CS & IT ENGINEERING

Data Structures

Tree

Lecture No.- 08

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Topics to be Covered

Topic Tree Part-08

Recap of Previous Lecture

Topic Tree Part-07

Heap
- → insert
- → heapify
- → build-heap ⇒ $O(n)$
- → Find max , find-min
- → Extract-max , Extract-min
- → # of max heaps

Slide 2

Topic : Tree

Q1. Let A be a priority queue for maintaining a set of elements. Suppose A is implemented using a max-heap data structure. The operation EXTRACT-MAX(A) extracts and deletes the maximum element from A. The operation INSERT(A, key) inserts a new element key in A. The properties of a max-heap are preserved at the end of each of these operations. When A contains n elements, which one of the following statements about the worst case running time of these two operations is TRUE?

[GATE-2023-CS: 2M]

A. Both EXTRACT-MAX(A) and INSERT(A, key) run in O(1).
B. Both EXTRACT-MAX(A) and INSERT(A, key) run in O(log(n)).
C. EXTRACT-MAX(A) runs in O(1) whereas INSERT(A, key) runs in O(n).
D. EXTRACT-MAX(A) runs in O(1) whereas INSERT(A, key) runs in O(log(n)).

Q2. Which one of the following sequences when stored in an array at locations A[1], . . . , A[10] forms a max-heap?

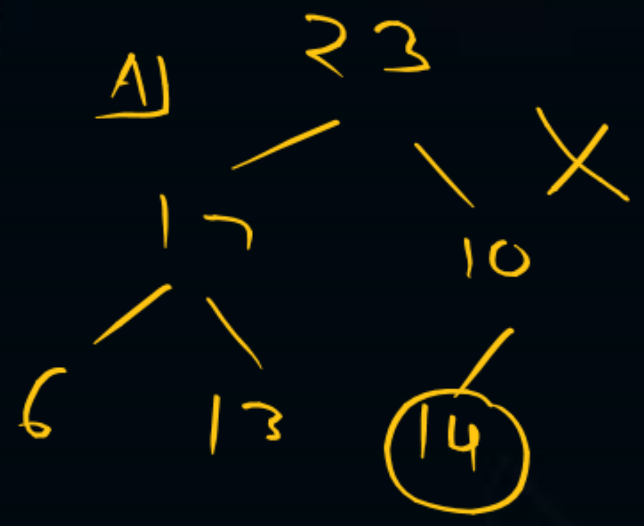
[GATE-2023-CS: 1M]

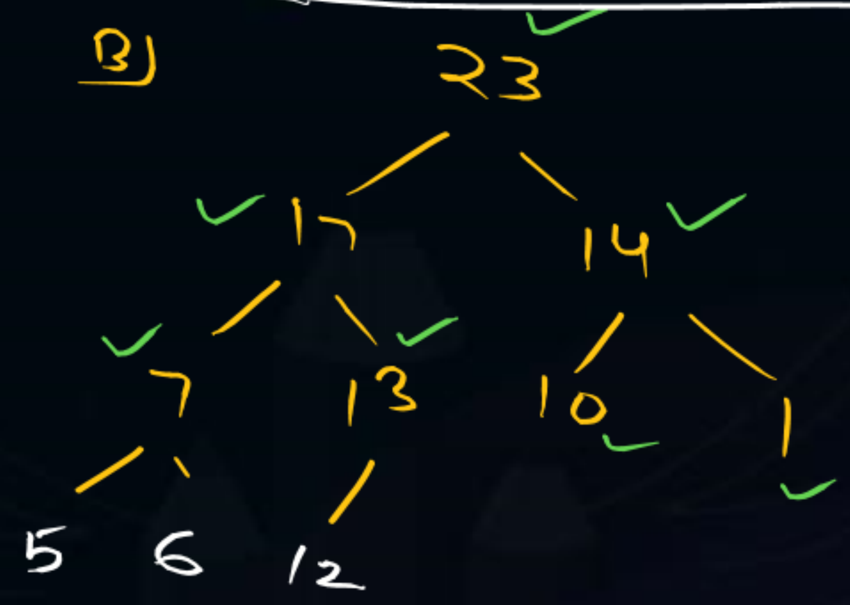
A. 23, 17, 10, 6, 13, 14, 1, 5, 7, 12

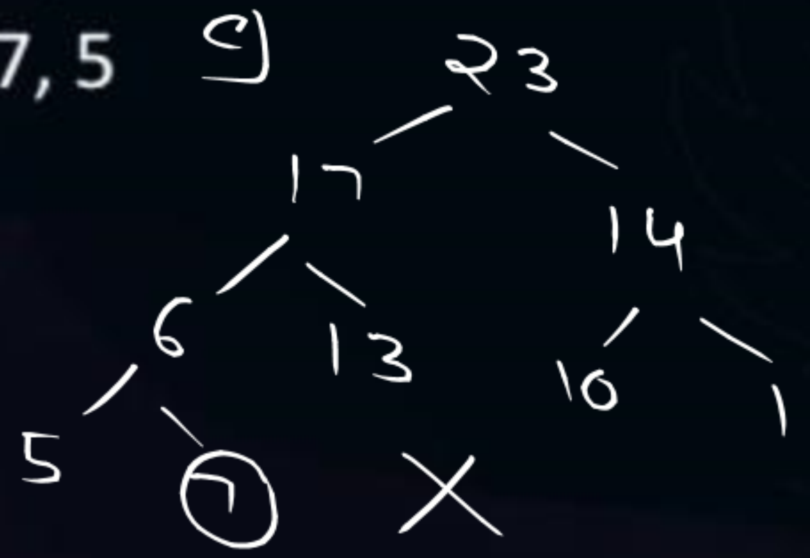
B. 23, 17, 14, 7, 13, 10, 1 ,5, 6, 12

C. 23, 17, 14, 6, 13, 10, 1, 5, 7, 15

D. 23, 14, 17, 1, 10, 13, 16, 12, 7, 5







Q3. Consider the array representation of a binary min-heap containing 1023 elements. The minimum number of comparisons required to find the maximum in the heap is_____.

[GATE-2020: 2M]


```
int max;
max = A[1];
for(i = 2; i<=100; i++)
  { if(A[i]>max)
        max = A[i];
  }
```

Topic : Tree

Q3. Consider the array representation of a binary min-heap containing 1023 elements. The minimum number of comparisons required to find the maximum in the heap is 511.

[GATE-2020: 2M]

10	1	3	4	- - - - - -	

1 2 ... 100

```
int max;
max = A[1];
for(i=2; i<=100; i++)
{ if(A[i]>max)
      max = A[i];
}
```

max — leaf node

$\#$ leaf node $= \lceil \frac{n}{2} \rceil = \lceil \frac{1023}{2} \rceil$

$= 512$

Q4. Consider the following array of elements ⟨89,19,50,17,12,15,2,5,7,11,6,9,100⟩. The minimum number of interchanges needed to convert it into a max-heap is

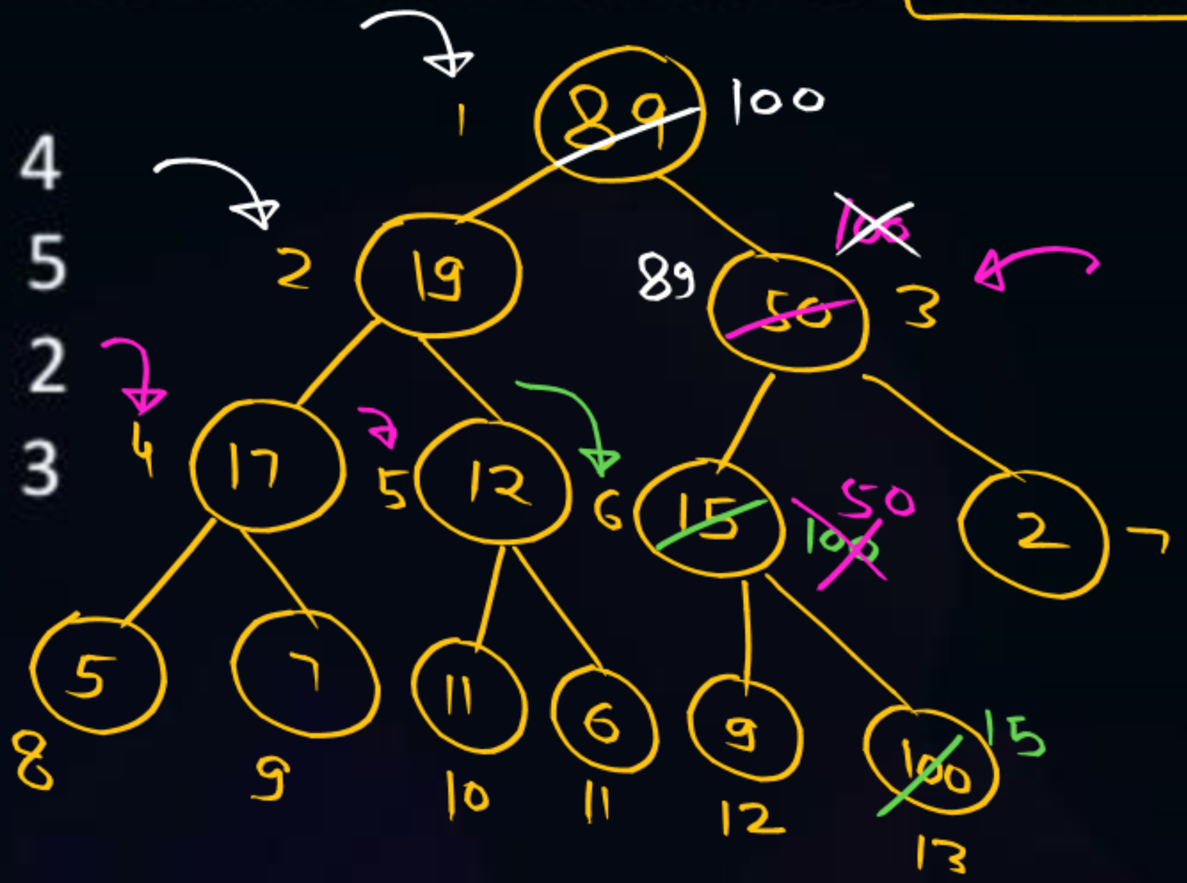
[GATE-2017: 1M]

A. 4

B. 5

C. 2

D. 3



1 to $\left\lfloor \frac{13}{2} \right\rfloor \Rightarrow 1, 2, 3, 4, 5, 6$

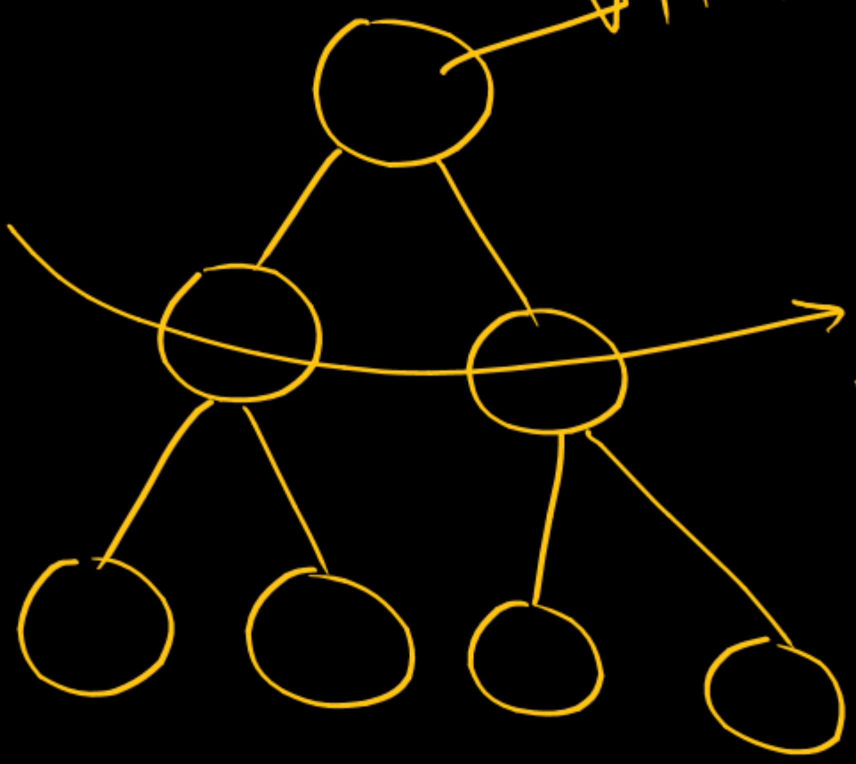
1 ✓
2 ✓
3 ✓ } 3 swap

Q5. A complete binary min-heap is made by including each integer in [1,1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is 8_____.

[GATE-2016: 2M]

$2^{7+1}-1$

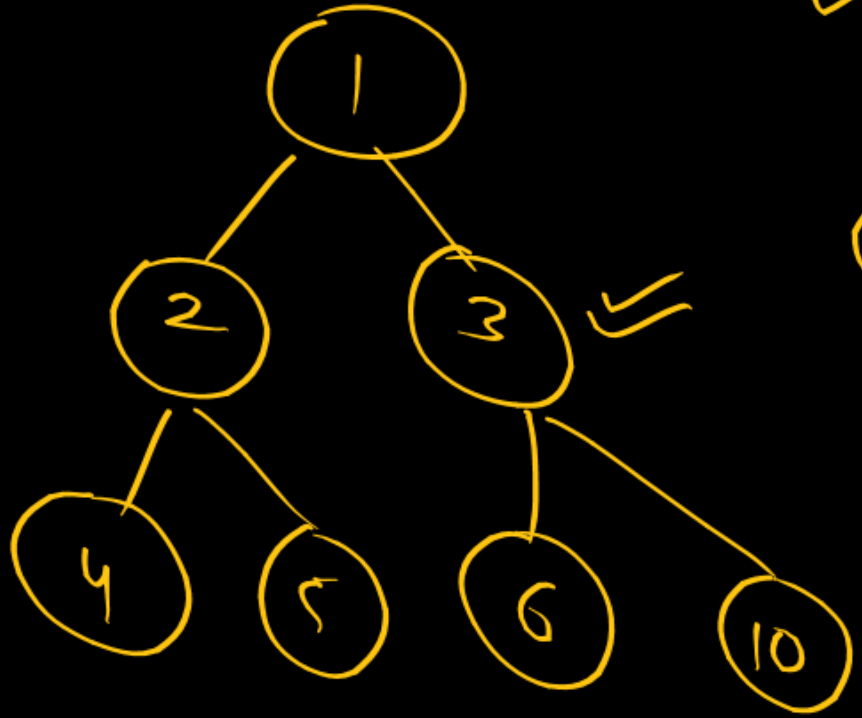
$\Rightarrow 2^{8}-1 \Rightarrow 255$ node

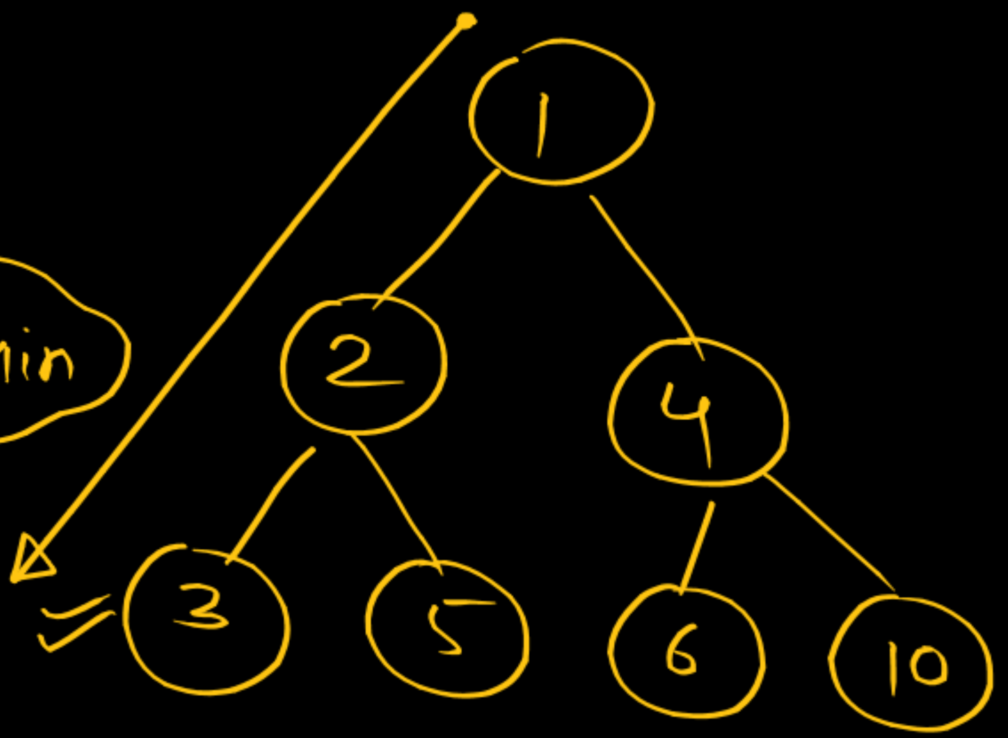


Min

2nd min (level-1)

3rd min

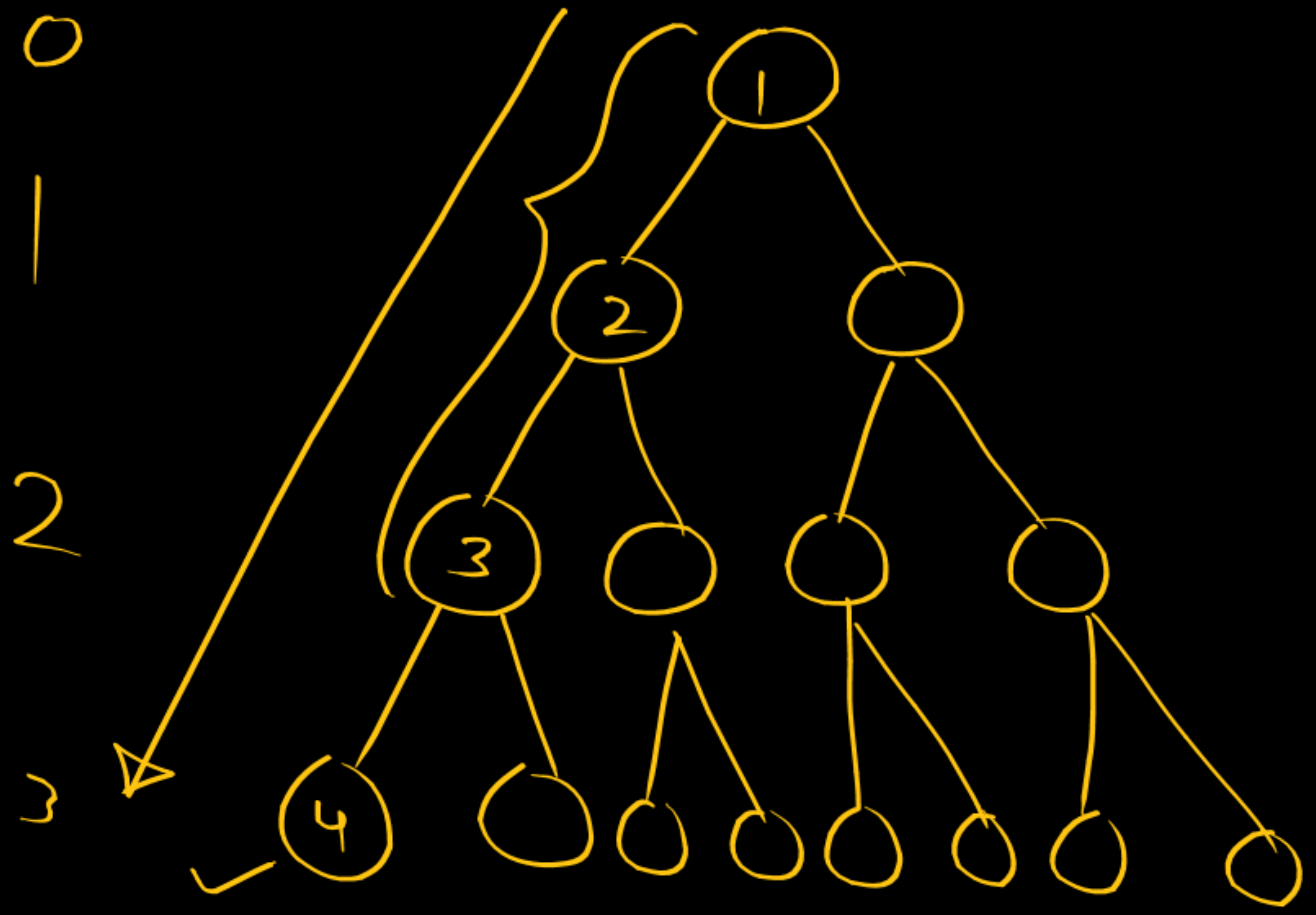




3rd min ⇒ level 2

K^{th} min

$\Rightarrow$ can not be beyond $(K-1)$ level



Topic : Tree

Q6. An operator delete(i) for a binary heap data structure is to be designed to delete the item in the i-th node. Assume that the heap is implemented in an array and i refers to the i-th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?

[GATE-2016: 2M]

A. $O(1)$

B. $O(d)$ but not $O(1)$

C. $O(2^d)$ but not $O(d)$

D. $O(d2^d)$ but not $O(2^d)$

delete →

re-heapify → $O(\log_2^n)$

Topic : Tree

Q7. Consider the C function given below. Assume that the array list A contains n (> 0) elements, sorted in ascending order.

int ProcessArray (int *list A, int x, int n){ **[GATE-2014: 1M]**

```
int i, j, k;
i = 0;
j = n - 1;
do {
k = ( i + j ) / 2;
if ( x <= listA[k] )
j = k - 1;
If ( listA [k] <= x )
i = k + 1;
}
while ( i <= j );
    if( listA[k] == x ) return ( k );
  else
  return - 1;}
```

Which one of the following statements about the function Process Array is CORRECT?

Slide 10

A. d

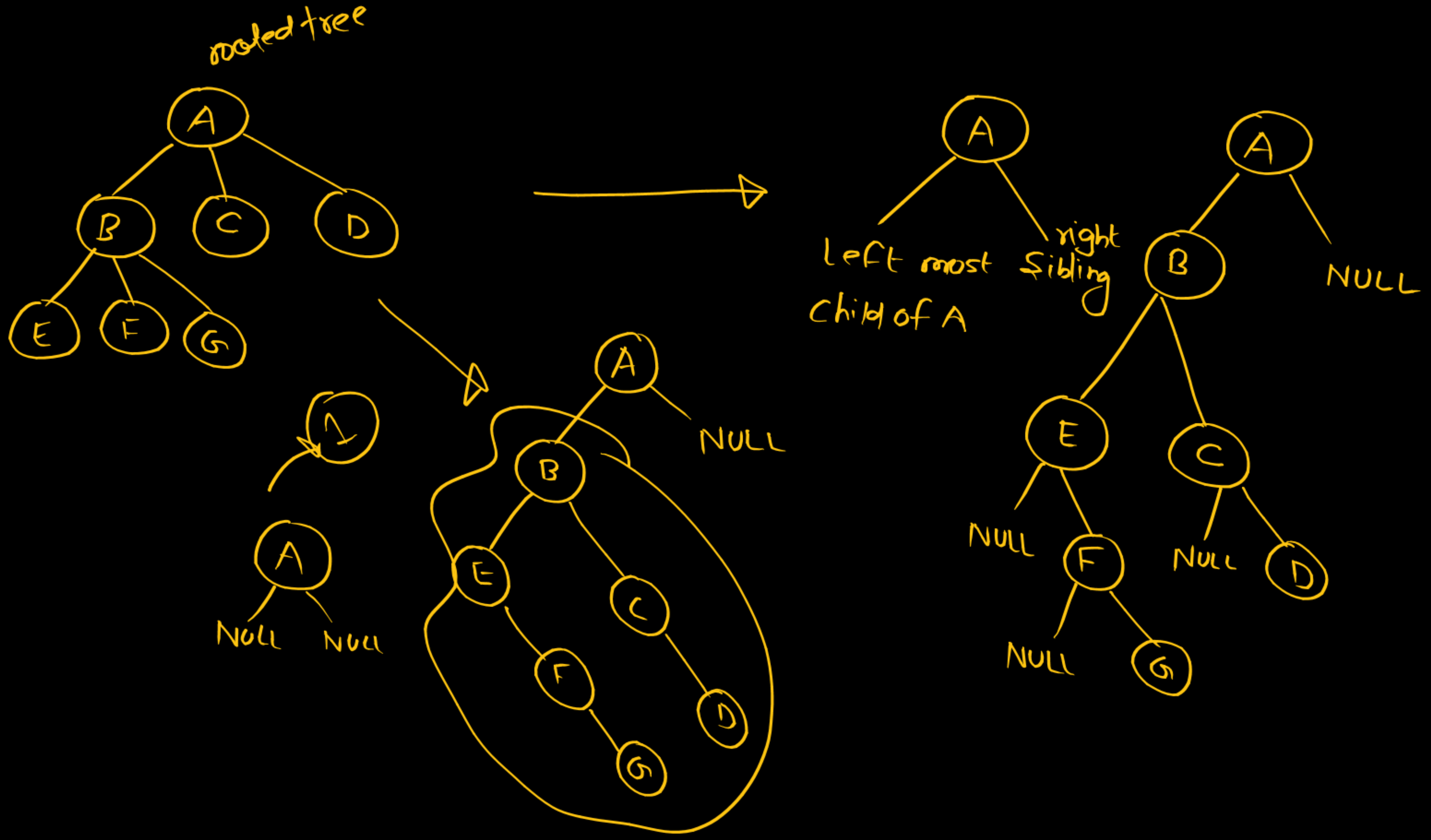
A. It will run into an infinite loop when x is not in list A.
B. It is an implementation of binary search.
C. It will always find the maximum element in list A
D. It will return – 1 even when x is present in list A.

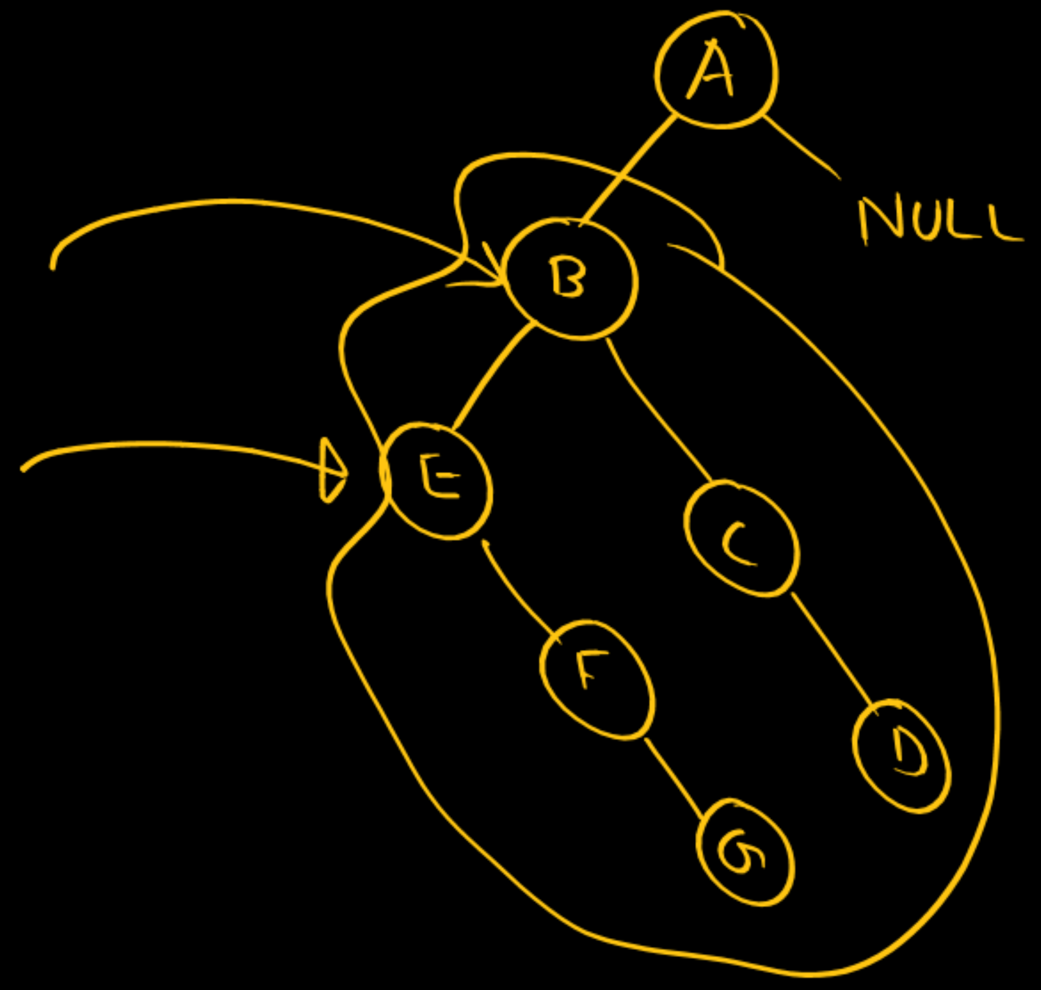
Topic : Tree

Q8.Consider the pseudocode given below. The function **DoSomething ( )** takes as a argument a pointer to the root of an arbitrary tree represented by the **leftMostChild - rightSibling** representation. Each node of the tree is of type treeNode.

```
typedef struct treeNode* treeptr;
struct treeNode
   {
treeptr leftMostChild, rightSibling;
};
int DoSomething (treeptr tree)
   {
int value = 0;
if (tree ! = NULL) {
   if (tree → left MostChild = = NULL) value = 1;
   else
   value = DoSomething(tree → leftMostChild);
   value = value + DoSomething(tree → rightSibling);
}
return (value);
}
```

When the pointer to the root of a tree is passed as the argument to **DoSomething,** the value returned by thye function corresponds to the


rooted tree
A
B
C
D
E
F
G
1
NULL
left most
child of A
right
Sibling


A
NULL
B
E
C
F
D
G

A. number of internal nodes in the tree. ×

B. height of the tree. ×

C. number of nodes without a right sibling in the tree.

D. number of leaf nodes in the tree. ✓

1+

Topic : Tree

Q9. A priority queue is implemented as Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is: 10, 8, 5, 3, 2. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:

[GATE-2014: 1M]

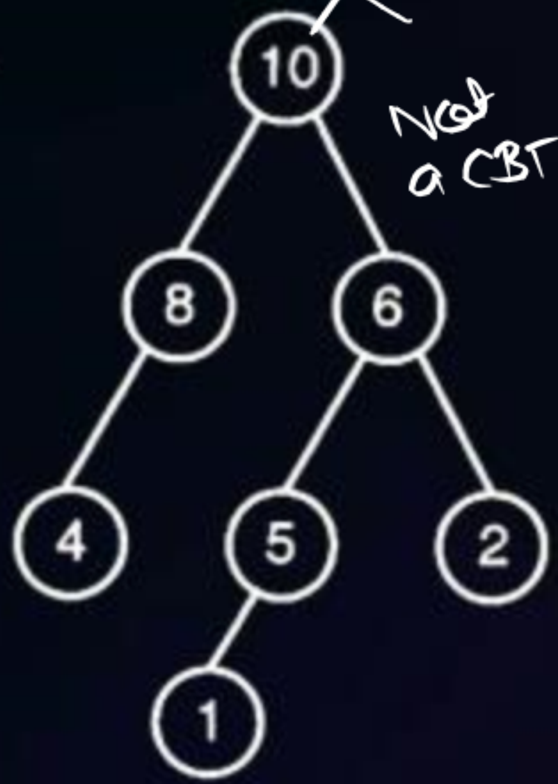
A. 10, 8, 7, 3, 2, 1, 5
B. 10, 8, 7, 2, 3, 1, 5
C. 10, 8, 7, 1, 2, 3, 5
D. 10, 8, 7, 5, 3, 2, 1

Topic : Tree

Q10. A max-heap is a heap where the value of each parent is greater than or equal to the value of its children. Which of the following is a max-heap?

[GATE-2011: 1M]

A.



Not a CBT

B.

C.

D.

Topic : Tree

Q11. Which one of the following array represents a binary max-heap?

[GATE-2009: 2M]

~~A.~~ {25, 12, 16, 13, 10, 8, 14}

~~B.~~ {25, 14, 13, 16, 10, 8, 12}

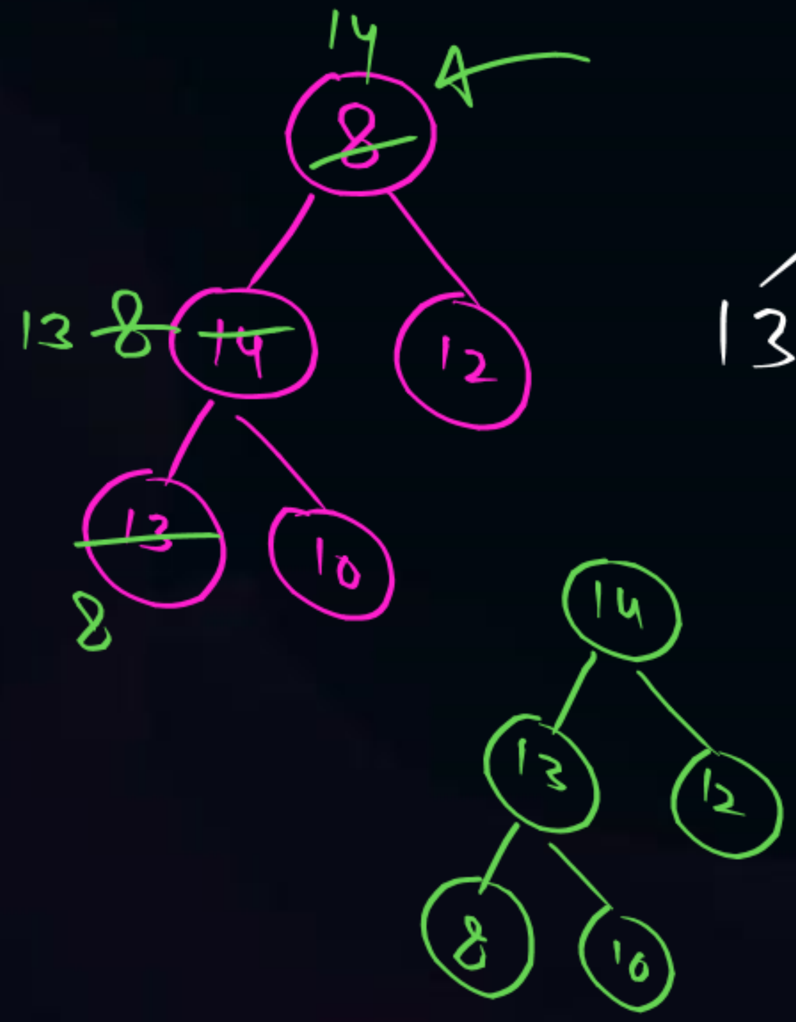
C. {25, 14, 16, 13, 10, 8, 12}

D. {25, 14, 12, 13, 10, 8, 16}

Q12. Consider a binary max-heap implemented using an array.
What is the content of the array after two delete operations on the correct answer to the previous question?

[GATE-2009: 2M]

A. { 14, 13, 12, 10, 8 }
B. { 14, 12, 13, 8, 10 }
C. { 14, 13, 8, 12, 10 }
D. { 14, 13, 12, 8, 10 }



Topic : Tree

Q13. We have a binary heap on n elements and wish to insert n more elements (not necessarily one after another) into this heap. The total time required for this is:

[GATE-2008: 1M]

$n + n \Rightarrow 2n \Rightarrow$ build - heap

$\Rightarrow \Theta(\cancel{2}n)$

$\Rightarrow \Theta(n)$

A. $\Theta(\log n)$

B. $\Theta(n)$

C. $\Theta(n \log n)$

D. $\Theta(n^2)$

foo(100)

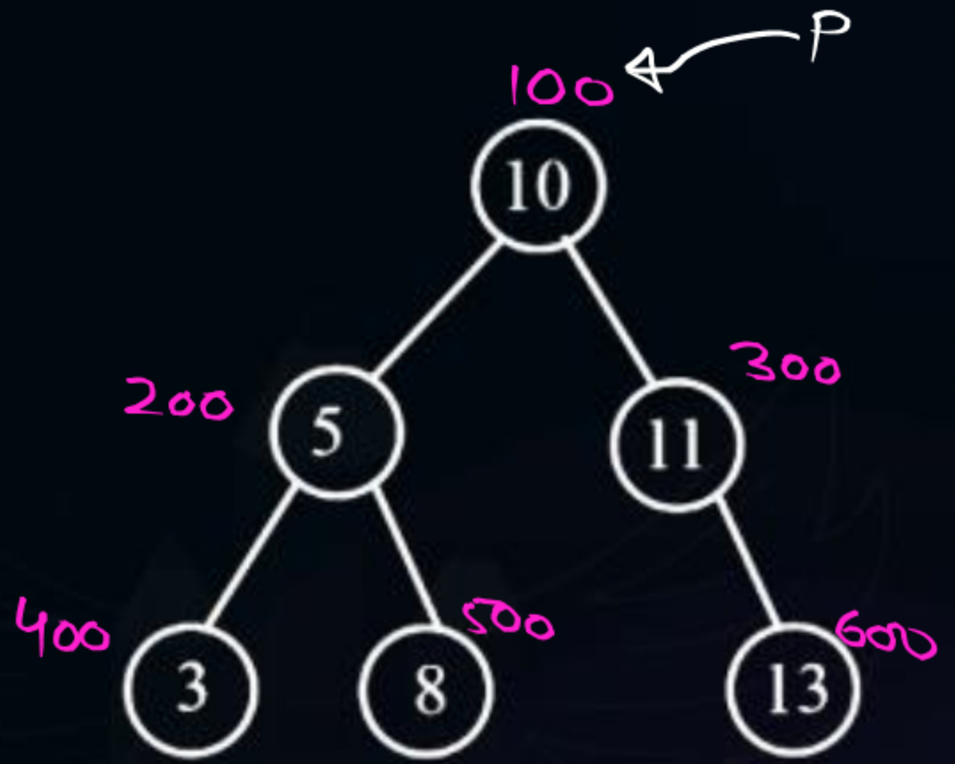
10 foo(200) foo(300) pf return

Topic : Tree

Q14. Consider the C function foo and the binary tree shown.

```
typedef struct node {
      int val;
      struct node *left, *right;
} node;

int foo( node *p ) {
      int retval;
      if ( p = = NULL )
         return 0;
      else {
      retval = p → val + foo( p  → left ) + foo( p →right );
      printf( "%d ", retval );
      return retval;
   }
}
```



Slide 19

When foo is called with a pointer to the root node of the given binary tree, what will it print?

A. 3 8 5 13 11 10
B. 3 5 8 10 11 13
C. 3 8 16 13 24 50
D. 3 16 8 50 24 13

Q15. Consider the following statements.

S_1: The sequence of procedure calls corresponds to a preorder traversal of the activation tree. True

S_2: The sequence of procedure returns corresponds to a postorder traversal of the activation tree. True

Which one of the following options is correct?

[GATE-2021-CS: 1M]

A. S_1 is true and S_2 is false

B. S_1 is false and S_2 is true

C. S_1 is true and S_2 is true

D. S_1 is false and S_2 is false

```
C(){
  =
}

B(){
  =
}

A(){
  ≡
  B() + C()
  ≡
}

main(){
  ≡
  A() + D()
  ≡
}
```


main
A( )
+
D( )
B( )
+
C( )

Topic : Tree

Q16. The preorder traversal of a binary search tree is 15 10, 12, 11, 20, 18, 16, 19. Which one of the following is the postorder traversal of the tree?

Inorder

[GATE-2020: 1M]

A. 20, 19, 18, 16, 15, 12, 11, 10
B. 11, 12, 10, 16, 19, 18, 20, 15
C. 19, 16, 18, 20, 11, 12, 10, 15
D. 10, 11, 12, 15, 16, 18, 19, 20

In: 10 11 12 15 16 18 19 20

Pre: 15 10 12 11 20 18 16 19

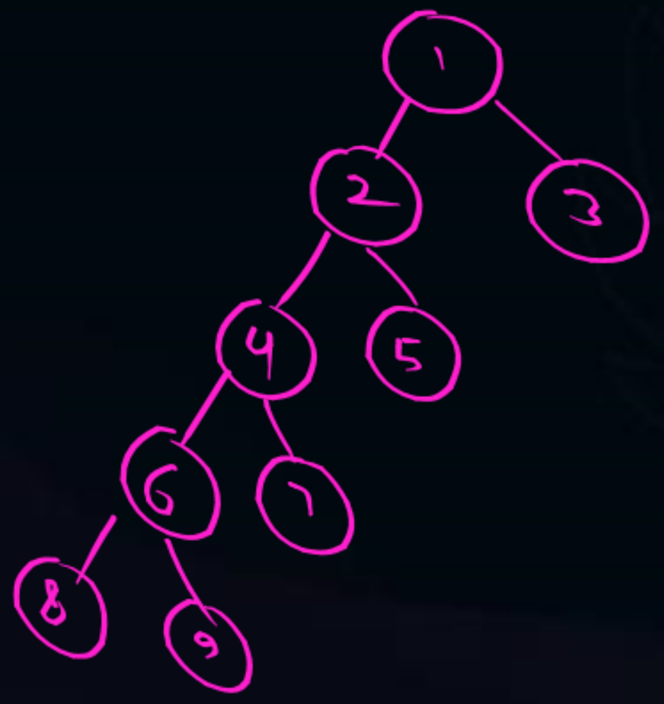
Topic : Tree

Q17. The postorder traversal of a binary tree is 8,9,6,7,4,5,2,3,1. The inorder traversal of the same tree is 8,6,9,4,7,2,5,1,3. The height of a tree is the length of the longest path from the root to any leaf. The height of the binary tree above is
__4__

Post : 8 9 6 7 4 5 2 3 1

In : 8 6 9 4 7 2 5 1 3

[GATE-2018: 1M]



Topic : Tree

Q18. The pre-order traversal of a binary search tree is given by 12,8, 6,2,7, 9,10, 16,15, 19,17, 20. Then the post-order traversal of this tree is:

[GATE-2017: 2M]

In: Inc. order of keys

Pre: Given

↓

tree

↓

Post-order

A. 2,6,7,8,9,10,12,15,16,17,19,20
B. 2,7,6,10,9,8,15,17,20,19,16,12
C. 7,2,6,8,9,10,20,17,19,15,16,12
D. 7,6,2,10,9,8,15,16,17,20,19,12

Topic : Tree

Q19. Which of the following is/are correct inorder traversal sequence(s) of binary search tree(s)?

1. 3, 5, 7, 8, 15, 19, 25 ✓
2. 5, 8, 9, 12, 10, 15, 25 ×
3. 2, 7, 10, 8, 14, 16, 20 ×
4. 4, 6, 7, 9, 18, 20, 25 ✓

[GATE-2015: 1M]

A. 1 and 4 only
B. 2 and 3 only
C. 2 and 4 only
D. 2 only

Topic : Tree

Q20. The preorder traversal sequence of a binary search tree is 30, 20, 10, 15, 25, 23, 39, 35, 42. Which one of the following is the postorder traversal sequence of the same tree?

[GATE-2013: 2M]

Pre : Given
In : inc-order of keys
→ tree → Post-order

A. 10, 20, 15, 23, 25, 35, 42, 39, 30
B. 15, 10, 25, 23, 20, 42, 35, 39, 30
C. 15, 20, 10, 23, 25, 42, 35, 39, 30
D. 15, 10, 23, 25, 20, 35, 42, 39, 30

Topic : Tree

Q21. The following three are known to be the preorder, inorder and postorder sequences of a binary tree. But it is not known which is which.

I. MBCAFHPYK

II. KAMCBYPFH

III. MABCKYFPH

Pick the true statement from the following.

[GATE-2008: 2M]

A. I and II are preorder and inorder sequences, respectively

B. I and III are preorder and postorder sequences, respectively

C. II is the inorder sequence, but nothing more can be said about the other two sequences

D. II and III are the preorder and inorder sequences, respectively

Q22. Suppose a binary search tree with 1000 distinct elements is also a complete binary tree. The tree is stored using the array representation of binary heap trees. Assuming that the array indices start with 0, the 3rd largest element of the tree is stored at index_____________.

[GATE-2022-CS: 1M]

Topic : Tree

Q23. Let T be a binary search tree with 15 nodes.
The minimum and maximum possible heights of T are:
Note: The height of a tree with a single node is 0.

[GATE-2017: 1M]

A. 4 and 15 respectively
B. 3 and 14 respectively
C. 4 and 14 respectively
D. 3 and 15 respectively

Topic : Tree

Q24. The number of ways in which the numbers 1,2,3,4,5,6,7 can be inserted in an empty binary search tree, such that the resulting tree has height 6, is ________.
Note: The height of a tree with a single node is 0.

[GATE-2016: 2M]

Topic : Tree

Q25. While inserting the elements 71,65,84,69,67,83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is

[GATE-2015: 1M]

A. 65
B. 67
C. 69
D. 83

Topic : Tree

Q26. A binary tree T has 20 leaves. The number of nodes in T having two children is ______

[GATE-2015: 1M]

Topic : Tree

Q27. Consider a max heap, represented by the array: 40, 30, 20, 10, 15, 16, 17, 8, 4.

Array Index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is

[GATE-2015: 1M]

A. 40, 30, 20, 10, 15, 16, 17, 8, 4, 35
B. 40, 35, 20, 10, 30, 16, 17, 8, 4, 15
C. 40, 30, 20, 10, 35, 16, 17, 8, 4, 15
D. 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

Topic : Tree

Q28. The height of a tree is the length of the longest root-to-leaf path in it. The maximum and minimum number of nodes in a binary tree of height 5 are

[GATE-2015: 1M]

A. 63 and 6, respectively
B. 64 and 5, respectively
C. 32 and 6, respectively
D. 31 and 5, respectively

Q29. Consider a rooted n node binary tree represented using pointers. The best upper bound on the time required to determine the number of subtrees having exactly 4 nodes is $O(n^a \log^b n)$. Then the value of a + 10b is____.

[GATE-2014: 1M]

Topic : Tree

Q30. The height of a tree is defined as the number of edges on the longest path in the tree. The function shown in the pseudo-code below is invoked as the height (root) to compute the height of a binary tree rooted at the tree pointer root.

```
int height (treeptr n)
{
 if ( n = = NULL) return –1;
 if ( n → left = = null )
 if ( n → right = = NULL ) return 0;
    else return  B1 ;//Box1
 else {
    h1 = height (n → left);
    if (n → right = = NULL ) return (1 + h1);
   else {
h2 = height ( n → right );
return  B2 ;//Box2
   }
 }
}
```

The appropriate expression for the two boxes B1 and B2 are

A. B1: (1 + height (n → right))
B2: (1 + max (h1, h2))

A. B1: (height (n → right))
B2: (1 + max (h1, h2))

C. B1: height (n → right))
B2: max (h1, h2)

D. B1: (1 + height (n → right))
B2: max (h1, h2))

Q31. We are given a set of n distinct elements and an unlabeled binary tree with n nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree?

[GATE-2011: 2M]

A. 0

B. 1

C. n!

D. $\frac{1}{(n+1)}{}^{2n}C_n$

15 years PYQs ✓

DPP/PS ✓

Test-Series → T_1 → TWT → {Silly mistake, new concept}

Test-Series → T_2 → TWT_1 → { }

2 Subjects

Pankaj sir PW

10:00 AM ⇒ PYQs

$\boxed{\text{1:00 PM} \Rightarrow \text{Graph}}$

10:00 AM ⇒ Hashing - 1

1:00 PM ⇒ Hashing - 2

Mon, Tue ⇒ Question

THANK - YOU