All Branches — Hinglish

Subject: Engineering Mathematics

DPP-01

Chapter : Complex Analysis

1. [MCQ]
The value of the integral $\oint_C \frac{e^2 \sin(z)}{z^2} dz$, where the contour C is the unit circle: $|z - 2| = 1$, is
(a) $2\pi i$ (b) $4\pi i$
(c) πi (d) 0

2. [MCQ]
Which of the following statements is TRUE for the function $f(z) = \frac{z \sin z}{(z-\pi)^2}$?
(a) $f(z)$ is analytic everywhere in the complex plane
(b) $f(z)$ has zero at $z = \pi$
(c) $f(z)$ has a pole of order at $z = \pi$
(d) $f(z)$ has a simple pole at $z = \pi$

3. [MCQ]
Consider a counter clockwise circular contour $|z| = 1$ about the origin. Let $f(z) = \frac{z \sin z}{(z-\pi)^2}$, then the integral $\oint f(z)dz$ over this contour is
(a) $-i\pi$ (b) zero
(c) $i\pi$ (d) $2i\pi$

4. [NAT]
For the function $f(z) = \frac{16z}{(z+3)(z-1)^2}$, the residue at the pole $z = 1$ is (your answer should be an integer) _______.

5. [MCQ]
The value of the integral $\oint_C \frac{z^2}{e^z + 1} dz$, where C is the circle $|z| = 4$, is
(a) $2\pi i$ (b) $2\pi^2 i$
(c) $4\pi^3 i$ (d) $4\pi^2 i$

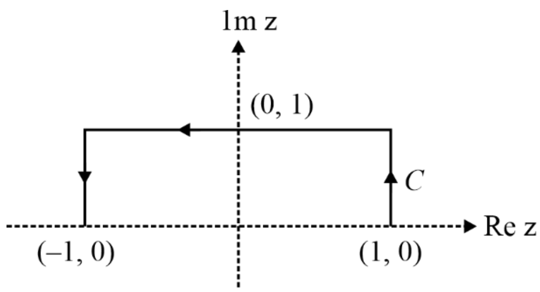
6. [MCQ]
Consider a complex function $f(z) = \frac{1}{z\left(z + \frac{1}{2}\right)\cos(zx)}$. Which one of the following statements is correct?
(a) $f(z)$ has simple poles at $z = 0$ and $z = -\frac{1}{2}$
(b) $f(z)$ has second order pole at $z = -\frac{1}{2}$
(c) $f(z)$ has infinite number of second order poles
(d) $f(z)$ has all simple poles

7. [MCQ]
Consider $w = f(z) = u(x, y) + iv(x, y)$ to be an analytic function in a domain D. Which one of the following options is NOT correct?
(a) $u(x, y)$ satisfies Laplace equation in D
(b) $v(x, y)$ satisfies Laplace equation in D
(c) $\int_{z_1}^{z_2} f(z)dz$ is dependent on the choice of the contour between z_1 and z_2 in D
(d) $f(z)$ can be Taylor expended in D

8. [NAT]
The contour integral $\oint \frac{dz}{1+z^2}$ evaluated along a contour going from $-\infty$ to $+\infty$ along the real axis and closed in the lower half-plane circle is equal to ……… (up to two decimal places).

9. [NAT]
The imaginary part of an analytic complex function is $v(x, y) = 2xy + 3y$. The real part of the function is zero at the origin. The value of the real part of the function at $1 + i$ is ……… (up to two decimal places).

10. [MCQ]
The $\int_c dz\, z^2 e^z$, where C is an open contour in the complex z-plane as shown in the figure below, is:



(a) $\frac{5}{e}+e$ (b) $e-\frac{5}{e}$

(c) $\frac{5}{e}-e$ (d) $-\frac{5}{e}-e$

11. [MCQ]

Which of the following is an analytic function of the complex variable $z = x + iy$ in the domain $|z| < 2$?

(a) $(3 + x - iy)^7$

(b) $(1 + x + iy)^4 (7 - x - iy)^3$

(c) $(1 - x - iy)^4 (7 - x + iy)^3$

(d) $(x + iy - 1)^{1/2}$

12. [MCQ]

Let $u(x, y) = x + \frac{1}{2}(x^2 - y^2)$ be the real part of analytic function $f(z)$ of the complex variable $z = x + iy$. The imaginary part of $f(z)$ is

(a) $y + xy$ (b) xy

(c) y (d) $y^2 - x^2$

13. [MCQ]

The value of the integral $\int_c \frac{z^3 dz}{(z^2 - 5z + 6)}$, where C is a closed contour defined by the equation $2|z| - 5 = 0$, traversed in the anti-clockwise direction, is

(a) $-16\pi i$ (b) $16\pi i$

(c) $8\pi i$ (d) $2\pi i$

14. [MCQ]

With $z = x + iy$, which of the following functions $f(x, y)$ is NOT a (complex) analytic function of z?

(a) $f(x, y) = (x + iy - 8)^3 (4 + x^2 - y^2 + 2ixy)^7$

(b) $f(x, y) = (x + iy)^7 (1 - x - iy)^3$

(c) $f(x, y) = (x^2 - y^2 + 2ixy - 3)^5$

(d) $f(x, y) = (1 - x + iy)^4 (2 + x + iy)^6$

15. [MCQ]

Which of the following functions cannot be the real part of a complex analytic function of $z = x + iy$?

(a) $x^2 y$ (b) $x^2 - y^2$

(c) $x^3 - 3xy^2$ (d) $3x^2y - y - y^3$

16. [MCQ]

Given that the integral $\int_0^\infty \frac{dx}{y^2 + x^2} = \frac{\pi}{2y}$, the value of $\int_0^\infty \frac{dx}{(y^2 + x^2)^2}$ is

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$

(c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

17. [MCQ]

If C is the contour defined by $|z| = \frac{1}{2}$, the value of integral $\oint_c \frac{dz}{\sin^2 z}$ is

(a) ∞ (b) $2\pi i$

(c) 0 (d) πi

18. [MCQ]

The principal value of the integral $\int_{-\infty}^{\infty} \frac{\sin(2x)}{x^3} dx$ is

(a) -2π (b) $-\pi$

(c) π (d) 2π

19. [MCQ]

The value of integral $\int_{-\infty}^{\infty} \frac{dx}{1 + x^4}$

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2}$

(c) $\sqrt{2}\pi$ (d) 2π

20. [MCQ]

The function $\dfrac{z}{\sin \pi z^2}$ of a complex variable z has

(a) a simple pole at 0 and poles of order 2 at $\pm\sqrt{n}$ for $n = 1, 2, 3 \ldots$

(b) a simple pole at 0 and poles of order 2 at $\pm\sqrt{n}$ and $\pm i\sqrt{n}$ for $n = 1, 2, 3 \ldots$

(c) poles of order 2 at $\pm\sqrt{n}$, $n = 0, 1, 2, 3 \ldots$

(d) poles of order 2 at $\pm n$, $n = 0, 1, 2, 3 \ldots$

21. [MCQ]

The radius of convergence of the Taylor series expansion of the function $\dfrac{1}{\cosh(x)}$ around $x = 0$, is

(a) ∞ (b) π

(c) $\dfrac{\pi}{2}$ (d) 1

22. [MCQ]

The value of the contour integral $\dfrac{1}{2\pi i}\oint\limits_C \dfrac{e^{4z}-1}{\cosh(z)-2\sinh(z)}dz$ around the unit circle C transversed in the anti-clockwise direction, is

(a) 0 (b) 2

(c) $\dfrac{-8}{\sqrt{3}}$ (d) $-\tanh\left(\dfrac{1}{2}\right)$

23. [MCQ]

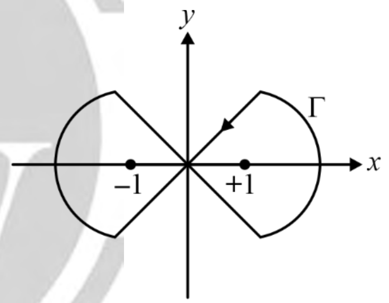
Let $u(x, y) = e^{ax}\cos(by)$ be the real part of a function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$, where a, b are real constants and $a \neq 0$. The function $f(z)$ is complex analytic everywhere in the complex plane if and only if

(a) $b = 0$ (b) $b = \pm a$

(c) $b = \pm 2\pi a$ (d) $b = a \pm 2\pi$

24. [MCQ]

The integral $\oint\limits_\Gamma \dfrac{ze^{i\pi z\backslash 2}}{z^2-1}dz$ along the closed contour Γ shown in the figure is



(a) 0 (b) 2π

(c) -2π (d) $4\pi i$

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Answer Key

1. (d)
2. (c)
3. (b)
4. (3)
5. (c)
6. (a)
7. (c)
8. (π)
9. (3)
10. (c)
11. (b)
12. (a)
13. (a)
14. (d)
15. (a)
16. (b)
17. (c)
18. (a)
19. (a)
20. (b)
21. (c)
22. (c)
23. (b)
24. (c)