CS & IT ENGINEERING

Discrete Maths
Graph theory

Lecture No. 08

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TOPICS TO BE COVERED

01 Annalysis In Connectivity

02 Various definition in Connectivity

03 Edge Connectivity

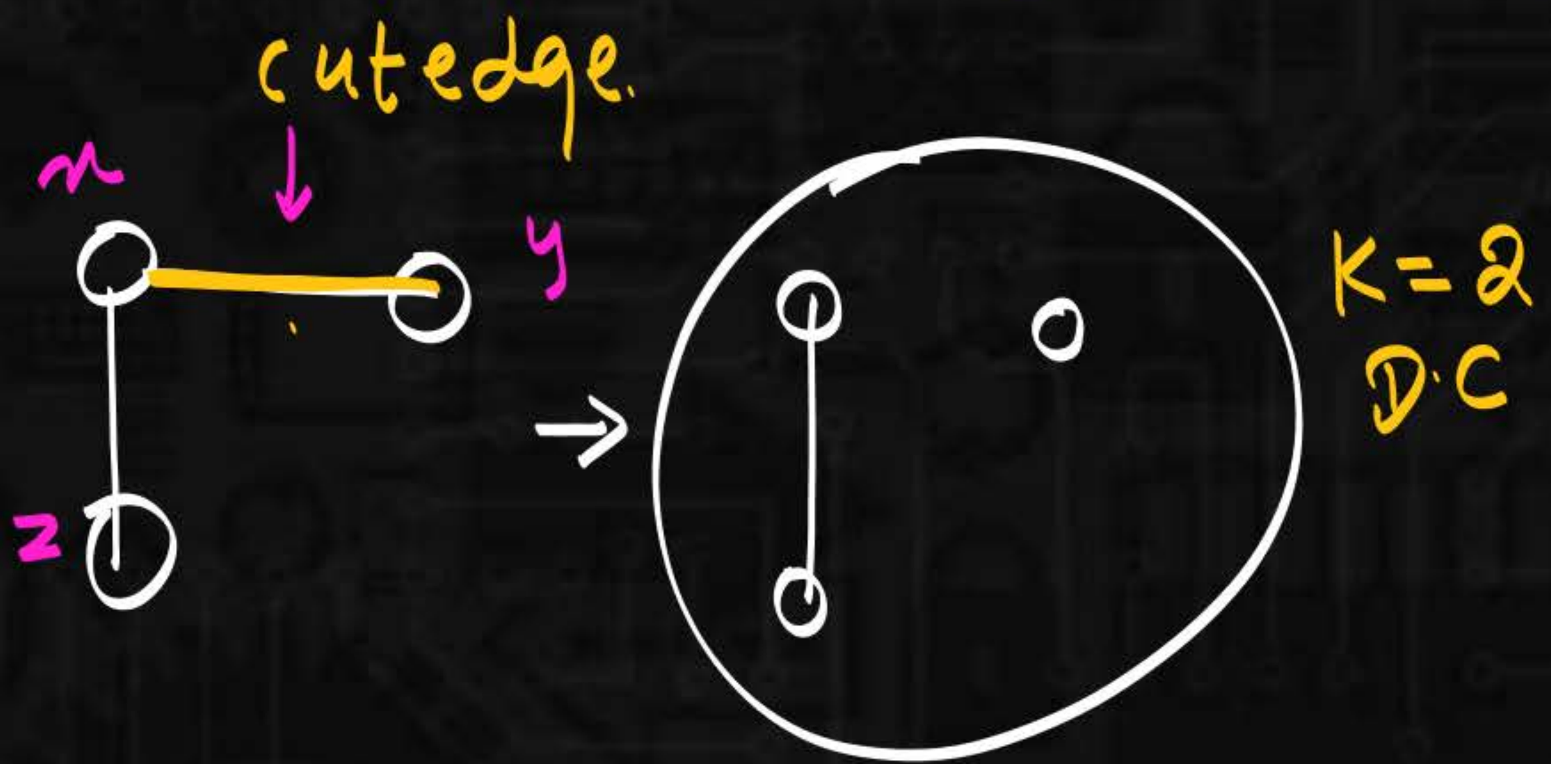
04 Vertex Connectivity

05 Largest inequality theorem

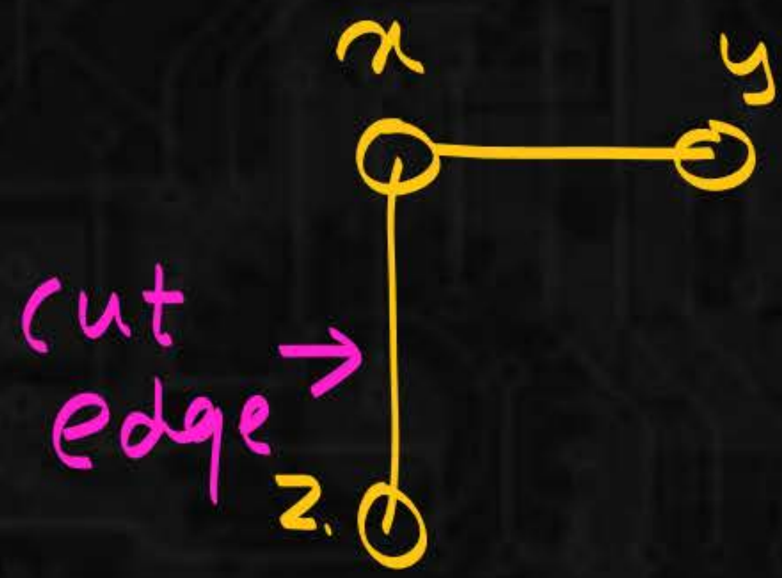
Connectivity in Graphs

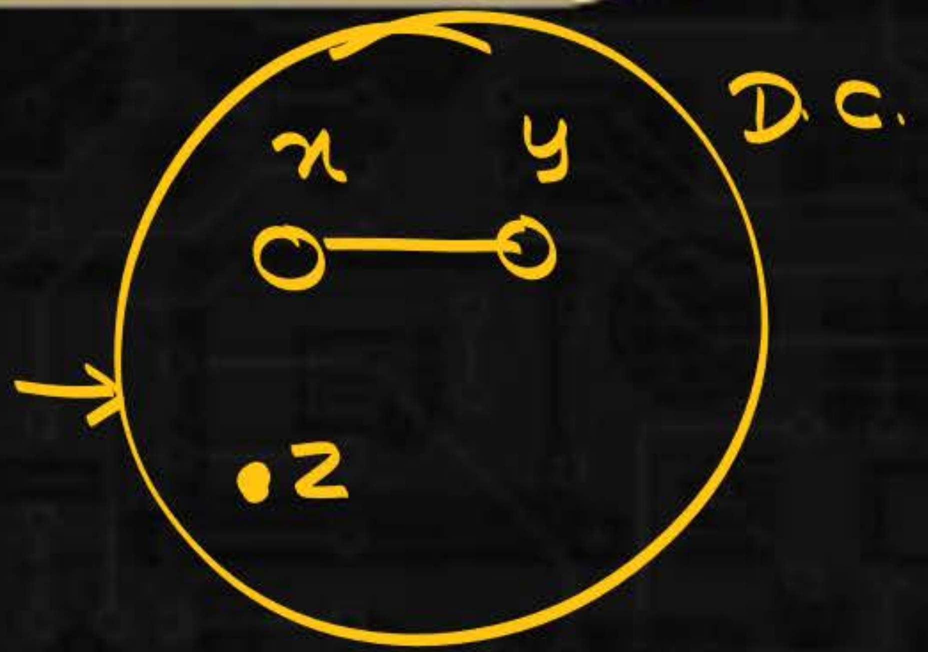
Cut edge / bridge :

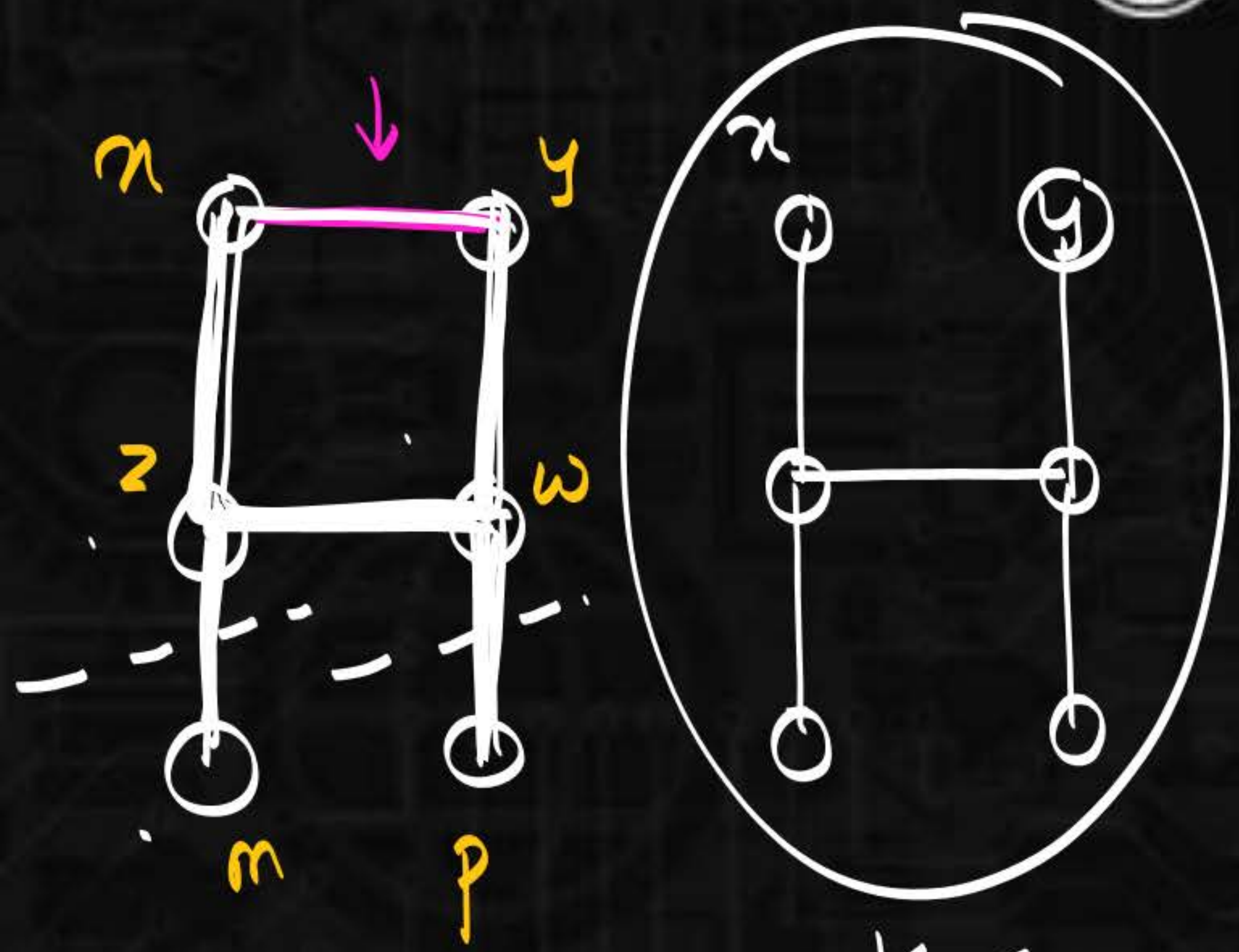
Removal of single edge from a graph will make graph as a disconnected.



Connectivity in Graphs



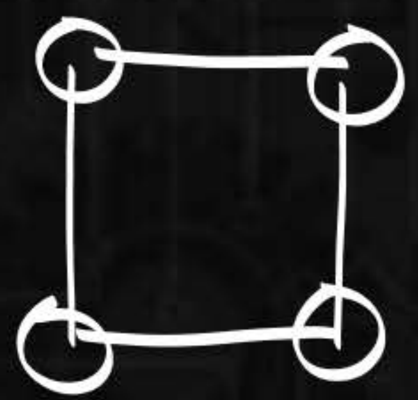




→ K = 1

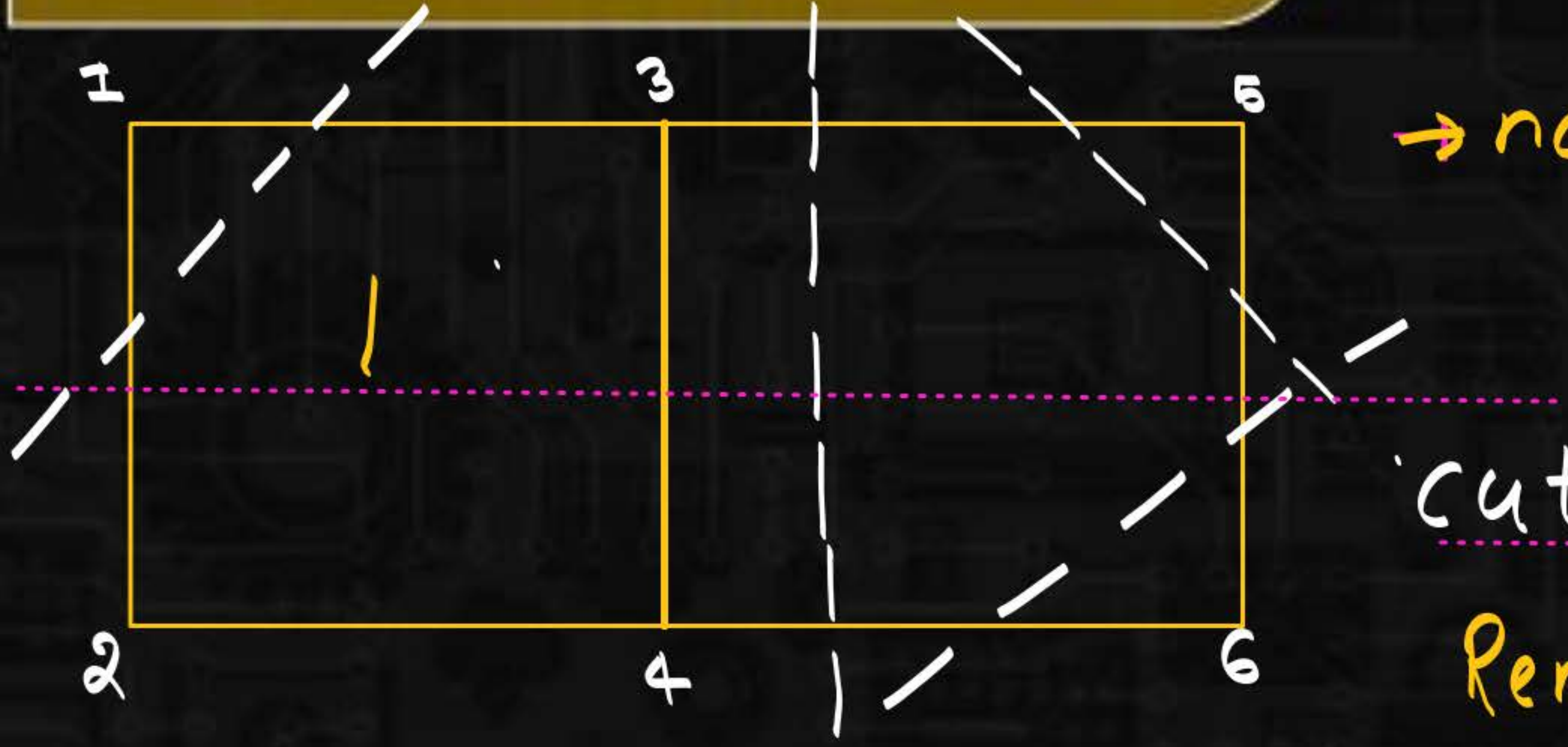
→ C

Connectivity in Graphs



if cut edge exist then it does not belongs cycle.

Connectivity in Graphs

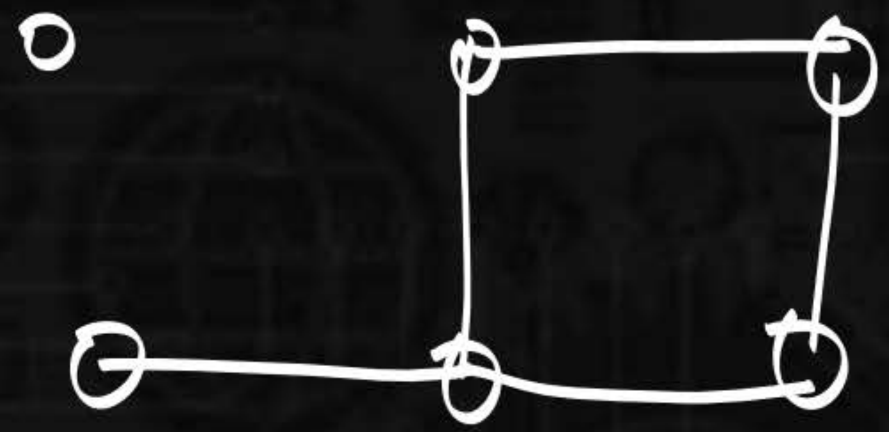


→ no $\underbrace{\text{cut edge}}$.

↳ 1 edge.

cut edgeset / cutset:

Removal of $\boxed{\text{set of edges}}$ from a graph will make graph as a disconnected

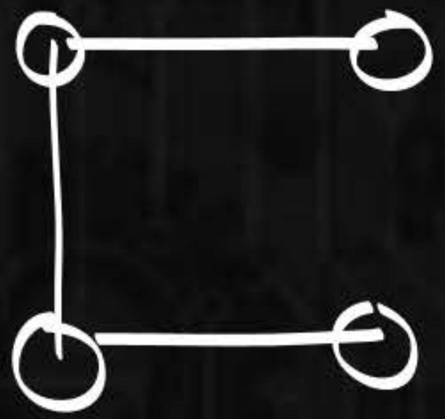


Connectivity in Graphs

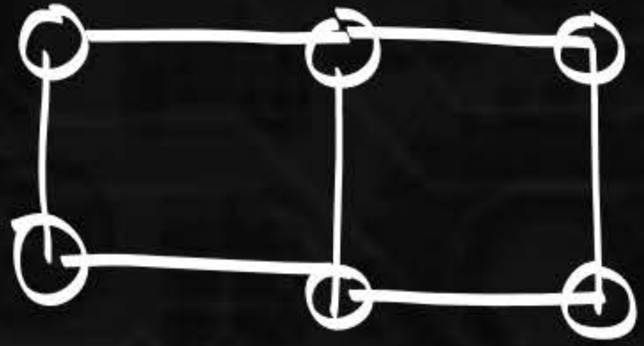
edge connectivity / min-cut : $\lambda(G)$

Removal of $\boxed{\text{min. no of edges}}$ from a graph will make graph as a disconnected.

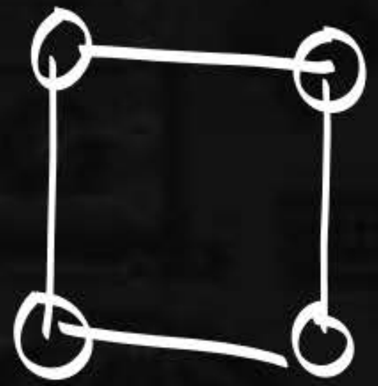
Connectivity in Graphs



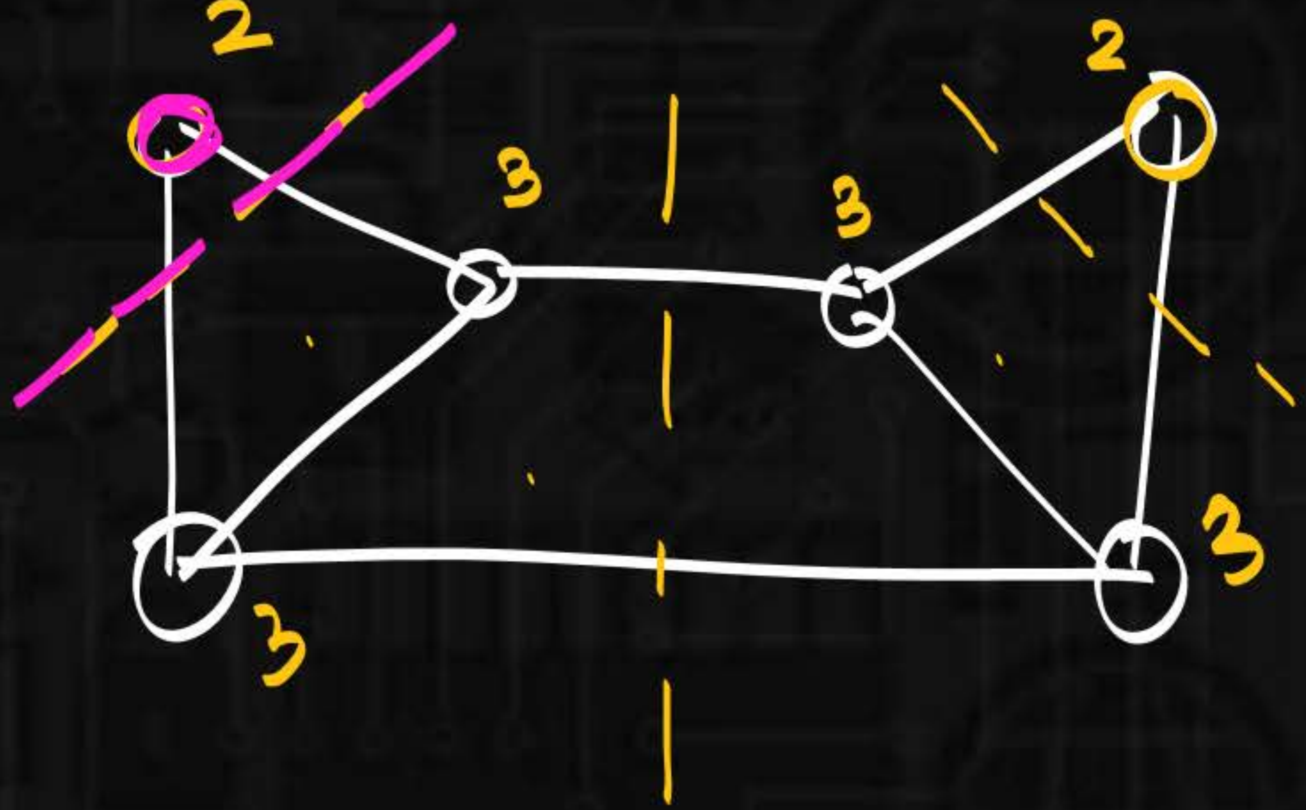
$\lambda(G) = 1.$



$\lambda(G) = 2.$

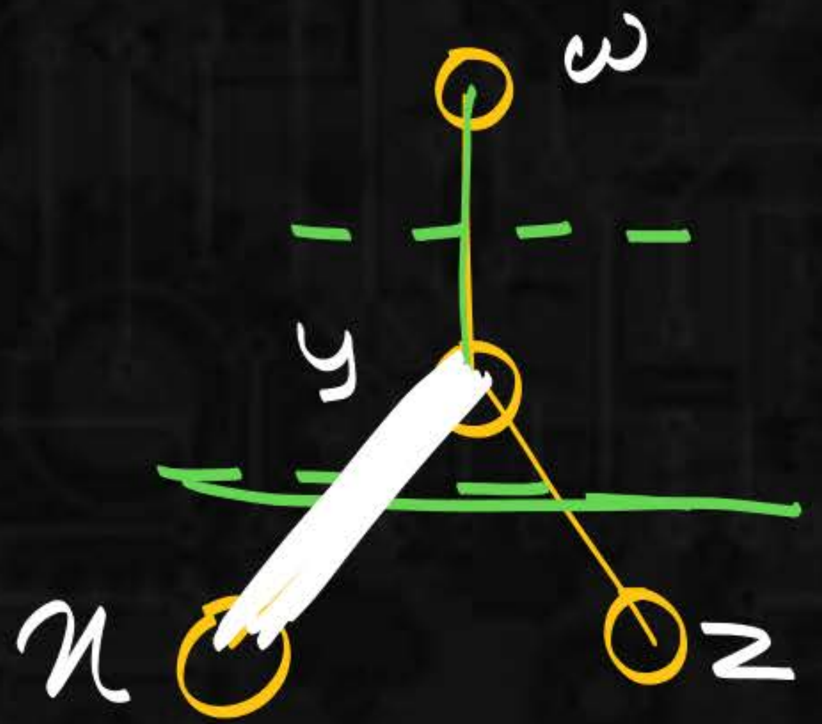


$\lambda(G) = 2.$



$\lambda(G) = 2.$

Connectivity in Graphs



{xy, yz} cutset

{e1, e2, e3}

if subset of any set is cutset then wholeset is not cutset

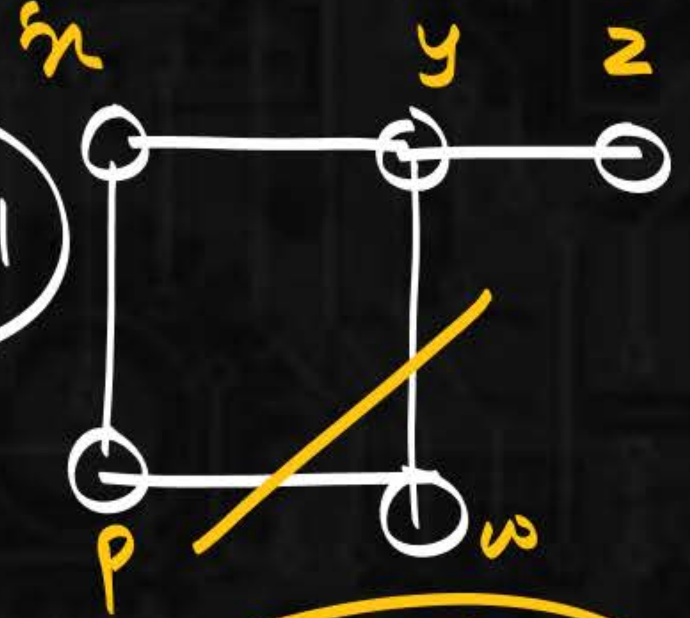
Connectivity in Graphs

cut edge → cutset
∴ set = 1

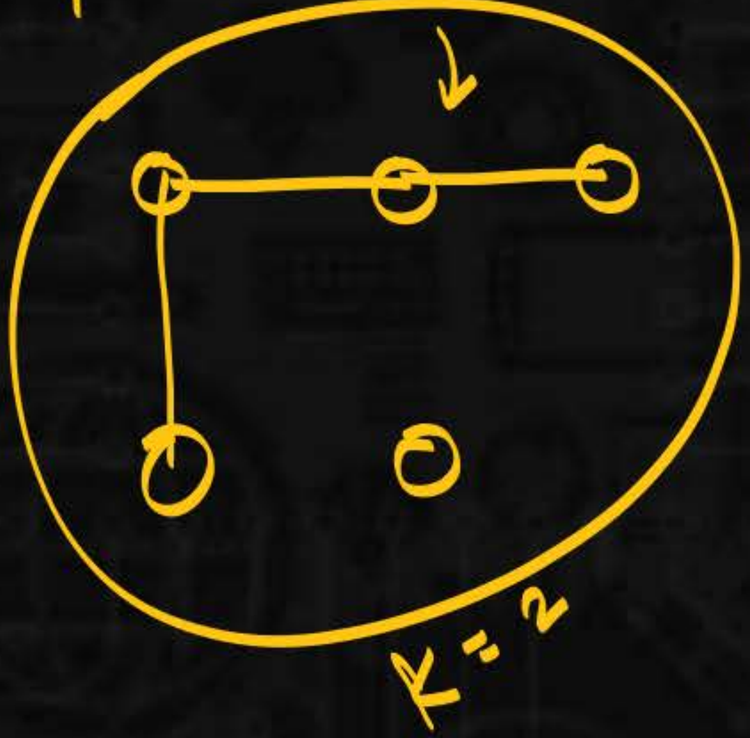
$\boxed{2 \text{ ——}}$

Doubt :

$\lambda(G) = 1$



→ (−yz)



cutset: {yz}

{pw, wy}

not cutset :

A = { $\boxed{pw, wy}$, $\boxed{yz}$ } → cutset

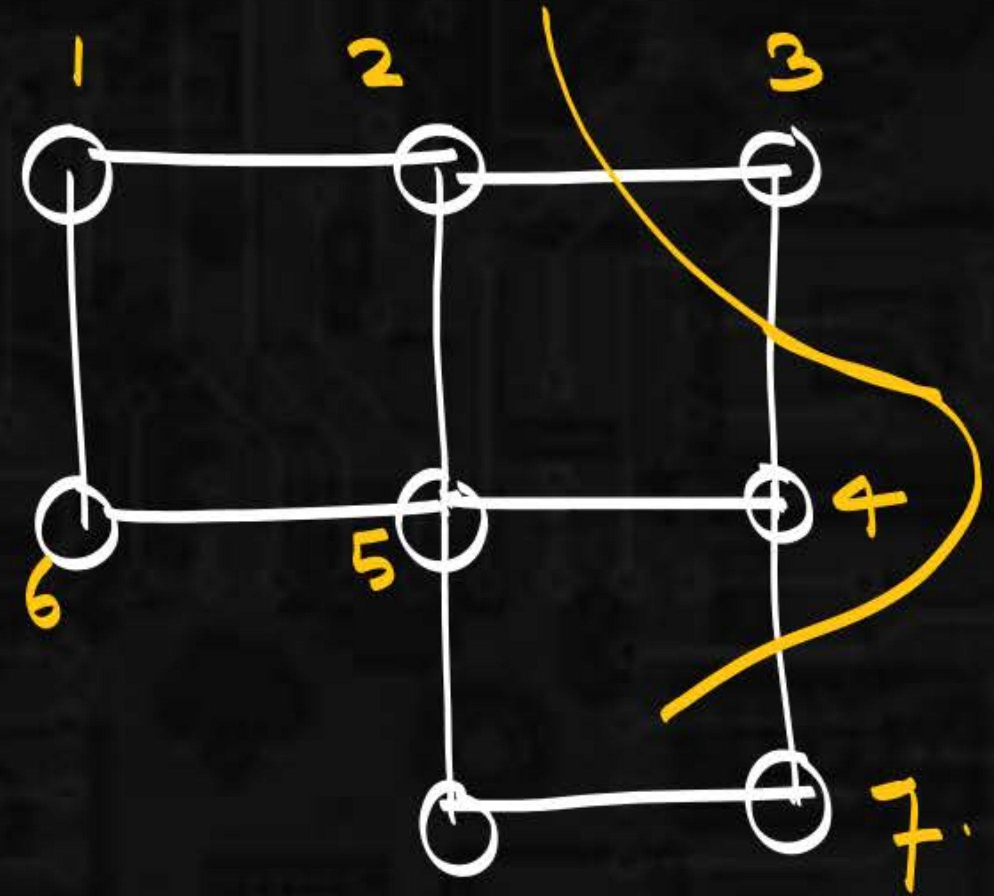
↓

cutset

Subset is cutset.

A → not cutset.

Connectivity in Graphs

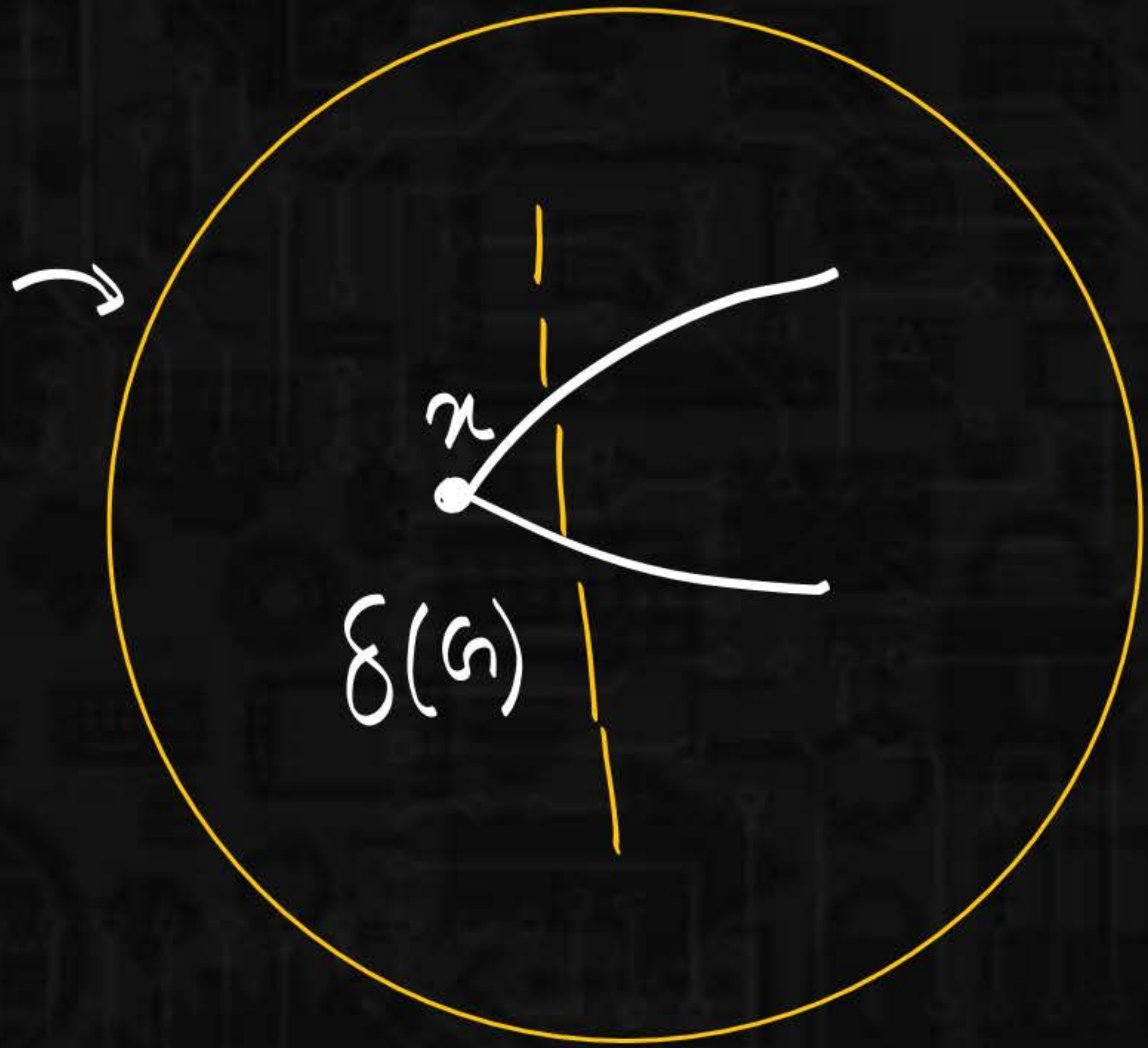


{23, 34} → cutset

{$\boxed{23, 34}$, 47} → not cutset
↓
cutset

Cutset {
$\boxed{\text{size} \to 1}$ → cutedge
size → 2
size → 3

Connectivity in Graphs

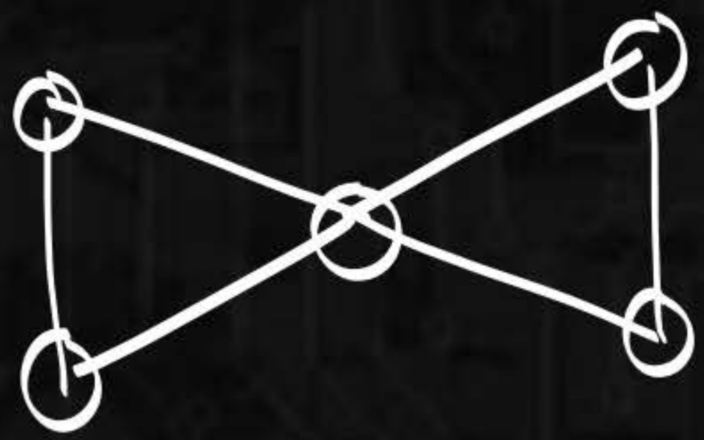


Every Graph $\rightarrow \delta(G)$

$\downarrow$

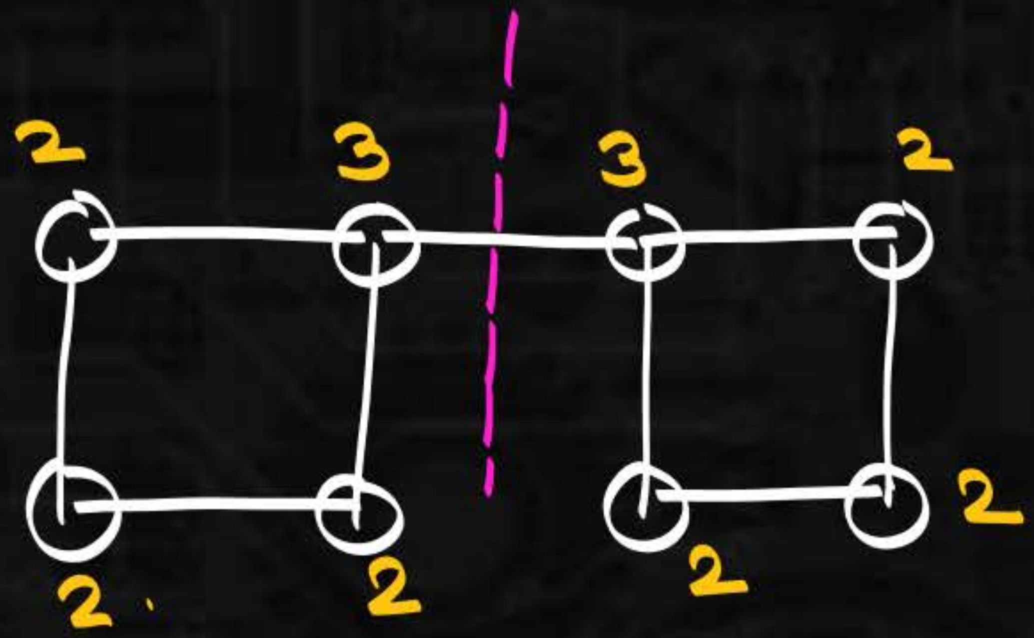
edge connectivity ~~min no of edges.~~

Connectivity in Graphs



$\delta(G) = 2 \quad \lambda(G) = 2.$

$$\lambda(G) = \delta(G) - \mathrm{I}.$$

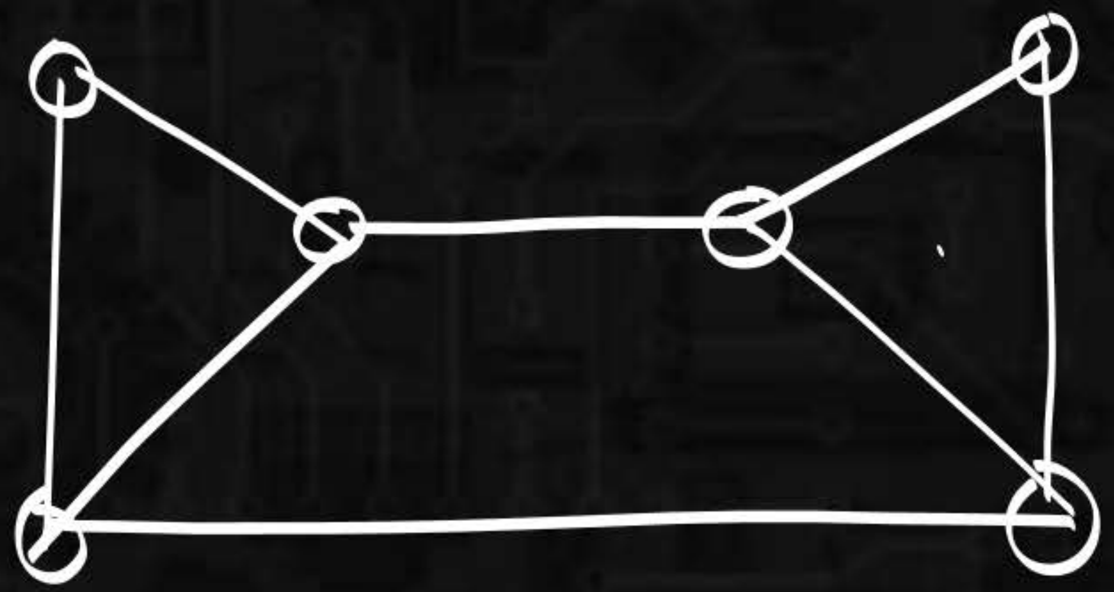


$\delta(G) = 2 \quad \lambda(G) = 1.$

$$\lambda(G) < \delta(G) - \mathrm{II}.$$

$$\boxed{\lambda(G) \leq \delta(G)}$$

Connectivity in Graphs



$\lambda(G) \leq \delta(G)$ $\delta(G) = 2$.

$\lambda(G) \leq 2$.

$\cancel{0}$, 1, 2.

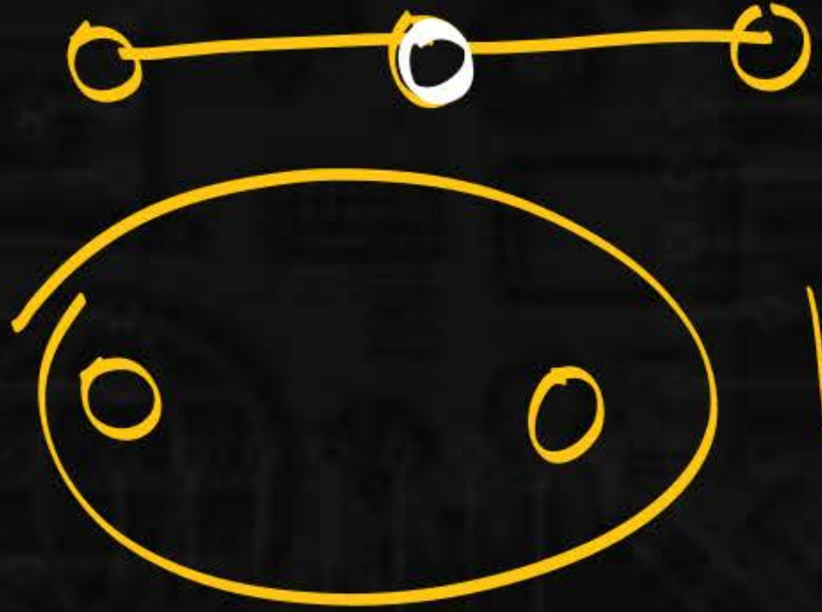
$\lambda(G) = 2$.

Connectivity in Graphs

Cut vertex/cutpoint/Articulation point.

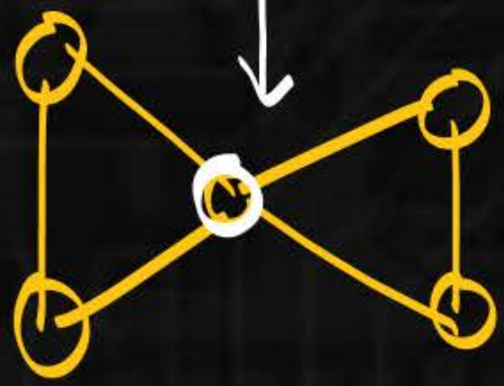
Removal of single vertex from a graph will make graph as a disconnected graph.

cut vertex.



$k = 2.$

cut vertex.



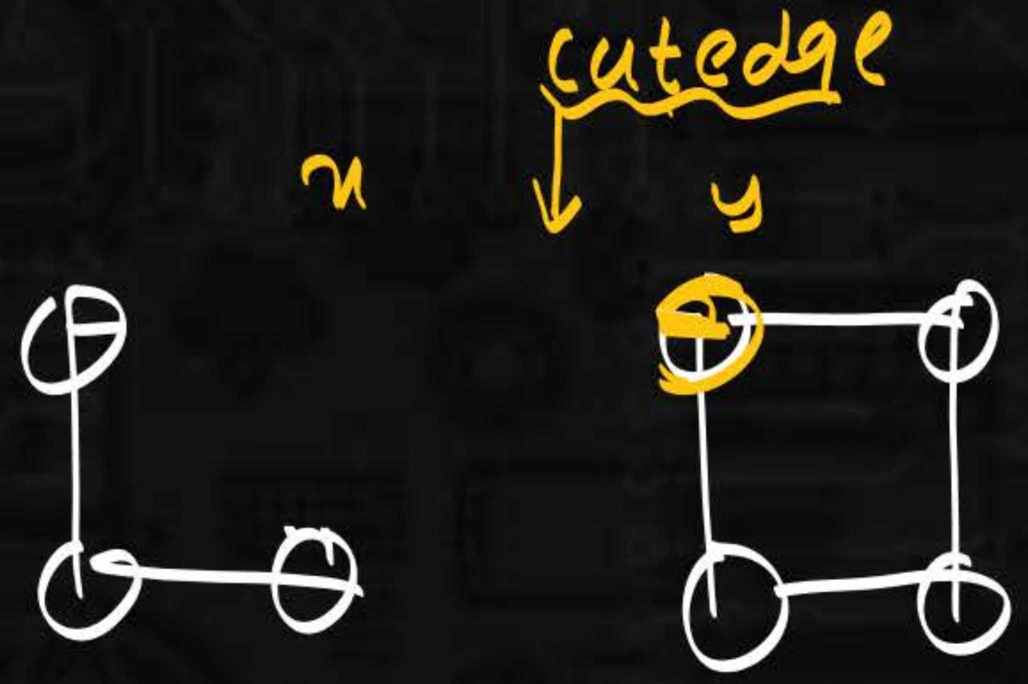
Connectivity in Graphs

[Stmt + $n \geq 3$]

Stmt (false)

cut edge

cut vertex



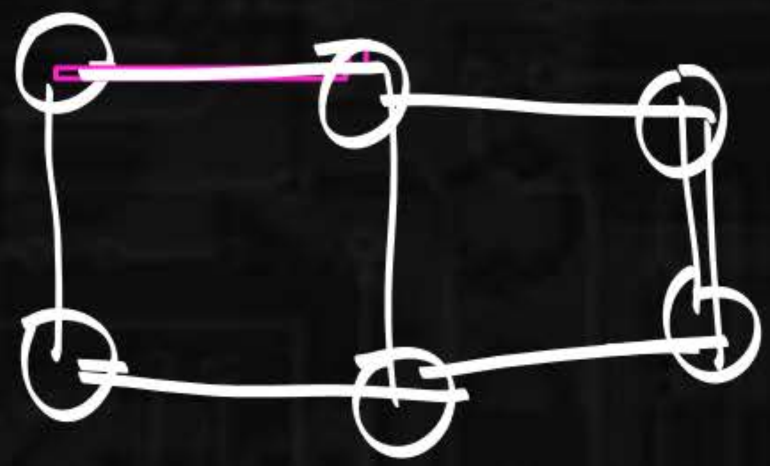
if cut edge exist then cut vertex also exist.

(false)

Connectivity in Graphs

Cut vertex set :

Removal of $\boxed{\text{set of vertices}}$ from a graph will make graph as a disconnected.



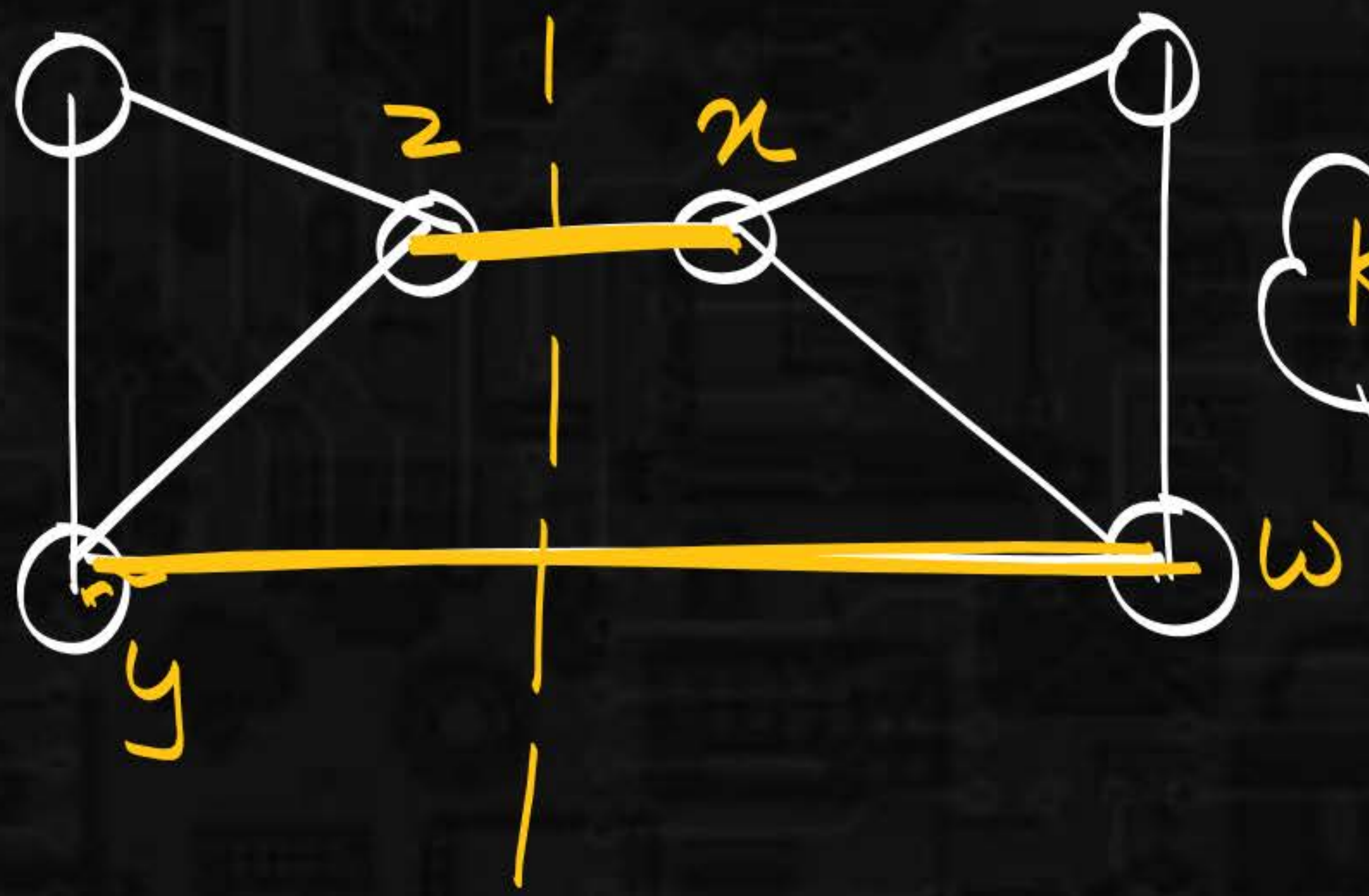
$K(G) = 2.$

Connectivity in Graphs

vertex connectivity $(K(G))$

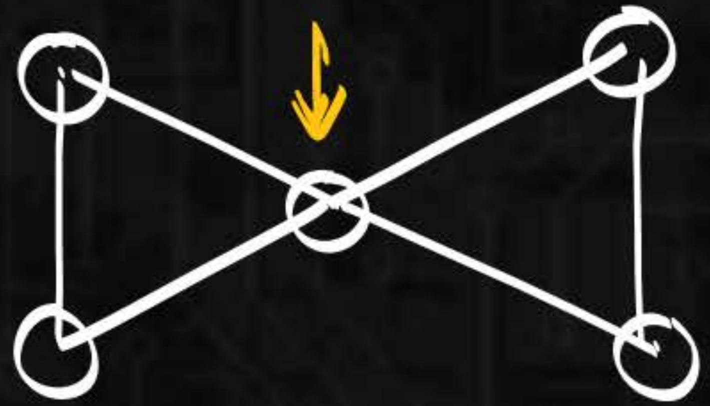
Removal of $\boxed{\text{min. no. of vertices}}$ from a graph will make graph as a disconnected.

Connectivity in Graphs



$K(G) = 2$

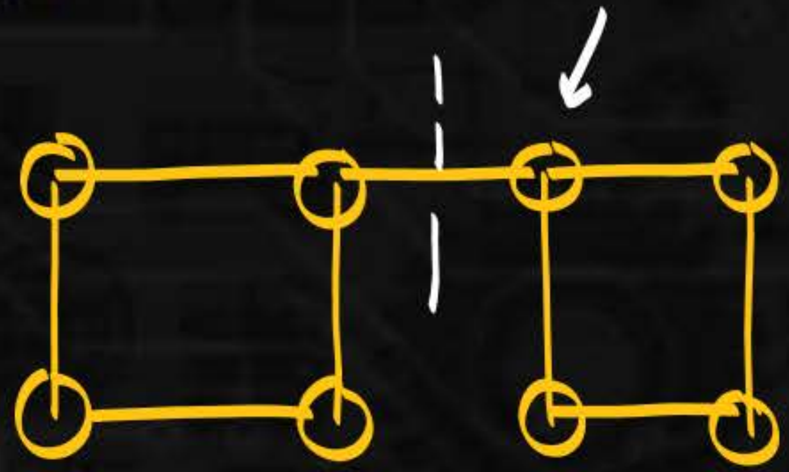
Connectivity in Graphs



$\rightarrow \lambda(G) = 2$

$\rightarrow K(G) = 1$

$\boxed{K(G) < \lambda(G)}$



$\lambda(G) = 1$

$K(G) = 1$

$\boxed{K(G) = \lambda(G)}$

$K(G) \leq \lambda(G)$

Connectivity in Graphs

edge connectivity ≤ min degree.
vertex cn ≤.

edge c. $\lambda(G) \le \delta(G)$

vertex c. $K(G) \le \lambda(G)$

$K(G) \le \lambda(G) \le \delta(G)$

Thm: $\delta(G) \le 2e/n \le \Delta(G) \le n-1$

$$\boxed{K(G) \le \lambda(G) \le \delta(G) \le 2e/n \le \Delta(G) \le n-1}$$

Connectivity in Graphs

G is connected graph with 10 vertices & vertex connectivity 3

min. no. of edges in G ?

$n = 10 \quad K(G) = 3$

$e = ?$

$$K(G) \le \lambda(G) \le \delta(G) \le \frac{2e}{n} \le \Delta(G) \le n-1.$$

$$K(G) \le \frac{2e}{n}$$

$$3 \le \frac{2e}{10}$$

$$30 \le 2e$$

$$\boxed{15 \le e}$$

$$15 \le 15, 16, 17, 18 \ldots$$

G is bipartite Graph of 9 vertices & min no of edges & $K(G) = ?$

$K_{1,8}$

$K_{m,n}$

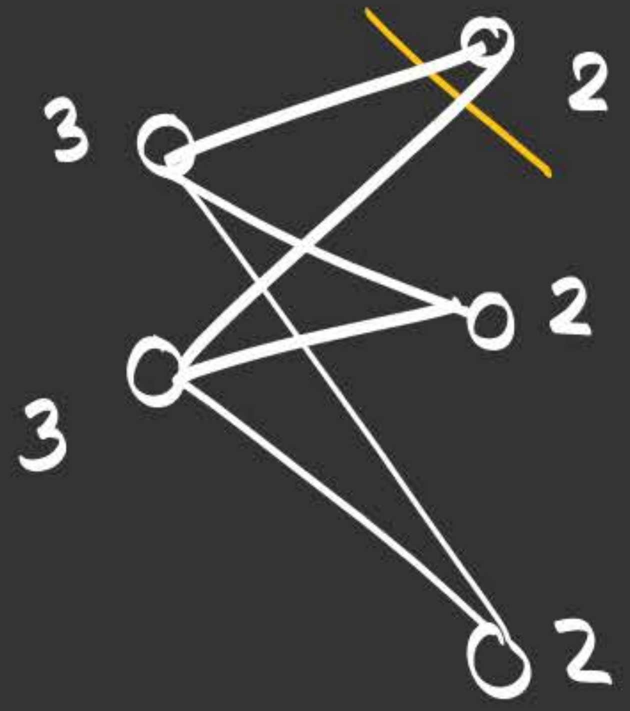
9 vertices

G is bipartite Graph + max no of edges

$K_{4,5}$ $\begin{cases} \lambda(K_{4,5}) = 4 \\ K(K_{4,5}) = 4 \end{cases}$

$$\boxed{\begin{aligned} K(K_{m,n}) &= \min(m,n) \\ \lambda(K_{m,n}) &= \min(m,n) \end{aligned}}$$

$K_{2,3}$



$\lambda(K_{2,3}) = 2$

$\lambda(K_{m,n}) = \min(m,n)$

$K(K_{2,3}) = \min(2,3)$

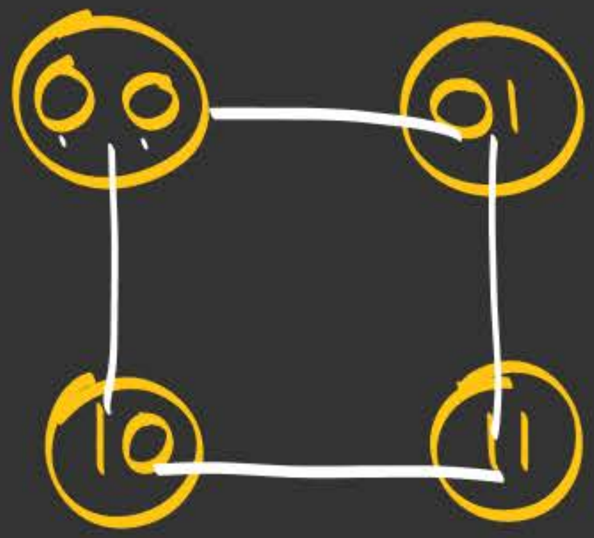
av

$K_{1,8}$ $e=8$

$K_{2,7}$ $e=14$

$K_{4,5}$ $e=20$

$Q_2 \rightarrow$ 2bit



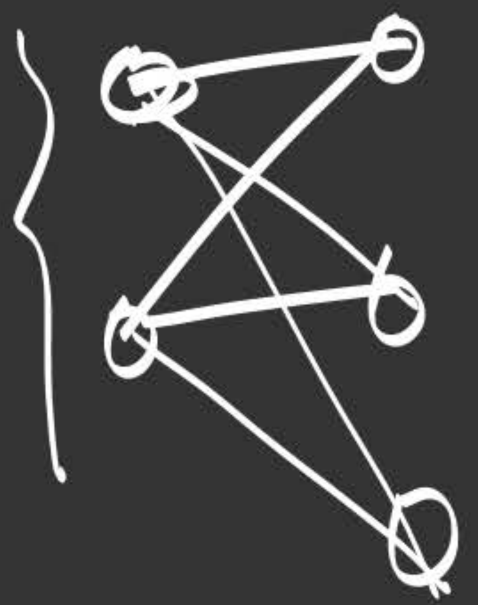
$2^2 \cdot 2 = 2e$

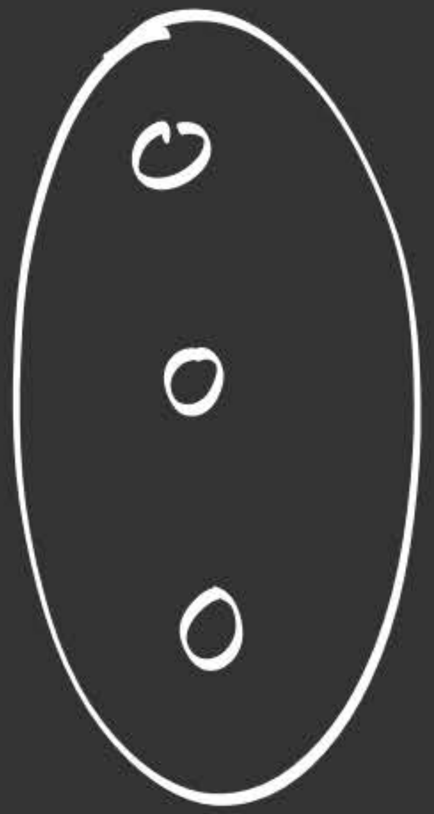
10

00

01

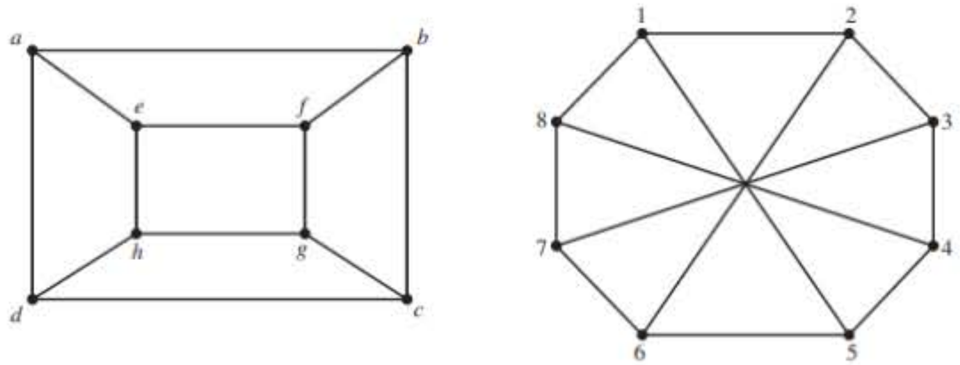
$K_{2,3}$



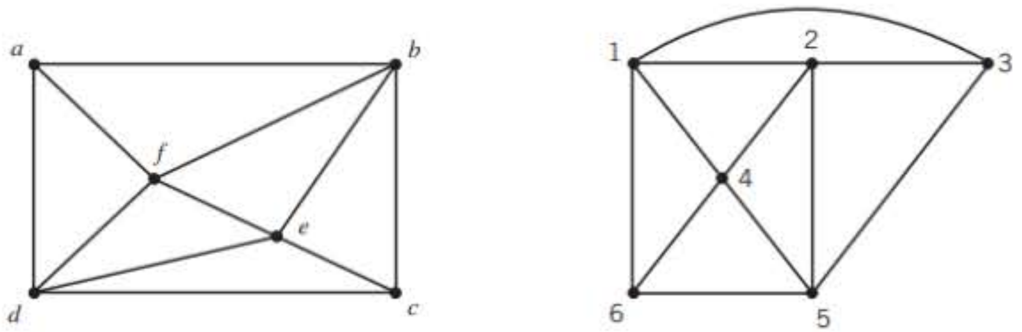


5. Which of the following pairs of graphs are isomorphic? Explain carefully.

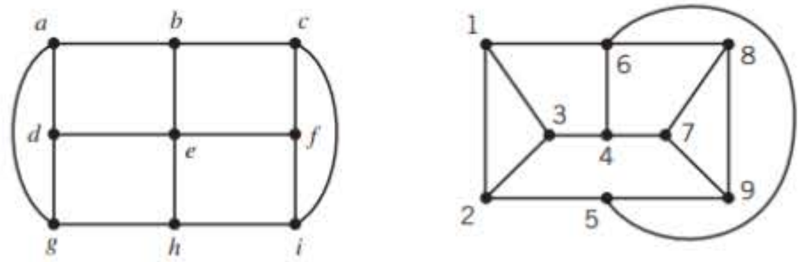
(a)



(b)



(c)

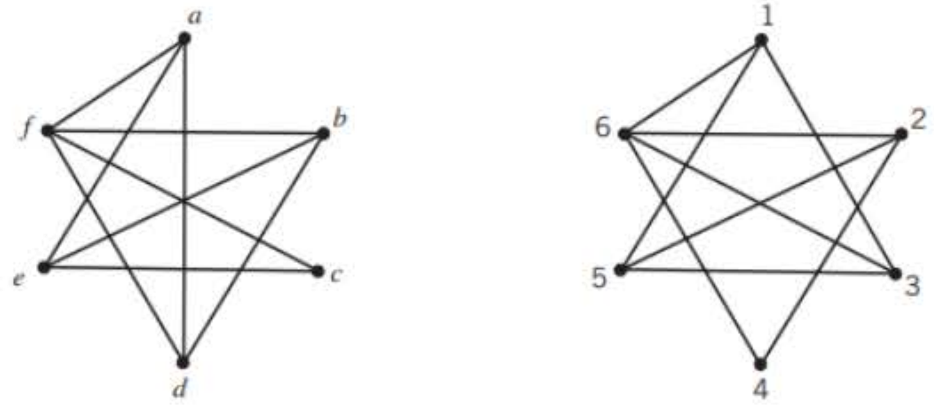


$\lambda(G) = ?$

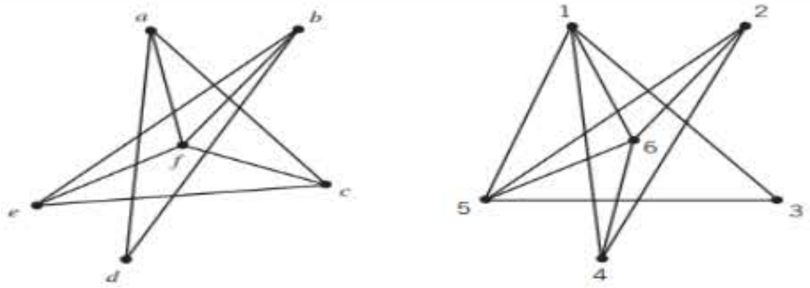
$\kappa(G) = ?$

for all

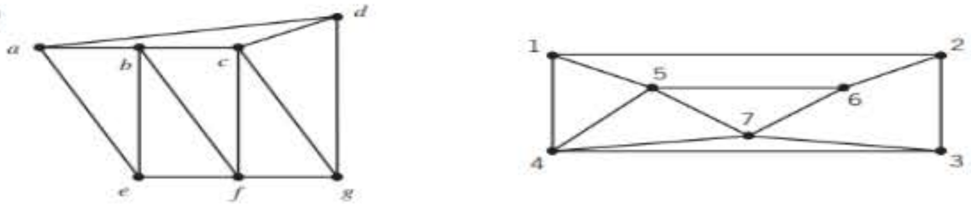
(d)



(e)



(f)



(g)


THANK YOU GW
SOLDIERS !