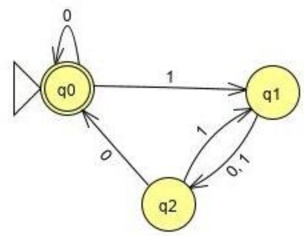
CSE 355: Intro to Theoretical Computer Science
Recitation #12 Solution (20 pts)

1. [6 pts] Answer questions for the following DFA M and give reasons for your answers.



1.1) Is $< M, 0100 > \in A_{DFA}$? (Note: A_{DFA} represents acceptance problem of DFA)

Yes, since M accepts string 0100.

1.2) Is $< M, 011 > \in A_{DFA}$?

No, since M does not accept string 011.

1.3) Is $< M > \in A_{DFA}$?

No, since <M> is not in the correct format and $\notin A_{DFA}$

1.4) Is $< M, 0100 > \in A_{REX}$?

No, since M is not a regular expression. It is a DFA.

1.5) Is $< M > \in E_{DFA}$?

No, since $L(M) \neq \phi$. *i.e.* the language M accepts is not empty.

1.6) Is $< M, M > \in EQ_{DFA}$?

Yes. Since they are the same machine and accept the same language.

2. [7 pts] Assume Σ={0,1}. Given a DFA C, does there exist an algorithm to decide whether $L(C) = \Sigma^*$? Express this problem as a language denoted as ALL_{DFA} and prove that ALL_{DFA} is decidable (Hints: build a Turing decider).

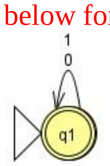
$$ALL_{DFA} = \{< C >|\ C \text{ is a DFA and } L(C) = \Sigma^*\}$$

Claim: ALL_{DFA} is decidable

Proof-by-construction:

TM **W** = "on input $< C >$"

1. Create a DFA **D** that accepts Σ^* (see below for such construction)



2. Run TM **F** on input $< C, D >$. Note: TM **F** decides EQ_{DFA} problem

   If F accepts, **W** ACCEPT

   If F rejects, **W** REJECT

3. [7 pts] Given two DFAs A and B, we consider the problem of deciding whether $L(A)$ (language of A) is a subset of $L(B)$. Express this problem as a language denoted as SUB_{DFA} and prove that SUB_{DFA} is decidable.

$$SUB_{DFA} = \{< A, B >| \ A, B \ are \ DFAs \ and \ L(A) \subseteq L(B)\}$$

Claim: SUB_{DFA} is decidable (Hint: $L(A) \subseteq L(B) \ if \ and \ only \ if \ L(A) \cap L(B) = L(A)$)

Proof-by-construction:

TM **X** = "on input < *A*, *B* >"

1. Since regular languages are closed under ∩, create a new DFA **C** such that C accepts $L(A) \cap L(B)$. (Note: we can build C by using cross-product construction)
2. Run TM **F** on input < *A*, *C* >. Note: TM **F** decides EQ_{DFA} problem

   If F accepts, **X** ACCEPT

   If F rejects, **X** REJECT