CS & IT ENGINEERING

Data Structures

Tree

Lecture No.- 09

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Recap of Previous Lecture

Topic Tree Part-08

Topics to be Covered

Topic Tree Part-09

Topic : Tree

Q18. The pre-order traversal of a binary search tree is given by 12,8, 6,2,7, 9,10, 16,15, 19,17, 20. Then the post-order traversal of this tree is:

[GATE-2017: 2M]

A. 2,6,7,8,9,10,12,15,16,17,19,20
B. 2,7,6,10,9,8,15,17,20,19,16,12
C. 7,2,6,8,9,10,20,17,19,15,16,12
D. 7,6,2,10,9,8,15,16,17,20,19,12

Topic : Tree

Q19. Which of the following is/are correct inorder traversal sequence(s) of binary search tree(s)?

1. 3, 5, 7, 8, 15, 19, 25
2. 5, 8, 9, 12, 10, 15, 25
3. 2, 7, 10, 8, 14, 16, 20
4. 4, 6, 7, 9, 18, 20, 25

[GATE-2015: 1M]

A. 1 and 4 only
B. 2 and 3 only
C. 2 and 4 only
D. 2 only

Topic : Tree

Q20. The preorder traversal sequence of a binary search tree is 30, 20, 10, 15, 25, 23, 39, 35, 42. Which one of the following is the postorder traversal sequence of the same tree?

[GATE-2013: 2M]

A. 10, 20, 15, 23, 25, 35, 42, 39, 30
B. 15, 10, 25, 23, 20, 42, 35, 39, 30
C. 15, 20, 10, 23, 25, 42, 35, 39, 30
D. 15, 10, 23, 25, 20, 35, 42, 39, 30

Pre : Root, L_T, R_T
In : L_T, Root, R_T
Post : L_T, R_T, Root

Q21. The following three are known to be the preorder, inorder and postorder sequences of a binary tree. But it is not known which is which.

I. MBCAFHPYK — Pre ⇒ M root node

II. KAMCBYPFH

III. MABCKYFPH

Pick the true statement from the following.

[GATE-2008: 2M]

A. I and II are preorder and inorder sequences, respectively

B. I and III are preorder and postorder sequences, respectively

C. II is the inorder sequence, but nothing more can be said about the other two sequences

D. II and III are the preorder and inorder sequences, respectively

Pre : Root, L_T, R_T
In : L_T, Root, R_T
Post : L_T, R_T, Root

Q21. The following three are known to be the preorder, inorder and postorder sequences of a binary tree. But it is not known which is which.

I. MBCAFHPYK → post
II. KAMCBYPFH → let Preorder ⇒ K root
III. MABCKYFPH → Inorder

Pick the true statement from the following.

[GATE-2008: 2M]

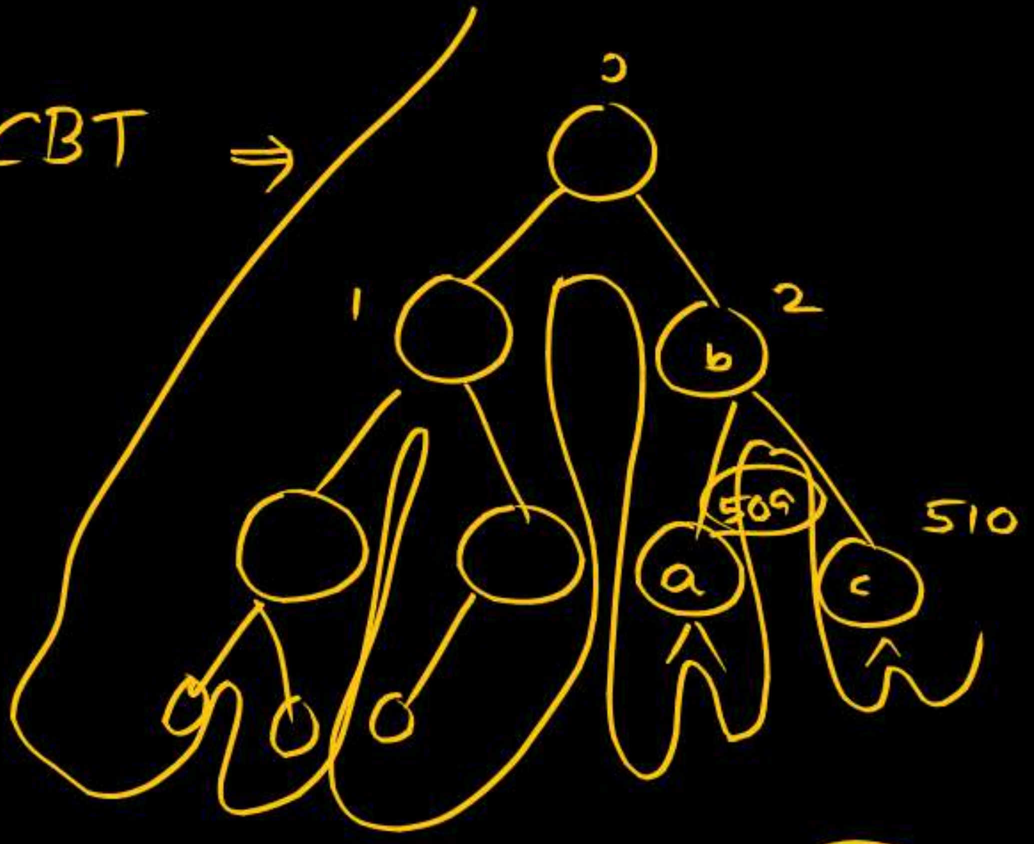
A. I and II are preorder and inorder sequences, respectively
B. I and III are preorder and postorder sequences, respectively
C. II is the inorder sequence, but nothing more can be said about the other two sequences
D. II and III are the preorder and inorder sequences, respectively

Topic : Tree

Q22. Suppose a binary search tree with 1000 distinct elements is also a complete binary tree. The tree is stored using the array representation of binary heap trees. Assuming that the array indices start with 0, the 3rd largest element of the tree is stored at index______________.

[GATE-2022-CS: 1M]

CBT ⇒



Inorder: abc

509

$2^3-1=7$

0

1

2

1

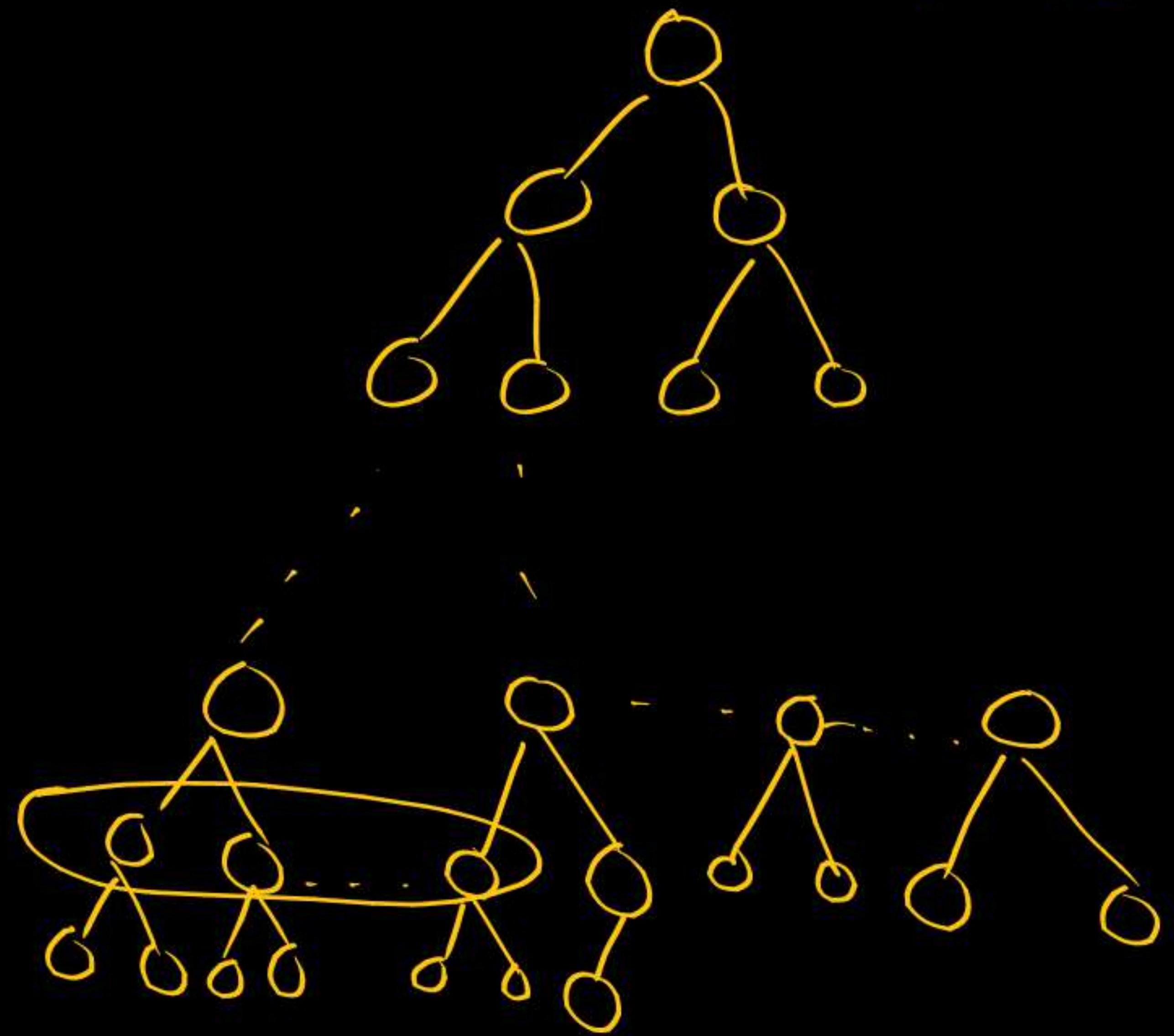
1

8

$\underset{\sim}{1000}$ ⇒ $2^{8+1}-1$ ⇒ 511 elem.

$2^{9+1}-1 = 1023$ elem

489 ele



Q23. Let T be a binary search tree with 15 nodes.
The minimum and maximum possible heights of T are:
Note: The height of a tree with a single node is 0.

[GATE-2017: 1M]

A. ~~4 and 15 respectively~~
B. 3 and 14 respectively
C. ~~4 and 14 respectively~~
D. 3 and 15 respectively

15 node

min height

$15 = 2^{h+1} - 1$

$16 = 2^{h+1}$

$h+1 = 4$

$\boxed{h = 3}$

max height

$n_{min} = h+1$

$15 = h+1$

$\boxed{h = 14}$

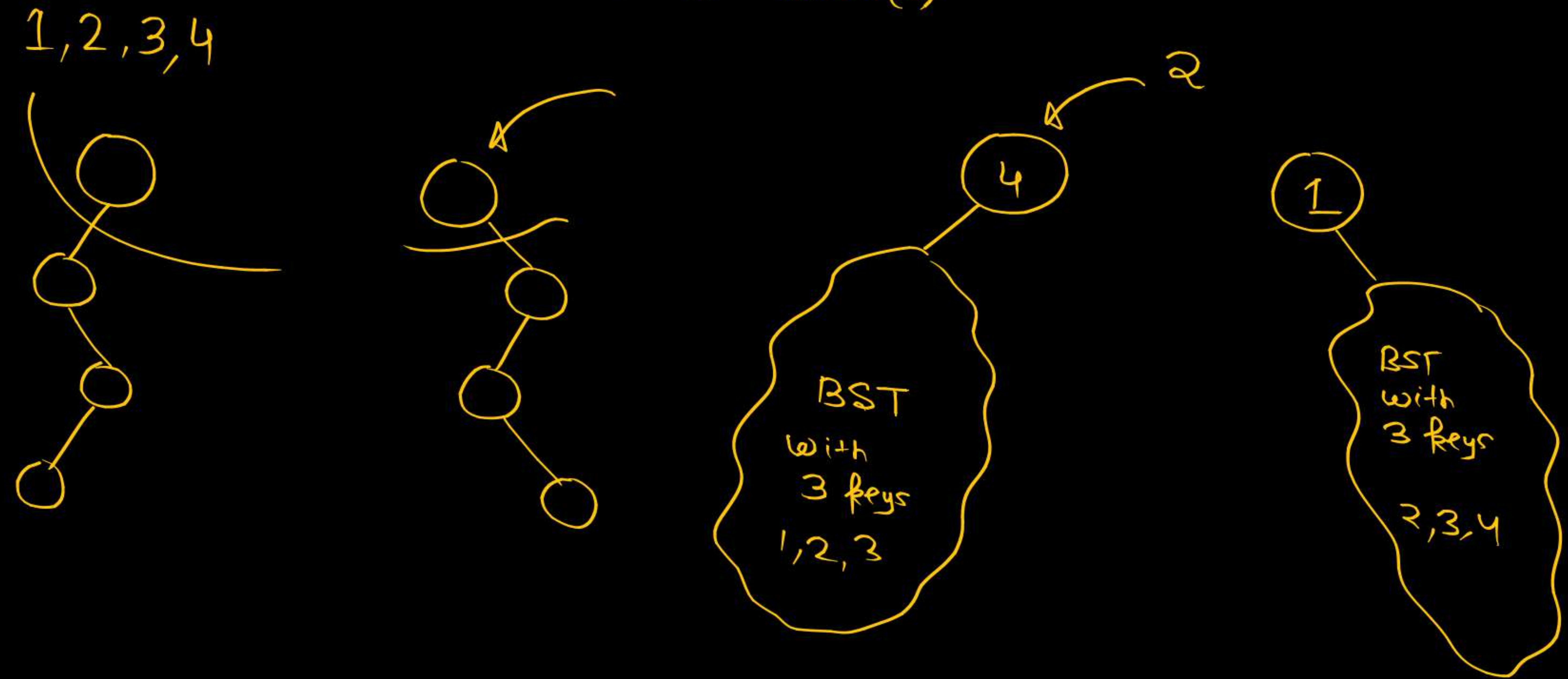
Q24. The number of ways in which the numbers 1,2,3,4,5,6,7 can be inserted in an empty binary search tree, such that the resulting tree has $\boxed{\text{height 6}}$ is ________.
Note: The height of a tree with a single node is 0.

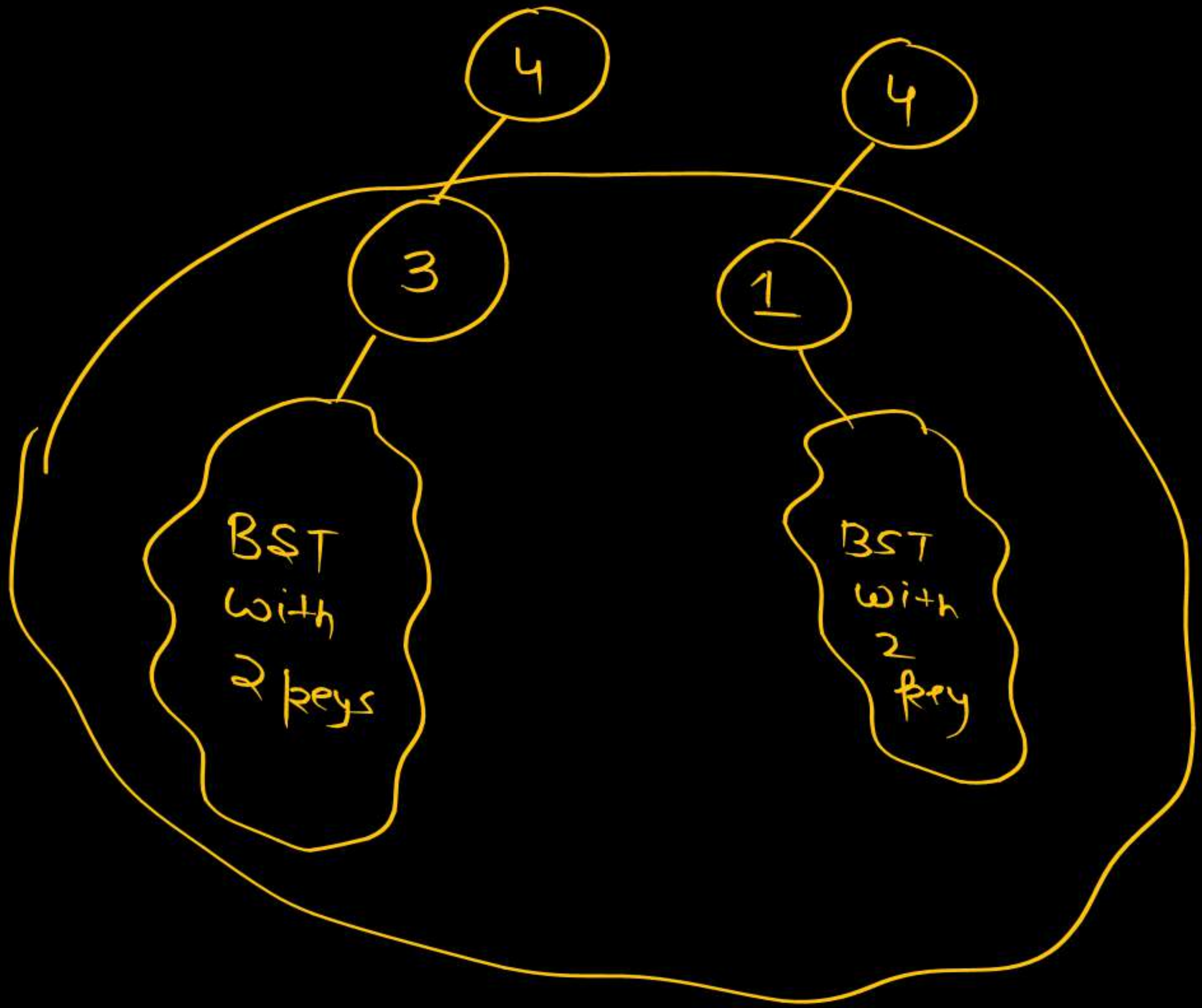
[GATE-2016: 2M]

Max. height possible with 7 nodes

Only when there is min no. of nodes at each level.
i,e 1 node at each level.

$BST(4) = 2 \times BST(3)$





$BST(4) = 2 \times BST(2)$

$BST(3) = 2 \times BST(2)$

$BST(4) = 2^2\, BST(2)$

$= 2^3\, BST(1)$

$\boxed{BST(4) = 2^3}$

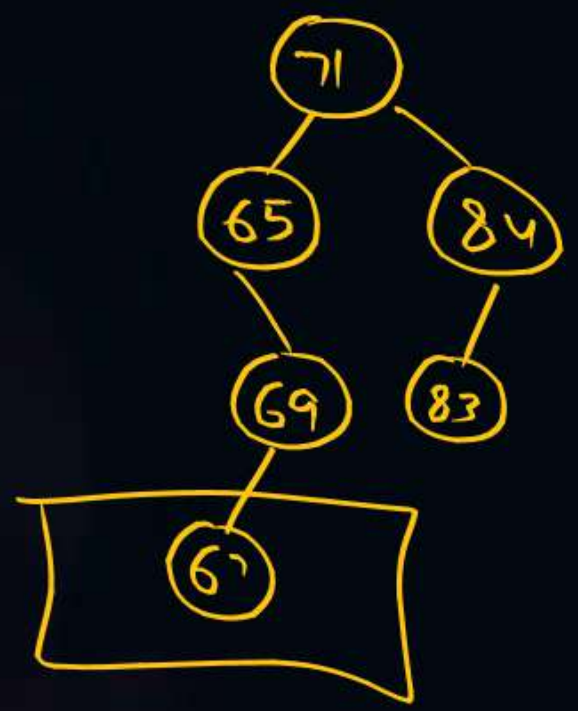
$$7 \text{ Keys} = 2^6$$

$$\Rightarrow \underline{64}$$

Topic : Tree

Q25. While inserting the elements 71,65,84,69,67,83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is

[GATE-2015: 1M]



A. 65
B. 67
C. 69
D. 83

Q26. A binary tree T has 20 leaves. The number of nodes in T having two children is _______

[GATE-2015: 1M]

$$\boxed{n_2 = n_0 - 1}$$

$n_0 \Rightarrow$ no. of node with 0-degree

$n_1 \Rightarrow$ no. of nodes with 1-degree

$n_2 \Rightarrow$ " " " 2-degree

$$N = 2 \times n_2 + 1 \times n_1 + \overset{(\text{root})}{1}$$

$$\downarrow$$

$$(n_0 + \cancel{n_1} + n_2) = 2n_2 + \cancel{n_1} + 1$$

$$n_0 + n_2 = 2n_2 + 1$$

$$\boxed{n_0 - 1 = n_2}$$

$$\boxed{n_2 = n_0 - 1}$$

Topic : Tree

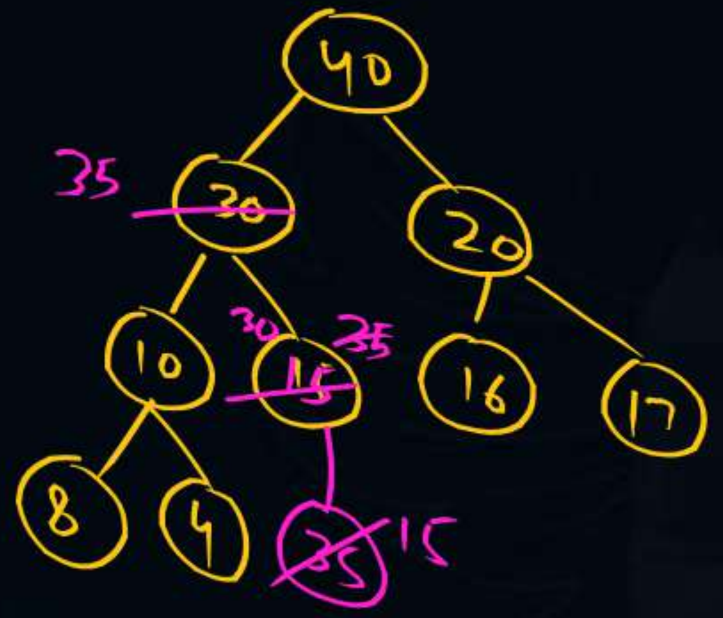
~~30~~ 40 sec — 1

Q27. Consider a max heap, represented by the array: 40, 30, 20, 10, 15, 16, 17, 8, 4.

Array Index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is

[GATE-2015: 1M]



~~A.~~ 40, 30, 20, 10, 15, 16, 17, 8, 4, 35

B. 40, 35, 20, 10, 30, 16, 17, 8, 4, 15

~~C.~~ 40, 30, 20, 10, 35, 16, 17, 8, 4, 15

D. 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

Topic : Tree

[Hashing-1
Hashing-2]

Monday
Tuesday

Q28. The height of a tree is the length of the longest root-to-leaf path in it. The maximum and minimum number of nodes in a binary tree of height 5 are

[GATE-2015: 1M]

PS

A. 63 and 6, respectively
B. 64 and 5, respectively
C. 32 and 6, respectively
D. 31 and 5, respectively

$n_{min} = 5+1 = h+1$

$n_{max} = 2^{h+1}-1 = 2^{5+1}-1 = 64-1$

$= 63$

Q29. Consider a rooted n node binary tree represented using pointers. The best upper bound on the time required to determine the number of subtrees having exactly 4 nodes is $O(n^a \log^b n)$. Then the value of a + 10b is 1 .

[GATE-2014: 1M]

$n = n^a \log^b n$

$a = 1, \ b = 0$

$a + 10 \times b$

$= 1 + 10 \times 0$

$= 1$

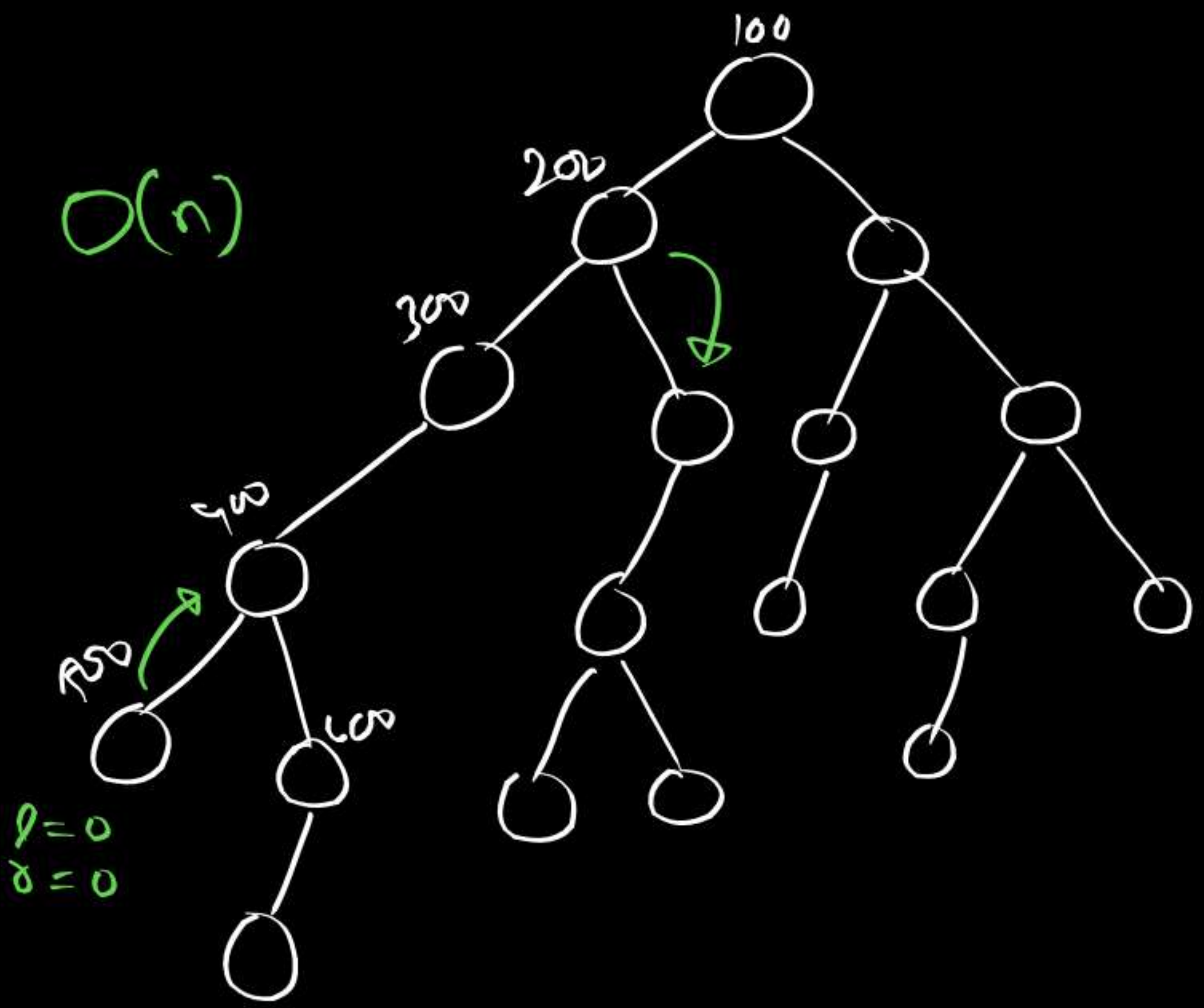
```
int count = 0;

int f(struct node *ptr)
{   int l=0, r=0;
    if(ptr==NULL)
            return 0;

    l = f(ptr->left);
    r = f(ptr->Right);
    if(1+l+r==4)
            count++;

    return 1+l+r;
}
```

$O(n)$



f(100)

l = f(200)

l = f(300)

l = f(400)

l = f(500)

l = ~~f(NULL)~~ 0

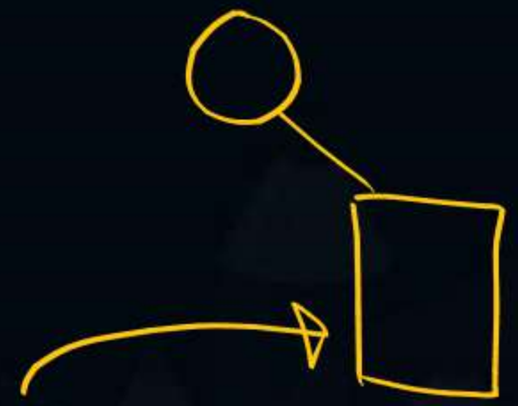
Topic : Tree

Q30. The height of a tree is defined as the number of edges on the longest path in the tree. The function shown in the pseudo-code below is invoked as the height (root) to compute the height of a binary tree rooted at the tree pointer root.

[GATE-2012: 2M]

```
int height (treeptr n)
{
 if ( n = = NULL) return –1;
 if ( n → left = = null )
 if ( n → right = = NULL ) return 0;
   else return B1 ;//Box1
 else {
   h1 = height (n → left);
   if (n → right = = NULL ) return (1 + h1);
   else {
h2 = height ( n → right );
return B2;//Box2
   }
 }
}
```

1+



$h = 1 + \max(h_1, h_2)$

Slide 36

The appropriate expression for the two boxes B1 and B2 are

A. B1: (1 + height (n → right)) ✓

B2: (1 + max (h1, h2))

A. B1: (height (n → right))

B2: (1 + max (h1, h2))

C. B1: height (n → right))

B2: max (h1, h2)

D. B1: (1 + height (n → right))

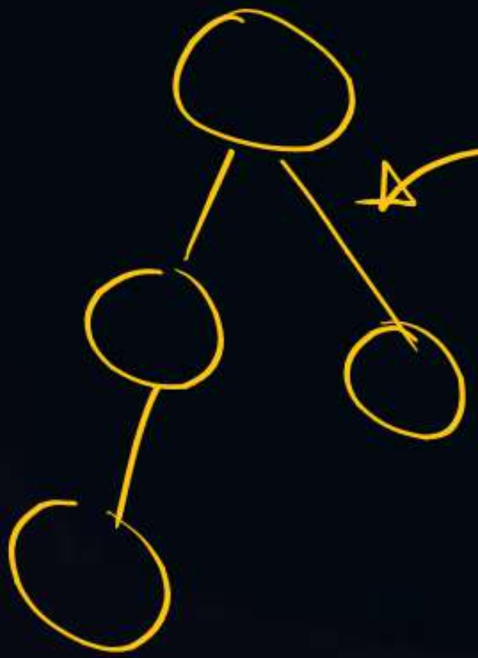
B2: max (h1, h2))

Topic : Tree

Q31. We are given a set of n distinct elements and an unlabeled binary tree with n nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree?

[GATE-2011: 2M]

40 %



1, 2, 3, 4

A. 0

B. 1

C. n!

D. $\frac{1}{(n+1)}\,{}^{2n}C_n$

Q32. You are given the postorder traversal, P, of a binary search tree on the n elements 1, 2, ..., n. You have to determine the unique binary search tree that has P as its postorder traversal. What is the time complexity of the most efficient algorithm for doing this?

[GATE-2008: 2M]

Divide & conquer Algorithm

A. $\Theta(\log n)$

B. $\Theta(n)$

C. $\Theta(n \log n)$

D. none of the above, as the tree cannot be uniquely determined.

Topic : Tree

Q33. A binary tree with n > 1 nodes has n_1, n_2 *and* n_3 nodes of degree one, two and three respectively. The degree of a node is a defined as the number of its neighbours.

Starting with the above tree, while there remains a node v of degree two in the tree, add an edge between the two neighbors of v and then remove v from the tree.

How many edges will remain at the end of the process?

[GATE-2008: 2M]

A. $2 * n_1 - 3$
B. $n_2 + 2 * n_1 - 2$
C. $n_3 - n_2$
D. $n_2 + n_1 - 2$

Topic : Tree

Q34. A binary tree with n > 1 nodes has n_1, n_2 *and* n_3 nodes of degree one, two and three respectively. The degree of a node is a defined as the number of its neighbours.

n_3 can be expressed as:

[GATE-2008: 2M]

A. $n_1 + n_2 - 1$

B. $n_1 - 2$

C. $[((n_1 + n_2)/2)]$

D. $n_2 - 1$

Topic : Tree

Q35. A Binary Search Tree (BST) stores values in the range 37 to 573. Consider the following sequence of keys.

I. 81, 537, 102, 439, 285, 376, 305

II. 52, 97, 121, 195, 242, 381, 472

III. 142, 248, 520, 386, 345, 270, 307

IV. 550, 149, 507, 395, 463, 402, 270

439

Which of the following statements is TRUE?

[GATE-2008: 2M]

A. I, II and IV are inorder sequences of three different BSTs

B. I is a preorder sequence of some BST with 439 as the root

C. II is an inorder sequence of some BS where 121 is the root and 52 is a leaf

D. IV is a postorder sequence of some BST with 149 as the root

Topic : Tree

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Q36. A Binary Search Tree (BST) stores values in the range 37 to 573. Consider the following sequence of keys.

~~I.~~ 81, 537, 102, 439, 285, 376, 305

II. 52, 97, 121, 195, 242, 381, 472

III. 142, 248, 520, 386, 345, 270, 307

IV. 550, 149, 507, 395, 463, 402, 270

Suppose the BST has been unsuccessfully searched for key 273. Which all of the above sequence list nodes in the order in which we could have encountered them in the search?

[GATE-2008: 2M]

A. II and III only

B. I and III only

C. III and IV only

D. III only

Topic : Tree

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Suppose the BST has been unsuccessfully searched for key 273. Which all of the above sequence list nodes in the order in which we could have encountered them in the search?

[GATE-2008: 2M]

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C. III and IV only

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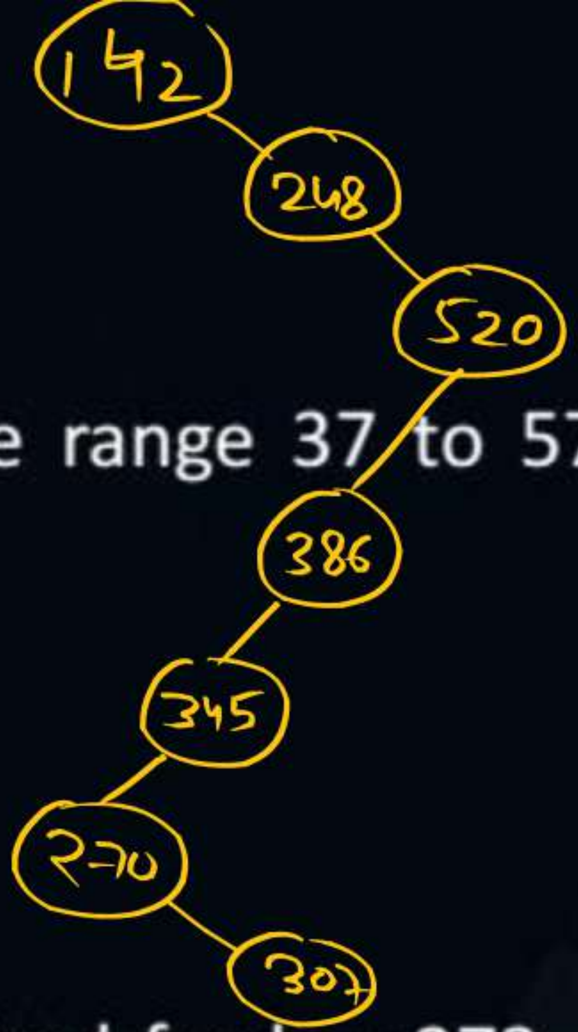
Topic : Tree



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Suppose the BST has been unsuccessfully searched for key 273. Which all of the above sequence list nodes in the order in which we could have encountered them in the search?

[GATE-2008: 2M]

A. II and III only

B. I and III only

C. III and IV only

D. III only

Topic : Tree

Q36. A Binary Search Tree (BST) stores values in the range 37 to 573. Consider the following sequence of keys.

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III. 142, 248, 520, 386, 345, 270, 307

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Suppose the BST has been unsuccessfully searched for key 273. Which all of the above sequence list nodes in the order in which we could have encountered them in the search?

[GATE-2008: 2M]

A. II and III only

B. I and III only

C. III and IV only

D. III only

Topic : Tree

Q37. What is the maximum height of any AVL-tree with 7 nodes? Assume that the height of a tree with a single node is 0.

[GATE-2009: 2M]

h	0	1	2	3	4
n(h)	1	2	4	7	12

A. 2
B. 3
C. 4
D. 5

3

Topic : Tree

Q38. Which of the following is TRUE? Balanced tree

[GATE-2008: 1M]

A. The cost of searching an AVL tree is $\Theta(\log n)$ but that of a binary search tree is $O(n)$ $\rightarrow O(n)$

B. The cost of searching an AVL tree is $\Theta(\log n)$ but that of a complete binary tree is $\Theta(n \log n)$

C. The cost of searching a binary search tree is $O(\log n)$ but that of an AVL tree is $\Theta(n)$

D. The cost of searching an AVL tree is $\Theta(n \log \llbracket(n)) \rrbracket$ but that of a binary search tree is $O(n)$

THANK - YOU