



CS & IT ENGINEERING

Data Structures

Stack and Queues

Lecture No.- 02

By- Pankaj Sharma Sir



Recap of Previous Lecture



Topic

Stack and Queues Part - 01



✓ Stack $\left\{ \begin{array}{l} \text{Push} \\ \text{Pop} \\ \text{IsEmpty} \end{array} \right.$

✓ Structure

Topics to be Covered



Topic

Stack and Queues Part - 02

Stack permutation
infix to postfix





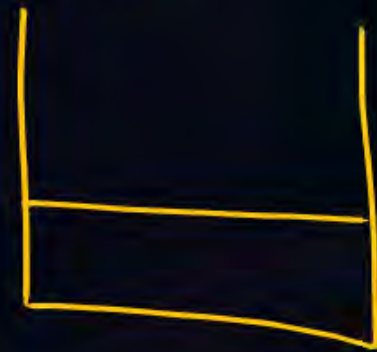
Topic : Stack and Queues



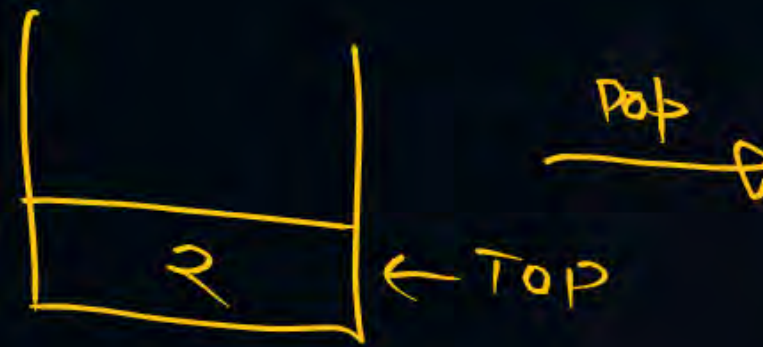
Stack Permutation

insertion order : $\overrightarrow{1, 2, 3}$ (fix)

1) $\checkmark \checkmark \checkmark$
 $1, 2, 3$
pop order



Push(2)



Pop



- (i) Push(1)
- (ii) Pop()
- (iii) Push(2)
- (iv) Pop()
- (v) Push(3)
- (vi) Pop()

$1, 2, 3 \Rightarrow$ valid stack permutation

2) $\checkmark \checkmark \checkmark$ \rightarrow valid stack permutation

- (i) Push(1) (iii) Push(2) v) Pop()
(ii) Pop (iv) Push(3)



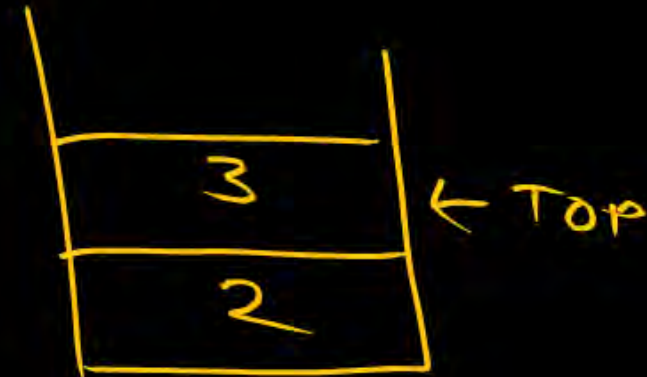
↓ Pop



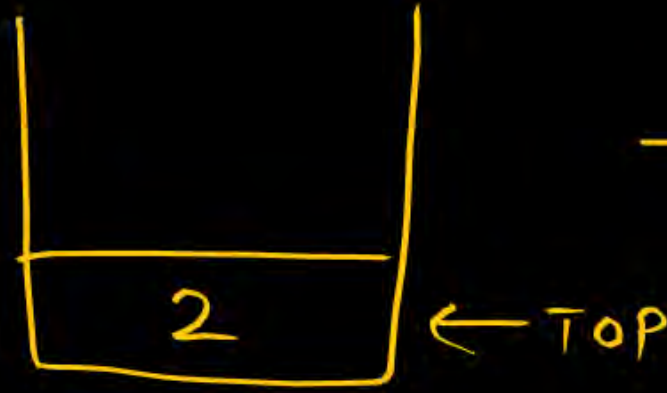
Push(2) \rightarrow



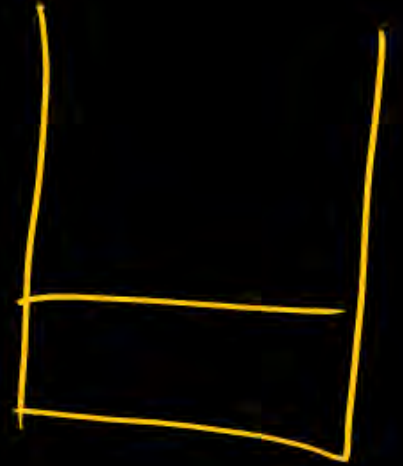
Push(3) \rightarrow



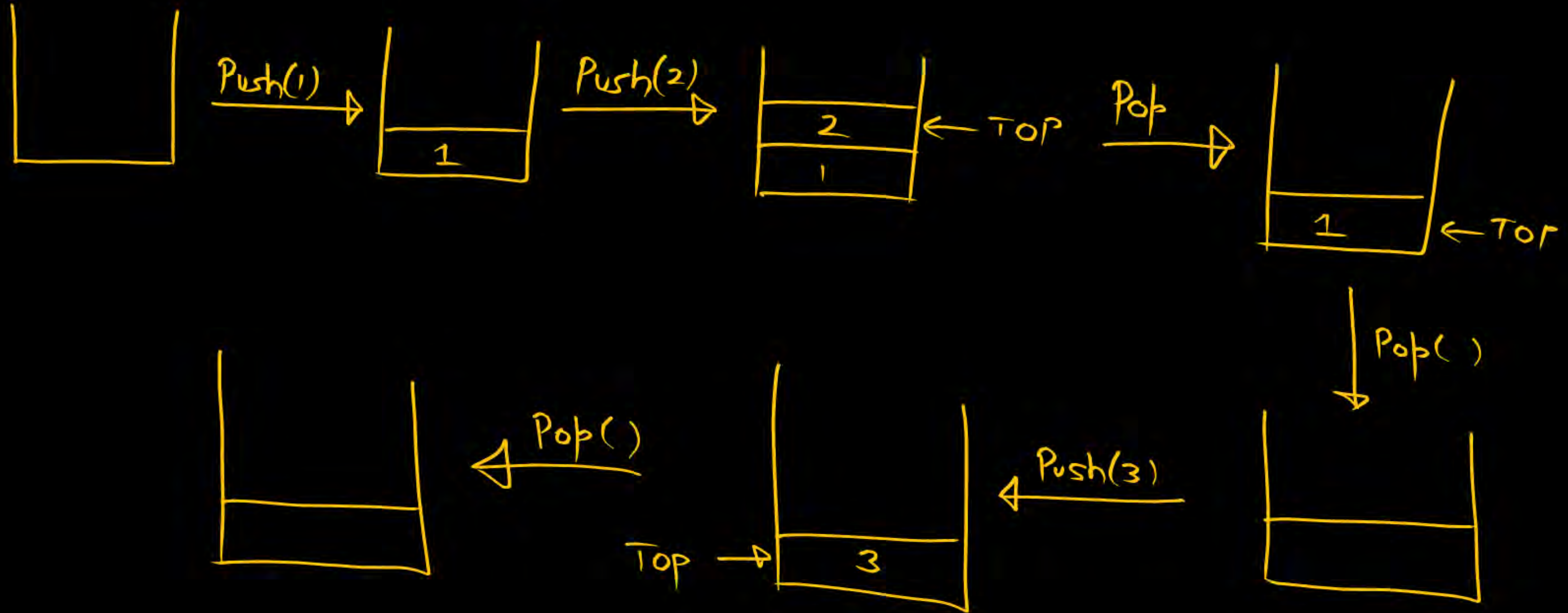
↑ Pop



Pop \rightarrow

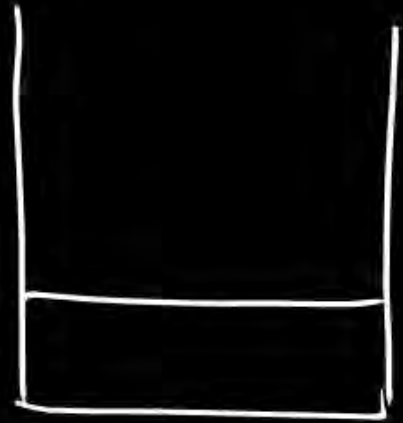


3.) $\checkmark \checkmark \checkmark$ 2,1,3 valid stack permutation

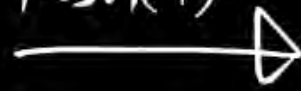


4.) $\checkmark \checkmark \checkmark$
2, 3, 1

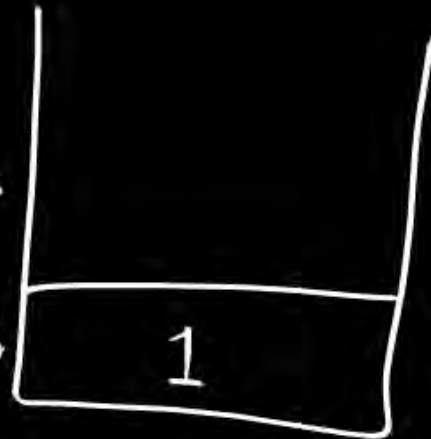
valid stack permutation



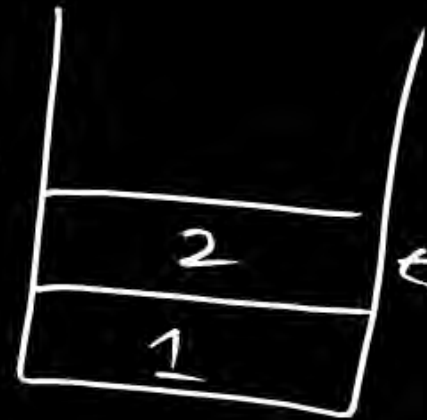
Push(1)



TOP →



Push(2)

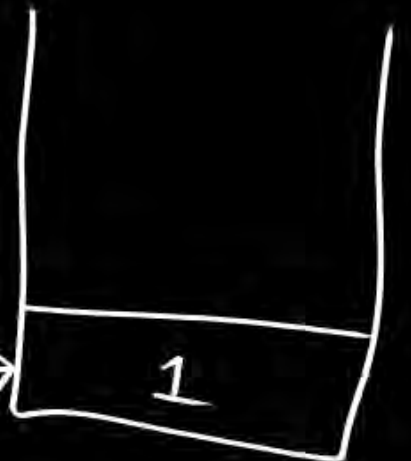


← TOP

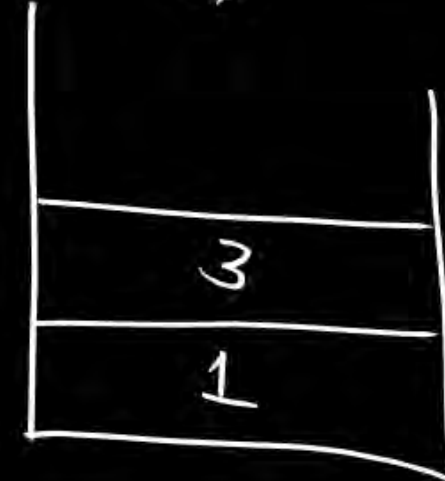
Pop()



TOP →



Push(3)

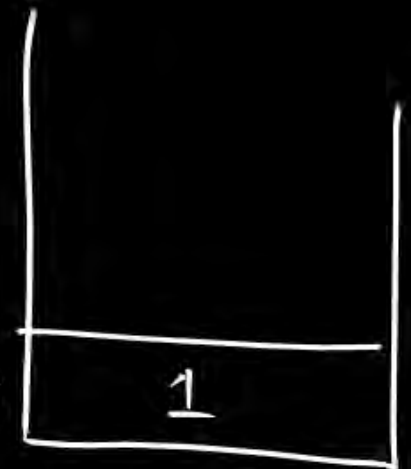


← TOP

Pop()



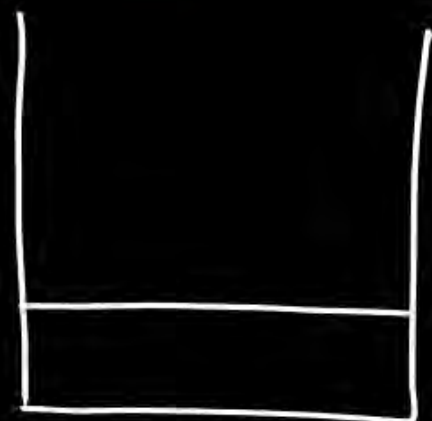
TOP →



Pop()



5) 3,1,2 Not a valid stack permutation

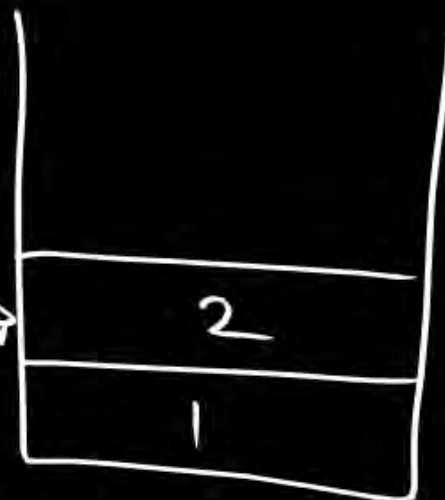


Push(1) →

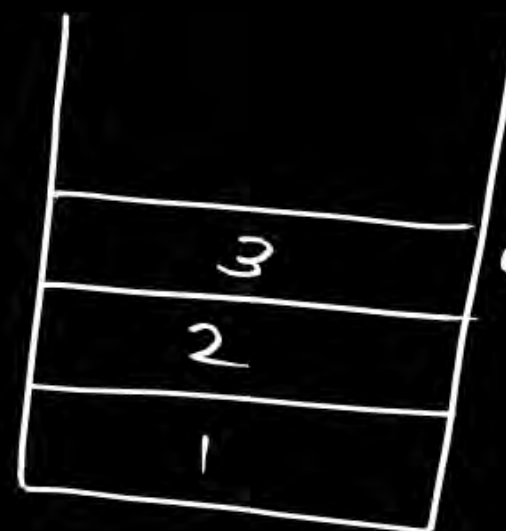


Push(2) →

TOP →

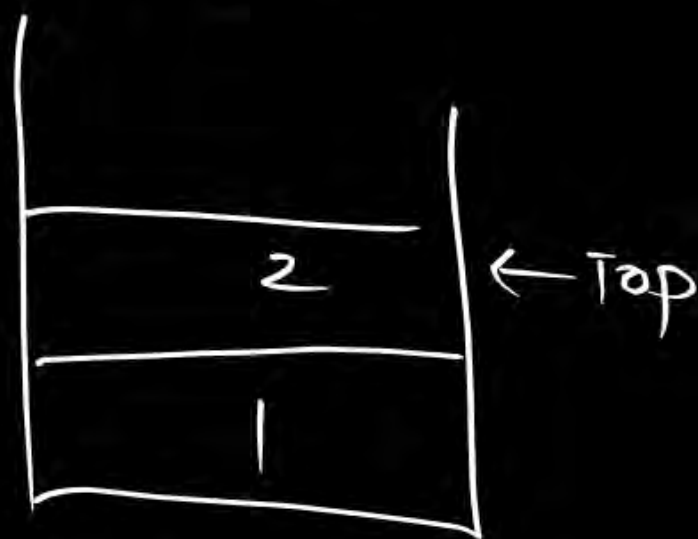


Push(3) →



↓ Pop

1 can not be popped



6) 3, 2, 1 valid stack permutation

- (i) Push(1)
- (ii) Push(2)
- (iii) Push(3)
- (iv) Pop()
- (v) Pop()
- (vi) Pop()

$$n = 3$$

Total permutation = 6

1, 2, 3
1, 3, 2
2, 1, 3
2, 3, 1
3, 1, 2
3, 2, 1

out of 6
permutation

5
valid
stack
permutation

n element
fix insertion
order

Stack Permutation

$$\frac{2n}{n+1} C_n$$

(Catalyn numbers)

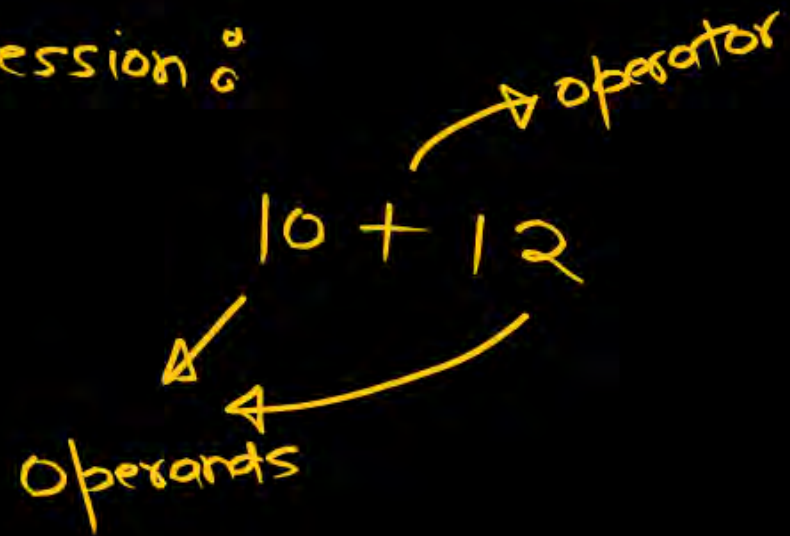
$$\frac{n=3}{}$$

$$\frac{6C_3}{4} = \frac{1}{4} \times \frac{6!}{3!3!}$$

$$= \frac{\cancel{4} \times \cancel{6} \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{3} \times \cancel{2} \times \cancel{1}} = 5$$

Infix, prefix and postfix

(i) Infix expression :



Operator is in b/w Operands

(ii) Prefix expression :

↓
before

Operator is before operands

infix : $a + b$
prefix : $+ab$

(iii) Postfix expression :

↓
after

Operator is after operands :

infix : $a + b$
postfix : $ab +$

Why postfix?

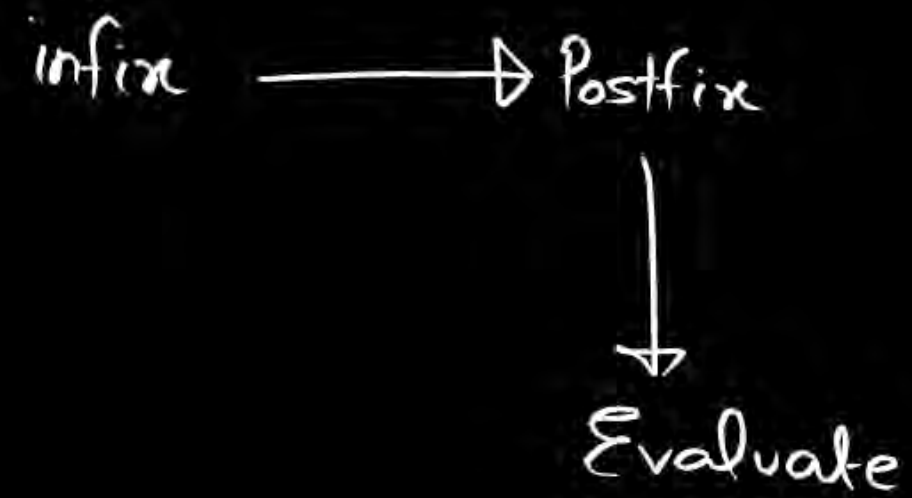
Computer \rightarrow Eyes X
scan
 \rightarrow

infix: $3 + 4 \times 3 - 15 / 5^2$

multiple scan
 \downarrow

time complexity

$3 + 4 \times 3 - 15 / 25$



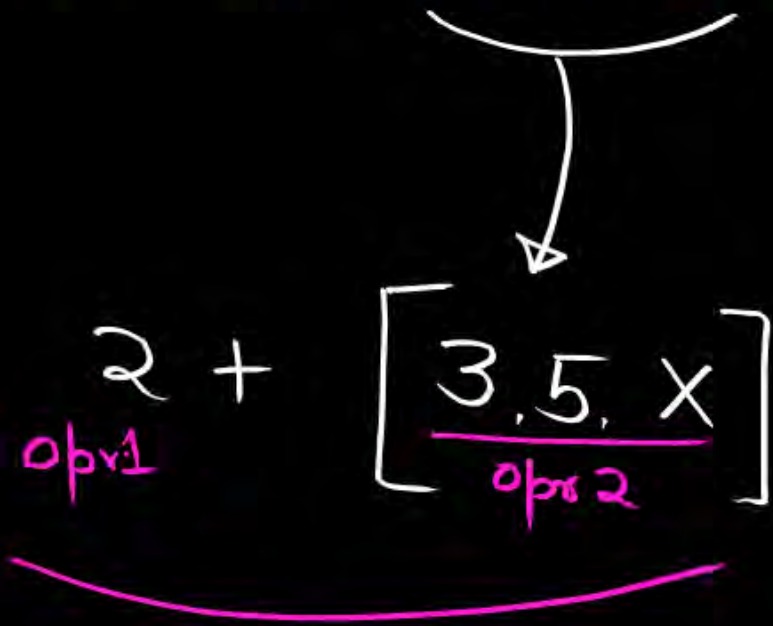
- (i) infix to postfix expression
- (ii) postfix evaluation

Infix to postfix



Priority, associativity

infix: $2 + 3 \times 5$



high
↓
low

postfix: $2\ 3\ 5\ x\ +$

$$\text{infix} : 3 + 5 \times 6 / 2^4$$

$$3 + 5 \times 6 / [2^4]$$

$$3 + \frac{[5 \ 6 \ X]}{\text{opr1}} / \frac{[2 \ 4^{\wedge}]}{\text{opr2}}$$

$$\text{opr1} \quad 3 + \frac{[5 \ 6 \ X \ 2 \ 4^{\wedge} /]}{\text{opr2}}$$

postfix : 3 5 6 X 2 4 ^ / +

theory
^ : Power

^	high	R to L
X, /		L to R
+	low	L to R

infix: $(a+b) \times c/d - e^{\wedge} f^{\wedge} g/h$

$$[ab+] \times c/d - e^{\wedge} f^{\wedge} g/h$$

$$[ab+] \times c/d - e^{\wedge} [fg^{\wedge}] / h$$

$$[ab+] \times c/d - [efg^{\wedge\wedge}] / h$$

$$[ab+cx]/d - [efg^{\wedge\wedge}] / h$$

$$[ab+cx d/] - [efg^{\wedge\wedge}] / h$$

$$[ab+cx d/] - [efg^{\wedge\wedge} h/]$$

$$ab+cx d/efg^{\wedge\wedge} h/-$$

high

low

$$\left. \begin{array}{cc} \wedge & \\ \times & / \\ + & + \end{array} \right\} \begin{array}{l} R \text{ to } L \\ L \text{ to } R \end{array}$$

2 min

Infix : $2+3$
Prefix : $+23$
Postfix : $23+$

Infix to postfix using stack

Infix : $2+3 \times 5$
Prefix : $+2 \times 35$
Postfix : $235 \times +$

Infix to postfix

Ex1

scan
↓
infix : 2+3

operand → output

o/p: 2



Infix to postfix

Ex1

infix : $2+3$

operator is
encounter

operand \longrightarrow output

o/p : 2



stack is empty

Push encountered
op. onto stack

Infix to postfix

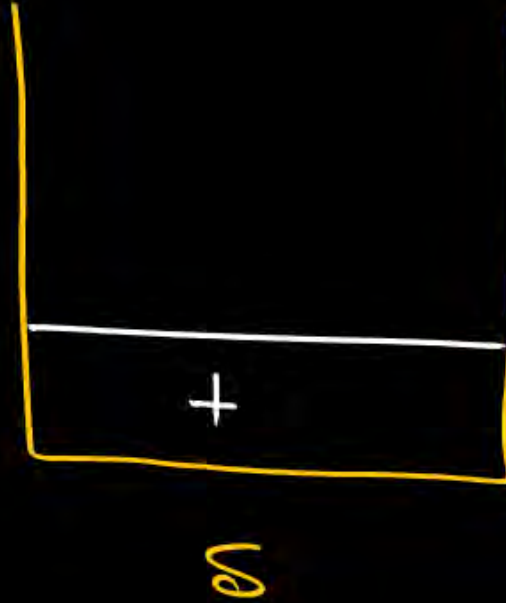
Ex 1

infix : $2+3$

operator is
encounter

operand \longrightarrow output

o/p : 2

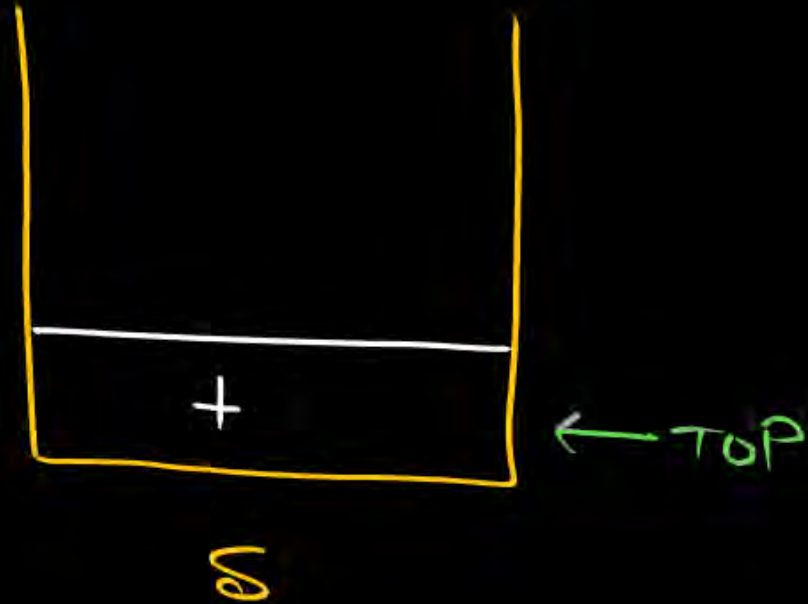


Infix to postfix

Ex1 infix : 2+3 

operand \longrightarrow output

o/p: 2 3



Infix to postfix

Ex1 infix: 2+3

operand \rightarrow output

o/p: 23



Pop everything
from stack &
add to o/p

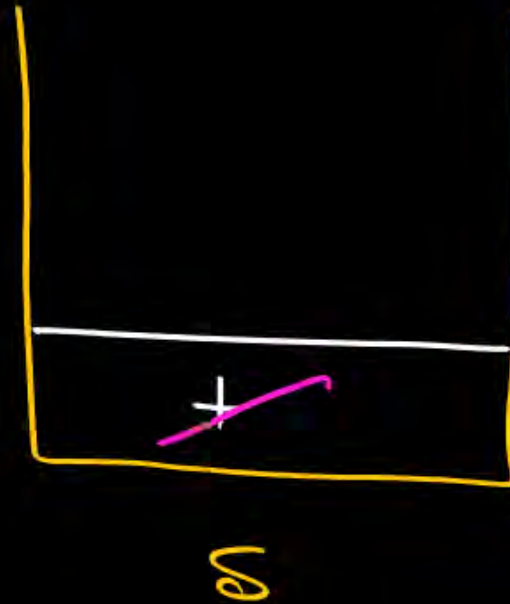
Infix to postfix

Ex1 infix: 2+3

operand \rightarrow output

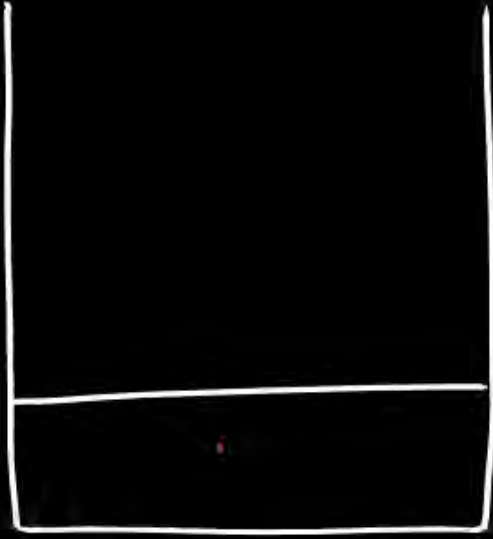
o/p: 23+

Postfix: 23+



Pop everything
from stack &
add to o/p.

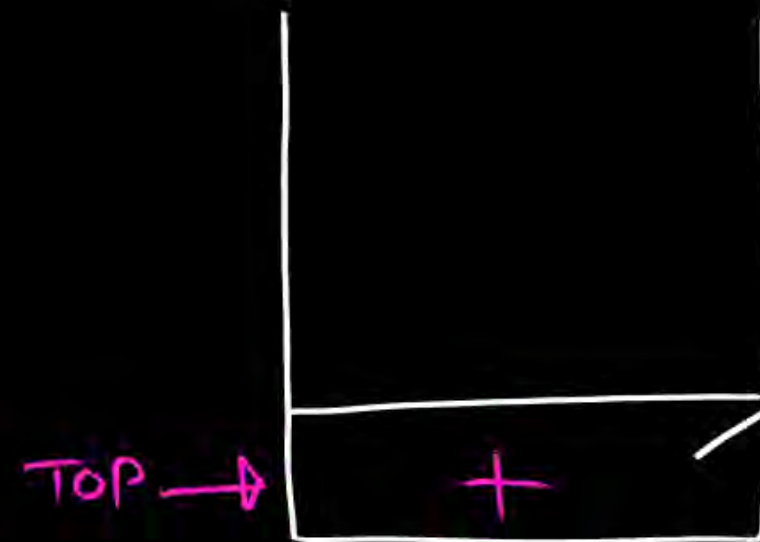
Ex 2: infix: $3 + 2 - 1$
o/p: 3



stack empty

Ex 2: infix: $3 + 2 - 1$

o/p: $3 2$

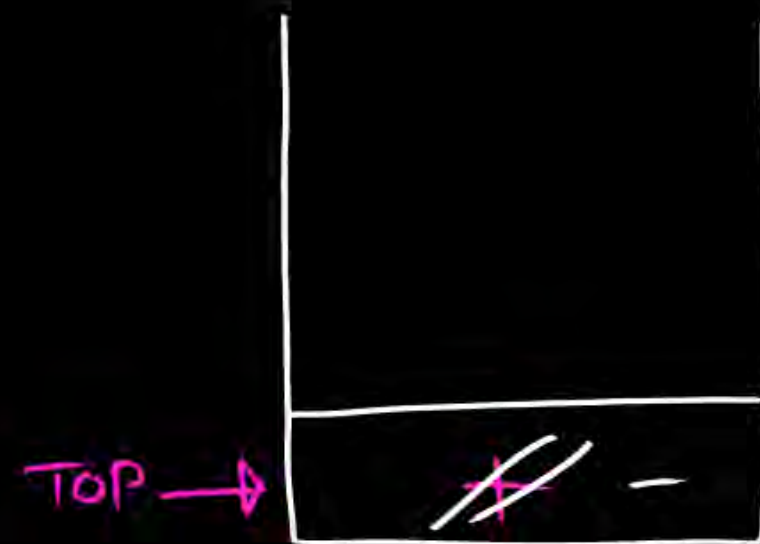


wait for 2nd & 1st
stack empty
Same + -
L to R
(Pop and add to o/p)

op. found $\Rightarrow -$

op. at top of stack $\Rightarrow +$

Ex 2: infix: $3 + 2 - 1$
o/p: $3 2 +$



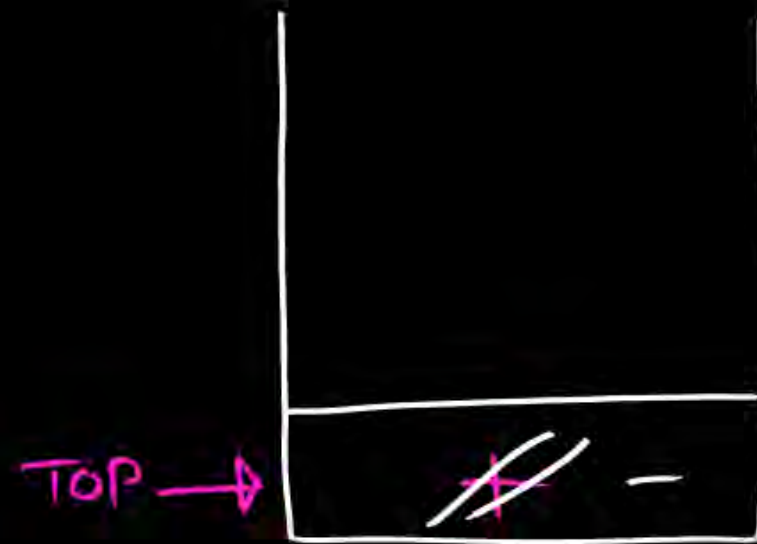
L to R
 $3 + 2 - 1$

(Pop and add to o/p)

Push -

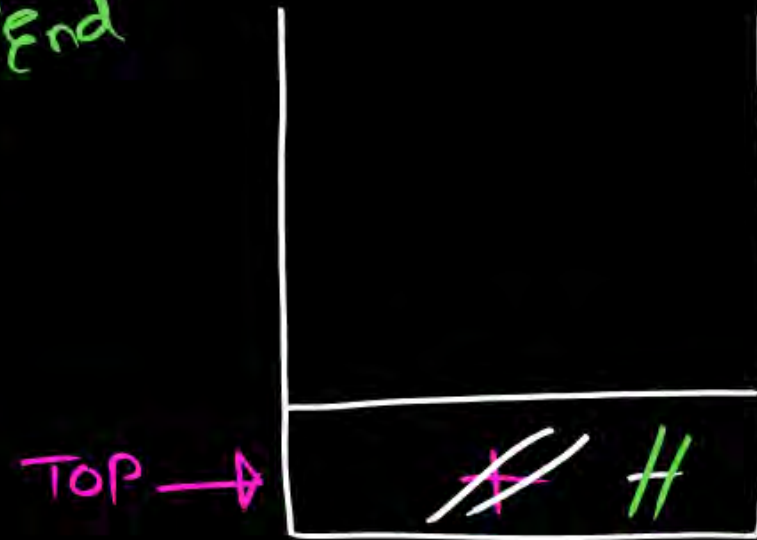
Ex 2: infix: $3+2-1$

o/p: $32+1$



L to R

Ex 2: infix: $3 + 2 - 1$
o/p: $3 2 + 1 -$



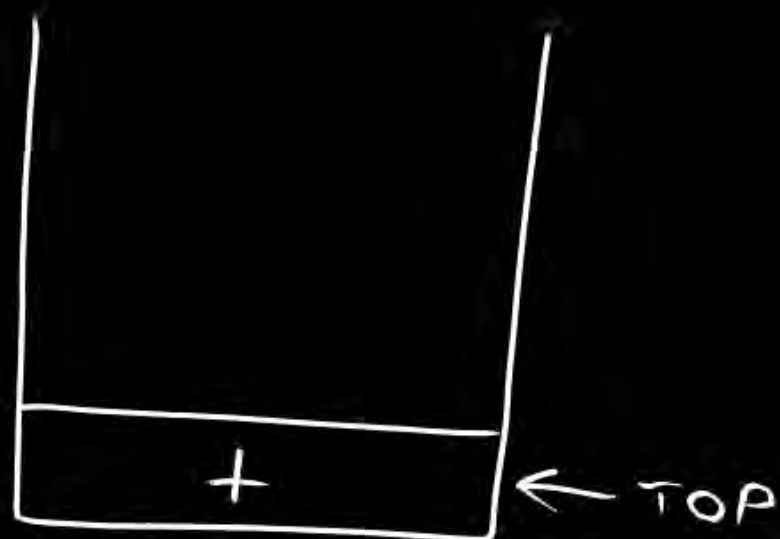
postfix: $3 2 + 1 -$

pop everything from stack
→ o/p

Ex3

infix : 2 + 3 x 5

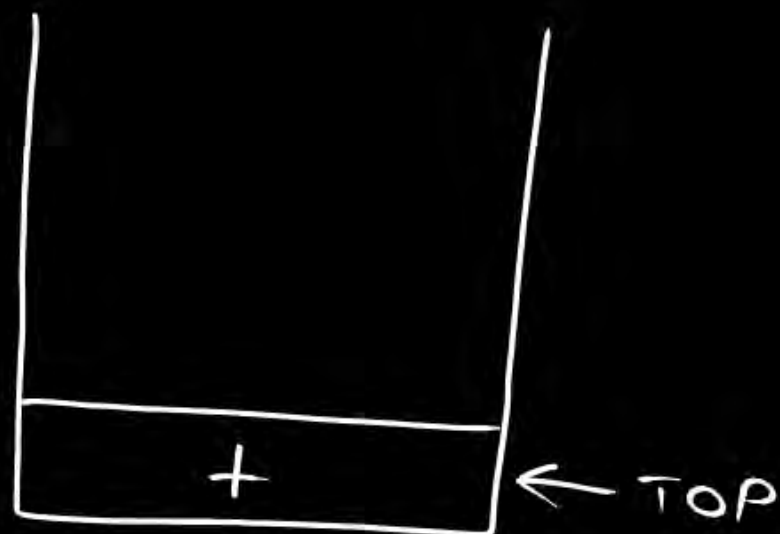
o/p : 2 3



Ex3

Infix : $2 + 3 \times 5$

O/P : 2 3



X (VIP)
↓
+ (गरीब)

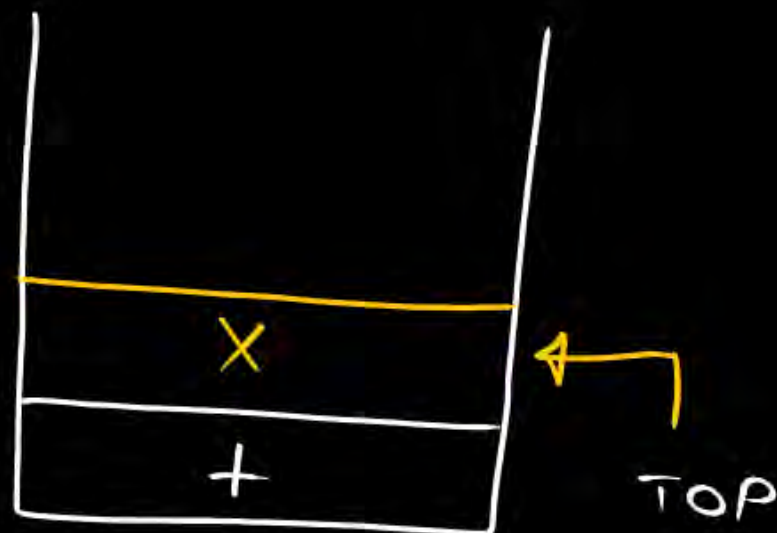
Op found : X

Push it

Ex3

Infix : $2 + 3 \times 5$

O/P : 2 3



X (VIP)
↓
+ (गरीब)

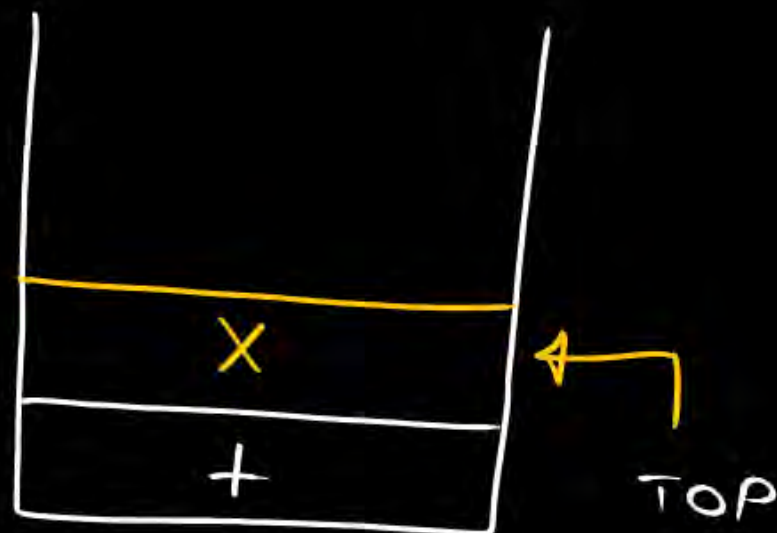
Op found : X

Push it

Ex3

Infix : $2 + 3 \times 5$

O/P : $2 \ 3 \ 5$



\downarrow X (VIP)
 \downarrow + (गरीब)

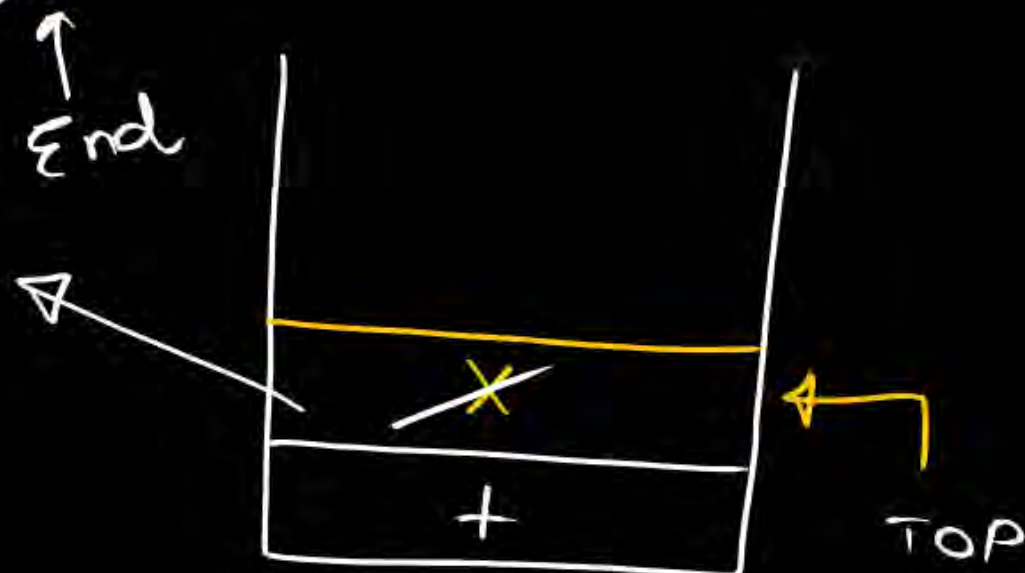
Op found : X

Push it

Ex3

Infix: $2 + 3 \times 5$

O/P: $2 3 5 \times$



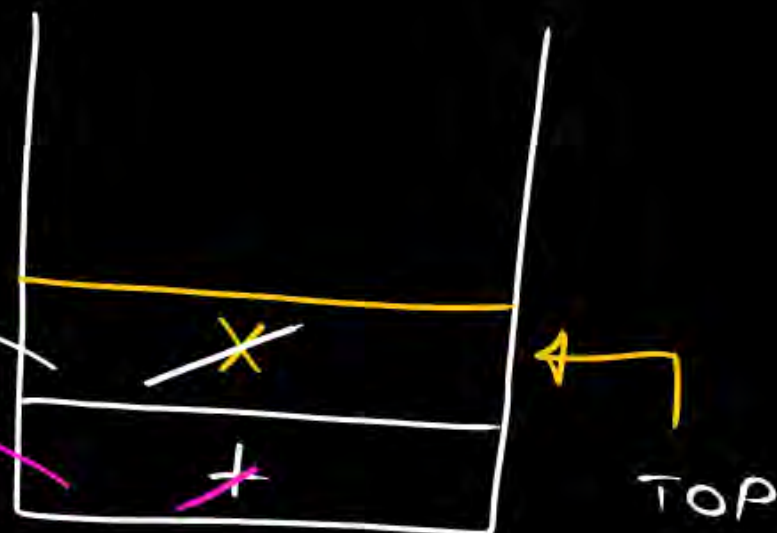
pop everything (one by one)

↓
O/P

Ex3

Infix: $2 + 3 \times 5$

O/P: $2\ 3\ 5\ \times\ +\ \text{End}$



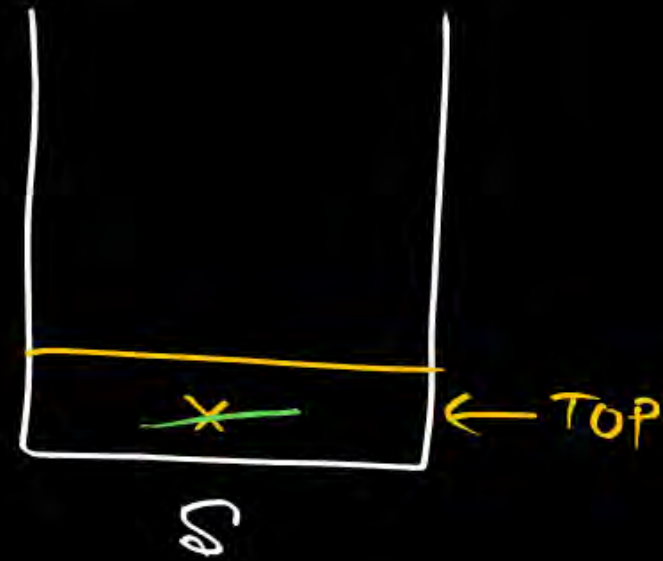
Postfix: $2\ 3\ 5\ \times\ +$

pop everything (one by one)

↓
O/P

Ex 4.

infix : $2 \times 3 + 4$
o/p : $23x$

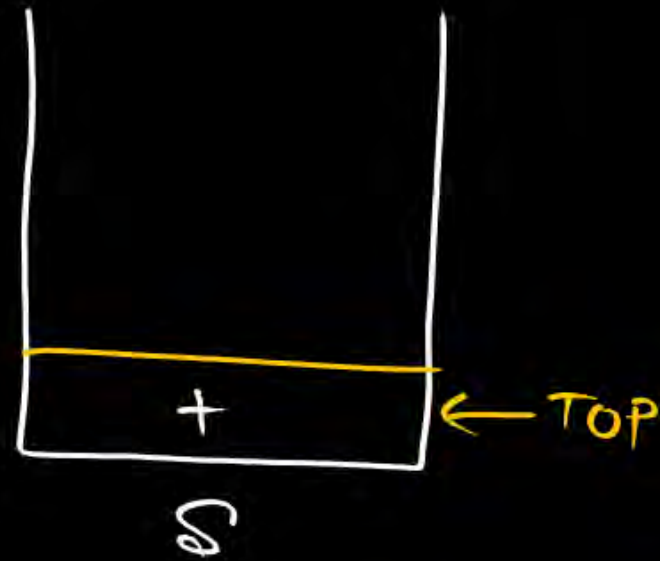


pop $x \rightarrow$ add o/p

$x > + (\text{scanned symbol})$

Ex 4.

infix: $2 \times 3 + 4$
o/p: $23x$



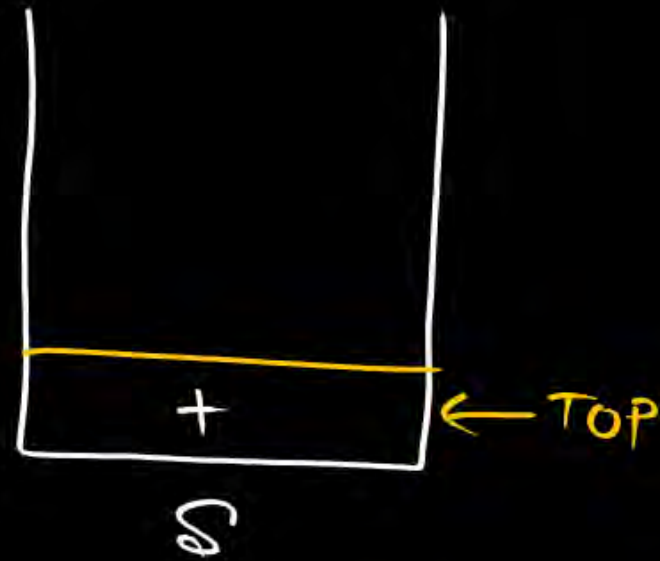
pop $\times \rightarrow$ add o/p

Push $+$ \rightarrow stack

$x > + (\text{scanned symbol})$

Ex 4.

infix : $2 \times 3 + 4$
o/p : 23×4



pop $\times \rightarrow$ add o/p

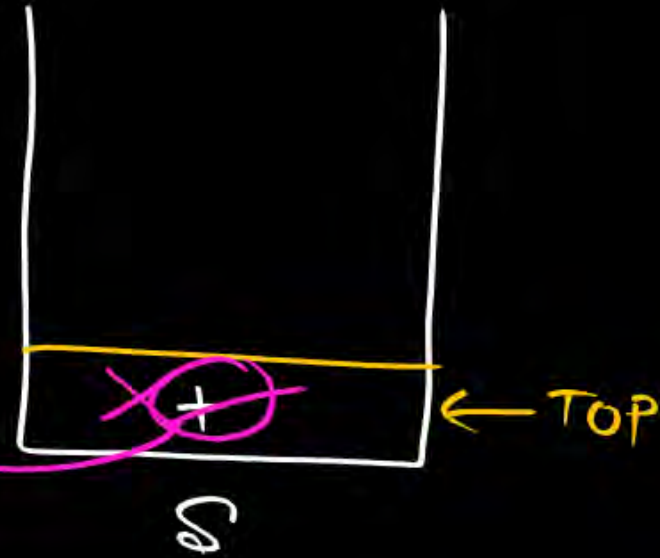
Push + \rightarrow stack

$\times > + (\text{scanned symbol})$

Ex 4.

infix : 2x3+4
o/p : 23x4+ End

23x4+

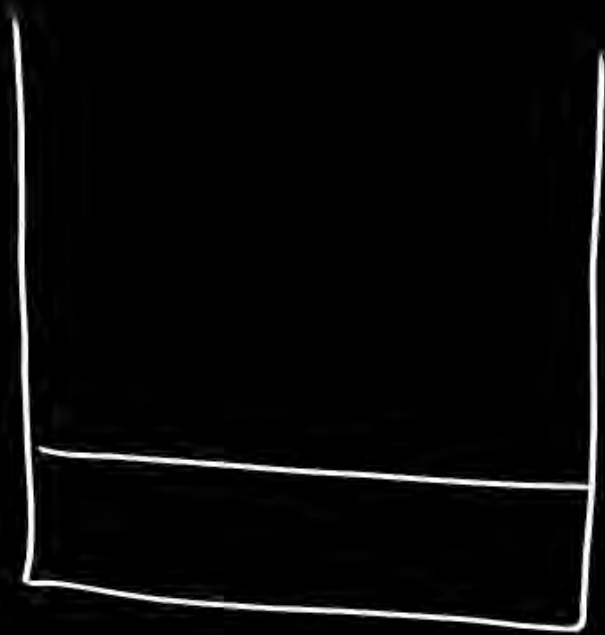


pop $x \rightarrow$ add o/p

Push $+$ \rightarrow stack

$x > + (\text{scanned symbol})$

if optr is found



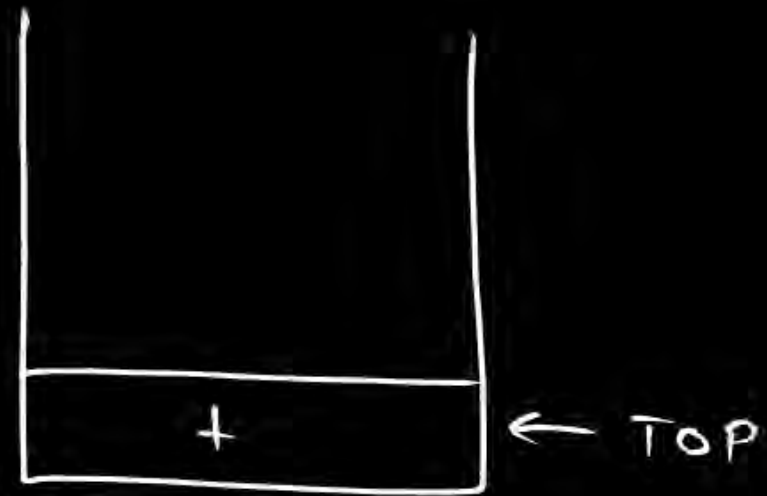
→ stack is Empty
Push

→ if optr at top of
stack \Rightarrow is of same/high priority
then pop it & add to o/p.
Push encountered optr onto stack.

→ otherwise, Push encountered optr onto stack.

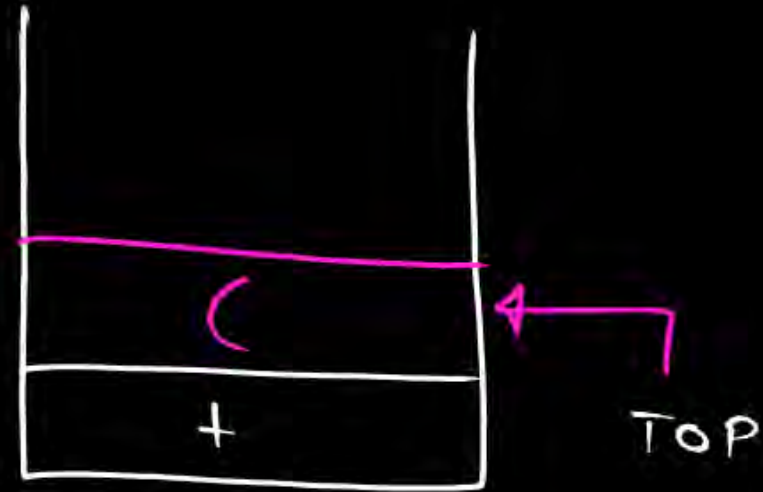
infix : $\overset{\curvearrowright}{2} \overset{\curvearrowright}{+} (3 \times 4 - 6/2)$

o/p : 2



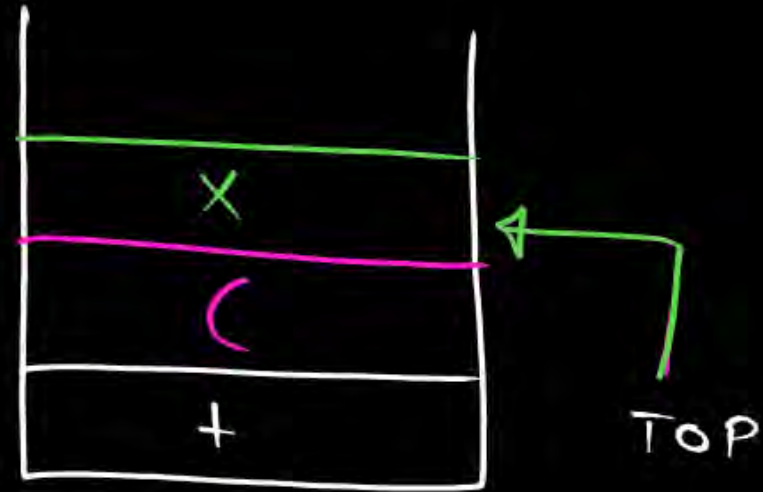
infix : $2 + (3 \times 4 - 6 / 2)$

O/P : 2



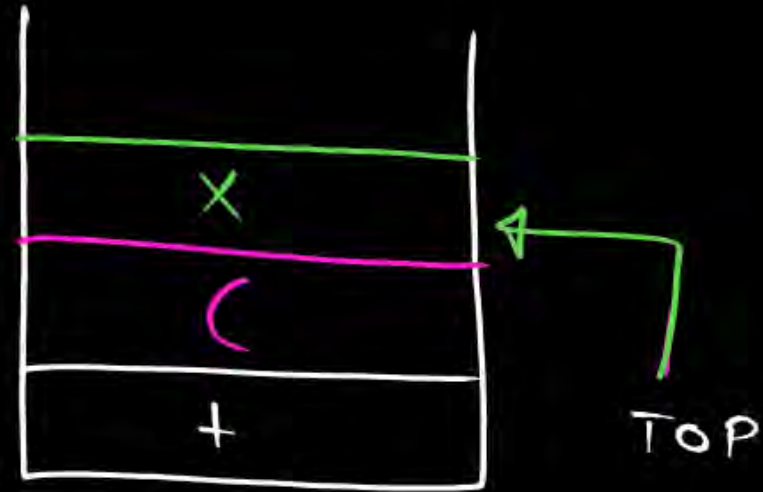
infix: $2 + (3 \times 4 - 6 / 2)$

o/p: 23



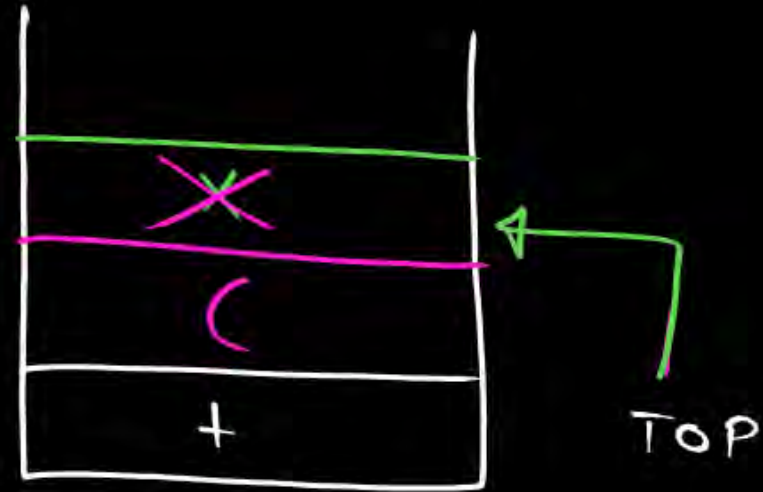
infix: $2 + (3 \times 4 - 6 / 2)$

o/p: 234



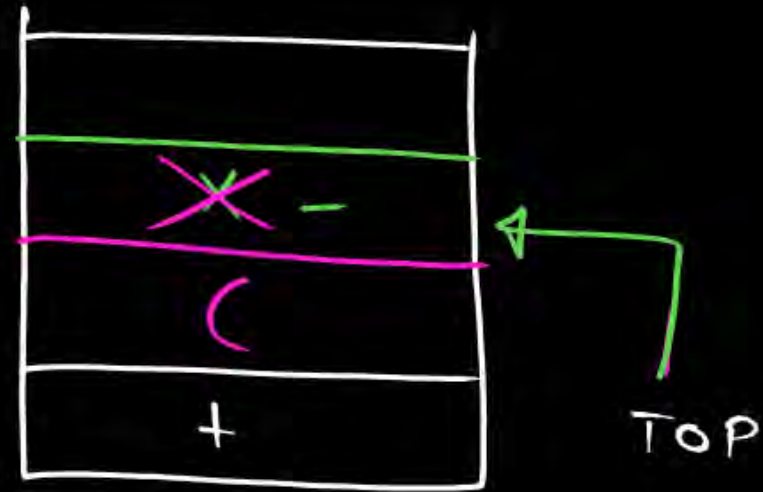
infix: $2 + (3 \times 4 - 6 / 2)$

o/p: 234x



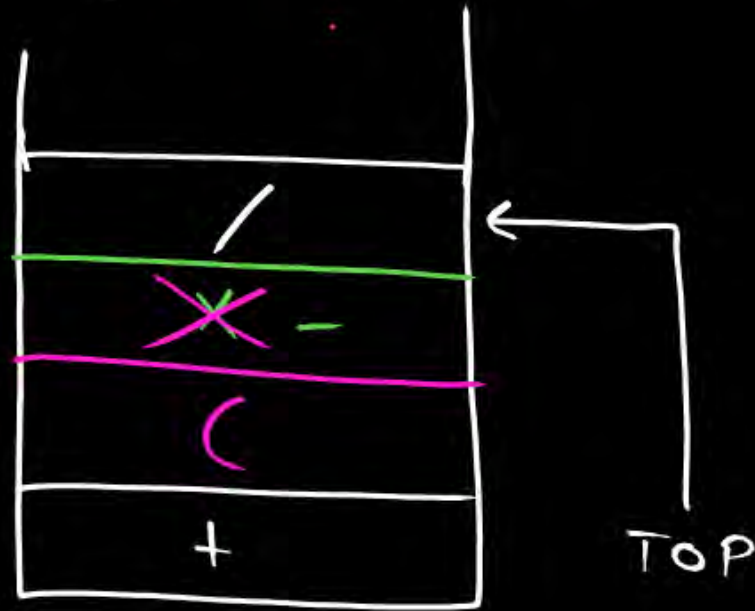
infix: $2 + (3 \times 4 - 6 / 2)$

o/p: 234x6



infix : $2 + (3 \times 4 - 6 / 2)$ Right par.

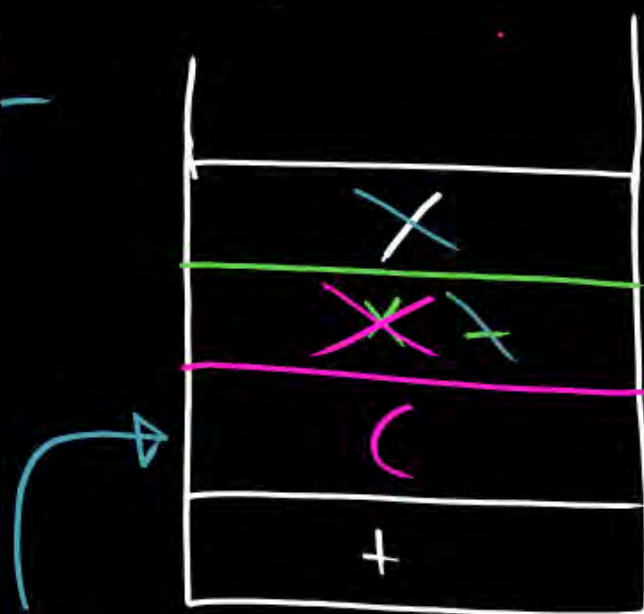
o/p : 2 3 4 \times 6 2



⇒ Pop everything & from st. & add to o/p until (is found

infix : $2 + (3 \times 4 - 6 / 2)$ Right par.

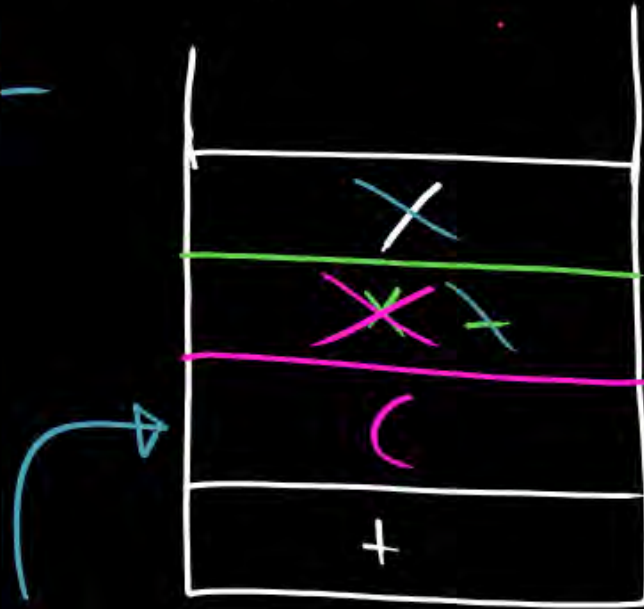
o/p : 2 3 4 x 6 2 / -



Pop everything &
from st. &
add to o/p
until (is found

infix : $2 + (3 \times 4 - 6 / 2)$ Right par.

o/p : 2 3 4 x 6 2 / -

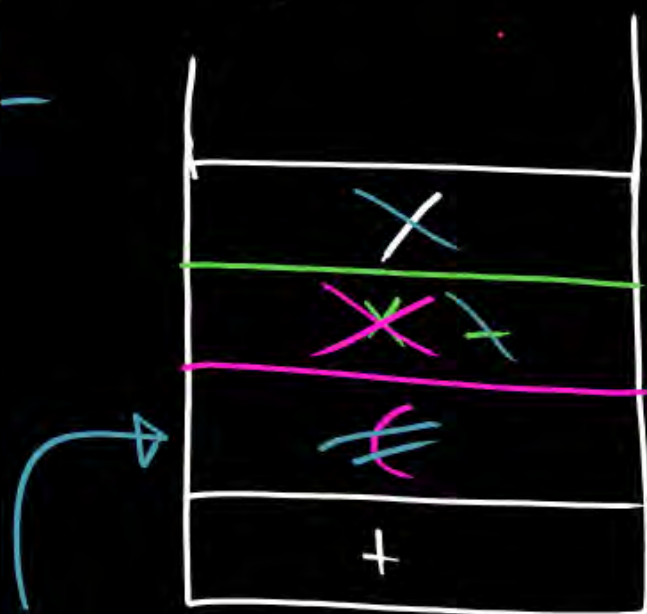


\Rightarrow Pop everything & from st. & add to o/p until (is found

(\Rightarrow pop

infix : $2 + (3 \times 4 - 6 / 2)$ Right par.

o/p : 2 3 4 x 6 2 / -

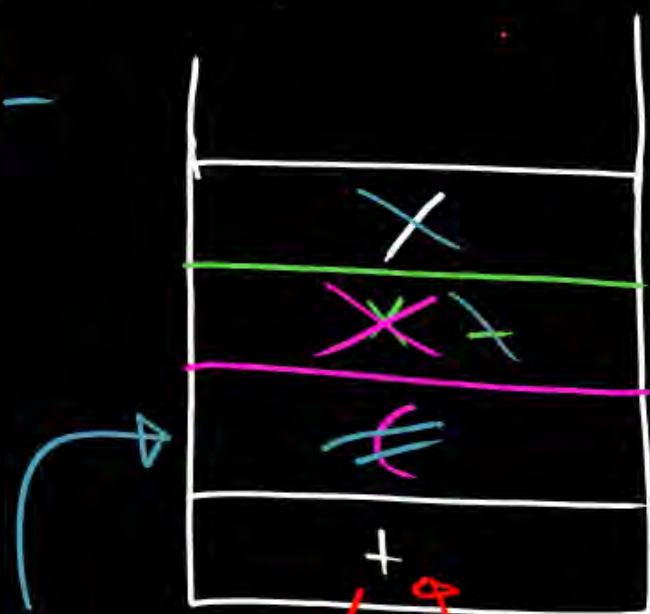


\Rightarrow Pop everything & from st. & add to o/p until (is found

(\Rightarrow pop

infix: $2 + (3 \times 4 - 6 / 2)$ ← End

O/P: 2 3 4 x 6 2 / -



O/P
pop

O/P: 2 3 4 x 6 2 / - +

$$2 + (3 \times 4 - 6 / 2)$$

$$2 + ([3 4 x] - 6 / 2)$$

$$2 + ([3 4 x] - [6 2 /])$$

$$2 + [3 4 x 6 2 / -]$$

$$2 3 4 x 6 2 / - +$$

Sol

$$\left((a + b \times c) - d / e \right)$$

O/p. $a b c \times +$

\times
$+$
$\frac{c}{2}$
$\frac{c}{1}$

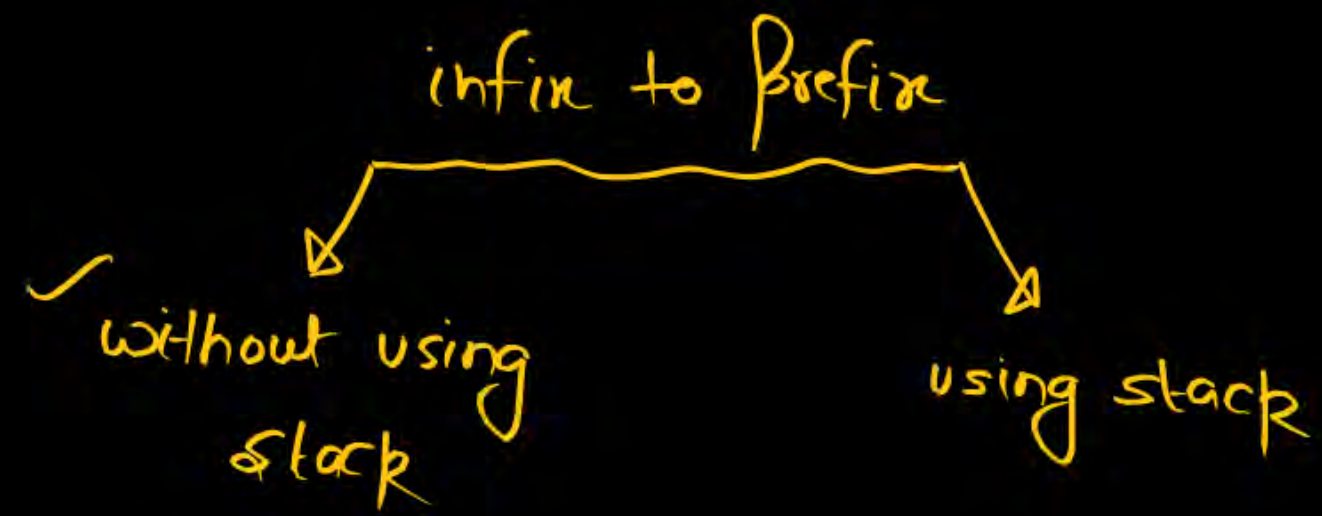
L to R

Sol

$$\left((a + b \times c) - d / e \right)$$

o/p: $a b c \times + d e / -$

\times
$+$ $/$
$($ $-$
$)$



infix : $2 + 3 \times 5$

$2 + \left[\frac{\times 3 5}{\text{op}_2} \right]$

op_1 op_2

\downarrow

$+ 2 \times 3 5$

Ex2

$$a + b \times c / d - e^{\wedge} f^{\wedge} g / h$$

$$a + b \times c / d - \underbrace{e^{\wedge} [f^{\wedge} g]} / h$$

$$a + \underbrace{b \times c / d} - [e^{\wedge} f^{\wedge} g] / h$$

$$a + [\underbrace{x b c}] / d - [e^{\wedge} f^{\wedge} g] / h$$

$$a + [/ x b c d] - [e^{\wedge} f^{\wedge} g] / h$$

$$\underbrace{a + [/ x b c d]} - [\underbrace{/ e^{\wedge} f^{\wedge} g h}]$$

$$[+ a / x b c d] - [/ e^{\wedge} f^{\wedge} g h]$$

$$- + a / x b c d / e^{\wedge} f^{\wedge} g h$$

infix to postfix ✓

infix to prefix →

→

using stack

update

R to L

THANK - YOU