CS & IT ENGINEERING

DISCRETE MATHS GRAPH THEORY

Lecture No. 2

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01 Definition of Graph

02 Handshaking Lemma

03 Types of Graphs

04 No of Graphs

05 Simple Graphs theorem

Basics of Graph

Thm 1: $\sum d(v_i) = 2e$.

Thm 2: no. of odd degree vertices should be even.

Thm 3: max. degree $\leq n-1$. $\Delta(G) \leq n-1$.

Thm 4: max edges in simple $\leq \frac{n(n-1)}{2}$.

Total Graphs

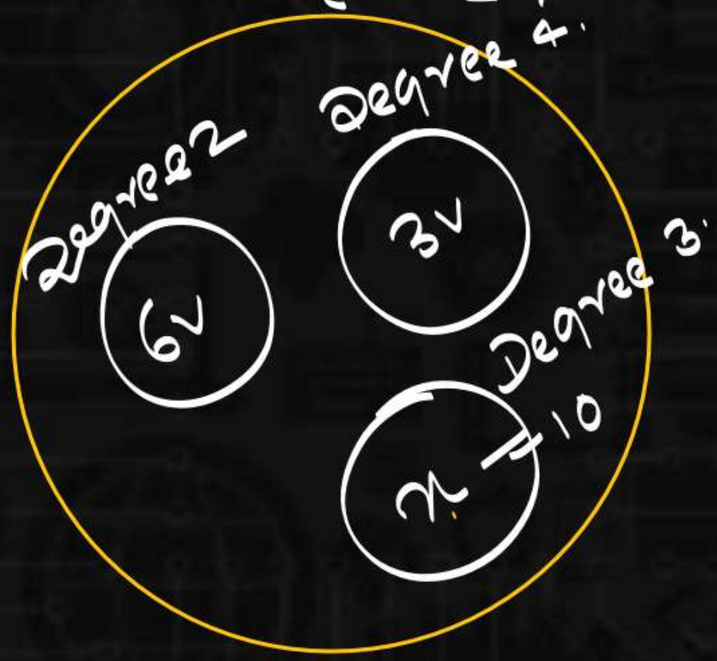
$= 2^{\frac{n(n-1)}{2}}$

$\rightarrow {}^{\frac{n(n-1)}{2}}C_e$.

Basics of Graph

$\Sigma d(v_i) = 2e$.

$E = 27$



Total vertices $= 6 + 3 + 10 = 19$.

Consider a Graph having 27 edges.

$6v \rightarrow$ degrees $\rightarrow 2$

$3v \rightarrow$ degree 4

Remaining vertices $\rightarrow$ Degree 3

Total vertices = ?

A) ~~10~~

B) 11

C) 18

D) 19.

$\Sigma d(v_i) = 2e$.

$6 \times 2 + 3 \times 4 + n \times 3 = 2 \cdot 27$.

$12 + 12 + 3n = 54$

$3n = 54 - 24 = 30$

$3n = 30$

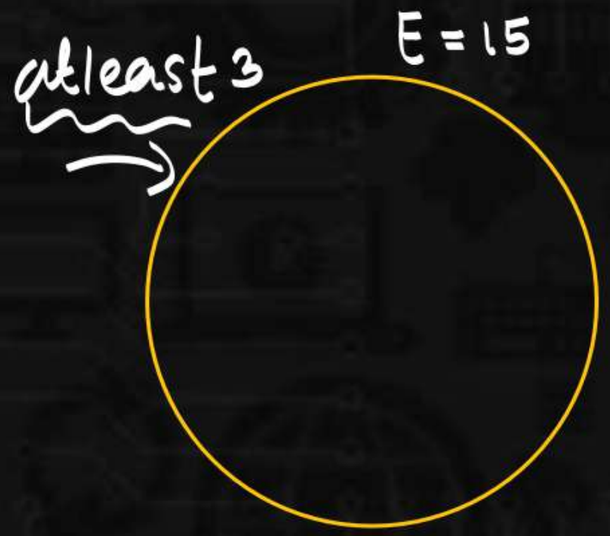
$n = 10$

Basics of Graph

→ Consider a Graph having 15 edges, degree of each vertex is atleast 3

→ what will be max no. of vertices ?

$E = 15$. $\delta(G) = 3$. $n = ?$



$$\delta(G) \leq \frac{2e}{n} \leq \Delta(G) \leq n-1.$$

$$\delta(G) \leq \frac{2e}{n} \qquad 3 \leq \frac{2 \cdot 15}{n} \qquad n \leq \frac{30}{3} \qquad n \leq 10$$

Basics of Graph

$$\delta(G) = \frac{2e}{n} = \Delta(G) \rightarrow \text{Case I :}$$

minimum degree ($\delta(G)$)

maximum degree ($\Delta(G)$)

$\delta(G) = 2$

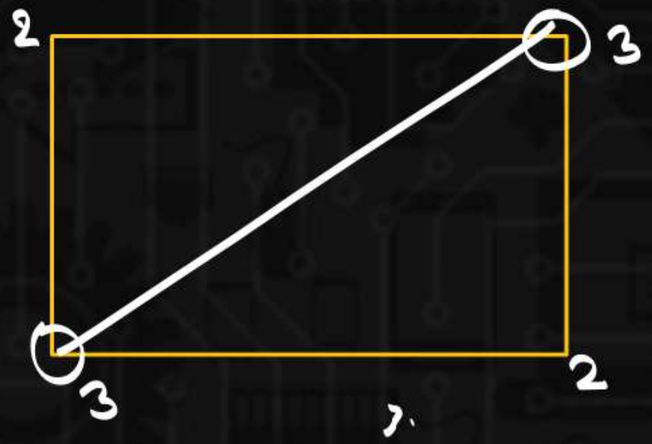
$\Delta(G) = 2.$

$\frac{2e}{n} = 2.$

$\frac{2+2+2+2}{4} = 2.$

avg. degree $= \frac{2+2+2+2}{\text{Total vertices}} = \frac{\sum d(v)}{n.}$

$= \frac{2e}{n.}$



$\delta(G) = 2$

$\Delta(G) = 3$

$$\delta(G) < \frac{2e}{n} < \Delta(G) \quad \text{— II.}$$

Thm 3: $\Delta(G) \leq n-1$.

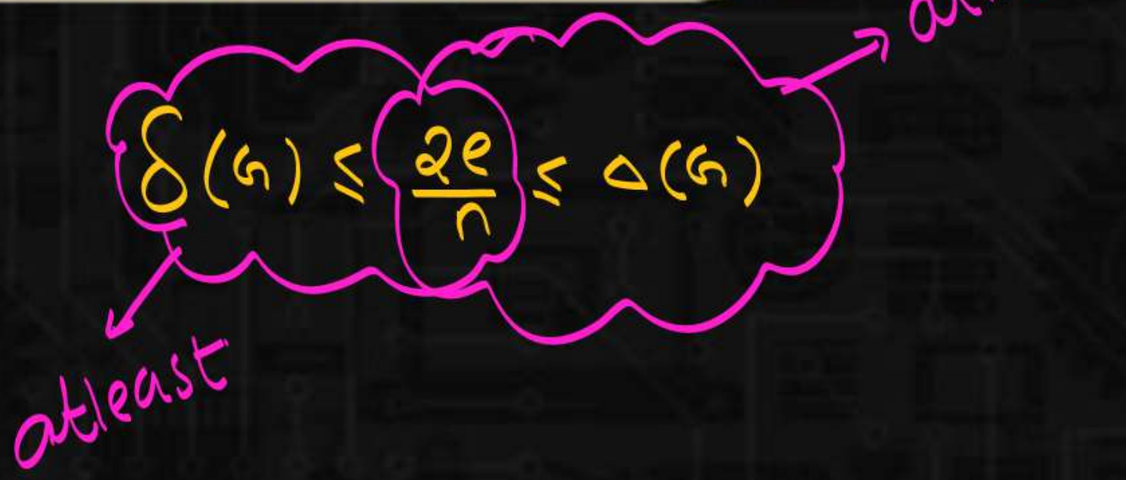
(I & II)

$$\delta(G) \leq \frac{2e}{n} \leq \Delta(G)$$

Thm 5:

$$\delta(G) \leq \frac{2e}{n} \leq \Delta(G) \leq n-1.$$

Basics of Graph



Degree
atleast
atmost

Basics of Graph

G is Graph 25 edges, each vertex is having degree atleast 3

maximum value of n — Ans: 16 (GATE-17)

$E = 25$ $\delta(G) = 3$

$\delta(G) \leq \frac{2e}{n}$

$3 \leq \frac{2.25}{n}$

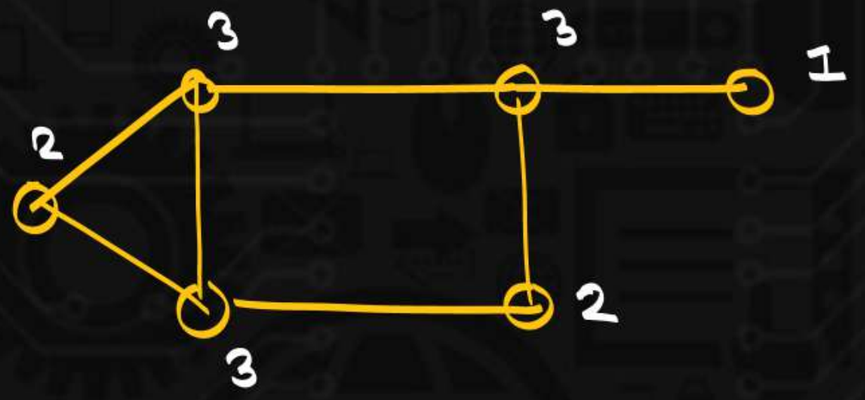
$n \leq \frac{50}{3}$

$n \leq 16.6....$

$n = 16$

Degree sequence :

writing degrees of all vertices either in increasing or decreasing order is called Degree sequence.



→ 3, 3, 3, 2, 2, 1.

OR.

→ 1, 2, 2, 3, 3, 3.

Basics of Graph

→ what will be no. of edges in @.S 5, 2, 2, 2, 2, 1 ?

m1:

$\sum d(v_i) = 2e$.

$5+2+2+2+2+1 = 2e$

$14 = 2e$

e = 7.

m2: 5, 2, 2, 2, 2, 1. Total vertices = 6.

e = 7

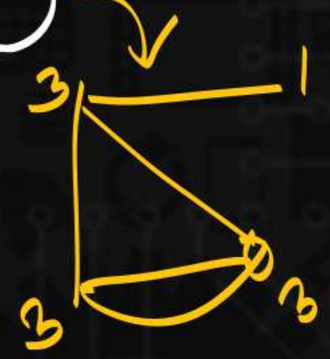
What will be edges in G.S of 3, 3, 3, 1 ? → no simple Graph.

$\sum d(v_i) = 2e$.

$3 + 3 + 3 + 1 = 2e$.

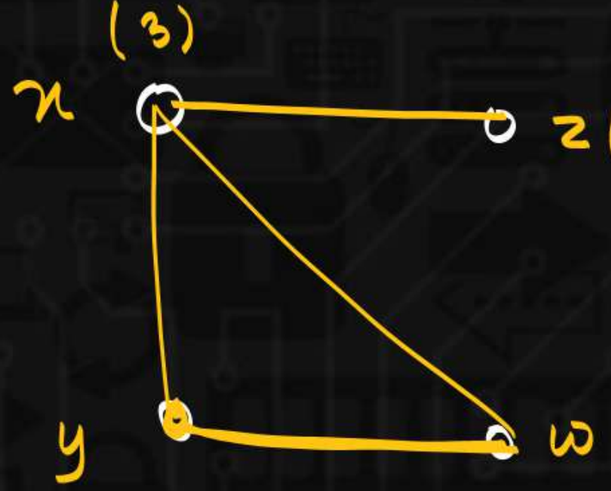
$10 = 2e$

$e = 5$



m2: 3, 3, 3, 1 (x, y, z, w)

Total vertices = 4.



y → (1)

Remaining + 2

Degree sequence → Simple Graph.

↳ Graphical sequence.

5, 2, 2, 2, 2, 1 → Graphical sequence.

3, 3, 3, 1 → no simple Graph.

Graphical ?.

A) 5, 4, 3, 2, 1.

B) 4; 4, 3, 2, 1.

C) 2, 2, 2, 2, 2, 2.

D) 1, 1, 1, 1, 1, 1.


THANK YOU GW
SOLDIERS !