Definition 1. An object 0 is an initial object if for every object A, there is a unique map $0 \to A$

Proposition 1. Initial and terminal objects are unique up to isomorphism.

Proof. Suppose that 0 and $0'$ are both terminal or initial objects in some category C; this diagram states that 0 and $0'$ are uniquely isomorphic.

For terminal objects, apply the previous to C^{op}. □

Definition 2. Disjoint Union The disjoint union of two sets A and B is the set

$$A \sqcup B = (0, a) : a \in A \cup (1, b) : b \in B.$$

Definition 3. Coproduct Let A and B be objects in a category. Then a sum (or coproduct) of A and B is an object $A+B$ together with maps $i_0 : A \to A+B$ and $i_1 : B \to A+B$ such that whenever we have an object C and maps $f_0 : A \to C$ and $f_1 : B \to C$, there is a unique map $f : A+B \to C$ such that $f_0 = fi_0$ and $f_1 = fi_1$

$$\begin{tikzcd}
A \arrow[r, "i_0"] \arrow[dr, "f_0"'] & A+B & B \arrow[l, "i_1"'] \arrow[dl, "f_1"] \\
& C &
\end{tikzcd}$$

Theorem 0.1. Let A and B be objects and let $A \to ["i_0"]P \leftarrow ["i_1"]B$ and $A \to ["j_0"]Q \leftarrow ["j_1"]B$ be two sums of A and B. Then there exists a unique isomorphism $f : P \to Q$ such that the following diagram commutes:

$$\begin{tikzcd}
& P \arrow[dd, "f"] & \\
A \arrow[ur, "i_0"] \arrow[dr, "j_0"] & & B \arrow[ul, "i_1"'] \arrow[dl, "j_1"] \\
& Q &
\end{tikzcd}$$