

ENGINEERING MATHEMATICS

ALL BRANCHES



Linear Algebra

DPP – 01

Discussion Notes (Part-02)



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✓ Weekly TEST
Calculus TEST

✓ ✓ Subject Test
 Linear +
 Prob +
 Calculus

#Q. One of the eigen values of the matrix

$$\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

→ eigenvalue = 5
 $\lambda_1 = 5$

is 5. The other two eigen values are

$\lambda_1 + \lambda_2 + \lambda_3 = \text{Trace of matrix } A$

(a) 0 and 0 $\Rightarrow \text{SUM} = 0$

(b) 1 and 1 $\text{SUM} = 2$

$$5 + (\lambda_2 + \lambda_3) = 2 + 2 + 1$$

✓ (c) 1 and -1 $\text{SUM} = 0$

(d) -1 and -1
 X $\text{SUM} = -2$

$$\lambda_2 + \lambda_3 = 5 - 5 = 0$$

$\lambda_2 + \lambda_3 = 0$ ✓

© 1 and -1

(SELF-assignment)

#Q. The normalized eigen vector corresponding to the eigen value 5 is

(Do yourself)

(a) $\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$A = \begin{bmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(c) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

(d) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

Normalized eigen vector
 → eigen values $\rightarrow -1, 1, 5$
 → eigen vector
 → Normalized eigen vector
 → $-1, 1, 5$

— 80%

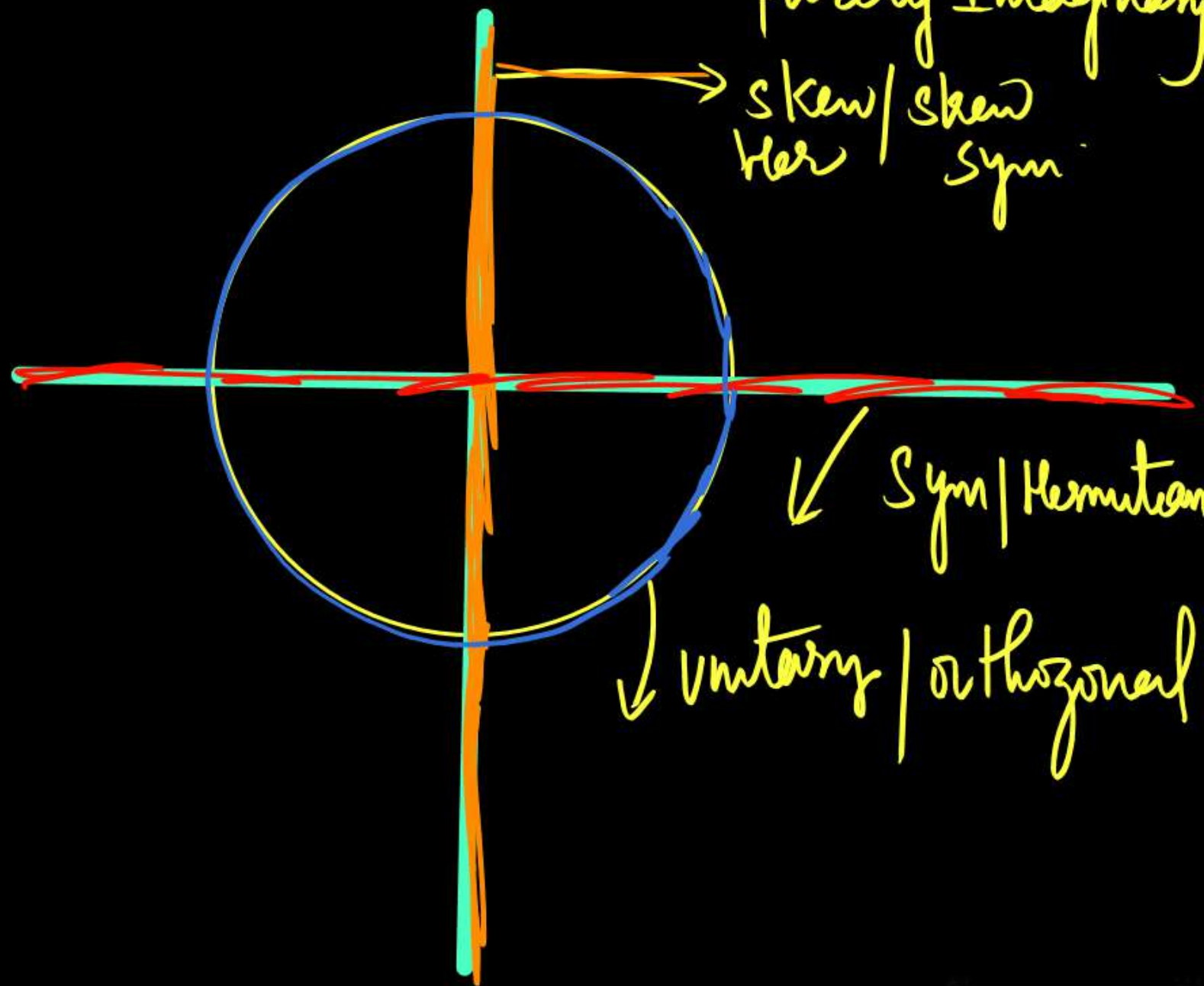
#Q. The eigenvalues of a matrix are i , $-2i$ and $3i$. The matrix is

- (a) Unitary
- (b) Anti-Unitary
- (c) Hermitian
- (d) ✓ Anti-hermitian

Skew-Hermitian $(\overline{A})^T = -A$

Eigen values $\rightarrow i, -2i, 3i$

purely Imaginary
skew / skew
Her / Sym



#Q. The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

SEFF arrangement

Do yourself

→ eigenvalues
→ eigenvectors

- (a) ✓ 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
- (c) ✓ 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

SAME
Ⓐ option correct

- (b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
- (d) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\lambda^2 - (5+2)\lambda + (10-4) = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda^2 - 6\lambda - \lambda + 6 = 0$$

$$\lambda(\lambda - 6) - 1(\lambda - 6) = 0$$

$$\lambda = 1, \lambda = 6$$

#Q. Consider a vector $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$.

The axes are rotated anti-clockwise about the Y axis by an angle of 60° .

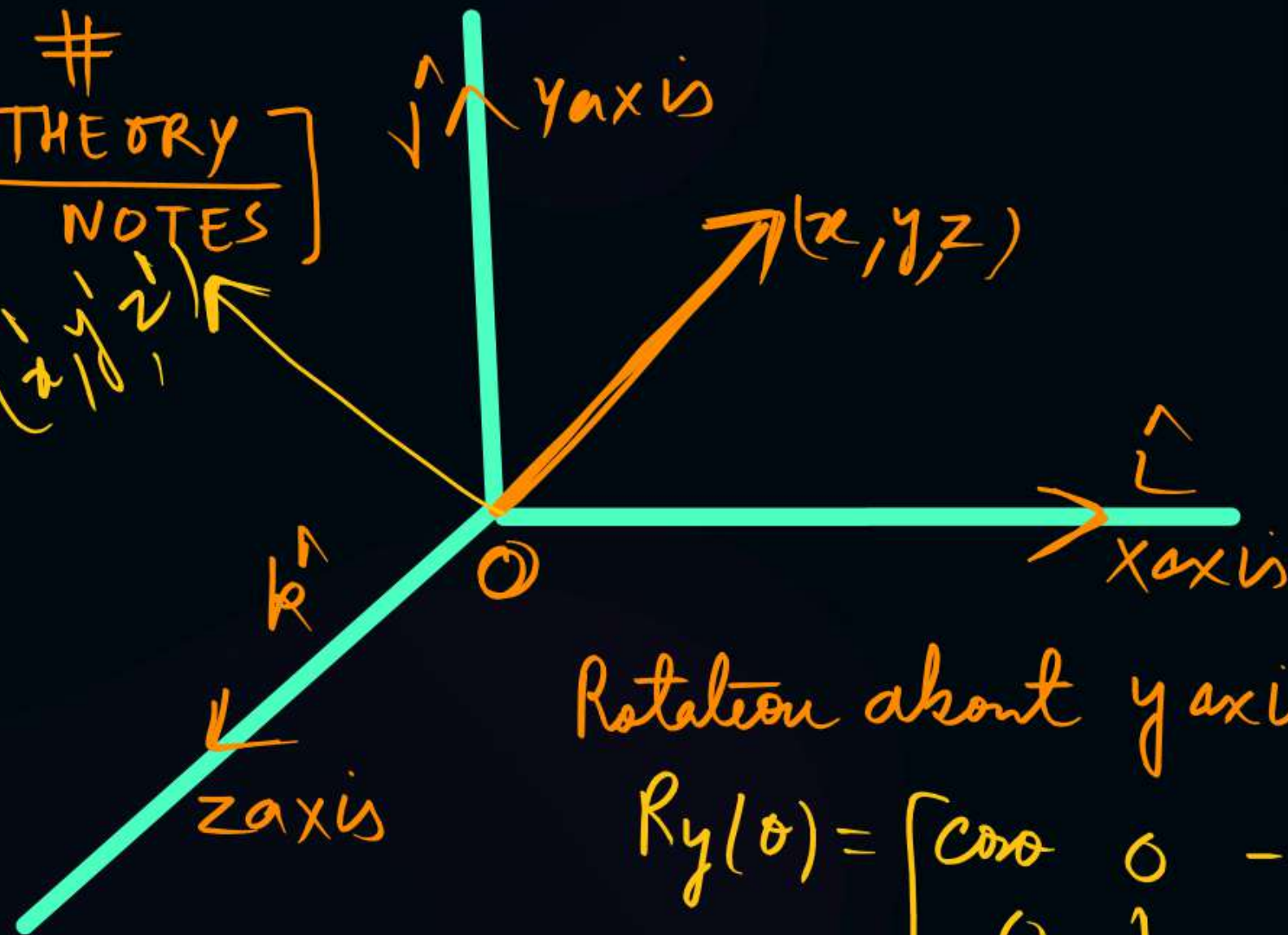
The vector \vec{p} in the rotated coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ is

(a) $(1 - \sqrt{3})\hat{i}' + 3\hat{j}' + (1 + \sqrt{3})\hat{k}'$ (b) $(1 + \sqrt{3})\hat{i}' + 3\hat{j}' + (1 - \sqrt{3})\hat{k}'$

(c) $(1 - \sqrt{3})\hat{i}' + (3 + \sqrt{3})\hat{j}' + 2\hat{k}'$ (d) $(1 - \sqrt{3})\hat{i}' + (3 - \sqrt{3})\hat{j}' + 2\hat{k}'$

THEORY NOTES

(x, y, z)
 $(\hat{i}, \hat{j}, \hat{k})$



$$\theta = 60^\circ$$

Rotation about y axis

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

Rotation

- x axis Rotation
- y axis "
- z axis Rotation

x-axis — along \hat{i}

y-axis — along \hat{j}

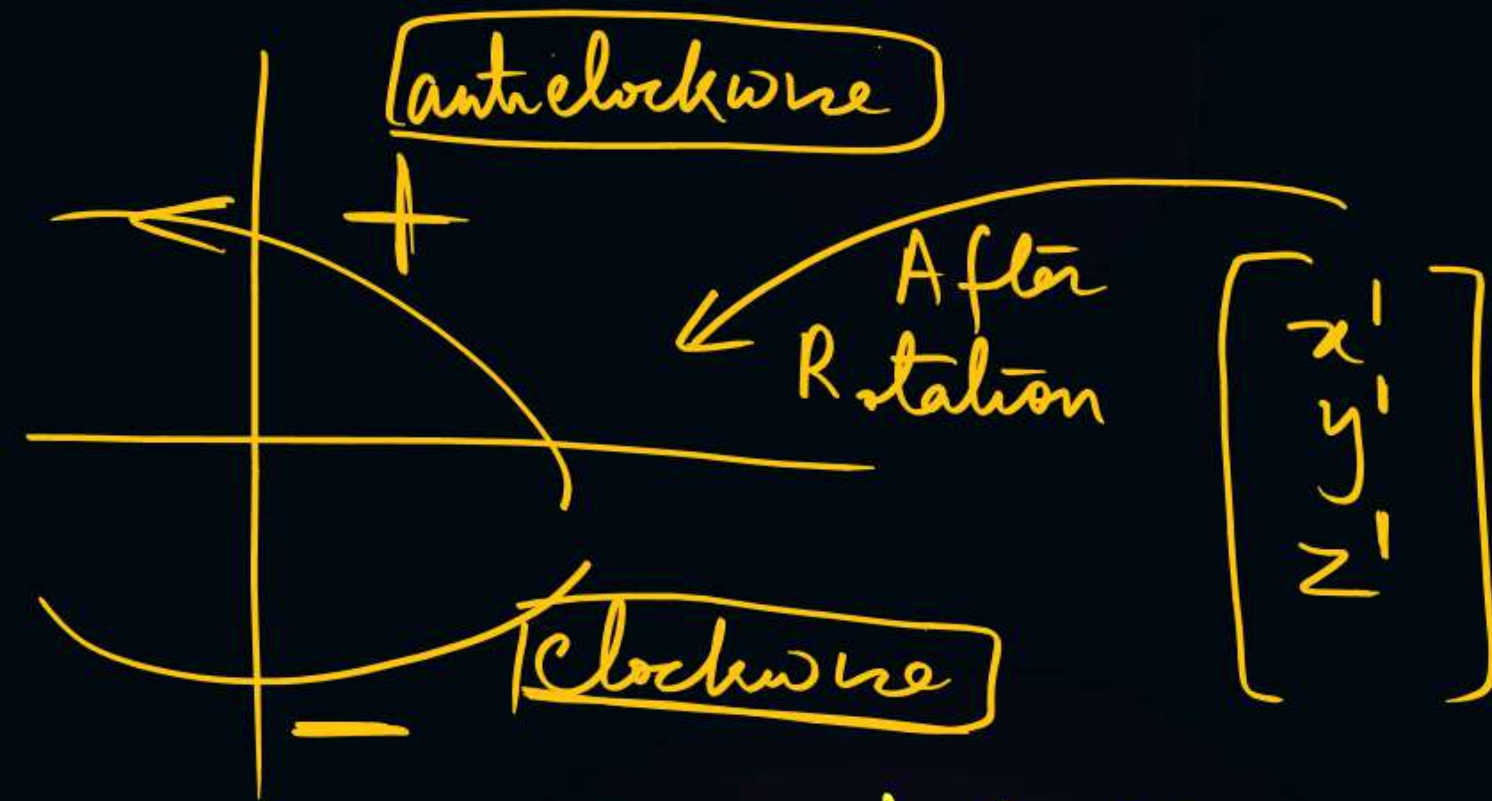
z-axis — along \hat{k}

$$(x, y, z) \rightarrow (2, 3, 2)$$

$$= 2\hat{i} + 3\hat{j} + 2\hat{k}$$

$\theta = 60^\circ$ Rotate

y axis about
 (x', y', z')



Rotation about y axis $\theta = 60^\circ$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}}_{\text{y axis}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos 60^\circ & 0 & -\sin 60^\circ \\ 0 & 1 & 0 \\ \sin 60^\circ & 0 & \cos 60^\circ \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \sqrt{3} \\ 3 \\ 1 + \sqrt{3} \end{bmatrix}$$

$$\begin{aligned} x' &= (1 - \sqrt{3}) \\ y' &= 3 \\ z' &= (1 + \sqrt{3}) \end{aligned}$$

vector

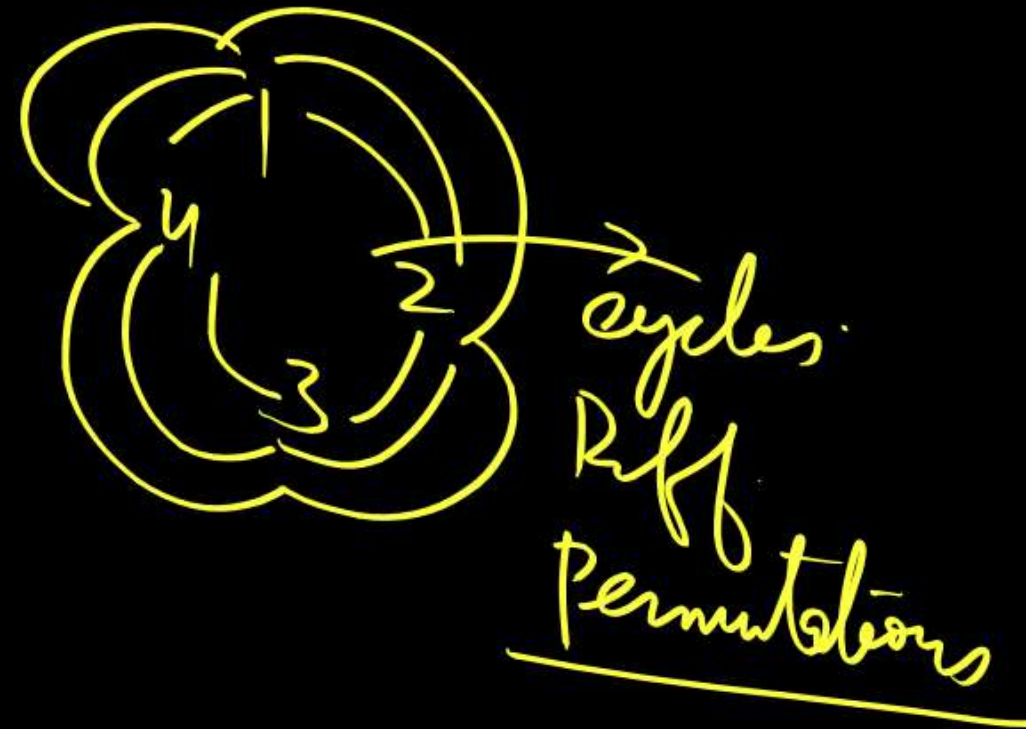
$$= (1 - \sqrt{3})\hat{i} + 3\hat{j} + (1 + \sqrt{3})\hat{k}$$

#Q. For arbitrary matrices E, F, G and H, if $EF - FE = 0$, then Trace (EFGH) is equal

✓ Notes

$$\begin{aligned} \text{Trace}(1234) &= \text{Trace}(4123) \\ &= \text{Trace}(3412) \\ &= \text{Trace}(2341) \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Trace}(1234) &= \text{Trace}(4123) \\ &= \text{Trace}(3412) \\ &= \text{Trace}(2341) \end{aligned}} \right\} \text{cycle}$$

- (a) ✓ Trace (HFEG)
- (b) Trace (E), Trace(F), Trace(G), Trace(H)
- (c) Trace (GFEH)
- (d) Trace (EGHF)



$$\begin{aligned} \text{Trace}(EFGH) &= \text{Trace}(HEFG) \\ &= \text{Trace}(GHFE) \\ &= \text{Trace}(FHGE) \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Trace}(EFGH) &= \text{Trace}(HEFG) \\ &= \text{Trace}(GHFE) \\ &= \text{Trace}(FHGE) \end{aligned}} \right\}$$

Trace(HEFG) = Trace(HFEH)
 $EF = FE$

#Q. An unitary matrix $\begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$ is given, where a, b, c, d, α and β are real. The inverse of the matrix is

$$\Rightarrow \frac{1}{ade^{i\alpha} - bce^{i\beta}} \begin{bmatrix} d & -b \\ -ce^{i\beta} & ae^{i\alpha} \end{bmatrix} \quad V = \begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$$

(a) $\begin{bmatrix} ae^{i\alpha} & -ce^{i\beta} \\ b & d \end{bmatrix}$

(b) $\begin{bmatrix} ae^{i\alpha} & ce^{i\beta} \\ b & d \end{bmatrix}$

(c) $\begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$

✓ (d) $\begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$

If V is Unitary matrix

$$V(\overline{V})^T = I$$

V^{-1} multiply

$$\boxed{(\overline{V})^T = V^{-1}}$$

$$(\overline{V})^T = \begin{bmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{bmatrix} = \begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$$

#Q. The eigenvalue of the matrix $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ are 2×2

$$\lambda^2 - (0+0)\lambda + (0-i^2) = 0$$

$$\Rightarrow \lambda^2 - 0 + (+1) = 0$$

$$\lambda^2 + 1 = 0$$

$$\boxed{\lambda = \pm i}$$

eigen values
Are
Complex
and Distinct

- (a) Real and distinct
- (b) ☒ Complex and distinct
- (c) Complex and coinciding
- (d) Real and coinciding

#Q. The eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are

Eigen values of The matrix
✓ $\lambda = 5, 1, -1$

↙ $\det = \text{product of eigen value} = -5$

(a) 5, 2, -2

(b) -5, -1, 1

✓ (c) 5, 1, -1

(d) -5, 1, 1

#Q. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following statements is NOT TRUE?

→ Diagonalization

If A and B matrix ARE SIMILAR

- (a) Det A = Det B
- (b) Trace of A = Trace of B
- (c) ✓ A and B have the same eigenvectors
- (d) A and B have the same eigenvalues

det A = det B

$\chi(A) = \chi(B)$
eigen values SAME

eigen vector Are Not SAME

Trace A = Trace B

#Q. A 3×3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalues of the matrix is:

- (a) 18
- (b) 12
- (c) 9
- (d) 6

$$\begin{aligned} \lambda_1 + \lambda_2 + \lambda_3 &= 11 \\ \lambda_1 \lambda_2 \lambda_3 &= 36 \end{aligned}$$

$$\left. \begin{array}{l} \lambda_1 = 6 \\ \lambda_2 = 3 \\ \lambda_3 = 2 \end{array} \right\} \begin{array}{l} 6 \times 3 \times 2 \\ = 36 \\ 6 + 3 + 2 \\ = 11 \end{array}$$

$$\begin{aligned} \text{Trace} &= 11 \\ \det A &= 36 \\ \text{Largest eigen value} &= 6 \checkmark \quad \underline{\underline{\text{Ans}}} \end{aligned}$$

#Q. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

(a) 0, 1, 1

(b) ✓ 0, $-\sqrt{2}$, $\sqrt{2}$

(c) $1/\sqrt{2}$, $1/\sqrt{2}$, 0

(d) $\sqrt{2}$, $\sqrt{2}$, 0

eigen values of The
matrix
 $= 0, -\sqrt{2}, \sqrt{2}$

#Q. The degenerate eigenvalues of the matrix $M = \begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$ is 5 = degenerate eigen value OR repeated eigen values

Eigen values = 2, 5, 5
 Degenerate eigen value = 5

#Q. The matrix $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is

- (a) Orthogonal
- (b) Symmetric
- (c) Anti-symmetric
- (d) Unitary

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

$A(\bar{A})^T = I$

Unitary
matrix

D

#Q. Which of the following is INCORRECT for the matrix $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

- (a) It is its own inverse
- (b) It is its own transpose
- (c) ✓ It is non-orthogonal
- (d) It has eigen values ± 1

Weekly TEST
06

Already
discuss

#Q. The symmetric pair of $P = \begin{pmatrix} a \\ b \end{pmatrix} (a - 2b)$ is :

(a) $\begin{pmatrix} a^2 - 2 & ba - 1 \\ ba - 1 & b^2 - 2 \end{pmatrix}$

(b) $\begin{pmatrix} a(a - 1) & b \\ b & b^2 \end{pmatrix}$

(c) $\begin{pmatrix} a(a - 1) & b(a - 1) \\ b(a - 1) & b^2 \end{pmatrix}$

✓ (d) $\begin{pmatrix} a(a - 2) & b(a - 1) \\ b(a - 1) & b^2 \end{pmatrix}$

SEE solⁿ →
Weekly TEST 06
 already discuss
 $P = \frac{P + P^T}{2} + \frac{P - P^T}{2}$

#Q. $(x \ y) \begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 15$

SEE The
✓ solⁿ

Weekly TEST
06
already Discussed

The matrix equation above represents

(a) A circle of radius $\sqrt{15}$

✓ (b) An ellipse of semi major axis $\sqrt{5}$

(c) A ellipse of semi major axis 5

(d) A hyperbola

#Q. The product PQ of any two real symmetric matrices P and Q is:

- (a) Symmetric for all P and Q
- (b) Never symmetric
- (c) ✓ Symmetric if $PQ = QP$
- (d) Antisymmetric for all P and Q

If product PQ of Any
Two real sym matrices P and
 Q is

Symmetric
If $\boxed{PQ = QP}$

✓ multiplication
is commutative

#Q. A matrix given by $M = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$ The eigenvalues of the M are

- (a) Real and positive ✗
- (b) Purely imaginary with modulus 1 ✗
- (c) ✓ Complex with modulus 1
- (d) Real and negative ✗

$$M = \frac{1}{\sqrt{2}} \begin{bmatrix} i & 1 \\ 1 & i \end{bmatrix}$$

eigen values

$$\Rightarrow \frac{1}{\sqrt{2}} \pm i \frac{1}{\sqrt{2}}$$

$$\lambda_1 = \frac{1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}} = \lambda_2$$

complex

Modular: $z = a + ib$
 $|z| = \sqrt{a^2 + b^2}$

$$|\lambda_1| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$$|\lambda_2| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

#Q. The inverse of the matrix $M = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is ✓

Inverse of matrix
 $\Rightarrow \underline{M^2 - I}$

(a) $M - I$

(b) ✓ $M^2 - I$

(c) $I - M^2$

(d) $I - M$

↙ Characteristic eqnⁿ
Step ② $\lambda = M$

Step ③ Multiply with M^{-1}

$M^{-1} = M^2 - I$

✓

#Q The normalized eigenvectors of the matrix $N = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ are β_1 and β_2 with the eigenvalues λ_1 and λ_2 , respectively and $\lambda_1 > \lambda_2$. If the eigenvector $\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is expressed as $\alpha = P\beta_1 + Q\beta_2$. Find the constant P and Q

(a) $\frac{1+i}{2}, \frac{1-i}{2}$

(b) $\frac{2+i}{2}, \frac{1+i}{2}$

(c) $\frac{1+i}{3}, \frac{1-i}{4}$

(d) $\frac{i}{2}, \frac{1+i}{2}$

$$N = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \beta_1, \beta_2$$

λ_1, λ_2

$$\alpha = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\alpha = P\beta_1 + Q\beta_2$$

$$N = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\lambda = 1, -1$$

$$\checkmark \beta_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

$$\checkmark \beta_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} i \\ 1 \end{bmatrix}$$

Eigen values

$$\lambda^2 - (0)\lambda + (0 + i^2) = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda = \pm 1$$

Linear combination

$$P = \alpha x + \beta y$$

$$\alpha = \beta_1 P + \beta_2$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{P}{\sqrt{2}} \begin{bmatrix} -i \\ 1 \end{bmatrix} + Q \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$P = \frac{1+i}{2} \quad Q = \frac{1-i}{2}$$

#Q. The trace of a 2×2 matrix is 4 and its determinant is 8. If one of the eigenvalues is $2(1 + i)$, the other eigenvalue is

- (a) ✓ $2(1 - i)$
- (b) $2(1 + i)$
- (c) $(1 + 2i)$
- (d) $(1 - 2i)$

$$\left. \begin{array}{l} \lambda_1 + \lambda_2 = 4 \\ \lambda_1 \lambda_2 = 8 \end{array} \right\} \begin{array}{l} \text{eigen value} \\ \lambda_1 \\ = 2(1+i) \end{array}$$

other Eigen value = $2(1-i)$ ✓

#Q. The eigenvalues of the matrix representing the following pair of linear equation $x + iy = 0$, $ix + y = 0$ are

$$\begin{aligned} x + iy &= 0 \\ ix + y &= 0 \end{aligned} \quad A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

$$\lambda^2 - (1+i)\lambda + (1-i^2) = 0$$

$$\lambda^2 - 2\lambda + 2 = 0$$

$$\lambda = 1+i, 1-i \quad \checkmark$$

$$i^2 = -1$$

(a) $1+i, 1+i$

(b) $1-i, 1-i$

(c) $1, i$

(d) $1+i, 1-i$ ✓

#Q. For the given set of equations:

$$x + y = 1$$

$$y + z = 1$$

$$x + z = 1$$

$$\frac{AX=B}{\left\{ \begin{array}{l} x = \frac{1}{2} \\ y = \frac{1}{2} \\ z = \frac{1}{2} \end{array} \right\} \text{Non Trivial sol}^n}$$

$$C = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{array} \right]$$

rank A = 3
rank C = 3
No. of variables = 3

This is condition consistent

$$R(A) = R(C) = n = \text{Unique sol}^n$$

OR
Non Trivial sol

Which one of the following statements is correct?

- (a) Equation are inconsistent
- (b) ✓ Equations are consistent and a single non-trivial solution exists
- (c) Equation are consistent and many solutions exist
- (d) Equation are consistent and only a trivial solution exists

Trivial = (0, 0, 0)
Non Trivial = (a, b, c)



**Thank
You !**

Next Test
- CALCULUS

✓ +
Next Test
- Subject Test

💡 full EM

≈ 20 questions