Data Science and Artificial Intelligence

Probability and Statistics

Continuous Probability Distribution

Lecture No. -04

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Topics to be Covered

30 min

Topic	**Problems based on Normal Distribution**
Topic	**Problems based on exponential Distribution**
Topic	**exponential Distribution**

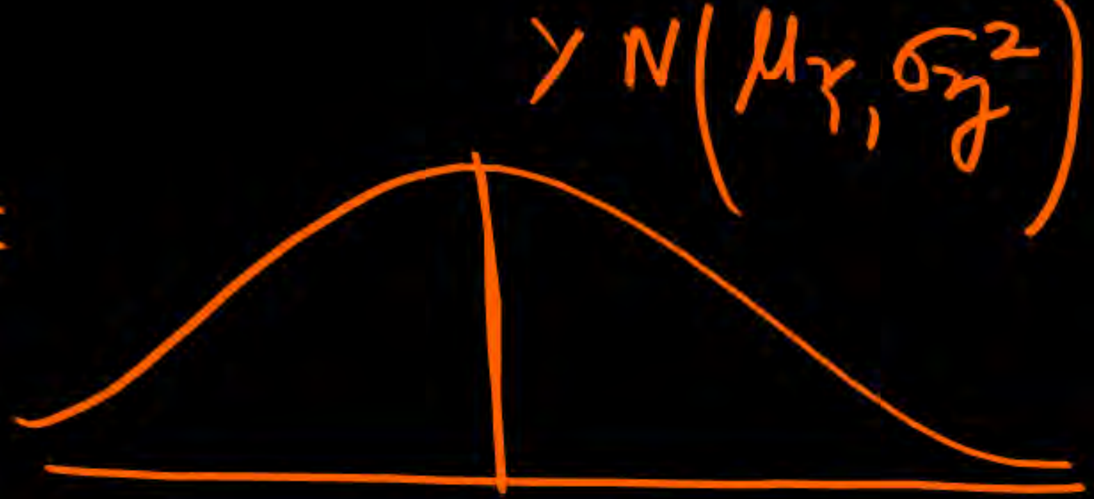
NCERT → 7th chapter (12th)
→ D.E
→ Diff + graphs
→ Equⁿ

$\boxed{\text{Environment}}$

Unlearning
Learning

Properties of Normal Distribution / Guassian Dist.

If X, Y Are Two Normally Distributed Random var.



If $\left.\begin{matrix} X+Y \\ X-Y \end{matrix}\right] = Z$ is also Normally Distributed

→ Sum of Two R.V

→ Diff. of Two R.V

$Z = X + Y$

$N(\mu_X, \sigma_X^2)$ X

$+$

y

$N(\mu_Y, \sigma_Y^2)$

$=$

$Z(\mu_Z, \sigma_Z^2)$

$\mu_X = 1$ $\sigma_X^2 = 2$

$\mu_Z = \mu_X + \mu_Y$ $\sigma_Z^2 = \sigma_X^2 + \sigma_y^2$ combined mean and S.D

$N(1, 2)$

$N(3, 4)$ $\sigma_y^2 = 4$

$\mu_Y = 3$

$Z \approx N(\mu_Z, \sigma_Z^2)$

$+$

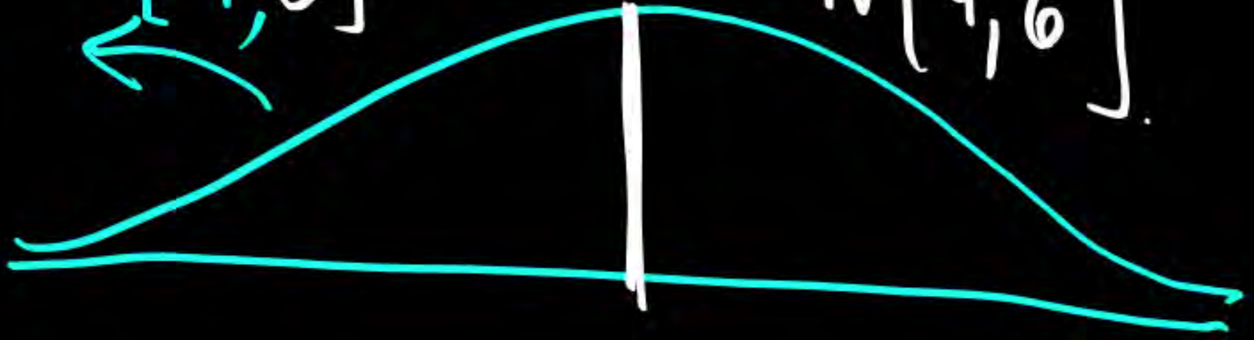
$=$

$\mu_Z = \mu_x + \mu_y = 1 + 3 = 4$

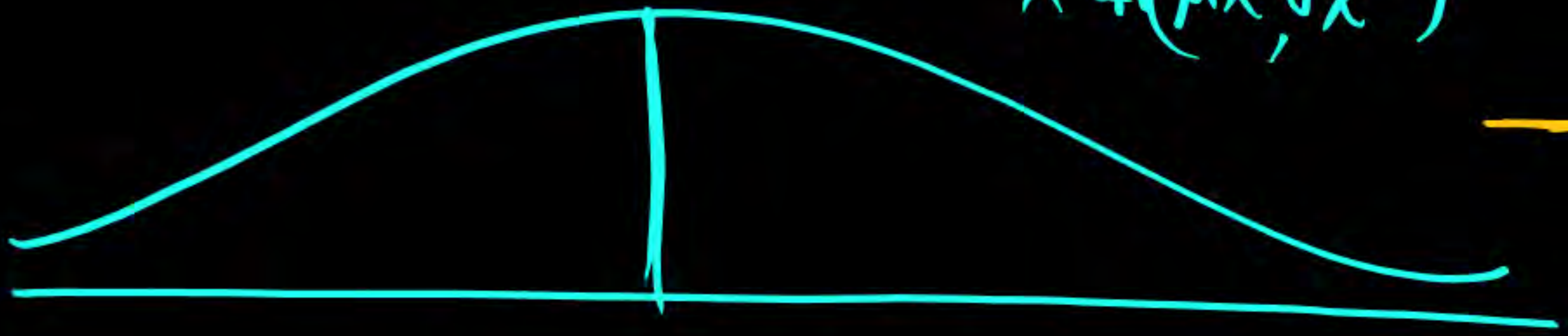
$\sigma_Z^2 = \sigma_x^2 + \sigma_y^2 = 2 + 4 = 6$

$Z[4, 6]$

$N[4, 6]$



$X \sim N(\mu_x, \sigma_x^2)$



$-$

$Y \sim N(\mu_y, \sigma_y^2)$

$\mu_z = \mu_x - \mu_y$

$\sigma_z^2 = \sigma_x^2 - \sigma_y^2$ $\quad V(x) > 0$

$$\boxed{\sigma_z^2 = \sigma_x^2 + \sigma_y^2}$$

$\Rightarrow$ $Z[\mu_z, \sigma_z^2]$

$N(5, 6)$

$-$

$N(3, 8)$

$=$

$N(2, 14)$

Normally

Generalization

If X, Y, Z, T... Are Normally Distributed Random var

$X \sim N(\mu_x, \sigma_x^2)$ + $Y \sim N(\mu_y, \sigma_y^2)$ + $Z(\mu_z, \sigma_z^2)$ + ...

$$\begin{cases} \mu_s = \mu_x + \mu_y + \mu_z - - - - - - - \\ \sigma_s^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 + - - - - \end{cases}$$

= $S(\mu_s, \sigma_s^2)$

→ Normally Distributed

Q1. Let x_1, x_2 and x_3 be independent and identically distribution random variables with the uniform distribution on [0, 1]. The probability p {x_1 is the largest} is _____.

x_1, x_2, x_3 (Independent)

$$\left\{\begin{array}{l} \boxed{x_1 > x_2 > x_3} \\ \boxed{x_1 > x_3 > x_2} \\ x_2 > x_3 > x_1 \\ x_2 > x_1 > x_3 \\ x_3 > x_1 > x_2 \\ x_3 > x_2 > x_1 \end{array}\right.$$

$P(x_1 \text{ is Largest}) = \frac{2}{6} = \frac{1}{3}$

$P(x_2 \text{ is Largest}) = \frac{2}{6} = \frac{1}{3}$

$P(x_3 \text{ is Largest}) = \frac{2}{6} = \frac{1}{3}$

$P(x_1) = \frac{1}{3}$ $P(x_3) = \frac{1}{3}$

$P(x_2) = \frac{1}{3}$

$N(\mu, \sigma^2)$ $\downarrow$ variance $N(\mu, \sigma)$ $\downarrow$ S.D $= \sqrt{5}$

Q2. Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P\{3V \geq 2U\}$ is

A. $\frac{4}{9}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{5}{9}$

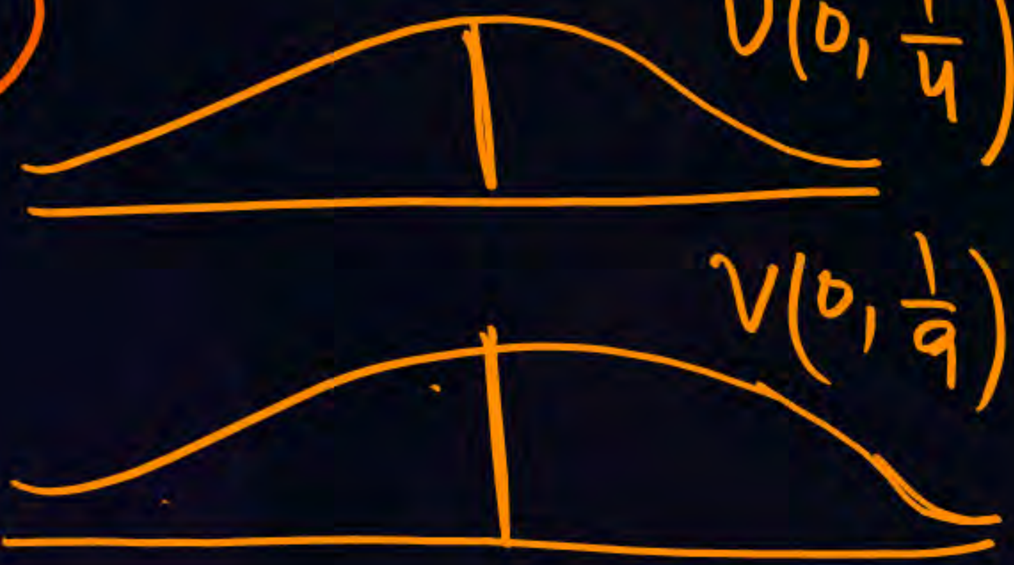
$P(3V - 2U \geq 0)$

$X = 3V - 2U$

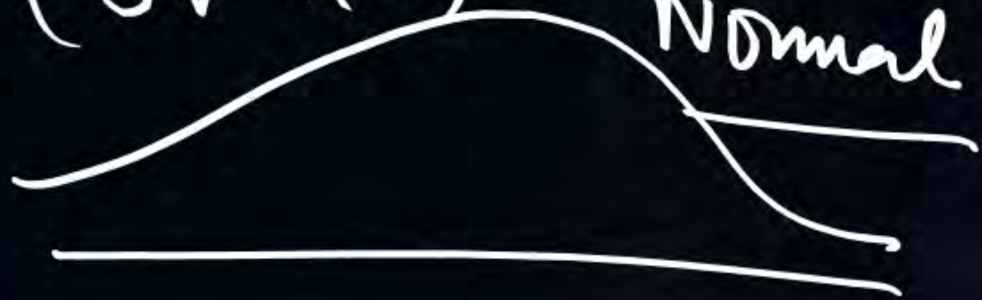
U, V Normally Dist.

$U(0, \frac{1}{4})$

$V(0, \frac{1}{9})$



$= (3V - 2U) \rightarrow$ Normal



$$P(\underbrace{3V-2U}\geq 0) = P[X\geq 0] = P\left[\frac{X-\mu}{\sigma}\geq\frac{0-\mu}{\sigma}\right]$$

$$= P\left[Z\geq\frac{0-\mu}{\sigma}\right]$$

$$X = 3V-2U$$

$$\mu_X = 3\mu_V - 2\mu_U$$

$$\sigma_X^2 = (3)^2\sigma_V^2 + (2)^2\sigma_U^2$$

$$\mu_X = 3\times 0 - 2\times 0 = 0$$

$$\sigma_X^2 = \cancel{9}\times\frac{1}{\cancel{9}} + 4\times\frac{1}{4}$$

$$\boxed{\sigma_X^2 = 2}$$

$$\sigma_X = \sqrt{2}$$

$$\text{var}(aX) = a^2\,\text{var}(X)$$

$$\text{var}(-X) = (-1)^2\,\text{var}(X)$$

$$\text{var}(aX+bY) = a^2\,\text{var}(X) + b^2\,\text{var}(Y)$$

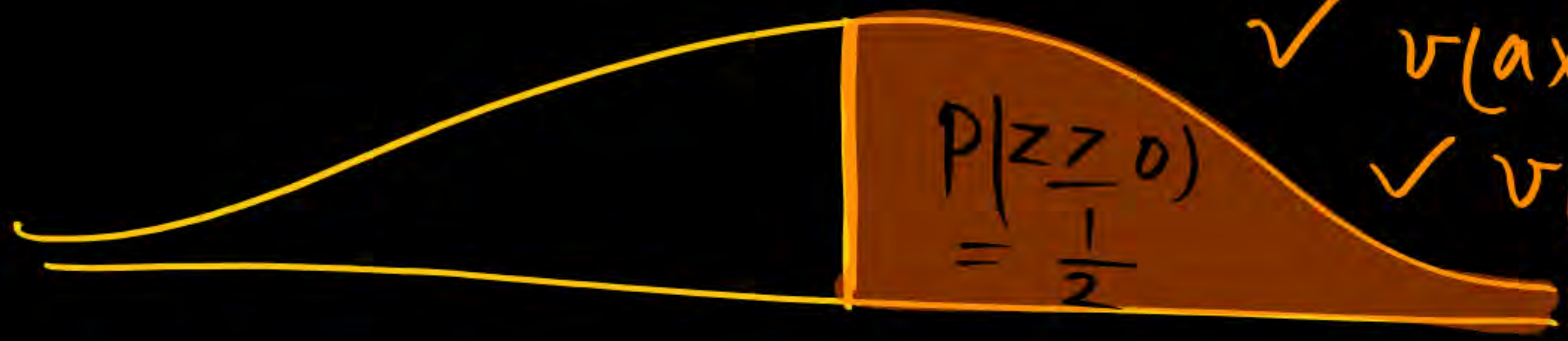
$$V(aX-bY) = a^2 V(X) + b^2 V(Y)$$

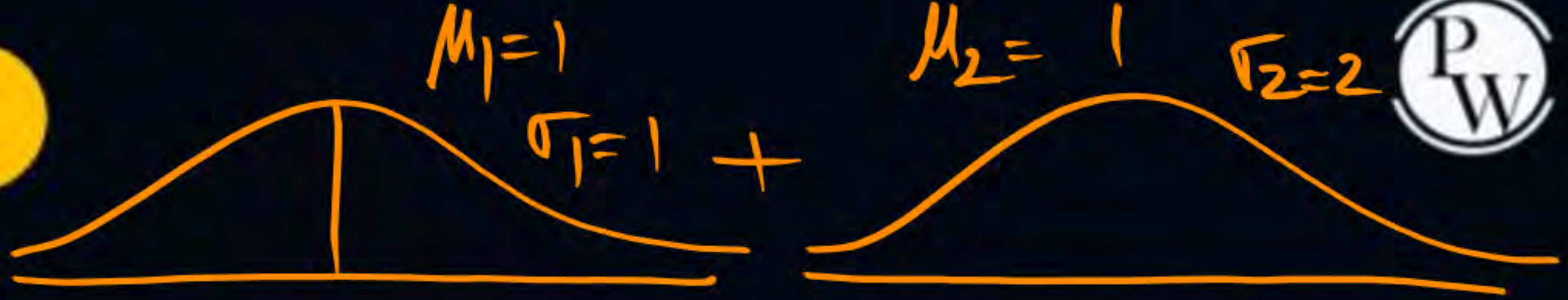
$$= P\left(Z \geq \frac{0-0}{\sqrt{2}}\right) = P(Z \geq 0)$$

$$Var(aX+bY) = a^2 V(X) + b^2 V(Y)$$

$$V(aX) = a^2 V(X)$$

$$V(-X) = (-1)^2 V(X)$$





Q3. let X_1, X_2 be two independent normal random variables with means μ_1, μ_2 and standard deviation σ_1, σ_2 respectively .consider $Y=X_1-X_2$;

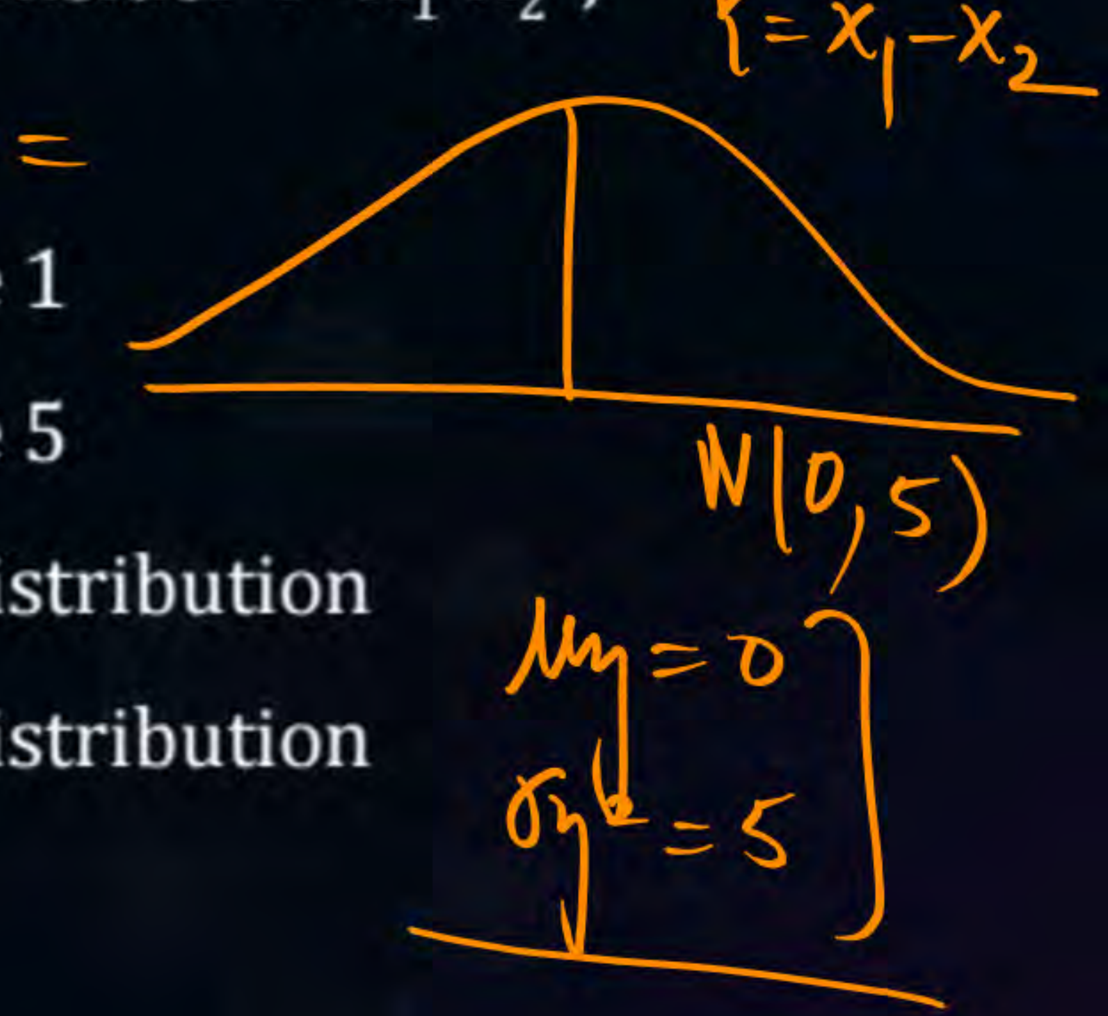
$\mu_1 = \mu_2 = 1$, , $\sigma_1 = 1$, $\sigma_2 = 2$

A. Y is normally distribution with mean 0 and variance 1

B. Y is normally distribution with mean 0 and variance 5

C. Y has mean 0 and variance 5, but is NOT normally distribution

D. Y has mean 0 and variance 1, but is NOT normally distribution



$\mu_y = \mu_{x_1} - \mu_{x_2} = 1 - 1 = 0$

$\sigma_y^2 = 1 + 4 = 5$

Exponetial Distribution:

OR Waiting Time Discrebution

Time $\longrightarrow$ continuous Process

$\longrightarrow$ ATM queue

| H | T H | T H | T H |

Trials $\longrightarrow$

$\longrightarrow$ Theatre

$\longrightarrow$ Loss of Large quantity

1 Phone call $\xrightarrow{\text{wait}}$

(Waiting time)

Poisson Distribution

$\longrightarrow$ Average No. of success In A Particular time

(Arrival)

$$P[X=x] = \frac{e^{-\lambda t}(\lambda t)^x}{x!}$$

$$\mu = \lambda t$$

$$P[X=x] = \frac{e^{-\mu}(\mu)^x}{x!}$$

μ = mean

$x = 0, 1, 2, 3, \ldots$

(No. of success)

If X is exponetial Distribution

Exponetial (λ) (Single Loss]

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

H Trial

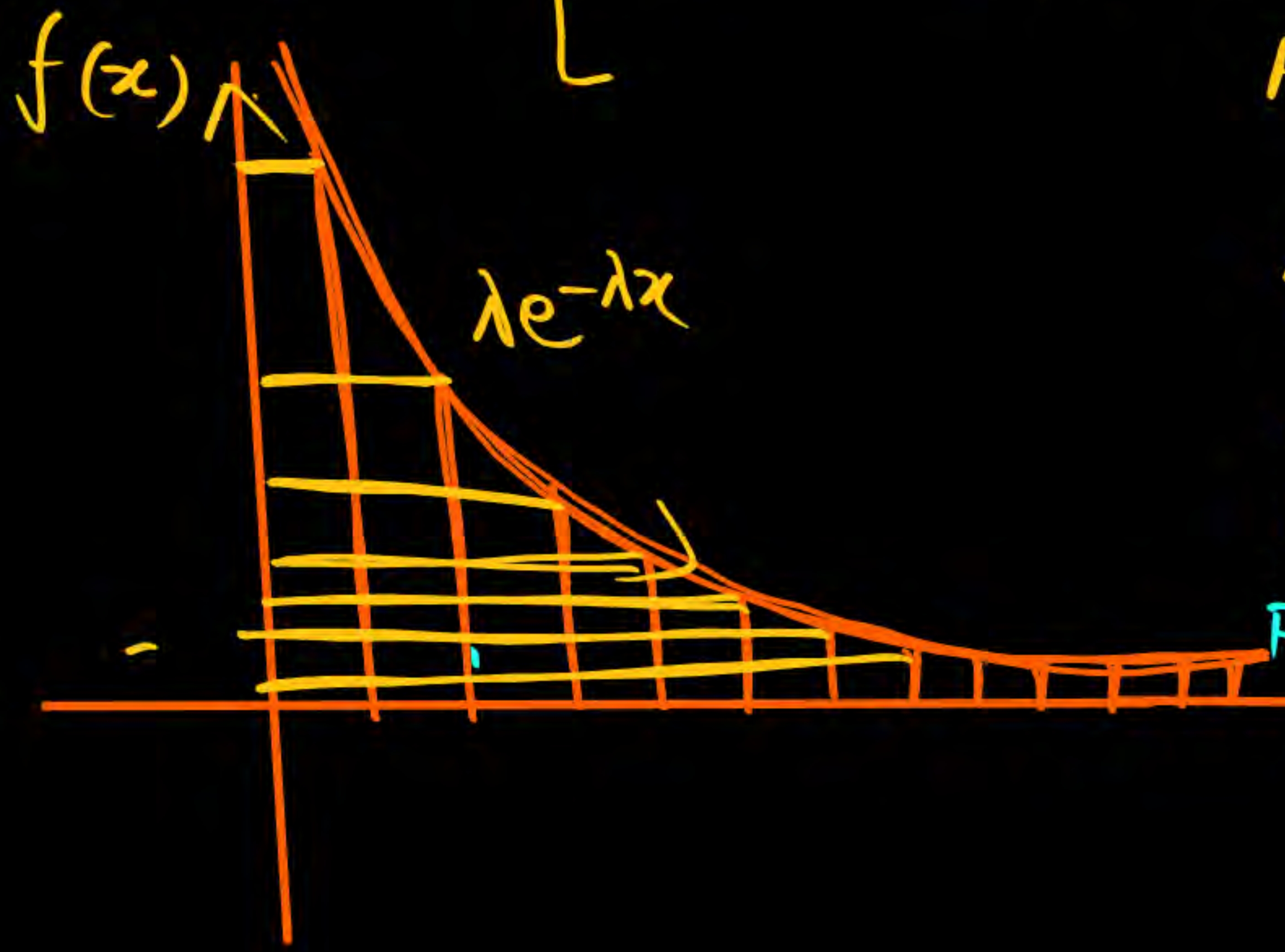
H] Loss time

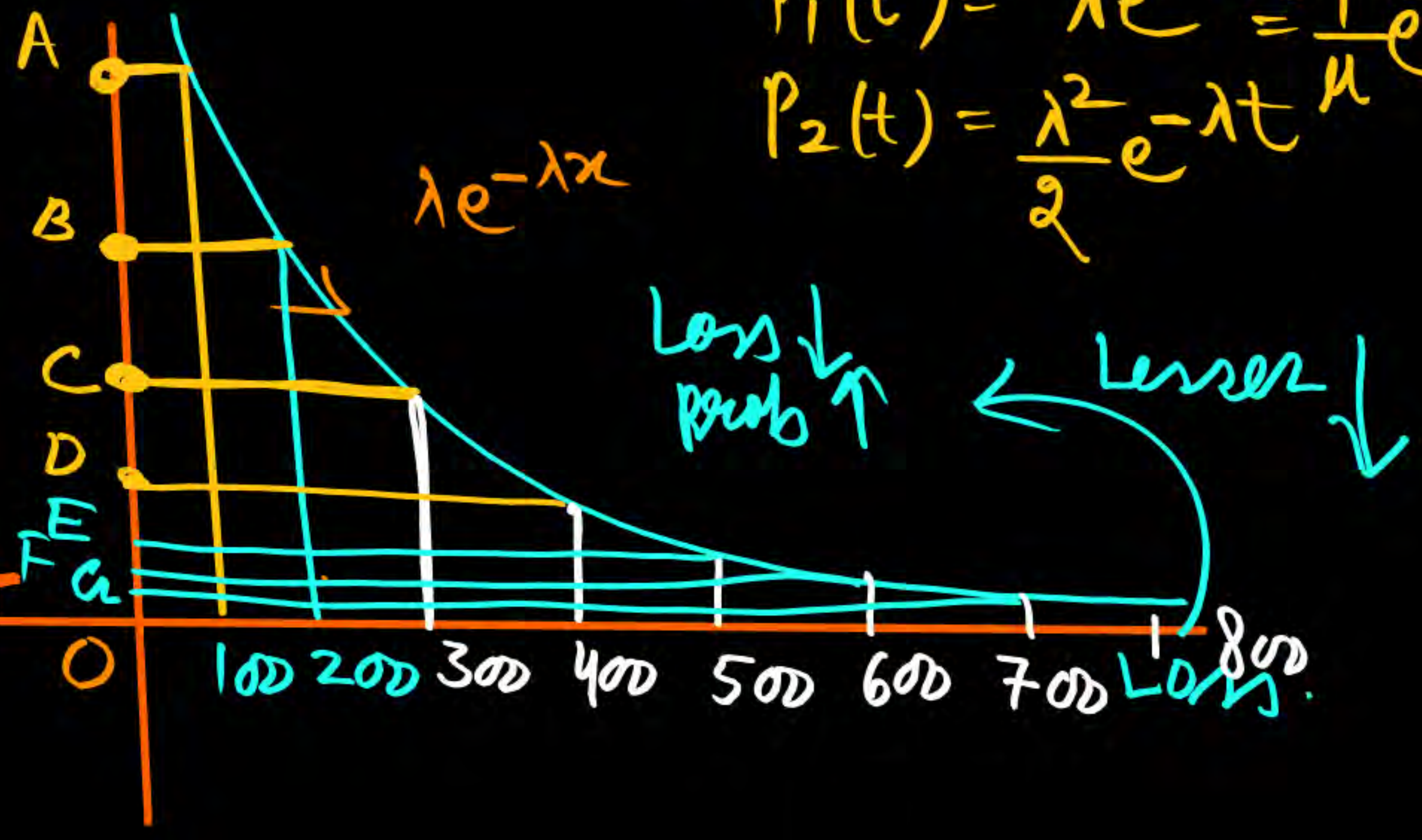
In Poisson Distribution

$$P_0(t) = e^{-\lambda t} = e^{-\mu}$$

$$P_1(t) = \lambda e^{-\lambda t} = \frac{1}{\mu} e^{-\mu}$$

$$P_2(t) = \frac{\lambda^2}{2} e^{-\lambda t}$$





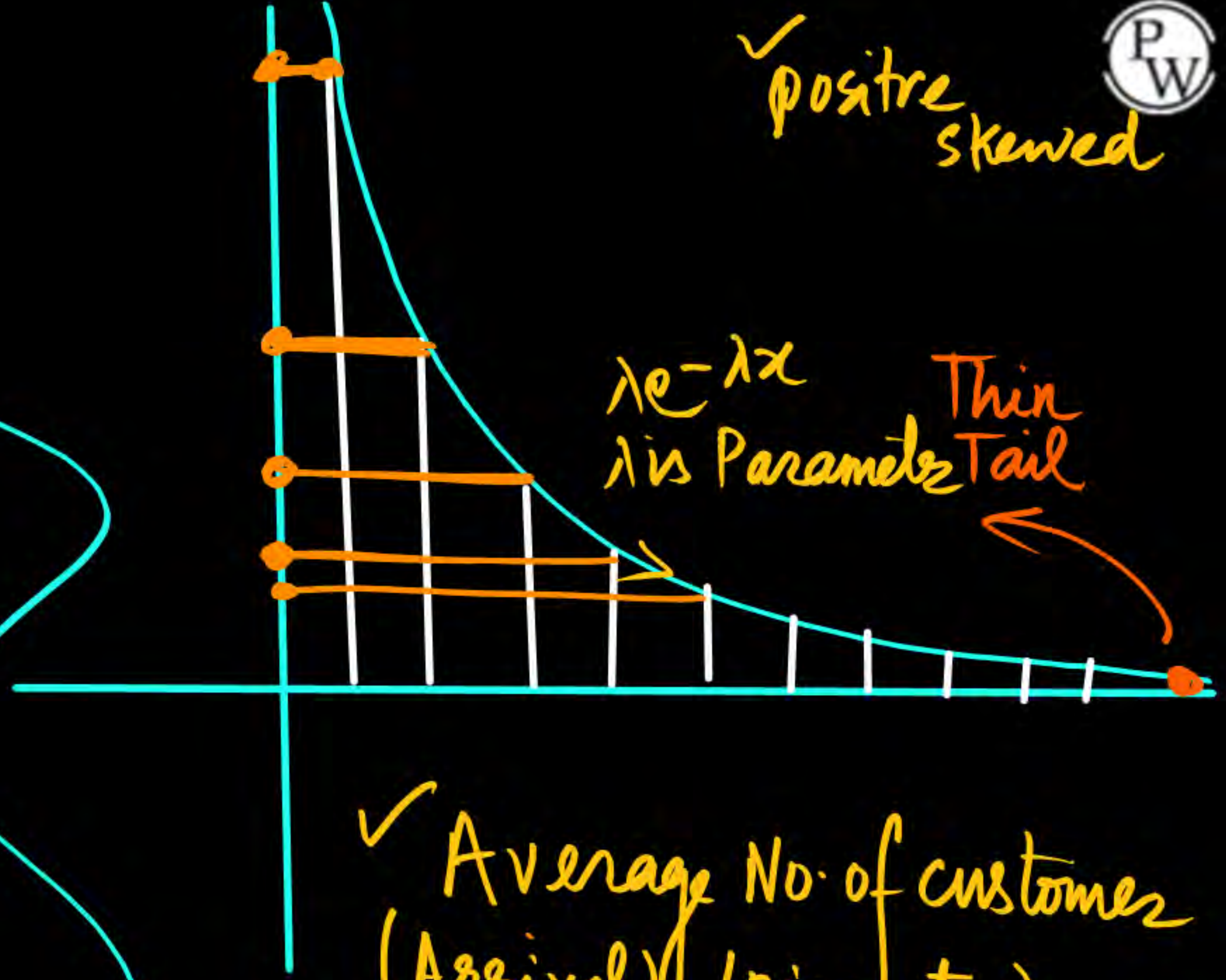
$$f(x)=\begin{cases}\lambda e^{-\lambda x} & x\ge 0\\ 0 & \text{otherwise}\end{cases}$$

In continuous form

How much Time to wait

Average No. customer

positive skewed



Average No. of customer (Arrival) (Discrete)

Poisson

$\lambda = 5$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

✓ MEAN $= \int_0^\infty \lambda e^{-\lambda x} . x \, dx = \lambda \int_0^\infty e^{-\lambda x} . x \, dx = \boxed{\frac{1}{\lambda} = \text{MEAN}}$

✓ variance $= \int_0^\infty x^2 \lambda e^{-\lambda x} dx - \left[\frac{1}{\lambda}\right]^2 = \boxed{\frac{1}{\lambda^2} = \text{variance}}$

✓ It is a valid pdf $= \int_0^\infty \lambda e^{-\lambda x} dx =$ YES $= \boxed{\text{valid pdf}}$

✓ S.tandard deviation $= \boxed{\frac{1}{\lambda} = \text{S.D}}$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

cdf = cumulative Distribution function

$$F_X(x) = \int_0^x \lambda e^{-\lambda x}\, dx = 1 - e^{-\lambda x} = \text{cdf}$$

✓ $P(X \leq x) = 1 - e^{-\lambda x} = \text{cdf}$

✓ $P(X \geq x) = 1 - (1 - e^{-\lambda x})$

$= e^{-\lambda x}$

✓ DEATH Prob. $P(X \leq x)$

✓ SERVIVAL Prob. $P(X \geq x)$

✓

$\boxed{\text{SERVIVAL } P(X \geq x) = e^{-\lambda x}}$

$\boxed{\text{DEATH Prob } P(X \leq x) = 1 - e^{-\lambda x}}$

✓A) Mode does Not exist

✓B) What is The Median

Median ⇒

$$P(X \le M) = \frac{1}{2}$$

$$\Rightarrow \quad 1 - e^{-\lambda M} = \frac{1}{2}$$

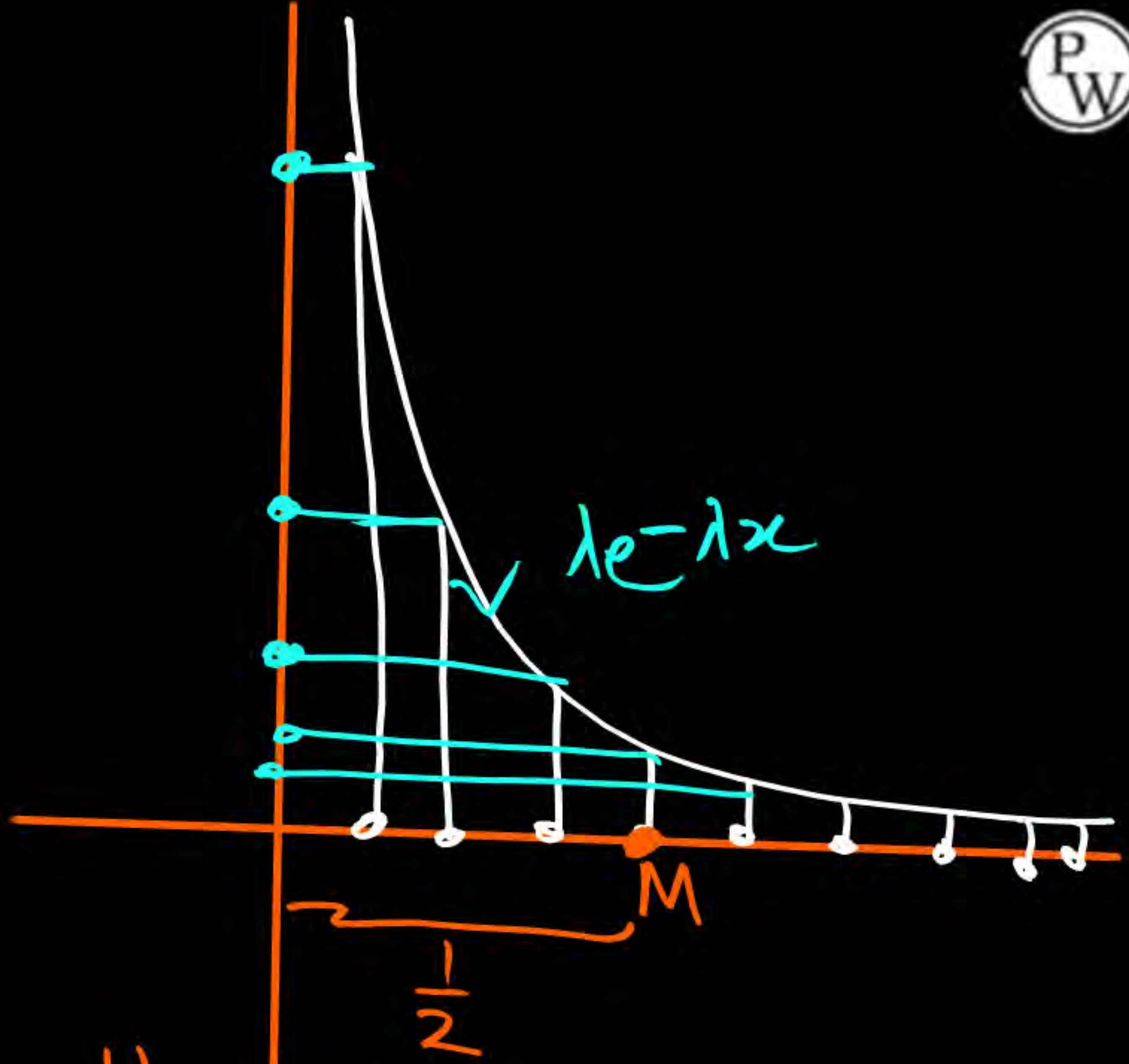
$$= \quad 1 - \frac{1}{2} = e^{-\lambda M}$$

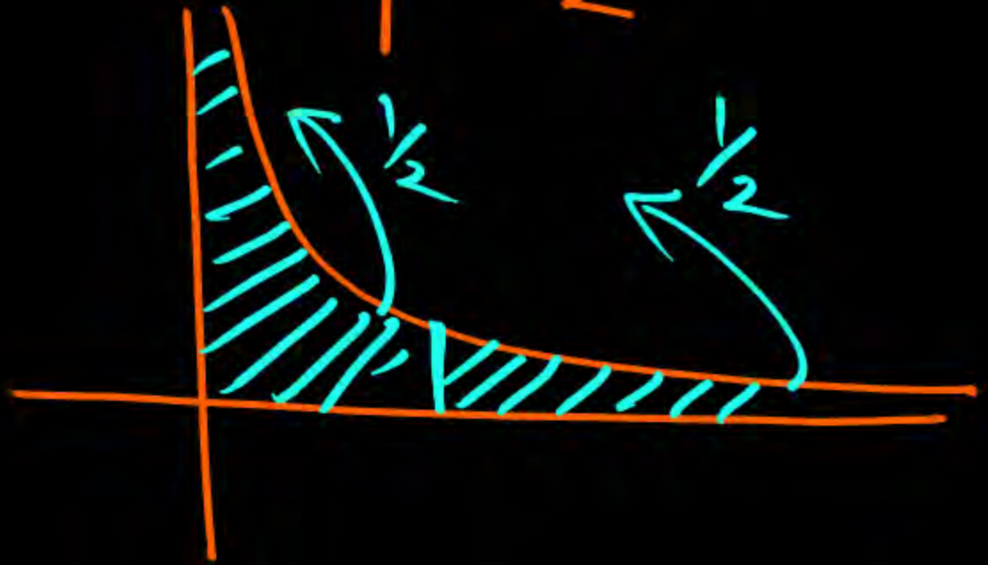
$$= \quad \frac{1}{2} = e^{-\lambda M}$$

$$\boxed{M = \frac{1}{\lambda}\ln 2}$$

$$\lambda m = \ln 2$$

$$m = \frac{1}{\lambda}\ln 2$$





Exponetial Dist(λ)

1) $f(x) = \lambda e^{-\lambda x} \quad x \geq 0$

MEAN $= \frac{1}{\lambda}$ $\quad V(X) = \frac{1}{\lambda^2}$

$S.D = \frac{1}{\lambda}$

$P(X \leq x) = 1 - e^{-\lambda x}$

$P(X \geq x) = e^{-\lambda x}$

median $= \frac{1}{\lambda} \ln 2$

Exponetial Distribution Parameter μ

✓ $f(x) = \frac{1}{\mu} e^{-\mu x}$ $\quad \mu$ = MEAN $= \frac{1}{\lambda}$

✓ $f(x) = \frac{1}{\mu} e^{-\frac{1}{\mu} x}$ $\quad \lambda = \frac{1}{\mu}$

✓ $E[X] = \mu$

✓ $V(X) = \mu^2$

✓ $S.D = \mu$

✓ median $= \mu \ln 2$

✓ cdf $= 1 - e^{-\frac{1}{\mu} x}$ = death

$P(X \geq x) = e^{-\frac{1}{\mu} x}$ = SERVIVAL

THANK - YOU

✓ Exponential
✓ Uniform (Discrete)

✓ GAMMA Distribution
✓ Beta Distribution

Problem Solving