ENGINEERING MATHEMATICS

LAPLACE TRANSFORM

DPP: 1

Q1 If the Laplace transform of a function f(t) is given by $\frac{s+3}{(s+1)(s+2)}$ then f(0) is.

(A) 0 (B) $\frac{1}{2}$

(C) 1 (D) $\frac{3}{2}$

Q2 The Laplace transform of sin h (at) is

(A) $\frac{s}{s^2+a^2}$ (B) $\frac{s}{s^2-a^2}$

(C) $\frac{a}{s^2-a^2}$ (D) $\frac{a}{s^2+a^2}$

Q3 The Laplace transform F(s) of the exponential function f(t) = e^{at} when t is greater than equal to 0, where a is a constant and (s– a) > 0, is

(A) $\frac{1}{s+a}$ (B) $\frac{1}{s-a}$

(C) $\frac{1}{a-s}$ (D) ∞

Q4 The value of $\int_0^\infty \frac{1}{1+x^2}dx + \int_0^\infty \frac{\sin x}{x}dx\, is$

(A) $\frac{\pi}{2}$ (B) π

(C) $\frac{3\pi}{2}$ (D) 1

Q5 The value of the integral $\int_{-\infty}^{\infty} 12\cos(2\pi t)\frac{\sin(4\pi t)}{4\pi t}\,dt$ is ______

Q6 Laplace transform of $\cos(\omega t) is \frac{s}{s^2+\omega^2}$. The Laplace transformation of e^{-2t} cos (4t) is

(A) $\frac{s-2}{(s-2)^2+16}$

(B) $\frac{s+2}{(s-2)^2+16}$

(C) $\frac{s-2}{(s+2)^2+16}$

(D) $\frac{s+2}{(s+2)^2+16}$

Q7 The function f(t) satisfies the differential equation $\frac{d^2f}{dt^2}+f=0$ and the auxiliary conditions, $f(0)=0, \frac{df}{dt}(0)=4$ .The Laplace transform of f(t) is given by

(A) $\frac{2}{s+1}$ (B) $\frac{4}{s+1}$

(C) $\frac{4}{s^2+1}$ (D) $\frac{2}{s^4+1}$

Q8 The inverse Laplace transform of the function $F(s)=\frac{1}{s(s+1)}$ is given by :

(A) $f(t)=\sin t$

(B) $f(t)=e^{-t}\sin t$

(C) $f(t)=e^{-t}$

(D) $f(t)=1-e^{-t}$

Q9 The Laplace transform of a function f(t) is $\frac{1}{s^2(s+1)}$. The function f(t) is

(A) $t-1+e^{-t}$ (B) $t+1+e^{-t}$

(C) $-1+e^{-t}$ (D) $2t+e^t$

Q10 The inverse Laplace transform of $\frac{1}{(S^2+S)}$ is

(A) $1+e^t$ (B) $1-e^t$

(C) $1-e^{-t}$ (D) $1+e^{-t}$

Q11 Laplace transform for the functions f (x) = cos h (ax) is

(A) $\frac{a}{s^2-a^2}$ (B) $\frac{s}{s^2-a^2}$

(C) $\frac{a}{s^2+a^2}$ (D) $\frac{s}{s^2+a^2}$

Q12 The solution of $\frac{d^2y}{dt^2}-y=1,$ which additionally satisfiles $y\big|t=0=\frac{dy}{dt}\Big|_{t=0}=0$ in the Laplace s-domain is

(A) $\frac{1}{s(s+1)(s-1)}$

(B) $\frac{1}{s(s+1)}$

(C) $\frac{1}{s(s-1)}$

(D) $\frac{1}{(s-1)}$

Q13 The value of the integral is $2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)dt$ equal to

(A) 0 (B) 0.5

(C) 1 (D) 2

Q14 Let $X(s)=\frac{3s+5}{s^2+10s+21}$ be the Laplace Transform of a signal x (t). Then, x (0^+) is

(A) 0 (B) 3

(C) 5 (D) 21

Q15 Consider the differential equation :

$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$ with

$y(t)\Big|_{t=0} = -2 \, and \, \frac{dy}{dt}\Big|_{t=0^+} = 0$

The numerical value of $\frac{dy}{dt}\Big|_{t=0}$ is

(A) -2 (B) -1

(C) 0 (D) 1

Answer Key

Q1	(C)
Q2	(C)
Q3	(B)
Q4	(B)
Q5	(3 to 3)
Q6	(D)
Q7	(C)
Q8	(D)
Q9	(A)
Q10	(C)
Q11	(B)
Q12	(A)
Q13	(D)
Q14	(B)
Q15	(D)

Hints & Solutions

Q1 Text Solution:

Given

$L\{f(t)\} = \frac{s+3}{(s+1)(s+2)}$

$\Rightarrow L\{f(t)\} = \frac{2}{s+1} - \frac{1}{s+2}$

Applying L^{-1} on both sides

$\Rightarrow L^{-1}\{L\{f(t)\}\} = L^{-1}\left\{\frac{2}{s+1} - \frac{1}{s+2}\right\}$

$\Rightarrow f(t) = 2e^{-t} - e^{-2t}$

$(\because L\{e^{at}\} = \frac{1}{s-a})$

$\therefore f(0) = 2.e^{-0} - e^{-2(0)} = 2 - 1 = 1$

$\Rightarrow f(0) = 1$

Q2 Text Solution:

Sin h at $= \frac{e^{at}-e^{-at}}{2}$

Taking Laplace Transform, we get

$= \frac{1}{2}\left[L\{e^{at}\} - L\{e^{-at}\}\right]$

$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right]$

$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right]$

$= \frac{1}{2}\left[\frac{2a}{s^2-a^2}\right]$

$\boxed{L[\sin hat] = \frac{a}{s^2-a^2}}$

Q3 Text Solution:

$L[f(t)] = \int\limits_0^\infty e^{-st} \cdot f(t)\, dt$

$L[e^{at}] = \int\limits_0^\infty e^{-st} \cdot e^{at}\, dt = \int\limits_0^\infty e^{-(s-a)t} dt$

$\left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty = -\frac{1}{s-a}(0-1)$

$\boxed{L(e^{at}) = \frac{1}{s-a}}$

Q4 Text Solution:

$I = \int_0^\infty \frac{1}{1+x^2} dx + \int_0^\infty \frac{\sin x}{x} dx \;=?$

$I = I_1 + I_2$

Where,

$I_1 = \int\limits_0^\infty \frac{1}{1+x^2} = \left[\tan^{-1} x\right]\Big|_0^\infty$

$I_2 = \int\limits_0^\infty \frac{\sin x}{x}\, dx$

Considering Laplace Transform of Sin at

$\Rightarrow L\{\sin at\} = \frac{a}{s^2+a^2}$

By the Property of division with ' t '

$\Rightarrow L\left\{\frac{\sin at}{t}\right\} = \int\limits_s^\infty \frac{a}{s^2+a^2} \cdot ds$

$\Rightarrow \int\limits_s^\infty e^{-st} \cdot \frac{\sin at}{t} dt = \frac{1}{a} \cdot \mathrm{Tan} - 1\left(\frac{s}{a}\right)\Big|_s^\infty$

$\Rightarrow \int\limits_s^\infty e^{-st} \cdot \frac{\sin at}{t} dt = \frac{1}{a} \cdot \left\{\frac{\pi}{2} - \mathrm{Tan}^{-1}\left(\frac{s}{a}\right)\right\}$

Substituting s = 0 and a = 1 in both sides of above equations :

$\Rightarrow \int\limits_s^\infty \frac{\sin at}{t} dt = 1\left\{\frac{\pi}{2} - 0\right\} = \frac{\pi}{2}$

$\Rightarrow \int\limits_s^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$

Therefore,

$I = \left[\tan^{-1} x\right]_0^\infty + \int_0^\infty \frac{\sin x}{x} dx$

$\left[\because \int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}\right]$

$I = \tan^{-1}\infty - \tan^{-1} 0 + \frac{\pi}{2}$

$I = \frac{\pi}{2} - 0 + \frac{\pi}{2} = \pi$

$\therefore \boxed{\int_0^\infty \frac{1}{1+x^2} dx + \int_0^\infty \frac{\sin x}{x}\, dx = \pi}$

Q5 Text Solution:

Let the given integral,

$I = \int\limits_{-\infty}^{\infty} 12 \cdot \cos(2\pi t) \frac{\sin(4\pi t)}{4\pi t}\, dt$

$= 2 \times \int\limits_0^\infty 12 \cdot \cos(2\pi t) \frac{\sin(4\pi t)}{4\pi t}\, dt$

$\left(\because \int\limits_{-\infty}^{\infty} f(t) dt = 2 \times \int\limits_0^\infty f(t) dt, If\, f(-t) = f(t)\right)$

$\Rightarrow I = 3 \times \int\limits_0^\infty \frac{2\sin(4\pi t)\cdot\cos(2\pi t)}{\pi t}\, dt$

$\Rightarrow I = \frac{3}{\pi} \times \int\limits_0^\infty \frac{\sin(6\pi t)+\sin(2\pi t)}{t}\, dt$

$\left(\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)\right)$

$\Rightarrow I = 3 \times \left\{\int\limits_0^\infty \frac{\sin(6\pi t)}{\pi t} dt + \int\limits_0^\infty \frac{\sin(2\pi t)}{\pi t} dt\right\}$

$I = I_1 + I_2$

$I_1 = 3 \times \int\limits_0^\infty \frac{\sin 6\pi t}{\pi t}\, dt$

Let

$6\pi t = u$

$6\pi dt = du$

$dt = \frac{du}{6\pi}$

when

$\left| \begin{array}{l} t = 0, u = 0 \\ t = \infty, u = \infty \end{array} \right.$

$\therefore I_1 = 3\int_0^\infty \frac{\sin u}{u/6} \cdot \frac{du}{6\pi} = \frac{3}{\pi}\int_0^\infty \frac{\sin u}{u} du$

We know that $\int_0^\infty \frac{\sin t}{t} dt = \frac{\pi}{2}$

$\therefore \quad I_1 = \frac{3}{\pi}\int_0^\infty \frac{\sin u}{u} dv = \frac{3}{\pi} \times \frac{\pi}{2} = \frac{3}{2}$

Now, $I_2 = 3\int_0^\infty \frac{\sin 2\pi t}{\pi t} dt$

Let

$2\pi t = v$

$2\pi dt = dv$

$dt = \frac{dv}{2\pi}$

when

$\left| \begin{array}{l} t = 0, u = 0 \\ t = \infty, v = \infty \end{array} \right.$

$\therefore \; I_2 = 3\int_0^\infty \frac{\sin v}{v/2} \cdot \frac{dv}{2\pi} = \frac{3}{\pi}\int_0^\infty \frac{\sin v}{v} dv$

Similarly,

$I_2 = \frac{3}{\pi}\int_0^\infty \frac{\sin v}{v} dv = \frac{3}{\pi} \times \frac{\pi}{2} = \frac{3}{2}$

$\therefore \quad I = I_1 + I_2 = \frac{3}{2} + \frac{3}{2} = 3$

Q6 Text Solution:

$L\{\cos \omega t\} = \frac{s}{s^2+\omega^2}$

By first shift property,

$L(e^{at} \cdot \cos \omega t) = \frac{(s-a)}{(s-a)^2+\omega^2}$

Substitute a = -2 & $\omega = 4$

$\Rightarrow L\{e^{-2t} \cdot \cos 4t\} = \frac{(s-(-2))}{(s-(-2))^2+4^2}$

$= \frac{(s+2)}{(s+2)^2+16}$

$\Rightarrow \boxed{L\{e^{-2t} \cdot \cos 4t\} = \frac{(s+2)}{(s+2)^2+16}}$

Q7 Text Solution:

Given :

$\frac{d^2 f}{dt^2} + f = 0; \quad f(0) = 0, \frac{df}{dt}(0) = 4$

The auxiliary equations is $m^2 + 1 = 0$

$\Rightarrow m = \pm i$

The solution is $f(t) = c_1 \cos t + c_2 \sin t$

Given

$f(0) = 0 \Rightarrow 0 = c_1 \cos 0 + c_2 \sin 0 \Rightarrow c_1 = 0$

$\therefore \quad f(t) = c_2 \sin t$

$\Rightarrow \quad \frac{d\,f(t)}{dt} = c_2 \cos t$

Given $\frac{df}{dt}(0) = 4 \Rightarrow 4 = c_2.(1) \Rightarrow c_2 = 4$

$\therefore \; f(t) = 4.\ \mathrm{Sin}\, t$

$\Rightarrow \quad L\{f(t)\} = 4.L\{\sin t\} = 4$

$\cdot\left(\frac{1}{s^2+1^2}\right)$

$= \frac{4}{s^2+1} \qquad \left(\because L(\sin at) = \frac{a}{s^2+a^2}\right)$

$\therefore \quad \boxed{L\left\{f(t) = \frac{4}{s^2+1}\right\}}$

Q8 Text Solution:

Given :

$F(s) = \frac{1}{s(s+1)}$

$\Rightarrow \quad F(s) = \frac{1}{s} - \frac{1}{s+1}$

Applying inverse Laplace on both sides

$\Rightarrow \; L^{-1}\left\{F(s)\right\} = L^{-1}\left\{\frac{1}{s}\right\}$

$-L^{-1}\left\{\frac{1}{s+1}\right\} = 1 - e^{-t}$

$\left(\because L\left\{t^n\right\} = \frac{n!}{s^{n+1}}; L\left\{e^{at}\right\} = \frac{1}{s-a}\right)$

$\Rightarrow \; \boxed{f(t) = 1 - e^{-t}}$

Q9 Text Solution:

Given :

$\bar{F}(s) = \frac{1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$

$\Rightarrow \quad \bar{F}(s) = \frac{As(s+1)+B(s+1)+Cs^2}{s^2(s+1)} = \frac{1}{s^2(s+1)}$

$\Rightarrow \quad A + C = 0; A + B = 0;\ B = 1$

$\Rightarrow \quad A = -1;\ B = 1;\ C = 1$

$\Rightarrow \quad \frac{1}{s^2(s+1)} = \frac{-1}{s} + \frac{1}{s^2} + \frac{1}{(s+1)}$

Applying inverse Laplace on both sides :

$L^{-1}\left\{\frac{1}{s^2(s+1)}\right\} = L^{-1}\left\{\frac{-1}{s}\right\} + L^{-1}\left\{\frac{1}{s^2}\right\}$

$+ L^{-1}\left\{\frac{1}{(s+1)}\right\}$

$L^{-1}\left\{\frac{1}{s^2(s+1)}\right\} = -1 + t + e^{-t}$

$\Rightarrow \quad \boxed{f(t) = -1 + t + e^{-t}}$

$\left\{\because L\left\{e^{at}\right\} = \frac{1}{s-a}\right\}$

Q10 Text Solution:

Given :

$$\overline{F}(S) = \frac{1}{(S^2+S)} = \frac{1}{S(S+1)} = \frac{1}{S} - \frac{1}{S+1}$$

$$\Rightarrow \quad \overline{F}(S) = \frac{1}{S} - \frac{1}{S+1}$$

$$\therefore L^{-1}\left\{\vec{F}(S)\right\} = L^{-1}\left\{\frac{1}{S}\right\} - L^{-1}\left\{\frac{1}{S+1}\right\}$$

$$= L^{-1}\left\{\frac{1}{S-0}\right\} - L^{-1}\left\{\frac{1}{S-(-1)}\right\}$$

$$= e^{0.t} - e^{-1.t} = 1 - e^{-t}$$

$$\left\{\because L(e^{at}) = \frac{1}{S-a}\right\}$$

$$\Rightarrow \quad \boxed{L^{-1}\left\{\frac{1}{S^2+S}\right\} = 1 - e^{-t}}$$

Q11 Text Solution:

Given :

Cos h ax = $\frac{e^{ax}+e^{-ax}}{2}$

Taking Laplace transform, we get

$$= \frac{1}{2}L\{e^{ax}\} + L\{e^{-ax}\}$$

$$= \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right]$$

$$\frac{1}{2}\left[\frac{2s}{s^2-a^2}\right]$$

L (cos h ax) = $\frac{s}{s^2-a^2}$

Q12 Text Solution:

Given

$$\frac{d^2y}{dt^2} - y = 1; \quad y(0) = 0\& \frac{dy}{dt}\Big|_{t=0} = 0$$

Appling Laplace transform on both sides of DE

$$\Rightarrow \quad \left(s^2 \cdot F(s) - s \cdot y(0) - y'(0)\right) - F(s)$$

$$= \frac{1}{s}$$

$$\Rightarrow \left(s^2 - 1\right)F(s) = \frac{1}{s}$$

$$\Rightarrow F(s) = \frac{1}{s(s^2-1)}$$

$$\Rightarrow F(s) = \frac{1}{s(s-1)(s+1)}$$

Q13 Text Solution:

Let f(t) = $\frac{\sin 2\pi t}{\pi t}$

f(t) = f (–t) = $\frac{\sin 2\pi t}{t}$

f(t) is even :

$$I = 2\int_{-\infty}^{\infty} \frac{\sin 2\pi t}{\pi t} = 4\int_0^{\infty} \frac{\sin 2\pi t}{\pi t}$$

$$= \frac{4}{\pi}\boxed{\int_0^{\infty} \frac{\sin 2\pi t}{t}}$$

From Laplace transform $L\left[\frac{f(t)}{t}\right] = \int_0^{\infty} f(s)ds$

We know, $\int_0^{\infty} \frac{\sin at}{t} = \frac{\pi}{2}$

$$\int_0^{\infty} \frac{\sin 2\pi t}{t} = \frac{\pi}{2}$$

Hence, $I = \frac{4}{\pi} \cdot \frac{\pi}{2} = 2$

Q14 Text Solution:

Using initial value theorem,

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

$$= \lim_{s\to\infty} s\left(\frac{3s+5}{s^2+10s+21}\right)$$

$$\lim_{s\to\infty} \frac{3+\frac{5}{s}}{1+\frac{10}{s}+\frac{21}{s^2}} = 3$$

Q15 Text Solution:

Given data,

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$$

$$y(t)|_{t=0} = -2 \Rightarrow y(0) = -2$$

$$\frac{dy}{dt}\Big|_{t=0} = 0 \Rightarrow y'(0) = 0$$

Differential equation can be written as :-

$$y'' + 2y' + y = \delta(t)$$

Taking Laplace transform on both sides

$$\left[s^2Y(s) - sy(0) - y'(0)\right]$$

$$+2\left[sY(s) - y(0)\right] + Y(s) = 1$$

$$\left[s^2 + 2s + 1\right]Y(s) + 2s + 4 = 1$$

$$Y(s) = \frac{-3-2s}{(s+1)^2}$$

$$Y(s) = -\frac{3}{(s+1)^2} - 2\left[\frac{(s+1)-1}{(s+1)^2}\right]$$

$$Y(s) = -\frac{3}{(s+1)^2} - \frac{2}{(s+1)} + \frac{2}{(s+1)^2} = -\frac{1}{(s+1)^2}$$

$$-\frac{2}{(s+1)}$$

$$y(t) = L^{-1}Y(s) = L^{-1}\left[-\frac{1}{(s+1)^2} - \frac{2}{(s+1)}\right]$$

y = $-te^{-t} - 2e^{-t}$

$$\frac{dy}{dt} = te^{-1} - e^{-t} + 2e^{-t}$$

$$= e^{-t}(t - 1 + 2)$$

$$\frac{dy}{dt} = e^{-t}(t+1)$$

At t = 0 ; $\boxed{\frac{dy}{dt} = 1}$