CSE 355: Intro. to Theoretical Computer Science
Recitation #5 Solution

Identify whether the following languages are regular or not. If it's regular, justify your answer; if not, use pumping Lemma to prove it. [5 pts each].

1. $L_1 = \{0^n 1^n \mid 0 \leq n \leq 1000\}$

Since $0 \leq n \leq 1000$, L_1 is a finite set, so L_1 must be regular.

2. $L_2 = \{a^{2^n} \mid n \geq 0, here\ a^{2^n}\ means\ a\ string\ of\ 2^n\ a's\}$

L_2 is non-regular, and we will prove it below.

Proof by contradiction:

Step #1: Let us assume L_2 is regular, then according to pumping lemma, there must exist a pumping length p such that, for any string $s \in L_2$, as long as $|s| \geq p$, s can be broken into three strings, $s = x \cdot y \cdot z$, such that:
1) $y \neq \varepsilon$
2) $|xy| \leq p$
3) for all $k \geq 0, the\ string\ xy^k z\ must\ also \in L_2$

Step #2: We will pick one special string $s \in L_2$ and make sure $|s| \geq p$ so that we can apply above rules. The special string we pick is $s = a^{2^p}$, notice $s \in L_2\ and\ |s| = 2^p\ which\ is > p$, so we should be able to apply pumping lemma to this string.

Step #3: We will get a contradiction from the string we picked in step #2. According to above pumping lemma condition #2, $|xy| \leq p$, since $p < 2^p$, so $|y| < 2^p$. If we pump y once, the new string we get will be: $xyyz$ and $|xyyz| = |xyz| + |y| = 2^p + |y| < 2^p + 2^p = 2^{p+1}$. *i.e.* the new string $xyyz$ cannot have a length of exactly power of 2, and this means $xyyz \notin L_2$, so we get a contradiction from above condition #3.

Step #4: from the result of step #3, we conclude that our assumption in step #1 must be wrong and L_2 cannot be regular!

3. $L_3 = \{\omega\omega^R\beta \mid \omega, \beta \in \{0,1\}^+\}$

L_3 is non-regular.

Sorry, this language cannot be proved to be non-regular by using pumping lemma since it always pumpable by re-drawing the boundary for string β. We must use Myhill-Nerode theorem to prove it, but we didn't learn it in this course, so we will give everyone the points in this question.

4. $L_4 = \{1^i 0^j 1^k \mid i > j\ and\ i < k\ and\ i, j, k > 0\}$

L_4 is non-regular, and we will prove it as below.

Proof by contradiction:

Step #1: Let us assume L_4 is regular, then according to pumping lemma, there must exist a pumping length p such that, for any string $\omega \in L_4$, as long as $|\omega| \geq p$, it can be broken into three strings, $\omega = xyz$, such that:
1) $y \neq \varepsilon$

2) $|xy| \leq p$
3) for all $k \geq 0, the\ string\ xy^k z\ must\ also \in\ L_4$

Step #2: We will pick one special string $\omega \in L_4$ and make sure $|\omega| \geq p$ so that we can apply above rules. The special string we pick is $\omega =\ 1^{p+1}0^p1^{p+2}$, notice here *i* = *p+1*, *j* = *p* and *k* = *p*+2 satisfy the condition that $i > j\ and\ i < k\ and\ i, j, k > 0, aslo\ \omega \in L_4\ and\ |\omega| = 3p + 3\ which\ is > p$, so we should be able to apply pumping lemma to this string.

Step #3: We will get a contradiction from the string we picked in step #2. According to pumping lemmas, $\omega =\ 1^{p+1}0^p1^{p+2}$ can be broke into three strings, $\omega = xyz$, such that:
1) $y\ \neq\ \varepsilon$
2) $|xy| \leq p$
3) for all $k \geq 0, the\ string\ xy^k z\ must\ also \in\ L_1$

From above condition #2, we conclude that for ω ($1^{p+1}0^p1^{p+2}$), *y* must contain all 1s because $|xy| \leq p$, then apply condition #3, if we pump *y* just once (*i.e.* pick *k* =1), the resulting string will be $1^{p+2}0^p1^{p+2}$, must also $\in\ L_4$, but this cannot be true since now *i* = *p+2*, *j* = *p* and *k* = *p*+2, *i* is NOT less than *k* anymore, the resulting string $\notin L_4$, , so we get a contradiction here.

Step #4: from the result of step #3, we conclude that our assumption in step #1 must be wrong and L_4 cannot be regular!