# Data Science and Artificial Intelligence

## Probability and Statistics

### Discrete Probability Distribution

Lecture No.- 05

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# Topics to be Covered

**Topic** Negative binomial Distribution

**Topic** Hyper geometric  Distribution

# Poisson Distribution

$$P_0(t) = P[0 \text{ events in } (t, t+dt)]$$

$$P_0(t+dt) = P_0(t)\, \underline{P_0(t+dt)}$$

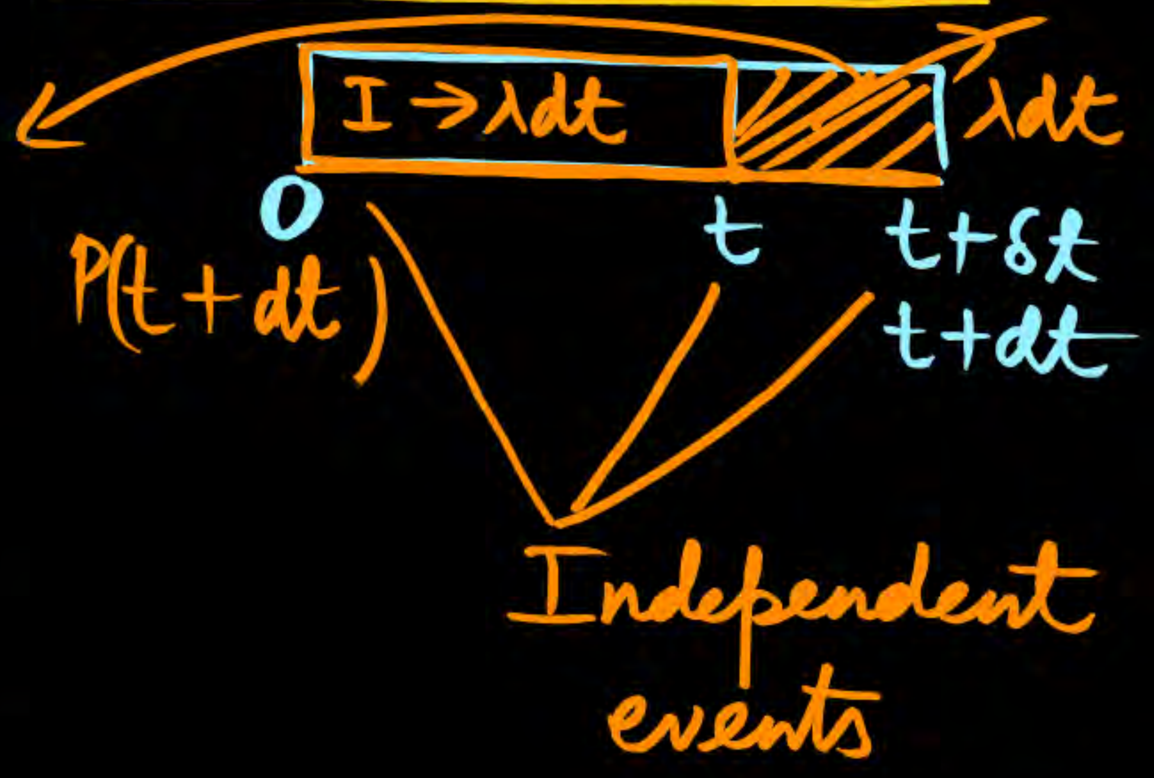
both events Are Indep.

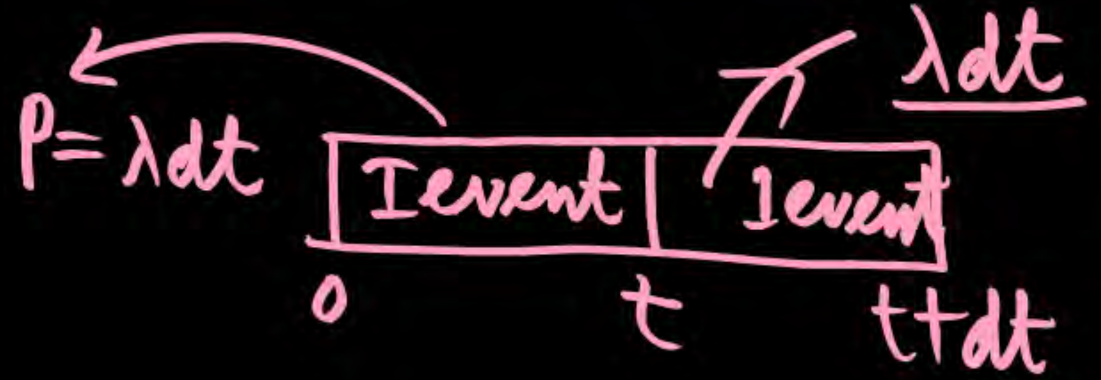
$$P_0(t+dt) = P_0(t)\Big[1 - [1 \text{ event occur}] - P[2 \text{ event occur}] - P[3 \text{ event occur}]\Big]$$

$$P_0(t+dt) = P_0(t)\left[1 - \lambda dt - 0 - 0\right]$$

$$P_0(t+dt) = P_0(t)\left[1 - \lambda dt\right]$$

$$= P_0(t) - \lambda P_0(t)\, dt$$



$$P_0(t+dt) - P_0(t) = -\lambda P_0(t)\,dt$$

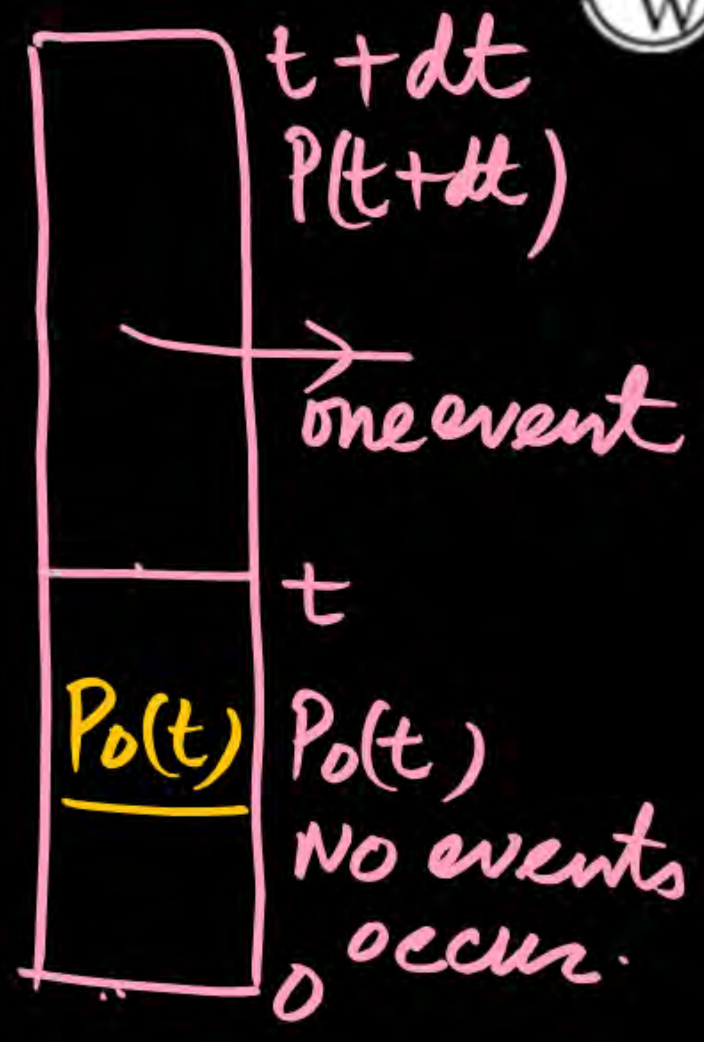
$$= \frac{P_0(t+dt) - P_0(t)}{dt} = -\lambda P_0(t)\frac{dt}{dt}$$

$$\Rightarrow \lim_{dt \to 0} \frac{P_0(t+dt) - P_0(t)}{dt} = -\lambda P_0(t)$$

$$\Rightarrow \frac{d}{dt} P_0(t) = -\lambda P_0(t)$$

$$\Rightarrow \boxed{\frac{dP_0}{dt} = -\lambda P_0(t)}$$

Differential eqn




$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Using variable seprable

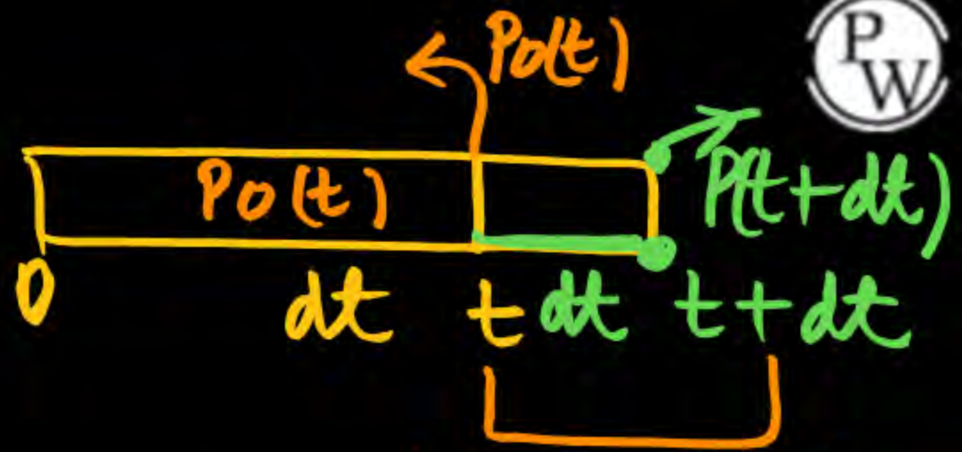
$$\frac{d P_0}{dt} = -\lambda P_0$$

$$= \int \frac{dP_0}{P_0} = \int -\lambda \, dt$$

= both sides Integrating

$$= \ln P_0 = -\lambda t + c$$

$$\boxed{P_0 = e^{-\lambda t}} \qquad \boxed{P_0(t) = e^{-\lambda t}}$$



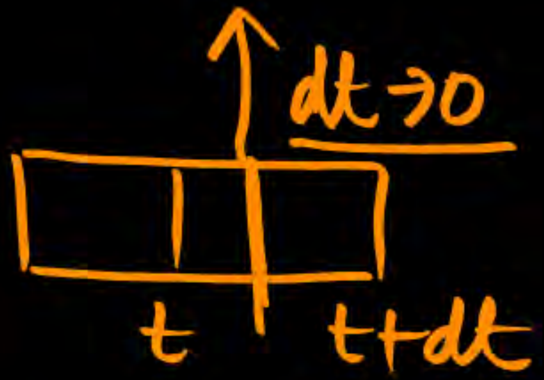

both events Are Independent (Bernoulli Trials)

$$\underline{P_0(t+dt)} = P_0(t)\, P_0(t+dt)$$

$$P_0(t+dt) = P_0(t)\left[1 - P[\text{1 event occur}] - P[\text{2 event occur}]\right]$$

$$P_0(t+dt) = P_0(t)\left[1 - \lambda dt - 0\right]$$

$$\lim_{dt\to 0} \frac{P_0(t+dt)-P_0(t)}{dt} = -\lambda P_0(t)$$




$$\Rightarrow \frac{d}{dt} P_0(t) = -\lambda P_0(t)$$

$$\Rightarrow \frac{d\,P_0(t)}{dt} = -\lambda P_0(t)$$ this is D.E

$$\Rightarrow \int \frac{d\,P_0}{P_0} = \int -\lambda\, dt$$

$$= \boxed{P_0(t) = e^{-\lambda t}}$$

→ Intial Prob.

$\lambda t = \mu$

$$\boxed{P_0 = e^{-\mu}}$$

$$\frac{dy}{dx} = y$$

variable seprable

$$\int \frac{dy}{y} = \int dx$$

both sides Integrate It

$$\ln y = x + c$$

$$y = e^x \cdot e^c$$

$$= Ae^x$$

$$P(X=x) = \frac{e^{-\mu}(\mu)^x}{x!}$$

$\mu$ = Average/mean of The Distribution

$x = 0, 1, 2 - - - -$

$$\left\{ \begin{array}{l} P_0 = e^{-\mu} \\ P_1 = \mu e^{-\mu} \\ P_2 = \dfrac{\mu^2 e^{-\mu}}{2!} \\ P_3 = \dfrac{\mu^3 e^{-\mu}}{3!} \end{array} \right.$$

$$P(X=x) = \frac{e^{-\mu}(\mu)^x}{x!}$$

## GEOMETRIC-Distribution:

Tossing A coin

GATE 1 X
2 X
3 X
4 X
5 X
6
7
8

(Re 1, Re 2, Re 3, Re 4, Re 5, Re 6, Re 7, Re 8)

Starting Trials

$\boxed{H}$

T T T T T

(F) (F) (F) (F) (F)

(No Success)

Remove The Trials

X

3 Trials more (SUCCESS)

Prob. of success.

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$P\left(\frac{X \geq (7+3)}{X \geq 7}\right) = P(X \geq 3)$$

## Memoryless Property:

X X X X X X

starting

+ Trials

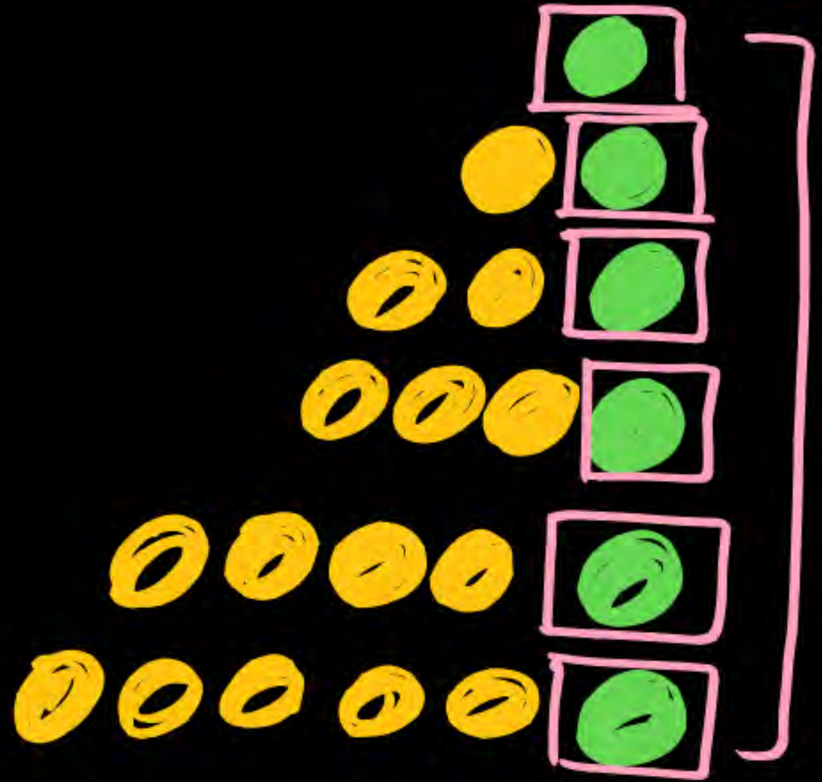
$$P\left[\frac{X \geq (a+b)}{X \geq a}\right]$$

$$= P\left[\frac{X \geq 5+3}{X \geq 5}\right]$$

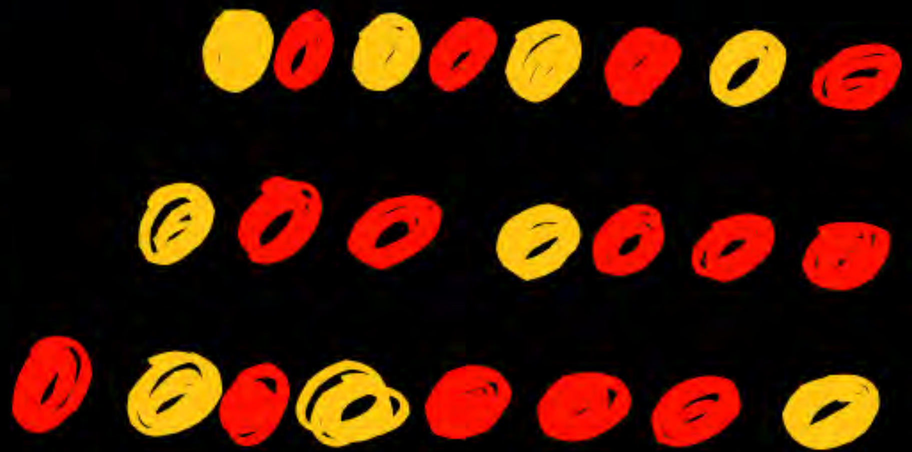
$$= P[X \geq b]$$

Ans

# Negative Binomial Distribution :



GEOMETRIC Distribution $G(p)$



$B(n, p)$

$X$ = No. of red balls.

$X = 0, 1, 2, 3, 4, 5,$

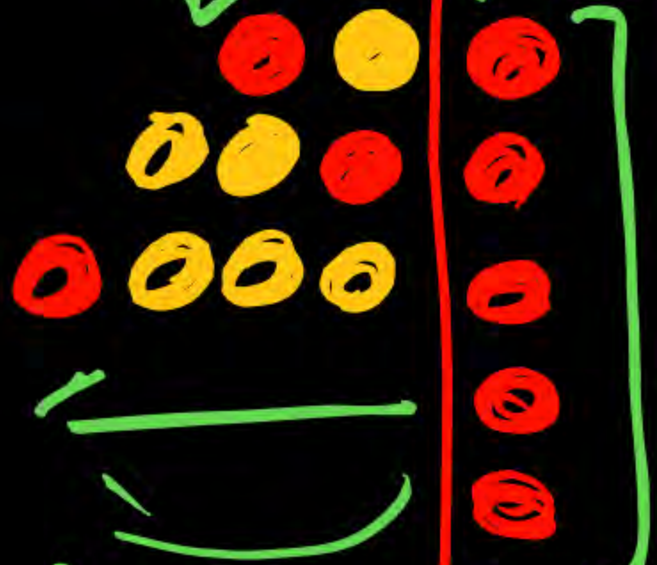
B
G B
GG B
GG B

B (SUCCESS)

Ist red

IInd SUCCESS

IInd red



Last

IInd SUCCESS

SUCCESS → SUCCESS 2nd

A coin is Tossed Infinite No. of times $4^{th}$ HEAD appear at The $10^{th}$ Toss.

Binomial Trials $\times p$

| H | H | H | T | T | T | T | T | T | | H |

Binomial $\times p$

| H | T | T | H | T | T | H | T | T | | H |

$B(n,p) \times p$

| T | T | T | H | H | H | T | T | T | | H |

$B(n,p)$

| H | H | T | T | T | T | T | T | H | | H |

| H | T | H | T | H | T | H | T | T | T | H |

many Sequences Are HERE $B(n,p) \times p$

$10^{th}$ Toss. ($4^{th}$ Head)

$r^{th}$ SUCCESS $= \boxed{B(n,p) \times p}$

$B(n,p) \rightarrow X = 0, 1$

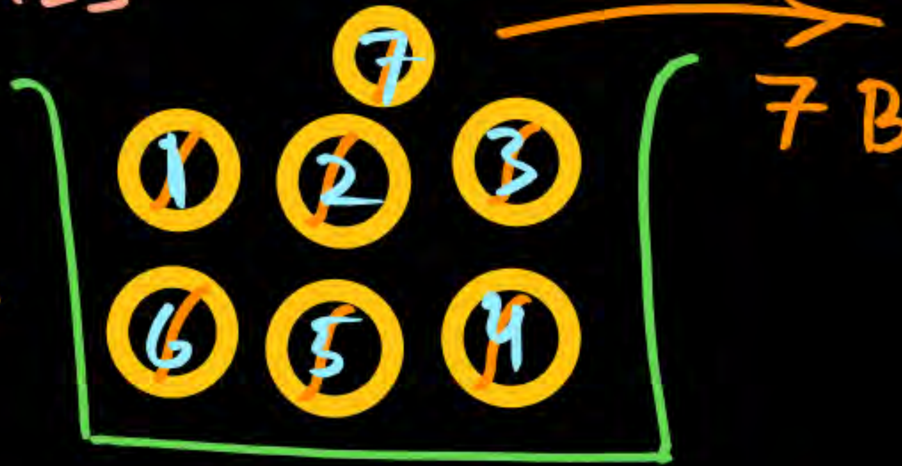
$G(p) \rightarrow$ only one success

Neg $B(n,p) \rightarrow k^{th}$ success

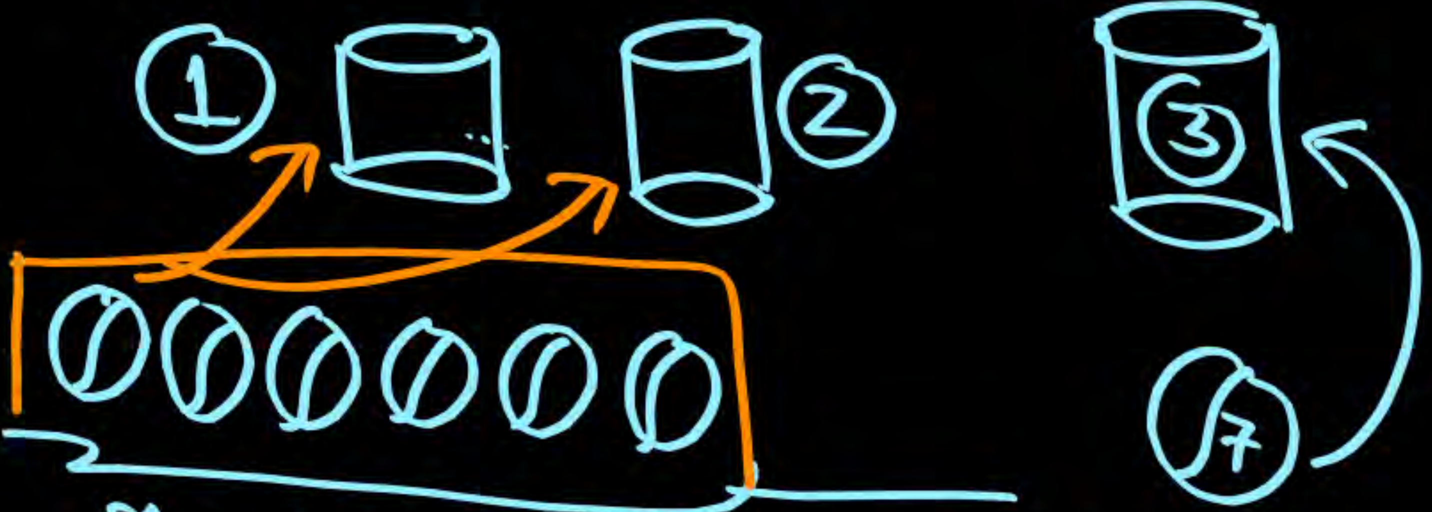
# Negative Binomial: for $r^{th}$ success

3 glass

$r^{th}$ success
1) 6 Balls $\rightarrow$ $\boxed{2 \text{ glass}}$ $\rightarrow$ Ist work
2) 7 balls $\rightarrow$ 3 glass.




7 Balls.




$n = 6 \quad r = 2 \quad \left(\boxed{B(n,p) \times p}\right)$

7 balls $\rightarrow$ $\boxed{3}$ glass

$X$ = No. of glass.

$\boxed{X = 0, 1, 2, 3.}$

$$P(X = r^{th}\ \text{success}) = {}^6C_2\, p^2 q^4 \cdot p$$

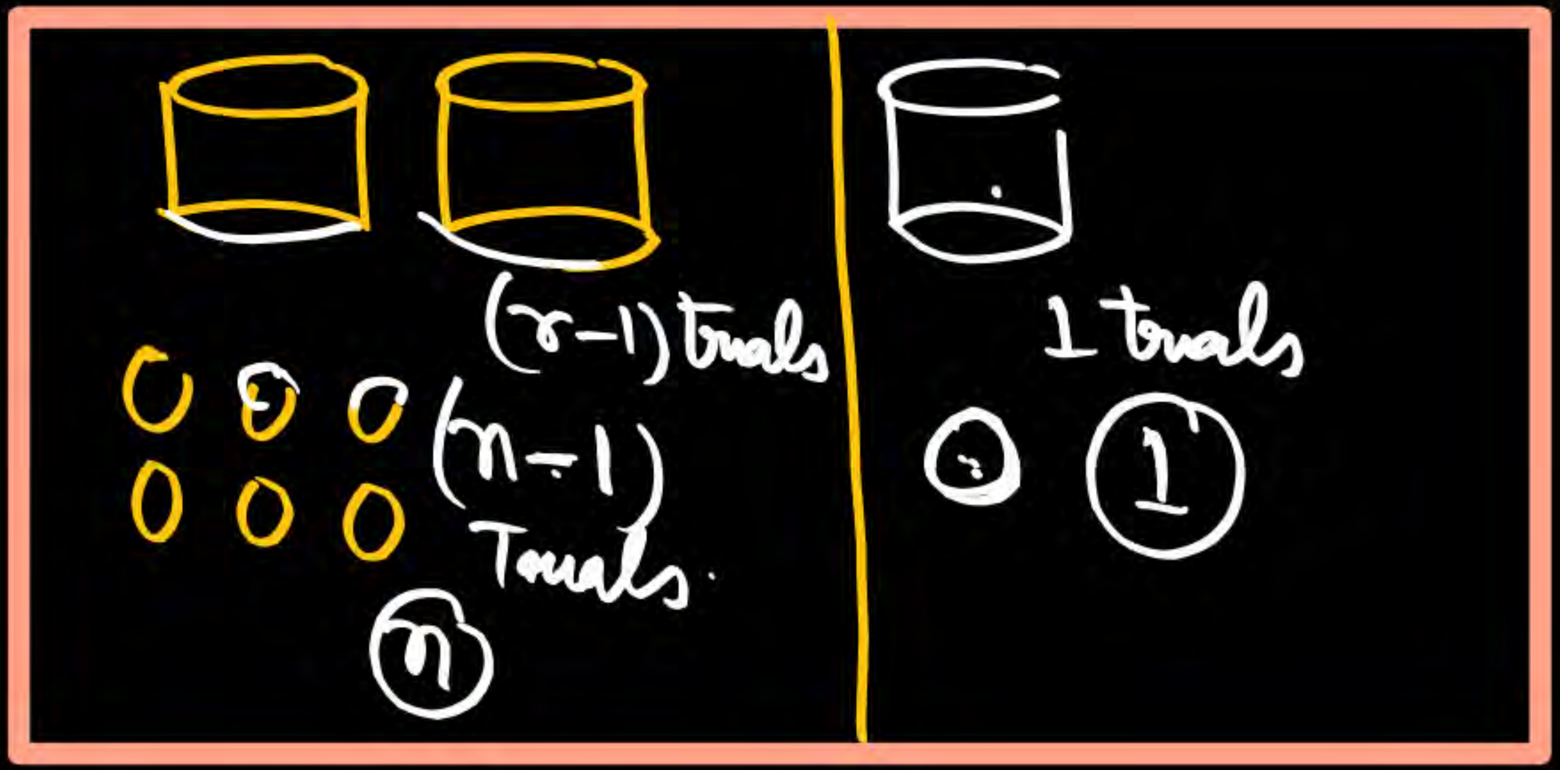

$$= \underbrace{{}^6C_2\, p^2 q^4}_{\text{Binomial Dis}} \times p$$

Negative Binomial

$r^{th}$ success

## Negative Binomial Distribution

$$NB(n, p) = B(n, p) \times p$$



# Negative Binomial Distribution

$$P(X=r) = {}^{n-1}C_{r-1}\, p^{r-1}\, q^{(n-1)-(r-1)} \cdot (p)$$

No. of success

$$P(X=r) = {}^{n-1}C_{r-1}\, p^{r-1} \cdot p \cdot q^{(n-1-r+1)}$$

$$\boxed{P(X=r) = {}^{n-1}C_{r-1}\, p^{r}\, q^{n-r}}$$

using for $r^{th}$ success for $n^{th}$ trials.

| $B(n,p)$ | $P$ |
|---|---|
| $n-1$ | $1$ |
| $r-1$ | $1$ |

$$B(n,p) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$$

$$NB(n,p) = \boxed{B(n,p)} \times p$$

## Topic :

Q1. Find the probability that third head turns up in 5 tosses of an unbiased coin.

Using Negative Binomial

$$= {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \times \frac{1}{2}$$

$$= \frac{3}{16} \text{ Ans}$$

| | | | | |
|---|---|---|---|---|
| H | H | T | T | H |

| | | | | |
|---|---|---|---|---|
| T | T | H | H | H |

| | | | | |
|---|---|---|---|---|
| H | T | H | T | H |

$\underbrace{H\ T\ H\ T}_{\text{4 trials, 2 HEAD}}\ \underbrace{H}_{\frac{1}{2}}$

Topic :

Q2. Find the probability that a third child in a family is the family's second daughter, assuming the male and female are equally probable.

$$= {}^2C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \times \frac{1}{2}$$

$$= \frac{2}{2\times2}\times\frac{1}{2} = \frac{1}{4} \text{ Ans}$$

| | | III rd child |
|---|---|---|

Binomial

SECOND DAUGTHER

$P(\text{boy}) = \frac{1}{2}$

$P(\text{Daughter}) = \frac{1}{2}$

1) H·W

Negative Binomial

moment generating Function

→ E[X]

→ V(X)

→ S·D

+ Uniform.

✓ 2) Arrival Discrete

LEO,
Bio,
Bernoulli,
Poisson
Negative Bi

→ Revise

①

PMF ✓ ①

mean
Var(X)
S·D

②

③

④

⑤

[redacted]

# THANK - YOU