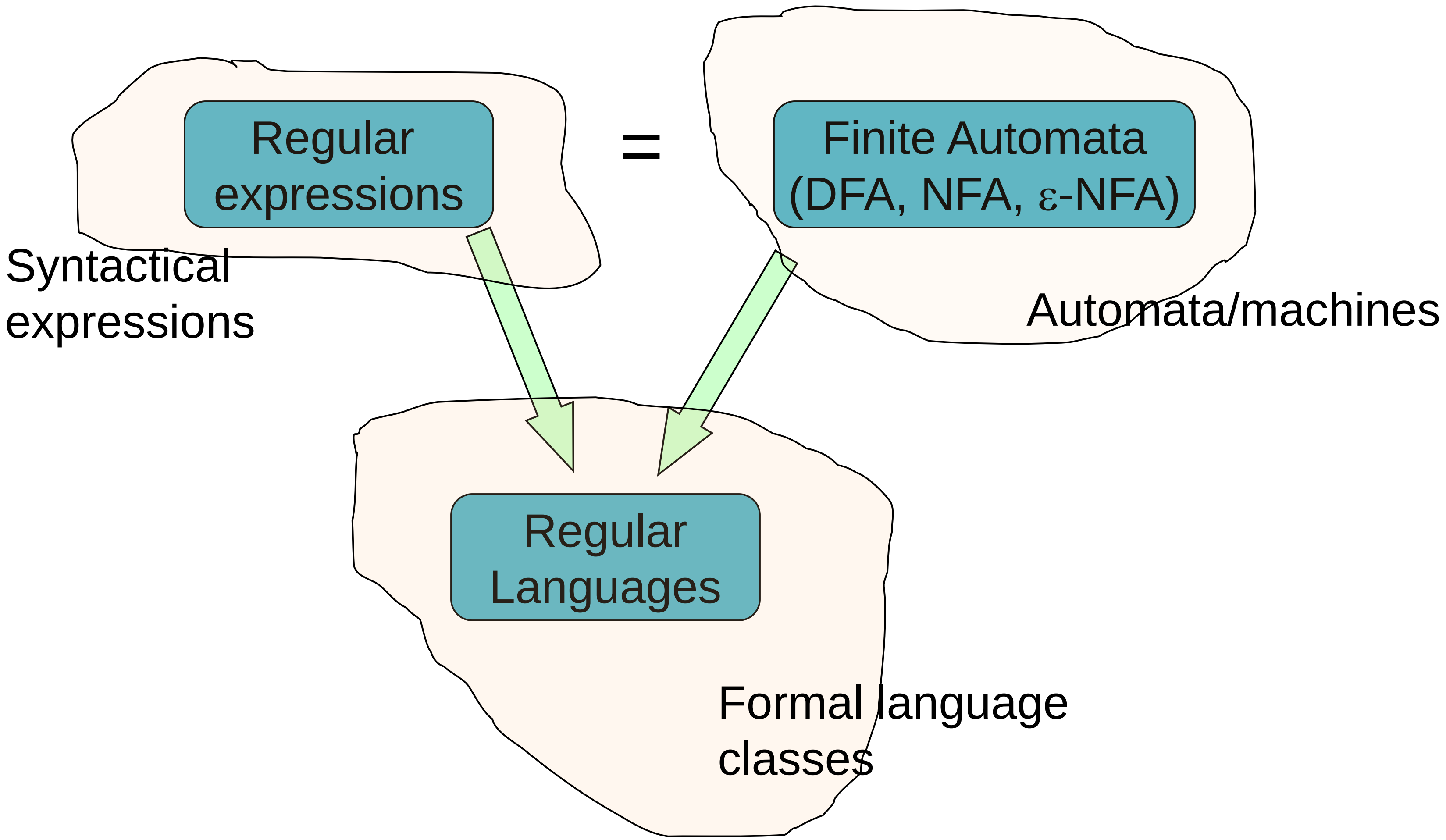
Regular Languages
&
Regular Expressions

Regular Expressions



Language Operators

<u>Union</u> of two languages:

- **L U M** = all strings that are either in L or M
- <u>Note:</u> A union of two languages produces a third language

Other symbols: L/M and L+M

<u>Concatenation</u> of two languages:

- **L . M** = all strings that are of the form *xy*
  s.t., $x \in L$ and $y \in M$
- The *dot* operator is usually omitted
  - *i.e.,* ***LM*** *is same as L.M*

“i” here refers to how many strings to concatenate from the parent language L to produce strings in the language L^i

Kleene Closure (the * operator)

Kleene Closure of a given language L:

$L^0 = \{\varepsilon\}$

$L^1 = \{w \mid \text{for some } w \in L\}$

$L^2 = \{ w_1 w_2 \mid w_1 \in L, w_2 \in L \text{ (duplicates allowed)}\}$

$L^i = \{ w_1 w_2 \ldots w_i \mid \text{all } w\text{'s chosen are} \in L \text{ (duplicates allowed)}\}$

(Note: the choice of each w_i is independent)

L^* $= \bigcup_{i \geq 0} L^i$ (arbitrary number of concatenations)

Example:

Let L = { 1, 00}

$L^0 = \{\varepsilon\}$

$L^1 = \{1,00\}$

$L^2 = \{11,100,001,0000\}$

$L^3 = \{111,1100,1001,10000,000000,00001,00100,0011\}$

L^* $= L^0 \cup L^1 \cup L^2 \cup \ldots$

Building Regular Expressions

Let E be a regular expression and the language represented by E is L(E)

Then:

$(E) = E$

$L(E + F) = L(E) \cup L(F)$

$L(E\ F) = L(E)\ L(F)$

$L(E^*) = (L(E))^*$

Example: how to use these regular expression properties and language operators?

L = { w | w is a binary string which does not contain two consecutive 0s or two consecutive 1s anywhere)

E.g., w = 01010101 is in L, while w = 10010 is not in L

Goal: Build a regular expression for L

Four cases for w:

Case A: w starts with 0 and |w| is even

Case B: w starts with 1 and |w| is even

Case C: w starts with 0 and |w| is odd

Case D: w starts with 1 and |w| is odd

Regular expression for the four cases:

Case A: (01)*

Case B: (10)*

Case C: 0(10)*

Case D: 1(01)*

Since L is the union of all 4 cases:

Reg Exp for L = (01)* + (10)* + 0(10)* + 1(01)*

If we introduce ε then the regular expression can be simplified to:

Reg Exp for L = (ε +1)(01)*(ε +0)

Precedence of Operators

Highest to lowest

* * operator (star)
* . (concatenation)
* \+ operator

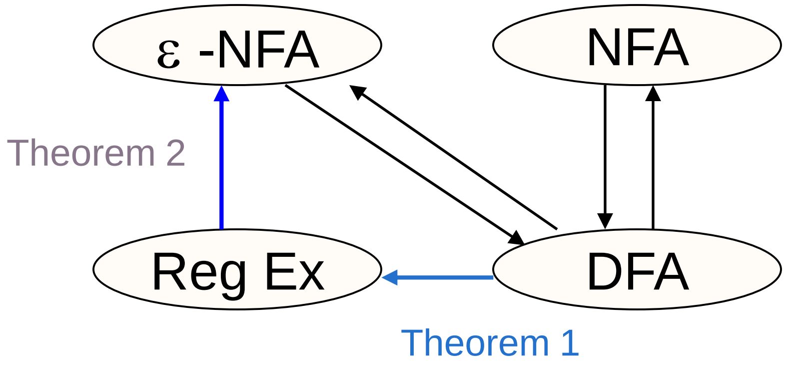
Example:

$01^* + 1 \quad = \quad (\,0\,.\,((1)^*)\,) + 1$

Finite Automata (FA) & Regular Expressions (Reg Ex)

- To show that they are interchangeable, consider the following theorems:
  - *Theorem 1: For every DFA A there exists a regular expression R such that L(R)=L(A)*
  - *Theorem 2: For every regular expression R there exists an ε-NFA E such that L(E)=L(R)*

Proofs in the book

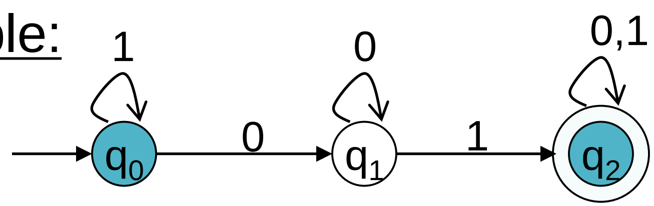


Kleene Theorem

DFA to RE construction

Informally, trace all distinct paths (traversing cycles only once)
from the start state to *each of the* final states
and enumerate all the expressions along the way

<u>Example:</u>



$$(1^*)\quad 0\quad (0^*)\quad 1\quad (0+1)^*$$

$$\underbrace{(1^*)}_{1^*}\ \underbrace{0\ (0^*)}_{00^*}\ \underbrace{1}_{1}\ \underbrace{(0+1)^*}_{(0+1)^*}$$

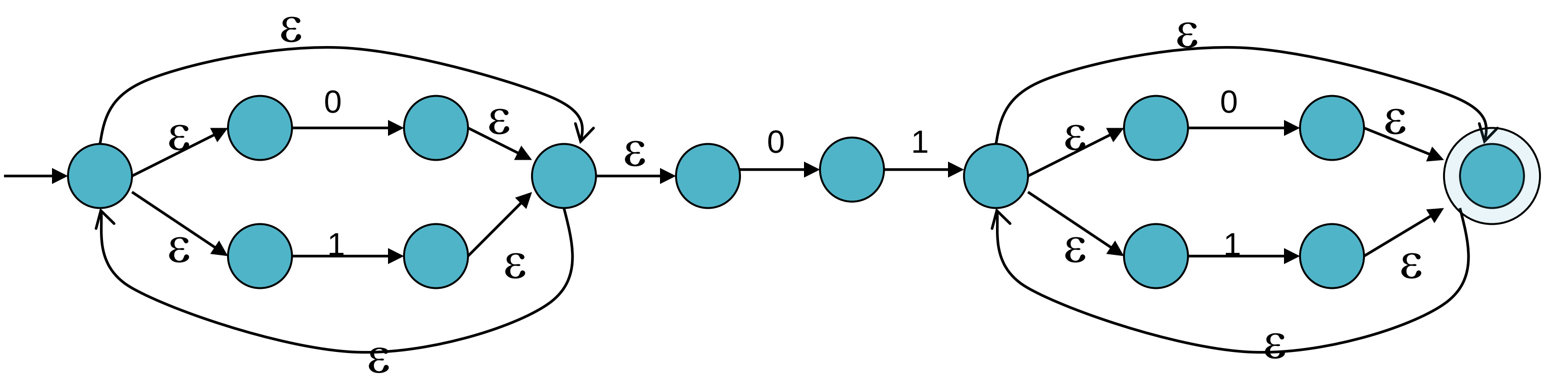
$$1^*00^*1(0+1)^*$$

Q) What is the language?

RE to ε-NFA construction

<u>Example:</u> (0+1)*01(0+1)*

(0+1)* 01 (0+1)*



Algebraic Laws of Regular Expressions

Commutative:

$E+F = F+E$

Associative:

$(E+F)+G = E+(F+G)$

$(EF)G = E(FG)$

Identity:

$E+\Phi = E$

$\varepsilon E = E \varepsilon = E$

Annihilator:

$\Phi E = E\Phi = \Phi$

Algebraic Laws...

<u>Distributive:</u>

$E(F+G) = EF + EG$

$(F+G)E = FE+GE$

<u>Idempotent:</u> $E + E = E$

<u>Involving Kleene closures:</u>

$(E^*)^* = E^*$

$\varepsilon^* = \varepsilon$

$E^+ = EE^*$

$E^* = \varepsilon + EE^*$

$EE^* = E^* E$

$E^* E^* = E^*$

Let R and S be two regular expressions. Then:

1. $((R^*)^*)^* = R^*$

2. $(R+S)^* = (R^* + S^*)^* = (R^* S^*)^*$

3. $(S R)^* S = S(RS)^*$