Data Science and Artificial Intelligence

Probability and Statistics

Bivariate Random Variable

Lecture No.- 05

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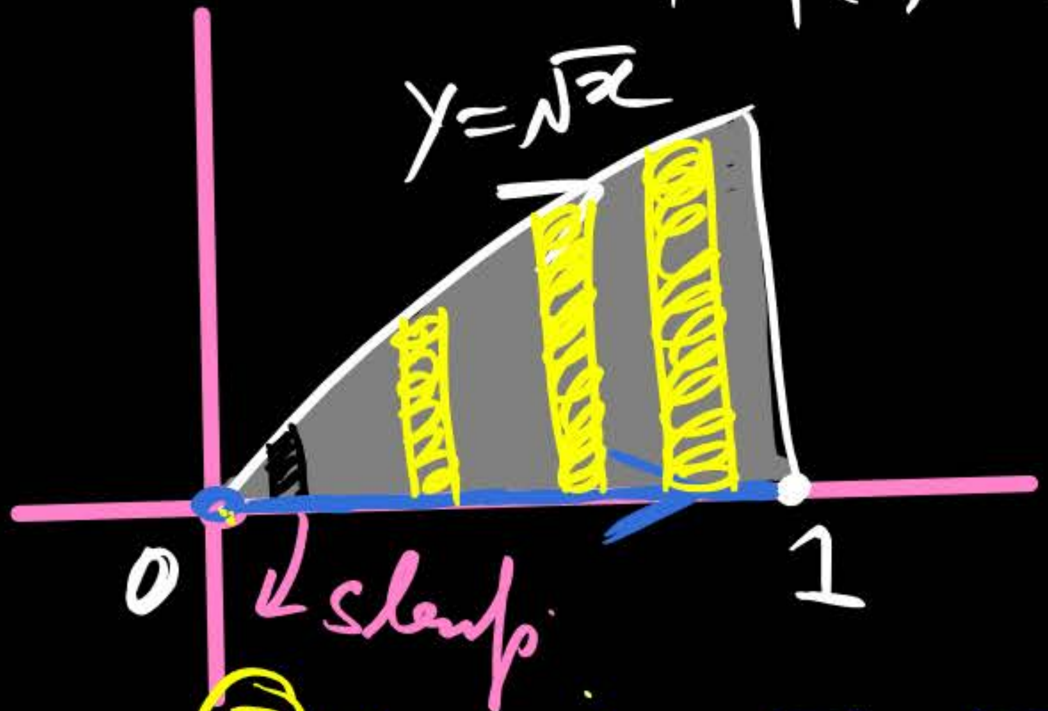
Topics to be Covered

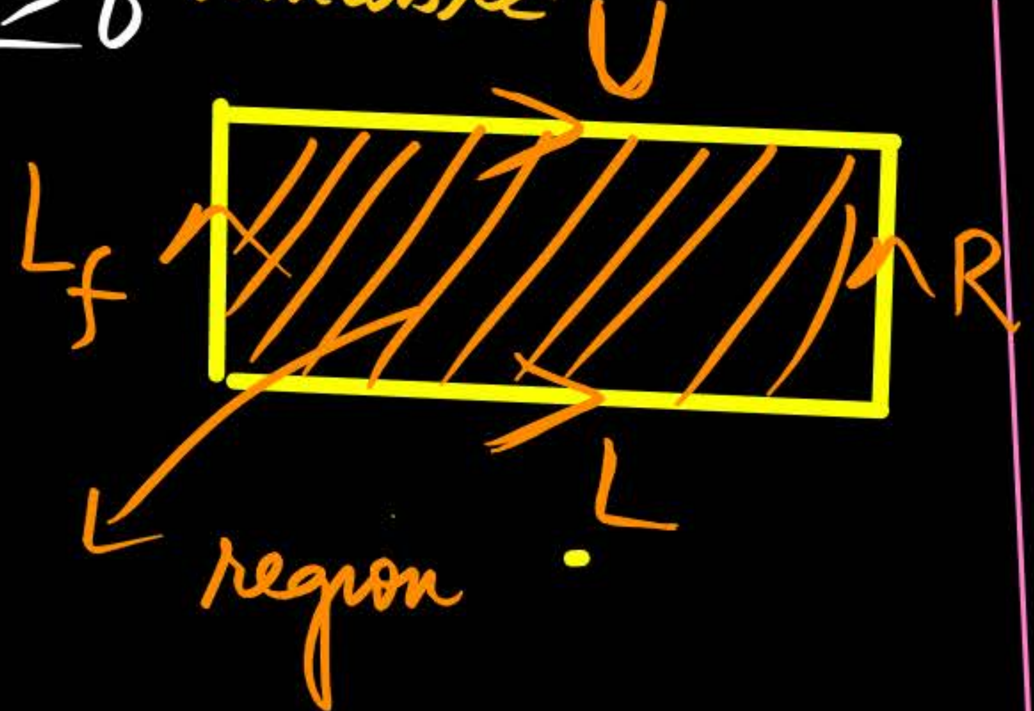
- **Topic**: Volume Via Double Integral
- **Topic**: Change the Order of Integration
- **Topic**: Bivariate Continuous Random Variables

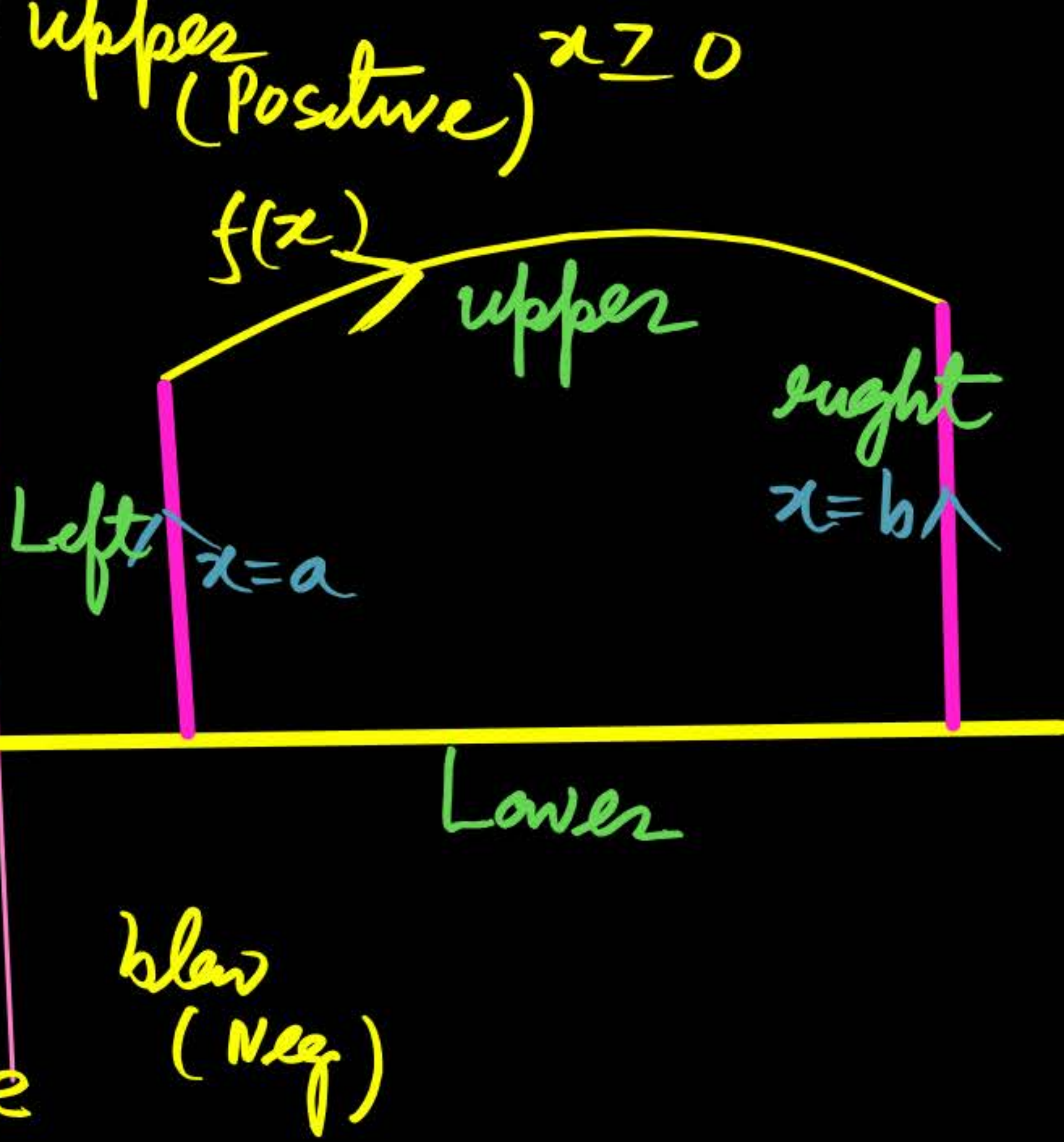
Area Bounded Regions →

If $y=f(x)$ $x=$ Independent variable $y=$ dependent variable.

$y=f(x)$ $x\geq 0$







(I) $x=0$ $y=\sqrt{x}=\sqrt{0}=0=x\cdot y=0\cdot 0=0$

$x=0.1$ $y=\sqrt{0.1}=\sqrt{0.1}=0.1\times\sqrt{0.1}=\mathrm{II}^{nd}$

sum of all rectangles = Area under a curve

$y = f(x) \ \forall \ x \in [a, b]$

Area Bounded region Above The x-axis

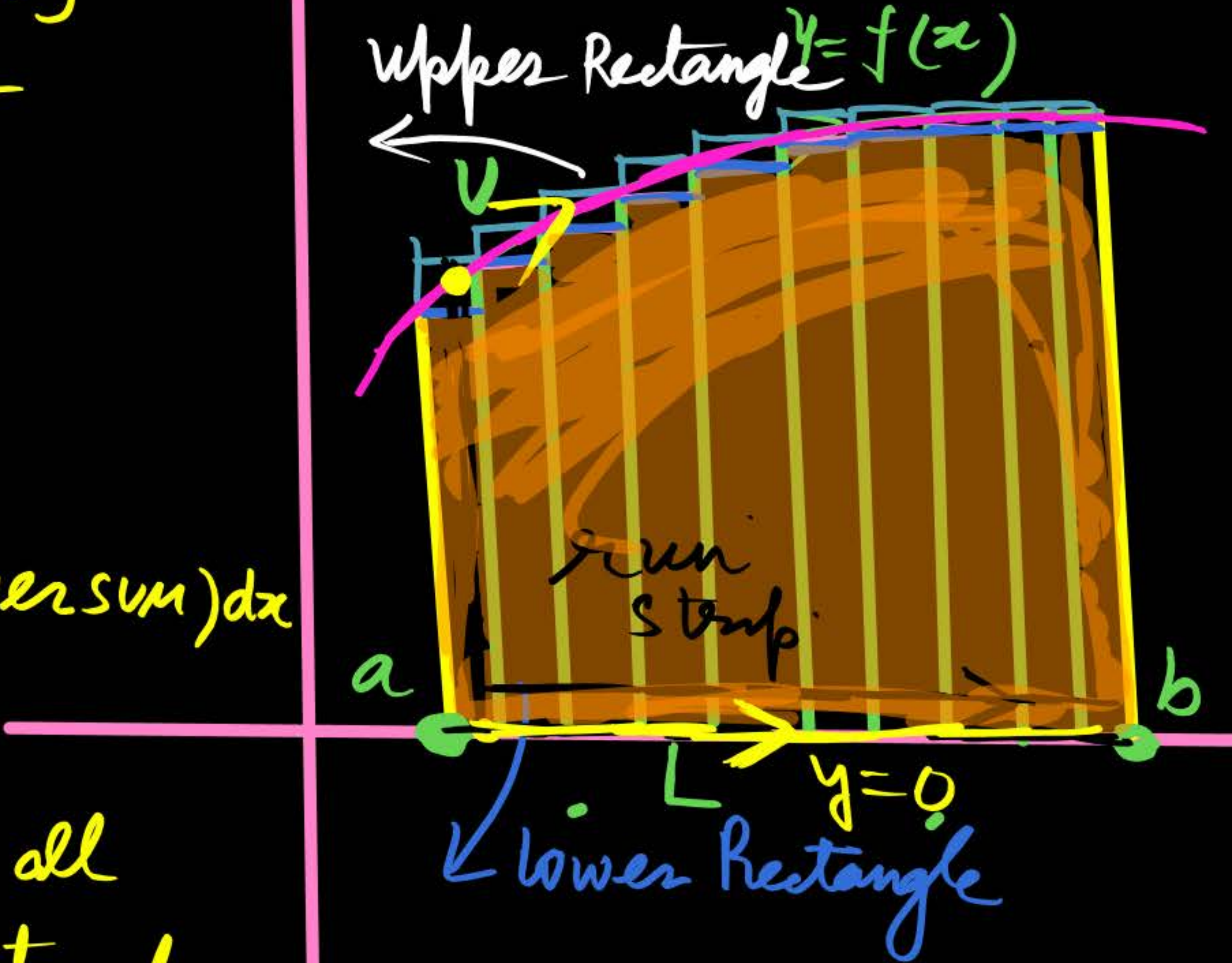
$$\text{Area} = \int_a^b (U - L)\, dx$$

$$= \int_a^b (\text{upper sum} - \text{Lower sum})\, dx$$

$$\text{Area} = \int_a^b [f(x) - 0]\, dx$$

$$\boxed{\text{Area} = \int_a^b f(x)\, dx}$$

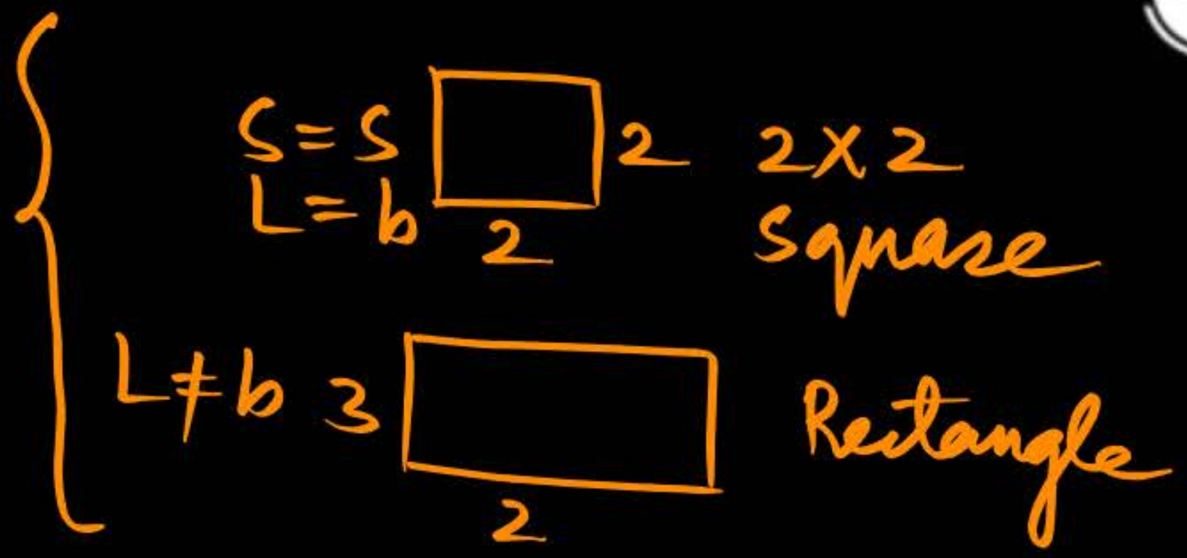
all Rectangle sum

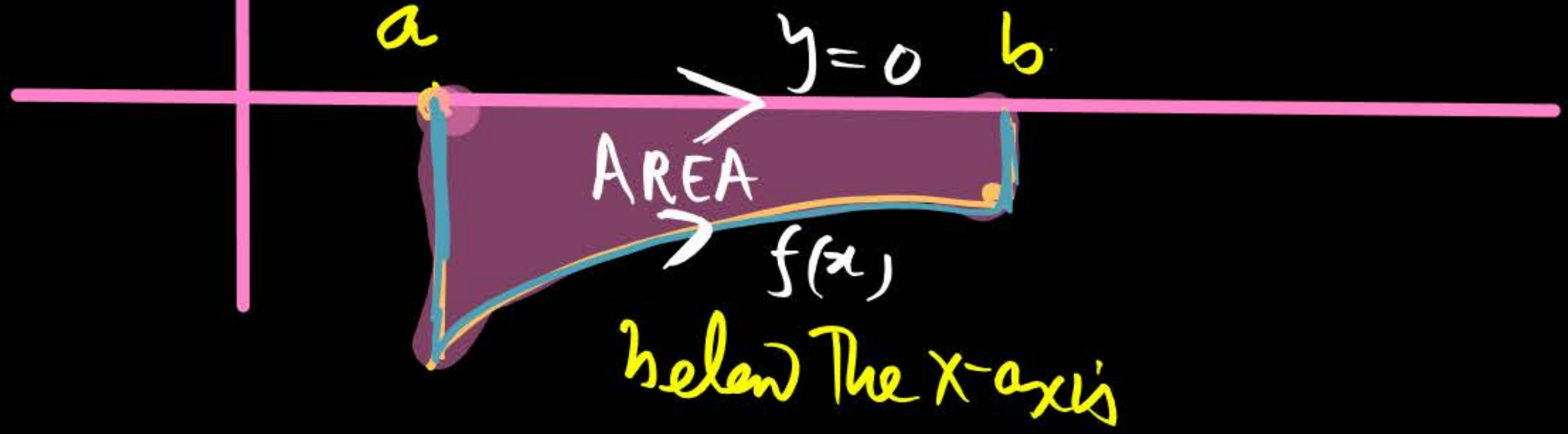


$y = f(x)\ \forall x \in [a, b)$

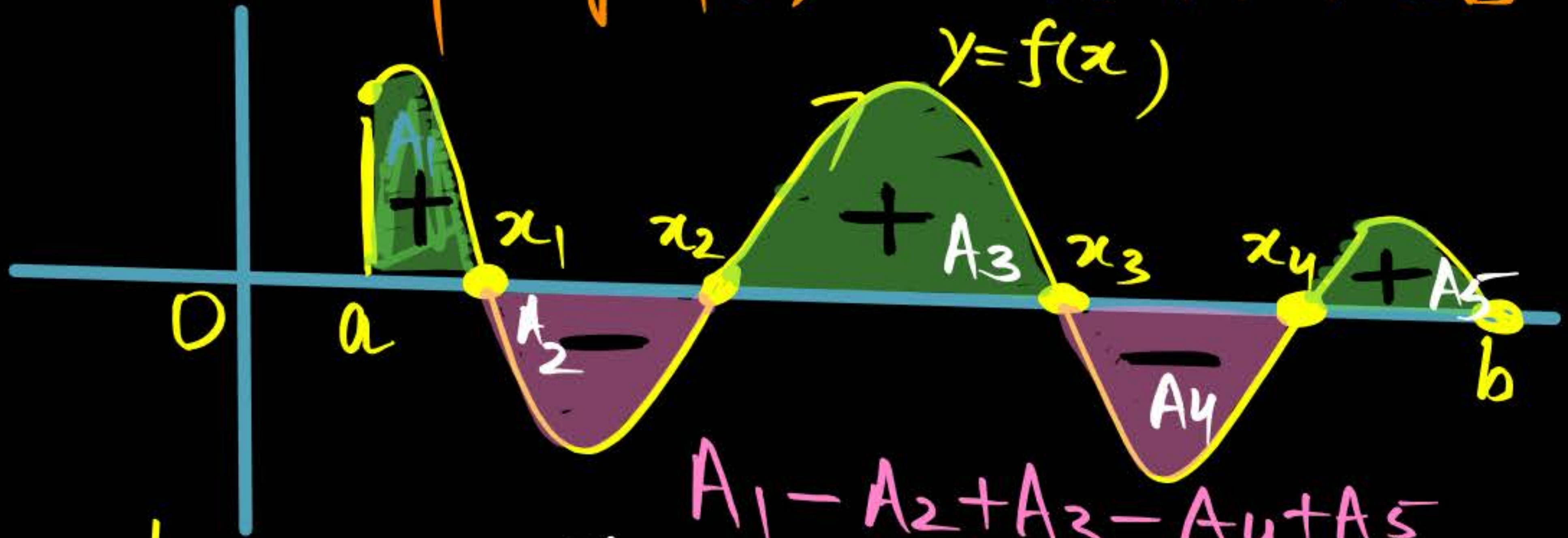
$f(x) \le 0$

Area Bounded via region $= \left| \int_a^b f(x)\,dx \right|$





(3) If $y=f(x)$ crosses the x-axis



$A_1 - A_2 + A_3 - A_4 + A_5$

$$\int_a^b f(x)\,dx = \underbrace{\int_a^{x_1} f(x)\,dx}_{\text{above the x-axis}} + \underbrace{\left|\int_{x_1}^{x_2} f(x)\,dx\right|}_{\text{below the x-axis}} + \int_{x_2}^{x_3} f(x)\,dx + \left|\int_{x_3}^{x_4} f(x)\,dx\right| + \int_{x_4}^{x_5 = b} f(x)\,dx$$

✓ CASE 04

If Two functions Are given $y_1 = f(x)$

$y_2 = g(x)$

AREA BOUNDED via region

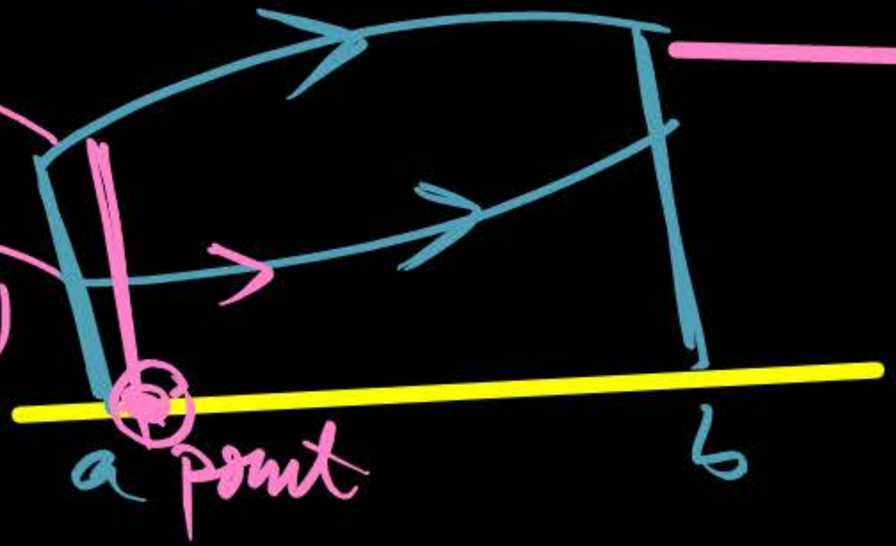
$$= \int_a^b [f(x) - g(x)]\,dx$$

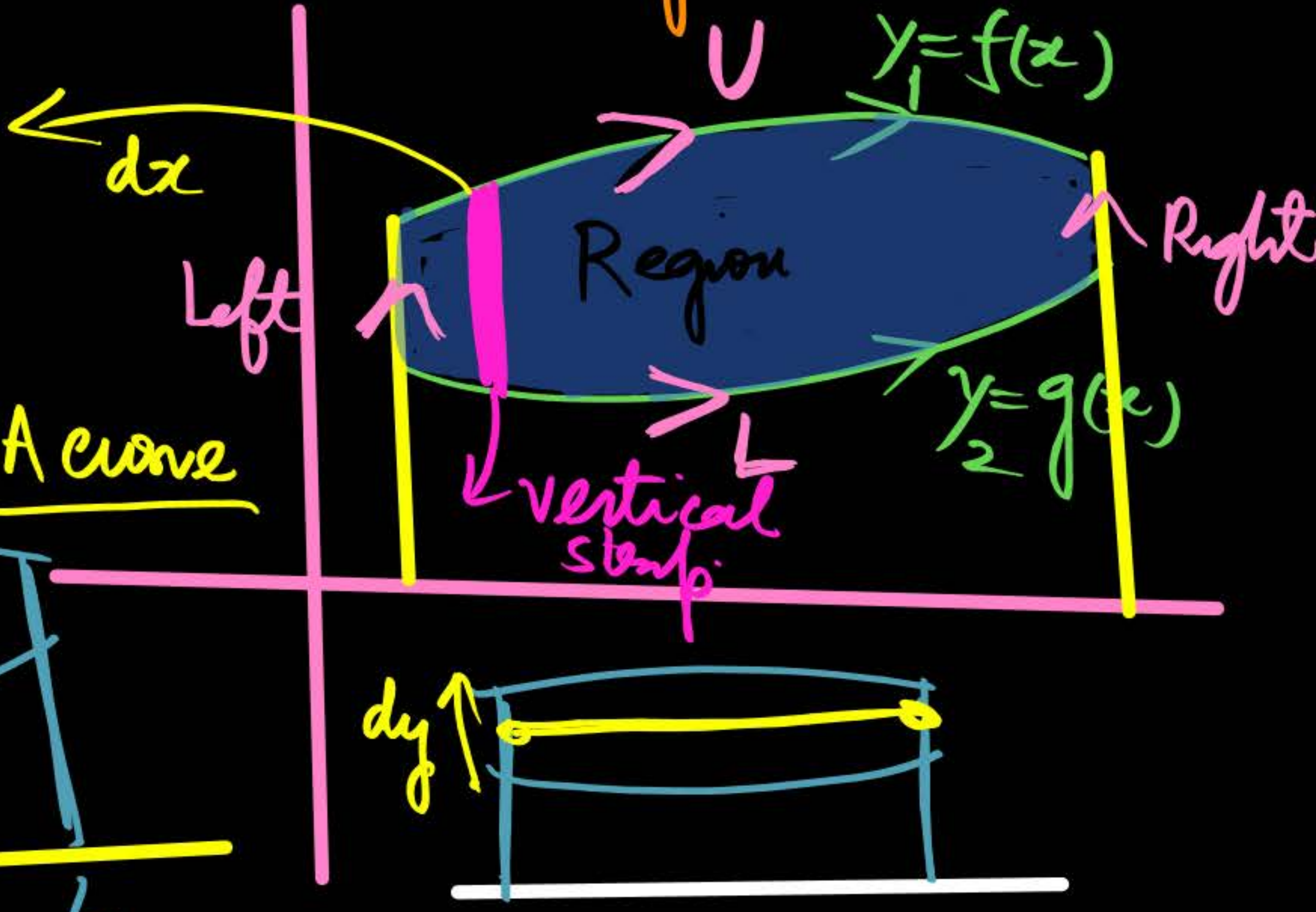
$$A = \int_a^b (y_1 - y_2)\,dx = \text{Area under A curve}$$

$(y_1 - y_2)\left\{\begin{array}{l} \square\ y_1 \leftarrow f(x) \\ \square\ y_2 \leftarrow g(x) \end{array}\right.$

$= y$

$y \cdot x$ = rectangle AREA





VOLUME VIA Double Integration:

Double Integral $\boxed{\int_x \boxed{\int_y dy} \, dx}$ → outer Integral = Area Bounded region in 2 dimension

↑ generalization

2 Integrals (Area)

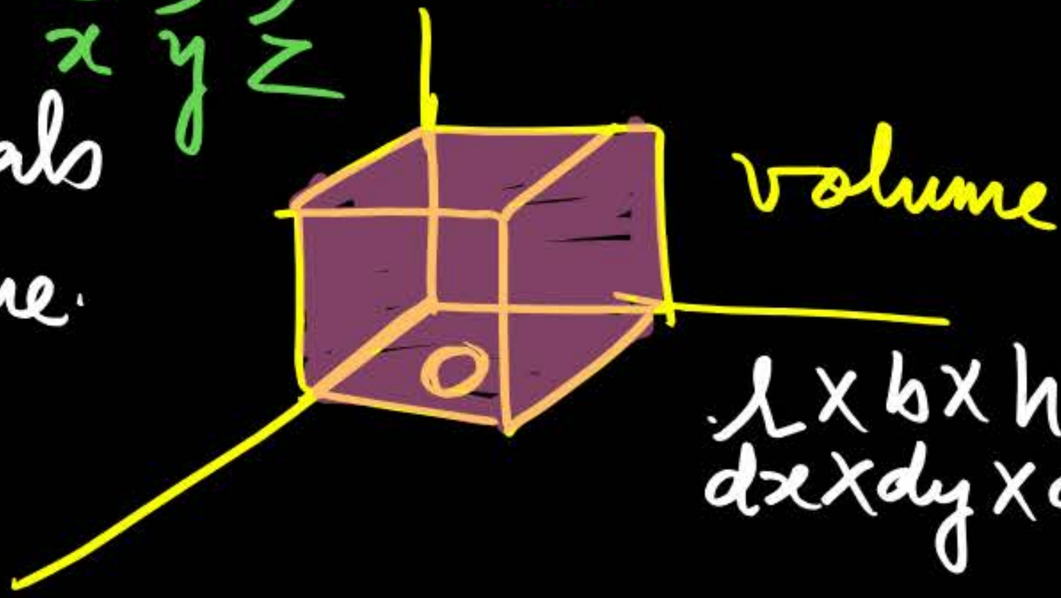
Inner Integrals

$\boxed{\text{2d space}}$ = Double Integral OR Single Integral

$A = dy\,dx$ OR $dx\,dy$

$$\int_x \int_y \int_z dz\,dy\,dx \longrightarrow \text{volume of region}$$

3 Integrals volume.



$l \times b \times h$

$dx \times dy \times dz$

Volume Via Double Integrals:

$$\boxed{\int_x \int_y \underbrace{f(x,y)}_{\text{surface}}\, dy\, dx}$$

= volume via double Int

✓ $\iint dy\, dx =$ Area

✓ $\int_x \int_y f(x,y)\, \underline{dy\, dx} =$ volume via double Integrals

OR

$\int_y \int_x f(x,y)\, dx\, dy$

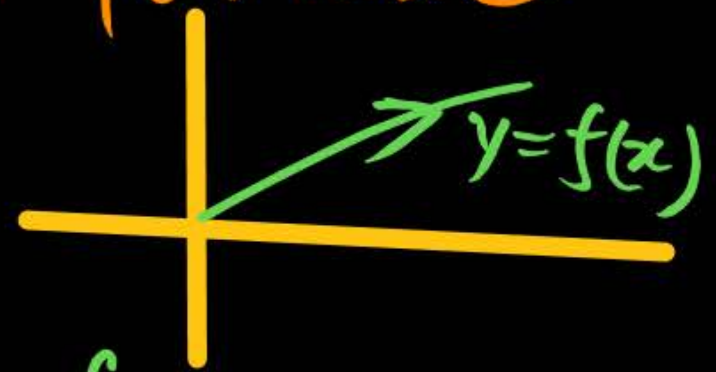
Volume via Double Integral

↓

work

surface is given

$y = f(x)$ curve



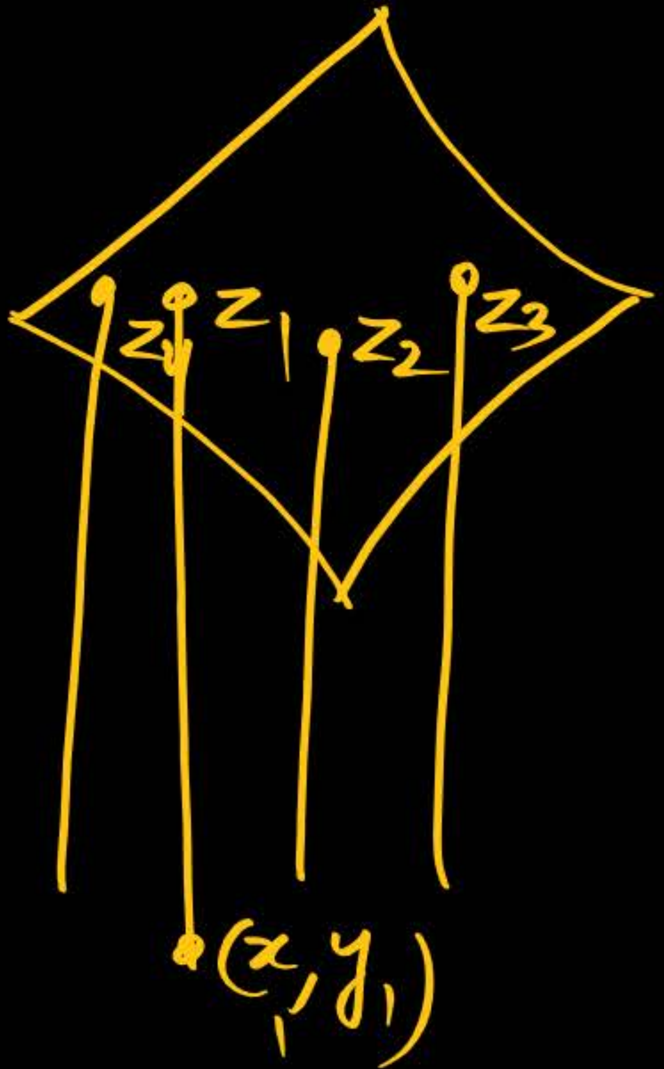
✓ $Z = f(x, y) = x + y$

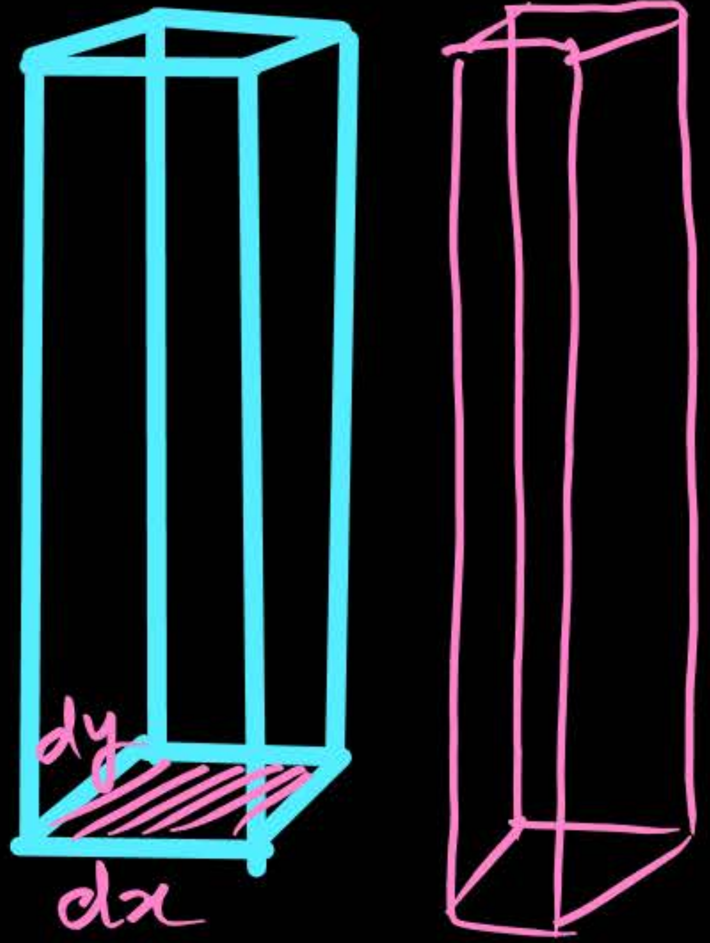
$x = 1$ $Z = 2$
$y = 1$

volume $= 1 \times 1 \times 2$
$= 2\,m^3$

$x = 2$
$y = 3$ $Z = 5$

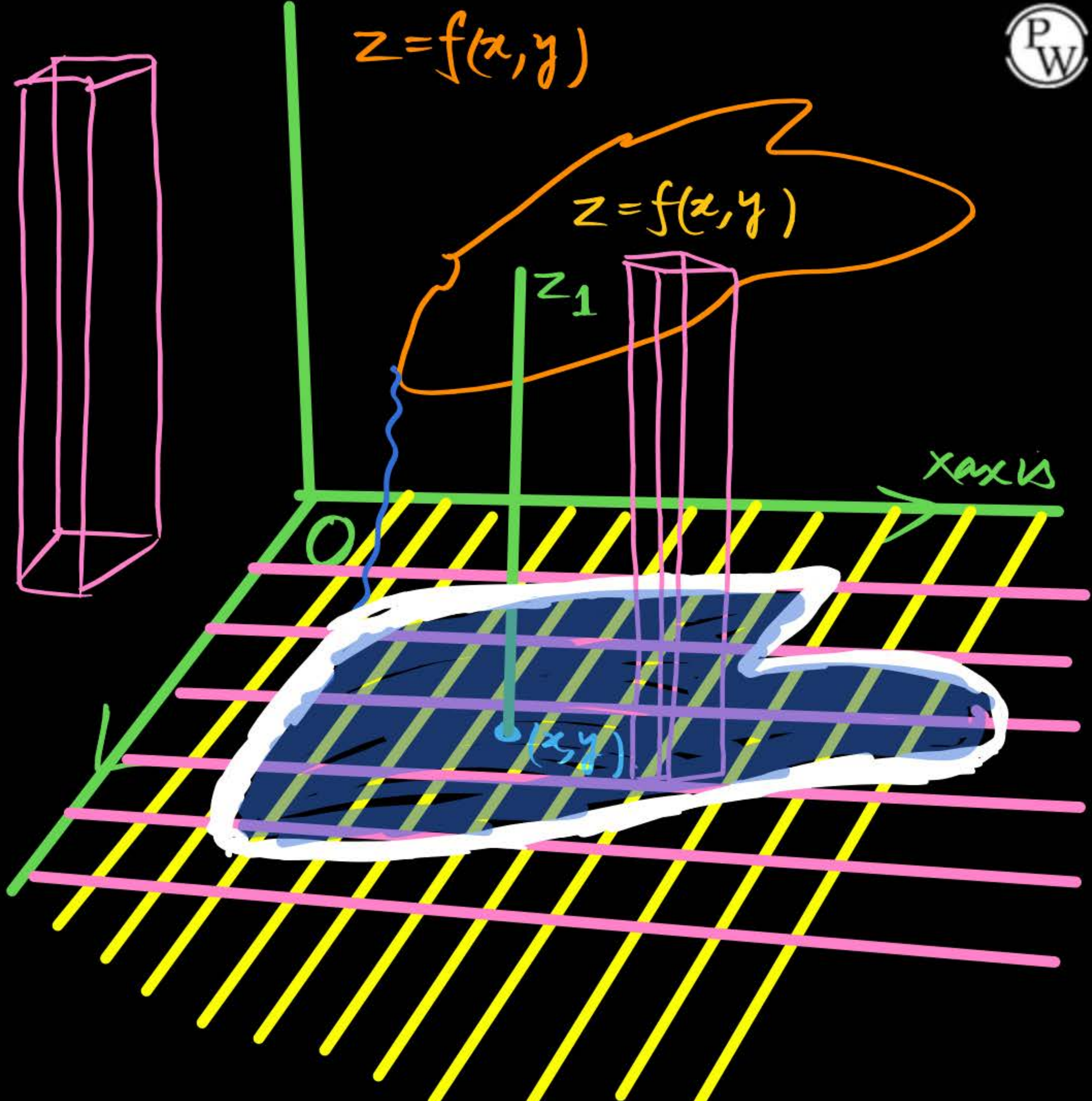
volume $= 2 \times 3 \times 5$
$= \underline{30\,m^3}$





all blocks - volume

$$= \int_x \int_y f(x,y)\,dy\,dx$$



Topic : Double Integrals

volume via double Integrals

$\iint (x+y)\,dy\,dx$

Where R is the region

Q1. Illustration

$\iint (x + y)dydx$ where R is the region bounded by

$$\left[\begin{array}{l} x = 0 \\ x = 2 \\ y = x \\ y = x + 2 \end{array}\right.$$

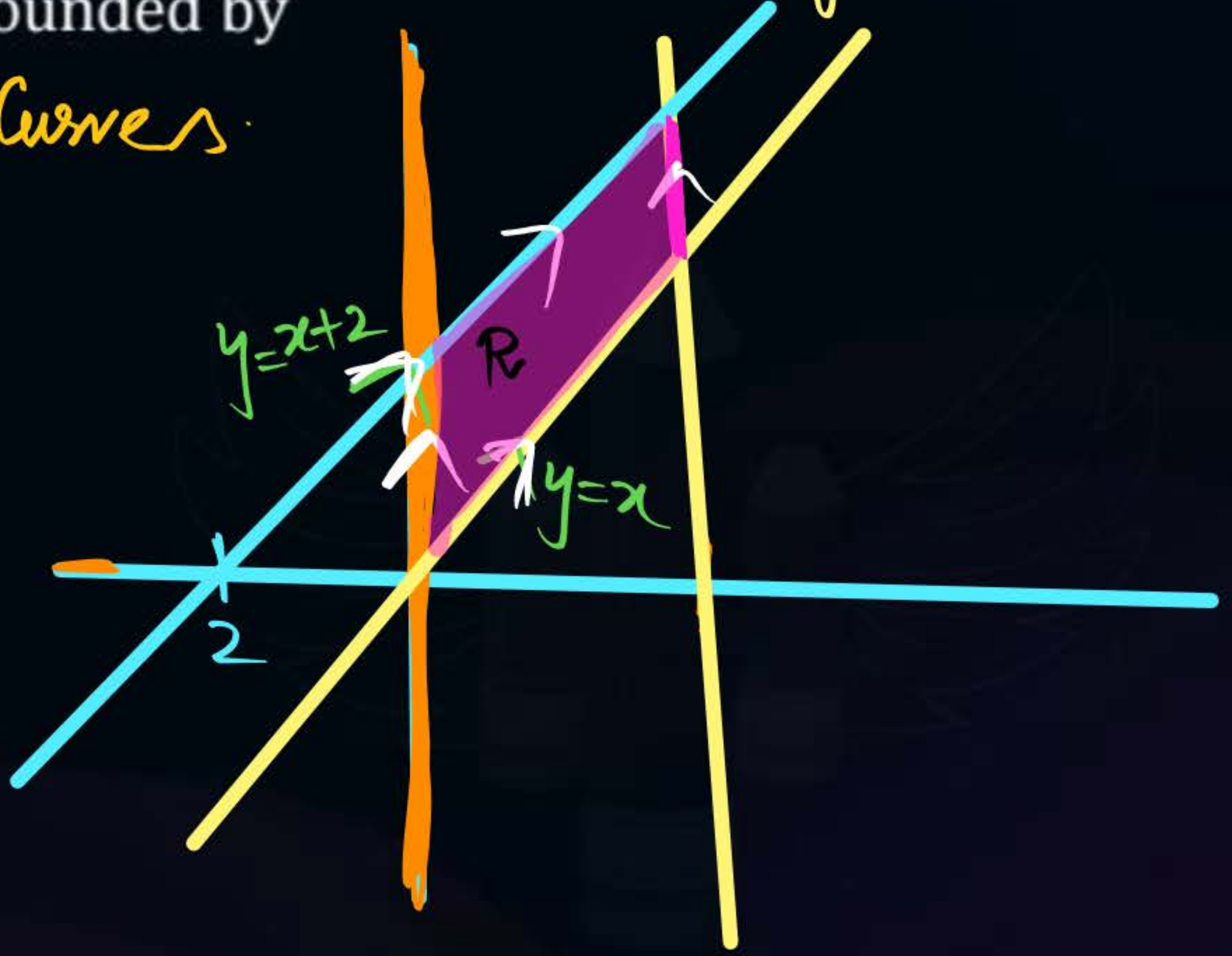
✓ Step (1) Plot The Curves.

$c_1 = \boxed{x=0}$

$c_2 = \boxed{x=2}$

$c_3 = \boxed{y=x}$

$c_4 = \boxed{y=(x+2)}$

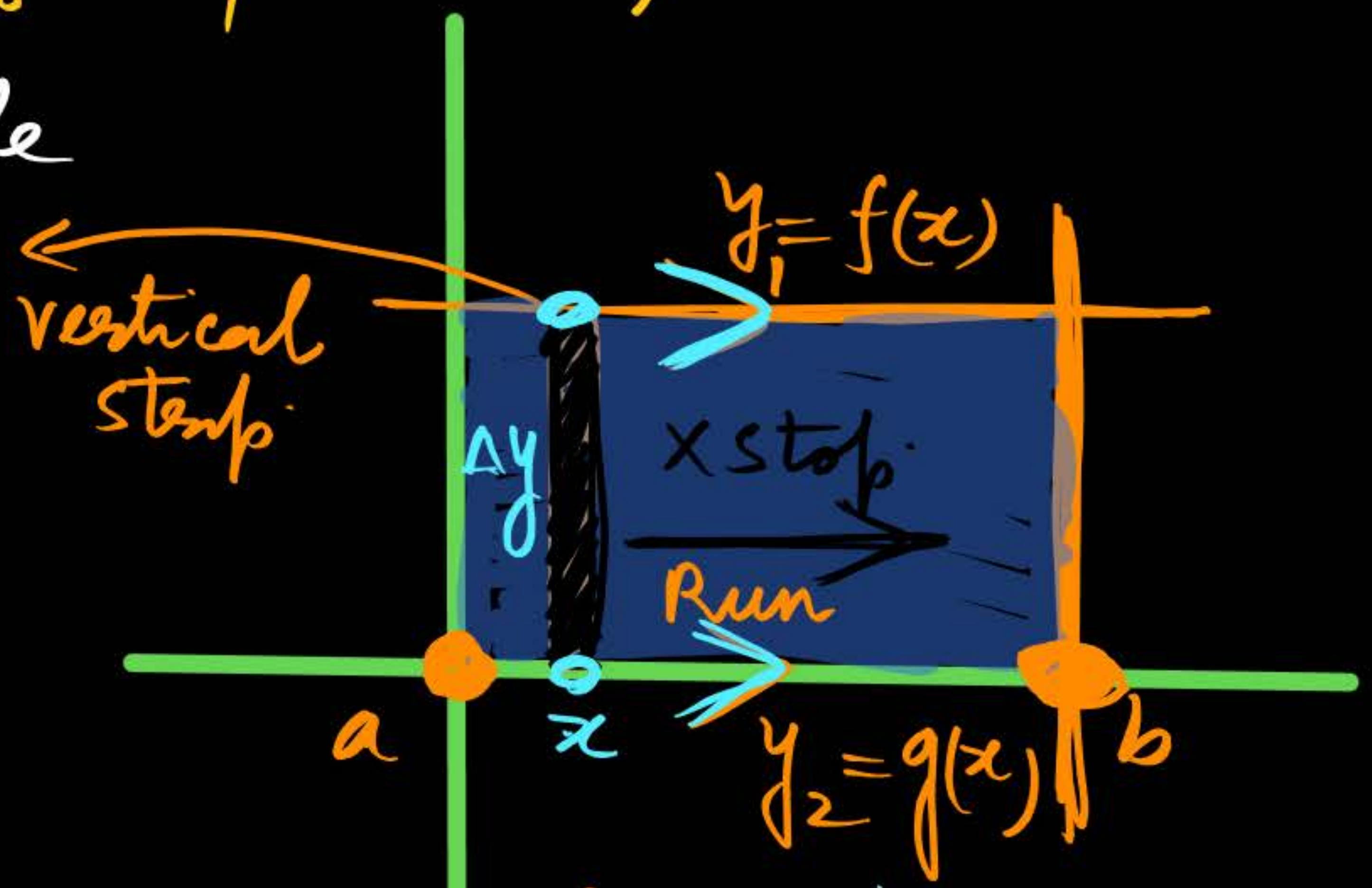


Step 02: find The strip (Horizontal / vertical)

✓ Using Vertical strip Y variable

✓ X constant a to b

$$\Rightarrow \int_{x=a}^{b} \int_{y=g(x)}^{f(x)} \boxed{f(x,y)}\, dy\, dx$$



X constant

X a to b move

Using

| vertical strip ⟶ X → a to b constant Limit Y → variable Limit g(x) to f(x)

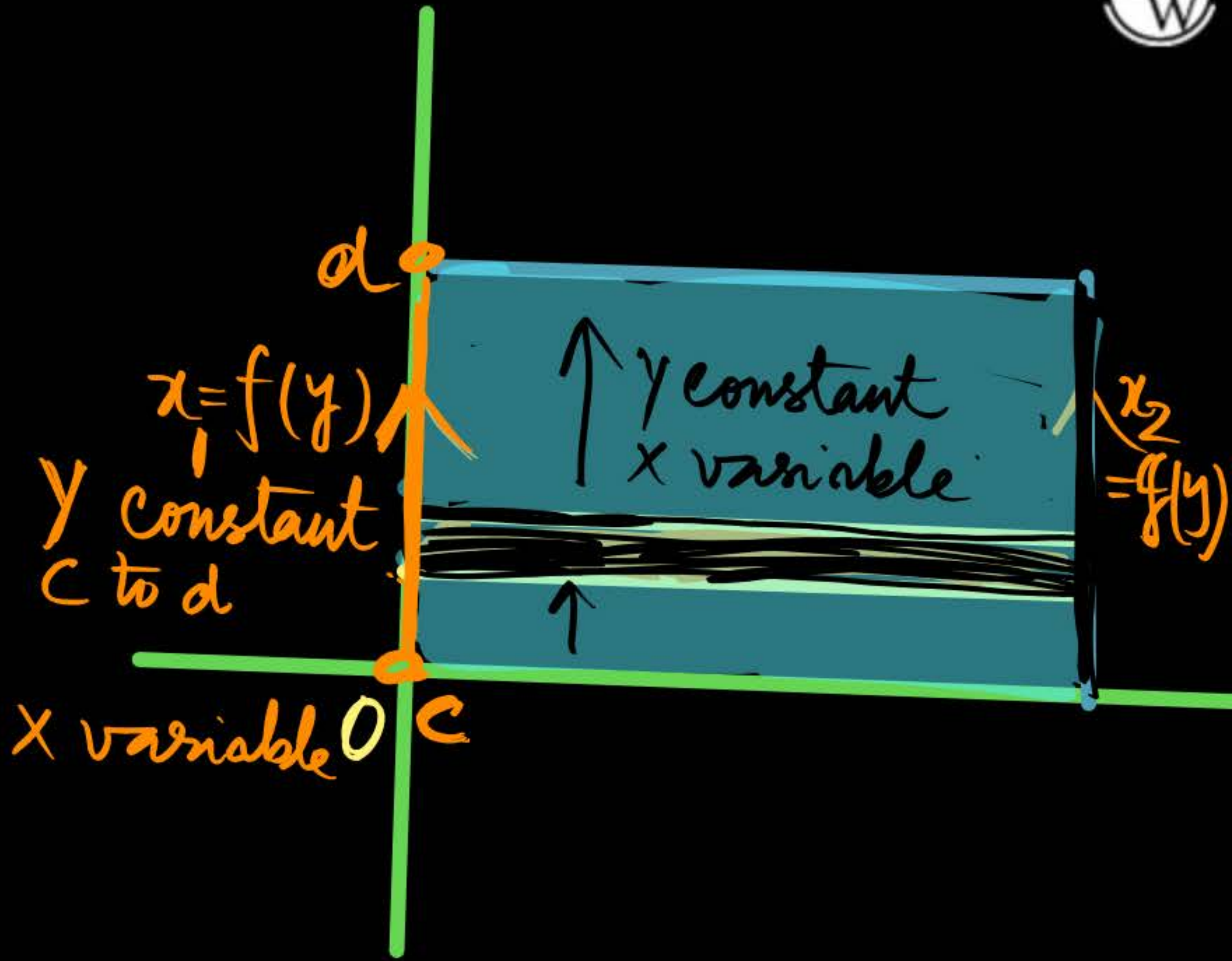
Using Horizontal strip:

$X \to$ variable $f(y)$ to $g(y)$

$Y \to$ constant c to d

volume via Double Integral

$$\Rightarrow \int_{y=c}^{d} \int_{x=f(y)}^{g(y)} f(x,y)\, dx\, dy$$

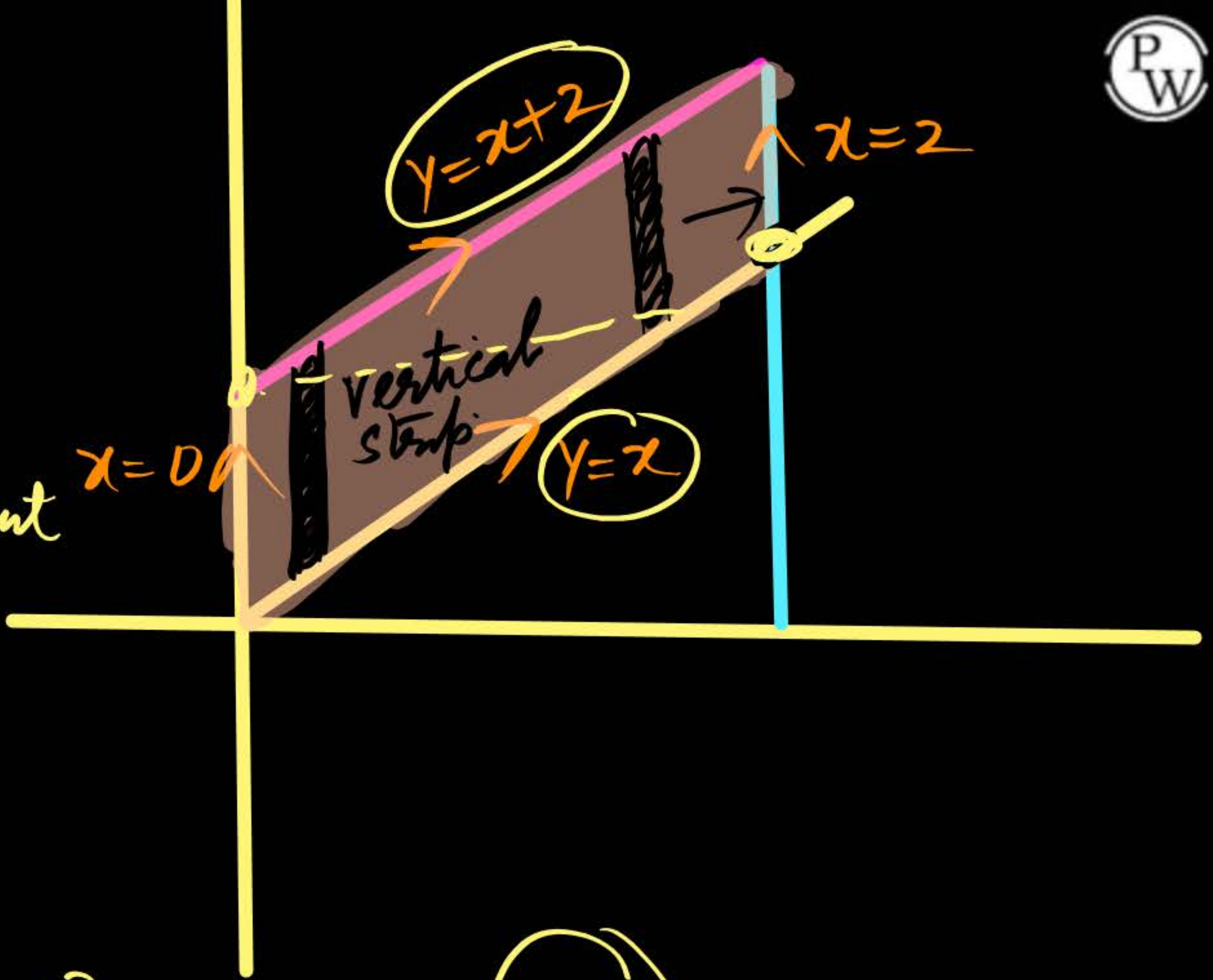


$$\Rightarrow \int_{x=0}^{2}\left[\int_{x}^{x+2}(x+y)\,dy\,dx\right]$$

$$\Rightarrow \int_{0}^{2}dx\left[\int_{x}^{x+2}(x+y)\,dy\right] \longrightarrow x \text{ constant}$$

$$\Rightarrow \int_{0}^{2}dx\left[x\cdot y+\frac{y^2}{2}\right]_{y=x}^{x+2}$$

$$\Rightarrow \int_{0}^{2}\left[x(x+2)+\frac{(x+2)^2}{2}\right]-\left[x\cdot x+\frac{x^2}{2}\right]dx = 12$$



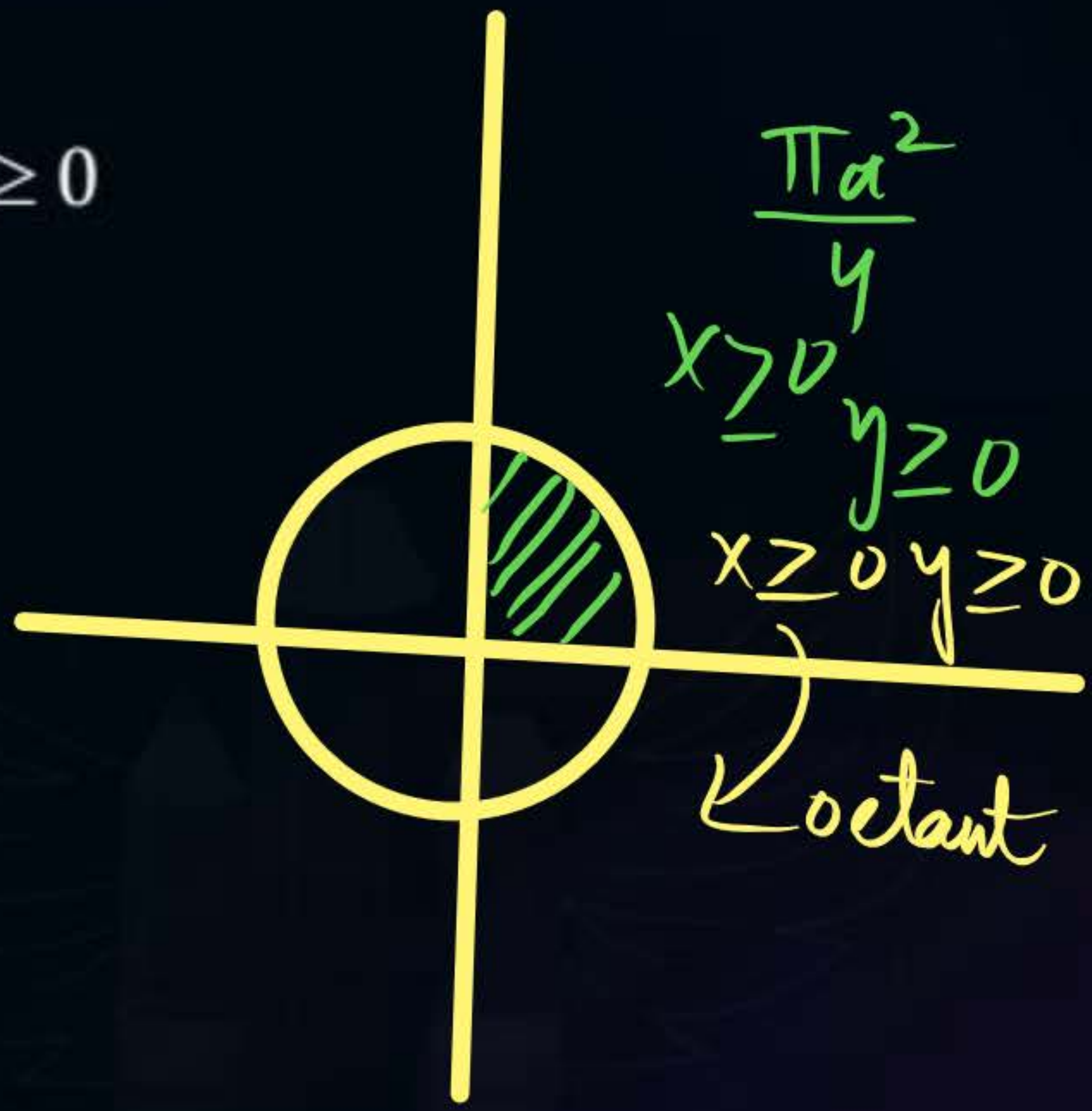
Topic : Double Integrals

$\iint dy\,dx = \frac{\pi a^2}{4}$ $\qquad$ $\iint xy\,dy\,dx$

Q2. Illustration

$\iint xy\, dy\, dx$ where R is the region $x^2 + y^2 = a^2$ $x \geq 0, y \geq 0$

volume via
Double Integrals.

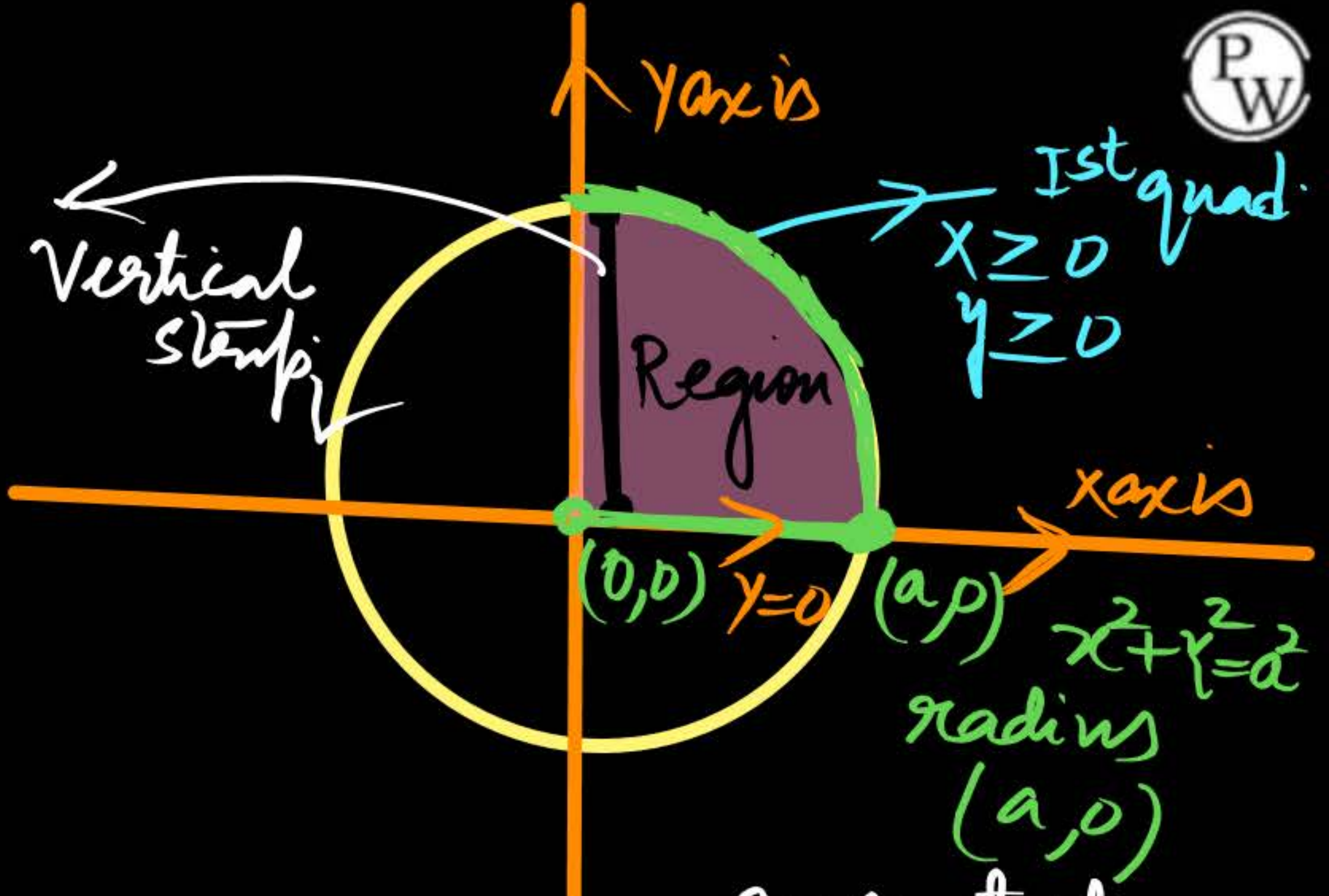


$$\iint_{x\;y} x y\,dy\,dx = \int_0^a x\,dx \int_0^{\sqrt{a^2-x^2}} y\,dy$$

$$= \int_0^a x\,dx \left[\frac{y^2}{2}\right]_0^{\sqrt{a^2-x^2}}$$

$$\Rightarrow \int_0^a x\,dx \left[\frac{(a^2-x^2)}{2}\right]$$

$$\Rightarrow \frac{1}{2}\int_0^a x(a^2-x^2)\,dx = \frac{a^4}{8}$$ Ans



a = constant

$$x^2+y^2=a^2$$

$$y=\pm\sqrt{a^2-x^2}$$

Q4. Illustration

Consider the shaded triangular region, the value of $\iint xy \, dx \, dy$

$\boxed{\text{H.W}}$ Do yourself

Ans $= \frac{1}{6}$

THANK - YOU