

# 1 Natural Transformations

$$\mathcal{C} \begin{array}{c} \xrightarrow{F} \\ \downarrow \eta \\ \xrightarrow{G} \end{array} \mathcal{D}$$

$\eta$  is a natural transformation  $\forall C \in \text{obj}(\mathcal{C})$  we have a morphism

$$F(C) \xrightarrow{\eta_C} G(C)$$

such that given  $f : A \rightarrow B$  in  $\mathcal{C}$

$$\begin{array}{ccc} F(A) & \xrightarrow{\eta_A} & G(A) \\ \downarrow F(f) & & \downarrow G(f) \\ F(B) & \xrightarrow{\eta_B} & G(B) \end{array}$$

We also have the identity:  $\mathcal{C} \begin{array}{c} \xrightarrow{F} \\ \downarrow 1 \\ \xrightarrow{F} \end{array} \mathcal{D}$

Composition and commutativity work here too!

Exponential of Categories of Categories:  $\mathcal{D}^{\mathcal{C}}$  -wow-