ENGINEERING MATHEMATICS

NUMERICAL METHODS

DPP: 1

Q1 The quadratic equation $2x^2 - 3x + 3 = 0$ is to be solved numerically starting with an initial guess as $x_0 = 2$. The new estimate of x after the first iteration using Newton Raphson method is ________________.

Q2 Starting with x =1 , the solution of the equation $x^3 + x = 1$, after two iterations of Newton-Raphson's method (***up to two decimal places***) is ____________.

Q3 Solve the equation x = 10 cos(x) using the Newton-Raphson method. The initial guess is x = pi/4. The value of the predicted root after the first iteration, up to second decimal is ______________.

Q4 The root of the function $f(x) = x^3 + x - 1$ obtained after first iteration on application of Newton-Raphson scheme using an initial guess of $x_0 = 1$ is

(A) 0.682 (B) 0.686

(C) 0.750 (D) 1.000

Q5 Newton-Raphson method is used to find the roots of the equation, $x^3+2x^2+3x - 1 = 0$. If the initial guess is $x_0 = 1$, then the value of x after 2nd iteration is ____________.

Q6 The Newton-Raphson method is used to solve the equation $f(x) = x^3 - 5x^2 + 6x - 8 = 0$. Taking the initial guess as x = 5, the solution obtained at the end of the first iteration is ______________.

Q7 The quadratic equation $x^2 - 4x + 4 = 0$ is to be solved numerically, starting with the initial guess $x_0 = 3$. The Newton-Raphson method is applied once to get a new estimated and then the Secant method is applied once using the initial guess and this new estmate. The estimated value of the root after the application of the Secant method is _______________.

Q8 In Newton-Raphson iterative method, the initial guess value (X_{ini}) is considered as zero while finding the roots of the equation : $f(x) = -2 + 6x - 4x^2 + 0.5x^3$.

The correction Δx, to be added to X_{ini} in the first iteration is _____________.

Q9 When the Newton-Raphson method is applied to solve the equation $f(x) = x^3 + 2x - 1 = 0$, the solution at the end of the first iteration with the initial guess value as $x_0 = 1.2$ is

(A) - 0.82 (B) 0.49

(C) 0.705 (D) 1.69

Q10 A numerical solution of the equation $f(x) = x + \sqrt{x} - 3 = 0$ can be obtained using Newton Raphson method. If the starting value is x = 2 for the iteration, the value of x that is to be used in the next step is

(A) 0.306 (B) 0.739

(C) 1.694 (D) 2.306

Q11 Let $x^2 - 117 = 0$. The iterative steps for the solution using Newton-Raphson's Method is given by

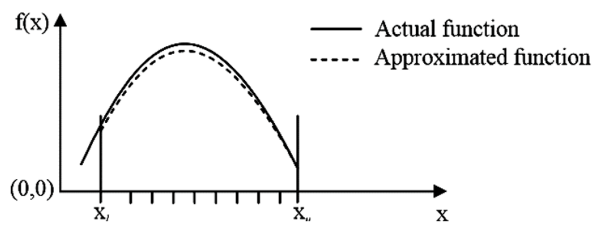
(A) $x_{k+1} = \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$

(B) $x_{k+1} = x_k - \frac{117}{x_k}$

(C) $x_{k+1} = x_k - \frac{x_k}{117}$

(D) $x_{k+1} = x_k - \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$

Q12 A function f(x), that is smooth and convex-shaped between interval (x_l, x_u) is shown in the figure. This function is observed at odd number of regularly spaced points. If the area under the function is computer numerically, then.



(A) the numerical value of the area obtained using the trapezoidal rule will be less than the actual

(B) the numerical value of the area obtained using the trapezoidal rule will be more than the actual

(C) the numerical value of the area obtained using the trapezoidal rule will be exactly equal to the actual

(D) with the given details, the numerical value of area cannot be obtained using trapezoidal rule

Q13 Consider the definite integral $\int_1^2 (4x^2 + 2x + 6)dx$

Let Ie be the exact value of the integral. If the same integral is estimated using Simpson's rule with 10 equal subintervals, the value is I_s. The percentage error is defined as e = 100 × (I_e - I_s)/ I_e. The value of I_e is

(A) 2.5 (B) 3.5

(C) 1.2 (D) 0

Q14 Numerically integrate f(x) = 10x - 20 x^2 from lower limit a = 0 to upper limit b = 0.5. Using Trapezoidal rule with five equal subdivision. The value (in units round off to two decimal places) obtained is _____________ .

Q15 The integral $\int_0^1 \left(5x^3 + 4x^2 + 3x + 2\right) dx$is estimated numerically using three alternative methods namely the rectangular, trapezoidal and Simpson'srules with a common step size . In the context, which one of the following statements is TRUE ?

(A) Simpson's rule as well as rectangular rule of estimation will give non-zero error.

(B) Only Simpson's rule of estimation will give zero error.

(C) Simpson's rule, rectangular rule as well as trapezoidal rule of estimation will give non-zero error

(D) Only the rectangular rule of estimation will give zero error.

Q16 For the intergral $\int_0^{\pi/2} (8 + 4\cos x)dx$ the absolute percentage error in numerical evaluation with the Trapezoidal rule, using only the end points, is ____________. (***round off to one decimal place***)

Q17 P(0,3), Q(0.5, 4), and R(1,5) are three points on the curve defined by f(x). Numerical integration is carried out using both Trapezoidal rule and Simpson's rule with in limits x = 0 and x = 1 for the curve. The difference between the two results will be.

(A) 0 (B) 0.25

(C) 0.5 (D) 1

Q18 The integral $\int_{x_1}^{x_2} x^2 dx \text{ with } x_2 > x_1 > 0$ is evaluated analytically as well as numerically using a single application of the trapezoidal rule. If I is the exact value of the integral obtained analytically and J is the approximate value obtained using the trapezoidal rule, which of the following statements is correct their relationship ?

(A) J > I

(B) J < I

(C) J = I

(D) Insufficient data to determine the relationship.

Q19 For step-size, Δx = 0.4, the value of following integral using Simpson's 1/3 rule is _________________.

$$\int_0^{0.8} \left(0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5\right) dx$$

Q20

Find the magnitude of error (Correct to two decimal places) in the estimation of following integral using Simpson 1/3 rule. Take the step length as 1.

$$\int_0^4 (x^4 + 10)\,dx$$

Q21 Torque exerted on a flywheel over a cycle is listed in the table Flywheel energy (in J per unit cycle) using Simpson's rule is

Angle (degree)	0	60	12	180	240	300	360
Torque (N m)	0	1066	-323	0	323	-355	0

(A) 542 (B) 993
(C) 1444 (D) 1986

Q22 The Table below gives values of a function F (x) obtained for values of x intervals of 0.25.

The value of the intergral of the function between the limits 0 to 1 using Simpson's rule is

X	0	0.25	0.5	0.75	1.0
F(x)	1	0.9412	0.8	0.64	0.50

(A) 0.7854 (B) 2.3562
(C) 3.1416 (D) 7.5000

Q23 Consider the differential equation $\frac{dy}{dx} = 4(x+2) - y$For the initial condition y = 3 at x = 1, the value of y at x = 1.4 obtained using Euler's method with a step- size of 0.2 is _______.

(***round off to one decimal place***)

Q24 Consider an ordinary differential equation $\frac{dx}{dt} = 4t + t$. If x = x_0 at t = 0, the increment in x calculated using Runge-kutta fourth order multi-step method with a step size of Δt = 0.2 is :

(A) 0.22 (B) 0.44
(C) 0.66 (D) 0.88

Answer Key

Q1	(1 to 1)	Q13	(D)
Q2	(0.65 to 0.72)	Q14	(0.38 to 0.42)
Q3	(1.53 to 1.59)	Q15	(B)
Q4	(C)	Q16	(5.1 to 5.5)
Q5	(0.29 to 0.31)	Q17	(A)
Q6	(4.25 to 4.35)	Q18	(A)
Q7	(2.32 to 2.34)	Q19	(1.36 to 1.37)
Q8	(0.3 to 0.4)	Q20	(0.50 to 0.53)
Q9	(C)	Q21	(B)
Q10	(C)	Q22	(A)
Q11	(A)	Q23	(6.3 to 6.5)
Q12	(A)	Q24	(D)

Hints & Solutions

Q1 Text Solution:

Given: $f(x) = 2x^2 - 3x + 3$, $x_0 = 2$

$f'(x) = 4x - 3$

By Newton-Raphson method,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{(2\times2^2-3\times2+3)}{4\times2-3} = 2 - \frac{5}{5}$$

$$= 1$$

Q2 Text Solution:

Given: Equation is,

$x^3 + x = 1$

$\Rightarrow x^3 + x - 1 = 0$

and initial guess,

$x_0 = 1$

The iterative scheme for f(x) = 0 using Newton-Raphson iterative scheme is,

$$\Rightarrow x_{k+1} = x_k - \frac{(x_k^3+x_k-1)}{(3x_k^2+1)}$$

$$\Rightarrow x_{k+1} = \frac{2x_k^3+1}{3x_k^2+1}$$

For k = 1:

$$\Rightarrow \quad x_1 = \frac{2x_0^3+1}{3x_0^2+1} = \frac{2(1)+1}{3(1)+1} = \frac{3}{4} = 0.75$$

For k = 2:

$$\Rightarrow \quad x_2 = \frac{2x_1^3+1}{3x_1^2+1} = \frac{2(0.75)^3+1}{3(0.75)^2+1} = \frac{1.84375}{2.6875} = 0.686$$

∴ The value after 2nd iteration is 0.69.

Q3 Text Solution:

Given: $x = 10\cos x$ and $x_0 = \frac{\pi}{4}$

$\Rightarrow \quad x - 10\cos x = 0$

For f(x) = 0, Newton – Raphson scheme is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\Rightarrow \quad x_{k+1} = x_k - \frac{(x_k - 10\cos x_k)}{(1+10\cdot\sin x_k)}$$

$$\Rightarrow \quad x_{k+1} = \frac{10x_k\cdot\sin x_k + 10\cos x_k}{(1+10\cdot\sin x_k)}$$

For k = 0,

$$x_1 = \frac{10x_0\cdot\sin x_0 + 10\cos x_0}{(1+10\cdot\sin x_0)}$$

$$= \frac{10\left(\frac{\pi}{4}\cdot\frac{1}{\sqrt{2}}\right)+10\left(\frac{1}{\sqrt{2}}\right)}{\left(1+10\left(\frac{1}{\sqrt{2}}\right)\right)} = = \frac{10\left(\frac{\pi}{4}+1\right)}{(10+\sqrt{2})} = 1.564$$

∴ First iteration value = x_1 = 1.564

Q4 Text Solution:

Given: $f(x) = x^3 + x - 1$ and $x_0 = 1$

By Newton-Raphson scheme,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\Rightarrow \quad x_{k+1} = x_k - \frac{({x_k}^3+x_k-1)}{(3{x_k}^2+1)}$$

$$\Rightarrow \quad x_{k+1} = \frac{2{x_k}^3+1}{3{x_k}^2+1}$$

For k = 0

$$\Rightarrow \quad x_1 = \frac{2{x_0}^3+1}{3{x_0}^2+1} = \frac{2(1)^3+1}{3(1)^2+1} = \frac{3}{4} = 0.75$$

$$\therefore \quad \boxed{\text{First Iteration value} = 0.750}$$

Q5 Text Solution:

Given: $x^3 + 2x^2 + 3x - 1 = 0$ and $x_0 = 1$

For *f*(x) = 0, Newton-Raphson method is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)},$$

$$\Rightarrow \quad x_{k+1} = x_k - \frac{\{{x_k}^3+2{x_k}^2+3x_k-1\}}{3{x_k}^2+4\cdot x_k+3}$$

$$\Rightarrow \quad x_{k+1} = \frac{2{x_k}^3+2{x_k}^2+1}{3{x_k}^2+4.x_k+3}$$

For k = 0, $x_1 = \frac{2{x_0}^3+2{x_0}^2+1}{3{x_0}^2+4.x_0+3} = \frac{5}{10} = 0.5$

For k =1;

$$x_2 = \frac{2{x_1}^3+2{x_1}^2+1}{3{x_1}^2+4.x_1+3} = \frac{2(0.5)^3+2(0.5)^2+1}{3(0.5)^2+4(0.5)+3} =$$

0.3043

∴

$$\boxed{\text{Value after 2}^{\text{nd}}\text{ iteration} = x_2 = 0.304}$$

Q6 Text Solution:

Given equation is $f(x) = x^3 - 5x^2 + 6x - 8 = 0$

Initial guess is $x_0 = 5$

For any function f(x) = 0, newton-Raphson iterative scheme is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\Rightarrow \quad x_{k+1} = x_k - \frac{(x_k^3-5x_k^2+6x_k-8)}{(3x_k^2-10x_k+6)}$$

$$\Rightarrow\ x_{k+1} = \frac{2x_k^3-5x_k^2+8}{3x_k^2-10x_k+6}$$

For k = 0; $x_1 = \frac{2x_0^3-5x_0^2+8}{3x_0^2-10x_0+6}$

$= \frac{2(125)-5(25)+8}{3(25)-10(5)+6} = \frac{133}{31} = 4.29$

∴ The value at the end of 1st iteration is 4.29.

Q7 Text Solution:

f(x) = $x^2 - 4x + 4$

f'(x) = 2x – 4

$x_0 = 3$

f(3) = 1, f'(3) = 2

By Newton Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{1}{2} = \frac{5}{2} = 2.5$$

$$(\because\ \ x_0 = 3,\ f(x_0) = 1,\ f'(x_0) = 2)$$

$$x_1 = \frac{5}{2}\quad f(x_1) = \left(\frac{5}{2}\right)^2 - 4\left(\frac{5}{2}\right) + 4 = \frac{1}{4}$$

By secant method.

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{3\times\frac{1}{4} - \frac{5}{2}\times 1}{\frac{1}{4} - 1} = \frac{7}{3} = 2.33$$

Q8 Text Solution:

Given: f(x) = $-2 + 6x - 4x^2 + 0.5x^3$, $x_{ini} = 0$

f'(x) = $6 - 8x + 1.5x^2$

By Newton Raphson method,

$$x_1 = x_{ini} - \frac{f(x_{ini})}{f'(x_{ini})}$$

$$x_1 = 0 - \frac{(-2)}{6}\qquad [\because f(0) = -2,\ f'(0) = 6]$$

$$x_1 = \frac{1}{3}$$

$$\Delta x = x_1 - x_{ini} = \frac{1}{3} - 0$$

$$\boxed{\Delta x = \frac{1}{3} = 0.33}$$

Q9 Text Solution:

Given: f(x) = $x^3 + 2x - 1$, $x_0 = 1.2$

f'(x) = $3x^2 + 2$

By Newton Raphson method;

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1.2 - \frac{[1.2^3 + 2\times1.2 - 1]}{3\times1.2^2 + 2} = 0.705$$

Q10 Text Solution:

Given: $f(x) = x + \sqrt{x} - 3 = 0$ and initial guess $x_0 = 2$.

As per Newton Raphson iteration scheme,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{(x_k + \sqrt{x_k} - 3)}{\left(1 + \frac{1}{2\sqrt{x_k}}\right)}$$

$$\Rightarrow\ x_{k+1} = \frac{x_k + \frac{\sqrt{x_k}}{2} - x_k - \sqrt{x_k} + 3}{\left(1 + \frac{1}{2\sqrt{x_k}}\right)}$$

$$\Rightarrow\ x_{k+1} = \frac{3 - \frac{\sqrt{x_k}}{2}}{\left(1 + \frac{1}{2\sqrt{x_k}}\right)}$$

For k = 0

$$x_1 = \frac{3 - \frac{\sqrt{x_0}}{2}}{\left(1 + \frac{1}{2\sqrt{x_0}}\right)} = \frac{3 - \frac{\sqrt{2}}{2}}{\left(1 + \frac{1}{2\sqrt{2}}\right)} = 1.6939 \approx 1$$

$.694$

∴ Value after 1st iteration = 1.694

Q11 Text Solution:

Let f(x) = $x^2 - 117$

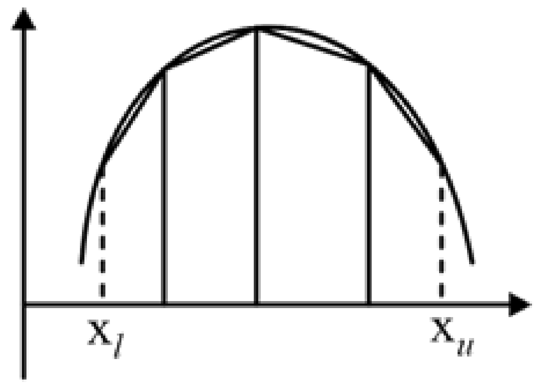
By Newton Raphson method, $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

$$x_{k+1} = x_k - \frac{(x_k^2 - 117)}{2x_k}$$

$= x_k - \frac{x_k}{2} + \frac{117}{2x_k} = \frac{x_k}{2} + \frac{117}{2x_k}$

$$x_{k+1} = \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$$

Q12 Text Solution:



Area under approximated curve will be less as compared to actual smooth function.

Numerically computed area < Actual area (Trapezoidal Rule)

Q13 Text Solution:

Given: $I = \int_1^2 (4x^2 + 2x + 6)\,dx$

For $\int_a^b f(x)\,dx$, if f(x) is polynomial of degree ≤ 2, the integration by Simpson's $\frac{1}{3}^{rd}$ rule will be same as the exact value.

∵ $f(x) = 4x^2 + 2x + 6$ is a polynomial of degree '2'

⇒ $I_e = I_s$

∴ $\left|\frac{I_e - I_s}{I_e}\right| \times 100 = 0$

Q14 Text Solution:

Given: , n = 5, a = 0, b = 0.5

Then $h = \frac{b-a}{n} = \frac{0.5-0}{5} = 0.1$

So,

x	0	0.1	0.2	0.3	0.4	0.5
f(x)	0	0.8	1.2	1.2	0.8	0
	y_0	y_1	y_2	y_3	y_4	y_n

Using Trapezoidal rule, $I = \int_0^{0.5} (10x - 20x^2)\,dx$

$= \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + y_3 + y_4)]$

$= \frac{0.1}{2}[0 + 2(0.8 + 1.2 + 1.2 + 0.8) + 0]$

∴ $\boxed{\int_0^{0.5} (10x - 20x^2)\,dx = 0.40}$

Q15 Text Solution:

The following rules will give zero error for polynomials up to the following degree:

Given, Polynomial is of 3 degree

Rectangular Rule	0 degree (x^0)
Trapezoidal Rule	1 degree (x^1)
Simpson $1/3^{rd}$ Rule	2 degree (x^2)
Simpson $3/8^{th}$ Rule	3 degree (x^3)

Since function is a polynomial of 3 degree hence Only Simpson's rule will give zero error.

Q16 Text Solution:

Given: Integral $\int_0^{\frac{\pi}{2}} (8 + 4\cos x)\,dx$ = I (say)

(i) Analytic Value = $\int_0^{\frac{\pi}{2}} (8 + 4\cos x)\,dx$

$= 8x + 4\sin x\Big|_0^{\frac{\pi}{2}} = 8\left(\frac{\pi}{2}\right) + 4\sin\left(\frac{\pi}{2}\right)$

= 4(1 + π) = A (say)

(ii) Using Trapezoidal rule by considering end points,

$\Rightarrow T = \frac{h}{2}[(y_0 + y_n)]$ =

$\frac{\left(\frac{\pi}{2}\right)}{2}\left[(8 + 4\cos x)|_{x=0} + (8 + 4\cos x)|_{x=\frac{\pi}{2}}\right]$

$= \frac{\pi}{4}[16 + 4] = 5\pi$

∴ T = 5pi

% error = $\left|\frac{A-T}{A}\right| \times 100$

$= \left|\frac{4+4\pi-5\pi}{4+4\pi}\right| \times 100 = 5.182\ \%$

$\boxed{\text{Absolute \% error} = 5.2\ \%}$

Q17 Text Solution:

Given:

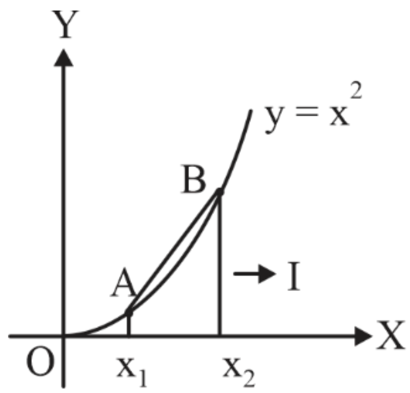
The Points are P (0, 3), Q (0.5, 4), R (1, 5) are collinear.

The difference in the values of $\int_0^1 f(x)\,dx$ will be zero in between Trapezoidal rule, Simpson's rule and analytical integration.

% Error between Trapezoidal and Simpson's rule = 0

Q18 Text Solution:

For the function $y = x^2$ between x_1 and x_2



I = exact value of the integral is given by area under the curve $y = x^2$ between x_1 and x_2 bounded by X-axis.

J = Approximate value is obtained by the area under the straight-line AB between x_1 and x_2 bounded by X-axis.

So, J > 1

Q19 Text Solution:

$f(x) = 0.2 + 2.5x - 200x^2 + 675x^3 - 900x^4 + 400x^5$

Step size, h = 0.4

x	0	0.4	0.8
f(x)	0.2	2.456	0.232

By Simpson's 1/3rd Rule,

$$I = \int_0^{0.8} f(x)dx = \frac{h}{3}[y_0 + 4y_1 + y_n]$$

$= \frac{0.4}{3}[0.2 + 4 \times 2.456 + 0.232]$

$\boxed{I = 1.367}$

Q20 Text Solution:

Given: h = 1

Exact value of integral $\int_0^4 (x^4 + 10)\,dx$

$$= \left[\frac{x^5}{5} + 10x\right]_0^4 = \frac{4^5}{5} + 10 \times 4 = 244.8$$

x	0	1	2	3	4
f(x)	10	11	26	91	266

Using Simpson's 1/3rd rule

$$\int_0^4 f(x)dx = \frac{h}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + y_n]$$

$$= \frac{1}{3}[10 + 4 \times 11 + 2 \times 26 + 4 \times 91 + 266]$$

= 245.33

Approximate value = 245.33

∴ Magnitude of error = 245.33 – 244.8 = 0.53

Q21 Text Solution:

Given: Step size $h = \frac{360-0}{6} = 60^\circ = \frac{\pi}{3}$

Angle (degree)	0	60	12	180	240	300	360
Torque (N m)	0	1066	–323	0	323	–355	0

Flywheel energy F.E. = $\int_{\theta=0^\circ}^{\theta=360^\circ} T \cdot d\theta$

Using Simpson's rule, $\int_a^b f(x)\,dx$

$= \frac{h}{3}$

$[(y_0 + y_n) + 4(y_1 + y_3 + \cdots)$

$+ 2(y_2 + y_4 + \cdots)]$

F.E. = $\frac{\pi}{3(3)}[(0 + 0) + 4(1066 + 0 - 355)]$

$+ 2\left(-323 + 323\right)$

= 992.74 J ⇒ $\boxed{\int T.\,d\theta \cong 993\text{ J}}$

Q22 Text Solution:

Given: h = 0.25

x	0	0.25	0.5	0.75	1.0
f(x)	1	0.9412	0.8	0.64	0.50

According to Simpson's 1/3rd Rule,

$\int_0^1 f(x)dx$

$= \frac{h}{3}[(y_o + y_n) + 2y_2 + 4(y_1 + y_3)]$

$= \frac{0.25}{3}$

$[(1 + 0.5) + 2 \times 0.8 + 4 \times (0.9412 + 0.64)]$

$\boxed{I = 0.7854}$

Q23 Text Solution:

Given: $\frac{dy}{dx} = 4(x + 2) - y$,

$f(x, y) = 4(x + 2) - y$

h = 0.2

For finding y_2 two iterations has to be followed

	x_0	x_1	x_2
x	1	1.2	1.4
y	3	?	?

By Euler's method

$y_1 = y_0 + hf(x_0, y_0)$

= 3 + 0.2 f(1,3) $\because x_0 = 1, y_0 = 3$

= 3 + 0.2 [4 (1+ 2) −3]

$y_1 = 4.8$

$\therefore x_1 = 1.2, y_1 = 4.8$

$y_2 = y_1 + h f(x_1, y_1)$

= 4.8 + 0.2 f(1.2, 4.8)

= 4.8 + 0.2 [4(1.2 + 2) − 4.8] = 4.8 + 1.6

$y_2 = 6.4$

Q24 Text Solution:

Given: $\frac{dx}{dt} = 4t + 4 = f(t, x)$

At t = 0, x = x_0; h = Δt = 0.2

From Rk – 4th order method

$x_{i+1} = x_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = \boldsymbol{f}(t_0, x_0)$

$k_2 = f\left(t_0 + \frac{h}{2}, x_0 + \frac{k_1}{2}\right)$

$k_3 = f\left(t_0 + \frac{h}{2},\ x_0 + \frac{k_2}{2}\right)$

$k_4 = \boldsymbol{f}(t_0 + h, x + k)$

$\therefore \quad k_1 = \boldsymbol{f}(0, x_0) = 4(0) + 4 = 4$

$$k_2 = f\left(0 + \frac{0.2}{2},\ x_0 + \frac{k_1}{2}\right) = 4\left(0.1\right) + 4$$
$$= 4.4$$

$$k_3 = f\left(0 + \frac{0.2}{2},\ x_0 + \frac{k_2}{2}\right) = 4\left(0.1\right) + 4$$
$$= 4.4$$

$$k_4 = f(0 + 0.2,\ x_0 + k_3) = 4(0.2) + 4 = 4$$
$$.8$$

The increment is given by

$x_1 - x_0 = \frac{0.2}{6}(4 + 8.8 + 8.8 + 4.8) = 0.88$