The Definition of the Definite Integral – Examples

Recall: $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$; where $\Delta x = \dfrac{b-a}{n}$ and $x_k = a + k\Delta x$

I. Which of the following is equal to $\int_2^5 x^4 dx$?

(A) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{k}{n}\right)^4\left(\frac{1}{n}\right)$

(B) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{k}{n}\right)^4\left(\frac{3}{n}\right)$

(C) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{3k}{n}\right)^4\left(\frac{3}{n}\right)$

(D) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{3k}{n}\right)^4\left(\frac{3}{n}\right)$

II. For the continuous function f, the right Riemann sum approximation for $\int_0^2 f(x)dx$ is given by the expression $\dfrac{4(n+1)(3n+2)}{n^2}$. What is the value of $\int_0^2 f(x)dx$?

(A) 1 (B) 3 (C) 6 (D) 12

III. The closed interval [a, b] is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, ..., x_n$ where $a = x_0 < x_1 < ... < x_{n-1} < x_n = b$. What is $\lim_{n \to \infty} \sum_{i=1}^{n} \sqrt{x_i} \Delta x$?

(A) $b^{\frac{1}{2}} - a^{\frac{1}{2}}$ (B) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$ (C) $\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$ (D) $\frac{3}{2}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$