DATA SCIENCE

Probability and Statistics

Counting Techniques

Lecture No.- 02

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Topics to be Covered : Counting Techniques

#Q. How many six-digit numbers divisible by 25 can be formed using digits 0,1, 2, 3, 4, 5? "without Replacement"

A. 24

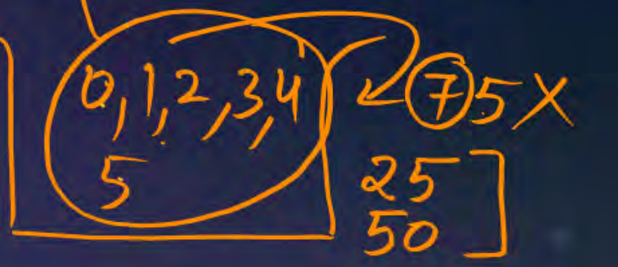
B. 42

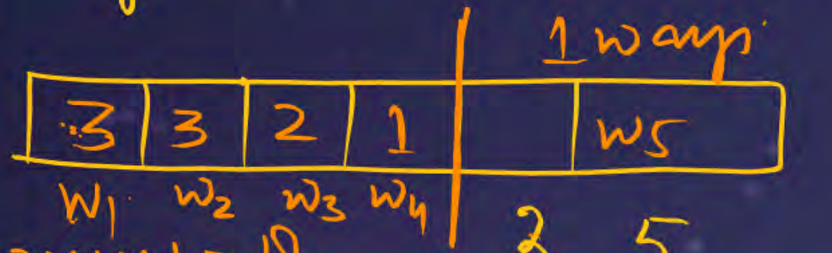
C. 256

D. 100

Six Digit Number

If any Number Divisible via 25





$= 3 \times 3 \times 2 \times 1 \times 1 = 18$

$w_1 \times w_2 \times w_3 \times w_4 \times w_5$

$= 4 \times 3 \times 2 \times 1 \times 1$

$= 24$

Total ways $= 18 + 24$

$= 42$

LAST (5)

25
50

$= 4$

0 Never comes first Place

$w_2 \to 0$

00 Repetation Not allowed

#Q. Find number of different words which can be formed using all the letters of the word 'HISTORY'.

A. 720

B. 5040

C. 2520

D. 360

Box method

n Diff. n Select
$= {}^nP_n = n! = {}^7P_7$ Ans

HISTORY

n Different Items ⟶ Taken all the Time

$= n!$

Total No. of Letters $n = 7$

$= 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

without replacement

H I S T O R Y

7	6	5	4	3	2	1
w_1	w_2	w_3	w_4	w_5	w_6	w_7

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

#Q. In how many ways 5 different red balls, 3 different black balls and 2 different white balls can be arranged along a row?

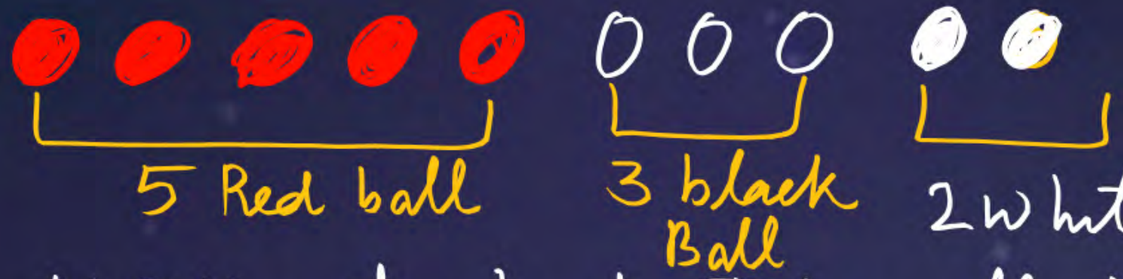
A. $10!$

B. 10^{10}

C. $10^{10} - 10!$

D. None of these

"n" Different Balls"



N Different objects Taken all at a time

$= 10!$

#Q. In how many ways can be letters of the word 'DELHI' be arranged so that the vowels occupy only even places?

(A) 6

(B) 12

(C) 24

(D) 48

Order Matter EI IE Diff

DELHI

V E, I

D, H, L

$W = W_1 \times W_2$

W_1 Job = 2!

Even Even

Counting ⇒ Restrictions

W_2 Job

D	H	L
H	D	L
H	L	D
D	L	H
L	D	H
L	H	D

n Different Taken all at a time = 3!

Total No. of ways = 3! × 2!

= 12

Slide : 14

DELHI EI = 2!
DNI = 3! = 2! × 3!

#Q. In how many of these words do the vowels and the consonants occupy the same relative positions as in 'COMBINE'?

COMBINE
C V C C V C V
SAME Relative Position

Vowel
Consonants

C V C C V C V
C_1 C_2 C_3 C_4
① ② ③ ④

A ✓	144
B	720
C	5040
D	360

$C_1 C_2 C_3 C_4$
$C_2 C_1 C_3 C_4$
$C_2 C_3 C_1 C_4$
$C_3 C_2 C_1 C_4$

W1 Job = 4! ways

$V_1 V_2 V_3$ — 3! ways — W2 Job

Total ways = 3! × 4!
= 24 × 6
= 144

#Q. How many words can be formed using letters of the word EQUATION taken all at a time?

= E Q U A T I O N
① ② ③ ④ ⑤ ⑥ ⑦ ⑧

EQUATION
Taken all at time

Total No. of ways = 8!

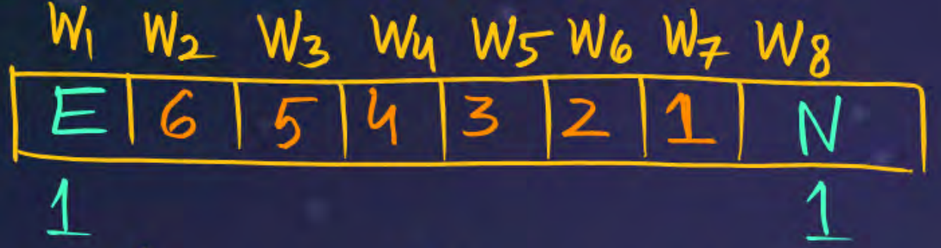
A. 8!

B. 8 × 7!

C. 7!

D. 4 × 7!

#Q. How many of these begin with E and end with N?



Total No. of ways = $1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 1$

$= 6!$ ✓

EQUATION

Restriction → counting

Q, U, A, T, I, O

A. $2 \times 6!$

B. $7!$

C. $2 \times 7!$

D. $6!$

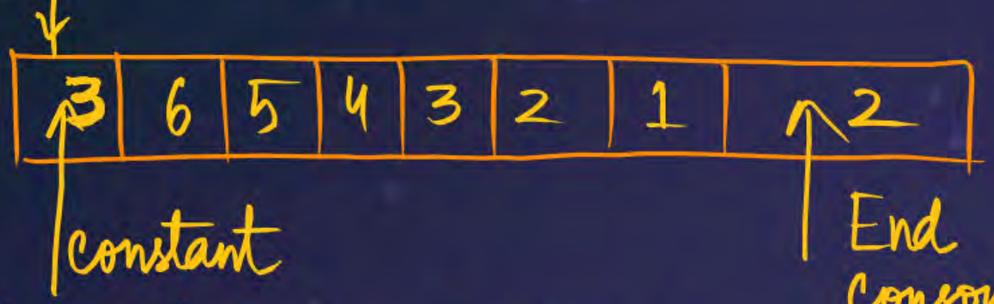
#Q. How many of these end and begin with a consonant?

A. 4320 ✓

B. 720

C. 1440

D. 2880

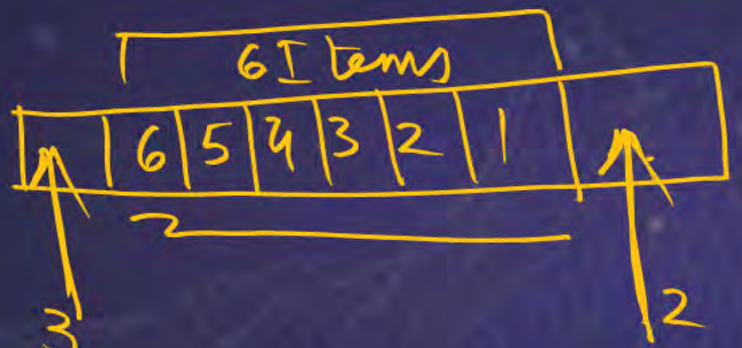


Total No. of ways = $3 \times 6! \times 2$

$= 4320$

EQUATION
(Restriction - counting)
→ vowel
E, U, A, O, I
T, Q, N - constants

$3 \times 2 \times {}^{6}P_{6}$

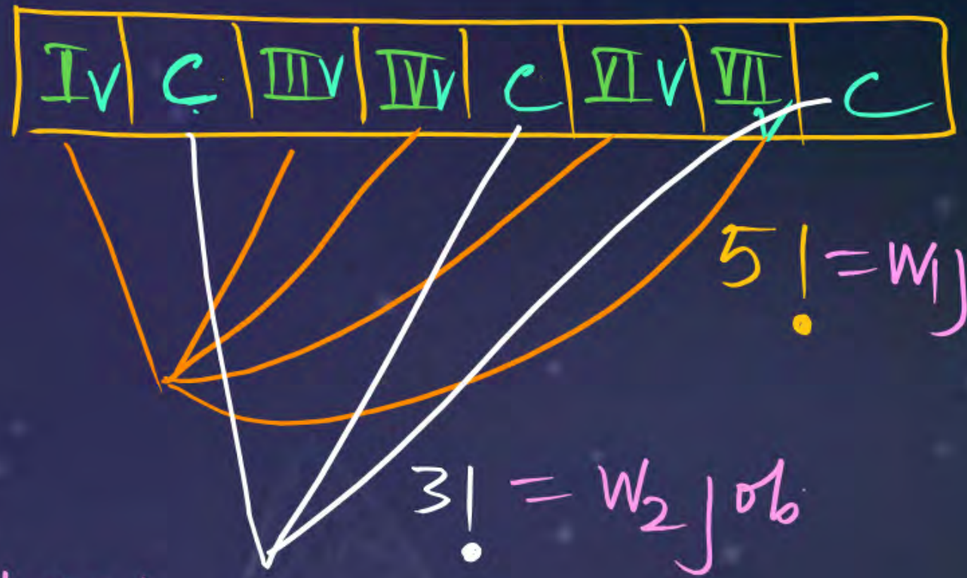


$= 4320$

EQUATION
5 vowels
3 constants

5 vowels, 1 consonants ⑥

#Q. In how many of these, vowels occupy the first, third, fourth, sixth & seventh positions?



A. 360

B. 720

C. 120

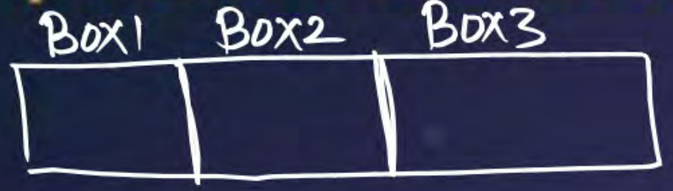
D. None of these

Total No. of ways = 5! × 3!
= 720

box $\rightarrow$ ${}^{n}P_r$ / ${}^{n}C_r$ $\rightarrow$ $n!$; n^N

Non Identical

#Q. In how many ways 5 different red balls, 3 different black balls and 2 different white balls can be placed in 3 different boxes such that each box contains only 1 ball.



$= {}^{10}P_3 = \frac{\lfloor 10}{\lfloor 7}$

$= 720$

$= 720$

5 Different Red

R R R R R

3 Diff. black

B1 B1 B1

2 White

W W

n Different Items Taken r at a time

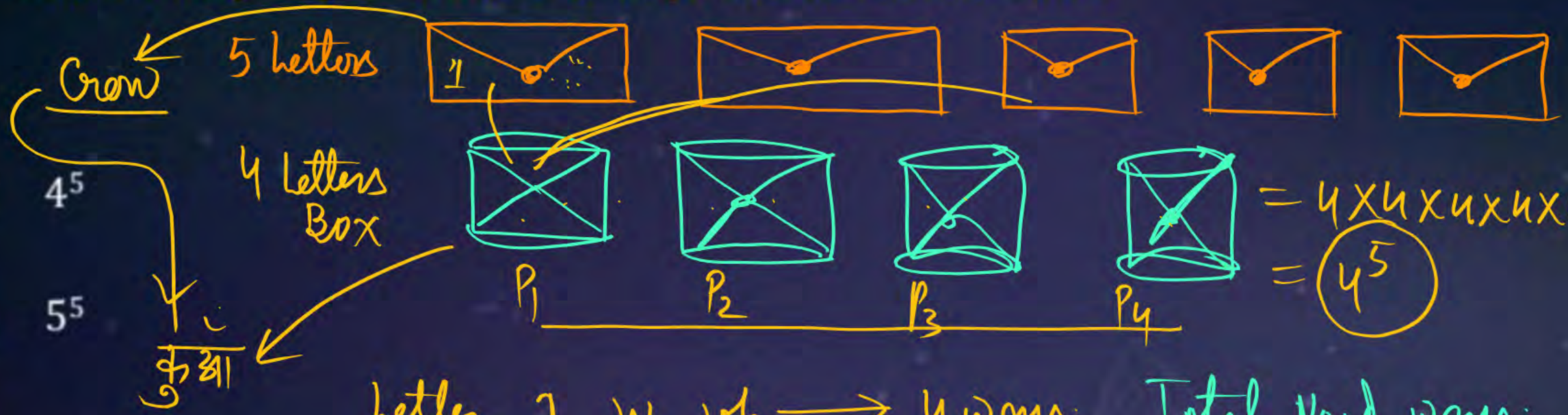
(A) 360

(B) 720

(C) 1440

(D) 1080

#Q. In how many ways can 5 letters be posted in 4 letter boxes?



A. 4^5

B. 5^5

C. 5!

D. 4!

Letter 1 W_1 Job ⟶ 4 ways
2 W_2 Job ⟶ 4 ways
3 W_3 Job ⟶ 4 ways
4 W_4 Job ⟶ 4 ways
W_5 Job ⟶ 4 ways

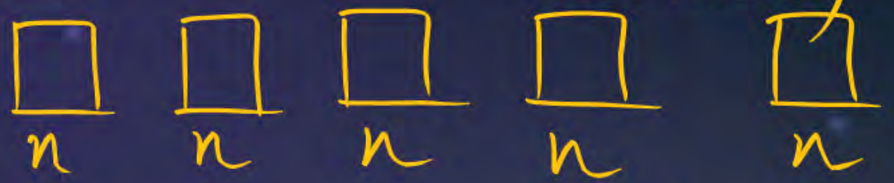
Total No. of ways = 4x4x4x4x4 = 4^5 Ans

All letter Box Are Non Identical

n Different Items Taken all at a time (Repetation is allowed)

$= n \times n \times n \times n$ --- N times

$\Rightarrow \underline{\underline{n^N}}$



$= n \times n \times n$ -- n times

$= \boxed{n^n}$

#Q. Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose each of them can leave the cabin independently at any floor beginning with the first. Find the total number of ways in which each of the five persons can leave the cabin:

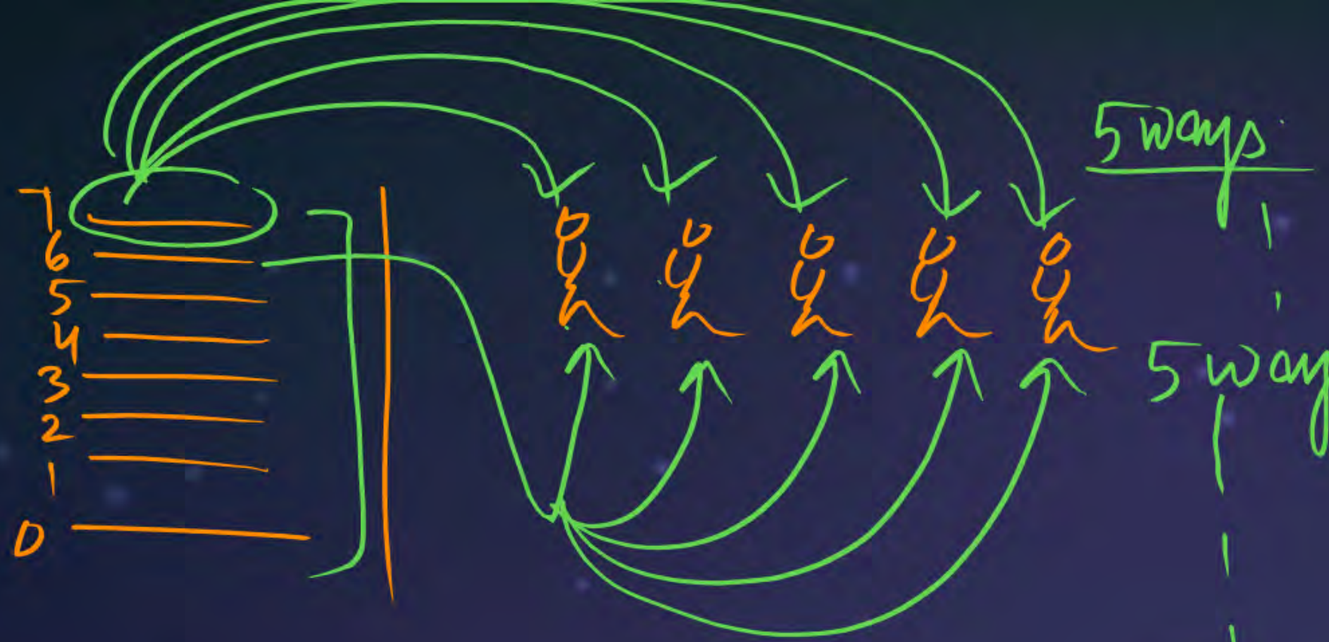
(i) at any one of the 7 floors

A. 5^7

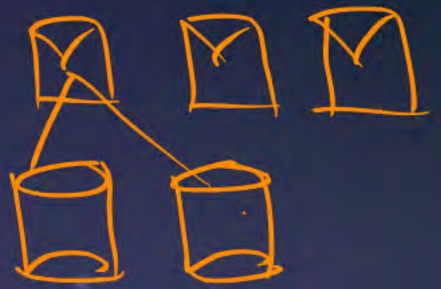
B. 7^5

C. $5!$

D. $7!$



5^7



$2 \times 2 \times 2 = 2^3$

$\neq 3^2$ ✗

✗

✓

Wrong approach

$= 7 \times 7 \times 7 \times 7 \times 7$

$= 7^5$

5^7

#Q. There are 6 single choice questions in an examination. How many sequence of answers are possible, if the first three questions have 4 choices each and the next three have 5 each?

6 Single choice Question

$$\left[\begin{array}{lll} W_1 & (1) & \rightarrow 4 \\ W_2 & (2) & \rightarrow 4 \\ W_3 & (3) & \rightarrow 4 \end{array}\right] = 4^3$$

$$\left[\begin{array}{lll} W_4 & (4) & \rightarrow 5 \\ W_5 & (5) & \rightarrow 5 \\ W_6 & (6) & \rightarrow 5 \end{array}\right] = 5^3$$

$= 4^3 \times 5^3$

$= 16 \times 125 \times 4$

$= 8000$

A. 15625

B. 8000

C. 4000

D. 4096

#Q. How many triangles can be formed by joining the vertices of a hexagon?

A 10

B 20

C 30

D 60

#Q. How many diagonals are there in a polygon with n sides?

A $\frac{n(n-1)}{2}$

B $\frac{n(n+1)}{2}$

C $\frac{n(n-3)}{2}$

D $\frac{n(n+3)}{2}$

#Q. In how many ways can a cricket team be selected from a group of 25 players containing 10 batsmen, 8 bowlers, 5 all-rounders and 2 wicketkeepers? Assume that the team of 11 players requires 5 batsmen, 3 all-rounders, 2-bowlers and 1 wicketkeeper.

A. $\frac{10!}{5!}$

B. $\frac{14\times10!}{3\times5!}$ ✓

C. $\frac{14\times10!}{5!}$

D. $\frac{10!}{3\times5!}$

10 Batsman | 8 Bowler | 5 all | 2 WK

25 Players group

11 Players

5 Batsman
3 all rounders
2 Bowlers
1 WK

25 group $\begin{cases} 10 \\ 5 \\ 8 \\ 2 \end{cases}$

11 Player
5 B
3 all
2 Bow
1 WK

W_1, W_2, W_3, W_4 all working Together

W_1 Job (5 batsman) $\Rightarrow {}^{10}C_5$ (n Different r choose)
W_2 Job $\Rightarrow {}^{5}C_3$ (3 all rounder)
W_3 Job $\Rightarrow {}^{8}C_2$ (2 bowler)
W_4 Job $\Rightarrow {}^{2}C_1$ (1 WK)

Groups / selection / Committee

Total No. of ways $= {}^{10}C_5 \times {}^{5}C_3 \times {}^{8}C_2 \times {}^{2}C_1$

$\Rightarrow \dfrac{14 \times 10!}{3 \times 5!}$ ways.

${}^{10}C_5 = \dfrac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}$

#Q. A man has 7 relatives, 4 of them are ladies and 3 gentlemen; his wife has 7 relatives, 3 of them are ladies and 4 gentlemen. In how many ways can he invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of wife's relatives?

A. 144

B. 720

C. 485 ✓

D. 340

Man Relative (3)		wife Relative (3)		Total case
(4) Ladies	3 gentleman	3 (ladies)	4 (gentleman)	$= 16+324+144+1 = 485$
3 (N_1)	0 (W_2)	0 (W_3)	3 (W_4)	${}^4C_3 \times {}^3C_0 \times {}^3C_0 \times {}^4C_3 = 16$
2	1	1	2	${}^4C_2 \times {}^3C_1 \times {}^3C_1 \times {}^4C_2 = 324$
1	2	2	1	${}^4C_1 \times {}^3C_2 \times {}^3C_2 \times {}^4C_1 = 144$
0	3	3	0	${}^4C_0 \times {}^3C_3 \times {}^3C_3 \times {}^4C_0 = 1$

#Q. A tea party is arranged for 16 people along two sides of a long table with 8 chairs on each side. Four men wish to sit on one particular side and two on the other side. In how many ways can they be seated?

M.W

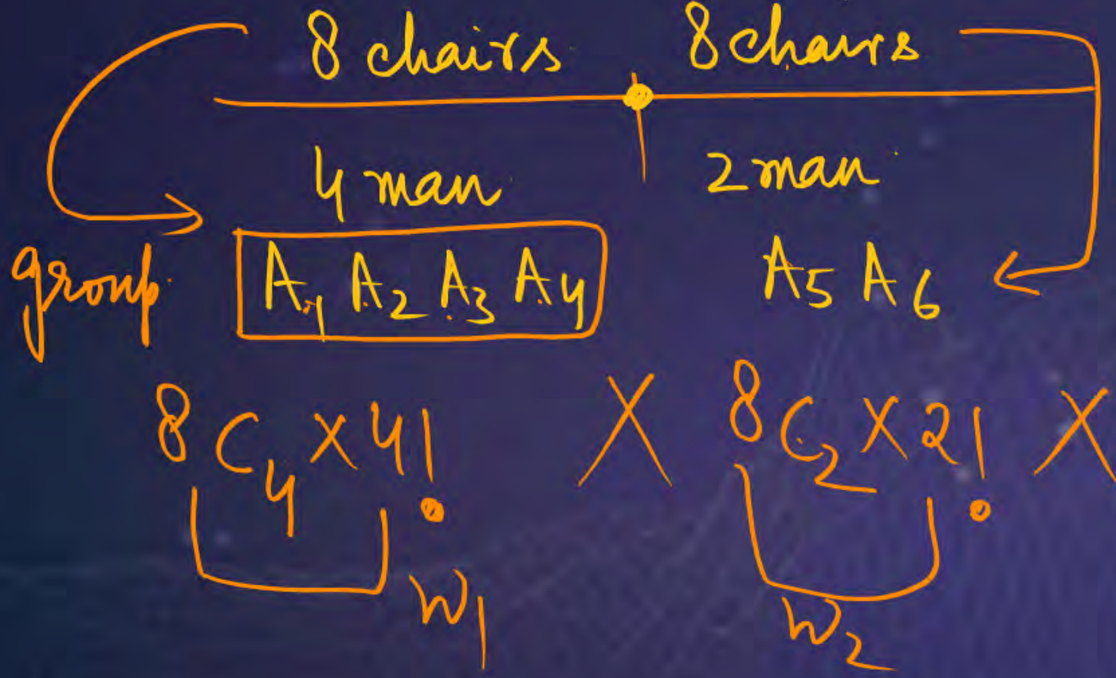
TEA PARTY ⟶ 16 People

A. $\frac{8!10!}{4!6!}$

B. $\frac{8!8!10!}{4!6!}$

C. $\frac{8!8!}{4!6!}$

D. $\frac{8!8!}{6!}$



Lot Type	Selection Type	No. of arrangement	No. of Selection
n Different Items	Taken all at Time	$n! = n(n-1)\cdots 1$	1
n Different Items	Taken r at a time	${}^{n}P_{r} = \frac{n!}{(n-r)!}$	${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$
n Different Items p alike q alike r alike	Taken all at a time	$\frac{(p+q+r)!}{p!\,q!\,r!}$	1

n

BANANA

AAA — p alike

NN — q alike

B — r alike

$= \frac{6!}{3!\,2!\,1!}$

1

Order

$\frac{PPPPRRR}{1}$

n Different Items	Taken (R) at a time	No. of Permutation (X)	No. of combination (X)
p alike q alike r alike			

ALLAHABAAD

n Different Items

p alike
q alike
r alike

5 alike
2 alike

AAAAA = 5A

LL = 2L

No. of arrangement $= \frac{10!}{5!\,2!}$

⇒ CALCULUS

1

AAR → $\frac{3!}{2!}$

CALCULAS

$\frac{8!}{2!\,2!\,2!}$


Thank You