6.6 Chapter Review

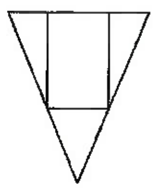
The following questions are suitable for reviewing sections 6.1 to 6.3.

1. The population of a bacteria culture after t hours is given by the function $P(t)=600+100t+8t^3$.
   Answer each of the following questions, providing appropriate units.
   (a) What was the initial size of the population?
   (b) What was the size of the population after 6 hours?
   (c) What was the average rate of growth of the population between $t=3$ and $t=7$?
   (d) What was the instantaneous rate of growth of the population when $t=5$?
2. The value, V, of a new automobile t years after it has been purchased is given by the function
   $V(t)=\dfrac{30000}{1+0.1t+0.05t^2}$. Round your answer to each question to the nearest cent.
   (a) Find the purchase price.
   (b) Find the value of the car after 3 years.
   (c) What was the average rate of depreciation during those three years?
   (d) Find the instantaneous rate of depreciation when $t=3$.
3. A particle moves along the x-axis so that after t seconds its position is given by the function
   $s(t)=2t^3-21t^2+72t$.
   (a) Find the velocity and acceleration of the particle at any time, t.
   (b) Find the position, velocity, and acceleration of the particle when $t=8$.
   (c) Find the time intervals during which the particle is moving to the right.
   (d) Find the time intervals during which the particle is moving to the left.
   (e) How far has the particle moved during between $t=0$ and $t=5$?
   (f) Find the average velocity of the particle between $t=6$ and $t=10$.
4. The height of a ball above ground level, measured in metres, is given by the function
   $s(t)=-5t^2+60t+8$, where t is time in seconds.
   (a) What was the initial height of the ball?
   (b) When did the ball reach its maximum height?
   (c) What was the maximum height reached by the ball?
   (d) When did the ball hit the ground? Round your answer to two decimal places.
   (e) With what velocity did the ball hit the ground? Round your answer to two decimal places.
5. A snowmobile operator, travelling along a narrow bush trail, came over a hill and saw another machine stalled 18 metres ahead. The operator immediately cut the throttle and travelled a distance of $s(t)=12t-t^3$ metres thereafter, where t was the time in seconds. Was there a collision? Explain.
6. The height (in metres) of a bullet above ground level, t seconds after being fired from a gun, is given by the function $h(t)=400t-5t^2$.
   (a) Find the maximum height reached by the bullet.
   (b) If the fuselage of an airplane can withstand bullets travelling at a rate of 50m/s, what is the lowest height at which the plane can safely fly?
7. Two nonnegative numbers have a sum of 180. Find these numbers if the product of one of the numbers with the square of the other is to be a maximum. Find that maximum product.
8. Two nonnegative numbers have a product of 72. Find these numbers if the sum of one of them and twice the other is to be a minimum.
9. Two nonnegative numbers have a sum of 4. Find these numbers if the sum of the cube of one of them and the square of the other is to be a minimum. What is that minimum sum?

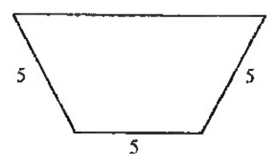
10. An open-topped box is to be made from a square piece of tin that is 54 cm by 54 cm by cutting out squares of equal size from each of the corners and folding up the flaps. Find the size of the square if the volume of the box is to be a maximum.
11. A dog kennel operator has 72 m of fencing and wants to enclose six congruent rectangular pens as shown below. What dimensions should be used for each pen in order to maximize the total area enclosed?

12. The sum of the lengths of two legs in a right triangle is 20 cm. Find the length of each leg if the length of the hypotenuse is to be minimized.
13. What is the shortest vertical distance between the graphs of the functions $f(x)=x^2+2$ and $g(x)=-(x-2)^2-1$?
14. A storage box with square ends and no open sides is to be built to have a volume of $50\ \text{m}^3$. If the material for the square ends costs $\$80/\text{m}^2$ while the material for the rectangular sides costs $\$200/\text{m}^2$, find the dimensions of the box in order to minimize its cost.

15. Find the dimensions of the cylinder of greatest volume that can be inscribed in a cone of height 12 cm and radius 4 cm.



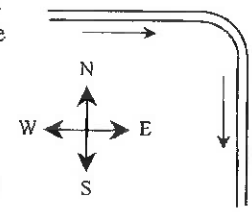
16. The shorter base and the legs of an isosceles trapezoid are each 5 cm in length. Find the length of the longer base if the area of the trapezoid is to be maximized.



The following questions are suitable for reviewing sections 6.4-6.5.

17. The base of a ladder 13 m long is pushed towards the wall at a rate of 10 cm/s. At what rate is the top of the ladder moving up the wall when the base is 5 m from the wall?
18. A right triangle has a hypotenuse of constant length 25 cm. One leg of the right triangle increases at a rate of 1.4 cm/s. When that leg is 24 cm long, find:
(a) the rate at which the other leg is decreasing in length.
(b) how the area is changing when the increasing leg is 24 cm.
19. A truck is parked 100 m directly south of an intersection. A car is travelling east at a rate of 20 m/s. At what rate is the distance between the car and the truck increasing 12 seconds after the car has passed through the intersection?
20. A bicyclist is approaching an intersection travelling south at a rate of 20 km/h. A motorcyclist is leaving the same intersection travelling west at a rate of 60 km/h. How is the distance between them changing when the cyclist is 42 m from the intersection and the motorcyclist is 40 m from the intersection?

21. A train of length 700 m maintains a constant speed of 100 km/h as it travels through a right-angled turn as shown. Let z denote the distance between the front and rear of the train at any time.



(a) Why is $\frac{dz}{dt}=0$ before the engine reaches the corner?

(b) Find $\frac{dz}{dt}$ at the moment when the engine is 300 m past the corner and is travelling south.

(c) When, if ever, is $\frac{dz}{dt}$ positive?

22. A fictitious piece of pie is in the shape of a sector with radius r and arc length s. If the radius is increasing at a rate of 4 cm/s while the arc length is decreasing at a rate of 3 cm/s, what is happening to the area of the piece of pie when the radius is 8 cm and the arc length is 7 cm? Recall that the area of such a sector is $\frac{1}{2}rs$.

23. A metal ball bearing is heating which causes the radius to grow at a rate of 2 mm/min. At what rate is the volume of the ball increasing when the radius of the ball is 1.5 cm? Give your answer in mm^3/min.

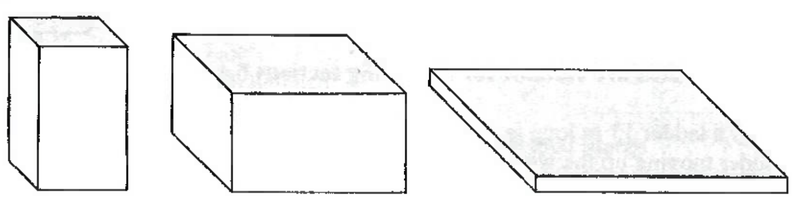
24. A spherical balloon is being inflated at a rate of $20\pi\ \text{m}^3/\text{h}$. At what rate is the radius increasing when the radius is 5 m?

25. A conical paper cup has a radius of 4 cm and a height of 12 cm. If water is leaking from the cup at a rate of 6 cm^3/s, at what rate is the water level falling when the height of the water is 8 cm?

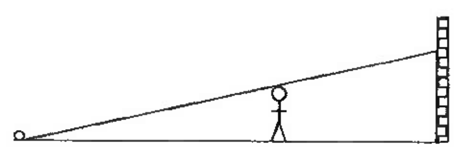
26. Sand leaves a conveyor belt at a rate of $0.72\ \text{m}^3/\text{min}$, forming a conical pile whose radius is the same as its height. At what rate is the height of the pile rising when the height of the pile is 6 m?

27. A conical wading pool has a diameter of 12 m and a depth of 1 m. Water is draining from the pool at a rate of $2\ \text{m}^3/\text{minute}$. At what rate is the height of the water decreasing when it is 25 cm deep above the vertex of the cone? Give your answer in cm/min.

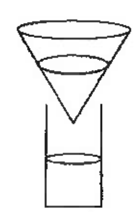
28. Each side of the square top of a box is growing at a rate of 8 cm/s while the height of the box is shrinking at a rate of 5 cm/s. What is happening to the volume of the box when it measures 60 cm by 60 cm by 10 cm?



29. A spotlight on the ground shines towards a wall 12 m away. A man 2 m tall walks directly towards the wall at a rate of 1.6 m/s. At what rate is his shadow length against the wall decreasing when he is 4 m from the wall?



30. Water is leaking from a cone with radius 6 cm and height 12 cm into a cylindrical glass jar with radius 5 cm. It is observed that the height of the water in the jar is increasing at a rate of 3 cm/min. At what rate is the height of the water in the cone decreasing when the height of the water in the cone is 5 cm?



Answers

ANSWERS 6.6

1. (a) 600 (b) 2100 (c) 732 bacteria/h (d) 700 bacteria/h 2. (a) $\$30\,000$ (b) $\$17\,142.86$ (c) $\$4285.71$/yr (d) $\$3918.37$/yr 3. (a) $v(t)=6t^2-42t+72$; $a(t)=12t-42$ (b) $s(8)=256$ u; $v(8)=120$ u/s; $a(8)=54$ u/s^2 (c) $t\in(0,3)\cup(4,\infty)$ (d) $t\in(3,4)$ (e) 87 u (f) 128 u/s 4. (a) 8 m (b) 6 s (c) 188 m (d) 12.13 s (e) −61.32 m/s 5. No. The operator's velocity was 0 in 2 seconds. During that time the snowmobile travelled 16 m, thus stopping 2 m before the stalled machine. 6. (a) 8000 m (b) 7875 m 7. 120 (the number to square) and 60; 864 000 8. 6 (the number to double) and 12 9. 4/3 (the number to cube) and 8/3 (the number to square); the minimum sum is 256/27 10. 9 cm by 9 cm 11. 4m by 4.5 m 12. each leg is 10 cm 13. 5 u 14. 5 m by 5 m by 2 m 15. $r=8/3$ cm, $h=4$ cm 16. 10 cm 17. 25/6 cm/s 18. (a) 4.8 cm/s (b) decreasing at 52.7 cm^2/s 19. 240/13 or 18.46 m/s 20. increasing at 780/29 or 26.90 km/h 21. (a) the distance between the front of the train and the back of the train is constant at that time (b) decreasing at 20 km/h (c) when the engine is more than 350 m south of the corner 22. increasing at 2 cm^2/s 23. 1800π or 5654.87 mm^3/min 24. 1/5 m/h 25. $27/32\pi$ or 0.27 cm/s 26. $1/50\pi$ or 0.0064 m/min 27. $800/9\pi$ cm/min 28. decreasing at 8400 cm^3/s 29. 3/5 m/s 30. 12 cm/min

ANSWERS 7.1

1. (a) 2 (b) 4 (c) 3 (d) −2 (e) 1 (f) 0 (g) 13 (h) −20 (i) 0 (j) 1 (k) 5 (l) −7 (m) −1 (n) −3
2. (a) $\log_3 120$ (b) $\ln 10$ (c) $\log_3 121$ (d) $\ln\left(\frac{1}{9}\right)$ (e) $\log_9 2$ (f) $\ln 10$ (g) $\log_5 4$ (h) $\ln 2$ (i) $\ln\left(\frac{1}{3}\right)$ (j) $\log_{14}\left(\frac{a^3}{bc}\right)$ (k) $\ln\left(\frac{s^3w^5}{rt^2}\right)$ (l) $\log_7\left(\frac{x^{2/3}}{y^{3/4}}\right)$ 3. (a) $10\log_2 x$ (b) $-7\ln y$ (c) $\frac{1}{2}\log_3 x$ (d) $\frac{1}{5}\ln x$ (e) $-3\log_6 x$ (f) $\frac{4}{5}\ln x$ (g) $\log_5(x+2)-\log_5(x-2)$ (h) $\ln x-\ln(x+1)-\ln(x+2)$ (i) $3\log_8 x+\log_8(x+2)$ (j) $\frac{1}{2}\ln(x+1)+\ln(2x-1)$ (k) $\frac{1}{2}\log_3 x-\frac{1}{2}\log_3(x+4)$ (l) $\frac{2}{3}\ln x-\frac{1}{3}\ln(2x+1)$
4. (a) 8.63345 (b) 6.40192 (c) 5.82728 (d) 0.91024 (e) −11.73607 (f) −0.24153 (g) −2.99573 (h) 20.08554 (i) 3.46718 (j) 1.85527 5. (a) 125 (b) e (c) e^2+1 (d) $\frac{1}{2e}$ 6. (a) 0.36067 (b) 1.24267 (c) 0.04343 (d) 0.05057 (e) 0.25 (f) 2 (g) 0.1 (h) 0.05556 7. (a) $\frac{5\log_4 e}{5x+6}$ (b) $\frac{8(2x-5)\log_6 e}{x^2-5x}$ (c) $\frac{2\log e}{(x-1)(x+1)}$ (d) $\frac{60x^2\log_{12} e}{2x^3+5}$ 8. (a) $\frac{1}{x}$ (b) $\frac{1}{x}$ (c) $\frac{6}{x}$ (d) $-\frac{2}{x}$ (e) $\frac{3}{4x}$ (f) $\frac{8}{2x+7}$ (g) $\frac{10}{6-5x}$ (h) $\frac{8}{x}$ (i) $-\frac{1}{x}$ (j) $\frac{5(2x+5)}{x^2+5x+3}$ (k) $-\frac{1}{x+3}$ (l) $-\frac{3x^2}{x^3+1}$ (m) $\frac{4}{6x-1}$ (n) $-\frac{3x}{2(3x^2+4)}$ (o) $\frac{6x-5}{(x-2)(3x+1)}$ (p) $\frac{11x-10}{2x(x-1)}$ (q) $\frac{x^2+12}{x(x^2+4)}$ (r) $\frac{-13}{(x+6)(2x-1)}$ 9. (a) $x^2(3\ln x+1)$ (b) $\frac{2x^2\ln x+x^2-4}{x}$ (c) $\frac{2(\ln x-1)}{(\ln x)^2}$ (d) $2x\ln(x+1)+x-1$ (e) $\frac{6x^2\left[(1-x^3)\ln(1-x^3)-x^3\right]}{1-x^3}$ (f) $4(x-1)^3\left[\ln(x-1)^4+1\right]$ (g) $\frac{72x\left[\ln(3x^2+8)\right]^5}{3x^2+8}$ (h) $\frac{72x}{3x^2+8}$ (i) 0 (j) 1 (k) $\frac{2}{x\ln x}$ (l) $\frac{15}{(3x+1)\ln(3x+1)}$ 12. $y=3x-1$ 13. $y=2x-e$ 14. (a) increasing for $x\in(0,2)$; decreasing for $x\in(2,\infty)$; relative maximum at $(2,\ln 2-1)$ (b) decreasing for $x\in(0,2)$; increasing for $x\in(2,\infty)$; relative minimum at $(2,4-8\ln 2)$ 15. concave down for $x\in(0,1/e)$; concave up for $x\in(1/e,\infty)$ 16. maximum temperature of 40.17°C after 8.09 hours