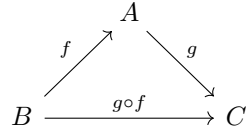


**Definition 1.** A category  $C$  consists of objects  $obj(C)$  and arrows  $hom(C)$  which satisfy

Categories focus on the relation between themselves, the arrows are the important parts. Here is an example category showing function composition with a commutative diagram.



And a quick check of why these are called commutative diagrams

$$h \circ (g \circ f) = (h \circ g) \circ f$$

*Remark.* For any category  $C$  there is always an identity arrow  $1_C$  though it would clutter diagrams if it were written every time.  $C \curvearrowright id_C$

*Proposition 1.* There is always one unique identity homomorphism.  $\exists! 1_A : A \rightarrow A$

*Proof.* Assume there are two unique identity morphisms from category  $A$ ,  $1$  and  $1'$  as shown in the diagram below.

$$A \begin{array}{c} \xrightarrow{1} \\ \xrightarrow{1'} \end{array} A$$

Then composing these two homomorphisms makes a contradiction.

$$1 = 1 \circ 1' = 1'$$

□

*Remark.* A homomorphism moving between a category and itself is also known as an endomorphism.

**Definition 2.** Small and Locally small categories Let  $C$  be a category

- if all  $hom(C)$ 's together form a set, the category is small
- if  $hom$  are all sets, the category is locally small

Some examples of categories:

- SET - The category of all sets with mappings between them is locally small but not small

$$\mathcal{P}(\mathbb{X}) = \{A \subseteq \mathbb{X}\} = 2^{\mathbb{X}}$$

$B^A$  :: All functions from  $A \rightarrow B$

- Grp - An object is a group and a map  $G \rightarrow H$  is a group homomorphism

- Ab - Abelian groups under homomorphism
- Top - Topological Spaces with continuous maps
- Vect - An object is a vector space and a map  $V \rightarrow W$  is a linear map

**Definition 3.** Let  $A$  and  $B$  be objects in a category. Then a map  $f : A \rightarrow B$  is an isomorphism if there is a map  $f^{-1} : B \rightarrow A$  (the inverse of  $f$ ) such that  $f^{-1} \circ f = Id_A$  and  $f \circ f^{-1} = Id_B$ .

If there exists an isomorphism between  $A$  and  $B$ , we say that  $A$  and  $B$  are isomorphic and write  $A \cong B$ .

*Proposition 2.* In Set, a map is an isomorphism iff it is a bijection. Two sets are isomorphic iff they have the same cardinality.