



# CS & IT ENGINEERING

## Data Structures

Tree

Lecture No.- 04

By- Pankaj Sharma Sir





# Recap of Previous Lecture



Topic

Tree Part-03



# Unlabelled binary trees

# Labelled binary tree

Pre  $\rightarrow$

Post  $\rightarrow$

In  $\rightarrow$

Pre, In

Post, In  $\rightarrow$

# Topics to be Covered



Topic

Tree Part-04

BST





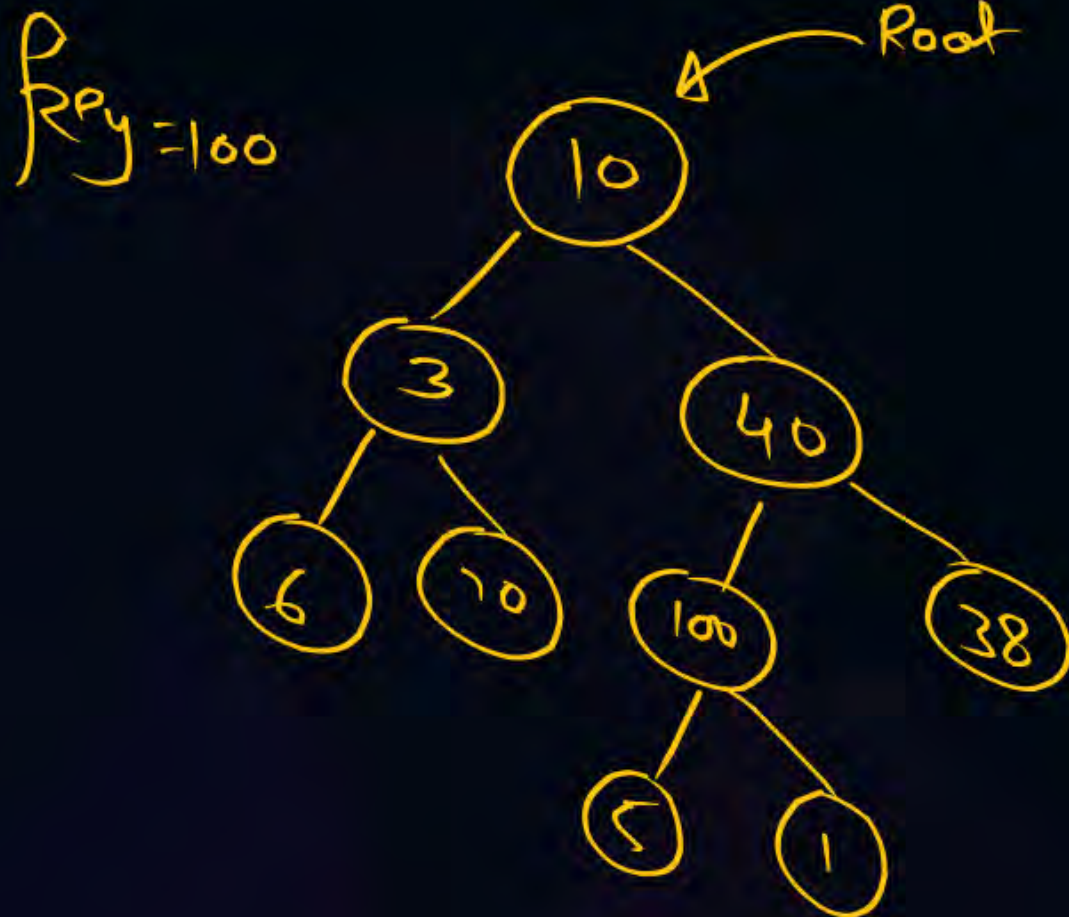


## Topic : Tree



BST

① why?



Given a key & a binary tree, find whether the key is present in the tree or not?

↓  
Traverse →  $O(n)$  time

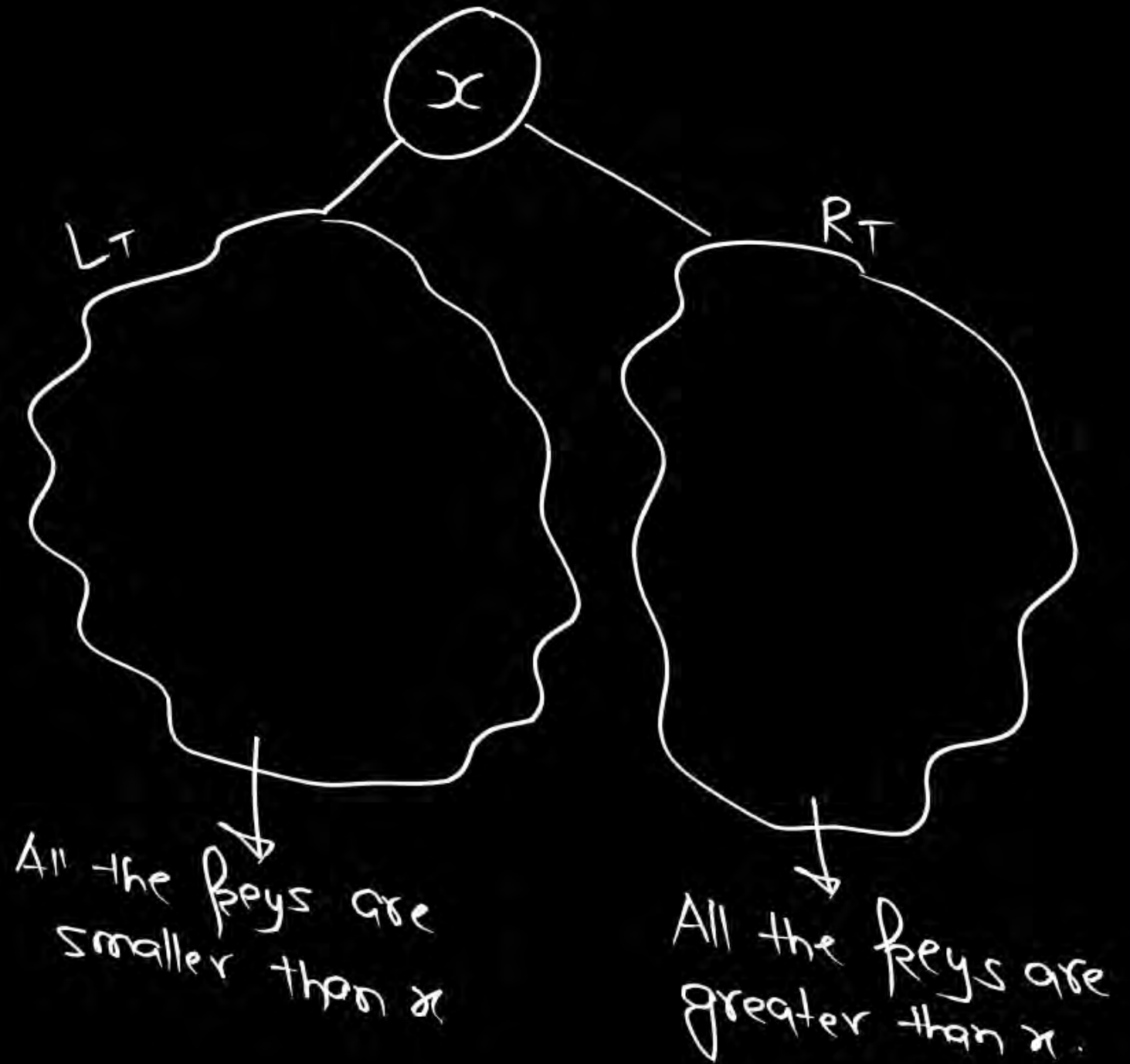
# BST

A binary search tree is a binary tree in which every node satisfies the following property:

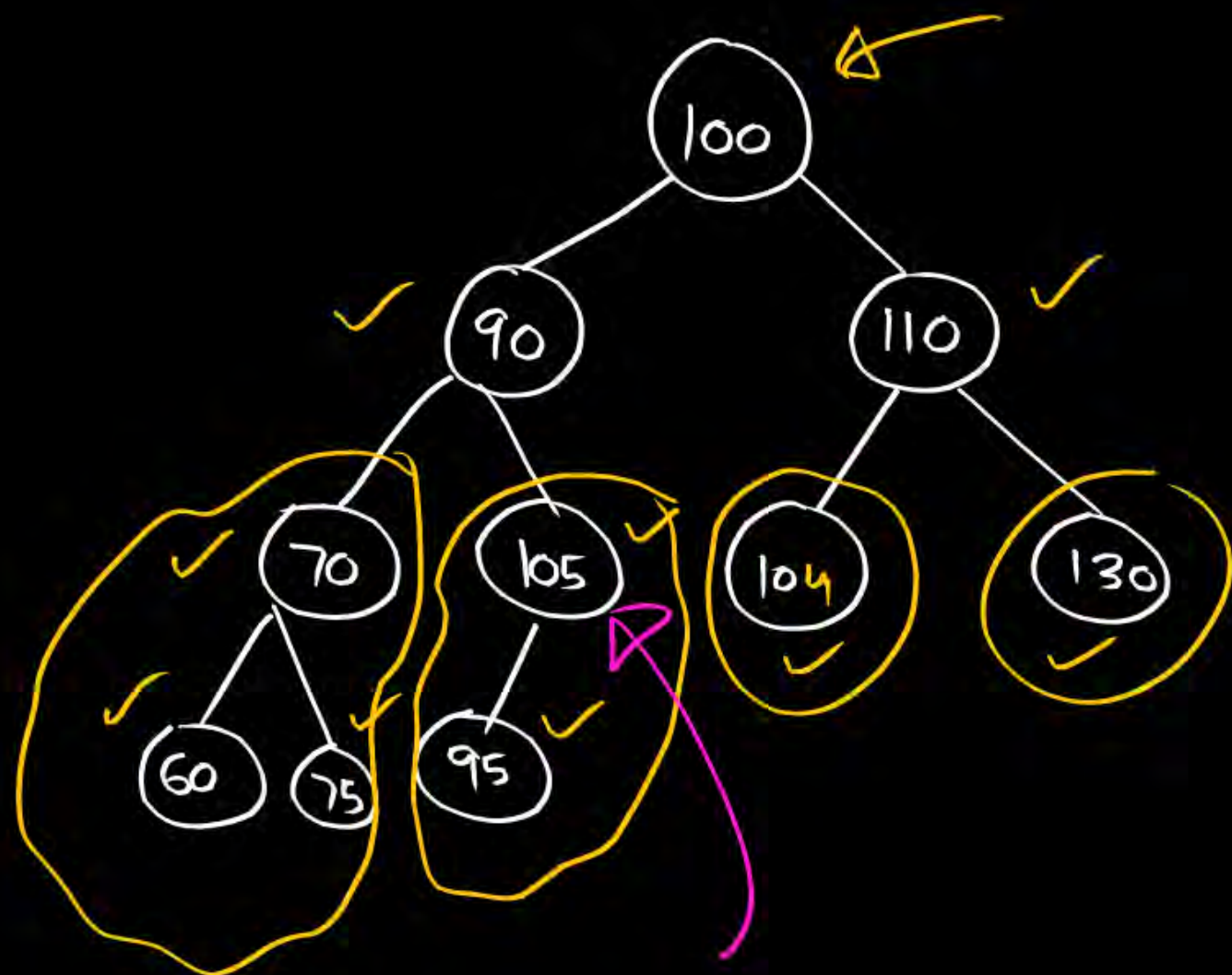
All the keys in the left subtree of a node are smaller than the key in the node.

&

All the keys in the right subtree of a node are greater than the key in the node.

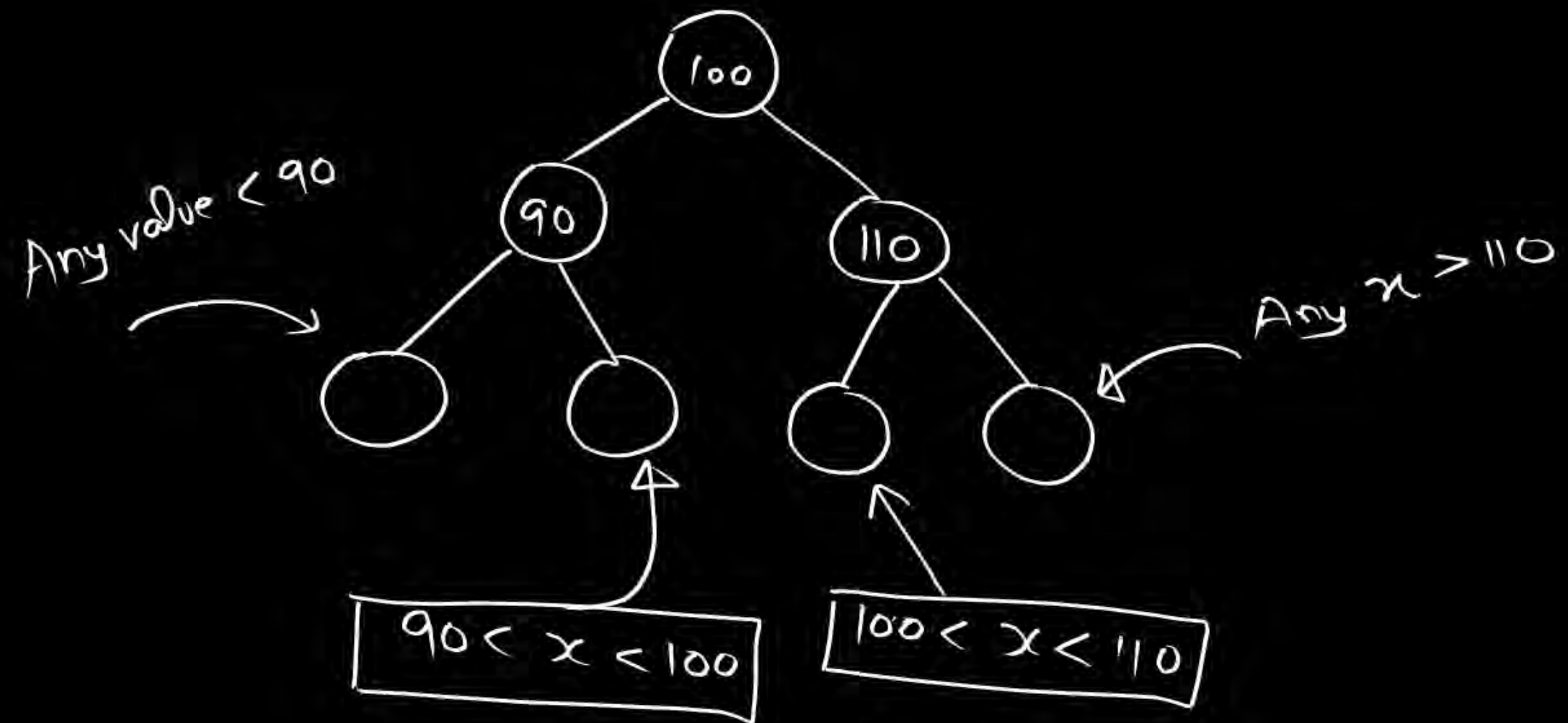






10

Binary tree  
BST



Q Construct a BST by inserting keys  $\overrightarrow{10, 20, 30}$  in order into initially empty BST?

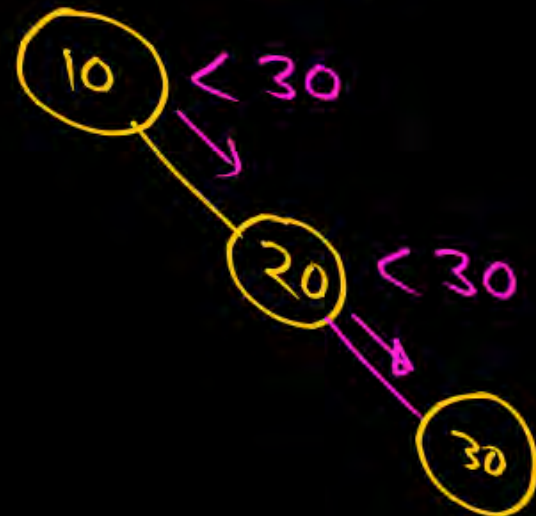
(i) Insert 10



(ii) Insert 20



(iii) Insert 30

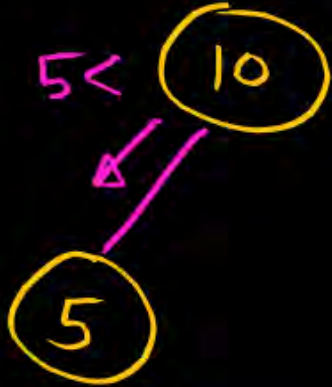


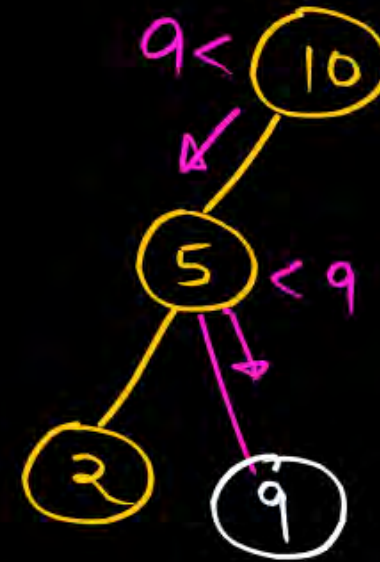


Q Construct a BST by inserting keys 10, 5, 2, 9, 3, 4 in order.

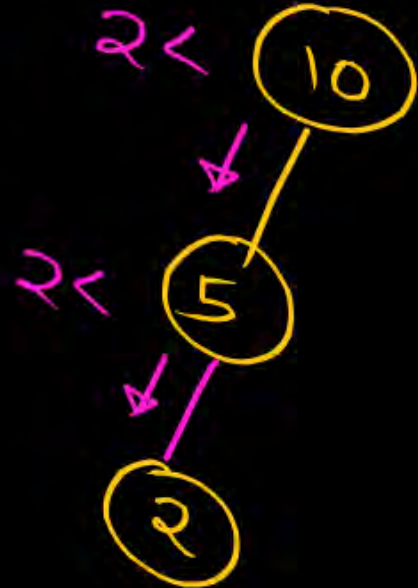
(i) Insert 10 

(iv) Insert 9

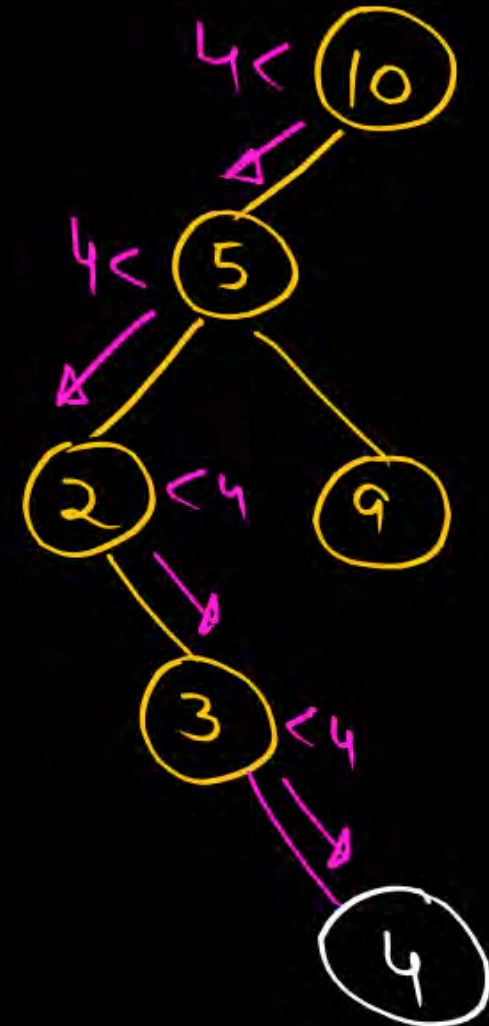
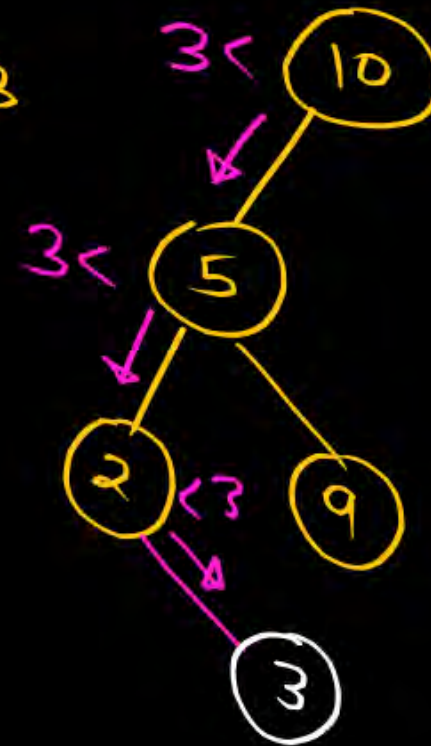
(ii) Insert 5 



vi) Insert 4

(iii) Insert 2 

(v) Insert 3



No. of BST, when the insertion order of keys is fixed  $\rightarrow 1$

a) <sup>Const.</sup> BST by inserting keys 10, 20, 30  $\rightarrow 1$

b) Const. BST by inserting keys 10, 5, 2, 9, 3, 4  $\rightarrow 1$



Const. BST by inserting keys 1, 2, 3? (in any order)  $\rightarrow$  5 BST

$n=3$  Keys  $\rightarrow$  5 BSTs

order could be

a) 1, 2, 3

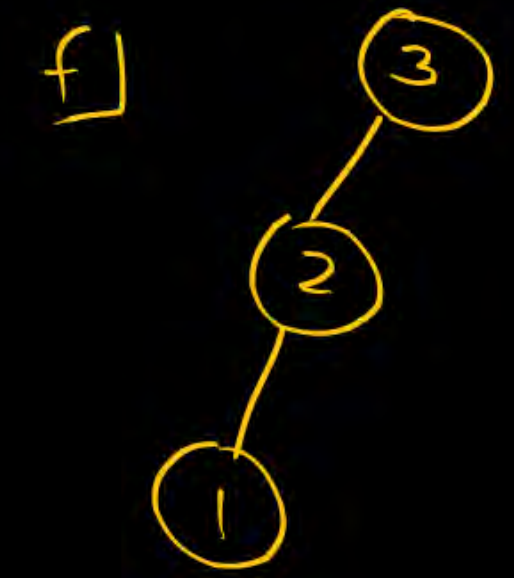
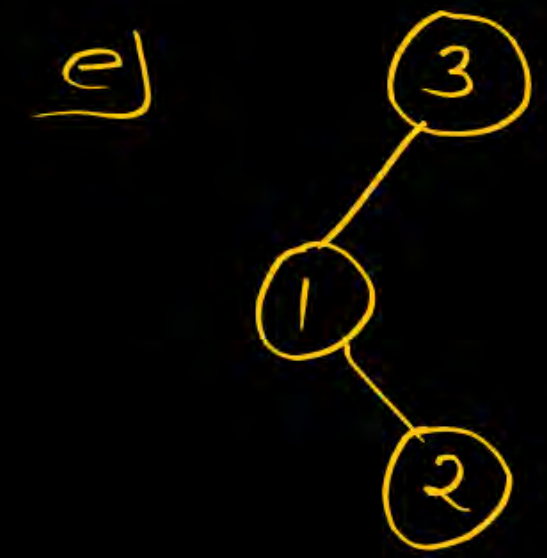
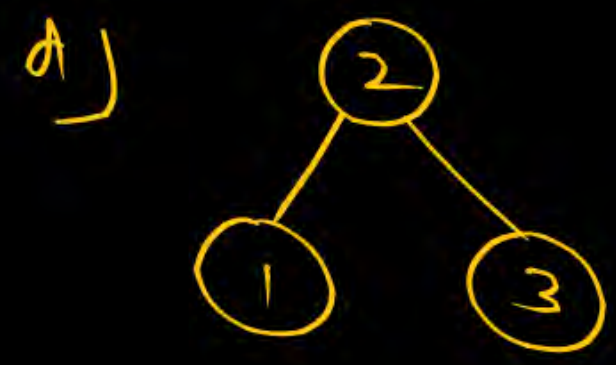
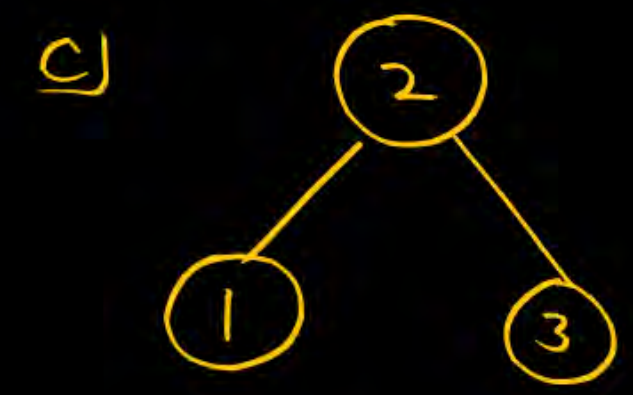
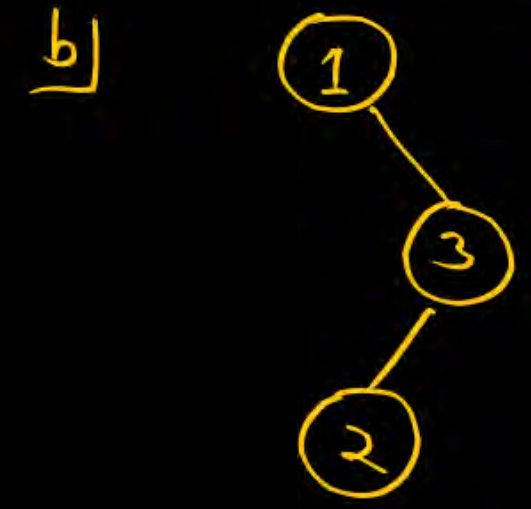
b) 1, 3, 2

c) 2, 1, 3

d) 2, 3, 1

e) 3, 1, 2

f) 3, 2, 1

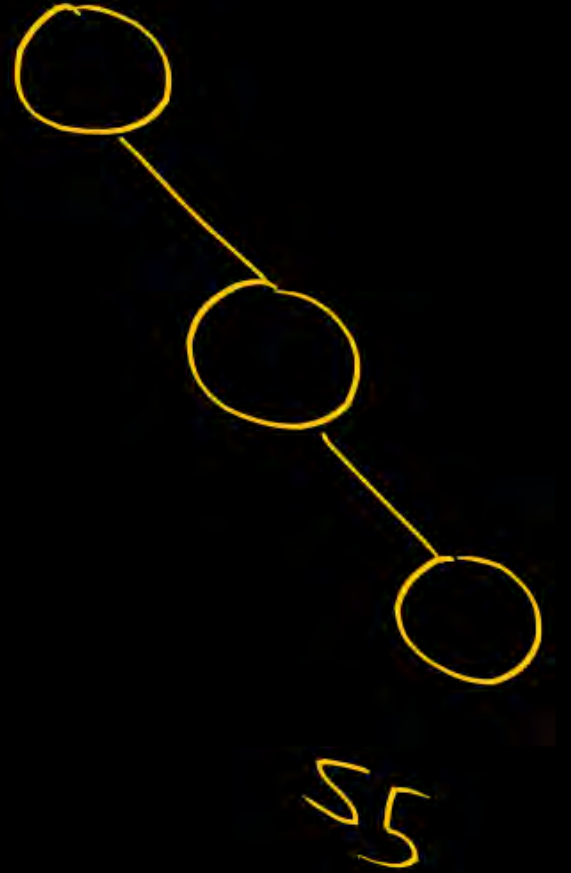
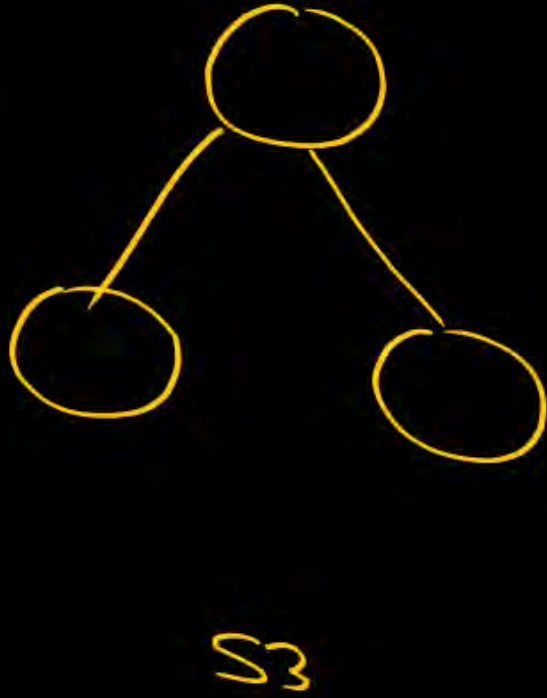
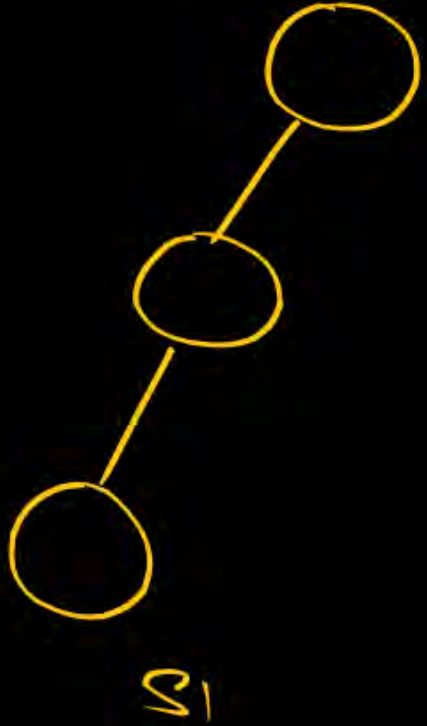


$$\# \text{BSTs with } n \text{ keys} = \frac{2^n C_n}{n+1}$$



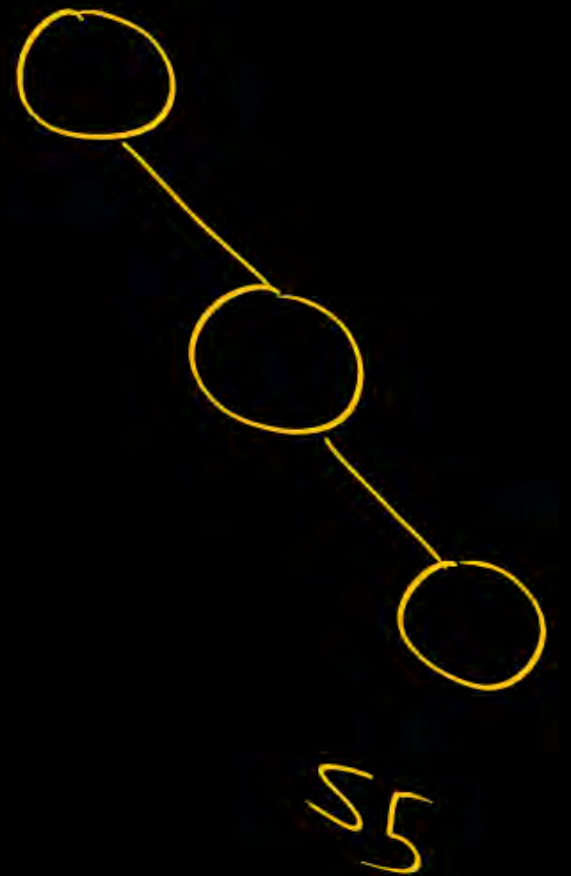
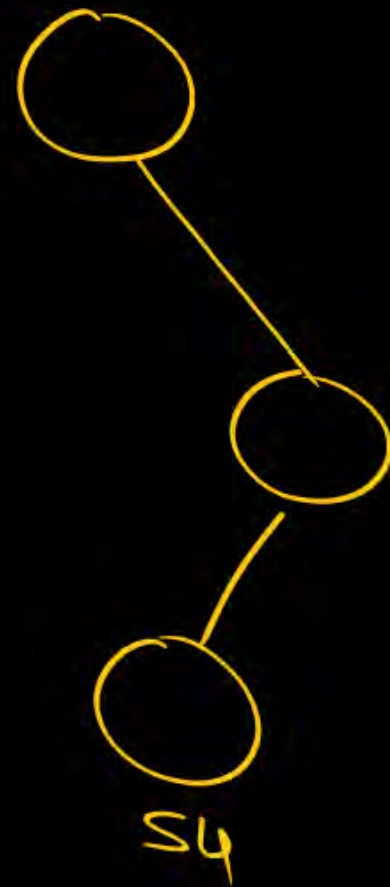
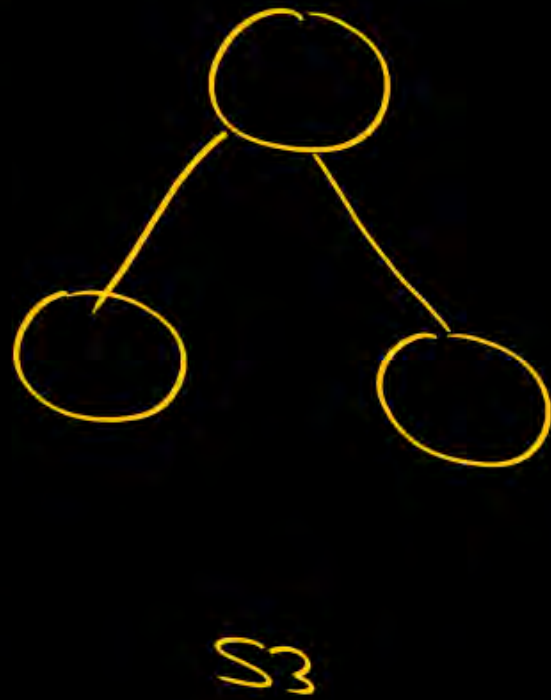
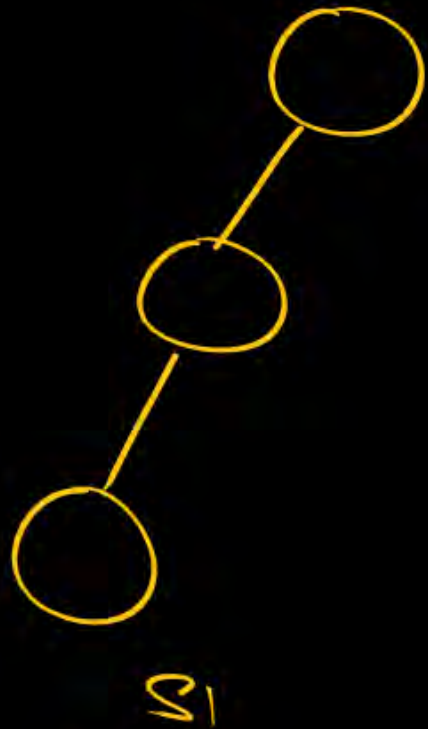
$n=3$  structure  $\Rightarrow \frac{2n(n-1)}{n+1} = 5$

keys  $\Rightarrow 1, 2, 3$



$n=3$     Structure  $\Rightarrow \frac{2n(n-1)}{n+1} = 5$

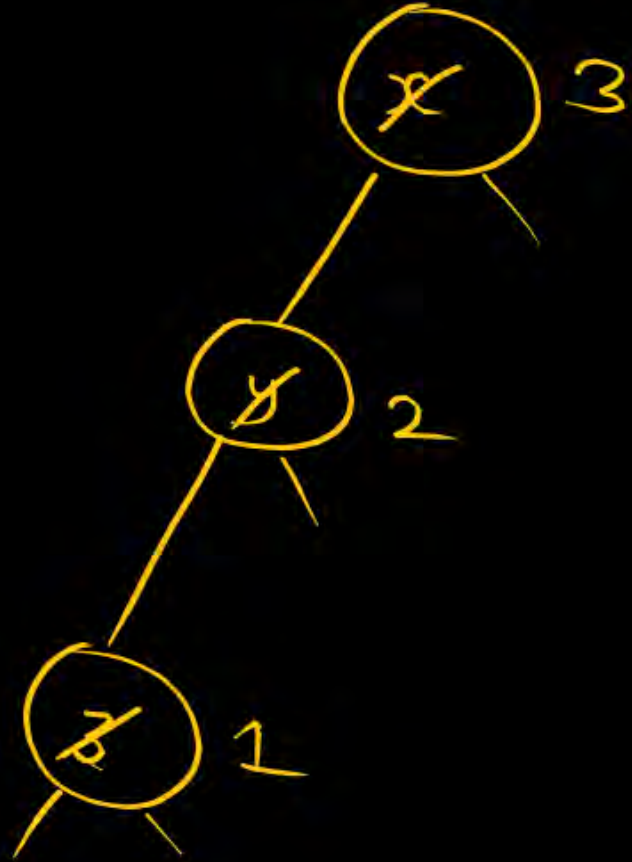
Keys  $\Rightarrow 1, 2, 3$



↓  
How to fill 1, 2, 3  
so that this structure  
will become a BST



1, 2, 3

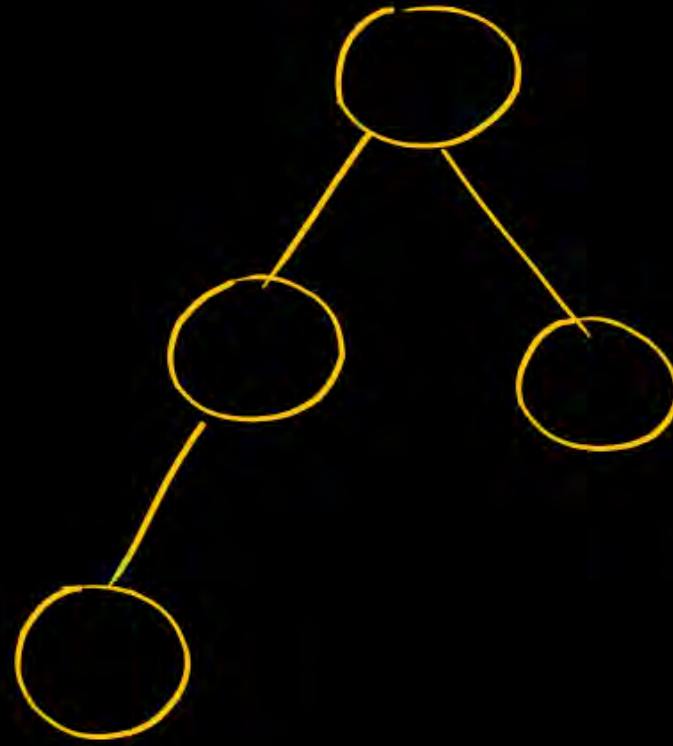


$$z < y < x$$

Given a  $n$ -node  
unlabelled structure  
&

also  $n$ -distinct  
keys are given.

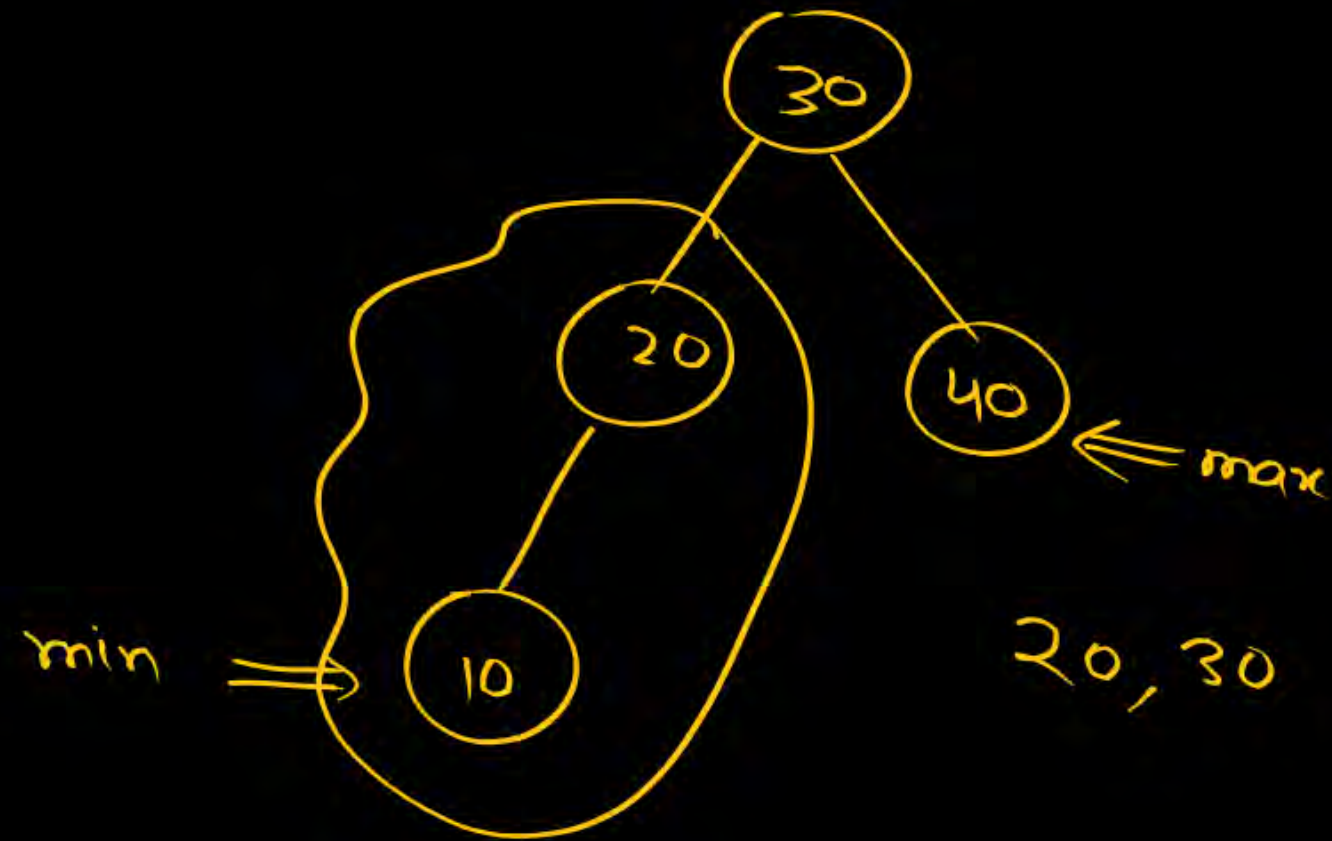
# ways to fill the structure  
so that we get BST



Keys

10, 20, 30, 40





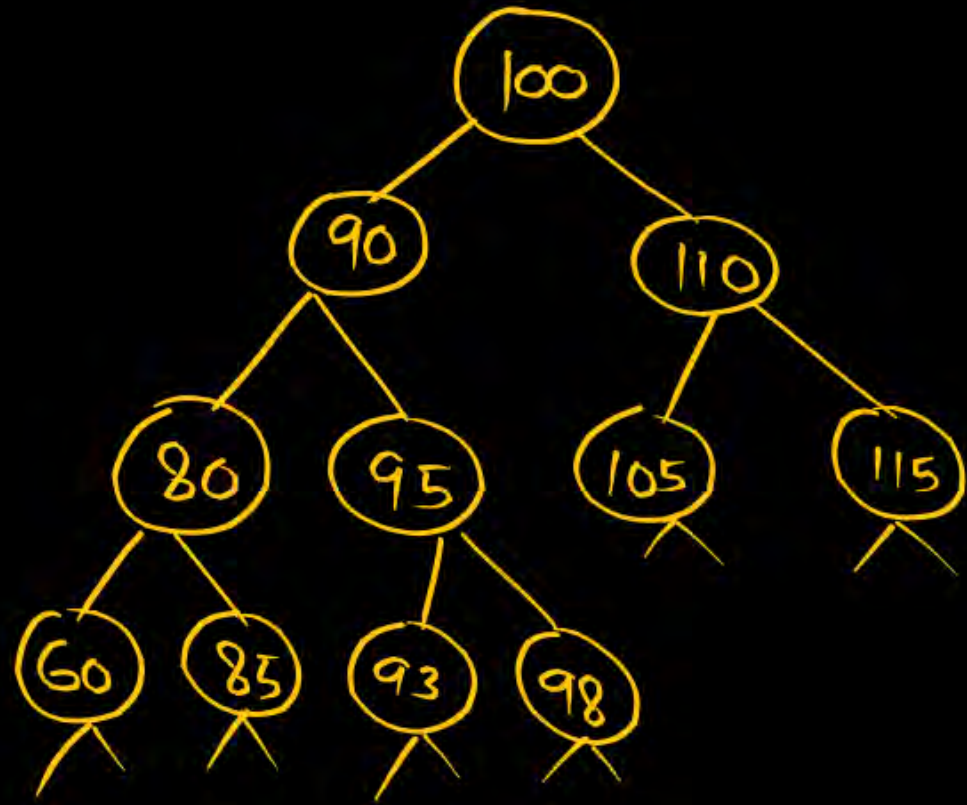
Keys  
10, 20, 30, 40

Q # BSTs with  $n$  keys  $\Rightarrow$   $\binom{\text{structure with } n \text{ nodes}}{n \text{ nodes}} \times 1$

$$= \frac{2^n C_n}{n+1}$$

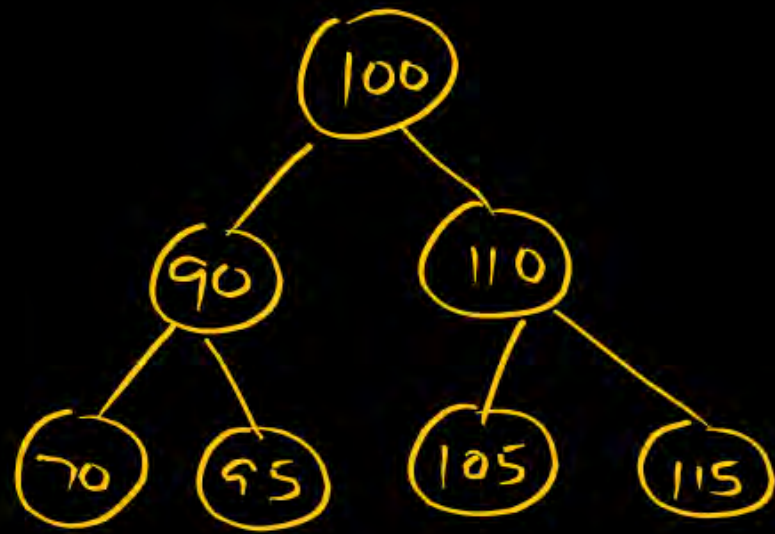
2

The inorder traversal of a BST is always increasing order of keys.



In: 60, 80, 85, 90, 93, 95, 98, 100, 105, 110, 115





Pre: 100, 90, 70, 95, 110, 105, 115

Pre: Given

In: 70, 90, 95, 100, 105, 110, 115

Unique BST

Post-order

Given the preorder of some BST:

Pre: 100, 90, 70, 95, 110, 105, 115

then the postorder traversal of the BST is: —

- a)
- b)
- c)
- d)

Preorder

→ unique tree

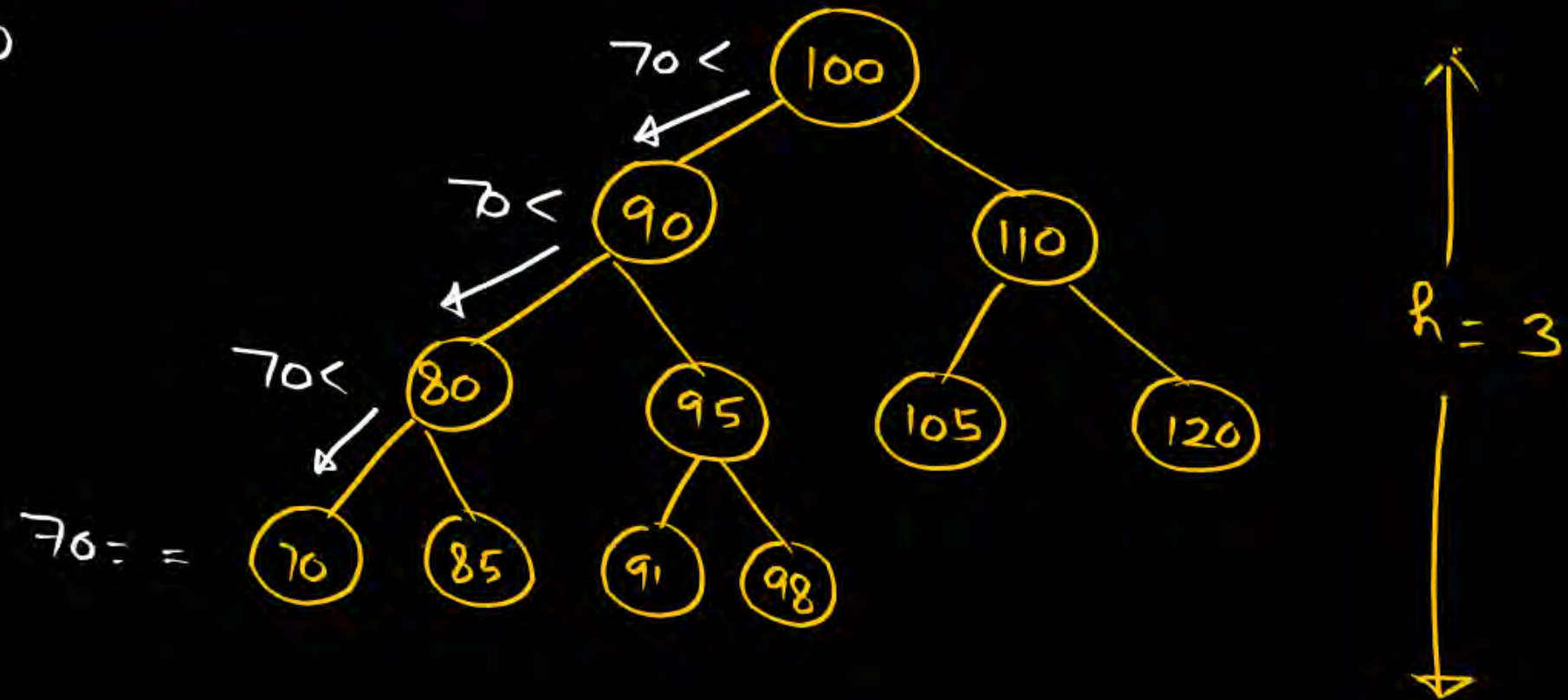
Pankaj sir PW



## Search in a BST

Key = 70

$$\begin{aligned}\# \text{comp} &= 4 \\ &= (h+1) \\ &= O(h)\end{aligned}$$



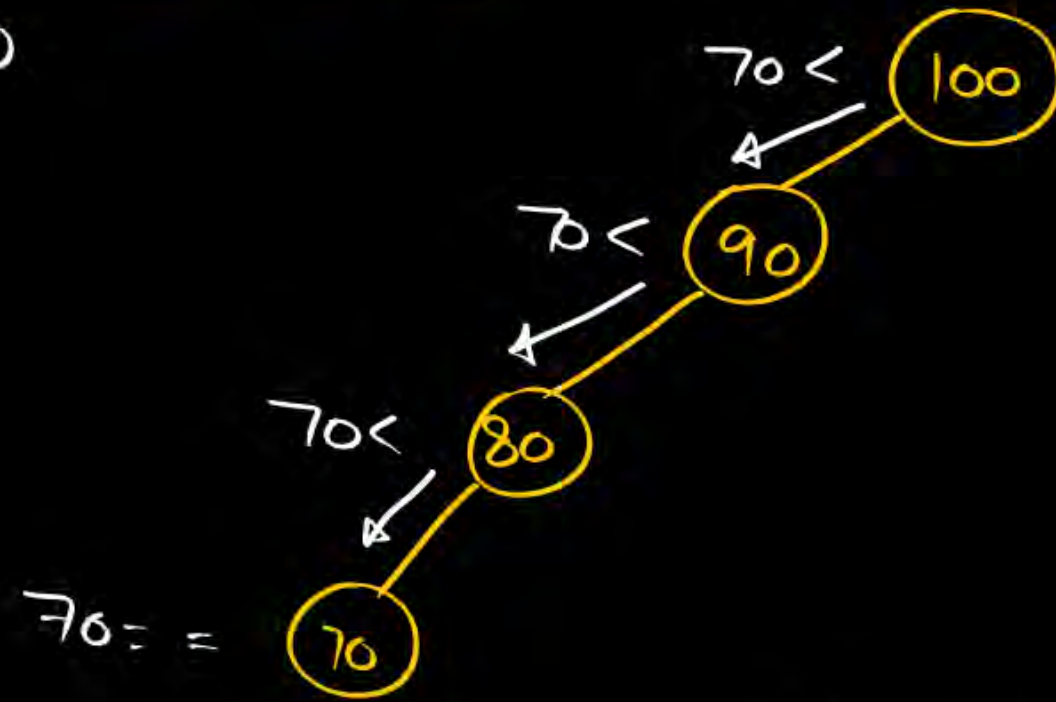
## Search in a BST

Key = 70

$$\# \text{ comp} = 4$$

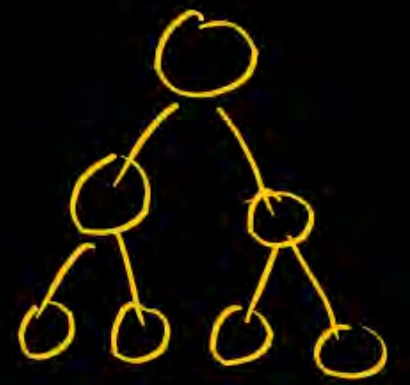
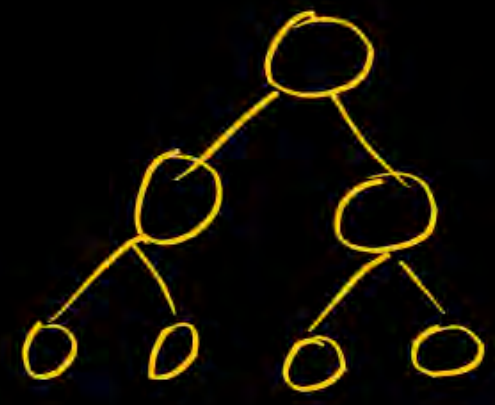
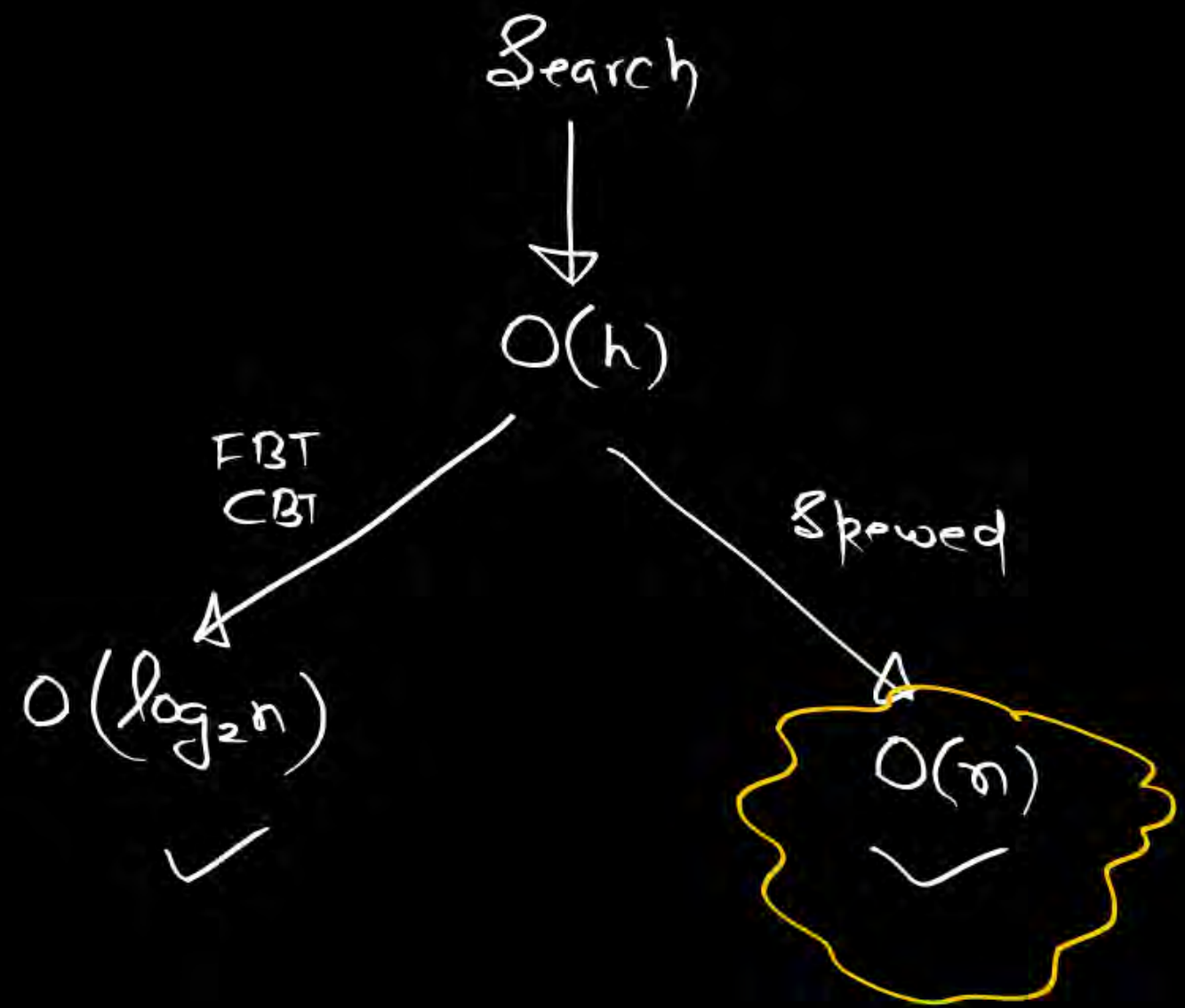
$$= (h + 1)$$

$$= O(h)$$



Skewed binary tree

$h = 3$





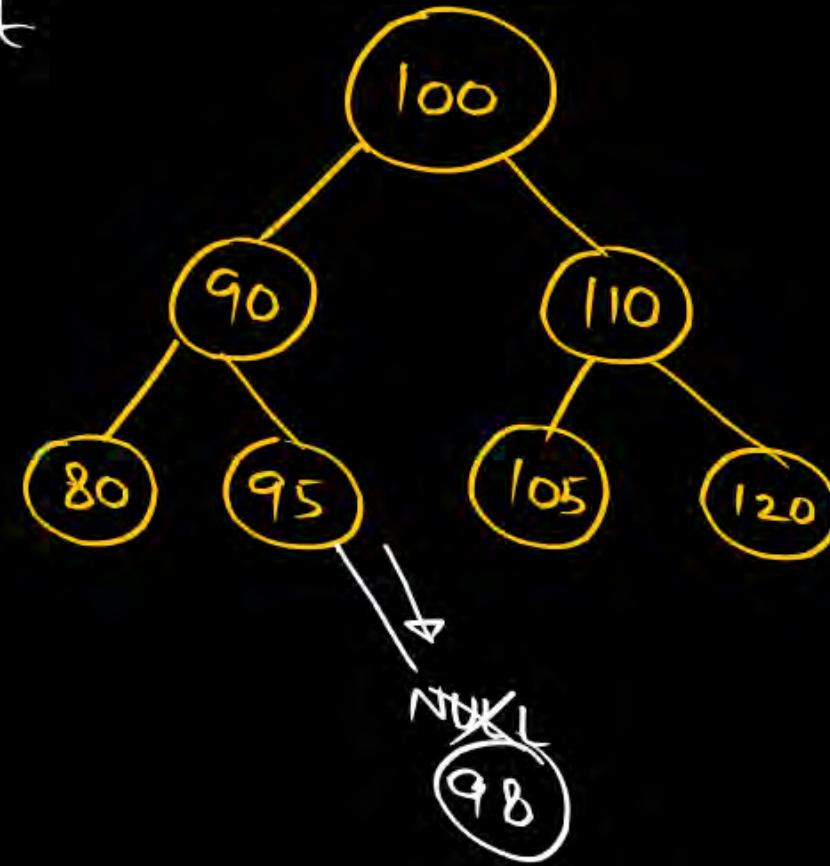
# Insertion in a BST

insert : 98

~~a) Search~~  
b) Insert

right of 95

Complexity

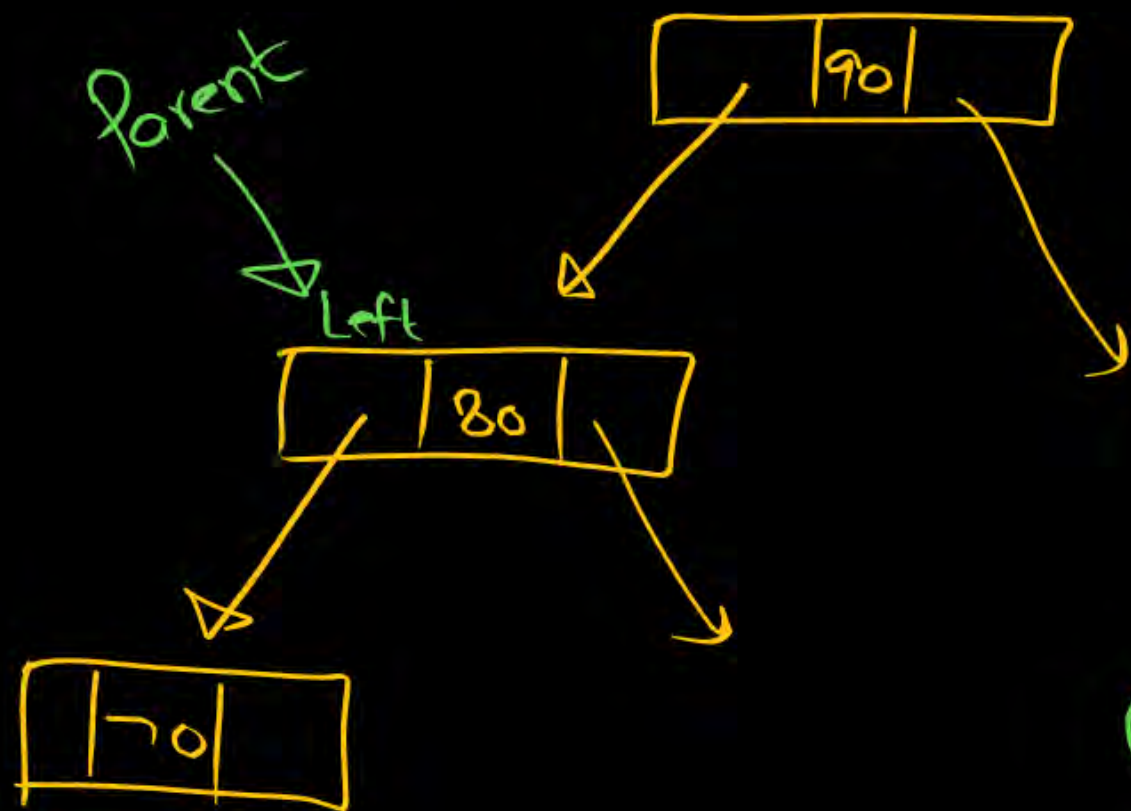


## Deletion from BST

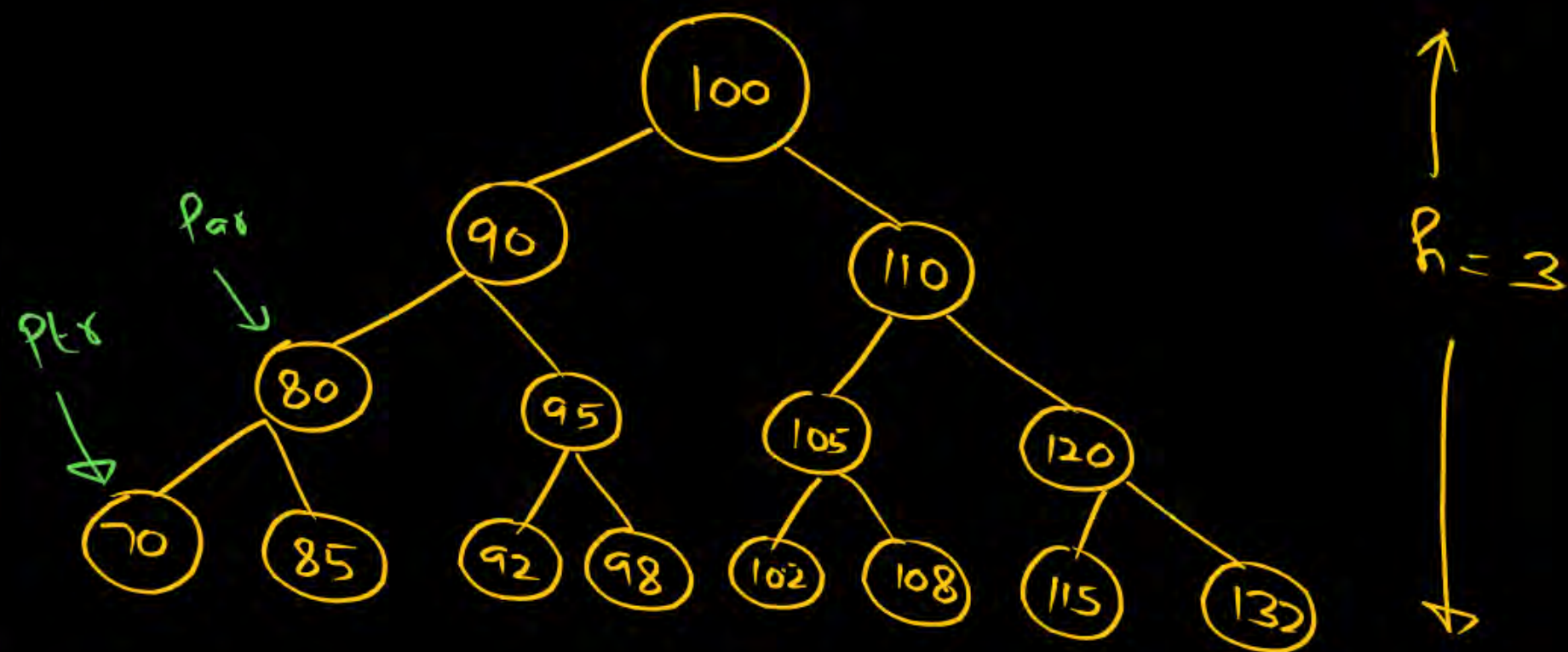
- a) Delete a node with 0-child (leaf node)
- b) Delete a node with 1-child
- c) Delete a node with 2-child

## Deletion of leaf node

delete 70



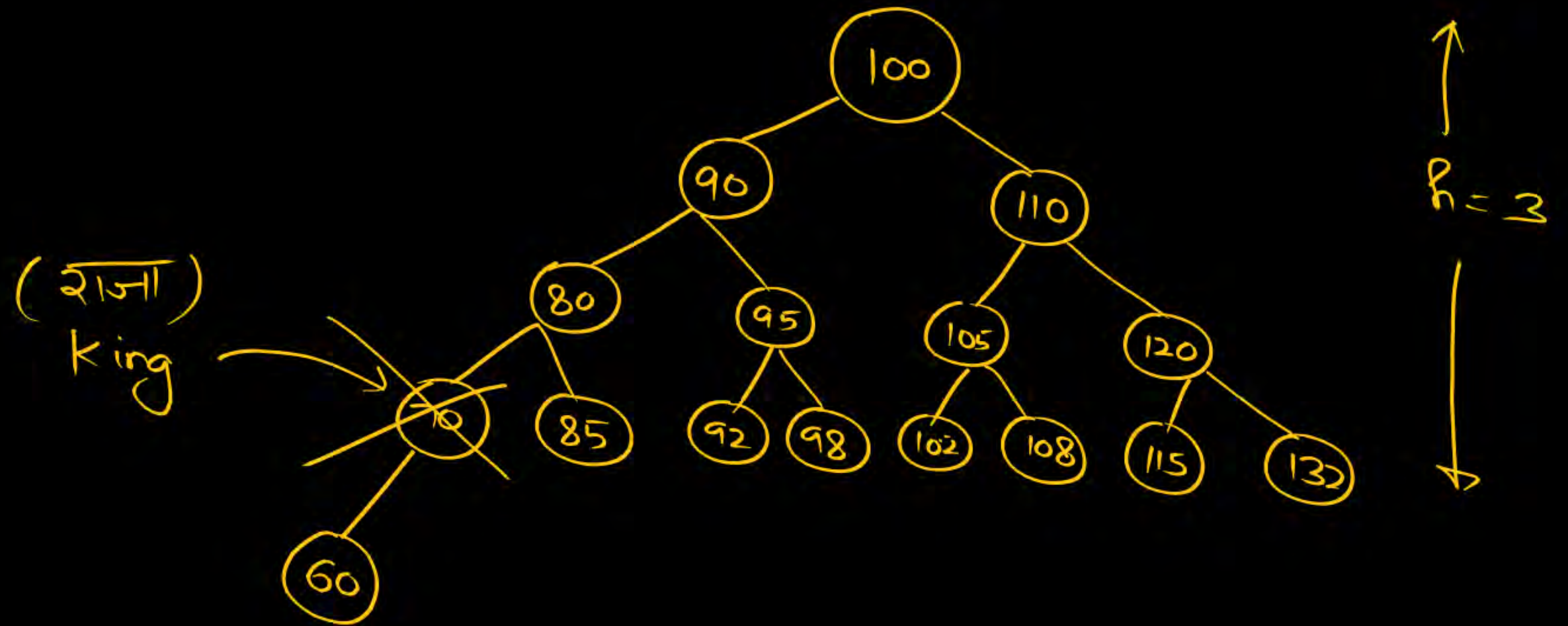
Node to be  
deleted



- ① we need to identify the parent pointer (of node to be deleted), which is pointing to deleted node
- ② Set this parent pointer to NULL.

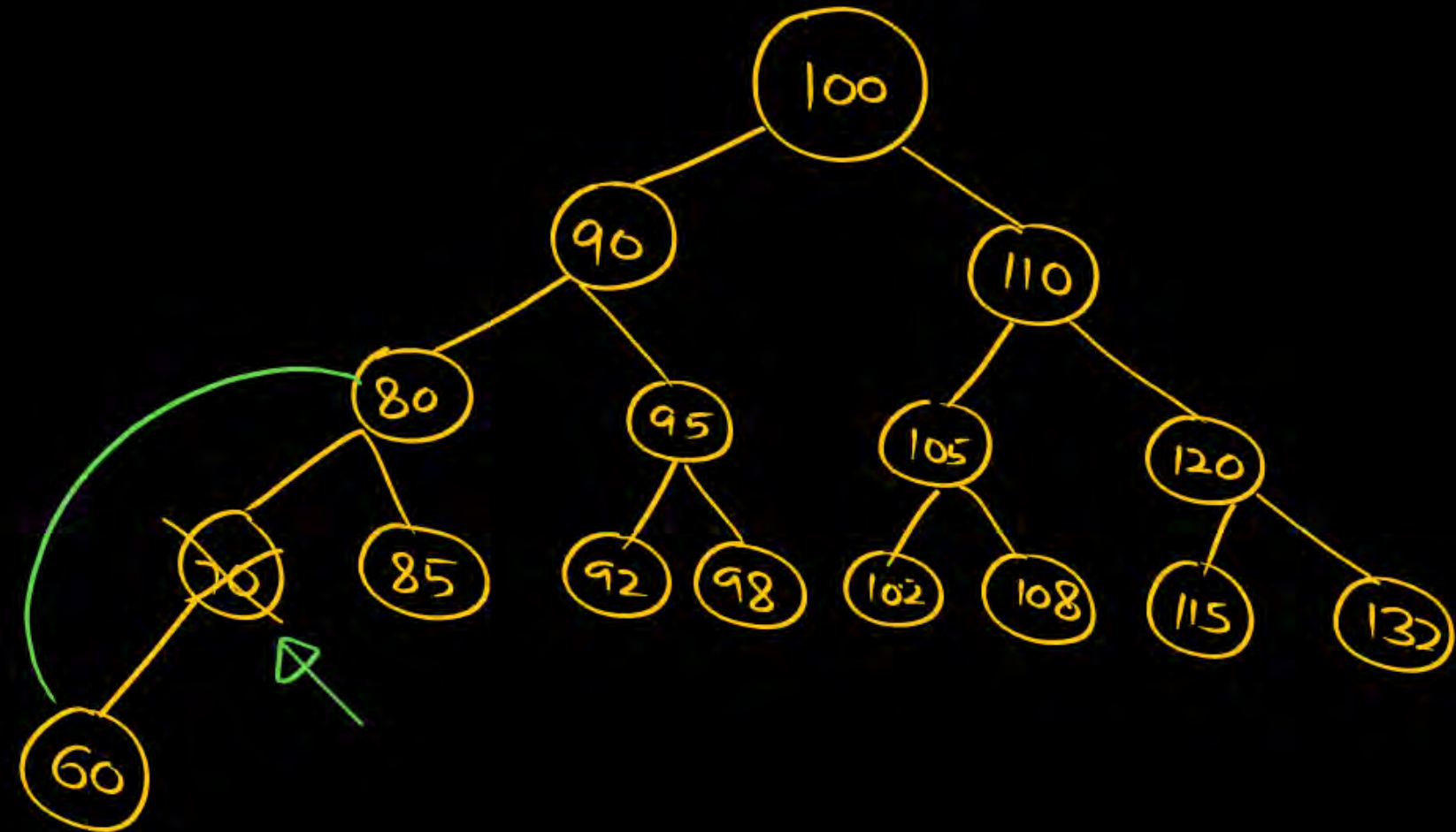


## Deletion of node with exactly 1-child

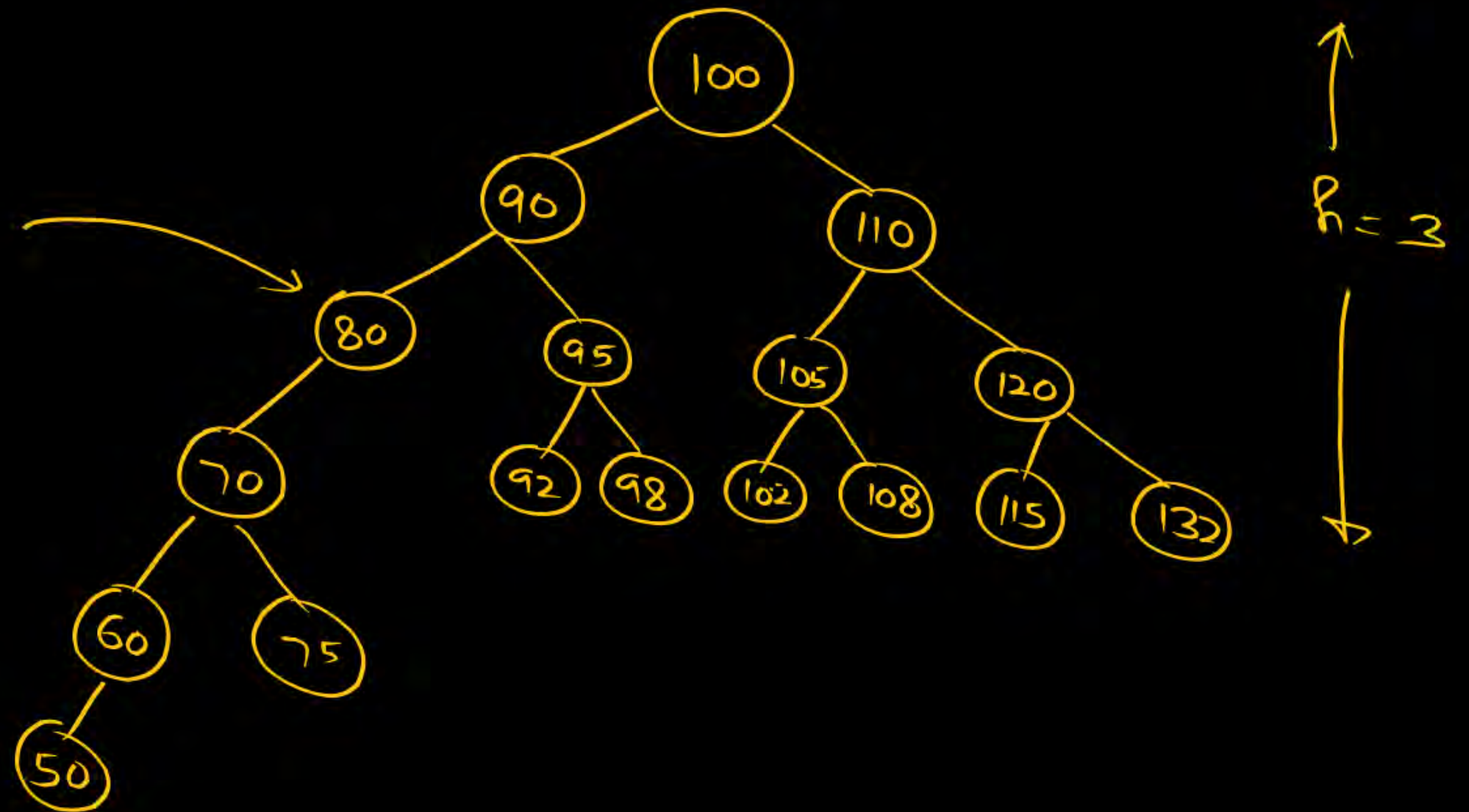


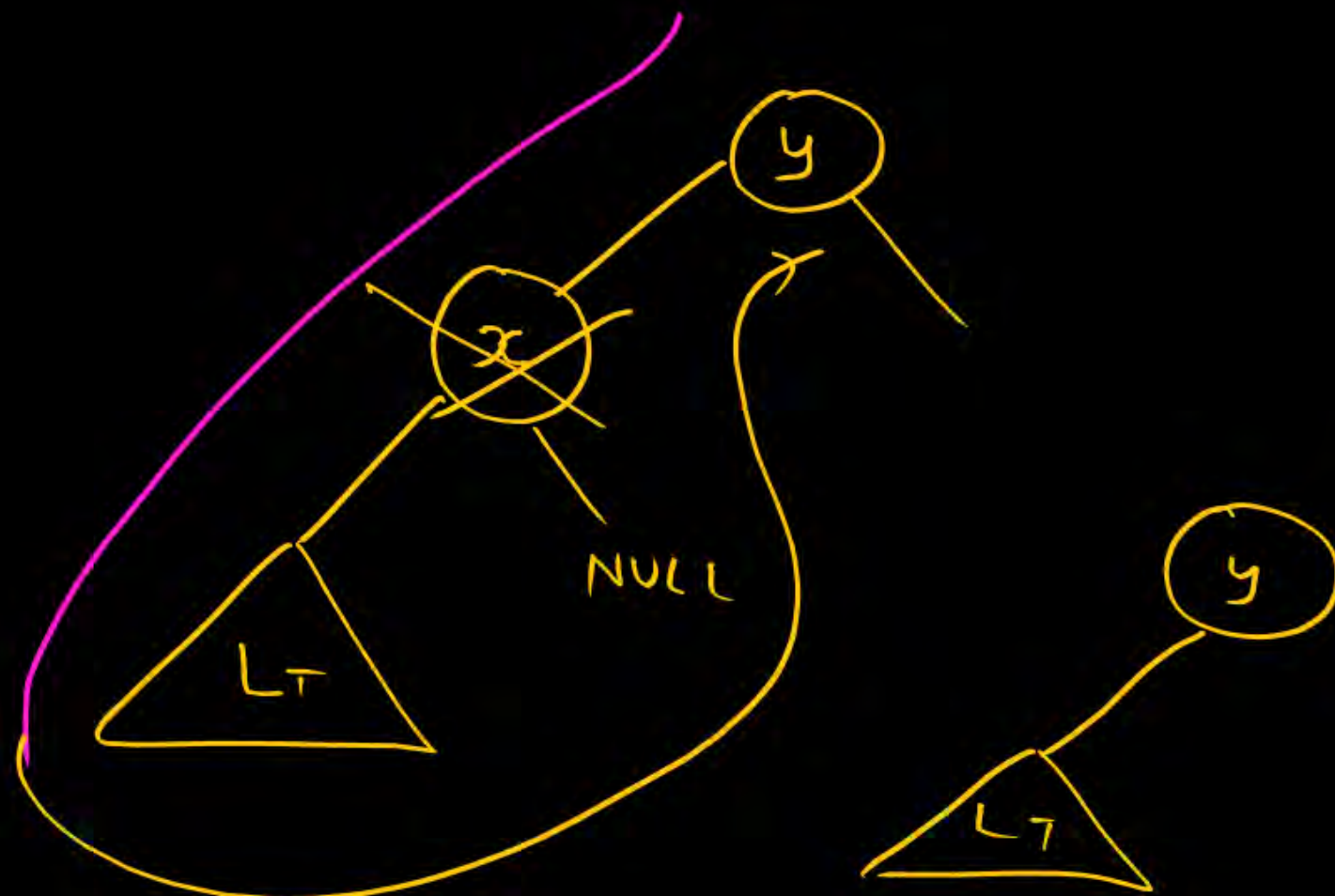
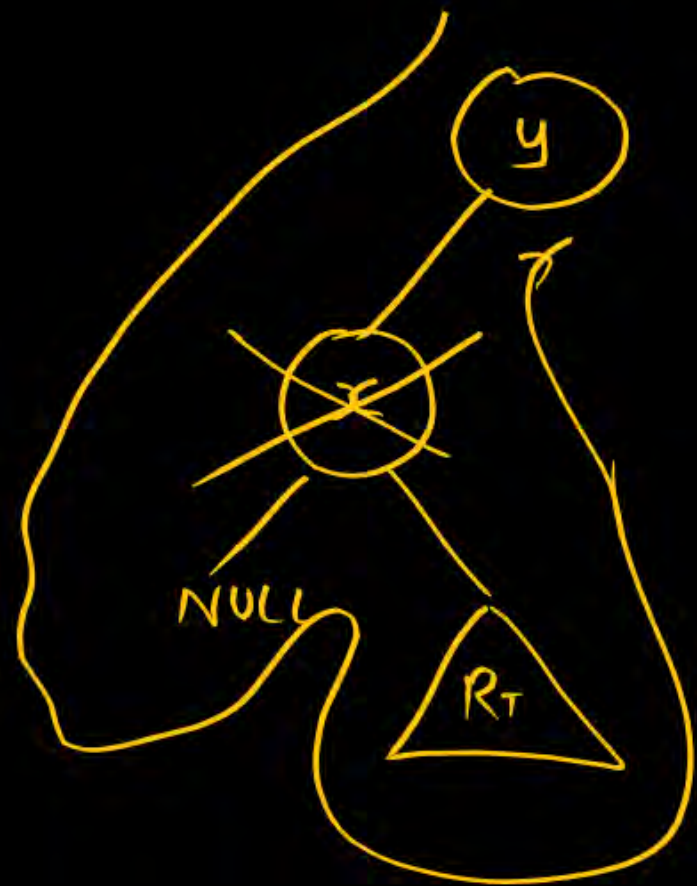
## Deletion of node with exactly 1-child

(21511)  
King

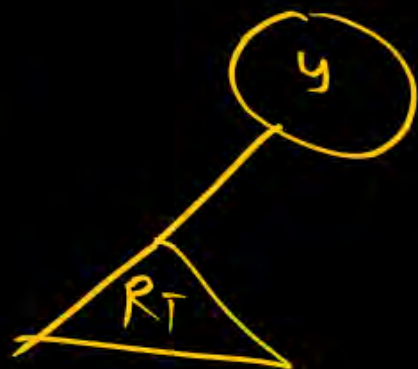


## Deletion of node with exactly 1-child





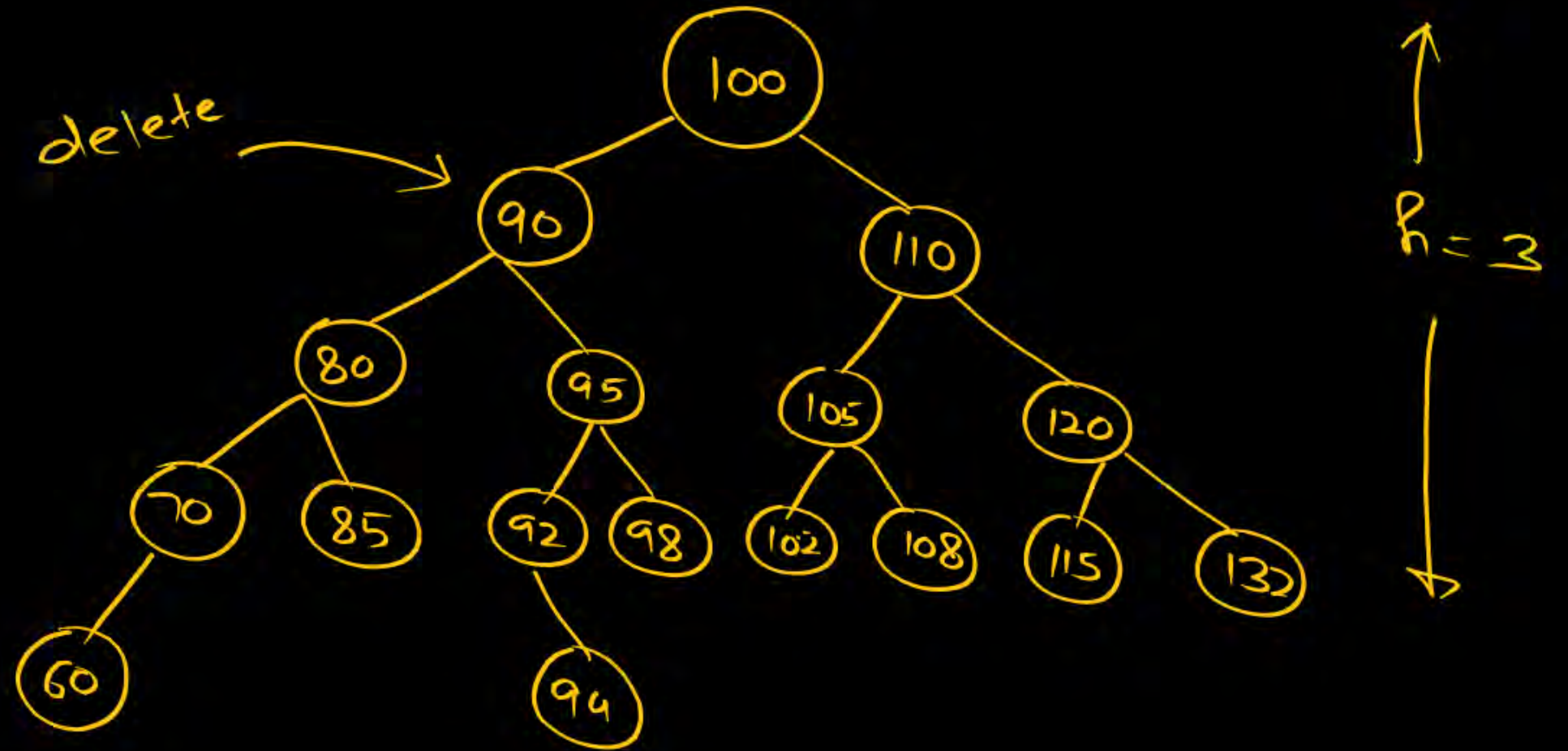
~~$x$~~ ,  $(R_T)$ ,  $y$



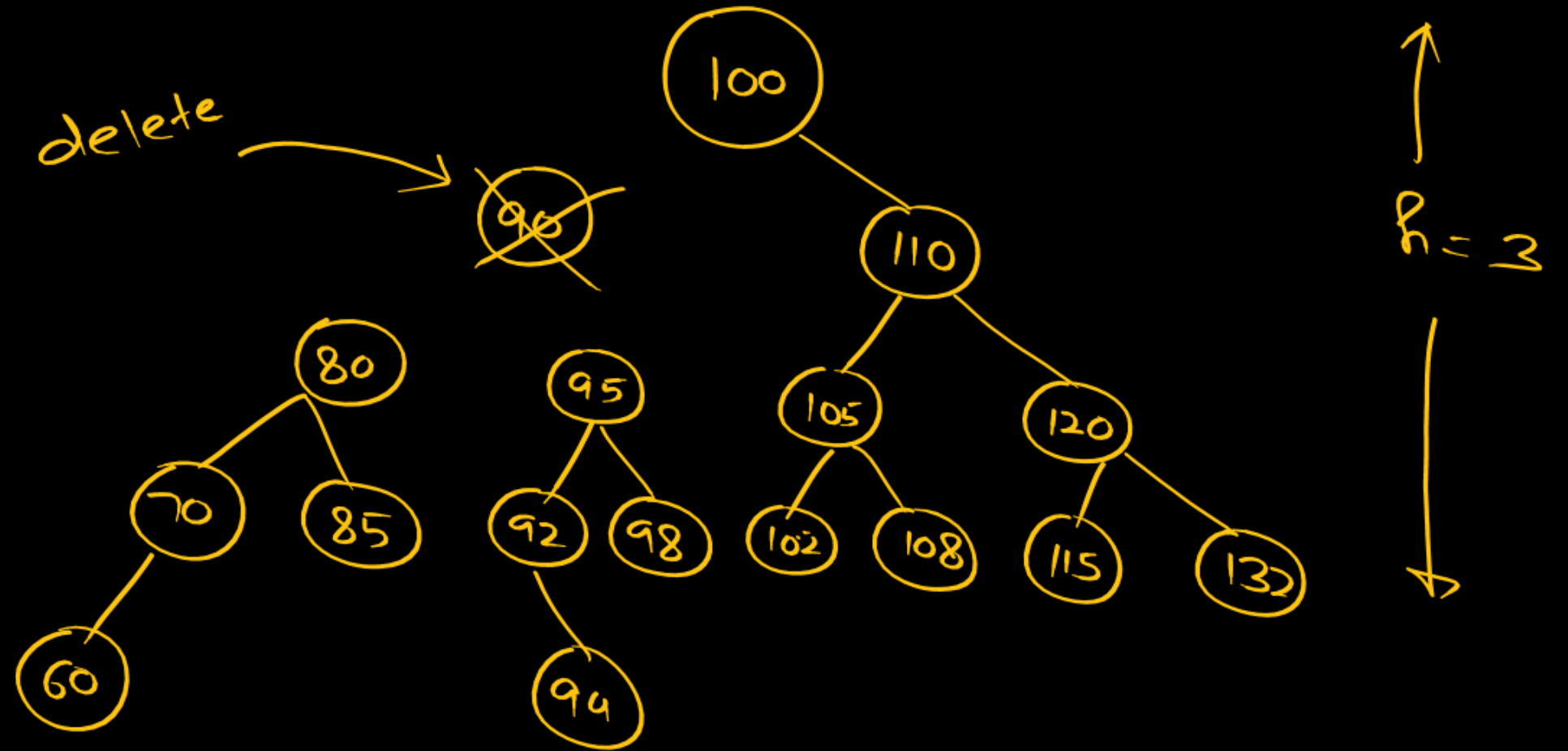
$L_T(x), x, y$



## Deletion of node with 2-children



## Deletion of node with 2-children



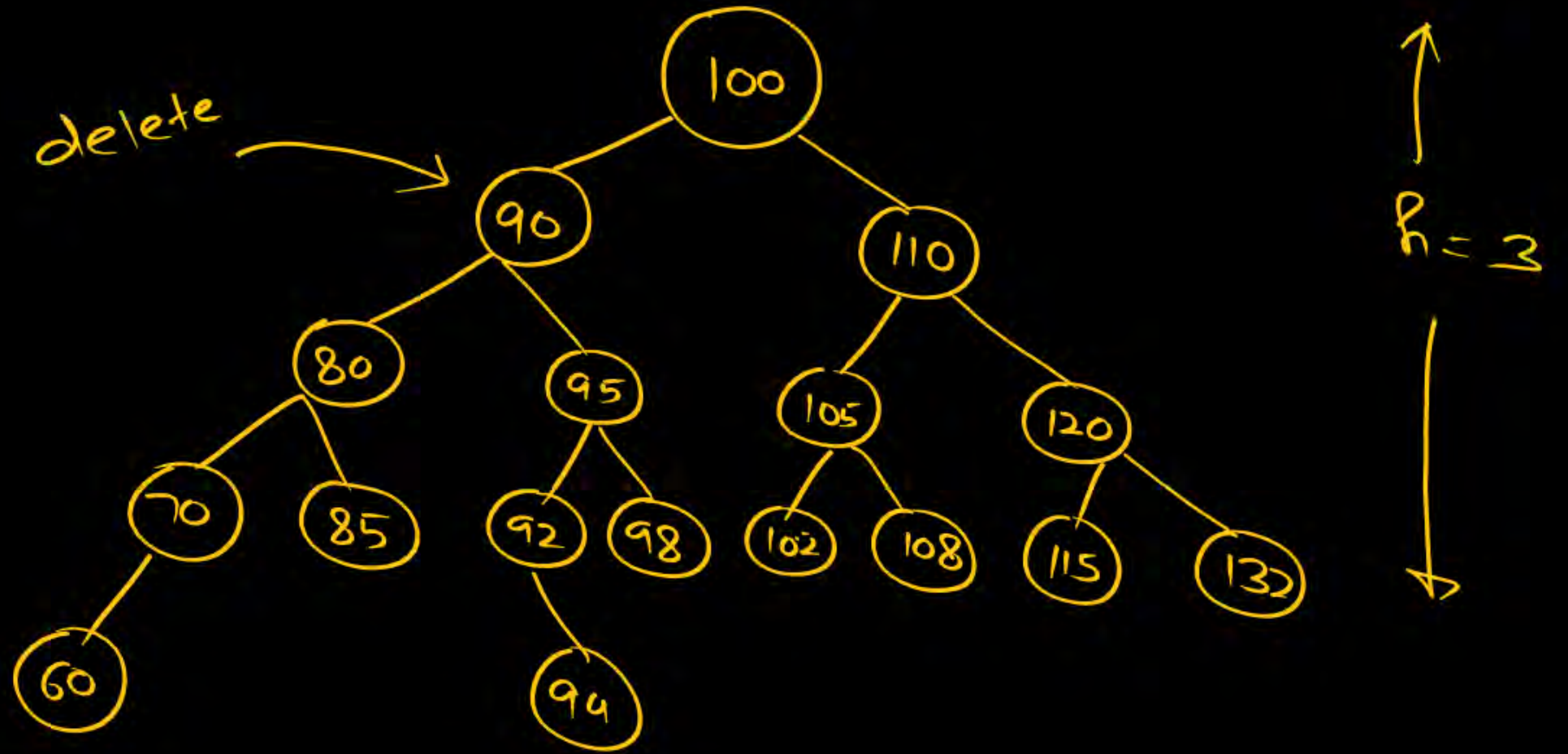
## Deletion of node with 2-children

(i) 1<sup>st</sup> way: Replace the key

to be deleted by the  
largest key in Left-subtree &  
of node to be deleted

then perform deletion  
OR

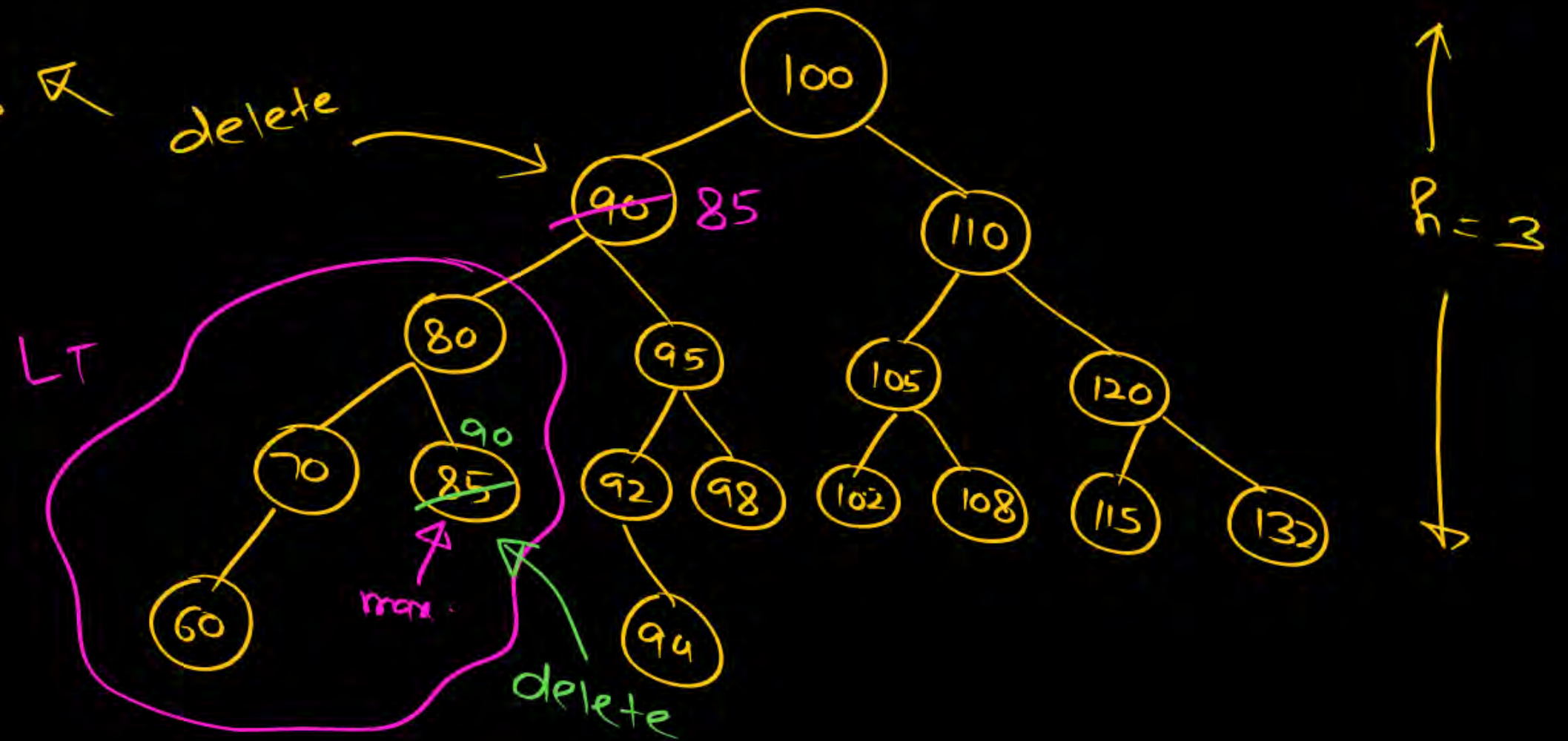
(ii) 2<sup>nd</sup> way: Replace the key  
to be deleted by the  
smallest key in the right  
subtree of node to be deleted  
& then perform deletion.





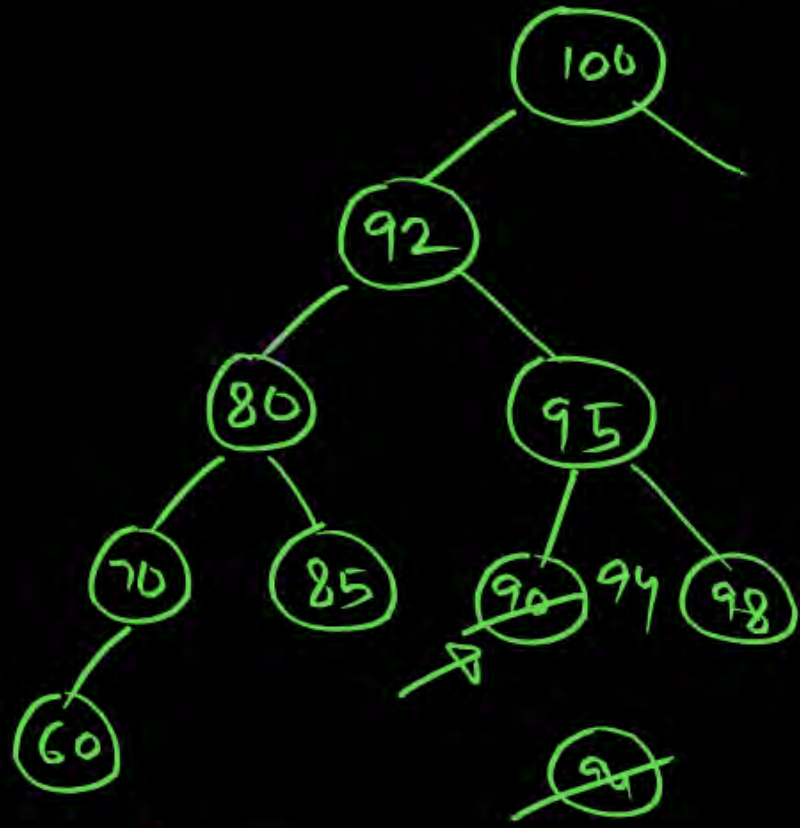
## Deletion of node with 2-children

(i) 1st way: Replace the key to be deleted by the largest key in Left subtree of node to be deleted & then perform deletion

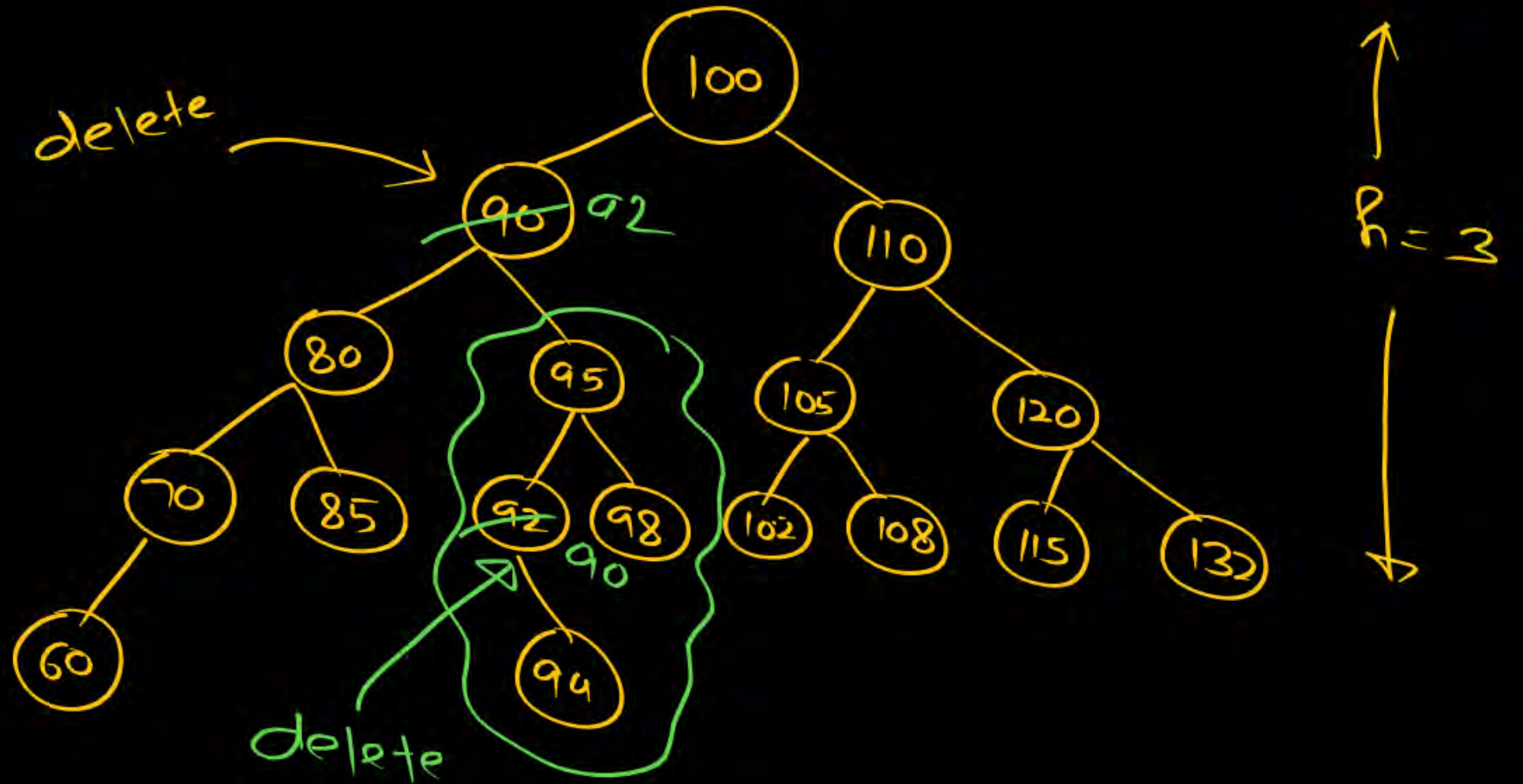




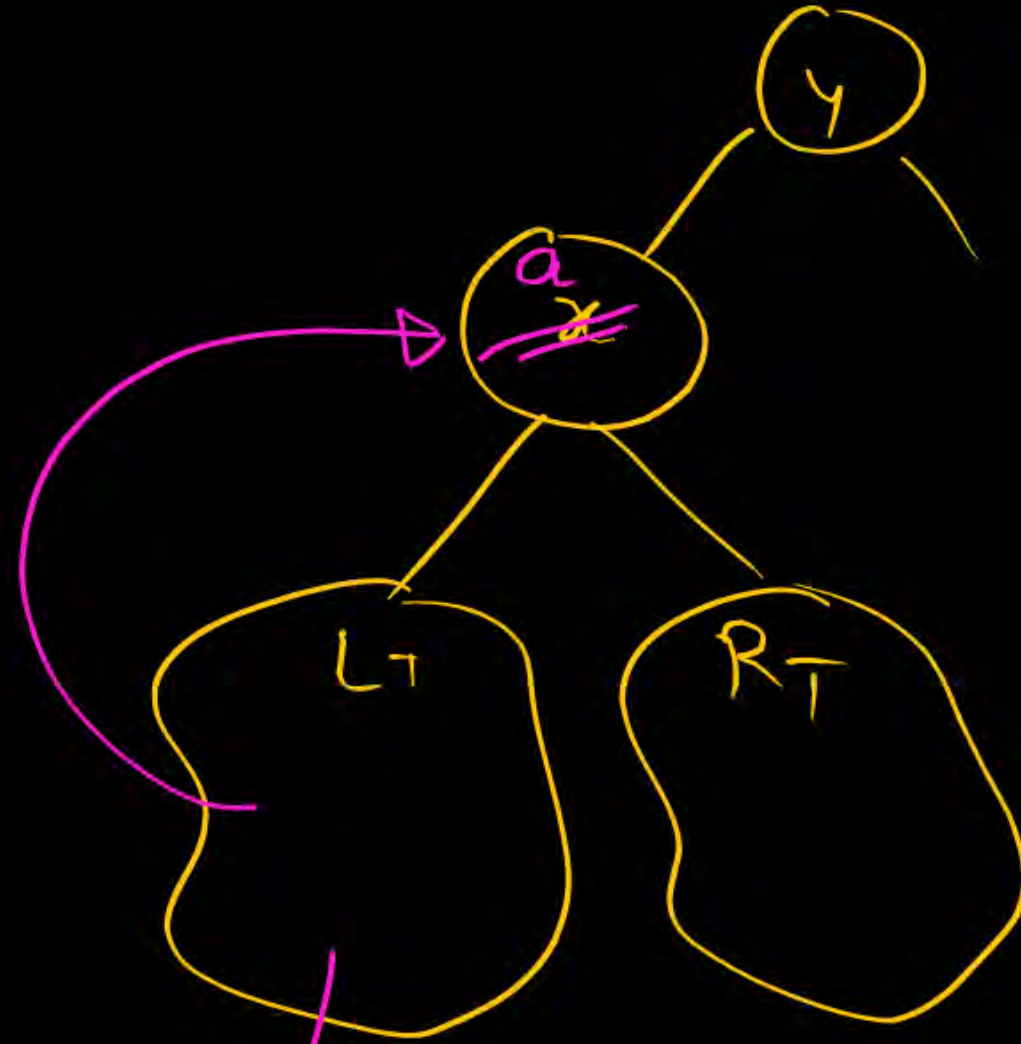
## Deletion of node with 2-children



ii) 2<sup>nd</sup> way: Replace the key to be deleted by the smallest key in the right subtree of node to be deleted & then perform deletion.

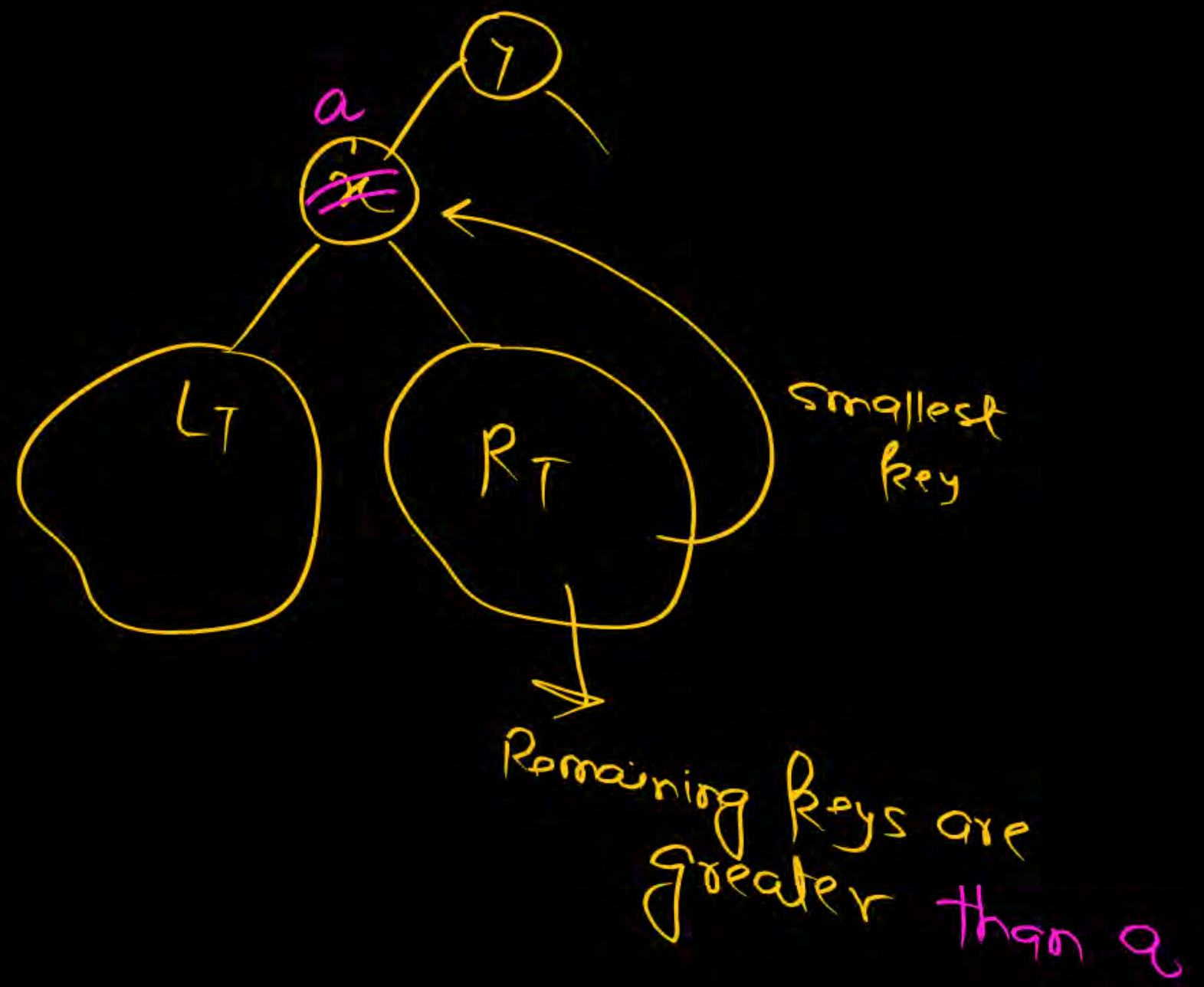


Case 1



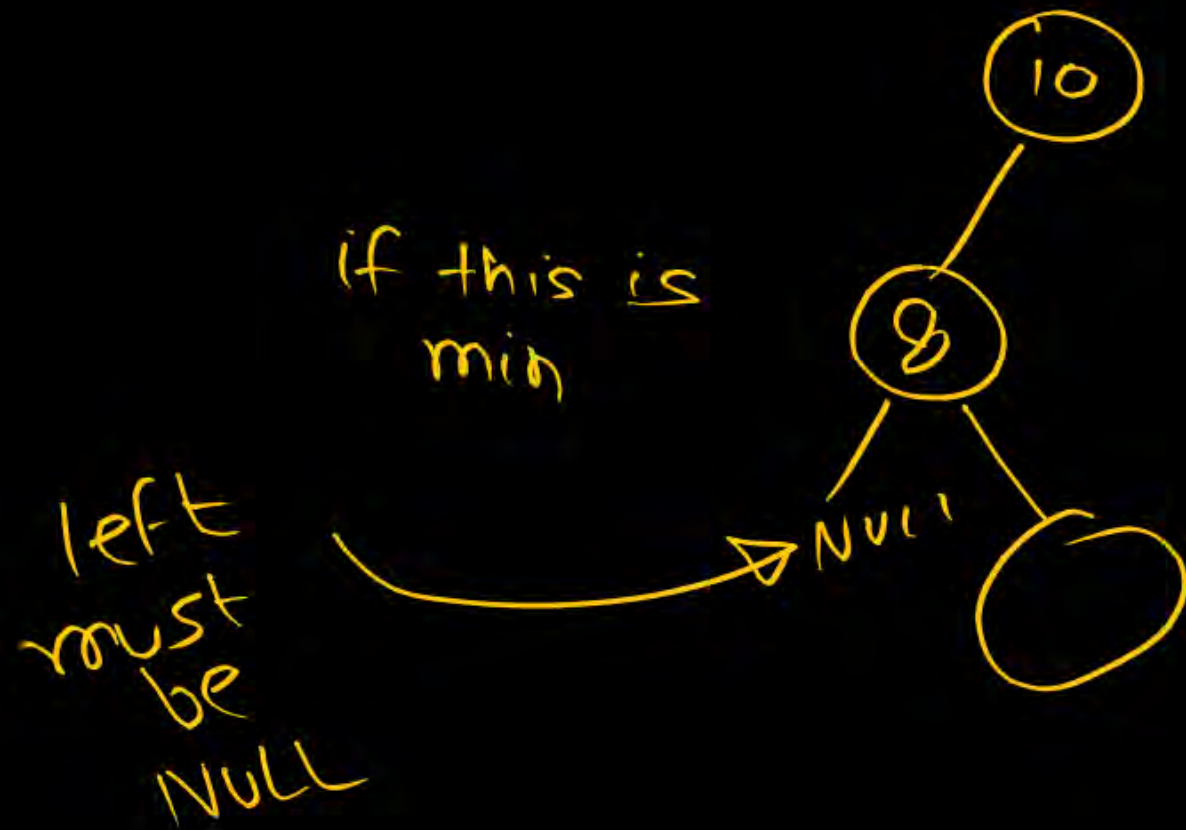
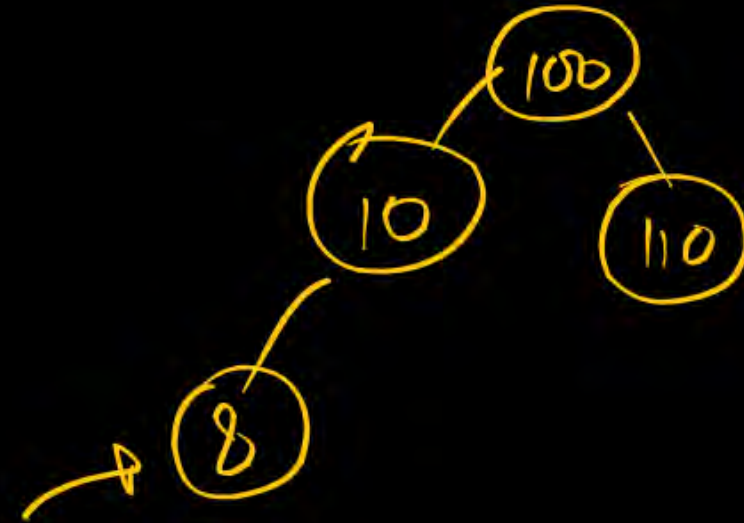
Rem. keys are  
smaller than a

Case 2:





Node with min value can have 0 or 1 child (No child or Right child)





Node with max. value can have 0 or 1 child



deletion of node  
with  
2-child



Converted into  
deletion of node  
with 0/1  
child

BST

10 node ↗ max  
↘ min



○ ○ ○ ○

○



C, DS

C++/STL

Python

Coding Problems 300+

→ review



**THANK - YOU**