Data Science and Artificial Intelligence

Probability and Statistics

Discrete Probability Distribution

Lecture No.-03

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Topics to be Covered

Discrete + conti

I
II
IIIrd exclusive - mix

End - bivariate

7 question - 10 question

DPP

Topic	Moment Generating Function
Topic	Uniform Distribution
Topic	Geometric Distribution

75 question

50 question

TEXT solution

70 questions
Problem - discrete Distribution
4 Hrs.

75 question

70 questions (Dis + cont)

Moment-generating Function : MGF

Transform of Random Variable

$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx \quad \text{(continuous)}$$

$$E[X] = \sum_{i=0}^{n} x_i P(x_i) \quad \text{(Discrete)}$$

Discrete Random Variable

$$E[X] = \sum_{x=0}^{\infty} x_i P(x_i)$$

$X \to$ Transform $\quad X = s^X \qquad X = s^X$

X^r

$r = 0$
$= 1$
$= 2$
$= 3$

generating Function

$$E[s^X] = \sum_{x=0}^{\infty} s^x P(X=x)$$

$\Pi_X(s)$ = moment genrating Function

$$\Pi_X(s) = \sum_{x=0}^{\infty} s^x P(X=x)$$

$$\Pi_X(s) = s^0 P(X=0) + s^1 P(X=1) + s^2 P(X=2) + P(X=3)s^3 + \cdots$$

$$\boxed{\Pi_X(s) = s^0 P(0) + s^1 P(1) + s^2 P(2) + s^3 P(3) + \cdots}$$

$$E[X] = \sum_{x=0}^{\infty} x P(x)$$

$$E[s^X] = \sum_{x=0}^{\infty} s^x P(x)$$

Generating Function

$X = s^X$ → Power

s^0
s^1
s^2
s^3
s^4
s^5

generating function

algebraic function

Discrete

$$\boxed{\Pi_X(s) = s^0 P(0) + s^1 P(1) + s^2 P(2) + \cdots s^x P(X=x)}$$

s^0 coefficient — $P(X=0)$

s^1 — $P(X=1)$

s^2 — $P(X=2)$

s^3 — $P(X=3)$

⋮

$s^{(x=x)}$ — $P(X=x)$

s^x – cofficient $\longrightarrow P(X=x)$

✓ s^0 — $P(X=0)$

✓ $s^1 \longrightarrow P(X=1)$

$s^3 \longrightarrow P(X=3)$

X	1	2
$P(X=x)$	0.1	0.9

genrating function $= \underbrace{0.1}_{P(X=1)} s + \underbrace{0.9}_{P(X=2)} s^2$

What is $M_G(F)$

X	1	2	3
$P(X=x)$	0.1	0.2	0.7

$$\Pi_X(s) = \underbrace{0.1}_{P(X=1)} s + \underbrace{0.2}_{P(X=2)} s^2 + \underbrace{0.7}_{P(X=3)} s^3$$

$$s^X \longrightarrow \underline{P(X=x)}$$

$$\Pi_X(s) \Rightarrow \sum_{X=0}^{\infty} s^X P(X=x) \longrightarrow 1^0 P(0) + 1^1 P(1) + 1^2 P(2) + 1^3 P(3) + \cdots$$

$\longrightarrow s \to 0$

$\underline{(0)}$

$$\Rightarrow s^0 P(0) + s^1 P(1) + s^2 P(2) + s^3 P(3) + \cdots + \cdots$$

$$s = 0 = 0^0 P(0) + 0^1 P(1) + 0^2 P(2) + 0^3 P(3) + \cdots$$

$$\Pi_X(0) = P(0) + 0 + 0 + 0 + 0 + 0 + \cdots$$

$\boxed{\Pi_X(0) = 0}$ $\boxed{\Pi_X(1) = 1}$

$S = 1$

$$\Pi_X(1) = P(0) + P(1) + P(2) + P(3) + P(4) + \cdots = 1$$

$M_G(F)$

$$\Pi_X(s) = \sum_{x=0}^{\infty} s^x P(X=x)$$

$\Pi_X(0) = 0$

$\Pi_X(1) = 1$

$$\Pi_X(s) = P_0 s^0 + P_1 s^1 + P_2 s^2 + \dots + P_x s^x$$

$s = 0$

$$\Pi_X(0) = P_0(0)^0 + P_1(0)^1 + P_2(0)^2 + \dots + P_x(0)^0$$

$\boxed{\Pi_X(0) = 0}$ $\quad s^0 \longrightarrow \underline{P(X=0)}$

$\Pi_X(1)$

$$\Pi_X(1) = P_0 1^0 + P_1 1^1 + P_2 1^2 + P_3 1^3 + \dots$$

$$= P_0 + P_1 + P_2 + P_3 + \dots + \underline{P(X=n)}$$

$$= 1$$

In Binomial Distribution

$$B(n,p) = {}^nC_x\, p^x q^{n-x}$$

n = No. of Trials.

x = No. of success.

$p = P(S)$ $\quad q = P(F)$

$(x+a)^n$

T_r

$= {}^nC_r\, x^{n-r} a^r$

$$M_G(F) = \Pi_X(s) = \sum_{x=0}^{\infty} s^x P(X=x)$$

$$= \sum_{x=0}^{\infty} s^x p^x q^{n-\cancel{x}}\, {}^nC_x$$

$$= \sum_{x=0}^{\infty} {}^nC_x (ps)^x q^{n-x}$$

$$\boxed{\Pi_X(s) = (q+ps)^n}$$

moment generation function

$(x+a)^n$

$= T_r = {}^nC_r\, x^{n-r} a^r$

$(q+ps)^n$

${}^nC_x (ps)^x q^{n-x}$

m. Imp ✓

Moment generation function

find $E[X]$, $V(X)$

$x^n = nx^{n-1}$

$= s^x = xs^{x-1}$

$$\Pi_X(s) = E[s^X] = \sum_{x=0}^{\infty} s^x P(X=x)$$ in Discrete manner.

$$E[X] = \sum_{x=0}^{\infty} x P(X=x)$$

$x + x^2 + x^3$

$= 1 + 2x + 3x^2$

↓ vanish

$P(0)$

both sides Differentiate It

$$\boxed{\Pi_X'(s) = \sum_{x=0}^{\infty} x s^{x-1} P(X=x)}$$

$E[X]$

$E[X^2]$

$E[X^3]$

$E[X^4]$

put $s = 1$

$$\Pi_X'(1) = 0 \times P(0) + P_1 \times 1 + P_2 \times 2 + P_3 \times 3 + P_4 \times 4 + - - -$$

$s = 1$

$$\boxed{\Pi'(1) = E[X]}$$

$$\boxed{\Pi_X'(1) = P_1 + 2P_2 + 3P_3 + 4P_3 + - - -}$$

$$\boxed{\Pi_X'(1) = E[X]}$$

moment generation $\xrightarrow[\text{Diff}]{s=1} E[X]$

Moment generation function

$$E[X^2] = \sum_{x=0}^{\infty} x^2 P(X=x)$$

$$\Pi_X(s) = \sum_{x=0}^{\infty} P(X=x)\, s^x$$

$$\Pi_X'(1) = E[X]$$

→ Discrete Distribution

In Binomial Distribution

$$\Pi_X'(1) = E[X]$$

$$\Pi_X'(s) = \sum_{x=0}^{\infty} x\, s^{x-1} P(X=x)$$

Again Differentiation

$$\Pi_X''(s) = \sum_{x=0}^{\infty} x(x-1)\, s^{x-2} P(X=x)$$

$$\boxed{s=1} \quad \Pi_X''(1) = \sum_{x=0}^{\infty} x(x-1) P(X=x)$$

$$\Pi_X''(1) = \sum_{x=0}^{\infty} x^2 P(X=x) - \sum_{x=0}^{\infty} P(X=x)\, x$$

$$\Pi_X''(1) = E[X^2] - E[X]$$

$$E[X^2] = \Pi_X''(1) + \mu$$

$$V(X) = E[X^2] - \left[E[X]\right]^2$$

$$\boxed{V(X) = \Pi_X''(1) + \mu - \mu^2}$$

In Moment generating Function

$\Pi_X'(1) = E[X]$

$$\boxed{Var(X) = \Pi_X''(1) + \mu - \mu^2}$$

$\rightarrow$ variance

Standard deviation $= \sqrt{\Pi''_X(1) + \mu - \mu^2}$

✓ $\Pi_X(s) = \sum_{x=0}^{\infty} s^x P(X=x)$ s^x coefficient $P(X=x)$

✓ (Discrete)

✓ $\Pi_X(s) = \sum_{x=0}^{\infty} s^x P(X=x)$ — $\Pi_X(0) = 0$, $\Pi_X(1) = 1$

✓ $\Pi_X(s) = M_G(F) = s^0 P(0) + s^1 P(1) + s^2 P(2) + \cdots s^x P(X=x)$

✓ $\left[\Pi_X'(s)\right]_{s=1} = E[X]$ = expectation

✓ $V(X) = \left[\Pi_X''(s)\right]_{s=1} + \mu - \mu^2$

✓ $S.D = \sqrt{\text{var}(X)}$

✓ $B(n, p)$ ✓ Two parameters

$M_G(F) = \Pi_X(s) = (q + ps)^n$

$\Pi_X'(1) = np = E[X]$

$$\Pi_X(s) = (q+ps)^n$$

$$\Pi_X'(s) = n(q+ps)^{n-1}p$$

for $s=1$

$$\Pi_X'(s) = n\underline{(q+p)}^{n-1}\cdot p$$

$$\boxed{\Pi_X'(1) = np}$$

$$\Pi_X''(s) = np\cdot(n-1)(q+ps)^{n-2}\cdot p$$

$$\Pi_X''(1) = np(n-1)\underline{(q+p)}^{n-2}p$$

$$\Pi_X''(1) = np(n-1)p$$

$$= np^2(n-1)$$

$$V(X) = \Pi''_X(1) + \mu - \mu^2$$

$$= \underline{npq}$$

$B(n, p)$

$$\Pi_X(s) = \sum_{x=0}^{\infty} s^x \cdot \underbrace{\left({}^nC_x \, p^x q^{n-x}\right)}_{\text{moment generating Function}}$$

$$\Pi_X(s) = (q+ps)^n \quad \text{Binomial}$$

$\longrightarrow$ one time Diff

$$\Pi_X'(s) = n(q+ps)^{n-1} \cdot p$$

$$\Pi_X'(s) = np(q+ps)^{n-1}$$

$s = 1$

$$\Pi_X'(1) = np(\underbrace{q+p})^{n-1}$$

$$= np(1)^{n-1} = np$$

1) $B(n, p) = \Pi_X(s) = (q + ps)^n$

$E[X] \rightarrow np$

$V(X) \rightarrow npq$

$S.D \rightarrow \sqrt{npq}$

$E[X] = np$
$V(X) = npq$
$S.D = \sqrt{npq}$

2) geometric Function

$\boxed{G(p) = \Pi_X(s) = q^{x-1}p}$

mean
variance

$\frac{q}{p}$

geometric Distribution

$$\left\{\begin{array}{ll} p & H \\ qp & TH \\ q^2 p & TTH \\ q^3 p & TTTH \\ q^4 p & TTTTH \end{array}\right.$$

$X = 1, 2 \ldots\ldots$

$$\Pi_X(s) = \sum_{X=1}^{\infty} q^{x-1} p \cdot s^X$$

$$= \sum_{X=1}^{\infty} (qs)^x \cdot p \cdot \frac{1}{q}$$

$$= p \sum_{x=1}^{\infty} (qs)^x \frac{1}{q}$$

$$= \frac{p}{q}\left[qs + q^2 s^2 + q^3 s^3 + q^4 s^4 + \ldots \right]$$

$$= \frac{p}{\cancel{q}} \times \cancel{q} \left[s + q s^2 + q^2 s^3 + q^3 s^4 + \ldots \right]$$

$$\Pi_X(s) = \sum_{X=1}^{\infty} s^X q^{x-1} \cdot p$$

$$= p\,[\underbrace{s + qs^2 + q^2s^3 + q^3s^4 + \cdots}_{G.P}]$$

$$S_\infty = \frac{s}{(1-qs)}$$

geometric Dis

$$= p \cdot \frac{s}{(1-qs)}$$

$$\boxed{\Pi_X(s) = \frac{ps}{(1-qs)}} \longrightarrow \Pi_X'(s) = \frac{p}{(1-qs)^2}$$

$$E[X] = \Pi_X'(s)$$

$$\Pi_X'(1) = \frac{p \times 1}{(1-q)^2}$$

$$= \frac{p}{(1-q)^2} = \frac{1}{p} \qquad \boxed{E[X] = \frac{1}{p}}$$

$$\boxed{V(X) = \frac{q}{p^2}}$$

GEOMETRIC DISTRIBUTION:

Tossing A coin

H

T $\boxed{H}$

T T $\boxed{H}$

T T T $\boxed{H}$

T T T T $\boxed{H}$

T T T T T $\boxed{H}$

GEOMETRIC Distribution

Geometric Distribution Intersted in only one SUCCESS

Until / final

Experiment
- SUCCESS (p)
- failure (q)

(Bernoulli Trials)

Trials Are Involving → SUCCESS

Red

S

F S

F F S

F F F S

F F F F S

F F F F F S

F F F F F F S — STOP.

S

I am Intersted only one SUCCESS

GEOMETRIC Distribution $\longrightarrow$ Trials Are Not fixed

$\downarrow$ till

SUCCESS (only one Success)

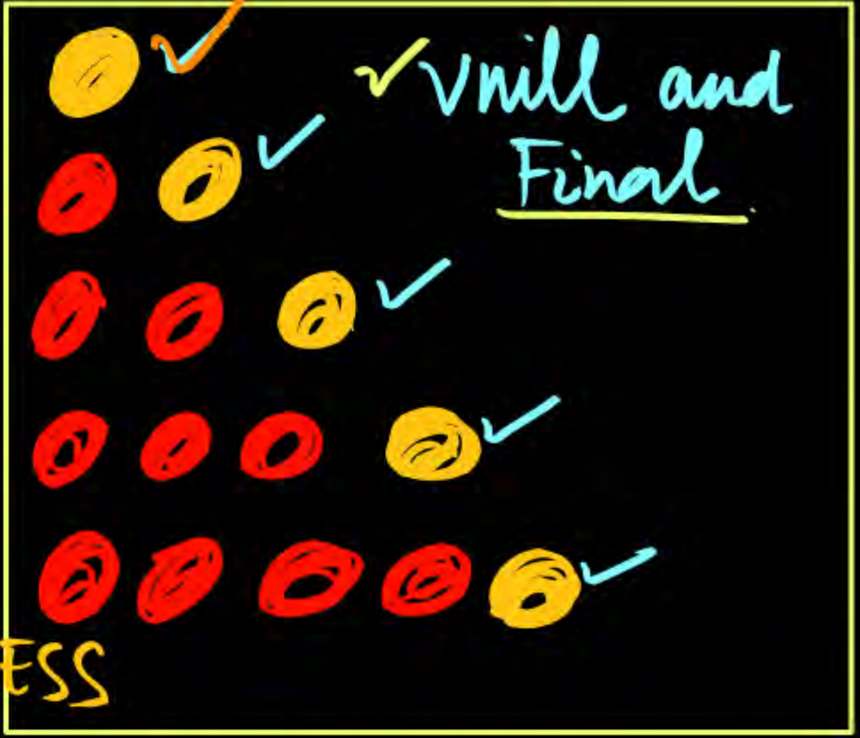
S

FS

FFS

FFFS

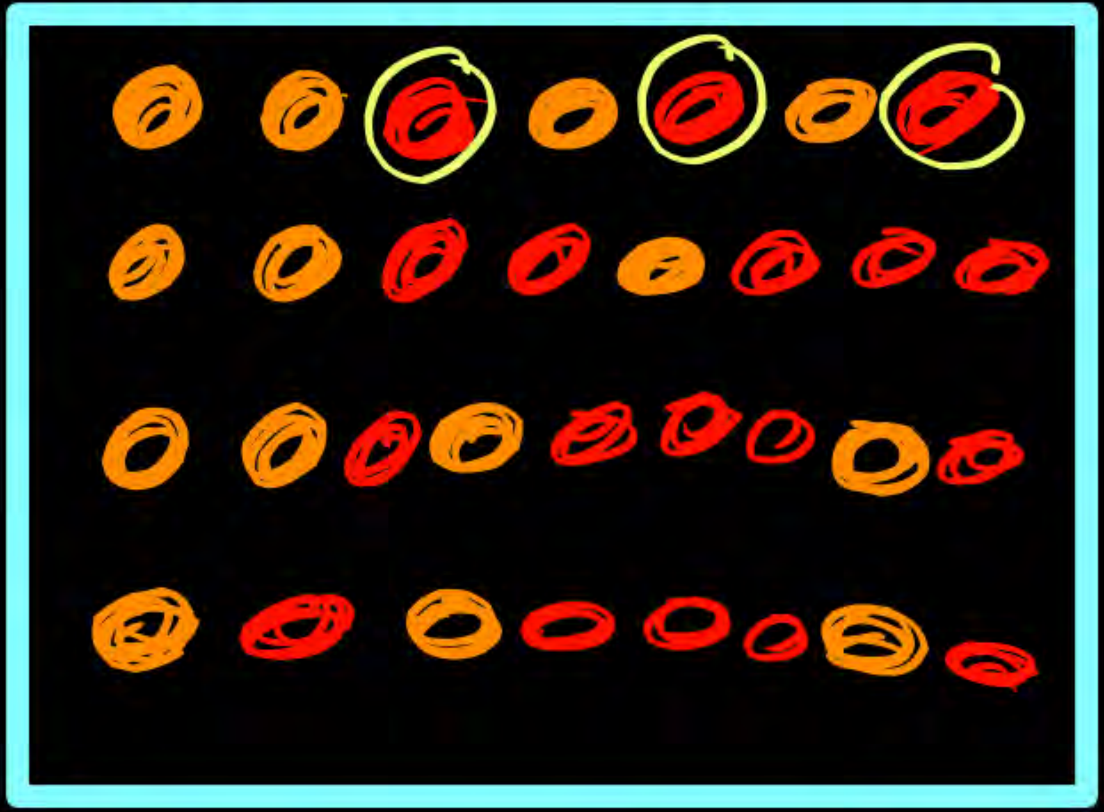
Failure SUCCESS



$X \sim GEO(P)$

Discrete Random variable

$B(n, p)$



$X =$ 1 red ball

2 red ball

3 red ball

4 red ball

$X = 0, 1, 2, 3, 4$

$X = 0, 1, 2, 3 \ldots$

If this is a Discrete Distribution

$X \sim GEO(P)$ $PMF = P(X=x) = \begin{cases} q^{x-1}\, p & x = 1, 2, 3 \text{ - - -} \\ 0 & \text{otherwise} \end{cases}$

failure

Pass (SUCCESS)

Moment generating Function

$M_G(s) = \pi_X(s) = \dfrac{ps}{(1-qs)}$

Statistical Average

$MEAN = \dfrac{1}{p}$

$V(X) = \dfrac{q}{p^2}$

Where $\left.\begin{matrix} q = p(F) \\ p = p(S) \end{matrix}\right]$ both Are Independent

THANK - YOU