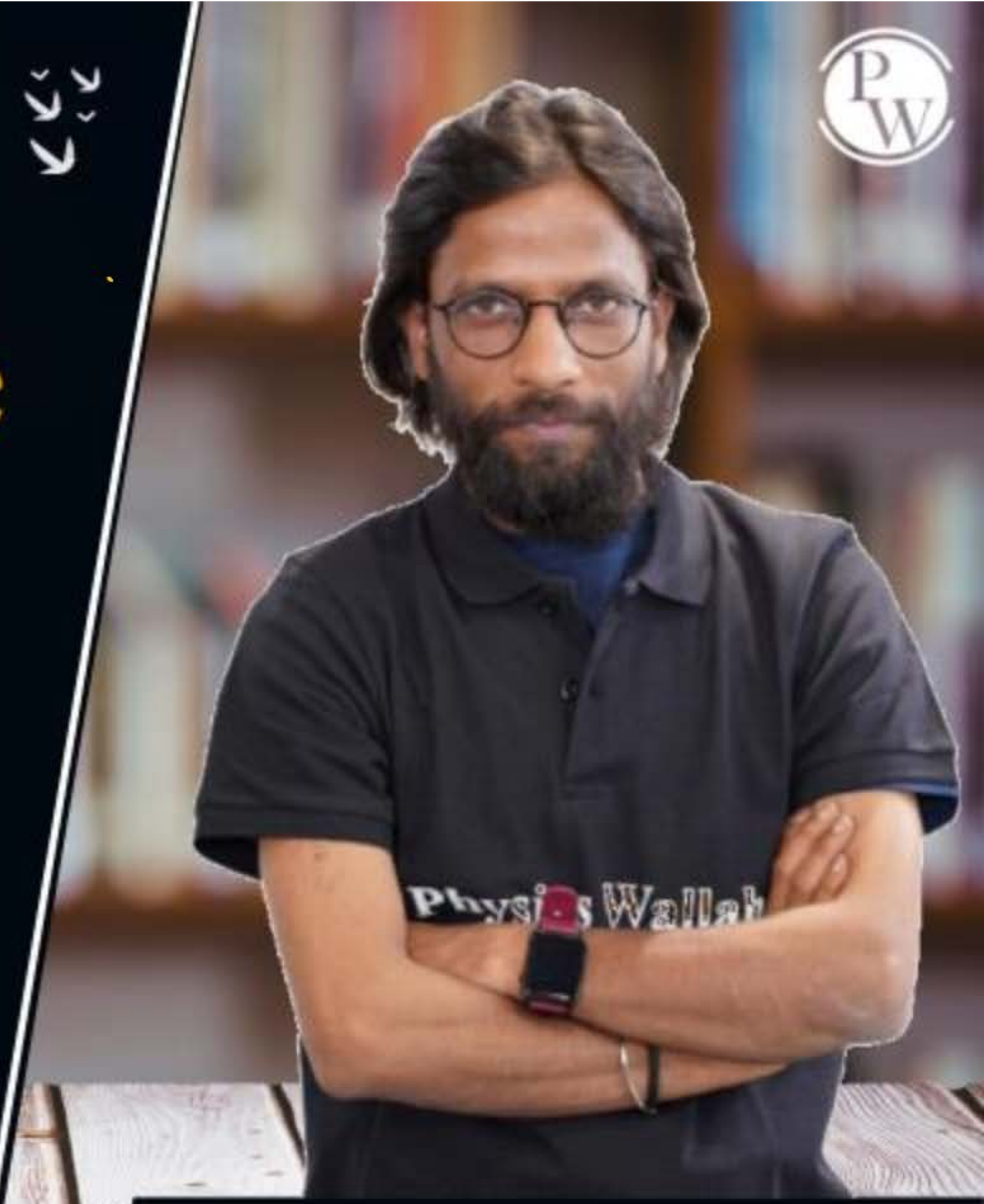
Data Science and Artificial Intelligence

Probability and Statistics

Discrete Probability Distribution

Lecture No.- 04



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Topics to be Covered

Topic	Question Based on Geometric Distributions
Topic	Poisson's distribution
Topic	Question Based on Poisson's distribution

GEOMETRIC DISTribution

$$P(X=x) = q^{x-1} \cdot p$$

GEOMETRIC Distribution

II type – Premium / Insurance

$$P(X=x) = q^{x} \cdot p \qquad X = 0, 1, 2, 3 \ldots$$

SUCCESS

$x = 0$ — SUCCESS

geometric Distribution

Prob. Mass Function

PMF

$X = 1, 2, 3 \ldots$

S $\quad X = 1$

FS $\quad X = 2$

FFS

I am Interested Untill / final Last SUCCESS

Trials - stop.

SUCCESS

Premium - 500

$\boxed{X=0}$ Assumed Insurance

$P(X=x) = q^x \cdot p \quad x = 0, 1, 2, 3, 4 \ldots$

Tossing A coin

HEAD - SUCCESS

Prob. of SUCCESS $= p$

Prob. of failure $= q$

H	$X=0$ — 0th trial → SUCCESS
TH	$X=1$
TTH	$X=2$
TTTH	$X=3$
TTTTH	$X=4$
⋮	⋮

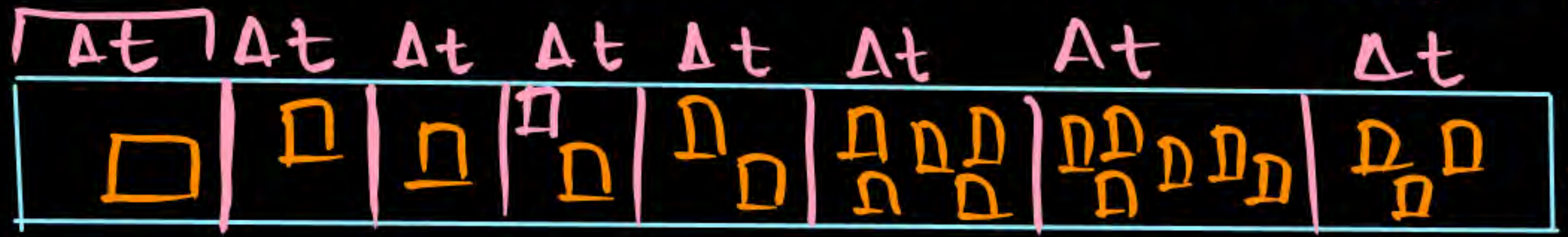
$$\boxed{P(X=x) = q^x \cdot p} \quad x = 0, 1, 2, 3$$

geometric Distribution

Type 02

Imp: **Poisson Distribution** :

Arrival Pattern → $\boxed{\text{Average No. of SUCCESS}}$ in a time Interval



No of trials — Increase.

✓ Average No. of SUCCESS in a given Time Interval

✓ Average No. of claims.

✓ Average No. doubts

✓ Average No. of HEADS.

✓ Average No. of calls.

✓ Average No. of customer Entering A shop.

✓ Average No. of Vehicles coming traffic light.

1 (SUCCESS) / 0 failure

Δt

SMALL Intervals

1/0

1/0

1/0

1/0

1/0

1/0

(1/0) - - -

1/0

1 Hr

1/0

30 min

1/0

15 min

n Trials

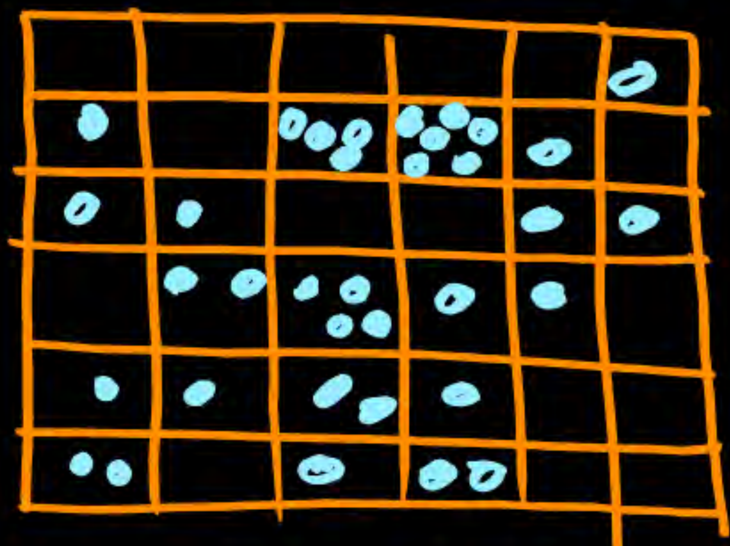
P ↓

1/0

1/0

given Small time Interval

Average No. of SUCCESS



Random

rice seeds - Scattering

4 vehicles
PASS
SEC

100 vehicles → How much time wait
Continuous - waiting

No. of arrival
No. of arrival
No. of arrivals

Average No. of vehicle

6 vehicles — 1 min

4 vehicles — 30 SEC — Average

2 vehicles — 15 SEC — Average

1 vehicles — 5 SEC

No vehicles — No vehicles — 1 SEC

Integer
Discrete

only one event

S/F YES/NO

t $\quad dt \quad$ $t+dt$

✓ $P[\text{multiple events } (t, t+dt)] = 0$

✓ $P[\text{1 event occur}] = \lambda \cdot dt$

✓

$t \qquad (t+\delta t)$

t event
t+dt event
] Independent
YES

$P(t \cap t+dt) = P(t)P(t+dt)$

Poisson Model

A) Poisson Model

Average

Dada = $\boxed{\text{Average}}$ × time = λ

✓ $\boxed{P = \lambda \cdot dt}$

time = dt

B) No multiple events occur

1/0

$t \quad dt \quad t+dt$

only one Happen

NO/YES ✓ 1 (SUCCESS)

X 0 (failure)

✓ 1

X 0

✓ 1

✓ 1

✓ 1

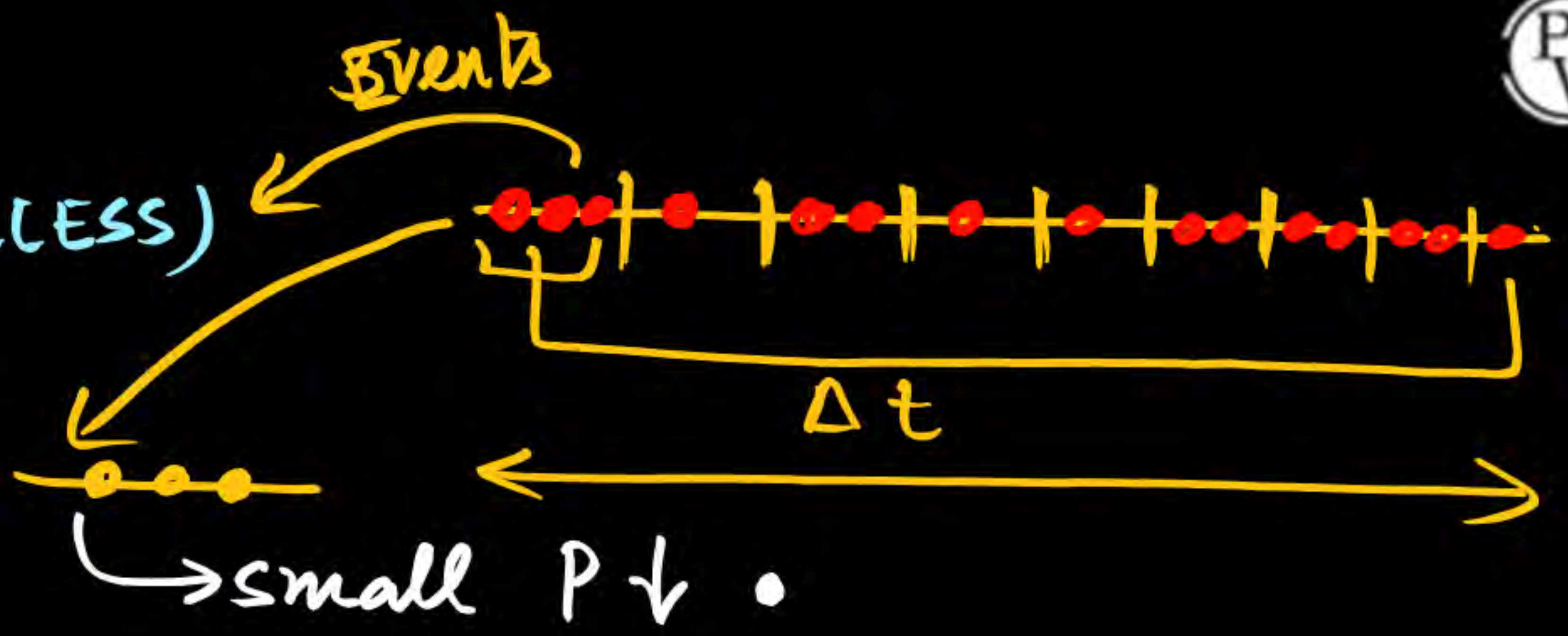
X 0

X 0

X 0

X 0

X 0



SEQUENCE

1 0 1 0 1 1 1 0 0 0 0 0 0

S F S F S S S F F F F F F

Average SUCCESS → 5 times

$$P[X=x] = \frac{e^{-\lambda t}\cdot(\lambda t)^x}{x!} \quad X = 0, 1, 2, 3 \ldots$$

Discrete Random variable

λ = Average

t = time

Where $\lambda t = \mu$ = Average → Per question

$$\boxed{P[X=x] = \frac{e^{-\mu}(\mu)^x}{x!}}$$

Poisson Distribution

Statistical Averages: MEAN =

variance =

moment generating function

Moment generating Function

$$\Pi_X(s) = \sum_{x=0}^{\infty} s^x \underbrace{P(X=x)}$$

PMF

$$P(X=x) = \frac{e^{-\mu}\mu^x}{x!}$$

μ = mean of The Distribution

$$= \sum_{x=0}^{\infty} s^x \cdot \frac{e^{-\mu}\mu^x}{x!}$$

$$= \sum_{x=0}^{\infty} e^{-\mu} \frac{(\mu s)^x}{x!}$$

$$e^{x \to \mu s} = e^{\mu s}.$$

$$= e^{-\mu} \sum_{x=0}^{\infty} \frac{(\mu s)^x}{x!} = e^{-\mu}\left[1 + \frac{\mu s}{1!} + \frac{\mu^2 s^2}{2!} + \frac{\mu^3 s^3}{3!} + \cdots\right]$$

$$\Rightarrow e^{-\mu} \cdot e^{\mu s} = e^{-\mu+\mu s} = e^{\mu(-1+s)} = \underline{e^{\mu(s-1)}}$$

Moment generating Function

$$MGF = \Pi_X(s) = e^{\mu(s-1)}$$

$$MEAN = \Pi_X'(s) = e^{\mu(s-1)} \cdot \frac{d}{ds}\mu(s-1)$$

$$\boxed{\Pi_X'(s) \Rightarrow e^{\mu(s-1)} \cdot \mu}$$

Put $s = 1$

$$\Pi_X'(1) = E[X] = \text{expected value}$$

Mean

$$\boxed{\left[\Pi_X'(s)\right]_{s=1} = E[X] = \mu}$$

$$\frac{d}{dx}\left(e^{\boxed{ax}}\right) = e^{ax} \cdot a$$

$$\boxed{\text{Variance} = \mu}$$

$$\text{Standard deviation} = \sqrt{\mu}$$

Poisson distribution

$$\rightarrow Po(\mu) = P(X=x) = \frac{e^{-\mu}\mu^x}{x!}$$

Topic :

GEO $P(X \geq 5) = 5$ ✓ $P(X<5)$ 5Ex

✓ 6 $\frac{1}{6}$ $n=0$

56 $\left(\frac{5}{6}\right)\frac{1}{6}$ $n=1$

556 $=\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}$ $n=2$

5556 $=\left(\frac{5}{6}\right)^3 \frac{1}{6}$ $n=3$

Q1. An unbiased die is cast until 6 appear. What is the probability that it must be cast more than five times?

$X = 0, 1, 2, 3, 4, 5 - - -$

$$\boxed{PMF = q^x \cdot p \quad x = 0, 1, 2 - - -}$$

$$PMF = (1-p)^x p \quad x = 0, 1, 2 - - -$$
$$= 0$$

$$P(X>5) = P(X=5) + P(X=6) + P(X=7) + - - -$$
$$= \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^7 \cdot \frac{1}{6} + - - -$$
$$= \frac{1}{6}\left[\underbrace{\left(\frac{5}{6}\right)^5 + \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^7 + - -}\right]$$
$$= \left(\frac{5}{6}\right)^5 \text{ Ans}$$

G.P in Infinite terms

$S_\infty = \frac{a}{1-r}$

Topic :

$X = 0$ start ✓ → Every start

Q2. Probability of hitting a target in any attempt is 0.6, what is the probability that it would be hit on fifth attempt?

$P(\text{Target}) = 0.6$ $P(\text{fail}) = 0.4$

Using geometric Distribution $P(\text{fifth attempt}) =$

S → $n = 0$
FS → $n = 1$
FFS → $n = 2$
FFFS → $n = 3$
FFFFS → $n = 4$

Prob. of Hit The Target

$= (0.4)^4 \cdot 0.6$

Topic :

→ Do yourself

Q3. Determine the geometric distribution for which the mean is 3 and variance is 4.

GEOMETRIC Distribution

Slide 5

Topic :

Do yourself

Q4. If a random variable X satisfies the poission's distribution with a mean value of 2, then the probability that $x > 2$ is.

Poisson Distribution

A. $2e^{-2}$

B. $1 - 2e^{-2}$

C. $3e^{-2}$

D. $1 - 3e^{-2}$

Topic :

5 PM to 6 PM

P = Poisson Distribution
μ = 3 cars per minute

Q5. Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and P has a poission's distribution with the $\boxed{\text{mean 3.}}$ What is the probability of observing fewer than 3 cars during any given minute in this interval?

X = No. of cars.

$P(X<3) = P(X=0)+P(X=1)+P(X=2)$

fewer Than 3 — 0 car, 1 car, 2 car.

A. $\frac{8}{(2e^3)}$

B. $\frac{8}{(2e^3)}$

C. $\frac{17}{(2e^3)}$

D. $\frac{26}{(2e^3)}$

$\lambda t = \mu$
$t = \delta t$

$P(X<3) = P(X=0\,CAR) + P(X=1\,car) + P(X=2\,car)$

$P(X=x) = \dfrac{e^{-\mu}(\mu)^x}{x!}$

$x = 0, 1, 2, 3 \,--$
No. of success

$P(X=0) = \dfrac{e^{-3}(3)^0}{0!} = e^{-3}$

$x=0$
$\mu=3$

$P(X<3) = e^{-3} + 3e^{-3} + \dfrac{9}{2}e^{-3}$

$\boxed{P(X<3) = \dfrac{17}{2}e^{-3}}$ Ans

$P(X=1) = \dfrac{e^{-3}(3)^1}{1!} = 3e^{-3}$

$x=1$
$\mu=3$

$P(X=2) = \dfrac{e^{-3}(3)^2}{2!} = \dfrac{9}{2}e^{-3}$

Topic :

$\mu = 5.2 \quad P(X<2)$

Q6. The number of accidents occurring in a plant in a month follows Poisson distribution with $\boxed{\text{mean as 5.2.}}$ The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is

A. 0.029

Using Poisson distribution $\rightarrow P(X=x) = \frac{e^{-\mu}\mu^x}{x!}$

B. 0.034

$P(X<2) = P(X=0) + P(X=1)$

$= \frac{e^{-5.2}(5.2)^0}{0!} + \frac{e^{-5.2}(5.2)^1}{1!}$

C. 0.039

$= e^{-5.2}[1 + 5.2]$

$= 6.2e^{-5.2} = \underline{0.0034}$

D. 0.044

Topic :

mean of Poisson distribution

$\mu > 0$ $\mu = 1$ is satisfied

$\to \mu$ μ = Average No. of (success)

Q7. The second moment of a poisson-distributed random variables is 2. The mean of the random variable ______ is

Imp:

Moment generating Function $= \sum_{x=0}^{\infty} s^x P(X=x)$

$E[X^1]$ = MEAN = First moment

$E[X^2]$ = SECOND moment

$E[X^3]$ = Third moment

$E[X^n]$ = n^{th} moment

$\boxed{E[X] = 1, -2}$

$E[X] = \mu$

$V(X) = \mu$

$E[X^2] = 2$ $E[X] = ✓$

Using $var(X) = E[X^2] - [E[X]]^2$

$\mu = 2 - \mu^2$

$\mu^2 + \mu - 2 = 0$

$\mu^2 + 2\mu - \mu - 2 = 0$

$\mu(\mu+2) - 1(\mu+2) = 0$

$\boxed{\begin{array}{l}\mu = 1 \\ \mu = -2\end{array}}$

Topic :

Q8. If a random variable X has a poisson distribution with mean 5, then the expectation $E[(X+2)^2]$ equals ______.

$\mu = 5$

$$E[(x+2)^2] = E[(x^2+4+4x)]$$
$$= E[x^2]+4+4E[x]$$
$$= 30+4+4\times5$$
$$= 54 \ \underline{\text{Ans}}$$

$\mu = 5$

variance $= 5$

$Var = E[x^2] - [E[x]]^2$

$var(x) + [E[x]]^2 = E[x^2]$

$\underline{5+25} = E[x^2]$

$= 30 = E[x^2]$

Topic :

$\mu = 5$ (No. of Penalties)

Q9. A traffic office imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days is independent and follows a poisson distribution.

$P(X=x) = \frac{e^{-\mu}\mu^{x}}{x!}$ $x = 0, 1, 2 \ldots$

The probability that there will be less than 4 penalties in a day is ________.

$$P(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \frac{e^{-5}(5)^0}{0!} + \frac{e^{-5}(5)^1}{1!} + \frac{e^{-5}(5)^2}{2!} + \frac{e^{-5}(5)^3}{3!} = 0.265$$

Topic :

$\boxed{240\text{veh}}$
1 hr

240veh/hr

Q10. An observer counts 240veh/h at a specific highway location. Assume that the vehicle arrival at the location is poisson distributed, the probability of having one vehicle arriving over a 30-second time interval is

15	15	15	15

240veh

45 minute

$\underline{240\text{veh}}$
Average

50

1	1	1	1	1

5 min
50 vehicles.

1 vehicle 3 min
49 veh 2 min

MESS

500 students

$\mu = \underline{240\ veh/hr} = \frac{240}{60}\ veh/min$

$= 4\ veh/min$

$\boxed{\mu = 2\ veh/30\ SEC} = 2\ veh/\underline{30\ SEC} = \mu$

$$P[x=1] = \frac{e^{-\mu}(\mu)^x}{x!}$$

$$= \frac{e^{-2}(2)^1}{1!}$$

$$= 2e^{-2} = \underline{0.27}$$

Topic :

{Poisson distribution

Q11. The average number of traffic accidents on a certain section of highway is two per week assume that the number of accidents follows a poisson distribution.

1. Find the probability of no accidents on this section of highway during a 1- week period.
2. Find the probability of at most three accidents on this section of highway during a 1-week.
3. Find the probability of at least four accidents during a 1-week
4. Find variance and standard deviation.

THANK - YOU