Data Science and Artificial Intelligence

Probability and Statistics

Continuous Probability Distribution

Lecture No.- 02

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Topics to be Covered

Guassian Distribution / Normal

Topic Problems based on uniform Distribution and Gaussian Distribution

Guassian Distribution:

$f(x) = N(\mu, \sigma^2)$

$$= \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

MEAN $= \mu$

standard deviation σ

variance $= \sigma^2$

median $= \mu$

mode $= \mu$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$f'(x) = 0$ $\quad f''(x) < 0$ $\quad$ max value / highest occur value $= \mu$

Mode :- $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Mode → Highest occured value.

1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 4

mode = 4

$f'(x) = 0$

$f''(x) < 0$ $\boxed{\text{max value} = \mu}$

Normal Dist. $\begin{cases} \boxed{\text{Mode} = \mu} \\ \boxed{\text{mean} = \mu} \\ \boxed{\text{median} = \mu} \end{cases}$ → mid value

$$\int_{-\infty}^{\mu} f(x)\,dx + \int_{\mu}^{\infty} f(x)\,dx = \frac{1}{2}$$

OR Mode.

How to find the max/min

A) $f'(x) = 0$ (stationary pt)

B) $f''(x) < 0$ (max)

> 0 (min)

$= 0$ (Neither max or min)

C) max value

Moment genrating function:

$$\Pi_X(s) = M_G(F) = \int_{-\infty}^{\infty} e^{sx} \cdot f(x)\,dx$$

$$f(x) = N(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\Pi_X(s) = \int_{-\infty}^{\infty} e^{sx} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{2\sigma^2}\right)^2}$$

$$= \int_{-\infty}^{\infty} e^{s(\mu+\sigma\sqrt{2}t)} \frac{1}{\cancel{\sigma\sqrt{2}}\sqrt{\pi}} e^{-t^2} \cdot \cancel{\sigma\sqrt{2}}\,dt$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{s(\mu+\sigma\sqrt{2}t)} \cdot e^{-t^2} dt$$

$$\frac{x-\mu}{\sigma\sqrt{2}} = t$$

$$\boxed{x = \sigma t\sqrt{2} + \mu}$$

$$\frac{x-\mu}{\sigma\sqrt{2}} = t \qquad t = -\infty \quad t = \infty$$

$$x = \mu + \sigma\sqrt{2}t$$

$$dx = 0 + \sigma\sqrt{2}\,dt$$

$$dx = \sigma\sqrt{2}\,dt$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \boxed{e^{\mu s + s\sigma\sqrt{2}t}} e^{-t^2} dt$$

$$= e^{\mu s} \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \boxed{e^{(s\sigma\sqrt{2}t - t^2)} dt}$$

$$N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$Z \text{ SCORE} = Z = \frac{\text{Random var} - \mu}{\text{S.D}}$$

$$\boxed{Z = \frac{X-\mu}{\sigma}}$$

statistical Parameter

$$M_X(s) = \int_{-\infty}^{\infty} e^{sx} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

$$\boxed{\frac{X-\mu}{\sigma} = z}$$

statistical Parameter

$$= \int_{-\infty}^{\infty} e^{s(\mu+\sigma z)} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}} \cdot \sigma\, dz$$

$$= \int_{-\infty}^{\infty} e^{\mu s + s\sigma z} \cdot \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \cdot \cancel{\sigma}\, e^{-\frac{z^2}{2}} dz$$

$dx = \sigma dz$

$x = \mu + \sigma z$

$$= \frac{1}{\sqrt{2\pi}} e^{\mu s} \int_{-\infty}^{\infty} e^{s\sigma z - \frac{z^2}{2}} dz$$

$$= \frac{1}{\sqrt{2\pi}} e^{\mu s} \int_{-\infty}^{\infty} e^{\frac{2s\sigma z - z^2}{2}} dz$$

$$e^{\mu s}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\frac{2s\sigma z - z^2}{2}}$$

$$= e^{\mu s}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\left[-\frac{1}{2}(z-\sigma s]^2 - \sigma^2 s^2\right]}\,dz$$

$$= e^{\mu s+\frac{\sigma^2 s^2}{2}}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-\sigma s)^2}\,dz$$

$$= e^{\mu s+\frac{\sigma^2 s^2}{2}}\frac{1}{\sqrt{2\pi}}\,2\int_{0}^{\infty} e^{-\frac{t^2}{2}}\,dt$$

$$\boxed{M_g f = e^{\mu s+\frac{\sigma^2 s^2}{2}}}$$

$$-\frac{1}{2}\left[z^2+\sigma^2 s^2-2\sigma z s\right]-\sigma^2 s^2$$

$s\sigma z - \frac{z^2}{2}$ → complete The square form.

$$-\frac{1}{2}(z^2 - s\sigma z)$$

$$= -\frac{1}{2}(z-\sigma s]^2$$

$$= -\frac{1}{2}\left[z^2+\sigma^2 s^2 - 2s\sigma z\right]$$

$$= \frac{z^2}{2} - s\sigma z + \frac{\sigma^2 s^2}{2}$$

Moment genrating Function

$$\boxed{M_G(F) = e^{\mu s + \frac{\sigma^2 s^2}{2}}}$$

$$N(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\frac{x-\mu}{\sigma} = z$$

$$N(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2/2}$$

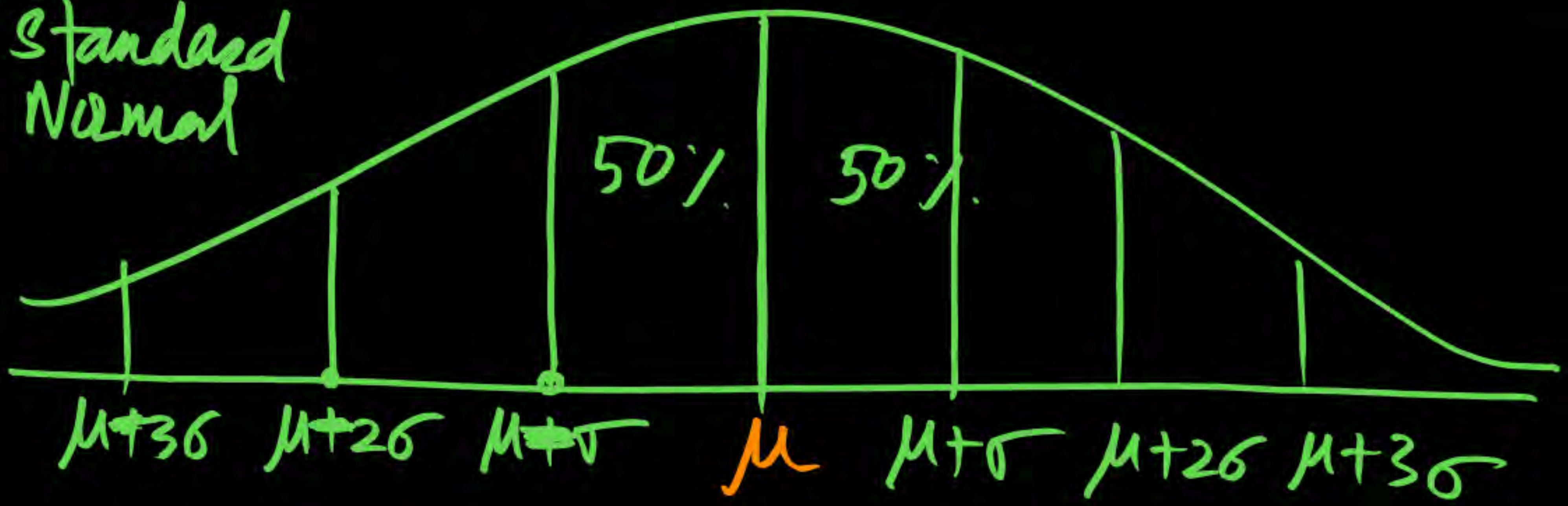
AREA

$$\boxed{\begin{gathered}\mu = 0 \quad \sigma = 1 \\ N(0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\end{gathered}}$$

Standardized Normal Distribution (ZERo mean and unit vari)

$N(\mu, \sigma^2)$ Transform → standard Normal

$N(0,1)$



convert

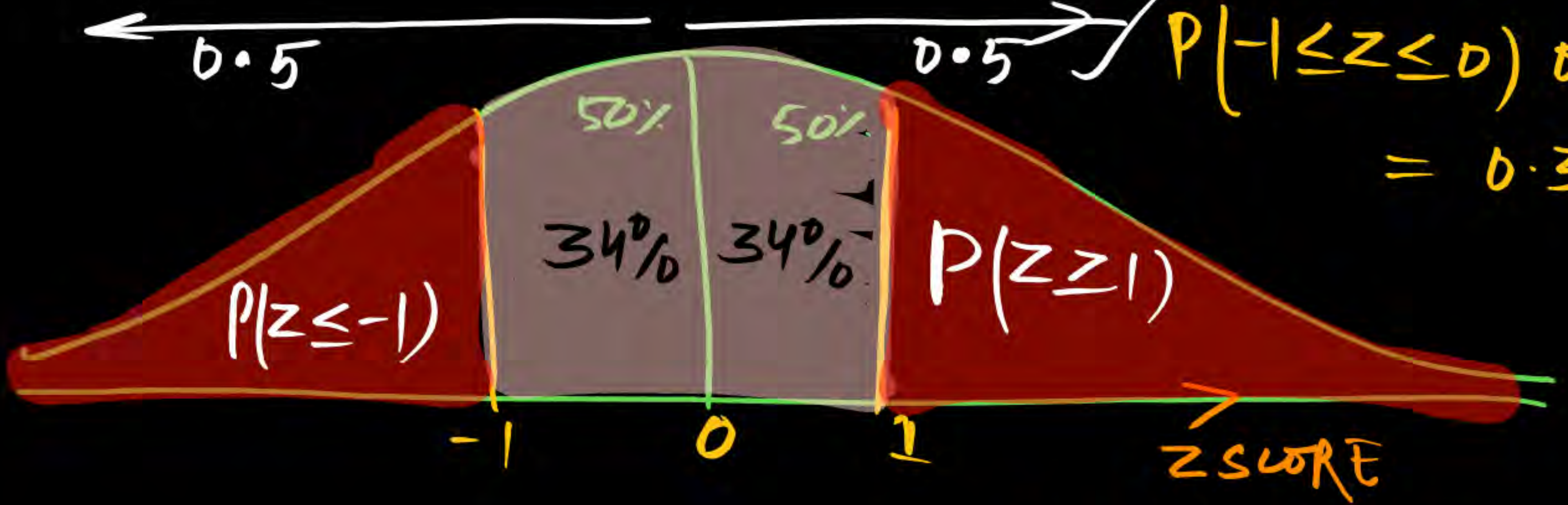
$N(0,1)$

Standard Normal.

Standard Normal Curve:

$$P(-1 \le z \le 1) = 68\% = 0.6834$$

$$P(-1 \le z \le 0) \text{ OR } P(0 \le z \le 1) = 0.3417$$



$$P(z \ge 1) \text{ OR } P(z \le -1)$$

$$P(z \ge 1) = 0.5 - P(0 \le z \le 1)$$

$$= 0.5 - 0.3417$$

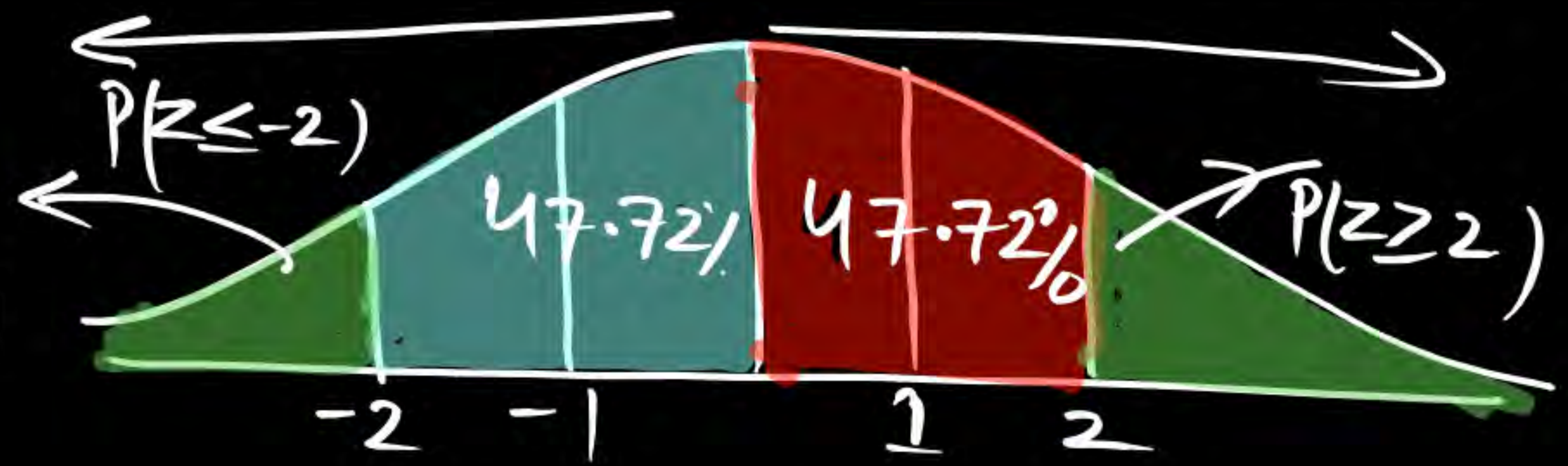
$$= \underline{0.1583}$$

2)

$P(-2 \le Z \le 2) = 0.9545$

$P(0 \le Z \le 2) = 0.4772$

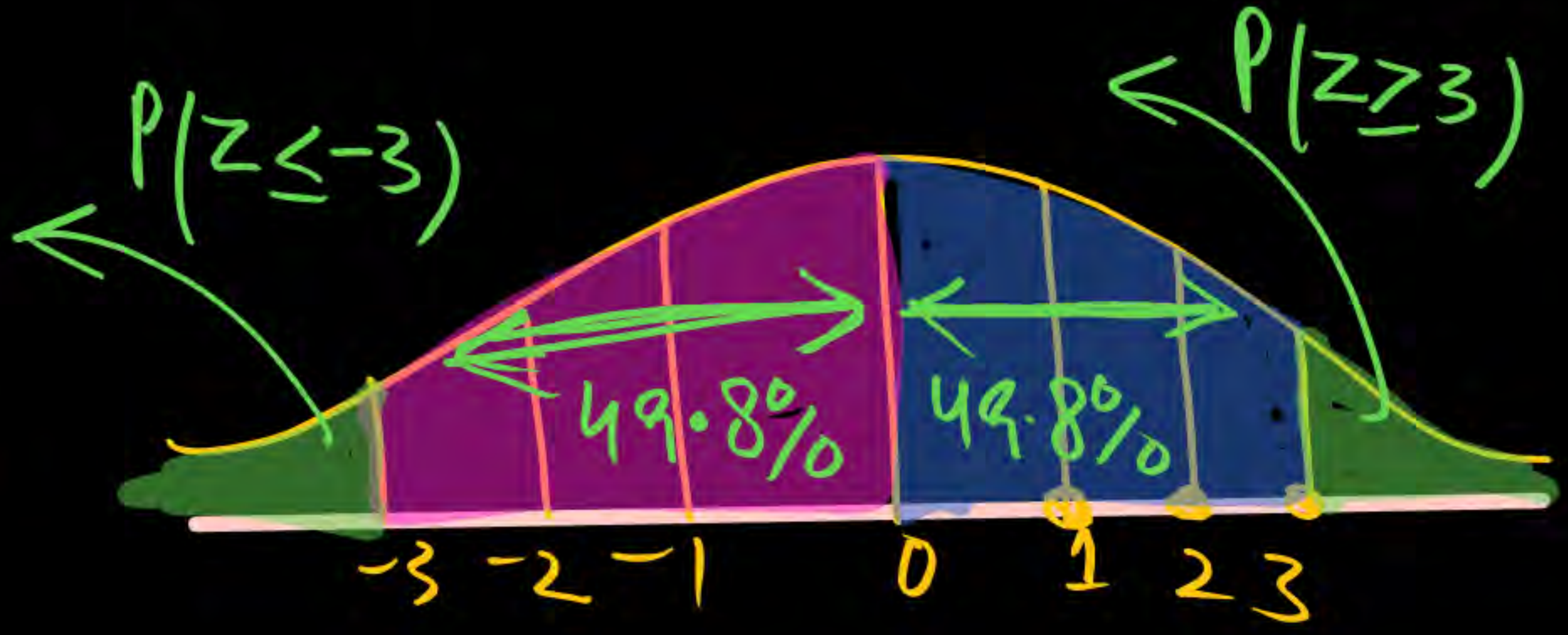
$P(Z \ge 2) = 0.5 - P(0 \le Z \le 2)$



3) $P(-3 \le Z \le 3) = 0.9971$

$P(0 \le Z \le 3) = 0.4985$

$P(Z \ge 3) = 0.5 - P(0 \le Z \le 3)$



Question always — Random vari

$$= a \le X \le b$$

$$P(a \le \underline{X} \le b) = P\left[\frac{a-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma}\right]$$

$$= P\left[\underline{\frac{a-\mu}{\sigma}} \le Z \le \underline{\frac{b-\mu}{\sigma}}\right]$$

$$N(\mu, \sigma^2)$$

convert

$\sin(x+3)$ $x+3=t$

$x+3=0$

$t=0$

Rs $500=\mu$

Rs $50=\sigma$

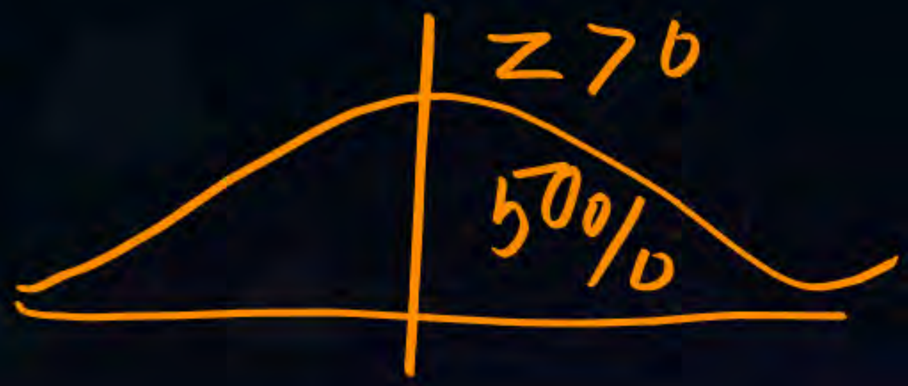
Q4. A nationalized bank has found the daily balance available in its saving accounts follows a normal distribution with a mean of $\boxed{\text{Rs. } 500}$ and a standard deviation of Rs.50. The percentage of saving account holders, who maintain an average daily balance more than Rs. 500 is

$$P(X>500)=P\left[\frac{X-\mu}{\sigma}>\frac{500-\mu}{\sigma}\right]$$

Z score

$$=P\left[Z>\frac{500-500}{50}\right]=P\left(Z>\frac{500-500}{50}\right)$$

$$=P(Z>0)=\frac{1}{2}$$



$n = 180$ times

Q7. A Die is rolled 180 times using Gaussian random variable. Find the Probability that faces 4 will turn up at least 35 times.

$\rightarrow$ success (4)
$\rightarrow$ failure ($\overline{4}$)

Bernoulli Trials

$n = 180$ (Binomal Dis)

$$P(X \geq 35) = P(X=35) + P(X=36) + P(X=37) + \cdots + P(X=180)$$

$$= [1 - P(X=0) - P(X=1) - \cdots - P(X=34)]$$

Using Binomial Distribution

Binomal Distribution $\xrightarrow{n \text{ Large}}$ Normal / Guassian

$$P(X \geq 35) = P\left[\frac{X-\mu}{\sigma} \geq \frac{35-\mu}{\sigma}\right]$$

binomial s.D

$$= P\left[Z \geq \frac{35-np}{\sqrt{npq}}\right]$$

mean $= np$

var $= npq$

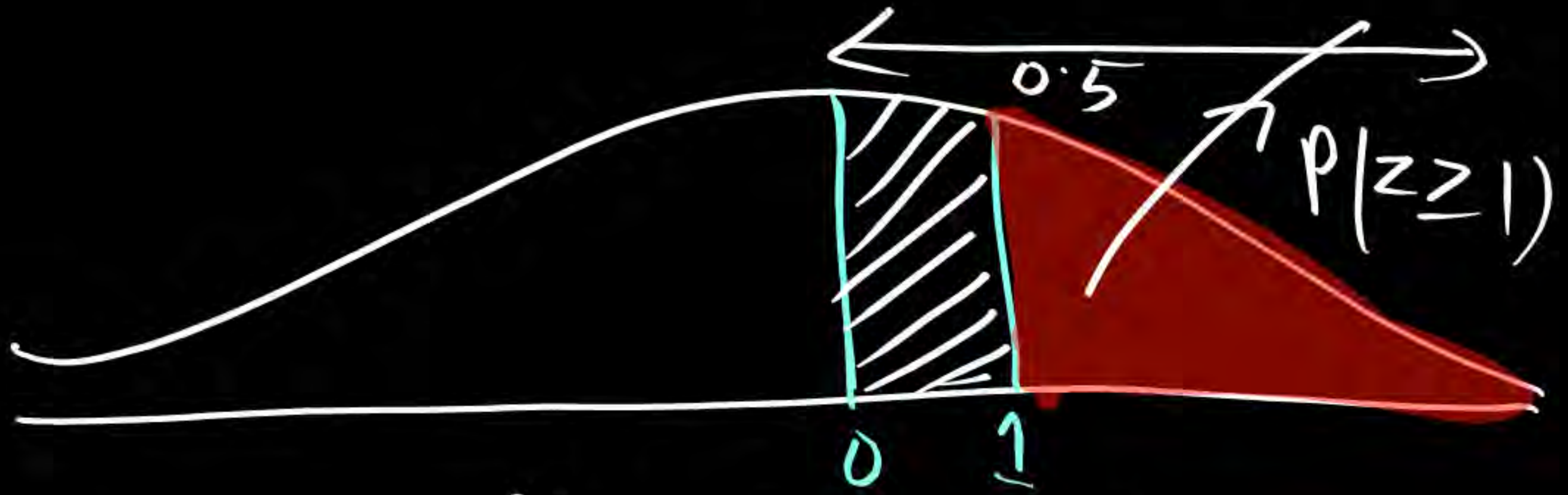
$$\Rightarrow P\left[Z \geq \frac{35-30}{5}\right] = P(Z \geq 1)$$

mean $= 180 \times \frac{1}{6} = 30$

var $= \cancel{180}^{30} \times \frac{1}{\cancel{6}} \times \frac{5}{6}$

$= 25$

$SD = \sqrt{25} = \underline{5}$



$$P(Z \geq 1) = 0.5 - P(0 \leq Z \leq 1)$$

$$= 0.5 - 0.3417 = \underline{0.1583}$$

$\boxed{\text{Daddy Dis}}$ $\int_{72}^{\infty}$

$\mu = 68.22$ inches. $\sigma^2 = 10.8$ inches^2

Q8. Assume Mean Height of the soldiers is 68.22 inches with the variance 10.8 inches. How many soldiers in Regiment of 1000 would you expected to be over $\boxed{\text{(6 feet tall).}}$ Given that the Standard Normal Curve X = 0 to 1.15 = 0.3746.

$P(Z > 1.15)$

$P(X > 6\text{ feet}) = P(X > 72)$

1 feet = 12 inches.

6 feet = 12 × 6

= 72 inches

$= P\left(\frac{X-\mu}{\sigma} > \frac{72-\mu}{\sigma}\right)$

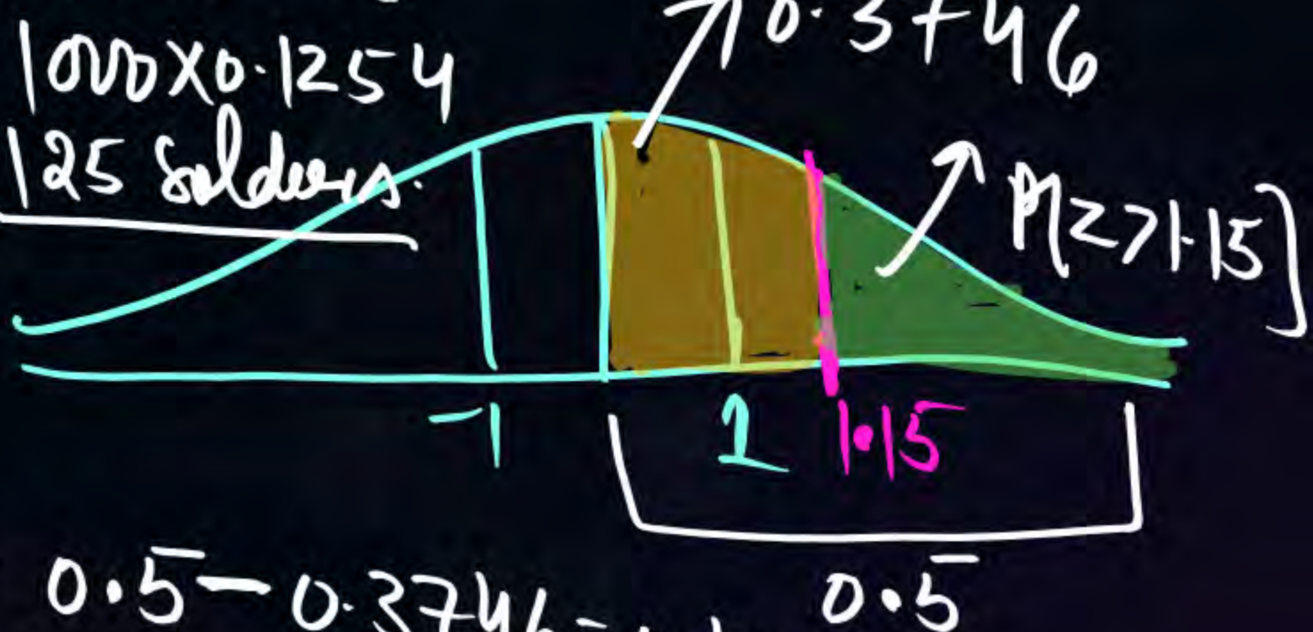
$= P\left(Z > \frac{72-68.22}{\sqrt{10.8}}\right)$

$= 0.5 - 0.3746 = 0.1254$

Total soldiers

= 1000 × 0.1254

= 125 soldiers.



Imp. (Data Science)

Q9. Let X_1, X_2, X_3 be three independent and identically distribution random variables with Uniform Distribution on [0, 1]. Find the probability $P[X_1 + X_2 \leq X_3]$

$V(-X) = (-1)^2 V(X)$

$\underbrace{X_1, X_2, X_3}$ Are Uniformly dist $[0,1]$

$X = X_1 + X_2 - X_3$

$P[X_1 + X_2 \le X_3] \Rightarrow P[X_1 + X_2 - X_3 \le X_3 - X_3]$

$\Rightarrow P[\underline{X_1 + X_2 - X_3} \le 0] = P[X \le 0] = P\left[\frac{X-\mu}{\sigma} \le \frac{0-\mu}{\sigma}\right]$

$\Rightarrow P\left[Z \le \frac{0-\mu}{\sigma}\right]$

X_1, X_2, X_3 Are $U(\underline{0,1})$

$E[X] = \mu = \frac{a+b}{2}$ $\quad V(X) = \frac{(b-a)^2}{12}$

$X = X_1 + X_2 - X_3$

$E[X_1] = \frac{0+1}{2} = \frac{1}{2}$ $\quad V(X) = \frac{(b-a)^2}{12}$

$E[X_2] = \frac{0+1}{2} = \frac{1}{2}$ $\quad V(X_1) = \frac{(1-0)^2}{12} = \frac{1}{12}$

$E[X_3] = \frac{0+1}{2} = \frac{1}{2}$ $\quad V(X_2) = \frac{1}{12}$

$V(X_3) = \frac{1}{12}$

$X = X_1 + X_2 - X_3$

$E[X] = E[X_1] + E[X_2] - E[X_3]$

$\mu = E[X] = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2}$

$V(X) = V(X_1) + V(X_2) + V(-X_3)$

$V(X) = V(X_1) + V(X_2) + (-1)^2 V(X_3)$

$V(X) = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{12}$

$$= P\left[z \le \frac{0-\mu}{\sigma}\right] = P\left[z \le \frac{0-\frac{1}{2}}{\sqrt{\frac{1}{4}}}\right]$$

$$\Rightarrow P\left[z \le \frac{-\frac{1}{2}}{\frac{1}{2}}\right] = P(z \le -1)$$

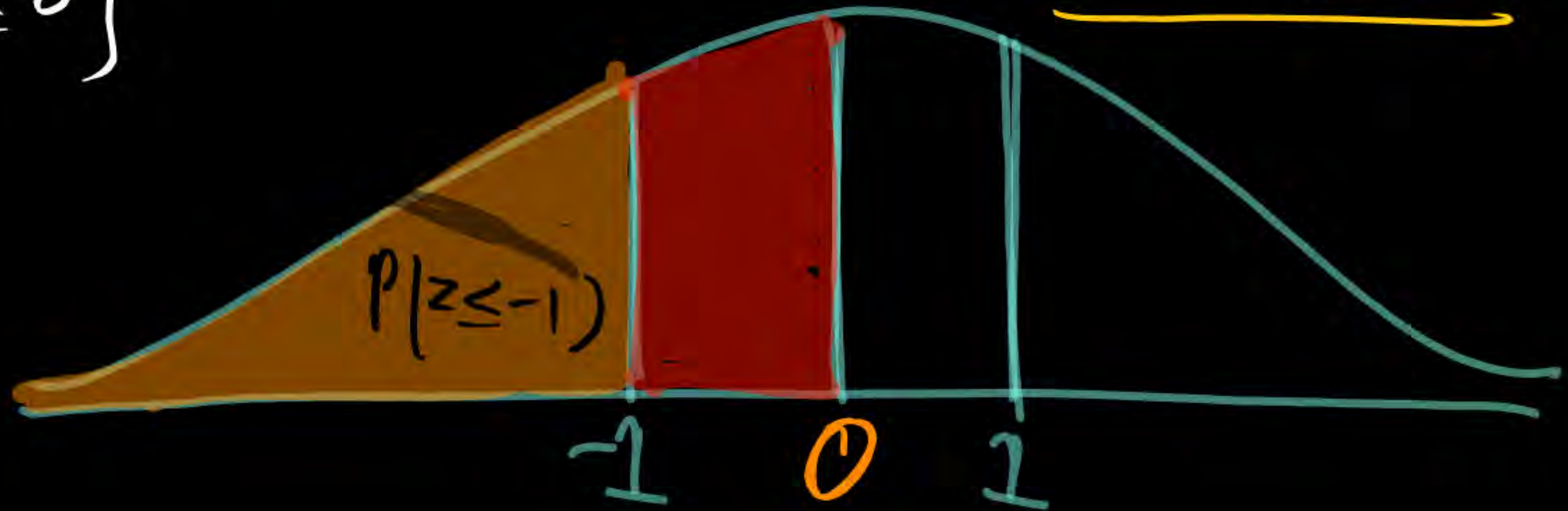
$$M_X(t) = e^{\mu s + \frac{\sigma^2 s^2}{2}}$$

$$\Rightarrow 0.5 - P[-1 \le z \le 0]$$

$$= 0.5 - 0.3417$$

$$= \underline{0.1583}$$

Ans



THANK - YOU