FA to Regular Expressions

$$(00)^*(\varepsilon+0)1^*+(0^*+1^*)^*+(01)^*$$

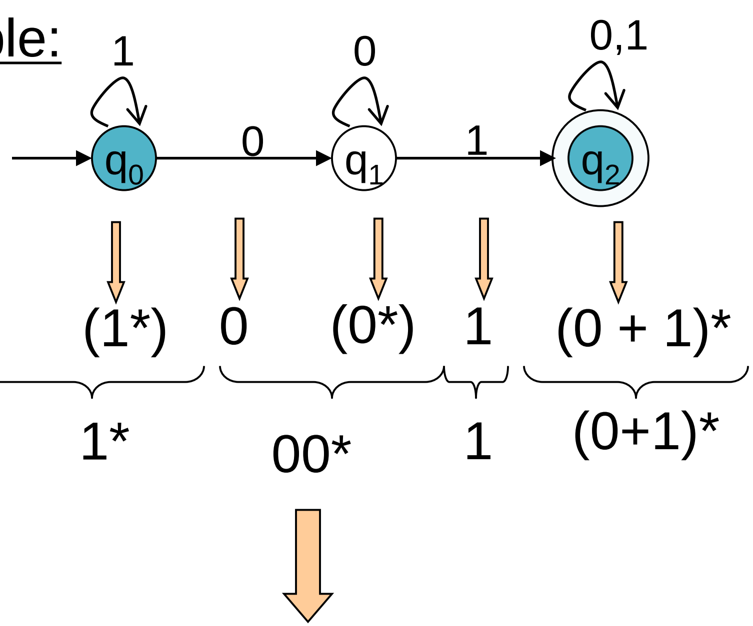
Write RE for all strings contain at least two occurrences of the sub string 00

Write RE for all strings with even number of a’s followed by odd number of b’s.

FA to RE construction

Informally, trace all distinct paths (traversing cycles only once)
from the start state to *each of the* final states
and enumerate all the expressions along the way

Example:



1*00*1(0+1)*

Algorithm for FA to RE

1) FA does not contain $\in$
2) only one initial state
3) v_1 - - - - - v_n
   → initial states.

a_{ij} → edge lable

$V_1 = V_1 a_{11} + V_2 a_{21} + \dots +$

$V_2 = V_1 a_{12} + V_2 a_{22} + \dots$

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$V_n = V_1 a_{1n} + V_2 a_{2n} + \dots$

$q_0 = q_0 1 + \varepsilon$ ....(i)

$q_1 = q_0 0 + q_1 0$ ....(ii)

$q_2 = q_1 1 + q_2 0 + q_2 1$...........(ii)

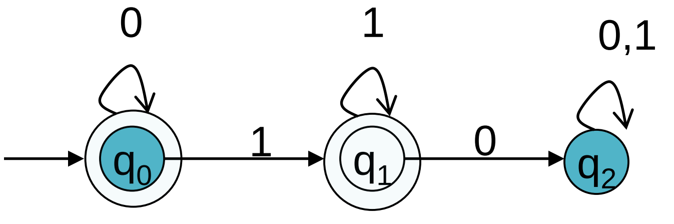
From (i)-> $q_0 = 1^*$ (apply arden's theorem)

Now, $q_1 = 1^*$(replace q_0)$0+ q_1 0 = 1^*00^*$ (apply arden's theorem)

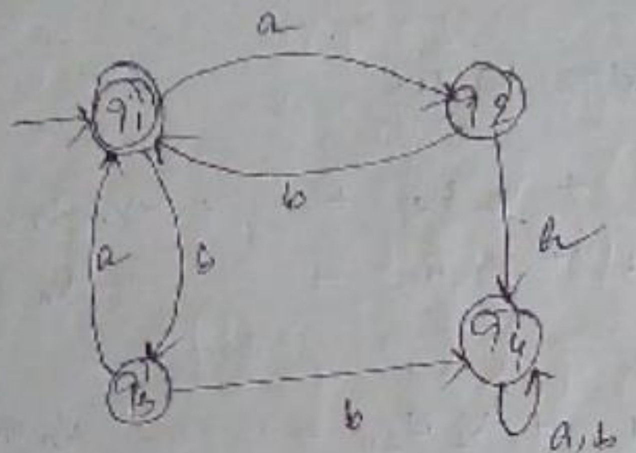
Similarly, $q_2 = q_1 1 + q_2 0 + q_2 1 = 1^*00^*$ (replace q_1) $1 + q_2 (1+0)$

$= 1^*00^* 1(1+0)^*$

Example:



Ex.2



$$q_1 = \cancel{q_2 b A/\epsilon} \quad q_2 b + q_3 a + \epsilon$$

$$q_2 = q_1 a$$

$$q_3 = q_1 b$$

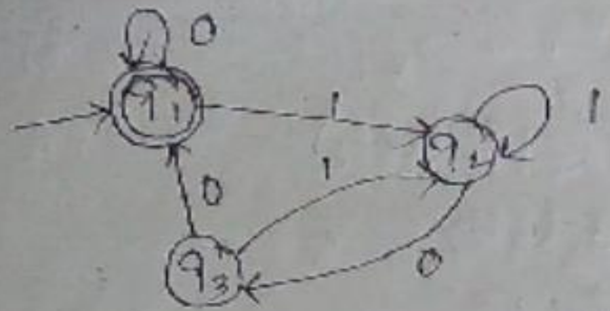
$$q_4 = q_4(a+b) + q_3 b + q_2 a$$

$$= q_4 a + q_4 b + q_3 b + q_2 a$$

$$q_1 = q_1 ab + q_1 ba + \epsilon$$

$$= q_1(ab+ba) + \epsilon$$

$$= (ab+ba)^*$$



$$\rightarrow q_1 = q_1 0 + q_3 0 + \epsilon$$

$$q_2 = q_2 1 + q_1 0 + q_3 1$$

$$q_3 = q_2 0$$

$$q_2 = q_2 1 + q_1 1 + q_2 0 \cdot 1$$

$$= q_1 1 + q_2 (1 + 01)$$

$$= q_1 1 (1 + 01)^*$$

$$q_1 = q_1 0 + q_2 0 + \epsilon$$

$$= q_1 0 + q_1 1 (1 + 0)^* 0 + \epsilon$$

$$= q_1 (0 + 1 (1 + 01)^* 0) + \epsilon$$

$$= (0 + 1 (1 + 01)^* 0)^*$$