GATE-All BRANCHES

Engineering Mathematics

Differential Equation + Partial Differential

Discussion Notes (Part-01)

DPP 01

By- Rajshree Mam

#Q. In R^2, the family of trajectories orthogonal to the family of $x^{2/3} + y^{2/3} = a^{2/3}$ is given by

$m_1 \cdot m_2 = -1$

$x^{2/3} + y^{2/3} = a^{2/3}$

A $x^{4/3} + y^{4/3} = c^{4/3}$

B $x^{4/3} - y^{4/3} = c^{4/3}$

C $x^{5/3} - y^{5/3} = c^{5/3}$

D $x^{2/3} - y^{2/3} = c^{2/3}$

Step ①:-

$$x^{2/3}+y^{2/3}=a^{2/3}$$

diff w.r.t to x on both hand sides —

$$\frac{2}{3}x^{-\frac{1}{3}}+\frac{2}{3}y^{-\frac{1}{3}}\cdot\frac{dy}{dx}=0$$

orthogonal trajectory hai !!

$$\boxed{\frac{dx}{dy}\times-\frac{dy}{dx}=-1}$$

$$\frac{dy}{dx} \rightarrow -\frac{dx}{dy}$$

$$\cancel{\frac{2}{3}}x^{-\frac{1}{3}} + \cancel{\frac{2}{3}}y^{-\frac{1}{3}}\left(-\frac{dx}{dy}\right) = 0$$

$$dy\, x^{-\frac{1}{3}} - y^{-\frac{1}{3}}dx = 0$$

$$dy\, y^{\frac{1}{3}} - x^{\frac{1}{3}}dx = 0 \quad \text{——} \quad (3)$$

$$M dx + N dy = 0$$

Integrate –

$-x^{\frac{1}{3}}dx + y^{\frac{1}{3}}dy = 0$

so after integrating we will get –

$$-\frac{x^{4/3}}{\cancel{4}} \times \cancel{3} + \frac{y^{4/3}}{\cancel{4}} \times \cancel{3} = a$$

$$-x^{4/3} + y^{4/3} = a^{4/3}$$

$$\boxed{x^{4/3} - y^{4/3} = a^{4/3}}$$

#Q. A particular integral of the differential equation

$$y'' + 3y' + 2y = e^{e^x} \text{ is}$$

A $e^{e^x}e^{-x}$

B $e^{e^x}e^{-2x}$

C $e^{e^x}e^{2x}$

D $e^{e^x}e^{x}$

$$y'' + 3y' + 2y = e^{e^x}$$

P.I. $\dfrac{1}{(D-\alpha)} \cdot X = e^{\alpha x} \int e^{-\alpha x} \cdot X \cdot dx$

$$\boxed{f(D)\, y = X}$$

$$m^2 + 3m + 2 = 0$$

$$D^2 + 3D + 2 = 0$$

$$D^2+3D+2$$

$$(D+2)(D+1)$$

$$P(I) = \frac{e^{e^x}}{(D+2)(D+1)}$$

$$= \left\{\frac{1}{D+1} - \frac{1}{D+2}\right\} e^{e^x}$$

$$\left\{\underbrace{\frac{1}{D+1}\cdot e^{e^x}}-\underbrace{\frac{1}{D+2}e^{e^x}}\right\}$$

$$\left\{e^{-x}\int e^{e^x}\cdot e^x\cdot dx\right\}-\left\{e^{-2x}\int e^{e^x}\cdot(e^x)^2\cdot dx\right.$$

$e^x=t\to e^x dx=dt$

$e^{2x}=(e^x)^2$

$$e^{-x}\int e^t\, dt=\underbrace{e^{-x}\left\{e^{e^x}\right\}}-\underbrace{\left\{e^{-2x}\int e^t\cdot t\cdot dt\right.}$$

$$\cancel{e^{-x}\cdot e^{e^x}}\qquad -\cancel{e^{-x}\cdot e^{e^x}}+\underbrace{e^{-2x}\cdot e^{e^x}}$$

$$\int e^{e^x}(e^x)^2\cdot dx \longrightarrow \underbrace{e^x\cdot e^x dx}$$

$$e^x = t \qquad \underbrace{t\cdot dt}$$

$$e^x\cdot dx = dt$$

$$\int e^t\cdot t\cdot dt$$

$\underline{I}$LATE Rule

$$e^t\cdot t - e^t = \left\{e^{e^x}\cdot e^x - e^{e^x}\right\}$$

$$e^{-2x}\left\{e^{e^x}\cdot e^x - e^{e^x}\right\} = e^{e^x}\cdot e^{-x} - e^{e^x}\cdot e^{-2x}$$

#Q. An integrating factor of the differential equation $\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx+\frac{1}{4}(x+xy^2)dy=0$ is

A. x^2

B. 3 log eX

C. x^3

D. 2 log eX

$$\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx+\frac{1}{4}(x+xy^2)\,dy=0$$

$$M\,dx+N\,dy=0$$

$$M(x,y)\,dx+N(x,y)\,dy=0$$

$$\frac{\partial M}{\partial y}=1+y^2 \qquad \frac{\partial N}{\partial x}=\frac{1}{4}(1+y^2)$$

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=\frac{3}{4}(1+y^2)$$

$$\frac{\left\{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right\}}{N}=\frac{3}{x}=f(x)$$

$$\text{I.F.}=e^{\int f(x)\cdot dx}$$

$$=e^{\int \frac{3}{x}dx}$$

$$=e^{3\ln x}=e^{\ln x^3}=\underline{\underline{x^3}}$$

#Q. Let y (x) be the solution of the differential equation

$(xy + y + e^{-x})\, dx + (x + e^{-x})\, dy = 0$

Satisfying y (0) = 1. Then, y (–1) is equal to

A $\dfrac{e}{e-1}$

B $\dfrac{2e}{e-1}$

C $\dfrac{e}{1-e}$

D 0

$$(xy+y+e^{-x})\,dx+(x+e^{-x})\,dy=0$$

$$M(x,y)\,dx+N(x,y)\,dy=0$$

$$\frac{\partial M}{\partial y}=x+1 \qquad \frac{\partial N}{\partial x}=1-e^{-x}$$

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=x+1-1+e^{-x}=x+e^{-x}$$

$$\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=1$$

$$I.F=e^{\int 1\cdot dx}=e^{x}$$

$$e^{x}(xy+y+e^{-x})dx+e^{x}(x+e^{-x})dy=0$$

$$\left(e^{x}xy+ye^{x}+1\right)dx+\left(e^{x}\cdot x+1\right)dy=0$$

Now, after multiplying the I.F we get it to be exact

$$\underbrace{\int M \cdot dx + \int N \cdot dy = 0}$$

$$\int \left(e^x xy + e^x \cdot y + 1\right) dx + \int \left(e^x \cdot x + 1\right) dy = 0$$

$$y\left(e^x \cdot x - \cancel{e^x} + \cancel{e^x} + x\right) + e^x \cdot x \cdot y + y$$

$$\boxed{xye^x + x + y = c}$$

$xye^x + x + y = c$

$y(0) = 1$ & find $y(-1) = ?$

$1 \times 0 \times e^0 + 0 + 1 = c \rightarrow \boxed{c = 1}$

$y(-1) = \quad -1 \times y \times e^{-1} + (-1) + y = 1$

$y = \dfrac{2}{1 - e^{-1}} = \dfrac{2e}{e-1}$

#Q. If $y(x) = \lambda e^{2x} + e^{\beta x}$, $\beta \neq 2$, is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$. Satisfying $\frac{dy}{dx}(0) = 5$ then y (0) equal to.

homogeneous $\longrightarrow$ C.F & N.o.P.?

as $x = 0$

A 1

B 4

C 5

D 9

$\lambda e^{2x} + e^{\beta x} \quad , \beta \neq \underset{\sim}{2}$

$D^2 + D - 6 = 0$

$m^2 + m - 6 = 0 \Rightarrow (m+3)(m-2) = 0$

$y(x) = c_1 \cdot e^{-3x} + c_2 \cdot e^{2x}$

$\boxed{c_2 = \lambda}$ & $\boxed{c_1 = 1}$ & $\boxed{\beta = -3}$

$\frac{dy}{dx}(0) = 5$ & find $y(0) = ?$

$$y(x) = c_1 e^{-3x} + c_2 \cdot e^{2x}$$

$$y'(x) = -3c_1 e^{-3x} + 2c_2 e^{2x}$$

$$y'(0) = 5 = -3c_1 + 2c_2$$

$$5 = -3 + 2c_2$$

$$\boxed{c_2 = 4}$$

$$y(x) = e^{-3x} + 4e^{2x}$$

$$y(0) = e^0 + 4 \cdot e^0$$

$$= 5$$

#Q. If $x^h y^k$ is an integrating factor of the differential equation $y(1+xy)\,dx + x(1-xy)\,dy = 0$, then the ordered pair (h, k) is equal to

A (–2, –2)

B (–2, –1)

C (–1, –2)

D (–1, –1)

$$y(1+xy)\,dx + x(1-xy)\,dy = 0$$

$x^h y^k \rightarrow$ I.F over here

$$M\,dx + N\,dy = 0 \xleftarrow{\text{I.F}}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$x^h y^{k+1}(1+xy)\,dx + x^{h+1}y^k(1-xy)\,dy = 0$$

$$\left(x^h y^{k+1} + x^{h+1}y^{k+2}\right)dx + \left(x^{h+1}y^k - x^{h+2}y^{k+1}\right)dy = 0$$

$$M\,dx + N\,dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$(k+1)x^h y^k + (k+2)x^{h+1}y^{k+1}$$

$$= (h+1)x^h y^k - (h+2)x^{h+1}y^{k+1}$$

$$\boxed{h=k}$$

$$k+2=-h+2$$

$$k+2=-k+2$$

$$\boxed{k=-2}$$

$$\boxed{h=-2,\Rightarrow k=-2}$$

#Q. The general solution of the differential equation with constant coefficients $\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ approaches zero as $x \to \infty$, if:

A. b is negative and c is positive

B. b is positive and c is negative

C. Both b and c are positive

D. Both b and c are negative

$$a=1$$

$$1.m^2+bm+c=0$$

$$D=\sqrt{b^2-4c}$$

$$\frac{-b\pm\sqrt{b^2-4c}}{2\times 1}$$

real & distinct

2

$$c_1 \cdot e^{\alpha x}+c_2 e^{\beta x}$$

$$\frac{-b\pm\sqrt{b^2-4c}}{2} \rightarrow \text{equal}$$

$$b^2-4c<0$$

$$b^2-4c>0$$

imaginary roots $\alpha \pm i\beta$

$$e^{\alpha x}(c_1 \cdot \underbrace{\cos\beta x}_{(-1,1)} + c_2 \underbrace{\sin\beta x}_{(-1,1)})$$

real

$-b \pm \sqrt{b^2-4c}$

imaginary

$-b \pm \sqrt{b^2-4c}$

equal

$-b \pm \sqrt{b^2-4c}$

$$\rightarrow C_1 \cdot e^{\left(\frac{-b+\sqrt{b^2-4c}}{2}\right)x} + C_2 e^{\left(\frac{-b-\sqrt{b^2-4c}}{2}\right)x}$$

$C.F \rightarrow 0$ as $x \rightarrow \infty$

$$\boxed{e^{c_1 x}} \rightarrow \infty = \frac{1}{e^{c_1 x}} \rightarrow 0$$

$$\left\{\frac{-b \pm \sqrt{b^2-4c}}{2}\right\} \times -ve$$

$b \rightarrow +ve$

$$-b \pm \sqrt{b^2-4c}$$

-ve

$$-b + \sqrt{b^2-4c} \rightarrow \sqrt{b^2-4c}$$

always be +ve

$$-b - \sqrt{b^2-4c}$$

$c > 0$

$c < 0$

$h > 0$ $c < 0$

#Q. Let y(x) be the solution of differential equation:
$\frac{d}{dx}\left(x\frac{dy}{dx}\right) = x; y(1) = 0, \frac{dy}{dx}|_{x=1} = 0$ Then y (2) is:

A $\frac{3}{4}+\frac{1}{2}\ln 2$

B $\frac{3}{4}-\frac{1}{2}\ln 2$

C $\frac{3}{4}+\ln 2$

D $\frac{3}{4}-\frac{1}{2}\ln 2$

$\frac{d}{dx}\left(x\frac{dy}{dx}\right) = x$ and $y(1) = 0$; $y'(1) = 0$

Integrate -

$d\left(x\frac{dy}{dx}\right) = x\,dx$

$x\frac{dy}{dx} = \frac{x^2}{2} + A = 1 \times 0 = \frac{1}{2} = A$

$\boxed{A = -1/2}$

$$x\frac{dy}{dx}=\frac{x^2}{2}+A$$

$$x\,dy=\frac{x^2}{2}dx-\frac{1}{2}dx$$

$$dy=\frac{x}{2}dx-\frac{1}{2x}dx$$

$$\boxed{y=\frac{x^2}{4}-\frac{1}{2}\ln x+B}$$

$$y(2)=1-\frac{1}{2}\ln 2-\frac{1}{4}=\boxed{\frac{3}{4}-\frac{1}{2}\ln 2}$$

$$y(1)=0$$

$$0=\frac{1}{4}-\frac{1}{2}\times 0+B$$

$$\boxed{B=-\frac{1}{4}}$$

#Q. One of the points which lies on the solution curve of the differential equation $(y - x)\,dx + (x + y)\,dy = 0$ with the condition $y(0) = 1$, is:

A $(1, -2)$

B $(2, -1)$

C $(2, 1)$

D $(-1, 2)$

$$(y-x)\,dx+(x+y)\,dy=0$$

$$\underbrace{y\,dx+x\,dy}+y\,dy-x\,dx=0$$

Integrating them we get —

$$\boxed{2xy+y^2-x^2=c}$$

$$y(0)=1$$

$$2\times 0\times 1+1-0=c$$

$$\boxed{c=1}$$

$$\cancel{2\times 2}\times 1+1-\cancel{4}=c=1$$

$$\boxed{1=1}$$

#Q. The non-zero value of the n for which the differential equation $(3xy^2 + n^2x^2y)\,dx + (nx^3 + 3x^2y)\,dy = 0,\ x \neq 0$, becomes exact is:

A) -3

B) -2

C) 2

D) 3

$$M\,dx + N\,dy = 0$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$M = 3xy^2 + n^2x^2y$$

$$N = nx^3 + 3x^2y$$

$$\frac{\partial M}{\partial y} = 6xy + n^2x^2$$

$$\frac{\partial N}{\partial x} = 3nx^2 + 6xy$$

$$n^2x^2 = 3nx^2$$

$$n^2 = 3n$$

$$n^2 - 3n = 0$$

#Q. If y(t) is a solution of the differential equation $y^n + 4y = 2e^t$, then $\lim_{t \to \infty} e^{-t}y(t)$ is equal to

A $\frac{2}{3}$

B $\frac{2}{5}$

C $\frac{2}{7}$

D $\frac{2}{9}$

$y^{(n)} + 4y = 2e^t$ where $y(t)$ is the solⁿ of the given differential eqⁿ.

$\boxed{n=1}$

$e^{-t}\, y(t)$

$\lim_{t\to\infty}$

$$\boxed{y' + 4y = 2e^t}$$

$$y' + 4y = 2e^{t}$$

$G.S = C.F + P.I$

$C.F \rightarrow$ Auxiliary eqn

$$m + 4 = 0$$

$$\boxed{m = -4}$$

$$C.F = C_1 e^{-4t}$$

$$P.I = \frac{2e^{t}}{f(D)} = \frac{2e^{t}}{D+4}$$

$$\boxed{\alpha = 1}$$

$$= \frac{2e^{t}}{5}$$

$$P.I = \frac{e^{\alpha x}}{f(D)} = \frac{e^{\alpha x}}{f(\alpha)}$$ provided that $f(\alpha) \neq 0$

$$G.S = 4e^{-4t} + \frac{2e^{t}}{5}$$

$$\lim_{t\to\infty} e^{-(t)} y(t) = e^{-t}\left\{4 e^{-4t} + \frac{2}{5}e^{t}\right\}$$

$$= \left\{4 \cdot e^{-5t} + \frac{2}{5}\right\}$$

$$\lim_{t \to \infty} \left\{ 4e^{-5t} + \frac{2}{5} \right\}$$

$$= 4 \times 0 + \frac{2}{5} = 2/5$$

#Q. Let x, $x + e^x$ and $1 + x + e^x$ be solutions of a linear second order ordinary differential equation with constant coefficients. If $y(x)$ is the solution of the same equation satisfying $y(0) = 3$ and $y'(0) = 4$, then $y(1)$ is equal to

A $e + 1$

B $2e + 3$

C $3e + 2$

D $3e + 1$

x, $x+e^x$, $1+x+e^x$ sol^n of the D.E

second order

$1, x, e^x$

r_1 r_2

$c_1 e^{r_1 x}$ $c_2 e^{r_2 x}$

$$\boxed{c_1 e^{r_1 x} + c_2 e^{r_2 x}}$$

x , $x+e^x \longrightarrow$ DE ka solⁿ hai;

iska matlab $\quad \underbrace{c_1 x + c_2(x+e^x)}$

$y(x) = c_1 x + c_2(x+e^x)$

$y(0) = 3$ & $y'(0) = 4$

$y(0) = 3 = c_1 \times 0 + c_2(0+e^0) \Rightarrow \boxed{c_2 = 3}$

$$y'(x) = c_1 + c_2(1 + e^x)$$

$$y'(0) = 4 = c_1 + c_2(1 + e^0)$$

$$4 = c_1 + c_2(1+1)$$

$$4 = c_1 + 3 \times 2 \qquad \boxed{c_1 = -2}$$

$$y(1) = c_1 \times 1 + c_2(1 + e^1) = -2 + 3(1 + e)$$

$$= \underline{\underline{3e + 1}}$$

#Q. A partial differential equation:

$$z\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xyz$$

$z \times xy$ $z = f(x,y)$

1

$\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$

A is of order 1, and is non-linear

B is of order 1, and is linear

C is of order 2, and is non-linear

D is of order 2, and is linear

#Q. The solution of homogeneous differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$ is (c being constant) :

A $x = c\sin\left(\frac{y}{x}\right)$

B $x = c\sin\left(\frac{x}{y}\right)$

C $x = c\tan\left(\frac{y}{x}\right)$

D $x = c\cot\left(\frac{y}{x}\right)$

$$\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right) = v + \tan v$$

$\frac{y}{x} = v$ Steps :- Assume $y/x = v$

Secondaly $y = xv$

Now, $\frac{dy}{dx}$, thus we will diffn $y = xv$ wrt to x

$$\frac{dy}{dx} = x \cdot \frac{dv}{dx} + \cancel{\frac{dx}{dx}}\ 1 \cdot v$$

$$\frac{dy}{dx} = x \cdot v' + v$$

$$xv' + \cancel{v} = \cancel{v} + \tan v$$

$$x\frac{dv}{dx} = \tan v$$

$$\frac{dv}{\tan v} = \frac{dx}{x}$$

Integrating —

$$\ln \sin v = \ln x + \ln a$$

$$\boxed{\sin\left(\frac{y}{x}\right) = x \cdot A}$$

#Q. The orthogonal trajectory of the family of curves $ay^2 = x^3$, where a is an arbitrary constant, is

$$\boxed{m_1 \cdot m_2 = -1}$$

A. $3y^2 + 2x^2 =$ constant

B. $2y^2 - 3x^2 =$ constant

$$\boxed{\frac{dy}{dx} \times -\frac{dx}{dy} = -1}$$

C. $3y^2 - 2x^2 =$ constant

D. $2y^2 + 3x^2 =$ constant

$$ay^2 = x^3$$

Step 1:- $a \times 2y \cdot y' = 3x^2$

Step 2:- $a = \dfrac{3x^2}{2y \cdot y'}$

Step 3:- $\dfrac{3\cancel{x^2}}{2\cancel{y} \cdot y'} \times y^{\cancel{2}} = \cancel{x^3}\ x$

Step:- $3y = 2xy'$

Step:- $3y = -2x \times \dfrac{dx}{dy}$

$$3y = -2x\frac{dx}{dy}$$

$$3y\,dy + 2x\,dx = 0$$

Integrating –

$$\frac{3y^2}{2} + \frac{2x^2}{2} = c$$

$$\boxed{3y^2 + 2x^2 = c}$$

#Q. The differential equation of a family of parabolas with foci at origin and axis along x-axis is

$y^2 = 4ax$

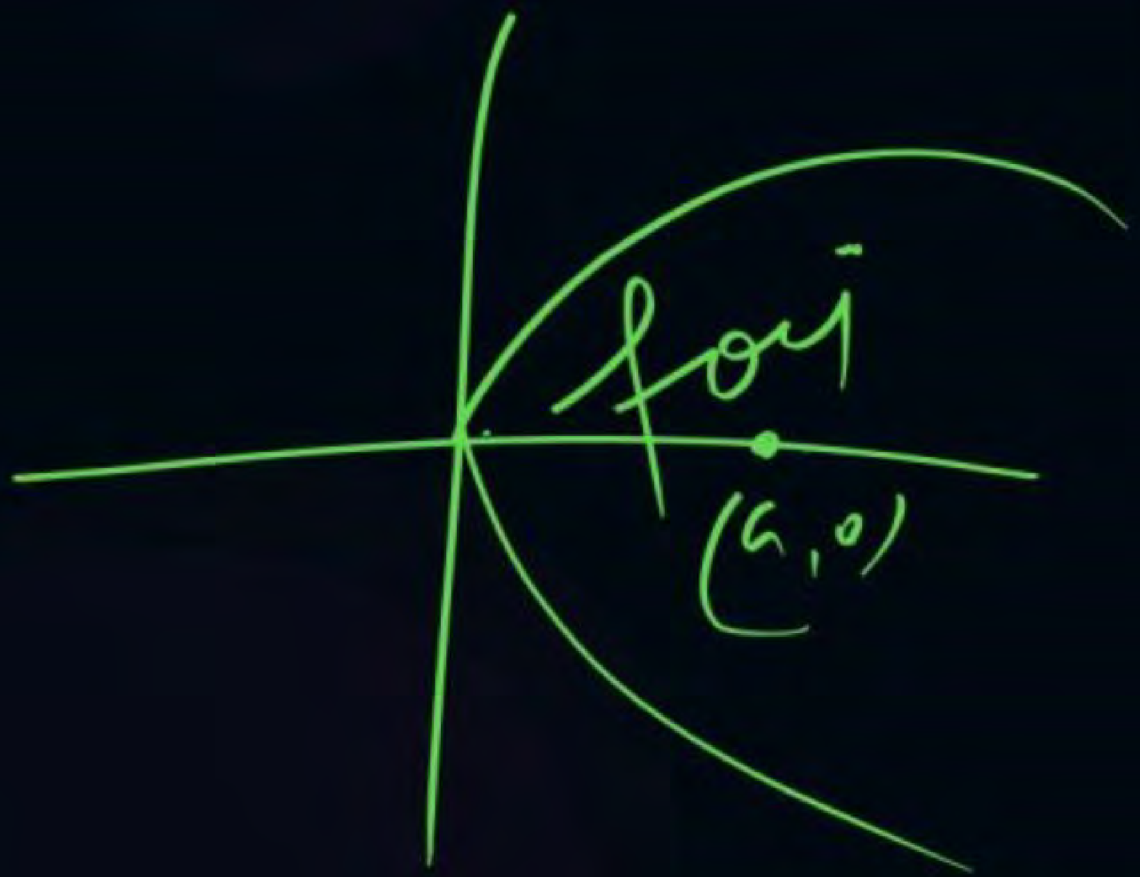
$y^2 = 4a(x+a)$

A $y\left(\frac{dy}{dx}\right)^2 + 2x^2\frac{dy}{dx} + y = 0$

B $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$

C $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} + y = 0$

D $y^2\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y^2 = 0$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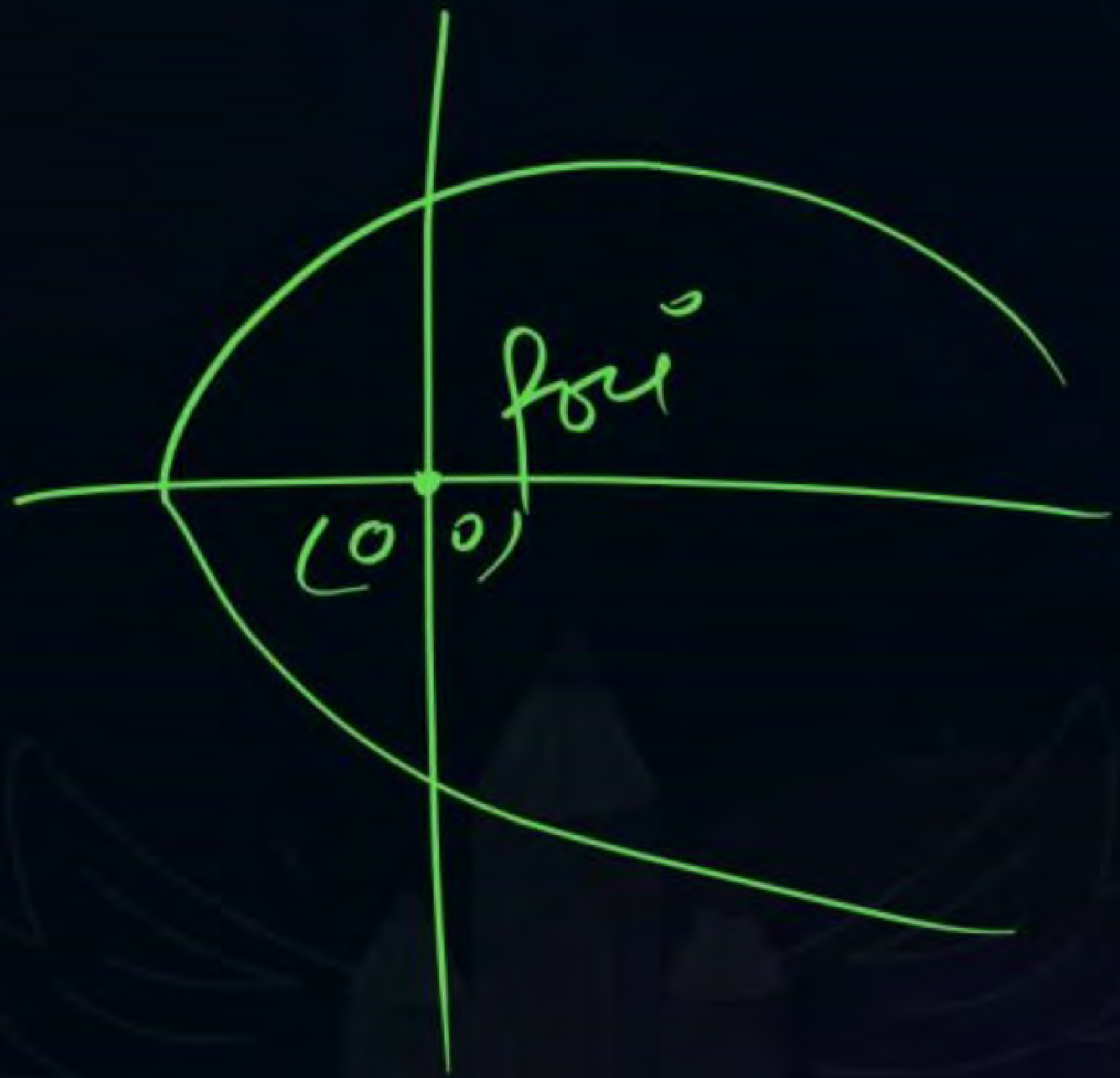
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$$y^2 = 4a(a+x)$$

$$2y\,y' = 4a(1)$$

$$4a = 2y\cdot y'$$

$$y^2 = 2y\cdot y'\left(\frac{\cancel{2}y\cdot y'}{\cancel{4}\,2} + x\right)$$

$$y = 2\cdot y'\left(\frac{y\cdot y'}{2} + x\right)$$

$$y = \cancel{2}y'\left(\frac{y\,y' + 2x}{\cancel{2}}\right)$$

$$y\cdot(y')^2 - y + 2xy' = 0$$

#Q. For $a, b, c \in \mathrm{R}$ if the differential equation $(ax^2 + bxy + y^2)\,dx + (2x^2 + cxy + y^2)\,dy = 0$ is exact, then

A $b = 2, c = 2a$

B $b = 4, c = 2$

C $b = 2, c = 4$

D $b = 2, a = 2c$

$$(ax^2+bxy+y^2)dx+(2x^2+cxy+y^2)dy=0$$

$$M(x,y)\,dx + N(x,y)\cdot dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$bx+2y = 4x+cy$$

$$b=4,\ c=2$$

#Q. The general solution of the equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{2x}$ is (c1, c2 being constants):

A $y = c_1 e^{-2x} + c_2 e^{-3x} + \frac{1}{10} e^{2x}$

B $y = c_1 e^{-2x} + c_2 e^{-3x} + \frac{1}{5} e^{2x}$

C $y = c_1 e^{-2x} + c_2 e^{-3x} + \frac{1}{20} e^{2x}$

D $y = c_1 e^{-2x} + c_2 e^{-3x} + \frac{1}{4} e^{2x}$

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{2x}$$

$G.S = P.I + C.F$

(1) $C.F \longrightarrow$ Auxiliary eqn

$= m^2 + 5m + 6 = 0$

$= m^2 + 2m + 3m + 6 = 0 \longrightarrow (m+2)(m+3) = 0$

$$C.F = c_1 e^{-2x} + c_2 e^{-3x}$$

$$P.I = \frac{e^{2x}}{f(m)} = \frac{e^{2x}}{f(D)} = \frac{e^{2x}}{D^2+5D+6} = \frac{e^{2x}}{4+10+6}$$

$$= \frac{e^{2x}}{20}$$

$$f(D)\cdot y = X = e^{\alpha x}$$

$$P.I = \frac{e^{\alpha x}}{f(\alpha)} \Rightarrow f(\alpha) \neq 0$$

#Q. The general solution of the equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 4y = 0$ is (c1, c2 being constants)

A. $y = c_1e^{-x} + c_2e^{-4x}$

B. $y = c_1e^{x} + c_2e^{-4x}$

C. $y = c_1e^{-x} + c_2e^{4x}$

D. $y = c_1e^{2x} + c_2e^{-4x}$

$G.S = C.F + P.I \rightarrow 0$ as $X = 0$

$f(D) \cdot y = X$

$X = 0$

$P.I = \dfrac{X}{f(D)} = 0$

$C.F = ??$

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 4y = 0$$

$$m^2 + 3m - 4 = 0$$

$$m^2 + 4m - m - 4 = 0$$

$$m(m+4) - 1(m+4) = 0$$

$$\boxed{m = -4, 1}$$

$$C.F = C_1 e^{\alpha_1 x} + C_2 e^{\alpha_2 x}$$

$$= C_1 e^{-4x} + C_2 e^{x}$$

#Q. The solution of the differential equation :

$$\frac{d^2y}{dx^2} + 4y = \cos 2x,$$ given by

A $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{4} \sin 2x$

B $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{2} \sin 2x$

C $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{4} \cos 2x$

D $c_1 \cos 2x + c_2 \sin 2x + x \cos 2x$

$$\frac{d^2y}{dx^2} + 4y = \cos 2x$$

$$G.S = \underbrace{C.F}_{} + \underbrace{P.I}_{} \longrightarrow \neq 0 \qquad \boxed{X \neq 0}$$

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$\boxed{m = \pm 2i}$$

$$C.F = e^{0.x}[C_1\cos 2x + C_2\sin 2x]$$

$$\alpha \pm i\beta \longrightarrow C.F \longrightarrow e^{\alpha x}[C_1\cos\beta x + C_2\sin\beta x]$$

$C.F \longrightarrow$ ✓

P.I

$$P.I = \frac{\cos 2x}{f(D)} = \frac{\cos 2x}{D^2+4} \rightarrow \times$$

$$P.I = \frac{\cos ax}{f(D^2)} = \frac{\cos ax}{f(-a^2)}$$

$$P.I = \frac{Q}{f(D^2)} \rightarrow \cos ax$$

$$= \frac{\cos ax}{f(-a^2)}$$

$$f(-a^2) = 0$$

$$f(-a^2) \neq 0$$

THANK - YOU