Asset Prices and Risk Sharing in Open Economies

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This paper proposes a two-country model that features time-varying heterogeneity in conditional risk aversion across countries, endogenously arising from the interaction between external habit formation and preference home bias. The model generates high international correlation of state prices along with modest cross-country consumption growth correlation and matches the empirical disconnect between exchange rate changes and consumption growth rate differentials. The key mechanism is endogenous time variation in conditional consumption growth volatility: the conditionally less risk averse country insures the more risk averse one, offsetting cross-country heterogeneity in conditional risk aversion and leading to significant international comovement in marginal utility growth. (*JEL* G12, G15, F31)

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International data document a strong dichotomy between asset prices and macroeconomic quantities. Although consumption growth rates are weakly correlated internationally, the empirically modest volatility of real exchange rate changes implies highly correlated state prices across countries under the assumption of complete markets (Brandt, Cochrane, and Santa-Clara 2006). Standard preferences, under which state prices solely reflect exposure to consumption growth shocks, are unable to reconcile low consumption growth correlations with high stochastic discount factor (SDF) correlations. Moreover, standard preferences are unable to explain the empirically observed lack of correlation between real exchange rate changes and consumption growth rate differentials across countries (Backus and Smith 1993).

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This paper addresses those stylized empirical findings by considering a two-good, two-country model that features time-varying heterogeneity in conditional risk aversion across countries, and exploring its implications for asset prices and macroeconomic aggregates. Heterogeneity in conditional risk aversion is endogenously induced by the combination of home-biased preferences and external habit formation. In particular, each of the two countries in the model is represented by a stand-in agent endowed with a stream of a single differentiated perishable good. Each agent has external habit preferences of the Menzly, Santos, and Veronesi (2004) type, with each agent forming habit on an agent-specific consumption basket, which is a home-biased basket of the two goods. International trade in goods is frictionless, and financial markets are frictionless and complete, so the two countries are able to achieve optimal international risk sharing.

The key attribute of the model is the interaction between preference home bias and external habit formation. The two countries have to optimally trade off consuming their preferred good and sharing risk with each other: the more home-biased equilibrium consumption is, the weaker the ability of countries to insure each other against adverse endowment shocks. In the absence of external habit formation, the two countries would be willing to fully accommodate their home-biased preferences, sacrificing some insurance. With external habit formation, however, consumption home bias also induces heterogeneity in the two countries' conditional risk aversion. This is due to the fact that conditional risk aversion is driven by past consumption shocks: given consumption home bias, consumption growth innovations are imperfectly correlated and lead to different conditional risk aversion paths. As a result, although the two countries have the same steady-state conditional risk aversion, in some states the domestic country is more risk averse than the foreign country and in other states the reverse occurs.

Crucially, the trade-off between insurance and home bias is state-dependent. When the domestic country is conditionally more risk averse than the foreign country, it is willing to sacrifice home bias in favor of insurance: in those states, the domestic country not only consumes a higher than usual share of the global endowment, but also has lower than usual consumption growth volatility. This is because the foreign country insures the domestic country in order to limit the (costly, in terms of welfare) future cross-sectional dispersion in conditional risk aversion; naturally, foreign consumption growth is more volatile than usual in those states. Exactly the reverse occurs in states characterized by relatively high foreign conditional risk aversion. In short, the conditional consumption growth volatility of each country is inversely related to the level of that country's risk aversion, as the conditionally more risk averse country holds less of the global endowment risk. Importantly, this risk sharing scheme leads to moderate unconditional correlation of consumption growth rates, as the two countries are exposed to different amounts of consumption risk at any given point in time.

Despite the low consumption growth correlation, the risk sharing mechanism generates stochastic discount factors characterized by high unconditional international correlation. This is because cross-country differences in consumption growth volatility are almost perfectly offset by cross-country differences in the conditional sensitivity of marginal utility growth to consumption risk. This follows from the fact that in the Menzly, Santos, and Veronesi (2004) habit specification the volatility of conditional risk aversion, which is the first-order component of SDF volatility, is increasing in the level of conditional risk aversion. Thus, when one country has relatively high conditional risk aversion and, due to the risk sharing scheme, has relatively low conditional consumption growth volatility, its marginal utility is very sensitive to consumption growth shocks. Conversely, the other country undertakes relatively high consumption risk, but its marginal utility is not very sensitive to that risk. The two effects, amount of consumption risk and sensitivity of marginal utility to consumption risk, almost fully offset each other. As a result, marginal utility growth is highly correlated across countries. Thus, the high correlation of SDFs across countries in the model is not generated by highly correlated consumption growth (which is a second-order SDF component in our economy), but by highly correlated risk aversion changes, which is exactly the SDF component the two countries want to align for welfare purposes.

The same mechanism can also explain the Backus and Smith (1993) exchange rate disconnect results. Real exchange rate changes, which reflect SDF heterogeneity across countries, are primarily driven by cross-country differences in risk aversion changes, not country differences in consumption growth rates. Thus, the unconditional correlation between real exchange changes and consumption growth rate differentials can be low in our model economy.

The model also addresses another key issue in international macroeconomics, the "remarkable," in the words of Obstfeld and Rogoff (2000), volatility of real exchange rate changes. From the perspective of international macroeconomics, the volatility of real exchange rate changes should be quantitatively similar to consumption or output growth volatility; it is, instead, almost an order of magnitude higher. In the model, real exchange rate volatility is generated by two economic mechanisms: time variation in relative endowment levels and time variation in relative conditional risk aversion. The latter, valuation-related mechanism greatly amplifies the effects of the endowment mechanism, resulting in real exchange rate changes that are significantly more volatile than macroeconomic fundamentals. Thus, the failure of most standard international macroeconomic models to generate substantial real exchange rate volatility can be traced to their inability to generate a valuation mechanism and is, therefore, linked to the inability of standard preferences to match asset price volatility.

Finally, the model illustrates the effect of foreign preferences on domestic asset prices. The price-dividend ratio of each country's endowment claim is determined by a weighted average of the two countries' conditional risk aversion, with the weights depending on the steady-state wealth share and the degree of preference home bias of the two countries. This implies that the effect of a country's preferences on foreign asset prices is related to the country's size and openness: risk aversion fluctuations in large, open economies affect foreign asset prices more than those in small, closed ones. The model also addresses the high correlation of equity returns across countries: the comovement of state prices across countries that arises from the international risk sharing scheme implies that equity returns are much more internationally correlated than their cash flows due to significant discount rate comovement.

The model is calibrated using quarterly data from the United States (domestic country) and the United Kingdom (foreign country) from 1975 to 2012, and it generates highly internationally correlated conditional risk aversion levels and changes, SDFs, and equity excess returns, despite the fact that consumption growth correlation is moderate. Moreover, the correlation between consumption growth differentials and real exchange rate changes is modest, in line with the data, as exchange rates are primarily driven by cross-country differences in conditional risk aversion fluctuations.

The key empirical implications of the model are tested by proxying the log surplus consumption ratio of each country with a weighted average of past consumption growth rates, as in Wachter (2006). We show that there is very strong empirical evidence of a negative relationship between relative conditional consumption growth volatility and relative conditional risk aversion, supporting the key international risk sharing mechanism of the model. Moreover, we find that, empirically, conditional consumption growth correlation and conditional real exchange rate volatility are increasing in both countries' conditional risk aversion, as suggested by the model.

This paper is part of the international asset pricing literature that focuses on the interactions between asset prices and exchange rates. The model in this paper builds on Pavlova and Rigobon (2007). They propose a Lucas (1982) two-country model with exogenous demand shocks and examine the effects of terms of trade fluctuations on asset prices and exchange rates. Despite its success in addressing puzzles in the international real business cycle literature, the ability of their model to match asset prices and returns is limited. In our model, external habit formation generates time-variation in conditional risk aversion, with risk aversion shocks effectively acting as endogenous demand shocks that are able to generate empirically plausible asset pricing dynamics.

In response to the limitations of standard preferences, strands of the international finance literature feature more complex preference specifications. Specifically, Verdelhan (2010) proposes a two-country, one-good model in which each country has an exogenously specified i.i.d. consumption growth process and Campbell and Cochrane (1999) external habit preferences. The model is able to explain the forward premium puzzle, but it generates real exchange rates that are both highly volatile, implying low SDF correlation, and excessively correlated with consumption growth rate differentials.

Colacito and Croce (2011) propose a two-country, two-good model where agents have Epstein and Zin (1989) preferences and are endowed with consumption processes that feature Bansal and Yaron (2004) long-run risk. The key assumption is that long-run consumption growth shocks are very highly correlated across countries, despite moderately correlated short-term consumption growth innovations. The first assumption generates high correlation in the two countries' SDFs, while the second assumption leads to moderate cross-country consumption growth correlation. In Colacito and Croce (2013), they employ recursive preferences, assume exogenous output subject to short-run and long-run shocks, and endogenize consumption by allowing for international trade. In their model, the Backus and Smith (1993) puzzle is addressed due to the effect of long-run news, which generate a negative correlation between real exchange rate changes and consumption growth rate differentials, partly offsetting the positive correlation generated by short-run news.

Farhi and Gabaix (2016) consider a model with multiple countries exposed to rare disaster risk, as in Rietz (1988) and Barro (2006), in order to explain the joint dynamics of exchange rates, stock prices and option prices. The model features a stochastically varying probability of global disasters, as well as exogenous heterogeneity across countries regarding average exposure to such disasters. Because of time variation in both the disaster probability and the countries' exposure to disasters, exchange rates are both more volatile than macroeconomic fundamentals and disconnected from them.

Other work that embeds complex preferences in an open economy context includes Bekaert (1996); Shore and White (2006); Lustig and Verdelhan (2007); Aydemir (2008); Moore and Roche (2010); Bansal and Shaliastovich (2013); and Heyerdahl-Larsen (2014).

The two main ingredients of the model are habit formation and preference home bias. Habit formation has been used extensively in the asset pricing literature. For example, see Sundaresan (1989); Abel (1990); Constantinides (1990); Detemple and Zapatero (1991); Ferson and Constantinides (1991); Heaton (1995); Jermann (1998); Boldrin, Christiano, and Fisher (2001); and Chan and Kogan (2002). The present paper uses Menzly, Santos, and Veronesi (2004) external habit preferences, which share many features with Campbell and Cochrane (1999) preferences. Menzly, Santos, and Veronesi (2004) model the inverse surplus consumption ratio — similar specifications have also been used in Buraschi and Jiltsov (2007); Bekaert, Engstrom, and Grenadier (2010); and Santos and Veronesi (2010). Preference home bias has been used, *inter alia*, in Cole and Obstfeld (1991); Zapatero (1995); and Serrat (2001).

1. The Model

The global economy comprises two countries, Domestic and Foreign, each populated by a representative agent who receives an endowment stream of a single perishable good: the domestic agent is endowed with the domestic good, whereas the foreign agent is endowed with the foreign good. Economic activity takes place in the time interval $[0,\infty)$. Uncertainty in this economy is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, on which is defined a standard two-dimensional Brownian motion. All stochastic processes introduced in the remainder of the paper are assumed to be adapted to $\{\mathcal{F}_t; t \in [0,\infty)\}$, the filtration generated by the Brownian motion, augmented by the null sets, and to satisfy all the necessary regularity conditions for them to be well-defined. All (in)equalities that involve random variables hold P-almost surely. All starred variables are the foreign counterparts of the similarly named domestic variables.

1.1 Endowments and markets

The endowment streams of the domestic and the foreign good are denoted by $\{\widetilde{X}_t\}$ and $\{\widetilde{Y}_t\}$, respectively, and satisfy

$$\begin{bmatrix} d\log \widetilde{X}_t \\ d\log \widetilde{Y}_t \end{bmatrix} = \begin{bmatrix} \mu_t^X \\ \mu_t^Y \end{bmatrix} dt + \begin{bmatrix} \sigma^X dB_t^X \\ \sigma^Y dB_t^Y \end{bmatrix},$$

where dB^X and dB^Y are Brownian increments with instantaneous correlation ρ^{XY}. Therefore, the two endowment growth rates have constant conditional volatility, and constant conditional correlation. The assumption of conditional homoskedasticity is adopted for convenience and can be relaxed without affecting the main economic mechanisms that operate in the model. However, it has one important implication: any conditional heteroskedasticity in prices and quantities that arises in equilibrium is not due to endowment effects.

Goods markets and financial markets are frictionless: the two agents can trade any quantities of the two goods, as well as a complete set of Arrow-Debreu securities, without incurring any transaction costs.

1.2 Preferences

The domestic representative agent maximizes expected discounted utility, given by

$$E_0\left[\int_0^\infty e^{-\rho t} u(X_t, Y_t) dt\right] = E_0\left[\int_0^\infty e^{-\rho t} \log(C_t - H_t) dt\right],$$

where $\rho > 0$ is the agent's subjective discount rate; X and Y is the quantity of the domestic and foreign good, respectively, the agent consumes; and C is the domestic consumption basket defined as

$$C \equiv X^a Y^{1-a}.$$

The domestic consumption basket is Cobb-Douglas, so the elasticity of substitution between the two goods is constant at 1. The domestic agent's preferences over the two goods are described by the parameter $a \in [0,1]$, which denotes the degree of preference for the domestic good. Finally, H is the habit level associated with consumption basket C, to be defined next.

The domestic agent forms a habit over total consumption, not over the consumption of individual goods.[1] Specifically, the external habit is of the Menzly, Santos, and Veronesi (2004) form: the domestic inverse surplus consumption ratio $G=\frac{C}{C-H}$ satisfies

$$\frac{dG_t}{G_t}=k\left(\frac{\bar{G}-G_t}{G_t}\right)dt-\delta\left(\frac{G_t-l}{G_t}\right)\left(\frac{dC_t}{C_t}-E_t\left(\frac{dC_t}{C_t}\right)\right). \tag{1}$$

The local curvature of the utility function is given by

$$-\frac{u_{CC}(C,H)}{u_C(C,H)}C=G.$$

For that reason, in a slight abuse of terminology, I will often refer to the inverse surplus consumption G as conditional risk aversion.

The inverse surplus consumption ratio G is mean-reverting and bounded below, with unconditional mean equal to $\bar{G}$ and a lower bound of l. Crucially, the shocks in the inverse surplus consumption ratio are perfectly negatively correlated with consumption growth shocks. Intuitively, the habit level adjusts slowly to consumption, so the surplus consumption ratio is procyclical and conditional risk aversion countercyclical, achieving high levels after a series of negative consumption growth shocks.

As in the Campbell and Cochrane (1999) specification, the sensitivity of the surplus consumption ratio to consumption growth innovations is countercyclical, decreasing in the level of the surplus consumption ratio. In a closed economy setting, this assumption generates asset return predictability, as it produces time variation in the market price of risk. As we will see, this assumption is also critical in addressing the major price-quantity puzzles in international finance, as it generates time variation in the conditional volatility and correlation of key variables. Finally, an important parameter is sensitivity $\delta>0$, a scaling coefficient that affects the average level of risk aversion volatility. Higher values of δ imply a higher volatility of risk aversion changes and, thus, more volatile SDFs.

The preferences of the foreign agent are identical to those of the domestic agent, with one exception. In particular, I assume that the parameters that regulate conditional risk aversion (k, δ, $\bar{G}$ and l), as well as the subjective discount rate ρ, have equal values in the two countries. The only source of *ex ante* preference heterogeneity across countries is the consumption basket parameter a. In the remainder of the paper, I focus on the home bias case $0<a^*<a<1$; under this condition, each country has a stronger preference for its own good than the other country does. The interaction between preference home bias and external habit formation is the key mechanism for the determination of prices and quantities in the model.

[1] The latter would constitute "deep habits", as defined by Ravn, Schmitt-Grohe, and Uribe (2006). For international finance models that feature deep habits, see Ravn, Schmitt-Grohe, and Uribe (2007); Heyerdahl-Larsen (2014).

2. Equilibrium

Under the assumption of market completeness, the competitive equilibrium allocation is equivalent to the social planner's allocation under the constraint that the planner takes the laws of motion for G and G^* as given, so it is constrained Pareto optimal. The planner maximizes a weighted average of the two countries' expected utility,

$$\max_{\{X_t, Y_t, X_t^*, Y_t^*\}} E_0\left[\int_0^\infty e^{-\rho t}\left(\lambda \log(C_t - H_t) + \lambda^* \log(C_t^* - H_t^*)\right) dt\right],$$

subject to the resource constraints

$$X_t + X_t^* = \widetilde{X}_t, \quad Y_t + Y_t^* = \widetilde{Y}_t.$$

For the planner solution to be identical to the competitive solution, the welfare weights must be determined endogenously. Thus, I first solve the planner's problem for fixed welfare weights λ and λ^* and then solve for the values of the weights that equate the planner's solution with the competitive solution. The solution for the competitive equilibrium is provided in detail in Appendix A1.

Without loss of generality, it is assumed that the local numeraire in each country is the local consumption basket. Moreover, the global numeraire is taken to be the domestic local numeraire. Because both goods are frictionlessly traded internationally, the law of one price holds and the price of each good in terms of the global numeraire is the same in both countries.

2.1 Consumption and marginal utility

In equilibrium, the two countries equalize their marginal utility growth for each for the two goods,

$$\lambda a \frac{G_t}{X_t} = \lambda^* a^* \frac{G_t^*}{X_t^*}, \quad \lambda(1-a)\frac{G_t}{Y_t} = \lambda^*(1-a^*)\frac{G_t^*}{Y_t^*}, \tag{2}$$

so the marginal rate of substitution between the two goods is the same in both countries. Indeed, defining

$$\Xi_t \equiv e^{-\rho t}\left(a\lambda G_t + a^*\lambda^* G_t^*\right)\frac{1}{\widetilde{X}_t}, \quad \Xi_t^* \equiv e^{-\rho t}\left((1-a)\lambda G_t + (1-a^*)\lambda^* G_t^*\right)\frac{1}{\widetilde{Y}_t},$$

the equilibrium marginal utility of the domestic good equals $\frac{1}{\lambda}\Xi$ for the domestic agent and $\frac{1}{\lambda^*}\Xi$ for the foreign agent. Similarly, the equilibrium marginal utility of the foreign good equals $\frac{1}{\lambda}\Xi^*$ for the domestic agent and $\frac{1}{\lambda^*}\Xi^*$ for the foreign agent. Thus, the marginal utility of each good for each agent is increasing in the scarcity of the good and in the conditional risk aversion of both agents.

In optimizing, the two agents face a trade-off between consuming their preferred good and achieving cross-sectional efficiency in consumption allocation. In the absence of any preference heterogeneity, the only welfare concern would be to minimize the dispersion of marginal utility (adjusted for cross-country differences in the welfare weight) across counties. In that case, the two countries would pool their consumption, ensuring that their exposure to endowment shocks is identical and the cross-sectional dispersion of marginal utility growth rates is always zero. However, full cross-insurance is not optimal in the presence of preference home bias. Consider, for example, the domestic good: given that $a > a^*$, the domestic agent has a stronger preference for the domestic good than the foreign agent, so the domestic agent is willing to sacrifice some insurance to be able to consume a higher share of that good. In that case, the two agents have to balance the efficiency gains from reducing the cross-sectional dispersion of marginal utility with the efficiency losses from allocating goods to their less preferred consumers, and vice versa.

In the absence of habit formation, this efficiency trade-off does not vary over time, so each agent consumes a constant, home-biased share of the global endowment of each good. However, when preferences are characterized by external habit formation, the time variation in the relative conditional risk aversion of the two countries (G^*/G) induces time variation in the trade-off between insurance and home bias and, thus, in equilibrium consumption shares. The equilibrium consumption of the domestic agent is given by

$$X_t = \omega_t \widetilde{X}_t = \frac{a\lambda}{a\lambda + a^*\lambda^*\left(\frac{G_t^*}{G_t}\right)} \widetilde{X}_t \tag{3}$$

for the domestic good, and by

$$Y_t = \omega_t^* \widetilde{Y}_t = \frac{(1-a)\lambda}{(1-a)\lambda + (1-a^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)} \widetilde{Y}_t \tag{4}$$

for the foreign good, where ω (ω^*) is the share of the domestic (foreign) endowment consumed by the domestic agent. As a result of the efficiency trade-off, the domestic consumption shares ω and ω^* are decreasing, convex functions of the ratio G^*/G.

To understand the properties of the efficiency trade-off, it is instructive to consider the "share ratio" of the domestic agent, defined as the ratio of the agent's consumption share of the more preferred good (the domestic good) over the agent's share of the foreign good:

$$\frac{\omega_t}{\omega_t^*} = \frac{a}{1-a} \frac{(1-a)\lambda + (1-a^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)}{a\lambda + a^*\lambda^*\left(\frac{G_t^*}{G_t}\right)}.$$

Conversely, the foreign share ratio is defined as the foreign share of the foreign good over the foreign share of the domestic good. The two share ratios are plotted against the risk aversion ratio G^*/G in panel A of Figure 1. By definition,

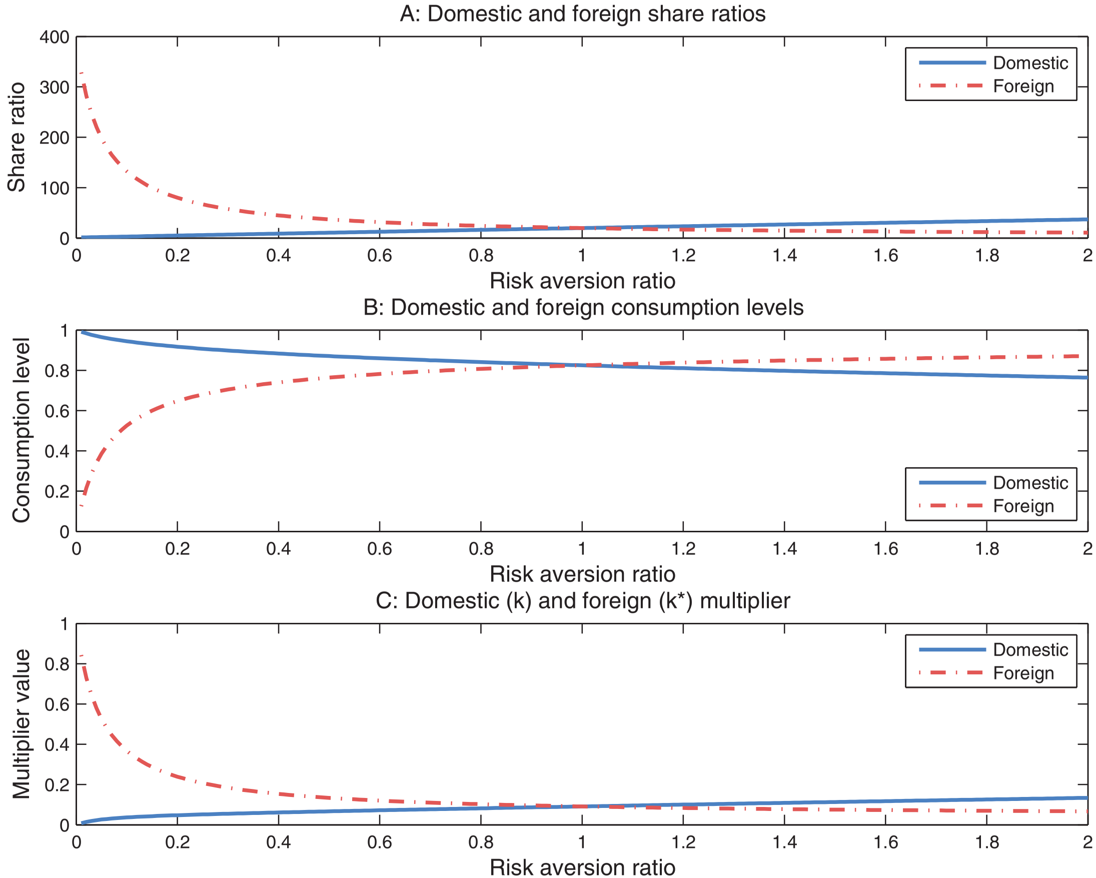

Figure 1
Share ratios, consumption levels, and multipliers
The figure shows model consumption share ratios, consumption levels, and multipliers as functions of the risk aversion ratio. Panel A presents the domestic and the foreign share ratio, $\frac{\omega_t}{\omega_t^*}$ and $\frac{1-\omega_t^*}{1-\omega_t}$, respectively, as a function of the risk aversion ratio G^*/G. Panel B presents the domestic and foreign consumption levels, C and C^*, respectively, as a function of the risk aversion ratio G^*/G. Panel C presents the domestic and foreign multiplier, k and k^*, respectively, as a function of the risk aversion ratio G^*/G. The calculations use the model parameter values shown in Table 1.

each share ratio reflects the degree of equilibrium consumption home bias in the corresponding country. As seen graphically, the domestic (foreign) share ratio is an increasing (decreasing) function of the risk aversion ratio G^*/G. In the absence of cross-sectional dispersion in conditional risk aversion ($G^*=G$), the two share ratios take their steady-state values, which are equal to their constant values in the log utility benchmark. When $G \neq G^*$, however, the share ratios deviate from their steady-state values. When $G^*/G \to 0$, the marginal utility of the domestic agent becomes much higher than that of the foreign agent, so maintaining the steady-state level of consumption home bias becomes very costly from a cross-sectional efficiency perspective. As a result, the domestic agent is allocated almost the entirety of the global endowment, the domestic consumption shares for both goods approach 1, and domestic consumption home bias is eliminated ($\frac{\omega_t}{\omega_t^*} \to 1$). As regards the foreign country, its share ratio is higher than its steady-state level, implying elevated consumption home bias, as the foreign country imports very little of the domestic good. Conversely, when $G^*/G \to \infty$ almost all of the global endowment is consumed by the foreign country, domestic consumption home bias is higher than its steady-state value and foreign consumption home bias is eliminated.

The previous discussion does not rely on a particular law of motion for the state variable G^*/G. Adding the assumption that the law of motion for G and G^* is given by Equation (1) and its foreign counterpart, respectively, generates a two-way interaction between consumption choices and conditional risk aversion. In that case, the optimality of time-varying consumption shares can be demonstrated by illustrating the insurance cost of constant consumption shares. To wit, consider the effect of a negative domestic endowment shock in our model, assuming that consumption shares were constant. Figure 2 traces the effects of such a shock for forty quarters, with the initial state being the steady state. Because both the domestic and the foreign agent consume the domestic good, the domestic endowment shock (panel A) reduces consumption in both countries. However, the effect is not symmetric: given home-biased preferences, the domestic country experiences a more adverse consumption growth shock than the foreign country (panel B), so consumption falls more (panel C) and conditional risk aversion increases more (panel D) in the domestic country than in the foreign country. Consider the marginal utility of consumption, given by

$$\Lambda_t = \left(\frac{\Xi_t}{a}\right)^a \left(\frac{\Xi_t^*}{1-a}\right)^{1-a} = \lambda e^{-\rho t} \frac{G_t}{C_t}$$

for the domestic agent, and

$$\Lambda_t^* = \left(\frac{\Xi_t}{a^*}\right)^{a^*} \left(\frac{\Xi_t^*}{1-a^*}\right)^{1-a^*} = \lambda^* e^{-\rho t} \frac{G_t^*}{C_t^*}$$

for the foreign agent. The combination of the consumption effect and the risk aversion effect results in domestic marginal utility increasing more than

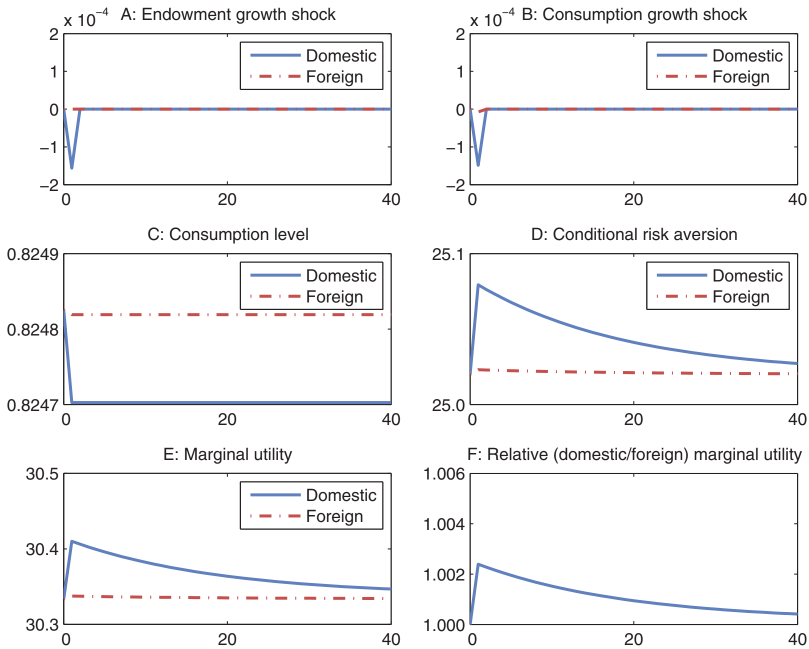

Figure 2
Impulse response functions, assuming constant consumption shares
The figure shows model impulse response functions, assuming constant consumption shares. In particular, it presents the impulse responses to a negative domestic endowment shock equal to one standard deviation. The horizontal axis measures time (in quarters), and the vertical axis the value of the variable of interest: endowment growth shocks (panel A), consumption growth shocks (panel B), consumption levels (panel C), conditional risk aversion levels (panel D), marginal utility levels (panel E), and relative (domestic/foreign) marginal utility levels (panel F). The initial state of the economy is its steady state, and the calculations use the model parameter values shown in Table 1.

foreign marginal utility does (panels E and F). Thus, external habit formation significantly amplifies the insurance cost of preference home bias, as the two components of marginal utility, consumption and risk aversion, move together.

Given the large insurance cost of constant consumption shares, the optimal risk sharing scheme conditions on the two countries' conditional risk aversion levels in a way that partly offsets the effects of consumption home bias. The impulse response functions are presented in Figure 3. Following a negative domestic endowment shock (panel A), the consumption share of the domestic agent increases for both goods, ameliorating the large negative consumption growth shock that would occur if consumption shares were constant. Conversely, the shock is amplified for the foreign agent, whose consumption shares decline. As a result, domestic consumption declines more than foreign consumption does (panel C), and domestic risk aversion increases more than foreign risk aversion does (panel D), but the disparity is significantly muted compared with the constant consumption shares case. Marginal utility

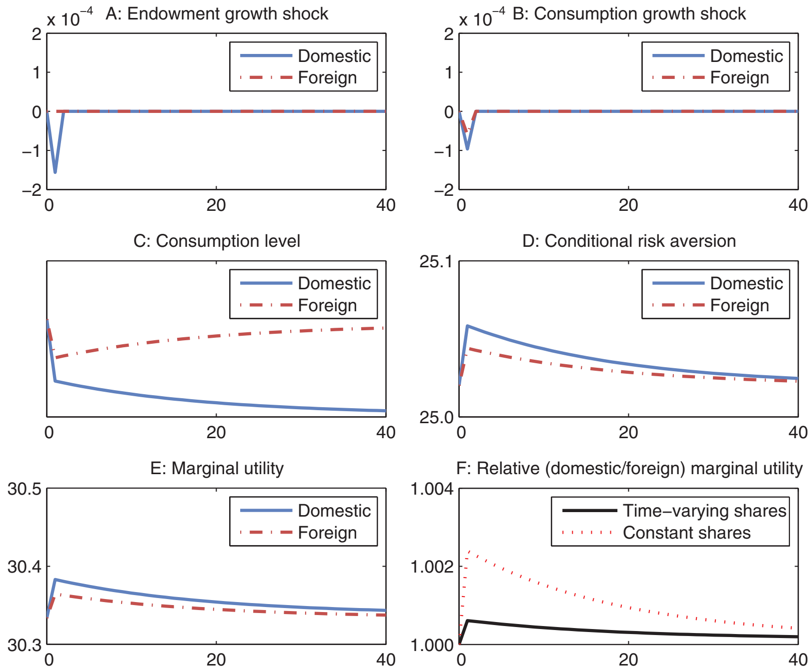

Figure 3
Impulse response functions: Consumption, risk aversion, and marginal utility
The figure shows model impulse response functions. In particular, it presents the impulse responses to a negative domestic endowment shock equal to one standard deviation. The horizontal axis measures time (in quarters), and the vertical axis the value of the variable of interest: endowment growth shocks (panel A), consumption growth shocks (panel B), consumption levels (panel C), conditional risk aversion levels (panel D), marginal utility levels (panel E), and relative (domestic/foreign) marginal utility levels (panel F). For comparison purposes, panel F also presents relative marginal utility levels under the assumption of constant consumption shares. The initial state of the economy is its steady state, and the calculations use the model parameter values shown in Table 1.

disparity is similarly blunted (panels E and F). In short, home bias introduces heterogeneity in marginal utility growth between the two countries that is amplified by habit formation via risk aversion effects, but the two countries optimally scale down this amplification by varying their consumption shares.

Notably, consumption levels do not immediately adjust to their long-run values; during the adjustment period, transitorily elevated domestic consumption requires positive net imports from the foreign country (panel C). The domestic positive net imports that occur in the adjustment period are financed by an increase in the share of global wealth owned by the domestic agent. In particular, the equilibrium share of domestic wealth as a proportion of global wealth is given by

$$\omega_t^W = \frac{W_t}{W_t + W_t^*} = \frac{\lambda\left(\rho G_t + k\bar{G}\right)}{\lambda\left(\rho G_t + k\bar{G}\right) + \lambda^*\left(\rho G_t^* + k\bar{G}\right)},$$

where W (W^*) is the wealth of the domestic (foreign) agent expressed in units of the global numeraire. The domestic wealth share is increasing in domestic risk aversion and decreasing in foreign risk aversion, so a transitorily high domestic consumption share (spurred by a relative increase in domestic risk aversion) is financed by a transitorily high domestic share of global wealth. The steady-state wealth share of the domestic country, $\bar{\omega}^W$, equals the domestic welfare weight λ, so I will often refer to welfare weights λ and λ^* as steady-state domestic and foreign wealth shares, respectively. In the absence of habit formation, the wealth shares would be constant at their steady-state values, as consumption would immediately adjust to its long-run level (panel C of Figure 2).

It should be stressed that external habit formation is unable to generate non-trivial consumption and marginal utility dynamics in the absence of *ex ante* heterogeneity in preferences. Without preference home bias, optimality implies identical consumption profiles, as the agents have no incentive to accept any deviation from the full cross-insurance of consumption pooling. In that case, consumption growth innovations are identical across countries, so G and G^* (assuming they have identical initial values) coincide state by state, resulting in identical consumption growth and marginal utility growth rates.

2.2 The conditional properties of consumption growth

The previous section discussed optimal consumption allocations taking the values of the state variables G and G^* as given. However, the realizations of conditional risk aversion are not exogenous, as they depend on endogenous consumption shocks: agents are able to affect the degree of costly cross-sectional dispersion in conditional risk aversion through their consumption decisions. Thus, understanding the dynamic evolution of domestic and foreign consumption requires explicitly solving for the two equilibrium consumption growth processes as functions of the two exogenous endowment shocks.

As will be seen, the properties of the conditional second moments of equilibrium consumption growth arise from the interaction of two effects: the curvature of the consumption level with respect to the ratio G^*/G, generated by the share mechanism described in the previous section, and the conditional heteroskedasticity of the inverse surplus consumption ratio, which is hard-wired in the Menzly, Santos, and Veronesi (2004) habit specification given in Equation (1) and its foreign counterpart.

We begin by exploring the properties of the two countries' consumption levels, C and C^*; their analytical expressions are provided in Appendix A1. Panel B of Figure 1 presents C and C^* as functions of the risk aversion ratio G^*/G. As a result of the share mechanism, C is a decreasing and convex function of G^*/G, while C^* is an increasing, concave function of the risk aversion ratio. Owing to the curvature of C and C^*, the sensitivity of consumption growth rates to fluctuations in the risk aversion ratio varies across the state space. Indeed, the domestic consumption growth rate satisfies

$$\frac{dC_t}{C_t}=a\frac{dX_t}{X_t}+(1-a)\frac{dY_t}{Y_t}=\left[a\frac{d\widetilde{X}_t}{\widetilde{X}_t}+(1-a)\frac{d\widetilde{Y}_t}{\widetilde{Y}_t}\right]-k_t\frac{d\left(G_t^*/G_t\right)}{G_t^*/G_t}+o(dt),$$

so it has two components: a component that purely reflects preference home bias, and a component induced by the concavity of the consumption level The first component is conditionally homoskedastic. The second component captures the time-varying effect of changes in the risk aversion ratio G^*/G on consumption shares, measured by the multiplier

$$k_t\equiv\lambda^*\frac{a(1-a)\lambda+a^*(1-a^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)}{\left(a\lambda+a^*\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)\left((1-a)\lambda+(1-a^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)}\left(\frac{G_t^*}{G_t}\right). \tag{5}$$

The multiplier k is graphed as a function of the risk aversion ratio in panel C of Figure 1: it is bounded in (0,1), so an increase in the risk aversion ratio always decreases domestic consumption growth. Moreover, k is increasing in G^*/G. When $G^*/G\to\infty$, in which case domestic consumption approaches zero and the foreign country consumes almost the entirety of the global endowment, domestic consumption growth is very sensitive to risk aversion fluctuations ($k\to 1$). Conversely, in the limit of $G^*/G\to 0$, when the domestic country consumes almost everything, domestic consumption growth sensitivity shuts down ($k\to 0$). Similarly, the foreign consumption growth rate satisfies

$$\frac{dC_t^*}{C_t^*}=a^*\frac{dX_t^*}{X_t^*}+(1-a^*)\frac{dY_t^*}{Y_t^*}=\left[a^*\frac{d\widetilde{X}_t}{\widetilde{X}_t}+(1-a^*)\frac{d\widetilde{Y}_t}{\widetilde{Y}_t}\right]+k_t^*\frac{d\left(G_t^*/G_t\right)}{G_t^*/G_t}+o(dt).$$

As seen in panel C of Figure 1, the foreign multiplier k^* is decreasing in G^*/G, as foreign consumption growth becomes less sensitive to risk aversion ratio fluctuations when G^*/G increases.

In short, the share mechanism induces time variation in consumption growth moments via consumption curvature, regardless of the law of motion for G^*/G. However, the specific law of motion of G^*/G is an important determinant of the properties of conditional consumption growth moments. In particular, for the share mechanism to be quantitatively first-order, the ratio G^*/G needs to be sufficiently volatile. Moreover, any time variation in the conditional volatility of G^*/G induces time variation in the magnitude of the curvature mechanism, *ceteris paribus*. Recall from Equation (1) and its foreign counterpart that the sensitivity of each agent's conditional risk aversion to her consumption growth shocks is increasing in the agent's conditional risk aversion; as a result, G^*/G has law of motion

$$\frac{d\left(G_t^*/G_t\right)}{G_t^*/G_t}=\frac{dG_t^*}{G_t^*}-\frac{dG_t}{G_t}+o(dt)=\delta\left(\frac{G_t-l}{G_t}\right)\frac{dC_t}{C_t}-\delta\left(\frac{G_t^*-l}{G_t^*}\right)\frac{dC_t^*}{C_t^*}+o(dt),$$

so it exhibits conditional heteroskedasticity.

Given that risk aversion innovations are scaled multiples of consumption growth innovations, the equilibrium consumption growth rate of the two countries is derived by solving for the fixed point for which the equilibrium consumption processes are consistent with the exogenous processes for the inverse surplus consumption ratios. The following proposition provides the expression for the domestic consumption growth rate. The expression for foreign consumption growth, as well as the proof of the proposition, can be found in Appendix A5.

Proposition 1. The equilibrium domestic consumption process satisfies

$$\frac{dC_t}{C_t} = E_t\left(\frac{dC_t}{C_t}\right) + \sigma_t^{CX} dB_t^X + \sigma_t^{CY} dB_t^Y,$$

where the exposure to domestic endowment shocks is given by

$$\sigma_t^{CX} = \frac{a + \left(ak_t^* + a^* k_t\right)\delta\left(\frac{G_t^* - l}{G_t^*}\right)}{1 + k_t \delta\left(\frac{G_t - l}{G_t}\right) + k_t^* \delta\left(\frac{G_t^* - l}{G_t^*}\right)}\sigma^X$$

and the exposure to foreign endowment shocks is given by

$$\sigma_t^{CY} = \frac{(1-a) + \left((1-a)k_t^* + (1-a^*)k_t\right)\delta\left(\frac{G_t^* - l}{G_t^*}\right)}{1 + k_t \delta\left(\frac{G_t - l}{G_t}\right) + k_t^* \delta\left(\frac{G_t^* - l}{G_t^*}\right)}\sigma^Y.$$

Conditional heteroskedasticity in consumption growth arises from the joint operation of two mechanisms: the share mechanism and the risk aversion sensitivity mechanism. The share mechanism, discussed previously, is captured by the time-varying multipliers k and k^* and solely reflects fluctuations in the ratio G^*/G. The risk aversion sensitivity mechanism is generated by the time variation that is hard-wired in the law of motion of conditional risk aversion: note that both diffusion arguments, σ_t^{CX} and σ_t^{CY}, of domestic consumption growth are decreasing in the sensitivity of domestic conditional risk aversion to consumption shocks, given by $\delta\left(\frac{G_t - l}{G_t}\right)$, and increasing in the consumption sensitivity of foreign risk aversion, given by $\delta\left(\frac{G_t^* - l}{G_t^*}\right)$. This second mechanism reflects fluctuations in G and G^*, not just in the ratio G^*/G.

The two mechanisms operate together owing to the fixed point solution for consumption growth, so it is instructive to examine their joint output. Figure 4 presents the conditional second moments of domestic and foreign equilibrium consumption growth as a function of G and G^*. Domestic consumption growth volatility is decreasing in domestic risk aversion and increasing in foreign risk aversion, while the reverse is true for foreign consumption growth volatility (panels A and B). Moreover, the conditional correlation between the two

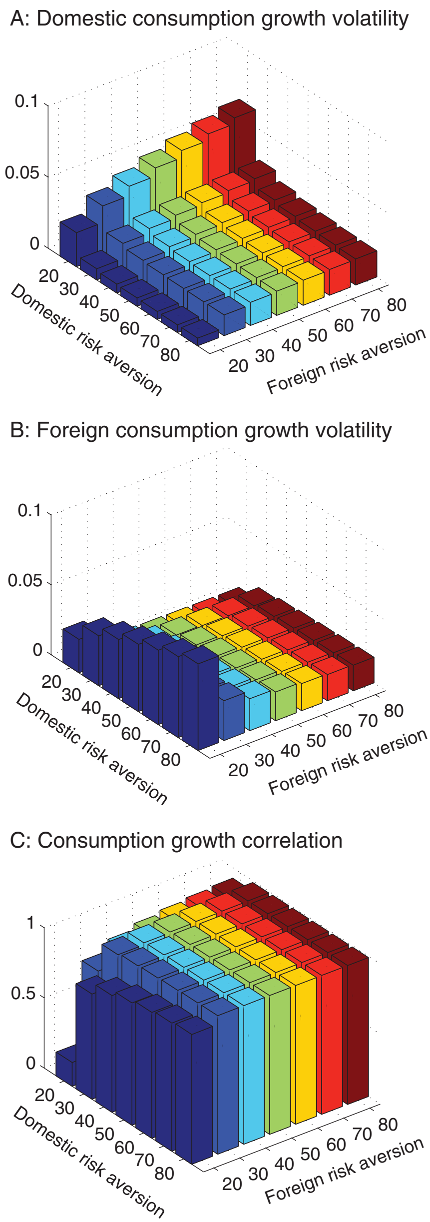

Figure 4
Conditional second moments of consumption growth
The figure shows the conditional standard deviation and the conditional correlation of consumption growth as functions of domestic and foreign risk aversion, in the model. The x-axis measures domestic conditional risk aversion G, the y-axis foreign conditional risk aversion G^*, and the z-axis the moment of interest. Panel A refers to the conditional standard deviation of domestic consumption growth, panel B to the conditional standard deviation of foreign consumption growth, and panel C to the conditional correlation between domestic and foreign consumption growth. The calculations use the model parameter values shown in Table 1.

consumption growth rates is increasing in either country's risk aversion (panel C). To understand the intuition, consider again the equilibrium allocation scheme discussed in the previous section: it optimally trades off home bias and cross-sectional dispersion in marginal utility. Given that scheme, the two countries have an incentive to implement consumption dynamics that reduce the likelihood of states characterized by large cross-country dispersion in conditional risk aversion, as those states entail costly deviations from steady-state consumption home bias.

First, consider the set of states characterized by equal conditional risk aversion across countries ($G = G^*$). In all those states, the two agents have no reason to deviate from the steady-state consumption allocation, so both countries exhibit their steady-state level of consumption home bias. However, the amount of consumption risk the two countries need to hold to achieve the optimal efficiency trade-off going forward differs across those states: the subset of high (low) conditional risk aversion states is characterized by low (high) conditional consumption growth volatility for both countries and high (low) conditional consumption growth correlation between the two countries. This is because in the high risk aversion states, both countries' conditional risk aversion is very sensitive to consumption growth shocks, so small differences in consumption growth realizations imply large differences in risk aversion realizations and, thus, costly sacrifices in home bias. As a result, the two countries have an incentive to trade in Arrow-Debreu markets in a way that ensures that the likelihood of large future cross-sectional dispersion in conditional risk aversion is low. This requires holding assets that aggressively cross-insure the two countries. Indeed, in the limit of infinite conditional risk aversion for both countries, consumption growth rates become perfectly positively correlated across countries. Conversely, when both countries have low conditional risk aversion, cross-country differences in marginal utility growth realizations are small, even when consumption growth shocks differ significantly. Therefore, countries are less incentivized to aggressively cross-insure in the securities market.

When the two countries differ in their conditional risk aversion ($G \neq G^*$), the allocation of global endowment risk is tilted towards the country that is less averse to it. When $G < G^*$, domestic consumption growth volatility is higher than its foreign counterpart, while the reverse occurs when $G > G^*$. Given that the more risk averse country has more volatile conditional risk aversion, it needs to insure more aggressively against consumption realizations that induce costly cross-sectional dispersion in conditional risk aversion and, thus, it has lower conditional consumption growth volatility. The more the two conditional risk aversion levels differ, the larger the disparity in cross-sectional volatility becomes; for example, for a given value of G^*, the disparity between domestic and foreign conditional consumption growth volatility is increasing in G, as the domestic country needs to insure itself more aggressively.

In contrast, those time-varying insurance effects are absent under log (and more generally CRRA) preferences. In the absence of state-contingent differences in the two countries' risk aversion, the efficiency trade-off between preference home bias and cross-insurance does not exhibit time variation. As a result, equilibrium consumption shares are constant, so homoskedastic endowment growth implies homoskedastic consumption growth and constant consumption growth correlation. For example, domestic consumption growth has diffusion arguments

$$\sigma_t^{CX} = a\sigma^X, \quad \sigma_t^{CY} = (1-a)\sigma^Y.$$

2.3 The conditional properties of marginal utility growth

We have seen that habit-induced time variation in conditional risk aversion introduces a novel dimension in international risk sharing: countries insure each other by dynamically reallocating global consumption risk depending on the cross-section of their conditional risk aversion levels. We now explore the implications of that insurance mechanism for marginal utility growth.

The growth rate of the marginal utility of consumption of the domestic agent has two parts, consumption growth and changes in conditional risk aversion:

$$\frac{d\Lambda_t}{\Lambda_t} = -\rho dt + \frac{dG_t}{G_t} - \frac{dC_t}{C_t} = o(dt) - \left(\sigma_t^{\Lambda X} dB_t^X + \sigma_t^{\Lambda Y} dB_t^Y\right),$$

where $\sigma^\Lambda \equiv [\sigma^{\Lambda X}, \sigma^{\Lambda Y}]'$ is the vector of the exposure of the domestic SDF to the two endowment shocks (i.e., the market price of risk vector). The market price of risk vector is given by

$$\sigma_t^\Lambda = \sigma_t^C - \sigma_t^G = \left(1 + \delta \frac{G_t - l}{G_t}\right)\sigma_t^C, \tag{6}$$

where $\sigma^C \equiv [\sigma^{CX}, \sigma^{CY}]'$ is the diffusion process of consumption growth, and $\sigma^G \equiv -\delta\left(\frac{G-l}{G}\right)\sigma^C$ is the diffusion process of risk aversion changes. The market price of risk is the product of two components: conditional SDF sensitivity to consumption growth shocks and conditional consumption growth volatility. As is standard in external habit models of the Campbell and Cochrane (1999) class, the conditional sensitivity term is time varying, as it is an increasing function of conditional risk aversion G by assumption (recall Equation [1]). In our model, consumption growth volatility is also time varying, not by assumption but as an endogenous outcome of international risk sharing, as discussed in the previous section.

The key insight here is that the international risk sharing mechanism tends to generate almost identical conditional SDF volatility across countries. Assume, for example, that conditional risk aversion is higher in the domestic country: $G > G^*$. As described in Equation (1) and its foreign counterpart, each country's SDF sensitivity to consumption growth shocks is increasing in its conditional risk aversion, so the condition $G > G^*$ implies that SDF sensitivity is higher in the domestic country. However, the domestic country, being the conditionally more risk averse one, has lower consumption growth volatility compared with the foreign country, owing to the international risk sharing mechanism. In short, the domestic country has high SDF sensitivity to consumption risk and smooth consumption growth, whereas the foreign country has low SDF sensitivity and volatile consumption growth. The reverse occurs when $G < G^*$: the relatively low SDF sensitivity of the domestic country is offset by its relatively high consumption growth volatility, and vice versa for the foreign country. Therefore, the insurance mechanism tends to offset the heterogeneity introduced by the habit-induced SDF sensitivity specification and, thus, to equalize the market price of risk in the two countries state by state.

Figure 5 presents the conditional second moments of marginal utility growth (panels A, B and C) and those of its first-order component, changes in conditional risk aversion (panels D, E and F). For both countries, risk aversion changes and marginal utility growth become more volatile when either country's risk aversion increases. For example, a domestic risk aversion increase increases the sensitivity of domestic risk aversion to consumption growth shocks, and this effect dominates the reduction in its consumption growth volatility. Moreover, it increases foreign consumption growth volatility owing to the risk sharing mechanism, increasing foreign SDF volatility. As regards conditional correlation, recall that risk aversion shocks are perfectly correlated with consumption growth shocks, so the conditional correlation of both risk aversion changes and marginal utility growth is identical to the conditional correlation of consumption growth and, thus, inherits its properties.

Although the conditional correlation of SDFs is equal to the conditional correlation of consumption growth rates state by state, this does not imply that unconditional SDF correlation equals unconditional consumption growth correlation. This is because unconditional correlation depends not only on conditional correlation, but also on conditional volatility. In the model, the risk sharing mechanism results in the two countries exhibiting negatively correlated conditional consumption growth volatility and positively correlated conditional SDF volatility. Indeed, the model simulation, to be discussed in detail in the next section, yields $corr(\sigma_t^C, \sigma_t^{C^*}) = -0.74$ and $corr(\sigma_t^\Lambda, \sigma_t^{\Lambda^*}) = 0.94$. As a result, consumption growth rates are less correlated unconditionally than their average conditional correlation would suggest, whereas SDFs strongly comove.

The key feature of the model is the international risk sharing mechanism, so it is useful to reiterate the importance of the assumption that the sensitivity of risk aversion changes to consumption growth shocks is increasing in conditional risk aversion. If risk aversion sensitivity was assumed to be constant, the conditional volatility of each country's marginal utility growth would be a constant multiple of the conditional volatility of its consumption growth, so $corr(\sigma_t^\Lambda, \sigma_t^{\Lambda^*}) = corr(\sigma_t^C, \sigma_t^{C^*})$. In that case, the unconditional SDF correlation would roughly equal the unconditional

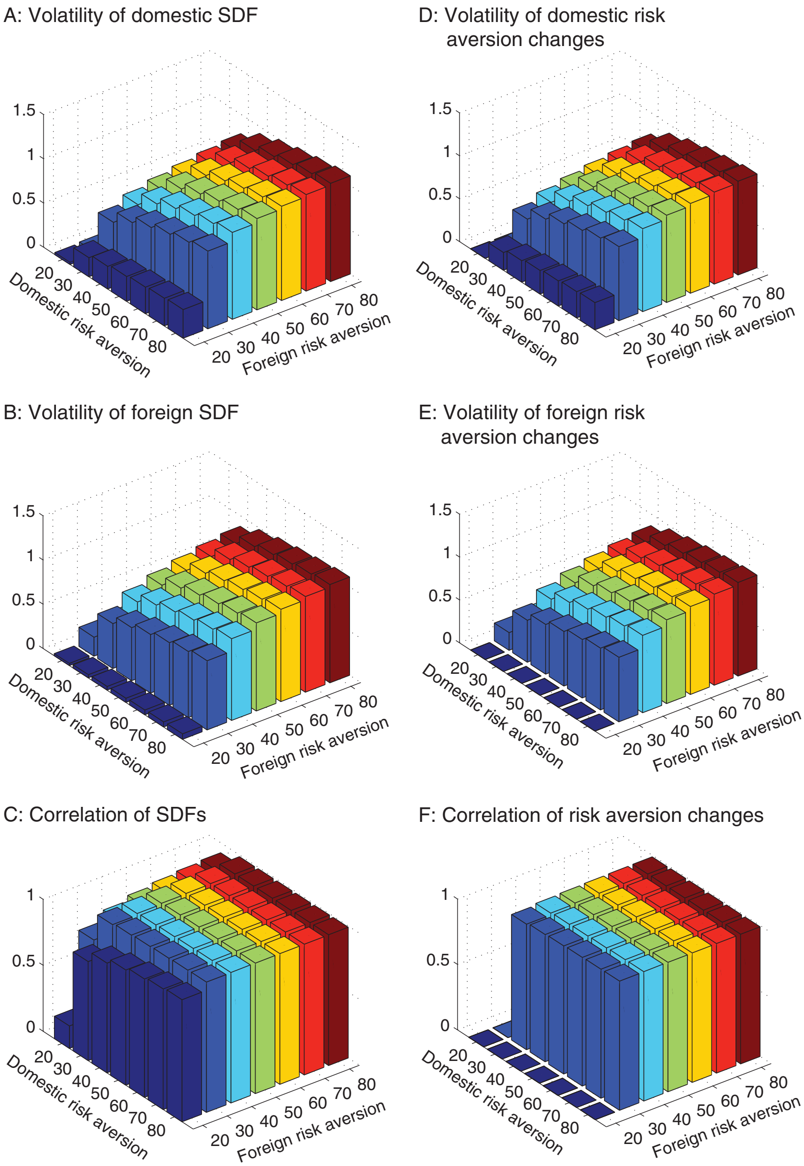

Figure 5
Conditional second moments of stochastic discount factors
The figure shows the conditional standard deviation and the conditional correlation of stochastic discount factors (SDFs) and changes in conditional risk aversion as functions of domestic and foreign risk aversion, in the model. The x-axis measures domestic conditional risk aversion G, the y-axis foreign conditional risk aversion G^*, and the z-axis the moment of interest. Panel A refers to the conditional standard deviation of the domestic SDF, panel B to the conditional standard deviation of the foreign SDF, panel C to the conditional correlation between the domestic and the foreign SDF, panel D to the conditional standard deviation of the change in domestic conditional risk aversion G, panel E to the conditional standard deviation of the change in the foreign conditional risk aversion G^*, and panel F to the conditional correlation between the change in the domestic and the foreign conditional risk aversion. The calculations use the model parameter values shown in Table 1.

consumption growth correlation (with perfect equality broken by small drift effects), and the Brandt, Cochrane, and Santa-Clara (2006) puzzle could not be addressed. In closed-economy settings, the assumption of countercyclical sensitivity of risk aversion changes to consumption growth shocks ensures that the model generates asset return predictability. In our open-economy model, this assumption also affects the countries' trade-off between home bias and cross-insurance, generating a particular risk sharing mechanism which helps address the Brandt, Cochrane, and Santa-Clara (2006) puzzle.

2.4 The real exchange rate

The price of each consumption basket is defined as the minimum expenditure required to buy a unit of the basket and is derived by minimizing the corresponding expenditure function. Given that the global numeraire is the domestic consumption basket, its price is $P_t = 1$ for all t by definition. The global numeraire price of the foreign consumption basket C^* is a scaled, home-biased geometric average of the prices of the two goods,

$$P_t^* = \left(\frac{Q_t}{a^*}\right)^{a^*} \left(\frac{Q_t^*}{1-a^*}\right)^{1-a^*},$$

and the real exchange rate is the relative price of the two consumption baskets,

$$E_t = \frac{P_t^*}{P_t} = \frac{a^a (1-a)^{1-a}}{(a^*)^{a^*}(1-a^*)^{1-a^*}} \left(\frac{Q_t^*}{Q_t}\right)^{a-a^*}.$$

Although the law of one price holds, the price of the two consumption baskets may differ because of differences in their composition. In particular, when $a \neq a^*$ the real exchange rate is time-varying and purchasing power parity fails. In that case, real exchange rate volatility is increasing in the volatility of the terms of trade Q^*/Q.

The equilibrium real exchange rate is increasing in the endowment ratio $\widetilde{X}/\widetilde{Y}$ and the risk aversion ratio G^*/G, the latter through the domestic share ratio ω/ω^*:

$$E_t = \left(\frac{a}{a^*}\right)^{a^*} \left(\frac{1-a}{1-a^*}\right)^{1-a^*} \left(\frac{\omega_t}{\omega_t^*}\right)^{a-a^*} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{a-a^*}. \tag{7}$$

Fluctuations in the relative supply of the two goods generate real exchange rate changes: this is the well-known endowment mechanism. In the presence of preference home bias, the effects of the endowment mechanism are significantly amplified by the risk aversion mechanism, so real exchange rate volatility is much higher than in the log utility benchmark. Panels A through D of Figure 6 illustrate the effects of a negative domestic endowment shock: it increases the relative price of the domestic good not only by lowering its relative supply, but also by increasing its relative demand. The demand effect is due to the fact

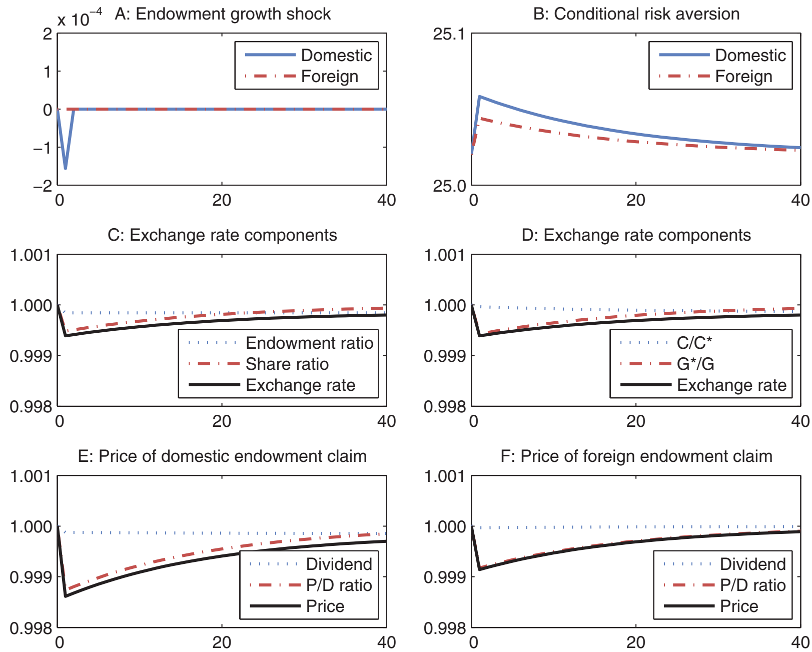

Figure 6
Impulse response functions: Real exchange rate and asset prices
The figure shows model impulse response functions. In particular, it presents the impulse responses to a negative domestic endowment shock equal to one standard deviation. The horizontal axis measures time (in quarters), and the vertical axis the value of the variable of interest: endowment growth shocks (panel A), consumption growth shocks (panel B), the components of the real exchange rate (panels C and D), the components of the price of the domestic endowment claim (panel E), and the components of the price of the foreign endowment claim (panel F). The initial state of the economy is its steady state, and the calculations use the model parameter values shown in Table 1.

that the shock increases domestic risk aversion more than foreign risk aversion (panel B), leading to a relative increase in domestic consumption, which is tilted toward the domestic good because of home bias. Panel C of Figure 6 presents both effects: the supply effect is reflected in the decrease of the endowment ratio, while the demand effect is reflected in the decrease of the domestic share ratio, which declines when domestic consumption increases in relative terms, as domestic consumption home bias goes down.

We can also consider an alternative decomposition of the real exchange rate, which is instructive for the discussion of the Backus and Smith (1993) finding that consumption growth differences appear to be disconnected from real exchange rate changes. Specifically, the real exchange rate equals to the ratio of the two countries' marginal utility of consumption,

$$E_t = \frac{\Lambda_t^*}{\Lambda_t} = \frac{\lambda^*}{\lambda} \frac{C_t}{C_t^*} \frac{G_t^*}{G_t}, \tag{8}$$

so the foreign currency appreciates in real terms when either domestic consumption growth exceeds its foreign counterpart or when foreign risk aversion changes exceed domestic risk aversion changes:

$$\frac{dE_t}{E_t} = o(dt) + \left[\frac{dC_t}{C_t} - \frac{dC_t^*}{C_t^*}\right] + \left[\frac{dG_t^*}{G_t} - \frac{dG_t}{G_t}\right]. \tag{9}$$

Panel D of Figure 6 suggests that the risk aversion effect amplifies the consumption effect: a negative domestic endowment shock not only lowers domestic consumption more than foreign consumption but also raises domestic risk aversion more than foreign risk aversion. If risk aversion is sufficiently sensitive to consumption growth shocks, the quantitatively first-order effect is the risk aversion one, so real exchange rate dynamics are largely determined by fluctuations in relative risk aversion.

This matters significantly for the resolution of the Backus and Smith (1993) puzzle. Panels A to C of Figure 7 present conditional moments relevant to real exchange rate changes. The first two panels illustrate the conditional correlation between real exchange rate changes and each of the two components in Equation (9), cross-country differences in consumption growth rates (panel A) and differences in risk aversion changes (panel B). Starting with panel A, the conditional correlation between consumption growth rate differences and real exchange rate changes varies considerably across the state state and can even turn negative. For example, a negative relationship exists when two countries' risk aversion levels are very different: in those states, positive consumption growth differentials are associated with negative marginal utility growth differentials. This is because of the disparity in the risk aversion sensitivity of the two countries to consumption shocks, which generates risk aversion change differentials that have a sign opposite to that of consumption growth differentials. On the contrary, there is a tight link between real exchange rate changes and risk aversion changes: given that the risk aversion component of the SDF is the dominant one, the conditional correlation between differences in risk aversion changes and real exchange rate changes is almost always close to perfect, as seen in panel B. As a result, real exchange rate changes are almost perfectly correlated unconditionally with differences in risk aversion changes and appear disconnected from consumption growth differentials.

The conditional volatility of real exchange rate changes is increasing in the volatility of the two countries' SDFs and decreasing in their correlation. To understand those properties, recall that, as implied from Equation (9), the conditional volatility of real exchange rate changes reflects the volatility of differences in marginal utility growth across countries. When either country's risk aversion increases, the two SDFs become more volatile and more correlated with each other; the first effect tends to increase real exchange rate volatility, whereas the second effect tends to reduce it, so the behavior of the real exchange rate volatility is determined by the relative strength of the two effects. As seen in panel C of Figure 7, the two effects have different relative strengths at

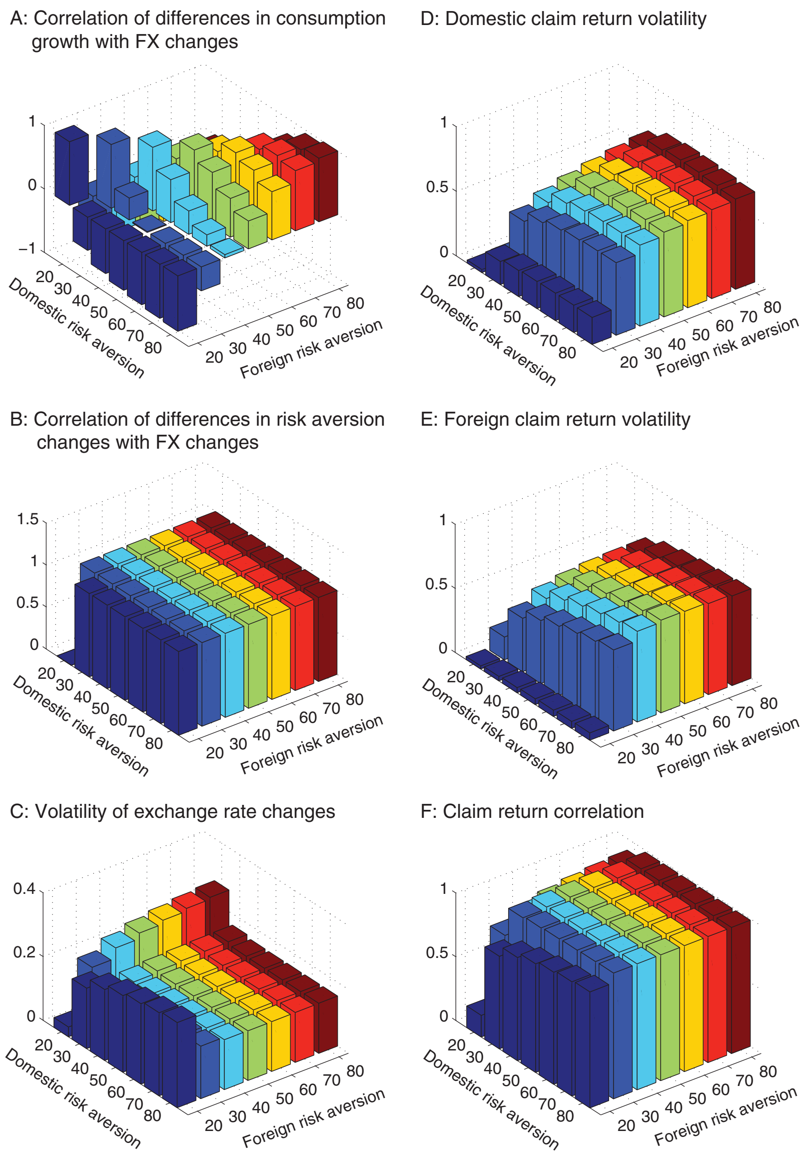

Figure 7
Conditional conditional second moments of real exchange rate changes and endowment claim returns
The figure shows conditional second moments of real exchange rate changes and endowment claim returns as functions of domestic and foreign risk aversion. The x-axis measures domestic conditional risk aversion G, the y-axis foreign conditional risk aversion G^*, and the z-axis the moment of interest. Panel A refers to the conditional correlation between the consumption growth differential of the two countries and real exchange rate changes, panel B to the conditional correlation between the conditional risk aversion change differential of the two countries and real exchange rate changes, panel C to the conditional standard deviation of real exchange rate changes, panel D to the conditional standard deviation of the return of the domestic endowment claim, panel E to the conditional standard deviation of the return of the foreign endowment claim, and panel F to the conditional correlation between the returns of the two endowment claims. The calculations use the model parameter values shown in Table 1.

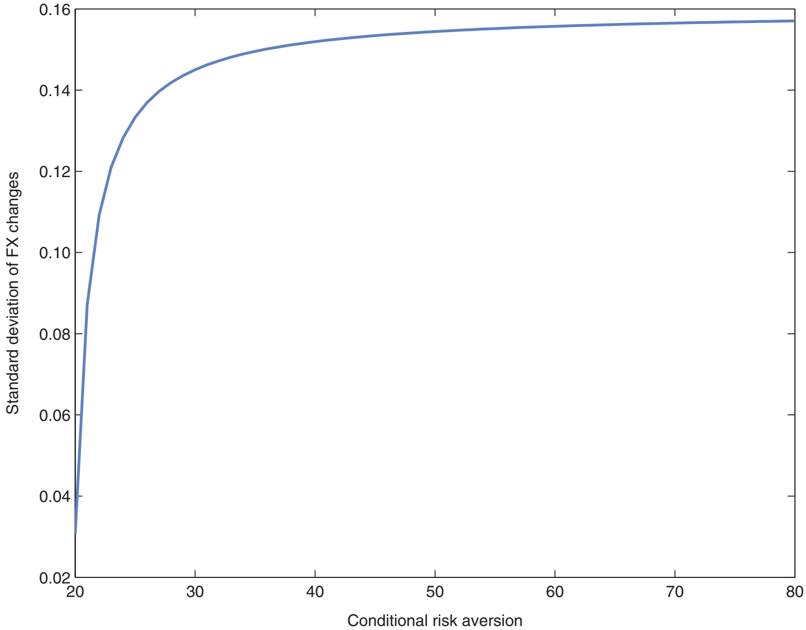

Figure 8
Conditional standard deviation of real exchange rate changes
The figure shows the conditional standard deviation of real exchange rate changes as a function of conditional risk aversion, in the model, assuming that conditional risk aversion is equal in the two countries. The calculations use the model parameter values shown in Table 1.

different regions of the state space. When conditional risk aversion does not differ much across the two countries, the dominant effect is the volatility one, so real exchange rate volatility is countercyclical: Figure 8 presents the conditional standard deviation of real exchange rate changes as a function of conditional risk aversion, assuming that risk aversion is equal across the two countries. As we will see in the simulation section, the two countries' conditional risk aversion levels are usually not far apart in equilibrium, so our model suggests that the conditional volatility of real exchange rate changes is countercyclical.[2]

2.5 Asset prices

We now turn to pricing the claims to each of the two countries' endowment streams; in the remainder of the paper, we will map endowment claims to equity. Asset prices are expressed in the local numeraire of each country, so V (V^*) denotes the price of the domestic (foreign) endowment claim in units of the

[2] This result provides a theoretical justification for the findings of Menkhoff, Sarno, Schmeling, and Schrimpf (2012), who argue that conditional exchange rate volatility is a risk factor with pricing power for the cross-section of currency returns. In our model, conditional exchange rate volatility should be priced in all asset classes because it proxies for global conditional risk aversion.

domestic (foreign) consumption good. The following proposition, the proof of which can be found in Appendix A5, provides the results.

Proposition 2. The price-dividend ratio of the domestic endowment claim is

$$\frac{V_t}{\widetilde{X}_t Q_t} = \frac{1}{\rho}\left(\frac{k}{\rho+k}\frac{(a\lambda+a^*\lambda^*)\bar{G}}{a\lambda G_t+a^*\lambda^* G_t^*}+\frac{\rho}{\rho+k}\right), \tag{10}$$

and the price-dividend ratio of the foreign endowment claim is

$$\frac{V_t^*}{\widetilde{Y}_t \frac{Q_t^*}{P_t^*}} = \frac{1}{\rho}\left(\frac{k}{\rho+k}\frac{((1-a)\lambda+(1-a^*)\lambda^*)\bar{G}}{(1-a)\lambda G_t+(1-a^*)\lambda^* G_t^*}+\frac{\rho}{\rho+k}\right). \tag{11}$$

The price-dividend ratio of each claim is determined by a weighted average of the two countries' risk aversion levels, with the weights depending on the home bias parameters and steady-state wealth shares. In particular, the weight on domestic (foreign) risk aversion for the domestic claim is $\frac{a\lambda}{a\lambda+a^*\lambda^*}$ ($\frac{a^*\lambda^*}{a\lambda+a^*\lambda^*}$), which equals $\bar{\omega}$ $(1-\bar{\omega})$, the steady-state share of the domestic endowment consumed by the domestic (foreign) country. Therefore, what matters for the valuation of each endowment claim is an average of risk aversion levels weighted by the steady-state consumption shares for the corresponding good: domestic asset prices are greatly affected by foreign risk aversion if either the foreign country is wealthy compared with the domestic country (i.e., λ^* is high relative to λ) or if it has a strong preference for the foreign good (i.e., a^* is high) and, as a result, consumes a large proportion of the foreign good on average. Naturally, this generates size effects: for example, since the U.S. economy is large compared with almost all other economies, U.S. conditional risk aversion should have a large effect on other countries' asset prices and returns, whereas foreign countries' risk aversion should have a relatively small effect on U.S. asset prices. An immediate corollary is that fluctuations in large countries' risk aversion are an important driver of asset valuations globally. Moreover, it follows that the asset prices of small countries and countries that export a large proportion of their output (high a^*) should be very sensitive to changes in global risk aversion. Country size has been studied before in international asset pricing, but not in the context of habit formation. For example, Martin (2011) and Hassan (2013) explore the effects of country size on asset prices in open economies, but their models feature standard preferences, so there are no effects that arise from fluctuations in conditional risk aversion.

To understand the effects of economy openness, it would be helpful to first consider the closed economy case ($a=1$ and $a^*=0$). Then, the price-dividend ratio of the domestic claim is the closed-economy expression in Menzly, Santos, and Veronesi (2004):

$$\frac{V_t}{\widetilde{X}_t} = \frac{1}{\rho}\left(\frac{k}{\rho+k}\frac{\bar{G}}{G_t}+\frac{\rho}{\rho+k}\right).$$

In the absence of external habit formation, the price-dividend ratio would be constant at $\frac{1}{\rho}$, so endowment shocks would have only direct, proportional effects on asset prices. With habit formation, however, endowment shocks affect the claim price not only through the current dividend, but also through the price-dividend ratio. The valuation effect reinforces the cash-flow effect, so asset price volatility is amplified.

In our open economy, both the cash-flow and the valuation channels operate, but the mechanics are more complex owing to terms-of-trade effects. For example, consider the effect of a negative domestic endowment growth shock. Figure 6 illustrates the response of the prices of the two endowment claims. Recall that both countries' risk aversion levels increase, but the increase is higher in the domestic country because of consumption home bias (panel B). The cash-flow and valuation effects for the domestic claim are presented in panel E and for the foreign claim in panel F. For the domestic claim, the cash-flow effect is negative, but weaker than in the closed economy benchmark: a decrease in the domestic endowment leads to a less than proportional decrease in the value of the domestic dividend $\widetilde{X}Q$, because Q, the price of the domestic good, increases. The cash-flow effect is also negative for the foreign claim: the price of the foreign good declines, so the value of the foreign dividend drops, even though the quantity of the foreign dividend does not change. As regards valuation effects, both claims' valuation ratios decline, but the valuation effect for each claim depends on an appropriately weighted average of domestic and foreign risk aversion. If preferences are home-biased and the countries have equal steady-state wealth shares, each valuation ratio is more exposed to local risk aversion fluctuations. In our example, domestic risk aversion falls more than foreign risk aversion does, so the decline in the domestic valuation ratio is steeper. In sum, the valuation effect reinforces the cash-flow effect for both claims, but both effects are more pronounced for the domestic claim, so its price falls proportionally more. Notably, our economy exhibits endogenous contagion: a negative endowment shock in the domestic country generates a stock market decline in the foreign country, even though its endowment process is unaffected.

Panels D, E and F of Figure 7 present the properties of the conditional moments of the two endowment claims' returns. Panels D and E illustrate the conditional volatility of the return of the domestic and the foreign claim, respectively, and show that return volatility typically increases when the risk aversion in either country increases. This is due to the increased sensitivity of risk aversion to consumption risk, which amplifies the volatility of valuation effects.[3] Panel F shows that the conditional correlation between the two claims' returns is increasing in both countries' conditional risk aversion. This follows from the SDF properties: as discussed previously, when either country's

[3] There is one exception to the return volatility pattern: when risk aversion is very high in either country, further increases in conditional risk aversion decrease the return volatility of both endowment claims, as valuation changes become smoother. This is because when conditional risk aversion becomes very large in either country, the P/D ratio of both claims approaches $\frac{1}{\rho+k}$, so the volatility of valuation changes shuts down. In that region, increases in conditional risk aversion also induce reductions in risk premiums, for the same reason.

risk aversion increases, marginal utility growth becomes more correlated internationally owing to the increased desire of the countries to cross-insure.

Finally, Figure 9 presents the properties of the risk premiums of the two endowment claims. As seen in panels A and B, both claims' premiums are increasing in domestic and foreign conditional risk aversion. This follows directly from previous results: both the price of risk and the conditional volatility of equity returns are increasing in both countries' conditional risk aversion, so investors require a higher compensation for investing in equity when either country's risk aversion is high.

Valuation effects generate novel interactions between asset prices and the real exchange rate. In particular, we can rewrite Equation (7) as a function of the endowment ratio and the valuation ratios of the two endowment claims:

$$E_t = \frac{a^a(1-a)^{1-a}}{(a^*)^{a^*}(1-a^*)^{1-a^*}}\left(\frac{(1-a)\lambda+(1-a^*)\lambda^*}{a\lambda+a^*\lambda^*}\right)^{a-a^*} \left(\frac{\frac{V_t}{\widetilde{X}_t Q_t}-\frac{1}{\rho+k}}{\frac{V_t^*}{\widetilde{Y}_t \frac{Q_t^*}{P_t^*}}-\frac{1}{\rho+k}}\right)^{a-a^*} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{a-a^*}. \tag{12}$$

The variability in valuation ratios is typically much higher than the variability in endowment levels, so real exchange rate volatility arises mostly from the valuation mechanism. In the absence of habit formation, the valuation mechanism would shut down; thus, standard preferences severely undershoot both asset price volatility and real exchange rate volatility for the same reason. Moreover, valuation effects are stronger in states of elevated risk aversion levels, so the model generates countercyclical volatility in both asset prices and exchange rates.

3. Simulation

To calibrate and simulate the model, I map the domestic country to the United States and the foreign country to the United Kingdom. To abstract from effects that arise from *ex ante* asymmetry between the two countries, the endowment and preference specification is symmetric. As regards endowments, symmetry is achieved by assuming that both endowment processes are geometric Brownian motions with identical drift and diffusion parameters:

$$\begin{bmatrix} d\log\widetilde{X}_t \\ d\log\widetilde{Y}_t \end{bmatrix} = \mu dt + \sigma \begin{bmatrix} dB_t^X \\ dB_t^Y \end{bmatrix}.$$

The assumption that the endowment drift is constant ensures that any time variation in conditional moments is not due to endowment effects.

The model is discretized and simulated in quarterly time increments. The sample period is 1975:Q1 to 2012:Q4, for a total of 152 quarterly observations.

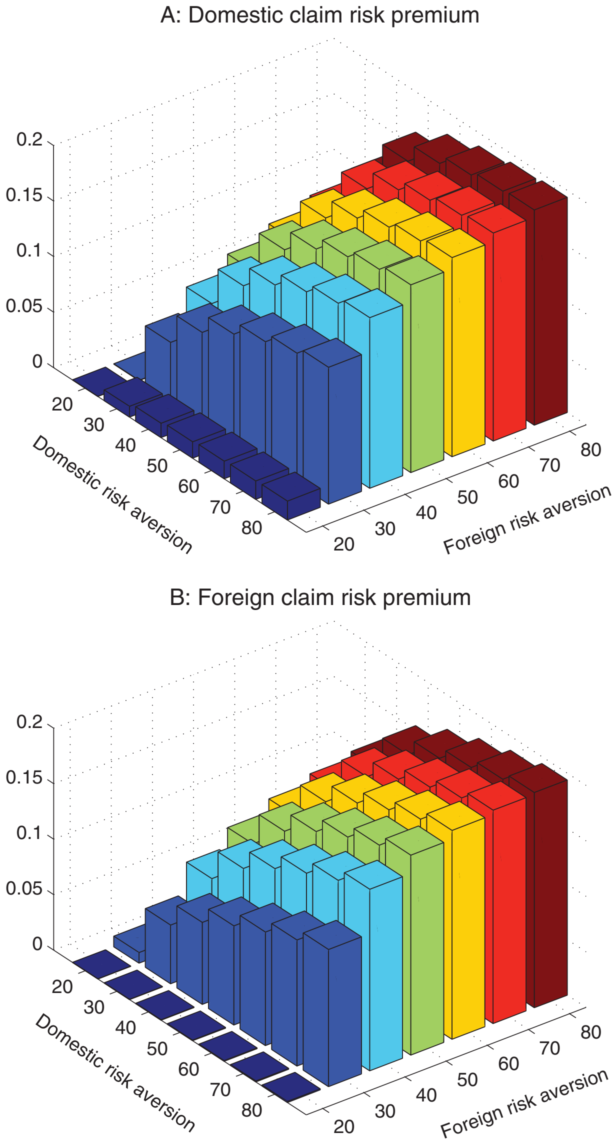

Figure 9
Endowment claim risk premiums
The figure shows the risk premiums of the endowment claims as functions of domestic and foreign risk aversion. The x-axis measures domestic conditional risk aversion G, the y-axis foreign conditional risk aversion G^{*}, and the z-axis the conditional risk premium. Panel A refers to the risk premium of the domestic endowment claim, and Panel B to the risk premium of the foreign endowment claim. The calculations use the model parameter values shown in Table 1.

Table 1
Model parameters

Parameter	Symbol	Value
Panel A: Endowment parameters		
Endowment growth mean	μ	0.0212
Endowment growth volatility	σ	0.0250
Endowment growth correlation	ρ^{XY}	0.07
Panel B: Preference parameters		
Domestic preference for the domestic good	a	0.952
Foreign preference for the domestic good	a^*	0.048
Subjective rate of time preference	ρ	0.04
Speed of G mean reversion	k	0.195
Steady-state value of G	$\bar{G}$	25.02
Lower bound of G	l	20
G sensitivity to consumption growth shocks	δ	79.39

Table 1 presents the annualized model parameters used to simulate the model with external habit formation. With the exception of the preference home bias parameters (a and a^*, for the domestic and foreign country, respectively), all other preference and endowment parameters are assumed to be identical in the two countries.

Real consumption is mapped to the sum of real *per capita* consumption of non-durable goods and services, whereas real endowment is mapped to the sum of real *per capita* consumption of non-durable goods and services and total net exports. Endowment claim returns are proxied by equity index returns. Other variable definitions, as well as the data sources, can be found in Appendix A2.

3.1 Parameter values

The model includes three endowment parameters and seven preference parameters. The endowment parameters (μ, σ and ρ^{XY}) are set using U.S. and U.K. endowment growth data, as proxied. In particular, μ and σ are set equal to the simple average of the endowment growth rate mean and standard deviation, respectively, of the two countries. The correlation coefficient ρ^{XY} is set equal to the correlation coefficient of the U.S. and U.K. endowment growth rates. As regards the preference parameters, the home bias parameters a and a^* are set to satisfy $a = 1 - a^*$ in order to ensure symmetry between the two countries. This reduces the number of free preference parameters to 6: the home bias parameter a, 4 habit-related parameters (k, $\bar{G}$, δ and l), and the time discount rate ρ. The selection details for the preference parameters are reported in Appendix A3. The annualized parameter values used in the simulation are reported in Table 1.

3.2 Simulation results

I simulate 10,000 sample paths of the model economy. For each path, the system is initialized at the steady state ($G_1 = G_1^* = \bar{G}$), and the endowment normalization $\widetilde{X}_1 = \widetilde{Y}_1 = 1$ is adopted. Each path consists of 192 quarterly observations, but the first 40 observations (10 years) are discarded to reduce the dependence on initial conditions, so moments for each sample path are calculated using 152 observations (38 years), to match the size of the empirical dataset. To explore the quantitative effects of external habit formation, I also perform an identical simulation exercise shutting down habit formation, in which case both economies have standard log preferences. The log economy model is a special case of the habit formation model for $k=0$, $\delta=0$, $\lambda=1$ and $\bar{G}=1$, which yield a constant habit level $H_t = H_t^* = 0$ for all t.

Table 2 presents key simulated moments, as well as the corresponding empirical moments for the United States and the United Kingdom. For each of the moments of interest, Table 2 reports the sample average across the 10,000 paths, as well as the 95% confidence interval (2.5 and 97.5 percentiles in the cross-section of the 10,000 paths). Given the symmetric parametrization of the model, simulated moments are identical for the two countries (allowing for trivially small differences owing to sampling), so only the domestic country moments are reported.

Panel A of Table 2 presents the properties of consumption growth rates and SDFs. Consumption is endogenous in the model, so generating empirically plausible moments for consumption growth rates is an important aim of the model. Compared with the log utility benchmark, the addition of external habit formation lowers consumption growth volatility and substantially increases consumption growth correlation (from 0.17 to 0.46) because of the stronger risk sharing mechanism between the two countries. Importantly, simulated consumption growth rate volatility is low and quite close to its empirical counterpart. Moreover, consumption growth rate correlation between the two countries is modest, with the empirical value of 0.38 being not far off the model value of 0.46 and comfortably within the 95% simulation confidence interval.

The habit model generates very high correlation between the two countries' levels of conditional risk aversion: on average, the correlation of G and G^* is 0.81, indicating that cross-sectional dispersion in conditional risk aversion is not large in equilibrium because of the desire of countries to align their marginal utility growth. Indeed, the mean of the risk aversion ratio G^*/G is 1.01, with its standard deviation being 0.11, suggesting that large cross-sectional disparity in conditional risk aversion is rare.[4] As a result of the international alignment of conditional risk aversion, the state prices of the two countries are very volatile, and they move together: not only is SDF correlation high on average (0.88), but it is also high across all simulated paths, with the 95% confidence interval being $[0.70, 0.98]$. Thus, the model is able to deliver highly correlated SDFs, along with moderately correlated consumption growth rates, addressing the Brandt, Cochrane, and Santa-Clara (2006) puzzle. As expected, despite the consumption growth smoothness, SDFs are very volatile

4 The kurtosis of the risk aversion ratio is 5.58 on average, suggesting that the likelihood of states characterized by large cross-country dispersion in conditional risk aversion, albeit small, is non-trivial. The average skewness of the risk aversion ratio is 0.47, but it varies a lot across simulation paths; its 95% confidence interval is $[-1.49, 2.63]$.

Table 2
Simulation results

Moment	Model (no habit)	Model	Data: United States	Data: United Kingdom
Panel A: Endowment, consumption, and marginal utility				
Endowment growth mean	2.12% [1.31%,2.91%]	2.12% [1.31%,2.91%]	1.95%	2.29%
Endowment growth rate st. dev.	2.50% [2.22%, 2.78%]	2.50% [2.22%,2.78%]	1.16%	3.84%
Endowment growth correlation	0.07 [−0.09,0.23]	0.07 [−0.09,0.23]	0.07	
Consumption growth mean	2.12% [1.35%,2.88%]	2.12% [1.36%,2.87%]	2.03%	2.47%
Consumption growth st. dev.	2.39% [2.12%,2.66%]	2.17% [1.78%,2.64%]	0.87%	1.71%
Consumption growth correlation	0.17 [0.01,0.32]	0.46 [0.23,0.61]	0.38	
G correlation	–	0.81 [0.36,1.00]	–	
G^*/G mean	–	1.01 [0.93,1.11]	–	
G^*/G st. dev.	–	0.11 [0.06,0.24]	–	
Log SDF st. dev.	2.39% [2.12%,2.66%]	27.90% [12.72%,63.12%]	–	–
Log SDF correlation	0.17 [0.01,0.32]	0.88 [0.70,0.98]	–	
Panel B: Real exchange rate				
Log terms of trade change st.dev.	3.40% [3.02%,3.79%]	12.84% [9.87%,17.34%]	9.88%	
Log real exchange rate change st.dev.	3.08% [2.73%,3.42%]	11.61% [8.93%,15.67%]	10.52%	
$corr(\Delta \log E_{t+1}, \Delta \log C_{t+1} - \Delta \log C^*_{t+1})$	1.00 [1.00,1.00]	0.23 [−0.33,0.58]	0.11	
Panel C: Asset prices and returns				
Equity P/D mean	25.00 [25.00,25.00]	27.33 [24.65,28.76]	41.89	25.63
Equity excess return mean	0.06% [−0.72%,0.82%]	5.39% [2.27%,8.81%]	5.62%	5.60%
Equity excess return st. dev.	2.39% [2.12%,2.66%]	20.59% [11.81%,31.61%]	16.54%	16.83%
Equity excess return correlation	0.17 [0.01,0.32]	0.88 [0.72,0.96]	0.75	
Sharpe ratio	0.02 [−0.30,0.34]	0.28 [0.09,0.47]	0.34	0.33
Risk-free rate mean	6.09% [6.09%,6.09%]	3.44% [2.16%,5.87%]	1.34%	2.58%
Risk-free rate st. dev.	0.00% [0.00%,0.00%]	1.57% [0.67%,3.15%]	1.89%	2.57%
Risk-free rate correlation	–	0.83 [0.52,0.98]	0.47	
$corr(\Delta \log E_{t+1}, R^e_{t+1})$	0.64 [0.54,0.73]	0.22 [−0.22,0.59]	0.14	
$corr(\Delta \log E_{t+1}, R^{e*}_{t+1})$	−0.64 [−0.73,0.54]	−0.22 [−0.59,0.22]	−0.02	

Table 2 presents a comparison of simulated and empirical annualized moments. The empirical moments are calculated using U.S. and U.K. data from 1975:Q1 to 2012:Q4. To calculate the moments for each of the two models (without habit formation and with habit formation), I simulate 10,000 sample paths of the model economy, with each sample path consisting of 152 quarterly observations, as many as available in the dataset. For each of the moments of interest, the table presents the sample average across the 10,000 simulations, as well as the 95% confidence interval (2.5 and 97.5 percentiles across simulations) in brackets.

owing to external habit formation. In contrast, the log utility model is by construction unable to generate SDF volatility higher than consumption growth volatility.

Panel B presents moments related to the real exchange rate. In the absence of habit formation, the volatility of real exchange rate changes is only 3.08%, significantly lower than the empirical value of 10.52%, illustrating the inability of standard preferences to generate realistic real exchange rate dynamics. When habit formation is added to the model, the average volatility of simulated log real exchange rate changes increases to 11.61%. Crucially, simulated real exchange rate changes are significantly more volatile compared with fundamentals (endowment and consumption growth rates), addressing the issue raised by Obstfeld and Rogoff (2000). Changes in the log terms of trade are more volatile in the model than in the data, suggesting that terms of trade fluctuations may not be the only driver of real exchange rates empirically. The last moment of panel B illustrates the ability of the habit model to resolve the Backus and Smith (1993) exchange rate disconnect puzzle: the correlation of consumption growth rate differentials with real exchange rate changes in the simulated data is 0.23, not only significantly below the log utility model correlation of 1, but also reasonably close to its empirical value (0.11).

Panel C evaluates the ability of the model to match the properties of asset prices and returns. The failure of the log utility model to match asset pricing dynamics is obvious and well-documented: average equity excess returns and Sharpe ratios are tiny, equity return volatility is counterfactually low, and real interest rates are constant and too high. In contrast, external habit formation allows for asset pricing moments to be successfully matched. Specifically, the model generates substantial equity premiums, volatile equity excess returns, and equity Sharpe ratios in line with their empirical values. The substantial decrease in average real interest rate levels compared with the log utility case is due to the very strong precautionary savings motive that operates in the habit economy. Importantly, and in contrast to the log utility model, the habit model is able to generate significant cross-country correlation in risk-free rates and equity excess returns, in line with the data.

Finally, the model is able to reproduce the low correlation between equity excess returns and real exchange rate changes. In the absence of habit formation, foreign currency appreciation is strongly positively associated with domestic equity excess returns and negatively associated with foreign returns, because of cash-flow effects: positive domestic (foreign) endowment shocks, which generate a positive domestic (foreign) equity excess return, increase (decrease) the relative scarcity, and thus the relative price, of the foreign good. Adding habit formation to the model reduces the comovement of real exchange rate changes with equity excess returns because of valuation effects: risk aversion changes are highly correlated across countries in equilibrium, so differences in risk aversion changes (which drive exchange rates) are small compared with overall risk aversion changes (which drive asset returns). Thus, asset returns move

Table 3
SDF components

Moment	Model (no habit)	Model
Panel A: SDF decomposition		
$std(d\log C)$	2.39%	2.17%
	[2.12%,2.66%]	[1.78%,2.64%]
$std(d\log G)$	–	26.73%
		[11.23%,62.07%]
$corr(d\log C, d\log G)$	–	-0.51
		[−0.70,−0.19]
$corr(d\log\Lambda, d\log C)$	-1.00	-0.57
	[−1.00,−1.00]	[−0.77,−0.23]
$corr(d\log\Lambda, d\log G)$	–	1.00
		[1.00,1.00]
Panel B: International comovement and real exchange rate decomposition		
$corr(d\log C, d\log C^*)$	0.17	0.46
	[0.01,0.32]	[0.23,0.61]
$corr(d\log G, d\log G^*)$	–	0.87
		[0.67,0.98]
$std(d\log C - d\log C^*)$	3.08%	2.27%
	[2.73%,3.42%]	[1.83%,2.99%]
$std(d\log G^* - d\log G)$	–	11.36%
		[8.05%,16.76%]
$corr(d\log C - d\log C^*, d\log G^* - d\log G)$	–	0.04
		[−0.48,0.40]
$corr(d\log E, d\log C - d\log C^*)$	1.00	0.23
	[1.00,1.00]	[−0.33,0.58]
$corr(d\log E, d\log G^* - d\log G)$	–	0.98
		[0.97,0.99]
Panel C: Marginal utility growth for each good		
$std(d\log\Xi)$	2.50 %	28.05 %
	[2.22%,2.78%]	[12.87%,63.16%]
$corr(d\log\Xi, d\log\Xi^*)$	0.07	0.85
	[−0.09,0.23]	[0.64,0.98]

Table 3 presents simulated annualized moments. To calculate the moments for each of the two models (without habit formation and with habit formation), I simulate 10,000 sample paths of the model economy, with each sample path consisting of 152 quarterly observations, as many as available in the dataset. For each of the moments of interest, the table presents the sample average across the 10,000 simulations, as well as the 95% confidence interval (2.5 and 97.5 percentiles across simulations) in brackets. Panels A, B, and C present moments pertaining to SDF components, international comovement and real exchange rate change components, and marginal utility growth for each good, respectively.

together internationally, being largely unaffected by the small cross-country valuation differences captured by real exchange rates.

Table 3 further explores the SDF properties in the model. Panel A focuses on the decomposition of the log SDF into two components: consumption growth and changes in conditional risk aversion. Changes in conditional risk aversion ($d\log G$) account for almost all SDF volatility: the annualized standard deviation of $d\log G$ is 26.73%, while the volatility of the other SDF component, consumption growth, is an order of magnitude lower. Moreover, although the conditional correlation between the two SDF components is -1 by construction, their unconditional correlation is only -0.51. Owing to the large contribution of $d\log G$ to total SDF volatility, $d\log G$ is perfectly correlated with the SDF, whereas consumption growth correlation with the SDF is much lower (-0.57).

Panel B examines how the two SDF components comove across countries and determine the properties of the real exchange rate. As expected, the high SDF correlation in the habit model (0.88) arises from the high correlation between the first-order SDF components, conditional risk aversion changes (0.87). Despite the high international correlation of risk aversion changes, their size is such that they still contribute the bulk of real exchange rate volatility: annualized $std(d\log G^{*}-d\log G)$ is 11.36%, whereas annualized $std(d\log C-\log C^{*})$ is only 2.27%. Given that those two exchange rate components are almost uncorrelated (their correlation is only 0.04), differences in conditional risk aversion changes account for about 96% of real exchange rate variance. It follows that real exchange rate changes are almost perfectly correlated with conditional risk aversion change differentials (correlation of 0.98), but have low correlation with consumption growth differentials, addressing the Backus and Smith (1993) puzzle.

Finally, panel C of Table 3 considers the decomposition of the log SDF into a weighted average of the log marginal utility growth rates of the two goods:

$$d\log\Lambda_t=\alpha(d\log\Xi_t)+(1-\alpha)(d\log\Xi_t^{*}),$$

and

$$d\log\Lambda_t^{*}=\alpha^{*}(d\log\Xi_t)+(1-\alpha^{*})(d\log\Xi_t^{*}).$$

Given that the two goods' marginal utility growth rates are very highly correlated (the correlation coefficient is 0.85), the standard deviation of each of the two growth rates (28.05%) is about the same as the standard deviation of each log SDF (27.90%).

We conclude with Table 4, which presents the properties of key conditional moments, contrasting the log utility model with the habit model. In the absence of external habit formation, the conditional volatility of consumption growth rates, SDFs, real exchange rate changes, and equity excess returns is constant, because of the conditional homoskedasticity of endowment growth. Moreover, consumption growth rates, SDFs, and asset returns have low and constant conditional correlation across countries. Adding external habit formation induces conditional heteroskedasticity: the standard deviation of the conditional volatility of consumption growth rates, SDFs, real exchange rate changes, and equity excess returns is 0.38%, 18.49%, 3.42%, and 12.99%, respectively, in annualized terms. As discussed in Section 2.3, the key for the resolution of the Brandt, Cochrane, and Santa-Clara (2006) puzzle lies in the cross-sectional properties of conditional consumption growth volatility: the international risk sharing mechanism generates a strongly negative association between the two countries' conditional consumption growth volatility (unconditional correlation of −0.74), whereas the risk aversion sensitivity mechanism offsets that negative association and generates an almost perfect correlation between the two countries' conditional SDF volatility, as well as between the two countries' conditional equity excess return volatility (0.94 for both). Moreover,

Table 4
Conditional moments

Moment	Model (no habit)	Model
Panel A: Consumption growth		
$E(\sigma_t^C)$	2.39%	2.05%
	[2.39%,2.39%]	[1.81%,2.29%]
$std(\sigma_t^C)$	0.00%	0.38%
	[0.00%,0.00%]	[0.27%,0.55%]
$corr(\sigma_t^C,\sigma_t^{C*})$	–	−0.74
		[−0.90,−0.48]
$E(\rho_t^C)$	0.17	0.64
	[0.17,0.17]	[0.56,0.73]
$std(\rho_t^C)$	0.00	0.21
	[0.00,0.00]	[0.18,0.25]
Panel B: Stochastic discount factor		
$E(\sigma_t^\Lambda)$	2.39%	18.00%
	[2.39%,2.39%]	[10.86%,32.09%]
$std(\sigma_t^\Lambda)$	0.00%	18.49%
	[0.00%,0.00%]	[7.64%,38.83%]
$corr(\sigma_t^\Lambda,\sigma_t^{\Lambda*})$	–	0.94
		[0.81,0.99]
$E(\rho_t^\Lambda)$	0.17	0.64
	[0.17,0.17]	[0.56,0.73]
$std(\rho_t^\Lambda)$	0.00	0.21
	[0.00,0.00]	[0.18,0.25]
Panel C: Real exchange rate		
$E(\sigma_t^E)$	3.08%	10.30%
	[3.08%,3.08%]	[8.86%,12.07%]
$std(\sigma_t^E)$	0.00%	3.42%
	[0.00%,0.00%]	[2.81%,4.15%]
Panel D: Equity excess returns		
$E(\sigma_t^R)$	2.39%	14.61%
	[2.39%,2.39%]	[9.58%,22.48%]
$std(\sigma_t^R)$	0.00%	12.99%
	[0.00%,0.00%]	[6.37%,21.00%]
$corr(\sigma_t^R,\sigma_t^{R*})$	–	0.94
		[0.81,0.99]
$E(\rho_t^R)$	0.17	0.63
	[0.17,0.17]	[0.55,0.73]
$std(\rho_t^R)$	0.00	0.22
	[0.00,0.00]	[0.18,0.25]

Table 4 presents simulated annualized conditional moments. To calculate the moments for each of the two models (without habit formation and with habit formation), I simulate 10,000 sample paths of the model economy, with each sample path consisting of 152 quarterly observations, as many as available in the dataset. For each of the moments of interest, the table presents the sample average across the 10,000 simulations, as well as the 95% confidence interval (2.5 and 97.5 percentiles across simulations) in brackets. Panels A, B, C, and D present moments pertaining to consumption growth, stochastic discount factors, real exchange rate changes, and equity excess returns, respectively.

consumption growth rates, SDFs, and equity excess returns exhibit much higher conditional correlation on average than in the log utility model (0.64 for consumption growth rates and SDFs and 0.63 for equity excess returns), with conditional correlation varying over time.

Table 5
Endowment and consumption growth rate regressions

Dep. variable	b	s.e.	b^*	s.e.	Adj. R^2	Wald p
			Panel A: Endowment growth rates			
$\Delta\log C$	0.32***	[0.06]	0.02	[0.02]	19.13%	0.00
$\Delta\log C^*$	0.20	[0.13]	0.11***	[0.04]	7.34%	0.00
$\Delta\log\left(\frac{C^*}{C}\right)$	−0.12	[0.13]	0.09**	[0.04]	3.89%	0.05
$\Delta\log G$	−0.38***	[0.08]	−0.03	[0.02]	22.21%	0.00
$\Delta\log G^*$	−0.21	[0.18]	−0.18***	[0.05]	10.89%	0.00
$\Delta\log\left(\frac{G^*}{G}\right)$	0.17	[0.18]	−0.15***	[0.05]	7.62%	0.01
r^e	3.27***	[1.04]	0.45	[0.30]	5.47%	0.00
$r^{e,*}$	1.36	[1.03]	0.05	[0.43]	−0.43%	0.40
			Panel B: Consumption growth rates			
$\Delta\log G$	−0.98***	[0.04]	0.02	[0.02]	88.58%	0.00
$\Delta\log G^*$	0.20***	[0.07]	−1.05***	[0.04]	90.87%	0.00
$\Delta\log\left(\frac{G^*}{G}\right)$	1.18***	[0.07]	−1.07***	[0.03]	90.01%	0.00
r^e	3.91*	[2.19]	−0.54	[0.90]	2.38%	0.19
$r^{e,*}$	3.25*	[1.93]	−1.30	[0.96]	1.58%	0.13

Table 5 presents the output of regressions of the form $\Delta y_{t+1} = b_0 + b\Delta\log x_{t+1} + b^*\Delta\log x^*_{t+1} + u_{t+1}$, where the independent variables x and x^* are the domestic and the foreign endowment growth rate (panel A) or the domestic and the foreign consumption growth rate (panel B), respectively. The dependent variable of each regression is given in the leftmost column of the table. Columns 2–5 present the coefficient estimates, along with their Newey-West standard errors with 2 lags. Column 6 presents the adjusted R^2 of each regression, and the rightmost column presents the p-value of the Wald test that both coefficients b and b^* are equal to zero. One, two, and three stars denote statistical significance at the 10%, 5%, and 1% level, respectively.

4. Empirical Results

In order to test the main empirical predictions of the model, I construct proxies for the key conditional moments and for the conditional risk aversion of each of the two countries.

To calculate the conditional risk aversion of each country, I follow Wachter (2006) and proxy the log surplus consumption ratio $\log S$ with a weighted average of past consumption growth realizations. In particular, $\log S_t = \sum_{j=1}^{40} 0.97^{j-1}\Delta\log C_{t-j+1}$, so the log conditional risk aversion proxy is $\log G_t = -\log S_t$.

Table 5 presents evidence that our empirical endowment, consumption, and risk aversion variables satisfy their model-implied relationships and, thus, constitute appropriate proxies for their model counterparts. In panel A, we regress consumption growth rates, conditional risk aversion measures, and equity excess returns on domestic and foreign endowment growth rates:

$$\Delta y_{t+1} = b_0 + b\Delta\log\widetilde{X}_{t+1} + b^*\Delta\log\widetilde{Y}_{t+1} + u_{t+1}, \tag{13}$$

where the dependent variable in each regression is given in the first column of Table 5. The table presents the coefficient estimates and their standard errors, the adjusted R^2 of each regression and the p-value of the Wald test that both coefficients b and b^* equal zero. For all regressions, Newey-West standard errors with two lags are reported. Crucially, we find that all statistically significant regression coefficients have the model-implied sign. Each country's endowment growth rate is positively and significantly associated with its consumption growth rate and negatively and significantly associated with the change in its conditional risk aversion. Cross-country interactions are not statistically significant, but they have the model-implied sign: endowment growth rates in each country are positively associated with consumption growth rates and negatively associated with changes in the conditional risk aversion of the other country. As a result, the difference in log consumption growth rates loads negatively (albeit not significantly) on domestic endowment growth and positively (and significantly) on foreign endowment growth, while the difference in log conditional risk aversion changes loads positively (but not significantly) on domestic endowment growth and negatively (and significantly) on foreign endowment growth. Equity excess returns in both countries have a positive loading on domestic endowment growth, but do not load significantly on foreign endowment growth; we can reject the null of both coefficients being zero only for the domestic equity excess return. Panel B of Table 5 employs the domestic and foreign consumption growth rates as the independent variables in Equation (13). Again, the empirical relationships are generally consistent with the model.

We can now explore the relationship between conditional moments and conditional risk aversion. For each conditional moment, three proxy measures are constructed: a measure that uses a rolling window of twenty quarters, an exponentially smoothed measure, and a model-based measure (GARCH(1,1) for conditional volatility and DCC(1,1) for conditional correlation). The details of the measure construction are reported in Appendix A4.

Table 6 presents unconditional correlations between conditional moments and log conditional risk aversion levels; the first column reports the simulated value in our habit model, whereas the three remaining columns present the corresponding empirical values using each of the conditional moment proxies (rolling estimate, exponentially smoothed estimate, and model—GARCH or DCC—estimate), as well as the bootstrap standard errors of those empirical correlations. Conditional risk aversion is very persistent (the one-quarter autocorrelation of the log conditional risk aversion proxy in each country is 0.99), so standard errors and confidence intervals are calculated using block bootstrapping with a block length of twenty quarters. This is a very conservative block length choice, aimed to eliminate the effects of serial dependence for the calculation of standard errors; indeed, the twenty-quarter autocorrelation of the risk aversion level is slightly negative both for the United States and the United Kingdom, suggesting that this block length choice achieves its objective.

We first consider the main risk allocation mechanism of the model, the inverse relationship between the ratio of conditional consumption growth volatilities and the ratio of conditional risk aversion coefficients. That novel prediction of the model is strongly confirmed in the data, with all three measures delivering statistically significant negative correlations. Using the rolling measure, the

Table 6
Conditional moments: Levels

Moment	Simulated value	Empirical value		
		Rolling	Smoothed	Model
$corr\left(\frac{\sigma_t^{C*}}{\sigma_t^{C}}, \log\frac{G_t^*}{G_t}\right)$	−0.82	−0.42**	−0.54***	−0.24**
s.e.		[0.15]	[0.11]	[0.12]
$corr\left(\rho_t^{C,C^*}, \log G_t\right)$	0.67	0.73***	0.84***	0.75**
s.e.		[0.11]	[0.17]	[0.20]
$corr\left(\rho_t^{C,C^*}, \log G_t^*\right)$	0.67	0.66***	0.76*	0.67
s.e.		[0.13]	[0.23]	[0.25]
$corr\left(\sigma_t^{E}, \log G_t\right)$	0.64	0.44**	0.30	0.07
s.e.		[0.18]	[0.22]	[0.26]
$corr\left(\sigma_t^{E}, \log G_t^*\right)$	0.64	0.30*	0.11	0.01
s.e.		[0.19]	[0.21]	[0.25]

Table 6 presents unconditional correlations of simulated conditional moments with simulated log risk aversion levels, and their empirical counterparts. As regards simulated data, I simulate 10,000 sample paths of the model economy, with each sample path consisting of 152 observations, as many as available in the dataset, and report the sample average across the 10,000 simulations for each correlation. As regards empirical data, I report unconditional correlations in the whole sample, as well as their bootstrap standard errors. Block bootstrapping with a block length of twenty quarters. One, two, and three stars denote statistical significance at the 10%, 5%, and 1% level, respectively, using the bootstrap distribution for inference.

correlation between the ratio of conditional consumption growth volatilities and the ratio of log conditional risk aversion levels is −0.42, statistically significant at the 5% level.[5] Using the exponentially smoothed measure, the correlation decreases to −0.54, significant at the 1% level, while it is −0.24 using the GARCH estimate, significant at the 5% level. Moreover, there is a positive and statistically significant relationship between the conditional consumption growth correlation of the two countries and each of the two countries' conditional risk aversion level, consistent with the model: all six point estimates are strongly positive, ranging from 0.66 to 0.84, and five out of six are statistically significant. Finally, the volatility of real exchange rate changes is increasing in both countries' conditional risk aversion, although the positive correlation is statistically significant only for the rolling correlation estimates.

A more stringent test of the model predictions involves changes, rather than levels. In particular, we regress changes in the aforementioned five moments on changes in log risk aversion levels:

$$y_{t+k} - y_t = b_0 + b(x_{t+k} - x_t) + u_{t,t+k}, \tag{14}$$

where y is the conditional moment of interest and x is the risk aversion measure of interest. We consider k=1-, 2-, 4-, and 8-quarter changes and present the results in Table 7. Our observations are quarterly, so we account for the overlap in the error terms by using Newey-West standard errors with $k+1$ lags.

[5] To evaluate statistical significance, I use the bootstrap distribution, which typically implies wider confidence intervals than intervals calculated using the bootstrap standard errors when the sample size is small.

Table 7
Conditional moments: Changes

Δy	Δx	Rolling	Smoothed	Model	Rolling	Smoothed	Model
		Panel A: One-quarter changes			Panel B: Two-quarter changes		
$\Delta\left(\frac{\sigma_t^{C*}}{\sigma_t^C}\right)$	$\Delta\log(\frac{G_t^*}{G_t})$	−2.55	−2.75	−1.93	−5.33	−3.35*	−13.77***
s.e.		[4.43]	[2.02]	[5.52]	[4.72]	[2.00]	[4.41]
Adj. R^2		−0.17%	3.76%	−0.80%	2.12%	5.55%	7.41%
$\Delta\left(\rho_t^{C,C*}\right)$	$\Delta\log(G_t)$	10.14***	4.47**	0.78	10.24***	4.84**	1.28*
s.e.		[2.69]	[1.93]	[0.68]	[2.72]	[2.11]	[0.71]
Adj. R^2		17.81%	17.29%	4.16%	25.71%	23.36%	16.41%
$\Delta\left(\rho_t^{C,C*}\right)$	$\Delta\log(G_t^*)$	2.74	1.32	0.25	4.09**	2.06	0.49
s.e.		[1.69]	[0.94]	[0.22]	[1.99]	[1.26]	[0.32]
Adj. R^2		5.89%	7.01%	1.78%	15.38%	15.95%	8.61%
$\Delta\left(\sigma_t^E\right)$	$\Delta\log(G_t)$	0.64	0.19	0.63	0.81	0.24	0.86*
s.e.		[0.42]	[0.26]	[0.40]	[0.59]	[0.35]	[0.45]
Adj. R^2		2.50%	−0.17%	5.96%	3.46%	0.01%	10.97%
$\Delta\left(\sigma_t^E\right)$	$\Delta\log(G_t^*)$	0.07	−0.01	0.03	0.12	−0.03	0.14
s.e.		[0.15]	[0.11]	[0.14]	[0.23]	[0.17]	[0.23]
Adj. R^2		−0.71%	−0.90%	−0.81%	−0.57%	−0.87%	0.28%
		Panel C: Four-quarter changes			Panel D: Eight-quarter changes		
$\Delta\left(\frac{\sigma_t^{C*}}{\sigma_t^C}\right)$	$\Delta\log\left(\frac{G_t^*}{G_t}\right)$	−7.38	−5.54***	−11.23***	−11.77***	−6.70***	−8.07*
s.e.		[4.85]	[1.89]	[4.35]	[4.33]	[1.62]	[4.44]
Adj. R^2		4.90%	19.45%	8.17%	14.20%	27.98%	5.81%
$\Delta\left(\rho_t^{C,C*}\right)$	$\Delta\log(G_t)$	10.96***	5.51**	1.51**	10.86***	5.24**	1.32**
s.e.		[2.50]	[2.15]	[0.66]	[1.97]	[2.07]	[0.62]
Adj. R^2		40.92%	37.20%	31.24%	50.42%	44.98%	34.29%
$\Delta\left(\rho_t^{C,C*}\right)$	$\Delta\log\left(G_t^*\right)$	5.49***	2.66*	0.72*	6.28***	2.67	0.66*
s.e.		[1.67]	[1.39]	[0.39]	[1.26]	[1.38]	[0.39]
Adj. R^2		32.69%	27.62%	22.36%	47.41%	32.70%	23.72%
$\Delta\left(\sigma_t^E\right)$	$\Delta\log(G_t)$	1.00	0.29	1.01**	1.29	0.34	1.10***
s.e.		[0.82]	[0.45]	[0.43]	[1.12]	[0.58]	[0.35]
Adj. R^2		4.45%	0.12%	15.21%	6.25%	0.22%	20.73%
$\Delta\left(\sigma_t^E\right)$	$\Delta\log\left(G_t^*\right)$	0.19	−0.02	0.22	0.36	−0.01	0.36
s.e.		[0.33]	[0.22]	[0.27]	[0.53]	[0.32]	[0.31]
Adj. R^2		−0.28%	−0.92%	1.60%	0.62%	−0.98%	5.61%

Table 7 presents the output of regressions of the form $y_{t+k}-y_t=b_0+b(x_{t+k}-x_t)+u_{t,t+k}$, for $k=\{1,2,4,8\}$ quarters. For each regression, the dependent and the independent variable are presented in the first and second column, respectively. The table presents the point estimates of the coefficients, their Newey-West standard errors with $k+1$ lags, and the adjusted R^2 of the regressions. One, two, and three stars denote statistical significance at the 10%, 5%, and 1% level, respectively.

Overall, we find strong support for the model predictions, with all statistically significant coefficients having the model-implied sign.[6] Importantly, the risk sharing mechanism of the model is validated in the data: changes in the ratio

[6] We also consider twelve-quarter changes, which are not reported in the interests of space as the results are qualitatively very similar to those for eight-quarter changes: all statistically significant coefficients (nine out of fifteen) have the model-implied sign.

of log conditional risk aversion levels are negatively associated with changes in the ratio of conditional consumption growth volatility and, although one-quarter coefficients are not statistically significant, the estimated negative coefficients become significant in most cases across different conditional moment measures when we consider two-, four-, and eight-quarter changes. Moreover, we find strong evidence that changes in the consumption growth correlation are positively associated with changes in both domestic and foreign conditional risk aversion, even when we consider short periods (one- and two-quarter changes). Results become stronger when we look at longer periods: when we consider 4- and 8-quarter changes, all coefficients are statistically significant for both countries and all conditional correlation measures. Finally, we find statistically significant evidence that changes in real exchange rate volatility positively comove with changes in domestic log risk aversion, but the association with foreign log risk aversion changes (albeit typically positive, as implied by the model) is not statistically significant.

Finally, we explore whether domestic (U.S.) and foreign (U.K.) equity excess returns can be predicted using conditional risk aversion levels. In the model, both countries' equity risk premiums are increasing in both domestic and foreign conditional risk aversion, as shown in Figure 9. Empirically, we consider regressions of the form

$$\overline{r^{e,i}_{t+k}} = b_0 + \mathbf{b}'\mathbf{z_t} + u_{t,t+k}, \tag{15}$$

where $\overline{r^{e,i}_{t+k}} \equiv \frac{\sum_{j=1}^{k} r^{e,i}_{t+j}}{k}$ is the k-quarter average excess equity return in country i, and $\mathbf{z_t}$ is the vector of predictive variables. We focus on three specifications: $\mathbf{z_t} = \{\log G_t\}$, $\mathbf{z_t} = \{\log G^*_t\}$ and $\mathbf{z_t} = \{\log G_t, \log G^*_t\}$. We consider horizons of k = 1, 2, 4, 8, and 12 quarters, and calculate Newey-West standard errors with $k+1$ lags. The results are presented in Table 8. For the third specification, which includes both countries' conditional risk aversion levels as regressors, the table also presents the p-value of the Wald test that both elements of the $\mathbf{b}$ vector equal zero.

We find that both U.S. and U.K. conditional risk aversion levels predict future U.S. equity excess returns with a positive sign in univariate regressions, consistent with the model; the coefficients are statistically significant at the two-, four-, eight-, and twelve-quarter horizon when the only predictive variable is U.S. risk aversion, and at the four-, eight-, and twelve-quarter horizon when the predictive variable is U.K. risk aversion. Similarly, in univariate regressions both U.S. and U.K. risk aversion levels predict future U.K. equity excess returns: the slope coefficient is positive and statistically significant at the two-, four-, eight-, and twelve-quarter horizon for U.S. risk aversion and at the four-, eight-, and twelve-quarter horizon for U.K. risk aversion.

We now turn to bivariate regressions. For both countries' equity excess returns, predictability increases with horizon: the adjusted R^2 starts at 0.75% for U.S. excess returns and at −0.17% for U.K. excess returns at the one-quarter

Table 8
Predictability regressions

	U.S. equity excess return			U.K. equity excess return		
	Panel A: One-quarter horizon					
$\log G$	0.39		0.89	0.36		0.51
s.e.	[0.25]		[0.70]	[0.25]		[0.61]
$\log G^*$		0.15	−0.31		0.17	−0.09
s.e.		[0.14]	[0.39]		[0.15]	[0.35]
Adj. R^2	0.99%	−0.10%	0.75%	0.69%	0.18%	−0.17%
Wald p			0.24			0.34
	Panel B: Two-quarter horizon					
$\log G$	0.43**		1.04	0.36*		0.55
s.e.	[0.20]		[0.70]	[0.21]		[0.58]
$\log G^*$		0.15	−0.39		0.16	−0.12
s.e.		[0.10]	[0.39]		[0.12]	[0.33]
Adj. R^2	3.27%	0.60%	4.34%	1.99%	0.89%	1.26%
Wald p			0.07			0.24
	Panel C: Four-quarter horizon					
$\log G$	0.48**		1.02	0.37*		0.51
s.e.	[0.20]		[0.67]	[0.21]		[0.57]
$\log G^*$		0.17**	−0.36		0.18*	−0.09
s.e.		[0.08]	[0.37]		[0.10]	[0.31]
Adj. R^2	8.58%	2.73%	10.98%	5.27%	3.06%	4.59%
Wald p			0.03			0.20
	Panel D: Eight-quarter horizon					
$\log G$	0.56**		1.19*	0.44*		0.61
s.e.	[0.24]		[0.63]	[0.23]		[0.55]
$\log G^*$		0.17**	−0.45		0.20**	−0.12
s.e.		[0.08]	[0.37]		[0.09]	[0.31]
Adj. R^2	18.90%	4.00%	27.48%	14.24%	7.24%	14.22%
Wald p			0.01			0.12
	Panel D: Twelve-quarter horizon					
$\log G$	0.63***		1.05***	0.50***		0.66**
s.e.	[0.22]		[0.39]	[0.17]		[0.28]
$\log G^*$		0.17*	−0.33		0.19***	−0.12
s.e.		[0.11]	[0.23]		[0.07]	[0.16]
Adj. R^2	25.39%	4.30%	32.17%	21.62%	7.80%	22.25%
Wald p			0.00			0.00

Table 8 presents the output of regressions of the form $\overline{r^{e,i}_{t+k}} = b_0 + \mathbf{b}'\mathbf{z_t} + u_{t,t+k}$, for $k = \{1,2,4,8,12\}$ quarters, where $\overline{r^{e,i}_{t+k}} \equiv \frac{\sum_{j=1}^{k} r^{e,i}_{t+j}}{k}$ is the k-quarter average equity excess return in country i. We consider three specifications for the vector of predictive variables $\mathbf{z_t}$: $\mathbf{z_t} = \{\log G_t\}$, $\mathbf{z_t} = \{\log G^*_t\}$, and $\mathbf{z_t} = \{\log G_t, \log G^*_t\}$. The table presents the point estimates of the coefficients, their Newey-West standard errors with $k+1$ lags, and the adjusted R^2 of the regressions. For the third specification, the table also presents the p-value of the Wald test that both elements of the vector $\mathbf{b}$ are equal to zero. One, two, and three stars denote statistical significance at the 10%, 5%, and 1% level, respectively.

horizon and reaches 32.17% and 22.25%, respectively, at the twelve-quarter horizon. For U.S. excess returns, the null hypothesis of zero predictability is rejected at the two-, four-, eight-, and twelve-quarter horizon (the p-value of the Wald test statistic is 0.07, 0.03, 0.01, and 0.00, respectively), and rejected at the twelve-quarter horizon for U.K. excess returns (Wald test p-value of 0.00).

Overall, there is considerable evidence that conditional risk aversion levels have predictive power for equity excess returns, consistent with the model.

5. Conclusion

This paper shows that a two-good, two-country general equilibrium model that incorporates preference home bias and external habit formation generates a novel international risk sharing mechanism that allows it to match several important SDF, exchange rate, and asset pricing moments, and to address key international finance puzzles, including the Brandt, Cochrane, and Santa-Clara (2006) international risk sharing puzzle and the Backus and Smith (1993) exchange rate disconnect puzzle. Moreover, the model shows that, in open economies, foreign preferences can have a significant effect on domestic asset prices and returns via valuation effects. The increased openness of most major economies over the last few decades suggests that foreign preferences are of increasing importance for domestic asset prices, even for large economies such as the U.S. economy.

The model proposed in this paper assumes a highly stylized environment that exhibits frictionless international trade in goods and assets. Richer models that incorporate frictions in international goods and asset markets have the potential to further enhance our understanding of joint consumption, asset price, and exchange rate determination in open economies by illustrating how international risk sharing is affected by the presence of frictions.

Appendix

The Appendix presents the detailed solution of the model, proofs omitted in the main body of the paper, and details of the model calibration and simulation.

A1. Model Solution

We solve for the competitive equilibrium. The domestic agent's intertemporal budget constraint can be written in static form as

$$E_0\left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0}\left(X_t Q_t + Y_t Q_t^*\right)dt\right] = E_0\left[\int_0^\infty \frac{\Lambda_t}{\Lambda_0}\tilde{X}_t Q_t dt\right],$$

where Λ is the state-price deflator process for cash flows expressed in units of the global numeraire; its existence is assured by the absence of arbitrage and its uniqueness by the completeness of financial markets. Λ is determined endogenously, as it is an equilibrium outcome of the economy.

The first order conditions for the domestic agent's problem are:

$$e^{-\rho t} a \frac{G_t}{X_t} = \zeta \Lambda_t Q_t \quad \text{and } e^{-\rho t}(1-a)\frac{G_t}{Y_t} = \zeta \Lambda_t Q_t^*,$$

where ζ is the Lagrange multiplier of the domestic agent's intertemporal budget constraint. Similarly, the FOCs of the foreign agent's problem are

$$e^{-\rho t} a^* \frac{G_t^*}{X_t^*} = \zeta^* \Lambda_t Q_t \quad \text{and } e^{-\rho t}\left(1-a^*\right)\frac{G_t^*}{Y_t^*} = \zeta^* \Lambda_t Q_t^*,$$

where ζ^* is the foreign Lagrange multiplier. Imposing the market clearing conditions for the two goods, we solve for the equilibrium allocation, Equations (3) and (4), where $\lambda \equiv \frac{1}{\zeta}$ and $\lambda^* \equiv \frac{1}{\zeta^*}$. Therefore, domestic equilibrium consumption is

$$C_t = \omega_t^a (\omega_t^*)^{1-a} \widetilde{X}_t^a \widetilde{Y}_t^{1-a}, \tag{A1a}$$

and foreign equilibrium consumption is

$$C_t^* = (1-\omega_t)^{a^*} (1-\omega_t^*)^{1-a^*} \widetilde{X}_t^{a^*} \widetilde{Y}_t^{1-a^*}. \tag{A1b}$$

As a result of frictionless international trade in goods, the two agents equalize their marginal utility growth for each of the two goods. Indeed, if we define

$$\Xi_t = e^{-\rho t} \left(a\lambda G_t + a^* \lambda^* G_t^* \right) \frac{1}{\widetilde{X}_t},$$

then the marginal utility of the domestic good equals $\frac{1}{\lambda} \Xi$ for the domestic agent and $\frac{1}{\lambda^*} \Xi^*$ for the foreign agent. Thus, the marginal utility of the domestic good is decreasing in the supply of the domestic good and increasing in the risk aversion of both countries. Note that G and G^* do not enter symmetrically: the sensitivity of Ξ to each country's conditional risk aversion fluctuations mirrors the steady-state equilibrium consumption pattern for the domestic good, being equal to $a\lambda$ for domestic risk aversion and $a^*\lambda^*$ for foreign risk aversion. Similarly, the marginal utility of the foreign good equals $\frac{1}{\lambda} \Xi^*$ for the domestic agent and $\frac{1}{\lambda^*} \Xi^*$ for the foreign agent, where Ξ^* is defined as

$$\Xi_t^* = e^{-\rho t} \left((1-a)\lambda G_t + \left(1-a^*\right) \lambda^* G_t^* \right) \frac{1}{\widetilde{Y}_t}.$$

In a slight abuse of terminology, I will refer to Ξ_t (Ξ_t^*) as the state-price deflator for the domestic (foreign) good.

The marginal rate of substitution between the two goods is identical for the two agents and determines the relative price of the two goods:

$$\frac{Q_t^*}{Q_t} = \frac{\Xi_t^*}{\Xi_t} = \frac{(1-a)\lambda G_t + (1-a^*)\lambda^* G_t^*}{a\lambda G_t + a^* \lambda^* G_t^*} \frac{\tilde{X}_t}{\widetilde{Y}_t}.$$

In order to pin down the absolute level of prices for the two goods, we need to define the global numeraire. Given that the price of the domestic consumption basket satisfies

$$P_t = \left(\frac{Q_t}{a} \right)^a \left(\frac{Q_t}{1-a^*} \right)^{1-a},$$

setting the global numeraire to be the domestic consumption basket implies that $P_t = 1$ for all t. Thus, we can use the normalization

$$\left(\frac{Q_t}{a} \right)^a \left(\frac{Q_t^*}{1-a} \right)^{1-a} = 1,$$

to solve for the prices of the two goods expressed in the global numeraire. In particular, the price of the domestic good is

$$Q_t = a^a (1-a)^{1-a} \left(\frac{a\lambda G_t + a^* \lambda^* G_t^*}{(1-a)\lambda G_t + (1-a^*)\lambda^* G_t^*} \frac{\widetilde{Y}_t}{\tilde{X}_t} \right)^{1-a}, \tag{A1c}$$

and the price of the foreign good is

$$Q_t^* = a^a (1-a)^{1-a} \left(\frac{(1-a)\lambda G_t + (1-a^*)\lambda^* G_t^*}{a\lambda G_t + a^* \lambda^* G_t^*} \frac{\tilde{X}_t}{\widetilde{Y}_t} \right)^a. \tag{A1d}$$

Finally, we need to solve for the values of the two Lagrange multipliers ζ and ζ^*. Given that different normalizations represent the same economy, only the ratio $\frac{\zeta}{\zeta^*}$ can be pinned down. Indeed, the aforementioned ratio is determined by the static budget constraint of either the domestic or the foreign agent. First, we solve for the global numeraire state-price deflator Λ. The domestic and foreign FOCs imply that

$$\Lambda_t = e^{-\rho t}\frac{1}{Q_t}\frac{a\lambda G_t + a^*\lambda^* G_t^*}{\tilde{X}_t} = e^{-\rho t}\frac{1}{Q_t^*}\frac{(1-a)\lambda G_t + (1-a^*)\lambda^* G_t^*}{\widetilde{Y}_t}.$$

Therefore, plugging the expressions for Q and Q^*, we get

$$\Lambda_t = \left(\frac{\Xi_t}{a}\right)^a \left(\frac{\Xi_t^*}{1-a}\right)^{1-a} = \lambda e^{-\rho t}\frac{G_t}{X_t^a Y_t^{1-a}} = \lambda e^{-\rho t}\frac{G_t}{C_t}, \tag{A1e}$$

so the numeraire state-price density is the marginal utility of the domestic consumption basket. We can similarly define the marginal utility of the foreign consumption basket by

$$\Lambda_t^* = \left(\frac{\Xi_t}{a^*}\right)^{a^*} \left(\frac{\Xi_t^*}{1-a^*}\right)^{1-a^*} = \lambda^* e^{-\rho t}\frac{G_t^*}{(X_t^*)^{a^*}(Y_t^*)^{1-a^*}} = \lambda^* e^{-\rho t}\frac{G_t^*}{C_t^*}. \tag{A1f}$$

Thus, despite the fact that the growth rate of the marginal utility for each of the two goods is equalized across countries, the growth rate of the marginal utility of the two consumption baskets is not equalized, as they have different composition because of home bias.

We can now plug the equilibrium expressions for the consumption allocation and the state-price deflator in either budget constraint and get

$$\frac{\zeta}{\zeta^*} = \frac{1-a}{a^*}\frac{\rho G_0 + k\bar{G}}{\rho G_0^* + k\bar{G}}.$$

Equilibrium prices and quantities are invariant to the specific values of ζ and ζ^*, as long as their ratio satisfies the expression above. We adopt the normalization

$$\frac{1}{\zeta} + \frac{1}{\zeta^*} = 1,$$

which generates the following values for the Lagrange multipliers:

$$\zeta = \frac{a^*\left(\rho G_0^* + k\bar{G}\right) + (1-a)\left(\rho G_0 + k\bar{G}\right)}{a^*\left(\rho G_0^* + k\bar{G}\right)} \text{ and } \zeta^* = \frac{a^*\left(\rho G_0^* + k\bar{G}\right) + (1-a)\left(\rho G_0 + k\bar{G}\right)}{(1-a)\left(\rho G_0 + k\bar{G}\right)}.$$

Finally, domestic wealth, in units of the global numeraire, equals the discounted present value of future domestic consumption:

$$W_t = E_t\left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} C_s P_s ds\right] = E_t\left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t}\left(X_s Q_s + Y_s Q_s^*\right) ds\right],$$

so, plugging in the equilibrium expressions for quantities and prices, the equilibrium value of domestic wealth is:

$$W_t = \frac{\rho G_t + k\bar{G}}{a\lambda G_t + a^*\lambda^* G_t^*}\frac{\lambda \widetilde{X}_t}{\rho(\rho+k)}.$$

Foreign wealth W^* is calculated in a similar fashion and is given by

$$W_t^* = E_t\left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} C_s^* P_s^* ds\right] = \frac{\rho G_t^* + k\bar{G}}{a\lambda G_t + a^*\lambda^* G_t^*} \frac{\lambda^* \widetilde{X}_t}{\rho(\rho+k)}.$$

A1.1 Solution to the planner's problem

For the planner's problem solution to coincide with the competitive equilibrium solution, the planner has to take into account the externality arising from external habit formation. Thus, the competitive solution will not be unconstrained Pareto optimal, but constrained Pareto optimal with the constraint being the specified law of motion for the two countries' habit processes. The planner takes the law of motion for each agent's inverse surplus consumption ratio as exogenous, so the first order conditions of the planner's problem are:

$$\lambda e^{-\rho t} \pi(\omega,t) a \frac{G_t}{X_t} = \Theta_t,$$

$$\lambda e^{-\rho t} \pi(\omega,t)(1-a) \frac{G_t}{Y_t} = \Theta_t^*,$$

$$\lambda^* e^{-\rho t} \pi(\omega,t) a^* \frac{G_t^*}{X_t^*} = \Theta_t,$$

$$\lambda^* e^{-\rho t} \pi(\omega,t)(1-a^*) \frac{G_t^*}{Y_t^*} = \Theta_t^*,$$

where Θ_t and Θ_t^* are the Lagrange multipliers associated with the time t resource constraint for the domestic and the foreign good, respectively, and $\pi(\omega,t)$ is the P measure probability that state ω occurs at time t. Using the FOCs, along with the two market clearing conditions, we get the sharing rules in Equations (3) and (4). It is trivial to show that the planner's problem solution is equivalent to the competitive solution if each country's welfare weight equals the inverse of its Lagrange multiplier,

$$\lambda = \frac{1}{\zeta} \text{ and } \lambda^* = \frac{1}{\zeta^*}.$$

A1.2 Endowment claim returns

As shown in the proof of Proposition 2 (Appendix A5), the price of the domestic endowment claim in units of the domestic numeraire, V, is given by

$$V_t = \frac{1}{\rho}\left[\frac{k}{\rho+k} \frac{a\lambda\bar{G} + a^*\lambda^*\bar{G}}{a\lambda G_t + a^*\lambda^* G_t^*} + \frac{\rho}{\rho+k}\right] \widetilde{X}_t Q_t.$$

Thus, its excess return is

$$dR_t^e = \frac{dV_t}{V_t} + \frac{\widetilde{X}_t Q_t}{V_t} dt - r_t^f dt = \mu_t^R dt + \sigma_t^{R\prime} \mathbf{dB}_t,$$

where r^f is the continuously compounded domestic risk-free rate, which satisfies

$$r_t^f dt = -E_t\left(\frac{d\Lambda_t}{\Lambda_t}\right) = \rho + \mu_t^C + k\left(\frac{G_t - \bar{G}}{G_t}\right) - \left(1 + \delta\left(\frac{G_t - l}{G_t}\right)\right) \sigma_t^{C\prime} \Sigma \sigma_t^C.$$

The domestic risk-free rate is increasing in μ^C, the drift of the domestic consumption growth process, and $\frac{G-\bar{G}}{G}$, the proportional deviation from steady-state risk aversion; both effects reflect the desire for intertemporal smoothing. Finally, r^f is decreasing in the conditional variance of domestic consumption growth, $\sigma^{C\prime}\Sigma\sigma^{C}$, which reflects the desire for precautionary savings, where Σ is the covariance matrix of the Brownian increments.

We can now characterize the excess return process of the domestic endowment claim. Its diffusion term σ^R is given by

$$\sigma_t^R = \sigma^X \mathbf{e}_1 - (1-a)\sigma_t^Q + \frac{(a\lambda + a^*\lambda^*)k\bar{G}\left(\omega_t \sigma_t^G + (1-\omega_t)\sigma_t^{G^*}\right)}{(a\lambda + a^*\lambda^*)k\bar{G} + \rho(a\lambda G_t + a^*\lambda^* G_t^*)},$$

where $\mathbf{e}_1 \equiv [1,\, 0]'$ and σ^Q is the diffusion term of the terms of trade,

$$\sigma_t^Q = \sigma^X \mathbf{e}_1 - \sigma^Y \mathbf{e}_2 - \left(\omega_t^* - \omega_t\right)\left(\sigma_t^G - \sigma_t^{G^*}\right).$$

The conditional risk premium of the domestic claim, μ^R, is equal to the conditional covariance between the excess return of the domestic claim and the domestic state-price density,

$$\mu_t^R = -E_t\left(dR_t^e \frac{d\Lambda_t}{\Lambda_t}\right) = \sigma_t^{R\prime}\Sigma\sigma_t^{\Lambda},$$

and, therefore, is time varying.

Similarly, the diffusion term of the foreign endowment claim excess return is given by

$$\sigma_t^{R^*} = \sigma^Y \mathbf{e}_2 + a^*\sigma_t^Q + \frac{((1-a)\lambda + (1-a^*)\lambda^*)k\bar{G}\left(\omega_t^* \sigma_t^G + (1-\omega_t^*)\sigma_t^{G^*}\right)}{((1-a)\lambda + (1-a^*)\lambda^*)k\bar{G} + \rho((1-a)\lambda G_t + (1-a^*)\lambda^* G_t^*)},$$

where $\mathbf{e}_2 \equiv [0,\, 1]'$, and the foreign claim risk premium is

$$\mu_t^{R^*} = -E_t\left(dR_t^{e*} \frac{d\Lambda_t^*}{\Lambda_t^*}\right) = \sigma_t^{R^*\prime}\Sigma\sigma_t^{\Lambda^*}.$$

A2. Data Sources and Definitions

Real endowment and real consumption, as defined in the main text, are derived from their nominal counterparts by deflation using the GDP deflator and the PCE deflator, respectively. The terms of trade are calculated using the implicit price deflator (IPD) for total U.K. exports and the IPD for total U.S. exports, with the export IPD of each country calculated as the ratio of its nominal exports over its real exports. Data on U.S. and U.K. consumption, total imports and total exports are from the U.S. Bureau of Economic Analysis and the U.K. Office for National Statistics, respectively. All data are seasonally adjusted; any time series not initially adjusted undergoes seasonal adjustment using the U.S. Census Bureau's X12 seasonal adjustment method.

The endowment claim for each country is proxied by the corresponding Datastream equity index. Nominal equity returns are constructed using the Datastream Total Return Index, whereas price-dividend ratios are constructed using the time series for the Price Index and the Total Return Index. The nominal U.S. and U.K. risk-free rate is proxied by the corresponding 3-month Treasury bill yield, provided by Global Financial Data. The GBP/USD nominal exchange rate is from WM/Reuters. Nominal interest rates, equity returns, and exchange rates are converted into their real counterparts using the OECD CPI time series for the two countries.

A3. Parameter Estimation

Out of the six preference parameters, three are estimated using Simulated Method of Moments (SMM), and three are set equal to their calibrated values in Menzly, Santos, and Veronesi (2004). In particular, we use SMM to estimate the three preference parameters a, k and $\bar{G}$. For the latter two parameters, we impose equality across the two countries, whereas for the home bias parameter we impose the symmetry condition $a^* = 1-a$. We target 9 moments: the mean and the variance of the real risk-free rate (domestic and foreign), the mean and the variance of the equity excess return (domestic and foreign), and the variance of changes of the real exchange rate between the two countries. We simulate 1,000 runs of 190 quarterly observations, each run initialized at $G = G^* = \bar{G}$ and $\widetilde{X} = \widetilde{Y} = 1$, and eliminate the first forty observations (10 years) in each run in order to reduce the dependence on initial conditions. Thus, each run consists of 150 quarterly observations, equal to the size of the empirical sample used (adjusting for observations eliminated for the calculation of growth rates and real interest rates). The point estimates of the annualized parameters are $a = 0.952$, $k = 0.195$, and $\bar{G} = 25.016$, with their 95% confidence intervals being [0.946, 0.958] for a, [0.168, 0.222] for k, and [24.061, 25.971] for $\bar{G}$.

A4. Conditional Moment Proxies

The conditional variance of domestic and foreign consumption growth rates and real exchange rate changes is calculated with three different methodologies: rolling variance, exponentially smoothed variance, and a GARCH(1,1) model variance estimate. The first measure is the sample variance of a rolling window of twenty quarterly observations (5 years): at quarter t, the conditional variance of variable $\Delta \log X$, for $X = \{C, C^*, E\}$, is given by $\sigma_t^X = \frac{\sum_{j=1}^{20} (\Delta \log X_{t-j+1} - \overline{\Delta \log X})^2}{19}$. The second measure is an exponentially smoothed variance estimate with the smoothing parameter equal to 0.95, following Colacito and Croce (2011): $\sigma_t^X = 0.95(\sigma_{t-1}^X) + 0.05(\Delta \log X - \hat{\mu})^2$, where $\hat{\mu}$ is the sample mean of the series. The initial value, σ_{20}^X, is the sample variance calculated using the first twenty quarterly observations. For the third measure, I fit a GARCH(1,1) model to the demeaned time series of $\Delta \log X$. For each measure, the conditional standard deviation is the square root of the conditional variance.

The conditional correlation of the two countries' consumption growth rates is also calculated with three different methods. The rolling conditional correlation is the sample correlation using a rolling window of twenty quarterly observations. The exponentially smoothed conditional correlation is given by $\rho_t^C = \frac{cov_t^{C,C^*}}{\sigma_t^C \sigma_t^{C^*}}$, where cov_t^{C,C^*} is the exponentially smoothed covariance of the two consumption growth rates, given by $cov_t^{C,C^*} = 0.95(cov_{t-1}^{C,C^*}) + 0.05(\Delta \log C - \hat{\mu})(\Delta \log C - \hat{\mu})$ and initialized at $t = 20$ with the sample covariance of the first twenty observations. Finally, the third conditional correlation measure is derived from a Dynamic Conditional Correlation (DCC) model, as proposed in Engle (2002). In particular, the conditional correlation estimate is the fitted value of a DCC(1,1) model, with the variance estimates given by a GARCH(1,1) model.

A5. Proofs

This section presents the proofs of Propositions 1 and 2.

Proof of Proposition 1. Consider the domestic consumption growth process

$$\frac{dC_t}{C_t} = \mu_t^C dt + \sigma_t^{C'} \mathbf{dB}_t \equiv \mu_t^C dt + \sigma_t^{CX} dB_t^X + \sigma_t^{CY} dB_t^Y, \tag{A5a}$$

where μ^C is the drift of the process, and σ^{CX} (σ^{CY}) is the exposure of domestic consumption growth to the domestic (foreign) endowment growth shock. The drift and diffusion terms μ^C, σ^{CX}, and σ^{CY} are determined endogenously. Similarly, the foreign consumption growth process can be written as

$$\frac{dC_t^*}{C_t^*} = \mu_t^{C^*} dt + \sigma_t^{C^*\prime} \mathbf{dB}_t \equiv \mu_t^{C^*} dt + \sigma_t^{C^*X} dB_t^X + \sigma_t^{C^*Y} dB_t^Y. \tag{A5b}$$

Our aim is to solve for the four diffusion terms (σ^{CX}, σ^{CY}, σ^{C^*X}, and σ^{C^*Y}) in terms of the state variables $\widetilde{X}$, $\widetilde{Y}$, G, and G^*. We apply Itô's lemma to Equation (A1a) and derive the following expression for the domestic consumption growth process:

$$\frac{dC_t}{C_t} = \left[a\frac{d\tilde{X}_t}{\tilde{X}_t} + (1-a)\frac{d\widetilde{Y}_t}{\widetilde{Y}_t} \right] + \left[a\frac{d\omega_t}{\omega_t} + (1-a)\frac{d\omega_t^*}{\omega_t^*} \right] + o(dt).$$

Thus, domestic consumption growth has two first-order components: a component that reflects endowment growth rates, and a component that reflects changes in the equilibrium domestic consumption shares. After some algebra, the expression above can be written as follows:

$$\frac{dC_t}{C_t} = \mu_t^C dt + \left(a\sigma^X dB_t^X + (1-a)\sigma^Y dB_t^Y\right) - k_t \left(\frac{d\left(G_t^*/G_t\right)}{G_t^*/G_t} - E_t\left(\frac{d\left(G_t^*/G_t\right)}{G_t^*/G_t} \right) \right).$$

Thus, the stochastic term of domestic consumption growth is the sum of a home-biased weighted average of the two endowment growth shocks and a term that captures the shocks in the domestic consumption shares. The latter terms depends on the sensitivity of domestic consumption growth to shocks in the risk aversion ratio $\frac{G^*}{G}$, given by multiplier k, defined in Equation (5).

Using the law of motion for the domestic and foreign inverse surplus consumption ratios, we get the law of motion for the risk aversion ratio:

$$\frac{d\left(G_t^*/G_t\right)}{G_t^*/G_t} = E_t\left(\frac{d\left(G_t^*/G_t\right)}{G_t^*/G_t} \right) + \varphi_t^X dB_t^X + \varphi_t^Y dB_t^Y,$$

where the exposure to domestic endowment growth shocks is given by

$$\varphi_t^X \equiv \delta\left(\frac{G_t - l}{G_t}\right)\sigma_t^{CX} - \delta\left(\frac{G_t^* - l}{G_t^*}\right)\sigma_t^{C^*X},$$

and the exposure to foreign endowment growth shocks is given by

$$\varphi_t^Y \equiv \delta\left(\frac{G_t - l}{G_t}\right)\sigma_t^{CY} - \delta\left(\frac{G_t^* - l}{G_t^*}\right)\sigma_t^{C^*Y}.$$

Thus, domestic consumption growth can be written as follows:

$$\frac{dC_t}{C_t} = \mu_t^C dt + \left(a\sigma^X - k_t\varphi_t^X\right)dB_t^X + \left((1-a)\sigma^Y - k_t\varphi_t^Y\right)dB_t^Y. \tag{A5c}$$

Similarly, foreign consumption growth satisfies

$$\frac{dC_t^*}{C_t^*} = \mu_t^{C^*} dt + \left(a^*\sigma^X dB_t^X + (1-a^*)\sigma^Y dB_t^Y\right) + k_t^* \left(\frac{d\left(G_t^*/G_t\right)}{G_t^*/G_t} - E_t\left(\frac{d\left(G_t^*/G_t\right)}{G_t^*/G_t} \right) \right),$$

where the sensitivity term k^* is defined as

$$k_t^* \equiv \lambda \frac{a(1-a)\lambda + a^*(1-a^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)}{\left(a\lambda + a^*\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)\left((1-a)\lambda + (1-a^*)\lambda^*\left(\frac{G_t^*}{G_t}\right)\right)}.$$

After some algebra, foreign consumption growth can be written as

$$\frac{dC_t^*}{C_t^*} = \mu_t^{C^*} dt + \left(a^*\sigma^X + k_t^*\varphi_t^X\right)dB_t^X + \left((1-a^*)\sigma^Y + k_t^*\varphi_t^Y\right)dB_t^Y. \tag{A5d}$$

We can solve for the diffusion terms of domestic and foreign consumption growth by solving for the fixed point that ensures that the equilibrium consumption processes are consistent with the exogenous processes for the inverse surplus consumption ratios. In particular, matching the domestic consumption growth diffusion terms in (A5a) and (A5c), we get:

$$\sigma_t^{CX} = a\sigma^X - k_t\varphi_t^X \quad \text{and } \sigma_t^{CY} = (1-a)\sigma^Y - k_t\varphi_t^Y. \tag{A5e}$$

Similarly, we match the foreign consumption growth diffusion terms in (A5b) and (A5d) to get:

$$\sigma_t^{C^*X} = a^*\sigma^X + k_t^*\varphi_t^X \text{ and } \sigma_t^{C^*Y} = (1-a^*)\sigma^Y + k_t^*\varphi_t^Y. \tag{A5f}$$

Note that (A5e) and (A5f) comprise a system of four equations in the four unknowns σ^{CX}, σ^{CY}, σ^{C^*X}, and σ^{C^*Y}. Solving this system of equations, we get the expressions for the diffusion terms of the domestic and the foreign consumption growth. The solution for the domestic equilibrium consumption growth diffusion terms is presented in Proposition 1. As regards foreign consumption growth, its exposure to domestic endowment shocks is given by the diffusion term

$$\sigma_t^{C^*X} = \frac{a^* + \left(ak_t^* + a^*k_t\right)\delta\left(\frac{G_t - l}{G_t}\right)}{1 + k_t\delta\left(\frac{G_t - l}{G_t}\right) + k_t^*\delta\left(\frac{G_t^* - l}{G_t^*}\right)}\sigma^X,$$

whereas its exposure to foreign endowment shocks is given by the diffusion term

$$\sigma_t^{C^*Y} = \frac{(1-a^*) + \left((1-a)k_t^* + (1-a^*)k_t\right)\delta\left(\frac{G_t - l}{G_t}\right)}{1 + k_t\delta\left(\frac{G_t - l}{G_t}\right) + k_t^*\delta\left(\frac{G_t^* - l}{G_t^*}\right)}\sigma^Y.$$

Proof of Proposition 2. The price of the domestic endowment claim in units of the domestic numeraire equals the present value of its future dividends, expressed in units of the domestic numeraire, discounted by the state-price density corresponding to the domestic numeraire:

$$V_t = E_t\left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t}\left(\widetilde{X}_s Q_s\right)ds\right].$$

To evaluate this expression, we first use Fubini's theorem in order to interchange of the order of integration,

$$V_t = \int_t^\infty E_t\left[\frac{\Lambda_s}{\Lambda_t}\widetilde{X}_s Q_s\right]ds = \frac{1}{\Lambda_t}\int_t^\infty E_t\left[\Lambda_s\widetilde{X}_s Q_s\right]ds,$$

and then use Equations (A1c) and (A1e) to evaluate the expression inside the expectation:

$$\Lambda_s\widetilde{X}_s Q_s = e^{-\rho s}\left(a\lambda G_s + a^*\lambda^* G_s^*\right). \tag{A5g}$$

Thus, the expectation inside the integral is a linear function of the conditional expectations of G and G^*,

$$E_t\left[\Lambda_s\widetilde{X}_s Q_s\right] = e^{-\rho s}\left[a\lambda E_t(G_s) + a^*\lambda^* E_t\left(G_s^*\right)\right],$$

and the latter can be easily calculated using the law of motion for the two state variables:

$$E_t(G_s) = e^{-k(s-t)}G_t + (1 - e^{-k(s-t)})\bar{G},$$

$$E_t(G_s^*) = e^{-k(s-t)}G_t^* + (1 - e^{-k(s-t)})\bar{G}.$$

After some algebra, we get

$$V_t = \frac{1}{\rho}\frac{e^{-\rho t}}{\Lambda_t}\left(a\lambda G_t + a^*\lambda^* G_t^*\right)\left[\frac{\rho}{\rho + k} + \frac{k}{\rho + k}\frac{a\lambda\bar{G} + a^*\lambda^*\bar{G}}{a\lambda G_t + a^*\lambda^* G_t^*}\right],$$

and using Equation (A5g) to substitute for Λ, we derive the expression in Proposition 2.

Similarly, the price of the foreign endowment claim in units of the foreign numeraire equals the present value of its future dividends, expressed in units of the foreign numeraire, discounted by the foreign state-price density:

$$V_t^* = E_t\left[\int_t^\infty \frac{\Lambda_s^*}{\Lambda_t^*}\left(\frac{\widetilde{Y}_s Q_s^*}{P_s^*}\right)ds\right].$$

We can evaluate this expression, and derive the solution given in Proposition 2, in a similar fashion as described previously.

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