Claremont Graduate University, M453, Financial Time Series
Homework 1
Instructor: Qidi Peng
Sept. 2, 2016

This assignment is due Friday 9/9/16 before class begins. You can either work on Latex (then print your PDF) or hand write the solutions. I only accept hard copies.

CH 0: Basic Concepts

1. **(Sample space: double die rolling)** Two friends are playing a board game that requires each of them to roll a die. Each player uses her/his own die.
   Questions.

   (a) Determine the sample space Ω and the set of events $\mathcal{F}$ for a single roll if their dice are painted two different colors.

   (b) What are the sample space Ω and the set of events $\mathcal{F}$ if both dice are white - does this change anything?

   (c) What if one person rolls both dice - does this change anything?

2. **(Probability: maximum of three dice)** Roll three distinguishable dice (e.g., assume that there is a way to tell them apart, for instance, that the dice are three different colors). There are $6^3 = 216$ possible outcomes.

   Let B_k be the event that the maximum value that appears on all three dice when they are rolled is less than or equal to k. Find $P(B_1)$, $P(B_2)$,...,$P(B_6)$. If you prefer, you are welcome to just give a general formula that covers all these 6 cases, i.e., you are welcome to just give a formula for $P(B_k)$, for $k = 1, \ldots, 6$.

3. **(Expected value of sum of random variables)** A student draws cards from a standard deck of playing cards until the ace of spades appears for the first time. After every unsuccessful draw, the student replaces the card and shuffles the deck thoroughly before selecting a new card. How many cards does the student expect to draw until the ace of spades appears?
   (Hint: first define a sequence of independent random variables corresponding to this experiment.)

4. **(Covariance and correlation of random variables)** Each day, Amy eats lunch at the cafeteria. She chooses pizza as her main dish with probability 40%, and her behavior each day is independent of all the other days. Let X denote the number of days she chooses pizza in a 10-day period. Let $Y = 10 - X$ denote the number of days in which Amy does not eat pizza.
   Questions.

   (a) Are X and Y dependent or independent?

   (b) Find the covariance $Cov(X, Y)$ of X and Y.

   (c) Find the correlation $\rho(X, Y)$ of X and Y.