

1. Let  $E = \{a,b\}$ . As the following example shows, for each regular expression, write the formal English description of the set described by the regular expression. 20 points

a)  $a^+(bb)^* \cup b^*a(aa)^*$

$L = \{x \mid x \text{ starts with at least one 'a' character and zero or more instances of 'bb' or } x \text{ starts with zero or more occurrences of 'b' and then an 'a' and then zero or more occurrences of the string 'aa'}\}$

b)  $(\Sigma^*aba\Sigma^* \cup \Sigma^*bab\Sigma^*)\Sigma^*abb\Sigma^*$

$L = \{x \mid x \text{ either contains 'aba' or 'bab' in the string, followed by zero or more occurrences of the substring 'abb'}\}$

c)  $a^* \cup (a^*ba^*ba^*ba^*)^*$

$L = \{x \mid x \text{ consists of zero or more 'a's in the string or it consists of zero or more repetitions of 'a' followed by zero or more occurrences of 'ba'}\}$

d)  $ba\Sigma^*a\Sigma^*\Sigma^*$

$L = \{x \mid x \text{ starts with 'ba' and then zero or more occurrences of a or b then contains 'a' then zero or more occurrences of a or b}\}$

2. Let  $\Sigma = \{0,1\}$ . For each part below, write the regular expression that describes the language.

a)  $L_1 = \{w \mid w \text{ is of even length and } |w| > 0\}$

$L_1 = (\Sigma\Sigma)^{+}$

b)  $L_2 = \{w \mid w \text{ has an even number of 0s and an odd number of 1s}\}$

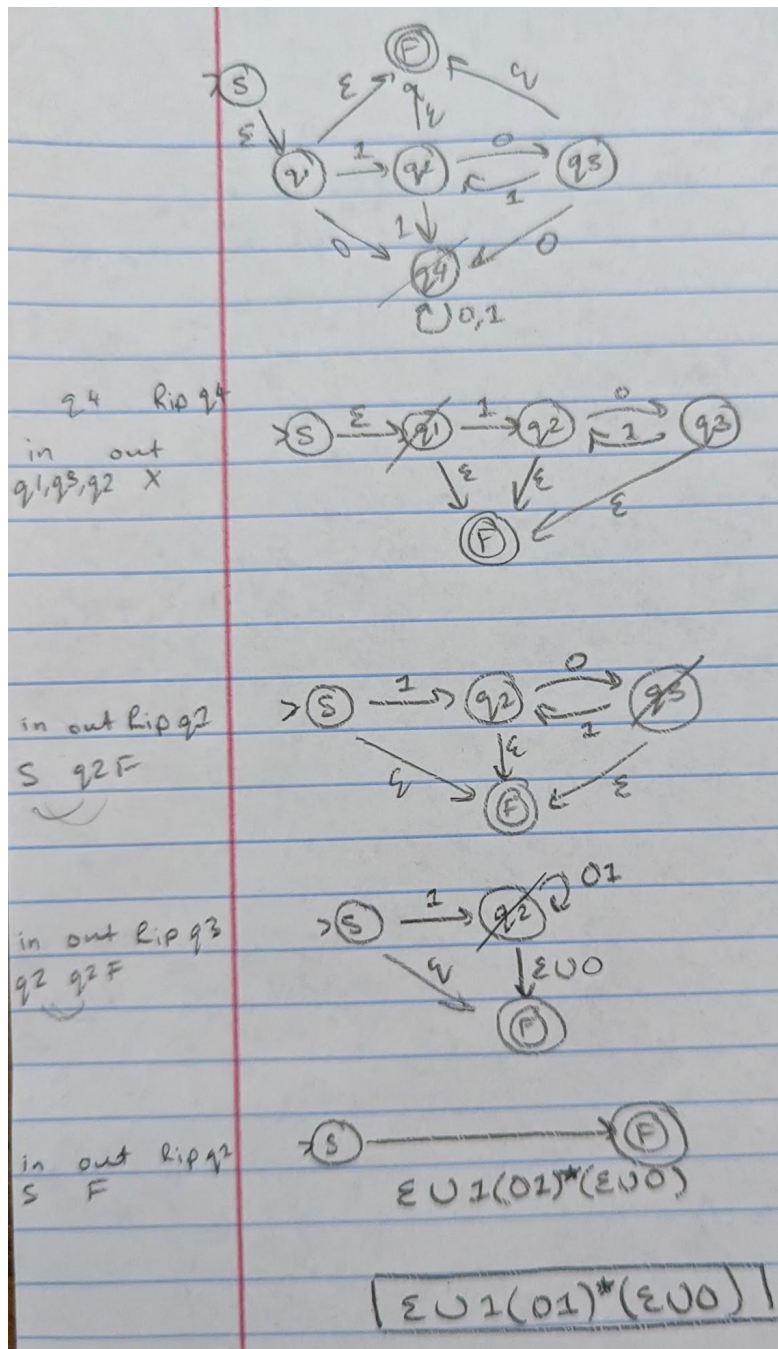
$L_2 = (00 \cup (11)^*1)^*$

c)  $L_3 = \{w \mid w \text{ is any string except } 0, 01, 11, 1100, 1101, 1110, 1111\}$

$L_3 = (\Sigma^*(0, 01, 11, 1100, 1101, 1110, 1111)\Sigma^*)^*$

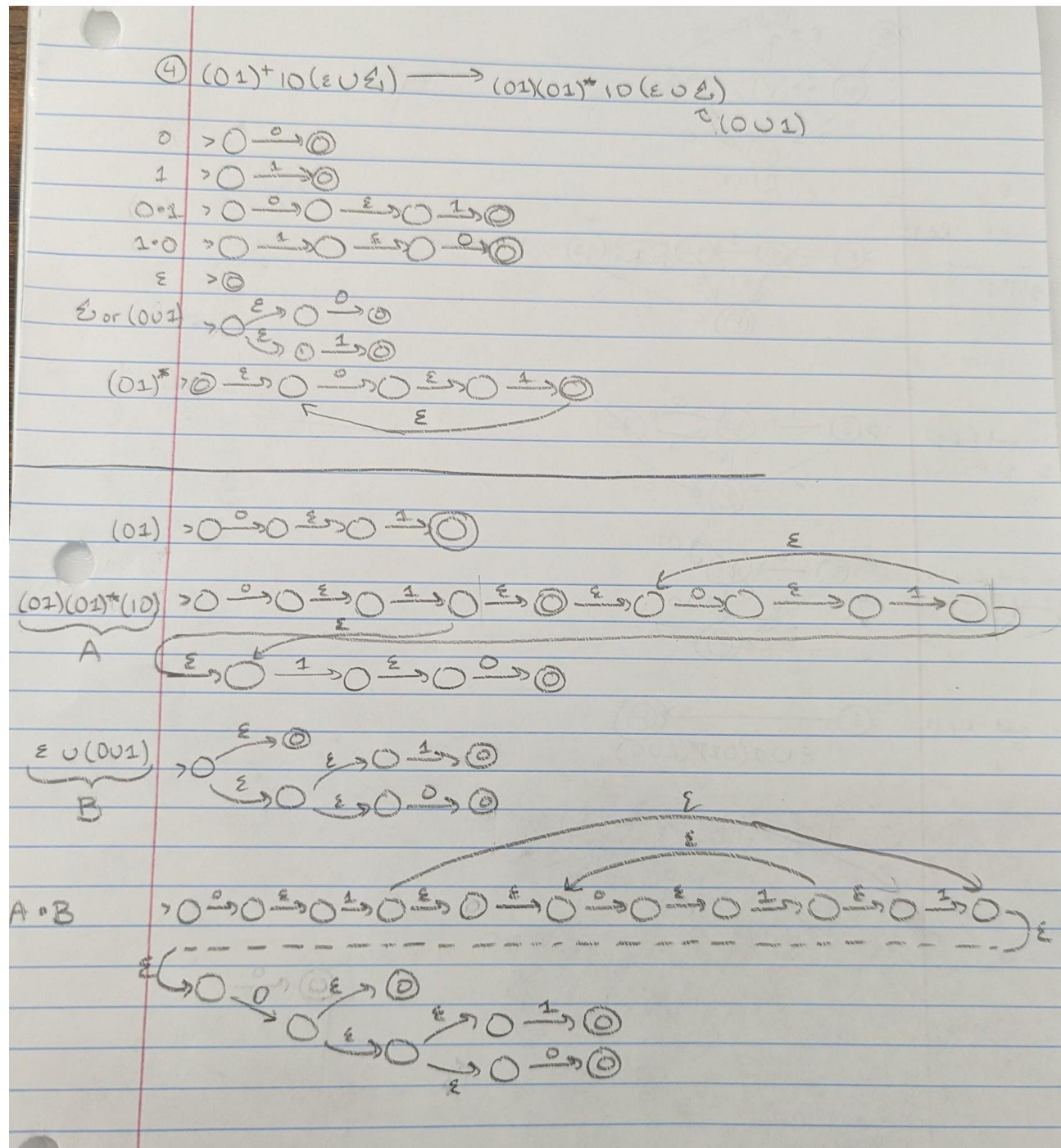
3. Use the GNFA method discussed in class to convert the following DFA into an equivalent regular expression. Note: 1) eliminate states in order of  $q_1, q_2, q_3, \dots$ , 2) you must show all steps to receive full credit.

Let DFA  $M = (\{q_1, q_2, q_3\}, \{0, 1\}, S, q_1, \{q_1, q_2, q_3\})$  where the transition function table is listed below. Draw its state diagram first before eliminating states.



4. Let  $0, 1 = \{0, 1\}$ . Use the procedure given in Lemma 1.55 to convert the following regular expression into an NFA. You must show all steps to receive full credit. 10 points

$(01)^+ 10(\epsilon \cup \Sigma)$



5. For each part, first, identify whether the language is regular or not, you must justify your answers to receive full credit. If the language is non-regular, use the pumping lemma to prove it. 40 points, 10 pts each

a)  $L4 = \{\omega\omega^R \mid \omega \in \{0, 1\}^*, \text{ and } \omega^R \text{ is the reversal of } \omega\}$

This language consists of strings that are either palindromes or contain their reversal. It is not regular. To prove this using the pumping lemma, assume for the sake of contradiction that  $L4$  is regular. Then, by the pumping lemma, there exists a pumping length  $p$  such that any string  $s$  in  $L4$  with length at least  $p$  can be split into three parts  $s = xyz$ , satisfying the pumping lemma conditions.

Now, consider the string  $\omega = 0^p 1^p 0^p 1^p$ . This string is in  $L4$  because it's a palindrome, and its length is greater than or equal to  $p$ . According to the pumping lemma, we can split  $\omega$  into  $xyz$  in such a way that the pumped string  $xy^n z$  is also in  $L4$  for all  $n \geq 0$ .

However, no matter how we split  $\omega$ , either  $x$  or  $z$  must contain characters from both the 0's and 1's, because pumping  $y$  will not change the balance of 0's and 1's. Therefore, the pumped string  $xy^n z$  cannot be a palindrome, nor can it contain its reversal. This contradicts the definition of  $L4$ , so  $L4$  is not regular.

b)  $L5 = \{xy \mid |x| = |y| \text{ and } x, y \in \{0, 1\}^*\}$

This language consists of pairs of strings  $x$  and  $y$ , where the length of  $x$  is equal to the length of  $y$ . It is regular because it can be recognized by a regular expression or a finite automaton. For example, a regular expression for  $L5$  is  $(0+1)00(0+1)^*$ .

c)  $L6 = \{a^i b^j c^k \mid j = i + k \text{ and } i, j, k > 0\}$

Handwritten proof for the non-regularity of  $L6$  using the pumping lemma:

$L6 = \{a^i b^j c^k \mid j = i + k \text{ and } i, j, k > 0\}$

Let  $L6$  be a regular language  
 Assume:  $|xy| \leq p$   
 $|y| > 0$   
 For any  $i \geq 0$ , the str  $xy^i z$  in  $L6$

$w = a^p b^{2p} c^p$

$xyz =$   
 $- x = a^x$   
 $- y = a^B$   
 $- z = a^{p-x-B} b^{2p} c^p$

$xy^i z = (a^x)(a^{iB})(a^{p-x-B} b^{2p} c^p)$   
 $= a^{p-B-iB} b^{2p} c^p$   
 $= a^{p-B(1+i)} b^{2p} c^p$

Because  $p + p - B(1+i) = 2p$   
 $2p - B(1+i) \neq 2p$   
 $\neq 0$

For all  $i \neq -1$ ,  $-B(1+i) \neq 0$  & this means that the following equation  $2p - n \neq 2p$  ∴ the language  $L6$  is non-regular

d)  $L_7 = \{\omega \mid \omega \in \{0, 1\}^*, \text{ and } N_0(\omega) \% 3 > N_1(\omega) \% 3\}$ .

This language consists of strings  $\omega$  where the number of 0's is greater than the number of 1's when considering the modulo 3 of their counts. It is regular because it can be recognized by a finite automaton. For example, a finite automaton can keep track of the counts of 0's and 1's modulo 3 and accept strings that satisfy the given condition.