Identify whether the following languages are regular or not. If it's regular, justify your answer; if not, use pumping Lemma to prove it. [5 pts each].

1. L1 = $\{0^n 1^n \mid 0 \le n \le 1000\}$
    - L1 represents a language where the number of "0"s followed by the same number of "1"s can vary from 0 to 1000, inclusive. This language is, in fact, regular because a finite automaton can recognize it
    - With no closed set, this language is irregular, but because it is in a closed set, it makes this language regular.
    - We can construct a simple deterministic finite automaton (DFA) for L1. The DFA can have states keep track of the number of "0"s encountered and ensure that they are followed by an equal number of "1"s.
    - The DFA would have 2n states with the max number of states being 2000. If there was no closed range, this number would be infinity and there after irregular
    - The accepting state is reached when all "0"s are matched by an equal number of "1"s.
    - This DFA will successfully recognize strings in L1 by ensuring that for every "0," there is a corresponding "1" following it. Since DFAs can recognize regular languages, this demonstrates that L1 is regular.

2. L2 = $\{a^{2^n} \mid n \ge 0,\ here\ a^{2^n}\ means\ a\ string\ of\ 2^n\ a's\}$ }
    - We must suppose that L2 is regular. There exists a pumping constant "p" for L2. We can choose a w that is inside the language.
    - For this example, we will select w = a^2^p.
    - We must look at all decomps of the within xyz.
        - X = 0^a
        - Y = 0^b
        - Z = 0
    - X Y^i Z = (0^a) (0^iB) (0^2^(p)-a-b)
    - Simplification = (0^2^(p)+(i-1)B)
    - Choose an i such that xy^i is not in the set of L
        - i such that 2^p + (i-1)B is a power of 2
        - Try i = 2
        - 2^p   <   2^p + B   <=   2^p + p   <   2^p+2^p = 2^p+1
        - | | |
        - B >=1   B<=p   p<2^p
    - For i = 2 the L is not regular
    - Therefore the language L4 is not regular

3. L3 = $\{\omega\omega^R\beta \mid \omega, \beta \in \{0,1\}^+\}$
    - Suppose L3 is regular and has a pumping length p. Let's choose the string w = "01^p10^p," which is in L3 (ω = "0," ω^R = "1," β = "0^p"). According to the pumping lemma, we can write w as xyz, where:
        - x = "0"
        - y = "1^q" for some q > 0

- z = "0^(p-q)10^p"
- Now, consider xy^2z:
  - xy^2z = "01^q1^q0^(p-q)10^p"
- This string is not in L3 because the ω and ω^R parts are no longer equal. Therefore, L3 cannot be regular.

4. L4 = {1^i 0^j1^k | i > j and i < k and i, j, k > 0}
   - Suppose L4 is regular and has a pumping length p.
   - Let's choose the string w = "1^p0^(p+1)1^(p+2)" for this language. According to the pumping lemma, we can write w as xyz, where:
     - x = "1^r" for some r ≥ 0
     - y = "1^s" for some s > 0
     - z = "1^(p-r-s)0^(p+1)1^(p+2)"
   - Now, consider xy^2z:
     - xy^2z = "1^(r+s)0^(p-r-s)1^(p+1)1^(p+2)"
   - The number of "1"s in the first part is r + s, and the number of "1"s in the last part is p+1+p+2=2p+3. Since r + s ≠ 2p + 3, this string is not in L4.
   - Therefore, L4 cannot be regular.