Claremont Graduate University, M453, Financial Time Series
Homework 2
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This assignment is due Friday 9/23/16 before class begins. You can either work on Latex (then print your PDF) or hand write the solutions. I only accept hard copies.

CH 1: Basic Stochastic Models

$(w_t)_t$, if not specified, denotes a white noise with $Var(w_t) = \sigma^2$.

1. **(Stationarity)** Consider a time series $(X_t)_t$ defined by

$$X_t = -0.5w_{t-1} + w_t,$$

where $(w_t)_t$ is a white noise of variance σ^2. Is $(X_t)_t$ second order stationary?
(Hint: calculate $E(X_t)$ and $Cov(X_t, X_{t+k})$ for all $k \in \mathbb{Z}$.)

2. **(Stationarity)** Consider a time series $(Y_t)_t$ defined by

$$Y_t = X_t - X_{t-1},$$

where $X_t = 0.9X_{t-1} + w_t$, with $(w_t)_t$ being a white noise of variance σ^2. The goal of this exercise is to show $(X_t)_t$ and $(Y_t)_t$ are both second order stationary.
Questions.

(a) Show that $X_t = \sum_{i=0}^{\infty} 0.9^i w_{t-i}$.

(b) Calculate $E(X_t)$ and $Var(X_t)$ for all $t \in \mathbb{Z}$.

(c) For $k \in \mathbb{Z}$, calculate $Cov(X_t, X_{t+k})$. Is $(X_t)_t$ second order stationary?
(Hint: consider two cases $k \geq 0$ and $k < 0$.)

(d) For $k \in \mathbb{Z}$, calculate $Cov(Y_t, Y_{t+k})$. Is $(Y_t)_t$ second order stationary?
(Hint: replace Y_t by $X_t - X_{t-1}$, then use the result from (c).)

3. **(Backward shift operator)** Express the following autoregressive model in its usual form:

$$(1 - 0.5B)(1 - 0.7B)(1 - 0.2B)X_t = w_t.$$

Justify whether $(X_t)_t$ is second order stationary.

4. **(Coefficients of AR)** An $AR(1)$ model with a non-zero mean μ can be expressed by either

$$X_t - \mu = \alpha(X_{t-1} - \mu) + w_t, \text{ or } X_t = a_0 + a_1 X_{t-1} + w_t.$$

Questions.

(a) What is the relationship between the parameters μ and α, and the parameters a_0 and a_1?

(b) Deduce a similar relationship for an $AR(2)$ process with mean μ.