All Branches Hinglish

Engineering Mathematics

Multivariable Calculus & Vector Calculus

DPP-01

1. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, then the value of div $\left(\frac{\vec{r}}{r^3}\right)$ is

(a) 1 (b) 2
(c) 0 (d) −1

2. Let $\vec{F}(x,y,z) = 2y\hat{i} + x^2\hat{j} + xy\hat{k}$ and let C be the curve of intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=1$. Then the value of $\left|\oint_C F \cdot d\vec{r}\right|$ is

(a) π (b) $\frac{3\pi}{2}$
(c) 2π (d) 3π

3. The flux of $\vec{F} = y\hat{i} - x\hat{j} + z^2\hat{k}$ along the outward normal, across the surface of the solid $\{(x,y,z) \in \mathbb{R}^3$ $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le \sqrt{2-x^2-y^2}\}$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$
(c) $\frac{8}{3}$ (d) $\frac{4}{3}$

4. For $a > 0, b > 0$ let $\vec{F} = \frac{x\hat{j} - y\hat{i}}{b^2x^2 + a^2y^2}$ be a planner vector field. Let $C = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = a^2 + b^2 \mid$ be the circle oriented anti-clockwise. Then $\oint \vec{F} \cdot d\vec{r} =$

(a) $\frac{2\pi}{ab}$ (b) 2π
(c) $2\pi ab$ (d) 0

5. Let $\vec{F} = (3+2xy)\hat{i} + (x^2 - 3y^2)\hat{j}$ and let L be the curve $\vec{r} = e^t \sin t\hat{i} + e^t \cos t\hat{j},\ 0 \le t \le \pi$. Then $\int_L \vec{F} \cdot d\vec{r} =$

(a) $e^{-3\pi} + 1$ (b) $e^{-6\pi} + 2$
(c) $e^{6\pi} + 2$ (d) $e^{3\pi} + 1$

6. The flux of the vector field

$$\vec{F} = \left(2\pi x + \frac{2x^2y^2}{\pi}\right)\hat{i} + \left(2\pi xy - \frac{4y}{\pi}\right)\hat{j}$$

along the outward normal, across the ellipse $x^2 + 16y^2 = 4$ is equal to

(a) $4\pi^2 - 2$ (b) $2\pi^2 - 4$
(c) $\pi^2 - 2$ (d) 2π

7. Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a scalar field, $\vec{v}: \mathbb{R}^3 \to \mathbb{R}^3$ be vector field and let $\vec{a} \in \mathbb{R}^3$ be a constant vector. If $\vec{r}$ represents the position vector $x\hat{i} + y\hat{j} + z\hat{k}$, then which one of the following is FALSE ?

(a) curl($f\ \vec{v}$) = grad $(f) \times \vec{v} \times f$ curl$(\vec{v})$

(b) div (grad)$(f) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) f$

(c) curl $(\vec{a} \times \vec{r}) = 2|\vec{a}|\vec{r}$

(d) div $\left(\frac{r^3}{|\vec{r}|^3}\right) = 0$ for $\vec{r} \ne \vec{0}$

8. Let S be the surface of the cone $z = \sqrt{x^2+y^2}$ bounded by the planes $z = 0$ and $z = 3$. Further, let C be the closed curve forming the boundary of the surface S. A vector field F is such that $\nabla \times \vec{F} = -x\hat{i} - y\hat{j}$. Then absolute value of the line integral $\oint_c \vec{F} \cdot d\vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, is

(a) 0 (b) 9π
(c) 15π (d) 18π

9. The line integral of the vector field $\vec{F} = zx\hat{i} + xy\hat{j} + yz\hat{k}$ along the boundary of the triangle with vertices (1, 0, 0), (0, 1, 0) and (0, 0, 1) oriented anticlockwise, when viewed from the point (2, 2, 2) is

(a) $-\frac{1}{2}$ (b) -2
(c) $\frac{1}{2}$ (d) 2

10. If $\vec{F}(x,y)=(3x-8y)\hat{i}+(4y-6xy)\hat{j}$ for $(x,y)\in\mathbb{R}^2$, then $\oint_C \vec{F}\cdot d\vec{r}$, where C is the boundary of the triangular region bounded by the lines $x=0$, $y=0$ and $x+y=1$ oriented in the anti-clockwise direction is

(a) $\frac{5}{2}$ (b) 3

(c) 4 (d) 5

11. If $F(x,y,z)=xy^2+3x^2-z^3$, then the value of $\nabla F(x,y,z)$ at (2, –1, 4) is equal to

(a) $13i-4j-48k$ (b) $i-4j-k$

(c) $13i+j-6k$ (d) $-13i+4j-6k$

12. Let F be a vector field given by

$$\vec{F}(x,y,z)=-y\hat{i}+3xy\hat{j}+z^3\hat{k} \text{ for } (x,y,z)\in\mathbb{R}^3.$$

If C is the curve of intersection of the surfaces $x^2+y^2=1$ and $x+z=2$, then which of the following is (are) equal to $\left|\oint_C \vec{F}\cdot d\vec{r}\right|$?

(a) $\int_0^{2\pi}\int_0^1 (1+2r\sin\theta)r\,dr\,d\theta$

(b) $\int_0^{2\pi}\left(\frac{1}{2}+\frac{2}{3}\sin\theta\right)d\theta$

(c) $\int_0^{2\pi}\int_0^1 (1+2r\sin\theta)dr\,d\theta$

(d) $\int_0^{2\pi}(1+\sin\theta)d\theta$

13. If $\vec{F}(x,y,z)=(2x+3yz)\hat{i}+(3xz+2y)\hat{j}+(3xy+2z)\hat{k}$ for $(x,y,z)\in\mathbb{R}^3$, then which among the following is (are) true ?

(a) $\nabla\times\vec{F}=\vec{0}$

(b) $\oint_C \vec{F}\cdot d\vec{r}=0$ along any simple closed curve C

(c) There exist a scalar function $\phi:\mathbb{R}^3\to\mathbb{R}$ such that $\nabla\cdot\vec{F}=\phi_{xx}+\phi_{yy}+\phi_{zz}$

(d) $\nabla\cdot\vec{F}=0$

14. Let T be the smallest positive real number such that the tangent to the helix $\cos t\vec{i}+\sin t\vec{j}+\frac{t}{\sqrt{2}}\hat{k}$ at $t=T$ is orthogonal to the tangent at $t=0$. Then the line integral of $\vec{F}=x\hat{j}-y\hat{i}$ along the section of the helix from $t=0$ to $t=T$ is _____

15. Let $\vec{F}=-y\hat{i}+x\hat{j}$ and let C be the ellipse

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

oriented counter clockwise. Then the value of $\oint_C \vec{F}\cdot d\vec{r}$ (round off to 2 decimal place) is _____

16. If the triple integral over the region bounded by the planes $2x+y+z=4$, $x=4$, $y=0$, $z=0$ is given by $\int_0^2\int_0^{\lambda(x)}\int_0^{\mu(x,y)} dzdydz$, then the function

(a) $x+y$ (b) $x-y$

(c) x (d) y

17. The value of the integral $\int_{y=0}^{1}\int_{x=0}^{1-y^2} y\sin(\pi(1-x)^2)dxdy$ is

(a) $\frac{1}{2\pi}$ (b) 2π

(c) $\frac{\pi}{2}$ (d) $\frac{2}{\pi}$

18. The area of the surface generated by rotating the curve $x=y^3$, $0\le y\le 1$. about the y-axis is

(a) $\frac{\pi}{27}10^{3/2}$ (b) $\frac{4\pi}{3}(10^{3/2}-1)$

(c) $\frac{\pi}{27}(10^{3/2}-1)$ (d) $\frac{4\pi}{3}10^{3/2}$

19. The area of the part of surface of the paraboloid $x^2+y^2+z=8$ lying inside the cylinder $x^2+y^2=4$ is

(a) $\frac{\pi}{2}(17^{\frac{3}{2}}-1)$ (b) $\pi(17^{\frac{3}{2}}-1)$

(c) $\frac{\pi}{6}(17^{\frac{3}{2}}-1)$ (d) $\frac{\pi}{3}(17^{\frac{3}{2}}-1)$

20. Let C be the circle $(x-1)^2+y^2=1$, oriented counter clockwise. Then the value of the line integral $\oint_C -\frac{4}{3}xy^3dx+x^4dy$ is

(a) 6π (b) 8π

(c) 12π (d) 14π

21. Length of the arc of the curve

$y = \log \sec x$ from $x = 0$ to $x = \frac{\pi}{3}$ is equal to

(a) $\log(2-\sqrt{3})$ (b) $\log(1-\sqrt{3})$

(c) $\log(1+\sqrt{3})$ (d) $\log(2+\sqrt{3})$

22. If $x = v(1+u)$, $y = u(1+v)$, then $\frac{\partial(x,y)}{\partial(u,v)} =$

(a) $1 + u + v$ (b) $-1 - u - v$

(c) $1 - u + v$ (d) 0

23. Consider the surface $S = \{(x, y, xy) \in \mathbb{R}^3 : x^2 + y^2 \le 1\}$.

Let $\vec{F} = y\hat{i} + x\hat{j} + \hat{k}$. If $\hat{n}$ is the continuous unit normal field to the surface S with positive z-component, then $\iint_S \vec{F} \cdot \hat{n}\, ds$ equals

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) 2π (d) π

24. The volume of the solid

$$\left\{(x, y, z) \in \mathbb{R}^3 \mid 1 \le x \le 2,\ 0 \le y \le \frac{2}{x},\ 0 \le z \le x\right\}$$

is/are expressible as

(a) $\int_1^2 \int_2^{2/x} \int_0^x dz\, dy\, dx$

(b) $\int_1^2 \int_0^x \int_0^{2/x} dy\, dz\, dx$

(c) $\int_0^2 \int_1^2 \int_0^{2/x} dy\, dx\, dz$

(d) $\int_0^2 \int_{\max(z,1)}^2 \int_0^{2/x} dy\, dx\, dz$

25. Let R be the planar region bounded by the lines $x = 0$, $y = 0$ and the curve $x^2 + y^2 = 4$ in the first quadrant. Let C be the boundary of R, oriented counter-clockwise. Then the value of $\oint_C x(1-y)dx + (x^2 - y^2)dy$ is ____

26. Let R be the region enclosed by $x^2 + 4y^2 \ge 1$ and $x^2 + y^2 \le 1$. Then the value of $\iint_R |xy|\, dx\, dy$ is ____

27. For a real number x, define $[x]$ to be the smallest integer greater than or equal to x. Then

$$\int_0^1 \int_0^1 \int_0^1 ([x] + [y] + [z])\, dx\, dy\, dz =$$

28. The value of the integral $\int_0^1 \int_x^1 y^4 e^{xy^2}\, dy dx$ is ____ (correct up to three decimal places)

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Answer Key

1. **(c)**
2. **(c)**
3. **(d)**
4. **(a)**
5. **(d)**
6. **(b)**
7. **(c)**
8. **(d)**
9. **(c)**
10. **(b)**
11. **(a)**
12. **(a, b)**
13. **(a, b, c)**
14. **(2.09)**
15. **(75.36)**
16. **(d)**
17. **(a)**
18. **(c)**
19. **(c)**
20. **(b)**
21. **(d)**
22. **(b)**
23. **(b)**
24. **(a, b, d)**
25. **(8)**
26. **(0.375)**
27. **(3)**
28. **(0.239)**

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