GATE (ALL BRANCHES)

Engineering Mathematics

Differential Equation +
Partial differential

Lecture No. 01

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TOPICS TO BE COVERED

01 Variable Separable Method

02 Reduced to Variable separable

03 Problems based on Variable Separable Method

Differential Equations :

Fundamental Topic

- ✓ Differential Equⁿ + type
- ✓ Order and Degree
- ✓ Solution of D.E
- ✓ Solution of first order D.E
- ✓ Solution of SECOND order D.E
- ✓ Solution of Cauchy Euler Equⁿ.
- ✓ Partial Diff Equⁿ

Differential eqn:

$y = f(x)$

x = Independent variable.

y = Dependent Variable

x = Ind. variable → Assign

y = Dependent variable → obtain

$y = \sin x$ $\quad$ $A = \pi r^2$

x = Indep. $\quad$ r = Indep. (Assign)

y = dep $\quad$ A = Dependent "

$x = f(y)$

Independent variable.

Dependent variable.

$V = \frac{4}{3}\pi r^3$

r = Assign

V = obtain

"A Equation Contains dependent Variable + Independent and with Its derivative is called Differential Eqn"

✓ $\frac{dy}{dx} = x^2 y$, ✓ $\frac{d^2y}{dx^2} + w^2 x = 0$

✓ $\frac{d^2q}{dt^2} + 5\frac{dq}{dt} + 6q = 0$

✓ $\frac{dy}{dx} + Py = Q$

P and Q Are function of x only

Types of D.E :- If single Dependent variable + single indep variable + ordinary derivative.

[Ordinary D.E)

Indep.

Dep.

1) $\frac{dy}{dx}+5y=0$ $y=f(x)$ Ordinary D.E

2) $\frac{d^2y}{dx^2}+0.5\frac{dy}{dx}+0.6y=0$ " " $y=f_1(x)$

3) $\frac{d^2y}{dx^2}+w^2x=0$ " " $y=f(x)$

4) $\frac{dy}{dt}+ky=0$ " " $y=f(t)$

CASE 02 :: If Two or more Independent variables + partial derivative

Partial Diff. Eqn

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad u = f(x, y)$$

Two or More

$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}$

$$\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

Partial derivatives ARE involved

P.D.E

Classification of D.E (Order And DEGREE)

$y_2 = \frac{d^2y}{dx^2} = y^{(2)} = \ddot{y}$

$y_1 = \frac{dy}{dx} = y^{(1)} = \dot{y}$

$y_2^{3/2} - y_1^{1/2} - 4 = 0$

$$\boxed{\left(\frac{d^2y}{dx^2}\right)^{3/2} - \left(\frac{dy}{dx}\right)^{\frac{1}{2}} - 4 = 0}$$

Remove The Radical

In Uses of algebra Rules:

✓ $\sqrt{x} + x^{3/2} = 0$

DEGREE → both sides Remove The Radical Power

$$\left(\frac{d^2y}{dx^2}\right)^{3/2} - \left(\frac{dy}{dx}\right)^{1/2} - 4 = 0$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^{3/2} = 4 + \left(\frac{dy}{dx}\right)^{1/2}$$

both sides Square It

$$\left[\left[\frac{d^2y}{dx^2}\right]^{3/2}\right]^2 = \left[4+\left(\frac{dy}{dx}\right)^{1/2}\right]^2$$

Highest derivative In present in Eqn $\left(\frac{d^0y}{dx^0}\right]$ DEGR

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^3 = 16+\left(\frac{dy}{dx}\right)+8\left(\frac{dy}{dx}\right)^{\frac{1}{2}}$$

Again both sides Square It

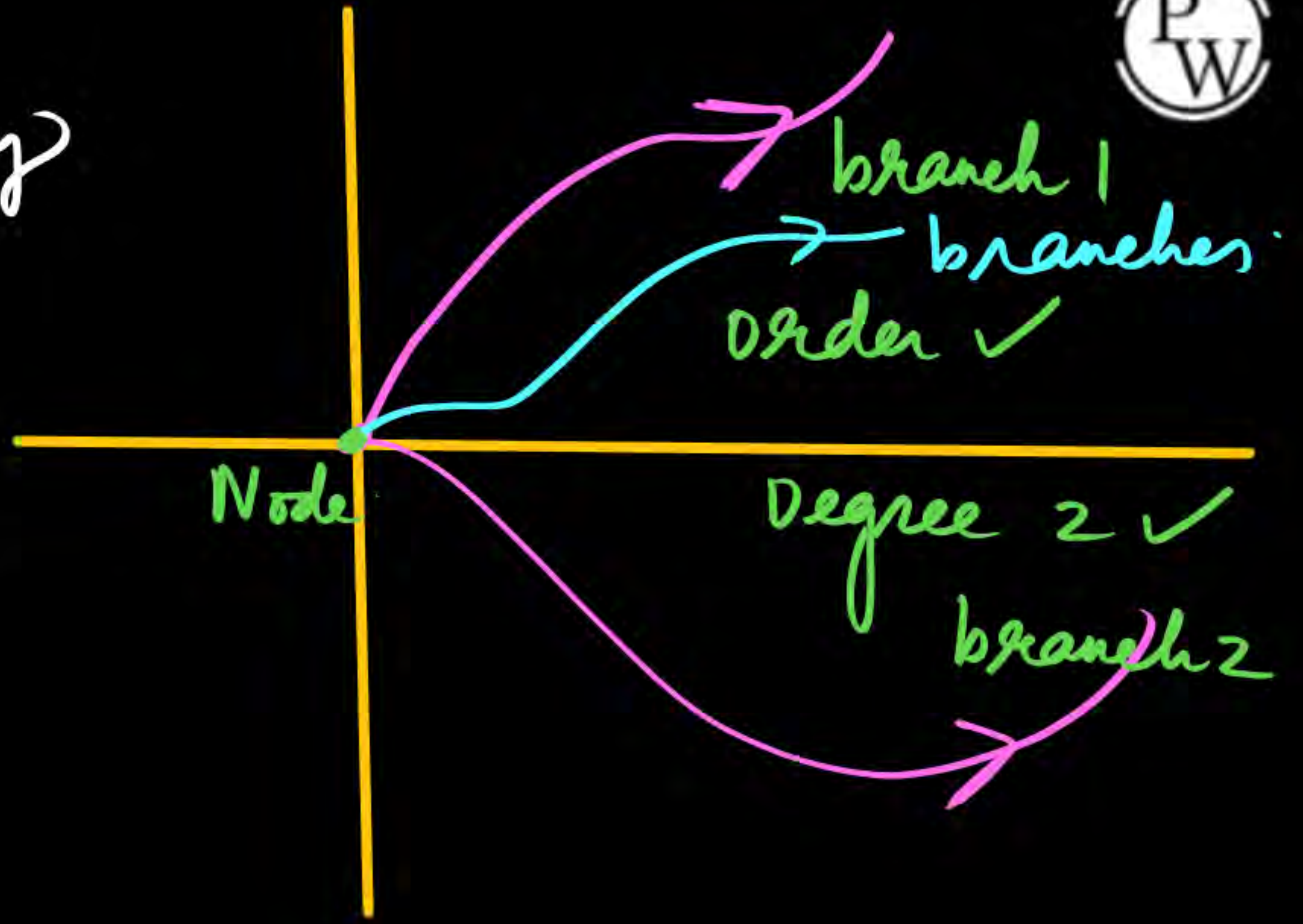
$$\Rightarrow \left[\left[\frac{d^2y}{dx^2}\right]^3 - 16 - \frac{dy}{dx}\right]^2 = \left[8\left(\frac{dy}{dx}\right)^{1/2}\right]^2$$

$$= \left[\left(\frac{d^2y}{dx^2}\right)^3 - 16 - \frac{dy}{dx}\right]^2 = 64\,\frac{dy}{dx}$$

Order = 2
Degree = 6

✓ <u>Order</u> → No. of Arbitrary Parameters

✓ <u>DEGREE</u> → No. of branches ARE connected in Node.



Solution of Differential Equation :

First order Differential Equation :

$$\frac{dy}{dx} = f(x) \text{ or } f(y)$$

Order must be Equal 1

Degree – Anything

OR

$$M\,dx + N\,dy = 0$$

Where M and N function x, y only.

$$M\,dx = -N\,dy$$

$$\boxed{\frac{dy}{dx} = -\frac{M}{N}}$$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

M and N ARE functions of x, y.

Solution of First order Differential Equⁿ :-

general Type. $\frac{dy}{dx} = f(x)$ or $f(y)$ or $\frac{f(x)}{f(y)}$ or $f(x) \cdot f(y)$ or $f(x) + f(y)$ or $f(x) - f(y)$

Type 01

$$\frac{dy}{dx} = \boxed{\frac{X(x)}{Y(y)}} \text{ or } \boxed{X(x)\, Y(y)}$$

Where $X(x)$ is a Function of x only.
$Y(y)$ is a function of y only.

Type 02

$$\boxed{\frac{dy}{dx} = X(x) + Y(y),\ X(x) - Y(y)}$$

First order D.E

$G = 6.67 \times 10^{-11}\ N \cdot m^2 / k$

$g =$

Solution of Ist order D.E :-

(A) Variable seprable method

$$\frac{dy}{dx} = X(x) \cdot Y(y) \quad \text{or} \quad \frac{X(x)}{Y(y)}$$

X is a function of x only

Y ——— of y only

Seprate the Variable

$$\int \frac{dy}{Y(y)} = \int X(x)\, dx$$

solution

$$\boxed{F(y) = F(x) + c}$$

→ solution of D.E

c = arbitrary constant

Q. Questions

#Q. A differential equation $\frac{di}{dt} - 2i = 0$ is applicable over -10 < t < 10.

If i(4) = 10, then i(-5) is ________.

$\boxed{\frac{di}{dt} - 2i = 0}$ $i(4) = 10$ $i(-5) = $ ✓

First order
D.E

$\frac{di}{dt} = 2i$ variable seprable form STEP01

$\Rightarrow \int \frac{di}{i} = \int 2\,dt$ both sides Integrate It STEP02

$\Rightarrow \ln i = 2t + c$

$i = e^{2t+c} = e^{2t} \cdot e^{c}$ → A STEP03

$\boxed{i(t) = Ae^{2t}}$ where A = arbitrary constant

apply Initial conditions $i(4) = 10$ $t = 4$ $i = 10$ STEP04

$10 = Ae^{2\times 4}$

$\boxed{10e^{-8} = A}$

Put the value of constant $A = 10e^{-8}$

STEP 05

$$i(t) = Ae^{2t}$$
$$i(t) = 10e^{2t-8}$$

Solution of this D.E $i(t) = 10e^{2t-8}$

$$i(-5) = 10e^{2\times-5-8}$$

STEP 06

$$= 10e^{-18}$$ Ans

Q. Questions

#Q. For the differential equation $\frac{dy}{dt} + 5y = 0$ with $y(0) = 1$, the general solution is

(a) e^{5t}

(b) e^{-5t}

(c) $5e^{-5t}$

(d) $e^{\sqrt{-5t}}$

$y = Ae^{-5t}$

$\boxed{y = e^{-5t}}$

$\frac{dy}{dt} + 5y = 0$

$y(0) = 1$

$\int \frac{dy}{y} = \int -5\,dt$

$y = e^{-5t+c} = Ae^{-5t}$

$1 = Ae^{0}$

$A = 1$

Q. Questions

$\frac{dy}{dx} = \frac{-2x}{y}$ variable seprable.

#Q. If at every point of a certain curve, the slope of the tangent equals $\frac{-2x}{y}$, the curve is ________.

Ellipse

$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

$\int y\,dy = \int -2x\,dx$

$= \frac{y^2}{2} + x^2 = C$

$= \frac{x^2}{1} + \frac{y^2}{2} = C$ ellipse.

(a) A straight line

(b) A parabola

(c) A circle

(d) An Ellipse

Q. Questions

$\frac{dx}{dt} = -3x \quad x(0) = x_0$

using V.S method

#Q. The solution of the first order differential equation $\dot{x}(t) = -3\,x(t)$, $x(0) = x_0$ is

(a) $x(t) = x_0 e^{-3t}$

(b) $x(t) = x_0 e^{-3}$

(c) $x(t) = x_0 e^{-t/3}$

(d) $x(t) = x_0 e^{-t}$

$\int \frac{dx}{x} = -3\,dt$

$= \ln x = -3t + c$

$x = Ae^{-3t}$

$x(0) = x_0$

$x_0 = Ae^{+0}$

$\boxed{A = x_0}$

$x(t) = x_0 e^{-3t}$

Q. Questions

#Q. Biotransformation of an organic compound having concentration (x) can be modeled using an ordinary differential equation $\boxed{\frac{dx}{dt} + k x^2 = 0,}$ where k is the reaction rate constant. If $x = a$ at $t = 0$ then solution of the equation is

(a) $x = a e^{-kt}$

(b) $\frac{1}{x} = {}^{1}/_{a} + kt$

(c) $x = a(1 - e^{-kt})$

(d) $x = a + kt$

Do yourself.

$$\frac{dx}{dt} = -Kx^2$$

If $x = a$ at $t = 0$

Q. Questions

$\frac{dy}{dx} = y^2$ variable-seprable method

#Q. The solution of $\frac{dy}{dx} = y^2$ with initial value $y(0) = 1$ is bounded in the interval is

$y = \frac{1}{(1-x)}$

$\int \frac{dy}{y^2} = \int dx$ $\quad y = \frac{1}{(1-x)}$

Solution

Bounded in The Interval

(a) $-\infty \le x \le \infty$

(b) $-\infty \le x \le 1$

(c) $x < 1, x > 1$

(d) $-2 \le x \le 2$

A) $-\infty \le x \le \infty$ Unbounded

$-\infty \quad 1 \quad \infty$

B) Unbounded

C) $x > 1, x < 1$ Bounded.

D) Unbounded.

Bounded = finite

Unbounded = Infinite

Slide-8

Q. Questions

$\frac{dy}{dx} = \frac{y}{x} = \frac{dy}{y} = \frac{dx}{x}$ $\ln y = \ln x + \ln c$

✓ $\boxed{y = cx}$

straight line

#Q. Match each differential equation in Group I to its family of solution curves from Group II. P(2) Q(3) R(3) S(1)

P:	$\frac{dy}{dx} = \frac{y}{x}$	(1)	Circles
Q:	$\frac{dy}{dx} = \frac{-y}{x}$	(2)	Straight lines
R:	$\frac{dy}{dx} = \frac{x}{y}$	(3)	Hyperbolae
S:	$\frac{dy}{dx} = \frac{-x}{y}$		

Codes:

(a) P – 2, Q – 3, R – 3, S – 1 ✓

(b) P – 1, Q – 3, R – 2, S – 1

(c) P – 2, Q – 1, R – 3, S – 3

(d) P – 3, Q – 2, R – 1, S – 2

3) $\frac{dy}{dx} = \frac{x}{y}$

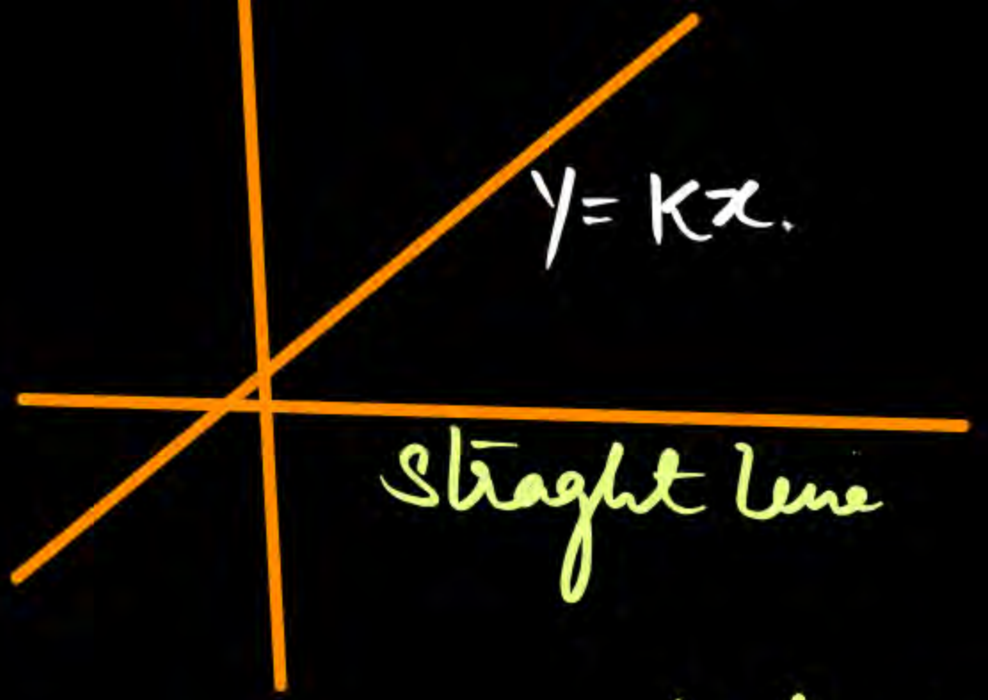
$= \underline{y^2 - x^2 = c}$

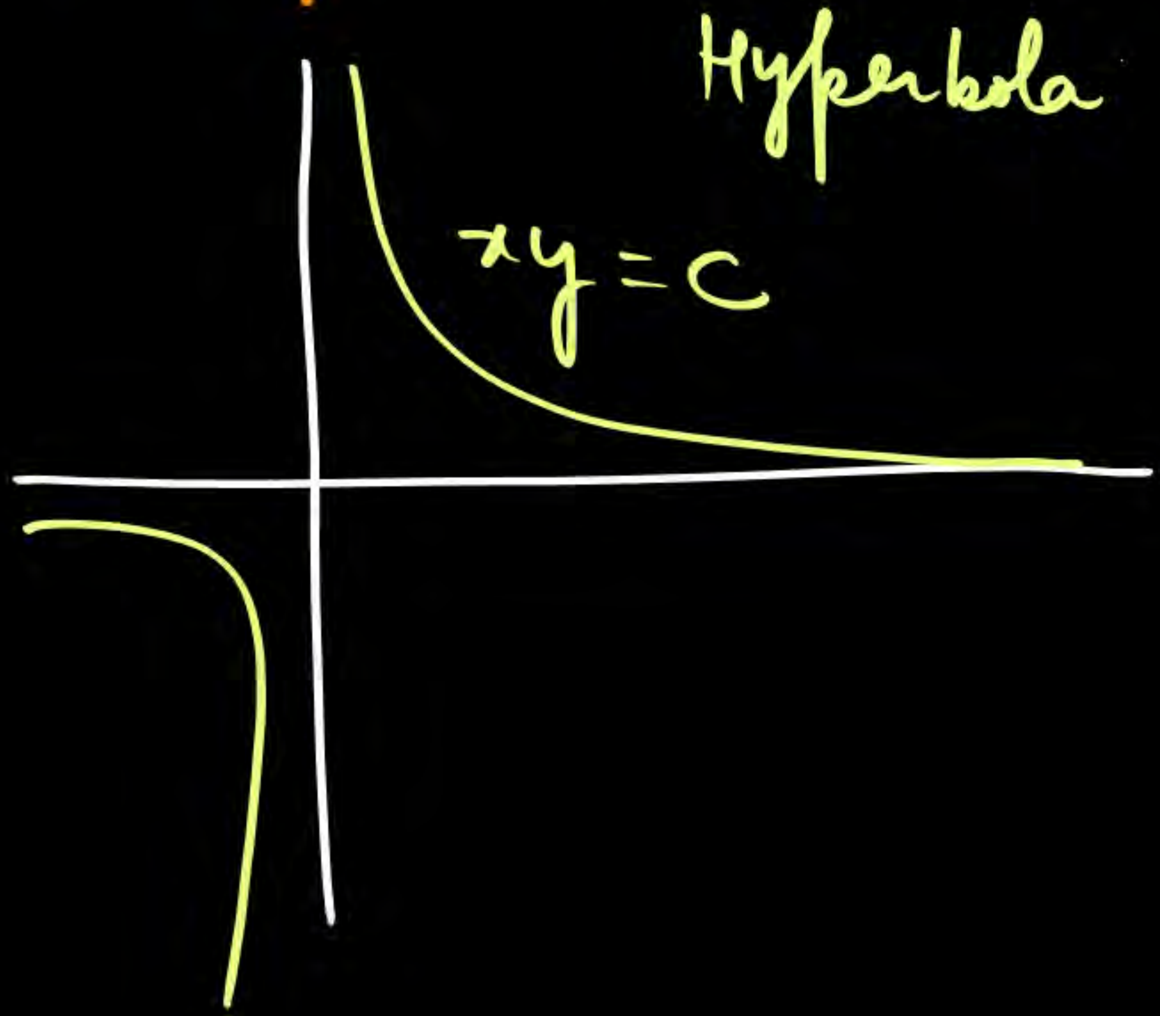
Hyperbola.

4) $\frac{dy}{dx} = \frac{-x}{y}$

$= y^2 + x^2 = c$

Circle.





$\frac{dy}{dx} = \frac{-y}{x}$

$\int \frac{dy}{y} = \int -\frac{dx}{x}$

$\ln y = -\ln x + \ln c$

$\ln y = \ln\left(\frac{c}{x}\right)$

$y = \frac{c}{x}$

$\boxed{xy = c}$ Hyperbola

Q. Questions

#Q. The solution of the differential equation $\frac{dy}{dx} - y^2 = 1$ satisfying the condition $y(0) = 1$ is

(a) $y = e^{x^2}$

(b) $y = \sqrt{x}$

(c) $y = \cot(x + \pi/4)$

(d) $y = \tan(x + \pi/4)$

$\frac{dy}{dx} = 1 + y^2$

variable seprable form

$\int \frac{dy}{1+y^2} = \int dx$

$\tan^{-1} y = x + c$

$y = \tan(x + c)$

$1 = \tan(0 + c)$

$c = \pi/4$

$\underline{y = \tan\left(x + \frac{\pi}{4}\right)}$

Q. Questions

Do yourself

#Q. The order and degree of a differential equation $\frac{d^3y}{dx^3} + 4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2} = 0$ are respectively

H.W

(a) 3 and 2

(b) 2 and 3

(c) 3 and 3

(d) 3 and 1

Q. Questions

#Q. The solution of the ordinary differential equation $\frac{dy}{dx} + 2y = 0$ for the boundary condition, $y = 5$ at $x = 1$ is

H.W

Do yourself

(a) $y = e^{-2x}$

(b) $y = 2e^{-2x}$

(c) $y = 10.95e^{-2x}$

(d) $36.95e^{-2x}$

Thank You!

PW Soldiers