Data Science and Artificial Intelligence

Probability and Statistics

Introduction to Probability

Lecture No.- 03

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Recap of Previous Lecture

Topic Problems based on Basic Probability

Topics to be Covered

Topic | **Classification of events**

Classification of events

A

B

A

B

C

A

B

C

.

.

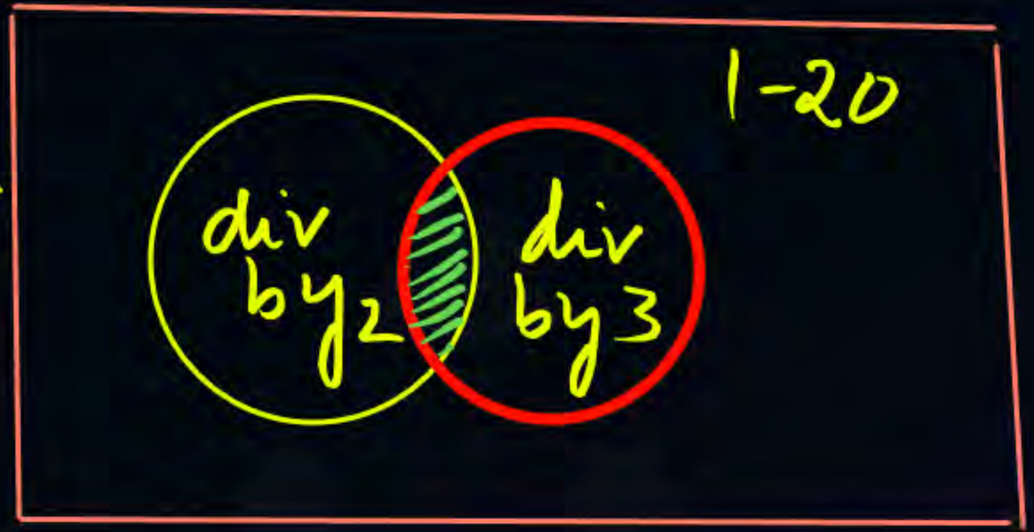
n

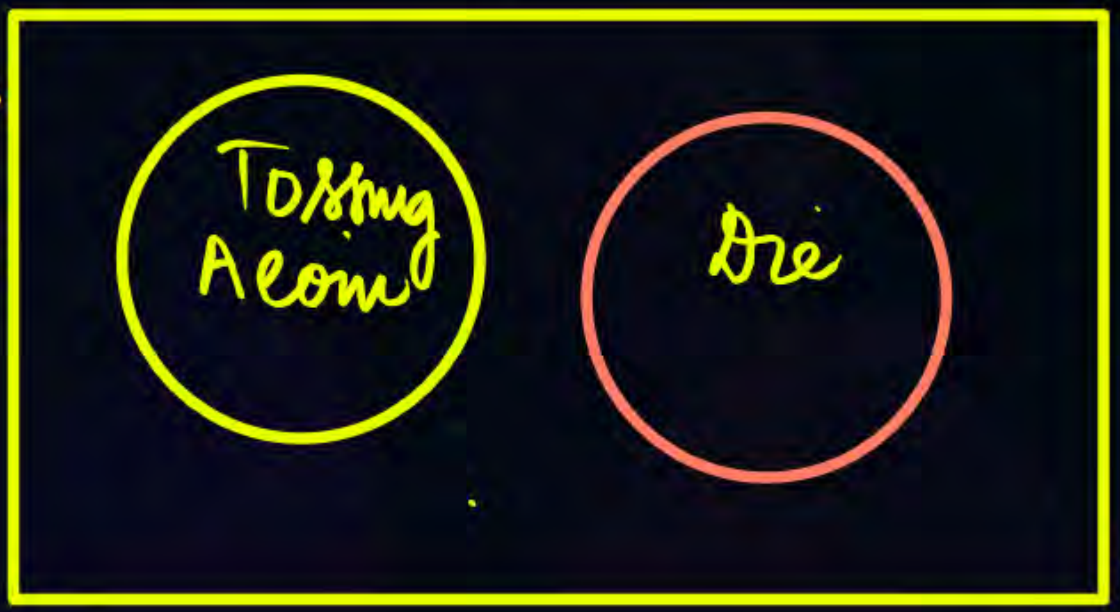
Topic : Classification of events

Single event $P(E) = \frac{n(E)}{n(S)}$

Compound Events:- min Two events OR More Than Two events → compound events

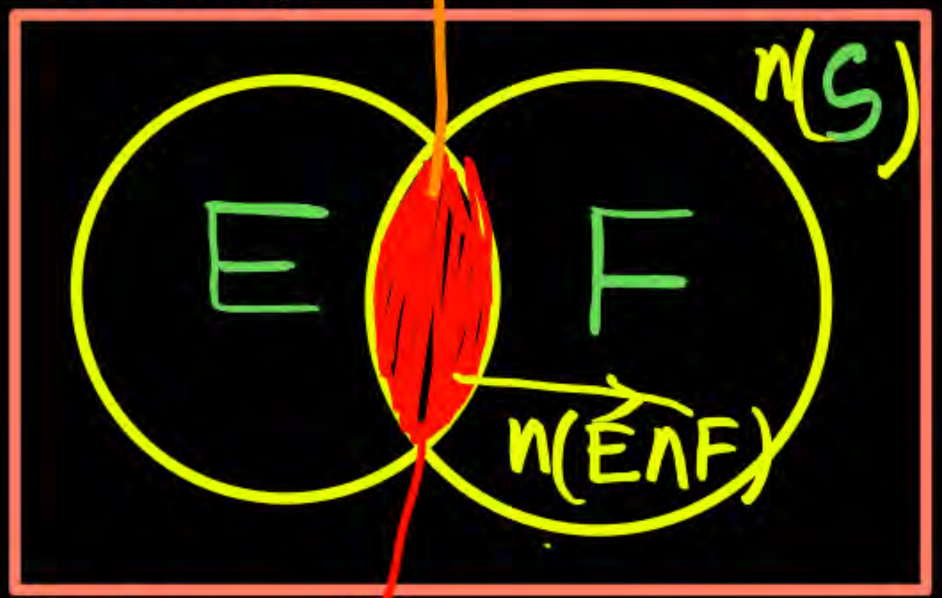
Something is common





Nothing is common

E and F (both occur) $\uparrow (E \cap F)$



✓ CASE No-1 :

Something is common

A) What is the Prob. (common region)

$$P(E \cap F) = P(E \text{ and } F)$$

$$= P(\text{both occur}) = P(\text{Shaded region})$$

common region

$$P(E \cap F) = \frac{\text{No. of favourable region}}{\text{Total region}}$$

✓ $$\boxed{P(E \cap F) = \frac{n(E \cap F)}{n(S)}}$$

$n(E \cap F)$ = Elements of Favourable region

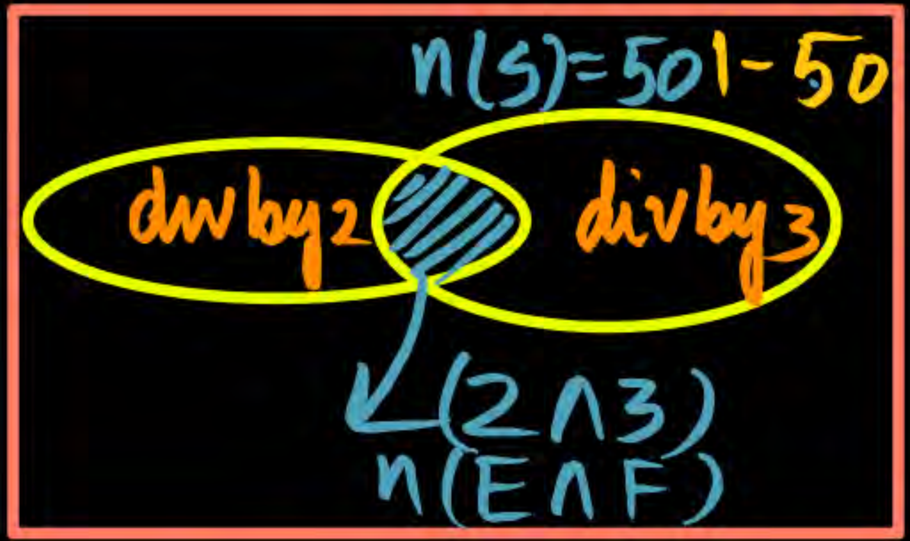
$n(S)$

$P(E \cap F) = \frac{\text{common}}{\text{Total}}$

$P(\text{Div by 2 and 3})$

$= P(2 \cap 3) = P(\text{both Happening})$

$= P(\text{Common region})$



$\boxed{P(E \cap F) = \frac{n(E \cap F)}{n(S)}} = \frac{8}{50}$

What is the Prob. $P(\text{div by 2 and 3})$

div by 2 = 2, 4, 6 ---- 50

div by 3 = 3, 6 --- 48

$n[\text{div}(2 \cap 3)] \Rightarrow 6, 12, 18, 24, 30, 36, 42, 48$

$n(2 \cap 3) = 8.$

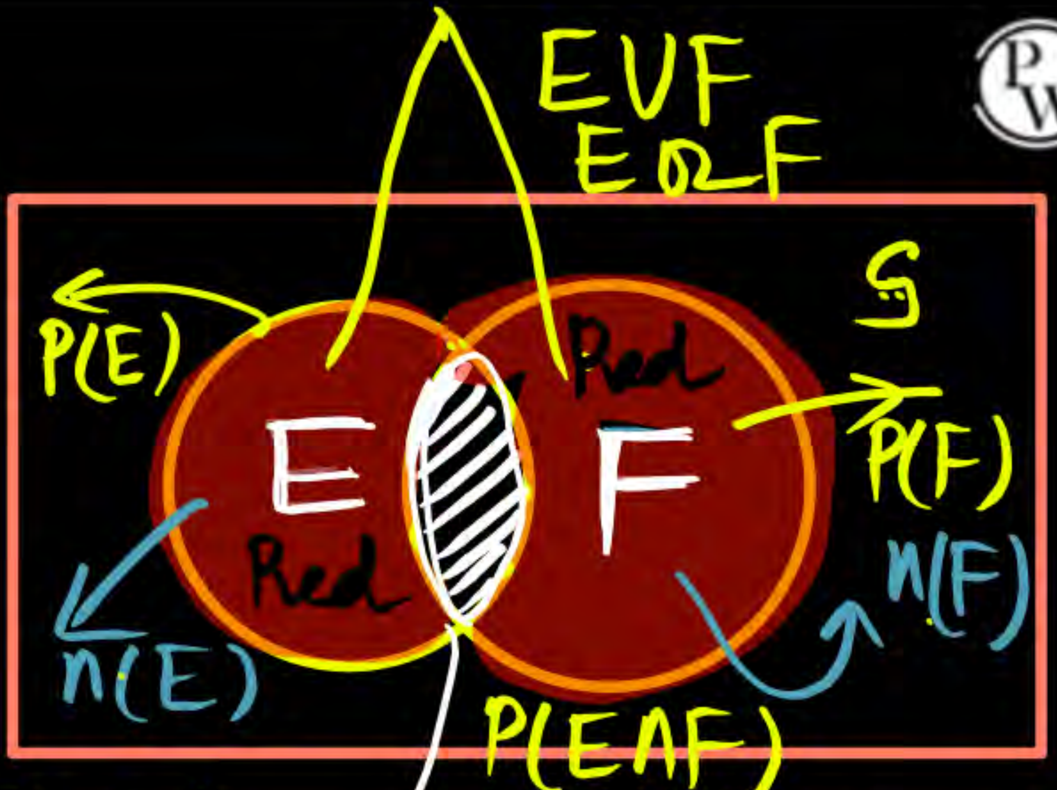
E and F, [common region]

Ans. $\boxed{P(E \cap F) = P(E \text{ and } F) = \frac{4}{25}}$

✓ CASE 02

$P(E \cup F) = P(\text{either A or B})$

$P(\text{red Portion})$



$n(E \cap F)$

common region ✓

Using Venn Diagram:

$$P(E \cup F) = \frac{\text{No. of Favourable}}{\text{Total outcomes}} = \frac{n(E \cup F)}{n(S)}$$

$$P(E \cup F) = \frac{n(E) + n(F) - n(E \cap F)}{n(S)}$$

$\cup$ = or

$\cap$ = And

$$P(\text{either A or B}) = P(E \cup F) = \frac{n(E)}{n(S)} + \frac{n(F)}{n(S)} - \frac{n(E \cap F)}{n(S)}$$

$$\boxed{P(E \cup F) = P(E) + P(F) - P(E \cap F)} \rightarrow \text{prob Addition THEOREM}$$

$$P(E \cup F) = P(\text{Div by } 2) + P(3) - P(2 \cap 3)$$

$$= \frac{25}{50} + \frac{16}{50} - \frac{8}{50}$$

$$\Rightarrow \frac{33}{50} \quad \underline{\text{Ans}}$$

$$\boxed{P(\text{either } A \text{ or } B) = \frac{33}{50}}$$

What is the Prob.

$P(\text{div by } 2 \text{ or } 3)$

$P(2 \cup 3)$

1-50 div by 2 $P(2) = \frac{25}{50}$

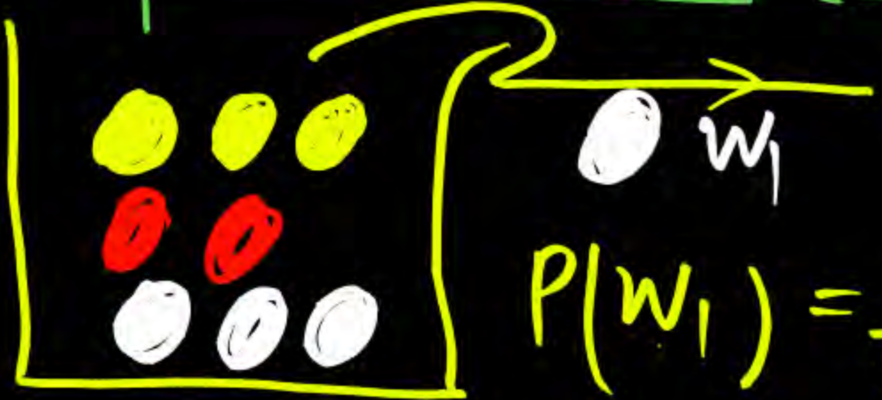
div by 3 $P(3) = \frac{16}{50}$

$(2 \cap 3) = \frac{8}{50}$

✓ CASE 03 Dependent Events / Independent events:

Dependent Events (without Replacement):- What is the Prob: P(3 White ball are drawn at random one at a time)

Job 1

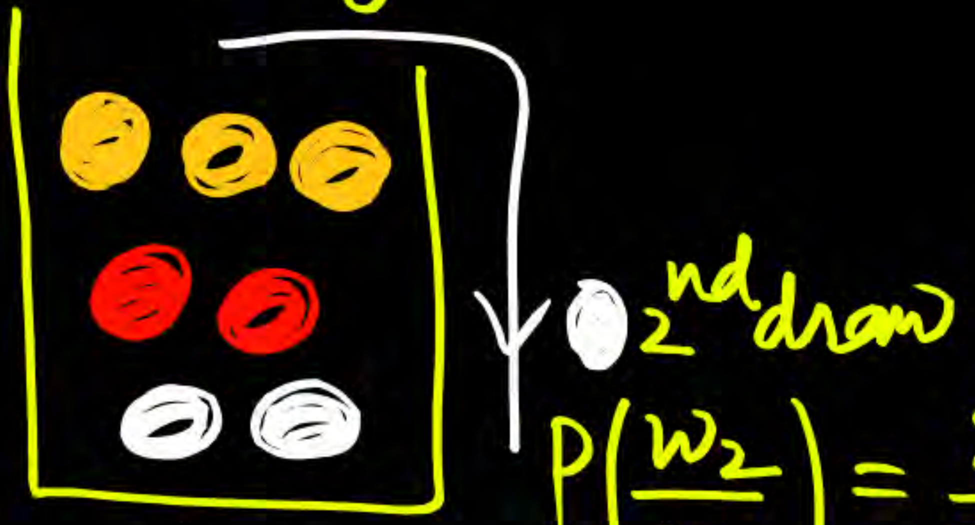


$P(W_1) = \frac{3}{8}$

$P\left[\frac{W_2}{W_1}\right]$

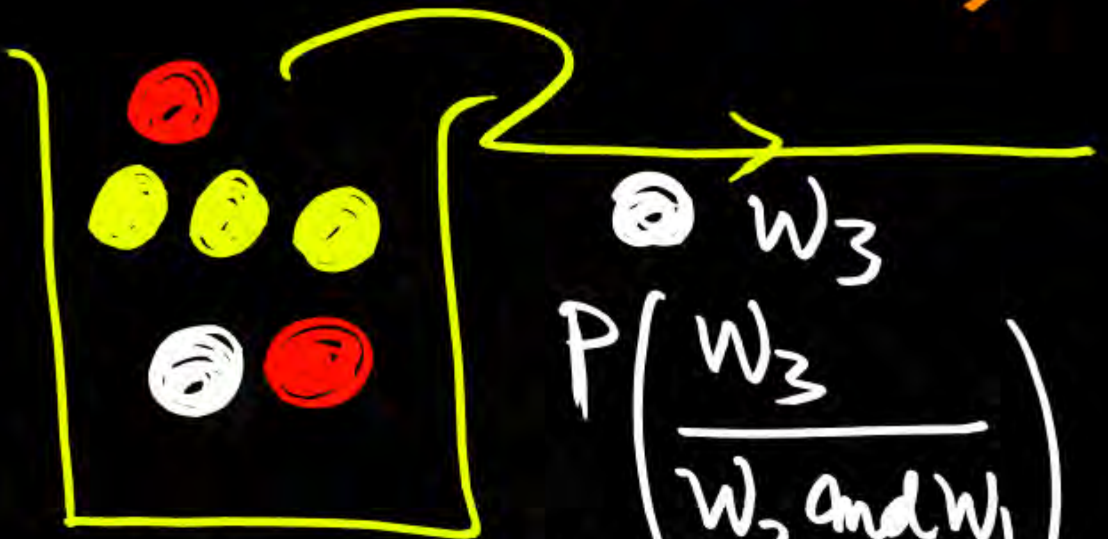
✓ W_1 is already occured

$\frac{W_2}{W_1}$ is occuring.



(Job 2)

$P\left(\frac{W_2}{W_1}\right) = \frac{2}{7}$



$P\left(\frac{W_3}{W_2 \text{ and } W_1}\right)$

(Job 3) $P\left(\frac{W_3}{W_1 \cap W_2}\right) = \frac{1}{6}$

$P(3 \text{ White ball}) =$

$$\Rightarrow \boxed{P(W_1 \cap W_2 \cap W_3) \Rightarrow P(W_1)\, P\left(\frac{W_2}{W_1}\right) P\left(\frac{W_3}{W_1 \cap W_2}\right)}$$

$\Rightarrow$ Dependent events

$\Rightarrow$ chances - CHANGE

$\Rightarrow$ Next event Effected on Previous one

(Dependence of events)

White 1 $\rightarrow$ A   White 2 $\rightarrow$ B   White 3 $\rightarrow$ C

$$\boxed{P(A \cap B \cap C) = P(A)\, P\left(\frac{B}{A}\right) P\left(\frac{C}{A \cap B}\right)}$$

For dependent Events

For n Events

$$\boxed{P(A \cap B \cap C \cap D \cap E \ldots) = P(A)\, P\left(\frac{B}{A}\right) P\left(\frac{C}{A \cap B}\right) P\left(\frac{D}{A \cap B \cap C}\right) \ldots}$$

$\boxed{\begin{matrix} 10\,I \\ 10\,T \end{matrix}}$ $P(IIT) = P(I_1)\, P\left(\frac{I_2}{I_1}\right) P\left(\frac{T}{I_1 \cap I_2}\right)$ Ans

Independent Events :

" with Replacement"

$P(W_1) = \frac{3}{8}$ $\quad P\left(\frac{W_2}{W_1}\right) = \frac{3}{8}$ $\quad P\left(\frac{W_3}{W_1 \cap W_2}\right) = \frac{3}{8}$

What is The Prob.
P(3 white ball Drawn at random)

If all Are working Together

$$P(W_1 \cap W_2 \cap W_3) = P(W_1)\, P\left(\frac{W_2}{W_1}\right) P\left(\frac{W_3}{W_1 \cap W_2}\right)$$

$$P\left(\frac{W_2}{W_1}\right) = P(W_2)$$

$$P\left(\frac{W_3}{W_1 \cap W_2}\right) = P(W_3)$$

$$\boxed{P(W_1 \cap W_2 \cap W_3) = P(W_1)\, P(W_2)\, P(W_3)}$$

→ for Independent events

$W_1 \to A$
$W_2 \to B$
$W_3 \to C$

For Two events condition for Independence

$$\boxed{P(A \cap B) = P(A)\,P(B)} = \text{Independent}$$

For THREE EVENTS

$$\boxed{P(A \cap B \cap C) = P(A)\,P(B)\,P(C)}$$ # Independent

For n events

$$P[A \cap B \cap C \cap D \cap E \cdots] = P(A)\,P(B)\,P(C)\,P(D)\,P(E) \cdots\cdots$$

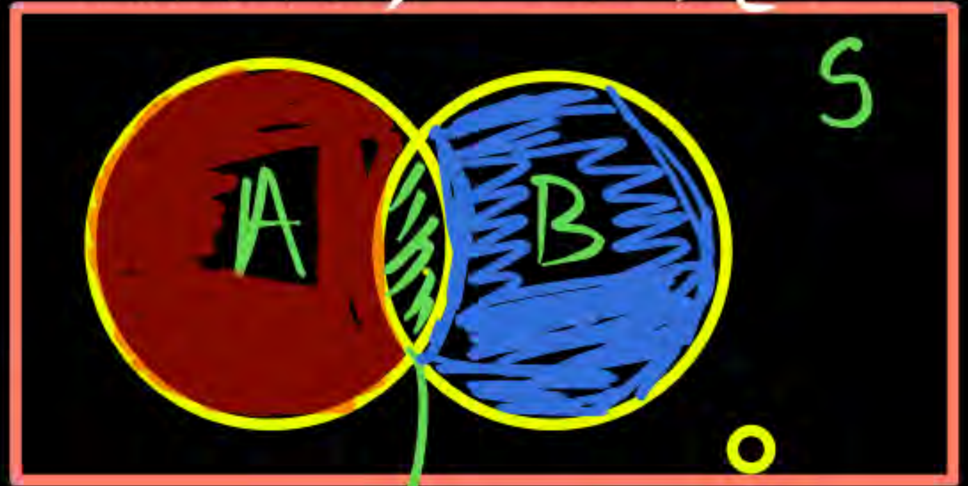
(Independence of events)

CASE-04

P(only A), P(only B), P(exactly one)

$$\begin{cases} P(\text{only } A) \Rightarrow P(A) - P(A \cap B) = P(A) - P(A)P(B) = P(A)[1 - P(B)] \\ P(\text{only } B) \Rightarrow P(B) - P(A \cap B) \end{cases}$$

"A and B Are Independent events"



A, B Independent

$\bar{A}, \bar{B}$ Indep.

$\bar{A} = 1 - A$

$\bar{B} = 1 - B$

$$P(\boxed{\text{only } A}) = P(A \cap \bar{B}) = P(A)P(\bar{B}) = P(A)[1 - P(B)]$$

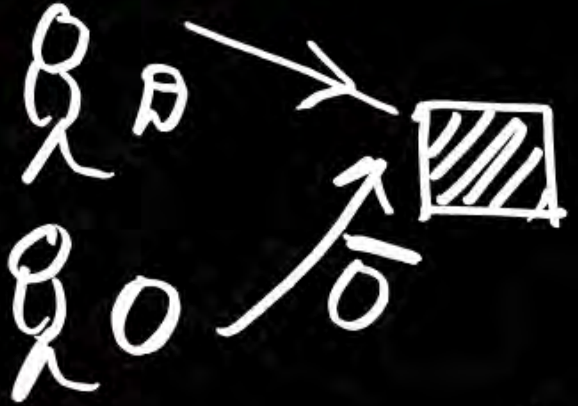
A work, $\bar{B}$ Not work

$P(A \cap \bar{B})$

$P(\text{only } A)$

$D \cap \bar{O}$

$O \cap \bar{D}$



D pass Om kar fail

O Pass Deepanshu fail.

$$P(\text{only } B) = P(B \cap \bar{A}) = P(B)\,P(\bar{A})$$

$$\boxed{P(\text{only } B) = P(B)[1 - P(A)]}$$

$P(\text{exactly one}) = P(\text{only } A) + P(\text{only } B)$

$$= P(A \cap \bar{B}) + P(B \cap \bar{A})$$

$$+ \quad A \; \bar{B}$$
$$\quad B \; \bar{A}$$

$$\boxed{P(\text{exactly one}) = P(A) + P(B) - 2P(A)P(B)}$$

$$\Rightarrow P(A)[1 - P(B)] + P(B)[1 - P(A)]$$

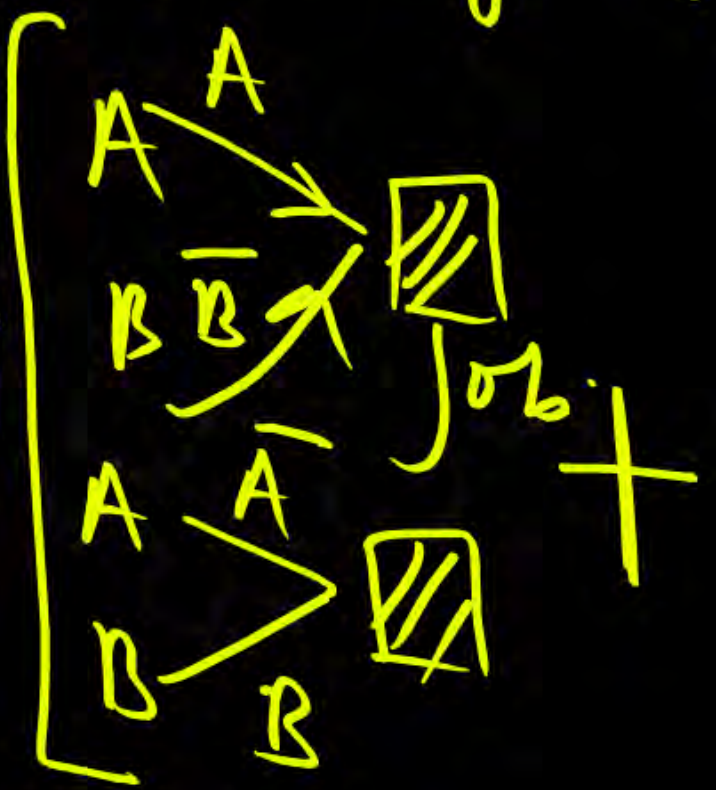
$$\Rightarrow \underline{P(A) + P(B) - 2P(A)P(B)}$$

Exactly one.

$$\underbrace{(A \cap \bar{B}) \cup (B \cap \bar{A})}$$

Exactly one (A, B)

$P(\text{exactly one})$



Diagram

Two Events , [Something is common)

E and F independent events)

E F

$$E \begin{cases} E \text{ (Happen)} \\ \bar{E} \text{ (does Not Happen)} \end{cases}$$

$$F \begin{cases} F \text{ (Happen)} \\ \bar{F} \text{ (does Not Happen)} \end{cases}$$

$$E \begin{cases} F = E \wedge F \\ \bar{F} = E \wedge \bar{F} \end{cases}$$

$$\bar{E} \begin{cases} F = \bar{E} \wedge F \\ \bar{F} = \bar{E} \wedge \bar{F} \end{cases}$$

	F (Happen)	$\bar{F}$ (does Not Happen)
E (Happen)	$E \wedge F$ (both occur)	$E \wedge \bar{F}$ (only E)
$\bar{E}$ (does Not Happen)	$F \wedge \bar{E}$ (only F)	$(\bar{E} \wedge \bar{F})$ Neither E Nor F)

P(exactly one)

	H	T
H		
T		

THANK - YOU