Unit 5: Graph Theory

Topic 4: Tree

Outline

1. Introduction

2. Tree
   - Properties of a tree
   - Rooted tree
   - Properties of rooted tree

3. Spanning Tree
   - Construction of spanning tree
   - Minimum spanning tree

Introduction

The concept of trees began in 1857 with the counting of certain types of chemical compounds. Trees are particularly useful in computer science, where they are employed in a wide range of algorithms.

Procedures for building trees containing every vertex of a graph, including depth-first search and breadth-first search, can be used to systematically explore the vertices of a graph. Exploring the vertices of a graph via depth-first search allows for the systematic search for solutions to a wide variety of problems.

Outline

1. Introduction

2. Tree
   - Properties of a tree
   - Rooted tree
   - Properties of rooted tree

3. Spanning Tree
   - Construction of spanning tree
   - Minimum spanning tree

Tree

Definition

A **tree** is a connected undirected graph with no simple circuits.

An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

Definition

Graphs containing no simple circuits that are not necessarily connected are called **forests**.

In a forest, each of their connected components is a tree.

Properties of a tree

Theorem

A tree with n vertices has $n-1$ edges.

Rooted Tree

The are different types of graph or tree models are used to solve physical problems. One of such important class of trees is rooted tree.

Definition

A **rooted tree** is a tree in which one vertex has been designated as the root and every edge is directed away from the root.

Terminologies

Suppose that T is a rooted tree. If v is a vertex in T other than the root, the **parent** of v is the unique vertex u such that there is a directed edge from u to v. When u is the parent of v, v is called a **child** of u. Vertices with the same parent are called **siblings**. The **ancestors** of a vertex other than the root are the vertices in the path from the root to this vertex, excluding the vertex itself and including the root (that is, its parent, its parent's parent, and so on, until the root is reached). The **descendants** of a vertex v are those vertices that have v as an ancestor. A vertex of a rooted tree is called a **leaf** if it has no children. Vertices that have children are called **internal vertices**. The root is an internal vertex unless it is the only vertex in the graph, in which case it is a leaf.

Properties of rooted Tree

If a is a vertex in a tree, the subtree with a as its root is the subgraph of the tree consisting of a and its descendants and all edges incident to these descendants.

Definition

A rooted tree is called an m-**ary tree** if every internal vertex has no more than m children. The tree is called a **full m-ary tree** if every internal vertex has exactly m children.

An m-ary tree with $m = 2$ is called a **binary tree**.

Properties of rooted tree

Theorem

A full m-ary tree with

1. n *vertices has* $i = \frac{n-1}{m}$ *internal vertices and* $l = \frac{[(m-1)n+1]}{m}$ *leaves,*
2. i *internal vertices has* $n = mi + 1$ *vertices and* $l = (m-1)i + 1$ *leaves,*
3. l *leaves has* $n = \frac{ml-1}{m-1}$ *vertices and* $i = \frac{l-1}{m-1}$ *internal vertices.*

Problems

1. What is the chromatic number of an *m*-ary tree?

Problems

1. What is the chromatic number of an m-ary tree?
2. Find all regular trees.

Problems

1. What is the chromatic number of an m-ary tree?
2. Find all regular trees.
3. How many vertices, edges and leaves does a full 5-ary tree with 100 internal vertices have?

Problems

1. Suppose that someone starts a chain letter. Each person who receives the letter is asked to send it on to four other people. Some people do this, but others do not send any letters. How many people have seen the letter, including the first person, if no one receives more than one letter and if the chain letter ends after there have been 100 people who read it but did not send it out? How many people sent out the letter?

Problems

1. Suppose that someone starts a chain letter. Each person who receives the letter is asked to send it on to four other people. Some people do this, but others do not send any letters. How many people have seen the letter, including the first person, if no one receives more than one letter and if the chain letter ends after there have been 100 people who read it but did not send it out? How many people sent out the letter?
2. Suppose there are seven coins, all with the same weight, and a counterfeit coin that weighs less than the others. How many weighings are necessary using a balance scale to determine which of the eight coins is the counterfeit one?

There are many practical applications of trees in different computer science related problems.

1. Binary Search Tree
2. Decision Making Tree
3. Game Tree

Outline

1. Introduction
2. Tree
   - Properties of a tree
   - Rooted tree
   - Properties of rooted tree
3. Spanning Tree
   - Construction of spanning tree
   - Minimum spanning tree

Spanning Tree

Consider the problem of keeping connection between cities of an area clear in winter when roads full of ice. Such system of roads connecting cities can be represented by a simple graph. The only way the roads can be kept open in the winter is by frequently plowing them. The highway department wants to plow the fewest roads so that there will always be cleared roads connecting any two towns.

This problem was solved with a connected subgraph with the minimum number of edges containing all vertices of the original simple graph. Such a graph must be a tree.

Spanning tree

Definition

Let G be a simple graph. A **spanning tree** of G is a subgraph of G that is a tree containing every vertex of G.

Theorem

A simple graph is connected if and only if it has a spanning tree.

Spanning Tree

There are many algorithms to find spanning tree from a connected graph. Following are some of them.

1. **Removal of Circuits**
2. **Depth-First Search (Backtracking)**
3. **Breadth- first Search**

Minimum spanning tree

A company plans to build a communications network connecting its five computer centers. Any pair of these centers can be linked with a leased telephone line. Consider the problem of links to be made to ensure that there is a path between any two computer centers so that the total cost of the network is minimized. Such problems are modelled by an weighted graph, and solved by finding a spanning tree so that the sum of the weights of the edges of the tree is minimized.

Definition

A **minimum spanning tree** in a connected weighted graph is a spanning tree that has the smallest possible sum of weights of its edges.

Minimum Spanning Tree

The commonly used algorithms to find minimum spanning tree of a graph are the following:

1. **Prim's algorithm:** Begin by choosing any edge with smallest weight, putting it into the spanning tree. Successively add the tree edges of minimum weight that are incident to a vertex already in the tree, never forming a simple circuit with those edges already in the tree. Stop when $n - 1$ edges have been added.

Minimum Spanning Tree

The commonly used algorithms to find minimum spanning tree of a graph are the following:

1. **Prim's algorithm:** Begin by choosing any edge with smallest weight, putting it into the spanning tree. Successively add the tree edges of minimum weight that are incident to a vertex already in the tree, never forming a simple circuit with those edges already in the tree. Stop when $n-1$ edges have been added.
2. **Kruskal's algorithm:** Start with an edge in the graph with minimum weight. Successively add edges with minimum weight that do not form a simple circuit with those edges already chosen. Stop after $n-1$ edges have been selected.

Thank You

Any Question!!!