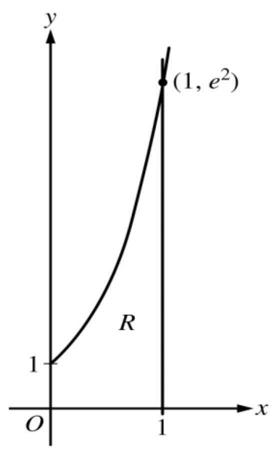
Volumes, Area and L'Hoptial's Rule Review

Exam Date:

Volumes

I. A vase has the shape obtained by revolving the curve y = 2 + sinx from x = 0 to x = 5 about the x-axis, where x and y are measured in inches. Determine the volume of the vase. (Calculator)

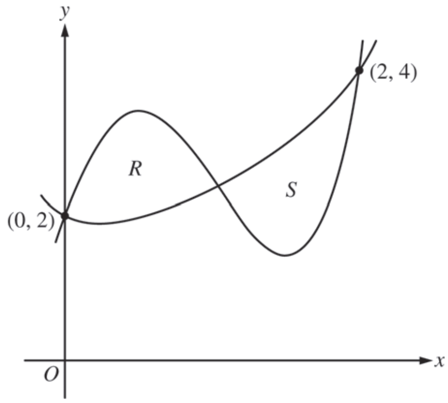
II.



Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of y = f(x) and the vertical line x = 1, as shown in the figure above (No Calculator)

1. Determine the volume of the solid obtained when the region R is revolved about the x – axis.
2. Write, but do not evaluate, an expression containing one or more integrals to determine the volume of the solid obtained when the region R is rotated about the y-axis.
3. Write, but do not evaluate, an integral expression to determine the volume of the solid obtained when the region R is rotated about the line y = -3.
4. Write, but do not evaluate, an expression containing one or more integrals to determine the volume of the solid obtained when the region R is rotated about the line x = 4.
5. The region R forms the base of a solid whose cross sections perpendicular to the x – axis are squares. Write, but do not evaluate, an integral expression that gives the volume of this solid.
6. The region R forms the base of a solid whose cross sections perpendicular to the y-axis are semi-circles. Write, but do not evaluate, an expression containing one or more integrals that gives the volume of this solid.

III.

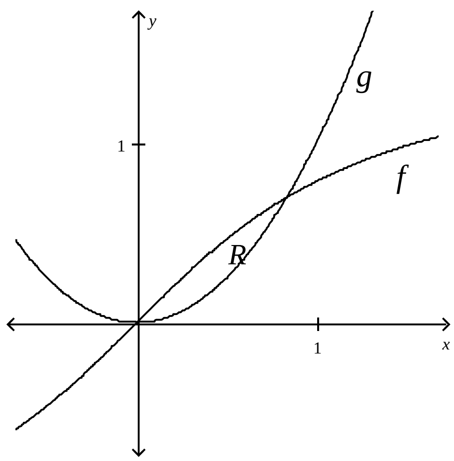


Let f and g be the functions defined by $f(x)=1+x+e^{x^2-2x}$ and $g(x)=x^4-6.5x^2+6x+2$. Let R and S be the two regions enclosed by the graphs of f and g as shown in the figure above. (Calculator)

1. Find the volume of the solid generated when R is rotated about the horizontal line $y=7$.

2. Region S is the base of a solid whose cross sections perpendicular to the x – axis are squares. Find the volume of the solid.

IV. Let R be the region in the first quadrant bounded by $f(x)=\tan^{-1}x$ and $g(x)=x^2$, as displayed in the figure below.



1. Determine the area of R

2. Calculate the following volumes of revolution:

a) about the x – axis	b) about the y – axis	c) about the line y = – 2
d) about the line x = 2	e) about the line y = 3	f) about the line x = – 7

3. The region R is the base of a solid. For the solid, each cross section perpendicular to the x – axis is a square. Determine the volume of this solid.

4. The region R is the base of a solid. For the solid, each cross section perpendicular to the x – axis is a semi – circle. Determine the volume of this solid.

5. The region R is the base of a solid. For the solid, each cross section perpendicular to the y – axis is a rectangle whose height is twice the base. Determine the volume of this solid.

L'Hopital's Rule

I. Evaluate the following limits if they exist.

1. $\lim_{x\to 0}\frac{\sin(x^2)}{x}$ 2. $\lim_{x\to 0}\frac{\ln(x+1)}{x}$ 3. $\lim_{x\to 2}\frac{x^2-4}{x^5-32}$ 4. $\lim_{x\to 0}\frac{3x^2+\sin x-x}{x^2e^{2x}}$

Answers: Volumes

I. $V=\int_0^5 \pi(2+\sin x)^2\,dx=80.115$

II. 1. $V=\int_0^1 \pi\left[e^{2x}\right]^2 dx=\frac{\pi}{4}\left(e^4-1\right)$

2. $y=e^{2x}\Leftrightarrow x=\frac{\ln y}{2}$

$V=\int_0^1\left(\pi[1]^2-\pi[0]^2\right)dy+\int_1^{e^2}\left(\pi[1]^2-\pi\left[\frac{\ln y}{2}\right]^2\right)dy$

3. $V=\int_0^1\left[\pi\left[3+e^{2x}\right]^2-\pi[3-0]^2\right]dx$

4. $V=\int_0^1\left[\pi[4-0]^2-\pi[4-1]^2\right]dy+\int_1^{e^2}\left[\pi\left[4-\frac{\ln y}{2}\right]^2-\pi[4-1]^2\right]dy$

5. $V=\int_0^1\left[e^{2x}\right]^2dx$

6. $V=\int_0^1\frac{\pi}{2}\left[\frac{1-0}{2}\right]^2dy+\int_1^{e^2}\frac{\pi}{2}\left[\frac{1-\frac{\ln y}{2}}{2}\right]^2dy$

III. 1. $V=\int_0^{1.0328319}\left[\pi[7-f(x)]^2-\pi[7-g(x)]^2\right]dx=27.614$

2. $V=\int_{1.0328319}^{2}[f(x)-g(x)]^2\,dx=1.283$

IV.

$y=\tan^{-1}x$ $\quad$ $y=x^2$

$x=\tan y$ $\quad$ $x=\sqrt{y}$

Intersection points

(0, 0) and (0.83360619, 0.69489929)

1. $A=\int_0^{0.83360619}\left(\tan^{-1}x-x^2\right)dx=0.122$

2. a) $V = \int_0^{0.83360619} \pi\left[\left(\tan^{-1} x\right)^2 - \left(x^2\right)^2\right]dx = 0.229 \; or \; 0.230$

b) $V = \int_0^{0.69489929} \pi\left[\left(\sqrt{y}\right)^2 - \left(\tan y\right)^2\right]dy = 0.322 \; or \; 0.323$

c) $V = \int_0^{0.83360619} \pi\left[\left(2 + \tan^{-1} x\right)^2 - \left(2 + x^2\right)^2\right]dx = 1.767 \; or \; 1.768$

d) $V = \int_0^{0.69489929} \pi\left[\left(2 - \tan y\right)^2 - \left(2 - \sqrt{y}\right)^2\right]dy = 1.215$

e) $V = \int_0^{0.83360619} \pi\left[\left(3 - x^2\right)^2 - \left(2 - \tan^{-1} x\right)^2\right]dx = 2.076 \; or \; 2.077$

f) $V = \int_0^{0.69489929} \pi\left[\left(7 + \sqrt{y}\right)^2 - \left(7 + \tan y\right)^2\right]dy = 5.704 \; or \; 5.705$

3. $V = \int_0^{0.83360619} \left(\tan^{-1} x - x^2\right)^2 dx = 0.021 \; or \; 0.022$

4. $V = \int_0^{0.83360619} \frac{\pi}{2}\left(\frac{\tan^{-1} x - x^2}{2}\right)^2 dx = 0.008$

5. $V = \int_0^{0.69489929} 2\left[\sqrt{y} - \tan y\right]^2 dy = 0.049 \; or \; 0.050$

Answers: Limits

I.

1. $\lim_{x\to 0} \frac{\sin\left(x^2\right)}{x} = \frac{0}{0}$

$$= \lim_{x\to 0} \frac{\cos\left(x^2\right)\left(2x\right)}{1}$$
$$= \cos(0)(0)$$
$$= 0$$

2. $\lim_{x\to 0} \frac{\ln\left(x+1\right)}{x} = \frac{0}{0}$

$$= \lim_{x\to 0} \frac{\frac{1}{x+1}}{1}$$
$$= \frac{1}{0+1}$$
$$= 1$$

3. $\lim_{x\to 2} \frac{x^2 - 4}{x^5 - 32} = \frac{0}{0}$

$$= \lim_{x\to 2} \frac{2x}{5x^4}$$
$$= \frac{4}{80}$$
$$= \frac{1}{20}$$

4. $\lim_{x\to 0} \frac{3x^2 + \sin x - x}{x^2 e^{2x}} = \frac{0}{0} = 3$