Unit 6: Algebraic Structures

Topic 2: Cyclic Group

Outline

Cyclic group

Definition

A group G is called cyclic if there is an element a in G such that $G = \langle a \rangle = \{a^k : k \in \mathbb{Z}\}$. Such an element a is called a generator of G.

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Remark

If G is finite group of order n, then G is cyclic if and only if G has an element of order n.

Outline

Lagrange' Theorem

Theorem (Lagrange's Theorem)

The order of a subgroup of a finite group divides the order of the group.

Outline

Lagrange's Theorem and Cyclic group

Theorem

Every subgroup of a cyclic group is cyclic. If $|\langle a\rangle| = n$, then the order of any subgroup of $\langle a\rangle$ is a divisor of n; and, for each divisor k of n, the group $\langle a\rangle$ has exactly one subgroup of order k, namely, $\langle a^{\frac{n}{k}}\rangle$.

Consequences of Lagrange's Theorem

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Consequences of Lagrange's Theorem

- The order of each element of a finite group divides the order of the group.
- If G is a cyclic group of order n with generator a, then a^k is also a generator of G if and only if $\gcd(k, n) = 1$.
- A group of prime order is always cyclic.

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Problems

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2. If g is a generator of the cyclic group $U(49)$, then find all generators of this group.

Thank You

Any Question!!!