CSE 355: Intro to Theoretical Computer Science
Recitation #6 Solution (20 pts)

1. [10 pts] Answer the questions for the following CFG G.

$$R \longrightarrow XRX \mid S$$
$$S \longrightarrow aTb \mid bTa$$
$$T \longrightarrow XTX \mid X \mid \varepsilon$$
$$X \longrightarrow a \mid b$$

1.1) What are the variables of G?

R, S, T* and *X

1.2) What are the terminals of G?

a, b

1.3) Which is the start variable of G?

R

1.4) Give three strings in L(G)

aabb, abbaa, ba

1.5) Give three strings **NOT** in L(G)

ε , a, bb

1.6) For the following derivations, circle **True** or **False**.

$T \Rightarrow aba$	True	False
$T \overset{*}{\Rightarrow} aba$	True	False
$T \Rightarrow T$	True	False
$XXX \overset{*}{\Rightarrow} aba$	True	False
$X \overset{*}{\Rightarrow} aba$	True	False
$T \overset{*}{\Rightarrow} XX$	True	False
$T \overset{*}{\Rightarrow} XXX$	True	False
$S \overset{*}{\Rightarrow} \varepsilon$	True	False

2. [5 pts] Give context free grammas that generate the following languages. Alphabet Σ = {0, 1}.

2.1) $L = \{\omega \mid \omega \; contains \; at \; least \; three \; 1s\}$

$$\boldsymbol{S \longrightarrow T1T1T1T}$$
$$\boldsymbol{T \longrightarrow 0T \mid 1T \mid \varepsilon}$$

2.2) $L = \{\omega \mid \omega \; starts \; and \; ends \; with \; the \; same \; symbol\}$

$\boldsymbol{S \longrightarrow 0T0 \mid 1T1 \mid 0 \mid 1 \mid \varepsilon}$

$\boldsymbol{T \longrightarrow 0T \mid 1T \mid \varepsilon}$

2.3) $L = \{\omega \mid the \; length \; of \; \omega \; is \; odd\}$

$\boldsymbol{S \longrightarrow 0T \mid 1T}$

$\boldsymbol{T \longrightarrow 0S \mid 1S \mid \varepsilon}$

2.4) $L = \{\omega \mid the \; length \; of \; \omega \; is \; odd \; and \; its \; middle \; is \; a \; 0\}$

$\boldsymbol{S \longrightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0}$

or $\boldsymbol{S \longrightarrow TST \mid 0}$

$\boldsymbol{T \longrightarrow 0 \mid 1}$

2.5) $L = \{\omega \mid \omega = \omega^R, that \; \omega \; is \; a \; palindrome\}$

$\boldsymbol{S \longrightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon}$

3. [5 pts] Give CFG that generates the language $A = \{a^i b^j c^k \mid i = j \; or \; j = k \; where \; i, j, k \geq 0\}$. Is your grammar ambiguous? Why or why not?

First identify the following two simpler languages L_1, and L_2. Notice that $A = L_1 \cup L_2$

$\boldsymbol{L_1 = \{a^i b^j c^k \mid i = j, where \; i, j, k \geq 0\}}$

$\boldsymbol{L_2 = \{a^i b^j c^k \mid j = k, where \; i, j, k \geq 0\}}$

CFG for generating L_1 is:

$$\boldsymbol{S_1 \longrightarrow X_1 T_1}$$
$$\boldsymbol{X_1 \longrightarrow aX_1 b \mid \varepsilon}$$
$$\boldsymbol{T_1 \longrightarrow cT_1 \mid \varepsilon}$$

CFG for generating L_2 is:

$$\boldsymbol{S_2 \longrightarrow T_2 X_2}$$
$$\boldsymbol{X_2 \longrightarrow bX_2 c \mid \varepsilon}$$
$$\boldsymbol{T_2 \longrightarrow aT_s \mid \varepsilon}$$

Then by adding a new starting symbol S, we can have the following CFG for A:

$$\boldsymbol{S \longrightarrow S_1 \mid S_2}$$
$$\boldsymbol{S_1 \longrightarrow X_1 T_1}$$
$$\boldsymbol{X_1 \longrightarrow aX_1 b \mid \varepsilon}$$
$$\boldsymbol{T_1 \longrightarrow cT_1 \mid \varepsilon}$$
$$\boldsymbol{S_2 \longrightarrow T_2 X_2}$$
$$\boldsymbol{X_2 \longrightarrow bX_2 c \mid \varepsilon}$$
$$\boldsymbol{T_2 \longrightarrow aT_s \mid \varepsilon}$$

(Note: above grammar is ambiguous since for string abc we can have two different derivations)