# DA5401 - Assignment 4

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[ ]:
```
# Importing necessary libraries

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import random
```

## 1 Task 1

First, we have to create our dummy binary classifier. We will build upon `sklearn`'s `BaseEstimator` class to create our classifier. Our dummy classifier randomly returns a prediction based on one of 3 methods - **Bernoulli**, **normal**, and **uniform**. The prediction also depends on a pre-defined **threshold** $p \in [0, 1]$.

**Bernoulli** - Returns `True` with probability $p$ and `False` with probability $1 - p$.

**Normal** - Randomly samples a number $n$ from a Gaussian distribution with $\mu = 0.5$ and $\sigma = 0.2$ and returns `True` if $n < p$ and `False` otherwise. This value of $\sigma$ was chosen to ensure that the classifier almost always returns `False` when $p = 0$ and almost always returns `True` when $p = 1$.

**Uniform** - Randomly samples a number $n$ from a uniform distribution between 0 and 1 and returns `True` if $n < p$ and `False` otherwise.

[ ]:
```
# Importing sklearn's BaseEstimator class

from sklearn.base import BaseEstimator
```

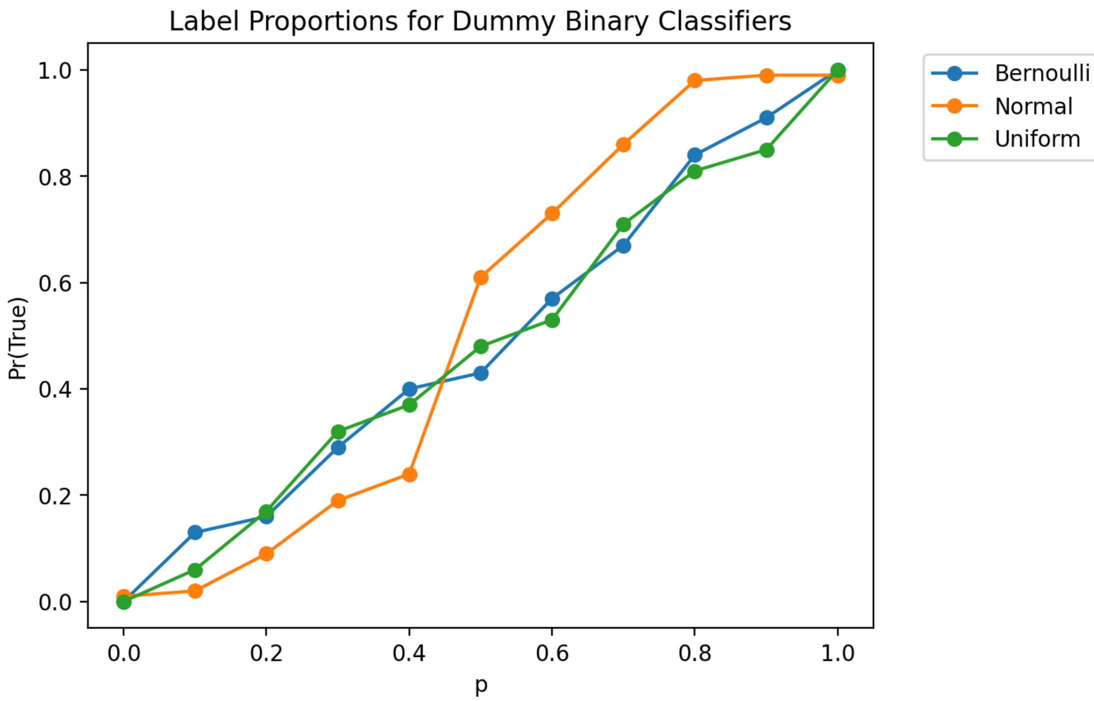
[ ]:
```
# Setting a random seed for replicability

random.seed(5401)

# Defining the DummyBinaryClassifier class

class DummyBinaryClassifier(BaseEstimator):
```

```python
def __init__(self, method="bernoulli", p=0.5):

  # Defining the attributes of the classifier

  self.method = method      # Method
  self.p = p                # Threshold

def fit(self, data):

  # Defining a dummy fit function

  pass

def predict(self, data):

  # Returning a prediction (with the same size as the input data) based on
↪the defined method and threshold

  # Bernoulli

  if self.method == "bernoulli":
    return random.choices(population=[True,False],weights=[self.p,1-self.
↪p],k=len(data))

  # Normal

  elif self.method == "normal":
    predictions = []
    for i in range(len(data)):
      if random.gauss(mu=0.5,sigma=0.2) < self.p:
        predictions.append(True)
      else:
        predictions.append(False)
    return predictions

  # Uniform

  elif self.method == "uniform":
    predictions = []
    for i in range(len(data)):
      if random.uniform(0,1) < self.p:
        predictions.append(True)
      else:
        predictions.append(False)
    return predictions
```

Now we wish to check the probability with which each of the methods returns `True` for different values of $p$. For this, we create dummy data with 100 instances. We scan through values of $p$ from 0 to 1 in steps of 0.1 and compute the fraction of times the classifier returns `True`. These fractions are then visualized as a line plot.

[ ]:
```
# Defining the dummy data

dummy_data = np.zeros(100)

# Defining the range of p values

p_values = np.arange(0,1.1,0.1)

# Storing the values of Pr(True) for different classifiers and for different␣
 ↪values of p

bernoulli_trues = []
normal_trues = []
uniform_trues = []

# Iterating through the different p values

for p in p_values:

  # Instantiating the classifiers

  bernoulli_classifier = DummyBinaryClassifier(method="bernoulli",p=p)
  normal_classifier = DummyBinaryClassifier(method="normal",p=p)
  uniform_classifier = DummyBinaryClassifier(method="uniform",p=p)

  # Making predictions on the dummy data and recording the fraction of Trues␣
 ↪returned

  bernoulli_trues.append(np.mean(bernoulli_classifier.predict(data=dummy_data)))
  normal_trues.append(np.mean(normal_classifier.predict(data=dummy_data)))
  uniform_trues.append(np.mean(uniform_classifier.predict(data=dummy_data)))

# Visualizing the fractions as a line plot

plt.figure(dpi=200)
plt.plot(p_values,bernoulli_trues,label="Bernoulli",marker="o")
plt.plot(p_values,normal_trues,label="Normal",marker="o")
plt.plot(p_values,uniform_trues,label="Uniform",marker="o")
plt.legend(bbox_to_anchor=(1.05, 1))
plt.xlabel("p")
plt.ylabel("Pr(True)")
plt.title("Label Proportions for Dummy Binary Classifiers")
```

```
plt.show()
```




We can see that all the three classifiers show a similar behaviour. When $p = 0$, they always return `False`. When $p = 1$, they always return `True`. It can be expected that they will all become straight lines if the number of samples is increased beyond 100.

# 2 Task 2

We now test our Bernoulli classifier on the Iris dataset. As all 3 classes have the same number of instances, we randomly pick 'Class 0' as `True` and the rest as `False`.

[ ]:
```
from sklearn.datasets import load_iris

iris_dataset = load_iris()

data = iris_dataset["data"]
features = iris_dataset["feature_names"]
labels = iris_dataset["target"]

df = pd.DataFrame(data,columns=features)
df["class"] = labels
df["class"] = df["class"].apply(lambda x: True if x == 0 else False)
```

[ ]:
```
df.head()
```

```
[ ]:    sepal length (cm)  sepal width (cm)  petal length (cm)  petal width (cm)  \
     0                5.1               3.5                1.4               0.2
     1                4.9               3.0                1.4               0.2
     2                4.7               3.2                1.3               0.2
     3                4.6               3.1                1.5               0.2
     4                5.0               3.6                1.4               0.2

        class
     0   True
     1   True
     2   True
     3   True
     4   True
```

**1.**

The distribution of labels in the modified Iris dataset is shown below:

```
[ ]: df["class"].value_counts()
```

```
[ ]: class
     False    100
     True      50
     Name: count, dtype: int64
```

50 instances are labelled `True` and 100 instances are labelled `False`.

**2.**

We compute the performance of the Bernoulli classifier on the binary IRIS dataset for different values of $p$ using the following metrics - precision, recall, and F1-score. These metrics are then reported as a line plot.

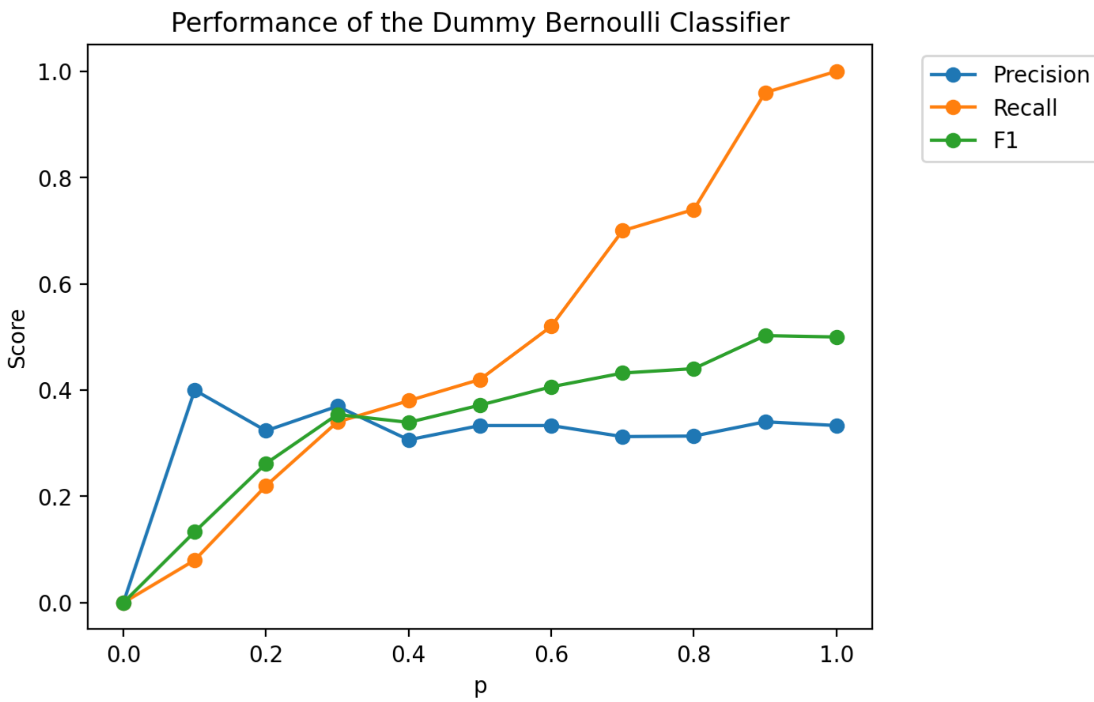
We also compute the true positive rate and false positive rate for subsequent tasks.

```
[ ]: # Defining the features and the label

     X = df.drop(columns=["class"])
     y = df["class"]
```

```
[ ]: # Importing the performance metrics

     from sklearn.metrics import precision_score, recall_score, f1_score,␣
      ↪confusion_matrix
```

```
[ ]: # Computing the metrics for different values of p

     precisions = []
     recalls = []
     f1s = []
```

```
tprs = []
fprs = []

for p in p_values:

  # Precision, recall, F1-score

  bernoulli_classifier = DummyBinaryClassifier(method="bernoulli",p=p)
  bernoulli_classifier.fit(data=X)
  predictions = bernoulli_classifier.predict(data=X)
  precisions.
 ↪append(precision_score(y_true=y,y_pred=predictions,zero_division=0))
  recalls.append(recall_score(y_true=y,y_pred=predictions))
  f1s.append(f1_score(y_true=y,y_pred=predictions))

  # TPR and FPR

  conf_matrix = confusion_matrix(y_true=y,y_pred=predictions)
  tp = conf_matrix[0,0]
  tn = conf_matrix[1,1]
  fp = conf_matrix[1,0]
  fn = conf_matrix[0,1]
  tpr = tp/(tp+fn)
  fpr = fp/(fp+tn)
  tprs.append(tpr)
  fprs.append(fpr)

# Visualizing the performance metrics

plt.figure(dpi=200)
plt.plot(p_values,precisions,label="Precision",marker="o")
plt.plot(p_values,recalls,label="Recall",marker="o")
plt.plot(p_values,f1s,label="F1",marker="o")
plt.title("Performance of the Dummy Bernoulli Classifier")
plt.legend(bbox_to_anchor=(1.05, 1))
plt.xlabel("p")
plt.ylabel("Score")
plt.show()
```

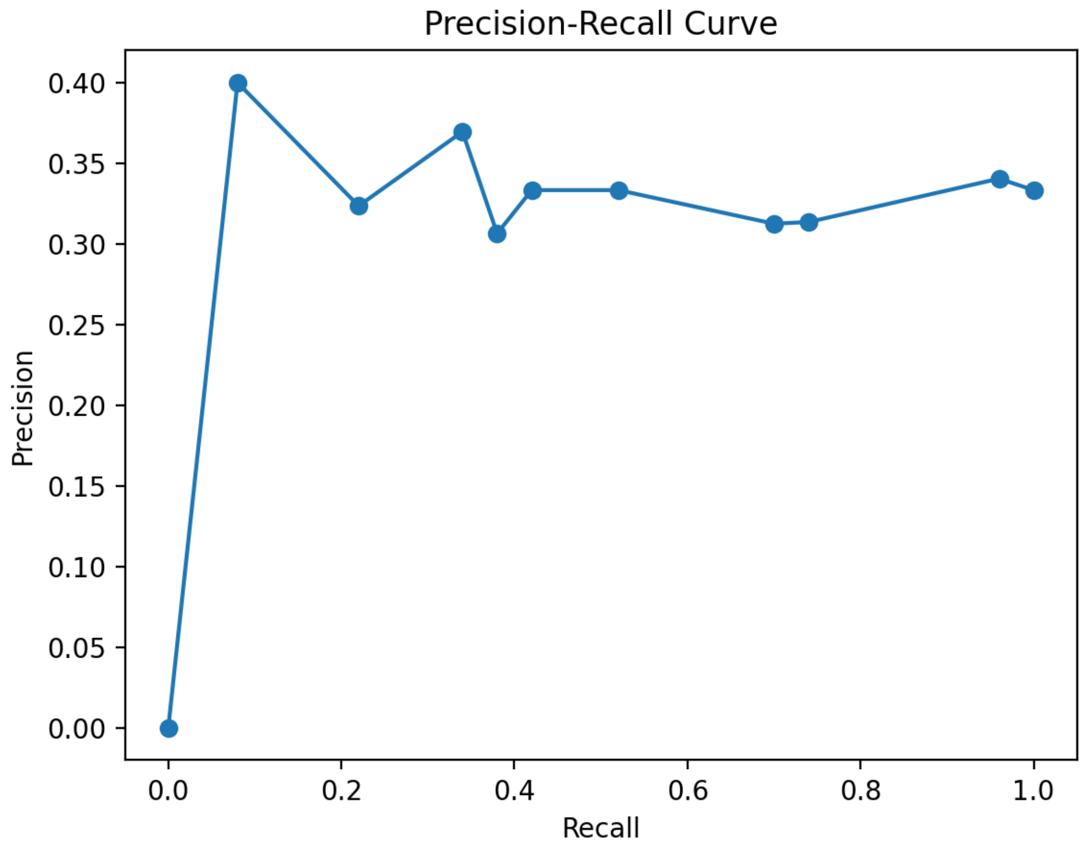



We can see that the recall $= \frac{TP}{TP+FN}$ increases from 0 to 1 as $p$ is increased. This is because all the predictions are `True` and there are no false negatives when $p = 1$. The precision does not increase beyond a point as the ratio $\frac{TP}{TP+FP}$ remains constant ($TP$ decreases and $FP$ increases). The F1-score is a function of the precision and recall.

**3.**

We plot the relation between the precision and recall based on the values computed earlier.

[ ]:
```
# Visualize the PRC

plt.figure(dpi=200)
plt.title("Precision-Recall Curve")
plt.plot(recalls,precisions,marker="o")
plt.xlabel("Recall")
plt.ylabel("Precision")
plt.show()
```

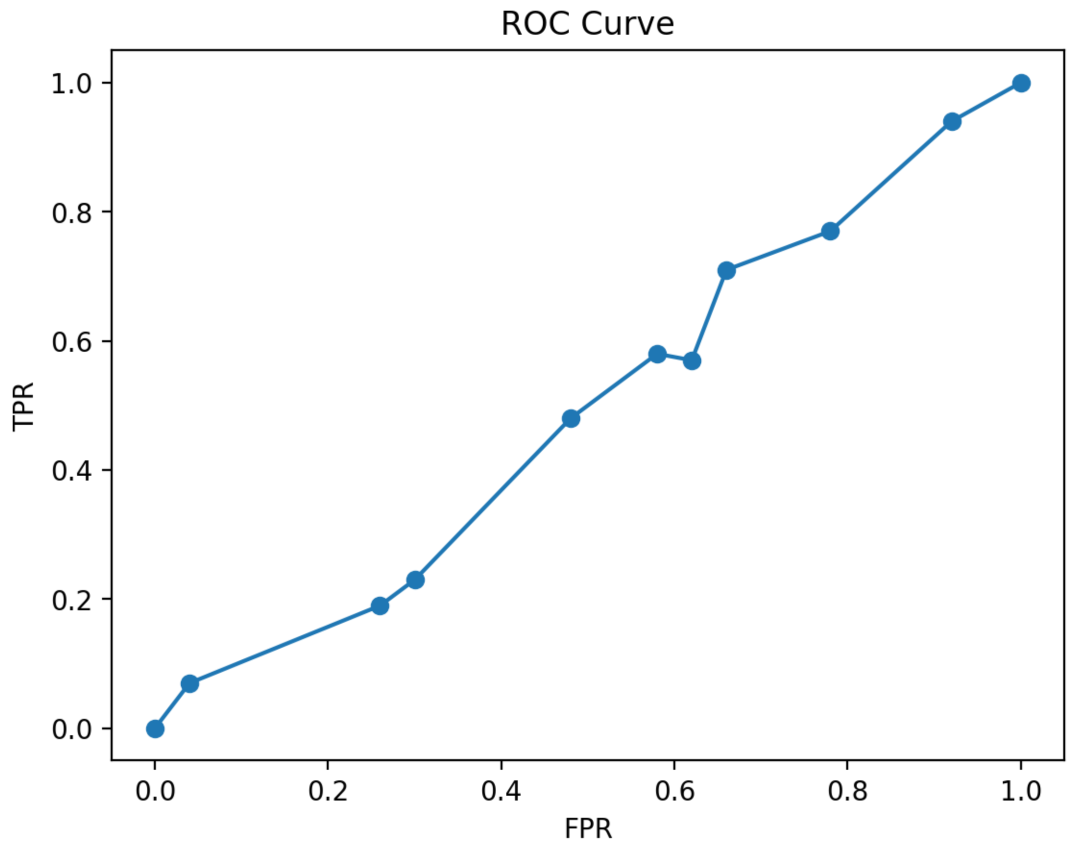



We can see that the precision stays constant even as the recall increases.

**4.**

We plot the ROC curve, the relation between the TPR and FPR.

[ ]:
```
# Visualize the ROC curve

plt.figure(dpi=200)
plt.plot(fprs,tprs,marker="o")
plt.xlabel("FPR")
plt.ylabel("TPR")
plt.title("ROC Curve")
plt.show()
```




We can see that the TPR and FPR increase in tandem from 0 to 1.

**5.**

Finally, we compute the area under the PRC and the ROC curve.

[ ]:
```
from sklearn.metrics import auc
```

[ ]:
```
auprc = auc(recalls,precisions)
auroc = auc(fprs,tprs)
print(f"AUPRC: {auprc}")
print(f"AUROC: {auroc}")
```

```
AUPRC: 0.3239418031168548
AUROC: 0.49
```

The AUPRC in this case is almost equal to the fraction of True's in the dataset $= \frac{50}{50+100} = 0.33$. This confirms the fact that our classifier returns predictions randomly.

The AUROC in this case is almost equal to 0.5. This again indicates that the classifier returns predictions based on random chance.

# 3 Task 3

We now visualise the **decision boundaries** of the 3 classifiers for different values of the threshold $p$. We convert the first 2 features of the IRIS dataset into a 2D grid and have the classifiers make predictions for these points. Points classified as `True` are coloured in red and those classified as `False` are coloured in blue. For reference, we also plot the original datapoints with their corresponding colour based on their class.

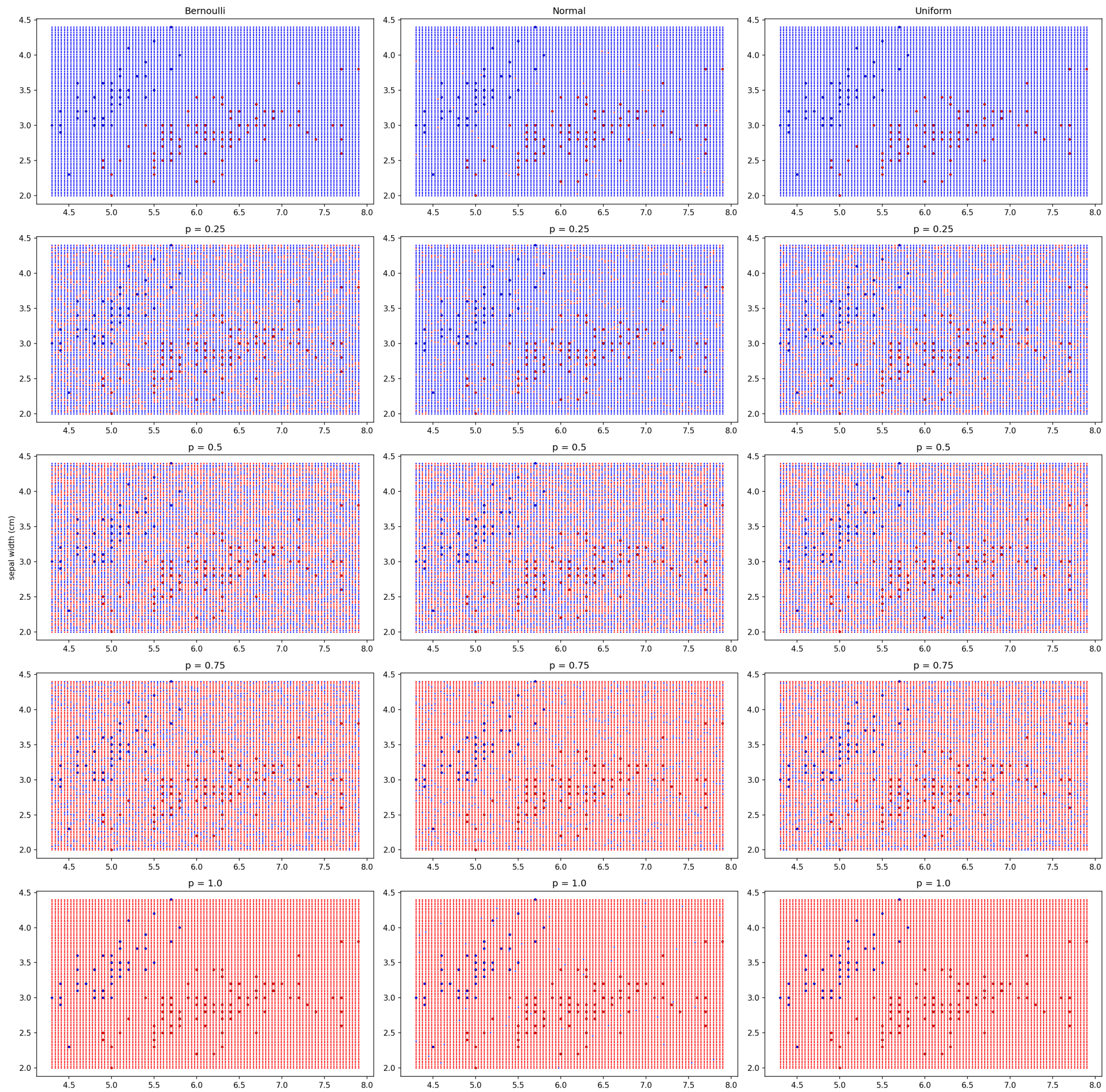
[ ]:
```
# Importing the colors library to visualise the decision boundary

import matplotlib.colors as colors
```

[ ]:
```
# Creating a grid of datapoints

X_1, X_2 = np.meshgrid(np.linspace(data[:, 0].min(), data[:, 0].max(),100),
                       np.linspace(data[:, 1].min(), data[:, 1].max(),100))
grid = np.array([X_1.reshape(-1), X_2.reshape(-1)]).T
```

[ ]:
```
# Creating a range of p values from 0 to 1 in steps of 0.25

p_values = np.arange(0,1.1,0.25)
```

[ ]:
```
# Visualising the decision boundaries for all 3 classifiers for the different␣
 ↪values of p

fig, ax = plt.subplots(5,3,dpi=150,figsize=(20,20))

i = 0
for method in ["bernoulli","normal","uniform"]:
  j = 0
  for p in p_values:
    classifier = DummyBinaryClassifier(method=method,p=p)
    classifier.fit(data=grid)
    y_pred = np.reshape(classifier.predict(grid), X_1.shape)
    ax[j,i].scatter(X_1[y_pred == True], X_2[y_pred == True], marker='.', s=2,␣
 ↪c="red")
    ax[j,i].scatter(X_1[y_pred == False], X_2[y_pred == False], marker='.',␣
 ↪s=2, c="blue")
    ax[j,i].scatter(data[:, 0], data[:, 1], marker='.', c=y, cmap=colors.
 ↪ListedColormap(['red','blue']), edgecolors='black', linewidth=0.25)
    ax[j,i].set_title(f"p = {p}")
    j += 1
  i += 1

for axis, method in zip(ax[0],["Bernoulli","Normal","Uniform"]):
    axis.set_title(method)
```

```
ax[4][1].set_xlabel(features[0])
ax[2][0].set_ylabel(features[1])

plt.tight_layout()
plt.show()
```




When $p = 0$, all classifiers return `False`. When $p = 1$, all classifiers return `True`. As $p$ increases, so does the number of points predicted as `True`. However, there is no clear decision boundary for intermediate values of $p$. The classifiers randomly predict points as `True` or `False`. This is especially clear when $p = 0.5$.