Average Value of a Function

The average (mean) value of a function f(x) on a closed interval [a , b] is:

$$\text{Average Value of } f(x) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Find the average value of each of the following functions on the given interval.

1. $f(x) = 3x^2 - 2x$ [0,3]
2. $f(x) = -2x^3 + 3x - 1$ [-1,2]
3. $f(x) = e^x$ [0,2]
4. $f(x) = \sin x$ [0, π]
5. $f(x) = \frac{1}{x}$ $\left[\frac{1}{2}, 3\right]$
6. $f(x) = \frac{-x^2}{4}$ [-2,3]

Free Response

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)dt$ in the context of the problem.

(b) Determine the value of $\frac{1}{6}\int_0^6 C'(t)dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C'(t)dt$ in the context of the problem.

ANSWERS:

1. 6
2. -2
3. $\frac{1}{2}\left[e^2 - 1\right]$
4. $\frac{2}{\pi}$
5. $\frac{2}{5}\left[\ln x\right]_{\frac{1}{2}}^{3} = \frac{2}{5}\left[\ln 3 - \ln\frac{1}{2}\right] = \frac{2}{5}\ln 6$
6. $\frac{-7}{12}$

Free Response

a) 10.1 ounces. $\frac{1}{6}\int_0^6 C(t)dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \le t \le 6$ minutes.

b) $\frac{1}{6}\int_0^6 C'(t)dt = \frac{1}{6}\left[C(6) - C(0)\right] = 2.417$ ounces/minute. $\frac{1}{6}\int_0^6 C'(t)dt$ is the average rate at which coffee is filling the coffee cup in ounces/minute over the time interval of $0 \le t \le 6$ minutes.