GATE-All BRANCHES
Engineering Mathematics

Multivariable calculus

Lecture No.- 02

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Recap of Previous Lecture

Topic Double integration

Topic Volume via double integrals

Topic Question based on double integrals

Topics to be Covered

- **Topic** Change the order of integration
- **Topic** Change the variables
- **Topic** Question based on change of variables

Topic :Double integration

$\iint x^2\, dy\, dx \longrightarrow$ volume via double integrals

#Q. Illustration

$\iint x^2\, dx\, dy$ where A is the region in the Ist quadrant bounded by

Hyperbola

$xy = 16$

$x = 8$

$x = y$

$y = 0$

$xy = 16$ (Hyperbola)

$x = 8$ (line)

$x = y$ (line)

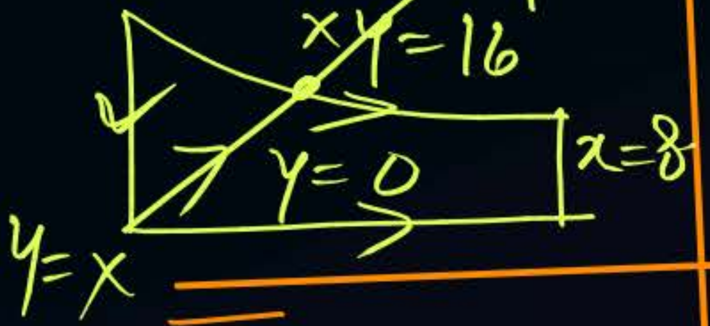
$y = 0$ (line)

$$\iint x^2\,dy\,dx$$

$xy = 16$

$y = \frac{16}{x}$

Vertical Strip.

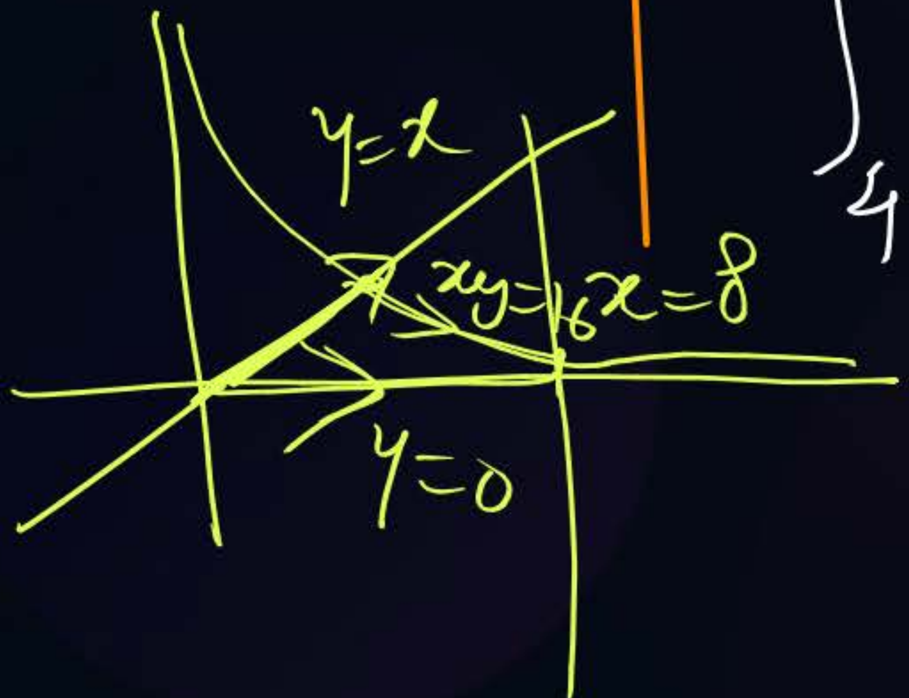


Ist region

$$= \int_0^4 \int_0^x x^2\,dy\,dx \quad (1)$$

IInd region

$$= \int_4^8 \int_0^{\frac{16}{x}} x^2\,dy\,dx \quad (2)$$



$xy = 16$

$y = x$

$y = 0$

$x = 8$

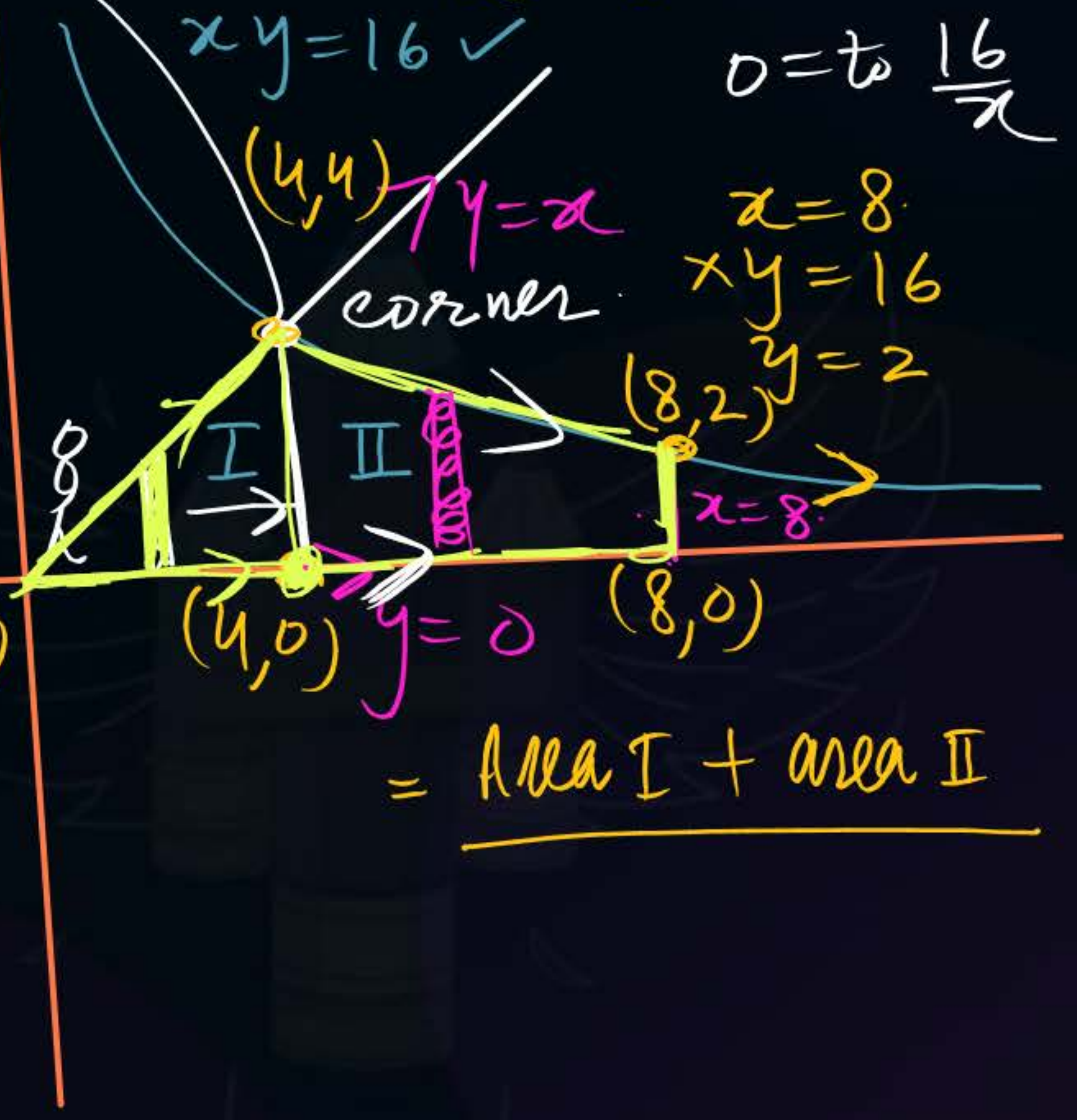
Non Diff.

$xy = 16$

$x = y$

$x = \pm 4$

$0 = \text{to } \frac{16}{x}$



= Area I + area II

$$I_1 = \int_0^4 \int_0^x x^2 \, dy \, dx$$

$$I = \int_0^4 x^2 dx \int_0^x dy$$

$$I = \int_0^4 x^2 dx \left[x \right]$$

$$= \int_0^4 x^3 dx$$

$$I = 64$$

$$I_2 = \int_4^8 x^2 dx \int_0^{\frac{16}{x}} y \, dy$$

$$= \int_4^8 \cancel{x^2} dx \left[\frac{16}{\cancel{x}} \right]$$

$$= 16 \int_4^8 x \, dx$$

$$= 16 \left(\frac{8^4 - 4^2}{2} \right)$$

$$= 384$$

Total Area $= 64 + 384$

$= 448$

Topic : Double integration

#Q. The value of the integral $\int_0^1 \int_y^1 x^2 e^{xy} dx dy$ is

H.W

A $\frac{e+2}{2}$

B $\frac{e-2}{2}$

C $\frac{e-1}{2}$

D $\frac{e+1}{2}$

Topic : Double integration

#Q. The value of the integral $\int_{y=0}^{1}\int_{x=y}^{1}\frac{x}{(x^2+y^2)}dxdy$ is

A $\pi/4$

B $\pi/2$

C $\pi/3$

D $\pi/5$

$\frac{\pi}{4}$

H.W

Topic : Integration

H.W

#Q. The solution of $\int_1^a \int_1^b \frac{dxdy}{xy}$ is

A ln (ab)

B ln (a/b)

C ln (a) + ln (b)

D ln (a) ln (b)

Topic : Integration

#Q. $I = \int_{y=0}^{2} \int_{x=y/2}^{1} e^{x^2}\, dx\, dy$

✓ Plot The Limits

- $y = 0$
- $y = 2$
- $x = y/2$
- $x = 1$

✓ SECOND step

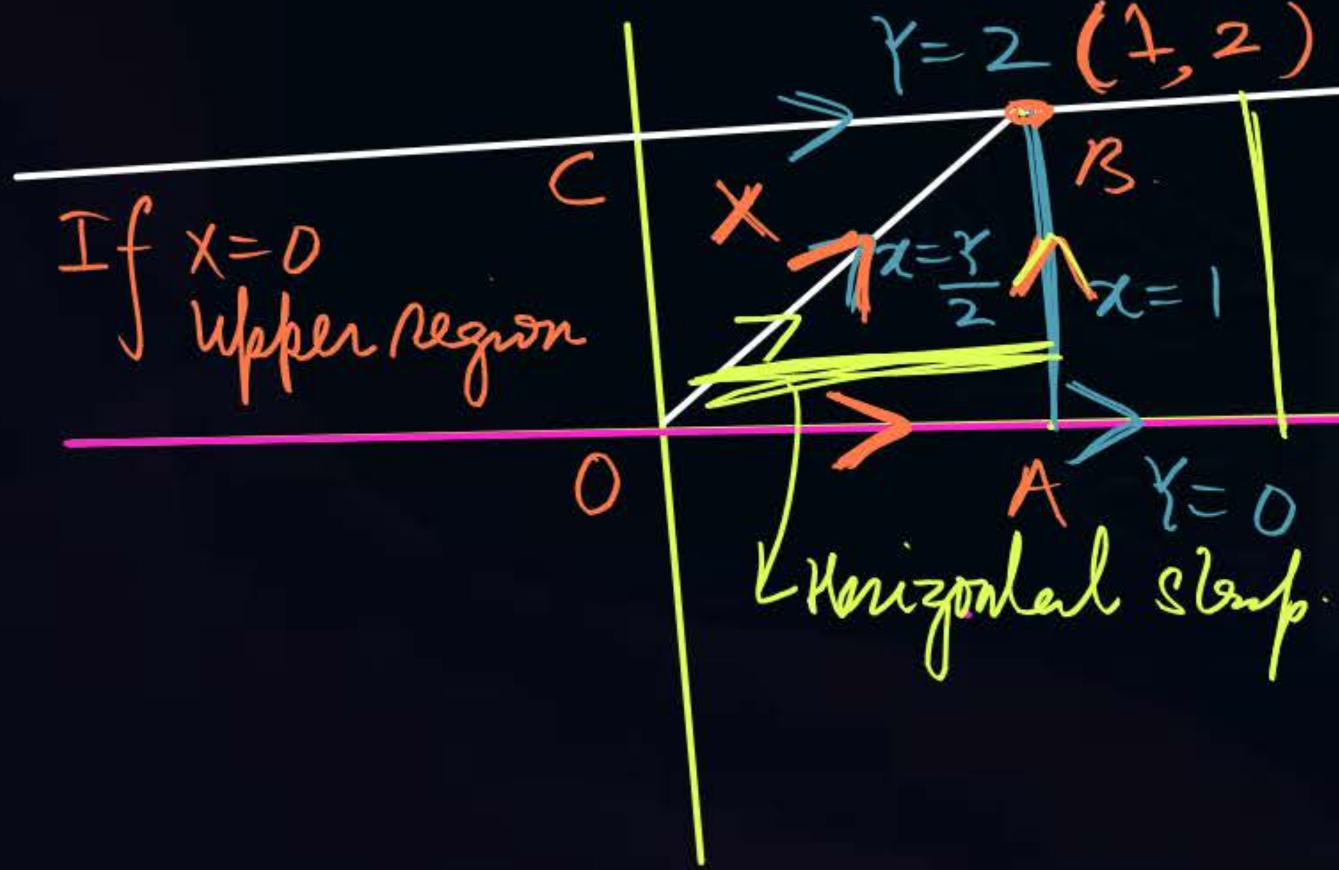
Check The strip

fields medal 2023

✓ $I = \int_{y=0}^{2} \int_{x=\frac{y}{2}}^{1} e^{x^2}\, dx\, dy$

✓ e^{x^2} is Not Integrable

✓ constant value



$\int_{y=0}^{2} \int_{x=y/2}^{1} e^{x^2}\, dx\, dy$

$y \to 0$ to 2 constant (fixed)

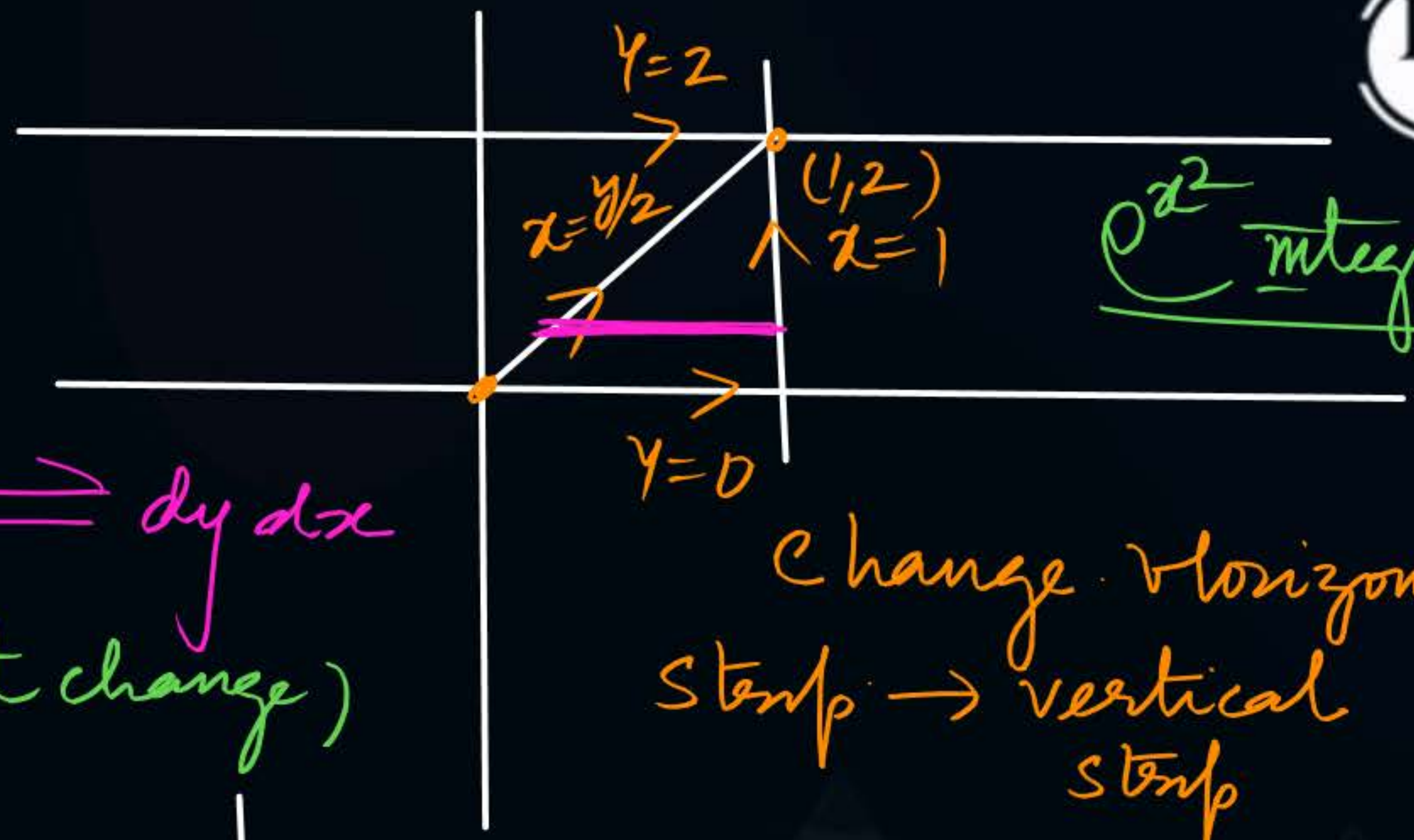
$x \to \frac{y}{2}$ to 1

$$= \int_{y=0}^{2}\int_{x=\frac{y}{2}}^{1} e^{x^2}\,\boxed{dx\,dy}$$

Change

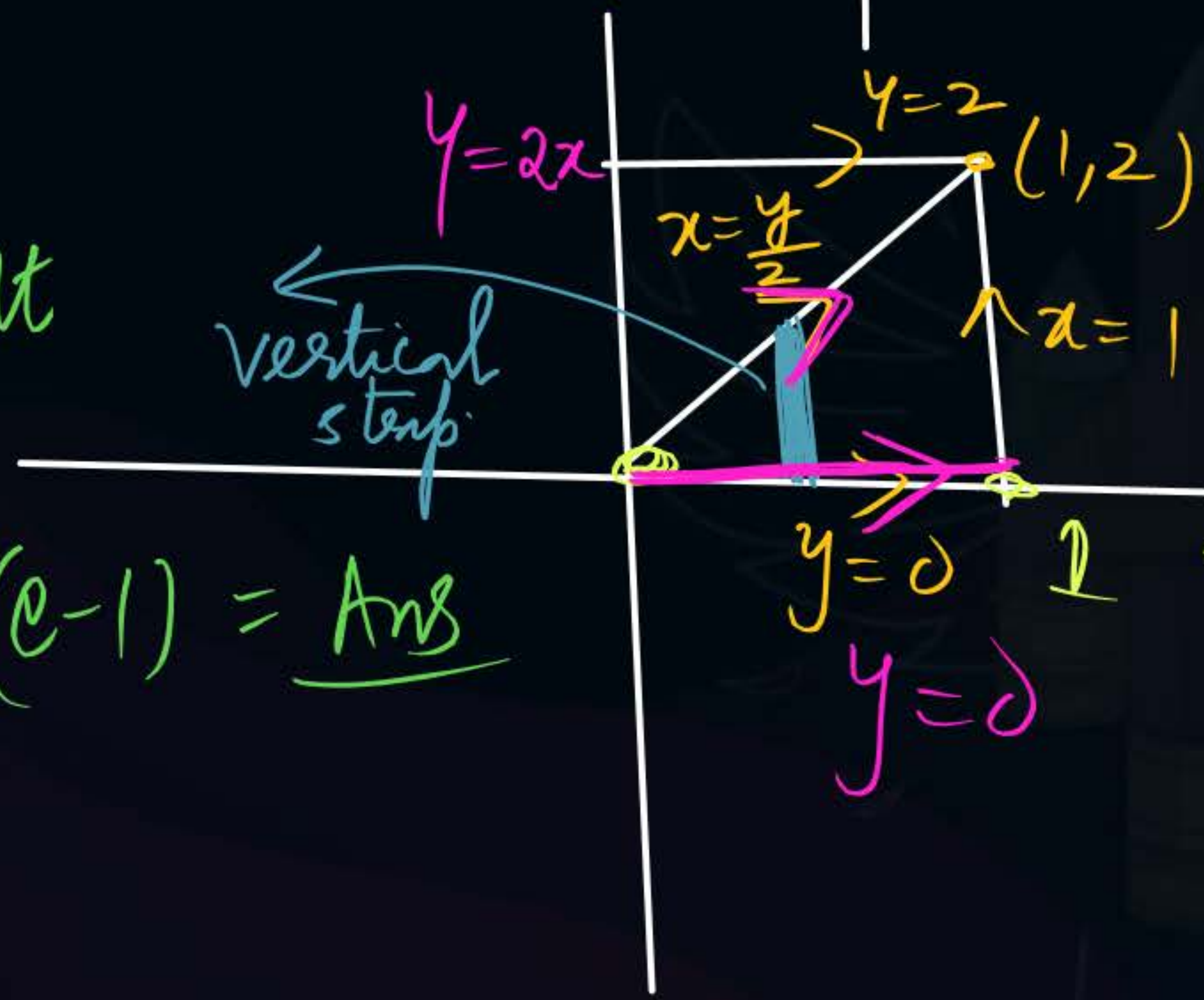
$$dx\,dy \rightleftharpoons dy\,dx$$

$$= \int_{x=0}^{1}\int_{y=0}^{2x} e^{x^2}\,dy\,dx$$ (element change)



e^{x^2} integrate

Change horizontal strip → vertical strip



"CHANGE the ORDER of integration"

$x^2 = t$

$2x\,dx = dt$

$$= \int_0^1 e^{x^2}dx \int_0^{2x} dy$$

$$= \int_0^1 e^{x^2}\cdot 2x\,dx = \int_0^1 e^t\,dt = (e-1) = \text{Ans}$$

$x \to 0$ to 1

CHANGE The Order of Integration :

(A) Plot The Limits

(B) find The strip — Horizontal / Vertical

(C) Horizontal $\rightleftarrows$ vertical

y constant $\rightleftarrows$ x constant
x variable   y variable

(D) Change The element

$$dy\,dx = dx\,dy$$

Topic : Integration

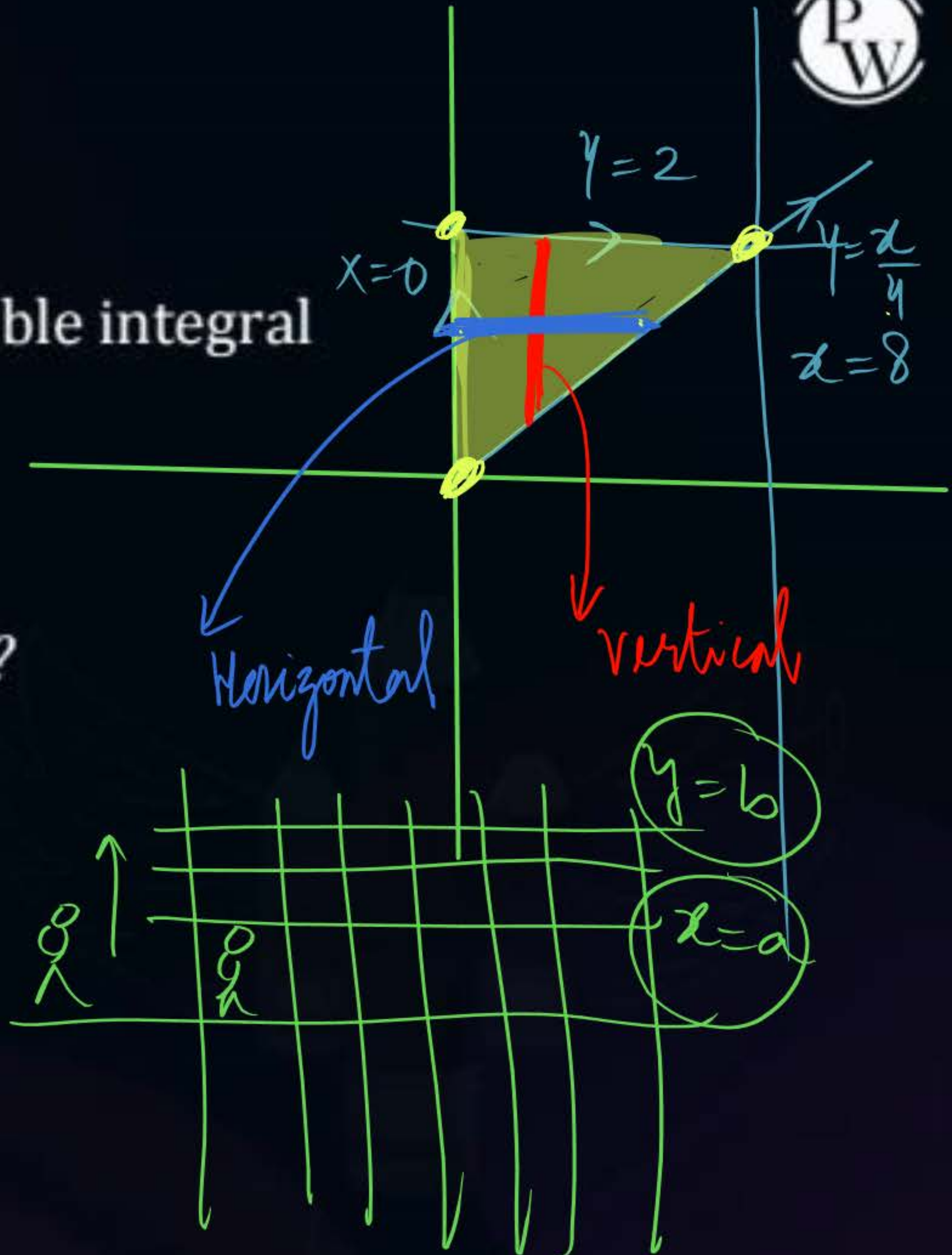
#Q. Changing the order of integration in double integral

$$I=\int_{x=0}^{8}\int_{y=\frac{x}{4}}^{2} f(x,y)\,dy\,dx$$

leads to $I=\int_{r}^{s}\int_{p}^{q} f(x,y)\,\underline{dx\,dy}$. What is q?

$$=\int_{y=0}^{2}\int_{x=0}^{4y} f(x,y)\,dx\,dy$$

$q=4y$



Topic : Integration

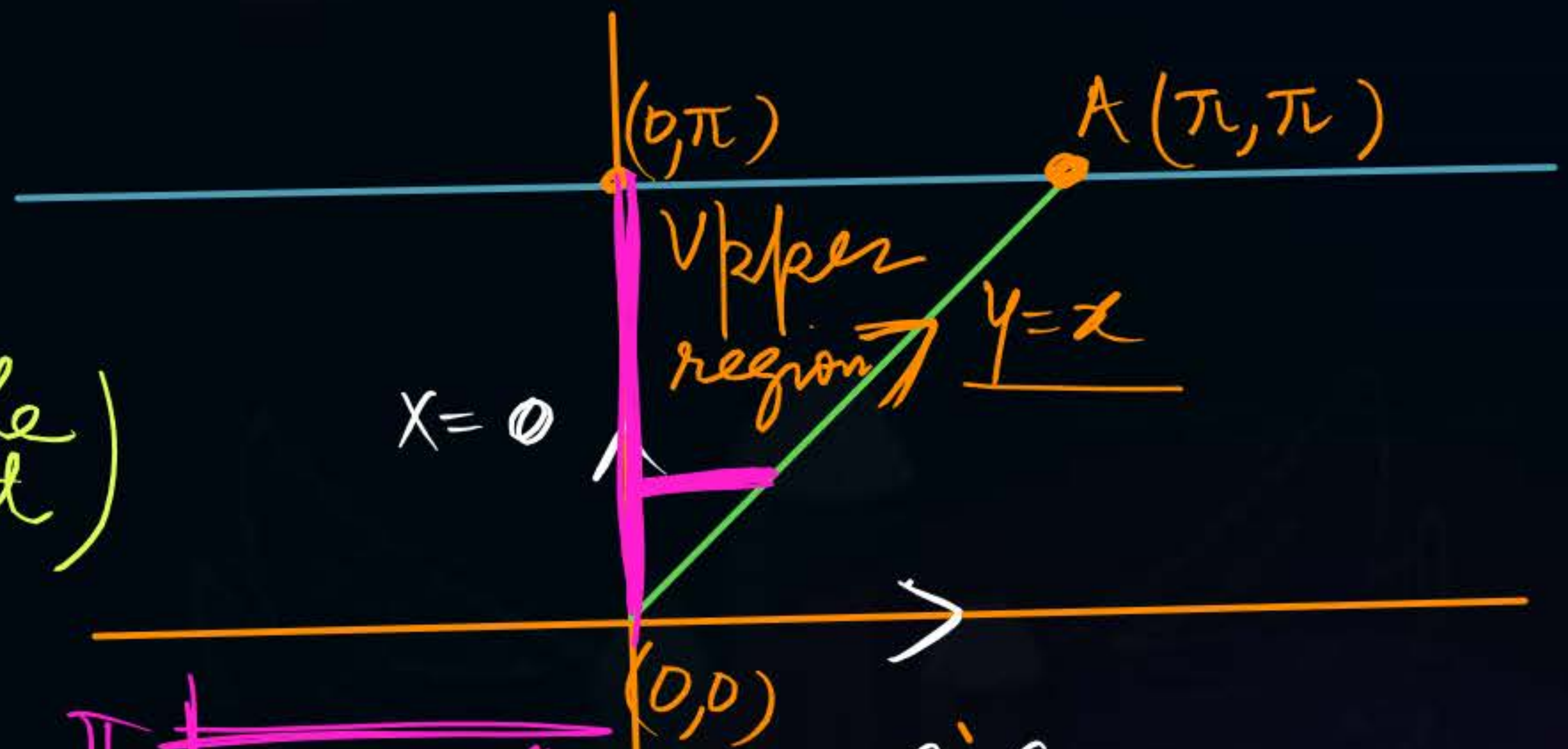
$y = \pi = \text{constant}$

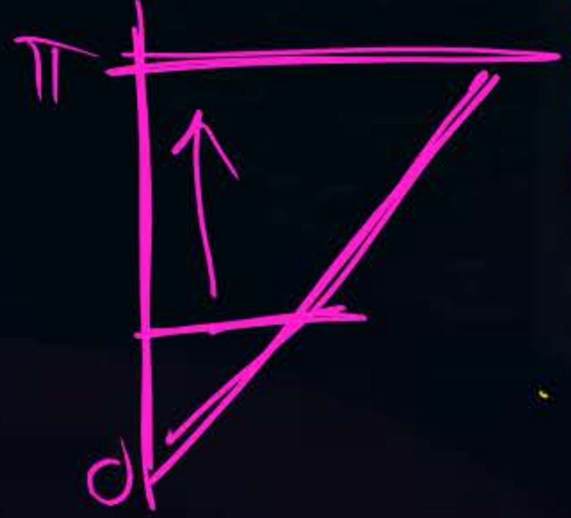
#Q. $\boxed{\iint \cos(x+y)\,dxdy \text{ over the region enclosed by } y = x,\ y = \pi,\ x = 0}$

$$= \int_{y=0}^{\pi}\int_{x=0}^{x=y} \cos(x+y)\,dx\,dy$$

$$= \int_0^{\pi} dy \int_{x=0}^{x=y} \cos(x+y)\,dx \quad \begin{pmatrix} x \text{ variable} \\ y \text{ constant} \end{pmatrix}$$

$$= \int_0^{\pi} dy \Big[+\sin(x+y)\Big]_{x=0}^{y}$$





$$\int_{y=}\int_{x=} \cos(x+y)\,dx\,dy$$

$$= \int_0^{\pi} \int_0^{y} \cos(x+y)\, dx$$

$$= \int_0^{\pi} \left[\sin(x+y)\right]_{0=x}^{y}$$

$$= \int_0^{\pi} \left(\sin 2y - \sin y\right]$$

$$= -2$$

$\sin(180+\theta)$

$=$

Topic : Integration

5 min
Electronics

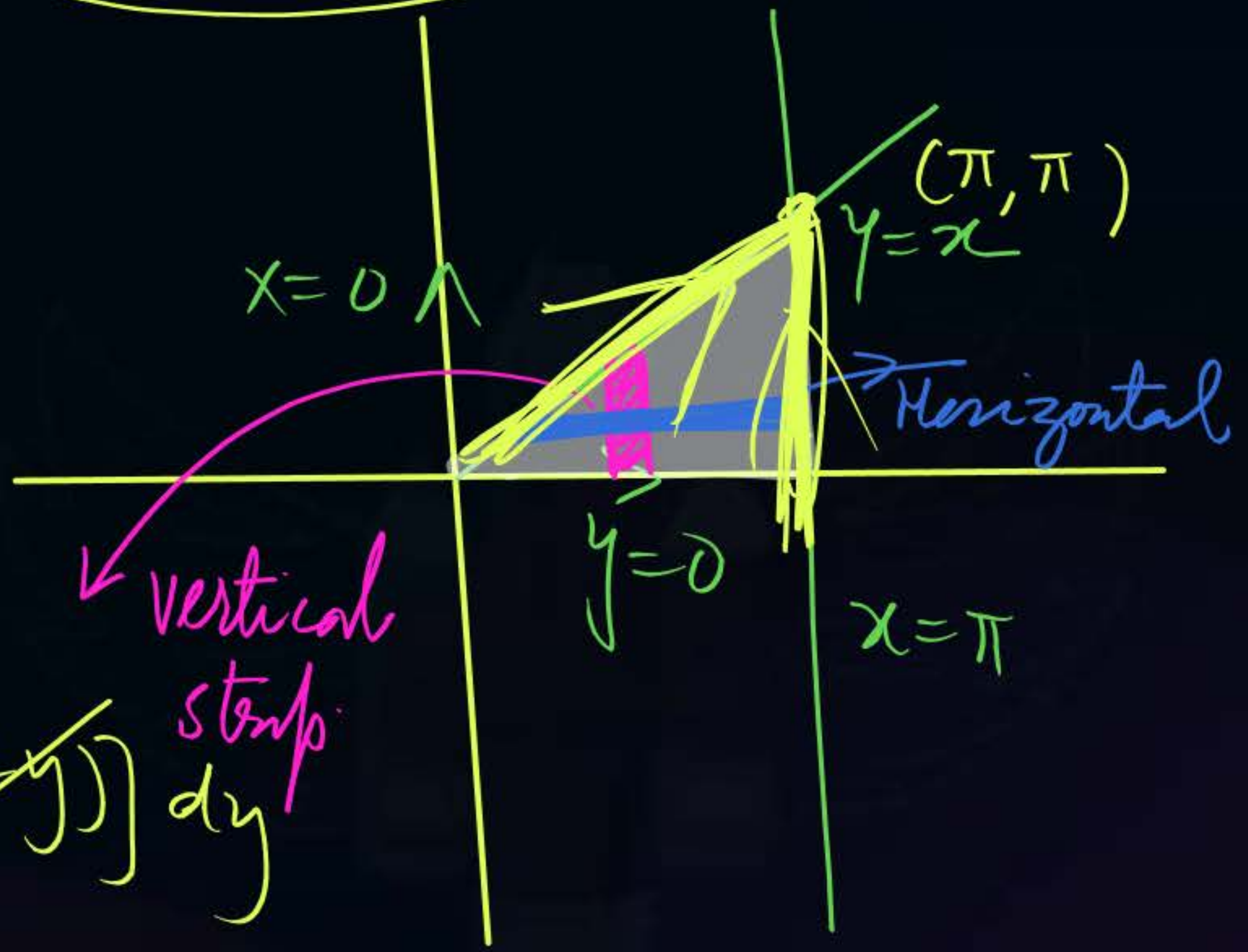
#Q. The value of double integral $\int_{x=0}^{\pi}\int_{y=0}^{x}\frac{\sin y}{(\pi-y)}\,dy\,dx$ $= 2$

$$\Rightarrow \int_{y=0}^{\pi}\int_{x=y}^{\pi}\frac{\sin y}{(\pi-y)}\,dx\,dy$$

$$= \int_{y=0}^{\pi}\frac{\sin y}{(\pi-y)}\int_{y}^{\pi}dx$$

$$= \int_{0}^{\pi}\frac{\sin y}{\cancel{(\pi-y)}}\left[\cancel{(\pi-y)}\right]dy$$

$$= \int_{0}^{\pi}\sin y\,dy = \underline{2}$$



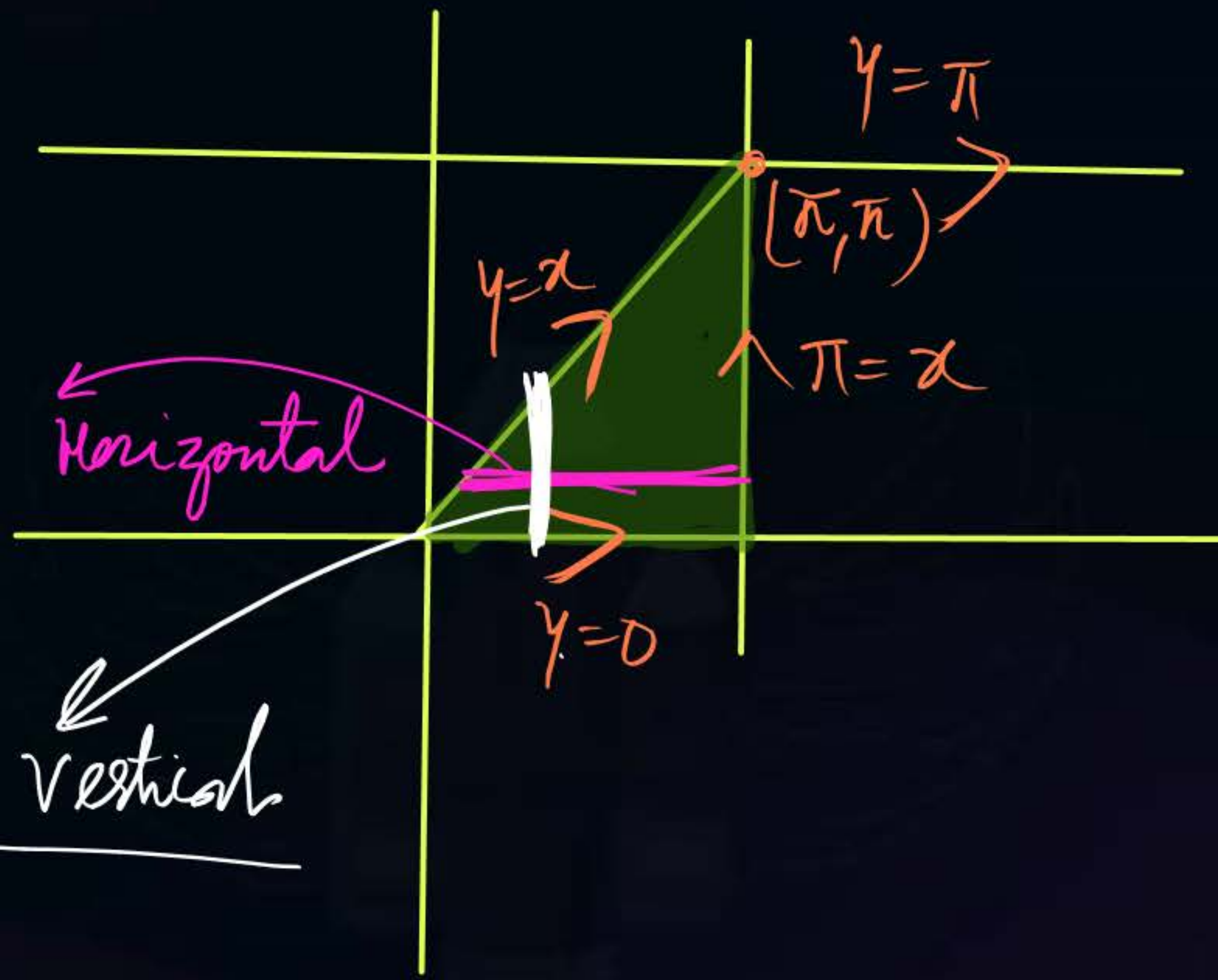
Topic : Integration

#Q. The value of the integral $\int_{y=0}^{\pi}\int_{x=y}^{\pi} \frac{\sin x}{x}\,dx\,dy$ is equal to

$$= \int_{x=0}^{\pi}\int_{y=0}^{x} \frac{\sin x}{x}\,dy\,dx$$

$$= \int_{x=0}^{\pi} \frac{\sin x}{x}\left[\int_{y=0}^{x} dy\right]$$

$$= \int_{0}^{\pi} \frac{\sin x}{\cancel{x}}\left[\cancel{x}-0\right] = \int_{0}^{\pi} \sin x\,dx = 2$$



Topic : Integration

#Q. The value of integral $\int_0^2\int_0^x e^{x+y}\,dx\,dy$ is

ME

A $\frac{1}{2}(e-1)$

B $\frac{1}{2}(e^2-1)^2$

C $\frac{1}{2}(e^2-e)$

D $\frac{1}{2}\left(e-\frac{1}{e}\right)^2$

$= \int_0^2\int_0^x e^x e^y\,dx\,dy$

$= \int_0^2 e^x\,dx \int_0^x e^y\,dy$

$\Rightarrow \int_0^2 e^x\,dx\,\left[e^y\right]_0^x$

$= \int_0^2 e^x\left[e^x - e^0\right]dx$

$= \int_0^2 \left(e^{2x} - e^x\right)dx$

Topic : Integration

H.W

#Q. The double integral $\int_0^a \int_0^y f(x,y)dx\,dy$ is equivalent to

A $\int_0^x \int_0^y f(x,y)dx\,dy$

B $\int_0^a \int_x^y f(x,y)dx\,dy$

C $\int_0^a \int_x^a f(x,y)dy\,dx$

D $\int_0^a \int_0^a f(x,y)dx\,dy$

Topic : Integration

M.W

#Q. Let $I = c\iint_R xy^2 \; dx\,dy$, where R is the region shown in the figure and $c = 6 \times 10^{-4}$. The value of I equals ____.

(Give the answer up to two decimal places)

THANK - YOU