Definition 1. A category consists of objects $obj(C)$ and arrows $hom(C)$

Categories focus on the relation between themselves, the arrows are the important parts. Here is an example category showing function composition with a communative diagram.

$$\begin{tikzcd} & A \arrow[dr, "g"] & \\ B \arrow[ur, "f"] \arrow[rr, "g \circ f"] & & C \end{tikzcd}$$

And a quick check of why these are called communative diagrams

$$h \circ (g \circ f) = (h \circ g) \circ f$$

Remark. For any category C there is always an identity arrow 1_C though it would clutter diagrams if it were written every time. $C \circlearrowleft_{id_C}$

There is always one unique identity homomorphism. $\exists! 1_A : A \to A$

Proof. Assume there are two unique identity morphisms from category A, 1 and $1'$ as shown in the diagram below.

$$\begin{tikzcd} A \arrow[r, shift left, "1"] \arrow[r, shift right, "1'"'] & A \end{tikzcd}$$

Then composing these two homomorphisms makes a contradiction.

$$1 = 1 \circ 1' = 1'$$

□

Remark. A homomorphism moving between a category and itself is also know as an endomorphism.

Definition 2. Small and Locally small categories
Let C be a category

- if all $hom(C)$'s together form a set, the category is small
- if hom are all sets, the category is locall small

Some examples of categories:

- SET - The category of all sets is locally small but not small

$$\mathcal{P}(\mathbb{X}) = \{A \subseteq \mathbb{X}\} = 2^{\mathbb{X}}$$

B^A :: All functions from $A \to B$