Frequency domain concepts

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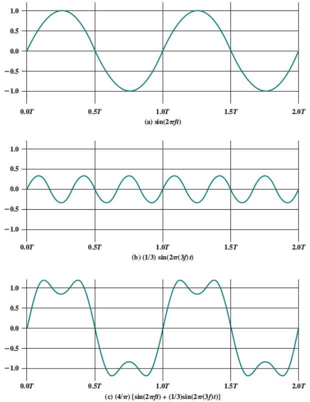
- An electromagnetic signal consists of many frequencies.
- Consider the signal $s(t) = (4/\pi) * [sin(2\pi ft) + (1/3)sin(2\pi(3f)t)]$



Note: $T = 1/f$ in the figure

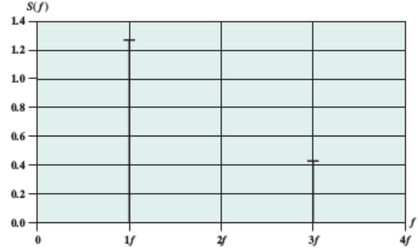
Frequency domain

- The second frequency is an integer multiple of the first frequency
- When all of the frequency components of a signal are integer multiples of one frequency, the latter frequency is referred to as the **fundamental frequency**
- Each multiple of the fundamental frequency is referred to as a **harmonic frequency** of the signal
- *Any electromagentic signal* is made up of components at various frequencies, in which each component is a sinusoid.

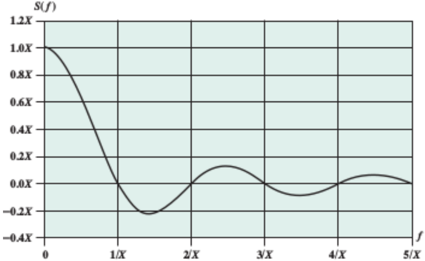
Spectrum of periodic and aperiodic signals

- The **spectrum** of a signal is the range of frequencies that it contains. Example: Spectrum of signal $s(t)$ extends from f to 3f
- For a periodic signal, its spectrum consists of discrete frequency components, at the fundamental frequency and its harmonics.
- A periodic signal can be decomposed into a combination of a finite number of sine waves with frequencies that have integer values
- The technique of **Fourier analysis** is used to find the frequencies of a periodic signal
- For an aperiodic signal, the spectrum consists of a continuum of frequencies.
- An aperiodic signal can be decomposed into a combination of an infinite number of sine waves with frequencies that have real values
- **Fourier transform** is used to find the frequencies of an aperiodic signal

Frequency domain representations of periodic and aperiodic signals



(a) $s(t) = (4/\pi)[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t)]$



(b) $s(t) = 1 \quad -X/2 \le t \le X/2$

- (a) is a periodic signal
- (b) is an aperiodic signal, a square pulse

The S(f) values shown are the maximum amplitudes of the component at that frequency

Question

How many radians are there in a circle?

(A) 360

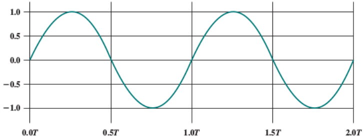
(B) 2π

(C) π

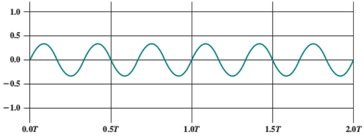
(D) 0

Frequency domain

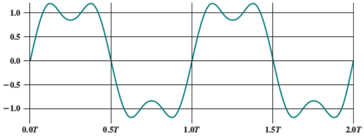
- The *absolute bandwidth* of a signal is the width of the spectrum



(a) sin(2πft)



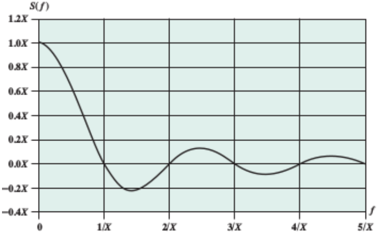
(b) (1/3) sin(2π(3f) t)



(c) (4/π) [sin(2πft) + (1/3)sin(2π(3f)t)]

- What is the bandwidth of the signal in (c)?

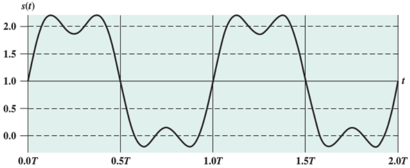
Frequency domain



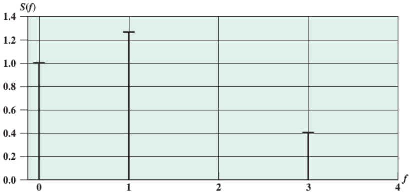
(b) $s(t) = 1 \quad -X/2 \leq t \leq X/2$

- What is the bandwidth of the signal in (b)?
- Most of the energy in the signal is contained in a relatively narrow band of frequencies. This band is referred to as the effective bandwidth, or just bandwidth.

Frequency domain



(a) $s(t) = 1 + (4/\pi)\,[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t)]$



(b) $S(f)$

- If a signal includes a component of zero frequency, that component is a direct current (dc) or constant component.
- With no dc component, a signal has an average amplitude of zero, as seen in the time domain.
- With a dc component, it has a frequency term at $f = 0$ and a nonzero average amplitude.

Data rate and bandwidth

- A given waveform may contain frequencies over a very broad range
- But any transmission system (transmitter plus medium plus receiver) will be able to accommodate only a limited band of frequencies
- This limits the data rate that can be carried on the transmission medium

Data rate and bandwidth

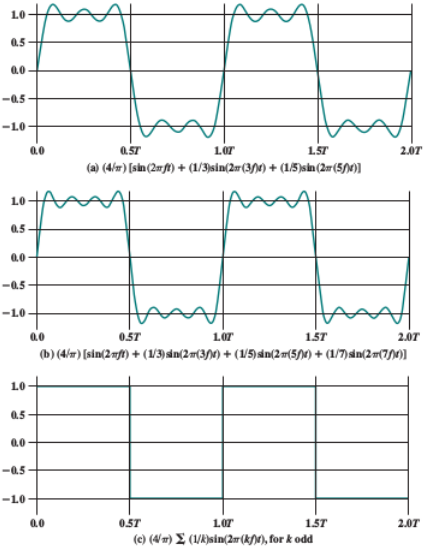


Figure 3.7 Frequency Components of Square Wave ($T = 1/f$)

- (c) represents 01010...
- the duration of each pulse is T/2=1/(2f) -
  the data rate is 2f
  bits per second (bps)
- it can be shown that the frequency components of the square wave with amplitudes A and -A can be expressed as follows:

$$s(t) =$$

$$A * \frac{4}{\pi} * \sum_{k\ odd,\ k=1}^{\infty} \frac{sin(2\pi kft)}{k}$$

Question

How many frequency components does the waveform in the previous slide have? What is its bandwidth?

(A) 4, infinite

(B) infinite, infinite

(C) 4, 6

(D) 0,0

Data rate and bandwidth

- The peak amplitude of the kth frequency component, kf, is only $1/k$, so most of the energy in this waveform is in the first few frequency components
- Limit the bandwidth to just the first three frequency components . The waveform we get is reasonably close to the square wave

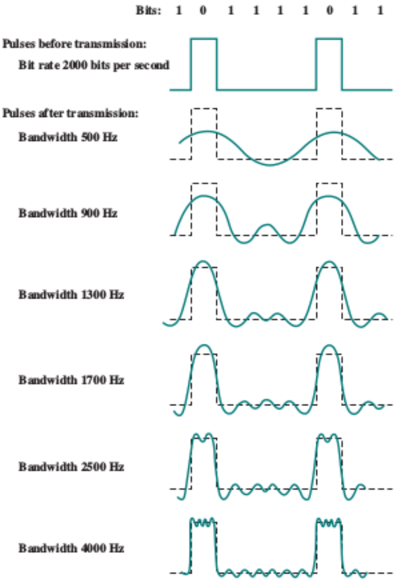
Data rate and bandwidth

- Case I : We take f and the first two harmonics, 3f and 5f
  Bandwidth = 5f-f= 4f = 4 MHz, assuming f=1 MHz.
  Period T = 1/f = 1 μs. Assuming we need to transmit 1010.., 1 bit occurs every 0.5μs. Data rate = 1 bit /0.5 μs= 2Mbps.
  For a bandwidth of 4MHz, a data rate of 2Mbps is achieved.
- Case II . Suppose we have bandwidth = 8 MHz. If f=2MHz, bandwidth of the signal is $(5 * 2 * 10^6 - 2 * 10^6)$ = 8 MHz. Now T=1/f=1/$(2 * 10^6)$ = 0.5μs. A bit occurs every 0.25μs for a data rate of 4Mbps.**By doubling the bandwidth, we double the potential data rate**.

Data rate and bandwidth

- In general, any digital waveform will have infinite bandwidth.
- If we attempt to transmit this waveform as a signal over any medium, the transmission system will limit the bandwidth that can be transmitted.
- For any given medium, the greater the bandwidth transmitted, the greater the cost.
- So we need to approximate digital information by a signal of limited bandwidth, which may create distortions
- Distortions cause a problem at the receiver. What is the problem?

Effect of bandwidth on a digital signal



Effect of bandwidth on a digital signal

- If the data rate of the digital signal is W bps, then a very good representation can be achieved with a bandwidth of 2W Hz.
- Unless noise is very severe, the bit pattern can be recovered with less bandwidth than this

Analog and digital data transmission

- Analog and digital — used in data communication in three contexts: data, signalling and transmission
- data: entities that convey meaning or information
- signals: electric or electromagnetic representations of data
- signalling: the physical propagation of the signal along a suitable medium
- transmission: the communication of data by the propagation and processing of signals

Analog and digital data transmission

- Analog data take on continuous values in some interval. Examples: Voice, video, temperature and pressure sensors
- Digital data: text (character strings)
- Human speech: Frequency components between 100 Hz and 7 kHz, dynamic range about 25dB

Decibels and signal strength

- As a signal propagates, it gets attenuated, often exponentially.
- $$G_{dB} = 10 \log_{10} \frac{P_{out}}{P_{in}}$$

  G_{dB} = gain, in decibels; P_{in}= input power level; P_{out} = output power level
- Power levels are expressed in ______
- Decibel loss

  $$L_{dB} = -10 \log_{10} \frac{P_{out}}{P_{in}} = 10 \log_{10} \frac{P_{in}}{P_{out}}$$
- If amplifiers are connected in series (cascaded), the overall gain (expressed as a ratio) is the product of the gain of individual stages

Question

1. Consider a series in which the input is at a power level of 4 mW, the first element is a transmission line with a 12-dB loss (-12-dB gain), the second element is an amplifier with a 35-dB gain, and the third element is a transmission line with a 10-dB loss. What is the output power?
2. Dynamic range of human speech is 25dB. What does this mean?

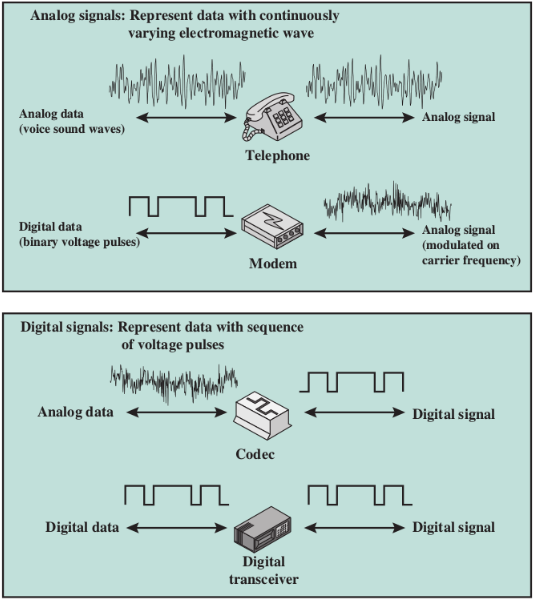
Analog and digital data transmission

- Textual data has to be stored as 1s and 0s - there are several codes to do this
- ASCII (American Standard Code for Information Interchange) is one of them (also called International Reference Alphabet — IRA)
- Each character in this code is represented by a unique 7-bit pattern. How many characters can be represented?
- Transmitted using 8 bits with 1 bit as parity bit
- This bit is set such that the total number of binary 1s in each octet is always odd (odd parity) or always even (even parity).
- Setting the parity bit is useful for ______

Analog and digital data transmission

- Video transmission carries sequences of pictures in time.
- Video can be captured by either analog or digital video recorders.
- Video that is captured can be transmitted using continuous (analog) or discrete (digital) signals, can be received by either analog or digital display devices, and can be stored in either analog or digital file formats.

Analog and digital signaling of analog and digital data



Analog transmission

- Analog transmission is a means of transmitting analog signals without regard to their content; the signals may represent analog data (e.g., voice) or digital data (e.g., binary data that pass through a modem).
- To transmit over long distances, signals are amplified
- Amplifiers also boost the noise components
- With amplifiers cascaded to achieve long distances, signals become distorted
- For voice, this is tolerable, but not for digital data

Digital transmission

- In contrast with analog transmission, assumes a binary content to the signal
- To transmit over long distances, repeaters are used
- A repeater receives the digital signal, recovers the pattern of 1s and 0s, and retransmits a new signal. Thus the attenuation is overcome
- Possible to use repeaters for an analog signal too (usually required when carrying digital data) : the repeater recovers the digital data from the analog signal and generates a new, clean analog signal.

Analog and digital transmission

(a) Data and Signals

	Analog Signal	Digital Signal
Analog Data	Two alternatives: (1) signal occupies the same spectrum as the analog data; (2) analog data are encoded to occupy a different portion of spectrum.	Analog data are encoded using a codec to produce a digital bit stream.
Digital Data	Digital data are encoded using a modem to produce analog signal.	Two alternatives: (1) signal consists of two voltage levels to represent the two binary values; (2) digital data are encoded to produce a digital signal with desired properties.

(b) Treatment of Signals

	Analog Transmission	Digital Transmission
Analog Signal	Is propagated through amplifiers; same treatment whether signal is used to represent analog data or digital data.	Assumes that the analog signal represents digital data. Signal is propagated through repeaters; at each repeater, digital data are recovered from inbound signal and used to generate a new analog outbound signal.
Digital Signal	Not used	Digital signal represents a stream of 1s and 0s, which may represent digital data or may be an encoding of analog data. Signal is propagated through repeaters; at each repeater, stream of 1s and 0s is recovered from inbound signal and used to generate a new digital outbound signal.

Preference for digital transmission

- Digital technology: drop in cost compared to analog ciruitry
- Data integrity: it is possible to transmit data longer distances and over lower quality lines by digital means using repeaters while maintaining the integrity of the data
- Capacity utilization: To use the high bandwidth provided by satellite channels and optical fiber, a high degree of multiplexing is required. This is easily achieved using digital techinques than analog
- Security and privacy: Easy to apply encryption techniques to digital data
- Integration: Can achieve economies of scale

Therefore the preferred method of transmission is digital