GATE-All BRANCHES
Engineering Mathematics

Multivariable calculus

Lecture No.- 01

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Recap of previous lecture

Topic

Properties of eigen values

Problems based on eigen values

LU decomposition -

Slide 2

Topics to be Covered

Topic Double integration

Double Integration

Topic Volume via double integrals

Topic Question based on double integrals

Double Integral

$$\int_{x=a}^{b} \underbrace{\int_{y=c}^{d} dy}_{\text{Inner Integral}} \, \underline{dx}$$

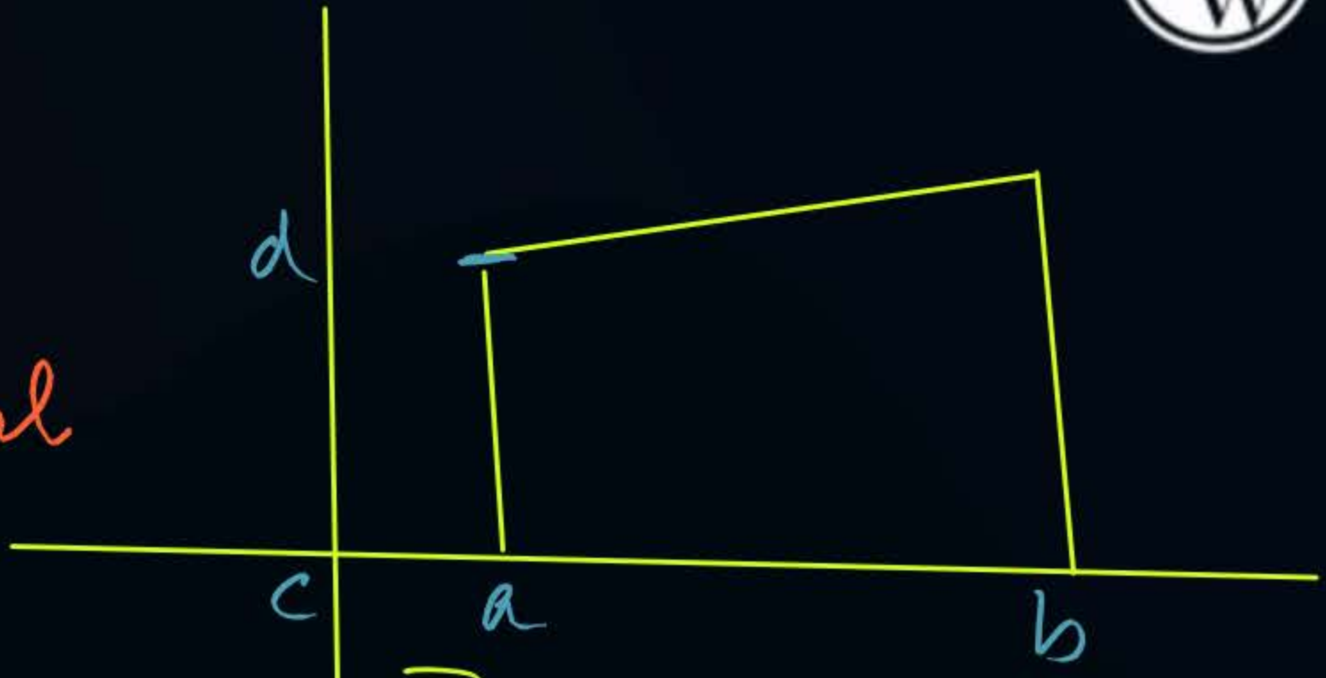
outer Integral

OR

$$\int_{y=c}^{d} \underbrace{\left[\int_{x=a}^{b} dx\right]}_{\text{Inner Integral}} dy$$

outer Integral

= <u>AREA</u>



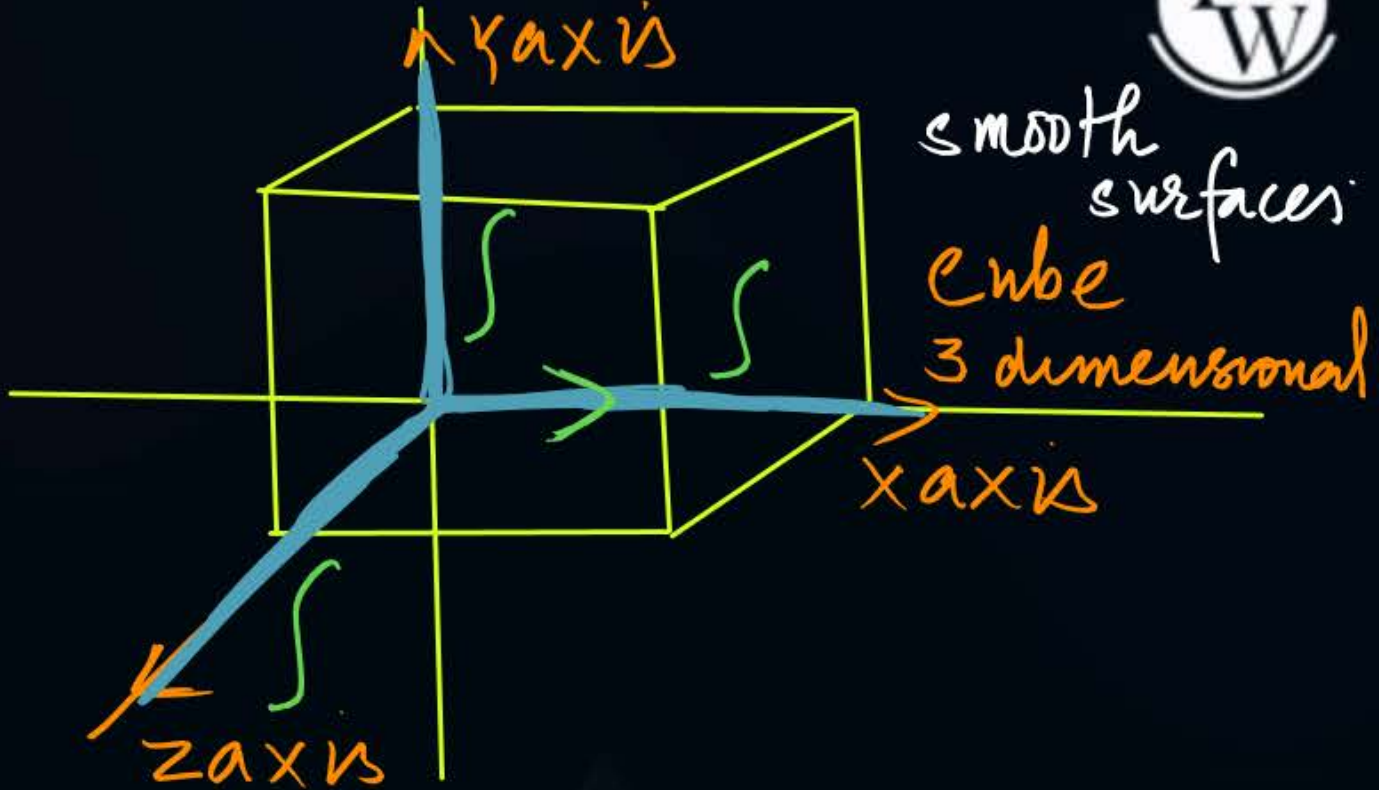
generalization

$x \longrightarrow$

$y \longrightarrow$

Volume Via Double Integral:

$$\text{volume} = \int_x \int_y \int_z dz\, dy\, dx$$



Volume via Double Integral

For given surface: $y = f(x) = \text{curve}$

$$f(x, y) = (x+y) \longrightarrow 3d$$

Two Independent variable

$f(x,y) = (x+y)$

$z = (x+y)$

Independent var.					
x	1	1	2		Two variable input
y	1	0	3		

$z = 2,\ z = 1,\ z = 5$ — One var. output

$\underline{x_1 y_1 z_1}$

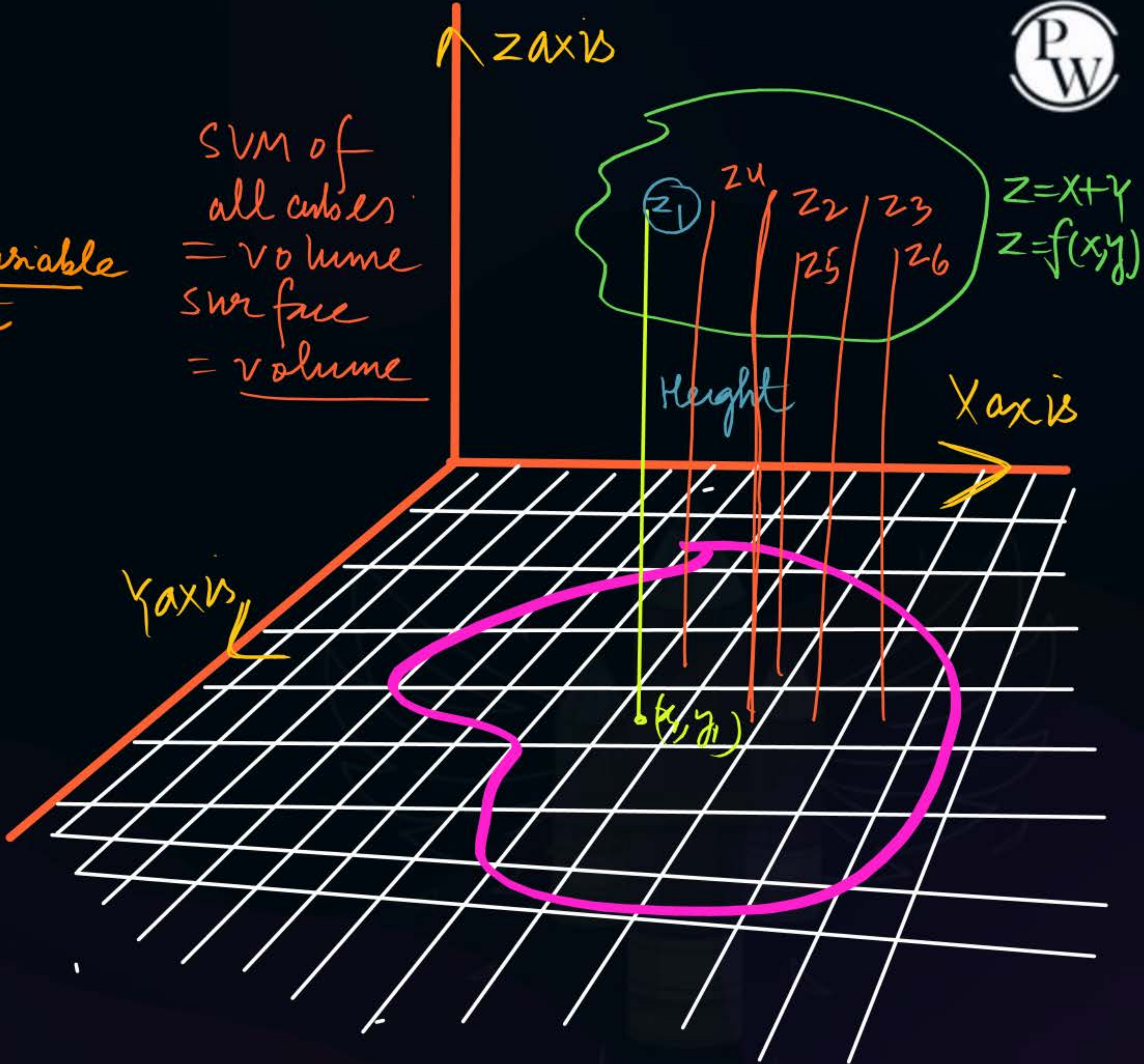
$V_1 \quad z_1$

$\underbrace{x_2 y_2 z_2}$

$\underline{\text{volume } V_2}$

SUM of all cubes = volume

surface = $\underline{\text{volume}}$



If $f(x,y)$ is given surface

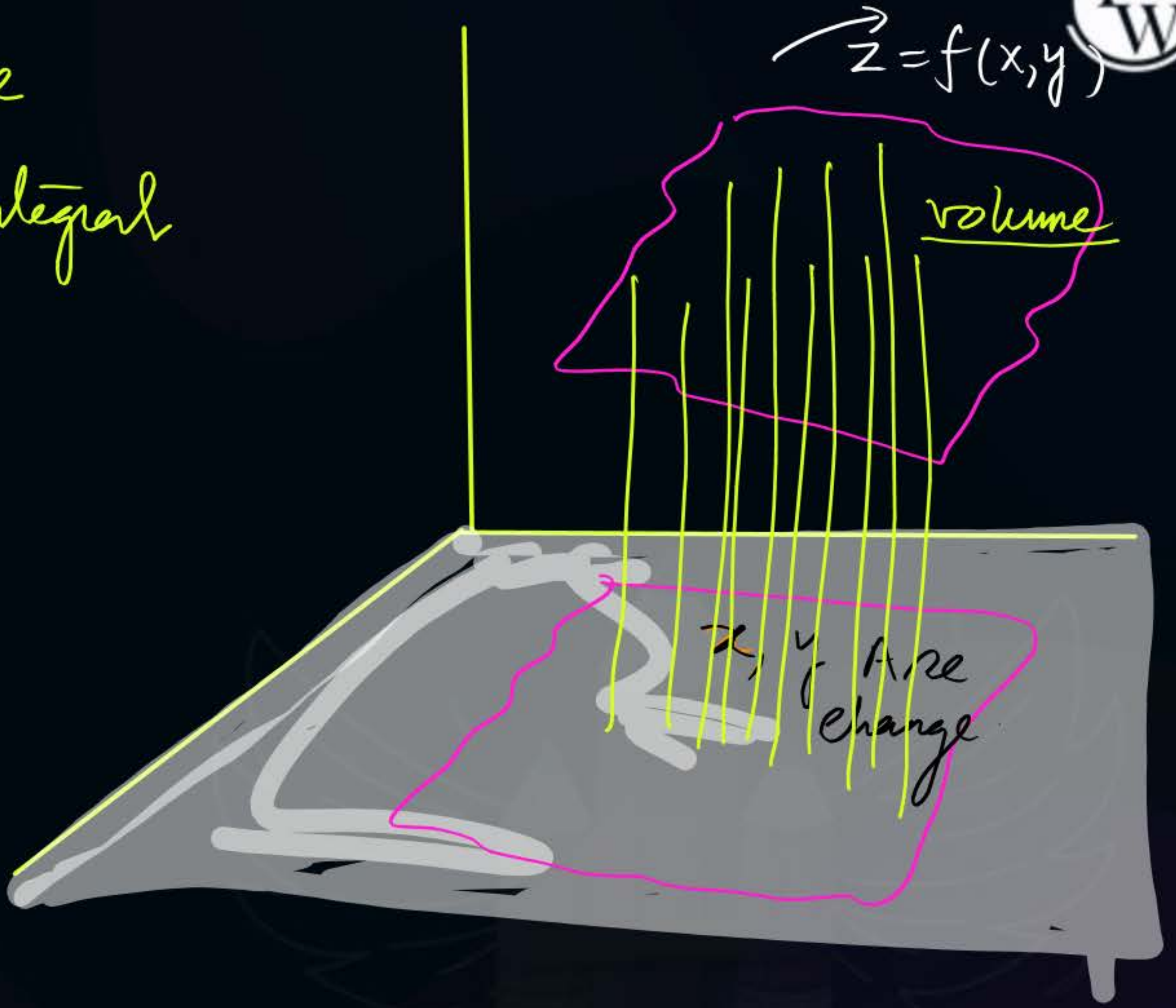
volume via double Integral

$$V = \int_{x=}\int_{y=} \boxed{f(x,y)}\, dy \cdot dx$$

If $f(x,y)$ is given surface.

$$V = \int_{y=}\int_{x} f(x,y)\, dx\, dy$$

$\iint dy\, dx = \underline{\text{Area}}$ $\quad$ $\underline{\iint f(x,y)\, dy\, dx = \text{volume}}$



How to evaluate Limit (x, y) :

$$V = \int_x \int_y f(x,y)\, dy\, \underline{dx}$$

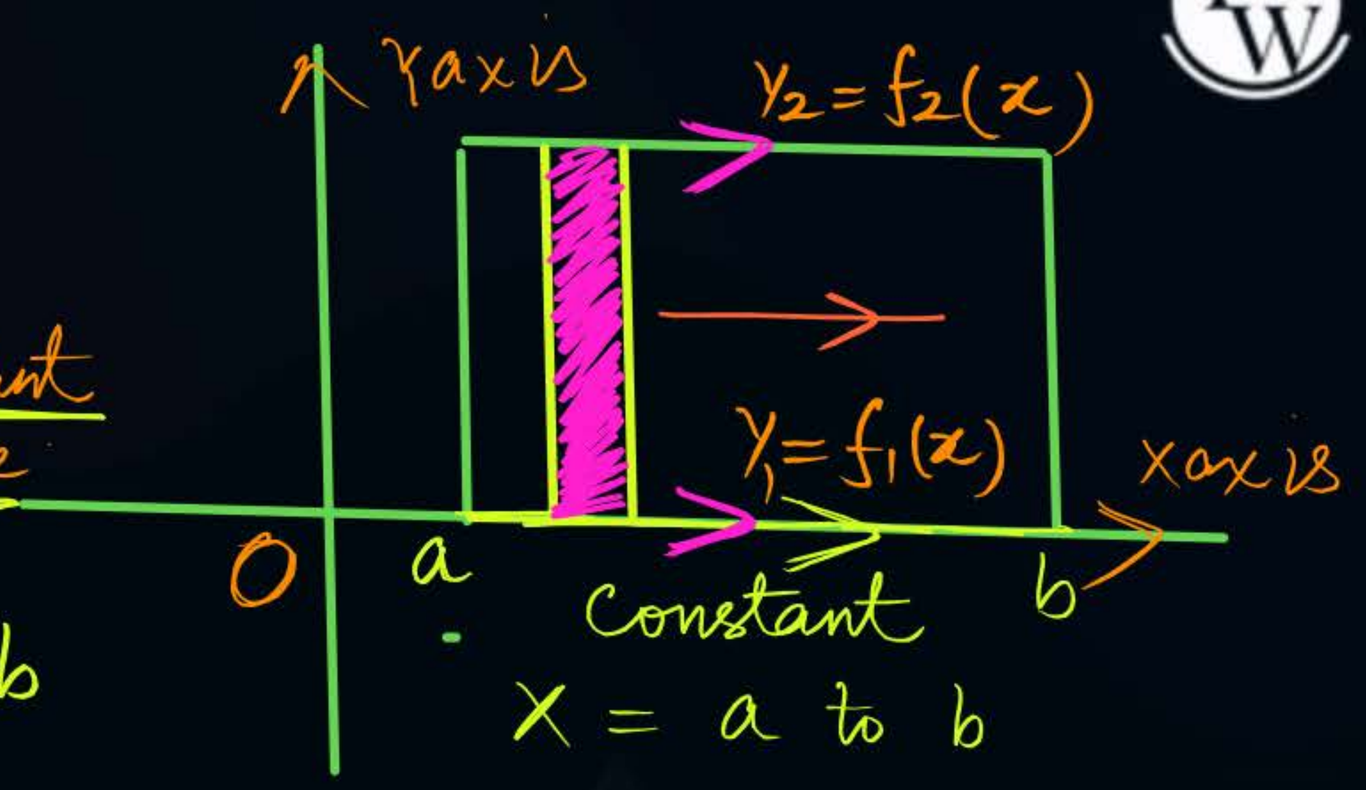
Outer Integral always constant value

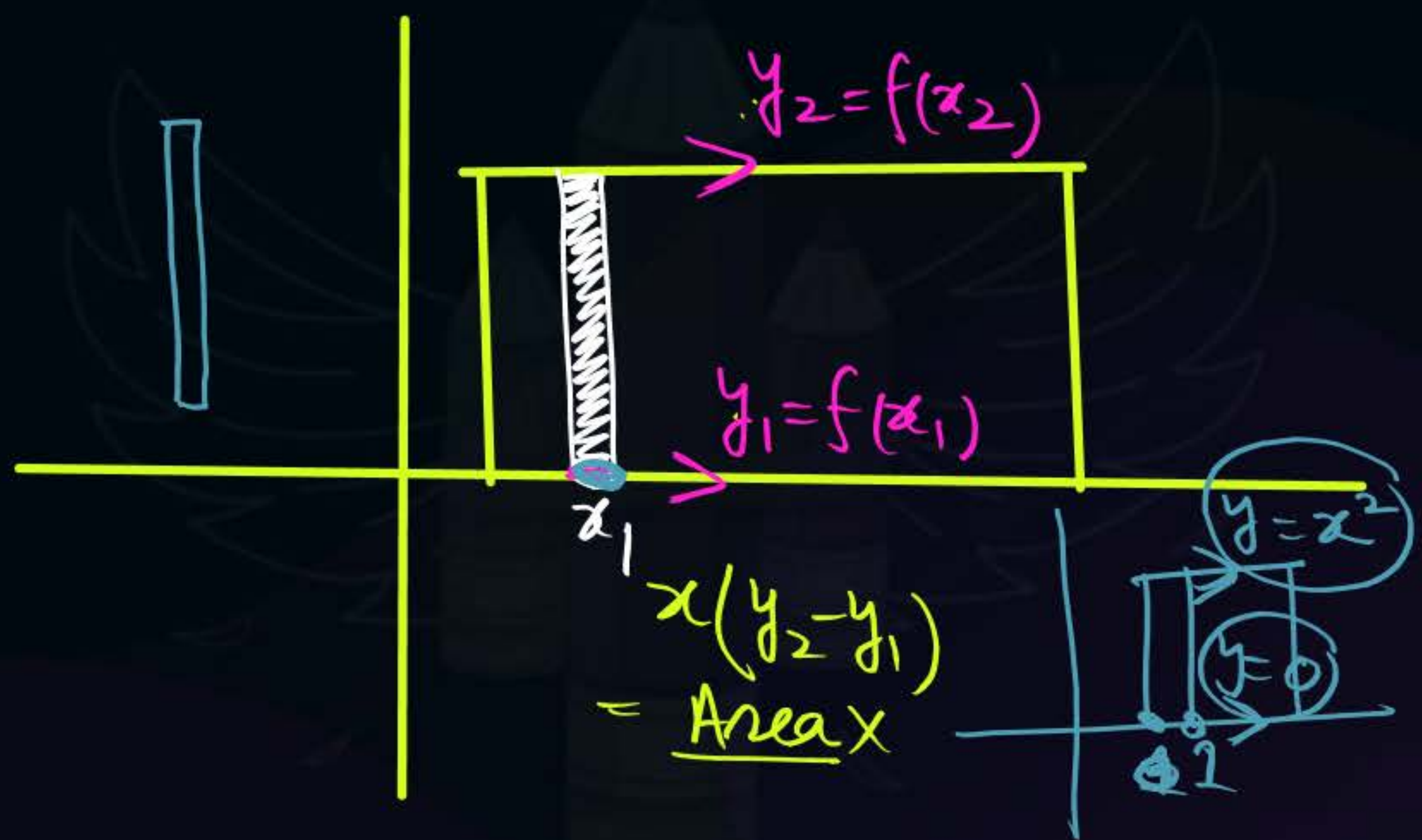
Vertical strip

X — Fixed - constant value a to b

y — variable - $y_1 = f_1(x)$ to $y_2 = f_2(x)$

$$\int_{x=a}^{b} \int_{y_1=f_1(x)}^{y_2=f_2(x)} f(x,y)\, dy\, dx = \text{volume via double Integral}$$



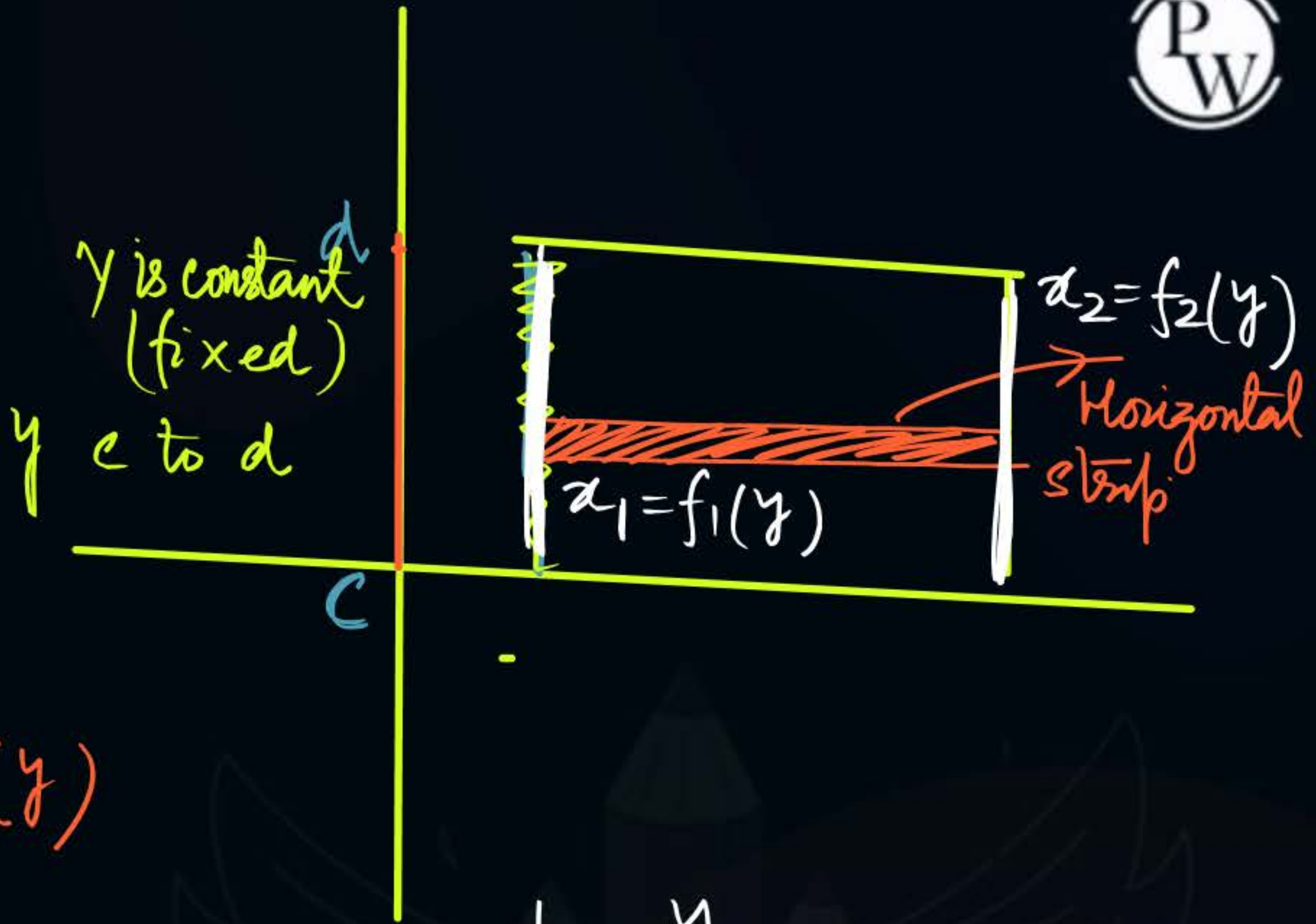


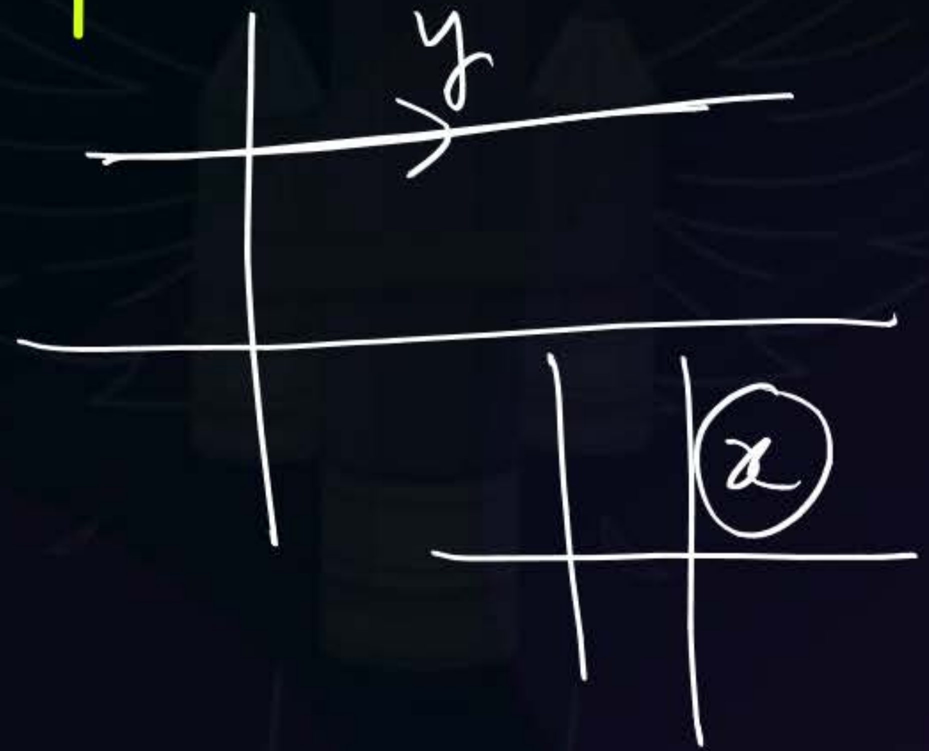
$$V = \int_{c}^{d} \int_{f_1(y)}^{f_2(y)} f(x,y)\, dx\, dy$$

Horizontal strip:

$y \rightarrow$ constant c to d

$x \rightarrow$ variable $x_1 = f_1(y)$ to $x_2 = f_2(y)$





Topic :Double integration

$$V = \iint f(x,y)\,dy\,dx$$

$(x+y)$

Region $x=0$ $y=x+2$
$x=2$
$y=x$

#Q. Illustration

$\iint(x+y)dydx$ where R is the region bounded by

x = 0

x = 2

y = x

y = x + 2

Step 01 Plot The Limit

$x=0,\ x=2,\ y=x,\ y=x+2$

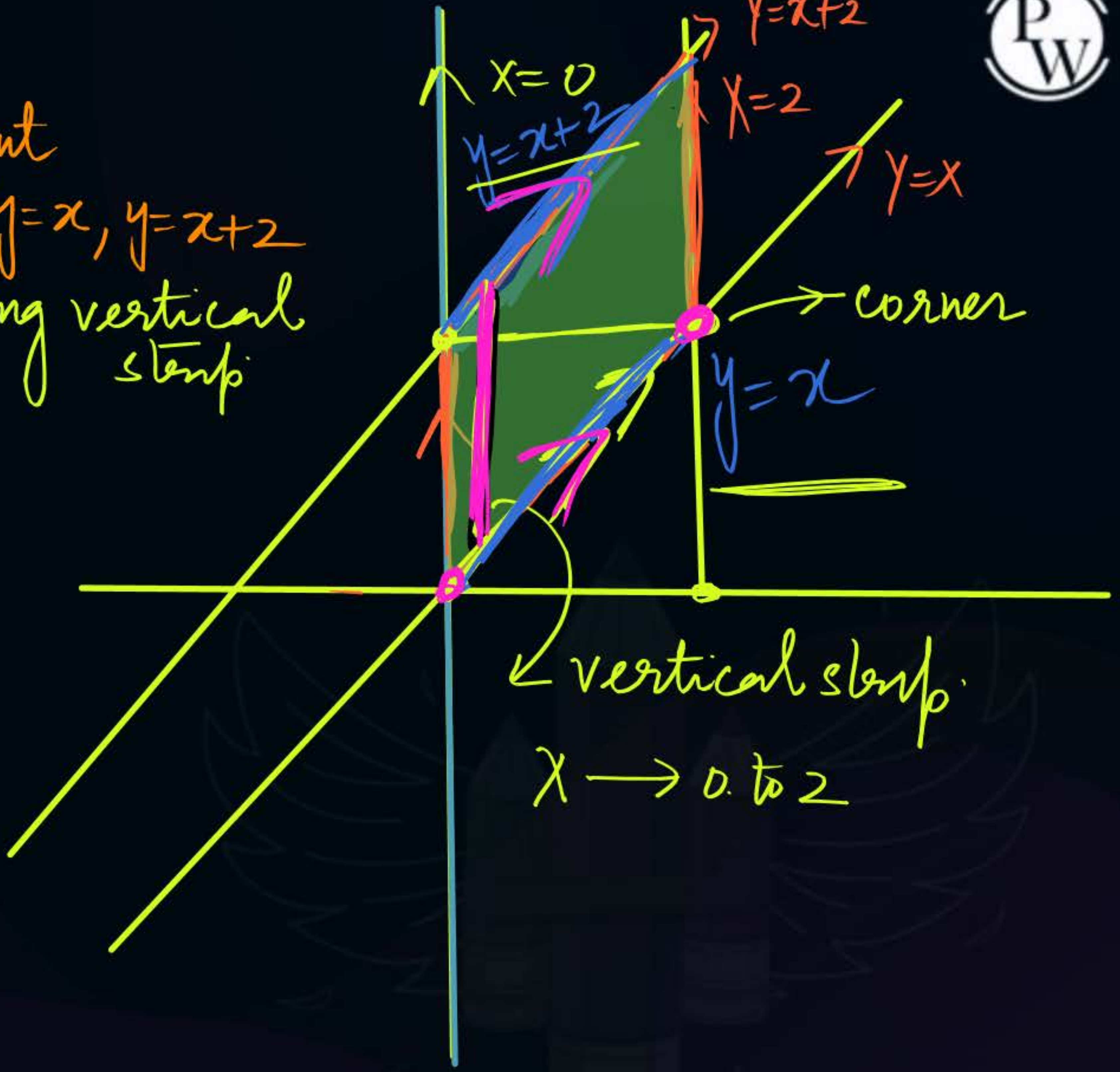
Step ② Check The strip — using vertical strip

Step ③ find The Limits x, y

$x \longrightarrow 0$ to 2

$y \longrightarrow x$ to $(x+2)$

$$V = \int_0^2 \int_x^{x+2} (x+y)\,dy\,dx$$



$$V = \int_0^2 \boxed{\int_x^{x+2} (x+y)\,dy}\,dx$$

$$= \int_0^2 dx \left[\int_x^{x+2} (x+y)\,dy \right]$$

$$\Rightarrow \int_0^2 dx \left[xy + \frac{y^2}{2} \right]_{y=x}^{x+2}$$

$$\Rightarrow \int_0^2 \left[x(x+2) + \frac{(x+2)^2}{2} - \left[x \cdot x + \frac{x^2}{2} \right] \right] dx$$

$$= \underline{\underline{12}}$$

Topic :Double integration

$\iint xy\,dy\,dx$ OR $I = \underbrace{\iint xy\,dx\,dy}_{\text{Volume via Double Integrals}}$

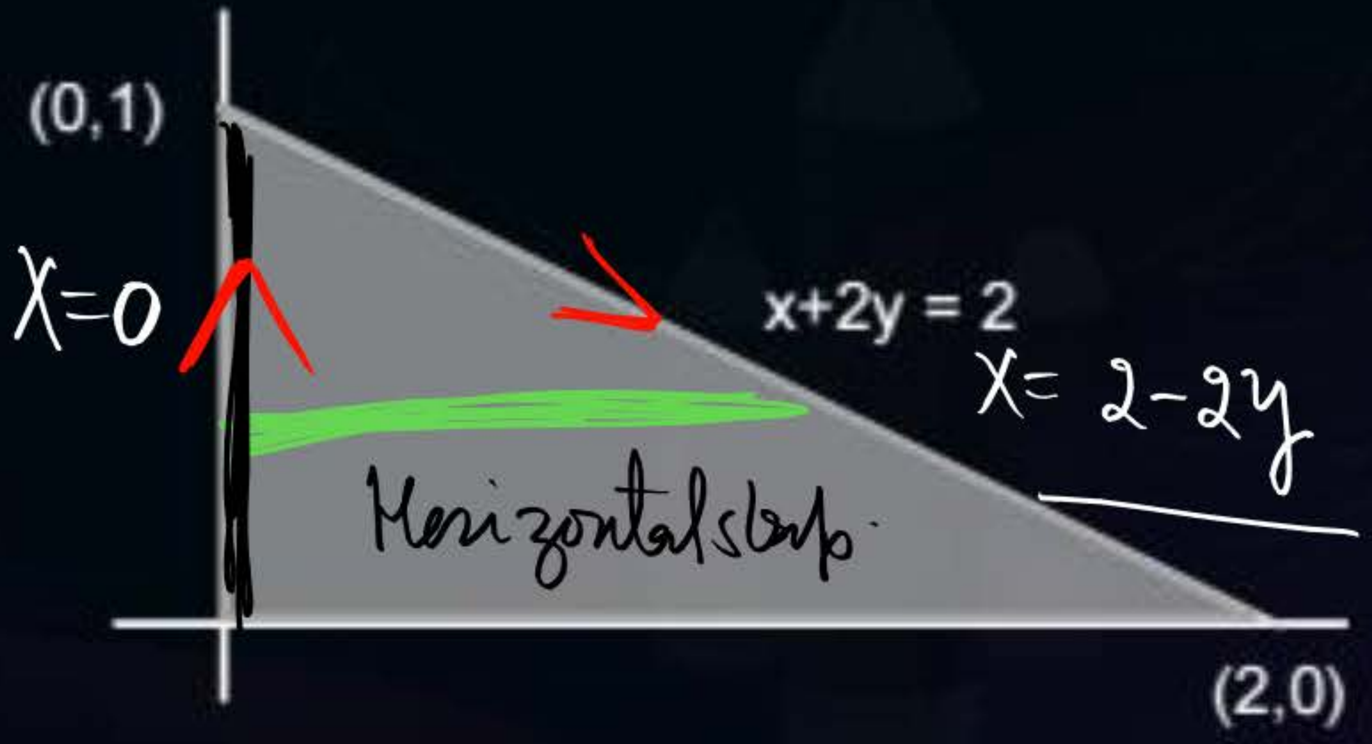
Change The order

#Q. Illustration

Consider the shaded triangular region, the value of $\iint xy \ dx\, dy$

$$\int_y \int_x xy\,dx\,dy$$

$$= \int_{y=0}^{1} y\,dy \int_{x=0}^{2-2y} x\,dx$$



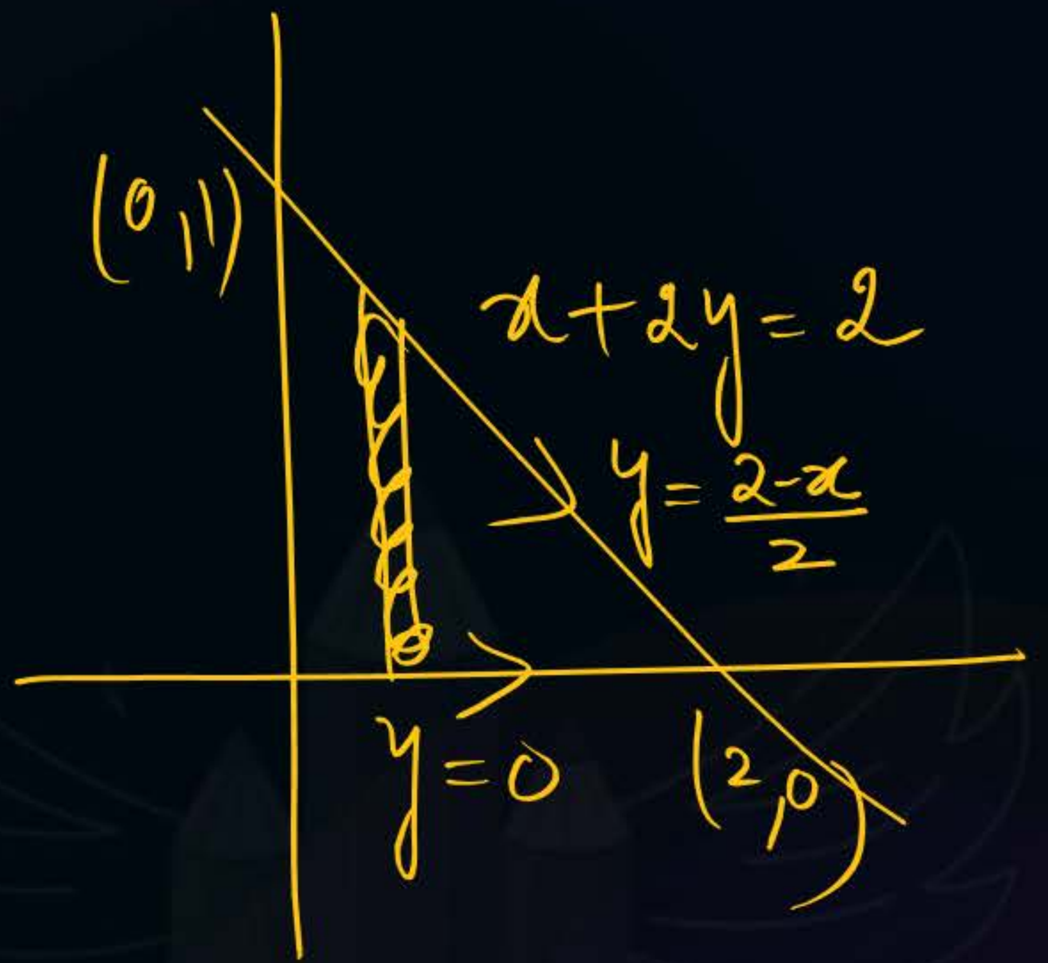
$$\int_0^1 y\,dy \int_0^{2-2y} \boxed{x}\,dx$$

$$= \int_0^1 y\,dy \left[\frac{x^2}{2}\right]_0^{2-2y}$$

$$= \int_0^1 \frac{y(2-2y)^2}{2}\,dy$$

$$= \frac{1}{6}$$

$$\int_a^b f(x)\,dx = F(b)-F(a)$$



$$\int_{x=0}^{2}\int_{y=0}^{\frac{2-x}{2}} xy\,dy\,dx$$

Topic :Double integration

GATE

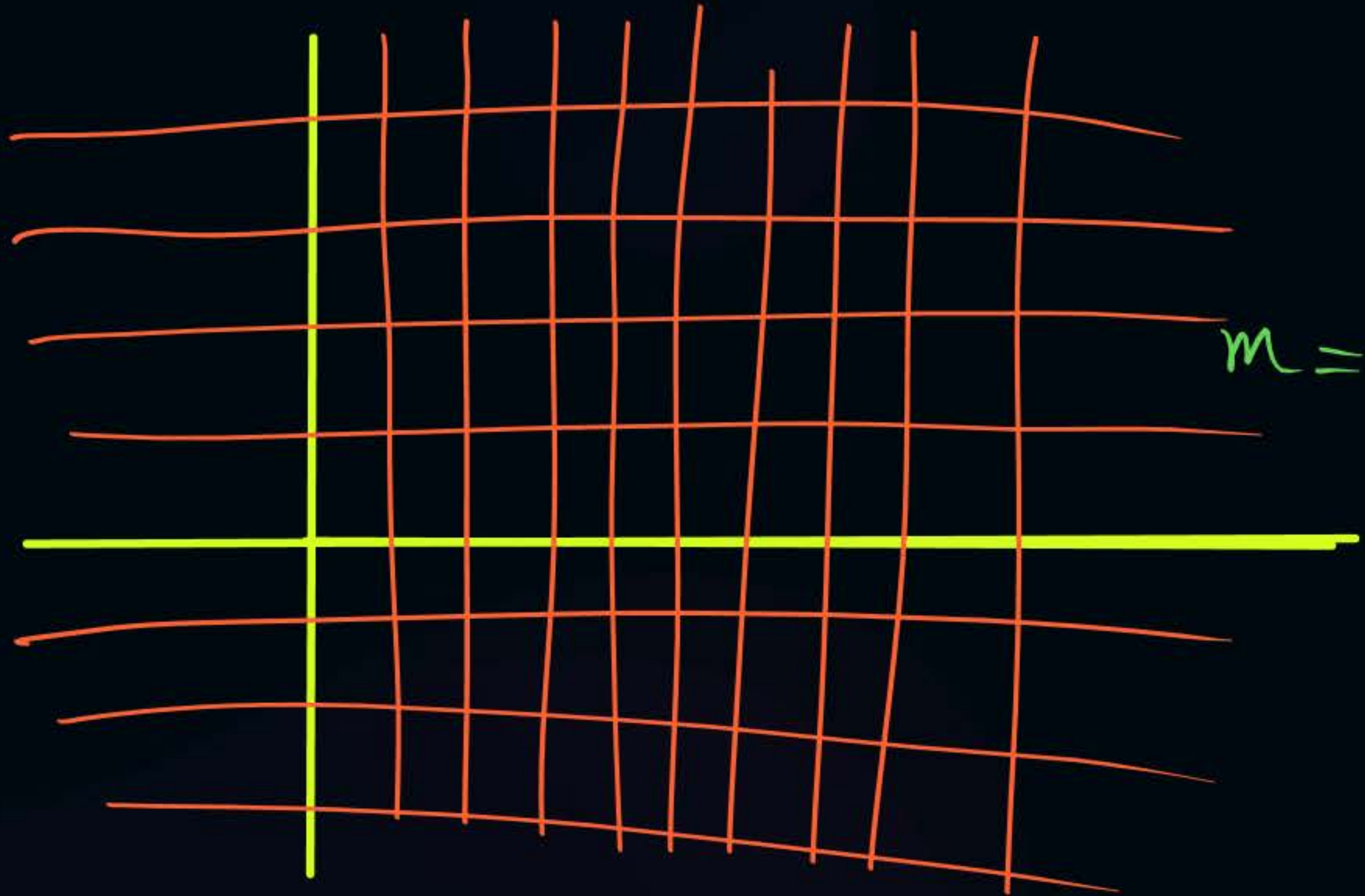
#Q. Illustration

The volume enclosed by the surface $f(x, y) = e^x$ over the triangle bounded by lines

$x = y$

$x = 0$

$y = 1$ in the xy plane



$y = x$

$y = mx$

$m = 1 = 45^\circ$

$m = \theta = \tan 45^\circ$

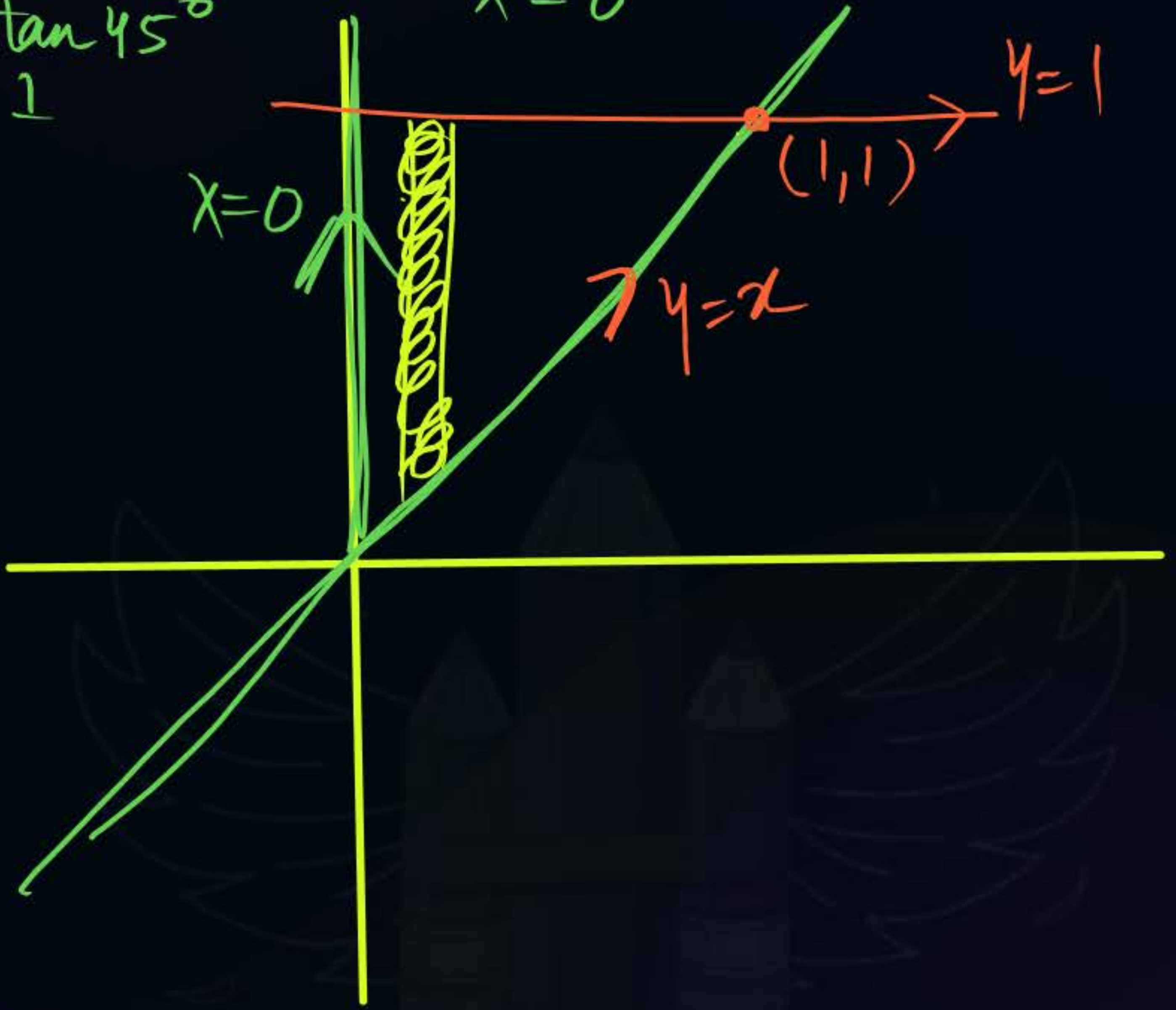
$= 1$

$x = y$

$y = 1$

$x = 0$

$\iint e^{x}\,dy\,dx$



$$= \int_{x=0}^{1} \int_{y=x}^{1} e^{x}\,dy\,dx$$

$$= \int_{x=0}^{1} e^{x}\,dx \int_{x}^{1} dy$$

$$= \int_{0}^{1} e^{x}\,dx \left[y \right]_{x}^{1}$$

$$= \int_{0}^{1} e^{x}(1-x)\,dx$$

$$= \int_{0}^{1} e^{x}\,dx - \int_{0}^{1} x e^{x}\,dx$$

$$= e-2$$

Topic :Double integration

#Q. $\int\int(x^2 + y^2)\, dx\, dy$ over the region bounded by $y = x^2$ & $y^2 = x$ H.W

A 6/32

B 6/35

C 6/30

D 1

THANK - YOU