CS & IT ENGINEERING

Data Structures

Tree

Lecture No.- 06

By- Pankaj Sharma Sir

Recap of Previous Lecture

Topic Tree Part-05

AVL tree → a) BST Property
b) AVL tree property
Bal. factor

Inserted → LL
RR
LR
RL

Topics to be Covered

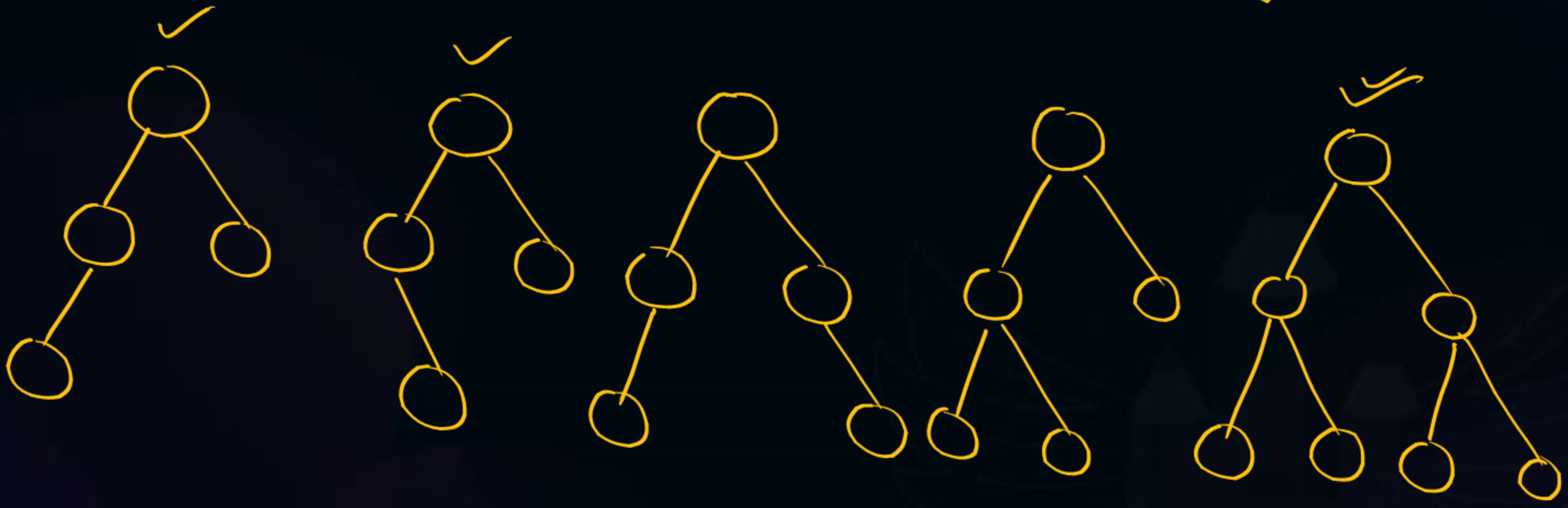
Topic Tree Part-06

Problems on AVL tree
results on AVL tree

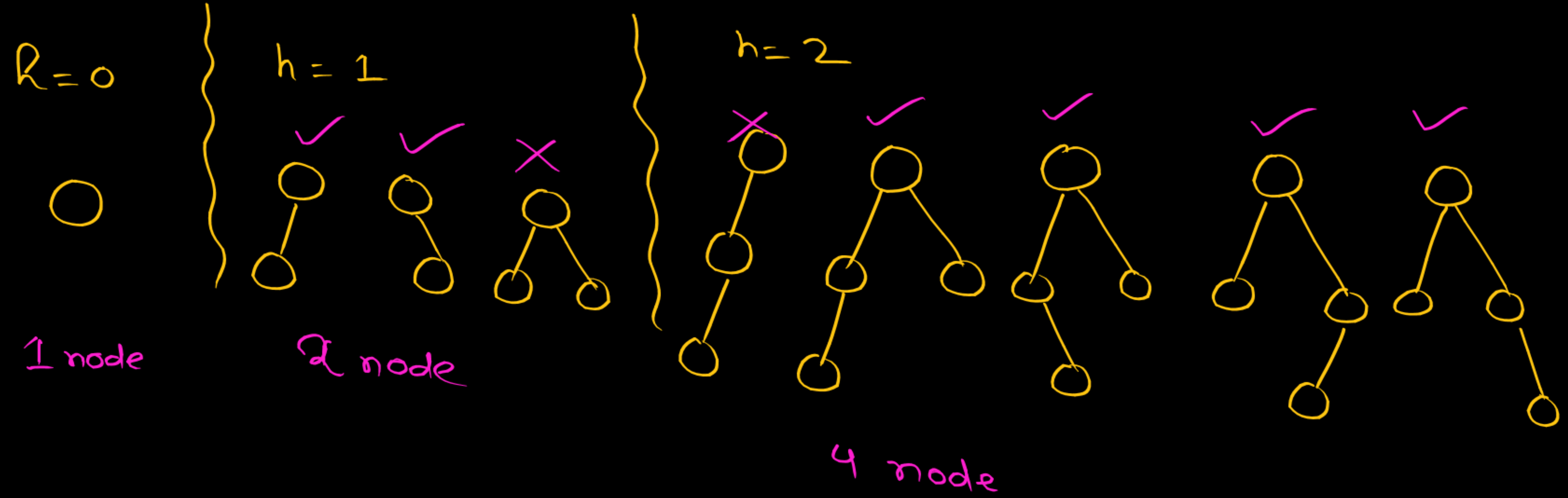
Topic : Tree

$$\boxed{n_{max} = 2^{h+1} - 1}$$

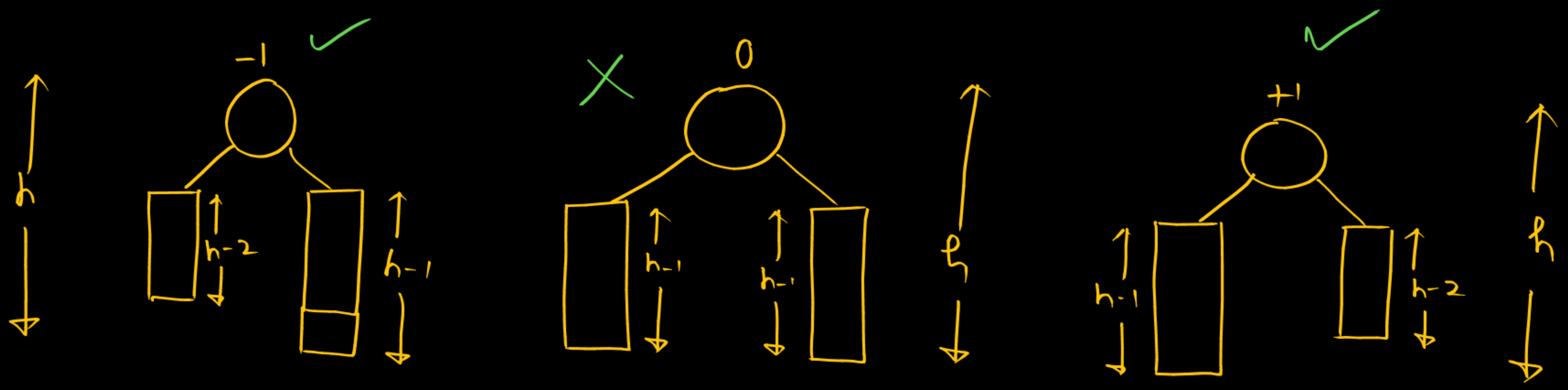
Q) What is the max. no. of nodes in an AVL tree of height h ?



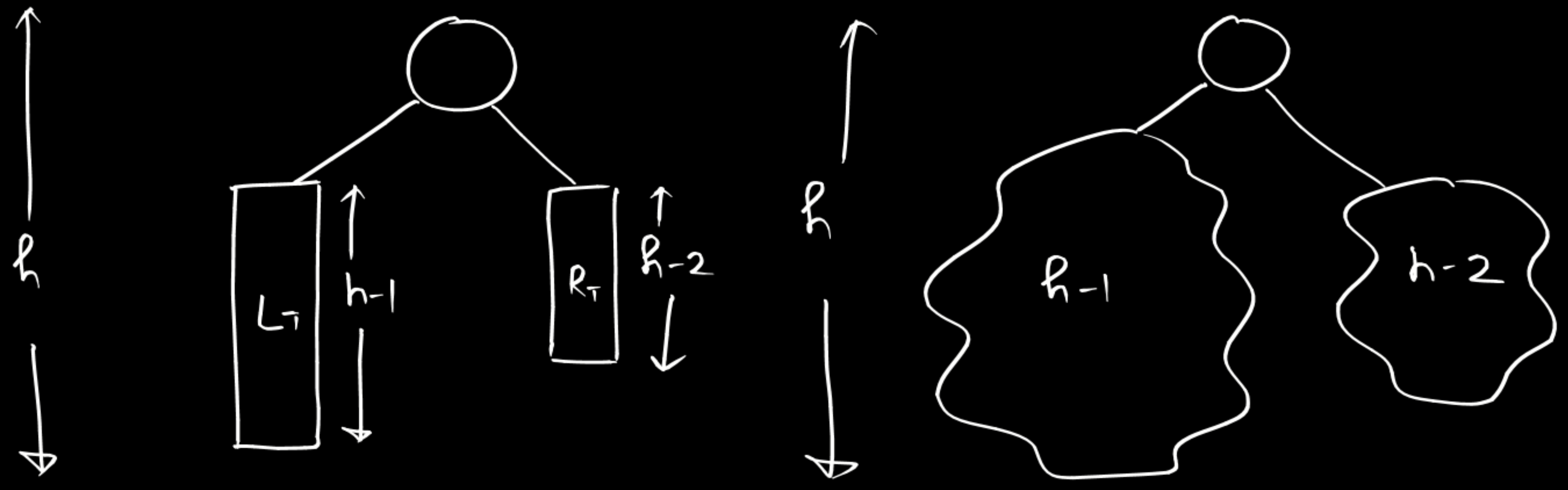
Q) What is the min. no. of nodes in an AVL tree of height h?



AVL tree of h height

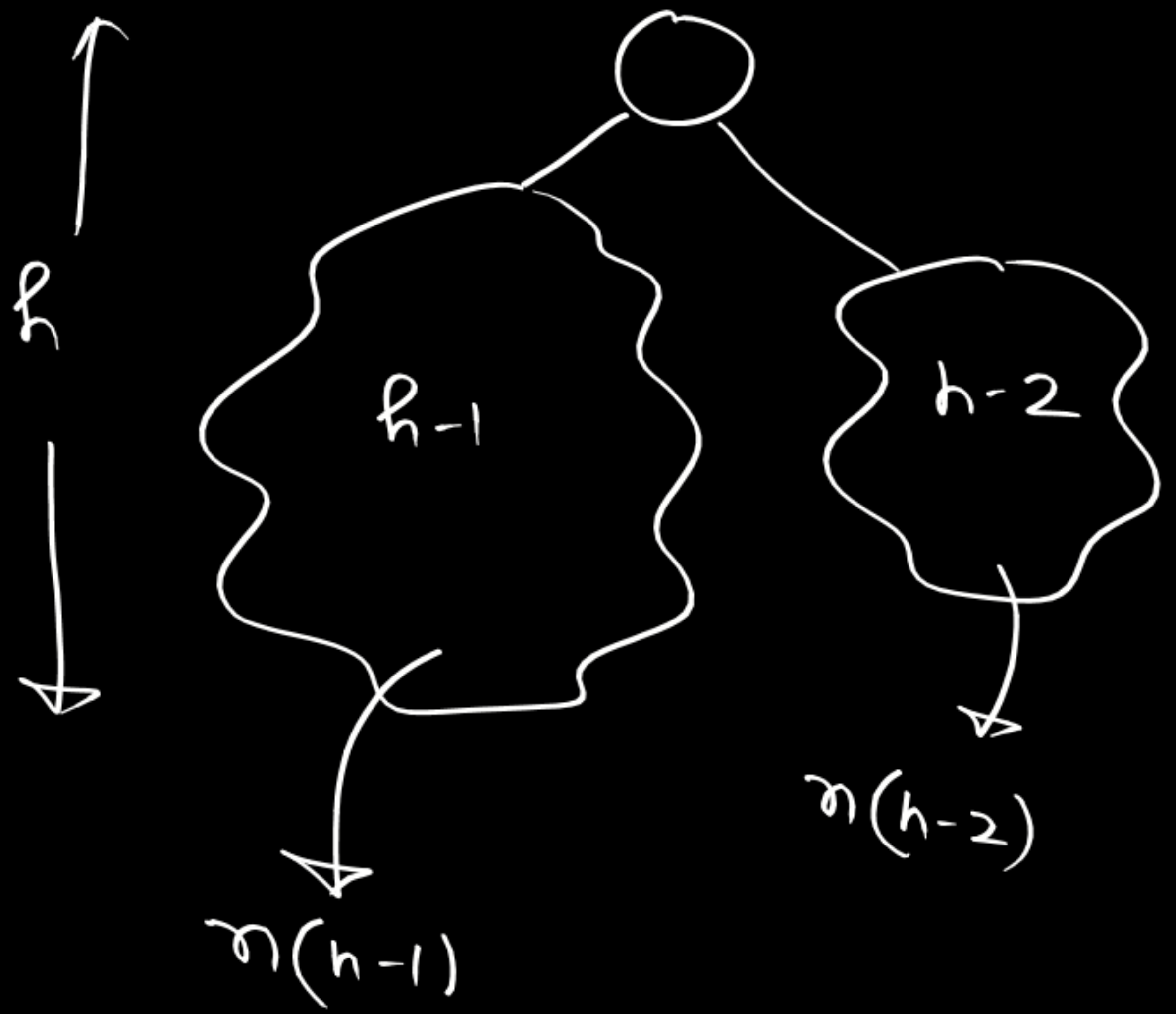


let $n(h)$: min. no. of nodes in a AVL tree of height h.


h
L_T
h-1
R_T
h-2
h
h-1
h-2

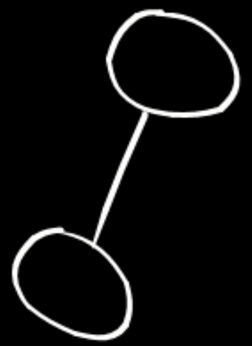
$n(h)$: min. no. of nodes in AVL tree of height h

$$\boxed{n(h) = 1 + n(h-1) + n(h-2)} \qquad h \geq 2$$



$n(0) = 1$

$n(1) = 2$



$$n(h) = 1 + n(h-1) + n(h-2)$$

$$n(0) = 1$$

$$n(1) = 2$$

$$n(2) = 1 + n(1) + n(0) = 1 + 2 + 1 = 4$$

$$n(3) = 1 + n(2) + n(1) = 1 + 4 + 2 = 7$$

h	0	1	2	3	4	5	6	7	8
$n(h)$	1	2	4	7	12	20	33	54	88

Q] Consider a binary tree in which every node satisfies the property:

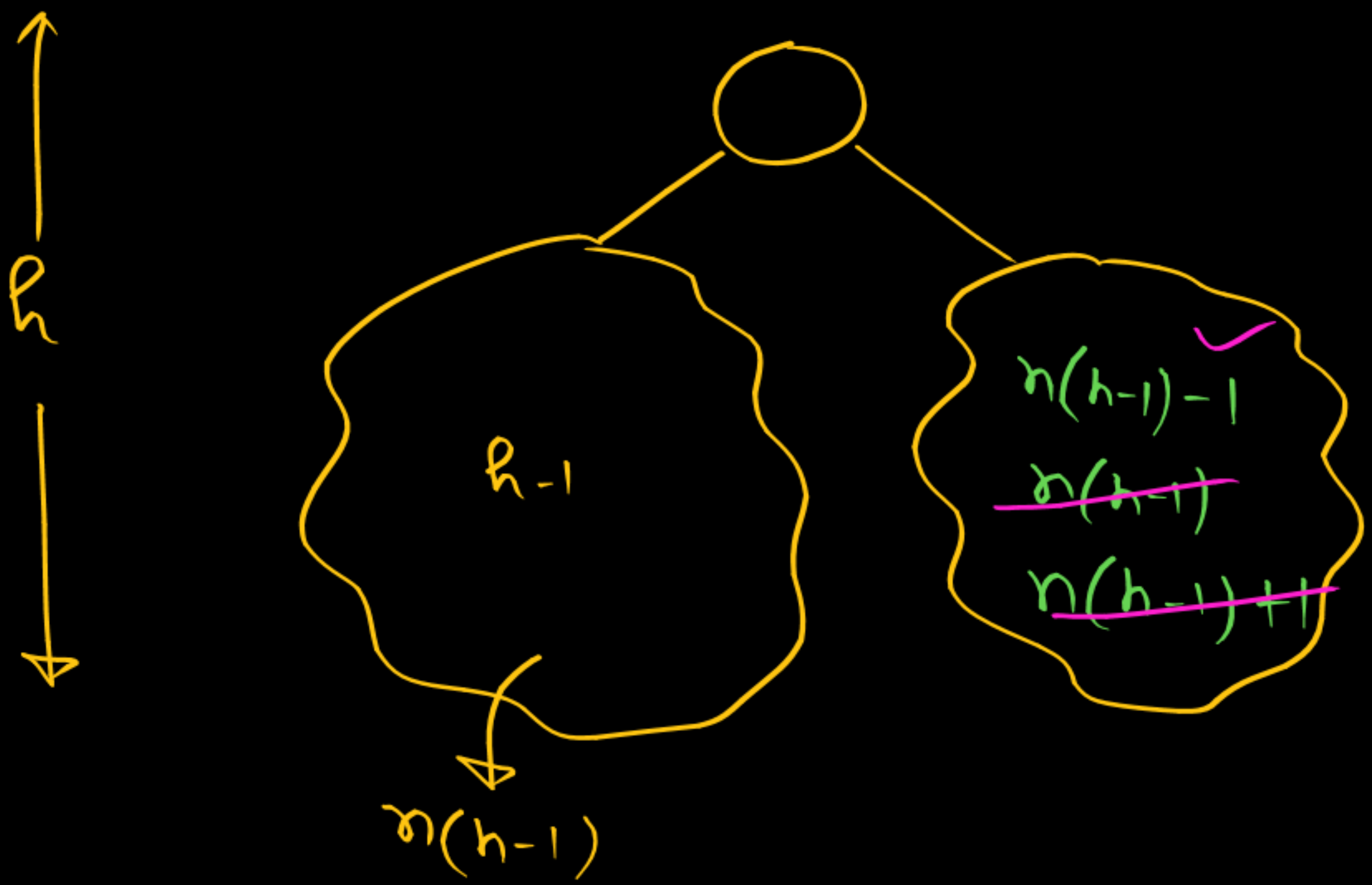
the diff. b/w no. of nodes in L_T of a node

& no. of nodes in R_T of node

is atmost 1. $\boxed{|n(L_T) - n(R_T)| \leq 1}$

What is the min. no. of nodes in such tree of height 5?

~~height 10~~



$$n(h) = \cancel{1+}\, n(h-1) + n(h-1) \cancel{-1}$$

$$\boxed{n(h) = 2n(h-1)}$$

n(h) : Min. no. of nodes in such tree of height h.

$h = 0$

$n(0) = 1$

$n(1) = 2 \times n(0) = 2$

$n(2) = 2n(1) = 2^2$

$n(3) = 2 \times n(2) = 2^3$

$n(4) = 2^4$

$\boxed{n(5) = 2^5}$

What is max. height possible of an AVL tree with 7 node.

h	0	1	2	3
$n(h)$	1	2	4	7

3

What is max. height possible of an AVL tree with 7 node.

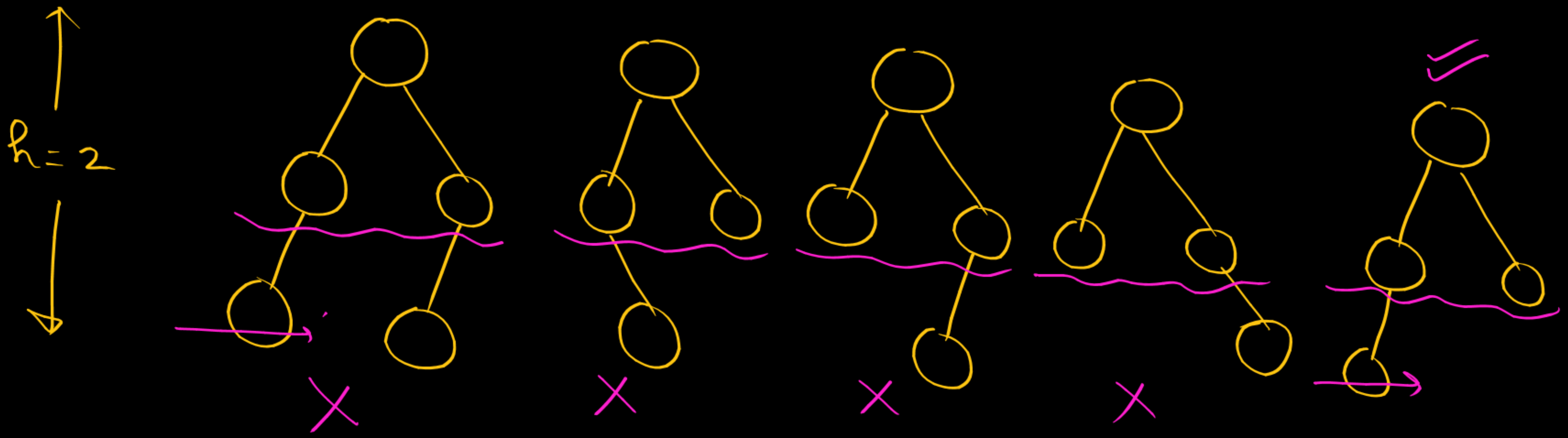
h	0	1	2	3	4	5
$n(h)$	1	2	4	7	12	20

$$\begin{cases} 7 \text{ to } 11 \text{ node} \rightarrow 3 \\ 19 \text{ node} \rightarrow \end{cases}$$

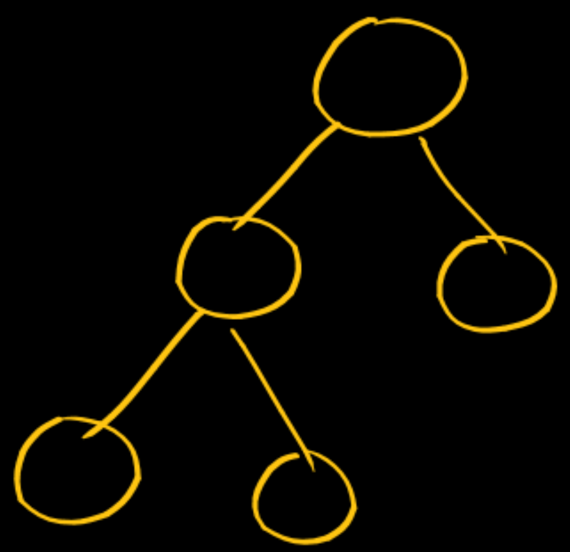
Heap

Heap is a CBT

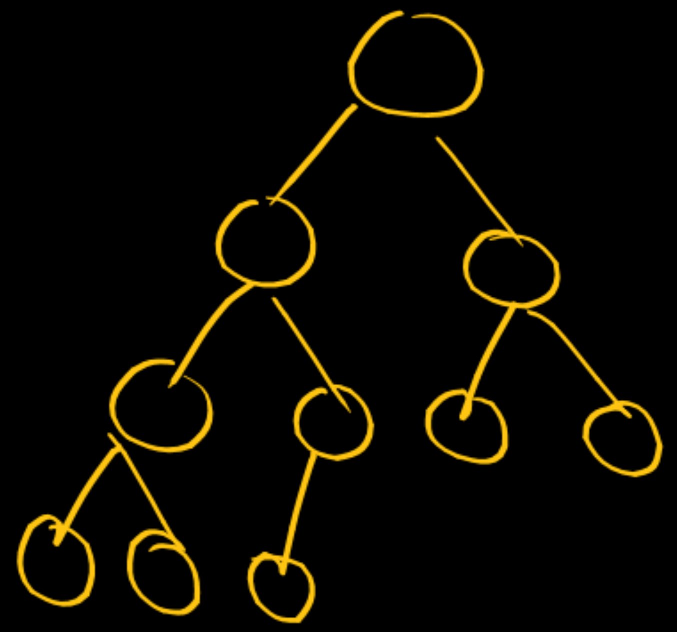
Complete Binary Tree :



CBT with 5 node $\Rightarrow$ Only 1 structure is possible



CBT with 10 node $\Rightarrow$ Only 1 structure is possible



Heap

Max heap

A CBT in which every node satisfies the property that :

The value of a node is greater than its children.

Min heap

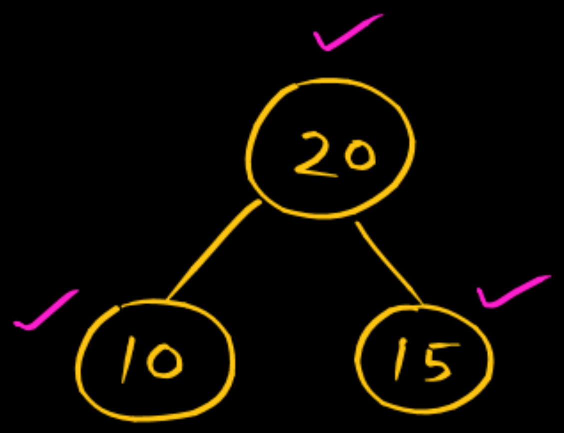
A CBT in which every node satisfies the property that :

The value of a node is smaller than its children.

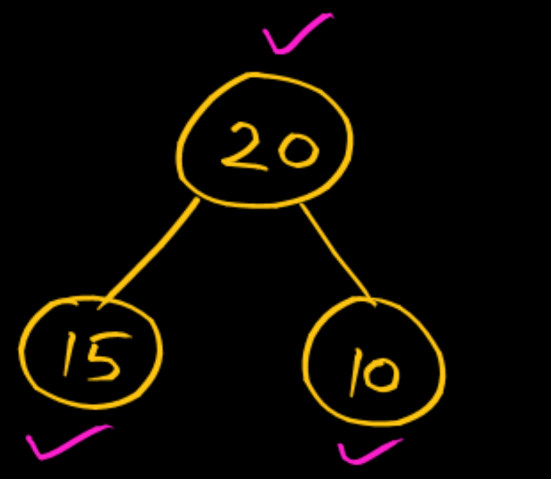
① Single node

Max heap ✓
Min heap ✓

Every leaf node
↳ satisfies ✓



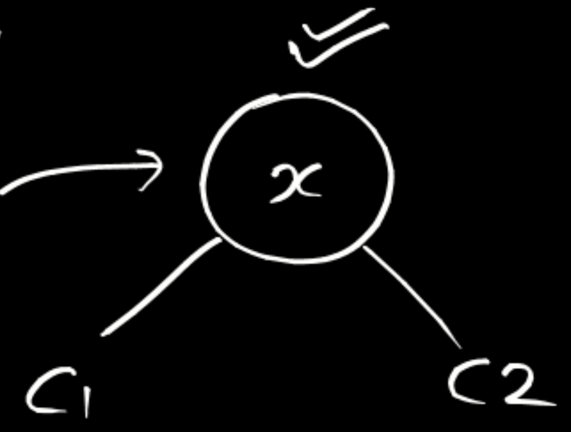
Max - heap ✓
Min - heap ✗



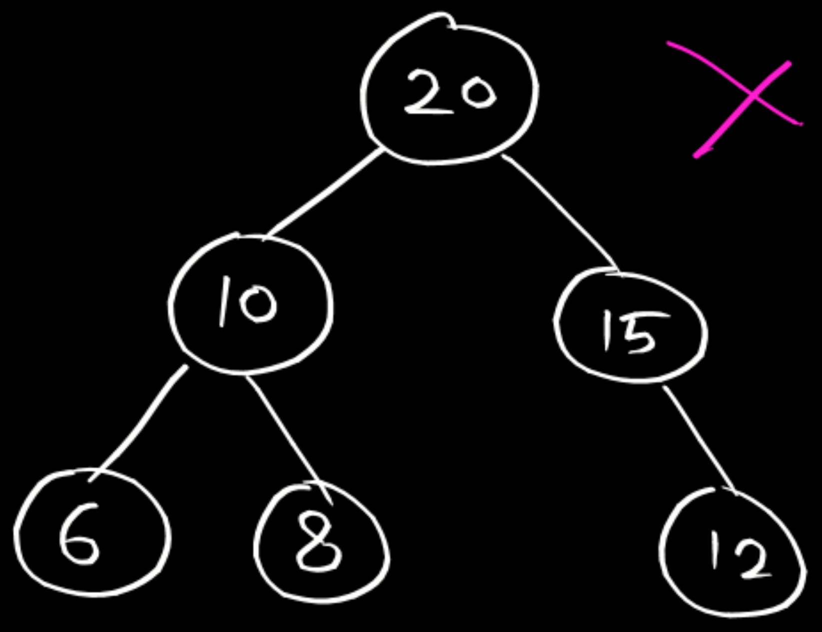
Max - heap ✓
Min - heap ✗

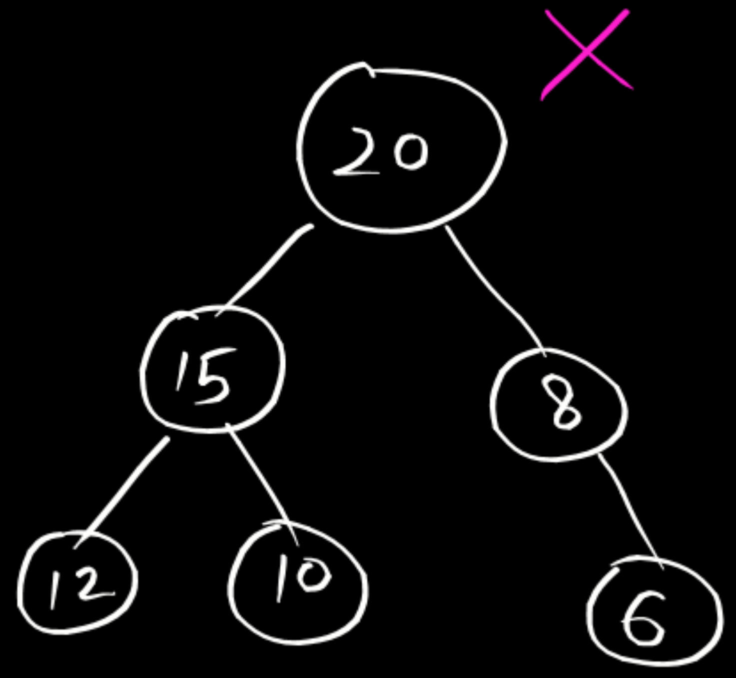
Max heap

max value



$\boxed{\text{Min value} \Rightarrow c_1 \text{ or } c_2}$





Not a CBT

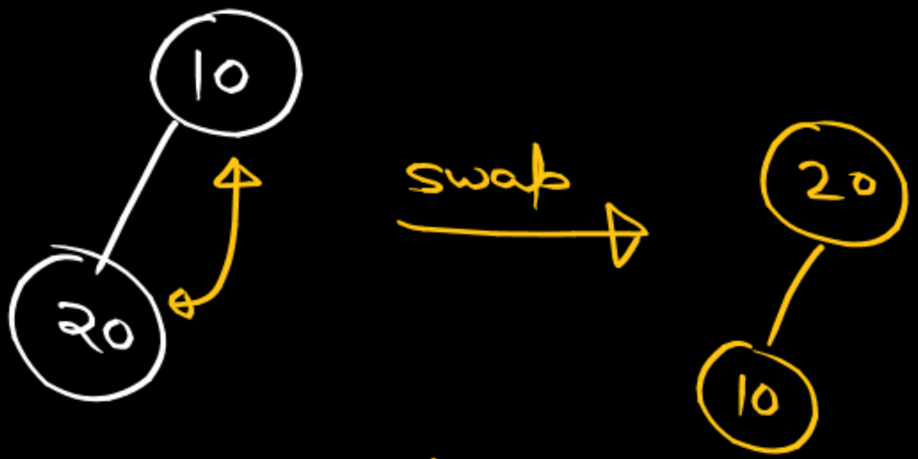
not a max_heap

Const. of heap by inserting keys in order one after another.

Const. max-heap by inserting keys 10, 20, 30, 40, 50, 60, 70 one after another respectively.

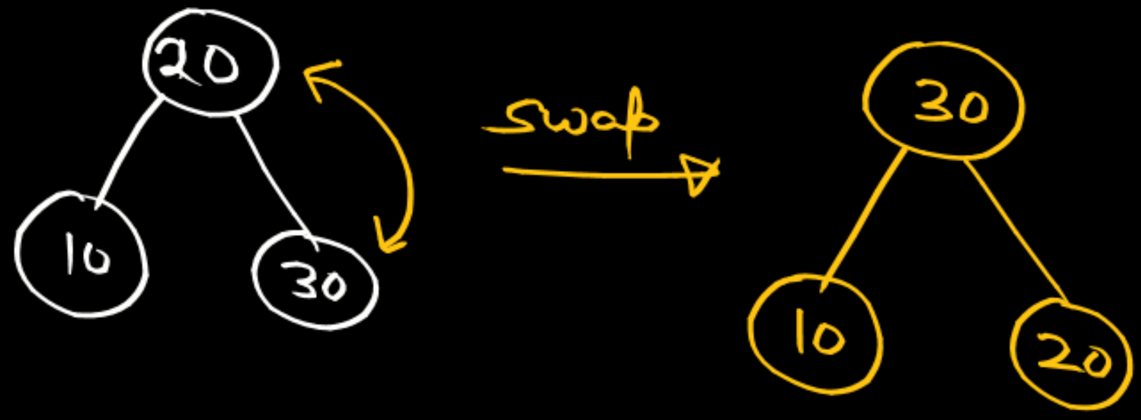
(i) 

(ii) Insert 20

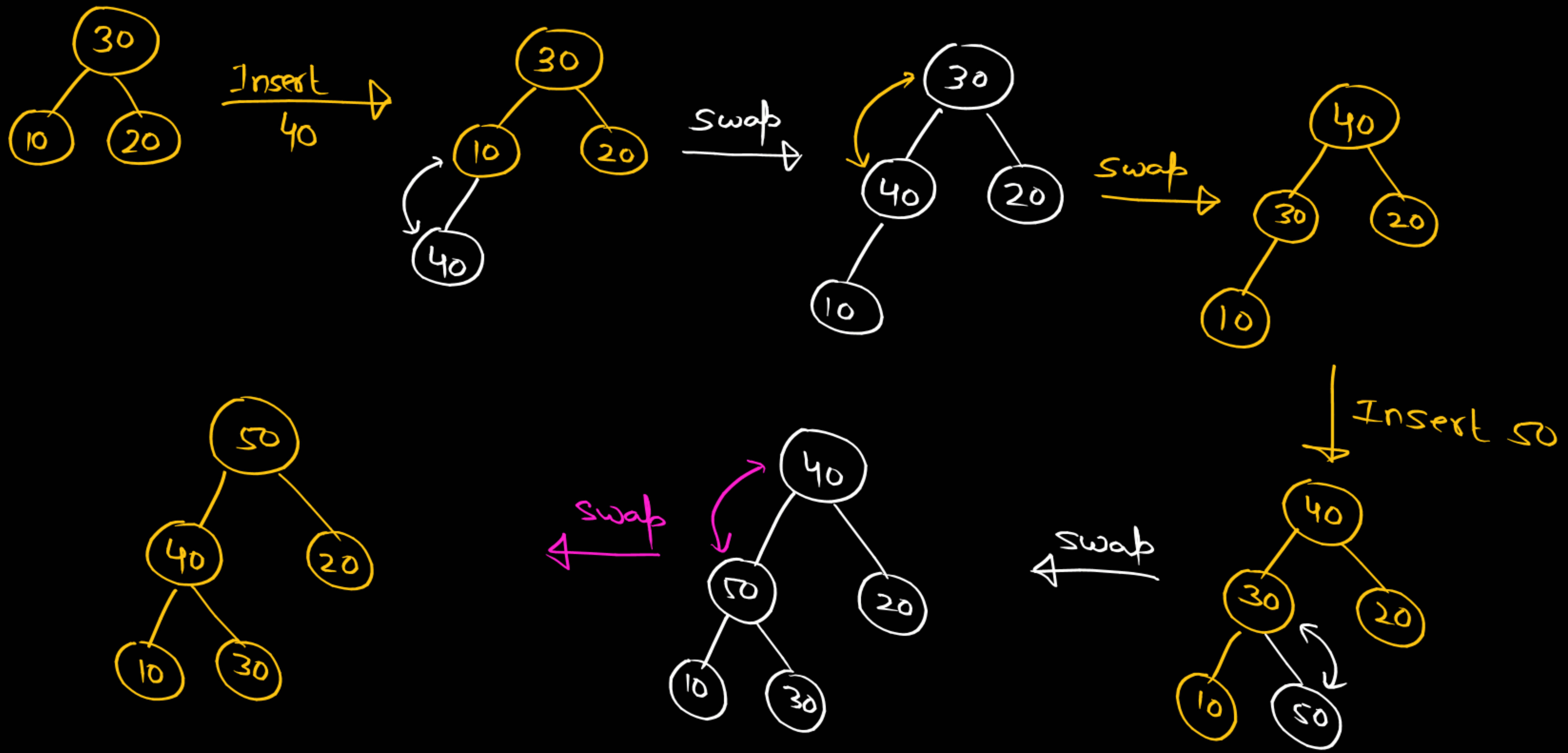


Is it a max-heap?

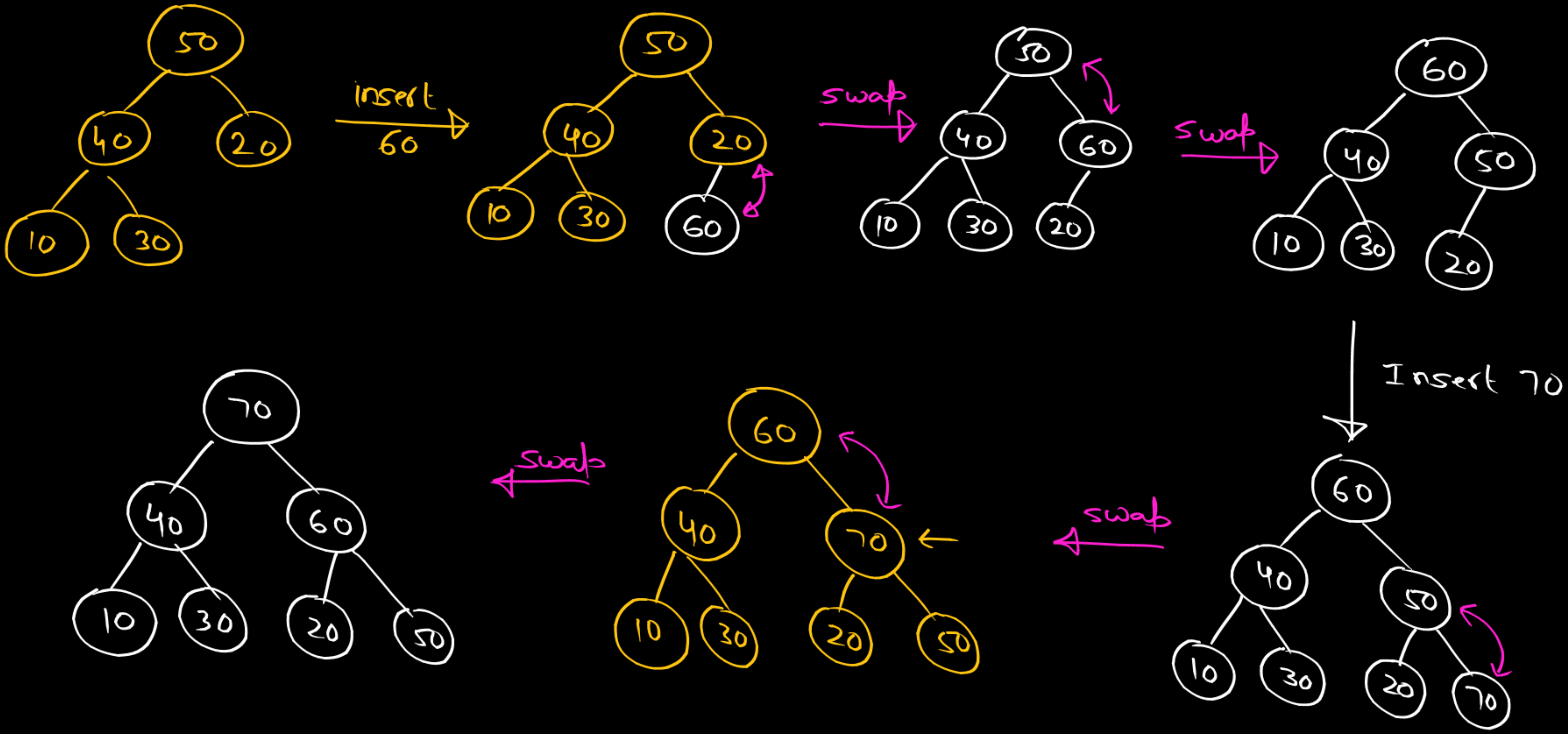
(iii) Insert 30



Is it a max-heap?



30
10
20
Insert 40
30
10
20
40
swap
30
40
20
10
swap
40
30
20
10
Insert 50
40
30
20
10
50
swap
40
50
20
10
30
swap
50
40
20
10
30


insert
60
swap
swap
Insert 70
swap
swap

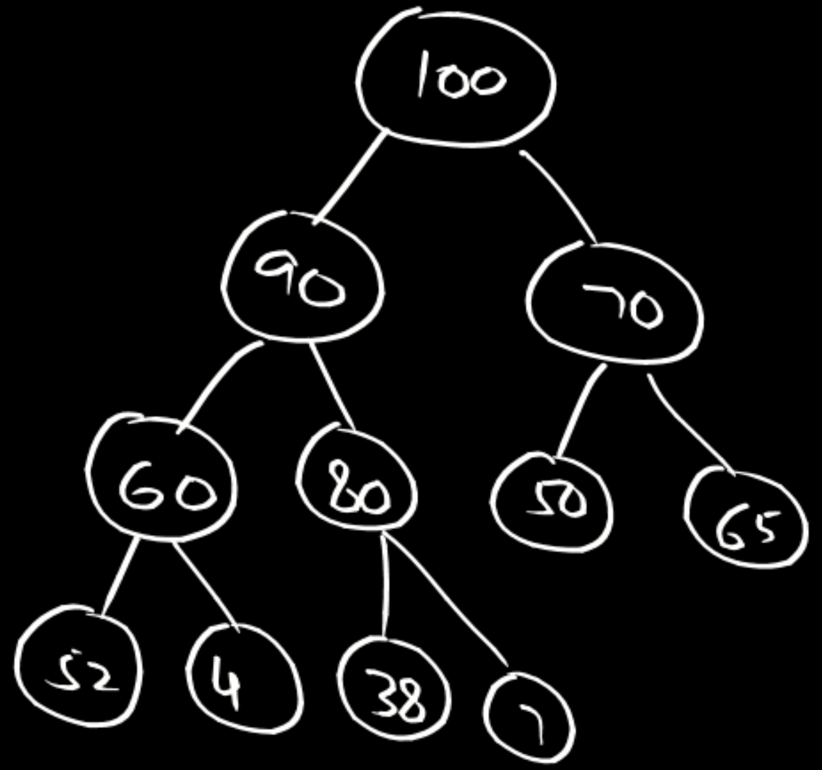
Existing heap with n nodes

Key insert

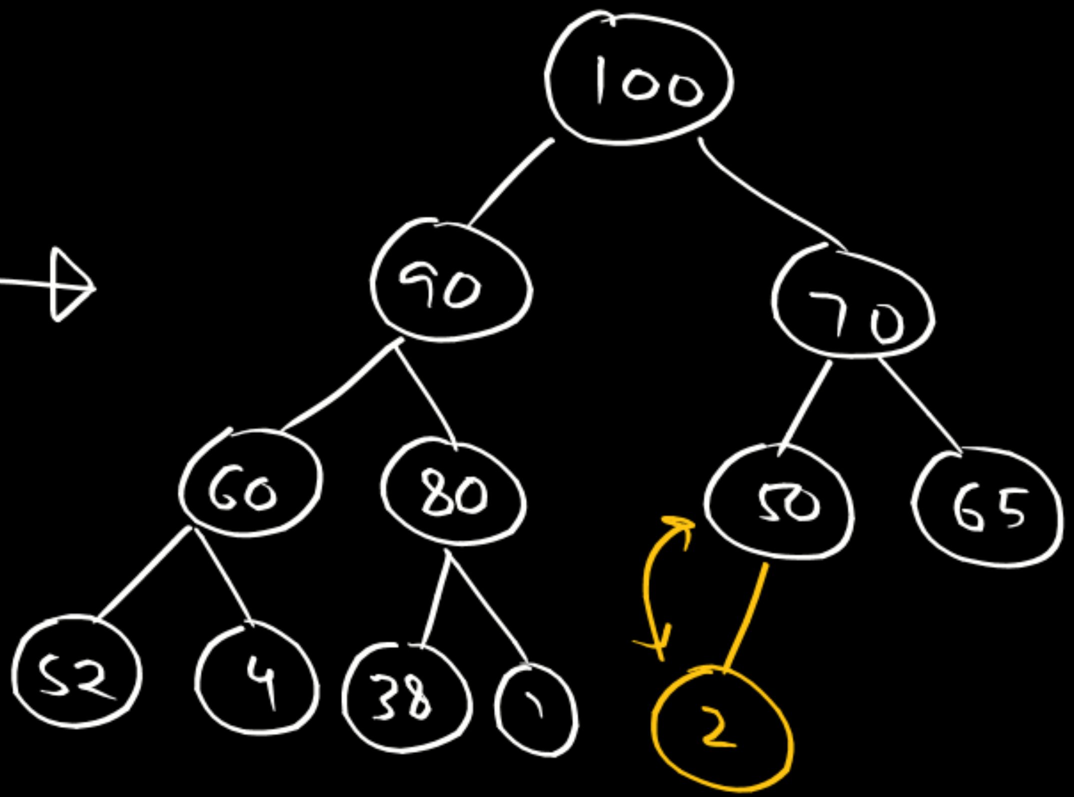
best case

1 comparision

Ex 1



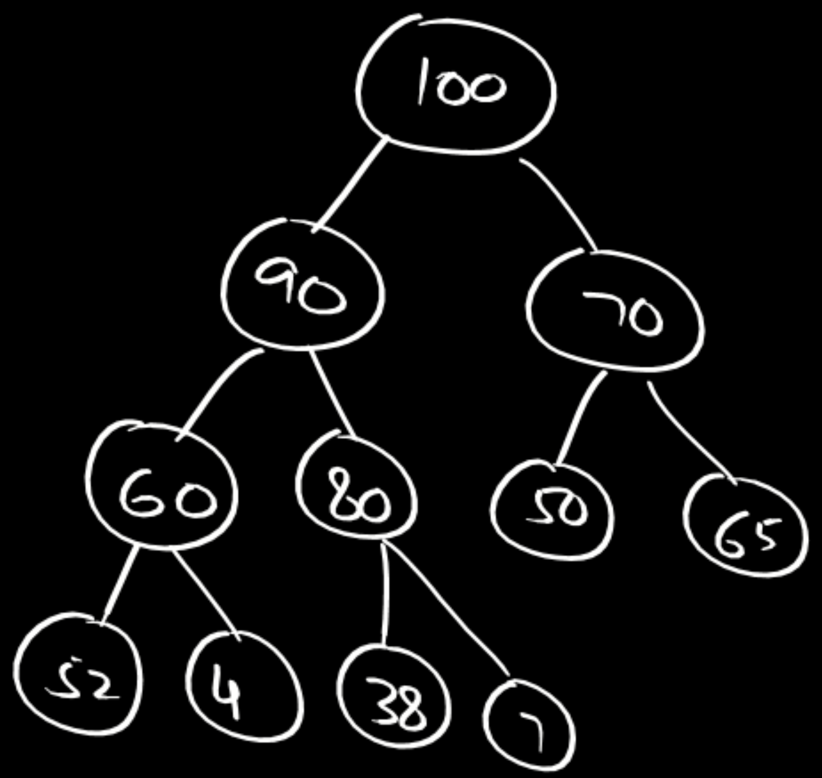
Insert 2

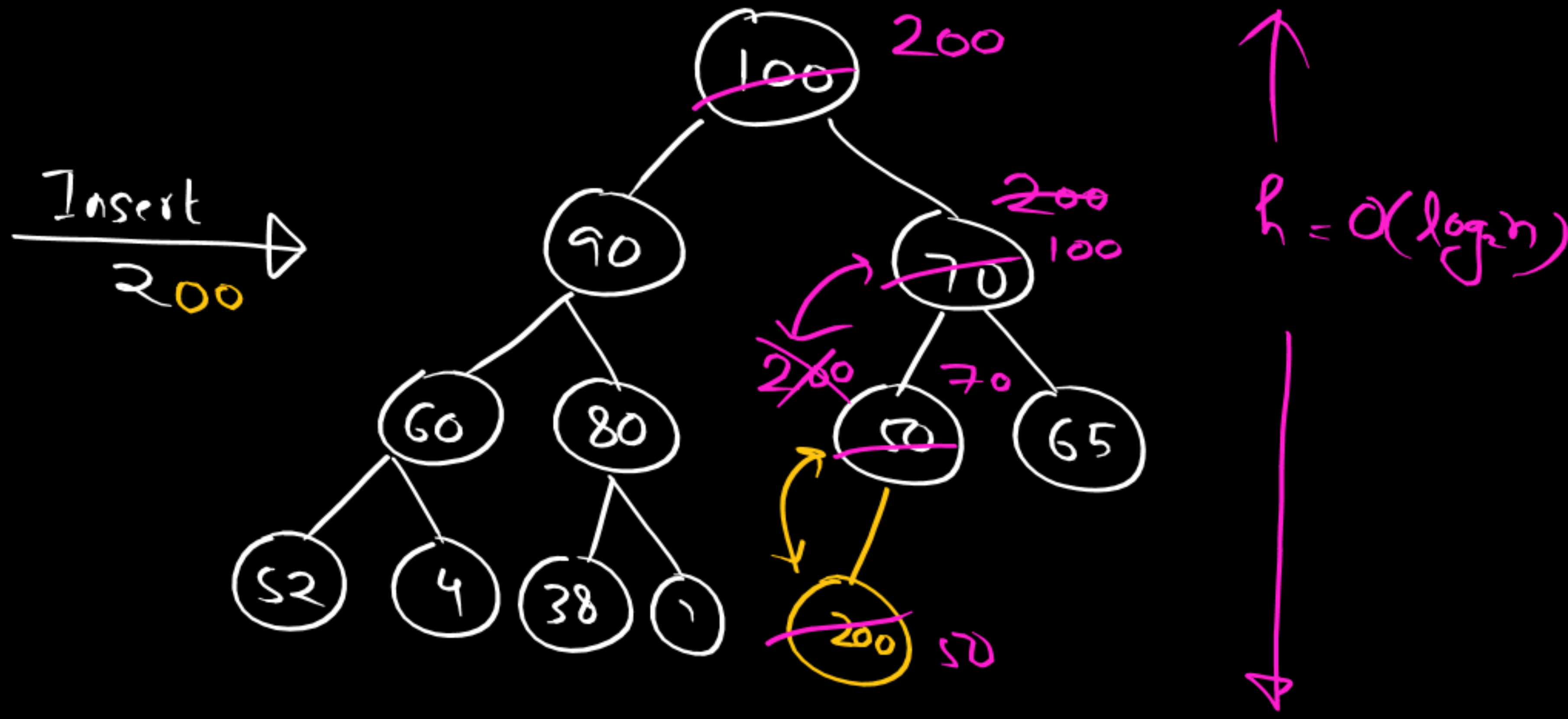


Existing heap with n nodes

Key insert

Ex 1





heap const by inserting keys one after another

$\Rightarrow O(n \cdot \log_2 n)$

Const. of heap using build_heap method

Given an array rep. of a CBT, Convert it into a max_heap.

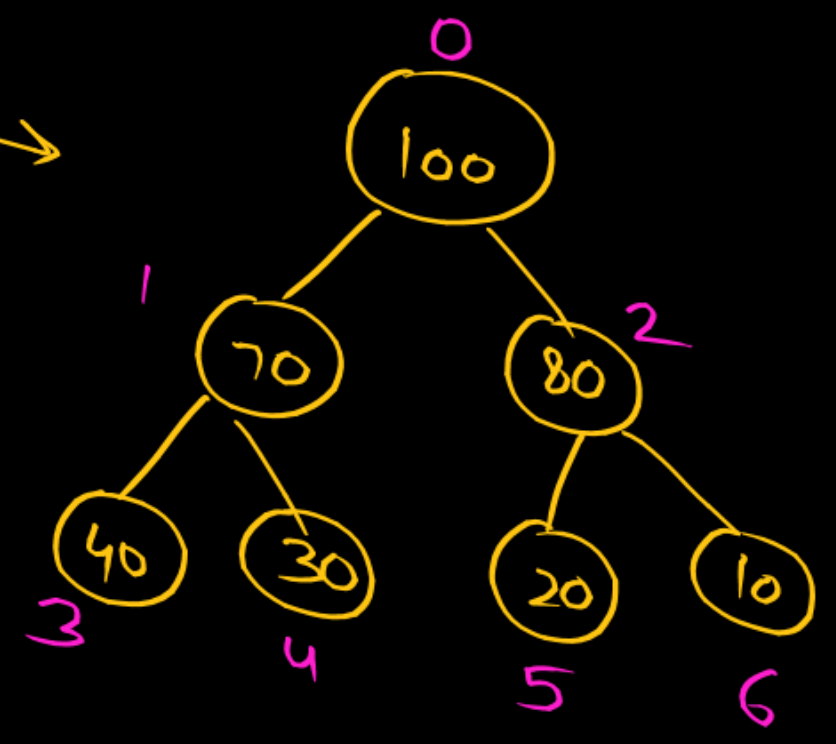
Heapify method/algo

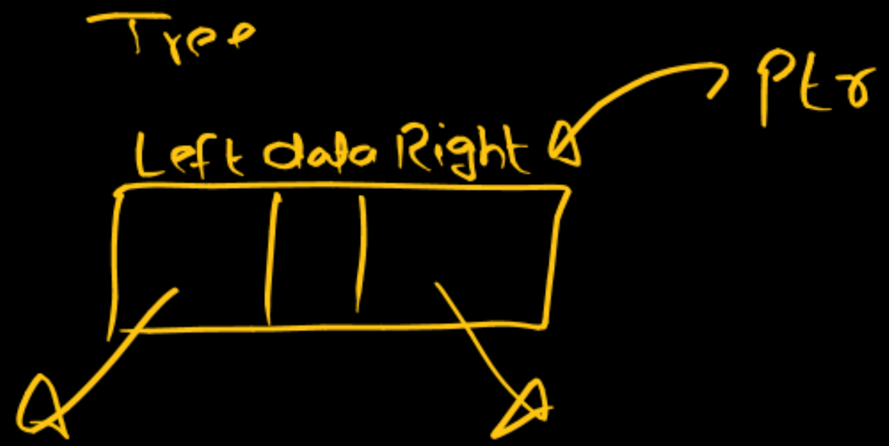
Heap → CBT

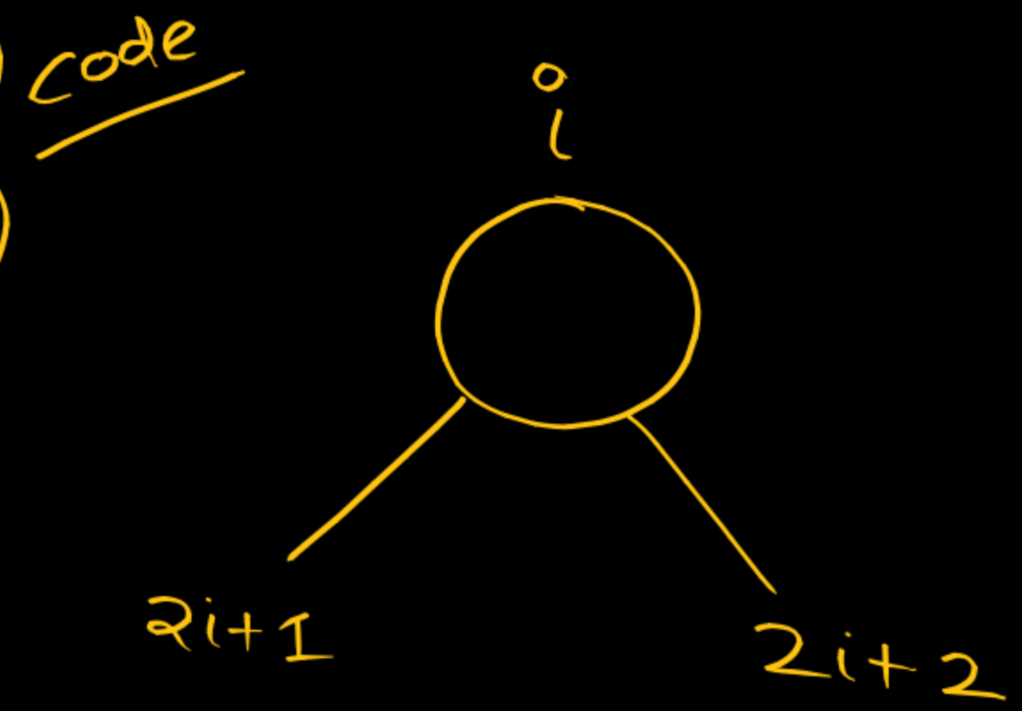
L.L.

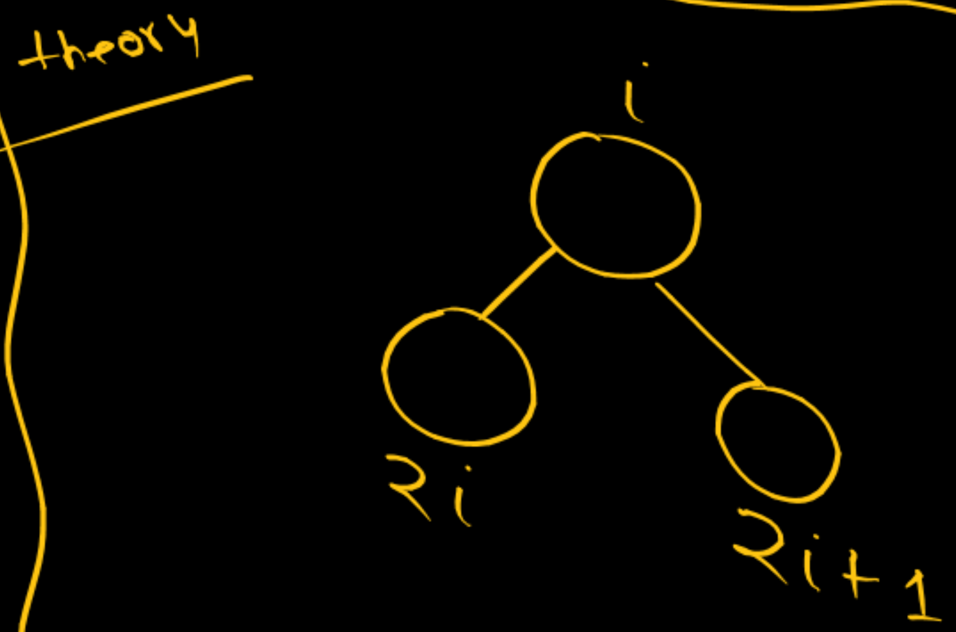
array representation

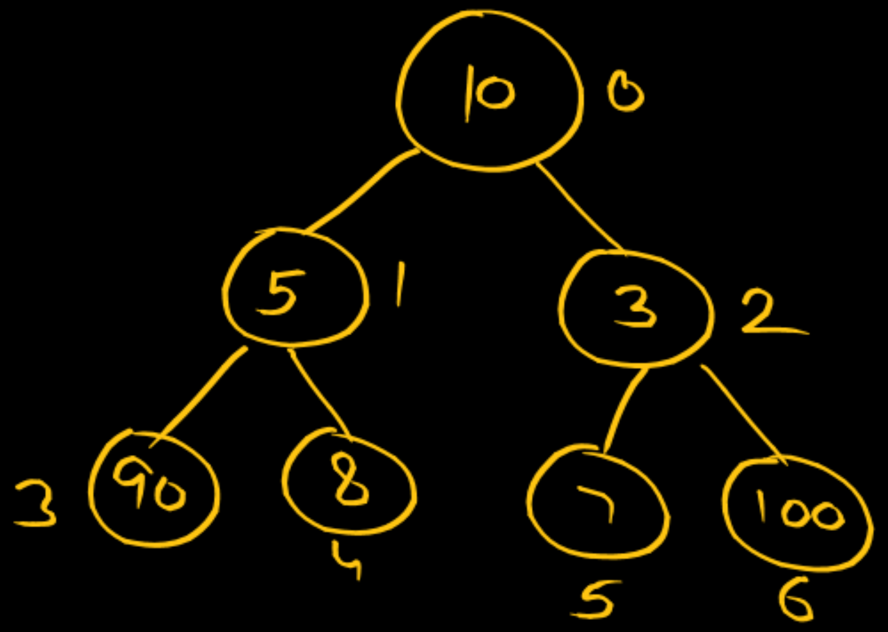
100	70	80	40	30	20	10
0	1	2	3	4	5	6



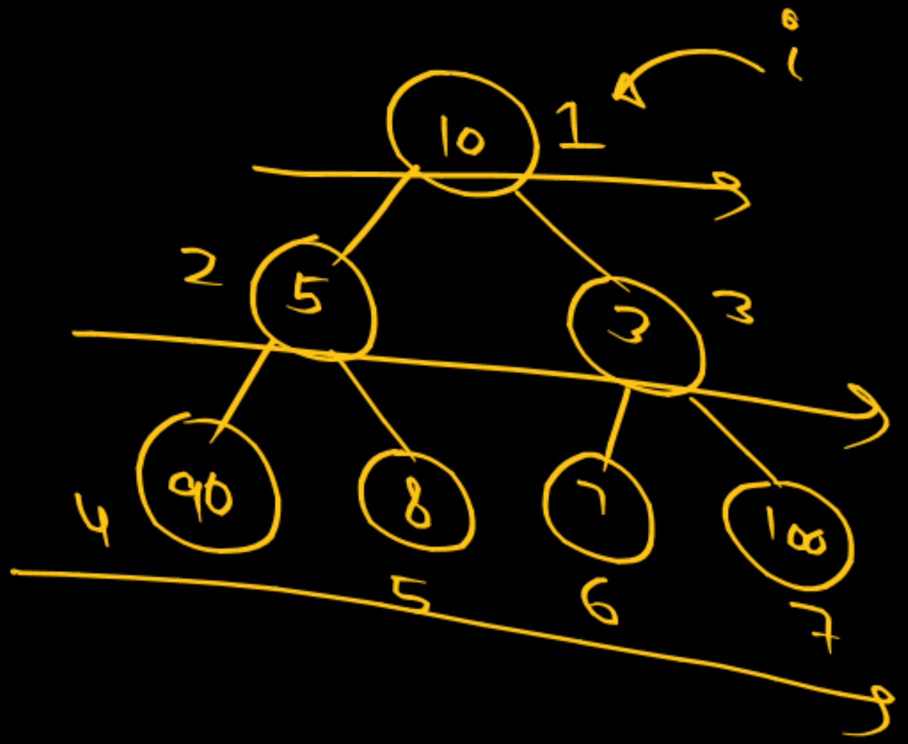

Tree
Left data Right
ptr


code
0
i
2i+1
2i+2


theory
i
2i
2i+1



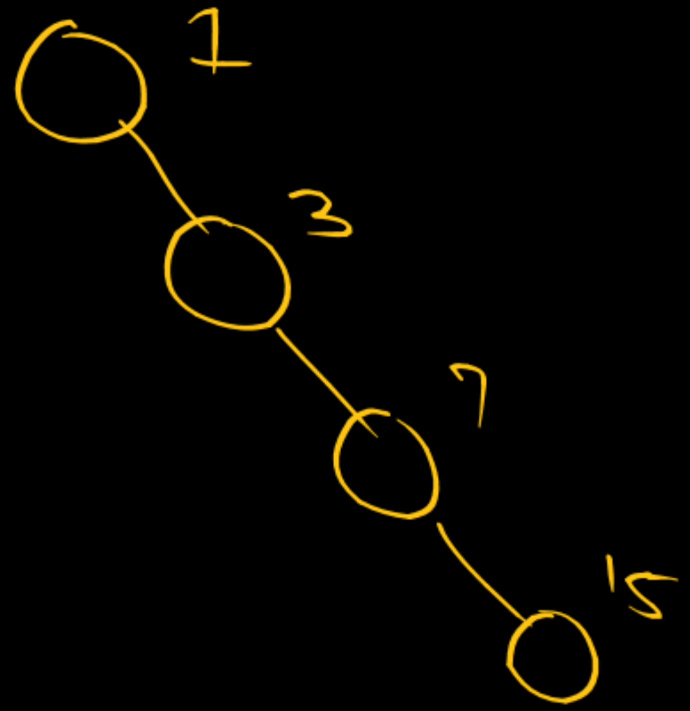
10	5	3	90	8	7	6
0	1	2	3	4	5	6

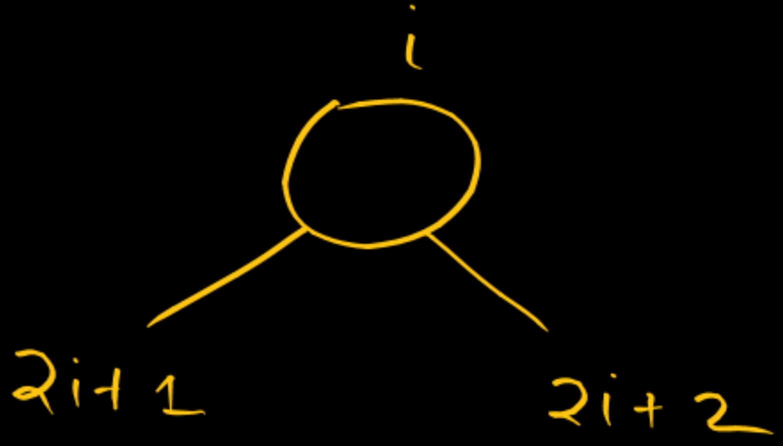


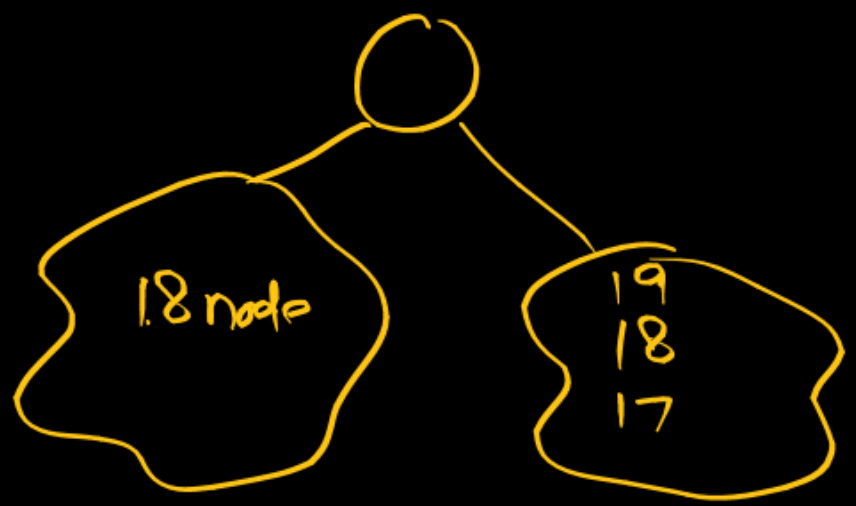
10	5	3	90	8	7	100
1	2	3	4	5	6	7

$$2^n$$

$O(2^n)$?






18 node
19
18
17

THANK - YOU