ENGINEERING MATHEMATICS

FOURIER SERIES

DPP: 1

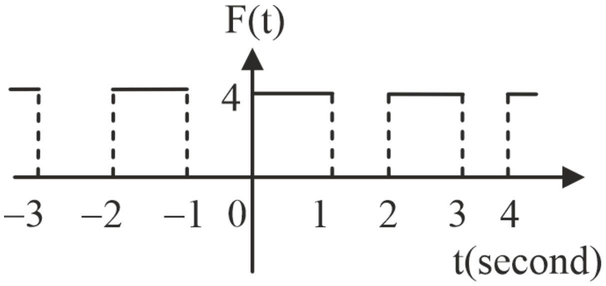
Q1 The following function is defined over the interval [− L, L]:
$f(x) = px^4 + qx^5$
If it is expressed as a Fourier series,
$f(x) = a_0 + \sum_{n=1}^{\infty} \left\{a_n \sin\left(\frac{\pi x}{L}\right) + b_n \cos\left(\frac{\pi x}{L}\right)\right\}$,
Which options amongst the following are true?
(A) a_n, n = 1,2,... , ∞ depend on p
(B) a_n, n = 1,2,... , ∞ depend on q
(C) b_n, n = 1,2,... , ∞ depend on p
(D) b_n, n = 1,2,... , ∞ depend on q

Q2 The Fourier cosine series of a function is given by :

$f(x) = \sum_{n=0}^{\infty} f_n \cos nx$
For f(x) = $\cos^4 x$, the numerical value of ($f_4 + f_5$) is ____________.
(round off to three decimal places).

Q3 The Fourier series expasion of x^3 in the interval $-1 \leq x < 1$ with periodic continuation has
(A) Only sine terms
(B) Only cosine terms
(C) Both sine and cosine terms
(D) Only sine terms and a non-zero constant

Q4 F(t) is a periodic square wave function as shown. It takes only two values, 4 and 0, and stay at each of these values for 1 second before changing. What is the constant term in the Fourier series expansion of F(t)?



(A) 1 (B) 2
(C) 3 (D) 4

Q5 Let f(t) be an even function, i.e. f(−t) = f(t) for all t. Let the Fourier transform of f(t) be defined as $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$.
Suppose $\frac{dF(\omega)}{d\omega} = -\omega F\left(\omega\right)$ for all ω ,and F(0) = 1.
Then
(A) f(0) < 1 (B) f(0) > 1
(C) f(0) = 1 (D) f(0) = 0

Q6 The Fourier series to represent x $-x^2$ for $-\pi \leq x \leq \pi$ is given by
$x - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$
The value of a_0 (round off to two decimal places), is ____________.

Q7 A periodic function f(t), with a period of 2π , is represented as its Fourier series,
$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt \sum_{n=1}^{\infty} b_n \sin nt$
If $f(t) = \begin{cases} A\sin t, & 0 \leq t \leq \pi \\ 0, & \pi < t < 2\pi \end{cases}$, the Fourier series coefficients a_1 and b_1 of f (t) are.
(A) $a_1 = \frac{A}{\pi}$; $b_1 = 0$
(B) $a_1 = \frac{A}{2}$; $b_1 = 0$
(C) $a_1 = 0$; $b_1 = \frac{A}{\pi}$
(D) $a_1 = 0$; $b_1 = \frac{A}{2}$

Q8 The fourier cosine series for an even function f (x) is given by $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$. The value of the coefficient a_2 for the function f (x) = $\cos^2$ (x) in [0, π]
(A) -0.5 (B) 0.00
(C) 0.5 (D) 1.0

Q9 For the function $f(x) = \begin{cases} -2 & -\pi < x < 0 \\ 2 & 0 < x < \pi \end{cases}$. The value of a_n in the Fourier Series expansion of f(x) is

(A) 2 (B) 4

(C) 0 (D) -2

Q10 The Fourier series of the function

$$f(x) = \begin{cases} 0 & -\pi < x \leq 0 \\ \pi - x & 0 < x < \pi \end{cases}$$

In the interval $[-\pi, \pi]$ is

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \ldots\right] + \left[\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \ldots\right]$$

The convergence of the above Fourier series at x = 0 gives.

(A) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

(B) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

(C) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}$

Q11 Let g: $[0, \infty] \rightarrow [0, \infty)$ be a function defined by g(x) = x $-$[x], where [x] represents the integer part of x. (That is the largest integer which is less than or equal to x).The value of the constant term in the Fouier Series expansion of g(x) is __________.

Q12 The period of the signal x(t) = 8 sin $\left(0.8\pi t + \frac{\pi}{4}\right)$

(A) 0.4 π s

(B) 0.8 π s

(C) 1.25 s

(D) 2.5 s

Q13 The Fourier series of a real periodic function has only

P. cosine terms if it is even

Q. sine terms if it is even

R. cosine terms if it is odd

S. sine terms if it is odd

Which of the above statements are correct?

(A) P and S (B) P and R

(C) Q and S (D) Q and R

Q14 Choose the function f(t); $-\infty < t < \infty$, for which a Fourier series cannot be defined.

(A) 3 sin (25 t)

(B) 4 cos (20 t + 3) + 2 sin (710 t)

(C) exp (−|t|) sin (25 t)

(D) 1

Q15 For the function e^{-x}, the linear approximation around x = 2 is

(A) (3 – x) e^{-2}

(B) 1 – x

(C) $\left[3+2\sqrt{2-}\left(1+\sqrt{2}\right)x\right]e^{-2}$

(D) e^{-2}

Q16 Which of the following functions would have only odd powers of x in its Taylor series expansion about the point x = 0?

(A) sin (x^3) (B) sin (x^2)

(C) cos (x^3) (D) cos (x^2)

Q17 In the Taylor series expansion of exp(x) + sin(x) about the point x = π, the coefficient of $(x - \pi)^2$ is

(A) exp (π)

(B) 0.5 exp(π)

(C) exp (π) + 1

(D) exp(π) – 1

Q18 The Taylor series expansion of $\frac{\sin x}{x-\pi}$ at x = π is given by

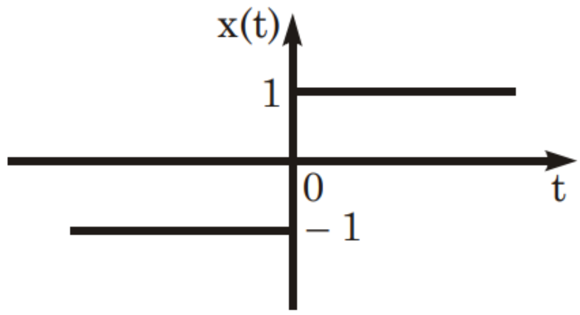
(A) $1 + \frac{(x-\pi)^2}{3!} + \ldots$

(B) $-1 - \frac{(x-\pi)^2}{3!} + \ldots$

(C) $1 - \frac{(x-\pi)^2}{3!} + \ldots$

(D) $-1 + \frac{(x-\pi)^2}{3!} + \ldots$

Q19 The function x(t) is shown in the figure. Even and odd parts of a unit-step function u(t) are respectively ,



(A) $\frac{1}{2}, \frac{1}{2}\ x\left(t\right)$ (B) $-\frac{1}{2}, \frac{1}{2}\ x\left(t\right)$

(C) $\frac{1}{2}, -\frac{1}{2}\ x\left(t\right)$ (D) $-\frac{1}{2}, -\frac{1}{2}x\left(t\right)$

Answer Key

Q1	(B, C)
Q2	(0.120 to 0.130)
Q3	(A)
Q4	(B)
Q5	(A)
Q6	(-6.28 to -6.68)
Q7	(D)
Q8	(C)
Q9	(C)
Q10	(C)
Q11	(0.5 to 0.5)
Q12	(D)
Q13	(A)
Q14	(C)
Q15	(A)
Q16	(A)
Q17	(B)
Q18	(D)
Q19	(A)

Hints & Solutions

Q1 Text Solution:

f(x) = $px^4 + qx^5$

using Fourier coefficient formula,

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot \sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L}\left[px^4 + qx^5\right] sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} px^4 \sin\left(\frac{n\pi x}{L}\right)dx + \frac{1}{L}$$

$$\int_{-L}^{L} qx^5 \sin\left(\frac{n\pi x}{L}\right)dx$$

$$a_n = 0 + \frac{1}{L}\int_{-L}^{L} qx^5 \sin\left(\frac{n\pi x}{L}\right)dx$$

$$\left[\because \quad px^4 \sin\left(\frac{n\pi x}{L}\right) is\, an\, odd\, function\right]$$

So a_n depends on q.

f(x) = $px^4 + qx^5$

Using Fourier coefficient formula,

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot \cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L}\left[px^4 + qx^5\right] \cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} px^4 \cos\left(\frac{n\pi x}{L}\right) dx + \frac{1}{L}\int_{-L}^{L} qx^5$$

$$\cos\left(\frac{n\pi x}{L}\right)dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} px^4 \cos\left(\frac{n\pi x}{L}\right)dx + 0$$

$$\left[\because qx^5 \cos\left(\frac{n\pi x}{L}\right)\ is\ an\ odd\ function\right]$$

So, b_n depends on p.

Q2 Text Solution:

$$f(x) = \cos^4 x = \left(\cos^2 x\right)^2 = \left[\frac{1+\cos 2x}{2}\right]^2$$

$$\Rightarrow \quad f(x) = \frac{1}{4}\left[1 + \cos^2 2x + 2\cos 2x\right]$$

$$= \frac{1}{4}\left[1 + \frac{1+\cos 4x}{2} + 2\cos\ 2x\right]$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{\cos 4x}{8} + \frac{\cos 2x}{2}$$

$$= \frac{3}{8} + 0\cos x + \frac{1}{2}\cos 2x + 0\cos 3x$$

$$+\frac{1}{8}\ cos4x +\ 0\ \cos 5x + \ldots$$

$\therefore$ From cosine Fourier series

$$f_4 = \frac{1}{8}\ \ and\ \ f_5 = 0$$

$$f_4 + f_5 = \frac{1}{8} = 0.125$$

Q3 Text Solution:

Given data :

f(x) = x^3 [−1,1]

The General Fourier Series Expansion is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos\left(\frac{n\pi x}{c}\right) + \sum_{n=1}^{\infty} b_n \cdot \sin\left(\frac{n\pi x}{c}\right)$$

Comparing $(\alpha, \alpha + 2c)\ with\ [-1, 1]$

$$\Rightarrow \quad \alpha = -1;\ c = 1$$

$$a_0 = \frac{1}{c}\cdot\int_{-1}^{1} f(x)dx = \frac{1}{c}\int_{-1}^{1} x^3 dx = 0$$

{ $\because$ f(x) is an odd function}

$$a_n = \frac{1}{c}\cdot\int_{-1}^{1} f(x)\ \cos nx dx = \frac{1}{c}\int_{-1}^{1} x^3$$

$$\cos nx dx = 0$$

$$b_n = \frac{1}{c}\cdot\int_{-1}^{1} f(x)\sin nx dx = \frac{1}{c}$$

$$\int_{-1}^{1} x^3 \sin nx dx \neq 0$$

$\therefore$ The Fourier Series expansion of x^3 in [−1, 1] has only sine terms in it.

Q4 Text Solution:

Given data : f(t) takes two values : 4 and 0.

Let us consider [−1, 1]

($\because$ since the period of function is 2 seconds)

The general Fourier Series expansion is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos\left(\frac{n\pi x}{c}\right) + \sum_{n=1}^{\infty} b_n \cdot \sin\left(\frac{n\pi x}{c}\right)$$

Where ,

$$a_0 = \frac{1}{c}\cdot\int_{\alpha}^{\alpha+2c} f(x)dx = \frac{1}{1}\cdot\int_{-1}^{1} f(x)dx$$

$$= \int_{-1}^{0} 0.dx + \int_{0}^{1} 4.\ dx = 4$$

Since a_0 = 4

$$\Rightarrow \quad \frac{a_0}{2} = 2$$

Q5 Text Solution:

Given function is even

$\frac{d}{d\omega}F(\omega) = -\omega F(\omega)$ ... (1)

From differentiation property,

$t\ f(t) = j\frac{d}{d\omega}F(\omega)$

Applying inverse Fourier transfer to the above equation,

$-j\ t\ f(t) = j\frac{d}{d\omega}f(t)$

$\frac{d}{dt}f(t) = -tf(t)$ ... (2)

From equation (1) and (2) it is clear that f(t) is Gaussian function, it can be written as,

$f(t) = \frac{1}{\sqrt{2\pi}}e^{-t^2/2}$

$\therefore\quad f(0) = \frac{1}{\sqrt{2\pi}} = 0.3989$

$\therefore\quad f(0) < 1$

Q6 Text Solution:

$a_0 = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} f(x).\,dx$

$= \frac{1}{\pi}\int\limits_{-\pi}^{\pi}(x - x^2)dx$

$= \frac{1}{\pi}\cdot 2\int\limits_{0}^{\pi} -x^2 dx$ [∵ x is an odd function.]

$= -\frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} = -\frac{2}{\pi}\cdot\frac{\pi^3}{3}$

$= -\frac{2\pi^2}{3} = -6.58$

Q7 Text Solution:

Given data :

$f(t) = \begin{cases} A.\sin t; & 0 \le t \le \pi \\ 0; & \pi < t < 2\pi \end{cases}$

$a_1 = \frac{1}{\pi}\cdot\int\limits_{0}^{2\pi} f(t).\cos\ t.\,dt$

$\Rightarrow\quad a_1 = \frac{1}{\pi}\cdot\left\{\int\limits_0^{\pi} A.\sin t.\cos.\,t.\ dt\right\}$

$\Rightarrow\quad a_1 = \frac{1}{\pi}\times\frac{A}{2}\times\int\limits_0^{\pi}\sin 2t.\,dt$

$\Rightarrow\quad a_1 = \frac{A}{2\pi}\times\left\{\frac{-\cos 2t}{2}\Big|_0^{\pi}\right\} = \frac{-A}{4\pi}\{1-1\}$

$= 0$

$b_1 = \frac{1}{\pi}\cdot\int\limits_0^{2\pi} f(t).\sin t.\,dt$

$\Rightarrow\quad b_1 = \frac{1}{\pi}\cdot\left\{\int\limits_0^{\pi} A.\sin t.\sin t.\,dt\right\}$

$\Rightarrow\quad b_1 = \frac{A}{\pi}\times\int\limits_0^{\pi}\sin^2 t.\,dt$

$\Rightarrow\quad b_1 = \frac{2A}{\pi}\times\int\limits_0^{\pi/2}\sin^2 t.\,dt = \frac{2A}{\pi}\times\frac{1}{2}\times\frac{\pi}{2}$

$\Rightarrow\quad \boxed{b_1 = \frac{A}{2}\&a_1 = 0}$

Q8 Text Solution:

Given Data : f(x) = $\cos^2$ x, [0, π]

$f(x) = a_0 + \sum_{n=1}^{\infty} a_n.\cos nx$

We know that

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\Rightarrow\quad \cos^2 x = \frac{1}{2} + \frac{1}{2}.\cos 2x = a_0 + a_2$

$.\cos 2x$

On comparing ⇒ $\boxed{a_2 = \frac{1}{2} = 0.5}$

Q9 Text Solution:

Given data :

$f(x) = \begin{cases} -2; & -\pi < x < 0 \\ 2; & 0 < x < \pi \end{cases}$

The general Fourier Series expansion of f(x) is given by

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n\sin nx$

Where $a_n = \frac{1}{\pi}\cdot\int\limits_{-\pi}^{\pi} f\ \ (x)\cos.\,nx.\,dx$

$\Rightarrow\quad a_n = \frac{1}{\pi}$

$\times\left\{\int\limits_{-\pi}^{0} -2.\cos nx.\,dx + \int\limits_0^{\pi} 2.\cos nx.\,dx\right\}$

$\Rightarrow\quad \frac{1}{\pi}\cdot\left\{-2.\frac{\sin nx}{n}\Big|_{-\pi}^{0} + 2.\frac{\sin nx}{n}\Big|_0^{\pi}\right\}$

$\Rightarrow \frac{1}{\pi}\cdot\left\{\frac{-2}{n}(0) + 2.(0)\right\} = 0$

∴ $a_n = 0$

Q10 Text Solution:

The function

$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \pi - x & 0 \le x < \pi \end{cases}$

And Fourier series is

$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + ...\right]$

$+\left[\frac{\sin x}{1}+\frac{\sin 2x}{2}+\frac{\sin 3x}{3}+\ldots\right]$

At x = 0, it is a point of discontinuity, the Fourier series converges to $\frac{1}{2}\left[f\left(0^{-}\right)+f\left(0^{+}\right)\right]$

Where $f\left(0^{-}\right)=\lim_{x\to 0}(\pi-x)=\pi$

$f\left(0^{+}\right)=0$

Put x = 0 in fourier series,

$f(0) \ = \ \frac{\pi}{4}+\frac{2}{\pi}\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right]$

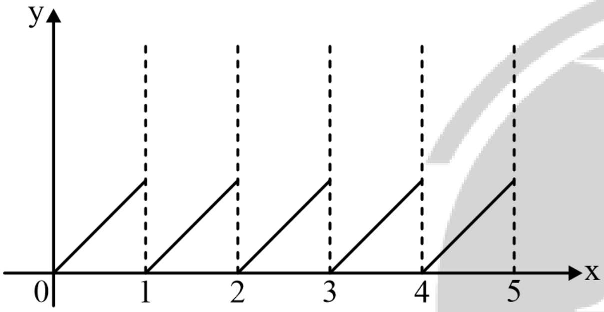
$\frac{\pi}{2}=\frac{\pi}{4}+\frac{2}{\pi}\ \left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right]$

$\frac{\pi}{2}\ \left(\frac{\pi}{2}-\frac{\pi}{4}\right) \ = \ \left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right]$

$\boxed{\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}}$

Q11 Text Solution:

Given data : f(x) = x – [x] = {x}



From the graph, period of f(x) = 1

The general Fourier Series expansion of f (x) in the interval $(\alpha,\alpha+2c)$ is given by

$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n.\cos\left(\frac{n\pi x}{c}\right)+\sum_{n=1}^{\infty}b_n$

$.\sin\left(\frac{n\pi x}{c}\right)$

Where $a_0=\frac{1}{c}\int_{\alpha}^{\alpha+2c}f(x)dx$

Considering the interval [0, 1]

$\Rightarrow \quad \alpha=0\ and\ c=\frac{1}{2}$

$\frac{a_0}{2}=\frac{1}{2}\left\{\frac{1}{c}\int_{\alpha}^{\alpha+2c}f(x)dx\right\}$

$=\frac{1}{2}\times\frac{1}{\left(\frac{1}{2}\right)}\times\int_0^1(x-[x])dx$

$\Rightarrow \quad \frac{a_0}{2}=\int_0^1(x-0)dx=\frac{1}{2}$

$\Rightarrow \quad \frac{a_0}{2}=\frac{1}{2}=0.5$

Q12 Text Solution:

The given signal is

x(t) = 8. sin $\left(0.8\pi t+\frac{\pi}{4}\right)$

Comparing with the standard from of the signal

x(t) = A. sin (ωt + ϕ)

$\Rightarrow \quad \omega=0.8\ \ \pi$

$\Rightarrow \quad \frac{2\pi}{T}=0.8\pi$

$\Rightarrow \quad T=\frac{2}{0.8}=2.5\ \ s$

$\therefore$ T = 2.5 s

Q13 Text Solution:

Because sine function is odd and cosine is even function.

Q14 Text Solution:

Option C is correct.

Q15 Text Solution:

$f(x)=f(x_0)+\frac{(x-x_0)f'(x_0)}{\lfloor 1}+\frac{(x-x_0)^2f''(x_0)}{\lfloor 2}$

$+\ldots\ldots.$

$=\ e^{-2}+(x-2)\left(-e^{-2}\right)$

$+\frac{(x-2)^2}{\lfloor 2}\left(+e^{-2}\right)\ldots\ldots$

$=e^{-2}+\left(2-x+\frac{(x-2)^2}{\lfloor 2}\right)e^{-2}+\ldots\ldots.$

$= \quad (3-x)e^{-2}$

(neglecting higher power of x)

Q16 Text Solution:

We know, sin x = $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots\ldots$

$\therefore \quad \sin x^3=x^3-\frac{x^9}{3!}+\frac{x^{15}}{5!}-\frac{x^{21}}{7!}+\ldots\ldots$

Q17 Text Solution:

Let f(x) = e^x + sin x

Taylor's series is

$f(x)=f(a)+(x-a)f'(a)+\frac{(x-a)^2}{2!}f''(a)$

Where a = π

$f(x) \ = \ f(\pi)+(x-\pi)\ f'(\pi)$

$+\frac{(x-\pi)^2}{2!}f''(\pi)$

$\therefore$ coefficient of $(x-\pi)^2\ is\frac{f''(\pi)}{2}$

Now, $f''\ \ (\pi)=e^x-\sin x|_{x=\pi}=e^{\pi}$

$\therefore$ coefficient of $(x-\pi)^2 \ = \ 0.5\exp(\pi)$.

Q18 Text Solution:

we know.

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots.$

$\therefore \quad \sin(x-\pi) = (x-\pi) - \frac{(x-\pi)^3}{3!} + \frac{(x-\pi)^5}{5!}$

$- \frac{(x-\pi)^7}{7!} + \ldots\ldots.$

$\Rightarrow \quad \frac{-\sin x}{x-\pi} = 1 - \frac{(x-\pi)^2}{3!} + \frac{(x-\pi)^4}{5!} - \frac{(x-\pi)^6}{7!}$

$+ \ldots\ldots$

$\Rightarrow \quad \frac{\sin x}{x-\pi} = -1 + \frac{(x-\pi)^2}{3!} - \frac{(x-\pi)^4}{5!} + \frac{(x-\pi)^6}{7!}$

$- \ldots\ldots$

Q19 Text Solution:

Even part $= \frac{u(t)+u(-t)}{2}$

Now u(t) = 0; t < 0

= 1, t ≥ 0

∴ u(−t) = 0, −t < 0

= 1, − t ≥ 0

i.e., u(−t) = 1, t ≤ 0

= 0, t > 0

$\therefore \frac{u(t)+u(-t)}{2} = \frac{1}{2}; \qquad t \leq 0$

=1/2;t>0

∴ Even [u(t)] = $\frac{1}{2}$

Odd (u(t)) = $\frac{u(t)+u(-t)}{2} \quad \begin{bmatrix} -\frac{1}{2}, & t \leq 0 \\ \frac{1}{2}, & t > 0 \end{bmatrix}$

$= \frac{x(t)}{2}$ from given figure.