

CSE 355: Intro to Theoretical Computer Science Recitation #7 Solution

1.[5 pts] For each of the following CFG, use set notation to define the language generated by the grammar

1.1)
$$\begin{aligned} S &\rightarrow aSbb \mid A \\ A &\rightarrow cA \mid c \end{aligned}$$

$$L = \{a^i c^j b^{2i} \mid \text{where } i \geq 0, j > 0\}$$

1.2)
$$\begin{aligned} S &\rightarrow aS \mid bB \mid \epsilon \\ B &\rightarrow aB \mid bS \mid bC \\ C &\rightarrow aC \mid \epsilon \end{aligned}$$

$$L = \{\omega \in \{a, b\}^* \mid \omega \text{ contains even number of } bs\}$$

2. [5 pts] Let G be the following grammar:

$$\begin{aligned} S &\rightarrow SAB \mid \epsilon \\ A &\rightarrow aA \mid a \\ B &\rightarrow bB \mid \epsilon \end{aligned}$$

2.1) Give a leftmost derivation of $abbaab$

Derivation	Rule applied
$S \Rightarrow SAB$	$S \rightarrow SAB$
$\Rightarrow SABAB$	$S \rightarrow SAB$
$\Rightarrow ABAB$	$S \rightarrow \epsilon$
$\Rightarrow aBAB$	$A \rightarrow a$
$\Rightarrow abBAB$	$B \rightarrow bB$
$\Rightarrow abbBAB$	$B \rightarrow bB$
$\Rightarrow abbAB$	$B \rightarrow \epsilon$
$\Rightarrow abbaAB$	$A \rightarrow aA$
$\Rightarrow abbaabB$	$A \rightarrow a$
$\Rightarrow abbaabB$	$B \rightarrow bB$
$\Rightarrow abbaab$	$B \rightarrow \epsilon$

2.2) Give two leftmost derivation of aa

First Derivation	Rule applied
$S \Rightarrow SAB$	$S \rightarrow SAB$
$\Rightarrow AB$	$S \rightarrow \epsilon$
$\Rightarrow aAB$	$A \rightarrow aA$
$\Rightarrow aaB$	$A \rightarrow a$
$\Rightarrow aa$	$B \rightarrow \epsilon$

Second Derivation	Rule applied
$S \Rightarrow SAB$	$S \rightarrow SAB$
$\Rightarrow SABAB$	$S \rightarrow SAB$
$\Rightarrow ABAB$	$S \rightarrow \epsilon$
$\Rightarrow aBAB$	$A \rightarrow a$
$\Rightarrow aAB$	$B \rightarrow \epsilon$
$\Rightarrow aaB$	$A \rightarrow a$
$\Rightarrow aa$	$B \rightarrow \epsilon$

2.3) Is this grammar ambiguous?

Since for aa , we have above two different left-most derivation, the grammar is ambiguous.

2.4) Is this language $L(G)$ regular? If yes, give a regular expression for the language.

Yes. rule $A \rightarrow aA | a$ generates language a^+ , rule $B \rightarrow bB | \epsilon$ generates language b^* , so rule $S \rightarrow SAB | \epsilon$ will generate $(a^+b^*)^*$.

3. [10 pts] Follow the procedure discussed in class (or in textbook pp.128), convert the following CFG into Chomsky Normal Form.

$$\begin{aligned} S &\rightarrow aTc \mid bc \\ T &\rightarrow ST \mid \epsilon \end{aligned}$$

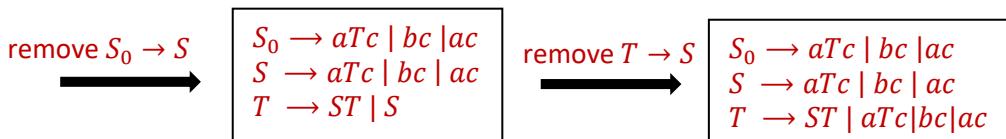
Step #1: add a new start variable S_0 and a new rule $S_0 \rightarrow S$ inside the grammar, where S is the original start variable.

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow aTc \mid bc \\ T &\rightarrow ST \mid \epsilon \end{aligned}$$

Step #2: eliminate the ϵ – rule. i.e. if there's a rule $T \rightarrow \epsilon$, then for any rule where T showing on the right hand side, add a rule with T being removed.

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow aTc \mid bc \mid ac \\ T &\rightarrow ST \mid S \end{aligned}$$

Step #3: remove the unit rule.



Step #4: take care of rules with more than two variables/terminals on the righthand side.

