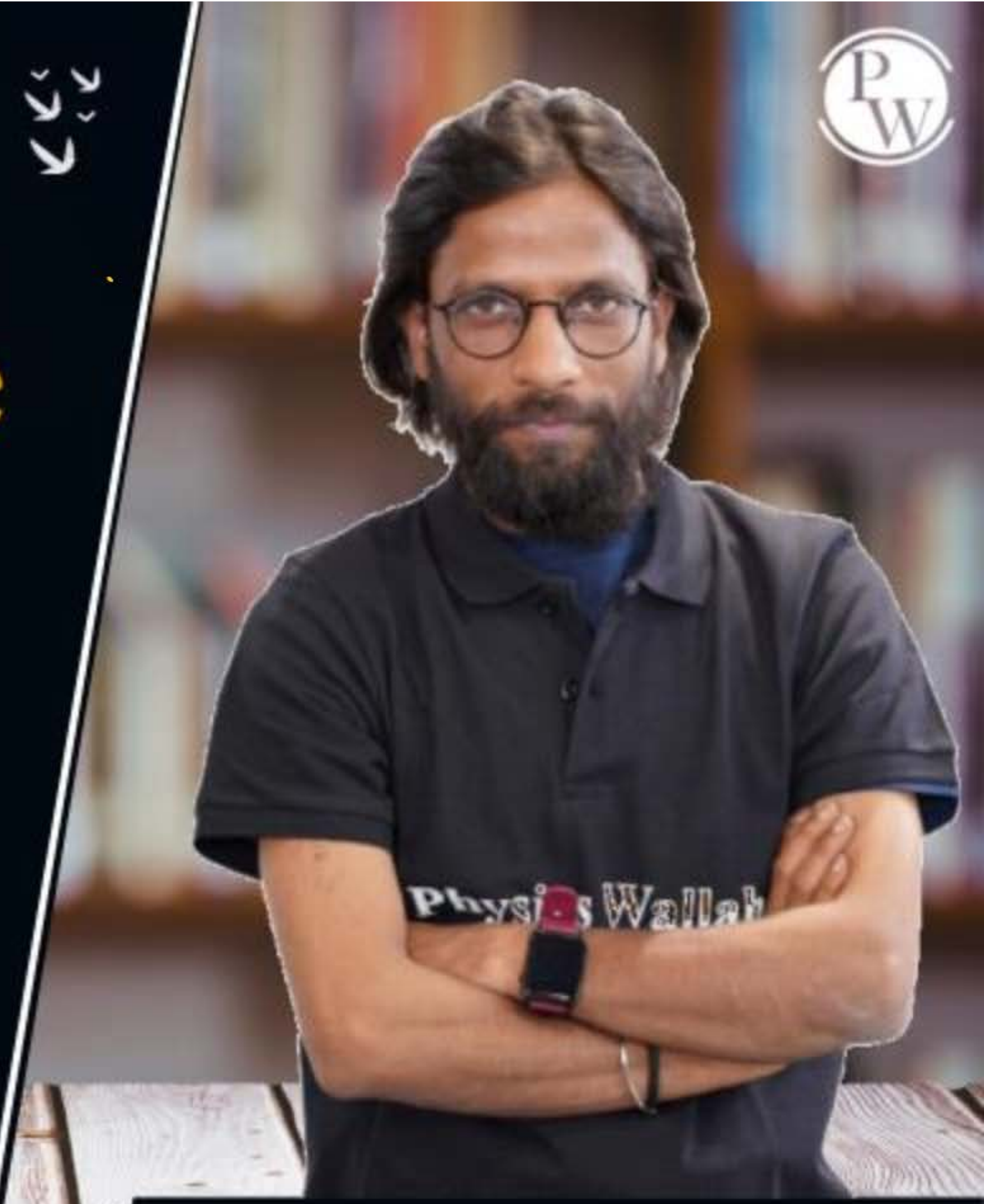
Data Science and Artificial Intelligence

Probability and Statistics

Discrete Probability Distribution

Lecture No.- 07



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Topics to be Covered

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Poisson Distribution

Topic Problem based on discrete random variable part 2

$$P(\mu) = \frac{e^{-\mu}\mu^{x}}{x!}$$

Mean $= \mu$

$var(X) = \mu$

$S.D = \sqrt{\mu}$

$$M_X(s) = e^{-\mu(1-s)}$$

$\boxed{\text{How to Solve It}}$ — George Polya

Probability Distribution

Q1. $X \sim P(\lambda = 1)$, Find $P[X \geq 2 / X \leq 4]$

$\frac{17}{65}$ ✓

$\lambda \to \mu$

MEAN

X is a Discrete Random var.

$X \sim P(\lambda = 1)$ $\lambda = 1$ (mean) $= \mu$

$$P\left[\frac{X \geq 2}{X \leq 4}\right] = \frac{P[X \geq 2 \cap X \leq 4]}{P[X \leq 4]}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P[X \geq 2 \cap X \leq 4]}{P[X \leq 4]}$$

$$P[X = x] = \frac{e^{-\lambda t} \cdot (\lambda t)^x}{x!}$$

$\lambda t = \mu$ mean

$$P[X = x) = \frac{e^{-\mu} \mu^x}{x!}$$

Slide 3

$$P\left(\frac{X \geq 2}{X \leq 4}\right) = \frac{P[X \geq 2 \wedge X \leq 4]}{P[X \leq 4]}$$

$\lambda = 1$

$$P[X \geq 2 \wedge X \leq 4] = \frac{P(X=2)+P(X=3)+P(X=4)}{P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)}$$

✓ $\lambda = 1$

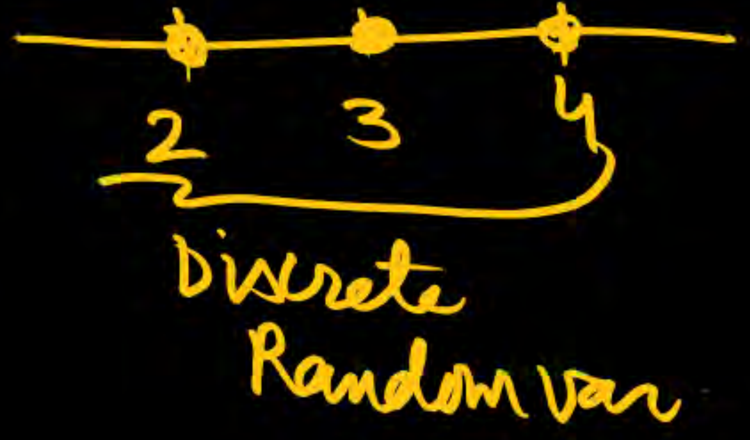
$$= \frac{\frac{e^{-1}(1)^2}{2!}+\frac{e^{-1}(1)^3}{3!}+\frac{e^{-1}(1)^4}{4!}}{\frac{e^{-1}(1)^0}{0!}+\frac{e^{-1}(1)^1}{1!}+\frac{e^{-1}(1)^2}{2!}+\frac{e^{-1}(1)^3}{3!}+\frac{e^{-1}(1)^4}{4!}}$$

$$= \frac{17}{65}$$

We Know That

$$P[X=x] = P[X=x] = \frac{e^{-\lambda}\lambda^x}{x!}$$

λ = MEAN



$\boxed{0.9817}$

Level
Poisson Distribution

Q2. Assume that the number of hits, X $\boxed{\text{per}}$ baseball game, has a Poisson distribution. If the probability of a no-hit game is $\frac{1}{10{,}000}$, find the probability of having $\boxed{\text{4 or more}}$ hits in a particular game.

$P[X \geq 4]$

P(Non Hit) $= \frac{1}{10000}$

$P(X=0) = \frac{1}{10000}$

No Hit

$e^{-\lambda} = \frac{1}{10000}$

$\lambda = \ln[10000]$

$\frac{e^{-\lambda}(\lambda)^0}{0!} = \frac{1}{10000}$

Using Poisson Distribution formula

$\mu = \lambda$

Mean $= \lambda$

$e^{x} = A$

$x = \ln(A)$

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$P[X = \text{No Hit}] = \frac{1}{10{,}000}$$

$$\frac{e^{-\lambda}(\lambda)^{0}}{0!} = \frac{1}{10{,}000}$$

$$e^{-\lambda} = \frac{1}{10000}$$

$$\Rightarrow e^{\lambda} = 10000 \qquad \boxed{\lambda = \ln(10000)} \quad \text{MEAN}$$

$$P[X \geq 4] = 1 - \left[P[X=0] + P[X=1] + P[X=2] + P[X=3]\right]$$

$$= 1 - \left[\frac{e^{-\lambda}(\lambda)^{0}}{0!} + \frac{e^{-\lambda}(\lambda)^{1}}{1!} + \frac{e^{-\lambda}(\lambda)^{2}}{2!} + \frac{e^{-\lambda}(\lambda)^{3}}{3!}\right] \Rightarrow \ln$$

$\lambda = (10000)$

$$\boxed{P[X = n+1] = P[X = n] \cdot \frac{\lambda}{(n+1)}}$$

$$= 1 - \left[\frac{e^{-\ln(10000)}}{0!} + \frac{e^{-\ln 10000} \cdot (\ln 10000)}{0+1} + \frac{e^{-\ln 10000} \cdot (\ln(10000))^{2}}{2} + - -\right]$$

$$= 0.9817$$

Probability Distribution

e^{10} ✓

Poisson

per 1 week = 3
2 week = 3×2 = 6

Q3. The number of traffic accident per week in a small city has a Poisson distribution with mean equal to 3. What is the probability of exactly 2 accidents occurs in 2 weeks?

$$P[x = 2\text{ Accident}] = \frac{e^{-\lambda}\lambda^2}{2!} \qquad \lambda = 6$$

$$= \frac{e^{-6}(6)^2}{2!} = \boxed{18e^{-6}} \text{ Ans}$$

$$= 0.044$$

Q4. Is the real valued function defined by $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$, x = 0, 1, 2, ... ∞, where $0 < \lambda < \infty$ is a parameter, a probability density function?

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, 2 \dots \infty$$

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \dots\right] = 1 \text{ valid pdf}$$

$$= e^{-\lambda}e^{\lambda} = e^{-\lambda+\lambda} = e^0 = 1$$

$12e^{-3}$

$\mu = \lambda = 3$

Q5. A random variable X has a Poisson distribution with a mean of 3.

Find $P(1 \le X \le 3)$ = ...

$P[1 \le x \le 3]$

$$P[1 \le x \le 3] = P[x=1] + P[x=2] + P[x=3]$$

$$= \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^3}{3!}$$

$$= 12e^{-3}$$

$154+2=156$

$\lambda=4$

Q6. On his tour of Jaipur, the number of selfies Rishabh takes per day is modelled by a Poisson distribution with mean 4. The number of selfies taken on different days are mutually independent. His trip lasts for three days. Calculate the second moment of the number of selfies $\boxed{\text{Rishabh}}$ takes for his entire trip.

$\lambda=4$ | $\lambda=4$ | $\lambda=4$

Day 1 | Day 2 | Day 3

mean (3 days) $= 4\times3=12$

$\lambda=12$

$var(X)=12$

$V(X)=E[X^2]-[E[X]]^2$

$V(X)+[E[X]]^2=E[X^2]$

$=12+(12)^2=E[X^2]$

$=\boxed{156=E[X^2]}$

$6!e^{-6} \Rightarrow \underline{0.1512}$

Q7. The number of calls $\boxed{\text{per minutes}}$ a service centre receives follows a Poisson distribution $\boxed{\text{with mean 0.3.}}$ The numbers of calls in different minutes are independent. Calculate the probability that fewer than four calls are received in $\boxed{\text{20 minutes.}}$

$\lambda = 20 \times 0.3 = 6$

$$P[X<4] = P[X=0] + P[X=1] + P[X=2] + P[X=3]$$

$$= \sum_{x=0}^{3} \frac{e^{-\lambda}(\lambda)^x}{x!} = \sum_{x=0}^{3} \frac{e^{-6}(6)^x}{x!} = 6!e^{-6}$$

$$= \underline{0.1512}$$

Q9. Let X be a Poisson random variable mean λ. If P [X = 1 | X ≤ 1] = 0.8.

What is the value of λ?

A. 4

B. –ln2

C. 0.8

D. 0.25

$$P\left[\frac{X=1}{X\le 1}\right] = 0.8$$

find λ

$$P\left[\frac{X=1}{X\le 1}\right] = \frac{P(X=1 \wedge X\le 1)}{P(X\le 1)}$$

$$= \frac{P(X=1)}{P(X=0)+P(X=1)}$$

$$= \frac{\frac{e^{-\lambda}\lambda^1}{1!}}{\frac{e^{-\lambda}\lambda^0}{0!}+\frac{e^{-\lambda}\lambda^1}{1!}} = \frac{\cancel{e^{-\lambda}}}{\cancel{e^{-\lambda}}}\left[\frac{\lambda}{1+\lambda}\right] = 0.8$$

$$\boxed{\lambda = 4}$$

Q10. Let X have the Poisson distribution with mean $\lambda = 1$. What is the $p[X \geq 2/X \leq 4]$

A. $\frac{5}{65}$

B. $\frac{5}{41}$

C. $\frac{17}{65}$

D. $\frac{17}{41}$

$$P\left[\frac{X \geq 2}{X \leq 4}\right] = \frac{P[X \geq 2 \wedge X \leq 4]}{P[X \leq 4]}$$

for $\lambda = 1$

$$= \frac{17}{65}$$

Probability Distribution

$P[X=2] = 3P[X=4]$
$\downarrow$ 2 com. $\swarrow$ 4 comp.

Q11. Teacher has discovered that students are three times as likely to file two complaints as to file four complaints. The number of complaints ~~filed~~ filled has a Poisson distribution. Calculate the variance of the number of complaints filed.

$P[X=2] = 3P[X=4]$

$$\frac{\cancel{e^{-\lambda}}(\lambda)^2}{2!} = 3\frac{\cancel{e^{-\lambda}}(\lambda)^4}{4!}$$

$$= \frac{\lambda^2}{2} = 3\frac{\lambda^4}{4!}$$

A. $\frac{1}{\sqrt{3}}$

B. 1

C. $\sqrt{2}$

D. 2

$$\frac{\lambda^2}{2} = \cancel{3}\cdot\frac{\lambda^4}{4\times 3\times 2\times 1}$$

$$\frac{\lambda^2}{\cancel{2}} = \frac{\lambda^4}{8_2}$$

$$= 4\lambda^2 - \lambda^4 = 0$$

$$\Rightarrow \lambda^2(4-\lambda^2) = 0$$

$\lambda^2 = 0$ $\quad$ $\lambda^2 = 4$

$\boxed{\lambda = 0, 0}$ $\quad$ $\boxed{\lambda = +2}$

Ans

Poisson distribution – mean/var/S.D $\longrightarrow$ +

$P[X=x] = \frac{\lambda}{x^2(x+1)}$

$E[X] =$

Q12. $\boxed{\text{Let } P(X = x) = \frac{\lambda}{x^2(x+1)}}$, where λ is an appropriate constant. Then E(X) is

A. $2\lambda + 1$

B. λ

C. ∞

D. 2λ

$$E[X] = \sum_{x=0}^{\infty} x_i \, P(x_i)$$

in Discrete Random var.

$$= \sum_{x=0}^{\infty} \frac{\lambda}{x^2(x+1)} \cdot x = \sum_{L=0}^{\infty} \lambda \cdot \frac{x}{x^2(\cancel{x}+1)}$$

$$= \lambda \sum_{x=0}^{\infty} \boxed{\frac{1}{x(x+1)}}$$

$$\frac{1}{\underbrace{x(x+1)}_{}} = \frac{A}{x} + \frac{B}{(x+1)}$$

Product Term

$+, -$

Using Partial Fractions

$$A = \frac{1}{(x+1)}\bigg|_{x=0} = 1$$

$$B = \frac{1}{x}\bigg|_{x=-1} = -1$$

$$\frac{1}{L_1(x)\,L_2(x)} = \frac{A}{L_1(x)} + \frac{B}{L_2(x)}$$

$$A = \frac{1}{[L_2(x)]}$$

$$B = \left[\frac{1}{L_1(x)}\right]$$

$$\frac{1}{x(x+1)} = \left[\frac{1}{x} - \frac{1}{(x+1)}\right]$$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{(x+1)}$$

$$A = \frac{1}{(x+1)}\bigg|_{x=0}$$

$$B = \frac{1}{x}\bigg]_{x=-1}$$

0 – remove

SERIES

Undefined

$$= \lambda \sum_{x=0}^{\infty} \left[\frac{1}{x} - \frac{1}{(x+1)}\right]$$

$$= \lambda \sum_{x=1}^{\infty} \left[\left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \cancel{-} - \cancel{-}\right]$$

$$= \lambda \times 1$$

$$= \lambda$$

Probability Distribution

Q14. A random variable X has poisson distribution.

if 2P(X = 2) = P(X = 1) + 2P (X = 0), then the variance of X is

A. $\frac{3}{2}$

B. $\boxed{2}$

C. 1

D. $\frac{1}{2}$

$$2P[X=2] = P[X=1] + 2P[X=0]$$

$$\cancel{2}\frac{e^{-\lambda}(\lambda)^2}{\cancel{2}!} = \frac{e^{-\lambda}(\lambda)^1}{1!} + \frac{2e^{-\lambda}(\lambda)^0}{0!}$$

$$\lambda^2 e^{-\lambda} = e^{-\lambda}\lambda^1 + 2e^{-\lambda}$$

$$\lambda^2 = \lambda + 2$$

$$\lambda^2 - \lambda - 2 = 0$$

$$\lambda^2 - \lambda - 2 = 0$$

$$\lambda = \frac{+1 \pm \sqrt{1+8}}{2}$$

$$\lambda = \frac{1 \pm \sqrt{9}}{2}$$

$$= \frac{1 \pm \sqrt{9}}{2}$$

$$= 2, -1$$

$$\lambda = 2, -1$$

$P(\lambda) > 0$

Slide 16

Q15. The number of calls coming per minutes into a customer call centre is Poisson random variable with mean 5. Assume that the number of calls arriving in two different minutes are independent. What is the probability that at least two calls will arrive in a given period of two minutes?

A. 11^{e-10}

B. $1 - 10e^{-10}$

C. $1 - 11e^{-10}$

D. None of these

$\lambda = 5$ $\lambda = 5$

0 A B C

combined mean $= (\lambda_1 + \lambda_2)$

$= 5 + 5 = 10$

$\lambda = 10$

$P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$= 1 - \frac{e^{-\lambda}(\lambda)^0}{0!} - \frac{e^{-\lambda}(\lambda)^1}{1!}$

$\lambda = 10$

$P(X \geq 2) = 1 - 11e^{-10}$

Do Yourself

Q16. The number of misprints per page of a book (X) follows the Poisson distribution such that $P(X = 1) = P(X = 2)$. If the book contains 500 pages, the expected number of pages containing at most one misprint is

A. $500e^{-2}$

B. $1000e^{-2}$

C. $1500e^{-2}$

D. $500(1 - 3e^{-2})$

Probability Distribution

Do yourself

Q17. A certain kind of sheet metal has, no the average 2 defects per 5 square-foot. It is assumed that the number of defects follows the Poisson distribution. Then the probability that a 10 square-foot sheet of the metal will have at most two defect is

A. $5e^{-2}$

B. $2e^{-2}$

C. $6e^{-4}$

D. $\boxed{13e^{-4}}$

Slide 19

Q21. A fair die is tossed until a 2 is obtained. If X is the number of trials required to obtain the first 2 What is the smallest value of x for which $p[X \leq x]$?

A. 2

B. 3

C. 4

D. 5

Q22. Two people take turns rolling a fair die. Person X rolls first, then Person Y, then X, and so on. The winner is the first one to roll a 6. What is the probability that person X wins?

A. $\frac{5}{11}$

B. $\frac{1}{2}$

C. $\frac{6}{11}$

D. $\frac{3}{5}$

Probability Distribution

Q23. If a random variable X assume only positive integral values, with the probability $P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}$, x = 1, 2, 3, …. , then E[x] is

A. $\frac{2}{9}$

B. $\frac{2}{3}$

C. 1

D. $\frac{3}{2}$

geometric Distribution

$\text{mean} = \frac{1}{p} = \frac{1}{2/3} = \frac{3}{2}$

$= 1.5$

$P(X=1) = \frac{2}{3}\left(\frac{1}{3}\right)^0$

$P(X=2) = \frac{2}{3}\left(\frac{1}{3}\right)^1$

$= \frac{2}{3}\left(\frac{1}{3}\right)^2$

$= \frac{2}{3}\left(\frac{1}{3}\right)^3$

JAM-Statistics.

0.086

Q24. Let X be a Geom (0.4) random variable. Then P(X = 5| X ≥ 2) = ……… .

$p = 0.4$

$P\left(\frac{X=5}{X\geq 2}\right) = \frac{P(X=5 \cap X\geq 2)}{P(X\geq 2)} = \frac{(0.4)(0.6)^{5-1}}{1-P[X=1]}$

geometric distribution $P(X=5) = (0.4)(0.6)^{5-1} = (0.4)(0.6)^4$

$=$

$$\begin{cases} P(X\geq 2) = P(X=2) + P(X=3) + \cdots \\ P(X\geq 2) = 1 - P(X=1) \end{cases}$$

$$P\left(\frac{X=5}{X\geq 2}\right) = \frac{P(X=5)}{1-P(X=1)} = \frac{0.0864}{}$$

THANK - YOU