DA5401 - Assignment 8

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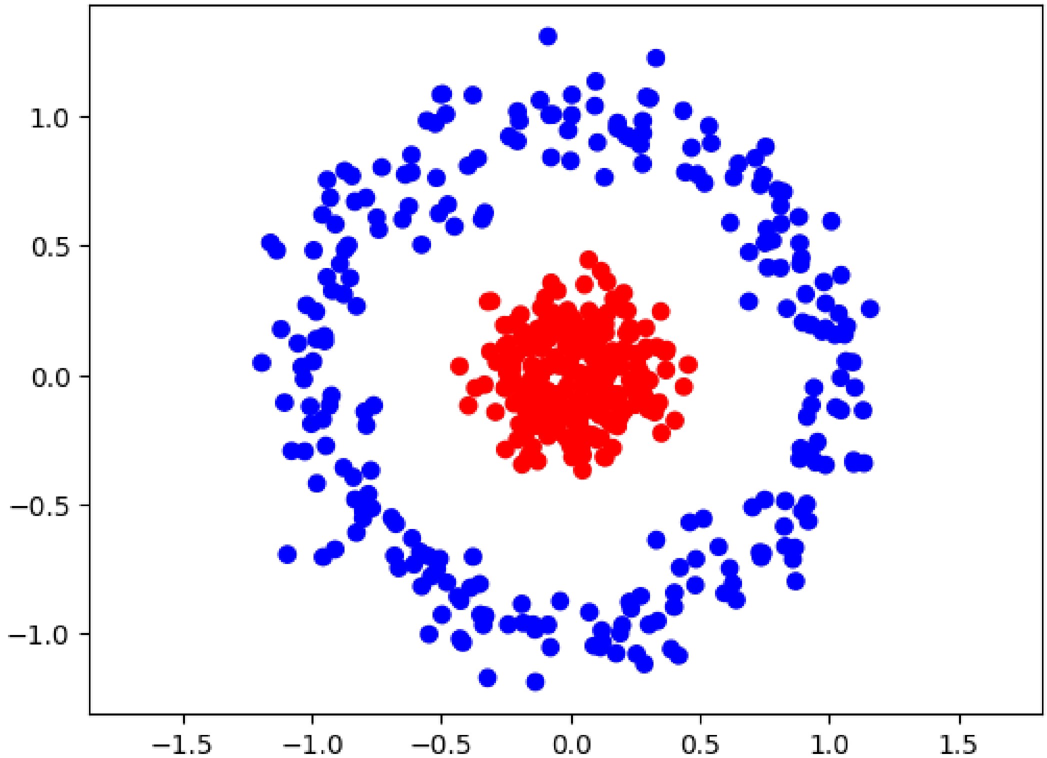
[1]:
```python
# Importing necessary libraries

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

We create the dataset using `sklearn`'s `make_circles` function. We replace the label 0 with -1 to make the AdaBoost implementation mathematically convenient.

[2]:
```python
from sklearn.datasets import make_circles
from sklearn.model_selection import train_test_split
import matplotlib.colors as colors

X, y = make_circles(n_samples=500, noise=0.1, random_state=42, factor=0.2)
y = np.where(y == 0, -1, 1)
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=42)
plt.scatter(X[:,0], X[:,1], c=y, cmap=colors.ListedColormap(['blue', 'red']))
plt.axis('equal')
plt.show()
```



1 Task 1

[3]:
```
from sklearn.base import clone
from sklearn.metrics import accuracy_score
```

We define the AdaBoost classifier using the algorithm given in the question. To incorporate the hyperparameter η, we modify the weight update rule to be $w_{i,t+1} = w_{i,t}\exp(-\eta y_i \alpha_t h_t(x))$. This controls how the weights are updated. We also define a function to plot the decision boundaries of the individual estimators and the ensemble.

[4]:
```
class AdaboostClassifier:

  def __init__(self, base_estimator, n_estimators=50, eta=0.5):

    # Initialising the classifier

    self.base_estimator = base_estimator
    self.n_estimators = n_estimators
    self.eta = eta
```

```
    self.ensemble = []
    self.estimator_weights = []

  def fit(self, X, y):

    # Training the ensemble classifier with adaptive boosting

    sample_weights = [1/len(X)] * len(X)

    for t in range(self.n_estimators):
      model = clone(self.base_estimator)
      model.fit(X, y, sample_weight=sample_weights)
      self.ensemble.append(model)
      y_pred = model.predict(X)
      error = np.sum(sample_weights * (y_pred != y))
      estimator_weight = 0.5 * np.log((1 - error) / error)
      self.estimator_weights.append(estimator_weight)
      sample_weights = sample_weights * np.exp(-self.eta * estimator_weight * y␣
↪* y_pred)
      sample_weights = sample_weights / np.sum(sample_weights)

  def predict(self, X):

      # Making a weighted prediction using the estimators in the ensemble

      y_pred = np.zeros(len(X))
      for model, estimator_weight in zip(self.ensemble, self.estimator_weights):
        y_pred += estimator_weight * model.predict(X)
      y_pred = np.sign(y_pred)
      return y_pred

  def score(self, X, y):

    # Defining the accuracy score

    y_pred = self.predict(X)
    return accuracy_score(y, y_pred)

  def visualize_boundary(self, X, y):

    # Plotting the decision boundaries of the individual estimators and the␣
↪ensemble

    fig, ax = plt.subplots(1, 2, figsize=(10, 5), dpi=150)

    X1, X2 = np.meshgrid(np.linspace(X[:, 0].min(), X[:, 0].max()),
                         np.linspace(X[:, 1].min(), X[:, 1].max()))
```

```
    grid = np.vstack([X1.ravel(), X2.ravel()]).T
    y_pred = self.predict(grid).reshape(X1.shape)
    ax[1].contourf(X1, X2, y_pred, alpha=0.3, cmap=colors.
↪ListedColormap(['blue', 'red']))
    ax[1].scatter(X[:,0], X[:,1], c=y, cmap=colors.ListedColormap(['blue',␣
↪'red']), edgecolors='k')
    ax[1].axis('equal')
    ax[1].set_xlabel('X1')
    ax[1].set_ylabel('X2')
    ax[1].set_title('Ensemble')

    for i, estimator in enumerate(self.ensemble):
      ax[0].contour(X1, X2, estimator.predict(grid).reshape(X1.shape),␣
↪cmap=colors.ListedColormap(['black', 'black']),levels=[0])
      ax[0].scatter(X[:, 0], X[:, 1], c=y, cmap=colors.ListedColormap(['blue',␣
↪'red']), edgecolors='k')
    ax[0].axis('equal')
    ax[0].set_xlabel('X1')
    ax[0].set_ylabel('X2')
    ax[0].set_title(f'Individual Estimators')

    plt.setp(ax, xlim=(X[:, 0].min(), X[:, 0].max()), ylim=(X[:, 1].min(), X[:,␣
↪1].max()))
    plt.suptitle(f'{self.base_estimator}\n n_estimators = {self.n_estimators},␣
↪ ={self.eta}')
    plt.tight_layout()
    plt.show()
```

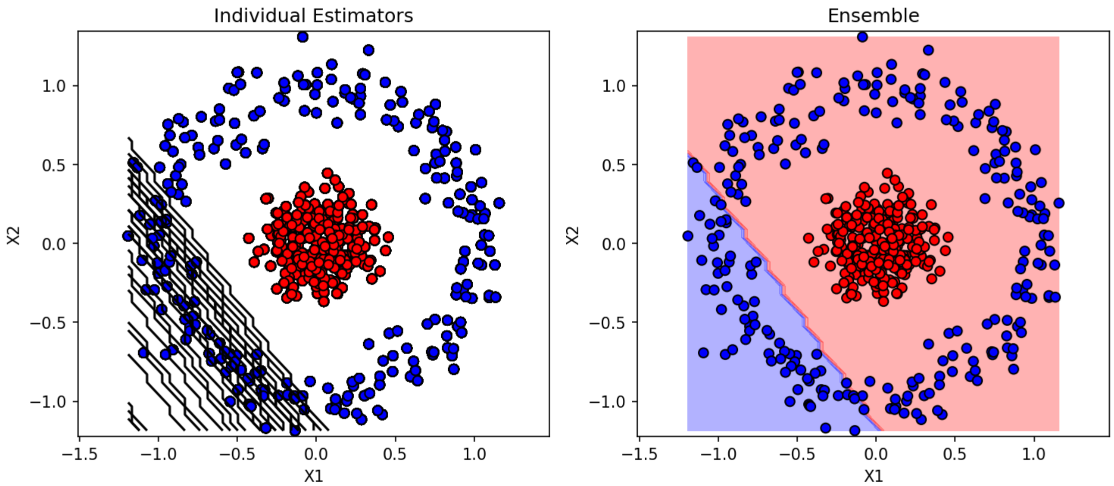
2 Task 2

We test the ensemble classifier using different base estimators.

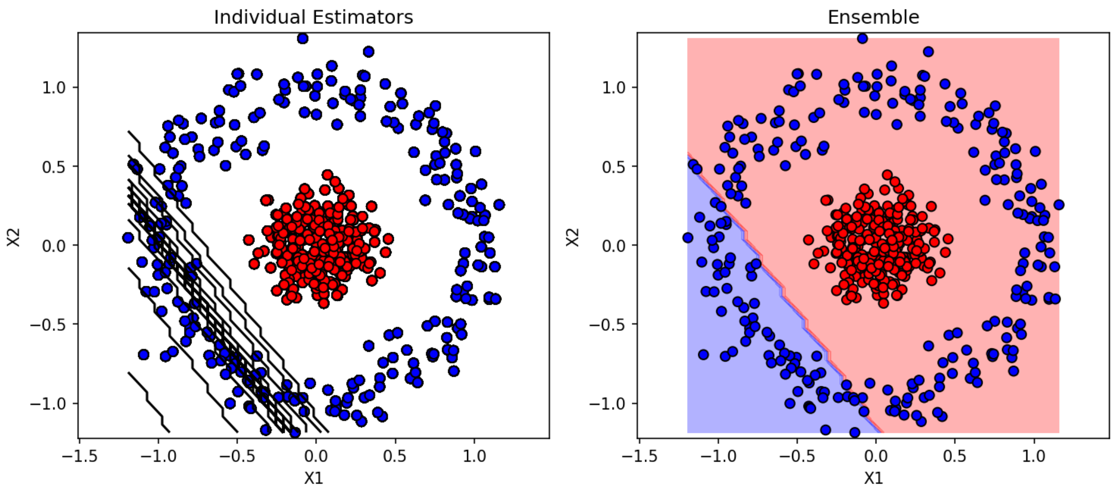
[5]:
```
from sklearn.linear_model import LogisticRegression
from sklearn.tree import DecisionTreeClassifier
from sklearn.svm import LinearSVC
```

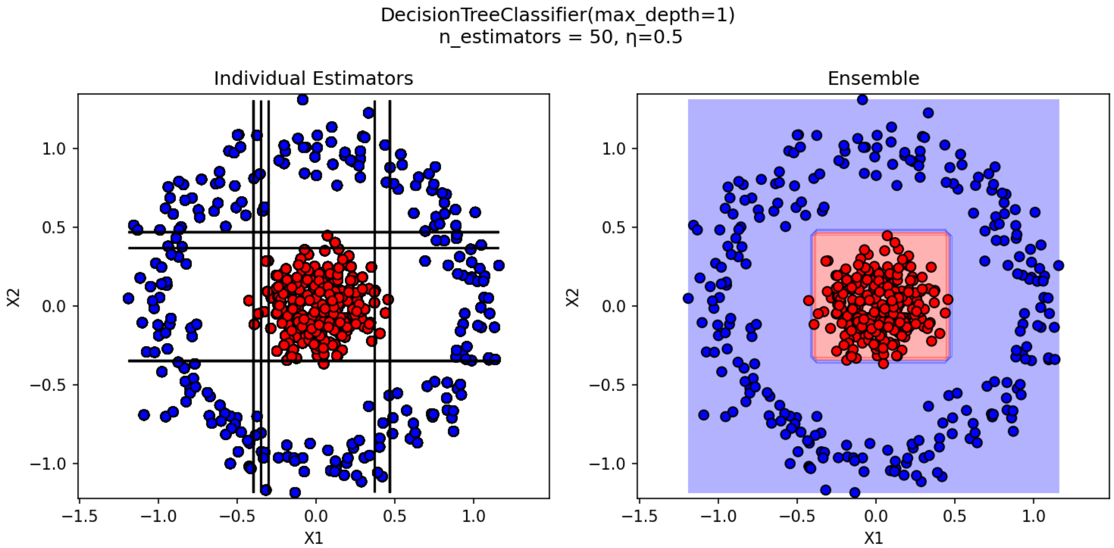
[6]:
```
for base_model in [LogisticRegression(), LinearSVC(),␣
↪DecisionTreeClassifier(max_depth=1), DecisionTreeClassifier(max_depth=3)]:
  adaboost_model = AdaboostClassifier(base_model)
  adaboost_model.fit(X_train, y_train)
  adaboost_model.visualize_boundary(X, y)
```

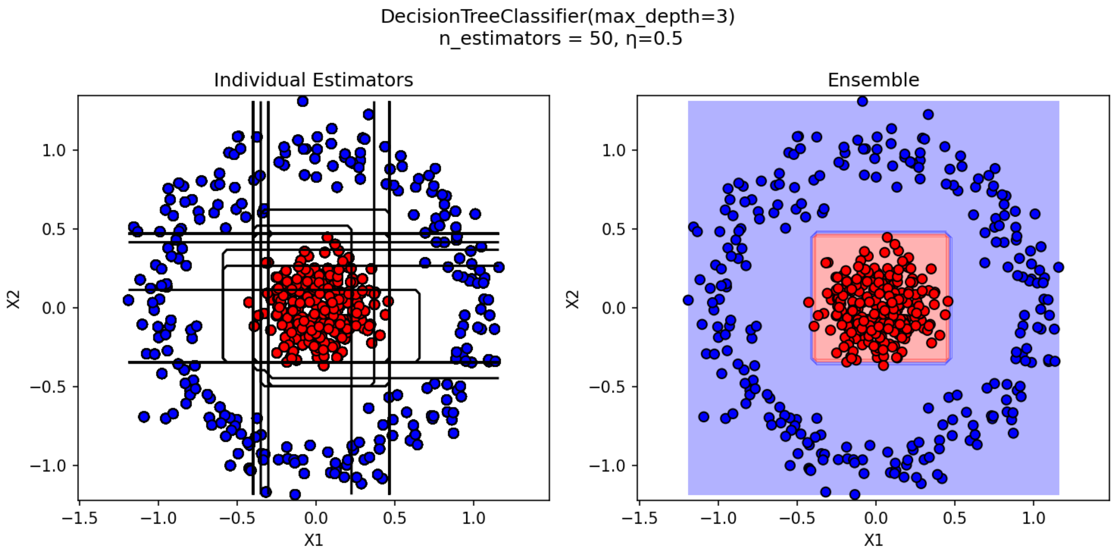
LogisticRegression()
n_estimators = 50, η=0.5



LinearSVC()
n_estimators = 50, η=0.5







We can see that the decision boundaries are different for different estimators. `LogisticRegression` and `LinearSVC` don't seem to be performing well. This can be seen by computing their baseline accuracy scores.

[7]:
```
for base_model in [LogisticRegression(), LinearSVC(), DecisionTreeClassifier(max_depth=1), DecisionTreeClassifier(max_depth=3)]:
    base_model.fit(X_train, y_train)
    print(f'Training accuracy of {base_model}: {base_model.score(X_train, y_train):.3f}')
```

```
print(f'Test accuracy of {base_model}: {base_model.score(X_test, y_test):.
↪3f}')
print()
```

```
Training accuracy of LogisticRegression(): 0.328
Test accuracy of LogisticRegression(): 0.264

Training accuracy of LinearSVC(): 0.331
Test accuracy of LinearSVC(): 0.264

Training accuracy of DecisionTreeClassifier(max_depth=1): 0.693
Test accuracy of DecisionTreeClassifier(max_depth=1): 0.712

Training accuracy of DecisionTreeClassifier(max_depth=3): 0.939
Test accuracy of DecisionTreeClassifier(max_depth=3): 0.912
```

As `DecisionTreeClassifier(max_depth=3)` has the best baseline performance, we will take it forward for hyperparameter tuning.

[8]:
```
from sklearn.model_selection import GridSearchCV
```

[9]:
```
# Defining the hyperparameter grid

tree_param_grid = {'criterion': ['gini', 'entropy'],
                   'min_samples_split': [2, 5, 10, 15, 20],
                   'min_samples_leaf': [1, 2, 4],
                   'max_leaf_nodes': [None, 10, 20, 30, 50],
                   'min_impurity_decrease': [0.0, 0.01, 0.1]}
```

[10]:
```
# Finding the best hyperparameters

grid_model = GridSearchCV(DecisionTreeClassifier(max_depth=3), tree_param_grid,␣
↪cv=5)
grid_model.fit(X_train, y_train)
```

[10]:
```
GridSearchCV(cv=5, estimator=DecisionTreeClassifier(max_depth=3),
             param_grid={'criterion': ['gini', 'entropy'],
                         'max_leaf_nodes': [None, 10, 20, 30, 50],
                         'min_impurity_decrease': [0.0, 0.01, 0.1],
                         'min_samples_leaf': [1, 2, 4],
                         'min_samples_split': [2, 5, 10, 15, 20]})
```

[11]:
```
grid_model.best_params_
```

[11]:
```
{'criterion': 'gini',
 'max_leaf_nodes': None,
 'min_impurity_decrease': 0.0,
```

```
 'min_samples_leaf': 1,
 'min_samples_split': 10}
```

[12]:
```
best_base_model = DecisionTreeClassifier(max_depth=3,**grid_model.best_params_)
best_base_model.fit(X_train,y_train)
print(f'Training accuracy of {best_base_model}: {best_base_model.score(X_train,␣
↪y_train):.3f}')
print(f'Test accuracy of {best_base_model}: {best_base_model.score(X_test,␣
↪y_test):.3f}')
```

```
Training accuracy of DecisionTreeClassifier(max_depth=3, min_samples_split=10):
0.936
Test accuracy of DecisionTreeClassifier(max_depth=3, min_samples_split=10):
0.912
```

This has a similar performance to the estimator without any tuning. We will take this optimized estimator and ensemble it with adaptive boosting.

[13]:
```
base_estimator = DecisionTreeClassifier(**best_base_model.get_params())
adaboost_model = AdaboostClassifier(base_estimator=base_estimator)
adaboost_model.fit(X_train, y_train)
print(f'Training accuracy of AdaBoost: {adaboost_model.score(X_train, y_train):.
↪3f}')
print(f'Test accuracy of AdaBoost: {adaboost_model.score(X_test, y_test):.3f}')
```

```
Training accuracy of AdaBoost: 1.000
Test accuracy of AdaBoost: 0.992
```

This has achieved nearly 100% accuracy. We will then fine-tune the number of estimators in the ensemble and the hyperparameter η.

[14]:
```
# Fine-tuning the AdaBoost hyperparameters

num_estimators = [5, 10, 25, 50, 100, 200, 500]
eta = [0, 0.1, 0.25, 0.5, 0.75, 0.9, 1]
train_scores = dict()
test_scores = dict()

for n in num_estimators:
    for e in eta:
        base_estimator = clone(DecisionTreeClassifier(**best_base_model.
↪get_params()))
        adaboost_model = AdaboostClassifier(base_estimator=base_estimator,␣
↪n_estimators=n, eta=e)
        adaboost_model.fit(X_train, y_train)
        train_scores[(n, e)] = adaboost_model.score(X_train, y_train)
        test_scores[(n, e)] = adaboost_model.score(X_test, y_test)
```

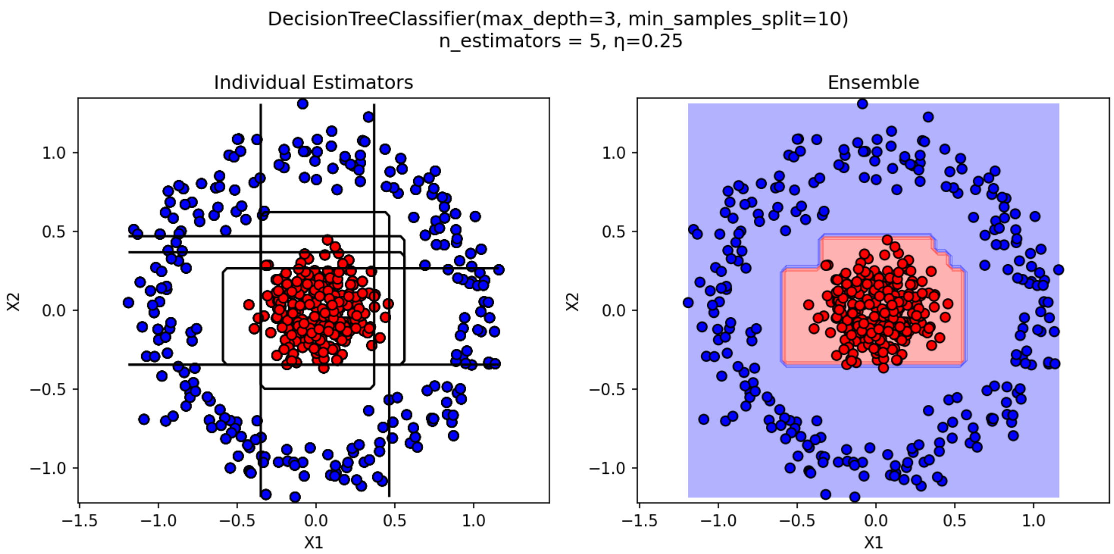
```
best_n_estimators = max(test_scores, key=lambda x: test_scores[x])[0]
best_eta = max(test_scores, key=lambda x: test_scores[x])[1]

print(f'Best training accuracy: {train_scores[(best_n_estimators,best_eta)]:.
↪3f}')
print(f'Best test accuracy: {max(test_scores.values()):.3f}')
print(f'Best n_estimators: {best_n_estimators}')
print(f'Best eta: {best_eta}')
```

```
Best training accuracy: 1.000
Best test accuracy: 1.000
Best n_estimators: 5
Best eta: 0.25
```

We see that when 5 estimators are used with $\eta = 0.25$, we get 100% accuracy on both the train and test sets. We can plot the decision boundary for this ensemble.

[15]:
```
best_adaboost_model = AdaboostClassifier(base_estimator=base_estimator,␣
↪n_estimators=best_n_estimators, eta=best_eta)
best_adaboost_model.fit(X_train, y_train)
best_adaboost_model.visualize_boundary(X, y)
```



We can observe that this ensemble classifier performs extremely well on the dataset. The decision boundary is more complex than when 50 estimators were used with $\eta = 0.5$.