Rates and Integrals as Net Change Examples

1. A function $f(t)$ gives the rate of evaporation of water, in liters per hour, from a pond, where t is measured in hour since 12 noon. Which of the following gives the meaning of $\int_4^{10} f(t)dt$ in the context described.

(A) The total volume of water, in liters, that evaporated from the pond during the first 10 hours after 12 noon.

(B) The total volume of water, in liters, that evaporated from the pond between 4 p.m. and 10 p.m.

(C) The net change in the rate of evaporation, in liters per hour, from the pond 4 p.m. and 10 p.m.

(D) The average rate of evaporation, in liters per hour, from the pond between 4 p.m. and 10 p.m.

2. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an artic weather station. Which of the following is the best interpretation of $H'(24)$?

(A) The change in temperature during the first day.

(B) The average rate at which the temperature changed during the 24th hour.

(C) The rate at which the temperature is changing during the first day.

(D) The rate at which the temperature is changing at the end of the 24th hour.

3. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit. If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at a rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175 °F (B) 130 °F (C) 95 °F (D) 70 °F

The penguin population on an island is modeled by a differentiable function P of time t, where $P(t)$ is the number of penguins and t is measured in years for $0 \leq t \leq 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

$$B(t) = 1000e^{0.06t} \text{ penguins per year}$$

and the death rate for the penguins on the island is modeled by

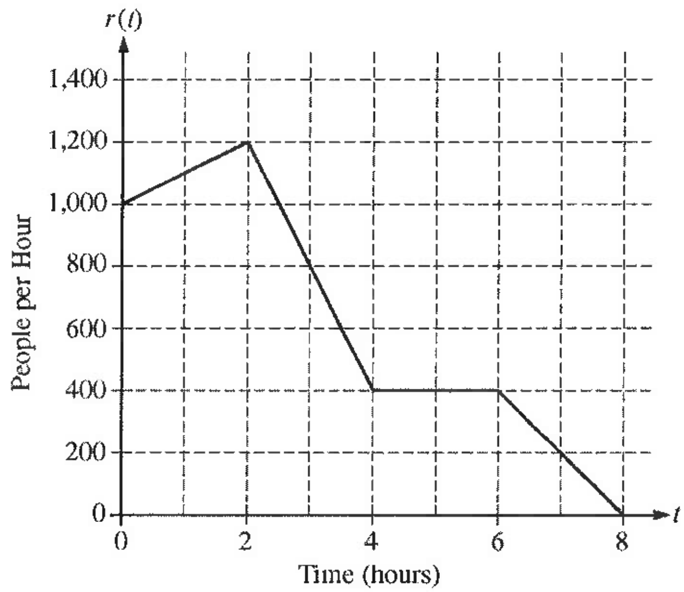
$$D(t) = 250e^{0.1t} \text{ penguins per year.}$$

(a) What is the rate of change of the penguin population on the island at time $t = 0$?

(b) To the nearest whole number, what is the penguin population on the island at time $t = 40$?

(c) To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \le t \le 40$?

(d) To the nearest whole number, find the absolute minimum penguin population and the absolutely maximum penguin population on the island for $0 \le t \le 40$. Show the analysis that leads to your answers.



There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

(c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

(d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a long for the ride.

Question 2

The penguin population on an island is modeled by a differentiable function P of time t, where $P(t)$ is the number of penguins and t is measured in years, for $0 \le t \le 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

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(a) What is the rate of change of the penguin population on the island at time $t = 0$?

(b) To the nearest whole number, what is the penguin population on the island at time $t = 40$?

(c) To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \le t \le 40$?

(d) To the nearest whole number, find the absolute minimum penguin population and the absolute maximum penguin population on the island for $0 \le t \le 40$. Show the analysis that leads to your answers.

(a) $P'(0) = B(0) - D(0) = 1000 - 250 = 750$ penguins per year

1 : answer

(b) $P(40) = 100000 + \int_0^{40} (B(t) - D(t))\, dt$

$= 100000 + 33057.56459$

There are 133,058 penguins on the island.

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{1}{40}\int_0^{40} (B(t) - D(t))\, dt = 826.439$

OR

$\frac{P(40) - P(0)}{40 - 0} = \frac{133058 - 100000}{40} = 826.45$

The average rate of change is 826 penguins per year.

1 : answer

(d) $B(t) - D(t) = 0$

$1000e^{0.06t} = 250e^{0.1t} \Rightarrow t = A = \frac{\ln 4}{0.04} = 34.657359$

The absolute minimum and absolute maximum occur at a critical point or at an endpoint.

$P(0) = 100000$

$P(A) = 100000 + \int_0^{A} (B(t) - D(t))\, dt = 139166.667$

$P(40) = 133058$

The minimum population is 100,000 and the maximum population is 139,167 penguins.

$4 : \begin{cases} 1 : B(t) - D(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{minimum value} \\ 1 : \text{maximum value} \end{cases}$

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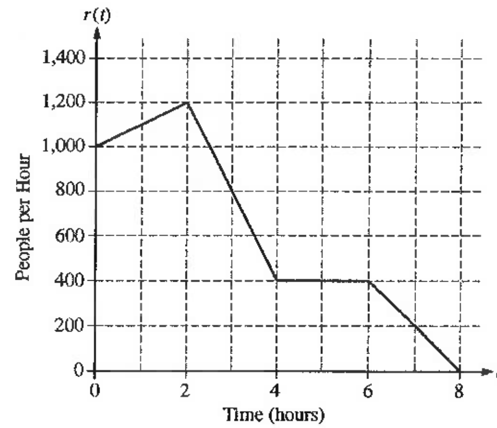
Question 3

There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.



(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

(c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

(d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

(a) $\int_0^3 r(t)\,dt = 2\cdot\frac{1000+1200}{2} + \frac{1200+800}{2} = 3200$ people

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(b) The number of people waiting in line is increasing because people move onto the ride at a rate of 800 people per hour and for $2 < t < 3$, $r(t) > 800$.

1 : answer with reason

(c) $r(t) = 800$ only at $t = 3$
For $0 \le t < 3$, $r(t) > 800$. For $3 < t \le 8$, $r(t) < 800$.
Therefore, the line is longest at time $t = 3$.
There are $700 + 3200 - 800\cdot 3 = 1500$ people waiting in line at time $t = 3$.

$3: \begin{cases} 1: \text{identifies } t = 3 \\ 1: \text{number of people in line} \\ 1: \text{justification} \end{cases}$

(d) $0 = 700 + \int_0^t r(s)\,ds - 800t$

$3: \begin{cases} 1: 800t \\ 1: \text{integral} \\ 1: \text{answer} \end{cases}$

Rates of Change & Integrals as Net Change

Practice
(All Calculator Active)

1. The temperature of a room, in degrees Fahrenheit, is modeled by H, a differentiable function of the number of minutes after the thermostat is adjusted. Of the following, which is the best interpretation of $H'(5) = 2$?

(A) The temperature of the room is 2 degrees Fahrenheit, 5 minutes after the thermostat is adjusted.

(B) The temperature of the room increases by 2 degrees Fahrenheit during the first 5 minutes after the thermostat is adjusted.

(C) The temperature of the room is increasing at a constant rate of $\frac{2}{5}$ degree Fahrenheit per minute.

(D) The temperature of the room is increasing at a rate of 2 degrees Fahrenheit per minute, 5 minutes after the thermostat is adjusted.

2. The cost, in dollars, to shred the confidential documents of a company is modeled by C, a differentiable function of the weight of documents in pounds. Of the following, which is the best interpretation of $C'(500) = 80$?

(A) The cost to shred 500 pounds of documents is $80.

(B) The average cost to shred documents is $\frac{80}{500}$ dollar per pound.

(C) Increasing the weight of documents by 500 pounds will increase the cost to shred the documents by approximately $80.

(D) The cost to shred documents is increasing at a rate of $80 per pound when the weight of the documents is 500 pounds.

3. A particle travels along a straight line with a velocity of $v(t) = 3e^{(-t/2)}\sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835
(B) 1.850
(C) 2.055
(D) 2.261
(E) 7.025

4. A race car is traveling on a straight track at a velocity of 80 meters per second when the brakes are applied at time $t = 0$ seconds. From time $t = 0$ to the moment the race car stops, the acceleration of the race car is given by $a(t) = -6t^2 - t$ meters per second per second. During this time period, how far does the race car travel?

(A) 188.229 m

(B) 198.766 m

(C) 260.042 m

(D) 267.089 m

5. Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil leaks out of the tanker from time $t = 0$ to $t = 10$?

(A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons

6. At time t, a population of bacteria grows at the rate of $5e^{0.2t} + 4t$ grams per day, where t is measured in days. By how many grams has the population grown from time $t = 0$ days to $t = 10$ days?

(A) $5e^2 + 40$

(B) $5e^2 + 195$

(C) $25e^2 + 175$

(D) $25e^2 + 375$

7. A rain barrel collects water off the roof of a house during three hours of heavy rainfall. The height of the water in the barrel increases at the rate of $r(t) = 4t^3e^{-1.5t}$ feet per hour, where t is the time in hours since the rain began. At time $t = 1$ hour, the height of the water is 0.75 foot. What is the height of the water in the barrel at time $t = 2$ hours?

(A) 1.361 ft

(B) 1.500 ft

(C) 1.672 ft

(D) 2.111 ft

8. The temperature in a room at midnight is 20 degrees Celsius. Over the next 24 hours, the temperature changes at a rate modeled by the differentiable function H, where $H(t)$ is measured in degrees Celsius per hour and time t is measured in hours since midnight. Which of the following is the best interpretation of $\int_0^6 H(t)\,dt$?

(A) The temperature of the room, in degrees Celsius, at 6:00 A.M.

(B) The average temperature of the room, in degrees Celsius, between midnight and 6:00 A.M.

(C) The change in the temperature of the room, in degrees Celsius, between midnight and 6:00 A.M.

(D) The rate at which the temperature in the room is changing, in degrees Celsius per hour, at 6:00 A.M.

9. The number of bacteria in a container increases at the rate of $R(t)$ bacteria per hour. If there are 1000 bacteria at time $t = 0$, which of the following expressions gives the number of bacteria in the container at time $t = 3$ hours?

(A) $R(3)$ (B) $1000 + R(3)$ (C) $\int_0^3 R(t)\,dt$ (D) $1000 + \int_0^3 R(t)\,dt$

Answers:

1. D 2. D 3. D 4. B 5. D 6. C

7. D 8. C 9. D

Practice Free Response (ALL CALCULATOR ACTIVE)

1. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \quad \text{for} \quad 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

(d) How many pounds of bananas are on the display table at time $t = 8$?

3. The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

4. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

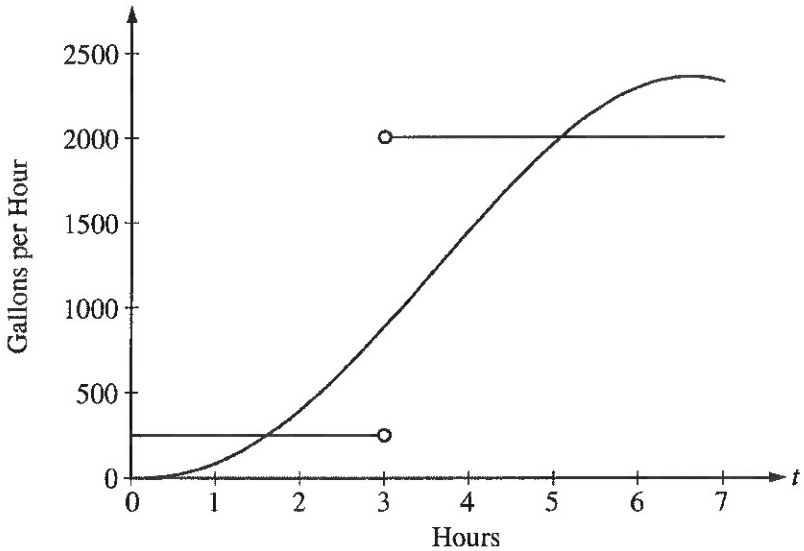
5. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.



6. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

7. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

8. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

Question #1

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Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(a) $\int_0^{30} F(t)\,dt = 2474$ cars	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $F'(7) = -1.872$ or -1.873 Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.	1 : answer with reason
(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2	1 : answer
Units of cars/min in (c) and cars/min^2 in (d)	1 : units in (c) and (d)

Question #2

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Question 2

(a) $\int_0^2 f(t)\,dt = 20.051175$ 20.051 pounds of bananas are removed from the display table during the first 2 hours the store is open.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) $f'(7) = -8.120$ (or -8.119) After the store has been open 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 (or 8.119) pounds per hour per hour.	$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning} \end{cases}$
(c) $g(5) - f(5) = -2.263103 < 0$ Because $g(5) - f(5) < 0$, the number of pounds of bananas on the display table is decreasing at time $t = 5$.	$2 : \begin{cases} 1 : \text{considers } f(5) \text{ and } g(5) \\ 1 : \text{answer with reason} \end{cases}$
(d) $50 + \int_3^8 g(t)\,dt - \int_0^8 f(t)\,dt = 23.347396$ 23.347 pounds of bananas are on the display table at time $t = 8$.	$3 : \begin{cases} 2 : \text{integrals} \\ 1 : \text{answer} \end{cases}$

Question #3

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t)\,dt = 76.570$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$

Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

$2 : \begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t, $0 \le t \le 8$, is $30 + \int_0^t [R(x) - D(x)]\,dx$.

$R(t) - D(t) = 0 \Rightarrow t = 0,\ 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

$3 : \begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $30 + \int_0^w [R(t) - D(t)]\,dt = 50$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

Question #4

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Question 2

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

(a) $E'(6) \approx \dfrac{E(7) - E(5)}{7 - 5} = 4$ hundred entries per hour

1 : answer

(b) $\frac{1}{8}\int_0^8 E(t)\,dt \approx$

$$\frac{1}{8}\left(2\cdot\frac{E(0)+E(2)}{2} + 3\cdot\frac{E(2)+E(5)}{2} + 2\cdot\frac{E(5)+E(7)}{2} + 1\cdot\frac{E(7)+E(8)}{2}\right)$$

$= 10.687$ or 10.688

$\frac{1}{8}\int_0^8 E(t)\,dt$ is the average number of hundreds of entries in the box between noon and 8 P.M.

$3 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \\ 1 : \text{meaning} \end{cases}$

(c) $23 - \int_8^{12} P(t)\,dt = 23 - 16 = 7$ hundred entries

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $P'(t) = 0$ when $t = 9.183503$ and $t = 10.816497$.

t	$P(t)$
8	0
9.183503	5.088662
10.816497	2.911338
12	8

Entries are being processed most quickly at time $t = 12$.

$3 : \begin{cases} 1 : \text{considers } P'(t) = 0 \\ 1 : \text{identifies candidates} \\ 1 : \text{answer with justification} \end{cases}$

Question #5

AP® CALCULUS AB
2013 SCORING GUIDELINES

Question 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

(a) $G'(5) = -24.588$ (or -24.587)

The rate at which gravel is arriving is decreasing by 24.588 (or 24.587) tons per hour per hour at time $t = 5$ hours.

$$2 : \begin{cases} 1 : G'(5) \\ 1 : \text{interpretation with units} \end{cases}$$

(b) $\int_0^8 G(t)\,dt = 825.551$ tons

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(c) $G(5) = 98.140764 < 100$

At time $t = 5$, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed. Therefore, the amount of unprocessed gravel at the plant is decreasing at time $t = 5$.

$$2 : \begin{cases} 1 : \text{compares } G(5) \text{ to } 100 \\ 1 : \text{conclusion} \end{cases}$$

(d) The amount of unprocessed gravel at time t is given by $A(t) = 500 + \int_0^t (G(s) - 100)\,ds$.

$A'(t) = G(t) - 100 = 0 \Rightarrow t = 4.923480$

t	$A(t)$
0	500
4.92348	635.376123
8	525.551089

The maximum amount of unprocessed gravel at the plant during this workday is 635.376 tons.

$$3 : \begin{cases} 1 : \text{considers } A'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

Question #6

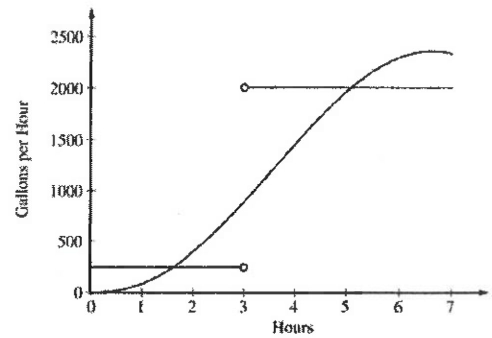
AP® CALCULUS AB
2007 SCORING GUIDELINES

Question 2

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is $g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases}$ gallons per hour.



The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a) $\int_0^7 f(t)\,dt \approx 8264$ gallons

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(b) The amount of water in the tank is decreasing on the intervals $0 \le t \le 1.617$ and $3 \le t \le 5.076$ because $f(t) < g(t)$ for $0 \le t < 1.617$ and $3 < t < 5.076$.

$2: \begin{cases} 1: \text{intervals} \\ 1: \text{reason} \end{cases}$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0$, 3, and 7.

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t)\,dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t)\,dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

$5: \begin{cases} 1: \text{identifies } t = 3 \text{ as a candidate} \\ 1: \text{integrand} \\ 1: \text{amount of water at } t = 3 \\ 1: \text{amount of water at } t = 7 \\ 1: \text{conclusion} \end{cases}$

Question #7

AP® CALCULUS AB/CALCULUS BC
2016 SCORING GUIDELINES

Question 1

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

(a) $R'(2) \approx \dfrac{R(3) - R(1)}{3 - 1} = \dfrac{950 - 1190}{3 - 1} = -120$ liters/hr^2

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The total amount of water removed is given by $\int_0^8 R(t)\,dt$.

$$\begin{aligned}\int_0^8 R(t)\,dt &\approx 1 \cdot R(0) + 2 \cdot R(1) + 3 \cdot R(3) + 2 \cdot R(6) \\ &= 1(1340) + 2(1190) + 3(950) + 2(740) \\ &= 8050 \text{ liters}\end{aligned}$$

This is an overestimate since R is a decreasing function.

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{estimate} \\ 1 : \text{overestimate with reason} \end{cases}$

(c) $$\begin{aligned}\text{Total} &\approx 50000 + \int_0^8 W(t)\,dt - 8050 \\ &= 50000 + 7836.195325 - 8050 \approx 49786 \text{ liters}\end{aligned}$$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{estimate} \end{cases}$

(d) $W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous.

Therefore, the Intermediate Value Theorem guarantees at least one time t, $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

For this value of t, the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

$2 : \begin{cases} 1 : \text{considers } W(t) - R(t) \\ 1 : \text{answer with explanation} \end{cases}$

Question #8

AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

(a) $\int_0^6 f(t)\,dt = 142.274$ or 142.275 cubic feet	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.	1 : answer
(c) $h(0) = 0$ For $0 < t \le 6$, $h(t) = h(0) + \int_0^t g(s)\,ds = 0 + \int_0^t 0\,ds = 0.$ For $6 < t \le 7$, $h(t) = h(6) + \int_6^t g(s)\,ds = 0 + \int_6^t 125\,ds = 125(t-6).$ For $7 < t \le 9$, $h(t) = h(7) + \int_7^t g(s)\,ds = 125 + \int_7^t 108\,ds = 125 + 108(t-7).$ Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \le t \le 6 \\ 125(t-6) & \text{for } 6 < t \le 7 \\ 125 + 108(t-7) & \text{for } 7 < t \le 9 \end{cases}$	$3 : \begin{cases} 1 : h(t) \text{ for } 0 \le t \le 6 \\ 1 : h(t) \text{ for } 6 < t \le 7 \\ 1 : h(t) \text{ for } 7 < t \le 9 \end{cases}$
(d) Amount of snow is $\int_0^9 f(t)\,dt - h(9) = 26.334$ or 26.335 cubic feet.	$3 : \begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$