



Engineering Mathematics

Differential Equation + Partial differential



By-Rahul Sir

Lecture No. 08



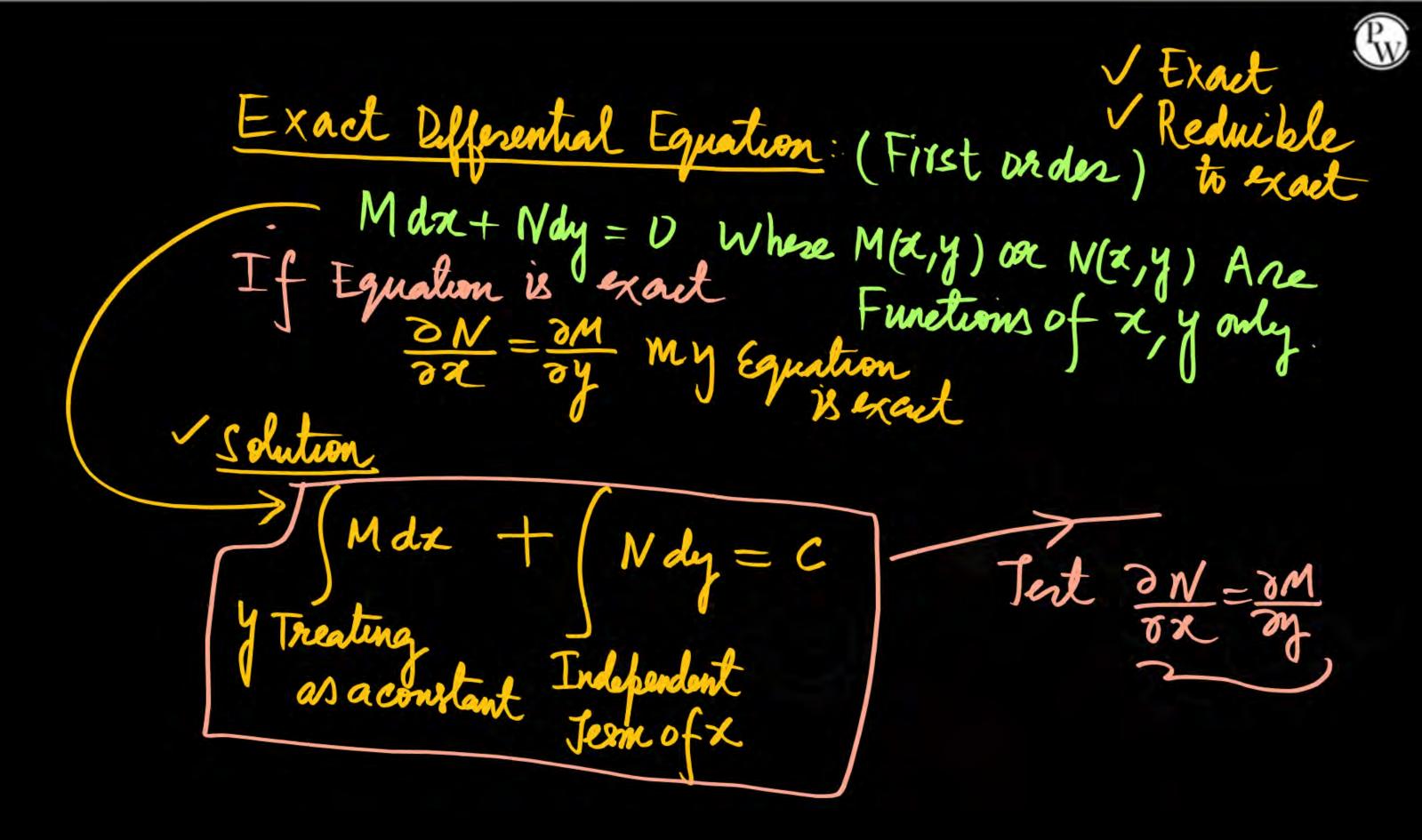




Exact D.E & Orthogonal Trajectory



Problems based on Exact D.E.







#Q. Solve the exact differential equation $(y^2 + 3)dx + (2xy - 4)dy = 0$

$$M(x,y) = y^2 + 3$$
 $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
 $N(x,y) = 2xy - 4$

Solution of this exact Robb. Egan

Slide-3





#Q. Solve the exact differential equation $(2xy + 1)dx + (x^2 + 4y)dy = 0$ $M(x, y) \qquad M(x, y)$

$$\int (2xy+1) dx + \int (x^2+4y) dy = c$$
Treating
as a const

Independent
of x





#Q. Solve the exact differential equation $(3x^2 + 2)dx - (x^2 + y)dy = 0$

$$M(x,y) = 3x^2 + 2$$
 Reducible to
 $N(x,y) = -(x^2 + y)$ Exact
 $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ fail
 $0 = -2x$ Kathen
Non exact





(ysecretana) da 10 Treatment + (tacx+2y) dy Solve the exact differential equation Treating #Q.

$$(ysec^2x + (secx)(tanx))dx + (tanx + 2y)dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial L}{\partial x} + 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial L}{\partial x} + 0$$





#Q. Solve the exact differential equation $(3xy^2 + 2y)dx + (2x^2y + x)dy = 0$

-> Non exact D.E

-- convert to said DE





#Q. Solve the exact differential equation
$$(2xy^2 - y)dx + (2x - x^2y)dy = 0$$

CASE 03
$$\sqrt{\frac{1}{M}} \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right] = f(y) \text{ Inlegaling} \qquad \frac{1}{N}$$

$$= \frac{1}{(2\pi y^2 - y)} \left[2 - 2\pi y - 4\pi y - 1 \right] e^{f(y)} dy = f(y)$$

$$= \frac{1}{(2\pi y^2 - y)} \left[2 - 1 - 6\pi y \right]$$

$$= \frac{1}{(2\pi y^2 - y)} \left[3 - 6\pi y \right] = -\frac{1}{y} \left(\frac{3}{2\pi y} \right) = -\frac{3}{y}$$



Inlegnaling facter = $(2\pi y^2-\pi)d\pi+(2\pi-\pi^2y)dy=0$ Non exact D. E lny Exact DM = 3



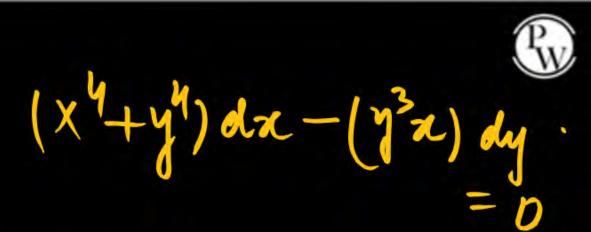
#Q. Solve the exact differential equation $(x^4 + y^4)dx - (x^3y)dy = 0$

Case of Max+Ndy=D M(x,y) or N(x,y) Are Honnogenons Function

am + aw = Non exact

Change Kart Ruff Egnation Kart n

SEARCH or find The Integrating factor



If M(x,y) and N(x,y) both ase Honrogenous function

Integrating factes = \frac{1}{MX+Ny.}

MX+Ny + D

 $MX+Ny = (X^{4}+y^{4})x - (y^{3}x)y$ $= x^{5}+xy^{4}-y^{4}=x^{5}$ Integrating factor = $\frac{1}{2}$ $= \frac{1}{2}$ $= \frac{1}{2}$



(x4+y4) dx-(y3x) dy=0 This is want Mulliply I.F factor 1 MX Reff. Equation





Solve the exact differential equation $y(x^2y^2 + 2)dx + x(2 - 2x^2y^2)dy = 0$ #Q. N(x, y) りfi(は,y)+xf2(2,y)=0 M(x,y)=y(x2y2+2) M(x,y) = y3x2+24 Test for Exactness.

3M = 3W

77. $N(x,y) = 2x - 2x^3y^2$ Non exact defferential equation 3y22+2 + 2-62242 > Reduced to Exactness.



CASE02 Integrating factes yf(x,y)dx+xf(x,y)dy = Ny where MX-Ny + D $MX-Ny = (y^3x^2+2y)x - (2x-2x^3y^2)y$ = y3x3+2xy-(2xy-2x3y3) = 4323+2x4-2x4+22343 $Mx-Ny=3x^3y^3$ I. Factor = 3x3y3 MX-NY



$$\begin{aligned}
& | (x^2y^2+2) dx + x(2-2x^2y^2) dy = 0 \\
& = (x^2y^3+2y) dx + (2x-2x^2y^2) dy = 0
\end{aligned}$$

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$$\end{aligned}$$





#Q. A solution of differential equation

$$\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$$
 Satisfying y(1) = 1 is given by

- (a) A system of hyperbolas
- (b) A system of Circles

(c)
$$y^2 = x(1+x)-1$$

(d) All the above

$$y(1) = 1 \text{ is given by}$$

$$y = \sqrt{x} \text{ with }$$

$$y = \sqrt{x} \text{ wit$$



$$\frac{2xy \, dy - y^{2} dx}{x^{2}} = \frac{(x^{2}+1)}{x^{2}} dx$$

$$= \frac{2xy \, dy - y^{2} dx}{x^{2}} = \left(1 + \frac{1}{x^{2}}\right) dx$$

$$= \left(1 + \frac$$

