# Derivation of Fokker-Planck Equation of GECIR Model

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## 1 Fokker-Planck Equation

The generalized extended CIR model (GECIR) we proposed is

$$dr(t) = (\theta(t) - b(t)r(t))dt + \sigma(t)r(t)^{\beta}dW(t).$$

where all the model parameters except the power term $\beta$ are known deterministic function of time. $\beta$ is a known constant. $\tilde{W}(t)$ is a Brownian modtion.

Let $f(r,t)$ be the density of the random variable $r(t)$. The Fokker-Planck (Kolmogorov forward equation) of the GECIR model is:

$$\begin{aligned}
\frac{\partial f(r,t)}{\partial t} &= -\frac{\partial}{\partial r}\left[(\theta(t) - b(t)r(t))f(r,t)\right] + \frac{1}{2}\frac{\partial^2}{\partial r^2}\left[\sigma(t)^2 r(t)^{2\beta} f(r,t)\right] \\
&= -\frac{\partial}{\partial r}\left[\theta(t)f(r,t) - b(t)r(t)f(r,t)\right] + \frac{1}{2}\sigma(t)^2\frac{\partial}{\partial r}\left[2\beta r(t)^{2\beta-1}f(r,t) + r(t)^{2\beta}\frac{\partial f(r,t)}{\partial r}\right] \\
&= -\theta(t)\frac{\partial f(r,t)}{\partial r} + b(t)f(r,t) + b(t)r(t)\frac{f(r,t)}{\partial t} \\
&\quad + \frac{1}{2}\sigma(t)^2\left[(2\beta-1)2\beta r(t)^{2\beta-2}f(r,t) + 2\beta r(t)^{2\beta-1}\frac{\partial f(r,t)}{\partial r}\right] \\
&\quad + \frac{1}{2}\sigma(t)^2\left[2\beta r(t)^{2\beta-1}\frac{\partial f(r,t)}{\partial r} + r(t)^{2\beta}\frac{\partial^2 f(r,t)}{\partial r^2}\right] \\
&= [b(t) + \sigma(t)^2(2\beta-1)\beta r(t)^{2\beta-2}]f(r,t) + \left[-\theta(t) + b(t)r(t) + 2\sigma(t)^2\beta r(t)^{2\beta-1}\right]\frac{\partial f(r,t)}{\partial r} \\
&\quad + \frac{1}{2}\sigma(t)^2 r(t)^{2\beta}\frac{\partial^2 f(r,t)}{\partial r^2}
\end{aligned}$$