DATA SCIENCE

Probability and Statistics

Counting Techniques

Lecture No.- 01

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Topics to be Covered : Counting Techniques

Permutation:

Arrangement
OR
Change The Relative Position
OR
Arrangement Number.

A B C - 3 Letters

$$\left\{\begin{array}{ccc} A & B & C \\ A & C & B \\ B & C & A \\ B & A & C \\ C & A & B \\ C & B & A \end{array}\right]$$

Arrangement

3 Letters

→ Box method
Pigeon Holes method

3 Letters

3 Items = A, B, C

3 balls

3 coupons

Relative Position

Different type

W_1	W_2	W_3
3	2	1

Box

A B C

A B C D

Change the Relative Position

A B C

A C B

B C A

B A C

C A B

C B A

Total No. of ways arrangement (order matters)

Using Fundamental Principal of counting

$= W_1 \times W_2 \times W_3$ (Working Together)

$= 3 \times 2 \times 1$

$= 6$

] Diff. Arrang.

$\boxed{\begin{matrix} A & B \\ B & A \end{matrix}}$] Different order

A B
B A] SAME

A B C
B C A] Different order — Arrangement — order matter

n Different Items or objects

n Items

- n
- n-1
- n-2
- n-4
- 1

without replacement

Place Item (SAME)

n Items

with Replacement

W_1	W_2	W_3	W_4	W_5	W_6		W_{Last}
n	(n-1)	n-2	n-3	n-4	n-5		1

boxES

A, B, C, D, E — 1st Draw 5 (without)

A, B, C, D, E — 4

A, B, C, D, E — 3

n Items - decrease

Total No. of ways $= W_1 \times W_2 \times W_3 \times \cdots \times W_n$

✓ n Different Items $= n \times (n-1) \times (n-2) \times \cdots 1$

Taken all at a time

$= n!$ [n Factorial]

4 objects

$\boxed{O_1 \; O_2 \; O_3 \; O_4}$

$= 4 \times 3 \times 2 \times 1$

Total No. of arrangement $= 4! = 24$

n Items
Taken r
at time

A B C

3 Letters Taken 2 at time

Diffrent arrangement (Order matters)

A B	B C	A C
B A	C B	C A

6 cases

2 balls select

$r-1$

W_1	W_2	W_3	W_4		W_r
$n-0$	$(n-1)$	$(n-2)$	$(n-3)$		$n-(r-1)$

rth box

Total No. of ways (Total arrangement)

$$= n(n-1)(n-2)(n-3)---n-(r-1)$$

$$= \frac{n(n-1)(n-2)(n-3)---n-(r-1)(n-r)!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!} = {}^{n}P_r$$

Permulate (arrange)

CASE 03

n Different Items $\longrightarrow$ group | Selection

$\longrightarrow$ order (doesn't) matter

$\boxed{A}$ $\boxed{B}$ $\boxed{C}$ $\longrightarrow$ 3 compo.

$\longrightarrow$ Taken 2 at a time

SAME $\begin{bmatrix} A & B \\ B & A \end{bmatrix}$ $\begin{bmatrix} B & C \\ C & B \end{bmatrix}$ $\begin{bmatrix} C & A \\ A & C \end{bmatrix}$

SAME SAME

$= \text{arrangement} = {}^3P_2$

$\longrightarrow$ repeat $\rightarrow$ divide

$= \dfrac{{}^3P_2}{2!}$

Total No. of Selection

$$= \frac{{}^nP_r}{r!} = {}^nC_r = \frac{n!}{(n-r)!\,r!}$$

5 Persons → 2 Person Select → Commite

A, B, C, D, E

Committee

Repeation Not allowed ('without replacement') = AA, BB, CC, DD, EE

$N^2 - N$

$= \frac{N^2 - N}{2}$

~~AA~~	BA	CA	DA	EA
AB	~~BB~~	CB	DB	EB
AC	BC	~~CC~~	DC	EC
AD	BD	CD	~~DD~~	ED
AE	BE	CE	DE	~~EE~~

$\frac{N^2 - N}{2}$ ✗

$\boxed{\frac{{}^5P_2}{2!} = \frac{20}{2} = 10}$

$= N^2 - 5$

$= 5^2 - 5$

$= 20$

No. of combination $= \frac{N^2 - N}{2}$

$= {}^5C_2 = 10$

$= \frac{N(N-1)}{2} = \frac{\cancel{20}}{\cancel{2}} = 10$

$\boxed{5 \text{ Items}}$ $\qquad \underline{3 \text{ select}}$ $= \dfrac{\boxed{{}^5P_3}}{3!} = \boxed{{}^5C_3} = \dfrac{5 \times \cancel{4}^{2} \times \cancel{3}}{\cancel{3} \times \cancel{2} \times 1}$

$= 10$ Items

A B C D E

Slide : 3

#Q. How many n-digit numbers can be formed using 1, 2, 3, 7, 9 without any repetition of digits when:

(i) n = 5

A 120

B 15

C 6^5

D 5^6

5 Digit
Taken all at a time
without replacement

5	4	3	2	1
W_1	W_2	W_3	W_4	W_5

$= 5 \times 4 \times 3 \times 2 \times 1$
$= 5!$
$= 120$
$= {}^5P_5 = 120$

n Items Diff. Taken all at a time

(ii) n = 3

A 12

B 5^3

C 60

D 3^5

n - Digit → 3 Digit

3 Digit → 1, 2, 3, 7, 9 → n Items Taken r at time

without replacement

5	4	3
W_1	W_2	W_3

$= 5 \times 4 \times 3$
$= 60$

$${}^5P_3 = 60$$

n Diff. Taken r at a time

#Q. How many 3-letter words can be formed using a, b, c, d, e if: (i) repetition is not allowed

(A) 60

(B) 5^3

(C) 3^5

(D) 12

1) Repetation is Not allowed
(without replacement)

a, b, c, d, e → n Items

3 Letter word

$w_1 \times$	$w_2 \times$	w_3
5	4	3

r cells

Total No. of ways $= 5 \times 4 \times 3$
$= \underline{60}$

a, b, c, d, e → without replacement Taken 3 at a time

5 Items

$^nP_r = {}^5P_3 = \underline{60}$

n Items r — Arrang $\longrightarrow {}^{n}P_{r}$

#Q. How many $\boxed{\text{3-letter words}}$ can be formed using a, b, c, d, e if:

(ii) repetition is allowed?

W_1	W_2	W_3
5	5	5

= Total No. of choice

$= 5 \times 5 \times 5 = 5^3 = 125$

Letter Place

a, b, c, d, e

box

→ with replacement

5 ways.

letter

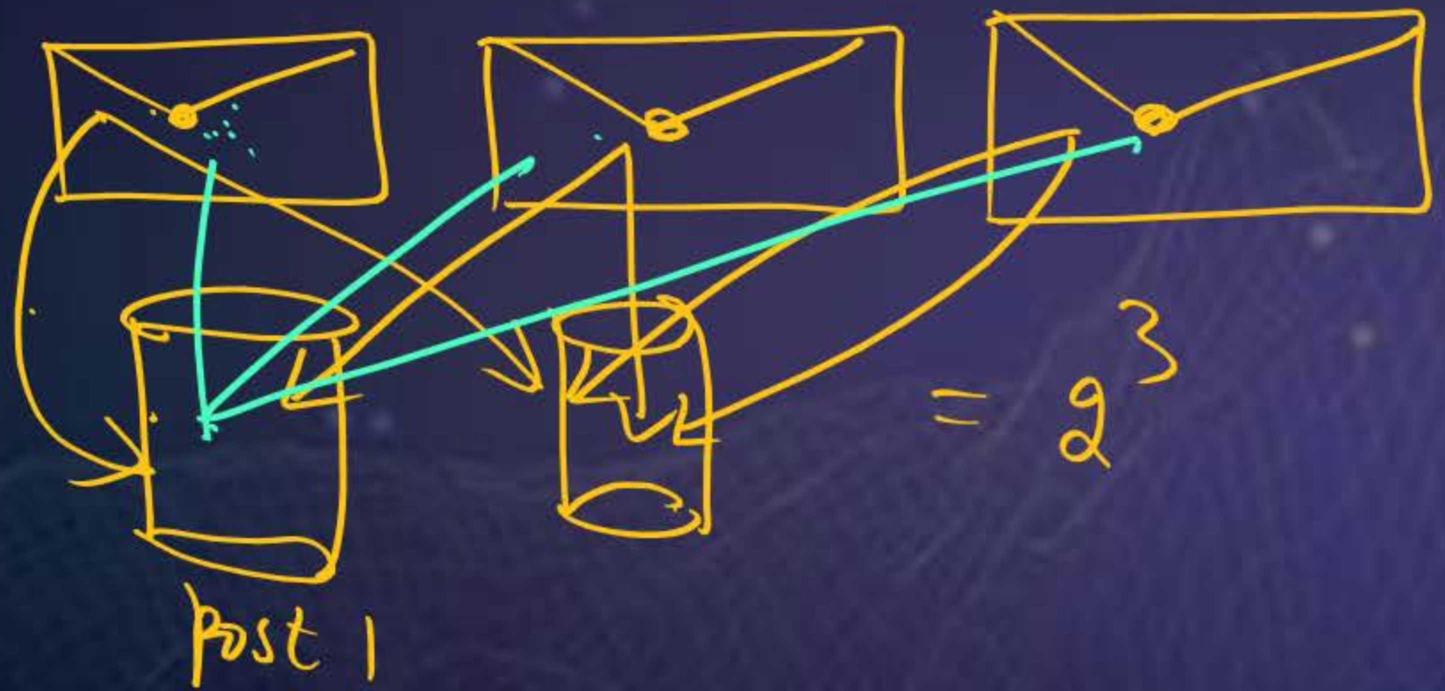
(A) 60

(B) 5^3 ✓

(C) 3^5

(D) 12

3 letters 2 boxes



$= 2^3$

with replacement Without replacement - default

#Q. How many four-digit numbers can be formed using the digits 0, 1, 2, 3, 4, 5?

4 Digit Number

W_1	W_2	W_3	W_4

0

$= 5 \quad 5 \quad 4 \quad 3$

Total No. of ways $= 5 \times 5 \times 4 \times 3$

$= 300$ Ans

with replacement $\Rightarrow 5 \times 6 \times 6 \times 6 =$ Ans

Without Replacement

1, 2, 3, 4, 5

~~0~~

W_1 5 ways

0, 1, 2, 3, 4, 5

W_2 job 5 ways

(A) 5^4

(B) 4^5

(C) 300

(D) 150

Slide : 6

#Q. In how many ways can six persons be arranged in a row?

without replacement
n Items
Taken all at a time

(A) 6!

(B) 6^6

(C) 6^5

(D) 5^6

Total No. of ways =

6	5	4	3	2	1

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 6!$

$= \boxed{720}$

#Q. How many 5-digit odd numbers can be formed using digits 0,1, 2, 3, 4, 5 without repeating digits?

(A) $4 \times 4!$

(B) 288 ✓

(C) $5!$

(D) 300

W_1	W_2	W_3	W_4	W_5

0, 1, 2, 3, 4, 5

W_1: $0 \neq$

$W_1\ W_2\ W_3\ W_4\ W_5$ Are Independent

W_5: (1, 3, 5) Last Digit (odd No)

4	4	3	2	3
W_1	W_2	W_3	W_4	W_5

Total No. of ways $= 4 \times 4 \times 3 \times 2 \times 3$

$= 288$ Ans

#Q. How many 5-digit numbers divisible by 2 can be formed using digits 0, 1, 2, 3, 4, 5 without repetition of digits.

(A) 120

(B) 192

(C) 312

(D) 208

W_1	W_2	W_3	W_4	W_5

0, 1, 2, 3, 4, 5 → divisible via 2

div by 2 → 0, | 2, 4

case A | case B

× 0, 1, 2, 3, 4, 5

5	4	3	2	0 1
				1 ways

$= 5 \times 4 \times 3 \times 2 \times 1 = 120$ ZERO

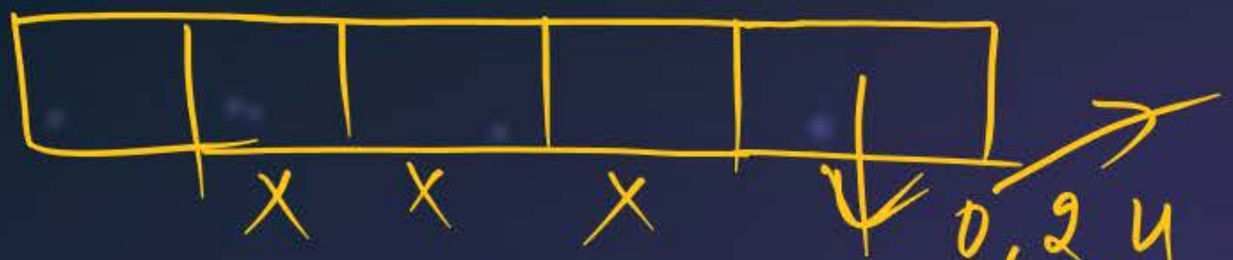
4	4	3	2	(2, 4) 2
				2 ways

Non-ZERO $= 4 \times 4 \times 3 \times 2 \times 2$

$= 192$

$= 120 + 192 = 312$

1,2,3,4,5



$\underbrace{0, 2, 4}_{\text{3 ways}}$

Divisible via 2

✓0 | 2 | 4

Last

W1 X

0, $\boxed{1, 2, 3, 4, 5}$

#Q. How many 5-digit numbers divisible by 4 can be formed using digits 0, 1, 2, 3, 4, 5?

ZERO 04, 40, 20

NONZERO

4	3	2		

04
40
20

(3 ways)

Total No. of ways
= 4×3×2×3
= 72

04
40
20

divisible via 4

0, 1, 2, 3, 4, 5 ✓

Divisible via 4

24, 52, 32, 12

3	3	2		

= 3×3×2×4
= 72

24
52
32
12

(A) 144

(B) 72

(C) 288

(D) 312

44
without
replacement

3 ways

4	3	2		

04
40
20

$= 4 \times 3 \times 2 \times 3$
$= \underline{\underline{72}}$

~~0~~
~~1~~
2
3
4
5

Last digit
04
40
20

24
32
12
52

ways

3	3	2	0	0

$= 3 \times 3 \times 2 \times 4$
$= \underline{\underline{72}}$

24
32
12
52

div by 4

$\frac{42}{4}, \frac{53}{4}$


Thank You