DATA SCIENCE

Probability and Statistics

Introduction to Probability

Lecture No.- 01

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Topics to be Covered : Introduction to Probability

Probability :

study of uncertainty

Random experiment

Results are Different

Random experiment — Results are Different

Tossing A coin — HEAD / TAIL

Tossing A unfair coin (Biased coin) — H T / H T

Results are Different or Not

Biased coin (Un fair coin)	Fair coin (Unbiased coin)
mass Distribution Different	mass Distribution SAME

Coins
- Fair coin T H (Mass Distribution SAME)
- Unfair coin (Mass Distribution Different)

H weight
T due to gravity
always show HEAD.

Random Experiment → Must

Unbiased → Results Are Different

Biased → Results ARE SAME X

 → Balanced Die

All surfaces Are SAME mass.

 → biased → 6 → Prob. X

everytime comes - 6

P and C ← # Random Experiment
Results / outcomes / SAMPLE Point
← # Events (subSET)

Tossing A coin

SAMPLE POINT $S = \{H, T\}$

✓ Heads appears on the coin
✓ Tail appears on the coin

A = Head appears
B = Tail appears

Total No. of elements

$$n(S) = n(A) + n(B) = 1 + 1$$

$$\boxed{n(S) = 2}$$

$$P(\text{Event}) = P(E) = \frac{\text{No. of Favourable}}{\text{Total outcomes}}$$

= Relative frequency

$$P(E) = \frac{\text{No. of desired outcomes}}{\text{Total No. of Possible outcomes}}$$

$$P(\text{HEAD}) = \frac{n(H)}{n(S)} = \frac{1}{2}$$

$$P(\text{TAIL}) = \frac{n(T)}{n(S)} = \frac{1}{2}$$

50% chance

H T

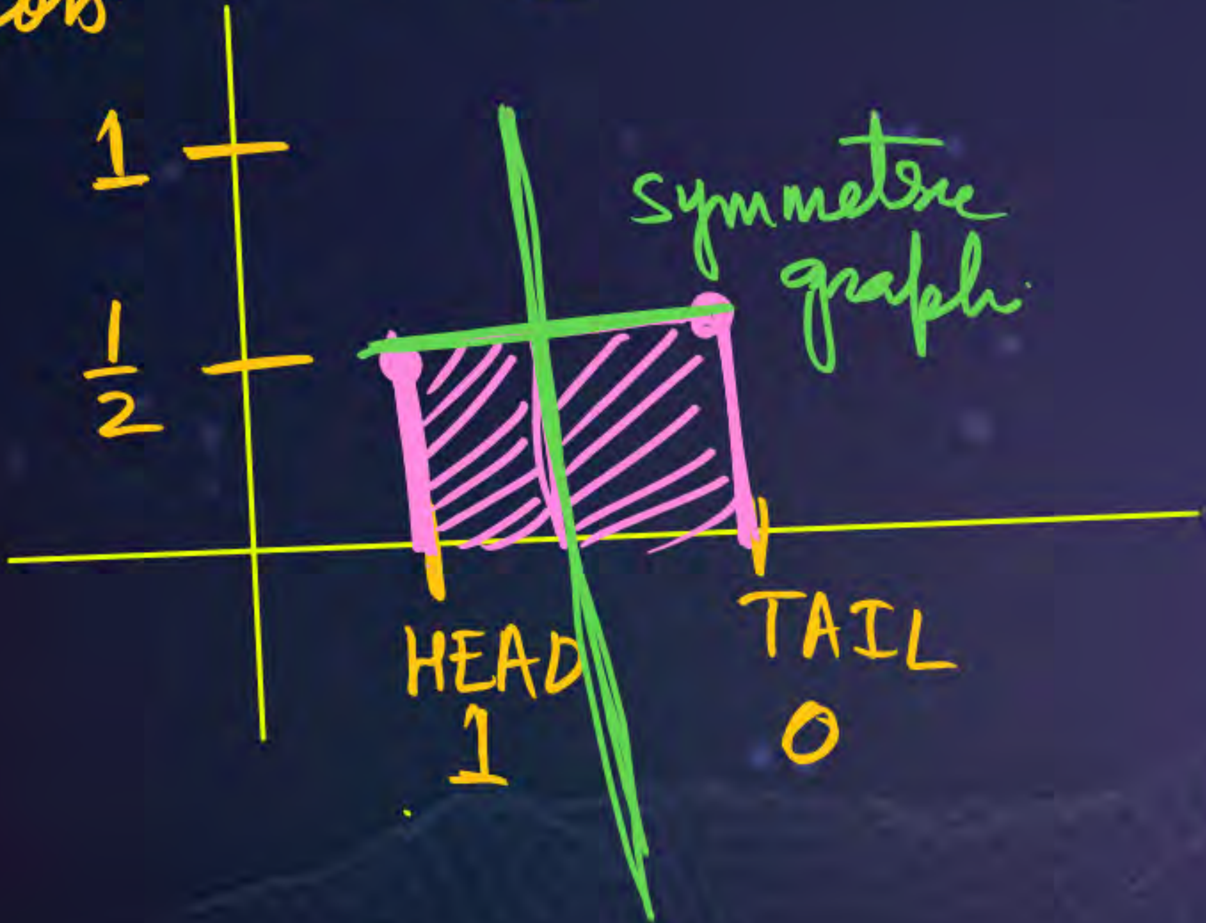
Tossing A coin

Re 1 < H, T

$P(H) = \frac{1}{2}$

$P(T) = \frac{1}{2}$

Prob.



Tossing A Two Coin

$W_1 \to 2$

$W_2 \to 2$

Total No $\to 2^2 = 4$

Simultaneously (Re 1, Re 2)

Draw

H T H T

Using Tree Diagram

OR using matrix method

Re2 \ Re1	H	T
H	HH	HT
T	TH	TT

$P(TT) = \frac{1}{4}$

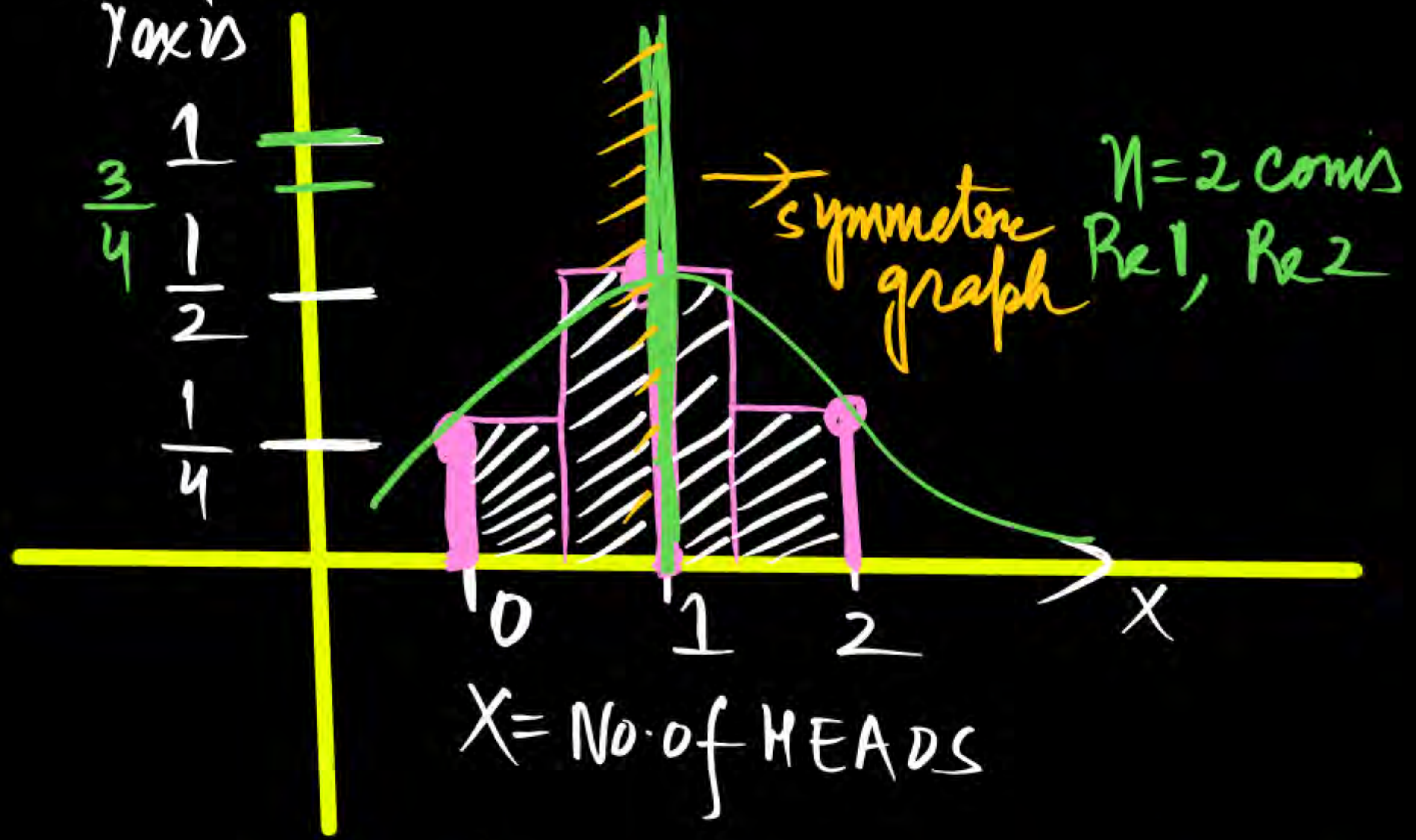
$S = \{\underset{1/4}{HH}, \underset{1/4}{HT}, \underset{1/4}{TH}, \underset{1/4}{TT}\}$

$P(HH) = \frac{1}{4}$

P(TH or HT)

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Slide : 3



$S=\{HH, HT, TH, TT\}$

$P(1\text{ HEAD exactly}) = \frac{2}{4} = \frac{\{HT, TH\}}{\{HH, HT, TH, TT\}} = \frac{1}{2}$

$P(2\text{ HEAD}) = \frac{(HH)}{(HH, HT, TH, TT\}} = \frac{1}{4}$

$P(0\text{ HEAD}) = \frac{\{TT\}}{\{HH, HT, TH, TT\}} = \frac{1}{4}$

for n = 3 coins Re1, Re2, Re5

n^N

$= (2)^3 =$

$= 8$ outcomes

= HHH

= HHT

= HTH

= HTT

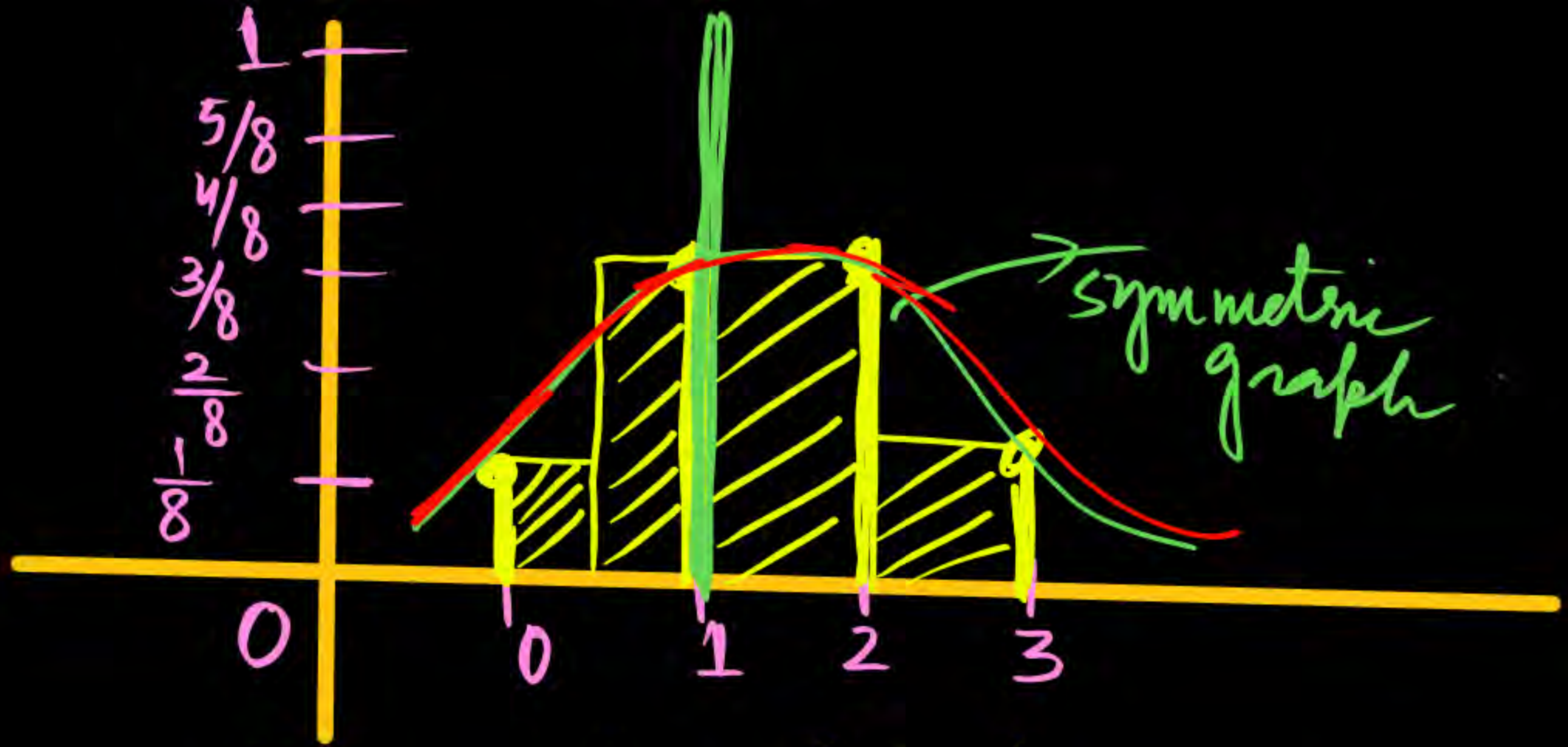
= THH

= THT

TTH

TTT

P(0H)	P(0T)
P(1H)	P(1T)
P(2H)	P(2T)
P(3H)	P(3T)



$P(0H) = \frac{1}{8}$

$P(1H) = \frac{3}{8}$

$P(2H) = \frac{3}{8}$

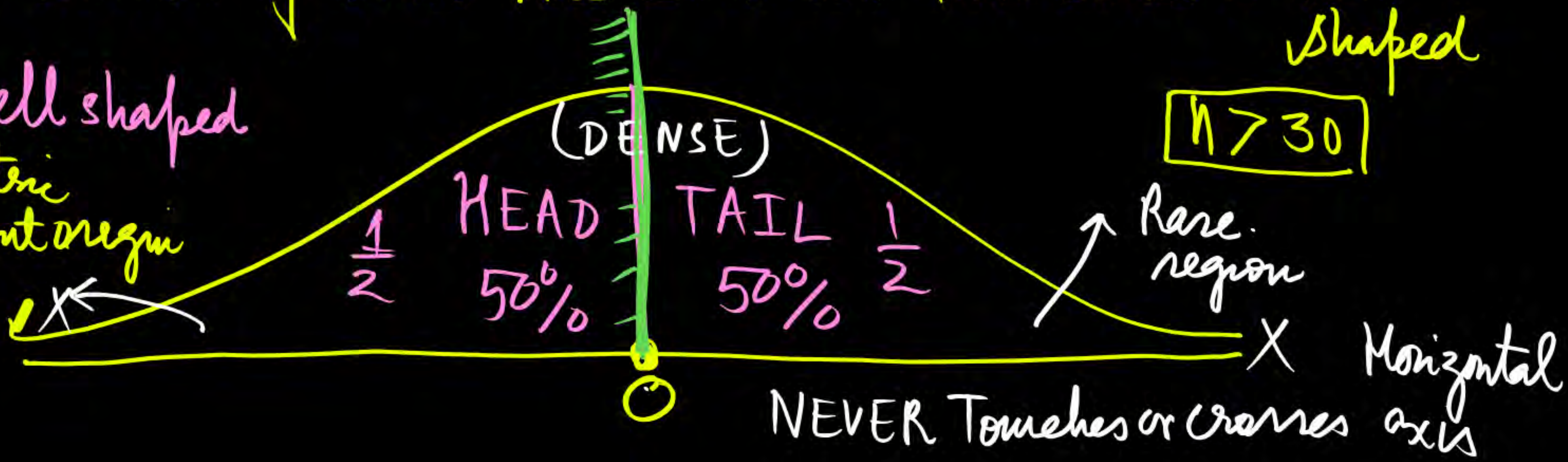
$P(3H) = \frac{1}{8}$

If Number of trials Are Increased Then curve is bell shaped

$\boxed{n > 30}$

✓ bell shaped

✓ symmetric about origin



New Defination

If Number of trials are sufficiently large

Then Prob. of event $= \lim_{n\to\infty} \frac{n(A)}{n} = P(A) = \text{constant}$

prob.

n is sufficiently large No.

$$\boxed{= \lim_{n\to\infty} \frac{n(A)}{n} = \text{constant No} = \text{Prob} = 0 \le P(A) \le 1}$$

statistical view

Infinite No. of Trials.

OR

$$\boxed{\frac{n(A)}{n} = P(A) = \text{prob. of event } A}$$

✓ Throwing A Die:

$n = 1$

$P(1) = \frac{1}{6}$ $P(3) = \frac{1}{6}$ $P(5) = \frac{1}{6}$

$P(2) = \frac{1}{6}$ $P(4) = \frac{1}{6}$ $P(6) = \frac{1}{6}$

✓ Throwing A Two Die

Die A, Die B

$2 \rightarrow \frac{1}{36}$

$3 \rightarrow$

SUM

Die A + Die B

graph → bell curve

+ (Die A \ Die B)	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(2) = P(\text{SUM}=2) = \frac{1}{36}$

$P(\text{SUM}=3) = \frac{2}{36}$

$P(\text{SUM}=4) = \frac{3}{36}$

$P(\text{SUM}=5) = \frac{4}{36}$

$P(\text{SUM}=6) = 5/36$

$P(\text{SUM}=7) = 6/36$

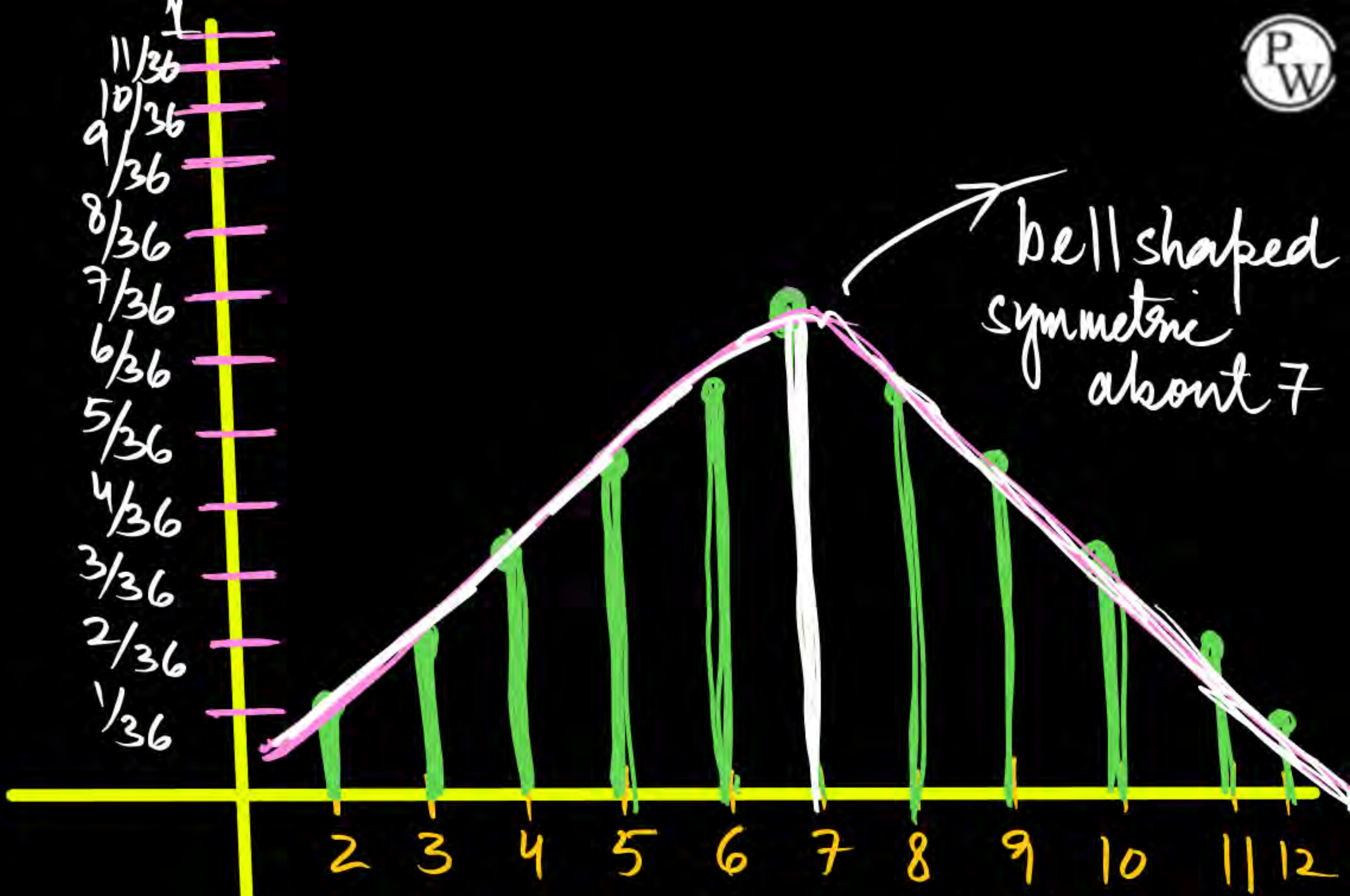
$P(\text{SUM}=8) = 5/36$

$P(\text{SUM}=9) = 4/36$

$P(\text{SUM}=10) = 3/36$

$P(\text{SUM}=11) = 2/36$

$P(\text{SUM}=12) = 1/36$



If trials Are large → bell shaped

∞

Normal curve.

✓ Imp **Counting strategies:**

P(3 green balls Are drawn one at a time)] without replacement

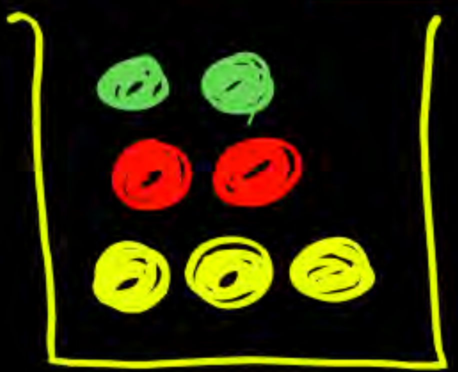
Working Together

W_1 Job: fav

Ist draw ✓

$P(\text{green }1) = \frac{3}{8}$

W_2 Job:



SECOND DRAW

$$P\left(\frac{\text{green}2}{\text{green }1}\right) = \frac{2}{7}$$

W_3 Job:

Third Draw

$$P\left(\frac{\text{green }3}{\text{green}1,\ \text{green}2}\right) = \frac{1}{6}$$

Without Replacement
= multiplication all the Prob
= dependent events

$P(3 \text{ green balls}) \Rightarrow \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$

SAMPLE space reduced (Decrease)

chances Are change.
Next prob Effected on Previous one.
Dependent events — without replacement

Illustration

3 coupons

10 I
10 T

What is the Prob making word IIT

$$\begin{matrix}10I\\10T\end{matrix} \xrightarrow[\frac{10}{20}]{I_1} \begin{matrix}9I\\10T\end{matrix} \xrightarrow[\frac{9}{19}]{I_2} \begin{matrix}8I\\10T\end{matrix} \xrightarrow[\frac{10}{18}]{T}$$

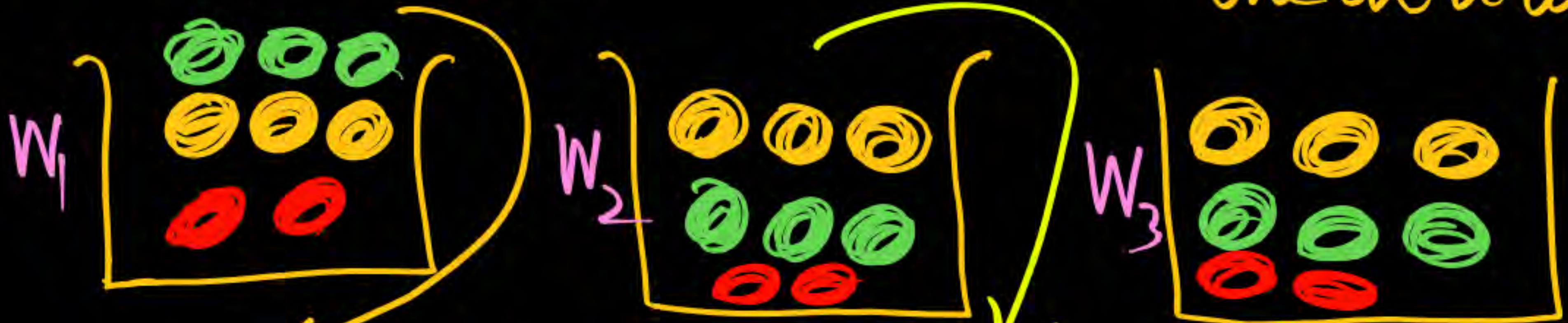
$$P(IIT) = \frac{10}{20} \times \frac{9}{19} \times \frac{10}{18} \longrightarrow \text{Every Event} \rightarrow \left.\frac{{}^{10}C_2 \times {}^{10}C_1}{{}^{20}C_3}\right\} \text{Ans}$$

$$W_{job} = W_1 \times W_2 \times W_3$$

CASE 02

with replacement

"What is the Prob. P(3 green balls are drawn one at a time)



$P(\text{green}_1) = \frac{3}{8}$ 1st draw

$P\left(\frac{\text{green}_2}{\text{green}_1}\right) = \frac{3}{8}$ 2nd green $= \frac{3}{8}$

$P\left(\frac{\text{green}_3}{\text{green}_1, \text{green}_2}\right)$

$$P(\text{3 green balls}) = P(\text{green 1})\, P\left(\frac{\text{green 2}}{\text{green 1}}\right) P\left(\frac{\text{green 3}}{\text{green 1 green 2}}\right)$$

$$= \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8}$$

- # SAMPLE spaces Not reduced
- # Chances don't change
- # follow the multiplication Rule
- # Independent events

(With replacement)
(Independent Events)

✓ without replacement

Illustration: THREE cards Are drawn at random one at a time

$\boxed{\text{one of THOSE cards. J, Q, K.}}$

4K, 4Q
4A
52 cards

J Q K ✓ $= \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$

J K Q ✓ $= \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$

K J Q ✓

K Q J ✓

Q J K ✓

Q K J ✓

1

52 cards

26 red → Diamond, HEART

26 black → Spade, club.

4A
4Q, 4K
52 cards

→ J, Q, K
order fixed

$= \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$

$JQK = \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} \times 3! = \underline{Ans}$

"n Different Items Taken all at Time" $= n!$

(2)

If 2 balls Are drawn at random one at a time one of Those balls Are orange and yellow.

2! ways [=

$+ = \frac{2}{4} \times \frac{2}{3} \times 2!$

$= \frac{8}{12} = \frac{2}{3}$ $\underline{Ans}$


Thank You