Introduction

Discrete Mathematics

Andrei Bulatov

Course Info

- **Instructor: Andrei Bulatov**
  - Email: abulatov@sfu.ca
  - Room: TASC 8013
  - Office hours (tentative):
    Monday 11:00 – 12:00 (from Sep 13th) ONLINE
    Wednesday 11:00 – 12:00 (from Sep 22nd) ONLINE

- **Teaching Assistants:**
  - Aditya Bhadreshkumar Panchal, email: abp3@sfu.ca
  - Mona Shahsavari, email: mshahsav@sfu.ca

- **Course webpage**
  - https://canvas.sfu.ca/courses/66056
  - Common email (instructor+TAs): macm101-fall2021-d100@sfu.ca
    (accessible from your SFU email)

Remote instruction

- **Lectures:**
  - Mo 8:30 – 9:20 in AQ 3182
  - Th 8:30 – 10:20 in RCB Image Theater
  - Old recordings will be available online
- **Tutorials:**
  - Problems will be posted in advance. Try to solve them
- **Office hours**
  - Live through Zoom. Links will be posted
- **Quizzes**
  - Online on Canvas
- **Midterm and final exam**
  - In person

Course Info

- **Course objective:**

  To introduce basic concepts and applications of discrete mathematics.

- **Syllabus:**
  - Logic and Formal Reasoning
  - Set Theory, Functions and Relations
  - Mathematical Induction
  - Combinatorics
  - Number Theory

Course Info

- **Textbook:**

  R. P. Grimaldi, *Discrete and Combinatorial Mathematics (an Applied Introduction)*, Addison-Wesley, 2004.

  - It is impossible to finish studying all the contents of the textbook in one semester. The contents not covered in lectures/slides are not required.
  - The content and order of topics, as presented in the class, do not one-to-one correspond to any part of the book. Use of Subject Index is advised.
  - In few cases the notation and terminology in the class differs from that in the book

Course Info

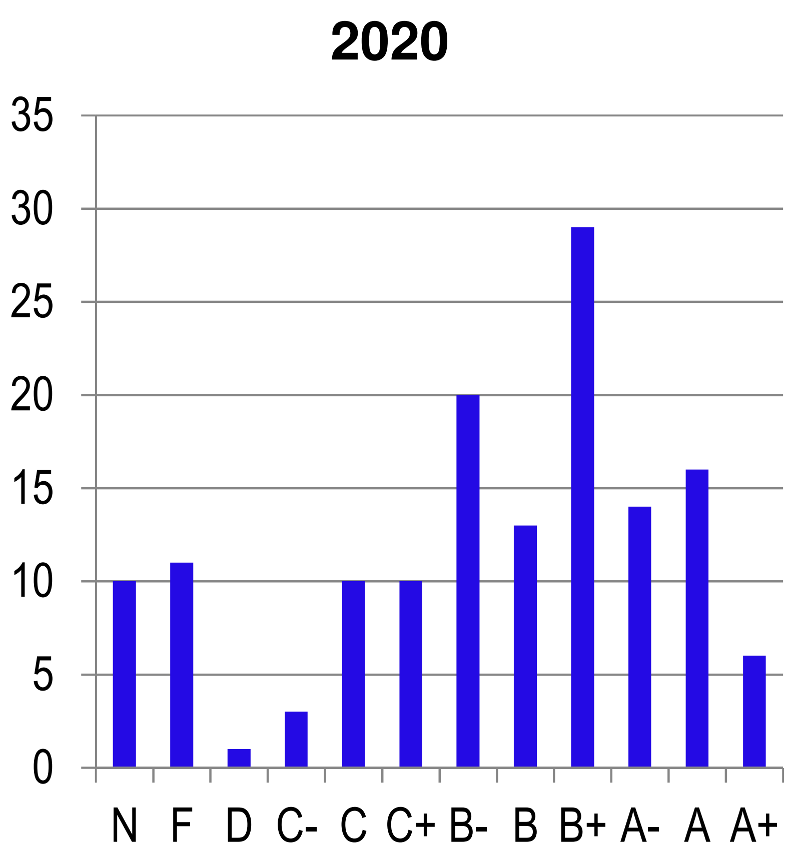
- **References:**
  - H. Rosen, *Discrete Mathematics and Its Applications*, 7/E, McGraw-Hill, 2012.
  - R. L. Graham; D. E. Knuth; and O. Patashnik, *Concrete Mathematics*, Addison-Wesley, Reading, MA, 1994
  - T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms*, 2nd Edition, MIT Press, Cambridge, MA, 2001.
  - G. Andrews, *Number theory*, Saunders or Dover Publications, Inc.
  - H. Enderton, *A Mathematical Introduction to Logic*, Harcourt/Academic Press, 2001

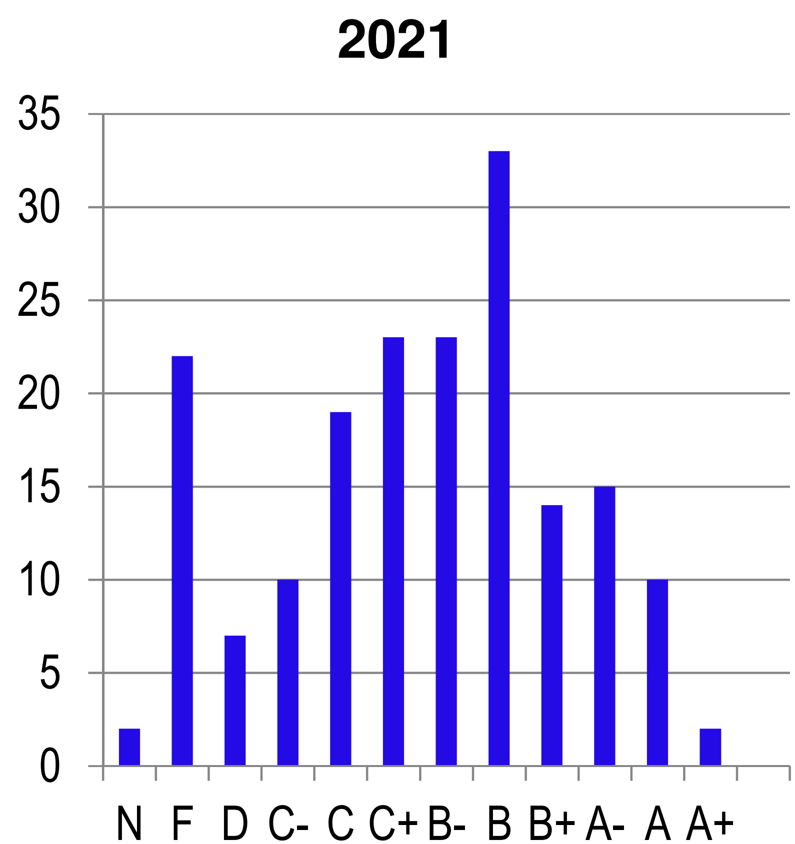
Course Info

- **Grading:**
  - 12 Tutorials $(12 \times 0.5\%)$
  - 10 Assignments $(10 \times 2\%)$
  - 4 Quizzes $(4 \times 8\%)$
  - 1 Midterm 12%
  - 1 Final Exam 30%

Course Info

- **Previous results:**





Prerequisites

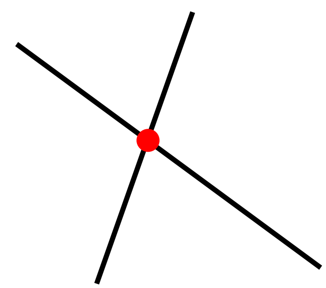
- Not much of specific knowledge
- Some general knowledge is needed, as there will be examples
- Modest math erudition (e.g., 5th Euclid's postulate, see next slide)
- Basics

$$2 \times 2 = 4$$

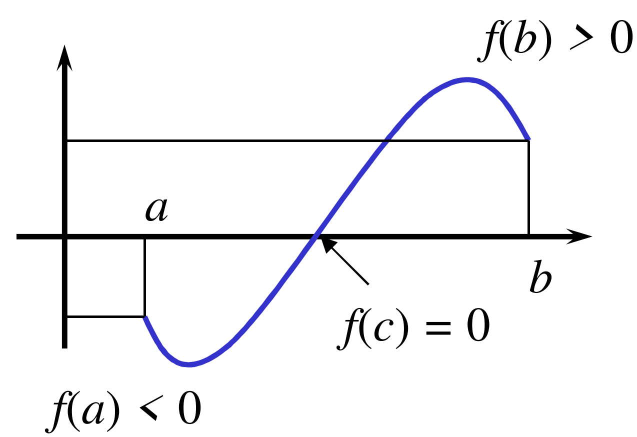
$$7 \times 8 = \cancel{55}\ 56$$

Two Mathematics

- **Continuous Mathematics**
  - Fifth Euclid's Postulate



  - Intermediate value theorem



Continuous Mathematics (cntd)

- Laws of Physics

$$\vec{F} = m\frac{d^2X}{dt^2}$$ Newton's second law of motion

$$\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}$$ Maxwell's law of electromagnetism

- Disciplines: geometry, calculus, differential equations, topology, …

- Applications: physics, engineering, astronomy, …

Discrete Mathematics

- Discrete Mathematics comprises all branches of mathematics that do not use the idea of continuity.

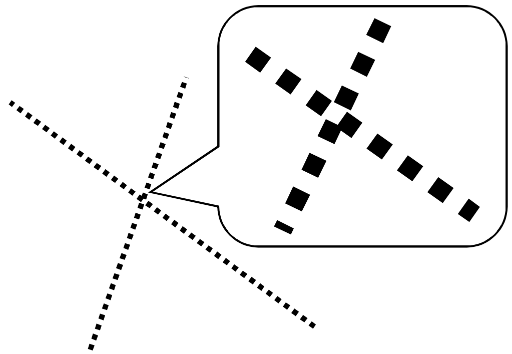
- `Formal' definition (Wikipedia):

  Discrete mathematics, sometimes called **finite mathematics**, is the study of mathematical structures that are fundamentally discrete, in the sense of not supporting or requiring the notion of continuity. Most, if not all, of the objects studied in finite mathematics are countable sets, such as the integers.

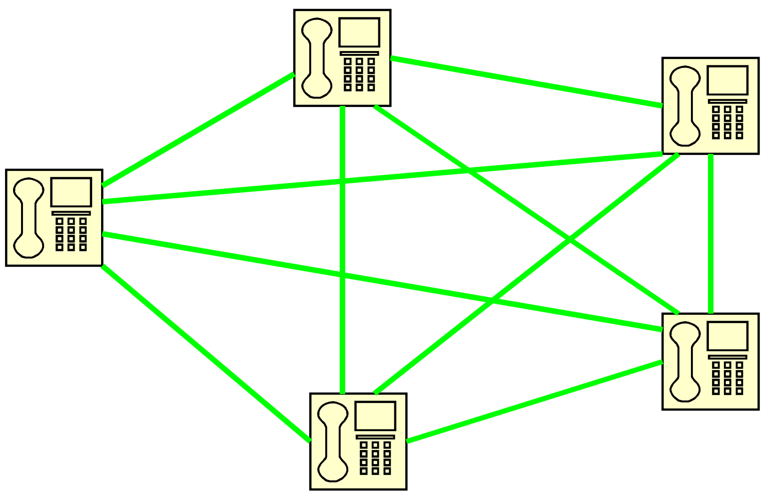
  For contrast, see continuum, topology, and mathematical analysis

Discrete Mathematics (cntd)

- Removing continuity
  - Discrete fifth Euclid's Postulate (???)

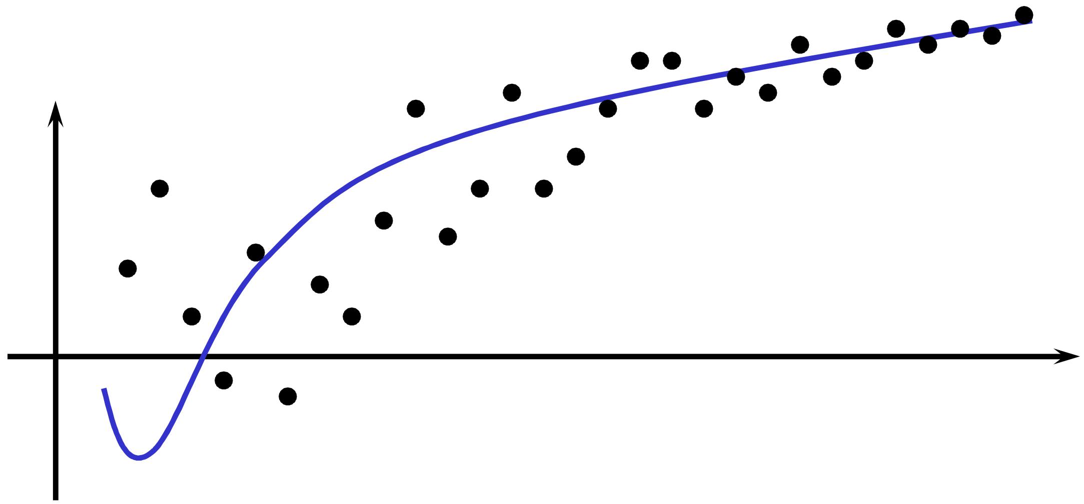


  - Graphs



Discrete Mathematics (cntd)

- Asymptotics



Properties of a discrete objects are `approximated' using a continuous function

- Laws of discrete mathematics:

$$((\forall x\, F(X) \to G(X)) \wedge F(a)) \to G(a)$$

The rule of universal specification

$$X^n + Y^n = Z^n$$ does not hold for any $n > 2$ and integer X, Y, Z

Great Fermat's Theorem

Topics in DM

- Wikipedia says that Discrete mathematics usually includes:
  - logic - a study of reasoning
  - set theory - a study of collections of elements
  - number theory
  - combinatorics - a study of counting
  - graph theory
  - algorithmics - a study of methods of calculation
  - information theory
  - the theory of computability and complexity - a study on theoretical limitations on algorithms …
- algebra – a study of algebraic systems (Bulatov)
- discrete probability theory (Grimaldi)

This is too much for us !!

DM at SFU:

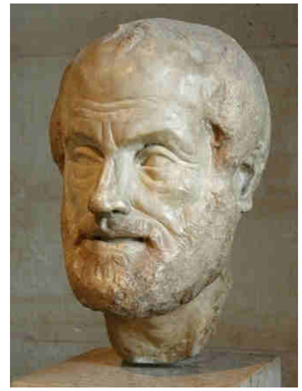
- MATH-232 Elementary linear algebra
- CMPT-413 Computational linguistics
- CMPT-379 Principles of compiler design
- CMPT-384 Symbolic computing
- CMPT-307 Data structures and algorithms
- CMPT-308 Computability and complexity
- CMPT-405 Design and analysis of computing algorithms
- CMPT-406 Computational geometry
- CMPT-407 Computational complexity
- CMPT-408 Theory of computer networks / communications
- MACM-300 Introduction to formal languages and automata
- MACM-401 Symbolic computation
- CMPT-477 Introduction to formal verification
- CMPT-705 Design and analysis of algorithms
- CMPT-706 Parallel algorithms
- CMPT-710 Computational complexity
- CMPT-725 Logical methods in computational intelligence
- CMPT-813 Computational geometry
- CMPT-815 Algorithms of optimization
- CMPT-816 Theory of communication networks
- CMPT-721 Knowledge representation and reasoning
- CMPT-814 Algorithmic graph theory

Our goal

is to learn basic concepts and terminology that provide basis and common language for those and many other courses.

Logic

- Formal logic, syllogisms



Aristotle
384 - 322 B.C.

- Mathematical logic, formal reasoning

George Boole
1815 - 1864

- Computational logic, formal verification

Pentium FDIV bug
1994

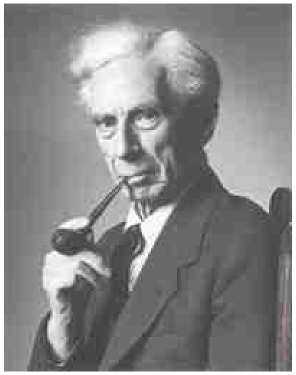
- Other applications: artificial intelligence, robotics, software verification, automated theorem proving, …

Set theory

- Naïve set theory

Georg Cantor
1845 - 1918

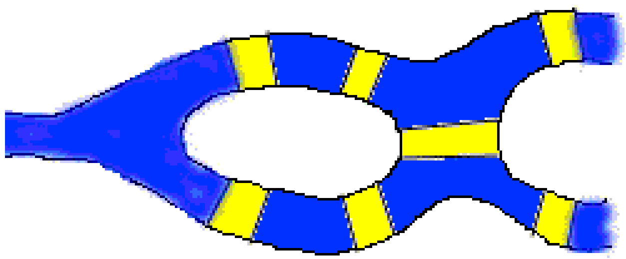
- Axiomatic set theory

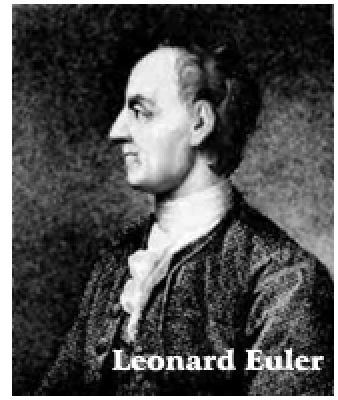


Bertrand Russell
1872 - 1970

Graphs

- Toy graph theory
  Konigsberg 7-bridge problem





Leonard Euler
1707 - 1783

- Other applications: modeling of nearly everything,
  electric circuits, networking, linguistics, data storage, coding theory, games, scheduling, combinatorial algorithms, …

- One more face

Paul Erdös
1913 - 1996

Number Theory

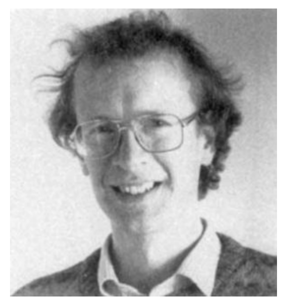
- Arithmetic (*Arithmetica*)

?

Diophantus
200 - 284

Pierre de Fermat
1608 - 1672

- Number theory



Andrew Wiles
1953 - ?

- Algebraic geometry
- Other applications: cryptography