This report will utilize the power method to compute the dominant eigenvector of various matrices to figure out their dominant eigenvalues using an algorithm (Figure 1).

> For $A \in M_{n\times n}(\mathbb{R})$, pick an initial vector $\vec{x}_0 \in \mathbb{R}^n$ and calculate $\vec{y}_0 = \frac{1}{||\vec{x}_0||}\vec{x}_0$.
>
> Let $\vec{x}_1 = A\vec{y}_0$ and then calculate $\vec{y}_1 = \frac{1}{||\vec{x}_1||}\vec{x}_1$.
>
> Let $\vec{x}_2 = A\vec{y}_1$ and then calculate $\vec{y}_2 = \frac{1}{||\vec{x}_2||}\vec{x}_2$.
>
> and so on.
>
> We seek convergence of $\vec{y}_m$ to some limiting vector; if such a vector exists, it must be $\vec{v}_1$, a unit eigenvector for the largest eigenvalue λ_1. We can then calculate $A\vec{v}_1$ to determine λ_1.

Figure 1: Algorithm 6.1.1 in *An Introduction to Linear Algebra For Science and Engineering*

To test and observe results, the algorithm was coded into MATLAB to find the dominant eigenvalues of the matrices in Section 6.1 A19, A20, A21, and A22 (Figure 2).

$$A19 = \begin{bmatrix} 0 & -5 & 3 \\ -2 & -6 & 6 \\ -2 & -7 & 7 \end{bmatrix} \quad A20 = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} \quad A21 = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \quad A22 = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

Figure 2: Matrices A19, A20, A21, A22

The results are as follows, with eigenvectors truncated at two decimal places.

For A19, after 500 iterations $x_{500} = \begin{pmatrix} -1.15 & -1.15 & -1.15 \end{pmatrix}$ and dominant eigenvector $y_{500} = \begin{pmatrix} 0.57 & 0.57 & 0.57 \end{pmatrix}$, which results in the dominant eigenvalue -2. However, since A19 also has eigenvalue 2, and $|-2| = 2$, it cannot satisfy the definition of a dominant eigenvalue being larger than all other eigenvalue absolute values.

For A20, after 500 iterations $x_{500} = \begin{pmatrix} 3.46 & 3.46 & 3.46 \end{pmatrix}$ and dominant eigenvector $y_{500} = \begin{pmatrix} 0.57 & 0.57 & 0.57 \end{pmatrix}$, which results in the dominant eigenvalue 6.

For A21, after 500 iterations $x_{500} = \begin{pmatrix} 2.88 & 2.88 & 2.88 \end{pmatrix}$ and dominant eigenvector $y_{500} = \begin{pmatrix} 0.57 & 0.57 & 0.57 \end{pmatrix}$, which results in the dominant eigenvalue 5.

For A22, after 500 iterations $x_{500} = \begin{pmatrix} 2.44 & 1.22 & 1.22 \end{pmatrix}$ and dominant eigenvector $y_{500} = \begin{pmatrix} 0.81 & 0.40 & 0.40 \end{pmatrix}$, which results in the dominant eigenvalue 3.

Now, it's time to apply the power method on the provided special matrix in R^{3x3} with a starting vector of $x = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$. When repeatedly multiplying the initial vector by the matrix, there is a continuous pattern where the new vector is $\frac{\pi}{6}$ degrees from the previous location. Geometrically, this forms a disc in R^3, with a unit circle based on $\frac{\pi}{6}$ on the z axis, which is rotated by $\frac{\pi}{4}$. This pattern repeats and does not converge to anything due to its circular nature, hence why there can be no dominant eigenvalue found.

From these experiments and observations, one can learn more about the applications of the power method regarding its effectiveness, limitations, and results.