Minimization of a DFA $M=(Q,\sum,\delta,q_0,F)$

A DFA M_0 with a minimum number of states can be found equivalent to M. M_0 is unique except for a possible renaming of states. The process of constructing M_0 is called the minimization of M.

The states of M_0 are the equivalence classes of Q under the following relation ρ_M on Q, which can be easily proved to be an equivalence relation. $q\ \rho_M\ q'$ if for every $w \in \sum^*$ either both $\delta(q,w)$ and $\delta(q',w)$ are in F or both are in F^c.

Using this notion of equivalent states, the minimization can be done by the following algorithm.

Step 1. Drop unreachable states.

Step 2. Carry out the following Table Filling algorithm.

2.1 In a blank table of unordered pairs of states mark all pairs of states s.t. one is in F and the other is in F^c.

2.2 repeat

2.2.1 For each unmarked pair (q,q') if the pair (δ(q,a),δ(q',a) is marked for some a Є ∑ then mark (q,q')

Until no new pairs get marked.

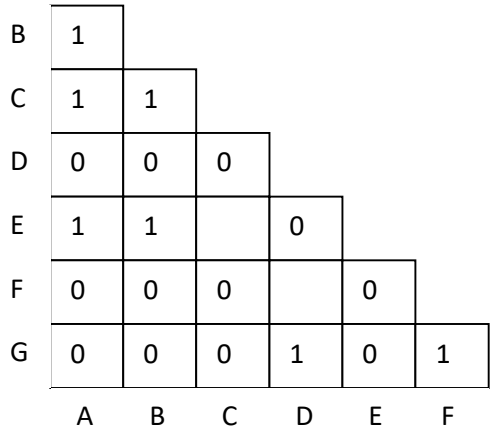
It is convenient to put mark 0 in step 2.1 and mark i in the i-th iteration of 2.2.1. If in the i-th iteration no mark i appears then Step 2 ie the marking phase is over.

Step 3. The unmarked pairs are equivalent.

Step 4. Obtain the equivalence classes and construct the minimum DFA.

Example 1 :

		0	1
→	A	B	C
	B	D	E
	C	F	G
*	D	D	E
	E	F	G
*	F	D	E
*	G	F	G



All states reachable

Itr 1			Itr 2		
	0	1		0	1
A	B		C	F	G
B	D		E	F	G
A	B		D	D	E
C	F		F	D	E
A	B	No mark 2			
E	F		in Itr 2		
B	D	E	end of marking		
C	F	G	Equiv Classes		
B	D	E	(A), (B), (C,E), (D,F), (G)		
E	F	G			
C	F	G			
E	F	G			

D D E

F D E

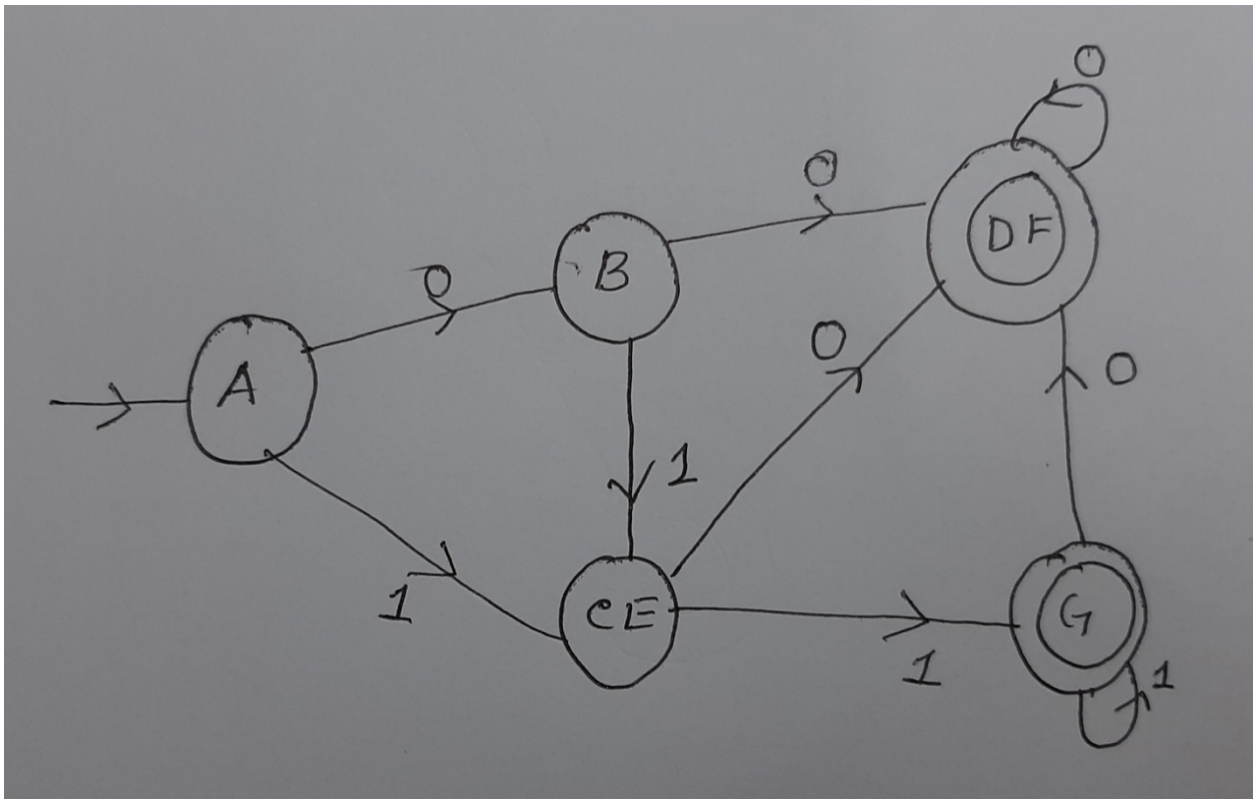
D D E

G F G

F D E

G F G

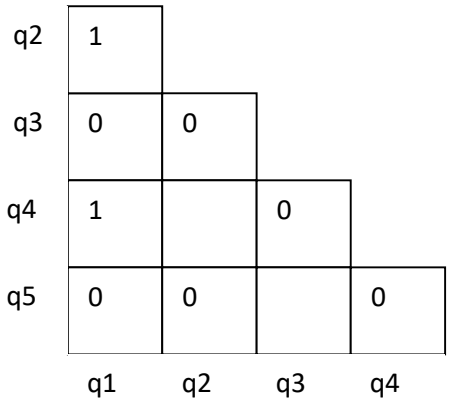
Minimized DFA



Should end with either 00 or 10 or 11

Example 2

	0	1
→ q1	q2	q3
q2	q3	q5
*q3	q4	q3
q4	q3	q5
*q5	q2	q5



All states reachable

Itr 1

	0	1
q1	q2	
q2	q3	
q1	q2	
q4	q3	

Itr 2

	0	1
q2	q3	q5
q4	q3	q5
q3	q4	q3
q5	q2	q5

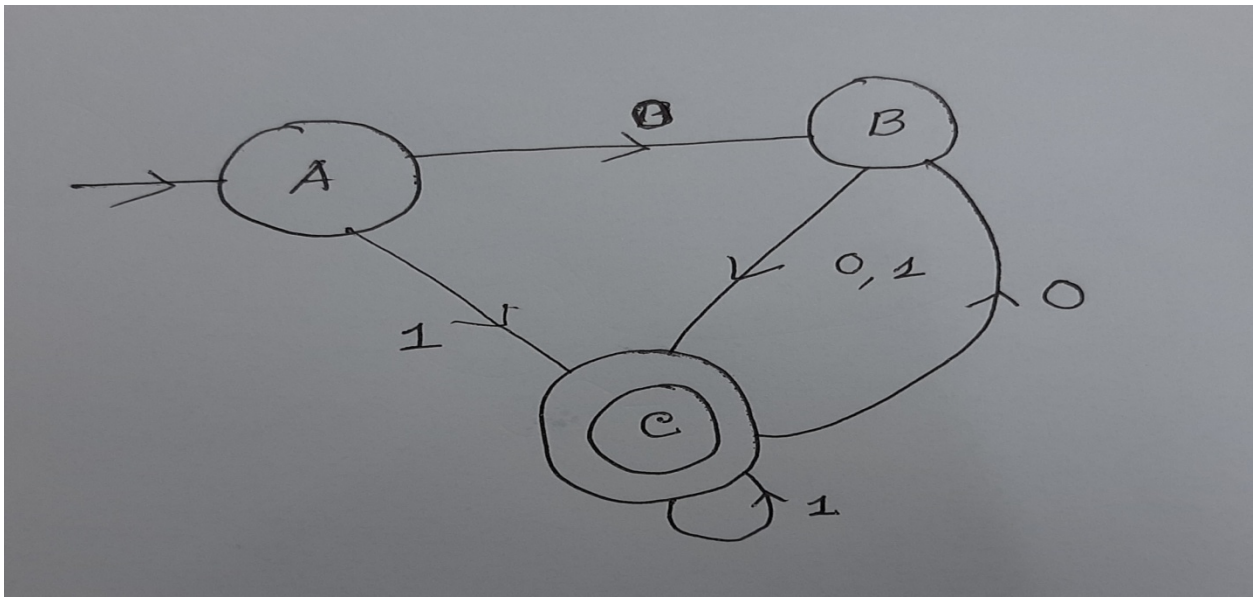
q2 q3 q5 No mark 2 in Itr 2

q4 q3 q5 Marking over

q3 q4 q3 equiv classes (q1),(q2,q4),(q3,q5)

q5 q2 q5 Minimized DFA

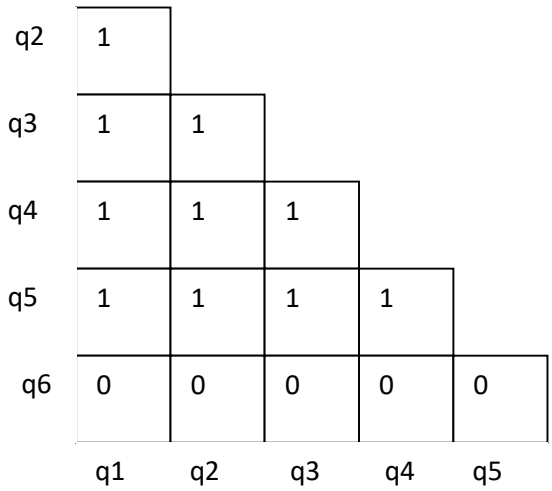
	0	1
→ A (q1)	(q2,q4)	(q3,q5)
B (q2,q4)	(q3,q5)	(q3,q5)
*C (q3,q5)	(q2,q4)	(q3,q5)



Ends with 1 or even number of 0's

Example 3

	0	1
→ q1	q2	q6
q2	q1	q3
q3	q2	q4
q4	q4	q2
q5	q4	q5
* q6	q5	q4



All states are reachable

Itr 1	0	1		0	1
q1	q2	q6		q3	q2
q2	q1	q3		q5	q4
q1	q2	q6		q4	q4	q2
q3	q2	q4		q5	q4	q5

Itr 1 over

q1 q2 q6 All pairs are marked

q4 q4 q2 No new pairs can be

q1 q2 q6 marked in Itr 2

q5 q4 q5 Marking over

q2 q1 Equivalence classes are

q3 q2 singletons ie original

q2 q1 DFA is minimal

q4 q4

q2 q1

q5 q4

q3 q2

q4 q4