GATE-All BRANCHES
Engineering Mathematics

NUMERICAL METHODS

Lecture No.- 02

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Recap of previous lecture

Topic Numerical methods

Topic Newton Raphson method

Topic Problems based on newton Raphson method and bisection rule

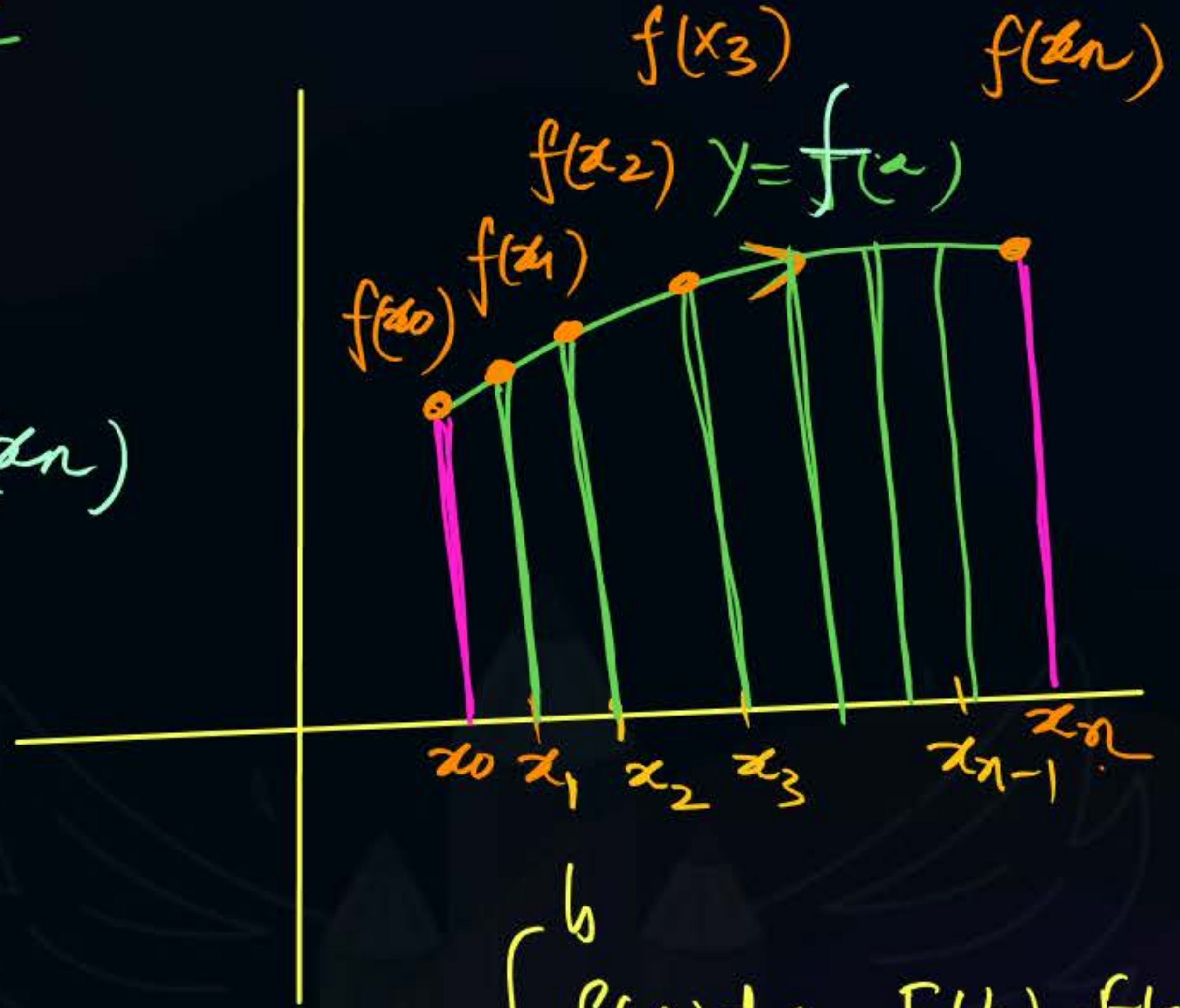
Topics to be covered

- Topic: Numerical integration
- Topic: Simpsons 1/3 rule
- Topic: Simpsons 3/8 rule
- Topic: Trapezoidal rule
- Topic: Problems based on numerical integration

Numerical integration : $\int_a^b f(x)\,dx$

Area = SUM of all rectangles
(Limit of SUM)

$$= \underbrace{x_0 f(x_0)}_{\substack{\text{Rectangle} \\ \text{Area}}} + x_1 f(x_1) + - - + x_n f(x_n)$$



x_0 = Initial guess.

x_n = final value.

$$\int_{x_0}^{x_n} f(x)\,dx = \underline{\text{Numerical integration}}$$

$$\int_a^b f(x)\,dx = \underline{F(b) - F(a)}$$

Newton's method

$x_0 = x_0$

$x_1 = x_0 + h$

$x_2 = x_0 + 2h$

$x_3 = x_0 + 3h$

$\vdots$

$\boxed{x_n = x_0 + nh}$

$$\frac{\text{final value} - \text{Initial value}}{\text{No. of Intervals}}$$

h = step size

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \text{Numerical integral}$$

Calculation for $f(x_i)$ $i = 0, 1, 2, 3 \ldots$

$y_i = f(x_i)$ → Trapezoidal method (more errors)

→ Simpson's $\frac{1}{3}$

→ $\frac{3}{8}$th Rule

Trapezoidal Rule:

Total AREA

$A_1 + A_2 + A_3 + A_4 + A_5$

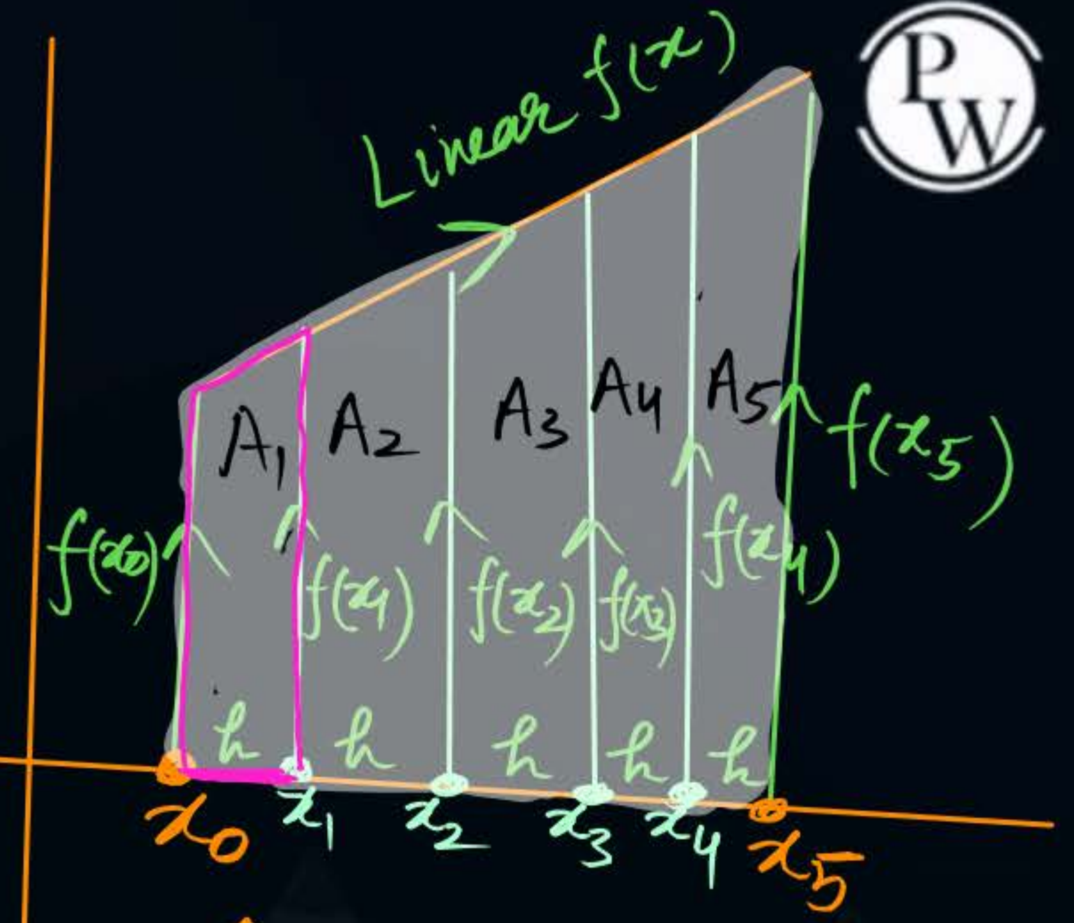
$A_1 = \frac{h}{2}\left[f(x_0) + f(x_1)\right]$ $\quad A_2 = \frac{h}{2}\left[f(x_1) + f(x_2)\right]$

$A_3 = \frac{h}{2}\left[f(x_2) + f(x_3)\right]$ $\quad A_4 = \frac{h}{2}\left[f(x_3) + f(x_4)\right]$

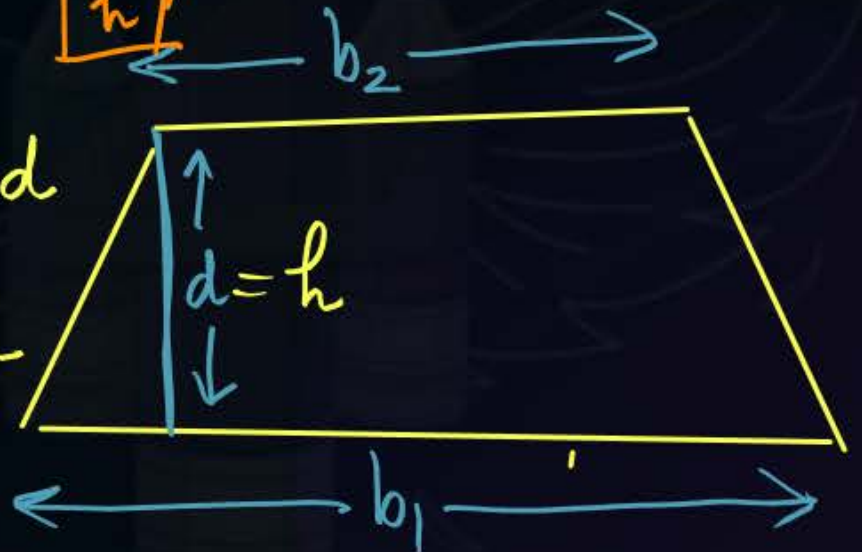
$A_5 = \frac{h}{2}\left[f(x_4) + f(x_5)\right]$

Total Area

$$\int_{x_0}^{x_5} f(x)\,dx = \frac{h}{2}\left[\left[f(x_0) + f(x_5)\right] + 2\left[f(x_1) + f(x_2) + - + f(x_4)\right]\right]$$



$$\text{AREA} = \frac{1}{2}(b_1 + b_2)d$$
$$= \frac{1}{2}(b_1 + b_2)h$$



$$\int_{x_0}^{x_5} f(x)\,dx = \frac{h}{2}\left[\left(f(x_0)+f(x_5)\right) + 2\left(f(x_1)+f(x_2)+f(x_3)+f(x_4)\right)\right]$$

$$\int_{x_0}^{x_5} f(x)\,dx = \frac{h}{2}\left[(y_0+y_5) + 2(y_1+y_2+y_3+y_4)\right]$$

For n Trapezoidal Rule:

$$\boxed{\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+\cdots+y_{n-1})\right]}$$

Rule: ✓

$x_0 = 0$ ✓ [possible case]

Trapezoidal Rule

Simpson $\frac{1}{3}$rd Rule: any Polynomial ✓

Parabola

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+--y_{n-1})+2(y_2+y_4+--y_{n-2})\right]$$

$$(F+L)+4(\text{odd})+2(\text{even})$$

FLOE

$$h = \frac{\text{Last}-\text{final}}{n}$$

Simpson $\frac{3}{8}$th Rule:

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n)+4(y_1+y_3+---y_{n-1})+2(y_2+y_4+--+y_{n-2})\right.$$

$$(F+L)+4(\text{odd})+2(\text{even})$$

Topic : Numerical Methods

$\int_0^1 e^x dx = e - e^0 = \underline{e-1}$

#Q. The value of $\int_0^1 e^x\,dx$ using the $\boxed{\text{trapezoidal rule}}$ with $\underbrace{\text{four equal}}_{}$ subintervals is

For equal sub.

$\int_0^1 e^x dx \longrightarrow$ Numerically compute

A. 1.718

B. 2.192

C. 1.727

D. 2.718

$$\int_0^1 e^x \, dx$$

c) corresponding value of y

$$y = f(x) = e^x$$

$$f(x_0) = y_0 = e^{x_0} = e^0 = 1$$

$$f(x_1) = y_1 = e^{x_1} = e^{\frac{1}{4}} = e^{\frac{1}{4}}$$

$$f(x_2) = y_2 = e^{x_2} = e^{1/2} = e^{1/2}$$

$$f(x_3) = y_3 = e^{x_3} = e^{3/4} = e^{3/4}$$

$$f(x_4) = y_4 = e^{x_4} = e^1 = e$$

Step 1 $\qquad n = 4$

A) Calcute h

$$h = \frac{1-0}{4} = \frac{1}{4} = 0.25$$

(B) Calulate Iteration of x

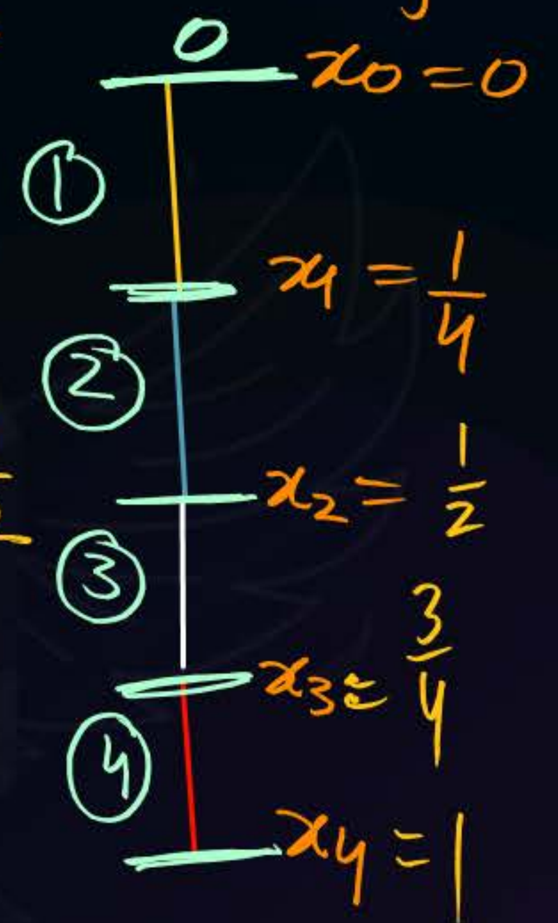
$$x_0 = 0$$

$$x_1 = x_0 + h = 0 + \frac{1}{4} = \frac{1}{4}$$

$$x_2 = x_1 + h = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$x_3 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$x_4 = \frac{4}{4} + \frac{4}{4} = 1$$



Using Trapezoidal Rule

$$\int_0^1 e^x\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

$$\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$$

$$= \frac{0.25}{2}\left[(e^0+e^1)+2(e^{0.25}+e^{.50}+e^{.75})\right]$$

$$\Rightarrow 1.727$$

$$\text{Absolute Error} = |W-T|$$

$$= 8.918\times10^{-3}$$

C

$$\int_0^1 e^x\,dx$$

$$= e-1$$

$$=$$

$\int_{x_0}^{x_0+nh}$ final $h = 0.3$ Initial

#Q. Value of $\int_{\boxed{4}}^{5.2} \ln x\, dx$ using Simpson's one-third rule with interval size 0.3 is

using simpson

$= \frac{0.3}{3}\left[\left[(\ln 4) + \ln(5.2)\right] + 4(\ln 4.3 + \ln 4.9)\right] + 2\left[\ln(4.6)\right]$

$= \underline{1.83}$

4 | 4.3 | 4.6 | 4.9 | 5.2

$x_0 = 4 \quad x_1 = 4.3 \quad x_2 = 4.6$

$x_3 = 4.9 \quad x_4 = 5.2$

$y_0 = \ln 4 \quad y_2 = \ln 4.6$

$y_1 = \ln 4.3 \quad y_3 = \ln 4.9$

$y_4 = \ln(5.2)$

A 1.83 ✓

B 1.60

C 1.51

D 1.06

Topic : Numerical Methods

#Q. The evaluation of the definite integral $\int_{-1}^{1.4} x|x|dx$ by using Simpson's $1/3^{rd}$ (one-third) rule with step size $h = 0.6$ yields.

H.W

A 0.914

B 1.248

C 0.581

D 0.592

$h = \frac{(4-2)}{2} = 1$

#Q. Evaluation of $\int_2^4 x^3 dx$ using a 2-equal- segment trapezoidal rule gives a value of ________.

$\int_2^4 x^3 dx$

$y_0 = (2)^3 = 8$

$y_1 = (3)^3 = 27$

$y_2 = (4)^3 = 64$

$x_0 = 2$

$3 = x_1$

$x_2 = 4$

Exm $= \frac{1}{2}\left[(8+64) + 2(27)\right]$

$= 63$

Topic : Numerical Methods

#Q. Numerical integration using trapezoidal rule gives the best results for a single variable function, which is

A. Linear

B. Parabolic

C. Logarithmic

D. Hyperbolic

$h = \frac{\pi - 0}{3} = \frac{\pi}{3}$

#Q. The error in numerically computing the intergral $\int_0^{\pi}(\sin x + \cos x)dx$ using the trapezoidal rule with three intervals of equal length between 0 and π is ________.

$x_0 = 0 \quad y_0 = \sin 0 + \cos 0$

$x_1 = \frac{\pi}{3} \quad y_1 = \sin\frac{\pi}{3} + \cos\frac{\pi}{3}$

$x_2 = \frac{2\pi}{3} \quad y_2 = \sin\left(\frac{2\pi}{3}\right) + \cos\left(\frac{2\pi}{3}\right)$

$x_3 = \pi \quad y_3 = \sin\pi + \cos\pi$

$\underbrace{0 \quad \frac{\pi}{3}}_{1} \quad \underbrace{\frac{\pi}{3} \quad \frac{2\pi}{3}}_{2} \quad \underbrace{\frac{2\pi}{3} \quad \pi}_{3}$

$I = \int_0^{\pi}(\sin x + \cos x)\,dx = 2$

$\text{Trapezoidal} = \frac{h}{2}\left[(y_0 + y_3) + 2(y_1 + y_2)\right] = \underline{1.8137}$

$\text{Error} = |2 - 1.8137|$

$= \underline{.1863}$

Topic : Numerical Methods

$\checkmark \int_0^1$

#Q. Simpson's $\frac{1}{3}$ rule is used to integrate the function $f(x) = \frac{3}{5}x^2 + \frac{9}{5}$ between x = 0 and x = 1 using the least number of equal sub-intervals. The values of the intergral is ______.

H.W

Topic : Numerical Methods

N.W

#Q. The value of function f(x) at 5 discrete points are given below :

X	0	0.1	0.2	0.3	0.4
f(x)	0	10	40	90	160

Using Trapezoidal rule with step size of 0.1, the value of $\int_0^{0.4} f(x)dx$ is _____.

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(H.W)

#Q. Using a unit step size, the value of integral $\int_1^2 x \ln x \, dx$ by trapezoidal rule is________

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#Q. The definite integral $\int_1^3 \frac{1}{x} dx$ is evaluated using Trapezoidal rule with a step size of 1. The correct answer is ______.

H.W

#Q. The value of $\int_{2.5}^{4} \ln x \, dx$ calculate using the Trapezoidal rule with five subintervals is ______.

H.W

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#Q. Using the trapezoidal rule, and dividing the interval of integration into three equal subintervals, the definite integral $\int_{-1}^{+1} |x| dx$ is____.

H.W

Topic : Numerical Methods

#Q. The estimated of $\int_{0.5}^{1.5} \frac{dx}{x}$ Obtained using Simpson's rule with three-point function evaluation exceeds the exact value by

H.W

A 0.235

B 0.068

C 0.024

D 0.012

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#Q. The integral $\int_1^3 \frac{1}{x}\,dx$, when evaluated by using Simpson's 1/3 rule on two equal subintervals each of length 1, equals

H.W

A 1.000

B 1.098

C 1.111

D 1.120

2 mins Summary

Topic	One	Trapezoidal
Topic	Two	Simpson $\frac{1}{3}$rd
Topic	Three	Simpson $\frac{3}{8}$th
Topic	Four	
Topic	Five	

THANK - YOU

Topics to be Covered