8.5 Chapter Review

1. If $y = x^4$, and if x changes from 5 to 5.1, find each of the following:
   (a) dx (b) dy (c) Δy (d) Explain why $dy \neq \Delta y$.
2. Use differentials to find an approximation to each of the following. Leave your answer in fractional form. Do not use a calculator.
   (a) 10.1^3 (b) $\sqrt[4]{80}$ (c) $1001^{2/3}$
3. Find an expression for dy.
   (a) $y = -3(2x+1)^4$ (b) $y = \cos\left(e^{4x}\right)$ (c) $y = \ln\left(x^2+16\right)$
4. Evaluate each of the following integrals by sight. You may have to simplify the integrand.
   (a) $\int\left(x^2+4x-5\right)dx$ (b) $\int e^{9x}dx$ (c) $\int 3\sin 6x\,dx$ (d) $\int -\frac{2}{3}\cos 2x\,dx$
   (e) $\int\frac{8}{x}dx$ (f) $\int x^5 dx$ (g) $\int\cos 3x\,dx$ (h) $\int\left[(x-5)(x+4)\right]dx$
   (i) $\int\sin\frac{1}{2}x\,dx$ (j) $\int dx$ (k) $\int\frac{x^{10}-9x^7}{x^5}dx$ (l) $\int\frac{1}{20}dx$
   (m) $\int\sin\pi x\,dx$ (n) $\int\left(1+\sqrt{x}\right)^2 dx$ (o) $\int 5^x dx$ (p) $\int -6\cos\frac{1}{2}x\,dx$
   (q) $\int\left(x-\frac{1}{x}\right)dx$ (r) $\int\sin\frac{1}{4}x\,dx$ (s) $\int e^{x/2}dx$ (t) $\int 10^x \ln 10\,dx$
   (u) $\int\frac{x+2}{x}dx$ (v) $\int\frac{12}{x^2}dx$ (w) $\int\frac{x-10}{\sqrt[3]{x}}dx$ (x) $\int x^{-7/5}dx$
5. Evaluate each integral by using u substitution.
   (a) $\int x^2\sqrt{x^3-1}dx$ (b) $\int(2x+3)\left(x^2+3x+1\right)^{11}dx$ (c) $\int e^{\cos x}\sin x\,dx$
   (d) $\int\frac{x^{-1}}{1+\ln x}dx$ (e) $\int e^{2x}\cos\left(e^{2x}\right)dx$ (f) $\int\frac{x}{\sqrt{x^2+1}}dx$
   (g) $\int\frac{\ln x}{x}dx$ (h) $\int\frac{4x+12}{x^2+6x+1}dx$ (i) $\int\sin^2 x\cos x\cos\left(\sin^3 x\right)dx$
   (j) $\int\frac{7}{5-x}dx$ (k) $\int\sin^2 x\cos x\,dx$ (l) $\int\frac{\cos\sqrt{x}}{\sqrt{x}}dx$
   (m) $\int(x+1)e^{3x^2+6x-4}dx$ (n) $\int\frac{\left(1+\frac{1}{x}\right)^5}{x^2}dx$ (o) $\int\cot 2x\,dx$
   (p) $\int\frac{x}{\left(5x^2+2\right)^3}dx$ (q) $\int 12x\sqrt[5]{1-x^2}dx$ (r) $\int\frac{\sqrt{\ln x}}{x}dx$

5. (Continued)

(s) $\int \frac{\cos x}{\sin^3 x}dx$ (t) $\int \frac{1+\cos x}{(x+\sin x)^2}dx$ (u) $\int\left(\frac{1}{x-2}+\frac{1}{x+2}\right)dx$

(v) $\int \frac{x}{x^4+2x^2+1}dx$ (w) $\int e^{(2x+1)^3}(2x+1)^2dx$ (x) $\int \frac{3x+2\cos x}{3x^2+4\sin x}dx$

6. Evaluate each of the following definite integrals.

(a) $\int_4^5 (4x-1)dx$ (b) $\int_0^{\pi/2} \sin\frac{1}{2}x dx$ (c) $\int_{-4}^{-2} e^{-2x}dx$ (d) $\int_e^{e^4} \frac{1}{x}dx$

(e) $\int_1^9 \sqrt{x}dx$ (f) $\int_0^{\pi/3} \frac{\sin x}{\cos^3 x}dx$ (g) $\int_{-3}^{-2} \frac{dx}{(x+5)^2}$ (h) $\int_1^{2\sqrt{2}} x\sqrt{x^2+8}dx$

7. Suppose that the rate at which water is draining from a tank (in litres per minute) is a function of t, the number of minutes that have passed since the drain valve was opened. If this function is $w(t)=100-\frac{1}{9}t^2$ for $0\le t\le 30$, then $\int_a^b w(t)dt$ gives the total number of litres of water that have drained in the time period $a\le t\le b$. How many litres of water drained from the tank between $t=6$ and $t=12$?

8. Evaluate each integral using the hint provided.

(a) $\int \frac{1}{x+\sqrt{x}}dx$ (Hint: let $u=\sqrt{x}$. Then $u^2=x$ and thus $dx=2udu$.)

(b) $\int \frac{x}{\sqrt{2x-1}}dx$ (Hint: let $u=\sqrt{2x-1}$.)

Attempt questions 9 to 11 if you studied sections 7.5 and 7.6.

9. Integrate by sight.

(a) $\int \frac{4}{1+x^2}dx$ (b) $\int \frac{2}{\sqrt{1-x^2}}dx$ (c) $\int \sec^2\frac{1}{2}x dx$ (d) $\int 60\csc 30x\cot 30x dx$

(e) $\int \csc^2 5x dx$ (f) $\int \frac{dx}{1+4x^2}$ (g) $\int 4\csc^2\frac{1}{3}x dx$ (h) $\int 2\sec 3x\tan 3x dx$

10. Integrate by u substitution.

(a) $\int \frac{dx}{9+4x^2}$ (b) $\int \frac{dx}{\sqrt{1-100x^2}}$ (c) $\int x^2\csc^2\left(x^3\right)dx$

(d) $\int e^{\tan 2x}\sec^2 2x dx$ (e) $\int \frac{\cot^3 x}{\sin^2 x}dx$ (f) $\int \sqrt[3]{\tan x}\sec^2 x dx$

11. Evaluate each of the following definite integrals.

(a) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ (b) $\int_{\sqrt{3}/3}^{\sqrt{3}} \frac{dx}{1+x^2}$ (c) $\int_0^{\pi/8}\sec^2 2x dx$ (d) $\int_{\pi/6}^{\pi/3}\sec x\tan x dx$

9.4 Chapter Review

Find the area of the region enclosed by the given lines and/or curves.

1. $y=\frac{12}{x^2}$, $x=1$, $x=3$, the x-axis.
2. $y=x^2-x-6$, the x-axis.
3. $y=5$, $y=4-x^2$, $x=0$, $x=3$
4. $y=2\sqrt[3]{x}$, the x-axis, $x=1$, $x=8$
5. $y=3x^2+2$, $x=-1$, $x=2$, the x-axis
6. $y=11-x^2$, $y=x^2-4x+5$
7. $y=\sqrt{\sin x}\cos x$, $x=0$, $x=\pi/2$, the x-axis
8. $y=\sqrt{x+1}$, $x=0$, $x=8$, the x-axis
9. $y=2-x^2$, $y=-x$
10. $y=x^4-1$, the x-axis
11. $y=e^{x/2}$, $x=2$, $x=6$, the x-axis
12. $y=e^{2x}$, $y=e^{3x}$, $x=0$, $x=1$
13. $y=x^3+4x^2$, $y=4x+16$
14. $y=x-2$, $x=y^2$
15. Find the area of the region in the first quadrant below $y=x^2$, below $y=12-4x$, and above the x-axis.
16. (a) Find the area bounded by the curve $y=e^{-x}$, the x-axis, the lines $x=1$ and $x=b$ where $b>1$.
    (b) What is the limit of the area in part (a) as $b\to\infty$?
17. Suppose that your hourly rate of pay is given by the function $r(x)=6\sqrt{x+1}$ where x is the number of hours you have worked. The area under this curve and above the x-axis, between $x=0$ and $x=k$, would give you the total amount of money you have earned after working k hours.
    (a) What is your hourly rate of pay after working 15 hours?
    (b) How much money would you have earned after working 15 hours?
    (c) How much longer would you have to work to double the amount you have earned after 15 hours? Round your answer to two decimal places.

ANSWERS 8.3 (Continued)

44. $\frac{1}{2}\sin^2 x + C$, also $-\frac{1}{2}\cos^2 x + C$ **45.** $\ln|\sin x| + C$ **46.** $-e^{1/x} + C$ **47.** $-\frac{1}{3}\left(x^2+1\right)^{-3/2} + C$

48. $\sin x^3 + C$ **49.** $\frac{1}{4}e^{x^4-20x} + C$ **50.** $\frac{1}{2}e^{\sin 2t} + C$ **51.** $\frac{1}{5}(1+\sin x)^5 + C$ **52.** $\frac{1}{2}\left(x^{4/3}+1\right)^{3/2} + C$

53. $\ln|1+\sin x| + C$ **54.** $\ln\left(e^x - e^{-x}\right) + C$ **55.** $\tan^{-1}\left(e^x\right) + C$ **56.** $\frac{1}{5}\sin^{-1}(5x) + C$ **57.** $\frac{1}{3}\tan x^3 + C$

58. $-\frac{1}{4}\cot^4 x + C$ **59.** $\frac{1}{6}\sec^3 2x + C$ **60.** $\frac{2}{5}\tan^{-1}\left(\frac{1}{5}x\right) + C$ **61.** $-\frac{1}{2}\cot^2 e^x + C$ **62.** $\tan^{-1}(\sin x) + C$

63. $\frac{2}{5}(x-2)^{5/2} + \frac{4}{3}(x-2)^{3/2} + C$ or $\frac{2(x-2)^{3/2}(3x+4)}{15} + C$

64. $\frac{1}{7}\left(1+x^2\right)^{7/2} - \frac{2}{5}\left(1+x^2\right)^{5/2} + \frac{1}{3}\left(1+x^2\right)^{3/2} + C$ or $\frac{\left(x^2+1\right)^{3/2}\left(15x^4-12x^2+8\right)}{105} + C$ **65.** $-\frac{\sqrt{1+x^2}}{x} + C$

ANSWERS 8.4

1. 30 **2.** $\frac{27}{2}$ **3.** $\frac{208}{3}$ **4.** 24 **5.** $-\frac{80}{3}$ **6.** $\frac{3}{5}$ **7.** $\frac{1}{4}$ **8.** $\ln 3 + 4$ **9.** $\frac{46}{15}$ **10.** 1 **11.** $\frac{\sqrt{2}}{2} + 1$ **12.** $e^6 - e^2$ **13.** 4

14. 3 **15.** $\frac{1}{4}$ **16.** $\ln 6 - \ln 3 = \ln\left(\frac{6}{3}\right) = \ln 2$ **17.** 2 **18.** $\frac{14}{3}$ **19.** $\frac{1076}{15}$ **20.** $\frac{e^{-6}-e^{-9}}{3}$ **21.** $\frac{\pi}{4}$ **22.** $\frac{\pi}{6}$ **23.** $\frac{1}{2}$

24. $\frac{2\sqrt{3}}{9}$ **25.** $2-\sqrt{2}$ **26.** $\sqrt{3}-1$ **27.** $\frac{\pi}{2}$ **28.** $4-2\sqrt{2}$ **29.** $\frac{21}{4}$ **30.** $\frac{1}{2}e^3 - \frac{1}{2}e^{-1}$ **31.** $\frac{1}{4}$ **32.** $\frac{13}{81}$ **33.** 14

34. $\ln\left(\frac{3}{7}\right)$ **35.** $e-1$ **36.** $\sin 1 + \sin\left(\frac{1}{2}\right)$; recall $\sin(-\theta) = -\sin\theta$ **37.** The function is not continuous on the interval $[-2,8]$, and thus definite integration is invalid. Even though an answer was obtained, you will see that it has no meaning as we study areas under curves in chapter 9. **38.** (a) 21 (b) 19.3°C

39. (a) \$8400/year (b) \$1200/year (c) \$23 400 (d) \$12 600 **40.** $\frac{1}{8}$

ANSWERS 8.5

1. (a) 0.1 (b) 50 (c) 51.5201 (d) dy is the change in y as you move from $x=5$ to $x=5.1$ along the tangent line drawn at $x=5$. Δy is the change in y as you move from $x=5$ to $x=5.1$ along the function $y=x^4$. Since the tangent line will not coincide with the function, dy is just an approximation to Δy.

2. (a) 1030 (b) $3-\frac{1}{108}$ or $2\frac{107}{108}$ (c) $100\frac{1}{15}$ **3.** (a) $-24(2x+1)^3\,dx$ (b) $-4e^{4x}\sin\left(e^{4x}\right)dx$

(c) $\frac{2x}{x^2+16}dx$ **4.** (a) $\frac{1}{3}x^3 + 2x^2 - 5x + C$ (b) $\frac{1}{9}e^{9x} + C$ (c) $-\frac{1}{2}\cos 6x + C$ (d) $-\frac{1}{3}\sin 2x + C$

(e) $8\ln|x| + C$ (f) $\frac{1}{6}x^6 + C$ (g) $\frac{1}{3}\sin 3x + C$ (h) $\frac{1}{3}x^3 - \frac{1}{2}x^2 - 20x + C$ (i) $-2\cos\frac{1}{2}x + C$ (j) $x + C$

(k) $\frac{1}{6}x^6 - 3x^3 + C$ (l) $\frac{1}{20}x + C$ (m) $-\frac{1}{\pi}\cos\pi x + C$ (n) $x + \frac{4}{3}x^{3/2} + \frac{1}{2}x^2 + C$ (o) $\frac{5^x}{\ln 5} + C$

(p) $-12\sin\frac{1}{2}x + C$ (q) $\frac{1}{2}x^2 - \ln|x| + C$ (r) $-4\cos\frac{1}{4}x + C$ (s) $2e^{x/2} + C$ (t) $10^x + C$ (u) $x + 2\ln|x| + C$

(v) $-12x^{-1} + C$ (w) $\frac{3}{5}x^{5/3} - 15x^{2/3} + C$ (x) $-\frac{5}{2}x^{-2/5} + C$ **5.** (a) $\frac{2}{9}\left(x^3-1\right)^{3/2} + C$

Answers

Answers

ANSWERS 8.5 (Continued)

5. (b) $\frac{1}{12}\left(x^2+3x+1\right)^{12}+C$ (c) $-e^{\cos x}+C$ (d) $\ln\left(1+\ln x\right)+C$ (e) $\frac{1}{2}\sin\left(e^{2x}\right)+C$ (f) $\sqrt{x^2+1}+C$

(g) $\frac{1}{2}\left(\ln x\right)^2+C$ (h) $2\ln\left|x^2+6x+1\right|+C$ (i) $\frac{1}{3}\sin\left(\sin^3 x\right)+C$ (j) $-7\ln\left|5-x\right|+C$ (k) $\frac{1}{3}\sin^3 x+C$

(l) $2\sin\sqrt{x}+C$ (m) $\frac{1}{6}e^{3x^2+6x-4}+C$ (n) $-\frac{1}{6}\left(1+\frac{1}{x}\right)^6+C$ (o) $\frac{1}{2}\ln\left|\sin 2x\right|+C$ (p) $-\frac{1}{20}\left(5x^2+2\right)^{-2}+C$

(q) $-5\left(1-x^2\right)^{6/5}+C$ (r) $\frac{2}{3}\left(\ln x\right)^{3/2}+C$ (s) $-\frac{1}{2}\left(\sin x\right)^{-2}+C$ (t) $-\left(x+\sin x\right)^{-1}+C$

(u) $\ln\left|x-2\right|+\ln\left|x+2\right|+C$ or $\ln\left|x^2-4\right|+C$ (v) $-\frac{1}{2}\left(x^2+1\right)^{-1}+C$ (w) $\frac{1}{6}e^{(2x+1)^3}+C$

(x) $\frac{1}{2}\ln\left|3x^2+4\sin x\right|+C$ 6. (a) 17 (b) $2-\sqrt{2}$ (c) $\frac{e^8-e^4}{2}$ (d) 3 (e) $\frac{52}{3}$ (f) $\frac{3}{2}$ (g) $\frac{1}{6}$ (h) $\frac{37}{3}$ 7. 544

8. (a) $2\ln\left|\sqrt{x}+1\right|+C$ (b) $\frac{1}{6}\left(2x-1\right)^{3/2}+\frac{1}{2}\left(2x-1\right)^{1/2}+C$ or $\frac{1}{3}\sqrt{2x-1}\left(x+1\right)+C$ 9. (a) $4\tan^{-1}\left(x\right)+C$

(b) $2\sin^{-1}x+C$ (c) $2\tan\frac{1}{2}x+C$ (d) $-2\csc 30x+C$ (e) $-\frac{1}{5}\cot 5x+C$ (f) $\frac{1}{2}\tan^{-1}2x+C$

(g) $-12\cot\frac{1}{3}x+C$ (h) $\frac{2}{3}\sec 3x+C$ 10. (a) $\frac{1}{6}\tan^{-1}\left(\frac{2}{3}x\right)+C$ (b) $\frac{1}{10}\sin^{-1}10x+C$ (c) $-\frac{1}{3}\cot\left(x^3\right)+C$

(d) $\frac{1}{2}e^{\tan 2x}+C$ (e) $-\frac{1}{4}\cot^4 x+C$ (f) $\frac{3}{4}\left(\tan x\right)^{4/3}+C$ 11. (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{1}{2}$ (d) $2-\frac{2\sqrt{3}}{3}$

ANSWERS 9.1

1. $26/3$ u^2 2. 8 u^2 3. $\ln 3$ u^2 4. e^2-e^{-2} u^2 5. 1 u^2 6. $14/3$ u^2 7. $\frac{3}{2}$ u^2 8. $1/2$ u^2 9. $125/3$ u^2

10. $\frac{e^4-1}{2}$ u^2 11. $10\ln 5$ u^2 12. 2π u^2 13. 6 14. The parabola has equation $y=-\frac{4h}{b^2}x^2+h$. The area is $\int_{-b/2}^{b/2}\left(-\frac{4h}{b^2}x^2+h\right)dx=\left[-\frac{4h}{3b^2}x^3+hx\right]_{-b/2}^{b/2}=\frac{2}{3}bh$ 15. $\frac{1}{2}\pi r^2$ 16. 400 km

ANSWERS 9.2

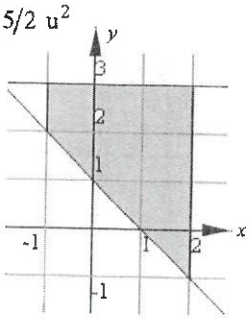
1. 25 u^2 2. 63 u^2 3. $28/3$ u^2 4. $4/3$ u^2 5. $\ln 2$ u^2 6. $3/2$ u^2 7. $\frac{e^2-e^{-2}}{2}$ u^2 8. $13/3$ u^2 9. 6 u^2

10. $2\ln 7$ u^2 11. $1562/5$ u^2 12. $e-e^{-1}$ u^2 13. 13 u^2 14. $25/2$ u^2 15. $34/3$ u^2 16. $5/4$ u^2

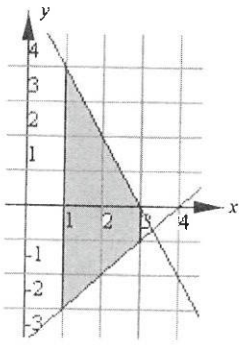
17. $e^{-1}+e^3-4$ u^2 18. $31/3$ u^2 19. $1/6$ u^2 20. $131/4$ u^2 21. 8 u^2

ANSWERS 9.3

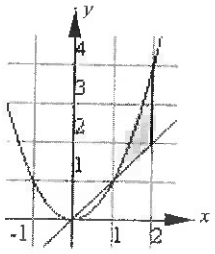
1. $15/2$ u^2



2. 8 u^2

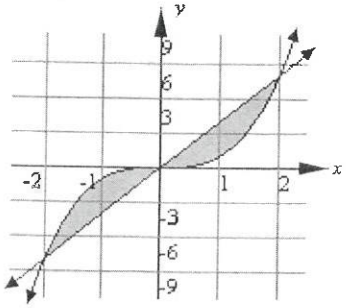


3. $5/6$ u^2

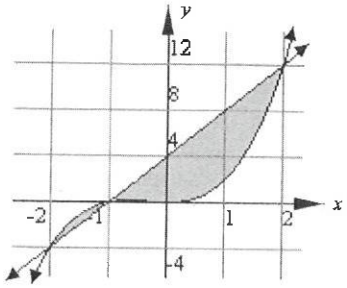


ANSWERS 9.3 (Continued)

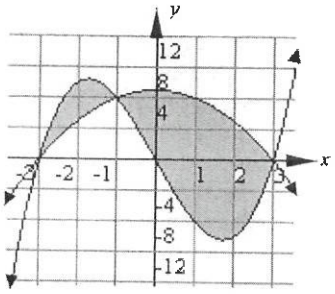
16. $8\ u^2$



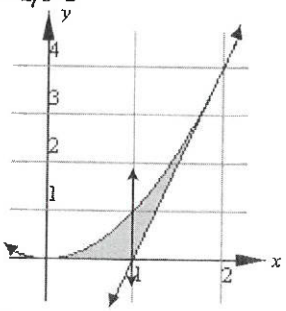
17. $71/6\ u^2$



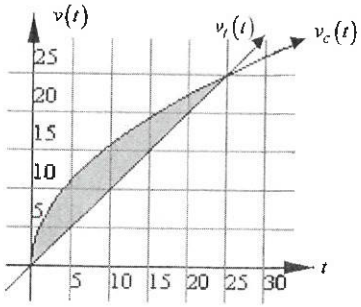
18. $148/3\ u^2$



19. $\int_0^1 x^2\,dx + \int_1^2 \left[x^2 - (4x-4)\right]dx$
$= 2/3\ u^2$



20. **(a)** 25 s **(b)** The car is 625/6 metres ahead of the truck.



ANSWERS 9.4

1. $8\ u^2$ **2.** $125/6\ u^2$ **3.** $12\ u^2$ **4.** $45/2\ u^2$ **5.** $15\ u^2$ **6.** $64/3\ u^2$ **7.** $2/3\ u^2$ **8.** $52/3\ u^2$ **9.** $9/2\ u^2$

10. $8/5\ u^2$ **11.** $2e^3 - 2e\ u^2$ **12.** $\frac{e^3}{3} - \frac{e^2}{2} + \frac{1}{6}\ u^2$ **13.** $148/3\ u^2$ **14.** $9/2\ u^2$ **15.** $14/3\ u^2$

16. **(a)** $\frac{1}{e} - \frac{1}{e^b}\ u^2$ **(b)** $\frac{1}{e}\ u^2$ **17.** **(a)** \$24/h **(b)** \$252 **(c)** 9.27 hours