0.1 Adjoints

Definition 1 (Adjoint)**.** A adjunction between categoriies C and D consists of functors

$$F : C \rightleftarrows D : C$$

with a natural transformation

$$\eta : 1_C \to U \circ F$$

with the property indicated in the diagram below

$$F(C) \xrightarrow{\quad g \quad} D$$

$$\begin{array}{ccc} U(F(C)) & \xrightarrow{U(g)} & U(D) \\ {\scriptstyle \eta_C}\uparrow & \nearrow_{f} & \\ C & & \end{array}$$

Every adjoint pair $F \dashv U$ with $U : D \to C$, unit $\eta : UF \to 1_C$ and counit $\epsilon : 1_D \to FU$ gives rise to a monad (T, η, μ) on C with

$$T = U \circ F : C \to C$$

$$\eta : 1 \to T$$

$$\mu = U_\epsilon F : T^2 \to T$$