

0.1 Duality

Proposition 1. Formal Duality For any sentence Σ in the language of category theory, if Σ follows from the axioms for categories, then so does its dual Σ^* :

$$CT \Rightarrow \Sigma \text{ implies } CT \Rightarrow \Sigma^*$$

Taking a diagram to illustrate, if this is a statement Σ

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ & \searrow g \circ f & \downarrow g \\ & & C \end{array}$$

then this is the dual statement Σ^*

$$\begin{array}{ccc} A & \xleftarrow{f} & B \\ & \nwarrow f \circ g & \uparrow g \\ & & C \end{array}$$

Note how close this is to the idea of an opposite category C^{op} .

Proposition 2. Conceptual duality For any statement Σ about categories, if Σ holds for all categories, then so does the dual statement Σ^* .