Claremont Graduate University, M453, Financial Time Series
Midterm
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Only an A3 size cheating sheet is allowed. No computers, no smart phones, no wifi.
Degree of difficulty: * easy, * medium, ***** hard**

Financial Time Series

1. **(Time series modeling)** Find a time series corresponding to each of the following sequence of observations (you should estimate all the parameters of your model, except the variance σ^2 of white noise):

   (a) * The daily number of sold items of a product: $2, 6, 12, 20, 30, 42, \ldots$;

   (b) * The yearly stock price of a startup company: $4, 10, 20, 36, 62, 104, \ldots$;

   (c) *** The weekly balance of an individual's bank account:
   $3270, 2160, 1490, 6490, 3330, 2240, 1390, 6390, \ldots$.

   (Hint: draw the trajectory of the sample first.)

2. **(Time series modeling)** The daily log returns $\{\log(S_t)\}_t$ of some asset price $\{S_t\}_t$ are observed to behave as follows:

   (1) The log return at day t linearly depends on its value at day $t-1$;

   (2) If this log return at day t is 1.6, then it is predicted to be 1.3 at day $t+1$ and 1.15 at day $t+2$.

   (a) *** Propose a time series model to fit this log return. Your model should satisfy (1) and (2).

   (b) *** Estimate all parameters in your model.

   (c) * Compute $E[\log(S_t)]$ and $Var(\log(S_t))$. Is your model second-order stationary? Why?

3. **(Autoregressive model: Part 1)** Consider an $AR(3)$ model $\{X_t\}_t$:

$$X_t = -\frac{1}{2}\left(X_{t-1} + X_{t-2} - X_{t-3}\right) + w_t, \tag{1}$$

   where $\{w_t\}_t$ is some white noise.

   (a) * Let B be the backward shift operator, i.e. $BX_t = X_{t-1}$. Determine a polynomial $\theta(B)$ such that

$$\theta(B)X_t = w_t.$$

   (b) *** Is $\{X_t\}_t$ second-order stationary? Why?
   (Hint: guess one root of $\theta(B)$.)

4. **(Autoregressive model: Part 2)** We will show that the $AR(3)$ defined in (1) CAN'T have period 3. Recall that a time series $\{Y_t\}_t$ has period 3 means that $Y_{t-3} = Y_t$ for all t.

   (a) *** Show that if the time series $\{X_t\}_t$ has period 3, then $X_t + X_{t-1} + X_{t-2}$ does not depend on t.

   (b) *** Using the above result to show that $\theta(B)$ given in Question (a) Exercise 3 transforms every time series of period 3 to a constant, i.e., if $\{X_t\}_t$ has period 3, then

$$\theta(B)X_t = constant.$$

   (Remark: this result contradicts the fact that $\theta(B)X_t = w_t$, as a consequence X_t can't have period 3.)

5. ***** A sample of a stationary time series $\{X_t\}_t$ has been observed: $X_1, \ldots, X_n$. The sample statistics show that

$$\overline{X} = 0.0013,\ corr(X_t, X_{t+1}) \approx 0.493,\ corr(X_t, X_{t+2}) \approx 0.248,\ corr(X_t, X_{t+3}) \approx 0.125,$$
$$corr(X_t, X_{t+4}) \approx 0.062,\ corr(X_t, X_{t+5}) \approx 0.031,\ corr(X_t, X_{t+6}) \approx 0.016.$$

According to your opinion, which kind of time series is $\{X_t\}_t$ most likely to be? (You should provide estimates of all parameters.)
(Hint: draw a correlogram, guess which time series does it represent.)