CSE 355: Intro to Theoretical Computer Science
Recitation #7 Solution

1.[5 pts] For each of the following CFG, use set notation to define the language generated by the grammar

1.1)

$$S \longrightarrow aSbb \mid A$$
$$A \longrightarrow cA \mid c$$

$$\boldsymbol{L = \{a^i c^j b^{2i} \mid where\ i \geq 0, j > 0\}}$$

1.2)

$$S \longrightarrow aS \mid bB \mid \varepsilon$$
$$B \longrightarrow aB \mid bS \mid bC$$
$$C \longrightarrow aC \mid \varepsilon$$

$$\boldsymbol{L = \{\omega \in \{a, b\}^* \mid \omega\ contains\ even\ number\ of\ bs\}}$$

2. [5 pts] Let G be the following grammar:

$$S \longrightarrow SAB \mid \varepsilon$$
$$A \longrightarrow aA \mid a$$
$$B \longrightarrow bB \mid \varepsilon$$

2.1) Give a leftmost derivation of *abbaab*

Derivation	**Rule applied**
$S \Rightarrow SAB$	$S \longrightarrow SAB$
$\Rightarrow SABAB$	$S \longrightarrow SAB$
$\Rightarrow ABAB$	$S \longrightarrow \varepsilon$
$\Rightarrow aBAB$	$A \longrightarrow a$
$\Rightarrow abBAB$	$B \longrightarrow bB$
$\Rightarrow abbBAB$	$B \longrightarrow bB$
$\Rightarrow abbAB$	$B \longrightarrow \varepsilon$
$\Rightarrow abbaAB$	$A \longrightarrow aA$
$\Rightarrow abbaabB$	$A \longrightarrow a$
$\Rightarrow abbaabB$	$B \longrightarrow bB$
$\Rightarrow abbaab$	$B \longrightarrow \varepsilon$

2.2) Give two leftmost derivation of *aa*

First Derivation	**Rule applied**
$S \Rightarrow SAB$	$S \longrightarrow SAB$
$\Rightarrow AB$	$S \longrightarrow \varepsilon$
$\Rightarrow aAB$	$A \longrightarrow aA$
$\Rightarrow aaB$	$A \longrightarrow a$
$\Rightarrow aa$	$B \longrightarrow \varepsilon$

Second Derivation	**Rule applied**
$S \Rightarrow SAB$	$S \longrightarrow SAB$
$\Rightarrow SABAB$	$S \longrightarrow SAB$
$\Rightarrow ABAB$	$S \longrightarrow \varepsilon$
$\Rightarrow aBAB$	$A \longrightarrow a$
$\Rightarrow aAB$	$B \longrightarrow \varepsilon$
$\Rightarrow aaB$	$A \longrightarrow a$
$\Rightarrow aa$	$B \longrightarrow \varepsilon$

2.3) Is this grammar ambiguous?

Since for *aa*, we have above two different left-most derivation, the grammar is ambiguous.

2.4) Is this language $L(G)$ regular? If yes, give a regular expression for the language.

Yes. rule $A \longrightarrow aA \mid a$ generates language a^+, rule $B \longrightarrow bB \mid \varepsilon$ generates language b^*, so rule $S \longrightarrow SAB \mid \varepsilon$ will generate $(a^+b^*)^*$.

3. [10 pts] Follow the procedure discussed in class (or in textbook pp.128), convert the following CFG into Chomsky Normal Form.

$$S \longrightarrow aTc \mid bc$$
$$T \longrightarrow ST \mid \varepsilon$$

Step #1: add a new start variable S_0 and a new rule $S_0 \longrightarrow S$ inside the grammar, where S is the original start variable.

$S_0 \longrightarrow S$
$S \longrightarrow aTc \mid bc$
$T \longrightarrow ST \mid \varepsilon$

Step #2: eliminate the $\varepsilon - rule$. i.e. if there's a rule $T \longrightarrow \varepsilon$, then for any rule where T showing on the right hand side, add a rule with T being removed.

$S_0 \longrightarrow S$
$S \longrightarrow aTc \mid bc \mid ac$
$T \longrightarrow ST \mid S$

Step #3: remove the unit rule.

remove $S_0 \to S$ $\Longrightarrow$

$S_0 \longrightarrow aTc \mid bc \mid ac$
$S \longrightarrow aTc \mid bc \mid ac$
$T \longrightarrow ST \mid S$

remove $T \to S$ $\Longrightarrow$

$S_0 \longrightarrow aTc \mid bc \mid ac$
$S \longrightarrow aTc \mid bc \mid ac$
$T \longrightarrow ST \mid aTc \mid bc \mid ac$

Step #4: take care of rules with more than two variables/terminals on the righthand side.

$S_0 \longrightarrow aTc \mid bc \mid ac$
$S \longrightarrow aTc \mid bc \mid ac$
$T \longrightarrow ST \mid aTc \mid bc \mid ac$

$\Longrightarrow$ $U \to a, V \longrightarrow b, W \longrightarrow c$

$S_0 \longrightarrow UTW \mid VW \mid UW$
$S \longrightarrow UTW \mid VW \mid UW$
$T \longrightarrow ST \mid UTW \mid UW \mid VW$
$U \longrightarrow a$
$V \longrightarrow b$
$W \longrightarrow c$

$\Longrightarrow$ $X \to TW$

$S_0 \longrightarrow UX \mid VW \mid UW$
$S \longrightarrow UX \mid VW \mid UW$
$T \longrightarrow ST \mid UX \mid UW \mid VW$
$X \longrightarrow TW$
$U \longrightarrow a$
$V \longrightarrow b$
$W \longrightarrow c$