Data Science and Artificial Intelligence

Probability and Statistics

Introduction to Probability

Lecture No.- 02

Recap of Previous Lecture

Topic Introduction to Probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{favourable}}{\text{Total}} = \text{Relative Frequency.}$$

Topics to be Covered

Topic Problems based on Basic Probability

Topic : Basic Probability

2-3 min

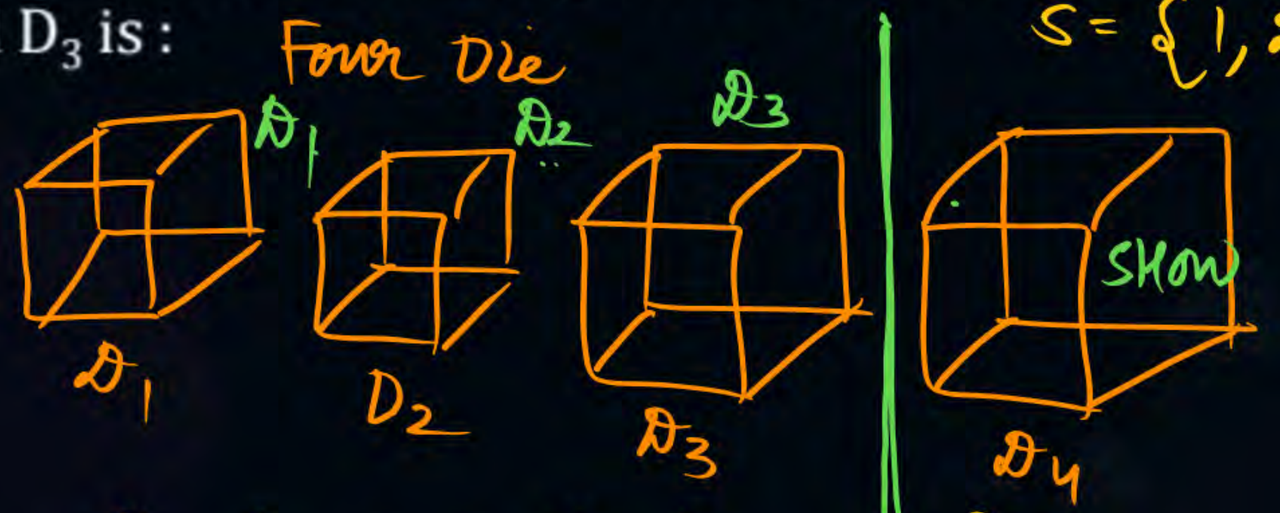
Q1. Four fair dice D_1, D_2, D_3, D_4, each having six faces numbered 1, 2, 3, 4, 5, 6 are rolled simultaneously. The Probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is :

A. ✓ 91/216

B. 108/216

C. 125/216

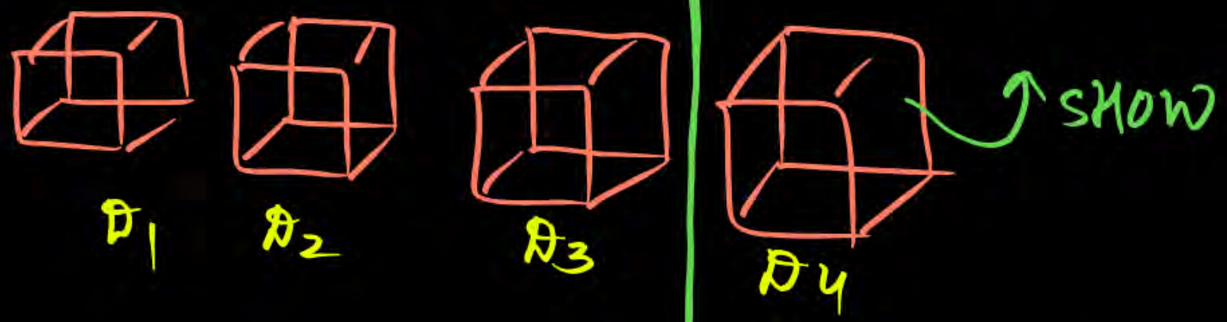
D. 127/216



$S = \{1, 2, 3, 4, 5, 6\}$

D_4 shows a Number ← D_1, D_2, D_3 already appeared

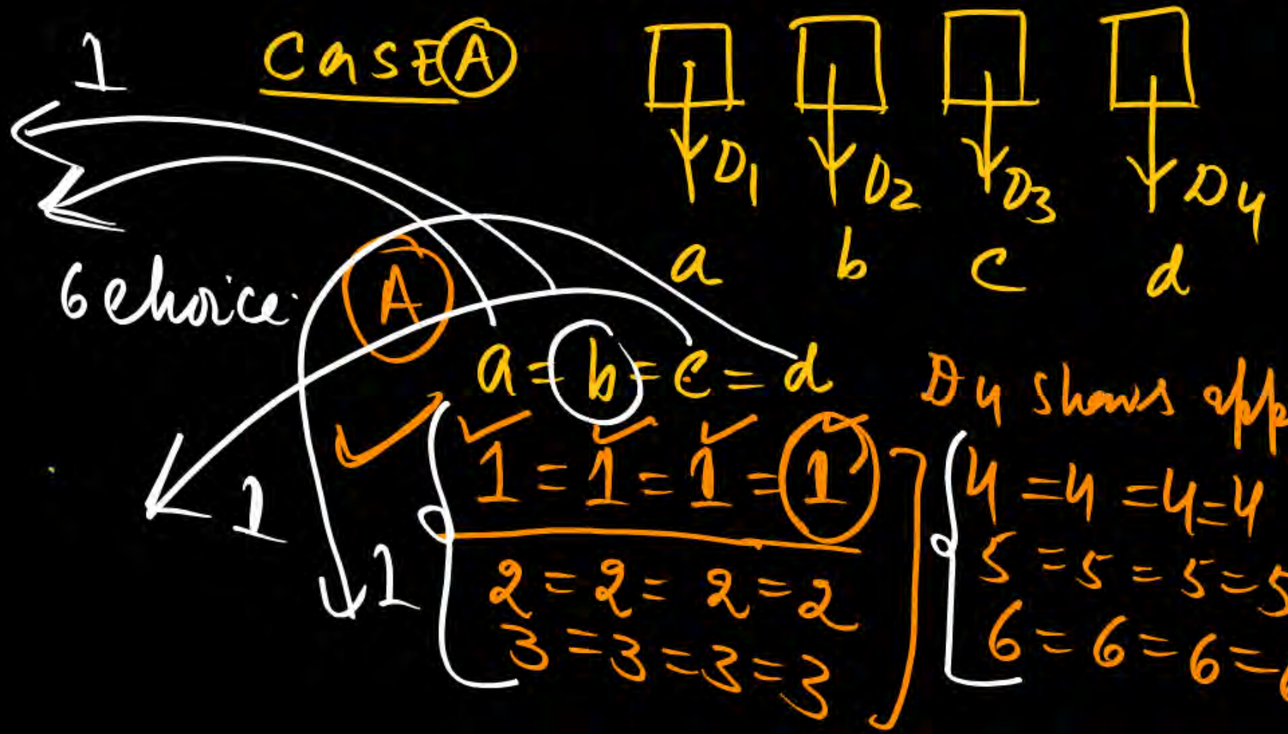
D_1 D_2 D_3	D_4
(5)	5



$a = D_1$ $c = D_3$
$b = D_2$ $d = D_4$

$$\text{Prob}(E) = \frac{n(E)}{n(S)}$$

Total choices $= 6 \times 6 \times 6 \times 6 = 6^4$



No. of ways $\Rightarrow$

$\Rightarrow 6 \times 1 \times 1 \times 1$

$= \underline{6 \text{ choices}}$

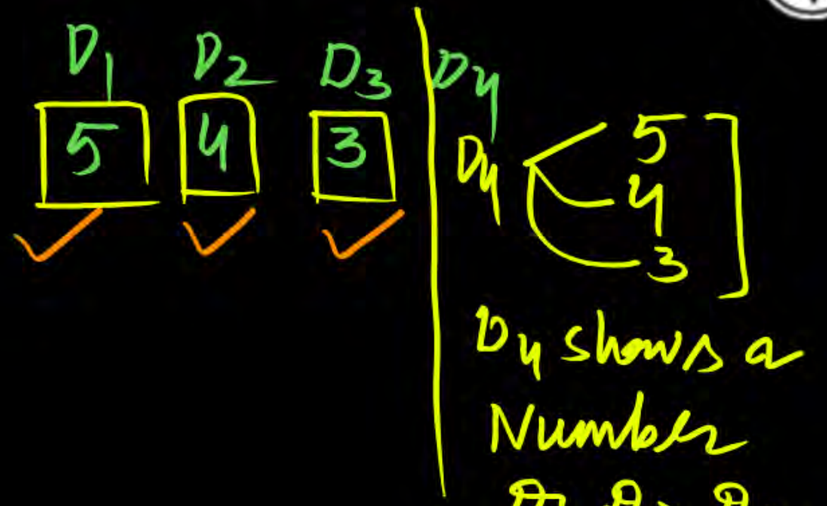
CASE ② $a \neq b \neq c$

$d \leftarrow a, b, c$

Total No. of choices $= 6 \times 5 \times 4 \times 3$

$= 30 \times 12$

$= 360$



CASE 03 ↖

$a = b \neq c$ $\quad d \leftarrow a, c$

$b = c \neq a$ $\quad d \leftarrow a, b$

$c = a \neq b$ $\quad d \leftarrow c, b$

$= 6 \times 1 \times 5 \times 2 \times 3$

$= 90 \times 2 = 180$

$D_1 = D_2 \neq D_3$

$D_2 = D_3 \neq D_1$

$D_3 = D_1 \neq D_2$

$5 = 5 \neq 6$ $\quad D_4 \leftarrow a, c$

$$P(E) = \frac{6+360+180}{6^4} = \frac{91}{216}$$ Ans

$D_1 = D_2 = D_3 = D_4$

$= \boxed{a=b=c=d} + \boxed{a\neq b\neq c \; d \leftarrow \begin{matrix} a \\ b \\ c \end{matrix}} + a=b\neq c$

$\checkmark\checkmark 1 \checkmark \checkmark$

$\checkmark \boxed{D_1=D_2=D_3=D_4}$

$D_1\neq D_2\neq D_3 \; D_4 \leftarrow \begin{matrix} D_1 \\ D_2 \\ D_3 \end{matrix}$

$D_1=D_2\neq D_3 \; D_4 \leftarrow \begin{matrix} D_1 \\ D_2 \end{matrix}$

$D_1=D_3\neq D_2 \; D_4 \leftarrow \begin{matrix} D_1 \\ D_3 \end{matrix}$

$D_2=D_3\neq D_1 \; D_4 \leftarrow \begin{matrix} D_2 \\ D_3 \end{matrix}$

$$\frac{6\times1\times1\times1}{6\times6\times6\times6} + \frac{6\times5\times4}{6\times6\times6\times6}\times3 + \frac{6\times1\times5\times2}{6\times6\times6\times6}\times3 = \text{Ans}$$

Topic : Basic Probability

$\frac{62}{100}$ final Answer.

Q2. If P and Q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} with replacement. Determine the probability that the roots of the equation $x^2 + px + q = 0$ are real.

$x^2 + px + q = 0$

If Roots Are Real

$D \geq 0 \quad b^2 - 4ac \geq 0$

If Roots Are complex

$D < 0 \quad b^2 - 4ac < 0$

P → 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

q → 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Roots Are Real

$P[p^2 - 4q \geq 0] = \frac{\text{No. of fav outcomes}}{\text{Total outcomes}}$

Total outcomes $= 10 \times 10$

$= 100$

$$P(p^2 - 4q \geq 0) = \frac{\bigcirc}{100}$$

p^2	1	4	9	16	25	36	49	64	81	100
$4q$	(4)	8	12	16	20	24	28	32	36	40

$n(p^2 - 4q \geq 0) = 0 + 1 + 2 + 4 + 6 + 9 + 10 + 10 + 10 + 10$

$= 62$ choices

$$P(\text{Event}) = \frac{62}{100} = \underline{\text{Ans}}$$

p \ q	1	2	3	4	5	6	7	8	9	10
1	(1,1)	(1,2)	(1,3)	(1,4)						
2										
3										
4										
5										
6										
7										
8										
9										
10										

Topic : Basic Probability

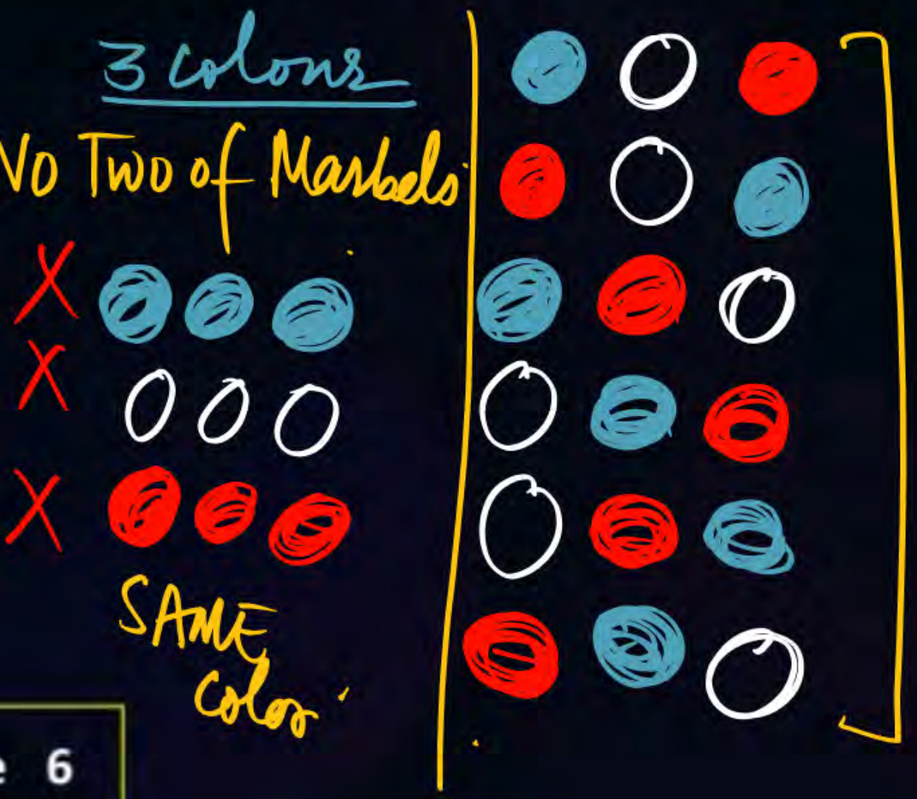
With Replacement

Q3. A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its colour is recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same colour___.

10 Blue
20 black
30 red
→ Bag.

3 colors

No Two of Marbels



SAME Color.

$$\Rightarrow \frac{10}{60} \times \frac{20}{60} \times \frac{30}{60} \times 6$$

$$= \frac{1}{6}$$ Ans

Topic : Basic Probability

(Kua) ✗ Pyasa

Q5. Twelve balls are distributed among three axes. The probability that the first box contains 3 balls is

A. $\frac{110}{9}\left(\frac{2}{3}\right)^9$

B. ✓ $\frac{9}{110}\left(\frac{2}{3}\right)^{10}$

C. $\frac{{}^{12}C_3}{12^3}.2^9$

D. $\frac{{}^{12}C_3}{3^{12}}$

Total outcomes
$= 3\times3\times3\times3\times3\times\text{--}$ 12 times
$= 3^{12}$

favourable outcomes $\Rightarrow {}^{12}C_3\ 2^9$

$$P(E) = \frac{{}^{12}C_3\ 2^9}{3^{12}}$$

Boxes

11 ⊔⊔ 4 ⊔⊔ 6 ⊔⊔
12 ⊔⊔ 5 ⊔⊔ 7 ⊔⊔
6 ⊔⊔ 8 ⊔⊔
9 ⊔⊔ 10 ⊔⊔

1 2 3 4 5 6 7 8 9 10 11 12

12 balls

B1 B2 B3

000

⊔⊔⊔ ⊔⊔⊔
⊔⊔⊔ ⊔⊔⊔
⊔⊔⊔ ⊔⊔⊔
⊔⊔⊔ ⊔⊔⊔
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⊔⊔⊔

Slide 8

Topic : Basic Probability

Q6. A cricket club has 15 members of whom only 5 can bowl. If the names of 15 members are put into a box and 11 are drawn at random. Then the probability of obtaining an eleven containing at least 3 bowlers is:

A. 7/13

B. 6/13

C. 11/15

D. 12/13

Do yourself

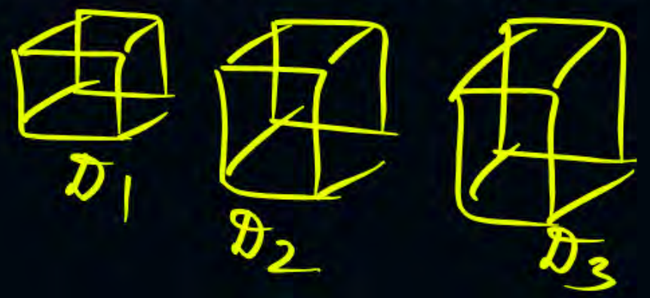
15 Members

5 can bowl

Topic : Basic Probability

THREE $D_1\ D_2\ D_3 \in \{1,2,3,4,5,6\}$

Q7. There six faced dice are tossed together, then the probability that exactly two of the three numbers are equal is:



$D_1 = D_2$ D_3 ✓
$D_2 = D_3$ D_1 ✓
$D_1 = D_3$ D_2 ✓

A. 165/216

B. 177/216

C. 51/216

D. ✓ 90/216

$$P(E) = \frac{6\times1\times5}{6\times6\times6} + \frac{6\times5\times1}{6\times6\times6} + \frac{6\times5\times1}{6\times6\times6}$$

$$= \frac{90}{216}$$

Topic : Basic Probability

Q11. Three chairs are arranged in a line, and three people randomly take seats. What is the probability that the person with the middle height ends up in the middle seat?

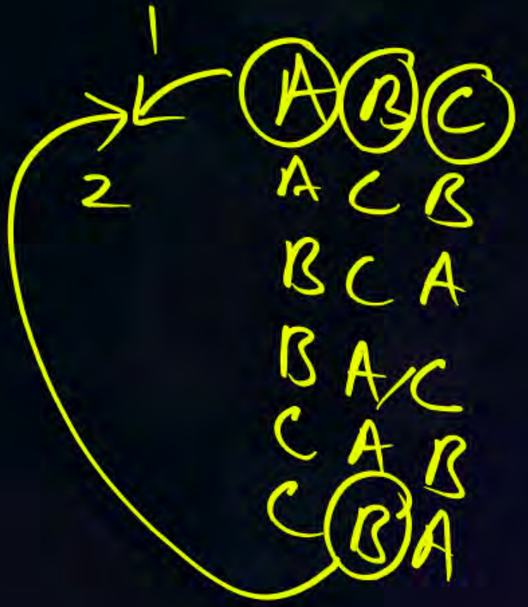
A (B) C

Person Increasing order

$$P(E) = \frac{\text{No. of favourable}}{\text{Total outcomes}}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$



Topic : Basic Probability

Q11. Six dice are rolled. What is the probability of getting three pairs, that is, three different numbers that each appear twice?

→ do yourself

Topic : Basic Probability

Q12. A coin is flipped five times. Calculate the probabilities of getting the various possible numbers of Heads (0 through 5).

→ Do yourself

Topic : Basic Probability

Q13. A drawer contains four blue socks and two red socks, as shown in fig. If you randomly pick two socks, what is the probability that you obtain a matching pair?

A box with four blue socks and two red socks.

→ Do yourself

Topic : Basic Probability

(1 x 1) square choose

Q14. 2 squares are chosen on a chessboard. Find the probability that they have a side in common.

$$P(E) = \frac{\text{favourable outcomes}}{\text{Total outcomes}}$$

n Different Items
Taken r at a time $= {}^nC_r$

$= \boxed{{}^{64}C_2} =$

favourable → | A | B | C | D | E | F | G | H |

AB CD EF GH = 7X8
BC DE FG

Total squares = 64

$$P(E) = \frac{n(E)}{n(S)}$$

Favourable outcomes $= 7 \times 8$ vertically

Total favourable outcomes $= 7 \times 8 + 7 \times 8$
$= 56 + 56$
$= 112$

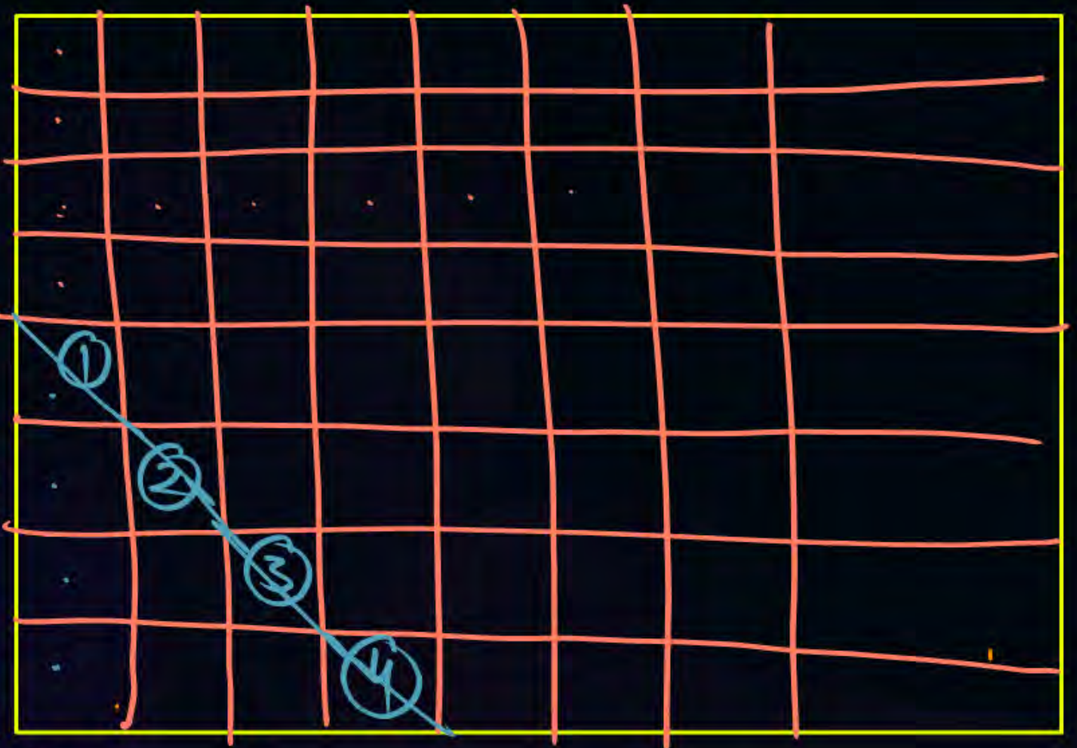
$$P(E) = \frac{n(E)}{n(S)} = \frac{112}{{}^{64}C_2}$$

Topic : Basic Probability

$2\left[2\left[{}^{4}C_4 + {}^{5}C_4 + {}^{6}C_4 + {}^{7}C_4 + {}^{8}C_4\right]\right]$ Favourable

(1 x 1) Squares

Q15. 4 squares are chosen at random on the chessboard. Find the probability that they lie along the same diagonal line.



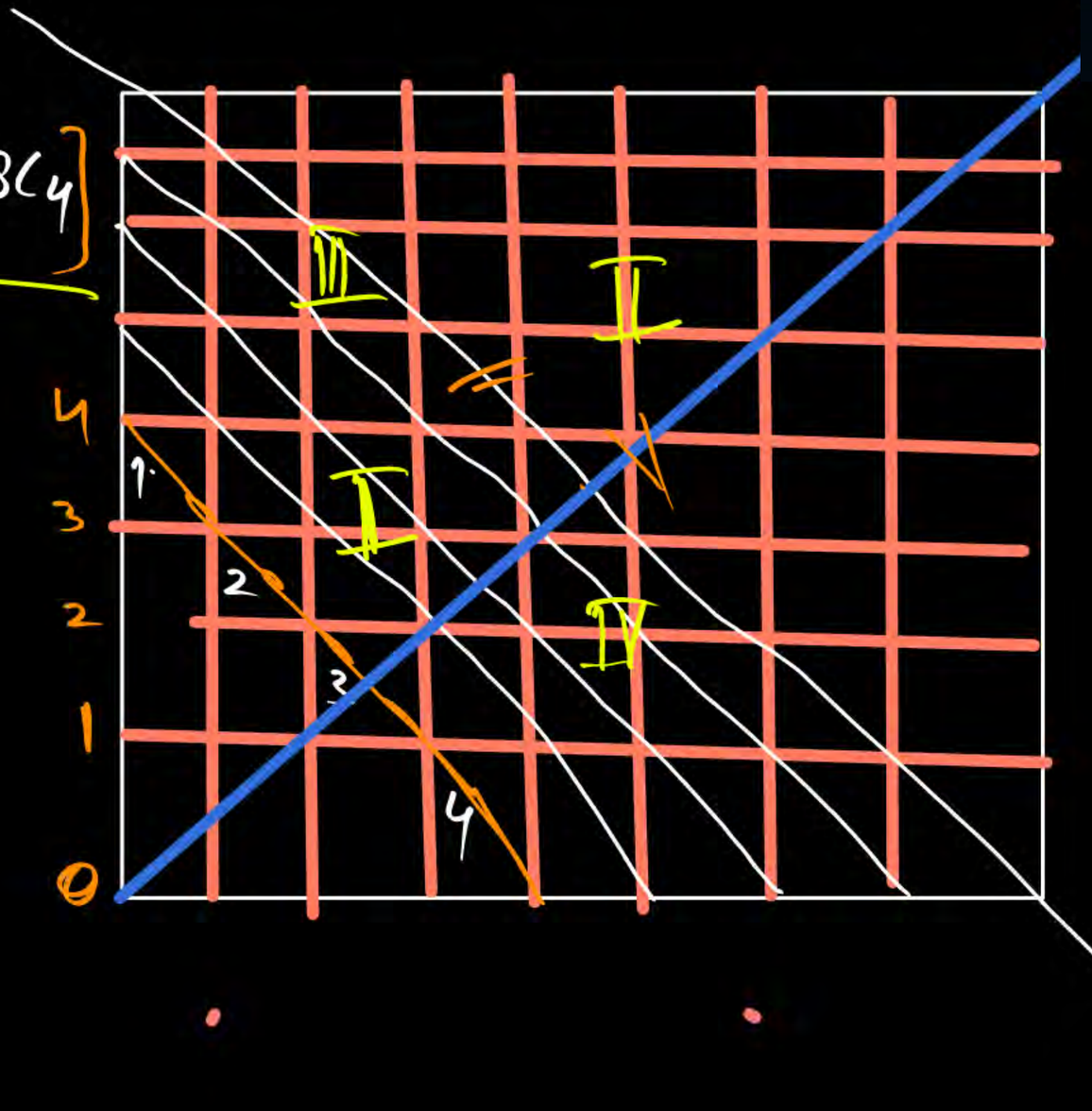
4 Squares and Diagonal ARE SAME

n Different Items and r is deleted

$= {}^{64}C_4$

$$P(E) = \frac{2\left[2\left[{}^{4}C_4 + {}^{5}C_4 + {}^{6}C_4 + {}^{7}C_4\right] + {}^{8}C_4\right]}{{}^{64}C_4}$$

Ans




Thank You