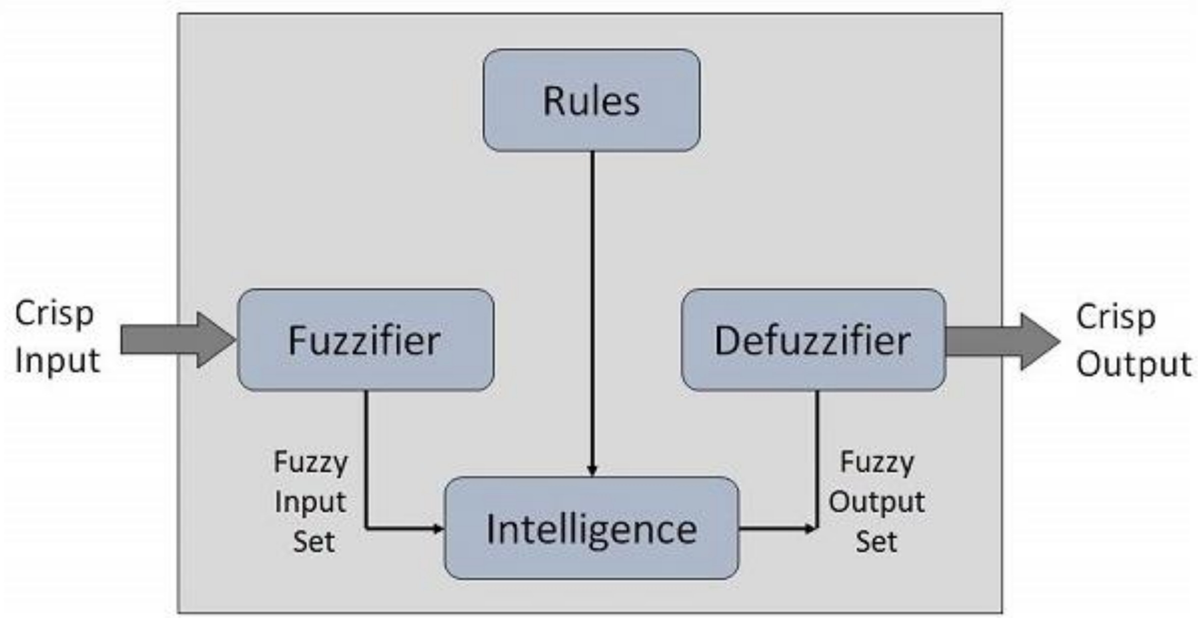
Fuzzy Logic in AI

- Fuzzy Logic (FL) is a method of reasoning that resembles human reasoning.
- The approach of FL imitates the way of decision making in humans that involves all intermediate possibilities between digital values YES and NO.
- The inventor of fuzzy logic, **Lotfi Zadeh**, observed that unlike computers, the human decision making includes a range of possibilities between YES and NO, such as ,

CERTAINLY YES
POSSIBLY YES
CANNOT SAY
POSSIBLY NO
CERTAINLY NO

- **Fuzzy logic is useful for commercial and practical purposes.**
    - ❑ It can control machines and consumer products.
    - ❑ It may not give accurate reasoning, but acceptable reasoning.
    - ❑ Fuzzy logic helps to deal with the uncertainty in engineering.
    - ❑ To understand fuzzy logic, we need to study the concept of fuzzy set.

- **Fuzzy Logic Systems Architecture**



Fuzzy Set:

We start our discussion with classical set

- Classical set is a collection of distinct objects. For example, a set of students, a set of even numbers etc.
- Each individual entity in a set is called a member or an element of the set.
- The classical set is defined in such a way that the universe of discourse is spitted into two groups: members and non-members. Hence, in the case of classical sets, no partial membership exists.
- This set is also called crisp set. For example,
  - $A = \{a_1, a_2, a_3, a_4\}$
  - $A = \{2, 4, 6, 8\}$

  If we want to express this using formula:

  $A = \{ x \mid x \text{ is an even natural number}\}$

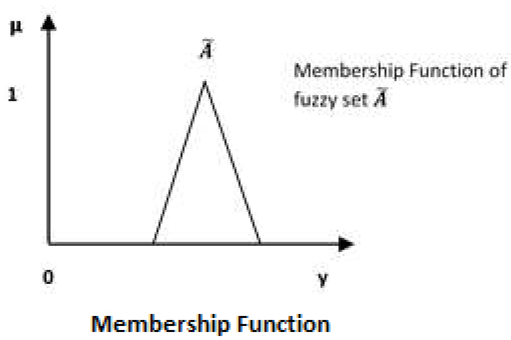
- Here, if we introduce membership function, then the membership value of x is 1 if it belongs to A ; otherwise the value is 0.

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

Fuzzy Set:

- Fuzzy set theory introduces vagueness (ambiguity) using the concept of membership function
- Membership value permits any value between [0,1]
- Formal representation of fuzzy set:

$$\widetilde{A} = \{(y, \mu_{\widetilde{A}}(y)) \,|\, y \in U\}$$



Here, *U* is universe of discourse

$\mu_{\widetilde{A}}(y)$ = membership value of y in fuzzy set Ã

The triangular membership function are most common among various other shapes of membership function such as trapezoidal, singleton, and Gaussian.

Example for fuzzy set:

. Let us assume,

$U=\{a_1,a_2,a_3, a_4,a_5\}$

The followings are the some examples of the fuzzy sets:

1. $A=\{(a_1,0.4),(a_2,0.7),(a_3,0),(a_4,0.1),(a_5,1)\}$

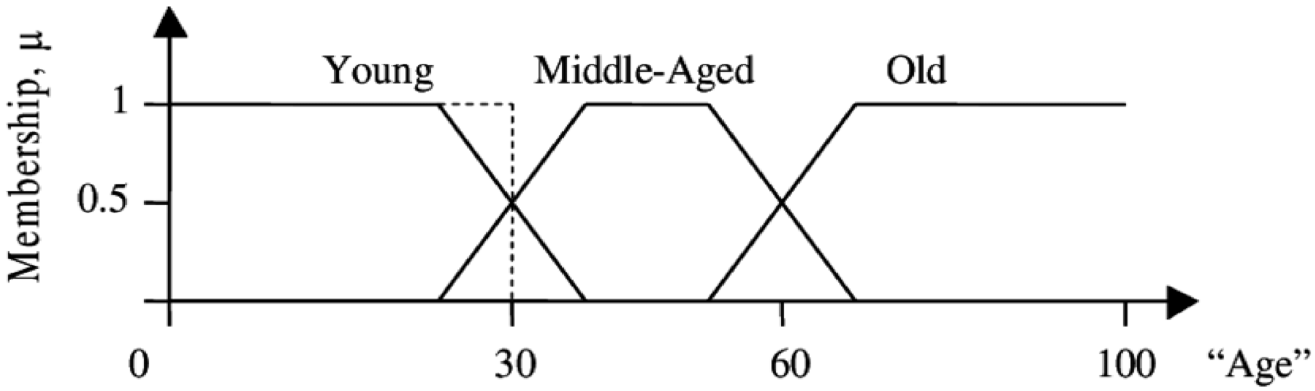
2. $B=\{(a_1,0.2),(a_2,0.6),(a_3,0.2),(a_4,0.1),(a_5,0.8)\}$

3. $C=\{(a_1,0),(a_2,1),(a_3,0),(a_4,1),(a_5,0)\}$

What is the corresponding crisp set representation for fuzzy set C?

$C=\{a_2,a_4\}$ [only two elements fully belong to the set]

Well-known example for fuzzy set:



•Human age groups illustrated as fuzzy sets and membership functions.

•"Young," "Middle-Aged," and "Old" are examples of fuzzy sets.

•In this illustration, the set "Young" includes ages from 0 to 37. "μ" represents the value of the membership function. Membership in the set "Young" decreases gradually from ages 23 to 37. An age of 30 has a membership value μ of 0.5 in the set "Young."

•The solid lines represent the fuzzy membership functions for each set. The dashed lines illustrate how "Young" could be made into a crisp, non-fuzzy set. In this illustration, the crisp, non-fuzzy set "Young" includes ages from 0 to 30.

• Here, any age may belongs to the three different sets, but with different membership values

Operations on fuzzy set:

Let A and B be fuzzy sets that $A,B \subseteq U$ and u is any element in the universe of discourse U i.e. $u \in U$

Standard complement ⇨ $\mu_{\neg A}(u) = 1 - \mu_A(u)$

Standard intersection ⇨ $\mu_{A\cap B}(u) = \min\{\mu_A(u), \mu_B(u)\}$

Standard union ⇨ $\mu_{A\cup B}(u) = \max\{\mu_A(u), \mu_B(u)\}$

Example: $\boldsymbol{A = \{(x_1, 0.5), (x_2, 0.7)\}}$ $\boldsymbol{B = \{(x_1, 0.8), (x_2, 0.2)\}}$

Union: $AUB = \{(x_1, 0.8), (x_2, 0.7)\}$

Intersection: $A \cap B = \{(x_1, 0.5), (x_2, 0.2)\}$

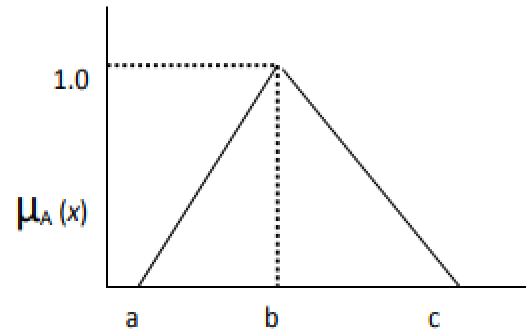
Complement: $\tilde{A} = \{(x_1, 0.5), (x_2, 0.3)\}$

Fuzzy membership function:

1. Triangular 2. Gaussian

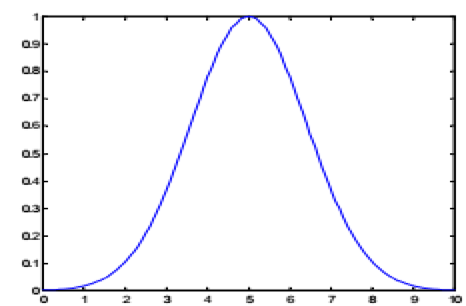
Triangular: Defined by three parameters: a, b, c

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ \dfrac{c-x}{c-b} & \text{if } b \le x \le c \\ 0 & \text{if } x \ge c \end{cases}$$



Gaussian:

$$\mu_A(x,c,s,m) = \exp\left[-\frac{1}{2}\left|\frac{x-c}{s}\right|^m\right]$$



c=5, s=0.5, m=2

Where, C = Center

S = Width

m = fuzzification factor

- **At this moment, want to introduce the terms like ‘young’, ‘not young’, ‘middle age’, ‘possibly yes’, ‘possibly no’ etc. in our knowledge-base/rule-base**

- **linguistic variables help us to include these vagueness/ambiguity in rules**

Linguistic variable:

A linguistic variable is a variable where values are words or sentences in a natural languages.

Suppose:

1. **Height** is a linguistic variable.

   Linguistic values are tall, very tall, very vary tall etc.

   Where

   Tall is linguistic value or primary term.

2. **Age** is a linguistic variable

   T(age) = {young, not young, less old etc.}

Every member of this set is a linguistic term and it can cover some portion of overall values.

Fuzzy rules:

- Fuzzy rules are helpful for modeling human thinking perception.
- Fuzzy if-then rule is in the form:

  Like, “if x is A then y is B”

where, A and B are linguistic values, defined by fuzzy sets over universe of discourse x and y respectively.

“x is A” is “antecedent”

“y is B” is “consequent”

Example:

A rule is

“If pressure is high, then volume is small”

•Here, high corresponds to fuzzy set A and small corresponds to fuzzy set B.

Then, the rule may be rewritten as, If x is A then y is B

• Here, if we have value for x (i.e. pressure), then we can calculate the membership value of x in the set A using pre-defined membership function.

Fuzzy inference system:

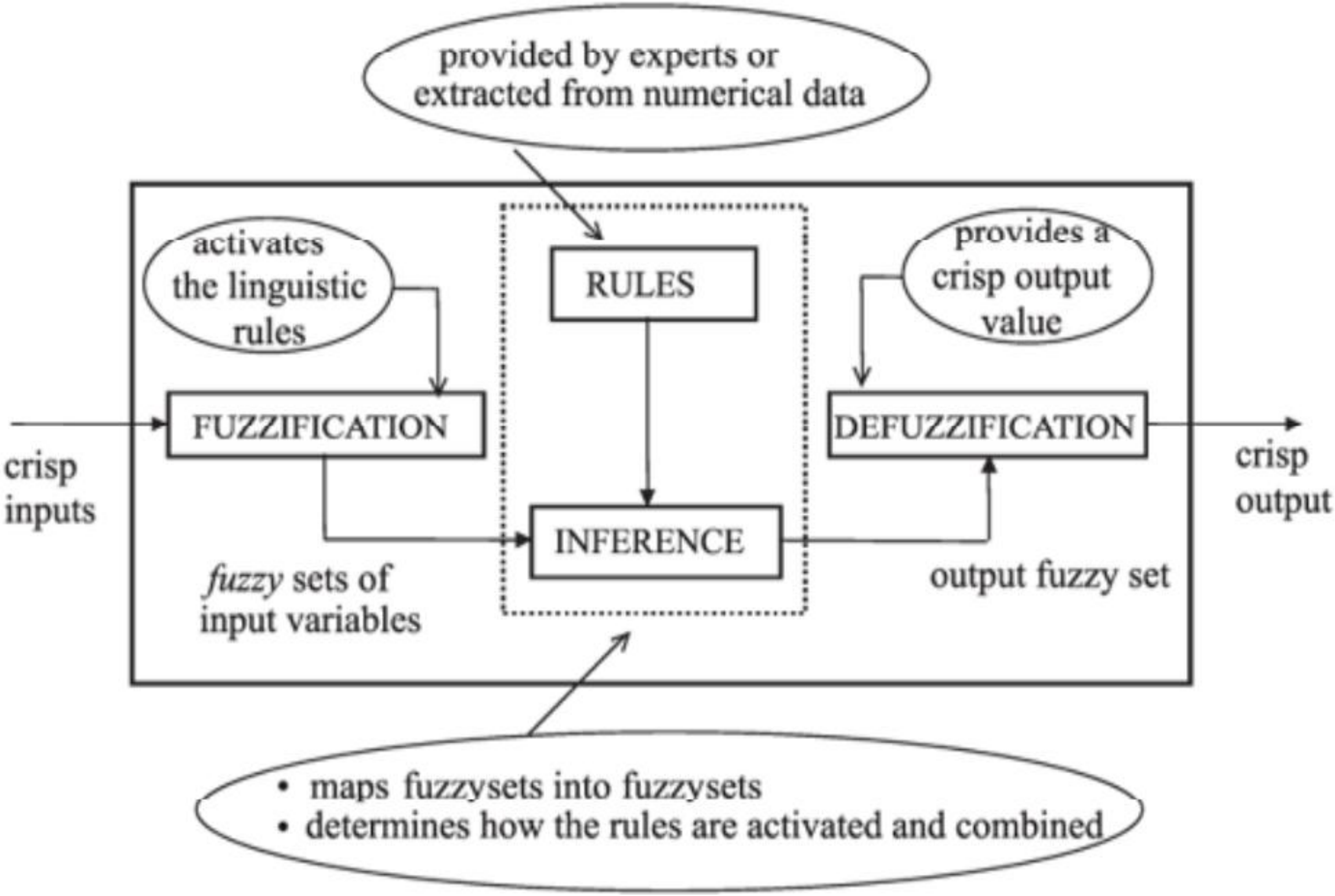
•Fuzzy Inference Systems (FIS) take inputs and process them based on fuzzy reasoning and a set of pre-specified fuzzy if-then rules to produce the outputs

•Both the inputs and outputs are real valued, whereas the internal processing is based on fuzzy rules and fuzzy arithmetic

• The basic structure of a fuzzy inference system consists of three conceptual components:

1. A rule base, which contains a selection of fuzzy rules
2. A database, which defines the membership functions used in the fuzzy rules
3. A reasoning mechanism, which performs the inference procedure upon the rules and given facts to derive a reasonable output or conclusion

Structure of FIS:



Three popular models of FIS:

- Mamdani Fuzzy Models
- Sugeno Fuzzy Models
- Tsukamoto Fuzzy Models

The differences between these three FISs lie in the consequents of their fuzzy rules, and thus their aggregation and defuzzification procedures differ accordingly . Here, we only discuss the Mamdani fuzzy model.

Mamdani inference system:

- The most commonly used fuzzy inference technique.
- It performs in four steps:
  - ➢ Fuzzification of input variable
  - ➢ Rule evaluation (inference)
  - ➢ Aggregation of rule outputs (composition)
  - ➢ Defuzzification

For example,

Design a FIS using Mamdani model for automatic prediction of risk in a project funding system. It has two input values: project funding (p_f) and project staffing (p_s). The output value is the risk prediction either in terms of linguistic values (low/high/normal) or by applying defuzzification to get crisp output.

Knowledge base for the FIS :

- Here, x is the crisp input for p_{f}, y is the crisp input for p_s and z is the crisp output.

- The fuzzy sets corresponding to the linguistic values of adequate, marginal, inadequate for linguistic variable, project funding are A_3, A_2, A_1, respectively.

- The fuzzy sets corresponding to the linguistic values of small, large for linguistic variable, project staffing are B_1, B_2, respectively.

- The fuzzy sets corresponding to the linguistic values of low, normal, high for linguistic variable, risk are C_1, C_2, C_3, respectively.
- There is pre-specified membership function corresponding to each fuzzy set.

- The rules are as follows:

- If p_f is adequate or p_s is small then risk is low.

same as, If x is A_3 **OR** y is B_1 then Z is C_1

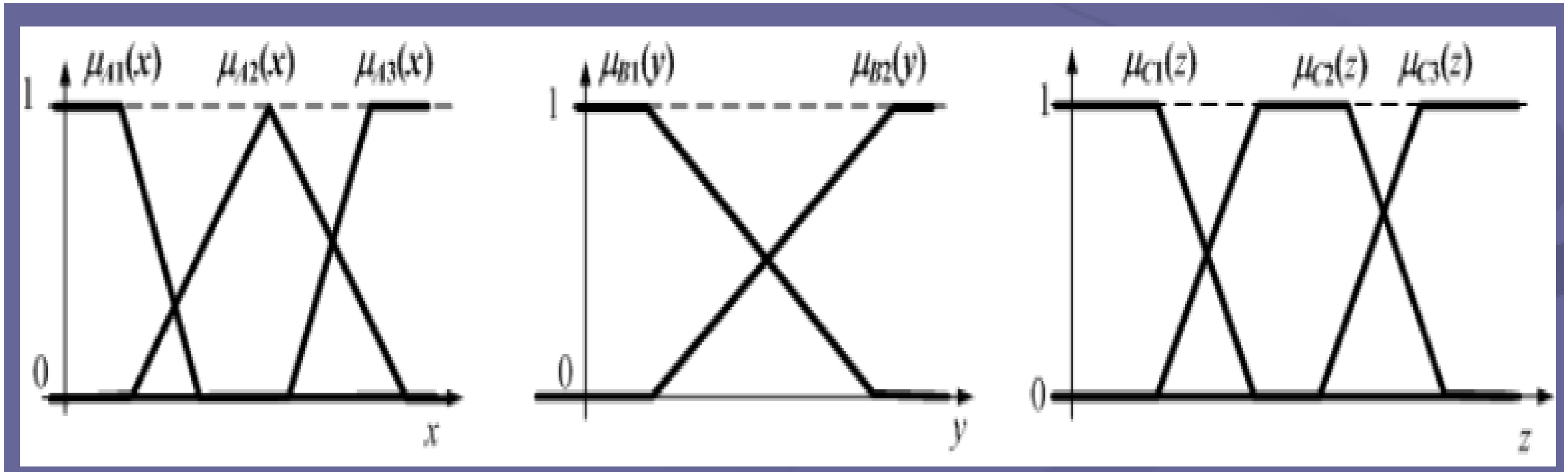
- If p_f is marginal and p_s is large then risk is normal.

same as, If x is A_2 **AND** y is B_2 then Z is C_2.

- If p_f is inadequate then risk is high.

Same as, If x is A_1 then Z is C_3.

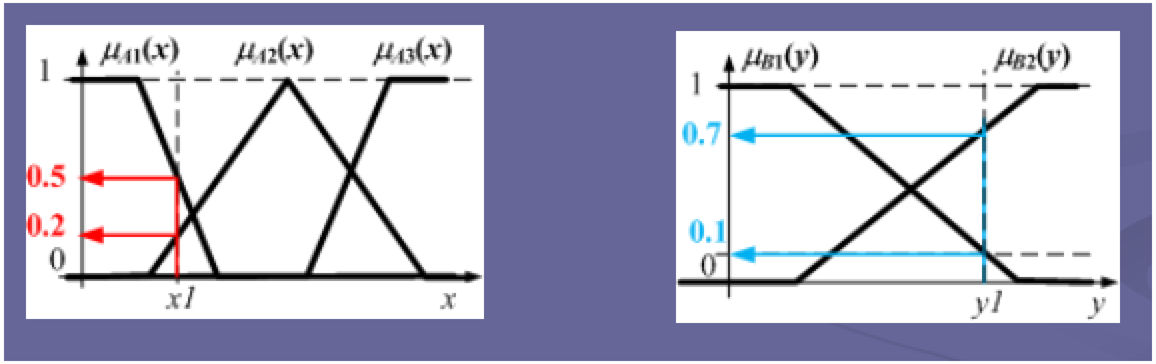
Fuzzy sets defined over the input and output variable are as follows:



Designing of FIS based on fuzzy reasoning:

Step 1 **(Fuzzification)*:***

- Convert the crisp input into a fuzzy one.
- Here, we have to take two crisp input values, x1 and y1 for project funding and project staff, respectively .
- put these values in the membership function of the corresponding fuzzy sets to calculate the membership values



$\mu_{A1}(x1)=0.5$, $\mu_{A2}(x1)=0.2$, $\mu_{A3}(x1)=0.0$

$\mu_{B1}(y1)=0.1$, $\mu_{B2}(y1)=0.7$

Step 2 (Rule evaluation):

•After fuzzification, we know the degree to which each part of the antecedent has been satisfied for each rule.

• If the antecedent of a given rule has more than one part, the fuzzy operator is applied to obtain one value that represents the result of the antecedent for that rule.

•The input to the fuzzy operator (AND/OR) is two or more membership values from fuzzified input variables ; whereas the output is a single value which is applied to the consequent membership function.

➢To evaluate the disjunction of the rule antecedents, we use the OR fuzzy operation. Typically, fuzzy expert systems make use of the classical fuzzy operation

union:

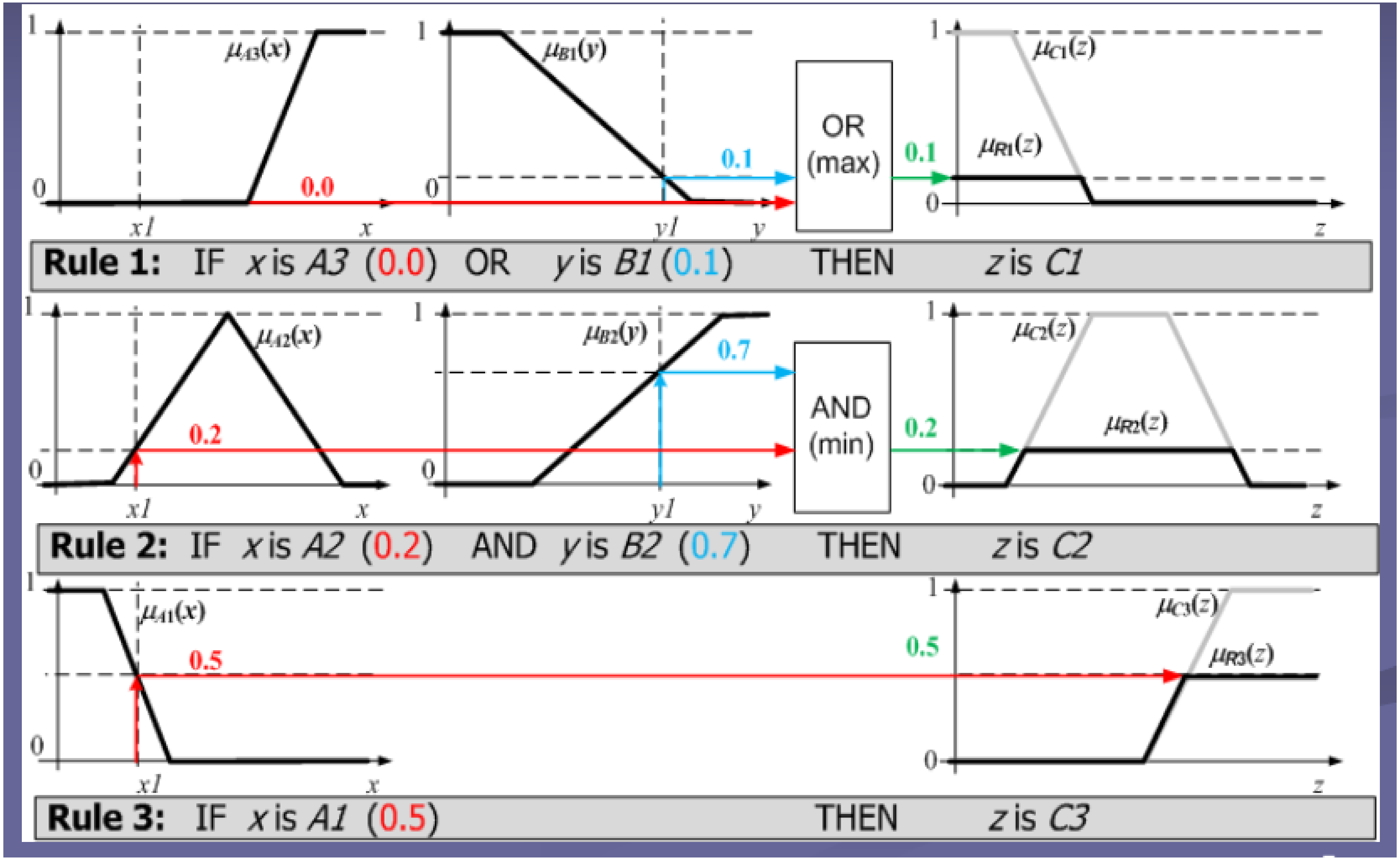
$$\mu_{A \cup B}(x) = \max\left[\mu_A(x),\ \mu_B(x)\right]$$

➢Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the AND fuzzy operation

intersection:

$$\mu_{A \cap B}(x) = \min\left[\mu_A(x),\ \mu_B(x)\right]$$

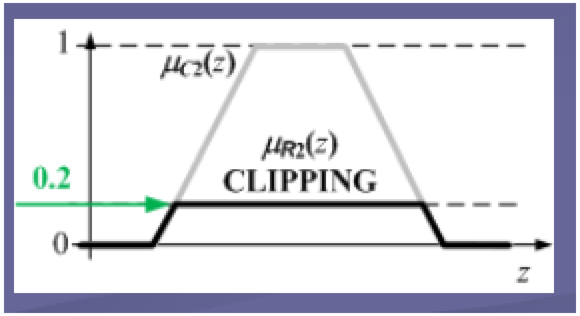
Mamdani style of rule evaluation for project system:



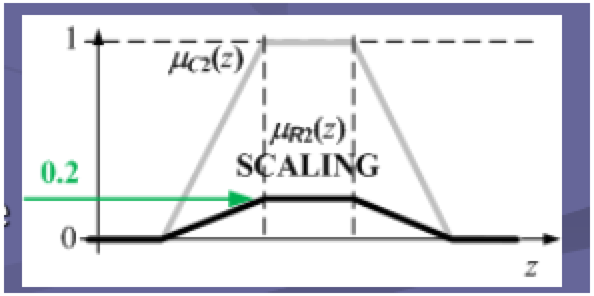
Fuzzy implication:

•Now the result of antecedent evaluation can be applied to the membership function of consequent (fuzzy implication)

➢ The most common method is to cut the consequent membership function at the level of antecedent evaluation. The method is called clipping



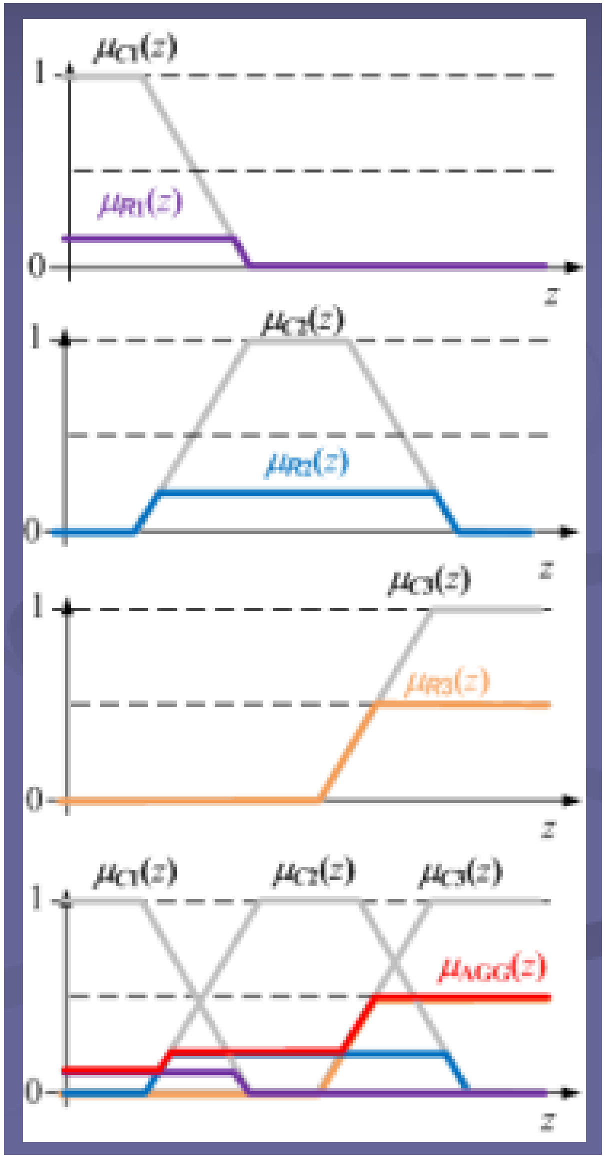
➢ Another method is called scaling where the membership function of consequent is multiplied by the evaluation value of antecedent.



Step 3: Aggregation of rules

- Aggregation is the process of unification of output of all rules.
- The input of the process is the clipped and scaled consequent membership functions and the output is one fuzzy set for each output variable.
- The most common method is the maximum of rules'
- output membership functions, i.e.,

$$\mu_{AGG}(z) = \max\,[\mu_{R1}(z), \mu_{R12}(z), \mu_{R3}(z)\,]$$



Step 4: Defuzzification

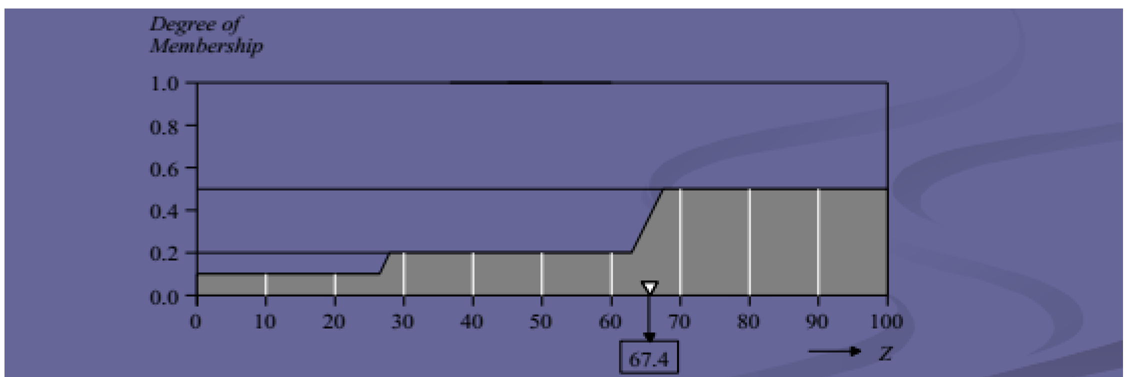
•Fuzziness over the inputs help to evaluate the fuzzy rules. However, the final output of FIS has to be a crisp number.

• The input of the defuzzification process is the aggregate output fuzzy set and the output is single number.

• There are various defuzzification methods, the most popular one is the centroid technique. It finds the points where the vertical line would slice the aggregate set into two equal masses.

•The technique finds the point representing the centre of gravity of the aggregated fuzzy set, on an interval [a,b].

•The reasonable estimation can obtained by calculating it over a sample of points.



$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

Thank you…………