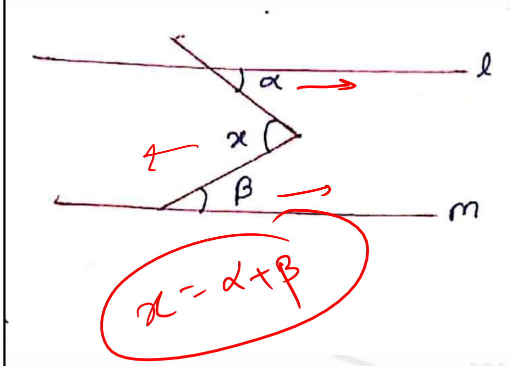
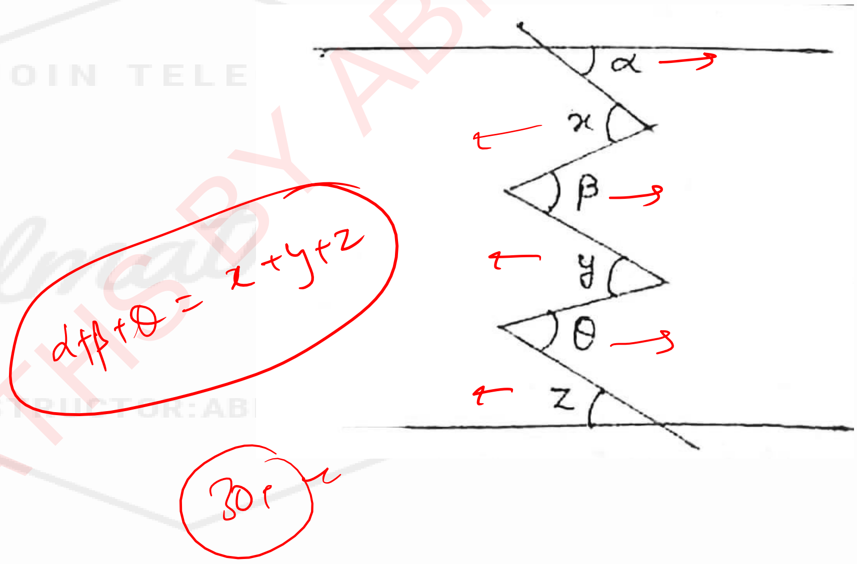
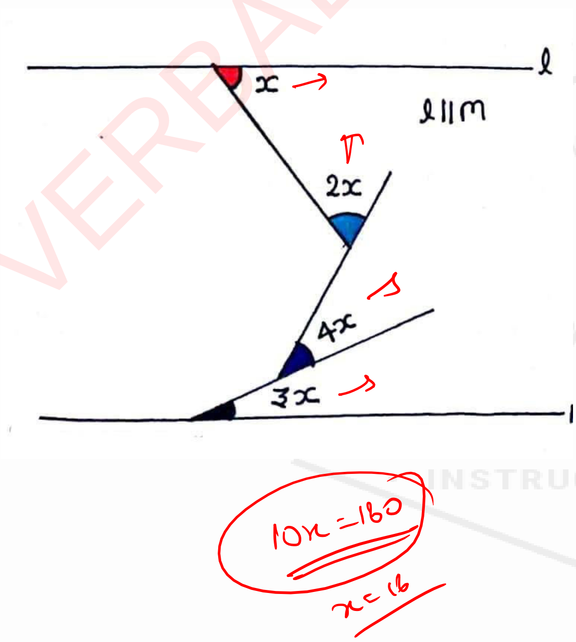
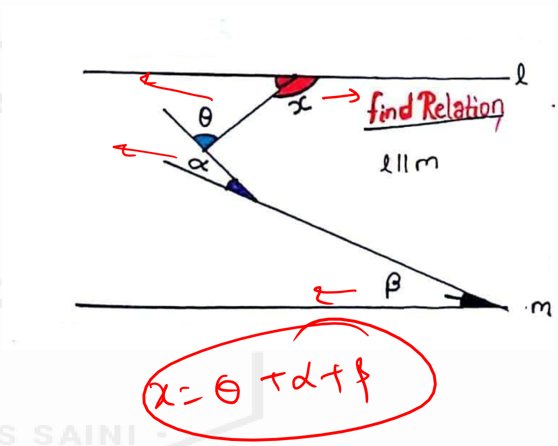
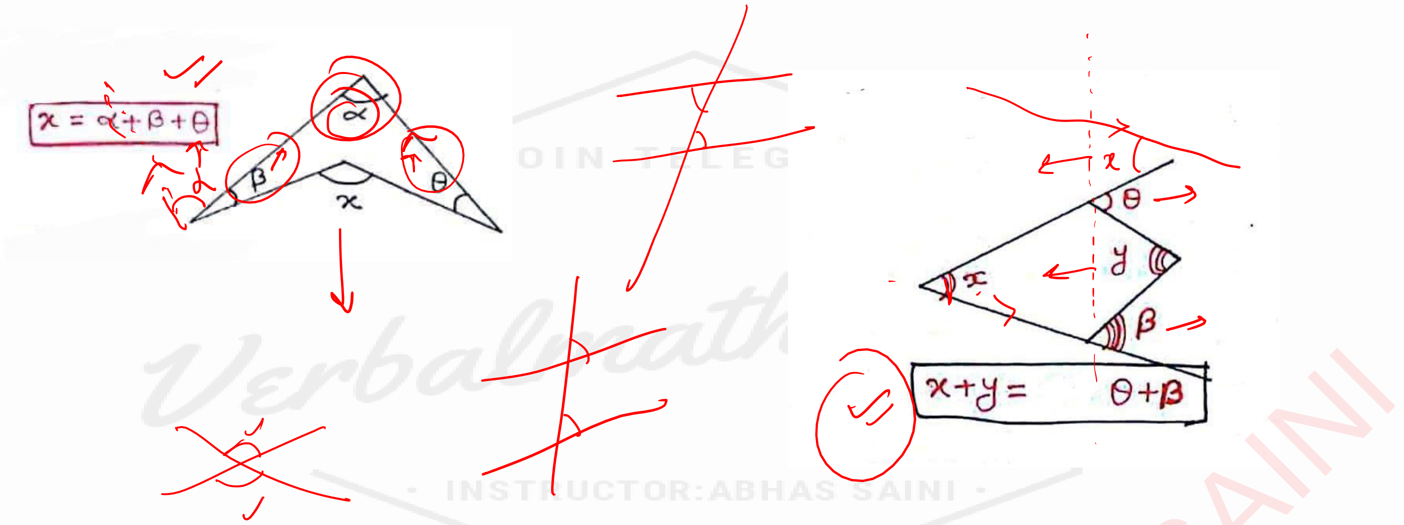
ARROW CONCEPTS BY ABHAS SAINI





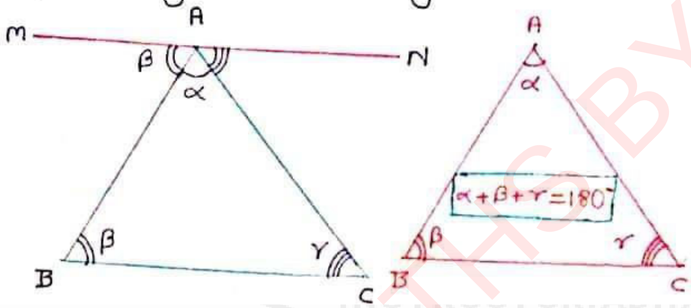






TRAINGLE

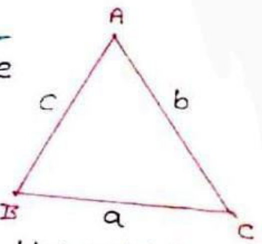
* Sum of all angles in a triangle = 180°



Properties of Triangle :—

If two sides of a triangle are given then range of third side will be

$$\boxed{|a-b| < c < |a+b|}$$



|| → modulus

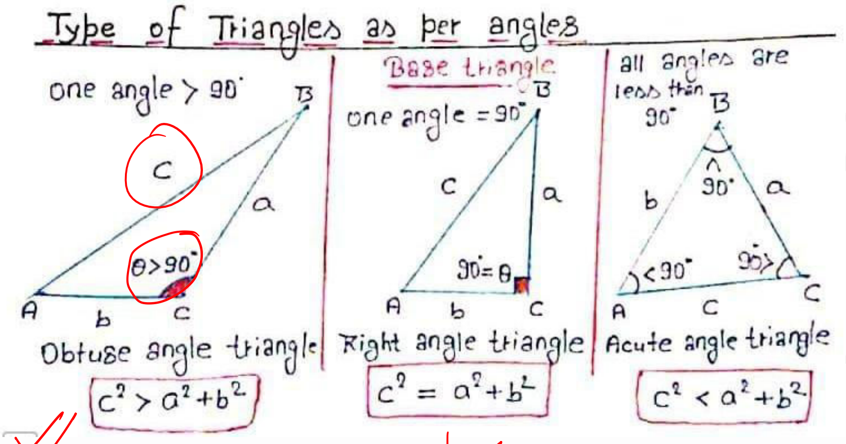
|Difference of two sides| < c < |Sum of two sides|

a, b, c

3, 2, 7

$|3-2| < 7 < |3+2|$

× fail

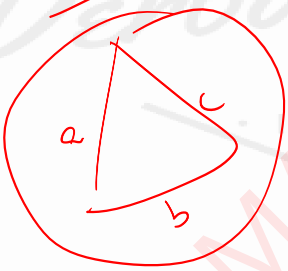


PHY.

TYPES OF TRIANGLES (AS PER SIDES)

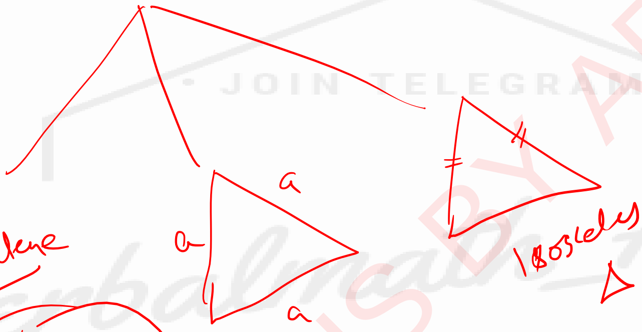
Scalene

$a \neq b \neq c$



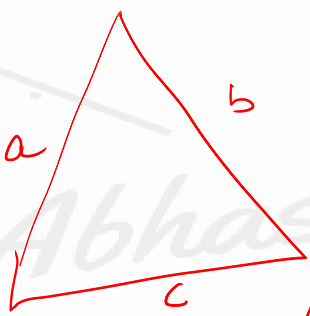
Isosceles

Equilateral



AREA OF TRIANGLE

General Scalene

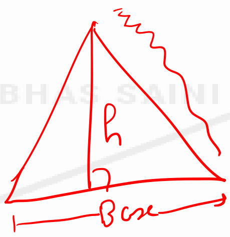


Heron's Formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

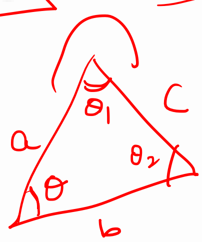
$$S = \frac{a+b+c}{2}$$

$$\boxed{A = \frac{1}{2} \times b \times h}$$



$\frac{1}{2} ab \sin\theta_1$

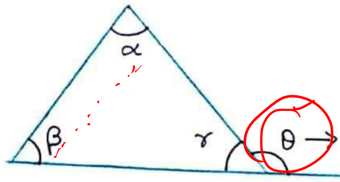
$\frac{1}{2} bc \sin\theta_2$



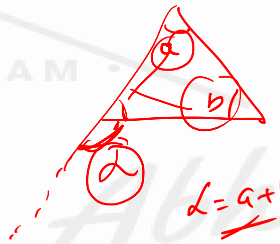
$$\boxed{\frac{1}{2} ab \sin\theta = A}$$

Exterior Angle

Exterior Angle Property :—

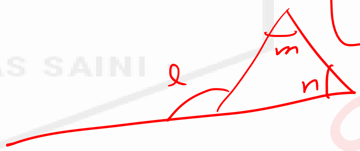


$\theta \rightarrow$ Exterior angle



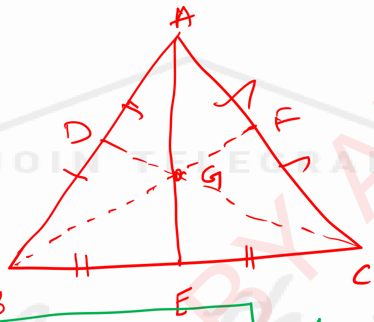
$d = a + b$

$\theta = \alpha + \beta$



$l = m + n$

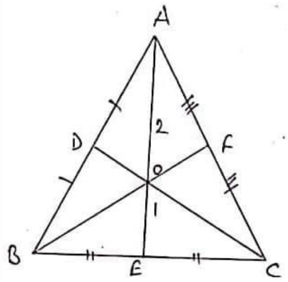
MEDIAN → Centroid



AE - median

G → Centroid

2 : 1 → median



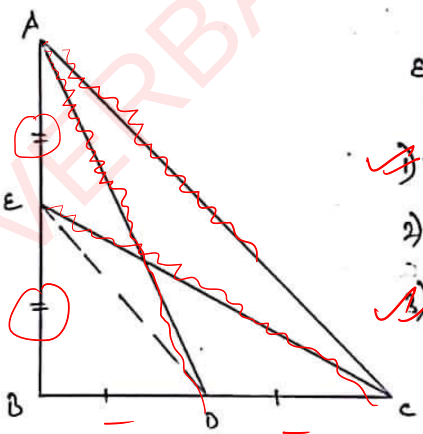
$\text{Area of } \triangle ABC = \frac{4}{3}(\triangle \text{ medians})$

AE, BF, CD

a, b, c → Area

$$AE^2 + BF^2 + CD^2 = \frac{3}{4}(AB^2 + BC^2 + AC^2)$$

$$2(AE^2 + BE^2) = AB^2 + AC^2$$



EC & AD are Medians

1) $4(AD^2 + CE^2) = 5AC^2$

2) $AD^2 + CE^2 = 5ED^2$

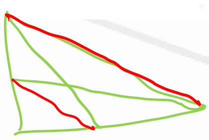
3) $AD^2 + CE^2 = AC^2 + ED^2$

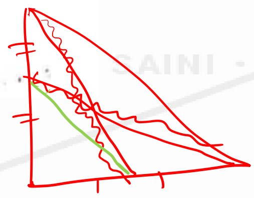
EC & AD

$$4(AD^2 + CE^2) = 5AC^2$$

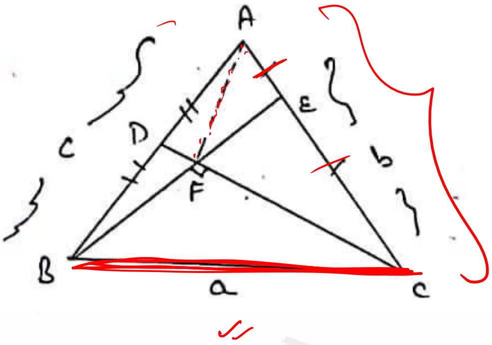
$$AD^2 + CE^2 = 5ED^2$$

$$AD^2 + CE^2 = AC^2 + ED^2$$





If the Medians are Perpendicular.



$$b^2 + c^2 = 5a^2$$

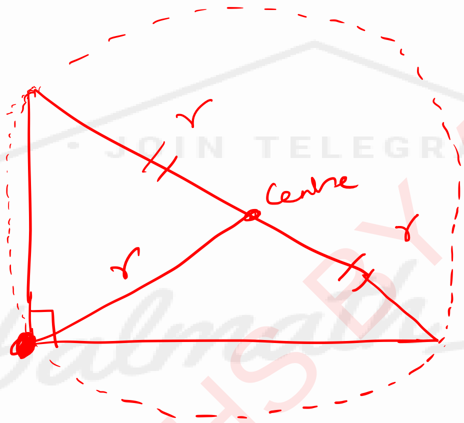
AF = BC

CD & BE → medians ⊥ to each other

$$AB^2 + AC^2 = 5BC^2$$

AF = BC

Right Angle △

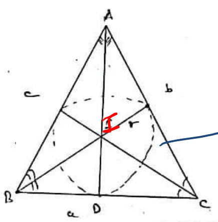


INCENTRE (ANGLE BISECTOR) →

INCENTRE (ANGLE BISECTOR)

$r = \frac{\Delta}{\text{Semi Perimeter}}$

$\angle BIC = 90 + \frac{A}{2}$



$\frac{AI}{ID} = \frac{b+c}{a}$

r → inradius

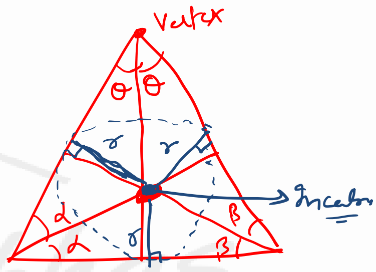
For Equilateral Triangle.

$r = a/2\sqrt{3}$

$r = R/2$

For Right Angle Triangle

$r = \frac{P+B-H}{2}$



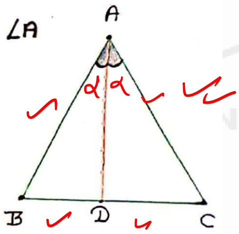
$$r = \frac{\triangle \text{ (Area of } \triangle)}{\text{Semi perimeter of } \triangle}$$

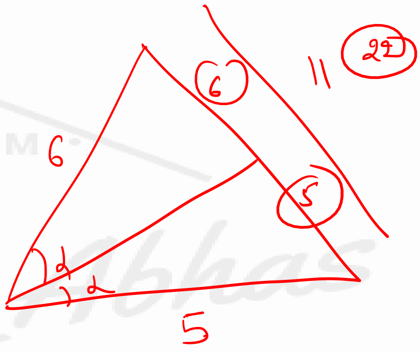
General

Angle bisector theorem :–

If AD is angle bisector of $\angle A$ in ΔABC then

$$\boxed{\frac{AB}{BD} = \frac{AC}{CD}}$$

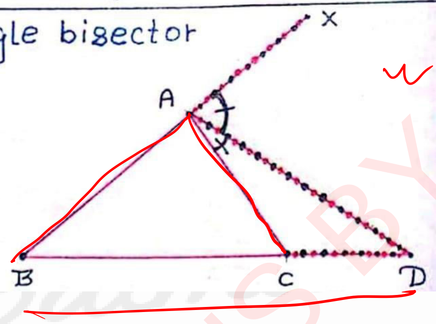




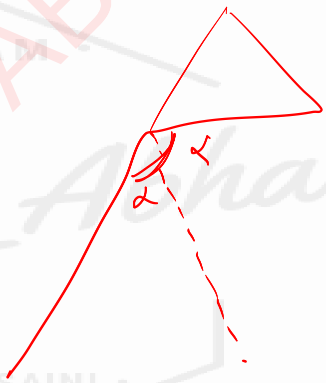
ANGLE BISECTOR THEOREM (EXTERNAL CASE) :–

If AD is external angle bisector of $\angle A$ then

$$\boxed{\frac{BD}{CD} = \frac{AB}{AC}}$$



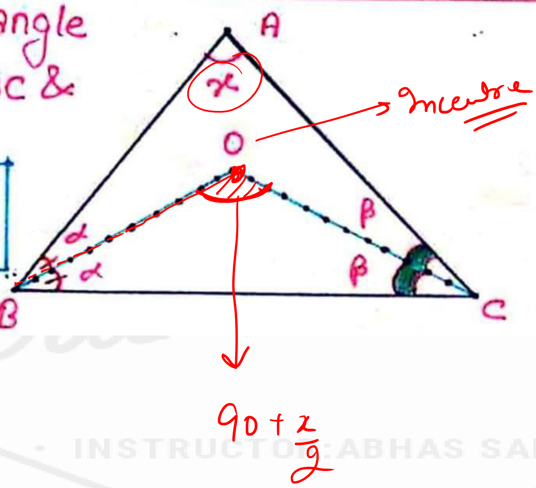
$$\boxed{\frac{AB}{AC} = \frac{BD}{CD}}$$

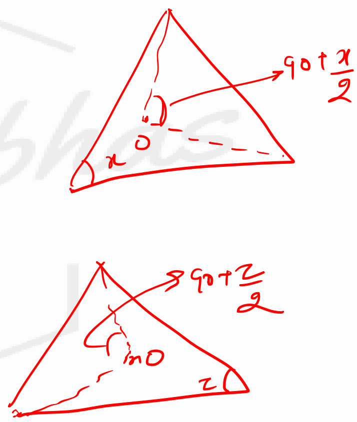


Generalisation – II

If BO & CO are angle bisectors of $\angle ABC$ & $\angle ACB$ then

$$\boxed{\angle BOC = 90° + \frac{\angle BAC}{2}}$$

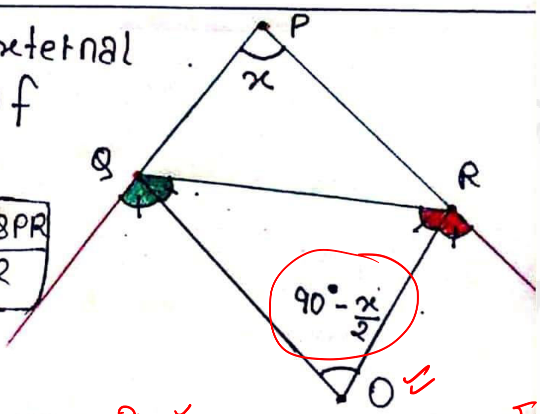




Generalisation – III

If QO & RO are external angle bisectors of ∠PQR & ∠PRQ then

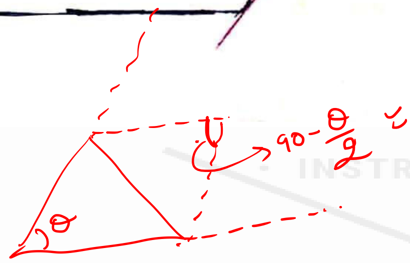
$$\boxed{\angle QOR = 90 - \frac{\angle QPR}{2}}$$



Internal A.B

↓ Incentre

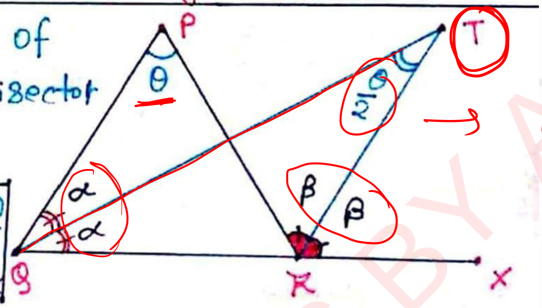
Excentre



Some Generalisations related to angle bisectors :–

If QT is bisector of ∠PQR & RT is bisector of ∠PRX then

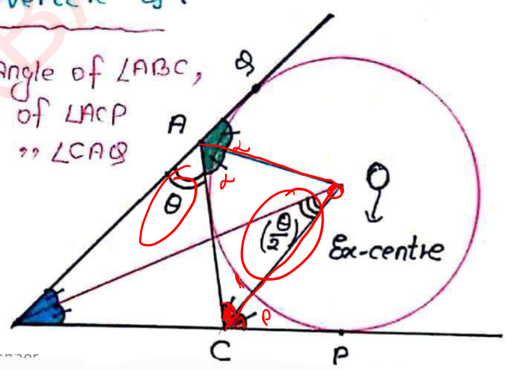
$$\boxed{\angle QTR = \frac{\angle QPR}{2} = \frac{\theta}{2}}$$

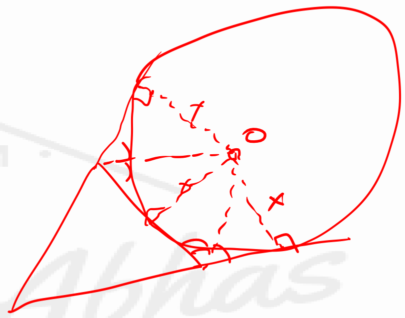


Imp.

Point 'T' is also the ex-centre of ΔPQR opposite to vertex 'Q'.

Here BO is angle of ∠ABC, CO is bisector of ∠ACP & AD is " " ∠CAQ



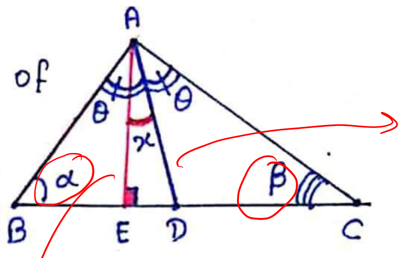


∠BOC

Generalisation – IV

In ΔABC AD is bisector of $\angle BAC$ & $AE \perp BC$

then $\boxed{\angle EAD = \dfrac{\angle ABC - \angle ACB}{2} \qquad x = \dfrac{\alpha - \beta}{2}}$



$\angle BAD = \angle CAD = \theta$

& $\angle EAD = x$

$\angle BAE = \theta - x$

Altitude

Class Notes

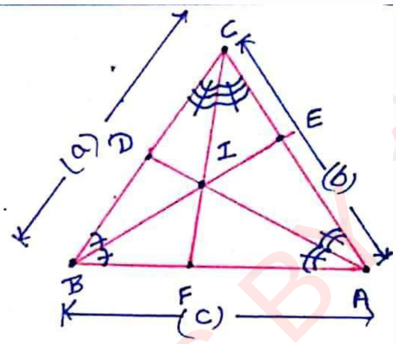
30-40 → 400

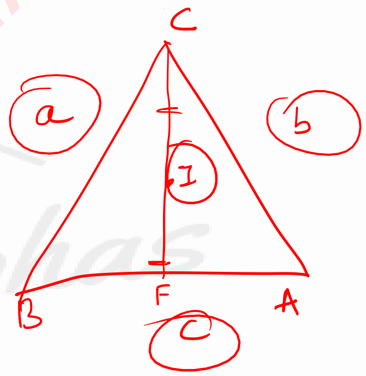
Generalisation – V

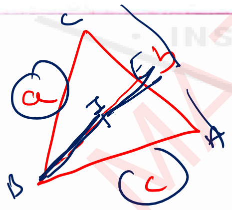
If AD, BE & CF are angle bisectors of $\angle A$, $\angle B$ & $\angle C$ respectively then

(i) $\dfrac{CI}{IF} = \dfrac{a+b}{c}$

(ii) $\dfrac{BI}{IE} = \dfrac{a+c}{b}$ & (iii) $\dfrac{AI}{ID} = \dfrac{b+c}{a}$







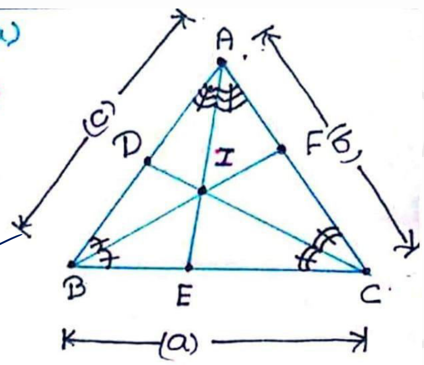
$\dfrac{BI}{IE} = \dfrac{a+c}{b}$

$\dfrac{CI}{IF} = a+b$

Generalisation :– VI (a)

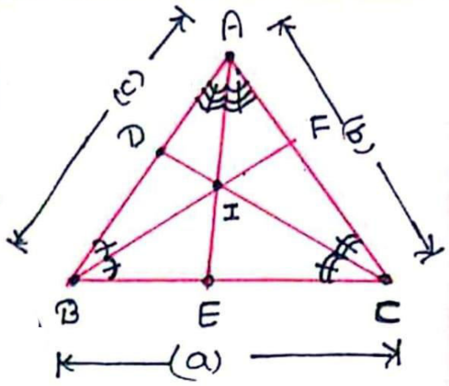
If AE, BF, & CD are angle bisectors then,

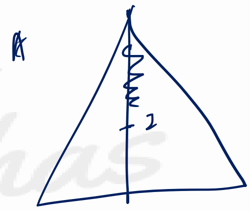
$\boxed{\dfrac{IE}{AE} + \dfrac{IF}{BF} + \dfrac{ID}{CD} = 1}$



Generalisation :- VI (b)

$$\boxed{\frac{AI}{AE} + \frac{BI}{BF} + \frac{CI}{CD} = 2}$$



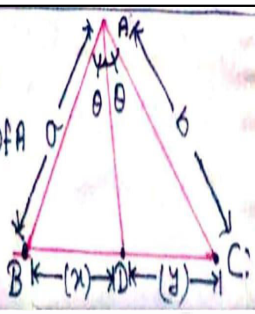


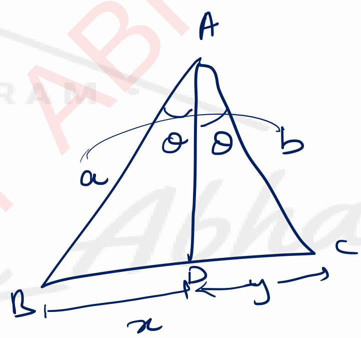
MPG

Generalisation :- VII (a)

AB = a, AC = b, BD = x, CD = y & AD is angle bisector of A

then $\boxed{(AD)^2 = ab - xy}$

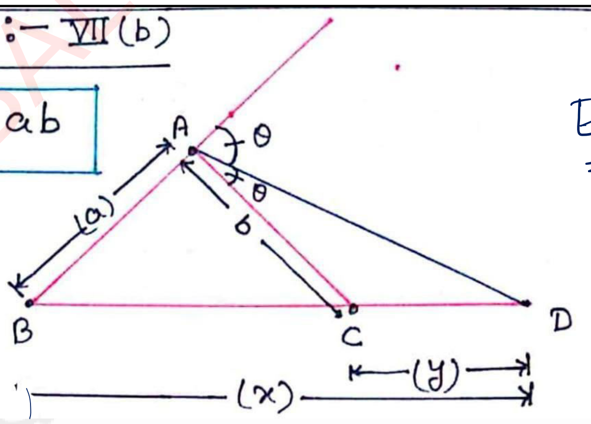




$$\boxed{AD^2 = ab - xy}$$

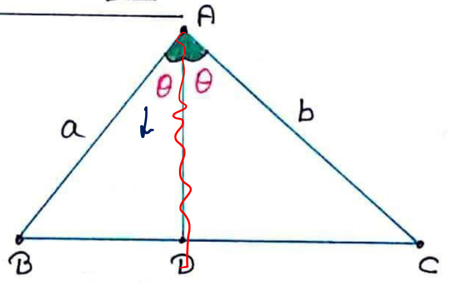
Generalisation :- VII (b)

$$\boxed{AD^2 = xy - ab}$$

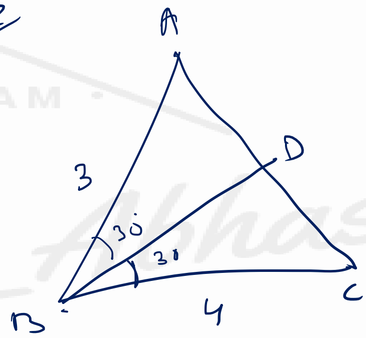


Extend A.B

Generalisation :— VIII



$$AD = \boxed{\frac{2ab\cos(\theta)}{(a+b)}}$$



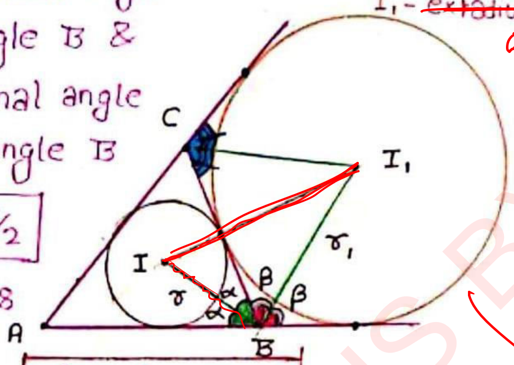
$$BD = \frac{2\times3\times4\cos30^\circ}{7}$$

here IB is internal angle bisector of angle B & I_1B is external angle bisector of angle B

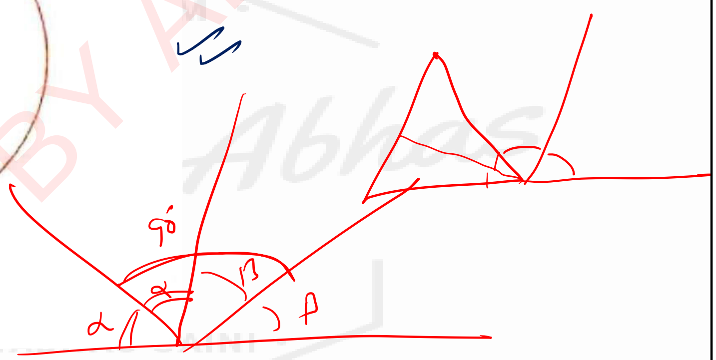
$\therefore \boxed{\alpha+\beta = \pi/2}$

if 'r' is inradius & 'r_1' is exradius then $\boxed{r^2+r_1^2=(II_1)^2}$

I – ~~inradius~~ Incentre
I_1 – ~~exradius~~ Excentre

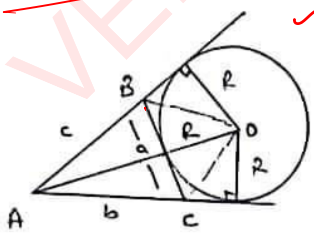


pg 4...



$2\alpha+2\beta = 180^\circ$
$\alpha+\beta = 90^\circ$

EXCENTER



Exradius

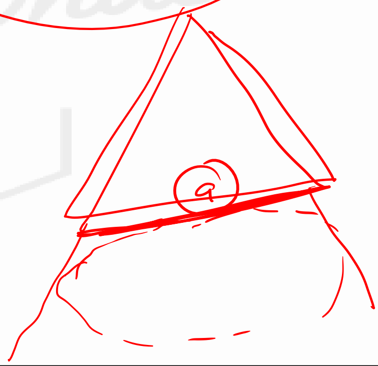
$X \; R = \frac{\Delta}{s-a}$

Area of Δ

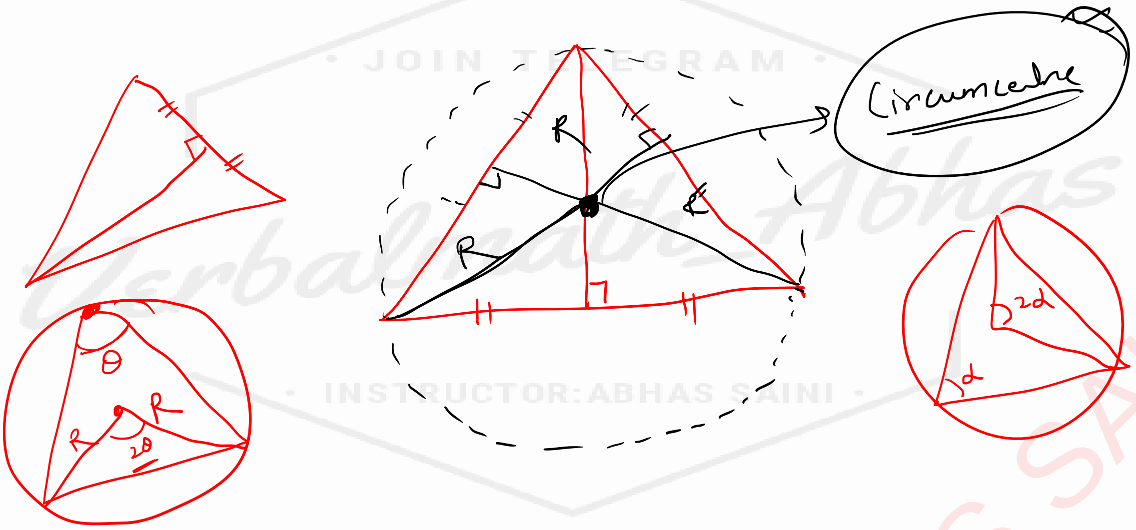
Semiperimeter of Δ

Circle for

$R = \frac{\Delta}{S-a}$



PERPENDICULAR BISECTORS



CIRCUM CIRCLE

PERP. BISECTOR

Radius = R

$$\boxed{R = \frac{abc}{4\Delta}}$$

Δ - Area

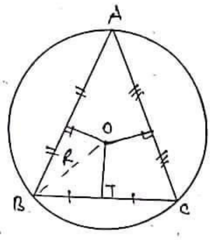
Area = $\sqrt{s(s-a)(s-b)(s-c)}$

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R}$$

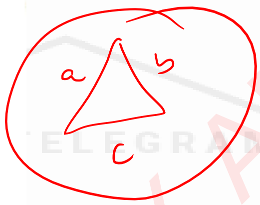
Sine law

circum

2R

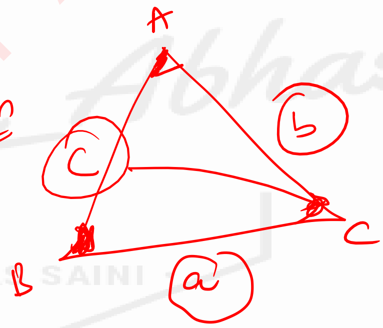


∠BOC = 2∠BAC



$$R = \frac{a \times b \times c}{4\Delta}$$

Δ → Area of Δ



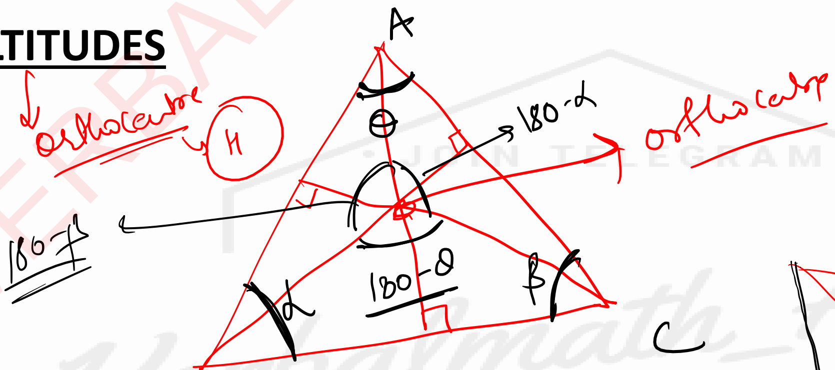
EQUILATERAL TRIANGLE $\boxed{R = a/\sqrt{3}}$

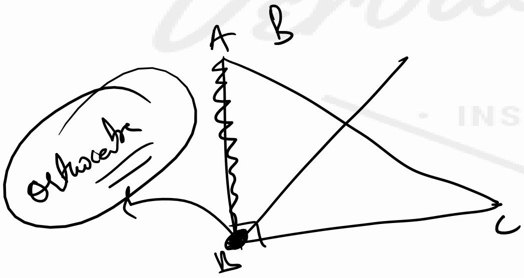
RIGHT TRIANGLE TRIANGLE $\boxed{R = HYP/2}$

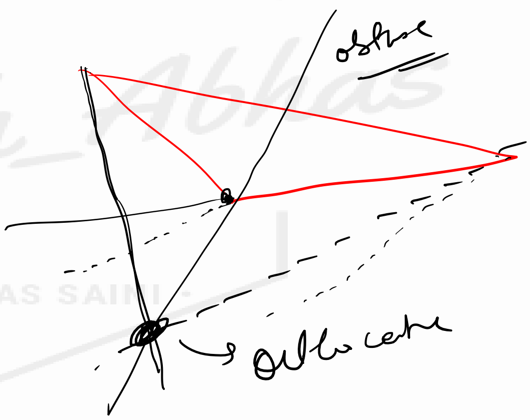
ALTITUDES

Orthocentre

H







ORTHOCENTER

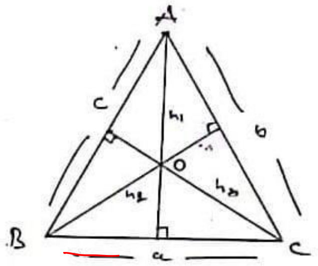
(PERPENDICULAR)

$\angle BOC = 180 - A$

$h_1 : h_2 : h_3 = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}$

$\frac{1}{r} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}$ ✗

inradius



$h_1 : h_2 : h_3 = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}$

Formulae $\frac{1}{\Delta} = \sqrt{\left(\frac{1}{h_1}+\frac{1}{h_2}+\frac{1}{h_3}\right)\left(\frac{1}{h_1}+\frac{1}{h_2}-\frac{1}{h_3}\right)\left(\frac{1}{h_1}-\frac{1}{h_2}+\frac{1}{h_3}\right)\left(-\frac{1}{h_1}+\frac{1}{h_2}+\frac{1}{h_3}\right)}$

When Altitudes are Given

$h_1 \quad h_2 \quad h_3$

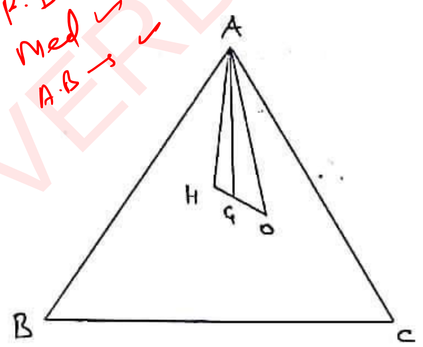
$$\frac{1}{\Delta} = \sqrt{\left(\frac{1}{h_1}+\frac{1}{h_2}+\frac{1}{h_3}\right)\left(\frac{1}{h_1}+\frac{1}{h_2}-\frac{1}{h_3}\right)\left(\frac{1}{h_1}-\frac{1}{h_2}+\frac{1}{h_3}\right)\left(-\frac{1}{h_1}+\frac{1}{h_2}+\frac{1}{h_3}\right)}$$

Alt

P.B

Med

A.B



Euler's line

$HG : GO = 2 : 1$

$$\boxed{\frac{HG}{GO} = \frac{2}{1}}$$

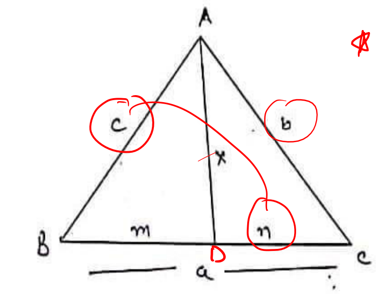
G : → centroid

H : → orthocentre

O → Circumcentre

I → incentre

STEWART'S THEOREM AND ITS EXTENSION



$c^2n + b^2m = a(x^2 + mn)$

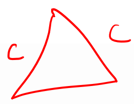
$$\boxed{c^2n + b^2m = (m+n)(x^2+mn)}$$

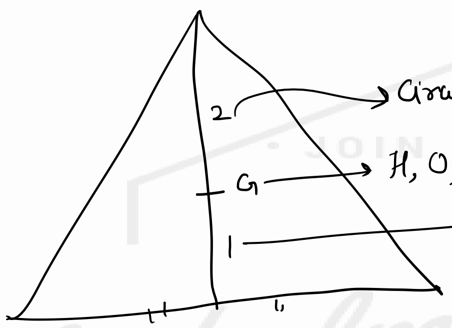
Apollonius (AD → median) [m=n]

$$c^2 + b^2 = 2(x^2+m^2)$$

Isosceles [c=b]

$$\boxed{c^2 = x^2+mn}$$

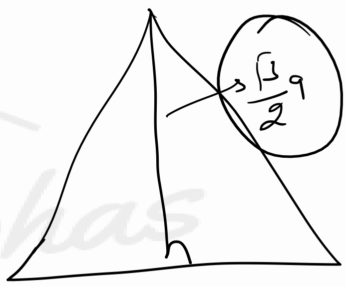




2 → Circumradius

G → H, O, I

1 → r → Inradius

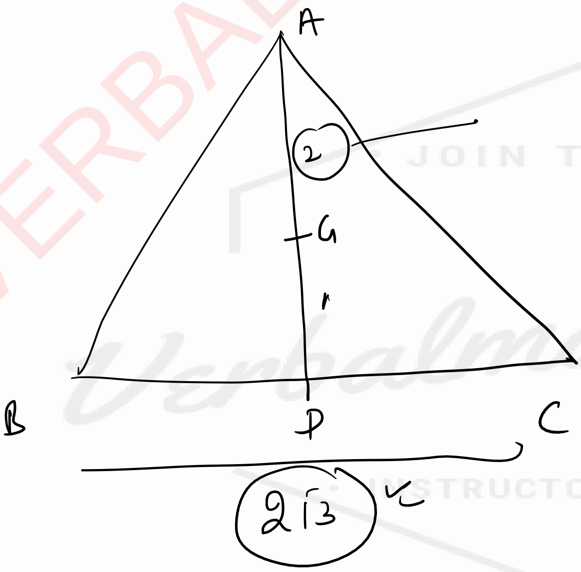


$\frac{\sqrt{3}}{2}a$

$3 \rightarrow \frac{\sqrt{3}}{2}a$

$2 \rightarrow \frac{a}{\sqrt{3}}$

$1 \rightarrow \frac{a}{2\sqrt{3}}$

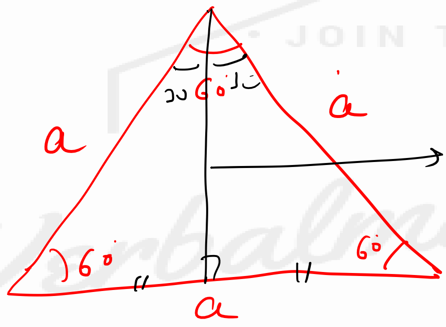


Special Analysis of equilateral

R → 4

Area $= \frac{\sqrt{3}}{4} \times 4\sqrt{3} \times 4\sqrt{3}$

EQUILATERAL TRIANGLE



$A = \frac{\sqrt{3}}{4} a^2$

A.B / Alt / median / P.B → I H G O

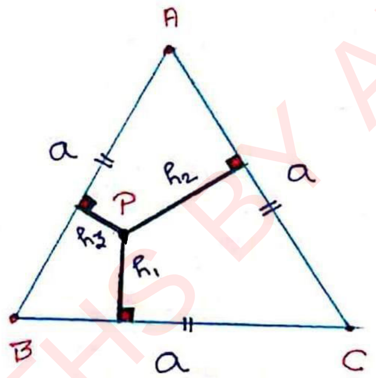
Style → Proof

Some Generalizations :—

If 'P' is any point inside the triangle and 'H' is height of the equilateral triangle then

$\boxed{H = h_1 + h_2 + h_3}$

$\frac{\sqrt{3}}{2}a$ ✓



1

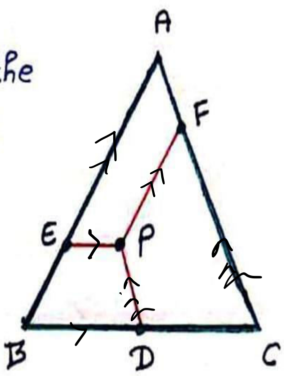
Generalisation – 2 :—

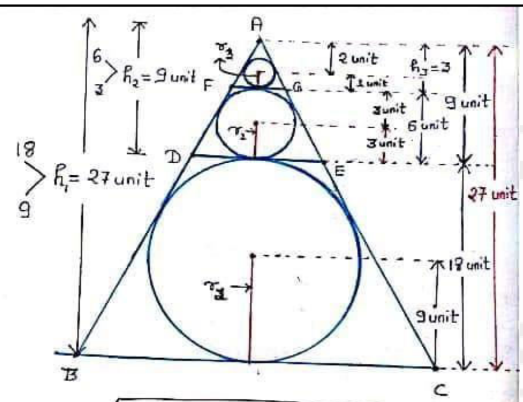
Suppose 'P' is a point inside the equilateral triangle such that

PE || BC , PF || AB &

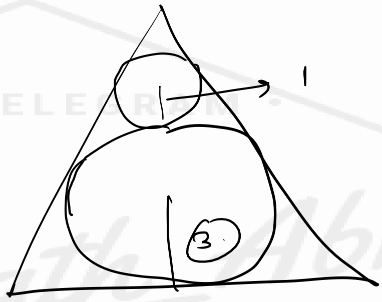
PD || AC then

$\boxed{PE + PF + PD = AB = AC = BC}$ ✓





$$\boxed{r_3 : r_2 : r_1 = 1 : 3 : 9}$$



1 : 3 : 9 : ......

* Similarly if there are infinite circles and there inradii are r_1, r_2, r_3 .... then they form a G.P. with common ratio $(1/3)$.

$r_1 : r_2 : r_3 : \ldots\ldots$

$= 1 : \frac{1}{3} : \frac{1}{9} : \frac{1}{27} : \ldots\ldots$

G.P

Area

G.P

⇒ Relation between Inradius, Circumradius and Ex-radius :—

ΔABC is an Equilateral triangle

if $xm = x$

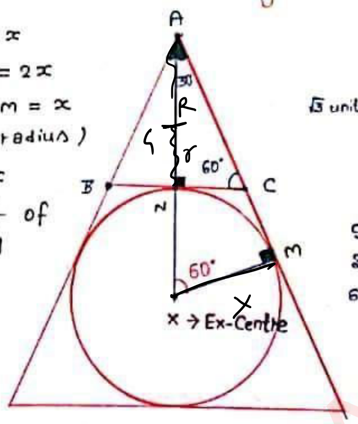
then $Ax = 2x$

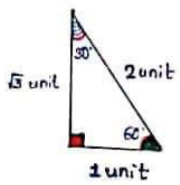
∵ $Nx = xm = x$ (Ex-radius)

⇒ $AN = x$

AN = height of equilateral triangle

then inradius $= \frac{x}{3}$





90° → 2 unit
30° → 1 unit
60° → $\sqrt{3}$ unit

Circumradius $= \frac{2x}{3}$, Ex-radius $= x$

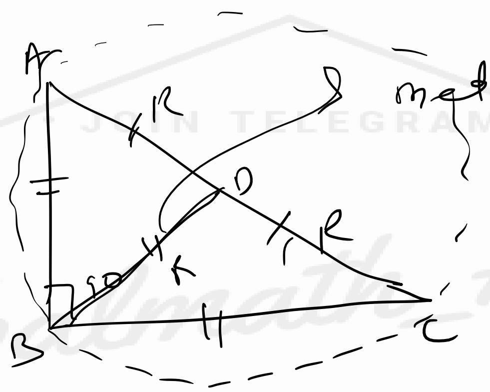
$$\boxed{\text{Inradius : Circumradius : Ex-radius} = 1 : 2 : 3}$$

Equilateral

$r : R : X$

$1 : 2 : 3$

Imp

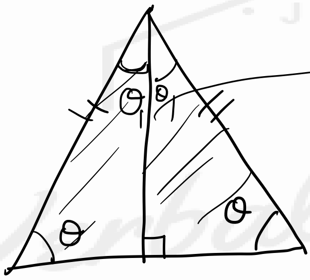


med / Alt / P.B

A.B

R

Isosceles Triangle



AB / H / P.B / med

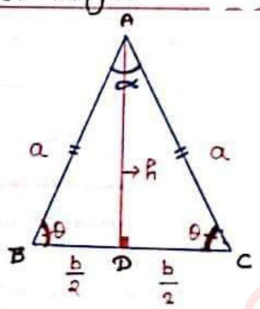
$\boxed{I, H, G, O}$ → Collinear

ISOSCELES TRIANGLE

Properties :—

Two sides are equal

Two angles are equal



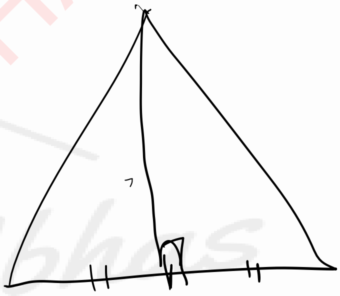
Height 'h' $= \sqrt{a^2 - \frac{b^2}{4}}$

$= \frac{\sqrt{4a^2 - b^2}}{2}$

Area $= \frac{1}{2} \times b \times h = \frac{b\sqrt{4a^2-b^2}}{4}$

Or AREA $= \frac{1}{2} a^2 \sin(\alpha)$

Note:— In an isosceles triangle Incentre, Circumcentre, orthocentre & centroid are colinear (i.e. lies on line AD)

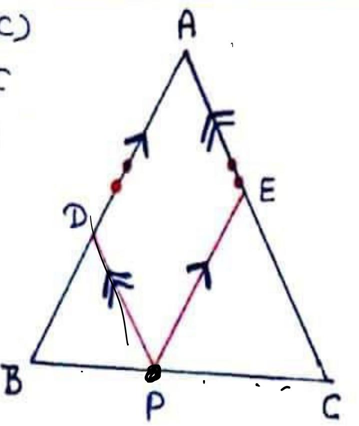


$\frac{1}{2} \times b \times h$

If ΔABC is isosceles (AB = AC) and P is any point on BC such that DP∥AC & EP∥AB

then, $\boxed{DP + EP = AB = AC}$

equal side



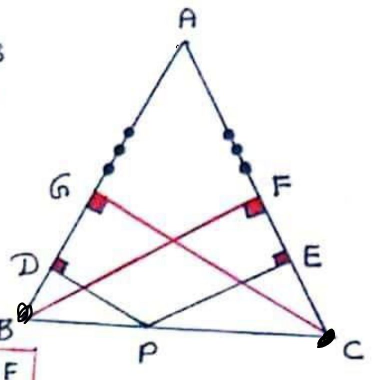
1

Some Generalizations :—

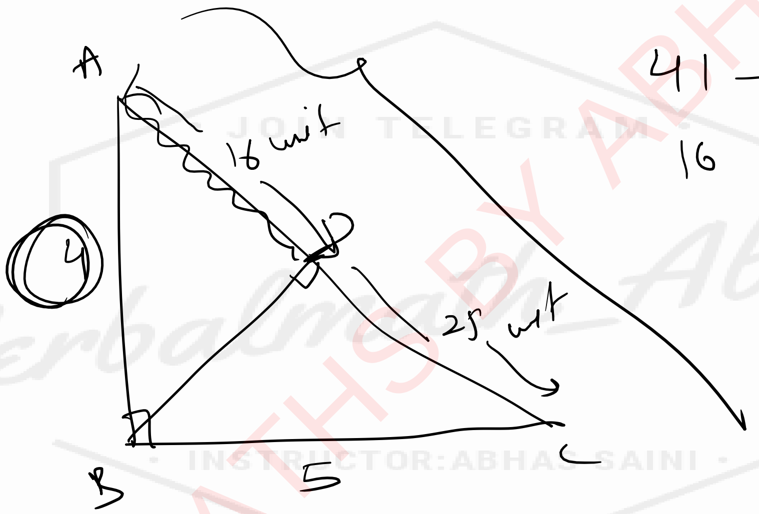
If ΔABC is an isosceles triangle ($AB = AC$) & P is an point on base BC such that

$PD \perp AB$, $PE \perp AC$ & $CG \perp AB$, $BF \perp AC$

then $\boxed{DP + PE = GC = BF}$



$$\begin{cases} DP + PE = GC \\ DP + PE = BF \end{cases}$$



$41 \longrightarrow 7.5$

$16 \longrightarrow 4$

RIGHT TRIANGLE

Right Angled Triangle

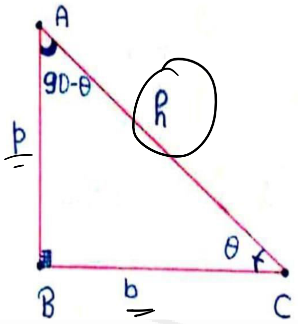
p → perpendicular length

b → base length

h → hypotenuse

Pythagorus Theorem

$$\boxed{p^2 + b^2 = h^2}$$



Tool – 1 :– $BD = \dfrac{AB \times BC}{AC}$ similarity

Tool – 2 :– $AB^2 = AD \times AC$

Tool – 3 :– $BC^2 = CD \times AC$

Tool – 4 :– $\dfrac{AB}{BC} = \sqrt{\dfrac{AD}{CD}}$

Tool – 5 :– $BD^2 = AD \times CD$

Tool – 6 :– $\dfrac{1}{BD^2} = \dfrac{1}{AB^2} + \dfrac{1}{BC^2}$

Tool – 7 :– $BC = \dfrac{AB \times BD}{AD}$

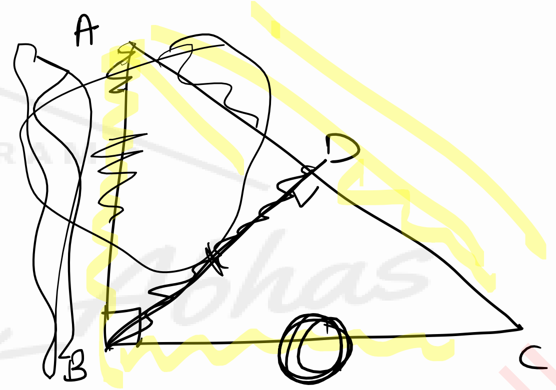
Tool – 8 :– $AB = \dfrac{BC \times BD}{AD}$

all

extra

Similarity

Hybrid



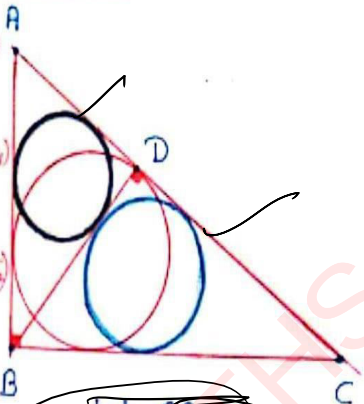
generalization – I :—

Red circle (inradius = r)
⇒ Incircle of $\triangle ABC$

Blue circle (inradius = r_1)
⇒ Incircle of $\triangle BDC$

Black Circle (inradius = r_2)
⇒ Incircle of $\triangle ADC$

$$\boxed{r_1^2 + r_2^2 = r^2}$$



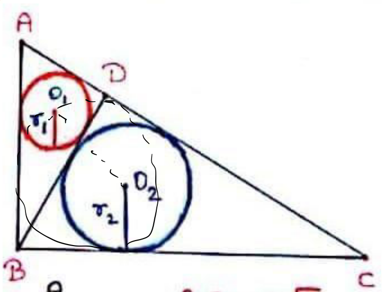
~~Let AB = a~~
~~BC = b~~

Generalization – 2 :—

Red circle
⇒ Incircle of $\triangle ADB$

Blue circle
⇒ Incircle of $\triangle BDC$

Black circle
⇒ Incircle of $\triangle ABC$

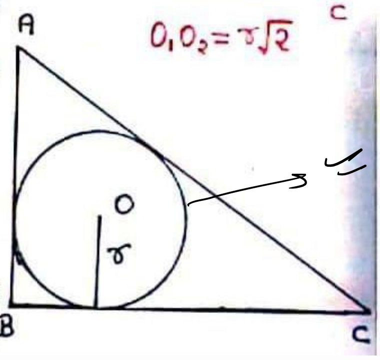


$O_1O_2 = r\sqrt{2}$

$O_1 \rightarrow$ Centre of Red circle
$O_2 \rightarrow$,, ,, Blue ,,
$O \rightarrow$,, ,, Black ,,

$$\boxed{O_1O_2 = \sqrt{2}\, r}$$

r = inradius of black circle



$\sqrt{2}\, r$

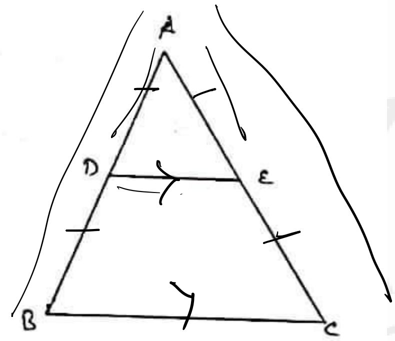
SIMILARITY

Thales

DE || BC

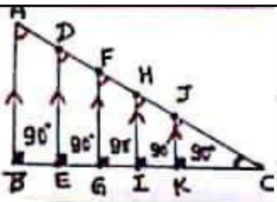
$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$$

$$\boxed{\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}}$$

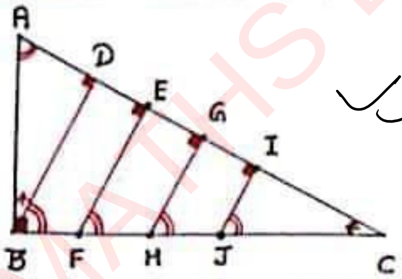


$$\boxed{\frac{DB}{AB} = \frac{EC}{AC}}$$

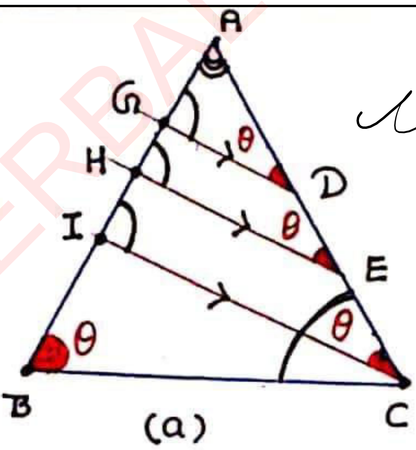
ABC is a right angle triangle at B
If AB || DE || FG || HI || JK
then $\Delta ABC \sim \Delta DEC \sim \Delta FGC \sim \Delta HIC \sim \Delta JKC$

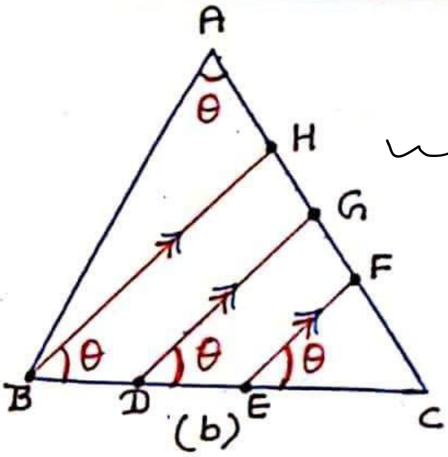


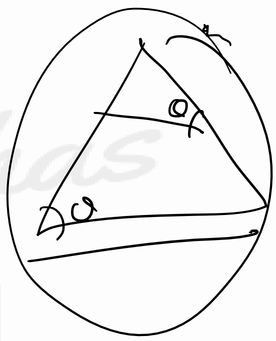
ΔABC is a right-angle triangle at B. if
BD ⊥ AC &
BD || EF || GH || IJ
then ;



$\Delta ABC \sim \Delta BDC \sim \Delta FEC \sim \Delta HGC \sim \Delta JIC \sim \Delta ADB$



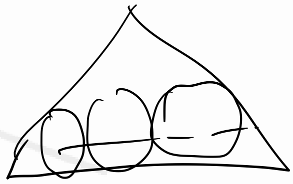




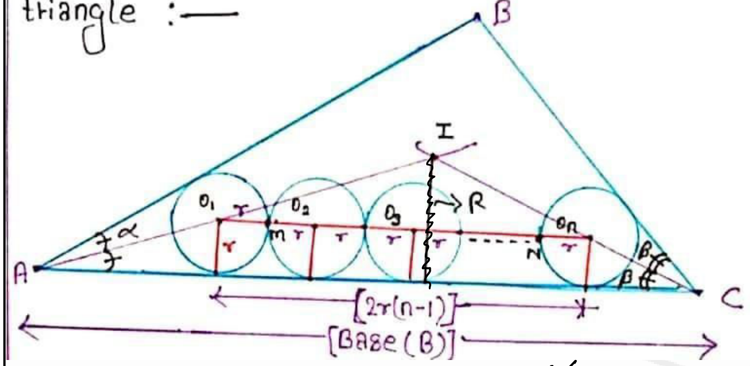
for figure (a) ; $\Delta ABC \sim \Delta ADG \sim \Delta AEH \sim \Delta ACI$

for figure (b) ; $\Delta ABC \sim \Delta BHC \sim \Delta DGC \sim \Delta EFC$

SOME IMPORTANT RESULTS



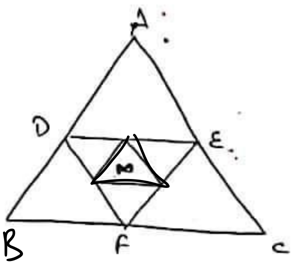
If there are 'n' circles on the base of any triangle :—



n → no. of circles

R → Inradius

$$\boxed{r = \frac{BR}{2R(n-1) + B}}$$



(Midpoints D-E-F)

Sum of Areas of triangle
$= \frac{4}{3}$ (Area of Bigger)

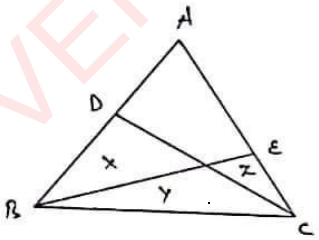
G.P

$\frac{4}{3}$ (Area of Bigger △)

Sum of Perimeter of all triangle
= 2 (Perimeter of bigger)

2 (Perimeter of bigger)

LADDER THEOREM



$$\frac{1}{\Delta} + \frac{1}{y} = \frac{1}{x+y} + \frac{1}{y+z}$$

Area

MPG

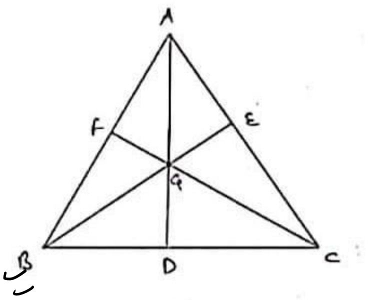
CEVIANS

$$\frac{AF}{BF} \times \frac{BD}{DC} \times \frac{CE}{EA} = 1$$

$$\frac{GD}{AD} + \frac{GE}{BE} + \frac{GF}{CF} = 1$$

$$\frac{AG}{AD} + \frac{BG}{BE} + \frac{CG}{CF} = 2$$

$$\frac{AF}{FB} + \frac{AE}{EC} = \frac{AG}{GD}$$

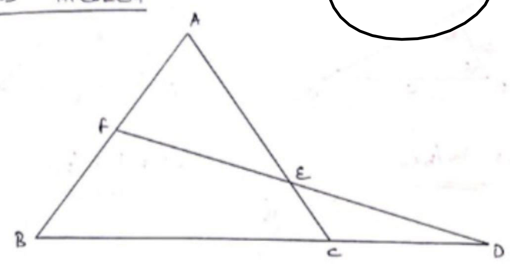


$$\frac{AF}{BF} \times \frac{BD}{CD} \times \frac{CE}{EA} = 1$$

$$\frac{\sin BAD}{\sin CAD} \times \frac{\sin ACF}{\sin BCF} \times \frac{\sin CBE}{\sin EBA} = 1$$ {Sine Rule of Cevians}

MENELAUS THEOREM

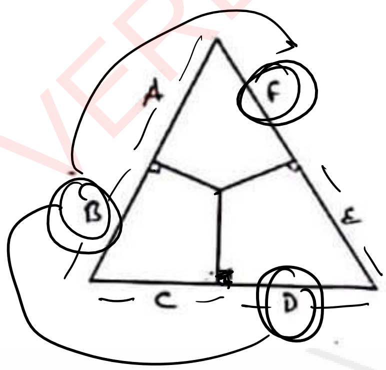
MPS



$$\frac{BF}{FA} \times \frac{AE}{EC} \times \frac{CD}{BD} = 1$$

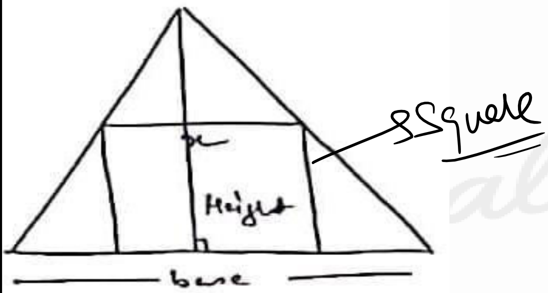
$$\frac{BF}{FA} \times \frac{AE}{EC} \times \frac{CD}{BD} = 1$$

CARNOT'S THEOREM

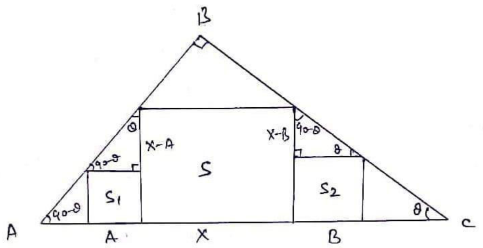


$$A^2 + C^2 + E^2 = B^2 + D^2 + F^2$$

$$\boxed{A^2 + C^2 + E^2 = B^2 + D^2 + F^2}$$



$$\frac{1}{\text{Side}} = \frac{1}{\text{base}} + \frac{1}{\text{height}}$$



Proof :-

$$\frac{x-A}{A} = \frac{B}{x-B}$$

$$\cancel{AB} = x^2 - Bx - Ax + \cancel{AB}$$

$$x^2 = Ax + Bx$$

$$\boxed{x = A+B}$$

$$\sqrt{x^2} = \sqrt{A^2} + \sqrt{B^2}$$

$$\boxed{\sqrt{S} = \sqrt{S_1} + \sqrt{S_2}}$$

$$\boxed{\sqrt{S} = \sqrt{S_1} + \sqrt{S_2}}$$

POLYGON

: Polygon :

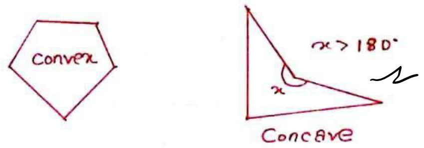
Smallest polygon :— Triangle (△)

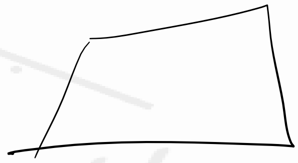
Largest polygon :— Circle (○)

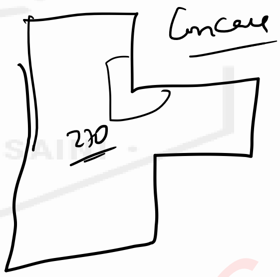
Types of polygon :—

Convex: — All angles are less than 180°

Concave: — At least one angle is greater than 180°.







Regular polygons :— Equal Sides & Angles

Irregular polygons :— Unequal sides & Angles

Name of some polygons :—

3 - Sided → Triangle △

4 - Sided → Quadrilateral ▭

5 - Sided → Pentagon ⬠

6 - Sided → Hexagon ⬡

7 - Sided → Heptagon

8 - Sided → Octagon

9 - Sided → Nonagon

10 - Sided → Decagon

Perimeter of regular polygon = (no. of sides × length of one side)

Sum of all internal angles of a polygon

$= (n-2) \times 180°$

where $n =$ no. of sides.

Sum of external angles of polygon = 360

$n \times \theta = 360$

θ

$\boxed{\frac{360}{\theta} = n}$

No. of diagonals in a polygon

$$\boxed{= \frac{n(n-3)}{2}} = {}^{n}C_{2} - n$$

n → no. of sides

Area of a regular polygon

$$= \frac{na^2}{4} \cot\left(\frac{\pi}{n}\right)$$

where
n = no. of sides

Proof :-

OM = Inradius = $\frac{a}{2}\cot\left(\frac{\pi}{n}\right)$

OP = Circumradius = $\frac{a}{2}\operatorname{cosec}\left(\frac{\pi}{n}\right)$

Relation between interior & Exterior :-

Polygon

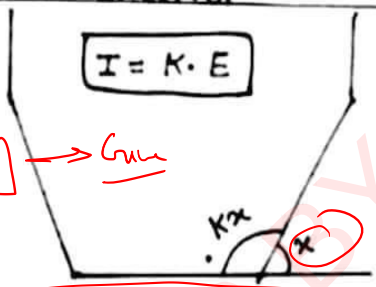
for $n \geq 5$

if Exterior angle = x

& interior angle = kx → Given

then $\boxed{n = 2k+2}$

$\boxed{I = K \cdot E}$



proof :- $n \times (kx) = (n-2)\times 180$ & $x = \frac{360}{n}$

$\Rightarrow n \times k \times \frac{360^\circ}{n} = (n-2)\times 180^\circ \Rightarrow (n-2) = 2k$

$\Rightarrow \boxed{n = 2k+2}$

$n = 2 \times k + 2$

Inter = 3E

$3 \times 2 + 2 = 8$

Quadrilateral

→ 4 Sides , $n = 4$

→ 4 Angles

→ Sum of Angles of Quadrilateral $= (n-2)\times 180 = (4-2)\times 180$
$= 2 \times 180 = 360$

→ No. of Diagonals $= \frac{n\times(n-3)}{2} = \frac{4\times 1}{2} = 2$

★ Types of Quadrilaterals :—

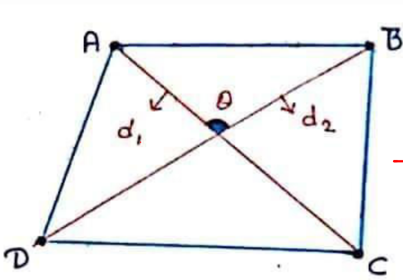
(1) Parallelogram → Rhombus, Rectangle → Square

(2) Trapezium (Trapezoid) → Isosceles Trapezium

(3) Kites

General

$$\boxed{\text{Area} = \frac{1}{2} \times d_1 \times d_2 \times \sin(\theta)}$$

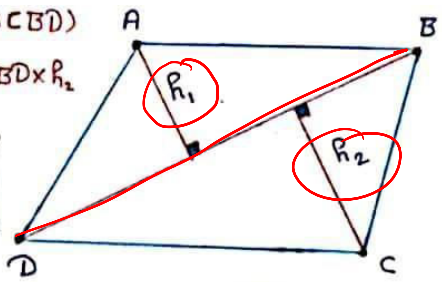


Area

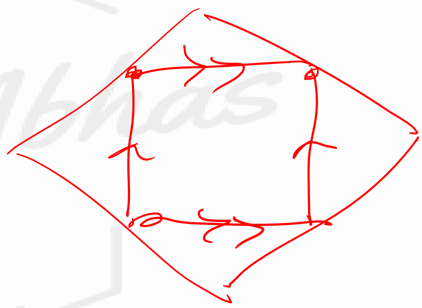
) Area = Ar($\triangle ABD$) + Ar($\triangle CBD$)

$$= \frac{1}{2} \times BD \times h_1 + \frac{1}{2} \times BD \times h_2$$

$$\boxed{\text{Area} = \frac{1}{2} \times BD \times (h_1 + h_2)}$$

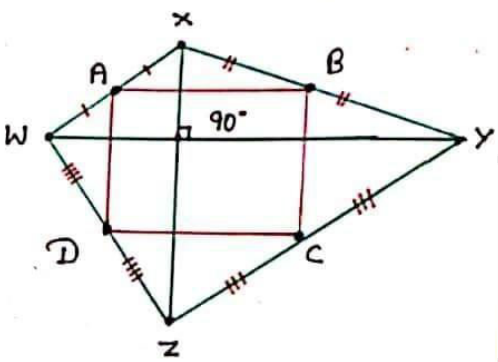


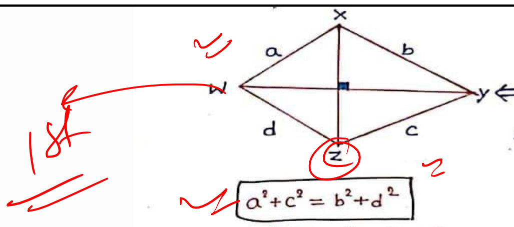
Lines joining the mid-points of the adjacent Sides of a quadrilateral (whose diagonals intersect at 90°) form a rectangle.

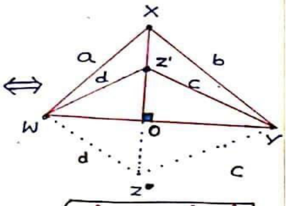


Lines joining the mid-points of the adjacent Sides of a quadrilateral form a parallelogram.

Lines joining the mid-points of the adjacent Sides of a quadrilateral (whose diagonals intersect at 90°) form a rectangle.







$a^2+c^2=b^2+d^2$

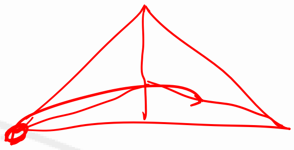
$a^2+c^2=b^2+d^2$

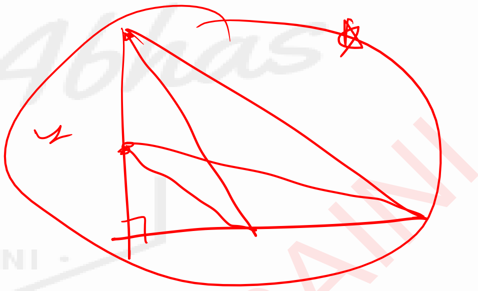
OZ = OZ' (Construction)

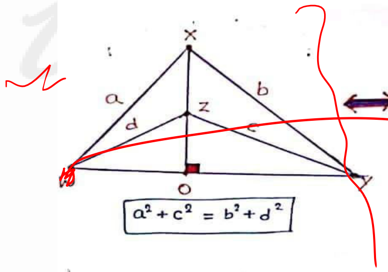
OW = OW (Common)

$\angle WOZ' = \angle WOZ = 90^\circ \Rightarrow WZ' = WZ = d$

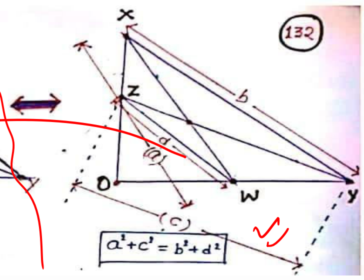
(CPCT) & $Z'Y = ZY = c$



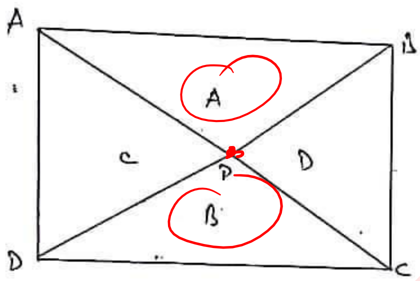




$a^2+c^2=b^2+d^2$

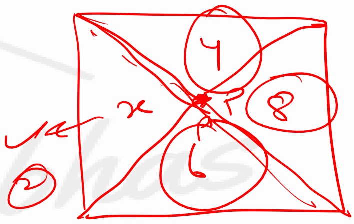


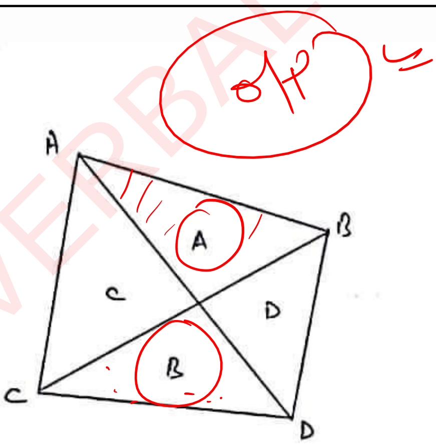
$a^2+c^2=b^2+d^2$



P - Any Point inside Rectangle / Square.

$\boxed{A+B = C+D}$





ANY QUADRILATERAL

A, B, C, D are Areas

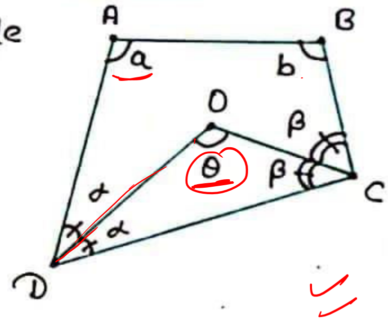
$\boxed{AB = CD}$

$\boxed{A \times B = C \times D}$

Generalizations :—

If DO & CO are angle bisectors then

$$\boxed{\theta = \frac{a+b}{2}}$$



2018 Mains

Gen - 2

from previous Generalization

$x = \frac{2\delta + 2\omega}{2}$

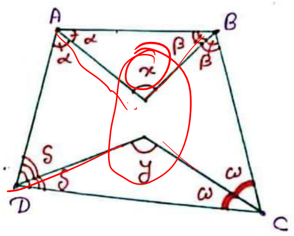
$\Rightarrow \quad x = \delta + \omega$

Similarily $\quad y = \alpha + \beta$

$\because \quad 2(\alpha+\beta+\delta+\omega) = 2\pi = 360^\circ$

$\Rightarrow \quad x+y = \alpha+\beta+\delta+\omega = \pi = 180^\circ$

$\therefore \quad \boxed{x+y = \pi = 180^\circ}$



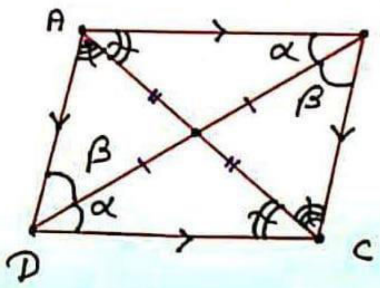
$\boxed{x+y=180^\circ}$

—: PARALLELOGRAM :—

⇒ AB ∥ CD & BC ∥ AD

⇒ opposite angles of a parallelogram are equal.

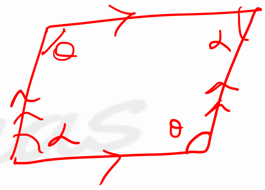
⇒ opposite sides are equal.

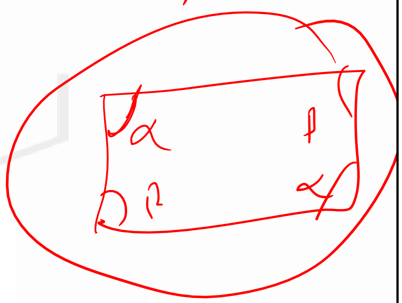


⇒ Diagonals bisects each other.

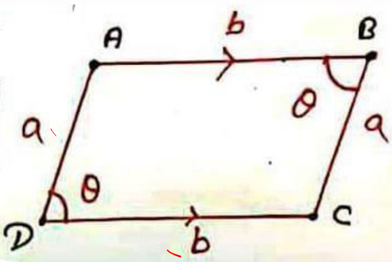
Vice-Versa (Maharaza) :— (i) If each pair of opposite angles are equal then it is a parallelogram.

(ii) If diagonals of a quadrilateral bisects each other then it is a parallelogram.





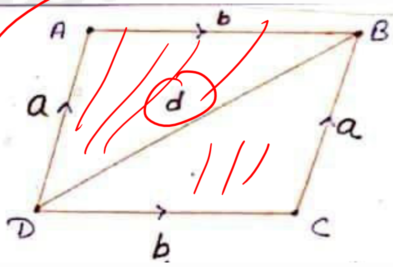
$$\boxed{\text{Area} = \frac{1}{2} \times a \times b \times \sin(\theta)}$$



Area of Quad [ABCD]

$$= 2\sqrt{s(s-a)(s-b)(s-d)}$$

where $s = \frac{a+b+d}{2}$

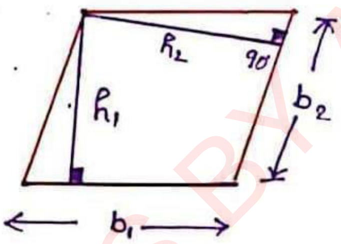


Area :—

Area of parallelogram = Base × Height

$\boxed{\text{Area} = b_1 \times h_1} = b_2 \times h_2$

Area = Height × Base



⇒ Height (h) × Base (b)

= Constant (A)

⇒ $h \propto \frac{1}{b}$ ⇒ $\boxed{\text{height} \propto \frac{1}{\text{Base}}}$

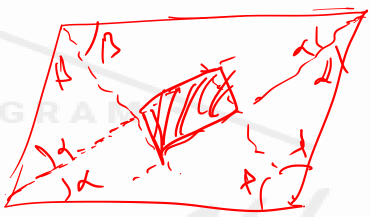
⇒ Height is inversly proportional to Base. //

$h \propto \frac{1}{b}$

$\frac{h_1}{h_2} = \frac{b_2}{b_1}$

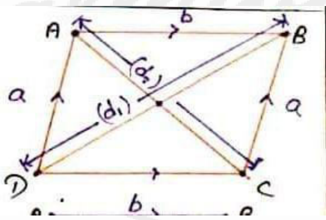
Gen – I

(i) Bisectors of angles of a parallelogram form a Rectangle. //



Gen – II :—

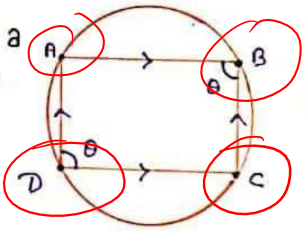
If ABCD is a parallelogram then $\boxed{2(a^2+b^2) = d_1^2 + d_2^2}$



$2(a^2+b^2) = d_1^2+d_2^2$

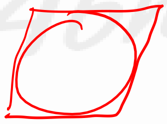
Gen - III :-

Parallelogram inscribed in a Circle is a rectangle.//

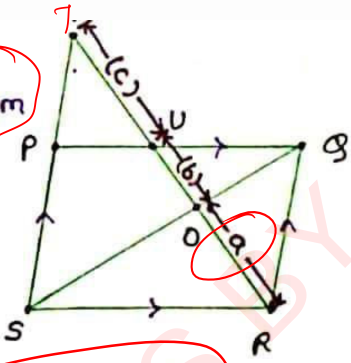


Gen - IV :-

Parallelogram that circumscribes a circle is a rhombus. //



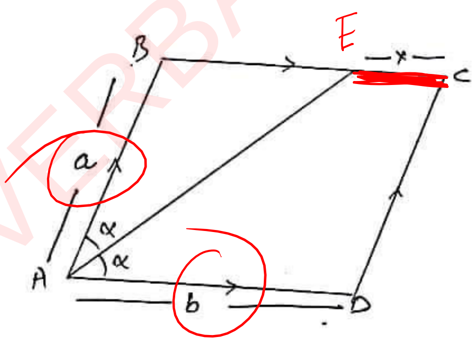
If PQRS is a parallelogram then $\boxed{a^2 = b(b+c)}$



$OR^2 = UO(UO+UT)$

Similarity

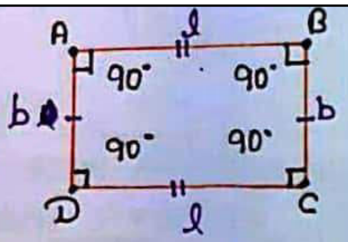
1 — 1.5



$\boxed{x = b - a}$

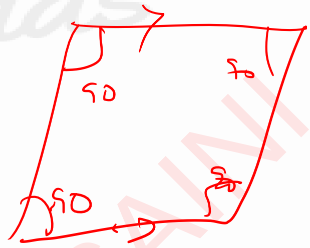
AE → Angle Bisector

[Rectangle]

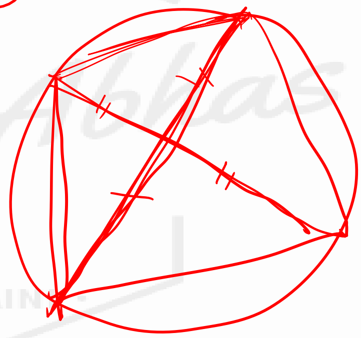


⇒ All Angles are 90°
⇒ Opposite sides are equal
⇒ Diagonals are equal & bisect each other
⇒ Area = $l \times b$
⇒ Perimeter = $2(l+b)$

Rectangle is a parallelogram which one angle is 90°.



Note(iii) :— If two chords of a circle bisect each other, then they are the diameters of the circle & if we join all the four points then they will form a Rectangle.//



If parallelogram is cyclic then it is a rectangle. //

(i) For equal Perimeter Area of rectangle is always greater than Area of parallelogram.//

$P_{rec} = P_{\|gm}$

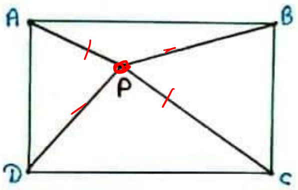
Note :— If one side & Area of a rectangle and a parallelogram are equal then in this case —

Perimeter of parallelogram > Perimeter of Rectangle. //

British Flag Theorem :—

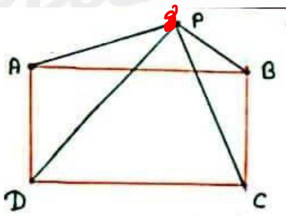
If 'P' is any point inside the Rectangle then

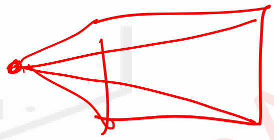
$$\boxed{PA^2 + PC^2 = PB^2 + PD^2}$$



If 'P' is any point Outside the Rectangle then also

$$\boxed{PA^2 + PC^2 = PB^2 + PD^2}$$





RHOMBUS

⇒ A parallelogram in which all sides are equal

⇒ A parallelogram in which diagonals are perpendicular

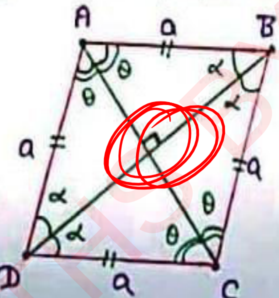
$4(\theta+\alpha) = 2\pi \Rightarrow \theta+\alpha = \frac{\pi}{2}$

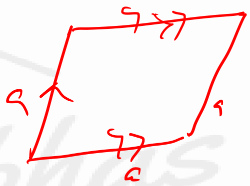
⇒ diagonals are perpendicular.//

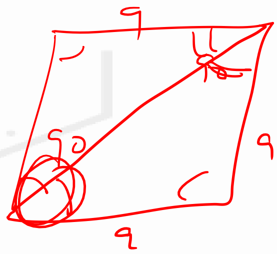
⇒ Diagonals bisects each other.

⇒ Diagonals of rhombus bisect the Vertex angle (interior angle)

⇒ If one angle of a rhombus is 90° then it is a square.







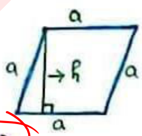
Perimeter of Rhombus = 4a

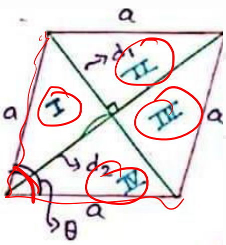
Area $= \frac{1}{2} \times d_1 \times d_2 \times \sin(90°)$

$= \frac{d_1 d_2}{2}$

or

Area $= a^2 \sin(\theta)$





Or, Area = 4 × I = 4 × II = 4 × III = 4 × IV

Or, Area = a × h

4a

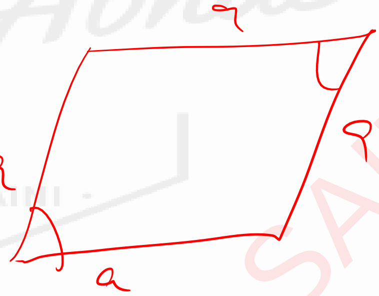
$\frac{1}{2} \times d_1 \times d_2$

a^2

Vice-versa :— If in a parallelogram a diagonal bisects an interior angle then it is rhombus.//

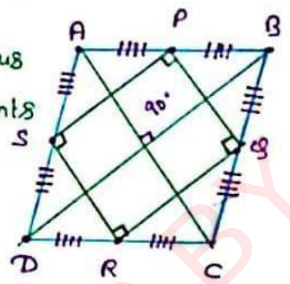
or

In in //-gm in which at least two consecutive sides are equal in length then it is a rhombus.



—: Generalizations :—

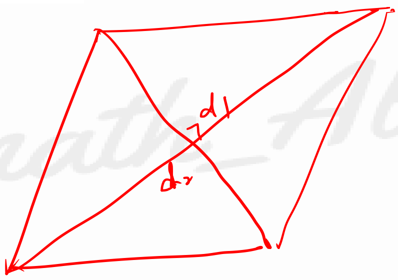
Gen - I :— ABCD is a Rhombus & P, Q, R, S are the mid-points of sides AB, BC, CD, DA then Quad PQRS is a rectangle.//



Gen - II :—

Prove that the sum of squares of the sides of a rhombus is equal to sum of the squares of its diagonals.

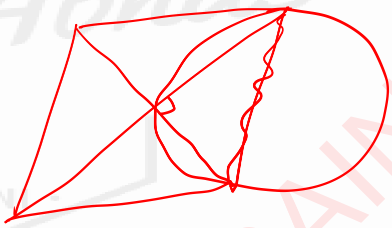
$$\boxed{d_1^2 + d_2^2 = 4a^2}$$



Gen – III :— Parallelogram circumscribing a circle is a rhombus.

(Already proved ; check at page no. –140)

Gen – IV :— Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.



Gen :— V :—

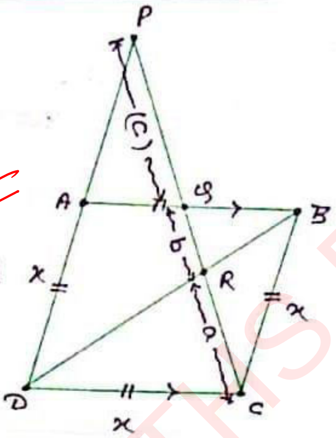
ABCD is a rhombus

AB = BC = CD = DA = x

$$\boxed{a^2 = b(b+c)}$$

Check at page no.– 141

$PQ = c$, $QR = b$, $RC = a$



SQUARE (वर्ग)

:— A Rhombus which all angles are 90°

Properties :—

(i) All Sides are equal

(ii) Each angle = 90°

(iii) Diagonals are equal & bisects each other at 90°

(iv) If AB = BC = CD = DA = a

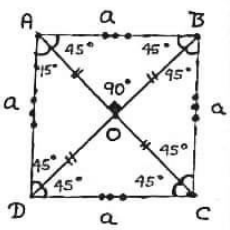
then AC = BD = $a\sqrt{2}$

AO = BO = CO = DO = $\frac{a}{\sqrt{2}}$

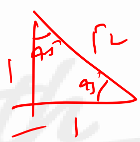
(v) Area = a^2 , Perimeter = $4a$

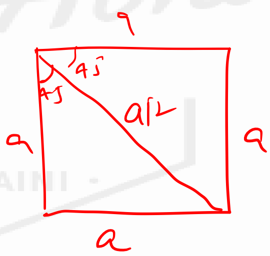
Diagonals of square bisects vertex angles

Note :- If In a Quadrilateral Diagonals are equal & bisects each other at 90° ⟹ It is a Square



$d_1 = d_2$





Some Generalizations :-

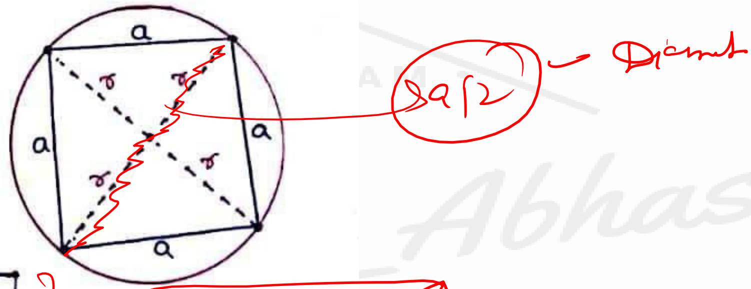
(1) Circumradius :-

$2r$ = Diameter

= Diagonal of square

$\Rightarrow \quad 2r = \sqrt{2}a$

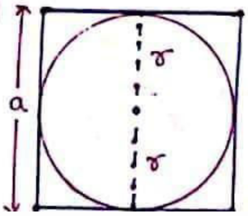
$\boxed{a = r\sqrt{2}}$



(2) Inradius :-

$a = 2r$

$\boxed{r = a/2}$



$a = 2r = D$

(i) $BD = \dfrac{AB \times BC}{AC}$

(ii) $AB^2 = AD \times AC$

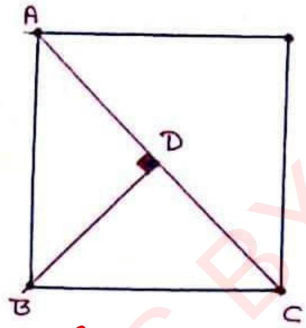
(iii) $BC^2 = CD \times AC$

(iv) $\dfrac{AB}{BC} = \sqrt{\dfrac{AD}{CD}}$

(v) $BD^2 = AD \times CD$

(vi) $\dfrac{1}{BD^2} = \dfrac{1}{BC^2} + \dfrac{1}{AB^2}$

(vii) $BC = \dfrac{AB \times BD}{AD}$ (viii) $AB = \dfrac{BC \times BD}{CD}$



$\boxed{a^2 = b(b+c)}$

$\because \dfrac{a}{b} = \dfrac{2}{1} \Rightarrow a = 2b$

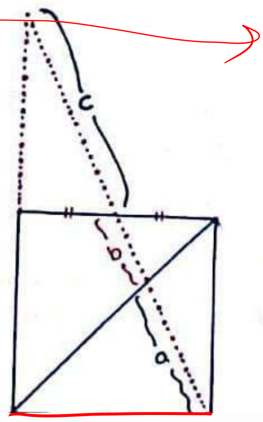
and $b + a = c$

$\Rightarrow c = 3b = \frac{3a}{2}$

$a^2 = b(b+3b)$

$= 4b^2 = (2b)^2$

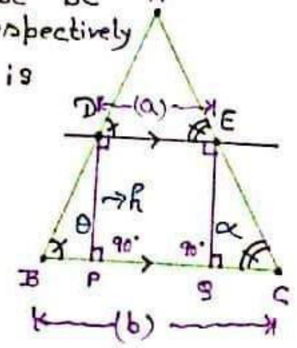
$= a^2$



$\boxed{a^2 = b(b+c)}$

TRAPEZIUM

Suppose a line parallel to base 'BC' cut AB & AC at D & E respectively then Quadrilateral 'DECB' is called 'Trapezium'



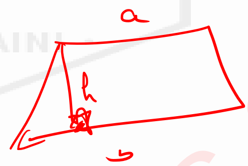
one pair of opposite sides are parallel. (DE ∥ BC)

& Two other sides are non-parallel. (DB ∦ EC)

In an isosceles trapezium $\boxed{\theta = \alpha}$ & $\boxed{DB = EC}$

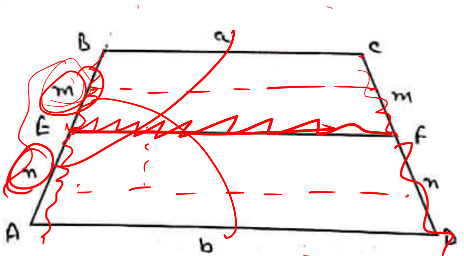
If we draw DP ⊥ BC & EQ ⊥ BC, then Quad. DEQP is always rectangle.

Area of Trapezium = $\frac{1}{2} \times h \times (a+b)$



$\frac{1}{2} \times (a+b) \times h$

(+)



$$\frac{AE}{EB} = \frac{CF}{FD} = \frac{m}{n}$$

Alligation

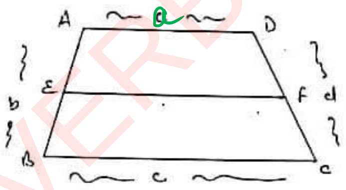
weighted average

$$EF = \frac{mb + na}{m+n}$$

alligation

$$\frac{3 \times 10 + 4 \times 7}{7} = \frac{30 + 28}{7} = \frac{58}{7}$$

(+) Trapezium divided into 2 - Equal Perimeter



If $\frac{AE}{EB} = \frac{DF}{FC} = x$

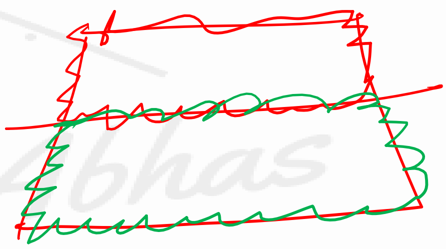
find x.

EF bisect it into 2 equal Perimeter.

Direct Result.

$$x = \frac{S-a}{S-c}$$

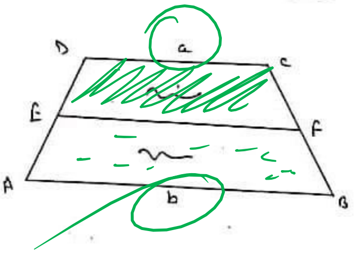
here $S = \frac{a+b+c+d}{2}$



$$S = \frac{a+b+c+d}{2}$$

$$x = \frac{S-a}{S-c}$$

Trapezium divided in two Equal Areas.



Ar. AEFB = Ar. DEFC

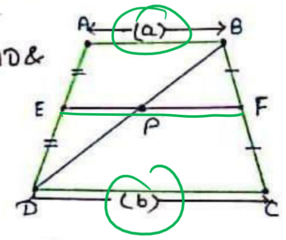
$$EF = \sqrt{\frac{a^2+b^2}{2}}$$

$$EF = \sqrt{\frac{a^2+b^2}{2}}$$

Generalization :— (1)

If E&F are mid-points of AD & BC respectively then

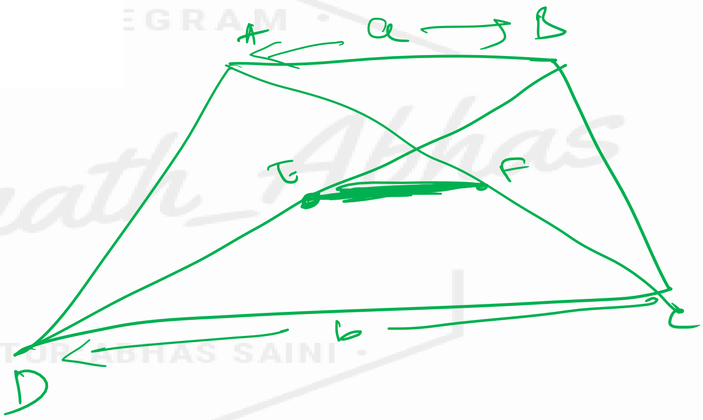
$$\boxed{EF = \left(\frac{a+b}{2}\right)}$$



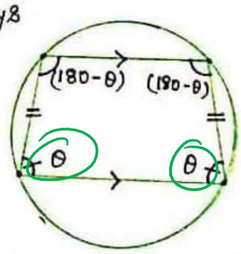
ABCD is a trapezium & (AB||CD)

E and F are mid-points of diagonals BD & AC respectively then

$$\boxed{EF = \frac{(b-a)}{2}}$$



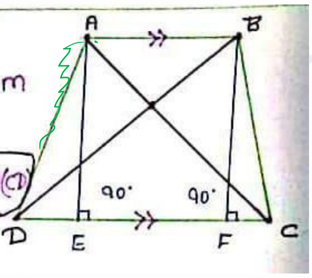
Cyclic Trapezium is always Isosceles Trapezium & vice-versa. //



Gen - IV :-

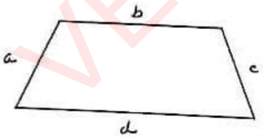
If ABCD is a Trapezium then,

$$\boxed{AC^2 + BD^2 = AD^2 + BC^2 + 2(AB)(CD)}$$



$$\underline{AD^2 + BC^2 + 2AB\,CD}$$

Area of trapezium.



$$s = \left(\frac{a+b+c+d}{2}\right)$$

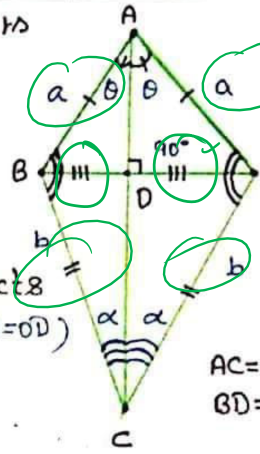
$$\text{Area} = \frac{D+B}{D-B}\sqrt{(s-B)(s-D)(s-B-A)(s-B-C)}$$

$$s = \frac{a+b+c+d}{2}$$

$$\boxed{\text{Area} = \frac{d+b}{d-b}\sqrt{(s-b)(s-d)(s-b-a)(s-b-c)}}$$

Kite

⇒ A kite has two distinct pairs of equal adjacent sides.
(here AB = AD & CB = CD)
⇒ Diagonals [redacted] cut at 90°
(AC ⊥ BD)
and one of the diagonals bisects the other (AC bisects BD; BO = OD)

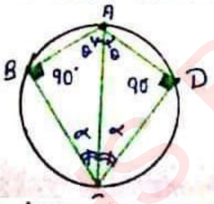


Tools

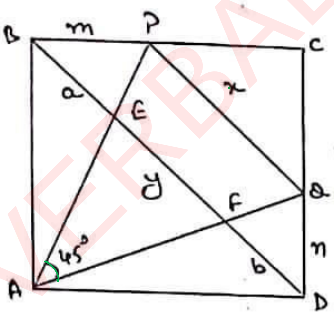
$AC = d_1$
$BD = d_2$

⇒ Perimeter = 2(a+b)
⇒ Area = $\frac{1}{2} \times AC \times BD = \frac{1}{2} \times d_1 \times d_2$
Or, Area = $ab\sin(\theta)$ [here $\theta = \angle ABC = \angle ADC$]
⇒ If $\angle ABC = \angle ADC = 90°$ the kite ABCD is a cyclic quadrilateral
⇒ AC = diameter of the circle

$2(\theta+\alpha) = 180° \Rightarrow \theta + \alpha = 90°$



If a = b (i.e. AB = BC) then kite become rhombus



ABCD – SQUARE
APQ – 45° triangle

$x = m + n$

$x^2 = 2(a^2 + b^2)$

$y^2 = a^2 + b^2$

Ignore

circle

CIRCLES

Angle = arc/radius

Area of Sector = $\frac{\theta}{360}(\pi r^2)$

$a = 2r \sin\theta/2$

circumference = $2\pi r$

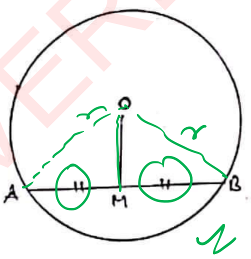
Sector = $\frac{\theta}{360}(2\pi r) + 2r$

$\theta = \frac{arc}{radius}$

$l = r\theta$

$\frac{\theta}{360} \times \pi r^2$

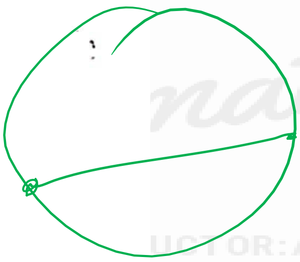
CHORD

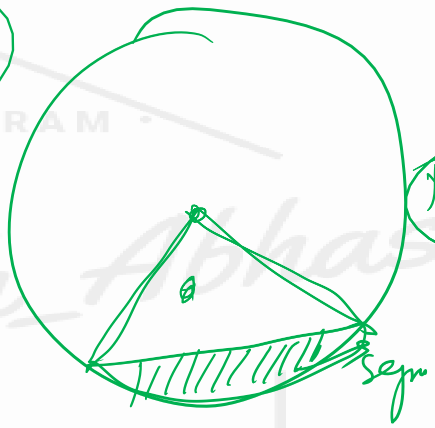


$OM \perp AB$

$\boxed{AM = MB}$

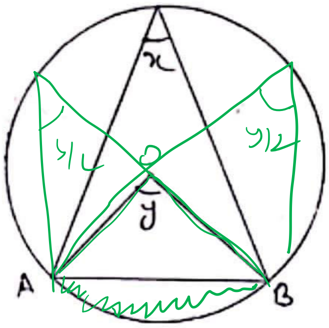
diameter

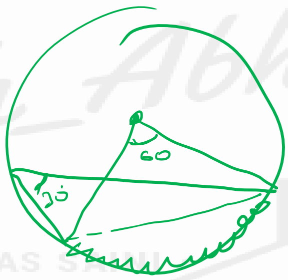




Mensuration

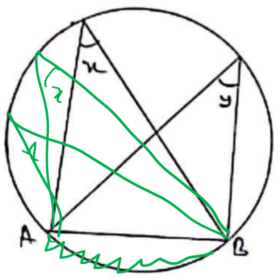
(i)





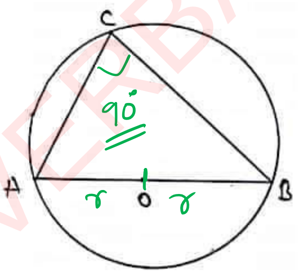
$\boxed{y = 2x}$

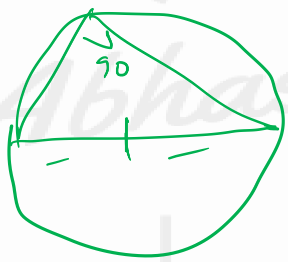
(ii)



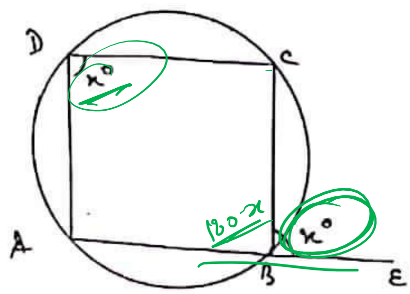
Angle subtended by same chord are Equal

$\boxed{x = y}$

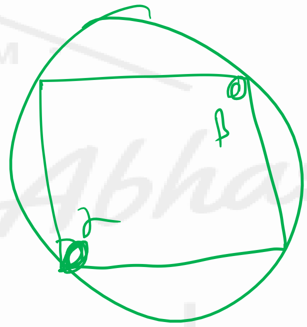




$\boxed{\angle ACB = 90^\circ}$

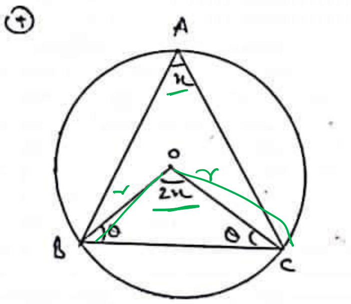


$$\boxed{\angle CBE = \angle ADC}$$



$\alpha + \beta = 180$

④



$\angle BAC + \angle OBC = 90^\circ$

$\angle BAC + \angle OCB = 90^\circ$

$$\left\{ \begin{array}{l} \angle BAC + \angle OBC = 90^\circ \\ \angle BAC + \angle OCB = 90^\circ \end{array} \right\}$$

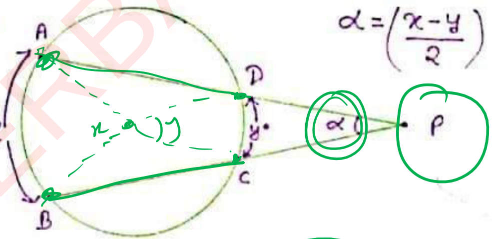
Intersecting Chords

If $\widehat{AB} = x^\circ$

$\widehat{CD} = y^\circ$

then

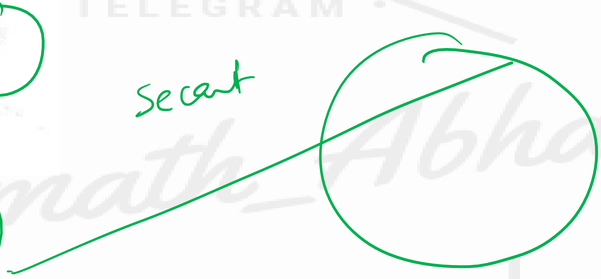
$\alpha = \frac{(x-y)}{2}$



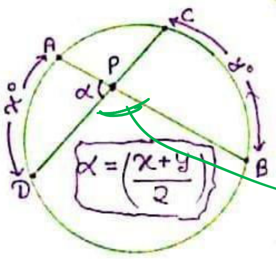
$\alpha = \left(\frac{x-y}{2}\right)$

$\alpha = \frac{x-y}{2}$

Secant

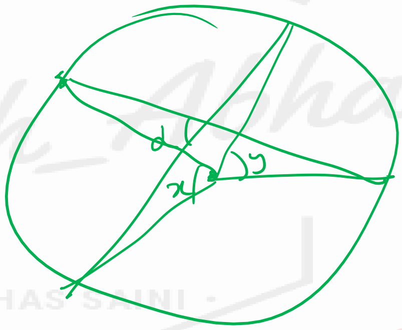


Tangent



$\frac{x+y}{2}$

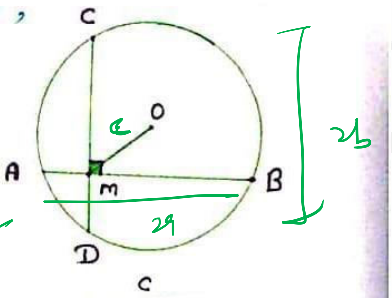
Complex



If $CD \perp AB$, $AB = 2b$, $CD = 2a$ & $OM = c$

then, r = radius

$$\boxed{r = \sqrt{\frac{a^2+b^2+c^2}{2}}}$$

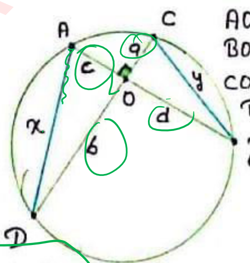


Intersecting Chords

$r =$

If $AB \perp CD$, $AD = y$, $CB = x$ & Radius of the circle $= R$, then

$$\boxed{x^2+y^2 = 4R^2}$$



$AO = c$
$BO = d$
$CO = a$
$DO = b$

$$x^2+y^2 = 4R^2$$

or

$$a^2+b^2+c^2+d^2 = 4R^2$$

$R =$

If ABCD is a rectangle/Square then $PQ = RS = x$.

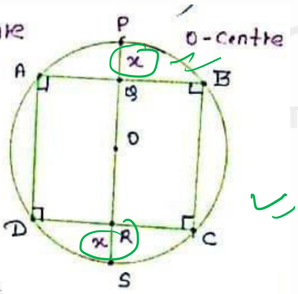
proof :—

$\because$ $PO = SO = r$ (radius

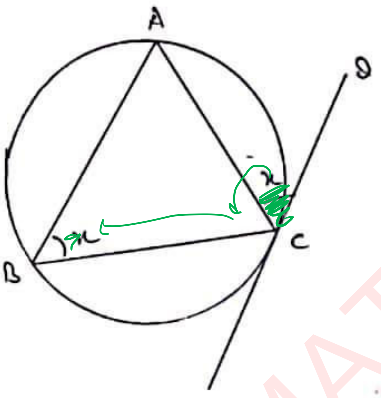
& $QO = RO$ ($\because$ O is centre)

$\Rightarrow$ $PO - QO = SO - RO$

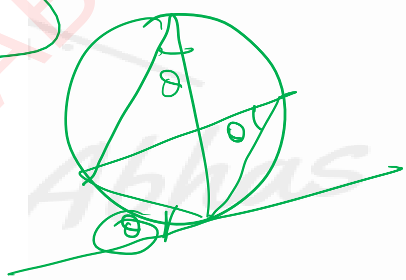
$\Rightarrow$ $\boxed{PQ = RS = x}$ proved

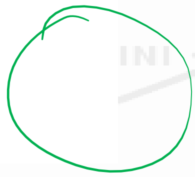


ALTERNATE SEGMENT THEOREM

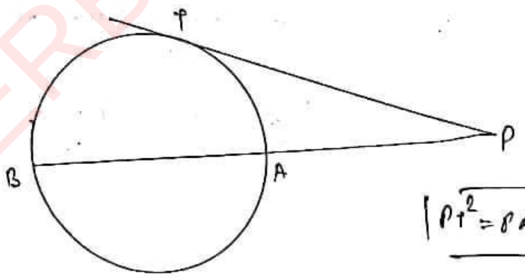


$\angle ABC = \angle ACQ$



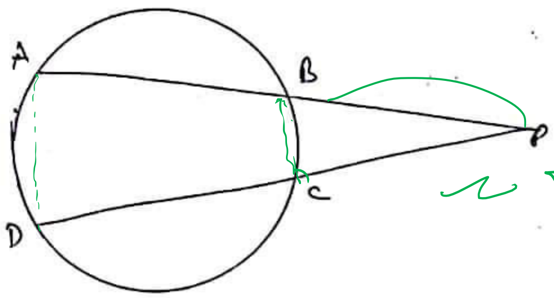


(7)



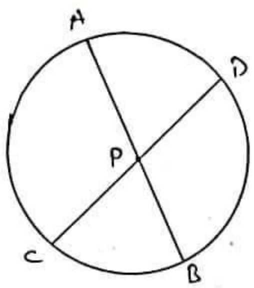
$\boxed{PT^2 = PA \times PB}$

$$\boxed{PT^2 = PA \times PB}$$



$PB \times PA = PC \times PD$

$\boxed{PA \times PB = PC \times PD}$

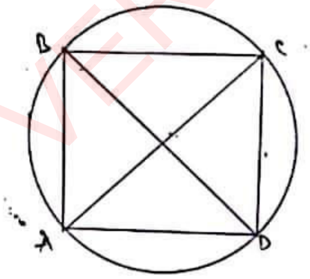


Two chords intersect at Point P then

$\boxed{PA \times PB = PC \times PD}$

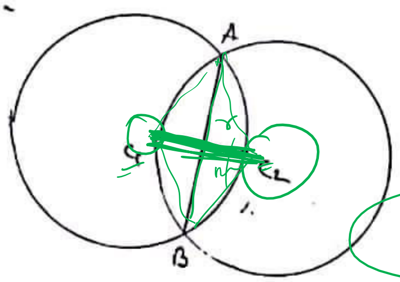
$PA \times PB = PC \times PD$

Ptolemy



$\boxed{AC \times BD = (AB \times CD) + (BC \times AD)}$

$AC \times BD = AB \times CD + BC \times AD$

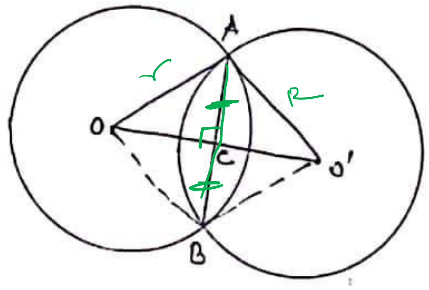


Radius Same

Radius = Dis/centers

$\boxed{AB = r\sqrt{3}}$

$AB = r\sqrt{3}$



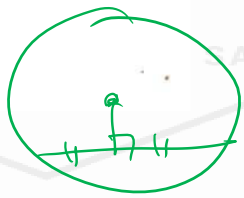
Radius Not Equal

$AC \perp OO'$

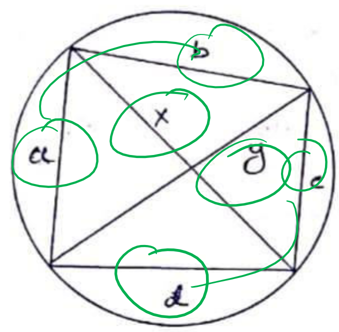
$\boxed{AC = CB}$

$\boxed{AB = 2Rr/OO'}$

$$AB = \frac{2Rr}{OO'}$$



$$\frac{x}{y} = \frac{ad + bc}{ab + dc}$$

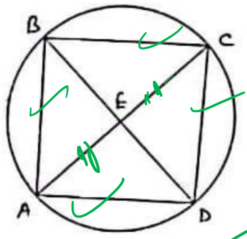


CYCLIC QUADRILATERAL

$\frac{AE}{EC} = \frac{AB * AD}{BC * CD}$

IF $AE = EC$

$\boxed{AB * AD = BC * CD}$

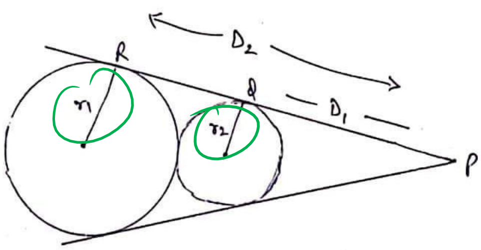


2019 → Pre

$\boxed{AE = EC}$

$AB \times AD = BC \times CD$

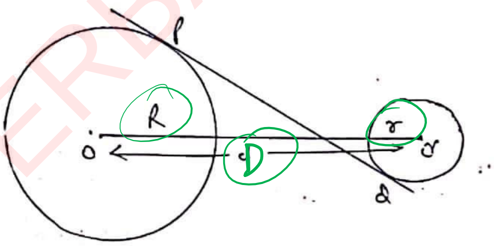
$\frac{AB \times AD}{BC \times CD} = \frac{AE}{EC}$



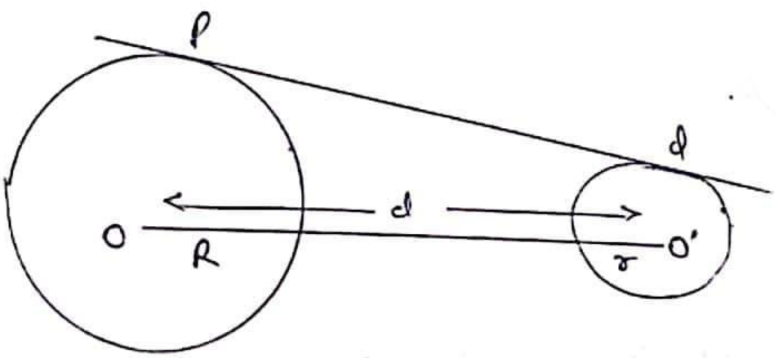
$D_1 = \frac{2r_2\sqrt{r_1 \times r_2}}{r_1 - r_2}$

$D_2 = \frac{2r_1\sqrt{r_1 r_2}}{r_1 - r_2}$

Traverse common tangent

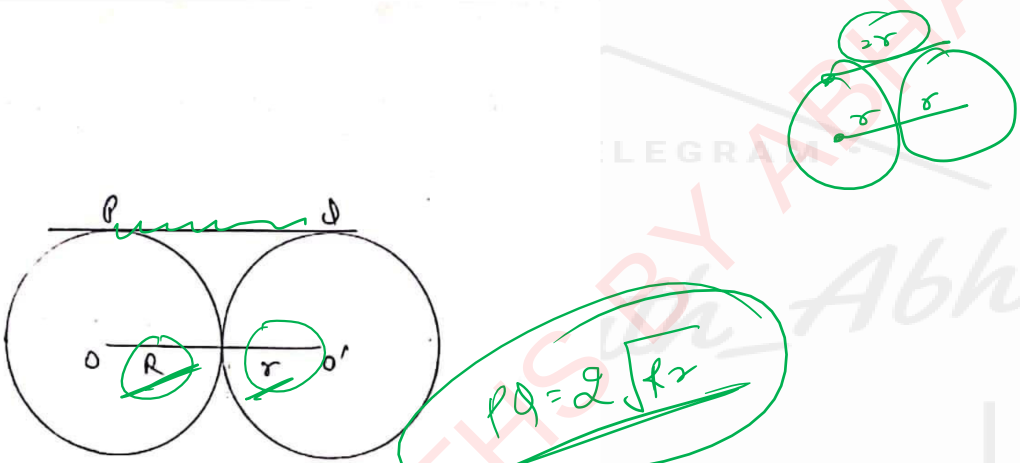


$PQ = \sqrt{D^2 - (R+r)^2}$



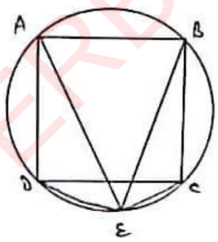
Common Tangent

Length of Tangent $PQ = \sqrt{d^2 - (R - r)^2}$ $= \sqrt{d^2 - (R-r)^2}$



$PQ = 2\sqrt{Rr}$

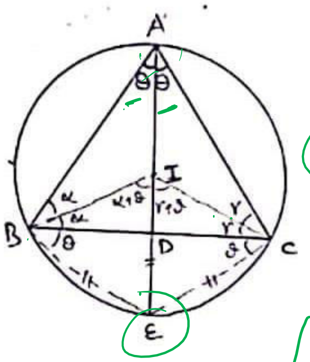
$PQ = 2\sqrt{Rr}$



ABCD is a SQUARE

$AE^2 + BE^2 + CE^2 + DE^2 = 4(\text{side})^2$

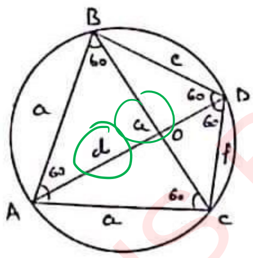
$AE^2 + BE^2 + CE^2 + DE^2 = 4(\text{side})^2$



$AB \times AC + AE \times DE = AE^2$ skip

$BE = IE = EC$

$BE = IE = EC$

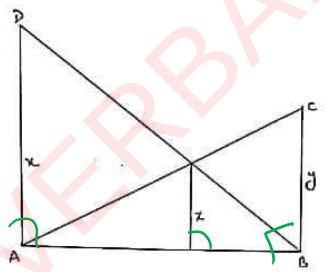


ABC is Equi triangle

$D = E + F$

$D^2 + E^2 + F^2 = 2A^2$

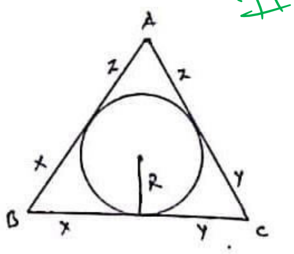
$OD = \dfrac{EF}{E+F}$



$\dfrac{1}{z} = \dfrac{1}{x} + \dfrac{1}{y}$

$\dfrac{1}{z} = \dfrac{1}{x} + \dfrac{1}{y}$

#

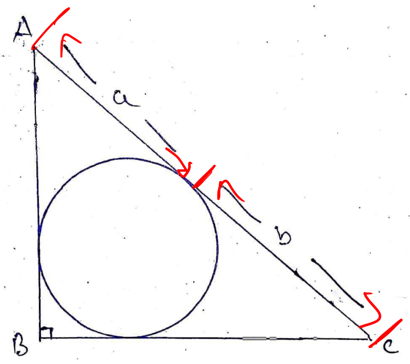


$$R = \sqrt{\frac{xyz}{x+y+z}}$$

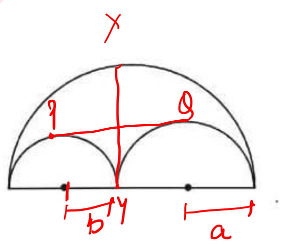
Area $= \sqrt{xyz(x+y+z)}$

$$R = \sqrt{\frac{xyz}{x+y+z}}$$

$$A = \sqrt{xyz(x+y+z)}$$



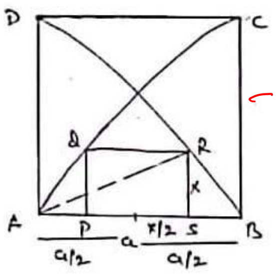
Area of triangle $= a \times b$



2017

$$XY = PQ$$

$$XY = 2\sqrt{ab}$$



Proof:-

$AR = a$

$RS = x$

$AS = \frac{a}{2} + \frac{x}{2} = \frac{a+x}{2}$

$$x^2 + \left(\frac{a+x}{2}\right)^2 = a^2$$

$$x^2 + \frac{1}{4}\left(a^2 + x^2 + 2ax\right) = a^2$$

$$4x^2 + a^2 + x^2 + 2ax = 4a^2$$

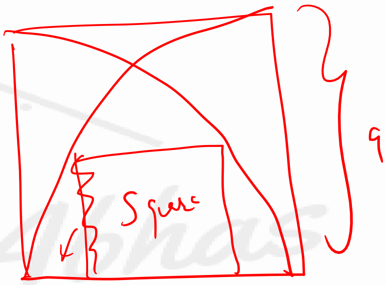
$$5x^2 + 2ax - 3a^2 = 0$$

$$5x(x+a) - 3a(x+a) = 0$$

$$5x = 3a$$

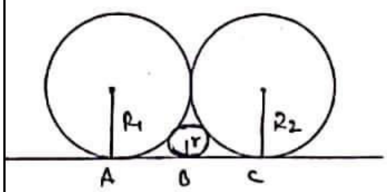
$$a = \frac{5}{3}x$$

$$\boxed{x = \frac{3a}{5}}$$



$x = \frac{3}{5}a$

THREE TANGENT CIRCLE



$AB = 2\sqrt{rR_1}$

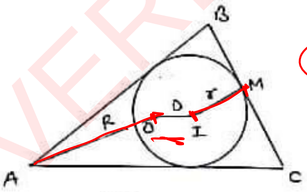
$BC = 2\sqrt{rR_2}$

$AC = 2\sqrt{R_1R_2}$

$$2\sqrt{rR_1} + 2\sqrt{rR_2} = 2\sqrt{R_1R_2}$$

$$\boxed{\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}}}$$

Distance between circumcentre & incentre.



$AO = R$

$IM = r$

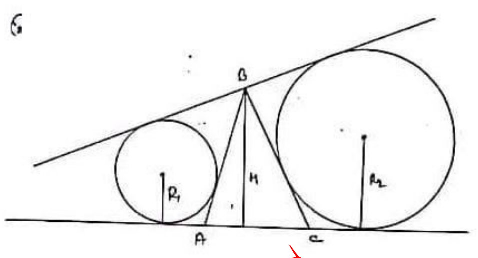
$$\boxed{D^2 = R^2 - 2Rr}$$

O → circum

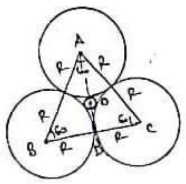
I →

$$\boxed{D^2 = R^2 - 2Rr}$$

6



$$\boxed{R_1 + R_2 = H}$$



All circle ... with radius R, find radius of smallest one

Height of Equi = $\frac{\sqrt{3}}{2}$ side.

$H = \frac{2\sqrt{3}}{2} R$

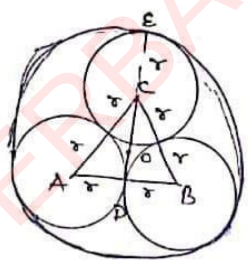
AO : OD = 2 : 1

$AO = \frac{2}{3} \times \frac{\sqrt{3}}{2} \times 2R = 2\sqrt{3}R/3$

$AO = R + r = \frac{2\sqrt{3}R}{3}$

$r = \frac{2\sqrt{3}R}{3} - R = R\left(\frac{2\sqrt{3}}{3} - 1\right)$

$$\boxed{r = R\left(\frac{2\sqrt{3}}{3} - 1\right)}$$



Three circle inside circle.

Find R of Bigger circle.

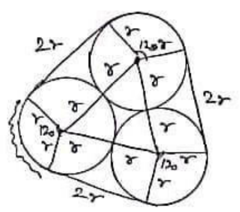
$CD = \frac{\sqrt{3}}{2} \times 2r = r\sqrt{3}$

$CO = \frac{2}{3} r\sqrt{3} = \frac{2\sqrt{3}r}{3}$

$EO = R = r + \frac{2\sqrt{3}r}{3}$

$R = r\left(\frac{3+2\sqrt{3}}{3}\right)$

What is length of rubber band if $r_1 = r_2 = r_3 = 10$ cm.



$$\text{Sector (Perimeter)} = \frac{120°}{360°_3} \times 2 \times \pi \times r$$

$$= 2\pi r/3$$

$$\text{Total} = \frac{2\pi r}{\cancel{3}} \times \cancel{3} + 6r$$

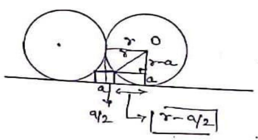
$$= \pi d + 6r$$

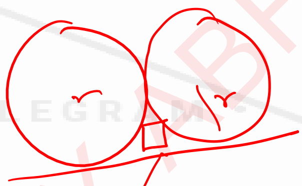
$$\boxed{\text{Total} = 2\pi r + nd} \text{ — Generalise.}$$

$$\boxed{2\pi r + nd}$$

d

find side of square if radius is r.





ϕ —

$$r^2 = \left(r - \frac{a}{2}\right)^2 + (r-a)^2$$

$$\cancel{r^2} = r^2 + \frac{a^2}{4} - ar + \cancel{r^2} + a^2 - 2ar$$

$$r^2 + \frac{5a^2}{4} - 3ar = 0$$

$$4r^2 + 5a^2 - 12ar = 0$$

$$4r^2 - 10ar - 2ar + 5a^2 = 0$$

$$2r(2r - 5a) - a(2r - 5a) = 0$$

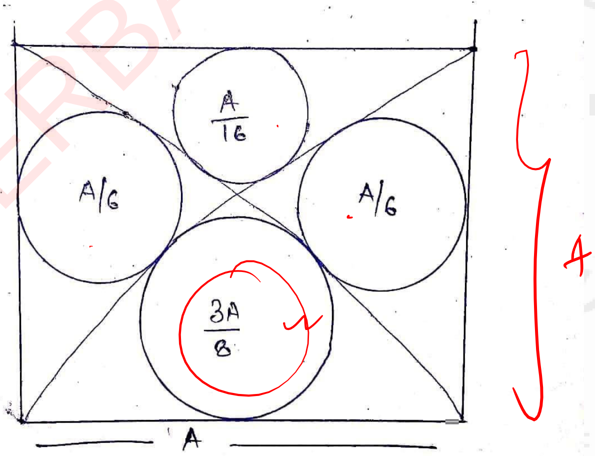
$$(2r - a)(2r - 5a) = 0$$

$$a = 2r \times$$

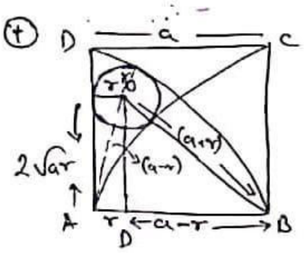
$$a = 2r/5$$

$$\boxed{a = \frac{2}{5} r}$$

$$\boxed{a = \frac{2}{5} r}$$



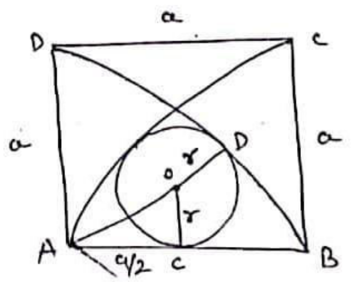
(4)



$$OD = \sqrt{(a+r)^2 - (a-r)^2}$$
$$= 2\sqrt{ar}$$

$$AO = a - r$$

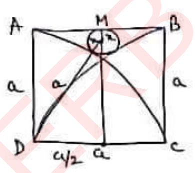
$$(a-r)^2 = r^2 + (2\sqrt{ar})^2$$
$$a^2 + \cancel{r^2} - 2ar = \cancel{r^2} + 4ar$$
$$a^{\cancel{2}} = 6\cancel{a}r$$
$$\boxed{r = a/6}$$



$$AO = a - r$$

$$(a-r)^2 = r^2 + (a/2)^2$$
$$a^2 + \cancel{r^2} - 2ar = \cancel{r^2} + a^2/4$$
$$\frac{3a^{\cancel{2}}}{4} = 2\cancel{a}r$$
$$\boxed{r = 3a/8}$$

(7)



first way.

$\because$ $AM = 2\sqrt{ax}$ Tangential dist.

$MB = 2\sqrt{ax}$

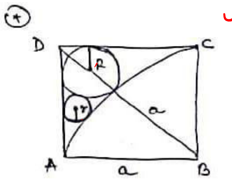
$$2\sqrt{ax} + 2\sqrt{ax} = a$$
$$4\sqrt{ax} = a$$
$$16ax = a^2$$
$$16x = a$$
$$\boxed{x = a/16}$$

second.

$$\left(\frac{a}{2}\right)^2 + (a-x)^2 = (a+x)^2$$
$$\frac{a^2}{4} = (a+x)^2 - (a-x)^2$$
$$\frac{a^2}{4} = (a+\cancel{x}+a-\cancel{x})(\cancel{a}+x-\cancel{a}+x)$$
$$\frac{a^2}{4} = 2a \cdot 2x$$
$$a^{\cancel{2}} = 16\cancel{a}x$$
$$\boxed{x = a/16}$$

Radius of circle = side of square/16

Extra

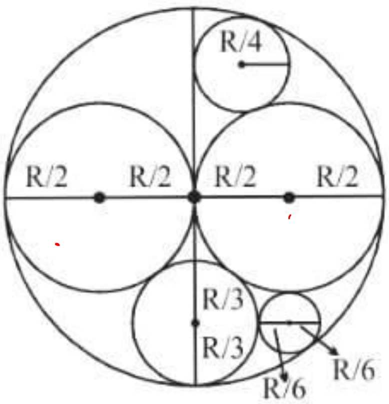


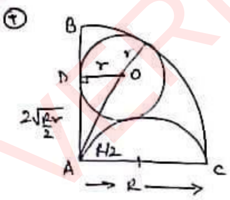
$$a\sqrt{2} = a + R + R\sqrt{2}$$
$$R(\sqrt{2}+1) = a(\sqrt{2}-1)$$
$$R = \frac{a(\sqrt{2}-1)}{(\sqrt{2}+1)} = a(\sqrt{2}-1)^2$$
$$R = a(\sqrt{2}-1)^2$$

$$\boxed{\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{R}} + \frac{1}{\sqrt{a}}}$$

$$\frac{1}{\sqrt{r}} = \frac{1}{(\sqrt{2}-1)\sqrt{a}} + \frac{1}{\sqrt{a}}$$
$$\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{a}}\left(\frac{1}{\sqrt{2}-1} + 1\right)$$
$$\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{a}}\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right) \Rightarrow \sqrt{r} = \frac{\sqrt{a}(\sqrt{2}-1)}{\sqrt{2}}$$
$$r = \frac{a(\sqrt{2}-1)^2}{2}$$

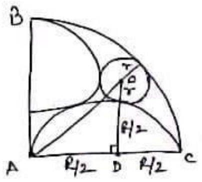
$$\boxed{r = R/2}$$





$$AO = R - r$$
$$AD = \sqrt{2Rr}$$
$$DO = r$$

$$r^2 + (\sqrt{2Rr})^2 = (R-r)^2$$
$$\cancel{r^2} + 2Rr = R^2 + \cancel{r^2} - 2Rr$$
$$R^{\cancel{2}} = 4\cancel{R}r$$
$$\boxed{r = R/4}$$



$AO = (R-r)$

$OD = (r + R/2)$

$AD = R/2$

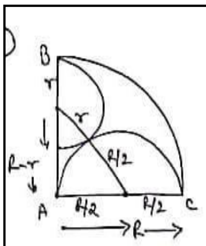
$$\left(\frac{R}{2}\right)^2 + \left(r + R/2\right)^2 = (R-r)^2$$

$$R^2/4 + \cancel{r^2} + \frac{R^2}{4} + Rr = R^2 + \cancel{r^2} - 2Rr$$

$$R^{\cancel{2}}/2 = 3\cancel{R}r$$

$$\boxed{\frac{R}{6} = r}$$

$$\boxed{r = R/6}$$

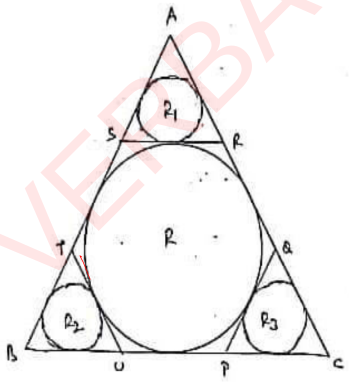


$$\left(\frac{R}{2}\right)^2 + (R-r)^2 = \left(\frac{R}{2} + r\right)^2$$

$$\cancel{\frac{R^2}{4}} + R^2 + \cancel{r^2} - 2Rr = \cancel{\frac{R^2}{4}} + \cancel{r^2} + Rr$$

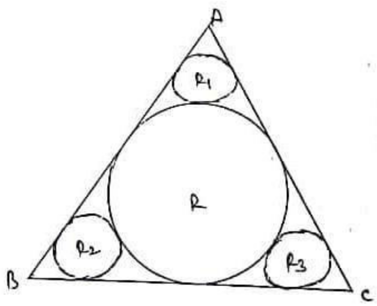
$$R^{\cancel{2}} = 3\cancel{R}r$$

$$\boxed{r = R/3}$$



$$\boxed{R = R_1 + R_2 + R_3}$$

$$R = R_1 + R_2 + R_3$$



$$\boxed{R = \sqrt{R_1R_2} + \sqrt{R_2R_3} + \sqrt{R_3R_1}}$$

$$R = \sqrt{R_1R_2} + \sqrt{R_2R_3} + \sqrt{R_3R_1}$$