Trigonometry

Important Formulas
&
Results

VERBAL MATH BY ABHAS SAINI

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TRIGONOMETRY

$$\sin^2 x + \cos^2 x = 1$$
$$\sec^2 x - \tan^2 x = 1$$
$$\operatorname{cosec}^2 x - \cot^2 x = 1$$

BASIC IDENTITIES

1.) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

2.) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

3.) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

4.) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

5.) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

6.) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

SUMMATION IDENTITIES

7.) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$

8.) $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot A - \cot B}$

9.) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

10.) $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

11.) $\cot\theta - \tan\theta = 2\cot 2\theta$

12.) $\tan\theta + \cot\theta = 2\operatorname{cosec} 2\theta = \sec\theta \times \operatorname{cosec}\theta$

13.) $\sin 3A = 3\sin A - 4\sin^3 A$

14.) $\cos 3A = 4\cos^3 A - 3\cos A$

15.) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

TRIPLE ANGLE FORMULAS

DOUBLE ANGLE FORMULAS

1.) $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

2.) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$= \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

3.) $1 + \cos 2\theta = 2\cos^2\theta$

4.) $1 - \cos 2\theta = 2\sin^2\theta$

5.) $\tan 2\theta = \dfrac{\sin 2\theta}{\cos 2\theta} = \dfrac{2\tan\theta}{1-\tan^2\theta}$

6.) $\tan\theta = \dfrac{\sin 2\theta}{1+\cos 2\theta} = \dfrac{1-\cos 2\theta}{\sin 2\theta}$

SUMMATION TO PRODUCT IDENTITIES

$$\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

$$= 2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{D-C}{2}\right)$$

$$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$$

$$\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$$

$$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$$

$$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$$

$$\sin\theta \sin(60-\theta) \sin(60+\theta) = \frac{1}{4}\sin 3\theta$$

$$\cos\theta \cos(60-\theta) \cos(60+\theta) = \frac{1}{4}\cos 3\theta$$

$$\tan\theta \tan(60-\theta) \tan(60+\theta) = \tan 3\theta$$

$$\tan\theta + \tan(60+\theta) + \tan(120+\theta) = 3\tan 3\theta$$

$$\cos\theta + \cos(120-\theta) + \cos(120+\theta) = 0$$

$$\cos^2\theta + \cos^2(120-\theta) + \cos^2(120-\theta) = \frac{3}{2}$$

$$\cos^3\theta + \cos^3(120-\theta) + \cos^3(120+\theta) = \frac{3}{4}\cos 3\theta$$

$$\cot\alpha + \cot(60+\alpha) - \cot(60-\alpha) = 3\cot 3\alpha$$

If $A+B = 60^\circ$

$$\cos^2 A + \cos^2 B - \cos A \cos B = 3/4$$

$$\sin^2 A + \sin^2 B + \sin A \sin B = 3/4$$

If $A-B = 60^\circ$

$$\cos^2 A + \cos^2 B + \cos A \cos B = 3/4$$

$$\sin^2 A + \sin^2 B - \sin A \sin B = 3/4$$

If $A+B+C = 90^\circ$

$$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$$

$$\cot A + \cot B + \cot C = \cot A \cdot \cot B \cdot \cot C$$

If $A+B+C = 180^\circ$

$$\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

$$\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C \cdot \cot A = 1$$

$$\sin A + \sin B + \sin C = 4 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$$

$$\sin A + \sin B - \sin C = 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

$$\cos A + \cos B + \cos C = 1 + 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

$$\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$$

$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$$

$$\cos 2A + \cos 2B + \cos 2C = 1 - 4 \cos A \cos B \cos C$$

$$\cos 2A + \cos 2B - \cos 2C = 1 - 4 \sin A \sin B \cos C$$

$$\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C$$

$$\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$$

$$\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2 \sin A \sin B \cos C$$

DERIVED RESULTS

$$\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$$

$$\sin^4\theta - \cos^4\theta = \sin^2\theta - \cos^2\theta$$

$$\sec^4\theta - \tan^4\theta = \sec^2\theta + \tan^2\theta$$

$$\operatorname{cosec}^4\theta - \cot^4\theta = \operatorname{cosec}^2\theta + \cot^2\theta$$

$$\frac{\sin(A-B)}{\cos A - \cos B} = \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B} = \tan A - \tan B$$

$$\frac{\cos(A+B)}{\sin A\cos B} = \frac{\cos A\cos B - \sin A\sin B}{\sin A\cos B} = \cot A - \tan B$$

SOME STANDARD ANGLES

1.) $\sin 18 = \cos 72 = \frac{\sqrt{5}-1}{4}$

2.) $\sin 54 = \cos 36 = \frac{\sqrt{5}+1}{4}$

3.) $\sin 72 = \cos 18 = \frac{\sqrt{10+2\sqrt{5}}}{4}$

4.) $\sin 36 = \cos 54 = \frac{\sqrt{10-2\sqrt{5}}}{4}$

5.) $\sin 15 = \cos 75 = \frac{\sqrt{3}-1}{2\sqrt{2}}$

6.) $\sin 75 = \cos 15 = \frac{\sqrt{3}+1}{2\sqrt{2}}$

7.) $\tan 15 = \cot 75 = 2-\sqrt{3}$

8.) $\tan 75 = \cot 15 = 2+\sqrt{3}$

9.) $\tan 22.5 = \cot 67.5 = \sqrt{2}-1$

10.) $\tan 67.5 = \cot 22.5 = \sqrt{2}+1$