

Data Science and Artificial Intelligence

Probability and Statistics

Random Variable

Lecture No.- 06



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Recap of Previous Lecture



Topic

Max and Min of Random Variables



Topics to be Covered



Topic

Max and Min of Random Variables
(Part-02)





Topic : Transformations of Random Variables 02

Transformation of Random Variables:

If x is a random variable $\longrightarrow f_x(x)$ prob. density Function
What is Transformation $\xrightarrow{\text{X is a random variable}}$

$Y = \phi(x) = \text{one-one Function}$

$x = \phi^{-1}(y)$ Invertible function

$$P[(x, x+dx)] \longrightarrow P[X \in (x, x+dx)]$$

Random var.

$$P[(y, y+dy)]$$

$$P[Y \in (y, y+dy)]$$

$$\begin{aligned} y &= x^2 & y &= x^3 \\ y &= \sqrt{x} \end{aligned}$$

Transformation
Transmitter
Receiver

$$P[X \in (x, x+dx)] = P[Y \in (y, y+dy)]$$

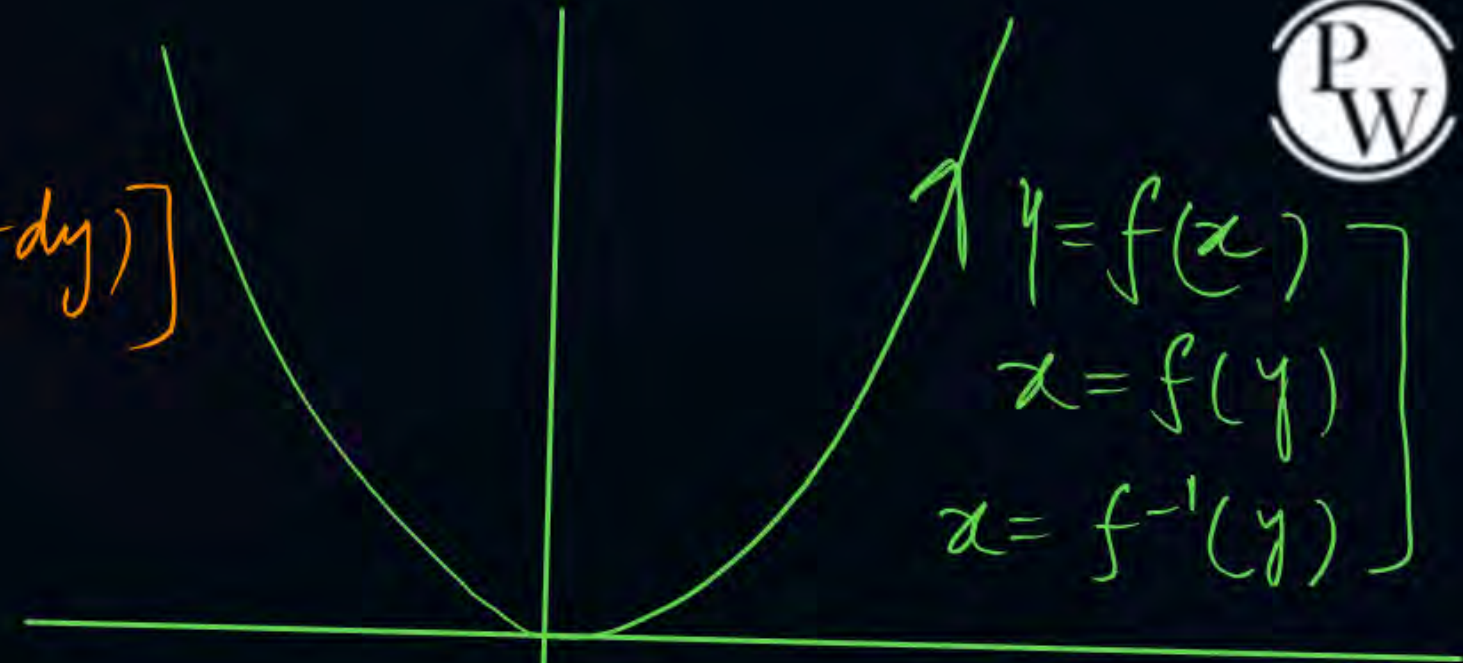
cdf Form $\Rightarrow \frac{F_X(x+dx) - F_X(x)}{dx} \cdot dx$

$$F_X'(x) \cdot dx$$

pdf $\rightarrow f_X(x) dx = f_Y(y) dy$

$$F_Y(y) = \left[f_X(x) \right]_{x=\phi^{-1}(y)}$$

magnitude $\left| \frac{dy}{dx} \right|_{x=\phi^{-1}(y)}$



both are
one-one
Function

$$F_Y(y) = \left[f_X(x) \right]_{x=\phi^{-1}(y)}$$

$$\left| \frac{dy}{dx} \right|_{x=\phi^{-1}(y)}$$

$$f_X(x) = e^{-x} \quad x \geq 0$$

$$f_Y(y) = \frac{f_X(x) |_{x=\phi^{-1}(y)}}{\left| \frac{dy}{dx} \right|_{x=\phi^{-1}(y)}}$$

Exponential random variable
find the $Y = \sqrt{X}$

Transform The
random variable.

step ① $y = \sqrt{x}$

$$x = y^2$$

$$x = \phi^{-1}(y)$$

$$x = y^2$$

$$f_Y(y) = e^{-x} |_{x=y^2} = e^{-y^2}$$

$$\frac{1}{2\sqrt{x}} |_{x=y^2} = \frac{1}{2\sqrt{y^2}} = 2ye^{-y^2}$$

$$f_Y(y) = 2ye^{-y^2}$$

step ② $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \quad x \geq 0$

✓ step ①

$$f_Y(y) = \frac{f_X(x) |_{x=\phi^{-1}(y)}}{\left| \frac{dy}{dx} \right|_{x=\phi^{-1}(y)}}$$

$f_X(x) = \text{given}$

$$y = \sqrt{f_X(x)}, f_X(x)$$

$$f_X(x) \Rightarrow 2e^{2x} \quad x \geq 0$$

$$y = \sqrt{x}$$

$$y = \sqrt[3]{f_X(x)}$$

\swarrow x is a Random variable

$$= (2e^{2x})_{x=\phi^{-1}(y)}$$

$$\frac{1}{2\sqrt{x}} \Big|_{x=\phi^{-1}(y)}$$

$$= \frac{2e^{2y^2}}{\frac{1}{2\sqrt{y^2}}} = 2 \cdot 2y e^{2y^2} = \underline{4ye^{2y^2}}$$

one-one Function

Invertible function

$$y = \sqrt{x} \quad \underline{\underline{x \geq 0}}$$

$$\underline{x = y^2}$$

✓ don't discuss - solution

prob → P & C
 → Equations X
 → Calculus
 (basic Integration)

15-20 days

→ Binary memory

✓ Time Bound Practice

✓ Mock TEST

✓ Topic - question Solved
 (Practice)

✓ DPP → 50% at least

Workout
 → Logic
 Previous concept
fundamental
weakness

50% { Teacher
 group
 Internet
 Friends
Leave

THANK - YOU