GATE
DATA SCIENCE AND AI

CALCULUS AND OPTIMIZATION
SINGLE VARIABLE CALCULUS

Lecture No.- 06

By- Rahul sir

Recap of previous lecture

Topic Evaluation of limits

Topic Evaluation of limits, Mean value theorem

Topics to be covered

After Calculus

Esh

correl

99%

OR Simplex

Topic — Continuity of the function

questions

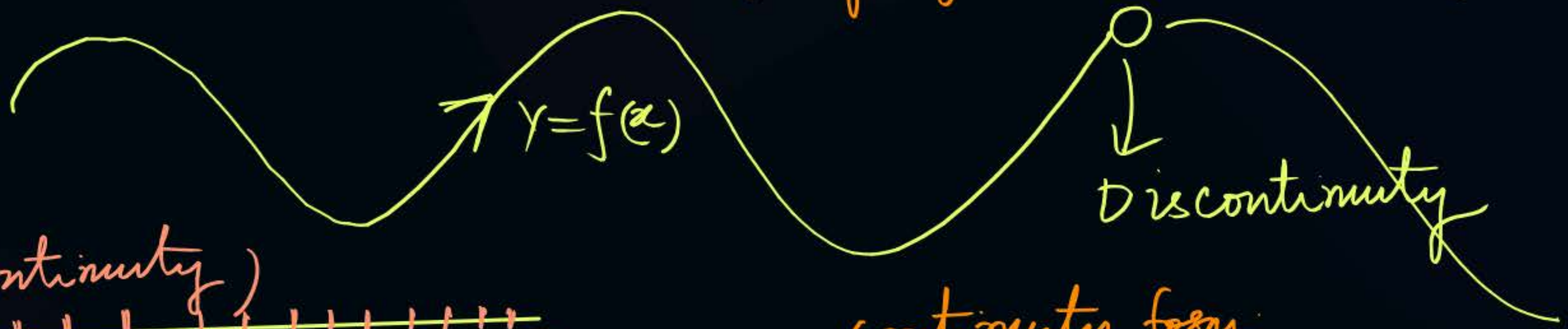
Topic — Differentiability of the functions

Topic — Mean value theorem

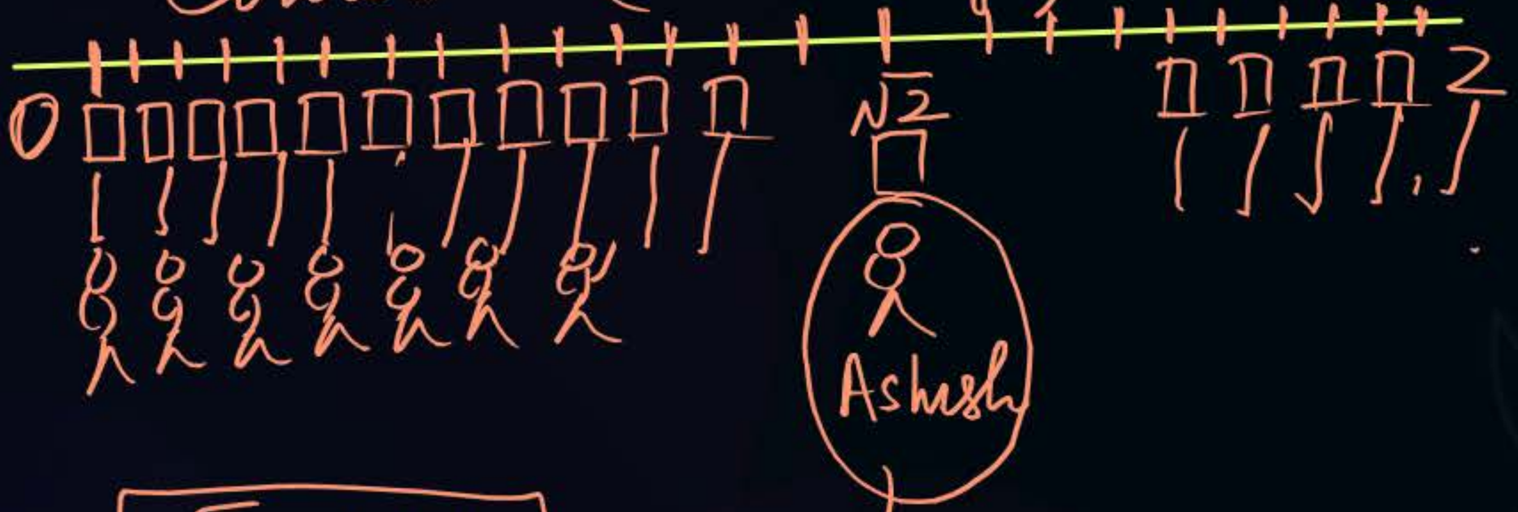
2-3 LECTURES

Taylor SERIES + Optimization (max or min) one variable

Continuity of a function : without lifting my Pen is continuity



Continuum (continuity)

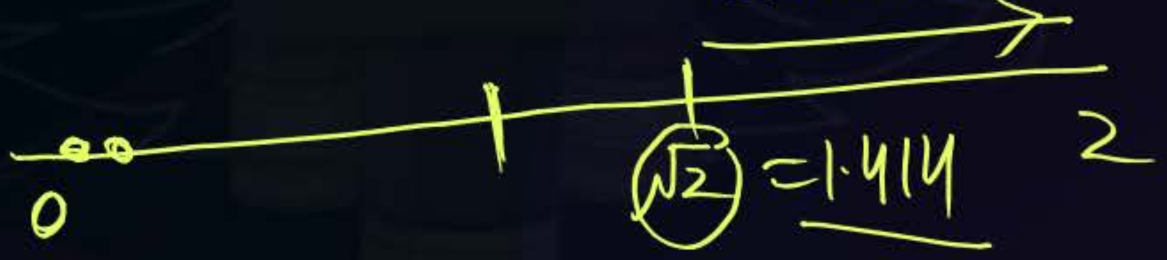


$\boxed{\sqrt{2}=1.414\ldots}$ - - -

Sit
approximation

continuity form

Real No Line – Continuum
follow
continuum



If $y=f(x)$

condition for continuity

$$\boxed{LHL = RHL = f(a)}$$

$$\lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a+h) = f(a)$$

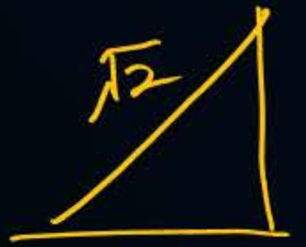
<u>Condition</u>

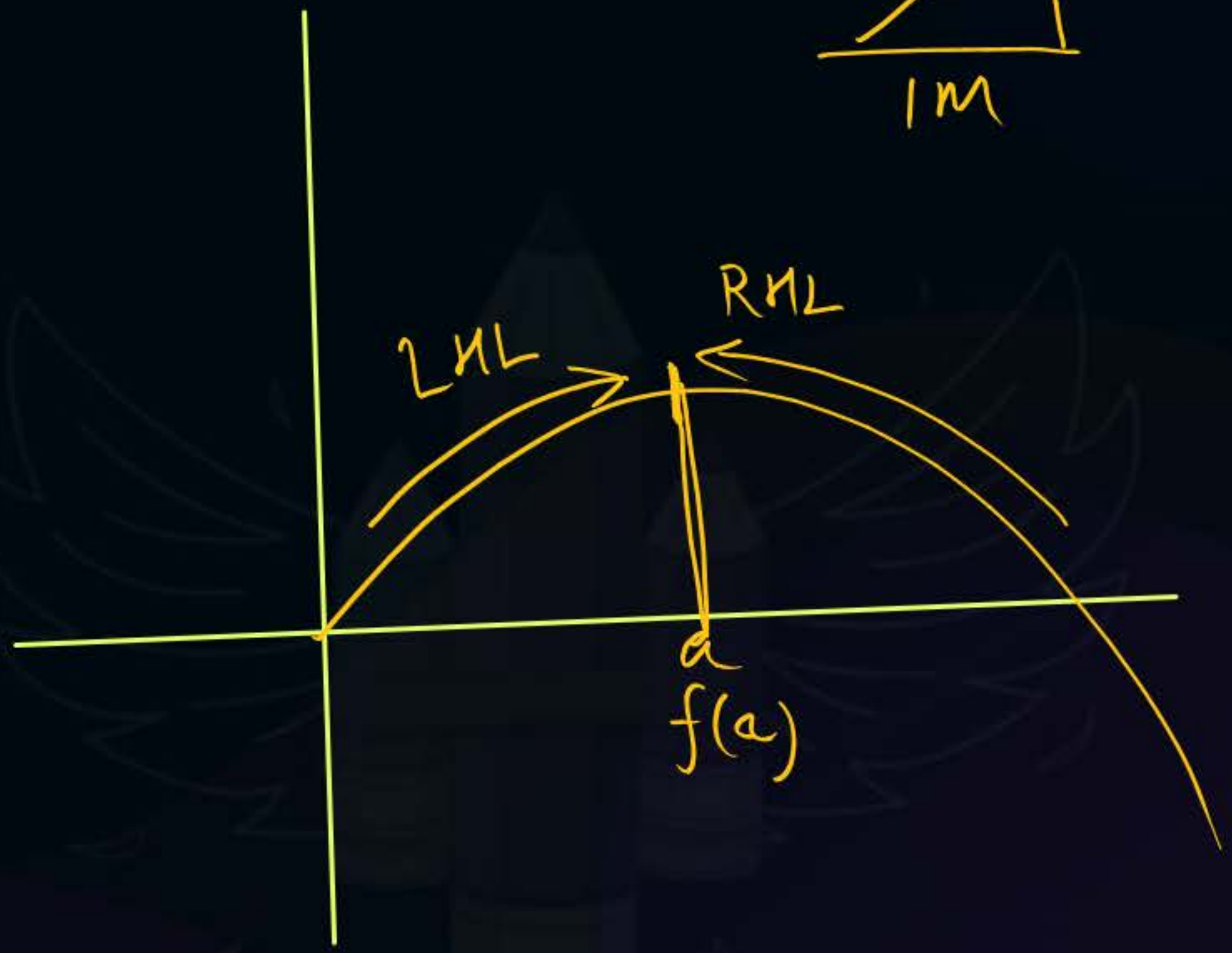
$= \pi r^2 = A$

$V = \frac{4}{3}\pi r^3$

$\sqrt{2} = 1.414\ldots$

uncertain

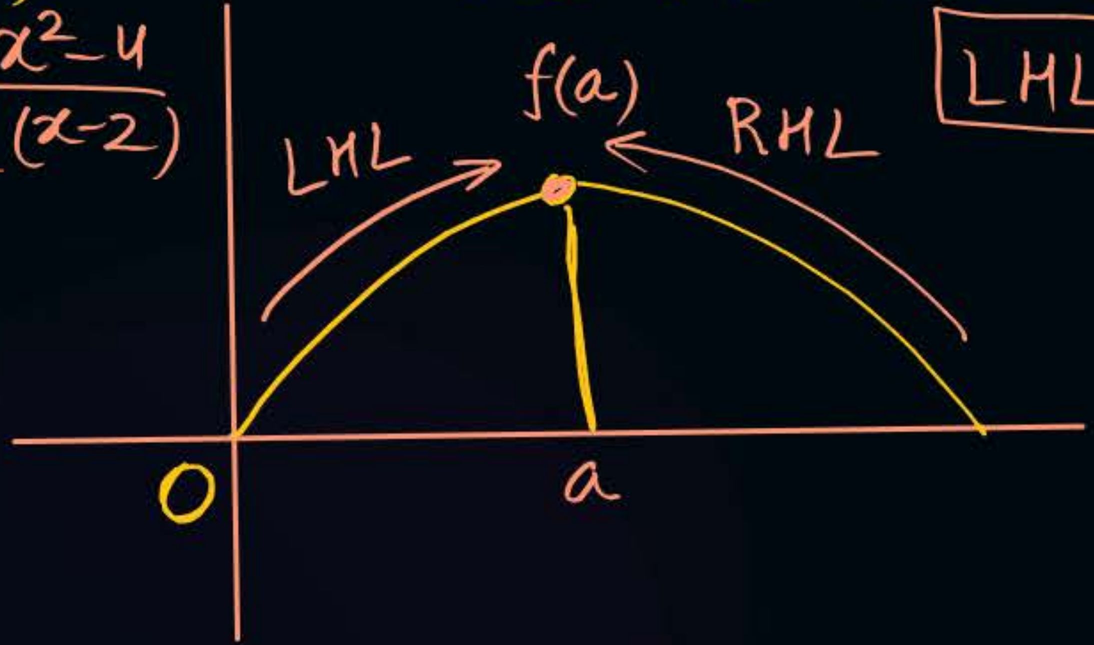




Types of Discontinuity:

A) Removable Discontinuity

$f(x) = \lim_{x \to 2} \frac{x^2 - 4}{(x-2)}$

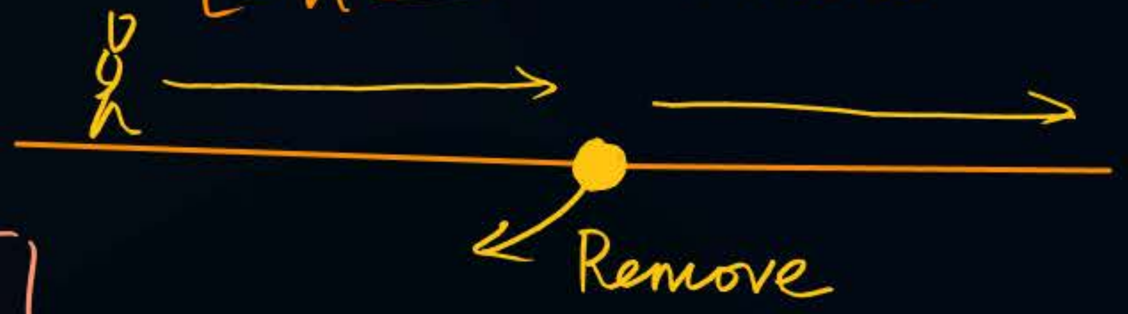


$\boxed{LHL = RHL \neq f(a)}$

fact
ral
L-H

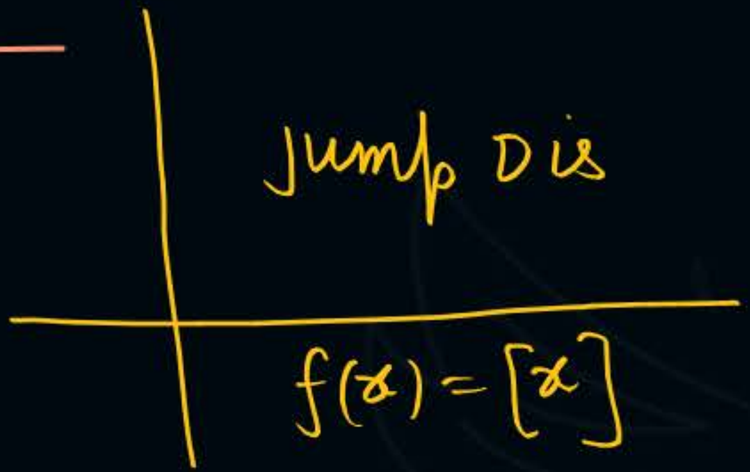
$\frac{\to 0}{\to 0}, \frac{\to \infty}{\to \infty}$

I)

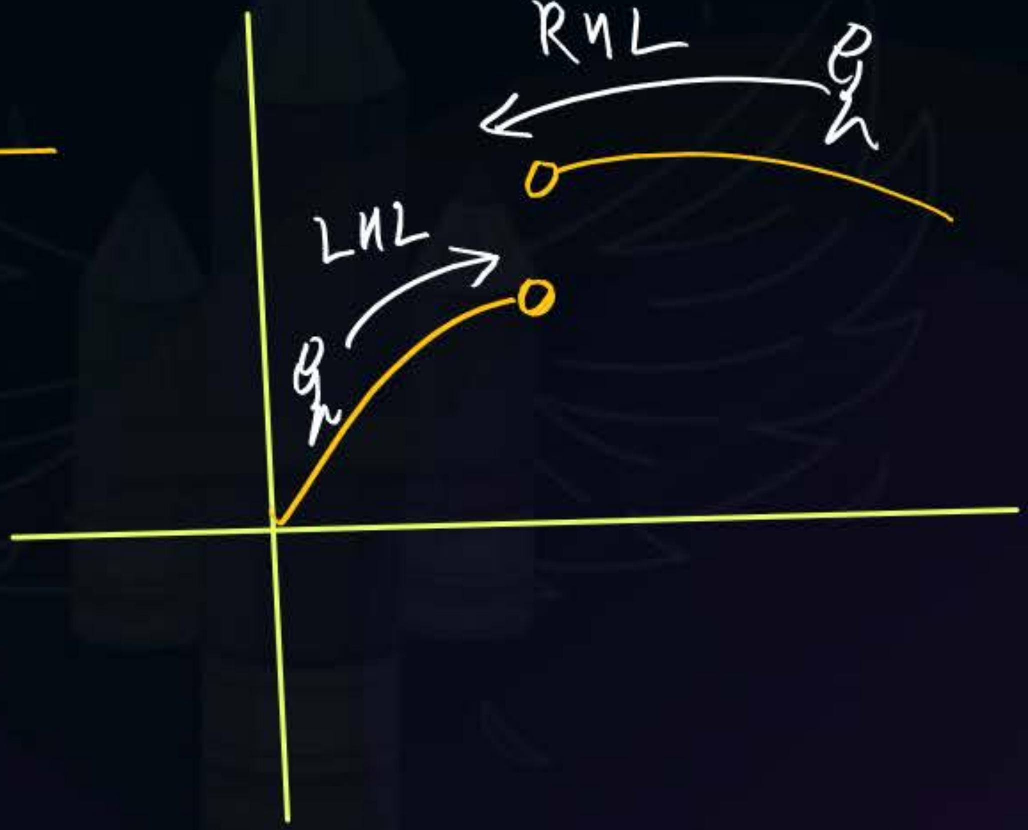


II)

factorization $\lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)} = 4$



B) Irremovable Discontinuity → Jump Discontinuity



Jump Discontinuity

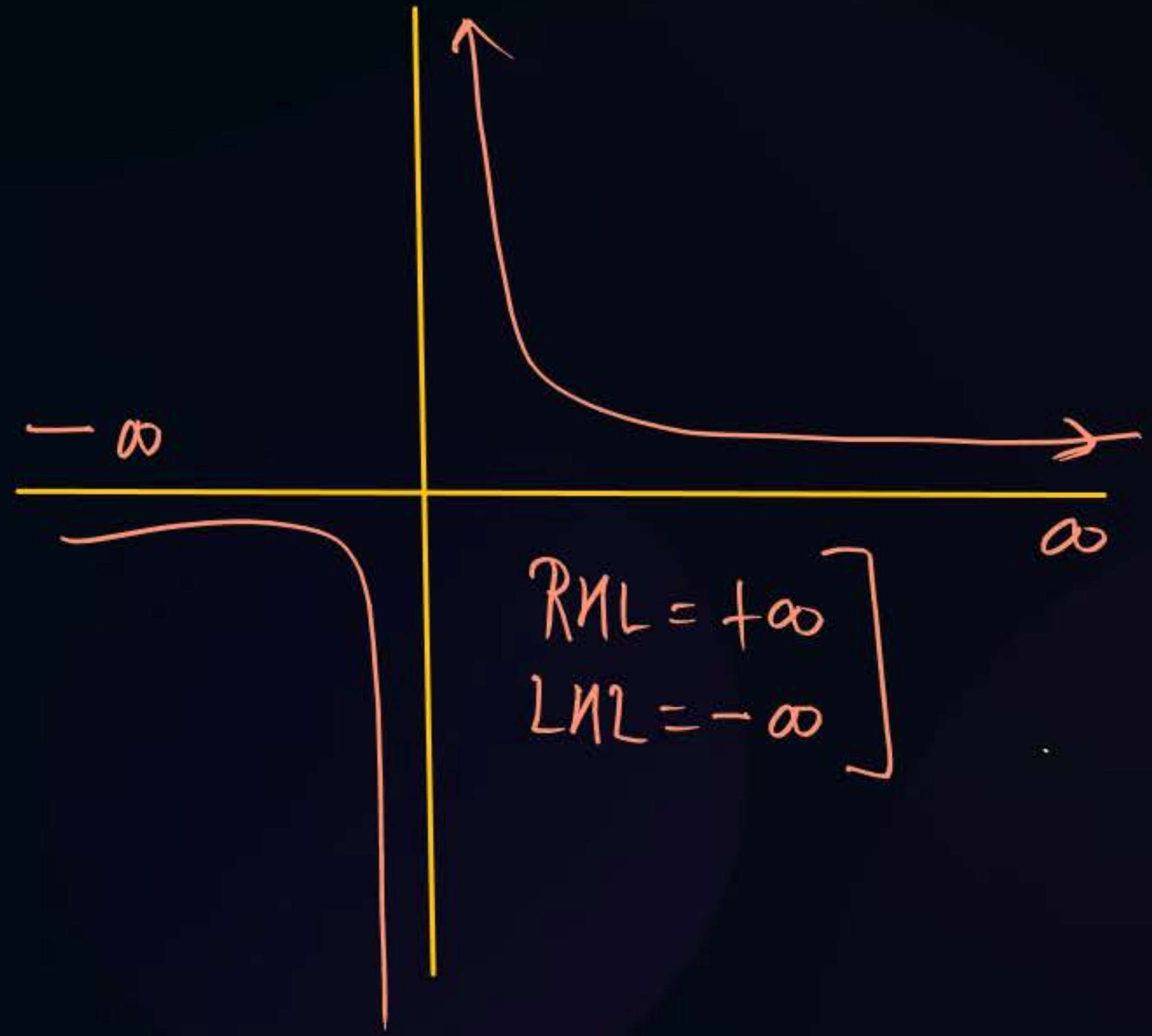
LHL $\neq$ RHL

$$\lim_{h\to 0} f(a-h) \neq \lim_{h\to 0} f(a+h)$$

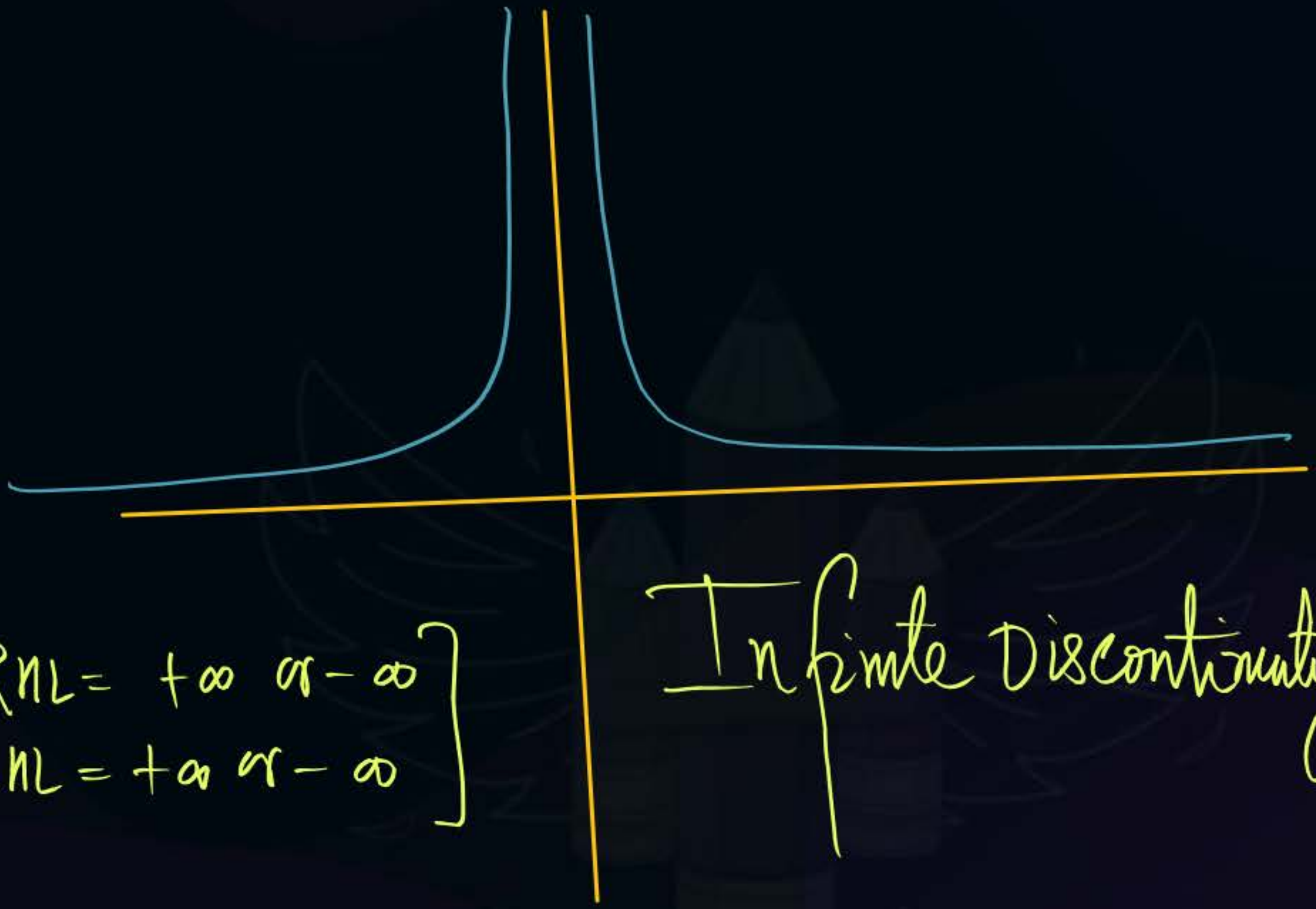
Jump $= |\text{RHL} - \text{LHL}|$

A) Irremovable Dis

B) Infinite Discontinuity:



RHL $= +\infty$
LHL $= -\infty$



RHL $= +\infty$ or $-\infty$
LHL $= +\infty$ or $-\infty$

Infinite Discontinuity

Topic : Single Variable Calculus

#Q. JEE Advanced

$$\text{Let } f(x) = \begin{cases} (1+|\sin x|)^{\frac{a}{|\sin x|}} & ; \quad \frac{-\pi}{6} < x < 0 \\ b & ; \quad x = 0 \\ e^{\left(\frac{\tan 8x}{\tan 3x}\right)} & ; \quad 0 < x < \frac{\pi}{6} \end{cases}$$

$$e^{\frac{\tan 8x}{\tan 3x} \cdot \frac{3x}{3x} \cdot \frac{8x}{8x}}$$

$$\boxed{b = e^{\frac{8}{3}}}$$

$$LHL = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} f(-h) = \lim_{h \to 0} \left[1 + |\sin(0-h)|\right]^{\frac{a}{|\sin(-h)|}}$$

$$= \lim_{h \to 0} \left[1 + \sin h\right]^{\frac{a}{\sin h}} = \to 1^{\to \infty} = e^{A}$$

LHL = RHL

$$e^{8/3} = e^{a}$$

$$a = \frac{8}{3}$$

$$b = e^{\frac{8}{3}}$$

The value a and b such that f(x) is continuous at x = 0 is :

Where

$$A = \lim_{h \to 0} \left[\cancel{1} + \sin h - \cancel{1}\right] \frac{a}{\sin h}$$

$$\boxed{A = a}$$

$$L = e^{a}$$

A $a = 8, b = e^{8}$

B $a = \frac{8}{3}, b = e^{-8}$

C $a = \frac{8}{3}, b = e^{8/3}$

D None of these

Topic : Single Variable Calculus

#Q. Let

$$f(x)=\begin{cases} \dfrac{1-\cos 4x}{x^2} & ; \quad x<0 \\ a & ; \quad x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & ; \quad x>0 \end{cases}$$

The value of a, if possible, so that the functions is continuous at x=0 is

A 6

B 8

C -6

D None of these

$\frac{1-\cos 4x}{x^2}$ $\quad a \quad$ $\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$

$x<0 \quad 0 \quad x>0$

Calculate $\rightarrow a$

Function is continuous

$LHL = RHL = f(0) = a$

$$LHL = \lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} f(-h)$$

$$= \lim_{h\to 0} \frac{1-\cos 4(-h)}{(-h)^2}$$

$$f(x) = \frac{1-\cos 4x}{x^2}$$

$$\Rightarrow \lim_{h\to 0} \frac{1-\cos 4h}{h^2} \left(\frac{\to 0}{\to 0}\right) \text{ Form}$$

Using L-Hospital Rule

$$= \lim_{h\to 0} \frac{+\sin 4h \cdot 4 \times 2}{2\times 2h} = \lim_{h\to 0} \left(\frac{\sin 4h}{4h}\right) \times 8$$

$$\boxed{LHL = 8}$$ ✓

$$RHL = \lim_{h\to 0} f(a+h) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} f(h)$$

$$= \lim_{h\to 0} \frac{\sqrt{h}}{\sqrt{16+\sqrt{h}}-4} = \underline{8}$$

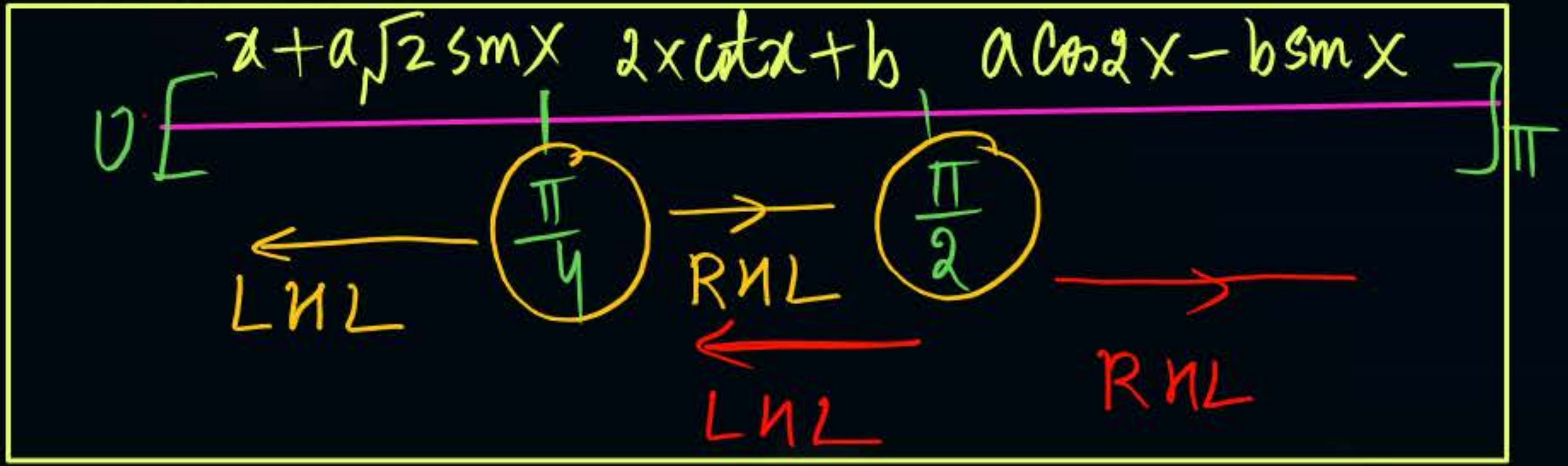
$$LHL = RHL = f(0)$$

$$\boxed{8 = 8 = a}$$

$$a = 8$$

#Q. The values a and b so that the functions :

$$f(x)=\begin{cases} x+a\sqrt{2}\sin x & ; \ 0\le x<\dfrac{\pi}{4} \\ 2x\cot x+b & ; \ \dfrac{\pi}{4}\le x\le\dfrac{\pi}{2} \\ a\cos 2x-b\sin x & ; \ \dfrac{\pi}{2}<x\le\pi \end{cases}$$



is continuous $x\in[0,\pi]$ is :

(A) $a=\dfrac{\pi}{6}, b=\dfrac{-\pi}{12}$

(B) $a=\dfrac{\pi}{3}, b=\dfrac{-\pi}{12}$

(C) $a=\dfrac{\pi}{6}, b=\dfrac{\pi}{12}$

(D) None of these

LHL at $x=\frac{\pi}{4}$

$x+a\sqrt{2}\sin x$ LHL

$2x\cot x+b$ RHL

$$\text{LHL}=\lim_{h\to 0} f(a-h)=\lim_{h\to 0} f\left(\frac{\pi}{4}-h\right) \quad x\to\frac{\pi}{4}-h$$

$$=\lim_{h\to 0}\left(\frac{\pi}{4}-h\right)+a\sqrt{2}\sin\left(\frac{\pi}{4}-h\right)$$

$$=\frac{\pi}{4}+a\sqrt{2}\sin\frac{\pi}{4}=\boxed{\frac{\pi}{4}+a} \text{ LHL}$$

$$\text{RHL}=\lim_{h\to 0} f(a+h)=\lim_{h\to 0} f\left(\frac{\pi}{4}+h\right)=\lim_{h\to 0} 2\left(\frac{\pi}{4}+h\right)\cot\left(\frac{\pi}{4}+h\right)+b$$

$$=\left(\frac{\pi}{2}\right)\times 1+b=\boxed{\frac{\pi}{2}+b} \text{ RHL}$$

Condition for continuity

$$\text{LHL}=\text{RHL}$$

$$\frac{\pi}{4}+a=\frac{\pi}{2}+b$$

$$a-b=\frac{\pi}{2}-\frac{\pi}{4}$$

$$\boxed{a-b=\frac{\pi}{4}} \quad \text{—(1)}$$

at point $x=\frac{\pi}{2}$

$$LHL = \lim_{h\to 0} f(a-h) = \lim_{h\to 0} f\left(\frac{\pi}{2}-h\right) = \lim_{h\to 0} 2\left(\frac{\pi}{2}-h\right)\cot\left(\frac{\pi}{2}-h\right)+b$$

$$= \pi \times 0 + b = \underline{\underline{b}}$$

$2x\cot x + b = LHL$
$= RHL$

$a\cos 2x - b\sin x$
$= LHL$

$$RHL = \lim_{h\to 0} f(a+h) = \lim_{h\to 0} f\left(\frac{\pi}{2}+h\right) = \lim_{h\to 0} a\cos 2\left(\frac{\pi}{2}+h\right) - b\sin\left(\frac{\pi}{2}+h\right)$$

$$= a\cos\pi - b\sin\frac{\pi}{2}$$

Condition for conti

$$RHL = \underline{-a-b}$$

$LHL = RHL$

$b = -a-b$

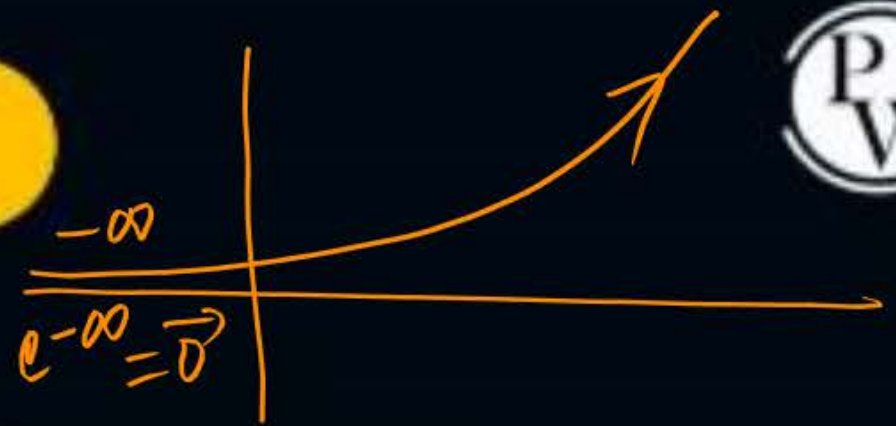
$2b+a=0$ — (2)

Solve the Equ^n (1) and (2) and get the value a and b

$a = \pi/6$

$b = -\pi/12$

Topic : Single Variable Calculus



#Q. Discuss the continuity of $f(x) = \begin{cases} \dfrac{e^{1/x}-1}{e^{1/x}+1} & ; \quad x \neq 0 \\ 0 & ; \quad x = 0 \end{cases}$ at the point x =0

LHL at Point $x = 0$

$$LHL = \lim_{h \to 0} f(a-h) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} f(-h)$$

$$= \lim_{h \to 0} \frac{\cancel{e^{-1/h}}^{\,0} - 1}{\cancel{e^{-1/h}}^{\,0} + 1} = -1$$

$$RHL = \lim_{h \to 0} f(a+h) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{e^{\frac{1}{h}} - 1}{e^{1/h} + 1}$$

$$= \lim_{h \to 0} \frac{e^{\frac{1}{h}} - 1}{e^{1/h} + 1} = \lim_{h \to 0} \frac{1 - \cancel{e^{-\frac{1}{h}}}^{\,0}}{1 + \cancel{e^{-1/h}}^{\,0}} = 1 \quad \underline{\text{Ans.}}$$

$e^{-\frac{1}{h}}$

" Using L-Hospital Rule "

$$\left.\begin{aligned} LHL &= -1 \\ RHL &= +1 \end{aligned}\right] \quad LHL \neq RHL$$

<u>Jump Discontinuity</u>

2 mins Summary

SAT – Q – Extra

Topic One — continuity

Topic Two — Discontinuity

Topic Three

Topic Four

Topic Five

THANK - YOU

[redacted]