## 0.1 Equalizers

**Definition 1.** Given a pair of categories and parallel morphisms of the shape

$$A \underset{g}{\overset{f}{\rightrightarrows}} B$$

The equalizer is the limit.

This means that for $f : A \to B$ and $g : A \to B$ in a category $C$, their equalizer is, if it exists

- an object $eq(f, g) \in C$
- a morphism $eq(f, g) \to x$
- such that
  - pulled back to $eq(f, g)$ both morphisms become equal
  - and $eq(f, g)$ is the universal object with this property

Examples : In $C = \text{SET}$, the equalizer of two function of sets is the subset of elements of $c$ on which both functions coincide

$$eq(f, g) = s \in c | f(s) = g(s)$$

For $C$ a category with a zero object the equalizer of a morphism $f : c \to d$ with the corresponding zero morphism is the kernel of $f$.

*Proposition* 1. A category has equalizers if it has products and pullbacks.

*Proposition* 2. If a category has products and equalizers, then it has limits.