MA 203

Introduction to Queueing System
Characteristics of Queueing System

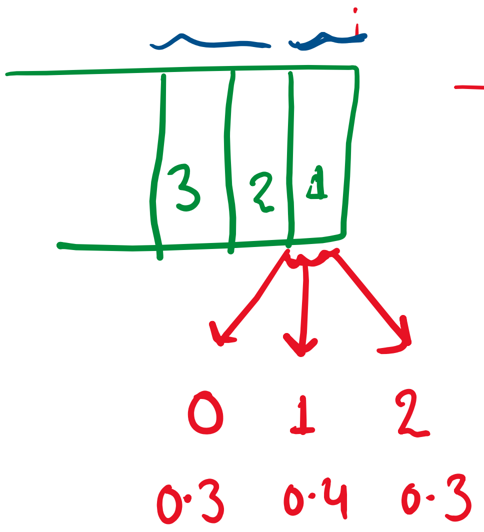
Example 3: The owner of a one-chair barber shop is thinking of expanding the shop capacity because there seem to be too many people are turned away. Observations indicate that in the time required to cut one person's hair there may be 0, 1, and 2 arrivals with probability 0.3, 0.4, and 0.3, respectively. The shop has a fixed capacity of 3 people including the one whose hair is being cut. Any new arrivals who finds 3 people in the barber shop is denied entry. Let X_n be the number of people in the shop at the completion of the nth person's hair cut. $\{X_n\}$ is a Markov chain assuming IID arrival. Find transition probability matrix?

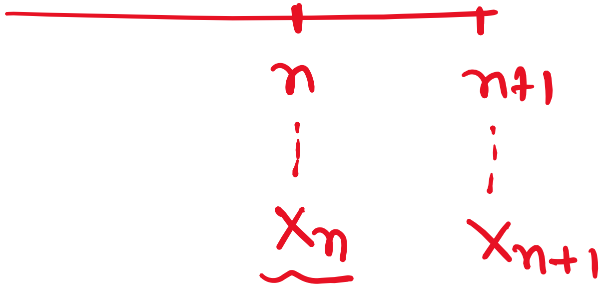
Sol:

Step 1:- $\Phi = \{0, 1, 2\}$

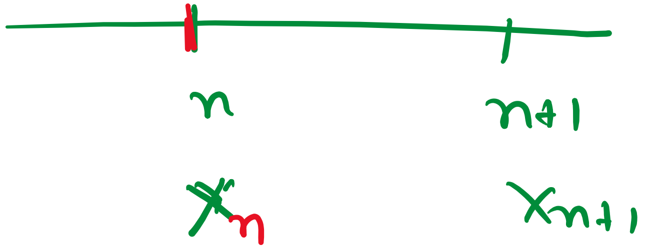
Step 2:-

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{bmatrix}_{3\times 3}$$





$$\underline{p_{00}} = P\{X_{n+1}=0 \mid X_n=0\} = 0.3$$



$$p_{01} = P\{X_{n+1}=1 \mid X_n=0\} = 0.4$$

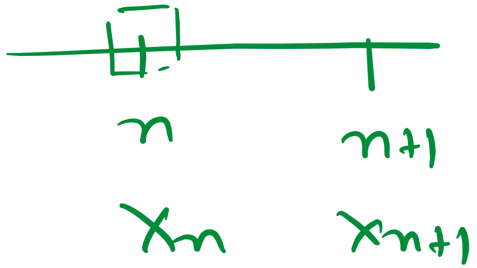
$$p_{02} = P\{X_{n+1}=2 \mid X_n=0\} = 0.3$$

$$p_{10} = P\{X_{n+1}=0 \mid X_n=1\} = 0.3$$

$$p_{11} = P\{X_{n+1}=1 \mid X_n=1\} = 0.4$$

$$p_{12} = P\{X_{n+1}=2 \mid X_n=1\} = 0.3$$

$$p_{20} = P\{X_{n+1}=0 \mid X_n = 2\} = 0$$



$$p_{21} = P\{X_{n+1}=1 \mid X_n = 2\} = 0.3$$

$$p_{22} = P\{X_{n+1}=2 \mid X_n = 2\} = 0.4$$

Step 4 :-

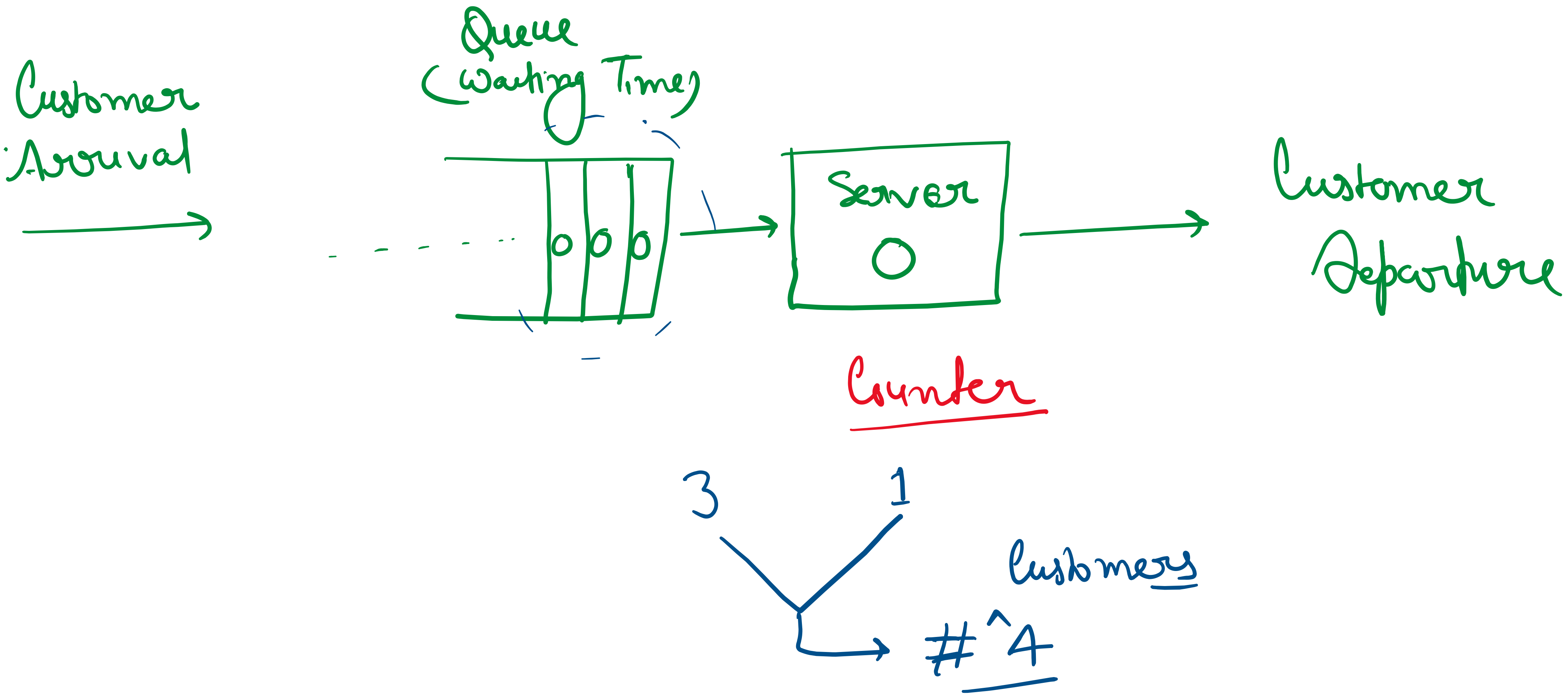
$$P = \begin{bmatrix} 0.3 & 0.4 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0 & 0.3 & 0.4 \end{bmatrix}$$

Queueing System

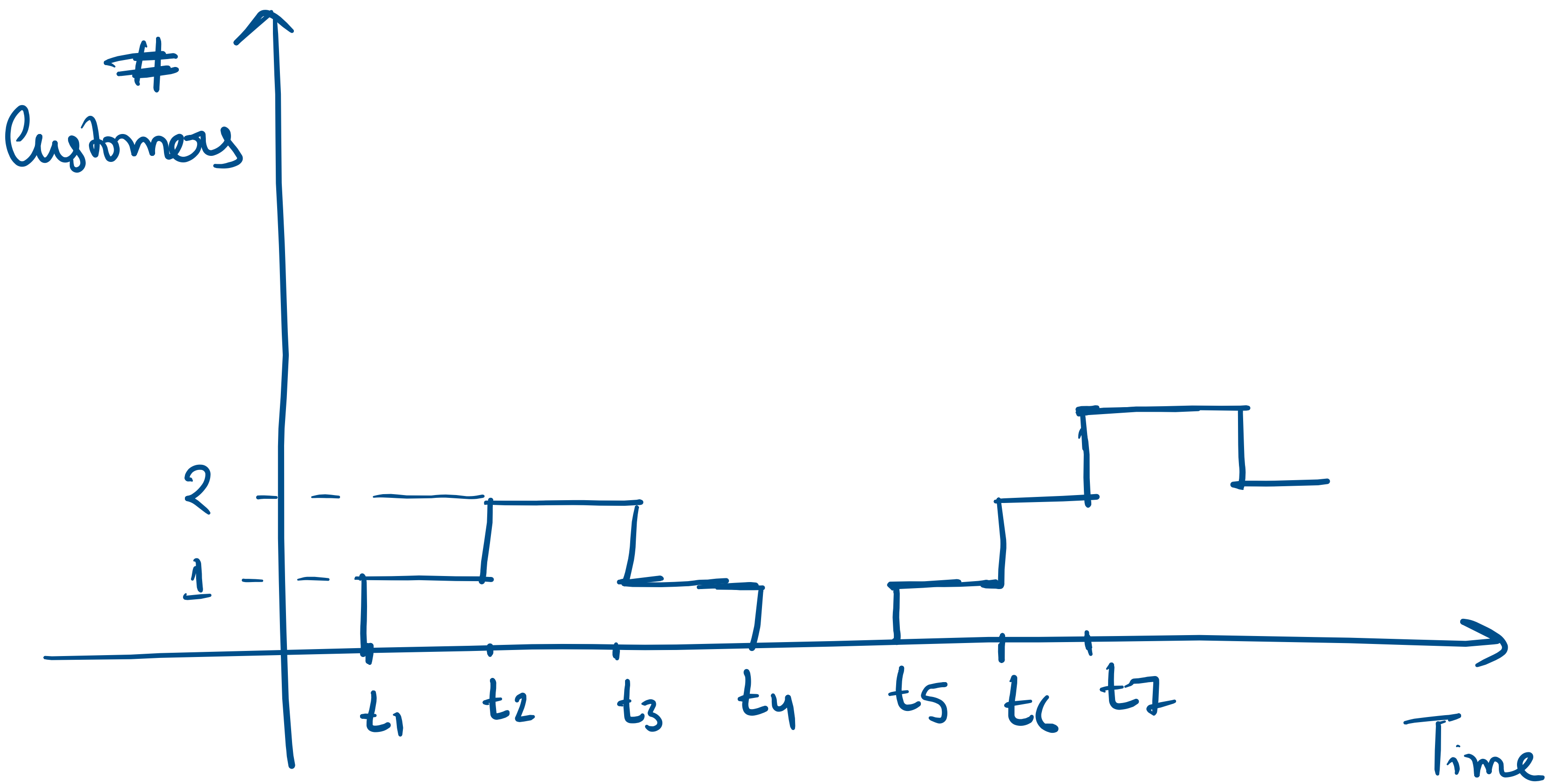
Definition:- A system is called queueing s/m if it follows following process:

1. Customers/Jobs arrive in the s/m.
2. Wait for their turn to receive the service
3. After getting service, they leave the s/m.

Ex:- A railway reservation counter.


Customer Arrival
Queue
(Waiting Time)
Server
Customer Departure
Counter
3
1
customers
4

Diagramatic Representation



Characteristics of Queueing Sym

1. Arrival Pattern:
   - → Const. → Deterministic
   - → Probabilistic → It follow a distribution

2. Service Pattern:
   - → Const. → Deterministic
   - → Probabilistic → It follow a distribution

3. No. of Service channel:

4. System Capacity: Maximum number of customers / Jobs the Sym can hold.

5. Queue Discipline: Method follow for providing service to the customers in queue

FCFS / Randomly

Kendall's Notation : A | B | X | Y | Z

A: Distribution of inter-arrival time

B: Distribution of service time

X: Number of servers

Y: Maximum # customers in the sym → ∞

Z: ~~Quue~~ Queue discipline → FCFS

A, B are chosen from a set

M: Exponential

D: Deterministic

E_k : Erlang Type k $(k = 0, 1, 2, \ldots)$

H_k : Hyper Exponential Type k

G: General

Markovian Queueing s/m

M/M/1

- Exponential inter-arrival time
- Exponential service-time
- One Server
- Queueing capacity is ∞.
- Queue discipline is FCFS

Ex2– $M/M/c$

– No. of server is c

Ex3– $M/M/c/K$

– Queue capacity is finite {K}

Ex4: $\underline{M/G/1}$: Non-Markovian Queueing s/m

– Inter-arrival time: Exponentially distributed

– Service Time: General distribution

– # servers is 1

– Queueing capacity is ∞

– Queue discipline is FCFS

Ex: 5:-

G/M/1

- Inter-arrival time is General distribution
- Service time: Exponentially distributed
- # Server: 1
- Queue capacity: ∞
- Queue discipline: FCFS

Non-Markovian Queueing Sym

Ex: 6:

$M/E_k/1$: Non-Markovian Queueing Sym

Service time distribution : Erlang.