# Data Science and Artificial Intelligence

## Probability and Statistics

Introduction to Probability

Lecture No.- 06

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# Recap of Previous Lecture

$$\frac{\frac{N}{2}\times 2}{\frac{N}{2}\times 2+\frac{N}{2}\times 1} = \frac{2}{3} \checkmark$$

**Topic** Bayes Theorem $\rightarrow$ BAYES THEOREM $\downarrow$ multi stage

**Topic** Problem Based on Bayes Theorem

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i)\,P\left(\frac{A}{E_i}\right)}{\sum_{i=1}^{n} P(E_i)\,P\left(\frac{A}{E_i}\right)}$$

# Topics to be Covered

**Topic** Probability Challenging Problem Part 1

$^3C_0\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 =$

Q1. If A and B each toss three coins. The probability that both get the same number of heads is:

A. 1/9

B. 3/16

C. 5/16

D. 3/8

| A | B |
|---|---|
| (TTT) 0H | 0H (TTT) |
| 1H | 1H |
| 2H | 2H |
| 3H | 3H |

A — Re 1, Re 2, Re 5

B → Re 1, Re 2, Re 5

SAME No. of HEAD

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

$\Rightarrow {}^3C_0\left(\frac{1}{2}\right)^6 + {}^3C_1\left(\frac{1}{2}\right)^6 + {}^3C_2\left(\frac{1}{2}\right)^6 + {}^3C_3\left(\frac{1}{2}\right)^6$

0H, 1H, 2H, 3H

$\Rightarrow \frac{5}{16}$ Ans

$0H = {}^3C_0\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ ✓ 0H (TTT)

$\Rightarrow \frac{3C_0}{2^5 \times 2} = \frac{1}{32 \times 2} = \frac{1}{64}$

$1H = {}^3C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{64}$ ✓

$2H = {}^3C_2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3 \times \cancel{2}}{\cancel{2} \times 1} = \frac{3}{64}$

Re 1, Re 2, Re 3

8 outcomes

HTT
HHT
HHH

| | |
|---|---|
| 0 | 0 TTT 0H |
| 1 | 1 THT |
| 2 | 2 THH |
| 3 | 3 HHH |
| 4 | |

$3H = {}^3C_3\left(\frac{1}{2}\right)^6 = \frac{1}{64}$

=

$X = \text{HEAD}_A \quad Y = \text{HEAD}_B$

$= P(X=0)P(Y=0) + P(X=1)P(Y=1) + P(X=2)P(Y=2) + P(X=3)P(Y=3)$

$= \left[{}^3C_0\left(\frac{1}{2}\right)^3\right]^2 + \left[{}^3C_1\left(\frac{1}{2}\right)^3\right]^2 + \left[{}^3C_2\left(\frac{1}{2}\right)^3\right]^2 + \left[{}^3C_3\left(\frac{1}{2}\right)^3\right]^2$

$= \frac{5}{16}$

Q2. If A and B are two independent events such that $P(\bar{A} \cap B) = 2/15$ and $P(A \cap \bar{B}) = 1/6$, then P(B) is:

$P(\bar{A} \cap B) = \frac{2}{15}$ $P(B) =$

If A and B Are Ind. events $P(A \cap \bar{B}) = \frac{1}{6}$

A. 1/5

$P(\bar{A})\,P(B) = \frac{2}{15}$

B. 1/6

$[1 - P(A)]\,P(B) = \frac{2}{15}$ — 1

C. 4/5

$P(A)\,[1 - P(B)] = \frac{1}{6}$ — 2

D. 5/6

$$\Rightarrow [1-P(A)]\,P(B) = \frac{2}{15}$$

$$= P(B) - \cancel{P(A)P(B)} = \frac{2}{15}$$

$$P(A) - \cancel{P(A)P(B)} = \frac{1}{6}$$

$$- \qquad + \qquad -$$

$$[P(B) - P(A)] = \frac{2}{15} - \frac{1}{6}$$

$$= \frac{12-15}{90}$$

$$= -\frac{3}{90} = -\frac{1}{30}$$

$$P(B) - P(A) = -\frac{1}{30}$$

$$P(A) - P(B) = \frac{1}{30}$$

$$\boxed{P(A) = \frac{1}{30} + P(B)}$$

$$P(A)\,[1-P(B)] = \frac{1}{6}$$

$$\Rightarrow \left[\frac{1}{30} + P(B)\right][1-P(B)] = \frac{1}{6}$$

$\Rightarrow$ quadratic equ$^n$

$$\boxed{P(B) = \frac{1}{6}, \frac{4}{5}}$$

Ans B C

## Topic : Probability Challenging Problem

Q3. If A and B are two events, the probability that exactly one of them occurs is given by:

Do yourself

H.W

A. $P(A) + P(B) - 2P(A \cap B)$

B. $P(A \cap \overline{B}) + P(\overline{A} \cap B)$

C. $P(A \cup B) - P(A \cap B)$

D. $P(\overline{A}) + P(\overline{B}) - 2P(\overline{A} \cap \overline{B})$

# Topic : Probability Challenging Problem

Q10. If A and B are events at the same experiments with $P(A) = 0.2$, $P(B) = 0.5$, then maximum value of $P(A' \cap B)$ is

H.W Do yourself

A. 1/4

B. 1/2

C. 1/8

D. 1/16

## Topic : Probability Challenging Problem

GATE

Q11. The probabilities that a student passes in mathematics, physics and chemistry are m, p and c respectively. Of these subjects, a student has a 75% chance of passing in at least one, a 50% chance of passing in at least one, 50% chance of passing in at least two and a 40% chance of passing in exactly two subjects. Which of the following relations are true?

Do yourself

A. $p + m + c = \frac{19}{20}$

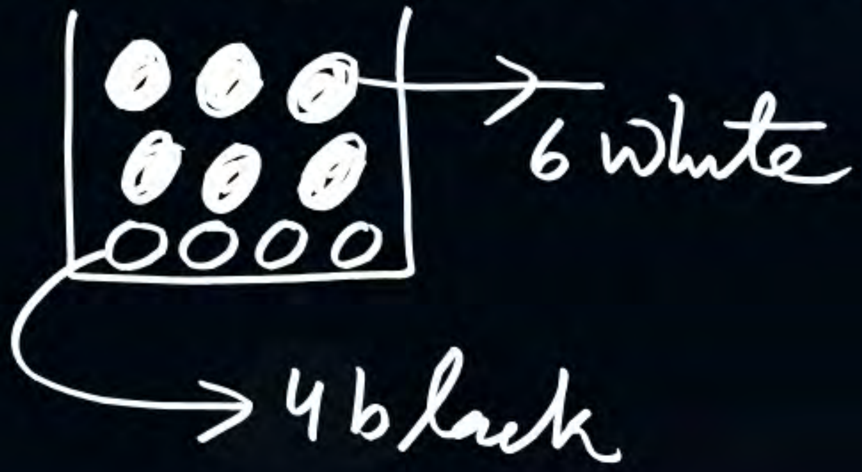
B. $p + m + c = \frac{27}{20}$

C. $pmc = \frac{1}{10}$

D. $pmc = \frac{1}{4}$

## Topic : Probability Challenging Problem

Q16. An urn contains 6 white and 4 black balls. A fair die is rolled, and that number of balls are chosen from the urn. The probability that the balls selected are white is:




A. 1/5

B. 1/6

C. 1/7

D. 1/8

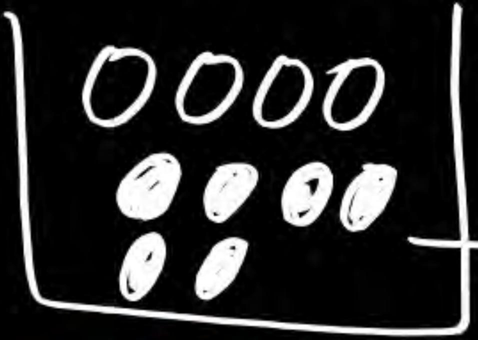
$$\Rightarrow \underset{①\times 1W}{\frac{1}{6}\times\left(\frac{6}{10}\right)} + \underset{②}{\frac{1}{6}} * \underbrace{\left(\frac{6}{10}\times\frac{5}{9}\right)}_{2W} + \underset{③}{\frac{1}{6}}\underbrace{\left(\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\right)}_{3W}$$

$$+ \underset{④}{\frac{1}{6}}\underbrace{\left(\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\right)}_{4W} + \underset{⑤}{\frac{1}{6}}\underbrace{\left(\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{2}{6}\right)}_{5W}$$

$$+ \frac{1}{\cancel{6}}\underbrace{\left[\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{2}{\cancel{6}}\times\frac{1}{\cancel{5}}\right]}_{6\,\text{White}}$$

$6$

$$= \frac{1}{5} \ \underline{\text{Ans}}$$



4 black

→ 6W

Die Rolled

1 → White (one) +

2 → 2 White +

3 → 3 White +

4 → 4 White +

5 → 5 White +

6 → 6 White +

$W_1$ Job

$W_2$ Job

# Topic : Probability Challenging Problem

$\rightarrow TH = (1-p)p$

$\rightarrow TTTH = p(1-p)^3$

Q18. A biased coin with probability p, $0 < p < 1$ of head is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p equals:

Imp

A. 1/3

B. 2/3

C. 2/5

D. 3/5

$P(2) = p(1-p)$

$P(4) = p(1-p)(1-p)(1-p)$

$P(6) = p(1-p)(1-p)(1-p)(1-p)$

$= P(2H) + P(4H) + P(6H) + \text{----}$

$= (TH) + TH(HH) + THTT(HH) + \text{--}$

$\frac{2}{5} \Rightarrow p(1-p) + p(1-p)^3 + p(1-p)^5 + \text{- -} \infty$

$\boxed{\frac{2}{5} = \frac{p(1-p)}{1-(1-p)^2}}$

$p = \frac{1}{3}$

$S_\infty = \frac{a}{(1-r)}$

$r = (1-p)^2$

$a = p(1-p)$

$$p(1-p) \quad = 2 \text{———} TH$$
$$(1-p)^3 p + \quad 4 \text{———} TTTH$$
$$(1-p)^5 p + \quad 6 \text{———} TTTTTH$$
$$(1-p)^7 p + \quad 8 \text{———} TTTTTTTH$$

$$\frac{2}{5} = \underline{p(1-p)} + p(1-p)^3 + \text{ - ———}$$

$$\frac{2}{5} = \frac{p(1-p)}{1-(1-p)^2}$$

$$\frac{2}{5} = \frac{p-p^2}{1-(1+p^2-2p)}$$

$$= \boxed{p = \frac{1}{3}}$$

## Topic : Probability Challenging Problem

formula 3-5 min

Q20. A signal which can be green or red with probability 4/5 and 1/5 respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is 3/4. If the signal received at station B is ~~given~~, then the probability that the original signal was green is

green

A. 3/5

B. 6/7

C. $\boxed{20/23}$

D. 9/20

$P(\text{green}) = \frac{4}{5}$

$P(\text{Red}) = \frac{1}{5}$

$P(\text{correct}) = \frac{3}{4}$

multi stage

$O \xrightarrow{\text{Received}} A \xrightarrow{\text{Transmitted}} B$

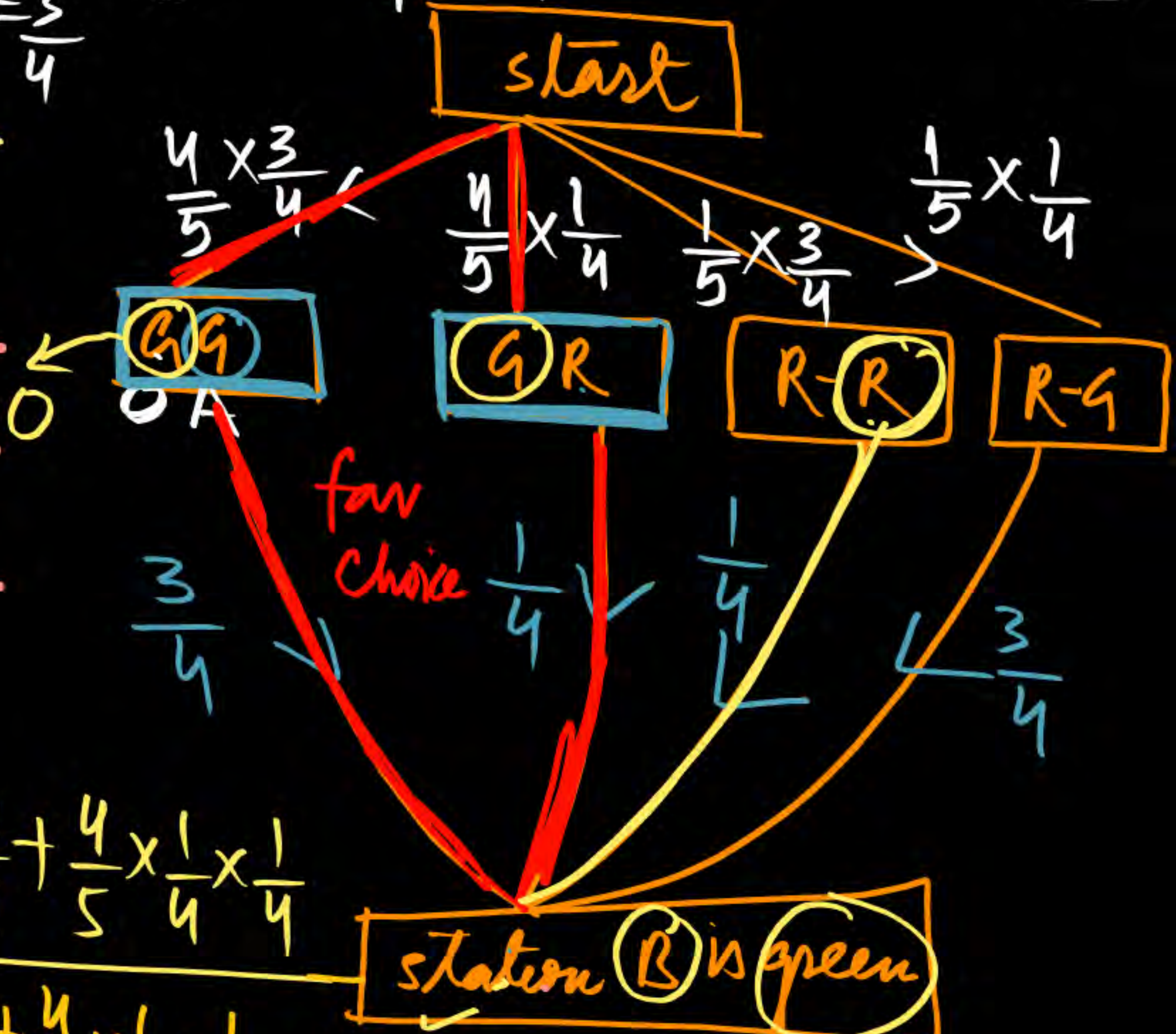
original

Bayes THEOREM

How to Solve It — George Polya

$P(G) = \frac{4}{5}$ $P(R) = \frac{1}{5}$ $P(C) = \frac{3}{4}$ $P(\text{Not } C) = 1 - \frac{3}{4} = \frac{1}{4}$

| Original | station A | station B |
| --- | --- | --- |
| Green | Green | Green |
| Green | Red | Green |
| Red | Green | Green |
| Red | Red | Green |




Original 

→ Pass

station A

Green — Green  Red — Green

Green — Red  Red — Red

$$\Rightarrow \frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}}$$

# Topic : Probability Challenging Problem

MSQ
5–8 min

Q21. A ship is fitted with three engines $E_1$, $E_2$ and $E_3$. The engines function independently of each other with respective probabilities 1/2, 1/4, 1/4. For the ship to be operational, at least two of its engines must function. Let X denote the event that the ship is operational and let $X_1$, $X_2$, and $X_3$ denote respectively the events that the engines $E_1$, $E_2$, and $E_3$ are functioning. Which of the following is/are true?

work

A. $P\left(\frac{X_1^C}{X}\right) = \frac{3}{16}$

B. P (exactly two engines of the ship are functioning/X) = 7/8

C. $P\left(\frac{X}{X_2}\right) = \frac{5}{16}$

D. $P\left(\frac{X}{X_1}\right) = \frac{7}{16}$

$E_1, E_2, E_3$ (THREE engines)

$$P(X_1) = \frac{1}{2} \quad P(X_2) = \frac{1}{4} \quad P(X_3) = \frac{1}{4}$$

$E_1 \; E_2 \; E_3 = \boxed{E_1 \cap E_2 \cap E_3}$ ✓

$\bar{E}_3 = \boxed{E_1 \cap E_2 \cap \bar{E}_3}$ ✓

$\bar{E}_1 \; \bar{E}_2 \; E_3 = \boxed{E_1 \cap \bar{E}_2 \cap E_3}$ ✓

$\bar{E}_3 = E_1 \cap \bar{E}_2 \cap \bar{E}_3$ ✗

$E_2 \; E_3 = \boxed{\bar{E}_1 \cap E_2 \cap E_3}$

$\bar{E}_3 = \bar{E}_1 \cap E_2 \cap \bar{E}_3$ ✗

$\bar{E}_2 \; E_3 = \bar{E}_1 \cap \bar{E}_2 \cap E_3$ ✗

$\bar{E}_3 = \bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3$ ✗

2 Engines Are working

✓ $E_1 \cap E_2 \cap E_3$

✓ $E_1 \cap E_2 \cap \bar{E}_3$

✓ $E_1 \cap \bar{E}_2 \cap E_3$

✓ $\bar{E}_1 \cap E_2 \cap E_3$

atleast Two Engines working

$X_1 \cap X_2 \cap X_3$ $\quad$ $X_1 \cap \bar{X}_2 \cap X_3$

$X_1 \cap X_2 \cap \bar{X}_3$ $\quad$ $\bar{X}_1 \cap X_2 \cap X_3$

$P(X_2) = \frac{1}{4}$ $\quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{4}$

$\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}$ $\quad \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}$ $\quad \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}$ $\quad \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4}$

✓ $\boxed{X_1 \cap X_2 \cap X_3}$ ✓ $\boxed{\overline{X_1} \cap X_2 \cap X_3}$ $\boxed{X_1 \cap X_2 \cap \overline{X_3}}$ $\boxed{X_1 \cap \overline{X_2} \cap X_3}$

1 $\quad$ 1 $\quad$ 1 $\quad$ 1

fav choice

$\boxed{\text{Ship is operational}}$

$$P\left[\frac{X_1^c}{X}\right] = P\left(\frac{X_1 \text{ is Not working}}{\text{Ship is operational}}\right)$$

$$\Rightarrow \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{1}{32} + \frac{1}{32} + \frac{3}{32} + \frac{3}{32}} = \frac{\frac{1}{32}}{8/32} = \frac{1}{8}$$

$$P\left(\frac{\text{Exactly Two engines}}{X}\right) \Rightarrow \frac{\frac{1}{32}+\frac{3}{32}+\frac{3}{32}}{\frac{8}{32}} = \frac{7}{8}$$

B

Right

$$P\left(\frac{X}{X_2}\right) = \text{conditional Prob}$$

$$= \frac{P(\overline{X} \cap X_2)}{P(X_2)} = \frac{P(\text{ship is operation} \cap X_2 \text{ is working})}{P(X_2 \text{ is working})}$$

$$= \frac{\frac{1}{32}+\frac{3}{32}+\frac{1}{32}}{\frac{1}{4}}$$

$$= \frac{\frac{5}{32}}{\frac{1}{4}} = \frac{20}{32} = \frac{10}{16} = \frac{5}{8}$$

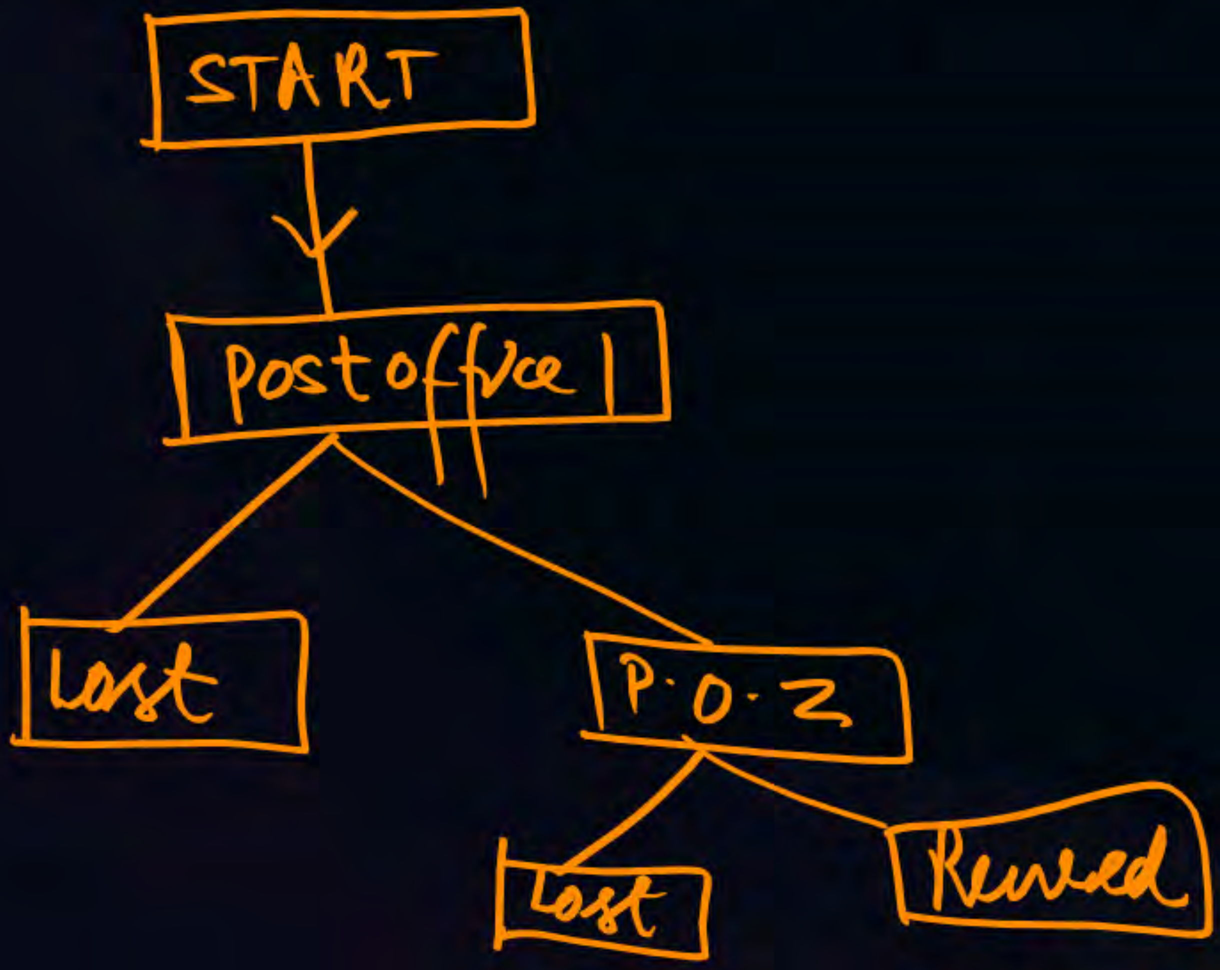
Option C wrong

$$P\left(\frac{X}{X_1}\right) = \frac{P(X \cap X_1)}{P(X_1)} \Rightarrow \frac{7}{8 \times 2} = \frac{7}{16} \text{ Ans}$$

# Topic : Probability Challenging Problem

Q22. Parcels from sender S to receiver R pass sequentially through two post-offices. Each post-offices has a probability 1/5 of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post-offices is _____.

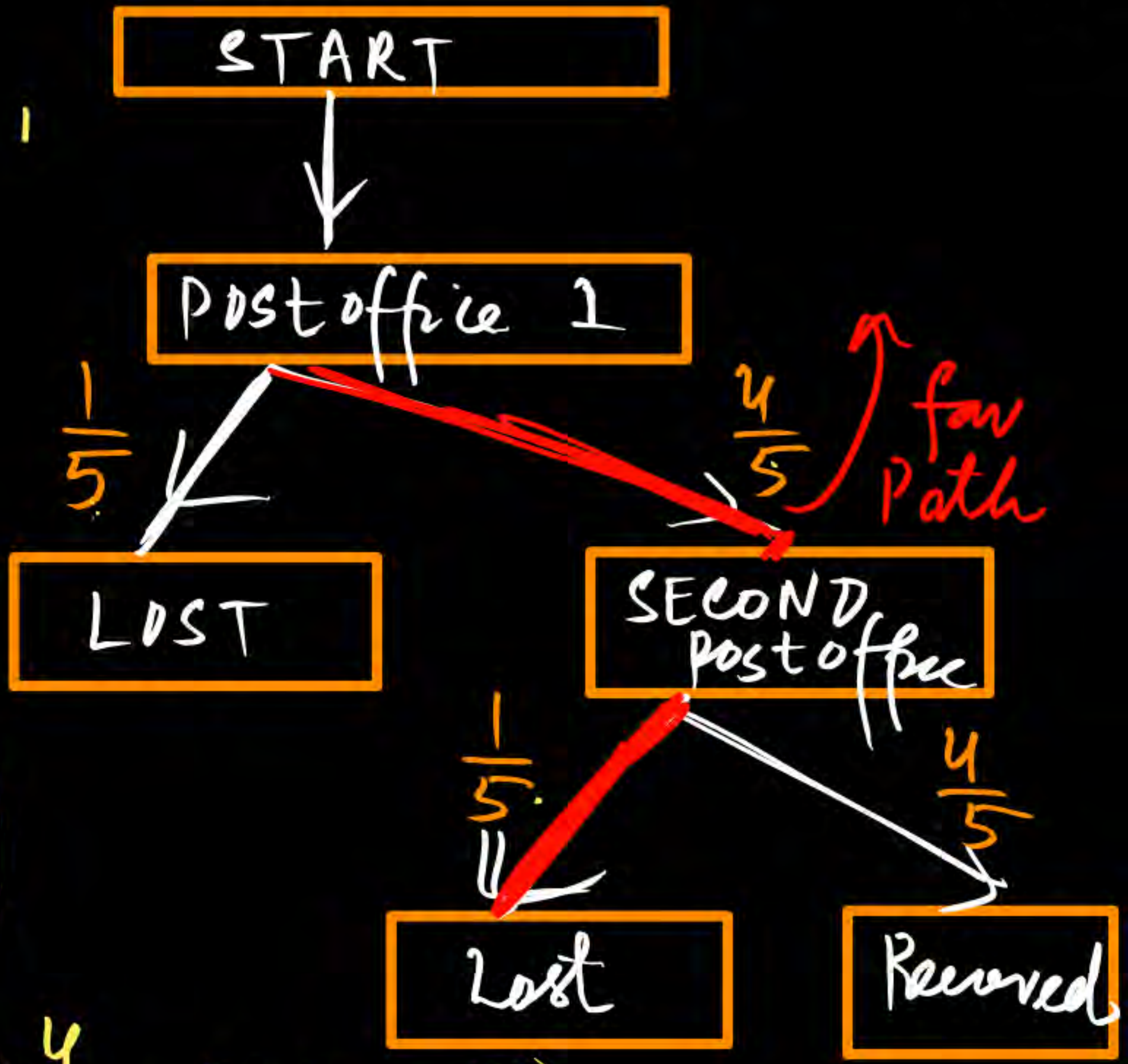
$\frac{4}{9}$ Ans



$$= \frac{\frac{4}{5} \times \frac{1}{5}}{\frac{1}{5} + \frac{4}{5} \times \frac{1}{5}} = \frac{4}{9}$$

$$P\left(\frac{\text{SECOND Post office}}{\text{Lost}}\right) = \frac{4}{25}$$

$$P(\text{Lost}) = \frac{1}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{1}{5} + \frac{4}{25} = \frac{5+4}{25} = \frac{9}{25}$$



# THANK - YOU