ENGINEERING MATHEMATICS

LAPLACE TRANSFORM

DPP: 1

- Q1 If the Laplace transform of a function f(t) is given by $\frac{s+3}{(s+1)(s+2)}$ then f(0) is.

(C)1

- Q2 The Laplace transform of sin h (at) is

- Q3 The Laplace transform F(s) of the exponential function $f(t) = e^{at}$ when t is greater than equal to 0, where a is a constant and (s-a) > 0, is
- (B) $\frac{1}{s-a}$

- **Q4** The value of $\int_0^\infty \frac{1}{1+x^2} dx + \int_0^\infty \frac{\sin x}{x} dx$ is

- Q5 The value of the integral $\int_{-\infty}^{\infty} 12\cos{(2\pi t)} \frac{\sin(4\pi t)}{4\pi t} dt$ is _____
- **Q6** Laplace transform of $\cos{(\omega t)} is \frac{s}{s^2 + \omega^2}$. The Laplace transformation of $e^{-2t}\cos(4t)$ is
- Q7 The function f(t) satisfies the differential equation $rac{d^2f}{dt^2}+f=0$ and the auxiliary
 - $f\left(0
 ight)=0,rac{df}{dt}\left(0
 ight)=4$.The Laplace transform of f(t) is given by

- (B) $\frac{4}{s+1}$ (D) $\frac{2}{a^4+1}$

Q8

The inverse Laplace transform of the function $F\left(s
ight) =rac{1}{s\left(s+1
ight) }$ is given by :

- (A) $f(t) = \sin t$
- (B) $f(t) = e^{-t} \sin t$
- (C) $f(t) = e^{-t}$
- (D) $f(t) = 1 e^{-t}$
- Q9 The Laplace transform of a function f(t) is $\frac{1}{s^2(s+1)}$. The function f(t) is
 - $\begin{array}{ll} \text{(A) } t-1+e^{-t} & \text{(B) } t+1+\\ \text{(C) } -1+e^{-t} & \text{(D) } 2t+e^{t} \end{array}$
- Q10 The inverse Laplace transform of $\frac{1}{(S^2+S)}$ is (A) $1+e^t$ (B) $1-e^t$ (C) $1-e^{-t}$ (D) $1+e^{-t}$
- (C) $1 e^{-t}$
- (D) $1 + e^{-t}$
- **Q11** Laplace transform for the functions $f(x) = \cos h$
 - (A) $\frac{a}{s^2 a^2}$ (B) $\frac{s}{s^2 a^2}$ (C) $\frac{a}{s^2 + a^2}$ (D) $\frac{s}{s^2 + a^2}$

- Q12 The solution of

$$rac{d^2y}{dt^2}-y=1,$$

additionally $yig|t=0=rac{dy}{dt}ig|_{t=0}=0$ in the Laplace s-domain

- (A) $\frac{1}{s(s+1)(s-1)}$ (B) $\frac{1}{s(s+1)}$ (C) $\frac{1}{s(s-1)}$ (D) $\frac{1}{(s-1)}$

- **Q13** The value of the integral is $2\int_{-\infty}^{\infty} \left(\frac{\sin 2\pi t}{\pi t}\right) dt$ equal to
 - (A) O

(B) 0.5

(C) 1

- (D) 2
- **Q14** Let $X(s)=rac{3s+5}{s^2+10s+21}$ be the Laplace Transform of a signal x (t). Then, x (0^+) is

- (A) O
- (B) 3
- (C) 5

(D) 21

Q15 Consider the differential equation:

$$rac{d^{2}y(t)}{dt^{2}}+2rac{dy(t)}{dt}+y\left(t
ight) =\delta\left(t
ight) ag{5}$$

with

- $\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y\left(t\right) = \delta\left(t\right)$ $y\left(t\right)\Big|_{t=0} = -2\,and\,\frac{dy}{dt}\Big|_{t=0^+} = 0$ The numerical value of $\frac{dy}{dt}\Big|_{t=0}$ is
- (A)-2

(C) 0

(D) 1



Answer Key

- Q1 (C)
- Q2 (C)
- Q3 (B)
- (B) Q4
- (3 to 3) Q5
- (D) Q6
- (C) Q7
- Q8 (D)

- (A) Q9
- (C) Q10
- (B) Q11
- Q12 (A)
- Q13 (D)
- Q14 (B)
- Q15 (D)



Hints & Solutions

Q1 Text Solution:

Given

$$L\left\{f\left(t
ight)
ight\}=rac{s+3}{(s+1)(s+2)}$$
 $\Rightarrow L\left\{f\left(t
ight)
ight\}=rac{2}{s+1}-rac{1}{s+2}$ Applying L^{-1} on both sides

$$\Rightarrow L^{-1}\left\{L\left\{f\left(t
ight)
ight\}
ight\} = L^{-1}\left\{rac{2}{s+1} - rac{1}{s+2}
ight\}$$

$$\Rightarrow f(t) = 2e^{-t} - e^{-2t}$$

$$\left(:: L\left\{ e^{at} \right\} = rac{1}{s-a}
ight)$$

$$\left(: L\left\{e^{at}\right\} = \frac{1}{s-a} \right)$$
$$: f(0) = 2 \cdot e^{-0} - e^{-2(0)} = 2 - 1 = 1$$

$$\Rightarrow f(0) = 1$$

Q2 Text Solution:

Sin h at
$$=rac{e^{at}-e^{-at}}{2}$$

Taking Laplace Transform, we get

$$= \frac{1}{2} \left[L \left\{ e^{at} \right\} - L \left\{ e^{-at} \right\} \right]$$

$$= \frac{1}{2} \left[\frac{1}{s-a} - \frac{1}{s+a} \right]$$

$$= \frac{1}{2} \left[\frac{1}{s-a} - \frac{1}{s+a} \right]$$

$$= \frac{1}{2} \left[\frac{2a}{s^2 - a^2} \right]$$

$$L\left[\sin\,hat
ight]=rac{a}{s^2-a^2}$$

Q3 Text Solution:

$$egin{aligned} L\left[f\left(t
ight)
ight] &= \int\limits_{0}^{\infty} e^{-st} \cdot f\left(t
ight) dt \ L\left[e^{at}
ight] &= \int\limits_{0}^{\infty} e^{-st} \cdot e^{at} \, dt = \int\limits_{0}^{\infty} e^{-(s-a)t} dt \ \left[rac{e^{-(s-a)t}}{-(s-a)}
ight]_{0}^{\infty} &= -rac{1}{s-a} \left(0-1
ight) \ \left[L\left(e^{at}
ight) &= rac{1}{s-a}
ight] \end{aligned}$$

Q4 Text Solution:

$$I = \int_0^\infty \frac{1}{1+x^2} dx + \int_0^\infty \frac{\sin x}{x} dx = ?$$

$$I_1 = \int\limits_0^\infty rac{1}{1+x^2} = \left[an^{-1} \, x
ight]igg|_0^\infty \ I_2 = \int\limits_0^\infty rac{\sin x}{x} \, dx$$

Considering Laplace Transform of Sin at

$$\Rightarrow L\left\{\sin\,at
ight\} = rac{a}{s^2+a^2}$$

By the Property of division with ' t '

$$\begin{split} &\Rightarrow L\left\{\frac{\sin at}{t}\right\} = \int\limits_{s}^{\infty} \frac{a}{s^2 + a^2} \cdot ds \\ &\Rightarrow \int\limits_{s}^{\infty} e^{-st} \cdot \frac{\sin at}{t} \, dt = \left. \frac{1}{a} \cdot \operatorname{Tan} - 1\left(\frac{s}{a}\right) \right| \frac{\infty}{s} \\ &\Rightarrow \int\limits_{s}^{\infty} e^{-st} \cdot \frac{\sin at}{t} \, dt = \left. \frac{1}{a} \cdot \left\{ \frac{\pi}{2} - \operatorname{Tan}^{-1}\left(\frac{s}{a}\right) \right\} \end{split}$$

Substituting s = 0 and a = 1 in both sides of above equations :

$$\Rightarrow \int\limits_{s}^{\infty} rac{\sin at}{t} dt = 1\left\{rac{\pi}{2} - 0
ight\} = rac{\pi}{2}$$
 $\Rightarrow \int\limits_{s}^{\infty} rac{\sin x}{x} dx = rac{\pi}{2}$

$$\begin{split} I &= \left[\tan^{-1}x\right]_0^\infty + \int_0^\infty \frac{\sin x}{x} dx \\ \left[\because \int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}\right] \\ I &= \tan^{-1}\infty - \tan^{-1}0 + \frac{\pi}{2} \\ I &= \frac{\pi}{2} - 0 + \frac{\pi}{2} = \pi \\ \therefore \left[\int_0^\infty \frac{1}{1+x^2} dx + \int_0^\infty \frac{\sin x}{x} dx = \pi\right] \end{split}$$

Q5 Text Solution:

Let the given integral,

$$I = \int\limits_{-\infty}^{\infty} 12 \cdot \cos{(2\pi t)} rac{\sin(4\pi t)}{4\pi t} \, dt \ = 2 imes \int\limits_{0}^{\infty} 12 \cdot \cos{(2\pi t)} rac{\sin(4\pi t)}{4\pi t} \, dt \ igg($$

$$\displaystyle \because\int\limits_{-\infty}^{\infty}f\left(t
ight)dt=2 imes\int\limits_{0}^{\infty}f\left(t
ight)dt,Iff\left(-t
ight)$$

$$A \Rightarrow I = 3 imes \int\limits_0^\infty rac{2\sin(4\pi t)\cdot\cos(2\pi t)}{\pi t} \; dt \ \Rightarrow I = rac{3}{\pi} imes \int\limits_0^\infty rac{\sin(6\pi t)+\sin(2\pi t)}{t} \; dt \ \Big($$

$$egin{aligned} \because \sin A + \sin B &= 2 \sin \left(rac{A+B}{2}
ight) \cdot \cos \left(rac{A-B}{2}
ight) \ \Rightarrow I &= 3 imes \left\{\int\limits_0^\infty rac{\sin(6\pi t)}{\pi t} dt + \int\limits_0^\infty rac{\sin(2\pi t)}{\pi t} dt
ight\} \ I &= I_1 + I_2 \end{aligned}$$

$$I=I_1+I_2 \ I_1=3 imes\int\limits_0^\infty rac{\sin 6\pi t}{\pi t} \; dt$$

$$6\pi t = u$$



$$6\pi dt = du \ dt = rac{du}{6\pi}$$

when

$$egin{aligned} t=0, u=0 \ t=\infty, u=\infty \end{aligned}$$

$$\therefore I_1=3\int\limits_0^\infty rac{\sin u}{u/6}\cdotrac{du}{6\pi}=rac{3}{\pi}\int\limits_0^\infty rac{\sin u}{u}du$$

We know that
$$\int\limits_0^\infty {\frac{\sin t}{t}} dt = {\frac{\pi }{2}}$$

$$\therefore I_1 = \frac{3}{\pi} \int_0^\infty \frac{\sin u}{u} dv = \frac{3}{\pi} \times \frac{\pi}{2} = \frac{3}{2}$$

Now,
$$I_2=3\int\limits_0^\infty rac{\sin 2\pi t}{\pi t}dt$$

Let

$$2\pi t = v$$

$$2\pi dt = dv$$

$$dt = \frac{dv}{2\pi}$$

$$t=0, u=0$$

$$ig| \, t = \infty, v = \infty$$

$$\therefore \ I_2=3\int\limits_0^\infty rac{\sin v}{v/2}.rac{dv}{2\pi}=rac{3}{\pi}\int\limits_0^\infty rac{\sin v}{v}dv$$

Similarly,

$$I_2=rac{3}{\pi}\int\limits_0^\inftyrac{\sin v}{v}dv=rac{3}{\pi} imesrac{\pi}{2}=rac{3}{2}$$

$$\therefore I = I_1 + I_2 = \frac{3}{2} + \frac{3}{2} = 3$$

Q6 Text Solution:

$$L\left\{\cos\omega t\right\} = rac{s}{s^2 + \omega^2}$$

By first shift property

$$L\left(e^{at}\cdot\cos\omega t
ight)=rac{(s-a)}{(s-a)^2+\omega^2}$$

Substitute a = $-2~\&~\omega=4$

$$\Rightarrow L\left\{e^{-2t} \cdot \cos 4t\right\} = \frac{(s-(-2))}{(s-(-2))^2+4^2}$$

$$= \frac{(s+2)}{(s+2)^2 + 16}$$

$$\Rightarrow \left[L\left\{ e^{-2t} \cdot \cos 4t \right\} = \frac{(s+2)}{(s+2)^2 + 16} \right]$$

Q7 Text Solution:

Given:

$$rac{d^{2}f}{dt^{2}}+f=0;\;\;f\left(0
ight) =0,rac{df}{dt}\left(0
ight) =4$$

The auxiliary equations is $m^2 + 1 = 0$

$$\Rightarrow$$
m = $\pm i$

The solution is
$$f(t) = c_1 \cos t + c_2 \sin t$$

Given

$$f\left(0
ight)=0\Rightarrow0=c_{1}\cos0+c_{2}\sin0\Rightarrow c_{1}=0$$

$$\therefore f(t) = c_2 \sin t$$

$$\Rightarrow \frac{df(t)}{dt} = c_2 \cos t$$

$$ightarrow rac{df(t)}{dt} = c_2 \cos t$$
 Given $rac{df}{dt}(0) = 4 \Rightarrow 4 = c_2. (1) \Rightarrow c_2 = 4$

$$\therefore f(t) = 4. \sin t$$

$$\Rightarrow L\{f(t)\} = 4.L\{\sin t\} = 4$$

$$\cdot \left(\frac{1}{s^2+1^2}\right)$$

$$egin{aligned} &=rac{4}{s^2+1} \qquad \left(\because L\left(\sin at
ight) = rac{a}{s^2+a^2}
ight) \ dots \qquad \left[L\left\{f\left(t
ight) = rac{4}{s^2+1}
ight\}
ight] \end{aligned}$$

$$\therefore \left[L\left\{ f\left(t
ight)=rac{4}{s^{2}+1}
ight\}
ight]$$

Q8 Text Solution:

Given:

$$F\left(s
ight)=rac{1}{s\left(s+1
ight)}$$

$$egin{aligned} F\left(s
ight) &= rac{1}{s\left(s+1
ight)} \ \Rightarrow & F\left(s
ight) &= rac{1}{s} - rac{1}{s+1} \end{aligned}$$

Applying inverse Laplace on both sides

$$\Rightarrow L^{-1}\left\{ F\left(s
ight)
ight\} =L^{-1}\left\{ rac{1}{s}
ight\}$$

$$-L^{-1}\left\{rac{1}{s+1}
ight\}=1-e^{-t}$$

$$\left(:: L\left\{t^{n}\right\} = \frac{n!}{s^{n+1}}; L\left\{e^{at}\right\} = \frac{1}{s-a}\right)$$
 $\Rightarrow f(t) = 1 - e^{-t}$

$$\Rightarrow [f(t) = 1 - e^{-t}]$$

Q9 Text Solution:

$$egin{aligned} \overline{F} \; (s) &= rac{1}{s^2(s+1)} = rac{A}{s} + rac{B}{s^2} + rac{C}{s+1} \ &\Rightarrow \qquad \overline{F} \; (s) = rac{As(s+1) + B(s+1) + Cs^2}{s^2(s+1)} = rac{1}{s^2(s+1)} \end{aligned}$$

$$\Rightarrow A + C = 0; A + B = 0; B = 1$$

$$\Rightarrow$$
 $A=-1$; $B=1$; $C=1$

$$\Rightarrow \frac{1}{s^2(s+1)} = \frac{-1}{s} + \frac{1}{s^2} + \frac{1}{(s+1)}$$

Applying inverse Laplace on both sides:

$$L^{-1}\left\{\frac{1}{s^2(s+1)}\right\} = L^{-1}\left\{\frac{-1}{s}\right\} + L^{-1}\left\{\frac{1}{s^2}\right\}$$

$$+L^{-1}\left\{rac{1}{(s+1)}
ight\}$$

$$L^{-1}\left\{\frac{1}{s^2(s+1)}\right\} = -1 + t + e^{-t}$$

$$\Rightarrow f(t) = -1 + t + e^{-t}$$
 $\left\{ \therefore L\left\{e^{at}\right\} = \frac{1}{s-a}\right\}$

$$\left\{ :: L\left\{e^{at}\right\} = \frac{1}{s-a} \right\}$$

Q10 Text Solution:



Given:

$$\begin{split} \overline{F} \; (S) &= \frac{1}{(S^2 + S)} = \frac{1}{S(S + 1)} = \frac{1}{S} - \frac{1}{S + 1} \\ \Rightarrow \quad \overline{F} \; (S) &= \frac{1}{S} - \frac{1}{S + 1} \\ \therefore \; L^{-1} \left\{ \overrightarrow{F} \; (S) \right\} = L^{-1} \left\{ \frac{1}{S} \right\} - L^{-1} \left\{ \frac{1}{S + 1} \right\} \\ &= L^{-1} \left\{ \frac{1}{S - 0} \right\} - L^{-1} \left\{ \frac{1}{S - (-1)} \right\} \\ &= e^{0.t} - e^{-1.t} = 1 - e^{-t} \\ \left\{ \because L \left(e^{at} \right) = \frac{1}{S - a} \right\} \\ \Rightarrow \qquad \left[L^{-1} \left\{ \frac{1}{S^2 + S} \right\} = 1 - e^{-t} \right] \end{split}$$

Q11 Text Solution:

Given:

$$Cos h ax = \frac{e^{ax} + e^{-ax}}{2}$$

Taking Laplace transform, we get

$$\begin{split} &= \frac{1}{2} L \left\{ e^{ax} \right\} + L \left\{ e^{-ax} \right\} \\ &= \frac{1}{2} \left[\frac{1}{s-a} + \frac{1}{s+a} \right] \\ &\frac{1}{2} \left[\frac{2s}{s^2 - a^2} \right] \\ &\text{L (cos h ax)} = \frac{s}{s^2 - a^2} \end{split}$$

Q12 Text Solution:

Given

$$\left. rac{d^2y}{dt^2} - y = 1 \, ; \quad y\left(0
ight) = 0 \& rac{dy}{dt}
ight|_{t=0} = 0$$

Appling Laplace transform on both sides of DE $\Rightarrow (s^2 \cdot F(s) - s \cdot y(0) - y'(0)) - F(s)$

$$= \frac{1}{s}$$

$$\Rightarrow (s^2 - 1)F(s) = \frac{1}{s}$$

$$\Rightarrow F(s) = \frac{1}{s(s^2 - 1)}$$

$$\Rightarrow F(s) = \frac{1}{s(s - 1)(s + 1)}$$

Q13 Text Solution:

Let
$$f(t) = \frac{\sin 2\pi t}{\pi t}$$

 $f(t) = f(-t) = \frac{\sin 2\pi t}{t}$
 $f(t)$ is even:

$$I = 2 \int_{-\infty}^{\infty} \frac{\sin 2\pi t}{\pi t} = 4 \int_{0}^{\infty} \frac{\sin 2\pi t}{\pi t}$$
 $= \frac{4}{\pi} \left[\int_{0}^{\infty} \frac{\sin 2\pi t}{t} \right]$

From Laplace transform
$$L\left[rac{f(t)}{t}
ight]=\int\limits_{0}^{\infty}f\left(s
ight)ds$$

We know,
$$\int\limits_0^\infty rac{\sin at}{t} = rac{\pi}{2}$$
 $\int\limits_0^\infty rac{\sin 2\pi t}{t} = rac{\pi}{2}$ Hence, $I = rac{4}{\pi} \cdot rac{\pi}{2} = 2$

Q14 Text Solution:

Using initial value theorem,

$$egin{aligned} x\left(0^{+}
ight) &= \lim_{s o \infty} sX\left(s
ight) \ &= \lim_{s o \infty} s\left(rac{3s+5}{s^2+10s+21}
ight) \ &\lim_{s o \infty} rac{3+rac{5}{s}}{1+rac{10}{s}+rac{21}{s^2}} &= 3 \end{aligned}$$

Q15 Text Solution:

Given data,

$$egin{aligned} rac{d^2y(t)}{dt^2} + 2rac{dy(t)}{dt} + y\left(t
ight) = \delta\left(t
ight) \ y(t)|_{t=0} = -2 \Rightarrow y\left(0
ight) = -2 \ rac{dy}{dt}\Big|_{t=0} = 0 \Rightarrow y'\left(0
ight) = 0 \end{aligned}$$

Differential equation can be written as:-

$$y'' + 2y' + y = \delta(t)$$

Taking Laplace transform on both sides

$$egin{aligned} \left[s^2 Y(s) - sy\left(0
ight) - y'\left(0
ight)
ight] \ + 2 \left[sY(s) - y\left(0
ight)
ight] + Y(s) = 1 \ \left[s^2 + 2s + 1
ight] Y(s) + 2s + 4 = 1 \ Y(s) = rac{-3 - 2s}{\left(s + 1
ight)^2} \end{aligned}$$

$$egin{align} Y\left(s
ight) &= -rac{3}{\left(s+1
ight)^2} - 2\left[rac{\left(s+1
ight)-1}{\left(s+1
ight)^2}
ight] \ Y\left(s
ight) &= -rac{3}{\left(s+1
ight)^2} - rac{2}{\left(s+1
ight)} + rac{2}{\left(s+1
ight)^2} = -rac{1}{\left(s+1
ight)^2} \ -rac{2}{\left(s+1
ight)} \ \end{array}$$

$$y\left(t
ight)=L^{-1}\,Y\left(s
ight)=L^{-1}\left[-rac{1}{\left(s+1
ight)^{2}}-rac{2}{\left(s+1
ight)}
ight]$$

$$egin{aligned} \mathbf{y} &= -te^{-t} - 2e^{-t} \ rac{dy}{dt} &= te^{-1} - e^{-t} + 2e^{-t} \ &= e^{-t} \left(t - 1 + 2
ight) \end{aligned}$$

$$rac{dy}{dt}=e^{-t}\left(t+1
ight)$$

At t = 0 ;
$$\left[\frac{dy}{dt} = 1\right]$$