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Idiosyncratic Cash Flows and Systematic Risk

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ABSTRACT

We show that unpriced cash flow shocks contain information about future priced risk. A positive idiosyncratic shock decreases the sensitivity of firm value to priced risk factors and simultaneously increases firm size and idiosyncratic risk. A simple model can therefore explain book-to-market and size anomalies, as well as the negative relation between idiosyncratic volatility and stock returns. Empirically, we find that anomalies are more pronounced for firms with high idiosyncratic cash flow volatility. More generally, our results imply that any economic variable correlated with the history of idiosyncratic shocks can help to explain expected stock returns.

IT IS WELL ESTABLISHED that, under standard asset pricing assumptions, only systematic risk is priced. In this paper, we argue that unpriced idiosyncratic cash flow shocks can also be important for asset prices as they contain valuable conditioning information in a dynamic asset pricing framework. In particular, we show that the conditional beta with respect to any priced source of risk depends directly on the history of firm-specific shocks. We use this insight to provide risk-based explanations for several anomalies in the cross-section of equity returns, including the widely documented value and size effects, the negative relation between idiosyncratic volatility and stock returns, and the underperformance following investment and equity issuance.[1]

To understand why firm-specific shocks are useful as conditioning information, consider a firm with two divisions. Suppose the profit of the first division depends exclusively on idiosyncratic profitability shocks and the profit of the

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[1] In the cross-section, firms with small market capitalization and a high ratio of fundamentals to price tend to have high stock returns (Banz (1981), Graham and Dodd (1934)). Fama and French (1992) provide a detailed analysis of both the value and the size premium. Ang et al. (2006) document that high idiosyncratic volatility predicts low returns. Among others, Loughran and Ritter (1995), Daniel and Titman (2006), and Pontiff and Woodgate (2008) show that stocks underperform following equity issuance.

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second division is driven only by systematic shocks. This firm can be viewed as a portfolio of a zero-beta asset and a risky asset. When a positive idiosyncratic shock occurs, the size of the zero-beta asset increases, making it a larger fraction of the total portfolio value. As a result, overall firm beta decreases, as do expected stock returns. Therefore, any firm characteristic correlated with the history of idiosyncratic cash flow shocks can help explain expected stock returns.

In a more general setting, we show that beta is invariant with respect to idiosyncratic shocks only in the special case in which profits are the product of idiosyncratic and systematic profitability shocks. Multiplicative production functions of this type are used extensively in the literature because of their tractability properties (see, for example, Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), and Cooper (2006)).[2] Therefore, without additional features such as operating leverage, time-varying price of risk, or investment options, market betas are independent of firm-specific shocks.

Using this insight, we build a simple model in which firm value is additive in two types of shocks and only systematic risk is priced. We first consider a firm consisting entirely of assets in place. We show that, in the simple benchmark model, firm characteristics are related to expected returns in the cross-section. All else being equal, firms with larger idiosyncratic cash flows have larger market capitalization and lower book-to-market, and at the same time have lower equity betas. As a result, large firms and growth firms have low expected returns.

Similarly, we obtain a negative relation between idiosyncratic volatility and expected stock returns, a puzzling empirical finding in Ang et al. (2006) that presents a challenge to risk-based explanations of expected stock returns. In our one-factor model, a history of favorable idiosyncratic shocks decreases the relative magnitude of the systematic profit component, thereby increasing idiosyncratic stock return volatility and lowering beta. Importantly, idiosyncratic risk is not priced in our framework, but it is negatively correlated with systematic risk and can therefore predict returns.

The model, which incorporates systematic and idiosyncratic cash flow shocks, also adds to our understanding of the relation between growth options and risk. Since investment options are levered claims on assets in place, they are usually considered more risky than installed capital.[3] We demonstrate that the relation between options and risk depends on the type of investment option. In particular, while growth options linked to systematic shocks increase a firm's risk, growth options linked to idiosyncratic profitability shocks have the opposite effect. Somewhat surprisingly, however, the exercise of both systematic

[2] Notable exceptions are Brennan (1973) and Bossaerts and Green (1989), who model dividends as the sum of persistent idiosyncratic shocks and a single systematic shock. In particular, Bossaerts and Green (1989) derive two-factor arbitrage pricing theory restrictions on dynamic equilibrium asset returns to explain the abnormally high January returns of small stocks.

[3] The existing literature shows that this relation can be reversed in the presence of operating leverage or adjustment costs. See, among others, Carlson, Fisher, and Giammarino (2004), Zhang (2005), Cooper (2006), and Novy-Marx (2011).

and idiosyncratic growth options always leads to a decline in a firm's systematic risk as long as the firm finances new investment with equity. Thus, our model also accounts for the observed poor stock return performance following seasoned equity offerings (Loughran and Ritter (1995)).

Furthermore, growth options magnify the value and size premiums in the model and give rise to time-varying price-earnings ratios. There are two reasons for these effects. First, options make firm value and conditional beta more sensitive to profitability shocks, as irreversible investment options grow in value exponentially (Dixit and Pindyck (1994)). Second, firms optimally exercise their investment options, and as a result lower their risk, only when their market capitalization is high. We show that the nonlinear exposure of growth options to the underlying profitability shock can generate price-earnings ratios that negatively predict returns.

The intuition developed in this paper applies to any setting with a single source or multiple sources of priced risk. As in previous studies, size and value effects are not anomalous relative to the correctly specified asset pricing model and appear only when not all sources of priced risk are accounted for correctly, as in Berk (1995). Reconciling the predictions of our model with the empirical evidence on value, size, and idiosyncratic volatility anomalies relative to the capital asset pricing model (CAPM) thus relies on imperfect measurement of risk, and in particular on differences between conditional and unconditional betas, as discussed in Gomes, Kogan, and Zhang (2003). Lewellen and Nagel (2006) argue that the conditional CAPM cannot match the magnitude of observed anomalies because the variation in estimated betas is not sufficiently large. However, betas are likely to be mismeasured because either asset pricing tests fail to use all conditioning information (Hansen and Richard (1987)) or the proxy for the market portfolio is imperfect (Roll (1977)).

We use analytical solutions from the model to simulate firms' stock returns and examine the fit between the model-generated and empirically observed data. Our analysis of the simulated data indicates that the model can produce reasonable value and size effects in cross-sectional Fama and MacBeth (1973) regressions even when we explicitly control for empirically estimated betas. For example, we find a size premium of 0.48% per month for the decile of stocks with the smallest market capitalization relative to the largest decile. Sorting based on book-to-market ratio, price-earnings ratio, and idiosyncratic volatility yields return differentials of similar magnitudes. Value and size anomalies are more pronounced when growth options are valuable. These results are consistent with empirical evidence in Da, Guo, and Jagannathan (2012) and Grullon, Lyandres, and Zhdanov (2012), who argue that the poor empirical performance of the unconditional CAPM is largely attributable to real options.

In addition to explaining established asset pricing anomalies, our model produces novel empirical predictions. In particular, the model suggests that asset pricing anomalies are stronger when a significant part of cash flows is idiosyncratic in nature. We test this prediction using portfolios of stocks sorted by the volatility of their idiosyncratic cash flows and by firm characteristics such as size and book-to-market. We find that that size and book-to-market effects are present within each idiosyncratic cash flow volatility group. The magnitude of the effects, however, increases sharply with idiosyncratic cash flow volatility. For example, small stocks outperform big stocks by 0.10% per month in the low volatility quintile, and by 1.23% in the high volatility quintile. The difference is economically large and statistically significant. Controlling for market risk or the three Fama and French (1993) factors has little effect on these return differences. We find similar patterns for the value premium. In sum, as predicted by the theory, we find that anomalies are stronger among firms with higher idiosyncratic cash flow risk.

The paper is organized as follows. In the next section, we provide a brief summary of the related literature. Section II presents a simple framework to develop the intuition. Section III builds the continuous-time model with investment, and Section IV discusses asset pricing implications. Section V presents simulation results and compares them with observed empirical regularities, and Section VI tests the model predictions empirically. Section VII concludes.

I. Literature

We contribute to the theoretical literature by showing that conditional betas are always affected by idiosyncratic cash flow shocks, except for the special case of multiplicative value functions. Models with financial or operating leverage, investment irreversibility, adjustment costs, or decreasing returns to scale typically produce value functions that fall outside of this special case. Therefore, the new results presented in our paper also help reinterpret previous theoretical studies that do not focus specifically on idiosyncratic shocks.

For example, in Merton's (1973) framework with risky debt, a positive idiosyncratic profitability shock decreases a firm's leverage and in turn equity risk. Therefore, conditional betas in his framework depend on past idiosyncratic shocks. Similarly, Carlson, Fisher, and Giammarino (2004) model a firm with operating leverage that can expand its business by investing in new projects. Operating leverage makes assets in place more risky than growth options, giving rise to book-to-market and size anomalies. Sagi and Seasholes (2007) allow for negative leverage (savings) and provide a risk-based explanation for momentum.[4]

Likewise, idiosyncratic cash flows have been shown to affect systematic risk in investment-based asset pricing models such as Zhang (2005), Cooper (2006), and Lin and Zhang (2013).[5] In this class of models, positive idiosyncratic

[4] Sagi and Seasholes (2007) model a firm as a portfolio that is long in both a risk-free asset and a risky asset. They show that savings result in a positive return autocorrelation, as high returns on the risky asset increase portfolio risk. In contrast, we model a risky zero-beta asset instead of a risk-free asset. The volatility embedded in the zero-beta asset changes the intuition of Sagi and Seasholes (2007) since positive shocks can now be either systematic or idiosyncratic in nature, and thus can either increase or decrease future risk.

[5] Zhang (2005) models costly investment reversibility and a countercyclical price of risk. Specifically, he shows that in bad times, when the price of risk is high, assets in place are riskier than growth options because they are costly to reduce. This effect leads to an unconditional value

productivity shocks increase contemporaneous investment but decreasing returns to scale lower the return on future investment. Expected returns also decline since investment returns equal stock returns. Importantly, our mechanism does not require either leverage or investment because firm beta can change with the relative weight of an idiosyncratic profit component in firm value.

Other explanations of cross-sectional anomalies rely on the variation in discount rates, additional risk factors, or time-varying prices of risk, and can therefore be viewed as complementary to ours. We show that size-related asset pricing anomalies can appear in any factor model if risk is not perfectly accounted for. In this respect, our study is similar to Berk (1995), who shows that anomalies arise from differences in firms' unobservable discount rates. In contrast, our results are driven by a new mechanism related to variation in firms' cash flows. Berk, Green, and Naik (1999) are among the first to link the theory of investment under uncertainty to determinants of the cross-section of stock returns in a two-factor model. Kogan and Papanikolaou (2013) build an equilibrium model with two aggregate sources of risk that have different pricing implications for growth options and assets in place.

A number of studies, including Gomes, Kogan, and Zhang (2003), Menzly, Santos, and Veronesi (2004), Lettau and Wachter (2007), and Da (2009), model mean-reverting processes for cash flows in an economy with an upward-sloping term structure of equity. Under these assumptions, a positive idiosyncratic shock shortens the cash flow duration and leads to a decrease in systematic risk. In contrast, our benchmark model assumes permanent cash flow shocks with a constant price of risk.

II. Idiosyncratic Shocks and Firm Risk

Here, we develop a simple framework to highlight the main economic mechanism in the paper. The shock y is the single source of systematic risk in the economy. We assume a linear stochastic discount factor (SDF)

$$m = a + bR_y, \tag{1}$$

where the factor return is $R_y = dy/y$. Conditional expected excess returns are determined by assets' covariance with the SDF,

$$\mathrm{E}\left(R_i\right) - r = -\frac{\mathrm{cov}\left(R_i, m\right)}{\mathrm{E}\left(m\right)} = \beta_i \lambda, \tag{2}$$

where r is the risk-free rate, $\lambda = -\frac{1}{b}\mathrm{var}(m)/\mathrm{E}(m)$ is the constant price of risk, and beta is defined as

$$\beta_i = \frac{\mathrm{cov}\left(R_i, R_y\right)}{\mathrm{var}\left(R_y\right)}. \tag{3}$$

premium. Cooper (2006) develops similar intuition in a model with lumpy investment and a constant price of risk.

Now consider a firm with value $V(x_i, y)$ that depends on both an idiosyncratic shock, x_i, and the systematic shock, y. We assume that $V(x_i, y)$ is a continuous, twice differentiable function, and that a higher idiosyncratic shock indicates a better state of the world, that is, $V_x(x_i, y) > 0$. From equation (3), the firm beta is the sensitivity of relative changes in value to relative changes in the systematic shock,[6]

$$\beta_i = \frac{V_y\,(x_i, y)\, y}{V\,(x_i, y)}. \tag{4}$$

Clearly, equity beta, and therefore the expected excess return, depends on idiosyncratic shock x_i. Differentiating expression (4) with respect to x_i, we show that beta is independent of idiosyncratic shocks only in the "knife-edge" case in which

$$V\,(x_i, y) = \frac{V_x\,(x_i, y)\, V_y\,(x_i, y)}{V_{xy}\,(x_i, y)}. \tag{5}$$

This partial differential equation is satisfied by multiplicatively separable functions of the form

$$V\,(x_i, y) = f\,(x_i)\, g\,(y)\,, \tag{6}$$

which are used extensively in the previous literature. It is worth examining criterion (5) with care. For the majority of value functions, this condition will not be satisfied and beta will depend on the current realization of shock x_i. This result implies that firm characteristics correlated with the history of idiosyncratic shocks, such as firm size, book-to-market, and volatility, must be related to expected returns.

As an example, consider a firm with additive value function

$$V\,(x_i, y) = f\,(x_i) + g\,(y) - c, \tag{7}$$

where the first term captures the value derived from idiosyncratic shocks, the second term captures the value from systematic shocks, and c is the firm's long or short position in the risk-free asset.[7]

From (4), firm beta is given by

$$\beta_i = \frac{g_y\,(y)\, y}{V\,(x_i, y)}. \tag{8}$$

[6] To obtain this result, substitute $R_i = dV_i/V_i$ in equation (3),

$$\beta_i = \frac{\operatorname{cov}\left(\frac{dV(x_i, y)}{V(x_i, y)}, \frac{dy}{y}\right)}{\operatorname{var}\left(\frac{dy}{y}\right)} = \frac{y}{V\,(x_i, y)} \frac{\operatorname{cov}\left(V_x\,(x_i, y)\, dx_i + V_y\,(x_i, y)\, dy, dy\right)}{\operatorname{var}\,(dy)},$$

and note that dx_i and dy are uncorrelated.

[7] The restriction to one purely idiosyncratic and one purely systematic division is made for simplicity. In the Internet Appendix, which may be found in the online version of this article, we show that similar intuition applies when two divisions have different sensitivities to a risk factor.

It is easy to see that β_i in this case is decreasing in the idiosyncratic shock since the numerator in expression (8) is independent of x_i and the denominator is increasing in x_i. Intuitively, although the idiosyncratic component is independent of priced risk and therefore does not affect the covariance between future firm value and the SDF, it does affect the covariance between firm stock returns and the SDF. For example, if the idiosyncratic component is very large, changes in the systematic component contribute little to firm returns, making them practically insensitive to the priced factor.

This result implies that a positive idiosyncratic shock simultaneously increases firm value and decreases beta, giving rise to value and size effects in the cross-section of stock returns. To link our results to prior literature, expression (8) can be rewritten as follows:

$$\beta_i = 1 - \frac{f(x_i)}{V(x_i, y)} + \frac{g_y(y)\,y - g(y)}{V(x_i, y)} + \frac{c}{V(x_i, y)}. \tag{9}$$

The first term in (9) is normalized to one. The second term is new to the literature and is responsible for the size effect since a higher value of shock x_i will simultaneously lead to higher firm value and lower beta. The third term appears only if function $g(y)$ is nonlinear in the systematic shock y. Whether this term increases or decreases the overall firm beta depends on the concavity/convexity of function $g(y)$. For example, growth options linked to systematic profitability shocks induce convexity in the value function and therefore tend to increase systematic risk. In contrast, decreasing returns to scale can limit the firm's profits and induce concavity in the value function, thereby decreasing firm beta. The last term in (9) can represent operating or financial leverage ($c > 0$) or cash savings ($c < 0$). This term has received considerable attention in previous literature (e.g., Carlson, Fisher, and Giammarino (2004) and Sagi and Seasholes (2007)).

III. The Model

This section lays out a model that extends the simple example to a dynamic setting and incorporates investment options. The model facilitates comparison of our results with those of previous studies and enables us to evaluate the economic importance of asset pricing anomalies in simulated data. We deliberately do not model operating leverage or a time-varying price of risk since previous work already shows that these features can help generate size and value premiums.

A. Model Setup

Each firm in the economy generates profit Π_i, which is driven by the idiosyncratic demand shock x_i, for example, tastes for the differentiated firm's product, and the systematic shock y,

$$\Pi_i = x_i + \rho_i y, \tag{10}$$

where ρ_i is the cash flow's sensitivity to the systematic shock y. Time subscripts are omitted throughout. The profitability shocks follow geometric Brownian motions in the risk-neutral measure

$$dx_i/x_i = \mu_x dt + \sigma_x dz_i, \tag{11}$$

$$dy/y = \mu_y dt + \sigma_y dz_y, \tag{12}$$

where dz_i and dz_y are increments of uncorrelated standard Wiener processes, $\mathrm{E}[dz_i dz_y] = 0$ for all i.

Note that our specification assumes that profitability shocks are permanent. This assumption is made for tractability and contrasts with, for example, Gomes, Kogan, and Zhang (2003), Menzly, Santos, and Veronesi (2004), Lettau and Wachter (2007), and Da (2009), who use mean-reverting stochastic processes. Note, however, that the main economic mechanism in our paper does not depend on the choice of stochastic process.[8]

In addition to receiving continuous profits (10), each firm has an opportunity to irreversibly expand production (or improve technology) by paying a fixed cost. For tractability purposes, we assume that a firm can separately exercise growth options linked to idiosyncratic shocks (x-options) and systematic shocks (y-options).[9] Specifically, by paying an investment cost I_x a firm can increase the idiosyncratic component of its cash flows x_i by a factor $1 + \gamma_x$, and by spending $\rho_i I_y$ it can increase the systematic component of cash flows $\rho_i y$ by a factor $1 + \gamma_y$. We make the exercise cost of the y-option proportional to ρ_i to ensure that the cost of exercising options scales up appropriately with the size of the firm's assets. Investment is irreversible and indivisible, and, unlike Ai and Kiku (2013), we do not allow the investment cost to change with the state of the economy.

B. Firm Value

The value of the firm $V_i = V(x_i, y)$ is given by

$$V_i = \mathrm{E} \int \Pi_i e^{-rt} dt, \tag{13}$$

where r is the constant discount rate and Π_i is the firm's instantaneous profit. Firm value is obtained by solving the partial differential equation

$$rV_i = \Pi_i + \mu_x x_i \frac{\partial V_i}{\partial x_i} + \mu_y y \frac{\partial V_i}{\partial y} + \frac{\sigma_x^2 x_i^2}{2} \frac{\partial^2 V_i}{\partial x_i^2} + \frac{\sigma_y^2 y^2}{2} \frac{\partial^2 V_i}{\partial y^2}, \tag{14}$$

[8] Specifically, in Section II, no assumptions on the dynamics of process of x_i were required. In addition, in the Internet Appendix we show that the main effect of idiosyncratic shocks on firm beta remains in a model with a mean-reverting shock process.

[9] Alternatively, growth options could be modeled to depend on both idiosyncratic and systematic profits, and to increase *total* firm cash flows. Such an approach does not change the intuition developed in this paper, but creates significant complications due to a two-dimensional option exercise policy.

with appropriate boundary conditions. In particular, we use both the value-matching conditions specifying that firm value changes by the amount of external financing at the time of growth option exercise and the smooth-pasting conditions on the first derivatives of firm value that are required for optimal option exercise (Dumas (1991) and Dixit (1993)).

The solution for firm value is summarized by the following proposition.

PROPOSITION 1: *Denote by ι_x and ι_y indicator functions equal to one if the respective growth option has been exercised. Then the market value of the firm is given by*

$$V_i = V_i^{AX} + V_i^{GX} + \rho_i(V^{AY} + V^{GY}), \tag{15}$$

where the value components V_i^{AX}, V_i^{GX}, V^{AY}, and V^{GY} are given by

$$V_i^{AX}(x_i) = \frac{(1 + \iota_x \gamma_x)\, x_i}{r - \mu_x}, \tag{16}$$

$$V_i^{GX}(x_i) = \frac{(1 - \iota_x)\, \gamma_x x^*}{(r - \mu_x)\, d_2} \left(\frac{x_i}{x^*}\right)^{d_2}, \tag{17}$$

$$V^{AY}(y) = \frac{\left(1 + \iota_y \gamma_y\right) y}{r - \mu_y}, \tag{18}$$

$$V^{GY}(y) = \frac{\left(1 - \iota_y\right) \gamma_y y^*}{\left(r - \mu_y\right) b_2} \left(\frac{y}{y^*}\right)^{b_2}, \tag{19}$$

and the constants $b_2 > 1$ and $d_2 > 1$ as well as the option exercise thresholds x^* and y^* are given in Appendix A.

PROOF: See Appendix A. □

Having derived firm value and the optimal investment strategies, we now turn to the analysis of systematic risk.

C. Equity Betas

Since the systematic shock y represents aggregate uncertainty in the model, the firm's equity beta is the elasticity of firm value with respect to this shock. We derive the factor beta in the following proposition. Appendix B discusses the relation between factor betas with respect to the systematic shock y and beta with respect to the aggregate market.

PROPOSITION 2: *The factor beta of the firm is given by*

$$\beta_i = 1 - \frac{V_i^{AX}}{V_i} - \frac{V_i^{GX}}{V_i} + \rho_i\,(b_2 - 1)\,\frac{V^{GY}}{V_i}. \tag{20}$$

PROOF: See Appendix A. □

The first term in (20) is normalized to one. The second term appears because part of firm value is derived from profits uncorrelated with aggregate demand uncertainty, reducing the firm's overall exposure to systematic risk. The third term shows that beta decreases with growth options linked to idiosyncratic profitability shocks (x-options) because such options increase the value of the firm without increasing the firm's sensitivity to the factor. Finally, the last term captures the well-known effect that beta is larger for growth options linked to systematic shocks (y-options). Such options are more sensitive to systematic shocks than assets in place and therefore increase firm risk.

D. Book-to-Market and Price-Earnings Ratios

To relate firm characteristics to risk, we now specify the evolution of book value and earnings. We calculate book value based on the cost incurred per unit of installed capital. Since, at the time of option exercise, γ_x units of x-assets are added at cost I_x and $\rho_i \gamma_y$ units of y-assets are added at cost $\rho_i I_y$, the initial book value is set to

$$B_i = \frac{I_x}{\gamma_x} + \rho_i \frac{I_y}{\gamma_y}. \tag{21}$$

Book value increases by I_x and $\rho_i I_y$ at the exercise of the corresponding idiosyncratic and systematic growth options. Since firms in our model have no leverage, the book-to-market ratio is given by B_i / V_i.

The price-earnings ratio PE_i is computed using the firm's profits (10) with an adjustment for asset expansion,

$$PE_i = \frac{V_i}{(1 + \iota_x \gamma_x) x_i + \left(1 + \iota_y \gamma_y\right) \rho_i y}. \tag{22}$$

The price-earnings ratio is a function of both shocks x_i and y, and therefore varies over time and in the cross-section. Finally, idiosyncratic volatility can be calculated as

$$IVol_i^2 = \mathrm{var}\left(\frac{dV_i}{V_i}\right) - \beta_i^2 \mathrm{var}\left(\frac{dy}{y}\right) = \frac{\sigma_x^2}{V_i^2}\left(V_i^{AX} + d_2 V_i^{GX}\right)^2, \tag{23}$$

where the last equality follows by substituting β_i from (20) and calculating variances.

IV. Asset Pricing Implications

Next, we use Propositions 1 and 2 to evaluate the ability of the model to explain asset pricing anomalies. To facilitate discussion, we first obtain the sensitivity of a firm's market value, factor beta, price-earnings ratio, and idiosyncratic volatility to the idiosyncratic profitability shock x_i,

$$\frac{\partial V_i}{\partial x_i} = \frac{1}{x_i}\left(V_i^{AX} + d_2 V_i^{GX}\right) > 0, \tag{24}$$

$$\frac{\partial \beta_i}{\partial x_i} = -\frac{\rho_i}{x_i V_i^2}\left(V_i^{AX} + d_2 V_i^{GX}\right)\left(V^{AY} + b_2 V^{GY}\right) < 0, \tag{25}$$

$$\frac{\partial PE_i}{\partial x_i} = \frac{\rho_i}{\Pi_i^2}\left((\mu_x - \mu_y)\frac{V_i^{AX} V^{AY}}{x_i} - V^{GY}\right) + \frac{V_i^{GX}}{\Pi_i^2}\left(d_2 - 1 + d_2\frac{\rho_i y}{x_i}\right), \tag{26}$$

$$\frac{\partial IVol_i}{\partial x_i} = \frac{\sigma_x}{x_i V_i^2}\left((d_2-1)^2 V_i^{AX} V_i^{GX} + \rho_i (V^{AY} + V^{GY})\left(V_i^{AX} + d_2^2 V_i^{GX}\right)\right) > 0. \tag{27}$$

Since a positive idiosyncratic shock represents good news and $d_2 > 1$, it is evident from (24) that firm value is increasing in the idiosyncratic shock. Furthermore, beta is decreasing in shock x_i. Therefore, it follows that a positive idiosyncratic shock simultaneously increases firm value and decreases beta, leading to the size and book-to-market anomalies. The effect of idiosyncratic shocks on the price-earnings ratio is in general ambiguous, as can be seen from (26). Finally, a positive idiosyncratic shock increases idiosyncratic volatility, resulting in the negative correlation between expected returns and idiosyncratic volatility. We now discuss how different ingredients of the model affect the magnitudes of the asset pricing anomalies.

A. The Benchmark Model

Consider first the benchmark model with no real options ($V_i^{GX} = V^{GY} = 0$). As outlined earlier, the negative relation between the market capitalization of the firm and beta leads to the size and book-to-market effects. However, since there is no time-series variation in book values, it is difficult to distinguish the two anomalies in this case.

Furthermore, as suggested by equation (26), the price-earnings ratio is constant in the benchmark model when the effective risk-neutral drifts are identical ($\mu_x = \mu_y$). If the risk-adjusted drift of the idiosyncratic component is higher than the drift of the systematic component, the price-earnings ratio is time-varying and negatively related to expected stock returns.

The benchmark model is also able to generate a negative relation between stock returns and idiosyncratic volatility. A positive idiosyncratic shock results in a larger idiosyncratic share of profits and hence higher idiosyncratic volatility of stock returns, while it also lowers systematic firm risk.

B. Growth Options

The general form of our profit function suggests a role for growth options that derive their value from the idiosyncratic profit component. We therefore analyze the effect of growth options on firm risk and their importance for asset pricing.

Since growth options can be viewed as levered claims on assets in place, prior literature typically finds a positive relation between the value of options and overall firm risk.[10] We show that this intuition breaks down once we allow for options to depend on idiosyncratic cash flow shocks. Specifically, all growth options increase firm value. In line with the previous literature, a higher value of the systematic option increases the firm's exposure to systematic risk. However, larger idiosyncratic options imply a smaller overall beta, as shown in Proposition 2. Therefore, depending on their nature, growth options can lead to either lower or higher firm risk.

Prior empirical literature suggests that growth options are related to asset pricing anomalies. In particular, Da, Guo, and Jagannathan (2012) and Grullon, Lyandres, and Zhdanov (2012) show that the unconditional CAPM performs better in the absence of growth options. To see how growth options affect anomalies in our model, we again look at the sensitivity of firm value and beta to idiosyncratic profit shocks in equations (24) to (27).

Since b_2 and d_2 are greater than one, it is easy to see that, with x-options, firm value and hence book-to-market become more sensitive to idiosyncratic shocks. The systematic options have no effect on this sensitivity. At the same time, the sensitivity of beta to the x_i shocks increases with both systematic and idiosyncratic options. In particular, it follows from (25) that, conditional on holding total firm value fixed, a firm that derives more value from growth options will have a higher sensitivity of beta to idiosyncratic profitability shocks. Similarly, idiosyncratic volatility is more sensitive to idiosyncratic shocks when the firm has large investment options. This can be seen by noting that $d_2 > 1$ in equation (27). Overall, these results imply that growth options, particularly those linked to idiosyncratic shocks, magnify value, size, and idiosyncratic volatility anomalies in the model.

Finally, growth options add to our understanding of the predictive ability of the price-earnings ratio. When firms have expansion options, their price-earnings ratio fluctuates with idiosyncratic and systematic shocks. Since betas decrease with idiosyncratic shocks, equation (26) shows that the relation between firms' price-earnings ratio and risk is in general ambiguous. The empirically observed negative relation obtains when idiosyncratic options are large in comparison with systematic options or when risk-neutral drift μ_x is large relative to μ_y.

[10] Note that operating or financial leverage can change this relation, as pointed out by Zhang (2005), Carlson, Fisher, and Giammarino (2006), and Novy-Marx (2011).

C. Option Exercise

We now show that option exercise in our model predicts low future returns. This is consistent with empirical evidence on underperformance following share issuances (Loughran and Ritter (1995), Daniel and Titman (2006), Pontiff and Woodgate (2008)) and the negative relation between asset growth and stock returns in the cross-section (Cooper, Gulen, and Schill (2008)). The following proposition shows that any option exercise (either x- or y-type) leads to a decline in equity beta, provided that the new investment is financed by equity issuance.

PROPOSITION 3: *The beta of the firm declines at the exercise of the idiosyncratic or systematic growth options if investment is financed by new equity issuance.*

PROOF: See Appendix A. □

As in Carlson, Fisher, and Giammarino (2006), systematic risk decreases following the exercise of options linked to systematic shocks because such options are more sensitive to the priced factor than are assets in place. The proposition shows that risk also decreases at the exercise of idiosyncratic options, albeit for a different reason. Since the firm raises external financing and uses it to invest in idiosyncratic assets, systematic profits become a smaller fraction of total firm value, reducing beta. If the firm finances investment by taking on new debt or using its own cash reserves, then firm beta does not change at the time of growth option exercise. Financial leverage increases with debt issuance or the reduction in cash holdings, and this effect exactly offsets the reduction in leverage from exchanging options for assets in place.

V. Simulation Results

In this section, we use simulations to evaluate the ability of our framework to reproduce the key features of stock return data. Since closed-form solutions to the model are available, we use them directly to generate a panel of firms over time. Because we model idiosyncratic and systematic profitability shocks as geometric Brownian motions and each firm is endowed with only two growth options, we allow for random firm exit and entry to ensure a stationary distribution of firm values over time. We first discuss calibration of the model parameters and then examine the properties of the generated data.

A. Calibration

Table I summarizes the parameters used in the calibration. We simulate monthly data for $N = 100$ economies of $n = 2,000$ firms over 1,050 years. The first 50 years of each simulation are discarded to ensure that the results are not affected by initial conditions. The result is a time series of 1,000 years that we use in our tests.

We assume that both systematic and idiosyncratic profitability shocks grow at 3% annually, in line with observed aggregate earnings growth rates. In

Table I
Parameter Values Used in Simulations

This table lists the parameters used in simulations.

Parameter	Notation	Value
Initial profitability shocks	(x_0, y_0)	(1, 1)
Distribution of factor sensitivity	ρ_i	$U[0, 2]$
Volatility of profitability shocks	(σ_x, σ_y)	(0.25, 0.15)
Drift of profitability shocks	(μ_x, μ_y)	(0.03, 0.01)
Cost of exercising options	$(I_x, \rho_i I_y)$	$(20, \rho_i 20)$
Investment scale	(γ_x, γ_y)	(1, 1)
Firm turnover rate	δ	0.02
Simulation horizon (in years)	T	1,000
Number of simulated firms	n	2,000
Number of simulated economies	N	100
Price of risk for y-factor	λ	0.10
Risk-free rate	r	0.04

the risk-neutral measure, we account for the risk premium associated with the systematic profitability shocks by reducing the growth rate of y-shocks to $\mu_y = 1\%$, while leaving μ_x unchanged at 3%.[11] The volatilities are set to $\sigma_y = 0.15$ and $\sigma_x = 0.25$. The risk-free rate is $r = 0.04$, and the price of risk associated with the y-factor is $\lambda = 0.10$. Under our parameterization, this price of risk implies an average equity risk premium of approximately 4.5% per year.

Firms' exposure to the systematic shock, ρ_i, is uniformly distributed on the interval $[0, 2]$. The exercise of systematic and idiosyncratic growth options doubles the corresponding cash flows, $\gamma_x = \gamma_y = 1$. The initial values of profitability shocks are normalized to $x_0 = y_0 = 1$ and exercise costs to $I_x = I_y = 20$.

Firms randomly exit and enter at a rate of $\delta = 2\%$. Upon entry, firms are rescaled versions of the initial firms, with new idiosyncratic shock x_i and exercise costs I_x and I_y scaled by y_t/y_0. As a result, while the economy as a whole can grow, the distribution of relative firm sizes and the proportion of unexercised growth options remains stationary over time.

Realized returns are obtained as follows:

$$r_{i,t} = \frac{V_{i,t} + \Pi_{i,t} - V_{i,t-1} - I_x \iota_x - \rho_i I_y \iota_y}{V_{i,t-1}} + \lambda \beta_{i,t}, \tag{28}$$

where $\Pi_{i,t}$ denotes the dividend process. The first term represents the realized return in the risk-neutral measure, adjusted for external financing and dividends. It has an expectation equal to the risk-free rate. The second term is the risk premium computed as the individual firm's beta multiplied by the price of risk.

[11] The risk adjustment of 2% is consistent with an annual market Sharpe ratio of 0.35, our assumed volatility of y-shocks of 0.15, and a correlation of cash flow growth rates and market returns of 0.4.

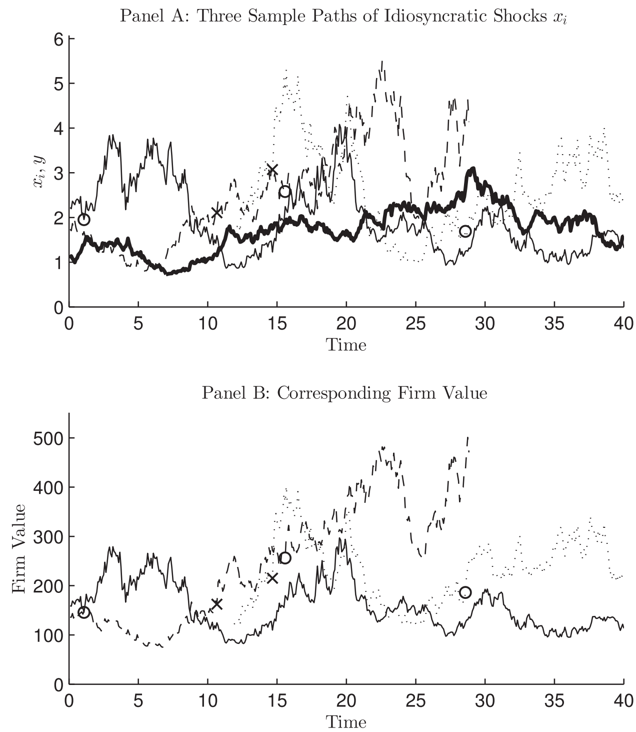

Figure 1. Sample firm dynamics. This figure plots for three randomly selected firms: sample paths of the x-shocks (Panel A), firm value (B), book-to-market (C), the price-earnings ratio (D), and factor beta (E). Panel A also depicts the sample path of the systematic shock (solid line). The exercise of idiosyncratic and systematic growth options is indicated with "×" and "○," respectively.

B. Anomalies

Figure 1 illustrates the dynamics of the main variables in the model. Specifically, in Panel A we display a sample path of the systematic component of cash flows, y (thick solid line), and three sample paths of x-shocks. One of the three firms exists during the entire 40-year observation period (solid line), the second firm is born in year 12 (dotted line), and the third firm dies in year 30 (dashed line). The circle (cross) markers on the graph indicate when the systematic (idiosyncratic) investment options are exercised. The first firm had already exercised its idiosyncratic option before entering the observation period. Evidently, the options are exercised when the respective profitability shocks reach new historical highs. The exercise of systematic options happens at different times for different firms because firms' exercise cost is rescaled at entry.

We next compute firm value at each point in time and plot the results in Panel B. Observe that firm value jumps at the point of option exercise. This is caused by an inflow of external funds to finance firm expansion, and does not

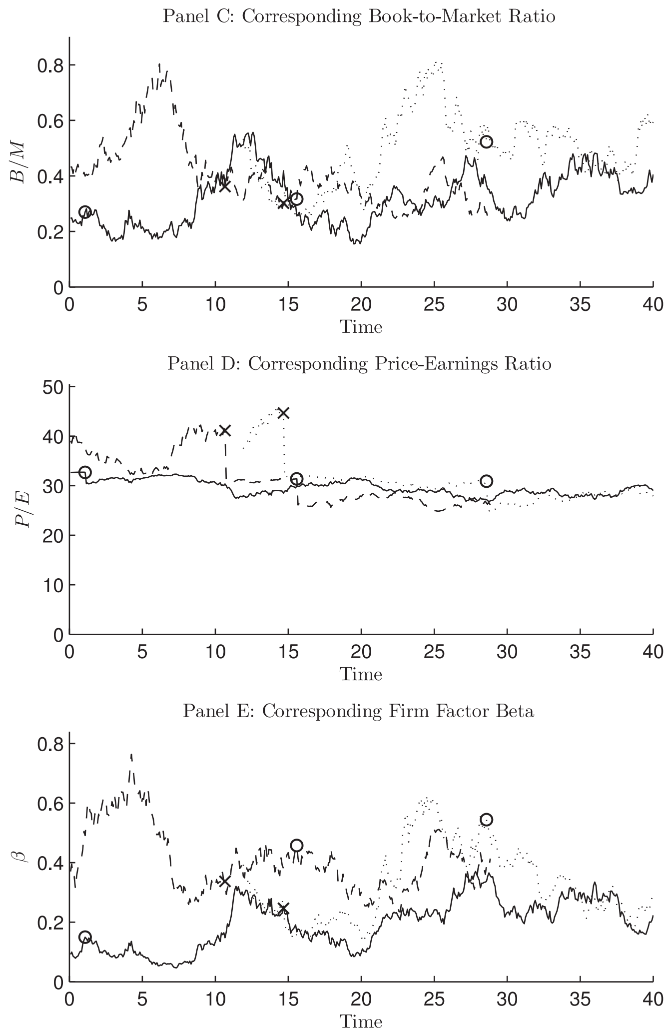

Figure 1. Continued

represent an investable return from holding the firm. The book-to-market ratio in Panel C fluctuates as x- and y-shocks evolve. In addition, book-to-market jumps discontinuously when investment options are exercised. At exercise, both market value and book value increase by the cost of investment. Since book-to-market ratios in our model are typically below one, exercise usually increases the book-to-market ratio.

In Panel D, the price-earnings ratio fluctuates with profitability shocks because the shocks have different drifts and also because the ratio of option value to assets in place changes. The price-earnings ratio generally increases with

x-shocks and drops sharply at option exercise because options are replaced with newly installed earnings generating assets in place.

Panel E illustrates that a firm's factor beta can change over time in response to several effects. First, for a fixed systematic shock, beta decreases with idiosyncratic shocks. This effect drives most of the gradual changes in beta in the graph. Beta also changes with systematic shocks, with larger effects for firms with higher factor sensitivity ρ_i. Second, when systematic investment options (y-options) are exercised and converted into assets in place, there is an immediate drop in beta because assets in place have lower sensitivity to the value of the shock. Beta also declines with the exercise of idiosyncratic options (x-options) because the part of firm value that is unrelated to market risk increases by the amount of new equity financing.

We conduct asset pricing tests using the simulated panel of data. Every month, we form 10 portfolios based on beginning-of-period market capitalization, book-to-market, the price-earnings ratio, and idiosyncratic volatility. Idiosyncratic volatility is estimated as the residual standard deviation from time-series regressions of stock returns on changes in the systematic profitability shock over the previous 24 months. Within each portfolio, we weight stocks equally. We rebalance the portfolios every month and calculate time-series average returns.

In Table II, we document the magnitudes of the cross-sectional effects generated by the model across decile portfolios. The relation between firm size and returns is negative and nearly linear. The difference in average stock returns between the top and bottom deciles by market capitalization amounts to 0.48% per month. This premium is fully explained by the difference in betas of 0.58 ($0.58 \times 10\% = 5.8\%$ annually, or 0.48% monthly). Similarly, returns of portfolios sorted on the book-to-market (price-earnings) ratio differ by 0.49% (0.59%) monthly. Consistent with empirical evidence, high idiosyncratic risk is associated with low returns. In particular, the difference in average stock returns between the top and bottom deciles by idiosyncratic volatility is 0.44% monthly.

Our evidence suggests that the model has the potential to explain four common asset pricing anomalies in a univariate setting. We now turn to cross-sectional Fama-MacBeth regressions to evaluate the multivariate performance of the model. Table III reports average Fama-MacBeth coefficients across 100 simulated economies and the corresponding average t-statistics for each coefficient. In each month t, realized stock returns are regressed on the theoretical factor beta (β), the estimated factor beta ($\widehat{\beta}$), log firm value (*Size*), the log book-to-market ratio (B/M), the log price-earnings ratio (P/E), and the log of idiosyncratic volatility ($IVol$). Factor beta and idiosyncratic volatility are estimated, respectively, as the slope coefficient and residual standard deviation from time-series regressions of stock returns on changes in the systematic profitability shock from month $t-24$ to $t-1$.

As a reference, the first row in the table shows that, not surprisingly, theoretical factor betas are highly significant, with a factor risk premium equal to $10\%/12 = 0.83\%$ per month. The R^2 indicates that approximately 4% of the

Table II

Characteristic-Sorted Portfolios on Simulated Data

This table reports average returns, estimated unconditional portfolio betas, and characteristics of 10 portfolios formed by market capitalization (*Size*, Panel A), book-to-market ratio (B/M, Panel B), price-earnings ratio (P/E, Panel C), and idiosyncratic volatility ($IVol$, Panel D). $IVol$ is estimated as the residual standard deviation from time-series regressions of stock returns on changes in the systematic profitability shock from month $t-24$ to $t-1$. The data are generated from 100 simulations of a cross-section of 2,000 stocks over 1,000 years.

	Low	2	3	4	5	6	7	8	9	High	H−L
					Panel A: Size Portfolios						
Ret	0.84	0.84	0.83	0.79	0.74	0.67	0.58	0.50	0.43	0.36	−0.48
β	0.61	0.62	0.60	0.56	0.50	0.40	0.30	0.20	0.12	0.04	−0.58
Size	1.71	2.48	2.80	3.04	3.27	3.52	3.83	4.25	4.88	6.58	4.87
					Panel B: Book-to-Market Portfolios						
Ret	0.39	0.47	0.54	0.59	0.65	0.70	0.75	0.79	0.83	0.88	0.49
β	0.07	0.17	0.24	0.32	0.38	0.45	0.50	0.55	0.60	0.66	0.59
B/M	−3.23	−1.71	−1.22	−0.93	−0.70	−0.48	−0.26	−0.00	0.34	1.12	4.35
					Panel C: Price-Earnings Portfolios						
Ret	1.08	0.93	0.80	0.70	0.61	0.53	0.47	0.44	0.54	0.49	−0.59
β	0.90	0.72	0.57	0.44	0.33	0.24	0.16	0.13	0.24	0.19	−0.71
P/E	3.06	3.17	3.26	3.33	3.38	3.42	3.46	3.49	3.56	3.75	0.69
					Panel D: Idiosyncratic Volatility Portfolios						
Ret	0.85	0.87	0.84	0.77	0.70	0.62	0.56	0.50	0.46	0.41	−0.44
β	0.62	0.65	0.61	0.53	0.44	0.35	0.27	0.21	0.15	0.10	−0.53
$IVol$	−3.57	−3.38	−3.27	−3.18	−3.08	−2.98	−2.89	−2.78	−2.67	−2.48	1.10

cross-sectional variation in realized returns can be attributed to variation in risk premiums. Specification II replaces true factor betas with their estimated counterparts. While empirical betas are also strongly related to returns, the estimated risk premium and the regression R^2 drop because of measurement error in the explanatory variable. Specifications III to VI confirm the findings of Table II. In particular, there is a significant positive relation between book-to-market and realized returns in the simulated data, while firm size, the price-earnings ratio, and idiosyncratic volatility all predict returns negatively. Specification VII shows that both size and book-to-market remain significant return predictors after controlling for estimated beta. Since our model is a conditional one-factor model, the true theoretical betas in regression VIII drive out all other variables.

C. Market-Based Asset Pricing

In our model, a firm's beta with respect to a single observable priced factor fully determines expected stock returns. However, since true beta is unknown

Table III

Fama-MacBeth Regressions on the Simulated Data

This table reports average Fama-MacBeth coefficients and average t-statistics from 100 simulations of a cross-section of 2,000 stocks over 1,000 years. In each month t, the realized stock returns are regressed on the theoretical factor beta (β), the estimated factor beta ($\widehat{\beta}$), log firm value ($Size$), the log book-to-market ratio (B/M), the log price-earnings ratio (P/E), and the log of idiosyncratic volatility ($IVol$). Factor beta and idiosyncratic volatility are estimated, respectively, as slope coefficient and residual standard deviation from time-series regressions of stock returns on changes in the systematic profitability shock from month $t-24$ to $t-1$. The t-statistics provided in parentheses are based on standard errors that correspond to 40-year sample periods. The adjusted R^2 in the last column is the time-series average of cross-sectional R^2.

	β	$\widehat{\beta}$	$Size$	B/M	P/E	$IVol$	R^2 (%)
I	0.83						3.97
	(4.14)						
II		0.33					1.68
		(3.85)					
III			−0.11				1.40
			(−3.97)				
IV				0.12			1.28
				(3.90)			
V					−0.96		2.23
					(−4.06)		
VI						−0.21	2.44
						(−4.05)	
VII		0.24	−0.04	0.05			2.35
		(3.66)	(−2.29)	(2.31)			
VIII	0.83		−0.00	−0.00			3.99
	(4.12)		(−0.01)	(−0.01)			

and has to be estimated, firm characteristics can also predict returns, as we have seen in Table III. We next replicate a common empirical approach that is undertaken when the factor is unobservable and beta is obtained by regressing firm returns on the market return. To do so, we compare the performance of several market-based asset pricing models: (i) the CAPM, (ii) the Fama and French (1993) three-factor model, and (iii) the CAPM or Fama-French model augmented with size and book-to-market characteristics.

We compute the market return, R_M, as the value-weighted average return of all stocks in the economy. The size and value factors are constructed as $R_{SMB} = R_S - R_B$ and $R_{HML} = R_H - R_L$, where R_S, R_B, R_H, and R_L are the value-weighted average returns of the 30% of stocks with, respectively, the smallest and largest market capitalization and the highest and lowest book-to-market ratio. Using these factors, we estimate factor risk loadings for each stock from time-series regressions with 24 months of data. As a last step, we perform Fama-MacBeth regressions to see if the estimated risk loadings explain the cross-section of stock returns.

Panel A of Table IV reports the regression results. The CAPM beta (in specification I) is a significant predictor of stock returns and explains 1.09% of the

Table IV

Market-Based Asset Pricing Tests

This table reports Fama-MacBeth coefficients (Panel A) and factor correlations (Panel B) for market-based asset pricing tests on simulated data. The data are generated from 100 simulations of a cross-section of 2,000 stocks over 1,000 years. R_M is the value-weighted average return of all stocks in the economy, and R_S, R_B, R_H, and R_L are the value-weighted average returns of the 30% of stocks with, respectively, the smallest and largest market capitalization and the highest and lowest book-to-market ratio. R_{SMB} and R_{HML} are, respectively, $R_S - R_B$ and $R_H - R_L$. For Panel A, in each month t the realized stock returns are regressed on estimated Fama-French three-factor betas ($\widehat{\beta}^M$, $\widehat{\beta}^{SMB}$, and $\widehat{\beta}^{HML}$), log firm value (*Size*), and the log book-to-market ratio (B/M). Betas are estimated as slope coefficients from time-series regressions of stock returns on R_M, R_{SMB}, and R_{HML} from month $t-24$ to $t-1$. The coefficients, t-statistics (in parentheses), and adjusted R^2s are averaged over time and across simulations. The factor correlations in Panel B are averages over simulations of the realized time-series correlations.

Panel A: Fama-MacBeth Regressions

	$\widehat{\beta}^M$	$\widehat{\beta}^{SMB}$	$\widehat{\beta}^{HML}$	*Size*	B/M	R^2 (%)
I	0.14					1.09
	(3.44)					
II	0.03	0.28	0.25			2.38
	(0.88)	(3.55)	(3.30)			
III	0.10			−0.05	0.06	2.07
	(3.24)			(−2.44)	(2.34)	
IV	0.03	0.22	0.19	−0.03	0.04	2.77
	(1.03)	(3.29)	(3.04)	(−2.02)	(2.08)	

Panel B: Factor Correlations

	R_y	R_M	R_S	R_B	R_{SMB}	R_H	R_L
R_M	0.12						
R_S	0.98	0.09					
R_B	0.04	0.99	0.04				
R_{SMB}	0.58	−0.74	0.59	−0.78			
R_H	0.92	0.09	0.95	0.04	0.56		
R_L	0.03	0.99	0.04	0.99	−0.78	0.04	
R_{HML}	0.51	−0.77	0.52	−0.81	0.98	0.54	−0.81

cross-sectional variation. However, the explanatory power of the CAPM beta is lower than that of the theoretical factor beta (R^2 of 3.97% in Table III) or the estimated factor beta (R^2 of 1.68%).

In specification II, we test the Fama-French three-factor model. Interestingly, the size and value factors both appear positively priced, while the market beta becomes insignificant. The R^2 from this regression compares favorably with all specifications from Table III, except those that use the theoretical beta. Specification III shows that size and book-to-market characteristics appear in addition to market risk. However, the characteristic-based model performs worse than the three-factor model. Finally, specification IV suggests that there is a role for both firm characteristics and risk factors in explaining stock returns in our model.

This evidence contributes to our understanding of the important question in the empirical asset pricing literature of whether cross-sectional differences in returns are driven mostly by firm characteristics (Daniel and Titman (1997)) or by covariances with priced factors. In our model, firm characteristics correlate with firm systematic risk and can therefore help predict returns when betas are measured with error.

It is less obvious why additional factors appear to be priced in our single-factor model. To understand this result, in Panel B of Table IV we display correlations between the returns of the priced factor, the market, and the size- and value-sorted portfolios. The correlation between the returns of the market portfolio and the priced factor is only 0.12. This correlation is low because large stocks tend to have high idiosyncratic profits, and therefore their returns are driven primarily by idiosyncratic shocks. As in Gabaix (2011), since large stocks dominate the value-weighted market, their idiosyncratic shocks affect aggregate fluctuations. Similarly, the correlation between the big (low book-to-market) stock portfolio and the systematic factor is only 0.04 (0.03). Returns of small stocks and high book-to-market stocks, on the other hand, are driven mostly by systematic shocks and have high correlations with the systematic factor of 0.98 and 0.92, respectively. SMB and HML portfolio returns therefore serve as better proxies for the return of the systematic factor than the market.

D. Option Exercise

In Figure 2, we plot key firm characteristics before and after growth option exercise, averaged across firms and economies. Panels A and B show the dynamics of firm characteristics during the 48-month period centered on the exercise of y- and x-options, respectively. The two plots for market value show that options are typically exercised following high stock returns. Values jump at exercise due to the injection of additional cash in the firm. Market capitalization increases with the issuance of new shares. There are no arbitrage opportunities because the price per share remains unchanged. In contrast, book-to-market tends to decrease prior to option exercise. At the time of investment, both the book and market values increase by the same amount. Since the average book-to-market ratio is below one, the ratio typically increases at exercise.

Finally, we plot conditional factor betas around the exercise of systematic and idiosyncratic options. As we argue theoretically above, betas decrease at the time of x- and y-option exercise under external equity financing. Exercising y-options lowers the sensitivity to the systematic profitability shock since assets in place are less risky than growth options. In contrast, exercising x-options increases the value of assets derived from the idiosyncratic component, thereby lowering firm risk. The distinctive pattern in the dynamics of preexercise betas is informative. Consistent with prior literature (e.g., Carlson, Fisher, and Giammarino (2010)), the average beta tends to increase prior to y-option exercise. The increase in both the systematic shock and the value of the option prior to exercise leads to an increase in beta. Conversely, the average beta

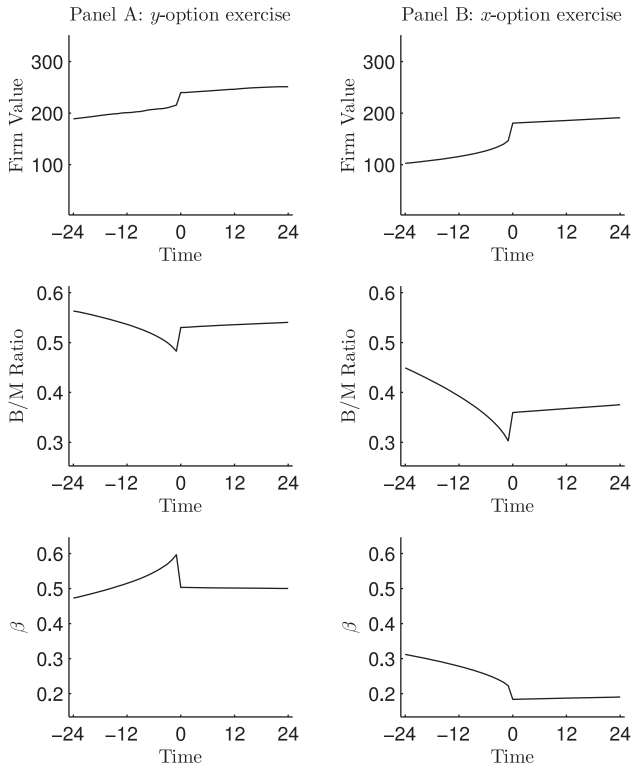

Figure 2. Firm characteristics around option exercise. This figure plots average firm value (top row), the book-to-market ratio (middle row), and beta (bottom row) in a 48-month window around exercise of the y-option (Panel A) and x-option (B).

decreases right before the x-option is exercised since this option depends on the idiosyncratic profit component.

VI. Empirical Analysis

The theoretical model developed above suggests that idiosyncratic cash flow shocks play an important role in driving asset pricing anomalies. Two ingredients are required for this result. First, the firm value function cannot have a simple multiplicative representation. This assumption is natural because most production functions do not fall in the category of simple, let alone multiplicative, functions. Second, a significant part of cash flows must be idiosyncratic in nature. This second requirement motivates our empirical test. Specifically, we examine whether size and value anomalies are more pronounced among firms with higher idiosyncratic cash flow risk.

Our methodology is similar to that in Sagi and Seasholes (2007), who demonstrate that the momentum anomaly is concentrated in firms with high sales growth volatility. To obtain a measure of idiosyncratic cash flow risk, we use quarterly COMPUSTAT data from 1963 to 2013.[12] We first compute cash flows as income before extraordinary items plus depreciation and amortization. We then obtain aggregate cash flows by summing cash flows for all firms in our sample. To account for potential seasonality in cash flows, we define the growth rate of cash flows in quarter t relative to the same quarter in the previous fiscal year as follows:

$$g_{i,t}^{CF} = \frac{CF_{i,t} - CF_{i,t-4}}{CF_{i,t-4}}. \tag{29}$$

Whenever observed cash flow is negative, the associated growth rates are omitted because the growth rate is undefined.

Since firms with high cash flow volatility are more likely to report negative earnings, our procedure tends to favor low volatility firms. Irvine and Pontiff (2009) propose an alternative cash flow shock measure that relies on differences in earnings per share rather than ratios and can therefore accommodate negative cash flow realizations. In the Internet Appendix, we show that our results are robust to using this alternative method.

Next, we regress firm-level cash flows on aggregate cash flows using 40-quarter rolling windows,

$$g_{i,\tau}^{CF} = \alpha_{i,t} + \beta_{i,t} g_{Agg,\tau}^{CF} + \varepsilon_{i,\tau}, \qquad \tau \in \{t-39, \ldots, t\}. \tag{30}$$

For each estimation, we require at least 30 valid quarterly observations. Our measure of idiosyncratic cash flow risk is the variance of the regression residuals, $\mathrm{var}(\varepsilon_{i,\tau})$.[13] We analyze the stock returns over the following three months, resulting in a time series from January 1973 to December 2013.

Tables V and VI show the relation between the volatility of idiosyncratic cash flows and the magnitude of the size and value anomalies, respectively. We sort all sample firms into quintiles by volatility of idiosyncratic cash flows, and within each quintile into five groups by their market capitalization and book-to-market ratio.

It is evident from Table V that the magnitude of the size premium is strongly increasing in idiosyncratic volatility. For example, Panel A shows that small stocks outperform big stocks by 0.10% per month in the low-volatility quintile, and by 1.23% in the high-volatility quintile. The difference is large at 1.13% and is statistically significant.

We further show that the standard risk adjustments using the CAPM or the Fama-French three-factor model have little effect on the observed differences in the size premiums. Specifically, in the next two rows we report alphas from time-series regressions of the small-minus-big portfolios. Adjustment for

[12] The data start with the last fiscal quarter ending in the first calendar quarter of 1963 and end in the third calendar quarter of 2013.

[13] We do not scale our measure by total volatility since a large proportion of total volatility is idiosyncratic, and therefore the ratio of the two is very close to one for most firms. Cross-sectional differences in the ratio are dominated by estimation noise.

Table V

Volatility of Idiosyncratic Cash Flows and the Size Anomaly

This table reports average monthly returns of value-weighted (Panel A) and equal-weighted (Panel B) portfolios sorted first by the volatility of idiosyncratic cash flows and then by market capitalization, as well as the difference between high and low idiosyncratic cash flow volatility and small and big market capitalization. Idiosyncratic cash flow volatility is computed as the volatility of residuals from rolling 40-quarter regressions of firm cash flow growth on aggregate cash flow growth. Details are provided in the main text. We also provide alphas from the CAPM and Fama-French three-factor models, and average log market capitalization in Panel C. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is January 1973 to December 2013.

	Volatility of Idiosyncratic Cash Flows					
	Low	2	3	4	High	H−L
Panel A: Value-Weighted Returns						
Small	1.16	1.27	1.48	1.48	2.05	0.89
	(5.96)	(4.95)	(5.36)	(4.76)	(6.31)	(3.47)
2	1.09	1.38	1.16	1.59	1.81	0.71
	(5.62)	(5.86)	(4.68)	(5.69)	(5.77)	(2.55)
3	1.10	1.15	1.28	1.39	1.33	0.24
	(5.81)	(5.86)	(5.49)	(5.67)	(5.20)	(1.22)
4	0.99	1.18	1.28	1.28	1.18	0.18
	(4.86)	(5.63)	(5.29)	(5.28)	(4.70)	(1.02)
Big	1.06	0.90	0.94	0.98	0.81	−0.25
	(6.21)	(4.01)	(4.19)	(3.80)	(3.23)	(−1.62)
S−B	0.10	0.36	0.54	0.50	1.23	1.13
	(0.57)	(1.41)	(2.34)	(1.84)	(4.37)	(4.50)
α_{S-B}^{CAPM}	0.15	0.44	0.64	0.56	1.26	1.11
	(0.84)	(1.66)	(2.73)	(1.99)	(4.46)	(4.50)
α_{S-B}^{FF3}	−0.13	0.14	0.35	0.18	1.05	1.18
	(−1.05)	(0.65)	(1.89)	(0.88)	(4.30)	(4.68)
Panel B: Equal-Weighted Returns						
Small	1.22	1.40	1.57	1.61	2.02	0.80
	(6.15)	(5.95)	(5.92)	(5.48)	(6.67)	(3.40)
2	1.17	1.41	1.27	1.53	1.72	0.55
	(5.87)	(6.14)	(5.13)	(5.59)	(6.41)	(2.41)
3	1.15	1.22	1.35	1.34	1.38	0.23
	(6.00)	(6.05)	(5.62)	(5.60)	(5.55)	(1.28)
4	1.04	1.25	1.35	1.35	1.27	0.23
	(5.04)	(5.90)	(5.66)	(5.73)	(4.96)	(1.29)
Big	1.05	1.01	1.03	1.11	0.99	−0.06
	(6.24)	(4.53)	(4.87)	(4.58)	(4.40)	(−0.43)
S−B	0.17	0.39	0.54	0.50	1.03	0.86
	(1.29)	(1.96)	(2.61)	(2.32)	(4.66)	(4.17)
α_{S-B}^{CAPM}	0.21	0.53	0.64	0.60	1.12	0.91
	(1.62)	(2.61)	(2.96)	(2.64)	(4.86)	(4.34)
α_{S-B}^{FF3}	0.05	0.30	0.38	0.34	0.91	0.87
	(0.45)	(1.91)	(2.13)	(1.86)	(4.83)	(4.19)
Panel C: Average Market Capitalization						
Small	11.65	10.93	10.48	10.06	9.79	−1.86
2	12.94	12.39	11.94	11.60	11.34	−1.61
3	13.78	13.35	12.97	12.74	12.37	−1.41
4	14.58	14.21	13.91	13.69	13.38	−1.20
Big	15.89	15.80	15.40	15.25	15.00	−0.90
S−B	−4.25	−4.87	−4.93	−5.19	−5.21	−0.96

Table VI

Volatility of Idiosyncratic Cash Flows and the Value Anomaly

This table reports average monthly returns of value-weighted (Panel A) and equal-weighted (Panel B) portfolios sorted first by the volatility of idiosyncratic cash flows and then by book-to-market ratios, as well as the difference between high and low idiosyncratic cash flow volatility and high and low book-to-market ratio. Idiosyncratic cash flow volatility is computed as the volatility of residuals from rolling 40-quarter regressions of firm cash flow growth on aggregate cash flow growth. Details are provided in the main text. We also provide alphas from the CAPM and Fama-French three-factor models, and average log book-to-market ratios in Panel C. t-statistics are based on Newey and West (1987) adjusted standard errors using 12 lags. The sample period is January 1973 to December 2013.

	Volatility of Idiosyncratic Cash Flows					
	Low	2	3	4	High	H−L
			Panel A: Value-Weighted Returns			
Low	1.02	0.78	1.05	1.08	0.78	−0.24
	(5.08)	(3.14)	(4.29)	(3.57)	(2.85)	(−1.37)
2	1.12	1.13	1.01	0.99	0.94	−0.18
	(5.75)	(4.80)	(4.47)	(3.70)	(3.89)	(−1.14)
3	1.13	1.09	0.83	1.14	1.00	−0.13
	(6.49)	(4.97)	(3.12)	(5.13)	(4.07)	(−0.66)
4	0.96	1.16	1.07	1.13	0.97	0.00
	(5.52)	(5.30)	(4.28)	(4.17)	(3.97)	(0.03)
High	1.08	1.14	1.33	1.53	1.71	0.63
	(5.39)	(4.81)	(5.45)	(6.32)	(5.25)	(1.99)
H−L	0.06	0.35	0.28	0.45	0.93	0.87
	(0.29)	(1.41)	(1.39)	(1.42)	(2.97)	(2.64)
α_{H-L}^{CAPM}	0.24	0.48	0.39	0.54	0.94	0.70
	(1.18)	(1.83)	(1.99)	(1.74)	(2.97)	(2.03)
α_{H-L}^{FF3}	−0.20	−0.12	−0.17	−0.02	0.30	0.50
	(−1.20)	(−0.60)	(−0.84)	(−0.09)	(1.08)	(1.46)
			Panel B: Equal-Weighted Returns			
Low	1.12	1.14	1.26	1.10	0.90	−0.23
	(5.69)	(5.11)	(4.99)	(4.17)	(3.27)	(−1.20)
2	1.14	1.24	1.26	1.27	1.55	0.41
	(5.74)	(5.67)	(5.36)	(4.88)	(6.93)	(2.47)
3	1.12	1.14	1.23	1.37	1.37	0.26
	(6.13)	(5.54)	(5.29)	(5.91)	(5.62)	(1.47)
4	1.07	1.30	1.28	1.48	1.51	0.44
	(5.48)	(6.67)	(5.92)	(5.93)	(5.89)	(2.19)
High	1.19	1.49	1.52	1.72	2.04	0.85
	(5.45)	(5.85)	(6.04)	(6.07)	(6.97)	(3.44)
H−L	0.07	0.35	0.26	0.62	1.14	1.08
	(0.40)	(1.70)	(1.40)	(2.60)	(5.79)	(5.19)
α_{H-L}^{CAPM}	0.21	0.48	0.35	0.71	1.23	1.02
	(1.28)	(2.26)	(1.92)	(3.01)	(6.24)	(4.64)
α_{H-L}^{FF3}	−0.16	−0.00	−0.07	0.20	0.79	0.95
	(−1.28)	(−0.03)	(−0.45)	(1.07)	(4.24)	(4.28)
			Panel C: Average Book-to-Market Ratios			
Low	−1.32	−1.21	−1.20	−1.24	−1.19	0.13
2	−0.71	−0.58	−0.55	−0.53	−0.48	0.23
3	−0.35	−0.26	−0.23	−0.18	−0.13	0.21
4	−0.08	−0.01	0.05	0.10	0.17	0.26
High	0.32	0.43	0.53	0.60	0.70	0.38
H-L	1.64	1.64	1.73	1.84	1.89	0.25

market risk using the CAPM model or the three Fama-French factors leaves all size premiums virtually unchanged.

The evidence on the value premium in Table VI mirrors the evidence on the size premium. In particular, value firms outperform growth firms for each volatility group. However, the value premium is larger for firms with high volatility of idiosyncratic cash flows. Specifically, there is no significant difference between value and growth portfolio returns for firms with low volatility of idiosyncratic cash flows, while the difference is large and significant at 0.93% for the high volatility portfolios. Controlling for the CAPM factor or the three Fama-French factors attenuates the difference in value premiums across idiosyncratic volatility quintiles slightly.

Panel B in both tables gives analogous results for equal-weighted portfolios. Similarly, Panel C shows that differences in size (value) premiums cannot be attributed to differences in the levels of average market capitalization (book-to-market ratio) across idiosyncratic volatility quintiles. Overall, we find that the level of idiosyncratic cash flow volatility is important for the magnitude of asset pricing anomalies.

VII. Conclusion

Cross-sectional anomalies in stock returns have long presented a challenge to standard asset pricing models. We show that, under general assumptions, firms' conditional betas depend directly on the history of idiosyncratic shocks and vary over time. Firm value is negatively related to risk because positive idiosyncratic shocks to cash flows increase market capitalization and simultaneously lead to a decrease in systematic risk. This size effect is distinct from the feedback of discount rates into market values analyzed by Berk (1995) and holds for both market- and accounting-based measures of firm size. Similarly, the model is able to replicate the value anomaly and the empirically observed negative relation between idiosyncratic volatility and expected returns.

We show that real options magnify the anomalies in the model, allow for separation of value and size effects, and can generate return predictability by price-earnings ratios. Since growth options can depend on systematic or idiosyncratic profits, they can increase or decrease the firm's factor risk. The analysis of the data generated by the model confirms that the model can produce reasonable magnitudes of value, size, price-earnings, and idiosyncratic volatility anomalies both in portfolio sorts and in Fama-Macbeth cross-sectional regressions. In addition, our model generates novel empirical predictions for which we find empirical support. In particular, we find that the magnitudes of the size and value anomalies are larger in firms with higher idiosyncratic cash flow risk. Overall, our results imply that any economic variable correlated with the history of idiosyncratic cash flow shocks can help explain expected stock returns.

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Appendix A: Proofs

PROOF OF PROPOSITION 1: Consider a trial solution

$$V(x_i, y) = v_1(x_i) + v_2(y). \tag{A1}$$

Since equation (14) is additively separable in variables x_i and y, the general solution to (14) is equal to the sum of the ODE solution for $v_1(x_i)$ and the ODE solution for $v_2(y)$. Consider the "continuation" problem of the firm that has exercised all of its options. Firm value after exercise is

$$\widehat{V}(x_i, y) = \frac{(1+\gamma_x)\, x_i}{r-\mu_x} + \frac{(1+\gamma_y)\, \rho_i y}{r-\mu_y}. \tag{A2}$$

Prior to the exercise of the options, the general solution for firm value is given by

$$V(x_i, y) = \frac{x_i}{r-\mu_x} + \frac{\rho_i y}{r-\mu_y} + Ay^{b_2} + Bx_i^{d_2}, \tag{A3}$$

where A and B are constants, b_2 is the positive root of the quadratic equation

$$b^2\sigma_y^2 + b\left(2\mu_y - \sigma_y^2\right) - 2r = 0, \tag{A4}$$

and d_2 is the positive root of a similar equation for x_i,

$$d^2\sigma_x^2 + d\left(2\mu_x - \sigma_x^2\right) - 2r = 0. \tag{A5}$$

At the time of the exercise, the value of the firm is equal to the value after exercise minus the investment cost (the value-matching conditions),

$$V\left(x^*, y\right) = \widehat{V}\left(x^*, y\right) - I_x, \tag{A6}$$

$$V\left(x_i, y^*\right) = \widehat{V}\left(x_i, y^*\right) - \rho_i I_y, \tag{A7}$$

where $\widehat{V}$ is given by (A2). Note that, since firm value is separable in the x_i and y components, the exercise of one option does not affect the exercise policy of another. For exercise to be optimal, additional conditions known as smooth-pasting or high-contact conditions (Dumas (1991), Dixit (1993)) have to be satisfied:

$$V_x\left(x^*, y\right) = \widehat{V}_x\left(x^*, y\right), \tag{A8}$$

$$V_y\left(x_i, y^*\right) = \widehat{V}_y\left(x_i, y^*\right). \tag{A9}$$

Using (A6) to (A9), we find constants A and B, and well as the preexercise firm value,

$$V(x_i, y) = \frac{(1+\iota_x\gamma_x)x_i}{r-\mu_x} + \frac{(1-\iota_x)\gamma_x x^*}{(r-\mu_x)d_2}\left(\frac{x_i}{x^*}\right)^{d_2}$$
$$+\frac{(1+\gamma_y\iota_y)\rho_i y}{r-\mu_y} + \frac{(1-\iota_y)\gamma_y\rho_i y^*}{\left(r-\mu_y\right)b_2}\left(\frac{y}{y^*}\right)^{b_2}. \tag{A10}$$

The thresholds for exercise x^* and y^* are then defined as

$$x^* = \frac{d_2}{d_2-1}\frac{(r-\mu_x)I_x}{\gamma_x}, \tag{A11}$$

$$y^* = \frac{b_2}{b_2-1}\frac{\left(r-\mu_y\right)I_y}{\gamma_y}. \tag{A12}$$

Note that y^* is identical for all firms since both investment benefits and costs are proportional to ρ_i. □

PROOF OF PROPOSITION 2: Using Ito's lemma, we first compute the expected changes in firm value in the physical and risk-neutral measures:

$$\mathrm{E}^P(dV_i) = \left(\mu_x x_i\frac{\partial V_i}{\partial x_i} + \frac{\sigma_x^2 x_i^2}{2}\frac{\partial^2 V_i}{\partial x_i^2} + \mu_y y\frac{\partial V_i}{\partial y} + \frac{\sigma_y^2 y^2}{2}\frac{\partial^2 V_i}{\partial y^2}\right)dt, \tag{A13}$$

$$\mathrm{E}^Q(dV_i) = \left(\mu_x x_i\frac{\partial V_i}{\partial x_i} + \frac{\sigma_x^2 x_i^2}{2}\frac{\partial^2 V_i}{\partial x_i^2} + ry\frac{\partial V_i}{\partial y} + \frac{\sigma_y^2 y^2}{2}\frac{\partial^2 V_i}{\partial y^2}\right)dt. \tag{A14}$$

Note that the drift of the x_i process is the same under both measures. By no arbitrage, we have

$$\mathrm{E}^Q(dV_i) = rV_i dt. \tag{A15}$$

Combining (A14) and (A15) and then substituting the result into (A13) produces

$$\mathrm{E}^P(dV_i) = rV_i dt + \left(\mu_y - r\right)y\frac{\partial V_i}{\partial y}dt. \tag{A16}$$

Finally, expected returns are given by

$$\mathrm{E}^P(R_i) = \frac{1}{dt}\mathrm{E}^P(dV_i/V_i) = r + \frac{\partial V_i}{\partial y}\frac{y}{V_i}\left(\mu_y - r\right), \tag{A17}$$

and therefore

$$\beta_i \equiv \frac{\partial V_i}{\partial y}\frac{y}{V_i}. \tag{A18}$$

Equation (20) in the proposition then follows from the differentiation of equation (15). □

PROOF OF PROPOSITION 3: Using (20), the claim follows by taking the difference in betas just *after* and just *prior* to the exercise of the x-option:

$$\beta_i(x_+^*) - \beta_i(x_-^*) = \frac{-\rho_i I_x \left(V^{AY} + b_2 V^{GY}\right)}{V(x^*, y)(V(x^*, y) + I_x)} < 0. \tag{A19}$$

Similarly, for the exercise of the y-option, the difference in postexercise and preexercise betas is

$$\beta_i(y_+^*) - \beta_i(y_-^*) = \frac{-\rho_i I_y \left(1 + \gamma_y\right)}{V(x_i, y^*)\left(V(x_i, y^*) + \rho_i I_y\right)} \frac{\rho_i y^*}{r - \mu_y} < 0. \tag{A20}$$

□

Appendix B: Relation between Factor and Market Betas

The analysis in the text is based on betas with respect to the common risk factor y,

$$\beta_i = \frac{\partial V_i / V_i}{\partial y / y}. \tag{B1}$$

We now connect our analysis to the commonly used "market beta" in the context of the capital asset pricing model (CAPM).

It is straightforward to show that

$$\beta_i^M = \beta_i \beta_y^M, \tag{B2}$$

where

$$\beta_y^M = \frac{\partial y / y}{\partial M / M} \tag{B3}$$

and M is the aggregate market value. We obtain the value of the stock market by summing all individual firm market values:

$$M \equiv \Sigma_i V_i = \Sigma_i \left(V_i^{AX} + V_i^{GX}\right) + (V^{AY} + V^{GY} t)\Sigma_i \rho_i. \tag{B4}$$

The beta of y with respect to the market can be found using implicit differentiation:

$$\beta_y^M = \frac{M}{\left(V^{AY} + b_2 V^{GY}\right) \Sigma_i \rho_i}. \tag{B5}$$

From Proposition 2, this beta simplifies to

$$\beta_i = \frac{\rho_i}{V_i} \left(V^{AY} + b_2 V^{GY}\right). \tag{B6}$$

Therefore, the equilibrium relation to the CAPM beta can be written as

$$\beta_i^M = \beta_i \beta_y^M = \frac{\rho_i}{V_i} \frac{\Sigma_i V_i}{\Sigma_i \rho_i}. \tag{B7}$$

In particular, when all firms have the same sensitivity to the systematic shock, we simply have

$$\beta_i^M = \frac{\overline{V}}{V_i}, \tag{B8}$$

where $\overline{V}$ is the average value of firms in the economy. It follows that firms smaller than average will have betas above one, while firms larger than average will have betas below one. By construction, the weighted sum of market betas is equal to one, $\Sigma_i V_i \beta_i^M / \Sigma_i V_i = 1$.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix