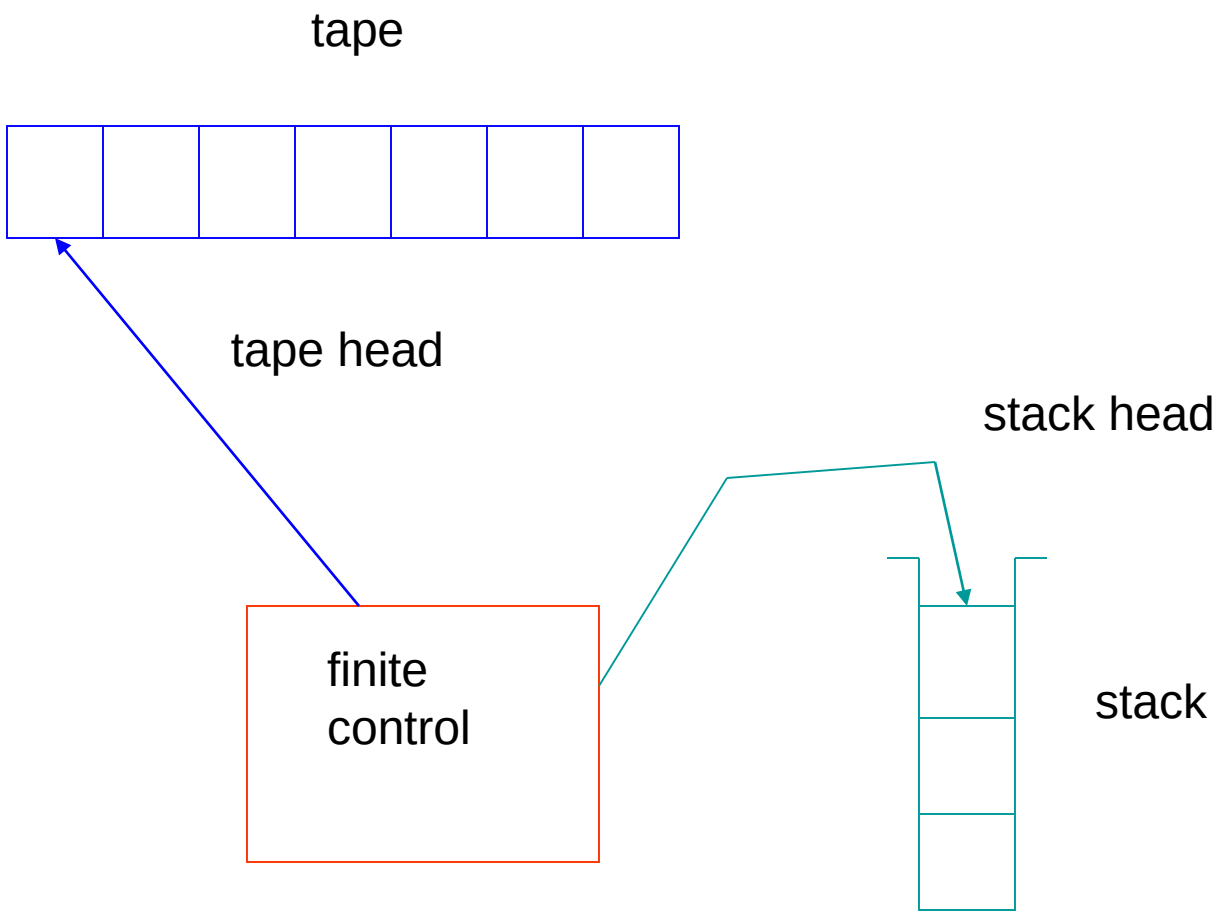
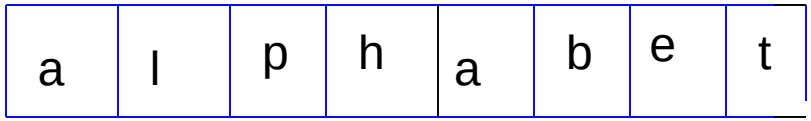
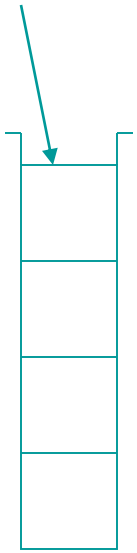
Pushdown Automata


tape
tape head
stack head
finite
control
stack



The tape is divided into finitely many cells. Each cell contains a symbol in an alphabet Σ.

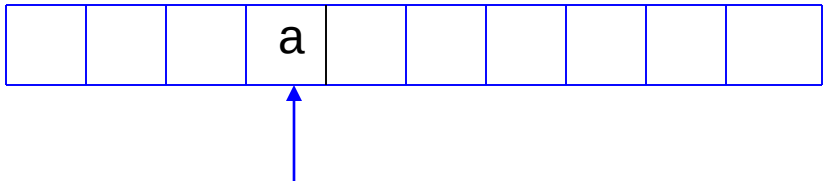


The stack head always scans the top symbol of the stack. It performs two basic operations:

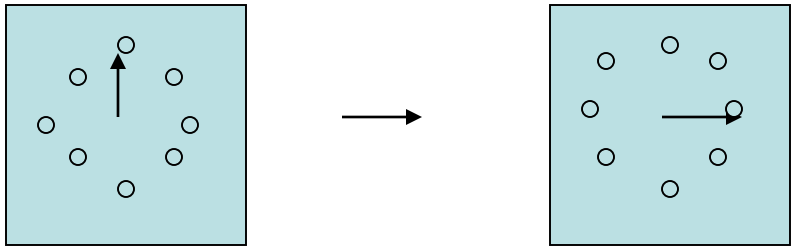
Push: add a new symbol at the top.

Pop: read and remove the top symbol.

Alphabet of stack symbols: Γ

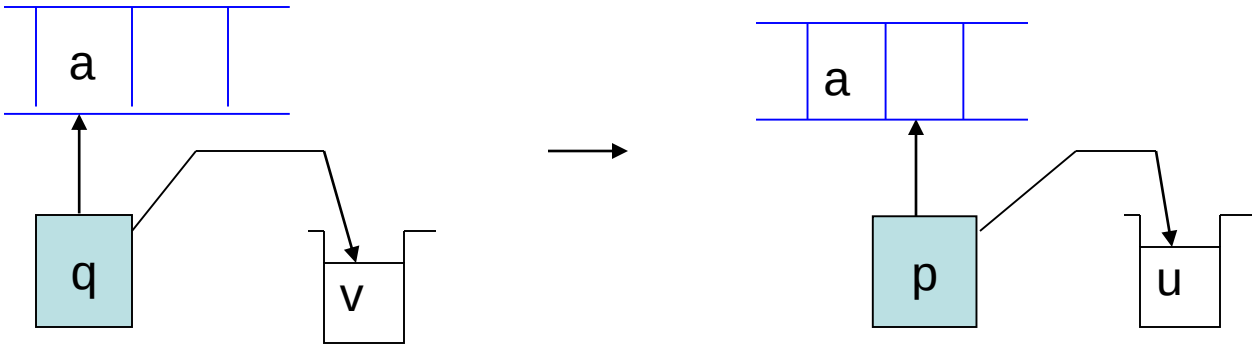


- The head scans at a cell on the tape and can *read* a symbol on the cell. In each move, the head can move to the right cell.

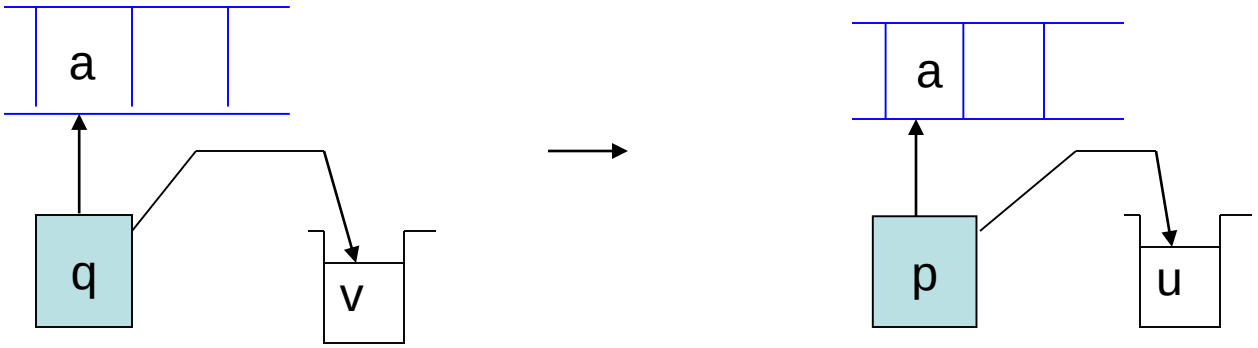


- The finite control has finitely many states which form a set Q. For each move, the state is changed according to the evaluation of a *transition function*

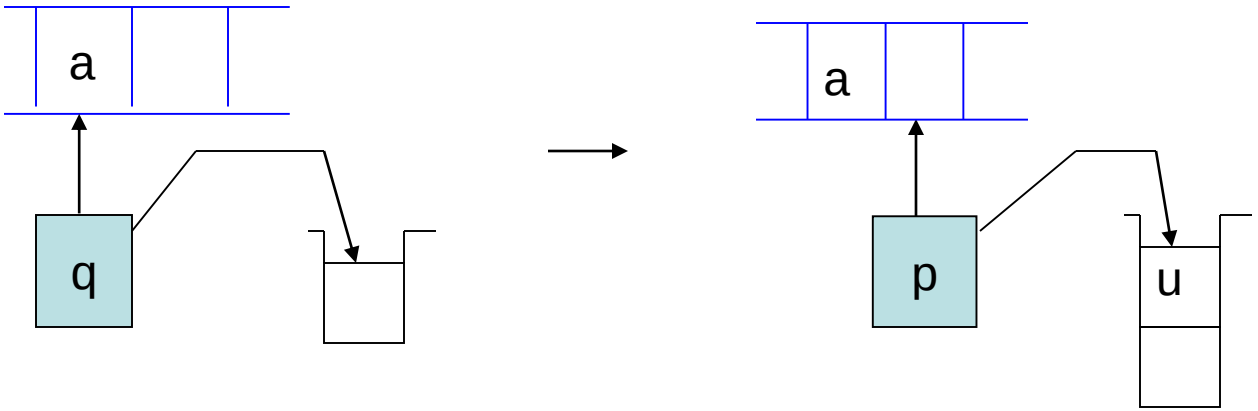
  $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \rightarrow 2^{Q \times (\Gamma \cup \{\varepsilon\})}$.



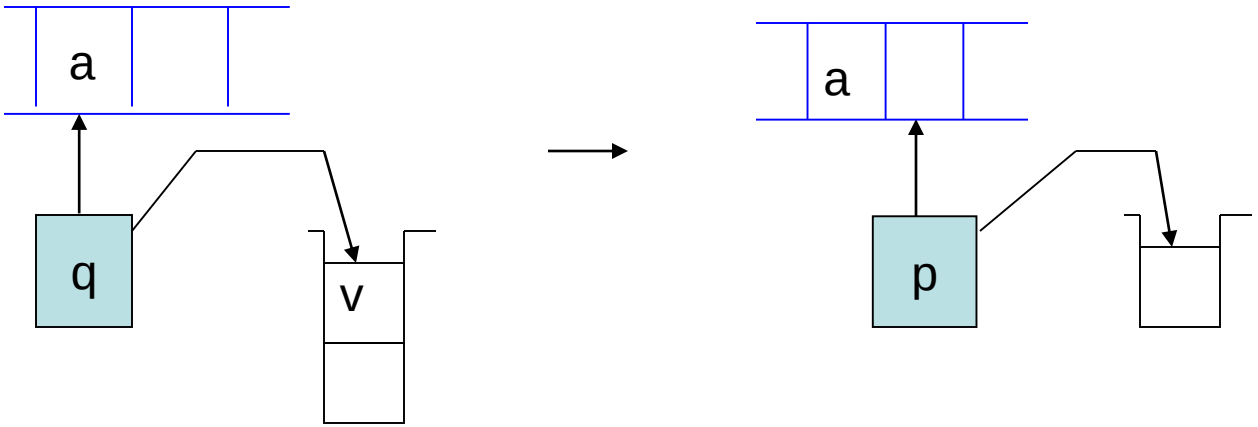
- *(p, u)* ∈ *δ(q, a, v)* means that if the tape head reads *a,* the stack head read *v*, and the finite control is in the state *q*, then one of possible moves is that the next state is *p*, *v* is replaced by *u* at stack, and the tape head moves one cell to the right.



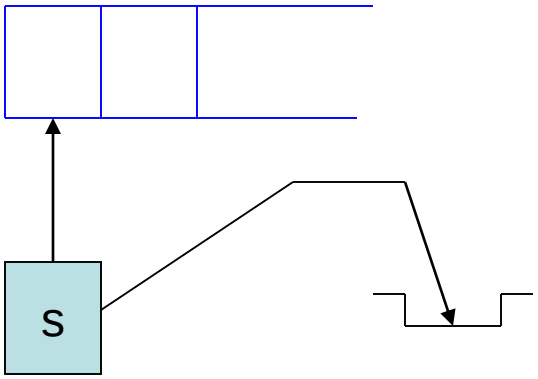
- $(p, u) \in \delta(q, \varepsilon, v)$ means that this a ε-move.



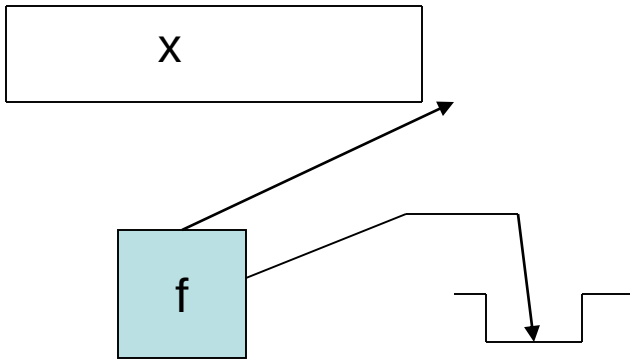
- $(p, u) \in \delta(q, a, \varepsilon)$ means that a push operation performs at stack.



- $(p, \varepsilon) \in \delta(q, a, v)$ means that a pop operation performs at stack



- There are some special states: an initial state s and a final set F of final states.
- Initially, the PDA is in the initial state s and the head scans the leftmost cell. The tape holds an input string. The stack is empty.



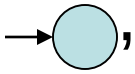
- When the head gets off the tape, the PDA stops. An input string *x* is accepted by the PDA if the PDA stops at a final state and the stack is empty.
- Otherwise, the input string is rejected.

- The PDA can be represented by

  $$M = (Q, \Sigma, \Gamma, \delta, s, F)$$

  where Σ is the alphabet of input symbols and Γ is the alphabet of stack symbols.
- The set of all strings accepted by a PDA M is denoted by $L(M)$. We also say that the language $L(M)$ is accepted by M.

- The transition diagram of a PDA is an alternative way to represent the PDA.
- For $M = (Q, \Sigma, \Gamma, \delta, s, F)$, the transition diagram of M is an edge-labeled digraph $G=(V, E)$ satisfying the following:

$V = Q$ ($s =$  , $f =$  for $f \in F$)

$E = \{ q \xrightarrow{a,\ v/u} p \mid (p,u) \in \delta(q, a, v) \}.$