DATA SCIENCE

Probability and Statistics

Counting Techniques

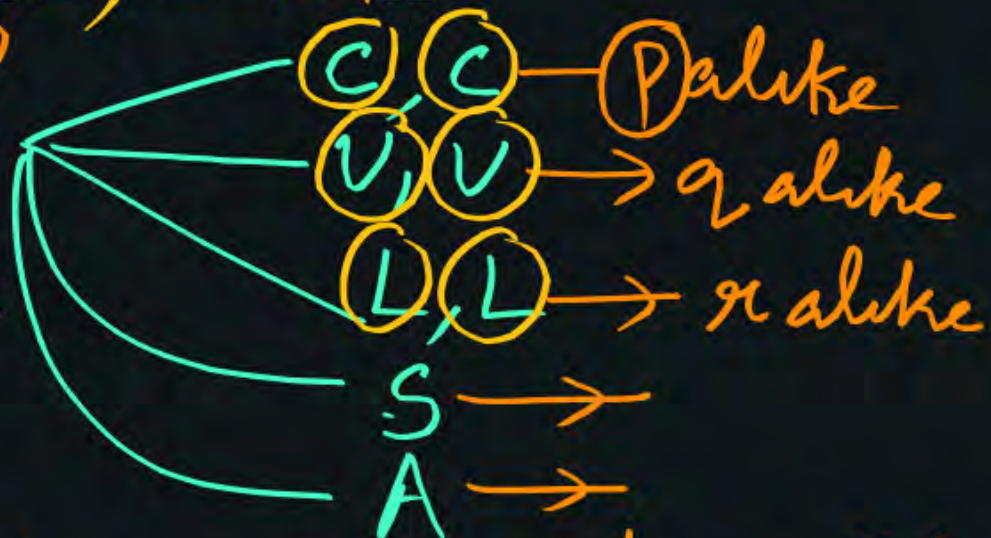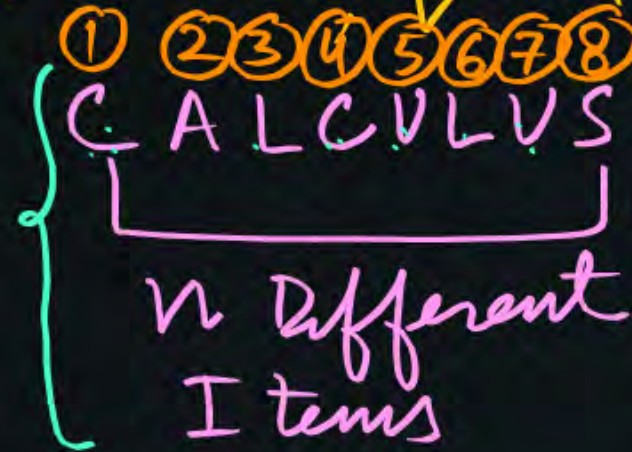Lecture No.- 03

RAHUL SIR

Topics to be Covered : Counting Techniques

\# n Different Items
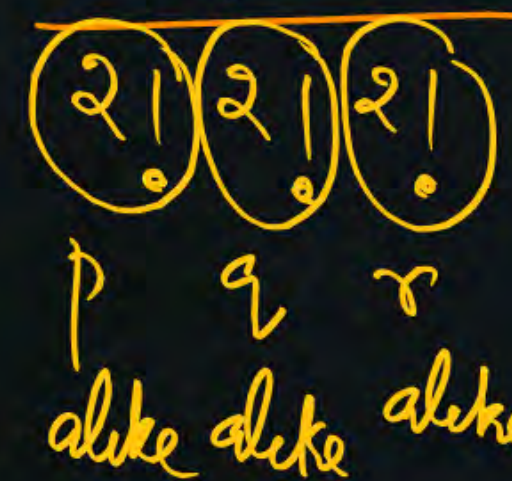
P alike q alike, r alike

① ②③④⑤⑥⑦⑧

C A L C U L U S

n Different Items

C C → P alike

V V → q alike

L L → r alike

S →

A →

Number of arrangement $]$ n Diff. Items

$\overleftrightarrow{C \; C}$ Relative Position Change — Arrangement Change (don't)

Taken all at time

Number of arrangement = 8!

n Different Items No. of arrangement $= \dfrac{n!}{P! \; q! \; r!}$

$$\dfrac{8!}{\boxed{2!}\boxed{2!}\boxed{2!}} = \dfrac{n \text{ Different Items}}{P! \; q! \; r!}$$

P q r

alike alike alike

M I S S I P P I
①②③④⑤⑥⑦⑧

Total No. of letters = 8

SS, IIT, PP, M

Palike &alike ralike

No. of arrangement
Taken all at a time
$\left(n\, \text{Diff.} < \begin{matrix} P \\ \xi \\ \gamma \end{matrix} \right)$

$\Rightarrow \dfrac{8!}{2!\,3!\,2!}$

MATHEMATICS — H.W

AAABBCCCCDD — H.W

ABRACADABRA
①②③④⑤⑥⑦⑧⑨⑩⑪

→ AAAAA, BB,
RR,

Ans
$= \dfrac{11!}{5!\,2!\,2!}$ → taken all at a time

10 balls.

5R 2Y 2B 1W

balls Are
Identical
Taken all
at a time

$$RRRRRYYBBW \Rightarrow \frac{10!}{5! \; 2! \; 2!}$$
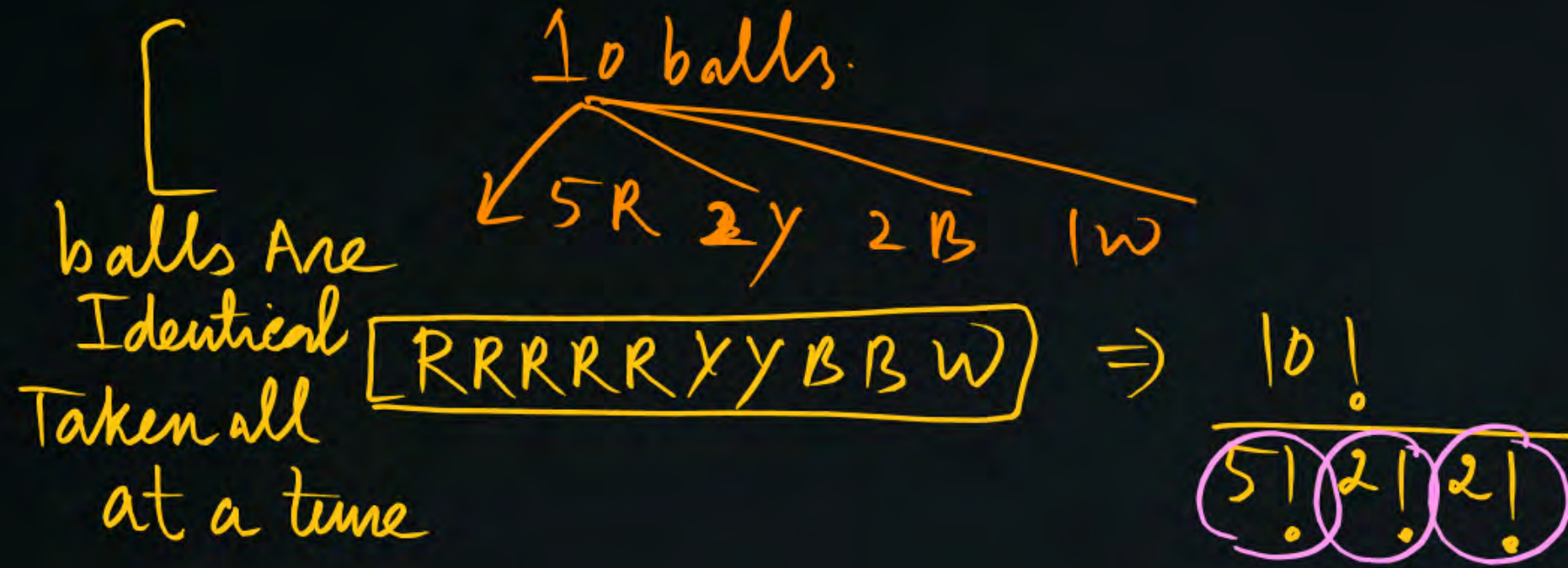
CASE 02

$\begin{cases} n \text{ Different Items} \\ P \text{ alike, } q \text{ alike, } r \text{ alike, } s \text{ alike} \\ \quad t \text{ alike } - - - - \end{cases}$

No. of groups/Selection

$$RRRRRYYBBW$$

$\rightarrow$ 1 Selection

$\begin{cases} RAHUL \\ AHRLA \\ ARLHA \end{cases}$

#Q. In how many ways we can arrange letter $\overbrace{A,A,}^{p}\overbrace{B,B,}^{q}B,\check{C}$ taken 3 at a time.

A A B B B C

P alike q alike r alike

✓ Case Ⓐ   3 alike

Case Ⓑ   2 alike 1 Different

Case Ⓒ   3 Different

$\begin{cases} n \text{ Different Items} \\ P \text{ alike, } q \text{ alike, } r \text{ alike} \\ \text{Taken 'R' at a time} \end{cases}$

arrangement = Selection × order

Selection — group.

Slide : 2

$$\boxed{^{n+r-1}C_r}$$

CASE A: 3 alike $= 1 \times \dfrac{3!}{3!}$ (BBB) = ① $\underline{AABBBC}$

Selection × order

2 alike 1 Diff $= ^2C_1 \times ^2C_1 \times \left(\dfrac{3!}{2!\,1!}\right)$

String → $\boxed{BBB}$ ⇒

3 alike

① $\left\{\dfrac{AA}{BB}\right\}]$

AA

$\boxed{\dfrac{BC}{AC}}$

BB

$\begin{bmatrix} AA & \boxed{BC} \\ BB & AC \end{bmatrix}$
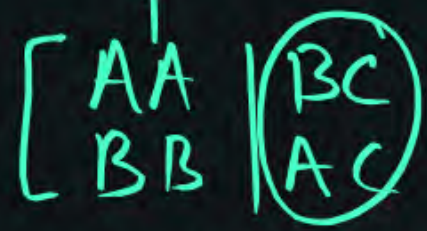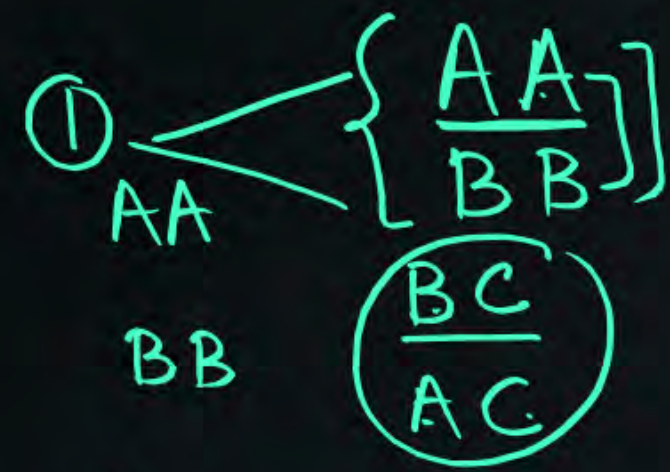
$= 2 \times 2 \times \dfrac{6}{2} = \boxed{12 \text{ ways}}$

Selection = 1

Order $= \boxed{\dfrac{\angle 3}{\angle 3}}$

$\rightarrow \boxed{BBB}$

3 alike

$\dfrac{3!}{3!} = ①$

Overcounting

AA

$\overset{AA}{\underset{2\text{alike}}{\left(\begin{matrix} \end{matrix}\right)}}$ I Different

BB

I Different

$^2C_1 \quad ^2C_1$
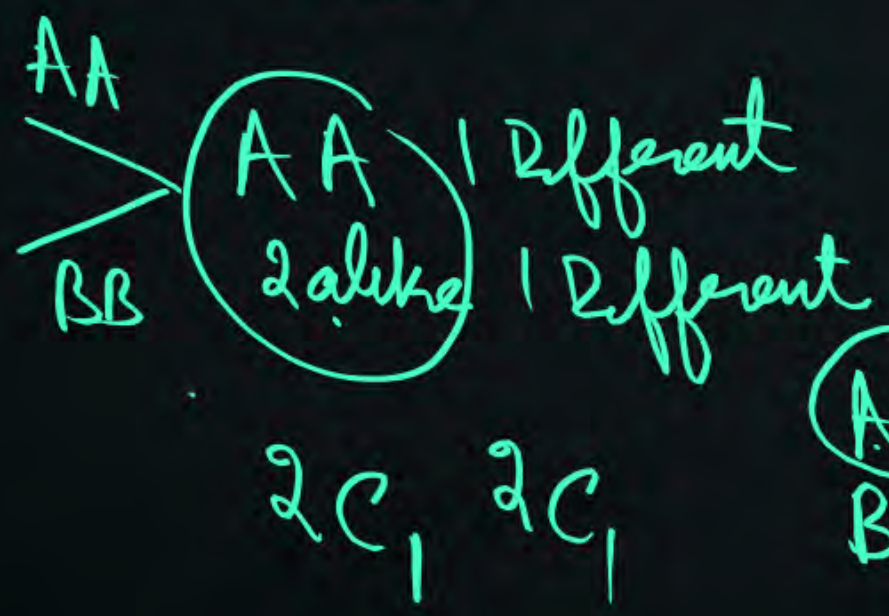
$\boxed{\dfrac{AAB}{BBC}} = \dfrac{\angle 3}{\angle 2 \angle 1}$

Case-3

All are Different

$\boxed{ABC}$ $1 \times \angle 3$

$= ⑥$

$= 18 \text{ ways}$

Arrangement = [Selection] × [Order]

$$= {}^nC_r \times r!$$

$$= \boxed{{}^nP_r}$$

AABBBC

n Different Items → Taken R at a time

✓ CASE ① 3 ball SAME colour

3 alike ⇒ ●●● $= 1 \times \dfrac{\lfloor 3}{\lfloor 3}$

Selection $\dfrac{}{\lfloor 3} = 1$ way

② 2 balls same + 1 different colour

colour $= 2C_1 \times 2C_1 \times \dfrac{\lfloor 3}{2!}$

$= 12$ ways.

③ All Are different colors

Total = 19 ways

③

BBB
●●●
●●● C
AA● → Taken
3 at a time

→ Identical balls.

= Selection ① × $\lfloor 3 = 6$.

2 alike ⇒ $\dfrac{\lfloor 3}{2!}$

BB Ⓒ Ⓐ
AA Ⓑ Ⓒ

#Q. How many four-letter words can be formed using the letters of the word

INEFFECTIVE

A) 840

B) 1380

C) 1422

D) None of these

II, N, FF, EEE, C, T, V,

EEE, II, FF, N, C, T, V,

$\rightarrow$ Taken 4 at time

A) 3 alike, 1 Diff EEE $= 1 \times 6C_1 = 6 \times \dfrac{4!}{3! \, 1!}$ 1 alike, 2 alike, 3 alike

$\rightarrow 6C_1$

B) 2 alike, 2 diff $\dfrac{EE/II/FF}{} = 3C_1 \times 6C_2 \times \dfrac{\angle 4}{\angle 2}$

C) 2 alike, 2 alike $= 3C_2 \times \dfrac{\angle 4}{\angle 2 \angle 2}$

D) 4 Different $= 7C_4 \times 4!$

$= 1422$

EEE ⟨ N C T I V E

$$= 1 \times {}^6C_1 \times \frac{\lfloor 4}{\lfloor 3 \lfloor 1}$$

$$= 3C_2 \times \frac{\lfloor 4}{\lfloor 2 \lfloor 2}$$

$$= 3C_1 \times 6C_2 \times \frac{\lfloor 4}{\lfloor 2}$$

$$= {}^7C_4 \times 4!$$

3 alike 1 Diff. = Select $\times$ order

2 alike 2 alike = Select $\times$ order

2 alike 2 diff. = Select $\times$ order

all Diff = Select $\times$ order

$$= 1422$$

<u>INEFFECTIVE</u>

EEE
II
FF
N
C
T
V

#Q. In how many ways 3 letters can be selected from letter A,A,B,B,B,C.

Try yourself

$\boxed{A A B B B C}$

#Q. In how many ways 4 letters can be selected from letter of the word 'INFEFECTIVE'?

(A) 80

(B) 89

(C) 51

(D) None of these

✓ 3 alike 1 diff
✓ 2 alike 2 diff
✓ 2 alike, 2 alike
✓ 4 Different

$= \underline{89}$

INFEFECTIVE

FF   N
EEE   C
II   T
      V

$\underline{89}$

Slide : 5

#Q. A man has 5 friends. In how many ways can be he invite one or more of them to a party?

(A) 32

(B) 31

(C) 30

(D) 16

$A \begin{cases} \\ \end{cases}$ friends 1

$B \begin{cases} \\ \end{cases} 2$

$C \begin{cases} \\ \end{cases} 3$

$D \begin{cases} \\ \end{cases} 4$
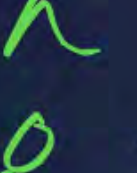
$E \begin{cases} \\ \end{cases} 5$

Order does Not matter

A, B, C

$\begin{bmatrix} A. \checkmark & {}^5C_1 + \\ A, B \checkmark & {}^5C_2 + \\ \textcircled{A}, \textcircled{B}, \textcircled{C} \checkmark & {}^5C_3 + \\ A, B, C, D \checkmark & {}^5C_4 + \\ A B, C, D, E & {}^5C_5 \end{bmatrix}$ Working Idiv.

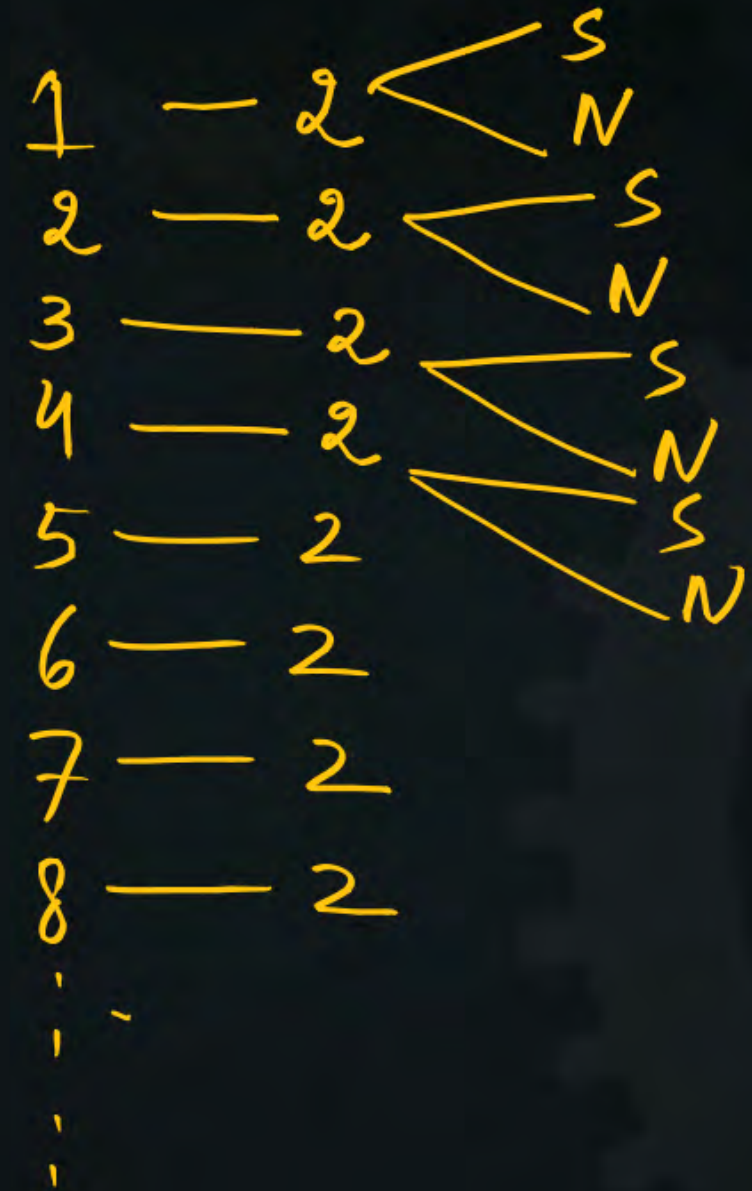$$\text{Total No. of ways} = {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$$
$$= \textcircled{31}$$

n Different Items — at least one select

$$\checkmark = \frac{{}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 + {}^nC_5 + \cdots + {}^nC_n}{}$$

$$= 2^n - 1 \qquad n = \text{Different Items}$$

$$\checkmark \boxed{\begin{array}{l} n \text{ Different Items} \\ 0 \text{ or more} \\ \quad \text{Select} \end{array} = {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n \\ \qquad\qquad = 2^n}$$

✓ n Different objects

0 or more select

$$1 - 2 \begin{cases} S \\ N \end{cases}$$
$$2 - 2 \begin{cases} S \\ N \end{cases}$$
$$3 - 2 \begin{cases} S \\ N \end{cases}$$
$$4 - 2 \begin{cases} S \\ N \end{cases}$$
$$5 - 2 \begin{cases} S \\ N \end{cases}$$
$$6 - 2$$
$$7 - 2$$
$$8 - 2$$
$$\vdots$$
$$n - n \text{ times}$$
$$= 2^n \text{ times}$$

#Q. The question paper in the examination contain three sections- *A*, *B*, *C*. Therefore are 6, 4, 3 questions in sections *A*, *B*, *C* respectively. A student has the freedom to answer any number of question attempting at least one from each section. In how many ways can the paper be attempted by a student ?
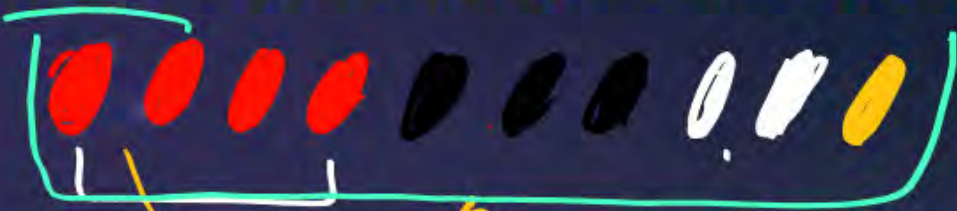
A 8192

SAME Idea
n Diff.
1 or more
Select

→ H·W

B 6615

C 7168

D None of these

#Q. In a box there are 10 balls; 4 red, 3 black, 2 white and 1 yellow. In how many ways can a child select 4 balls out of these 10 balls? (Assume that the balls of the same color are identical)

A 20

B 18

C 19

D 19

1) $\Rightarrow$ all alike $\rightarrow {}^1C_1 = 1$

2) 3 alike 1 Diff $\rightarrow {}^2C_1 \times {}^3C_1 = 2 \times 3 = 6$

$\rightarrow$ 4 balls selected

3) 2 alike, 2 alike $\rightarrow {}^3C_2 = \dfrac{3 \times 2}{2 \times 1} = 3$

All Balls Are Identical Taken R at a time

4) 2 alike, 2 diff $\rightarrow {}^3C_1 \times {}^3C_2 = 9$

5) all Different $\rightarrow {}^4C_4 = 1$

Total $= 6 + 3 + 9 + 1 + 1 = 18 + 1 = 20$

Lot

${}^3C_1$

#Q. **Illustrating the Concepts :**

A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be selected so that are at least two balls of each colour ?

H.W

#Q. In how many ways a team of 5 members can be selected from 4 ladies and 8 gentlemen such that selection includes at least 2 ladies ?

✓ combinations   ↑ H.W

A 336

B 448

C 449

D 456