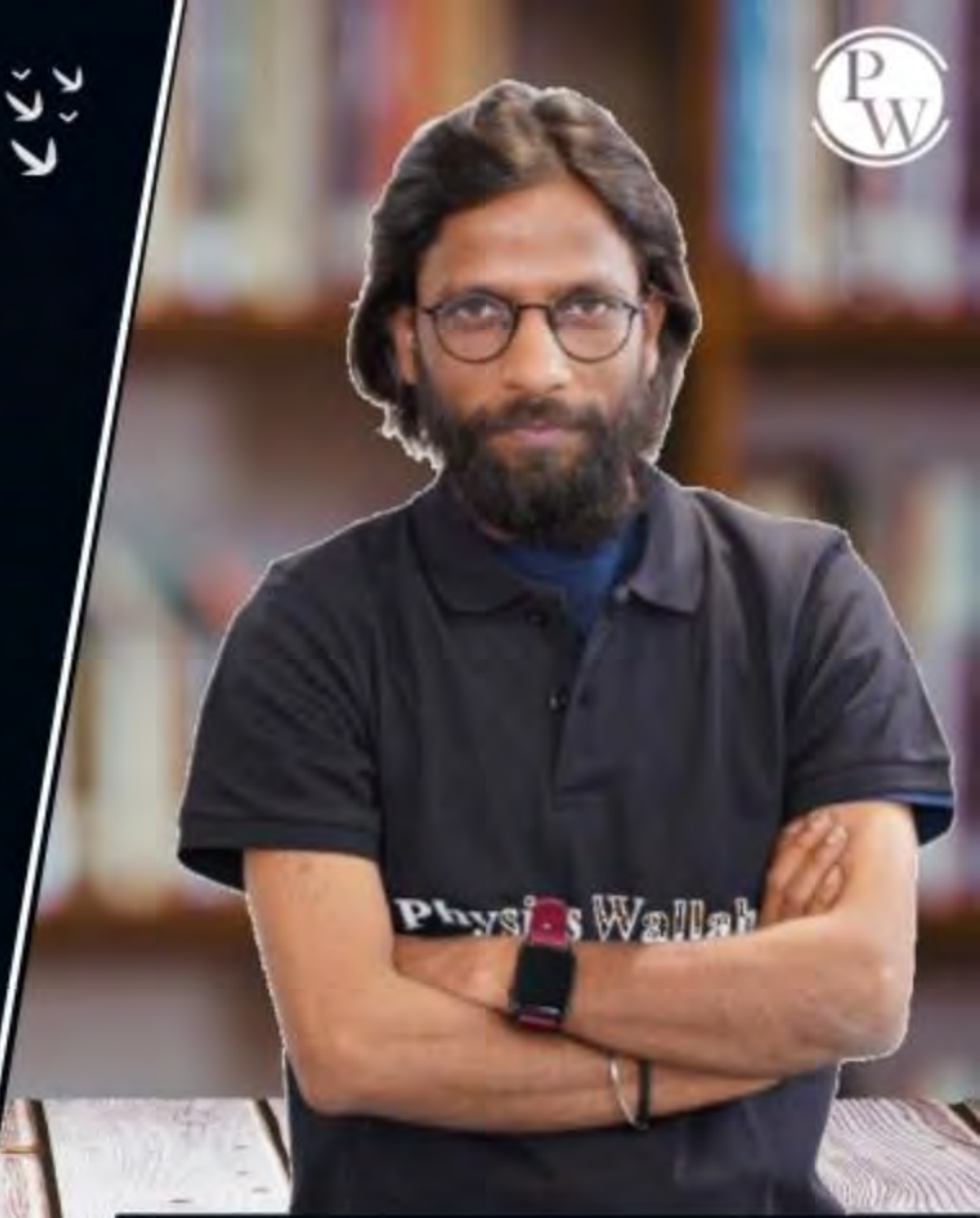
Data Science and Artificial Intelligence

Probability and Statistics

Continuous Probability Distribution

Lecture No.-03



By- Rahul Sir

Topics to be Covered

CHRIS
NAYAK
2 Hrs

Linear Algebra = vector form → machine Learning.

+ Linear Algebra (2 Hrs)

10 to 12

3rd october

Topic Problems based on Normal Distribution

Topic Problems based on Uniform Distribution

$$f(x) = \begin{cases} \frac{1}{10-2} & 2 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

Q1. A random variable is uniformly distribution over the interval 2 to 10. Its variance will be

$U[a, b]$ = Uniform Distribution

A. $\frac{16}{3}$

B. 6

C. $\frac{256}{9}$

D. 36

$$\text{var}(x) = \frac{(b-a)^2}{12}$$

$$\text{var}(x) = \frac{(10-2)^2}{12} = \frac{(8)^2}{12} = \frac{16}{3}$$

$$f(x) = \frac{1}{8}$$

$$v(x) = E[x^2] - [E[x]]^2$$

$$= \int_2^{10} \frac{1}{8} x^2 dx - \left[\int_2^{10} x \frac{1}{8} dx\right]^2$$

$$= \frac{16}{3} \text{ Ans}$$

$\rightarrow$ Rod of Length $\longrightarrow N(\mu, \sigma^2)$

Q6. The length of a large stock of titanium rods follow a normal distribution with a mean (μ) of 440 mm and a standard deviation (σ) of 1 mm. What is the percentage of rods $\boxed{\text{whose lengths lie between 438 mm and 441 mm?}}$

A. 81.85%

B. 68.4%

C. 99.75%

D. 86.64%

$P[438 \le X \le 441]$

$\mu = 440\,mm$

S.t dev $= 1\,mm$

$P[438 \le X \le 441]$

$= P\left[\frac{438-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{441-\mu}{\sigma}\right] = P\left[\frac{438-440}{1} \le \frac{X-\mu}{\sigma} \le \frac{441-440}{1}\right]$

$= P[-2 \le Z \le 1]$

{MESURE THEORY Infinite Theory}

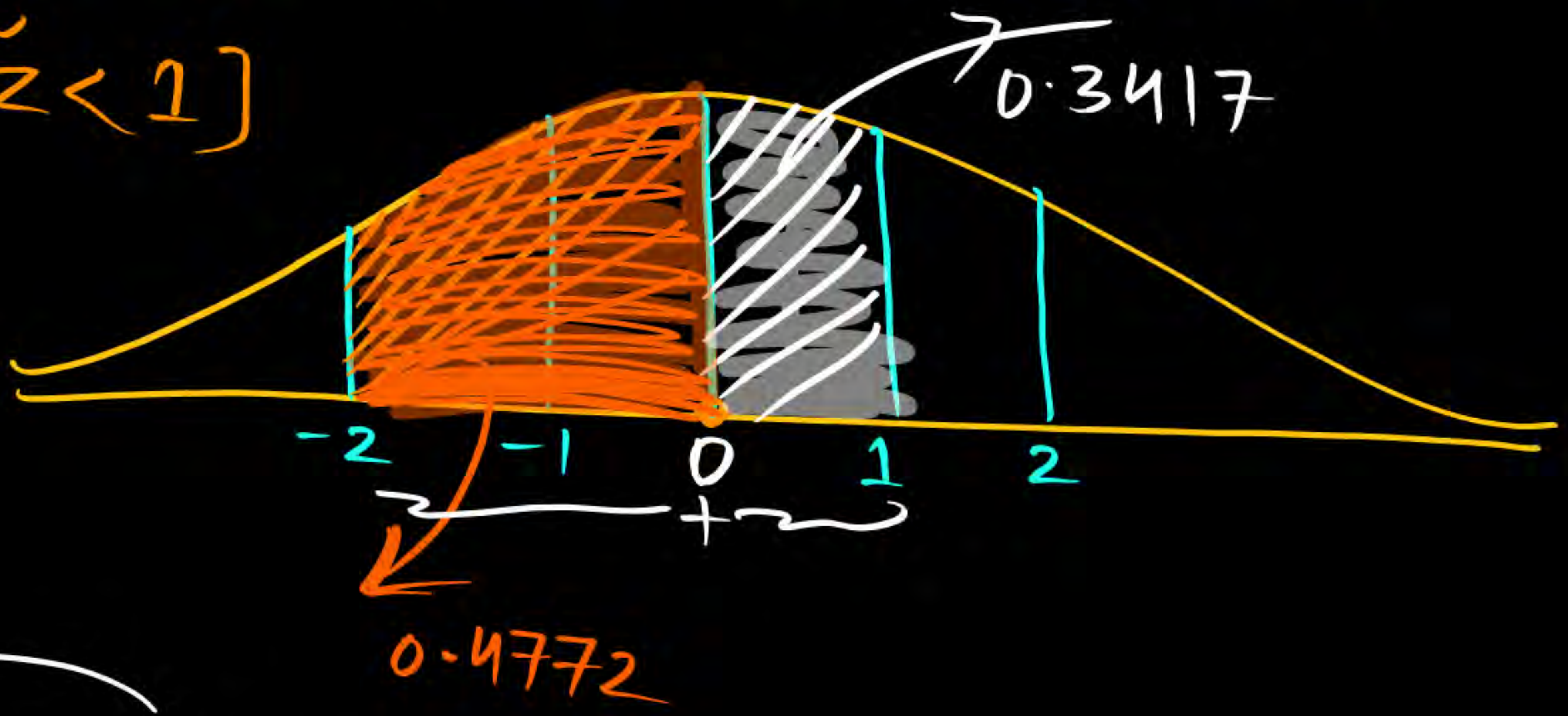
$P[-2 \le Z \le 1] = P[-2 \le Z < 0] + P[0 < Z < 1]$

OR

$P[-2 < Z < 1]$

$P(a < X < b)$

$= \int_a^b f(x)\,dx$



$= 0.3417 + 0.4772$

$= 0.8185$

$= 81\%$

solution

Q10. If X is a Gaussian Distributed Random variable with Mean = 30 and Standard Deviation = 5, then find $P(|X - 30| < 5|)$

$\boxed{2 \times 0.3413}$

$\boxed{\text{Do yourself}}$

Z SCORE

$P[|X-30|<5]$

0.8

Q-function

Error functions

Q11. Let x be zero mean unit variance Gaussian Random variable, find E (|X|).

$N(0,1) = \mu = 0 \quad \sigma = 1$

$N(\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$N(0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$E[|x|] = \int_{-\infty}^{\infty} |x| \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

$$E[|X|] = \int_{-\infty}^{\infty} \boxed{|x|} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx$$

$\to$ Even

$|x| \to$ compound function

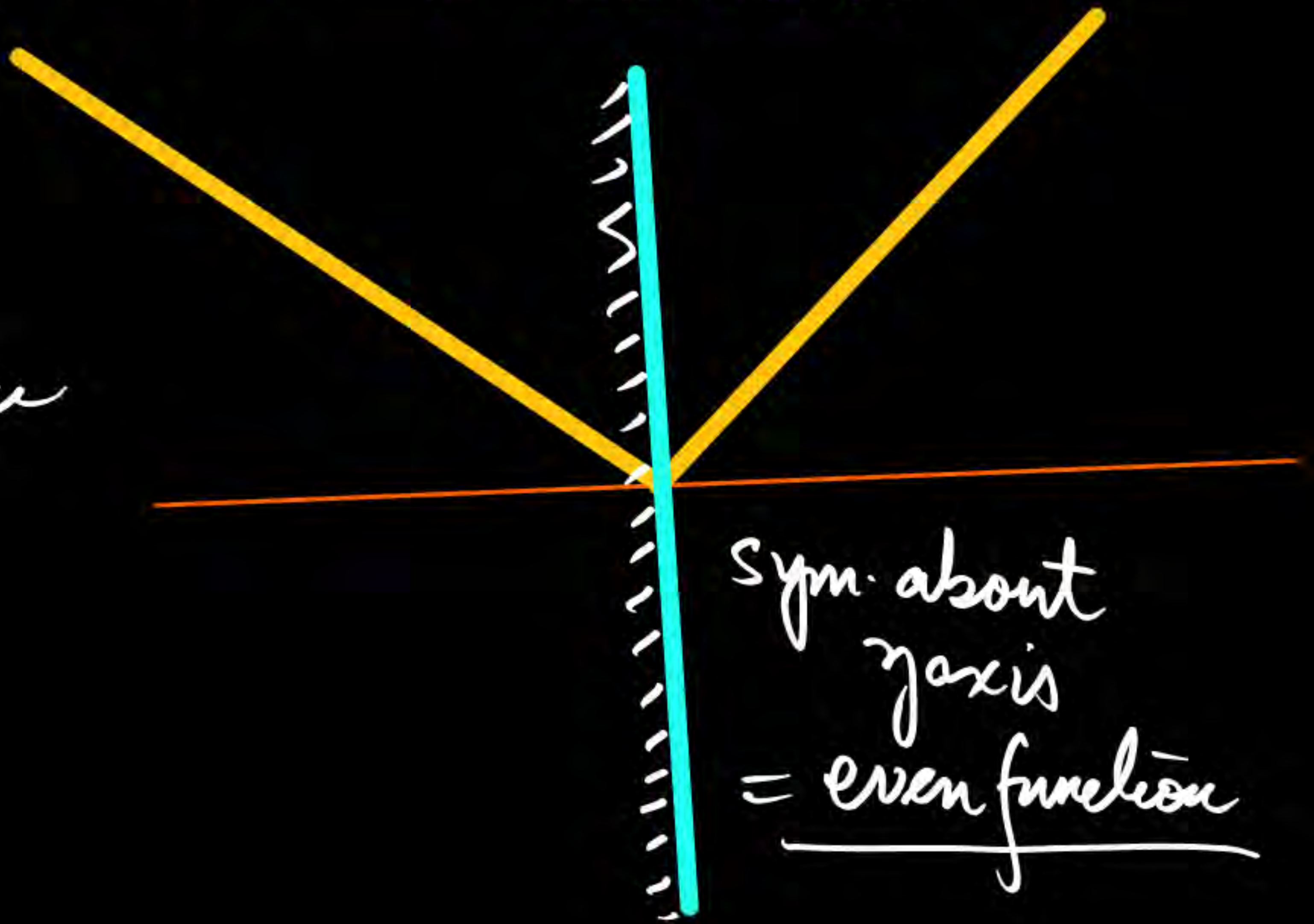
$\to$ Photo $\longrightarrow$ behaviour.

$$f(-x) = f(x)$$

$$-\text{input} \longrightarrow \boxed{f} \longrightarrow f(\text{input})$$

$$f(-x) = |-x| = \text{even function}$$

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$



$$E[|X|] = 2\int_0^\infty x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

STRATEGY

Formula List

$\frac{x^2}{2} = t$

$t = \frac{0^2}{2} = 0$

both sides Diff It

$t = \frac{\infty}{2} = \infty$

$\frac{2x}{2} dx = dt$

$x\,dx = dt$

$$\Rightarrow 2\int_{t=0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t} dt$$

$$= \frac{1\times 2}{\sqrt{2\pi}} \int_0^\infty e^{-t} dt$$

$$\boxed{\begin{aligned} e^0 &= 1 \\ e^{-\infty} &= 0 \end{aligned}}$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-t} dt = \sqrt{\frac{2}{\pi}} = 0.8$$

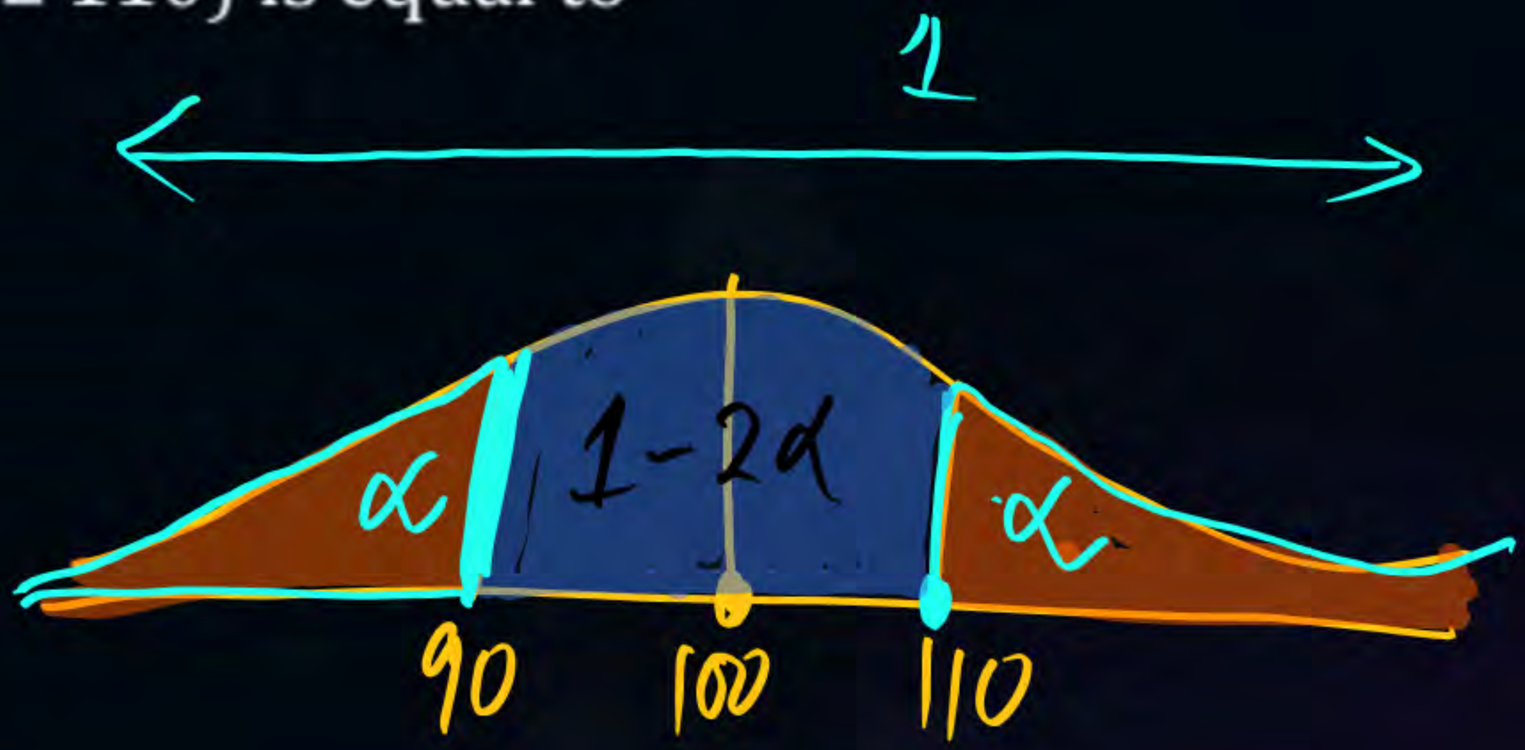
Q12. For a random variable $x(-\infty < x < \infty)$ following Normal distribution, the mean is $\mu = 100$. if the probability is $p = \alpha$ for $x \geq 110$. Then the probability x lying between 90 and 110 i.e, $P(90 \leq x \leq 110)$ is equal to

A. $1 - 2\alpha$

B. $1 - \alpha$

C. $1 - \frac{\alpha}{2}$

D. 2α



$\mu = 1000 \quad \sigma = 200$

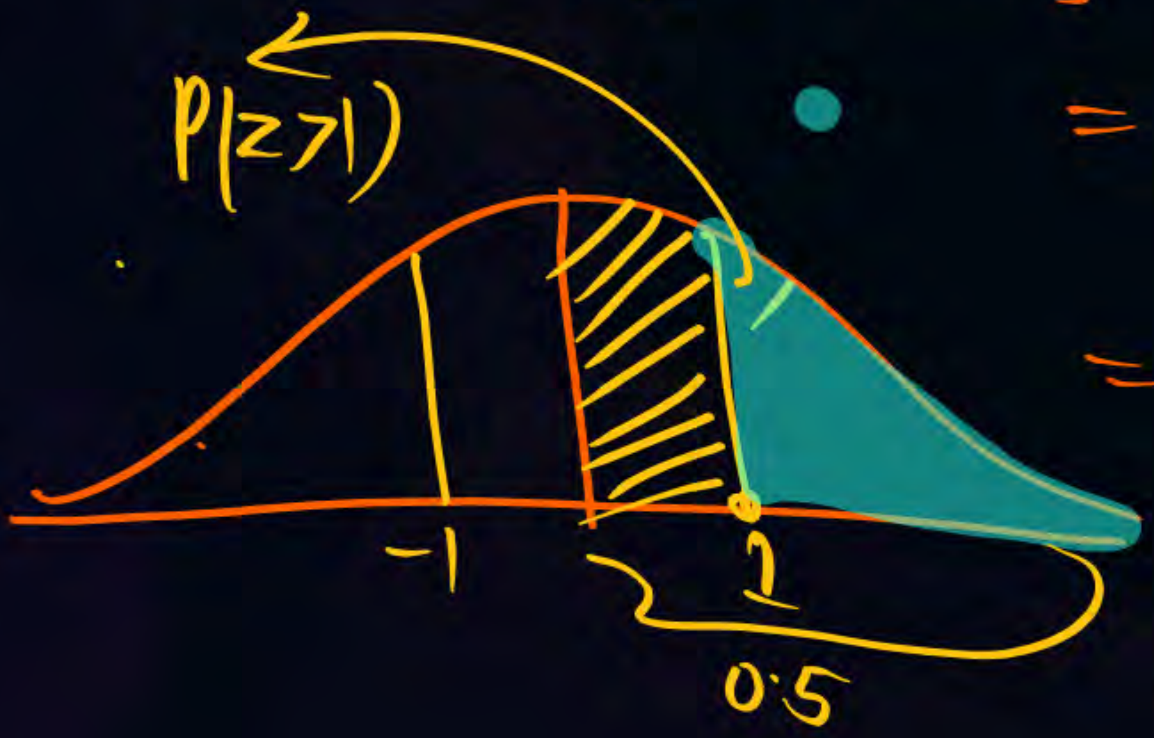
Q13. The annual precipitation data of a city is normally distribution with mean and standard deviation as 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is

A. <50%

B. 50%

C. 75%

D. 100%



$$P[X>1200] = P\left[\frac{X-\mu}{\sigma} > \frac{1200-\mu}{\sigma}\right]$$

$$= P\left[Z > \frac{1200-1000}{200}\right]$$

$$= P[Z>1]$$

$$= 0.5 - 0.3417$$

$$= \underline{0.1583}$$

$\left.\begin{matrix}X\\Y\end{matrix}\right] \; U[-1, 1]$

$P[\max[X, Y] \le \frac{1}{2}] = ✓$

Q14. Two independent random variable X and Y are uniformly distribution in the interval [–1, 1]. The probability that $\boxed{\max [X, Y]}$ is less than $\frac{1}{2}$ is

$U[a, b]$

$X \sim [-1, 1]$

$Y \sim [-1, 1]$

A. $\frac{3}{4}$

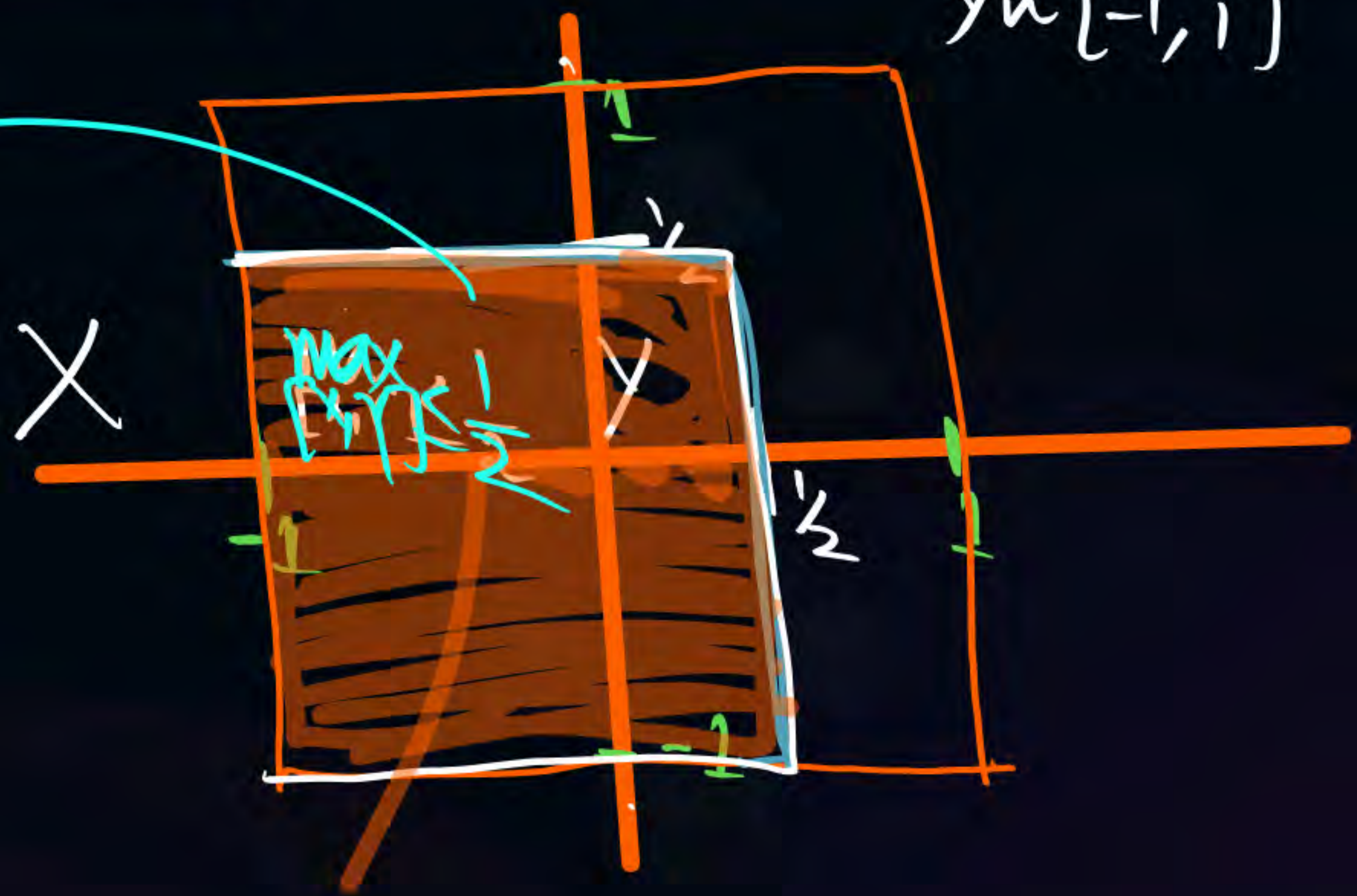
B. $\frac{9}{16}$

C. $\frac{1}{4}$

D. $\frac{2}{3}$

$P[\max[X, Y] \le \frac{1}{2}]$

for region



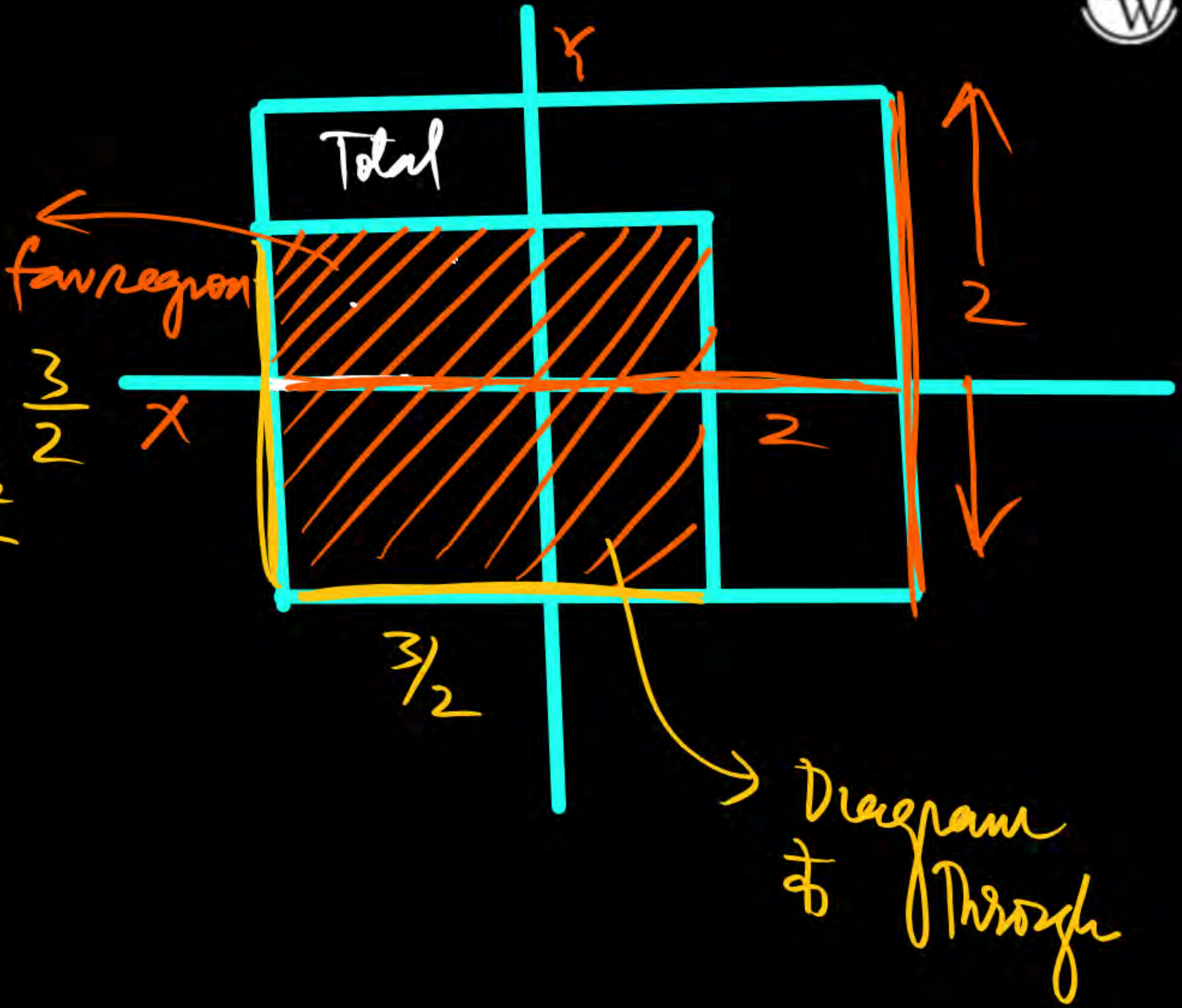
$$P\left[\max[X,Y] \le \tfrac{1}{2}\right]$$

$$= \frac{\text{fav region}}{\text{Total region}}$$

$$= \frac{\text{Area of shaded rectangle}}{\text{Area of big Rectangle}}$$

$$= \frac{\frac{3}{2} \times \frac{3}{2}}{2 \times 2} =$$

$$= \frac{9}{16}$$



H.W

Q15. Let X be a normal random variable with mean 1 and variance 4. The probability $P\{X < 0\}$ is

A. 0.5

B. Greater than zero less than 0.5

C. Greater than 0.5 less than 1.0

D. 1.0

Q16. The probability density function of a random variable X is $P_x(x) = e^{-x}$ for $x \geq 0$ and 0 otherwise. The expected value of the function $g_x(x) = e^{3x/4}$ is ____.

$f_X(x) = P_X(x) = e^{-x} \quad x \geq 0$

Expected value of function $g_X(x) = e^{\frac{3x}{4}}$

function of a Random var.

$E[x] = \int_{-\infty}^{\infty} x f(x)\,dx$

$g(x)$

$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x)\,dx$

$E\left[e^{\frac{3x}{4}}\right] = \int_0^{\infty} e^{\frac{3x}{4}} \cdot e^{-x}$

$= \int_0^{\infty} e^{-\frac{x}{4}}\,dx$

$= 4$

Q17. Subway trains on a certain line runs every half hour between midnight and six in the morning. What is the probability that the men entering the station at random time during the period will have to wait at least 20 minutes.

Subway Trains $\longrightarrow$ Every Half Hour.
12:00 o'clock
6:00 morg.

12	12:30 ✓
12:30	1:00 ✓
1:00	1:30 ✓
1:30	2:00 ✓
2:00	2:30 ✓
2:30	3
3	3:30
3:30	4

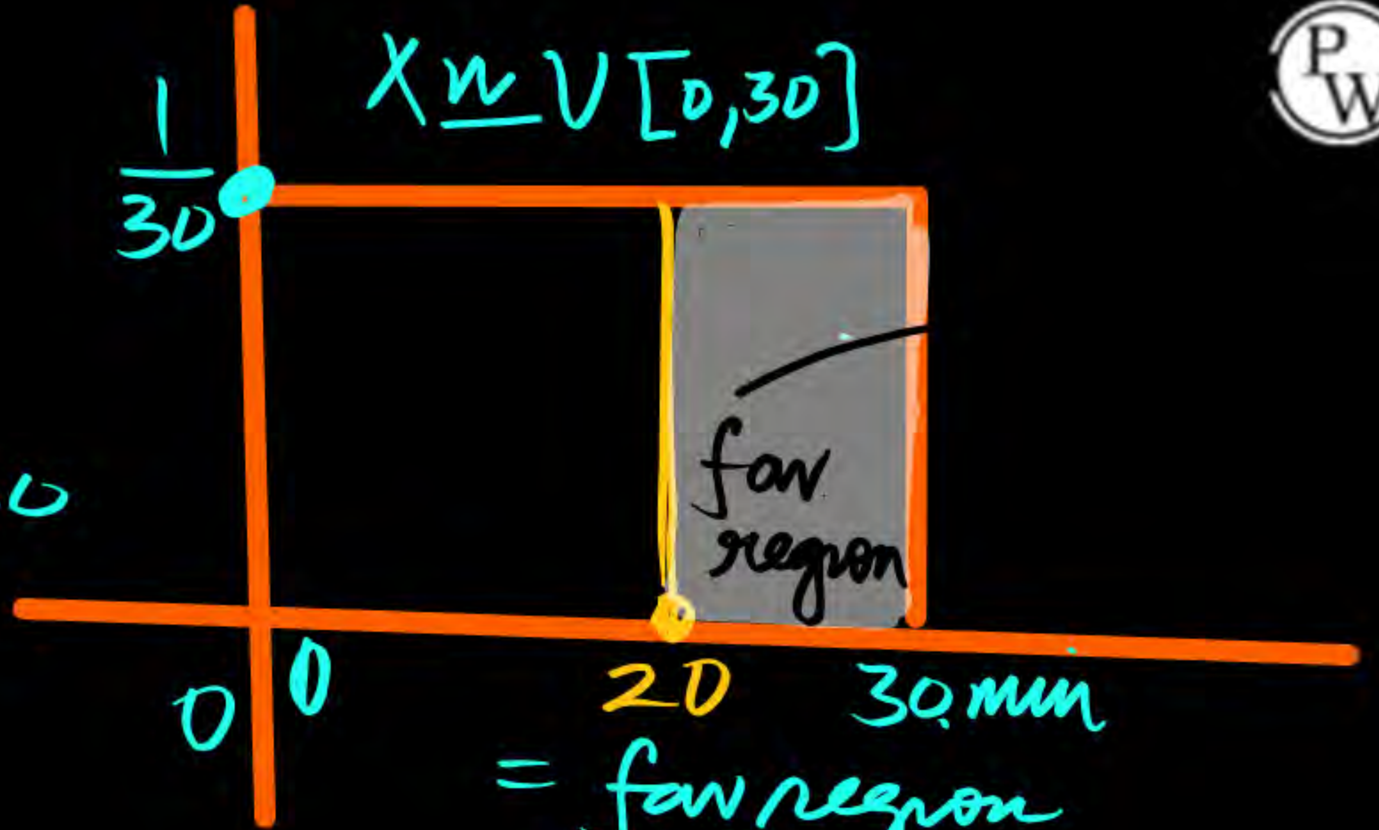
30 min

min 30

4:00	4:30
4:30	5:00
5:00	5:30
5:30	6:00

$$f(x) = \begin{cases} \dfrac{1}{(b-a)} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$X \sim U[0,30]$$



$$= \frac{\text{far region}}{\text{Total region}} = \frac{1}{3}$$

$$f(x) = \begin{cases} \dfrac{1}{30-0} = \dfrac{1}{30} & 0 < x < 30 \\ 0 & \text{otherwise} \end{cases}$$

$U[0,30]$

$$f(x) = \begin{cases} \dfrac{1}{30} & 0 < x < 30 \\ 0 & \text{otherwise} \end{cases}$$

$$P[X \geq 20) = \int_{20}^{30} \frac{1}{30}\,dx = \frac{30-20}{30} = \frac{10}{30} = \frac{1}{3}$$

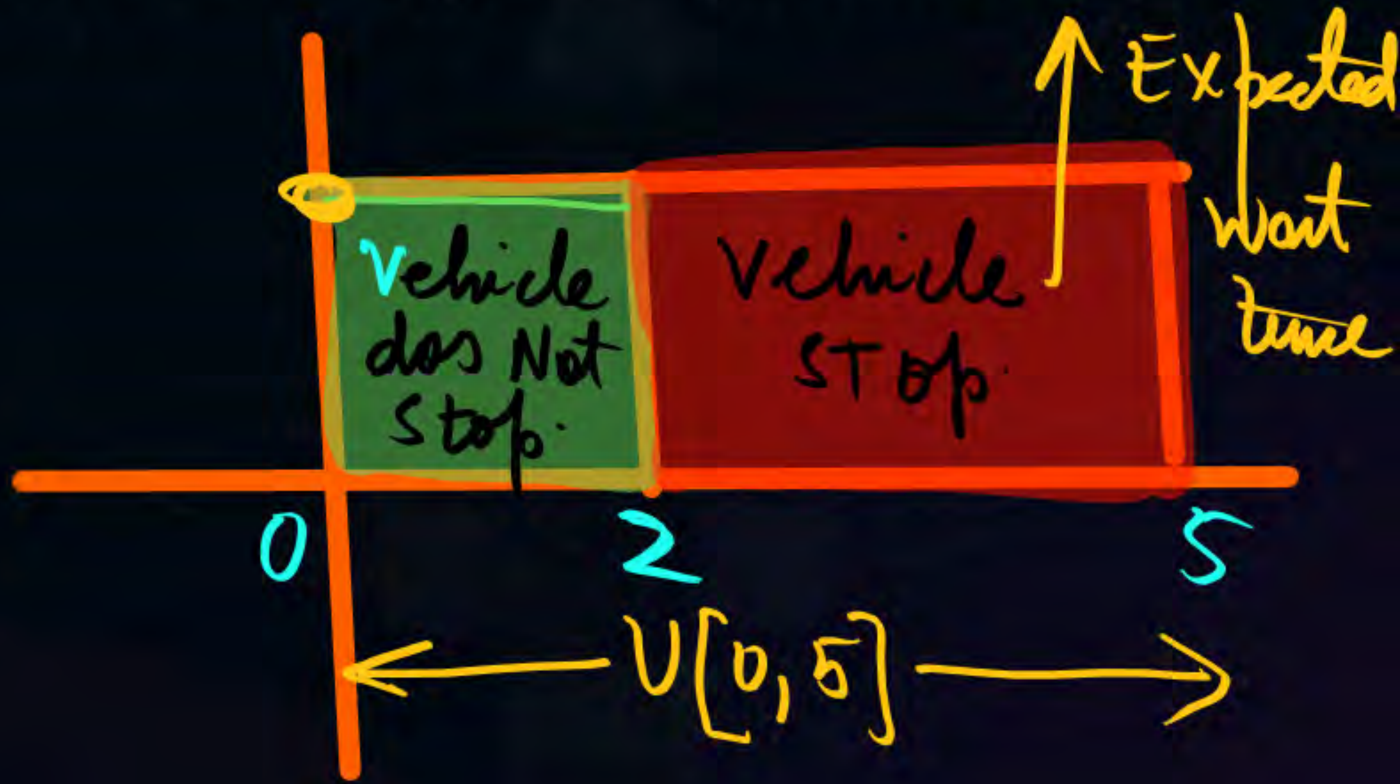
20 at least

Waiting Time - conti.

Q18. Assume that in a traffic junction, the cycle of the traffic signal lights is 2 minutes of green (vehicle does not stop) and 3 minutes of red (vehicle stops). Consider that the arrival time of vehicles at the junction is uniformly distribution over 5 minutes cycle. The expected waiting time (in minutes) for the vehicle at the junction is _____.

$$f(x) = \begin{cases} \frac{1}{5-0} & 0 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

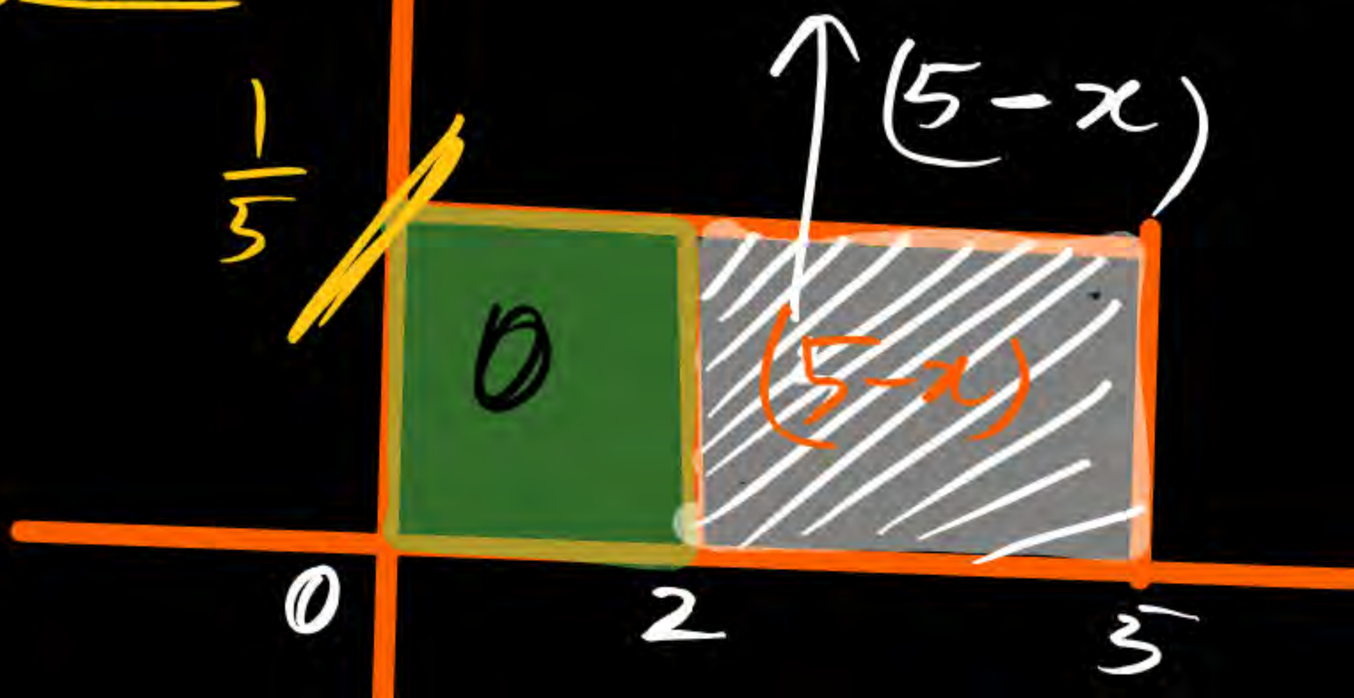
$$f(x) = \begin{cases} \frac{1}{5} & 0 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$



Expected waiting Time $E[X] \rightarrow \underline{W(X)}$

waiting function

$$W(x) = \begin{cases} 0 & 0 < x < 2 \\ 5 - x & 2 < x < 5 \end{cases}$$



$$E[\underset{\downarrow}{\overset{X}{W(X)}}] = \int_0^5 \underline{w(x)} \cdot \underline{f(x)}\, dx \qquad f(x) = \frac{1}{5-0} = \frac{1}{5}$$

$$= \int_0^2 0 \cdot \frac{1}{5}\, dx + \int_2^5 (5-x) \cdot \frac{1}{5}\, dx = \underline{0.9 \text{ min}}$$

$\frac{4}{5} = 0.8$

Q19. Suppose Y is distribution uniformly in the open interval (1, 6). The probability that the polynomial $\boxed{3x^2 + 6xy + 3y + 6}$ has only real roots is (rounded off to 1 decimal place)

$ax^2 + bx + c = 0$

$a = 3 \quad b = 6y$

$c = 3y + 6$

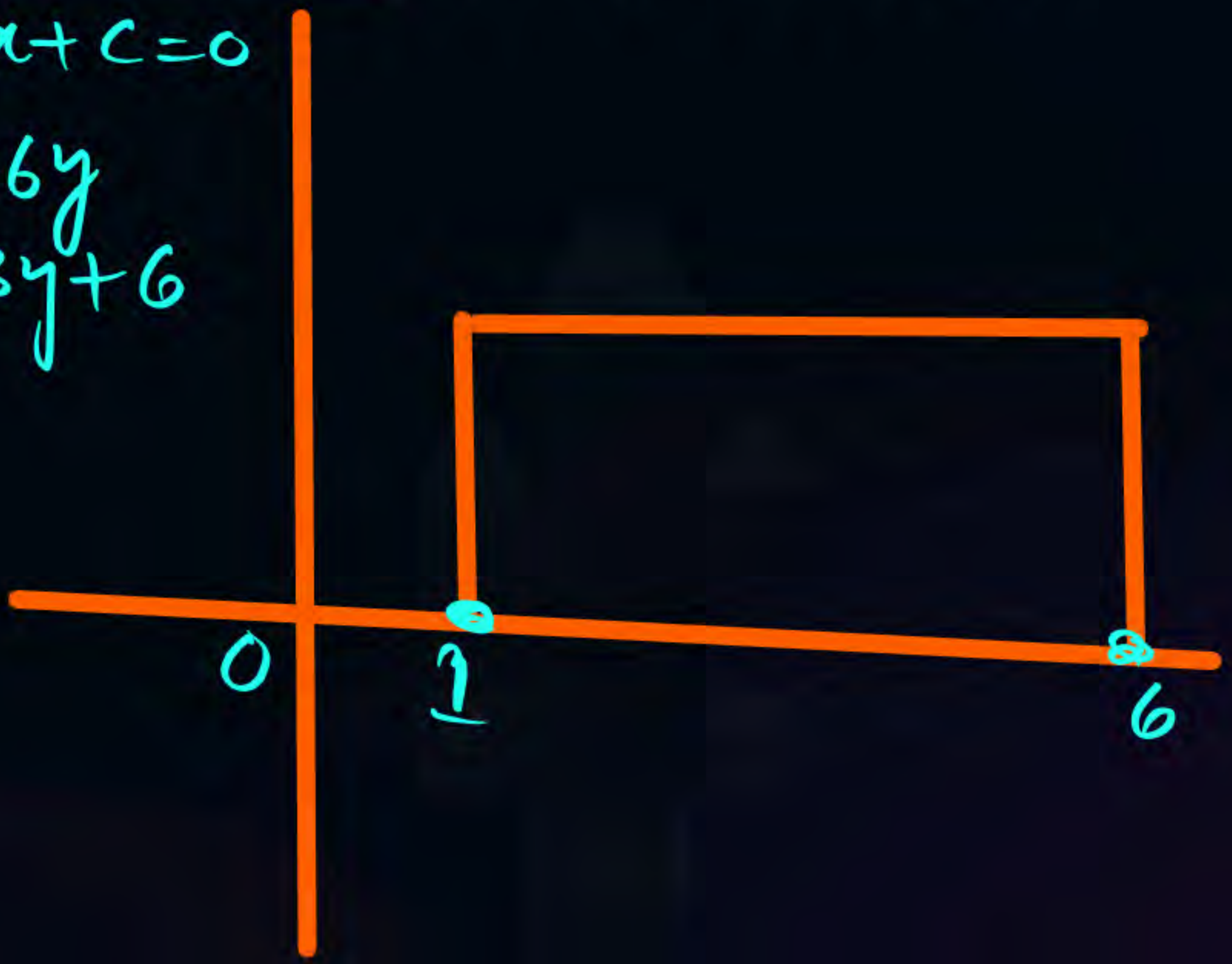
$Y \sim U[1, 6)$

$b^2 - 4ac \geq 0$ (Real Roots)

$(6y)^2 - 4 \times 3(3y + 6) \geq 0$

$36y^2 - 36y - 72 \geq 0$

$36(y^2 - y - 2) \geq 0$



$$y^2 - y - 2 \geq 0$$

$$\Rightarrow y^2 - 2y + y - 2 \geq 0$$

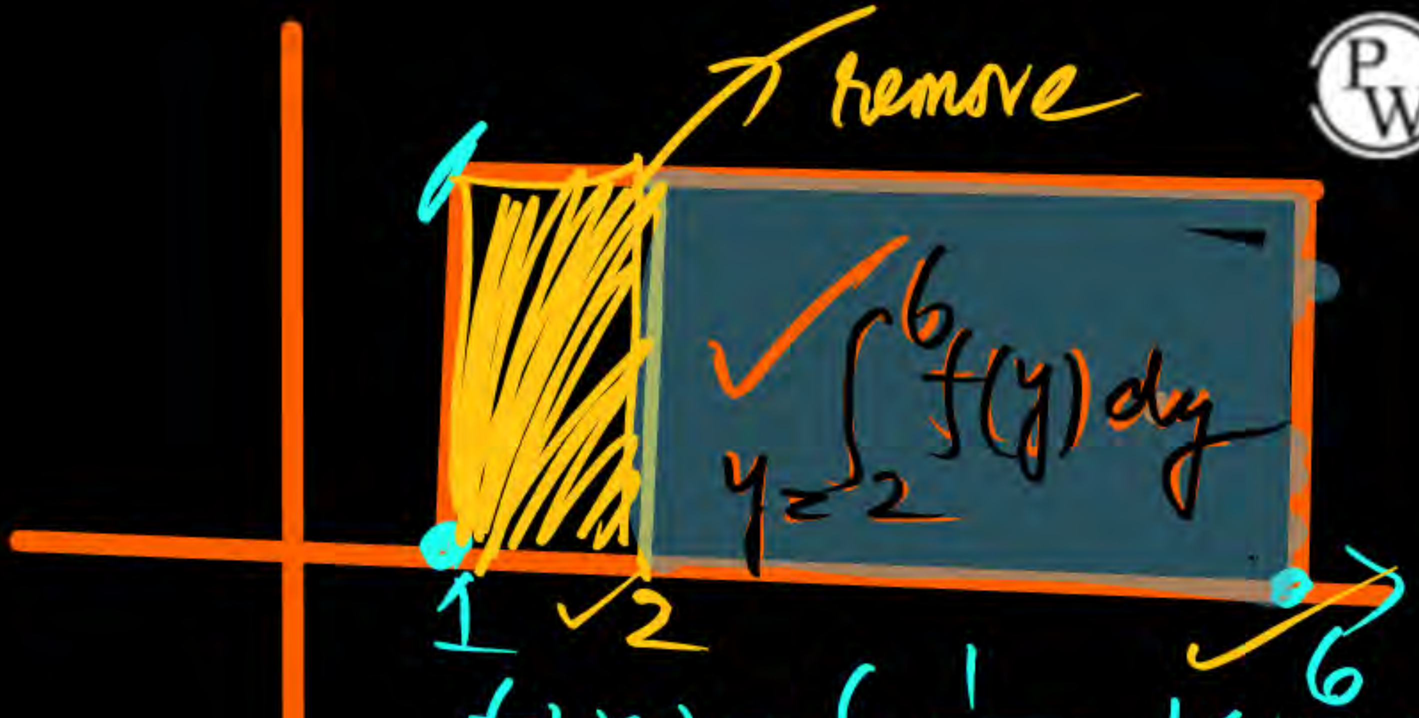
$$\Rightarrow y(y-2) + (y-2) \geq 0$$

$$= (y-2)(y+1) \geq 0$$

$$\Rightarrow y \geq 2$$

$$\Rightarrow y \geq -1$$

reject the roots



$$f(y) = \begin{cases} \frac{1}{6-1} & 1 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

$$f(y) = \begin{cases} \frac{1}{5} & 1 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

$$P(Y \geq 2) = \int_{2}^{6} \frac{1}{5}\,dy$$

$$= \frac{4}{5} \quad \underline{\text{Ans}}$$

$V(a,b)$ $\frac{1}{4}$ Ans

Q21. If X is a uniformly distributed random variable with mean 1 and variance $\frac{4}{3}$.

Find $P(X \le 0)$

$E[x] =$ $V(x) = \frac{4}{3}$ find $P[x \le 0]$

$E[x] = \frac{a+b}{2} = 1$, $a+b = 2$ — (1)

$V(x) = \frac{(b-a)^2}{\cancel{12}_{4}} = \frac{4}{\cancel{3}}$ $(b-a)^2 = 16$

$(b-a) = \pm 4$ — (2)

$a + b = 2$
$-b + a = \pm 4$
$2a = -2$
$a = -1$

$a + b = 2$
$-1 + b = 2$
$b = 3$

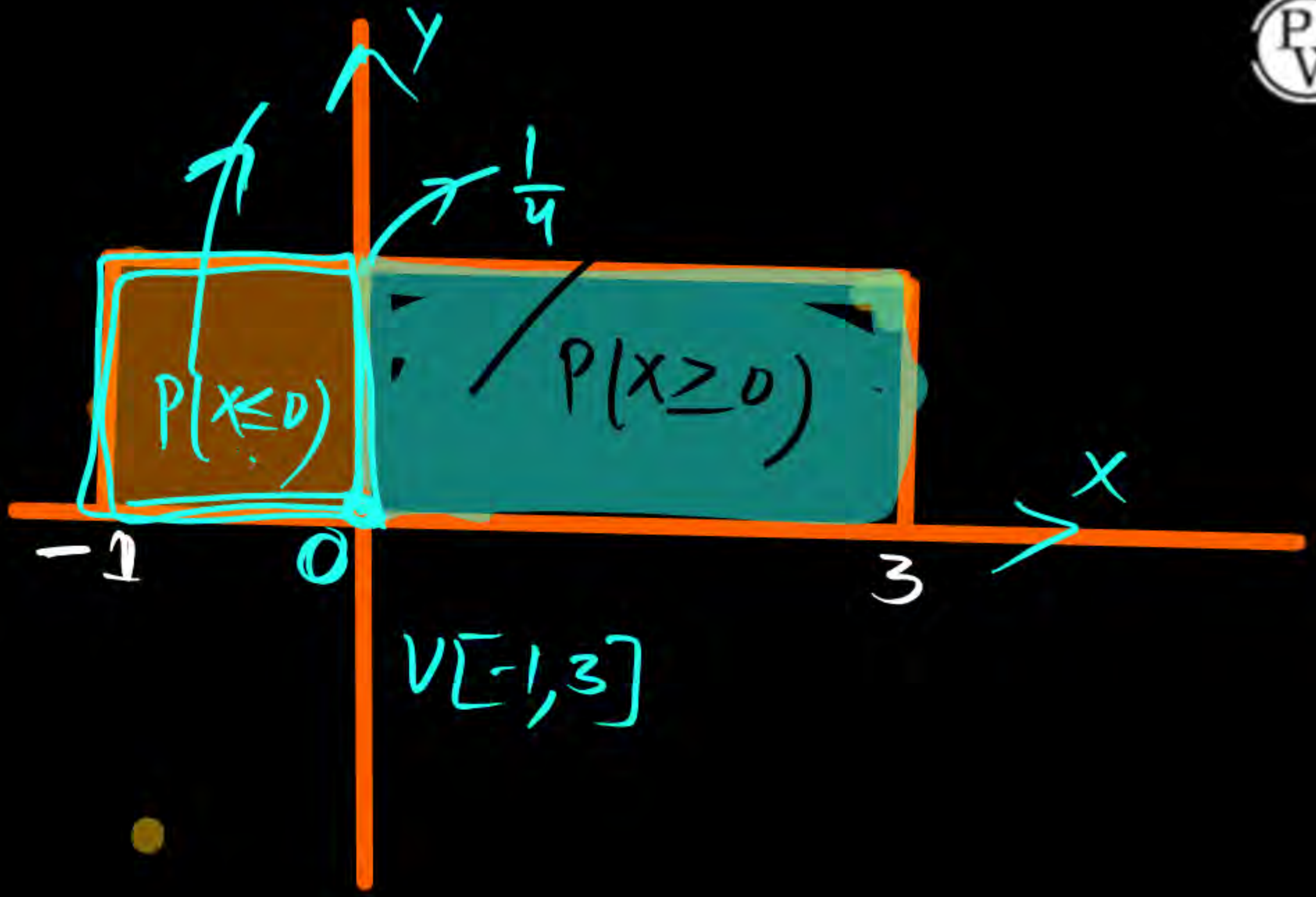
$a + b = 2$
$b - a = -4$
$2b = -2$
$b = -1$

$-1 + a = 2$
$\boxed{a = 3}$

$a=-1,\ b=3$ | $a=3,\ b=-1$ x

$$P(X \le 0) = \int_{-1}^{0} \frac{1}{4}\,dx$$

$$= \frac{1}{4} \quad \underline{\text{Ans}}$$



$\boxed{\text{Normal + Exponential}}$

THANK - YOU