CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 14

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TOPICS TO BE COVERED

01 Assignment Questions

02 GATE Questions

03 Doubts

Q.1

Number of prefixes of "n" length string is ___
(assume all the symbols in given string are different)

A. n
B. n+1
C. n+2
D. n-1

$w = \boxed{abc} = n = 3$

ε, a, ab, abc $\Rightarrow 4 = n+1$

Q.2

((ab)* $\varnothing$) is equivalent to

A. (ab)*
B. $\varnothing$
C. (a+b)*
D. None

$(ab)^* + \phi = (ab)^*$

$(ab)^* \cdot \phi = \phi$

Q.3

$(\varnothing^* \cup (bb^*)) =$

A. Epsilon
B. $\varnothing$
C. b^+
D. b^*

$\varepsilon + b^+ = b^*$

$\boxed{\phi^* = \varepsilon}$

$\boxed{\phi^+ = \phi}$

$\boxed{\phi^0 = \varepsilon}$

Q.4

Which of the following is TRUE?

a, aba, ababa, ...

a, aba, ababa, ...

A. $(ab)^*a = a(ba)^*$

B. $(aa)^*b = a(ab)^*$

b   a

C. $(ba)^*a = b(aa)^*$

a   b

D. All of the above

Q.5

OR operator in regular expression satisfies

A. Associative

B. Commutative

C. both A and B

D. Neither A nor B

Q.6

Concatenation operator in regular expression satisfies

A. Associative

B. Commutative

C. both A and B

D. Neither A nor B

Q.7

Which of the following distribution is valid in regular expressions?

A. OR over CONCATENATION
B. CONCATENATION over OR
C. Both A and B
D. Neither A nor B

$\cancel{a+(b.c)} =$

$a\cdot(b+c) = ab+ac$

Q.8

Match the following groups over Sigma={a,b}.

Group-1:

1. OR identity → b
2. OR dominator → c
3. Concatenation identity → a
4. Concatenation dominator → b

Group-2:

a. Epsilon

b. Empty Expression

c. (a+b)*

d. (aa)*

A. 1-a, 2-b, 3-c, 4-d

B. 1-b, 2-c, 3-a, 4-b

C. 1-d, 2-a, 3-b, 4-c

D. None

Q.9 If R1= a*, and R2=(aa)* then R1.R2=____

A. R1
B. R2.R1
C. R1+R2
D. All of these

$$a^* . (aa)^* = a^*$$

$$a^* \quad \varepsilon \rightarrow \boxed{a^*}$$

$$a^* . aa$$

$$a^* . a^4$$

Q.10

If R1= a(aa)*, and R2=(aa)* then R1+R2=____

A. R1

B. R2.R1

C. R1*

D. None

$R_1 + R_2 = a(aa)^* \cup (aa)^*$

$= a^*$

$R_2 . R_1 = (aa)^* . a(aa)^* \to a, a^3, a^5, \ldots$

$\to R_1$

$R_1^* = [a(aa)^*]^* = \{\varepsilon, a, a^2, \ldots\}$

$= a^*$

Home Work :

Find minimum string (shortest length string) in the following Expressions.

1) $\underbrace{(ab)^+}_{ab}\underbrace{aaa}_{aaa}\underbrace{(b+ab)}_{b} \longrightarrow$ min = abaaab

2) $\underbrace{(ab)^*}_{\varepsilon}\underbrace{aaa}_{aaa}\underbrace{(b+\varepsilon)}_{\varepsilon} \longrightarrow aaa$

3) $\underbrace{(a+ab)^+}_{a}\underbrace{(bb+aaa)^+}_{bb}\underbrace{(ab)^*}_{\varepsilon} \longrightarrow abb$

4) $\underbrace{(ab)^+ + \varepsilon}_{\varepsilon} \longrightarrow \varepsilon$

5) $\underbrace{(ab)(a+b)^*aaa} + \underbrace{ba}_{ba} \longrightarrow ba$

6) $\left[\underbrace{\underbrace{(ab^*+a^++ba)}_{a}\underline{ab}}_{aab}\right]^+ \cdot \underbrace{(a+b)^*}_{\varepsilon} \longrightarrow aab$

7) $\left[\underbrace{((ab)^+a)^*}_{\varepsilon}\,\underline{aaa}\right]^+ \longrightarrow aaa$

8) $\left[((aba)^+aba)^+ab\right]^* \longrightarrow \varepsilon$

9) $(a^*)^+ \rightarrow \varepsilon$

10) $(\underbrace{(ab)^*}_{\varepsilon}\underbrace{a^+}_{a})^+ \cdot \underbrace{b}_{b}\underbrace{b^+}_{b}\underbrace{a}_{a}\underbrace{a^*}_{\varepsilon} \rightarrow abba$

11) $[\underbrace{(ab)^+}_{ab} \cdot \underbrace{aba}_{aba}]^+ \cdot \underbrace{aaa}_{aaa} \rightarrow ababaaaa$

12) $(\underline{a}\,\underline{a}\,\underbrace{a^*}_{\varepsilon}\,\underline{a}\,\underline{b}\,\underbrace{a^*}_{\varepsilon})^+ \rightarrow aaab$

13) $(\underline{a}+aa+aaa)^+ \rightarrow a$

14) $\underbrace{a^+}_{a}\underbrace{b^+}_{b} \rightarrow ab$

15) $\underbrace{a^*}_{\varepsilon}\underbrace{b^*}_{\varepsilon} \rightarrow \varepsilon$

16) $\underbrace{a^*}_{\varepsilon}\underbrace{b^*}_{\varepsilon}\underbrace{c^+}_{c}\underbrace{d^*}_{\varepsilon} \rightarrow c$

17) $(a+\underbrace{b^*}_{\varepsilon}+c^+)^+ \rightarrow \varepsilon$

18) $(a+b+\underline{\varepsilon})^+ \rightarrow \varepsilon$

$(a+\varepsilon)^3$ $\Sigma = \{a\}$

1) $L = \{w \in \{a, b\}^* : \underbrace{\#_a(w) \le 3}_{\text{atmost } 3}\}$.

A. $b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^*$

B. $b^* (\underline{a}) b^* (\underline{a \cup} \varepsilon) b^* (\underline{a \cup} \varepsilon) b^* \rightarrow \varepsilon \times$

C. $b^* (\underline{a}) b^* (\underline{a}) b^* (\underline{a \cup \varepsilon}) b^* \rightarrow \varepsilon \times$

D. $b^* (a \cup \varepsilon) b^* (\underline{a}) b^* (\underline{a}) b^* \rightarrow \varepsilon \times$

$1 \le \#_a(w) \le 3$

$2 \le \#_a(w) \le 3$

2

$a \cup b$

$a + b$

$a | b$

$b^* (a+\varepsilon) b^* (a+\varepsilon) b^* (a+\varepsilon) b^*$

$\Sigma = \{a, b\}$

2) $L = \{w \in \{a, b\}^* : \underbrace{\#_a(w) >= 3}_{\text{Atleast 3}}\}$.

A. $b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* \rightarrow \varepsilon \rightarrow \#a(w) \leq 3$

B. $(a \cup b)^* (a) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* \rightarrow a \rightarrow \#a(w) \geq 1$

C. $b^* (a) b^* (a) b^* (a \cup \varepsilon) b^* \rightarrow aa \rightarrow 2 \leq \#a(w) \leq 3$

D. $(a \cup b)^* a (a \cup b)^* a (a \cup b)^* a (a \cup b)^*$ ✓

X a X a X a X

3) $L1 = a^*b^*$

$L2 = a^+b^+$

Find L2-L1.

A. a^*

B. b^*

C. $a^* + b^*$

D. None

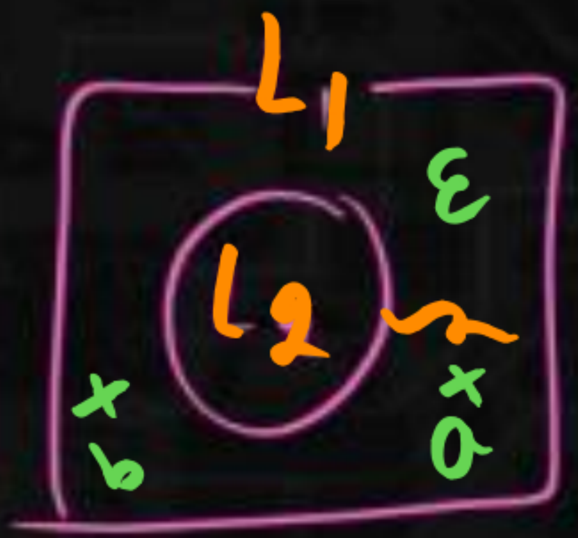
$L_2 \subset L_1$

$L_2 - L_1 = \phi$

$$L_2 - L_1 = a^+b^+ - a^*b^*$$

$$= \phi$$

$a^*b^* \rightarrow \underbrace{a^+b^+}_{L_2},\ a^+,\ b^+,\ \epsilon$



$$\boxed{L_2 \subset L_1}$$

Note: $L_1 - L_2 = a^*b^* - a^+b^+ = \underbrace{a^+ + b^+ + \epsilon} = a^* + b^*$

$a^*b^* \rightarrow a^+b^+,\ a^+,\ b^+,\ \epsilon$

$$a^*b^* \rightarrow \begin{cases} \varepsilon \\ a^+ \\ b^+ \\ a^+b^+ \end{cases}$$

$$a^+b^+$$

If $X \subset Y$ then

i) $X \cup Y = Y$

ii) $X \cap Y = X$

4) $L1 = a^*+b^*$ and $L2 = a^*b^*$.
Which of the following is TRUE?

A. $L1 = L2$ $\boxed{L_1 \subset L_2}$
B. $L1 \cup L2 = (a+b)^*$ $\boxed{L_1 \cup L_2 = L_2}$
C. $L1^* = L2^*$ $= (a+b)^*$
D. None

$L_1^* = (a^*+b^*)^* = (a+b)^*$

$L_2^* = (a^*b^*)^* = (a+b)^*$

$L_1 = a^* + b^*$

$L_2 = a^* b^*$

$\underbrace{\varepsilon \quad a^+ \quad b^+}_{L_1} \quad a^+b^+$

5) $L1 = a^*+b^*$ and $L2 = a^*b^*$.

Which of the following is TRUE?

A. L1 is subset of L2

B. L2 is subset of L1

C. L1 U L2 = L1

D. None

$L_1 \subset L_2$

$L_1 \cup L_2 = L_2$

$L_1 \cap L_2 = L_1$

$L_1 \subset L_2$

$L_2 \supset L_1$

$L_1 \cup L_2 = L_2$

$(a+b)^* = (a^+ + b^+)^*$

6) $L1 = a^+b^+$ and $L2 = a^*b^*$.

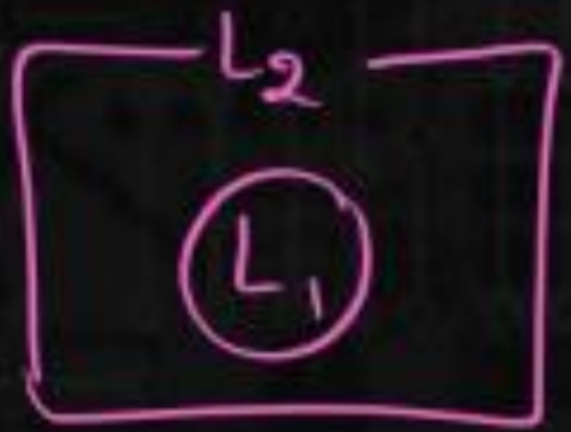
Which of the following is FALSE?

A. L1 is subset of L2 T

B. $L1^* = L2^*$ False

C. L1 U L2 = L2 T

D. None



$L_1 \subset L_2$

✗ $a \leftarrow L_1^* = (a^+b^+)^*$

✓ $a \leftarrow L_2^* = (a^*b^*)^*$

$\epsilon, \cancel{a}, \cancel{b}, ab, \ldots$

$(a^+b^+)^* \neq (a^*b^*)^* = (a+b)^*$

$(a^+ + b^+)^+ = (a+b)^+$

7) $L1 = a^+ + b^+$ and $L2 = a^* + b^*$.

Which of the following is TRUE?

$(a+b)^+$

A. $L1 = L2$ ✗

$(a+b)^*$

B. $\boxed{L1^+} = \boxed{L2^+}$ ✗

C. $L1 \cup L2 = L2$ ✓

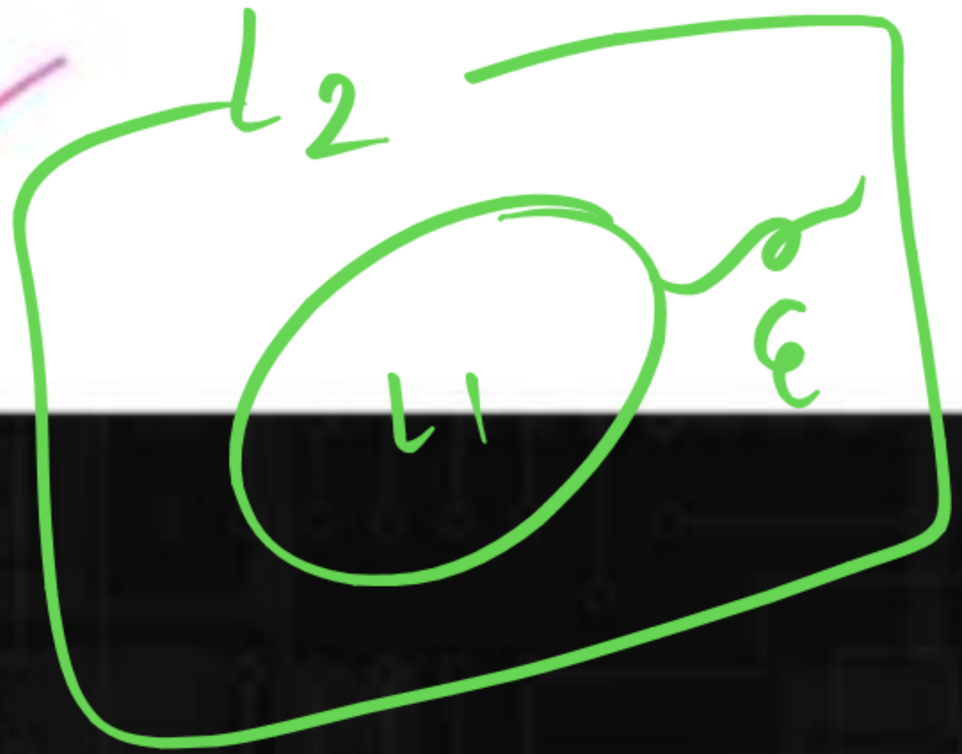
D. None

$L_1 + \varepsilon = L_2$

$L_1^+ = (a^+ + b^+)^+ = (a+b)^+$

$L_2^+ = (a^* + b^*)^+ = (a+b)^*$

$L_1 \subset L_2$



$L_2 = L_1 \cup \{\varepsilon\}$

8) $L1 = a^+$ and $L2 = a^*$

Which of the following is TRUE?

A. $\boxed{L1+} = L2^*$ $\qquad (a^+)^+ \neq (a^*)^*$

B. $L1^+ = L2^+$ $\qquad (a^+)^+ \neq (a^*)^+$

C. $L1^* = L2^+$ $\qquad (a^+)^* = (a^*)^+$

D. None

$a^+ \quad a^* \quad a^* \quad a^* \quad a^*$

i) $a^+ a^+ = aa^+ = aaa^*$

ii) $a^* . a^* = a^*$

iii) $a^+ + a^* = a^*$

iv) $(a^+)^+ = a^+$

v) $(a^+)^* = a^*$

vi) $(a^*)^+ = a^*$

vii) $(a^*)^* = a^*$

$$(a^+)^+ = (a^+)^1 \cup (a^+)^2 \cup (a^+)^3 \cup \cdots$$

$$= a^+ \cup a^+a^+ \cup a^+a^+a^+ \cup \cdots$$

$$= a^+$$

9) $A = a^*$ and $B = b^*$

$AB = ?$

A. $\{ a^n b^n \mid n>=0\}$

B. $\{ a^m b^n \mid m, n>=0\}$

C. $(a+b)^*$

D. None

$A = a^* = \{a^n \mid n \geq 0\}$

$B = b^* = \{b^n \mid n \geq 0\}$

$A.B = a^* b^*$

$= \{a^{n_1} b^{n_2} \mid n_1, n_2 \geq 0\}$

$\neq \{a^n b^n \mid n \geq 0\}$

$a^* = \{a^n \mid n \geq 0\}$

$b^* = \{b^n \mid n \geq 0\}$

$a^* . b^* \to \varepsilon$

$\to a$

$\{a^n\} \{b^n\}$

$\{a^{n_1} b^{n_2}\}$

$a^n b^n \neq a^* b^*$

$\{a^n\} = \{a^n \mid n \geq 0\}$

$$a^n b^n \Rightarrow \{\varepsilon, ab, a^2b^2, a^3b^3, \dots\}$$

$$a^* b^* \Rightarrow \{\varepsilon, a, aa, aaa, \dots, b, bb, bbb, \dots$$

$$aab, abb, \dots\}$$

$$a^* b^* = a^{n_1} b^{n_2} = a^m b^n$$

$$(ab)^* \neq a^n b^n$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \varepsilon & & \varepsilon \\ ab & & ab \\ \underline{ab}\,\underline{ab} & & a^2b^2 = aabb \\ \vdots & & \vdots \end{array}$$

10) $A = aa^*$ and $B = bb^*$

$(A \cup B)^* = ?$

A. $\{ a^n b^n \mid n >= 0 \}$

B. $\{ a^m b^n \mid m, n >= 0 \}$

C. $(a+b)^*$ ✓

D. None

$A = a^+$

$B = b^+$

$(A+B)^*$

$= (a^+ + b^+)^*$

$= (a+b)^*$

$a^* a = a a^* = a^+$

$b b^* = b^* b = b^+$

$(A \cup B)^* = (a^+ + b^+)^*$

$= (a+b)^*$

11) Given the language L = {ab, aa, baa}, which of the following strings are not in L*?

1) abaabaaabaa $\in L^5$
2) aaaabaaaa $\in L^4$
3) baaaaabaaaab $\notin L^*$

A. 1 only
B. 2 only
C. 3 only
D. None

$L = ab + aa + baa$

$L^* = (ab + aa + baa)^*$

1) ab aa baa ab aa $\in L^5$

$L = \{ab, aa, baa\}$

$L^* = (ab + aa + baa)^*$

2) aa aa baa aa

3) baa aaa baa aa b $\notin L^*$

12) The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is ______.

$a^*(ba)^*a^*$

ε ✓

a ✓

b ✗

A. 1 ✓

B. 2

C. 3

D. 4

$a^*(ba)^*a^*$

ε ✓ $a^0(ba)^0a^0$

a ✓

$\boxed{b}$ → 1 length

aa

ab

ba

bb

⋮

starts with a

13) $L = a(a+b)^*$ is equivalent to ________

A. $(ab^*)^+$

B. $(a^+b^*)^+$

C. $a^*(ab^*)^+$

D. All of the above

$= a(a+b)^*$

$a(a+b)^*$

$(ab^*)^+$

$(a^+b^*)^+$

14) $L = (a+b)^*b$ is equivalent to ________

~~A~~. $(ab^*)^+ \rightarrow a$

~~B~~. $(a^+b^*)^+ \rightarrow a$

C. $\boxed{b^*(ab^*)^*}\,b \rightarrow (a+b)^*$.

D. None

Note:

i) $b^*(ab^*)^*$

ii) $a^*(ba^*)^*$

iii) $(b^*a)^*b^*$

iv) $(a^*b)^*a^*$

$\Big\} = (a+b)^*$

$$(a+b)^*b = (a^*b)^+$$

$$= (a^*b^+)^+ = (a^*b)^+b^*$$

***

$$= (a+b)^*$$

$$\boxed{\begin{aligned} &= (b^*a)^* b^* \\ &= (a^*b)^* a^* \\ &= b^*(ab^*)^* \\ &= a^*(ba^*)^* \end{aligned}}$$

Imp forms

15) $(b + ba)(b + a)^*(ab + b)$ $=$ $b(\varepsilon + a)\,X\,(a+\varepsilon)b$

$$\underbrace{b(\varepsilon + a)\,X\,(a+\varepsilon)b}_{X}$$

$$\boxed{b\,X\,b}$$

A. $(a+b)^*$

B. $a(a+b)^*a$

C. ~~C.~~ $b(a+b)^*b$

D. None

$(a+b)^*(\varepsilon + \text{Any}) = (a+b)^*$

I) $a(a+b)^*a \longrightarrow$ starts and ends with 'a' & min 2 length

II) $b(a+b)^*b$

III) $a + a(a+b)^*a \Rightarrow$ starts & ends with 'a'

IV) $b + b(a+b)^*b$

$$(a+b)^* \cdot (\varepsilon + \underline{Any}) = (a+b)^*$$

$$(a+b)^* \cdot \underbrace{(ab)^*}_{\varepsilon + ab + abab} = (a+b)^*$$

16) $\{w \in \{a, b\}^* : \#_a(w) \equiv_3 0\}$.

A. $(b^*ab^*ab^*a)^*b^*$ ✓

B. $(b^*ab^*ab^*a)^*$ → b ✗

C. $(ab^*ab^*a)^*$ → b ✗

D. $(ab^*ab^*a)^*b^*$ → baaa ✗

$b^*(b^*ab^*ab^*ab^*)^*b^*$

17) $(a \cup b)^* \underbrace{(a \cup \varepsilon)}_{\varepsilon} \underbrace{b^*}_{\varepsilon} =$

$(a+b)^* \underbrace{(a+\varepsilon) b^*}_{\circ} = (a+b)^*$

A. $(a+b)(a+b)^* = (a+b)^+$

B. $(a+b)^*$

C. $(aa+b)^* \rightarrow \cancel{a}$

D. None

$(a+b)^* \cdot \varepsilon \cdot \varepsilon$

18) $L = \{w \in \{a, b\}^* \mid w$ has bba as a substring$\}$
Which of the following describes L ?

A. $(a \cup b)^*\, bba\, (a \cup b)^*$
B. $(a \cup b)^+\, bba\, (a \cup b)^*$
C. $(a \cup b)^+\, bba\, (a \cup b)^+$
D. $(a \cup b)^*\, bba\, (a \cup b)^+$

A = X bba X

~~bba~~

~~bba~~

~~bba~~

bba
valid
but

B C A
A
C
C A
C
D

19) $L = \{w \in \{a, b\}^*\}$ $= (a+b)^*$

same

1. $(a + b)^*$
2. $(a + b + \text{epsilon})^+$
3. Epsilon $+ (a + b)^+$
4. $(a^*b^*)^*$
5. $(b^*a^*)^*$

a $\leftarrow$ 6. $(a^+b^+)^*$ $\neq (a+b)^*$

How many of above are equivalent to given L ?

A. 4 B. 5 ✓ C. 6 D. 3

a^+b^+

$\rightarrow$ a ✗

$\rightarrow$ b ✗

$\rightarrow$ ab ✓

$\rightarrow$ aab ✓

$\rightarrow$ abb ✓

(1) Which Two of the following four regular expressions are equivalent?

(i) $(00)^*(\varepsilon + 0) = 0^*$ (ii) $\underbrace{(00)^*}_{\text{Even 0's}}$ (iii) $\underbrace{0^*}_{\text{All 0's}}$ (iv) $\underbrace{0\,(00)^*}_{\text{odd 0's}}$

ε, 0, 00, 000, 0^4, ...

(a) (i) and (ii) (b) (ii) and (iii)

(c) (i) and (iii) (d) (iii) and (iv)

(GATE - 96)

$$(00)^*(\varepsilon+0) = 0^*$$

ε, 0, 0^2, 0^3, 0^4

I) $\left((00)^*\right)^* = (00)^*$

II) $\left(0(00)^*\right)^* = 0^*$

(2) If the regular set A is represented by $A = (01+1)^*$ and the regular set 'B' is represented by $B = ((01)^*1^*)^*$, which of the following is true? **(GATE - 98)**

(a) $A \subset B$

(b) $B \subset A$

(c) A and B are incomparable

(d) $A = B$

$01 = x$

$1 = y$

$A = (x+y)^*$

$B = (x^* y^*)^*$

$\boxed{A = B}$

$(01+1)^* = (x+y)^*$

$((01)^* 1^*)^* = (x^* y^*)^*$

3 The string 1101 does not belong to the set represented by (GATE - 98)

(a) $110^*(0+1)$ ← $110^1(1)$

(b) $1(0+1)^*101$ ← $1()^0 101$

(c) $(10)^*(01)^*(00+11)^*$

$1101 \notin$

(d) $(00+(11)^*0)^*$ → $110110 \neq 1101$

$1101 \notin$

substring

string

1101 is string $\notin L$

$1101 \in (a)$

$1101 \in (b)$

(4) Let S and T be languages over $\sum = \{a, b\}$ represented by the regular expressions $(a + b^*)^*$ and $(a + b)^*$, respectively. Which of the following is true? **(GATE - 2000)**

(a) $S \subset T$ (b) $T \subset S$

(c) $S = T$ (d) $S \cap T = \phi$

$S \subseteq T$ ✓ $(a+b^*)^* = (a+b)^*$

$S \supseteq T$ ✓

$T \subseteq S$ ✓ $(a^*+b)^* = (a+b)^*$

× $S \subset T$

✓ $S \subseteq T$

× $S \supset T$

✓ $S \supseteq T$

✗ $S \cap T = \phi$

✗ $S \cup T = \phi$

✓ $S \cap T = T$

✓ $S = T$

✓ $S \cup T = S$

✓ $S \cup T = T$

✓ $S \cap T = S$

(5) H.W.

Consider the set Σ^* of all strings over the alphabet $\Sigma = \{0, 1\}$. Σ^* with the concatenation operator for strings **(GATE - 03)**

(a) Does not form a group

(b) Forms a non-commutative group

(c) Does not have a right identity element

(d) Forms a group if the empty string is removed from Σ^*

H.W. : Group ?

(In discrete maths)

Group Theory

closure ?

Associative ?

Identity ?

Inverse ?

Commutative ?

Group Theory

- Closure
- Associative
- Identity
- Inverse
- Commutative

Semigroup

Monoid

Group

Abelian

6) The regular expression $\underbrace{0^*(10^*)^*}_{=(0+1)^*}$ denotes the same set as **(GATE - 03)**

(a) $(1^*0)^*1^* = (0+1)^*$

(b) $0 + (0+10)^*$

(c) $(0+1)^*10\,(0+1)^*$

(d) None of the above

$(1^*0)^*1^*$

- $\to (1^*0)^+$ ends with 0
- $\to \varepsilon \leftarrow (\ )^0 1^0$
- $\to (1^*0)^* 1^+$ ends with 1

$0^*(10^*)^*$

- $(10^*)^+$ starts with 1
- $\varepsilon = 0^0(\ )^0$
- $0^+(10^*)^*$ starts with 0

$= (0+1)^*$

$(01^*)^+ \rightarrow 0X$

$(10^*)^+ \longrightarrow 1X$

$(1^*0)^+ \longrightarrow X0$

$(0^*1)^+ \longrightarrow X1$

$\underbrace{(01^*)^*}_{\text{Not starting with 1}} = 0X + \varepsilon$

$\underbrace{(10^*)^*}_{\text{not starting with 0}} = 1X + \varepsilon$

$\underbrace{(1^*0)^*}_{\text{not ending with 1}} = X0 + \varepsilon$

$\underbrace{(0^*1)^*}_{\text{not ending with 0}} = X1 + \varepsilon$

$$L = 0(0+1)^*$$

$$\overline{L} = \varepsilon + 1(0+1)^* = \underbrace{(10^*)^*}_{\text{Not starts with '0'}}$$

$$L \cup \overline{L} = \Sigma^*$$

$$L \cap \overline{L} = \phi$$

$$1^* \underbrace{(01^*)^*}_{\text{Not starting with 1}} = (0+1)^*$$

$$1^0 \longrightarrow \text{Not starting with 1}$$

$$1^+ \longrightarrow \text{starts with 1}$$

$$\left.\begin{array}{l} 1^*(01^*)^* \\ 0^*(10^*)^* \\ (1^*0)^*1^* \\ (0^*1)^*0^* \end{array}\right\} = (0+1)^*$$

$$0X + 1X + \varepsilon = X$$

$$X0 + X1 + \varepsilon = X$$

$$\boxed{X \text{ is } \Sigma^*}$$

7) Which one of the following languages over the alphabet {0, 1} is described by the regular expression $(0+1)^*0(0+1)^*0(0+1)^*$ **(GATE - 09)**

X 0 X 0 X

(a) The set of all strings containing the substring 00 $\Rightarrow$ X00X

(b) The set of all strings containing at most two 0's $\Rightarrow 1^*(0+\epsilon)1^*(0+\epsilon)1^*$

(c) The set of all strings containing at least two 0's

(d) The set of all strings that begin and end with either 0 or 1 $= (0+1)^+$

Except ε

8) Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$? **(GATE - 13)**

(a) $\{\varepsilon\}$ ✓ $\underbrace{\phi . L_2^*}_{} \cup \phi^*$ (b) ϕ

(c) a^* $\phi \cup \{\varepsilon\}$ (d) $\{\varepsilon, a\}$

$$L_1 L_2^* \cup L_1^*$$

$$\underbrace{\phi . a^*}_{} \cup \phi^*$$

$$\phi \cup \{\varepsilon\} \Rightarrow \{\varepsilon\}$$

$$\boxed{\phi^* = \varepsilon}$$

$$\boxed{\begin{array}{l} \phi^* = \phi^0 = \varepsilon \\ \phi^+ = \phi \end{array}}$$

(9) The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is ___3___.

$$a^*b^*(ba)^*a^*$$

(GATE – 14-SET3)

- ε ✓
- a ✓
- b ✓
- aa ✓
- ab ✓
- ba ✓
- bb ✓

- aaa ✓
- aab ✓
- aba ✓
- abb ✓
- baa ✓
- $\boxed{\text{bab}}$ ✗ → 3 length is shortest string not generated
- bba
- bbb

10. Which one of the following regular expressions represents the language: the set of all binary strings having $\underbrace{\text{two consecutive 0's}}_{00}$ and $\underbrace{\text{two consecutive 1's}}_{11}$? **(GATE – 16 – SET1)**

(a) $(0+1)^*\ \underbrace{0011}_{\text{contain } 0011}(0+1)^* + (0+1)^*\ \underbrace{1100}_{\text{contain } 1100}(0+1)^*$

(b) $(0+1)^*\ (00(0+1)^*\ 11 + 11(0+1)^*\ 00)(0+1)^*$

(c) $(0+1)^*\ 00(0+1)^* + (0+1)^*\ 11(0+1)^*$

OR

(d) $00(0+1)^*\ 11 + 11(0+1)^*\ 00$

$X00X11X + X11X00X$

$X[00X11 + 11X00]X$

contain 00 and 11

11)

Let $r = 1(1+0)^*$, $s = 11^*0$ and $t = 1^*0$ be three regular expressions. Which one of the following is true? **(GATE - 91)**

(a) $L(s) \subseteq L(r)$ and $L(s) \subseteq L(t)$

(b) $L(r) \not\subseteq L(s)$ and $L(s) \subseteq L(t)$

(c) $L(s) \subseteq L(t)$ and $L(s) \subseteq L(r)$

(d) $L(t) \not\subseteq L(s)$ and $L(s) \subseteq L(r)$.

$r = 1(1+0)^*$

$s = 1^+0$

$t = 1^*0 = 0 + 1^+0$

$s \subseteq r$

$s \subseteq t$

$s \subset t$

$\Downarrow$

$s \subseteq t$

12) Which of the following regular expression identities are true?

(GATE - 92)

(a) ~~$r(^*) = r^*$~~

(b) $(r^*s^*)^* = (r+s)^*$

(c) $(r+s)^* = r^* + s^*$

(d) $r^*s^* = r^* + s^*$

r^*

$(r)^*$

$r^{(*)}$

not reg exp

not wff

$$X = r + s$$

$$Y = r^* s^*$$

$$Z = s^* r^*$$

$$X^* = Y^* = Z^*$$

$$X \neq Y \neq Z$$

13) Which one of the following regular expressions represents the set of all binary strings with an odd number of $1'$s?

A. $((0+1)^*1(0+1)^*1)^*10^*$ → 10

B. $(0^*10^*10^*)^*0^*1$ → 01

C. $10^*(0^*10^*10^*)^*$

D. $(0^*10^*10^*)^*10^*$ → 0

$\in$ ⊂ Actual

GATE 2020

$L = \{1, 01, 10, 100, 010, 001, 111, \ldots\}$

$= 0^*(0^*1\,0^*\,1\,0^*)^*\,0^*\,1\,0^*$

14) Which one of the following regular expressions over {0, 1} denotes the set of all strings **not** containing 100 as a substring?

(GATE - 97)

(a) $0^*(1+0)^*$ 1 ✗

(b) ~~$0^* 1010^*$~~ ε

(c) $0^*1^*01^*$ ⊂ Actual

ε ✗

(d) ~~$0^* (10+1)^*$~~ = Actual

L = {ε, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 101, 110, 111, 0000, 0001, . . .}

Model-6:

H.W.

(5) $L = b^* a^*$

(6) $L = b^+ a^*$

(7) $L = b^* a^+$

(8) $L = b^* a^*$

$\Sigma = \{a, b\}$

(9) $L = a^+ b^+ c^+$

(10) $L = b^+ a^+ c^+$

(11) $L = c^+ a^+ b^+$

$\Sigma = \{a, b, c\}$

Model-7:

(10) $\{a^m b^n \mid m = \text{even}, n = \text{odd}\}$

Model-8:

(3) $\{\omega \mid$ '' $, \; n_a(\omega) \leq 1, \; n_b(\omega) = 2\}$

(4) $\{\omega \mid$ '' $, \; n_a(\omega) \leq 1, \; n_b(\omega) \leq 2\}$

(5) $\{\omega \mid$ '' $, \; n_a(\omega) \geq 1, \; n_b(\omega) \leq 2\}$

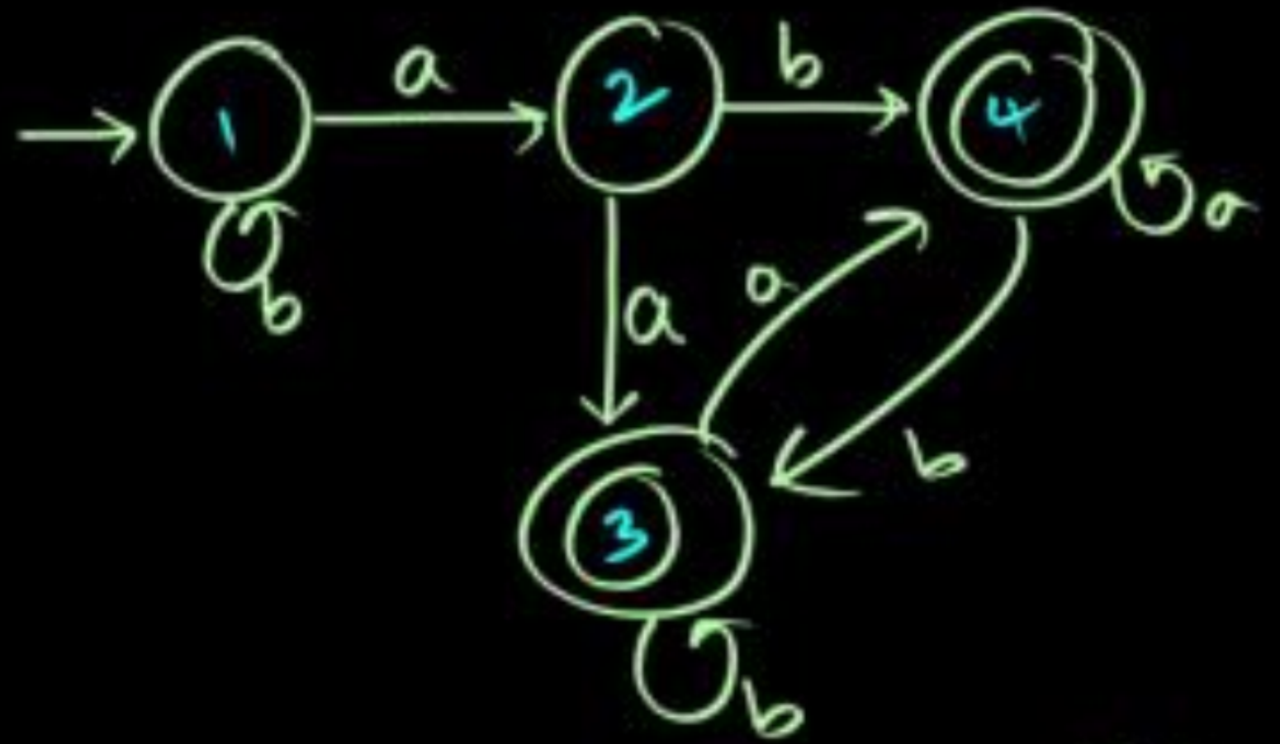
Model-12:

(4) $L = \{ \omega \mid$ " , 2^{nd} symbol is 'a' AND 4^{th} symbol is 'b' $\}$

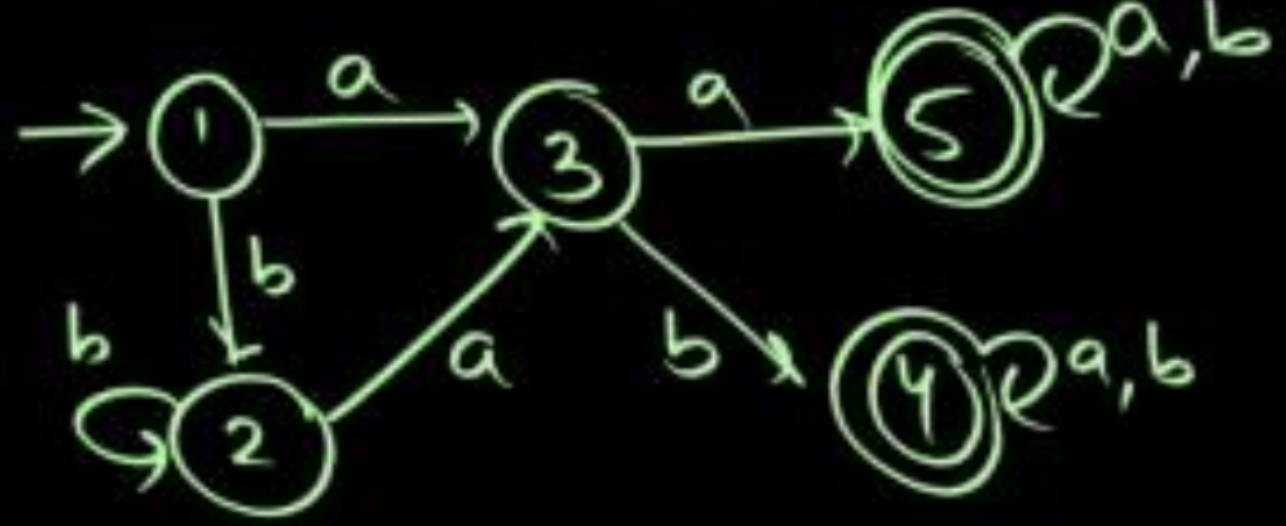
(5) $L = \{ w \mid$ " , 2^{nd} symbol is 'a' OR 4^{th} symbol is 'b' $\}$

DFA Minimization

H.W. 1)



2)

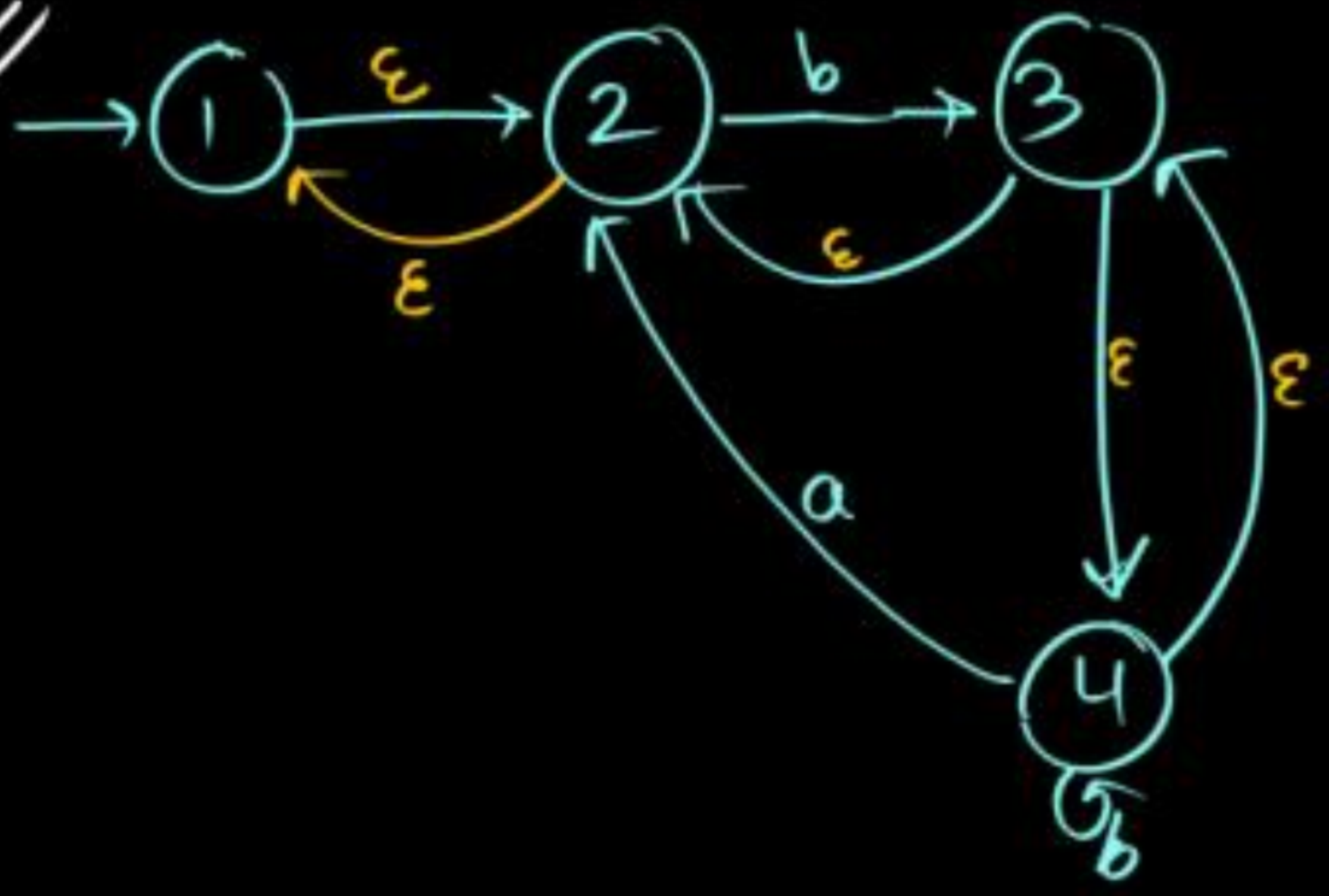


Model-13:

prefix ③ $\{w \mid w \in \{0,1\}^*$, every prefix P of w satisfies $|n_0(P) - n_1(P)| \leq 1\}$

$\{\epsilon, 0, 1, \cancel{00}, 01, 10, \cancel{11}, \ldots\}$

H.W.



$\hat{\delta}(1, aab) =$

$\hat{\delta}(2, bb) =$

$\hat{\delta}(3, bab) =$

$\hat{\delta}(4, ab) =$

Let L be the language represented by the regular expression $\Sigma^*0011\Sigma^*$ where $\Sigma=\{0,1\}$. What is the minimum number of states in a DFA that recognizes $\overline{L}$ (complement of L)?

(GATE – 15 – SET3)

(a) 4

(b) 5

(c) 6

(d) 8

Consider the language L given by the regular expression $(a+b)^*b(a+b)$ over the alphabet $\{a, b\}$. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is_______.   **(GATE – 17 – SET1)**

The minimum possible number of states of a deterministic finite automaton that accepts the regular language **(GATE – 17 – SET2)** $L=\{w_1aw_2 \mid w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is ________.

The lexical analysis for a modern computer language such as java needs the power of which one of the following machine models in a necessary and sufficient sense? **(GATE - 11)**

(a) Finite state automata

(b) Deterministic pushdown automata

(c) Non –deterministic pushdown automata

(d) Turing machine

The number of substrings (of all lengths inclusive) that can be formed from a character string of length n is **(GATE – 89 & 94)**

(a) n

(b) n^2

(c) $\frac{n(n-1)}{2}$

(d) $\frac{n(n+1)}{2}+1$

Consider the following language.

$L = \{x \in \{a, b\}^* | \text{number of } a\text{'s in } x \text{ divisible by } 2 \text{ but not divisible by } 3\}$

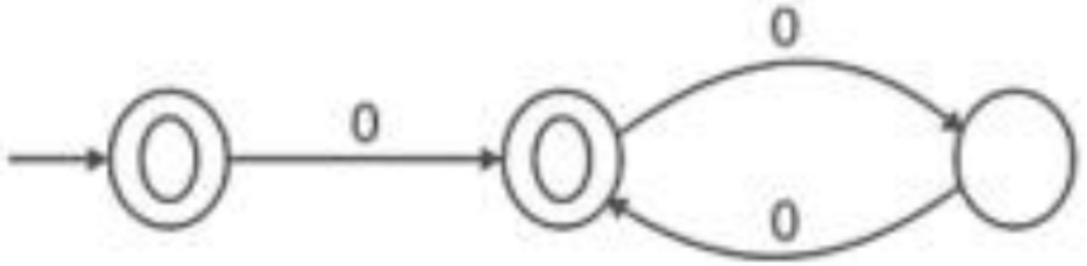
The minimum number of states in DFA that accepts L is ________

Given a language L, define L^i as follows:

$L^0 = \varepsilon$

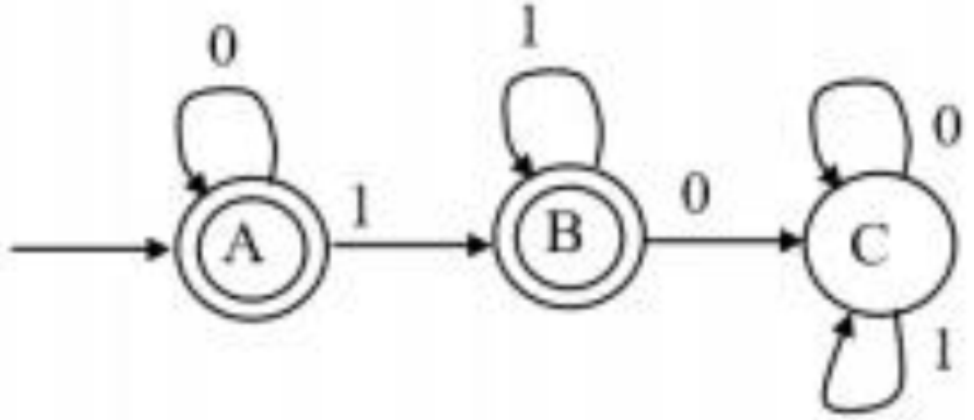
$L^i = L^{i-1} \cdot L \; For \; all \; i > 0$

The order of a language is defined as the smallest such that L^k = L^{k+1}.Consider the language L_1 (over alphabet 0) accepted by the following automaton:



The order of L_1 is ________

The regular expression for the language recognized by the finite state automation of the below figure is _____ **(GATE - 94)**



Let L be the set of all binary strings whose last two symbols are the same .The number of states in the minimum state deterministic finite-state automaton accepting L is

(GATE - 98)

What can be said about a regular language L over {a} whose minimal finite state automaton has two states? **(GATE - 2000)**

(a) L must be $\{a^n \mid n \text{ is odd}\}$

(b) L must be $\{a^n \mid n \text{ is even}\}$

(c) L must be $\{a^n \mid n \geq 0\}$

(d) Either L must be $\{a^n \mid n \text{ is odd}\}$ or L must be $\{a^n \mid n \text{ is even}\}$

Consider a DFA over $\sum = \{a, b\}$ accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of states that the DFA will have?

(GATE - 01)

(a) 8 (b) 14

(c) 15 (d) 48

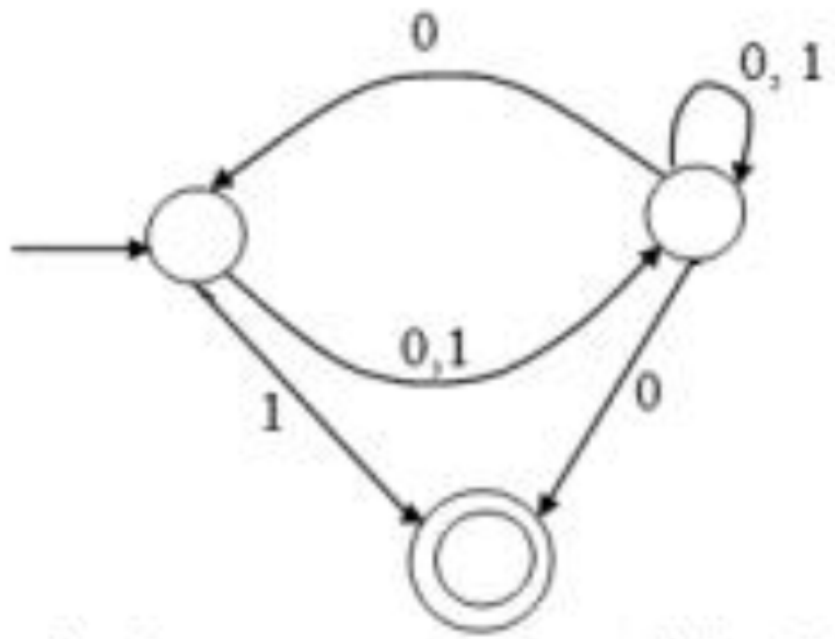
The smallest finite automation which accepts the language

$L = \{x \mid \text{length of } x \text{ is divisible by } 3\}$ has

(GATE - 02)

(a) 2 states (b) 3 states
(c) 4 states (d) 5states

Consider the NFA M shown below.



Let the language accepted by M be L. Let L_1 be the language accepted by the NFA, M_1 obtained by changing the accepting state of M to a non-accepting state and by changing the non-accepting state of M to accepting states. Which of the following statements is true?

(a) $L_1 = \{0, 1\}^* - L$

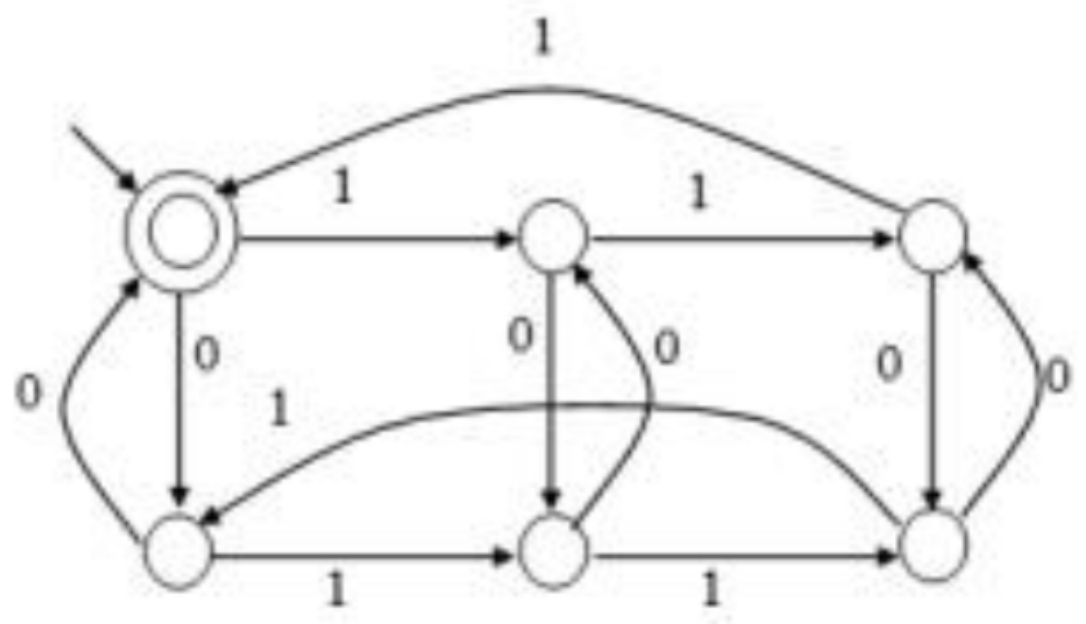
(b) $L_1 = \{0, 1\}^*$

(c) $L_1 \subseteq L$

(d) $L_1 = L$

The following finite state machine accepts all those binary strings in which the number of 1's and 0's are respectively

(GATE - 04)



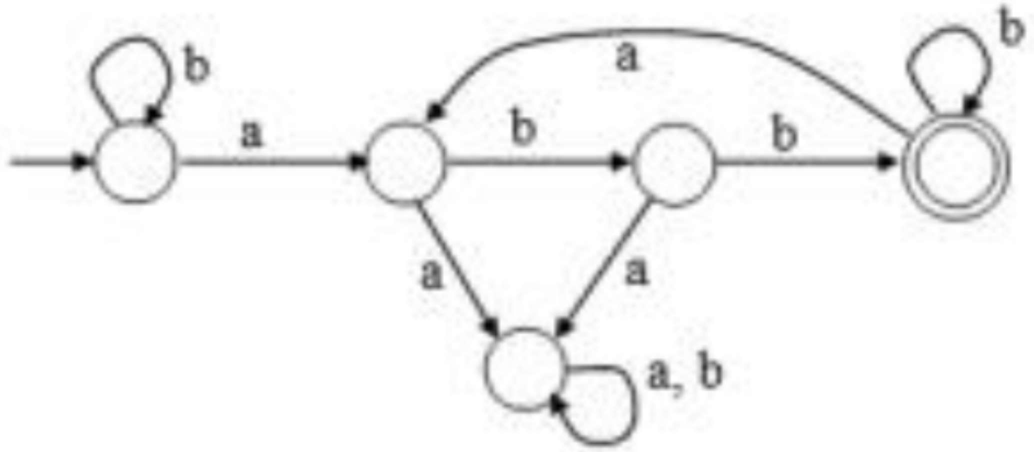
(a) Divisible by 3 and 2

(b) Odd and even

(c) Even and odd

(d) Divisible by 2 and 3

Consider the machine M:



The language recognized by M is:

(GATE - 05)

(a) $\{w \in \{a,b\}^* \mid$ every a in w is followed by exactly two b's$\}$

(b) $\{w \in \{a,b\}^* \mid$ every a in w is followed by at least two b's$\}$

(c) $\{w \in \{a,b\}^* \mid$ w contains the substring 'abb'$\}$

(d) $\{w \in \{a,b\}^* \mid$ w does not contain 'aa' as a substring$\}$

If s is a string over $(0+1)^*$ then let $n_0(s)$ denote the number of 0's in s and $n_1(s)$ the number of 1's in s. Which one of the following languages is not regular? **(GATE - 06)**

(a) $L = \{s \in (0+1)^* \mid n_0(s) \text{ is a 3-digit prime}\}$

(b) $L = \{s \in (0+1)^* \mid \text{for every prefix } s' \text{ of } s, |n_0(s') - n_1(s')| \leq 2\}$

(c) $L = \{s \in (0+1)^* \mid |n_0(s) - n_1(s)| \leq 4\}$

(d) $L = \{s \in (0+1)^* \mid n_0(s) \bmod 7 = n_1(s) \bmod 5 = 0\}$

Consider the regular language

$L = (111+11111)^*$. The minimum number of states in any DFA accepting this language is

(GATE - 06)

(a) 3 (b) 5 (c) 8 (d) 9

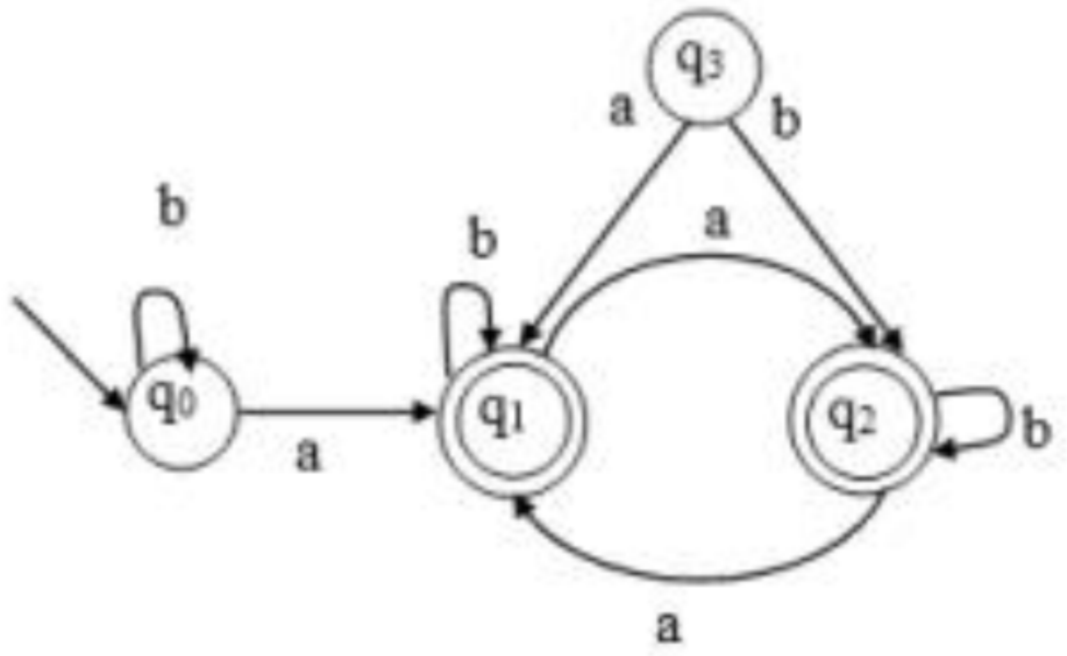
A minimum state deterministic finite automaton accepting the language

$L = \{w \mid w \in \{0, 1\}^*$, number of 0's and 1's in w are divisible by 3 and 5, respectively$\}$ has **(GATE - 07)**

(a) 15 states
(b) 11 states
(c) 10 states
(d) 9 states

Consider the following finite state automaton

(GATE - 07)



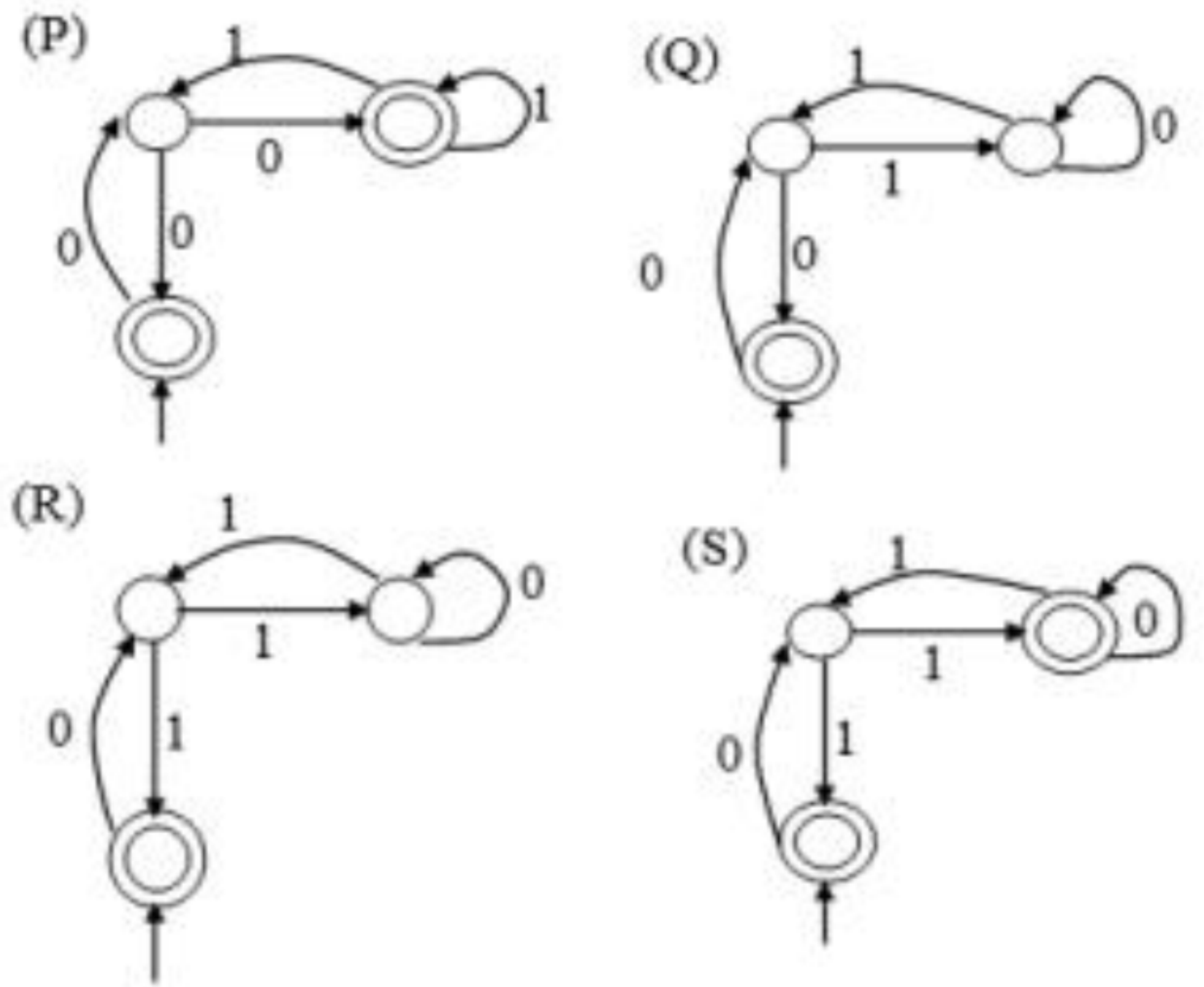
The language accepted by this automaton is given by the regular expression

(a) $b^*ab^*ab^*ab^*$ (b) $(a+b)^*$

(c) $b^*a(a+b)^*$ (d) $b^*ab^*ab^*$

Match the following NFAs with the regular expressions they correspond to

(GATE - 08)



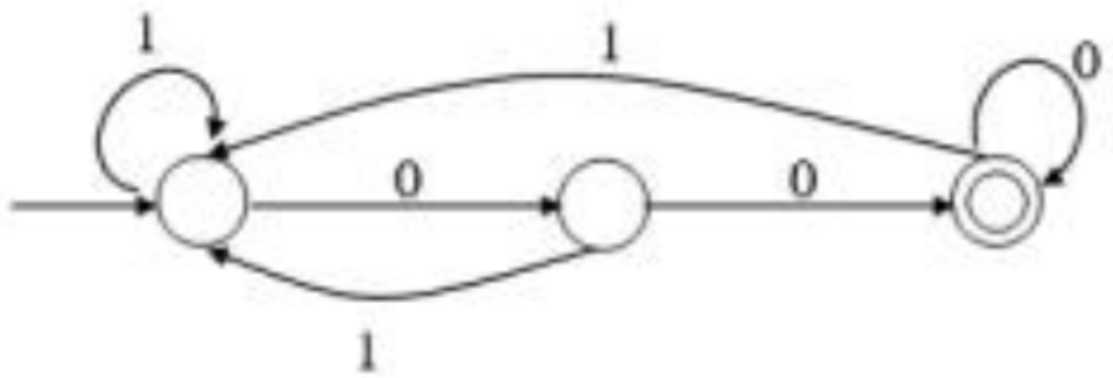
1. $\varepsilon + 0(01^*1 + 00)^*01^*$
2. $\varepsilon + 0(10^*1 + 00)^*0$
3. $\varepsilon + 0(10^*1 + 10)^*1$
4. $\varepsilon + 0(10^*1 + 10)^*10^*$

(a) P-2, Q-1, R-3, S-4

(b) P-1, Q-3, R-2, S-4

(c) P-1, Q-2, R-3, S-4

(d) P-3, Q-2, R-1,S-4



The above DFA accepts the set of all strings over {0, 1} that

(GATE - 09)

(a) Begins either with 0 or 1

(b) End with 0

(c) End with 00

(d) Contains the substring 00.

Let $L=\{w \in (0+1)^* \mid w$ has even number of 1's$\}$, i.e L is the set of all bit strings with even number of 1's. Which one of the regular expressions below represents L?

(GATE - 10)

(a) $(0^*10^*1)^*$

(b) $0^*(10^*10^*)^*$

(c) $0^*(10^*1)^*0^*$

(d) $0^*1(10^*1)^*10^*$

Let w be any string of length n in $\{0, 1\}^*$. Let L be the set of all substrings of w. What is the minimum number of states in a non-deterministic finite automation that accepts L?

(GATE-10)

(a) $n - 1$
(b) n
(c) $n+1$
(d) 2^{n-1}

Definition of the language L with alphabet {a} is given as following.

$L = \{a^{nk} | k > 0$, and n is a positive integer constant$\}$

What is the minimum number of states needed in a DFA to recognize L?

(GATE - 11)

(a) $k + 1$

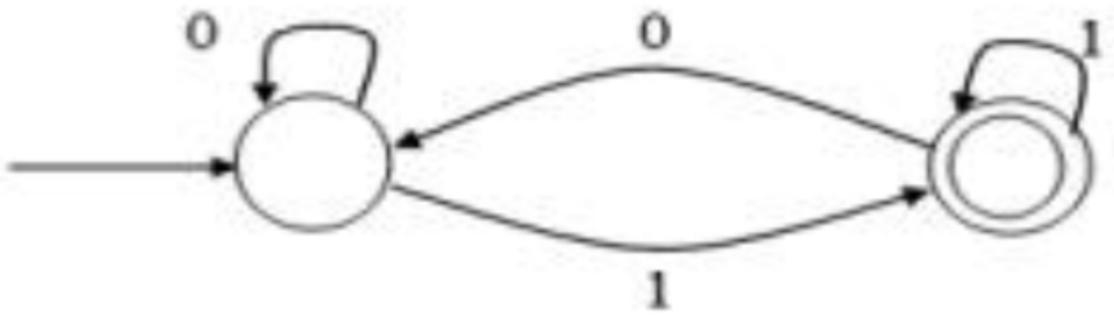
(b) $n + 1$

(c) 2^{n+1}

(d) 2^{k+1}

Which of the regular expression given below represent the following DFA?

(GATE – 14-SET1)

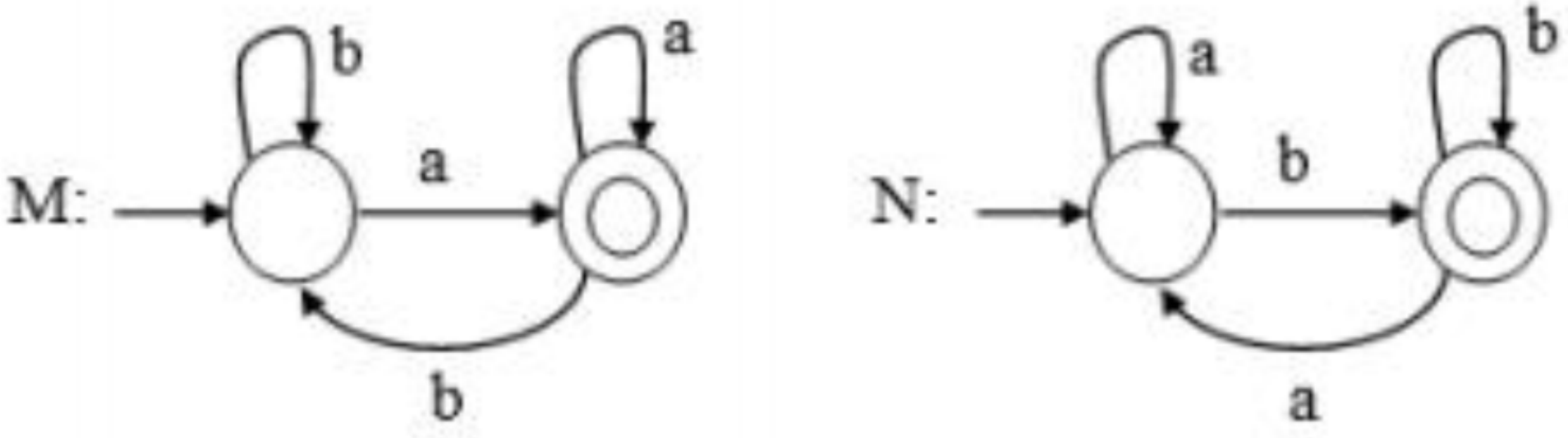


I. 0*1(1+00*1)*

II. 0*1*1+11*0*1

III. (0+1)*1

(a) I and II only
(b) I and III only
(c) II and III only
(d) I, II, and III



Consider the DFAs M and N given above. The number of states in a minimal DFA that accepts the languages $L(M) \cap L(N)$ is ________.   (GATE – 15 – SET1)

Consider the alphabet $\Sigma = \{0, 1\}$, the null/empty string λ and the set of strings X_0, X_1, and X_2 generated by the corresponding non-terminals of a regular grammar X_0, X_1, and X_2 are related as follows.

$$X_0 = 1\, X_1$$
$$X_1 = 0\, X_1 + 1\, X_2$$
$$X_2 = 0\, X_1 + \{\lambda\}$$

Which one of the following choices precisely represents the strings in X_0?

(GATE – 15- SET2)

(a) 10(0*+(10)*)1

(b) 10(0*+(10*))*1

(c) 1(0+10)*1

(d) 10(0+10)*1+110(0+10)*1

Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the ε–NFA whose transition table is give below:

δ	ε	a	b
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	ϕ	ϕ
q_3	ϕ	ϕ	$\{q_2\}$

Then $\hat{\delta}(q_2, aba)$ is **(GATE – 17 – SET2)**

(a) ϕ

(b) $\{q_0, q_1, q_3\}$

(c) $\{q_0, q_1, q_2\}$

(d) $\{q_0, q_2, q_3\}$

Let $L = (a+b)(a+b)(a+b)^*$ over alphabet $\Sigma = \{a, b\}$. **[NAT]**

If x is total number of final states and y is total number of non-final states, then $|x-y|$ = ________.

If x and y are number of equivalence classes for L1 and L2 respectively then L1+L2 = ____0.

L1 = aa(a+b)*

L2 = (a+b)* aa

GATE 1998

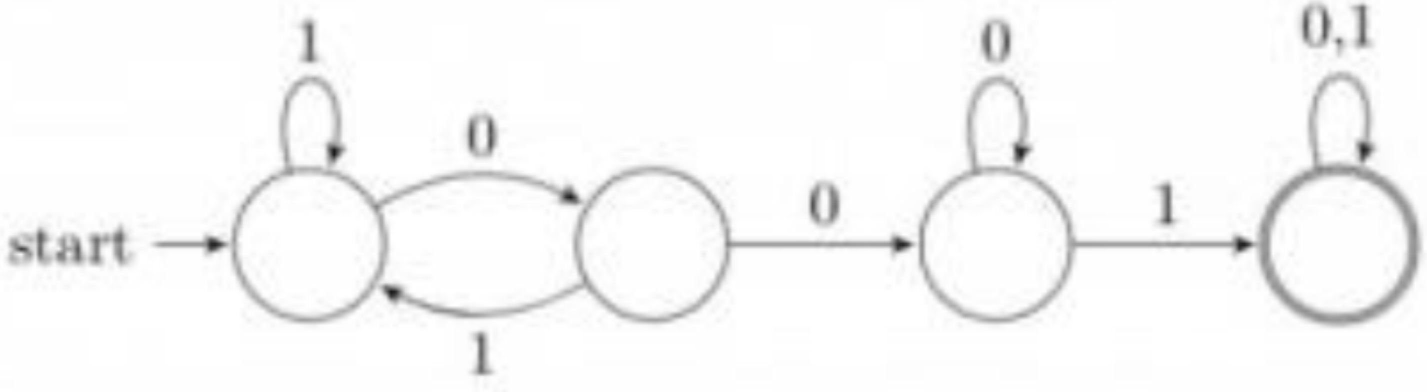
Which of the following set can be recognized by a Deterministic Finite state Automaton?

A. The numbers $1,2,4,8,\ldots 2^n,\ldots$ written in binary
B. The numbers $1,2,4,8,\ldots 2^n,\ldots$ written in unary
C. The set of binary string in which the number of zeros is the same as the number of ones.
D. The set $\{1,101,11011,1110111,\ldots\}$

Consider a DFA over $\Sigma=\{a,b\}$ accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of states that the DFA will have?

A. 8
B. 14
C. 15
D. 48

Consider the following deterministic finite state automaton M.



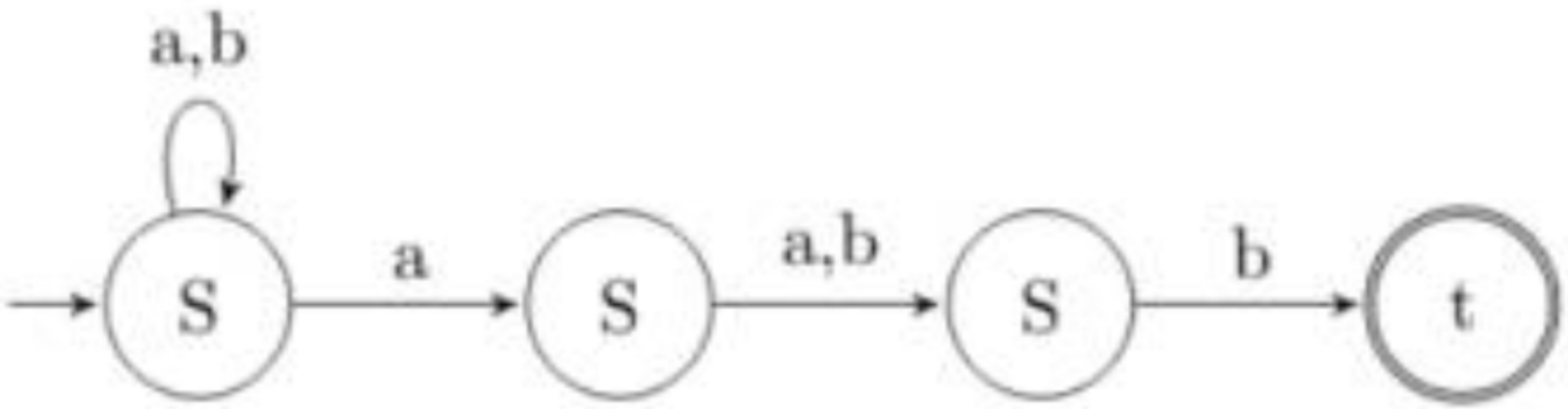
Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings in S that are accepted by M is

A. 1
B. 5
C. 7
D. 8

Which one of the following regular expressions is NOT equivalent to the regular expression $(a+b+c)^*$?

A. $(a^*+b^*+c^*)^*$
B. $(a^*b^*c^*)^*$
C. $((ab)^*+c^*)^*$
D. $(a^*b^*+c^*)^*$

Which regular expression best describes the language accepted by the non-deterministic automaton below?



A. $(a+b)^* a(a+b)b$

B. $(abb)^*$

C. $(a+b)^* a(a+b)^* b(a+b)^*$

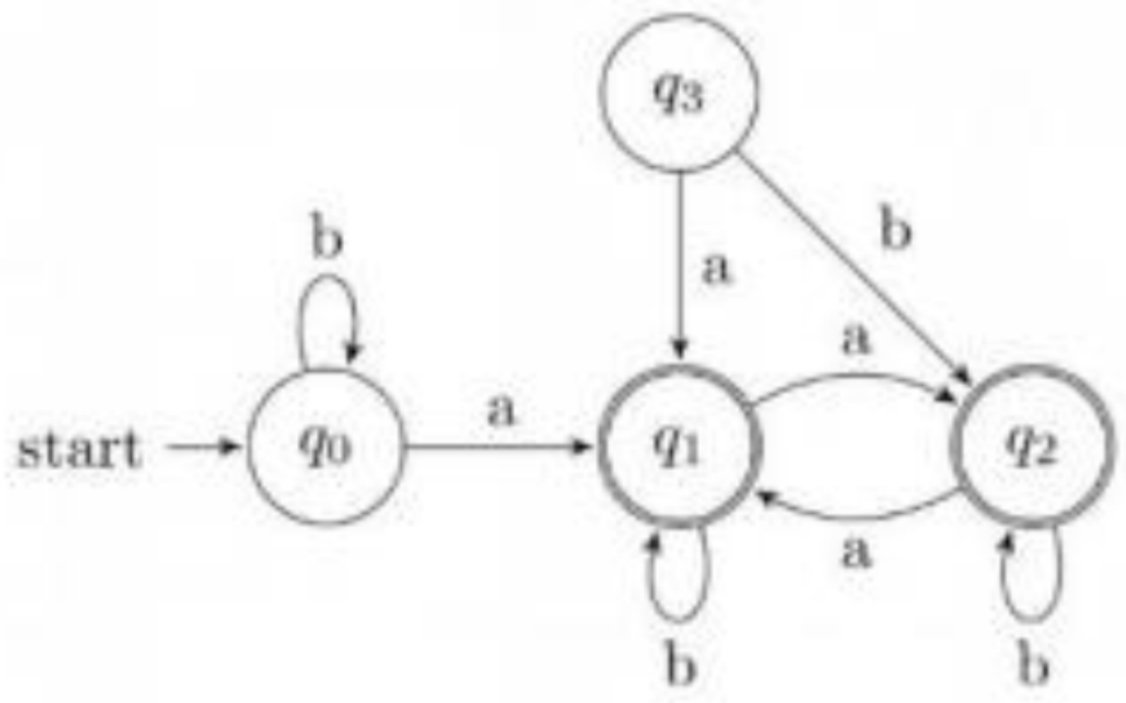
D. $(a+b)^*$

A minimum state deterministic finite automaton accepting the language

$L=\{w|w\in\{0,1\}^*$, number of 0s and 1s in w are divisible by 3 and 5, respectively $\}\}$ has

A. 15 states
B. 11 states
C. 10 states
D. 9 states

Consider the following Finite State Automaton:



The language accepted by this automaton is given by the regular expression

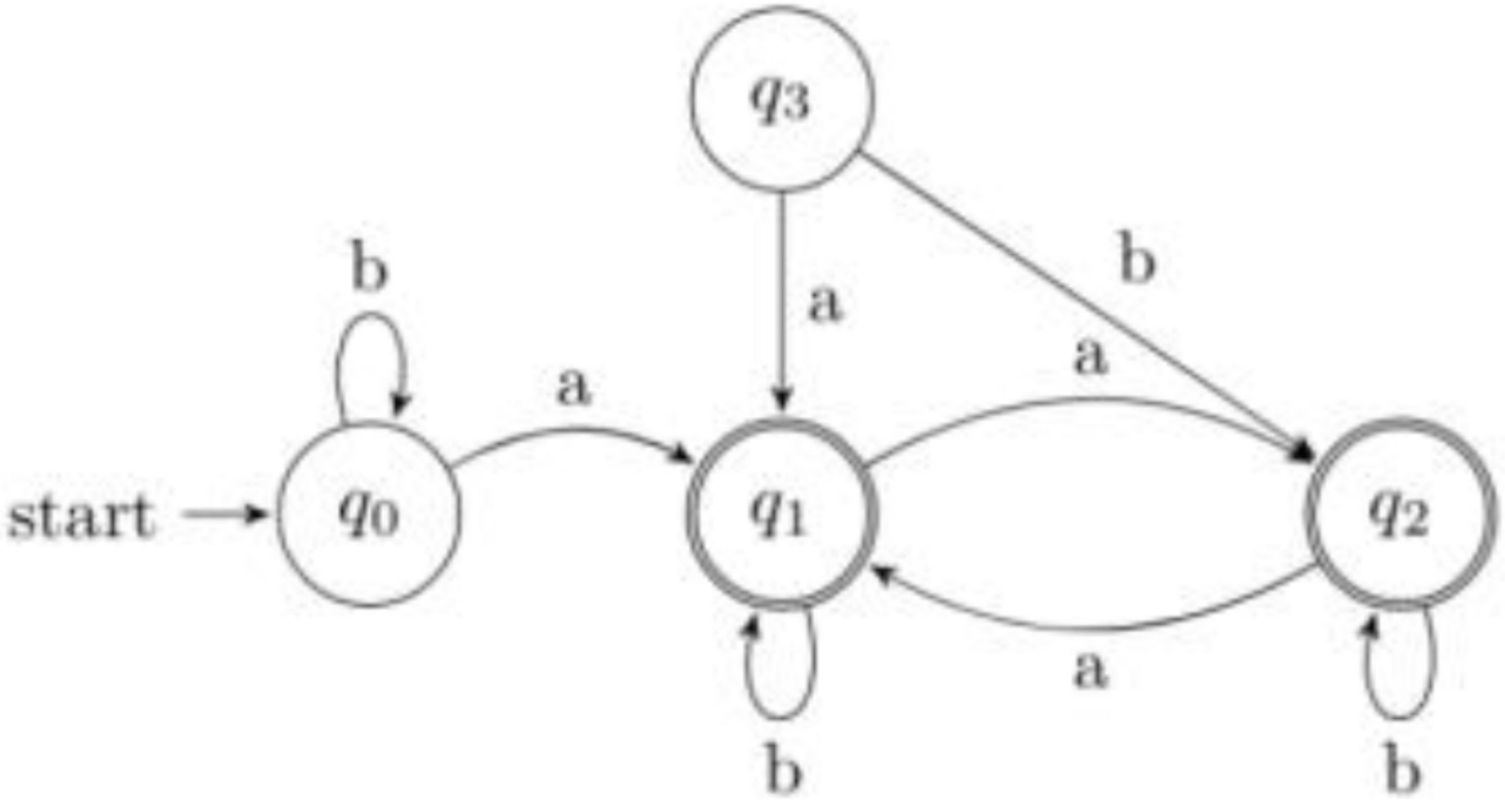
A. b*ab*ab*ab*

B. (a+b)*

C. b*a(a+b)*

D. b*ab*ab*

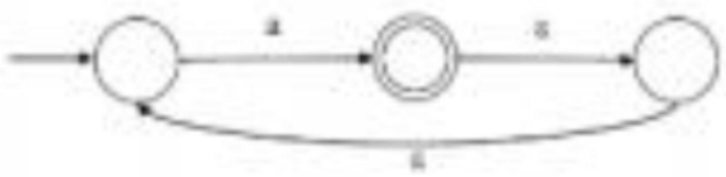
Consider the following Finite State Automaton:



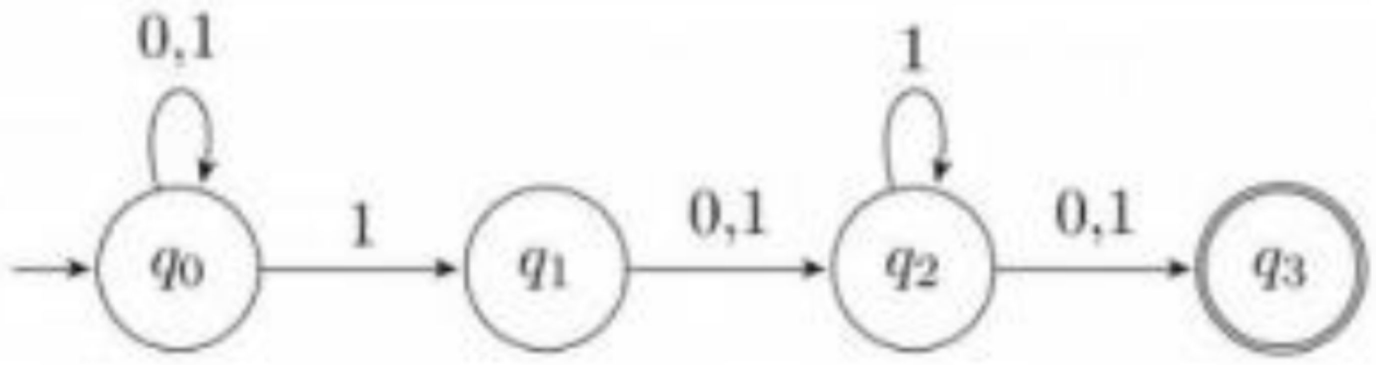
The minimum state automaton equivalent to the above FSA has the following number of states:

A. 1
B. 2
C. 3
D. 4

What is the complement of the language accepted by the NFA shown below? Assume $\Sigma=\{a\}$ and ϵ is the empty string.



A. ϕ
B. $\{\epsilon\}$
C. a^*
D. $\{a,\epsilon\}$



What is the set of reachable states for the input string 0011?

A. {q0,q1,q2}
B. {q0,q1}
C. {q0,q1,q2,q3}
D. {q3}

The number of states in the minimal deterministic finite automaton corresponding to the regular expression $(0+1)^*(10)$ is _____.

The number of states in the minimum sized DFA that accepts the language defined by the regular expression.

$(0+1)^*(0+1)(0+1)^*$ is ________.

Let Σ be the set of all bijections from $\{1, \ldots, 5\}$ to $\{1, \ldots, 5\}$, where id denotes the identity function, i.e. $id(j) = j, \forall j$. Let $\circ$ denote composition on functions. For a string $x = x_1 x_2 \cdots x_n \in \Sigma^n$, $n \geq 0$, let $\pi(x) = x_1 \circ x_2 \circ \cdots \circ x_n$.

Consider the language $L = \{x \in \Sigma^* \mid \pi(x) = id\}$. The minimum number of states in any DFA accepting L is ______.


THANK YOU
GW
SOLDIERS !