Deterministic Push Down Automaton (DPDA)

A PDA $P = (Q,\Sigma,\Gamma,\delta,q_0,Z_0,F)$ is a DPDA if at every configuration there is at most one next configuration. Thus in a DPDA

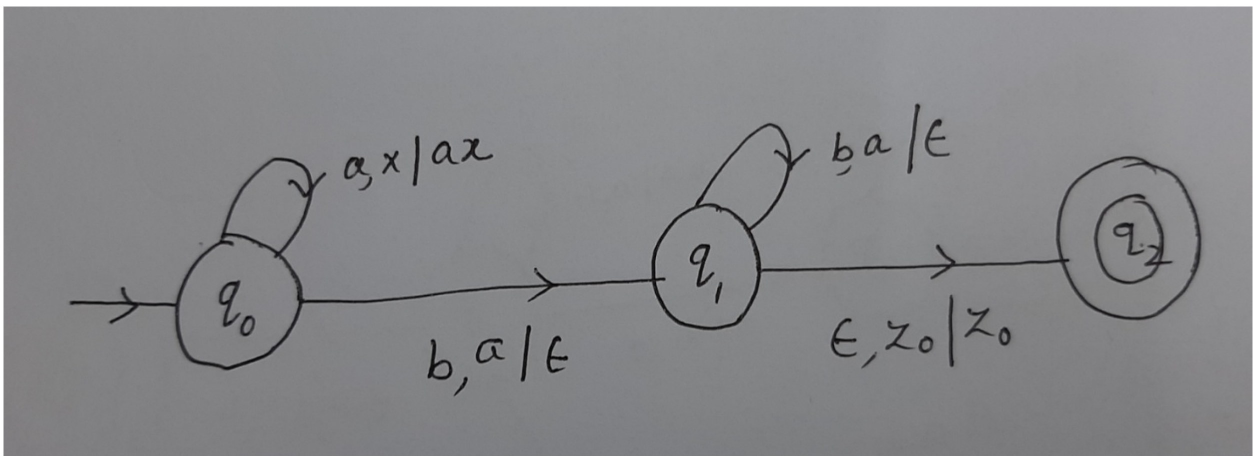
1) $\delta(q,a,X)$ has at most one pair for every $q \in Q$, $a \in \Sigma_e$ and $X \in \Gamma$.
2) If $\delta(q,a,X)$ is nonempty for some $a \in \Sigma$ and some $X \in \Gamma$, then $\delta(q,\varepsilon,X)$ is empty.

Condition 2) is required because if for example $\delta(q,a,\beta)$ contains (p_1,u_1) and $\delta(q,\varepsilon,\beta)$ contains (p_2,u_2) then the configuration $(q,aw,\beta u)$ will yield in one step either (p_1,w,u_1u) or (p_2,aw,u_2u) which are different since w and aw are different.

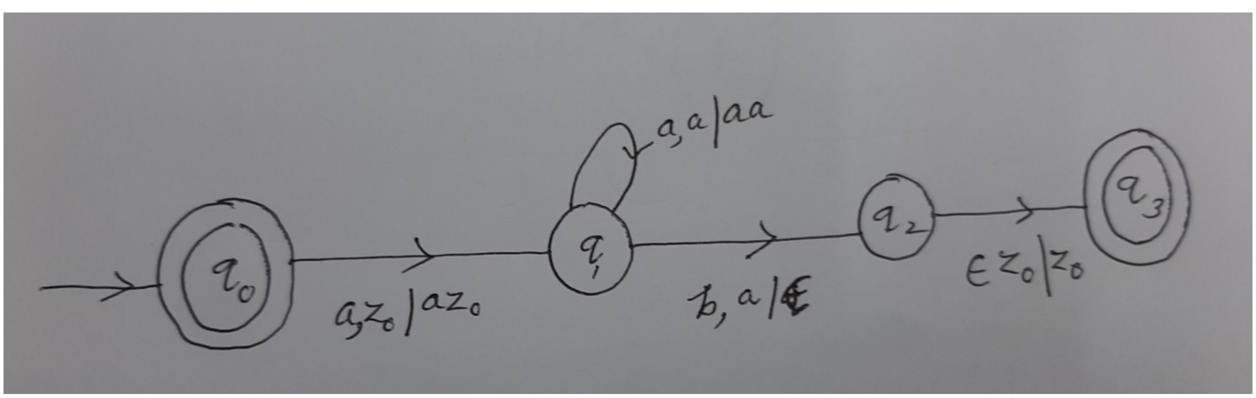
A language L which is L(P) for some DPDA P, is called a deterministic context free language(DCFL).

Examples :

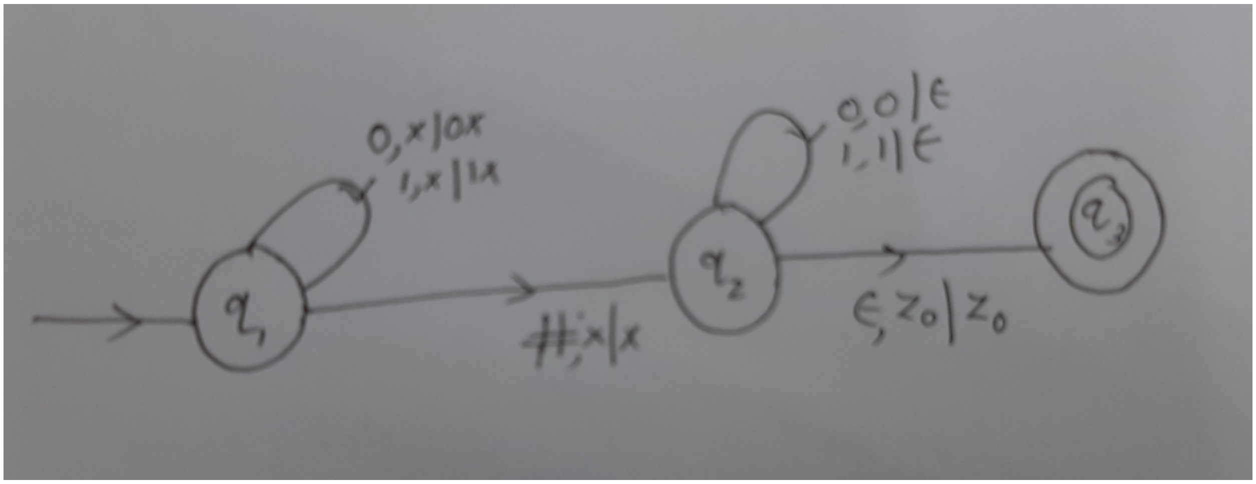
1) $L1 = \{a^n b^n \mid n \geq 1\}$ is a DCFL



2) $L2 = \{a^n b^n \mid n \geq 0\}$ is a DCFL. We cannot simply make the start state in 1) final because then a, aa,… etc will be accepted.

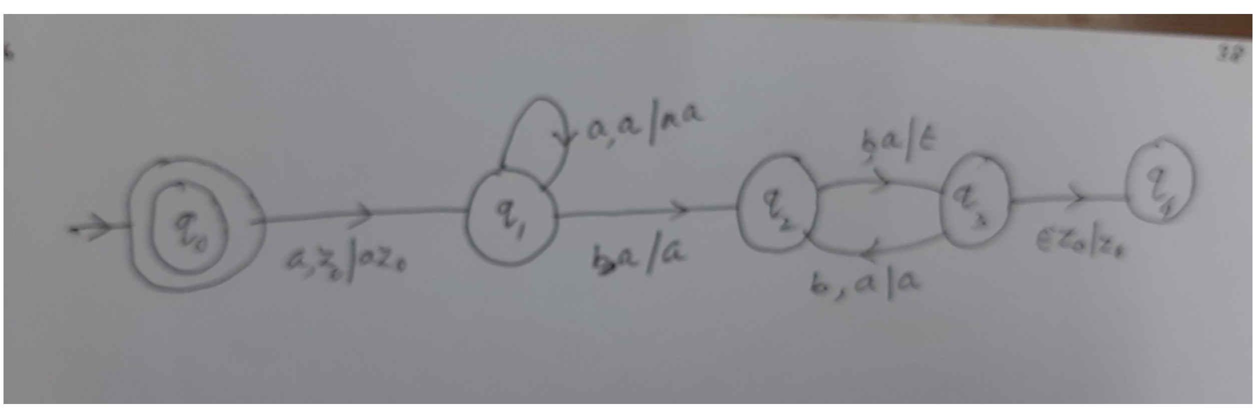


3) L3 = $\{w\#w^R \mid w \in \{0,1\}^*\}$ is a DCFL

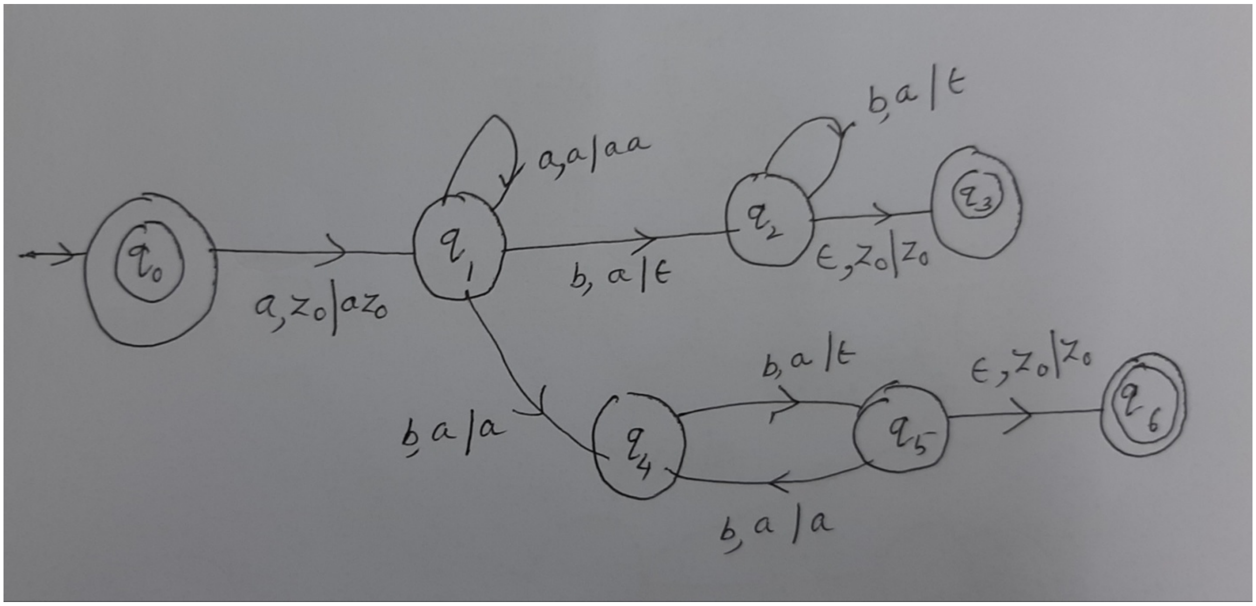


Strings of L3 are palindromes where the middle of the string is given by the # symbol – no guessing is required unlike in $L_{epal}=\{ww^R \mid w\in\{0,1\}^*\}$,$L_{opal}=\{w\in \{0,1\}^* \mid w=w^R , |w| \text{ odd}\}$,$L_{pal}=\{w\in\{0,1\}^* \mid w=w^R\}$ where guessing of the middle is required – can be proved to be not DCFL.

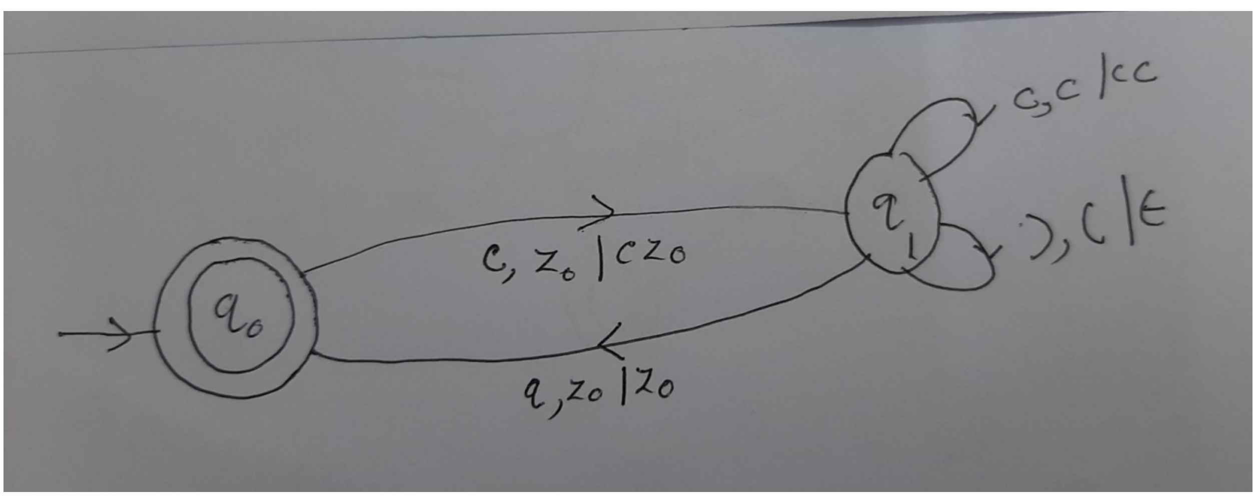
4) L4=$\{a^nb^{2n} \mid n\geq0\}$ is a DCFL



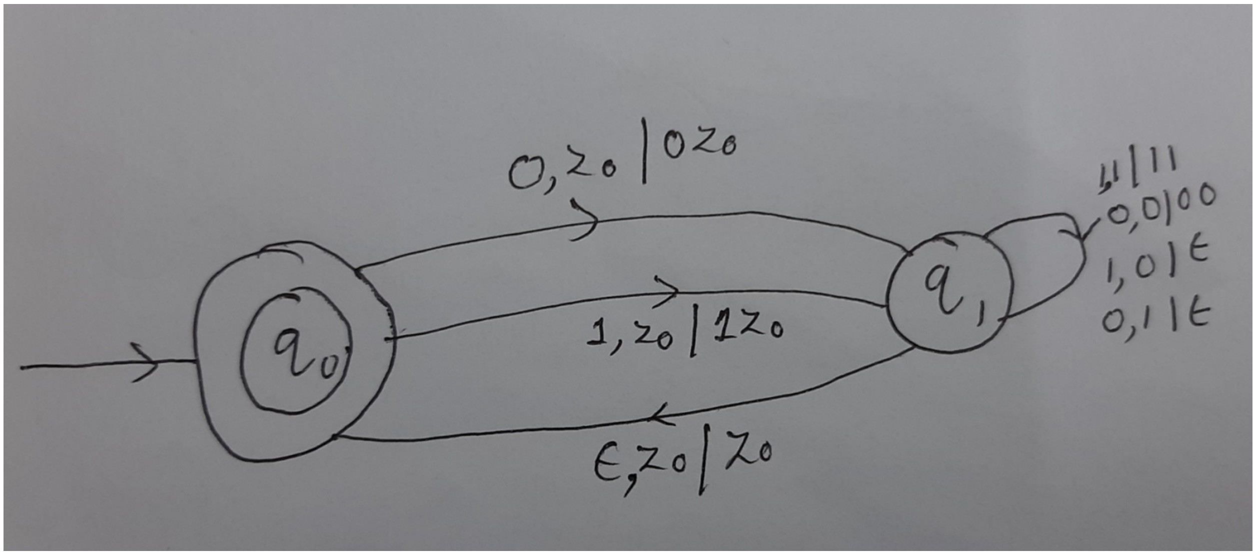
5) L5=L2 U L4 = $\{a^nb^n|n\geq0\}\cup\{a^nb^{2n}|n\geq0\}$ is not a DCFL.



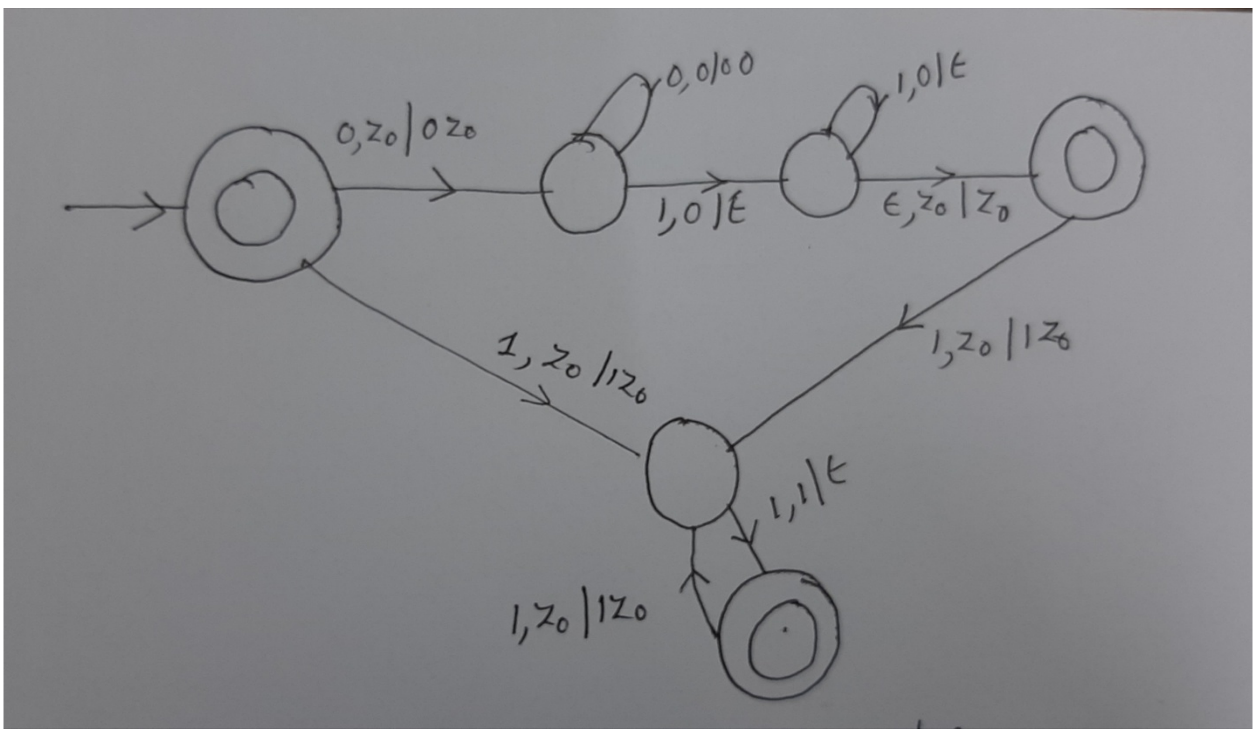
6) L6=L_{par}=Language of balanced parentheses is a DCFL



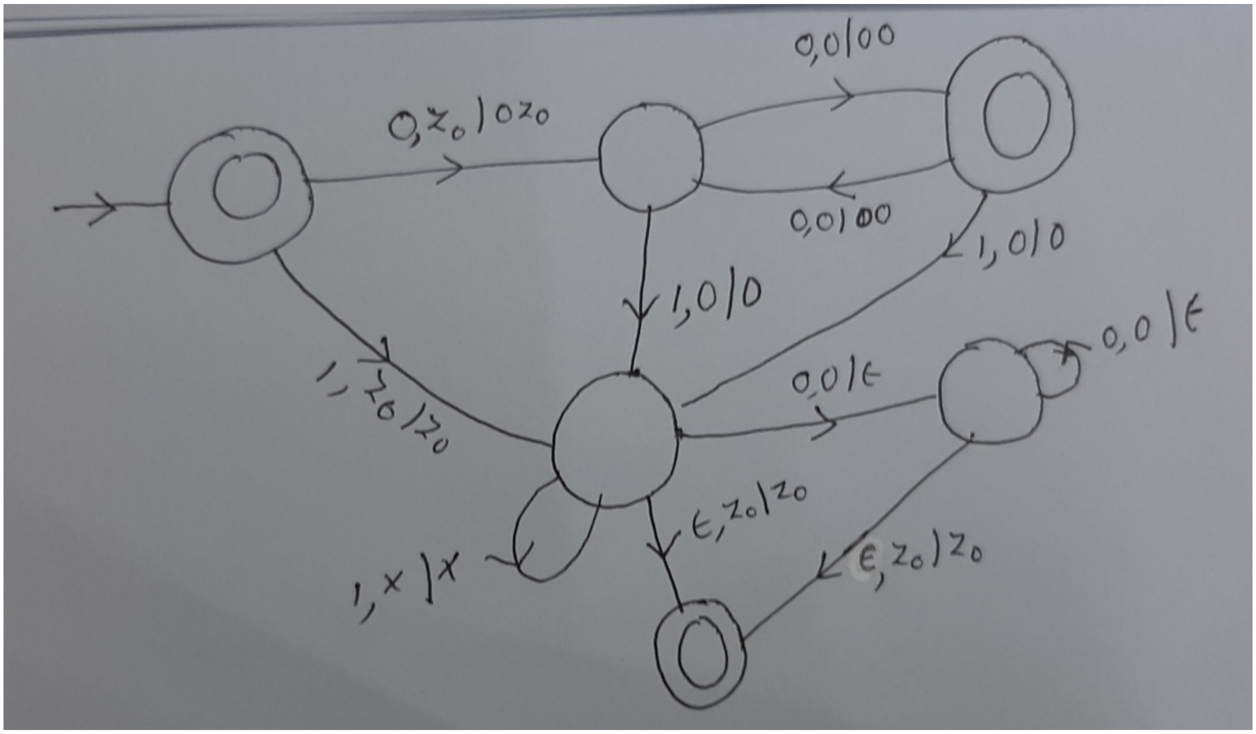
7) $L7 = \{u \in \{0,1\}^* \mid n_0(u)=n_1(u)\}$ is a DCFL



8) $L8=\{0^n1^{m+n}1^m \mid m,n\geq 0\}$ is a DCFL



9) $L9=\{0^n1^m0^n \mid m,n \text{ arbitrary}\}$ is a DCFL



We now explore the connection of various classes of DCFL's.

Theorem : If L is a DCFL ie if L = L(P) for some DPDA P, then L has an unambiguous Grammar.

We omit the proof. Thus an inherently ambiguous language cannot be a DCFL. For example $L_S = \{a^ib^jc^k \mid i=j \text{ or } j=k\}$ is linear but not a DCFL. Also $L_U = \{a^ib^jc^kd^l \mid i=j,k=l \text{ or } i=l,j=k\}$ is not linear and not a DCFL. We have already noted that the language of palindromes $L_{pal}=\{w\in\{0,1\}^* \mid w=w^R\}$ is linear but not a DCFL. Also the language of balanced parentheses L_{par} is a DCFL but not linear.

We have already seen that a regular language is linear. It is also a DCFL.

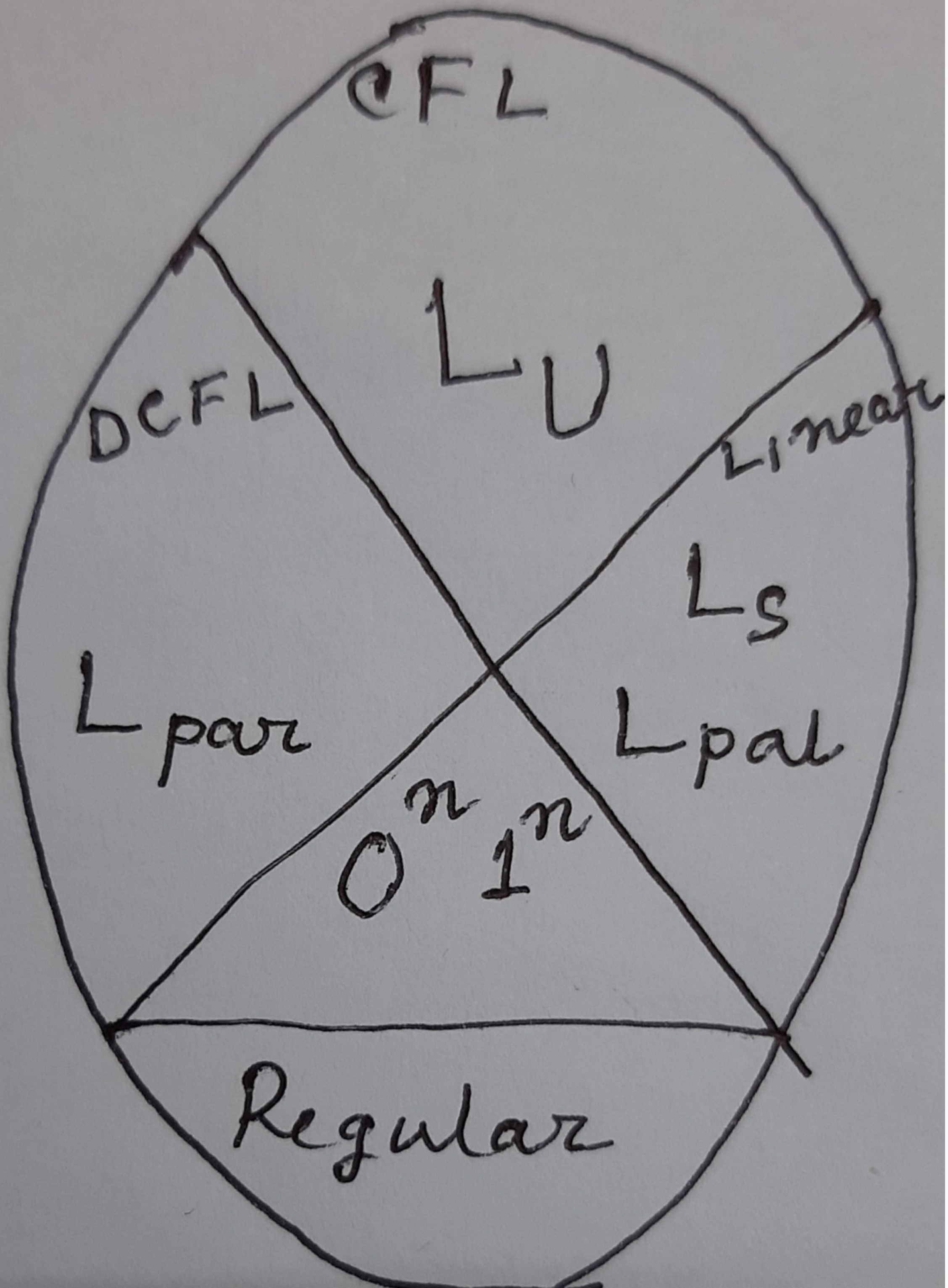
Theorem : If L is regular then L = L(P) for some DPDA P.

Let L be recognized by the DFA $M=(Q,\Sigma,\delta_M,,$

q_0,F). Construct $P=(Q,\Sigma,\{Z_0\},\delta_P,q_0,Z_0,F)$ where

$\delta_P(q,a,Z_0) = \{(p,Z_0)\}$ if $\delta_M(q,a)=p$ ie ignore the stack and change state like the DFA. Then clearly L=L(M)=L(P).

Finally the language $\{0^n1^n \mid n\geq 0\}$ is both linear and a DCFL but not regular. Thus we have the following Venn diagram :



CFL
L_U
DCFL
Linear
L_S
L_{par}
L_{pal}
$0^n 1^n$
Regular