Data Science and Artificial Intelligence

Probability and Statistics

Introduction to Probability

Lecture No.- 04

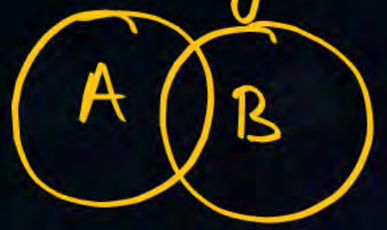
Recap of Previous Lecture

CHALENGING Problem. Problem Session - 1, 2 - 30 question

Topic Classification of events

- ✓ $A \cap B$
- ✓ $A \cup B$
- ✓ $A \cap \bar{B}$
- ✓ $\bar{A} \cap B$
- ✓ $\bar{A} \cap \bar{B}$
- ✓ $P(\bar{A})$
- ✓ exactly one.

A → something is common ← B



Topics to be Covered

Topic Problem Based on Events

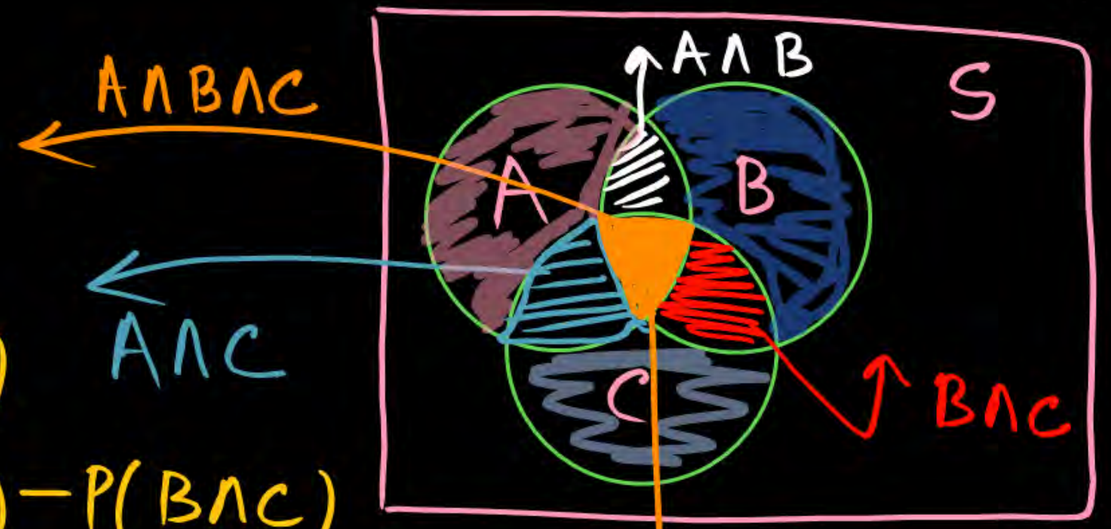
Topic Conditional Probability

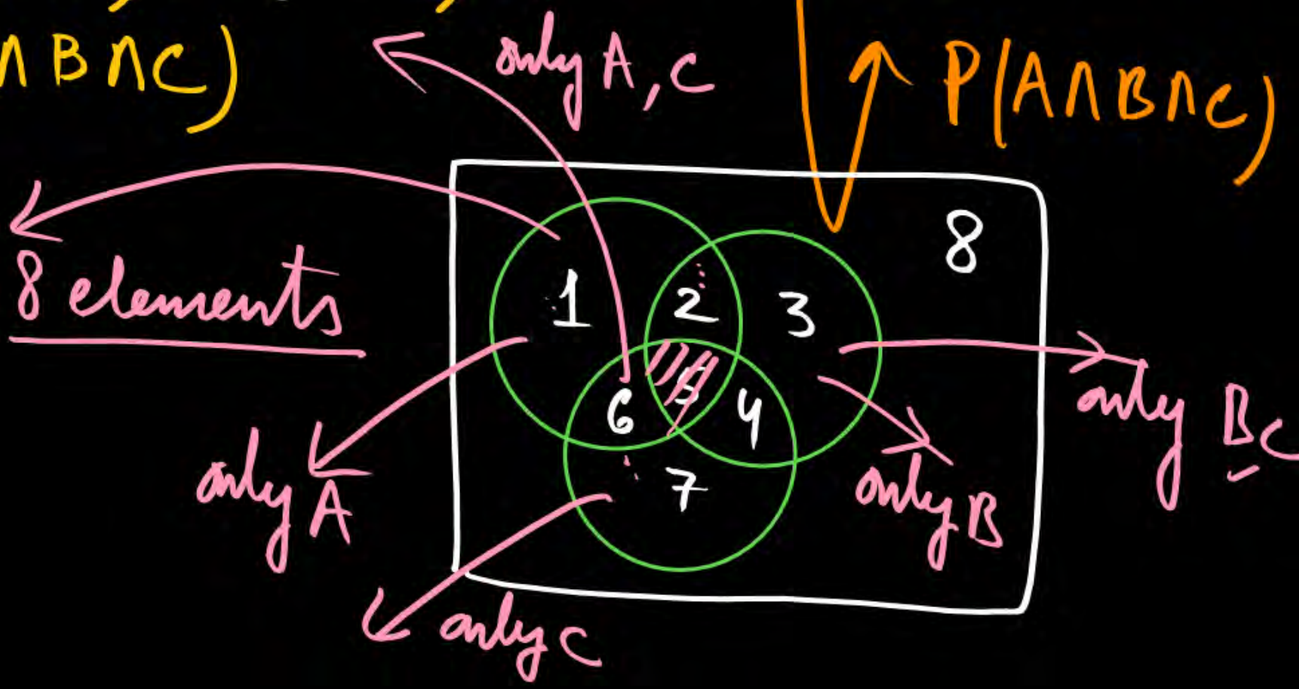
For Three Events:

A, B, C [something is common]

$$P(A \cup B \cup C) = P(\text{either } A \text{ or } B \text{ or } C)$$

$$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

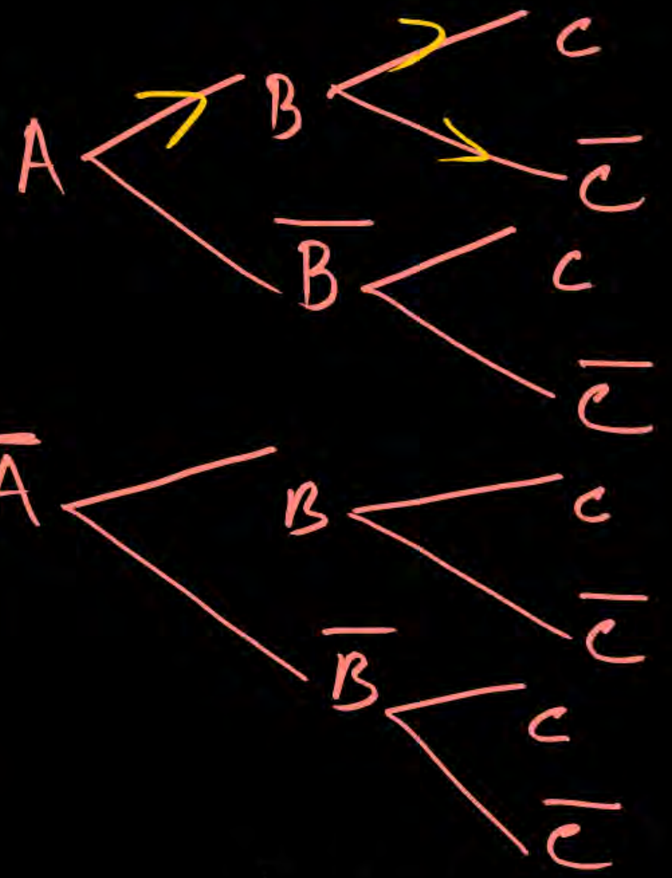




for THREE events A, B, C

all Happen
only A and B
only A and C
only A
only B, C
only B
only C
Neither A Nor B Nor C

$A \cap B \cap C$ (1)
$A \cap B \cap \bar{C}$ (2)
$A \cap \bar{B} \cap C$ (3)
$A \cap \bar{B} \cap \bar{C}$ (4)
$\bar{A} \cap B \cap C$ (5)
$\bar{A} \cap B \cap \bar{C}$ (6)
$\bar{A} \cap \bar{B} \cap C$ (7)
$\bar{A} \cap \bar{B} \cap \bar{C}$ (8)



A, B, C Are independent

$P(A \cap B \cap C) = P(A)P(B)P(C)$

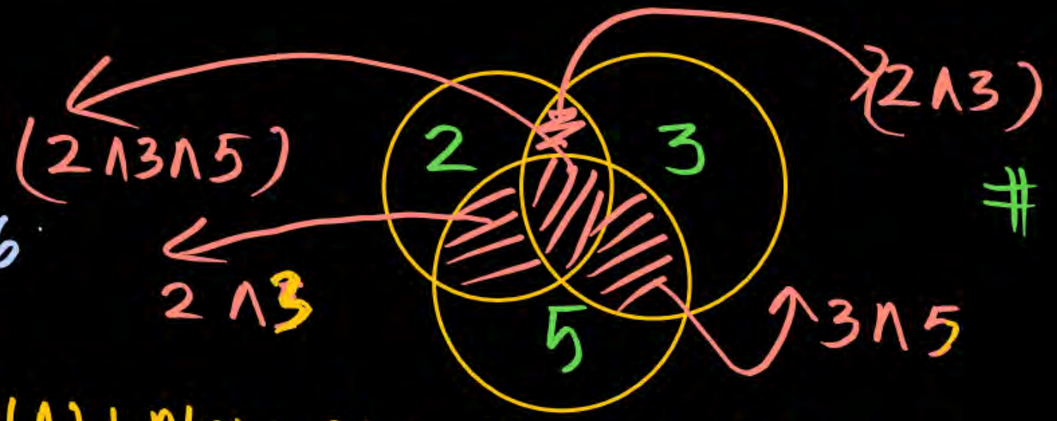
$P(A \cup B \cup C)$
$= P(\text{either } A \text{ or } B \text{ or } C)$

$\frac{100}{2}=50$ $\frac{100}{5}=20$

$\frac{100}{3}=33$ $\frac{100}{6}=16$

$\frac{100}{15}=6$

$\frac{100}{10}=10$



#

1 to 100

Digit

A = div by 2

B = div by 3

C = div by 5

What is the Prob. either div by 2 or 3 or 5

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$$

$$P(2\cup 3\cup 5)=\underline{P(2)}+\underline{P(3)}+\underline{P(5)}-\underline{P(2\cap 3)}-\underline{P(3\cap 5)}-\underline{P(2\cap 5)}+\underline{P(2\cap 3\cap 5)}$$

$$=\frac{n(2)}{100}+\frac{n(3)}{100}+\frac{n(5)}{100}-\frac{n(2\cap 3)}{100}-\frac{n(3\cap 5)}{100}-\frac{n(2\cap 5)}{100}+\frac{n(2\cap 3\cap 5)}{100}$$

$$=\frac{50+33+20-16-6-10+3}{100}=\frac{74}{100}=\frac{37}{50}$$

Nothing is common :: → Mutually exclusive events

A = Rolling A Die
B = flipping A coin
] Disjoint event

$(A \cap B) = \phi$ OR 0
(Disjoint event) → Null SET

If A and B Are simultaneously Not occur.

If A and B Are Disjoint event
→ Mutually exclusive events

$P(A \text{ OR } B) = P(A) + P(B) - P(A \cap B)$

If A and B Are exclusive

$P(A \cup B) = P(A) + P(B)$

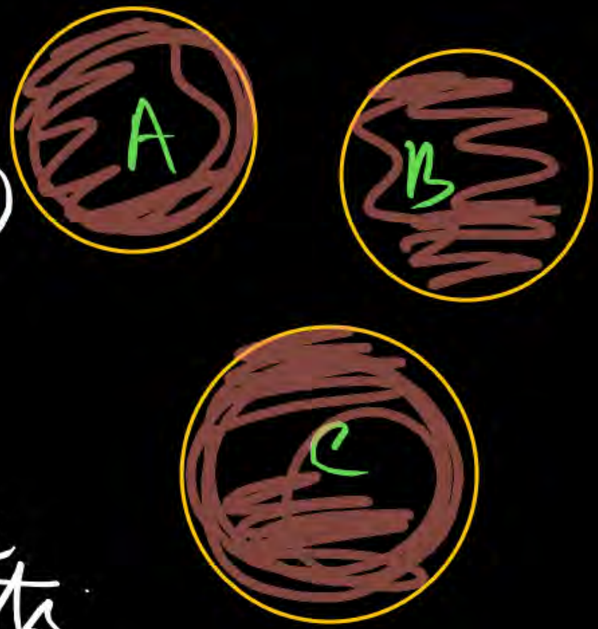
✓ LATE / TIME
Win / Loose
Draw Red OR / Not draw
HEAD / TAIL
5 / $\overline{5}$

] LATE ∩ time = 0

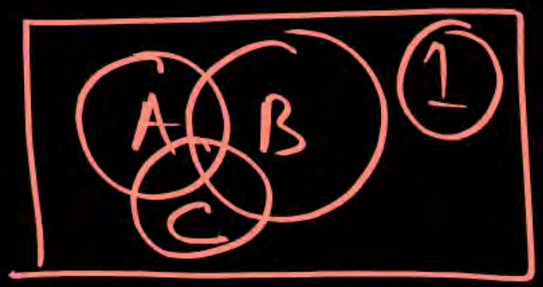
$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C)}$$

A, B, C Are Disjoint / mutually exclusive events.



2)



Neither A or B or C = $1 - [P(A \cup B \cup C)]$

$P(A \cap B) = P(B \cap C) = P(C \cap A) = P(A \cap B \cap C) = 0$ or ϕ

Topic : Problem Based on Events

$\frac{200}{8} = 25$ $\quad \frac{200}{6} = 33$ $\quad \frac{200}{24} = 8$

Q1. An integer is chosen at random from 200 positive integers. Find the probability that the integer chosen is divided by 6 or 8

$P(6 \cup 8)$

A. 1/4

B. 1/6

C. 1/2

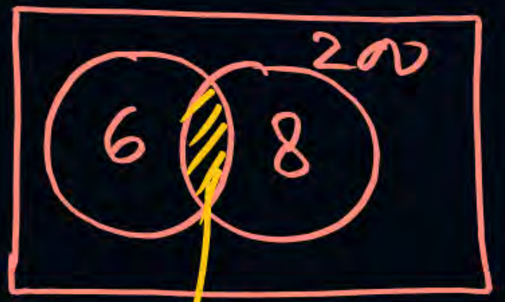
D. None of these

$$P(6 \cup 8) = P(6) + P(8) - P(6 \cap 8)$$

$$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200}$$

$$= \frac{50}{200} = \frac{1}{4}$$

$\cup$ or $\cap$ and



Algorithm – Computer
Digital – circuit
Logic Algebra

Topic : Problem Based on Events

$P(R \cup K) = P(R) + P(K) - P(R \cap K)$

$P(R \cup K) = \frac{26 \times 25}{52 \times 51} + \frac{4 \times 3}{52 \times 51} - \frac{2 \times 1}{52 \times 51}$

Q2. Two cards are drawn from pack of 52 cards. What is the probability that either of the cards, both are red or both are kings.

A. 660/2652

B. 660/1352

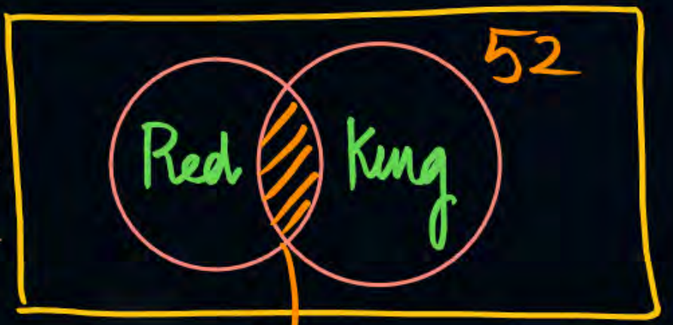
C. 330/2652

D. $1/52 \times 53$

R
26 cards
52 cards
2 cards
$P(R) = \frac{26}{52} \times \frac{25}{51}$
$\frac{{}^{26}C_2}{{}^{52}C_2}$

4 cards
K
52 cards
2 cards
$P(K) = \frac{4}{52} \times \frac{3}{51}$
$\frac{{}^{4}C_2}{{}^{52}C_2}$

R ∩ K 2 cards
52 cards
R or K
$= \frac{2}{52} \times \frac{1}{51}$



(Red ∩ King)

K Diamond
K
Heart

$P(R \cup K) = \frac{660}{2652}$

Topic : Problem Based on Events

$P(A \cap B \cap C) = P(A)P(B)P(C)$

A, B, C – Indep. $\bar{A}, \bar{B}, \bar{C}$ – indep.

Q3. 3 person A, B, C independently try to hit a target, if probability of hitting a target by A, B, C are 3/4, 1/2, 5/8 then the probability that target hit by A or B but not C

$P(A) = \frac{3}{4}$

$P(B) = \frac{1}{2}$

$P(C) = \frac{5}{8}$

$P(A \cup B \cap \bar{C}) = P(A \text{ or } B \text{ but Not } C)$

$\Rightarrow P(A \cup B)\, P(\bar{C})$

$\Rightarrow (P(A) + P(B) - P(A \cap B))\, P(\bar{C})$

$\Rightarrow [P(A) + P(B) - P(A)P(B)]\, P(\bar{C})$

$\Rightarrow \left[\frac{3}{4} + \frac{1}{2} - \frac{3}{4} \times \frac{1}{2}\right]\left[1 - \frac{5}{8}\right] = \frac{21}{64}$

A or B but Not C

$(A \cup B) \cap \bar{C}$

Working Together

A. 21/64

B. 1/64

C. 7/64

D. 3/64

Topic : Problem Based on Events

Q4. Let E and F be two independent events. The probability of both E and F happens is 1/12 and neither E nor F happens is 1/2 then the value $\frac{P(E)}{P(F)} = ?$

$P(\text{both Happens}) = \frac{1}{12}$ $P(E \cap F) = \frac{1}{12}$

$P(\bar{E} \cap \bar{F}) = P(\text{Neither E Nor F}) = \frac{1}{2}$

$P(E \cap F) = \frac{1}{12}$ E, F independent events

$\bar{E}, \bar{F}$

$P(E)\,P(F) = \frac{1}{12}$

$P(\bar{E})\,P(\bar{F}) = \frac{1}{2}$

A. 4/5

B. 3/2

C. 1/5

D. None of these ✓

$P(E)\,P(F)=\frac{1}{12}$

$P(\bar{E})\,P(\bar{F})=\frac{1}{2}$

$\Rightarrow \boxed{x\cdot y=\frac{1}{12}}$ — ①

$P(E)$ $P(F)$

$(1-x)(1-y)=\frac{1}{2}$

$1-y-x+xy=\frac{1}{2}$

$1+\frac{1}{12}-\frac{1}{2}=x+y$

$\boxed{\frac{7}{12}=x+y}$ — ②

$\begin{bmatrix}\alpha+\beta\\ \alpha\beta\\ \alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}\end{bmatrix}$

$x-y=\sqrt{(x+y)^2-4xy}$

$=\sqrt{\frac{49}{144}-4\frac{1}{12}}$

$\boxed{x-y=\pm\frac{1}{12}}$

$x+y=\frac{7}{12}$

$x-y=\frac{1}{12}$

$2x=\frac{8}{12}$ $\quad x=\frac{8}{24}=\frac{1}{3}$

$y=\frac{1}{4}$

$\frac{P(E)}{P(F)}=\frac{\frac{1}{3}}{\frac{1}{4}}=\frac{4}{3}$ $\quad P(E)=\frac{1}{3}$ ✓ $\quad P(F)=\frac{1}{4}$ ✓

$\boxed{\binom{x+y}{xy}\ \frac{x-y}{k}}$ ✓

If something is common

$P(E)=x$

$P(F)=y$]

If Nothing is common

$P(E)=x$

$P(F)=(1-x)$

Topic : Problem Based on Events

Q5. Let S be the sample space with two mutually exclusive events A and B and $A \cup B = S$. If P denotes probability of events, then the maximum value of $P(A) \cdot P(B) = ?$

A. 1/2

B. 1/4

C. 1/6

D. None of these

$P(A \cup B) = P(S)$ ↳ sure event

$P(A \cup B) = 1$

$= P(A) + P(B) = 1$

What is the max value $[P(A)P(B)]$

mutually exclusive $P(A) = \frac{1}{2}$

max value $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $P(B) = \frac{1}{2}$

A and B Are Mutually Exclusive Events

$P(A \cap B) = 0$

Using A.M G.M Inequality

$$A.M \geq G.M$$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$\Rightarrow \frac{P(A)+P(B)}{2} \geq \sqrt{P(A)\,P(B)}$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 \geq \left(P(A)\,P(B)\right]_{max}$$

$$\Rightarrow \boxed{\frac{1}{4} \geq \left[P(A)P(B)\right]_{max}}$$

✓ $A.M = \frac{a+b}{2}$ arithmetic Mean

✓ $G.M = \sqrt{ab}$ geometric MEAN

≠ Using Calculus:

$P(B)+P(A)=1$

$[P(A)\,P(B)]_{max} = [P(A)(1-P(A)]$

$P(A)=x$

$f(x) = x(1-x) = x-x^2$

$f'(x) = 0$

$f'(x) = 1-2x$

$1-2x=0$

$\boxed{x=\frac{1}{2}}$

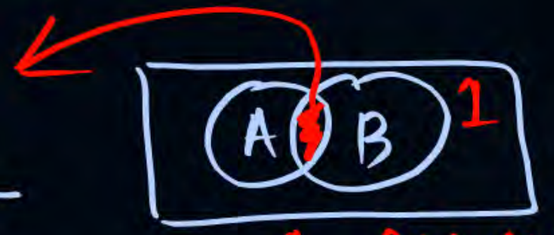
$f''(x) = -2 < 0$ (maxima) at $x=\frac{1}{2}$

Max value $f(x) = x(1-x) = \frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4}$

$[P(A)\,P(B)]_{max} = \frac{1}{4}$ Ans

Topic : Problem Based on Events

MSQ



1 − P(A∪B) = None

Q6. Let E and F be two independent events. The probability that exactly one of them occurs is 11/25 and the probability of none of them occurring is 2/25. If P(T) denotes the probability of occurrence of the event T, then

A. P(E) = 4/5, P(F) = 3/5

B. P(E) = 1/5, P(F) = 2/5

C. P(E) = 2/5, P(F) = 1/5

D. P(E) = 3/5, P(F) = 4/5

$P(A) + P(B) - 2P(A \cap B) = P(\text{exactly one})$

If A and B Are Independent

$P(A \cap B) = P(A)\,P(B)$

$\frac{11}{25} = P(A) + P(B) - 2P(A)\,P(B)$

$P(A) = x$

$P(B) = y$

$x + y - 2xy = \frac{11}{25}$

quadratic eqn

$P(\text{Neither A NOR B}) = \frac{2}{25}$

$= (1-x)(1-y) = \frac{2}{25}$ — (2)

$= 1 - y - x + xy = \frac{2}{25}$

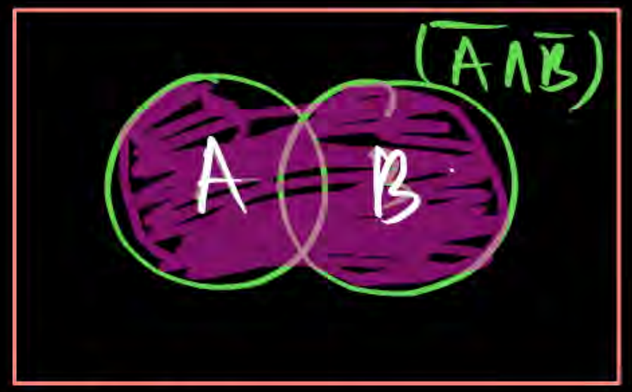
$\Rightarrow 1 - \frac{2}{25} = x + y - xy$

$= \frac{23}{25} = x + y - xy$ — (2)

$x + y - 2xy = \frac{11}{25}$ — (1)

$x + y - xy = \frac{23}{25}$ — (2)

Neither Nor None



$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$P(\bar{A})\,P(\bar{B}) = [1 - P(A)][1 - P(B)]$

Solving The eqn Now get The

$x = \frac{3}{5}$ $y = \frac{4}{5}$ | $x = \frac{4}{5}$ $y = \frac{3}{5}$

THANK - YOU