Data Science and Artificial Intelligence

Probability and Statistics

Random Variable

Lecture No.- 02

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Recap of Previous Lecture

Topic Random Variable Part-1

- ✓ Random Variable
- ✓ Types of Random Variable
- ✓ cdf (Distribution)
- ✓ Cumulative Dist. function

Topics to be Covered

Topic Random Variable Part-2

Tossing A THREE coins:

Integer value ← X = No. of HEADS

$F(x_i) = P(X \le x_i)$

X	0	1	2	3
$P(X=x)$	$1/8$	$3/8$	$3/8$	$1/8$

→ No. of customer

Integer { No. of HEADS, No. of balls, No. of shops }

valid prob. dis

$\sum_{i=0}^{n} P[X=x_i] = 1$

$F(3) = P(X \le 3)$

$= P_0 + P_1 + P_2 + P_3$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{8}{8} = 1$

Re1, Re2, Re5

Plot The cdf.

(Discrete Distribution)

Input = Integer value

($X = 0, 1, 2, 3, 4 \ldots$)

$F(0) = P(X \le 0) = P_0 = \frac{1}{8}$

$F(1) = P(X \le 1) = P_0 + P_1$

$= \frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

$F(2) = P(X \le 2)$

$= P_0 + P_1 + P_2$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

cdf (cumulative Distribution Function):

$F_X(0) = \frac{1}{8}$ $F_X(1) = \frac{4}{8}$ $F_X(2) = \frac{7}{8}$ $F_X(3) = \frac{8}{8}$

$$\sum_{i=0}^{n} P[X = x_i] = 1$$

$$P_0 + P_1 + P_2 + \cdots + P_n = 1$$

$$P[X=0] + P[X=1] + P[X=2] + \cdots P[X=x_n] = 1$$

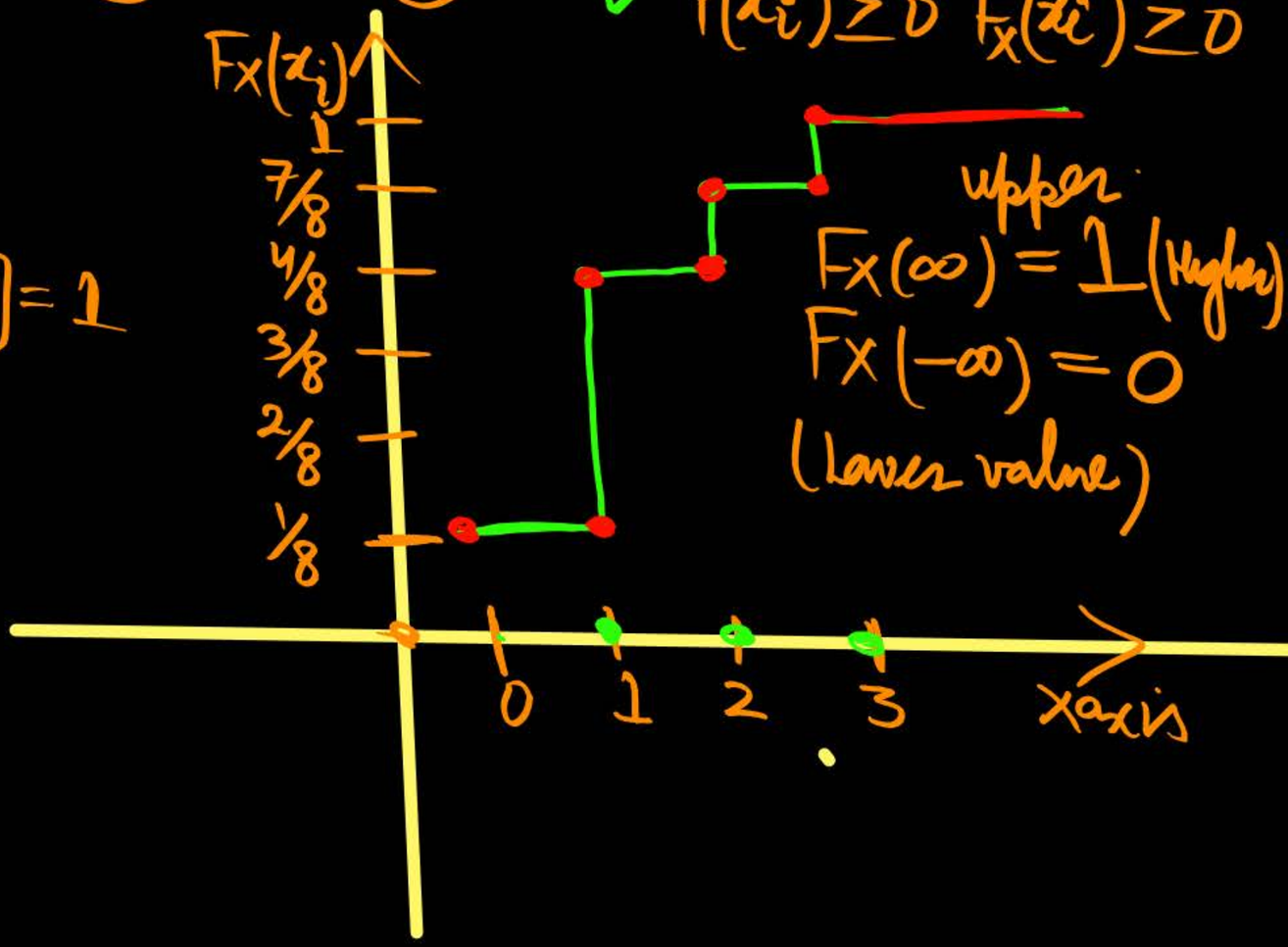
(Discrete Random variable)

$$F_X(x) = cdf = P(X \le x)$$

$F_X(\infty) = 1$

$F_X(-\infty) = 0$

- ✓ step Function
- ✓ Monotonic behaviour
- ✓ staircase function
- ✓ $P(x_i) \ge 0$ $F_X(x_i) \ge 0$

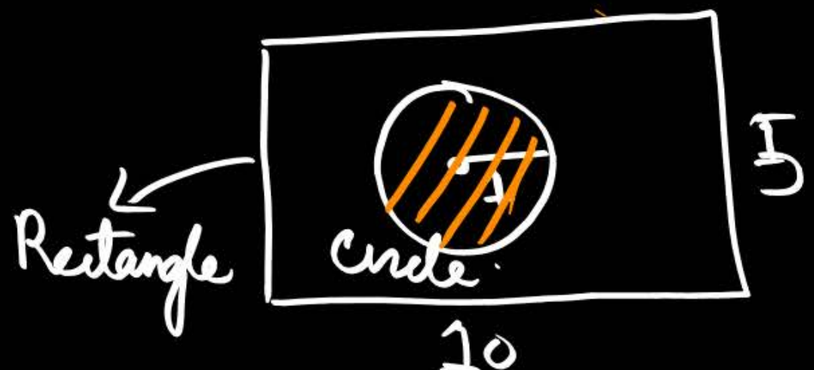


Continuous Probability Distribution:

Uncountable Infinite Number

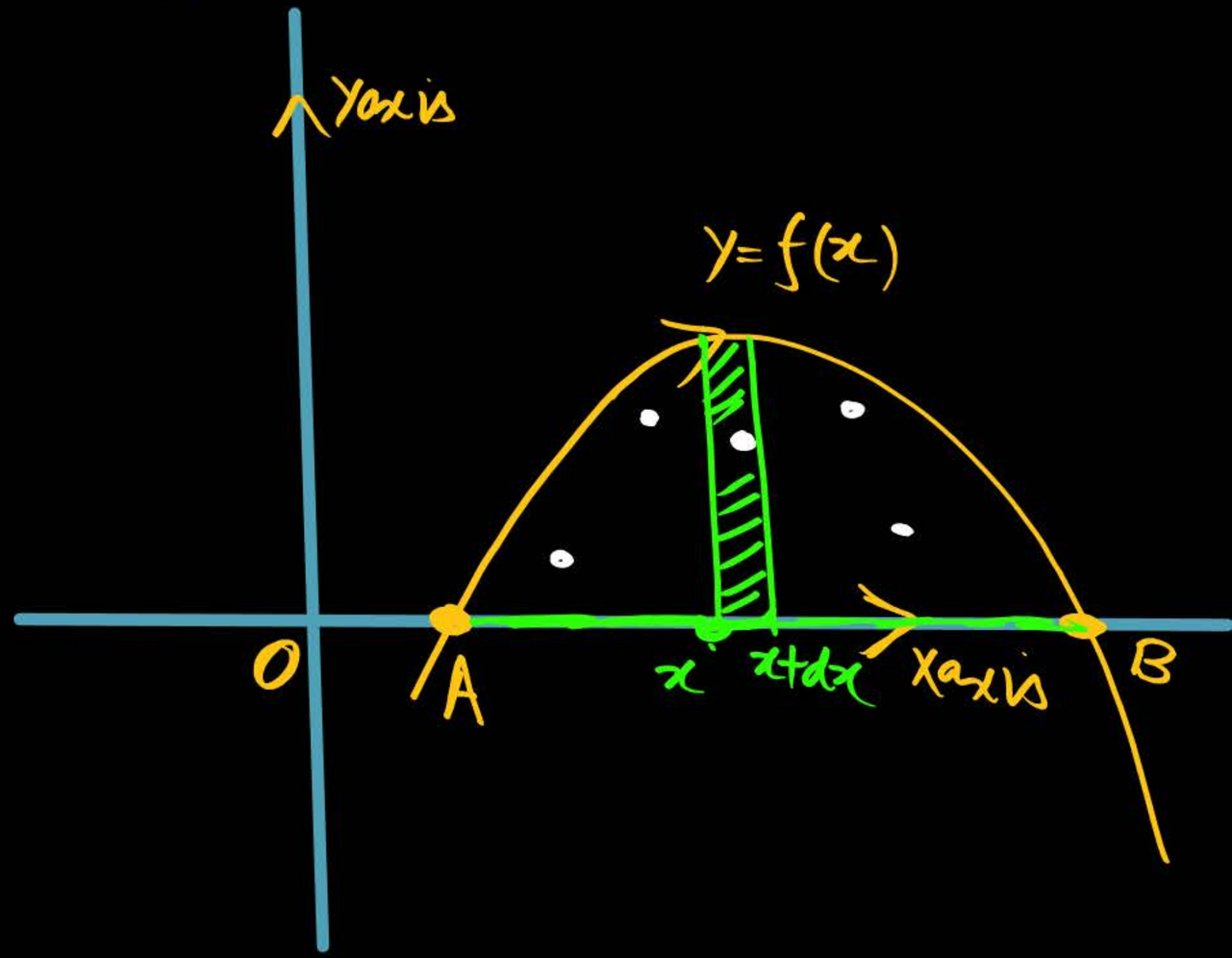
always defined In Interval

- ✓ Discharging A battery
- ✓ Rainfall
- ✓ Height of Person



geometrically

$$\text{Prob}(E) = \frac{\text{fav region}}{\text{Total region}}$$



dx = small element

$$P(x \le X \le x+dx) = P(x \le X \le x+\Delta x)$$

= Area of Rectangle

$$P(x \le X \le x+dx) = f(x) \cdot dx$$

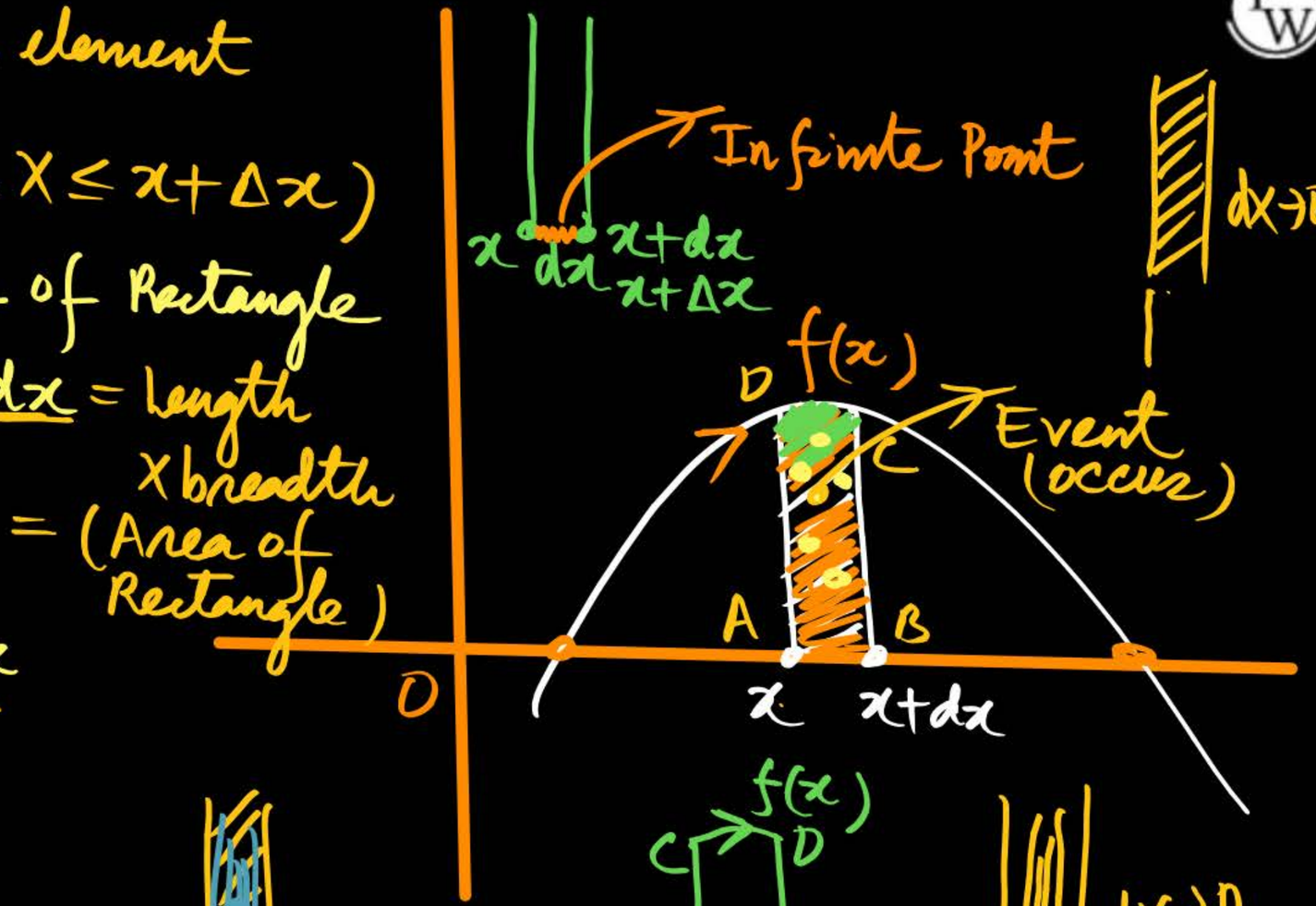
= Length × breadth

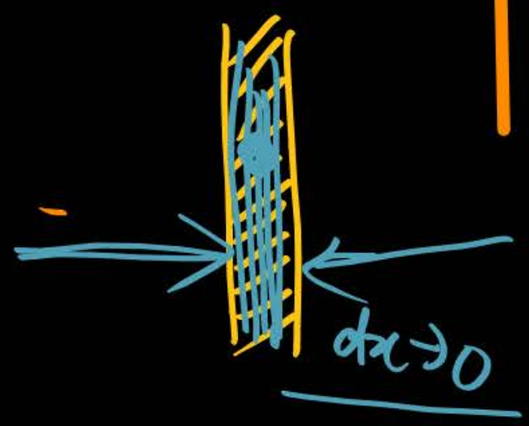
= (Area of Rectangle)

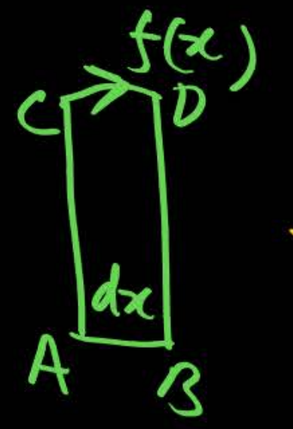
$$F_X(x_i) = P(X \le x_i)$$

$$\frac{F_X(x+dx) - F_X(x)}{dx} = \frac{f(x)\,dx}{dx}$$

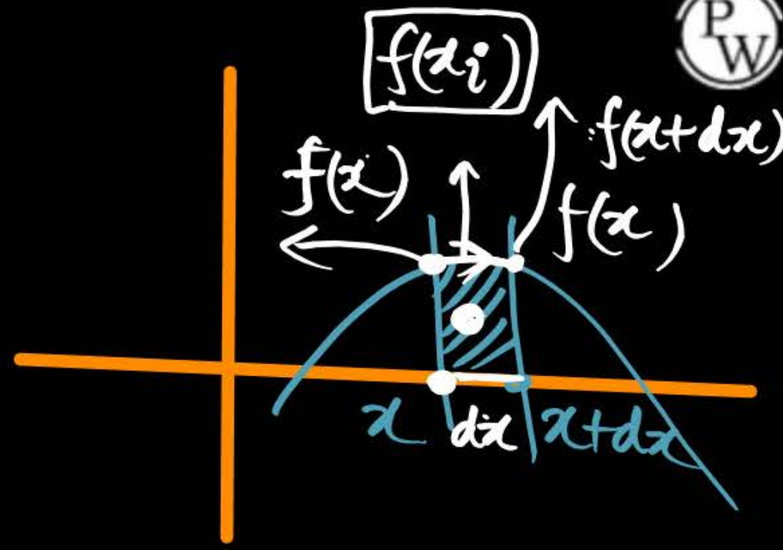
$$\boxed{\frac{F_X(x+dx) - F_X(x)}{dx} = f(x)}$$

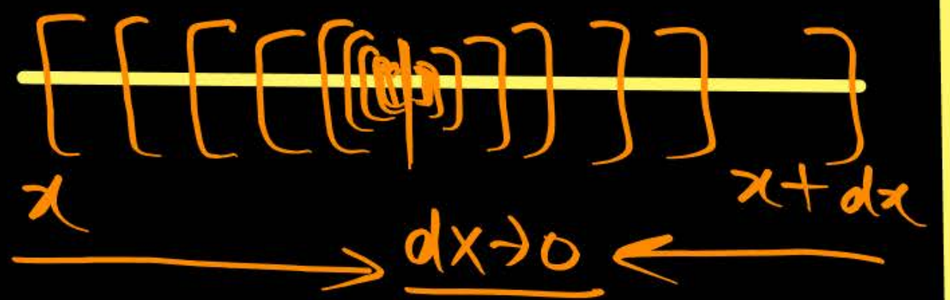


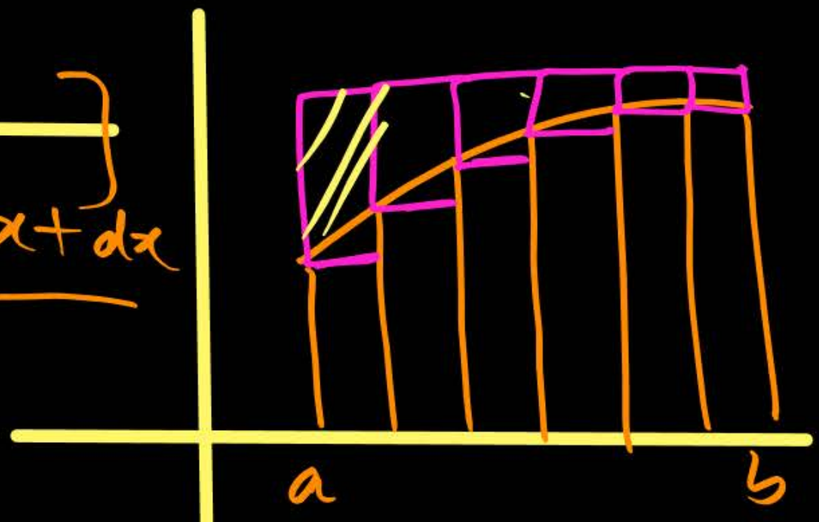


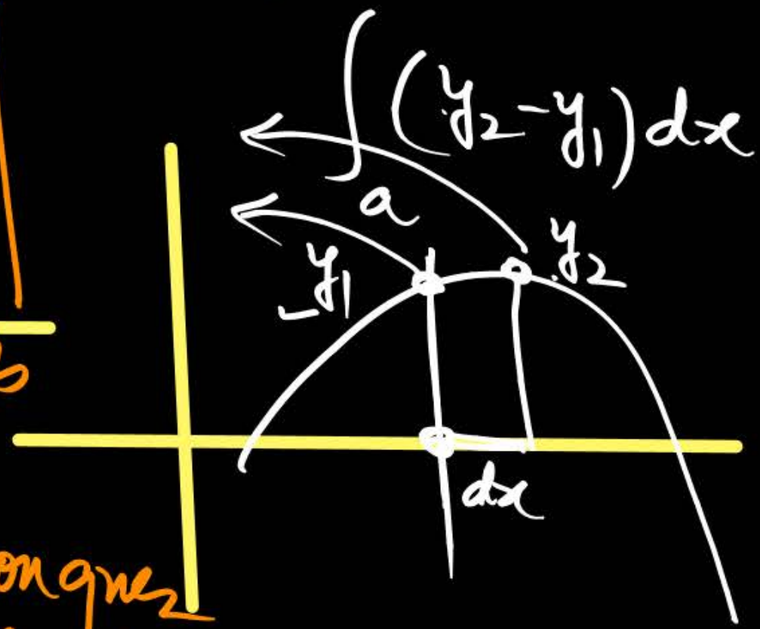


$$\lim_{dx \to 0} \frac{Fx(x+dx) - Fx(x)}{dx} = f(x)$$









✓ Binary SEARCH algorithms.

✓ divide and conquer Rule

$$P(x \le X \le x+dx) = f(x)\,dx$$

$$= \lim_{dx \to 0} \frac{F_X(x+dx) - F_X(x)}{dx} = f(x)$$

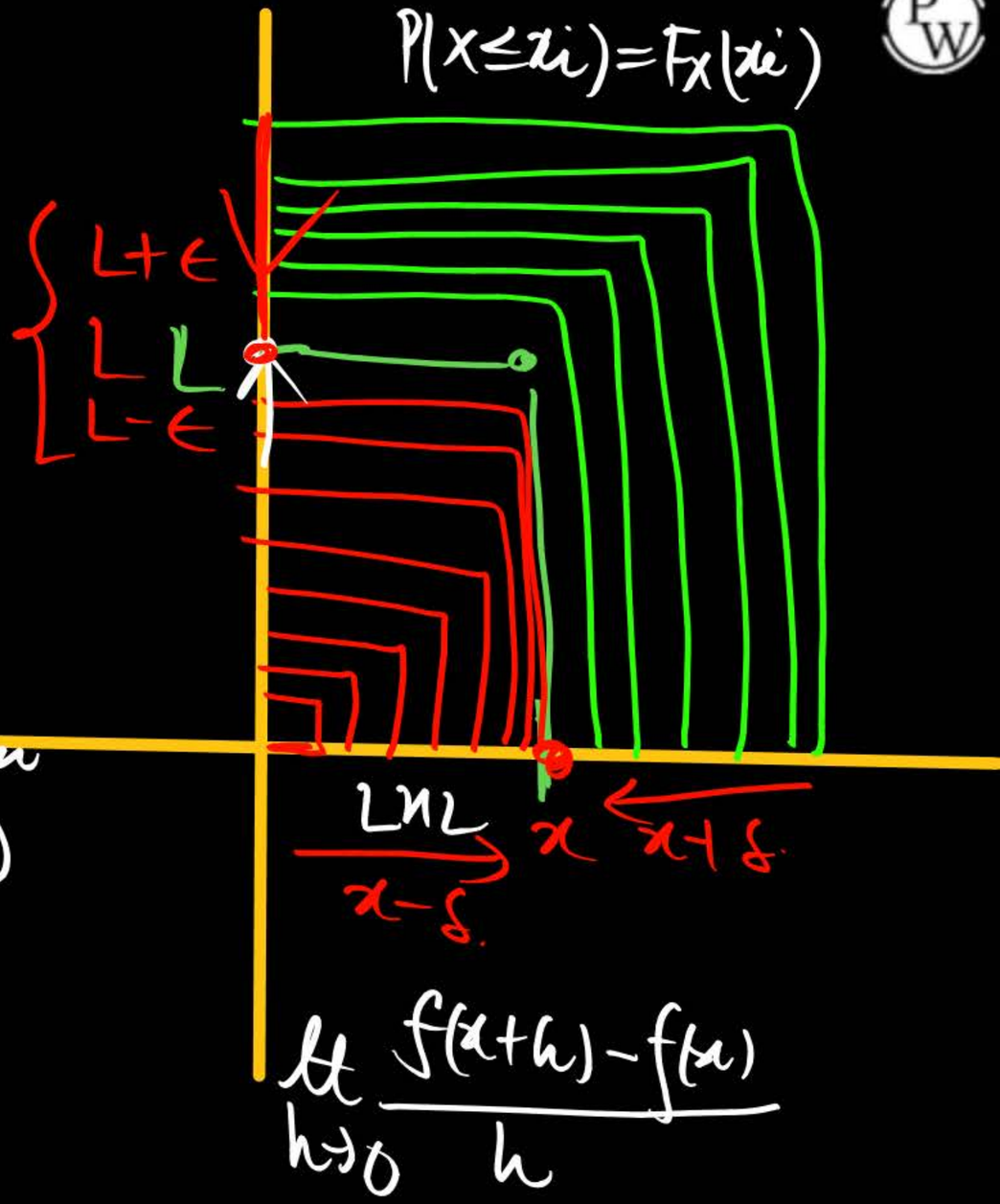
$$\Rightarrow F_X'(x) = f(x)$$

In continuous Probability distribution

$$F_X'(x) = f(x)$$

$$\frac{d}{dx} F(x) = f(x)$$

$$\boxed{F_X(x) = cdf = \int_R f(x)\,dx}$$



$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$F_X(x) = \int_{\text{Region}} f(x)\,dx$$

$$P_X(x) = \int_{\text{region}} f(x)\,dx$$

Probability in continuous random variable

\# $P_X(b) - P_X(a) = F_X(b) - F_X(a) = \int_{a\,(\text{region})}^{b} f(x)\,dx$

Question limit दी होगी $f(x)$ की

\# $P(X \geq a) = \int_{a}^{\infty} f(x)\,dx$

\# $P(X \leq a) = \int_{-\infty}^{a} f(x)\,dx$

\# If $f(x)$ is prob. density Function or valid prob. density function

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Topic : Random Variable

$\frac{1}{2}+\frac{3}{4}$

$-\frac{1}{2}$] Study of belief

Prob can't Negative

Q1. State, giving reasons, which of the following are not probability distributions:

(i)

X	0	1
p(x)	$\frac{1}{2}$	$\frac{3}{4}$

$P(X=0)+P(X=1)=\frac{1}{2}+\frac{3}{4}=\frac{10}{8}=\frac{5}{4}$

$\neq 1$

(ii)

X	0	1	2
p(x)	$\frac{3}{4}$	$+\frac{1}{2}$	$\frac{3}{4}$

$P(X=0)+P(X=1)+P(X=2)=\frac{3}{4}+\frac{1}{2}+\frac{3}{4}$

$=\frac{1}{8}+\frac{3}{8}+\frac{1}{4}+\frac{1}{8}\neq 1$

$=\frac{1+3+2+1}{8}$

(iii)

X	0	1	2
p(x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

YES

$\frac{1}{4}+\frac{1}{2}+\frac{1}{4}$

$=1$

(iv)

X	0	1	2	3
p(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

"In Discrete distribution → Integer $x=0,1,2$, all prob = 1"

$\sum_{i=0}^{n} P[X=x_i]=1$

Topic : Random Variable

Q2. Find the probability distribution of the number of heads when three fair coins are tossed simultaneously.

Topic : Random Variable

"Prob. distribution Table"

Q3 2 bad articles are mixed with 5 good ones. Find the probability distribution of the number of bad articles, if 2 articles are drawn at random.

2 Bad Articles are mixed with 5 good ones.

X = No. of Bad Articles.

X = 0, 1, 2

X	0 (bad)	1 (bad)	2 (bad)
P(X = xi or x)	$\frac{10}{21}$	$\frac{10}{21}$	$\frac{1}{21}$

Prob. Distribution

P[X = 0 Bad Items]

0 bad | 2 bad, 5 good | $= \frac{5}{7} \times \frac{4}{6} = \frac{20}{42} = \frac{10}{21}$

1 bad | 2 bad, 5 good | $= \frac{5}{7} \times \frac{2}{6} \times 2 = \frac{10}{21}$

2 bad | 2 bad, 5 good | $= \frac{1}{21} \Rightarrow \frac{2}{7} \times \frac{1}{63}$

Topic : Random Variable

Q4. Given the probability distribution:

X	0	1	2	3
p(x)	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{1}{10}$

Let $Y = X^2 + 2X$. Find the probability distribution of Y?

$Y = (0)^2 + 2\times0 = 0$, $Y = (1)^2 + 2\times1 = 3$

$P(X=0) = \frac{1}{10}$

$P(Y=0) = \frac{1}{10}$

$Y = (2)^2 + 2\times2 = 8$

$Y = (3)^2 + 3\times2 = 15$

X	0	1	2	3
P(X=x)	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{1}{10}$

$Y = X^2 + 2X$

Y	0	3	8	15
P	$\frac{1}{10}$	$\frac{1}{10}$	0	0

Probability distribution valid or Not (No)

Topic : Random Variable

Prob. Distribution

Q5. An urn contains 3 white and 4 red balls. 3 balls are drawn one by one with replacement. Find the probability distribution of the number of red balls.

3 White, 4 Red

WWW → 0 Red, 3 White: $\frac{3}{7}\times\frac{3}{7}\times\frac{3}{7}$

RWW → 1 Red, 2 White: $\frac{4}{7}\times\frac{3}{7}\times\frac{3}{7}\times 3$

RRW → 2 Red, 1 White: $\frac{4}{7}\times\frac{4}{7}\times\frac{3}{7}\times 3$

RRR → 3 Red, 0 W: $\frac{4}{7}\times\frac{4}{7}\times\frac{4}{7}$

X	0	1	2	3
$P(X=x_i)$	$\frac{27}{343}$	$\frac{108}{343}$	$\frac{144}{343}$	$\frac{64}{343}$

WWR

p alike, q alike $= \frac{\lfloor p+q+r}{\lfloor p \lfloor q \lfloor r}$

Topic : Random Variable

Q6. A continuous random variable X has the probability density function:

$$f(x) = Ax^3, \quad 0 \le x \le 1.$$

$f(x) = Ax^3$, $0 \le x \le 1$

Determine

(i) $A = 4$

(ii) $P[0.2 < X < 0.5]$

(iii) $P\left[X > \frac{3}{4} \text{ given } X > \frac{1}{2}\right]$

If This Function is valid pdf

$$\int_{-\infty}^{\infty} f(x) = 1 \qquad -\infty \le x \le \infty$$

$$\int_0^1 f(x)\,dx = 1$$

$$\int_0^1 Ax^3\,dx = 1$$

$$A\left[\frac{x^4}{4}\right]_0^1 = 1$$

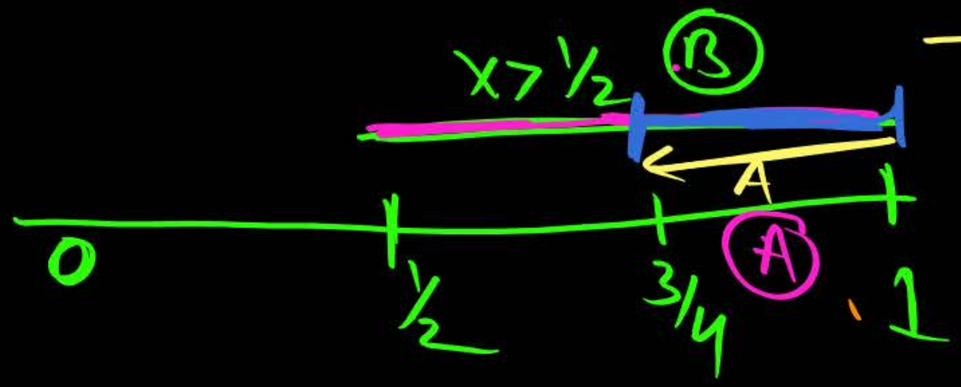
$$\frac{A}{4} = 1$$

$$A = 4$$

✓ $P(0.2 < x < 0.5) = \int_{0.2}^{0.5} Ax^3\,dx = \int_{0.2}^{0.5} 4x^3\,dx =$ Ans $= 0.0609$

✓ $P\left(x > \frac{3}{4} \text{ given } x > \frac{1}{2}\right) = P\left(\frac{x > \frac{3}{4}}{x > \frac{1}{2}}\right) =$ Conditional Prob.

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P\left(x > \frac{3}{4} \cap x > \frac{1}{2}\right)}{P\left(x > \frac{1}{2}\right)} = \frac{P\left(x > \frac{3}{4}\right)}{\int_{1/2}^{1} 4x^3\,dx}$



$= \int_{3/4}^{1} 4x^3\,dx \Big/ \int_{1/2}^{1} 4x^3\,dx = \frac{35}{48}$

Topic : Random Variable

Q7. The life (in hours) X of a certain type of light bulb may be supposed to be a continuous random variable with p.d.f.:

$$f(x) = \begin{cases} \frac{A}{x^3} & 1500 < x < 2500 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the constant A and compute the probability that $1600 \leq X \leq 2000$.

Do yourself

$A = 703125$

$Ans = \frac{2025}{4096}$]

Topic : Random Variable

Q8. The diameter 'X' of a cable is assumed to be a continuous random variable with p.d.f.

$$f(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1 \\ 0, & \textit{elsewhere} \end{cases}$$

Obtain the c.d.f. of X.

$$F(x) = \begin{cases} 0 & x \le 0 \\ 3x^2 - 2x^3 & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

Topic : Random Variable

Q9. A random variable X has the following probability function:

X	0	1	2	3	4	5	6	7
p(x)	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{1}{50}$	$\frac{17}{100}$

Determine the distribution function of X.

Topic : Random Variable

Q10. The p.d.f. of the different weights of a "1 litre pure ghee pack" of a company is given by:

Do yourself

$$f(x) = \begin{cases} 200(x-1) & \text{for } 1 \le x \le 1.1 \\ 0, & \textit{otherwise} \end{cases}$$

Examine whether the given p.d.f. is a valid one. If yes, find the probability that the weight of any pack will lie between 1.01 and 1.02.

Topic : Random Variable

Q12. A random variable X has the following probability distribution:

Do yourself

X	0	1	2	3	4	5	6	7	8
p(x)	K	3k	5k	7k	9k	11k	13k	15k	17k

(i) Determine the value of k. $K = \frac{1}{81}$

(ii) Find the distribution function of X.

THANK - YOU