CS & IT ENGINEERING

Theory of Computation

Miscellaneous Topics (Part 2)

Lecture No. 2

By- DEVA Sir

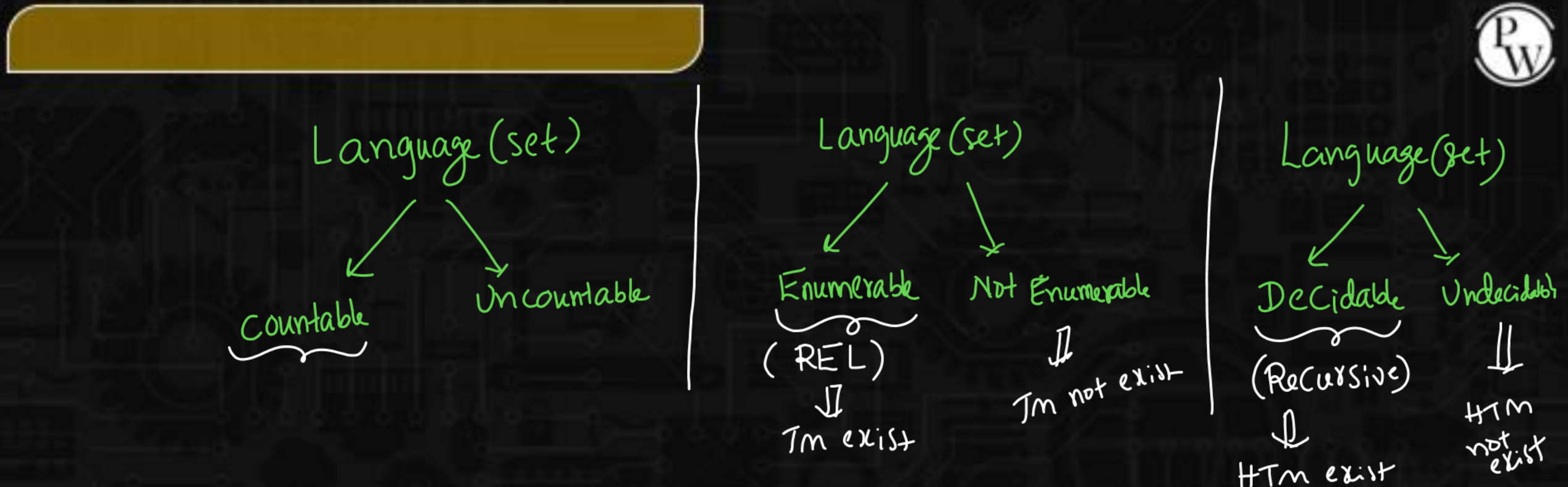
01 Countable Vs Enumerable Vs Decidable

02 Partial Function Vs Total Function

03 Undecidable Problems

04 Unrestricted Grammar

05 Doubts


Language (set)
Countable
Uncountable
Language (set)
Enumerable
(REL)
TM exist
Not Enumerable
TM not exist
Language (set)
Decidable
(Recursive)
HTM exist
Undecidable
HTM not exist

Countable Set

X is countable set

iff

$f: X \to Y$ is bijective

and Y is known countable set

Countable sets

$\mathbb{N}$

$2\mathbb{N}$

$2\mathbb{N}+1$

$\mathbb{Z}$

$\mathbb{Q}$

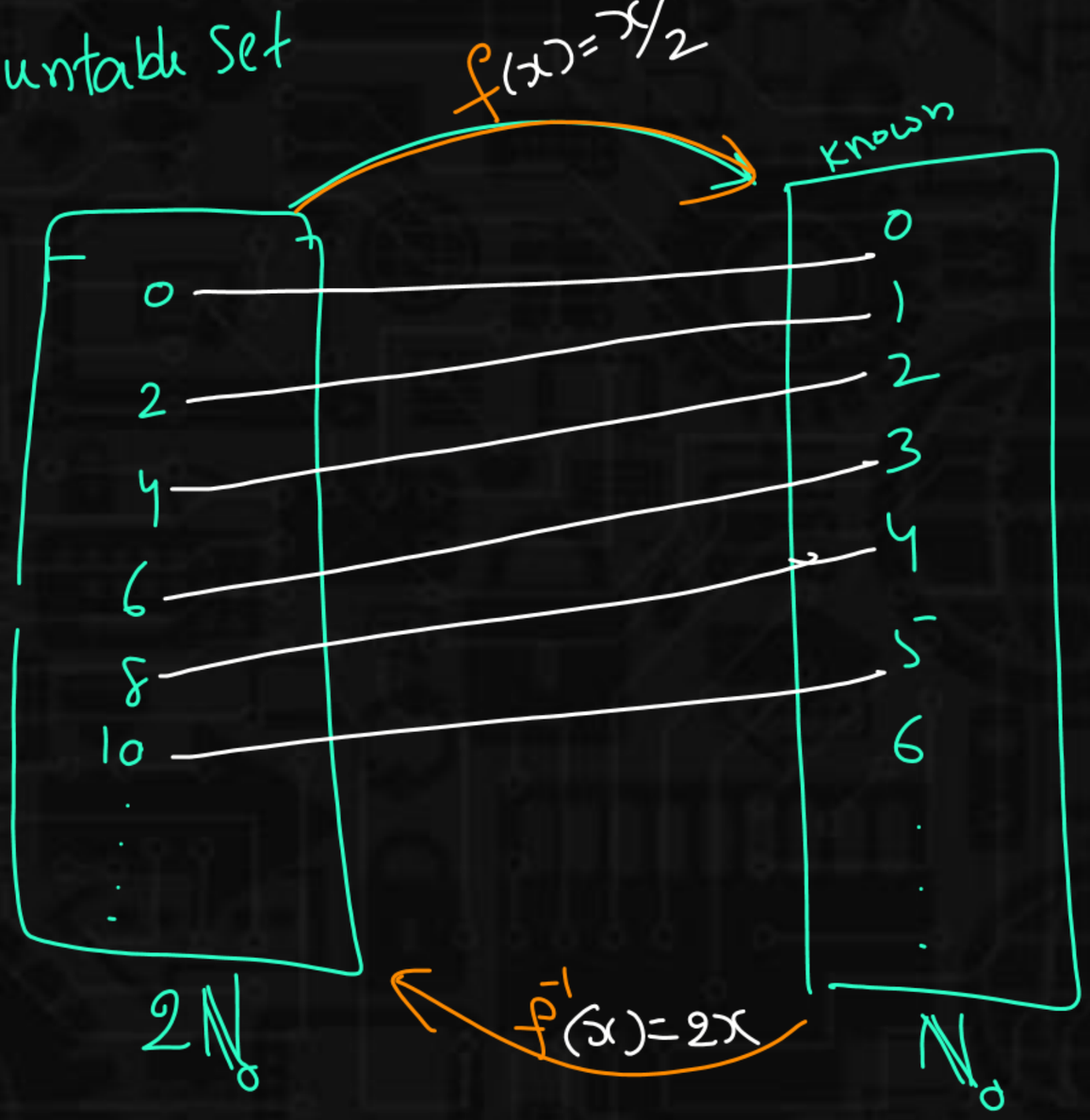
Uncountable sets

$\mathbb{R}$

$\mathbb{C}$

Set of real numbers $[0,1]$

$2\mathbb{N}_0$ is countable set



f exist
f^{-1} exist
$\Rightarrow$ f is bij

Countable Languages

1) Σ ⇒ Finite Set ⇒ Countable

2) Σ^* ⇒ Regular Set ⇒ Countable

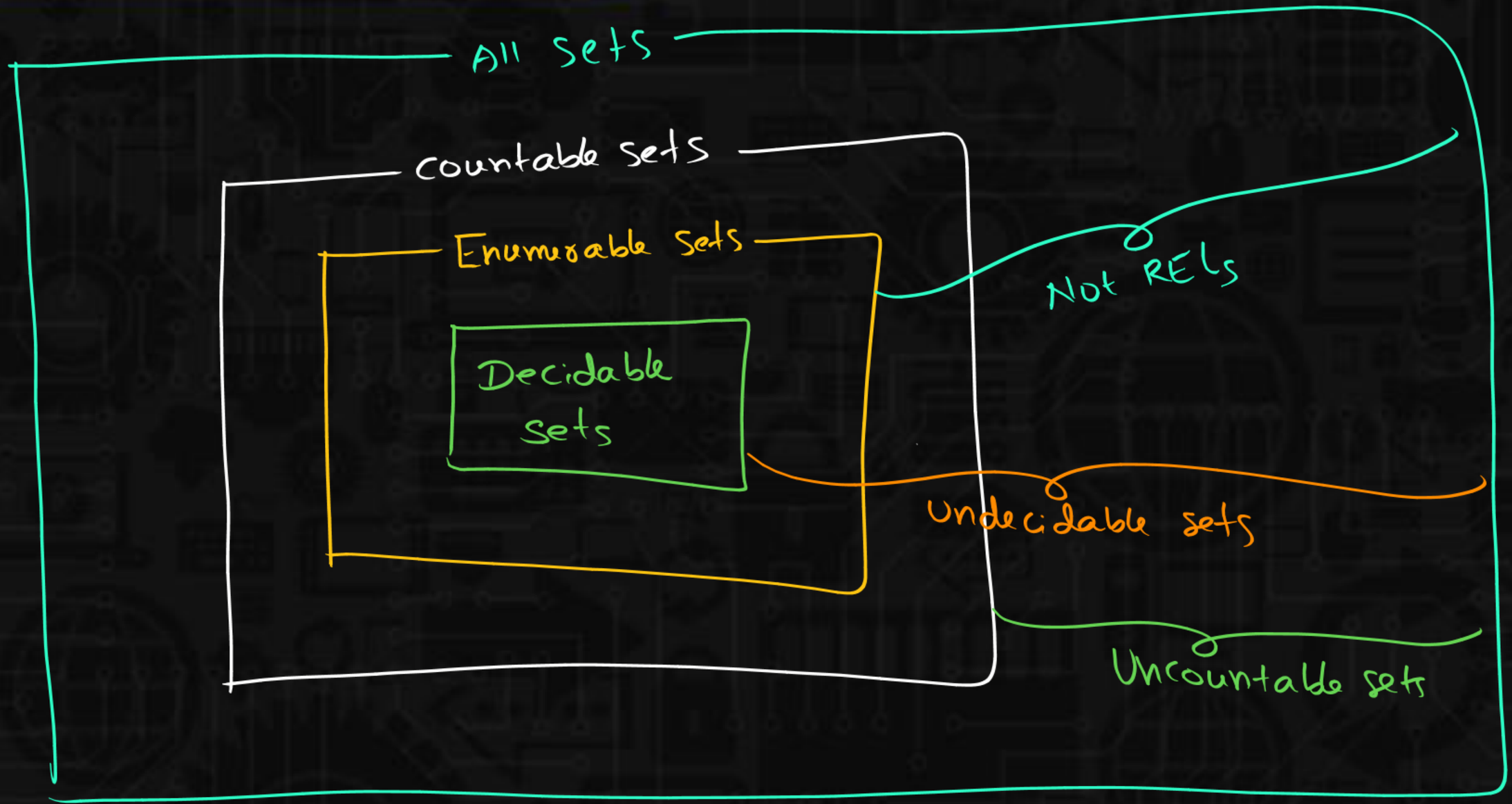
3) 2^{Σ^*} ⇒ Uncountable Set

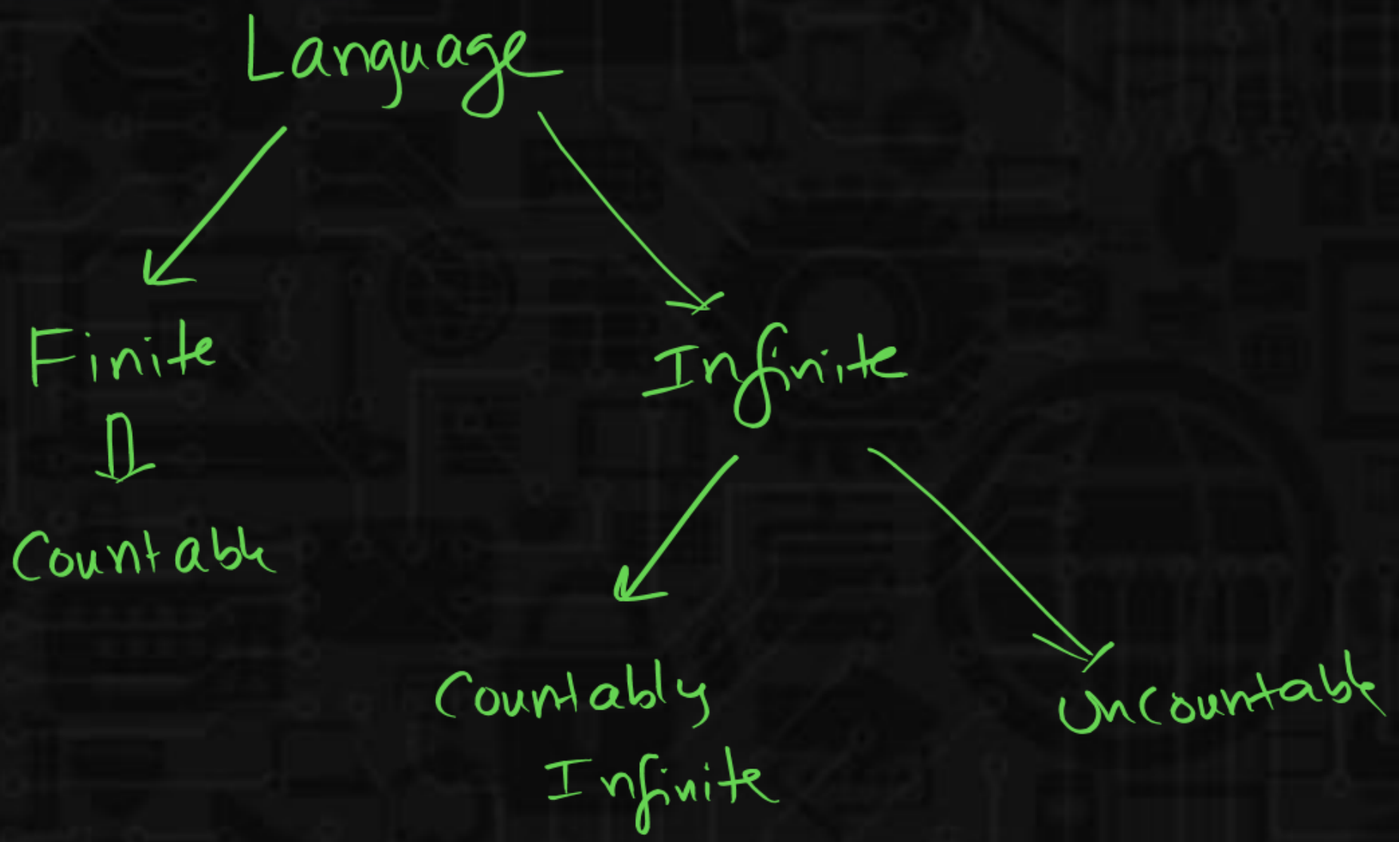
Σ = Set of all symbols

Σ^* = Set of all strings over Σ

2^{Σ^*} = Set of all languages over Σ

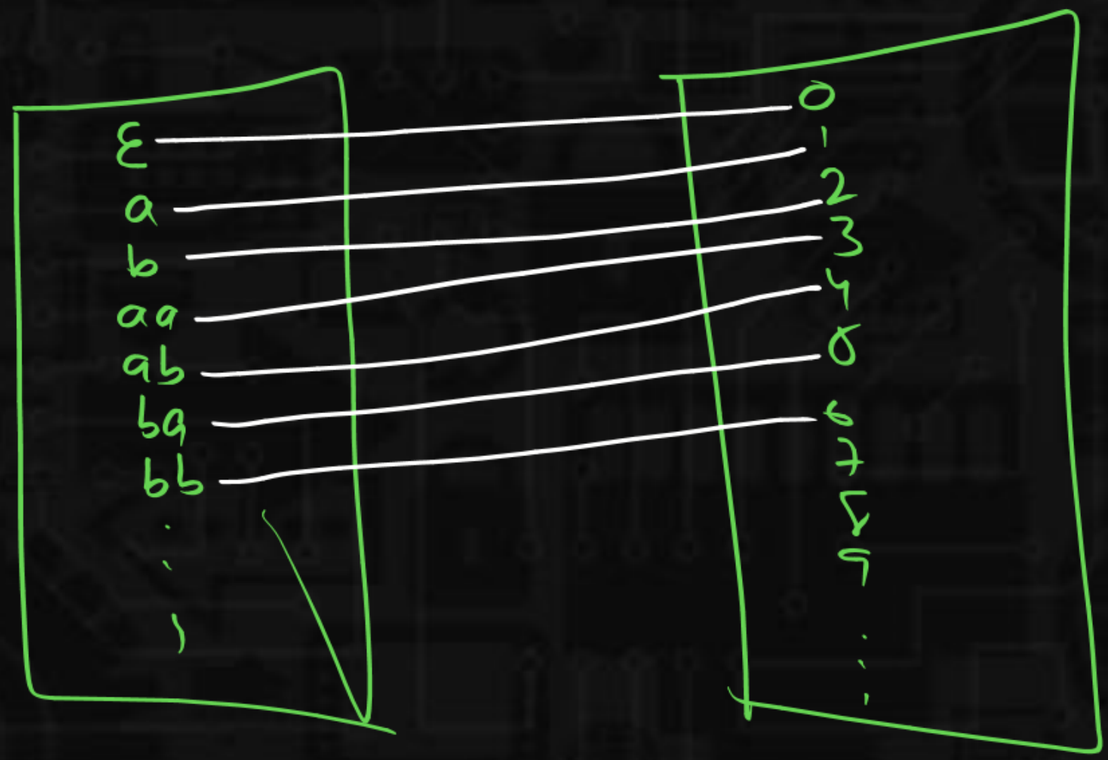
Countable Vs Enumerable Vs Decidable




Language
Finite
Countable
Infinite
Countably Infinite
Uncountable

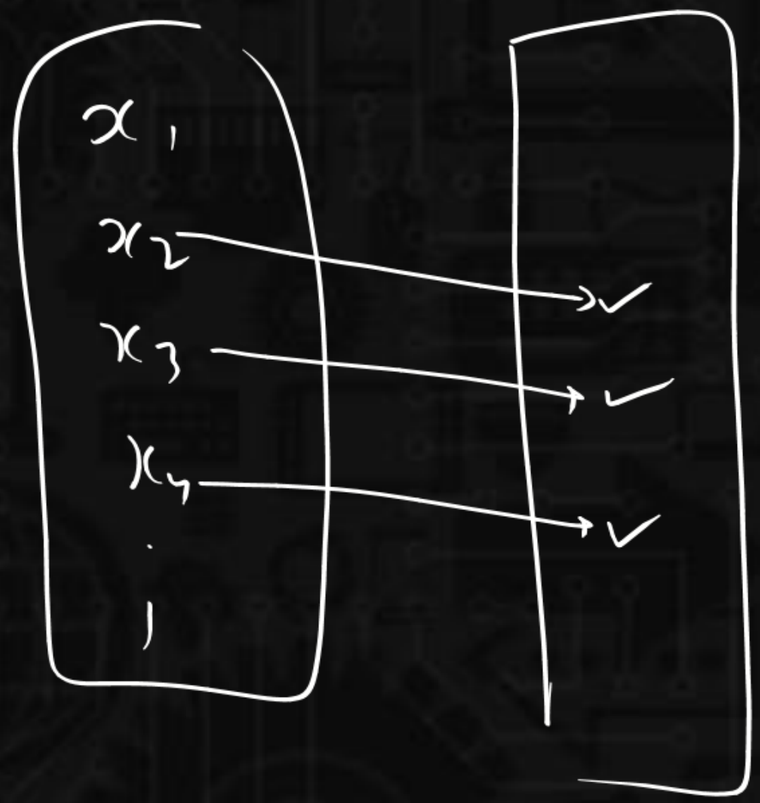
Σ^* is Countable set

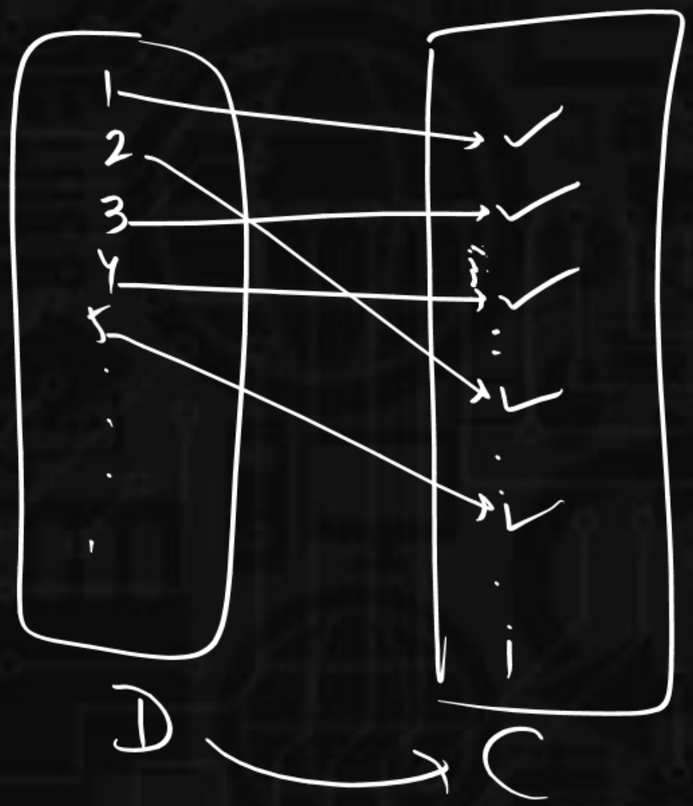
$f : \Sigma^* \longrightarrow N$ is bijective



Partial Function Vs Total Function

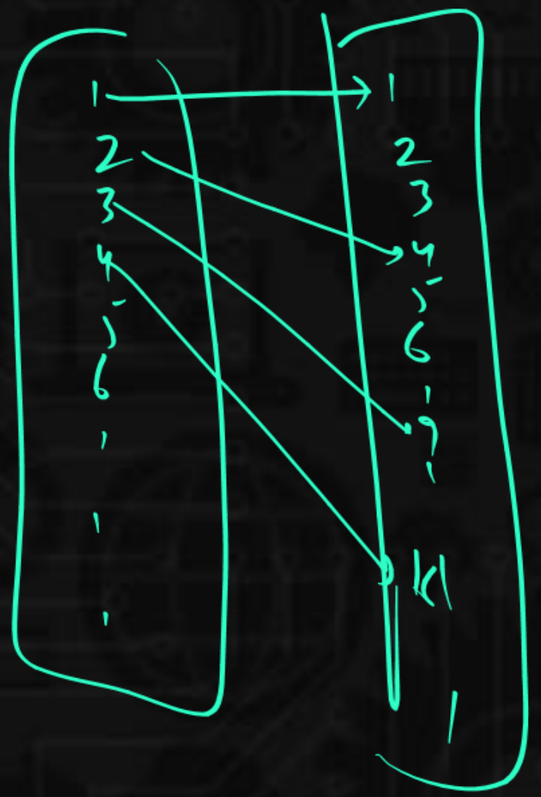
- Total Function → For every element in domain, mapping exist
- Partial Function → For some elements in domain, mapping exist





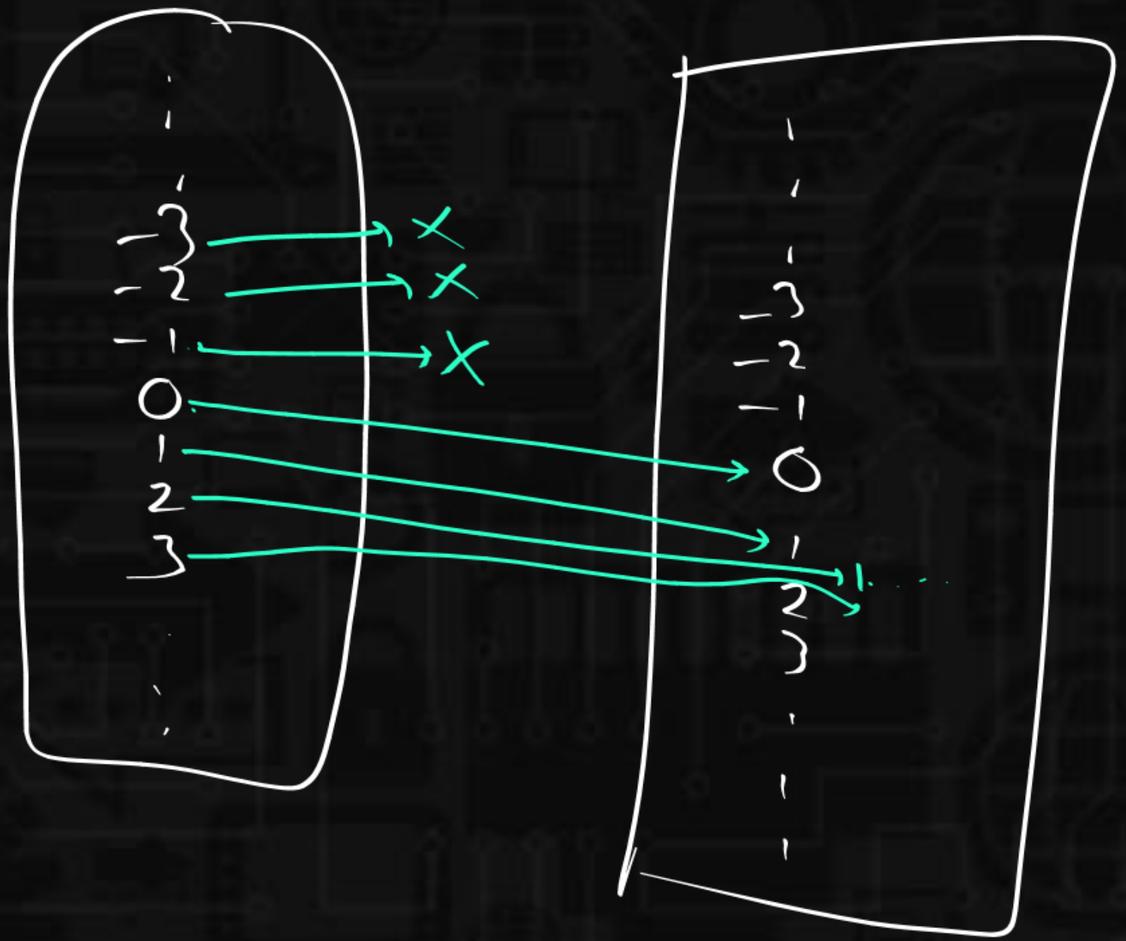
$f(x) = x^2$

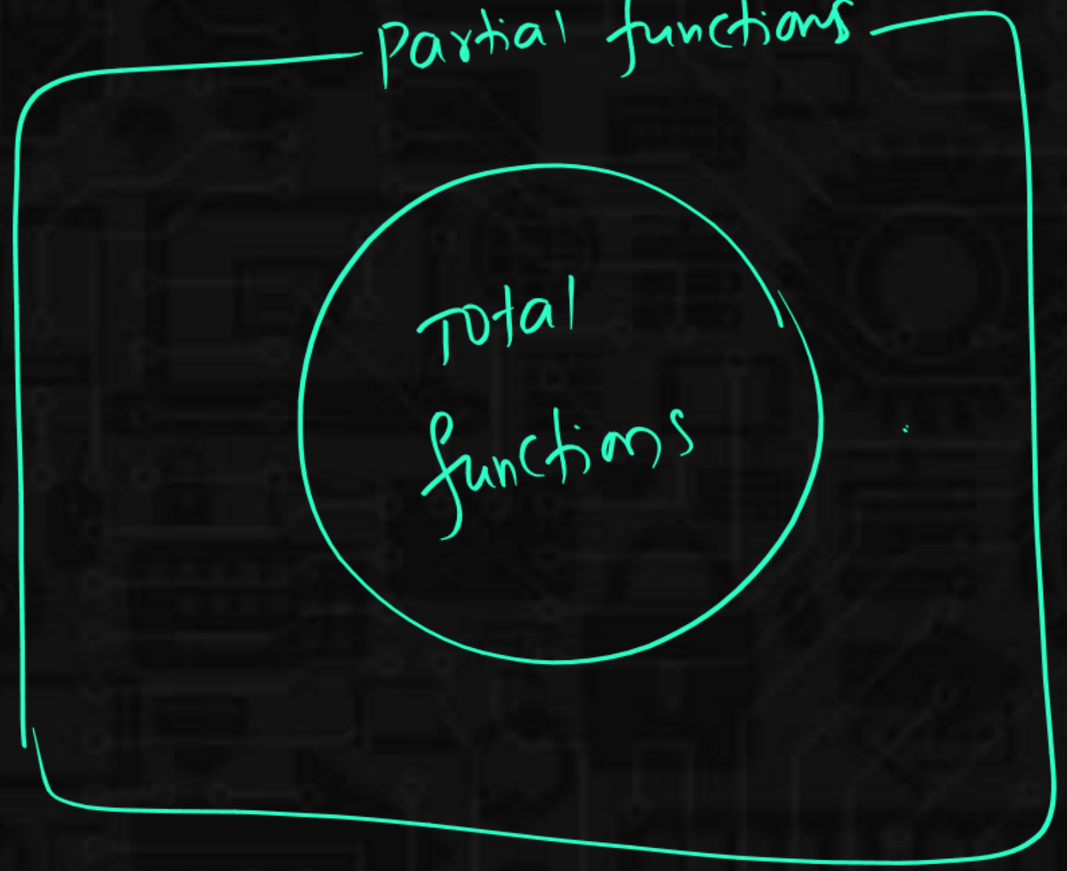
$f : N \to N$



$f(x) = \sqrt{x} \quad ; \quad x \in \mathbb{Z}$

$f : \mathbb{Z} \to \mathbb{R}$




Partial functions
Total functions

RG

- LLG: $V \to VT^*$, $V \to T^*$
- RLG: $V \to T^*V$, $V \to T^*$

LG

$V \to T^*VT^* / T^*$

CFG

(Syntax oriented)

$V \to (VUT)^*$

any

CSG

(Length restricted)

$LHS \to RHS$

$|LHS| \leq |RHS|$

$\underbrace{any\ V\ any}_{LHS} \to \underbrace{any}_{RHS}$

UG

(phrase grammars)

$LHS \longrightarrow RHS$

① $S \longrightarrow aS$

$aS \longrightarrow aaS \mid \varepsilon$

$L = ?$

ε ✓ ← $S \Rightarrow aS \Rightarrow \varepsilon$

a ✓ ← $S \Rightarrow aS \Rightarrow a\varepsilon = a$

aa ✓ ← $S \Rightarrow aS \Rightarrow aaS \Rightarrow aa\varepsilon = aa$

$aaS \Leftarrow aaaS \Leftarrow aaaaS \Leftarrow \ldots aa$

A) $(aa)^*$

B) a^+

C) a^*

D) none

(2) $S \rightarrow aAb$

$aA \rightarrow aaA \mid \varepsilon$

b ✓
ab ✓
aab ✓
aaab ✓
.
.
.

$L = a^* b$

b: $S \Rightarrow (aA)b \Rightarrow b$

ab: $S \Rightarrow (aA)b \Rightarrow a(aA)b \Rightarrow ab$

aab: $S \Rightarrow (aA)b \Rightarrow a(aA)b \Rightarrow a(a(aA)b \Rightarrow aaab$

Problems

Decidable
(1) Whether given grammar is CFG.

Decidable
(2) Whether given grammar is LLG.

RE but not rec
(3) Whether given grammar is generating ab [Membership for TM]

Not RE
(4) " " " is " regular language

(5) " " " is " a^*

Decidable

(6) Whether given CFG is Regular grammar

Not REL

(7) " " " is generating regular language

Decidable

(8) " " " " " ab (membership for PDA)

CYK Algo

(9) " " " " " CFL

Decidable

Undecidable

RE but not REL

1) TM accepts ε

membership

RE but not REL

2) Tm accepts some string

non emptiness

Not REL

3) Tm accepts only ε

Is $L(Tm) = \{\varepsilon\}$?

Not REL

1) TM accepts exactly 50 strings [IS $|L(TM)| = 50$?]

RE but not rec

2) TM accepts atleast 50 strings → YES: Logic exist

→ NO: logic not exist

3) TM accepts atmost 50 strings

Not REL

Decidable
1) Whether TM has 100 states

RE but not rec
2) Whether TM reaches state q on some string
- YES : logic exist
- NO : logic not exist

Decidable
3) Whether TM reaches state q within 100 steps

Undecidable Problems

1. Post Correspondence Problem
2. Ambiguity Problem for CFG
3. Equivalence problem for PDAs/LBAs/TMs
4. Equivalence of two programs
5. Checking program's behavior

Summary

- UG
- C vs E vs D
- Undecidable problems


THANK YOU GW
SOLDIERS !