Data Science and Artificial Intelligence

Probability and Statistics

Random Variable

Lecture No.- 01

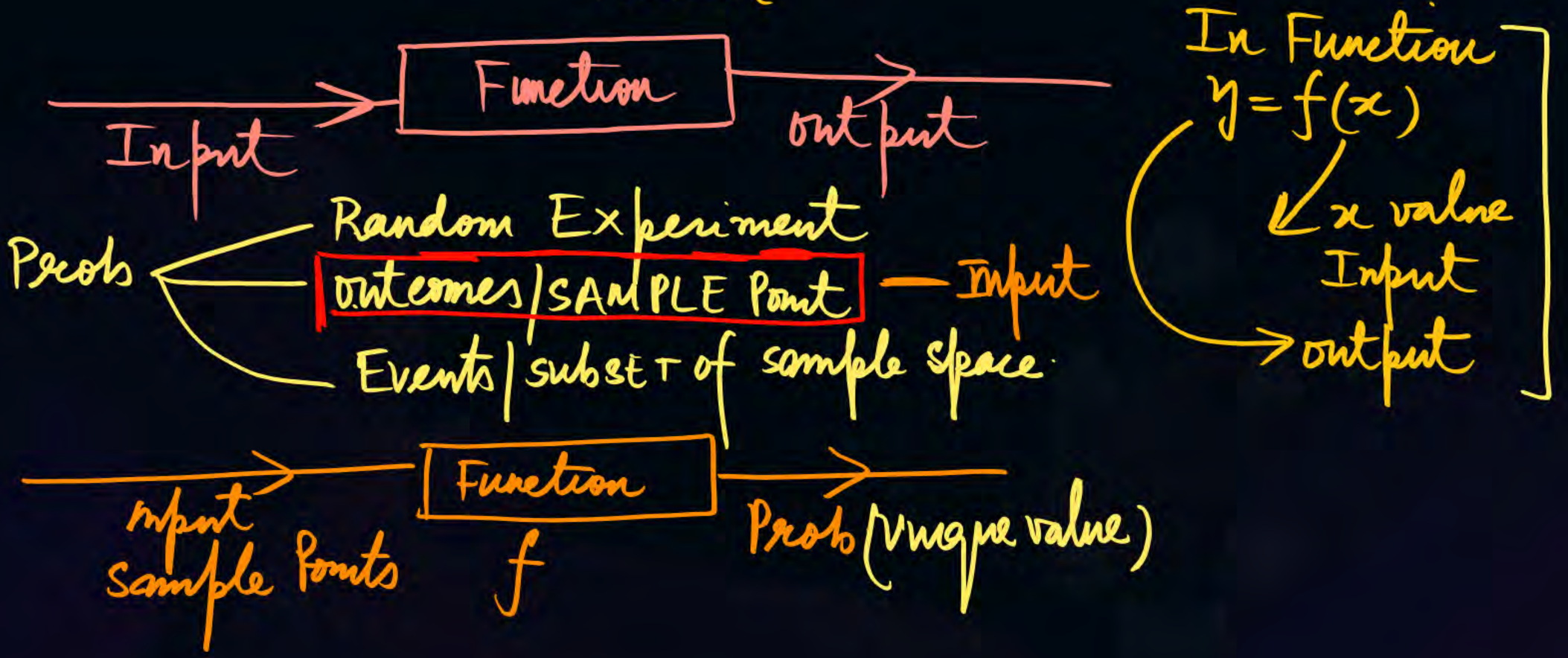
Topics to be Covered

Topic	Random Variable Part 1

lengthy

Topic : Random Variable

Random Variable: Random Variable is a Mathematical Function.



Input	HH	HT,	T.H	TT
X	2	1	1	0

X = No. of HEADS / No. of tails

both variable Are change.

X = No. of HEADS

X = 0, 1, 2

Change

Tossing A Two Coin

Re 1, Re 2

S = {HH, TH, TT, HT}

Input

Simple Points

$P[X=0H] = \frac{1}{4}$ $P[X=1H] = \frac{2}{4}$ $P(X=2) = \frac{1}{4}$

random variable

✓ Random variable real valued function $\left\{\frac{1}{4}, \frac{2}{4}, \frac{1}{4}\right\}$

Tossing A THREE coins

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

Re 1, Re 2, Re 5

HHH	0T
HHT	1T
HTH	1T
HTT	2T
THH	1T
THT	2T
TTH	2T
TTT	3T

X = No. of tails

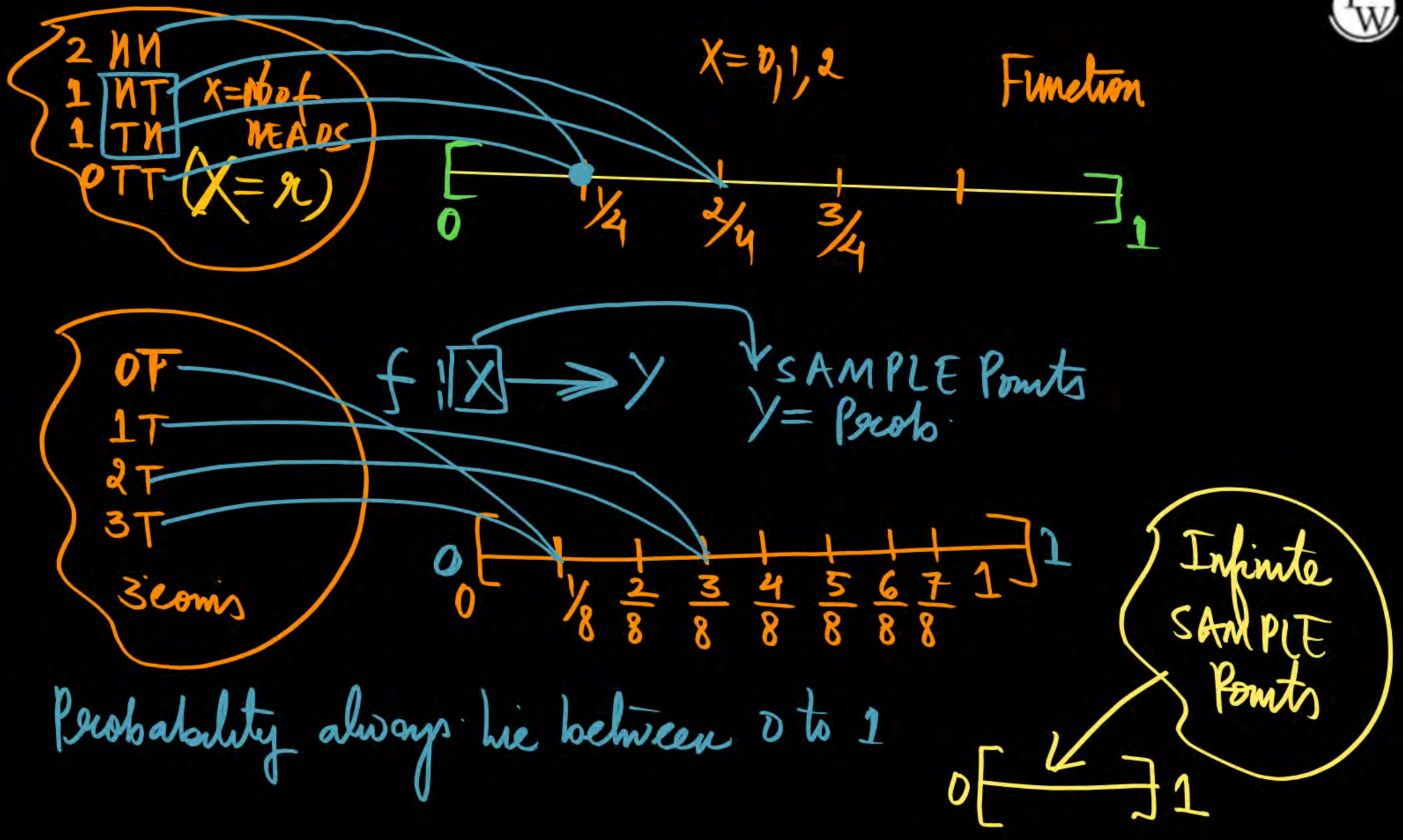
X = 0, 1, 2, 3

P(X = x) = Random variable

$P(X=0T)=\frac{1}{8}$ $P(X=1T)=\frac{3}{8}$ $P(X=2T)=\frac{3}{8}$ $P(X=3T)=\frac{1}{8}$

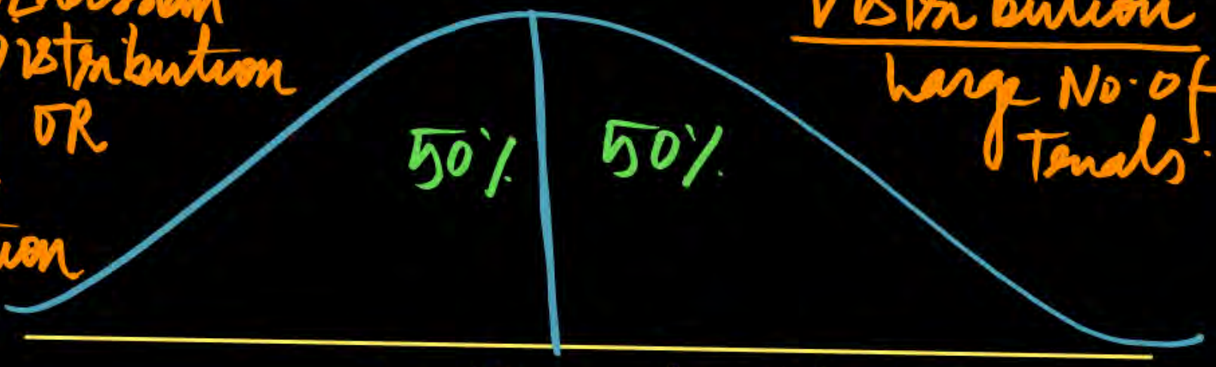
all Sum (prob) = 1

$$\sum_{i=1}^{n} P_i = 1$$


2 HH
1 HT
1 TH
0 TT
X = No of HEADS
(X = x)
X = 0, 1, 2
Function
0
1/4
2/4
3/4
1
OF
1T
2T
3T
3 coins
f : X → Y
SAMPLE Points
Y = Probb
0
1/8
2/8
3/8
4/8
5/8
6/8
7/8
1
Probability always lie between 0 to 1
Infinite SAMPLE Points
0
1

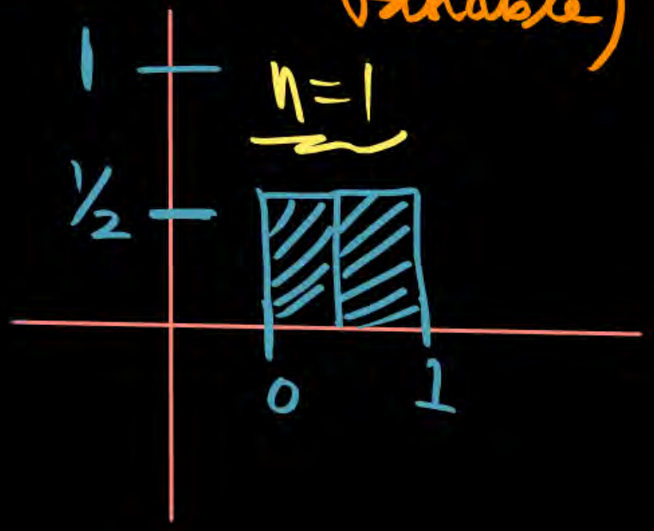
Large No. of Trials

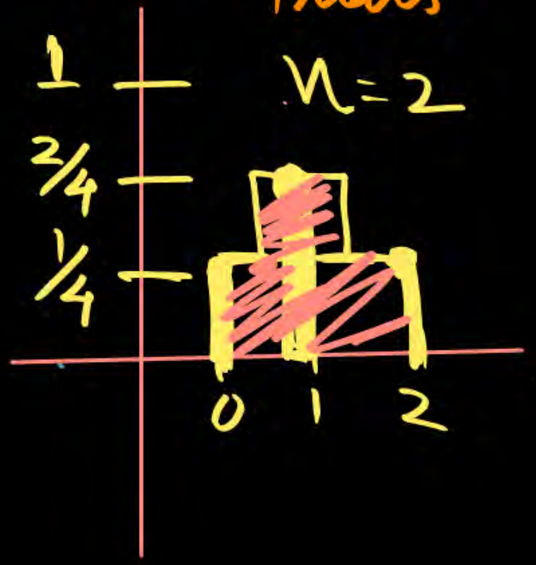
Gaussian Distribution OR Normal Distribution

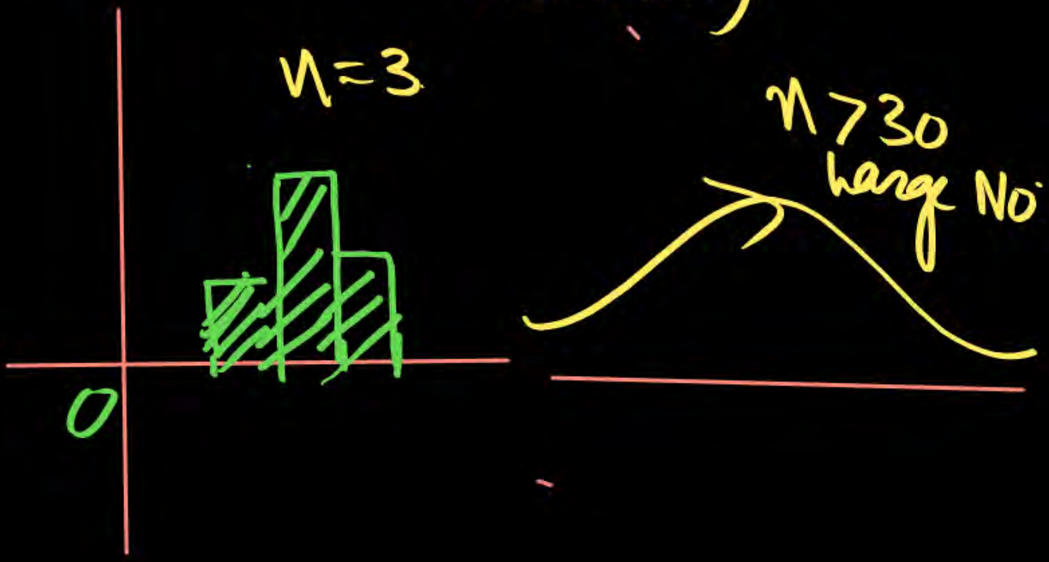


Distribution / Large No. of Trials

If any Experiment (Normal random variable) → n Large No. of Trials → Normal Distribution / (Pattern)







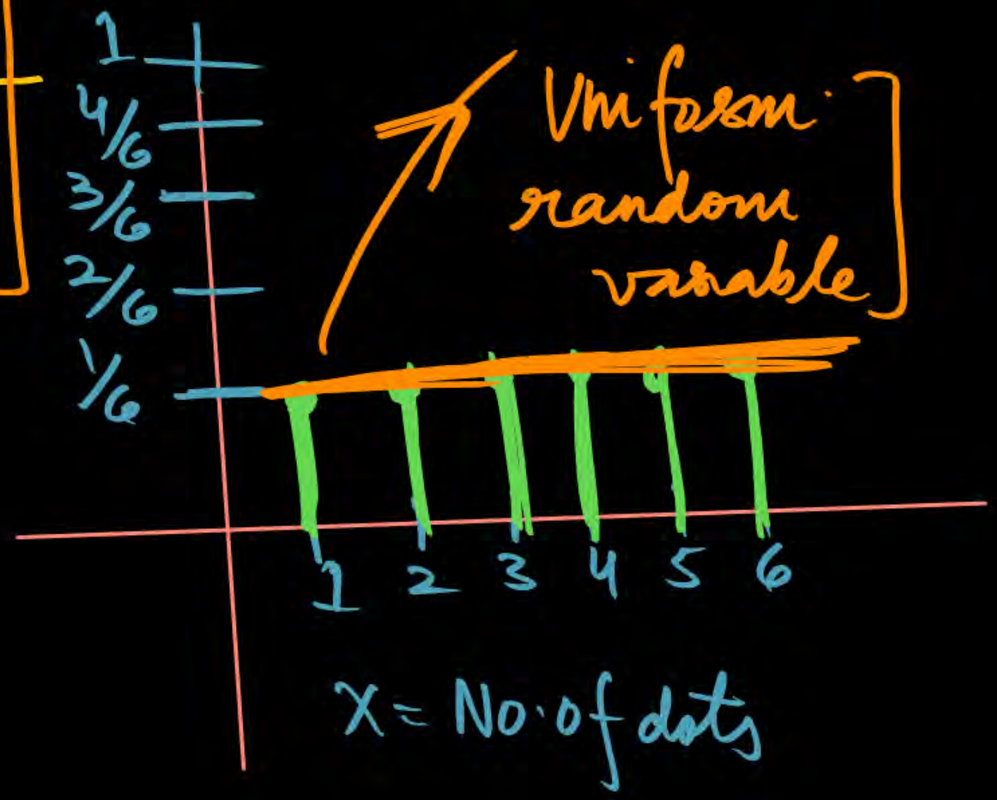
Rolling A Balaced Die.

$S = \{1, 2, 3, 4, 5, 6\}$

X = No. of dots

Make The Table:

X	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$



Random Variable

always defined Interval

[Continuous random variable]

Countable
[Infinite countable SET]
(Discrete)

(measurable)
(measure)

Uncountable
(Infinite Uncountable SET)

Integer value $X = 0, 1, 2 \ldots$

- ⟶ Rolling A Dice
- ⟹ Pick a ball
- ⟹ flipping A coin
- ⟶ Pick A card

$(a \leq X \leq b)$

- ✓ Discharging A Battery
- ✓ Person Height $1 \leq x \leq 2$
- ✓ Distance-length $1 \leq x \leq 2$ $a \leq x \leq b$

Uncountable
(Infinite
Uncountable
SET)

Infinite Point

Measure (माप)

✓ Help of Integration

DENSE

Infinite Point

Infinite Point

Integration (Infinite
Uncountable
SET)

Add

Cumulative Distribution Function: (cdf)

$$F_X(x_i) = P[X \le x_i]$$

Throwing A Fair Die:

X = No. of dots 1, 2, 3, 4, 5, 6 (Discrete Random variable)

↑ Input

X	1	2	3	4	5	6
$F_X(x_i)$	$F_X(1)$	$F_X(2)$	$F_X(3)$	$F_X(4)$	$F_X(5)$	$F_X(6)$

↑ output

Discrete (Countable Infinite)

$F_X(0) = P[X \le 0] = P_0$

$F_X(1) = P[X \le 1] = P[X=0] + P(X=1)$
$= P_0 + P_1$

$F_X(2) = P(X \le 2) = P_0 + P_1 + P_2$

$F_X(3) = P(X \le 3) = P_0 + P_1 + P_2 + P_3$

$F_X(4) = P(X \le 4) = P_0 + P_1 + P_2 + P_3 + P_4$

$F_X(x_i) = P(X \le x_i)$
$= P_0 + P_1 + P_2 + P_3 + \dots$

$$F_X(1) = P(X \le 1) = P_1 = \frac{1}{6}$$

$$F_X(2) = P(X \le 2) = P_1 + P_2 = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

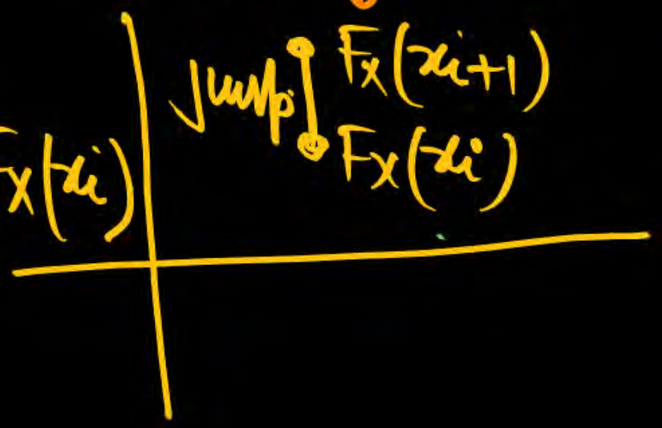
$$F_X(3) = P(X \le 3) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$

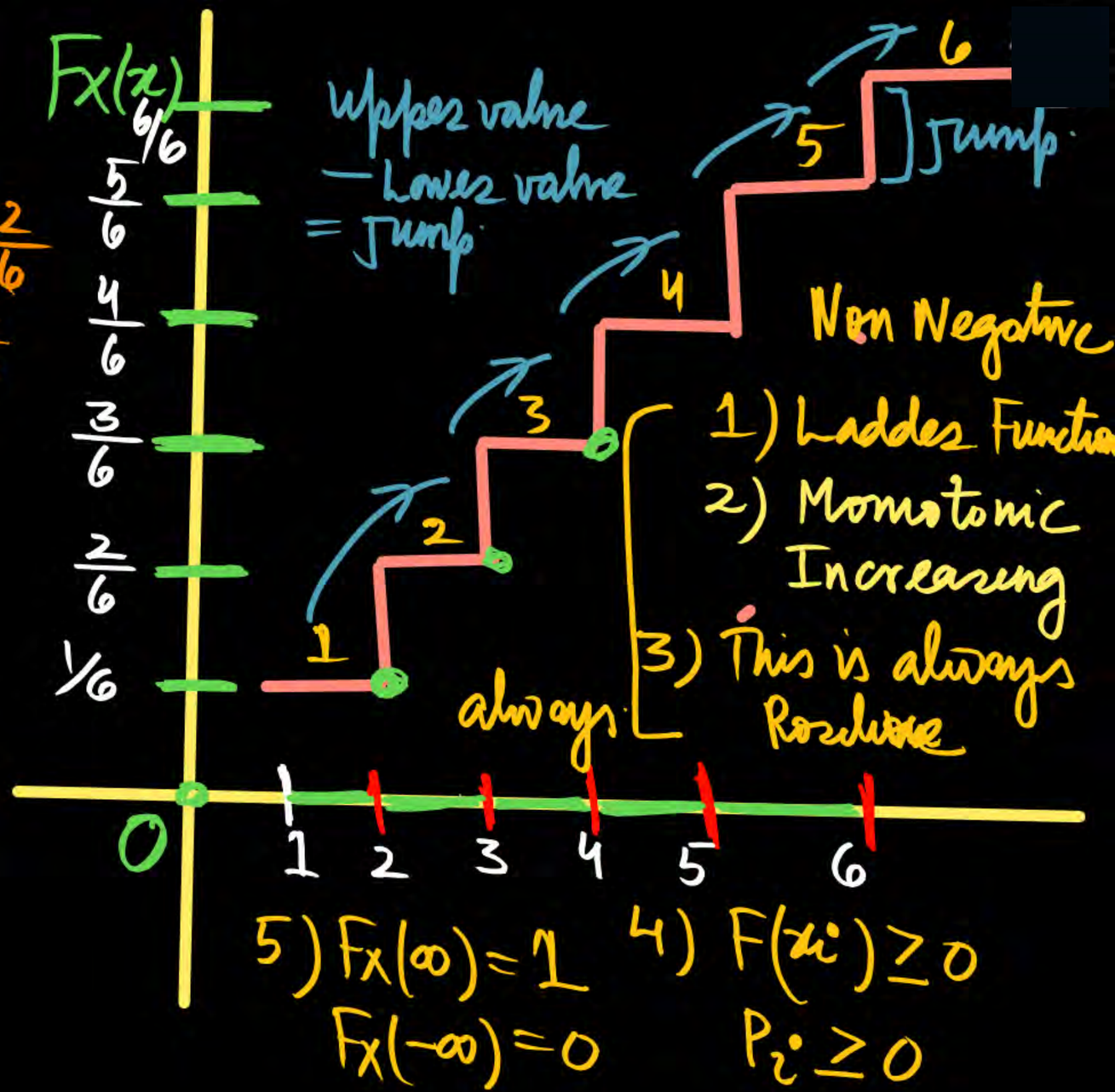
$$F_X(4) = P(X \le 4) = \frac{4}{6}$$

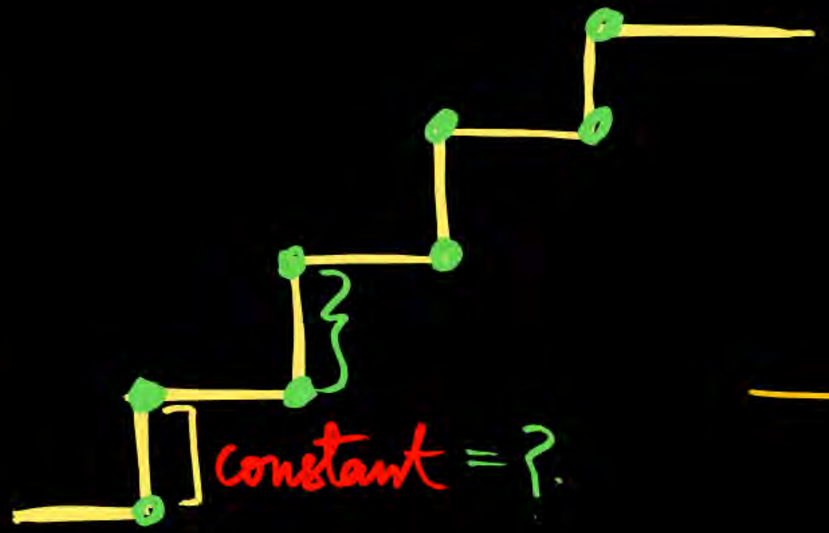
$$F_X(5) = P(X \le 5) = \frac{5}{6}$$

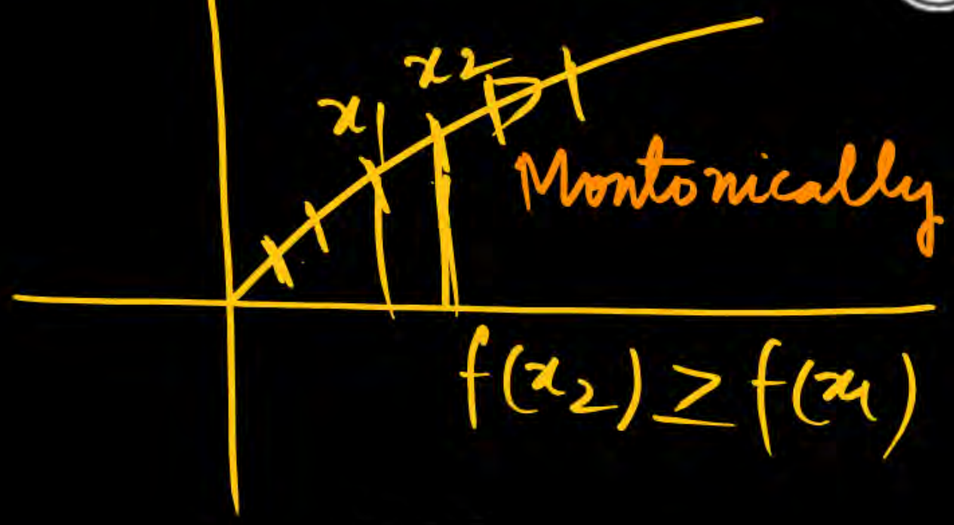
$$F_X(6) = P(X \le 6) = \frac{6}{6} = 1$$

Jump $= F_X(x_{i+1}) - F_X(x_i)$

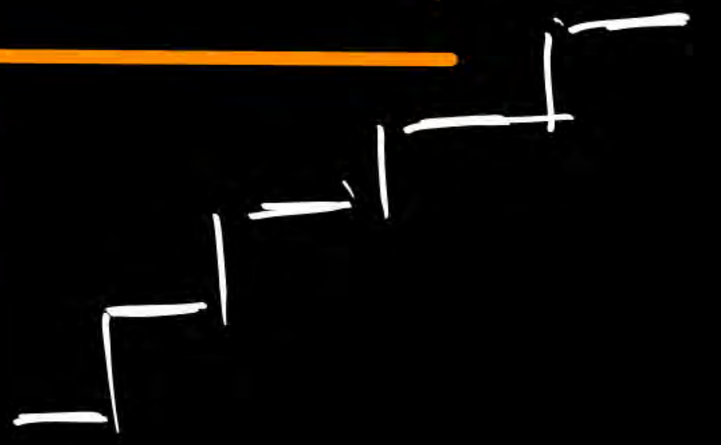








$f'(x) = 0$



THANK - YOU