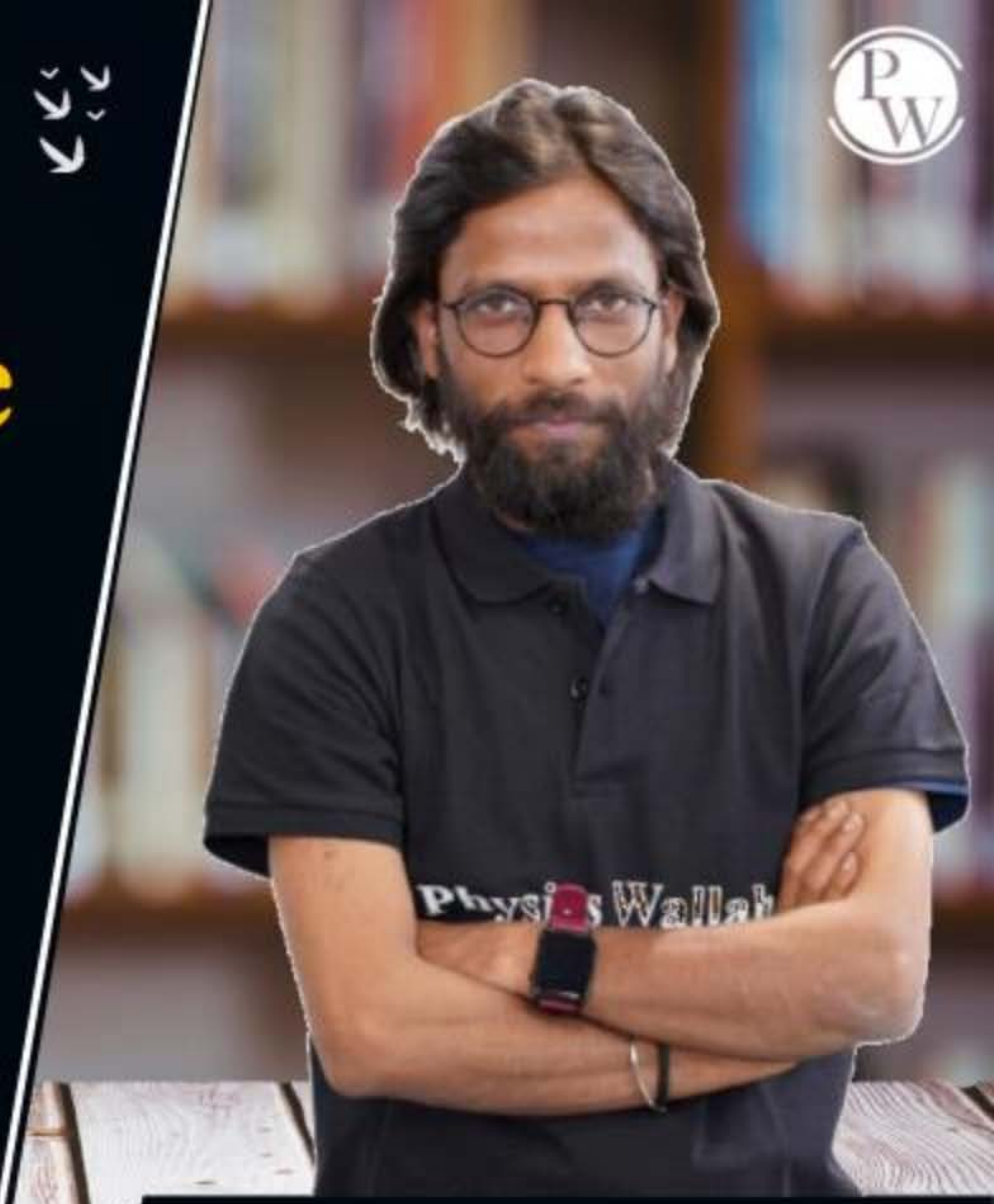
Data Science and Artificial Intelligence

Probability and Statistics



Bivariate Random Variable

Lecture No.- 01

By- Rahul Sir

Recap of Previous Lecture

Topic Random Variable Part-2

Event A

$P(A) = \frac{n(A)}{n(S)}$ ——function——> Random variable

cdf, pdf

Event A, B, C ... ——> Random variable

Two or more event simultaneously work

?

Topics to be Covered

Topic Bivariate Random Variable Part-1

Bivariate Random Variables:

X, Y } Two dimensional Random Variable.

3 Balls. b_1 b_2 b_3

3 balls put 3 cells.

cell 1, cell 2, cell 3 — 3 cells.

cell 1, cell 2, cell 3 } → No ball in any cell (Repetation allowed)

b_1	b_2	b_3
$b_1 b_2$	—	b_3
—	b_3	$b_1 b_2$
$b_1 b_2 b_3$	—	—

X = random variable.

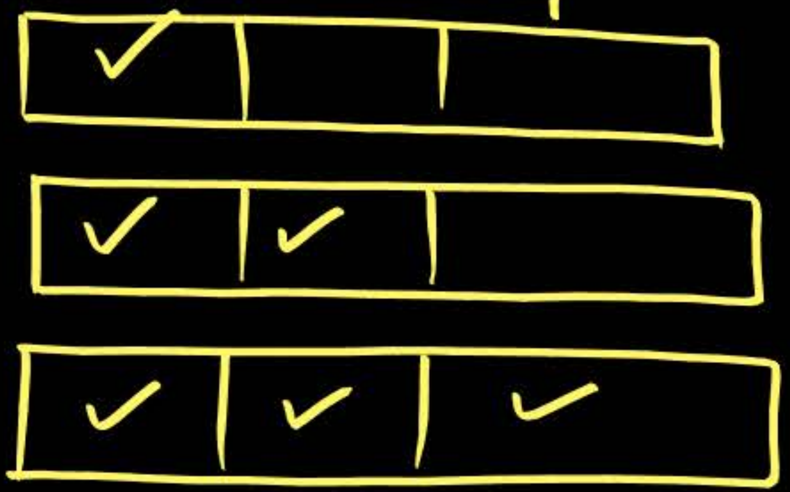
Y = random variable

Simultaneously jointly work {
X = No. of balls in cell 1
Y = No. of cells Are occupied

Y = No. of cells Are occupied

Y = 1 cell
= 2 cell
= 3 cell

$Y = 1, 2, 3$

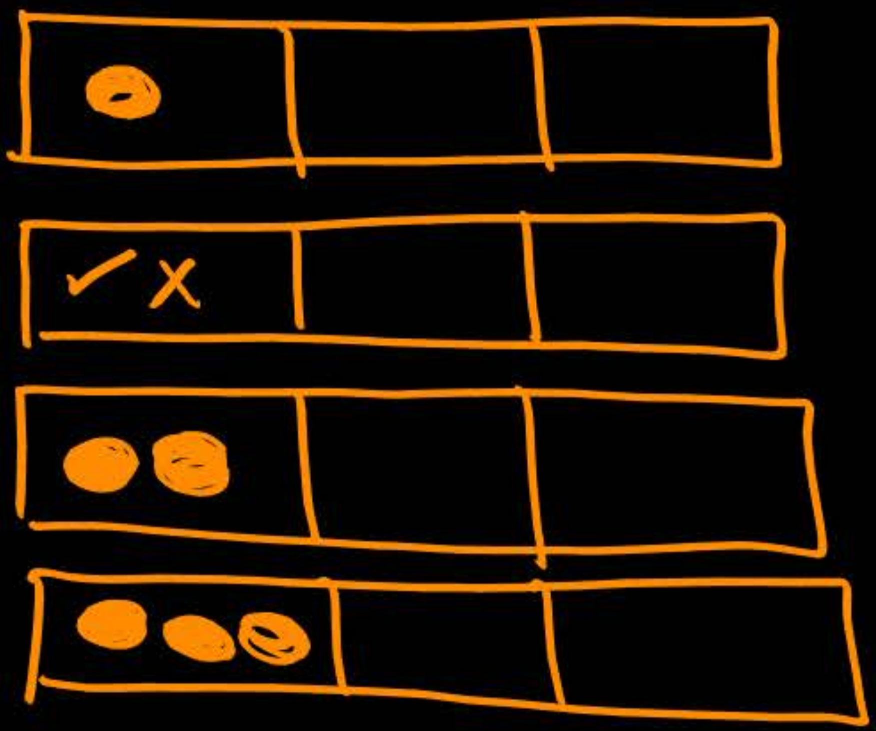


X = No of balls in cell 1

X — 0 ball, 1 ball, 2 ball, 3 ball

$X = 0, 1, 2, 3$

1
0
2
3



P[X = No. of balls $\cap$ Y = No. of cells occupied]

$= \underbrace{P[X = x_i, Y = y_j]}$ = Two dimensional random variable.

bivariate random variable [simultanously together)

$X = 0, 1, 2, 3$ (No. of balls in cell 1)
$Y = 1, 2, 3$ (No. of cells occupied)

$P[X=0, Y=1] = P[X = \text{No ball in cell 1}, Y = \text{1 cell occupied}]$

$P[X=0, Y=2] = P[X = \text{No ball in cell 1}, Y = \text{2 cell occupied}]$

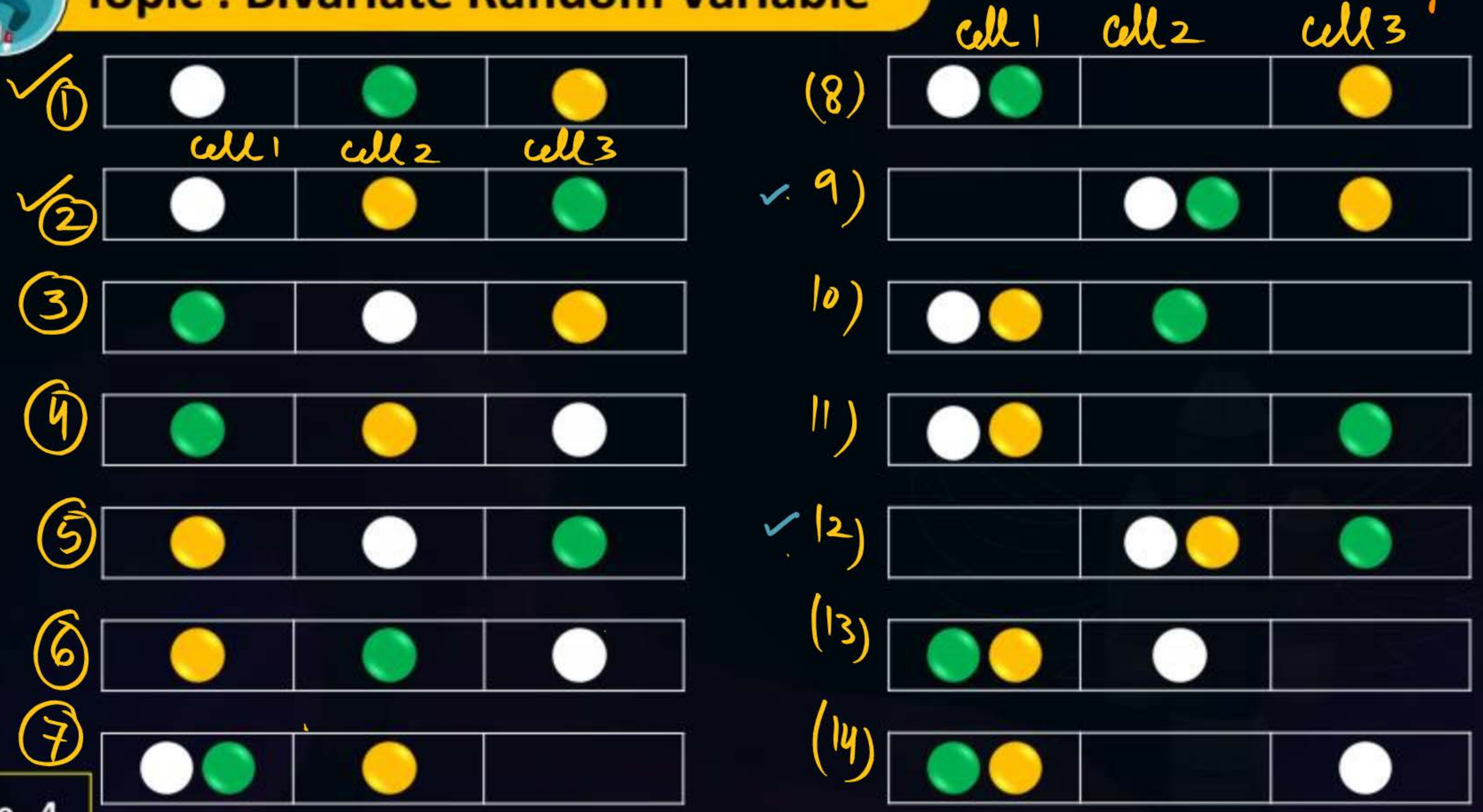
$P[X=1, Y=1] = P[X = \text{one ball in cell 1}, Y = \text{1 cell occupied}]$

X \ Y	1	2	3
0	(0,1)	(0,2)	(0,3)
1	(1,1)	(1,2)	(1,3)
2	(2,1)	(2,2)	(2,3)
3	(3,1)	(3,2)	(3,3)

(0,1) (0,2) (0,3)
(1,1) (1,2) (1,3)
(2,1) (2,2) (2,3)
(3,1) (3,2) (3,3)

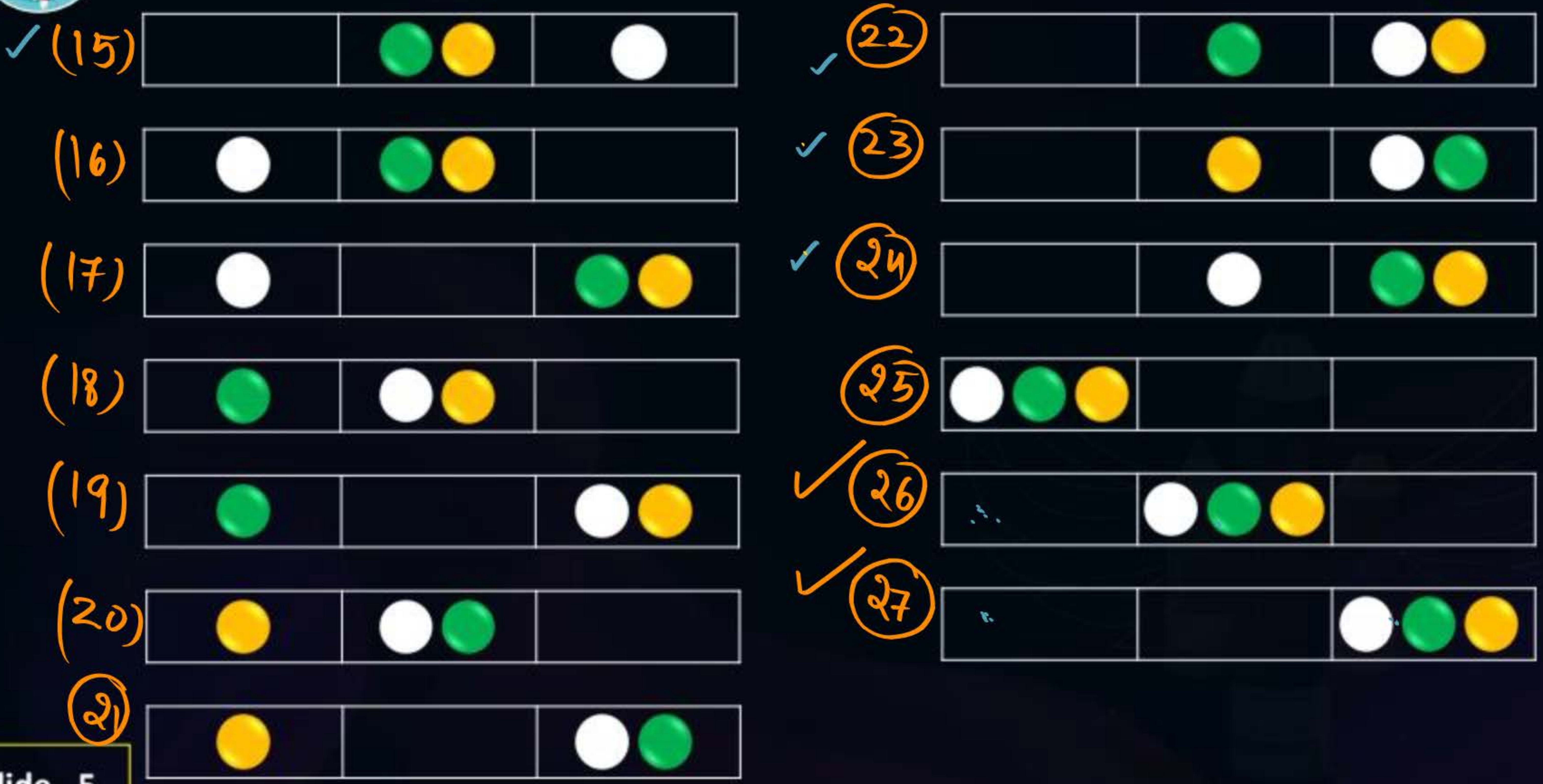
Topic : Bivariate Random Variable

$x = (0, 2)$ No ball in cell 1 and 2 cells occupied



Slide 4

Topic : Bivariate Random Variable



X = No. of ball in cell 1

Y = No. of cells occupied

X \ Y	1	2	3
0	(0,1)	(0,2)	(0,3)
1	(1,1)	(1,2)	(1,3)
2	(2,1)	(2,2)	(2,3)
3	(3,1)	(3,2)	(3,3)

$P[X=0, Y=1] \Rightarrow \frac{2}{27}$

$P[X=0, Y=2] \Rightarrow \frac{6}{27}$

$P[X=0, Y=3] \Rightarrow 0$

$P[X=1\ Y=1] \Rightarrow 0$

$P[X=1, Y=2] \Rightarrow \frac{6}{27}$

$P[X=1\ Y=3] \Rightarrow 6/27$

$P[X=2, Y=1] \Rightarrow 0$

$P[X=2, Y=2] = \frac{6}{27}$

$P[X=2, Y=3] = 0$

$P[X=3, Y=1] = \frac{1}{27}$

$P(X=3, Y=2) = 0$

$P[X=3, Y=3) = 0$

Let X, Y be Two Random Variable Then:

Joint Prob: (Discrete bivariate Random Variable]

Joint Probability Mass Function $= P[X=x_i, Y=y_j] = P[X=x_i \wedge Y=y_j] =$ both Simultaneously occur

Joint Prob: Table:

A) $P[X=x_i, Y=Y_j] \geq 0$

B) $\sum_{i=0, j=0}^{n} P[X=x_i, Y=Y_j]$ Column Add $= 1$

X \ Y	1	2	3	Total
0	$\frac{2}{27}$ (0,1)	$\frac{6}{27}$ (0,2)	0 (0,3)	8/27
1	0 (1,1)	6/27 (1,2)	6/27 (1,3)	12/27
2	0 (2,1)	6/27 (2,2)	0 (2,3)	6/27
3	1/27 (3,1)	0 (3,2)	0 (3,3)	1/27
Total	$\frac{3}{27}$	$\frac{18}{27}$	$\frac{6}{27}$	1

$(0,1)\ (0,2)\ (0,3)$

$P[X=0] = \frac{8}{27} = \frac{2}{27} + \frac{6}{27} + 0$

$P[X=1] = \frac{12}{27}$

$P[X=2] = \frac{6}{27}$

$P[X=3] = \frac{1}{27}$

$P[Y=1] = \frac{3}{27}$

$P[Y=2] = \frac{18}{27}$

$P[Y=3] = \frac{6}{27}$

X \ Y	1	2	3	
0	$\frac{2}{27}$ (0,1)	$\frac{6}{27}$ (0,2)	0 (0,3)	8/27
1	0 (1,1)	6/27 (1,2)	6/27 (1,3)	$\frac{12}{27}$
2	0 (2,1)	6/27 (2,2)	0 (2,3)	6/27
3	$\frac{1}{27}$ (3,1)	0 (3,2)	0 (3,3)	1/27
	3/27	18/27	6/27	1

$\boxed{\text{Marginal Probability}}$

only Target with one value.

$P[x_i, y_j] \rightarrow P[x_i]$

$P[x_i, y_j] \rightarrow P[y_i]$

X static

Y Dynamic

Marginal Prob:

$$P[X=x_i] = P[X=x_i \cap Y=y_j] \Rightarrow P[X=x_1 \cap Y=y_1] + P[X=x_1 \cap Y=y_2) + P[X=x_1 \cap Y=y_3) + \cdots$$

$$\begin{cases} P[X=x_i] = \sum_j P[X=x_i, Y=y_j] \\ P[Y=y_j] = \sum_i P[X=x_i, Y=y_j] \end{cases}$$

Marginal Probability Distribution

for X

X	0	1	2	3
$P[X=x_i]$	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$

marginal prob. Dis for Y.

Y	1	2	3
$P[Y=y_j]$	$\frac{3}{27}$	$\frac{18}{27}$	$\frac{6}{27}$

THANK - YOU