CS & IT ENGINEERING

Discrete Maths
Graph Theory

Lecture No. 15

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01 Matching number

02 Perfect Matching

03 planarity

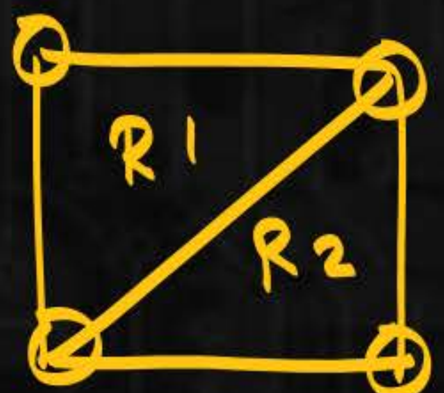
04 Eulers Formula

05 Practice

Graph Theory

$\Sigma d(R_i) = 2e.$

each Region is made up of atleast 3 edges.



$$\left\{\begin{array}{l} deg(R_1) = 3 \\ deg(R_2) = 3 \\ deg(R_3) = 4 \end{array}\right\} \longrightarrow \begin{array}{l} deg(R_1) \geq 3 \\ deg(R_2) \geq 3 \\ deg(R_3) \geq 3. \end{array}$$

$$\deg(R_1)+\deg(R_2)+\deg(R_3) \geq 3+3+3$$

$\sum d(R_i) \geq 3 \cdot 3 \rightarrow$ no of Regions/faces

$$\boxed{2e \geq 3f}$$

$n-e+f=2$

$f = 2+e-n$

$2e \geq 3(2+e-n)$

$2e \geq 6+3e-3n$

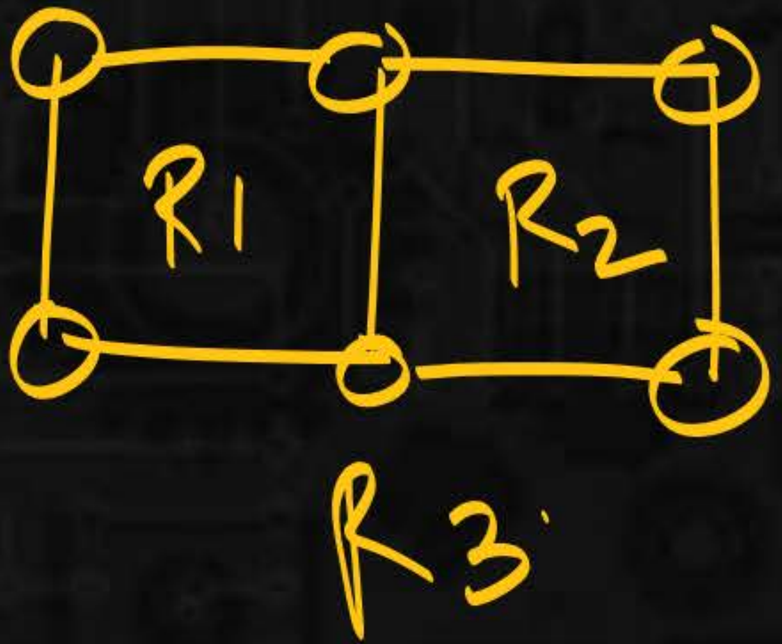
$3n-6 \geq 3e-2e$

$3n-6 \geq e$

Thm: if G is planar then $e \leq 3n-6$

Graph Theory

Region is madeup of atleast 4 edges.
$(n \geq 4)$



$\deg(R_1) \geq 4$

$\deg(R_2) \geq 4$

$\deg(R_3) \geq 4$

$\deg(R_1) + \deg(R_2) + \deg(R_3) \geq 4 + 4 + 4$

$\sum d(R_i) \geq 4 \cdot 3 \rightarrow$ no. of $\frac{\text{region}}{\text{faces}}$

$$\boxed{2e \geq 4f}$$

Graph Theory

$2e \geq 4f$ $\quad n - e + f = 2$

$f = 2 + e - n$

$2e \geq 4(2+e-n)$

$2e \geq 8 + 4e - 4n$

$4n - 8 \geq 4e - 2e$

$4n - 8 \geq 2e$

$2n - 4 \geq e$

<u>Thm</u>: if G is planar then $e \leq 2n-4$.

$2e \geq 5f$

$2e \geq 5(2+e-n)$

$2e \geq 10+5e-5n$

$5n-10 \geq 5e-2e$

$5n-10 \geq 3e$

$3e \leq 5n-10$

$e \leq \left(\frac{5}{3}\right)n - \left(\frac{10}{3}\right)$

Degree of each region is made up of atleast 5 edges.

$n-e+f=2$

$f=2+e-n$

Thm:

if G is planar then $e \leq \left(\frac{5}{3}\right)n - \left(\frac{10}{3}\right)$

Graph Theory

if G is planar then $e \leq 3n-6$.

$\delta(G) \leq 2e/n \leq \Delta(G) \leq n-1$.

Connected
+
planar
Simple

Connected
discon
Simple.
planar
non planar

Simple.
Connected.
planar

Graph Theory

$e \leq 3n-6$

$\delta(G) \leq \frac{2e}{n} \leq \Delta(G) \leq n-1$

$\delta(G) \leq \frac{2e}{n}$

$\delta(G) \leq \frac{2(3n-6)}{n} \leq \frac{6n-12}{n} \leq \frac{6n}{n} - \frac{12}{n} \leq 6 - \frac{12}{n}$

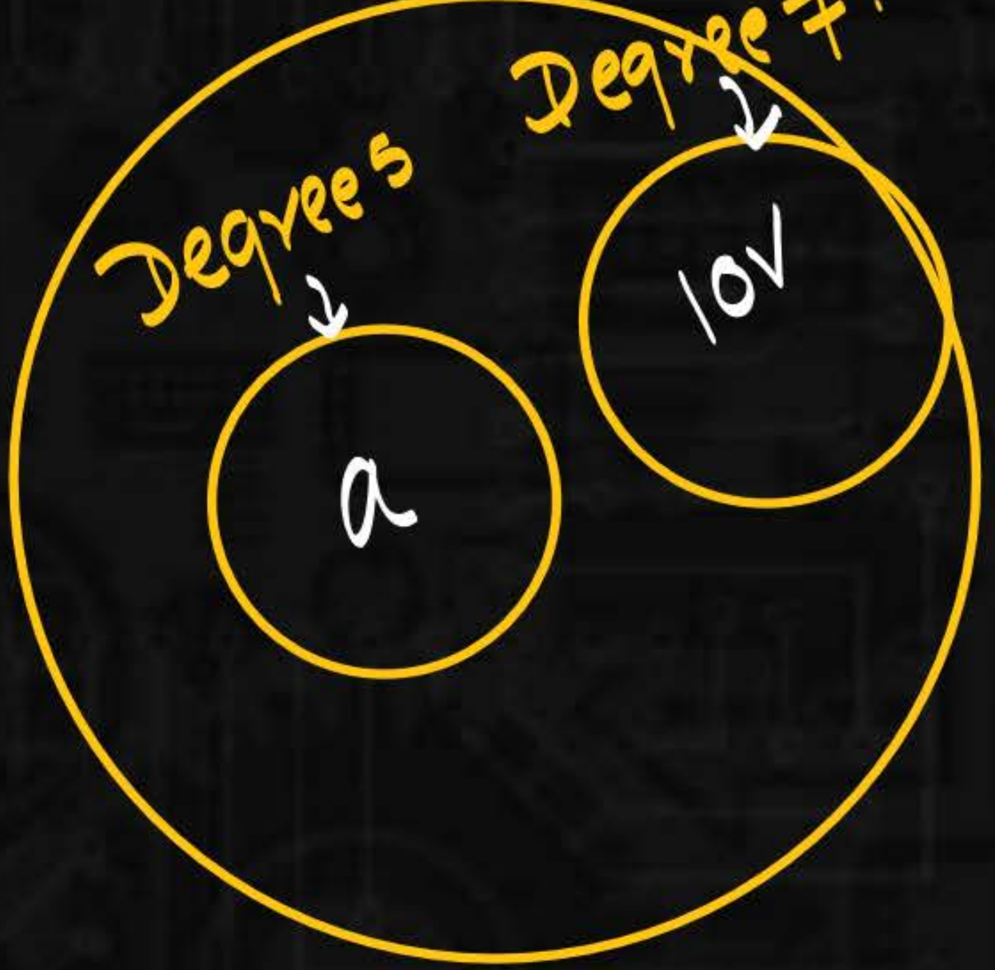
$\delta(G) \leq 6 - \frac{12}{n}$ $\left(\delta(G) \leq 5 \right)$ $\begin{cases} \text{connected} \\ \text{planar} \\ \text{simple} \end{cases}$

Thm: if G is connected planar Graph then $\delta(G) \leq 5$.

Graph Theory

A planar Graph has vertices degree 5 & 7. if there are 10 vertices of degree 7. prove that atleast 22 vertices of degree 5.

Total vertices $= 10 + a$.



$\sum d(v_i) = 2e$

$5 \cdot a + 10 \cdot 7 = 2e$

$e = \frac{5}{2}a + 35$

$22 \le a$

$e \le 3n-6$

$\frac{5}{2}a + 35 \le 3(10+a) - 6$

$\frac{5}{2}a + 35 \le 30 + 3a - 6$

$35 - 30 + 6 \le 3a - \frac{5}{2}a$

$11 \le \frac{1}{2}a$

Graph Theory

$\delta(G) \leq \frac{2e}{n}$

$3 \leq \frac{2e}{n}$ $\quad 3n \leq 2e$

$\delta \to$ minimum degree $(\delta \geq 3)$

$n - e + f = 2$

for all the planar of n vertices.

f $\quad$ n

a) In any planar embedding, the no. of faces is at least $\frac{n}{2} + 2$.

$n - e + f = 2$

$n + f - 2 = e$.

less than $\frac{n}{2} + 2$

$$3n \leq 2e. \qquad n - e + f = 2.$$

$$3n \leq 2(n + f - 2) \qquad n + f - 2 = e.$$

$$3n \leq 2n + 2f - 4.$$

$$3n - 2n + 4 \leq 2f$$

$$n + 4 \leq 2f$$

$$\frac{n}{2} + 2 \leq f$$

Subgraph:

Subgraph.

edge disjoint subgraph.

vertex disjoint subgraph.

induced subgraph.

Spanning subgraph (Spanning Tree)

Clique.

Graph Theory

Subgraph ($\subseteq$)

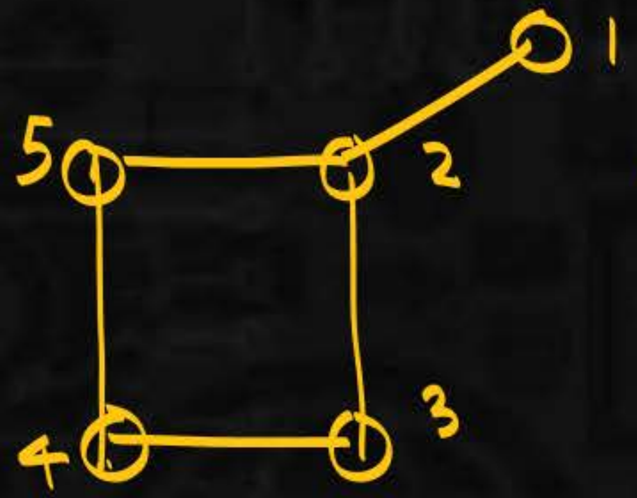
G_1 is subgraph of G.

$\phi \neq V_1 \subseteq V$ $(V \subseteq V)$

$E_1 \subseteq E$. $(E \subseteq E)$

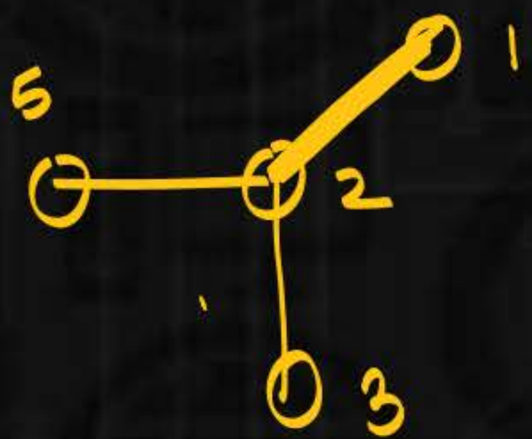
each of edge E_1 is indicent on G_1 is same as in G.

$G = (V, E)$



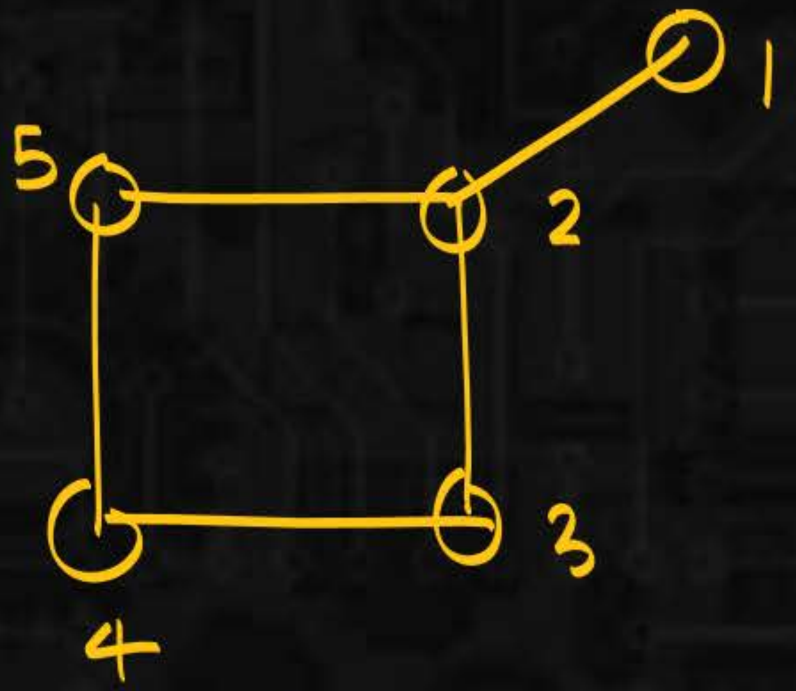
$V = \{1, 2, 3, 4, 5\}$

$G_1 = (V_1, E_1)$ Subgraph :



$V = \{1, 2, 3, 5\}$

Graph Theory



$G = (V, E)$

$G_2 = (V_2, E_2)$

Graph Theory

1) Every Graph is subgraph of itself.

2) $G_2 \subseteq G_1 \subseteq G$. $G_2 \subseteq G$.

Subgraph of a subgraph of a Graph is subgraph of a Graph.

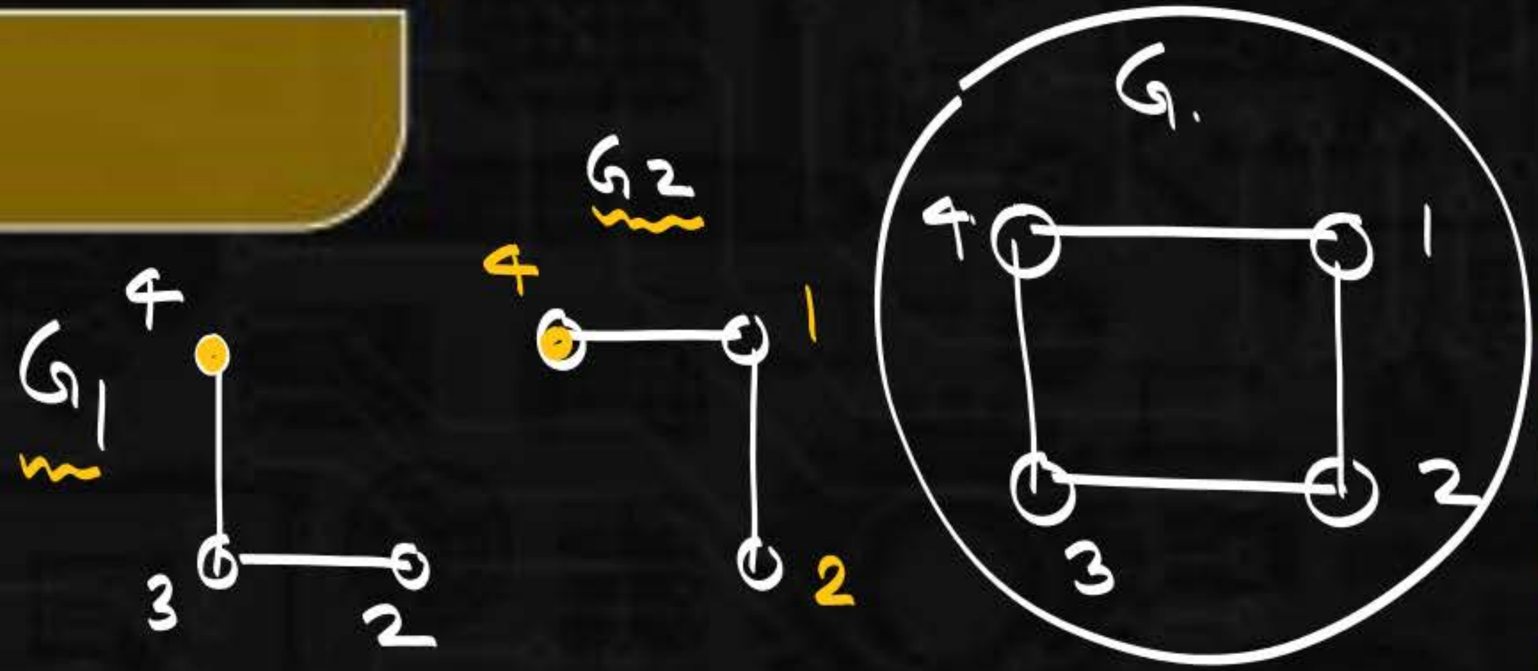
3) Every single vertex of a Graph is subgraph of a Graph.

4) Every single edge of a Graph is subgraph of a Graph.

Graph Theory

edge disjoint subgraph

→ Two graphs which does not have any common edges.

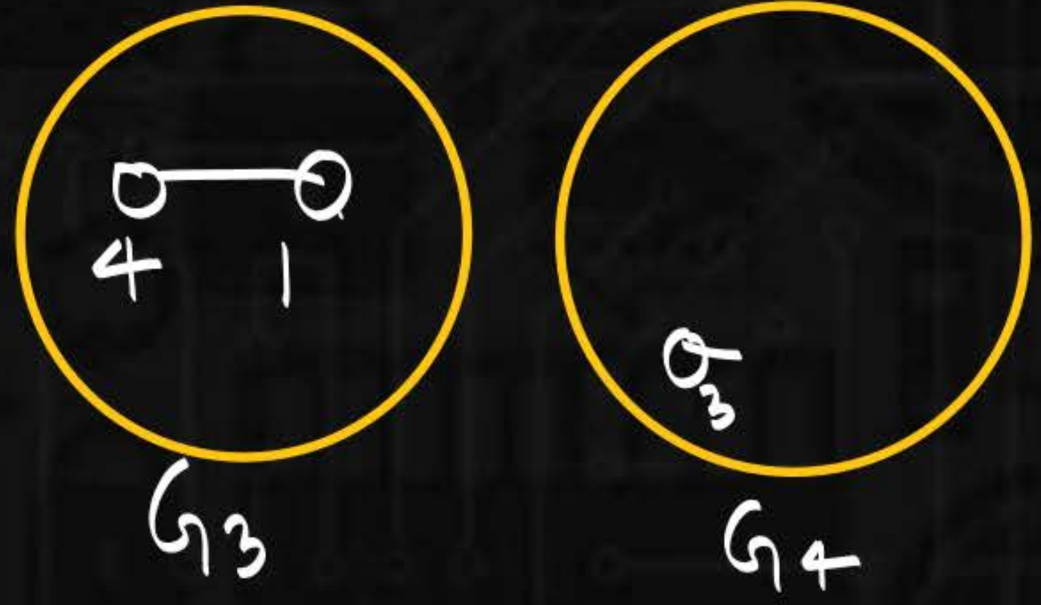


$E_1 = \{43, 32\}$ $E_2 = \{14, 12\}$

(we may get common vertex)

vertex disjoint subgraph:

two graphs, which does not have any common vertex



(it will not have any common vertex so no point of having common edge)

Graph Theory

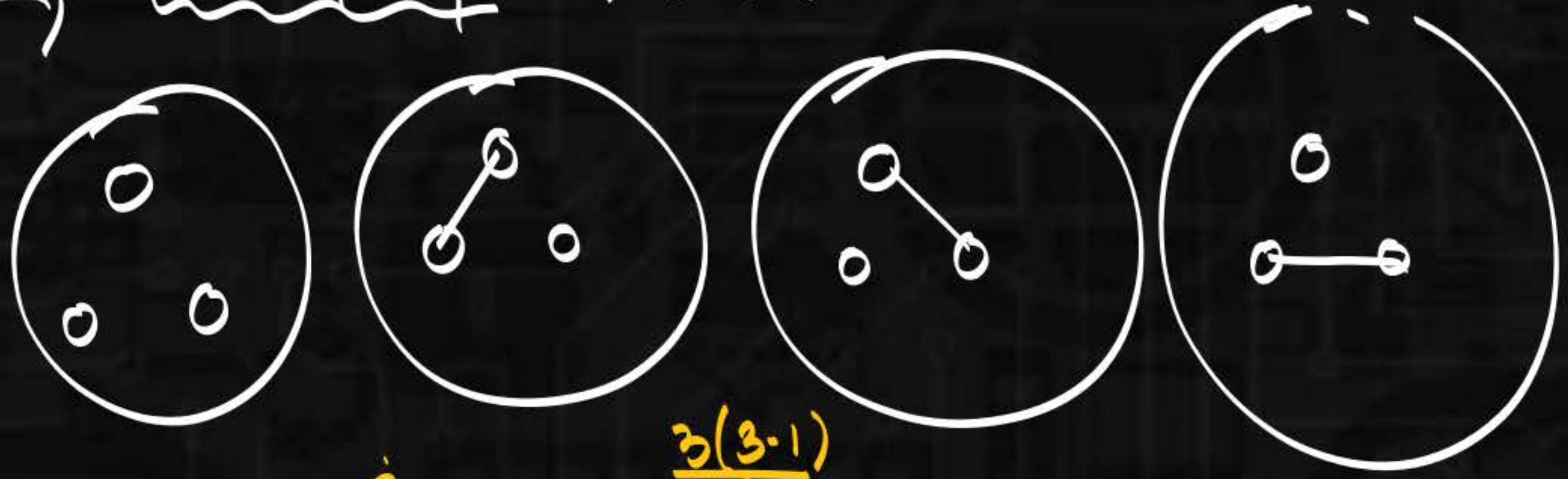
How many subgraph of K_6 satisfy $|V| = 3$.

$2^{\frac{3(3-1)}{2}}$

no. of subgraphs with 3vertices.

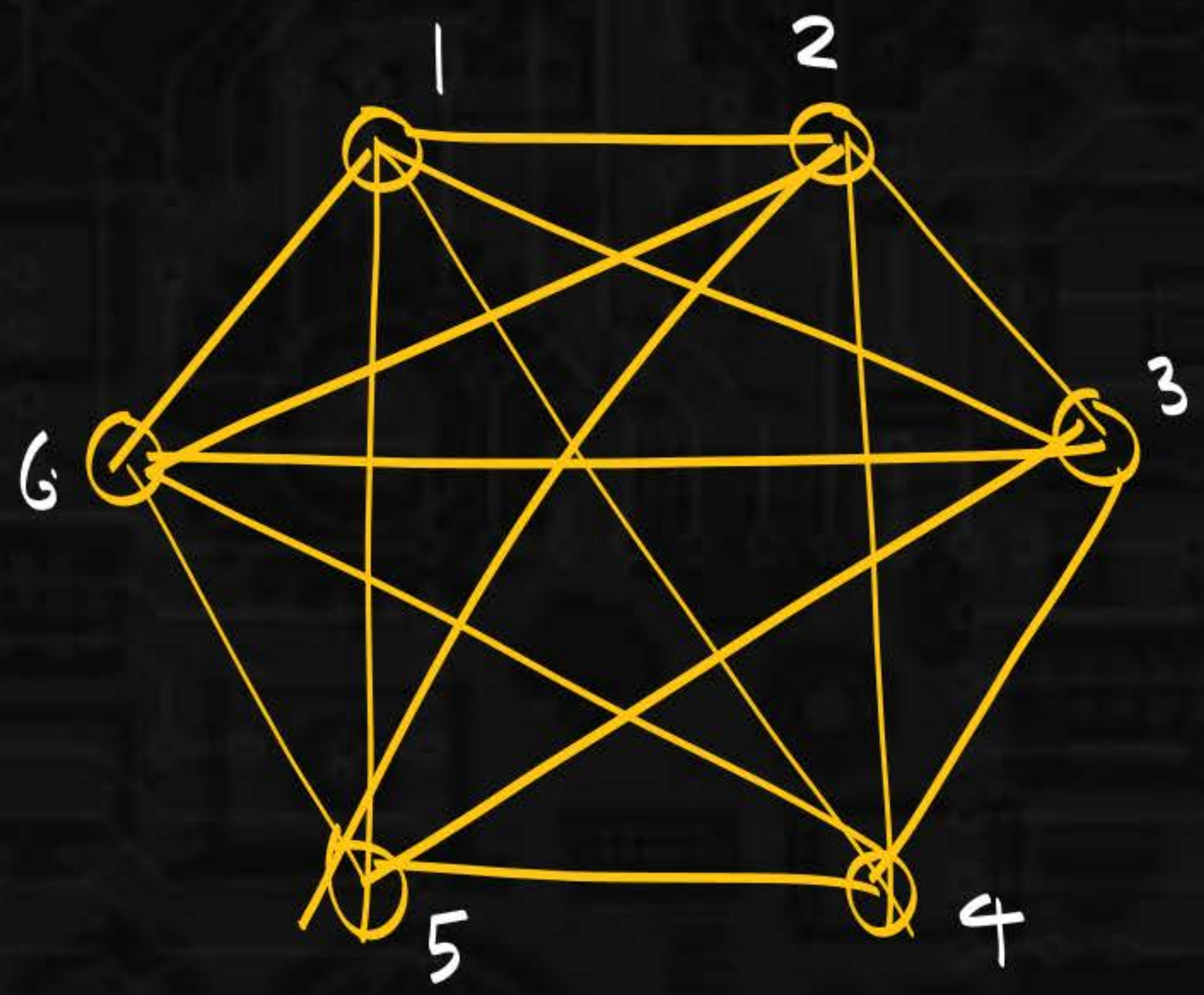
K_6.

subgraph → 3vertices.



$^{6}C_{3} \times 2^{\frac{3(3-1)}{2}}$

Graph Theory



$V = \{1, 2, 3, 4, 5, 6\}$.

In a class of 6 students, how many ways we can select 3 students.

$$^6C_3 \times 2^{\frac{3(3-1)}{2}}$$

How many subgraphs are possible with K_6 ?.

previous que

1 vertex   2 vertices   3 vertices   4 vertices   5 vertices   6 vertices.

$$6_{C_1} \times 2^{\frac{1(1-1)}{2}} + 6_{C_2} \times 2^{\frac{2(2-1)}{2}} + 6_{C_3} \times 2^{\frac{3(3-1)}{2}} + 6_{C_4} \times 2^{\frac{4(4-1)}{2}} + 6_{C_5} \times 2^{\frac{5(5-1)}{2}} + 6_{C_6} 2^{\frac{6(6-1)}{2}}$$

$$2^{15}$$

$$\sum_{i=1}^{n} n_{C_i} \times 2^{\frac{i(i-1)}{2}}$$

$$\sum_{i=1}^{6} 6_{C_i} \times 2^{i(i-1)/2}$$

Graph Theory

How many subgraph of K6 with 1 vertex

——————— " ——————— 2 vertex.

——————— " ——————— 3 vertex $\rightarrow {}^6C_3 \times 2^{\frac{3(3-1)}{2}}$

——————— " ——————— 4 vertex.

——————— " ——————— 5 vertex

——————— " ——————— 6 vertex.

Graph Theory

Spanning subgraph:

G_1 is spanning subgraph of G.

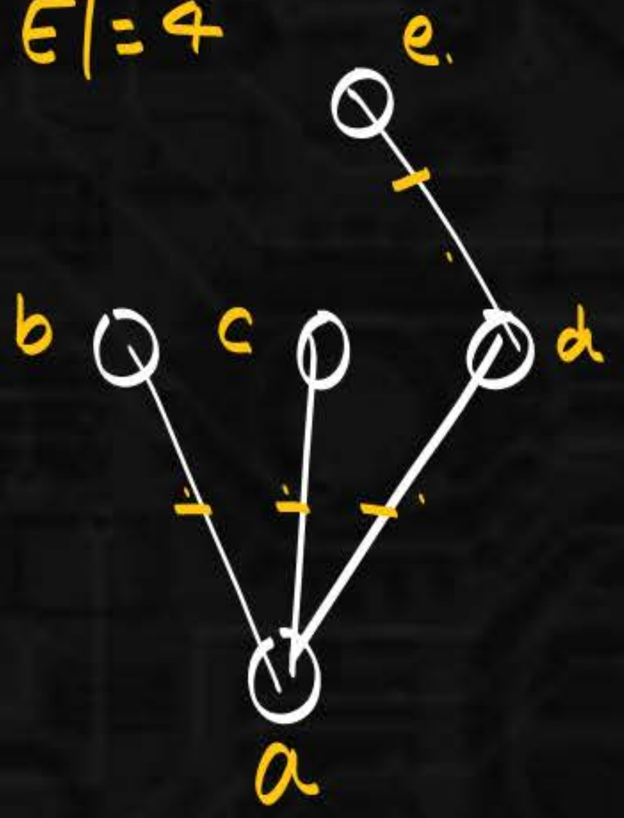
$G_1 = (V_1, E_1)$ $\quad$ $G = (V, E)$

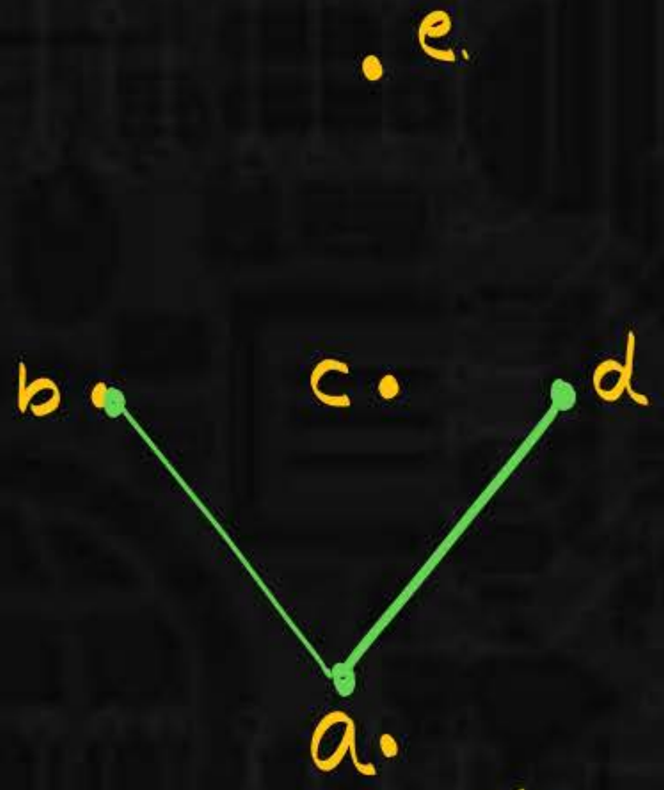
1) Subgraph.
2) $\boxed{V_1 = V}$

$(E_1 \subseteq E)$

$(V_1 = V)$

$|E| = 4$

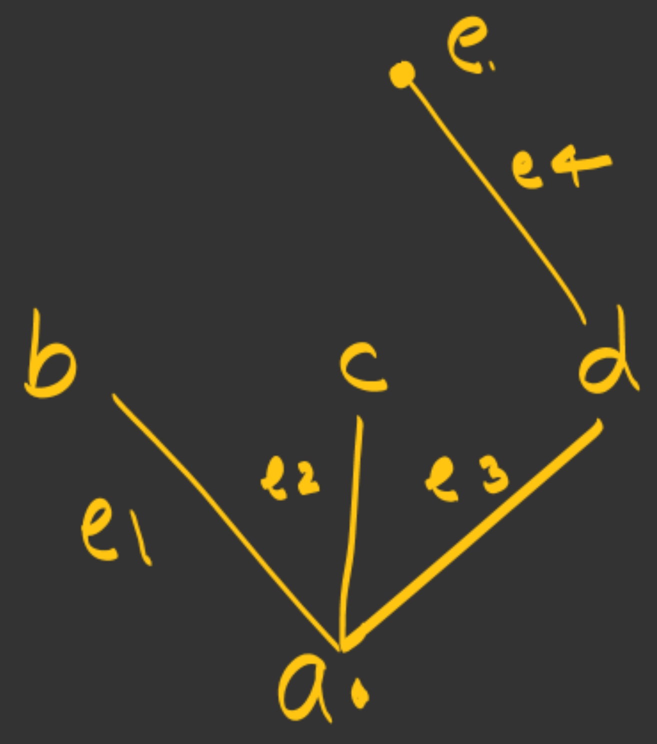




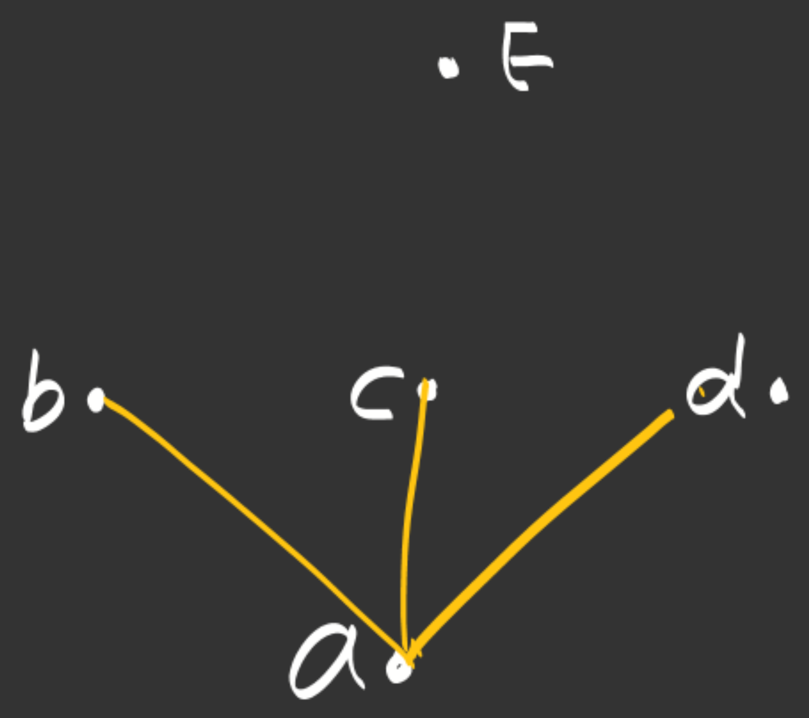
not spanning subgraph.

2^4

(spanning subgraphs are possible)



$|E| = 4$



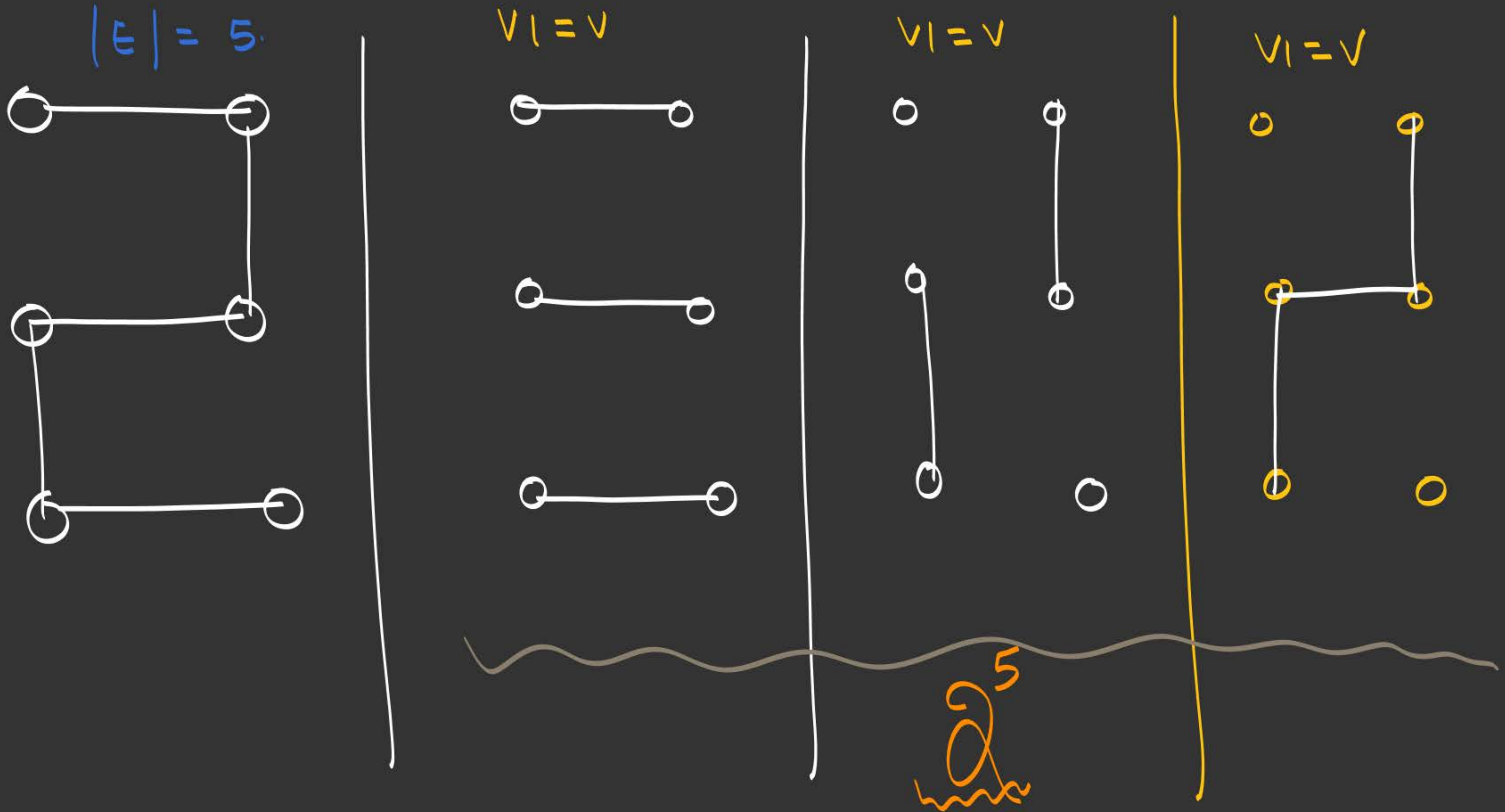
all vertices must be present.

edges may or may no.

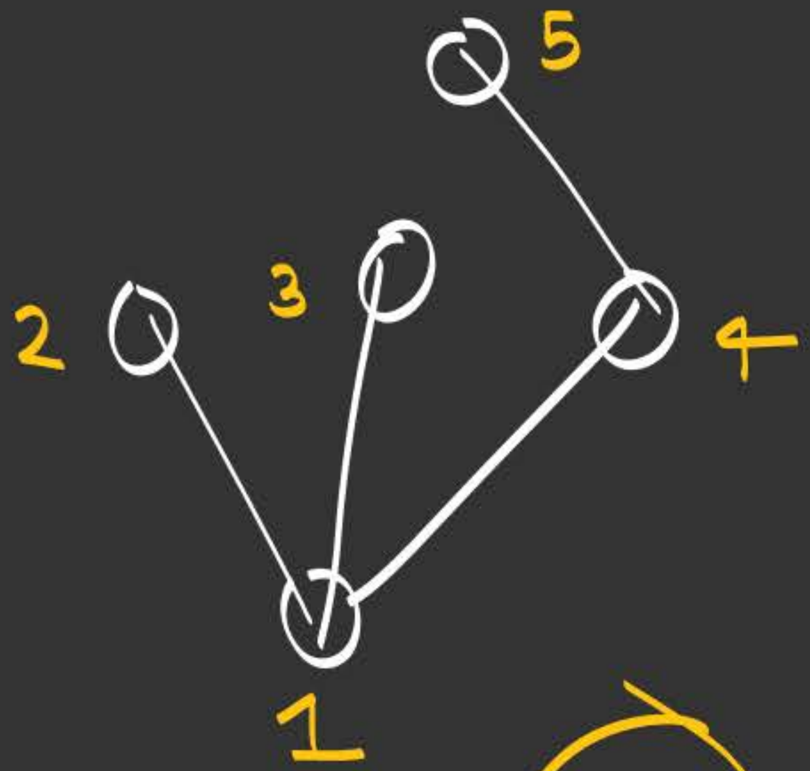
2 choices.

$e_1 \; e_2 \; e_3 \; e_4$

2^4

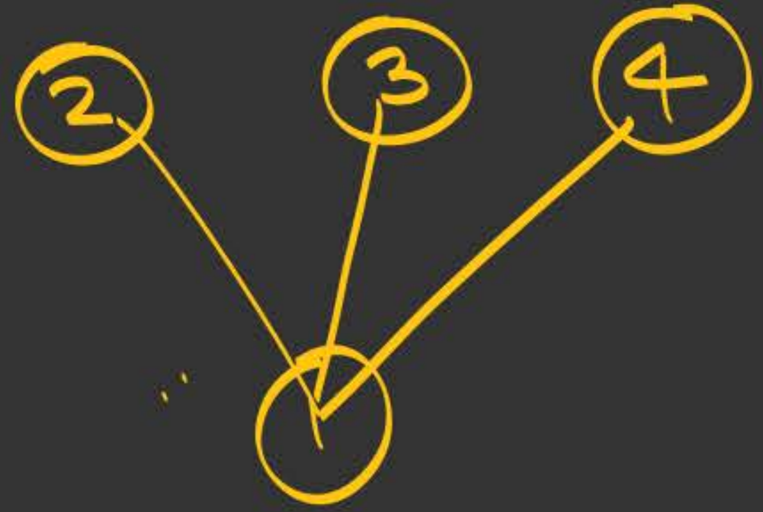

$|E| = 5$
$V' = V$
$V' = V$
$V' = V$
2^5

induced subgraph:

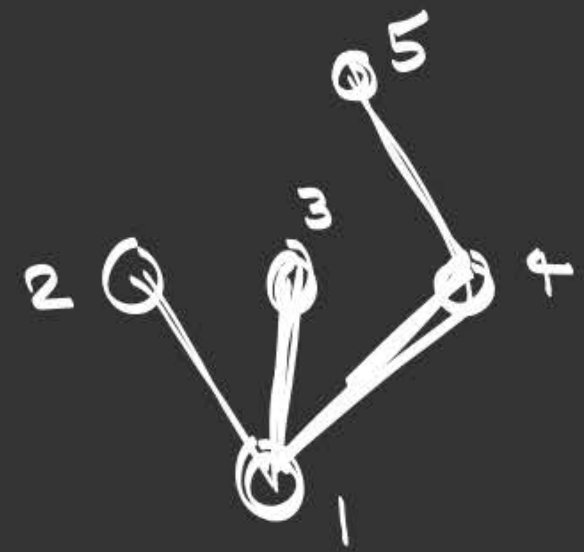


2^V

$V_1 = \{1, 2, 3, 4\}$



if we take any vertex set.
all edges related.
to those vertices
must be present.



2^V

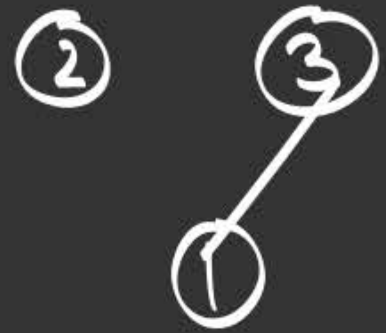
$V_3 = \{1, 4, 5\}$

$V_1 = \{1, 2, 3\}$.

induced subgraph.

✓

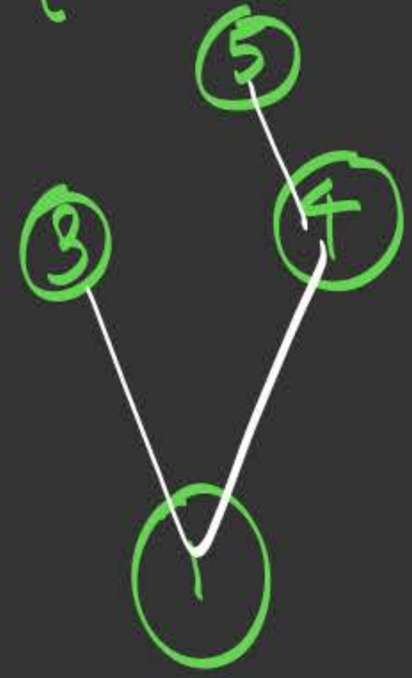
$V_1 = \{1, 2, 3\}$

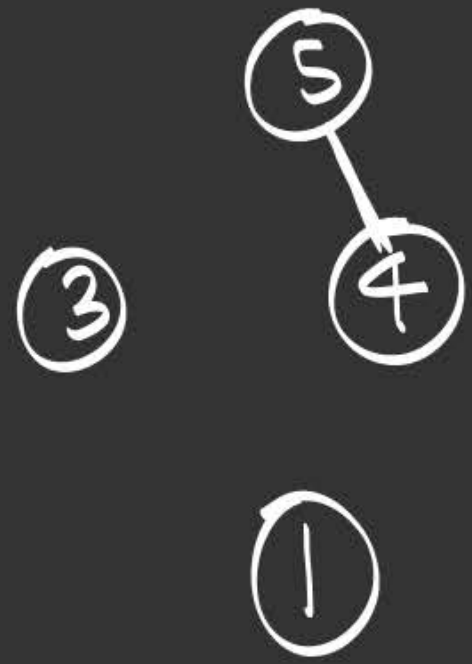


not induced subgraph.

$\{12\}$ is missing.

$V_4 = \{1, 3, 4, 5\}$





not induced.

G be a graph of 10v & 15 edges.

\# of induced subgraph $\rightarrow \underbrace{2^{10}}$

\# of spanning subgraph $\rightarrow 2^{15}$


THANK YOU GW
SOLDIERS !