Context Sensitive Grammar (CSG) and Context Sensitive Language (CSL)

A Grammar G = (N, ∑, R, S) is a CSG if productions are of the form

$$\alpha A \delta \rightarrow \alpha \beta \delta$$

where A is in N, α, β and δ are in (N U ∑)*, |β| ≥ 1 ie A replaced by β under the left context α and the right context δ.

In order to have ε in L(G) we are allowed to have the production S -> ε, but then S should not be in the body of any production.

A language is context sensitive (CSL) if L = L(G) for a CSG G.

A monotonic Grammar is a grammar where every production is of the form α -> β

α containing at least one variable and $| \alpha | \leq | \beta |$. S -> ε may be allowed but then S should not appear in the body of any production.

A CSG is obviously monotonic and conversely it can be proved that every monotonic Grammar is equivalent to a CSG. Hence the language generated by a monotonic Grammar is a CSL.

Example 1 : $L = \{ a^n b^n c^n \mid n \geq 0\}$

Take the monotonic grammar G:

$S \rightarrow \varepsilon \mid S_1$

$S_1 \rightarrow a\, S_1\, B\, C \mid a\, B\, C$

$C\, B \rightarrow B\, C$

$a\, B \rightarrow a\, b$, $b\, B \rightarrow b\, b$, $b\, C \rightarrow b\, c$, $c\, C \rightarrow c\, c$

It can be proved that L =L(G). Thus L is a CSL since a monotonic Grammar has an equivalent CSG. One can see that

For a b c : S -> S_1 -> a B C -> a b C -> a b c

For a^2 b^2 c^2 : S -> S_1 -> a S_1 B C

-> a a B C B C

-> a a b C B C

-> a a b B C C

-> a a b b C C

-> a a b b c C

> a a b b c c

HW Generate a^3 b^3 c^3 using only one production at one step.

Example 2 : L = { w w | w is in {0, 1}*}

Take the monotonic Grammar G :

$S \rightarrow \varepsilon \mid S_1$

$S_1 \rightarrow F_0 M_0 \mid F_1 M_1$

$F_0 \rightarrow F_0 0 A \mid F_0 1 B$, $F_1 \rightarrow F_1 0 A \mid F_1 1 B$

$A 0 \rightarrow 0 A$, $A 1 \rightarrow 1 A$, $B 0 \rightarrow 0 B$, $B 1 \rightarrow 1 B$

$A M_0 \rightarrow M_0 0$, $B M_0 \rightarrow M_0 1$, $A M_1 \rightarrow M_1 0$, $B M_1 \rightarrow M_1 1$

$F_0 \rightarrow 0$, $M_0 \rightarrow 0$, $F_1 \rightarrow 1$, $M_1 \rightarrow 1$

It can be proved that L = L(G). hence L is a CSL since a monotonic grammar is equivalent to a CSG. One can see that

For 1 1 , $S \rightarrow S_1 \rightarrow F_1 M_1 \rightarrow 1 M_1 \rightarrow 1 1$

For 1 0 1 0, $S \rightarrow S_1 \rightarrow F_1 M_1$

$\rightarrow F_1 0 A M_1$

$\rightarrow F_1 0 M_1 0$

$\rightarrow 1\ 0\ M_1\ 0$

$\rightarrow 1\ 0\ 1\ 0$

For 1 0 1 1 0 1 : $S \rightarrow S_1 \rightarrow F_1\ M_1$

$\rightarrow F_1\ 1\ B\ M_1$

$\rightarrow F_1\ 1\ M_1\ 1$

$\rightarrow F_1\ 0\ A\ 1\ M_1\ 1$

$\rightarrow F_1\ 0\ 1\ A\ M_1\ 1$

$\rightarrow F_1\ 0\ 1\ M_1\ 0\ 1$

$\rightarrow 1\ 0\ 1\ M_1\ 0\ 1$

$\rightarrow 1\ 0\ 1\ 1\ 0\ 1$

HW : Generate the following strings using only one production at every step.

i) 1 1 0 1 1 1 0 1
ii) 1 0 1 1 1 0 1 1