Analysis of the placement of Marine Protected Areas and their efficiency to safeguard Whale Shark populations

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1. **Introduction/Rationale**:

Whale sharks, the largest living fish species, are known as megafauna and serve as indicators of the health of our oceans. The conservation of megafauna, such as whale sharks, has become an growing aspect of global efforts to preserve marine ecosystems. In the face of increasing anthropogenic threats, such as climate change and overfishing, the establishment and optimization of MPAs emerge as strong prospective solutions to preserving these species and the ecosystems they inhabit. As an effort to protect these endangered gentle giants, this investigation delves into the placement of Marine Protected Areas (MPAs) that safeguard habitats of whale sharks, and their efficiency in protecting these species using mathematical analysis and interpretation of data.

2. **Aim**:

The aim of this investigation is to employ mathematical methodologies, including the largest empty circle methodology within Voronoi diagrams, to identify optimal locations for the establishment of a new Marine Protected Area (MPA). This approach considers the natural distribution of whale sharks as aggregation sites recorded and currently existing MPAs around the world. An exponential growth model and regression is used to test the efficiency of these Marine Protected Areas as a solution to preserving populations of Whale Sharks. Also, graph theory, specifically the travelling salesman problem is incorporated to optimize a route for an on-site inspection of each of aggregation sites in East Africa for future research purposes to gather more data for further optimization of MPAs.

3. **Exploration**:

Mapping Existing MPAs

The exploration phase of this study initiated with the acquisition of data from *MPAtlas*, focusing on very large Marine Protected Areas (MPAs) spanning 100,000 km² or more. The rationale behind this criterion lies in the emerging trend of designating larger MPAs that cover expansive open ocean regions. These extensive protected areas play a crucial role in safeguarding large-scale marine ecosystems, particularly critical habitats of marine megafauna such as the majestic whale shark.

Using the geographic information obtained from MPAtlas, the locations of these substantial MPAs were meticulously plotted on a world map derived from *Google Maps*. To facilitate this visual representation, Geogebra was employed, leveraging its functions to overlay geographical data onto the world map.

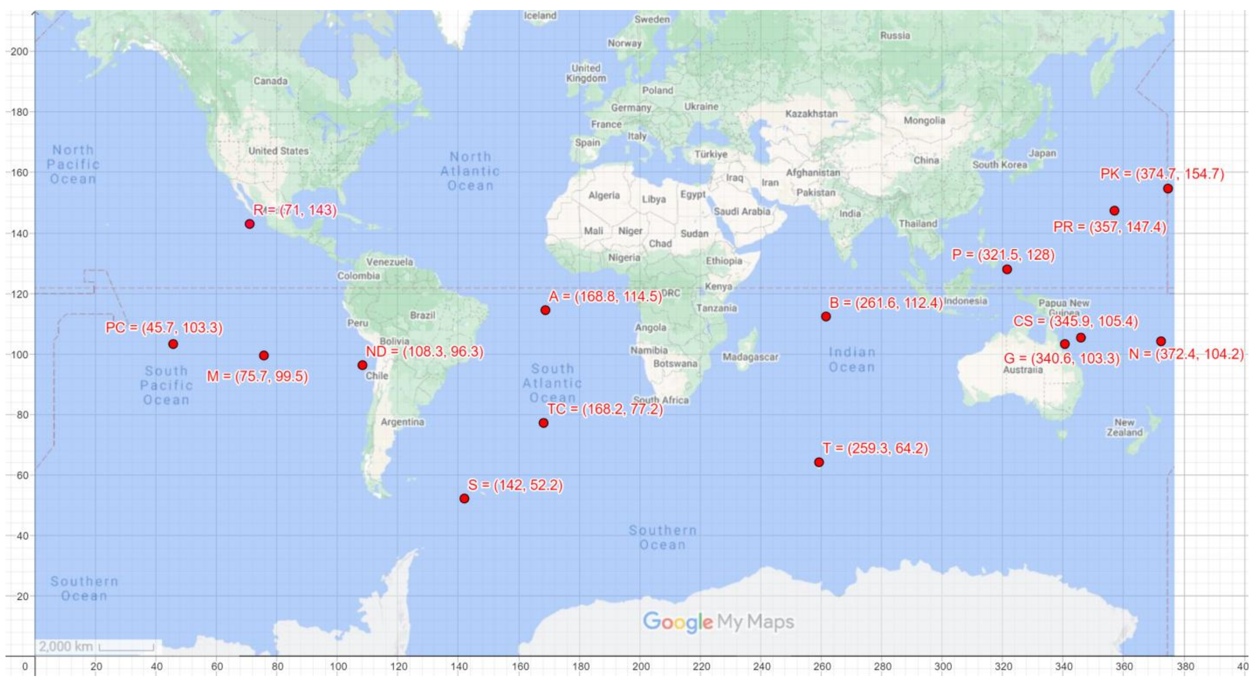

Fig. 1 Map (derived from *Google Maps*) with plotted existing MPAs on *Geogebra*

Table 1 MPAs and corresponding site name and coordinates

MPA Name	Site Name	Site Coordinates
Revillagigedo	R	(71, 143)
Motu Motiro Hiva	M	(75.7, 99.5)
Pitcairn	PC	(45.7, 103.3)
Nazca-Desventuradas	ND	(108.3, 96.3)
Ascension Exclusive Economic Zone	A	(168.8, 114.5)
Tristan da Cunha	TC	(168.2, 77.2)
South Georgia and South Sandwich Islands	S	(142, 52.2)
British Indian Ocean Territory MPA	B	(261.6, 112.4)
Terres Australes Françaises	T	(259.3, 64.2)
Palau	P	(321.5, 128)
Great Barrier Reef	G	(340.6, 103.3)
Coral Sea	CS	(345.9, 105.4)
Pacific Remote Islands	PR	(357, 147.4)
Niue Moana Mahu	N	(372.4, 104.2)
Papahānaumokuākea	PK	(374.7, 154.7)

Aggregation Sites Mapping

Aggregation sites denote locations where the whale sharks gather in significant numbers for specific activities such as feeding or mating. In the context of this investigation, the mapping of aggregation sites is crucial as it sheds light on the spatial distribution of key habitats where populations of these whale sharks peak. By incorporating this data, areas are identified where the establishment of new Marine Protected Areas (MPAs) could contribute to the preservation of critical habitats. (Data derived from *Copping, Joshua P., et al*.)

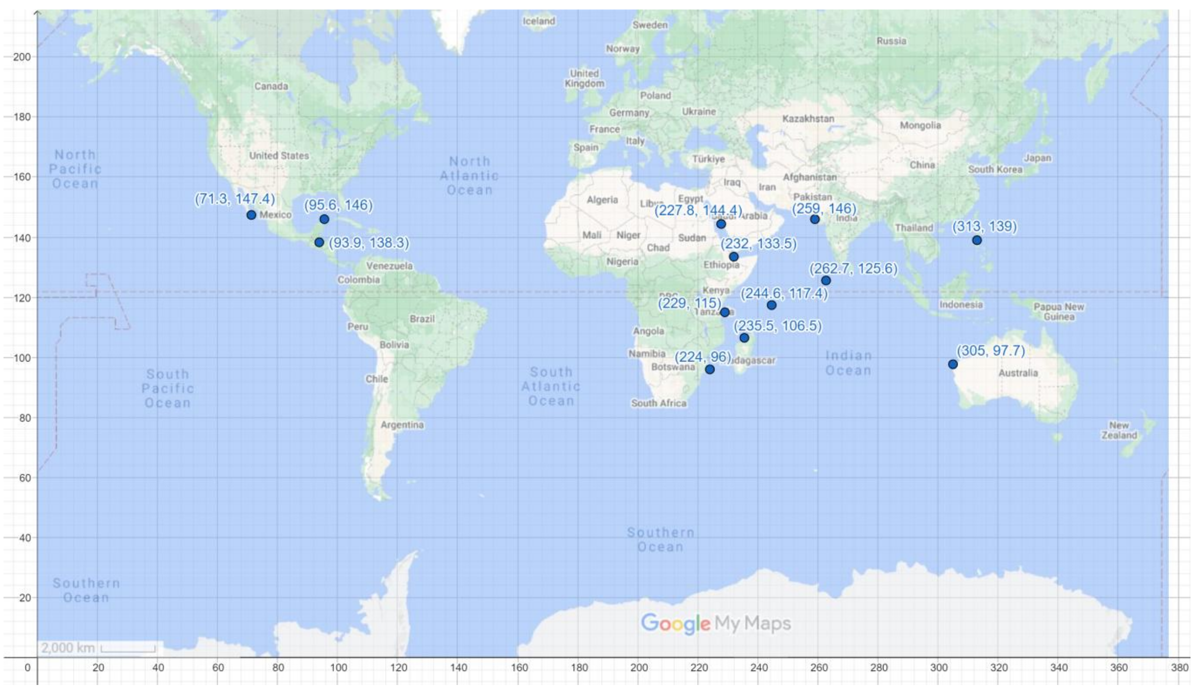

Fig. 2. Map (derived from *Google Maps*) with plotted Whale Shark aggregation sites on *Geogebra*

3.1 Analysis of the Placement of a new Marine Protected Area

Assessing Placement of new MPA from a glance

With the mapped data of whale shark aggregation sites and the locations of existing large Marine Protected Areas (MPAs), the analysis now shifts towards identifying strategic opportunities for the addition of a new MPA. To choose a site for the new MPA, it is needed to take into consideration a region from which most sites of aggregation would benefit from as this is where most whale sharks are observed to be found. This can be done on observation of the map depicting aggregation sites and marine protected areas. Since the most number of aggregation sites in close proximity (3) without any presence of an MPA are in the circled region marked in Fig. 3. below, it is sensible to make it a new MPA for the purpose of safeguard whale shark populations.

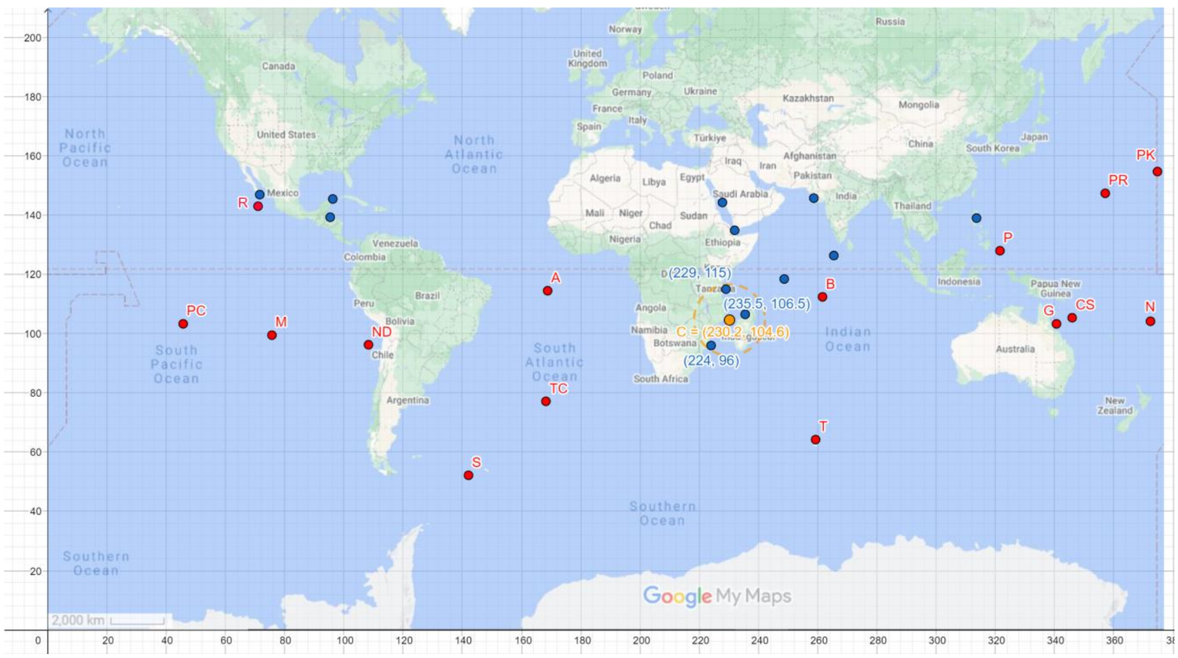

Fig. 3. Map (derived from *Google Maps*) with both MPAs and aggregation sites plotted on *Geogebra*

The assumed new MPA site is localized with all other sites on the map above. The detailed coordinates of the new MPA site C in the Indian Ocean on the West Coast of Madagascar are presented in the table below.

MPA Name	Site Name	Site Coordinates
Assumed new MPA Site	C	(230.2, 104.6)

According to the observations of the aggregation sites of the whale sharks, site C is the best place to designate a new MPA. However, other methods can be used to consider the optimal place for the new MPA. One of them is constructing a Voronoi Diagram and looking for the center of the largest empty circle, which will be the place most distant from every other MPA site on the world map.

Assessing Placement of new MPA with the help of Voronoi Diagrams (Largest Empty Circle)

To determine which MPA a given whale shark from an aggregation site would most benefit from, a Voronoi diagram is needed to be constructed. From such a diagram, it is known that each point is closest to the site of the cell it is in. When a point lies on the edge of two cells, it is equidistant from both of them. Every cell of the Voronoi diagram can be constructed by finding the perpendicular bisectors of each neighboring site. Since the greatest precision of calculations is not important in the given situation, all results will be rounded to one decimal place.

To determine the equation of a perpendicular bisector of two given points it is needed to find its gradient, as well as the midpoint of these points. The calculations to find the equation of the edge of the cells containing R (71, 143) and PC (45.7, 103.3) are presented below.

$$\text{Midpoint of [R-PC]} = \frac{x1+x2}{2}, \frac{y1+y2}{2}$$

$$= \left(\frac{71 + 45.7}{2}, \frac{143 + 103.3}{2}\right) \approx (58.4, 123.2)$$

The slope of the perpendicular bisector is the negative reciprocal of the slope of the line segment connecting points R and PC. Hence, the gradient of [R-PC] is found first.

$$\text{Gradient m of [R-PC]} = \frac{y2-y1}{x2-x1} = \frac{103.3-143}{45.7-71} = \frac{397}{253} \approx 1.6$$

$$\text{Gradient of the perpendicular bisector of [R-PC]} = -\frac{1}{m} = -\frac{253}{397} \approx -0.6$$

To determine the equation of the perpendicular bisector of [R-PC] in the form of $y = mx + c$, it is needed to find c. To do so, we substitute y and x from midpoint and m of the perpendicular bisector.

$$y = mx + c$$

$$123.2 = -0.6 \times 58.4 + c$$

$$c \approx 158.2$$

Therefore the equation of the perpendicular bisector of [R-PC] is

$$y = -0.6x + 158.2$$

These steps were repeated for each pair of sites to determine the equations of all edges of the cells in the Voronoi Diagram. The calculations to find midpoints and gradients were carried out in an Excel spreadsheet. The results can be seen in the Table below.

Table 2. Finding perpendicular bisectors for respective sites for formation of Voronoi Diagram

Cell edge	$x1$	$y1$	$x2$	$y2$	m	$-\frac{1}{m}$	$\frac{x1+x2}{2}$	$\frac{y1+y2}{2}$	c	$y = mx + c$
PC-R	45.7	103.3	71	143	1.6	-0.6	58.4	123.2	158.2	$y = -0.6x + 158.2$
PC-M	45.7	103.3	75.7	99.5	-0.1	7.9	60.7	101.4	-377.8	$y = 7.9x - 377.8$
R-M	71	143	75.7	99.5	-9.3	0.1	73.4	121.3	113.3	$y = 0.1x + 113.3$
R-ND	71	143	108.3	96.3	-1.3	0.8	89.7	119.7	48.0	$y = 0.8x + 48$
R-A	71	143	168.8	114.5	-0.3	3.4	112	128.8	-282.7	$y = 3.4x - 282.7$
M-ND	75.7	99.5	108.3	96.3	-0.1	10.2	92	97.9	-839.4	$y = 10.2x - 839.4$
M-S	75.7	99.5	142	52.2	-0.7	1.4	108.9	75.9	-76.7	$y = 1.4x - 76.7$
ND-A	108.3	96.3	168.8	114.5	0.3	-3.3	138.6	105.4	565.9	$y = -3.3x + 565.9$
ND-TC	108.3	96.3	168.2	77.2	-0.3	3.1	138.3	86.8	-346.8	$y = 3.1x - 346.8$
ND-S	108.3	96.3	142	52.2	-1.3	0.8	125.2	74.3	-21.4	$y = 0.8x - 21.4$
A-TC	168.8	114.5	168.2	77.2	62.2	-0.01	168.5	95.9	98.5	$y = -0.01x + 98.5$
A-B	168.8	114.5	261.6	112.4	-0.02	44.2	215.2	113.5	-9396.3	$y = 44.2x - 9396.3$
TC-S	168.2	77.2	142	52.2	0.9	-1.04	155.1	64.7	227.2	$y = -1.04x + 227.2$
TC-T	168.2	77.2	259.3	64.2	-0.1	7.0	213.8	70.7	-1427.2	$y = 7x - 1427.2$
TC-B	168.2	77.2	261.6	112.4	0.4	-2.7	214.9	94.8	665.0	$y = -2.7x + 665$
B-T	261.6	112.4	259.3	64.2	20.9	-0.04	260.5	88.3	100.7	$y = -0.04x + 100.7$
B-P	261.6	112.4	321.5	128	0.3	-3.8	291.5	120.2	1239.6	$y = -3.8x + 1239.6$
B-G	261.6	112.4	340.6	103.3	-0.1	8.7	301.1	107.9	-2506.1	$y = 8.7x - 2506.1$
T-S	259.3	64.2	142	52.2	0.1	-9.7	200.7	58.2	2019.6	$y = -9.7x + 2019.6$
T-G	259.3	64.2	340.6	103.3	0.5	-2.1	300	83.8	707.4	$y = -2.1x + 707.4$
P-PR	321.5	128	357	147.4	0.5	-1.8	339.3	137.7	758.5	$y = -1.8x + 758.5$
P-CS	321.5	128	345.9	105.4	-0.9	1.08	333.7	116.7	-243.6	$y = 1.08x - 243.6$
P-G	321.5	128	340.6	103.3	-1.3	0.8	331.1	115.7	-140.3	$y = 0.8x - 140.3$
G-CS	340.6	103.3	345.9	105.4	0.4	-2.5	343.3	104.4	970.6	$y = -2.5x + 970.6$
G-N	340.6	103.3	372.4	104.2	0.02	-35.3	356.5	103.8	12700.0	$y = -35.3x + 12700$
CS-PR	345.9	105.4	357	147.4	3.8	-0.3	351.5	126.4	219.3	$y = -0.03x + 219.3$
CS-N	345.9	105.4	372.4	104.2	-0.04	22.08	359.2	104.8	-7826.4	$y = 22.08x - 7826.4$
PR-PK	357	147.4	374.7	104.2	-2.4	0.4	365.9	125.8	-24.1	$y = 0.4x - 24.1$
PR-N	357	147.4	372.4	104.2	-2.8	0.4	364.7	125.8	-4.2	$y = 0.4x - 4.2$
N-PK	372.4	104.2	374.7	104.2	0	0	373.6	104.2	104.2	$y = 104.2$

After calculating the equations of each perpendicular bisector, they were graphed on the world map with the MPAs. Then, the Voronoi diagram is constructed using parts of the perpendicular bisectors. The complete diagram can be seen below:

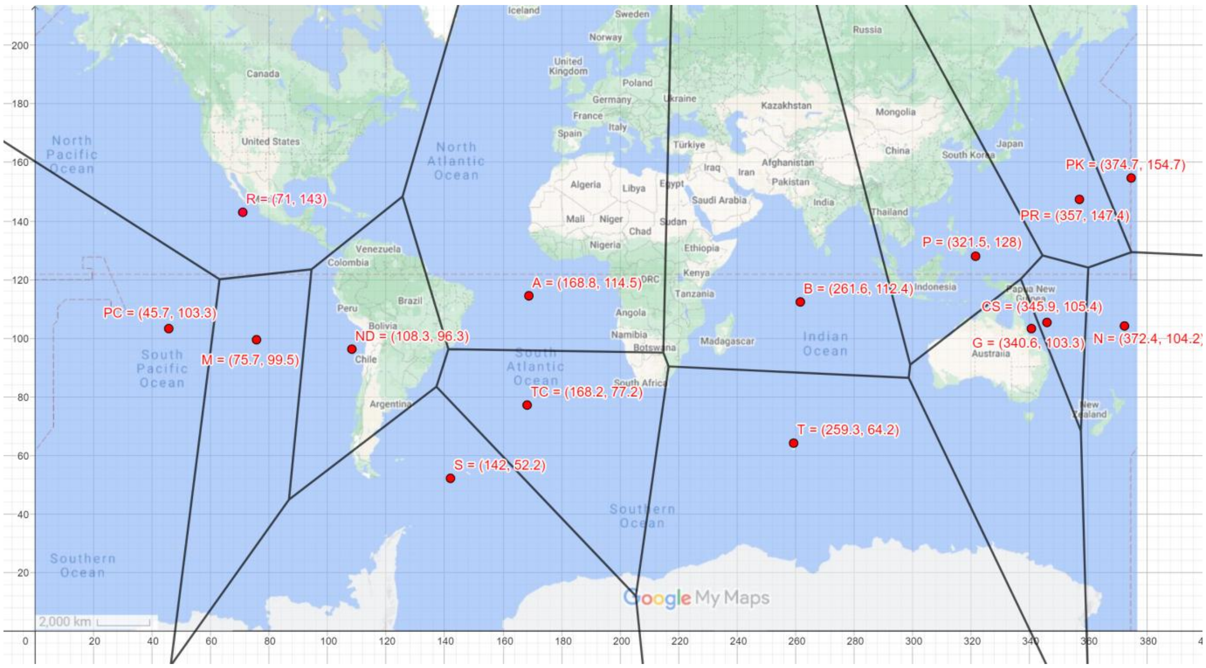

Fig. 4. Voronoi Diagram depicting sites surrounding existing MPAs (*Geogebra*)

After the Voronoi diagram is constructed, a place for the new MPA to be designated is needed to be found. One way of finding the place most distant from every other site is to find the center of the Largest Empty Circle. This approach is applicable because the sites form a convex hull, a prerequisite for addressing the Largest Empty Circle problem.

The Largest Empty Circle in the diagram can be centered on one of the three intersections of selected edges of the Voronoi diagram. The largest 3 circles on observation have been marked in the Figure below:

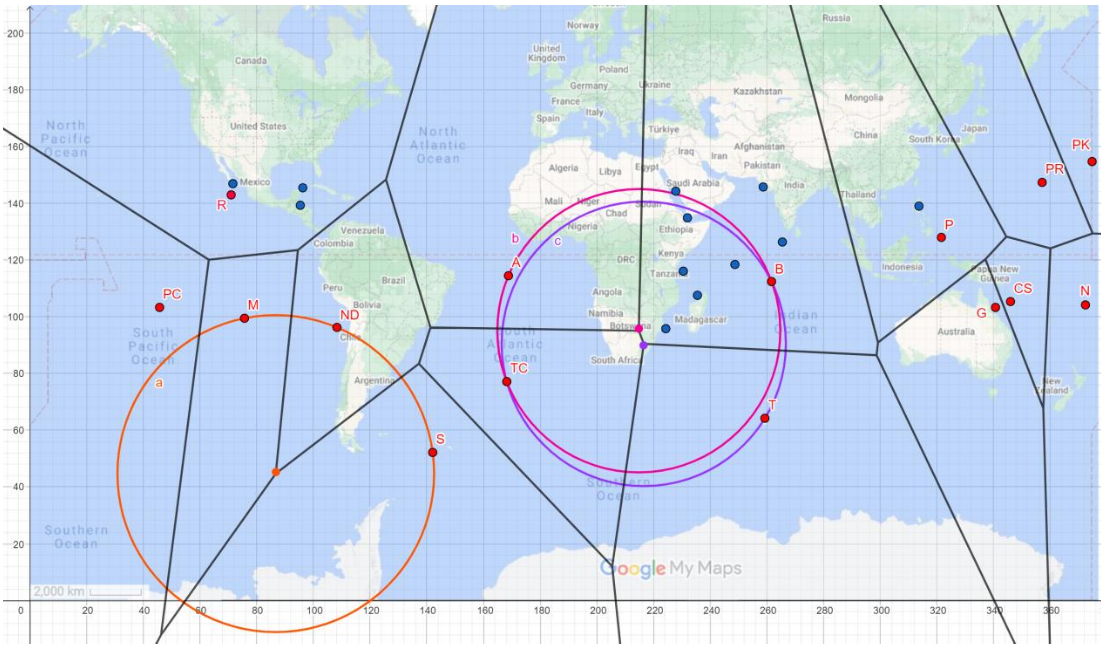

Fig. 5. Centres of three largest empty circles plotted (Circle a M-ND-S, Circle b TC-A-B, Circle c TC-T-B) on *Geogebra*

In order to determine which of the three empty circles is the largest, it is necessary to compute the radius of each circle. Firstly, the coordinates of the center of each circle must be identified. To find the x-coordinate of the intersections of lines of the Voronoi Diagram at point a, the equation of lines is considered:

$$10.2x - 839.4 = 0.8x - 21.4 \qquad x \approx 87.02$$

After that, the y-coordinate can be found:

$$y = 10.2(87.02) - 839.4 \approx 48.2$$

The coordinates of the intersection *a* are (87.02, 48.2)

Similarly, the coordinates for intersections b and c are found:

For intersection b, $\quad 44.2x - 9396.3 = -0.01x + 98.5 \qquad x \approx 214.8$

$$y = -0.01(214.8) + 98.5 \approx 96.4$$

The coordinates of the intersection *b* are (214.8, 96.4)

For intersection c, $7x - 1427.2 = -0.04x + 100.7$ $x \approx 217.03$

$$y = -0.04(217) + 100.7 \approx 92.02$$

The coordinates of the intersection *c* are (217.03, 92.02)

Now, to find the radius of each circle, the distance between its center and the site next to it is considered.

$$distance\ formula = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

$$radius\ of\ circle\ a = \sqrt{(87.02 - 75.7)^2 + (48.2 - 99.5)^2} \approx 52.5$$

$$radius\ of\ circle\ b = \sqrt{(214.8 - 261.6)^2 + (96.4 - 112.4)^2} \approx 49.5$$

$$radius\ of\ circle\ c = \sqrt{(217.03 - 261.6)^2 + (92.02 - 112.4)^2} \approx 49.0$$

Thus, the Largest Empty Circle is centered at *a*, since its radius is the largest amongst all other intersections, and *a* is the place where the new MPA site should be designated.

After solving the problem of the Largest Empty Circle, it can be concluded that the new MPA site should be designated in the South Pacific Ocean because it corresponds to the center of the largest empty circle on the world map (*a* (87.02, 48.2)). This is the place where, according to the calculations, the new MPA should be established because this is a region furthest away from all other existing MPAs.

While such a designation holds potential benefits for the migratory patterns of whale sharks, it is crucial to note that the primary focus of this investigation centers around whale shark aggregation sites. Unlike migration routes, aggregation sites are often situated near coastal areas or other specific habitats. Consequently, despite the South Pacific Ocean being identified as the optimal location based on the Largest Empty Circle methodology, its distance from coastal regions raises considerations regarding its suitability for protecting critical whale shark aggregation habitats. An alternative decision could be to opt for intersection *c*, forming the third largest empty circle. If moved slightly below towards the ocean rather than being on the land, it would be optimal as it is not only close to an aggregation site but also far from most other MPA sites.

It would be more beneficial to consider the actual aggregation sites and already existing MPAs when deciding where to designate a new MPA. For that reason, the conclusion from the first part of the investigation, that the new MPA should be designated in the Indian Ocean on the West Coast of Madagascar in site C, is more reasonable.

3.2 Exponential Model for Population Growth of Whale Sharks to determine efficiency of MPAs

To assess the impact of marine protected areas, an exponential growth model of population of Whale Sharks at one of the aggregation sites mentioned before, Bahía de La Paz, Mexico with data from identification studies ranging from 2003-2009 is formulated.

Exponential growth occurs when a population's increase or decrease is directly proportional to its current size. This pattern is described mathematically through a separable differential equation.

$$\frac{dW}{dt} = kP$$

Where

$t - time,$

$W - population\ of\ whale\ sharks,$

$k - growth\ constant\ describing\ proportionality\ with\ respect\ to\ time$

$\frac{dW}{dt} - rate\ of\ change\ of\ a\ population\ of\ whale\ sharks\ with\ respect\ to\ time,$

To transform this equation into an exponential growth model, both sides of the equation are divided by W and multiplied by *dt*:

$$\frac{dW}{dt} = kW * \frac{dt}{W}$$

$$dW * \frac{1}{W} = kW * \frac{dt}{W}$$

Next, both sides should be integrated to isolate the variables with P from all other terms:

$$\int \frac{1}{P} * dP = \int k * dt$$

$$\ln|P| = k * t + c$$

where c is another constant. Now, the derived equation is rearranged to express the absolute value of W:

$$|W| = e^{k*t+c}$$

$$|W| = e^{k*t+c}$$

$$|W| = e^{k*t+c} > 0$$

Given that e^c is a constant, it can be represented as c, and considering that population values are always positive, the absolute value for W is unnecessary. Thus, the final equation is:

$$W = c * e^{k*t}$$

The general equation of the exponential growth model is:

$$W = W_0 * e^{k*t}$$

Where W_0 represents the population when $t=0$. Upon comparing this model with the equation derived, it becomes evident that W_0 is equivalent to the constant c:

$$c * e^{k*t} = W_0 * e^{k*t}$$

$$c = W_0$$

Fitting data to the Exponential Growth Model

To establish the Exponential Model for the population of Whale Sharks in Bahía de La Paz, the growth constant 'k' must be determined by fitting a regression line to the dataset that is derived from a research paper (Ramírez-Macías et al.). To derive the equation of the line, the formula for the model is rearranged by incorporating natural logarithms to both sides:

$$\ln W = \ln W_0 * e^{k*t}$$

Letting $\ln W = Y$ and using logarithmic properties, the equation of a line is obtained:

$$Y = k * t + \ln W_0$$

Table 3. Data depicting size population of Whale Sharks in Bahía de La Paz in years 2003-2009 as derived from (Ramírez-Macías et al.) and its natural logarithms from which the regression line of the data set will be found

Year	Number of the year *(t)*	Whale Shark Population in Bahía de La Paz (*W*)	$Y = \ln W$
2003	0	1	0
2004	1	8	2.079441542
2005	2	30	3.401197382
2006	3	19	2.944438979
2007	4	20	2.995732274
2008	5	33	3.496507561
2009	6	54	3.988984047

The regression line obtained is (by inputting number of year and ln *W* on *CASIO fx-CG50* graphic display calculator using statistics linear regression function):

$$Y = 0.51412925t + 1.15851249$$

$$\ln W = 0.51412925t + 1.15851249$$

Therefore, the value of 'k' is approximately equal to *0.51412925*. By applying the exponential function to both sides of the equation of the regression line, the formula for the Exponential Growth Model for the population is:

$$W = 3.185191746 * e^{0.51412925t}$$

To evaluate the accuracy of the provided regression line, Pearson's product-moment correlation coefficient between time and ln(*W*) must be computed. Pearson's product-moment correlation coefficient is a statistical measure utilized to ascertain the strength of linear correlation between two variables. Its value can be either positive, meaning that an increase in one variable results in an increase in the other one. Values of this correlation range from -1 to 1.

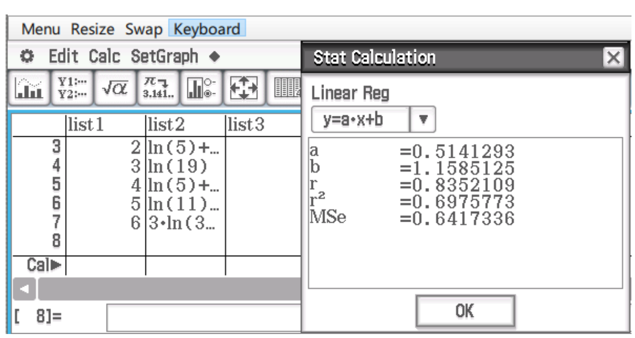

Fig. 6. Screenshot of statistical linear regression and correlation coefficient obtained of Casio fx-CG50

Calculating the correlation:

$$r = 0.8352109$$

The outcome indicates a strong positive correlation between time and $\ln(W)$. Thus, it can be inferred that the established model can be used to describe and predict the population growth of Whale Sharks in Bahía de La Paz with good accuracy.

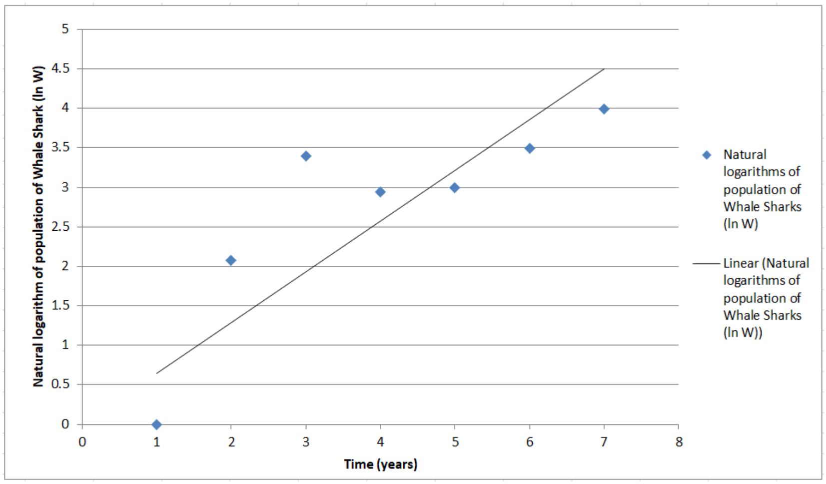

Fig. 7. Natural logarithms of the whale shark population over time and calculated regression line

The table below compares values of predictions of the population from the Exponential Growth Model created from the regression line and the actual values.

Table 4. Comparison of real-time data and calculated data when exponential model is obtained

Year	Number of the year (t)	Population of Whale Sharks in Bahía de La Paz	Predictions of the model $W = 3.185191746 * e^{0.51412925t}$
2003	0	1	≈ 3
2004	1	8	≈ 6
2005	2	30	≈ 9
2006	3	19	≈ 15
2007	4	20	≈ 25
2008	5	33	≈ 42
2009	6	54	≈ 70

According to the table, from the first year, population increases at an exponential rate, and is higher than the observed values. Beginning from 2004, the predictions gradually become less accurate as they follow a calculated pattern. In reality, there exists no flawless model to forecast population growth accurately.

3.3 Solving the Traveling Salesman Problem:

In order to further gain a better understanding of factors affecting Whale Shark populations to dedicate more marine protected areas suited to their requirements and ensure their protection, it is a necessity to visit these aggregation sites and collect data on this unique species.

As a future marine biologist, the realization strikes of the importance to examine each of these aggregation sites and gain more data for tagging purposes as well. The requirement of more data was expressed in the evaluation for the exponential growth model above as well.

Thus, as a start, I would like to use the Traveling Salesman Problem to optimize a route to visit aggregation sites in East Africa.

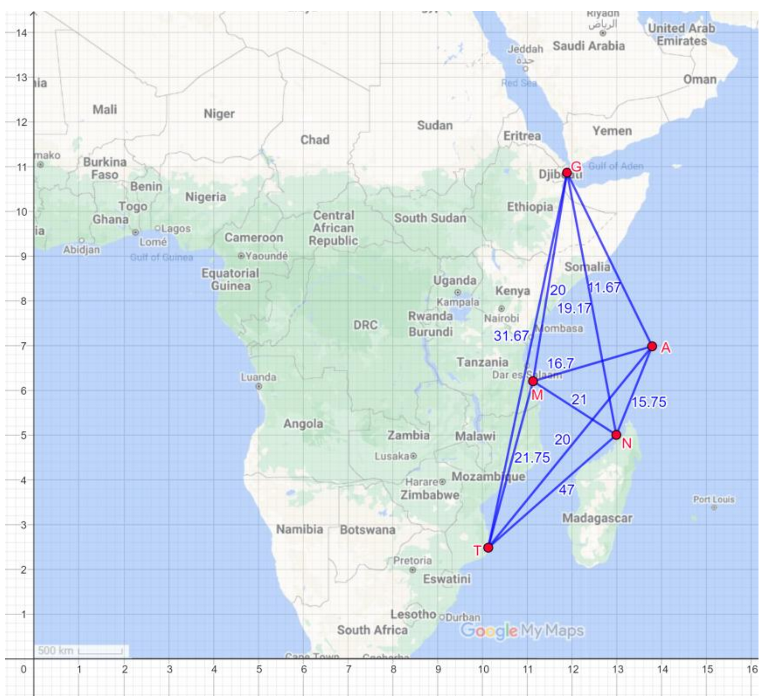

Fig. 8. Close-up map of aggregation sites near East Africa with edges depicting travel time in hours

The Hamiltonian cycle having least weight would be most likely to be a solution to this practical sales man problem. A Hamiltonian cycle is “a graph cycle (i.e., closed loop) through a graph that visits each node exactly once”. The nodes of the graph are represented by the aggregation sites in East Africa.

Using google maps, shortest time (in hours) between each site was obtained to create a weighted graph to discover shortest time for this journey.

To simplify this, a weighted adjacency table is constructed. (Least weights between points as done through inspection)

Table 5.

	To					
From	Vertex	G	M	T	N	A
	G	0	20	31.67	19.17	11.67
	M	20	0	21.75	21	16.7
	T	31.67	21.75	0	47	20
	N	16.7	21	47	0	15.75
	A	11.67	16.7	20	15.75	0

Now, the minimum weighted journey that visits each vertex can be determined. As no algorithm has been devised to find the Hamiltonian cycle of the least weight, an estimation of the least weighted Hamiltonian cycle is calculated using upper and lower bounds. To establish an upper bound for the minimum Hamiltonian cycle, the nearest neighbor algorithm is followed. This algorithm consists of four steps:

1) A starting vertex is selected.
2) The edge with the least weight to an unvisited vertex is added.
3) Step 2 is repeated until every vertex has been visited.
4) The starting vertex is returned to by including the corresponding edge.

The starting vertex chosen is G, as it is closest to my origin.

Table 6.

	To					
		1	4	5	3	2
From	Vertex	G	M	T	N	A
	~~G~~	~~0~~	~~20~~	~~31.67~~	~~16.7~~	~~11.67~~
	~~M~~	~~20~~	~~0~~	~~21.75~~	~~21~~	~~16.7~~
	~~T~~	~~31.67~~	~~21.75~~	~~0~~	~~47~~	~~20~~
	~~N~~	~~16.7~~	~~21~~	~~47~~	~~0~~	~~15.75~~
	~~A~~	~~11.67~~	~~16.7~~	~~20~~	~~15.75~~	~~0~~

The Hamiltonian cycle formed as the upper bound is:

$G \rightarrow A \rightarrow N \rightarrow M \rightarrow T \rightarrow G$

Let **m** represent the minimum weight (**minimum travel time in hours**) of a Hamiltonian cycle in Fig. 3. The upperbound of m is expressed as:

$m \leq 11.67 + 15.75 + 21 + 21.75 + 31.67$

$m \leq 101.84$ hours

With the upper bound *m* calculated, the deleted vertex algorithm is used to find the lower bound of *m*. These are the steps followed to find the lower bound:

1) The specified vertex (can be any) is removed, along with all its connected edges, from the original graph.
2) The minimum spanning tree for the remaining graph is found.
3) The lengths of the two shortest deleted edges are added to the length of the minimum spanning tree.

The vertex G is chosen to be deleted. Table 6 below shoes the weighted adjacency table without vertex G and the edges connected to it.

Table 7.

	To				
From	Vertex	M	T	N	A
	M	0	21.75	21	16.7
	T	21.75	0	47	20
	N	21	47	0	15.75
	A	16.7	20	15.75	0

With vertex G deleted, the minimum spanning tree will be found using Prim's algorithm, following these steps:

1) Any vertex randomly is chosen.
2) The row corresponding to the selected vertex is eliminated.
3) Numbers to the columns of the chosen vertex following the respective order in which they are chosen are written.
4) The lowest undeleted entry in any of the numbered columns is circled.
5) Steps 2 to 4 are repeated until all rows are deleted.

The circles entries give the edges of the minimum spanning tree. The starting vertex chosen is A.

Table 8.

		To			
		3	4	2	1
From	Vertex	M	T	N	A
	M	0	21.75	21	16.7
	T	21.75	0	47	20
	N	21	47	0	15.75
	A	16.7	20	15.75	0

Using the weight of the edges circled, the weight of the minimum spanning tree = 15.75 + 21 + 21.75 = 58.5

The edges with minimum weight from the deleted vertex G were GA and GN, of 11.67 and 16.7 hours, respectively. These weights are added to the summed weight of the minimum spanning tree, which is 58.5 hours, to thus find the lower bound of the minimum Hamiltonian cycle.

$11.67 + 16.7 + 58.5 = 86.87$

$m \geq 86.87$

To improve this lower bound, the deleted vertex algorithm is iterated with different vertices to ascertain the highest possible lowest bound. For each iteration, G is selected as the starting vertex.

Table 9.

Deleted Vertex	Weight of minimum spanning tree	Sum of weight of two shortest edges of deleted vertex	Lower bound
M	74.2	36.7	110.9
T	48.42	41.75	90.17
N	50.12	32.45	82.57
A	59.45	28.37	87.82

The greater the value of the lower bound, the more ideal it is. Hence, based on the provided table, the best lower bound is 110.9 hours.

$110.9 \leq m \leq 101.84$ hours

This range of values gives an approximate of the minimum time required to visit all five sites starting and ending at vertex G, in Djibouti, Gulf of Tadjou.

4. Conclusion and Evaluation:

To successfully analyze the placement of Marine Protected Areas to safeguard Whale Shark populations and their efficiency, various mathematical methodologies were applied to identify optimal locations for establishing Marine Protected Areas (MPAs) aimed at conserving whale shark populations.

While this investigation was successful, areas for improvement in terms of both practicality and the mathematical modeling exist, which would improve the applicability and reliability of the results.

The use of Voronoi diagrams had a spatial approach to identifying potential MPA locations. However, this method depends on the availability and accuracy of whale shark aggregation data. So, incorporating more up-to-date datasets in future research could improve the precision of the results.

While the exponential growth model is a straightforward approach to modeling population and assessing MPA effectiveness in preserving whale shark populations, its simplicity may overlook important factors such as carrying capacity. As an improvement, a logistic growth model would account for long-term sustainability of populations within MPAs.

In the traveling salesman problem, it is assumed that there is a direct connection (edge) between two aggregation sites. However, in reality aggregation sites may not have feasible direct connections due to geographical barriers, restricted access, or other logistic constraints.

All in all, this investigation demonstrates the application of mathematical methods to contribute to conservation efforts for whale sharks and the marine ecosystems they are part of, by focusing on the effectiveness and practicality of MPAs.

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