# FTC Part 1 and Part 2 Applications Review

Question 1

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where $t$ is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$ ? Why or why not?

(b) For what value of $t$ will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time $t$ does this model predict that the lake becomes safe?

Question 2

The rate at which people enter an amusement park on a given day is modeled by the function $E$ defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function $L$ defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time $t$ is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time $t$, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

Question 3

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \leq t \leq 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time $t$, for $0 \leq t \leq 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

Question 4

For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time $t$ days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.

Question 5

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time $t$ minutes, where $v$ is a differentiable function of $t$. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

| $t$ (min) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|---|---|---|
| $v(t)$ (mpm) | 7.0 | 9.2 | 9.5 | 7.0 | 4.5 | 2.4 | 2.4 | 4.3 | 7.3 |

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function $f$, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

(d) According to the model $f$, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

Question 6

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function $F$ defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and $t$ is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

Question 7

The tide removes sand from Sandy Point Beach at a rate modeled by the function $R$, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function $S$, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and $t$ is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

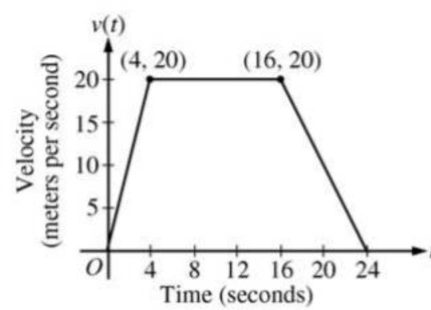
(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time $t$.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time $t$ is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

Question 8




A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time $t$, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of $v$ over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of $c$, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

Question 9

A particle moves along the $x$-axis so that its velocity $v$ at time $t$, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times $t$ in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \le t \le 2$.

Question 10

| $t$ (seconds) | 0 | 8 | 20 | 25 | 32 | 40 |
|---|---|---|---|---|---|---|
| $v(t)$ (meters per second) | 3 | 5 | $-10$ | $-8$ | $-4$ | 7 |

The velocity of a particle moving along the $x$-axis is modeled by a differentiable function $v$, where the position $x$ is measured in meters, and time $t$ is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

Question 11

For $0 \le t \le 6$, a particle is moving along the $x$-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$ ? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

Answers:

## Question 1

(a) $P'(9) = 1 - 3e^{-0.6} = -0.646 < 0$

so the amount is not increasing at this time.

(b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$

$t = (5\ln 3)^2 = 30.174$

$P'(t)$ is negative for $0 < t < (5\ln 3)^2$ and positive for $t > (5\ln 3)^2$. Therefore there is a minimum at $t = (5\ln 3)^2$.

(c) $P(30.174) = 50 + \int_0^{30.174} \left(1 - 3e^{-0.2\sqrt{t}}\right) dt$

$= 35.104 < 40$, so the lake is safe.

(d) $P'(0) = 1 - 3 = -2$. The lake will become safe when the amount decreases by 10. A linear model predicts this will happen when $t = 5$.

## Question 2

(a) $\int_9^{17} E(t)\,dt = 6004.270$

6004 people entered the park by 5 pm.

(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$

The amount collected was \$104,048.

or

$\int_{17}^{23} E(t)\,dt = 1271.283$

1271 people entered the park between 5 pm and 11 pm, so the amount collected was

$\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$

(c) $H'(17) = E(17) - L(17) = -380.281$

There were 3725 people in the park at $t = 17$.

The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.

(d) $H'(t) = E(t) - L(t) = 0$

$t = 15.794$ or $15.795$

## Question 3

(a) $\int_0^{12} H(t)\,dt = 70.570$ or $70.571$

(b) $H(6) - R(6) = -2.924$,

so the level of heating oil is falling at $t = 6$.

(c) $125 + \int_0^{12} (H(t) - R(t))\,dt = 122.025$ or $122.026$

(d) The absolute minimum occurs at a critical point or an endpoint.

$H(t) - R(t) = 0$ when $t = 4.790$ and $t = 11.318$.

The volume increases until $t = 4.790$, then decreases until $t = 11.318$, then increases, so the absolute minimum will be at $t = 0$ or at $t = 11.318$.

$125 + \int_0^{11.318} (H(t) - R(t))\,dt = 120.738$

Since the volume is 125 at $t = 0$, the volume is least at $t = 11.318$.

## Question 4

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.

(b) $R'(6) = -1.913$

Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.

(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$

To the nearest whole number, there are 964 mosquitoes.

(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$

$R(t) > 0$ on $0 < t < 2.5\pi$

$R(t) < 0$ on $2.5\pi < t < 7.5\pi$

$R(t) > 0$ on $7.5\pi < t < 31$

The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$.

$1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357,$

There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.

## Question 5

(a) Midpoint Riemann sum is
$10 \cdot [v(5) + v(15) + v(25) + v(35)]$
$= 10 \cdot [9.2 + 7.0 + 2.4 + 4.3] = 229$
The integral gives the total distance in miles that the plane flies during the 40 minutes.

(b) By the Mean Value Theorem, $v'(t) = 0$ somewhere in the interval $(0, 15)$ and somewhere in the interval $(25, 30)$. Therefore the acceleration will equal 0 for at least two values of $t$.

(c) $f'(23) = -0.407$ or $-0.408$ miles per minute$^2$

(d) Average velocity $= \dfrac{1}{40}\displaystyle\int_0^{40} f(t)\,dt$
$= 5.916$ miles per minute

## Question 6

(a) $\displaystyle\int_0^{30} F(t)\,dt = 2474$ cars

(b) $F'(7) = -1.872$ or $-1.873$
Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.

(c) $\dfrac{1}{5}\displaystyle\int_{10}^{15} F(t)\,dt = 81.899$ cars/min

(d) $\dfrac{F(15) - F(10)}{15 - 10} = 1.517$ or $1.518$ cars/min$^2$

## Question 7

(a) $\displaystyle\int_0^{6} R(t)\,dt = 31.815$ or $31.816$ yd$^3$

(b) $Y(t) = 2500 + \displaystyle\int_0^{t} (S(x) - R(x))\,dx$

(c) $Y'(t) = S(t) - R(t)$
$Y'(4) = S(4) - R(4) = -1.908$ or $-1.909$ yd$^3$/hr

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.
The only value in $[0, 6]$ to satisfy $S(t) = R(t)$ is $a = 5.117865$.

| $t$ | $Y(t)$ |
|---|---|
| 0 | 2500 |
| $a$ | 2492.3694 |
| 6 | 2493.2766 |

The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.

## Question 8

(a) $\displaystyle\int_0^{24} v(t)\,dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$
The car travels 360 meters in these 24 seconds.

(b) $v'(4)$ does not exist because
$$\lim_{t \to 4^-} \left( \frac{v(t) - v(4)}{t - 4} \right) = 5 \neq 0 = \lim_{t \to 4^+} \left( \frac{v(t) - v(4)}{t - 4} \right).$$
$v'(20) = \dfrac{20 - 0}{16 - 24} = -\dfrac{5}{2}$ m/sec$^2$

(c)
$$a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$$
$a(t)$ does not exist at $t = 4$ and $t = 16$.

(d) The average rate of change of $v$ on $[8, 20]$ is
$\dfrac{v(20) - v(8)}{20 - 8} = -\dfrac{5}{6}$ m/sec$^2$.
No, the Mean Value Theorem does not apply to $v$ on $[8, 20]$ because $v$ is not differentiable at $t = 16$.

## Question 9

(a) $a(4) = v'(4) = \frac{5}{7}$

(b) $v(t) = 0$

$t^2 - 3t + 3 = 1$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 1, 2$

$v(t) > 0$ for $0 < t < 1$

$v(t) < 0$ for $1 < t < 2$

$v(t) > 0$ for $2 < t < 5$

The particle changes direction when $t = 1$ and $t = 2$.
The particle travels to the left when $1 < t < 2$.

(c) $s(t) = s(0) + \int_0^t \ln\left(u^2 - 3u + 3\right) du$

$s(2) = 8 + \int_0^2 \ln\left(u^2 - 3u + 3\right) du$

$= 8.368$ or $8.369$

(d) $\frac{1}{2}\int_0^2 |v(t)|\, dt = 0.370$ or $0.371$

## Question 10

(a) $a(36) = v'(36) \approx \frac{v(40) - v(32)}{40 - 32} = \frac{11}{8}$ meters/sec$^2$

(b) $\int_{20}^{40} v(t)\, dt$ is the particle's change in position in meters from time $t = 20$ seconds to time $t = 40$ seconds.

$\int_{20}^{40} v(t)\, dt \approx \frac{v(20) + v(25)}{2} \cdot 5 + \frac{v(25) + v(32)}{2} \cdot 7 + \frac{v(32) + v(40)}{2} \cdot 8$

$= -75$ meters

(c) $v(8) > 0$ and $v(20) < 0$

$v(32) < 0$ and $v(40) > 0$

Therefore, the particle changes direction in the intervals $8 < t < 20$ and $32 < t < 40$.

(d) Since $v'(t) = a(t) > 0$ for $0 < t < 8$, $v(t) \geq 3$ on this interval.

Therefore, $x(8) = x(0) + \int_0^8 v(t)\, dt \geq 7 + 8 \cdot 3 > 30$.

## Question 11

(a) $v(5.5) = -0.45337$, $a(5.5) = -1.35851$

The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.

(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\, dt = 1.949$

(c) Distance $= \int_0^6 |v(t)|\, dt = 12.573$

(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$.

$v(t)$ changes sign from positive to negative at time $t = b$.

$x(b) = 2 + \int_0^b v(t)\, dt = 14.134$ or $14.135$