Optics

Text books

1. F.A. Jenkins & H.E. White, **Fundamentals of Optics**, Tata McGraw-Hill, 1981

1. Ajoy Ghatak, **Optics**, Tata McGraw-Hill, New Delhi, 1992

What is the Nature of Light?

René Descartes
(1596– 1650)

The Corpuscular model of light:

Originally proposed by Descartes in 1637, and later on developed by Isaac Newton in his famous book **OPTICKS**.

According to it, a luminous body emits a stream of particles in all directions.

It satisfactorily explained the phenomena of reflection & refraction, the rectilinear propagation of light, and the fact that light could propagate through vacuum.

Isaac Newton
(1642– 1726)

Drawbacks:

a) This model predicts that light should travel faster in an optically denser medium like water than in a less dense medium like air. Later on, this was disproved experimentally.

b) The phenomenon of interference, diffraction and polarization could not be explained by this model.

a) The explanation of Newton's rings using this model was found unsatisfactory.

The Wave Model of light

In 1678, wave model was first put forward by Huygens.

According to this model, light is composed of waves, similar to water or sound waves.

Using this model, he could explain the laws of reflection, refraction and the phenomenon of double refraction.

Huygens' wave theory required light to travel slowly in the more optically denser medium.

Drawback:
Huygens tried to invoke unrealistic assumptions to explain the rectilinear propagation of light using his wave theory. This led to the nonacceptance of the wave model, until 1801.

- In 1801, Thomas Young performed the famous interference experiment which could be explained only on the basis of a wave model of light.
- Showed that the wavelength of light waves is small. Hence diffraction effects are small and therefore light travels approximately in straight lines.
- Gave satisfactory explanation of Newton's rings.


Thomas Young
(1773-1829)

Both interference & diffraction phenomena could only be explained by assuming a wave model of light.

1n 1850, Foucault measured the speed of light in water. He found that the speed of light to be less in water than in air.

A strong confirmation of the wave theory.

Huygens did not know whether the light waves were longitudinal or transverse and also how they propagate through vacuum. Only when Maxwell propounded his famous electromagnetic theory, the nature of light waves could be understood properly.

Jean-Bernard-Leon Foucault (1819-1868)

How Huygens' wave theory could be used to understand light wave propagation?

Huygens' wave theory is based on a geometrical construction that allows us to tell where a given wavefront will be at any time in the future, if we know its present position.

What is a Wave front?

A **wavefront** is the **locus of the points** which are in the **same phase**

Types of a Wave front:

1. Circular wave fronts

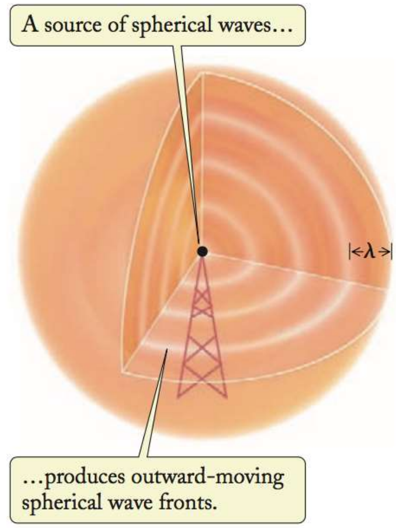
If we drop a small stone in a calm pool of water, circular ripples spread out from the point of impact, each point on the circumference of the circle (whose center is at the point of contact) oscillates with the same amplitude and same phase.

In this case we have **circular wave fronts.**

2. spherical wave fronts

For a point source emanating waves in a uniform isotropic medium, the locus of points which have the same amplitude and are in the same phase is spheres.

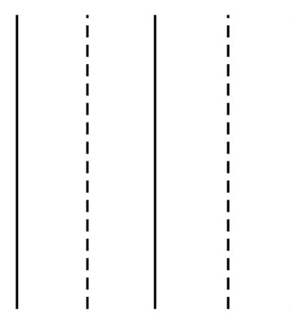
In this case we have **spherical wave fronts**.



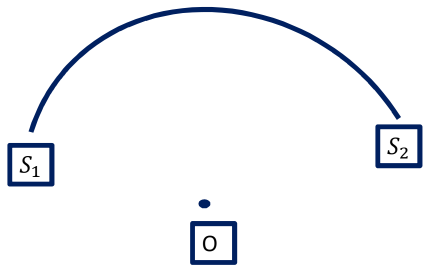
3. Plane wave fronts

At large distances from the source, a small portion of the sphere can be considered as a plane, and we have what is known as a plane wave.

So, a spherical wave front can be approximated to a **plane wave front.**



Consider a homogeneous and isotropic medium. Let S_1S_2 represent the shape of the wave front emanating from the point O, at a particular time (say $t = 0$), moving with speed v in the medium.



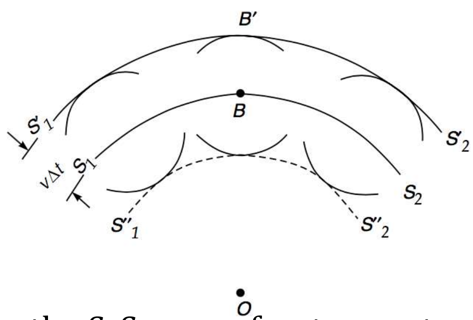
What is the shape of the wave front after a time interval Δt?

Huygens' principle

- All points on a wavefront serve as point sources of secondary wavelets.
- The wavelets emanating from these points spread out in all directions with the speed of the wave.
- After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.
- The envelope of these wavelets gives the shape of the new wave front.

Christiaan Huygens
(1629 – 1695)

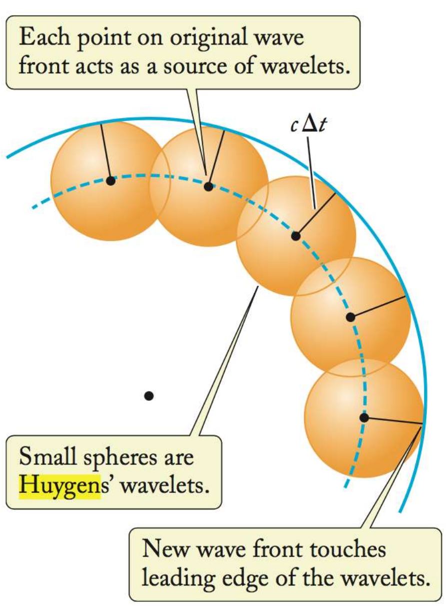
Huygens' Construction



With each point on the S_1S_2 wave front as center, draw spheres of radius $v\,\Delta t$.

Draw common tangent to all these spheres, then the envelope is again a sphere centered at O.

Thus the **shape of the wave front at a later time is the sphere $S_1'S_2'$**.

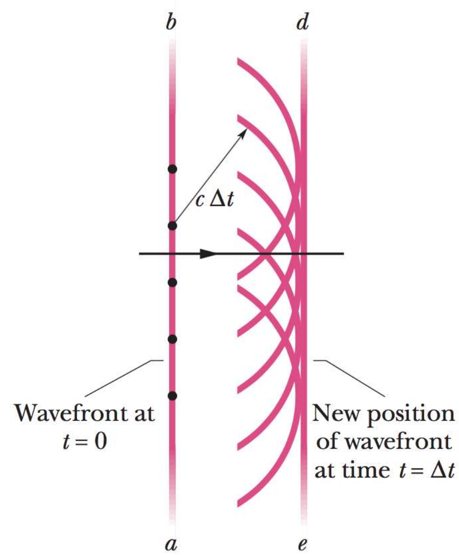


Drawbacks: With Huygens' model, we also obtain a back wave $S_1''S_2''$ (dashed curve in the figure) .

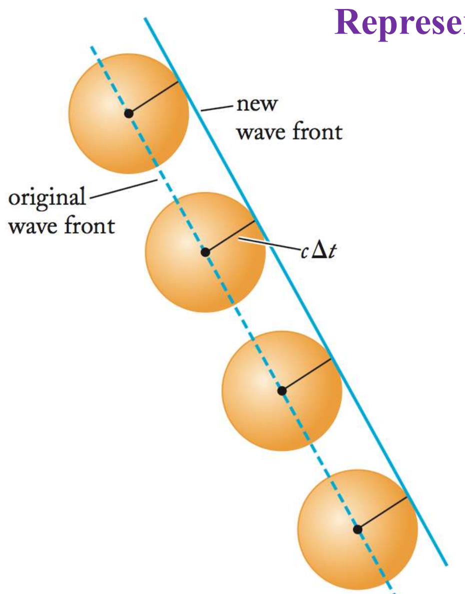
In Huygens' theory, the presence of the back wave is avoided by assuming that the amplitude of the secondary wavelets is not uniform in all directions. It is maximum in the forward direction and zero in the backward direction.

Huygens' Principle Applied to Plane Wavefronts

- Consider a plane wavefront, represented by plane ab, travelling to the right in vacuum.
- Let several points on plane ab (dots) serve as sources of spherical secondary wavelets that are emitted at $t = 0$.
- At time Δt, the radius of all these spherical wavelets will have grown to $c\ \Delta t$.
- Draw plane de tangent to these wavelets ---- **represents plane wavefront at Δt.**



Represents plane wavefront at Δt



From Huygens' theory of light waves, one can determine the shape of the wave front at any time, if shape of the initial wave front is known.

Now, what happens if these light waves superimpose?

Superposition of two waves

- The modification of intensities by superposition of two waves is called **Interference.**
- According to the *superposition principle*, when two light waves superimpose, the net displacement is simply the vector or the algebraic sum of the individual displacements.

Let us consider the superposition of two sinusoidal waves (having the same frequencies, different amplitudes and different initial phases) at a particular point.
Here, we are assuming that the displacements are in the same direction.

Suppose, the displacements produced by each of the disturbances are:

$$\left.\begin{aligned} x_1(t) &= a_1 \cos(\omega t + \theta_1) \\ \text{and} \quad x_2(t) &= a_2 \cos(\omega t + \theta_2) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots 1$$

According to the superposition principle, the resultant displacement *x(t)* is given by

$$x(t) = x_1(t) + x_2(t)$$

$$= a_1 \cos(\omega t + \theta_1) + a_2 \cos(\omega t + \theta_2)$$

which can be written in the form

$$x(t) = a \cos(\omega t + \theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots 2$$

where

$$\left.\begin{aligned} a \cos\theta &= a_1 \cos\theta_1 + a_2 \cos\theta_2 \\ \text{and} \quad a \sin\theta &= a_1 \sin\theta_1 + a_2 \sin\theta_2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

Thus the resultant disturbance is also simple harmonic in character having the same frequency but different amplitude and different initial phase.

On squaring and adding above equations, we obtain,

$$\boldsymbol{a} = [a_1^2 + a_2^2 + 2a_1 a_2\, cos(\theta_1 - \theta_2)]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots 4$$

and

$$\boldsymbol{\theta} = \tan^{-1} \frac{a_1 \sin\theta_1 + a_2 \sin\theta_2}{a_1 \cos\theta_1 + a_2 \cos\theta_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots 5$$

The angle θ is not uniquely determined from Eq.5, however, if we assume a to be always positive, then cos θ and sin θ can be determined from Eq.3 which will uniquely determine θ.

From Eq. 4, we find that if $\theta_1 \sim \theta_2 = 0, 2\pi, 4\pi, \ldots.$

then $a = a_1 + a_2$

Thus, if the two displacements are in phase, then the resultant amplitude will be the sum of the two amplitudes; this is known as *constructive interference*.

Similarly, if $\theta_1 \sim \theta_2 = \pi, 3\pi, 5\pi, \ldots.$

then $a = a_1 \sim a_2$

then the resultant amplitude will be the difference of the two amplitudes. The two displacements are out of phase and this is known as *destructive interference*.

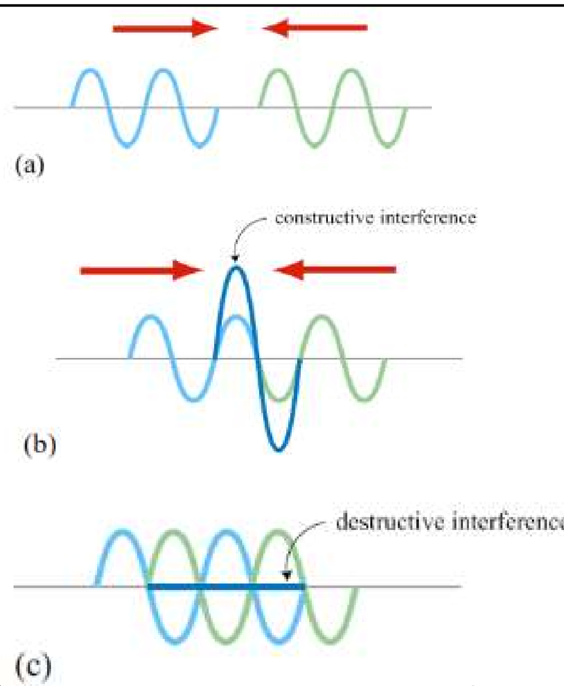
When constructive and destructive interferences occur, there is no violation of the principle of conservation of energy; the energy is just redistributed.

Superposition of waves:

(a) Travelling wave pulses approach each other.

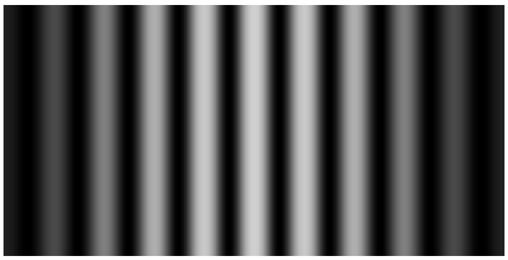
(b) Constructive interference,

(c) destructive interference,

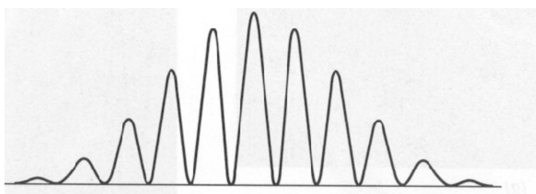


Interference pattern

Whenever two waves superimpose, one obtains an intensity distribution which is known as the interference pattern.

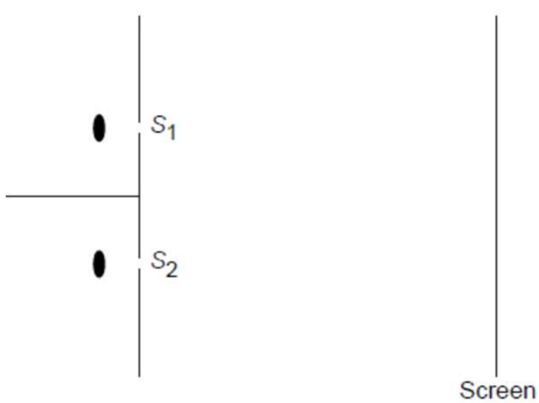


Interference pattern.



Intensity distribution

If two sodium lamps illuminate two pinholes S_1 and S_2 , no interference pattern will be observed on the screen. why?



- In a conventional light source, light comes from a large number of independent atoms, each atom emitting light for about 10^{-10} s, i.e., light emitted by an atom is essentially a pulse lasting for only 10^{-10} s. However, since the optical frequencies are of the order of 10^{15} s^{-1}, such a short pulse consists of about 1 million oscillations; thus it is almost monochromatic. Even if the atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases.

- Consequently, light coming out from holes S_1 and S_2 will have a fixed phase relationship for about 10^{-10} s, hence the interference pattern will keep on changing every billionth of a second. The eye can notice intensity changes which last at least for 0.1 s, and hence we will observe a uniform intensity over the screen. However, if we have a camera whose time of shutter opening can be made less than 10^{-10} s, then the film will record an interference pattern.

- Thus, light beams from two independent sources do not have any fixed relationship, as such, they do not produce any stationary interference pattern.

- For sound waves and for microwaves, the interference pattern can be observed without much difficulty because the two interfering waves maintain a constant phase relationship.

- However, for light waves, due to the very process of emission, one cannot observe interference between the waves from two independent sources.(although the interference does take place).

- In order to form an interference pattern on superposition two light waves, the light sources must be monochromatic and coherent (same phase).

- Thus, one should derive two interfering waves from a single wave to maintain phase relation.

The methods to achieve this can be classified under two broad categories.

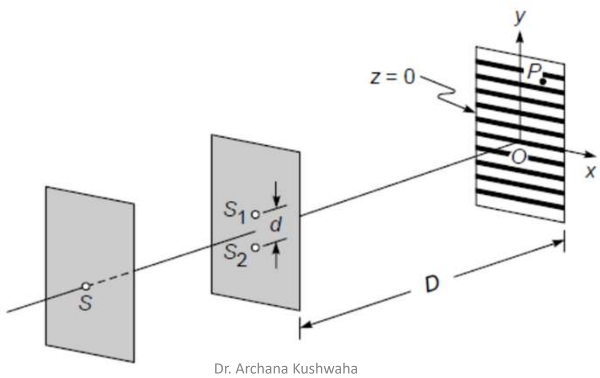
1. Division of wave front:
   Two coherent beams are obtained by dividing the wave front, originating from a point source.
   Examples:
   - Young's double slit Experiment
   - Fresnel Biprism

2. Division of amplitude:
   A beam is divided at two or more reflecting surfaces and the reflected beams interfere.
   Examples:
   - Plane parallel films illuminated by a plane wave
   - Newton's Rings

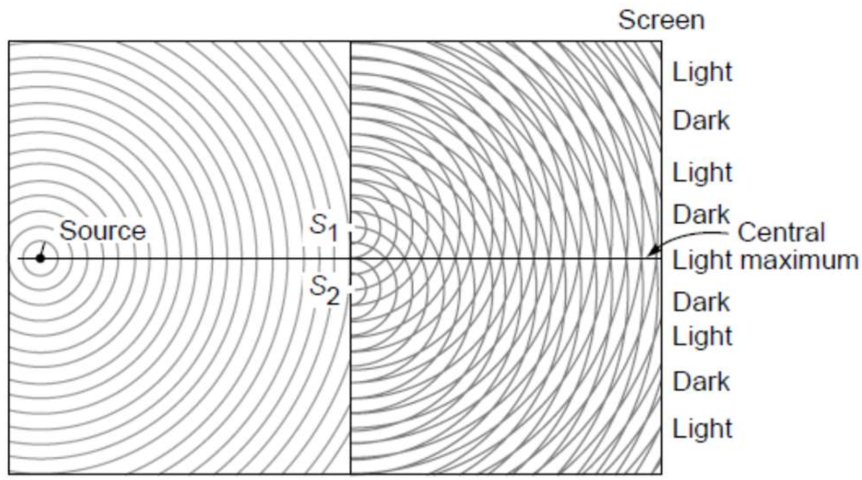
Division of wave front: Young's two-slit experiment

- Thomas Young in 1801 devised an ingenious but simple method to lock the phase relationship between the two sources.
- The trick lies in the division of a single wave front into two.
- These two split wave fronts act as if they emanated from two sources having a fixed phase relationship, and therefore when these two waves were allowed to interfere, a stationary interference pattern was obtained.

- A monochromatic light source is incident on the first screen that illuminates a pinhole S .
- Light diverging from this pinhole fell on a second screen which contained two pinholes S_1 and S_2 that were very close to each other and were located equidistant from S.

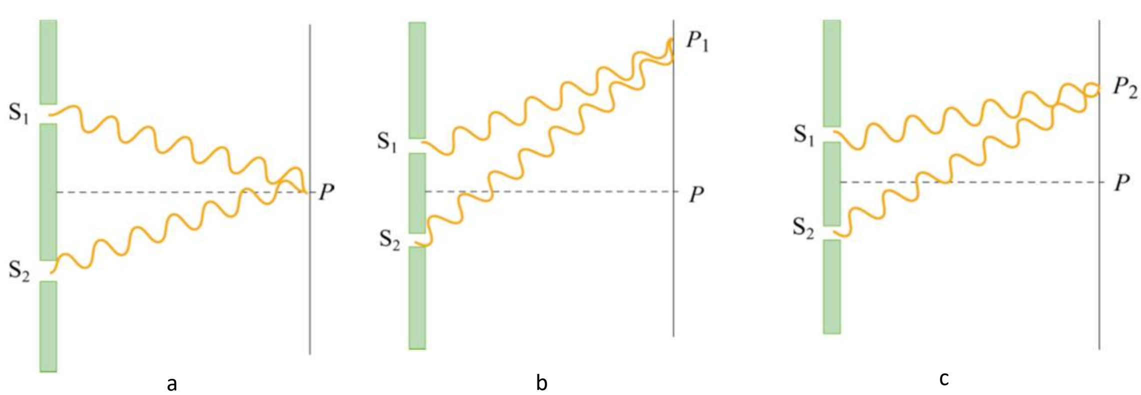


- Spherical waves emanating from S_1 and S_2 are coherent.
- These coherent light waves interfere and form beautiful interference fringes on the viewing screen.
- To show that this was indeed an interference effect, Young showed that the fringes on the screen disappear when S_1 (or S_2) is covered up.
- Young explained the interference pattern by considering the principle of superposition, and by measuring the distance between the fringes he calculated the wavelength.



The spherical wave fronts emanating from S, S_1, and S_2

- When a wave crest meets to a wave peaks/crest, constructive interference occurs and will get a maximum.
- When a wave peak meets to a wave trough, destructive interference occurs and will get a minimum.

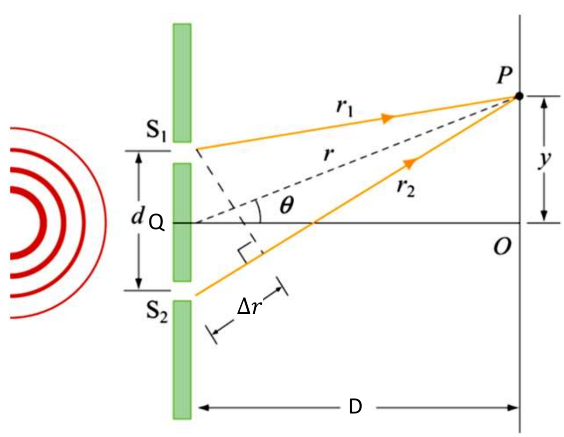


Constructive interference (a) at P , and (b) at P_1 . (c) Destructive interference at P_2 .

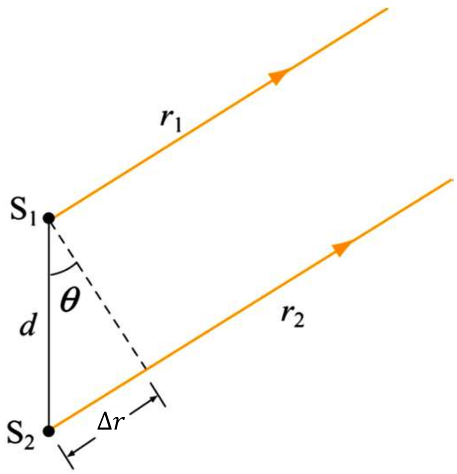
The Interference Pattern

Geometry of the double-slit interference

- To determine the positions of maxima and of minima on the screen.
- To observe the interference pattern consists of a series of dark and bright fringes on the screen.



In the limit ***D*** >> ***d*** , where the distance to the screen is much greater than the distance between the slits, the two rays $r_1$and r_2 are essentially treated as parallel



$$Path\ difference, \qquad \Delta r = r_1 - r_2 = d \sin\theta$$

Constructive interference means Bright fringes occur when δ is zero or an integer multiple of the wavelength λ ,

$$\Delta r = d \sin\theta_{bright} = n\lambda \qquad n = 0, \pm 1, \pm 2, \ldots$$

where *n* is called the *order* number.

The zeroth-order (*n*= 0) maximum corresponds to the central bright fringe at θ = 0

The first-order maxima (*n*= ±1) are the bright fringes on either side of the central fringe.

Destructive interference with a dark fringe on the screen will be seen when δ is equal to an odd integer multiple of $\lambda / 2$, the waves will be 180° out of phase at *P*.

The condition for destructive interference is given by

$$d \sin \theta_{dark} = \left(n + \frac{1}{2}\right)\lambda \qquad n = 0, \pm 1, \pm 2, \ldots$$

Positions of the fringes

Assumptions:

- Distance between slits and screen is too large than the distance between Slits, $D \gg d$
- The distance between the slits is much greater than the wavelength of the monochromatic light, $d \gg \lambda$
- These conditions imply that the angle θ is very small, so from Δ POQ

$$\sin \theta \approx \tan \theta = y/D$$

Substituting this into the constructive and destructive interference conditions eqs., we get the positions of the bright and dark fringes

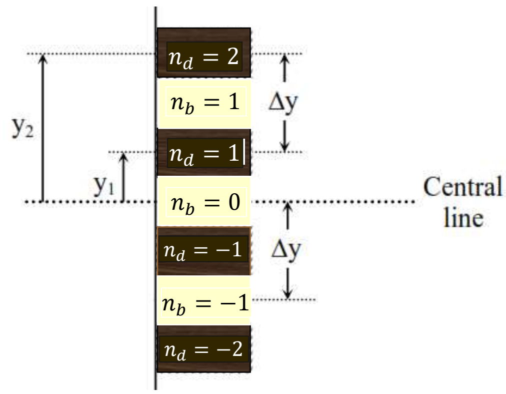
$$y_{bright} = \frac{\lambda \mathrm{D}}{d} n \qquad n = 0, \pm 1, \pm 2, \ldots$$

$$y_{dark} = \frac{\lambda \mathrm{D}}{d}\left(n + \frac{1}{2}\right) \qquad n = 0, \pm 1, \pm 2, \ldots$$

Fringe Width(Δy)

Since there is no definite boundaries between dark and bright fringes, we take the region between two absolute darks (at the center of the dark fringe) as the width of a bright fringe

$$\Delta y = y_{n+1} - y_n$$
$$= (n+1)\frac{\lambda D}{d} - n\frac{\lambda D}{d}$$
$$= \frac{\lambda D}{d}$$
$$\boldsymbol{\Delta y = \frac{\lambda D}{d}}$$



Problem1:

In the double-slit arrangement, d = 0.150 mm, D=120 cm, λ = 833 nm, and y = 2.00 cm . What is the path difference Δr for the rays from the two slits arriving at point P ?

Solution:

The path difference is given by $\Delta r = d \sin\theta$

When $D >> y$, Then, θ is small, and we can make the approximation

$$\sin\theta \approx \tan\theta = y / D$$

Thus, path difference $\Delta r = d\, y/D$

$$= (1.50 \times 10^{-4}\ \text{m})\ 2.00 \times 10^{-2}\ \text{m}/\ 1.20\text{m}$$

$$= 2.50 \times 10^{-6}\ \text{m}$$

Problem 2: In Young's double-hole experiment, the distance between the two holes is 0.5 mm, λ=5x10^{-5} cm, and D= 50 cm. What will be the fringe width?

Solution: The fringe width is given by

$$\Delta \boldsymbol{y} = \frac{\boldsymbol{\lambda D}}{\boldsymbol{d}}$$

Putting, $\lambda = 5 \times 10^{-5} cm = 5 \times 10^{-7}$m

d= 0.5 mm=0.5x10^{-3}m

D =50 cm =50x10^{-2} m

we have

$$\Delta y = \frac{5 \times 10^{-7} \times 0.50}{0.5 \times 10^{-3}} = 50 \text{ mm}$$

The Intensity Distribution

Let $\mathbf{E}_1$ and $\mathbf{E}_2$ be the electric fields produced at point P by S_1 and S_2, respectively. The electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ will, in general, have different directions and different magnitudes. However, if the distances S_1P and S_2P are very large in comparison to the distance S_1S_2, the two fields will almost be in the same direction. Thus, we may write

$$E_1 = \hat{\imath}E_{01}cos\left(\frac{2\pi}{\lambda}S_1P - \omega t\right) \quad ....1$$

$$E_2 = \hat{\imath}E_{02}cos\left(\frac{2\pi}{\lambda}S_2P - \omega t\right) \quad ....2$$

where $\hat{\imath}$ represents the unit vector along the direction of either of the electric fields. The resultant field is given by

$$E = E_1 + E_2$$

$$= \hat{\imath}\left[E_{01}cos\left(\frac{2\pi}{\lambda}S_1P - \omega t\right) + E_{02}cos\left(\frac{2\pi}{\lambda}S_2P - \omega t\right)\right]$$

The intensity I is proportional to the square of the electric field and is given by

$$I = KE^2$$

$$I = K\begin{bmatrix} E_{01}^2cos^2\left(\frac{2\pi}{\lambda}S_1P - \omega t\right) + E_{02}^2\,cos^2\left(\frac{2\pi}{\lambda}S_2P - \omega t\right) \\ +E_{01}E_{02}\left\{cos\left[\frac{2\pi}{\lambda}(S_2P - S_1P)\right] + cos\left[2\omega t - \frac{2\pi}{\lambda}(S_2P + S_1P)\right]\right\} \end{bmatrix} \quad ....3$$

where K is a proportionality constant

For an optical beam the frequency is very large ($\omega \approx 10^{15} s^{-1}$), and all the terms depending on ωt will vary with extreme rapidity (10^{15} times per second); consequently, any detector would record an average value of various quantities . Now

$$\langle cos^2(\omega t - \theta)\rangle = \frac{1}{2\tau}\int_{-\tau}^{+\tau}\frac{1 + cos[2(\omega t - \theta)]}{2}dt$$

where τ represents the time over which the averaging is carried out

$$= \frac{1}{2} + \frac{1}{16\pi}\frac{T}{\tau}\{[sin2(\omega t - \theta)]_{-\tau}^{+\tau}\}$$

where $T = 2\pi/\omega(\approx 2\pi \times 10^{-15} s$ for an optical beam).

For any practical detector $T\ /\ \tau <<<\ 1$, and since the quantity within the curly braces will always be between –2 and +2, we may write

$$\langle cos^2(\omega t - \theta)\rangle \approx \frac{1}{2}$$

For the normal eye,$\tau \approx 0.1$ s; thus $T/\tau \approx 6 \times 10^{-14}$ even for a detector having 1 ns as the resolution time, $T/\tau \approx 6 \times 10^{-5}$.

The factor cos $(2\omega t - \phi)$will oscillate between +1 and –1, and its average will be zero as can indeed be shown mathematically. Thus the intensity, that a detector will record, will be given by

$$I = I_1 + I_1 + 2\sqrt{I_1 I_2}\ cos\delta \qquad \text{.........................4}$$

where, $$\delta = \frac{2\pi}{\lambda}(S_2P - S_1P)$$

represents the phase difference between the displacements reaching point P from S_1 and S_2.

Further,

$$I_1 = \frac{1}{2} K E_{01}^2$$

represents the intensity produced by source S_1 if no light from S_2 is allowed to fall on the screen.

Similarly, $$I_2 = \frac{1}{2} K E_{02}^2$$

represents the intensity produced by source S_2 if no light from S_1 is allowed to fall on the screen.

From eq. 4 we may deduce the following:

1. The maximum and minimum values of cos δ are +1 and –1, respectively; as such, the maximum and minimum values of I are given by

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \qquad \text{..........................5}$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

The maximum intensity occurs when

$$\delta = 2n\pi \qquad n = 0, 1, 2, \ldots$$

or $$S_2P - S_1P = n\,\lambda$$

and the minimum intensity occurs when

$$\delta = (2n+1)\pi \qquad n = 0, 1, 2, \ldots$$

or $$S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda$$

When $I_1 = I_2$, the intensity minimum is zero. In general, $I_1 \neq I_2$ and the minimum intensity is not zero.

2. If holes S_1 and S_2 are illuminated by different light sources, then the phase difference δ will remain constant for about 10^{-10} s and thus δ would also vary with time in a random way. If we now carry out the averaging over time scales which are of the order of 10^{-8} s, then

$$\langle \cos \delta \rangle = 0$$

and we obtain

$$I = I_1 + I_2$$

Thus, for two incoherent sources, the resultant intensity is the sum of the intensities produced by each one of the sources independently, and no interference pattern is observed.

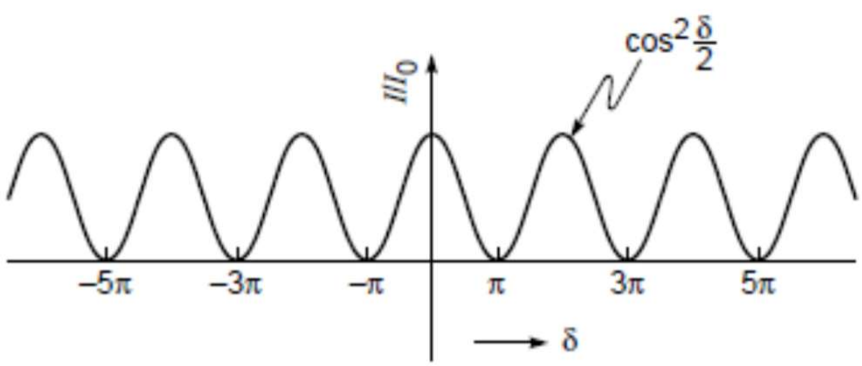
3. In the arrangement, if the distances S_1P and S_2P are extremely large in comparison to d, then

$$I_1 \approx I_2 = I_0 \text{ (say)}$$

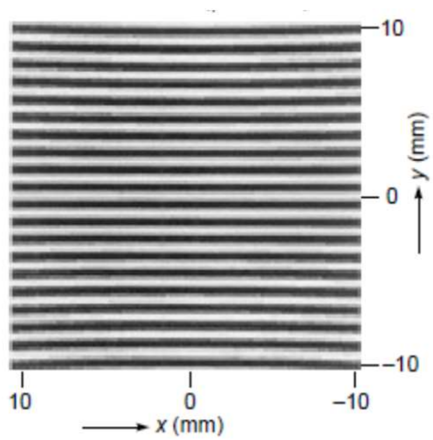
and

$$I = 2I_0 + 2I_0\, cos\delta = 4I_0 cos^2\frac{\delta}{2}$$

The intensity distribution with δ (often termed the $\cos^2$ pattern)



The actual fringe pattern (as it will appear on the screen)

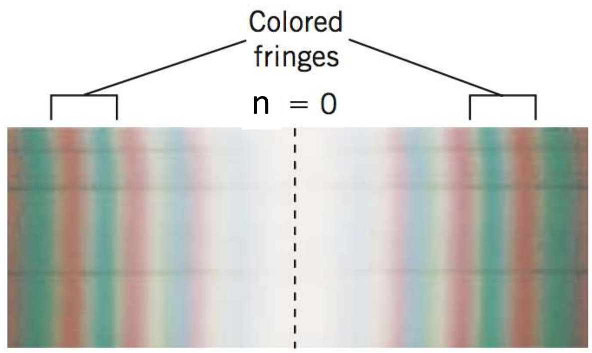


Interference with white light

We discussed Young's double slit experiment using monochromatic light source.

Suppose the slit is illuminated by white light, what is the interference pattern?

Interference with white light



Interference pattern:
The **central fringe** (for n=0) is **white**.
Except for the central fringe, other **bright fringes** are a **rainbow of colors**.

Central fringe:

The central fringe is different from the other fringes, because it is the only one for which $n = 0$.
Since $\sin\theta = n\lambda/d$,

$$\therefore \ \boldsymbol{\theta = 0^\circ} \ \textbf{(for all } \boldsymbol{\lambda}\textbf{)}$$

all wavelengths will constructively interfere.

"Zeroth-order bright fringe is located at the same place for all wavelengths"

Reason for color separation:

- Each color corresponds to a different wavelength λ
- constructive and destructive interference depend on the wavelength.
- Since $\sin\theta = n\lambda/d$, there is a different angle that locates a bright fringe for each value of λ.

"Different angles lead to the separation of colors on the screen"

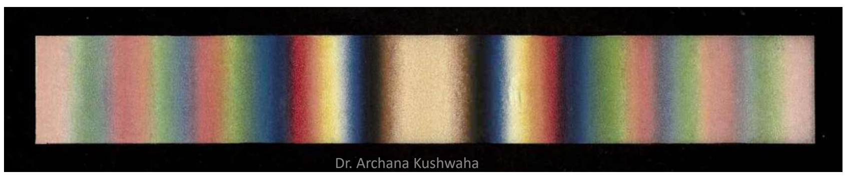
Order of colours (within any single group of fringes):

The wavelength of red (λ_{red} : 660 nm) is larger than green (λ_{green} : 550 nm)

Hence red light has a larger angle θ than green light does. (as $\sin\theta \propto \lambda$)

"Red is farther out from the central fringe than green"

Far away from zeroth-order fringe, the **colored fringes will soon disappear.**
Because many wavelengths (in the visible region) will constructively interfere resulting in **uniform white illumination**.



Example: Consider a point R in the interference pattern, such that $S_2R \sim S_1R = 30 \times 10^{-5}\ cm$,

constructive interference for wavelengths corresponding to

$$30 \times 10^{-5}/n:$$

7.5×10^{-5} - red
6×10^{-5} - yellow
5×10^{-5} - greenish yellow
4.3×10^{-5} - violet

destructive interference for wavelengths corresponding to

$$30 \times 10^{-5}/(n + \tfrac{1}{2})$$

6.67×10^{-5} - orange
5.5×10^{-5} - yellow
4.6×10^{-5} - indigo
4.0×10^{-5} - violet

Summary

Thus with white light, one gets

- A white central fringe at the point of zero path difference
- Along with a few colored fringes on both the sides, the color soon fading off to white.
- If we put a red (green) filter in front of our eye, the fringe pattern corresponding to red (green) color will suddenly appear.

Advantage of white light:

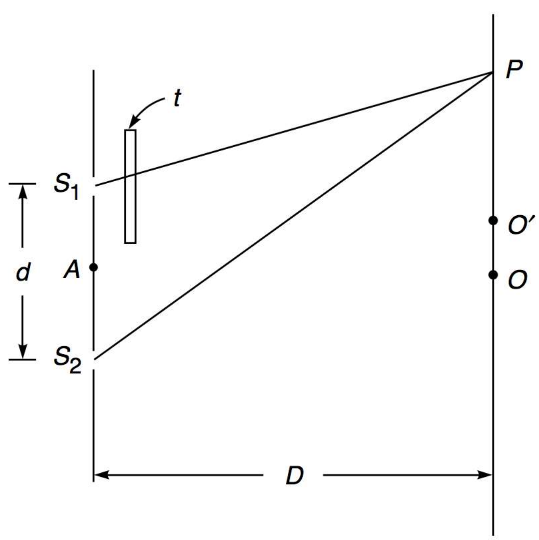
With a nearly monochromatic source, a large number of interference fringes are obtained—so it is extremely difficult to determine the position of the central fringe.

In many interference experiments, it is necessary to determine the position of the central fringe, and this can be easily done by using white light as a source.

Displacement of Fringes

What if a thin plate in the path of one of the two interference beams is inserted?

Displacement of Fringes



- Consider a plate of thickness t and refractive index n introduced in the path of the beam emanating from slit S_1.
- The speed of light in the plate is $\boldsymbol{v = c/n}$

From the figure, it is clear that the light reaching point P from S_1 has to traverse a distance t in the plate and a distance $S_1P - t$ in air.

The time required for the light to reach from S_1 to point P is given by

$$\underbrace{\frac{S_1P - t}{c}}_{\text{In air}} + \underbrace{\frac{t}{v}}_{\text{In plate}} = \frac{1}{c}(S_1P - t + nt)$$

$$= \frac{1}{c}(S_1P + \underbrace{(n-1)t}_{\text{Increase in effective optical path}}) \ldots\ldots\ldots(1)$$

With the increase in effective optical path, the central fringe is formed at point O' where

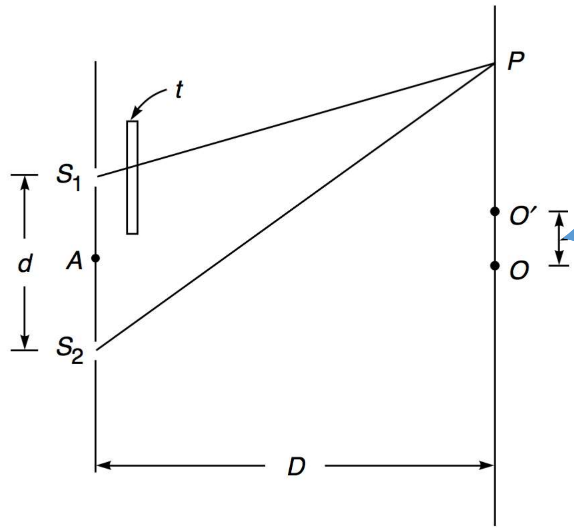
$$S_1O' + (\mathrm{n}-1)\mathrm{t} = S_2O' \text{ (equal optical path from } S_1 \,\&\, S_2)$$

Since $\quad S_2O' - S_1O' \approx \frac{d}{D}OO'.$

Therefore $\quad (\mathrm{n}-1)\mathrm{t} = \frac{d}{D}OO'$

Thus **the fringe pattern gets shifted by a distance Δ** given be

$$\Delta = \frac{D(\mathrm{n}-1)\mathrm{t}}{d}$$



Shift in the fringe pattern by a distance

$$\frac{D(n-1)t}{d}$$

Application:

Using the above principle, we can determine the thickness of extremely thin transparent sheets (eg: Mica) by measuring the displacement of the central fringe.

If white light is used as a source, the displacement of the central fringe is easy to measure.

Problem: In a double-slit interference arrangement one of the slits is covered by a thin mica sheet whose refractive index is 1.58. The distance S_1S_2 and AO are 0.1 and 50 cm , respectively. Due to the introduction of the mica sheet the central fringe gets shifted by 0.2 cm. Determine the thickness of the mica sheet.

Solution: $\Delta = 0.2\ cm$, d $= 0.1\ cm,\ \ D = 50\ cm$

Hence

$$t = \frac{d\,\Delta}{D(n-1)}$$

$$= \frac{0.1 \times 0.2}{50 \times 0.58}$$

$$\approx 6.9\ \times\ 10^{-4}\ cm$$

Fresnel’s Biprism

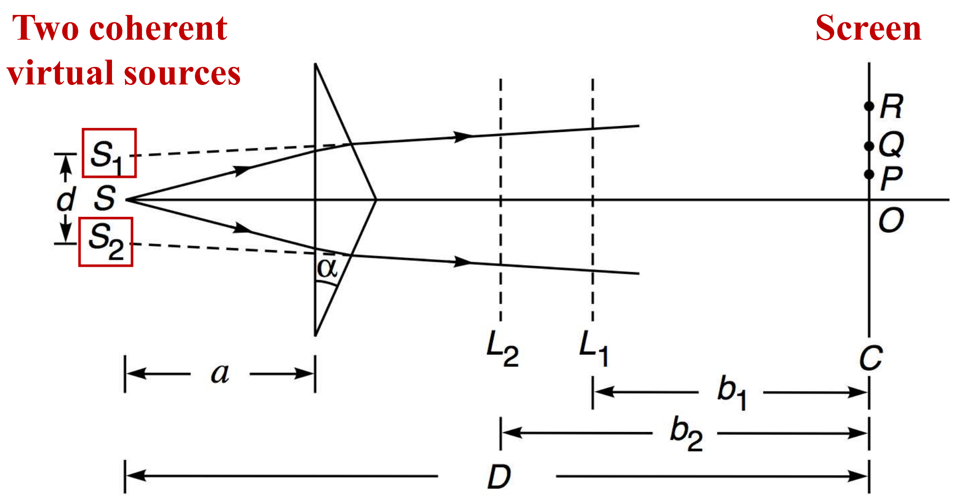
Objection faced byYoung’s double slit experiment

Are the bright fringes due to some complicated modification of the light by the edges of the slits and not due to interference?

One of the simple arrangements to produce interference pattern by Fresnel-

Fresnel’s biprism.

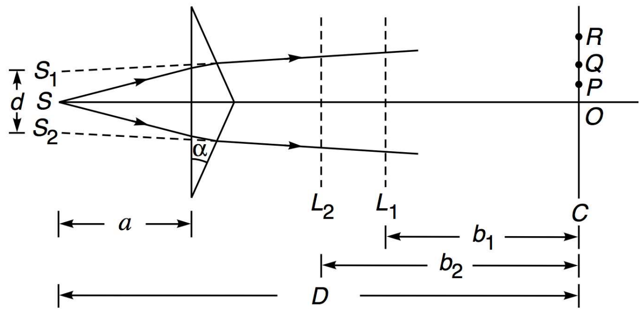
Fresnel's Biprism



Fresnel's Biprism : Experimental Setup

- Biprism is a simple prism with the base angles extremely small ($\sim 20'$).
- The prism is assumed to stand perpendicular to the plane of the paper.
- Point S represents the slit which is also placed perpendicular to the plane of the paper.
- Light from slit S gets refracted by the prism and produces two **virtual images S_1 and S_2.**
- These **virtual images act as coherent sources** and produced interference fringes on the right of the biprism.

Determination of distance between virtual sources (d)



Place a convex lens between the biprism and the eyepiece. For a fixed position of the eyepiece there will be two positions of the lens (L_1 *and* L_2) where the images of S_1 and S_2 can be seen clearly at the eyepiece.

Let d_1: distance between the two images when the lens is at position L_1($\boldsymbol{b_1}$ from eyepiece)
d_2: distance between the two images when the lens is at position L_2 ($\boldsymbol{b_2}$ from eyepiece)

Magnification :

when the lens is at L_1 : $\frac{d_1}{d} = \frac{b_1}{b_2}$ ....(1)

when the lens is at L_2 : $\frac{d_2}{d} = \frac{b_2}{b_1}$ ....(2)

Multiplying equations (1) & (2), we get

$$\boldsymbol{d = \sqrt{d_1 d_2}}$$

From the figure, the distance between object the sources and eyepiece is given by,

$$\boldsymbol{D = b_1 + b_2}$$

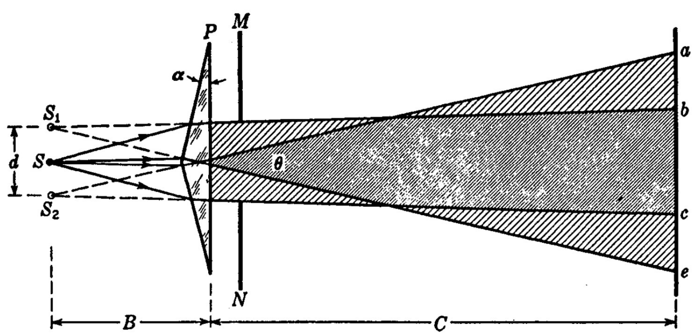
Determination of wavelength of a monochromatic light

Consider the light from the sodium lamp illuminating slit S. The fringe width Δy can be determined by means of a micrometer attached to the eyepiece.

Then the wavelength of the source λ can be determined by using the following relation

$$\boldsymbol{\lambda = \frac{d\,\Delta y}{D}}$$

Consider the overlapping beams ac and be produced due to refraction of light from the biprism.

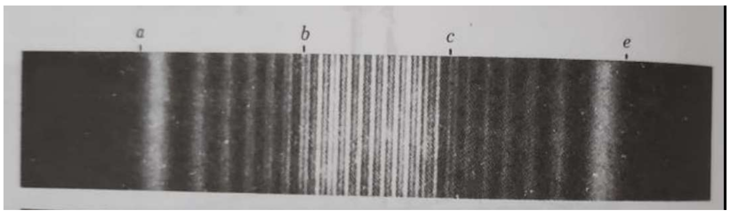


If screens M and N are placed as shown in the figure, interference fringes are observed only in the region bc.

If the screens are removed, the two beams ac and be will overlap over the whole region ae.

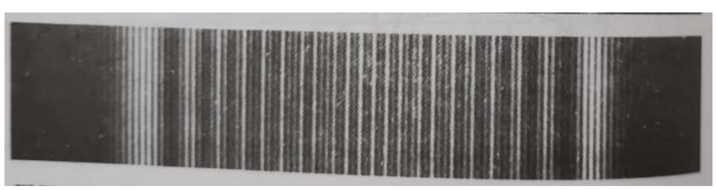
Interference and diffraction fringes: When screens M,N are present

The closely spaced fringes in the center of the photograph are due to interference, while the wide fringes at the edge of the pattern are due to diffraction.



diffraction pattern(without interference fringes)

Interference and diffraction fringes: When screens M,N are removed



The equally spaced interference fringes superposed on the diffraction pattern of a wide aperture.

Interference by Division of amplitude

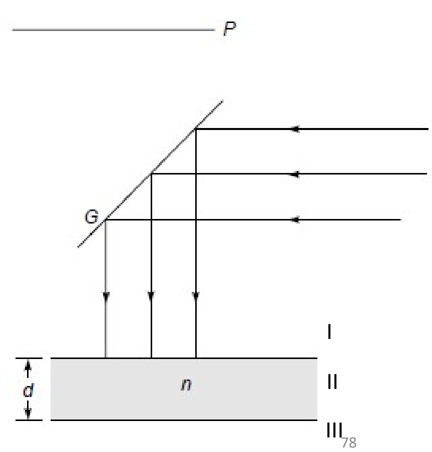
- We have discussed the interference pattern produced by division of wavefront.
- In this part , we will study the formation of interference pattern by division of amplitude.
- If a plane wave falls on a thin film, then the wave reflected from the upper surface interferes with the wave reflected from the lower surface.
- This study has many practical applications and also explain phenomena such as the formation of beautiful colors produced by a soap film illuminated by white light.

Interference by plane parallel film when illuminated by a plane wave

Normal incidence:

If a plane wave is incident normally on a thin film of uniform thickness d, then the waves reflected from the upper surface interfere with the waves reflected from the lower surface.

- The normal incidence of a parallel beam of light on a thin film of refractive index n and thickness d.
- To observe the interference pattern without obstructing the incident beam, we use a partially reflecting plate *G*
- This arrangement also enables us to eliminate the direct beam from reaching the photographic plate *P* (or the eye).
- The plane wave may be produced by placing an illuminated pinhole at the focal point of a corrected lens; alternatively, it may just be a beam coming out of a laser.



The wave reflected from the lower surface of the film traverses an additional optical path of $2nd$, where n represents the refractive index of the material of the film.

Further, if the film is placed in air, then the wave reflected from the upper surface of the film will undergo a sudden change in phase of π

The conditions for destructive or constructive interference will be given by

$$2nd = m\lambda \qquad \text{destructive interference} \qquad \ldots\ldots\ldots\ldots 1a$$

$$= \left(m + \frac{1}{2}\right)\lambda \qquad \text{constructive interference} \qquad \ldots\ldots\ldots\ldots 1b$$

where $m = 0, 1, 2, \ldots$ and λ represents the free space wavelength.

Thus, if we place a photographic plate at P , then the plate will receive uniform illumination; it will be dark when

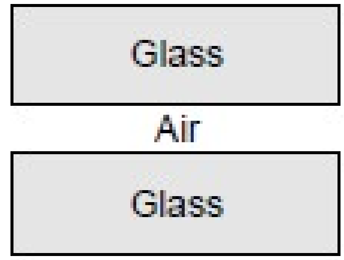
$$2nd = m\lambda$$

and bright when , $$2nd = \left(m + \frac{1}{2}\right)\lambda$$ for $m = 0, 1, 2, \ldots$.

Instead of placing the photographic plate, if we try to view the film (from the top) with the naked eye, then the film will appear to be uniformly illuminated.

The amplitudes of the waves reflected from the upper and lower surfaces will, in general, be slightly different; and as such the interference will not be completely destructive. However, with appropriate choice of the refractive indices of media II and III, the two amplitudes can be made very nearly equal.

Air film between two glass plates.



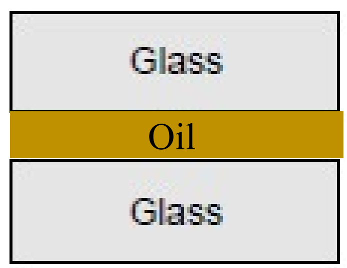
For an air film between two glass plates , no phase change will occur on reflection at the glass-air interface; but a phase change of π will occur on reflection at the air-glass interface and the conditions for maxima and minima will remain the same.
i.e.

$$2nd = m\lambda \qquad \text{destructive interference} \quad \ldots\ldots\ldots\ldots 1a$$

$$= \left(m + \frac{1}{2}\right)\lambda \qquad \text{constructive interference} \quad \ldots\ldots\ldots\ldots 1b$$

where $m = 0, 1, 2, \ldots$ and λ represents the free space wavelength.

Oil film between two glass plates.



If medium I is crown glass (n = 1.52), medium II is an oil of refractive index 1.60, and medium III is flint glass (n = 1.66), then a phase change of π will occur at both the reflections and the conditions for maxima and minima will be

$$2nd = \left(m + \frac{1}{2}\right)\lambda \qquad \text{minima} \quad \ldots\ldots\ldots\ldots 2a$$

$$= m\lambda \qquad \text{maxima} \quad \ldots\ldots\ldots\ldots 2b$$

where $m = 0, 1, 2, \ldots$ and λ represents the free space wavelength.

In general, whenever the refractive index of medium II lies in between the refractive indices of medium I and medium III, then the conditions of maxima and minima are given by Eqs. (2a) and (2b).

Interference by plane parallel film when illuminated by a plane wave

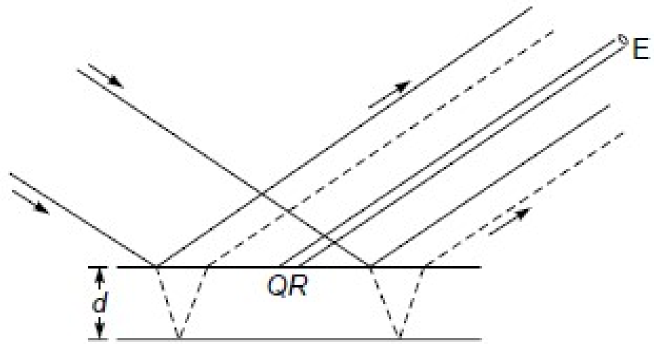
The oblique incidence

Again, the wave reflected from the upper surface of the film interferes with the wave reflected from the lower surface of the film.

The oblique incidence of a plane wave on a thin film.

The solid and dashed lines denote the boundary of the wave reflected from the upper surface and from the lower surface of the film.

The eye E receives the light reflected from the region QR.



The wave reflected from the lower surface of the film traverses an additional optical path Δ, which is given by

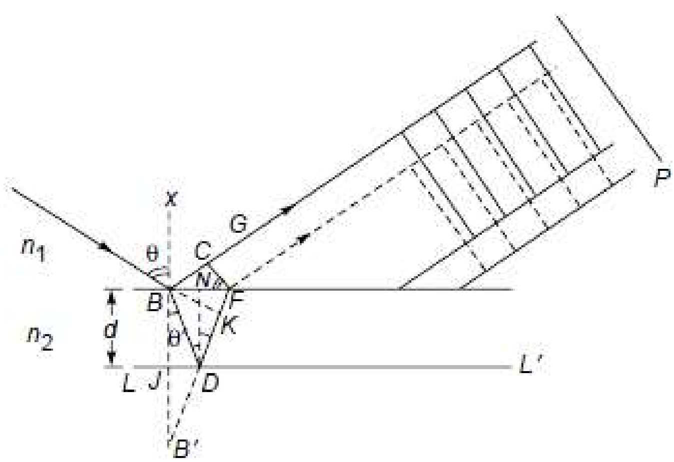
$$\Delta = n_2(BD + DF) - n_1 BC$$

where C is the foot of the perpendicular from point F on BG.

From cosine law,

$$\Delta = 2n_2 d \cos\theta'$$

where θ' is the angle of refraction.



For a film placed in air, a phase change of π will occur when reflection takes place at point B, and as such, the conditions of destructive and constructive interference are given by

$$\Delta = 2n_2 d \cos\theta' = m\lambda \qquad \text{minima}$$

$$= \left(m + \frac{1}{2}\right)\lambda \qquad \text{maxima}$$

If we place a photographic plate at P , it will receive uniform illumination; if we try to view the film with the naked eye, then only light rays reflected from a small position QR of the film will reach the eye. The image formed at the retina will be dark or bright depending on the value of Δ

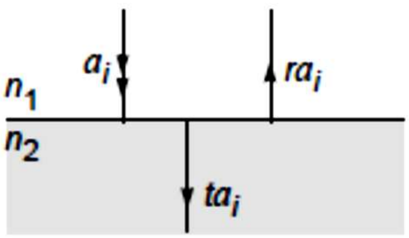
Non-Reflecting Films

One of the important applications of the thin film interference phenomenon lies in reducing the reflectivity of lens surfaces.

For a quantitative understanding of the phenomenon, we will have to assume that when a light beam (propagating in a medium of refractive index n_1) is incident normally on a dielectric of refractive index n_2, then the amplitudes of the reflected and the transmitted beams are related to that of the incident beam through the following relations:

$$a_r = \frac{n_1 - n_2}{n_1 + n_2} a_i$$

$$a_t = \frac{2n_1}{n_1 + n_2} a_i$$



amplitude of incident wave $=a_i$
amplitude of reflected wave $=a_r$
amplitude of transmitted wave $=a_t$,
refractive index of first medium$=n_1$
refractive index of first medium$=n_2$
r= reflection coefficient
T=transmission coefficient

The amplitude reflection and transmission coefficients r and t are given by

$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t = \frac{2n_1}{n_1 + n_2}$$

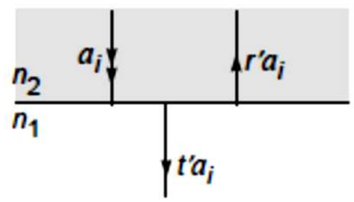
when $n_2 > n_1$, amplitude a_r is negative, showing that when a reflection occurs at a denser medium, a phase change of π occurs.

If r' and t' are the reflection and transmission coefficients where light propagating in a medium of refractive index n_2 is incident on a medium of refractive index n_1 ,then

$$r' = \frac{n_2 - n_1}{n_2 + n_1} = -r$$

$$t' = \frac{2n_2}{n_1 + n_2}$$

$$1 - tt' = 1 - \frac{4n_1 n_2}{(n_1 + n_2)^2} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = r^2$$



These Equations represent Stokes' relations

Now, we will discuss the application of the thin film interference phenomenon in reducing the reflectivity of lens surfaces.

We all know that in many optical instruments (such as a telescope) there are many interfaces, and the loss of intensity due to reflections can not be ignored.

For example

for near-normal incidence, the reflectivity of the crown glass surface in air is

$$\left(\frac{n-1}{n+1}\right)^2 = \left(\frac{1.5-1}{1.5+1}\right)^2 \approx 0.04$$

i.e., 4% of the incident light is reflected.

For a dense flint glass $n \sim 1.67$, and about 6% of light is reflected.

Thus, if we have a large number of surfaces, the losses at the interfaces can not be ignored.

In order to reduce these losses, lens surfaces are often coated with a $\lambda/4n$ thick *non-reflecting film*; the refractive index of the film is less than that of the lens.

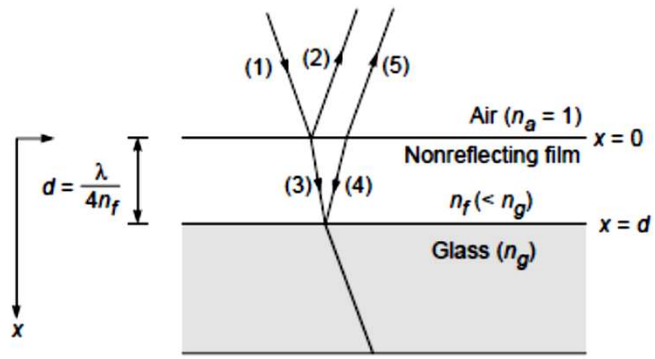
For example

Glass ($n = 1.5$) may be coated with a MgF_2 film, and the film thickness d should be such that

$$2n_f d = \frac{1}{2}\lambda$$

$$d = \frac{\lambda}{4n_f}$$

where we have assumed near-normal incidence and n_f represents the refractive index of the film; for MgF_2, $n_f = 1.38$.



If a film (having a thickness of $\frac{\lambda}{4n_f}$ and having refractive index less than that of the glass) is coated on the glass, then waves reflected from the upper surface of the film destructively interfere with the waves reflected from the lower surface of the film. Such a film is known as a nonreflecting film.

The main Observations:

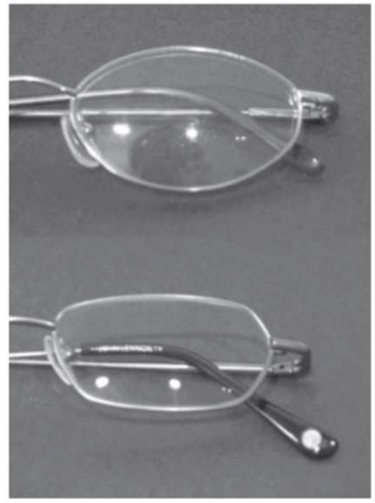
1. By putting a NR film of MgF_2 (n=1.38), the reflectivity will be about 1.3%, which is 4% without film.

   This technique of reducing the reflectivity is known as *blooming*.

2. The film is non reflecting only for a particular value of λ (~ 5000 Å)

3. As in the case of Young's double-slit experiment there is no loss of energy; there is merely a redistribution of energy. The energy appears mostly in the transmitted beam.

Comparison between an eyeglass lens without antireflective coating (top) and a lens with antireflective coating (bottom). Note the reflection of the photographer in the top lens and the tinted reflection in the bottom.



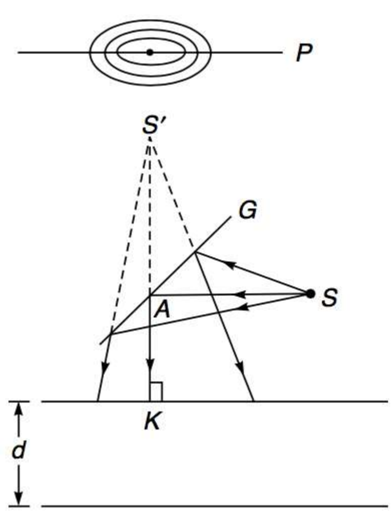
Problem: Consider a NR film of r.i. 1.38. Assume that its thickness is $9x10^{-6}$ cm. Calculate the wavelength(in VIS) for which the film will be reflecting.

Ans= 496.8 nm

Interference by a plane parallel film: illuminated by a point source

Let us consider interference pattern produced due to the illumination of a plane parallel film by a point source of light.

Experimental set up



Consider the illumination of the thin film (thickness d, refractive index n_2) by a point source of light S.

The partially reflecting plate G is used to observe the film without obstructing the incident beam.

P represents the photographic plate. On the plate, **circular fringes** are obtained.

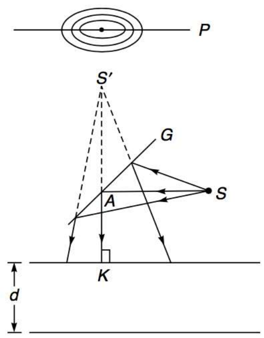


Fig (i)
KA is normal to the film

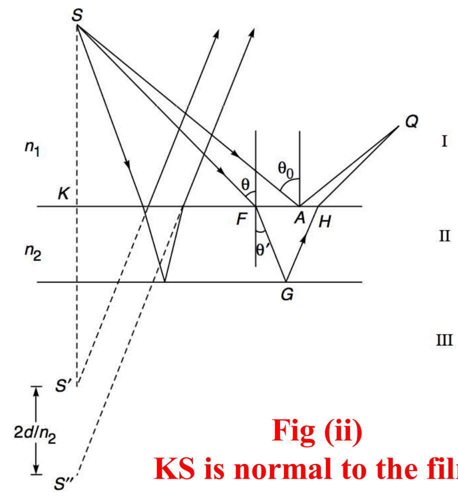
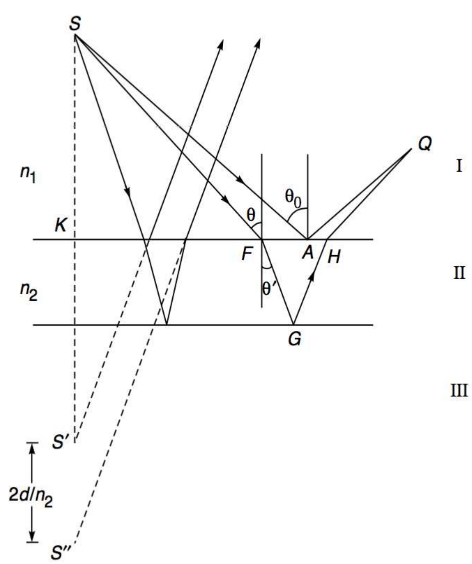


Fig (ii)
KS is normal to the film

To study the interference pattern, let us assume the point source S to be right above the film such that **SK** (in Fig (ii)) = **SA + AK** (in Fig (i))



The waves reflected from the upper surface of the film will appear to emanate from the point S' where

$$KS' = KS. \ldots\ldots\ldots..(1)$$

From geometrical considerations, the waves reflected from the lower surface appear to emanate from point S'' where,

$$KS'' \cong KS + \underbrace{2d/n_2}_{\text{Distance between } S' \text{ and } S''} \text{----------}(2)$$

Equation 2 is valid only for near-normal incidence—as the image of a point source produced by a plane refracting surface is not perfect.

For near-normal incidence:

The interference pattern produced in region I will be very nearly the same as that produced by two point coherent sources S' and S''---This is not identical to Young's pattern as S'' is not a perfect image of point S.

For large angles of incidence:

The waves reflected from the lower surface will appear to emanate from a point which will be displaced from S''. Thus, if we put a photographic plate P, we will obtain interference fringes.

Let n_1 and n_2 : refractive indices of media I and II.

Let us assume that in one of the reflections, an abrupt phase change of π occurs.
The intensity of an arbitrary point Q will be determined by the optical path difference:

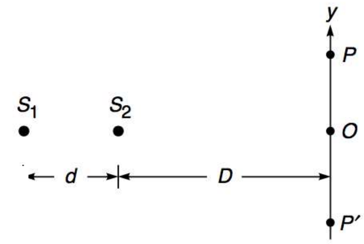
$$\Delta = \left(m + \frac{1}{2}\right)\lambda \qquad \text{maxima}$$
$$= m\,\lambda \qquad \text{minima}$$

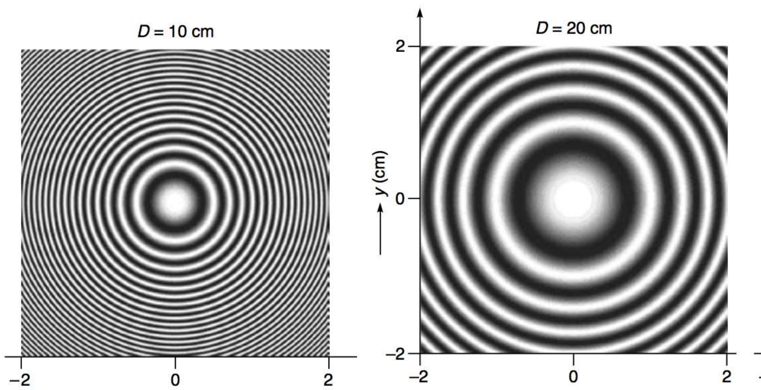
where $\Delta = [n_1 SF + n_2(FG + GH) + n_1 HQ] - [n_1(SA + AQ)]$.
The above conditions are rigorously correct, valid even for large angles of incidence.

For **near-normal incidence:**

$$\Delta \cong 2n_2 d \cos\theta'$$

If we put a photographic plate parallel to the surface of the film, we will obtain dark and bright concentric rings.



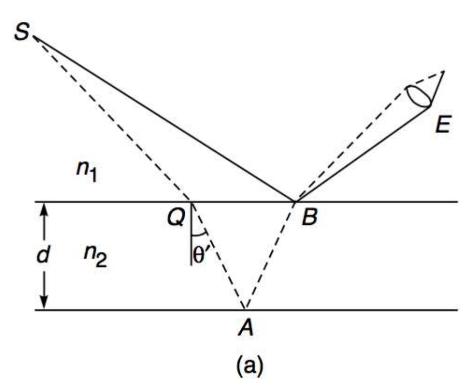


If the point source is taken far away, then the rings will spread out, in the limit of point source being taken to infinity, the photographic plate will be uniformly illuminated.

If we view the film with the naked eye, with the eye at the position E and the point source at S, only a portion of the film around point B will be visible and this point will appear to be bright or dark as the

Optical path difference:

$$\Delta = \left(m + \frac{1}{2}\right)\lambda \quad \text{maxima}$$
$$= m\,\lambda \quad \text{minima}$$

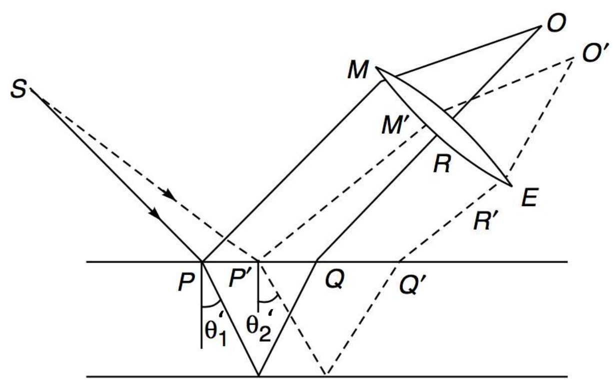


where

$$\Delta = n_1 SQ + n_2(QA + AB) - n_1\, SB$$

Instead of looking at the film, if the **eye is focused at infinity**, then the interference is between the rays which are derived from a single incident ray by reflection from the upper and lower surfaces of the film.

For example, rays PM and QR Which focus at the point O Of the retina, are derived from The single ray SP, and rays $P'M'$ and $Q'R'$, which focus at a different point O' on the retina, are derived from ray SP'.



(b)

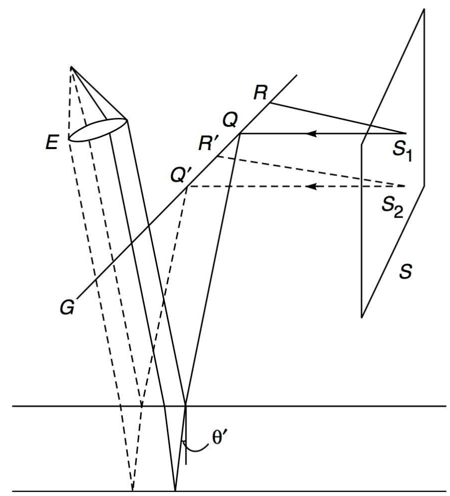
∴ **if the eye is focused for infinity,**

- Then it receives parallel rays from different directions corresponding to **different values of the angles of refraction θ'** -- hence **different values of the optical path difference.**

- Since the angles of refraction θ_1' and θ_2' will be different, points O and O' will not have the same intensity.

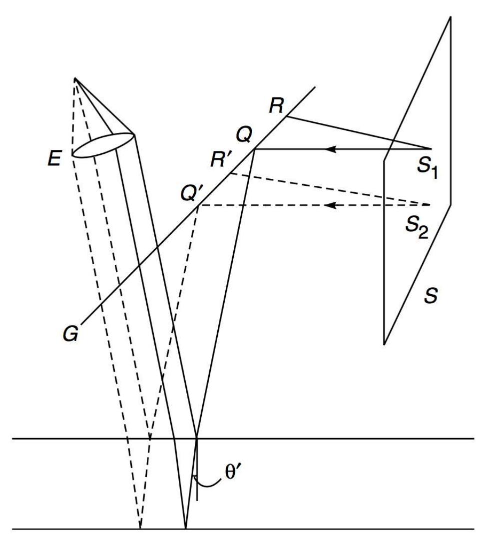
Illumination by an extended source:

- Consider a ground glass plate illuminated by sodium lamp, producing an extended source.
- Each point on the extended source will produce its own interference pattern, displaced with respect to one another, on the photographic plate P.
- Consequently no definite fringe pattern will appear on the photographic plate. If viewed with the naked eye, rays from all points of the film will reach the eye.



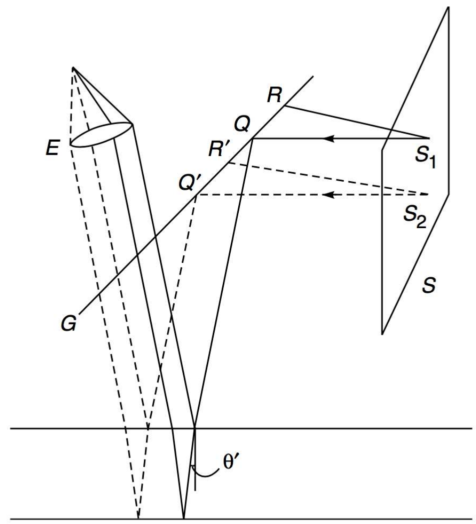
Eye focused at infinity:

If the eye is focused at infinity, then parallel light coming in a particular direction reaching the eye would have originated from nearby points of the extended source and the intensity produced on the retina would depend on the value of 2nd cos θ' which is the same for all parallel rays such as S_1Q, S_2Q' etc.

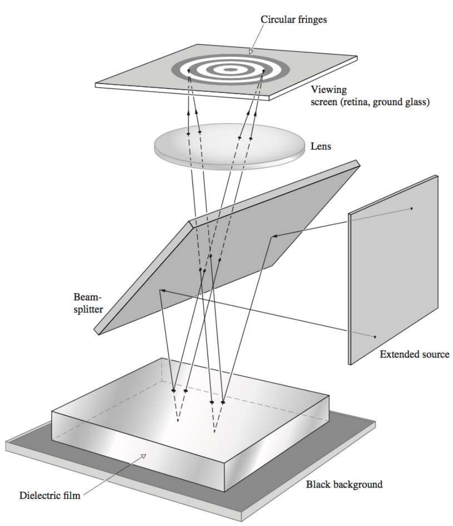


Rays emanating in a different direction (such as S_1R, S_2R' etc.,) would correspond to a different value of θ' and would focus at a different point on the retina.

Since $\boldsymbol{\theta'}$ **is constant** over the circumference of a cone (whose axis is normal to the film and whose vertex is at the eye) the eye will see **dark and bright concentric rings.**



Circular Haidinger fringes centered on the lens axis

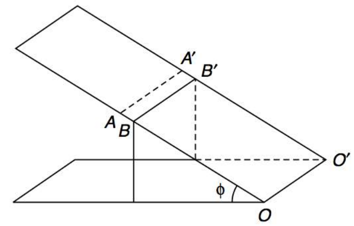


- The center is lying along the direction of $\theta' = 0$.
- The changes in the optical path are due to the changes in the direction of incidence and hence in the value of θ'.
- Such fringes, produced by a film of uniform thickness, are known as "**Haidinger fringes**" –also known as **"fringes of equal inclination"**.

Haidinger fringes are fringes seen with thick plates near normal incidence

Interference by a film with two nonparallel reflecting surfaces

Consider a thin film produced by a wedge which consists of two nonparallel plane surfaces. Let the light from an extended monochromatic source be incident normally on the wedge.



The path difference $\Delta = \boldsymbol{2nd\cos(\theta' + \phi)}$, where ϕ: wedge angle

Interference fringes consists of **equally spaced dark and bright straight-line fringes- "Fringes of equal thickness".** They will be parallel to the edge of the wedge.

The distance between two consecutive bright (dark) fringes is determined by the wedge angle(ϕ), wavelength of light and the refractive index of the film.

Colours of thin films: soap bubble & oil on water

- If instead of a wedge, we have a film of arbitrarily varying thickness. If we use polychromatic source, we will observe fringes--each **fringe represents the locus of constant film thickness**.
- This is indeed what happens when sunlight falls on a soap bubble or on a thin film of oil on water.

Thick vs thin films

If the optical path difference between the waves reflected from the upper surface of the film and from the lower surface of the film exceeds a few wavelengths, the interference pattern will be washed out due to the overlapping of interference patterns of many colors and no fringes will be seen.

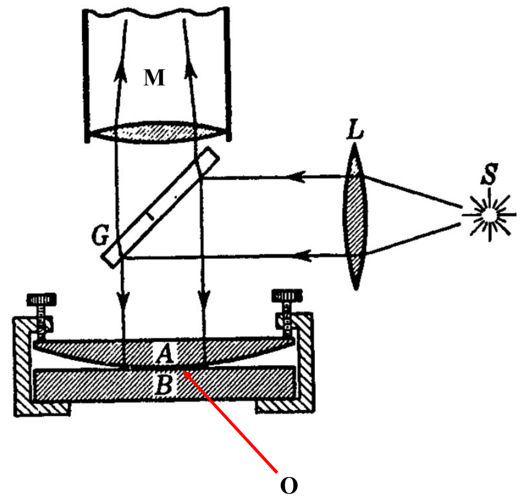
Thus, **to see the fringes with white light, the film should not be more than few wavelengths thick**.

Newton's Rings

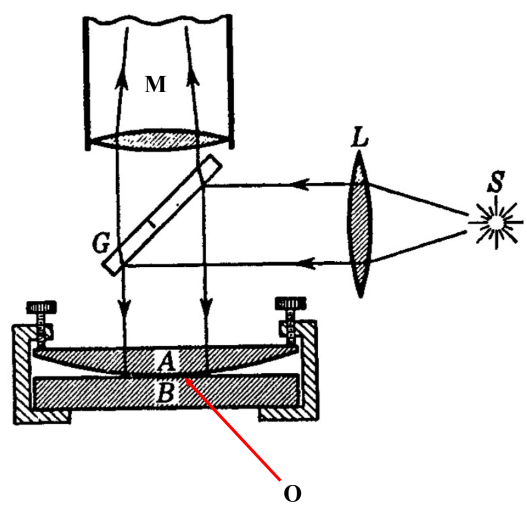
Experimental Set Up

Consider a planoconvex lens placed on a plane glass surface. A thin film of air is formed between the curved surface of the lens (A) and the plane glass plate (B).

- Light from an extended source S is allowed to fall on the thin film of air.
- M represents a traveling microscope.



- The thickness of air film is zero at the point of contact O and increases as one moves away from the point of contact.
- If we allow monochromatic light to fall on the surface of the lens, then the light reflected from the bottom surface of lens A interferes with the light reflected from the top surface of the glass plate B.



Near-normal incidence:

If we consider points very close to the point of contact the optical path difference between the two waves is very nearly equal to 2nt, where n is the refractive index of the film and t is the thickness of the film.

Thus whenever the thickness of the air film satisfies the condition

$2nt = \left(m + \frac{1}{2}\right)\lambda.$ m=0, 1, 2 ……… (**maxima**) ….(1)

$2nt = m\lambda$ (**minima**). …..(2)

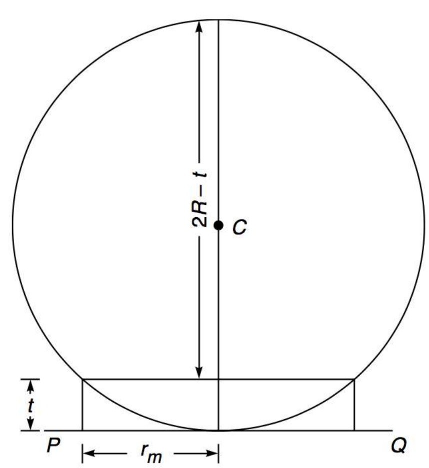
Since the convex side of the lens is a spherical surface, the thickness of the air film will be constant over a circle (whose center will be at O), and we will obtain **concentric dark and bright fringes**. These rings are known Newton's rings. Note that to observe the fringes, the microscope has to be focused on the upper surface of the film.

Calculation of the radii of rings:

The thickness of the air film will be constant over a circle whose center is at the point of contact O. Let the radius of the m^{th} dark ring be r_m, and if t is the thickness of the air film where m^{th} dark ring appears to be formed, then

$$r_m^2 = t(2R - t)\text{....(3)}$$

R: radius of curvature of the convex surface of The lens.

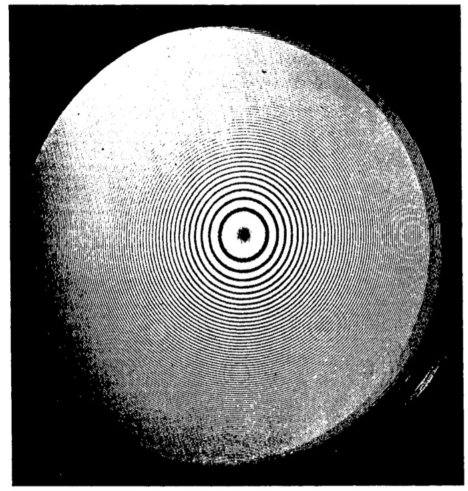


For $R \approx 100\ cm\ \ and\ t \leq 10^{-3} cm$, we may neglect t in comparison to 2R.

Thus,

$$r_m{}^2 \approx 2Rt$$

$$2t \approx \frac{r_m^2}{R} \quad \text{.........(4)}$$



Substituting 4 in equation 2,

$$r_m^2 \approx m\lambda R \qquad \text{m=0, 1, 2 .....(5)}$$

The radii of the rings vary as the square rootof natural numbers for dark rings.

- Typically for $\lambda = 6 \times 10^{-5} cm$ and R = 100cm

$$r_m = 0.0774\sqrt{m}\ cm$$

Thus the radii of the first, second and third dark rings are approximately 0.0774, 0.110 and 0.134 cm respectively.

Note: The spacing between the second third dark rings is smaller than the spacing between the first and second dark rings.

- As the **radius increases**, the **rings** will become **close** to **each other**.

The radius of bright rings:

$$r_m^2 \approx (m + \frac{1}{2})\lambda R$$

- Between the two dark rings, there will be a bright ring whose radius will be $\sqrt{(m+\frac{1}{2})\lambda R}$.

- To observe Newton's rings one really need not have a planoconvex lens; the rings will be visible even if a biconvex lens is used.

- Equation (2) predicts that **the central spot should be dark**. Normally, with the presence of minute dust particles the point of contact is really not perfect and the central spot may not be perfectly dark.

- Thus, while carrying out the experiment, one should measure the radii of the m^{th} and $(m+p)^{th}$ ring $(p \approx 10)$ and calculate,

$$r_{m+p}^2 - r_m^2 = p\lambda R \qquad \textbf{(independent of } \boldsymbol{m}\textbf{)} \ \ldots\ldots\ldots(6)$$

- In an interesting modification of the experiment, due to Thomas Young, the lower plate has a higher index of refraction than the lens, and the film between is filled with an oil of intermediate index.

- Then both reflections are at "rare-to-dense" surfaces, no relative phase change occurs, and the **central fringe of the reflected system is bright**.

- A ring system is also observed in the light transmitted by the Newton's-ring plates. These rings are exactly complementary to the reflected ring system, so that the central spot is now bright.

Applications

(i) Determination of λ:

From the accurate measurement of diameters and radius of curvature,

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}. \quad \ldots\ldots\ldots\ldots\ldots(7)$$

(ii) Determination of refractive index of a liquid:

If a liquid of refractive index n is introduced between the lens and the glass plate, the radii of the dark rings are given by

$$r_m = \sqrt{\frac{m\lambda R}{n}} \ldots\ldots\ldots\ldots(8)$$

Compare equation (8) with (5).

If the refractive indices of the material of the lens and of the glass plate are different and if the refractive index of the liquid lies in between the two values, the central spot will be bright ---equation (8) gives the radii of the bright rings.

$$n = \frac{\left(D_{m+p}^2 - D_m^2\right)_{air}}{\left(D_{m+p}^2 - D_m^2\right)_{liquid}}$$

where D is the diameter of the ring.

(iii) Determination of the optical flatness of a glass plate

Consider a glass surface placed on another surface whose flatness is known. If a monochromatic light beam is allowed to fall on this combination and the reflected light is viewed by a microscope, then, in general, dark and bright patches will be seen.

The space between the two glass surfaces forms an air film of varying thickness and whenever this thickness

$$t = m\ \lambda/2. \qquad \text{(dark spot)}$$

$$t = \left(m + \frac{1}{2}\right)\lambda/2 \qquad \text{(bright spot)}$$

- Two consecutive dark fringes will be separated by the air film whose thickness will differ by $\lambda/2$.
- Consequently, by measuring the distance between consecutive dark and bright fringes, one can calculate the optical flatness of a glass plate.

- A typical fringe pattern produced by an air film formed between two glass surfaces (which are not optically flat) and placed in contact with each other.
- Each fringe describes a locus of equal thickness of the film.

When we observe Newton's rings by using a white light source, we will see only a few colored fringes. However, if we put a red filter in front of the naked eye, the fringe pattern (corresponding to the red color) will suddenly appear. If we replace the red filter by a green filter in front of the eye, the fringe pattern corresponding to the green color will appear.

Problem: In the Newton rings arrangement, the radius of curvature of the curved surface is 50 cm. The radii of the 9th and 16^{th} dark rings are 0.18 and 0.2235 cm, respectively. Calculate the wavelength.

[**Ans:** 5015 Å]