CS & IT ENGINEERING

Discrete maths
Mathematical logic

Lecture No. 03

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TOPICS TO BE COVERED

01 Inference Rule

02 Type 3 Questions in logic

03 Type 3 with Type 1

04 GATE QUESTIONS on 3 &1

05 Practice

Inference Rule : (Type - 3)

premises, conclusion
prepositional stmt

premises $\longrightarrow$ conclusion OR

True   Check

$$\frac{\text{premises}}{\therefore \text{ conclusion}}$$

take premises as True, check the conclusion $\rightarrow$ True $\rightarrow$ I.R

$\rightarrow$ False $\rightarrow$ ~~I~~R eg: premises $\rightarrow$ conclusion OR $\dfrac{\text{premises.}}{\therefore \text{ conclusion.}}$ $\underline{a} \rightarrow b$

eg2: $\boxed{\overset{\text{True}}{p_1 \wedge p_2 \wedge p_3 \wedge \cdots p_n}} \longrightarrow \dfrac{\text{Check.}}{\text{conclusion}}$

OR

$\rightarrow \underline{p_1}$

$\rightarrow \underline{p_2}$

$\vdots$

$p_n.$

Conclusion

P1: mobile phone it is in left OR Right pocket (True)

P2: it is not in left pocket (True).

Q: it will be in right pocket (True)

P1: $L \vee R$

P2: $\neg L$

Q: R.

$$\Big[\overset{T.\ P_1}{\overline{(L \vee R)}} \wedge \overset{T.\ P_2}{\overline{\neg L}}\Big] \rightarrow \overline{R}$$

L	R	$L \vee R$	$\neg L$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T.

$$[(L \vee R) \wedge \neg L] \rightarrow R.$$

$$\begin{array}{l} L \vee R \\ \neg L \\ \hline \therefore R. \end{array}$$

$$((P \vee Q) \wedge \neg P) \rightarrow Q.$$

$$\begin{array}{l} P \vee Q \\ \neg P \\ \hline \therefore Q. \end{array}$$

$$((P \vee Q) \wedge \neg Q) \rightarrow P.$$

$$\begin{array}{l} P \vee Q \\ \neg Q \\ \hline \therefore P. \end{array} \qquad \begin{array}{l} Q \vee P \\ \neg Q \\ \hline \therefore P. \end{array}$$

$$\Big[\Big(\underbrace{(a \to b)}_{X} \vee (c \to d)\Big) \wedge \neg \underbrace{(c \to d)}_{Y}\Big] \to \underbrace{(a \to b)}_{X}$$

$$\begin{array}{l} L \vee R \\ \neg L \\ \hline \therefore R \end{array}$$

$$\begin{array}{c} (a \to b) \vee (c \to d) \\ \neg (c \to d) \\ \hline (a \to b) \end{array}$$

P_1: if Perfect matching exist then no. of vertices will be even (T)

P_2: P.m exist. (True) 

Q: no. of vertices will be even (True)

P_1: P.M → even.

P_2: p.m.

Q: even.

P_1: $P \to Q$

P_2: P

$\therefore$ Q.

$$\Big[\underbrace{\overbrace{(\underbrace{P \to Q}_{T}) \wedge \underbrace{P}_{T}}^{\text{True}}\Big] \to \overbrace{Q}^{\text{Check}}}_{}$$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

True Check

$$\left[(P \to Q) \wedge P\right] \to Q$$

$$\begin{array}{l} P \to Q \\ P \\ \hline \therefore Q \end{array}$$

P	Q	$P \to Q$	$(P \to Q) \wedge P$	$((P \wedge Q) \wedge P) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

exp → I.R. → satisfy → tautology (valid)

P_1 : True

P_2 : True

⋮

Con :

P_1 : if p.m exist then no of vertices will be even. (True)

P_2 : no. of vertices even.

Q : p.m will exist

$$\left((P \to Q) \land Q\right) \to P$$

$\overset{P}{\text{p.m}} \to \overset{Q}{\text{even}}$ (T) $P \to Q$

even (T) Q

p.m. $\therefore P$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

$$\begin{array}{l} P \vee Q \\ \neg P \\ \hline \therefore Q \end{array} \quad \text{OR} \quad \begin{array}{l} P \vee Q \\ \neg Q \\ \hline \therefore P \end{array} \quad \text{OR} \quad \begin{array}{l} \neg P \vee Q \\ P \\ \hline \therefore Q \end{array} \equiv \begin{array}{l} P \to Q \\ P \\ \hline Q \end{array}$$

$$\begin{array}{l} P \to Q \\ Q \\ \hline \therefore P \end{array} \quad \text{(fallacy)}$$

P1: if it rains then cricket match will not played.

P2: cricket match was played

$$R \rightarrow \neg P \equiv P \rightarrow \neg R$$

$$\frac{P}{\neg R} \qquad \frac{P}{\neg R} \text{ (valid)}$$

There was no rain.

if it rains then cricket match will not be played.

it did not rain.

Cricket match was played

$$R \rightarrow \neg P \equiv P \rightarrow \neg R$$

$$\frac{\neg R}{P} \quad \times \quad \frac{\neg R}{P}$$

1) $P \to Q$
P
$\therefore Q$

OR $((P \to Q) \wedge P) \to Q$

(modus ponens)

Fallacy:

$$\boxed{\begin{array}{c} P \to Q \\ Q \\ \hline \therefore P \end{array}}$$

2) $P \to Q \equiv \neg Q \to \neg P$
$\neg Q$
$\therefore \neg P$

$\neg Q$
$\neg P$

$((P \to Q) \wedge \neg Q) \to \neg P$

(modus tollens)

3) $P \to Q$
$Q \to R$
$P \to R$

$((P \to Q) \wedge (Q \to R)) \to (P \to R)$

(Hypothetical syllogism)

4) $P \vee Q$
$\neg P$
$\therefore Q$

$((P \vee Q) \wedge \neg P) \to Q$

(disjunctive syllogism)

5) $\dfrac{P \wedge Q}{P}$

$P \wedge Q \to P$
OR
$P \wedge Q \to Q$
(simplification)

6) $\dfrac{P}{\therefore P \vee Q}$

$P \to P \vee Q$
(Addition)

(Resolution)

7) $P \vee Q$
$\neg P \vee R$
$Q \vee R$

R

$((P \vee Q) \wedge (\neg P \vee R)) \to Q \vee R$

$$(a \wedge (a \to b) \wedge (\neg b \vee c)) \to c.$$

1: a.

2: $a \to b$ (modus ponens)

$$\underline{\hspace{6em}}$$

b

3. $\neg b \vee c$ (d s)

$$\underline{\hspace{6em}}$$

c.

$$(a \wedge (a \to b) \wedge (\neg b \vee c) \wedge (c \to x)) \to x.$$

1. a.

2. $a \to b$. (m.p)

b

3. $\neg b \vee c$. (D.s)

c.

4. $c \to x$. (m.p)

x

$$
\begin{array}{c}
x \to m \\
a \\
\neg b \vee x \\
a \to b \\
\hline
m
\end{array}
$$

$x \to m$

x

$\neg b \vee x$

$a \to b$

a

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

Establish the validity of the following arguments.

a) $[p \wedge (p \to q) \wedge (\neg q \vee r)] \to r$

b)
$$\begin{array}{l} p \to q \\ \neg q \\ \neg r \\ \hline \therefore \neg(p \vee r) \end{array}$$

$\equiv \neg p \vee q$

$$\begin{array}{c} \neg q \\ \hline \neg p \\ \neg r \end{array}$$

$\neg p \wedge \neg R$

$\neg(p \vee R)$

$p \to q \equiv \neg p \vee q$

$p \to q \equiv \neg q \to \neg p$

c)
$$\begin{array}{l} p \to q \\ r \to \neg q \\ r \\ \hline \therefore \neg p \end{array}$$

$= \neg q \to \neg p$

$$\begin{array}{c} \neg q \\ \hline \therefore \neg p \end{array}$$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?(GATE - 04)

(a) P and Q only

(b) *P* and *R* only

(c) *P* and *S* only

(d) P, Q, R and S

d) $p \wedge q$

$p \rightarrow (r \wedge q)$

$r \rightarrow (s \vee t)$

$\neg s$

$\therefore t$

$p \wedge q$

$\downarrow$

p

$$\frac{2.\ p \rightarrow (r \wedge q)}{r \wedge q} \longrightarrow r$$

$$\frac{3.\ r \rightarrow (s \vee t)}{s \vee t}$$

$$\frac{4.\ \neg s}{t}$$

e) 1. $p \rightarrow (q \rightarrow r)$

2. $p \vee s$

3. $t \rightarrow q$

4. $\neg s$

$\therefore \neg r \rightarrow \neg t$

2. $p \vee s$

4. $\neg s$

p

1. $p \rightarrow (q \rightarrow R)$

$q \rightarrow R \longrightarrow t \rightarrow R \equiv \neg R \rightarrow \neg t$

3. $t \rightarrow q$

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(GATE-15-Set1)

(a) $(\sim p \vee q) \wedge (p \vee \sim q)$

(b) $(\sim p \vee q) \wedge (q \rightarrow p)$

(c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

The Simplest form of $(p \wedge (\sim r \vee q \vee \sim q)) \vee ((r \vee t \vee \sim r) \wedge \sim q)$ is

(a) $p \wedge \sim q$

(b) $p \vee \sim q$

(c) t

(d) $(p \rightarrow \sim q)$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \to s) \wedge (p \vee r)] \to (\sim s \to q)$

Q: $[(\sim p \wedge q) \wedge [q \to (p \to r)]] \to \sim r$

R: $[[(q \wedge r) \to p] \wedge (\sim q \vee p)] \to r$

S: $[p \wedge (p \to r) \wedge (q \vee \sim r)] \to q$

Which of the above arguments are valid?

(GATE - 04)

(a) P and Q only

(b) P and R only

(c) P and S only

(d) P,Q,R and S

The Simplest form of

$(p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)) \wedge ((p \wedge r \wedge t) \vee t)$ is

(a) $p \wedge t$

(b) $q \wedge t$

(c) $p \wedge r$

(d) $p \wedge q$

$S_1: \{(\sim p \rightarrow (q \rightarrow \sim W)) \wedge (\sim S \rightarrow q) \wedge \sim t \wedge (\sim p \vee t)\} \rightarrow (w \rightarrow s)$

$S_2: \{(q \rightarrow t) \wedge (s \rightarrow r) \wedge (\sim q \rightarrow s)\} \rightarrow (\sim t \rightarrow r)$

S_2: $q \rightarrow t \equiv \neg q \vee t$

$s \rightarrow R \equiv \neg s \vee R$

$\neg q \rightarrow s \equiv q \vee s$

$\neg t \rightarrow R$

$t \vee R$

$\neg q \vee t$

$q \vee R$

$t \vee R$

$S_1: \{(\sim p \rightarrow (q \rightarrow \sim W)) \wedge (\sim S \rightarrow q) \wedge \sim t \wedge (\sim p \vee t)\} \rightarrow (w \rightarrow s)$

$S_2: \{(q \rightarrow t) \wedge (s \rightarrow r) \wedge (\sim q \rightarrow s)\} \rightarrow (\sim t \rightarrow r)$

$\neg p \rightarrow (q \rightarrow \neg w)$

$\neg s \rightarrow q$

$\neg t$

$\neg p \vee t$

$\neg p \rightarrow (q \rightarrow \neg w)$

$\neg p$

$q \rightarrow \neg w$

God's Rule:

$\neg s \rightarrow \neg w$

$w \rightarrow s$

The statement formula $\{(a \to c) \wedge (b \to d) \wedge (c \to \sim d)\} \to (\sim a \vee \sim b)$ is

(a) satisfiable but not-valid

(b) valid

(c) not satisfiable

(d) none of these

$a \to c \equiv \neg a \vee c$

$b \to d \equiv \neg b \vee d$

$c \to \neg d \equiv \neg c \vee \neg d$

$\neg a \vee \neg b$

$\neg a \vee c$

$\neg b \vee \neg c$

$\neg a \vee \neg b$

(i)
$$\begin{array}{l} p \vee (q \wedge r) \\ \underline{p \rightarrow s} \\ \therefore r \vee s \end{array}$$

(ii)
$$\begin{array}{l} p \\ \underline{p \leftrightarrow q} \\ \therefore q \end{array}$$

(iii)
$$\begin{array}{l} p \vee q \\ p \rightarrow r \\ \underline{r \rightarrow s} \\ \therefore q \vee s \end{array}$$

(iv)
$$\begin{array}{l} \neg p \vee q \vee r \\ \neg q \\ \underline{\neg r} \\ \therefore \neg p \end{array}$$

(v)
$$\begin{array}{l} \neg p \vee s \\ \neg t \vee (s \wedge r) \\ \neg q \vee r \\ \underline{p \vee q \vee t} \\ \therefore r \vee s \end{array}$$

$$\begin{array}{l} p \\ p \vee q \\ q \rightarrow (r \rightarrow s) \\ \underline{t \rightarrow r} \\ \therefore \neg s \rightarrow \neg t \end{array}$$

$$\begin{array}{l} u \rightarrow r \\ (r \wedge s) \rightarrow (p \vee t) \\ q \rightarrow (u \wedge s) \\ \underline{\neg t} \\ \therefore q \rightarrow p \end{array}$$

c) $\begin{array}{l} p \rightarrow q \\ \neg q \\ \underline{\neg r} \\ \therefore \neg(p \vee r) \end{array}$

d) $\begin{array}{l} p \rightarrow q \\ r \rightarrow \neg q \\ \underline{r} \\ \therefore \neg p \end{array}$

e) $\begin{array}{l} p \rightarrow (q \rightarrow r) \\ \neg q \rightarrow \neg p \\ \underline{p} \\ \therefore r \end{array}$

f) $\begin{array}{l} p \wedge q \\ p \rightarrow (r \wedge q) \\ r \rightarrow (s \vee t) \\ \underline{\neg s} \\ \therefore t \end{array}$

g) $\begin{array}{l} p \rightarrow (q \rightarrow r) \\ p \vee s \\ t \rightarrow q \\ \underline{\neg s} \\ \therefore \neg r \rightarrow \neg t \end{array}$

h) $\begin{array}{l} p \vee q \\ \neg p \vee r \\ \underline{\neg r} \\ \therefore q \end{array}$

a) $[(p \wedge \neg q) \wedge [p \rightarrow (q \rightarrow r)]] \rightarrow \neg r$

b) $[[(p \wedge q) \rightarrow r] \wedge (\neg q \vee r)] \rightarrow p$

c) $\begin{array}{l} p \leftrightarrow q \\ q \rightarrow r \\ r \vee \neg s \\ \underline{\neg s \rightarrow q} \\ \therefore s \end{array}$

d) $\begin{array}{l} p \\ p \rightarrow r \\ p \rightarrow (q \vee \neg r) \\ \underline{\neg q \vee \neg s} \\ \therefore s \end{array}$

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$$
\begin{array}{l}
p \to q \\
q \to (r \wedge s) \\
\neg r \vee (\neg t \vee u) \\
p \wedge t \\
\hline
\therefore u
\end{array}
$$

(d) none of these

(i)
$$
\begin{array}{l}
p \vee (q \wedge r) \\
p \to s \\
\hline
\therefore r \vee s
\end{array}
$$

(ii)
$$
\begin{array}{l}
p \\
p \leftrightarrow q \\
\hline
\therefore q
\end{array}
$$

(iii)
$$
\begin{array}{l}
p \vee q \\
p \to r \\
r \to s \\
\hline
\therefore q \vee s
\end{array}
$$

(iv)
$$
\begin{array}{l}
\neg p \vee q \vee r \\
\neg q \\
\neg r \\
\hline
\therefore \neg p
\end{array}
$$

(v)
$$
\begin{array}{l}
\neg p \vee s \\
\neg t \vee (s \wedge r) \\
\neg q \vee r \\
p \vee q \vee t \\
\hline
\therefore r \vee s
\end{array}
$$

$$
\begin{array}{l}
u \to r \\
(r \wedge s) \to (p \vee t) \\
q \to (u \wedge s) \\
\neg t \\
\hline
\therefore q \to p
\end{array}
$$

$$
\begin{array}{l}
p \\
p \vee q \\
q \to (r \to s) \\
t \to r \\
\hline
\therefore \neg s \to \neg t
\end{array}
$$

$$
\begin{array}{l}
p \\
p \vee q \\
q \rightarrow (r \rightarrow s) \\
t \rightarrow r \\
\hline
\therefore \neg s \rightarrow \neg t
\end{array}
$$

$s \wedge \sim u) \wedge ($

c)
$$
\begin{array}{l}
p \rightarrow q \\
\neg q \\
\neg r \\
\hline
\therefore \neg(p \vee r)
\end{array}
$$

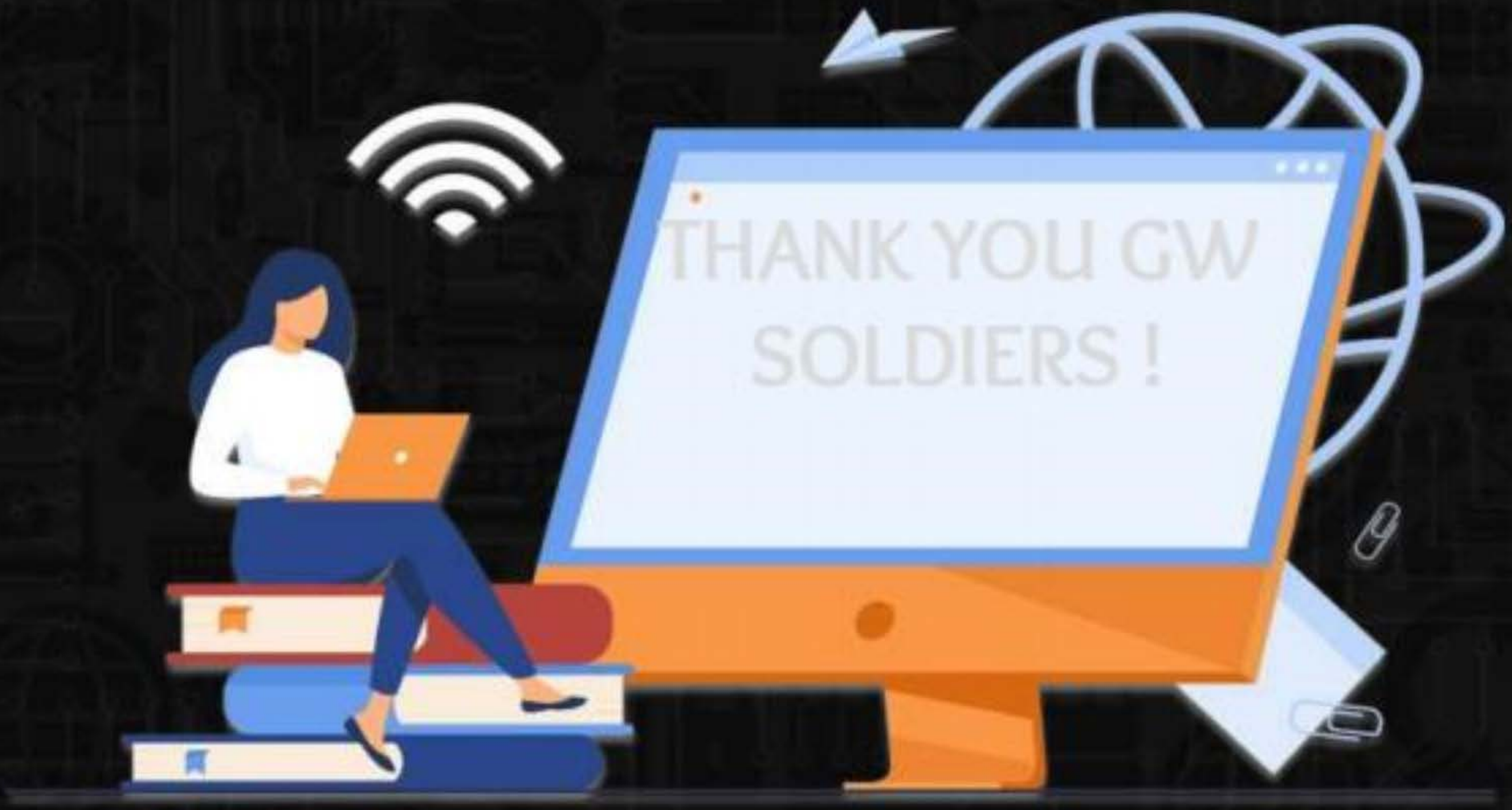
d)
$$
\begin{array}{l}
p \rightarrow q \\
r \rightarrow \neg q \\
r \\
\hline
\therefore \neg p
\end{array}
$$

e)
$$
\begin{array}{l}
p \rightarrow (q \rightarrow r) \\
\neg q \rightarrow \neg p \\
p \\
\hline
\therefore r
\end{array}
$$

f)
$$
\begin{array}{l}
p \wedge q \\
p \rightarrow (r \wedge q) \\
r \rightarrow (s \vee t) \\
\neg s \\
\hline
\therefore t
\end{array}
$$

g)
$$
\begin{array}{l}
p \rightarrow (q \rightarrow r) \\
p \vee s \\
t \rightarrow q \\
\neg s \\
\hline
\therefore \neg r \rightarrow \neg t
\end{array}
$$

h)
$$
\begin{array}{l}
p \vee q \\
\neg p \vee r \\
\neg r \\
\hline
\therefore q
\end{array}
$$


THANK YOU GW
SOLDIERS !