CS & IT ENGINEERING

Discrete Mathematics

Mathematical Logic

DPP 06 Discussion notes

SATISH YADAV SIR

01 Question

02 Discussion

Q.1 Consider a function, $P(x, y, z) = x + y + z = 15$ and domain = z, then which of the following is correct?

[MCQ]

A. $\forall x \exists y \exists z\ P(x, y, z)$ ✓

B. $\exists z \forall x \forall y\ P(x, y, z)$

C. $\forall x \exists z \forall y\ P(x, y, z)$

D. $\exists z\ \exists y \forall x\ P(x, y, z)$

$x + y + z = 15$

$\forall x \quad \exists y \quad \exists z$

$y + z = 15 - x$

$y + z = 16 - 1$

$= 14$

$y + z = 14$

Q.2 Consider an asymmetric function $P(x, y) = x^2 + y^2 = 10$ on domain integer, then which of the following is correct? [MCQ]

A. $\exists x \exists y\ P(x, y)$

B. $\forall x \exists y\ P(x, y)$

C. $\forall y \exists x\ P(x, y)$

D. None of these

$x^2 + y^2 = 10$

$x = 1 \quad y = 3$

$1^2 + 3^2 = 10$

Q.3 Which of the following is/ are negation of [MSQ]

$[\forall x \exists y \forall z (P(x, y, z) \oplus Q(x, y, z))]$

A. $\exists x \forall y \exists z\ (\sim P(x, y, z) \oplus \sim Q(x, y, z))$ ✗

B. $\exists x \forall y \exists z\ (P(x, y, z) \Rightarrow \sim Q(x, y, z))$ ✗

C. $\exists x \forall y \exists z\ (P(x, y, z) \Leftrightarrow Q(x, y, z))$ ✓

D. $\exists x \forall y \exists z\ (\sim P(x, y, z) \Leftrightarrow \sim Q(x, y, z))$ ✓

P	q	$P \oplus q$	$\neg A$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

Q.4 [NAT]

Consider the following logical expressions

(a) $\forall x \forall y\, P(x, y) \leftrightarrow \exists y \forall x\, P(x, y)$ (false)

(b) $[\forall x P(x)] \vee Q \leftrightarrow \forall x [P(x) \vee Q]$ (valid)

(c) $\forall x [P(x) \wedge Q] \leftrightarrow [\forall x\, P(x)] \wedge Q$ (valid) / Invalid

(d) $\exists x [P(x) \vee Q] \leftrightarrow [\exists x\, P(x)] \wedge Q$

Total invalid expressions are ②

b) $\forall x P(x) \vee Q$ $\quad \forall x (P(x) \vee Q)$

$\left(\begin{matrix} P_1 \\ \wedge \\ P_2 \end{matrix}\right) \vee Q$ $\quad (P_1 \vee Q) \wedge (P_2 \vee Q)$

$(P_1 \wedge P_2) \vee Q$

c) $\forall x (P(x) \wedge Q) \leftrightarrow (\forall x P(x)) \wedge Q$

$(P_1 \wedge Q) \wedge (P_2 \wedge Q)$ $\quad \left(\begin{matrix} P_1 \\ \wedge \\ P_2 \end{matrix}\right) \wedge Q$

$(P_1 \wedge P_2) \wedge Q$

d) $\exists x (P(x) \vee Q)$ $\quad (\exists x P(x)) \vee Q$

$(P_1 \vee Q)$

$\vee$

$(P_2 \vee Q)$

$\left(\begin{matrix} P_1 \\ \vee \\ P_2 \end{matrix}\right) \wedge Q$

Q.5

[MCQ]

Consider the following statements

S_1: There is someone who is loved by everyone.

S_2: Every real number has its corresponding negative.

Here L(x, y) denotes "x loves y"

P(x, y) denotes "x + y = 0"

Which of the following represent the correct predicate logic of the given statement?

A. $S_1: \exists x \forall y\ L(x, y)$, $S_2: \exists y \forall x\ p(x, y)$

B. $S_1: \forall x \exists y\ L(x, y)$, $S_2: \forall x \forall y\ p(x, y)$

C. $S_1: \exists y \forall x\ L(x, y)$, $S_2: \forall x \exists y\ p(x, y)$ (True)

D. None of these.


THANK YOU GW
SOLDIERS !