Engineering Mathematics

DPP-01

Differential Equation + Partial Differential

1. In R^2, the family of trajectories orthogonal to the family of $x^{2/3} + y^{2/3} = a^{2/3}$ is given by
(a) $x^{4/3} + y^{4/3} = c^{4/3}$ (b) $x^{4/3} - y^{4/3} = c^{4/3}$
(c) $x^{5/3} - y^{5/3} = c^{5/3}$ (d) $x^{2/3} - y^{2/3} = c^{2/3}$

2. A particular integral of the differential equation
$y'' + 3y' + 2y = e^{e^x}$ is
(a) $e^{e^x} e^{-x}$ (b) $e^{e^x} e^{-2x}$
(c) $e^{e^x} e^{2x}$ (d) $e^{e^x} e^{x}$

3. An integrating factor of the differential equation
$\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \frac{1}{4}\left(x + xy^2\right)dy = 0$ is
(a) x^2 (b) $3 \log_e x$
(c) x^3 (d) $2 \log_e x$

4. Let y (x) be the solution of the differential equation
$(xy + y + e^{-x})\, dx + (x + e^{-x})\, dy = 0$
Satisfying y (0) = 1. Then, y (–1) is equal to
(a) $\frac{e}{e-1}$ (b) $\frac{2e}{e-1}$
(c) $\frac{e}{1-e}$ (d) 0

5. If $y(x) = \lambda e^{2x} + e^{\beta x}$, $\beta \neq 2$, is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$. Satisfying $\frac{dy}{dx}(0) = 5$, then y (0) equal to
(a) 1 (b) 4
(c) 5 (d) 9

6. If $x^h y^k$ is an integrating factor of the differential equation $y\,(1 + xy)\,dx + x\,(1 - xy)\,dy = 0$, then the ordered pair (h, k) is equal to
(a) (–2, –2) (b) (–2, –1)
(c) (–1, –2) (d) (–1, –1)

7. The general solution of the differential equation with constant coefficients $\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$
Approaches zero as $x \to \infty$, if:
(a) b is negative and c is positive
(b) b is positive and c is negative
(c) Both b and c are positive
(d) Both b and c are negative

8. Let y(x) be the solution of differential equation:
$\frac{d}{dx}\left(x\frac{dy}{dx}\right) = x;\ y(1) = 0, \frac{dy}{dx}|_{x=1} = 0$. Then y (2) is:
(a) $\frac{3}{4} + \frac{1}{2}\ln 2$ (b) $\frac{3}{4} - \frac{1}{2}\ln 2$
(c) $\frac{3}{4} + \ln 2$ (d) $\frac{3}{4} - \ln 2$

9. One of the points which lies on the solution curve of the differential equation (y – x) dx + (x + y) dy = 0 with the condition y (0) = 1, is:
(a) (1, –2) (b) (2, –1)
(c) (2, 1) (d) (–1, 2)

10. The non-zero value of the n for which the differential equation $(3xy^2 + n^2x^2y)\, dx + (nx^3 + 3x^2y)dy = 0$, $x \neq 0$, becomes exact is:
(a) –3 (b) –2
(c) 2 (d) 3

11. If y(t) is a solution of the differential equation
$y'' + 4y = 2e^t$, then $\lim_{t\to\infty} e^{-t}y(t)$ is equal to
(a) $\frac{2}{3}$ (b) $\frac{2}{5}$
(c) $\frac{2}{7}$ (d) $\frac{2}{9}$

12. A particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^{2x}\sin x \text{ is}$$

(a) $\frac{e^{2x}}{10}(3\cos x - 2\sin x)$

(b) $-\frac{e^{2x}}{10}(3\cos x - 2\sin x)$

(c) $-\frac{e^{2x}}{5}(2\cos x + \sin x)$

(d) $-\frac{e^{2x}}{5}(2\cos x - \sin x)$

13. Let $y(x) = u(x)\sin x + v(x)\cos x$ be a solution of the differential equation $y'' + y = \sec x$. Then $u(x)$ is

(a) $\ln|\cos x| + c$ (b) $-x + c$

(c) $x + c$ (d) $\ln|\sec x| + c$

14. Let x, $x + e^x$ and $1 + x + e^x$ be solutions of a linear second order ordinary differential equation with constant coefficients. If $y(x)$ is the solution of the same equation satisfying $y(0) = 3$ and $y'(0) = 4$, then $y(1)$ is equal to

(a) $e + 1$ (b) $2e + 3$

(c) $3e + 2$ (d) $3e + 1$

15. A partial differential equation:

$$z\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xyz$$

(a) is of order 1, and is non-linear

(b) is of order 1, and is linear

(c) is of order 2, and is non-linear

(d) is of order 2, and is linear

16. Particular integral of ordinary differential equation $y'' + 2y + y = xe^{-x}\sin x$ is:

(a) $-e^{-x}(x\sin x + 2\cos x)$

(b) $e^{x}(x\cos x + 2\sin x)$

(c) $xe^{-x}(\cos x + \sin x)$

(d) $xe^{-x}(\sin x - \cos x)$

17. The solution of homogeneous differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$ is (c being constant):

(a) $x = c\sin\left(\frac{y}{x}\right)$ (b) $x = c\sin\left(\frac{x}{y}\right)$

(c) $x = c\tan\left(\frac{y}{x}\right)$ (d) $x = c\cot\left(\frac{y}{x}\right)$

18. The orthogonal trajectory of the family of curves $ay^2 = x^3$, where a is an arbitrary constant, is

(a) $3y^2 + 2x^2 = \text{constant}$

(b) $2y^2 - 3x^2 = \text{constant}$

(c) $3y^2 - 2x^2 = \text{constant}$

(d) $2y^2 + 3x^2 = \text{constant}$

19. The differential equation of a family of parabolas with foci at origin and axis along x-axis is

(a) $y\left(\frac{dy}{dx}\right)^2 + 2x^2\frac{dy}{dx} + y = 0$

(b) $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$

(c) $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} + y = 0$

(d) $y^2\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y^2 = 0$

20. For $a, b, c \in \mathbb{R}$ if the differential equation $(ax^2 + bxy + y^2)\,dx + (2x^2 + cxy + y^2)\,dy = 0$ is exact, then

(a) $b = 2, c = 2a$ (b) $b = 4, c = 2$

(c) $b = 2, c = 4$ (d) $b = 2, a = 2c$

21. Let f_1 and f_2 be two solutions of

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$$

where a_0, a_1 and a_2 are continuous on [0, 1] and $a_0(x) \neq 0$ for all $x \in [0, 1]$. Moreover, let $f_1\left(\frac{1}{2}\right) = f_2\left(\frac{1}{2}\right) = 0$.

Then,

(a) one of f_1 and f_2 must be identically zero

(b) $f_1(x) = f_2(x)$ for all $x \in [0,1]$

(c) $f_2(x) = cf_2(x)$ for some constant c

(d) None of the above

22. The general solution of the equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{2x}$ is (c_1, c_2 being constants):

(a) $y = c_1e^{-2x} + c_2e^{-3x} + \frac{1}{10}e^{2x}$

(b) $y = c_1e^{-2x} + c_2e^{-3x} + \frac{1}{5}e^{2x}$

(c) $y = c_1e^{-2x} + c_2e^{-3x} + \frac{1}{20}e^{2x}$

(d) $y = c_1e^{-2x} + c_2e^{-3x} + \frac{1}{4}e^{2x}$

23. The general solution of the equation

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}-4y=0$$ is (c_1, c_2 being constants)

(a) $y = c_1e^{-x} + c_2e^{-4x}$
(b) $y = c_1e^{x} + c_2e^{-4x}$
(c) $y = c_1e^{-x} + c_2e^{4x}$
(d) $y = c_1e^{2x} + c_2e^{-4x}$

24. The singular solution of the Clairaut's differential equation $y = px+\frac{a}{p}$, where $p=\frac{dy}{dx}$, is:

(a) $y^2 = 4x$ (b) $y^2 = 4ax$
(c) $y^2 = ax$ (d) $y^2=\frac{x}{a}$

25. Consider the second order differential equation
$x^2y''(x) + xy'(x) - 9y(x) = 0$ for $x > 0$
If the solution satisfies the initial conditions $y(1) = 0$, $y'(1) = 2$, then $y(2)$ is

(a) $\frac{21}{8}$ (b) $\frac{63}{8}$
(c) $\frac{7}{16}$ (d) $\frac{63}{4}$

26. The eigenvalues associated with the BVP
$y''(x) - 2y'(x) + (1-\lambda)\,y(x) = 0$, $y(0) = 0$, $y(1) = 0$
js/are

(a) $\lambda = 0$
(b) $\lambda = \pi^2n^2$, $n = 1, 2, 3, \ldots$
(c) $\lambda = -\pi^2n^2$, $n = 1, 2, 3, \ldots$
(d) $\lambda = \pi n$, $n = 1, 2, 3, \ldots$

27. If the partial differential equations

$$(x-2)^2\frac{\partial^2u}{\partial x^2}-(y-3)^3\frac{\partial^2u}{\partial y^2}+2x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u$$

is parabolic in the region $S \subseteq R^2$ but not in $R^2\backslash S$, then S is
S is

(a) $\{(x, y) \in R^2 : x = 2 \text{ or } y = 3\}$
(b) $\{(x, y) \in R^2 : x = 2 \text{ and } y = 3\}$
(c) $\{(x, y) \in R^2 : x = 2\}$
(d) $\{(x, y) \in R^2 : y = 3\}$

28. Let $u(x, y)$ be the solution of the Cauchy problem $x^2\frac{\partial u}{\partial x}-y^2\frac{\partial u}{\partial y}=0$, $u \to e^x$ as $y\to\infty$, then $u(1, 1)$

(a) -1 (b) 0
(c) 1 (d) $e^{1/2}$

29. The general solution of the differential equation

$$x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=0 \text{ is}$$

(a) $y=\left(c_1+c_2x^2\right)e^x$
(b) $y=\left(c_1+c_2x\right)e^{2x}$
(c) $y=\left(c_1+c_2\log x\right)x$
(d) $y=\left(c_1+c_2\log x\right)x^2$

30. The initial value problem $x\frac{dy}{dx}=2y, y(a)=b$ has

(a) infinitely many solutions through $(0, b)$ if $b \neq 0$
(b) unique solution for all a and b
(c) no solution if $a = b = 0$
(d) infinitely many solution if $a = b = 0$

31. The solution of the differential equation $\frac{d^2y}{dx^2}+4y=\cos 2x$, given by

(a) $c_1\cos 2x+c_2\sin 2x+\frac{x}{4}\sin 2x$
(b) $c_1\cos 2x+c_2\sin 2x+\frac{x}{2}\sin 2x$
(c) $c_1\cos 2x+c_2\sin 2x+\frac{x}{4}\cos 2x$
(d) $c_1\cos 2x+c_2\sin 2x+x\cos 2x$

32. The following initial value problem of a first order linear system

$x' = 3x - 2y$, $x(0) = 1$
$y' = -3x + 4y$, $y(0) = -2$

can be converted into an initial value problem of a 2nd order differential equation for $x(t)$. It is

(a) $x'' - 7x' + 6x = 0$; $x(0) = 1$, $x'(0) = -2$
(b) $x'' - 7x' + 6x = 0$; $x(0) = 1$, $x'(0) = 0$
(c) $x'' - 7x' + 6x = 0$; $x(0) = 1$, $x'(0) = 7$
(d) $x'' - x' + 6x = 0$; $x(0) = 1$, $x'(0) = -2$

33. The solution of the differential equation $x(x-y)\,dy + y^2\,dx = 0$ is (c being constant):

(a) $y = ce^{y/x}$ (b) $y = ce^{x/y}$
(c) $y = x + ce^{y/x}$ (d) $y = x^2 - ce^{y/x}$

34. Let $u(x, t)$ be the solution of the wave equation $u_{xx} = u_{tt}$, $u(x, 0) = \cos(5\pi x)$, $u_t(x, 0) = 0$.
Then, the value of $u(1, 1)$ is:

(a) -1 (b) 0
(c) 2 (d) 1

35. For the wave equation $u_{tt} = 16u_{xx}$, the characteristic co-ordinates are
(a) $\xi = x + 16t$, $\eta = x - 16t$
(b) $\xi = x + 4t$, $\eta = x - 4t$
(c) $\xi = x + 256t$, $\eta = x - 256t$
(d) $\xi = x + 2t$, $\eta = x - 2t$

36. If the differential equation $2t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} - 3y = 0$ associated with the boundary conditions $y(1) = 5$, $y(4) = 9$, then $y(9) =$
(a) 27.44 (b) 13.2
(c) 19 (d) 11.35

37. The initial value problem
$x\frac{dy}{dx} = y + x^2, x > 0, y(0) = 0$
(a) infinitely many solutions
(b) exactly two solutions
(c) a unique solution
(d) no solution

38. Consider the one dimensional wave equation:
$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, x > 0, t > 0$
subject to the initial conditions:
$u(x, 0) = |\sin x|, x \geq 0$
$u_t(x, 0) = 0, x \geq 0$
and the boundary condition:
$u(0, t) = 0, t \geq 0$.
Then $u\left(\pi, \frac{\pi}{4}\right)$ is equal to
(a) 1 (b) 0
(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

39. The ordinary differential equation:
$\frac{dy}{dx} = \frac{2y}{x}$ with the initial condition y (0) = 0, has
(a) infinitely many solutions
(b) no solution
(c) more than one but only finitely many solutions
(d) unique solution

40. If $y = a\cos(\log x) + b\sin(\log x)$, then
$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y$ is equal to:
(a) –1 (b) 1
(c) 0 (d) None of these

41. Solution of the differential equation
$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is
(a) $e^y = x + e^x + c$ (b) $e^y = \frac{x^2}{2} + e^x + c$
(c) $e^y = \frac{x^3}{3} + e^x + c$ (d) $e^y = \frac{x^4}{4} + e^x + c$

42. Let y be the solution of
$(1+x)y''(x) + y'(x) - \frac{1}{1+x}y(x) = 0, x \in (-1, \infty)$
$y(0) = 1, y'(0) = 0$
Then
(a) y is bounded on $(0, \infty)$
(b) y is bounded on $(-1, 0]$
(c) $y(x) \geq 2$ on $(-1, \infty)$
(d) y attains it minimum at $x = 0$

43. The Wronskian of $\cos x$, $\sin x$ and e^{-x} at $x = 0$ is
(a) 1 (b) 2
(c) –1 (d) –2

44. The initial value problem $y' = \sqrt{y}, y(0) = \alpha, \alpha \geq 0$ has
(a) at least two solutions if $\alpha = 0$
(b) no solution if $\alpha > 0$
(c) at least one solution if $\alpha > 0$
(d) a unique solution if $\alpha = 0$

45. Consider the differential equation
$\sin 2x\frac{dy}{dx} = 2y + 2\cos x, y\left(\frac{\pi}{4}\right) = 1 - \sqrt{2}$
Then which of the following statements(s) is (are) TRUE?
(a) The solution is unbounded when $x \to 0$
(b) The solution is unbounded when $x \to \pi/2$
(c) The solution is bounded when $x \to 0$
(d) The solution is bounded when $x \to \pi/2$

46. Let $y(x)$ be the solution of the differential equation $\frac{dy}{dx} = (y-1)(y-3)$ satisfying the condition y (0) = 2.
Then which of the following is/are TRUE?
(a) The function $y(x)$ is not bounded above
(b) The function $y(x)$ is bounded
(c) $\lim_{a \to +\infty} y(x) = 1$
(d) $\lim_{a \to -\infty} y(x) = 3$

47. Let k, l, $\in$ R be such that every solution of $\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + ly = 0$ satisfies $\lim_{x \to \infty} y(x) = 0$.Then

(a) $3k^2 + l < 0$ and $k > 0$
(b) $k^2 + l > 0$ and $k < 0$
(c) $k^2 - l \le 0$ and $k > 0$
(d) $k^2 - l > 0$, $k > 0$ and $l > 0$

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48. The Solution(s) of the differential equation $\frac{dy}{dx} = (\sin 2x) y^{1/3}$ satisfying $y(0) = 0$ is (are)

(a) $y(x) = 0$

(b) $y(x) = -\sqrt{\frac{8}{27}} \sin^3 x$

(c) $y(x) = \sqrt{\frac{8}{27}} \sin^3 x$

(d) $y(x) = -\sqrt{\frac{8}{27}} \cos^3 x$

49. Let $y(x)$ be the solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0,\ y(0) = 1, \left.\frac{dy}{dx}\right|_{x=0} = -1$$

Then, $y(x)$ attains its maximum value at x.....

50. If the orthogonal trajectories of the family of ellipse $x^2 + 2y^2 = c_1, c_1 > 0$ are given by $y = c_2 x^{\alpha}, c_2 \in \mathbb{R}$, then $\alpha = \ldots$

51. Let $y(x)$, $x > 0$ be the solution of the differential equation $x^2 \frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = 0$ satisfying the conditions $y(1) = 1$ and $y'(1) = 0$. Then, the value of $e^2 y(e)$ is …

52. If $y(x)$ is the solution of the initial value problem

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0, y(0) = 2, \frac{dy}{dx}(0) = 0$$

Then $y(\ln 2)$ is (round off to 2 decimal places) equal to

53. If x(t) is the solution to the differential equation $\frac{dz}{dt} = x^2 t^3 + xt$, for $t > 0$, satisfying $x(0) = 1$, then the value of $x(\sqrt{2})$ is … (correct up to two decimal places.)

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Answer Key

1. (b)
2. (b)
3. (c)
4. (b)
5. (c)
6. (a)
7. (c)
8. (b)
9. (c)
10. (d)
11. (b)
12 (c)
13. (c)
14. (d)
15. (a)
16. (a)
17. (a)
18. (a)
19. (b)
20. (b)
21. (c)
22. (c)
23. (b)
24. (b)
25. (a)
26. (c)
27 (a)
28. (d)
29. (d)
30. (d)
31. (b)
32. (c)
33. (a)
34. (d)
35. (b)
36. (a)
37. (a)
38. (a)
39. (a)
40. (c)
41. (c)
42. (d)
43. (d)
44. (a, c)
45. (c, d)
46. (b, c, d)
47. (c, d)
48. (a, b, c)
49. (–0.28)
50. (1)
51. (3)
52. (1.19)
53. (2.712)

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