GATE
DATA SCIENCE AND AI

CALCULUS AND OPTIMIZATION
SINGLE VARIABLE CALCULUS

Lecture No.- 02

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Recape of previous lecture

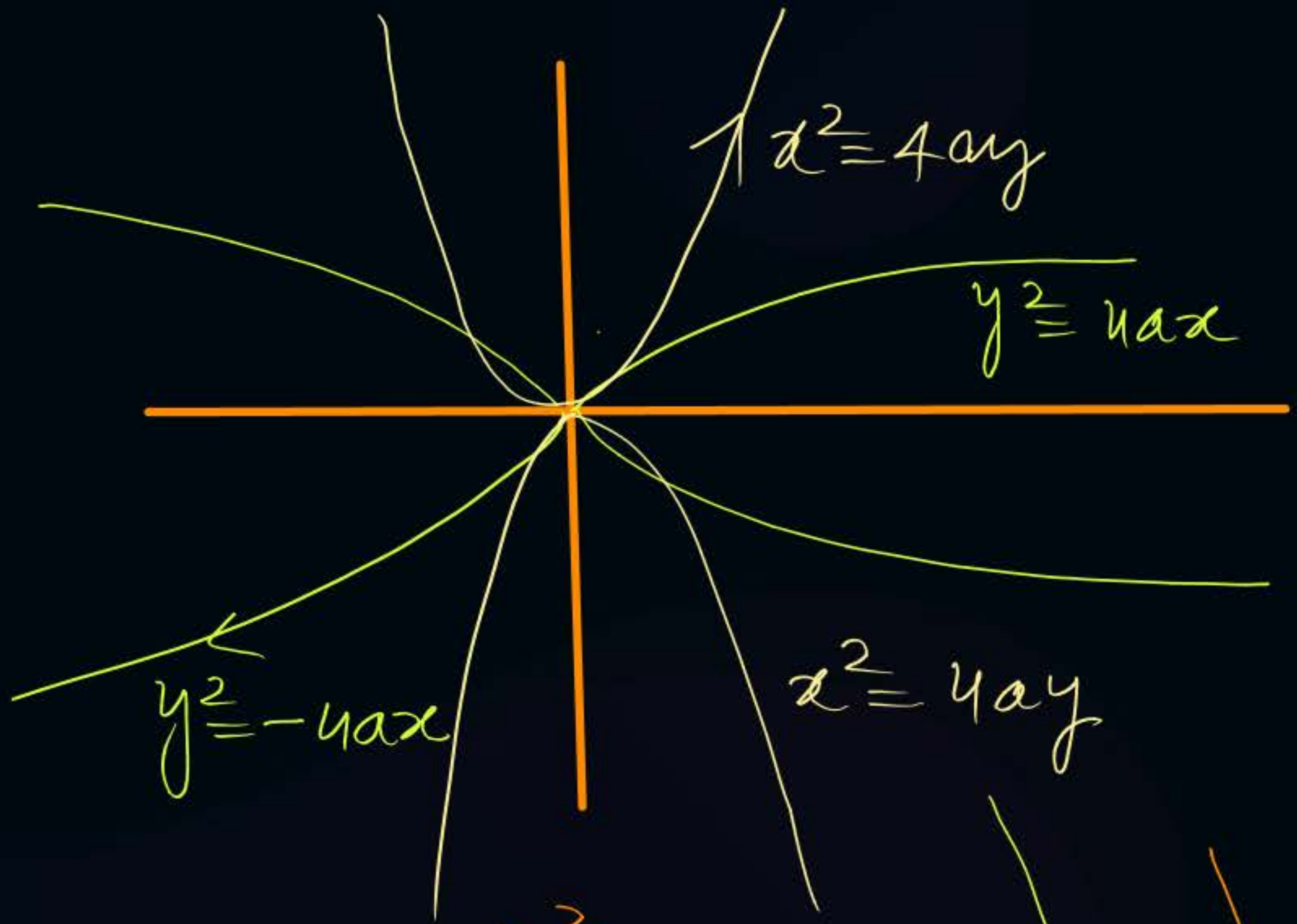
Topic Sketching graphs

Topics to be Covered

Topic Some transformation of graphs

Topic Existence of limits

Topic How to evaluate limits



Plot The curve

$y^2 = 4ax$ #

$y^2 = -4ax$ #

$4ay = x^2$ #

$x^2 = -4ay$ #

C_1 $y = (x+1)^2$

C_2 $y = (x-1)^2$

C_3 $y = x^2+1$

C_4 $y = x^2-1$

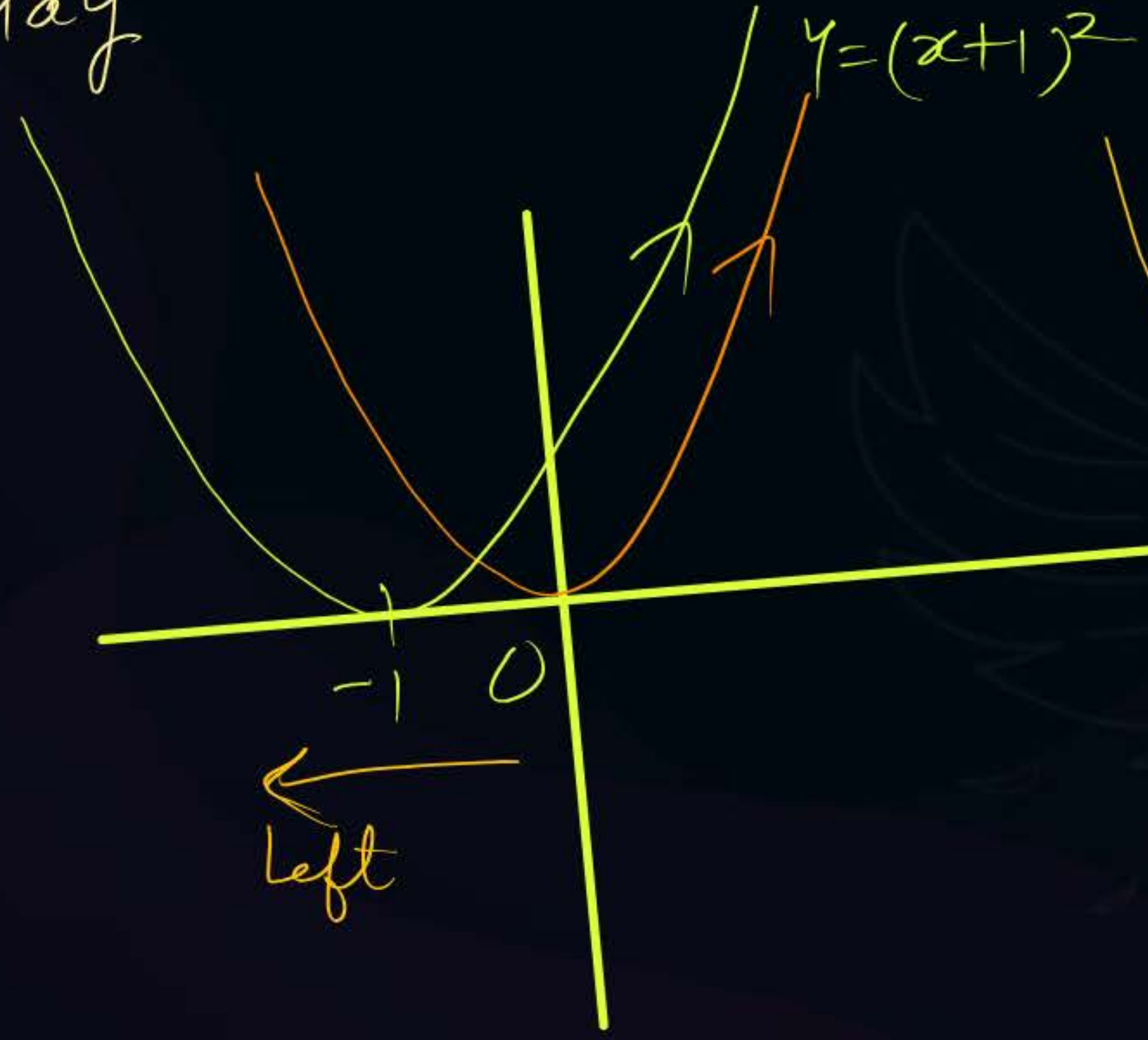
x^2

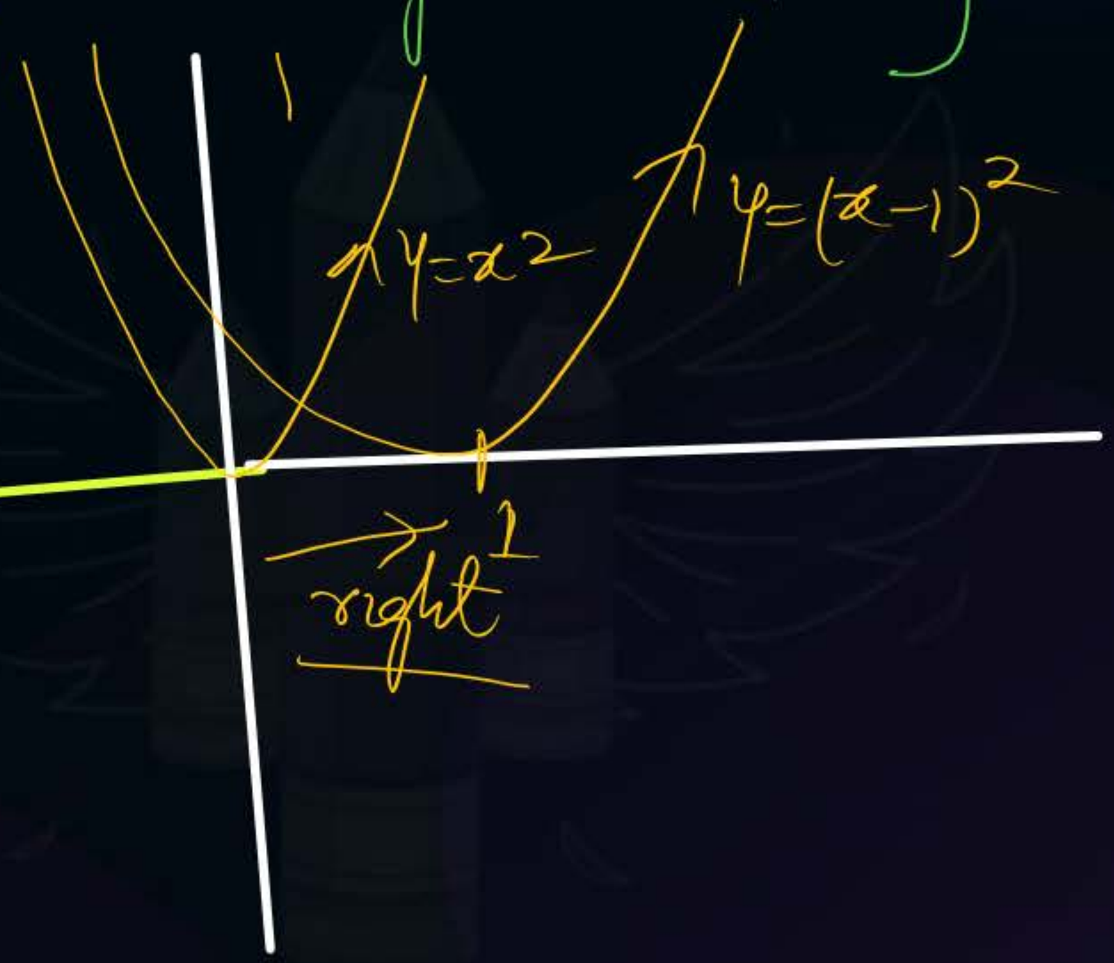
$y = (x+1)^2$

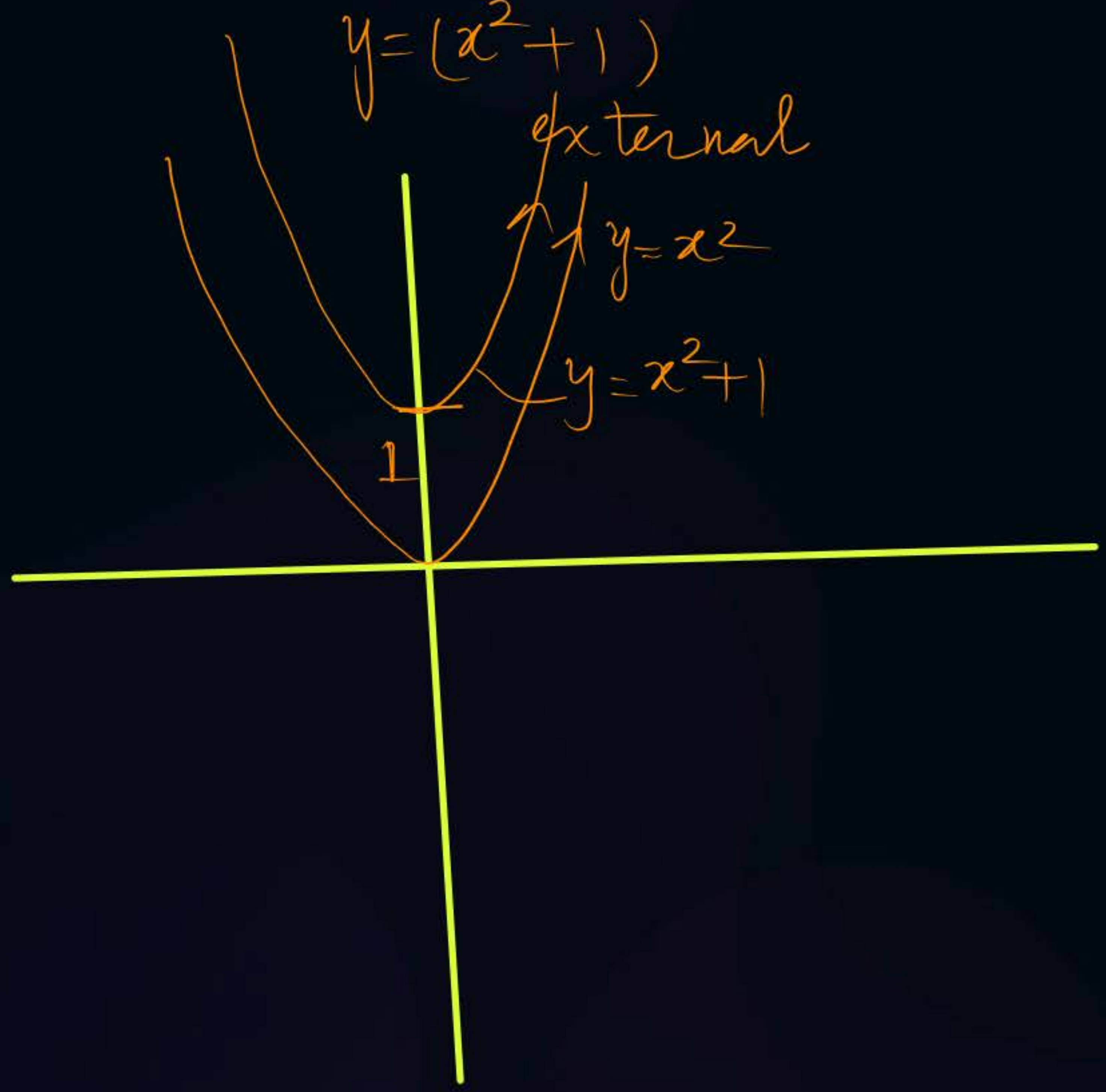
Internal Adjusment

$x+1=0$

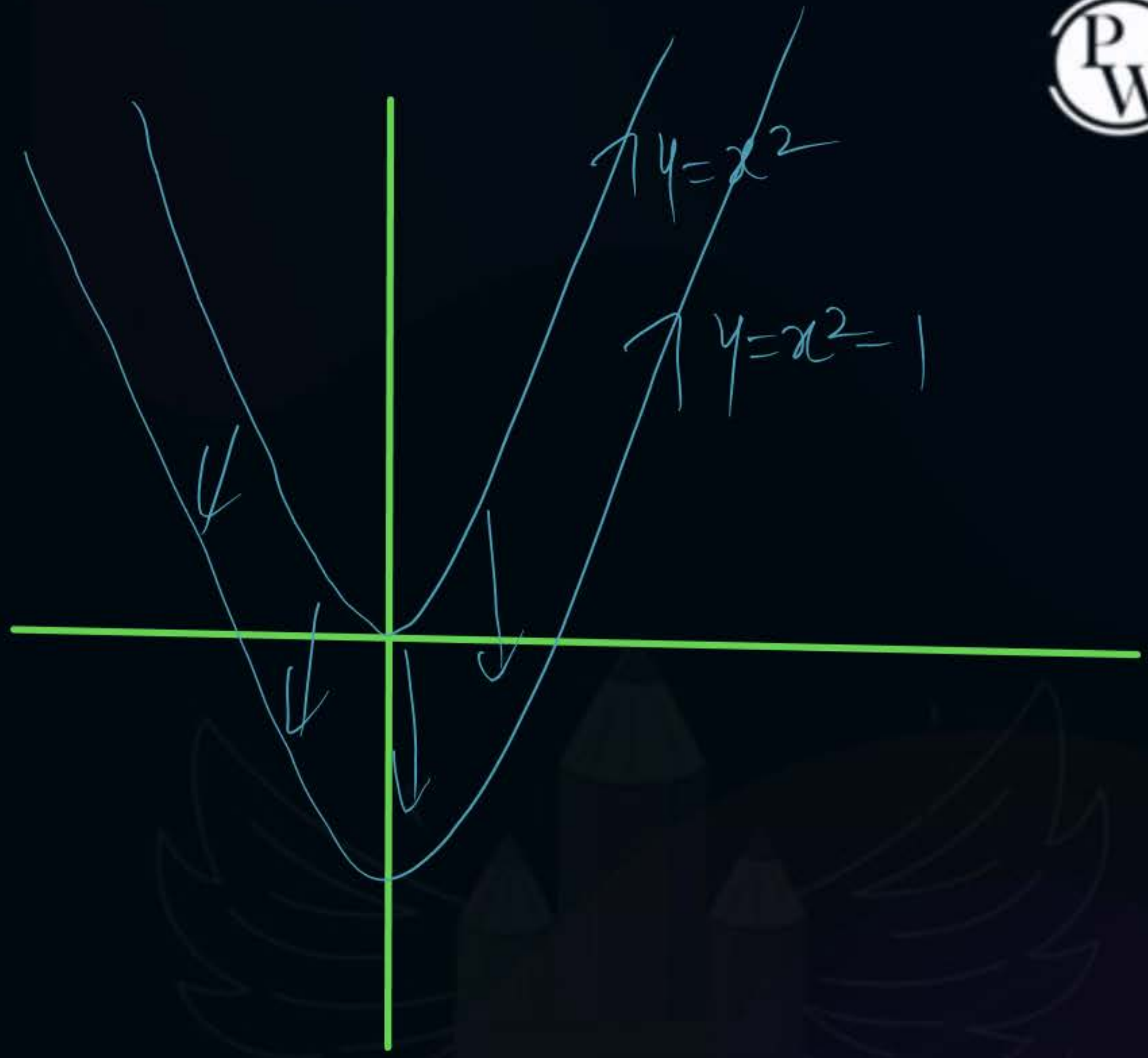
$x=-1$





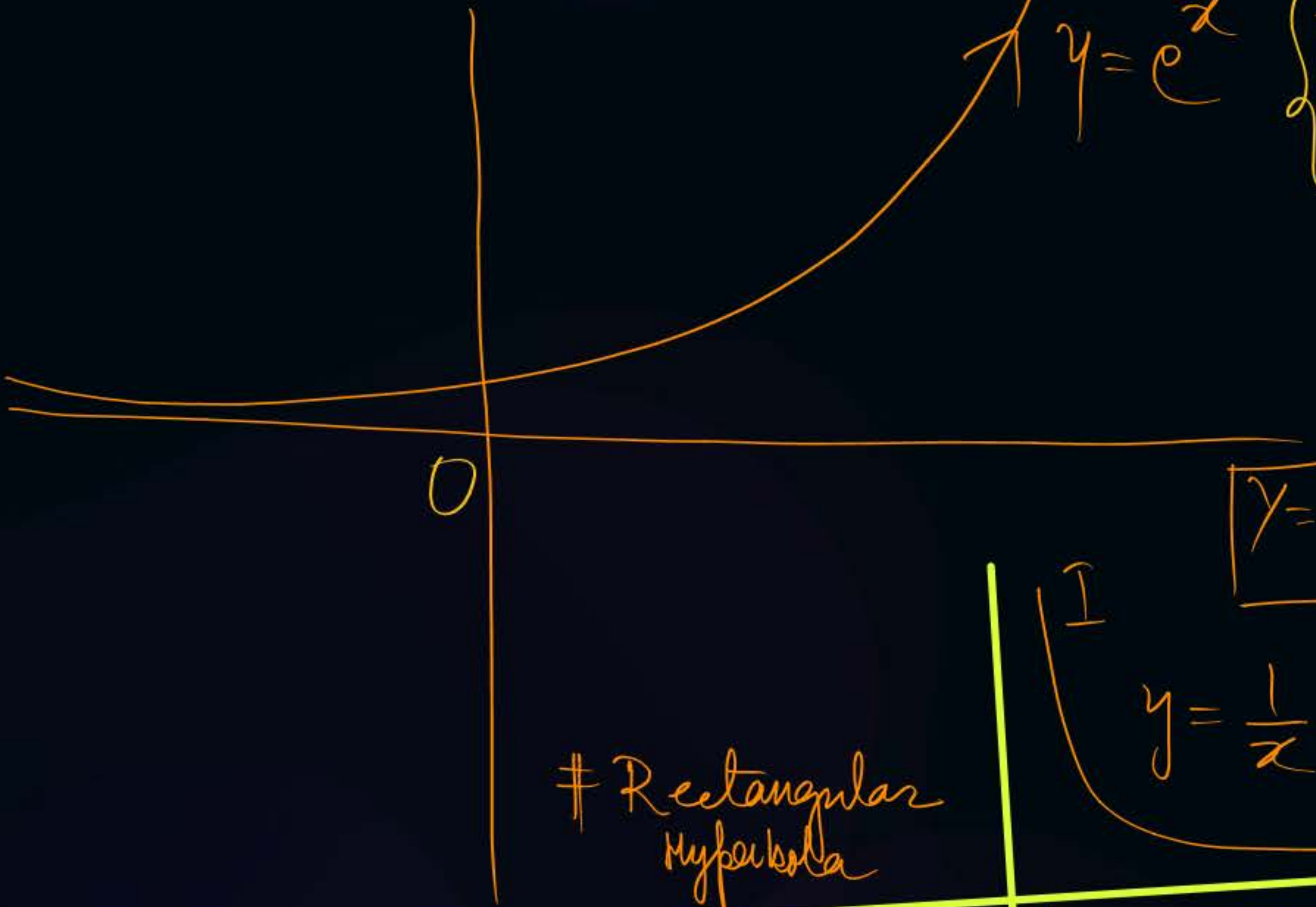


y=(x²+1)
external
y=x²
y=x²+1
1



y=x²
y=x²-1

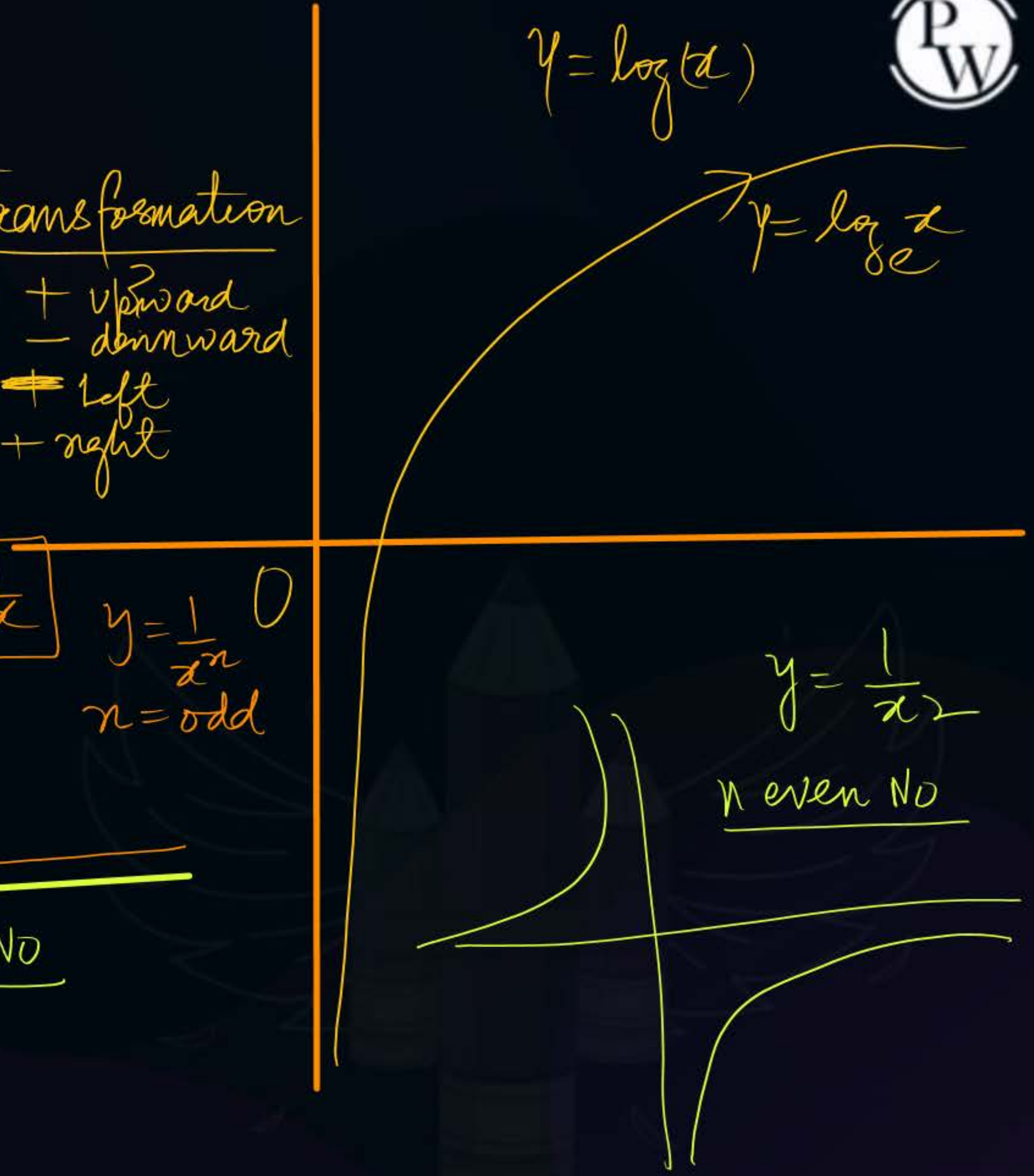
$f(x) = e^x$



$y = e^x$

Transformation
- $+$ upward
- $-$ downward
- ~~+~~ left
- $+$ right

$y = \log(x)$



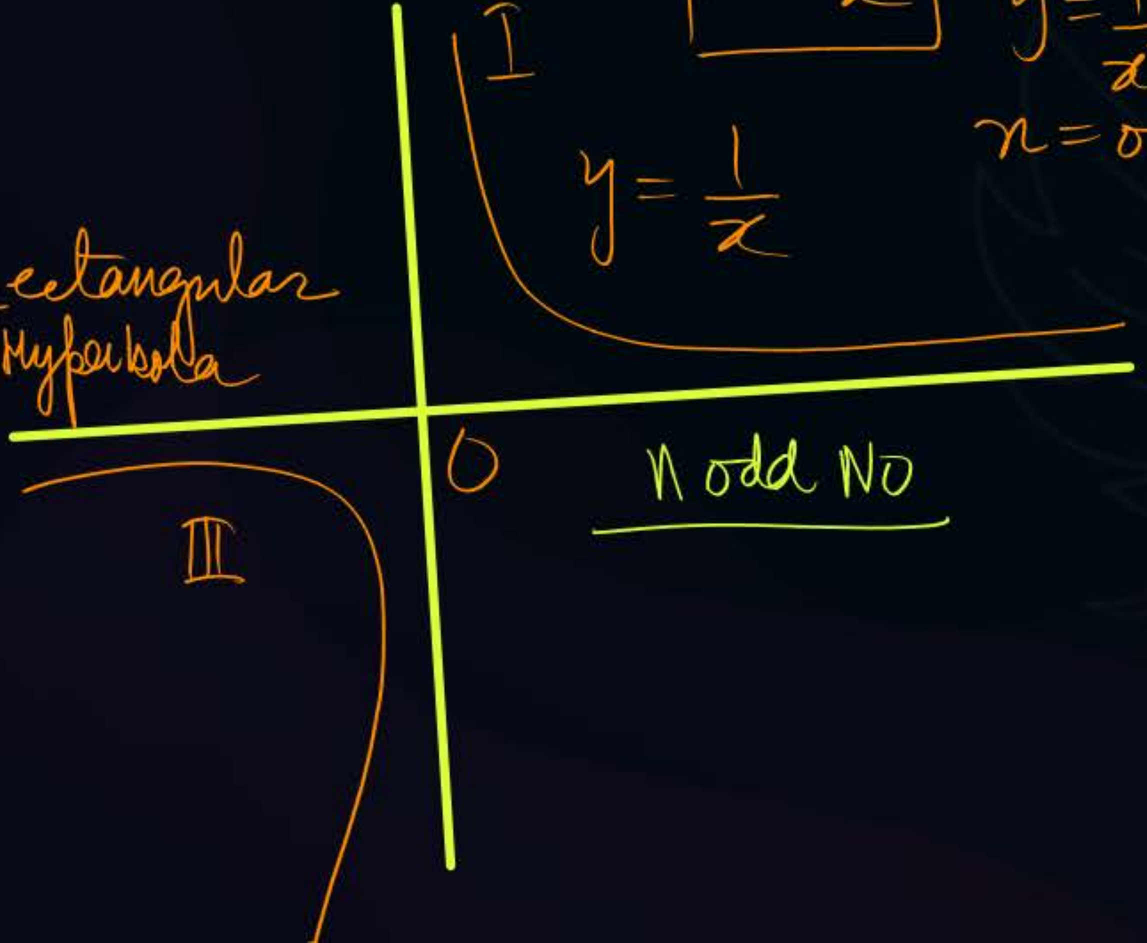
$y = \log_e x$

$\boxed{y = \frac{1}{x}}$

$y = \frac{1}{x^n}$

$n =$ odd

Rectangular Hyperbola



I

$y = \frac{1}{x}$

III

n odd No

$y = \frac{1}{x^2}$

n even No

Existence of Limit:

$$\underset{x \to a}{\text{Lt}} \, f(x) = \text{Limiting value of } f(x)$$

x tends a

'x towards a'

variable   constant No

$y = f(x)$ = Limit find

x = understand

No-line — DENSE   Real line

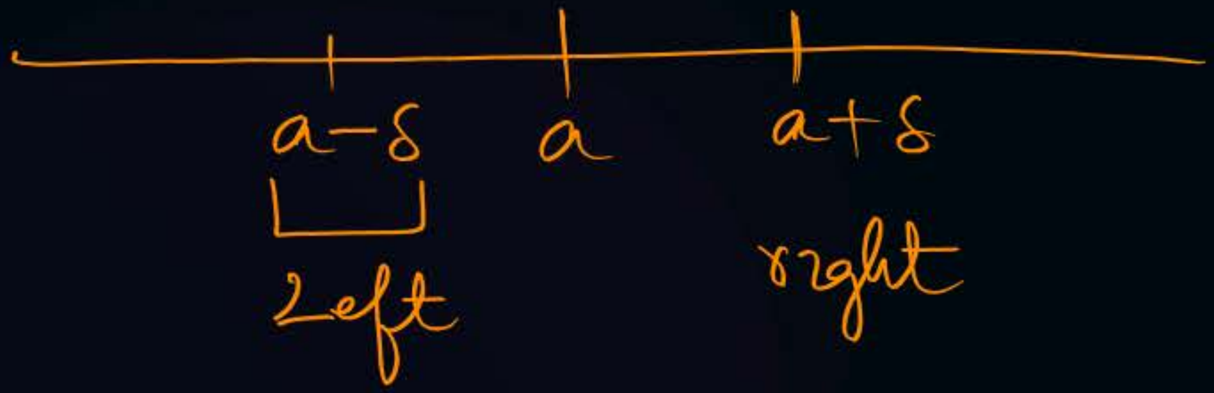
Infinite Points

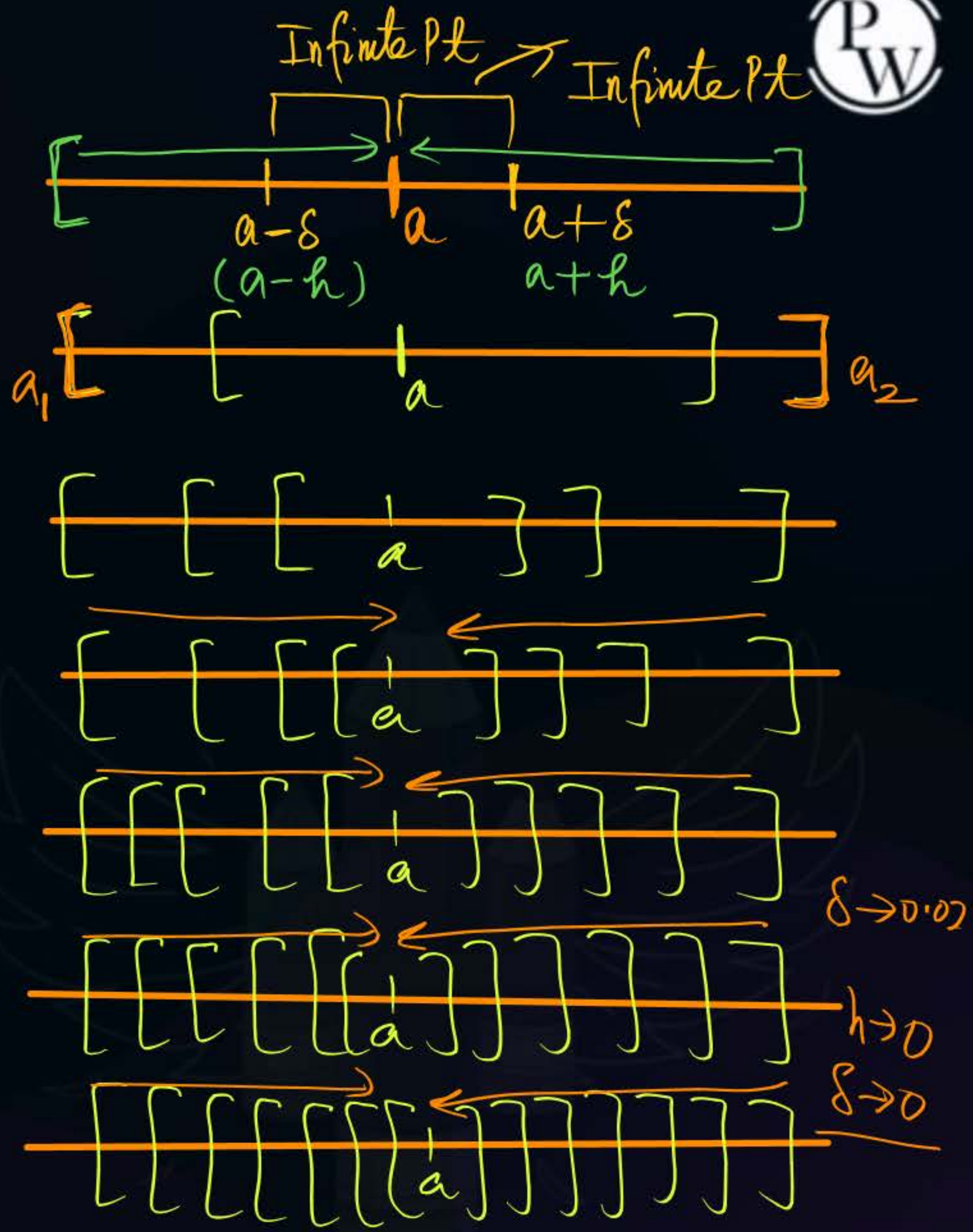
distance (h) > 0

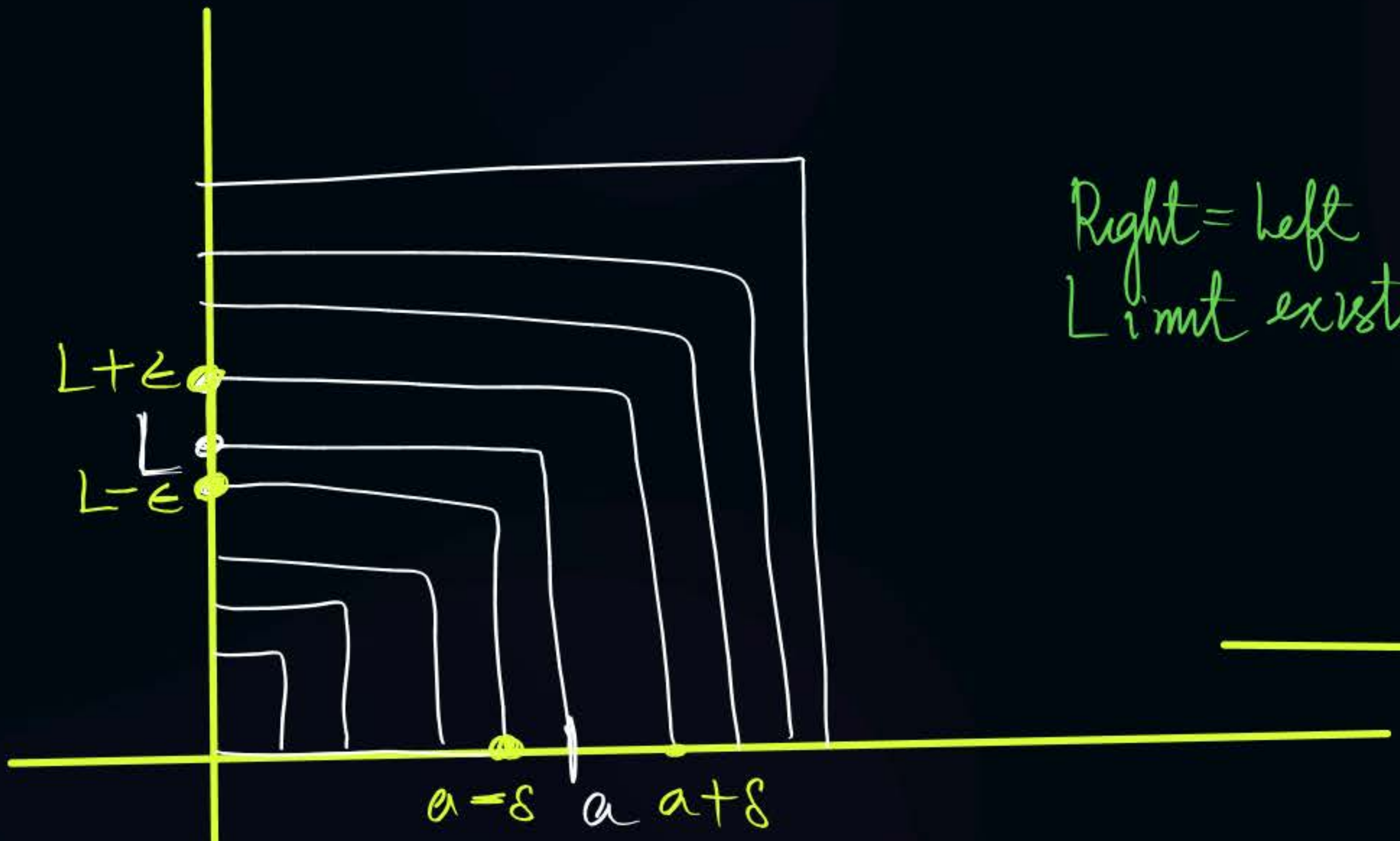
step size h > 0

$h \longrightarrow \delta$ (some No)

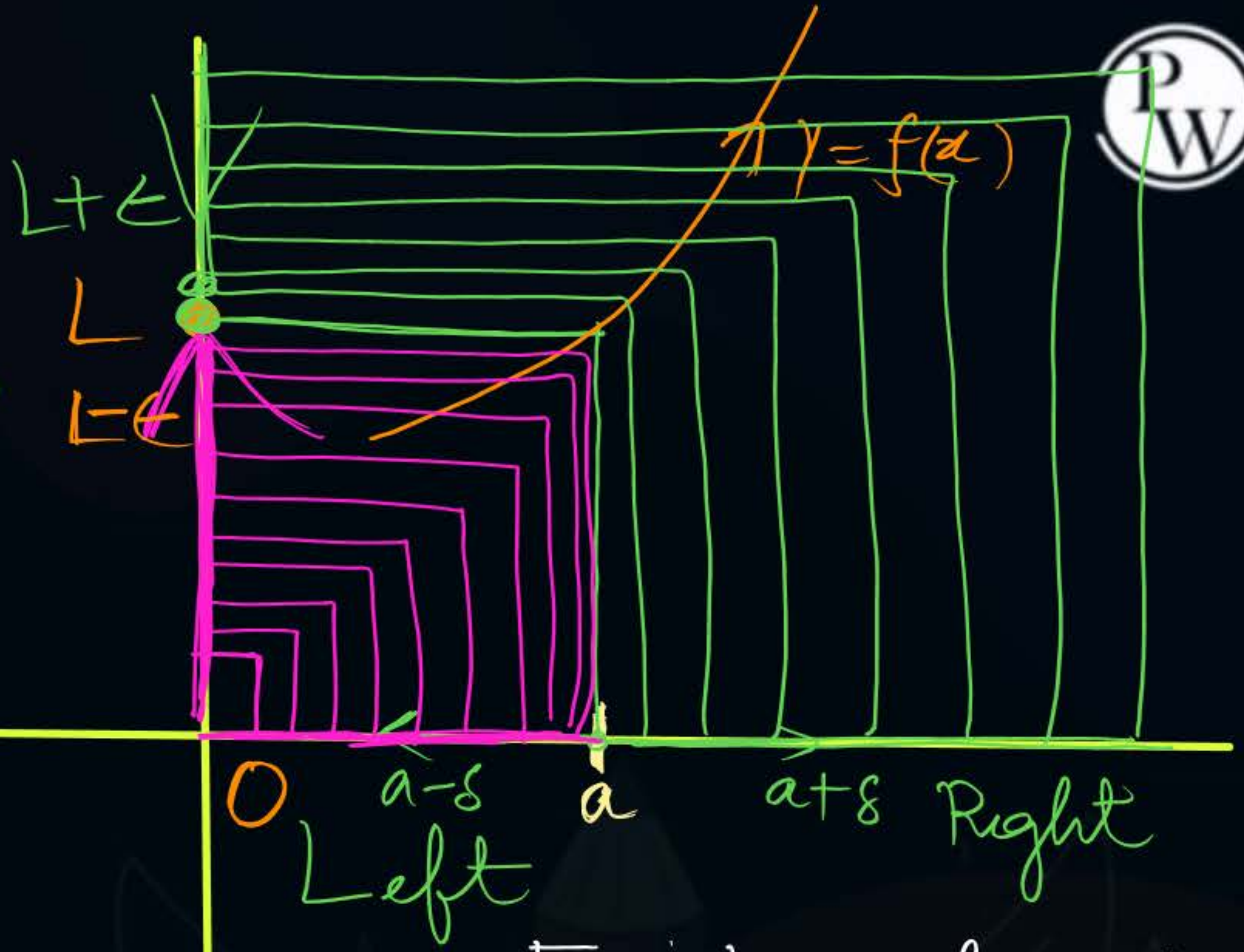
✓ binary SEARCH
✓ bisection Rule
✓ NEST Interval







Right = Left
Limit exists



Existence of Limit

Left Hand limit = Right Hand limit = finite

$$\boxed{LHL = RHL = \text{finite}}$$

existence of limit

$a-( ) \leftarrow a \rightarrow a+( )$

$$f(x) = (x+2)$$

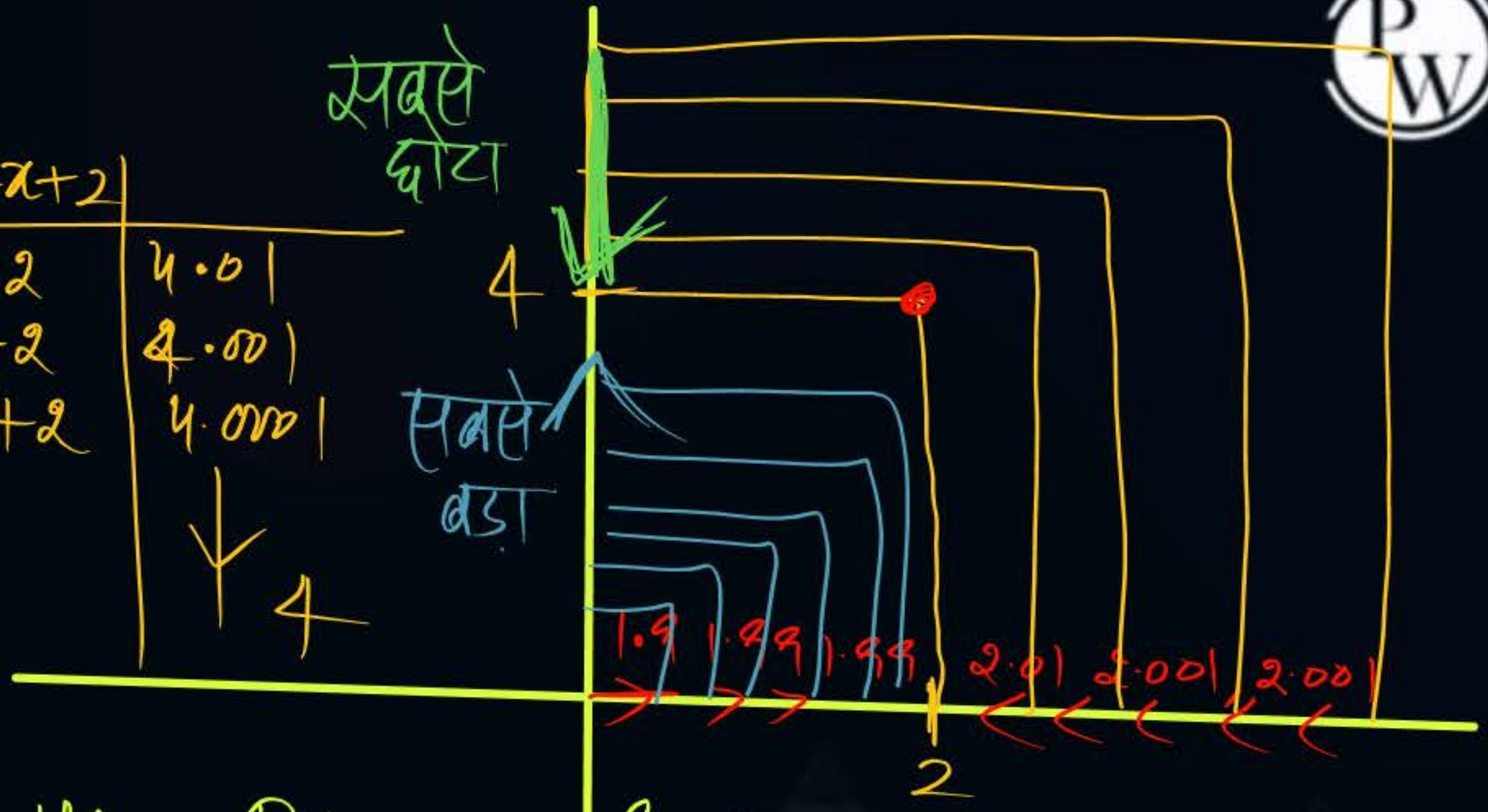
$$\underset{x\to 2}{Lt}\,(x+2) = 4$$

$x \to 2^-$ Left Hand

$x \to 2^-$	$f(x) = (x+2)$
1.9	$1.9 + 2 = 3.9$
1.99	$1.99 + 2 = 3.99$
1.999	$1.999 + 2 = 3.999$
1.9999	$1.9999 + 2 = 3.9999$
⋮	⋮ $\downarrow$ 4

$x \to 2^+$

$x \to 2^+$	$f(x) = x+2$	
2.01	$2.01 + 2$	4.01
2.001	$2.001 + 2$	4.001
2.0001	$2.0001 + 2$	4.0001
		$\downarrow$ 4



If $\underline{LHL = RHL} = \text{finite}$

Limit exists

Existence of Limit:

LHL = RHL = finite

LHL = $\lim_{h\to 0} f(a-h)$ RHL = $\lim_{h\to 0} f(a+h)$

$$\boxed{\lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a+h) = \text{finite}}$$

at $x=a$ point

Topic : Single Variable Calculus

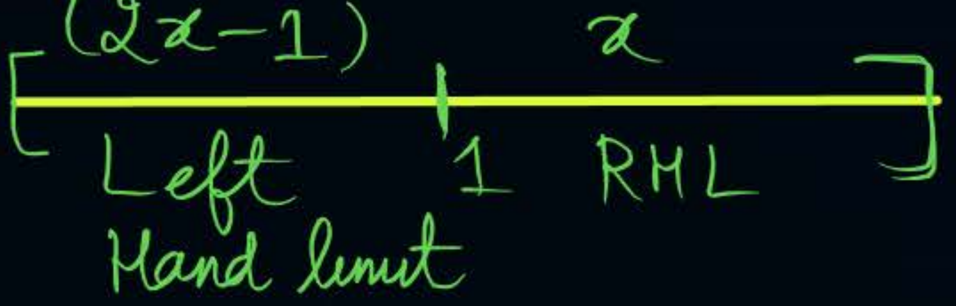
#Q. Show that the limit of: $f(x)=\begin{cases} 2x-1 & ; \quad x\le 1 \\ x & ; \quad x>1 \end{cases}$ at x = 1 exists.

Left Hand limit LHL $x = 1$

$= \lim\limits_{h\to 0} f(a-h) = \lim\limits_{h\to 0} f(1-h)$

$= \lim\limits_{h\to 0} (1-2h)$

$= 1$ = finite



$f(x) = 2x-1$

$f(1-h) = 2(1-h)-1$

$= 2-2h-1$

$= \underline{1-2h}$

$f(x) = x$

$f(1+h) = 1+h$

RHL

$= \lim\limits_{h\to 0} f(a+h)$

$= \lim\limits_{h\to 0} f(1+h) = \lim\limits_{h\to 0} (1+h) = 1$ = finite

$\boxed{\text{LHL} = \text{RHL} \quad x=1 \text{ exists}}$

Topic : Single Variable Calculus

x^2 $\quad$ x^3 $\quad$ $4-x$

LHL $\quad$ 1 $\quad$ RHL $\quad$ 4

LHL $\quad$ RHL

$\boxed{\begin{array}{l} x = 1 \text{ exists} \\ x = 4 \text{ Does Not exists} \end{array}}$

Ans

#Q. Evaluate the left hand and right hand limits of the function defined by

$$f(x)=\begin{cases} x^2 & , \quad x<1 \\ x^3 & , \quad 1<x<4 \\ 4-x & , \quad x>4 \end{cases}$$

at $x = 1, 4$ and hence check existence of limit at x = 1, 4.

Topic : Single Variable Calculus

$$\left.\begin{matrix} 3+h \\ 3-h \end{matrix}\right]$$

#Q. Evaluate the left hand and right-hand limits of the function

$$f(x)=\begin{cases} \dfrac{\sqrt{(x^2-6x+9)}}{(x-3)}, & x\neq 3 \\ 0, & x=3 \end{cases}$$

$$f(x)=\begin{cases} \dfrac{\sqrt{(x-3)^2}}{(x-3)} & x\neq 3 \\ 0 & x=3 \end{cases}$$

$$f(x)=\begin{cases} \dfrac{|x-3|}{(x-3)} & x\neq 3 \\ 0 & x=3 \end{cases}$$

at $x = 3$ and hence comment on the existence of limit at x = 3.

$$f(x) = \begin{cases} \dfrac{|x-3|}{(x-3)} & x \neq 3 \\ 0 & x = 3 \end{cases}$$

Left Hand Limit

$$LHL = \lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0} \frac{|\cancel{3}-h-\cancel{3}|}{(\cancel{3}-h-\cancel{3})}$$

$$= \lim_{h\to 0} \frac{|-h|}{-h} = \lim_{h\to 0} \frac{h}{-h}$$

$$= -1$$

$$RHL = \lim_{h\to 0} f(3+h) = \lim_{h\to 0} \frac{|\cancel{3}+h-\cancel{3}|}{[\cancel{3}+h-\cancel{3}]}$$

$$= \lim_{h\to 0} \frac{|h|}{h} = 1$$

$LHL = -1$
$RHL = +1$

$LHL \neq RHL$

does Not exists

Key pt $\longrightarrow$ Square Root, modulas $\Big]$ Limit - always - does Not exists

Topic : Single Variable Calculus

H.W

Do yourself

#Q. If $f(x)=\begin{cases} \dfrac{x-|x|}{x} & , \quad x\neq 0 \\ 2 & , \quad x=0 \end{cases}$ the $\lim\limits_{h\to 0} f(x)$ is

A 2

B 0

C 1

D Does not exist

2 mins Summary

Topic One

Topic Two

Topic Three

Topic Four

Topic Five

Limit existence

$\underline{LHL = RHL} = \text{finite}$

THANK - YOU

Topics to be Covered