GATE-All BRANCHES
Engineering Mathematics

Vector Calculus

Lecture No.- 01

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Recap of Previous Lecture

Topic

Concept of length of curve

Volume of solid revolution

Topic

Maxima and minima with two variables

Topic

Problems based on length of curve, Volume of solid revolution, Maxima and minima

Topics to be Covered

Topic Concept of line integral

Topic Problem based on line integral

Topic Concept and problems based of curl

Line Integral :

Field $\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{K}$

Vector is always one-dimension

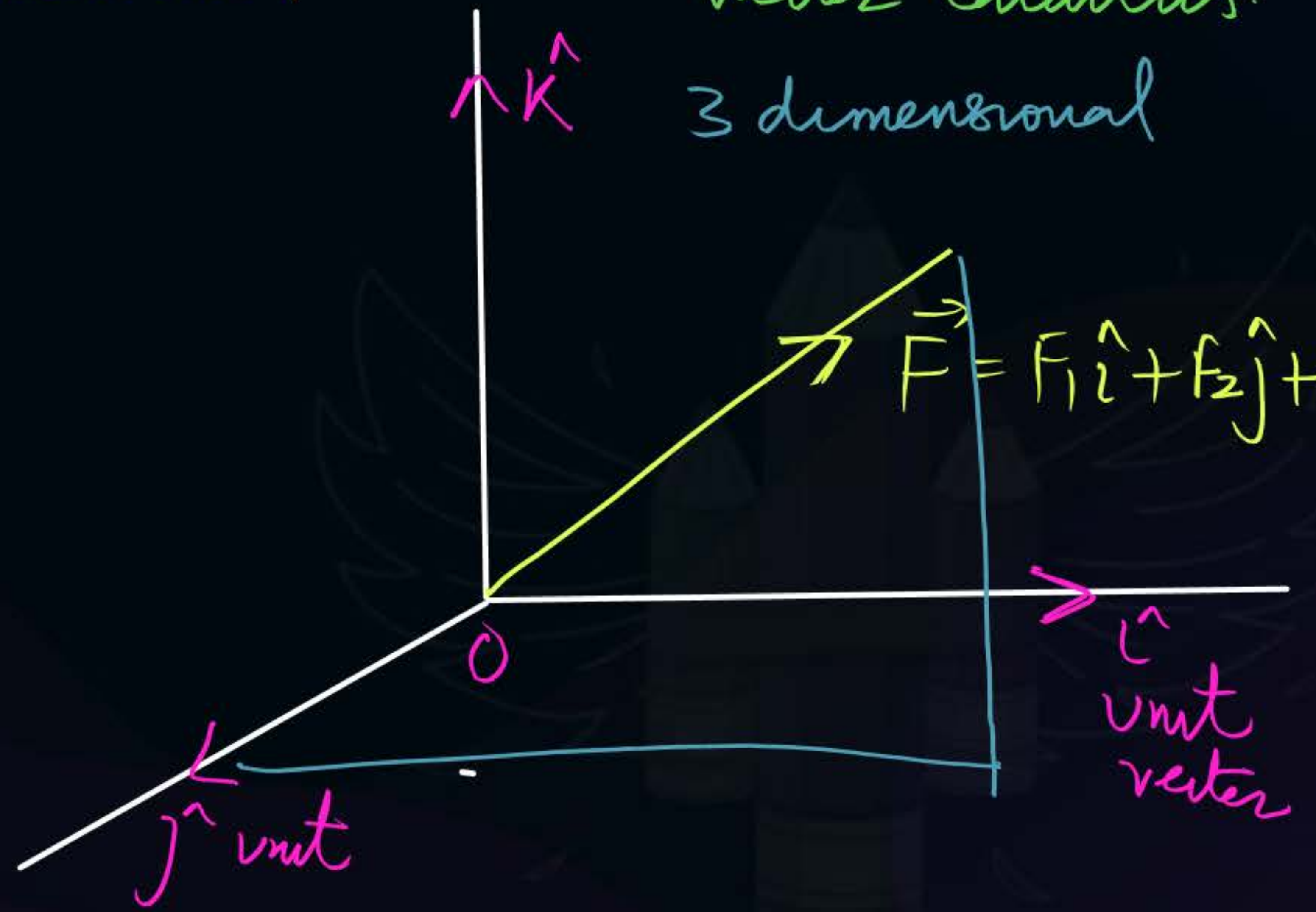
but Projection — n-dimensional

Function — Scaler Function
— Vector Function

$\boxed{\text{Vector Function} \rightarrow \text{mag} + \text{Direction}}$

Vector Calculus.

3 dimensional



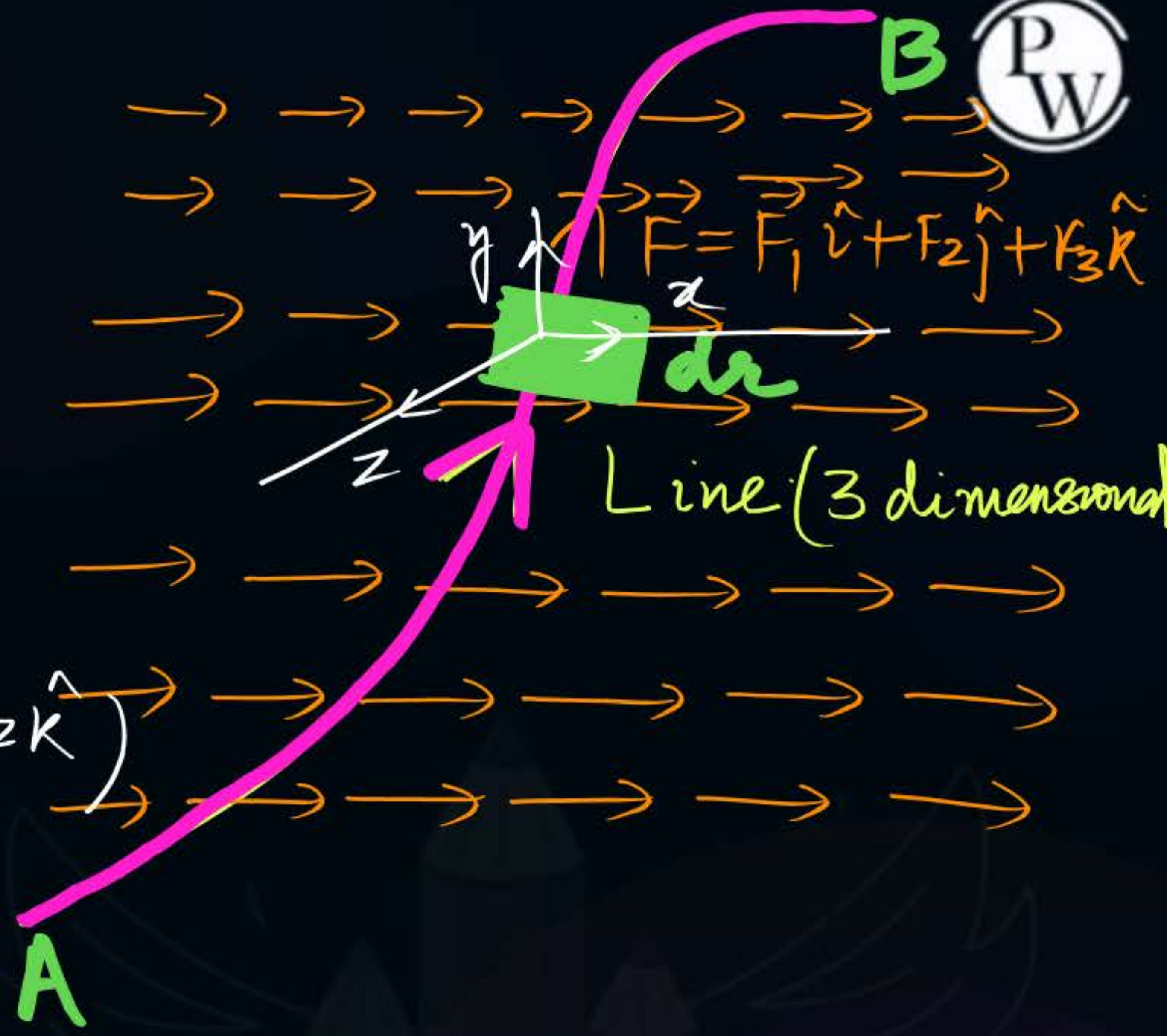
Work done or Line integral

$\oint \vec{F}\, d\vec{r}$ = Line Integral

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$$\oint \vec{F}\, d\vec{r} = \oint (F_1\hat{i} + F_2\hat{j} + F_3\hat{k})(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$



Line integral

$$\oint \vec{F}\, d\vec{r} = \oint_A^B F_1\,dx + F_2\,dy + F_3\,dz$$

$x \to y \to z$

cartesian form — convert x OR convert y OR convert z

Topic : Vector calculus

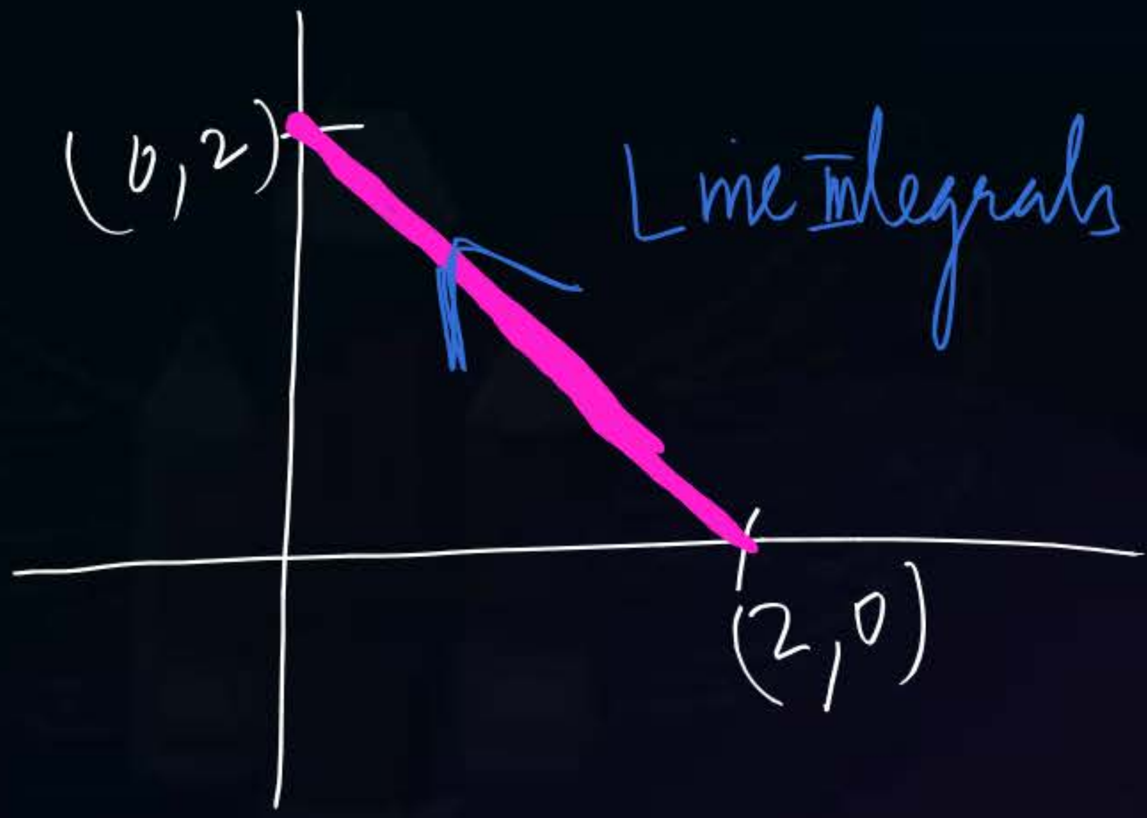
#Q. $F(x, y) = (x^2 + xy)\,\hat{a}_x + (y^2 + xy)\,\hat{a}_y$. Its line integral over the straight line from $(x, y) = (0, 2)$ to $(x, y) = (2, 0)$ evaluates to

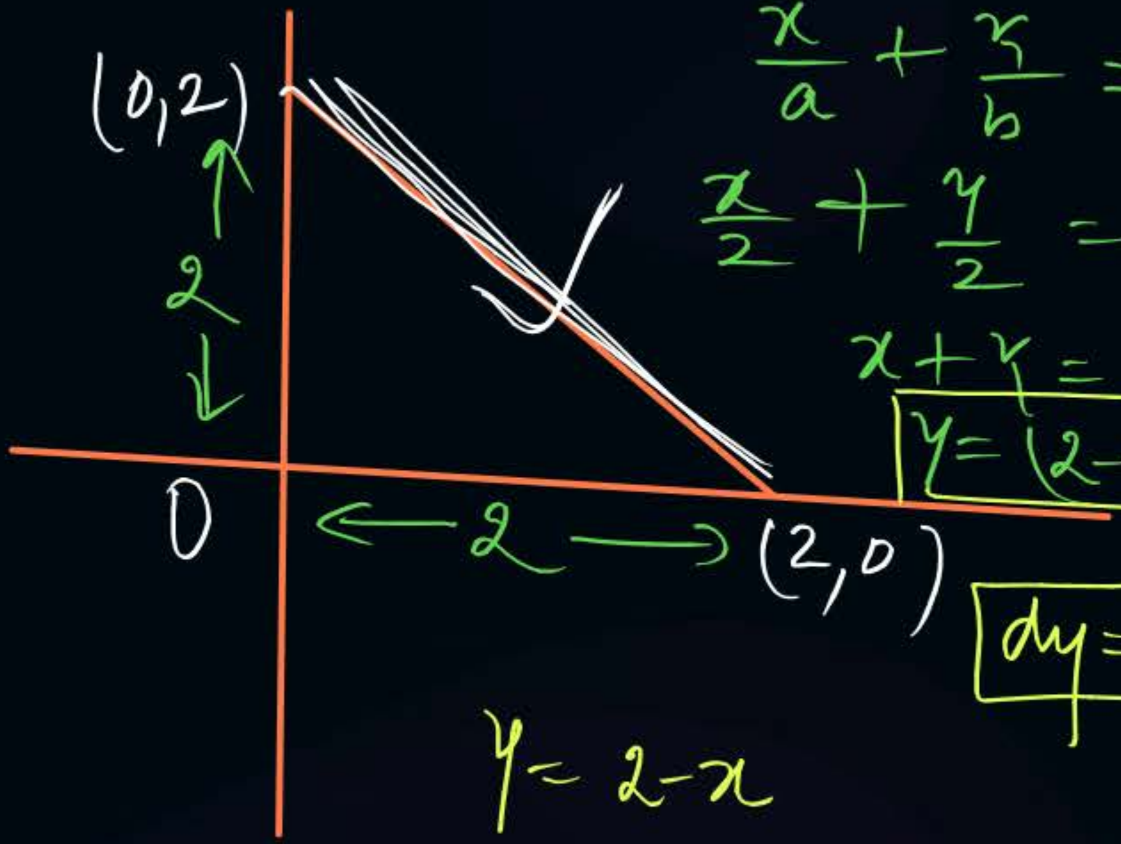
A. −8

B. 4

C. 8

D. 0





$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{2} + \frac{y}{2} = 1$$

$$x + y = 2$$

$$\boxed{y = (2-x)}$$

$$\boxed{dy = -dx}$$

$$y = 2 - x$$

element change

$$\boxed{dy = -dx}$$

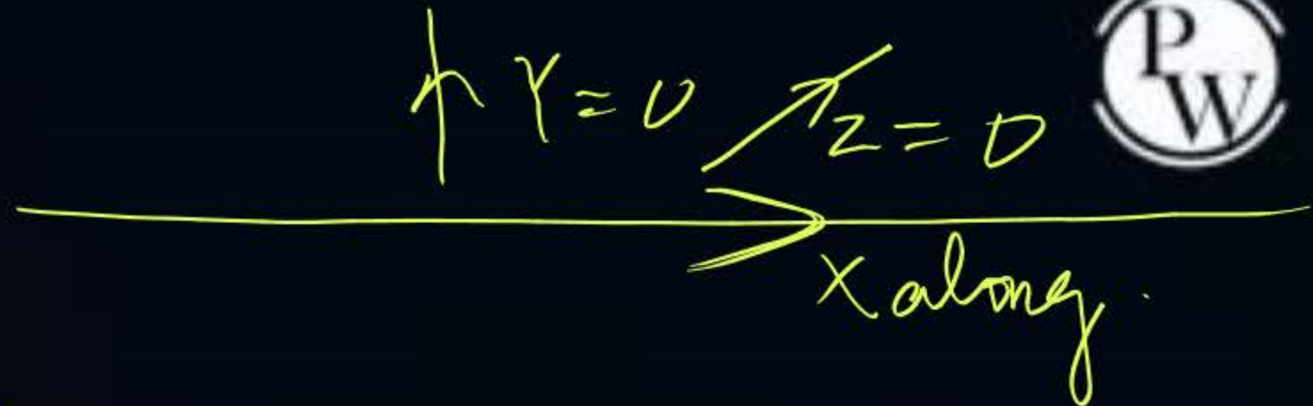
$$\vec{F} = (x^2+xy)\hat{i} + (y^2+xy)\hat{j}$$

$$\oint \left[(x^2+xy)\,dx + (y^2+xy)\,dy\right]$$

$$= \int_0^2 \left[(x^2 + x(2-x))\,dx + \left((2-x)^2 + x(2-x)\right)(-dx)\right]$$

$$= 8$$ **Ans**

Topic : Vector calculus



#Q. Given a vector field $\overline{F} = y^2x\hat{a}_x - yz\hat{a}_y - x^2\hat{a}_z$, the line integral $\int F.dl$ evaluated along a segment on the x-axis from x = 1 to x = 2 is

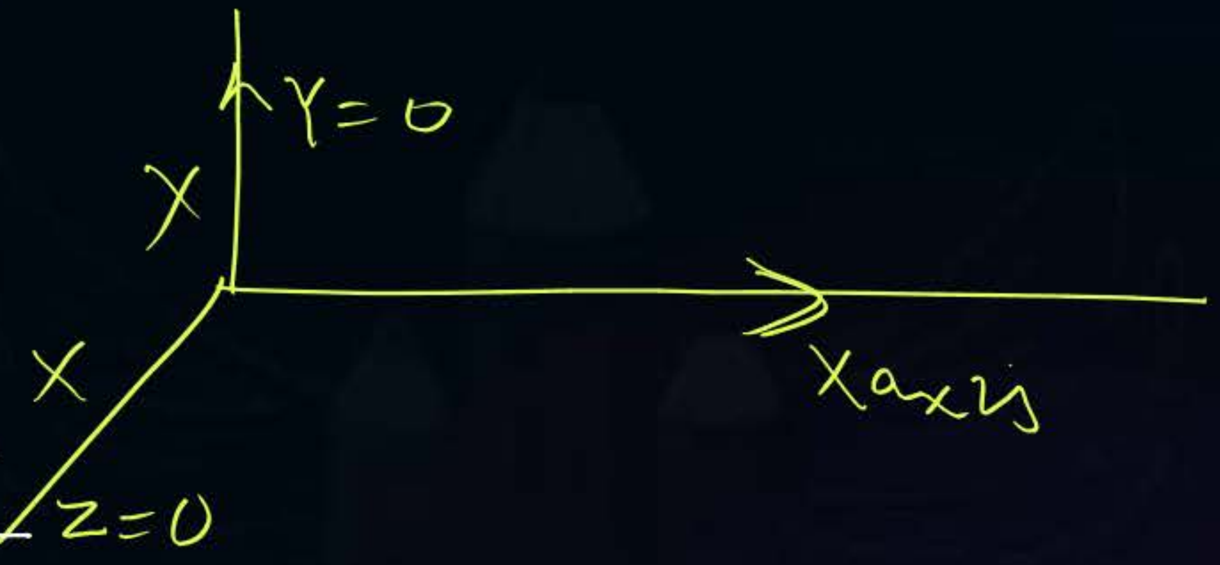
$\vec{F} = y^2x\,\hat{i} - yz\,\hat{j} - x^2\hat{k}$

$y=0 \quad z=0$

$= \oint \vec{F}\,d\vec{r} = \oint y^2x\,dx - yz\,dy - x^2dz$

x-axis $= 0$

$y=0$
$z=0$



A 2.33

B ✓ 0

C 2.47

D 7

Topic : Vector calculus

$\oint (2xy^2 dx + 2x^2 y\, dy + dz)$

$\left.\begin{matrix} x = t \\ y = t \\ z = t \end{matrix}\right]$

#Q. The value of line integral $\int_c \left(2xy^2 dx + 2x^2 y dy + dz\right)$ along a path joining the origin $\underset{A}{(0, 0, 0)}$ and the point $\underset{B}{(1, 1, 1)}$ is

$A(0,0,0)\ B(1,1,1)$

$\frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0}$

$\frac{x}{1} = \frac{y}{1} = \frac{z}{1} = t$

$\begin{cases} x = t & dx = dt \\ y = t & dy = dt \\ z = t & dz = dt \end{cases}$

$t = 0$ $x = 0$, $y = 0$, $z = 0$ $t = 1$ $x = 1$ $z = 1$, $y = 1$

given pts

$(x_1, y_1, z_1)\ (x_2, y_2, z_2)$

$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

$\Rightarrow \int (2t^3 + 2t^3 + 1)\, dt$

$= \int_0^1 (4t^3 + 1)\, dt = 2$

A. 0

B. 2

C. 4

D. 4

Topic : Vector calculus

$\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

$x = t \quad y = t^2 \quad z = t$

#Q. The line integral of the vector field $F = 5xz\hat{i} + (3x^2 + 2y)\hat{j} + x^2z\hat{k}$ along a path from (0, 0, 0) to (1, 1, 1) parameterized by (t, t^2, t) is

$= \frac{43}{12}$

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Topic : Vector calculus

H.W

#Q. Given $\overline{F} = (x^2 - 2y)\overline{i} - 4yz\overline{j} + 4xz^2\overline{k}$, the value of the line integral $\int_c \overline{F}.d\overline{l}$ along the straight line c from (0, 0, 0) to (1, 1, 1) is

A 3/16

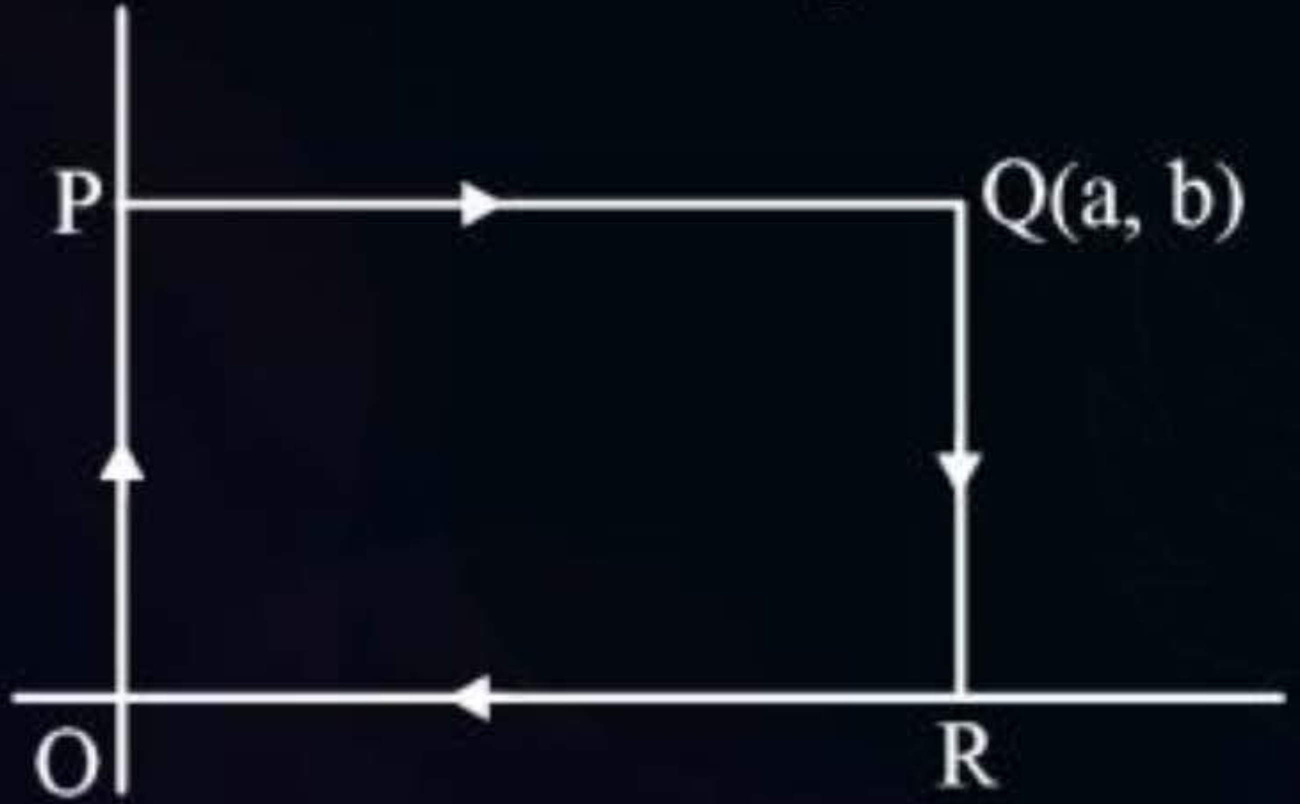
B 0

C -5/12

D -1

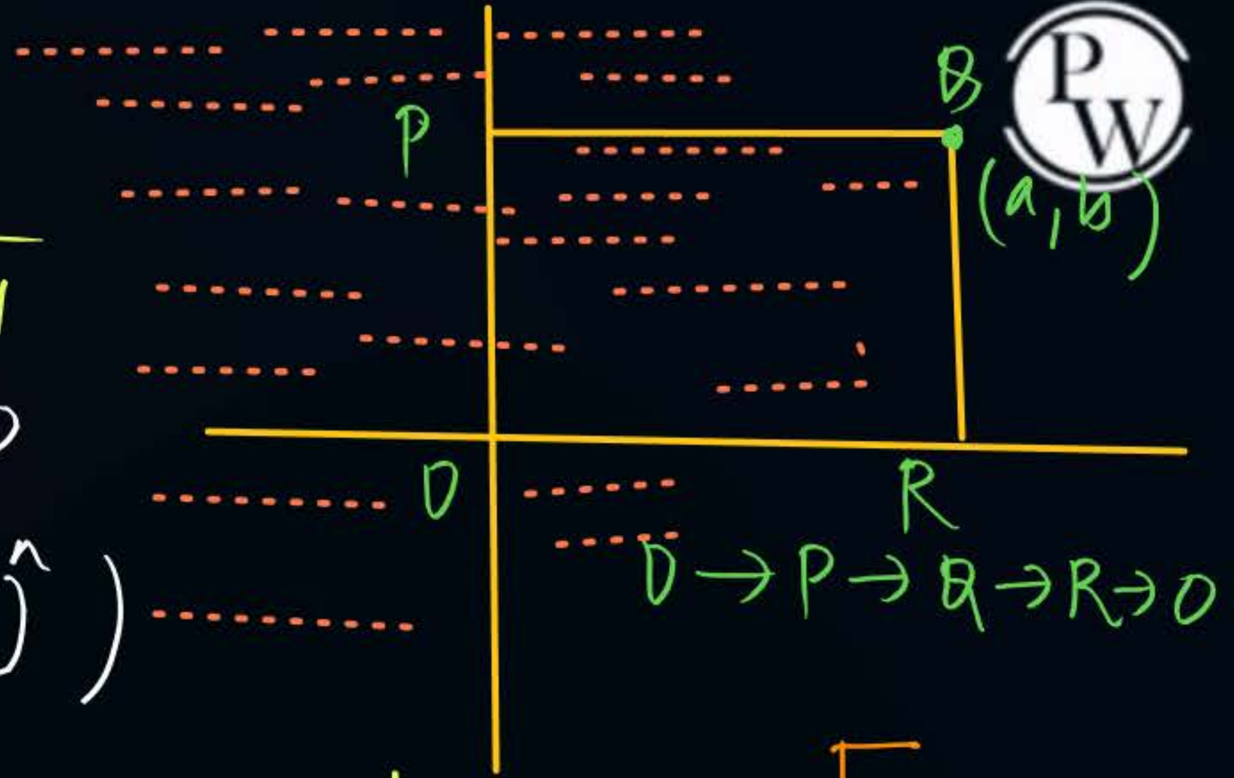
Topic : Vector calculus

#Q. When an object $O \rightarrow P \rightarrow Q \rightarrow R \rightarrow O$ in a Force field $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ along the line integral Calculate $\oint \vec{F}.dl$



$\oint \vec{F}\cdot d\vec{r}$ $\quad \vec{F} = (x^2-y^2)\hat{i} + 2xy\hat{j}$

Γ = total line integral $= \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4$



path $O \to P$ $\quad x = 0$ (path) $dx = 0$

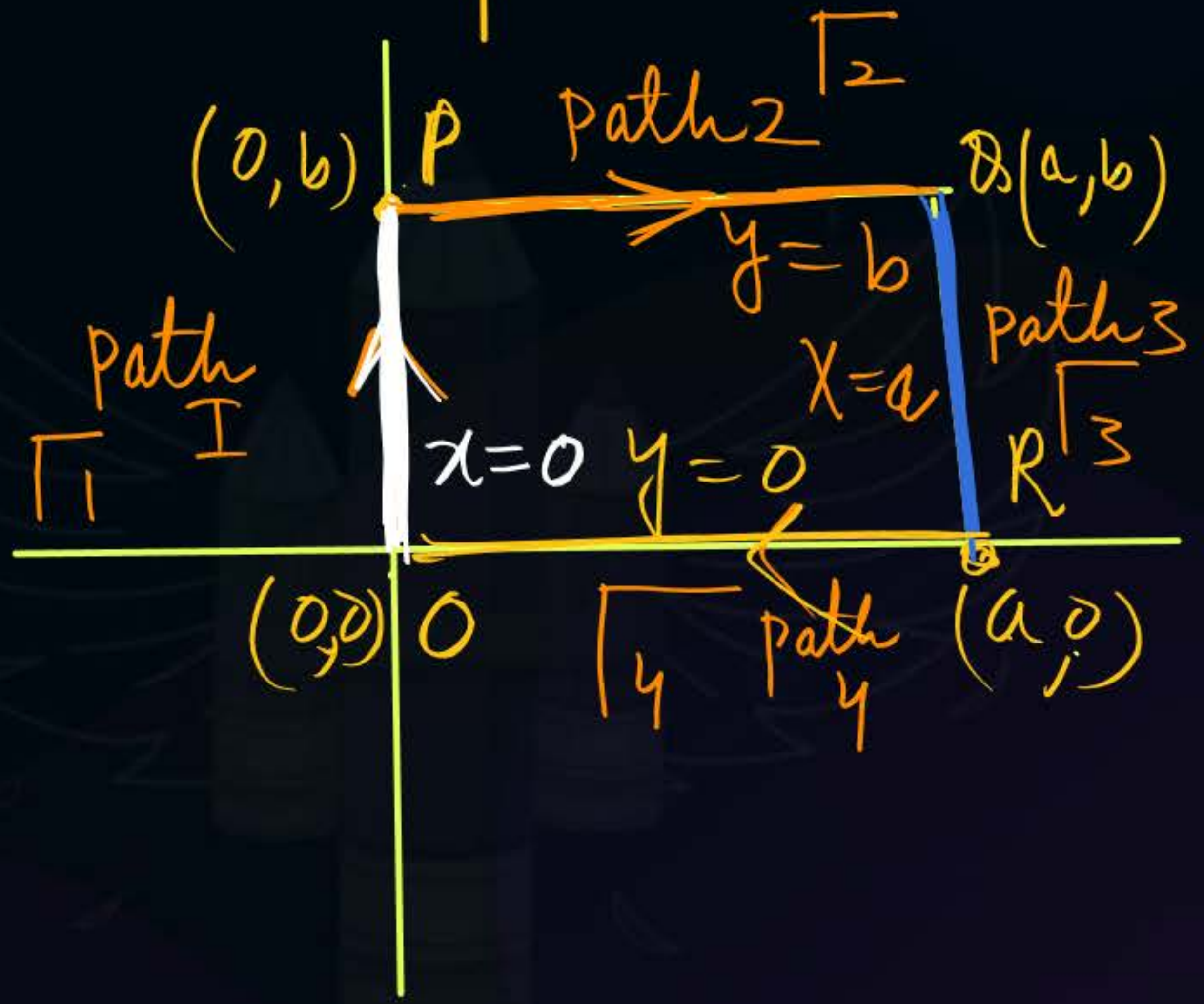
$$\int \vec{F}\cdot d\vec{r} = \int \left((x^2-y^2)\hat{i} + 2xy\hat{j}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j}\right)$$

$$= \int \underline{(x^2-y^2)\,dx} + 2xy\,dy = 0$$

Path $P \to Q$ $\quad y = b \quad dy = 0$

$$\int \vec{F}\cdot d\vec{r} = \int_a (x^2-y^2)\,dx + 2xy\,dy$$

$$= \int_0 (x^2-b^2)\,dx = \frac{a^3}{3} - b^2 a$$



path $Q \to R$ $\quad x = a \quad dx = 0$

$$\int_{Q\to R} \vec{F}\,d\vec{r} = \int \underline{(x^2 - y^2)\,dx} + 2xy\,dy = \int 2xy\,dy = 2a\int_b^0 y\,dy$$

$$= \underline{-ab^2}$$

path $R \to O$ $\quad y = 0 \quad dy = 0$

$$\int_{R\to O} \vec{F}\,d\vec{r} = \int (x^2 - y^2)\,dx + \underline{2xy\,dy}$$

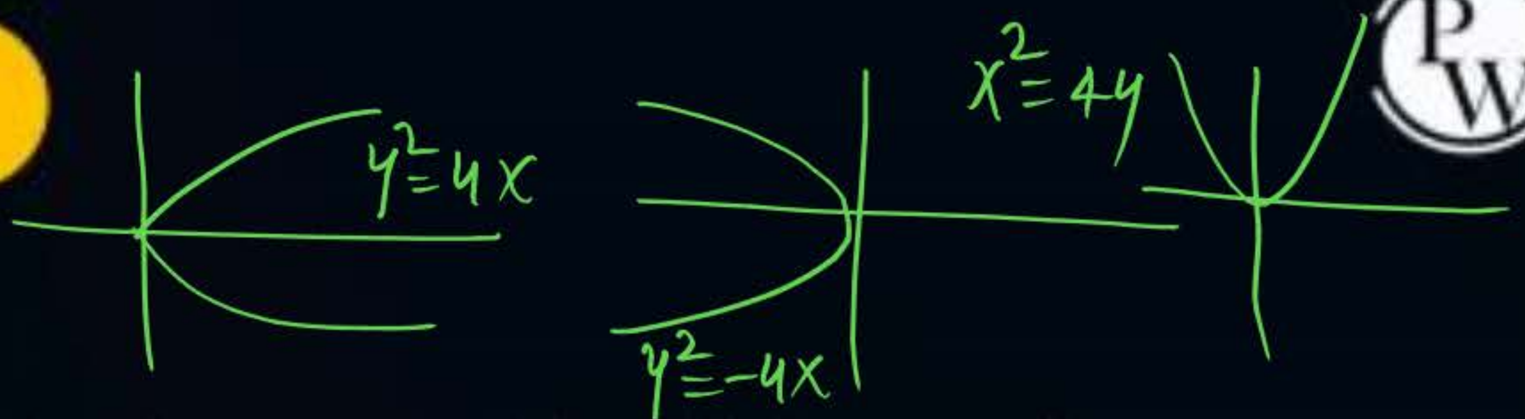
$$= \int_a^0 x^2\,dx = -\frac{a^3}{3}$$

$$\oint \vec{F}\,d\vec{r} = \int_{O\to P} + \int_{P\to Q} + \int_{Q\to R} + \int_{R\to O} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4$$

$$= 0 + \cancel{\frac{a^3}{3}} - ab^2 + (-ab^2) - \cancel{\frac{a^3}{3}} = \underline{-2ab^2}$$

Topic : Vector calculus

$\oint \vec{F}\, d\vec{r}$



#Q. $\oint \vec{F}.dl$ where $\vec{F} = x^2y^2\hat{i} + y\hat{j}$ along C is the curve $y^2 = 4x$ is xy plane (0, 0) to (4, 4)

$\vec{F} = x^2y^2\hat{i} + y\hat{j}$

$y^2 = 4x$

$\oint \vec{F}\,d\vec{r} = \oint (x^2y^2\,dx + y\,dy)$

convert in X

convert to Y

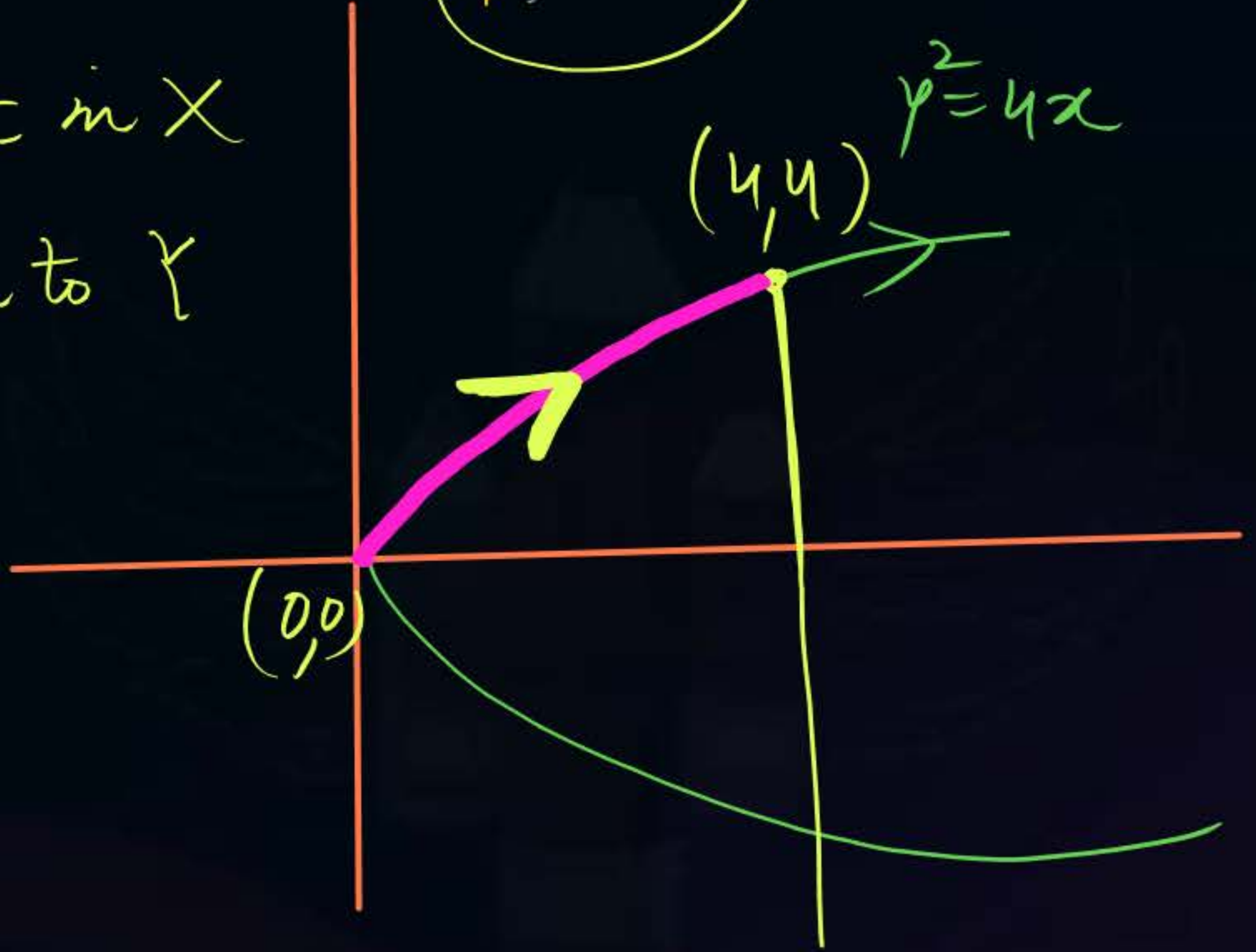
$y^2 = 4x$

$2y\,dy = 4\,dx$

$y\,dy = 2\,dx$

$= \int_0^4 (x^2 \cdot 4x + 2)\,dx$

$= \int_0^4 (4x^3 + 2)\,dx = 264$



THANK - YOU