

1.[5 pts] For each of the following CFG, use set notation to define the language generated by the grammar

1.1)  $S \rightarrow aSbb \mid A$   
 $A \rightarrow cA \mid c$   
 $L = \{a^n c^m b^{2n} \mid n \geq 0, m \geq 1\}$

1.2)  $S \rightarrow aS \mid bB \mid \epsilon$   
 $B \rightarrow aB \mid bS \mid bC$   
 $C \rightarrow aC \mid \epsilon$   
 $L = \{w \in \{a, b\}^*\}$

2. [5 pts] Let G be the following grammar:

$$\begin{aligned} S &\rightarrow SAB \mid \epsilon \\ A &\rightarrow aA \mid a \\ B &\rightarrow bB \mid \epsilon \end{aligned}$$

2.1) Give a leftmost derivation of abbaab (fill in the table to show both derivation and the rules)

Derivation Rule applied

Derivation	Rule Applied
$S \Rightarrow SAB$	$S \rightarrow SAB$
$SABAB$	$S \rightarrow SAB$
$ABAB$	$S \rightarrow \epsilon$
$aBAB$	$A \rightarrow a$
$abBAB$	$B \rightarrow bB$
$abbBAB$	$B \rightarrow bB$
$abbAB$	$B \rightarrow \epsilon$
$abbaAB$	$A \rightarrow aA$
$abbaaB$	$A \rightarrow a$
$abbaabB$	$B \rightarrow bB$
$abbaab$	$B \rightarrow \epsilon$

2.2) Give two leftmost derivation of aa Derivation Rule applied

ONE:

Derivation	Rule Applied
$S \Rightarrow SAB$	$S \rightarrow SAB$
AB	$S \rightarrow e$
aAB	$A \rightarrow aA$
aaB	$A \rightarrow a$
aa	$B \rightarrow e$

TWO:

Derivation	Rule Applied
$S \Rightarrow SAB$	$S \rightarrow SAB$
SABAB	$S \rightarrow SAB$
ABAB	$S \rightarrow e$
aBAB	$A \rightarrow a$
aAB	$B \rightarrow e$
aaB	$A \rightarrow a$
aa	$B \rightarrow e$

2.3) Is this grammar ambiguous?

**The grammar is ambiguous because there are 2 different leftmost derivations of the string 'aa'. Several other strings can be taken to prove this idea that the grammar is in fact, ambiguous**

2.4) Is this language L(G) regular? If yes, give a regular expression for the language.

**No, this language is not regular. This language could also be stated as  $a^n b^n$  where  $n \geq 0$ . This is not regular because a dfa/nfa could not be created that demonstrates this language because it would require an infinite number of states. N in this case can be infinite and there would be no way to count the number of a's that are there to make sure that they are equal to the number of b characters.**

3. [10 pts] Follow the procedure discussed in class (or in textbook pp.128), convert the following CFG into Chomsky Normal Form.

$$S \rightarrow aTc \mid bc$$

$$T \rightarrow ST \mid \epsilon$$

Converting to CNF:

- Make a new start state
  - $X \rightarrow S$
  - $S \rightarrow aTc \mid bc$
  - $T \rightarrow ST \mid \epsilon$
- Find Nullable Vars and Remove Epsilons
  - $X \rightarrow S \mid \epsilon$
  - $S \rightarrow aTc \mid bc \mid ac$
  - $T \rightarrow ST \mid T \mid S$
- Eliminate unit rules
  - $X \rightarrow aTc \mid bc \mid ac \mid \epsilon$
  - $S \rightarrow aTc \mid bc \mid ac$
  - $T \rightarrow ST \mid aTc \mid bc \mid ac$
- Ensure RHS is all Vars, or Single Terminals
  - $X \rightarrow ATC \mid BC \mid AC \mid \epsilon$
  - $S \rightarrow ATC \mid BC \mid AC$
  - $T \rightarrow ST \mid ATC \mid BC \mid AC$
  - $A \rightarrow a$
  - $B \rightarrow b$
  - $C \rightarrow c$
- Reduce RHS so only 2 Vars
  - $X \rightarrow JC \mid BC \mid AC \mid \epsilon$
  - $S \rightarrow JC \mid BC \mid AC$
  - $T \rightarrow ST \mid JC \mid BC \mid AC$
  - $A \rightarrow a$
  - $B \rightarrow b$
  - $C \rightarrow c$
  - $J \rightarrow AT$