sense, hopefully not encountering a rejection. It can be easily shown that in a model where a denied customer leaves never to return, the system can be analyzed as two independent subsystems with poissonian stream of arrivals to each one, an M/M/1/1 and an M/M/1, and that there exists a unique Nash equilibrium. As opposed to such simple models, we assume in our model that customers that are rejected from service after sensing are routed to the shared queue. We model this process as a queueing system comprising of two servers working in parallel, one is an M/M/1/1 loss system and the other is a G/M/1 with heterogeneous arrivals. We suggest a lightweight algorithm for computing the transition probabilities and the stationary probabilities of the Markov chain describing the process, analyze the system's capacity and utilization and show, using the technique of Sample Path Analysis, that the Nash equilibrium strategy in the system is well-defined.

1 Introduction

2 System-model

We consider a system composed of two identical servers, S_Q and S_L (Q for queue, L for loss), each one's service-duration is exponential with rate μ, and a single FCFS unobservable queue (see Fig.1). Customers are identical, and arrive at the system following a Poisson process with rate Λ. Upon arrival, a costumer chooses one of two options: pay a *sensing price* and try to attain service in S_L (a.k.a, *Sense*), or join the queue and wait until accepted to service in S_Q. A costumer who chooses to sense S_L and finds it idle is immediately accepted to service without waiting. If he finds S_L occupied, he is sent to the queue and waits his turn to service by S_Q.

Denote p the probability that a costumer senses S_L, and denote $(X(t), Y(t))$ the state of the system at time t, consisting of $X(t) \in \{0, 1, 2, \ldots\}$ the number of customers in the queue (including the one in service at S_Q), and $Y(t) \in \{0, 1\}$ the state of S_L (where $Y(t) = 0$ means that S_L is idle at time t). Thus, the system can be described as a bi-dimensional Markov process, as shown in Fig.2, and the stationary probabilities are computed through the following set of equations:

$$\begin{cases} \Lambda P_{0,0} - \mu P_{1,0} - \mu P_{0,1} = 0 \,, \\ (\mu + \Lambda) P_{0,1} - p\Lambda P_{0,0} - \mu P_{1,1} = 0 \,. \end{cases} \tag{2.1}$$

$$\forall n \in \{1, 2, \ldots\} : \begin{cases} (\mu + \Lambda) P_{n,0} - (1-p)\Lambda P_{n-1,0} - \mu P_{n+1,0} - \mu P_{n,1} = 0 \,, \\ (2\mu + \Lambda) P_{n,1} - p\Lambda P_{n,0} - \Lambda P_{n-1,1} - \mu P_{n+1,1} = 0 \,. \end{cases} \tag{2.2}$$

where $P_{i,j}$ is the stationary probability that the system is in state (i, j). These equations lead to the following relationship:

$$\forall n \in \{0, 1, 2, \ldots\} : \; P_{n+1,0} + P_{n+1,1} = \frac{\Lambda}{\mu}((1-p)P_{n,0} + P_{n,1}) \,, \tag{2.3}$$

By the assumption that each individual chooses randomly and independently whether or not to sense, we deduce that the sub-system S_L can be considered as an M/M/1/1 queue (Erlang's Loss Model) with arrival rate $p\Lambda$ and service rate μ. Therefore, the *loss-probability* of S_L (i.e., the probability that a customer sensing S_L will find it occupied) is:

$$\Pr(Y = 1) = \sum_{i=0}^{\infty} P_{i,1} = \frac{p\Lambda}{\mu + p\Lambda} \,, \tag{2.4}$$

and its complement:

$$\Pr(Y = 0) = 1 - \Pr(Y = 1) = \sum_{i=0}^{\infty} P_{i,0} = \frac{\mu}{\mu + p\Lambda} \,. \tag{2.5}$$

One can also derive the above formula by summing the right-hand-sides of the equations in (2.3) for $n = 0, 1, 2, \ldots$ and equating the sum of this infinite series to $1 - (P_{0,0} + P_{0,1})$, considering that in steady state of the system:

$$p\Lambda \sum_{i=0}^{\infty} P_{i,0} - \mu \sum_{i=0}^{\infty} P_{i,1} = 0 \,.$$

Summing the equations in (2.3) for $n = 0, 1, 2, \ldots$ and utilizing (2.4) and (2.5) we get:

$$P_{0,0} + P_{1,0} = 1 - \frac{\Lambda}{\mu}\left(1 - \frac{p}{1 + p\frac{\Lambda}{\mu}}\right) \tag{2.6}$$

which induces a linear relationship between $P_{0,0}$ and $P_{0,1}$.

Subtracting the second equation from the first one in (2.2) and combining with (2.3) we deduce the following formulae:

$$\forall n \in \{0, 1, \ldots\} : \begin{cases} P_{n+1,0} = (1-p)\frac{\Lambda}{\mu} P_{n,0} + p\frac{\Lambda}{\mu} \sum\limits_{i=0}^{n} P_{i,0} - \sum\limits_{i=0}^{n} P_{i,1} \,, \\ P_{n+1,1} = \frac{\Lambda}{\mu} P_{n,1} + \sum\limits_{i=0}^{n} P_{i,1} - p\frac{\Lambda}{\mu} \sum\limits_{i=0}^{n} P_{i,0} \,. \end{cases} \tag{2.7}$$

It can be seen from (2.7) that, for given constants p and ${}^{\Lambda}/_{\mu}$, every value $P_{n+1,k}$ ($k \in \{0,1\}$) is expressed as a linear combination of $\{P_{0,0}, P_{1,0}, \ldots, P_{n,0}, P_{0,1}, P_{1,1}, \ldots, P_{n,1}\}$. Hence, with (2.6), it follows by induction that for each $n \in \{0, 1, \ldots\}$ and for each $k \in \{0, 1\}$, $P_{n,k}$ is a linear function of $P_{0,0}$, as a countable sum of linear functions. Thus, for N sufficiently large, one can bound the queue length by N, express the finite series $\sum_{i=0}^{N} P_{i,0} + P_{i,1}$ as a linear function of $P_{0,0}$, equate to 1 and find a fixed value of $P_{0,0}$ (and all other $P_{n,k}$) in $O(N)$ time complexity.

3 System-utilization

We now intend to find the maximum utilization of the system. For convenience we use the notation $\rho := {}^{\Lambda}/_{\mu}$. The *effective-arrival-rate* to the queue, $\hat{\lambda}(p, \rho)$, consists of two streams of arrivals: customers who join the queue without sensing at all (whose proportion is $1-p$) and customers who sense S_L and find it busy (whose proportion is $p \cdot \Pr(Y = 1)$). Therefore,

$$\hat{\lambda}(p, \rho) = (1-p)\Lambda + \Pr(Y = 1) \cdot p\Lambda \, .$$

Using (2.4) and dividing by μ we get the *effective-utilization*, $\hat{\rho}(p, \rho)$:

$$\hat{\rho}(p, \rho) := \frac{1}{\mu}\hat{\lambda}(p, \rho) = \rho - p\rho + \frac{p\rho}{1+p\rho}p\rho = \rho - \frac{1}{1+p\rho}p\rho \, , \tag{3.1}$$

which is monotone decreasing in p for every $\rho \in (0, \infty)$.

Note, by (2.6), that $\hat{\rho}(p, \rho)$ is equal to $1 - (P_{0,0} + P_{0,1})$, for it represents the *busy fraction* of subsystem S_Q.

The stochastic process representing the stream of arrivals over time to the queue served by S_Q is not necessarily poissonian. This is because the inter-arrival times to the queue for $p \neq 0$ are not i.i.d., and the arrival-rate increases when S_L is busy in comparison to the arrival-rate in its idle periods. Thus, we model this subsystem as a G/M/1 with heterogeneous inter-arrival times, and for the system to remain stable, as implied by [5], we assume $\hat{\rho}(p, \rho) < 1$.

Recall the *golden ratio*, $\varphi = {}^{1}/_{2} \cdot (1 + \sqrt{5}) \approx 1.618$. Then:

Proposition 3.1. *For each $\rho \in (0, \varphi)$, there exists a lower bound for p, denoted $\underline{p}$, such that the system is stable iff $p \in (\underline{p}, 1]$.*

Proof. As discussed, a necessary and sufficient condition for stability is: $\hat{\rho}(p, \rho) < 1$. Substitute (3.1) and isolate p in the inequality to get an equivalent stability criterion:

$$p > \frac{\rho - 1}{\rho(2-\rho)} =: \underline{p}$$

from which it is clear that, if the system is stable for some $p_0 > \underline{p}$, it is also stable for all $p \in [p_0, 1]$.

Finding the maximum throughput we assume that $p = 1$, which means everyone senses S_L before they join the queue. This is followed by the monotonicity of $\hat{\rho}(p, \rho)$. Then, from (3.1), the stability criterion is:

$$\hat{\rho}(1, \rho) = \frac{\rho^2}{1+\rho} < 1 \quad \Leftrightarrow \quad \rho^2 - \rho - 1 < 0 \quad \Leftrightarrow \quad \rho < \frac{1+\sqrt{5}}{2} = \varphi$$

where φ is the golden ratio, concluding that for every $\rho \in (0, \varphi)$ and $p \in (\underline{p}, 1]$, the system is stable. $\square$

4 Equilibrium Strategy

In this section we discuss the states and (Nash) equilibrium strategies in the system. Let $c_w > 0$ be the waiting cost per unit time of an individual customer waiting in line, and $c_s > 0$ be the cost of sensing (incurred by each customer who chooses to sense, regardless of the consequences of his action). Since all customers are identical, and each one arriving at the system is eventually attained to service, neither the reward from service nor the time spent in service is relevant. Of course, the state of the system, including the length of the queue and the status of the servers, is unobservable as mentioned, and unknown to the customers at the time they make their decisions. We assume the system is not overloaded (i.e., $\rho < \varphi$) and that all customers act to reduce their expected cost to minimum. Denote $C_S(p)$ and $C_N(p)$ the expected cost of an individual who chooses to sense and an individual who chooses not to sense, respectively, given that the others' probability of sensing is p.

$$\begin{cases} C_N(p) = \frac{c_w}{\mu}\mathrm{E}\left[L(p)\right] \ , \\ C_S(p) = c_s + \Pr(Y=1)\cdot\frac{c_w}{\mu}\mathrm{E}\left[L(p) \mid Y=1\right] \ , \end{cases} \tag{4.1}$$

where $L(p)$ is a random variable representing the queue length (including the customer in service) upon arrival to the system.

Prior to discussion of equilibrium strategies we assert some attributes of $\mathrm{E}\left[L(p) \mid Y=0\right]$ which appear to be essential in this context.

Proposition 4.1. *The function* $\mathrm{E}\left[L(p) \mid Y=0\right]$ *is continuous and monotone non-increasing in* p.

Proof. Proving the continuity of $\mathrm{E}\left[L(p) \mid Y=0\right]$ is immediate, as, referring to (2.5), (2.6) and (2.7),

$$\mathrm{E}\left[L(p) \mid Y=0\right] = \frac{1}{\Pr(Y=0)}\sum_{i=0}^{\infty} iP_{i,0} = (1+p\rho)\sum_{i=0}^{\infty} iP_{i,0} \ ,$$

which is a countable sum of compositions of continuous functions in $p \in [0,1]$.

In order to prove monotonicity, we examine the transitions between states in two systems, $\Omega := \{S_Q, S_L\}$ and $\Omega' := \{S'_Q, S'_L\}$, under the same sequence of events. Systems Ω and Ω' are identical except that in Ω the sensing probability is p, as opposed to Ω', where the sensing probability is p', and $p < p'$.

Denote by $(X(t), Y(t))$ and $(X'(t), Y'(t))$ the states at time t of systems Ω and Ω' respectively. Since that for all $p \in [0,1]$ the state $(0,0)$ in the Markov chain is recurrent, we can assume without loss of generality, that at time $t = 0$,

$$(X(0), Y(0)) = (X'(0), Y'(0)) = (0,0) \ .$$

Showing that

$$\forall t \in [0,\infty) : \mathrm{E}[X(t) \mid Y(t) = 0] \geq \mathrm{E}[X'(t) \mid Y'(t) = 0] \tag{4.2}$$

will complete the proof.

We begin by attaching three variables to each customer: Let $\{(T_i, \tau_i, u_i)\}_{i\in\mathbb{N}}$ represent the *arrival time*, *service duration* and *sensing aversion* of the i-th customer respectively, and for all $i \in \mathbb{N}$:

$$T_{i+1} - T_i \sim \exp(\Lambda) \ , \quad \tau_i \sim \exp(\mu) \ , \quad u_i \sim \mathrm{U}[0,1] \ .$$

Each customer arrives to both systems simultaneously. Customer i arriving at Ω senses if and only if $u_i \leq p$, and similarly, arriving at Ω' he/she senses if and only if $u_i \leq p'$. This ensures that the sensing probability is p in system Ω and p' in system Ω', and as a fact, implies that customers who sense in Ω also sense in Ω'. Note that the value u_i is only relevant when $Y(T_i) = 0$ or $Y'(T_i) = 0$.

We assign each customer one of three types, based on the values of their sensing aversion: Customer i will be assigned

- *type-L* (stands for "Low") if $u_i \in [0, p]$,
- *type-M* (stands for "Medium") if $u_i \in (p, p']$,
- *type-H* (stands for "High") if $u_i \in (p', 1]$.

We reiterate that one's type is only significant if S_L or S'_L are empty by the time of customers' arrival. By definition, upon arrival, an H-type customer joins both S_Q and S'_Q, an M-type customer joins S_Q but senses S'_L, whereas an L-type customer senses both S_L and S'_L.

Without loss of generality, we modify the service regime as follows:

(i) Arriving at Ω (Ω') when S_L (S'_L) is busy, customer i preempts the customer in service in S_L (S'_L) with no regard to type, and the preempted customer is routed to continue the service in S_Q (S'_Q). Arriving at Ω (Ω') when S_L (S'_L) is empty, customer-i's actions are followed by type.

(ii) Subsystem S_Q (S'_Q) is a preemptive resume LCFS queue, meaning that customers are served following a preemptive LCFS discipline, and the preempted service will be resumed from the point it is stopped.

Lemma 4.1.1. *Assumptions* (i) *and* (ii) *do not affect the stationary probabilities of the systems.*

Proof. We shall show that Lemma 4.1.1 holds for system Ω and the proof for Ω' is identical.

Let i and $i+1$ be a successive pair of customers and suppose that customer $i+1$ preempts customer i in subsystem S_L. Denote $\Delta_{i,i+1} := T_{i+1} - T_i$ the time difference between their arrivals. Then, as a result of the memoryless property of the exponential distribution, the service duration of customer $i+1$ (i.e., τ_{i+1}) and the residual service of customer i (i.e., $\tau_i - \Delta_{i,i+1} \mid \tau_i > \Delta_{i,i+1}$) are independent and identically distributed. This incident of preemption is equivalent to the joining of the new comer to subsystem S_Q when S_L is busy. Hence, assumptions (i) does not influence the transition probabilities and the stationary probabilities remain as before.

The validity of assumption (ii) can be explained by the fact that in a work-conserving preemptive resume system with exponential i.i.d services duration, as sustains in S_Q, the length of the queue is independent of the service regime. Consequently, assumptions (i) and (ii) do not restrict the generality of the model. $\square$

Exploiting assumptions (i) and (ii) we immediately achieve several properties that will be later used in the proof:

(a) At any moment, if there is a customer in S_L (or S'_L), then it must be the last customer to arrive at the system until that moment.

(b) All customers begin service at the moment of their arrival (whether they sense or not).

Observing the system at an arbitrary point in time, suppose the last customer has joined both S_L and S'_L. By assumption (i), this tagged customer can either complete service in S_L and S'_L simultaneously or be preempted in the two subsystems by the next customer to come. As for the latter case, it holds that the preempting customer joins both S_L and S'_L upon arrival and the scenario repeats. Thus, in a successive sequence of preemption-incidents beginning with a customer joining S_L and S'_L, all customers join S_L and S'_L. Note that the population of customers that join S_L is consisted of L-type customers and successively preempting customers. We recall that upon arrival, L-type customers join S_L and S'_L simultaneously, and at time $t = 0$ both systems are in the same state. Combining these all together we achieve:

(c) Customers joining S_L upon their arrival, attend S'_L as well.

(d) Customers joining S'_Q upon their arrival, attend S_Q as well (the *modus tollens* form of (c)).

(e) The sojourn time of customer i in S'_Q is no greater then the sojourn time of i in S_Q (from (d), customers who arrive at S'_Q after i, simultaneously arrive at S_Q).

With property (c) and the simultaneousness of occurrences we get:

$$\forall t \in [0, \infty) : \{Y(t) = 1\} \Rightarrow \{Y'(t) = 1\} ,$$

or, in an analogous fashion:

$$\forall t \in [0, \infty) : \{Y'(t) = 0\} \Rightarrow \{Y(t) = 0\} . \tag{4.3}$$

Moreover, from properties (d) and (e), we see that at an arbitrary point in time, each customer in S'_Q, must as well linger in S_Q, not leaving Ω before leaving Ω'. Therefore,

$$\forall t \in [0, \infty) : X(t) \geq X'(t) , \tag{4.4}$$

which implies that the process representing the number of customers in S_Q over time stochastically dominates the one that represents the number of customers in S'_Q.

For simplicity, we denote $(X(t), Y(t))$ by (X, Y), and $(X'(t), Y'(t))$ by (X', Y') (all equations are to be understood at an arbitrary time sample). On utilizing (4.4), we get

$$\mathrm{E}[X' \mid Y' = 0] \leq \mathrm{E}[X \mid Y' = 0] .$$

To prove our claim (4.2) it suffices to prove:

$$\mathrm{E}[X \mid Y' = 0] \leq \mathrm{E}[X \mid Y = 0] . \tag{4.5}$$

Denote i the last customer that arrived by time t, and let

$$A(t) := \{i \text{ joined } S_Q \text{ and } T_i + \tau_i > t\}$$

express the event "i joined S_Q and had not left until t". Because of property (b), this is possible if and only if i was still in service in S_Q by time t. By definition,

$$A(t) \Rightarrow \{Y(t) = 0\} . \tag{4.6}$$

Clearly, given that i has not left Ω by time t, the residual service of customer i at t is exponential with parameter μ. Thus, we can assume w.l.o.g that T_i occurred a moment before t and $A(t)$ can be rewritten as the event when a new busy period in S_Q began right before t (i.e., $t = T_i^+ := T_i + \varepsilon$, where ε is a small positive infinitesimal quantity).

Recall that, by property (a), if there is a customer in S_L at time t, then it must be i (the last customer arrived). So, if $t \in \mathbb{R}$ satisfies $Y(t) = 0$ and $Y'(t) = 1$, customer i joined S_Q at the same time beginning service in S'_L, and, together with property (b), had not left the system before t. Hence,

$$\{\{Y'(t) = 1\} \wedge \{Y(t) = 0\}\} \Rightarrow A(t)\,. \tag{4.7}$$

Lemma 4.1.2. *Conditioned on $A(t)$, $X(t)$ and $Y'(t)$ are independent random variables.*

Proof. Denote T_n^+ the moment after customer-n's arrival. We shall show by induction that $\forall n \in \mathbb{N}$, $X(T_n^+)$ and $Y'(T_n^+)$ are conditionally independent given $A(T_n^+)$.

Note that as $A(T_n^+)$ occurs, the values of $Y'(T_n^+)$ can be specified one to one by customer-n's actions:

$$\{Y'(T_n^+) \mid A(T_n^+)\} = \begin{cases} 0, \text{ iff } n \text{ joins } S'_Q; \\ 1, \text{ iff } n \text{ joins } S'_L. \end{cases} \tag{4.8}$$

Presume that $A(T_n^+)$ is satisfied. Having that n joined S_Q means that n is not of type-L, and, by (4.6), $Y(T_n^+) = 0$. Define $T_i^- := T_i - \varepsilon$, where ε is a small positive infinitesimal quantity, the moment before i's arrival. Regarding n's arrival at Ω', there are two possibilities:

- Upon n's arrival, S'_L was empty (i.e., $Y'(T_n^-) = 0$).
- Upon n's arrival, S'_L was busy (i.e., $Y'(T_n^-) = 1$).

To the extent that $Y'(T_n^-) = 0$, n's action is determined by type (M or H), which is chosen randomly and independently, thus (4.8) can be rephrased to

$$\{Y'(T_n^+) \mid A(T_n^+)\} = \begin{cases} 0, \text{ iff } n \text{ is of type-H}; \\ 1, \text{ iff } n \text{ is of type-M}. \end{cases}$$

and $X(T_n^+)$ and $Y'(T_n^+)$ are independent.

To the extent that $Y'(T_n^-) = 1$, then, with assumption (i), n must have preempted his/her predecessor $(n-1)$ in S'_L simultaneously as joining S_Q. This is possible only as customer $n-1$ remained in service until customer-n's arrival, and indicates that

- Customer $n-1$ joined S_Q as arriving at Ω, and $Y(T_{n-1}^+) = 0$.
- Customer $n-1$ joined S'_L as arriving at Ω', and $Y'(T_{n-1}^+) = Y'(T_n^+) = 1$.
- Customer n had arrived before $n-1$ left S_Q, and $X(T_n^+) = X(T_{n-1}^+) + 1$.

Supported by the first two statements and (4.7) we infer that $A(T_{n-1}^+)$ holds. Followed by the induction hypothesis, $X(T_{n-1}^+)$ and $Y'(T_{n-1}^+)$ are independent under these conditions, and combining the second and the third statements we deduce that $X(T_n^+)$ and $Y'(T_n^+)$ are also independent.

It is left to show that $X(T_1^+)$ and $Y'(T_1^+)$ are conditionally independent given $A(T_1^+)$. We can assume, w.l.o.g, that $Y'(T_1) = 0$ (otherwise we will observe the last customer that arrived before T_1). Thus, as discussed, cusotmer-1's action is equivalent to type, which is a random variable independent of $X(T_1^+)$. □

Define operator $\mathrm{E}_{Y=0}[\bullet] := \mathrm{E}[\bullet \mid Y = 0]$. Combining Lemma 4.1.2 with (4.7), we arrive at

$$\mathrm{E}_{Y=0}[X \mid A] = \mathrm{E}_{Y=0}[X \mid A, Y' = 1] = \mathrm{E}_{Y=0}[X \mid Y' = 1] \,, \tag{4.9}$$

and since

$$\mathrm{E}_{Y=0}[X] \leq \mathrm{E}_{Y=0}[X \mid A] \,,$$

upon substituting (4.9) we get

$$\mathrm{E}_{Y=0}[X] \leq \mathrm{E}_{Y=0}[X \mid Y' = 1] \,. \tag{4.10}$$

By the law of total expectation we have

$$\begin{aligned} \mathrm{E}_{Y=0}[X] &= \mathrm{E}_{Y=0}[X \mid Y' = 0] \cdot \Pr(Y' = 0 \mid Y = 0) + \\ &\quad \mathrm{E}_{Y=0}[X \mid Y' = 1] \cdot \Pr(Y' = 1 \mid Y = 0) \\ &= \mathrm{E}[X \mid Y' = 0] \cdot \Pr(Y' = 0 \mid Y = 0) + \\ &\quad \mathrm{E}_{Y=0}[X \mid Y' = 1] \cdot \Pr(Y' = 1 \mid Y = 0) \,, \end{aligned} \tag{4.11}$$

where the second equality evolves from (4.3). Let us mention that the right-hand side of (4.11) is a convex combination of $\mathrm{E}[X \mid Y' = 0]$ and $\mathrm{E}_{Y=0}[X \mid Y' = 1]$. Amalgamating (4.10) and (4.11) we attain

$$\mathrm{E}[X \mid Y' = 0] \leq \mathrm{E}_{Y=0}[X]$$

thus, confirming (4.5) to complete the proof of Proposition 4.1. □

Denote $\gamma := {}^{c_w}/_{\mu c_s}$. The value of ${}^1/_\gamma$ can be interpreted as the normalized cost of sensing in terms of the cost of waiting a single service period, that is to say, for how many service completions one has to wait, on average, in order to pay a total time expense of c_s. Then:

Proposition 4.2. *For every $\rho \in (0, \varphi)$, and for every value $\gamma > 0$, a unique equilibrium strategy $p_e \in [0, 1]$ exists.*

Proof. p is an equilibrium strategy if no individual can benefit from choosing any alternative strategy. Thus, for $p \in (0, 1)$ to be an equilibrium, the following criterion must hold:

$$C_N(p) = C_S(p) \,. \tag{4.12}$$

Note that:

$$\mathrm{E}\left[L(p)\right] = \Pr(Y = 1) \cdot \mathrm{E}\left[L(p) \mid Y = 1\right] + \Pr(Y = 0) \cdot \mathrm{E}\left[L(p) \mid Y = 0\right] \,.$$

Divide both sides of (4.12) by c_s and substitute the values in (4.1), concluding that for $p \in (0, 1)$, $p_e = p$ if and only if:

$$\gamma \Pr(Y = 0) \cdot \mathrm{E}\left[L(p) \mid Y = 0\right] = 1 \,, \tag{4.13}$$

or,

$$\gamma \mathrm{E}\left[L(p) \mid Y=0\right] = 1 + p\rho\,. \tag{4.14}$$

Note that the function $1+p\rho$ is continuous and strictly monotone increasing in p, and therefore, using Proposition 4.1, we deduce that if there exists a solution p^* to (4.14) then it is unique, and $p_e = p^*$ is a unique equilibrium strategy.

Suppose that:

$$\forall p \in (0,1) : \gamma \mathrm{E}\left[L(p) \mid Y=0\right] > 1 + p\rho\,. \tag{4.15}$$

Following Proposition 4.1 and the monotonicity of $1+p\rho$, this inequality holds if and only if

$$\gamma \mathrm{E}\left[L(1) \mid Y=0\right] \geq 1 + \rho\,,$$

or equivalently: $C_N(1) \geq C_S(1)$. This means that $p_e = 1$, and followed by (4.15), sensing is a dominant strategy for each individual, and therefore the equilibrium is unique. Similarly, if:

$$\forall p \in (0,1) : \gamma \mathrm{E}\left[L(p) \mid Y=0\right] < 1 + p\rho$$

then $C_N(0) \leq C_S(0)$, and $p_e = 0$, which is unique. □

This result is not as intuitive as it seems, since the function $C_S(p)$ is not necessarily monotonic (as shown in Fig. 4a and 4b). This fact can be explained as follows: an increase in the proportion of customers that sense, results in a decrease of the probability to attain service in S_L on the first hand. It also decreases the expected queue length, and shortens the waiting time of the customers who failed to attain S_L on the other hand. Nevertheless, the equilibrium probability p_e is always unique.

Proposition 4.3. *The pure strategy $p=0$ is an equilibrium strategy (in other words $p_e = 0$) iff:*

$$\rho \leq \frac{1}{1+\gamma}\,.$$

Proof. Assuming that $\hat{\rho}(0,\rho) = \rho < 1$, from (4.12), $p=0$ is an equilibrium strategy if and only if:

$$\gamma \mathrm{E}[L(0) \mid Y=0] \leq 1\,.$$

Note that in this case the queue is a regular M/M/1 queue, and

$$\mathrm{E}[L(0) \mid Y=0] = \mathrm{E}[L(0)] = \frac{\rho}{1-\rho}\,.$$

Thus, a necessary and sufficient condition for $p_e = 0$ is:

$$\gamma \frac{\rho}{1-\rho} \leq 1 \quad \Leftrightarrow \quad \rho \leq \frac{1}{1+\gamma}\,.$$

□

5 Social Optimization

Hereupon, we turn our attention to social optimization. The social objective function, $C(p)$, is defined as

$$C(p) := (1-p)C_N(p) + pC_S(p) \ , \tag{5.1}$$

and represents the average cost of a customer, where the population's probability of sensing is p. Using (5.1) and (4.1), after some development we obtain

$$\frac{1}{c_s}C(p) = p + \gamma\left(L(p) - \frac{p}{1+p\rho}\mathrm{E}[L(p) \mid Y=0]\right) \ . \tag{5.2}$$

Denote p^* the socially optimal strategy. Accordingly,

$$p^* := \operatorname*{arg\,min}_{p\in[0,1]} C(p) = \operatorname*{arg\,min}_{p\in[0,1]} \left\{p + \gamma\left(L(p) - \frac{p}{1+p\rho}\mathrm{E}[L(p) \mid Y=0]\right)\right\} \ . \tag{5.3}$$

Due to empirical results shown in Fig.6, it can be observed that for all examined pair of constants, $\gamma > 0$ and $\rho \in (0, \varphi)$, the following inequality holds:

$$p_e \le p^* \ , \tag{5.4}$$

which means that in an equilibrium state, customer's willing to sense is less than that in the socially optimal case.

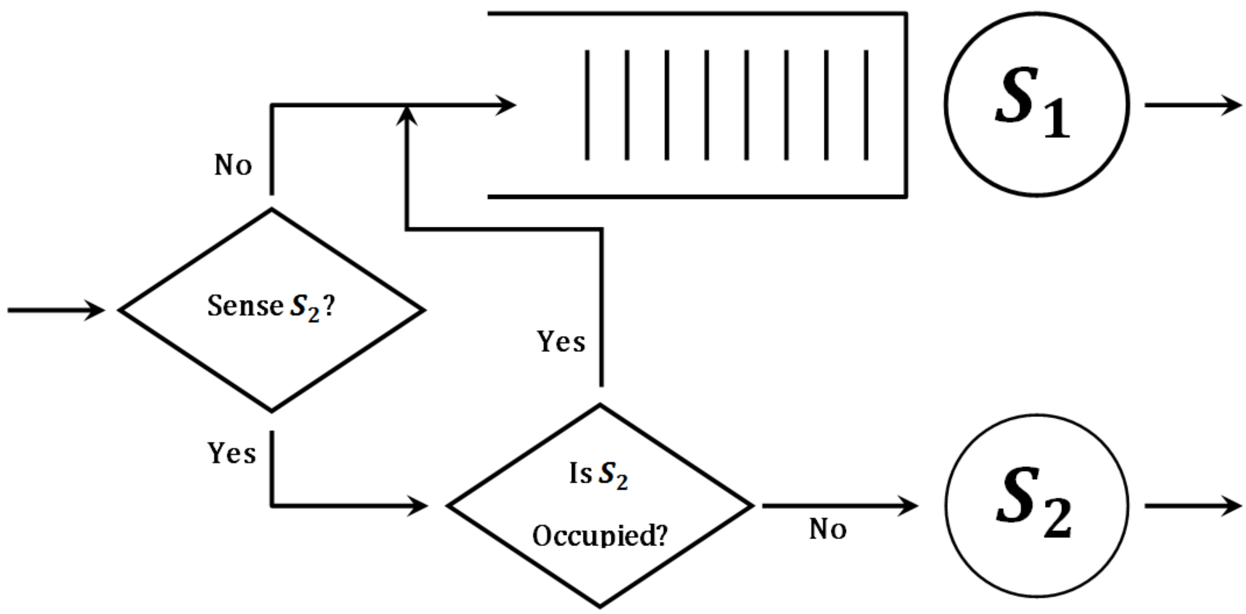


Figure 1: Customers' flow chart of the system with an opportunistic sensing

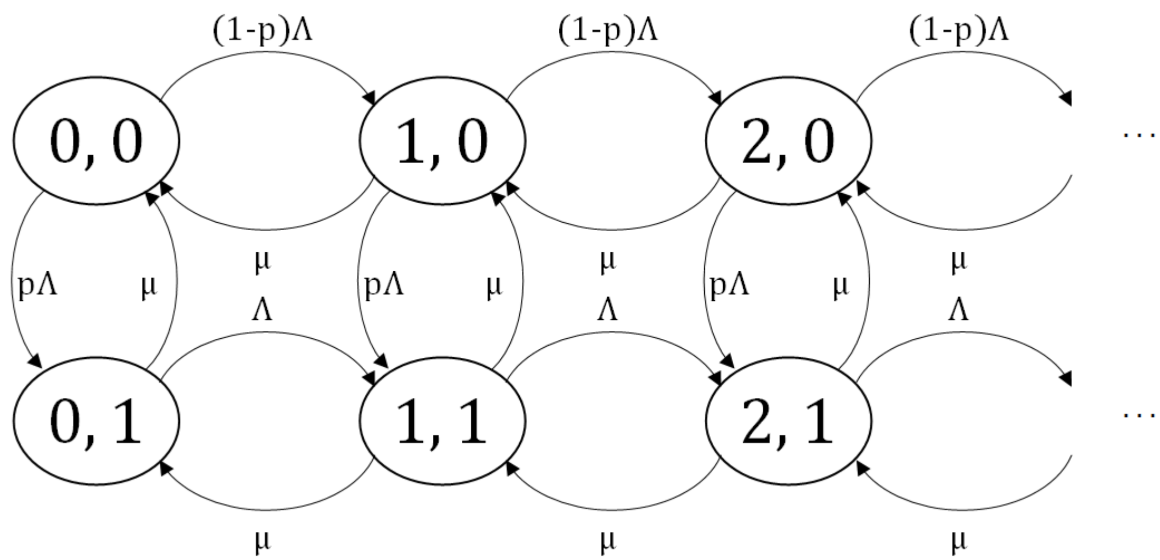


Figure 2: The Markov chain describing the transitions between states in the system

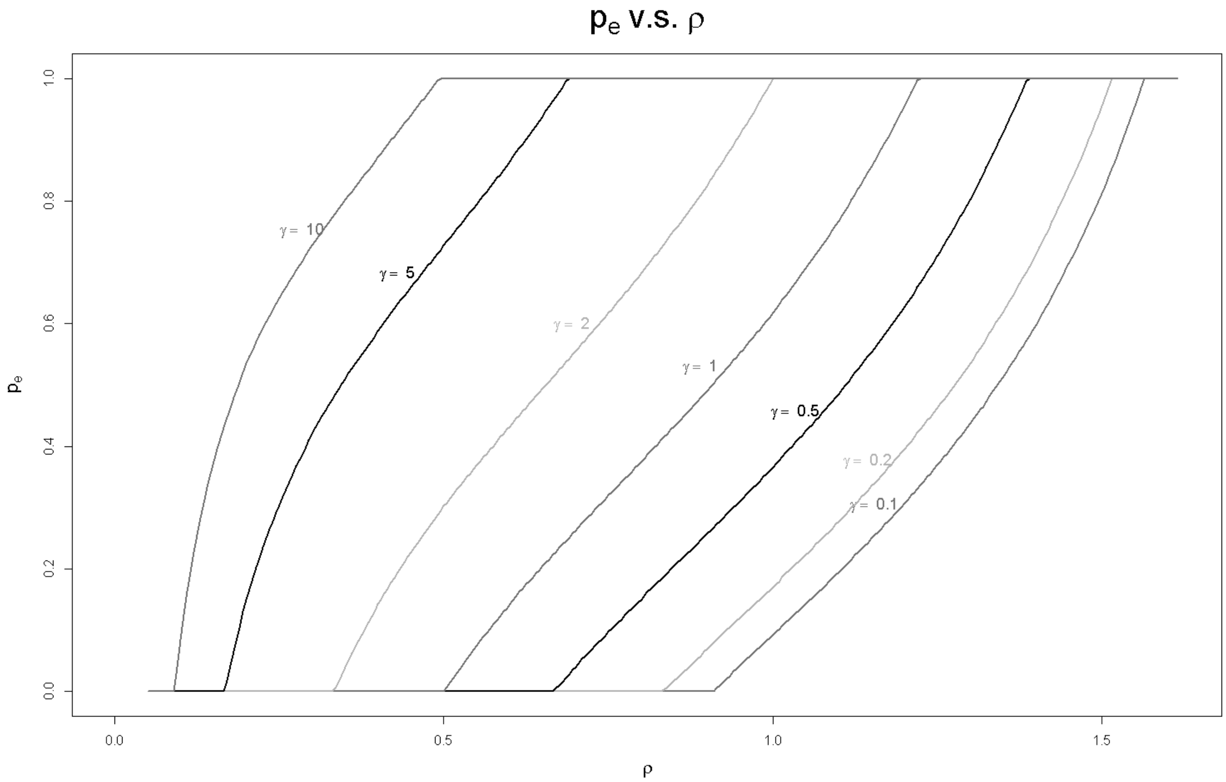
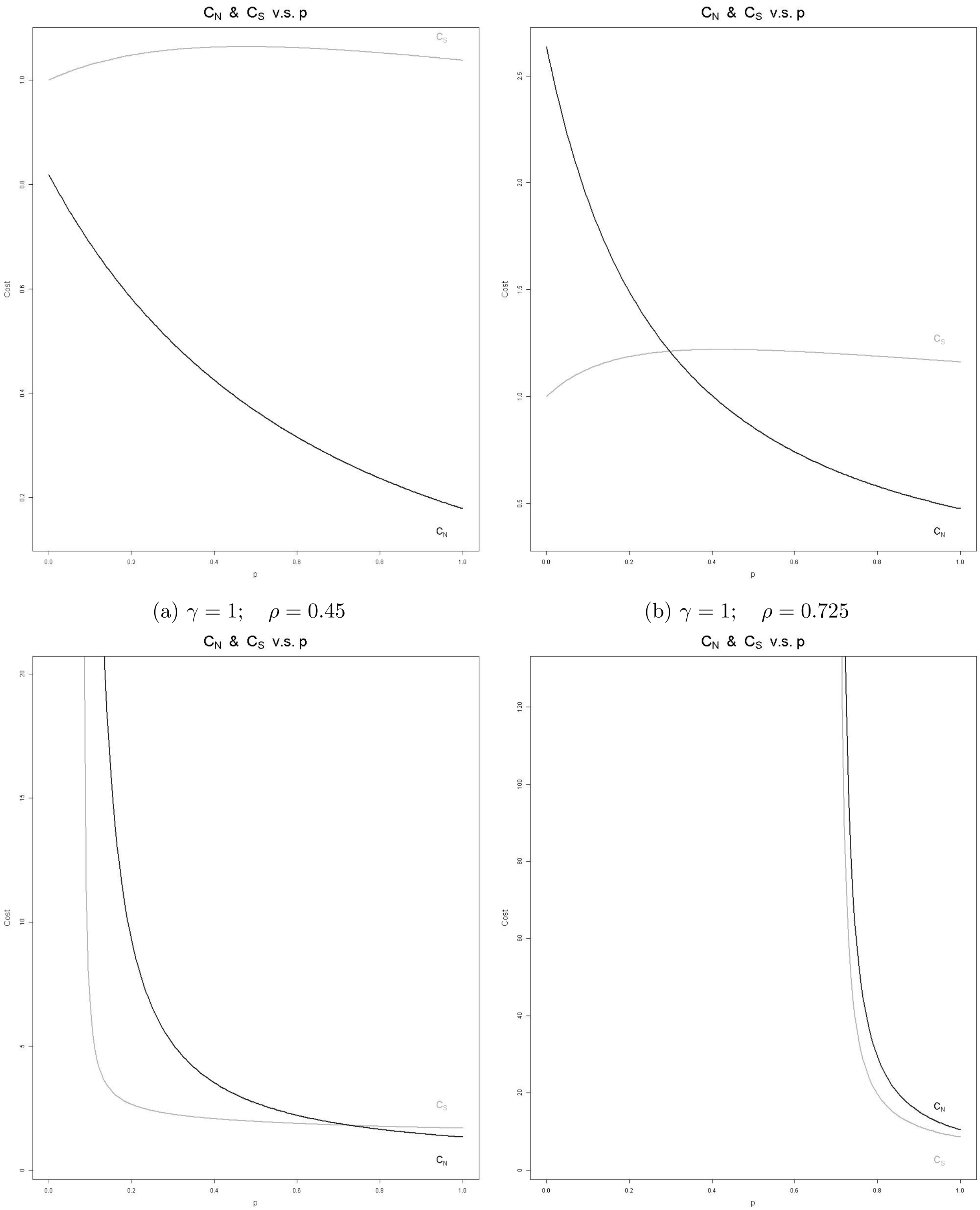


Figure 3: The equilibrium strategy p_e as a function of ρ for a various values of γ



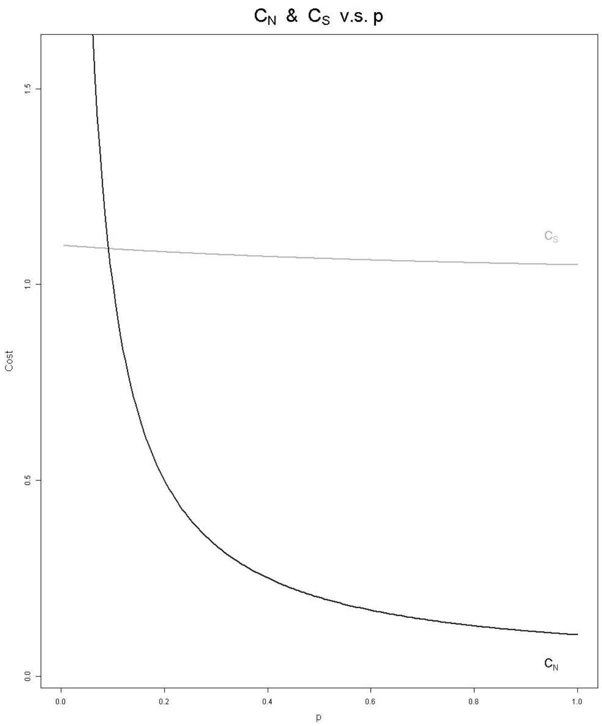
(a) $\gamma = 1; \quad \rho = 0.45$

(b) $\gamma = 1; \quad \rho = 0.725$

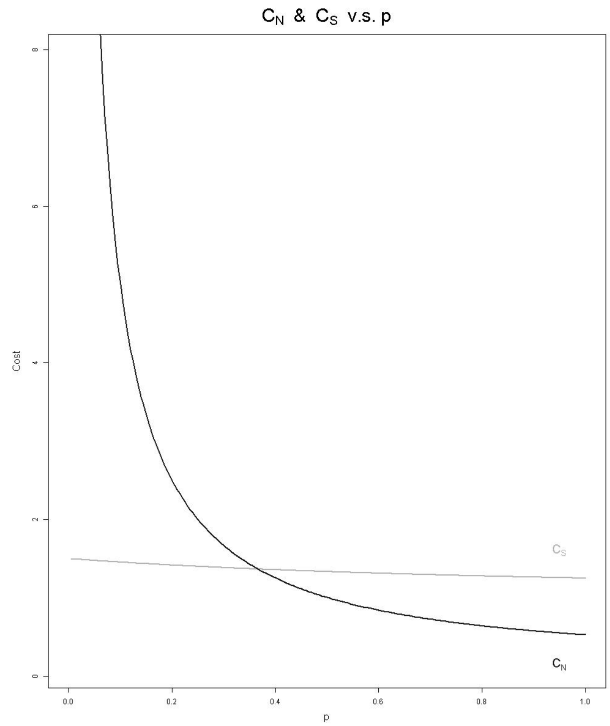
(c) $\gamma = 1; \quad \rho = 1.08$

(d) $\gamma = 1; \quad \rho = 1.515$

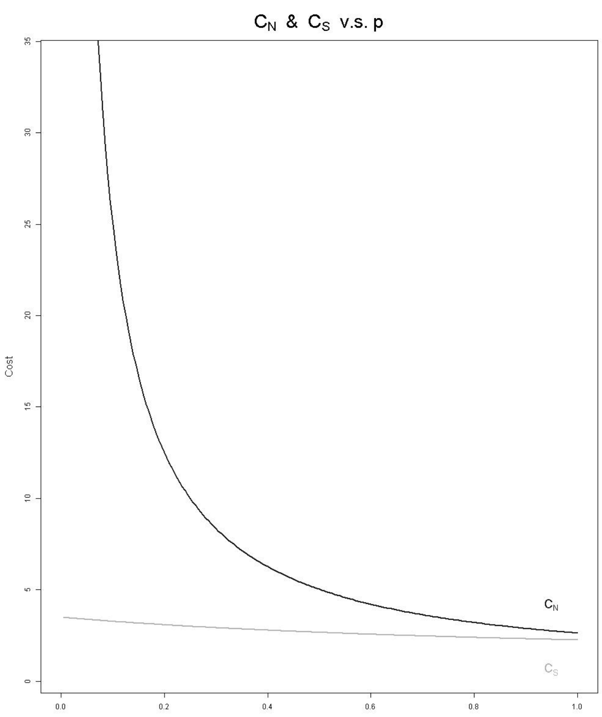
Figure 4: The expected costs of sensing and not sensing as a function of p cosidering $\gamma = 1$ and various values of ρ.



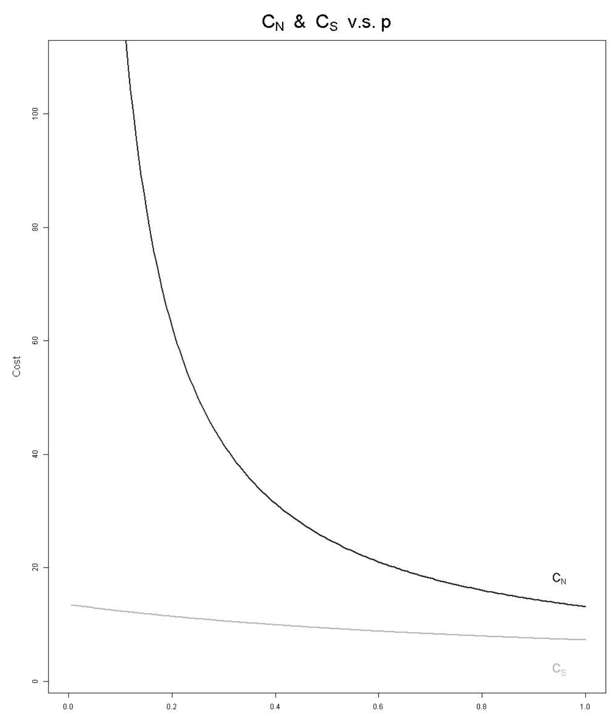
(a) $\gamma = 0.1; \quad \rho = 1$



(b) $\gamma = 0.5; \quad \rho = 1$

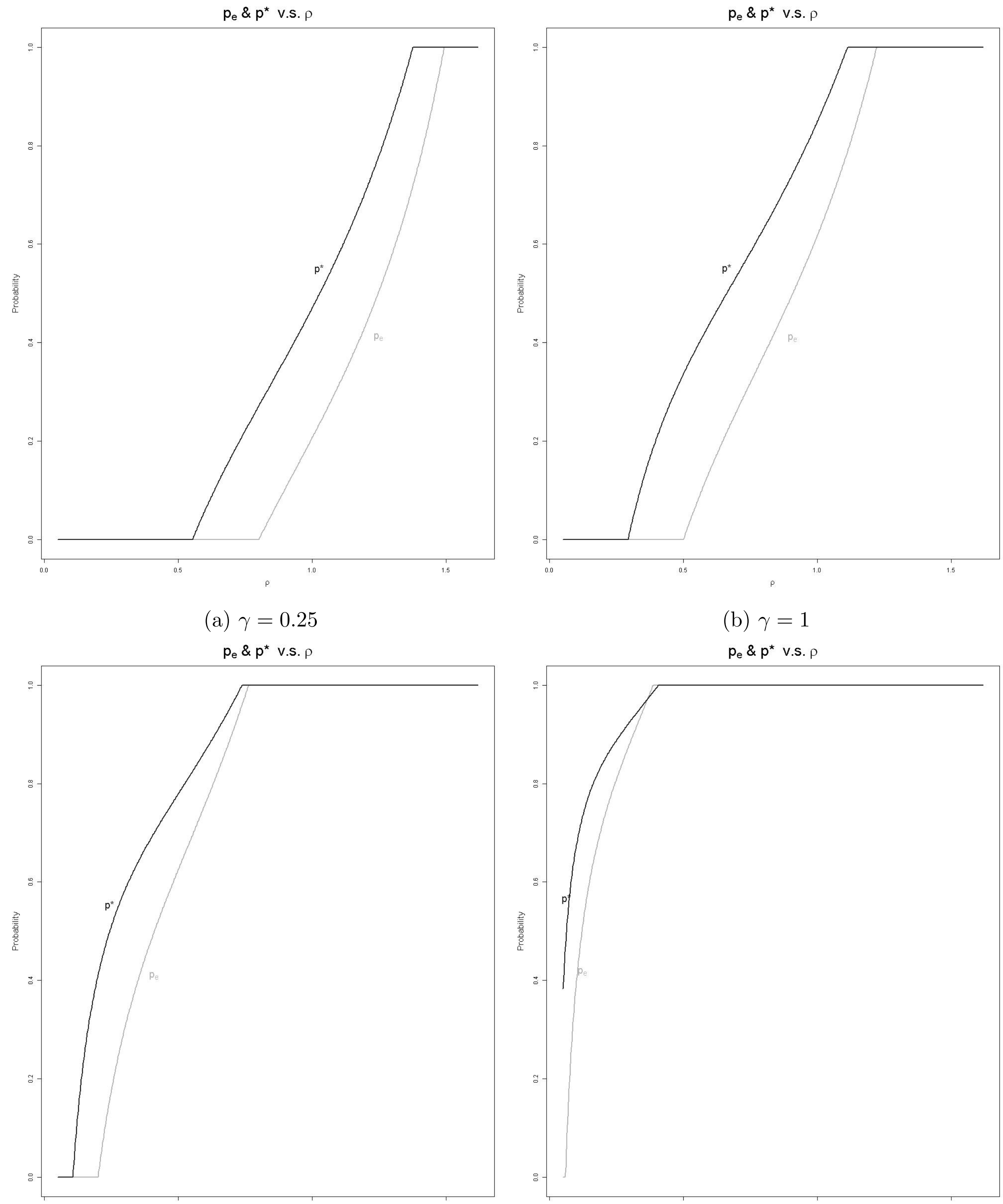


(c) $\gamma = 2.5; \quad \rho = 1$



(d) $\gamma = 12.5; \quad \rho = 1$

Figure 5: The expected costs of sensing and not sensing as a function of p cosidering $\rho = 1$ and various values of γ.



(a) $\gamma = 0.25$

(b) $\gamma = 1$

(c) $\gamma = 4$

(d) $\gamma = 16$

Figure 6: The expected costs of sensing and not sensing as a function of p cosidering various values values of ρ and γ.

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