EE & ECE

Engineering Mathematics

Probability and Statistics

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Lecture No. 4

01 Concept of Classification of events

02 Problems based on Classification of Events

03 Concept of Conditional Probability & Bayes Theorem

04 Problems based on Conditional Probability & Bayes Theorem

Classification of Events :

compound

2 Items mix

Simple Event / Single event

$$P(\text{Event}) = \frac{\text{No. of Favourable}}{\text{Total Possible outcomes}}$$

Compound Events

Compound Events :

min 2 Events or more Than 2 Events

Compound Events → Two Events, Three events, |, n events

case 01

E and F Are Two Events

\# E and F → something is common

$P(E \cap F) = P(E \text{ and } F)$

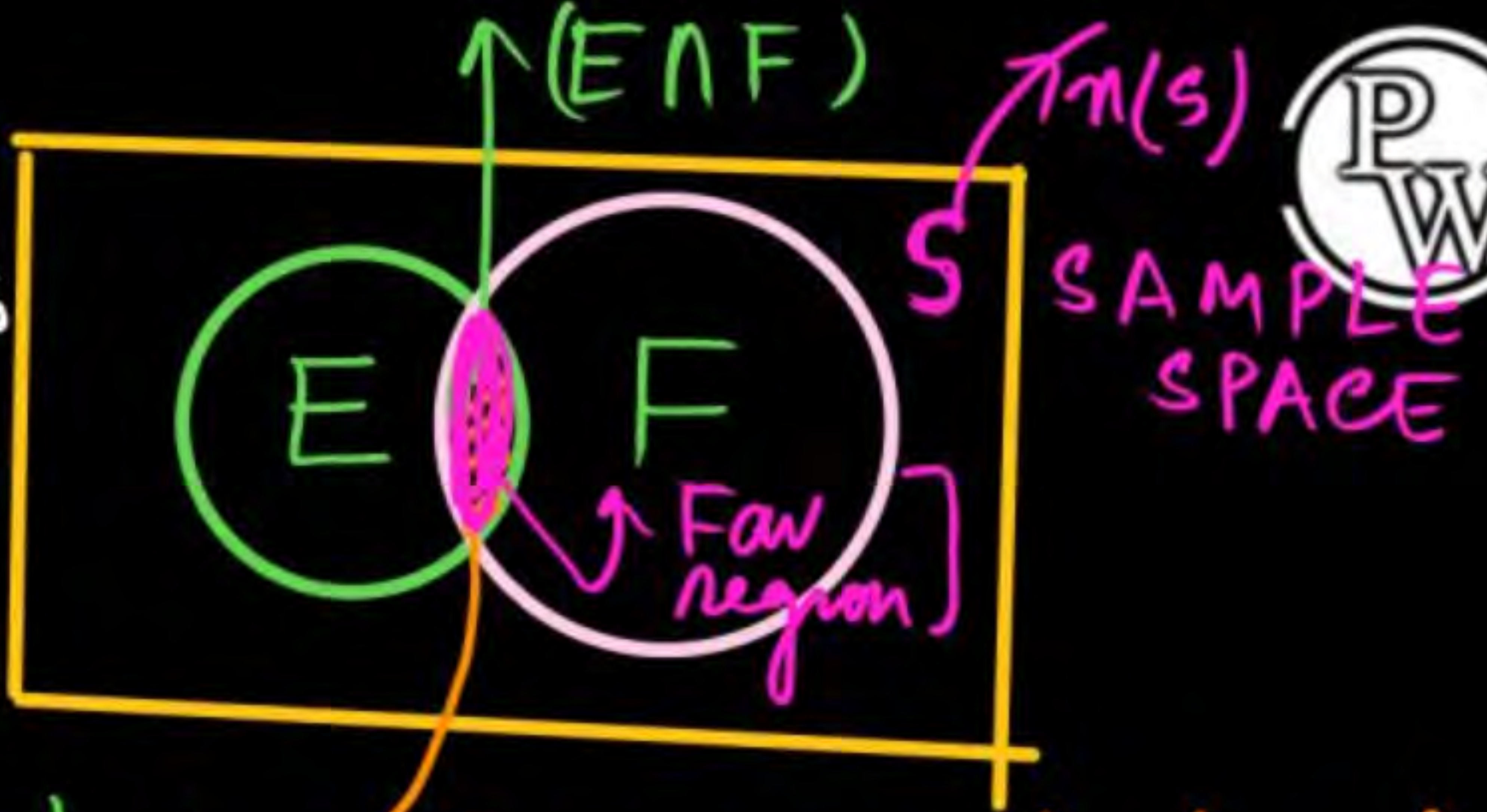
$= P(\text{both occur})$

$= P(\text{common region})$

$P(E \text{ and } F) = \dfrac{\text{fav region}}{\text{Total region}}$

$(E \cap F)$

$= \boxed{\dfrac{n(E \cap F)}{n(S)}}$



'E and F' → common ∩ (Intersection)

$b_2 \cap b_1 = \{2, 4\}$

$$P(E \text{ and } F) = P(E \cap F) = P(\text{both occur}) = \frac{n(E \cap F)}{n(S)}$$

Illustrations

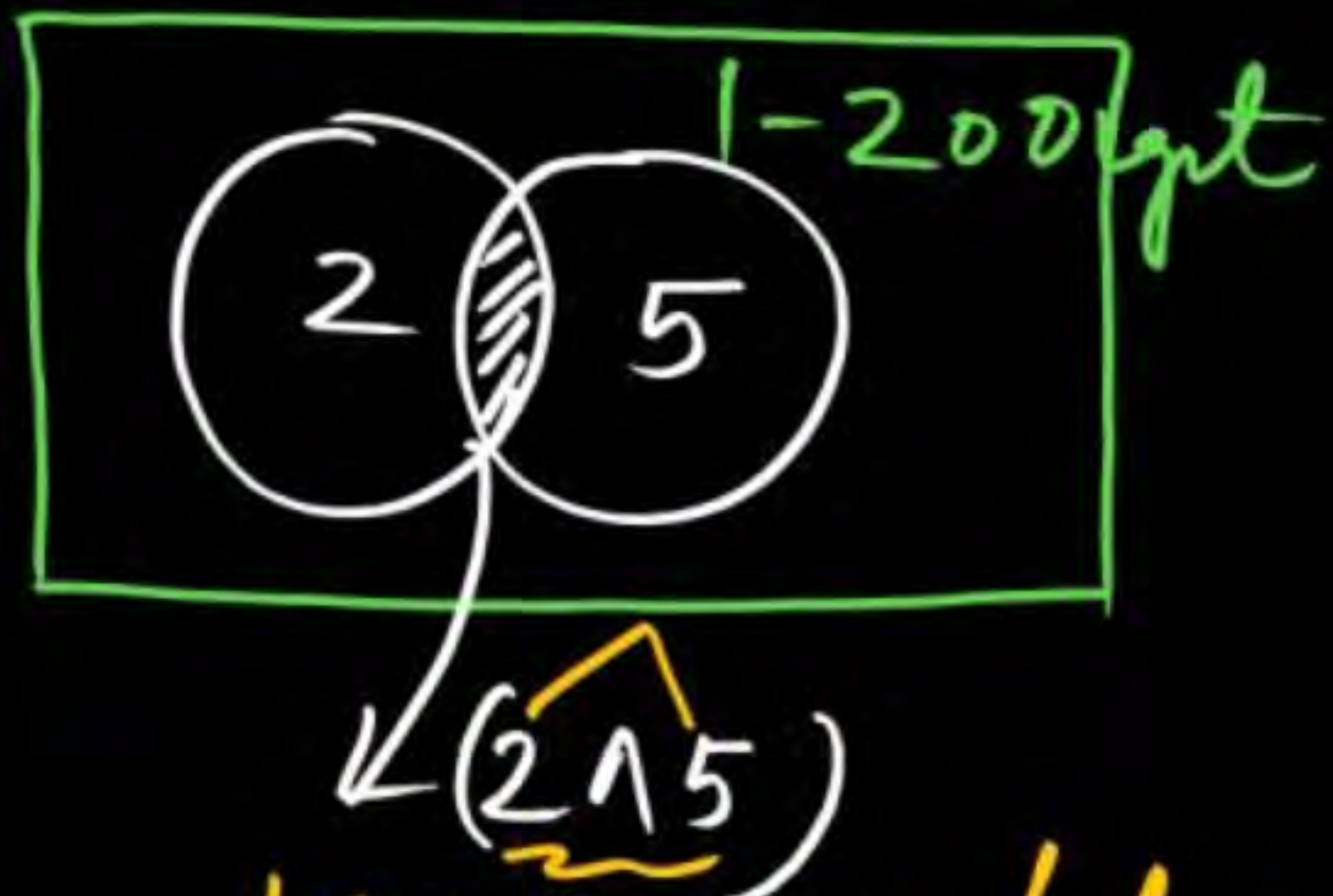
1-20 Digit

$2 \rightarrow$ Div by 2

$5 \rightarrow$ Div by 5

What is the Prob $P(\text{div by 2 and 5})$

$P(\text{both occur 2 and 5})$



Lcm 10 divisible.

$$P(2 \cap 5) = \frac{n(2 \cap 5)}{n(S)} = \frac{2}{20} = \frac{1}{10}$$

$$n(2 \cap 5) = \{10, 20\}$$

CASE 02 :

✓ $\boxed{\text{something is common}}$

$P(E \text{ or } F) = P(E \cup F)$

$= P(\text{either } E \text{ or } F)$

$P(E \text{ or } F) = \dfrac{\text{No. of Favourable}}{\text{Total outcomes}}$

$\Rightarrow ✓ \dfrac{\text{White region \#}}{\text{rectange region \#}}$

$\Rightarrow \dfrac{n(E)+n(F)-n(E \wedge F)}{n(S)}$

$\Rightarrow \dfrac{n(E)}{n(S)} + \dfrac{n(F)}{n(S)} - \dfrac{n(E \wedge F)}{n(S)} = P(E) + P(F) - P(E \wedge F)$

$\underbrace{n(E \cup F)}_{\text{fav region}} = \underbrace{n(E) + n(F) - n(E \wedge F)}_{}$

White region

$\#_1 + \#_2 - \#_{\text{blue}}$

(Prob Add Theorem)

OR $\cup$

AND $= \cap$ (both)

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

(something is comm

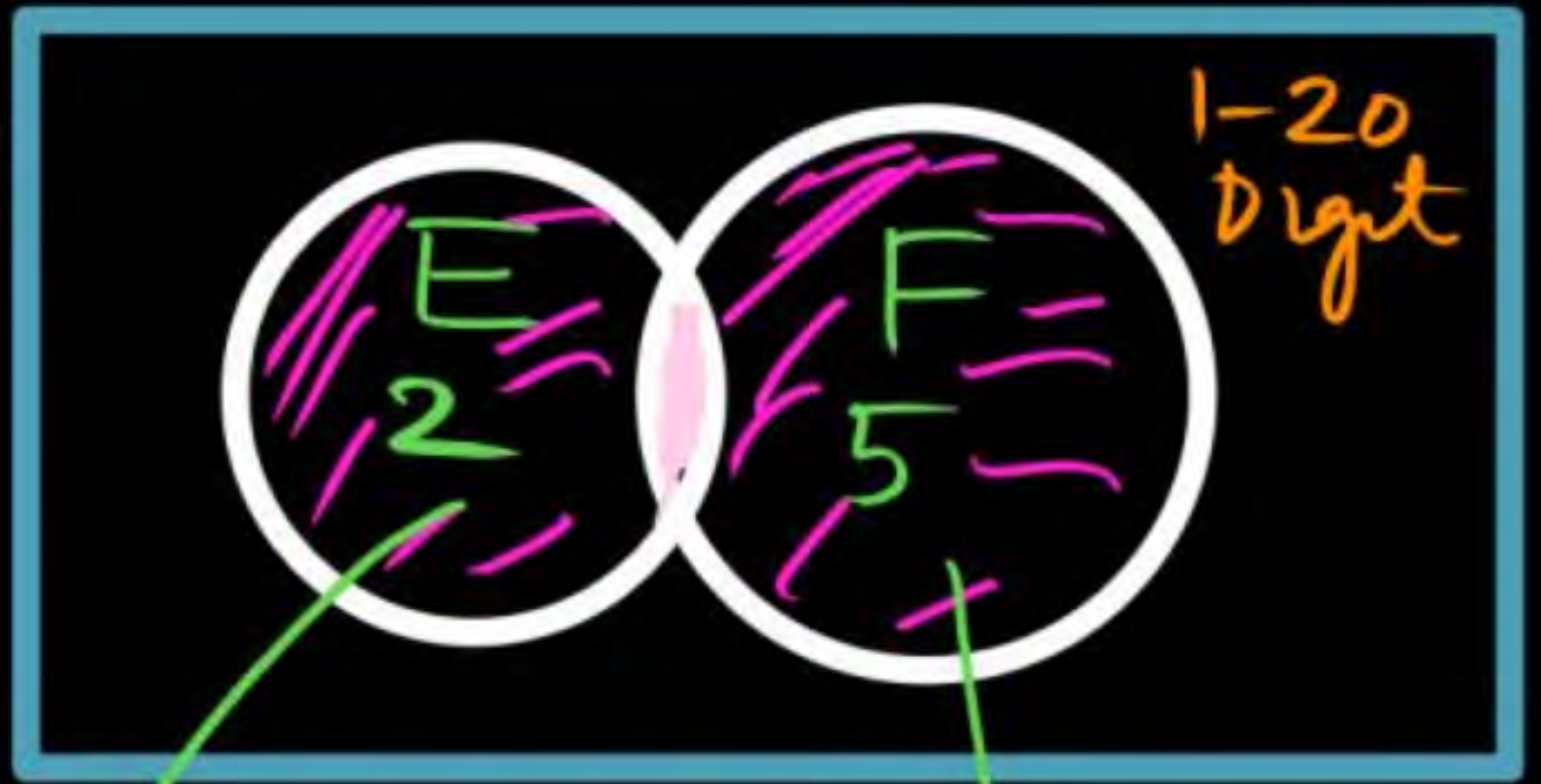
(2) What is the Probability

P(div by 2 or 5)

$P(2 \cup 5) = P(2) + P(5) - P(2 \cap 5)$

$= \frac{10}{20} + \frac{4}{20} - \frac{2}{20}$

$= \frac{12}{20} = \frac{6}{10} = \frac{3}{5}$



2, 4, 6, 8, 10, 12, 16, 14, 18, 20

5, 10, 20, 15

Dependent Events / Independent events :

Occuring / occured ← Without Replacement | With replacement

Dependent Events : (without replacement)

3 White Ball Draw

box W_1 W_2

What is the Prob 3 White balls Are Drawn one at a random

$P\left(\frac{\text{White } 2}{\text{White } 1}\right)$

W_1 is already Happend.

$\frac{W_2}{W_1}$ Happening

Ist white ball

$P(\text{White}_1) = \frac{3}{6}$

IInd white ball

$P\left(\frac{W_2}{W_1}\right) = \frac{2}{5}$

$P\left(\frac{W_3}{W_1 \text{ and } W_2}\right) = \frac{1}{4}$

$P(W_1) = \frac{3}{6}$ $P\left(\frac{W_2}{W_1}\right) = \frac{2}{5}$ $P\left(\frac{W_3}{\underbrace{W_1 \text{ and } W_2}_{W_1 \cap W_2}}\right) = \frac{1}{4}$ $P\left(\frac{W_3}{W_1 \cap W_2}\right) = \frac{1}{4}$

$W_1\, W_2\, W_3$ all working Together

✓ Dependent

✓ without replacement

$P(W_1 \cap W_2 \cap W_3) \Rightarrow P(W_1)\, P\left(\frac{W_2}{W_1}\right) P\left(\frac{W_3}{W_1 \cap W_2}\right)$

White 1 → E
White 2 → F
White 3 → G

$$\boxed{P(E \cap F \cap G) \Rightarrow P(E)\, P\left(\frac{F}{E}\right) P\left(\frac{G}{E \cap F}\right)}$$

✓ For Two Events $P(E \cap F) = P(E)\, P\left(\frac{F}{E}\right)$

Dependence of events

for n Events

$$P[E \cap F \cap G \cap H \cap I \ldots] \Rightarrow P(E)\, P\left(\frac{F}{E}\right) P\left(\frac{G}{E \cap F}\right) P\left(\frac{H}{E \cap F \cap G}\right) \ldots$$

INDEPENDENT EVENTS :

What is the Prob 3 white balls Are Drawn one at a time

with replacement

Place

SAMPLE SPACE DONT Change.

Ist draw

W_1

$P(W_1) = \frac{3}{7}$

SECOND DRAW

W_2

$P\left(\frac{W_2}{W_1}\right) = \frac{3}{7}$

Third Draw

W_3

$P\left(\frac{W_3}{W_1 \cap W_2}\right) = \frac{3}{7}$

$$P(W_1 \cap W_2 \cap W_3) = P(W_1)\, P\left(\frac{W_2}{W_1}\right) P\left(\frac{W_3}{W_1 \cap W_2}\right)$$

$$P\left(\frac{W_2}{W_1}\right) = P(W_2) \qquad P\left(\frac{W_3}{W_1 \cap W_2}\right) = P(W_3)$$

$$\boxed{P(W_1 \cap W_2 \cap W_3) = P(W_1)\, P(W_2)\, P(W_3)}$$

Independent events

White $\left.\begin{array}{l} W_1 \to E \\ W_2 \to F \\ W_3 \to G \end{array}\right\}$

$P(E \cap F \cap G) = P(E)P(F)P(G)$ (Stochastic Indep.)

For Two Events $\boxed{P(E \cap F) = P(E)P(F)}$ (Independence of Events)

For n Events

$$P(E \cap F \cap H \cap G \cap I \ldots) = P(E)P(F)P(G)P(H) \ldots$$

INDEPENDENCE of Events

✓ CASE 05 : P(only E), P(only F), P(exactly one E)

'SOMEthing is COMMON'

P(only E) "E and F Are Indep."

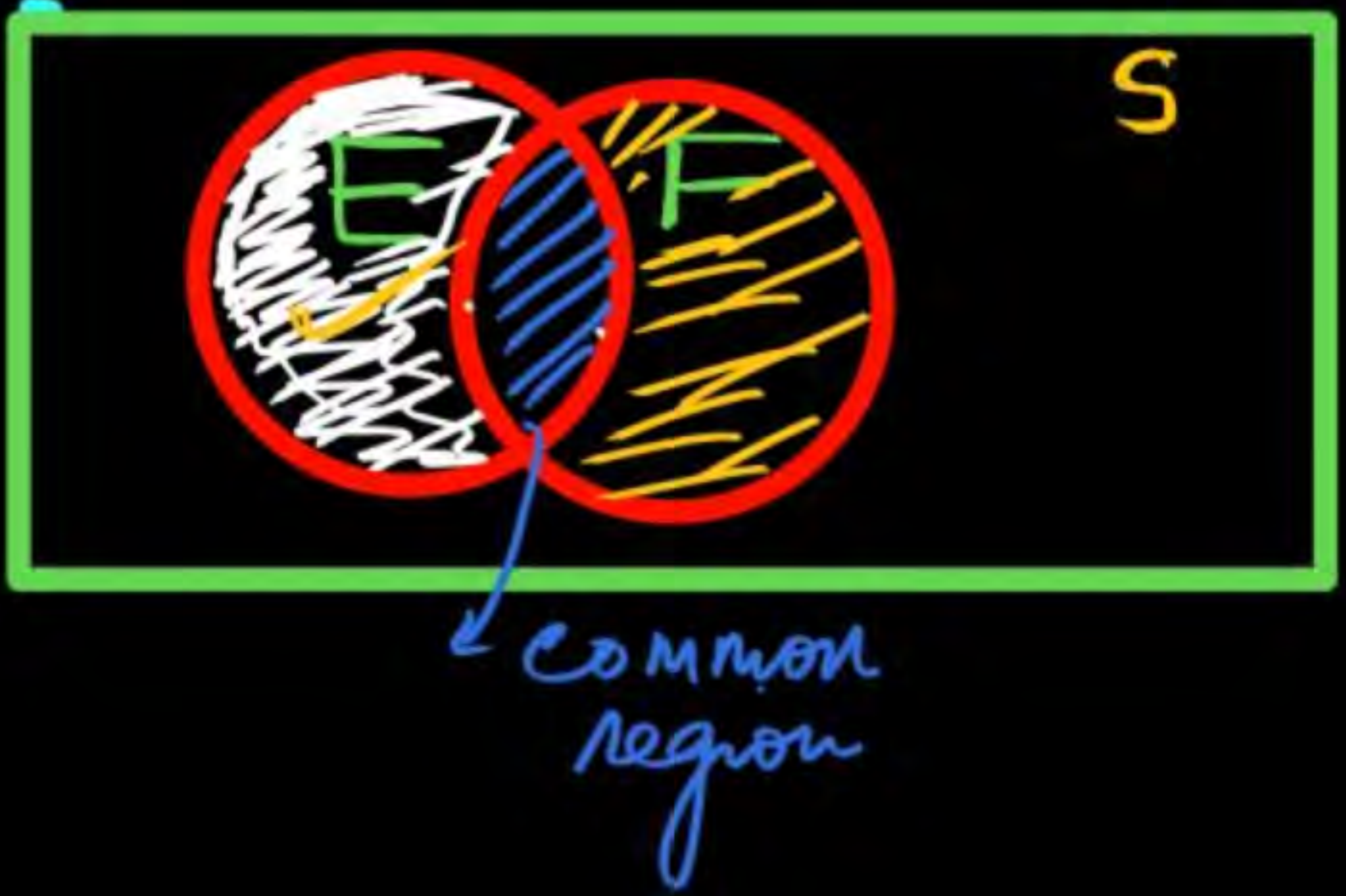
White region

$= P(E) - P(E \cap F)$

$P(\text{only } E) = P(E) - P(E)P(F)$

$$\boxed{P(\text{only } E) = P(E)[1 - P(F)]}$$

$$\boxed{P(\text{only } F) = P(F)[1 - P(E)]}$$



$E, F \rightarrow$ Indep.
$\bar{E}, \bar{F} \rightarrow$ Indep.

$$P(\text{only } E) = P(E \cap \bar{F}) = P(E)P(\bar{F}) = P(E)[1 - P(F)]$$

$$\boxed{P(\text{only } E) = P(E) - P(E)P(F)}$$

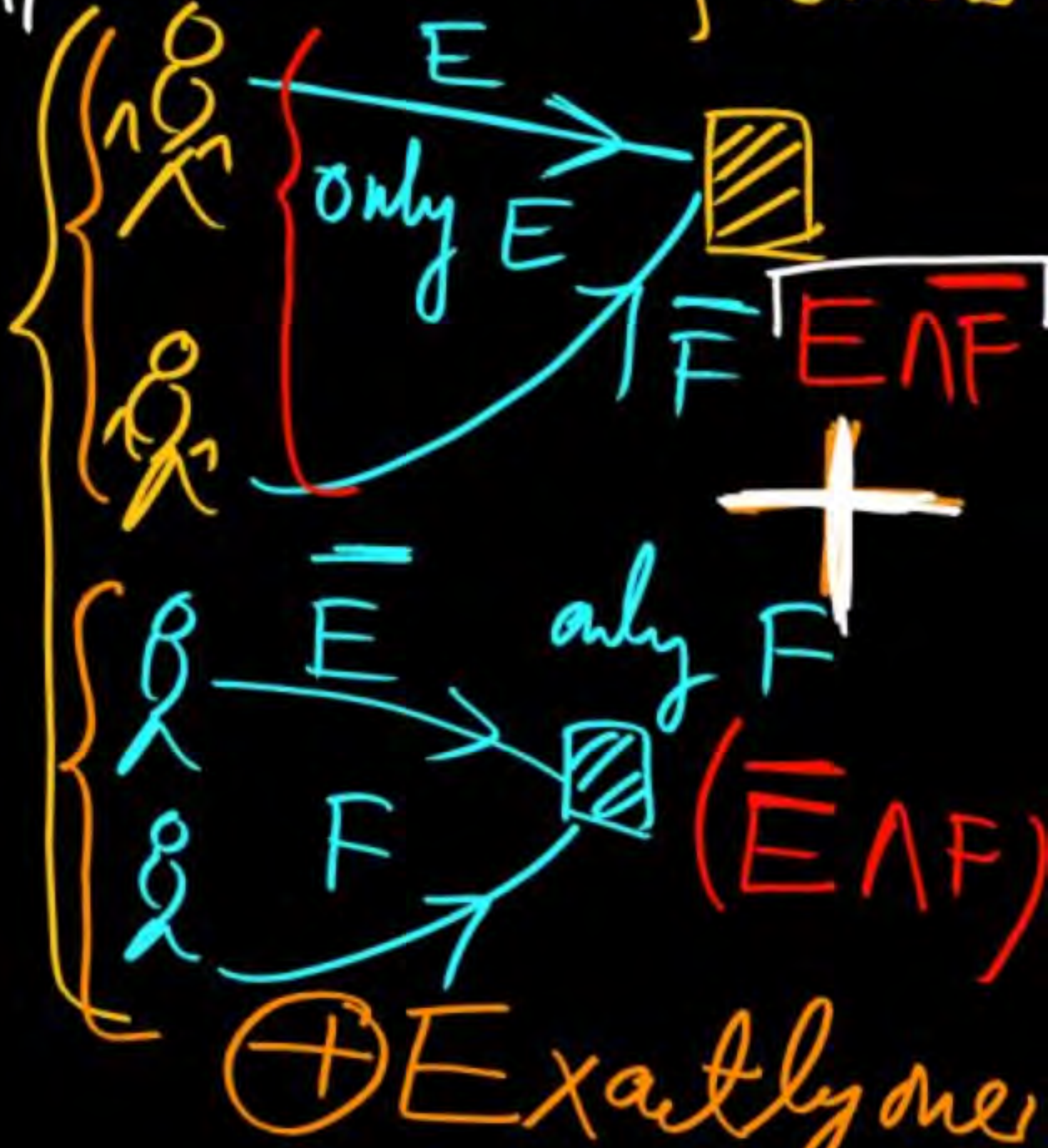
$$P(F \cap \bar{E}) = \boxed{P(\text{only } F) = P(F) - P(E)P(F)}$$

$$P[\text{Exactly one occur}]$$
$$\Rightarrow P(\text{only } E) + P(\text{only } F)$$
$$\Rightarrow P(E \cap \bar{F}) + P(F \cap \bar{E})$$

$$\boxed{P(\text{exactly one}) = P(E) + P(F) - 2P(E)P(F)}$$

E and F Are Independent of Events

#



$\rightarrow E$: E Happen; $\bar{E}$ Does Not Happen

$\rightarrow F$: F (Happen); $\bar{F}$ (Does)

"Classification of TWO Events"

	E	$\bar{E}$
F	$E \cap F$ (both occur)	$\bar{E} \cap F$ (only F)
$\bar{F}$	$E \cap \bar{F}$ (only E)	$\bar{E} \cap \bar{F}$ [Neither E Nor F]

Prob of Exactly one

$P(\bar{E} \cap F)$
$+ P(E \cap \bar{F})$

10:35 – Class resume

Q. Questions

A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even' then find whether E and F are independent?

Q. Questions

6 or 8 $P(6 \vee 8)$

Div by 6 → 6, 12, ... 198

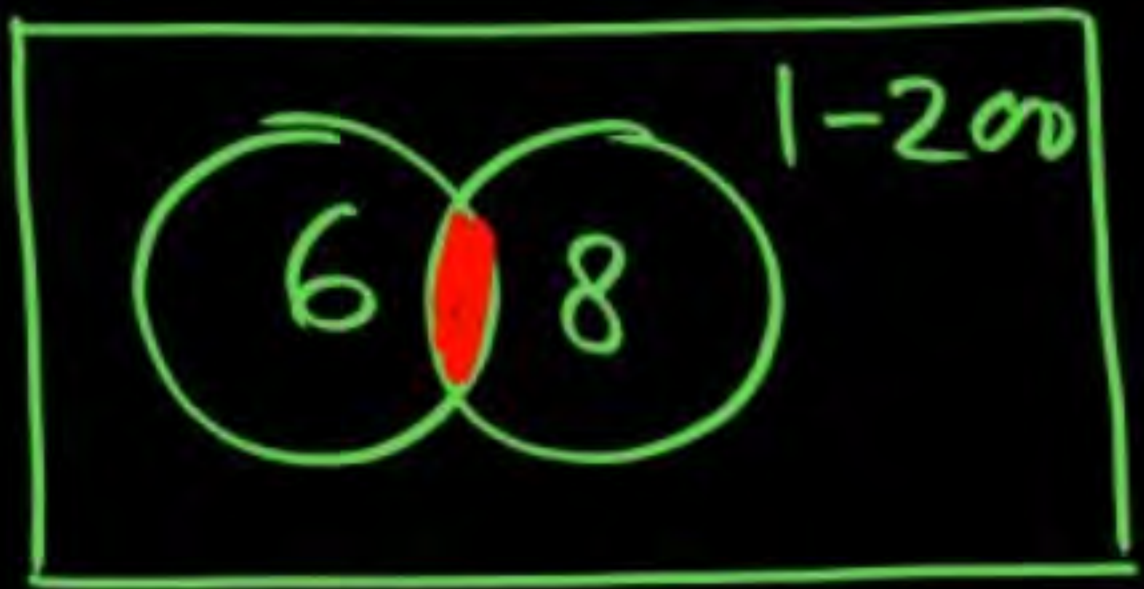
$\frac{200}{6} = 33$ 6, 8 LCM 24

$\frac{200}{8} = 25$ $\frac{200}{24} = 8$

something is common

An integer is chosen at random from 200 positive integers. Find the probability that the integer chosen is divided by 6 or 8 6 or 8

$$\boxed{P(6 \vee 8) = P(6) + P(8) - P(6 \wedge 8)}$$



(a) ✓ 1/4

(b) 1/6

(c) 1/2

(d) N.O.T.

$$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200}$$

$$= \frac{50}{200}$$

$$P(6 \text{ or } 8) = \frac{1}{4}$$

$$\left.\begin{aligned} n(6) &= 33 \\ n(8) &= 25 \\ n(6 \wedge 8) &= 8 \end{aligned}\right\}$$

Q. Questions

2 Minutes

Two cards are drawn from pack of 52 cards. What is the probability that either of the cards, both are red or both are kings

(a) 660/2652

(b) 660/1352

(c) 330/2652

(d) 1/52×53

52 cards → 2 cards → Red R or V King K

52 → 2card — red R or V King K

26 cards red
52 cards

$P(R) = \frac{26}{52} \times \frac{25}{51}$

I II

$(R_1) \left(\frac{R_2}{R_1}\right)$

4 card king
52 cards

$P(K) = \frac{4}{52} \times \frac{3}{51}$

I II

2 cards
52 cards

$P(R \cap K) = \frac{2}{52} \times \frac{1}{51}$

I II

$$P(R \cup K) = \frac{26 \times 25 + 4 \times 3 - 2 \times 1}{52 \times 51} = \frac{660}{2652}$$

Ans

$\Rightarrow P(\text{red or King})$

$\Rightarrow P(R \cap K)$

$\Rightarrow P(R) + P(K) - P(R \cap K)$

red $\rightarrow$ 26 $\rightarrow$ Diamond, Heart

King $\rightarrow$ 4 $\rightarrow$ Diamond, HEART, Club, Spade

red And King

Diamond: R, K
Heart: R, K

Q. Questions

3 person A, B, C independently try to hit a target, if probability of hitting a target by A, B, C are 3/4, 1/2, 5/8 then the probability that target hit by A or B but not C

$P((A \text{ or } B) \cap \overline{C})$

Hit — Not Hit

System × ∩ System

$P(A \cup B \cap \bar{C})$

$(A \cup B) \cap \overline{C}$

(a) 21/64

(b) 1/64

(c) 7/64

(d) 3/64

$$P(A \cup B \cap \bar{C}) = P(A \cup B) \cdot P(\bar{C})$$

$\because A, B, C$ Are Ind

$$P(A) = \frac{3}{4} \quad P(B) = \frac{1}{2} \quad P(\bar{C}) = 1 - \frac{5}{8} = \frac{3}{8} \checkmark$$

$$P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$$

$$\Rightarrow [P(A) + P(B) - P(A \cap B)]\,P(\bar{C})$$

$$\Rightarrow [P(A) + P(B) - P(A)P(B)]\,P(\bar{C})$$

$$\Rightarrow \left[\frac{3}{4} + \frac{1}{2} - \frac{3}{4} \times \frac{1}{2}\right]\left(1 - \frac{5}{8}\right)$$

$$\Rightarrow \frac{21}{64} \checkmark \ \underline{\text{Ans}}$$

$0 \le P(E) \le 1$

Q. Questions

2 Min

$P(E \cap F) = P(E)\,P(F)$

Let E and F be two independent events. The probability of both E and F happens is 1/12 and neither E nor F happens is 1/2 then the value $\frac{P(E)}{P(F)} = ?$

$P(E \cap F) = \frac{1}{12}$ — (1)

$P(\bar{E} \cap \bar{F}) = \frac{1}{2}$ — (2)

$P(E) = ?$

$P(F) = ?$

Ratio of $\frac{P(E)}{P(F)} =$ ✓

(a) 4/5

(b) 3/2

(c) 1/5

(d) ✓ N.O.T.

$P(E \cap F) = \frac{1}{12}$ $\quad$ $P(\bar{E} \cap \bar{F}) = \frac{1}{2}$ $\quad$ "E and F are Independent events"

$P(E)\,P(F) = \frac{1}{12}$ $\quad$ $P(\bar{E})P(\bar{F}) = \frac{1}{2}$

$[1-P(E)][1-P(F)] = \frac{1}{2}$

"something is common"

$$\begin{cases} P(E) = x \\ P(F) = y \end{cases}$$

"Nothing is common"

$$\begin{cases} P(E) = x \\ P(F) = 1-x \end{cases}$$

$x \cdot y = \frac{1}{12}$ — ① ✓

$(1-x)(1-y) = \frac{1}{2}$ — ② ✓

$\Rightarrow 1 - y - x + xy = \frac{1}{2}$

$\Rightarrow -x - y + 1 + xy = \frac{1}{2}$

$\Rightarrow -x - y + 1 - \frac{1}{2} + \frac{1}{12} = 0$

$\Rightarrow \frac{1}{2} + \frac{1}{12} = x + y$

$$\begin{cases} x + y = \frac{7}{12} & \text{— ①} \\ xy = \frac{1}{12} & \text{— ②} \end{cases}$$

$x+y=\frac{7}{12}$ $xy=\frac{1}{12}$ $\longrightarrow$ quadratic Equ^n

IInd method : $(x-y)=\sqrt{(x+y)^2-4xy}$

$=\sqrt{\frac{49}{144}-4\cdot\frac{1}{12}}=\sqrt{\frac{49-48}{144}}$

$=\frac{1}{12}$ ✓

$$x+\cancel{y}=\frac{7}{12}$$
$$x-\cancel{y}=\frac{1}{12}$$
$$2x=\frac{8}{12}$$

$\boxed{x=\frac{1}{3}}$ $\boxed{y=\frac{1}{4}}$

$P(E)=\frac{1}{3}$ $P(F)=\frac{1}{4}$

$$\frac{P(E)}{P(F)}=\frac{\frac{1}{3}}{\frac{1}{4}}=\frac{4}{3}$$

$$\boxed{\frac{P(E)}{P(F)}=\frac{4}{3}}$$ ✓

Quadratic

$\frac{3}{4}$ $\frac{4}{3}$

Mutually Exclusive events: (Nothing is common)

E and F are Disjoint events

↙ No connection Between E, F

$n(E \cap F) = 0$ ✓

or

ϕ ✓

E and F does Not occur Simultaneously.

E F S L W

F× E× F× T× L×

at a time does Not simultaneously occur

$$P(E \text{ or } F) = P(E)+P(F)-P(E\cap F)$$

$$P(E\cap F) = 0$$

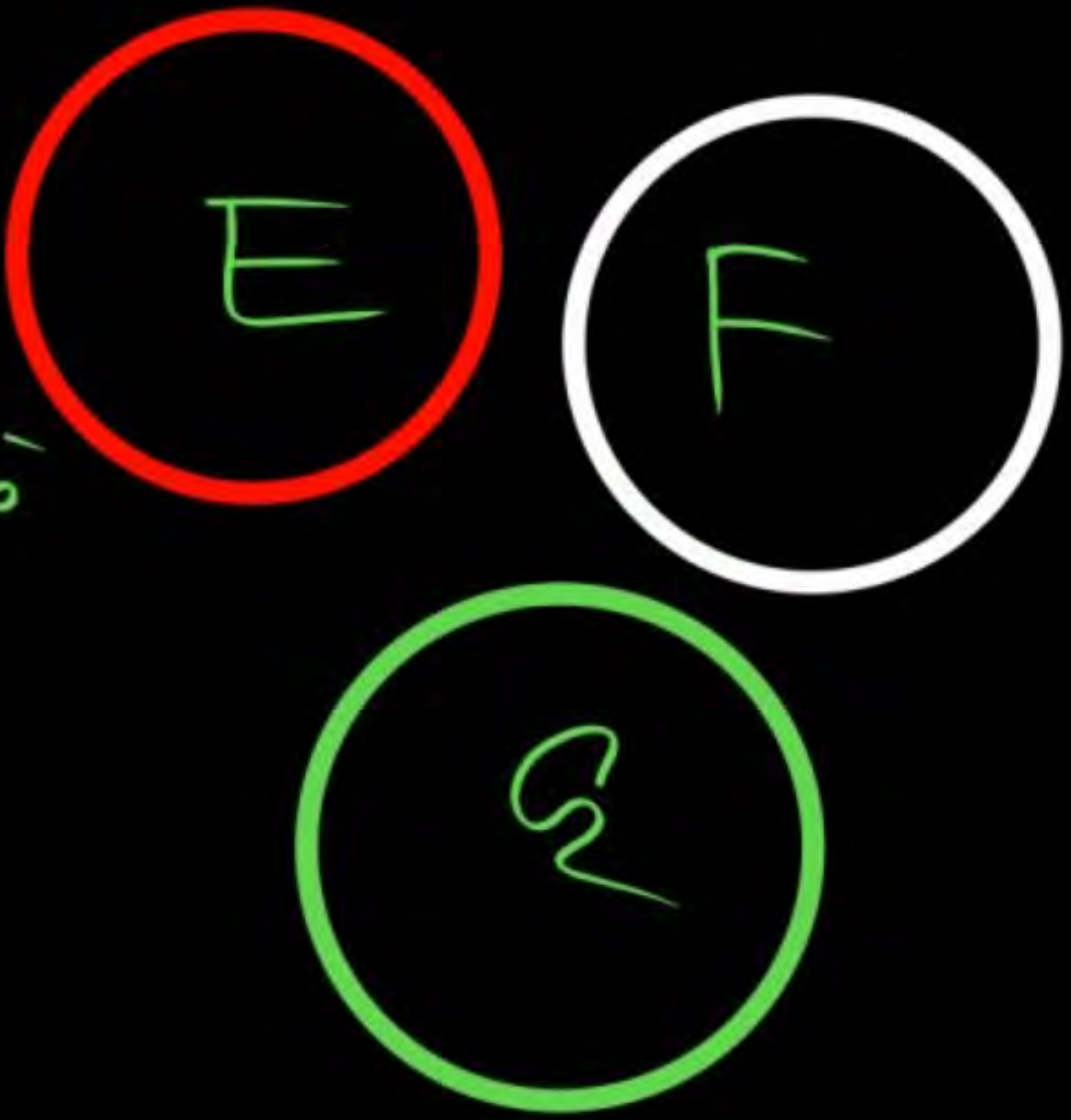
$$\Rightarrow \boxed{P(E \text{ or } F) = P(E\cup F) = P(E)+P(F)}$$

For Three Events (E, F, G)

$$P(E \text{ or } F \text{ or } G) = P(E\cup F\cup G)$$

E, F, G is a mutually exclusive events

$$= P(E)+P(F)+P(G)$$



Q. Questions

□ 1E

50 50

A B

$\frac{1}{2}A$ $\frac{1}{2}B$ $A \cap B = 0$

Let S be the sample space with two mutually exclusive events A and B and $\boxed{A \cup B = S.}$ If P denotes probability of events, then the maximum value of $P(A) \cdot P(B) = ?$

$P(A \cup B) = P(S)$

A and B Are mutually exclusive events

$\boxed{P(A) + P(B) = 1}$

$A \cup B = S =$ sure event

$P(S) = 1$

$P(A)_{max} = \frac{1}{2}$

$P(B)_{max} = \frac{1}{2}$

$[P(A)P(B)]_{max} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(a) 1/2

(b) 1/4

(c) 1/6

(d) N.O.T.

II^{nd} method

$P(A)+P(B)=1$

$$A.M \geq G.M$$

$$\Rightarrow \frac{P(A)+P(B)}{2} \geq \sqrt{P(A)P(B)}$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 \geq [P(A)P(B)]_{max}$$

$$\Rightarrow \frac{1}{4} \geq [P(A)P(B)]_{max}$$

$$A.M = \frac{P(A)+P(B)}{2}$$

$$G.M = \sqrt{P(A)P(B)}$$

Max value $(P(A)P(B))$

$$= \frac{1}{4}$$

Q. Questions

Homework

Let E and F be two independent events. The probability that exactly one of them occurs is 11/25 and the probability of none of them occurring is 2/25. If P(T) denotes the probability of occurrence of the event T, then

H.W

(a) $P(E) = 4/5, P(F) = 3/5$

(b) $P(E) = 1/5, P(F) = 2/5$

(c) $P(E) = 2/5, P(F) = 1/5$

(d) $P(E) = 3/5, P(F) = 4/5$

Thank You!

GW Soldiers