GATE-All BRANCHES
Engineering Mathematics

NUMERICAL METHODS

Lecture No.- 01

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Recap of previous lecture

Topic

Solution of differential equation using laplace transforms

Problems based on solution of differential equations

Topics to be Covered

Calculator

Topic Numerical methods

Topic Newton Raphson method

80%
Chance

Topic Problems based on newton Raphson method and bisection rule

X EE/EC Syllabus

Numerical method

CE/ME →

SERVEYING

IES/XE ✓

Syllabus 2023 GATE

✓ 1) N-R method 80%

✓ 2) Numerical integration 20%

1) Trapezoidal Rule

2) Simpson Rule $\frac{1}{3}$

3) $\frac{3}{8}$th

MTech (SVD)

Numerical method

3) Numerical differential Equ^n 1st — Euler method

1) Euler method

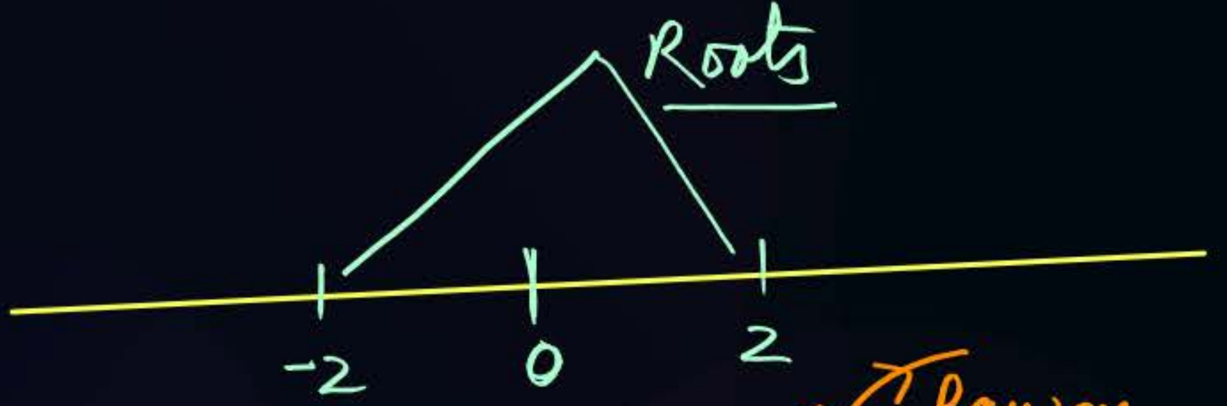
2) Runge-Kutta method 2nd, 3rd, 4th order

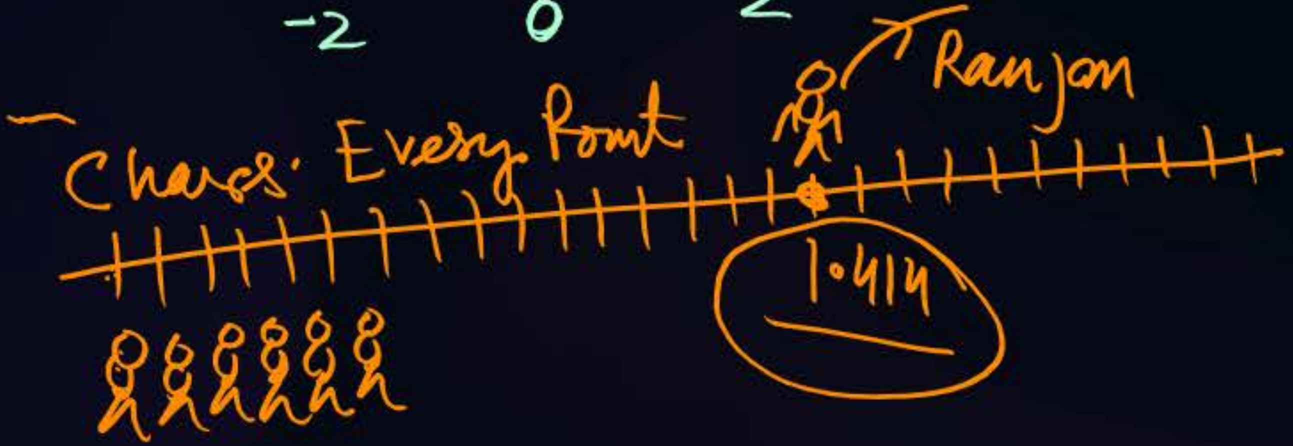
Newton Raphson method :

Newton Raphson method apply on Transcendental equation

(A) Transcendental Equation :

$$x^2 - 4 = 0$$
$$x = \pm 2$$





$$x^2 - 2 = 0$$
$$x = \pm\sqrt{2}$$

(1.414)
exact
approximated
Root

Not
Location
Identified

Uncertain

Recursing

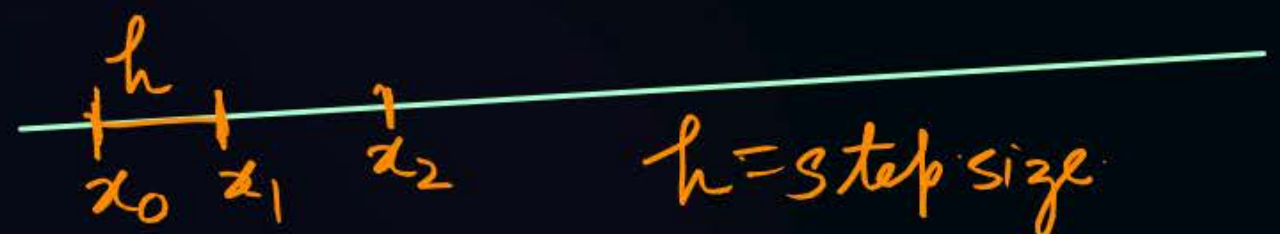
↑ Irrational No → →∞

Transcendental Equⁿ → $\sqrt{2}, \sqrt{3}, e^{x}, \log x$ - - - -

$f(x) = 0$ (Approximation of Roots)

Using Newton Raphson method

x_0 = Initial guess (Problem given)
= Initial Iteration



h = step size

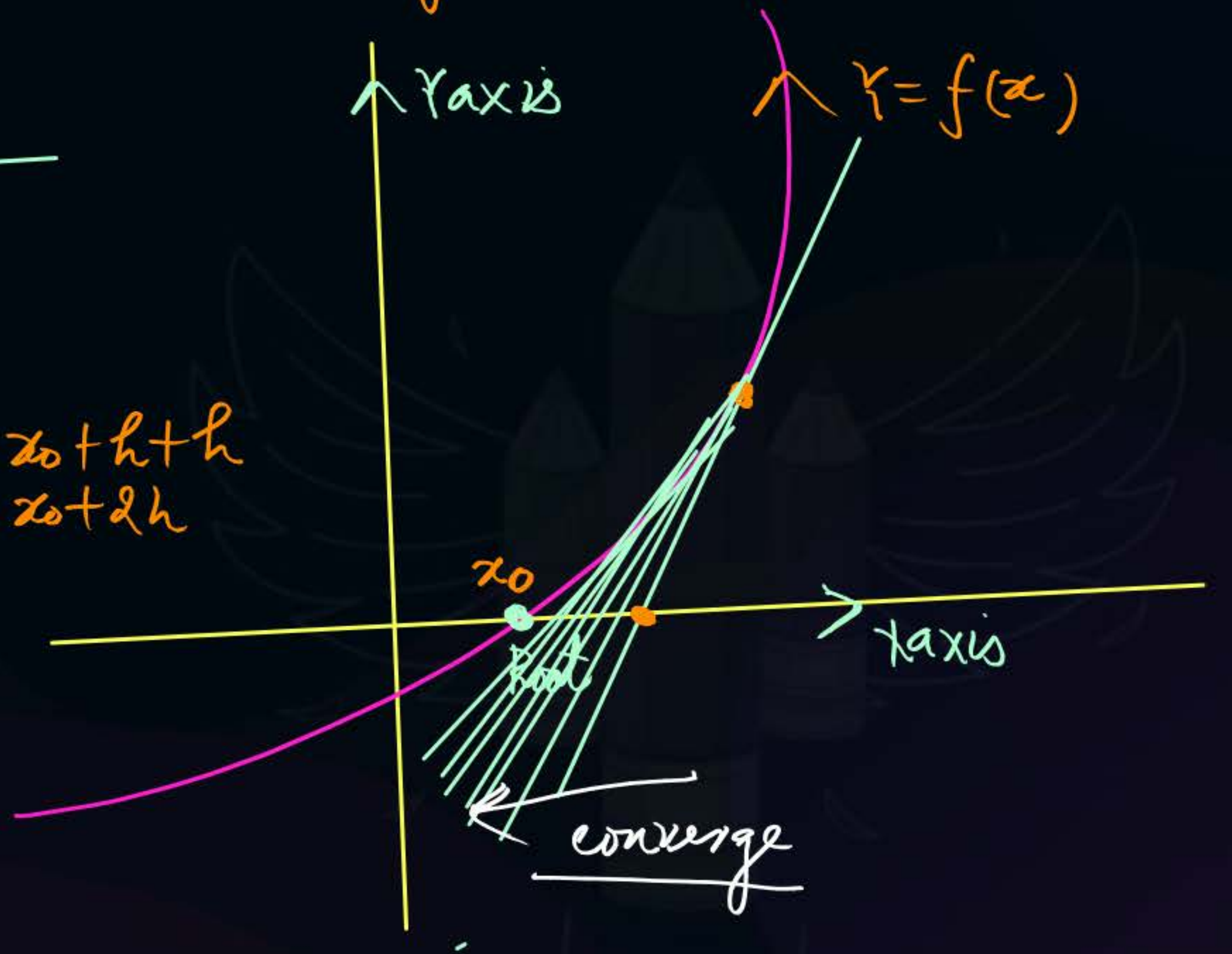
First approximation $x_1 = x_0 + h$

SECOND approximation $x_2 = x_1 + h = x_0 + h + h = x_0 + 2h$

Third approximation $x_3 = x_0 + 3h$

fourth approximation $x_4 = x_0 + 4h$

$$\boxed{x_n = x_0 + nh}$$



$x_n = x_0 + nh$

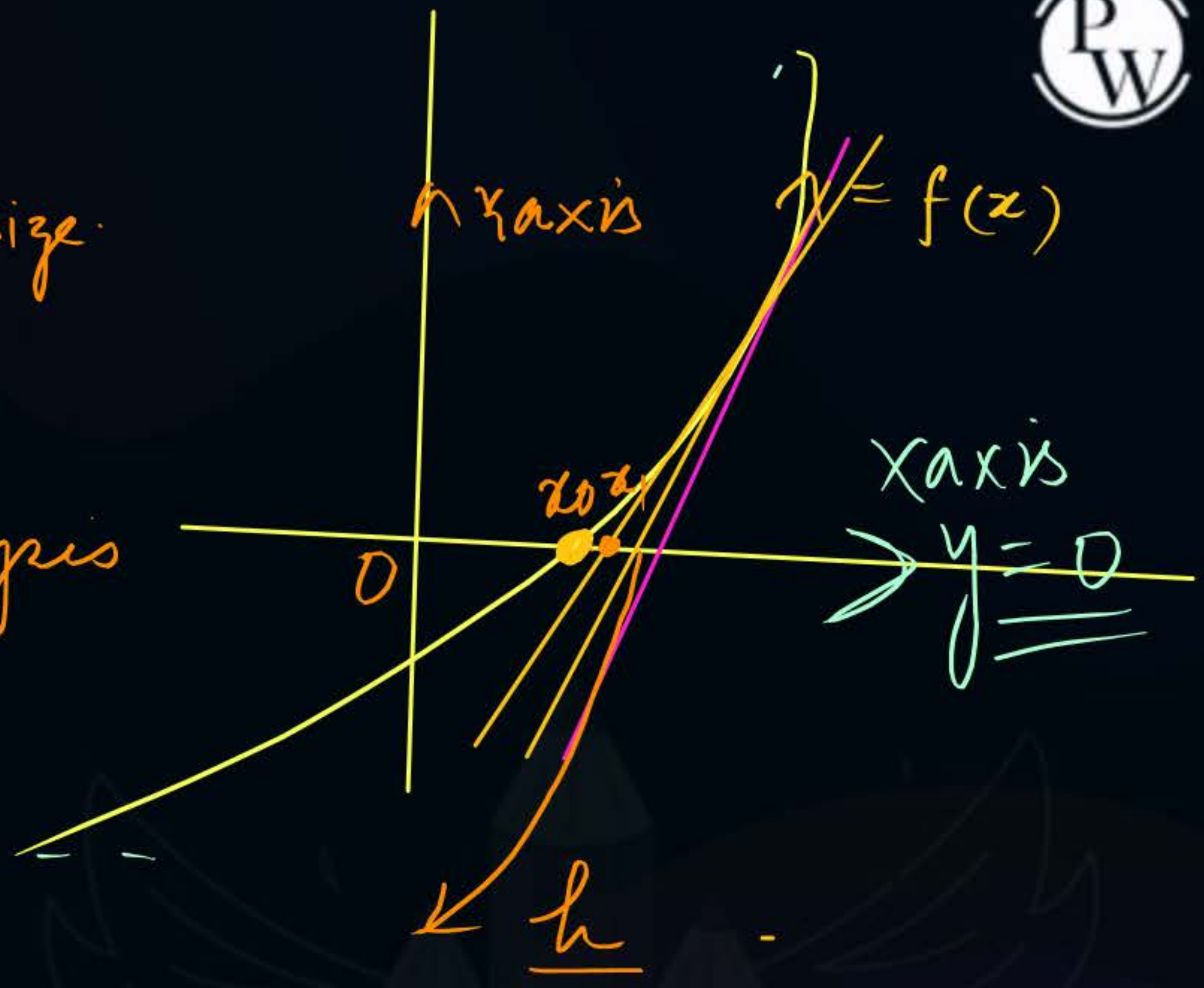
x_0 = Initial guess

x_n = n^{th} Iteration

h = step size

h = step size

Taylor SERIES — Neighbourhood Point — analysis



Using Taylor SERIES (Tangent)

$$f(x+h) = \underbrace{f(x) + hf'(x)}_{\substack{\text{Linear approximation} \\ \text{(Tangent)}}} + \frac{h^2}{2!} f''(x) + \cdots$$

$$\boxed{f(x+h)} = f(x) + hf'(x) + \cdots$$

$$0 = f(x) + hf'(x) + \cdots$$

$$0 = f(x) + h f'(x)$$

$$\boxed{h = \frac{-f(x)}{f'(x)}}$$ step size

n^{th} Iteration (approximation)

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

N-R method
for Any Transcendental equ^n $f(x) = 0$

Initial guess $= x_0$

First Iteration $x_1 = x_0 + h$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

SECOND approximation

$$x_2 = x_0 + 2h = x_1 + h$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Third approximation

$$x_3 = x_2 + h$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

#Q. Find the positive real root of $x^3 - x - 3 = 0$ using Newton-Raphson method. If the starting guess (x_0) is 2,the numerical value of the root after two iterations (x_2) is _____ (*round off to two decimal places*).

$$f(x) = x^3 - x - 3 = 0 \qquad x_0 = 2 \longrightarrow x_1 \longrightarrow x_2$$

$$x_1 = x_0 + h \qquad x_2 = x_0 + 2h$$

N-R method

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$$x_{n+1} = x_n - \frac{[x_n^3 - x_n - 3]}{3x_n^2 - 1}$$

$$x_{n+1} = \frac{(3x_n^3 - \cancel{x_n}) - (x_n^3 - \cancel{x_n} - 3)}{3x_n^2 - 1}$$

$$x_{n+1} = \frac{2x_n^3 + 3}{3x_n - 1}$$

$$x_{n+1} = \frac{2x_n^3 + 3}{3x_n^2 - 1}$$

$x_0 = 2$

$x_1 = \checkmark$

$x_2 = \checkmark$

$n = 0$

$$x_1 = \frac{2x_0^3 + 3}{3x_0^2 - 1} = \frac{2 \times (2)^3 + 3}{3(2)^2 - 1} = 1.727$$

$n = 1$

$$x_2 = \frac{2x_1^3 + 3}{3x_1^2 - 1} = \frac{2 \times (1.727)^3 + 3}{3 \times (1.727)^2 - 1} = \underline{1.67}$$

$x_1 = 1.727$

$x_2 = 1.67$

$\underline{n = 2}$

#Q. Newton-Raphson method is to be used to find root of equation $3x - e^x + \sin x = 0$. If the initial trial value for the roots is taken as 0.333, the next approximation for the root would be ________ (*answer up to three decimal places*)

N.R method

$$f(x) = 3x - e^x + \sin x$$

$$x_0 = 0.333$$

$$\boxed{x_1 = 0.36}$$

$$x_1 = x_0 - \frac{(3x_0 - e^{x_0} + \sin x_0)}{(3 - e^{x_0} + \cos x_0)}$$

$$x_0 = 0.333$$

$$\boxed{x_1 = 0.36}$$

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#Q. The function $f(x) = e^x - 1$ is to be solved using Newton-Raphson method. If the initial value of x_0 is taken as 1.0, then the absolute error observed at 2^{nd} iteration is ________.

$f(x) = e^x - 1$

$x_0 = 1.0$

SECOND Iteration = 0.06

$f(x) = e^x - 1$

$f(0) = e^0 - 1 = 0$

Actual Root

$x_0 = 1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_1)} = \frac{1}{e}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)} = 0.06$

Topic : Numerical Methods

5 times

$x^2 - K = 0$

#Q. The square root of a number N is to be obtained by applying the Newton Raphson iterations to the equation $x^2 - N = 0$. If i denotes the iteration index , the correct iterative scheme will be :

$x^2 - N = 0$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = x_i - \frac{(x_i^2 - N)}{2x_i}$$

$$x_{i+1} = \frac{2x_i^2 - (x_i^2) + N}{2x_i} = \frac{x_i^2 + N}{2x_i}$$

$$x_{i+1} = \frac{1}{2}\left[x_i + \frac{N}{x_i}\right]$$

A $x_{i+1} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)$

B $x_{i+1} = \frac{1}{2}\left(x_i^2 + \frac{N}{x_i^2}\right)$

C $x_{i+1} = \frac{1}{2}\left(x_i + \frac{N^2}{x_i}\right)$

D $x_{i+1} = \frac{1}{2}\left(x_i - \frac{N}{x_i}\right)$

#Q. Equation $e^x - 1 = 0$ is required to be solved using Newton's method with an initial guess $x_0 = -1$. Then after one step of Newton's method, estimate, x_1 the solution will be the given by :

$e^x - 1 = 0$

$x_0 = -1$

$x_1 = ✓$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{(e^{x_0} - 1)}{e^{x_0}}$

$x_1 = -1 - \frac{(e^{-1} - 1)}{e^{-1}} = 0.71828$

A. $\boxed{0.71828}$

B. 0.36784

C. 0.20587

D. 0.00000

2 mins Summary

Topic One

Topic Two

Topic Three

Topic Four

Topic Five

✓ Transform of derivative

✓ Problems

Numerical methods

THANK - YOU