GATE-All BRANCHES
Engineering Mathematics

2024

Fourier series

one shot

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Recap of previous lecture

Topic Complex Analysis

Topic Problems.

Topics to be covered

Topic FOURIER SERIES

Topic Problems.

Topic

Topic

EC ✗
EE ✗
ME ✓
CE ✓

Fourier Series

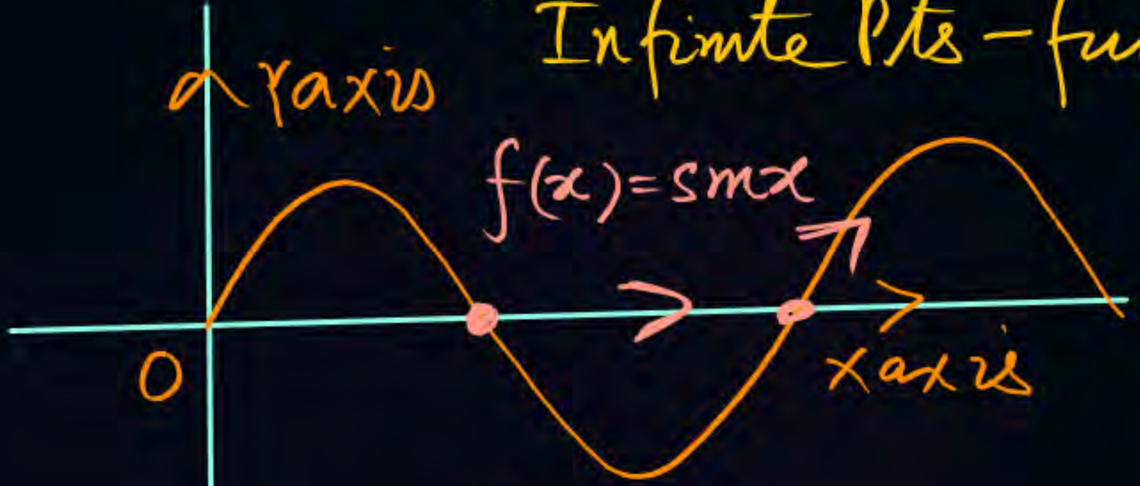
YOUTUBE - fourier series
04 DEC
YOUTUBE
CRASH COURSE

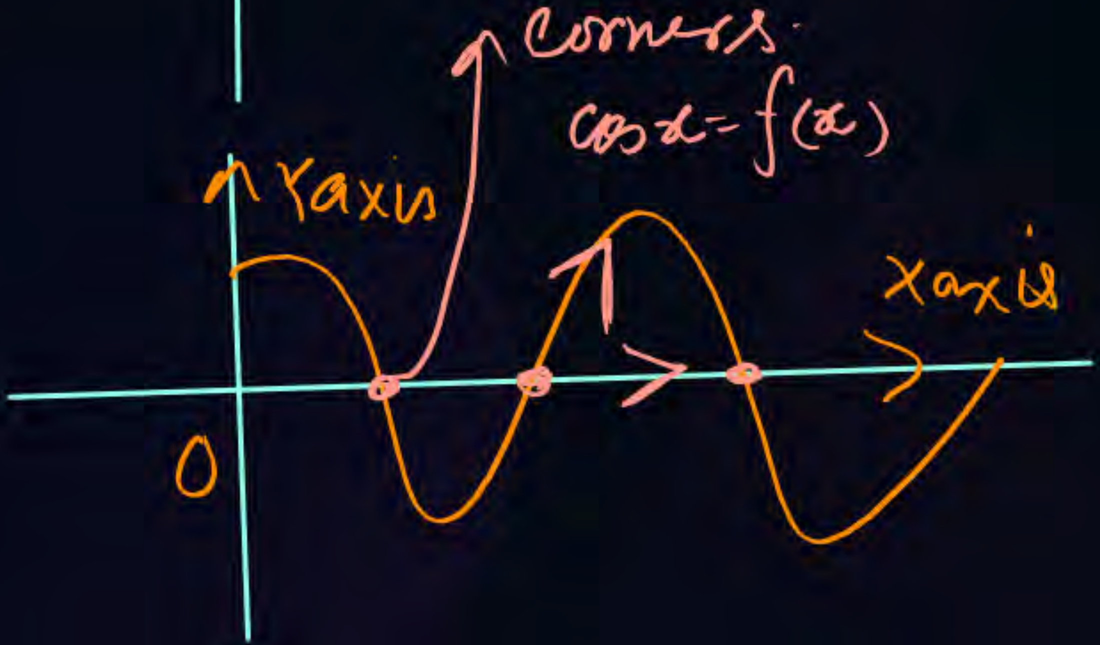
Problems

Taylor Series apply on Differentiable Functions

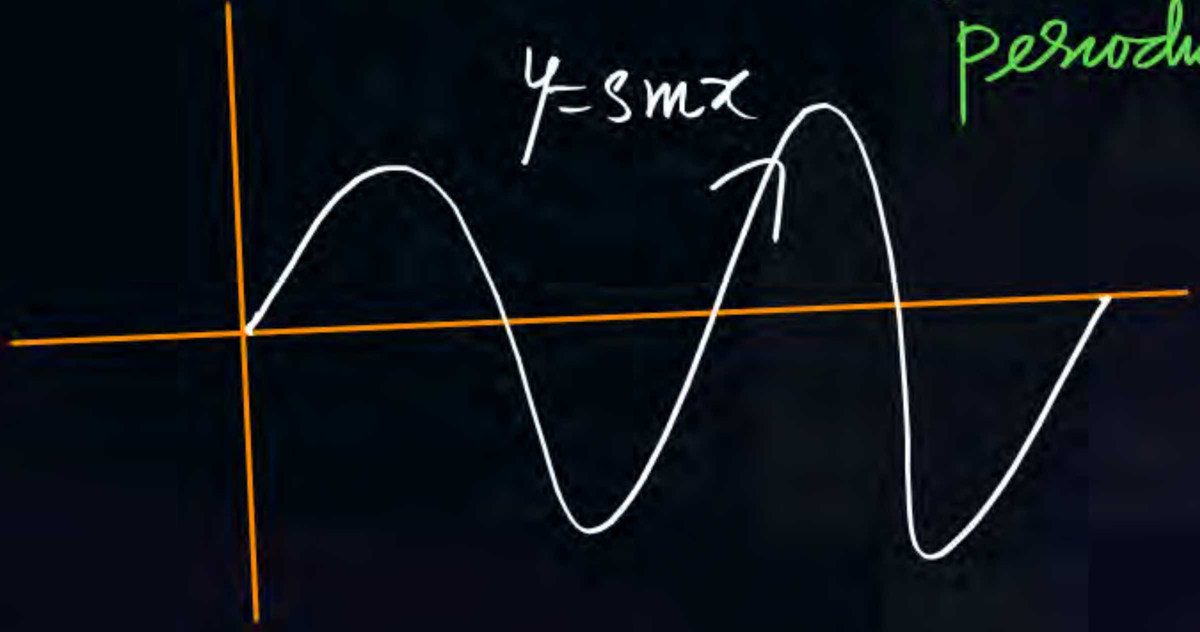
sin/cos

Infinite Pts - function Non Differentiable
(corner)





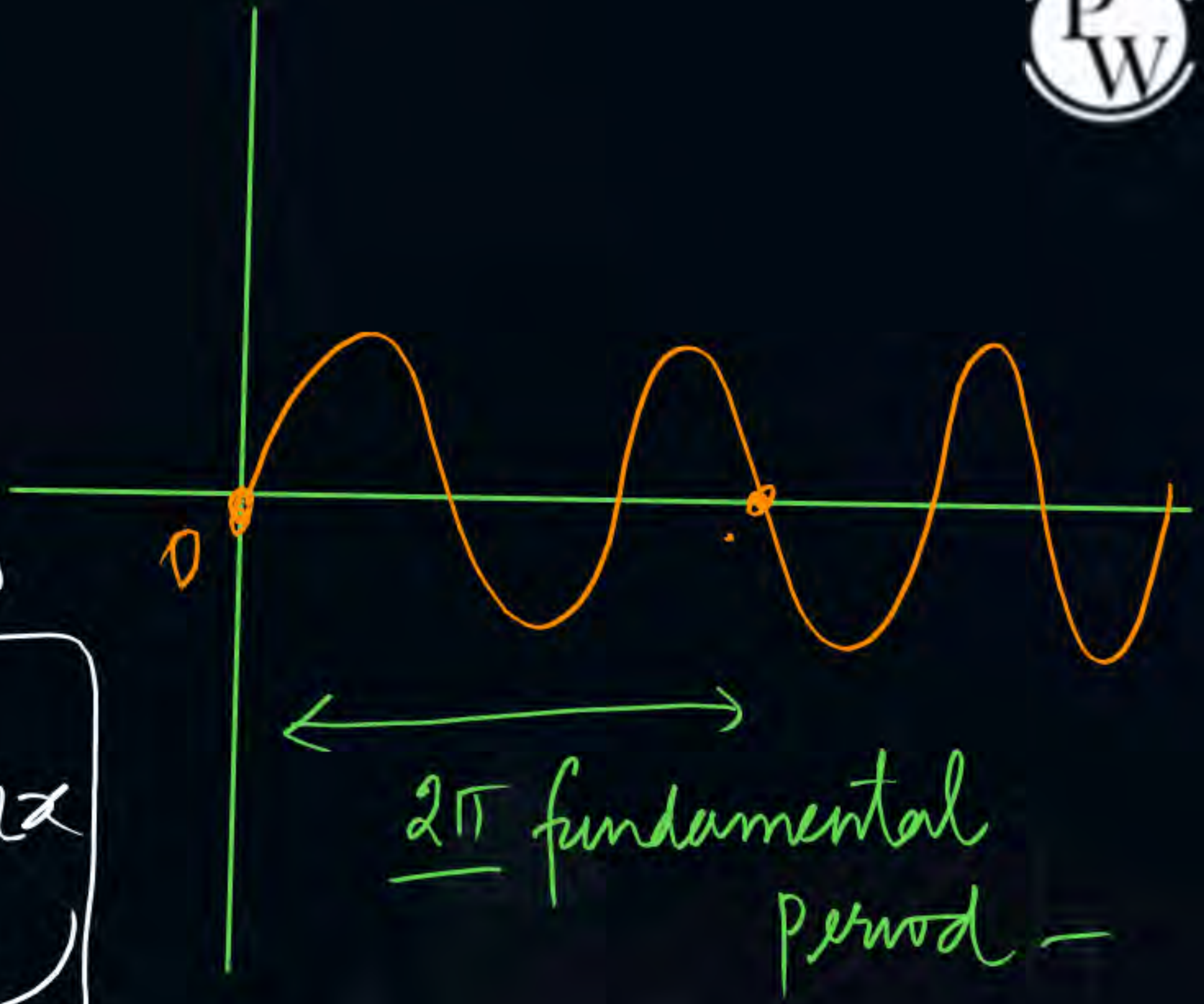
$\Rightarrow$ Fourier Series ($\sin/\cos x$)
periodic functions



If T is fundamental Period

$$f(x+T) = f(x)$$

$$\sin(x+2\pi) = \sin x$$



Fourier SERIES = fourier Series

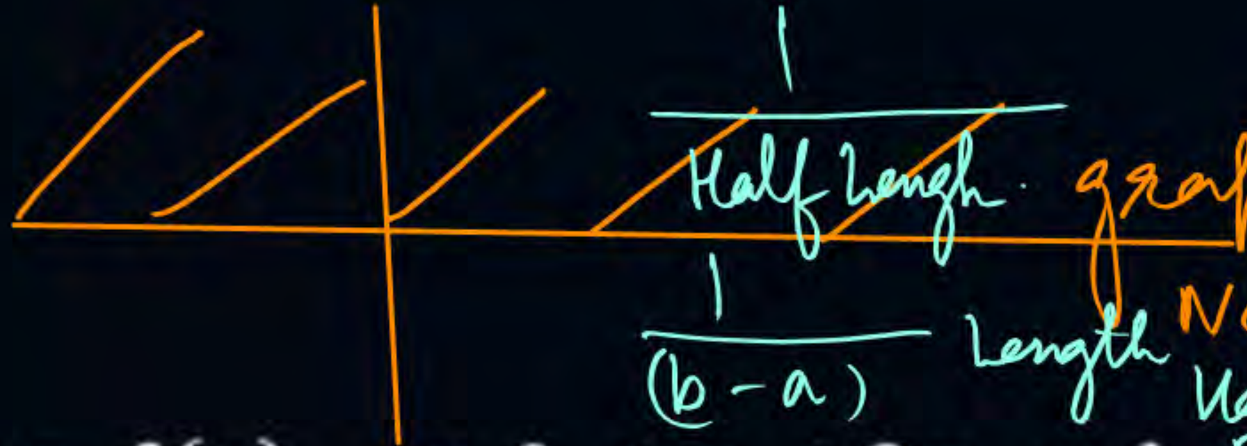
$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx}$$

SVM

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx \qquad b_n \qquad b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

fourier Coefficients only This



$\frac{1}{\text{Half Length}}$ graph

$\frac{1}{(b-a)}$ Length

Nature
Half
Periodic

#Q. Find the fourier series representing, $f(x) = x, 0 < x < 2\pi$, and sketch its graph from $x = -4\pi$ to $x = 4\pi$.

$f(x) = x \quad 0 < x < 2\pi$

✓ Fourier Series $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

"Fourier coefficients" $[0 \text{ to } 2\pi]$

on Infinite Discontinuity

$$\begin{cases} a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx \\ a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx \\ b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} x\,dx = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} = \frac{1}{\pi}\left[\frac{\cancel{4}^{\,2}\pi^2}{\cancel{2}}\right] = 2\pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{2\pi} x\cos nx\,dx$$

D		I
x	$+$	$\cos nx$
1	$-$	$+\frac{\sin nx}{n}$
0		$-\frac{\cos nx}{n^2}$

$$= \frac{1}{\pi}\left[x\frac{\sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{2\pi}{n}\sin 2n\pi + \frac{\cos 2n\pi}{n^2}\right] - \left[0 + \frac{\cos 0}{n^2}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{1}{n^2} - \frac{1}{n^2}\right] = 0$$

$f(x) = x$

odd function

$f(-x) = -x = -f(x)$

$= \underline{\text{odd}}$

$= 0$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left[-x\frac{\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-\frac{2\pi}{n}\cos 2n\pi + \frac{\sin 2n\pi}{n^2}\right] - \left[0\right] =$$

$$= \frac{1}{\cancel{\pi}}\left[-\frac{2\cancel{\pi}}{n} \times 1\right]$$

$$= -\frac{2}{n}$$

x	$\sin nx$
1	$-\frac{\cos nx}{n}$
0	$-\frac{\sin nx}{n^2}$

$\cos n\pi = (-1)^n$

$\cos 2n\pi = 1$

$$\begin{cases} a_0 = 2\pi \\ b_n = -\frac{2}{n} \end{cases} \quad \begin{cases} a_n = 0 \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \cancel{a_n \cos nx}^{\,0} + \sum_{n=1}^{\infty} b_n \sin nx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\boxed{f(x) = \frac{2\pi}{2} + \sum_{n=1}^{\infty} \left(-\frac{2}{n}\right) \sin nx} = \pi + \sum_{n=1}^{\infty} \left(-\frac{2}{n}\right) \sin nx$$

for $n = 1, 2, 3 \ldots$

$x \sin(x)$

$-x \sin(-x)$

$= -x(-\sin x)$

$= \boxed{x \sin x}$ even

$x \cos x$

$-x \cos(-x) =$ odd

2π

#Q. Obtain a fourier expression for $f(x) = x^3$ for $-\pi < x < \pi$ H.W

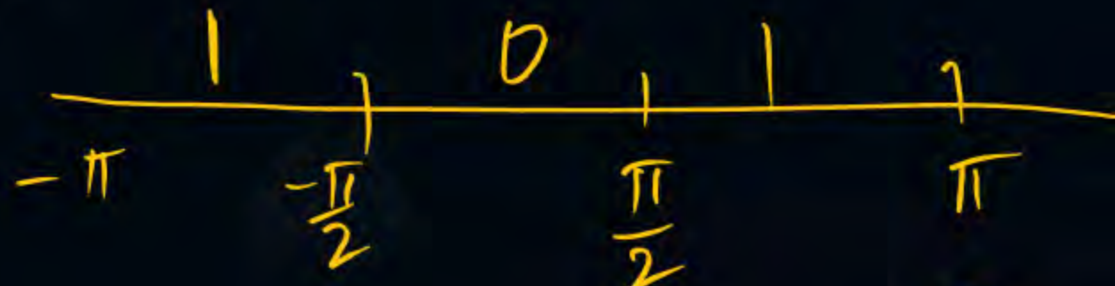
$\boxed{\text{Fourier Series}}$

$$\left\{\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \end{aligned}\right]$$

Topic : Linear Algebra

#Q. Find the fourier series of the function-

$$f(x) = \begin{cases} -1, & -\pi < x < -\pi/2 \\ 0, & -\frac{\pi}{2} < x < \pi/2 \\ +1, & \frac{\pi}{2} < x < \pi \end{cases}$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{-\frac{\pi}{2}} f(x)\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx + \int_{\frac{\pi}{2}}^{\pi} f(x)\,dx\right]$$

Fourier Cofficients

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{-\frac{\pi}{2}} -1\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 0\,dx + \int_{\frac{\pi}{2}}^{\pi} 1\,dx\right] = \boxed{0}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{-\frac{\pi}{2}} -1\cos nx\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 0\cdot\cos nx\,dx + \int_{\frac{\pi}{2}}^{\pi} 1\cos nx\,dx\right] = \boxed{0}$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{-\frac{\pi}{2}} -1\sin nx\,dx + \int_{-\frac{\pi}{2}}^{\pi/2} 0\sin nx\,dx + \int_{\frac{\pi}{2}}^{\pi} 1\sin nx\,dx\right] = \frac{2}{n\pi}\left[\cos\frac{n\pi}{2} - \cos n\pi\right]$$

fourier SERIES S

$n=1$ $\quad \frac{2}{\pi}[0+1] = \frac{2}{\pi}$ ✓

$n=2$ $\quad \frac{1}{\pi}[-1-(-1)^1] = -\frac{2}{\pi}$ $\quad = \frac{2}{n\pi}\left[\underbrace{\cos\frac{n\pi}{2} - (-1)^n}\right]$ generalized Term

$n=3$ $\quad \frac{2}{3\pi}$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx =$$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{n\pi}\left[\cos\frac{n\pi}{2} - \cos n\pi\right]$$

$$a_0 = 0 \qquad a_n = 0$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

Fourier coefficients

$$b_1 = \frac{2}{\pi} \quad b_2 = \frac{-2}{\pi} \quad b_3 = \frac{2}{3\pi}$$

$$= b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + b_4 \sin 4x + \cdots$$

$$\boxed{f(x) = \frac{2}{\pi}\sin x - \frac{2}{\pi}\sin 2x + \frac{2}{3\pi}\sin 3x + \cdots}$$

fourier series

Topic : Linear Algebra

$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$

Fourier SERIES

#Q. Find the fourier series of the function

$$f(x) = \begin{cases} -1, & -\pi < x < -\pi/2 \\ 0, & -\dfrac{\pi}{2} < x < \pi/2 \end{cases}$$

$a_0 = 2$

fourier series coefficients

$a_n =$

$b_n = \frac{2}{n\pi}\left[\cos\frac{n\pi}{n} - 1\right]$

2 mins Summary

Topic One

Topic Two

Topic Three

Topic Four

Topic Five

fourier series ✓

THANK - YOU