Data Science and Artificial Intelligence

Probability and Statistics

Discrete Probability Distribution

Lecture No.- 02

Topics to be Covered

Short Notes

Prob (Data Science) Schaum Series MSV — Prob and Random

Topic	Question Based on Binomial Distributions
Topic	Moment Generating Function
Topic	Uniform Distribution

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Uwei MSV

1400

Topic : Question Based on Binomial Distribution

Homework

Q1. Let X denote the number of times head occur in n tosses of an unbiased coin. If P(X = 4), P(X = 5) and P (X = 6) are in AP ; the value of n is:

A. 7

B. 10

C. 14

D. 12

P(x=4), P(x=5) P(x=6)

A.P

a, b, c

$2b = a + c$

$2P(X=5) = P(X=6) + P(X=4)$

Topic : Question Based on Binomial Distribution

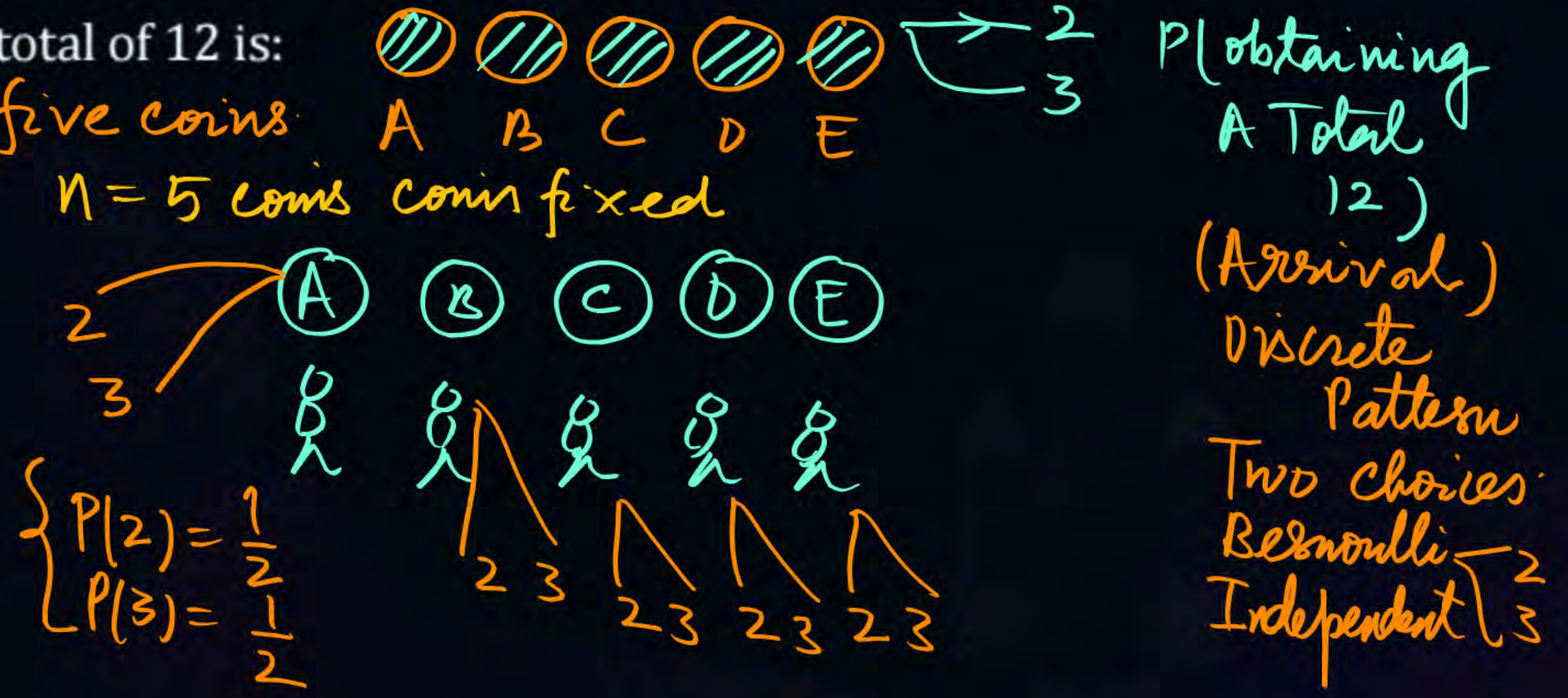
Q2. Five coins whose faces are marked 2, 3 are thrown. The probability of obtaining a total of 12 is:

A. 5/32

B. 11/16

C. 5/16

D. 10/16



P(obtaining A Total 12)
(Arrival)
Discrete Pattern
Two choices
Bernoulli — 2, 3
Independent

bernoulli $P(\text{SUM } 12) = P(2+2+2+3+3)$

2 SUCCESS
3 Failure

12 | 1

SUCCESS | failure

$2+2+2+[3+3]$

SUCCESS

$\Rightarrow$ SUM = 12

00000

$\frac{2}{3}$ $\frac{3}{5}$ total

$\rightarrow$ coins 2, 3

$n = 5$

$P = \frac{1}{2}$ $q = \frac{1}{2}$ $r = 3$

SUCCESS $r = 2$

$P(X = r) \Rightarrow {}^{n}C_{r}\, p^{r} q^{n-r}$

SUCCESS random variable

$\Rightarrow {}^{5}C_{3} \left(\frac{1}{2}\right)^{3} \left(\frac{1}{2}\right)^{5-3}$

$= \frac{5 \times 4 \times 3}{3 \times 2 \times 1} \times \frac{1}{2^{3}}$

$= \frac{5}{16}$

$\frac{1}{16}$

Topic : Question Based on Binomial Distribution

6=P

Read The Prob.

Trial fixed

Q3. Six persons try to swim across a wide river. It is known that on an average, only three persons out of ten are successful in crossing the river. 0.98

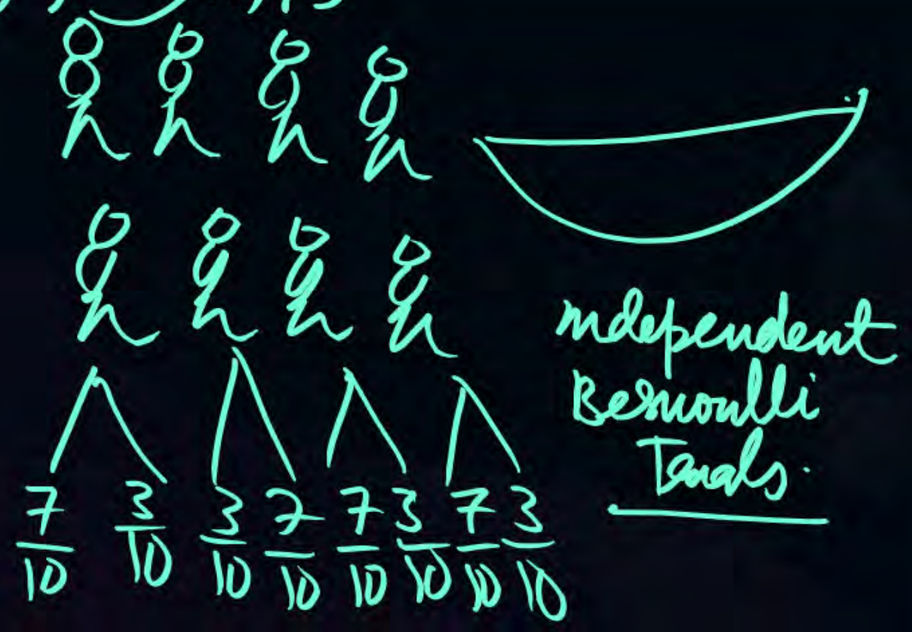
What is the probability that at most four of the six persons will cross safely ?

$P(S) = \frac{3}{10}$ $p = \frac{3}{10}$ $q = 3/10 \nearrow 7/10$

$P(X \le 4) \Rightarrow$

Crossing

$P(\text{Not crossing}) = \frac{7}{10}$



Independent Bernoulli Trials.

Cross Not Cross

Single Trial Bernoulli Trials

5 trials fixed

p - fixed

q → fixed

boat crossing

Arrival

$p = \frac{3}{10}$

$P(\text{Crossing}) = \frac{3}{10}$

$P(\text{Not}) = \frac{7}{10}$

X = random variable $= 0, 1, 2, 3, 4, 5, 6$

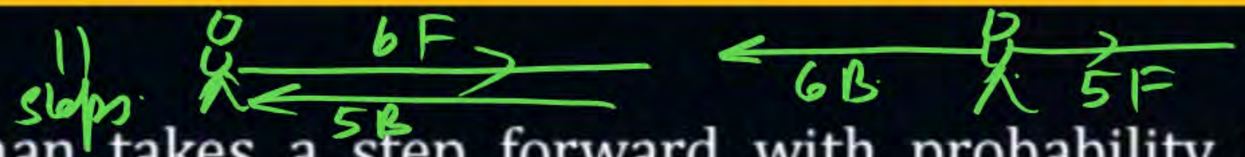
$P(X \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 1 - P(X=5) - P(X=6)$

$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

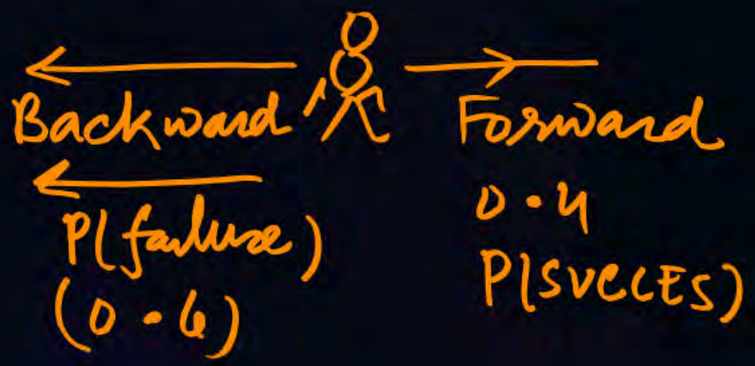
$P(X \ge 4) = 0.98$

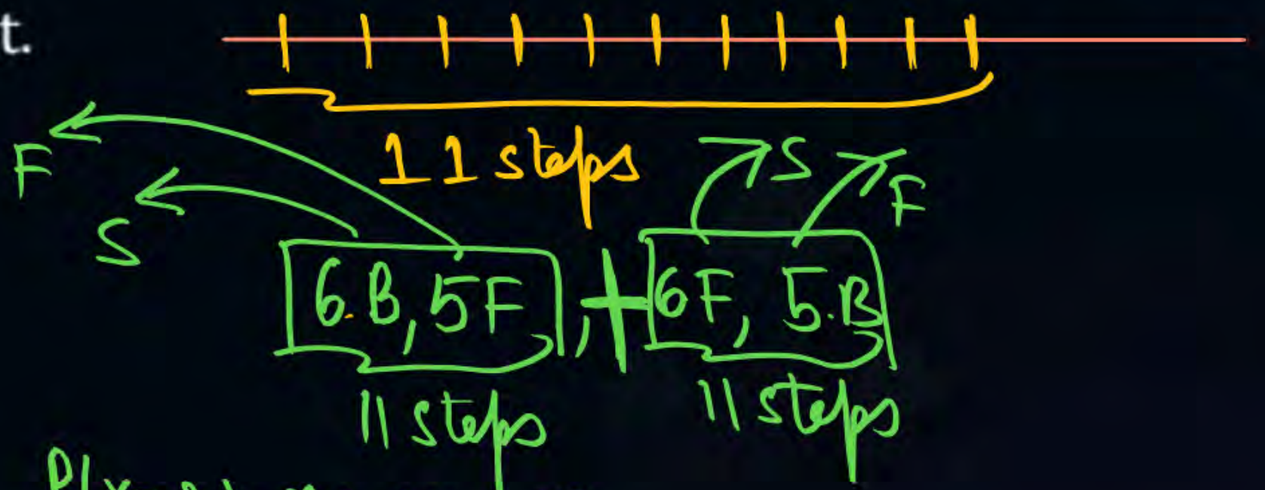
Topic : Question Based on Binomial Distribution



Imp

Q4. A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.





$P(X=r) = {}^{n}C_r\, p^r q^{n-r}$

$P(X=6) =$

$$P(X=r) = {}^{n}C_r\, p^r q^{n-r}$$

$$\left[\begin{array}{l} P(X=6F) = {}^{11}C_6 (0.4)^6 (0.6)^5 \\ P(X=5F) = {}^{11}C_5 (0.4)^5 (0.6)^5 \end{array}\right] +$$

$$= 0.37$$

Topic : Question Based on Binomial Distribution

50% ✓

Probability (Revise) → CS, EC, PYQ-GATE,

Q5. In an experiment, positive and negative values are equally likely to occur.

The probability of obtaining at most one negative value in five trials is

A. 1/32

B. 2/32

C. 3/32

D. 6/32

$(+,-)$

$P(+) = \frac{1}{2} \quad P(-1) = \frac{1}{2}$

$n = 5$

$P(x \le 1) = P(x=0) + P(x=1)$

$= {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 + {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4$

$= \frac{1}{32} + \frac{5}{32} = \frac{6}{32} = \frac{3}{16}$

Topic : Question Based on Binomial Distribution

0.082

Q6. An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is

A. 0.067

B. 0.073

C. 0.082

D. 0.091

10th toss HEAD

$P(10^{th}\ \text{toss}) = \frac{1}{2}$

4th HEAD

$\frac{1}{2}$

$S = \{H, T\}$

$P(E) = \frac{1}{2}$

$\{H\}$

Bernoulli Trial

S/F

$P(H) = \frac{1}{2}$ $P(T) = \frac{1}{2}$

number of trials $n = 9$

✓ 3 HEADS

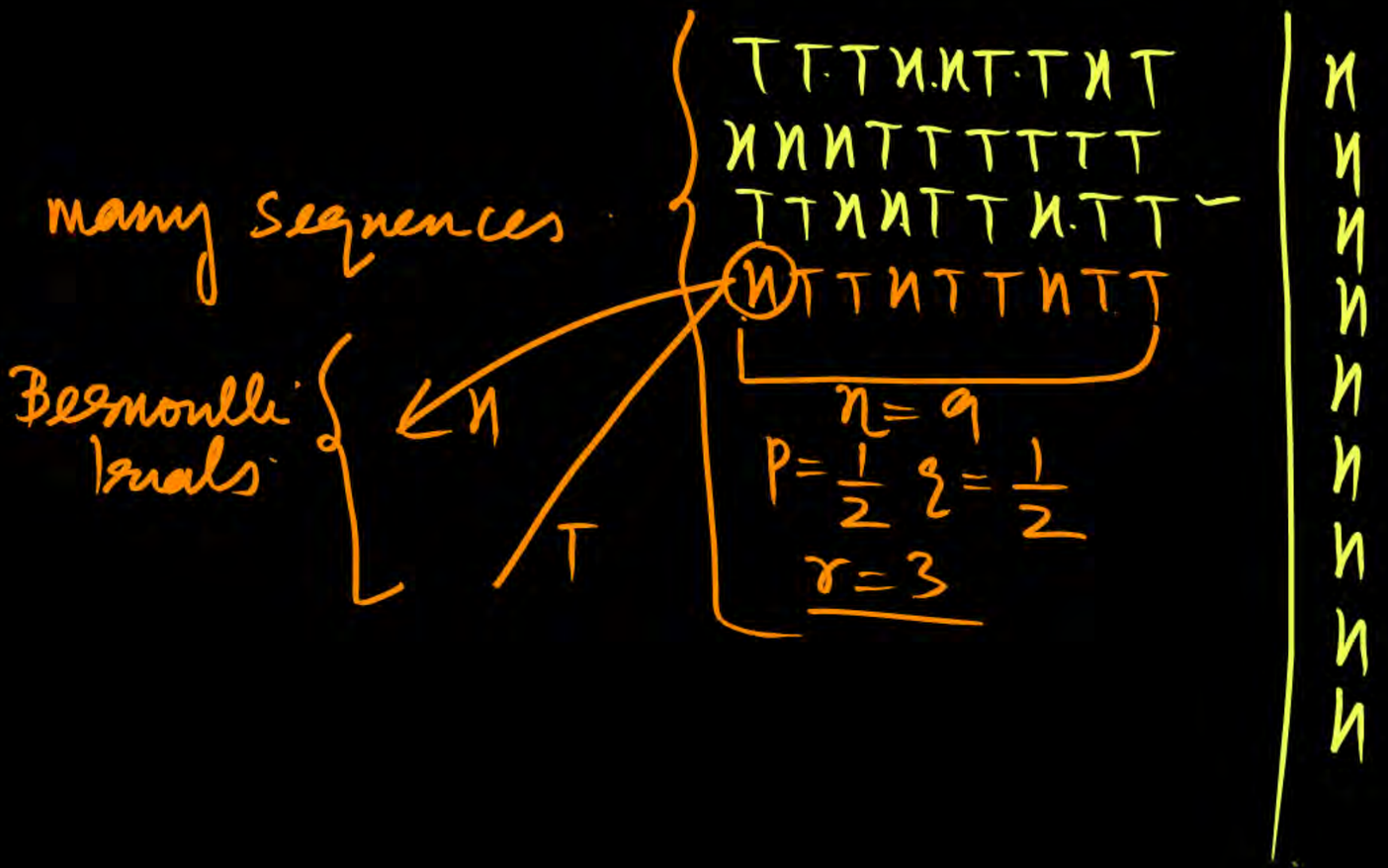
Past

$P(S_H) = \frac{1}{2}$ $P(T) = \frac{1}{2}$ $n = 9$ $r = 3$

$P(X = r) = {}^{n}C_r p^r q^{n-r}$

$= {}^{9}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^6$

$P(4^{th}\ \text{HEAD upper}\ 10^{th}\ \text{toss}) = \frac{{}^{9}C_3}{2^9} \times \frac{1}{2}$


many sequences
Bernoulli trials
H
T
TTTHHTTHTTHTT
HHHTTTTTTT
TTHHTTTHTTT
HTTTHTTHTTT
n = 9
p = 1/2 q = 1/2
r = 3
HHHHHHHHH

Topic : Question Based on Binomial Distribution

Expectation

Q7. Passengers try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40% chance of getting reservation in any attempt by a passenger, then the average number of attempts that passengers need to make to get a seat reserved is ____.

↓ vntll and final

X ✓ 1 (S) = 0·4

2 F(S) = 0·6 × 0·4

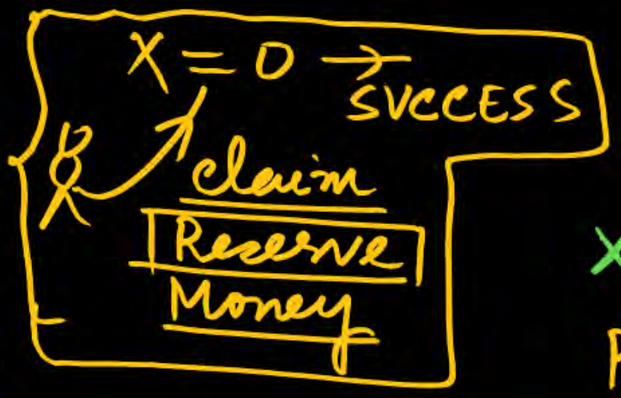
3 FF(S) = $(0·6)^2$ 0·4

4 FFF(S) = $(0·6)^3$ 0·4

5 FFFFS = $(0·6)^4$ 0·4

$E[X] = 1 \times 0·4 + 2 \times 0·6 \times 0·4 + 3 \times (0·6)^2 0·4 + (0·6)^3 0·4 + \dots$

SERIES → A.G.P



X = No. of attempts = SUCCESS Trials fixed.

X = 1, 2, 3, 4, 5, 6

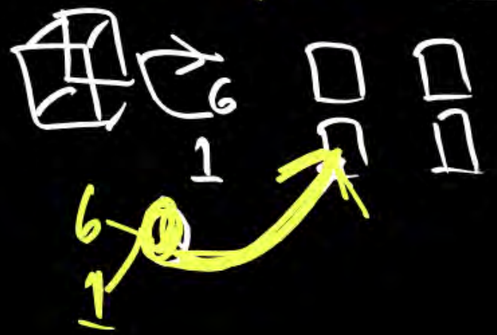
P(Tkt / Reserved) = 0·4

P(Not) = 0·6

$E[X] = 2·5$ → Expectation

On Average 3 Tkts Are Reserved

✓ Ludo



vntill / finally SUCCESS Dice-Throw

Topic : Question Based on Binomial Distribution

$M - V = 1$ $\quad M^2 - V^2 = 11$

Q8. Difference between mean and variance of a binomial random variable is 1 and difference between their squares is 11. Find the probability of getting exactly three success.

$B(n, p)$ binomial $\begin{cases} n = 3 \\ n = ? \\ P = ? \\ q = ? \end{cases}$

MEAN

$\mu = E[X] = np$

variance

$\sigma_X^2 = V[X] = npq$

$np - npq = 1$ — (1)

$\Rightarrow np(1-q) = 1$ — (1)

$np = \frac{1}{(1-q)}$

$n^2p^2 - n^2p^2q^2 = 11$

$n^2p^2(1-q^2) = 11$ — (2)

$\frac{n^2p^2(1-q^2)}{np(1-q)} = \frac{11}{1}$

$= np\frac{(1-q)(1+q)}{(1-q)} = \frac{11}{1}$

$= np(1+q) = \frac{11}{1}$

$= \frac{1}{(1-q)}(1+q) = \frac{11}{1}$

$\frac{1+q}{1-q} = \frac{11}{1}$

$\boxed{q = \frac{5}{6}}$ $p = \frac{1}{6}$

$np(1-q) = 1$

Put the value of p, q.

$n \times \frac{1}{6}\left(1-\frac{5}{6}\right) = 1$

$n = 36$

$n = 36 \quad p = \frac{1}{6} \quad q = \frac{5}{6} \quad r = 3$

Using $B(n, p)$

$$P(X = r) = {}^{n}C_r \, p^r q^{n-r}$$

$$= {}^{36}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{33}$$

Ans

Topic : Question Based on Binomial Distribution

Q9. In an Examination of 10 Multiple choice question (one or more can be correct) out of 4 options. A student decided to mark the answer at random. Find the probability he gets the exact two questions correct?

SINGLE question

IIT

A → ${}^4C_1$ ✓ W_1

B → ${}^4C_2$ ✓ W_2

C → ${}^4C_3$ ✓ W_3

D → ${}^4C_4$ ✓ W_4

] Individ work

Ist | IInd. Advanced | III

Ist	IInd. Advanced	III	
A	A B	A B C	A B
B	B C	B C D	C D
C	C D	C D A	1 ways
D	D A	A B D	
4 ways	A C	4 ways	
	B D		
	6 ways		

Total choices $= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

$= 4 + 6 + 4 + 1$

$= 15$ ways

Bernoulli Trials

$P(\text{Single correct}) = \frac{1}{15}$

$P(\text{Not correct}) = 1 - \frac{1}{15} = \frac{14}{15}$

$n = 10$ (binomial)

$r = 2$

$$P(X=r) = {}^{n}C_r\, p^r q^{n-r}$$

$$= {}^{10}C_2 \left(\frac{1}{15}\right)^2 \left(\frac{14}{15}\right)^{10-2}$$

$$= {}^{10}C_2 \frac{1}{15\times 15} \times \left(\frac{14}{15}\right)^8$$

Topic : Question Based on Binomial Distribution

Q10. In a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total of 900 bolts are respectively

$P(\text{Def}) = 0.1$ $\quad E[X] =$

$P(\text{Not def}) = 0.9$ $\quad \sigma_X =$

$n = 900$

A. 90 and 9

B. 9 and 90

C. 81 and 9

D. 9 and 81

$E[X] = np = 900 \times 0.1 = 90$ ✓

$V(X) = npq = 900 \times 0.1 \times 0.9$

$\sigma_X = \sqrt{\text{Var}(X)}$

$= \sqrt{900 \times 0.1 \times 0.9} = 9$ ✓

THANK - YOU