Data Science and Artificial Intelligence

Probability and Statistics

Random Variable

Lecture No.- 05

Recap of Previous Lecture

Topic Expectation of Random Variables (One Dimensional)

Topics to be Covered

Topic

Max and Min of Random Variables

- ✓ Transformation of Random variable
  - Pdf to cdf
  - cdf to pdf
- ✓ Max/min Problems → max/min
- ✓ Inequalities —
- ✓ Bayes theorem

Topic : Probability and statistics

#Q. X is a random variable with probability density function

$$f_x(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Let $Y = X^2$

$f_Y(y) = ?$

Determine the probaility density function for Y.

cdf method → pdf

$$f(x)=\begin{cases}\frac{1}{2} & -1<x<1\\ 0 & \text{otherwise}\end{cases}$$

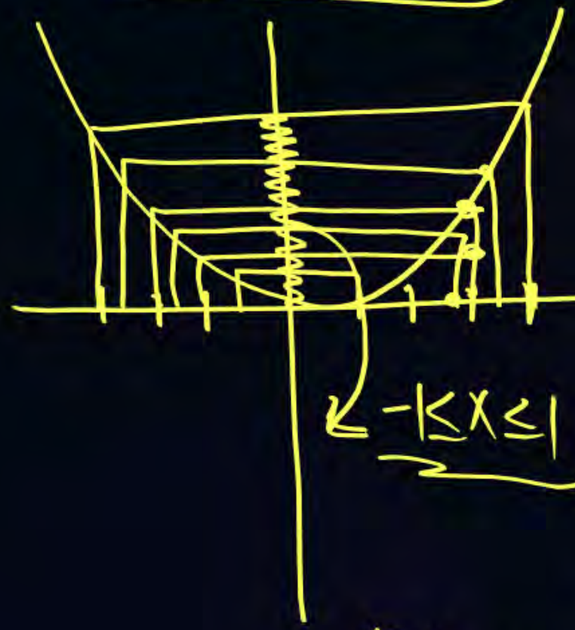
Let $Y=X^2$

Determine The pdf of Y.

step ① Solve The Inequality $Y=X^2$

step ② $\text{cdf}=F_Y(y)=P[Y\le y]=P[X^2\le y]$

$=P[-\sqrt{y}\le X\le\sqrt{y}]$ Interval



$$=\int_{X=-\sqrt{Y}}^{\sqrt{Y}}\frac{1}{2}\,dx$$

$$=\frac{1}{2}\left[\sqrt{Y}+\sqrt{Y}\right]=\frac{2\sqrt{Y}}{2}=\sqrt{Y}$$

$$F_Y(y)=\sqrt{y}$$

$F_Y(y)=\text{cdf}$ — strategy — $f_Y(y)$

$\frac{d}{dx}F_X(x)=f_X(x)$

M.Imp

Using defination of pdf

$$\frac{d}{dy}[F_Y(y)] = f_Y(y)$$

$$\frac{d}{dy}[\sqrt{y}] = f_Y(y)$$

$$= \frac{1}{2\sqrt{y}} = f_Y(y)$$

$$f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}} & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{pdf = \frac{1}{2\sqrt{y}} \quad 0<y<1}$$

one-one Function

$$\boxed{y^{1/3} < x < y^{1/3}}$$

$$\frac{F_Y(y)}{f_Y(y)} =$$

Topic : Probability and statistics

#Q. A random variable , X, has the following cummulative distribution function:

cdf $F_X(x) = 1 - e^{-x}, \quad 0 < x < \infty$

You are given $Y = \sqrt{6X}$.

Determine the probability dentsity function of Y.

pdf to cdf
cdf to pdf
cdf/pdf

pdf method

1) $g^{-1}(y) = x$
2) $\frac{d}{dy}[g^{-1}(y)]$
3) $f_Y(y) = f_X[g^{-1}(y)] \frac{d}{dy}[g^{-1}(y)]$

$Y = \sqrt{6X}$

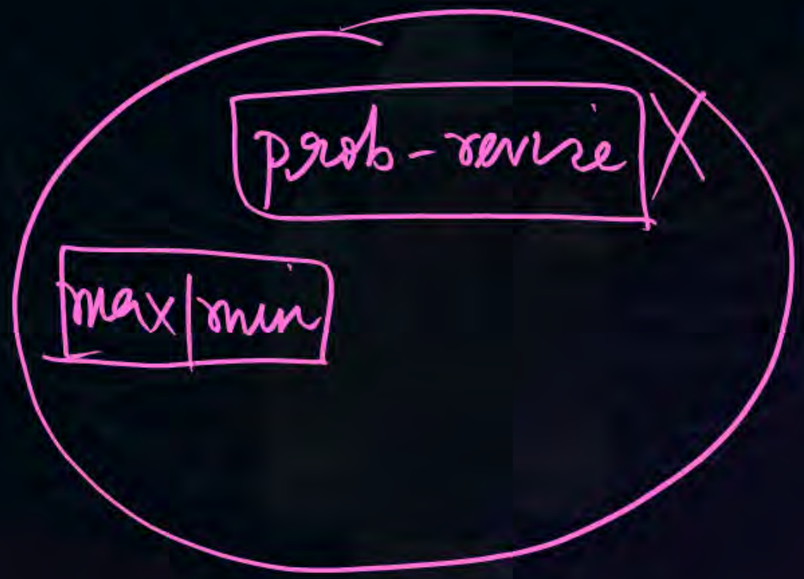
$\boxed{X = \frac{Y^2}{6}} = g^{-1}(y)$

$\frac{d}{dy}\left[\left(\frac{y^2}{6}\right)\right] = \frac{d}{dy}[g^{-1}(y)]$

$= \frac{y}{3}$

$f(y) = f_X[g^{-1}(y)] \frac{d}{dy} g^{-1}(y)$

$\boxed{f_Y(y) = e^{-y^2/6} \cdot \left(\frac{y}{3}\right)}$



Topic : Probability and statistics

$\frac{3}{4}$ ✓

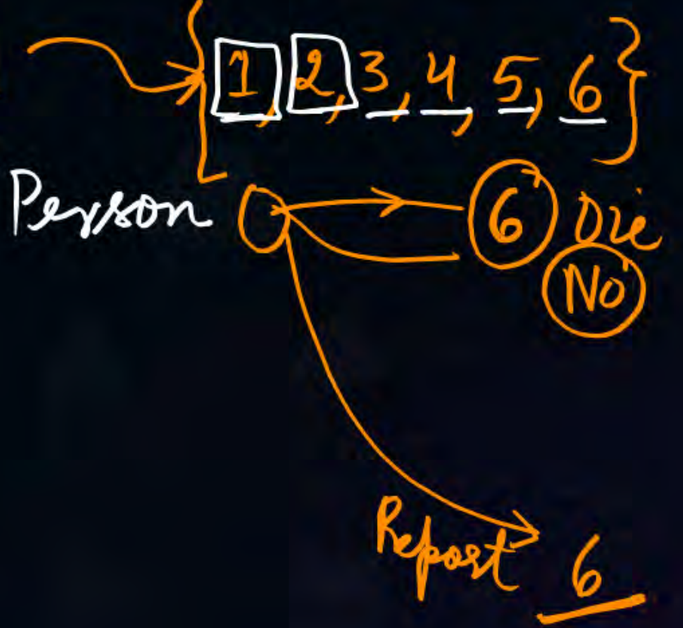
Wrong $-\frac{3}{8}$ ✗

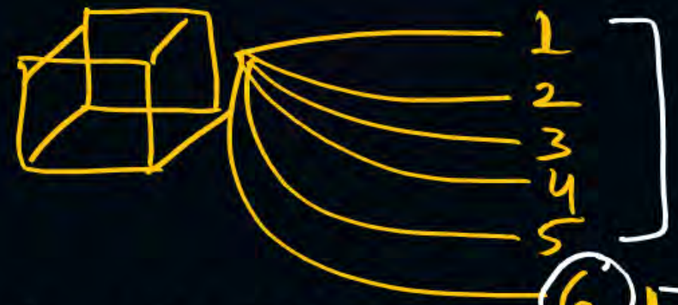
Weekly TEST 01

#Q. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

$P(\text{speaking Truth}) = \frac{3}{4}$

1 2 3 4 5 → It report Is 1,2,3,4,5





$P(\bar{6}) = 1 - \frac{1}{6} = \frac{5}{6}$

Report $P(6) = \frac{1}{6}$

① ② ③ ④ ⑤ ⑥

Prob (right No) = P

Prob (wrong) = 1 − P

① ② ③ ④ ⑤ ⑥ Are mutually exclusive events

Particular: $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$P(1) + P(2) + P(3) + P(4) + P(5)$ n times $+ P(6) = 1$

$x + x + x + x + x + P = 1$

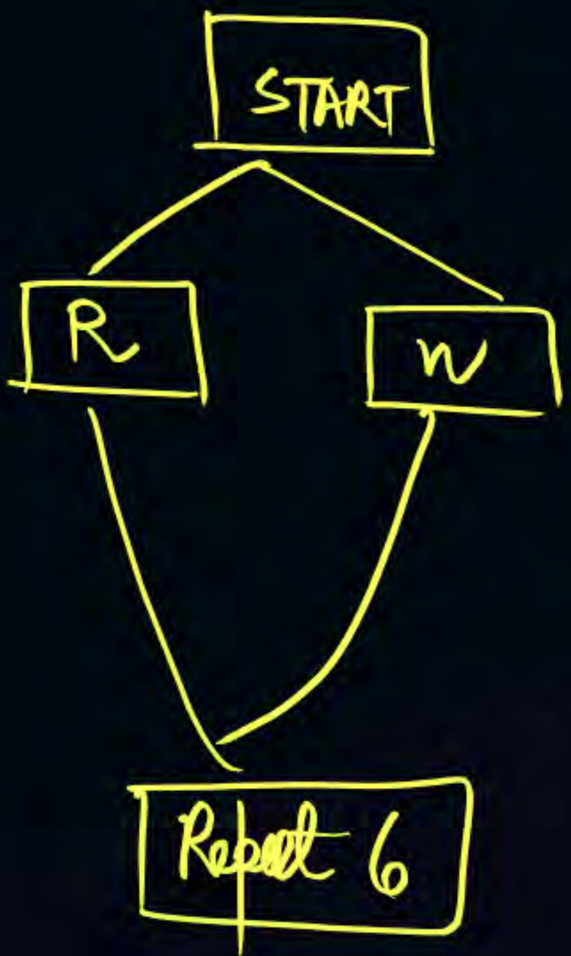
$$x = \frac{1-P}{5}$$

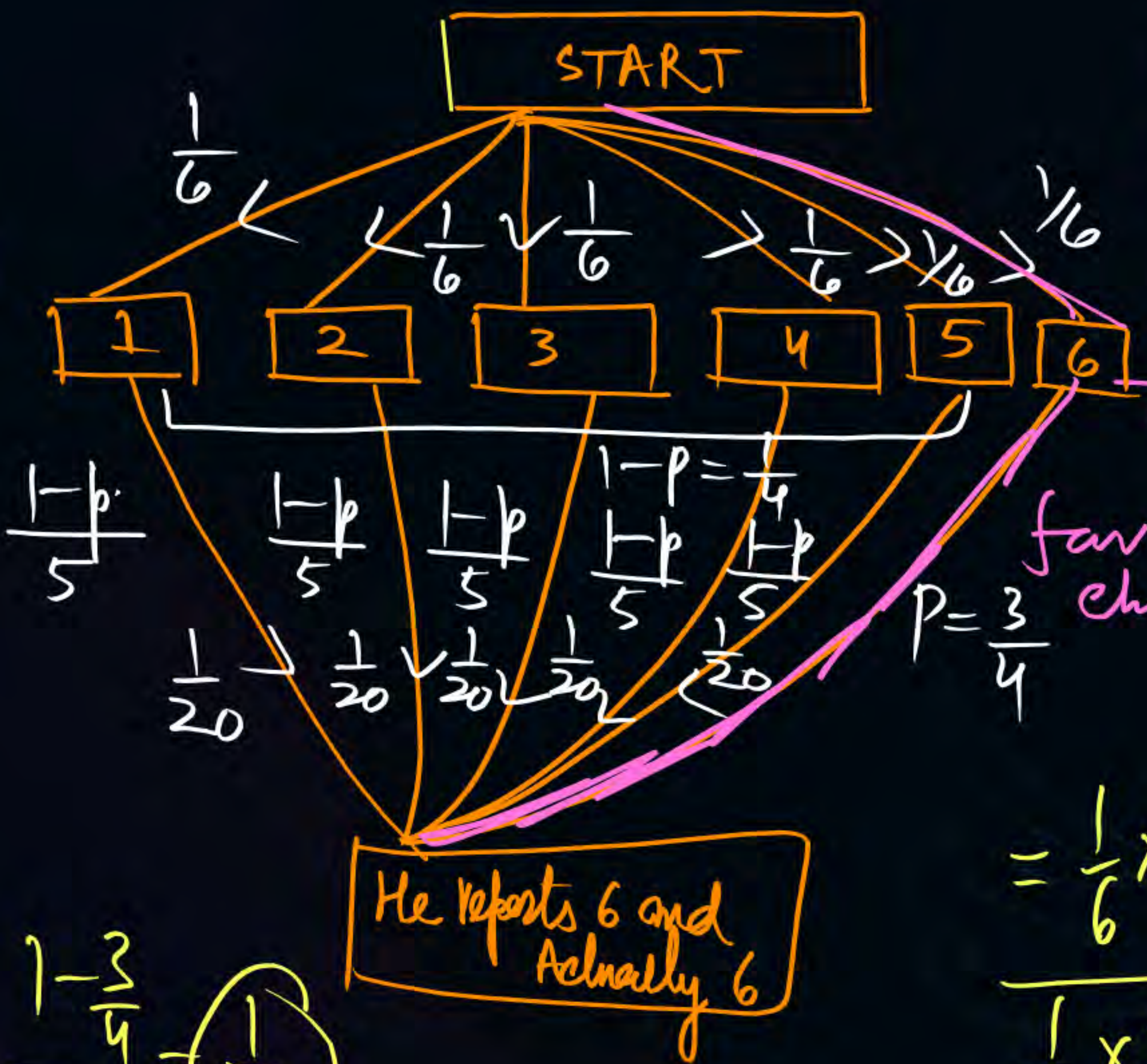
6 → 1/6 | ① ② ③ ④ ⑤

5/6 Single wrong Number

x

particular wrong





01
02
03
04
05

$\Big\} \; 1-p \cdot \frac{1-p}{5} \quad \frac{1-p}{5} \quad \frac{1-p}{5} \quad \frac{1-p}{5} \quad \frac{1-p}{5}$

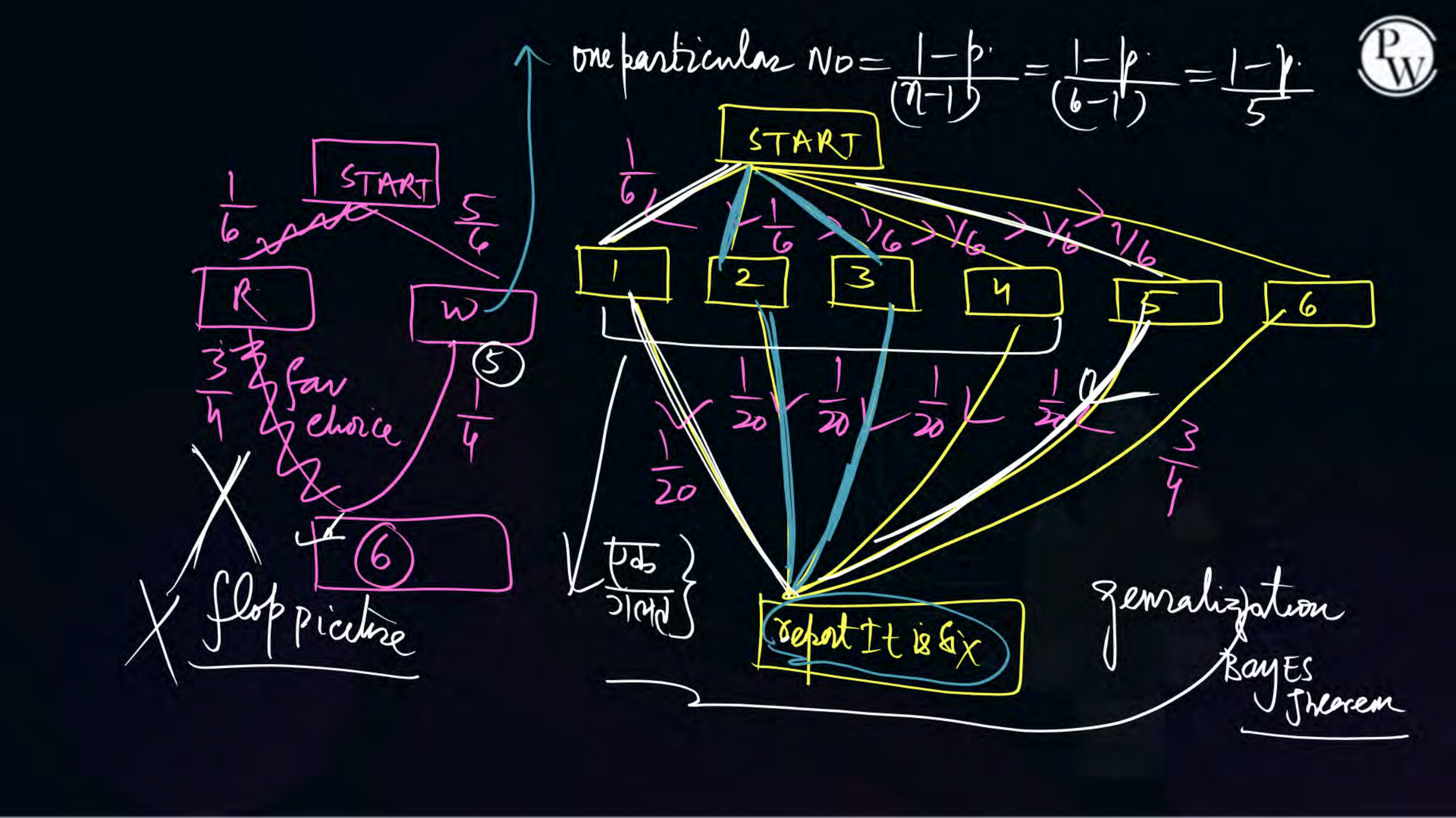
06 p

far choice

Particular wrong Letter $= \frac{1-p}{5} = \frac{1-\frac{1}{6}}{5} = \frac{1}{6}$

$$\frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{1}{6}\times\frac{1}{20}\times 5} = \frac{3}{4}$$

$$\frac{1-\frac{3}{4}}{6-1} = \frac{1}{20}$$


one particular No = $\frac{1-p}{(n-1)} = \frac{1-p}{(6-1)} = \frac{1-p}{5}$
START
$\frac{1}{6}$
$\frac{5}{6}$
R
W
5
$\frac{3}{4}$
$\frac{1}{4}$
fav choice
6
flop picture
START
$\frac{1}{6}$
1
2
3
4
5
6
$\frac{1}{20}$
$\frac{3}{4}$
report It is Six
generalization
BAYES Theorem

THANK - YOU