Data Science and Artificial Intelligence

Probability and Statistics

Bivariate Random Variable

Lecture No.- 07

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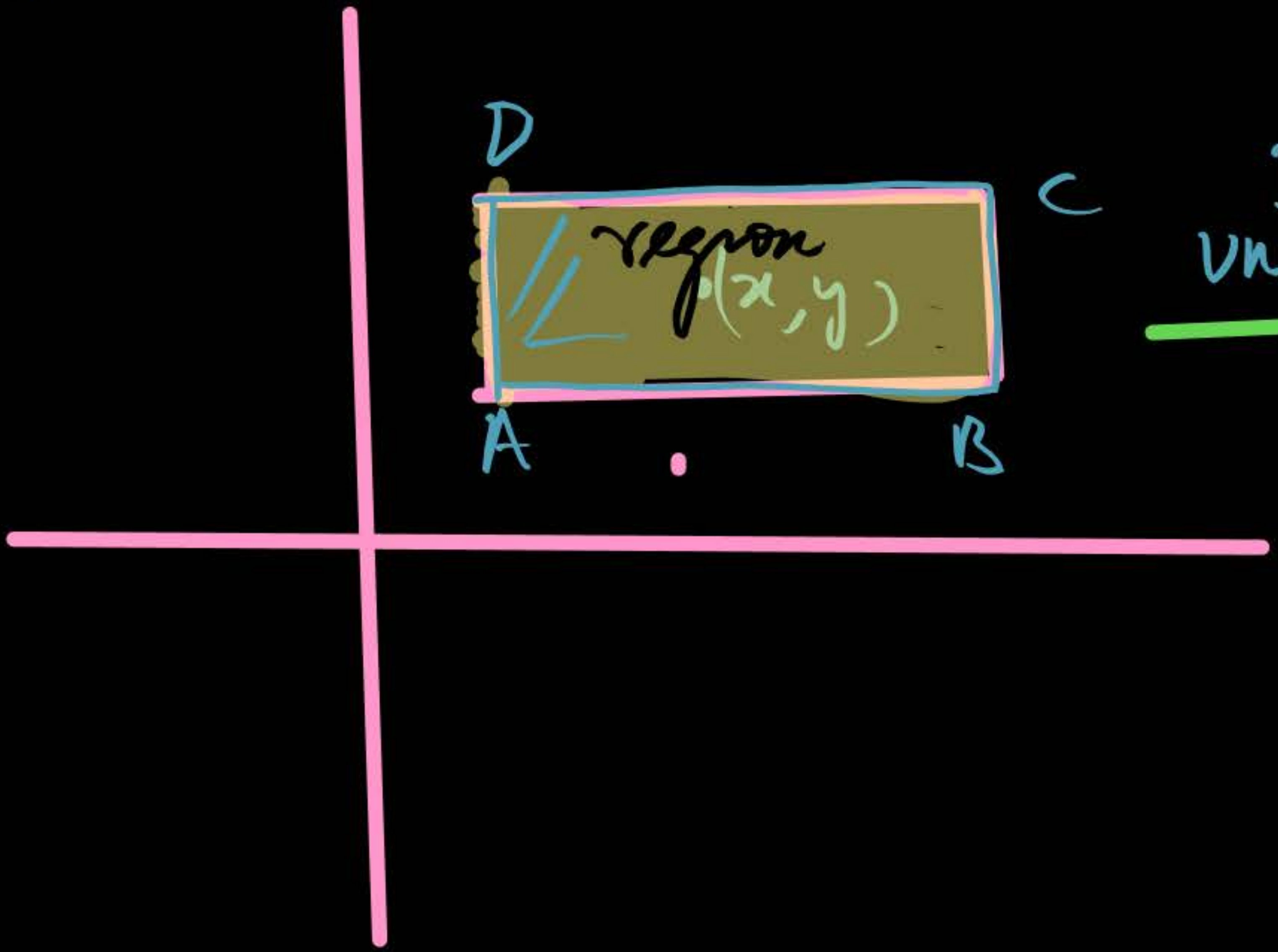
Topics to be Covered

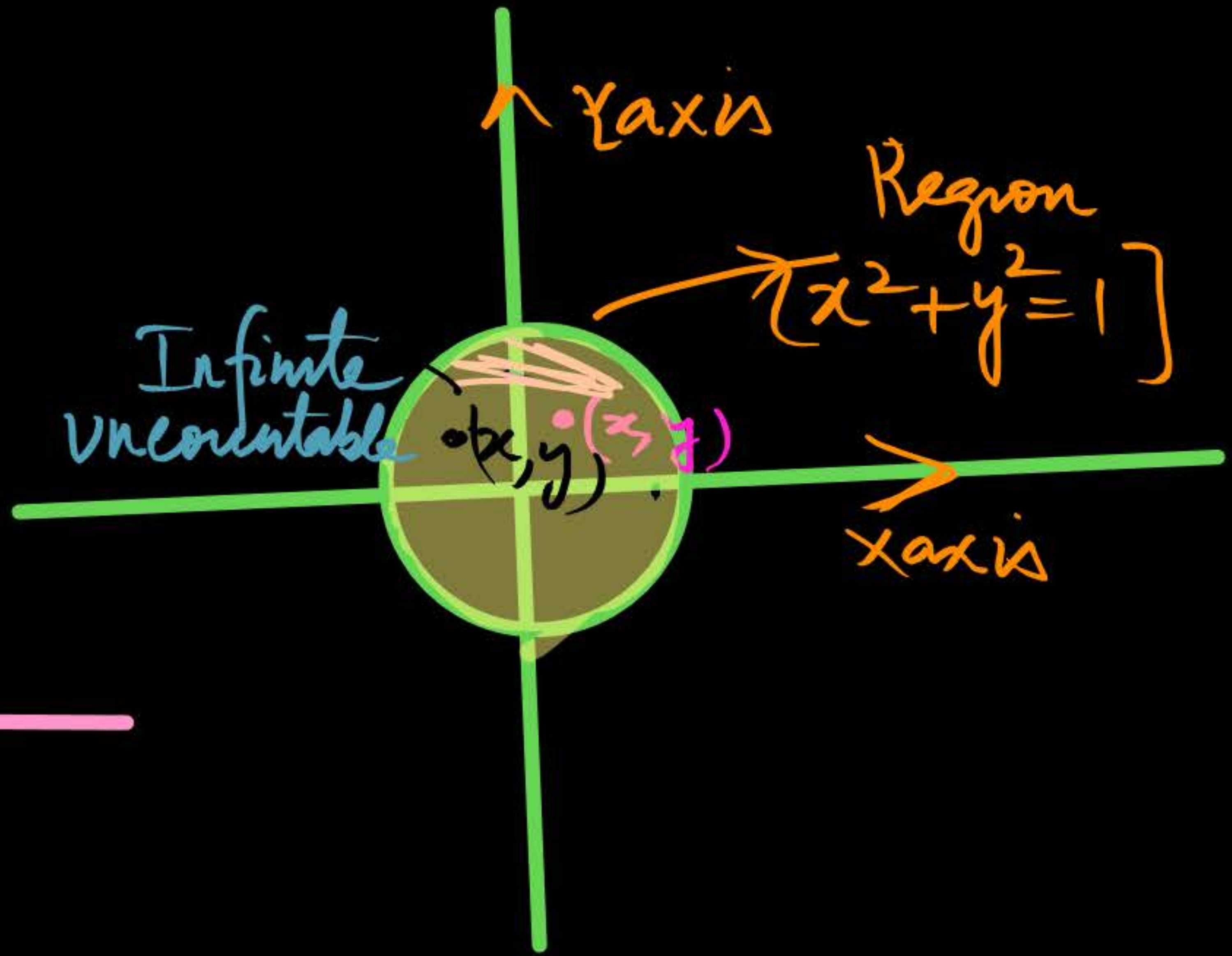
Topic Bivariate Continuous Random Variable

Bivariate Continuous Random Variable:

If X and Y be Two continuous Random variable.

If X and Y —— both conti:





If X and Y Are conti: Random variable.

Distribution Function — In one Dimension:

cdf $F_X(x) = P(X \le x)$ (Number Line)

Joint Prob. density Function

$$\boxed{F_{XY}(x,y) = P[X \le x, Y \le y]}$$

2 dimensional continuous. Distribution function

$$\Rightarrow \int_{-\infty}^{x} \int_{-\infty}^{y} f(x,y)\, dy\, dx$$

For Discrete

$F_{XY}(x,y) = P[X \le x, Y \le y]$

marginal cdf

$F(x) = P[X \le x, Y \le \infty]$ Y take value.

$F(y) = P[X < \infty, Y \le y]$ X take any value.

In bivariate continuous Random variable:

$$F_{XY}(x,y) = P[X \le x, Y \le y]$$

marginal cdf for X

$$F_X(x) = P[X \le x, \; Y \le \infty]$$

Y Take any value.

Y take any value.

$F_X(x)$ Single value.

$$\Rightarrow \int_{-\infty}^{x} \left[\int_{-\infty}^{\infty} f(x,y)\, dy \right] dx$$

Using

$$F(x) = \int_{-\infty}^{x} f(x)\, dx$$

cdf

OR

$$= \int_{x}^{\infty} f(x)\, dx$$

$\left\{ \begin{array}{l} x \text{ fix} \\ Y \text{ moving} \end{array} \right.$

X \ Y	0	1	2
0	(0,0)	(0,1)	(0,2)
1			
2			

$F_X(x) = P[X \le x, Y < \infty] = \int_{-\infty}^{x} \left[\int_{-\infty}^{\infty} f(x,y)\,dy\right] dx$

marginal cdf
[continuous]

"marginal cdf of f" $F_Y(y) = P[X < \infty, Y \le y]$
[continuous]

$$\Rightarrow \int_{y=-\infty}^{y} \left[\int_{x=-\infty}^{\infty} f(x,y)\,dx\right] dy$$

joint Prob. Density function

$F(x) = \int_{R} f(x)\,dx$. 2 dimension. $F(x,y) = \int_{-\infty}^{x} \left[\int_{-\infty}^{y} f(x,y)\,dy\right] dx$

Marginal joint Density Function :-

Density Function of (X, Y)

$$\left. \begin{aligned} f(x) &= \int_{-\infty}^{\infty} f(x,y)\,dy \\ f(y) &= \int_{-\infty}^{\infty} f(x,y)\,dx \end{aligned} \right]$$ Important formulae

$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) = 1$ OR $\int_{-\infty}^{\infty} f(x) = 1$

valid prob. density function

Conditional prob. Density Function:

X given Y

$$p\left(\frac{x}{y}\right) = P\left[\frac{X=x}{Y=y}\right] = \frac{f(x,y)}{f(y)}$$

$f(Y) > 0$

$f(y)$ = marginal Prob. Density function

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

Y given X

$$p\left(\frac{y}{x}\right) = P\left[\frac{Y=y}{X=x}\right] = \frac{f(x,y)}{f(x)}$$

$f(x) > 0$

marginal prob. Density function

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

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PW

$\therefore F(x) = \int_{\mathbb{R}} f(x)\,dx \qquad \frac{d}{dx}F(x) = f(x)$

$f(x, y) = \begin{cases} k(2x+y) & 0<x<1,\ 0<y<2 \\ 0 & \end{cases}$

Q1. Let X and Y be two random variables. Then for

$$f(x,y)=\begin{cases}\boxed{k(2x+y), \quad 0<x<1, 0<y<2} \\ 0, \quad \text{otherwise}\end{cases}$$

If $f(x,y)$ is a joint Prob. Density function

to be a joint density function, what must be the value of k ?

If $f(x,y)$ is a valid prob. Density function

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx = 1$$

$$= \int_{x=0}^{1} \int_{y=0}^{2} f(x,y)\, dy\, dx = 1$$

$$\boxed{K = \frac{1}{4}}$$

$$\Rightarrow \int_{x=0}^{1} \int_{y=0}^{2} K(2x+y)\, dy\, dx = 1$$

$$\Rightarrow K \int_{x=0}^{1} dx \left[2xy + \frac{y^2}{2} \right]_0^2$$

$$= K \int_{x=0}^{1} dx \left[4x + 2 \right]$$

$$= K \int_0^1 (4x+2)\, dx = 1$$

$$= K \left[\frac{4x^2}{2} + 2x \right]_0^1 = 1$$

$$= K \left[\frac{4}{2} + 2 \right] = 1$$

$$\boxed{K = \frac{1}{4}}$$

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Q2. Let X and Y be two random variables. Then for

$$f(x,y)=\begin{cases} kxy & \text{for } 0<x<4 \text{ and } 1<y<5 \\ 0, & \text{otherwise} \end{cases}$$

to be a joint density function, what must be the value of k?

If f(x,y) is valid prob. Density function $\boxed{K=\frac{1}{96}}$

$$\int_0^4\int_1^5 kxy = 1$$

$$= K\int_0^4 x\,dx \int_1^5 y\,dy = 1$$

$$K\int_0^4 x\,dx\,[12] = 1$$

$$= 12K\left[\frac{x^2}{2}\right]_0^4 = 1$$

$$\boxed{k=\frac{1}{96}}$$

Topic : Bivariate Continuous Random Variable

Q4. If (X, Y) be two-dimensional random variable having joint density function.

$$f(x,y)=\begin{cases}\frac{1}{8}(6-x-y); & 0<x<2,\ 2<y<4\\ 0, & \text{elsewhere}\end{cases}$$

$P\left(\frac{X<1}{Y<3}\right)$

$=\frac{P(X<1 \cap Y<3)}{P(Y<3)}$

Find (i) $P[X<1, Y<3] = \frac{3}{8}$

(ii) $\boxed{P[X<1 \mid Y<3]} = \frac{3}{5}$ $= \frac{\frac{3}{8}}{}$

$$f(x,y) = \frac{1}{8}(6-x-y) \quad \left.\begin{matrix} 0<x<2 \\ 2<y<4 \end{matrix}\right]$$

$$P(X\leq 1, Y<3) = \int_{-\infty}^{1}\int_{-\infty}^{3} \frac{1}{8}(6-x-y)\,dy\,dx \qquad = \frac{3}{8} \checkmark$$

$$\Rightarrow \int_{0}^{1}\int_{2}^{3} \frac{1}{8}(6-x-y)\,dy\,dx$$

$$\Rightarrow \frac{1}{8}\int_{0}^{1} dx \int_{2}^{3} (6-x-y)\,dy$$

$$\Rightarrow \frac{1}{8}\int_{0}^{1} dx \left[6y - \frac{xy}{2} - \frac{y^2}{2}\right]_{2}^{3} = \frac{3}{8}$$

$$P(Y<3) = \int_{x=0}^{2}\int_{y=2}^{3} \frac{1}{8}(6-x-y)\,dy\,dx$$

$$\Rightarrow \quad \frac{5}{8}$$

$$P\left(\frac{X<1}{Y<3}\right) = \frac{P(X<1,\ Y<3)}{P(Y<3)} = \frac{\frac{3}{\cancel{8}}}{\frac{5}{\cancel{8}}} = \frac{3}{5}$$

THANK - YOU