Data Science and Artificial Intelligence

Probability and Statistics

Introduction to Probability

Lecture No.- 05

By- Rahul Sir

Recap of Previous Lecture

Topic — Problem Based on Events ✓ Done

Topic — Conditional Probability + Bayes THEOREM

Challenging Problem
01
02

Topics to be Covered

Telegram
Esp — Data Science
Sunday — Data
2 Hrs

Postcard

- **Topic** Bayes Theorem
- **Topic** Problem Based on Bayes Theorem

Phython
1-Hrs

✓ Seldom Ross.
✓ Miller freuend
✓ Moods and graybill.
✓ Ant. Popoulis

✓ Gupta Kapoor.
✓ T. Veerarajan.

Conditional Probability:

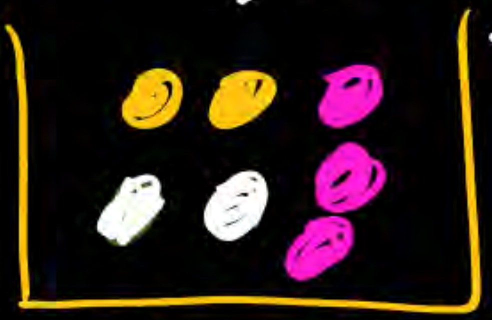
Relative Frequency / Brownian motion $\rightarrow \frac{n(E)}{n(S)}$

What is the Prob. 2 White Ball Are Drawn at random one at a time)

Past

stage 0

Ist white $P(W_1) = \frac{3}{8}$



2nd draw $P\left(\frac{W_2}{W_1}\right) = \frac{2}{7}$

W_1

W_2 working Together

$P(W_1 \cap W_2)$

$P(W_1 \cap W_2) = P(W_1) \boxed{P\left(\frac{W_2}{W_1}\right)}$ conditional prob.

W_1 is condition given.

occured

given condition Present

stage 1

$P\left(\frac{W_2}{W_1}\right)$ W_1 is already occured

$\frac{W_2}{W_1}$ is occuring.

Conditional Prob.

$$P\left(\frac{W_2}{W_1}\right) = \frac{P(W_1 \cap W_2)}{P(W_1)}$$ conditional prob. $P(W_1) \neq 0$

OR $$P\left(\frac{W_1}{W_2}\right) = \frac{P(W_1 \cap W_2)}{P(W_2)}$$ $P(W_2) \neq 0$

without Replacent

White 2 $\rightarrow$ B White 1 $\rightarrow$ A

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$ $P(A) \neq 0$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$ $P(B) \neq 0$

Conditional prob.
OR
Relative Prob.

Topic : Problem Based on Bayes Theorem

Bag choose

& Tkt drawn

Imp.

Q1. A bag contains 4 ticket numbers (1, 2, 3, 4) and another bag contain 6 ticket numbers (2, 4, 6, 7, 8, 9). $\boxed{\text{One bag is chosen,}}$ and ticket is drawn. The probability that the ticket bears the number 4 is ______

$$\text{Total prob} = \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6} = \frac{5}{24}$$

Dependent events

Box1	Box2
1, 2, 3, 4	2, 4, 6, 7, 8, 9
4: $\frac{1}{4}$	$\frac{1}{6}$

START

$P(\text{bag }1)$ $\qquad$ $P(\text{bag }2)$

(E_1) Bag 1 $\qquad$ Bag 2 (E_2)

$P\left(\frac{\text{Tkt on }4}{\text{bag }1}\right)$ $\qquad$ $P\left(\frac{\text{Tkt on }4}{\text{bag }2}\right)$

Tkt on 4 (A)

$P(A)$

$P(\text{Target})$

"Bag is already occured $\frac{\text{Tkt on }4}{\text{bag }1}$ is occuring"

$$P(\text{Target}) = \underset{E_1}{P(\text{bag }1)}\,P\left(\frac{\overset{A}{\text{Tkt on }4}}{E_1\ \text{bag }1}\right) + \underset{E_2}{P(\text{bag }2)}\,P\left(\frac{\overset{A}{\text{Tkt on }4}}{\underset{E_2}{\text{bag }2}}\right)$$

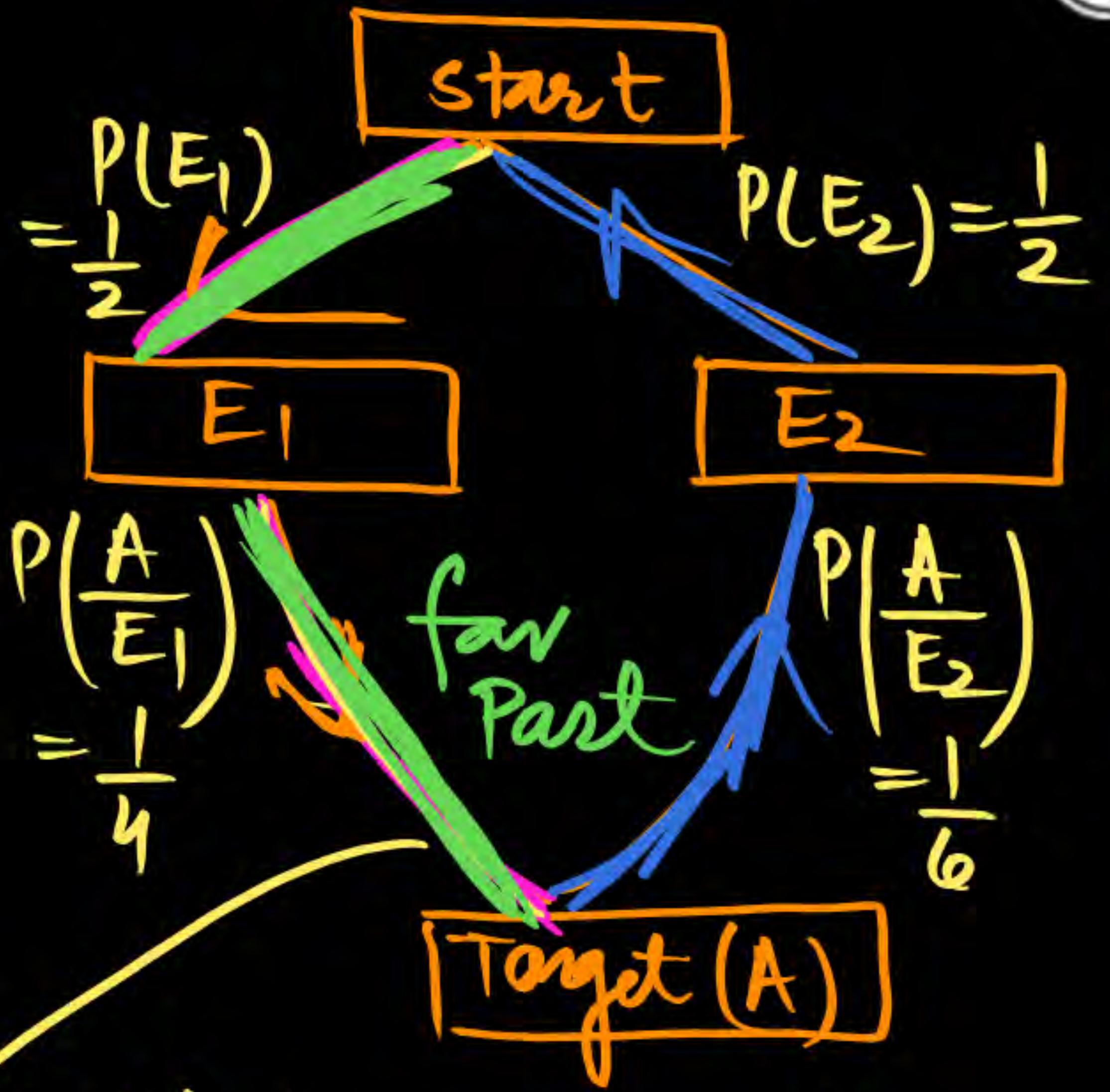
$$\boxed{P(A) = P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)}$$

→ Total Prob. for Dependent events

$$P(A) = P(\text{Target}) = P(\text{Tkt on } 4)$$

$$P(A) = P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)$$

$$P(A) = \frac{1}{2}\times\frac{1}{4} + \frac{1}{2}\times\frac{1}{6} = \frac{5}{24} \checkmark$$



✓ # $$P\left(\frac{\text{bag } 1}{\text{Tkt on } 4}\right) = \frac{\text{fav outcomes}}{\text{Total outcomes}}$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)}$$

$$P\left(\frac{\text{bag } 1}{\text{Tkt on } 4}\right) = \text{Inverse Prob.}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\,P\left(\frac{A}{E_2}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)}$$

Inverse Prob.

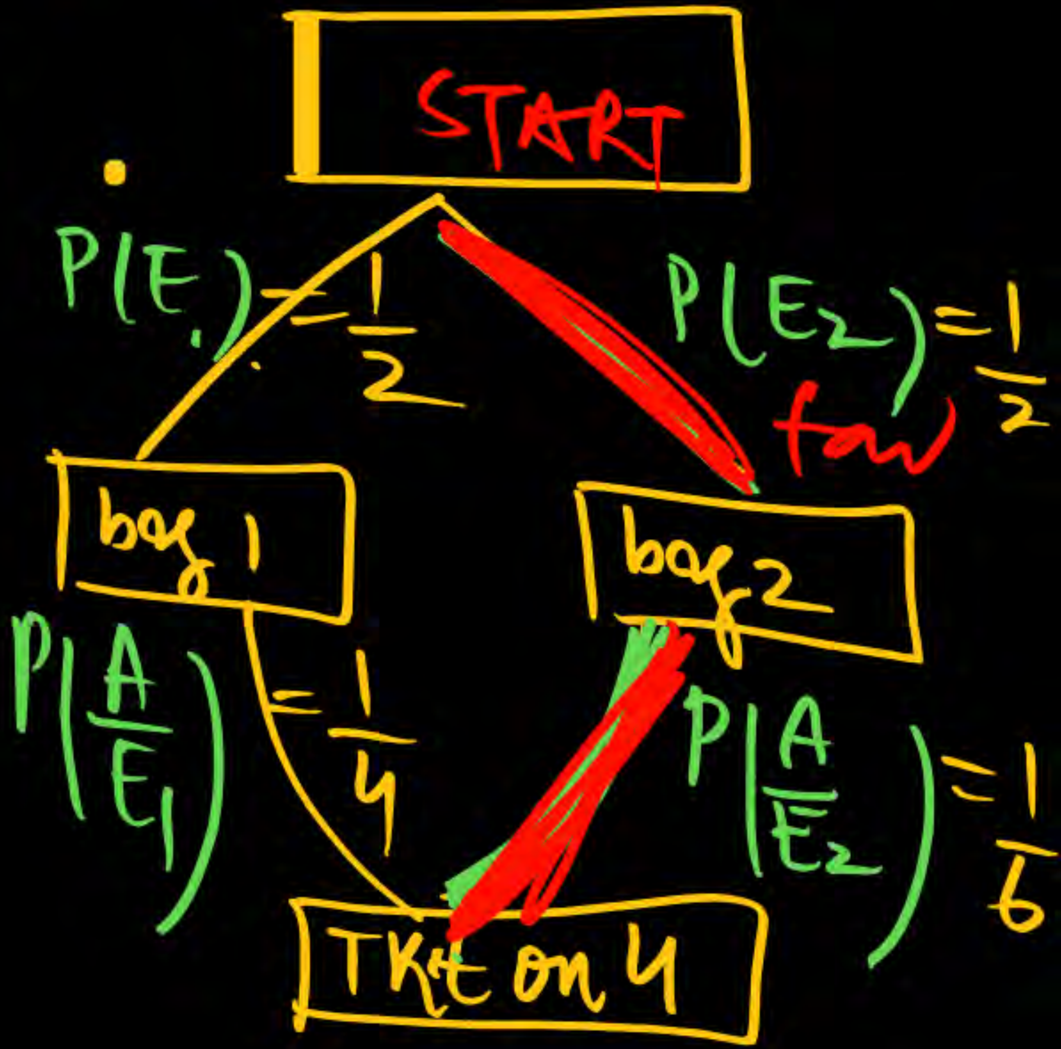
↑ Inverse Prob.

Bayes THEOREM

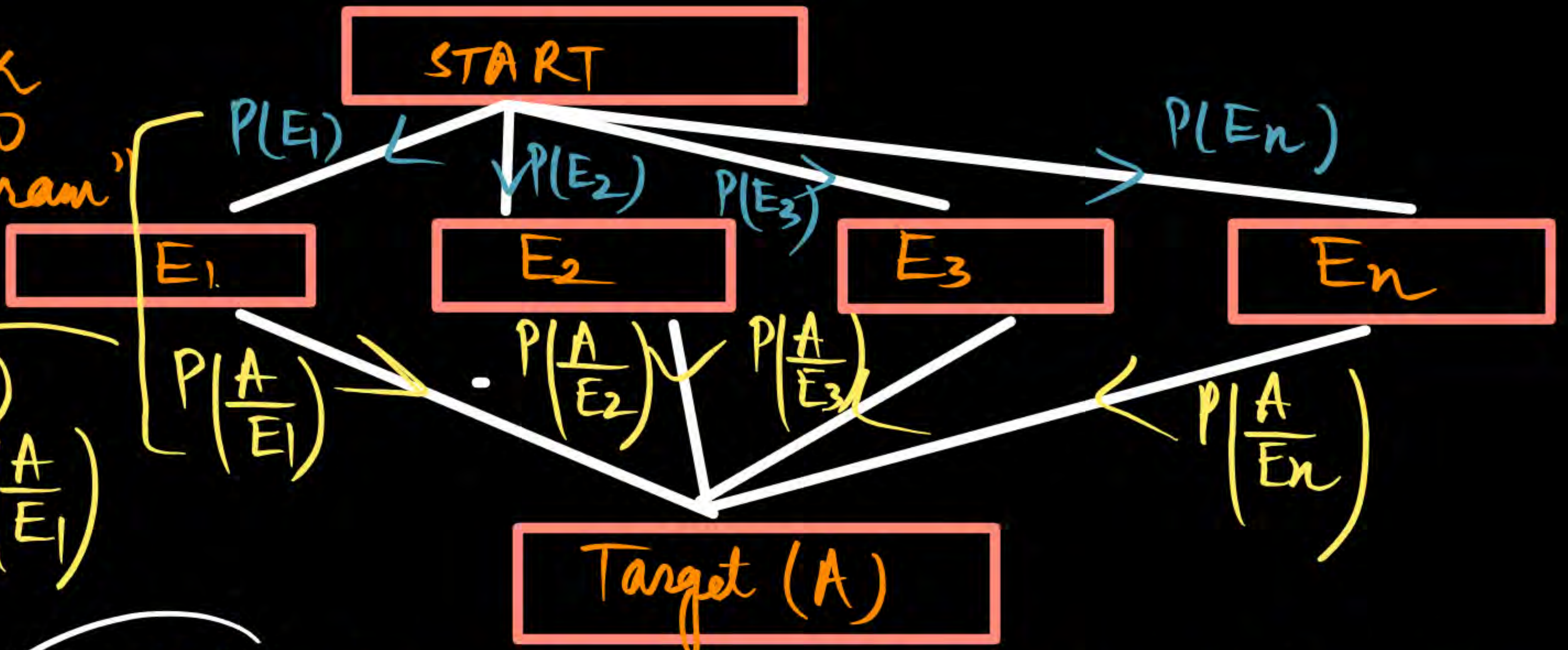
$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{2} \times \frac{1}{4}}{\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6}} = \frac{\frac{1}{8}}{\frac{5}{24}}$$

$$P\left(\frac{E_1}{A}\right) = \frac{3}{5}$$

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{1}{2} \times \frac{1}{6}}{5/24} = \frac{2}{5}$$



"Work Flow Diagram"



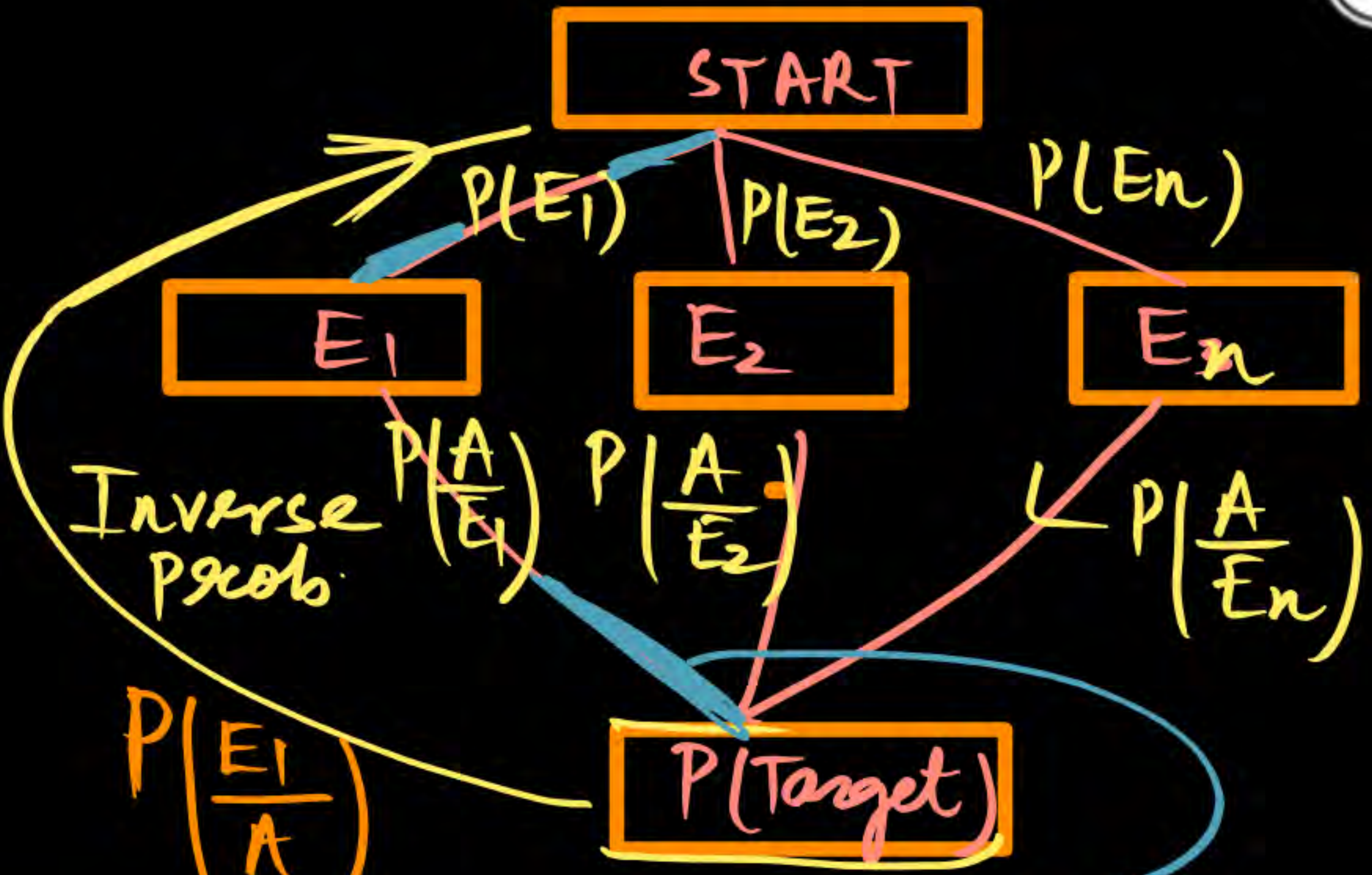
$$P(A \cap E_1) = P(E_1)\,P\left(\frac{A}{E_1}\right)$$

Total prob.

$$P(\text{Target}) = P(A) = P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right)$$

$$P\left(\frac{E_1}{A}\right) = \text{Inverse prob.}$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\, P\left(\frac{A}{E_1}\right)}{\sum_{i=1}^{n} P(E_i)\, P\left(\frac{A}{E_i}\right)}$$



$$P\left(\frac{E_1}{A}\right)$$

for general Rule.

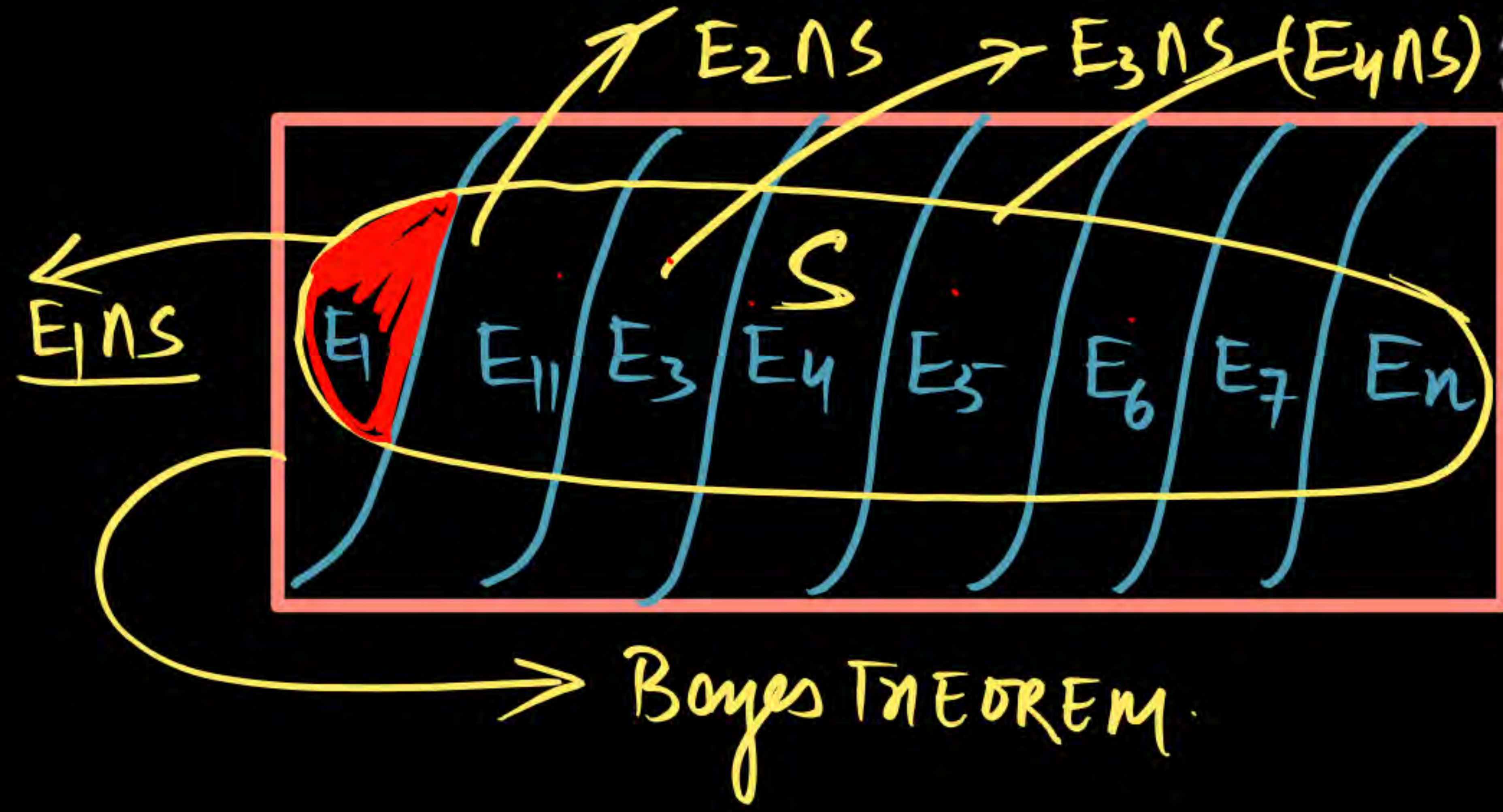
Bayes. THEOREM

$$\boxed{P\left[\frac{E_i}{A}\right]_{i=1 \text{ to } n} = \frac{P(E_i)\, P\left(\frac{A}{E_i}\right)}{\sum_{i=1}^{n} P(E_i)\, P\left(\frac{A}{E_i}\right)}}$$

for Path

$E_1, E_2, E_3 - -$

mutually exclusive events.



$E_2 \cap S$
$E_3 \cap S$
$(E_4 \cap S)$
$E_1 \cap S$
S
E_1
E_{11}
E_3
E_4
E_5
E_6
E_7
E_n
Bayes Theorem

Topic : Problem Based on Bayes Theorem

Q2. Three Boxes B_1 B_2 B_3 contains balls

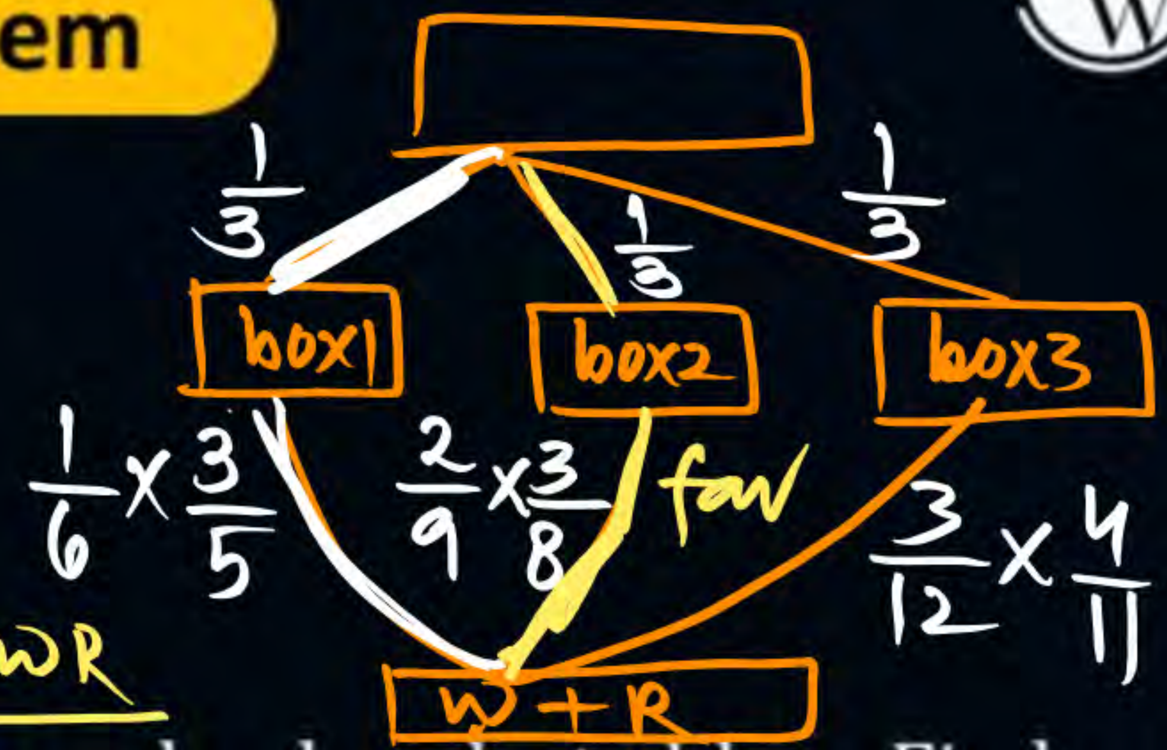
1W 2B 3R

B_1

$\frac{1}{6} \times \frac{3}{5}$

$$\begin{Bmatrix} B_1 \rightarrow 1W, 2B, 3R \\ B_2 \rightarrow 2W, 4B, 3R \\ B_3 \rightarrow 3W, 5B, 4R \end{Bmatrix}$$

RW, WR



Without replacement, if 2 balls are drawn from randomly selected box. Find the probability one of the ball drawn is white and other ball is red $\boxed{\text{from box 2}}$ order is specified.

(W + R)

2W 4B 3R $\quad \frac{2}{9} \times \frac{3}{8}$

3W 5B 4R $\rightarrow \frac{3}{12} \times \frac{4}{11}$

$\frac{55}{181}$

$$= \frac{\frac{1}{3} \times \frac{2}{9} \times \frac{3}{8}}{\frac{1}{3} \times \frac{1}{6} \times \frac{3}{5} + \frac{1}{3} \times \frac{2}{9} \times \frac{3}{8} + \frac{1}{3} \times \frac{3}{12} \times \frac{4}{11}}$$

Topic : Problem Based on Bayes Theorem

Q3. One ticket is selected at random from 100 ticket 00, 01, 02, ...99. Suppose A and B are the sum and product of digits found on the ticket. Then $P\left[\frac{A=7}{B=0}\right]$ is given by

A. 2/13

B. 2/19

C. 1/50

D. None of these

$A = \text{Sum} \quad B = \text{Product}$

$A = x+y \quad B = x \cdot y$

$P\left[\frac{A=7}{B=0}\right]$ = conditional Prob.

$P\left[\frac{A=7}{B=0}\right] = \frac{P[A \cap B]}{P(B)} = \frac{\frac{n(A\cap B)}{\cancel{n(S)}}}{\frac{n(B)}{\cancel{n(S)}}}$

$= \frac{n(A\cap B)}{n(B)}$

S, P, P

00, 01, 02, ---- 99

00, 01

x y

Slide 6

Common
Den

$$P\left(\frac{A=7}{B=0}\right) = \frac{n(A \cap B)}{n(B)}$$

$$= \frac{2}{19}$$

$$\boxed{P\left(\frac{A=7}{B=0}\right) = \frac{2}{19}}$$

SUM = 7 and Product = 0

00
01
02
03
04
05
06
07
08
09
10
20
30
40
50
60
70
80 — 90

00, 01
02, 03
04, 05 - -

7, 0 = 07
7, 0 70

34
61
16
43
52
25

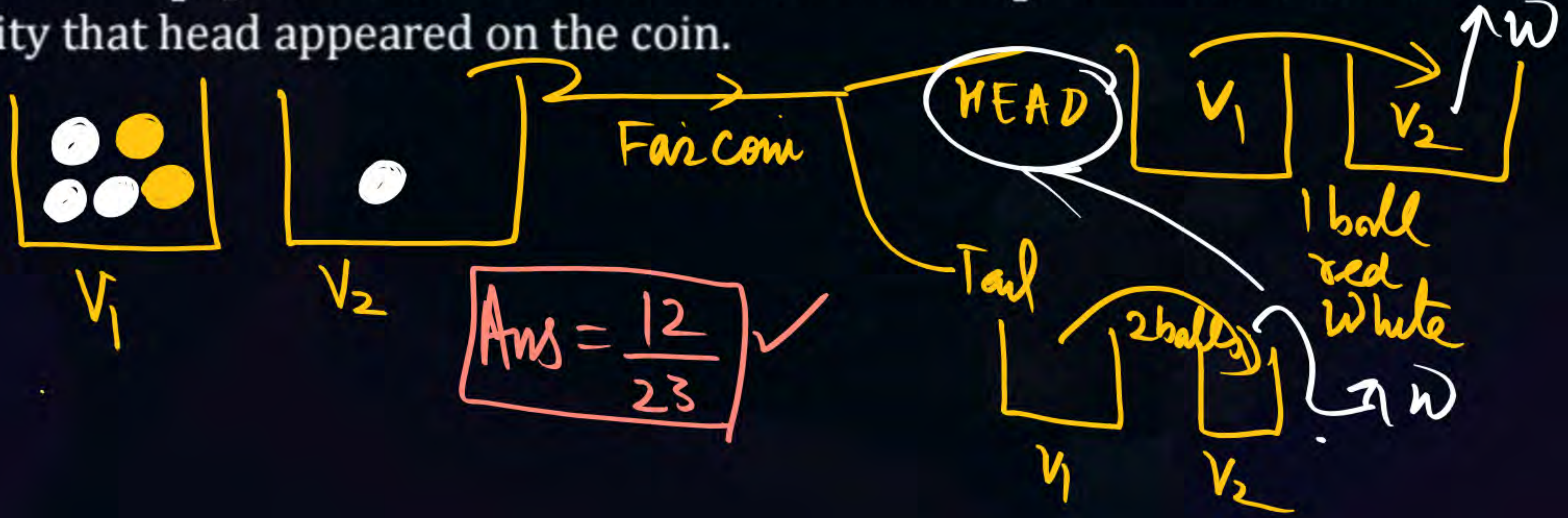
SUM
product ≠ 0

Product = 0

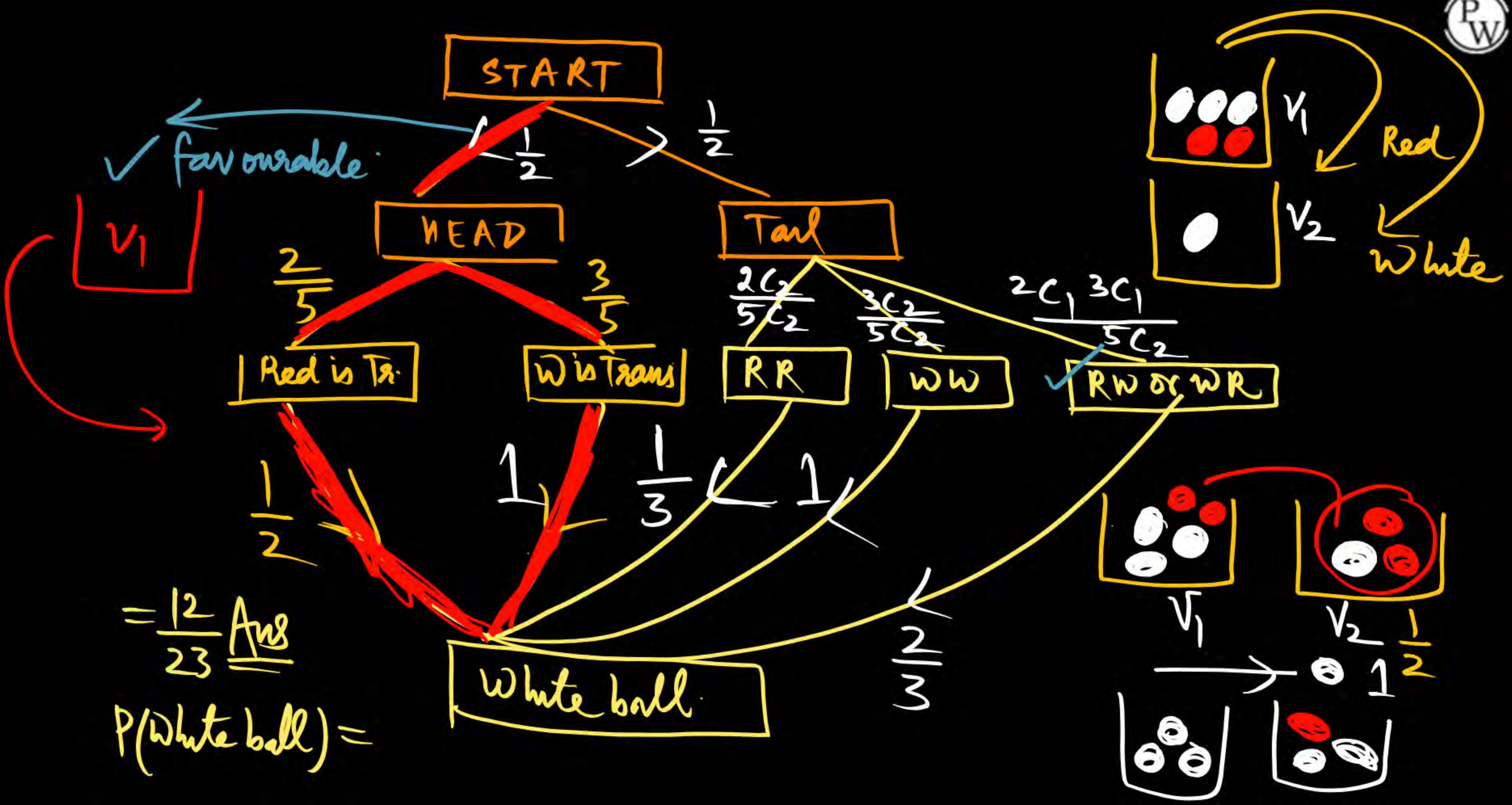
Topic : Problem Based on Bayes Theorem

$\frac{12}{23}$

Q4. Let V_1 and V_2 be two urns box such that v_1 contains 3 white and 2 Red balls and v_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from v_1 and put into v_2. However, If tail appears then 2 balls are drawn at random from v_1 and put into v_2. Now one ball is drawn at random from v_2 given that the drawn ball from v_2 is white then the probability that head appeared on the coin.



$\boxed{\text{Ans} = \frac{12}{23}}$ ✓



$$P(\text{White ball}) = \frac{12}{23}\ \text{Ans}$$

Q5. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. Then probability that a child picked at random, has a sibling is ______.

Doyourself

Topic : Problem Based on Bayes Theorem

Q9. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3, or 5 is ______.

Do yourself

Topic : Problem Based on Bayes Theorem

Q10. If $P(X) = 1/4$, $P(Y) = 1/3$, and $P(X \cap Y) = 1/12$, then value of $P(Y/X)$ is

— Do yourself

A. $1/4$

B. $4/25$

C. $1/3$

D. $29/50$

Topic : Problem Based on Bayes Theorem

Q13. The probabilities of occurrence of events F and G are $P(F) = 0.3$ and $P(G) = 0.4$ respectively. The probability that both events occur simultaneously is $P(F \cap G) = 0.2$. The probability of occurrence of at least one event $P(F \cup G)$ is ______.

✓ Do yourself

THANK - YOU