Data Transmission: Shannon Capacity

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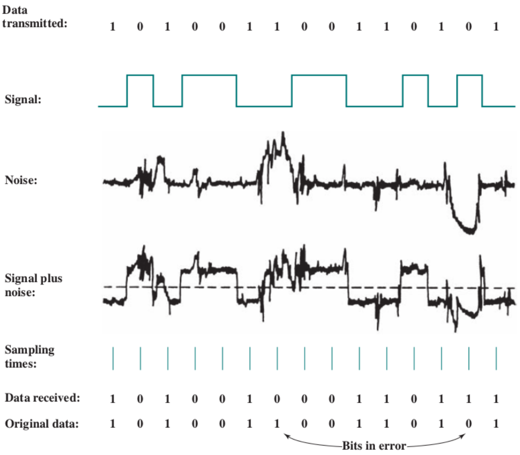
Shannon Capacity Formula

- Nyquist's formula indicates that, *all other things being equal*, doubling the bandwidth doubles the data rate.
- The presence of noise corrupts 1 or more data bits
- If the data rate is increased, bit duration reduces
- Then more bits get affected by the same pattern of noise
- How to connect all the above?

Effect of noise



Shannon Capacity Formula

- For a given level of noise, suppose we increase the signal strength so that we can send data at a higher rate - we improve SNR
- Shannon's formula: $C = B \log_2(1 + SNR)$
- C : Capacity of the channel in bps (also called the **error-free capacity**), B: bandwidth of the channel in Hz
- This presents the theoretical maximum that can be achieved
- In practice, only much lower rates are achieved: the formula assumes only thermal noise

Shannon Capacity Formula

- No matter how many signal levels we have, we cannot achieve a data rate higher than the capacity of the channel
- Shannon's formula defines the characteristics of the channel, not the method of transmission
- From Nyquist's formula, $C_r = 2B \log_2 M$, we can find out the maximum achievable data rate for a method of transmission, given a channel bandwidth and number of levels

Question

- What is the theroetical highest bit rate for a telephone line? The lowest frequency of the line is 300Hz and the highest is 3300 Hz. The signal to noise ratio is 3162.
- What should we do to send data faster than this?

Discussion

We have a channel with 1 MHz bandwidth. The SNR for this channel is 63. What is the maximum capacity of the channel? Give an appropriate number of levels.

Shannon's formula: $C = B\log_2(1 + SNR)$

$C = 10^6 \log_2(1 + 63) = 6Mbps$

Assuming we can reach the limit, suppose we choose 6Mbps as the desired rate.

Now let use the Nyquist formula to find the number of levels.

$C_r = 2B\log_2 M$

$6 * 10^6 bps = 2 * 1 * 10^6 Hz \log_2 M$

$M = 8$

Suppose we choose 4 Mpbs as the desired bit rate, for greater accuracy.

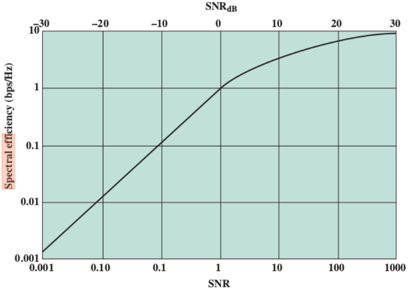
$4 * 10^6 = 2 * 1 * 10^6 Hz \log_2 M$

$M = 4$

Spectral efficiency

- Spectral efficiency or bandwidth efficiency is the number of bits per second of data that can be supported by each hertz of bandwidth
- $C/B = \log_2(1 + SNR)$

Spectral efficiency



- Below 0 dB SNR, noise is the dominant factor in the capacity of a channel. Communication is possible
- In the region of at least 6 dB above 0 dB SNR, noise is no longer the limiting factor in communications speed. Achieving a high-channel capacity depends on the design of the signal

Spectral efficiency

- For a given level of noise, if the bandwidth or the signal strength is increased, does the rate keep increasing? yes
- As signal strength increases, effects of nonlinearities increase, increasing intermodulation noise
- Noise is assumed white: so, the wider the bandwidth, the more the noise admitted. As B increases, SNR decreases!