Data Science and Artificial Intelligence

Probability and Statistics

Discrete Probability Distribution

Lecture No.- 01

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Topics to be Covered

Topic Bernoulli Distribution

Topic Binomial Distribution

Probability Distribution

Discrete case

D
I
S
C
R
E
T
E
Distri

- ✓ Bernoulli Distribution
- ✓ Binomial Distribution
- ✓ Uniform Distribution
- ✓ Geometric Distribution
- ✓ Hypergeometric Distribution
- ✓ Negative Binomial Distribution
- ✓ Poisson Distribution
- ✓ problem based on Discrete Distribution (2 lecture)

Continuous case

- ✓ Uniform Distribution
- ✓ Beta Distribution
- ✓ Gamma Distribution
- ✓ Normal Distribution
- ✓ St. Normal Distribution
- ✓ t-Distribution
- ✓ F-Distribution
- ✓ Chi-Square Distribution
- ✓ Pareto Distribution
- ✓ Log Normal Distribution

Weibull Distribution
Double exponential Dis
Exponential Distribution

Distribution — Model → Real life Problem — Inspire

→ Function

→ MEAN/st/variance

→ Moment generating function

Bernoulli Distribution

Prob. Distri (Discrete manner)

(A) Σ Prob. of all Possible outcomes = 1

(B) Random variable X Are evenly space (Discrete Distribution)

Prob. Discrete Distribution.

$X = \underbrace{0, 1}_{1}, \underbrace{1, 2}_{1}, \underbrace{2, 3}_{1}, \underbrace{3, 4}_{1}, \underbrace{4, 5}_{1} \ldots$

Discrete Distribution → Pattern of Arrival

✓ Tossing A coin → $\boxed{H}$ / T → S/F

✓ Accident → YES/NO F/S

∟ Pick a ball → Red / Not Red S/F

✓ Throwing A Die → $5/\overline{5}$ S/F

✓ Average No. of customer → NO

Arrival Pattern] { SUCCESS (1) Present | FAILURE (0) Not present

— A Single Trial

→ Indicator Function

$I = \begin{cases} 1 - \text{SUCCESS} \\ 0 - \text{failure} \end{cases}$

only Single Trial

[→ child — H (1), T (0)] H and T both Independent

coin Toss

[→ child — SUCCESS (1), failure (0)] Single Trials (Trials fixed)

✓ Trials Are fixed.

✓ Prob. Distribution Equally.

Single Trial — SUCCESS / Failure $\Longrightarrow$ $\boxed{\text{Bernoulli Trials. (Independent Trial)}}$

If one child Tossing A coin

S / F

Prob. of SUCCESS $= p$

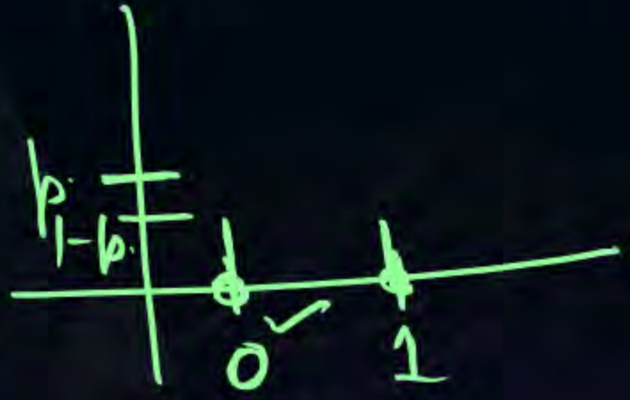
prob. of failure $= (1-p)$

X	SUCCESS(1)	Failure(0)
$P(X=x_i)$	p	$(1-p) = q$

Prob. mass function

$$P[X=x_i] = p^x q^{1-x}$$

$$\boxed{P[X=x_i) = p^x (1-p)^{1-x}}$$



$X = 0, 1$

{ bernoulli Distribution }

Single Trial

$\left\{ \begin{array}{l} X = 0, 1 \\ F \mid S \end{array} \right.$

$\boxed{\text{prob. Are equal Distributed}}$

$$P[X=0] = (1-p)$$

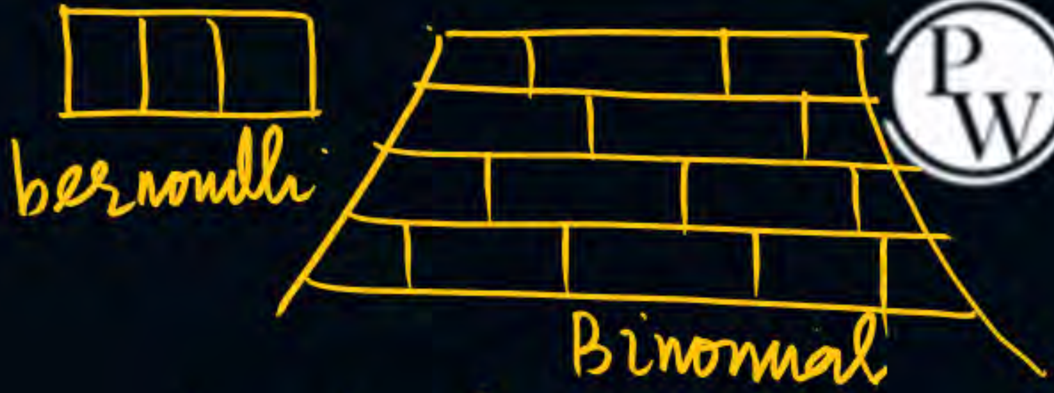
$$P[X=1] = p^1(1-p)^0 = p.$$

If number of trials ↑

Trials fixed

Independent Bernoulli Trials

Independent events



Prob. of success HEAD Are Also fixed (Single Trial)

SUCCESS Are Not fixed - This is Random

$\boxed{X = 0, 1, 2, 3, 4, 5}$

SUCCESS → 5 child

0H
1H
2H
3H
4H

X = 0H   X = 3H
X = 1H   X = 4H
X = 2H   X = 5H

SUCCESS

Brick

1	0

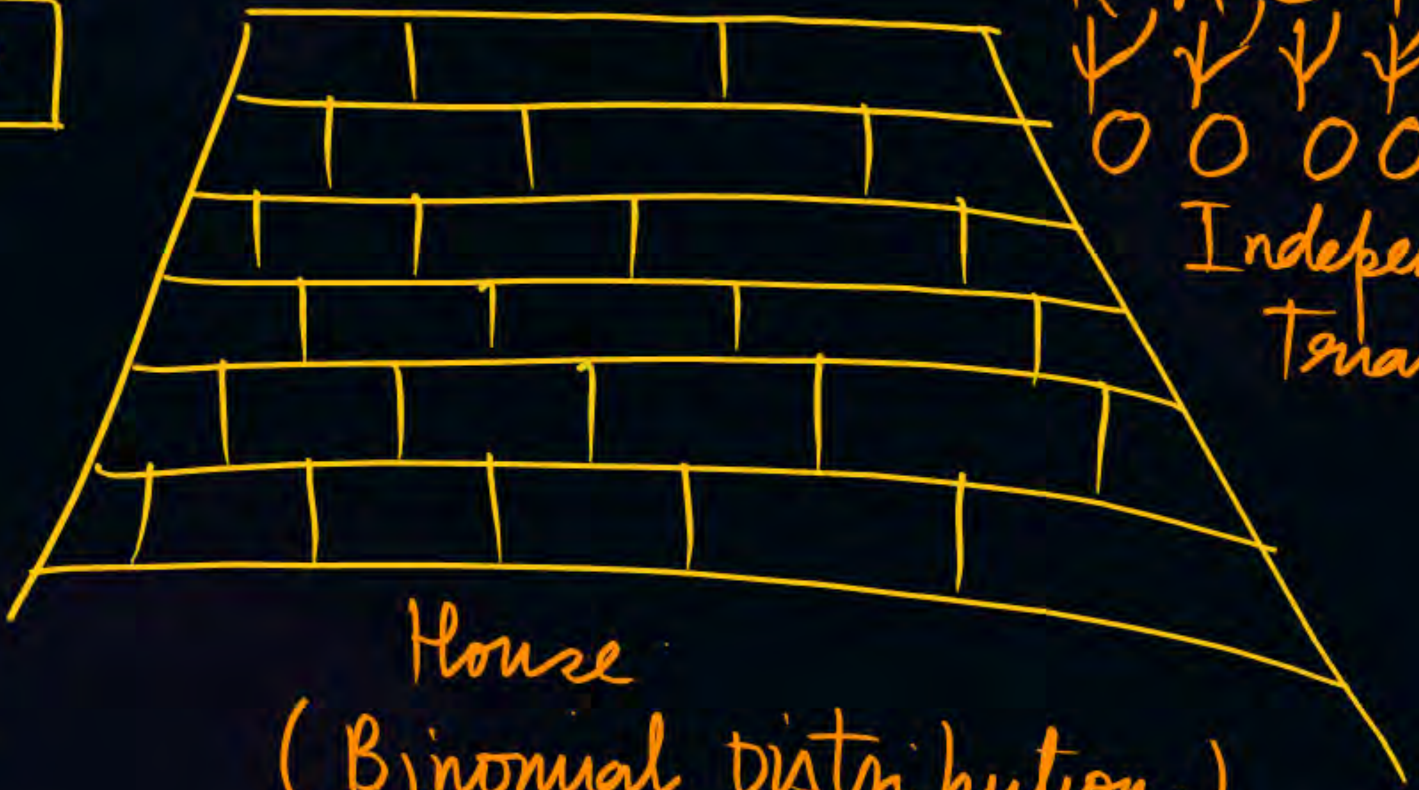
brick.

S/F 1/0

Single Trial.

(Bernoulli)

(Independent)



House.

(Binomial Distribution)

(Trials Are fixed)

Prob. are equal Distribution

S/F

R A S G N

O O O O O

Independent Trials.

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

Re1 Re2 Re3 Re4 Re5

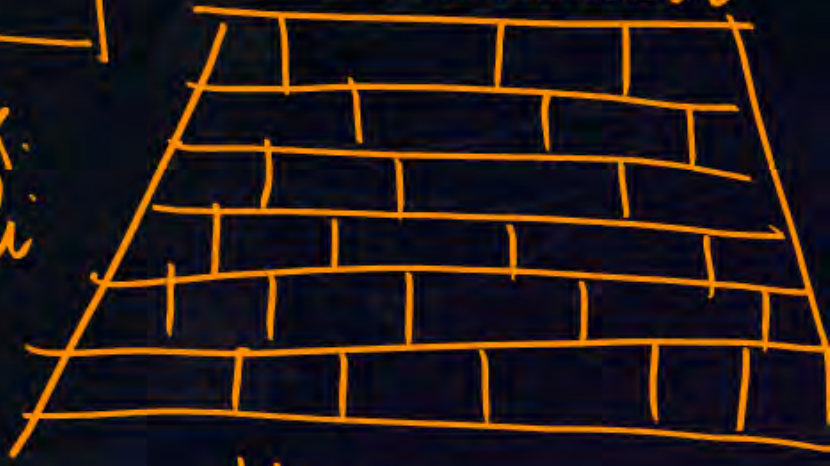
$X = 0, 1, 2, 3, 4, 5$

+1 +1 +1 +1

Discrete Random variable.

Prob. are also SAME

brick. bernoulli → binomial



House.

What is The SUCCESS

(Arrival Pattern

X = random variable.

X = 0N, 1N, 2N, 3N, 4N, 5N

X = random variable 0, 1, 2, 3, 4, 5

If n = No. of Trials

r = No. of success 0, 1, 2, 3, 4, 5 ....

Success	Faluze
'r' times	$(n-r)$ times
p^r	q^{n-r}

$$\boxed{P[X = r \text{ success}] = {}^nC_r\, p^r q^{n-r}}$$

$\boxed{1}$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Discrete Random

$P(1) = \frac{1}{6}$

$P(\bar{1}) = \frac{5}{6}$

$P(1) + P(\bar{1}) = 1$

$r = \underbrace{0, 1, 2, 3, 4, 5 ....}_{\text{No. of successess}}$

$$P(X=r) = {}^nC_r\, p^r q^{n-r}$$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4 \times 4 \times 2}$

$= \frac{1}{32}$

$$P(X=0) = {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0} \Rightarrow \frac{1}{32}$$

$$P(X=1) = {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1} \Rightarrow \frac{5}{1} \times \frac{1}{32} = \frac{5}{32}$$

$$P(X=2) = {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2} \Rightarrow \frac{5 \times \cancel{4}^{2}}{\cancel{2} \times 1} \times \frac{1}{32} = \frac{10}{32}$$

$$P(X=3) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} \Rightarrow \frac{5 \times \cancel{4}^{2} \times \cancel{3}}{\cancel{3} \times \cancel{2} \times 1} \cdot \frac{1}{32} = \frac{10}{32}$$

$$P(X=4) = {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} \Rightarrow \frac{5}{32}$$

$$P(X=5) = {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-5} \Rightarrow \frac{1}{32}$$

H, T   H, T   H, T   H, T   H, T

A   B   C   D   E

$n = 5$ trials

prob. of success $p = \frac{1}{2}$

Failure $q = \frac{1}{2}$

$X = 0, 1, 2, 3, 4, 5$

Arrival Pattern Discrete

Arrival	0	1	2	3	4	5
Pattern	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

$n = 3 \quad P(H) = \frac{1}{2} \quad P(T) = \frac{1}{2}$

no. of trials $= 3 \quad P(S) = \frac{1}{2} \quad P(F) = \frac{1}{2}$

Binomial Distribution

$$P[X = r] = {}^nC_r \, p^r q^{n-r}$$

r = No. of success
p = prob. of success
q = prob. of failure

$$P(X = 0) = {}^3C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{3-0} = \frac{1}{8}$$

$$P(X = 1) = {}^3C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{3-1} = \frac{3}{8}$$

$$P(X = 2) = {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{3-2} = \frac{3}{8}$$

#

A	B	C
Re 1	Re 2	Re 3

$n = 3$ Trials.

What is The Prob.

✓ $P[X = 0H, 1H, 2H]$

✓ $P[X \geq 1]$ at least one HEAD

✓ $P[X \leq 1]$ at most one HEAD.

X	0	1	2
$P(X = x_i)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$

Arrival Pattern

HHT
HTH
TTH

$\underline{1+2+3}$

$P(X \geq 1) = P(\text{atleast one HEAD}) = P(X=1) + P(X=2) + P(X=3)$

$= {}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{3-1} + {}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{3-2}$

$+ {}^3C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{3-3}$

$$\boxed{P(X \geq 1) = \frac{7}{8}}$$

$P(x \leq 1) = P(X=0) + P(X=1)$

$= {}^3C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{3-0} + {}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{3-1}$

$= \frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$

Bernoulli Trials (Bernoulli Dist)	Binomial Distribution $B(n, p)$
(A)  brick	(A)  all bernoulli Trials make the Binomial Dist.
(B) Arrival Pattern	B) Arrival
C) $P(X=x) = p^x (1-p)^{1-x}$, $x = (0, 1)$	C) $P[X=r] = {}^nC_r \, p^r q^{n-r}$ [Trials, prob $\to$ fixed
D) Independent	D) Independent
E)  $\xrightarrow{\text{Trans from}}$  Bernoulli $\to$ Bionomial	E)   $\Rightarrow$  Bionomial $\to$ binomial.

Bernoulli Distribution

$P(X=x) = p^x (1-p)^{1-x} \quad x = 0,1$

X	1 (S)	0 (F)
$P(X=x_i)$	p	$(1-p) = q$

✓ Expected value $E[X] = 1 \times p + (0)(1-p)$

$\mu = E[X]$

$$\boxed{E[X] = p}$$

✓ Variance $= E[X^2] - [E[X]]^2$

σ_X^2

$= (1)^2 p + (0)^2 (1-p) - [p]^2$

$= p - p^2 = p(1-p) = pq$

✓ $\boxed{\text{Variance} = pq}$

✓ Standard deviation $= \sqrt{pq}$

Binomial Distribution (For n Trials)

MEAN $E[X] = np$

Variance $V(X) = \sigma_x^2 = npq$

Standard deviation $= \sqrt{npq}$

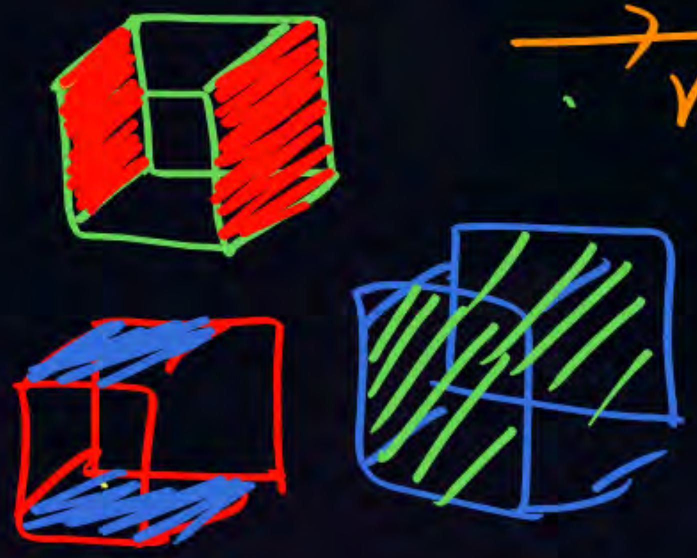
Where n = No. of trials

p = prob. of SUCCESS

q = prob. of failure.

Topic : Bernoulli Distribution & Binomial Distribution

Q7. Consider an unbiased cubic die with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the die. If the die is thrown thrice, the probability of obtaining red colour on top face of the die at least twice is_____.



$n = 3$ Die Thrown

Red ball SUCCESS

Not red Failure

$X = 0, 1, 2, 3$

Red
Not red

Bernoulli Trials

Red
Not red

Red
Not red

$P(X=0)+P(X=1)+P(X=2)+P(X=3) = 1$

$\boxed{X = 0, 1, 2, 3}$

No. of SUCCESSES

$P(X \geq 2) = P(X=2) + P(X=3)$

$= 1 - P(X=0) - P(X=1)$

$P(\text{red}) = \frac{2}{6} = \frac{1}{3}$

$P(\text{Not red}) = 1 - \frac{1}{3} = \frac{2}{3}$

$p = \frac{1}{3} \quad q = \frac{2}{3} \quad n = 3$

$P(X=r) = {}^nC_r \, p^r q^{n-r}$

$P(X=2) = {}^3C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{3-2}$

$P(X=3) = {}^3C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{3-3}$

$P(X \geq 2) = {}^3C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{3-2} + {}^3C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{3-3} = \frac{7}{27}$

$P(6) = \frac{1}{6}$ $P(\bar{6}) = \frac{5}{6}$ $n = 4$

Q8. The probability of obtaining at least two SIX' in throwing a fair dice 4 times is ____.

$P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$

$= 1 - P(X=0) - P(X=1)$

A. 425/432

B. 19/144

C. 13/144

D. 125/432

$P(X=0) = {}^4C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{4-0}$

$P(X=1) = {}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{4-1}$

$= 1 - {}^4C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^4 - {}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3$

$= \frac{19}{144}$

D_1 D_2 D_3 D_4

$6\ \bar{6}$ $6\ \bar{6}$ $6\ \bar{6}$ $6\ \bar{6}$

min 2 — 6

$X = 0, 1, 2, 3, 4$

THANK - YOU