Rakesh Yadav Sir's

ADVANCE MATHS

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- Co-ordinate Geometry
- Mensuration
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- Trigonometry
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- Simplification

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by Rakesh Yadav
Selected
Excise Inspector

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Dedicated to

My Beloved Daughters
Unnati Yadav & Pragati Yadav

जीतने की इच्छा सभी में होती है, मगर जीतने के लिए तैयारी करने की इच्छा बहुत कम लोगों में होती है।

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Editor-in-chief
Karan Chaudhary

Preface

Nothing thrills a writer more than the success of his book. With this book, I hope to reach a much wider section of the student community and others, who relentlessly compete for various Government – jobs.

I am thankful to Almighty and my family (My parents, brother, wife, daughters and son), who extended their help in various invisible ways. I sincerely hope, the book **ADVANCE MATHS** will meet a good response. I would humbly appreciate suggestions, doubt, etc. concerned with this book at the following.

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Content

CHAPTER
01

UNIT DIGIT

Last Digit of number is called Unit Digit

1 2 3 4
↓
unit digit

In This no. 4 is unit digit.

The unit digit of the Resultant value depends upon The unit digits of all participating numbers.

Ex.1: 23 + 34 + 46 + 78 = 181, unit digit of 181.

Sol. ∴ unit digit = 1

It is clear that the unit digit of the Resultant value 181 depends upon the unit digits 3, 4, 6, 8

3 + 4 + 6 + 8 = 21

So, units digit = 1

Ex.2: What is the unit digit of ?
31 × 37 × 36 × 46 × 89

sol. 31 × 37 × 36 × 46 × 89
Unit digit = 1, 7, 6, 6, 9
multiply the unit digits = 1 × 7 × 6 × 6 × 9
⇒ 1 × 7 = 7
⇒ 7 × 6 = 42
⇒ 2 × 6 = 12
⇒ 2 × 9 = 18
unit digit = 8

Ex.3: What is the unit digit of?
31 × 33 × 37 × 39 × 43

Sol. 31 × 33 × 37 × 39 × 43
multiply the unit digits
= 1 × 3 × 7 × 9 × 3
unit digit = 7

Ex.4: What is the unit digit of ?
91 × 93 × 95 × 96 × 97 × 98

Sol. multiply the unit digit
1 × 3 × 5 × 6 × 7 × 8 = 0

Ex.5: Find the unit digit of 135 × 136 × 170

Sol. The unit digits = 5, 6, 0
multiply the units digit
= 5 × 6 × 0
= unit digit = 0

Ex.6: Find the unit digit at the product of all the odd prime numbers.

sol. The prime numbers are 3, 5, 7, 11, 13, 17, ... etc.
Now we know that if 5 is multiplied by any odd number it always gives the last digit 5. So the required unit digit will ne '5',

Ex.7: Find the unit digit of 584 × 328 × 547 × 613

Sol. The unit digits = 4, 8, 7, 3
multiplying the unit digits
= 4 × 8 × 7 × 3
= unit digit = 2

Ex.8: Find the unit digit of the product of all the even numbers

Sol. The even number are
2, 4, 6, 8, 10, 12, ... etc.
Now we know that if '0' is multiplied by any number it always gives the last digit 0. so the required unit digit will be 0.

Ex.9: Find the unit digit 4!

Sol. 4! = 4 × 3 × 2 × 1 = 2<u>4</u>
unit digit = 4

Ex.10: Find the unit digit 5!

Sol. 5! = 5 × 4 × 3 × 2 × 1 = 120
unit digit = 0

* Factorial 5 and more than 5 express gives unit digit 0.

Unit digit when 'N' is Raised to a power

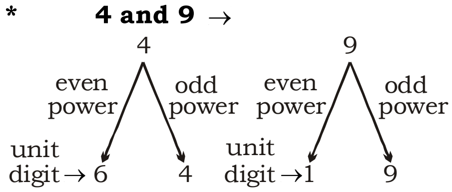
unit digit of 0, 1, 5 and 6 has any power (odd or even) no change

Ex.11: $(3765)^{437}$
unit digit = $(5)^{437} = 5$

Ex.12: $(6736)^{32567}$
unit digit = $(6)^{32567} = 6$

Ex.13: $(32541)^{325}$
unit digit = $(1)^{325} = 1$

* **4 and 9 →**



Ex.14: Find the unit place $(67354)^{1237}$

Sol. $(67354)^{1237}$
unit digit = $(4)^{1237} = (4)^{\text{odd power}}$
So, unit digit = 4

Ex.15: Find the unit place $(3259)^{1214}$

Sol. $(3259)^{1214}$
unit digit = $(9)^{1214} = (9)^{\text{even power}}$
unit digit = 1

Ex.16: Find the unit place $(6734)^{312}$

Sol. $(6734)^{312}$
unit digit = $(4)^{312} = (4)^{\text{even}}$
unit digit = 6

Rule of (2, 3, 7 and 8) →

unit digit when 'N' is raised to a power
If the value of the power is

Power →

unit digit ↓	1 or 4n+1	2 or 4n+2	3 or 4n+3	4 or 4n+4
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6

here n → Natural No.
If those number which unit digit 2, 3, 7 and 8. →
all unit digit have cyclicity 4

Ex.18: Find the unit place 3^{35}

Sol. $3^{35} = 3^{32} \times 3^3$
Break the power form of 4n
$(3^4)^8 \times 3^3$ = (......1) × (...7)
unit place = 1 × 7 = 7

Ex.19: Find the unit place $(127)^{39}$

Sol. $(127)^{39}$
unit place = $(7)^{39}$
= $(7)^{36} \times (7)^3 = (7^4)^9 \times (7)^3$
= (...1) × (....3)
unit place = 1 × 3 = 3

Ex.20: Find the unit place $(678)^{562}$

Sol. $(678)^{562}$
unit digit = $(8)^{562}$
= $(8)^{560} \times (8)^2$
= $(8^4)^{140} \times (8)^2$
= (...6) × (...4)
unit digit = 6 × 4 = 2<u>4</u> = **4**

Ex.21: Find the unit place $(327)^{640}$

Sol. $(327)^{640}$

unit digit = $(7)^{640}$

640 is multiple of 4

then = $(7^4)^{160}$

unit digit = $(1)^{160}$ = **1**

Ex.22: Find the unit digit of $(2137)^{753}$

Sol. $(2137)^{753}$

unit digit = $(7)^{753}$

= $(7)^{752} \times 7^1$

= $(7^4)^{188} \times 7^1$

= $(....1) \times 7$

unit digit = 1×7 = **7**

Ex.23: Find the unit digit of $(13)^{2003}$

Sol. $(13)^{2003}$

unit digit = $(3)^{2003}$

= $3^{2000} \times 3^3$

= $(3^4)^{500} \times 3^3$

= $(....1)^{500} \times 27$

= $1 \times 27 = 27$

unit digit = **7**

Ex.24: Find the unit digit of $(22)^{23}$

Sol. $(22)^{23}$

unit digit = $(2)^{23}$

= $(2)^{20} \times 2^3 = (2^4)^5 \times 8$

= $(....6)^5 \times 8$

unit digit = $6 \times 8 = 48$ = **8**

Ex.25: Find the unit digit of $(37)^{105}$

Sol. $(37)^{105}$

unit digit = $(7)^{105}$

= $(7)^{104} \times 7^1$

= $(7^4)^{26} \times 7^1$

= $(...1)^{26} \times 7$

unit digit = 1×7 = **7**

Ex.26: Find the unit place

$(23)^{21} \times (24)^{22} \times (26)^{23} \times (27)^{24} \times (25)^{25}$

Sol. $(23)^{21} \times (24)^{22} \times (26)^{23} \times (27)^{24} \times (25)^{25}$

unit digit = $(3)^{21} \times (4)^{22} \times (6)^{23} \times (7)^{24} \times (5)^{25}$

Break the power multiple of 4

$$= \underbrace{3^{20} \times 3^1}_{} \times \underset{\substack{\downarrow \\ \text{even power}}}{4^{22}} \times \underset{\substack{\downarrow \\ \text{same digit}}}{6^{23}} \times (7^4)^6 \times \underset{\substack{\downarrow \\ \text{same digit}}}{5^{25}}$$

$= 3 \times 6 \times 6 \times 1 \times 5$

unit digit = 0

Note:- unit digit = even × 5 = '0'

Ex.27: Find the unit place

$(235)^{215} + (314)^{326} + (6736)^{213} + (3167)^{112}$

unit digit

$$\underset{\substack{\downarrow \\ \text{Same}}}{(5)^{215}} + \underset{\substack{\downarrow \\ \text{even power}}}{(4)^{326}} + \underset{\substack{\downarrow \\ \text{Same}}}{(6)^{213}} + \underset{\substack{\downarrow \\ (7^4)^{28}}}{(7)^{112}}$$

$= 5 + 6 + 6 + 1$

unit digit = 18 = 8

Ex.28: Find the unit place of

$$\frac{12^{55}}{3^{11}} + \frac{8^{48}}{16^{18}}$$

Sol. $\frac{12^{55}}{3^{11}} + \frac{8^{48}}{16^{18}}$

$= \frac{(3 \times 4)^{55}}{3^{11}} + \frac{(2^3)^{48}}{(2^4)^{18}}$

$= \frac{3^{55} \times 4^{55}}{3^{11}} + \frac{2^{144}}{2^{72}}$

$= 3^{44} \times 4^{55} + 2^{72}$

unit digt = $(.....1) \times (......4) + 6$

= 4 + 6 = 10,

unit digit = 0

EXERCISE

1. Find the unit digit of 584 × 389 × 476 × 786
(a) 7 (b) 3 (c) 4 (d) 6

2. Find the unit digit of
641 × 673 × 677 × 679 × 681
(a) 9 (b) 3 (c) 6 (d) 7

3. Find the unit digit of
$(5627)^{153} \times (671)^{230}$
(a) 7 (b) 9 (c) 3 (d) 1

4. Find the unit digit of
$(3625)^{333} \times (4268)^{645}$
(a) 6 (b) 3 (c) 4 (d) 0

5. Find the unit digit
$(3694)^{1793} \times (615)^{317} \times (841)^{941}$
(a) 5 (b) 3 (c) 4 (d) 0

6. Find the unit digit of
$(7^{95} - 3^{58})$
(a) 7 (b) 3 (c) 4 (d) 0

7. Find the unit place of
$(17)^{1999} + (11)^{1999} - (7)^{1999}$
(a) 0 (b) 1 (c) 2 (d) 7

8. Find the unit digit of
$3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$
(a) 6 (b) 9 (c) 0 (d) 2

9. Find the unit digit of 111! (factorial 111).
(a) 0 (b) 1 (c) 5 (d) 3

10. The last digit of the number obtained by multiplying the numbers 81 × 82 × 83 × 84 × 85 × 86 × 87 × 88 × 89 will be
(a) 0 (b) 9 (c) 7 (d) 2

11. Find the units digit of the expression $25^{6251} + 36^{528} + 22^{853}$
(a) 4 (b) 3 (d) 6 (d) 5

12. Find the units digit of the expression $55^{725} + 73^{5810} + 22^{853}$
(a) 4 (b) 0 (c) 6 (d) 5

13. Find the units digit of the expression $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$.
(a) 1 (b) 9 (d) 7 (d) 0

14. Find the last digit of the number $1^3 + 2^3 + 3^3 + 4^3 .... + 99^3$.
(a) 0 (b) 1 (d) 2 (d) 5

15. Unit digit in $(264)^{102}+(264)^{103}$ is:
(a) 0 (b) 4 (c) 6 (d) 8

16. Unit digit $[(251)^{98}+(21)^{29}-(106)^{100}+(705)^{35}-(16)^4+259]$ is
(a) 1 (b) 4 (c) 5 (d) 6

17. The unit digit in the expansion of $(2137)^{754}$ is
(a) 1 (b) 3 (c) 7 (d) 9

18. The digit in unit's place of the product $(2153)^{167}$ is :
(a) 1 (b) 3 (c) 7 (d) 9

19. The digit in unit's place of the product
$(2464)^{1793} \times (615)^{317} \times (131)^{491}$ is
(a) 0 (b) 2 (c) 3 (d) 5

20. What will be the unit digit in the product of 7^{105} ?
(a) 5 (b) 7 (c) 9 (d) 1

21. what is the number of unit place in $(329)^{78}$
(a) 1 (b) 7 (c) 9 (d) 3

22. unit digit of the number $(22)^{23}$ is:
(a) 4 (b) 6 (c) 6 (d) 8

23. The unit digit in the product $(122)^{173}$ is:
(a) 2 (b) 4 (c) 6 (d) 8

24. The unit digit in the sum of $(124)^{372} + (124)^{373}$ is :
(a) 5 (b) 4 (c) 2 (d) 0

25. The last digit of $(1001)^{2008} + (1002)$ is:
(a) 0 (b) 3 (c) 4 (d) 6

26. Find the unit digit in the product: $(4387)^{245} \times (621)^{72}$.
(a) 1 (b) 2 (c) 5 (d) 7

27. The unit digit of the expression $25^{6251} + 36^{528} + 73^{54}$ is
(a) 6 (b) 5 (c) 4 (d) 0

28. The unit's digit in the product $7^{71} \times 6^{63} \times 3^{65}$ is
(a) 1 (b) 2 (c) 3 (d) 4

29. The last digit of 3^{40} is
(a) 1 (b) 3 (c) 7 (d) 9

30. The digit in unit's place of the number $(1570)^2 + (1571)^2 + (1572)^2 + (1573)^2$ is :
(a) 4 (b) 1 (c) 2 (d) 3

31. The unit digit in $3 \times 38 \times 537 \times 1256$ is
(a) 4 (b) 2 (c) 6 (d) 8

32. The unit digit in the product $(2467)^{153} \times (341)^{72}$ is
(a) 1 (b) 3 (c) 7 (d) 9

33. The unit digit in the product $(6732)^{170} \times (6733)^{172} \times (6734)^{174} \times (6736)^{176}$
(a) 1 (b) 3 (c) 4 (d) 5

34. Find the unit digit of the product of all the prime number between 1 and 99999
(a) 9 (b) 7 (c) 0 (d) N.O.T.

35. Find the unit digit of the product of all the elements of the set which consists all the prime numbers greater than 2 but less than 222.
(a) 4 (b) 5 (c) 0 (d) N.O.T.

36. Find the last digit of $222^{888} + 888^{222}$
(a) 2 (b) 6 (c) 0 (d) 8

37. Find the last digit of $32^{32^{32}}$
(a) 4 (b) 8 (c) 6 (d) 2

38. Find the last digit of the expression:
$1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + 100^2$.
(a) 0 (b) 4 (c) 6 (d) 8

39. Find the unit digit of $1^1 + 2^2 + 3^3 + \ldots\ldots 10^{10}$.
(a) 9 (b) 7 (c) 0 (d) N.O.T.

40. Find the unit digit of $13^{24} \times 68^{57} + 24^{13} \times 57^{68} + 1234 + 5678$.
(a) 4 (b) 7 (c) 0 (d) 8

41. The unit digit of the expression
$$\frac{1\times2\times3\times4\times5\times6\times7\times8\times9\times10}{100}$$
(a) 7 (b) 9 (c) 8 (d) N.O.T.

42. Find the unit digit of the expression $888^{9235!} + 222^{9235!} + 666^{2359!} + 999^{9999!}$.
(a) 5 (b) 9 (c) 3
(d) None of these

43. The last digit of the following expression is: $(1!)^1 + (2!)^2 + (3!)^3 + (4!)^4 + \ldots. + (10!)^{10}$
(a) 4 (b) 5 (c) 6 (d) 7

44. The last 5 digits of the following expression will be
$(1!)^5 + (2!)^4 + (3!)^3 + (4!)^2 + (5!)^1 + (10!)^5 + (100!)^4 + (1000!)^3 + (10000!)^2 + (100000!)$
(a) 45939 (b) 00929
(c) 20929
(d) Can't determined

45. The unit digit of the following expression $(1!)^{99} + (2!)^{98} + (3!)^{97} + (4!)^{96} + \ldots\ldots (99!)^1$ is:
(a) 1 (b) 3 (c) 7 (d) 6

46. The unit digit of $(12345k)^{72}$ is 6. The value of k is:
(a) 8 (b) 6 (c) 2
(d) all of these

47. The unit digit of the expression $(1!)^{1!} + (2!)^{2!} + (3!)^{3!} + \ldots. + (100!)^{100!}$
(a) 0 (b) 1 (c) 2 (d) 7

48. The last digit of the expressioin $4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \times \ldots\ldots \times 4^{99} \times 9^{100}$ is :
(a) 4 (b) 6 (c) 9 (d) 1

49. The last digit of the expression $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + \ldots\ldots + 4^{99} + 9^{100}$ is:
(a) 0 (b) 3 (c) 5
(d) None of these

50. The unit digit of $2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 7^{8^9}$ is:
(a) 0 (b) 5
(c) Can't be determined
(d) None of these

ANSWER KEY

1. (d)	6. (c)	11. (b)	16. (b)	21. (a)	26. (d)	31. (d)	36. (c)	41. (c)	46. (d)
2. (a)	7. (b)	12. (c)	17. (d)	22. (d)	27. (d)	32. (c)	37. (c)	42. (b)	47. (d)
3. (a)	8. (a)	13. (b)	18. (c)	23. (a)	28. (d)	33. (c)	38. (a)	43. (d)	48. (b)
4. (d)	9. (a)	14. (a)	19. (a)	24. (d)	29. (a)	34. (c)	39. (b)	44. (b)	49. (a)
5. (d)	10. (a)	15. (a)	20. (b)	25. (b)	30. (a)	35. (b)	40. (a)	45. (c)	50. (a)

SOLUTION

1. (d) $584 \times 389 \times 476 \times 786$
unit digit 4, 9, 6, 6
Multiplying the unit digit
$= 4 \times 9 \times 6 \times 6$
unit digit = 6

2. (a) $641 \times 673 \times 677 \times 679 \times 681$
unit digit = 1, 3, 7, 9, 1
Multiply the unit digit
$= 1 \times 3 \times 7 \times 9 \times 1$
$= 21 \times 9 = 189$
unit digit = 9

3. (a)$(5627)^{153} \times (671)^{230}$
unit digit $(7)^{153} \times (1)^{230}$
$= (7)^{152} \times 7^1 \times 1$
$= (7^4)^{38} \times 7 \times 1$
$= (\ldots.1)^{38} \times 7$
unit digit $= 1 \times 7 = 7$

4. (d) $(3625)^{333} \times (4268)^{645}$
unit digit $(5)^{333} \times (8)^{645}$
$= 5 \times (8)^{644} \times 8^1$
$= 5 \times (8^4)^{161} \times 8^1$
$= 5 \times (6)^{161} \times 8$
unit digit $= 5 \times 6 \times 8 = 240 =$ **0**

5. (d)$(3694)^{1793} \times (615)^{317} \times (841)^{941}$
unit digit $(4)^{1793} \times (5)^{317} \times (1)^{941}$
$4^{\text{odd power}} = 4$
$5^n = 5$
$4 \times 5 \times 1 = 20$
Hence, unit digit = 0

6. (c) $7^{95} - 3^{58}$
$= 7^{92} \times 7^3 - 3^{56} \times 3^2$
$= (7^4)^{23} \times 343 - (3^4)^{14} \times 9$
$= (.....1)^{23} \times 3 - (.....1)^{14} \times 9$
unit digit = (.....3) – (......9)
= 13 – 9 = **4**

7. (b) $(17)^{1999} + (11)^{1999} - (7)^{1999}$
unit digit $= (7)^{1999} + (1)^{1999} - (7)^{1999}$
$\because (7)^{1999} - (7)^{1999}$ gives = 0
Then, unit digit = 1

8. (a) Unit digit $= 3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$
The unit digit of $3^6 = 3^4 \times 3^2 = 9$
The unit digit of $4^7 = 4$
The unit digit of $6^3 = 6$
The unit digit of $7^4 = 1$
The unit digit of $8^2 = 4$
The unit digit of $9^5 = 9^4 \times 9^1 = 9$
multiply the unit digits = 9 × 4 × 6 × 1 × 4 × 9
unit digit = 6

9. (a) 111! = 1 × 2 × 3 × 4 × 5 × .... × 100 × 111
Since there is product of 5 and 2 hence it will give zero as the unit digit.
Hence the unit digit of 111! is 0 (zero).

10. (a) 81 × 82 × 83 × 84 × 85 × 86 × 87 × 88 × 89
Unit digits = 1, 2, 3, 4, ...,9
Product of unit digits
= 1 × 2 × 3 ×...........×9
Because 5 multiply any even no. Then
we gets unit digit = 0

11. (b) $25^{6251} + 36^{528} + 22^{853}$
unit digit $= (5)^{6251} + (6)^{528} + (2)^{853}$
unit digit $= (....5) + (....6) + (2)^{852} \times 2^1$
$= (....5) + (....6) + (2^4)^{213} \times 2$
$= 5 + 6 + (6)^{213} \times 2$
Sum of unit digit = 5 + 6 + 6 × 2
= 5 + 6 + 12 = 23
Hence, unit digit = 3

12. (c) $55^{725} + 73^{5810} + 22^{853}$
unit digit $= (5)^{725} + (3)^{5810} + (2)^{853}$
$= (...5) + (3^4)^{1452} \times 3^2 + (2^4)^{213} \times 2^1$
$= 5 + (1)^{1452} \times 9 + (16)^{213} \times 2^1$
Sum of unit digit = 5 + 1 × 9 + 6 × 2 = 5 + 9 + 12 = 26
unit digit = 6

13. (b) $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$
unit digit $= (1)^1 + (2)^2 + (3)^3 + (4)^4 + (5)^5 + (6)^6$
Sum of unit digit = 1 + 4 + 7 + 6 + 5 + 6 = 29
unit digit = 9

14. (a) $1^3 + 2^3 + 3^3 + 4^3 .... + 99^3$
Sum of cube of natural no.

$$= \left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{99(99+1)}{2}\right)^2$$

$$= \left(\frac{99 \times 100}{2}\right)^2 = (99 \times 50)^2$$

$= (4850)^2$
Unit digit = 0

15. (a) $(264)^{102} + (264)^{103}$
unit digit
$4^1 \rightarrow 4 \rightarrow 4$
$4^2 \rightarrow 16 \rightarrow 6$
$4^3 \rightarrow 64 \rightarrow 4$

Rule: When 4 has odd power, then unit digit is: 4
When 4 has even power, then unit digit is 6

$$\underset{\substack{\downarrow \\ \textbf{(even power)}}}{(4)^{102}} + \underset{\substack{\downarrow \\ \textbf{(odd power)}}}{(4)^{103}}$$

$(264)^{102} + (264)^{103}$
$\downarrow \qquad \downarrow$
$(4)^{102} + (4)^{103}$
$\downarrow \qquad \downarrow$
(even power) (odd power)
unit digit 6 + 4 = 10 → **0**

Alternate :
$\Rightarrow (264)^{102} + (264)^{103}$
$\Rightarrow (264)^{102}(1 + 264)$
$\Rightarrow (264)^{102} \times 265$
Multiplication of 5 & 2 = 0
Hence, unit digit is 0.

16 (b) $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + (259)]$
unit place of 1,5 and 6 will remain same
$= [(1)^{98} + (1)^{29} - (6)^{100} + (5)^{35} - (6)^4 + 9]$
= [1 + 1 – 6 + 5 – 6 + 9]
⇒ 16 – 12 = 4
Hence, unit digit = 4

17. (d) $(2137)^{754}$
$= (7)^{754}$ will give unit digit

	unit digit
$7^1 = 7$	→ 7
$7^2 = 49$	→ 9
$7^3 = 343$	→ 3
$7^4 = 2401$	→ 1
$7^5 = 16807$	→ 7

4
& will repeat

754 divide by $4 = \frac{754}{4}$
So, remainder is 2
$7^2 =$ **9**

Unit Place = 9

18. (c) $(2153)^{167}$
unit digit $= 3^{167}$
unit digit
$3^1 \rightarrow 3 \rightarrow 3$
$3^2 \rightarrow 9 \rightarrow 9$
$3^3 \rightarrow 27 \rightarrow 7$
$3^4 \rightarrow 81 \rightarrow 1$
This cycle will continue
⇒ divide the power of 3 by 4
$\frac{167}{4} \Rightarrow$ remainder is 3
$3^3 \Rightarrow 7$
Unit digit = 7

19. (a) $(2464)^{1793} \times (615)^{317} \times (131)^{491}$
$4^1 \rightarrow 4 \rightarrow 4$
$4^2 \rightarrow 16 \rightarrow 6$
$4^3 \rightarrow 64 \rightarrow 4$
So odd power of 4 will have 4 as unit digit and even power will have 6 as unit digit 5 and 1 have same unit digits respectively

$(2464)^{1793} \times (615)^{317} \times (131)^{491}$
odd power
unit digit → 4 × 5 × 1 = 20

⇒ 20 ⇒ 0 unit digit

20. (b) 7^{105}
⇒ $7^1 \rightarrow 7 \rightarrow 7$
⇒ $7^2 \rightarrow 49 \rightarrow 9$
⇒ $7^3 \rightarrow 343 \rightarrow 3$
⇒ $7^4 \rightarrow 2401 \rightarrow 1$
Divide power of 7 by 4
$\frac{105}{4} \rightarrow$ remainder $= 1 \Rightarrow 7^1$ is left
unit digit = 7

21. (a) $(329)^{78}$
⇒ If power of 9 is odd, then unit digit number be 9. If power is even then unit digit number be 1.
Hence, unit digit = 1

22. (d) $(22)^{23}$

	Result	unit digit	
2^1	2	2	Cycle completes
2^2	4	4	
2^3	8	8	
2^4	16	6	
2^5	32	2	

So divide power of 22 by 4

$\frac{23}{4}$ = remainder 3

$2^3 = 8$

unit digit = 8

23. (a) $(122)^{173}$

		Unit digit	
$2^1 \rightarrow$	2	$\rightarrow$ 2	Cycle
$2^2 \rightarrow$	4	$\rightarrow$ 4	
$2^3 \rightarrow$	8	$\rightarrow$ 8	
$2^4 \rightarrow$	16	$\rightarrow$ 6	
$2^5 \rightarrow$	32	$\rightarrow$ 2	

$2^{173} = 2^{4\times43+1} = 2^{4\times43}\times2 = 16^{43}\times2$

$= 6^{43}\times2 = 6\times2 = 12$

unit digit = 2

24. (d) $(124)^{372}$ $(124)^{373}$

$\downarrow$ $\downarrow$

4^{372} 4^{373}

When 4 has odd power then unit digit is 4 when 4 has even power then unit digit is 6

$4^1 \rightarrow 4 \rightarrow 4$

$4^2 \rightarrow 16 \rightarrow 6$

$4^3 \rightarrow 64 \rightarrow 4$

$4^4 \rightarrow 256 \rightarrow 6$

4^{372} 4^{373}

$\downarrow$ $\downarrow$

6 + 4 = 10

last (unit) digit = 0

25. (b) $(1001)^{2008} + 1002$

$\downarrow$

Unit digit $\rightarrow 1^{2008} + 1002$

Unit digit will be 1 in case of 1 respective of power

$\Rightarrow$ 1+ 1002 = 1003

unit digit (last digit) = 3

26. (d)

		unit place	
$7^1 \rightarrow$	7	$\rightarrow$ 7	Cycle
$7^2 \rightarrow$	49	$\rightarrow$ 9	
$7^3 \rightarrow$	343	$\rightarrow$ 3	
$7^4 \rightarrow$	2401	$\rightarrow$ 1	

$(4387)^{245} \times (621)^{72}$

$\downarrow$

$(7)^{245} \times (1)^{72}$

$\downarrow$

$(7)^{4\times61+1} \times 1$

$\downarrow$

$(7)^{4\times61}\times7 \times 1$

$\downarrow$

$(1)^{61}\times7 \times 1$

unit digit = 7

27. (d) 5 always gives unit digit 5 and 6 always gives unit digit 6

		unit digit	
$3^1 \rightarrow$	3	$\rightarrow$ 3	Cycle
$3^2 \rightarrow$	9	$\rightarrow$ 9	
$3^3 \rightarrow$	27	$\rightarrow$ 7	
$3^4 \rightarrow$	81	$\rightarrow$ 1	

$25^{6251} + 36^{528} + 72^{54}$

$\downarrow$ $\downarrow$ $\downarrow$

$5^{6251} + 6^{528} + 3^{54}$

$\downarrow$ $\downarrow$ $\downarrow$ $54/4$ = r = 2

unit digit $\rightarrow$ 5 + 6 + 3^2

= 5 + 6 + 9 = 20 = 0

Hence, unit digit = 0

28. (d) $7^{71} \times 6^{63} \times 3^{65}$

$\downarrow$ $\downarrow$ $\downarrow$

unit place 7^3 6^3 3^1

$\downarrow$ $\downarrow$ $\downarrow$

unit digit $\Rightarrow 3 \times 6 \times 3 = 54$

$\Rightarrow 4$

29. (a) 3^{40} :

Divide = $\frac{40}{4} \Rightarrow$ remainder = 0

		Unit digit	
3^1	$\rightarrow$ 3	$\rightarrow$ 3	Cycle
3^2	$\rightarrow$ 9	$\rightarrow$ 9	
3^3	$\rightarrow$ 27	$\rightarrow$ 7	
3^4	$\rightarrow$ 81	$\rightarrow$ 1	

Hence, unit digit of 3^{40} of completing all cycle = 1

30. (a)

$(1570)^2 + (1571)^2 + (1572)^2 + (1573)^2$

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

unit digit $\rightarrow$ $0^2 + 1^2 + 2^2 + 3^2$

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

0 + 1 + 4 + 9 = 14

unit digit = 4

31. (d) 3 × 38 × 537 × 1256 × 24 × 28 × 48

Note:- Always multiply only unit digit of first no. to second and product's unit digit no. with 3rd no. Again product of last's unit digit to fourth and so on.

Hence, unit digit = 8

32. (c) $(2467)^{153} \times (341)^{72}$

$\downarrow$ $\downarrow$

$(7)^{153} \times (1)^{72}$

$\downarrow$

[153/4= remainder = 1]

$\Rightarrow$ $7^1 \times 1 = 7$

	Result	Unit digit
7^1 =	7	7
7^2 =	49	9
7^3 =	343	3
7^4 =	2401	1

Hence, unit digit = 7

33. (c) $(6732)^{170} \times (6733)^{172} \times (6734)^{174} \times (6736)^{176}$

unit digit = $(2)^{170} \times (3)^{172} \times (4)^{174} \times (6)^{176}$

$= (2^4)^{42} \times 2^2 \times (3^4)^{43} \times (4)^{174} \times (6)^{176}$

$= (\ldots6) \times 4 \times (\ldots1) \times (\ldots6) \times (\ldots6)$

Multiplication of unit digit

$= 6 \times 4 \times 1 \times 6 \times 6 = 864$

Hence, unit digit = **4**

34. (c) The set of prime number S = {2, 3, 5, 7, 11, 13, ...}

Since there is one 5 and one 2 which gives 10 after multiplying mutually, it means the unit digit will be zero.

Hence, unit digit = 0

35. (b) The set of required prime number = The set of reqired prime number

= {3, 5, 7, 11, ...}

Since there is no any even number is the set so when 5 will multiply with any odd number, it will always give 5 as the last digit.

Hence the unit digit will be 5.

36. (c) The last digit of the expression will be same as the last digit of $2^{888} + 8^{222}$.

Now the last digit of 2^{888} is 6 and the last digit of the 8^{222} is 4.

$\therefore$ 6 + 4 = 10.

Hence, unit digit = 0

37. (c) Find the last digit of $2^{32^{32}}$

But $2^{32^{32}} = 2^{32 \times 32 \times 32 \ldots\ldots \times 32 \text{ times}}$

$\Rightarrow 2^{32^{32}} = 2^{4 \times 8 \times (32 \times 32 \ldots\ldots \times 31 \text{ times})}$

$\Rightarrow 2^{32^{32}} = 2^{4n}$

where n = 8 × (32 × 32 .... × 32 times)

Again $2^{4n} = (16)^n \Rightarrow$ unit digit is 6, for every $n \in N$

Hence, the required unit digit = 6

38. (a) Sum of square natural number = $\frac{n(n+1)(2n+1)}{6}$

Here, n = 100

$= \frac{100\times101\times201}{6} = 338350$

Then, Unit digit = 0

39. (b) Find the unit digit of $1^1 + 2^2 + 3^3 + ...10^{10}$.

The unit digit of $1^1 = 1$
The unit digit of $2^2 = 4$
The unit digit of $3^3 = 7$
The unit digit of $4^4 = 6$
The unit digit of $5^5 = 5$
The unit digit of $6^6 = 6$
The unit digit of $7^7 = 3$
The unit digit of $8^8 = 6$
The unit digit of $9^9 = 9$
The unit digit of $10^{10} = 0$

Thus the unit digit of the given expression will be 7.
($\therefore$ 1 + 4 + 7 + 6 + 5 + 6 + 3 + 6 + 9 = 47)

40. (a) The unit of 3^{24} is 1
The unit digit of 8^{57} is 8
The unit digit of 4^{13} is 4
The unit digit of 7^{68} is 1
So the resultant value of the unit digits
= 1 × 8 + 4 × 1 + 4 + 8
= 8 + 4 + 4 + 8 = 24
Thus the unit digit of the whole expression is 4.

41. (c) Since in the numerator of the product of the expression there will be 2 zeros at the and thase two zeros will be cancelled by 2 zeros of the denominator. Hence finally we get a non-zero unit digit in the expression.

Now, $\dfrac{1\times2\times3\times4\times5\times6\times7\times8\times9\times10}{100}$

$$= \frac{1\times2^1\times3^1\times2^2\times5^1\times2^1\times 3^1\times7^1\times2^3\times3^2\times2^1\times5^1}{5^2\times2^2}$$

$$= \frac{1\times2^8\times3^4\times5^2\times7^1}{5^2\times2^2}$$

$= 1 \times 2^6 \times 3^4 \times 7$

Therefore, the unit digit of the given expression will be same as that of $1 \times 2^6 \times 3^4 \times 7$.

Now, The unit digit of $1 \times 2^6 \times 3^4 \times 7$ is 8.
($\therefore$ the product of unit digits of 1, 2^6, 3^4, 7 si 1 × 4 × 1 × 7 = 28)

Hence, the unit digit of $\dfrac{10!}{100}$ is 8.

42. (b) First of all we find the unit digit individually of all the four terms,

So, the unit digit of $888^{9235!}$ is equal to the unit digit of $8^{9235!}$
Now, the unit digit of $8^{9235!}$ equal to the unit of 8^4 (since 9235! is divisible by 4), which is 6.

unit digit of $(888)^{9235!} = (8)^{4n} = 6$
unit digit of $(222)^{9235!} = (2)^{4n} = 6$
unit digit of $(666)^{2359!} = (6)^{\text{anypower}} = 6$
unit digit of $(999)^{9999!} = (9)^{\text{even power}} = 1$

Thus the unit digit of the expression is 9. ($\therefore$ 6 + 6 + 6 + 1 = 19)

43. (d) The unit digit of the given expression will be equal to the unit digit of the sum of the unit digits of every term of the expression.

Now, The unit digit of $(1!)^2 = 1$
The unit digit of $(2!)^2 = 4$
The unit digit of $(3!)^3 = 6$
The unit digit of $(4!)^4 = 6$
The unit digit of $(5!)^5 = 0$
The unit digit of $(6!)^6 = 0$
Thus the last digit of the $(7!)^7$, $(8!)^8$, $(9!)^9$, $(10!)^{10}$ will be zero.
So, the unit digit of the given expression = 7
($\therefore$ 1+4+6 + 6 + 0 + 0 + 0 + 0 + 0 = 17)

44. (b) The last digit of $(1!)^5 = 1$
The last digit of $(2!)^4 = 16$
The last digit of $(3!)^3 = 216$
The last digit of $(4!)^2 = 576$
The last digit of $(10!)^5 = 00000$
The last digit of $(100!)^4 = 00000$
$(1000!)^3 = 00000$
$(10000!)^2 = 00000$
$(100000!)^1 = 00000$
Thus the last 5 digits of the given expression = 00929
[$\therefore$ 1 + 16 + 216 + 576 + 120 + 00000 + 00000 + 00000 + 00000 + 00000 + = 00929]

45. (c) $(1!)^{99} + (2!)^{98} + (3!)^{97} + (4!)^{96} +...... (99!)^1$
unti digit $(1!)^{99} = (1!)^{99} = 1$
unit digit $(2!)^{98} = 1 \times 2 = (2)^{98} = 4$
unit digit $(3!)^{97} = 1 \times 2 \times 3 = (6)^{97} = 6$
unit digit$=(4!)^{96} = 1 \times 2 \times 3 \times 4 = (4)^{96} = 6$
unit digit$=(5!)^{95} = 1\times2\times3\times4\times 5 = (0)^{95} = 0$
....
....
same unit digit $(99!)^1 = (1\times2....99) = (0)^1 = 0$
Then, Sum of unit digit = 1 + 4 + 6 + 6 + 0 + 0 +.... 0 = 17
unit digit = **7**

46. (d) unit digit $(12345k)^{72} = 6$
if we put the value of k = 2, 6, 8
Then we get unit digit = 6

47. (d) $(1!)^{1!} + (2!)^{2!} + (3!)^{3!} + ... +(100!)^{100!}$
unit digit $(1!)^{1!} = 1^1 = 1$
unit digit $(2!)^{2!} = (2)^2 = 4$
unit digit $(3!)^{3!} = (6)^6 = 6$
unit digit $(4!)^{4!} = (4)^{24} = 6$
unit digit $(5!)^{5!} = (0)^{120} = 0$
....
....
unit digit $(100!)^{100!}$
$\Rightarrow$ $= (0)^{1\times2\times3\times.........\times100} = 0$
Sum of unit digit = 1 + 4 + 6 + 6 + 0 + 0 + 0 = 17
unit digit = 7

48. (b) $4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \times ...... \times 4^{99} \times 9^{100}$
unit digit $4^1 = 4$
unit digit $9^2 = 1$
unit digit $4^3 = 4$
unit digit $9^4 = 1$
unit digit $4^5 = 4$
....
....
unit digit $4^{99} = 4$
unit digit $9^{100} = 1$
Then multiply the unit digit
$\underline{4} \times 1 \times \underline{4} \times 1 \times \underline{4} \times 1$ ...... $\underline{4} \times 1$
Pair of 4 × 1 (4) is equal 50
we can say this expression = 4^{50}
Then, unit place = 6

49. (a) $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + ... + 4^{99} + 9^{100}$
unit digit $4^1 = 4$
unit digit $9^2 = 1$
unit digit $4^3 = 4$
unit digit $9^4 = 1$
unit digit $4^5 = 4$
....
....
unit digit $4^{99} = 4$
unit digit $9^{100} = 1$
Then,
Sum of unit place
4 + 1 + 4 + 1 + 4+ 1.......4 + 1
Pair of 4 + 1 (5) is equal to 50
We can say this expression
= 50 × 5 = 250
unit digit = 0

50. (a) $2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 7^{8^9}$
We know unit digit of
$2^{3^4} = 2^{81} = 2$
unit digit of $5^{6^7} = 5$
Then we know that when even number is multplied by 5 then we get unit place = '0'
So, last digit = 0

CHAPTER 02

NUMBER OF ZEROES

Number of zeroes in an Expression zero:- zero will be formed by 2 and 5

Ex. $10 = 2 \times 5$

$100 = 2^2 \times 5^2$

$1000 = 2^3 \times 5^3$

$\Rightarrow$ We can say that for 'n' number of zeroes at end of the product. We need exactly 'n' combinations of 5 and 2

Ex.1 Find the number of zeroes at the end of the product:-
$5 \times 7 \times 9 \times 2 \times 11$

Sol. $5 \times 7 \times 9 \times 2 \times 11$

In this product we see

Number of 2's = 1

Number of 5's = 1

Number of pair 2's and 5's = 1

$\therefore$ Number of zero = 1

Ex.2 Find the number of zeroes at the end of the product:- $12 \times 27 \times 63 \times 113 \times 1250 \times 24 \times 650$

Sol. $\underline{12} \times 27 \times 63 \times 113 \times \underline{1250} \times \underline{24} \times \underline{650}$

Break the numbers form of 2 and 5 multiple

In this series 27, 63 & 113 are not multiple of 2 & 5.

$\therefore$ The multiple of 2 & 5 are 12, 1250, 24 & 650

$\Rightarrow$ $12 = 2 \times 2 \times 3 = 2^2 \times 3$

$\Rightarrow$ $1250 = 2 \times 5 \times 5 \times 5 \times 5 = 2^1 \times 5^4$

$\Rightarrow$ $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$\Rightarrow$ $650 = 2 \times 5 \times 5 \times 13 = 2^1 \times 5^2 \times 13$

$\underline{2^2} \times 3 \times 27 \times 63 \times 113 \times \underline{2^1} \times \underline{5^4} \times \underline{2^3} \times 3 \times \underline{2^1} \times \underline{5^2} \times 13$

Number of 2's = 7

Number of 5's = 6

$\Rightarrow$ Number of pair 2's and 5's = 6 there are 7 two's and 6 fives. Hence we shall be able to form only 6 pairs of 2 and 5, Hence there will be 6 zeroes at the end of the product of numbers.

Ex.3 Find the number of zeroes at the end of the product:-
$1 \times 3 \times 5 \times 7 \times 9 \,........\, 97 \times 99$

Sol. $1 \times 3 \times 5 \times 7 \times 9 \,........\, 97 \times 99$

In this series the number of zero and the end of the product is "0". Because there is no even number present in this series while it is necessary to be 2 and 5 for the Zero

The highest power of k that can exactly divided n! we divide n by k, n by k^2, n by k^3 and so on till we get $\left[\frac{n}{k^x}\right]$ equal to 1 an then add up as.

$$\left[\frac{n}{k}\right]+\left[\frac{n}{k^2}\right]+\left[\frac{n}{k^3}\right]+\left[\frac{n}{k^4}\right]+...+\left[\frac{n}{k^x}\right]$$

Ex.4 Find the largest power of 5 contained in 124!

Sol. $\left[\frac{124}{5}\right]+\left[\frac{124}{5^2}\right] = 24 + 4 = 28$

[We cannot do it further since 124 is not divisible by 5^3]

Hence, there are 28 times 5 alternate as a factor in 124!

Alternate:-

Divide successive quotients till we get 0 as the last quotient

$$\begin{array}{r|l} 5 & 124 \\ \hline 5 & 24 \rightarrow \\ \hline & 4 \rightarrow \end{array} \Big\} \text{ 28 (add up all the quotients)}$$

Ex.5 Find the largest power of 3 that can divide 270!

Sol. $\left[\frac{270}{3}\right]+\left[\frac{270}{3^2}\right]+\left[\frac{270}{3^3}\right]+\left[\frac{270}{3^4}\right]$

$+\left[\frac{270}{3^5}\right] = 90 + 30 + 10 + 3 + 1 = 134$

Hence, there are 134 times 3 involved as a factor in 270!

Alternate:-

Divide successive quotients till we get 0 as the last quotient

$$\left.\begin{array}{r|l} 3 & 270 \\ \hline 3 & 90 \rightarrow \\ \hline 3 & 30 \rightarrow \\ \hline 3 & 10 \rightarrow \\ \hline 3 & 3 \rightarrow \\ \hline 3 & 1 \rightarrow \\ \hline & 0 \end{array}\right\} \text{ 134 (add up all the quotients)}$$

* Alternate method is easier than first.

Ex.6 Find the largest power of 2 that can contained in:-
$1 \times 2 \times 3 \times 4 \,.........\, 22$?

Sol. $1 \times 2 \times 3 \times 4 \,.........\, 22$

$$\left.\begin{array}{r|l} 2 & 22 \\ \hline 2 & 11 \rightarrow \\ \hline 2 & 5 \rightarrow \\ \hline 2 & 2 \rightarrow \\ \hline 2 & 1 \rightarrow \\ \hline & 0 \end{array}\right\} 19$$

Number of 2's = 11 + 5 + 2 + 1= 19

Hence, there are 19 times 2 involved as a factor in 22!

Ex.7 Find the largest power of 5 that can contained in
$1 \times 2 \times 3 \times 4 \,.........\, 41 \times 42$

Sol.

$$\left.\begin{array}{r|l} 5 & 42 \\ \hline 5 & 8 \rightarrow \\ \hline 5 & 1 \rightarrow \\ \hline & 0 \end{array}\right\} 9$$

Hence, there are 9 times 5 involved as a factor in 42!

Ex.8 Find the largest power of 7 that can exactly divide 777!

Sol.

$$\left.\begin{array}{r|l} 7 & 777 \\ \hline 7 & 111 \rightarrow \\ \hline 7 & 15 \rightarrow \\ \hline 7 & 2 \rightarrow \\ \hline & 0 \end{array}\right\} \text{128 (add up all the quotients)}$$

Thus the highest power of 7 is 128 by which 777! can be divided.

Ex.9 Find the number of zeroes at the end of the product 10!

Sol. $10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times \underline{10}$

2×5

* In this type expression it is clear that in any factorial value, the number of 5's will always be lesser then the number of 2's In this condition. all we need to do is to count the number of 5's

Number of 5's = 2

Number of 2's = 5

But pair of 2's and 5's are =2

Then,

Number of zero's = 2

Alternate:-

Highest power of 5's in 10!

$\frac{10}{5} = 2$

Number of 5's = 2

Then number of zeroes = 2

Ex.10 Find the number of zeroes at the end of the product 100!

Sol.
$$\begin{array}{r|l} 5 & 100 \\ 5 & 20 \rightarrow \\ 5 & 4 \rightarrow \\ & 0 \end{array} \Big\} \text{ 24 (add up all the quotients)}$$

Number of 5's = 20 + 4 = 24

then number of zeroes = 24

Ex.11 Find the number of zeroes at the end of the product 1000!.

Sol. 1 × 2 × 3 × .... 999 × 1000
$$\begin{array}{r|l} 5 & 1000 \\ 5 & 200 \rightarrow \\ 5 & 40 \rightarrow \\ 5 & 8 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Big\} 249$$

The highest power of Number 5's = 200 + 40 + 8 + 1= 249

then number of zeroes at the end of the product = 249

Ex.12 Find the number of zeroes at the end of the product

1 × 3 × 5 × 7 .... 73 × 1024

Sol. 1 × 3 × 5 × 7 .... 73 × 1024

Number of 5's from 1 to 73
$$\begin{array}{r|l} 5 & 73 \\ 5 & 14 \rightarrow \\ 5 & 2 \rightarrow \\ & 0 \end{array} \Big\} \text{ 16 (add up all the quotients)}$$

Total number of 5's = 14 + 2 = 16

we know that

$1024 = 2^{10}$

number of 2 = 10

number of pairs (2 and 5) = 10

then number of zeroes 10

Ex.13 Find the number of zeroes at the end of the product

12 × 13 × 14 84

Sol. In this types expression first for us complete the sereis

$\underline{1} \times \underline{2} \times \underline{3}$ .... $\underline{11}$ × 12 × 13 .... 84 $- \underline{1} \times \underline{2} \times \underline{3} ......\underline{11}$

Number of zero (1 to 84)
$$\begin{array}{r|l} 5 & 84 \\ 5 & 16 \rightarrow \\ 5 & 3 \rightarrow \\ & 0 \end{array} \Big\} 19$$

Number of zero (1 to 11)
$$\begin{array}{r|l} 5 & 11 \\ 5 & 2 \\ & 0 \end{array} \Big\} 2$$

Number of zeroes = 19 – 2 = 17

Ex.14 Find the number of zeroes at the end of the product

512 × 513 1120

Sol. $\underline{1} \times \underline{2} \times \underline{3}$ .... $\underline{511}$ × 512 × 513 ....1120 – $\underline{1} \times \underline{2} \times \underline{3}$ .... $\underline{511}$
$$\begin{array}{r|l} 5 & 1120 \\ 5 & 224 \rightarrow \\ 5 & 44 \rightarrow \\ 5 & 8 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Big\} 277$$
$$\begin{array}{r|l} 5 & 511 \\ 5 & 102 \rightarrow \\ 5 & 20 \rightarrow \\ 5 & 4 \rightarrow \\ & 0 \end{array} \Big\} 126$$

Number of zeroes = 277 - 126 = 151

Ex.15 Find the number of zeroes at the end of the product

$1^5 \times 2^5 \times 3^5$ .... 32^5

Sol. In this type every second terms has power of 2's. It means power of 2's more than that of 5 So count the power of 5's

power of 5's = total power of 5's

$5^5 = (1\times 5)^5 = 1^5 \times 5^5 = 5$

$10^5 = (2 \times 5)^5 = 2^5 \times 5^5 = 5$

$15^5 = (3 \times 5)^5 = 3^5 \times 5^5 = 5$

$20^5 = (4 \times 5)^5 = 4^5 \times 5^5 = 5$

$25^5 = (5 \times 5)^5 = 5^5 \times 5^5 = 10$

$30^5 = (6 \times 5)^5 = 6^5 \times 5^5 = 5$

Number of 5's power = 35

then number of zeroes at the end of the product = 35

Ex.16 Find the number of zeroes at the end of the product

$1^1 \times 2^2 \times 3^3 \times 4^4$........ 28^{28}

Sol. count the numbe of 5's

power of 5's = total power of 5's

$5^5 = (1\times 5)^5 = 1^5 \times 5^5 = 5$

$10^{10} = (2 \times 5)^{10} = 2^{10} \times 5^{10} = 10$

$15^{15} = (3 \times 5)^{15} = 3^{15} \times 5^{15} = 15$

$20^{20} = (4 \times 5)^{20} = 4^{20} \times 5^{20} = 20$

$25^{25} = (5 \times 5)^{25} = 5^{25} \times 5^{25} = 50$

Number of 5's power

= 5 + 10 + 15 + 20 + 50

= 100

Then number of zeroes at the end of product = 100

Ex.17 Find the number of zeroes at the end of the product

$a = 1^3, b = 2^4, c = 3^5, ..., z = 26^{28}$

a × b × c × d .... × z

Sol. Count the number of 5's

power of 5's = total power of 5's

$5^7 = (1\times 5)^7 = 1^7 \times 5^7 = 7$

$10^{12} = (2 \times 5)^{12} = 2^{12} \times 5^{12} = 12$

$15^{17} = (3 \times 5)^{17} = 3^{17} \times 5^{17} = 17$

$20^{22} = (4 \times 5)^{22} = 4^{22} \times 5^{22} = 22$

$25^{27} = (5 \times 5)^{27} = 5^{27} \times 5^{27} = 54$

Number of 5's power

= 7 + 12 + 17 + 22 + 54 = 112

Then number of zeroes at the end of product = 112

Ex.18 Find the number of zeroes at the end of the product

$1^1 \times 2^2 \times 3^3 \times 4^4$........ 100^{100}

Sol. Count the power of 5's

$5^5 = 5$

$10^{10} = 10$

$15^{15} = 15$

. .

. .

. .

. .

$100^{100} = 100$

5 + 10 + 15 .... + 100

it is an a.p.series

we use a.p. formula

number of term = $\boxed{\frac{l - a}{d} + 1}$

l = last term of a.p.

a = first term of a.p.

d = common difference

number of term = $\frac{100-5}{5} + 1$

= 20

sum of nth term of a.p.

$= \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{20}{2}\left[2 \times 5 + (20-1) \times 5\right]$

$= 10\left[10 + 19 \times 5\right]$

$= 10\left[105\right] = 1050$

but $25^{25} = (5 \times 5)^{25} = 5^{25} \times 5^{25}$

$50^{50} = (2 \times 5 \times 5)^{50} = 2^{50} \times 5^{50} \times 5^{50}$

$75^{75} = (3 \times 5 \times 5)^{75} = 3^{75} \times 5^{75} \times 5^{75}$

$100^{100} = (4 \times 5 \times 5)^{100} = 4^{100} \times 5^{100} \times 5^{100}$

then number of 5's power

= 25 + 50 + 75 + 100

= 250

then number of total zeroes at the end of product

= 1050 + 250 = 1300

Ex.19 Find the number of zeroes at the end of the product

10 × 20 × 30 80

Sol. $10^1 \times 1 \times 10^1 \times 2 \times 10^1 \times 3....10^1 \times 8 = 10^8$ [1 × 2 × 3 .... 8]

from 1 to 8, number of 0's = 1
∴ Only one pair (2 & 5)
then total number of 0's
= 1 + 8 = 9

Ex.20 Find the number of zeroes at the end of the product
10 × 20 × 30 1000

Sol. $10^1 \times 1 \times 10^1 \times 2 \times 10^1 \times 3 \ldots 10^1 \times 100$
$= 10^{100}[1 \times 2 \times 3 \times \ldots\ldots 100]$
from 1 to 100 number of 0's

$$\begin{array}{r|l} 5 & 100 \\ 5 & 20 \rightarrow \\ 5 & 4 \rightarrow \\ & 0 \end{array} \Bigg\} \text{24 (add up all the quotients)}$$

number of 0's = 20 + 4 = 24
and 10^{100} , here number of zero = 100
total number of 0's
= 24 + 100 = 124
then number of zeroes = 124

EXERCISE

1. Find the number of zeroes at the end of the product 47!
(a) 8 (b) 9 (c) 10 (d) 11

2. Find the number of zeroes at the end of the product 125!
(a) 25 (b) 30 (c) 31 (d) 28

3. Find the number of zeroes at the end of the product 378!
(a) 93 (b) 90 (c) 75 (d) 81

4. Find the number of zeroes at the end of the product 680!
(a) 163 (b) 169 (c) 170 (d) 165

5. Find the number of zeroes at the end of the product 1000!
(a) 200 (b) 249 (c) 248 (d) 250

6. Find the number of zeroes at the end of the product 500!
(a) 100 (b) 124 (c) 120 (d) 125

7. Find the number of zeroes at the end of the product 1132!
(a) 280 (b) 271 (c) 281 (d) 272

8. Find the number of zeroes at the end of the product 1098!
(a) 280 (b) 270 (c) 271 (d) 262

9. Find the number of zeroes at the end of the product 2346!
(a) 580 (b) 583 (c) 575 (d) 580

10. Find the number of zeroes at the end of the product 2700!
(a) 673 (b) 670 (c) 669 (d) 675

11. Find the number of zeroes at the end of the product 10 × 15 × 44 × 28 × 70
(a) 2 (b) 3 (c) 4 (d) 5

12. Find the number of zeroes at the end of the product 12×5×15×24×13×30×75
(a) 4 (b) 5 (c) 2 (d) 3

13. Find the number of zeroes at the end of the product 2×4×6×.......48×50
(a) 6 (b) 12 (c) 7 (d) 5

14. Find the number of zeroes at the end of the product 1×3×5×7×9×11........99×101
(a) 24 (b) 5 (c) 2 (d) 0

15. Find the number of zeroes at the end of the product 21×22×23.......59×60
(a) 14 (b) 4 (c) 10 (d) 12

16. Find the number of zeroes at the end of the product 35 × 36 × 37 × ......89 × 90
(a) 21 (b) 7 (c) 14 (d) 20

17. Find the number of zeroes at the end of the product 41 × 42 ....109 × 110
(a) 26 (b) 9 (c) 17 (d) 25

18. Find the number of zeroes at the end of the product 140!×5×15×22×11×44×135
(a) 34 (b) 35 (c) 36 (d) 37

19. Find the number of zeroes at the end of the product 25! × 32! × 45!
(a) 10 (b) 23 (c) 22 (d) 7

20. Find the number of zeroes at the end of the product 1140! × 358! × 171!
(a) 282 (b) 325 (c) 411 (d) 370

21. Find the number of zeroes at the end of the product $1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \ldots\ldots 49^{49}$
(a) 225 (b) 250 (c) 240 (d) 245

22. Find the number of zeroes at the end of the product $100^1 \times 99^2 \times 98^3 \times 97^4 \ldots 1^{100}$
(a) 970 (b) 1124 (c) 875 (d) 975

23. Find the number of zeroes at the end of the product $1^{1!} \times 2^{2!} \times 3^{3!} \times 4^{4!} \ldots\ldots 10^{10!}$
(a) 51 (b) 10 (c) 5! + 10!
(d) N.O.T

24. Find the number of zeroes at the end of the product $2^2 \times 5^4 \times 4^2 \times 10^8 \times 6^{10} \times 15^{12} \times 8^{14} \times 20^{16} \times 10^{18} \times 25^{20}$
(a) 98 (b) 90 (c) 94 (d) 100

25. Find the number of zeroes at the end of the product 3200+1000+40000+32000+15000
(a) 15 (b) 13 (c) 2 (d) 3

26. Find the number of zeroes at the end of the product 3200×1000×40000×32000×15000
(a) 15 (b) 2 (c) 14 (d) 16

27. Find the number of zeroes at the end of the product 20×40×7600×600×300×1000
(a) 11 (b) 10 (c) 2 (d) 3

28. Find the number of zeroes at the end of the product 100! + 200!
(a) 24 (b) 25 (c) 49
(d) N.O.T

29. Find the number of zeroes at the end of the product $1^1 \times 2^2 \times 3^3 \times 4^4 \ldots\ldots 10^{10}$
(a) 10 (b) 15 (c) 5
(d) N.O.T

30. Find the number of zeroes at the end of the product 100! × 200!
(a) 49 (b) 24 (c) 73
(d) N.O.T

31. Find the number of zeroes at the end of the product 5×10×15×20×25×30×35×40×45×50
(a) 8 (b) 12 (c) 10 (d) 14

32. Find the number of zeroes at the end of the product 2 × 4 × 6 × 8 × 10 200
(a) 49 (b) 24 (c) 25 (d) 50

33. Find the number of zeroes at the end of the product 1 × 3 × 5 × 7 ....99 × 64
(a) 23 (b) 6 (c) 0 (d) 5

34. Find The No. zero at the end of the product of $2^{222} \times 5^{555}$
(a) 222 (b) 555 (c) 777 (d) 333

35. Find the number of zeroes at the end of the product 10 + 100 + 1000 + ...100000000
(a) 8 (b) 28 (c) 0 (d) 1

36. Find the number of zeroes at the end of the product $10^1 \times 10^2 \times 10^3 \times 10^4 \ldots\ldots 10^{10}$
(a) 10 (b) 55 (c) 50 (d) 45

37. Find the number of zeroes at the end of the product
$2^1\times5^2\times2^3\times5^4\times2^5\times5^6\times2^7\times5^8\times2^9\times5^{10}$
(a) 30 (b) 25
(c) 55 (d) 50

38. Find the number of zeroes at the end of the product
$(3^{123}-3^{122}-3^{121})\ (2^{121}-2^{120}-2^{119})$
(a) 1 (b) 0 (c) 119 (d) 120

39. Find the number of zeroes at the end of the product
$(8^{123}-8^{122}-8^{121})\ (3^{223}-3^{222}-3^{221})$
(a) 1 (b) 2 (c) 0 (d) 3

40 Find the number of zeroes at the end of the product
$5\times 10\times 15$ 75
(a) 11 (b) 15 (c) 10 (d) 18

ANSWER KEY

1. (c)	5. (b)	9. (b)	13. (a)	17. (c)	21. (b)	25. (c)	29. (b)	33. (b)	37. (b)
2. (c)	6. (b)	10. (a)	14. (d)	18. (d)	22. (b)	26. (d)	30. (c)	34. (a)	38. (a)
3. (a)	7. (c)	11. (b)	15. (c)	19. (b)	23. (c)	27. (a)	31. (a)	35. (d)	39. (b)
4. (b)	8. (c)	12. (b)	16. (c)	20. (c)	24. (a)	28. (a)	32. (b)	36. (b)	40. (a)

SOLUTION

1. (c)
5 | 47
5 | 9
5 | 1
  0

No. of zeroes = 9 + 1 = 10

2. (c)
5 | 125
5 | 25 →
5 | 5 →
5 | 1 →
  0
} 31

No of zeroes
= 25 + 5 + 1 = 31

3. (a)
5 | 378
5 | 75 →
5 | 15 →
5 | 3 →
  0
} 93

No. of zeroes = 75 + 15 + 3 = 93

4. (b)
5 | 680
5 | 136 →
5 | 27
5 | 5
5 | 1 →
  0
} 169

No. of zeroes = 136 + 27 + 5 + 1
= 169

5. (b)
5 | 1000
5 | 200 →
5 | 40 →
5 | 8 →
5 | 1 →
  0
} 249

No. of zeroes = 200 + 40 + 8+ 1
= 249

6. (b)
5 | 500
5 | 100 →
5 | 20 →
5 | 4 →
  0
} 124

No. of zeroes = 100 + 20 + 4
= 124

7. (c)
5 | 1132
5 | 226 →
5 | 45 →
5 | 9 →
5 | 1 →
  0
} 281

No. of zeroes = 226 + 45 + 9 + 1
= 281

8. (c)
5 | 1098
5 | 219 →
5 | 43 →
5 | 8 →
  | 1 →
  0
} 271

No. of zeroes = 219 + 43 + 8 + 1
= 271

9.(b)
5 | 2346
5 | 469 →
5 | 93 →
5 | 18 →
5 | 3 →
  0
} 583

No. of zeroes = 469 + 93 + 18 + 3
= 583

10.(a)
5 | 2700
5 | 540 →
5 | 108 →
5 | 21 →
5 | 4 →
  0
} 673

No. of zeroes = 540 + 108 + 21 + 4
= 673

11.(b) $10\times15\times44\times28\times70$
$\underline{2}\times\underline{5}\times3\times\underline{5}\times\underline{2}\times\underline{2}\times11\times\underline{2}$
$\times\underline{2}\times7\times\underline{2}\times5\times7$
In this expression
No of 2's = 6
No. of 5's = 3
Pair of 2's and 5's = 3
So, No of zeroes = 3

12.(b) $12\times5\times15\times24\times13\times30\times75$
$\underline{2}\times\underline{2}\times3\times\underline{5}\times3\times\underline{5}\times\underline{2}\times\underline{2}$
$\times\underline{2}\times3\times13\times\underline{2}\times3\times\underline{5}\times\underline{5}\times\underline{5}\times3$
No. of 2's → 6
No. of 5's → 5
Pair of 2's and 5's = 5
No. of zeroes = 5

13.(a) $2\times4\times6\times$.......48×50
⇒ $2\times1\times2\times2\times2\times3$ .... $2\times24\times2\times25$
⇒ $2^{25}\ (1\times2\times3\times4\times$.......$25)$
There are many 2's In This series we count 5's

5 | 25
5 | 5 →
5 | 1 →
  0
} 6

No. of 5's = 5 + 1 = 6
Then No. of zeroes = 6

14. (d) $1\times3\times5\times7\times9\times11$........$99\times101$
There is no 'zero' in this expression because there is no even present here.

15.(c) $21\times22\times23$.......59×60
$1\times2\times3$.....$19\times20\times21\times22\times23$......$59\times60$
$-\ 1\times2\times3$......20

$\begin{array}{r|l} 5 & 60 \\ \hline 5 & 12 \\ \hline 5 & 2 \\ \hline & 0 \end{array}\;\}14$ $\quad$ $\begin{array}{r|l} 5 & 20 \\ \hline 5 & 4 \\ \hline & 0 \end{array}\;\}4$

No. of zeroes 1 to 60 = 12 + 2 = 14

No. of zeroes 1 to 20 = 4

No. of zeroes 21 to 60 = 14 - 4 = 10

16. (c) 35 × 36 × 37 × ……89 × 90

1×2×3×4…..34×35×36……89×90

–1×2×3×………×33×34

No. of zeroes 1 to 90 = 18 + 3 = 21

$\begin{array}{r|l} 5 & 90 \\ \hline 5 & 18 \\ \hline 5 & 3 \\ \hline & 0 \end{array}\;\}21$

No. of Zeroes 1 to 34

$\begin{array}{r|l} 5 & 34 \\ \hline 5 & 6 \\ \hline 5 & 1 \\ \hline & 0 \end{array}\;\}7$

No. of Zeroes = 6 + 1 = 7

No. of zeroes 35 to 90 = 21 – 7 = 14

17.(c) 41 × 42 ………….109 × 110

1×2×3×4…..40×41×42……109×110

-1×2×3…..40

No. of zeroes 1 to 110 = 22 + 4 = 26

No. of zeroes 1 to 40 =8 + 1 = 9

$\begin{array}{r|l} 5 & 110 \\ \hline 5 & 22 \\ \hline 5 & 4 \\ \hline & 0 \end{array}\;\}26$ $\quad$ $\begin{array}{r|l} 5 & 40 \\ \hline 5 & 8 \\ \hline 5 & 1 \\ \hline & 0 \end{array}\;\}9$

No. of zeroes 41 to 110 = 26–9 = 17

18.(d) 140! would have 28 + 5 + 1 = 34

$\begin{array}{r|l} 5 & 140 \\ \hline 5 & 28 \\ \hline 5 & 5 \\ \hline 5 & 1 \end{array}\;\}34$

Now Remaining part

5 × 15 × 22 × 11 × 44 × 135

$5 \times 3 \times \underline{5} \times \underline{2} \times 11 \times 11 \times \underline{2} \times \underline{2} \times 11 \times \underline{5} \times 27$

No. of 2's = 3

No. of 5's = 3

Pair of (2 & 5) = 3

Remaining part of the Expression would have 3 zeroes

Total No. of zeroes = 34 + 3 = 37

19.(b) $\begin{array}{r|l} 5 & 25 \\ \hline 5 & 5 \\ \hline 5 & 1 \\ \hline & 0 \end{array}\;\}6$ $\quad$ $\begin{array}{r|l} 5 & 32 \\ \hline 5 & 6 \\ \hline 5 & 1 \\ \hline & 0 \end{array}\;\}7$

$\begin{array}{r|l} 5 & 45 \\ \hline 5 & 9 \\ \hline 5 & 1 \\ \hline & 0 \end{array}\;\}10$

No. of zeroes In 25! = 5 + 1 = 6

No of zeroes In 32! = 6 + 1 = 7

No. of zero In 45! 9 + 1 = 10

Total No. of zero = 6 + 7 + 10= 23

20.(c)

$\begin{array}{r|l} 5 & 1140 \\ \hline 5 & 228 \\ \hline 5 & 45 \\ \hline 5 & 9 \\ \hline 5 & 1 \\ \hline & 0 \end{array}\;\}283$ $\quad$ $\begin{array}{r|l} 5 & 358 \\ \hline 5 & 71 \\ \hline 5 & 14 \\ \hline 5 & 2 \\ \hline & 0 \end{array}\;\}87$

$\begin{array}{r|l} 5 & 171 \\ \hline 5 & 34 \\ \hline 5 & 6 \\ \hline 5 & 1 \\ \hline & 0 \end{array}\;\}41$

No. of zeroes in 1140!

= 228 + 45 + 9 + 1 = 283

No. of zeroes in 358!

= 71 + 14 + 2 = 87

No. of zeroes in 171!

= 34 + 6 + 1 = 41

Total No. of zeroes= 283+87+41 = 411

21.(b) The Fives will be less than the two's Hence, we need to count only the Fives

Thus,

5^5	=	$(5\times 1)^5$	=	5
10^{10}	=	$(5\times 2)^{10}$	=	10
15^{15}	=	$(3\times 5)^{15}$	=	15
20^{20}	=	$(4\times 5)^{20}$	=	20
25^{25}	=	$(5\times 5)^{25}$	=	50
30^{30}	=	$(5\times 6)^{30}$	=	30
35^{35}	=	$(5\times 7)^{35}$	=	35
40^{40}	=	$(5\times 8)^{40}$	=	40
45^{45}	=	$(5\times 9)^{45}$	=	45

5+10+15+20+50+30+35+40+45

No. of Fives = 250

Then,

Number of zeroes = 250

22.(b) The Five will be less than the two's Then count the number of five

$100^1\times 95^6\times 90^{11}\ldots\ldots\ldots 10^{91}\times 5^{96}$

(1+6+11……91+96) using sum of A.P.

a = 1, d = 5

No. of term = $\frac{96-1}{5}+1 = 20$

$S_n = \frac{20}{2}[2\times 1+19\times 5]$

= 10 [2 + 95] = 970

But

$100^1=(5\times 5\times 4)^1 = 5^1\times 5^1$

$75^{26} = (5\times 5\times 3)^{26} = 5^{26}\times 5^{26}$

$50^{51} = (5\times 5\times 2)^{51} = 5^{51}\times 5^{51}$

$25^{76} = (5\times 5)^{76} = 5^{76}\times 5^{76}$

Then no of zeroes

$= \underline{1}+\underline{26}+\underline{51}+\underline{76} = 154$

Total number of zeroes

= 154 + 970 = 1124

23 (c) Count the No. of 5's

$5^{5!}$ and $10^{10!}$

No. of 5's = 5! + 10!

Then ,

No. of zeroes = 5! + 10!

24.(a) Count the No. of 5's

Then

$5^4\times 10^8\times 15^{12}\times 20^{16}\times 25^{20}$

= 4 + 8 + 12 + 16 + 40

= 80

So, No. of zero = 80

25.(c)

$$\begin{array}{r} 3200 \\ 1000 \\ 40000 \\ 32000 \\ +\ 15000 \\ \hline 91200 \end{array}$$

No. or zero = 2

26.(d) 3200 × 1000 × 40000 × 32000 × 15000

No. of zero's 2 + 3 + 4 + 3 + 3

= 15

But 1500 = 3 × 5 × 100

Here 5 is present

When 5 is multiply by even number, then unit digit '0' is get.

Then,

No. of Total zero = 15 + 1 =16

27.(a) 20 × 40 × 7600 × 600 × 300 × 1000

No. of zeroes = 1 + 1 + 2 + 2 + 2 + 3 = 11

28.(a) 100! + 200!

No. of zeroes In 100! = 20 + 4 = 24

No. of zeroes In 200! = 40 + 8 + 1 = 49

When you add the two Number (One with 24 zeroes and the other with 49 zeroes at It's end)

The Total No. of zeroes = 24

29.(b) $1^1\times 2^2\times 3^3\times 4^4\ldots\ldots.10^{10}$

Count the Number of 5's

5^5 no of fives = 5

10^{10} No. of Fives = 10

No. of zeroes = 5 + 10 = 15

30.(c) 100! 200!

$$\begin{array}{r|l} 5 & 100 \\ \hline 5 & 20 \rightarrow \\ \hline 5 & 4 \rightarrow \\ \hline & 0 \end{array} \Big\} 24 \qquad \begin{array}{r|l} 5 & 200 \\ \hline 5 & 40 \rightarrow \\ \hline 5 & 8 \rightarrow \\ \hline 5 & 1 \rightarrow \\ \hline & 0 \end{array} \Big\} 49$$

No. of zeroes In 100! = 24

No. of zeroes In 200! = 49

When you multiply two numbers (One with 24 zeroes and the other with 49 zeroes at It's end). The Resultant Total No. of zeroes = 24 + 49 = 73

31.(a) $5 \times 10 \times 15 \times 20 \times 25 \times .....\times 50$

$5\times1\times5\times2\times5\times3\times5\times4 \; ......5\times10$

$5^{10}\,(1\times2\times3\times4 \; ....10)$

The two will be less than the fives hence we need to count only the two's

1 to 10 no of 2's

$$\begin{array}{r|l} 2 & 10 \\ \hline 2 & 5 \rightarrow \\ \hline 2 & 2 \rightarrow \\ \hline 2 & 1 \rightarrow \\ \hline & 0 \end{array} \Big\} 8$$

No. of 2's = 5 + 2 + 1 = 8

Then No. of zeroes = 8

32.(b) $2 \times 4 \times 6 \times 8 \times 10 \; .... \; 200$

$= 2\times1\times2\times2\times2\times2\times3 \; .... \; 2\times100$

$= 2^{100}\,(1\times2\times3\times.........100)$

We count No of 5

$$\begin{array}{r|l} 5 & 100 \\ \hline 5 & 20 \rightarrow \\ \hline 5 & 4 \rightarrow \\ \hline & 0 \end{array} \Big\} 24$$

No of 5's = 24

Then No. of zeroes = 24

33.(b) $1\times3\times5\times7.......99\times2^6$

Here No. of 5 is more than no.of 2 then count the number of 2

No. of 2's = 6

Now No. of zero = 6

34.(a) $2^{222} \times 5^{555}$

No. of 2's = 222

No. of 5's = 555

No. of 2's are less than Number of 5's

Pair (2's & 5's) = 222

No. of zero = 222

35.(d) 10+100+1000+ 100000000

$$\begin{array}{r} 10 \\ 100 \\ 1000 \\ \ldots\ldots\ldots \\ \ldots\ldots\ldots \\ 100000000 \\ \hline 111111110 \\ \hline \end{array}$$

This there is only one zero at the end of result

36.(b) $10^1 \times 10^2 \times 10^3 \times 10^4 \; .... 10^{10}$

$10^{(1+2+3........10)} = 10^{55}$

$$\boxed{\because\; 1+2+3....10 = \frac{10(10+1)}{2} = 55}$$

No. of zero = 55

37.(b) $2^1 \times 5^2 \times 2^3 \times 5^4 \times 2^5 \times 5^6 \times 2^7 \times 5^8 \times 2^9 \times 5^{10}$

$\Rightarrow 2^{(1+3+5+7+9)} \times 5^{(2+4+6+8+10)}$

$\Rightarrow 2^{25} \times 5^{30}$

Number of 2's are less than the Number of 5's

= Pair of (2×5) = 25

No of zero = 25

38.(a) $(3^{123} - 3^{122} - 3^{121})\,(2^{121} - 2^{120} - 2^{119})$

$\Rightarrow 3^{121}\,(3^2 - 3^1 - 3^0)\; 2^{119}\,(2^2 - 2^1 - 2^0)$

$\Rightarrow (3^{121})(2^{119})\;(9-3-1)\;(4 - 2 - 1)$

$\Rightarrow (3^{121})(2^{119})\;(5)\;(1)$

$= 5^1 \times 2^{119} \times 3^{121}$

No. of 5's = 1

No. of 2's = 119

Pair of (2 & 5) = 1

No. of zero = 1

39.(b) $(8^{123} - 8^{122} - 8^{121})\,(3^{223} - 3^{222} - 3^{221})$

$\rightarrow 8^{121}\,(8^2 - 8^1 - 1)\; 3^{221}\,(3^2 - 3^1 - 1)$

$\rightarrow 8^{121}\,(64 - 9)\; 3^{221}\,(9 - 4)$

$\rightarrow 8^{121} \times 55 \times 3^{221} \times 5$

$= 8^{121} \times 3^{221} \times 5 \times 11 \times 5$

$= (2^3)^{121} \times 3^{221} \times 5^2 \times 11$

$= 11 \times 5^2 \times 2^{363} \times 3^{221}$

No. of 2's = 363

No. of 5's = 2

Pair of (2 & 5) = 2

No . of zero = 2

40.(a) $5^1 \times 1 \times 5^1 \times 2 \times 5^1 \times 3 \; .... \; 5^1 \times 15$

$= 5^{15}\,(1 \times 2 \times 3 \; .... \; 15)$

each term multiple of 5 So power of 5's more than 2 then count the number of 2 from 1 to 15.

$$\begin{array}{r|l} 2 & 15 \\ \hline 2 & 7 \longrightarrow \\ \hline 2 & 3 \longrightarrow \\ \hline 2 & 1 \longrightarrow \\ \hline & 0 \end{array} \Big\} 11$$

number of (2 and 5) pairs = 11

then number of zeroes = 11

CHAPTER 03

FACTOR

Factor→ A number which divides a given number exactly is called factor (or divisor) of that given number and the given number is called a multiple of that number.

Ex. 1, 2, 4, and 8 are factors of 8 because 8 is perfectly divisible of 1, 2, 4 and 8

Factors and Multiple

Ex. Factors of 35 = 1, 5, 7, 35

Ex. Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Same,

Multiple of 2 = 2, 4, 6, 8, 10, ...

Multiple of 7 = 7, 14, 21, 28, 35 ....

* 1 is a factor of every number
* every number is a factor of itself
* every number, except 1 has at least 2 factor
* every number has infinite number of its multiples
* every number is a multiple of itself

Number of Factors

Let N be the composite number and a, b, c,.. be its prime factors and p, q, r be the indices (or powers) of a, b, c, i.e, if N can be expressed as $N = a^p.b^q.c^r$ then total number of factors of N = (p + 1) × (q + 1) × (r + 1)

If a is even prime factor, b and c are odd prime factors

The number of even factors

= (P) × (q + 1) × (r + 1)

The number of odd factors

= (1) × (q + 1) × (r + 1)

Ex.1 Find the total number of factors of 8.

Sol. 8 = 1, 2, 4 and 8 are Perfectly divisible

So number of factors = 4

* This method is easy for smaller number but for larger number its a probelm So use for alternate method

Alternate

$8 = 2 \times 2 \times 2 = 2^3$

Number of Total factors

= 3 + 1 = 4

Ex.2 Find the total number of factors of 240

Sol.

$$\begin{array}{r|l} 2 & 240 \\ 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$= 2^4 \times 3^1 \times 5^1$

Total Factors

= (4 + 1) × (1 + 1) × (1 + 1)

= 5 × 2 × 2 = 20

Ex.3 Find the total number of factors of 500.

Sol.

$$\begin{array}{r|l} 2 & 500 \\ 2 & 250 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$500 = 2 \times 2 \times 5 \times 5 \times 5$

$= 2^2 \times 5^3$

No. of factors = (2 + 1) × (3 + 1)

= 3 × 4 = 12

Number of even Factor

Ex4. Find the number of even factors of 24.

Sol. Factor of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Even Factor of 24 = 2, 4, 6, 8, 12, 24,

So,

Total number of even Factor of 24 = 6

Alternate

$$\begin{array}{r|l} 2 & 24 \\ 2 & 2 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$24 = 2^3 \times 3^1$

Number of even factor = 3×(1+1)

= 3 × 2 = 6

Ex.5 Find the number of even factor of 60.

Sol.

$$\begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$60 = 2 \times 2 \times 3 \times 5$

$= 2^2 \times 3^1 \times 5^1$

No. of even factor = 2 × (1+1) × (1+1)

= 2 × 2 × 2 = 8

No. of odd factor

Ex.6 Find the number of odd factors of 40.

Sol. 40 = 1, 2, 4, 5, 8, 10, 20, 40

Odd factors = 1, 5

Number of odd factors = 2

Alternate

$$\begin{array}{r|l} 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ 5 & 5 \\ & 1 \end{array}$$

$40 = 2 \times 2 \times 2 \times 5$

$= 2^3 \times 5^1$

No. of odd Factors= 1 × (1 + 1)

= 1 × 2

= 2

Ex.7 Find the number of factors, number of even factors and number of odd factors of 180

Sol. $180 = 2 \times 2 \times 3 \times 3 \times 5$

$= 2^2 \times 3^2 \times 5^1$

Total Number of factors

= (2 + 1) × (2 + 1) × (1 + 1)

= 3 × 3 × 2 = 18

Number of even factors
$= 2\,(2 + 1) \times (1 + 1)$
$= 2 \times 3 \times 2 = 12$
Number of odd factors
$= 1 \times (2 + 1) \times (1 + 1)$
$= 1 \times 3 \times 2 = 6$

Ex.8 Find the number of factors, number of even factors and number of odd factors of 360.

Sol.
$$\begin{array}{r|l} 2 & 360 \\ 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$
$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$= 2^3 \times 3^2 \times 5^1$
Total number of factors
$= (3 + 1) \times (2 + 1) \times (1 + 1)$
$= 4 \times 3 \times 2 = 24$
Number of even factors
$= 3 \times (2 + 1) \times (1 + 1)$
$= 3 \times 3 \times 2 = 18$
Number of odd factors
$= 1 \times (2 + 1) \times (1 + 1)$
$= 3 \times 2 = 6$

Ex.9 Find the number of factors, number of even factors and number of odd factors of 100

Sol.
$$\begin{array}{r|l} 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$
$100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$
Total no. of Factor
$= (2 + 1) \times (2 + 1) = 3 \times 3 = 9$
No. of even factor $= 2 \times (2 + 1)$
$= 2 \times 3 = 6$
No. of odd factor $= 1 \times (2 + 1)$
$= 1 \times 3 = 3$

Sum of factors

Let N be the composite number and a, b, c,... be its prime factors and p, q, r be the indices (or powers) of a, b, c, i.e. if N can be expressed as $N = a^p.b^q.c^r$

then the sum of all the divisors (or factors) of N
$= (a^o + a^1 + a^2 \ldots . a^p) \times (b^o + b^1 + b^2 \ldots . b^q) \times (c^o + c^1 + c^2 \ldots\ldots c^r)$

If a is even prime factor and b and c odd prime factors then

- Sum of even factors $= (a^1 + a^2 \ldots a^p) \times (b^o + b^1 + b^2 \ldots . b^q) \times (c^o + c^1 + c^2 \ldots\ldots c^r)$
- Sum of odd actor $= (a^o) \times (b^o + b^1 + b^2 \ldots . b^q) \times (c^o + c^1 + c^2 \ldots\ldots c^r)$

Ex10. Find the sum of all factors of 8.

Sol. factors of 8 = 1, 2, 4, 8
Sum of factors = 1 + 2 + 4 + 8
= 15

This method is easy for smaller number but for larger number its a probelm So use for alternate method

Alternate
$8 = 2^3$
sum of all factors $= (2^0 + 2^1 + 2^2 + 2^3)$
$= 1 + 2 + 4 + 8 = 15$

$\boxed{(a^o = 1, \text{ where } a = \text{real number})}$

Ex.11 find the sum of all factors , sum of even factors and sum of odd factors of 24.

Sol. factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24
sum of factors = 1 + 2 + 3 + 4 + 6 + 8 + 12 + 24 = 60
sum of even factors
= 2 + 4 + 6 + 8 + 12 + 24 = 56
Sum of odd factors = 1 + 3 = 4

Alternate
$$\begin{array}{r|l} 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array} \quad 24 = 2^3 \times 3^1$$
sum of all factors
$= (2^0 + 2^1 + 2^2 + 2^3) \times (3^0 + 3^1)$
$= (1 + 2 + 4 + 8) \times (1 + 3)$
$= 15 \times 4 = 60$
Sum of even factors
$= (2^1 + 2^2 + 2^3) \times (3^0 + 3^1)$
$= (2 + 4 + 8) \times (1 + 3)$
$= 14 \times 4 = 56$
Sum of odd factos
$= (2^0) \times (3^0 + 3^1) = 1 \times 4 = 4$

Ex12. find the sum of all factors , sum of even factors and sum of odd factors of 360.

Sol.
$$\begin{array}{r|l} 2 & 360 \\ 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array} \quad 360 = 2^3 \times 3^2 \times 5^1$$
sum of all factors
$= (2^0 + 2^1 + 2^2 + 2^3) \times (3^0 + 3^1 + 3^2) \times (5^0 + 5^1)$
$= 15 \times 13 \times 6 = 1170$
Sum of even factors
$= (2^1 + 2^2 + 2^3) \times (3^0 + 3^1 + 3^2) \times (5^0 \times 5^1)$
$= 14 \times 13 \times 6 = 1092$
sum of odd factors
$= 2^0 \times (3^0 + 3^1 + 3^2) \times (5^0 \times 5^1)$
$= 1 \times 13 \times 6 = 78$

Ex13. find the sum of all factors , sum of even factors and sum of odd factors of 100.

$$\begin{array}{r|l} 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array} \quad 100 = 2^2 \times 5^2$$
Sum of all factors
$= (2^0 + 2^1 + 2^2) \times (5^0 + 5^1 + 5^2)$
$= 7 \times 31 = 217$
Sum of even factors
$= (2^1 + 2^2) \times (5^0 + 5^1 + 5^2)$
$= 6 \times 31 = 186$
Sum of odd factors
$= (2^0) \times (5^0 + 5^1 + 5^2)$
$= 1 \times 31 = 31$

Prime Factorisation

Prime Factorisation : If a natural number is expressed as the product of prime numbers (factors) then the factorisation of the number is called its prime factorisation.

(i) **72**
$$\begin{array}{r|l} 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$
$72 = 2 \times 2 \times 2 \times 3 \times 3$
$72 = 2^3 \times 3^2$
number of prime factors = 3 + 2 = 5

(ii) **540**
$$\begin{array}{r|l} 2 & 540 \\ 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$
$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$= 2^2 \times 3^3 \times 5^1$

No. of prime factor = 2 + 3 + 1
= 6

(iii) find the number of prime factor **$2^3 \times 5^7 \times 21^4 \times 10^8$**

Sol. $2^3 \times 5^7 \times 21^4 \times 10^8$
$2^3 \times 5^7 \times (3 \times 7)^4 \times (2 \times 5)^8$
$2^3 \times 5^7 \times 3^4 \times 7^4 \times 2^8 \times 5^8$
$2^{11} \times 3^4 \times 5^{15} \times 7^4$
Total No. of prime factors
= 11 +4 + 15 + 4 = 34

Ex14. The Number of prime Factors In the expression $6^4 \times 8^6 \times 10^8 \times 12^{10}$ is
(a) 48 (b) 64
(b) 72 (d) 80

Sol. $6^4 \times 8^6 \times 10^8 \times 12^{10}$
$\Rightarrow (2\times3)^4 \times (2\times2\times2)^6 \times (2\times5)^8 \times (2\times2\times3)^{10}$
$\Rightarrow 2^4\times3^4 \times (2^3)^6 \times 2^8 \times 5^8 \times (2^2 \times 3)^{10}$
$\Rightarrow 2^4 \times 3^4 \times 2^{18} \times 2^8 \times 5^8 \times 2^{20} \times 3^{10}$
$\Rightarrow 2^{4+18+8+20} \times 3^{4+10} \times 5^8$
$= 2^{50} \times 3^{14} \times 5^8$
Total No. of prime factor
= 50 + 14 + 8 = **72**

EXERCISE

1 Find the number of Factors of 1728
(a) 28 (b) 29 (c) 30 (d) 31

2. Find the Number of Factor of 1420
(a) 12 (b) 13 (c) 14 (d) 15

3. Find the Number of Divisors of 10800
(a) 30 (b) 60 (c) 120 (d) 180

4. Find the No. of Prime Factor of 240.
(a) 4 (b) 5 (c) 6 (d) 8

5. Find the No. of prime factor. $(30)^{26} \times (25)^{51} \times (12)^{23}$
(a) 249 (b) 250 (c) 255 (d) 260

6. Find the No. of Prime Factor $(30)^{15} \times (22)^{11} \times (15)^{24}$
(a) 110 (b) 115 (c) 120 (d) 125

7. Find the No. of Prime Factor 180
(a) 4 (b) 5 (c) 6 (d) 7

8. Find the No. of Prime Factor of 536
(a) 4 (b) 5 (c) 6 (d) 3

9. Find the No. of prime Factor of 1044
(a) 4 (b) 5 (c) 10 (d) 9

10. Find The No. of prime factor of $(56)^{20} \times (36)^{31} \times (42)^{13} \times (13)^{21}$
(a) 240 (b) 242 (c) 264 (d) 248

11. Find the total Number of Prime Factors of
$2^{17} \times 6^{31} \times 7^5 \times 10^{11} \times 11^{10} \times 21^{12}$
(a) 142 (b) 144 (c) 140 (d) 146

12. Find the prime Factors 210
(a) 3 (b) 4 (c) 5 (d) 6

13. Find the sum of odd factors of544
(a) 16 (b) 18 (c) 20 (d) 22

14. For the Number 2450 find
(i) Number of all factors
(ii) Number of even factors
(iii) Number of odd factors
(a) 18,9,9 (b) 18,10,8
(c) 18,8,10 (d) 18,12,6

15. For the Number 760
(i) The sum and Number of all factors
(ii) The Sum and Number of even factors
(iii) The Sum and Number of odd factors

16. For The Number 96
(i) Sum and number of all factors
(ii) The sum and Number of even factors
(iii) The sum and Number of odd factors

17. For the Number 270
(i) The sum & Number of all Factor
(ii) The sum & Number of even factor
(iii) The sum & Number of odd Factor

ANSWER KEY

1. (a)	3. (b)	5. (a)	7. (b)	9. (b)	11. (c)	13. (b)
2. (a)	4. (c)	6. (b)	8. (a)	10. (c)	12. (b)	14. (a)

SOLUTION

1. **(a)**

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$1728 = 2\times2\times2\times2\times2\times2\times3\times3\times3$
$= 2^6 \times 3^3$
No. of factors = (6 + 1) × (3 + 1)
= 7 × 4 = 28

2 **(a)**

2	1420
2	710
5	355
71	71
	1

1420 = 2 × 2 × 5 × 71
$= 2^2 \times 5^1 \times 71^1$
No. of factors
= (2 + 1) × (1+1) × (1+1)
= 3 × 2 × 2 = 12

3. **(b)**

2	10800
2	5400
2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

$10800 = 2\times2\times2\times2\times3\times3\times3\times5\times5$
$= 2^4 \times 2^3 \times 5^2 = 2^4 \times 3^3 \times 5^2$

No. of factors = (4 + 1) (3 + 1) (2 + 1)
= 5 × 4 × 3 = 60

4. **(c)**
$$\begin{array}{r|l} 2 & 240 \\ 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$
$240 = 2\times2\times2\times2\times3\times5$
$= 2^4 \times 3^1 \times 5^1$
No. of prime factor = 4+1+1=6

5. **(a)** $(30)^{26} \times (25)^{51} \times (12)^{23}$
Break The form of prime factor
$\Rightarrow (2^1\times3^1\times5^1)^{26} \times (5\times5)^{51} \times (2\times2\times3)^{23}$
$\Rightarrow \underline{2^{26}} \times \underline{3^{26}} \times 5^{26} \times 5^{102} \times \underline{2^{46}} \times \underline{3^{23}}$
$\Rightarrow 2^{26+46} \times 3^{26+23} \times 5^{26+102}$
$\Rightarrow 2^{72} \times 3^{49} \times 5^{128}$
No. of prime factors
$\Rightarrow$ 72 + 49 + 128 = 249

6. **(b)** $(30)^{15} \times (22)^{11} \times (15)^{24}$
$\Rightarrow (2\times3\times5)^{15} \times (2\times11)^{11} \times (3\times5)^{24}$
$\Rightarrow 2^{15} \times 3^{15} \times5^{15} \times 2^{11} \times11^{11} \times3^{24}\times5^{24}$
$\Rightarrow 2^{15+11} \times3^{15+24} \times5^{15+24}+11^{11}$
$\Rightarrow 2^{26} \times3^{39} \times5^{39} \times11^{11}$
No. of Prime factor
26 + 39 + 39 + 11 = 115

7. **(b)**
$$\begin{array}{r|l} 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$
$180 = 2 \times 2 \times 3 \times3 \times 5$
$=2^2 \times 3^2 \times5^1$
No. of prime Factor = 2+2+1 = 5

8. **(a)**
$$\begin{array}{r|l} 2 & 536 \\ 2 & 268 \\ 2 & 134 \\ 67 & 67 \\ & 1 \end{array}$$
$536 = 2\times2\times2\times67 = 2^3 \times 67^1$
No. of prime factor = 3+1 = 4

9. **(b)**
$$\begin{array}{r|l} 2 & 1044 \\ 2 & 522 \\ 3 & 261 \\ 3 & 87 \\ 29 & 29 \\ & 1 \end{array}$$
$1044 = 2\times2\times3\times3\times29$
$= 2^2 \times 3^2 \times 29^1$

No. of prime factor
= 2 + 2 + 1 = 5

10. **(c)** $(56)^{20} \times (36)^{31} \times (42)^{13} \times (13)^{21}$
$\Rightarrow (2\times\underline{2}\times\underline{2}\times\underline{7})^{20} \times (\underline{2}^2\times\underline{3}^2)^{31} \times (\underline{2} \times \underline{3} \times\underline{7})^{13} \times (13)^{21}$
$\Rightarrow (2^3 \times7)^{20} \times (2^{62})\times(3)^{62} \times (\underline{2} \times \underline{3} \times \underline{7})^{13} \times (13)^{21}$
$\Rightarrow 2^{60} \times 7^{20} \times 2^{62} \times 3^{62}\times 2^{13} \times 3^{13}\times7^{13} \times13^{21}$
$\Rightarrow 2^{60+62+13} \times 3^{62+13} \times 7^{20+13} \times 13^{21}$
$\Rightarrow 2^{135} \times 3^{75} \times 7^{33} \times 13^{21}$
Number of prime factors
= 135 + 75 + 33 + 21 = 264

11. **(c)** $2^{17} \times6^{31} \times 7^5\times 10^{11} \times 11^{10} \times 21^{12}$
$\Rightarrow 2^{17} \times (2\times3)^{31} \times 7^5 \times (2\times5)^{11} \times11^{10} \times (3\times7)^{12}$
$\Rightarrow 2^{17} \times 2^{31} \times 3^{31} \times 7^5 \times 2^{11} \times 5^{11} \times 11^{10} \times 3^{12} \times 7^{12}$
$\Rightarrow 2^{17+31+11} \times 3^{31+12} \times 5^{11} \times 7^{5+12} \times 11^{10}$
$\Rightarrow 2^{59} \times 3^{43} \times5^{11} \times 7^{17} \times 11^{10}$
Total No. of Prime Factors
= 59 + 43 + 11 + 17 + 10
= 140

12. **(b)**
$$\begin{array}{r|l} 2 & 210 \\ 3 & 105 \\ 5 & 35 \\ 7 & 7 \\ & 1 \end{array}$$
$210 = 2^1 \times 3^1 \times 5^1 \times 7^1$
= 1 + 1 + 1 + 1 = 4

13. **(b)** $544 = 2\times2\times2\times2\times2\times17$
$= 2^5 \times 17^1$
Sum of odd factors
$= (2^0) \times (17^0 + 17^1)$
$= 1 \times (1 + 17)$
= 1 × 18 = **18**

14. (a)
$$\begin{array}{r|l} 2 & 2450 \\ 5 & 1225 \\ 5 & 245 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$
$2450 = 2 \times 5 \times 5 \times 7 \times 7$
$2450 = 2^1 \times 5^2 \times 7^2$
Number of Factor = (1 + 1) (2 + 1) (2 + 1)
= 2 × 3 × 3 = 18
Number of even Factor = 1 × (2 + 1) × (2 + 1)
= 1 × 3 × 3 = 9
Number of odd factor = 1(2+1)×(2+1)
= 3 × 3 = 9

15.
$$\begin{array}{r|l} 2 & 760 \\ 2 & 380 \\ 2 & 190 \\ 5 & 95 \\ 19 & 19 \\ & 1 \end{array}$$
$760 = 2\times2\times2\times5\times19$
$= 2^3\times5^1\times19^1$
(i) Numbe of factor
= (3+1)×(1+1)×(1+1)
= 4×2×2=16
Sum of factor
$= (2^0 + 2^1+2^2+2^3) \times (5^0 +5^1) \times (19^0 +19^1)$
= (1+2+4+8) × (1+5) × (1+19)
= 15×6×20 = 1800
(ii) Number of even factor
= 3 × (1 + 1) × (1 + 1)
= 3 × 2 × 2 = 12
Sum of even factor
$=(2^1 +2^2 + 2^3) \times (5^0 + 5^1) \times (19^0 +19^1)$
= 14 × 6 × 20 = 1680
(iii) Number of odd factors
= 1 × (1+1) × (1+1)
= 1 × 2 × 2 = 4
Sum of odd factors
$= (2^0) \times (5^0 + 5^1) \times (19^0 + 19^1)$
= 1 × 6 × 20 = 120

16. $96 = 2\times2\times2\times2\times2\times3$
$= 2^5 \times3^1$
(i) Number of all factor
= (5+1) × (1+1) = 6 × 2 = 12
Sum of all factor
$= (2^0 +2^1 + 2^2 +2^3 +2^4 +2^5)\times(3^0 +3^1)$
= (1+2+4+8+16+32)×(1+3)
= 63 × 4 = 252
(ii) Number of even factor
= 5 × (1+1) = 5 × 2 = 10
Sum of even factor
$= (2^1 + 2^2 +2^3 +2^4 +2^5)\times(3^0 +3^1)$
= (2+4+8+16+32) × (1+3)
= 62 × 4 = 248
(iii) Number of odd factor
= 1× (1 + 1) = 1 × 2 = 2
Sum of odd factor
$= (2^0) \times (3^0 + 3^1)$
= 1 × 4 = 4

17.Sol.

$$\begin{array}{r|l} 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$270 = 2\times3\times3\times3\times5$

$= 2^1\times3^3\times5^1$

(i) Number of all factor

$= (1+1)\times(3+1)\times(1+1)$

$= 2\times4\times2=16$

Sum of all factor

$= (2^0+2^1)\times(3^0+3^1+3^2+3^3)\times(5^0+5^1)$

$= 3 \times 40 \times 6 = 720$

(ii) Number of even factor

$= 1 \times (3+1) \times (1+1)$

$= 1 \times 4 \times 2 = 8$

Sum of even factor

$= (2^1)\times(3^0+3^1+3^2+3^3) \times (5^0 + 5^1)$

$= 2 \times 40 \times 6 = 480$

Number of odd factors

$= 1\times(3+1)\times(1+1)$

$= 4\times2 = 8$

Sum of odd factors

$=2^0\times(3^0+3^1+3^2+3^3)\times(5^0+5^1)$

$= 1\times40\times6 = 240$

CHAPTER 04

DIVISIBILITY

Rule of Divisibility

* **Divisibility by 2** ⟶ If Last digit of the number is divisible by 2

Divisibility by 4 ⟶ If Last two digits of the number are divisibile by 4

Divisibility by 8 ⟶ If Last three digits of the number are divisible by 8

Divisibility by 16 ⟶ If Last four digits of the number are divisible by 16

Divisibility by 32 ⟶ If Last five digits of the number are divisible by 32

* **Divisibility of 3** ⟶ All such numbers the Sum of whose digits are divisible by 3

Divisibility of 9 ⟶ All such numbers the Sum of whose digits are divisible by 9

* **Divisibility by 6** ⟶A number is divisible by 6 If it is simultaneously divisible by 2 and 3

* **Divisibility by 5** ⟶If Last digit (0 and 5) is divisible by 5

Divisibility by 25 ⟶If Last two digits of the number are divisibile by 25

Divisibility by 125 ⟶If Last three digits of the number are divisible by 125

* **Divisibility by 7** ⟶ Double the last digit and subtract it from the remaining leading truncated number. If the result is divisible by 7, then so was the original number.

* **Divisibility by 11** ⟶The difference of the sum of the digits in the odd places and the sum of digits in the even places is '0' or multiple of 11 is divisible

* **Divisibility by 3, 7, 11, 13, 21, 37 and 1001** ⟶

(i) If any number is made by repeating a digit 6 times the number will be divisible by 3, 7, 11, 13, 21, 37 and 1001 etc.

(ii) A six digit number if formed by repeating a three digit number; for example, 256, 256 or 678, 678 etc. Any number of this form is always exactly divisible by 7, 11, 13, 1001 etc.

Some important points

(a) If a is divisible by b then ac is also divisible by b.

(b) If a is divisible by b and b is divisible by c then a is divisible by c.

(c) If n is divisible by d and m is divisible by d then (m + n) and (m – n) are both divisible by d. This has an important implication. Suppose 48 and 528 are both divisible by 8. Then (528 + 48) as well as (528 – 48) are divisible by 8

Ex.1: Check to see if 203 is divisible by 7

Sol.

20	3
–6	×2
14	

Step I. Double the last digit = 3 × 2 = 6

Step II. Substract that from the rest of the Number = 20 – 6 = 14

Step III. Check to see if the difference is divisible by 7. 14 is divisible by 7 therefore 203 is also divisible by 7

Ex.2: Check to see if 68734 is divisible by 7

Sol.

6873	4
–8	×2
686	5
–10	×2
67	6
–12	×2
55	

55 is not divisible by 7 So, 68734 is not divisible by 7

Ex.3: Check to see if 24983 is divisible by 7

Sol.

2498	3
–6	×2
249	2
–4	×2
24	5
–10	×2
14	

14 is divisible by 7, therefore 24983 is also divisible by 7

Ex.4: Check to see if 65432577 is divisible by 7

Sol. When any number is made of more than five digits then we check divisiblity by 7 another rule

Step I. First for we make pair of 3 digits from right side (last)

<u>65</u> <u>432</u> <u>577</u>

Step II. Add alternate pairs = 65 + 577 = 642

Step III. Substract from remaning (3^{rd}) pair = 642 –432 = 210

If difference is divisible by 7 therefore number is also divisible by 7

Here difference = 210

210 is divisible by 7. Therefore 65432577 will be divisible by 7.

Note:- We can use First rule of divisibility by 7 but when a number has more than 5 digits this rule is easy for solve problem.

Ex.5: Check to see if 23756789765 is divisible by 7

Sol. <u>23</u> <u>756</u> <u>789</u> <u>765</u>

Step I. Add alternate pair

765 + 756 = 1521

23 + 789 = 812

Step II. Substract pairs

1521 – 812 = 709

709 is not divisible by 7 therefore 23756789765 is not divisible by 7

Ex.6: If 5432*7 is divisible by 9, then the digit in place of * is

(a) 0 (b) 1 (c) 6 (d) 9

Sol. (c) $\frac{5+4+3+2+x+7}{9} = \frac{21+x}{9}$

Put the value of 'x'. So the number is completely divisible by 9. Put $x = 6$

$= \frac{21+6}{9} = \frac{27}{9} =$ '0' remainder

Property: A number is completely divisible by 9 it the sum of the digits of the number is completely divisible by 9 and give no remainder.

Ex.7: When 335 is added to 5A7, the result is 8B2. 8B2 is divisible by 3. What is the largest posisble value of A?

(a) 8 (b) 2 (c) 1 (d) 4

Sol.
$$\begin{array}{ccc} 5 & A & 7 \\ 3 & 3 & 5 \\ \hline 8 & B & 2 \end{array}$$

$\Rightarrow$ A $\rightarrow$ 1, 2, 3, 4, 5 &

B $\rightarrow$ 5, 6, 7, 8, 9

8B2 is exactly $\therefore$ 8 + B + 2 = multiple of 3

$\therefore$ B = 5 or 8 $\Rightarrow$ A = 1 or 4

Ex.8: If * is a digit such that 5824* is divisible by 11, then * equals :

(a) 2 (b) 3 (c) 5 (d) 6

Sol. **(c)** 5 8 2 4 *

$\Rightarrow$ 5 +2 + * = 8 + 4

7 + * = 12

* = 12 - 7 = 5

Property: A number will be exactly divisible by 11 when the difference of the sum of odd place digits and even place digits is zero or divisible by 11.

Ex.9: Both the end digits of a 99 digit number N are 2 N is divisible by 11 then all the middle digits are:

(a) 1 (b) 2 (c) 3 (d) 4

Sol. (d) A number is divisible by 11 if the difference of the sum of digits at odd and even places by either zero or multiple of 11.

If the middle digit be 4, then 24442 or 244442 etc are divisible by 11.

Alternate:-

2............................2

This number has 99 digits. First (1^{st}) and last (99^{th}) term is 2 (given) middle terms (2^{nd} to 98^{th}) is assume 4.

3^{rd} to 98^{th} term = (4 .... 4)

difference between odd and even place of terms (3^{rd} to 98^{th}) = 0

Remaining Terms

1^{st}, 2^{nd} and 99^{th} (last)

Here,

1^{st} term = 2

2^{nd} terms = 4

Last terms = 2

Difference of odd and even place of the remaning term

(2 + 2) – 4 = 0

So, If the middle digit be 4, then 24442 or 244442 etc are divisible by 11.

Ex.10: Both the end digits of a 100 digit number N are 2. N is divisible by 11 then all the middle digits are:

(a) Only 4 (b) Only 2

(c) Only 3 (d) Any digit

Sol. (d) A number is divisible by 11 if the difference of the sum of digits at odd and even places by either zero or multiple of 11.

If the middle digit be any digit then 211112 or 23333332 etc are divisible by 11.

Alternate:-

2............................2

This number has 100 digits. First (1^{st}) and last (100^{th}) term is 2 (given) middle terms (2^{nd} to 99^{th}) is assume any digit.

2^{rd} to 99^{th} term = (n .... n)

difference between odd and even places (2^{rd} to 99^{th}) = 0

Remaining Terms

1^{st}, and 100^{th} (last)

Here,

1^{st} term = 2

Last terms = 2

Difference of odd and even place of the remaning terms

(2 – 2) = 0

So, If the middle digit be any digit, then (2..........2) is divisible by 11.

Ex.11: If the number 243x 51 is divisible by 9 then the value of the digit marked as x would be:

(a) 3 (b) 1 (c) 2 (d) 4

Sol. **(a)** 243x 51 is divisible by 9

divisiblty of 9 = sum of digit divisible by 9

$= 2 + 4 + 3 + x + 5 + 1 = \frac{15+x}{9}$

x would be $3 = \frac{18}{9}$

So, $x = 3$

Ex.12. $2^{71} + 2^{72} + 2^{73} + 2^{74}$) is divisible by

(a) 9 (b) 10 (c) 11 (d) 13

Sol.(b) Expression

$= 2^{71}(1 + 2 + 4 + 8)$

$= 2^{71} \times 15 = 2^{71} \times 3 \times 5$

Which is exactly divisible by 10.

Ex.13. A 4 digit number is formed by repeating a 2-digit number such of 2525, 3232, etc.Any number of this form is always exactly divisible by :

(a) 7 (b) 11 (c) 13

(d) Smallest 3-digit prime number

Sol.(d) Let the unit digit be x and ten's digit be y.

$\therefore$ Number

$= 1000y + 100x + 10y + x$

$= 1010y + 101x = 101(10y + x)$

Clearly, this number is divisible by 101, which is the smallest three-digit prime number.

Ex. 14: A six digit number is formed by repeating a three digit number; for example, 256, 256 or 678, 678 etc. Any number of this form is always exactly divisible by:

(a) 7 only (b) 11 only

(c) 13 only (d) 1001

Sol. (d) The number (x y z x y z) can be written, after given corresponding weightage of the places at which the digits occur, as 100000 x + 10000y + 1000z + 100+ 10y + z

= 100100x + 10010y + 1001z

= 1001 (100x + 10y + z)

Since 1001 is a factor, the number is divisible by 1001.

As the number is divisible by 1001, it will also be divisible by all three namely, 7, 11 and 13 and not by only one of these because all three are factors of 1001.

Ex.15: Which of the following number will always divide a six–digit number of the form *xyxyxy* (where $1 \le x \le 9$, $1 \le y \le 9$)?

(a) 1010 (b) 10101
(c) 11011 (d) 11010

Sol. (b) Number = $xy\ xy\ xy$
$= xy \times 10000 + xy \times 100 + xy$
$= xy\ (10000 + 100 + 1)$
$= xy \times 10101$

Ex.16. 47 is added to the product of 71 and an unknown number. The new number is divisible by 7 giving the quotient 98. The unknown number is a multiple of
(a) 2 (b) 5 (c) 7 (d) 3

Sol. (d) Let the unknown number be x.
$\therefore$ $71 \times x + 47 = 98 \times 7$
$\Rightarrow 71x = 686 - 47 = 639$
$\Rightarrow x = \frac{639}{71} = 9 = 3 \times 3$

Ex.17. When an integer K is divided by 3, the remainder is 1, and when K + 1 is divided by 5, there remainder is 0. Of the following, a possible value of K is
(a) 62 (b) 63 (c) 64 (d) 65

Sol. (c) Take option (d)
When 64 is divided by 3, remainder = 1
When 65 is divided by 5, remainder = 0

Ex.18. If n is a whole number greater than 1, then $n^2\ (n^2-1)$ is always divisible by:
(a) 16 (b) 12 (c) 10 (d) 8

Sol.(b) $n^2\ (n^2 - 1) = n^2\ (n+1)\ (n-1)$
Now, we put values n = 2,3.....
When n = 2
$\therefore$ $n^2\ (n^2-1) = 4 \times 3 \times 1 = 12$, which is a multiple of 12
When n= 3.
$n^2\ (n^2-1) = 9 \times 4 \times 2 = 72$,
Which is also a multiple of 12.
etc.

Ex.19. If n is even, $(6^n - 1)$ is divisible by
(a) 37 (b) 35 (c) 30 (d) 6

Sol.(b) When n = 2.
$6^n - 1 = 6^2 - 1 = 36 - 1 = 35$
When, n = an even number,
$a^n - b^n$ is always divisible by $(a^2 - b^2)$.

Relation among divisor, dividend quotient and remainder :

$\rightarrow$ 27 is divided by 6 then $\frac{27}{6}$

$$\begin{array}{r|l} \text{divisor}\ 6 & 27\ (4 \rightarrow \textbf{Quotient} \\ & \underline{24} \\ & 3 \rightarrow \textbf{Remainder} \end{array}$$
(27 = **dividend**)

$6 \times 4 + 3 = 27$
divisor × quotient + remainder = dividend

Ex.20: In a problem involving division, the divisor is eight times the quotient and four times the remainder. If the remainder is 12, then the dividend is:
(a) 300 (b) 288 (c) 512 (d) 524

Sol. (a) Remainder = 12
Divisor = 4 × 12 = 48
Quotient = $\frac{48}{8} = 6$
Dividend = divisor × quotient + remainder = 48 × 6 + 12
= 288 + 12 = 300

Ex.21: The divisor is 25 times the quotient and 5 times the remainder. If the quotient is 16, the dividend is :
(a) 6400 (b) 6480
(c) 400 (d) 480

Sol. (b) Dividend= divisor × quotient + Remainder
According to Question
Divisor = 16×25 = 5×R
$\Rightarrow R = \frac{1}{5} \times 16 \times 25$
Dividend
$= [(16 \times 25) \times 16] + \frac{1}{5} \times 16 \times 25$
$= [16 \times 25 \times 16] + 80 = 6480$

Ex.22: In a division problem, the divisor is 4 times the quotient and 3 times the remainder. If remainder is 4, the dividend is:
(a) 36 (b) 40 (c) 12 (d) 30

Sol. (b)
$$\begin{array}{r|l} \text{Divisor} & \text{Dividend}\ (\text{Quotient} \\ & \underline{-} \\ & \text{Remainder} \end{array}$$
According to the question
$$\begin{array}{r|l} 12 & \text{Dividend}\ (3 \\ & \underline{-} \\ & 4 \end{array}$$
Dividend is (Divisor × Quotient) + Remainder
= (12 × 3) + 4
= 40

Ex.23: In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, the dividend is:

Sol. Remainder = 46
Divisor = 5 × 46 = 230
Quotient = $\frac{230}{10} = 23$
Dividend = divisor × quotient + remainder
= 230 × 23 + 46
= 5290 + 46 = 5336

Ex.24: On dividing 397246 by a certain number, the quotient is 865 and the remainder is 211. Find the divisior.

Sol. Divisor = $\frac{397246 - 211}{865} = 459$

Ex.25: A number when divided by 14 leaves a remainder of 8, but when the same number is divided by 7, it will leave the remainder:
(a) 3 (b) 2 (c) 1
(d) Can't be determined

Sol.
$$\begin{array}{r|l} 14 & \text{N}\ (x \\ & \overline{\quad} \\ & 8 \rightarrow \text{Remainder} \end{array}$$
N = 14x + 8
According to the question,
$\frac{N}{7} = \frac{14x + 8}{7}$ (0 under $14x$, 1 under 8)
Remainder = $\frac{8}{7} = 1$

Ex.26: If a number is divided by 102 and leaves remainder 91. If this number is divided by 17 the remainder

Sol.
$$\begin{array}{r|l} \textbf{divisor}\ 102 & \text{M}\ (\text{P} \rightarrow \textbf{Quotient} \\ & \overline{\quad} \\ & 91 \rightarrow \textbf{Remainder} \end{array}$$
(M = **dividend**)
We know that

$\boxed{\text{divisor} \times \text{quotent} + \text{remainder} = \text{divided}}$

M = 102 × P + 91
Now = M = $\frac{17 \times 6 \times P + 91}{17}$
M = $\frac{17 \times 6P}{17} + \frac{91}{17}$

$R = \frac{91}{17} \Rightarrow R = 6$

Note: If we have to find remainder of those term which divide previous term we will take remainder of it and divide by this term and we have to get.

$\frac{91}{17}$ **R = 6**

Ex.27: If a number is divided by 84 and leaves remainder 37. If this number is divided by 12.

Sol. Then the remainder 84 is divisible by 12

So, remainder $= \frac{37}{12} = \mathbf{1}$

Ex.28: A number when divided by 899 gives a remainder 63. If the same number is divided by 29, the remainder will be:

(a) 10 (b) 5 (c) 4 (d) 2

Sol. (b) $\frac{\text{Remainder}}{29} = \frac{63}{29}$

$\Rightarrow$ remainder = 5

Ex.29: A number when divided by 296 gives a remainder 75. When the same number is divided by 37 the remainder will be

(a) 1 (b) 2 (c) 8 (d) 11

Sol. (a) $\frac{\text{Remainder}}{37} = \frac{75}{37}$

remaindder = 1

Ex.30: A number being divided by 52 gives remainder 45. If the number is divided by 13, the remainder will be:

(a) 5 (b) 6 (c) 12 (d) 7

Sol. (b) since 13 is factor of 52. So divide its remainder by 13

Remainder $= \frac{45}{13} = 6$

Ex.31: If A number is divided by 225 a remainder at 70. But when a square of the number is divided by 15. What is the remainder?

Sol. $225\overline{)\,N\,}(Q$

$\overline{\quad 70 \quad}$

N = 225 Q + 70

Square of number = N^2

$= (225\ Q + 70)^2$

Then $= \frac{(\overset{0}{\overset{\uparrow}{225\ Q}} + \overset{10}{\overset{\uparrow}{70}})^2}{15}$

$= \frac{(10)^2}{15} = \frac{100}{15}$

$\boxed{\text{Remainder} = 10}$

Alternate:-

$\frac{(\text{Remainder})^2}{15} = \frac{(70)^2}{15}$

$= \frac{(\overset{10}{\overset{\uparrow}{70}})^2}{15} = \frac{(10)^2}{15} = \frac{100}{15}$

Remainder = 10

Ex.32: If a number is divided 36 and leaves remainder 23. If cube of this number is divided by 12. Then what is the remainder.

Sol. $36\overline{)\,N\,}(Q$

$\overline{\quad 23 \quad}$

N = 36 Q + 23

cube of number

$= N^3 = (36\ Q + 23)^3$

Now, $\frac{(\overset{0}{\overset{\uparrow}{36\ Q}} + \overset{-1}{\overset{\uparrow}{23}})^3}{12}$

$= \frac{(-1)^3}{12} = \frac{-1}{12}$

Remainder = 12 – 1 = 11

Alternate:-

Remainder $= \frac{(\overset{-1}{\overset{\uparrow}{23}})^3}{12}$

$= \frac{(-1)^3}{12} = \frac{-1}{12}$

Remainder = 12 – 1 = 11

Ex.33: Two number when divided by 17. Leave remainder 13 and 11 respectively if the sum of those two numbers is divided by 17 the remainder will be

Sol. N_1 (First Number) = $17x+13$

N_2 (Second no.) = $17y + 11$

$\frac{(N_1+N_2)}{17} = \frac{17(x+y)}{17} + \frac{13+11}{17}$

Remainder $= \frac{24}{17} = 7$

Ex.34: When a number is divided certain divisor, remainder is 35 but another no. is divided by the same divisor remainder is 27. If the sum of both number is divided by the same certain divisor remainder is 20. Find the certain divisor

Sol. $N_1 = Dx + 35$ .... (i)

$N_2 = Dy + 27$ ......(ii)

Here N_1 = First no.

N_2 = Second no.

D = certain divisor

x & y = Quotient

(i) + (ii)

According to the question

$\frac{N_1+N_2}{D} = \frac{D(x+y)+62}{D}$

Here divisor is same

Then Remainder $= D\overline{)\,62\,}(1$

$\quad -42$

$\overline{\quad 20 \quad}$

Remainder = 20

Quotient = 1

Dividend = 62

Divisor = 62 – 20 × 1 = 42

$\boxed{\text{Divisor} = 42}$

Alternate:

$$\frac{N_1}{D} \qquad \frac{N_2}{D}, \qquad \frac{N_1+N_2}{D}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$R_1 \qquad R_2 \qquad R_3$$

$\boxed{D = R_1+R_2-R_3}$

Then divisor = 35 + 27 – 20

= **42**

Successive Division : If the quotient in a division is further used as a dividend for the next divisor and again the latest obtained divisor is used as a dividend for another divisor and so on, then it is called then " successive division" i.e, if we divide 150 by 4, we get 37 as quotient and 2 as a remainder then if 37 it divided by another divisior say 5 then we get 7 as a quotient and 2 remainder and again if we divide 7 by another divisior say 3 we get 2 as quotient and 1 as a remainder i.e, we can represent it as following

4	150		
5	37	→	2
3	7	→	2
	2	→	1

} Remainder

Now you can see that the quotient obtained in the first division behaves as a dividend for another divisor 5. Once again the quotient 7 is treated as a dividend for the next divisor 3. Thus it is clear from the above discussion as

Dividend	Divisior	Quotient	Remainder
150	4	37	2
37	5	7	2
7	3	2	1

So the 150 is successively divided by 4, 5, and 3 the corresponding remainders are 2, 2 and 1.

Ex.35: The least possible number when successively divided by 2, 5, 4, 3 gives respective remainders of 1, 1, 3, 1 is :

(a) 372 (b) 275 (c) 273 (d) 193

Sol. The problem can be expressed as

2	A		
5	B	→	1
4	C	→	1
3	D	→	3
	E	→	1

} Remainder

So it can be solved as

(((((E × 3) + 1)4 + 3) 5 + 1)2 + 1) =A (where A is the required number)

So for the least possible number E = 1 (the least positive integer)

then A= (((((1 × 3) + 1) × 4 + 3) 5 + 1)2 + 1)

[Since at E = 0, we get a two digit number]

So it can be solved as

2	**193**	1
5	**96**	1
4	**19**	3
3	**4**	1
	1	

D = 1 × 3 + 1 = 4

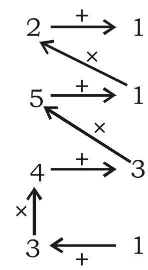
C = 4 × 4 + 3 = 19

B = 19 × 5 + 1 = 96

A = 96 × 2 + 1 = 193

So $\boxed{\text{Number}= 193}$

Alternate :



Step 1. (1 + 3) × 4 = 16

Step 2. (16 + 3) × 5 = 95

Step 3. (95 + 1) × 2 = 192

Step 4. (192 + 1) = **193**

Or

Number = (((1 + 3) × 4 + 3) × 5 + 1) × 2 + 1

= ((16 + 3) × 5 + 1) × 2 + 1

= 96 × 2 + 1

$\boxed{\text{Number} = 193}$

Ex.36: A number when divided successively by 4 and 5 leaves remainders 1 and 4 respectively. When it is successively divided by 5 and 4 the respective remainder will be

Sol. The least number × in this case will be determind as follows

4	X
5	y –1
	1 – 4

y = 5 × 1 + 4 = 9

X = 4 × y + 1 = 4 × 9 + 1 = 37

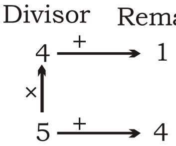
Now

5	37
4	7 –2
	1 – 3

Here, the respective remainder are 2, 3

Alternate:-

Successive Divisor Remainder



Number = (4 + 5) × 4 + 1 = 36 + 1 = 37

5	37
4	7 – 2
	1 – 3

Remainder = 2, 3

Alternate II.

4	37	1
5	9	4
	1	

1 × 5 + 4 = 9

9 × 4 + 1 = 37

Number = 37

Now, divided by 5 and 4 successively

5) 37 (7
35
2 → Remainder

4) 7 (1
4
3 → Remainder

Remainder = 2, 3

Ex.37: Find the smallest no. which one successive divided 5, 3 and 7 give remainder 2, 1 and 2 respectively

Sol.

5	**142**	2
3	**28**	1
7	**9**	2
	1	

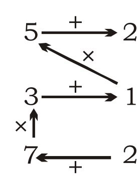
1 × 7 + 2 = 9

9 × 3 + 1 = 28

28 × 5 + 2 = 142

Number = 142

Alternate:-



$\left[\left((2+7)\times 3+1\right)\times 5\right]+2$

= (28 × 5) + 2

Number = 142

Ex.38: A least number when successively divided by 2, 3, 5 it leaves the respective remainder 1, 2 and 3. What will be the remainder if this number will be divided by 7 ?

Sol.

2	**53**	1
3	**26**	2
5	**8**	3
	1	

Step. I 5 × 1 + 3 = 8

Step. II 8 × 3 + 2 = 26

Step. III 26 × 2 + 1 = 53

So the least number = 53

According to the question, 53 is divided by 7 then remainder = **4**

Ex.39: Find the smallest no. which when successive divided by 4, 5 and 6 give remainder 2, 1 and 1. Also find sequence of remainder if the sequence of divisor is reverse.

Sol.

4	146	2
5	36	1
6	7	1
	1	

6 × 1 + 1 = 7
7 × 5 + 1 = 36
36 × 4 + 2 = 146
Number = 146
According to the question,
Now divisor is 6, 5 and 4
Then successive remainder

6	146
5	24 – 2
4	4 – 4
	1 – 0

Remainder = 2, 4 and 0

Ex.40: A number when divided successively by 6, 7 and 8, it leaves the respective remainders of 3, 5 and 4, what will be the last remainder when such a least possible number is divided successively by 8, 7 and 6.

Sol.

6	**537**	3
7	**89**	5
8	**12**	4
	1	

Step. I 1 × 8 + 4 = 12
Step. II 12 × 7 + 5 = 89
Step. III 89 × 6 + 3 = 537
least number = 537
Now we divide 537 successively by 8, 7 and 6.

8	**537**	
7	**67 —1**	**Remainder**
6	**9 —4**	
	1 —3	

So, 3 is the last Remainder.

Ex.41: A number when divided by 3 leaves a remainder1. When the quotient is divided by 2. It leaves a remainder1. What will be the remainder when the number is divided by 6?

Sol.

$$\begin{array}{ccc} 3 & \xrightarrow{+} & 1 \\ \times\uparrow & & \\ 2 & \xrightarrow{+} & 1 \end{array}$$

Number = $((1+2)\times 3)+1$

= 9 + 1 = 10

According to question,

Remainder = $\frac{10}{6}$ = 4

Ex.42: A number divided by 13 leaves a remainder 1 and if the quotient is divided by 5. We got a remainder of 3. What will be the remainder if the number is divided by 65?

Sol.

$$\begin{array}{ccc} 13 & \xrightarrow{+} & 1 \\ \times\uparrow & & \\ 5 & \xrightarrow[+]{} & 3 \end{array}$$

Number = $[(3+5)\times 13]+1$

= 8 × 13 + 1 = 105

According to the question,

Remainder = $\frac{105}{65}$ = 40

BINOMIAL THEOREMREM

* **Statement of the theorem:-**

According to the theorem, it is possible to expand any power of x +y into a sum of the form

$$\boxed{(x+y)^n = {}^nc_0\, x^n y^0 + {}^nc_1\, x^{n-1} y^1 + {}^nc_2\, x^{n-2} y^2 + \ldots.. + {}^nc_{n-1}\, x^1 y^{n-1} + {}^nc_n\, x^0 y^n}$$

Where each $\left({}^nc_k\right)$ is a specific positive integer known as **binomial coefficient.** (When an exponent is zero, the corresponding power expression is taken to be 1 and this multiplicative factor is often omitted from the term. Hence one often sees the right side written as $\left({}^nc_0\right) x^n + \ldots.)$ This formula is also referred to as the **binomial formula or the binomial identity.** Using **summation notation,** it can be written as

$$(x+y)^n = \sum_{k=0}^{n} \left({}^nc_k\right) x^{n-k} y^k = \sum_{k=0}^{n} \left({}^nc_k\right) x^k y^{n-k}$$

The final expression follows from the previous one by the symmetry of x and y in the first expression, and by comparison it follows that the sequence of binomial coefficients in the formula is symmetrical. A simple variant of the binomial formula is obtained by **substituting** 1 for y, so that it involves only a single **variable,** In this form, the formula reads

$(1 + x)^n = {}^nc_0\, x^0 + {}^nc_1\, x^1 + {}^nc_2\, x^2 + \ldots.. + {}^nc_{n-1}\, x^{n-1}$ ${}^nc_n\, x^n$

or equivalently

$$(1+x)^n = \sum_{k=0}^{n} \left({}^nc_k\right) x^k.$$

Ex. (i) $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$,

(ii) $(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + {}^{y4}y$

Some important points

1. The powers of x start at n and decrease by 1 in each term until they reach 0 (with {{{1}}} often unwritten);
2. The powers of y start at 0 and increase by 1 until they reach n;
3. The n^{th} row of pascal's Triangle will be the coefficients of the expanded binomial when the terms are arranged in this way;
4. The number of terms in the expansion before like terms are combined is the sum of the coefficients and is equal to 2^n, and
5. there will be **(n + 1) terms** in the expression after

combining like terms in the expansion.

The binomial theorem can be applied to the powers of any binomal. for example.

$(x + 2)^3$

$= x^3 + 3x^3 + 3x^2(2) + 3x(2)^2 + 2^3$

$= x^3 + 6x^2 + 12x + 8.$

* For a binomial involving subtraction, the theorem can be applied by using the form **$(x - y)^n = (x + (-y))^n$.** This has the effect of changing the sign of every other term in expansion:

$(x - y)^3 = (x + (-y))^3$

$= x^3 + 3x^2(-y) + 3x(-y)^2 + (-y)^3$

$= x^3 - 3x^2y + 3xy^2 - y^3$

* $(a^n + b^n)$ is always divisible by $(a + b)$ when $n \rightarrow$ odd power

HINT

$\boxed{a^3+b^3 = \underline{(a+b)}\,(a^2+ab+b^2)}$

Ex.43: Which of the following number will not completly divide the $(29)^{37} + (17)^{37}$?

(a) 2 (b) 11 (c) 23 (d) 46

Sol. (b) $(29^{37} + 17^{37})$, $(29 + 17) = 46$ Completely divisible by 46 = 1, 2, 23, 46

This will be completely divisible by all the factors of 46 So 11 will not divide the given number.

Ex.44: Which of the following will not completely divide $(3^{41} + 7^{82})$?

(a) 4 (b) 52 (c) 17 (d) 26

Sol. (c) $3^{41} + 7^{82}$

$\Rightarrow$ (Equalising the power)

$\Rightarrow 3^{41} + (7^2)^{41}$

$\Rightarrow 3^{41} + 49^{41}$, $3 + 49$ = Completely Divisible by 52

52 = 1, 2, 4, 13, 26, 52

So, 17 is not the factor of 52 hence this number will be completely divisible by 17

Ex.45. $(49)^{15} - 1$ is exactly divisible by:

(a) 50 (b) 51 (c) 29 (d) 8

Sol.(d) $x^n - a^n$ is exactly divisible by $(x - a)$ if n is odd.

$\therefore$ $(49)^{15} - (1)^{15}$ is exactly divisible by $49-1 = 48$, that is a multiple of 8.

Ex.46: Which of the following completely divide $(29^{47} + 23^{47} + 17^{47})$

(a) 21 (b) 22 (c) 23 (d) 24

Sol. (c) $\dfrac{29^{47}+17^{47}+23^{47}}{23}$

$29^{47} + 17^{47}$ will be completely divisible by 46 or its factor (2 and 23) and 23^{47} is completely divisible by 23 so 23 will completely divide this number

* $(a^n - b^n)$ is always divisible by $(a - b)$ where $n \rightarrow$ odd power

Hint

$\boxed{(a^3+b^3) = \underline{(a-b)}\,(a^2+ab+b^2)}$

* $(a^n - b^n)$ is always completely divisible by $(a - b)$, $(a + b)$ where $n \rightarrow$ (even power)

Hint

$\boxed{(a^2 - b^2) = (a-b)\,(a+b)}$

Ex.47 Which of the following will not divide $23^{10} - 1024$ complety.

(a) 3 (b) 5 (c) 7 (d) 4

Sol. 1024 is the value of 2^{10} and

$23^{10} - 2^{10} \rightarrow (23 - 2)$ and $(23 + 2)$ is completely divisible

$(23 - 2) = 21 = 1, 3, 7, 21$

$(23 + 2) = 25 = 1, 5, 25$

Hence this number is not divisibe by 4.

(a^n+b^n) $n \rightarrow$ odd (a^n+b^n) is perfectly divisble by $(a+b)$	(a^n-b^n) $n \rightarrow$ odd (a^n-b^n) is perfectly divisble by $(a-b)$	(a^n-b^n) $n \rightarrow$ even (a^n-b^n) is perfectly divisble by $(a+b),(a-b)$	(a^n+b^n) $n \rightarrow$ even It can't be determined
$\dfrac{(a^3+b^3)}{\underline{(a+b)}\,(a^2-ab+b^2)}$	$\dfrac{(a^3-b^3)}{\underline{(a-b)}\,(a^2+b^2+ab)}$	$\dfrac{(a^2-b^2)}{\underline{(a+b)\,(a-b)}}$	$a^2+b^2 = ...$

EXERCISE

1. In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 48, the dividend is:
(a) 808 (b) 5008
(c) 5808 (d) 8508

2. The divisor is 321, the quotient 11 and the remainder 260. Find the dividend.
(a) 3719 (b) 3971
(c) 3791 (d) 3179

3. In a division sum, the divisor is 5 times the remainder and the quotient is 6 times the remainder which is 73. What is the dividend ?
(a) 169943 (b) 159963
(c) 159943 (d) 159953

4. The sum of 20 odd natural number is equal to :
(a) 210 (b) 300 (c) 400 (d) 240

5. When a number is divided by 56, the remainder obtained is 29. What will be the remainder when the number is divided by 8 ?
(a) 4 (b) 5 (c) 3 (d) 7

6. A number when divided successively by 4 and 5 leave the remainder 1 and 4 respectively. When it is successively divided by 5 and 4 the respective remainders will be:
(a) 4,1 (b) 3,2 (c) 2,3 (d) 1,2

7. $4^{61}+4^{62}+4^{63}+4^{64}$ is divisible by :
(a) 3 (b) 10 (c) 11 (d) 13

8. $(3^{25} + 3^{26} + 3^{27} + 3^{28})$ is divisible by :
(a) 11 (b) 16 (c) 25 (d) 30

9. The least number, which must be added to 6709 to make it exactly divisible by 9, is
(a) 5 (b) 4 (c) 7 (d) 2

10. If 78*3945 is divisible by 11 where * is a digit, then * is equal to :
(a) 1 (b) 0 (c) 3 (d) 5
11. When a number is divided by 357 the remainder is 39. If same number is divided by 17, the remainder will be :
(a) 0 (b) 3 (c) 5 (d) 11
12. A number when divided by 6 leaves remainder 3. When the square of the same number is divided by 6, the remainder is :
(a) 0 (b) 1 (c) 2 (d) 3
13. When a number is divided by 893, the remainder is 193. What will be remainder when it is divided by 47 ?
(a) 3 (b) 5 (c) 25 (d) 33
14. A number divided by 13 leaves a remainder 1 and if the quotient, thus obtained, is divided by 5, we get a remainder of 3. What will be the remainder if the number is divided by 65 ?
(a) 28 (b) 16 (c) 18 (d) 40
15. Which of the following number is NOT divisible by 18 ?
(a) 54036 (b) 50436
(c) 34056 (d) 65043
16. If n is an integer, then $(n^3 - n)$ is always divisible by :
(a) 4 (b) 5 (c) 6 (d) 7
17. A 4 digit number is formed by repeating a 2 digit number such as 2525, 3232, etc. Any number of this form is always exactly divisible by:
(a) 7 only (b) 11 only
(c) 13 only (d) Smallest 3 digit prime number
18. If two numbers are each divided by the same divisor, the remainders are respectively 3 and 4. If the sum of the two numbers be divided by the same divisor, the remainder is 2. The divisor is :
(a) 9 (b) 7 (c) 5 (d) 3
19. A number when divided by 5 leaves remainder 3. What is the remainder when the square of the same number is divided by 5 ?
(a) 1 (b) 2 (c) 3 (d) 4
20. If the number 4 8 3 2 7 * 8 is divisible by 11, then the missing digit (*) is
(a) 5 (b) 3 (c) 2 (d) 1
21. A number, when divided by 136, leaves remainder 36. If the same number is divided by 17, the remainder will be
(a) 9 (b) 7 (c) 3 (d) 2
22. Two numbers, when divided by 17, leaves remainder 13 and 11 respectively. If the sum of those two numbers is divided by 17, the remainder will be :
(a) 13 (b) 11 (c) 7 (d) 4
23. A number, when divided by 221, leaves a remainder 64. What is the remainder if the same number is divided by 13?
(a) 0 (b) 1 (c) 11 (d) 12
24. When a number is divided by 387, the remainder obtained is 48. If the same number is divided by 43, the remainder obtained will be ?
(a) 0 (b) 3 (c) 5 (d) 35
25. When two number are separately divided by 33, the remainders are 21 and 28 respectively. If the sum of the two number is divided by 33, the remainder will be ?
(a) 10 (b) 12 (c) 14 (d) 16
26. $(2^{71} + 2^{72} + 2^{73} + 2^{74})$ is divisible by :
(a) 9 (b) 10 (c) 11 (d) 13
27. When 'n' is divisible by 5 the remainder is 2. What is the remainder when n^2 is divided by 5 ?
(a) 2 (b) 3 (c) 1 (d) 4
28. A number when divided by 49 leaves 32 as remainder. The number when divided by 7 will have the remainder as:
(a) 4 (b) 3 (c) 2 (d) 5
29. When a number is divided by 36, the remainder is 19. What will be the remainder when the number is divided by 12 ?
(a) 7 (b) 5 (c) 3 (d) 0
30. In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, then the dividend is :
(a) 4236 (b) 4306
(c) 4336 (d) 5336
31. When a number is divided by 24, the remainder is 16. The remainder when the same number is divided by 12 is
(a) 3 (b) 4 (c) 6 (d) 8
32. The expression $8^n - 4^n$, where n is a natural number is always divisible by
(a) 15 (b) 18 (c) 36 (d) 48
33. $(4^{61} + 4^{62} + 4^{63})$ is divisible by
(a) 3 (b) 11 (c) 13 (d) 17
34. When an integer K is divided by 3, the remainder is 1, and when K + 1 is divided by 5, the remainder is 0. Of the following, a possible value of K is:
(a) 62 (b) 63 (c) 64 (d) 65
35. A number when divided by 91 gives a remainder 17. When the same number is divided by 13, the remainder will be :
(a) 0 (b) 4 (c) 6 (d) 3
36. A number when divided by 280 leaves 115 as remainder. When the same number is divided by 35, the remainder is:
(a) 15 (b) 10 (c) 20 (d) 17
37. A certain number when divided by 175 leaves a remainder 132. When the same number is divided by 25, the remainder is:
(a) 6 (b) 7 (c) 8 (d) 9
38. Which one of the following will completely divide by $5^{71} + 5^{72} + 5^{73}$
(a) 150 (b) 160 (c) 155 (d) 30
39. Which of the following numbers will always divide a six-digit number of the form *xyxyxy* (where $1 \le x \le$, $1 \le y < 9$)?
(a) 1010 (b) 10101
(c) 11011 (d) 11010
40. A positive integer when divided by 425 gives remainder 45. When the same number is divided by 17, the remainder will be
(a) 5 (b) 2 (c) 11 (d) 13
41. A number x when divided by 289 leaves 18 as the remainder. The same number when divided by 17 leaves y as a remainder. The value of y is
(a) 5 (b) 2 (c) 3 (d) 1
42. When n is divided by 6, the remainder is 4. When 2n is divided by 6, the remainder is:
(a) 2 (b) 0 (c) 4 (d) 1
43. In a division sum, the divisor is 3 times the quotient and 6 times the remainder. If the remainder is 2, then the dividend is :
(a) 50 (b) 48 (c) 36 (d) 28

44. In a divison sum, the divisor is 12 times the quotient and 5 times the remainder. If the remainder is 36, then the dividend is :

(a) 2706 (b) 2726

(c) 2736 (d) 2262

45. For any integral value of n, $3^{2n} + 9n + 5$ when divide by 3 will leave the remainder

(a) 1 (b) 2

(c) 0 (d) 5

46. The quotient when 10^{100} is divided by 5^{75} is :

(a) 10^{25} (b) 2^{75}

(c) $2^{75}\times10^{25}$ (d) $2^{25}\times10^{75}$

47. The remainder obtained when $23^3 + 31^3$ is divided by 54

(a) 0 (b) 1

(c) 3 (d) C.N.D

48. $(19^5 + 21^5)$ is divisible by

(a) Only 10

(b) Only 20

(c) Both 10 & 20

(d) Neither 10 nor 20

49. If $(17)^{41} + (29)^{41}$ is divided by 23. Find the remainder

(a) 1 (b) 6 (c) 0 (d) 12

50. If $(3)^{41} + (7)^{82}$ always divisible by

(a) 10 (b) 49 (c) 52 (d) 44

51. If $m^n - n^m = (m + n)$; $(m, n) \in$ prime numbers, then what can be said about m and n:

(a) m, n are only even integers

(b) m, n are only odd integers

(b) m is even and n is odd

(d) none of these

ANSWER KEY

1. (c)	6. (c)	11. (c)	16. (c)	21. (d)	26. (b)	31. (b)	36. (b)	41. (d)	46. (c)
2. (c)	7. (b)	12. (d)	17. (d)	22. (c)	27. (d)	32. (d)	37. (b)	42. (a)	47. (a)
3. (c)	8. (d)	13. (b)	18. (c)	23. (d)	28. (a)	33. (a)	38. (c)	43. (a)	48. (c)
4. (c)	9. (a)	14. (d)	19. (d)	24. (c)	29. (a)	34. (c)	39. (b)	44. (c)	49. (c)
5. (b)	10. (d)	15. (d)	20. (d)	25. (d)	30. (d)	35. (b)	40. (c)	45. (b)	50. (c)
									51. (c)

SOLUTION

1. (c) Remainder = 48

Divisor = 48 × 5 = 240

Quotient $= \frac{240}{10} = 24$

Dividend = 240 × 24 + 48

= 5760 + 48

= 5808

2. (c) Dividend = Divisor × Quotient + Remainder

= 321 × 11 + 260

= 3531 + 260 = 3791

3. (c) Remainder = 73

Quotient = 6×73 = 438

Divisor = 5×73 = 365

Dividend = 365 × 438 + 73

= 159943

4. (c) 1, 3, 5, 7 ....20th term

a = 1, d = 2, n = 20

sum $= \frac{n}{2}[2a + (n - 1)d]$

$= \frac{20}{2}[2 \times 1+(20-1)2]$

= 10[2×1 + 19 ×2] = 400

Alternate :

The sum of first n odd natural numbers = $n^2 = 20^2 = 400$

5. (b) $\frac{\text{Remainder}}{8} = \frac{29}{8}$

Remainder = 5

6. (c) Number is divided successively

Remainder

4 | 37   1
5 | 9   4
  | 1

5×1+4=9

9×4+1= 37

Number is 37

5 | 37   2
4 | 7   3
  | 1

Remainder is 2, 3

7. (b) $4^{61} + 4^{62} + 4^{63} + 4^{64}$

$= 4^{61}(4^0 + 4^1 + 4^2 + 4^3)$

$= 4^{61}(1 + 4 + 16 + 64)$

$= 4^{61} \times 85$

$= 4^{60} \times 4 \times 85$

$= 4^{60} \times 340$

$= 4^{60} \times 34 \times 10$

Now, check with option

Only, check with option

Only 10 can divide this.

8. (d) $(3^{25} + 3^{26} + 3^{27} + 3^{28})$

$= 3^{35}(3^0 + 3^1 + 3^2 + 3^3)$

$= 3^{25}(1 + 3 + 9 + 27)$

$= 3^{25} \times 40 = 3^{24} \times 120$

Now, check with option

Only, check with option

Only 30 can divide this.

9. (a) 6709

$\Rightarrow$ 6+7+0+9=22

[9 - (divisibility property)

Sum of digits must be divisible by 9]

So 22 + 5 = 27 is divisible by 9

5 is answer

10. (d) 78 * 3945

Odd place : 7 + * + 9 + 5 = 21 + *

Even place : 8 + 3 + 4 = 15

(21+ *) –(15)= either 11 or 0

(21 + *) – 15 = 11

21 + * = 26

* = 5

11. (c) $\frac{\text{Remainder of number}}{17} = \frac{39}{17}$

$\Rightarrow$ remainder = 5

12. (d) Shorcut Method

Let number is: 9 (Gives remainder 3 when divided by 6)

Now $\frac{9^2}{6} = \frac{81}{6} \Rightarrow$ Remainder = 3

13. (b) $\frac{\text{Remainder of no.}}{47} = \frac{193}{47}$

$\Rightarrow$ remainder = 5

14. (d)

13	105	1
5	8	3
	1	

$5 \times 1 + 3 = 8$

$13 \times 8 + 1 = 105$

remainder = $105 \div 65$

Remainder = 40

15. (d) A number will be divisible by 18 if it is divisible by 2 and 9

Clearly we can see 65043 is not divisible by 2. Because unit digit of 65043 is 3 so this will not be divsible by 18

16. (c) $(n^3 - n)$ and n is any integer.

put n = 2 so, $2^3 - 2 = 6$

It will be always divisble by 6 (Put n = 2,3,4...)

17. (d) Smallest 3 digit prime number is '101'

xyxy is always divisible by 101

Hence, 101 Will be the divisor.

18. (c) **Shortcut Method**

divisor = Remainder 1 + Remainder 2 – Remainder 3

= 3 + 4 – 2 = 7 – 2 = 5

19. (d) Let no. be 8

$\Rightarrow \frac{8^2}{5} = \frac{64}{5}$

= 4 remainder

Alternate:-

Remainder = $\frac{(\text{Remainder})^2}{5}$

$= \frac{(3)^2}{5} = \frac{9}{5} = 4$

20. (d) 4 8 3 2 7 * 8

odd place $\Rightarrow$ 4 + 3 + 7 + 8 = 22

Even place $\Rightarrow$ 8 + 2 + * = 10 + *

Difference should be either zero or 11,22,33 ....etc.

$\Rightarrow$ 22 – (10 + *) = 11

22 – 10 – * = 11

12 – * = 11

* = 1

21. (d) $\frac{\text{Remainder of no.}}{17} = \frac{36}{17}$

$\Rightarrow$ remainder = 2

22. (c) (dividend = divisor × quotient + remainder)

First no. = (17 × n) +13

Let 'n' = 1

$\Rightarrow$ (17 × 1) + 13

$\Rightarrow$ 30

Second no. = (17 × n) + 11

= (17 × 1) + 11 = 28

According to question

$\frac{30+28}{17} = \frac{58}{17} \Rightarrow$ remainder = 7

Alternate:-

Divisor = Remainder 1 + Remainder 2 – Remainder 3

17 = 13 + 11 – Remainder 3

Remainder 3 = 24 – 17 = 7

23. (d) $\frac{\text{Remainder of no.}}{13} = \frac{64}{13}$

$\Rightarrow$ remainder = 12

24. (c) $\frac{\text{Remainder of no.}}{43} = \frac{48}{43}$

$\Rightarrow$ remainder = 5

25. (d) first no. = (33 × n) + 21

Let no = 1

= (33 × 1) + 21 = 54

Second no. = (33 × n) + 28

= (33 × 1) + 28 = 61

According to question

$\frac{54+61}{33} \Rightarrow \frac{115}{33}$

$\Rightarrow$ 16 Remainder

Alternate:-

Divisor = Remainder 1 + Remainder 2 – Remainder 3

33 = 21 + 28 – Remainder 3

Remainder 3 = 16

26. (b) $(2^{71} + 2^{72} + 2^{73} + 2^{74})$

$= 2^{71}(2^0 + 2^1 + 2^2 + 2^3)$

$= 2^{71}(1 + 2 + 4 + 8)$

$= 2^{71} \times 15 = 2^{70} \times 30$

It is divisible by 10

27. (d) $\frac{n}{5} \Rightarrow$ remainder 2

If we put n = 7 Then it satisfies above situation

So n = 7

$\frac{n^2}{5} = \frac{7^2}{5} = \frac{49}{5} \Rightarrow$ remainder= 4

28. (a) $\frac{\text{remainder of no.}}{7} = \frac{32}{7}$

$\Rightarrow$ Remainder = 4

29. (a) $\frac{\text{remainder of no.}}{12} = \frac{19}{12}$

$\Rightarrow$ remainder = 7

30. (d)

Quotient	:	Divisor	:	Remainder
1	:	10		
		5	:	1
1	:	10	:	2
$\downarrow \times 23$		$\downarrow \times 23$		$\downarrow \times 23$
23	:	230	:	46

Dividend = (Divisor × Quotient) + Remainder

= (230 × 23) + 46= 5336

31. (b) $\frac{\text{Remainder of no.}}{12} = \frac{16}{12}$

= 4 is remainder

32. (d) $8^n - 4^n$

n = 1,2,3.............(n is a natural number)

Put, n = 2,

expression = $8^2 - 4^2 = 64 - 16 = 48$

$\therefore$ $8^n - 4^n$ is divisible by 48

48 is completely divisible by 4 so f^n is divisible 4

33. (a) $(4^{61} + 4^{62} + 4^{63})$

$= 4^{61}(4^0 + 4^1 + 4^2)$

$= 4^{61}(1 + 4 + 16) = 4^{61} \times 21$

Now check the options

Only 3 divides it. So '3' is answer

34. (c) Always do these types of question by options to save time

Pick up the option and follow the question instruction

take option (c)

64 $\Rightarrow$ Divide 3 it gives remainder 1

Now add 1 to 64

$\frac{65}{5} \Rightarrow$ remainder '0' it satisfies

So, k = 64 this is answer

35. (b) $\frac{\text{Remainder of no.}}{13} = \frac{17}{13}$

remainder = 4

36. (b) $\frac{\text{Remainder of no.}}{35} = \frac{115}{35}$

Remainder = 10

37. (b) $\frac{\text{Remainder of no.}}{25} = \frac{132}{25}$
remainder = 7

38. (c) $5^{71} + 5^{72} + 5^{73}$
$= 5^{71}\ (5^0 + 5^1 + 5^2)$
$= 5^{71}\ (1 + 5 + 25)$
$= 5^{71} \times 31 = 5^{70} \times 155$
Check with option,
So 155 is answer

39. (b) Number = *xyxyxy*
= $xy \times 10000 + xy \times 100 + xy$
= $xy\ (10000+ 100 +1)$
= $xy(10101)$
Hence, option (B) will divide answer

Alternate:
You can assume (121212, 343434..........) any number divisible by option, So that number is divisible by exactly that's the answer

40. (c) $\frac{\text{Remainder of no.}}{17} = \frac{45}{17}$
$\Rightarrow$ remainder = 11

41. (d) $\frac{\text{Remainder of no.}}{17} = \frac{18}{17}$
$\Rightarrow$ remainder = 1

42. (a) $\frac{n}{6}$ = remainder 4

If n = 10 $\Rightarrow \frac{10}{6}$

$\Rightarrow$ remainder = 4 (matched) n = 10

$2n = 2 \times 10 \Rightarrow \frac{20}{6}$

$\Rightarrow$ remainder = 2

Note : Always put value in these type of questions.

43. (a)

Remainder : Divisor : Quotient

		3	:	1
1	:	6		
1	:	6	:	2
↓×2		↓×2		↓×2
Actual → 2		12		4

Dividend = (Divisor × Quotient) + remainder
$= (12 \times 4) + 2 = 50$

44. (c)

Remainder	:	Divisor	:	Quotient
		12	:	1
1	:	5		
12	:	60	:	5
↓×3		↓×3		↓×3
36		180		15

Dividend = (divisor × quotient) + Remainder
$= (180 \times 15) + 36$
$= 2736$

45. (b) $3^{2n} + 9n + 5$
Put n = 1
$\Rightarrow 3^{2\times1} + 9\times 1 + 5$
$\Rightarrow 9 + 9 + 5 \Rightarrow 23$
$\Rightarrow \frac{23}{3} \Rightarrow$ remainder = 2

Note: value of n can be 1,2,3,4, ...

46. (c) $10^{100} \div 5^{75}$
$\frac{2^{100} \times 5^{100}}{5^{75}} = 2^{100} \times 5^{25} = 2^{25}\ .2^{75}\ .5^{25}$
$= 2^{75} \times 10^{25}$

47. (a) We know that $(a^n + b^n)$ is always divisible (a + b) then.
where n $\rightarrow$ odd power
$(23^3 + 31^3)$ is Always divisible by (23 + 31) = 54
So remainder is '**0**'

48. (c) $(a^n + b^n)$, is always divisible by (a + b)
When n $\rightarrow$ odd power
(19 + 21) = 40
Factor of 40 (1, 2, 4, 5, 10, 20, 40) is divisible by $(19^5 + 21^5)$ then options 10 & 20 is divisible

49. (c) $(a^n + b^n)$, is always divisible (a + b)
When n is odd power
Then,
$(17^{41} + 29^{41})$ is always divisible by (17 + 29) = 46
factor of 46 (1, 2, 23, 46)
So, $(17^{41} + 29^{41})$ is perfecty divisible by 23
hence, Remainder '0'

50. (c) $3^{41} + 7^{82}$
Equalising the power
$3^{41} + (7^2)^{41} = 3^{41} + 49^{41}$
$3^{41} + 49^{41}$ is always divisible (3 + 49) = 52
So 52 is divisible by $(3^{41} + 7^{82})$

51. (c) $m^n - n^m = m + n$
Consider m = 2 and n = 5, then
$2^5 - 5^2 = 5 + 2$
7 = 7
Thus option (a) and (b) are wrong and option (c) is correct.

CHAPTER 05

REMAINDER THEOREM

Ex:- What remainders can be possible when 25 is divided by 7

$$+4 \nwarrow \frac{25}{7} \nearrow -3$$

$$7\,)\,25\,(\,3 \quad -21 \quad +4 \rightarrow \text{Actual Remainder}$$

or

$$7\,)\,25\,(\,4 \quad -28 \quad -3 \rightarrow \text{Negative Remainder}$$

Remainder is always positive but some times we use negative remainder for our convenience if 25 is divided by 7 then actual remainder will be + 4 but – 3 can be used for convenience for actual remainder multiple of 7, less than 25 is 21 hence actual remainder will be + 4 and for negative remainder we have to see the multiple of 7 greater than 25, which is 28 so – 3 will be the remainder

Ex:- What will be the remainder when 37 is divided by 9

$$9\,)\,37\,(\,4 \quad -36 \quad +1 \rightarrow \text{Actual Reminder}$$

or

$$9\,)\,37\,(\,5 \quad -45 \quad -8 \rightarrow \text{Negative Remainder}$$

When 37 is divided 9, then the multiple of 9 smaller than 37 is 36. Hence actual remainder will be +1 It we want a negative remainder we have to see the multiple of 9 greater than 37 which is 45, hence – 8 will be the negative Remainder.

$$+1 \nwarrow \frac{37}{9} \nearrow -8$$

Ex:- $$+6 \nwarrow \frac{55}{7} \nearrow -1$$

Ex:- $$+5 \nwarrow \frac{167}{6} \nearrow -1$$

* '0' is the smallest divisible number when 0 is divided by any number always remainder will be 0

Ex:- $\frac{0}{7}$ $$7\,)\,0\,(\,0 \quad -0 \quad 0$$

* when 0 is divided by 7, then 0^{th} multiple of 7 is $(7 \times 0 = 0)$ then 0 is subtracted from 0, we will get zero.

Ex:- When 45 is divided by 14 then

$$14\,)\,45\,(\,3 \quad -42 \quad +3 \rightarrow \text{Actual Remainder}$$

or

$$14\,)\,45\,(\,4 \quad -56 \quad -11 \rightarrow \text{Negative Remainder}$$

$$+3 \nwarrow \frac{45}{14} \nearrow -11$$

Ex:- $$+1 \nwarrow \frac{73}{8} \nearrow -7$$

Ex:- $$+3 \nwarrow \frac{111}{4} \nearrow -1$$

Ex:- $\frac{0}{100}$, Remainder = 0

$$100\,)\,0\,(\,0 \quad 0 \quad 0 \rightarrow \text{Remainder}$$

Ex:- $$+13 \nwarrow \frac{13}{15} \nearrow -2$$

$$15\,)\,13\,(\,0 \quad 0 \quad 13 \qquad \text{or} \qquad 15\,)\,13\,(\,1 \quad 15 \quad -2$$

* When 13 is divided by 15, then the multiple of 15 which is less than 13 is 0. which is 0^{th} multiple of 15. Hence actual remainder will be +13 and for the negative remainder we have to see the multiple of 15 which should be greater than 13, Now 15 is the multiple of 15 greater than 13, so remainder will be - 2

Ex:- $$+2 \nwarrow \frac{2}{3} \nearrow -1$$

$$3\,)\,2\,(\,0 \quad -0 \quad +2 \rightarrow \text{Actual Remainder}$$

$$3\,)\,2\,(\,1 \quad -3 \quad -1 \rightarrow \text{Negative Remainder}$$

Ex:-

$$+4 \nwarrow \frac{4}{7} \nearrow -3$$

$$7\,)\,4\,(\,0 \quad 0 \quad +4$$

$$7\,)\,4\,(\,1 \quad -7 \quad -3$$

TYPE – 1

Ex.1 what will be the remainder when 23×34 is divided by 9

Sol. $\frac{23 \times 34}{9}$

when 23 is divided by 9 the remainder is

$+5 \quad -4$

$\frac{23}{9}$

When 34 is divided by 9, the remainder is

$+7 \quad -2$

$\frac{34}{9}$

then $\frac{\overset{+5}{\uparrow}23 \times \overset{+7}{\uparrow}34}{9}$ $= 5 \times 7 = 35$

The sign will be the same between remainders as in the process. For Ex (23 × 34). Here we see that the sign b/w 23 and 34 is (×), So, the sign b/w remainders will be (×). If the product of remainder is greater than divisor, we have to divide it again to get the remainder

In this process when 23 is divided by 9, remainder +5 has been used and when 34 is divided by 9 remainder +7 has been used we can see that the sign between the proces is (×), then the product of remainders is (5 × 7) = 35,

Which is greater than 9. Now again we have to divide 35 by 9 we will get + 8 or - 1 as remainder. If the remainder is negative (-1) it should be deducted from divisor, so we will get positive (+ve) remainder)

$\frac{\overset{+5}{23} \times \overset{+7}{34}}{9} = \frac{5 \times 7}{9}$

$+8 \quad -1$

$= \frac{35}{9} = \boxed{+8 \text{ (Remander)}}$

or

$\boxed{9 - 1 = 8}$

Alternate II

$\frac{\overset{-4}{23} \times \overset{-2}{34}}{9} = \frac{-4 \times -2}{9} = 8$

$+5 \quad -4 \qquad +7 \quad -2$

$\frac{23}{9} \qquad \frac{34}{9}$

Now this time we have used negative remainder. If 23 is divided by 9, the remainder will be -4 and if 34 is divided by 9, the remainder will be - 2 As there is (×) sign in the process, the product of the remainders is (+8) As the product is less than divisor so there is no need to divide it again.

Alternate III

$\frac{\overset{-4}{23} \times \overset{+7}{34}}{9}$

$= -4 \times 7 = -28$

After dividing 23 by 9 remainder - 4 has taken and after dividing 34 by 9, remainder (+7) has taken. Now the product of the remainders are (-28). We will neglect the (-ve) sign and again will get the remainder by dividing fist process. After that we will put (-ve) sign. It the remainder is negative, then we will get (+ve) remainder by adding divisor into it.

$\frac{\overset{-4}{\uparrow}23 \times \overset{+7}{\uparrow}34}{9} = -28$

Negelecting (-ve) sign

$= \frac{\overset{+1}{\uparrow}28}{9}$ Now dividing by general process

= +1 (Again putting (-ve) sign)

= – 1 = 9 – 1

(To get (+ve) remainder) = 8

$\boxed{\text{Same Remainder in each process}}$

Ex.2 What will be the remainder when 43 × 83 is divided by 21?

Sol.

$\frac{43 \times 83}{21}$

$+1 \quad -20 \qquad +20 \quad -1$

$\frac{43}{21} \qquad \frac{83}{21}$

Wheather remainder is + ve or negative smaller remainder should be used for the easier calculation. If 43 is divided by 21, the smaller remainder will be (+1) and If 83 is divided by 21 the smaller remainder will be – 1,

$\overset{+1}{\uparrow}43 \times \overset{-1}{\uparrow}83 = \frac{1 \times -1}{21} = -1 + 21$

= 20

Ex.3 What will be the remainder when $\frac{121 + 93}{8}$

$+1 \quad -7 \qquad +5 \quad -3$

$\frac{121}{8} \qquad \frac{93}{8}$

By using smaller remainders

$\frac{\overset{+1}{\uparrow}121 + \overset{-3}{\uparrow}93}{8} = \frac{1 - 3}{8} = -2$

= 8 – 2 = 6

In this operation we have used (+ve) sign. So the same sign (+) will be used b/w the remainders. (1 –3) = (–2) the remainder is (-ve). So, to get actual remainder we have to add 8 hence actual remainder will be 6.

Ex.4 What will be the remainder

when $\frac{130 + 147}{11}$

Sol. $+9 \quad -2 \qquad +4 \quad -7$

$\frac{130}{11} \qquad \frac{147}{11}$

By using smaller remainder

$\frac{\overset{-2}{\uparrow}130 + \overset{+4}{\uparrow}147}{11} = \frac{-2 + 4}{11} = 2$

So, remainder is 2

Ex.5 When 127 × 139 × 12653 × 79 × 18769 is divided by 5, the remainder will be.

$$\frac{\overset{+2}{\uparrow}\!12\underline{7}\times \overset{-1}{\uparrow}\!13\underline{9}\times \overset{-2}{\uparrow}\!1265\underline{3}\times \overset{-1}{\uparrow}\!7\underline{9}\times \overset{-1}{\uparrow}\!1876\underline{9}}{5}$$

Divisiblity of 5 can be examined by dividing the last digit of the number

$$\Rightarrow \frac{2\times -1\times -2\times -1\times -1}{5}=\frac{4}{5}=4$$

Hence remainder is 4

Ex.6 What will be the remainder when $127+139+12653+79+18769$ is divided by 5

Sol.

$$\frac{\overset{+2}{12\underline{7}}+\overset{-1}{13\underline{9}}+\overset{-2}{1265\underline{3}}+\overset{-1}{7\underline{9}}+\overset{-1}{1876\underline{9}}}{5}$$

$$\frac{2-1-2-1-1}{5}=\frac{-3}{5}=-3$$

$= 5 - 3 = 2$

Ex.7 What will be the remainder when 195×1958×1975×170 is divided by 19.

$$=\frac{\overset{+5}{195}\times\overset{+1}{1958}\times\overset{-1}{1975}\times\overset{-1}{170}}{19}$$

$$=\frac{5\times 1\times -1\times -1}{19}=5$$

Ex.8 What will be the remainder when 1750 × 1748 × 1753 × 70 × 35 is divided by 17

Sol.

$$\frac{\overset{-1}{175\underline{0}}\times\overset{-3}{17\underline{48}}\times\overset{+2}{17\underline{53}}\times\overset{+2}{\underline{70}}\times\overset{+1}{\underline{35}}}{17}$$

$$\frac{-1\times -3\times 2\times 2\times 1}{17}=12$$

Hence Remainder is 12

Ex.9 What will be the remainder when (1750 + 1748 + 1752 + 70 + 35) is divided by 17 ?

Sol.

$$\frac{\overset{-1}{175\underline{0}}+\overset{-3}{17\underline{48}}+\overset{+1}{17\underline{52}}+\overset{+2}{70}+\overset{+1}{35}}{17}$$

$$=\frac{-1-3+1+2+1}{17}=\frac{0}{17}=0$$

Hence remainder is 0

Ex.10 When $\lfloor 1+\lfloor 2+\lfloor 3+\lfloor 4+\lfloor 5 \ldots\ldots+\lfloor 1000$ is divided by 10 the remainder will be

$\lfloor\ \rightarrow$ It is sign of Factorial.
$\lfloor 1 \rightarrow 1$
$\lfloor 2 \rightarrow 1 \times 2 = 2$
$\lfloor 3 \rightarrow 1 \times 2 \times 3 = 6$
$\lfloor 4 \rightarrow 1 \times 2 \times 3 \times 4 = 24$
$\lfloor 5 \rightarrow 1 \times 2 \times 3 \times 4 \times 5 = 120$
$\lfloor 0 \rightarrow$ value is 1

$\lfloor 1 = \frac{1}{10}$ (+1, −9)

$\lfloor 2 = 1\times 2 = \frac{2}{10}$ (+2, −8)

$\lfloor 3 = 1\times 2\times 3 = \frac{6}{10}$ (+6, −4)

$\lfloor 4 = 1\times 2\times 3\times 4 = \frac{24}{10}$ (+4, −6)

$\lfloor 5 = 5\times 4\times 3\times 2\times 1 = \frac{120}{10}$

=remainder =0

Value of $\lfloor 5$ is 120 which is completly divisible by 10, Hence the reamainder (In the same way) will be 0.

In the same way

$\lfloor 6 = 6\times 5\times 4\times 3\times 2\times 1 = \frac{720}{10}$

= Reamainder = 0

* $\lfloor 7$, $\lfloor 8$.........$\lfloor 1000$ is divided by 10, 0 will be the remainder in each case. So by using smaller remainder

$$\frac{\overset{+1}{\lfloor 1}+\overset{+2}{\lfloor 2}+\overset{-4}{\lfloor 3}+\overset{+4}{\lfloor 4}+\overset{0}{\lfloor 5}+\overset{0}{\lfloor 6}\ldots\ldots\ldots\overset{0}{\lfloor 1000}}{10}$$

$$=\frac{1+2-4+4}{10}=\frac{3}{10}=\boxed{3}$$

Ex.11 What will be the remainder when $\lfloor 1+\lfloor 2+\lfloor 3+\lfloor 4+\ldots\ldots\ldots\lfloor 1000$ is divided by 12 ?

$\lfloor 1 = 1 = \frac{1}{12}$ (1, −11)

$\lfloor 2 = 1\times 2 = \frac{2}{12}$ (+2, −10)

$\lfloor 3 = 1\times 2\times 3 = \frac{6}{12}$ (+6, −6)

$\lfloor 4 = 1\times 2\times 3\times 4 = \frac{24}{12}$

= Remainder = 0

Hence all the factorial next to will be completly divisible by 12 So, '0' will be the remainder is each case

$$\Rightarrow \frac{1+2+6}{12}=\frac{9}{12}=9$$

Remainder = 9

Ex.12 Which of the following will completly divide

$\lfloor 1+\lfloor 2+\lfloor 3+\lfloor 4+\lfloor 5+\lfloor 6\ldots\ldots\ldots\lfloor 1000$

(a) 10 (b) 9

(c) 12 (d) 8

Sol. In such type of question you can take the help of Options to save your valuable time

Option 'b'

$$\frac{\overset{+1}{\lfloor 1}+\overset{+2}{\lfloor 2}+\overset{-3}{\lfloor 3}+\overset{-3}{\lfloor 4}+\overset{+3}{\lfloor 5}+\overset{0}{\lfloor 6}+\overset{0}{\lfloor 7}\ldots\ldots\overset{0}{\lfloor 1000}}{9}$$

$$=\frac{1+2-3-3+3}{9}=0$$

Hence 0 is the remainder

Hence this number is divisible by 9

* The number is divided by 10 to get unit digit
* The number is divided by100 (10^2) to get last two digits
* The number is divided by[$(10)^3$]to get last three digit
* This process will continues as it is

Last Two Digit (अन्तिम 2 अंक) $\longrightarrow$

Ex.13 Find the last two digit of the product

23 × 13999 × 497 × 73 × 96

Sol. This number should be divided by 100 to get last two digit.

$$\frac{123\times 13999\times 497\times 73\times \cancel{96}^{\,24}}{\cancel{100}_{\,25}}$$

In such type of process we simplify the operation firstly. The number by which we simplify the operation, the same number is multiplied in the last. In this case 96 and 100 are simplified. So, we multiply by 4 in the last

$$\frac{1\overset{-2}{\underline{23}}\times 13\overset{-1}{\underline{99}}\times 4\overset{-3}{\underline{97}}\times \overset{-2}{\underline{73}}\times \overset{-1}{\underline{24}}}{25}$$

$$=\frac{-12}{25}=-12$$

$\Rightarrow \quad 25-12=13$

To get last two digit we multiply it by 4.

13 × 4 = 52

So, the last two digit is 52 (5 and 2)

* divisibility of 25 → last 2 digits divisible by 25

Ex.14 39 × 55 × 57 × 24 × 13872 × 9871 Find the last two digits

Sol. $$=\frac{39\times \overset{11}{\cancel{55}}\times 57\times \overset{6}{\cancel{24}}\times 13872\times 9871}{\cancel{100}\ \cancel{20}\ 5}$$

Simplifying two times by 4 and 5. So, to get last two digit we have to multiply 20 (4 × 5)

$$\frac{\overset{-1}{39}\times \overset{+1}{11}\times \overset{+2}{57}\times \overset{+1}{6}\times \overset{+2}{13872}\times \overset{+1}{9871}}{5}$$

* divisibility of 5 → last 1 digit divisible by 5

$$=\frac{-1\times 1\times 2\times 1\times 2\times 1}{5}=\frac{-4}{5}=-4$$

$=5-4=1$

So, actual last two digits

1 × 20 = 20 (2 and 0)

Ex.15 173 × 192 × 99 × 96 find the last two digits

Sol. $$\frac{173\times 192\times 99\times \cancel{96}^{24}}{\cancel{100}\ 25}$$

Simplifying by 4

$$\frac{\overset{-2}{173}\times \overset{-8}{192}\times \overset{-1}{99}\times \overset{-1}{24}}{25}$$

$$=\frac{-2\times -8\times -1\times -1}{25}=\frac{16}{25}=16$$

So, Actual last two digit

= 16 × 4 = 64 (6 and 4)

Ex.16 87× 92 × 194 × 44 Find the last two digits ?

Sol. $$\frac{87\times \cancel{92}^{23}\times 194\times 44}{\cancel{100}\ 25}$$

Simplifying by 4

$$\frac{\overset{-13}{87}\times \overset{-2}{23}\times \overset{-6}{194}\times \overset{-6}{44}}{25}$$

$$\frac{\underline{-13}\times \underline{-2}\times \underline{-6}\times \underline{-6}}{25}$$

$$\frac{\overset{+1}{26}\times \overset{+11}{36}}{25}$$

$$\frac{1\times 11}{25}=11$$

So, Actual last two digits

$=11\times 4=\boxed{44}$

Ex.17 What will be the remainder when 25 is divided by 13

Sol. 12 ← $\frac{25}{13}$ → -1 = -1

= 13 - 1 = 12

Remainder is always positive

Ex.18 Find the remainder $\frac{(25)^{48}}{13}$?

$$\frac{\overset{-1}{(25)}^{48}}{13}=(-1)^{48}=1$$

In such type of operations we try to get the multiple of divisor near to the dividends actual number So, that the difference b/w then will be 1. In this case the multiple of 13 near to 25 is 26. and the difference b/w 25 and 26 is 1 and power of even So, the remainder will be (+ve)

Ex.19 $\frac{(36)^{13}}{7}$ Find the remainder?

Sol. $$\frac{\overset{+1}{(36)}^{13}}{7}=(+1)^{13}=1$$

In this operation the multiple of 7 near to 36 is 35 and the difference between 36 and 35 is 1

Ex.20 $\frac{2^{18}}{9}$ Find the remainder?

Sol. In such type of Operations, the power is simplified in such a way that the difference b/w divisor and the number made by breaking of power is minimum So, the number near to 9 should be 8 or 10

$$9<\begin{matrix}8\\10\end{matrix}$$

$$\frac{2^{3\times 6}}{9}=\frac{(2^3)^6}{9}=\frac{\overset{-1}{(8)}^6}{9}$$

$$=\frac{(-1)^6}{9}=\frac{1}{9}=1$$

So Remainder = 1

Ex.21 What will be the remainder when 2^{21} is divided by 9

Sol. $$\frac{2^{21}}{9}=\frac{(2^3)^7}{9}=\frac{\overset{-1}{(8)}^7}{9}$$

$$=\frac{(-1)^7}{9}=-1=9-1=8$$

Ex.22 What will be the remainder when 2^{22} is divided by 9

Sol. $$=\frac{2^{22}}{9}=\frac{(2^3)^7\times 2}{9}=\frac{\overset{-1}{(8)}^7\times \overset{+2}{2}}{9}$$

$$=\frac{(-1)^7\times 2}{9}$$

$$\Rightarrow \quad \frac{-1\times 2}{9}=\frac{-2}{9}=-2$$

$\Rightarrow \quad 9-2=7$

Ex.23 What will be the remainder when $(35)^{37}$ is divided by 9 ?

Sol. $$\frac{(35)^{37}}{9}$$

The multiple of 9 near to 35 is 36

$$\Rightarrow \frac{\overset{-1}{(35)}^{37}}{9}=\frac{(1)^{37}}{9}=-1$$

Remainder = 9 - 1 = 8

Ex.24 What will be the remainder when 7^{40} is divided by 400

Sol.

$7^1 = 7$
$7^2 = 49$
$7^3 = 343$
$7^4 = 2401$

$\frac{7^{40}}{400}$ $\frac{(7^4)^{10}}{400}$

$\Rightarrow \frac{(2401)^{10}}{400} = \frac{(1)^{10}}{400} = 1$

(power has broken in such a way that $7^4 = 2401$, which is near to the 2400 a multiple of 400)

Ex.25 What will be the remainder when 2^{42} is divided by 33

Sol. $\frac{2^{42}}{33}$ $\quad 33 < \begin{matrix}32\\34\end{matrix}$

32 and 34 are near to the 33 the difference is 1. Hence Power is to be broken in such a way that we can a get 32 and 34

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$

$= \frac{2^2 \times 2^{40}}{33} = \frac{4 \times (2^5)^8}{33}$

$\Rightarrow \frac{\overset{+4}{4} \times (\overset{-1}{32})^8}{33}$

$= \frac{+4 \times (-1)^8}{33} = \frac{4 \times 1}{33} = 4$

remainder = 4

Ex.26 What will be the remainder when 3^{55} is divided by 82

Sol.

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$

$\frac{3^{55}}{82} \Rightarrow \frac{3^3 \times 3^{52}}{82}$ $\quad 82 < \begin{matrix}81\\83\end{matrix}$

$\Rightarrow \frac{3^3 \times (3^4)^{13}}{82}$

$= \frac{\overset{+27}{27} \times (\overset{-1}{81})}{82} = \frac{27 \times -1}{82} = -27$

remainder = 82 - 27 = 55

Ex.27 What will be the remainder when 2^{68} is divided by 65 ?

Sol. $65 < \begin{matrix}64\\66\end{matrix}$ $\quad \boxed{2^6 = 64}$

$\frac{2^{68}}{65}$

$= \frac{2^2 \times (2^6)^{11}}{65}$

$= \frac{\overset{+4}{4} \times (\overset{-1}{64})^{11}}{65} \Rightarrow \frac{+4 \times (-1)^{11}}{65}$

$\Rightarrow \frac{4 \times -1}{65} = \frac{-4}{65}$

remainder = 65 - 4 = 61

Ex.28 What will be the remainder when 4^{19} is divided by 33

Sol. $\frac{4^{19}}{33}$ $\quad 33 < \begin{matrix}32\\34\end{matrix}$

$4^{19} = (2^2)^{19} = 2^{38}$

So $\frac{2^{38}}{33}$ $\because \boxed{2^5 = 32}$

$\Rightarrow \frac{2^3 \times 2^{35}}{33} = \frac{8 \times (2^5)^7}{33}$

$\Rightarrow \frac{\overset{+8}{8} \times (\overset{-1}{32})^7}{33} = \frac{+8 \times (-1)^7}{33}$

$= \frac{8 \times -1}{33} = -8$

remainder $= 33 - 8 = \boxed{25}$

TYPE - 2

Ex.29 When 20 is divided by 8 the remainder will be

Sol. 8)20(2
-16
4 → Remainder

$= \frac{\cancel{20}^{\,5}}{\cancel{8}_{\,2}}$ $\quad = \frac{\overset{+1}{5}}{2} = 1$

When 20 is divided by 8 we get '4' remainder.

If $\frac{20}{8}$ is simplified by 4 we get $\frac{5}{2}$. Now 5 is divided by 2 we get remainder 1, In means that the divisor should be multiplied by remainder to get actual remainder

$= \frac{5}{2} = 1 \times \underline{4} = 4$

(Actual remainder)

Ex.30 What will be the remainder when 2^{35} is by 10 ?

Sol. $\frac{2^{35}}{10} = \frac{2^{35}}{2 \times 5} = \frac{\cancel{2} \times 2^{34}}{\cancel{2} \times 5}$

This Fraction is simplified by 2

$= \frac{2^{34}}{5} = \frac{(2^2)^{17}}{5}$

$= \frac{(\overset{-1}{4})^{17}}{5}$

$= \frac{(-1)^{17}}{5} = -1$ = 5 - 1 = 4

Actual Remainder = 4 × $\underline{2}$ = 8

As this number was simplified by 2, So to get actual remainder we have to multiply it by 2

Ex.31 What will be the remainder when 5^{500} is divided by 500

Sol. $\frac{5^{500}}{500} = \frac{5^3 \times 5^{497}}{125 \times 4}$

$= \frac{\cancel{5^3} \times 5^{497}}{\cancel{5^3} \times 4}$

5^3 = simplifying by 125

$\Rightarrow \frac{(\overset{+1}{5})^{497}}{4} = \frac{(+1)^{497}}{4} = 1$

Actual Remainder = 1 × 125 = 125

Ex.32 What will be the remainder when 37^{100} is divided by 7 ?

Sol. $\frac{(\overset{+2}{37})^{100}}{7} = \frac{(+2)^{100}}{7} = \frac{2 \times (2^3)^{33}}{7}$

$\boxed{2^3=8}$

2^{100} remainder is far greater than 7, So, we have to divide remainder again.

$$\Rightarrow \frac{\overset{\overset{+2}{\uparrow}}{2}\times\overset{\overset{+1}{\uparrow}}{(8)}^{33}}{7}$$

$$=\frac{+2\times(+1)^{33}}{7}$$

$$=\frac{2\times+1}{7}=\frac{+2}{7}$$

So, Remainder = 2

Cyclicity:- Happening again and again In the same order or period

Ex.33 Find the remainder when 11^{77} is divided by 7

Sol. $\frac{11^{77}}{7}$

The Remainder when 11^1 is divided by 7 = $=\frac{11}{7}=4$

$$11^2=\frac{\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}}{7}=\frac{4\times4}{7}=\frac{16}{7}=2$$

$$11^3=\frac{\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}}{7}=\frac{4\times4\times4}{7}=\frac{64}{7}=1$$

$$11^4=\frac{\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}}{7}=\frac{4\times4\times4\times4}{7}$$

$$\frac{\overset{\overset{2}{\uparrow}}{16}\times\overset{\overset{2}{\uparrow}}{16}}{7}=4$$

$$11^5=\frac{\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}}{7}=\frac{4\times4\times4\times4\times4}{7}$$

$$=\frac{\overset{\overset{2}{\uparrow}}{16}\times\overset{\overset{2}{\uparrow}}{16}\times\overset{\overset{+4}{\uparrow}}{4}}{7}=\frac{4\times4}{7}=2$$

$$11^6=\frac{\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}\times\overset{\overset{+4}{\uparrow}}{11}}{7}$$

$$=\frac{64\times64}{7}=\frac{1\times1}{7}=1$$

you are seeing that after three steps the cycle of remainders is repeating, which is generaly know as 'Pattern method So break the power of multiple of 3

$$\frac{(11)^{77}}{7}=\frac{(11)^{75}\times(11)^2}{7}$$

$$=\frac{\overset{\overset{+1}{\uparrow}}{(11^3)}^{25}\times\overset{\overset{+2}{\uparrow}}{121}}{7}$$

$$\frac{(+1)^{25}\times2}{7}=\frac{1\times2}{7}=2$$

Ex.34 Find the remainder when 5^{135} is divided by 7.

Sol. The Remainder when 5^1 is divided by 7 = $\frac{5^1}{7}=R=\mathbf{5}$.

$$\frac{5^2}{7}=\frac{\overset{\overset{+4}{\uparrow}}{25}}{7}=R=\mathbf{4}$$

$$\frac{5^3}{7}=\frac{\overset{\overset{+6}{\uparrow}}{125}}{7}=R=\mathbf{6}$$

$$\frac{5^4}{7}=\frac{5\times5\times5\times5}{7}$$

$$=\frac{\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{-3}{\uparrow}}{25}}{7}=\frac{9}{7}=R=\mathbf{2}$$

$$=\frac{5^5}{7}=\frac{5\times5\times5\times5\times5}{7}$$

$$=\frac{\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{+5}{\uparrow}}{5}}{7}=\frac{9\times5}{7}=R=\mathbf{3}$$

$$=\frac{5^6}{7}=\frac{5\times5\times5\times5\times5\times5}{7}$$

$$=\frac{\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{-3}{\uparrow}}{25}=}{7}\frac{-27}{7}=-6$$

$\therefore$ R = −6 + 7 = 1

$$\frac{5^7}{7}=\frac{5\times5\times5\times5\times5\times5\times5}{7}$$

$$\frac{\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{-2}{\uparrow}}{5}}{7}=\frac{9\times6}{7}=\mathbf{5}$$

$$\frac{5^8}{7}=\frac{5\times5\times5\times5\times5\times5\times5\times5}{7}$$

$$=\frac{\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{-3}{\uparrow}}{25}\times\overset{\overset{-3}{\uparrow}}{25}}{7}=\frac{9\times9}{7}=\mathbf{4}$$

So we see that the cyclic period of remainder is 6, since after 6 steps the remainder start repeating Now we divide the power by 6

$$=\frac{5^{135}}{7}=\frac{(5^{6\times22})\times5^3}{7}$$

$$=\frac{\overset{\overset{1}{\uparrow}}{(5^6)}^{22}\times125}{7}$$

$$=\frac{(1)^{22}\times6}{7}=\frac{6}{7}$$

Remainder = 6

Ex.35 Find the Remainder when 143^{321} is divided by 5

Sol. when 143 is divided by 5 we get remainder 3 thus 143^{321} is divided by 5 then remainder 3^{321}. this remainder is very large divisor so again divided

$$\frac{(143)^{321}}{5}=\frac{(3)^{321}}{5}$$

The remainder when 3^1 is divided by 5 $=\frac{3^1}{5}=R=3$

$$=\frac{3^1}{5}=R=3$$

$$\frac{3^2}{5}=\frac{9}{5}=R=4$$

$$\frac{3^3}{5}=\frac{27}{5}=R=2$$

$$\frac{3^4}{5}=\frac{81}{5}=R=1$$

$$\frac{3^5}{5}=\frac{243}{5}=R=3$$

$$\frac{3^6}{5}=\frac{729}{5}=R=4$$

$$\frac{3^{321}}{5}=\frac{(3)^{4\times80}\times3}{5}$$

$$=\frac{\overset{\overset{+1}{\uparrow}}{(3^4)}^{80}\times\overset{\overset{+3}{\uparrow}}{3}}{5}$$

$= \frac{(+1)^{80} \times 3}{5} = \frac{1 \times 3}{5} = 3$

Remainder = 3

Ex.36 find the remainder when 3^{6773} divide by 80

Sol. $= \frac{3^{6773}}{80}$

we know that = $3^4 = 81$

$\frac{3^{6773}}{80} = \frac{3^{6772} \times 3^1}{80}$

$= \frac{(3^4)^{1693} \times 3^1}{80}$

$= \frac{(\overset{+1}{81})^{1693} \times 3^1}{80}$

$= \frac{(1)^{1693} \times 3}{80} = \frac{1 \times 3}{80} = 3$

Hence Remainder = 3

Ex.37 Find the Remainder of $(32^{32})^{32}$ when divided by 7.

Sol. $\frac{(32^{32})^{32}}{7}$

$= \frac{(4^{32})^{32}}{7}$

$4 = 2^2$

$\Rightarrow \frac{(2^{2 \times 32})^{32}}{7}$

$\Rightarrow \frac{(2^{64})^{32}}{7}$

$\because \quad 2^3 = 8$

$= \frac{(2^{63} \times 2^1)^{32}}{7}$

$= \frac{((2^3)^{21} \times 2^1)^{32}}{7} = \frac{((\overset{+1}{8})^{21} \times 2^1)^{32}}{7}$

$= \frac{(1 \times 2)^{32}}{7} = \frac{2^{32}}{7}$

Again $\because \quad 2^3 = 8$

$\frac{2^{30} \times 2^2}{7} = \frac{(2^3)^{10} \times 4}{7}$

$= \frac{(\overset{+1}{8})^{10} \times 4}{7}$

$= \frac{(1)^{10} \times 4}{7} = \frac{1 \times 4}{7}$

Remainder = 4

Ex.38 What will be the remainder when $[48 + (62)^{117}]$ is divided by 9 ?

Sol. $\frac{\overset{+3}{48} + (\overset{-1}{62})^{117}}{9}$

$= \frac{+3 + (-1)^{117}}{9} = \frac{3-1}{9} = \frac{2}{9} = \boxed{2}$

Hence Remainder = 2

Ex.39 when $[51 + (67)^{99}]$ is divided by 68, find the remainder

Sol. $\frac{\overset{-17}{51} + (\overset{-1}{67})^{99}}{68}$

$\Rightarrow \frac{-17 + (-1)^{99}}{68} = \frac{-17-1}{68} = \frac{-18}{68}$

$= -18$

Remainder = 68 - 18 = 50

Remainder of Algebraic Function

When F (x) is divided by $(x$-a) the remainder is F (a)

$\because$ $(x$ - a) is a factor of F (x) then f (a) = 0

Ex.40 Is $(x - 2)$ a factor of f (x) $x^2 + x - 5$?

Sol. $(x - 2) = 0$

$\boxed{x = 2}$

x value f (x)

F (2) = $(2)^2 + (2) - 5$

= 4 + 2 - 5

= 6 - 5 = 1 $\neq$ 0

$(x$ - 2) is not a factor of $x^2 - x + 5$

If F (2) = 0, we can say $(x$ - 2), it is a factor of f (x)

Ex.41 $x^{29} - x^{26} - x^{23} + 1$

(a) $(x - 1)$ but not $(x + 1)$

(b) $(x + 1)$ but not $(x - 1)$

(c) both $(x + 1)$ & $(x - 1)$

(d) Neither $(x + 1)$ not $(x - 1)$

Sol. (c) If $(x$ - 1), is a factor then, $f(x) = 0$,

and $x - 1 = 0$

$x = 1$

f (1) = 0

f $(x) = x^{29} - x^{26} - x^{23} + 1$

$f(1) = 1 - 1 - 1 + 1 = 0$

$f(1) = 0$,

we can say $(x$ - 1) is a factor of f (x)

$x + 1 = 0$

$x = -1$

$x^{29} - x^{26} - x^{23} + 1$

$-1 - 1 + 1 + 1 = 0$

$(x + 1)$ is a factor of f (x)

Both $(x + 1)$ & $(x - 1)$ is a factor of $x^{29} - x^{26} - x^{23} + 1$

Ex.42 If $(x - 2)$ is a factor of Polynomial $x^2 + kx + 4$. Find the value of k.

Sol. $(x - 2)$ is a factor of $x^2 + kx + 4$

when $(x - 2) = 0$

$x = 2$

f (2) = $(2)^2$ + 2k + 4 = 0

2k = – 8

k = –4

Ex.43 If $(x + 1)$ & $(x - 1)$ are the Factor of the Polynomial $ax^3 + bx^2 + 3x + 5$. find the value of a and b

Sol. If $(x - 1)$ is factor of $f(x)$ then,

$x - 1 = 0$

$x = 1$

$f(x) = ax^3 + bx^2 + 3x + 5$

$f(1) = a(1)^3 + b(1)^2 + 3(1) + 5 = 0$

a + b = –8 ..... (i)

If $(x + 1)$, is a factor of f (x)

Then,

$(x + 1) = 0$

$x = -1$

$f(-1) = a(-1)^3 + b(-1)^2 + 3(-1) + 5 = 0$

– a + b – 3 + 5 = 0

–a + b = –2

a - b = 2 ..... (ii)

from (i) & (ii)

$\boxed{a = -3}$, $\boxed{b = -5}$

Ex.44 Find the remainder when $x^3 + 5x^2 + 7$ is divided by $(x - 2)$

Sol. $x - 2 = 0$

$x = 2$

f $(x) = x^3 + 5x^2 + 7$

f (2) = $(2)^3 + 5(2)^2 + 7$

= 8 + 20 + 7 = 35

Remainder = 35

Ex.45 Find the remainder when $x^2 - 7x + 15$ is divided by $x - 3$

Sol. $x - 3 = 0$

$x = 3$

Put the value of $x = 3$

F $(x) = x^2 - 7x + 15$

F (3) = $(3)^2 - 7(3) + 15$

= 9 – 21 + 15 = 3

Remainder 3

Ex.46 $x^{51} + 16$ when divided by $x + 1$ find the Remainder.

Sol. $(x + 1) = 0$

$x = -1$

$f(x) = x^{51} + 16$

$f(-1) = (-1)^{51} + 16 = -1 + 16 = 15$

Remainder = 15

Ex.47 If $x^2 + 4x + k$ when divided by $x - 2$ leave remainder $2x$. find the value of k.

Sol. $x^2 + 4x + k$

$x - 2 = 0$

$x = 2$

$f(x) = 2x$

$f(2) = 2 + 2 = 4$

$f(2) = (2)^2 + 4 \times 2 + k = 4$

$4 + 8 + k = 4$

$\boxed{K = -8}$

TYPE – 3

Ex.48 777777........... 129 Times is divided by 37 the remainder will be ?

Sol. If any number is made by repeating a digit 6 times the number will be divisible by 7, 11, 13 and 37.

So, 777777 .... 126 times is divisible by 37 because 126 is the multiple of 6. So, the remaining three digits will be divided by 37 to get the remainder

$$\Rightarrow \frac{77777777......126\text{ Times}, 777}{37}$$

$$37\overline{)777}(21$$
$$\underline{74}$$
$$37$$
$$\underline{37}$$
$$\times$$

$\frac{777}{37} = 0$

Remainder = 0

Hence, the number is divisible by 37.

Ex.49 When 444444444 is divided by 13 the remainder will be ?

Sol. 4 is repeating 9 times in this number As we know that any number repeating 6 times is divisible by 13. So the remaining three digit will be divided by 13 to get the will be divided by 13 to get the remainder

$$\frac{444444,444}{13} = \frac{444}{13}$$

$$= 13\overline{)444}(34$$
$$\underline{39}$$
$$54$$
$$\underline{52}$$
$$2 \longrightarrow \text{Remainder}$$

Ex.50 What will be the remainder when 123456789 is divided by 8 ?

Sol. (Divisibility Rule)

$2^1 = 2 \longrightarrow$ Last digit divisibile by 2

$2^2 = 4 \longrightarrow$ Last two digits divisibile by 4

$2^3 = 8 \longrightarrow$ Last three digits divisibile by 8

$2^4 = 16 \longrightarrow$ Last four digits divisibile by 16

$2^5 = 32 \longrightarrow$ Last five digits divisibile by 32

So, for the divisibility of 8 the last three digit of the number should be divisible by 8. In this way we get 5 as the remainder

$$\frac{123456\underline{789}}{8}$$

$$8\overline{)789}(98$$
$$\underline{72}$$
$$69$$
$$\underline{64}$$
$$5 \longrightarrow \text{Remainder}$$

So, the remainder is 5.

Ex.51 What will be remainder when 12345678910111213141516 divided by 16.

Sol. For the divisibility of 16, the last four digits of the number should be divisible by 16. In this way we get 12 as remainder

$$\frac{12345678910111213141\underline{516}}{16}$$

$$16\overline{)1516}(94$$
$$\underline{144}$$
$$76$$
$$\underline{64}$$
$$12 \longrightarrow \text{Remainder}$$

Hence the remainder is 12.

Ex.52 $10^1 + 10^2 + 10^3 + ... 10^{99} + 10^{100}$ when divided by 6, the remainder will be?

Sol. $$\frac{10^1 + 10^2 + 10^3 + ...10^{99} + 10^{100}}{6}$$

$$\longrightarrow \frac{\overset{+4}{10}}{6} = \text{Remainder} = 4$$

$$\longrightarrow \frac{\overset{+4}{10} + \overset{+4}{10^2}}{6} = \frac{4+4}{6} = \frac{8}{6}$$

Remainder = 8 - 6 = 2

$$\longrightarrow \frac{\overset{+4}{10^1} + \overset{+4}{10^2} + \overset{+4}{10^3}}{6} = \frac{4+4+4}{6} = \frac{12}{6}$$

Remainder = 0

The remainder will be zero (0) after each three number So the remainder is 0 upto the 99th term. So the remaining 10^{100} term will be divided by 6 to get the remainder

$$\frac{\overset{+4}{10^1} + \overset{+4}{10^2} + \overset{+4}{10^3} + \overset{+4}{10^1} + \overset{+4}{10^2} + \overset{+4}{10^3} \ldots \ldots \overset{+4}{10^{99}} + \overset{+4}{10^{100}}}{6}$$

$$\frac{\overset{+4}{10^{100}}}{6} = 4$$

Hence, the remainder is 4.

Ex.53 What will be the remainder when $10^1 + 10^2 + 10^3 + ... 10^{32}$ is divided by 6 ?

Sol. $$\frac{\overset{+4}{10^1} + \overset{+4}{10^2} + \overset{+4}{10^3} + \ldots \overset{+4}{10^{30}} + \overset{+4}{10^{31}} + \overset{+4}{10^{32}}}{6}$$

0 will be the remainder of ter each three term. So, o will be the remainder up to 30th term

$$\Rightarrow \frac{\overset{+4}{10^{31}} + \overset{+4}{10^{32}}}{6}$$

$$= \frac{4+4}{6} = \frac{8}{6} = 2$$

Hence remainder is 2

EXERCISE

1. Find the Remainder when 77 × 85 × 73 is divided by 9
(a) 1 (b) 2 (c) 4 (d) 7
2. Find the Remainder when 273 + 375 + 478 + 657 + 597 is divided by 25
(a) 5 (b) 10 (c) 9 (d) 8
3. Find the Remainder when 1330 × 1356 × 1363 × 1368 × 1397 is divided by 13
(a) 7 (b) 9 (c) 11 (d) 8
4. Find the Remainder when 2327 + 2372 + 2394 + 4624 + 4650 is divided by 23
(a) 12 (b) 14 (c) 13 (d) 10
5. Find the Remainder when 67^{32} is divided by 68
(a) 67 (b) 66 (c) 1 (d) 0
6. Find the Remainder when 99^{99} is divided by 100
(a) 99 (b) 98 (c) 1 (d) 3
7. Find the Remainder 197^{130} is divided by 196
(a) 1 (b) 195 (c) 7 (d) 5
8. Find the Remainder 6^{36} is divided by 215
(a) 214 (b) 6 (c) 5 (d) 1
9. Find the Remainder 75^{7575} is divided by 37
(a) 1 (b) 36 (c) 3 (d) 7
10. Find the Remainder 43^{197} is divided by 7
(a) 42 (b) 41 (c) 1 (d) 6
11. Find the Remainder when 17^{200} is divided by 18
(a) 17 (b) 16 (c) 1 (d) 4
12. Find the Remainder when $(12^{13} + 23^{13})$ is divided by 11
(a) 2 (b) 1 (c) 0 (d) 3
13. Find the remainder when $(7^{19} + 2)$ is divided by 6
(a) 3 (b) 1 (c) 5 (d) 2
14. Find the Remainder when 3^{21} is divided by 5 is
(a) 3 (b) 2 (c) 1 (d) 4
15. Find the Remainder when 2^{31} is divided by 5
(a) 1 (b) 2 (c) 3 (d) 4
16. Find the Remainder when 2^{591} is divided by 255
(a) 225 (b) 128 (c) 127 (d) 64
17. Find the Remainder when 51^{203} is divided by 7
(a) 4 (b) 2 (c) 1 (d) 6
18. The Remainder when $(2)^{243}$ is divided by 3^2 is
(a) 8 (b) 4 (c) 10
(d) None of these
19. Find the Remainder when $(59)^{28}$ is divided by 7
(a) 2 (b) 4 (c) 6 (d) 1
20. Find the Remainder when 41^{77} is divided by 17
(a) 2 (b) 1 (c) 6 (d) 4
21. Find the Remainder when 2^{49} is divided by 7
(a) 1 (b) 2 (c) 3 (d) 4
22. Find the Remainder when $(51^{203} + 2^{49})$is divided by 17
(a) 4 (b) 5 (c) 6
(d) None of these
23. Find the Remainder when 1234567891011121314is divided by 8
(a) 4 (b) 2 (c) 6 (d) 3
24. Find the Remainder when 41424344454647484950 is divided by 16
(a) 2 (b) 12 (c) 6 (d) 8
25. Find the Remainder when 21222324252627282930 is divided by 8
(a) 5 (b) 2 (c) 3 (d) 4
26. Find the Remainder when 919293949596979899 is divided by 16
(a) 3 (b) 13 (c) 11 (d) 8
27. Find the Remainder when 313233343536373839 is divided by 4
(a) 1 (b) 2 (c) 3 (d) N.O.T.
28. Find the Remainder when 1234.... 41digits is divided by 8
(a) 1 (b) 2 (c) 3 (d) 4
29. Find the Remainder when 1234....... 81digits is divided by 16
(a) 13 (b) 8 (c) 1 (d) 7
30. Find the Remainder when 8^{77} is divided by 17
(a) 8 (b) 9 (c) 13 (d) 7
31. Find the Remainder when $\lfloor\underline{1}+\lfloor\underline{2}+\lfloor\underline{3}+\lfloor\underline{4}.......\lfloor\underline{100}$ is divided by 5 is
(a) 0 (b) 1 (c) 2 (d) 3
32. Find the Remainder when $\lfloor\underline{1}+\lfloor\underline{2}+\lfloor\underline{3}+\lfloor\underline{4}.......\lfloor\underline{100}$ is divided by 6 is
(a) 3 (b) 4 (c) 2 (d) 1
33. Find the Remainder when $\lfloor\underline{1}+\lfloor\underline{2}+\lfloor\underline{3}+\lfloor\underline{4}.......\lfloor\underline{50}$ is divided by 12 is
(a) 2 (b) 8 (c) 7 (d) 9
34. Find the Remainder when 9^{111} is divided by 11
(a) 2 (b) 9 (c) 7 (d) 6
35. Find the Remainder when 5^{2450} is divided by 126
(a) 5 (b) 25 (c) 125 (d) 1
36. Find the Remainder when 40^{1012} is divided by 7
(a) 5 (b) 4 (c) 3 (d) 2
37. Find the Remainder when $10^1 + 10^2 + 10^3 + ... 10^{100}$ is divided by 6
(a) 4 (b) 6 (c) 2 (d) 3
38. Find the Remainder when $10^1 + 10^2 + 10^3 + ... 10^{1000} + 10^{1001}$ is divided by 6
(a) 4 (b) 6 (c) 2 (d) 3
39. Find the Remainder when 666666 .... 134times is divided by 13
(a) 1 (b) 3 (c) 11 (d) 9
40. Find the Remainder when 555555 .... 244times is divided by 37
(a) 18 (b) 5 (c) 36 (d) 0
41. Find the Remainder when 777777 .... 363 times is divided by 11
(a) 0 (b) 7 (c) 1 (d) 3
42. Find the Remainder when 888888 .... 184 times is divided by 37
(a) 1 (b) 8 (c) 36 (d) 7
43. Find the Remainder when 999999999 is divided by 13
(a) 8 (b) 11 (c) 5 (d) 12
44. Find the Remainder when 7^{99} is divided by 2400
(a) 1 (b) 49 (c) 343 (d) 7
45. Find the Remainder when 3^{1989} is divided by 7
(a) 2 (b) 6 (c) 4 (d) 5
46. Find the Remainder when 54^{124} is divided by 17
(a) 4 (b) 5 (c) 3 (d) 15
47. Find the Remainder when 21^{875} is divided by 17
(a) 8 (b) 13 (c) 16 (d) 9
48. Find the Remainder when 83^{261} is divided by 17
(a) 13 (b) 9 (c) 8 (d) 2

49. Find the Remainder when $(32^{32})^{32}$ is divided by 9
(a) 4 (b) 7 (c) 1 (d) 2

50. Find the Remainder when $(32^{32})^{32}$ is divided by 7
(a) 4 (b) 7 (c) 2 (d) 1

51. Find the Remainder when $(33^{34})^{35}$ is divided by 7
(a) 5 (b) 4 (c) 6 (d) 2

52. Find the Remainder when $888^{222} + 222^{888}$ is divided by 5
(a) 0 (b) 1 (c) 3 (d) 4

53. Find the Remainder when $2222^{5555} + 5555^{2222}$ is divided by 7
(a) 0 (b) 2 (c) 4 (d) 5

54. Find the Remainder when $50^{51^{52}}$ is divided by 11
(a) 6 (b) 4 (c) 7 (d) 3

55. The Remaidner when $(20)^{23}$ is divided by 17 is
(a) 11 (b) 3 (c) 6
(d) Can't determind

56. If $(x - 2)$ is a factor of $(x^2 + 3qx - 2q)$, then the value of q is :
(a) 2 (b) -2 (c) -1 (d) 1

57. If $x^3 + 6x^2 + 4x + k$ is exactly divisible by $(x + 2)$ then value of k is : –
(a) -6 (b) -7 (c) -8 (d) -10

58. Value of k for with $(x - 1)$ is a factor of $(x^3 - k)$ is :
(a) - 1 (b) 1 (c) 8 (d) - 8

59. If $x^{100} + 2x^{99} + k$ is divisible by $(x + 1)$, then the value of k is:
(a) 1 (b) -3 (c) 2 (d) -2

60. If $(x^3 - 5x^2 + 4p)$ is divisible by $(x + 2)$, then the value of p is
(a) 7 (b) -2 (c) 3 (d) -7

61. If (x - a) is a factor of $(x^3 - 3x^2 a + 2a^2x + b)$, then the value of b is:
(a) 0 (b) 2 (c) 1 (d) 3

62. If $x^3 + 3x^2 + 4x + k$ contains $(x + 6)$ as a factor, the value of k is:
(a) 66 (b) 33 (c) 132 (d) 36

63. If $(x + 2)$ and $(x - 1)$ are the factors of $(x^3 + 10x^2 + mx + n)$, the values of m and n are :
(a) m = 5, n = -3
(b) m = 17, n = -8
(c) m = 7, n = -18
(d) m = 23, n = -19

64. On dividing $(x^3 - 6x + 7)$ by $(x + 1)$, then remainder is :
(a) 2 (b) 12 (c) 0 (d) 7

65. If $(x^5 - 9x^2 + 12x - 14)$ is divided by $(x - 3)$, the remainder is :
(a) 184 (b) 56 (c) 2 (d) 1

66. When $(x^4 - 3x^3 + 2x^2 - 5x + 7)$ is divided by $(x-2)$, then remainder is :
(a) 3 (b) - 3 (c) 2 (d) 0

67. If $5x^3 + 5x^2 - 6x + 9$ is divided by $(x + 3)$, then remainder is :
(a) 135 (b) -135 (c) 63 (d) -63

68. If $(x^{11} + 1)$ is divided by (x + 1), then remainder is :
(a) 2 (b) 0 (c) 11 (d) 12

69. If $2x^3 + 5x^2 - 4x - 6$ is divided by $2x + 1$, then remainder is :
(a) $-\frac{13}{3}$ (b) 3
(c) -3 (d) 6

70. If $x^3 + 5x^2 + 10k$ leaves remainder -2x when divided by $x^2 + 2$, then the value of k is:
(a) -2 (b) -1 (c) 1 (d) 2

ANSWER KEY

1. (b)	8. (d)	15. (c)	22. (d)	29. (a)	36. (d)	43. (b)	50. (a)	57. (c)	64. (b)
2. (a)	9. (a)	16. (b)	23. (b)	30. (b)	37. (a)	44. (c)	51. (b)	58. (b)	65. (a)
3. (b)	10. (c)	17. (a)	24. (c)	31. (d)	38. (c)	45. (b)	52. (a)	59. (a)	66. (b)
4. (b)	11. (c)	18. (a)	25. (b)	32. (a)	39. (a)	46. (a)	53. (a)	60. (a)	67. (d)
5. (c)	12. (a)	19. (b)	26. (c)	33. (d)	40. (b)	47. (b)	54. (d)	61. (a)	68. (b)
6. (a)	13. (a)	20. (c)	27. (c)	34. (b)	41. (b)	48. (d)	55. (a)	62. (c)	69. (c)
7. (a)	14. (a)	21. (b)	28. (a)	35. (b)	42. (b)	49. (a)	56. (c)	63. (c)	70. (c)

SOLUTION

1. (b)
$$\frac{\overset{+5}{77}\times\overset{+4}{85}\times\overset{+1}{73}}{9}$$
$$=\frac{5\times4\times1}{9}=\frac{20}{9}=\mathbf{2}$$

2. (a)
$$\frac{\overset{-2}{273}+\overset{0}{375}+\overset{+3}{478}+\overset{+7}{657}+\overset{-3}{597}}{25}$$
$$\frac{-2+0+3+7-3}{25}=\frac{5}{25}=\mathbf{5}$$

3. (b)
$$\frac{\overset{+4}{1330}\times\overset{+4}{1356}\times\overset{-2}{1363}\times\overset{+3}{1368}\times\overset{+6}{1397}}{13}$$
$$\frac{4\times4\times-2\times3\times6}{13}=\frac{16\times\overset{+3}{-36}}{13}$$

* Avoid '–' (Negative) sign. Normally divided 36 by 13 remainder = -3. Now use '-' (Negative) sign

R = (-3) = 3
$$\frac{\overset{+3}{16}\times\overset{+3}{-36}}{13}=\frac{9}{13}=R=\mathbf{9}$$

4. (b)
$$\frac{\overset{+4}{2327}+\overset{+3}{2372}+\overset{+2}{2394}+\overset{+1}{4624}+\overset{+4}{4650}}{23}$$
$$\frac{4+3+2+1+4}{23}=\frac{14}{23}$$
R = **14**

5. (c) $\frac{\overset{-1}{67}^{32}}{68} = \frac{(-1)^{32}}{68} = \frac{1}{68}$

R = **1**

6. (a) $\frac{\overset{-1}{99}^{99}}{100} = \frac{(-1)^{99}}{100} = \frac{-1}{100}$

R = 100 - 1 = **99**

7. (a) $\frac{\overset{+1}{197}^{130}}{196} = \frac{(+1)^{130}}{196} = \frac{1}{196}$

R = **1**

8. (d) $\frac{6^{36}}{215}$

∵ We know that $6^3 = 216$

∵ So break the power multiple 3

$= \frac{(6^3)^{12}}{215} = \frac{(\overset{+1}{216})^{12}}{215}$

$= \frac{(+1)^{12}}{215} = \frac{1}{215}$

R = **1**

9. (a) $\frac{\overset{+1}{75}^{7575}}{37} = \frac{(+1)^{7575}}{37} = \frac{1}{37}$

R = **1**

10. (c)

$\frac{\overset{+1}{43}^{197}}{7} = \frac{1^{197}}{7} = \frac{1}{7}$

R = 1

11. (c) $\frac{\overset{+1}{17}^{200}}{18} = \frac{(-1)^{200}}{18} = \frac{1}{18}$

R = **1**

12. (a) $\frac{\overset{+1}{12}^{13} + \overset{+1}{23}^{13}}{11}$

$= \frac{(+1)^{13} + (+1)^{13}}{11} = \frac{1+1}{11} = \frac{2}{11}$

R = **2**

13. (a) $\frac{\overset{+1}{7}^{19} + \overset{+2}{2}}{6} = \frac{(+1)^{19} + 2}{6}$

$= \frac{1+2}{6} = \frac{3}{6}$

R = **3**

14.(a) $\frac{3^{21}}{5}$

∵ $3^2 = 9$

Break The power multiple of 2 form

$= \frac{(3^2)^{10} \times 3^1}{5} = \frac{(\overset{-1}{9})^{10} \times \overset{+3}{3}}{5}$

$= \frac{(-1)^{10} \times 3}{5}$

$= \frac{1 \times 3}{5} = \frac{3}{5}$

R = **3**

15. (c) $\frac{2^{31}}{5}$

∵ $2^2 = 4$

$\frac{(2^2)^{15} \times 2^1}{5} = \frac{(\overset{-1}{4})^{15} \times 2^1}{5}$

$= \frac{(-1)^{15} \times 2}{5} = \frac{-1 \times 2}{5} = \frac{-2}{5}$

R = 5 - 2 = **3**

16.(b) $\frac{2^{591}}{255}$

∵ $2^8 = 256$

Now $\frac{(2^8)^{73} \times 2^7}{255}$

$= \frac{(\overset{+1}{256})^{73} \times 128}{255}$

$= \frac{(1)^{73} \times 128}{255} = \frac{128}{255}$

Remainder = **128**

17.(a) $\frac{\overset{+2}{51}^{203}}{7} = \frac{(2)^{203}}{7}$

∵ $2^3 = 8$

$= \frac{(2^3)^{67} \times 2^2}{7} = \frac{(\overset{+1}{8})^{67} \times \overset{+4}{4}}{7}$

$\Rightarrow \frac{(+1)^{67} \times 4}{7}$

$\Rightarrow \frac{1 \times 4}{7} = \frac{4}{7}$

R = 4

18.(a) $\frac{2^{243}}{3^2} = \frac{2^{243}}{9}$

∵ $2^3 = 8$

$= \frac{(2^3)^{81}}{9} = \frac{(\overset{-1}{8})^{81}}{9} = \frac{(-1)^{81}}{9} = \frac{-1}{9}$

Remainder = 9 – 1 = **8**

19. (b) $\frac{\overset{+3}{59}^{28}}{7} = \frac{3^{28}}{7} = \frac{(3^3)^9 \times 3}{7}$

∵ $3^3 = 27$

$\Rightarrow \frac{(\overset{-1}{27})^9 \times \overset{+3}{3}}{7}$

$\Rightarrow \frac{(-1)^9 \times 3}{7} = \frac{-1 \times 3}{7} = \frac{-3}{7}$

R = 7 - 3 = 4

20. (c) $\frac{\overset{+7}{41}^{77}}{17} = \frac{7^{77}}{17} = \frac{(7^2)^{38} \times 7^1}{17}$

$\Rightarrow \frac{(\overset{-2}{49})^{38} \times 7}{17} = \frac{(-2)^{38} \times 7}{17}$

∵ There will be no effect of -ve sign because the power is even

$\rightarrow \frac{2^{38} \times 7}{17} = \frac{(2^4)^9 \times 2^2 \times 7}{17}$

∵ $2^4 = 16$

$\rightarrow \frac{(\overset{-1}{16})^9 \times 4 \times 7}{17} = \frac{(-1)^9 \times 28}{17} = \frac{-1 \times 28}{17}$

$= \frac{-28}{17} = \frac{\overset{\overset{-6}{\uparrow}}{(28)}}{17}$ (Avoid -ve sign)

Now use -ve sign
R = - (-6) = **6**

21. (b) $\frac{2^{49}}{7}$ $\because 2^3 = 8$

$$\Rightarrow \frac{(2^3)^{16}\times 2^1}{7} = \frac{(\overset{\overset{+1}{\uparrow}}{8})^{16}\times 2}{7}$$

$$= \frac{(+1)^{16}\times 2}{7} = \frac{1\times 2}{7} = \frac{2}{7}$$

R = 2

22. (d) $\frac{\overset{\overset{0}{\uparrow}}{51^{203}} + 2^{49}}{17}$

51 is divisible by 17 So $(51)^{203}$ is divisible by 17 then remainder '0', Now only divide 2^{49}

$$= \frac{2^{49}}{17} = \frac{(2^4)^{12}\times 2^1}{17} = \frac{(\overset{\overset{-1}{\uparrow}}{16})^{12}\times 2}{17}$$

$$= \frac{(-1)^{12}\times 2}{17} = \frac{1\times 2}{17} = \frac{2}{17}$$

R = **2**

23. (b) $\frac{1234567891011121314}{8}$

$\because$ divisibility by 8 $\rightarrow$ The Last Three digits are divisible by 8 So Now last 3 digits 314 divide by 8 we get remainder

$= \frac{314}{8}$, R = **2**

24.(c) $\frac{41424344.........4950}{16}$

divisibility by 16 $\rightarrow$ The last Four digits are divisible by 16
No : Last '4' digits $\underline{4}$ $\underline{9}$ $\underline{5}$ $\underline{0}$

$\frac{4950}{16}$ = R = **6**

25. (b) $\frac{21222324252627282930}{8}$

Last '3' digits 930

Remainder = $\frac{930}{8}$

R = **2**

26. (c) $\frac{919293949596979899}{16}$

Last '4' digits $\underline{9}$ $\underline{8}$ $\underline{9}$ $\underline{9}$

Remainder = $\begin{array}{r} 16\,)\,9899\,(\\ 96 \\ \hline 29 \\ 16 \\ \hline 139 \\ 128 \\ \hline 11 \end{array}$

Remainder = **11**

27. (c) $\frac{313233........3839}{4}$

divisibility by 4 $\rightarrow$ The last '2' digits divisible by 4
Last '2' digits 39

R = $\frac{39}{4}$

R = **3**

28. (a) $\frac{12345..........41 \text{ digits}}{8}$

From 1 to 9 = 9 digits
Remainder = 41 - 9 = 32 digits

Number = $\frac{32}{2}$ = 16

1, 2, 3, 4, ...9/ $\underline{10}$ $\underline{11}$ .... 41 digits
Total Number = 9 + 16 = 25
1 2 3 4 .... 23 $\underline{24}$ $\underline{25}$
Last '3' digits = 425

Remainder = $\frac{425}{8}$

R = **1**

29. (a) $\frac{1234.......81\text{digits}}{16}$

from 1 to 9 = 9 digits
Remainder digits = 81 - 9 = 72 digits

Number = $\frac{72}{2}$ = 36

1, 2, 3, ...9/ $\underline{10}$ $\underline{11}$ .. 81 digits
Total Number = 9 + 36 + 45
1, 2, 3, ... 43 44 45
Last '4' digits 4445 divide by 16 we get remainder

R = $\frac{4445}{16}$ = **13**

$\begin{array}{r} 16\,)\,4445\,(\\ 32 \\ \hline 124 \\ 112 \\ \hline 125 \\ 112 \\ \hline 13 \end{array} \rightarrow$ Remainder

30. (b) $\frac{8^{77}}{17}$

$\because 8^3 = 512$

$$\Rightarrow \frac{(8^3)^{25}\times 8^2}{17} = \frac{(\overset{\overset{+2}{\uparrow}}{512})^{25}\times 64}{17}$$

$$\Rightarrow \frac{(2)^{25}\times 64}{17}$$

$$= \frac{(2^4)^6\times 2^1\times 64}{17}$$

$$= \frac{(\overset{\overset{1}{\uparrow}}{16})^6\times 2\times \overset{\overset{-4}{\uparrow}}{64}}{17} = \frac{(-1)^6\times 2\times -4}{17}$$

$$= \frac{1\times 2\times -4}{17} = \frac{-8}{17}$$

R = 17 - 8 = 9

31. (d) $\frac{\lfloor 1 + \lfloor 2 + \lfloor 3 + ...\lfloor 100}{5}$

$\lfloor 1 = 1 = \frac{\overset{\overset{+1}{\uparrow}}{1}}{5}$, R = 1

$\lfloor 2 = 1\times 2 = \frac{\overset{\overset{+2}{\uparrow}}{2}}{5}$, R = 2

$\lfloor 3 = 1\times 2\times 3 = \frac{\overset{\overset{+1}{\uparrow}}{6}}{5}$, R = 1

$\lfloor 4 = 1\times 2\times 3\times 4 = \frac{\overset{\overset{-1}{\uparrow}}{24}}{5}$, R = -1

$\lfloor 5 = 5\times 4\times 3\times 2\times 1 = \frac{120}{5}$, R = 0

$\lfloor 5, \lfloor 6, \lfloor 7$ .... $\lfloor 100$ is all perfect divisible by 5.So remainder '0'

$$= \frac{\overset{\overset{+1}{\uparrow}}{\lfloor 1} + \overset{\overset{+2}{\uparrow}}{\lfloor 2} + \overset{\overset{+1}{\uparrow}}{\lfloor 3} + \overset{\overset{-1}{\uparrow}}{\lfloor 4} + \overset{\overset{0}{\uparrow}}{\lfloor 5} + \overset{\overset{0}{\uparrow}}{\lfloor 6} + ...\overset{\overset{0}{\uparrow}}{\lfloor 99} + \overset{\overset{0}{\uparrow}}{\lfloor 100}}{5}$$

$$\Rightarrow \frac{1+2+1-1}{5} = \frac{3}{5}$$

R = **3**

32. (a) $\dfrac{\lfloor 1+\lfloor 2+\lfloor 3+\dots\dots\lfloor 100}{6}$

$\lfloor 1 = 1 = \dfrac{\overset{+1}{\overset{\uparrow}{1}}}{6} = R = 1$

$\lfloor 2 = 1\times 2 = \dfrac{\overset{+2}{\overset{\uparrow}{2}}}{6} = R = 2$

$\lfloor 3 = 1\times 2\times 3 = \dfrac{\overset{0}{\overset{\uparrow}{6}}}{6} = R = 0$

$\lfloor 4 = 4\times 3\times 2\times 1 = \dfrac{24}{6} = R = 0$

$= \dfrac{\overset{+1}{\overset{\uparrow}{\lfloor 1}}+\overset{+2}{\overset{\uparrow}{\lfloor 2}}+\overset{0}{\overset{\uparrow}{\lfloor 3}}+\overset{0}{\overset{\uparrow}{\lfloor 4}}+\overset{0}{\overset{\uparrow}{\lfloor 5}}+\dots\dots\dots\overset{0}{\overset{\uparrow}{\lfloor 100}}}{6}$

$\dfrac{1+2}{6} = \dfrac{3}{6}$

R = **3**

33. (d) $\dfrac{\lfloor 1+\lfloor 2+\lfloor 3+\dots\dots\lfloor 50}{12}$

$\lfloor 1 = 1 = \dfrac{\overset{+1}{\overset{\uparrow}{1}}}{12} = R = 1$

$\lfloor 2 = 1\times 2 = \dfrac{\overset{+2}{\overset{\uparrow}{2}}}{12} = R = 2$

$\lfloor 3 = \dfrac{1\times 2\times 3}{12} = \dfrac{\overset{+6}{\overset{\uparrow}{6}}}{12} = R = 6$

$\lfloor 4 = \dfrac{1\times 2\times 3\times 4}{12} = \dfrac{\overset{0}{\overset{\uparrow}{24}}}{12} = R = 0$

$\lfloor 5 = \dfrac{1\times 2\times 3\times 4\times 5}{12} = \dfrac{\overset{0}{\overset{\uparrow}{120}}}{12} = R = 0$

$= \dfrac{\overset{+1}{\overset{\uparrow}{\lfloor 1}}+\overset{+2}{\overset{\uparrow}{\lfloor 2}}+\overset{+6}{\overset{\uparrow}{\lfloor 3}}+\overset{0}{\overset{\uparrow}{\lfloor 4}}+\overset{0}{\overset{\uparrow}{\lfloor 5}}+\dots\dots\dots\overset{0}{\overset{\uparrow}{\lfloor 50}}}{6}$

$\dfrac{1+2+6}{12} = \dfrac{9}{12} = 9$

R = 9

34.(b) $\dfrac{9^{111}}{11} = \dfrac{\overset{-2}{\overset{\uparrow}{(9)}}{}^{111}}{11}$

$= \dfrac{(-2)^{111}}{11} = -\dfrac{(2)^{111}}{11}$

Avoid -ve sign

$\dfrac{2^{111}}{11} = \dfrac{(2^5)^{22}\times 2^1}{11}$

$\because\ 2^5 = 32$

$= \dfrac{\overset{-1}{\overset{\uparrow}{(32)}}{}^{22}\times 2}{11}$

$= \dfrac{(-1)^{22}\times 2}{11} = \dfrac{1\times 2}{11} = \dfrac{2}{11}$

Now use -ve sign

R = – 2

Actual Remainder = 11 – 2 = **9**

35. (b) $\dfrac{5^{2450}}{126}$

$\because\ 5^3 = 125$

$\dfrac{(5^3)^{816}\times 5^2}{126} = \dfrac{\overset{-1}{\overset{\uparrow}{(125)}}{}^{816}\times 25}{126}$

$= \dfrac{(-1)^8\times 25}{126} = \dfrac{1\times 25}{126} = \dfrac{25}{126}$

R = 25

36. (d) $\dfrac{\overset{-2}{\overset{\uparrow}{(40)}}{}^{1012}}{7} = \dfrac{(-2)^{1012}}{7}$

- ve sign will be no effect because power is even

$= \dfrac{(2)^{1012}}{7}$

$\because\ 2^3 = 8$

$= \dfrac{(2^3)^{337}\times 2^1}{7}$

$= \dfrac{\overset{+1}{\overset{\uparrow}{(8)}}{}^{337}\times 2}{7} = \dfrac{(+1)^{337}\times 2}{7}$

$= \dfrac{1\times 2}{7} = \dfrac{2}{7}$

R = **2**

37. (a) $\dfrac{10^1+10^2+10^3+10^4\dots\dots.10^{100}}{6}$

$10^1 = \dfrac{10}{6} = R = 4$

$10^1+10^2 = \dfrac{\overset{4}{\overset{\uparrow}{10}}+\overset{4}{\overset{\uparrow}{100}}}{6} = \dfrac{\overset{+2}{\overset{\uparrow}{8}}}{6}$

R = 2

$10^1+10^2+10^3 = \dfrac{10+100+1000}{6}$

$= \dfrac{4+4+4}{6} = \dfrac{12}{6}$

Remainder = 0

'0' will be the remainder of each three terms

So '0' will be the remainder of 99th term.

$\Rightarrow \dfrac{\overset{+4}{\overset{\uparrow}{10}}{}^{100}}{6} \quad R = \mathbf{4}$

* **10^n is divided by 6. We always get remainder 4, where n = natural number**

38. (c) $\dfrac{10^1+10^2+10^3\dots\dots\dots10^{1000}+10^{1001}}{6}$

$10^1 = \dfrac{\overset{+4}{\overset{\uparrow}{10}}}{6}\ R = 4$

$10^1+10^2 = \dfrac{\overset{+4}{\overset{\uparrow}{10}}+\overset{+4}{\overset{\uparrow}{100}}}{6} = \dfrac{8}{6}$

R = 2

$10^1+10^2+10^3 = \dfrac{\overset{+4}{\overset{\uparrow}{10}}+\overset{+4}{\overset{\uparrow}{100}}+\overset{+4}{\overset{\uparrow}{1000}}}{6}$

$= \dfrac{4+4+4}{6} = \dfrac{12}{6}$

R = 0

'0' will be remainder of each three terms so '0' will be the remainder of 999th.

$\dfrac{\overset{+4}{\overset{\uparrow}{10}}{}^{1000}+\overset{4}{\overset{\uparrow}{10}}{}^{1001}}{6} = \dfrac{4+4}{6} = \dfrac{8}{6}$

R = 2

39.(a) $\dfrac{666666\dots\dots.134\,\text{times}}{13}$

* If any digit is made by repeating a 6 times. The Number will be divisible by 3, 7, 11, 13, 37, 39. So, 666666 .... 132 times is divisible by 13. because 132 is the multiple of 6. So, The Remaining 2 digits will be. divideded by 13 to get the Remainder

$$= \frac{666666........132\text{ times }66}{13}$$

$$= \frac{66}{13} = R = 1$$

40. (b) $\frac{555555......244\text{ times}}{37}$

555555......240 times is divisible by 37 becaused any digit is made by repeating 6 times. The number will be divisible by 37.

$$\frac{555555........240\text{ times},5555}{37}$$

Remainder $\frac{5555}{37}$

$$\begin{array}{r} 37\overline{)5555}(15 \\ \underline{37} \\ 185 \\ \underline{185} \\ 5 \end{array} \longrightarrow \text{Remainder}$$

R = 5

41. (b) $\frac{777777.............363\text{ Times}}{11}$

$$= \frac{777777.............360\text{ Times},777}{11}$$

$$= \frac{777}{11}$$

Remainder = 7

42. (b) $\frac{888888.........184\text{ times}}{37}$

$$\Rightarrow \frac{888888.........180\text{ times},8888}{37}$$

$$\Rightarrow \frac{8888}{37}$$

$$\begin{array}{r} 37\overline{)8888}(\\ \underline{74} \\ 148 \\ \underline{148} \\ 8 \end{array} \longrightarrow \text{Remainder}$$

∵ So Remainder = 8

43. (b) $\frac{999999999}{13}$

6 times '9' is divisible by 13, remainder will be 0, Remaining digits divide by 13 we get Remainder

$$\frac{999}{13}$$

R = **11**

44. (c) $\frac{7^{99}}{2400}$

$$\boxed{\begin{array}{l} 7^1 = 7 \\ 7^2 = 49 \\ 7^3 = 343 \\ 7^4 = 2401 \end{array}}$$

Break the power multiple of '4' form

$$\Rightarrow \frac{(7^4)^{24} \times 7^3}{2400}$$

$$\Rightarrow \frac{(2401)^{24} \times 343}{2400}$$

$$\Rightarrow \frac{(\overset{+1}{\overset{\uparrow}{2401}})^{24} \times 343}{2400} = \frac{(+1)^{24} \times 343}{2400}$$

$$= \frac{1 \times 343}{2400} = \frac{343}{2400}$$

R = 343

45. (b) $\frac{3^{1989}}{7}$

∵ $3^3 = 27$

$$\frac{(3^3)^{663}}{7} = \frac{(\overset{-1}{\overset{\uparrow}{27}})^{663}}{7}$$

$$= \frac{(-1)^{663}}{7} = -1$$

R = 7 – 1 = 6

46. (a)

$$\frac{54^{124}}{17} = \frac{(\overset{+3}{\overset{\uparrow}{54}})^{124}}{17}$$

$$= \frac{(3)^{124}}{17}$$

$$\boxed{\begin{array}{l} 3^1 = 3 \\ 3^2 = 9 \\ 3^3 = 27 \\ 3^4 = 81 \end{array}}$$

∵ $3^4 = 81$

$$= \frac{(3^4)^{31}}{17} = \frac{(\overset{-4}{\overset{\uparrow}{81}})^{31}}{17}$$

$$= \frac{(-4)^{31}}{17} = -\frac{(4)^{31}}{17}$$

Avoid -ve sing. $= \frac{4^{31}}{17}$

∵ $4^2 = 16$

$$= \frac{(4^2)^{15} \times 4^1}{17} = \frac{(16)^{15} \times 4}{17}$$

$$= \frac{(-1)^{15} \times 4}{17} = \frac{-1 \times 4}{17} = \frac{-4}{17} = -4$$

Now use -ve sign = – (–4)

Remainder = **4**

47. (b)

$$\frac{(21)^{875}}{17} = \frac{(\overset{+4}{\overset{\uparrow}{21}})^{875}}{17}$$

$$= \frac{(4)^{875}}{17} \quad \because 4^2 = 16$$

$$= \frac{(4^2)^{437} \times 4^1}{17}$$

$$= \frac{(\overset{-1}{\overset{\uparrow}{16}})^{437} \times 4^1}{17} = \frac{(-1)^{437} \times 4}{17}$$

$$= \frac{-1 \times 4}{17} = \frac{-4}{17}$$

R = 17 – 4 = 13

48. (d)

$$\frac{83^{261}}{17} = \frac{(\overset{-2}{\overset{\uparrow}{83}})^{261}}{17}$$

$$= \frac{(-2)^{261}}{17} = \frac{-(2)^{261}}{17}$$

Avoide -ve sign.

$$= \frac{2^{261}}{17} = \frac{(2^4)^{65} \times 2^1}{17}$$

∵ $2^4 = 16$

$$= \frac{(\overset{-1}{\overset{\uparrow}{16}})^{65} \times 2}{17} = \frac{(-1)^{65} \times 2}{17}$$

$$= \frac{-1 \times 2}{17} = -2$$

Now use sign -(-2) = 2

Remainder = 2

49. (a) $\frac{(32^{32})^{32}}{9}$

Cyclicity

$\frac{\overset{\overset{+5}{\uparrow}}{32^1}}{9} = R = 5$

$\frac{32^2}{9} = \frac{\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}}{9} = \frac{5\times5}{9} = \frac{25}{9} = R = 7$

$\frac{32^3}{9} = \frac{\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}}{9} = \frac{5\times5\times5}{9}$

$= \frac{125}{9} = R = 8$

$\frac{32^4}{9} = \frac{\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}}{9} = \frac{5\times5\times5\times5}{9}$

$= \frac{\overset{\overset{-2}{\uparrow}}{25}\times\overset{\overset{-2}{\uparrow}}{25}}{9} = \frac{-2\times-2}{9} = \frac{4}{9}$, R = 4

$\frac{32^5}{9} = \frac{\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}}{9} = \frac{\overset{\overset{-2}{\uparrow}}{25}\times\overset{\overset{-2}{\uparrow}}{25}\times\overset{\overset{5}{\uparrow}}{5}}{9}$

$= \frac{-2\times-2\times5}{9} = \frac{20}{9}$, R = 2

$\frac{32^6}{9} = \frac{\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}\times\overset{\overset{+5}{\uparrow}}{32}}{9} = \frac{\overset{\overset{-2}{\uparrow}}{25}\times\overset{\overset{-2}{\uparrow}}{25}\times\overset{\overset{-2}{\uparrow}}{25}}{9}$

$= \frac{-2\times-2\times-2}{9} = \frac{-8}{9}$, R = 9 − 8 = 1

After this It repeated so,
Cyclicity = 6

So, $\frac{(32)^6}{9}$ R = 1

Now $(32^{32})^{32} = \left[(32^6)^5\times32^2\right]^{32}$

$= \frac{\left((1)^5\times32^2\right)^{32}}{9} = \frac{(32^2)^{32}}{9}$

$\because \frac{32^2}{9} = R = 7$

(above explain In Solution)

$= \frac{(7)^{32}}{9} = \frac{(7^2)^{16}}{9} = \frac{(\overset{\overset{+4}{\uparrow}}{49})^{16}}{9}$

$= \frac{4^{16}}{9} = \frac{(4^3)^5\times4^1}{9} = \frac{(\overset{\overset{+1}{\uparrow}}{64})^5\times4}{9}$

$\frac{(+1)^5\times4}{9} = \frac{1\times4}{9} = \frac{4}{9}$

R = 4

50. (a) $\frac{(32^{32})^{32}}{7}$

When 32 is divided by 7 then Remainder 4
So, 32^{32} is divided by 7 remainder = 4^{32}

$= \frac{(4^{32})^{32}}{7}$

$4 = 2^2$

$\Rightarrow \frac{(2^{2\times32})^{32}}{7}$

$\Rightarrow \frac{(2^{64})^{32}}{7}$

$\because 2^3 = 8$

$= \frac{(2^{63}\times2^1)^{32}}{7}$

$= \frac{\left((2^3)^{21}\times2^1\right)^{32}}{7} = \frac{\left((\overset{\overset{+1}{\uparrow}}{8})^{21}\times2^1\right)^{32}}{7}$

$= \frac{(1\times2)^{32}}{7} = \frac{2^{32}}{7}$

Again $\because$ $2^3 = 8$

$\frac{2^{30}\times2^2}{7} = \frac{(2^3)^{10}\times4}{7}$

$= \frac{(\overset{\overset{+1}{\uparrow}}{8})^{10}\times4}{7} = \frac{(1)^{10}\times4}{7} = \frac{1\times4}{7}$

Remainder = 4

51. (b) $\frac{(33^{34})^{35}}{7}$

we solve $\frac{33^{34}}{7} = \frac{(\overset{\overset{-2}{\uparrow}}{33})^{34}}{7}$

$= \frac{(-2)^{34}}{7}$

No effect of -ve sign. Because power is even.

$= \frac{2^{34}}{7} = \frac{(2^3)^{11}\times2^1}{7}$

$= \frac{(\overset{\overset{+1}{\uparrow}}{8})^{11}\times2}{7} = \frac{(+1)^{11}\times2}{7} = \frac{1\times2}{7}$

$= \frac{2}{7}$

Now :- $\frac{(33^{34})^{35}}{7} = \frac{(2)^{35}}{7} = \frac{(2^3)^{11}\times2^2}{7}$

$= \frac{(\overset{\overset{+1}{\uparrow}}{8})^{11}\times4}{7} = \frac{1\times4}{7} = \frac{4}{7}$

R = 4

52. (a) $\frac{888^{222}+222^{888}}{5}$

$= \frac{888^{222}}{5} + \frac{222^{888}}{5}$

$= \frac{3^{222}}{5} + \frac{2^{888}}{5}$

$= \frac{(3^4)^{55}\times3^2}{5} + \frac{(2^4)^{222}}{5}$

$= \frac{1\times9}{5} + \frac{1}{5}$

$= \frac{4}{5} + \frac{1}{5} = \frac{4+1}{5} \Rightarrow \frac{5}{5}$

Thus the remainder is zero.

Alternatively:

[To check the divisibility by 5 just see the sum of the unit digits which is 10 (=4+6)

$\because$ $8^{222} \to 4(\text{units digit})$

and $2^{888} \to 6(\text{units digit})$

Hence it is divisible. So there is no remainder]

53.(a) $\frac{2222^{5555}+5555^{2222}}{7}$

$+3 \uparrow \quad +4 \uparrow$

$\frac{2222^{5555}}{7}+\frac{5555^{2222}}{7}$

$\frac{(3)^{5555}}{7}+\frac{(4)^{2222}}{7}$

$\frac{(3^3)^{1851}\times 3^2}{7}+\frac{(4^3)^{740}\times (4)^2}{7}$

$-1 \uparrow \quad +1 \uparrow$

$\frac{(27)^{1851}\times 9}{7}+\frac{(64)^{740}\times 16}{7}$

$\frac{(-1)^{1851}\times 9}{7}+\frac{(+1)^{740}\times 16}{7}$

$\frac{-9}{7}+\frac{16}{7}$

$\frac{-9+16}{7}=\frac{7}{7}$

Remainder = **0**

54. (d) $\frac{(50^{51})^{52}}{11}$,we break = 50^{51}

$6 \uparrow$

$\frac{(50^{51})}{11}=\frac{(6)^{51}}{11}$

$+3 \uparrow$

$=\frac{(6^2)^{25}\times 6^1}{11}=\frac{(36)^{25}\times 6}{11}$

$\frac{(3)^{25}\times 6}{11}=\frac{(3^5)^5\times 6}{11}$

∵ $3^5 = 243$

$+1 \uparrow$

$=\frac{(243)^5\times 6}{11}=\frac{(1)^5\times 6}{11}=\frac{6}{11}$

Now $\frac{50^{51}}{11}=\frac{6}{11}$

∵ $50^{51^{52}}=\frac{(6)^{52}}{11}$

$+3 \uparrow$

$=\frac{(6^2)^{26}}{11}=\frac{(36)^{26}}{11}$

$+1 \uparrow$

$=\frac{(3)^{26}}{11}=\frac{(3^5)^5\times 3^1}{11}$

$+1 \uparrow$

$=\frac{(243)^5\times 3}{11}$

$=\frac{(1)^5\times 3}{11}=\frac{1\times 3}{11}=\frac{3}{11}$

Remainder = **3**

55. (a)

$+3 \uparrow \qquad +10 \uparrow$

$\frac{(20)^{23}}{17}\to\frac{(3)^{23}}{17}\to\frac{(3^3)^7\times 3^2}{17}\to\frac{(27)^7\times 9}{17}$

$-2 \uparrow \quad +5 \uparrow$

$\to\frac{(10)^7\times 9}{17}\to\frac{(10^2)^3\times 10\times 9}{17}\to\frac{(100)^3\times 90}{17}$

$=\frac{(-2)^3\times 5}{17}=\frac{-8\times 5}{17}=\frac{-40}{17}$

∵ Avoid (-) sign normally divide 40 by 17, Remainder 6, Now use negative sign R = -6

∵ R = 17 - 6 = **11**

56. (c) $(x - 2)$ is factor of $x^2 + 3qx - 2q$
So, $x - 2 = 0 \Rightarrow x = 2$
Put, $x = 2$ in $x^2 + 3qx - 2q$ and equate to zero.
∵ $(2)^2 + 3 \times q \times 2 - 2q = 0$
$\Rightarrow 4 + 6q - 2q = 0 \Rightarrow 4 + 4q = 0$
$1 + q = 0 \Rightarrow$ **q = - 1**

57. (c)Factor $\Rightarrow x + 2 = 0 \Rightarrow x = -2$ (put)
∵ $(-2)^3 + 6(-2)^2 + 4(-2) + k = 0$
$\Rightarrow -8 + 24 - 8 + k = 0$
$\Rightarrow$ **k = -8**

58.(b) $x - 1 = 0 \Rightarrow x = 1$ (put)
$\Rightarrow 1^3 - k = 0$
$\Rightarrow 1 - k = 0 \Rightarrow$ **k = 1**

59.(a) $x + 1 = 0 \Rightarrow x = -1$ (put)
$\Rightarrow x^{100} + 2x^{99} + k = 0$
$\Rightarrow (-1)^{100} + 2(-1)^{99} + k = 0$
$\Rightarrow 1 - 2 + k = 0 \quad \Rightarrow -1 + k = 0$
$\Rightarrow$ **k = 1**

60.(a) $x + 2 = 0 \Rightarrow x = -2$ (put)
$\Rightarrow x^3 - 5x^2 + 4p = 0$
$\Rightarrow (-2)^3 - 5(-2)^2 + 4p = 0$
$\Rightarrow -8 - 20 + 4p = 0$
$\Rightarrow -28 + 4p = 0 \quad \Rightarrow 4p = 28$
$\Rightarrow p = 7$

61.(a) $x - a = 0 \Rightarrow x = a$
$\Rightarrow x^3 - 3x^2a + 2a^2x + b = 0$
$\Rightarrow a^3 - 3a^2a + 2a^2a + b = 0$
$\Rightarrow a^3 - 3a^3 + 2a^3 + b = 0$
$\Rightarrow b = 0$

62.(c) $(x + 6)$ is a factor of $f(x)$
The $f(x) = 0$
$x + 6 = 0$
$x = -6$
$f(x) = x^3 + 3x^2 + 4x + k$
$f(-6) = (-6)^3 + 3(-6)^2 + 4\times -6 + k = 0$
$= -216 + 108 - 24 + k = 0$
$k = 132$

63.(c) $(x + 2)$ & $(x - 1)$ are the factor of $x^3 + 10x^2 + mx + n$, so put $x = -2$ and $x = 1$ respectively
$\Rightarrow (-2)^3 + 10(-2)^2 + m(-2) + n = 0$
$\Rightarrow -8 + 40 - 2m + n = 0$
$\Rightarrow -2m + n + 32 = 0$
$\Rightarrow -2m + n = -32$ ......(i)
Put $x = 1$
$\Rightarrow (1)^3 + 10(1)^2 + m.1 + n = 0$
$\Rightarrow 1 + 10 + m + n = 0$
$\Rightarrow m + n = -11$ ....(ii)
solve (i) and (ii) we get
$\Rightarrow m = 7$
and
$\Rightarrow n = -18$

64. (b)$x + 1 = 0 \Rightarrow x = -1$
put the value of $x = -1$ in equation
$R = x^3 - 6x + 7 = (-1)^3 - 6(-1) + 7$
$= -1 + 6 + 7$
Remainder = 12

65. (a)$x - 3 = 0 \Rightarrow x = 3$
put of value of $x = 3$ in $x^5 - 9x^2 + 12x - 14$
$R = (3)^5 - 9(3)^2 + 12(3) - 14$
$R = 243 - 81 + 36 - 14$
$R = 279 - 95$
$R = 184$

66. (b)$x - 2 = 0 \Rightarrow x = 2$
put the value of $x = 2$ in $x^4 - 3x^3 + 2x^2 - 5x + 7$
$R = 2^4 - 3(2)^3 + 2(2)^2 - 5(2) + 7$
$R = 16 - 24 + 8 - 10 + 7$
$R = 31 - 34 = -3$

67. (d)$x + 3 = 0 \Rightarrow x = -3$
put te value of $x = -3$ in $5x^3 + 5x^2 - 6x + 9$
$R = 5(-3)^3 + 5(-3)^2 - 6(-3) + 9$
$R = -135 + 45 + 18 + 9$
$R = -135 + 72 = -63$

68. (b) $x + 1 = 0 \Rightarrow x = -1$
put the value of $x = -1$ in $x^{11} + 1$
$R = (-1)^{11} + 1 = -1 + 1 = 0$

69. (c) $2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

put the value of $x = -\frac{1}{2}$ in $2x^3 + 5x^2 - 4x - 6$

$$R = 2\left(-\frac{1}{2}\right)^3 + 5\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) - 6$$

$$R = -2 \times \frac{1}{8} + \frac{5}{4} + \frac{4}{2} - 6$$

$$R = -\frac{1}{4} + \frac{5}{4} + 2 - 6$$

$$R = \frac{4}{4} - 4 = -3$$

70. (c) $x^2 + 2 = 0 \Rightarrow x^2 = -2$

Put the value of $x^2 = -2$ in $x^3 + 5x^2 + 10k$

$R = x^2 . x + 5x^2 + 10k = -2x$ (given)

$\Rightarrow (-2)\, x + 5. (-2) + 10k = -2x$

$\Rightarrow -10 + 10k = 0$

$\Rightarrow k = 1$

CHAPTER

06

ARITHMETIC PROGRESSION & GEOMETRIC PROGRESSION

Arithmetic Progression

A.P :- Quantities are said to be In arithmetic progression when they Increase or decrease by a common difference.

Some more examples of Arithmetic Progression are as follows

1, 2, 3, 4, 5, 6, 7,...

3, 7, 11, 15, 19, 23 ....

19, 17, 15, 13, 11, ...

–10, -4, 2, 8, 14, 20, ...

40, 37.5, 35, 32.5, 30, ...

5 , 12 , 19 , 26 , 33..........

+7 +7 +7 +7

a_1 a_2 a_3 a_4 a_5

d = Common difference

$= a_2 - a_1 = 12 - 5 = 7$

$a_1 \rightarrow$ First Term

$a_2 \rightarrow$ Second Term

$\boxed{T_n=a+(n-1)\ d}$

$T_n \rightarrow$ nth term of A.P.

a $\rightarrow$ First Term of A.P.

d $\rightarrow$ common difference

$\boxed{L= a+(n-1)\ d}$

L $\rightarrow$ Last term/ nth term of A.P.

Sum of n term.

$$Sn = \frac{n}{2}\left[2a+(n-1)d\right]$$

or $Sn = \frac{n}{2}[a+l]$

Ex. 1 5, 12, 19, 26 .... T_{10}

(i) Find the value of t_{10}

(ii) Sum of 10 term.

Sol. 5, 12, 19, 26 .... T_{10}

d = 12 - 5 = 7

a = 5

n = 10

T_{10} = a + (n - 1) d

= 5 + (10 - 1) 7

= 5 + 63 = 68

(ii) $Sn = \frac{n}{2}\left[2a+(n-1)d\right]$

$= \frac{10}{2}\left[2\times5+(10-1)7\right]$

= 5 [10 + 63] = 5 × 73 = **365**

Ex. 2 Find the sum of the 10 terms of the following series -11, -8, -5, -2, ...

Sol. -11, -8, -5, -2, ...

a = - 11, d = 3, n = 10

$S_n = \frac{n}{2}$ [2a + (n-1)d]

$S_{10} = \frac{10}{2}$ [2×(-11)+(9)3]

= 5× (-22 +27) = 25

Ex. 3 If $t_1+t_5+t_{10}+t_{15}+t_{20}+t_{24}= 225$ (where t_1) $\rightarrow$ first term of A.P, T_5 $\rightarrow$ fifth term of A.P) find the sum of $t_1+t_2+t_3+$ .... t_{24}

$\boxed{Tn = a+(n-1)d}$

Sol. t_1 = first term = a

t_5 = a + (5 - 1) d = a + 4d

t_{10} = a + 9d

t_{15} = a + 14d

t_{20} = a + 19d

t_{24} = a + 23 d

a + a + 4d + a +9d + a + 14d + a + 19d + a + 23d = 225

6a + 69d = 225

$\boxed{2a+23d = 75}$

$S_{24} = \frac{24}{2}\left[2a+(24-1)d\right]$

$S_{24} = 12\left[2a+23d\right]$

Put value of 2a + 23d = 75

S_{24} = 12 [75] = **900**

Ex.4 Find the 33rd term of the sequence.

3, 8, 9, 13, 15, 18, 21, 23 ....

(a) 93 (b) 992 (c) 105 (d) 83

Sol. (b) 3, 8, 9, 13, 15, 18, 21, 23 ....

The above series is a combination af two APS

The Ist A.P (3, 9, 15, 21....) + 2nd AP (8, 13, 18, ...)

33rd Term lies, In first A.P which 17th term of first A.P Then

T_{17} = a + (n - 1) d

a = 3

d = 6

T_{17} = 3 + 16 × 6

T_{17} = 99

We can say that

$\mathbf{T_{33} = 99}$

Ex.5 Find the sum to 100 terms of the series

1 + 4 + 6 + 5 + 11 + 6 + ...

(a) 7400 (b) 7550

(c) 7600 (d) 7500

Sol. (c) 1 + 4 + 6 + 5 + 11 + 6 + ...

The series is a combination of two APS.

(1 + 6 + 11 + ... 50th term) + (4 + 5 + 6 + ...50th term)

Sum of series

$a_1 = 1$	a = 4
$d_1 = 5$	d = 1
n = 50	n = 50

$S_{100} = \frac{50}{2}[2 \times 1 + 49 \times 5] + \frac{50}{2}[2 \times 4 + 49 \times 1]$

= 25 [2 + 245] + 25 [8 + 49]

= 25 [247] + 25 [57]

= 25 [247 + 57]

= 25 × 304 = **7600**

Ex.6 Find the value of

1 - 2 - 3 + 2 - 3 - 4 + ...+up to 100 terms

(a) –626 (b) – 622

(c) – 624 (d) – 628

Sol.(a) Series is a combination of three APS.

(1 + 2 + 3 ...... 34 terms) -

a = 1

d = 1

(2 + 3 + 4 ......33 terms) -

a = 2

d = 1

(3 + 4 + 5 ....33terms)

a = 3

d = 1

Sum of series

$= \frac{34}{2}[2 \times 1 + 33 \times 1] - \frac{33}{2}[2 \times 2 + 32 \times 1] - \frac{33}{2}[2 \times 3 + 32 \times 1]$

$= 17[2 + 33] - \frac{33}{2}[4 + 32] - \frac{33}{2}[6 + 32]$

$= 17 \times 35 - \frac{33}{2} \times 36 - \frac{33}{2} \times 38$

$= 17 \times 35 - 33 \times 18 - 33 \times 19$

$= 595 - 594 - 627 = \mathbf{-626}$

Ex.7 It the sum of first 11 terms of A.P is equal to sum of first 19 terms of that A.P. find the sum of first 30 terms of that A.P.

Sol. According to question

$S_{11} = S_{19}$

$\frac{11}{2}[2a + 10d] = \frac{19}{2}[2a+18d]$

$22a + 110d = 38a + 342d$

$16a = -232d$

$2a = -29d$

$\boxed{2a+29d=0}$

$S_{30} = \frac{30}{2}[2a + (30-1)d]$

$= 15[2a + 29d]$

Put value

$\boxed{2a+29d=0}$

$S_{30} = 15 \times 0 = 0$

Sum of first 30 terms of that A.P = 0

$$\boxed{\text{Total No. of Term} = \frac{\text{last term} - \text{First term}}{\text{common difference}} + 1}$$

Ex.8 Find the Total No. of terms Between. 300 to 600 which are exactly divisible by 4.

Sol. 304, 308, 312 596

d = 4, a = 304, last terms = 596

No. of Terms $= \frac{l-a}{d} + 1$

$= \frac{596-304}{4} + 1$

$= \frac{292}{4} + 1$

$= 73 + 1 = 74$

Ex.9 The sum of the second and the fifth term of an AP is 8 and that of the third and the seventh term is 14. Find the eleventh term.

(a) 19 (b) 17 (c) 15 (d) 16

Sol. **(a)** $T_2 + T_5 = 8$

$T_3 + T_7 = 14$

$T_2 = a + d$

$T_5 = a + 4d$

$T_2 + T_5 = a + d + a + 4d = 8$

$\mathbf{2a + 5d = 8}$ ....(i)

$T_3 = a + 2d$

$T_7 = a + 6d$

$T_3 + T_7 = a + 2d + a + 6d = 14$

$\mathbf{2a + 8d = 14}$ ....(ii)

(ii) – (i)

$(2a + 8d) - (2a + 5d) = 14 - 8$

$3d = 6$

$\mathbf{d = 2}$

Put the value d equation (ii)

$2a + 8 \times 2 = 14$

$\mathbf{a = -1}$

$T_{11} = a + 10d$

$= -1 + 10 \times 2$

$= -1 + 20$

$= \mathbf{19}$

Ex.10 A number 20 is divided Into Four Parts that are in AP such that the product of the first and fourth is to the product of the second and third is 2 : 3. Find the Largest parts.

(a) 12 (b) 4 (c) 8 (d) 9

Sol. **(c)** Let the 4 terms of A.P.

$a - 3d, a - d, a + d, a + 3d$

$(a - 3d) + (a + 3d) + a - d + a + d = 20$

$4a = 20$

$\mathbf{a = 5}$

$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{2}{3}$

$\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{2}{3}$

Put the value of a

$\frac{25 - 9d^2}{25 - d^2} = \frac{2}{3}$

$75 - 27d^2 = 50 - 2d^2$

$25 = 25d^2$

$d^2 = 1$

$\mathbf{d = 1}$

$a - 3d = 5 - 3 \times 1 = 2$

$a - d = 5 - 1 = 4$

$a + d = 5 + 1 = 6$

$a + 3d = 5 + 3 \times 1 = 8$

Largest part = 8

Ex.11 How many term of an AP must be taken for their sum to be equal to 120 if its third term is 9 and the difference between the seventh and the second term is 20 ?

(a) 6 (b) 9 (c) 7 (d) 8

Sol. **(d)** $T_3 = a + 2d = 9$

$T_7 - T_2 = (a + 6d) - (a + d) = 20$

$5d = 20$

$\mathbf{d = 4}$

$T_3 = a + 2 \times 4 = 9$

$\mathbf{a = 1}$

$120 = \frac{n}{2}[2 \times 1 + (n - 1)4]$

$120 = \frac{n}{2} \times 2[1 + (n - 1)2]$

$120 = n[1 + 2n - 2]$

$120 = n(2n - 1)$

Take Option (d) , Put the value

n = 8

$= 8(2 \times 8 - 1) = 120$

$\because$ **n = 8**

Useful Results

* If the same quantity be added to, or subtracted from,all the terms of an AP, the resulting terms will from an AP, but with the same common difference as before.
* If all the terms of an AP be multiplied or divided by the same quantity, the resulting terms will form an AP, but with a new common difference, which will be the multiplication/division of the old common difference. (as the case may be)
* If $a_1, a_2, a_3, a_4, a_5, \ldots\ldots. a_n$ are in A.P then **the arithmetic mean (A.M)**

  $= \left(\frac{\mathbf{a_1 + a_2 + a_3 + \ldots. a_n}}{\mathbf{n}}\right)$
* The arithmetic mean (A.M) of odd numbers of consecutive terms is the **middle most term** itself.
* If a and b are in A.P then

  A.M. $= \frac{a+b}{2}$

* If you have to assume **3** terms in AP, assume them as **a -d, a, a +d** or **a,a + d, a +2d**
For assuming **4** terms of an AP we use; **a - 3d, a-d, a +d and a +3d**
For assuming **5** terms of an AP, take them as:
a-2d, a-d, a, a+d, a+2d.

Geometric Progression

Geometric Progression (G.P) GP
→ Guantitier are said to be In geometric progression when they Increase or decrease by a constant factor

Ex. 3, 6, 12, 24, ...
$\downarrow \; \downarrow$
$a_1 \; a_2$

Common ratio (r) $= \frac{a_2}{a_1}$

n^{th} term of G.P →

$\boxed{T_n = ar^{n-1}}$

Where a = First terms of G.P
r = common ratio
n → no. of Term.

Sum of G.P

$\boxed{s_n = a\frac{(r^n - 1)}{r-1}}$ where r > 1

or,

$\boxed{s_n = a\frac{(1 - r^n)}{1-r}}$ Where r <1

Ex. 1 3, 6, 12, 24 .... T_{10}
Find
(i) T_{10} (10^{th} term of G.P)
(ii) S_{10} (Sum of First 10 terms of G.P)

Sol. 3, 6, 12, 24, ...T_{10}

$r = \frac{6}{3} = 2$

a = 3

$T_{10} = 3.2^{(10-1)} = 3 \times 2^9$
$= 3 \times 512 = 1536$

$Sn = a\frac{r^n - 1}{r-1}$ r>1

$S_{10} = \frac{3(2^{10}-1)}{2-1} = \frac{3(2^{10}-1)}{1}$

= 3×(1024 – 1)
= 3 × 1023 = 3096

Sum of an Infinite geometric progression when r < 1

$\boxed{S_\infty = \frac{a}{1-r}}$

Ex. 2 16, 8, 4, 2, 1, $\frac{1}{2}$ ....∞
find sum of G.P

Sol. 16, 8, 4, 2 .... ∞
a = 16

$r = \frac{8}{16} = \frac{1}{2}$

r < 1

$S_\infty = \frac{a}{1-r} = \frac{16}{1-\frac{1}{2}} = \frac{16}{1/2} = 32$

Ex. 3 Find the value of

$25^{\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+.........\infty\right)}$

Sol. First Solve this part

$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}.........\infty$

It is an Inifinite G.P.

$a = \frac{1}{3}$

$r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}$

∵ where r < 1

$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

Then
Put the value of

$\frac{1}{3}+\frac{1}{4}+\frac{1}{27}......\infty = \frac{1}{2}$

$= (25)^{1/2} = 5$

Ex. 4 Find the Geometric mean of 3, 9, 27.
(a) 9 (b) 27 (c) 3 (d) 81

Sol. (a) G.M = $(a_1.a_2.a_3.)^{1/3}$
$= (3 \times 9 \times 27)^{1/3}$
$= (3^1 \times 3^2 \times 3^3)^{1/3}$
$= (3^6)^{1/3} = 3^2$ **= 9**

Ex. 5 Find the G.M of 2, 4, 8, 16,32
(a) 4 (b) 8 (c) 16 (d) 32

Sol. (b) $(2\times4\times8\times16\times32)^{1/5}$
$= (2^1 \times 2^2 \times 2^3 \times 2^4 \times 2^5)^{1/5}$
$= (2^{1+2+3+4+5})^{1/5} = 2^{15\times\frac{1}{5}}$
$= 2^3$ **= 8**

Ex.6 The seventh Term of a G.P 8 times the Fourth Term. What will be the first term when its fifth term is 48 ?
(a) 3 (b) 6 (c) 2 (d) 4

Sol. (a) $T_7 = 8\ T_4$
$ar^6 = 8 \times ar^3$
$r^3 = 8$
r = 2
$T_5 = ar^4 = 48$
$a \times (2)^4 = 48$
$a \times 16 = 48$
First term **a = 3**

Ex.7 What will be the sum of n terms of the series
8 + 88 + 888 + ... ?

(a) $\frac{8(10^{n+1} - 9n)}{81}$

(b) $\frac{8(10^{n+1} - 10 - 9n)}{81}$

(c) $8\ (10^{n-1} - 10)$
(d) $8\ (10^{n+1} - 10)$

Sol. (b) 8 + 88 + 888 + ... n
= 8 (1 + 11 + 111 + ...n)

$= \frac{8}{9}$ [9 + 99 + 999 ....n]

$= \frac{8}{9}$ [(10-1)+(100-1)+(100-1).....n]

$= \frac{8}{9}$ $[(10^1+10^2+10^3...10^n)$ – (1+1+1....n)]
↓
It is a G.P

$= \frac{8}{9}\left[10\frac{(10^n - 1)}{10-1} - n\right]$

$= \frac{8}{9}\left[\frac{10^{n+1} - 10}{9} - n\right]$

$= \frac{8}{9}\frac{[10^{n+1} - 10 - 9n]}{9}$

$= \frac{8[10^{n+1} - 10 - 9n]}{81}$

Ex.8 (666 ....n digit)^2 + (888.....n digit) is equal to

(a) $(10^n - 1)\frac{4}{9}$

(b) $(10^{2n} - 1)\frac{4}{9}$

(c) $\frac{4}{9}(10^n - 1)^{n-1} - 1)$

(d) $\frac{4}{9}(10^n + 1)$

Sol. **(b)** $(666.........n\ digit)^2 + (888....n\ digit)$

In this type question we take n = 1 then

$(6)^2 + 8 = 44$

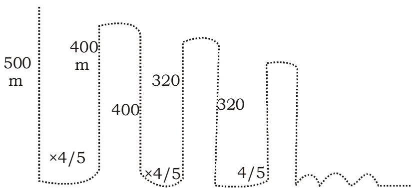
we take option (b) put the n = 1

$(10^2 - 1)\frac{4}{9} = 99\times\frac{4}{9} = 44$

So, $\mathbf{(10^{2n} - 1)\frac{4}{9}}$

Ex.9 A ball is thrown from a height of 500m on the ground. The ball bounce 4/5 times of I[st] every last bounce then calculate the total Distance the ball It stop.

Sol.



(500 + 400), (400 + 320), ...

900, 720, ... ∞

a = 900

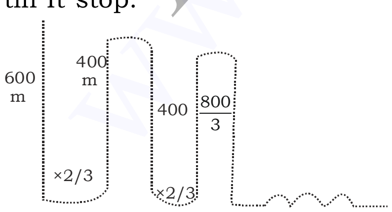
$r = \frac{720}{900} = \frac{4}{5}$

$S_\infty = \frac{a}{1-r} = \frac{900}{1-\frac{4}{5}} = \frac{900}{1/5}$

= 900 × 5 = 4500 m

Ex.10 A ball is thrown from a height of 600 m on the ground. the ball bounce 2/3 times of Its every Last Bounce then calculate the Total Distance the ball till It stop.

Sol.



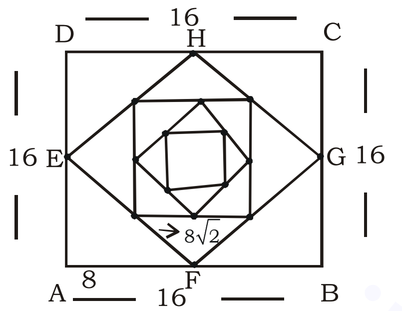
a = (600 + 400) = 1000

$1000,\ 400 + \frac{800}{3}, ...\ r = \frac{2}{3}$

$S_\infty = \frac{1000}{1-\frac{2}{3}} = \frac{1000}{1/3} = 3000\text{m}$

Ex.11 The side of a square is 16 cm Infinite squares are made by joining the mid points of the square. Calculate the area of all the squares made.

Sol.



Area of large square of ABCD $= (16)^2 = 256$

Square of made of mid-point (EFGH) $EF^2 = AE^2 + AF^2$

$AE = \frac{16}{2} = 8$ (E, midpoint of AD)

$AF = \frac{16}{2} = 8$ (F, mid point of AB)

$EF = \sqrt{(8)^2 + (8)^2} = 8\sqrt{2}$

Area of EFGH $= (8\sqrt{2})^2 = 128$

256 + 128 + ... ∞

a = 256

r = 128/256 = 1/2

$S_\infty = \frac{256}{1-1/2} = \frac{256}{1/2} = 512\ cm^2$

* The figure made by the joining the mid-points of a square and the area of thus made square is half of the actual square

Ex.12 A number of Squares are described whose perimetres are in G.P. Then their sides will be In

(a) A.P (b) G.P

(c) H.P

(d) None of these

Sol. **(b)** Perimeter of square = 4 × side

According to question

Let perimetres of square in G.P. 4, 8, 16, 32 ....

Then sides of square

$4a_1 = 4,\quad \mathbf{a_1 = 1}$

$4a_2 = 8,\quad \mathbf{a_2 = 2}$

$4a_3 = 16,\quad \mathbf{a_3 = 4}$

$4a_4 = 32,\quad \mathbf{a_4 = 8}$

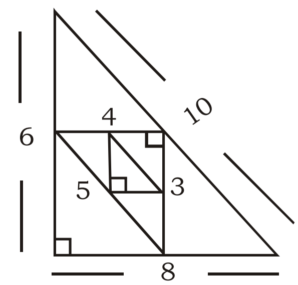
Where a_1, a_2, a_3, a_4 sides of square 1, 2, 4, 8

This is a G.P.

We can say that If perimeter of square In G.P. then their sides will be in G.P.

Ex.13 The sides of a right angle △ are 6, 8 and 10 cm respectively. A new right angle △ is made by joining the mid-points of all the sides . This process continues for infinite then calculate the area of all the △ so made.

Sol.



Area of Large right angle triangle $= \frac{1}{2}\times 6\times 8 = 24$

A_1 ↑ 24 $\quad A_2$ ↑ 6 $\quad A_3$ ↑ 3/2..........∞

$A_1 = 24 \rightarrow$ Area of first right angle triangle

$A_2 = 6 \rightarrow$ Area of second right angle triangle

$r = \frac{A_2}{A_1} = \frac{6}{24} = \frac{1}{4}$

$S_\infty = \frac{24}{1-\frac{1}{4}} = \frac{24}{3/4} = 24\times\frac{4}{3} = 32\text{cm}^2$

* The figure made by the joining the mid-points of a right angle triangle is right angle triangle and the area of thus made triangle is $\frac{1}{4}^{th}$ of the actual triangle.

Ex.14 If A is the sum of the n terms of the series 1 + 1/4 + 1/16 + ... and B is the sum of 2n terms

of the series 1 + 1/2 + 1/4 + ..., then find the value of A/B
(a) 1/3 (b) 1/2 (c) 2/3 (d) 3/4

Sol. (c) $A = 1 + \frac{1}{4} + \frac{1}{16} + \text{........n term}$

$a = 1$

$r = \frac{1}{4}$

$$A = S_n = \frac{1\left(1-\left(\frac{1}{4}\right)^n\right)}{1-\frac{1}{4}} = \frac{1-\left(\frac{1}{4}\right)^n}{\frac{3}{4}} \quad \text{....(i)}$$

$B = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ ....2n term

$a = 1$

$r = \frac{1}{2}$

$B = S_{2n}$

$$= \frac{1\left(1-\left(\frac{1}{2}\right)^{2n}\right)}{1-\frac{1}{2}} = \frac{1-\left(\frac{1}{4}\right)^n}{\frac{1}{2}} \quad \text{...(ii)}$$

(i) / (ii)

$A/B = \frac{2}{3}$

Ex.15 Find the sum to n terms of the series
11 + 103 + 1005 + ...

(a) $\frac{10}{9}(10^n + 1) + n^2$

(b) $\frac{10}{9}(10^n - 1) + n^2$

(c) $\frac{10}{9}(10^n + 1) + n$

(d) $\frac{10}{9}(10^n - 1) + n$

Sol. (b) 11 + 103 + 1005 + ...
10 + 1 + 100 + 3 + 1000 + 5 + ...
$(10^1 + 10^2 + 10^3 + ...10^n) + (1+3+5+......n)$
↓ ↓
It is a G.P It is sum of odd no.

$S_n = a\frac{(r^n - 1)}{r-1}$ $S = n^2$

Total sum = $\frac{10(10^n - 1)}{10-1} + n^2$

$= \mathbf{\frac{10}{9}(10^n - 1) + n^2}$

Ex.16 Two numbers A and B are such that their GM is 20% Lower than their AM. Find the Ratio B/W the numbers.
(a) 3 : 2 (b) 3 : 1
(c) 4 : 1 (d) 2 : 1

Sol. (c) GM of A and B = $(AB)^{1/2}$

AM of A and B = $\frac{A+B}{2}$

Let A.M = 100
Then G.M = 80
A.M : GM
100 : 80
5 : 4

$\frac{A.M}{G.M} = \frac{5}{4}$

$$\frac{\frac{(A+B)}{2}}{(AB)^{1/2}} = \frac{5}{4}$$

$\frac{A+B}{2\sqrt{AB}} = \frac{5}{4}$

Uses c & d form

$\frac{A+B+2\sqrt{ab}}{A+B-2\sqrt{ab}} = \frac{5+4}{5-4}$

$\frac{(\sqrt{A}+\sqrt{B})^2}{(\sqrt{A}-\sqrt{B})^2} = \frac{9}{1}$

$\frac{\sqrt{A}+\sqrt{B}}{\sqrt{A}-\sqrt{B}} = \frac{3}{1}$

Again Use c & d

$\frac{\sqrt{A}}{\sqrt{B}} = \frac{3+1}{3-1}$

$\frac{\sqrt{A}}{\sqrt{B}} = \frac{4}{2} = \frac{2}{1}$

Take Square Both sides

$\frac{A}{B} = \frac{4}{1}$

A : B = 4 : 1

Useful Result of G.P

* If all the terms of a GP be multiplied or divided by the same quantity, the resulting terms will form a GP with the same common ratio as before.
* If a, b, c, d, ...are in GP they are also in continued proportion, since, by definition,
  a/b =b/c =c/d = ...=1/r
  Conversely, a series of quantities in continued proportion may be represented by x, xr, xr^2......
* For assuming **3** terms of GP assume them as
  a, ar, ar² or **a/r, a, ar**
* for assuming **4** terms of GP assume them as
  $\mathbf{\frac{a}{r^3}, \frac{a}{r}, ar, ar^3}$
* If $a_1, a_2, a_3, a_4, a_5, ...a_n$ are in G.P **the geometric mean G.M** $= (a_1, a_2, a_3, a_4, ...a_n)^{1/n}$
* If a and b are two term in G.P then
  G.M = $\sqrt{ab}$
* To find the sum of first n natural numbers
  Let the sum be denoted by S; then
  S = 1 +2 + 3 + ...+n, is given by
  $\mathbf{S = \frac{n(n+1)}{2}}$
* To find the sum of the squares of the first n natural numbers
  Let the sum be denoted by S; then
  $S = 1^2 + 2^2 + 3^2 + ...n^2$
  This is given by:
  $\mathbf{S = \left\{\frac{n(n+1)(2n+1)}{6}\right\}}$
* To find the sum of the cubes of the first n natural numbers.
  Let the sum be denoted by S:then
  $S = 1^3 + 2^3 + 3^3 + ...+n^3$
  $\mathbf{S = \left[\frac{n(n+1)}{2}\right]^2}$
  Thus, the sum of the cubes of the first n natural numbers is equal to the square of the sum of these numbers.
* To find the sum of the first n odd natural numbers.
  S = 1 + 3 + 5 + ... + (2n - 1) → **n²**
* To find the sum of the first n even natural numbers.
  S = 2 + 4 + 6+....+2n → **n (n+1)**

EXERCISE

1. Find t_{10} and s_{10} for the following series.
1, 8, 15, 22..............
(a) 64,325 (c) 64,318
(c) 57,325 (d) 57,318

2. Find t_{20} and S_{20} for the following series
2, 8, 14, 20, ...
(a) 116,1172 (c) 110,1180
(c) 116,1180 (d) 110,1172

3. Find t_{24} and s_{24} for the following series.
3, 13, 23, 33 ....
(a) 233,2842 (b) 230,2832
(c) 230,2842 (d) 233,2832

4. Find t_{18} and s_{22} for the following series.
series – 3, 1, 5, 9,.........
(a) 65,858 (b) 60,850
(c) 60,868 (d) 65,850

5. Find t_{28} and s_{48} for the following series.
30, 33, 36, 39 ......
(a) 111,4834 (b) 111,4824
(c) 121,4824 (d) 121,4834

6. Find t_{30} and s_{30} for the following series.
If 36, 34, 32, 30......
(a) 22,210 (b) –22,210
(c) –22,220 (d) 22, –210

7. Find the t_{20} & S_{20} for the following series
2, 8, 32, ...

(a) $2^{39}, \frac{2}{3}\left(4^{20}-1\right)$

(b) $2^{40}, \frac{4}{3}\left(4^{20}-1\right)$

(c) $2^{38}, \frac{2}{3}\left(4^{21}-1\right)$

(d) $2^{41}, \frac{2}{3}\left(4^{20}-1\right)$

8. Find t_7 and s_7 for the following series.
1, 3, 9, 27 ......

(a) $729, \frac{1}{2}\left(3^{7}-1\right)$

(b) $243, \frac{1}{2}\left(3^{8}-1\right)$

(c) $729, \frac{1}{2}\left(3^{7}+1\right)$

(d) $243, \frac{1}{3}\left(3^{8}+1\right)$

9. Find t_{24} and s_{24} for the following series.

$18, 9, \frac{9}{2}, \ldots\ldots$

(a) $18\left(\frac{1}{2}\right)^{23}, 36\left(1-\frac{1}{2^{22}}\right)$

(b) $18\left(\frac{1}{2}\right)^{22}, 36\left(1-\frac{1}{2^{24}}\right)$

(c) $18\left(\frac{1}{2}\right)^{24}, 36\left(1-\frac{1}{2^{24}}\right)$

(d) $18\left(\frac{1}{2}\right)^{23}, 36\left(1-\frac{1}{2^{24}}\right)$

10. Find t_{30} and s_{30} for the following series.
64, 16, 4, 1 ....

(a) $\frac{1}{4^{26}}, \frac{64\left(1-\left(\frac{1}{4}\right)^{30}\right)}{1-\frac{1}{4}}$

(b) $\frac{1}{4^{26}}, \frac{64\left(1-\left(\frac{1}{4}\right)^{30}\right)}{1+\frac{1}{4}}$

(c) $\frac{1}{4^{26}}, \frac{64\left(1+\left(\frac{1}{4}\right)^{30}\right)}{1-\frac{1}{4}}$

(d) $\frac{1}{4^{26}}, \frac{64\left(1+\left(\frac{1}{4}\right)^{30}\right)}{1+\frac{1}{4}}$

11 Find the sum of all numbers divisible by 6 In 100 to 400
(a) 12450 (b) 12550
(c) 12400 (d) 12456

12. How many natural Numbers between 300 to 500 are nultiple of 7 ?
(a) 29 (b) 28 (c) 27 (d) 30

13. Find the value of the Expression 1 - 6 + 2 - 7 + 3 - 8 + ... to 100 terms
(a) -250 (b) -500
(c) -450 (d) -300

14. How many terms of the series -12, -9, -6, ...must be Taken that the sum may be 54 ?
(a) 15 (b) 14 (c) 18 (d) 12

15. How many terms are there In the A.P 20, 25, 30, ...130?
(a) 22 (b) 23 (c) 21 (d) 24

16. Find the Ist terms an A.P. whose 8th and 12th terms are respectively 39 & 59
(a) 5 (b) 6 (c) 4 (d) 3

17 There is an AP 1, 3, 5, ... which term of this AP is 55. ?
(a) 27th (b) 26th (c) 25th (d) 28th

18. Find the 15th term of the sequence 20, 15, 10, ...
(a) -45 (b) -50 (c) -55 (d) 0

19. A number 15 is divided In three parts which are In A.P and The sum of their squares is 83. Find the Smallest No.
(a) 5 (b) 3 (c) 6 (d) 8

20. The sum of the first 16 terms of an A.P. whose first terms and third term are 5 and 15 respectively is
(a) 600 (b) 765 (c) 640 (d) 680

21. The Number of terms of the series 54 + 51 + 48 + ... such that the sum is 513 is
(a) 18 (b) 19
(c) Both a and b (d) 15

22 A man receives ₹ 60 for the first week and ₹ 3 more each week than the preceding week. How much does he earn by the 20th week ?
(a) ₹ 1770 (b) ₹ 1620
(c) ₹ 1890 (d) ₹ 1790

23. How many terms are there In the G.P 5, 20, 80, 320 .... 20480?
(a) 5 (b) 6 (c) 7 (d) 8

24. A boy agrees to work at the rate of one rupee on the first day, two rupees on the second day, four rupees on the third day and does on. How much will the boy get if he start working on the 1st of February and finishes on the 20th of February ?
(a) 2^{20} (b) $2^{20}-1$
(c) $2^{19}-1$ (d) 2^{19}

25. If the Fifth term of a G.P. is 81 and first term is 16, what will be the 4th terms of the G.P?
(a) 36 (b) 18 (c) 54 (d) 24

26. The 4th and 10th term of a GP are 1/3 and 243 respectively. Find the 2nd term.
(a) 3 (b) 1
(c) 1/27 (d) 1/9

27. The 7th and 21th term of an AP are 6 and -22 respectively. Find the 26th term.
(a) -34 (b) -32 (c) -12 (d) -10

28. The sum of 5 number in AP is 30 and the sum of their squares is 220. Which of the following is the third term?
(a) 5 (b) 6 (c) 8 (d) 10

29. The sum of the first four terms of an AP is 28 and sum of the first eight terms of the same AP is 88. Find the sum of the first 16 terms of the AP?
(a) 346 (b) 340 (c) 304 (d) 268

30. Find the number of terms of the series $\frac{1}{81}, \frac{-1}{27}, \frac{1}{9}, \ldots\ldots\ 729$
(a) 11 (b) 12 (c) 10 (d) 13

31. A man saves ₹ 100 in January 2014 and increases his saving by ₹ 50 every month over the previous month. What is the annual saving for the man in the year 2014 ?
(a) ₹ 4200 (b) ₹ 4500
(c) ₹ 4000 (d) ₹ 4100

32. What is the maximum sum of the terms in the Arithmetic progression
$25, 24\frac{1}{2}, 24, \ldots\ldots\ 1, \frac{1}{2}$
(a) $637\frac{1}{2}$ (b) 625
(c) $662\frac{1}{2}$ (d) 650

33. An equilateral triangle is drawn by joining the midpoints of the sides of another equilateral triangle. A third equilateral triangle is drawn inside the second one joining the midpoints of the sides of the second equilateral triangle, and the process continues infinitely. Find the sum of the perimeters of all the equilateral triangles, if side of the largest equilateral triangle is 24 units.
(a) 288 units (b) 72 units
(c) 36 units (d) 144 units

34. Find the value of the expression
1 – 6 + 2 – 7 + 3 – 8 + ... to 100 terms
(a) –250 (b) –500
(c) –450 (d) –300

35. Find the sum of the Integers between 1 and 200 that are multiples of 7 ?
(a) 2742 (b) 2842
(c) 2642 (d) 2546

36. After striking a floor a rubber ball rebounds $(7/8)^{th}$ of the height from which it has fallen. Find the total distance that it travels before coming to rest, if it is gently dropped from a height of 420 meters.
(a) 2940 (b) 6300
(c) 1080 (d) 3360

37. Jack and Jil were playing mathematical puzzles with each other. Jill drew a square of sides 8 cm and then kept on drawing squares inside the squares by joining the mid points of the squares. She continued this process indefinitely. Jill asked jack to determine the sum of the areas of all the squares that she drew. If Jack answered correctly then what would be his answer ?
(a) 128 (b) 64 (c) 256 (d) 32

38. If the m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ then find the sum to mn^{th} term
(a) $\frac{1}{4}(mn-1)$ (b) $\frac{1}{4}(mn+1)$
(c) $\frac{1}{2}(mn+1)$ (d) $\frac{1}{2}(mn-1)$

39. The first and the Last terms of an A.P. are 107 and 253. If there are five terms in this sequence, find the sum of sequence
(a) 1080 (b) 720
(c) 900 (d) 620

40. The sum of an infinite GP whose common ratio is numerically less than 1 is 32 and the sum of the first two terms is 24. What will be the third term ?
(a) 2 (b) 16 (c) 8 (d) 4

41. What will be the value of $n^{1/2}. n^{1/4}. n^{1/8}. \ldots\ldots\ \infty$
(a) n^2 (b) n (c) $n^{3/2}$ (d) n^3

42. Determine the first term of the geometric progression, the sum of whose first term and third term is 40 and the sum of the second term and fourth term is 80.
(a) 12 (b) 8 (c) 16 (d) 4

43. It is G.P 32, 4, 8, n and 2 and G.M is 8, find the value of n
(a) 2 (b) 4 (c) 8 (d) 16

44. Find the Arithmetic mean of the following series
20, 23, 26, 29
(a) 49 (b) $\frac{59}{2}$ (c) $\frac{49}{2}$ (d) 59

45. If the arithmetic mean of the number $x_1, x_2, x_3 \ldots\ldots x_n$, is $\bar{x}$, then the arithmetic mean of the number ax_1+b, ax_2+b, ax_3+b, ...ax_n+b, where a and b are two constants, would be ?
(a) $\bar{x}$ (b) $na\bar{x}+nb$
(c) $a\bar{x}$ (d) $a\bar{x}+b$

46. A boy draws n squares with sides 1, 2, 3, 4, 5, ... in inches. The average area covered by these n squares will be :
(a) $\left(\frac{n+1}{2}\right)$
(b) $\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3}\right)$
(c) $\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3}\right)^{-1}$
(d) $\left(\frac{n+1}{2}\right)-1\left(\frac{2n+1}{3}\right)$

ANSWER KEY

1. (a)	6. (b)	11. (a)	16. (c)	21. (c)	26. (c)	31. (b)	36. (b)	41. (b)	46. (b)
2. (c)	7. (a)	12. (a)	17. (d)	22. (a)	27. (b)	32. (a)	37. (a)	42. (b)	
3. (d)	8. (a)	13. (a)	18. (b)	23. (c)	28. (d)	33. (d)	38. (c)	43. (d)	
4. (a)	9. (d)	14. (d)	19. (b)	24. (b)	29. (c)	34. (a)	39. (c)	44. (c)	
5. (b)	10. (a)	15. (b)	20. (d)	25. (c)	30. (a)	35. (b)	40. (d)	45. (d)	

SOLUTION

1. **(a)** 1, 8, 15, 22, ...

This is A.P series

a = 1

d = (8 - 1) = 7

T_{10} = 1 + (10 - 1) 7 = 1 + 63 = **64**

$S_{10} = \frac{10}{2}$ [2 × 1 + 9 × 7]

= 5 [2 + 63] = 5 × 65 = **325**

2. (c) 2, 8, 14, 20

a = 2

d = 6

T_{20} = 2 + 19 × 6 = **116**

S_{20} = 10 [2 × 2 + 19 ×6]

= 10 [4 + 114] **= 1180**

3. **(d)** 3, 13, 23, 33 ....

a = 3

d = 10

T_{24} = 3 + 23 × 10 = **233**

S_{24} = 12 [2 × 3 + 23 × 10]

= 12 [6 + 230]

= 12 × 236 **= 2832**

4. **(a)** – 3, 1, 5, 9,.........

a = – 3

d = 1 – (-3) = 1 + 3 = 4

T_{18} = -3 + 17 × 4 = **65**

S_{22} = 11 [2 × -3 + 21 × 4]

= 11 [-6 + 84] = 78 × 11 **= 858**

5. (b) 30, 33, 36, 39 ......

a = 30

d = 3

T_{28} = 30 + 27 × 3 = **111**

S_{48} = 24 [2 × 30 + 47 × 3]

= 24 [60 + 141] **= 4824**

6. **(b)** 36, 34, 32, 30......

a = 36

d = (34 - 36) = -2

T_{30} = 36 + 29 × -2

= 36 - 58 = **-22**

S_{30} = 15 [2 × 36 + 29 × -2]

= 15 [72 - 58]

= 15 [14] **= 210**

7. **(a)** 2, 8, 32, ...

a = 2

$r = \frac{8}{2} = 4$

$T_{20} = ar^{n-1} = 2(4)^{20-1} = 2 \times (4)^{19}$

$= 2 \times 2^{38} = \mathbf{2^{39}}$

$S_{20} = \frac{a(r^n - 1)}{r - 1}$ (r > 1)

$= \frac{2(4^{20} - 1)}{4 - 1} = \mathbf{\frac{2}{3}(4^{20} - 1)}$

8. **(a)** 1, 3, 9, 27 ......

a = 1

$r = \frac{3}{1} = 3$

$T_7 = 1(3)^{7-1} = 3^6 = \mathbf{729}$

$S_7 = \frac{1(3^7 - 1)}{3 - 1} = \mathbf{\frac{1}{2}}(3^7 - 1)$

9. **(d)** 18, 9, $\frac{9}{2}$, ...

a = 18

$r = \frac{9}{18} = \frac{1}{2}$

$T_{24} = 18\left(\frac{1}{2}\right)^{24-1} = 18\left(\frac{1}{2}\right)^{23}$

$S_{24} = a\left(\frac{1-r^n}{1-r}\right)$ r < 1

$= \frac{18\left[1-\left(\frac{1}{2}\right)^{24}\right]}{1-\frac{1}{2}}$

$= \frac{18\left[1-\left(\frac{1}{2}\right)^{24}\right]}{1/2} = \mathbf{36\left[1 - \frac{1}{2^{24}}\right]}$

10. **(a)** 64, 16, 4, 1 ....

a = 64

$d = \frac{16}{64} = \frac{1}{4}$

$T_{30} = 64\left(\frac{1}{4}\right)^{30-1} = 64\left(\frac{1}{4}\right)^{29}$

$= 4^3 \times \frac{1}{4^{29}} = \frac{1}{4^{26}}$

$S_{30} = \frac{64\left(1-\left(\frac{1}{4}\right)^{30}\right)}{1-\frac{1}{4}}$ r < 1

$= \frac{64\left(1-\left(\frac{1}{4}\right)^{30}\right)}{1-\frac{1}{4}}$

11. **(a)** Ist Term = a = 102 (where is the Ist term greater than 100 that is divisible by 6)

Last term less than 400, which is divisible by 6 is 396.

then,

102 + 108 + 114 396

a = 102, d = 6

No. of term

$= \frac{\text{Last term} - \text{First term}}{\text{difference}} + 1$

$= \frac{396 - 102}{6} + 1$

$= \frac{294}{6} + 1 = 49 + 1 = 50$

$S_{50} = \frac{50}{2}[2 \times 102 + 49 \times 6]$

= 25 [204 + 294] **= 12450**

12. **(a)** First multiple of 7 term (300 to 500) = 301

Last multiple of 7 term (300 to 500) = 497

301, 308, ... 497

No. of term = $\frac{497 - 301}{7} + 1$

$= \frac{196}{7} + 1$ = 28 + 1 = 29

13. (a) 1 - 6 + 2 - 7 + 3 - 8 + ... to 100

(1 + 2 + 3 .... to 50 term)

[Where a = 1, d = 1, n = 50]

- (6 + 7 + 8 ... to 50 term)

[Where a = 6, d = 1, n = 50]

Both series are In A.P.

Use the formula for sum of an A.P.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= 25\ (2 \times 1 + 24 \times 1) - 25\ (2 \times 6 + 24 \times 1)$

$= 25\ (2 + 24) - 25\ (12 + 24)$

$= 25 \times 26 - 25 \times 36$

$= 25\ (26 - 36) = 25 \times -10$

$= -250$

14. **(d)** $-12, -9, -6, \ldots\ldots$

$S_n = 54$

$a = -12$

$d = 3$

$n = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$54 = \frac{n}{2}\left[2 \times -12 + (n-1)3\right]$$

$108 = n\ [-24 + 3n - 3]$

$108 = n\ [3n - 27]$

$36 = n\ [n - 9]$

Take option (d) $n = 12$, satisfy

$= 12 \times 3 = 36$

So : **$n = 12$**

15. **(b)** $20, 25, 30, \ldots\ldots\ldots 130$

$a = 20$

$d = 5$

last term $= l = 130$

$l = a + (n + 1)\ d$

$130 = 20 + (n - 1)\ 5$

$110 = (n - 1)\ 5$

$22 = n - 1$

$n = 23$

16. **(c)** $T_8 = 39$

$T_{12} = 59$

$T_8 = a + 7d = 39$ ...(i)

$T_{12} = a + 11d = 59$ ...(ii)

(ii) - (i)

$4d = 20$

$d = 5$

Put the value of d (i) equation

$a + 7 \times 5 = 39$

$a = 4$

17. **(d)** $1, 3, 5, \ldots\ldots.$

$a = 1$

$d = 2$

n^{th} of AP $= 55$

$55 = 1 + (n - 1)\ 2$

$54 = (n - 1)\ 2$

$27 = n - 1$

$n = 28$th

18. **(b)** $20, 15, 10, \ldots\ldots\ldots$

$a = 20$

$d = -5$

$T_{15} = 20 + 14 \times -5$

$T_{15} = 20 - 70$

$T_{15} = -50$

19. **(b)** Let three term In A.P.

$a - d,\ a,\ a + d$

A.T.Q

$a - d + a + a + d = 15$

$3a = 15$

$a = 5$

and

$(a - d)^2 + (a)^2 + (a + d)^2 = 83$

$a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 83$

$3a^2 + 2d^2 = 83$

Put the value $a = 5$

$3 \times (5)^2 + 2d^2 = 83$

$75 + 2d^2 = 83$

$2d^2 = 8$

$d = 2$

Then $a - d = 5 - 2 = 3$

$a = 5$

$a + d = 5 + 2 = 7$

Smallest No. **= 3**

20. **(d)** $a_1 = 5 = a \rightarrow$ (i)

$a_3 = 15 = a + 2d \rightarrow$ (ii)

(ii) - (i)

$2d = 10$

$d = 5$

$5, 10, 15 \ldots\ldots.$

$a = 5$

$d = 5$

$n = 16$

$$S_{16} = \frac{16}{2}\left[2 \times 5 + 15 \times 5\right] = 8\left[10 + 75\right]$$

= 680

21. **(c)** $54 + 51 + 48 + \ldots\ldots.$

$a = 54, \quad d = -3$

$$S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$$

$$513 = \frac{n}{2}\ [2 \times 54 + (n - 1)(-3)]$$

$$513 = \frac{n}{2}\ [108 - 3n + 3]$$

$1026 = n\ (111 - 3n)$

$342 = n\ (37 - n)$

This is a quadratic equation n has two values.

In this condition we help option.

Option Both $n = 18$, And $n = 19$, Satisfy this eauation.

Then (c) option.

22. **(a)** First week $= a = 60$

second week $= 63$

Therefore

$60, 63, 66 \ldots\ldots 20$th week

$a = 60$

$d = 3$

$n = 20$

$S_{20} = 10\ [2 \times 60 + 19 \times 3]$

$= 10\ [120 + 57]$ **= 1770**

23. **(c)** $5, 20, 80, 320 \ldots\ldots\ldots 20480$

$a = 5$

$$r = \frac{20}{5} = 4$$

$T_n = 20480$

$T_n = ar^{n-1}$

$20480 = 5\ (4)^{n-1}$

$4096 = (4)^{n-1}$

$2^{12} = 2^{2(n-1)}$

$2^{12} = 2^{2n-2}$

same base comparision the power.

$12 = 2n - 2$

$n = 7$

24. **(b)** First day $= 1$

IInd $= 2$

IIIrd $= 4$

Boy does the work $= 20$ day

then $1, 2, 4, 8, \ldots\ldots\ldots$

$a = 1$

$r = 2$

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1} \qquad r > 1$$

$$= \frac{1\left(2^{20} - 1\right)}{2 - 1} = \mathbf{2^{20} - 1}$$

25. **(c)** $T_5 = ar^{5-1} = ar^4 = 81$

$\because a = 16(\text{given})$

$16 \times r^4 = 81$

$$r^4 = \frac{81}{16}$$

$$\mathbf{r = \frac{3}{2}}$$

$T_4 = ar^{4-1} = ar^3$

$$= 16 \times \left(\frac{3}{2}\right)^3 = 16 \times \frac{27}{8}$$

$T_4 = 54$

26. **(c)** $T_4 = ar^3 = \frac{1}{3}$ ....(i)

$T_{10} = ar^9 = 243$........(ii)

(ii) , (i)

$$\frac{ar^9}{ar^3} = \frac{243}{1/3}$$

$r^6 = 243 \times 3 = 729$

$\mathbf{r = 3}$

Put the value r First equation

$a \times (3)^3 = \frac{1}{3}$

$a \times 27 = \frac{1}{3}$

$\mathbf{a = \frac{1}{81}}$

$T_2 = ar = \frac{1}{81} \times 3 = \mathbf{\frac{1}{27}}$

27. (b) $T_7 = a + 6d = 6$ ......(i)

$T_{21} = a + 20d = -22$ ......(ii)

(ii) – (i)

$(a + 20d) - (a + 6d) = -22 - 6$

$14d = -28$

$\mathbf{d = -2}$

Put the value d (i) equation

$a + 6 \times -2 = 6$

$a - 12 = 6$

$\mathbf{a = 18}$

$T_{26} = a + 25d = 18 + 25 \times -2$

$= 18 - 50 = \mathbf{-32}$

28. (d) Let the 5 number of A.P

$a - 2d, a - d, a, a + d, a + 2d$

$a - 2d + a - d + a + a + d + a + 2d = 30$

$5a = 30$

$a = 6$

$(a-2d)^2 + (a-d)^2 + a^2 + (a+d)^2 + (a+2d)^2 = 220$

$a^2 + 4d^2 - 4ad + a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad + a^2 + 4d^2 + 4ad = 220$

$5a^2 + 10d^2 = 220$

$\mathbf{a^2 + 2d^2 = 44}$

Put the value a = 6

$(6)^2 + 2d^2 = 44$

$36 + 2d^2 = 44$

$2d^2 = 8$

$d^2 = 4$, $\mathbf{d = 2}$

$T_3 = a = 6$

29. (c) $S_4 = 28$

$S_8 = 88$

$S_4 = 2\,[2a + 3d] = 28$

$S_8 = 4\,[2a + 7d] = 88$

$2a + 3d = 14$ .....(i)

$2a + 7d = 22$ ......(ii)

(ii) - (i)

$4d = 8$

$\mathbf{d = 2}$

Put the value of d, equation (i)

$2a + 3 \times 2 = 14$

$2a = 8$

$\mathbf{a = 4}$

$S_{16} = 8\,[2 \times 4 + 15 \times 2]$

$= 8\,[8 + 30] = 8 \times 38 = \mathbf{304}$

30. (a) $a = \frac{1}{81}$

$d = \frac{-1/27}{1/81} = -3$

$T_n = ar^{n-1}$

$729 = \frac{1}{81}\,(-3)^{n-1}$

$729 \times 81 = (-3)^{n-1}$

$3^6 \times 3^4 = (-3)^{n-1}$

$3^{10} = (-3)^{n-1}$

comparison

$10 = n - 1$

$n = 11$

31. (b) January Saves = 100

Febury = 100 + 50 = 150

100 + 150 + ...

Sum of 12 months saving

$S_{12} = 6[2 \times 100 + 11 \times 50]$

$= 6\,[200 + 550]$

$= 6 \times 750 = ₹\,4500$

32. (a) The maximum sum of the Terms in the A.P. when all terms will be positive then.

A.P is

$25, 24\frac{1}{2}, 24, 23\frac{1}{2}, ...1, \frac{1}{2}$

No. of term $= \frac{l - a}{d} + 1$

$= \frac{\frac{1}{2} - 25}{-1/2} + 1 = 49 + 1 = 50$

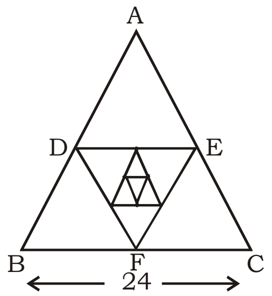
Sum of series

$S_{50} = \frac{50}{2}\,[2 \times 25 + 49 \times -\frac{1}{2}]$

$= 25\,[50 - 24.5] = 25 \times \frac{51}{2}$

$\mathbf{= 637\frac{1}{2}}$

33. (d)



Perimeter of $\triangle$ ABC

$= 3 \times \text{side} = 3 \times 24 = 72$

$\triangle$ DEF

Perimeter $\triangle$ DEF $= 3 \times 12 = 36$

DE || BC, D & E mid point of AB & AC

Then $DE = \frac{1}{2}BC$

Then $DF = \frac{1}{2}AC$

$EF = \frac{1}{2}AB$

Therefore,

$72, 36, 18, \ldots\ldots\ldots\ldots \infty$

$a = 72, r = \frac{1}{2}$

$S_\infty = \frac{72}{1 - \frac{1}{2}} = 72 \times 2 = \mathbf{144}$

34. (a) $1 - 6 + 2 - 7 + 3 - 8 + \ldots$ to 100 terms

The above series is a combination of Two APs.

$\Rightarrow$ (1 + 2 + 3 + ... + 50 terms) – (6 + 7 + 8 .... + 50 terms)

$\Rightarrow$ $25\,[2 \times 1 + 49 \times 1] - [25\,(2 \times 6 + 49 \times 1)]$

$\Rightarrow$ $25\,[51] - 25 \times 61$

$\Rightarrow$ $25\,[51 - 61] = 25 \times -10 = \mathbf{-250}$

35. (b) Multiple of 7 from 1 to 200

7, 14, ... 196

No. of Term $= \frac{l - a}{d} + 1$

$l = 196$

$a = 7$

$d = 7$

No. of term $= \frac{196 - 7}{7} + 1$

No. of term $= \frac{189}{7} + 1 = 27 + 1 = 28$

$S_{28} = \frac{28}{2}[2 \times 7 + 27 \times 7]$

$= 14[14 + 189] = \mathbf{2842}$

36. (b)

(420+367.5), (367.5+321. 5625), ...
0

787.5, 689.0625, ...

It is Infinite G.P

$S\infty = \frac{a}{1-r}$

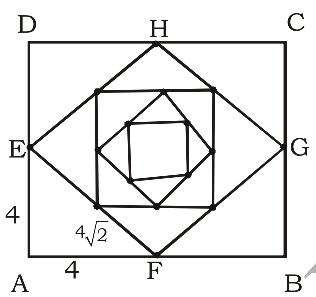
a = 787.5

$r = \frac{7}{8}$

$S\infty = \frac{787.5}{1-\frac{7}{8}} = \frac{787.5}{1/8}$

= 797.5 × 8 = **6300m**.

37. (a)



Area of ABCD = $(8)^2$ = 64

Area of square of EFGH

$= \left(4\sqrt{2}\right)^2 = 32$

Same

64, 32, 16.......... ∞

It is a G.P.

$S\infty = \frac{a}{1-r}$

$\because a = 64,\ r = \frac{1}{2}$

$= \frac{64}{1-\frac{1}{2}}$

= 128

38. (c) $T_m = a + (m-1)d = \frac{1}{n}$ .....(i)

$T_n = a + (n-1)d = \frac{1}{m}$ .....(ii)

(i) - (ii)

$(m-1)d - (n-1)d = \frac{1}{n} - \frac{1}{m}$

$d[m - 1 - n + 1] = \frac{m-n}{mn}$

$d \times (m-n) = \frac{m-n}{mn}$

$\mathbf{d = \frac{1}{mn}}$

Put the value of 'd' equation (i)

$a + (m-1) \times \frac{1}{mn} = \frac{1}{n}$

$a + m \times \frac{1}{mn} - \frac{1}{mn} = \frac{1}{n}$

$a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$

$\mathbf{a = \frac{1}{mn}}$

$S_{mn} = \frac{mn}{2}[2a + (mn-1)d]$

$= \frac{mn}{2}\left[2 \times \frac{1}{mn} + (mn-1) \times \frac{1}{mn}\right]$

$= \frac{mn}{2} \times \frac{1}{mn}\left[2 + (mn-1)\right]$

$= \frac{1}{2}(2 + mn - 1)$

$\mathbf{= \frac{1}{2}(mn+1)}$

39. (c) Let the 4 terms of A.P.

a - 2d, a - d, a, a + d, a + 2d

Sum of II^{nd} and IV^{th} term

= a - d + a + d

= 2a

A.T.Q.

$I^{st} + V^{th}$ = 107 + 253 = 2a

a = 180

Sum of all 5 terms

2a + 2a + a = 5a = 5 × 180

= 900

40. (d) $S\infty = \frac{a}{1-r} = 32$

a = 32 (1 - r) ....(i)

Let 2 Terms of G.P a, ar

a + ar = 24

a (1 + r) = 24

$a = \frac{24}{1+r}$ ....(ii)

put the value of a

$\frac{24}{1+r} = 32(1-r)$

24 = 32 (1 - r) (1 + r)

$\frac{3}{4} = (1 - r^2)$

$r^2 = 1 - \frac{3}{4} = \frac{1}{4}$

$\mathbf{r = \frac{1}{2}}$

A.T.Q.

a + ar = 24

a (1 + r) = 24

$a\left(1+\frac{1}{2}\right) = 24$

$a \times \frac{3}{2} = 24$

a = 16

III^{rd} Term of G.P = ar^2

$= 16 \times \left(\frac{1}{2}\right)^2$

$= 16 \times \frac{1}{4}$ **= 4**

41. (b) $n^{1/2}.\ n^{1/4}.\ n^{1/8}.$ .... ∞

$n^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\text{.........}\infty\right)}$

$a = \frac{1}{2},\ r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

$S\infty = \frac{a}{1-r}$

$S\infty = \left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)$

$S\infty = 1$

$n^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\text{.........}\infty\right)} = n^1 = n$

42. **(b)** Let the 4 terms of G.P

a, ar, ar^2, ar^3

A.T.Q.

$a + ar^2 = 40$, $a(1 + r^2) = 40$ .....(i)

$ar + ar^3 = 80$, $ar(1 + r^2) = 80$ .....(ii)

(i) / (ii)

$$\frac{a(1+r^2)}{ar(1+r^2)} = \frac{40}{80}$$

$$\frac{1}{r} = \frac{1}{2}$$

$r = 2$

Put the value r in equation (ii)

$a + ar^2 = 40$

$a(1 + r^2) = 40$

$a(1 + 2^2) = 40$

$a(5) = 40$

$\mathbf{a = 8}$

43. (d) G.M $= (a_1 \times a_2 \times a_3 \times a_4 \times a_5)^{1/5}$

$8 = (32 \times 4 \times 8 \times n \times 2)^{1/5}$

$8 = (2^5 \times 2^2 \times 2^3 \times n \times 2)^{1/5}$

$8 = (2^{10} \times 2 \times n)^{1/5}$

$8 = 2^{10\times\frac{1}{5}} \times 2^{1/5} \times n^{1/5}$

$8 = 2^2 \times 2^{1/5} \times n^{1/5}$

$(2n)^{1/5} = 2$

$2n = 32$

$\mathbf{n = 16}$

44. **(c)** A.M $= \frac{(a_1 + a_2 + a_3 + a_4)}{4}$

20, 23, 26, 29,

$$\text{A.M} = \frac{(20+23+26+29)}{4}$$

$$= \frac{98}{4} = \mathbf{\frac{49}{2}}$$

45. **(d)** AM of $ax_1 + b$, $ax_2 + b$, $ax_3 + b$, ...,$ax_n + b$ is

$$= \frac{(ax_1+b)+(ax_2+b)+(ax_3+b)+...+(ax_n+b)}{n}$$

$$= \frac{(ax_1+ax_2+ax_3+...+ax_n)+(b+b+..+b)}{n}$$

$$= \frac{a(x_1+x_2+....+x_n)+nb}{n}$$

$$= \frac{a(x_1+x_2+x_3+...+x_n)}{n} + \frac{nb}{n}$$

$$= a\bar{x} + b$$

46. **(b)** Average area

$$= \frac{1^2+2^2+3^2....n^2}{n}$$

$$= \frac{\left(\frac{n(n+1)(2n+1)}{6}\right)}{n}$$

$$= \frac{(n+1)(2n+1)}{6}$$

$$= \frac{(n+1)}{2} \cdot \frac{(2n+1)}{3}$$

Hence, (b) is the correct option.

CHAPTER 07

POWER, INDICES AND SURDS

- Before we proceed to exponents (Indices) and surds, it is proper to learn about Real numbers.

Number System

Natural Numbers: These are the numbers (1, 2, 3, ...etc) that are used for counting. In other words, all positive integers are natural numbers.

The least natural number is 1 but there is no largest natural number.

The set of natural number is denoted by N.

Thus, N = {1, 2, 3,......}

- **Whole Numbers :** The set of numbers that includes all natural numbers and the number zero are called whole numbers.

  The set of whole numbers is denoted by W.

  Thus, W = {0, 1, 2, 3, ...}

  Note : Whole numbers are also called as "Non-negative Integers".

  Integers : All the natural numbers, zero, and the negatives of natural numbers are called integers.

  I = {......, -3, -2, -1, 0, 1, 2, 3, ...}

(i) Set of negative integers = {-1, -2, -3, ...}

(ii) Set of non-negative integers = {0, 1, 2, 3...}

(iii) Set of positive integers = {1, 2, 3 ....}

(iv) Set of non-positive integers = {0, 1, 2, 3.....}

Note: '0' is definately a non-negative integer as well as a non-positive integer.

- **Rational numbers** : The numbers which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called rational numbers and their set is denoted by Q.

Ex. $\frac{1}{4}, \frac{2}{5}, -\frac{3}{7}, 6\left(as\ 6=\frac{6}{1}\right)$ etc. are rational numbers.

The set of rational numbers encloses the set of integers and fractions.

Representation of Rational Numbers as Decimals : The decimal form of a rational number is either terminating or non-terminating.

E.g. $\frac{17}{4}=4.25,\ \frac{21}{5}=4.2 \rightarrow$ terminating (or finite) decimal.

$\frac{16}{3}=5.\overline{3},\ \frac{2}{3}=0.\overline{6} \rightarrow$ Non-terminating (or Recurring) decimal.

Note: If the denominator of a rational number has no prime factors other than 2 or 5, then and only then it is expressible as a terminating decimal.

- **Irrational numbers** : The numbers which when expressed in decimal form are neither terminating nor repeating decimals are called " Irrational numbers".

  e.g. $\sqrt{2}, \sqrt{3}, \sqrt{50}, \sqrt{7}, \pi$ etc

Note: The exact value of π is not $\frac{22}{7}$, as $\frac{22}{7}$ is rational while π is irrational. $\frac{22}{7}$ is the approximate value of π. Similarly 3.14 is not an exact value of π.

- **Real numbers** : All rational and irrational numbers together form the set of real numbers, denoted by R.

  Thus, every natural number, every whole number, every integer, every rational number and every irrational number is a real number.

Note:-

(i) The sum (or difference) of a rational and an irrational number is irrational.

E.g. $(4+\sqrt{3}), (2-\sqrt{5}), \left(\frac{3}{2}-\sqrt{2}\right), 7+\pi$ etc. are all irrational.

(ii) The product of a rational and an irrational number is irrational, e.g. $4\sqrt{3}$, $-2\sqrt{5}$ etc. are all irrational.

- **Even and Odd numbers** : Integers divisible by 2 are called even numbers, while those which are not divisible by 2 are known as odd integers.

  Thus, ..., –6, –4,–2, 0, 2, 4, 6, ...etc are even integers.

  And –5, -3, -1, 1, 3, 5, ...etc are odd integers.

- **Prime numbers**:- A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and itself.

E.g.: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, etc.

→ 2 is the only even number which is prime.

- **Composite numbers**:- Composite numbers are the numbers greater than 1 which are not prime.

  e.g. 4, 6, 9, 14, 15, etc.

Note: 1 is neither prime nor composite.

There are 25 prime numbers between numbers 1 & 100.

Test for Prime Numbers : Let x be a given number and let k be an integer very near to $\sqrt{x}$ s.t. $k > \sqrt{x}$.

If x is not divisible by any prime number less than k, then x is prime, otherwise, it is not prime.

E.g.: Check whether 571 is prime or not ?

clearly, $24 > \sqrt{571}$

So, we divide 571 by each prime number less than 24 which are 2, 3, 5, 7, 9, 11, 17, 19 and 23 we find that 571 is not divisible by any of them. So, 571 is a prime number.

□ **Co-Prime Numbers** : Two numbers are co-prime, if their H.C.F (Highest common factor) is 1.

E.g. (2, 3), (3, 13), (5, 7) etc are co-prime numbers.

□ **Perfect Numbers** : If the sum of divisors of a number excluding N itself is equal to N, then N is called a perfect number.

E.g.: 6, 28, 496, 8128 etc.

For 6, divisors are 1, 2 and 3.

6 : 1 + 2 + 3 = 6

28 : 1 + 2 + 4 + 7 + 14 = 28

Note: The sum of the reciprocals of the divisors of a perfect number including that of its own is always equal to 2.

E.g. For 6, divisors are 1, 2 and 3.

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=\frac{6+3+2+1}{6}=\frac{12}{6}=2$$

Type – I

Indices and Surds

⇒ Let n be a positive integer and a be real number, then:

$$a^n = \frac{a \times a \times a \times ...... \times a}{(n\ factors)}$$

a^n is called "n[th] power of a" or "a raised to the power n"

where, a is called the base and n is called index or exponent of the power a^n.

E.g. 3^2 = square of 3, 3^3 = cube of 3 etc.

⇒ **Laws of Indices:**

1. $a^m \times a^n = a^{m+n}$ where $a \neq 0$ and $(m, n) \in I$
2. $a^m \times a^n \times a^p$ .... $= a^{m+n+p+.......}$
3. $$\frac{a^m}{a^n} = \begin{cases} a^{m-n} & if\ m>n \\ \dfrac{1}{a^{n-m}} & if\ n>m \\ 1 & if\ m=n \end{cases}$$
4. $(a^m)^n = a^{nm} = (a^n)^m$
5. $a^{m^n} = a^{m\times m\times ....n}$ times $\neq (a^m)^n$
6. $(ab)^n = a^n b^n$
7. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n},\ b \neq 0$
8. $(-a)^n = \begin{bmatrix} a^n, \text{when n is even} \\ -a^n, \text{ when n is odd} \end{bmatrix}$

Remark:- These rules are also true when n is negative or fraction.

9. $a^{-n} = a^{(-1)n} = \left(a^{-1}\right)^n = \left(\frac{1}{a}\right)^n$

$= \frac{1}{a}\times\frac{1}{a}\times\frac{1}{a}........n$ times

10. $a^{p/q} = a^{1/q \times p} = (a^{1/q})^p$ is positive integer, $q \neq 0$

$= a^{1/q} \times a^{1/q} \times$ .... p times

⇒ If the index of a power is until (i.e. 1) then the value of the power is equal to its base, i.e.

$a^1 = a,\ 0^1 = 0$

- $a^m = a^n \Rightarrow$ m = n when $a \neq 0$, 1
- $a^m = b^m \Rightarrow a = b$

Ex1. Solve the following:

(i) $(5)^3$

Sol. $5^3 = 5 \times 5 \times 5 = 125$

(ii) $(-6)^4$

Sol. $(-6)^4 = (-6)\times(-6)\times(-6)\times(-6) = 1296$

(iii) $(-2)^5$

Sol. $(-2)^5 = (-2)\times(-2)\times(-2)\times(-2)\times(-2) = -32$

Ex.2 Solve the following expression

(i) $(32)^{-1/5}$

Sol. $(32)^{-1/5} = (2^5)^{-1/5} = 2^{5\times-\frac{1}{5}} = 2^{-1}$

$= \frac{1}{2}$

(ii) $\left(-\frac{1}{343}\right)^{-2/3}$

Sol. $\left(-\frac{1}{343}\right)^{-2/3} = \left(-\frac{1}{7^3}\right)^{-2/3} = \left(-7^{-3}\right)^{-2/3}$

$= (-7)^{-3\times-2/3} = (-7)^2 = 49$

(iii) $3^{-3} + (-3)^3$

Sol. $3^{-3}+(-3)^3 = \frac{1}{3^3}+(-3)^3 = \frac{1}{27}-27$

$= \frac{1-729}{27} = -\frac{728}{27}$

Ex.3: Solve the following:

$(2^2 + 2^3 + 2^{-2} + 2^{-3})$

Sol. $(2^2 + 2^3 + 2^{-2} + 2^{-3})$

$= \left(4+8+\frac{1}{4}+\frac{1}{8}\right)$

$= 12+\frac{3}{8} = \frac{99}{8}$

Ex. 4 If $2^{2x-1} = \frac{1}{8^{(x-3)}}$, then $x = ?$

Sol. $2^{2x-1} = \frac{1}{8^{(x-3)}} \Rightarrow 2^{2x-1} = \frac{1}{2^{3(x-3)}}$

⇒ $2^{2x-1} = \frac{1}{2^{3x-9}}$

⇒ $(2^{2x-1})(2^{3x-9}) = 1$

⇒ $2^{(2x-1)+(3x-9)} = 1$

⇒ $2^{5x-10} = 1 \Rightarrow 2^{5(x-2)} = 1$

⇒ $2^{5(x-2)} = 2^0$

⇒ $x - 2 = 0 \Rightarrow x = 2$

Ex. 5 If $4^{2x+1} = 8^{x+3}$ then $x = ?$

Sol. $4^{2x+1} = 8^{x+3}$

⇒ $(2^2)^{(2x+1)} = (2^3)^{(x+3)}$

⇒ $2^{4x+2} = 2^{3x+9}$

⇒ $4x + 2 = 3x + 9$

($\because$ base in both side is same)

⇒ $x = 7$

Ex. 6 If $3^{x-1} + 3^{x+1} = 90$, then $x = ?$

Sol. $3^{x-1} + 3^{x+1} = 90$

⇒ $\frac{3^x}{3} + 3.3^x = 90$

⇒ $3^x\left(\frac{1}{3}+3\right) = 90$

⇒ $3^x \times \frac{10}{3} = 90$

⇒ $\frac{3^x}{3} = 9$

⇒ $3^x = 27$

⇒ $3^x = 3^3$

$x = 3$

TYPE – II

- **Surd:** If a is rational and n is a positive integer and $a^{1/n} = \sqrt[n]{a}$ is
- Irrational, then $\sqrt[n]{a}$ is called a "surd of order n" or "n[th] root of a"

For the surd $\sqrt[n]{a}$, n is called the surd-index or the order of the surd and "a" is called the radicand. The symbol '$\sqrt{\ }$' is called the surd sign or radical.

E.g. $\sqrt{5}$ is a surd of order 2 or square root of 5.

$\sqrt[3]{6}$ is a surd of order 3 or cube root of 6.

$\sqrt{6+\sqrt{5}}$ is not a surd as $6+\sqrt{5}$ is not a rational number.

Note: Every surd is an irrational number but every irrational number is not a surd.

→ In the surd $a\sqrt[n]{b}$, a and b are called factors of the surd.

(i) A surd which has unity as its rational factor (i.e. a = 1) is called "pure surd".

E.g.: $\sqrt[4]{3}$, $\sqrt{2}$, $\sqrt[3]{3}$ etc.

(ii) A surd which has a rational factor other than unity, the other irrational, is called "mixed surd". e.g. $3\sqrt{5}$, $2\sqrt{7}$, $5\sqrt[3]{7}$

→ If $\sqrt[n]{a}$ is a surd it implies.

(i) a is a rational number.

(ii) $\sqrt[n]{a}$ is an irrational number.

⇒ **Quadratic surd:** A surd of order 2 (i.e. $\sqrt{a}$) is called a quadratic surd.

E.g.: $\sqrt{2} = 2^{1/2}$ is a quadratic surd but $\sqrt{4} = 4^{1/2}$ is not a quadratic surd because $\sqrt{4} = 2$ is a rational number.

Therefore $\sqrt{4}$ is not a surd.

⇒ **Cubic Surd:** A surd of order 3(i.e. $\sqrt[3]{a}$) is called a cubic surd.

E.g.: $\sqrt[3]{9}$ is a cubic surd but $\sqrt[3]{27}$ is not a surd because $\sqrt[3]{27} = 3$ is a rational number.

⇒ **Quartic or Biquadratic surd:** A surd of order 4(i.e. $\sqrt[4]{a}$) is called a quartic surd.

E.g.: $\sqrt[4]{3}$ is a quartic surd but $\sqrt[4]{81}$ is not a quartic surd.

Note: Each surd can be represented on the number line.

Important Formulae Based on Surds:

(i) $\sqrt[n]{a^n} = a$

(ii) $\sqrt[n]{ab} = \sqrt[n]{a}\,.\sqrt[n]{b}$

(iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ and $\frac{k\sqrt[n]{a}}{l\sqrt[n]{b}} = \frac{k}{l}\sqrt[n]{\frac{a}{b}}$

(iv) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(v) $\left(\sqrt[n]{a^m}\right) = (a)^{m/n} = \left(a^m\right)^{1/n} = \sqrt[n]{a^m}$

(vi) $\sqrt{a}\times\sqrt{a} = a$

(vii) $\sqrt{a}\times\sqrt{b} = \sqrt{ab}$

and k. $\sqrt[n]{a}\times l.\sqrt[m]{b} = kl.\sqrt[n]{a}.\sqrt[m]{b} = kl.\sqrt[mn]{a^m b^n}$

(viii) $\sqrt{a^2 b} = a\sqrt{b}$

(ix) $\left(\sqrt{a}+\sqrt{b}\right)^2 = a+b+2\sqrt{ab}$

(x) $\left(\sqrt{a}-\sqrt{b}\right)^2 = a+b-2\sqrt{ab}$

(xi) $\left(\sqrt{a}+\sqrt{b}\right)\times\left(\sqrt{a}-\sqrt{b}\right) = a - b$, where a and b are positive rational numbers.

→ A surd is in its simplest form if:

(i) There is no factor which has n^{th} power of a rational number, under the radical sign whose index is n,

(ii) There is no fraction under the radical sign, and

(iii) The index of the surd is the smallest possible.

E.g. The surd $\sqrt[4]{3\times 5^4}$ is not in its simplest form since the number under the radical sign has factor 5^4 s.t. its index is equal to the order of the surd. It simplest form:

$\sqrt[4]{3\times 5^4} = \sqrt[4]{3}.\ \sqrt[4]{5^4} = \left(\sqrt[4]{3}\right)(5) = 5.\ \left(\sqrt[4]{3}\right)$

⇒ **Similar or like Surds:** Surds having same irrational factors are called "similar or like surd".

E.g.: $3\sqrt{3},\ 7\sqrt{3},\ \frac{2}{5}\sqrt{3},\ \sqrt{3}$ etc. are similar surds.

⇒ **Unlike surds:** Surds having non-common irrational factors are called "unlike surds".

E.g.: $3\sqrt{3},\ 5\sqrt{2},\ 6\sqrt{7}$, etc. are unlike surds.

Ex. 7. $\left[\left(x+\frac{1}{y}\right)^a\left(x-\frac{1}{y}\right)^b\right] \div \left[\left(y+\frac{1}{x}\right)^a\left(y-\frac{1}{x}\right)^b\right]$ is equal to:

Sol. $\left[\left(x+\frac{1}{y}\right)^a\left(x-\frac{1}{y}\right)^b\right] \div \left[\left(y+\frac{1}{x}\right)^a\left(y-\frac{1}{x}\right)^b\right]$

$$\Rightarrow \frac{\left(x+\frac{1}{y}\right)^a\left(x-\frac{1}{y}\right)^b}{\left(y+\frac{1}{x}\right)^a\left(y-\frac{1}{x}\right)^b}$$

$$\Rightarrow \frac{\left(\frac{xy+1}{y}\right)^a\left(\frac{xy-1}{y}\right)^b}{\left(\frac{xy+1}{x}\right)^a\left(\frac{xy-1}{x}\right)^b}$$

$$\Rightarrow \frac{\frac{(xy+1)^a}{y^a}.\frac{(xy-1)^b}{y^b}}{\frac{(xy+1)^a}{x^a}.\frac{(xy-1)^b}{x^b}}$$

$$\Rightarrow \frac{(xy+1)^a(xy-1)^b}{y^{(a+b)}}\times\frac{x^{(a+b)}}{(xy+1)^a(xy-1)^b}$$

$$\Rightarrow \frac{x^{a+b}}{y^{a+b}} = \left(\frac{x}{y}\right)^{a+b}$$

Ex.8. If $x^{x^{3/2}} = (x^{3/2})^x$, then the value of x is:

Sol. $x^{x^{3/2}} = (x^{3/2})^x$

$\Rightarrow x^{x^{3/2}} = x^{(3/2)x}$

$\Rightarrow x^{3/2} = \frac{3}{2}x$

$\Rightarrow x^{1/2} = \frac{3}{2} \Rightarrow x = \frac{9}{4}$

Ex.9 If $x^a = y^b = z^c$ and $y^2 = zx$ then the value of $\frac{1}{a}+\frac{1}{c}$ is:

Sol. Let $x^a = y^b = z^c = k$

$\Rightarrow x = k^{1/a},\ y = k^{1/b},\ z = k^{1/c}$

Now,

$\because y^2 = zx$

$\therefore (k^{1/b})^2 = (k^{1/c}).(k^{1/a})$

$\Rightarrow k^{2/b} = k^{\frac{1}{c}+\frac{1}{a}}$

⇒ $\frac{1}{a}+\frac{1}{c}=\frac{2}{b}$

Ex.10. $(a^{m-n})^l \times (a^{n-l})^m \times (a^{l-m})^n$

Sol. $(a^{m-n})^l \times (a^{n-l})^m \times (a^{l-m})^n$

$= a^{ml-nl} \times a^{nm-lm} \times a^{ln-mn}$

$= a^{ml-nl+nm-lm+ln-mn}$

$= a^0 = 1$

Ex.11. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of xyz is:

Sol. If $a^x = b$, $b^y = c$, $c^z = a$

$\because$ $a^x = b$

$\therefore$ $(c^z)^x = b$

⇒ $c^{zx} = b$ ⇒ $(b^y)^{zx} = b$

⇒ $b^{xyz} = b$ ⇒ $xyz = 1$

Ex.12. $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ is equal to:

Sol. $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

$= (x^{a-b})^{(a+b)} \times (x^{b-c})^{(b+c)} \times (x^{c-a})^{(c+a)}$

$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$

$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$

$= x^0 = 1$

Ex.13. The value of $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{c-b}+x^{a-b}} + \frac{1}{1+x^{a-c}+x^{b-c}}$ is:

Sol. $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{c-b}+x^{a-b}} + \frac{1}{1+x^{a-c}+x^{b-c}}$

⇒ $\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^c}{x^b}+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^a}{x^c}+\frac{x^b}{x^c}}$

⇒ $\frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^c+x^a} + \frac{x^c}{x^c+x^a+x^b}$

⇒ $\frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1$

Ex.14 $2^{2^x} = 16^{2^{3x}}$, then x is equal to

Sol. $2^{2^x} = 16^{2^{3x}}$

⇒ $2^{2^x} = (2^4)^{2^{3x}}$

⇒ $2^{2^x} = 2^{4.2^{3x}} = 2^{2^{3x+2}}$

⇒ $2^{2^x} = 2^{2^{3x+2}}$ (Since has base in both sides is equal)

⇒ $2^x = 2^{3x+2}$

⇒ $x = 3x + 2$

⇒ $2x = -2$

⇒ $x = -1$

Ex.15. If $2^x = 4^y = 8^z$ and $xyz = 288$ find the value of $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}$

Sol. $2^x = 4^y = 8^z$

$2^x = 2^{2y} = 2^{3z}$

[If $a^x = a^y = a^z$ then, $x = y = z$]

$x = 2y = 3z$

$\boldsymbol{x = 3z}$

$2y = 3z$

$\boldsymbol{y = \frac{3z}{2}}$

$xyz = 288$

Put the value of x,y in z form

$3z \times \frac{3z}{2} \times z = 288$

$z^3 = 32 \times 2$

$z = 4$

Then x = 3z = 4×3 = 12

y = $\frac{3z}{2} = \frac{3\times4}{2} = 6$

So,

$\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}$

Put the value x,y and z

$\frac{1}{2\times12}+\frac{1}{4\times6}+\frac{1}{6\times4}$

$\frac{1}{24}+\frac{1}{24}+\frac{1}{24}$

$= 3\times\frac{1}{24} = \frac{1}{8}$

Ex.16: If $2^x = 3^y = 6^{-z}$, Find the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$

Sol. $2^x = 3^y = 6^{-z} = k$

Then,

$2 = k^{\frac{1}{x}}$, $3 = k^{\frac{1}{y}}$, $6 = k^{\frac{1}{-z}}$

We know that

2 × 3 = 6

$k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{\frac{1}{-z}}$

Same base power will be add

$k^{\frac{1}{x}+\frac{1}{y}} = k^{\frac{1}{-z}}$

Now,

$\frac{1}{x}+\frac{1}{y} = \frac{-1}{z}$

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$

Ex.17: If $\left(\frac{9}{4}\right)^x \times \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$, Find the value of x.

Sol. $\left(\frac{9}{4}\right)^x \times \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$

$\left[a^x = \frac{1}{a^{-x}}\right]$

$\left(\frac{4}{9}\right)^{-x} \times \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$

$\left(\frac{2}{3}\right)^{-2x} \times \left(\frac{2}{3}\right)^{3(x-1)} = \frac{2}{3}$

$\left(\frac{2}{3}\right)^{-2x+3x-3} = \left(\frac{2}{3}\right)^1$

The power will be equal when the base is equal.

$-2x + 3x - 3 = 1$

$x - 3 = 1$

$x = 4$

Ex.18. If $2^x = 4^y = 8^z$, $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z} = \frac{24}{7}$, Find the value of z.

Sol. $2^x = 4^y = 8^z$,

$2^x = 2^{2y} = 2^{3z}$

When base is equal then power will be equal,

$x = 2y = 3z$

So, x = 3z, y = $\frac{3z}{2}$

$\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z} = \frac{24}{7}$

Put the value of x and y in z form

$$\frac{1}{2\times 3z}+\frac{1}{4\times\frac{3z}{2}}+\frac{1}{6z}=\frac{24}{7}$$

$$\frac{1}{6z}+\frac{1}{6z}+\frac{1}{6z}=\frac{24}{7}$$

$$\frac{1}{6z}\times 3=\frac{24}{7}$$

$$\frac{1}{2z}=\frac{24}{7}$$

$$z=\frac{7}{48}$$

Ex.19: The value of expression

$$\frac{4^n\times 20^{m-1}\times 12^{m-n}\times 15^{m+n-2}}{16^m\times 5^{2m+n}\times 9^{m-1}}\text{ is:}$$

Sol. $\dfrac{4^n\times 20^{m-1}\times 12^{m-n}\times 15^{m+n-2}}{16^m\times 5^{2m+n}\times 9^{m-1}}$

$$\Rightarrow \frac{2^{2n}\times 2^{2m-2}\times 5^{m-1}\times 2^{2m-2n}\times 3^{m-n}\times 3^{m+n-2}\times 5^{m+n-2}}{2^{4m}\times 5^{2m+n}\times 3^{2m-2}}$$

$\Rightarrow$ $2^{2n+2m-2+2m-2n-4m}\times 3^{m-n+m+n-2-2m+2}\times 5^{m-1+m+n-2-2m-n}$

$\Rightarrow$ $2^{-2}\times 3^0\times 5^{-3}$

$$\Rightarrow \frac{1}{4}\times\frac{1}{125}=\frac{1}{500}$$

Ex.20: The value of Expression

$$\frac{(0.3)^{1/3}.\left(\frac{1}{27}\right)^{1/4}.(9)^{1/6}.(0.81)^{2/3}}{(0.9)^{2/3}.(3)^{-1/2}.\left(\frac{1}{3}\right)^{-2}.(243)^{-1/4}}\text{ is}$$

Sol. $\dfrac{(0.3)^{1/3}.\left(\frac{1}{27}\right)^{1/4}.(9)^{1/6}.(0.81)^{2/3}}{(0.9)^{2/3}.(3)^{-1/2}.\left(\frac{1}{3}\right)^{-2}.(243)^{-1/4}}$

$$\Rightarrow \frac{\left(\frac{3}{10}\right)^{1/3}\left(\frac{1}{3}\right)^{3/4}(3)^{1/3}.\left(\frac{81}{100}\right)^{2/3}}{\left(\frac{9}{10}\right)^{2/3}\left(\frac{1}{3}\right)^{1/2}.(3)^2(3)^{-5/4}}$$

Break the power of 3 & 10 separately

$$\Rightarrow \frac{3^{\frac{1}{3}+\frac{1}{3}-\frac{3}{4}+\frac{8}{3}}\times 10^{-\frac{1}{3}-\frac{4}{3}}}{3^{\frac{4}{3}+2-\frac{1}{2}-\frac{5}{4}}\times 10^{-\frac{2}{3}}}$$

$$\Rightarrow \frac{3^{31/12}\times 10^{-\frac{5}{3}}}{3^{\frac{19}{12}}\times 10^{-\frac{2}{3}}}$$

$$\Rightarrow 3\times 10^{-1}=\frac{3}{10}=0.3$$

Ex.21: The value of expression

$$\frac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1}\left(\frac{3}{2}\right)^3+\left(-\frac{1}{3}\right)^{-1}}\text{ is}$$

Sol. $\dfrac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1}\left(\frac{3}{2}\right)^3+\left(-\frac{1}{3}\right)^{-1}}$

$$\Rightarrow \frac{1-\left(\frac{1}{10}\right)^{-1}}{(3^{-1})(2^3)\times 3^3\times 2^{-3}+(-3)^1}$$

$$=\frac{1-10}{3^{3-1}.2^0-3}=\frac{-9}{9-3}=\frac{-9}{6}=\frac{-3}{2}$$

Ex.22: If $\dfrac{9^n\times 3^2\times(3^{-n/2})^{-2}-(27)^n}{3^{3m}\times 2^3}=\dfrac{1}{27}$ then the value of $(m-n)$ is:

Sol. $\dfrac{9^n\times 3^2\times(3^{-n/2})^{-2}-(27)^n}{3^{3m}\times 2^3}=\dfrac{1}{27}$

$$\Rightarrow \frac{3^{2n}\times 3^2\times 3^n-3^{3n}}{3^{3m}\times 2^3}=\frac{1}{3^3}$$

$$\Rightarrow \frac{3^{2n+2+n}-3^{3n}}{8\times 3^{3m}}=\frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n+2}-3^{3n}}{3^{3m}\times 8}=3^{-3}$$

$$\Rightarrow \frac{3^{3n}(3^2-1)}{3^{3m}\times 8}=3^{-3}$$

$\Rightarrow$ $3^{3n-3m}=3^{-3}$

$\Rightarrow$ $3n-3m=-3$

$\Rightarrow$ $m-n=1$

TYPE – III

$\Rightarrow$ **Comparison of Surds:** (i) If two surds are of the same order, then the one whose radicand is larger, is the larger of the two.

E.g.: $\sqrt[3]{19}>\sqrt[3]{15},\ \sqrt{7}>\sqrt{5},\ \sqrt[3]{9}>\sqrt[3]{7}$ etc.

(ii) If two surds are **distinct order,** we change them into the surds of the same order.

This order is the L.C.M. of the orders of the given surds.

E.g.: Which is larger $\sqrt{2}$ of $\sqrt[3]{3}$?

Sol. Given surds are of order 2 & 3 respectively whose L.C.M is 6.

Convert each into a surd of order 6, as show below :

$\sqrt{2}=2^{1/2}=2^{\frac{1}{2}\times\frac{3}{3}}=2^{3/6}=\left(2^3\right)^{1/6}$

$=(8)^{1/6}=\sqrt[6]{8}$

$=3^{1/3}=3^{\frac{1}{3}\times\frac{2}{2}}=3^{2/6}=(9)^{1/6}$

$=\sqrt[6]{9}$

Clearly, $\sqrt[6]{9}>\sqrt[6]{8}$, so $\sqrt[3]{3}>\sqrt{2}$

Ex.23: Which one is smaller out of $\sqrt[3]{2}$ and $\sqrt[4]{3}$?

Sol. $\sqrt[3]{2}=(2)^{1/3}=2^{4/12}=(16)^{1/12}$

and $\sqrt[4]{3}=(3)^{1/4}=3^{3/12}=(27)^{1/12}$

(Taking the LCM of surd)

Hence $\sqrt[3]{2}<\sqrt[4]{3}$

Ex.24: Which one is greatest out of $\sqrt[3]{5},\ \sqrt[4]{3},\ \sqrt[5]{4}$?

Sol. $\sqrt[3]{5}=(5)^{1/3}=(5)^{20/60}=(5^{20})^{1/60}=\sqrt[60]{5^{20}}$

$\sqrt[4]{3}=(3)^{1/4}=(3)^{15/60}=(3^{15})^{1/60}=\sqrt[60]{3^{15}}$

$\sqrt[5]{4}=(4)^{1/5}=(4)^{12/60}=(4^{12})^{1/60}=\sqrt[60]{4^{12}}$

$\sqrt[3]{5}$ is the greatest.

Ex.25: Which of the following relation is correct A= $\sqrt{2}$, B= $\sqrt[3]{3}$,C= $\sqrt[4]{4}$

Sol. $(2)^{1/2}$, $(3)^{1/3}$, $(4)^{1/4}$

LCM 2, 3 and 4 = 12

convert each into a surd of order 12

$(2)^{\frac{1}{2}\times\frac{6}{6}}$, $(3)^{\frac{1}{3}\times\frac{4}{4}}$, $(4)^{\frac{1}{4}\times\frac{3}{3}}$

$(2)^{\frac{6}{12}}$, $(3)^{\frac{4}{12}}$, $(4)^{\frac{3}{12}}$

$(2^6)^{\frac{1}{12}}$, $(3^4)^{\frac{1}{12}}$, $(4^3)^{\frac{1}{12}}$

$(64)^{\frac{1}{12}}$, $(81)^{\frac{1}{12}}$, $(64)^{\frac{1}{12}}$

clearly $(81)^{\frac{1}{12}}>(64)^{\frac{1}{12}}=(64)^{\frac{1}{12}}$

so, $(3)^{\frac{1}{3}}>(2)^{\frac{1}{2}}=(4)^{\frac{1}{4}}$

B > A = C

Ex.26: Arrange the following in descending order.

$\sqrt{3}-\sqrt{2}, \sqrt{4}-\sqrt{3}, \sqrt{5}-\sqrt{4}, \sqrt{2}-1$

Sol. $\sqrt{3}-\sqrt{2} = \frac{\sqrt{3}-\sqrt{2}}{1} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \quad \frac{3-2}{\sqrt{3}+\sqrt{2}} = \frac{1}{\sqrt{3}+\sqrt{2}}$

Similarly, $\sqrt{4}-\sqrt{3} = \frac{1}{\sqrt{4}+\sqrt{3}}$,

$\sqrt{5}-\sqrt{4} = \frac{1}{\sqrt{5}+\sqrt{4}}$

and $\sqrt{2}-1 = \frac{1}{\sqrt{2}+1}$

As we know, if the numerator is same then the fraction whose denominator is larger the fraction will be lower. Hence the correct order of descending is.

$(\sqrt{2}-1) > (\sqrt{3}-\sqrt{2}) > (\sqrt{4}-\sqrt{3}) > (\sqrt{5}-\sqrt{4})$

Ex.27: Which is greater than $(\sqrt{8}-\sqrt{5})$ or $(\sqrt{13}-\sqrt{10})$

Sol. Rationalisation of Surds:

$(\sqrt{8}-\sqrt{5}) \times \frac{(\sqrt{8}+\sqrt{5})}{(\sqrt{8}+\sqrt{5})} = \frac{3}{\sqrt{8}+\sqrt{5}}$

$(\sqrt{13}-\sqrt{10}) \times \frac{(\sqrt{13}+\sqrt{10})}{(\sqrt{13}+\sqrt{10})} = \frac{3}{\sqrt{13}+\sqrt{10}}$

So, It is clear that

$\Rightarrow \quad \frac{3}{\sqrt{8}+\sqrt{5}} > \frac{3}{\sqrt{13}+\sqrt{10}}$

Then $(\sqrt{8}-\sqrt{5}) > (\sqrt{13}-\sqrt{10})$

Ex.28: Arrange the following in ascending order.

$\sqrt{8}+\sqrt{5}$, $\sqrt{6}+\sqrt{7}$, $\sqrt{9}+\sqrt{4}$, $\sqrt{11}+\sqrt{2}$, $\sqrt{10}+\sqrt{3}$

Sol. In this type questions we use square method

So, $(\sqrt{8}+\sqrt{5})^2 = 13+2\sqrt{40}$

$(\sqrt{6}+\sqrt{7})^2 = 13+2\sqrt{42}$

$(\sqrt{9}+\sqrt{4})^2 = 13+2\sqrt{36}$

$(\sqrt{11}+\sqrt{2})^2 = 13+2\sqrt{22}$

$(\sqrt{10}+\sqrt{3})^2 = 13+2\sqrt{30}$

So, It is cleared that

$13+2\sqrt{22} < 13+2\sqrt{30} < 13+2\sqrt{36} < 13+2\sqrt{40} < 13+2\sqrt{42}$

Then

$\sqrt{11}+\sqrt{2} < \sqrt{10}+\sqrt{3} < \sqrt{9}+\sqrt{4} < \sqrt{8}+\sqrt{5} < \sqrt{6}+\sqrt{7}$

Ex.29: Arrange the following in descending order.

$\sqrt{8}-\sqrt{5}$, $\sqrt{6}-\sqrt{7}$, $\sqrt{9}-\sqrt{4}$, $\sqrt{11}-\sqrt{2}$, $\sqrt{10}-\sqrt{3}$

Sol. We use square method

$(\sqrt{8}-\sqrt{5})^2 = 13-2\sqrt{40}$

$(\sqrt{6}-\sqrt{7})^2 = 13-2\sqrt{42}$

$(\sqrt{9}-\sqrt{4})^2 = 13-2\sqrt{36}$

$(\sqrt{11}-\sqrt{2})^2 = 13-2\sqrt{22}$

$(\sqrt{10}-\sqrt{3})^2 = 13-2\sqrt{30}$

It is cleared that

$13-2\sqrt{22} > 13-2\sqrt{30} > 13-2\sqrt{36} > 13-2\sqrt{40} > 13-2\sqrt{42}$

Then

$\sqrt{11}-\sqrt{2} > \sqrt{10}-\sqrt{3} > \sqrt{9}-\sqrt{4} > \sqrt{8}-\sqrt{5} > \sqrt{6}-\sqrt{7}$

Ex.30: Which is greater than $(\sqrt{23}-\sqrt{21})$ or $(\sqrt{21}-\sqrt{19})$

Sol. Rationalisation of Surds:

$(\sqrt{23}-\sqrt{21}) \times \frac{(\sqrt{23}+\sqrt{21})}{\sqrt{23}+\sqrt{21}}$

$= \frac{2}{\sqrt{23}+\sqrt{21}}$

$(\sqrt{21}-\sqrt{19}) \times \frac{(\sqrt{21}+\sqrt{19})}{(\sqrt{21}+\sqrt{19})}$

$= \frac{2}{(\sqrt{21}+\sqrt{19})}$

It is cleared that

$\frac{2}{(\sqrt{21}+\sqrt{19})} > \frac{2}{\sqrt{23}+\sqrt{21}}$

Then $(\sqrt{21}-\sqrt{19}) > (\sqrt{23}-\sqrt{21})$

Ex.31: Arrange the following in decendinding order.

$2^{350}, 5^{200}, 3^{300}, 4^{250}$

Sol. $2^{350}, 5^{200}, 3^{300}, 4^{250}$

Power in same form

$(2^7)^{50}, (5^4)^{50}, (3^6)^{50}, (4^5)^{50}$

$(128)^{50}, (625)^{50}, (729)^{50}, (1024)^{50}$

So,

$4^{250} > 3^{300} > 5^{200} > 2^{350}$

Ex.32: Arrange the following in descending order.

$2^{72}, 5^{36}, 4^{48}, 3^{60}$

Power in same form

$(2^6)^{12}, (5^3)^{12}, (4^4)^{12}, (3^5)^{12}$

$(64)^{12}, (125)^{12}, (256)^{12}, (243)^{12}$

Then

$(256)^{12} > (243)^{12} > (125)^{12} > (64)^{12}$

$\therefore 4^{48} > 3^{60} > 5^{36} > 2^{72}$

Ex.33: Arrange the following in descending order.

$\sqrt[3]{3}$, $\sqrt[4]{4}$, $\sqrt[6]{6}$, $\sqrt[12]{12}$

Sol. $3^{\frac{1}{3}}, 4^{\frac{1}{4}}, 6^{\frac{1}{6}}, 12^{\frac{1}{12}}$

LCM of 3,4,6 and 12 = 12

Then

$3^{\frac{1}{3}\times\frac{4}{4}}, 4^{\frac{1}{4}\times\frac{3}{3}}, 6^{\frac{1}{6}\times\frac{2}{2}}, 12^{\frac{1}{12}}$

$(3^4)^{\frac{1}{12}}, (4^3)^{\frac{1}{12}}, (6^2)^{\frac{1}{12}}, 12^{\frac{1}{12}}$

It is cleared that

$(81)^{\frac{1}{12}}, (64)^{\frac{1}{12}}, (36)^{\frac{1}{12}}, 12^{\frac{1}{12}}$

$(81)^{\frac{1}{12}} > (64)^{\frac{1}{12}} > (36)^{\frac{1}{12}} > 12^{\frac{1}{12}}$

So,

$3^{\frac{1}{3}} > 4^{\frac{1}{4}} > 6^{\frac{1}{2}} > 12^{\frac{1}{12}}$

TYPE – IV

$\Rightarrow$ **Rationalisation of Surds:** If the product of two surds is rational, then each of them is called the (R.F.) rationalising factor of the other.

E.g.: $5\sqrt{7} \times \sqrt{7} = 5\sqrt{7\times7} = 5 \times 7 = 35$

$\therefore \sqrt{7}$ is a rationalising factor of $5\sqrt{7}$.

→ Rationalising factor (R.F) of the surd $\sqrt[n]{a}$ is $a^{1-\frac{1}{n}}$

→ R.F. of the surd $\sqrt{a} \pm \sqrt{b}$ is $\sqrt{a} \mp \sqrt{b}$.

E.g.: $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)$

$= \left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2 = 3-2 = 1$

$\therefore$ $\left(\sqrt{3}-\sqrt{2}\right)$ is a R.F. of $\left(\sqrt{3}+\sqrt{2}\right)$

Note: The R.F. of a given surd is not unique.

Ex.34: If $x = 7+4\sqrt{3}$ find the value of $\frac{1}{x}$

Sol. $x = 7+4\sqrt{3}$

$\frac{1}{x} = \frac{1}{7+4\sqrt{3}}$

(Multiplying numberator and denominator by conjugate)

$\frac{1}{x} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$= \frac{1}{x} = \frac{7-4\sqrt{3}}{(7)^2 - \left(4\sqrt{3}\right)^2}$

$= \frac{1}{x} = \frac{7-4\sqrt{3}}{49-48}$

$= \frac{1}{x} = 7-4\sqrt{3}$

Alternate:-

R.F. of the surd $\sqrt{a} \pm \sqrt{b}$ is $\sqrt{a} \mp \sqrt{b}$.

when difference of square of both number $\sqrt{a}$ and $\sqrt{b}$ = 1

$x = 7+4\sqrt{3}$

difference $= (7)^2 - \left(4\sqrt{3}\right)^2$

$= 49 - 48 = 1$

Then $\frac{1}{x} = \frac{1}{7+4\sqrt{3}} = 7-4\sqrt{3}$

Ex.35: If $x = \sqrt{3}-\sqrt{2}$, find the vlaue of $\frac{1}{x}$ = ?

Sol. $x = \sqrt{3}-\sqrt{2}$

Difference $= \left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2$

$= 3 - 2 = 1$

Then, $\frac{1}{x} = \sqrt{3}+\sqrt{2}$

Ex.36: If $x = \frac{1}{5-2\sqrt{6}}$, Find the value of $\frac{1}{x}$ = ?

Sol. $x = \frac{1}{5-2\sqrt{6}}$

Difference of nos.

$= (5)^2 - \left(2\sqrt{6}\right)^2$

$= 25 - 24 = 1$

then, $\frac{1}{x} = \frac{1}{5+2\sqrt{6}} = 5-2\sqrt{6}$

Note:- (i) If $x = \sqrt{3}+\sqrt{2}$

then $\frac{1}{x} = \sqrt{3}-\sqrt{2}$

(ii) If $x = 5-2\sqrt{6}$

then $\frac{1}{x} = 5+2\sqrt{6}$

(iii) If $x = \frac{1}{7-4\sqrt{3}}$

then $\frac{1}{x} = \frac{1}{7+4\sqrt{3}}$

- When the difference between two numbers is equal to 1 then reciprocal of this number only change their sign. ("+" change into "–", and "–" change into "+")

Ex.37: Find the value of $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

Sol. $\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

(Multiplying numerator and denominator by conjugate)

$= \frac{\left(2+\sqrt{3}\right)^2}{4-3} = \frac{4+3+4\sqrt{3}}{1}$

$= 7+4\sqrt{3}$

Ex.38: The value of $\frac{\sqrt{5}-2}{\sqrt{5}+2}$ is:

Sol. $\frac{\sqrt{5}-2}{\sqrt{5}+2} = \frac{\left(\sqrt{5}-2\right)}{\left(\sqrt{5}+2\right)} \times \frac{\left(\sqrt{5}-2\right)}{\left(\sqrt{5}-2\right)}$

$= \frac{\left(\sqrt{5}-2\right)^2}{5-4}$

$= \frac{5+4-4\sqrt{5}}{1} = 9-4\sqrt{5}$

Ex.39: Find the value of $\frac{5}{6\sqrt{6}} \times \frac{12\sqrt{30}}{25\sqrt{5}}$.

Sol. $\frac{5}{6\sqrt{6}} \times \frac{12\sqrt{30}}{25\sqrt{5}} = \frac{5}{6\sqrt{6}} \times \frac{12\sqrt{6}\sqrt{5}}{25\sqrt{5}} = \frac{2}{5}$

TYPE – V
SOME USEFUL RESULTS

(a) If $y = \sqrt{x+\sqrt{x+\sqrt{x+\ldots\ldots\infty}}}$

then, $y = \frac{1+\sqrt{1+4x}}{2}$

Ex.40: $y = \sqrt{7+\sqrt{7+\sqrt{7\ldots\ldots\ldots\infty}}}$, then which of the following is true ?

(a) $y = 3$ (b) $3 < y < 3.5$
(b) $y = 7$ (d) Greater than 4

Sol. $y = \frac{1+\sqrt{1+4x}}{2}$

Here, $x = 7$

then $y = \frac{1+\sqrt{1+4\times7}}{2}$

$= \frac{1+\sqrt{29}}{2}$

$\therefore$ $\sqrt{29}$ lies between 5 or 6

So, $y = \frac{1+5}{2} = 3$

or, $y = \frac{1+6}{2} = 3.5$

So, $3 < y < 3.5$ is correct.

Ex.41:

If $y = \sqrt{12+\sqrt{12+\sqrt{12+\ldots\ldots\infty}}}$,

then y = ?

Sol. $y = \dfrac{1+\sqrt{1+4\times12}}{2}$

$y = \dfrac{1+\sqrt{49}}{2}$

$= \dfrac{1+7}{2} = 4$

Note:

if $y = \sqrt{x+\sqrt{x+\sqrt{x+.......\infty}}}$

and $x = n(n + 1)$ then $y = (n + 1)$

Alternate:

we have $x = 12 = 3 \times 4 = n(n + 1)$

$\therefore\ y = 4$

Ex.42: If $\sqrt{20+\sqrt{20+\sqrt{20+......\infty}}}$ then the value of y = ?

Sol. We have $x = 20 = 4 \times 5 = n(n + 1)$

$\therefore\ y = 5$

(b) If $y = \sqrt{x-\sqrt{x-\sqrt{x-........\infty}}}$

then $y = \dfrac{-1+\sqrt{1+4x}}{2}$

Ex.43: If $y = \sqrt{9-\sqrt{9-\sqrt{9........\infty}}}$ then which of the following is true ?

(a) $y = 3$ (b) $2.5 < y < 3$

(b) $y = 9$ (d) Greater than 4

Sol. $y = \dfrac{-1+\sqrt{1+4x}}{2}$

Here, $x = 9$

then $y = \dfrac{-1+\sqrt{1+4\times9}}{2}$

$= \dfrac{-1+\sqrt{37}}{2}$

$\therefore\ \sqrt{37}$ lies between 6 or 7

So, $y = \dfrac{-1+6}{2} = 2.5$

or, $y = \dfrac{-1+7}{2} = 3$

So, $2.5 < y < 3$ is correct.

Ex.44: If $y = \sqrt{12-\sqrt{12-\sqrt{12-.......\infty}}}$, then the value of y = ?

Sol. $y = \dfrac{-1+\sqrt{1+4\times12}}{2}$

$y = \dfrac{-1+\sqrt{49}}{2}$

$y = \dfrac{-1+7}{2} = 3$

Note:

if $y = \sqrt{x-\sqrt{x-\sqrt{x-.......\infty}}}$

and $x = n(n + 1)$ then $y = n$

Alternate:

we have, $x = 12 = 3 \times 4 = n(n+1)$

$\therefore\ y = 3.$

Ex.45: If $y = \sqrt{42-\sqrt{42-\sqrt{42......\infty}}}$, the value of y = ?

Sol. We have, $x = 42 = 6 \times 7 = n(n+1)$

$\therefore\ y = 6$

(c) **If $y = \sqrt{a\sqrt{a\sqrt{a......\infty}}}$ then $y = a$**

Ex.46: $y = \sqrt{5\sqrt{5\sqrt{5.......\infty}}}$

Sol. $y = 5$

(d) **If $y = \sqrt{a\sqrt{a\sqrt{a......n}}}$, then $y = a^{\frac{2^n-1}{2^n}}$**

Ex.47: If $y = \sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}}$, find the value of y.

Sol. $n = 4$, then $y = a^{\frac{2^4-1}{2^4}}$

$y = a^{\frac{15}{16}}$

Ex.48: If $y = \sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}}}$, find the value of y ?

Sol. $n = 5$

Then, $y = 3^{\frac{2^5-1}{2^5}}$

$y = 3^{\frac{31}{32}}$

(e) I $y = \left(\sqrt[m]{\left(\sqrt[p]{a^x}\right)^z}\right)^n$

$a^{\frac{x\times z\times n}{m\times p}}$

Ex.49: If $\left(\sqrt[12]{\left(\sqrt[6]{\left(\sqrt[3]{5^4}\right)^8}\right)^9}\right)^{18} \times \sqrt[9]{\left(\sqrt[8]{\left(\sqrt[4]{5^3}\right)^6}\right)^{12}}$

Sol. $5^{\frac{4\times8\times9\times18}{3\times6\times12}} \times 5^{\frac{3\times6\times12}{4\times8\times9}}$

$5^{8\times3} \times 5^{\frac{3}{4}}$

$5^{24+\frac{3}{4}} = 5^{\frac{99}{4}}$

Ex.50: $\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4.......}}}}}}$ will be equal to

Sol. Let $x = \sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4.......}}}}}}$

$x = \sqrt{2\sqrt[3]{4x}}$ (Squaring both sides)

$x^2 = 2\sqrt[3]{4x}$ (Cubing both sides)

$x^6 = 2^3.4x = 2^5.x \Rightarrow x^5 = 2^5$

$x = 2$

TYPE – VI

$\Rightarrow$ **Square-root of an Irrational number:**

As we know that,

$(a+b)^2 = (a^2+b^2)+2ab$

$\therefore\ \left(\sqrt{2}+\sqrt{3}\right)^2 = \underbrace{5}_{(a^2+b^2)} + \underbrace{2\sqrt{6}}_{(2ab)}$

$\therefore\ 5+2\sqrt{6} = 5+\underbrace{2\sqrt{2}\sqrt{3}}_{(2ab)}$

$\therefore$ a = $\sqrt{2}$ & $b = \sqrt{3}$

& $a^2 + b^2 = 5$

$\therefore\ 5+2\sqrt{6} = \left(\sqrt{2}+\sqrt{3}\right)^2$

$\Rightarrow\ a+b = \sqrt{5+2\sqrt{6}} = \sqrt{2}+\sqrt{3}$

Examples:

(i) $\sqrt{7-4\sqrt{3}} = \sqrt{7-2\times \underset{a}{2}\times \underset{b}{\sqrt{3}}}$

& $a^2 + b^2 = 2^2 + \sqrt{3}^2 = 7$

$= \sqrt{\left(2-\sqrt{3}\right)^2} = \left(2-\sqrt{3}\right)$

(ii) $\sqrt{5\quad \underbrace{\sqrt{21}}_{(2ab)}}$ for making it 2ab

$\sqrt{\dfrac{\left(5-\sqrt{21}\right)\times2}{2}} = \sqrt{\dfrac{10-2\times\sqrt{3}\times\sqrt{7}}{2}}$

$= \dfrac{1}{\sqrt{2}}\sqrt{\left(10-2\times\sqrt{3}\times\sqrt{7}\right)}$

$\left[\begin{array}{l}\therefore\ a=\sqrt{7}\ \&\ b=\sqrt{3}\\ \because\ a^2+b^2=10\end{array}\right]$

$$= \frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{7}-\sqrt{3}\right)^2} = \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}}$$

Ex.51: Find the square root of $\left(7+4\sqrt{3}\right)$.

Sol. $\underbrace{7}_{(a^2+b^2)}+\underbrace{4\sqrt{3}}_{(2ab)}$

$= \underbrace{7}_{4+3}+\underbrace{4\sqrt{3}}_{2\times2\sqrt{3}}$

$= \left(2+\sqrt{3}\right)^2$

$= \sqrt{\left(2+\sqrt{3}\right)^2} = 2+\sqrt{3}$

Ex.52: Find the square root of $13-4\sqrt{3}$

Sol. $\underbrace{13}_{(a^2+b^2)}-\underbrace{4\sqrt{3}}_{(2ab)}$

$= \underbrace{13}_{(12+1)}-\underbrace{4\sqrt{3}}_{2\times2\sqrt{3}\times1}$

$= \left(\sqrt{12}-1\right)^2$

Square root of

Then $\sqrt{\left(\sqrt{12}-1\right)^2} = \sqrt{12}-1$

Ex.53: Find the square root of $76+10\sqrt{3}$

Sol. $\underbrace{76}_{75+1}+\underbrace{10\sqrt{3}}_{2\times5\sqrt{3}\times1}$

$= \left(5\sqrt{3}+1\right)^2$

Square root

Then $= \sqrt{\left(5\sqrt{3}+1\right)^2}$

$= 5\sqrt{3}+1$

Ex.54: Find the square of $33-4\sqrt{35}$

Sol. $\underbrace{33}_{(28+5)}-\underbrace{4\sqrt{35}}_{2\times2\sqrt{7}\times\sqrt{5}}$

$= \left(2\sqrt{7}-\sqrt{5}\right)^2$

Square root

Then, $\sqrt{\left(2\sqrt{7}-\sqrt{5}\right)^2}$

$2\sqrt{7}-\sqrt{5}$

Ex.55: Find the square root of $139-80\sqrt{3}$.

Sol. $\underbrace{139}_{(75+64)}-\underbrace{80\sqrt{3}}_{2\times5\sqrt{3}\times8}$

$= \left(5\sqrt{3}-8\right)^2$

Square root

Then, $\sqrt{\left(5\sqrt{3}-8\right)^2}$

$5\sqrt{3}-8$

Ex.56: Find the square root of $8-4\sqrt{3}$

Sol. $8-4\sqrt{3}$

$= 2\left(4-2\sqrt{3}\right)$

$= 2\left(\underbrace{4}_{3+1}-\underbrace{2\sqrt{3}}_{2\sqrt{3}\times1}\right)$

$= 2\left(\sqrt{3}-1\right)^2$

Square root of

Then, $\sqrt{2\left(\sqrt{3}-1\right)^2}$

$= \sqrt{2}\left(\sqrt{3}-1\right)$

Ex.57: If $x=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$, then find the value of x.

Sol. $x=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$

$x=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{\left(2+\sqrt{3}\right)^2}}}$

$\Rightarrow x=\sqrt{-\sqrt{3}+\sqrt{3+8\left(2+\sqrt{3}\right)}}$

$\Rightarrow x=\sqrt{-\sqrt{3}+\sqrt{3+16+8\sqrt{3}}}$

$\Rightarrow x=\sqrt{-\sqrt{3}+\sqrt{19+8\sqrt{3}}}$

$\Rightarrow x=\sqrt{-\sqrt{3}+\sqrt{\left(4+\sqrt{3}\right)^2}}$

$\Rightarrow x=\sqrt{-\sqrt{3}+\left(4+\sqrt{3}\right)}$

$x = 2$

Ex.58: If $x=\frac{2\sqrt{10}}{7}$, find the value of $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$

Sol. $1+x = 1+\frac{2\sqrt{10}}{7} = \frac{7+2\sqrt{10}}{7}$

$1+x = \left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{7}}\right)^2$

Square root of both sides,

$\sqrt{1+x} = \sqrt{\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{7}}\right)^2}$

$\sqrt{1+x} = \left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{7}}\right)$

Same as $\sqrt{1-x} = \left(\frac{\sqrt{5}-\sqrt{2}}{\sqrt{7}}\right)$

$\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$

Put the value,

$$= \frac{\frac{\sqrt{5}+\sqrt{2}}{\sqrt{7}}+\frac{\sqrt{5}-\sqrt{2}}{\sqrt{7}}}{\frac{\sqrt{5}+\sqrt{2}}{\sqrt{7}}-\frac{\sqrt{5}-\sqrt{2}}{\sqrt{7}}}$$

$$= \frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}-\sqrt{5}+\sqrt{2}} = \frac{2\sqrt{5}}{2\sqrt{2}}$$

$$= \frac{\sqrt{5}}{\sqrt{2}}$$

Ex.59: If $x = 38+5\sqrt{3}$, Find the value of $\sqrt{x}$.

Sol. $x=38+5\sqrt{3}$

Multiply and divided by 2

$x=\frac{76+2\times5\times\sqrt{3}}{2}$

$x=\left(\frac{5\sqrt{3}+1}{\sqrt{2}}\right)^2$

Then, $\sqrt{x}=\sqrt{\left(\frac{5\sqrt{3}+1}{\sqrt{2}}\right)^2}$

$\sqrt{x}=\frac{5\sqrt{3}+1}{\sqrt{2}}$

Ex.60: If $x=\frac{\sqrt{3}}{2}$, Find the value of $\sqrt{1+x}$

Sol. $1+x=1+\frac{\sqrt{3}}{2}=\frac{(2+\sqrt{3})}{2}$

Multiply and divided by 2

$1+x=\frac{4+2\sqrt{3}}{4}$

$1+x=\left(\frac{\sqrt{3}+1}{2}\right)^2$

Square Root of Both Sides

Then, $\sqrt{1+x}=\sqrt{\left(\frac{\sqrt{3}+1}{2}\right)^2}$

$\sqrt{1+x}=\frac{\sqrt{3}+1}{2}$

Ex.61: If $x=\frac{\sqrt{3}}{2}$, Find the value of

$\frac{\sqrt{1+x}}{1+\sqrt{1+x}}+\frac{\sqrt{1-x}}{1-\sqrt{1-x}}$

Sol. When $x=\frac{\sqrt{3}}{2}$

Then, $\sqrt{1+x}=\frac{\sqrt{3}+1}{2}$

(Explanation above question 61)
Put the value:

$\Rightarrow \frac{\frac{\sqrt{3}+1}{2}}{1+\left(\frac{\sqrt{3}+1}{2}\right)}+\frac{\frac{\sqrt{3}-1}{2}}{1-\left(\frac{\sqrt{3}-1}{2}\right)}$

$\Rightarrow \frac{\frac{\sqrt{3}+1}{2}}{\frac{2+\sqrt{3}+1}{2}}+\frac{\frac{\sqrt{3}-1}{2}}{\frac{2-\sqrt{3}+1}{2}}$

$\Rightarrow \frac{\frac{\sqrt{3}+1}{2}}{\frac{(3+\sqrt{3})}{2}}+\frac{\frac{\sqrt{3}-1}{2}}{\frac{(3-\sqrt{3})}{2}}$

$\Rightarrow \frac{(\sqrt{3}+1)}{\sqrt{3}(\sqrt{3}+1)}+\frac{\sqrt{3}-1}{\sqrt{3}(\sqrt{3}-1)}$

$\Rightarrow \frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}=\frac{2}{\sqrt{3}}$

Ex.62: If $x=\frac{\sqrt{3}}{2}$, Find the value of

$\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}$

Sol. When $x=\frac{\sqrt{3}}{2}$

Then, $\sqrt{1+x}=\frac{\sqrt{3}+1}{2}$

(Explanation above question 61)
Put the value:

$\Rightarrow \frac{1+\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}+1}{2}}+\frac{1-\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}-1}{2}}$

$\Rightarrow \frac{\frac{2+\sqrt{3}}{2}}{\frac{2+\sqrt{3}+1}{2}}+\frac{\frac{2-\sqrt{3}}{2}}{\frac{2-\sqrt{3}+1}{2}}$

$\Rightarrow \frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2-\sqrt{3}}{3-\sqrt{3}}=1$

$=\frac{(2+\sqrt{3})(3-\sqrt{3})+(2-\sqrt{3})(3+\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$

$=\frac{6-2\sqrt{3}+3\sqrt{3}-3+6+2\sqrt{3}-3\sqrt{3}-3}{(3)^2-(\sqrt{3})^2}$

$=\frac{6}{9-3}=\frac{6}{6}=\mathbf{1}$

TYPE – VII

Useful Result:-

(a) If $x=\frac{4\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

$\frac{x+2\sqrt{a}}{x-2\sqrt{a}}+\frac{x+2\sqrt{b}}{x-2\sqrt{b}}=2$

(b) If $=x\frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

$\frac{x+\sqrt{a}}{x-\sqrt{a}}+\frac{x+\sqrt{b}}{x-\sqrt{b}}=2$

Ex.63: If $=x\frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$, the value of

$\frac{x+\sqrt{20}}{x-\sqrt{20}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}$

Sol. $x=\frac{2\times 2\times\sqrt{5}\times\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$\Rightarrow \frac{x}{2\sqrt{5}}=\frac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$\Rightarrow \frac{x}{\sqrt{20}}=\frac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ also

$\Rightarrow \frac{x}{\sqrt{12}}=\frac{2\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

Applying componendo & dividendo

$\therefore \frac{x+\sqrt{20}}{x-\sqrt{20}}=\frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}=-\frac{3\sqrt{3}+\sqrt{5}}{\sqrt{5}-\sqrt{3}}$

Similarly, $\frac{x+\sqrt{12}}{x-\sqrt{12}}=\frac{\sqrt{3}+3\sqrt{5}}{\sqrt{5}-\sqrt{3}}$

$\frac{x+\sqrt{20}}{x-\sqrt{20}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}$

$=\frac{-3\sqrt{3}-\sqrt{5}+\sqrt{3}+3\sqrt{5}}{\sqrt{5}-\sqrt{3}}$

$=\frac{2\sqrt{5}-2\sqrt{3}}{(\sqrt{5}-\sqrt{3})}=2$

Note: In these type of questions there is maximum possibility of having answer '2'.

EXERCISE

1. The value of $\sqrt{5+2\sqrt{6}} - \frac{1}{\sqrt{5+2\sqrt{6}}}$ is:
(a) $2\sqrt{2}$ (b) $2\sqrt{3}$
(c) $1+\sqrt{5}$ (d) $\sqrt{5}-1$

2. Simplify : $\left(\frac{\frac{3}{2+\sqrt{3}} - \frac{2}{2-\sqrt{3}}}{2-5\sqrt{3}}\right)$
(a) $\frac{1}{2}-5\sqrt{3}$ (b) $2-5\sqrt{3}$
(c) 1 (d) 0

3. The value of $(243)^{0.16} \times (243)^{0.04}$ is equal to :
(a) 0.16 (b) 3 (c) $\frac{1}{3}$ (d) 0.04

4. The simplification of
$$\frac{0.06\times0.06\times0.06-0.05\times0.05\times0.05}{0.06\times0.06+0.06\times0.05+0.05\times0.05}$$
(a) 1 (b) 0.1
(c) 0.01 (d) 0.001

5. Simplify :
$$\frac{0.05\times0.05\times0.05-0.04\times0.04\times0.04}{0.05\times0.05+0.002+0.04\times0.04}$$
(a) 1 (b) 0.1
(c) 0.01 (d) 0.001

6. Which one of the following is the least?
$\sqrt{3}, \sqrt[3]{2}, \sqrt{2}$ and $\sqrt[3]{4}$
(a) $\sqrt{2}$ (b) $\sqrt[3]{2}$ (c) $\sqrt{3}$ (d) $\sqrt[3]{3}$

7. Which one of the following is the biggest ?
$\sqrt[3]{4}$, $\sqrt[4]{6}$, $\sqrt[6]{15}$, and $\sqrt[12]{245}$.
(a) $\sqrt[3]{4}$ (b) $\sqrt[4]{6}$
(c) $\sqrt[6]{15}$ (d) $\sqrt[12]{245}$

8. Simplify :
$$\left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4 \left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4$$
(a) 5^2 (b) 5^4 (c) 5^8 d) 5^{12}

9. If $3^{x+8} = 27^{2x+1}$, the value of x is :
(a) 7 (b) 3 (c) –2 (d) 1

10. The simplified form of $\left(16^{3/2}+16^{-3/2}\right)$ is :
(a) 0 (b) $\frac{4097}{64}$
(c) 1 (d) $\frac{16}{4097}$

11. $(0.01024)^{1/5}$ is equal to :
(a) 4.0 (d) 0.04
(c) 0.4 (d) 0.00004

12. $(64)^{-\frac{2}{3}} \times \left(\frac{1}{4}\right)^{-2}$ is equal to :
(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{16}$

13. $\left(\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\right)$ simplifies to :
(a) $\sqrt{5}+\sqrt{6}$ (b) $2\sqrt{5}+\sqrt{6}$
(c) $\sqrt{5}-\sqrt{6}$ (d) $2\sqrt{5}-3\sqrt{6}$

14. $\left(\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ simplifies to:
(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $16-\sqrt{3}$ (d) $40-\sqrt{3}$

15. $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2 + \left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)^2$ is equal to:
(a) 64 (b) 62 (c) 66 (d) 68

16. (7.5 × 7.5 +37.5 +2.5 × 2.5) is equal to:
(a) 100 (b) 80 (c) 60 (d) 30

17. $\left(\frac{8}{125}\right)^{-\frac{4}{3}}$ simplifies to :
(a) $\frac{625}{16}$ (b) $\frac{625}{8}$
(c) $\frac{625}{32}$ (d) $\frac{16}{625}$

18. The value of
$$\sqrt{\frac{\left(\sqrt{12}-\sqrt{8}\right)\left(\sqrt{3}+\sqrt{2}\right)}{5+\sqrt{24}}}$$ is :
(a) $\sqrt{6}-\sqrt{2}$ (b) $\sqrt{6}+\sqrt{2}$
(c) $\sqrt{6}-2$ (d) $2-\sqrt{6}$

19. Simplify :
$$\left[64^{\frac{2}{3}} \times 2^{-2} \div 8^0\right]^{\frac{1}{2}}$$
(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

20. The value of $\frac{1}{\sqrt{\left(12-\sqrt{140}\right)}} - \frac{1}{\sqrt{\left(8-\sqrt{60}\right)}} - \frac{2}{\sqrt{10+\sqrt{84}}}$ is :
(a) 0 (b) 1 (c) 2 (d) 3

21. Which of the following number is the least ?
$(0.5)^2$, $\sqrt{0.49}$, $\sqrt[3]{0.008}$, 0.23
(a) $(0.5)^2$ (b) $\sqrt{0.49}$
(c) $\sqrt[3]{0.008}$ (d) 0.23

22. Arrange the following in descending order : $\sqrt[3]{4}, \sqrt{2}, \sqrt[6]{3}, \sqrt[4]{5}$
(a) $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$
(b) $\sqrt[3]{5} > \sqrt[3]{4} > \sqrt[6]{3} > \sqrt{2}$
(c) $\sqrt{2} > \sqrt[4]{5} > \sqrt[3]{4} > \sqrt[6]{3}$
(d) $\sqrt{2} > \sqrt[3]{4} > \sqrt[4]{5} > \sqrt[6]{3}$

23. The value of
$$\frac{(243)^{0.13} \times (243)^{0.07}}{(7)^{0.25} \times (49)^{0.075} \times (343)^{0.2}}$$
(a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) $1\frac{3}{7}$ (d) $2\frac{2}{7}$

24. The value of :
$\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$ is
(a) 1 (b) 2 (c) 3 (d) 8

25. $\sqrt{\sqrt[3]{0.004096}}$ is equal to
(a) 4 (b) 0.4
(c) 0.04 (d) 0.004

26. $\frac{2.3\times2.3\times2.3-1}{2.3\times2.3+2.3+1}$ is equal to
(a) 1.3 (b) 3.3 (c) 0.3 (d) 2.2

27. The ascending order of
$(2.89)^{0.5}, 2-(0.5)^2, \sqrt{3}$ and $\sqrt[3]{0.008}$ is
(a) $2-(0.5)^2$, $\sqrt{3}$, $\sqrt[3]{0.008}$, $(2.89)^{0.5}$
(b) $\sqrt[3]{0.008}$, $(2.89)^{0.5}$, $\sqrt{3}$, $2-(0.5)^2$
(c) $\sqrt[3]{0.008}$, $\sqrt{3}$, $(2.89)^{0.5}$, $2-(0.5)^2$
(d) $\sqrt{3}$, $\sqrt[3]{0.008}$, $2-(0.5)^2$, $(2.89)^{0.5}$

28. $\sqrt{2+\sqrt{2+\sqrt{2+\ldots\ldots}}}$ is equal to

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 2 (d) 3

29. The value of $\left(\sqrt[3]{3.5}+\sqrt[3]{2.5}\right)$ $\left\{\left(\sqrt[3]{3.5}\right)^2-\sqrt[3]{8.75}+\left(\sqrt[3]{2.5}\right)^2\right\}$ is :

(a) 5.375 (b) 1
(c) 6 (d) 5

30. The value of

$\left(3+2\sqrt{2}\right)^{-3}+\left(3-2\sqrt{2}\right)^{-3}$ is

(a) 189 (b) 180 (c) 108 (d) 198

31. $\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}}-\frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}}+\frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}}$ is equal to:

(a) 0 (b) $2\sqrt{15}$
(c) $2\sqrt{10}$ (d) $2\sqrt{6}$

32. $\frac{(0.96)^3-(0.1)^3}{(0.96)^2+0.096+(0.1)^2}$ is simplified to :

(a) 1.06 (b) 0.95
(c) 0.86 (d) 0.97

33. When $\left(4+\sqrt{7}\right)$ is presented in the form of perfect square it will be equal to :

(a) $\left(2+\sqrt{7}\right)^2$ (b) $\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)^2$

(c) $\left\{\frac{1}{\sqrt{2}}\left(\sqrt{7}+1\right)\right\}^2$ (d) $\left(\sqrt{3}+\sqrt{4}\right)^2$

34. The simplified form of

$\frac{2}{\sqrt{7}+\sqrt{5}}+\frac{7}{\sqrt{12}-\sqrt{5}}-\frac{5}{\sqrt{12}-\sqrt{7}}$ is :

(a) 5 (b) 2 (c) 1 (d) 0

35. $\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}$ $+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$ is

(a) $\sqrt{3}$ (b) $3\sqrt{3}$
(c) $3-\sqrt{3}$ (d) $5-\sqrt{3}$

36. Simplify :

$\frac{1}{\sqrt{100}-\sqrt{99}}-\frac{1}{\sqrt{99}-\sqrt{98}}$ $+\frac{1}{\sqrt{98}-\sqrt{97}}-\frac{1}{\sqrt{97}-\sqrt{96}}$ $+\ldots.+\frac{1}{\sqrt{2}-\sqrt{1}}$

(a) 10 (b) 9 (c) 13 (d) 11

37. $\left[\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}+\frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}\right]$ in simplified form equals to :

(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 0

38. $\left\{(-2)^{(-2)}\right\}^{(-2)}$ is equal to :

(a) 16 (b) 8 (c) −8 (d) −1

39. Which is the greatest among $\left(\sqrt{19}-\sqrt{17}\right)$, $\left(\sqrt{13}-\sqrt{11}\right)$, $\left(\sqrt{7}-\sqrt{5}\right)$ and $\left(\sqrt{5}-\sqrt{3}\right)$?

(a) $\sqrt{19}-\sqrt{17}$ (b) $\sqrt{13}-\sqrt{11}$
(c) $\sqrt{7}-\sqrt{5}$ (d) $\sqrt{5}-\sqrt{3}$

40. If x = $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and y = $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, then (x+y) equals:

(a) 8 (b) 16
(c) $2\sqrt{15}$ (d) $2\left(\sqrt{5}+\sqrt{3}\right)$

41. 0.75 × 0.75 − 2 × 0.75 × 0.25 + 0.25 × 0.25 is equal to

(a) 250 (b) 2500
(c) 2.5 (d) 0.25

42. $\left(3+\frac{1}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}+\frac{1}{\sqrt{3}-3}\right)$ is equal to

(a) 1 (b) 3
(c) $3+\sqrt{3}$ (d) $3-\sqrt{3}$

43. $\left[8-\left(\frac{4^{\frac{9}{4}}\sqrt{2.2^2}}{2\sqrt{2^{-2}}}\right)^{\frac{1}{2}}\right]$ is equal to

(a) 32 (b) 8 (c) 1 (d) 0

44. $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{2\sqrt{6}}{\sqrt{3}+1}+\frac{2\sqrt{3}}{\sqrt{6}+2}$ is equal to

(a) 3 (b) 2 (c) 0 (d) $\sqrt{3}$

45. Greatest among the numbers $\sqrt[3]{9},\sqrt{3},\sqrt[4]{16},\sqrt[6]{80}$ is

(a) $\sqrt[3]{9}$ (b) $\sqrt{3}$
(c) $\sqrt[4]{16}$ (d) $\sqrt[6]{80}$

46. Given that $\sqrt{3}$ =1.732, the value of

$\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ is :

(a) 4.899 (b) 2.551
(c) 1.414 (d) 1.732

47. $\sqrt{12+\sqrt{12+\sqrt{12+\ldots\ldots}}}$ is equal to

(a) 3 (b) 4 (c) 6 (d) 2

48. If a = $\frac{\sqrt{3}}{2}$, then the value of $\sqrt{1+a}+\sqrt{1-a}$ is :

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$
(c) $2+\sqrt{3}$ (d) $2-\sqrt{3}$

49. If a = $\frac{\sqrt{5}+1}{\sqrt{5}-1}$, b= $\frac{\sqrt{5}-1}{\sqrt{5}+1}$, the value of $\left(\frac{a^2+ab+b^2}{a^2-ab+b^2}\right)$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

50. $\left(\frac{2}{\sqrt{5}+\sqrt{3}}-\frac{3}{\sqrt{6}-\sqrt{3}}+\frac{1}{\sqrt{6}+\sqrt{5}}\right)$ is equal to

(a) $-2\sqrt{6}$ (b) $-2\sqrt{5}$
(c) $-2\sqrt{3}$ (d) 0

51. $\left(\sqrt{2}+\sqrt{7-2\sqrt{10}}\right)$ is equal to

(a) $\sqrt{2}$ (b) $\sqrt{7}$
(c) $\sqrt{5}$ (d) $2\sqrt{5}$

52. If x =1 + $\sqrt{2}+\sqrt{3}$, then the value of $\left(x+\frac{1}{x-1}\right)$ is

(a) $1+2\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $3+\sqrt{2}$ (d) $2\sqrt{3}-1$

53. If m and $n(n>1)$ are whole numbers such that m^n =121, the value of $(m-1)^{n+1}$ is

(a) 1 (b) 10
(c) 121 (d) 1000

54. $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$ =?

(a) 5 (b) 4 (c) 3 (d) 2

55. $\left(\frac{2}{\sqrt{6}+2}+\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{8}-\sqrt{7}}+2-2\sqrt{2}\right)$ is equal to

(a) 0 (b) $2\sqrt{2}$
(c) $\sqrt{2}$ (d) $2\sqrt{7}$

56. $\left[\left\{\left(-\frac{1}{2}\right)^2\right\}^{-2}\right]^{-1}$ is equal to :

(a) $\frac{1}{16}$ (b) 16 (c) $-\frac{1}{16}$ (d) −16

57. The greatest number among $2^{60}, 3^{48}, 4^{36}$ and 5^{24} is
(a) 2^{60} (b) 3^{48} (c) 4^{36} (d) 5^{24}

58. $\sqrt{3\sqrt{3\sqrt{3.....}}}$ is equal to
(a) $\sqrt{3}$ (b) 3
(c) $2\sqrt{3}$ (d) $3\sqrt{3}$

59. The greatest among the numbers $\sqrt{0.09}, \sqrt[3]{0.064}$, 0.5 and $\frac{3}{5}$
(a) $\sqrt{0.09}$ (b) $\sqrt[3]{0.064}$
(c) 0.5 (d) $\frac{3}{5}$

60. The greatest of the following numbers 0.16, $\sqrt{0.16}$, $(0.16)^2$, 0.04 is
(a) 0.16 (b) $\sqrt{0.16}$
(c) 0.04 (d) $(0.16)^2$

61. The greatest of the numbers $\sqrt[2]{8}$, $\sqrt[4]{13}$, $\sqrt[5]{16}$, $\sqrt[10]{41}$ is :
(a) $\sqrt[4]{13}$ (b) $\sqrt[5]{16}$
(c) $\sqrt[10]{41}$ (d) $\sqrt[2]{8}$

62. If $2^x = 3^y = 6^{-z}$ then $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ is equal to
(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

63. $2\sqrt[3]{40} - 4\sqrt[3]{320} + 3\sqrt[3]{625} - 3\sqrt[3]{5}$ is equal to
(a) $-2\sqrt[3]{340}$ (b) 0
(c) $\sqrt[3]{340}$ (d) $\sqrt[3]{660}$

64. The value of $\sqrt{40+\sqrt{9\sqrt{81}}}$ is
(a) $\sqrt{111}$ (b) 9
(c) 7 (d) 11

65. If $\frac{\left(x-\sqrt{24}\right)\left(\sqrt{75}+\sqrt{50}\right)}{\sqrt{75}-\sqrt{50}} = 1$, then the value of x is
(a) $\sqrt{5}$ (b) 5
(c) $2\sqrt{5}$ (d) $3\sqrt{5}$

66. Evaluate
$$\sqrt{20}+\sqrt{12}+\sqrt[3]{729}-\frac{4}{\sqrt{5}-\sqrt{3}}-\sqrt{81}$$
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) 0 (d) $2\sqrt{2}$

67. If a, b are rationals and $a\sqrt{2}+b\sqrt{3} = \sqrt{98}+\sqrt{108}-\sqrt{48}-\sqrt{72}$, then the values of a, b are respectively
(a) 1, 2 (b) 1, 3
(c) 2, 1 (d) 2 , 3

68. Let $\sqrt[3]{a} = \sqrt[3]{26}+\sqrt[3]{7}+\sqrt[3]{63}$ then
(a) $a < 729$ but $a > 216$
(b) $a < 216$
(c) $a > 729$
(d) $a = 729$

69. $2+\frac{6}{\sqrt{3}}+\frac{1}{2+\sqrt{3}}+\frac{1}{\sqrt{3}-2}$ equals to
(a) $+\left(2\sqrt{3}\right)$ (b) $-\left(2+\sqrt{3}\right)$
(c) 1 (d) 2

70. If $\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}} = A+\sqrt{B}$, then B – A is
(a) – 13 (b) $2\sqrt{13}$
(c) 13 (d) $3\sqrt{3}-\sqrt{7}$

71. The smallest among the numbers $2^{250}, 3^{150}, 5^{100}$ and 4^{200}
(a) 4^{200} (b) 5^{100}
(c) 3^{150} (d) 2^{250}

72. Find the value of
$$\sqrt{30+\sqrt{30+\sqrt{30+....}}}$$
(a) 5 (b) $3\sqrt{10}$
(c) 6 (d) 7

73. The value of $\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4 ....}}}}$ is
(a) 2 (b) 2^2 (c) 2^3 (d) 2^5

74. $55^3 + 17^3 - 72^3 + 201960$ is equal to
(a) – 1 (b) 0 (c) 1 (d) 17

75. The value of $\frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$ is
(a) 3 (b) 9 (c) 6 (d) 12

76. The simplified value of
$$\left(\sqrt{3}+1\right)\left(10+\sqrt{12}\right)\left(\sqrt{12}-2\right)\left(5-\sqrt{3}\right) \text{ is}$$
(a) 16 (b) 88 (c) 176 (d) 132

77. If $2^{n-1} + 2^{n+1} = 320$, then the value of n is
(a) 6 (b) 8 (c) 5 (d) 7

78. If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$, then the value of a is
(a) 4 (b) 5 (c) 6 (d) 8

79. The value of
$$\left(3+2\sqrt{2}\right)^{-3}+\left(3-2\sqrt{2}\right)^{-3} \text{ is}$$
(a) 198 (b) 180 (c) 108 (d) 189

80. A tap is dripping at a constant rate into a container. The level (L cm) of the water in the container is given by the equation $L = 2 - 2^t$, where t is time taken in hours . Then the level of water in the container at the level of water in the container at the start is
(a) 0cm (b) 1cm (c) 2cm (d) 4cm

81. $\frac{\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}}{\sqrt[3]{8}}$ =?
(a) 8 (b) 4 (c) 1/2 (d) 2

82. The Simplified value of
$$\frac{\sqrt{6}+2}{\sqrt{2}+\sqrt{2+\sqrt{3}}}-\frac{\sqrt{6}-2}{\sqrt{2}-\sqrt{2-\sqrt{3}}}-\frac{2\sqrt{2}}{2+\sqrt{2}}$$
(a) $2\sqrt{6}$ (b) 2 (c) $\sqrt{3}$ (d) 0

83. $\frac{6^2+7^2+8^2+9^2+10^2}{\sqrt{7+4\sqrt{3}}-\sqrt{4+2\sqrt{3}}}$ is equal to/
(a) 330 (b) 355
(c 305 (d) 366

84. $(3x-2y) : (2x+3y) = 5 : 6$, then one of the value of $\left(\frac{\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt[3]{x}-\sqrt[3]{y}}\right)^2$ is
(a) $\frac{1}{25}$ (b) 5 (c) $\frac{1}{5}$ (d) 25

85. The value of
$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$$
is
(a) 2 (b) 4 (c) 0 (d) 1

86. The value of
$$\sqrt{72+\sqrt{72+\sqrt{72+....}}} \text{ is}$$
(a) 9 (b) 18 (c) 8 (d) 12

87. The value of $\frac{1}{1+\sqrt{2}+\sqrt{3}}+\frac{1}{1-\sqrt{2}+\sqrt{3}}$ is:
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) 1 (d) $4\left(\sqrt{3}+\sqrt{2}\right)$

88. The value of the expression
$$\sqrt{6+\sqrt{6+\sqrt{6+...+\text{upto}\infty}}} \text{ is}$$
(a) 30 (b) 5
(c) 3 (d) 2

89. The value of
$$\frac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}}-\frac{5\sqrt{5}}{\sqrt{2}+\sqrt{7}}+\frac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}} \text{ is:}$$
(a) 1 (b) 0
(c) $2\sqrt{3}$ (d) $\sqrt{7}$

90. If $11\sqrt{n} = \sqrt{112}+\sqrt{343}$, then the value of n is:
(a) 3 (b) 11 (c) 13 (d) 7

ANSWER KEY

1. (a)	10. (b)	19. (c)	28. (c)	37. (c)	46. (d)	55. (d)	64. (c)	73. (a)	82. (d)
2. (c)	11. (c)	20. (a)	29. (c)	38. (a)	47. (b)	56. (a)	65. (b)	74. (b)	83. (a)
3. (b)	12. (a)	21. (c)	30. (d)	39. (d)	48. (a)	57. (b)	66. (c)	75. (b)	84. (d)
4. (c)	13. (c)	22. (a)	31. (a)	40. (a)	49. (b)	58. (b)	67. (a)	76. (c)	85. (a)
5. (c)	14. (c)	23. (a)	32. (c)	41. (d)	50. (c)	59. (d)	68. (a)	77. (d)	86. (a)
6. (b)	15. (b)	24. (b)	33. (c)	42. (b)	51. (c)	60. (b)	69. (d)	78. (a)	87. (c)
7. (a)	16. (a)	25. (b)	34. (d)	43. (d)	52. (a)	61. (d)	70. (c)	79. (a)	88. (c)
8. (b)	17. (a)	26. (a)	35. (c)	44. (c)	53. (d)	62. (a)	71. (b)	80. (b)	89. (b)
9. (d)	18. (c)	27. (b)	36. (d)	45. (a)	54. (a)	63. (b)	72. (c)	81. (d)	90. (d)

SOLUTION

1. (a) $\sqrt{5+2\sqrt{6}} - \dfrac{1}{\sqrt{5+2\sqrt{6}}}$

$\Rightarrow \left(\sqrt{3}+\sqrt{2}\right) - \dfrac{1}{\left(\sqrt{3}+\sqrt{2}\right)}$

$\left[\sqrt{5+2\sqrt{6}} = \sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2} \Rightarrow \sqrt{3}+\sqrt{2} \right.$
$\left. a^2 + b^2 + 2ab = (a+b)^2 \right]$

$\Rightarrow \sqrt{3}+\sqrt{2} - \left(\dfrac{1}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$

$\Rightarrow \sqrt{3}+\sqrt{2} - \left(\dfrac{\sqrt{3}-\sqrt{2}}{3-2}\right)$

$\Rightarrow \sqrt{3}+\sqrt{2}-\sqrt{3}+\sqrt{2} \Rightarrow \mathbf{2\sqrt{2}}$

2. (c) $\dfrac{\dfrac{3}{2+\sqrt{3}} - \dfrac{2}{2-\sqrt{3}}}{2-5\sqrt{3}}$

$\Rightarrow \dfrac{\dfrac{3\left(2-\sqrt{3}\right)-2\left(2+\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}}{2-5\sqrt{3}}$

$\Rightarrow \dfrac{6-3\sqrt{3}-4-2\sqrt{3}}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)\left(2-5\sqrt{3}\right)}$

$= \dfrac{2-5\sqrt{3}}{2-5\sqrt{3}} = 1$

3. (b) $(243)^{0.16} \times (243)^{0.04}$

$\Rightarrow (243)^{0.16+0.04} \left[a^m \times a^n = a^{m+n}\right]$

$\Rightarrow 243^{0.20}$

$\Rightarrow 243^{\frac{20}{100}}$

$\Rightarrow 243^{\frac{1}{5}} \Rightarrow \sqrt[5]{243} = 3$

4. (c)
$\dfrac{0.06 \times 0.06 \times 0.06 - 0.05 \times 0.05 \times 0.05}{0.06 \times 0.06 + 0.06 \times 0.05 + 0.05 \times 0.05}$

$\Rightarrow \dfrac{0.06^3 - 0.05^3}{0.06^2 + 0.06 \times 0.05 + 0.05^2}$

$\Rightarrow \dfrac{a^3 - b^3}{a^2 + ab + b^2}$

$\Rightarrow \dfrac{(a-b)\left(a^2+ab+b^2\right)}{\left(a^2+ab+b^2\right)}$

$\Rightarrow a - b$

So, $a = 0.06$, $b = 0.05$

$\Rightarrow 0.06 - 0.05 \Rightarrow 0.01$

5. (c) $\dfrac{0.05 \times 0.05 \times 0.05 - 0.04 \times 0.04 \times 0.04}{0.05 \times 0.05 + 0.002 + 0.04 \times 0.04}$

$\Rightarrow \dfrac{(0.05)^3 - (0.04)^3}{0.05^2 + 0.002 + 0.04^2}$

(Description: same as above question)

$a = 0.05$, $b = 0.04$

$\Rightarrow a - b \Rightarrow 0.05 - 0.04$

$\Rightarrow 0.01$

6. (b) $\sqrt{3}, \sqrt[3]{2}, \sqrt{2}, \sqrt[3]{4}$

$\Rightarrow 3^{\frac{1}{2}}, 2^{\frac{1}{3}}, 2^{\frac{1}{2}}, 4^{\frac{1}{3}}$

(take LCM of 3 &2)

$\Rightarrow 3^{\frac{3}{6}}, 2^{\frac{2}{6}}, 2^{\frac{3}{6}}, 4^{\frac{2}{6}}$

$\Rightarrow \sqrt[6]{3^3}, \sqrt[6]{2^2}, \sqrt[6]{2^3}, \sqrt[6]{4^2}$

$\Rightarrow \sqrt[6]{27}, \sqrt[6]{4}, \sqrt[6]{8}, \sqrt[6]{16}$

$\Rightarrow \mathbf{\sqrt[3]{2}}$ (Least)

7. (a) $\sqrt[3]{4}, \sqrt[4]{6}, \sqrt[6]{15}, \sqrt[12]{245}$

$\Rightarrow 4^{\frac{1}{3}}, 6^{\frac{1}{4}}, 15^{\frac{1}{6}}, 245^{\frac{1}{12}}$

(take LCM of 3, 4, 12 & 6)

$\Rightarrow 4^{\frac{4}{12}}, 6^{\frac{3}{12}}, 15^{\frac{2}{12}}, 245^{\frac{1}{12}}$

$\Rightarrow \sqrt[12]{4^4}, \sqrt[12]{6^3}, \sqrt[12]{15^2}, \sqrt[12]{245}$

$\Rightarrow \sqrt[12]{256}, \sqrt[12]{216}, \sqrt[12]{225}, \sqrt[12]{245}$

$\Rightarrow$ Biggest = $\sqrt[3]{4}$

8. (b) $\left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4 \left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4$

$\Rightarrow \left[5^{9\times\frac{1}{6}\times\frac{1}{3}}\right]^4 \left[5^{9\times\frac{1}{3}\times\frac{1}{6}}\right]^4$

$\Rightarrow \left[5^{\frac{1}{2}}\right]^4 \left[5^{\frac{1}{2}}\right]^4$

$\Rightarrow 5^2 \times 5^2$

$\Rightarrow 5^{2+2} = 5^4$

9. (d) $3^{x+8} = 27^{2x+1}$

$3^{x+8} = (3^3)^{2x+1}$

$3^{x+8} = 3^{6x+3}$

$x + 8 = 6x + 3$

$5x = 5, \ x = 1$

10. (b) $\left(16^{\frac{3}{2}} + 16^{\frac{-3}{2}}\right) \Rightarrow \left(16^{\frac{3}{2}} + \dfrac{1}{16^{\frac{3}{2}}}\right)$

$\Rightarrow \left(4^{2\times\frac{3}{2}} + \dfrac{1}{4^{2\times\frac{3}{2}}}\right) \Rightarrow 4^3 + \dfrac{1}{4^3}$

$\Rightarrow \dfrac{4097}{64}$

11. (c) $(0.01024)^{\frac{1}{5}}$

$\Rightarrow (0.4^5)^{\frac{1}{5}}$

$\Rightarrow 0.4^{5\times\frac{1}{5}} = 0.4$

12. (a) $64^{\frac{-2}{3}} \times \left(\dfrac{1}{4}\right)^{-2}$

$\Rightarrow (4^3)^{\frac{-2}{3}} \times \left(\dfrac{1}{4}\right)^{-2}$

$\Rightarrow\ 4^{-2}\times\left(\frac{1}{4}\right)^{-2}$

$\Rightarrow\ \left(\frac{1}{4}\right)^{2}\times\left(\frac{1}{4}\right)^{-2}=\left(\frac{1}{4}\right)^{2-2}$

$\Rightarrow\ \left(\frac{1}{4}\right)^{0}=1$

13. (c) $\left(\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\right)$

$\Rightarrow\ \frac{\left(1+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{3}\right)+\left(1-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{2}\right)}$

$\Rightarrow\ \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$

$\Rightarrow\ \frac{2\sqrt{5}-2\sqrt{6}}{2}\Rightarrow\frac{2\left(\sqrt{5}-\sqrt{6}\right)}{2}$

$\Rightarrow\ \left(\sqrt{\mathbf{5}}-\sqrt{\mathbf{6}}\right)$

14. (c)

$\Rightarrow\ \left(\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$

$\Rightarrow\ \left(\frac{\left(2+\sqrt{3}\right)^2+\left(2-\sqrt{3}\right)^2}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}+\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)$

$\Rightarrow\ \left(\frac{4+3+4\sqrt{3}+4+3-4\sqrt{3}}{4-3}+\frac{\left(\sqrt{3}-1\right)^2}{3-1}\right)$

$\Rightarrow\ \left(14+\frac{3+1-2\sqrt{3}}{2}\right)$

$\Rightarrow\ 14+\frac{2\left(2-\sqrt{3}\right)}{2}$

$\Rightarrow\ 14+2-\sqrt{3}=16-\sqrt{3}$

15. (b) $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2\Rightarrow\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}-\sqrt{3}\right)^2}$

$\Rightarrow\ \frac{5+3+2\sqrt{15}}{5+3-2\sqrt{15}}$

$\Rightarrow\ \frac{8+2\sqrt{15}}{8-2\sqrt{15}}$

$\Rightarrow\ \frac{4+\sqrt{15}}{4-\sqrt{15}}$

Similarly: $\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)^2=\frac{4-\sqrt{15}}{4+\sqrt{15}}$

Thus, the expression.

$\Rightarrow\ \frac{4+\sqrt{15}}{4-\sqrt{15}}+\frac{4-\sqrt{15}}{4+\sqrt{15}}$

$\Rightarrow\ \frac{16+15+8\sqrt{15}+16+15-8\sqrt{15}}{\left(16-15\right)}$

$\Rightarrow\ 62$

16. (a) a = 7.5 and b = 2.5

$\Rightarrow$ a × a +2ab + b × b

$\Rightarrow$ $a^2+2ab+b^2$

$\Rightarrow$ $(a+b)^2=(7.5+2.5)^2$

$\Rightarrow$ $(10)^2\Rightarrow 100$

17. (a) $\left(\frac{8}{125}\right)^{\frac{-4}{3}}$

$\Rightarrow\ \left(\frac{125}{8}\right)^{\frac{4}{3}}\Rightarrow\left[\left(\frac{5}{2}\right)^3\right]^{\frac{4}{3}}$

$\Rightarrow\ \left(\frac{5}{2}\right)^4\Rightarrow\frac{625}{16}$

18. (c) $\sqrt{\frac{\left[\left(\sqrt{12}-\sqrt{8}\right)\left(\sqrt{3}+\sqrt{2}\right)\right]}{5+\sqrt{24}}}$

$\Rightarrow\ \sqrt{\frac{\sqrt{36}+\sqrt{24}-\sqrt{24}-\sqrt{16}}{5+\sqrt{24}}}$

$\Rightarrow\ \sqrt{\frac{6-4}{5+\sqrt{24}}}$

$\Rightarrow\ \sqrt{\frac{2}{5+\sqrt{24}}\times\frac{5-\sqrt{24}}{5-\sqrt{24}}}$

$\Rightarrow\ \sqrt{\frac{2\left(5-\sqrt{24}\right)}{25-24}}$

$\Rightarrow\ \sqrt{2\left(5-2\sqrt{6}\right)}$

$\Rightarrow\ \sqrt{2\left(\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2-2\sqrt{3}\times\sqrt{2}\right)}$

$\Rightarrow\ \sqrt{2\left(\sqrt{3}-\sqrt{2}\right)^2}$

$\Rightarrow\ \sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)$

$\Rightarrow\ \sqrt{6}-2$

19. (c) $\left[64^{\frac{2}{3}}\times 2^{-2}\div 8^{0}\right]^{\frac{1}{2}}$

$\Rightarrow\ \left((4)^{3\times\frac{2}{3}}\times\left(\frac{1}{2}\right)^2\div 1\right)^{\frac{1}{2}}$

$\Rightarrow\ \left(4^2\times\frac{1}{4}\div 1\right)^{\frac{1}{2}}$

$\Rightarrow\ \left(16\times\frac{1}{4}\right)^{\frac{1}{2}}=\sqrt{4}=2$

20. (a) $\frac{1}{\sqrt{\left(12-\sqrt{140}\right)}}-\frac{1}{\sqrt{\left(8-\sqrt{60}\right)}}-\frac{2}{\sqrt{\left(10+\sqrt{84}\right)}}$

$\Rightarrow\ \frac{1}{\sqrt{12-\sqrt{4\times 35}}}-\frac{1}{\sqrt{8-\sqrt{4\times 15}}}-\frac{2}{\sqrt{10+\sqrt{4\times 21}}}$

$\Rightarrow\ \frac{1}{\sqrt{12-2\sqrt{35}}}-\frac{1}{\sqrt{8-2\sqrt{15}}}-\frac{2}{\sqrt{10+2\sqrt{21}}}$

$\Rightarrow\ \frac{1}{\sqrt{\left(\sqrt{7}\right)^2+\left(\sqrt{5}\right)^2-2\sqrt{7}\sqrt{5}}}-\frac{1}{\sqrt{\left(5\right)^2+\left(\sqrt{3}\right)^2-2\sqrt{5}\sqrt{3}}}-\frac{2}{\sqrt{\left(\sqrt{7}\right)^2+\left(\sqrt{3}\right)^2+2\sqrt{7}\sqrt{3}}}$

$\Rightarrow\ \frac{1}{\sqrt{\left(\sqrt{7}-\sqrt{5}\right)^2}}-\frac{1}{\sqrt{\left(\sqrt{5}-\sqrt{3}\right)^2}}-\frac{2}{\sqrt{\left(\sqrt{7}+\sqrt{3}\right)^2}}$

$\Rightarrow\ \frac{1}{\sqrt{7}-\sqrt{5}}-\frac{1}{\sqrt{5}-\sqrt{3}}-\frac{2}{\sqrt{7}+\sqrt{3}}$

Rationalizing in above equation.

$$\Rightarrow \frac{1}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} - \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} - \frac{2}{\sqrt{7}+\sqrt{3}} + \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{7}+\sqrt{5}}{2} - \frac{\left(\sqrt{5}+\sqrt{3}\right)}{2} - \frac{\left(\sqrt{7}-\sqrt{3}\right)}{2}$$

$$\Rightarrow \frac{\sqrt{7}+\sqrt{5}-\sqrt{5}-\sqrt{3}-\sqrt{7}+\sqrt{3}}{2}$$

$\Rightarrow 0$

21. (c)

$(0.5)^2$, $\sqrt{0.49}$, $\sqrt[3]{0.008}$, 0.23

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

0.25 0.7 $\boxed{0.2}$ 0.23

$\downarrow$

least

22. (a)

Descending order:

$$\sqrt[12]{256} > \sqrt[12]{125} > \sqrt[12]{64} > \sqrt[12]{9}$$

$$\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$$

23. (a) $$\frac{(243)^{0.13} \times (243)^{0.07}}{7^{0.25} \times 49^{0.075} \times 343^{0.2}}$$

$$\Rightarrow \frac{243^{0.13+0.07}}{7^{0.25} \times 7^{2\times.075} \times 7^{3\times0.2}}$$

$$\Rightarrow \frac{3^{5\times0.20}}{7^{0.25+0.150+0.6}} \Rightarrow \frac{3^1}{7^1} = \frac{3}{7}$$

24. (b) $$\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$$

$$\Rightarrow \sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{4+3+2\times2\times\sqrt{3}}}}$$

$$\Rightarrow \sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{\left(2+\sqrt{3}\right)^2}}}$$

$$\Rightarrow \sqrt{-\sqrt{3}+\sqrt{3+8\left(2+\sqrt{3}\right)}}$$

$$\Rightarrow \sqrt{-\sqrt{3}+\sqrt{3+16+8\sqrt{3}}}$$

$$\Rightarrow \sqrt{-\sqrt{3}+\sqrt{\left(\sqrt{3}\right)^2+(4)^2+2\times4\times\sqrt{3}}}$$

$$\Rightarrow \sqrt{-\sqrt{3}+\sqrt{\left(4+\sqrt{3}\right)^2}}$$

$$\Rightarrow \sqrt{-\sqrt{3}+4+\sqrt{3}}$$

$\Rightarrow \sqrt{4} = 2$

25. (b) $\sqrt{\sqrt[3]{0.004096}}$

$\Rightarrow \sqrt{0.16}$ $(16^3 = 4096)$

$\Rightarrow \sqrt{0.4 \times 0.4}$

$\Rightarrow 0.4$

26. (a)

$$\frac{2.3 \times 2.3 \times 2.3 - 1}{2.3 \times 2.3 + 2.3 + 1}$$

a = 2.3

b = 1

$$\Rightarrow \frac{a^3-b^3}{a^2+ab+b^2}$$

$$\Rightarrow \frac{(a-b)(a^2+ab+b^2)}{(a^2+ab+b^2)}$$

$\Rightarrow 2.3 - 1 = 1.3$

27. (b)

$(2.89)^{0.5} \rightarrow 2.89^{1/2} \rightarrow \sqrt{2.89} \rightarrow 1.7$

$2-(0.5)^2 \rightarrow 2-0.25 \rightarrow 1.75$

$\sqrt{3} \rightarrow 1.732$

$\sqrt[3]{0.008} \rightarrow 0.2$

Assending order :

$0.2 < 1.7 < 1.732 < 1.75$

$\sqrt[3]{0.008} < 2.89^{0.5} < \sqrt{3} < 2-(0.5)^2$

28. (c) $x = \sqrt{2+\sqrt{2+\sqrt{2+}}}$ .....

$x^2 = 2 + \sqrt{2+\sqrt{2+}}$ .....

$x^2 = 2 + x$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x+1)(x-2) = 0$

$x = 2$

Shortcut Method

When the question is in this form i.e $\sqrt{x+\sqrt{x+\sqrt{x}}}$ ....

Then factor the x into $n1$, $n2$ (mi.diff.) $n1 > n2$

So n1 is answer $\sqrt{2+\sqrt{2+2}}$

2×1

29. (c) $\left(\sqrt[3]{3.5}+\sqrt[3]{2.5}\right)$

$$\left\{\left(\sqrt[3]{3.5}\right)^2 - \sqrt[3]{8.75} + \left(\sqrt[3]{2.5}\right)^2\right\}$$

$x = \sqrt[3]{3.5}$

$y = \sqrt[3]{2.5}$

$\Rightarrow (x+y)(x^2-xy+y^2)$

$\Rightarrow x^3 + y^3$

$\Rightarrow \left(\sqrt[3]{3.5}\right)^3 + \left(\sqrt[3]{2.5}\right)^3$

$\Rightarrow 3.5 + 2.5 = 6$

30. (d) $$\left(3+2\sqrt{2}\right)^{-3} + \left(3-2\sqrt{2}\right)^{-3}$$

$$\Rightarrow \left(\frac{1}{3+2\sqrt{2}}\right)^3 + \left(\frac{1}{3-2\sqrt{2}}\right)^3$$

$$\Rightarrow \left(\frac{1}{\left(3+2\sqrt{2}\right)} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}\right)^3 + \left(\frac{1}{\left(3+2\sqrt{2}\right)} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}\right)^3$$

$$\Rightarrow \left(\frac{3-2\sqrt{2}}{9-8}\right)^3 + \left(\frac{3+2\sqrt{2}}{9-8}\right)^3$$

$$\Rightarrow \left(3-2\sqrt{2}\right)^3 + \left(3+2\sqrt{2}\right)^3$$

$a = 3 - 2\sqrt{2}$

$b = 3 + 2\sqrt{2}$

$a^3 + b^3 \Rightarrow (a+b)(a^2+b^2-ab)$

$\Rightarrow (3-2\sqrt{2} + 3+2\sqrt{2})(17+17-1)$

$\Rightarrow (6)(33)$

$\Rightarrow 198$

31. (a) $$\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} - \frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} + \frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} \times \frac{\left(\sqrt{3}-\sqrt{2}\right)}{\sqrt{3}-\sqrt{2}} - \left(\frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}\right) + \frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}} \times \frac{\left(\sqrt{5}-\sqrt{3}\right)}{\sqrt{5}-\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{15}-\sqrt{10}}{3-2} - \frac{3\sqrt{3}\left(\sqrt{5}-\sqrt{2}\right)}{5-2} + \frac{2\sqrt{2}\left(\sqrt{5}-\sqrt{3}\right)}{5-3}$$

$= \sqrt{15} - \sqrt{10} - \left(\sqrt{15}-\sqrt{6}\right) + \sqrt{10} - \sqrt{6}$

$\Rightarrow \sqrt{15} - \sqrt{10} - \sqrt{15} + \sqrt{6} + \sqrt{10} - \sqrt{6}$

$\Rightarrow 0$

32. (c) $$\frac{0.96^3 - 0.1^3}{0.96^2 + 0.096 + 0.1^2}$$

$\Rightarrow a = 0.96$

$\Rightarrow b = 0.1$

$$\Rightarrow \frac{a^3-b^3}{a^2+ab+b^2}$$

$\Rightarrow \dfrac{(a-b)(a^2+b^2+ab)}{(a^2+ab+b^2)}$

$\Rightarrow$ a – b

$\Rightarrow$ 0.96 – 0.1

= 0.86

33. (c) $4+\sqrt{7}$

$\Rightarrow \dfrac{8+2\sqrt{7}}{2}$

$\Rightarrow \dfrac{(\sqrt{7})^2+1+2.\sqrt{7}.1}{2}$

$\Rightarrow \dfrac{(\sqrt{7}+1)^2}{(\sqrt{2})^2} \Rightarrow \left\{\dfrac{1}{\sqrt{2}}(\sqrt{7}+1)\right\}^2$

34. (d) $\dfrac{2}{\sqrt{7}+\sqrt{5}}+\dfrac{7}{\sqrt{12}-\sqrt{5}}-\dfrac{5}{\sqrt{12}-\sqrt{7}}$

$\Rightarrow \dfrac{2}{\sqrt{7}+\sqrt{5}}\times\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}+$

$\dfrac{7}{\sqrt{12}-\sqrt{5}}\times\dfrac{(\sqrt{12}+\sqrt{5})}{(\sqrt{12}+\sqrt{5})}-$

$\left(\dfrac{5}{\sqrt{12}-\sqrt{7}}\times\dfrac{\sqrt{12}+\sqrt{7}}{\sqrt{12}+\sqrt{7}}\right)$

$\Rightarrow \dfrac{2(\sqrt{7}-\sqrt{5})}{2}+\dfrac{7(\sqrt{12}+\sqrt{5})}{7}-$

$\left(\dfrac{5(\sqrt{12}+\sqrt{7})}{5}\right)$

$\Rightarrow \sqrt{7}-\sqrt{5}+\sqrt{12}+\sqrt{5}-\sqrt{12}-\sqrt{7}$

$\Rightarrow$ 0

35. (c) $\dfrac{1}{\sqrt{3}+\sqrt{4}}$

$\Rightarrow \dfrac{1}{\sqrt{4}+\sqrt{3}}\times\dfrac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}}$

$\Rightarrow \dfrac{\sqrt{4}-\sqrt{3}}{1}$

$\Rightarrow \sqrt{4}-\sqrt{3}$

Similarly $\Rightarrow \dfrac{1}{\sqrt{4}+\sqrt{5}}=\sqrt{5}-\sqrt{4}$

$\Rightarrow \dfrac{1}{\sqrt{5}+\sqrt{6}} \Rightarrow \sqrt{6}-\sqrt{5}$

$\Rightarrow \dfrac{1}{\sqrt{6}+\sqrt{7}} \Rightarrow \sqrt{7}-\sqrt{6}$

$\Rightarrow \dfrac{1}{\sqrt{7}+\sqrt{8}}=\sqrt{8}-\sqrt{7}$

$\Rightarrow \dfrac{1}{\sqrt{8}-\sqrt{9}} \Rightarrow \sqrt{9}-\sqrt{8}$

Now put values

$\Rightarrow \sqrt{4}-\sqrt{3}+\sqrt{5}-\sqrt{4}+\sqrt{6}-\sqrt{5}+\sqrt{7}-$
$\sqrt{6}+\sqrt{8}-\sqrt{7}+\sqrt{9}-\sqrt{8}$

$\Rightarrow \sqrt{9}-\sqrt{3} \Rightarrow 3-\sqrt{3}$

36. (d) $\dfrac{1}{\sqrt{100}-\sqrt{99}}\times\dfrac{\sqrt{100}+\sqrt{99}}{\sqrt{100}+\sqrt{99}}$

$\Rightarrow \dfrac{\sqrt{100}+\sqrt{99}}{1} \Rightarrow \sqrt{100}+\sqrt{99}$

Similarly

$\Rightarrow \dfrac{1}{\sqrt{99}-\sqrt{98}} \Rightarrow \sqrt{99}+\sqrt{98}$

$\Rightarrow \dfrac{1}{\sqrt{98}-\sqrt{97}} \Rightarrow \sqrt{98}+\sqrt{97}$ ....and soon

Now : expression:

$\Rightarrow$

$\sqrt{100}+\sqrt{99}-\sqrt{99}-\sqrt{98}+\sqrt{98}+\sqrt{97}$

.. $+\sqrt{2}+1$

$\Rightarrow \sqrt{100}+1 \quad \Rightarrow 10+1=11$

37. (c) $\left[\dfrac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}+\dfrac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}\right]$

$\Rightarrow \dfrac{1}{(\sqrt{2}+\sqrt{3})-(\sqrt{5})}\times\dfrac{(\sqrt{2}+\sqrt{3})+(\sqrt{5})}{(\sqrt{2}+\sqrt{3})+(\sqrt{5})}$

$\Rightarrow \dfrac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{2+3+2\sqrt{6}-5} \Rightarrow \dfrac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{2\sqrt{6}}$

Similarly

$\Rightarrow \dfrac{1}{(\sqrt{2}-\sqrt{3}-(\sqrt{5}))}+\dfrac{(\sqrt{2}-\sqrt{3})+(\sqrt{5})}{(\sqrt{2}-\sqrt{3})+\sqrt{5}}$

$\Rightarrow \dfrac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{-2\sqrt{6}}$

Now put the value in question

$\Rightarrow \dfrac{(\sqrt{2}+\sqrt{3}+\sqrt{5})}{(2\sqrt{6})}-\dfrac{(\sqrt{2}-\sqrt{3}+\sqrt{5})}{(2\sqrt{6})}$

$\Rightarrow \dfrac{\sqrt{2}+\sqrt{3}+\sqrt{5}-\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}$

$\Rightarrow \dfrac{2\sqrt{3}}{2\sqrt{6}} \Rightarrow \dfrac{1}{\sqrt{2}}$

38. (a) $\left\{(-2)^{(-2)}\right\}^{-2}$

$\Rightarrow \dfrac{1}{\left\{(-2)^{(-2)}\right\}^2} \Rightarrow \dfrac{1}{(-2)^{-4}}$

$\Rightarrow (-2)^4 = 16$

39. (d) $(\sqrt{19}-\sqrt{17}) \Rightarrow (\sqrt{19}-\sqrt{17})\times$

$\dfrac{(\sqrt{19}+\sqrt{17})}{\sqrt{19}+\sqrt{17}} \Rightarrow \dfrac{19-17}{\sqrt{19}+\sqrt{17}}=\dfrac{2}{\sqrt{19}+\sqrt{17}}$

Similarly $(\sqrt{13}-\sqrt{11}) \Rightarrow \dfrac{2}{\sqrt{13}+\sqrt{11}}$

$\boxed{(\sqrt{5}-\sqrt{3})} \Rightarrow \boxed{\dfrac{2}{\sqrt{5}+\sqrt{3}}}$

Largest + (Because, Same Numerator is dividided by Smallest denominator)

40. (a) $x=\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \Rightarrow \dfrac{(\sqrt{5}+\sqrt{3})^2}{2}$

$\Rightarrow$ Similarly

$y=\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \Rightarrow \dfrac{(\sqrt{5}-\sqrt{3})^2}{2}$

$\Rightarrow$ x + y

$\Rightarrow \dfrac{5+3+2\sqrt{15}+5+3-2\sqrt{15}}{2}$

$\Rightarrow \dfrac{16}{2}=8$

41. (d) 0.75 = a, 0.25 = b

$\Rightarrow$ a × a – 2 × a × b + b × b

$\Rightarrow$ $a^2 - 2ab + b^2 \Rightarrow (a-b)^2$

$\Rightarrow$ $(0.75 - 0.25)^2$

$\Rightarrow$ $(0.50)^2 = 0.2500$

42. (b) $3+\dfrac{1}{\sqrt{3}}+\dfrac{1}{3+\sqrt{3}}+\dfrac{1}{\sqrt{3}-3}$

$\Rightarrow 3+\dfrac{1}{\sqrt{3}}+\dfrac{1}{3+\sqrt{3}}-\dfrac{1}{3-\sqrt{3}}$

$\Rightarrow 3+\dfrac{1}{\sqrt{3}}+\left[\dfrac{3-\sqrt{3}-3-\sqrt{3}}{9-3}\right]$

$\Rightarrow 3+\dfrac{1}{\sqrt{3}}-\dfrac{\sqrt{3}}{3} \Rightarrow 3+\dfrac{1}{\sqrt{3}}-\dfrac{1}{\sqrt{3}}=3$

43. (d) $\left[8-\left(\frac{4^{9/4}\sqrt{2\times2^2}}{2\sqrt{2^{-2}}}\right)^{\frac12}\right]$

⇒ $\left[8-\left(\frac{2^{2\times 9/4}\sqrt{2^{1+2}}}{2\sqrt{\frac14}}\right)^{1/2}\right]$

⇒ $\left[8-\left(\frac{2^{9/2}.2^{3/2}}{2\times\frac12}\right)^{1/2}\right]$

⇒ $\left[8-\left(2^{12/2}\right)^{1/2}\right]$

⇒ $\left[8-\left(2^{6\times\frac12}\right)\right]$

⇒ [8 – 8] = 0 Ans.

44. (c) $\frac{3\sqrt2}{\sqrt6+\sqrt3}-\frac{2\sqrt6}{\sqrt3+1}+\frac{2\sqrt3}{\sqrt6+2}$

⇒ $\left(\frac{3\sqrt2}{\sqrt6+\sqrt3}\times\frac{(\sqrt6-\sqrt3)}{\sqrt6-\sqrt3}\right)-\left(\frac{2\sqrt6}{\sqrt3+1}\times\frac{\sqrt3-1}{\sqrt3-1}\right)$

$+\left(\frac{2\sqrt3}{\sqrt6+2}\times\frac{\sqrt6-2}{\sqrt6-2}\right)$

⇒ $\frac{3\sqrt2\left(\sqrt6-\sqrt3\right)}{3}-\left(\frac{2\sqrt6\left(\sqrt3-1\right)}{2}\right)$

$+\frac{2\sqrt3\times\left(\sqrt6-2\right)}{2}$

⇒ $\sqrt{12}-\sqrt6-\sqrt{18}+\sqrt6+\sqrt{18}-2\sqrt3$

⇒ $\sqrt{12}-2\sqrt3$

⇒ $2\sqrt3-2\sqrt3=0$

45.(a)

$\sqrt[3]{9}$	$\sqrt{3}$	$\sqrt[4]{16}$	$\sqrt[6]{80}$
↓	↓	↓	↓
$9^{1/3}$	$3^{1/2}$	$16^{1/4}$	$80^{1/6}$
↓	↓	↓	↓
$9^{4/12}$	$3^{6/12}$	$16^{3/12}$	$80^{2/12}$
↓	↓	↓	↓
$\sqrt[12]{9^4}$	$\sqrt[12]{27^2}$	$\sqrt[12]{16^3}$	$\sqrt[12]{80^2}$

Square of 81 is largest . So Ans $\sqrt[3]{9}$

46. (d) $\frac{3+\sqrt6}{5\sqrt3-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$

⇒ $\frac{3+\sqrt6}{5\sqrt3-2\times2\sqrt3-4\sqrt2+5\sqrt2}$

⇒ $\frac{3+\sqrt6}{5\sqrt3-4\sqrt3-4\sqrt2+5\sqrt2}$

⇒ $\frac{3+\sqrt6}{\sqrt3+\sqrt2}$ ⇒ $\frac{\sqrt3\left(\sqrt3+\sqrt2\right)}{\sqrt3+\sqrt2}$

⇒ $\sqrt3$ = 1.732

47. (b) $\sqrt{12+\sqrt{12+\sqrt{12+}}}$.......

4 × 3

48. (a) $a=\frac{\sqrt3}{2}$

⇒ $a+1=\frac{\sqrt3}{2}+1$

⇒ $\frac{\sqrt3+2}{2}$

⇒ $\frac{4+2\sqrt3}{4}$ ⇒ $\frac{\left(\sqrt3+1\right)^2}{4}$

$a+1=\frac{\left(\sqrt3+1\right)^2}{4}$

⇒ $\sqrt{a+1}=\sqrt{\frac{\left(\sqrt3+1\right)^2}{4}}$

$\sqrt{1+a}=\frac{\sqrt3+1}{2}$

Similarly,

$\sqrt{1-a}=\frac{\sqrt3-1}{2}$

put values :

$\frac{\sqrt3+1}{2}+\frac{\sqrt3-1}{2}$

⇒ $\frac{\sqrt3+1+\sqrt3-1}{2}$

$=\frac{2\sqrt3}{2}$

⇒ $\sqrt3$

49. (b) $a+b=\frac{\sqrt5+1}{\sqrt5-1}+\frac{\sqrt5-1}{\sqrt5+1}$

⇒ $\frac{\left(\sqrt5+1\right)^2+\left(\sqrt5-1\right)^2}{\left(\sqrt5-1\right)\left(\sqrt5+1\right)}$

⇒ $\frac{2\left[\left(\sqrt5\right)^2+1\right]}{5-1}=\frac{2\left(5+1\right)}{4}=3$

$a.b=\frac{\sqrt5+1}{\sqrt5-1}\times\frac{\sqrt5-1}{\sqrt5+1}=1$

put value in expression

⇒ $\frac{a^2+ab+b^2}{a^2-ab+b^2}=\frac{(a+b)^2-ab}{(a+b)^2-3ab}$

$=\frac{3^2-1}{3^2-3}=\frac{9-1}{9-3}=\frac43$

50. (c) $\left[\frac{2}{\sqrt5+\sqrt3}-\frac{3}{\sqrt6-\sqrt3}+\frac{1}{\sqrt6+\sqrt5}\right]$

⇒ $\frac{2}{\sqrt5+\sqrt3}\times\frac{\sqrt5-\sqrt3}{\sqrt5-\sqrt3}-\frac{3}{\sqrt6-\sqrt3}\times$

$\frac{\sqrt6+\sqrt3}{\sqrt6+\sqrt3}+\frac{1}{\sqrt6+\sqrt5}\times\frac{\sqrt6-\sqrt5}{\sqrt6-\sqrt5}$

⇒ $\frac{2\left(\sqrt5-\sqrt3\right)}{5-3}-\frac{3\left(\sqrt6+\sqrt3\right)}{6-3}+\frac{\sqrt6-\sqrt5}{6-5}$

⇒ $\sqrt5-\sqrt3-\sqrt6-\sqrt3+\sqrt6-\sqrt5$

⇒ $-2\sqrt3$

51. (c) $\left(\sqrt2+\sqrt{7-2\sqrt{10}}\right)$

⇒ $\sqrt2+\sqrt{(5)^2+\left(\sqrt2\right)^2-2\sqrt5\sqrt2}$

⇒ $\sqrt2+\sqrt{\left(\sqrt5-\sqrt2\right)^2}$

⇒ $\sqrt2+\sqrt5-\sqrt2$ ⇒ $\sqrt5$

52. (a) x = $1+\sqrt2+\sqrt3$

⇒ $x+\frac{1}{x-1}$

⇒ $1+\sqrt2+\sqrt3+\frac{1}{1+\sqrt2+\sqrt3-1}$

⇒ $1+\sqrt2+\sqrt3+\frac{\sqrt3-\sqrt2}{3-2}$

⇒ $1+2\sqrt3$

53. (d) $m^n = 121 = 11^2$

$\Rightarrow m = 11$

$\Rightarrow n = 2$

$\Rightarrow (m-1)^{n+1}$

$\Rightarrow (11-1)^{2+1} \Rightarrow 10^3 \Rightarrow 1000$

54. (a) $\frac{1}{3-\sqrt{8}}$

$\Rightarrow \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} \Rightarrow \frac{3+\sqrt{8}}{9-8}$

$\Rightarrow 3+\sqrt{8}$

Similarly,

$\Rightarrow \frac{1}{\sqrt{8}-\sqrt{7}} = \sqrt{8}+\sqrt{7}$

$\Rightarrow \frac{1}{\sqrt{7}-\sqrt{6}} \Rightarrow \sqrt{7}+\sqrt{6}$

$\Rightarrow \frac{1}{\sqrt{6}-\sqrt{5}} = \sqrt{6}+\sqrt{5}$

$\Rightarrow \frac{1}{\sqrt{5}-2} = \sqrt{5}+2$

Put value in question

$\Rightarrow \left(3+\sqrt{8}\right)-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)$

$-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+2\right)$

$\Rightarrow 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$

$\Rightarrow 3 + 2 = 5$

55. (d)

$\frac{2}{\sqrt{6}+2}+\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{8}-\sqrt{7}}+2-2\sqrt{2}$

$\Rightarrow \frac{2}{\sqrt{6}+2}\times\frac{\sqrt{6}-2}{\sqrt{6}-2}+\frac{1}{\sqrt{7}+\sqrt{6}}\times\frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}-\sqrt{6}}$

$+\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}+2-2\sqrt{2}$

$\Rightarrow \frac{2\times\left(\sqrt{6}-2\right)}{6-4}+\frac{\left(\sqrt{7}-\sqrt{6}\right)}{7-6}+\frac{\sqrt{8}+\sqrt{7}}{8-7}$

$+2-2\sqrt{2}$

$\Rightarrow \sqrt{6}-2+\sqrt{7}-\sqrt{6}+\sqrt{8}+\sqrt{7}+2-2\sqrt{2}$

$\Rightarrow \sqrt{6}-2+\sqrt{7}-\sqrt{6}+2\sqrt{2}+\sqrt{7}+2-2\sqrt{2}$

$\Rightarrow 2\sqrt{7}$

56. (a)

$\left[\left\{\left(-\frac{1}{2}\right)^2\right\}^{-2}\right]^{-1}$

$\Rightarrow \left\{\left(-\frac{1}{2}\right)^2\right\}^{-2\times-1}$

$\Rightarrow \left(-\frac{1}{2}\right)^{2\times2} \Rightarrow \left(-\frac{1}{2}\right)^4 \Rightarrow \frac{1}{16}$

57. (b) $2^{60} \rightarrow (2^5)^{12} \rightarrow (32)^{12}$

$\Rightarrow 3^{48} \rightarrow (3^4)^{12} \rightarrow (81)^{12}$ (Greatest)

$\Rightarrow 4^{36} \rightarrow (4^3)^{12} \rightarrow (64)^{12}$

$\Rightarrow 5^{24} \rightarrow (5^2)^{12} \rightarrow (25)^{12}$

58. (b) $\sqrt{3\sqrt{3\sqrt{3}}}$ ....

Shortcut method

⇒ When the question in from

$\Rightarrow \sqrt{n\sqrt{n\sqrt{n}}}$ ....∞

⇒ So n is answer

⇒ 3

59. (d)

$\sqrt{0.09}$	$\sqrt[3]{0.064}$	0.5	3/5
↓	↓	↓	↓
0.3	0.4	0.5	**0.6**

60. (b)

0.16	$\sqrt{0.16}$	$(0.16)^2$	0.04
↓	↓	↓	↓
0.16	**0.40**	0.0256	0.04

61. (d)

$\sqrt[2]{8}$	$\sqrt[4]{13}$	$\sqrt[5]{16}$	$\sqrt[10]{41}$
↓	↓	↓	↓
$8^{1/2}$	$13^{1/4}$	$16^{1/5}$	$41^{1/10}$
$8^{10/20}$	$13^{5/20}$	$16^{4/20}$	$41^{2/20}$
↓	↓	↓	↓
$\sqrt[20]{8^{10}}$	$\sqrt[20]{13^5}$	$\sqrt[20]{16^4}$	$\sqrt[20]{41^2}$
↓	↓	↓	↓
$\sqrt[20]{64^5}$	$\sqrt[20]{13^5}$	$\sqrt[20]{16^4}$	$\sqrt[20]{41^2}$

Greatest = $\sqrt[2]{8}$

62. (a) $2^x = 3^y = 6^{-z} = k$

$\Rightarrow 2 = k^{1/x}$; $3 = k^{1/y}$ = 6 = $k^{-1/z}$

$\therefore$ 2 × 3 = 6

$k^{1/x} \times k^{1/y} = k^{-1/z}$

$\frac{1}{x}+\frac{1}{y}=-\frac{1}{z}$

$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

63. (b) $2\sqrt[3]{40} \Rightarrow 2\times\sqrt[3]{2\times2\times2\times5}$

$\Rightarrow 2\times2\sqrt[3]{5}$

$\Rightarrow 4\sqrt[3]{5}$

$\Rightarrow 4\sqrt[3]{320}$

$\Rightarrow 4\times\sqrt[3]{4\times4\times4\times5}$

$\Rightarrow 4\times4\sqrt[3]{5}$

$\Rightarrow 16\sqrt[3]{5}$

$\Rightarrow 3\sqrt[3]{625}$

$\Rightarrow 3\times\sqrt[3]{5\times5\times5\times5} \Rightarrow 3\times5\sqrt[3]{5}$

$\Rightarrow 15\sqrt[3]{5}$

Now put the value in question

$\Rightarrow 4\sqrt[3]{5}-16\sqrt[3]{5}+15\sqrt[3]{5}-3\sqrt[3]{5}$

$\Rightarrow 19\sqrt[3]{5}-19\sqrt[3]{5}$

$\Rightarrow 0$

64. (c) $\sqrt{40+\sqrt{9\sqrt{81}}}$

$\Rightarrow \sqrt{40+\sqrt{9\times9}}$

$\Rightarrow \sqrt{40+9} \Rightarrow \sqrt{49} \Rightarrow 7$

65. (b) $\frac{\left(x-\sqrt{24}\right)\left(\sqrt{75}+\sqrt{50}\right)}{\sqrt{75}-\sqrt{50}} = 1$

$\Rightarrow \left(x-\sqrt{24}\right) = \frac{\sqrt{75}-\sqrt{50}}{\sqrt{75}+\sqrt{50}}$

$\Rightarrow \left(x-\sqrt{24}\right) = \frac{\left(\sqrt{75}-\sqrt{50}\right)^2}{75-50}$

$\Rightarrow \left(x-\sqrt{24}\right) = \frac{75+50-2\sqrt{75}\sqrt{50}}{25}$

$\Rightarrow \left(x-\sqrt{24}\right) = \frac{125-2\times5\sqrt{3}\times5\sqrt{2}}{25}$

$\Rightarrow \left(x-\sqrt{24}\right) = \frac{125-50\sqrt{6}}{25}$

$\Rightarrow \left(x-\sqrt{24}\right) = \frac{25\left(5-2\sqrt{6}\right)}{25}$

$\Rightarrow x-2\sqrt{6} = 5-2\sqrt{6}$

$\Rightarrow x = 5$

66. (c) $\sqrt{20}+\sqrt{12}+\sqrt[3]{729}-\frac{4}{\sqrt{5}-\sqrt{3}}-\sqrt{81}$

$\Rightarrow 2\sqrt{5}+2\sqrt{3}+9-\left(\frac{4}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)-9$

$\Rightarrow 2\sqrt{5}+2\sqrt{3}+9-\left(\frac{4\left(\sqrt{5}+\sqrt{3}\right)}{2}\right)-9$

$\Rightarrow 2\sqrt{5}+2\sqrt{3}+9-2\sqrt{5}-2\sqrt{3}-9 \Rightarrow 0$

67. (a) $a\sqrt{2}+b\sqrt{3}$

$= \sqrt{98}+\sqrt{108}-\sqrt{48}-\sqrt{72}$

$\Rightarrow \sqrt{7\times7\times2}+\sqrt{3\times3\times3\times2\times2}$

$-\sqrt{2\times2\times2\times2\times3}-\sqrt{3\times3\times2\times2\times2}$

⇒ $7\sqrt{2} + 6\sqrt{3} - 4\sqrt{3} - 6\sqrt{2}$

$a\sqrt{2} + b\sqrt{3} = 1\sqrt{2} + 2\sqrt{3}$

a = 1

b = 2

68. (a) $\sqrt[3]{a} = \sqrt[3]{26} + \sqrt[3]{7} + \sqrt[3]{63}$

Take round figure

⇒ $\sqrt[3]{a} < \sqrt[3]{27} + \sqrt[3]{8} + \sqrt[3]{64}$

⇒ $\sqrt[3]{a} < 3 + 2 + 4$

⇒ $\sqrt[3]{a} < 9$

⇒ $a < 9^3$

⇒ $a < 729$

Option A is answer

69. (d) $2 + \frac{6}{\sqrt{3}} + \frac{1}{2+\sqrt{3}} + \frac{1}{\sqrt{3}-2}$

⇒ $2 + \frac{2\times 3\sqrt{3}}{\sqrt{3}\times\sqrt{3}} + \frac{1}{2+\sqrt{3}} - \frac{1}{2-\sqrt{3}}$

⇒ $2 + 2\sqrt{3} + \left(\frac{(2-\sqrt{3})-(2+\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}\right)$

⇒ $2 + 2\sqrt{3} + \left(\frac{2-\sqrt{3}-2-\sqrt{3}}{4-3}\right)$

⇒ $2 + 2\sqrt{3} - 2\sqrt{3} \Rightarrow 2$

70. (c) $\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}} = A + \sqrt{B}$

⇒ $\sqrt{7+4\sqrt{3}}$

⇒ $\sqrt{2^2 + (\sqrt{3})^2 + 2\times 2\sqrt{3}}$

⇒ $\sqrt{(2+\sqrt{3})^2}$

⇒ $(2+\sqrt{3})$

⇒ $\frac{4+3\sqrt{3}}{2+\sqrt{3}} = A+\sqrt{B}$

⇒ $\frac{4+3\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = A+\sqrt{B}$

⇒ $\frac{(4+3\sqrt{3})(2-\sqrt{3})}{4-3} = A+\sqrt{B}$

⇒ $8 - 4\sqrt{3} + 6\sqrt{3} - 9 = A+\sqrt{B}$

⇒ $2\sqrt{3} - 1 = A+\sqrt{B}$

$A = -1$ and $\sqrt{B} = 2\sqrt{3}$

$B = 2\sqrt{3}\times 2\sqrt{3} = 12$

$B - A = 12 - (-1) = 13$

71. (b) $2^{250} \to (2^5)^{50} \to (32)^{50}$

⇒ $3^{150} \to (3^3)^{50} \to (27)^{50}$

⇒ $5^{100} \to (5^2)^{50} \to \mathbf{(25)^{50}}$ **(Smallest)**

⇒ $4^{200} \to (4^4)^{50} \to (256)^{50}$

72. (c) $\sqrt{30+\sqrt{30+\sqrt{30}}}$ ....

6 × 5

73. (a) $x = \sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4}}}}$ ....

⇒ Squaring both sides

⇒ $x^2 = 2\sqrt[3]{4\sqrt{2\sqrt[3]{4}}}$ ......

Now cubing both sides

$x^6 = 8 \times 4x$

⇒ $x^5 = 32$

⇒ $x^5 = 2^5$

⇒ $x = 2$

74. (b) a = 55, b = 17

c = – 72

a + b + c = 55 + 17 – 72 = 0

∴ $a^3 + b^3 + c^3 - 3abc = 0$

If (a + b + c) = 0

answer = 0

75. (b) $\frac{243^{n/5}\times 3^{2n+1}}{9^n \times 3^{n-1}}$

⇒ $\frac{3^{5\times n/5}\times 3^{2n+1}}{3^{2n}\times 3^{n-1}}$

⇒ $\frac{3^{n}\times 3^{2n+1}}{3^{2n}\times 3^{n-1}}$

⇒ $\frac{3^{n+2n+1}}{3^{2n+n-1}}$ ⇒ $\frac{3^{3n+1}}{3^{3n-1}}$

⇒ $3^{(3n+1)-(3n-1)}$ ⇒ $3^{3n+1-3n+1}$

⇒ $3^2 = 9$

76. (c) $(\sqrt{3}+1)(10+\sqrt{12})(\sqrt{12}-2)(5-\sqrt{3})$

⇒ $(\sqrt{3}+1)(10+2\sqrt{3})(2\sqrt{3}-2)(5-\sqrt{3})$

⇒ $(\sqrt{3}+1)\times 2(5+\sqrt{3})\times 2(\sqrt{3}-1)(5-\sqrt{3})$

⇒ $4(\sqrt{3}+1)(\sqrt{3}-1)(5+\sqrt{3})(5-\sqrt{3})$

⇒ $4\left[(\sqrt{3})^2 - 1^2\right]\left[(5)^2 - (\sqrt{3})^2\right]$

⇒ 4 × 2 × 22 ⇒ 176

77.(d) $2^{n-1} + 2^{n+1} = 320$

⇒ $2^{n-1}(1 + 2^2) = 320$

⇒ $2^{n-1}(1 + 2^2) = 320$

⇒ $2^{n-1} \times 5 = 320$

⇒ $2^{n-1} = \frac{320}{5} = 64$

⇒ $(2)^{n-1} = (2)^6$

⇒ n = 7

78. (a) $5\sqrt{5} \times 5^3 \div 5^{-3/2} = 5^{a+2}$

⇒ $5^1 \times 5^{1/2} \times 5^3 \div 5^{-3/2} = 5^{a+2}$

⇒ $5^{1+1/2+3-(-3/2)} = 5^{a+2}$

⇒ $5^{1+1/2+3+3/2} = 5^{a+2}$

⇒ $5^6 = 5^{a+2}$

⇒ a + 2 = 6

⇒ a = 4

79. (a) $(3+2\sqrt{2})^{-3} + (3-2\sqrt{2})^{-3}$

⇒ $\left(\frac{1}{3+2\sqrt{2}}\right)^3 + \left(\frac{1}{3-2\sqrt{2}}\right)^3$

⇒ $(3-2\sqrt{2})^3 + (3+2\sqrt{2})^3$

⇒ $2 \times 27 + 6 \times 3 \times (2\sqrt{2})^2$

⇒ 2 × 27 + 18 × 8

⇒ 54 + 144 ⇒ 198

80. (b) At the start t = 0°

L = 2 – 2°

⇒ 2 – 1 = 1 cm

81. (d) $\frac{\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}}{\sqrt[3]{8}}$

⇒ $\frac{\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{169}}}}}{2}$

⇒ $\frac{\sqrt{10+\sqrt{25+\sqrt{121}}}}{2}$

⇒ $\frac{\sqrt{10+\sqrt{36}}}{2}$

⇒ $\frac{\sqrt{16}}{2}$ ⇒ $\frac{4}{2}$ ⇒ 2

82. (d)

$\frac{\sqrt{6}+2}{\sqrt{2}+\sqrt{2+\sqrt{3}}} - \frac{\sqrt{6}-2}{\sqrt{2}-\sqrt{2-\sqrt{3}}} - \frac{2\sqrt{2}}{2+\sqrt{2}}$

⇒ $\frac{\sqrt{6}+2}{\sqrt{2}+\frac{\sqrt{3}+1}{\sqrt{2}}} - \frac{\sqrt{6}-2}{\sqrt{2}-\frac{\sqrt{3}-1}{\sqrt{2}}} - \frac{2}{\sqrt{2}+1}$

⇒ $\frac{(\sqrt{6}+2)\sqrt{2}}{2+\sqrt{3}+1} - \frac{(\sqrt{6}-2)\sqrt{2}}{2-\sqrt{3}+1} - \frac{2}{\sqrt{2}+1}$

$$= \frac{\sqrt2}{\sqrt3}\left[\frac{\sqrt6+2}{\left(\sqrt3+1\right)} - \frac{\sqrt6-2}{\left(\sqrt3-1\right)}\right] -$$

$$\frac{2}{\sqrt2+1}\times\frac{\sqrt2-1}{\sqrt2-1}$$

$$\frac{\sqrt2}{\sqrt3}\left[\frac{\sqrt{18}+2\sqrt3-\sqrt6-2-\sqrt{18}+2\sqrt3-\sqrt6+2}{3-1}\right]$$

$$-\frac{2\left(\sqrt2-1\right)}{2-1}$$

$$\Rightarrow \frac{\sqrt2}{\sqrt3}\left[\frac{2\left(-\sqrt6+2\sqrt3\right)}{2}\right]-2\left(\sqrt2-1\right)$$

$$\Rightarrow -\sqrt3\times\sqrt2\times\frac{\sqrt2}{\sqrt3}+2\sqrt3\times\frac{\sqrt2}{\sqrt3}-2\left(\sqrt2-1\right)$$

$$\Rightarrow -2+2\sqrt2-2\sqrt2+2=0$$

83. (a) $\dfrac{6^2+7^2+8^2+9^2+10^2}{\sqrt{7+4\sqrt3}-\sqrt{4+2\sqrt3}}$

$$\Rightarrow \frac{6^2+7^2+8^2+9^2+10^2}{\sqrt{(2+\sqrt3)^2}-\sqrt{(\sqrt3+1)^2}}$$

$$\Rightarrow \frac{6^2+7^2+8^2+9^2+10^2}{2+\sqrt3-\sqrt3-1}$$

$$\Rightarrow 6^2+7^2+8^2+9^2+10^2$$

$\Rightarrow$ 36+49+64+81+100 $\Rightarrow$ 330 Ans.

84. (d) $\dfrac{3x-2y}{2x+3y}=\dfrac{5}{6}$

$$18x-12y=10x+15y$$

$$8x=27y$$

$$\frac{x}{y}=\frac{27}{8}$$

$$\left[\frac{\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt[3]{x}-\sqrt[3]{y}}\right]^2$$

$$\left(\frac{\sqrt[3]{27}+\sqrt[3]{8}}{\sqrt[3]{27}-\sqrt[3]{8}}\right)^2$$

$$\Rightarrow \left(\frac{3+2}{3-2}\right)^2 = (5)^2 = 25 \text{ Ans.}$$

85. (a)

$$\frac{1}{\sqrt2+1}+\frac{1}{\sqrt3+\sqrt2}+\frac{1}{\sqrt4+\sqrt3}+\frac{1}{\sqrt5+\sqrt4}+$$

$$\frac{1}{\sqrt6+\sqrt5}+\frac{1}{\sqrt7+\sqrt6}+\frac{1}{\sqrt8+\sqrt7}+\frac{1}{\sqrt9+\sqrt8}$$

After Rationalizing

$$= \left(\sqrt2-1\right)+\left(\sqrt3-\sqrt2\right)+\left(\sqrt4-\sqrt3\right)+\left(\sqrt5-\sqrt4\right)+$$

$$\left(\sqrt6-\sqrt5\right)$$

$$\left(\sqrt7-\sqrt6\right)+\left(\sqrt8-\sqrt7\right)+\left(\sqrt9-\sqrt8\right)$$

$$= \sqrt9-1 = 3-1 = 2$$

86. (a) $\sqrt{72+\sqrt{72+\sqrt{72+\ldots.}}}$

9 × 8

87. (c) $\dfrac{1}{1+\sqrt2+\sqrt3}+\dfrac{1}{1-\sqrt2+\sqrt3}$

$$\Rightarrow \frac{1}{1+\sqrt3+\sqrt2}+\frac{1}{1+\sqrt3-\sqrt2}$$

$$\Rightarrow \frac{1+\sqrt3-\sqrt2+1+\sqrt3+\sqrt2}{\left(1+\sqrt3\right)^2-\left(\sqrt2\right)^2}$$

$$\Rightarrow \frac{2+2\sqrt3}{4+2\sqrt3-2} \Rightarrow \frac{2+2\sqrt3}{2+2\sqrt3}$$

$$\Rightarrow 1$$

88. (c) $\sqrt{6+\sqrt{6+\sqrt{6+\ldots\ldots+\infty}}}$

(2, 3) are the factor of 6.

If there is '+' in '$\sqrt{\ }$' Answer is Highest value.

If there is '–' in '$\sqrt{\ }$', Answer is lowest value.

Alternate →

$$x = \sqrt{6+\sqrt{6+\sqrt{6+\ldots+\infty}}}$$

(squaring both side)

$$x^2 = 6+\sqrt{6+\sqrt{6+\ldots\infty}}$$

$$x^2 = 6+x$$

$$\left[\because \sqrt{6+\sqrt{6+\ldots\infty}} = x\right]$$

$$x^2-x-6 = 0$$

$$x^2-3x+2x-6 = 0$$

$$x(x-3)+2(x-3) = 0$$

$$(x+2)(x-3)$$

$x \neq 2$, & $x = 3$

So, Answer is = 3

89. (b)

$$\frac{3\sqrt7}{\sqrt5+\sqrt2}-\frac{5\sqrt5}{\sqrt2+\sqrt7}+\frac{2\sqrt2}{\sqrt7+\sqrt5}$$

$$= \frac{3\sqrt7}{\sqrt5+\sqrt2}\times\frac{\sqrt5-\sqrt2}{\sqrt5-\sqrt2}-\frac{5\sqrt5}{\sqrt7+\sqrt2}\times\frac{\sqrt7-\sqrt2}{\sqrt7-\sqrt2}+\frac{2\sqrt2}{\sqrt7+\sqrt5}\times\frac{\sqrt7-\sqrt5}{\sqrt7+\sqrt5}$$

$$= \frac{3\sqrt7\left(\sqrt5-\sqrt2\right)}{\left(\sqrt5\right)^2-\left(\sqrt2\right)^2}-\frac{5\sqrt5\left(\sqrt7-\sqrt2\right)}{\left(\sqrt7\right)^2-\left(\sqrt2\right)^2}$$

$$+\frac{2\sqrt2\left(\sqrt7-\sqrt5\right)}{\left(\sqrt7\right)^2-\left(\sqrt5\right)^2}$$

$$= \sqrt{35}-\sqrt{14}-\sqrt{35}+\sqrt{10}+\sqrt{14}-\sqrt{10} = 0$$

90. (d) $11\sqrt{n} = \sqrt{112}+\sqrt{343}$

$$11\sqrt{n} = \sqrt{2\times2\times2\times2\times7}+\sqrt{7\times7\times7}$$

$$11\sqrt{n} = 4\sqrt7+7\sqrt7$$

$$11\sqrt{n} = 11\sqrt7$$

$$\sqrt{n} = \sqrt7$$

$$n = 7$$

CHAPTER 08

SIMPLIFICATION

- **Simplification**:
  In simplification an expression, we must remove the brackets strictly in the order ( ), { }, [] and then we must apply the operations:-
  Of, division, Multiplication, Addition and Subtraction.
- **Remember**:- 'BODMAS' where B stands for bracket, O for of ; D for division; M for multiplication, A for addition and S for Substraction strictly in this order.
  Note: 'Of' means multiplication.
- **Division Algorithm:-**
  Dividend = (Divisor × Quotient) + Remainder

2. **Modulus or Absolute value** :
The absolute value of a real number X is denoted by the symbol $|x|$ and is defined as -

$$|x| = \begin{cases} x, & if \ \ x > 0 \\ -x, & if \ \ x < 0 \\ 0, & if \ \ x = 0 \end{cases}$$

E.x : $|5| = 5$, $|-5| = -(-5) = 5$

Note: In multiplication and division, when both the numbers carry similar sign, we get positive sign in the result, otherwise we get negative sign in the result i.e.

$(+) \times (+)$	=	+
$(+) \times (-)$	=	–
$(-) \times (+)$	=	–
$(-) \times (-)$	=	+
$(+) \div (+)$	=	+
$(+) \div (-)$	=	–
$(-) \div (+)$	=	–
$(-) \div (-)$	=	+

Important terms

1. **Identity element of Addition:** '0' (zero) is called identity element of addition as Addition of 'O' in any number does not affect that number.
e.g. $x + 0 = x$ ($x \in Q$)

2. **Identity element of Multiplication**: '1' is called identity element of multiplication as multiplication of '1' in any number does not affect that number.
e.g. $x \times 1 = x$

3. **Inverse Element of Addition/ Negative element of Addition/ Additive Inverse:**
The number is called "Additive inverse" of a certain number, when it is added to the certain number and result becomes '0' (zero).

E.x. (i) $x + (-x) = 0$
Here $(-x)$ is Additive inverse of x.
(ii) (9) + (-9) is Additive inverse of '9'

4. **Inverse Element of Multiplication/Reciprocal Element/Multiplicative Inverse**:
The number is called "Multiplicative inverse" of a certain number, when the product of number and multiplicative inverse is 1.

E.x. $x \times \frac{1}{x} = 1$

Here, $\frac{1}{x}$ is multiplicative inverse of 'x'

TYPE – I

Ex.1. The value of $9\frac{8}{9} \times 9$ is:

Sol. $9\frac{8}{9} \times 9$

$\left(9 + \frac{8}{9}\right) \times 9$

$9 \times 9 + \frac{8}{9} \times 9$

$81 + 8 = 89$

Ex.2. The value of $\left(99\frac{7}{9}\right)9$ is:

Sol. $\left(99\frac{7}{9}\right)9$

$\left(99 + \frac{7}{9}\right) \times 9$

$99 \times 9 + \frac{7}{9} \times 9$

$(100 - 1) \times 9 + 7$

$900 - 9 + 7$

$= 900 - 2 = 898$

Ex.3. The value of $\frac{1}{5} + 99\frac{44}{45} \times 9$ is:

Sol. $\frac{1}{5} + 99\frac{44}{45} \times 9$

$\frac{1}{5} + \left(99 + \frac{44}{45}\right) \times 9$

$\frac{1}{5} + \left[\frac{99 \times 45 + 44}{45}\right] \times 9$

$\frac{1}{5} + \left[\frac{(100-1)45 + 44}{45}\right] \times 9$

$\frac{1}{5} + \frac{4500 - 45 + 44}{5}$

$\frac{1}{5} + \frac{4500 - 1}{5}$

$\frac{1}{5} + \frac{4500}{5} - \frac{1}{5} = 900$

Ex.4. The value of $\frac{1}{8} + 999\frac{791}{792} \times 99$

Sol. $\frac{1}{8} + 999\frac{791}{792} \times 99$

$= \frac{1}{8} + \left(999 + \frac{791}{792}\right)99$

$= \frac{1}{8} + 999 \times 99 + \frac{791}{792} \times 99$

$= \frac{1}{8} + (1000-1)99 + \frac{791}{8}$

$= \frac{792}{8} + 99000 - 99$

$= 99 + 99000 - 99 = 99000$

TYPE – II

Series base question:-

Ex.5. The value of

$999\frac{1}{7} + 999\frac{2}{7} + 999\frac{3}{7} + 999\frac{4}{7}$ .... $999\frac{6}{7}$

Sol. $999\frac{1}{7} + 999\frac{2}{7} + 999\frac{3}{7} + 999\frac{4}{7}$ .... $999\frac{6}{7}$

$= 999 + \frac{1}{7} + 999 + \frac{2}{7} + 999 + \frac{3}{7}$ ...... $999 + \frac{6}{7}$

$= 999 \times 6 + \frac{1}{7} + \frac{2}{7} + \frac{3}{7} \dots \frac{6}{7}$

$5994 + \frac{1+2+3+4+5+6}{7}$

$5994 + \frac{21}{7} = 5994 + 3$

$= 5997$

Ex.6. The value of $3\frac{1}{3} + 33\frac{1}{3} + \dots\dots 333333\frac{1}{3}$ is:

Sol. $3\frac{1}{3} + 33\frac{1}{3} + \dots\dots 333333\frac{1}{3}$

$= 3 + \frac{1}{3} + 33 + \frac{1}{3} \dots\dots 333333 + \frac{1}{3}$

$= 3 + 33 + 333 + 3333 + 33333 + 333333 + \frac{1}{3} \times 6$

$= 370368 + 2$

$= 370370$

Ex.7. The value of 9 + 99 + 999 + ... + 20th term is:

Sol. 9 + 99 + 999 + ... + 20th term

$= (10^1 - 1) + (10^2 - 1) + (10^3 - 1) \dots\dots (10^{20} - 1)$

$= (10^1 + 10^2 + 10^3 \dots 10^{20}) - (1 + 1 + \dots\dots 20 \text{ times})$

we use G.P formula:

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{10(10^{20} - 1)}{10 - 1} - 20$

$= \frac{10(10^{20} - 1)}{9} - 20$

Ex.8. The value of $\left[1 - \frac{1}{n+1}\right] + \left[1 - \frac{2}{n+1}\right] + \dots\dots + \left[1 - \frac{n}{n+1}\right]$ is:

Sol. $\left[1 - \frac{1}{n+1}\right] + \left[1 - \frac{2}{n+1}\right] + \dots\dots + \left[1 - \frac{n}{n+1}\right]$

$= 1 - \frac{1}{n+1} + 1 - \frac{2}{n+1} \dots 1 - \frac{n}{n+1}$

$= (1 + 1 + \dots\dots n \text{ times}) - \frac{1}{n+1}(1 + 2 + 3 \dots n)$

$\because$ Sum of n natural no.$= \frac{n(n+1)}{2}$

$= n - \frac{1}{n+1} \times \frac{n(n+1)}{2}$

$= n - \frac{n}{2} = \frac{n}{2}$

Ex.9. The value of $\left(2 - \frac{1}{3}\right) \times \left(2 - \frac{3}{5}\right) \times \left(2 - \frac{5}{7}\right) \dots \left(2 - \frac{999}{1001}\right)$

Sol. $\left(2 - \frac{1}{3}\right) \times \left(2 - \frac{3}{5}\right) \times \left(2 - \frac{5}{7}\right) \dots\dots \left(2 - \frac{999}{1001}\right)$

$= \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \dots\dots \frac{1003}{1001} = \frac{1003}{3}$

Ex.10 The value of $999\frac{92}{99} \times 99$ is:

Sol. $999\frac{92}{99} \times 99$

$\left(999 + \frac{92}{99}\right)99$

$999 \times 99 + \frac{92}{99} \times 99$

$(1000 - 1) \times 99 + 92$

$99000 - 99 + 92$

$99000 - 7 = 98993$

Ex.11 Find the value of

$\frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \dots\dots + \frac{19}{8100}$

Sol. $\frac{3}{4} + \frac{5}{36} + \frac{7}{144} \dots + \frac{19}{8100}$

1×9 4×9 9×16 81×100

$= \frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{16} \dots \frac{1}{81} - \frac{1}{100}$

$= 1 - \frac{1}{100} = \frac{100-1}{100} = \frac{99}{100}$

Ex12. Find the value of

$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\left(1 - \frac{1}{5^2}\right) \dots\dots \times \left(1 - \frac{1}{9^2}\right)$

Sol. $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\left(1 - \frac{1}{5^2}\right) \dots\dots \times \left(1 - \frac{1}{9^2}\right)$

$\boxed{\because A^2 - B^2 = (A+B)(A-B)}$

$\left(1 - \frac{1}{2}\right)\left(1 + \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 + \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 + \frac{1}{4}\right) \dots\dots \left(1 - \frac{1}{9}\right)\left(1 + \frac{1}{9}\right)$

$= \frac{1}{2} \times \frac{3}{2} \times \frac{2}{3} \times \frac{4}{3} \times \frac{3}{4} \times \frac{5}{4} \dots\dots \frac{8}{9} \times \frac{10}{9}$

$\frac{1}{2} \times \frac{10}{9} = \frac{5}{9}$

Ex13. Find the value of

$$\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{2}-\frac{1}{3}+\frac{1}{4}+\frac{1}{2}+\frac{1}{3}-\frac{1}{4}....$$

34 terms

Sol.

$$\underbrace{\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}+\frac{1}{2}+\frac{1}{3}-\frac{1}{4}....34\text{terms}$$

After 6 Terms series repeat so, then

$$S_6=\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{2}-\frac{1}{3}+\frac{1}{4}=0$$

In the series sum of 6 terms is 0. so sum of 30 terms (multiple of 6) will be 0

$S_{30} = 0$

Now, solve the remaining 4 term

$$\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{2}=\mathbf{\frac{1}{12}}$$

Ex.14. Find the value of $\left(1-\frac{1}{4}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{2}{3}\right)\left(1-\frac{2}{5}\right)\left(1+\frac{6}{7}\right)\left(1-\frac{12}{13}\right)$

Sol. $\left(1-\frac{1}{4}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{2}{3}\right)\left(1-\frac{2}{5}\right)\left(1+\frac{6}{7}\right)\left(1-\frac{12}{13}\right)$

$$=\frac{3}{4}\times\frac{4}{3}\times\frac{5}{3}\times\frac{3}{5}\times\frac{13}{7}\times\frac{1}{13}=\frac{1}{7}$$

Ex.15. Find the value of $1^2-2^2+3^2-4^2+5^2-6^2+7^2-8^2.........90^2$

sol. $\underbrace{1^2-2^2}+\underbrace{3^2-4^2}+\underbrace{5^2-6^2}+\underbrace{7^2-8^2}$.

$.......\underbrace{89^2+90^2}$

(1 – 2) (1 + 2) + (3 – 4) (3 + 4) + (5 – 6) (5 + 6) + (7 – 8) (7 + 8) .... (89 –90) (89 + 90)

= (– 1) ×(3) + (–1) × (7) + (–1) ×(11) + (–1) × (15) ...... (–1) × (179)

= –(3 + 7 + 11 + 15 .... 179)

a = 3, d = 4

No. of term = $\frac{179-3}{4}+1$

= 44 + 1 = 45

$$S_{45}=-\left(\frac{45}{2}\left[2\times3+44\times4\right]\right)$$

$$=-\frac{45}{2}[6+176]$$

$$=-\frac{45}{2}\times182=-4095$$

So,
The value of series = –4095

Ex.16 Find the value of $\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+......\frac{1}{156}$

Sol. $\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+.....+\frac{1}{156}$

3 4 4 5 5 6 12 13

$$\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}\,.........\,\frac{1}{12}-\frac{1}{13}$$

$$=\frac{1}{3}-\frac{1}{13}=\frac{10}{39}$$

Ex17. Find the value of

$$\frac{1}{10}+\frac{1}{40}+\frac{1}{88}.......+\frac{1}{598}$$

Sol. $\frac{1}{10}+\frac{1}{40}+\frac{1}{88}.....+\frac{1}{598}$

2 5 5 8 8 11 23 26

3 3 3 3

$$=\frac{1}{3}\left(\frac{1}{2}-\frac{1}{5}+\frac{1}{5}-\frac{1}{8}+\frac{1}{8}-\frac{1}{11}...\;.......\frac{1}{23}-\frac{1}{26}\right)$$

$$=\frac{1}{3}\left(\frac{1}{2}-\frac{1}{26}\right)$$

$$=\frac{1}{3}\left(\frac{12}{26}\right)=\frac{2}{13}$$

Alternate:-

$$\frac{1}{\text{diff. b/w two no.}}\left[\frac{1}{\text{First term}}-\frac{1}{\text{Last term}}\right]$$

$$\frac{1}{3}\left[\frac{1}{2}-\frac{1}{26}\right]=\frac{2}{13}$$

Ex.18. Find the value of

$$\frac{1}{45}+\frac{1}{117}+.........+\frac{1}{3965}$$

Sol. $\frac{1}{45}+\frac{1}{117}+\frac{1}{221}.....+\frac{1}{3965}$

5 9 9 13 13 17 61 65

$$\frac{1}{5\times9}+\frac{1}{9\times13}+\frac{1}{13\times17}.....+\frac{1}{61\times65}$$

$$\Rightarrow\frac{1}{4}\left(\frac{1}{5}-\frac{1}{9}+\frac{1}{9}-\frac{1}{13}+\frac{1}{13}-\frac{1}{17}....\frac{1}{61}-\frac{1}{65}\right)$$

$$\Rightarrow\frac{1}{4}\left[\frac{1}{5}-\frac{1}{65}\right]$$

$$=\frac{1}{4}\times\frac{12}{65}=\frac{3}{65}$$

TYPE – III

CONTINUED FRACTION

A continued fraction consists of the fractional denominators

Ex.19 The value of $\cfrac{1}{2+\cfrac{1}{8+\cfrac{1}{5}}}$ is:

$$=\cfrac{1}{2+\cfrac{1}{41/5}}$$

$$=\frac{1}{2+\frac{5}{41}}=\frac{1}{\frac{87}{41}}=\frac{41}{87}$$

Note: These fractions are solved starting from the bottom towards upside.

Ex20. The value of

$$1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5}}}}\text{ is:}$$

Sol. $1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5}}}}$

$$\Rightarrow 1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{5}{21}}}$$

$$\Rightarrow 1+\cfrac{1}{2+\cfrac{1}{68/5}}$$

$$\Rightarrow 1+\cfrac{1}{2+\cfrac{5}{68}}$$

$$\Rightarrow 1+\cfrac{1}{\cfrac{141}{68}} \Rightarrow 1+\frac{68}{141}$$

$$x \Rightarrow \frac{209}{141}$$

Ex.21. The value of 2 $+\cfrac{3}{5+\cfrac{2}{7+\cfrac{6}{5+\cfrac{2}{3}}}}$ is:

Sol. $$2+\cfrac{3}{5+\cfrac{2}{7+\cfrac{6}{5+\cfrac{2}{3}}}}$$

$$= 2+\cfrac{3}{5+\cfrac{2}{7+\cfrac{6}{17/3}}}$$

$$= 2+\cfrac{3}{5+\cfrac{2}{7+\cfrac{18}{17}}}$$

$$= 2+\cfrac{3}{5+\cfrac{2}{137/17}}$$

$$= 2+\cfrac{3}{5+\cfrac{34}{137}} = 2+\cfrac{3}{\cfrac{719}{137}}$$

$$= 2+\frac{411}{719} = 2\frac{411}{719}$$

Ex 22. The value of

$$\cfrac{2}{2+\cfrac{2}{3+\cfrac{2}{3+\cfrac{2}{3}}}\times 0.39}$$ is :

Sol. $$\cfrac{2}{2+\cfrac{2}{3+\cfrac{2}{3+\cfrac{2}{3}}}\times 0.39}$$

$$= \cfrac{2}{2+\cfrac{2}{3+\cfrac{2}{11/3}}\times 0.39}$$

$$= \cfrac{2}{2+\cfrac{2}{3+\cfrac{6}{11}}\times 0.39}$$

$$= \cfrac{2}{2+\cfrac{2}{39/11}\times 0.39}$$

$$= \cfrac{2}{2+\cfrac{22}{39}\times 0.39}$$

$$= \cfrac{2}{2+\cfrac{22}{39}\times\cfrac{39}{100}} = \cfrac{2}{2+\cfrac{22}{100}}$$

$$= \cfrac{2}{\cfrac{222}{100}} = \frac{200}{222} = \mathbf{\frac{100}{111}}$$

Ex23. The value of 1 + $\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{6}}}$ is:

Sol. $$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{6}}} = 1+\cfrac{1}{1+\cfrac{1}{7/6}}$$

$$= 1+\cfrac{1}{1+\cfrac{6}{7}} = 1+\cfrac{1}{13/7}$$

$$= 1+\frac{7}{13} = \frac{20}{13} = 1\frac{7}{13}$$

Ex24.

(a) The vlaue of

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}}$$ is:

Sol. $$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}}$$

$$= 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}}$$

$$= 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cfrac{3}{2}}}} = 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{3}}}$$

$$= \cfrac{1}{1+\cfrac{1}{\cfrac{5}{3}}} = 1+\cfrac{1}{1+\cfrac{3}{5}} = 1+\cfrac{1}{\cfrac{8}{5}}$$

$$= 1+\frac{5}{8} = \frac{13}{8} = 1\frac{5}{8}$$

(b) If $\cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{c}}} = \cfrac{9}{26}$

Find the value of $a + b + c$

Sol. Convert the fraction same form of series

$$= \cfrac{1}{\cfrac{26}{9}} = \cfrac{1}{2+\cfrac{8}{9}}$$

$$= \cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{8}}}$$

Now, comparison both series

$$\cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{c}}} = \cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{8}}}$$

a = 2

b = 1

c = 8

So, a + b + c = 2 + 1 + 8 = 11

Alternate:-

$$\frac{26}{9} = 2+\frac{8}{9}$$

a = 2

$\frac{9}{8} = 1 + \frac{1}{8}$

b = 1

and $\frac{1}{c} = \frac{1}{8}$

c = 8

$\therefore$ a + b + c = 2 + 1 + 8 = 11

Ex25. If $\dfrac{1}{a+\dfrac{1}{b+\dfrac{1}{c+\dfrac{1}{d}}}} = \dfrac{17}{60}$

Find the value of a × c + b + d

Sol. Convert the fraction same form of series.

$$\frac{1}{\frac{60}{17}} = \frac{1}{3+\frac{9}{17}} = \frac{1}{3+\frac{1}{\frac{17}{9}}}$$

$$= \frac{1}{3+\dfrac{1}{1+\frac{8}{9}}}$$

$$= \frac{1}{3+\dfrac{1}{1+\dfrac{1}{9/8}}} = \frac{1}{3+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{8}}}}$$

Comparision both series

$$\frac{1}{a+\dfrac{1}{b+\dfrac{1}{c+\frac{1}{d}}}} = \frac{1}{3+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{8}}}}$$

a = 3, b = 1, c = 1 and d = 8

$\therefore$ a × c + b + d

= 3 × 1 + 1 + 8 = 12

Alternate:-

$\frac{17}{60}$

$\frac{60}{17} = 3 + \frac{9}{17}$

a = 3

$\frac{17}{9} = 1 + \frac{8}{9}$

b = 1

$\frac{9}{8} = 1 + \frac{1}{8}$

c = 1

and $\frac{1}{d} = \frac{1}{8}$

Then d = 8

Now a × c + b + d

= 3 × 1 + 1 + 8 = 12

TYPE – IV

Componendo and dividendo (C. & D) It is a theorem on proportions that allows for a quick way to perform calculations and **Reduce** the amount of expansions **needed**

It is particularly useful when dealing with equations involving fractions or rational functions.

Ex. $\frac{a}{b}, \frac{a+b}{a-b}, \frac{a+kb}{a-kb}$

If a, b, c and d are numbers such that b and d are non-zero and $\frac{a}{b} = \frac{c}{d}$, then

Some Points

1. Componendo $\frac{a+b}{b} = \frac{c+d}{d}$
2. Dividendo $\frac{a-b}{b} = \frac{c-d}{d}$
3. for $k \neq \frac{a}{b}$, $\frac{a+kb}{a-kb} = \frac{c+kd}{c-kd}$
4. for $k \neq \frac{-b}{a}$, $\frac{a}{b} = \frac{a+kc}{b+kd}$

Ex.26. If $\frac{a}{b} = \frac{16}{3}$

Find the value $\frac{a+b}{a-b}$

Sol. If $\frac{a}{b} = \frac{c}{d}$

Then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

The value. $\frac{a}{b} = \frac{16}{3}$

$\frac{a+b}{a-b} = \frac{16+3}{16-3} = \frac{19}{13}$

Ex.27. If $\frac{a}{b} = \frac{5}{3}$ Find $\frac{a+b}{a-b}$

Sol. $\frac{a}{b} = \frac{5}{3}$,

use c & d

$\frac{a+b}{a-b} = \frac{5+3}{5-3}$

$\frac{a+b}{a-b} = \frac{8}{2} = \frac{4}{1}$

Ex.28. If $\frac{a+b}{a-b} = \frac{25}{12}$

Find the value of $\frac{a}{b}$

Sol. $\frac{a+b}{a-b} = \frac{25}{12}$

use c & d

$\frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{25+12}{25-12}$

$\frac{2a}{2b} = \frac{37}{13}$

$\frac{a}{b} = \frac{37}{13}$

Ex.29. $\frac{a+b}{a-b} = \frac{4}{1}$, Find the vlaue of $\frac{a}{b}$

Sol. $\frac{a+b}{a-b} = \frac{4}{1}$

use C & D

$\frac{a+b+a-b}{a+b-a+b} = \frac{4+1}{4-1}$

$\frac{2a}{2b} = \frac{5}{3}$,

$\frac{a}{b} = \frac{5}{3}$

Ex.30. If $\frac{x+\sqrt{5}}{x-\sqrt{5}} = 2$

Find the value of x

Sol. $\frac{x+\sqrt{5}}{x-\sqrt{5}} = \frac{2}{1}$

use C & D.

$\frac{(x+\sqrt{5})+(x-\sqrt{5})}{(x+\sqrt{5})-(x-\sqrt{5})} = \frac{2+1}{2-1}$

$\frac{2x}{2\sqrt{5}} = \frac{3}{1}$

$\frac{x}{\sqrt{5}} = 3$

$\mathbf{x = 3\sqrt{5}}$

Ex31. If $\frac{x^3+1}{x+1}=\frac{x^3-1}{x-1}$, $(x\neq 1,-1)$
Find the value of x

Sol. $\frac{x^3+1}{x+1}=\frac{x^3-1}{x-1}$

$\frac{x^3+1}{x^3-1}=\frac{x+1}{x-1}$

We use C & d

$\frac{(x^3+1)+(x^3-1)}{(x^3+1)-(x^3-1)}=\frac{(x+1)+(x-1)}{(x+1)-(x-1)}$

$\frac{2x^3}{2}=\frac{2x}{2}$

$\frac{x^3}{1}=\frac{x}{1}$

$x^3 = x$

$x^3-x = 0$

$x(x^2-1) = 0$

However, since $x \neq 1, -1$,

So $\boldsymbol{x = 0}$

Ex.32. If $\frac{2x+y}{2x-y}=\frac{5}{4}$, find the value of $\frac{2x}{y}$

Sol. $\frac{2x+y}{2x-y}=\frac{5}{4}$

Use C & D

$\frac{(2x+y)+(2x-y)}{(2x+y)-(2x-y)}=\frac{5+4}{5-4}$

$\Rightarrow \frac{4x}{2y}=\frac{9}{1}$

So, $\Rightarrow \frac{2x}{y}=9$

Ex.33. If $\frac{2x-y}{x+2y}=\frac{1}{2}$,

Find the value of $\frac{3x+y}{3x-y}$

Sol. $\frac{2x-y}{x+2y}=\frac{1}{2}$

$4x-2y = x+2y$

$3x = 4y$

$\frac{\mathbf{3x}}{y}=\frac{\mathbf{4}}{\mathbf{1}}$

Use C & D

$\frac{3x+y}{3x-y}=\frac{4+1}{4-1}=\frac{5}{3}$

Ex.34. If $\frac{x^3+3x}{3x^2+1}=\frac{189}{61}$
Find the value of x

Sol. $\frac{x^3+3x}{3x^2+1}=\frac{189}{61}$

Use C & D

$\Rightarrow \frac{x^3+3x+3x^2+1}{x^3+3x-3x^2-1}=\frac{189+61}{189-61}$

$=\frac{250}{128}=\frac{125}{64}$

$\Rightarrow \left(\frac{x+1}{x-1}\right)^3=\frac{125}{64}$

$\left(\frac{x+1}{x-1}\right)^3=\left(\frac{5}{4}\right)^3$

$\frac{x+1}{x-1}=\frac{5}{4}$

$4x+4 = 5x-5$

$x = 9$

Ex.35. If a+b = 1
c+d = 1

a–b = $\frac{d}{c}$,

find the value of c^2-d^2

Sol. a+b = 1 ……(i)

a–b = $\frac{d}{c}$……(ii)

(i) / (ii)

$\frac{a+b}{a-b}=\frac{1}{d/c}$

$\frac{a+b}{a-b}=\frac{c}{d}$

Use. C & D

$\frac{c+d}{c-d}=\frac{a}{b}$

$\therefore$ c+d = 1 (Given)

$\frac{1}{c-d}=\frac{a}{b}$

then, (c – d) = $\frac{b}{a}$

Now, c^2-d^2 = (c+d) (c–d)
= 1×(c–d) = c – d

$\mathbf{c^2-d^2} = \frac{\boldsymbol{b}}{\boldsymbol{a}}$

Ex.36. If x = $\frac{\sqrt{m+3n}+\sqrt{m-3n}}{\sqrt{m+3n}-\sqrt{m-3n}}$

Find $3nx^2 + 3n$

Sol. $\frac{x}{1}=\frac{\sqrt{m+3n}+\sqrt{m-3n}}{\sqrt{m+3n}-\sqrt{m-3n}}$

Use C & D

$\frac{x+1}{x-1}=\frac{\sqrt{m+3n}}{\sqrt{m-3n}}$

square both sides

$\left(\frac{x+1}{x-1}\right)^2=\frac{m+3n}{m-3n}$

Again use c & d

$\frac{(x+1)^2+(x-1)^2}{(x-1)^2-(x-1)^2}=\frac{(m+3n)+(m-3n)}{(m+3m)-(m-3n)}$

$\frac{2(x^2+1)}{2\times 2x}=\frac{2m}{6n}$

$\frac{x^2+1}{2x}=\frac{m}{3n}$

$3x^2n+3n = 2mx$

Ex.37. If $(a+b):\sqrt{ab}$ = 4:1 a>b
Find a : b

Sol. $\frac{a+b}{\sqrt{ab}}=\frac{4}{1}$

multiply 2 both denominator sides

$\frac{a+b}{2\sqrt{ab}}=\frac{4}{1\times 2}$

$\frac{a+b}{2\sqrt{ab}}=\frac{2}{1}$

Use C & D

$\frac{a+b+2\sqrt{ab}}{a+b-\sqrt{ab}}=\frac{2+1}{2-1}$

$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2}=\frac{3}{1}$

$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{3}}{1}$

Again use C & D

$\frac{\sqrt{\mathbf{a}}}{\sqrt{\mathbf{b}}}=\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Square both sides

$\frac{a}{b}=\frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)^2}$

$\frac{a}{b}=\frac{4+2\sqrt{3}}{4-2\sqrt{3}}$

$\frac{\mathbf{a}}{\mathbf{b}}=\frac{2+\sqrt{3}}{2-\sqrt{3}}$

Ex.38. If a+b : $\sqrt{ab}$ = 4:1
find the value a

Sol. a +b = 4,

$\sqrt{ab}$ = 1

then, square both sides

ab = 1

$b = \frac{1}{a}$

Now, a +b = 4

Put the value b

$a + \frac{1}{a} = 4$

$a^2 - 4a + 1 = 0$

$ax^2 + bx + c$

we use,

$\therefore \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-(-4) \pm \sqrt{((-4)^2 - 4\times1\times1)}}{2\times1}$

$= \frac{4 \pm \sqrt{16-4}}{2}$

$= \frac{4 \pm \sqrt{12}}{2}$

$= \frac{4 \pm 2\sqrt{3}}{2}$

Take positive

then,

$a = 2 + \sqrt{3}$

Ex39. If $x = \frac{\sqrt[3]{m+1} + \sqrt[3]{m-1}}{\sqrt[3]{m+1} - \sqrt[3]{m-1}}$ Find the value of $x^2+3x-3mx^2-m$

Sol. $\frac{x}{1} = \frac{\sqrt[3]{m+1} + \sqrt[3]{m-1}}{\sqrt[3]{m+1} - \sqrt[3]{m-1}}$

Use C & D

$\frac{x+1}{x-1} = \frac{\sqrt[3]{m+1}}{\sqrt[3]{m-1}}$

cube both sides

$\frac{(x+1)^3}{(x-1)^3} = \frac{m+1}{m-1}$

$\left(\begin{matrix}(a+b)^3 = a^3+b^3+3a^2b+3b^2a \\ (a-b)^3 = a^3-b^3-3a^2b+3b^2a\end{matrix}\right)$

$\frac{(x^3+3x)+(3x^2+1)}{(x^3+3x)-(3x^2+1)}$

$= \frac{m+1}{m-1}$

Again use C & D

$\frac{x^3+3x}{3x^2+1} = \frac{m}{1}$

$3mx^2 + m = x^2 + 3x$

$\boldsymbol{x^2+3x-3mx^2-m = 0}$

Ex.40. If $x = \frac{2ab}{b^2+1}$ and b > 1

Find $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$

Sol. $\frac{x}{a} = \frac{2b}{b^2+1}$

Use C & D

$\frac{a}{x} = \frac{b^2+1}{2b}$

$\frac{a+x}{a-x} = \frac{b^2+1+2b}{b^2+1-2b}$

$\frac{a+x}{a-x} = \left(\frac{b+1}{b-1}\right)^2$

$\frac{\sqrt{a+x}}{\sqrt{a-x}} = \frac{b+1}{b-1}$

Again use C & D

Then, $\frac{(\sqrt{a+x})+(\sqrt{a-x})}{(\sqrt{a+x})-(\sqrt{a-x})} = \frac{b}{1}$

Ex.41. If $x = \frac{a-b}{a+b}$, $y = \frac{b-c}{b+c}$,

$z = \frac{c-a}{c+a}$

Find $\frac{1+x}{1-x} \times \frac{1+y}{1-y} \times \frac{1+z}{1-z}$

Sol. $x = \frac{a-b}{a+b}$

$\frac{1}{x} = \frac{a+b}{a-b}$

Use C & D

$\frac{1+x}{1-x} = \frac{a}{b}$ ......(I)

Same $\frac{1+y}{1-y} = \frac{b}{c}$ .....(II)

$\frac{1+z}{1-z} = \frac{c}{a}$ ......(III)

(I)×(II)×(III)

$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a} = 1$

So, $\frac{1+x}{1-x} \times \frac{1+y}{1-y} \times \frac{1+z}{1-z} = 1$

TYPE – V

Recurring number

- Pure Recurring decimal Number
- Impure Recurring decimal Number

1. Pure recurring decimals : These are recurring decimals where the recurrence starts immediately after the decimal point.

Ex: 0. 4444 ...... $= 0.\overline{4}$

3.232323.... $= 3.\overline{23}$

0.564564564 $= 0.\overline{564}$

2. Impure recurring decimals: Unlike pure recurring decimals, in these decimals, the recurrence occurs after a certain number of digits in the decimal.

Ex: 0.43542542........ $= 0.43\overline{542}$

0.546666......... $= 0.54\overline{6}$

recurring number

Pure recurring decimal number convert into fraction

0.333..... $= 0.\overline{3} = \frac{3}{9}$

0.444..... $= 0.\overline{4} = \frac{4}{9}$

0.454545..... = $0.\overline{45} = \frac{45}{99}$

3.232323.... $= 3.\overline{23}$

$= 3 + \frac{23}{99} = 3\frac{23}{99}$

5.564564564 $= 5.\overline{564}$

$= 5 + \frac{564}{999}$

* the number of of 9's in this group equals the number of digits in the recurring part of the decimal.

Impure recurring decimal number convert into fraction

0.4565656......... = $0.4\overline{56} = \frac{456-4}{990}$

expanding the meaning

Step - 1

Subtract the non-recurring initial part of the decimal (in this case, it is 4) from the number formed by writing down the starting digits of the decimal value upto the digit where the recurring decimals are written for the first time; (456 - 4)

Step - 2

As many 9's as the number of digits in the recurring part of the decimal. (in this case, since the recurring part '56' has 2 digits, we write down 2 9's.) These nines have to be followed by as many zeroes as the number of digits in the non recurring part of the decimal value. (In this case, the non recurring part of the decimal value is '4'. Since, 4 has 1 digit, attach one zeroes to the two nines to get the number to divide the result of the first step.)

$= \frac{456-4}{990}$

42. Convert to fraction.

(i) 0.22222.........
(ii) 0.5555 ....
(iii) 0.444747........
(iv) 0.456456456.........
(v) 0.57333333..........
(vi) 0.72666666..........
(vii) 9.868686
(viii) 0.783333......
(ix) 0.67777.......
(x) 4.717171........
(xi) $5.00\overline{73}$
(xii) 0.87373.........

Sol. (i) 0.222....= $0.\overline{2} = \frac{2}{9}$

(ii) 0.555...=$0.\overline{5} = \frac{5}{9}$

(iii) 0.474747...=$0.\overline{47} = \frac{47}{99}$

(iv) 0.456456....=$0.\overline{456} = \frac{456}{999} = \frac{152}{333}$

(v) 0.57333...=$0.57\overline{3} = \frac{573-57}{900}$

$= \frac{516}{900} = \frac{43}{75}$

(vi) 0.7266....=$0.72\overline{6} = \frac{726-72}{900}$

$= \frac{654}{900} = \frac{109}{150}$

(vii) 9.868686...9+$0.\overline{86}$ = $9+\frac{86}{99}$

(viii) 0.783333...$0.78\overline{3} = \frac{783-78}{900}$

$= \frac{705}{900} = \frac{47}{60}$

(ix) 0.6777...=$0.6\overline{7} = \frac{67-6}{90} = \frac{61}{90}$

(x) 4.7171...= $4.\overline{71} = 4+\frac{71}{99} = 4\frac{71}{99}$

(xi) $5.00\overline{73} = 5+\frac{73-00}{9900} = 5+\frac{73}{9900}$

$= 5\frac{73}{9900}$

(xii) 0.8737373....=$0.8\overline{73} = \frac{873-8}{990}$

$= \frac{865}{900} = \frac{173}{180}$

Ex. 43

0.56777........

Sol $0.56\overline{7}$

$= \frac{567-56}{900} = \frac{511}{900}$

Ex.44

0.43542542......

Sol $0.43\overline{542}$

$= \frac{43542-43}{99900} = \frac{43499}{99900}$

Ex.45 Find the value of $\sqrt[3]{0.\overline{037}}$

Sol. $0.\overline{037} = \frac{37}{999} = \frac{1}{27}$

$\sqrt[3]{0.\overline{037}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$

= 0.333..... = $\mathbf{0.\overline{3}}$

Ex.46 Find the value of

$0.\overline{37}+8.\overline{56}+1.\overline{23}$

Sol. $0.\overline{37}+8.\overline{56}+1.\overline{23}$

$\downarrow \quad \downarrow \quad \downarrow$

$\frac{37}{99}+8+\frac{56}{99}+1+\frac{23}{99}$

$= 8 + 1 + \frac{37}{99}+\frac{56}{99}+\frac{23}{99}$

$= 9 + \frac{37+56+23}{99}$

$= 9 + \frac{116}{99}$

$= 9 + 1 + \frac{17}{99}$

$= 10 + 0.\overline{17} = \mathbf{10.\overline{17}}$

Ex.47 Find the value of

$2.8\overline{56}+3.\overline{74}+5.875\overline{6}$

Sol. $2.8\overline{56}+3.\overline{74}+5.875\overline{6}$

I	II	III
× × ×	× ×	× × ×
2. 8 5 6	5 6	5 6 5
3. 7 4 7	4 7	4 7 4
5. 8 7 5	6 6	6 6 6
12.4 7 9	7 0	7 0 5

= $\mathbf{12.479\overline{70}}$

Where

I. → maximum digits non recurring [where ($5.875\overline{6}$) has 3 max. non recurring digit (875)]

II. → LCM of recurring digits of number (where 2,2 and 1 recurring digits)

III. → atleast 3 digits

Type – VI

Number of Digits

Ex.48 How many digits are required to write the counting number from 1 to 50 ?

Sol. No. × digit = total digit

From 1 to 9 = 9 × 1 = 9
From 10 to 50 = 41 × 2 = 82
total digit = 9 + 82 = 91

* There are 9 numbers from 1 to 9, and each number has 1 digit hence, the total number of digits are (1×9) = 9. In the same way there are 41 number (50 – 10 + 1) from 10 upto 50, and the number of digits are (41 ×2) = 82.

Hence,
The total number of digits
(9 + 82) = 91

Ex.49 How many digits are requried to write the counting number from 1 to 672?

Sol. No.× digit = total digit
From 1 to 9 = 9 × 1 = 9
From 10 to 99 = 90 × 2 = 180
From 100 to 672 = 573 × 3 = 1719
total digits = 1908

Ex.50 How many digits are requried to write the counting number from 1 to 8756?

Sol. No. × digit = total digit
From 1 to 9 = 9 × 1 = 9
From 10 to 99 = 90 × 2 = 180
From100 to 999 = 900 × 3 = 2700
From1000 to 8756 = 7757 × 4 = 31028
total digit = 9 + 180 + 2700 + 31028 = 33917

Ex.51 Calculate the number of digits in the product of $4^{11111} \times 5^{22222}$

Sol. $4 = 2^2$
$= (2^2)^{11111} \times (5)^{22222}$ **($a^m \times b^m = (ab)^m$)**
(By equalising power)
$= (2)^{22222} \times (5)^{22222} = (10)^{22222}$
So,
Number of digits = 22222 + 1
= 22223

No.			Digit
10^1	=	<u>1</u> <u>0</u>	2 digit
10^2	=	<u>1</u> <u>0</u> <u>0</u>	3 digit
10^3	=	<u>1</u> <u>0</u> <u>0</u> <u>0</u>	4 digit

Ex. 52 Calculate the number of digits in the product of $8^{232} \times 25^{348}$

Sol. $8^{232} \times 25^{348}$
$(2^3)^{232} \times (5^2)^{348}$
(By equalising power)
$(2)^{696} \times (5)^{696}$
$= (10)^{696}$
number of digits
= 696 + 1 = 697

SOLUTION

1. Simplify : $1+\cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{1+\cfrac{4}{5}}}}$

(a) $1\frac{11}{17}$ (b) $1\frac{5}{7}$
(c) $1\frac{6}{17}$ (d) $1\frac{21}{17}$

2. Simplify : $1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{4}{5}}}$

(a) $\frac{7}{4}$ (b) $\frac{4}{7}$ (c) $\frac{7}{5}$ (d) $\frac{3}{7}$

3. Simplify : $\dfrac{\frac{5}{3}\times\frac{7}{51}\text{ of }\frac{17}{5}-\frac{1}{3}}{\frac{2}{9}\times\frac{5}{7}\text{ of }\frac{28}{5}-\frac{2}{3}}$

(a) $\frac{1}{2}$ (b) 4 (c) 2 (d) $\frac{1}{4}$

4. Assume that
$\sqrt{13}$ = 3.605 (approximately)
$\sqrt{130}$ = 11.40 (approximately)
find the value of $\sqrt{1.3}+\sqrt{1300}+\sqrt{0.013}$:
(a) 36.164 (b) 36.304
(c) 37.304 (d) 37.164

5. On simplification of
$\dfrac{(2.644)^2-(2.356)^2}{0.288}$
(a) 1 (b) 4 (c) 5 (d) 6

6. What is the square root of 0.09 ?
(a) 0.3 (b) 0.03
(c) 0.003 (d) 3.0

7. Find the value of
$\dfrac{(0.75)^3}{1-0.75}+\left[0.75+(0.75)^2+1\right]$
(a) 4 (b) 1 (c) 2 (d) 0.25

8. Find the value of
$\sqrt{x}\div\sqrt{441} = 0.02$
(a) 1.64 (b) 2.64
(c) 1.764 (d) 0.1764

9. By which smallest number should 5808 be multiplied so that it becomes a perfect square ?
(a) 2 (b) 7 (c) 11 (d) 3

10. $\dfrac{\sqrt[3]{8}}{\sqrt{16}}\div\sqrt{\dfrac{100}{49}}\times\sqrt[3]{125}$ is equal to :
(a) 7 (b) $1\frac{3}{4}$ (c) $\frac{7}{100}$ (d) $\frac{4}{7}$

11. By which smallest number 1323 must be multiplied, so that it becomes a perfect cube ?
(a) 2 (b) 3 (c) 5 (d) 7

12. When simplified, the expression
$(100)^{\frac{1}{2}}\times(0.001)^{\frac{1}{3}}-(0.0016)^{\frac{1}{4}}\times3^0+\left(\frac{5}{4}\right)^{-1}$
is equal to :
(a) 1.6 (b) 0.8 (c) 1.0 (d) 0

13. When $\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\right)$ is divided by $\left(\frac{2}{5}-\frac{5}{9}+\frac{3}{5}-\frac{7}{18}\right)$, the result is:
(a) $5\frac{1}{10}$ (b) $2\frac{1}{18}$
(c) $3\frac{1}{6}$ (d) $3\frac{3}{10}$

14. The square root of $(272^2–128^2)$ is:
(a) 256 (b) 200 (c) 240 (d) 144

15. One-third of the square root of which number is 0.001?
(a) 0.0009 (b) 0.000001
(c) 0.00009
(d) None of the above

16. $\sqrt[3]{\dfrac{72.9}{0.4096}}$ is equal to :
(a) 0.5625 (b) 5.625
(c) 182 (d) 13.6

17. The value of $\dfrac{1}{3+\dfrac{1}{2-\dfrac{1}{\frac{7}{9}}}}+\dfrac{17}{22}$

(a) $\frac{12}{22}$ (b) $\frac{22}{5}$ (c) $\frac{5}{22}$ (d) 1

18. If $x=1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}}$, then the value of $2x+\frac{7}{4}$ is :

(a) 3 (b) 4 (c) 5 (d) 6

19. Simplify : $\dfrac{19}{43}+\dfrac{1}{2+\dfrac{1}{3+\dfrac{1}{1+\frac{1}{4}}}}$:

(a) 1 (b) $\frac{19}{43}$ (c) $\frac{43}{19}$ (d) $\frac{38}{43}$

20. Simplify:

$$8\frac{1}{2}-\left[3\frac{1}{4}+\left\{1\frac{1}{4}-\frac{1}{2}\left(1\frac{1}{2}-\frac{1}{3}-\frac{1}{6}\right)\right\}\right]$$

(a) $4\frac{1}{2}$ (b) $4\frac{1}{6}$ (c) $9\frac{1}{2}$ (d) $\frac{2}{9}$

21. If $\frac{50}{*}=\dfrac{*}{12\frac{1}{2}}$, then the value of * is:

(a) $\frac{25}{2}$ (b) $\frac{4}{25}$ (c) 4 (d) 25

22. Find the sum of the following :

$$\frac{1}{9}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}$$

(a) $\frac{1}{2}$ (b) 0 (c) $\frac{1}{9}$ (d) 1

23. The value of 25–5[2+3(2–2(5–3)+5)–10]÷4

(a) 5 (b) 23.25

(c) 23.75 (d) 25

24. If $x=\dfrac{1}{2+\frac{1}{2}}$ then $\frac{1}{x}=?$

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{3}{5}$ (d) $\frac{1}{2}$

25. $\frac{9}{20}-\left[\frac{1}{5}+\left\{\frac{1}{4}+\left(\frac{5}{6}-\overline{\frac{1}{3}+\frac{1}{2}}\right)\right\}\right]$ is equal to

(a) 0 (b) 1 (c) $\frac{9}{20}$ (d) $\frac{9}{10}$

26. The value of

$$\sqrt{\frac{(0.1)^2+(0.01)^2+(0.009)^2}{(0.01)^2+(0.001)^2+(0.0009)^2}}$$

(a) 10^2 (b) 10

(c) 0.1 (d) 0.01

27. The value of

$$\sqrt{\frac{(0.03)^2+(0.21)^2+(0.065)^2}{(0.003)^2+(0.021)^2+(0.0065)^2}}$$

(a) 0.1 (b) 10 (c) 10^2 (d) 10^3

28. If $(102)^2 = 10404$, then the value of $\sqrt{104.04}+\sqrt{1.0404}+\sqrt{0.010404}$ is equal to

(a) 0.306 (b) 0.0306

(c) 11.122 (d) 11.322

29. If $\sqrt{4096}=64$, then the value of $\sqrt{40.96}+\sqrt{0.4096}$ + $\sqrt{0.004096}$ + $\sqrt{0.00004096}$ up to two place of decimals is:

(a) 7.09 (b) 7.10

(c) 7.11 (d) 7.12

30. The least number that must be subtracted from 63522 to make the result a perfect square is :

(a) 18 (b) 20 (c) 24 (d) 30

31. By which smallest number should 20184 be multiplied so that it becomes a perfect square?

(a) 2 (b) 3 (c) 5 (d) 6

32. If $2=x+\dfrac{1}{1+\dfrac{1}{3+\frac{1}{4}}}$, then the value of x is :

(a) $\frac{18}{17}$ (b) $\frac{21}{17}$ (c) $\frac{13}{17}$ (d) $\frac{12}{17}$

33. Simplify:

$$\left[\left(1+\frac{1}{10+\frac{1}{10}}\right)\times\left(1+\frac{1}{10+\frac{1}{10}}\right)-\left(1-\frac{1}{10+\frac{1}{10}}\right)\times\left(1-\frac{1}{10+\frac{1}{10}}\right)\right]$$

$$\div\left[\left(1+\frac{1}{10+\frac{1}{10}}\right)+\left(1-\frac{1}{10+\frac{1}{10}}\right)\right]$$

(a) $\frac{100}{101}$ (b) $\frac{90}{101}$

(c) $\frac{20}{101}$ (d) $\frac{101}{100}$

34. $\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}+\frac{1}{110}=?$

(a) $\sqrt{2}\,\frac{2}{27}$ (b) $\frac{1}{9}$

(c) $\frac{5}{27}$ (d) $\frac{6}{55}$

35. The value of 1 ÷ [1 + 1 ÷ { 1 + 1 ÷(1 +1 ÷ 2)}]

(a) 1 (b) $\frac{5}{8}$ (c) 2 (b) $\frac{1}{2}$

36. The simplification of $3.\overline{36}-2.\overline{05}+1.\overline{33}$ equals :

(a) 2.60 (b) $2.\overline{61}$

(c) 2.64 (d) $2.\overline{64}$

37. $(0.2\times0.2+0.01)\,(0.1\times0.1+0.02)^{-1}$

(a) $\frac{5}{3}$ (b) $\frac{41}{12}$ (c) $\frac{41}{4}$ (d) $\frac{9}{5}$

38. The value of

$$\sqrt{5+\sqrt{11+\sqrt{19+\sqrt{29+\sqrt{49}}}}}$$

(a) 3 (b) 9 (c) 7 (d) 5

39. The value of $\sqrt[3]{\frac{7}{875}}$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{15}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

40. $\dfrac{2}{2+\dfrac{2}{3+\dfrac{2}{3+\dfrac{2}{3}}}\times 0.39}$ is simplified to

(a) $\frac{1}{3}$ (b) 2

(c) 6 (d) None of these

41. $\dfrac{-\frac{1}{2}-\frac{2}{3}+\frac{4}{5}-\frac{1}{3}+\frac{1}{5}+\frac{3}{4}}{\frac{1}{2}+\frac{2}{3}-\frac{4}{3}+\frac{1}{3}-\frac{1}{5}-\frac{4}{5}}$ is simplified to

(a) $-\frac{10}{3}$ (b) $-\frac{3}{10}$

(c) 1 (d) – 2

42. The simplification of $\left(0.\overline{63}+0.\overline{37}+0.\overline{80}\right)$ yields the result is:

(a) $1.\overline{80}$ (b) $1.\overline{81}$

(c) $1.\overline{79}$ (d) 1.80

43. The square root of $0.\bar{4}$ is :

(a) $0.\bar{8}$ (b) $0.\bar{6}$

(c) $0.\bar{7}$ (d) $0.\bar{9}$

44. The square root of $\left(7+3\sqrt{5}\right)\left(7-3\sqrt{5}\right)$

(a) 4 (b) $\sqrt{5}$ (c) $3\sqrt{5}$ (d) 2

45. $\left(4\frac{11}{15}+\frac{15}{71}\right)^2-\left(4\frac{11}{15}-\frac{15}{71}\right)^2$ is equal to:

(a) 1 (b) 2 (c) 3 (d) 4

46. $\frac{13}{48}$ is equal to

(a) $\dfrac{1}{3+\dfrac{1}{1+\dfrac{1}{16}}}$ (b) $\dfrac{1}{2+\dfrac{1}{1+\dfrac{1}{8}}}$

(c) $\dfrac{1}{3+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{8}}}}$ (d) $\dfrac{1}{3+\dfrac{1}{1+\dfrac{1}{2+\dfrac{1}{4}}}}$

47. $\left(\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\frac{1}{9.11}+\frac{1}{11.13}+\frac{1}{13.15}\right)$ is equal to

(a) $\frac{2}{45}$ (b) $\frac{2}{25}$ (c) $\frac{7}{45}$ (d) $\frac{2}{15}$

48. The number of digits in the square root of 625686734489 is

(a) 4 (b) 5 (c) 6 (d) 7

49. There are some boys and girls in a room. The square of the number of the girls is less than the square of the number of boys by 28. If there were two more girls, the number of boys would have been the same as that of the girls. The total number of the boys and girls in the room are

(a) 56 (b) 14 (c) 10 (d) 7

50. If $\sqrt[2]{0.014\times 0.14x}=0.014\times 0.14\sqrt[2]{y}$. find the value of $\frac{x}{y}$,

(a) 0.000196 (b) 0.00196

(c) 0.0196 (d) 0.196

51. $\left\{\dfrac{(0.1)^2-(0.01)^2}{0.0001}\right\}+1$ is equal to

(a) 1010 (b) 110

(c) 101 (d) 100

52. $\dfrac{(100-1)(100-2)(100-3)\ldots(100-200)}{100\times 99\times 98\times\ldots\times 3\times 2\times 1}$ is equal to

(a) $\dfrac{100}{99\times 98\times 97\times..\times 3\times 2\times 1}$

(b) $-\dfrac{1}{99\times 98\times 97\times\ldots\times 3\times 2\times 1}$

(c) 0

(d) $-\dfrac{2}{99\times 98\times 97\times\ldots\times 3\times 2\times 1}$

53. The value of $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{2}{3}}}}}$

(a) $\frac{21}{13}$ (b) $\frac{17}{3}$ (c) $\frac{34}{21}$ (d) $\frac{8}{5}$

54. The value of $\dfrac{2\frac{1}{3}-1\frac{2}{11}}{3+\dfrac{1}{3+\dfrac{1}{3+\dfrac{1}{3}}}}$

(a) $\frac{38}{109}$ (b) $\frac{109}{38}$ (c) 1 (d) $\frac{116}{109}$

55. $\sqrt{8+\sqrt{57+\sqrt{38+\sqrt{108+\sqrt{169}}}}}=?$

(a) 4 (b) 6 (c) 8 (d) 10

56. If the number p is 5 more than q and the sum of the squares of p and q is 55, then the product of p and q is

(a) 10 (b) – 10 (c) 15 (d) – 15

57. $\dfrac{4\frac{2}{7}-\frac{1}{2}}{3\frac{1}{2}+1\frac{1}{7}}\div\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{5-\dfrac{1}{5}}}}$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c)2 (d) $\frac{1}{3}$

58. If $\left[4-\dfrac{5}{1+\dfrac{1}{3+\dfrac{1}{2+\dfrac{1}{4}}}}\right]^{th}$ part of a journey takes ten minutes, then to complete $\frac{3}{5}^{th}$ of that journey. it will take

(a) 80 minutes (b) 50 minutes

(c) 48 minutes (d) 60 minutes

59. $\sqrt{\dfrac{4\frac{1}{7}-2\frac{1}{4}}{3\frac{1}{2}+1\frac{1}{7}}\div\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{5-\dfrac{1}{5}}}}}$ is equal to

(a) 1 (b) 4 (c) 3 (d) 2

60. $\dfrac{1}{1+2^{a-b}}+\dfrac{1}{1+2^{b-a}}$ is equal to

(a) a – b (b) b – a

(c) 1 (d) 0

61. Find the sum of

$\left(1-\frac{1}{n+1}\right)+\left(1-\frac{2}{n+1}\right)+\left(1-\frac{3}{n+1}\right)+.....+\left(1-\frac{n}{n+1}\right)$

(a) n (b) $\frac{1}{2}n$

(c) $(n+1)$ (d) $\frac{1}{2}(n+1)$

62. The square root of $33 - 4\sqrt{35}$ is :

(a) $\pm\left(2\sqrt{7}+\sqrt{5}\right)$ (b) $\pm\left(\sqrt{7}+2\sqrt{5}\right)$

(c) $\pm\left(\sqrt{7}-2\sqrt{5}\right)$ (d) $\pm\left(2\sqrt{7}-\sqrt{5}\right)$

63. What number must be added to the expression $16a^2 - 12a$ to make it a perfect square ?

(a) $\frac{9}{4}$ (b) $\frac{3}{2}$ (c) $\frac{13}{2}$ (d) 16

64. If $x\ [-2\{-4(-a)\}]+5\ [-2\{-2\ (-a)\}] = 4a$, then x = ?

(a) – 2 (b) – 3 (c) – 4 (d) – 5

65. If a = 64 and b =289, then the value of $\left(\sqrt{\sqrt{a}+\sqrt{b}}-\sqrt{\sqrt{b}-\sqrt{a}}\right)^{\frac{1}{2}}$ is

(a) $2^{1/2}$ (b) 2

(c) 4 (d) –2

66. The simplified value of

$\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{3}-\sqrt{2}$ (d) 0

67. The value of $\frac{4-\sqrt{0.04}}{4+\sqrt{0.4}}$ is close to

(a) 0.4 (b) 0.8 (c) 1.0 (d) 1.4

68. If $\sqrt{0.05\times0.5\times a} = 0.5 \times 0.05 \times \sqrt{b}$, then $\frac{a}{b}$ is equal to

(a) 0.0025 (b) 0.025

(c) 0.25 (d) 0.00025

69. The value of $(1001)^3$ is

(a) 1003003001

(b) 100303001

(c) 100300301

(d) 103003001

70. Find the Value of

$5.\bar{6} + 7.\bar{3} + 8.\bar{7} + 6.\bar{1}$

(a) $27.\bar{8}$ (b) $27.\bar{7}$

(c) $27.\overline{08}$ (d) $27.\overline{07}$

71. Find the value of $6.\overline{74} + 7.\overline{32}$

(a) $14.\overline{06}$ (b) $14.\overline{07}$

(c) $13.\overline{06}$ (d) $13.\overline{07}$

72. Which of the following number greatest of all ?

$0.9,\ 0.\bar{9},\ 0.0\bar{9},\ 0.\overline{09}$

(a) 0.9 (b) $0.\bar{9}$

(c) $0.0\bar{9}$ (d) $0.\overline{09}$

73. $0.\overline{001}$ is equal to

(a) $\frac{1}{1000}$ (b) $\frac{1}{999}$

(c) $\frac{1}{99}$ (d) $\frac{1}{9}$

74. $1.\overline{27}$ in the form $\frac{p}{q}$ is equal to

(a) $\frac{127}{100}$ (b) $\frac{73}{100}$

(c) $\frac{14}{11}$ (d) $\frac{11}{14}$

75. $8.3\bar{1} + 0.\bar{6} + 0.00\bar{2}$ is equal to

(a) $8.\overline{912}$ (b) $8.9\overline{12}$

(c) $8.97\bar{9}$ (d) $8.9\overline{79}$

76. The difference of $5.\overline{76}$ & $2.\bar{3}$

(a) $2.\overline{54}$ (b) $3.\overline{73}$

(c) $3.\overline{46}$ (d) $3.\overline{43}$

77. The value of $(0.\overline{63} + 0.\overline{37})$

(a) 1 (b) $\frac{100}{99}$

(c) $\frac{99}{100}$ (d) $\frac{100}{33}$

ANSWER KEY

1. (a)	9. (d)	17. (d)	25. (a)	33. (c)	41. (b)	49. (b)	57. (c)	65. (a)	73. (b)
2. (a)	10. (b)	18. (c)	26. (b)	34. (d)	42. (b)	50. (b)	58. (c)	66. (d)	74. (c)
3. (c)	11. (d)	19. (d)	27. (b)	35. (b)	43. (b)	51. (d)	59. (a)	67. (b)	75. (c)
4. (c)	12. (a)	20. (a)	28. (d)	36. (d)	44. (d)	52. (c)	60. (c)	68. (b)	76. (d)
5. (c)	13. (a)	21. (d)	29. (c)	37. (a)	45. (d)	53. (c)	61. (b)	69. (a)	77. (b)
6. (a)	14. (c)	22. (a)	30. (a)	38. (a)	46. (d)	54. (a)	62. (d)	70. (a)	
7. (a)	15. (d)	23. (c)	31. (d)	39. (d)	47. (d)	55. (a)	63. (a)	71. (b)	
8. (d)	16. (b)	24. (b)	32. (b)	40. (d)	48. (c)	56. (c)	64. (b)	72. (b)	

SOLUTION

1. (a) According to the question,

$$1+\cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{1+\cfrac{4}{5}}}}$$

$$\Rightarrow 1+\cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{\frac{5+4}{5}}}}$$

$$\Rightarrow 1+\cfrac{1}{1+\cfrac{2}{2+\frac{3\times 5}{9}}}$$

$$\Rightarrow 1+\cfrac{1}{1+\cfrac{2}{\frac{18+15}{9}}}$$

$$\Rightarrow 1+\cfrac{1}{1+\frac{18}{33}}$$

$$\Rightarrow 1+\cfrac{1}{\frac{33+18}{33}} \Rightarrow 1+\frac{33}{51}$$

$$\Rightarrow \frac{51+33}{51} \Rightarrow \frac{84}{51} \Rightarrow 1\frac{11}{17}$$

2. (a) According to the question,

$$1+\cfrac{2}{1+\cfrac{3}{1+\frac{4}{5}}} \Rightarrow 1+\cfrac{2}{1+\frac{3\times 5}{5+4}}$$

$$\Rightarrow 1+\cfrac{2}{1+\frac{15}{9}} \Rightarrow 1+\frac{18}{9+15}$$

$$\Rightarrow \frac{24+18}{24} \Rightarrow \frac{42}{24} \Rightarrow \frac{7}{4}$$

3. (c) According to the question,

$$\dfrac{\frac{5}{3}\times\frac{7}{51}\text{ of }\frac{17}{5}-\frac{1}{3}}{\frac{2}{9}\times\frac{5}{7}\text{ of }\frac{28}{5}-\frac{2}{3}}$$

$$\Rightarrow \dfrac{\frac{5}{3}\times\frac{7}{51}\times\frac{17}{5}-\frac{1}{3}}{\frac{2}{9}\times\frac{5}{7}\times\frac{28}{5}-\frac{2}{3}}$$

$$\Rightarrow \dfrac{\frac{5}{3}\times\frac{7}{15}-\frac{1}{3}}{\frac{2}{9}\times 4-\frac{2}{3}} \Rightarrow \dfrac{\frac{7}{9}-\frac{1}{3}}{\frac{8}{9}-\frac{2}{3}}$$

$$\Rightarrow \dfrac{\frac{7-3}{9}}{\frac{8-6}{9}} \Rightarrow \frac{4}{2}=2$$

4. (c) According to the question,

$$\sqrt{1.3}+\sqrt{1300}+\sqrt{0.013}$$

$$\Rightarrow \sqrt{\frac{130}{100}}+\sqrt{1300}+\sqrt{\frac{130}{10000}}$$

$$\Rightarrow \frac{11.4}{10}+36.05+\frac{11.4}{100}$$

$\Rightarrow$ 1.14 + 36.05 + 0.114

$\Rightarrow$ 37.304

5. (c) According to the question,

$$\frac{(2.644)^2-(2.356)^2}{0.288}$$

$$\Rightarrow \frac{(2.644+2.356)(2.644-2.356)}{0.288}$$

$$\left[\because a^2-b^2=(a+b)(a-b)\right]$$

$$\Rightarrow \frac{5\times 0.288}{0.288}=5$$

6. (a) According to the question,

Square root of 0.09 = $\sqrt{0.09}=0.3$

7. (a) According to the question,

$$\Rightarrow \frac{(0.75)^3}{1-0.75}+\left[0.75+(0.75)^2+1\right]$$

$$\Rightarrow \frac{(0.75)^3}{1-0.75}+\frac{1^3-(0.75)^3}{1-0.75}$$

$$\left[\because a^3-b^3=(a-b)\ (a^2+b^2+ab)\right]$$

$$\Rightarrow \frac{(0.75)^3+1-(0.75)^3}{0.25}$$

$$\Rightarrow \frac{1}{0.25} \Rightarrow 4$$

8. (d) According to the question,

$$\sqrt{x}\div\sqrt{441}=0.02$$

$$\Rightarrow \frac{\sqrt{x}}{\sqrt{441}}=0.02$$

$$\Rightarrow \frac{\sqrt{x}}{21}=\frac{2}{100}$$

$$\Rightarrow \sqrt{x}=\frac{42}{100}$$

Squaring both sides.

$\Rightarrow$ x = 0.1764

9. (d) According to the question,

3	5808
2	1936
2	968
2	484
2	242
11	121
11	11
	1

Factors are:

$\boxed{3}$ 2, 2, 2, 2, 11, 11

Smallest number is = 3

10. (b) According to the question,

$$\frac{\sqrt[3]{8}}{\sqrt{16}}\div\sqrt{\frac{100}{49}}\times\sqrt[3]{125}$$

$$\Rightarrow \frac{2}{4}\div\frac{10}{7}\times 5$$

$$\Rightarrow \frac{2}{4}\times\frac{7}{10}\times 5$$

$$\Rightarrow \frac{7}{4}=1\frac{3}{4}$$

11. (d) According to the question,

3	1323
3	441
3	147
7	49
7	7
	1

Factors are: 3, 3, 3, 7, 7, $\boxed{7}$

$\therefore$ Smallest number is = 7

12. (a) According to the question,

$$(100)^{\frac{1}{2}}\times(0.001)^{\frac{1}{3}}-(0.0016)^{\frac{1}{4}}\times 3^{0}+\left(\frac{5}{4}\right)^{-1}$$

$\Rightarrow 10 \times 0.1 - 0.2 \times 1 + \dfrac{4}{5}$

$\Rightarrow 10 \times 0.1 - 0.2 + \dfrac{4}{5}$

$\Rightarrow 1 - \dfrac{2}{10} + \dfrac{4}{5}$

$\Rightarrow \dfrac{5-1+4}{5}$

$\Rightarrow \dfrac{8}{5} = 1.6$

13. (a) According to the question,

$\Rightarrow \dfrac{\frac{1}{2}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}}{\frac{2}{5}-\frac{5}{9}+\frac{3}{5}-\frac{7}{18}}$

$\Rightarrow \dfrac{\frac{30-15+12-10}{60}}{\frac{36-50+54-35}{90}}$

$\Rightarrow \dfrac{17}{60}\times\dfrac{90}{5} = 5\dfrac{1}{10}$

14. (c) According to the question,

$\Rightarrow \sqrt{(272)^2-(128)^2}$

$\Rightarrow \sqrt{(272+128)(272-128)}$

$\left[\because a^2-b^2=(a+b)(a-b)\right]$

$\Rightarrow \sqrt{400\times 144}$

$\Rightarrow 20\times 12 = 240$

15. (d) According to the question,

$\dfrac{1}{3}\sqrt{x} = 0.001$

$\sqrt{x} = 0.003$

Squaring both sides

$x = 0.000009$

$\therefore$ Number is = 0.000009

16. (b) According to the question,

$\Rightarrow \sqrt[3]{\dfrac{72.9}{0.4096}}$

$\Rightarrow \sqrt[3]{\dfrac{729\times 10000}{4096\times 10}}$

$\Rightarrow \sqrt[3]{\dfrac{729}{4096}\times 1000}$

$\Rightarrow \dfrac{9}{16}\times 10$

$\Rightarrow 5.625$

17. (d) According to the question,

$\Rightarrow \dfrac{1}{3+\dfrac{1}{2-\dfrac{1}{\frac{7}{9}}}} + \dfrac{17}{22}$

$\Rightarrow \dfrac{1}{3+\dfrac{1}{2-\frac{9}{7}}} + \dfrac{17}{22}$

$\Rightarrow \dfrac{1}{3+\frac{7}{5}} + \dfrac{17}{22}$

$\Rightarrow \dfrac{5}{15+7} + \dfrac{17}{22}$

$\Rightarrow \dfrac{5}{22} + \dfrac{17}{22} \Rightarrow \dfrac{22}{22} = 1$

18. (c) According to the question,

If $x = 1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}}$

$\Rightarrow x = 1+\dfrac{1}{1+\dfrac{1}{1+\frac{2}{3}}}$

$\Rightarrow x = 1+\dfrac{1}{1+\frac{3}{5}}$

$\Rightarrow x = 1+\dfrac{5}{8}$

$\Rightarrow x = \dfrac{13}{8}$

$\therefore$ Value of $2x+\dfrac{7}{4}$ is

$\Rightarrow 2\times\dfrac{13}{8}+\dfrac{7}{4}$

$\Rightarrow \dfrac{13}{4}+\dfrac{7}{4}$

$\Rightarrow \dfrac{20}{4} = 5$

19. (d) According to the question,

$\Rightarrow \dfrac{19}{43}+\dfrac{1}{2+\dfrac{1}{3+\dfrac{1}{1+\frac{1}{4}}}}$

$\Rightarrow \dfrac{19}{43}+\dfrac{1}{2+\dfrac{1}{3+\frac{4}{5}}}$

$\Rightarrow \dfrac{19}{43}+\dfrac{1}{2+\frac{5}{19}}$

$\Rightarrow \dfrac{19}{43}+\dfrac{19}{43} \Rightarrow \dfrac{38}{43}$

20. (a) According to the question,

$\Rightarrow 8\dfrac{1}{2}-\left[3\dfrac{1}{4}+\left\{1\dfrac{1}{4}-\dfrac{1}{2}\left(1\dfrac{1}{2}-\dfrac{1}{3}-\dfrac{1}{6}\right)\right\}\right]$

$\Rightarrow \dfrac{17}{2}-\left[\dfrac{13}{4}+\left\{\dfrac{5}{4}-\dfrac{1}{2}\left(\dfrac{3}{2}-\dfrac{1}{3}-\dfrac{1}{6}\right)\right\}\right]$

$\Rightarrow \dfrac{17}{2}-\left[\dfrac{13}{4}+\left\{\dfrac{5}{4}-\dfrac{1}{2}\left(\dfrac{9-2-1}{6}\right)\right\}\right]$

$\Rightarrow \dfrac{17}{2}-\left[\dfrac{13}{4}+\left\{\dfrac{5}{4}-\dfrac{1}{2}\times 1\right\}\right]$

$\Rightarrow \dfrac{17}{2}-\left[\dfrac{13}{4}+\dfrac{5-2}{4}\right]$

$\Rightarrow \dfrac{17}{2}-\left[\dfrac{13}{4}+\dfrac{3}{4}\right]$

$\Rightarrow \dfrac{17}{2}-\dfrac{16}{4}$

$\Rightarrow \dfrac{34-16}{4} = \dfrac{18}{4}$

$\Rightarrow \dfrac{9}{2} = 4\dfrac{1}{2}$

21. (d) According to the question,

$\Rightarrow \dfrac{50}{x} = \dfrac{x}{12\frac{1}{2}}$

$\Rightarrow 50 \times \frac{25}{2} = x^2$

$\Rightarrow$ $x^2 = 25 \times 25$

$\therefore$ x = 25

22. (a) According to the question,

$$\Rightarrow \frac{1}{9}+\underset{2\ 3}{\frac{1}{6}}+\underset{3\ 4}{\frac{1}{12}}+\underset{4\ 5}{\frac{1}{20}}+\underset{5\ 6}{\frac{1}{30}}+\underset{6\ 7}{\frac{1}{42}}+\underset{7\ 8}{\frac{1}{56}}+\underset{8\ 9}{\frac{1}{72}}$$

$$\Rightarrow \frac{1}{9}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+\frac{1}{6}-\frac{1}{7}$$

$$+\frac{1}{7}-\frac{1}{8}+\frac{1}{8}-\frac{1}{9} \quad \Rightarrow \frac{1}{2}$$

23. (c) According to the question.

$\Rightarrow$ 25–5[2+3(2–2(5–3)+5)–10]÷4

$\Rightarrow$ 25–5[2+3(2–2×2+5)–10]÷4

$\Rightarrow$ 25 – 5 [2 + 3 (2 – 4 + 5) – 10] ÷ 4

$\Rightarrow$ 25 – 5 [2+3 × 3– 10] ÷ 4

$\Rightarrow$ 25 – 5 [11– 10] ÷ 4

$\Rightarrow 25 - \frac{5}{4}$

$\Rightarrow \frac{100-5}{4}$

$\Rightarrow \frac{95}{4} = 23.75$

24. (b) $x = \frac{1}{2+\frac{1}{2}} = \frac{2}{5} \quad \Rightarrow \frac{1}{x} = \frac{5}{2}$

25. (a) According to the question

$$\Rightarrow \frac{9}{20} - \left[\frac{1}{5} + \left\{\frac{1}{4} + \left(\frac{5}{6} - \overline{\frac{1}{3}+\frac{1}{2}}\right)\right\}\right]$$

$$\Rightarrow \frac{9}{20} - \left[\frac{1}{5} + \left\{\frac{1}{4} + \left(\frac{5}{6} - \frac{5}{6}\right)\right\}\right]$$

$$\Rightarrow \frac{9}{20} - \left[\frac{1}{5} + \left\{\frac{1}{4} + 0\right\}\right]$$

$$\Rightarrow \frac{9}{20} - \left[\frac{1}{5} + \frac{1}{4}\right]$$

$$\Rightarrow \frac{9}{20} - \frac{9}{20} = 0$$

26. (b) According to the question,

$$\Rightarrow \sqrt{\frac{(0.1)^2+(0.01)^2+(0.009)^2}{(0.01)^2+(0.001)^2+(0.0009)^2}}$$

$$\Rightarrow \sqrt{\frac{0.01+0.0001+0.000081}{0.0001+0.000001+0.00000081}}$$

$$\Rightarrow \sqrt{\frac{0.010181}{0.00010181}}$$

$\Rightarrow \sqrt{100} = 10$

27. (b) According to the question,

$$\Rightarrow \sqrt{\frac{(0.03)^2+(0.21)^2+(0.065)^2}{(0.003)^2+(0.021)^2+(0.0065)^2}}$$

$$\Rightarrow \sqrt{\frac{0.0009+0.0441+0.004225}{0.000009+0.000441+0.00004225}}$$

$$\Rightarrow \sqrt{\frac{0.049225}{0.00049225}} \quad \Rightarrow \sqrt{100} = 10$$

28. (d) According to the question,

$$\Rightarrow \sqrt{104.04} + \sqrt{1.0404} + \sqrt{0.010404}$$

$$\Rightarrow \sqrt{\frac{10404}{100}} + \sqrt{\frac{10404}{10000}} + \sqrt{\frac{10404}{1000000}}$$

$$\Rightarrow \frac{102}{10} + \frac{102}{100} + \frac{102}{1000}$$

$\Rightarrow$ 10.2+1.02+0.102 = 11.322

29. (c) According to the question,

$$\Rightarrow \sqrt{40.96}+\sqrt{0.4096}+\sqrt{0.004096}+\sqrt{0.00004096}$$

$$\Rightarrow \sqrt{\frac{4096}{100}}+\sqrt{\frac{4096}{10000}}+\sqrt{\frac{4096}{1000000}}+\sqrt{\frac{4096}{100000000}}$$

$$\Rightarrow \frac{64}{10}+\frac{64}{100}+\frac{64}{1000}+\frac{64}{10000}$$

$\Rightarrow$ 6.4+0.64+0.064+0.0064 = 7.11

30. (a) According to the question,

As we know that the square of 252 is which is near the value of 63522

$\therefore$ 63522 – x = 63504

x = 18

31. (d) According to the question,

$$\begin{array}{r|l} 4 & 20184 \\ \hline 3 & 5046 \\ \hline 2 & 1682 \\ \hline 29 & 841 \\ \hline 29 & 29 \\ \hline & 1 \end{array}$$

Factors are 2, 2 3, 2, 29, 29

$\therefore$ It should be multiplied by = 6

32. (b) According to the question,

$$\text{If } 2 = x + \cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4}}}$$

$$\Rightarrow 2 = x + \cfrac{1}{1+\cfrac{4}{12+1}}$$

$$\Rightarrow 2 = x + \cfrac{1}{1+\cfrac{4}{13}}$$

$$\Rightarrow 2 = x + \frac{13}{17}$$

$$\Rightarrow x = 2 - \frac{13}{17}$$

$$\Rightarrow x = \frac{34-13}{17} \quad \Rightarrow x = \frac{21}{17}$$

33. (c) According to the question

$$\left[\left(1+\cfrac{1}{10+\cfrac{1}{10}}\right)\times\left(1+\cfrac{1}{10+\cfrac{1}{10}}\right)-\left(1-\cfrac{1}{10+\cfrac{1}{10}}\right)\times\left(1-\cfrac{1}{10+\cfrac{1}{10}}\right)\right]$$

$$\div \left[\left(1+\cfrac{1}{10+\cfrac{1}{10}}\right)+\left(1-\cfrac{1}{10+\cfrac{1}{10}}\right)\right]$$

$$\text{Let } 1+\cfrac{1}{10+\cfrac{1}{10}} = \frac{111}{101} = a$$

$$1-\cfrac{1}{10+\cfrac{1}{10}} = \frac{91}{101} = b$$

$$\Rightarrow \frac{a^2-b^2}{a+b} \qquad [\because a^2-b^2 = (a+b)(a-b)]$$

$$\Rightarrow \frac{(a+b)(a-b)}{a+b}$$

$\Rightarrow$ (a – b)

$$\Rightarrow \frac{111}{101} - \frac{91}{101}$$

$$\Rightarrow \frac{20}{101}$$

34. (d) According to the Question

$$\Rightarrow \frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}+\frac{1}{110}$$

$$\Rightarrow \frac{1}{5}-\frac{1}{6}+\frac{1}{6}-\frac{1}{7}+\frac{1}{7}-\frac{1}{8}+\frac{1}{8}-\frac{1}{9}+\frac{1}{9}-\frac{1}{10}+\frac{1}{10}-\frac{1}{11}$$

$$\Rightarrow \frac{1}{5}-\frac{1}{11} \quad \Rightarrow \frac{11-5}{55} \quad \Rightarrow \frac{6}{55}$$

35. (b) According to the question,

$1 \div \left[1+1 \div \left\{1+1 \div (1+1 \div 2)\right\}\right]$

$\Rightarrow 1 \div \left[1+1 \div \left\{1+1 \div \left(1+\frac{1}{2}\right)\right\}\right]$

$\Rightarrow 1 \div \left[1+1 \div \left\{1+1 \div \frac{3}{2}\right\}\right]$

$\Rightarrow 1 \div \left[1+1 \div \left\{1+\frac{2}{3}\right\}\right]$

$\Rightarrow 1 \div \left[1+1 \div \frac{5}{3}\right]$

$\Rightarrow 1 \div \left[1+\frac{3}{5}\right]$

$\Rightarrow 1 \div \left[\frac{8}{5}\right] \quad \Rightarrow \frac{5}{8}$

36. (d) According to question,

$3.\overline{36} - 2.\overline{05} + 1.\overline{33}$

$\Rightarrow$ 3.363636...–2.050505... + 1.333333...

$\Rightarrow$ 2.646464...

$\Rightarrow 2.\overline{64}$

37. (a) According to the question,

$(0.2 \times 0.2 + 0.01)(0.1 \times 0.1 + 0.02)^{-1}$

$\Rightarrow \frac{0.2 \times 0.2 + 0.01}{0.1 \times 0.1 + 0.02}$

$\Rightarrow \frac{0.04 + 0.01}{0.01 + 0.02}$

$\Rightarrow \frac{0.05}{0.03} = \frac{5}{3}$

38. (a) According to the question,

$\sqrt{5+\sqrt{11+\sqrt{19+\sqrt{29+\sqrt{49}}}}}$

$\Rightarrow \sqrt{5+\sqrt{11+\sqrt{19+\sqrt{29+7}}}}$

$\Rightarrow \sqrt{5+\sqrt{11+\sqrt{19+\sqrt{36}}}}$

$\Rightarrow \sqrt{5+\sqrt{11+\sqrt{19+6}}}$

$\Rightarrow \sqrt{5+\sqrt{11+\sqrt{25}}}$

$\Rightarrow \sqrt{5+\sqrt{11+5}}$

$\Rightarrow \sqrt{5+\sqrt{16}}$

$\Rightarrow \sqrt{9} = 3$

39. (d) According to the question,

$\Rightarrow \sqrt[3]{\frac{7}{875}}$

$\Rightarrow \sqrt[3]{\frac{1}{125}} \Rightarrow \frac{1}{5}$

40. (d) According to the question,

$$\frac{2}{2+\frac{2}{3+\frac{2}{3+\frac{2}{3}}}} \times 0.39$$

$$\Rightarrow \frac{2}{2+\frac{2}{3+\frac{6}{11}}} \times 0.39$$

$$\Rightarrow \frac{2}{2+\frac{22}{39} \times 0.39}$$

$$\Rightarrow \frac{2}{2+\frac{22}{100}}$$

$$\Rightarrow \frac{200}{222} = \frac{100}{111}$$

41. (b) According to the question,

$$\Rightarrow \frac{-\frac{1}{2}-\frac{2}{3}+\frac{4}{5}-\frac{1}{3}+\frac{1}{5}+\frac{3}{4}}{\frac{1}{2}+\frac{2}{3}-\frac{4}{3}+\frac{1}{3}-\frac{1}{5}-\frac{4}{5}}$$

$$\Rightarrow \frac{\frac{-30-40+48-20+12+45}{60}}{\frac{15+20-40+10-6-24}{30}}$$

$$\Rightarrow \frac{15}{60} \times \frac{-30}{25} \Rightarrow \frac{-3}{10}$$

42. (b) According to the question,

$\Rightarrow 0.\overline{63} + 0.\overline{37} + 0.\overline{80}$

$\Rightarrow$ 0.6363...+0.3737....+0.8080...

$\Rightarrow 1.\overline{81}$

43. (b) According to the question,

$\Rightarrow \sqrt{0.\bar{4}}$

$\Rightarrow \sqrt{\frac{4}{9}}$

$\Rightarrow \frac{2}{3}$

$\Rightarrow$ 0.66666.....

$\Rightarrow 0.\bar{6}$

44. (d) According to the question,

$\Rightarrow \sqrt{(7+3\sqrt{5})(7-3\sqrt{5})}$

$\Rightarrow \sqrt{49-45} \Rightarrow \sqrt{4} = 2$

45. (d) According to the question,

$$\Rightarrow \left(4\frac{11}{15}+\frac{15}{71}\right)^2 - \left(4\frac{11}{15}-\frac{15}{71}\right)^2$$

$$\Rightarrow \left(4\frac{11}{15}+\frac{15}{71}+4\frac{11}{15}-\frac{15}{71}\right)\left(4\frac{11}{15}+\frac{15}{71}-4\frac{11}{15}+\frac{15}{71}\right)$$

$$\Rightarrow \left(2 \times 4\frac{11}{15}\right)\left(2 \times \frac{15}{71}\right)$$

$$\Rightarrow \left(2 \times \frac{71}{15} \times 2 \times \frac{15}{71}\right) \Rightarrow 4$$

46. (d) According to the question,

$$\Rightarrow \frac{1}{3+\frac{1}{1+\frac{1}{2+\frac{1}{4}}}} \Rightarrow \frac{1}{3+\frac{1}{1+\frac{4}{9}}}$$

$$\Rightarrow \frac{1}{3+\frac{9}{13}} \Rightarrow \frac{13}{48} \quad \text{Satisfied}$$

47. (d) According to the question,

$$\Rightarrow \left(\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\frac{1}{9.11}+\frac{1}{11.13}+\frac{1}{13.15}\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\frac{1}{7}-\frac{1}{9}+\frac{1}{9}-\frac{1}{11}+\frac{1}{11}-\frac{1}{13}+\frac{1}{13}-\frac{1}{15}\right)$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{3}-\frac{1}{15}\right] \Rightarrow \frac{1}{2}\left[\frac{5-1}{15}\right]$$

$$\Rightarrow \frac{1}{2} \times \frac{4}{15} = \frac{2}{15}$$

48. (c) According to the question,

$\Rightarrow \sqrt{\underline{6\,2}\;\underline{5\,6}\;\underline{8\,6}\;\underline{7\,3}\;\underline{4\,4}\;\underline{8\,9}} = 6$

NOTE: For counting the digits of square root we make pairs first. Then the digits will be equal to number of pairs.

49. (b) Let the number of boys = x

the number of girls = y

According to the question,

$\Rightarrow x^2 - y^2 = 28$ ....(i)

$\Rightarrow x = y + 2$

$\Rightarrow x - y = 2$ ....(ii)

From eq. (i)

$\Rightarrow x^2 - y^2 = 28$

$\Rightarrow (x + y)(x - y) = 28$

$\Rightarrow (x + y)\times 2 = 28$

$\Rightarrow x + y = 14$

50. (b) According to the question,

$\Rightarrow \sqrt[2]{0.014\times 0.14x} = 0.014\times 0.14\sqrt[2]{y}$

Squaring both sides

$\Rightarrow 0.014\times 0.14x = (0.014)^2 \times (0.14)^2 \times y$

$\Rightarrow \frac{x}{y} = 0.014 \times 0.14$

$\Rightarrow \frac{x}{y} = 0.00196$

51. (d) According to the question

$\Rightarrow \left[\frac{(0.1)^2 - (0.01)^2}{0.0001}\right]+1$

$\Rightarrow \left[\frac{(0.1+0.01)(0.1-0.01)}{0.0001}\right]+1$

$\Rightarrow \frac{0.11\times 0.09}{0.0001}+1$

$\Rightarrow 99 + 1 = 100$

52. (c) According to the question

$\Rightarrow \frac{(100-1)(100-2)(100-3)..........(100-200)}{100\times 99\times 98\times ....\times 3\times 2\times 1}$

$\Rightarrow \frac{99\times 98\times 97\times ....1\times 0\times(-1)\times(-2).......(-100)}{100\times 99\times 98\times .....3\times 2\times 1}$

$= 0$

53. (c) According to the question

$\Rightarrow 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{2}{3}}}}}$

$\Rightarrow 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{3}{5}}}} \Rightarrow 1+\cfrac{1}{1+\cfrac{1}{1+\frac{5}{8}}}$

$\Rightarrow 1+\cfrac{1}{1+\frac{8}{13}} \Rightarrow 1+\frac{13}{21} \Rightarrow \frac{34}{21}$

54. (a) According to the question

$= \cfrac{2\frac{1}{3}-1\frac{2}{11}}{3+\cfrac{1}{3+\cfrac{1}{3+\frac{1}{3}}}} \Rightarrow \cfrac{\frac{7}{3}-\frac{13}{11}}{3+\cfrac{1}{3+\frac{3}{10}}}$

$\Rightarrow \cfrac{\frac{77-39}{33}}{3+\frac{10}{33}} \Rightarrow \frac{38}{33}\times\frac{33}{109}$

$\Rightarrow \frac{38}{109}$

55. (a) According to the question

$\Rightarrow \sqrt{8+\sqrt{57+\sqrt{38+\sqrt{108+\sqrt{169}}}}}$

$\Rightarrow \sqrt{8+\sqrt{57+\sqrt{38+\sqrt{108+13}}}}$

$\Rightarrow \sqrt{8+\sqrt{57+\sqrt{38+\sqrt{121}}}}$

$\Rightarrow \sqrt{8+\sqrt{57+\sqrt{38+11}}}$

$\Rightarrow \sqrt{8+\sqrt{57+\sqrt{49}}}$

$\Rightarrow \sqrt{8+\sqrt{57+7}} \Rightarrow \sqrt{8+\sqrt{64}}$

$\Rightarrow \sqrt{8+8} \Rightarrow \sqrt{16} = 4$

56. (c) According to the question

$p - q = 5$ ......(i)

$p^2 + q^2 = 55$ ....(ii)

$\therefore (p-q)^2 = p^2 + q^2 - 2pq$

$(5)^2 = 55 - 2pq$

$25 = 55 - 2pq$

$2pq = 30 \Rightarrow pq = 15$

57. (c) Take a first part

$\cfrac{4\frac{2}{7}-\frac{1}{2}}{3\frac{1}{2}+1\frac{1}{7}} = \cfrac{\frac{30}{7}-\frac{1}{2}}{\frac{7}{2}+\frac{8}{7}} = \cfrac{\frac{60-7}{14}}{\frac{49+16}{14}} = \frac{53}{65}$

Second part

$\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{5-\frac{1}{5}}}} = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{\frac{24}{5}}}}$

$= \cfrac{1}{2+\cfrac{1}{2+\frac{5}{24}}} = \cfrac{1}{2+\cfrac{1}{\frac{53}{24}}} = \cfrac{1}{2+\frac{24}{53}}$

$= \cfrac{1}{\frac{130}{53}} = \frac{53}{130}$

According to question,

$= \frac{53}{65} \div \frac{53}{130}$

$= \frac{53}{65}\times\frac{130}{53} = \frac{130}{65} = 2$

58. (c) $4-\cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2+\frac{1}{4}}}}$

$= 4-\cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{\frac{9}{4}}}}$

$= 4-\cfrac{5}{1+\cfrac{1}{3+\frac{4}{9}}} = 4-\cfrac{5}{1+\cfrac{1}{\frac{31}{9}}}$

$= 4-\cfrac{5}{1+\frac{9}{31}} = 4-\cfrac{5}{\frac{40}{31}}$

$= 4-\frac{5\times 31}{40} = 4-\frac{31}{8} = \frac{1}{8}$

According to question

$\frac{1}{8}$ part = 10 minutes

1 part = 80 minutes

$\frac{3}{5}$ part = $80\times\frac{3}{5}$ = 48 minutes

59. (a) Take first part

$= \cfrac{4\frac{1}{7}-2\frac{1}{4}}{3\frac{1}{2}+1\frac{1}{7}} = \cfrac{\frac{29}{7}-\frac{9}{4}}{\frac{7}{2}+\frac{8}{7}}$

$$= \frac{\frac{116-63}{28}}{\frac{49+16}{14}} = \frac{53}{28} \times \frac{14}{65} = \frac{53}{130}$$

Take second part

$$\frac{1}{2+\frac{1}{2+\frac{1}{5-\frac{1}{5}}}} = \frac{1}{2+\frac{1}{2+\frac{1}{\frac{25-1}{5}}}}$$

$$= \frac{1}{2+\frac{1}{2+\frac{5}{24}}} = \frac{1}{2+\frac{1}{\frac{53}{24}}}$$

$$= \frac{1}{2+\frac{24}{53}} = \frac{1}{\frac{106+24}{53}} = \frac{53}{130}$$

According to the question

$$\sqrt{\frac{53}{130} \div \frac{53}{130}} = \sqrt{\frac{53}{130} \times \frac{130}{53}} = \sqrt{1} = 1$$

60. (c) $\frac{1}{1+2^{a-b}} + \frac{1}{1+2^{b-a}}$

$$= \frac{1}{1+2^{a-b}} + \frac{1}{1+2^{-(a-b)}}$$

$$= \frac{1}{1+2^{a-b}} + \frac{2^{a-b}}{2^{a-b}+1} = \frac{1+2^{a-b}}{1+2^{a-b}} = 1$$

61. (b) $\left(1-\frac{1}{n+1}\right)+\left(1-\frac{2}{n+1}\right)+\left(1-\frac{3}{n+1}\right)+..$

$.........+\left(1-\frac{n}{n+1}\right)$

$$= \left(\frac{n+1-1}{n+1}\right)+\left(\frac{n+1-2}{n+1}\right)+\left(\frac{n+1-3}{n+1}\right).........$$

$$+\left(\frac{n+1-n}{n+1}\right)$$

$$= \frac{n}{n+1}+\frac{n-1}{n+1}+\frac{n-2}{n+1}.......+\frac{1}{n+1}$$

$$= \frac{1}{n+1}(n+(n-1)+(n-2)...........1)$$

$$\left\{1+2+3.....n = \frac{n(n+1)}{2}\right\}$$

$$= \frac{1}{n+1}\left(\frac{n(n+1)}{2}\right) = \frac{n}{2}$$

62. (d) $33 - 4\sqrt{35}$

$= 33 - 2 \times 2\sqrt{35}$

$= 33 - 2 \times 2 \times \sqrt{7} \times \sqrt{5}$

$= 28 + 5 - 2 \times 2\sqrt{7} \times \sqrt{5}$

$= (2\sqrt{7})^2 + (\sqrt{5})^2 - 2 \times 2\sqrt{7} \times \sqrt{5}$

$= (2\sqrt{7} - \sqrt{5})^2$

$\sqrt{33-4\sqrt{35}} = \pm\sqrt{(2\sqrt{7}-\sqrt{5})^2}$

$= \pm(2\sqrt{7}-\sqrt{5})$

63. (a) $(a-b)^2 = a^2 + b^2 - 2ab$
$16a^2-12a$

$= (4a)^2 - 2\times 4a \times \frac{3}{2} + \left(\frac{3}{2}\right)^2$

Number be added $= \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

64. (b) $x\ [-2\{-4\ (-a)\}]+5\ [-2\{-2(-a)\}] = 4a$

$x\ [-2\{4a\}] + 5[-2\{2a\}] = 4a$

$x\ [-8a] + 5[-4a] = 4a$

$-8ax - 20a = 4a$

$-8ax = 24a$

$x = -\frac{24a}{8a}$

$x = -3$

65. (a) $a = 64$, $b = 289$

$\sqrt{a} = 8$, $\sqrt{b} = 17$

$\therefore \left(\sqrt{\sqrt{a}+\sqrt{b}} - \sqrt{\sqrt{b}-\sqrt{a}}\right)^{\frac{1}{2}}$

$= \left(\sqrt{8+17} - \sqrt{17-8}\right)^{\frac{1}{2}}$

$= \left(\sqrt{25}-\sqrt{9}\right)^{\frac{1}{2}} = (5-3)^{1/2} = 2^{\frac{1}{2}}$

66. (d) $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

$= \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} \times \frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

$+ \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{3} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{4}$

$+ \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{1}$

$= 2\sqrt{3} - \sqrt{6} - 3\sqrt{2} + \sqrt{6} + 3\sqrt{2} - 2\sqrt{3}$

$= 0$

67. (b) $\frac{4-\sqrt{0.04}}{4+\sqrt{0.4}}$

$= \frac{4-0.2}{4+0.6} = \frac{3.8}{4.6} = \frac{38}{46} = \frac{19}{23} = 0.8$

68. (b) $\sqrt{0.05 \times 0.5 \times a} = 0.5 \times 0.05 \times \sqrt{b}$

Squaring both sides

$0.05 \times 0.5 \times a = 0.5 \times 0.5 \times 0.05 \times 0.05 \times b$

$a = 0.5 \times 0.05 \times b$

$\frac{a}{b} = \frac{5 \times 5}{10 \times 100}$

$\frac{a}{b} = \frac{1}{40} = 0.025$

69. (a) $(1001)^3$

$= 1001 \times 1001 \times 1001$

$= 1002001 \times 1001$

$= 1003003001$

70. (a). $5.\bar{6} + 7.\bar{3} + 8.\bar{7} + 6.\bar{1}$

$= 5+\frac{6}{9}+7+\frac{3}{9}+8+\frac{7}{9}+6+\frac{1}{9}$

$= 26 + \left(\frac{6+3+7+1}{9}\right)$

$= 26 + \frac{17}{9}$

$= 26 + 1 + \frac{8}{9} = 27\frac{8}{9} = 27.\bar{8}$

71. (b) $6.\overline{74} + 7.\overline{32}$

$= 6+\frac{74}{99}+7+\frac{32}{99}$

$= 13+\frac{74+32}{99} = 13+\frac{106}{99}$

$= 13+1+\frac{7}{99} = 14\frac{7}{99}$

$= 14.\overline{07}$

72. (b) $0.9 = 0.9$

$0.\overline{9} = 0.999$ ....

$0.0\overline{9} = 0.09999$.......

$0.\overline{09} = 0.090909$........

$0.\overline{9}$ is the greatest of all

73. (b) $0.\overline{001} = \frac{1}{999}$

74. (c) $1.\overline{27} = 1 + \frac{27}{99}$

$= 1 + \frac{3}{11} = \frac{14}{11}$

75. (c) $8.3\overline{1} + 0.\overline{6} + 0.00\overline{2}$

xx	x	xx x
8. 31	1	1 1 1
0. 66	6	6 6 6
0. 00	2	2 2 2
8.97	9	9 9 9

$= 8.979999$

$= 8.97\overline{9}$

76. (d) $(5.\overline{76}) - (2.\overline{3})$

$= \left(5 + \frac{76}{99}\right) - \left(2 + \frac{3}{9}\right)$

$= 5 + \frac{76}{99} - 2 - \frac{3}{9}$

$= 3 + \left(\frac{76}{99} - \frac{3}{9}\right)$

$= 3.\overline{43}$

77. (b) $0.\overline{63} + 0.\overline{37}$

$= \frac{63}{99} + \frac{37}{99} = \frac{100}{99}$

CHAPTER 09

LINER EQUATIONS IN TWO VARIABLES

- **Linear Equations in Two Variables:** An equation of the form $ax + by + c = 0$ where a, b, c $\in$ R(real numbers) and $a \neq 0$, $b \neq 0$ and x, y are variables is called a linear equation in two variables.

  Examples : Each of the following equations is a linear equation :

  (i) $4x + 7y = 13$
  (ii) $2x - 5y = 36$
  (iii) $\sqrt{3}x - \sqrt{7}y = 2$

  The condition $a \neq 0$, $b \neq 0$, is often denoted by $a^2 + b^2 \neq 0$

Note: The graph of a linear equation $ax + by + c = 0$, is a straight line.

Solution of linear equation : Any pair of values of x and y which satisfy the equation $ax + by + c = 0$, is called its solution.

E.g.: show that $x = 2$ and $y = 1$ is a solution of $2x + 5y = 9$

Sol: Substituting $x = 2$ and $y = 1$ in the given equation, we get LHS $= 2 \times 2 + 5 \times 1 = 9 =$ RHS

$\therefore$ $x = 2$, $y = 1$ is a solution of $2x + 5y = 9$

- **System of Linear Equations:**

  Consistent System :- A system consisting of two simultaneous linear equations is said to be consistent, if it has at least one solutions.

  Inconsistent System : A system consisting of two simultaneous linear equations is said to be inconsistent, if it has no solution at all.

E.g.: Consider the system of equations: $x + y = 9$ & $3x + 3y = 5$. Clearly, there are no values of x and y which may simulatneously satisfy the given equations. So, the system given above is inconsistent.

Conditions for Solvability :

The system of equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ has :

(i) a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) an infinite number of solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iii) no solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Homogenous System of Equations:

The system of equations $a_1x + b_1y = 0$; $a_2x + b_2y = 0$ has

(i) only solution $x = 0$, $y = 0$, when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) an infinite number of solutions when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

- The graphs of $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + C_2 = 0$ will be :

(i) Parallel, if the system has no Solution ;
(ii) Coincident, if the system has infinite number of solutions ;
(iii) Intersecting, if the system has a unique solution.

EXERCISE

1. If $11x - 13 = -2x + 78$, then $x = ?$
   (a) 7 (b) 8 (c) 6 (d) 4
2. If $2x + 3y = 29$ and $y = x + 3$, what is the value of x ?
   (a) 5 (b) 6 (c) 4 (d) 7
3. If $2x + 3y = 5$ and $x = -2$, then the value of y is :
   (a) $\frac{1}{3}$ (b) 3 (c) 1 (d) 9
4. The value of $x + y$ in the solution of the equations $\frac{x}{4} + \frac{y}{3} = \frac{5}{12}$ and $\frac{x}{2} + y = 1$
   (a) $\frac{1}{2}$ (b) 2 (c) $\frac{5}{2}$ (d) $\frac{3}{2}$
5. If $2x + 3y = 12$ and $3x - 2y = 5$, then x and y must have the values :
   (a) 2 and 3 (b) 2 and -3
   (c) 3 and -2 (d) 3 and 2
6. The equations $ax + b = 0$ and $cx + d = 0$ are consistent, if :
   (a) ad = bc (b) ad + bc = 0
   (c) ab - cd = 0 (d) ab + cd = 0
7. The equations $2x + y = 5$ and $x + 2y = 4$ are
   (a) consistent and have infinitely many solutions
   (b) consistent and have a unique solution.
   (c) inconsistent
   (d) none of these
8. The cost of 2 sarees and 4 shirts is Rs. 16000 while 1 saree and 6 shirts cost the same. The cost of 12 shirts is :
   (a) Rs. 12,000 (b) Rs. 24,000
   (c) Rs. 48,000
   (d) Can't be determined
9. The system of equations $kx - y = 2$ and $6x - 2y = 3$ has a unique solution when :
   (a) K = 0 (b) K $\neq$ 0
   (c) K = 3 (d) K $\neq$ 3
10. The value of y in the solution of the equation $2^{x+y} = 2^{x-y} = \sqrt{8}$ is :
    (a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

11. The solutions of the equations $\frac{3x-y+1}{3}=\frac{2x+y+2}{5}=\frac{3x+2y+1}{6}$ is
(a) $x = 2, y = 1$ (b) $x = 1, y = 1$
(c) $x = -1, y = -1$ (d) $x = 1, y = 2$

12. If $x + 2y \le 3$, $x > 0$ and $y > 0$, then one of the solutions is :
(a) $x = -1, y = 2$
(b) $x = 2, y = 1$
(c) $x = 1, y = 1$
(d) $x = 0, y = 0$

13. A purse contains 25 paise and 10 paise coins. The total amount in the purse is ₹ 8.25. If the number of 25 paise coins is one- third of the number of 10 paise coins in the purse, then the total number of coins in the purse:
(a) 30 (b) 40 (c) 45 (d) 60

14. The value of k for which the system of equations $x + 2y = 5$, $3x + ky + 15 = 0$ has no solution, is:
(a) 6 (b) -6 (c) 2 (d) 4

15. The equations $2x - 5y = 9$ and $8x - 20y = 36$ have :
(a) no common solution
(b) exactly one common solution
(c) exactly two common solutions
(d) more than two common solutions

16. The difference between two numbers is 5 and the difference between their squares is 65. The larger number is :
(a) 9 (b) 10 (c) 11 (d) 12

17. The number of solutions of the equations $x+\frac{1}{y}=2$ and $2xy - 3y = -2$ is :
(a) 0 (b) 1 (c) 2
(d) None of these

18. If $2^a + 3^b = 17$ and $2^{a+2} - 3^{b+1} = 5$, then:
(a) a = 2, b = 3 (b) a= -2, b = 3
(c) a = 2, b = -3 (d) a = 3, b = 2

19. The solution to the system of equations $|x+y| = 1$ and $x - y = 0$ is given by:
(a) $x=y=\frac{1}{2}$ (b) $x=y=-\frac{1}{2}$
(c) $x=y=\frac{1}{2}$ or $x=y=-\frac{1}{2}$
(d) $x = 1, y = 0$

ANSWER KEY

1. (a)	3. (b)	5. (d)	7. (b)	9. (d)	11. (b)	13. (d)	15. (d)	17. (d)	19. (c)
2. (c)	4. (d)	6. (a)	8. (b)	10. (a)	12. (c)	14. (a)	16. (a)	18. (d)	

SOLUTION

1. (a) $11x - 13 = -2x + 78$
$\Rightarrow 11x + 2x = 78 + 13$
$\Rightarrow 13x = 91$
$\Rightarrow x=\frac{91}{13}=7$

2.(c) Putting $y = x + 3$ in $2x + 3y = 29$, we get,
$2x + 3(x + 3) = 29 \Rightarrow 2x + 3x + 9 = 29$
$\Rightarrow 5x = 29 - 9 = 20 \Rightarrow x=\frac{20}{5}=4$

3.(b) Putting $x = -2$ in $2x + 3y = 5$, we get ;
$-4 + 3y = 5 \Rightarrow 3y = 5 + 4 = 9$
$\Rightarrow y=\frac{9}{3}=3$

4.(d) Given equations are :
$3x + 4y = 5$ ________(i) and
$x + 2y = 2$ ________(ii)
(i) - 2× (ii): $x = 5 - 4 = 1$
$\therefore$ from (ii) $2y = 2 - x = 2 - 1 = 1$
$\Rightarrow y=\frac{1}{2}$

$\therefore x+y=1+\frac{1}{2}=\frac{3}{2}$

5.(d) $2x + 3y = 12$ ________(i)
$3x - 2y = 5$ ________(ii)
(i)× 2 + (ii)× 3, we get ; $x = 3$
putting $x = 3$ in (i), we get $2\times 3+3y = 12$
$\Rightarrow 3y = 6 \Rightarrow y = 2$
$\therefore x = 3$ and $y = 2$

6.(a) The equations are consistent if
$\frac{a}{c}=\frac{b}{d}$
i.e. ad = bc

7.(b) $2x + y = 5$ ________(i)
$x + 2y = 4$ ________(ii)
On solving we get, $x = 2, y = 1$
Thus (b) is true

8.(b) Let cost of 1 saree = Rs. x & cost of 1 shirt = Rs. y
$\therefore 2x + 4y = 16000$ ....(i)
and $x + 6y = 16000$ ....(ii)
Multiplying (ii) by 2 and substracting (i) from it, we get,
$8y = 16000 \Rightarrow y = 2000$

$\therefore$ cost of 12 shirts = (Rs.2000× 12)
= Rs. 24000

9.(d) For a unique solution, we must have
$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$
$\Rightarrow\frac{k}{6}\neq\frac{-1}{-2}\Rightarrow k\neq\left(6\times\frac{1}{2}\right)\Rightarrow k\neq3$

10.(a) $2^{x+y} = 2^{x-y} = \sqrt{8} = 2^{3/2}$
$\Rightarrow x + y = \frac{3}{2}$ ....(i)
and $x - y = \frac{3}{2}$ .... (ii)
(i) – (ii)
$2y = 0 \Rightarrow y = 0$

11.(b) $\frac{3x-y+1}{3}=\frac{2x+y+2}{5}$
$\Rightarrow 5(3x-y+1)=3(2x+y+2)$

$\Rightarrow$ $9x - 8y = 1$ ....(i)

and $\frac{3x-y+1}{3} = \frac{3x+2y+1}{6}$

$= 2(3x - y + 1) = (3x + 2y + 1)$

$\Rightarrow 3x - 4y = -1$ ....(ii)

(i) - 2× (ii):-

$(9 - 6)x - 8y + 8y = 1 - (-2) \Rightarrow x = 1$

putting $x = 1$ in (i) we get, $9\times 1 - 8y = 1$

$\Rightarrow 8y = 8 \Rightarrow y = 1$

$\therefore$ $x = 1, y = 1$.

12. (c) Here we will go through options.
in option (a) $x < 0$ and
in option (d) $x = 0$
hence (a) and (d) can't be the required answer because both does not satisfy the given condition i.e. $x > 0$.
Now option (b) $x = 2, y = 1$, then $x + 2y = 2 + 2(1) = 4$ which is > 3
clearly, values of option (b) do not satisfy $x + 2y \leq 3$
option (c) $x = 1, y = 1$, then $x + 2y = 1 + 2 = 3 \leq 3$

So, $x = 1$, y = 1 is one of the solutions.

13. (d) Let the number of 25 paise coins be x & that of 10 paise coins be y, then:

$$\frac{25}{100}x + \frac{10}{100}y = 8.25$$

$\Rightarrow$ $5x + 2y = 165$_______(i)

and $x = \frac{1}{3}y \Rightarrow y = 3x$ _______(ii)

putting $y = 3x$ in (i), we get :
$5x + 6x = 165 \Rightarrow 11x = 165 \Rightarrow x = 15$

$\therefore$ from (ii), $y = 3x = 3\times 15 = 45$

$\therefore$ Total number of coins in the purse
$= x + y = 15 + 45 = 60$

14.(a) $a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$ will have no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{1}{3} = \frac{2}{k} \Rightarrow k = 6$$

15.(d) The given equations are $2x - 5y = 9$ and $8x - 20y = 36 \Rightarrow 2x - 5y = 9$
Thus, there is one equation in two variables. So, the given equations have an infinite number of solutions.

16.(a) Let the numbers be x and y. Then,
$x - y = 5$ and $x^2 - y^2 = 65$

$$\therefore \frac{x^2 - y^2}{x - y} = \frac{65}{5} \Rightarrow x + y = 13$$

solving $x - y = 5$ and $x + y = 13$, we get;
$x = 9$ and y = 4

$\therefore$ larger number = 9

17.(d)

$$x + \frac{1}{y} = 2 \Rightarrow \frac{1}{y} = 2 - x \Rightarrow y = \frac{1}{2-x} ....(i)$$

and $2xy - 3y = -2$

$\Rightarrow y(2x - 3) = -2$ ......(ii)

putting $y = \frac{1}{2-x}$ in (ii)

$\frac{2x-3}{2-x} = -2 \Rightarrow 2x - 3 = -4 + 2x$. this gives 1=0
This is impossible So, there is no solution.

18.(b) $2^a + 3^b = 17$ and
$2^{a+2} - 3^{b+1} = 5 \Rightarrow 2^2.2^a - 3.3^b = 5$
$\Rightarrow 4.2^a - 3.3^b = 5$
let $2^a = x$ & $3^b = y$ then
$x + y = 17$ ....(i)
$4x - 3y = 5$ ....(ii)
3× (i) + (ii), we get
$7x = 56 \Rightarrow x = 8 \Rightarrow 2^a = 8 = 2^3 \Rightarrow$

$\boxed{a = 3}$

putting $x = 8$ in (i), we get
$y = 17 - 8 = 9 \Rightarrow 3^b = 9 = 3^2 \Rightarrow$

$\boxed{b = 2}$

$\therefore$ a = 3 and b = 2.

19.(c) Note that $|a| = 1$ means a = 1 or a = -1

So, $|x + y| = 1 \Rightarrow x + y = 1$ or $-(x + y) = 1$

$\Rightarrow$ $(x + y) = -1$

solving $x + y = 1$, $x - y = 0$, we get

$x = \frac{1}{2}$ and $y = \frac{1}{2}$

solving $x + y = -1$, $x - y = 0$, we get $x = -1/2$ and $y = -1/2$

$\therefore x = y = \pm\frac{1}{2}$

CHAPTER 10

POLYNOMIALS

- **Polynomials :** An expression of the form $p(x) = a_0 + a_1x + a_2x^2 + ...+ a_nx^n$, where $a_n \neq 0$, is called a polynomial in x of degree n.
  Here a_0, a_1, a_2, ...a_n are real numbers and each power of x is a non-negative integer.
  e.g.

(i) $2x + 7$ is a polynomial in x of degree 1.

(ii) $2y^2 - 5y + 7$ is a polynomial in y of degree 2.

(iii) $3u^3 + \frac{3}{7}u^2 - 8u + \sqrt{7}$ is a polynomial in u of degree 3.

(iv) $5t^4 - \frac{2}{7}t^3 + \sqrt{3}t^2 + \frac{3}{8}$ is a polynomial in t of degree 4.

(v) $\left(\sqrt{x}+5\right)$, $\frac{1}{x+3}$, $\frac{5}{x^2-3x+1}$ etc. are not polynomials.

- **Polynomials of Various Degrees :**

(1) **Linear Polynomial :** A polynomial of degree 1 is called a linear polynomial.
A linear polynomial is of the form $p(x) = ax + b$, where $a \neq 0$
e.g.

$(3x-7), \left(\sqrt{2}x+5\right), \left(x-\frac{7}{3}\right)$ etc.

(2) **Quadratic Polynomial :** A polynomial of degree 2 is called a quadratic polynomial.
It is of the form $p(x) = ax^2 + bx + c$, where $a \neq 0$

E.g. $(2x^2 + 7x - 9)$, $\left(3x^2-\sqrt{2}x+7\right)$, $\left(y^2-7y+\sqrt{5}\right)$ etc.

Nature of Roots

The value of x at which value of equation will be zero.

1. **Roots are imaginary :**
$\mathbf{b^2 - 4ac \leq 0}$

2. **Roots are real:**
$\mathbf{b^2 - 4ac \geq 0}$

$b^2 - 4ac \geq 0$

different ⇓ $b^2-4ac>0$ — equal ⇓ $b^2-4ac=0$

rational: b^2-4ac ⇓ perfect square — irrational: $a+\sqrt{b}$, $a-\sqrt{b}$

Sum & product of root:-

Let there are two roots named α & β, then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \& \quad \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Sum of root:-

$$\alpha+\beta = \frac{-b}{a}$$

Product of root:

$$\alpha\beta = \frac{c}{a}$$

then, $ax^2 + bx + c = 0$ can be written as:

$$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\Rightarrow x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$$

$\boxed{x^2 - (\text{sum of root})x + \text{product of root} = 0}$

- If the roots α & β be reciprocal to each other then a = c.
- If the two roots α & β be equal in magnitude and opposite in sign, then b = 0
- If a,b,c are rational number and $a+\sqrt{b}$ is one root of the quadratic equation, then the other root must be conjugate $a-\sqrt{b}$ and viceversa

Ex.1 Find the Quadratic equation whose one root is $3+\sqrt{3}$

Sol. If one root is $3+\sqrt{3}$ then second root will be $3-\sqrt{3}$

Sum of root

$= \left(3+\sqrt{3}\right)+\left(3-\sqrt{3}\right) = 6$

Product of root

$= \left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right) = 6$

using,
x^2 (sum of root) x + (product of root) = 0
$\Rightarrow x^2 - 6x + 6 = 0$

Ex.2: Two roots of equation $2x^2 - 7x + 12 =$ are α & β then, find $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = ?$

Sol. $2x^2 - 7x + 12 = 0$
On comparing with standard equation $ax^2 + bx + c = 0$
a = 2, b = –7, & c = 12

$\alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{7}{2}$

$\alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = 6$

$\Rightarrow \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{7}{2}\right)^2 - 2\times 6}{6}$

$$= \frac{\frac{49}{4}-12}{6} = \frac{49-48}{4\times6} = \frac{1}{24}$$

Ex.3: Find the product of the root of the equation $x^2 - \sqrt{3} = 0$

Sol. On comparing this equation with $ax^2 + bx + c = 0$

$a = 1,\ b = 0\ \&\ c = -\sqrt{3}$

Product of root $\alpha\beta = \frac{c}{a} = -\sqrt{3}$

- **Byquadratic Polynomial :** A polynomial of degree 4 is called a biquadratic polynomial. It is of the form $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ where $a \neq 0$

E.g. $(3x^4 + 7x^3 - 4x^2 + 6x + 11)$, $(4t^4 - 7t^3 + 6t^2 - 11t + 9)$ etc.

(3) **Cubic Polynomial :** A polynomial of degree 3 is called a cubic polynomial. It is of the form $P(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$

E.g. $(4x^3 - 2x^2 + 7x + 9)$, $\left(2\sqrt{2}y^3 - 5y^2 - 8\right)$ etc.

Value of a Polynomial at a given point: If $P(x)$ is a polynomial in x and if α is any real number, then the value obtained by putting $x = \alpha$ in $P(x)$ is called the value of $P(x)$ at $x = \alpha$

The value of P (x) at $x = \alpha$ is denoted by $p(\alpha)$.

e.g. Let $p(x) = 3x^2 - 2x + 7$. then

$p(2) = (3\times2^2 - 2\times2 + 7)$
$= (12 - 4 + 7)$
$= 15$

$p(-1) = [3\times(-1)^2 - 2(-1) + 7]$
$= (3+2+7) = 12$

Zeros of a Polynomial : A real number α is called a zero of the polynomial $p(x)$, if $p(\alpha) = 0$

Note : 1. If α and β are the zeros of $p(x) = ax^2 + bx + c$, $a \neq 0$, then.

(i) $\alpha + \beta = -\frac{b}{a}$

(ii) $\alpha\beta = \frac{c}{a}$

(2) A quadratic polynomial whose zeros are α and β is given by $p(x) = \{x^2 - (\alpha+\beta)x + \alpha\beta\}$

(3) If α, β and γ are the zeros of $p(x) = ax^3 + bx^2 + cx + d$, then,

(i) $\alpha + \beta + \gamma = -\frac{b}{a}$

(ii) $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{c}{a}$

(iii) $\alpha\beta\gamma = -\frac{d}{a}$

(4) A cubic polynomial whose zeros are α, β and γ is given by $p(x)$
$= \{x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$

Factor Theorem : The Condition that $(x - a)$ is a factor of a polynomial $f(x)$, if and only if $f(a) = 0$

Thus, $(x - a)$ is a factor of $f(x) \Leftrightarrow f(a) = 0$.

Remarks : (i) $(x + a)$ is a factor of polynomiual $p(x)$ if and only if $p(-a) = 0$

(ii) $(ax - b)$ is a factor of a polynomial $p(x)$, if $p\left(\frac{b}{a}\right) = 0$

(iii) $(ax + b)$ is a factor of a polynomial $p(x)$, if $p\left(-\frac{b}{a}\right) = 0$

(iv) $(x - a)(x - b)$ are factors of a polynomial $p(x)$ if $p(a) = 0$ and $p(b) = 0$.

- **Remainder Theorem :** If a polynomial $f(x)$ of degree $n \geq 1$, is divided by $(x - a)$, then the remainder is $f(a)$.

e.g. Let $f(x) = x^3 + 3x^2 - 5x + 4$ be divided by $(x - 1)$. Find the remainder.

Sol. Remainder $= f(1)$
$= 1^3 + 3\times1^2 - 5\times1 + 4 = 3$

Important Results :

(i) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

(ii) $(x^n + a^n)$ is divisible by $(x + a)$ only when n is odd.

(iii) $(x^n - a^n)$ is divisible by $(x + a)$ only for even values of n.

(iv) $(x^n + a^n)$ is never divisible by $(x - a)$

- **H.C.F & L.C.M of Polynomials :**

Divisor : A polynomial $p(x)$ is called a divisor of another polynomial $f(x) = p(x).g(x)$ for some polynomial $g(x)$.

- **H.C.F. or (G.C.D.) of Polynomials :** A polynomial $h(x)$ is called the H.C.F. or G.C.D of two or more given polynomials, if $h(x)$ is a polynomial of heighest degree dividing each one of the given polynomials.

- **Remark :** The coefficient of heighest degree term in H.C.F is always taken as positive.

e.g. What is the HCF of $(x + 3)^2(x - 2)^3$ and $(x - 1)(x + 3)(x - 2)^2$?

Sol. $p(x) = (x + 3)^2(x - 2)^3$
$q(x) = (x - 1)(x + 3)(x - 2)^2$

We see that $(x + 3)(x - 2)^2$ is such a polynomial that is a common divisor and whose degree is heighest among all common divisors.

- **L.C.M. of Polynomials :** A polynomial $p(x)$ is called the L.C.M. of two or more given polynomials, if it is a polynomial of smallest degree which is divided by each one of the given polynomials.

e.g. Find the L.C.M of $(x - 3)(x + 4)^2$ and $(x - 3)^3(x + 4)$:

Sol : $p(x) = (x - 3)(x + 4)^2$
$q(x) = (x - 3)^3(x + 4)$

we make a polynomial by taking each factor of $p(x)$ and $q(x)$.

If a factor is common in both, then we take the factor which has highest degree in $p(x)$ and $q(x)$.

$\therefore$ LCM $= (x - 3)^3(x + 4)^2$

Note : For any two polynomials $p(x)$ and $q(x)$

$p(x) \times q(x)$ = (Their H.C.F.) $\times$ (Their L.C.M.)

- **Factorisation of Polynomials :** To express a given polynomial as the product of polynomials, each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorisation.
- **Formulae for Factorisation:**

(i) $(x + y)^2 = x^2 + y^2 + 2xy$
(ii) $(x - y)^2 = x^2 + y^2 - 2xy$
(iii) $(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$
(iv) $(x + y)^2 - (x - y)^2 = 4xy$
(v) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
(vi) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
(vii) $x^2 - y^2 = (x + y)(x - y)$
(viii) $(x^3 + y^3) = (x + y)(x^2 + y^2 - xy)$
(ix) $(x^3 - y^3) = (x - y)(x^2 + y^2 + xy)$
(x) $(x + y + z)^2 = (x^2 + y^2 + z^2 + 2(xy + yz + zx))$
(xi) $(x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$
(xii) $x^2 + y^2 + z^2 - xy - yz - zx$
$= \frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$
(xiii) $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$

EXERCISE

1. If $f(x)$ is divided by $(3x + 5)$, the remainder is :
(a) $f\left(\frac{3}{5}\right)$ (b) $f\left(-\frac{3}{5}\right)$
(c) $f\left(\frac{5}{3}\right)$ (d) $f\left(-\frac{5}{3}\right)$
2. If $(x^{11} + 1)$ is divided by $(x + 1)$, the remainder is :
(a) 0 (b) 2
(c) 11 (d) 12
3. When $(x^4 - 3x^3 + 2x^2 - 5x + 7)$ is divided by $(x - 2)$, the remainder is :
(a) 3 (b) -3 (c) 2 (d) 0
4. If $(x - 2)$ is a factor of $(x^2 + 3qx - 2q)$, then the value of q is :
(a) 2 (b) - 2 (c) 1 (d) - 1
5. The value of λ for which the expression $x^3 + x^2 - 5x + \lambda$ will be divisible by $(x - 2)$ is :
(a) 2 (b) - 2 (c) - 3 (d) 4
6. If $(x + 1)$ and $(x - 2)$ be the factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, then the value of a and b will be :
(a) 2 and 8 (b) 1 and 7
(c) 5 and 3 (d) 3 and 7
7. The polynomial $(x^4 - 5x^3 + 5x^2 - 10x + 24)$ has a factor as :
(a) $x + 4$ (b) $x - 2$
(c) $x + 2$
(d) None of these
8. $(x^{29} - x^{25} + x^{13} - 1)$ is divisible by:
(a) both $(x - 1)$ & $(x + 1)$
(b) $(x - 1)$ but not by $(x + 1)$
(c) $(x + 1)$ but not by $(x - 1)$
(d) neither $(x - 1)$ nor $(x + 1)$
9. The value of expression $(9x^2 + 12x + 7)$ for $x = -\frac{4}{3}$ is :
(a) 7 (b) 0 (c) - 7 (d) 18
10. When $(x^3 - 2x^2 + px - q)$ is divided by $(x^2 - 2x - 3)$, the remainder is $(x - 6)$. The values of P and q are :
(a) p = - 2, q = -6
(b) p = 2, q = -6
(c) p = - 2, q = 6
(d) p = 2, q = 6
11. If $(x - a)$ is a factor of $(x^3 - 3x^2a + 2a^2x + b)$, then the value of b is :
(a) 0 (b) 2 (c) 1 (d) 3
12. If $x^{100} + 2x^{99} + K$ is divisible by $(x + 1)$, then the value of K is :
(a) – 3 (b) 2 (c) – 2 (d) 1
13. If the polynomial $f(x)$ is such that $f(-1) = 0$, then a factor of $f(x)$ is :
(a) -1 (b) $x - 1$
(c) $x + 1$ (d) $-1 - x$
14. If $x^3 + 5x^2 + 10K$ leaves remainder $-2x$ when divided by $x^2 + 2$, then the value of k is :
(a) - 2 (b) 1 (c) - 1 (d) 2
15. Which of the following is a polynomial ?
(a) $x^2 - 3x + 2\sqrt{x} + 7$
(b) $\sqrt{x} - \frac{1}{\sqrt{x}}$
(c) $x^{7/2} - x + x^{3/2}$
(d) None of these
16. If α and β are the zeros of $x^2 + 3x + 7$, then the vaue of $(\alpha + \beta)$ is :
(a) -3 (b) 3 (c) 7 (d) -7
17. If α and β are the zeros of $2x^2 + 3x - 10$, then the value of $\alpha\beta$ is:
(a) $-\frac{5}{2}$ (b) 5 (c) - 5 (d) $-\frac{3}{2}$
18. If common factor of $x^2 + bx + c$ and $x^2 + mx + n$ is $(x + a)$, then the value of a is :
(a) $\frac{c-n}{b-m}$ (b) $\frac{c-n}{b+m}$
(c) $\frac{c-n}{m-b}$ (d) $\frac{c+1}{b-m}$
19. $(x^4 + 5x^3 + 6x^2)$ is equal to :
(a) $x(x + 3)(x^2 + 2)$
(b) $x^2(x + 3)(x + 2)$
(c) $x^2(x - 2)(x - 3)$
(d) $x(x^2 + 3)(x + 2)$
20. The factors of $(x^4 + 625)$ are :
(a) $(x^2 - 25)(x^2 + 25)$
(b) $(x^2 + 25)(x^2 + 25)$
(c) $(x^2 - 10x + 25)(x^2 + 5x + 24)$
(d) do not exist
21. The factors of $(x^4 + 4)$ are :
(a) $(x^2 + 2)^2$
(b) $(x^2 + 2)(x^2 - 2)$
(c) $(x^2 + 2x + 2)(x^2 - 2x + 2)$
(d) None of these
22. $(x + y)^3 - (x - y)^3$ can be factorized as :
(a) $2y(3x^2 + y^2)$
(b) $2x(3x^2 + y^2)$
(c) $2y(3y^2 + x^2)$
(d) $2x(x^2 + 3y^2)$

23. The H.C.F. of $x^2 - xy - 2y^2$ and $2x^2 - xy - y^2$ is :
(a) $(x + y)$
(b) $(x - y)$
(c) $(2x - 3y)$
(d) None of these

24. The H.C.F. of $(x^3 + x^2 + x + 1)$ and $(x^4 - 1)$ is:
(a) $(x^2 - 1)(x^2 + 1)$
(b) $(x + 1)(x^2 - 1)$
(c) $(x + 1)(x^2 + 1)$
(d) $(x^2 + 1)(x + 1)(x^3 + 1)$

25. The L.C.M of the polynomials X and Y, where $X = (x + 3)^2(x - 2)(x + 1)^2$ and $Y = (x + 1)^2(x + 3)(x + 4)$ is given by :
(a) $(x - 2)(x + 4)(x + 3)^2(x + 1)^2$
(b) $(x + 1)(x - 2)(x + 3)(x + 4)$
(c) $(x - 2)(x + 1)(x + 3)^2(x + 4)$
(d) $(x - 2)(x + 1)^2(x + 3)(x + 4)$

26. The L.C.M of $(x + 2)^2(x - 2)$ and $(x^2 - 4x - 12)$ is :
(a) $(x + 2)(x - 2)$
(b) $(x + 2)(x - 2)(x - 6)$
(c) $(x + 2)(x - 2)^2$
(d) $(x + 2)^2(x - 2)(x - 6)$

27. The H.C.F. of $(x^2 - 4)$, $(x^2 - 5x - 6)$ and $(x^2 + x - 6)$ is :
(a) 1 (b) $(x - 2)$
(c) $(x + 2)$ (d) $(x^2 + x - 6)$

28. The H.C.F of $2(x^2 - y^2)$ and $5(x^3 - y^3)$ is :
(a) $2(x^2 - y^2)$ (b) $(x - y)$
(c) $(x + y)$ (d) $(x^2 + y^2)$

29. The L.C.M of $(2x^2 - 3x + 2)$ and $(x^3 - 4x^2 + 4x)$ is :
(a) $x(2x^2 + 1)(x^2 + 2)$
(b) $x(2x + 1)(x - 2)^2$
(c) $x(2x^2 + 1)(x - 1)^2$
(d) $x(2x + 1)(x^2 - 1)$

30. The L.C.M of $(a^3 + b^3)$ and $(a^4 - b^4)$ is :
(a) $(a^3 + b^3)(a^2 + b^2)(a - b)$
(b) $(a^3 + b^3)(a + b)(a^2 + b^2)$
(c) $(a + b)(a^2 + ab + b^2)(a^3 + b^3)$
(d) $(a^3 + b^3)(a^2 - b^2)(a - b)$

31. If Polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 2x + a$ are divided by $(x - 2)$, the same remainder are obtained. Find the value of a :
(a) 3 (b) - 9 (c) - 3 (d) - 5

32. If the polynomial $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $(x - 1)$ and $(x + 1)$, the remainders are 5 and 19 respectively. The values of a and b are:
(a) a = 8, b = 5
(b) a = 5, b = 6
(c) a = 6, b = 8
(d) a = 5, b = 8

33. Factories : $(x^8 + x^4y^4 + y^8)$
(a) $(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$
(b) $(x^2 + xy - y^2)(x^4 - x^4y^4 + y^4)$
(c) $(x^2 + xy + y^2)^2(x^4 - x^2y^2 + y^4)$
(d) $(x^2 - xy + y^2)^2(x^4 - x^4y^4 - y^4)$

34. Factorise : $\left(x^6 + \frac{y^6}{27}\right)$
(a) $\left(x^2 + \frac{y^2}{3}\right)\left(x^4 + \frac{x^2y^2}{3} + \frac{x^2y^6}{9}\right)$
(b) $\left(x^2 + \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{y^4}{9}\right)$
(c) $\left(x^2 - \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{x^2y^4}{9}\right)$
(d) $\left(x^2 - \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{y^4}{9}\right)$

35. Factorise : $(x^4 + x^2 + 25)$
(a) $(x^2 + 3x + 5)(x^2 + 3x - 5)$
(b) $(x^2 + 5 + 3x)(x^2 + 5 - 3x)$
(c) $(x^2 + x + 5)(x^2 - x + 5)$
(d) None of these

ANSWER KEY

1. (d)	5. (b)	9. (a)	13. (c)	17. (c)	21. (c)	24. (c)	27. (a)	30. (a)	33. (a)
2. (a)	6. (b)	10. (c)	14. (b)	18. (a)	22. (a)	25. (a)	28. (b)	31. (c)	34. (b)
3. (b)	7. (b)	11. (a)	15. (d)	19. (b)	23. (d)	26. (d)	29. (b)	32. (d)	35. (b)
4. (d)	8. (b)	12. (d)	16. (a)	20. (d)					

SOLUTION

1.(d) $3x + 5 = 0 \Rightarrow x = -\frac{5}{3}$

So, remainder is $f\left(-\frac{5}{3}\right)$

2.(a) Remainder = $f(-1)$
$= (-1)^{11} + 1$
$= -1 + 1 = 0$

3.(b) Remainder = $f(2)$
$= 2^4 - 3(2)^3 + 2(2)^2 - 5 \times 2 + 7$
$= 16 - 24 + 8 - 10 + 7 = -3$

4.(d) Since $(x - 2)$ is a factor of $f(x)$
$= x^2 + 3qx - 2q$
$\therefore f(2) = 0 \Rightarrow 2^2 + 3q \times 2 - 2q = 0$
$4q = -4 \Rightarrow q = -1$

5.(b) $(x - 2)$ is a factor of polynomial
$f(x) = x^3 + x^2 - 5x + \lambda$
$\therefore f(2) = 0 \Rightarrow 2^3 + 2^2 - 5 \times 2 + \lambda = 0$
$\Rightarrow 12 - 10 + \lambda = 0 \Rightarrow \lambda = -2$

6.(b) Since $(x + 1)$ & $(x - 2)$ are the factors of
$f(x) = x^3 + (a + 1)x^2 - (b - 2)x - 6$
$\therefore f(-1) = 0$ and $f(2) = 0$
or $-1 + (a + 1) + (b - 2) - 6 = 0$
and $8 + 4(a + 1) - (b - 2) \times 2 - 6 = 0$
or $a + b = 8$ ......(i)
and $2a - b = -5$ ......(ii)
(i) + (ii) $3a = 3 \Rightarrow a = 1$
From equation (i)
$b = 8 - 1 = 7$
$\therefore$ a = 1 & b = 7

7.(b) Since $x = 2$ makes the given expression zero, so, $(x - 2)$ is its factor.

8.(b) Since $x = 1$ makes $x^{29} - x^{25} + x^{13} - 1$ zero, so $(x - 1)$ is its factor. And $x = -1$ does not make it zero so $(x + 1)$ is not its factor.

9.(a) $f(x) = 9x^2 + 12x + 7$

$\therefore f\left(-\frac{4}{3}\right) = 9\left(-\frac{4}{3}\right)^2 + 12\left(-\frac{4}{3}\right) + 7$

$= 16 - 16 + 7 = 7$

10.(c)

$$x^2 - 2x - 3 \overline{) x^3 - 2x^2 + px - q} \; (x$$
$$\underline{\begin{array}{l} x^3 - 2x^2 - 3x \\ - \quad + \quad + \end{array}}$$
$$(p+3)x - q \Leftarrow \text{remainder}$$

$\therefore (p + 3)x - q = x - 6$

$\therefore p + 3 = 1$ and $q = 6$

or $p = -2$ and $q = 6$

11.(a) let $f(x) = x^3 - 3x^2a + 2a^2x + b$

$\because (x - a)$ is a factor of $f(x)$

$\therefore f(a) = 0 \Rightarrow a^3 - 3a^3 + 2a^3 + b = 0$

$\Rightarrow b = 0$

12.(d) $\because x^{100} + 2x^{99} + k$

$= f(x)$ (let) is divisible by $(x + 1)$

$\therefore f(-1) = 0$

$\Rightarrow 1 - 2 + k = 0 \Rightarrow k = 1$

13.(c) Since $x = -1$ makes $f(x)$ zero, So $(x + 1)$ is its factor.

14.(b)

$$x^2 + 2 \overline{) x^3 + 5x^2 + 10k} \; (x + 5$$
$$\underline{\begin{array}{l} x^3 + 2x \\ - \quad - \end{array}}$$
$$5x^2 - 2x + 10k$$
$$\underline{\begin{array}{l} 5x^2 \quad + 10 \\ - \qquad - \end{array}}$$
$$-2x + 10k - 10 \Leftarrow \text{Remainder}$$

but given, remainder $= -2x$

$\therefore -2x + 10k - 10 = -2x$

$\Rightarrow 10k = 10$

$\Rightarrow k = 1$

15.(d) For polynomial, each power of x must be a non-negative integer.

16.(a) $\alpha + \beta = -\frac{b}{a} = -\frac{3}{1} = -3$

17.(c) $\alpha\beta = \frac{c}{a} = -\frac{10}{2} = -5$

18.(a) Let $f(x) = x^2 + bx + c$ and $g(x) = x^2 + mx + n$

$\because (x + a)$ is a common factor of $f(x)$ and $g(x)$

$\therefore f(-a) = 0$ and $g(-a) = 0$

or $a^2 - ba + c = 0$ and $a^2 - ma + n = 0$

$\Rightarrow a^2 = ab - c$...(i) and $a^2 = ma - n$ .....(ii)

$\therefore$ from (i) and (ii)

$ab - c = ma - n$

$\Rightarrow a(b - m) = c - n$ or $a = \frac{c - n}{b - m}$

19. (b) $x^4 + 5x^3 + 6x^2 = x^2(x^2 + 5x + 6)$

$= x^2(x^2 + 3x + 2x + 6)$

$= x^2[x(x + 3) + 2(x + 3)]$

$= x^2(x + 3)(x + 2)$

20.(b) Do not exist

21. (c) $x^4 + 4 = (x^2)^2 + (2)^2 + 4x^2 - 4x^2$

$= (x + 2)^2 - (2x)^2$

$= (x^2 + 2x + 2)(x^2 - 2x + 2)$

22. (a) Using formuale, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$\therefore (x + y)^3 - (x - y)^3 = [(x + y) - (x - y)] + [(x + y)^2 + (x - y)^2 + (x + y)(x - y)]$

$= 2y[2(x^2 + y^2) + (x^2 - y^2)]$

$= 2y(3x^2 + y^2)$

23.(d) $x^2 - xy - 2y^2 = (x^2 - y^2) - (xy + y^2)$

$= (x + y)(x - y) - y(x + y)$

$= (x + y)(x - y - y)$

$= (x + y)(x - 2y)$

$2x^2 - xy - y^2 = (x^2 - xy) + (x^2 - y^2)$

$= x(x - y) + (x + y)(x - y)$

$= (x - y)(x + x + y)$

$= (x - y)(2x + y)$

Clearly, no factor is common,

So, H.C.F = 1

24.(c) $x^3 + x^2 + x + 1 = x^2(x + 1) + 1(x + 1)$

$= (x + 1)(x^2 + 1)$

$x^4 - 1 = (x^2 - 1)(x^2 + 1)$

$= (x + 1)(x - 1)(x^2 + 1)$

$\therefore$ Required H.C.F $= (x + 1)(x^2 + 1)$

25.(a) $X = (x + 3)^2(x - 2)(x + 1)^2$

$Y = (x + 1)^2(x + 3)(x + 4)$

So, LCM $= (x - 2)(x + 4)(x + 3)^2(x + 1)^2$

26.(d) $x^2 - 4x - 12 = x^2 - 6x + 2x - 12$

$= x(x - 6) + 2(x - 6)$

$= (x + 2)(x - 6)$

and other is $(x + 2)^2(x - 2)$

$\therefore$ L.C.M $= (x + 2)^2(x - 2)(x - 6)$

27.(a) $x^2 - 4 = (x + 2)(x - 2)$

$x^2 - 5x - 6 = x^2 - 6x + x - 6$

$= (x - 6)(x + 1)$

and $x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= (x + 3)(x - 2)$

Clearly, ther is no common factor.

So, H.C.F = 1.

28.(b) $2(x^2 - y^2) = 2(x - y)(x + y)$

and $5(x^3 - y^3) = 5(x - y)(x^2 + y^2 + xy)$

$\therefore$ H.C.F. $= (x - y)$

29.(b) $2x^2 - 3x + 2 = 2x^2 - 4x + x - 2 = 2x(x - 2) + 1(x - 2) = (x - 2)(2x + 1)$

$x^3 - 4x^2 + 4x = x(x^2 - 4x + 4)$

$= x(x - 2)^2$

$\therefore$ L.C.M. $= x(x - 2)^2(2x + 1)$

30.(a) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$

$\therefore$ L.C.M. $= (a - b)(a + b)(a^2 - ab + b^2)(a^2 + b^2)$

$= (a - b)(a^3 + b^3)(a^2 + b^2)$

31.(c) $f(x) = 2x^3 + ax^2 + 3x - 5$

$g(x) = x^3 + x^2 - 2x + a$

By remainder theorem,

$f(2) = 2(2)^3 + a(2)^2 + 3 \times 2 - 5$

$= 17 + 4a$

and, $g(2) = 2^3 + (2)^2 - 2 \times 2 + a$

$= 8 + a$

$\therefore 17 + 4a = 8 + a$

$\Rightarrow 3a = -9$ or $a = -3$

32.(d) By remainder theorem,

$f(1) = 5$ ....(i) $[\because x - 1 = 0 \Rightarrow x = 1]$

and $f(-1) = 19$ .....(ii) $[\because x + 1 = 0 \Rightarrow x = -1]$

Now, from (i) $1 - 2 + 3 - a + b = 5$

or $b - a = 3$ ...... (iii)

from (ii) $1 + 2 + 3 + a + b = 19$

or $a + b = 13$ ......(iv)

(iii) + (iv) $2b = 16$ or $b = 8$

Now from (iv), $a = 13 - 8 = 5$

$\therefore a = 5, \; b = 8$

33.(a) $x^8 + x^4y^4 + y^8$

$= x^8 + 2x^4y^4 + y^8 - x^4y^4$

$= (x^4 + y^4)^2 - (x^2y^2)^2$

$= (x^4 + y^4 + x^2y^2)(x^4 + y^4 - x^2y^2)$

$= [(x^2 + y^2)^2 - (xy)^2](x^4 - x^2y^2 + y^4)$

$= (x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$

34.(b) $x^6 + \frac{y^6}{27} = (x^2)^3 + \left(\frac{y^2}{3}\right)^3$

$= \left(x^2 + \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{y^4}{9}\right)$

35.(b) $x^4 + x^2 + 25 = (x^2)^2 + (5)^2 + 10x^2 - 9x^2$

$= (x^2 + 5)^2 - (3x)^2$

$= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$

CHAPTER 11

ALGEBRIC IDENTITIES

- An algebraic identity is an algebraic equation which is true for all values of the variable (s).

Important Formulae:

1. $(a+b)^2 = a^2 + b^2 + 2ab$
2. $(a-b)^2 = a^2 + b^2 - 2ab$
3. $(a+b)^2 = (a-b)^2 + 4ab$
4. $(a-b)^2 = (a+b)^2 - 4ab$
5. $a^2-b^2 = (a+b)(a-b)$
6. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
7. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
8. $(a^3+b^3) = (a+b)(a^2-ab+b^2)$
9. $(a^3-b^3) = (a-b)(a^2+ab+b^2)$
10. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$
11. $a^3+b^3+c^3-3abc$
 $= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
 $= \frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2 + (c-a)^2]$

Note:-

$a^3+b^3+c^3-3abc = 0$,
If {(i) $a+b+c = 0$ $(a \neq b \neq c)$
or
(ii) $a^2+b^2+c^2-ab-bc-ca = 0$

12. $a^2+b^2+c^2-ab-bc-ca = \frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$
13. If $ax^2+bx+c = 0$, then, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Some Important Results

1. If $x+\frac{1}{x} = a$, then
 (i) $x^2+\frac{1}{x^2} = a^2-2$
 (ii) $x^4+\frac{1}{x^4} = b^2-2$ where $b = a^2-2$ 0 e.g. $x+\frac{1}{x} = 3$,
 Then,
 $x^2+\frac{1}{x^2} = 3^2-2 = 7$, and $x^4+\frac{1}{x^4} = 49-2 = 47$
2. If $x-\frac{1}{x}-$ a, then
 (i) $x^2+\frac{1}{x^2} = a^2+2$
 (ii) $x^4+\frac{1}{x^4} = b^2-2$, where $b = a^2+2$
 e.g. $x-\frac{1}{x} = 3$, then
 $x^2+\frac{1}{x^2} = 3^2+2 = 11$ and
 $x^4+\frac{1}{x^4} = 11^2-2 = 119$.
3. If $x^4+\frac{1}{x^4} = a$, then
 (i) $x^2+\frac{1}{x^2} = \sqrt{a+2} = b$
 (ii) $x+\frac{1}{x} = \sqrt{b+2}$
 (ii) $x-\frac{1}{x} = \sqrt{b-2}$
 e.g. $x^4+\frac{1}{x^4} = 119$
 $\therefore x^2+\frac{1}{x^2} = \sqrt{119+2} = 11$
 $x+\frac{1}{x} = \sqrt{11+2} = \sqrt{13}$
 $x-\frac{1}{x} = \sqrt{11-2} = 3$
4. If $x+\frac{1}{x} = 2$, then $x = 1$
5. If $x+\frac{1}{x} = -2$, then $x = -1$
6. $x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$
7. $x^3-\frac{1}{x^3} = \left(x-\frac{1}{x}\right)^3 + 3\left(x-\frac{1}{x}\right)$
8. If $x+\frac{1}{x} = 1$, then $x^3 = -1$
9. If $x+\frac{1}{x} = -1$, then $x^3 = 1$,
10. If $x+\frac{1}{x} = \sqrt{3}$ then $x^3+\frac{1}{x^3} = 0 \Rightarrow x^6 = -1$ or $x^6+1 = 0$
11. If $ax + by = m$ and $bx - ay = n$ then, $(a^2+b^2)(x^2+y^2) = m^2+n^2$

Note : If the sum of squares of real numbers be zero, then each number is equal to zero i.e.
if $(x-a)^2 + (y-b)^2 + (z-c)^2 = 0$, then
$x-a = 0 \Rightarrow x = a$,
$y-b = 0 \Rightarrow y = b$ and
$z-c = 0 \Rightarrow z = c$
or if $x^2+y^2+z^2 = 0$, then $x = 0$, $y = 0$ & $z = 0$

Based on Increasing power

Ex.1 If $x+\frac{1}{x} = 3$, find the value of $x^2+\frac{1}{x^2} = ?$

Sol. $x+\frac{1}{x} = 3$

Squaring both sides,

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = (3)^2$

$\Rightarrow x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x} = 9$

$\Rightarrow x^2+\frac{1}{x^2} = 9-2 = 7$

(If $x+\frac{1}{x} = a$ then $x^2+\frac{1}{x^2} = a^2-2$)

Same $x^4+\frac{1}{x^4} = (a^2-2)^2-2$

Alternate:-

$x+\frac{1}{x} = 3$

$x^2+\frac{1}{x^2} = 3^2-2 = 7$

Ex.2 If $x+\frac{1}{x}=5$, find the (i) $x^2+\frac{1}{x^2}$

(ii) $x^4+\frac{1}{x^4}$

Sol.(i) $x+\frac{1}{x}=5$

$x^2+\frac{1}{x^2}$ = 5² – 2 = 23

(ii) $x+\frac{1}{x}=5$

$x^2+\frac{1}{x^2}=23$

Again squaring both sides

$\left(x^2+\frac{1}{x^2}\right)^2=(23)^2$

$x^4+\frac{1}{x^4}+2\times x^4\times\frac{1}{x^4}=529$

$x^4+\frac{1}{x^4}=529-2$

$x^4+\frac{1}{x^4}=527$

Ex.3 If $x+\frac{1}{x}=4$, find the (i) $x^2+\frac{1}{x^2}$

(ii) $x^4+\frac{1}{x^4}$

Sol.(i) $x+\frac{1}{x}=4$

$x^2+\frac{1}{x^2}$ = 4² – 2 = 14

(ii) $x+\frac{1}{x}=4$

$x^2+\frac{1}{x^2}$ = 14

Again squaring both sides

$x^4+\frac{1}{x^4}=196-2$

$x^4+\frac{1}{x^4}=194$

Ex.4 If $x+\frac{1}{x}=6$, Find the value of

(i) $x^2+\frac{1}{x^2}$ (ii) $x^4+\frac{1}{x^4}$

(iii) $x^8+\frac{1}{x^8}$

Sol. $x+\frac{1}{x}=6$

$x^2+\frac{1}{x^2}$ = 6² – 2 = **34**

$x^4+\frac{1}{x^4}$ = 34² – 2 = **1154**

$x^8+\frac{1}{x^8}$ = 1154²– 2 = **1331714**

Ex.5 If $x-\frac{1}{x}=3$, Find the value of

(i) $x^2+\frac{1}{x^2}$ (ii) $x^4+\frac{1}{x^4}$

Sol. $x-\frac{1}{x}=3$

squaring both side,

$\left(x-\frac{1}{x}\right)^2=3^2$

$x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x}=9$

$x^2+\frac{1}{x^2}=9+2$ = **11**

squaring both sides,

$\left(x^2+\frac{1}{x^2}\right)^2=(11)^2$

$x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}=121$

$x^4+\frac{1}{x^4}=121-2=$ **119**

(If $x-\frac{1}{x}=a$ then $x^2+\frac{1}{x^2}=a^2+2$

and $x^4+\frac{1}{x^4}=(a^2+2)^2-2$)

Ex.6 If $x-\frac{1}{x}=4$, Find the value of

(i) $x^2+\frac{1}{x^2}$ (ii) $x^4+\frac{1}{x^4}$

Sol. $x-\frac{1}{x}=4$

$x^2+\frac{1}{x^2}$ = 4² + 2 = **18**

$x^4+\frac{1}{x^4}$ = 18² – 2 = **322**

Ex.7 If $x+\frac{1}{x}=3$, Find the value of

$x^3+\frac{1}{x^3}$

Sol. $x+\frac{1}{x}=3$

Cube both sides,

$\left(x+\frac{1}{x}\right)^3=3^3$

$x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=27$

$\therefore$ Put the value of $x+\frac{1}{x}=3$

$x^3+\frac{1}{x^3}+3\times3$ = 27

$x^3+\frac{1}{x^3}=27-9=18$

(If $x+\frac{1}{x}=a$ then $x^3+\frac{1}{x^3}$ = a³ – 3a)

Alternate:

Here, $a=3$

$x^3+\frac{1}{x^3}=3^3-3\times3$

$x^3+\frac{1}{x^3}=18$

Ex.8 If $x+\frac{1}{x}=4$, Find the value of

$x^3+\frac{1}{x^3}$

Sol. $x+\frac{1}{x}=4$

$x^3+\frac{1}{x^3}$ = 4³ – 3×4 = 64 – 12

= **52**

- **Same as:**

⇒ If $x+\frac{1}{x}=3, 4, 5, 6$, then

$x^3+\frac{1}{x^3}$=18, 52, 110, 198

Ex.9 If $x-\frac{1}{x}=4$, then the value of

$x^3-\frac{1}{x^3}$

Sol. $x-\frac{1}{x}=4$

Cube both sides,

$\left(x-\frac{1}{x}\right)^3=4^3$

$$x^3-\frac{1}{x^3}-3\times x\times\frac{1}{x}\left(x-\frac{1}{x}\right)=4^3$$

Put the value of $x-\frac{1}{x}=4$

$$x^3-\frac{1}{x^3}-3\times4=64$$

$$x^3-\frac{1}{x^3}=64+12$$

$$x^3-\frac{1}{x^3}=76$$

(If $x-\frac{1}{x}$=a then $x^3-\frac{1}{x^3}$ = a^3+3a)

Alternate:

Here, a = 4

$x^3-\frac{1}{x^3}$ = $4^3+3\times4$ = 64+12

= 76

* Same as if $x-\frac{1}{x}$ = 3,4,5,6

Then $x^3-\frac{1}{x^3}$ = 36, 76, 140, 234

Ex.10 If $x+\frac{1}{x}$ = 5 find the vlaue of

$x^5+\frac{1}{x^5}$

Sol. $x+\frac{1}{x}$ = 5

$x^2+\frac{1}{x^2}$ = 5^2-2 = 23 ....(i)

$x^3+\frac{1}{x^3}$ = $5^3-3\times5$ = 110 ...(ii)

(i) × (ii)

$$\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)=23\times110$$

$$x^5+\frac{1}{x}+x+\frac{1}{x^5}=2530$$

Put the value of $x+\frac{1}{x}$ = 5

$x^5+\frac{1}{x^5}$ =2530–5 = 2525

* **If $x+\frac{1}{x}$ = a**

Then $x^5+\frac{1}{x^5}$ = $(a^2-2)(a^3-3a)-a$

Ex.11 If $x+\frac{1}{x}$ = 4,

Find the value of $x^5+\frac{1}{x^5}$

Sol. $x+\frac{1}{x}$ = 4

$x^2+\frac{1}{x^2}$ = 4^2-2 = 14 ....(i)

$x^3+\frac{1}{x^3}$ = $4^3-3\times4$ = 52 ...(ii)

(i) × (ii)

$x^5+\frac{1}{x^5}$ = 14×52 – 4 = 724

Ex.12 If $x-\frac{1}{x}$ = 4 find the value of

$x^5-\frac{1}{x^5}$

Sol. $x-\frac{1}{x}$ = 4

$x^2+\frac{1}{x^2}$ = 4^2+2 = 18 ......(i)

$x^3-\frac{1}{x^3}$ = $4^3+3\times4$ = 76 .....(ii)

(i) × (ii)

$$\left(x^2+\frac{1}{x^2}\right)\left(x^3-\frac{1}{x^3}\right)=18\times76$$

$$x^5-\frac{1}{x}+x-\frac{1}{x^5}=18\times76$$

Put the value of $x-\frac{1}{x}$ = 4

$x^5-\frac{1}{x^5}$ = $18\times76-\left(x-\frac{1}{x}\right)$

= 1368–4 = 1364

* **If $\left(x-\frac{1}{x}\right)$ = a, then**

$x^5-\frac{1}{x^5}$ = $(a^2+2)(a^3+3a)-a$

Ex.13 If $x-\frac{1}{x}$ =5 then find the value of

$x^5-\frac{1}{x^5}$

Sol. $x-\frac{1}{x}$ =5

$x^2+\frac{1}{x^2}$ = 5^2+2 = 27 ......(i)

$x^3-\frac{1}{x^3}$ = $5^3+3\times5$ = 140 .....(ii)

(i) × (ii)

$x^5-\frac{1}{x^5}$ = 27×140 – 5 = 3775

Ex.14 If $x+\frac{1}{x}=3$, Find the value of $x^6+\frac{1}{x^6}$.

Sol. $x+\frac{1}{x}=3$

$$x^3+\frac{1}{x^3}=3^3-3\times3=18$$

Squaring both side,

$$\left(x^3+\frac{1}{x^3}\right)^2=18^2$$

$$x^6+\frac{1}{x^6}+2\left(x^3\right)\left(\frac{1}{x^3}\right)=324$$

$$x^6+\frac{1}{x^6}=324-2$$

$$x^6+\frac{1}{x^6}=322$$

* **$x+\frac{1}{x}=a$, then $x^6+\frac{1}{x^6}$ = $(a^3-3a)^2-2$**

Ex.15. If $x+\frac{1}{x}=3$, Find the value of $x^7+\frac{1}{x^7}$.

Sol. $x+\frac{1}{x}=3$

$x^2+\frac{1}{x^2}$ = 3^2-2 = 7

$x^4+\frac{1}{x^4}$ = 7^2-2 = 47 ....(i)

$x^3+\frac{1}{x^3}$ = $3^3-3\times3$ = 18 ....(ii)

Multiply (i) and (ii)

$$\left(x^3+\frac{1}{x^3}\right)\times\left(x^4+\frac{1}{x^4}\right)$$

= 47 × 18

$\Rightarrow \quad x^7 + x + \frac{1}{x} + \frac{1}{x^7} = 18 \times 47$

Put the value of $x + \frac{1}{x} = 3$

$x^7 + \frac{1}{x^7} = 18 \times 47 - 3$

$x^7 + \frac{1}{x^7} = 843$

* $x + \frac{1}{x} = a, \text{then } x^7 + \frac{1}{x^7}$

$= \left((a^2-2)^2 - 2\right) \times \left(a^3 - 3a\right) - a$

Ex.16 If $x - \frac{1}{x} = 4$, Find the value of $x^7 - \frac{1}{x^7}$.

Sol. $x - \frac{1}{x} = 4$

$x^2 + \frac{1}{x^2} = 4^2 + 2 = 18$

$x^4 + \frac{1}{x^4} = 18^2 - 2 = 322 \quad \ldots(i)$

$x^3 - \frac{1}{x^3} = 4^3 + 3 \times 4 = 76 \quad \ldots(ii)$

Multiply (i) and (ii)

$\left(x^4 + \frac{1}{x^4}\right) \times \left(x^3 - \frac{1}{x^3}\right)$

$= 322 \times 76$

$x^7 - x + \frac{1}{x} - \frac{1}{x^7} = 24472$

$x^7 - \frac{1}{x^7} = 24472 + \left(x - \frac{1}{x}\right)$

$x^7 - \frac{1}{x^7} = 24472 + 4 = \mathbf{24476}$

* $\boldsymbol{x - \frac{1}{x} = a, \text{then } x^7 - \frac{1}{x^7}}$

$= \left((a^2+2)^2 - 2\right) \times \left(a^3 + 3a\right) + a$

Ex.17. If $x + \frac{1}{x} = 4$, find the value of $x^2 - \frac{1}{x^2}$

Sol. $x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$x + \frac{1}{x} = 4$

$x^2 + \frac{1}{x^2} = 4^2 - 2 = 14$

Subtract 2 both side

$x^2 + \frac{1}{x^2} - 2 = 14 - 2$

$\left(x - \frac{1}{x}\right)^2 = 12$

$x - \frac{1}{x} = \sqrt{12}$

We know that,

$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$x^2 - \frac{1}{x^2} = 4\sqrt{12}$

Ex.18. If $x - \frac{1}{x} = 3$ find the value of $x^3 + \frac{1}{x^3}$

Sol. $x - \frac{1}{x} = 3$

$x^2 + \frac{1}{x^2} = 3^2 + 2 = 11$

Adding 2 both sides

$x^2 + \frac{1}{x^2} + 2 = 11 + 2$

$\left(x + \frac{1}{x}\right)^2 = 13$

$x + \frac{1}{x} = \sqrt{13}$

Now,

$x^3 + \frac{1}{x^3} = \left(\sqrt{13}\right)^3 - 3 \times \sqrt{13}$

$x^3 + \frac{1}{x^3} = 13\sqrt{13} - 3\sqrt{13}$

$x^3 + \frac{1}{x^3} = 10\sqrt{13}$

Ex.19 If $x - \frac{1}{x} = 4$, Find the value of $x^6 - \frac{1}{x^6}$

Sol: $x - \frac{1}{x} = 4$

$x^2 + \frac{1}{x^2} = 4^2 + 2 = 18$

Adding 2 both sides,

$x^2 + \frac{1}{x^2} + 2 = 18 + 2$

$\left(x + \frac{1}{x}\right)^2 = 20$

$x + \frac{1}{x} = \sqrt{20}$

$x^3 + \frac{1}{x^3} = \left(\sqrt{20}\right)^3 - 3 \times \sqrt{20}$

$x^3 + \frac{1}{x^3} = 20\sqrt{20} - 3\sqrt{20}$

$x^3 + \frac{1}{x^3} = 17\sqrt{20} \quad \ldots(i)$

When $x - \frac{1}{x} = 4$

Then,

$x^3 - \frac{1}{x^3} = 4^3 + 3 \times 4 = 76 \ldots(ii)$

Multiply (i) and (ii)

$\left(x^3 + \frac{1}{x^3}\right) \times \left(x^3 - \frac{1}{x^3}\right)$

$= 17\sqrt{20} \times 76$

$x^6 - \frac{1}{x^6} = 1292\sqrt{20}$

Ex.20 If $x^2 + \frac{1}{x^2} = 27$, Find the value of $x + \frac{1}{x}$

Sol. $x^2 + \frac{1}{x^2} = 27$

Adding 2 both sides,

$x^2 + \frac{1}{x^2} + 2 = 27 + 2$

$\left(x + \frac{1}{x}\right)^2 = 29$

$x + \frac{1}{x} = \sqrt{29}$

Ex21. If $x^2 + \frac{1}{x^2} = 31$, find the value of $x - \frac{1}{x}$

Sol. $x^2+\frac{1}{x^2}=31$

Substract 2 both sides,

$\left(x-\frac{1}{x}\right)^2=29$

$x-\frac{1}{x}=\sqrt{29}$

Ex.22 If $x^4+\frac{1}{x^4}=23$, find the value of $x+\frac{1}{x}$

Sol. $x^4+\frac{1}{x^4}=23$

By adding (2) both side,
We get,

$x^4+\frac{1}{x^4}+2=23+2$

$\left(x^2+\frac{1}{x^2}\right)^2=(5)^2$

$x^2+\frac{1}{x^2}=5$ ....(i)

Adding 2 both side
We get,

$x^2+\frac{1}{x^2}+2=5+2$

$\left(x+\frac{1}{x}\right)^2=7$

$x+\frac{1}{x}=\sqrt{7}$

Ex.23 If $x^4+\frac{1}{x^4}=194$, find the value of $x^3+\frac{1}{x^3}$.

Sol. $x^4+\frac{1}{x^4}=194$

By adding (2) both side,
We get,

$x^4+\frac{1}{x^4}+2=194+2$

$x^4+\frac{1}{x^4}+2=196$

$\left(x^2+\frac{1}{x^2}\right)^2=(14)^2$

$x^2+\frac{1}{x^2}=14$

Again adding (2) both side
We get,

$x^2+\frac{1}{x^2}+2=14+2$

$\left(x+\frac{1}{x}\right)^2=(4)^2$

$x+\frac{1}{x}=4$

Now,

$x^3+\frac{1}{x^3}=4^3-3\times4$

$x^3+\frac{1}{x^3}=52$

Ex.24 If $x+\frac{1}{x}=5$, find the value of $x^2-\frac{1}{x^2}$

Sol. $x+\frac{1}{x}=5$

$x^2+\frac{1}{x^2}=5^2-2=23$

Subtract 2 both sides,

$x^2+\frac{1}{x^2}-2=23-2$

$\left(x-\frac{1}{x}\right)^2=21$

$x-\frac{1}{x}=\sqrt{21}$

We know that,

$x^2-\frac{1}{x^2}=\left(x-\frac{1}{x}\right)\times\left(x+\frac{1}{x}\right)$

$=5\sqrt{21}$

Ex.25 If $x^4+\frac{1}{x^4}=322$, Find the value of $x^3-\frac{1}{x^3}$

Sol. $x^4+\frac{1}{x^4}=322$

By adding (2) both side,
We get,

$x^4+\frac{1}{x^4}+2=322+2$

$x^4+\frac{1}{x^4}+2=324$

$\left(x^2+\frac{1}{x^2}\right)^2=(18)^2$

$x^2+\frac{1}{x^2}=18$ ....(i)

Again subtracting (2) from both side,
We get,

$x^2+\frac{1}{x^2}-2=18-2$

$x^2+\frac{1}{x^2}-2=16$

$\left(x-\frac{1}{x}\right)^2=(4)^2$

$x-\frac{1}{x}=4$

Now,

$x^3-\frac{1}{x^3}=4^3+3\times4$

$x^3-\frac{1}{x^3}=76$

Ex.26 If $2x+\frac{1}{7x}=4$, Find the value of $49x^2+\frac{1}{4x^2}$

Sol. $2x+\frac{1}{7x}=4$

Multiply by $\frac{7}{2}$ both sides

$\frac{7}{2}\left(2x+\frac{1}{7x}\right)=4\times\frac{7}{2}$

$7x+\frac{1}{2x}=14$

Squaring both sides,

$\left(7x+\frac{1}{2x}\right)^2=(14)^2$

$49x^2+\frac{1}{4x^2}+2\times7x\times\frac{1}{2x}$
$=196$

$49x^2+\frac{1}{4x^2}=196-7$

$49x^2+\frac{1}{4x^2}=189$

Ex.27 If $4x+\frac{1}{3x}=5$, Find the value of $9x^2+\frac{1}{16x^2}$

Sol. $4x+\frac{1}{3x}=5$

Multiply by $\frac{3}{4}$ both sides

$\frac{3}{4}\left(4x+\frac{1}{3x}\right)=5\times\frac{3}{4}$

$3x+\frac{1}{4x}=\frac{15}{4}$

Squaring both side,

$\left(3x+\frac{1}{4x}\right)^2=\left(\frac{15}{4}\right)^2$

$9x^2+\frac{1}{16x^2}+2\times3x\times\frac{1}{4x}=\frac{225}{16}$

$9x^2+\frac{1}{16x^2}=\frac{225}{16}-\frac{3}{2}$

$9x^2+\frac{1}{16x^2}=\frac{201}{16}$

Ex.28 If $a=x+\frac{1}{x}$ and $b=x-\frac{1}{x}$, find the value of $a^4-2a^2b^2+b^4$

(a) 10 (b) 4 (c) 16 (d) 8

Sol. $a^4-2a^2b^2+b^4 = \left(a^2-b^2\right)^2$

$= \left((a+b)(a-b)\right)^2$

$a + b = 2x$ .....(i)

$a-b=\frac{2}{x}$ ....(ii)

From (i) and (ii)

$a^4-2a^2b^2+b^4$

$= \left(2x\times\frac{2}{x}\right)^2 = 16$

Alternate:

Let x = 1

Then,

$a=2$, $b=0$

Put the value of a and b in equation,

$a^4-2a^2b^2+b^4$

Then,

$= 2^4-2\times2^2\times0+0^4 = 16$

Option c is correct.

Ex.29 If $x+y+z=7$, $x^2+y^2+z^2=19$ Find $xy + yz + zx$ = ?

Sol. $(x+y+z)^2 =$

$x^2+y^2+z^2+2(xy+yz+zx)$

$(7)^2=19+2(xy+yz+zx)$

$49 - 19 = 2(xy+yz+zx)$

$30 = 2(xy+yz+zx)$

$15 = xy+yz+zx$

Ex.30 If $x^2 + 2 = 2x$, Find $x^4 - x^3 + x^2 + 5$

Sol. $x^2 + 2 = 2x$

Squaring both sides,

$\left(x^2+2\right)^2=(2x)^2$

$x^4+4+4x^2=4x^2$

$\mathbf{x^4=-4}$

$x^2 + 2 = 2x$

$x^2=2x-2$

$x^2=2(x-1)$

$\mathbf{x-1=\frac{x^2}{2}}$

$x^4 - x^3 + x^2 + 5$

Put the value of of $x^4 = -4$

$-4 - x^2(x - 1)+5$

$1 - x^2(x - 1)$

Put the value of $x - 1$

$1-\frac{x^2\times x^2}{2}$

$1-\frac{x^4}{2} = 1 + \frac{4}{2}$

$= 3$

Ex.31 If $a^4+a^2b^2+b^4=8$ and $a^2+ab+b^2=4$ find the value of ab.

Sol. $a^4+a^2b^2+b^4=8$

$a^4+b^4=8-a^2b^2$ ....(i)

$a^2+b^2=4-ab$

Squaring both sides,

$\left(a^2+b^2\right)^2 = (4 - ab)^2$

$a^4 + b^4 + 2a^2b^2 = 16+a^2b^2- 8ab$

From equation (i),

$8-a^2b^2+2a^2b^2 = 16+a^2b^2- 8ab$

$8 = 8ab$

$ab = 1$

Alternate:-

We use formula,

$a^4+a^2b^2+b^4 =$

$\left(a^2+ab+b^2\right)\left(a^2-ab+b^2\right)$

$8 = 4 \times \left(a^2-ab+b^2\right)$

$\left(a^2-ab+b^2\right) = 2$ ....(i)

$\left(a^2+ab+b^2\right) = 4$ (As Given)..(ii)

(ii) – (i)

$2ab = 2$

$ab = 1$

Ex.32 If $a^4 + a^2b^2 + b^4 = 12$, $a^2 - ab + b^2 = 4$, Find the value of ab.

Sol. $a^4+a^2b^2+b^4 =$

$\left(a^2+ab+b^2\right)\left(a^2-ab+b^2\right)$

$12 = 4 \times \left(a^2+ab+b^2\right)$

$\left(a^2+ab+b^2\right) = 3$ ...(i)

$\left(a^2-ab+b^2\right) = 4$ ...(ii)

(i) – (ii)

$2ab = -1$

$ab = \frac{-1}{2}$

Ex.33 If $x = a^2 + b^2$, $y = \sqrt{2}ab$, find $\frac{a^4+b^4}{a^2-ab\sqrt{2}+b^2}$

(a) $x+y$ (b) $x-y$

(c) xy (d) $2xy$

Sol. $x = a^2 + b^2$

Squaring both side,

$x^2=\left(a^2+b^2\right)^2$

$x^2=a^4+b^4+2a^2b^2$ ...(i)

$y=\sqrt{2}ab$

$y^2=2a^2b^2$ ...(ii)

From (i) and (ii)

$x^2=a^4+b^4+y^2$

$x^2-y^2=a^4+b^4$

Now,

Put the value of

$\frac{a^4+b^4}{a^2-ab\sqrt{2}+b^2}$

$\left[(x-y)=a^2+b^2-2\sqrt{ab}\right]$

$=\frac{x^2-y^2}{x-y}$

$\frac{(x+y)(x-y)}{(x-y)} = (x+y)$

Ex.34 If $x + y = 1$, $x^4 + y^4 = -1$, Find $x^2 y^2 - 2xy$

Sol. $x + y = 1$
Squaring both side
$(x + y)^2 = (1)^2$
$x^2 + y^2 + 2xy = 1$
$x^2 + y^2 = 1 - 2xy$
Again Squaring both sides,
$(x^2 + y^2)^2 = (1 - 2xy)^2$
$x^4+y^4+2x^2y^2 = 1+ 4x^2y^2-4xy$
Put the value of
$x^4 + y^4 = -1$
$-1 + 2x^2 y^2 = 1+ 4x^2y^2-4xy$
$-1 = 1 + 2x^2 y^2 - 4xy$
$-2 = 2(x^2 y^2 - 2xy)$
$x^2 y^2 - 2xy = \mathbf{-1}$

Ex.35 If $x+y+z=3$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2$, $x^2+y^2+z^2=6$, Find xyz = ?

Sol. $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2$

$xy+yz+zx=2xyz$

$(x+y+z)^2 =$

$x^2+y^2+z^2+2(xy+yz+zx)$

$9 = 6 + 2\ (2xyz)$
$3 = 4xyz$

$xyz = \frac{3}{4}$

Ex.36 If $\frac{p}{x}+\frac{q}{y}+\frac{r}{z}=1$ &

$\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=0$ Find

$\frac{p^2}{x^2}+\frac{q^2}{y^2}+\frac{r^2}{z^2}=?$

Sol. Let $a = \frac{p}{x}$, $b = \frac{q}{y}$ and $c = \frac{r}{z}$
Then, $a + b + c = 1$

& $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

$\Rightarrow$ $ab + bc + ca = 0$

$\Rightarrow$ $(a+b+c)^2 =$

$a^2+b^2+c^2+2(ab+bc+ca)$
$1 = a^2 + b^2 + c^2 + 2(0)$
$a^2 + b^2 + c^2 = 1$

Hence, $\frac{p^2}{x^2}+\frac{q^2}{y^2}+\frac{r^2}{z^2}=1$

Ex.37 If $x^3 + y^3 = 0$ find $x + y$ = ?

(a) $\sqrt{3xy}$ (b) $\sqrt{2xy}$
(c) 3xy (d) $\sqrt{4xy}$

Sol. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
$\because x^3 + y^3 = 0$ (As given)
$(x + y)^3 = 3xy(x+ y)$
$(x + y)^2 = 3xy$

$x + y = \sqrt{3xy}$

Ex.38 If $x^4 + y^4 = x^2y^2$ find $x^6 + y^6$.

Sol. $x^6 + y^6 = (x^2)^3 + (y^2)^3$
$= (x^2 + y^2)(x^4 + y^4 - x^2y^2)$
$= (x^2 + y^2)\ (x^2y^2 - x^2y^2)$
$= 0$

Ex.39 If $(x-a)(x-b) = 1$ and $a - b + 5 = 0$ find $(x-a)^3 - \frac{1}{(x-a)^3}$ = ?

(a) 125 (b) –125
(c) 0 (d) 140

Sol. $(x - a)(x - b) = 1$

$\Rightarrow (x - b) = \frac{1}{(x-a)}$

$(x - a)\ (x - b) = 1$ ...(i)
$\because a - b + 5 = 0$
$-b = -a - 5$
Put the value (– b) In equation (i)
$(x - a)\ (x - a - 5) = 1$
let $(x - a) = M$
$M(M - 5) = 1$

$M - 5 = \frac{1}{M}$

$M - \frac{1}{M} = 5$

Now,

$M^3 - \frac{1}{M^3} = 5^3 + 3 \times 5$

$M^3 - \frac{1}{M^3} = 140$

Put the value of $M = x - a$
So,

$(x-a)^3 - \frac{1}{(x-a)^3} = 140$

Ex.40 If $x^2 + x = 5$ find the value of

$(x + 3)^3 + \frac{1}{(x+3)^3}$

Sol. Let $(x + 3) = m$
$x + 3 = m$
$\boldsymbol{x = m - 3}$
Put the value of 'x'
$\boldsymbol{x^2 + x = 5}$

$\Rightarrow$ $(m - 3)^2 + (m - 3) = 5$
$\Rightarrow$ $m^2 + 9 - 6m + m - 3 = 5$
$\Rightarrow$ $m^2 - 5m + 1 = 0$
$\Rightarrow$ $m^2 + 1 = 5m$

$\Rightarrow$ $m + \frac{1}{m} = 5$

Now,

$m^3 + \frac{1}{m^3} = 5^3 - 3 \times 5$

$m^3 + \frac{1}{m^3} = 110$

Put the value of $m = x + 3$

Then,

$(x + 3)^3 + \frac{1}{(x+3)^3} = 110$

Ex.41 If $x(x-3) = -1$ find the value of $x^3(x^3 - 18)$

Sol. $x(x-3) = -1$

$x - 3 = \frac{-1}{x}$

$x + \frac{1}{x} = 3$

$x^3 + \frac{1}{x^3} = 3^3 - 3 \times 3 = 18$

$\boldsymbol{x^3 - 18 = \frac{-1}{x^3}}$ ......(i)

Now,
$x^3(x^3 - 18)$
From Equation (i)

$= x^3 \times \frac{-1}{x^3} = -1$

$x^3(x^3 - 18) = \mathbf{-1}$

Ex.42 If $(a + b)^2 = 21 + c^2$, $(b + c)^2 = 32 + a^2$ and $(c + a)^2 = 28 + b^2$, find $a + b + c$ = ?

Sol. $(a + b)^2 - c^2 = 21$
$\Rightarrow (a + b+ c)(a + b - c) = 21$ ...(i)
$(b + c)^2 - a^2 = 32$
$\Rightarrow (b + c + a)\ (b + c - a) = 32$ ..(ii)
$(c + a)^2 - b^2 = 28$
$\Rightarrow (c +a + b)(c + a - b) = 28$..(iii)
Adding all three equations:-
$\Rightarrow$ $(a +b + c)\ [(a + b - c) + (b + c - a) + (c + a - b)] = 81$
$\Rightarrow$ $(a + b + c)^2 = 81$
$\Rightarrow$ $a + b + c = 9$

$a^3 + b^3 + c^3 - 3abc$
$= \frac{1}{2}(a + b + c)[(a - b)^2] + (b - c)^2 + (c - a)^2$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
(i) If $(a + b + c) = 0$
then $a^3 + b^3 + c^3 - 3abc = 0$
$a^3 + b^3 + c^3 = 3abc$
(ii) If $a^3 + b^3 + c^3 - 3abc = 0$
a, b and c are distinct no
then. $a + b + c = 0$
(iii) $a^3 + b^3 + c^3 - 3abc = 0$
a, b and c all are +ve integer no
then $a = b = c$.
(iv) $a^2 + b^2 + c^2 - ab - bc - ca = 0$
$a^2 + b^2 + c^2 = ab + bc + ca$
then $a = b = c$

Ex.43 If $a + b + c = 0$, then the value of $a^3 + b^3 + c^3$ is:
(a) 0 (b) abc
(c) $3abc$
(d) None of these

Sol. $\because a^3 + b^3 + c^3 - 3abc = (a + b + c)$
$(a^2 + b^2 + c^2 - ab - bc - ac)$
or $a^3 + b^3 + c^3 - 3abc = 0$
$\Rightarrow$ $a^3 + b^3 + c^3 = 3abc$
Hence (c) is the correct option.

Ex.44 If $a^3 + b^3 + c^3 - 3abc = 0$ and $a + b + c \neq 0$
Which statement is true
(a) a>b>c (b) a = b = c
(c) a > b < c (d) a < b < c

Sol.(b) We know that,
$a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)[(a - b)^2] + (b - c)^2 + (c - a)^2$
$\therefore$ $a^3 + b^3 + c^3 - 3abc = 0$ (As given)
When $a + b + c \neq 0$
Then $(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$
$(a - b)^2 = 0$
$(a - b) = 0$
Then a = b
Same b = c
c = a
So. **(a = b = c)**

Ex.45 If $a^3 + b^3 + c^3 - 3abc = 0$, $a + b + c \neq 0$ and a, b & c are natural number find the possible value of a + b + c
(a) 4 (b) 8 (c) 5 (d) 12

Sol.(d) We know that in this condition a = b = c and given a, b and c are natural no. we take option (d) because 12 is divide 3 equal natural part
$\frac{12}{3} = 4$
a = b = c = 4
Then 12 is possible a + b + c

Ex.46 If $a^3 + b^3 + c^3 - 3abc = 0$, $a + b + c \neq 0$ and a, b & c are natural number find the possible value of a × b × c
(a) 4 (b) 8 (c) 5 (d) 12

Sol.(b) We know that in this condition a = b = c and given a, b and c are natural no. we take option (b) because 8 = 2 × 2 × 2
We can say that **8** is possible value of a × b × c

Ex.47 Find the value of
$$\frac{(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3}{(x-y)^3 + (y-z)^3 + (z-x)^3}$$

Sol. Let, $a = x^2 - y^2$
$b = y^2 - z^2$
$c = z^2 - x^2$
$a + b + c = 0$
Then, $a^3 + b^3 + c^3 = 3abc$
Thus, $p = x - y$
$q = y - z$
$r = z - x$
$p + q + r = 0$
Then, $p^3 + q^3 + r^3 = 3pqr$
A.T.Q.
$$\frac{(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3}{(x-y)^3 + (y-z)^3 + (z-x)^3}$$
$$= \frac{3(x^2 - y^2)(y^2 - z^2)(z^2 - x^2)}{3(x-y)(y-z)(z-x)}$$
$\therefore$ **$(a^2 - b^2) = (a + b)(a - b)$**
$$\frac{(x + y)(x - y)(y + z)(y - z)(z + x)(z - x)}{(x-y)(y-z)(z-x)}$$
$= (x + y)(y + z)(z + x)$

Ex.48 If a + b + c = 0 Find the value of $a^3 + b^3 + c^3 + 3abc$
(a) 0 (b) 1
(c) abc (d) 6abc

Sol.(d) $(a + b + c) = 0$
Then $a^3 + b^3 + c^3 = 3abc$
So, $a^3 + b^3 + c^3 + 3abc$
$= 3abc + 3abc$
= 6abc

Ex.49 If $x = 1.235$
$y = 3.422$
$z = 4.377$
Find $x^3 + y^3 - z^3 + 3xyz$

Sol. $x + y = z$
$(x) + (y) + (-z) = 0$
$(x)^3 + (y)^3 + (-z)^3 = 3(x)(y)(-z)$
$x^3 + y^3 - z^3 = -3xyz$
$\therefore$ $x^3 + y^3 - z^3 + 3xyz = \mathbf{0}$

Ex.50 If $a^2 + b^2 = c^2$, Find the value of
$$\frac{a^6 + b^6 - c^6}{a^2 b^2 c^2}$$

Sol. $a^2 + b^2 = c^2$
Cube both side
$(a^2 + b^2)^3 = (c^2)^3$
$a^6 + b^6 + 3a^2 b^2 \underline{(a^2 + b^2)} = c^6$
$a^6 + b^6 + 3a^2 b^2 c^2 = c^6$
$a^6 + b^6 - c^6 = -3a^2 b^2 c^2$
A.T.Q.
$$\frac{a^6 + b^6 - c^6}{a^2 b^2 c^2}$$
$$= \frac{-3a^2 b^2 c^2}{a^2 b^2 c^2}$$
$= -3$

Ex.51 If $a^{1/3} + b^{1/3} = c^{1/3}$
Which statement is true
(a) $a^3 + b^3 - c^3 = 3abc$
(b) $a^3 + b^3 - c^3 + 3abc = 0$
(c) $(a + b - c)^3 - 27abc = 0$
(d) $(a + b - c)^3 + 27abc = 0$

Sol.(d) $(a)^{1/3} + (b)^{1/3} + (-c)^{1/3} = 0$
If (a + b + c) = 0
Then $a^3 + b^3 + c^3 = 3abc$
$(a) + (b) + (-c) = 3(a)^{1/3}(b)^{1/3}(-c)^{1/3}$
$a + b - c = -3a^{1/3} b^{1/3} c^{1/3}$
Cube both side
$(a + b - c)^3 = (-3a^{1/3} b^{1/3} c^{1/3})^3$
$(a + b - c)^3 = -27abc$
$\therefore$ **$(a + b - c)^3 + 27abc = 0$**

Ex.52 If $x + y + z = 2s$
Find the value of $(s - x)^3 + (s - y)^3 + 3(s - x)(s - y)z$
(a) y^3 (b) x^3 (c) z^3 (d) 0

Sol.(c) $x + y + z = 2s$
$x + y + z = s + s$
$s - x + s - y - z = 0$
$(s - x) + (s - y) + (-z) = 0$
$(s - x)^3 + (s - y)^3 + (-z)^3 = 3(s - x)(s - y)(-z)$
$(s - x)^3 + (s - y)^3 - z^3 = -3(s - x)(s - y)z$
$(s - x)^3 + (s - y)^3 + 3(s - x)(s - y)z$
$= z^3$

Ex.53 Find the value of
$$\sqrt[3]{2 \times 333^3 + 334^3 - 3 \times 333^2 \times 334}$$

Sol. $\sqrt[3]{2 \times 333^3 + 334^3 - 3 \times 333^2 \times 334}$
After describing
$= \sqrt[3]{333^3 + 333^3 + 334^3 - 3 \times 333 \times 333 \times 334}$

$a^3 + b^3 + c^3 - 3abc$
$= \frac{1}{2}(a + b + c)[(a - b)^2] + (b - c)^2 + (c - a)^2$

$$= \sqrt[3]{\frac{1}{2}(333+333+334)[(333-333)^2 + (333-334)^2+(334-333)^2]}$$

$$= \sqrt[3]{\frac{1}{2}(1000)(0+1+1)}$$

$$= \sqrt[3]{\frac{1}{2}\times 1000\times 2} = \mathbf{10}$$

Ex.54 If a = 20, b = 25, c = 15, Find

$$\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}$$

Sol. $\boxed{a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)}$

$$\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}$$

A.T.Q.

$$= \frac{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}{a^2+b^2+c^2-ab-bc-ca}$$

= a + b + c

Put the value

= 20 + 25 + 15 = **60**

Ex.55 If a + b + c = 3 and $a^2 + b^2 + c^2 = 6$, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$, Find **abc**

Sol. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$

$$\frac{ab+bc+ca}{abc}=1$$

ab + bc + ca = abc

$\boxed{(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)}$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2abc$

$(3)^2 = 6 + 2abc$

9 - 6 = 2abc

abc = $\mathbf{\frac{3}{2}}$

Ex.56 If a + b + c = 15 and $a^2 + b^2 + c^2 = 83$ find $a^3 + b^3 + c^3 - 3abc$

Sol.

$\boxed{(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)}$

$(15)^2 = 83 + 2(ab+bc+ca)$

225 - 83 = 2 (ab + bc + ca)

ab + bc + ca = 71

$\mathbf{a^3 + b^3 + c^3 - 3abc = (a + b + c)[a^2 + b^2 + c^2 -(ab + bc + ca)]}$

$a^3 + b^3 + c^3 - 3abc = 15(83-71)$

= 15 × 12 = 180

Ex.57 Find $(a - b)^3 + (b - c)^3 + (c - a)^3$ =?

Sol. ∵ (a - b) + (b - c) + (c - a) = 0

∴ $(a - b)^3 + (b - c)^3 + (c - a)^3$

= 3 (a - b) (b - c) (c - a)

Ex.58 The value of $a^3 + b^3 + c^3 - 3abc$, Where $a = 87$, $b = -126$ and $c = 39$ is:

(a) 0 (b) 1259

(c) –48

(d) None of these

Sol. ∵ $a + b + c = 0$

∴ $a^3 + b^3 + c^3 - 3abc = 0$

Thus (a) is the correct option.

Ex.59 Find the value of $a^3 + b^3 + c^3 - 3abc$ If $a + b + c = 12$ and $ab + bc + ac = 47$.

Sol. ∵ $a + b + c = 12$

∴ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac) = 144$

⇒ $a^2 + b^2 + c^2 + 2 \times 47 = 144$

⇒ $a^2 + b^2 + c^2 = 50$

Now, since $a^3 + b^3 + c^3 - 3ab = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Then,

$a^3 + b^3 + c^3 - 3abc$

= 12(50 – 47) = 12×3 = 36

Ex.60 If a = 997, b = 999 and c = 996 find the value of $a^3 + b^3 + c^3 - 3abc$

Sol.

$\boxed{a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)[(a - b)^2] + (b - c)^2 + (c - a)^2}$

$$= \frac{1}{2}(999+997+996)\begin{bmatrix}(999-997)^2 + \\ (997-996)^2 + \\ (996-999)^2\end{bmatrix}$$

$$= \frac{1}{2}\times 2992\times 14$$

= 2992 × 7 = **20944**

TYPE IV

$\ast$ $\mathbf{a^2 + b^2 + c^2 - ab - bc - ca}$

$$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

When,

$\mathbf{a^2 + b^2 + c^2 - ab - bc - ca = 0}$

$\mathbf{a^2 + b^2 + c^2 = ab + bc + ca}$

Then a = b = c

Ex.61 If a = 99, b = 97, and c = 96 Find the value of $a^2 + b^2 + c^2 - ab - bc - ca$

Sol. $a^2 + b^2 + c^2 - ab - bc - ca$

$$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

$$= \frac{1}{2}\left[(99-97)^2+(97-96)^2+(96-99)^2\right]$$

$$= \frac{1}{2}\left[(2)^2+(1)^2+(-3)^2\right]$$

$$= \frac{1}{2}[4+1+9] \quad 7$$

Ex.62 If a = 556, b = 558 and c =561 Find the value of $a^2 + b^2 + c^2 - ab - bc - ca$

Sol. $a^2 + b^2 + c^2 - ab - bc - ca$

$$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

$$= \frac{1}{2}\left[(556-558)^2+(558-561)^2+(561-556)^2\right]$$

$$= \frac{1}{2}[4+9+25] = \frac{1}{2}\times 38 = \mathbf{19}$$

Ex.63 If $a^2 = b+c$, $b^2 = c+a$, $c^2 = a+b$

$$\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}$$

Here a, b and c non zero number

Sol. $a^2 = b+c$

Divide by 3 both sides

$$\frac{a^2}{a}=\frac{b+c}{a}$$

Then,

$$a = \frac{b+c}{a}$$

Thus, $b = \frac{c+a}{b}$

$$c = \frac{a+b}{c}$$

Now put the value of a, b and c

$$\Rightarrow \frac{1}{1+\frac{b+c}{a}}+\frac{1}{1+\frac{c+a}{b}}+\frac{1}{1+\frac{a+b}{c}}$$

$$\Rightarrow \frac{a}{a+b+c}+\frac{b}{b+c+a}+\frac{c}{a+b+c}$$

$$\Rightarrow \frac{a+b+c}{a+b+c} = 1$$

Alternate:

We put the value

a = b = c = 2

$a^2 = b+c = (2)^2 = 2+2$

4 = 4

Then $\frac{1}{1+2}+\frac{1}{1+2}+\frac{1}{1+2}$

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$

Ex.64 If $\frac{x^2+y^2+z^2}{xy+yz+zx}=1$, then find the value of $\frac{x+y}{z}+\frac{y+z}{x}+\frac{z+x}{y}$

Sol. $\frac{x^2+y^2+z^2}{xy+yz+zx}=1$

$x^2+y^2+z^2 = xy+yz+zx$

In this condition $x = y = z$

Put the value of x = y = z = 1

Then, $\frac{x+y}{z}+\frac{y+z}{x}+\frac{z+x}{y}$

$= \frac{1+1}{1}+\frac{1+1}{1}+\frac{1+1}{1}$

= 2 + 2 + 2 = **6**

Ex.65 If $x^2+y^2+z^2 = xy+yz+zx$, find the value of $\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{y+x}$

Sol. $x^2+y^2+z^2 = xy+yz+zx$

Here,

$x = y = z$

Let put the value of x = y = z =1

Now,

$\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{y+x}$

$= \frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}$

$= \frac{1}{2}+\frac{1}{2}+\frac{1}{2} = \mathbf{\frac{3}{2}}$

Ex.66 If a + b + c = 0, Find the value of

$\frac{1}{a^2+b^2-c^2}+\frac{1}{b^2+c^2-a^2}+\frac{1}{c^2+a^2-b^2}$

Sol. a+b+c = 0

a+b = –c

Squaring Both side

$(a+b)^2 = (-c)^2$

$a^2+b^2+2ab = c^2$

$a^2+b^2-c^2 = -2ab$ ....(I)

Same, b+c = –a

$b^2+c^2-a^2 = -2bc$ ...(II)

Thus, c+a = –b

$c^2+a^2-b^2 = -2ac$ .....(III)

Put the value

$\frac{1}{a^2+b^2-c^2}+\frac{1}{b^2+c^2-a^2}+\frac{1}{c^2+a^2-b^2}$

$\Rightarrow \frac{1}{-2ab}+\frac{1}{-2bc}+\frac{1}{-2ab}$

$\frac{a+b+c}{-2abc} = \frac{O}{-2abc} = 0$

Alternate:-

We choose the value which we follow the condition.

If a+b+c = 0

a = 1, b = 1, c = –2

Put the value a,b,c

$= \frac{1}{a^2+b^2-c^2}+\frac{1}{b^2+c^2-a^2}+\frac{1}{c^2+a^2-b^2}$

$= \frac{1}{(1)^2+(1)^2-(-2)^2}+\frac{1}{(1)^2+(-2)^2-(1)^2}$

$+\frac{1}{(-2)^2+(1)^2-(1)^2}$

$= \frac{1}{1+1-4}+\frac{1}{1+4-1}+\frac{1}{4+1-1}$

$= -\frac{1}{2}+\frac{1}{4}+\frac{1}{4} = \mathbf{0}$

Ex.67 If $x + y + z = 0$ then the value of $\frac{x^2y^2+y^2z^2+z^2x^2}{x^4+y^4+z^4}$ is:

(a) 0 (b) 1/2

(c) 1 (d) 2

Sol. $\because (x + y + z) = 0$

$\Rightarrow x^2+y^2+z^2+2(xy+yz+zx) = 0$

$\Rightarrow x^2+y^2+z^2 = -2(xy+yz+zx)$

$\Rightarrow (x^2+y^2+z^2)^2 = 4(xy+yz+zx)^2$

$\Rightarrow x^4+y^4+z^4+2(x^2y^2+y^2z^2+z^2x^2) = 4[x^2y^2+y^2z^2+z^2x^2+2xyz(x+y+z)]$

$\therefore x^4+y^4+z^4 = 2(x^2y^2+y^2z^2+z^2x^2)$

$[\because (x+y+z) = 0]$

$\therefore \frac{x^2y^2+y^2z^2+z^2x^2}{x^4+y^4+z^4} = \frac{1}{2}$

Hence (b) is the correct option.

Alternate:

$x + y + z = 0$

Then, Let $x = 1,\ y = 1,\ z = -2$

$\frac{x^2y^2+y^2z^2+z^2x^2}{x^4+y^4+z^4}$

Now,

$\frac{(1^2)\times(1^2)+(1^2)\times(-2^2)+(-2^2)\times(1^2)}{(1)^4+(1)^4+(-2)^4}$

$\frac{1+4+4}{1+1+16} = \frac{9}{18} = \frac{1}{2}$

Ex.68 If pq+qr+rp = 0, Find the value

$\frac{p^2}{p^2-qr}+\frac{q^2}{q^2-pr}+\frac{r^2}{r^2-pq}$

Sol. pq + pr + rp = 0

pq + rp = – qr

Same, pq + qr = – pr

qr + rp = – pq

$\frac{p^2}{p^2-qr}+\frac{q^2}{q^2-pr}+\frac{r^2}{r^2-pq}$

$\Rightarrow \frac{p^2}{p^2+pq+rp}+\frac{q^2}{q^2+pq+qr}$

$+\frac{r^2}{r^2+qr+rp}$

$= \frac{p^2}{p(p+q+r)}+\frac{q^2}{q(p+q+r)}$

$+\frac{r^2}{r(p+q+r)}$

$= \frac{p}{p+q+r}+\frac{q}{p+q+r}+\frac{r}{p+q+r}$

$= \frac{p+q+r}{p+q+r} = 1$

Alternate:

Put the value $p = -1,\ q = 2$

$r = 2$

So,

$pq+qr+rp$

$= -1\times2+2\times2+2\times-1 = \mathbf{0}$

$= \frac{p^2}{p^2-pq}+\frac{q^2}{a^2-pr}+\frac{r^2}{r^2-pq}$

$= \frac{(-1)^2}{(-1)^2-(2\times2)}+\frac{(2)^2}{(2)^2-(-1\times2)}+$

$\frac{(2)^2}{(2)^2-(-1\times2)}$

$= \frac{1}{1-4}+\frac{4}{4+2}+\frac{4}{4+2}$

$= \frac{-1}{3}+\frac{4}{6}+\frac{4}{6}$

$= \frac{-2+4+4}{6} = \frac{6}{6} = 1$

Ex.69 If p×q×r = 1 Find $\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$

Sol. $\frac{1}{1+p+\frac{1}{q}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+\frac{1}{pq}+\frac{1}{p}}$

$(\because Pqr = 1,\ r = \frac{1}{pq})$

$\frac{q}{q+pq+1} + \frac{1}{1+q+pq} + \frac{pq}{pq+1+q}$

$\frac{pq+1+q}{q+pq+1} = 1$

Alternate

p = 1 = q = r

Then

$= \frac{1}{1+1+1} + \frac{1}{1+1+1} + \frac{1}{1+1+1}$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

Ex.70 If $\frac{x-a^2}{b+c} + \frac{x-b^2}{c+a} + \frac{x-c^2}{a+b}$ = 4(a+b+c)

Find the value of x

(a) $(a^2+b^2+c^2)$

(b) $(a+b+c)^2$

(c) $(a^2+b^2+c^2-ab-bc-ca)$

(d) $(ab+bc+ca)$

Sol. We take option (B)

Then $x = (a+b+c)^2$

$\frac{(a+b+c)^2-a^2}{(b+c)} + \frac{(a+b+c)^2-b^2}{(a+c)} + \frac{(a+b+c)^2-c^2}{(a+b)}$

$a^2-b^2 = (a+b)(a-b)$

$\Rightarrow \frac{(a+b+c-a)(a+b+c+a)}{(b+c)} + \frac{(a+b+c-b)(a+b+c+b)}{(a+c)} + \frac{(a+b+c-c)(a+b+c+c)}{(a+b)}$

$\Rightarrow \frac{(b+c)(2a+b+c)}{(b+c)} + \frac{(a+c)(2b+a+c)}{(a+c)} + \frac{(a+b)(2c+a+b)}{(a+b)}$

$\Rightarrow$ 2a+b+c+2b+a+c+2c+a+b

$\Rightarrow$ 4a+4b+4c = 4 (a+b+c)

L.H.S = R.H.S

So, $x = (a+b+c)^2$

Alternate:

Let, a = b = c =1

Then

$\frac{x-1}{2} + \frac{x-1}{2} + \frac{x-1}{2} = 4\times3$

$\frac{(x-1)\times3}{2} = 4\times3$

$x-1 = 8$

$\boldsymbol{x = 9}$

Option (B)

$x = (a+b+c)^2$

$= (1+1+1)^2$

$= (3)^2 = 9$

So, It is proofed

$\because x = (a+b+c)^2$

Ex.71 If $\frac{x-a^2}{b^2+c^2} + \frac{x-b^2}{c^2+a^2} + \frac{x-c^2}{a^2+b^2}$ = 3 Find the value of x

(a) $a^2+b^2+c^2$

(b) $a^2+b^2+c^2-ab-bc-ca$

(c) $(a+b+c)^2$

(d) $a^2+b^2+c^2+ab+bc+ca$

Sol. We take option (A) $a^2+b^2+c^2$

$\frac{a^2+b^2+c^2-a^2}{b^2+c^2} + \frac{a^2+b^2+c^2-b^2}{c^2+a^2} + \frac{a^2+b^2+c^2-c^2}{a^2+b^2}$

$\Rightarrow \frac{b^2+c^2}{b^2+c^2} + \frac{c^2+a^2}{c^2+a^2} + \frac{a^2+b^2}{a^2+b^2}$

1+1+1 = 3

L.H.S = R.H.S

Then, **$(x = a^2+b^2+c^2)$**

Ex.72 If $x + \frac{1}{y} = 1,\ y + \frac{1}{z} = 1$

Find the value of (i) $z + \frac{1}{x}$

(ii) $x + y + z + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

Sol. $x + \frac{1}{y} = 1$

$x = 1 - \frac{1}{y}$

$\boldsymbol{x} = \frac{y-1}{y}$ **....(i)**

$y + \frac{1}{z} = 1$

$\frac{1}{z} = 1-y$

$z = \boldsymbol{\frac{1}{1-y}}$ **....(ii)**

From (i) & (ii)

(i) $z + \frac{1}{x} = \frac{1}{1-y} + \frac{y}{y-1}$

$= \frac{1}{1-y} - \frac{y}{1-y} = \frac{1-y}{1-y} = 1$

(ii) $x + \frac{1}{y} + y + \frac{1}{z} + z + \frac{1}{x}$

= 1 + 1 + 1 = 3

So, $x + y + z + \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 3$

Alternate:-

(i) $x + \frac{1}{y} = 1$

Let, $\boldsymbol{x = \frac{1}{2}}$

Then y = 2

So, $\frac{1}{2} + \frac{1}{2} = 1$

Now, $y + \frac{1}{z} =$

If y = 2

Then z = **– 1**

$2 + \frac{1}{-1} = 2-1 = 1$

Now, $z + \frac{1}{x}$

Put the value z and x

$-1 + \frac{1}{1/2} = -1+2 = 1$

Ex.73 If $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$

Find $\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b}$

Sol. Divide and Multiply a + b + c

Now, $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$

$= \frac{1\times(a+b+c)}{(a+b+c)}$

$\Rightarrow \left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)$(a+b+c)

$= a+b+c$

$\Rightarrow \frac{a(a+b+c+)}{b+c}+\frac{b(a+b+c)}{c+a}$

$+\frac{c(a+b+c)}{a+b}$ = a+b+c

$\Rightarrow \frac{a^2+a(b+c)}{b+c}+\frac{b^2+b(a+c)}{c+a}+$

$\frac{c^2+c(a+b)}{a+b}$ = a+b+c

$\Rightarrow \frac{a^2}{b+c}+a+\frac{b^2}{c+a}+b+\frac{c^2}{a+b}+c$

= a + b + c

Then, $\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}=0$

Alternate:

$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ = 1

We break equation small step

Let, a = 0, b = 1

Put the value of a and b

Then,

$\frac{0}{1+c}+\frac{1}{c+0}+\frac{c}{0+1}$ = 1

$\frac{\mathbf{1}}{\mathbf{c}}$ **+ c = 1,**

1+ c^2 = c

c^2 = c – 1

Again put the value a, b

$\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}$

$\Rightarrow 0+\frac{1}{c+0}+\frac{c^2}{0+1}$

$\Rightarrow \frac{1}{c}+c^2$

Put the value of c^2

Then,

So, $\underline{\left|\frac{1}{c}+c\right|}-1$ = 1–1 = 0

Ex.74 If $\frac{b-c}{a}+\frac{a+c}{b}+\frac{a-b}{c}$ = 1 And a-b+c $\neq$ 0 which statement is true

(a) $\frac{1}{b}=\frac{1}{a}-\frac{1}{c}$ (b) $\frac{1}{a}=\frac{1}{b}-\frac{1}{c}$

(c) $\frac{1}{c}=\frac{1}{a}+\frac{1}{b}$ (d) N.O.T

Sol. $\frac{b-c}{a}+\frac{a+c}{b}+\frac{a-b}{c}$ = 1

We adding $\pm$ 1 in this equation

$\frac{b-c}{a}-1+\frac{a+c}{b}-1+\frac{a-b}{c}$ + 1 = 1–1

$\frac{b-c-a}{a}+\frac{a+c-b}{b}+\frac{a-b+c}{c}$ =0

$\frac{-(a-b+c)}{a}+\frac{(a-b-c)}{b}+\frac{(a-b+c)}{c}$

=0

$\frac{-1}{a}+\frac{1}{b}+\frac{1}{c}$ = 0

$\frac{\mathbf{1}}{\mathbf{b}}=\frac{\mathbf{1}}{\mathbf{a}}-$

Alternate:

Let, b = 1

c = 1

Then,

Put the value of b and c

$\frac{1-1}{a}+\frac{a+1}{1}+\frac{a-1}{1}$ = 1

a+1 + a–1 = 1

a = $\frac{1}{2}$

Option (A) $\frac{1}{b}=\frac{1}{a}-\frac{1}{c}$

= $\frac{1}{1/2}-\frac{1}{1}$ = 2 – 1 = 1

So, L.H.S = R.H.S

$\frac{1}{b}=\frac{1}{a}-\frac{1}{c}$

TYPE – V

When $a+\frac{1}{a}=2$ then a = 1

Proof $a+\frac{1}{a}=2$

$a^2+1=2a$

$a^2 - 2a + 1 = 0$

$(a-1)^2 = 0$

So, $a = 1$

Ex.75 If $a+\frac{1}{a}=2$, find the value of

$a^{100}+\frac{1}{a^{100}}$

Sol. $a+\frac{1}{a}=2$

Then, a = 1

Put the value of a,

$\Rightarrow 1^{100}+\frac{1}{1^{100}}$

$\Rightarrow 1+\frac{1}{1} \Rightarrow$ 1 + 1 = 2

Ex.76 If $a+\frac{1}{a}=2$, find the value of

(i) $a^{50}+\frac{1}{a^{50}}$ (ii) $a^{49}-\frac{1}{a^{49}}$

(iii) a^3+a^2+a+1

Sol. $a+\frac{1}{a}=2$

a = 1

(i) $a^{50}+\frac{1}{a^{50}}=1^{50}+\frac{1}{1^{50}}$

$= 1+\frac{1}{1}$ = **2**

(ii) $a^{49}-\frac{1}{a^{49}}=1^{49}-\frac{1}{1^{49}}$

$= 1-\frac{1}{1}$ = **0**

(iii) a^3+a^2+a+1

$1^3 + 1^2 + 1 + 1$ = **4**

Ex.77 If $p+\frac{1}{p-3}=5$, find the value of :-

(i) $(p-3)^{100}+\left(\frac{1}{p-3}\right)^{100}$

(ii) p^2-3p+4

Sol.(i) $p+\frac{1}{p-3}=5$

Subtract 3 both sides,

$(p-3)+\frac{1}{(p-3)}=5-3$

$(p-3)+\frac{1}{(p-3)}=2$

Let a = (P – 3)

Then,

a + $\frac{1}{a}$ = 2

$\because$ a = 1 = (P – 3)
Now,
$p-3=1$
$p=4$

$(p-3)^{100}+\left(\frac{1}{p-3}\right)^{100}$

$1^{100}+\frac{1}{1^{100}}=2$

(ii) p^2-3p+4
$4^2-3\times4+4$
$=8$

* **When $a+\frac{1}{a}=-2$, then a = –1**

Ex.78 If $a+\frac{1}{a}=-2$, Find the value of $a^{200}+\frac{1}{a^{200}}$

Sol. $a+\frac{1}{a}=-2$
Now, a = –1
$a^{200}+\frac{1}{a^{200}}$
$=(-1)^{200}+\frac{1}{(-1)^{200}}$
= 1 + 1 = 2

Ex.79 If $a+\frac{1}{a}=-2$, find the value of

(i) $a^{99}+\frac{1}{a^{99}}$ (ii) $a^{32}+\frac{1}{a^{31}}$

(iii) $a^4+a^3+a^2+a+1$

Sol. $a+\frac{1}{a}=-2$
So, a = –1

(i) $a^{99}+\frac{1}{a^{99}}$
$(-1)^{99}+\frac{1}{(-1)^{99}}$
(–1) + (–1) = –2

(ii) $a^{32}+\frac{1}{a^{31}}$
$(-1)^{32}+\frac{1}{(-1)^{31}}$
1 – 1 = 0

(iii) $a^4+a^3+a^2+a+1$
$(-1)^4+(-1)^3+(-1)^2+(-1)+(1)$
= 1

Ex.80 If p + $\frac{1}{p-3}$ = 1, Find the value of

(i) $(P-3)^{42}+\frac{1}{(p-3)^{42}}$

(ii) $(P-3)^{101}-\frac{1}{(p-3)^{101}}$

(iii) P^3+4P^2+5P+1

Sol. p + $\frac{1}{p-3}$ = 1
Subtract 3 both sides
$(P-3)+\frac{1}{(p-3)}=1-3$
$(P-3)+\frac{1}{(p-3)}=-2$
So,
(P – 3) = – 1
P = – 1 + 3 = 2
P = 2

(i) $(P-3)^{42}+\frac{1}{(p-3)^{42}}$
$(-1)^{42}+\frac{1}{(-1)^{42}}$
= 1 + 1 = 2

(ii) $(P-3)^{101}-\frac{1}{(p-3)^{101}}$
$(-1)^{101}-\frac{1}{(-1)^{101}}$
= (–1) – (–1)
= –1 + 1 = 0

(iii) P^3+4P^2+5P+1
Put the value of P = 2
$2^3+4\times2^2+5\times2+1$
= 8 + 16 + 10 + 1 = 35

TYPE – VI

(A). When $a+\frac{1}{a}=1$,
or
$a^2-a+1=0$
Then $a^3=-1$
or
$a^3+1=0$

Proof $a+\frac{1}{a}=1$
Squaring both sides
$a^2+\frac{1}{a^2}=1^2-2=-1$
$a^2+\frac{1}{a^2}=-1$
Multiply a both sides
$=a\left(a^2+\frac{1}{a^2}\right)-1\times a$
$a^3+\frac{1}{a}=-a$
$a^3+\frac{1}{a}+a=0$
$a^3+1=0$
or
$a^3=-1$

Ex.81 If a + $\frac{1}{a}$ = 1, Find the value of $a^{15}+\frac{1}{a^{15}}$

Sol. $a+\frac{1}{a}=1$
Then,
$a^3=-1$
$a^{15}+\frac{1}{a^{15}}=(a^3)^5+\frac{1}{(a^3)^5}$
$(-1)^5+\frac{1}{(-1)^5}=(-1)+(-1)$
= – 2

Ex.82 If $a^2-a+1=0$, Find the value of

(i) $a^{36}+\frac{1}{a^{36}}$ (ii) $a^{37}+\frac{1}{a^{37}}$

(iii) $a^{38}+\frac{1}{a^{38}}$

Sol. $a^2-a+1=0$
or
$a+\frac{1}{a}=1$
Then,
$a^3=-1$

(i) $a^{36}+\frac{1}{a^{36}}$
$(a^3)^{12}+\frac{1}{(a^3)^{12}}$
$=(-1)^{12}+\frac{1}{(-1)^{12}}=1+1=2$

(ii) $a^{37} + \frac{1}{a^{37}}$

Break the power multiple of 3

$a^{36} \times a + \frac{1}{a^{36} \times a}$

$= (-1)^{12} \times a + \frac{1}{(-1)^{12} \times a}$

$= a + \frac{1}{a}$ = **1 (As Given)**

So,

$a^{37} + \frac{1}{a^{37}} = 1$

(iii) $a^{38} + \frac{1}{a^{38}}$

Break the power multiple of 3

$a^{36} \times a^2 + \frac{1}{a^{36} \times a^2}$

$= (-1)^{12} \times a^2 + \frac{1}{(-1)^{12} \times a^2}$

$= a^2 + \frac{1}{a^2}$

$= a + \frac{1}{a}$ = **1 (As Given)**

Then,

$a^2 + \frac{1}{a^2} = 1^2 - 2 = -1$

$a^{38} + \frac{1}{a^{38}} = -1$

Ex.83 If $a + \frac{1}{a} = 1$ find the value of $a^{100} + a^{99} + a^{98} + a^{97} + a^{96} + a^{95}$

Sol. $a + \frac{1}{a} = 1$

Then,

$a^3 + 1 = 0$

$a^{100} + a^{99} + a^{98} + a^{97} + a^{96} + a^{95}$

$= a^{100} + a^{97} + a^{99} + a^{96} + a^{98} + a^{95}$

$= a^{97}(a^3 + 1) + a^{96}(a^3 + 1) + a^{95}(a^3 + 1)$

$= a^{97} \times 0 + a^{96} \times 0 + a^{95} = 0$

Note: When difference of the power is 3 then the value of both terms is = 0

Ex.84 If $\frac{x}{3} + \frac{3}{x} = 1$, Find the value of x^3

Sol. Let $a = \frac{x}{3}$

Then,

$a + \frac{1}{a} = 1$

We know that $a^3 = -1$

Put the value a

$\left(\frac{x}{3}\right)^3 = -1$

$\frac{x^3}{27} = -1$

$x^3 = -27$

(B). When $a + \frac{1}{a} = -1$

or

$a^2 + a + 1 = 0$

Then,

$a^3 = 1$

Proof :- $a + \frac{1}{a} = -1$

$a^2 + \frac{1}{a^2} = (-1)^2 - 2 = -1$

Multiply a both sides.

$a\left(a^2 + \frac{1}{a^2}\right) = -1 \times a$

$a^3 + \frac{1}{a} = -a$

$a^3 + a + \frac{1}{a} = 0$

$a^3 - 1 = 0$

$a^3 = 1$

Ex.85 If $a + \frac{1}{a} = -1$ Find the value of

(i) $a^{48} + \frac{1}{a^{48}}$ (ii) $a^{50} + \frac{1}{a^{50}}$

(iii) $a^{51} + \frac{1}{a^{51}}$

Sol. (i) $a^{48} + \frac{1}{a^{48}}$

$1^{16} + \frac{1}{1^{16}} = 1 + 1 = 2$

(ii) $a^{50} + \frac{1}{a^{50}}$

$a^{48} \times a^2 + \frac{1}{a^{48} \times a^2}$

$= 1^{16} \times a^2 + \frac{1}{1^{16} \times a^2}$

$= a^2 + \frac{1}{a^2}$

$= a + \frac{1}{a} = -1$

Then,

$= a^2 + \frac{1}{a^2} = (-1)^2 - 2$

$= -1$

$\because a^{50} + \frac{1}{a^{50}} = -1$

(iii) $a^{51} + \frac{1}{a^{51}}$

$(a^3)^{17} + \frac{1}{(a^3)^{17}}$

$= 1^{17} + \frac{1}{1^{17}} = 1 + 1 =$ **2**

Ex.86 If $a^2 + a + 1 = 0$ find the value of $(a^2 - a + 1)(a + 1 - a^2)$

Sol. $a^2 + a + 1 = 0$

or

$a + \frac{1}{a} = -1$

We know that

$a^3 = 1$

$a^2 + 1 = -a$ ....(i)

$a + 1 = -a^2$..........(ii)

$(a^2 - a + 1)(a + 1 - a^2)$

From Equation (i) and (ii)

$(-a - a)(-a^2 - a^2)$

$= -2a \times -2a^2$

$= 4a^3 = 4 \times 1 =$ **4**

Ex.87 If $a^2 + a + 1 = 0$, find the value of $a^5 + a^4 + 1$

Sol. $a^2 + a + 1 = 0$

or

$a + \frac{1}{a} = -1$

Then,

$a^3 = 1$

$a^5 + a^4 + 1$

$a^3 \times a^2 + a^3 \times a + 1$

Put the value of **$a^3 = 1$**

$a^2 + a + 1 =$ **0 (As Given)**

Ex.88 If $x^{1/4} + \frac{1}{x^{1/4}} = 1$ find the value of $x^{252} + \frac{1}{x^{252}}$

Sol. $x^{1/4} + \frac{1}{x^{1/4}} = 1$

Squaring both sides

$x^{1/2} + \frac{1}{x^{1/2}} = 1^2 - 2 = -1$

Again Squaring both side

$x + \frac{1}{x} = (-1)^2 - 2 = -1$

Then,

$x + \frac{1}{x} = -1$

$x^3 = 1$

Now,

$x^{252} + \frac{1}{x^{252}}$

$(x^3)^{84} + \frac{1}{(x^3)^{84}}$

$(1)^{84} + \frac{1}{(1)^{84}}$

$= 1 + 1 = 2$

TYPE – VII

When $x + \frac{1}{x} = \sqrt{3}$

Then

$x^6 + 1 = 0$

or

$x^6 = -1$

Proof $x + \frac{1}{x} = \sqrt{3}$

Cube both sides

$x^3 + \frac{1}{x^3} = \left(\sqrt{3}\right)^3 - 3 \times \sqrt{3}$

$x^3 + \frac{1}{x^3} = 0$

Multiply x^3 both sides

$x^6 + 1 = 0$

$x^6 = -1$

Ex.89 If $x + \frac{1}{x} = \sqrt{3}$, Find the value of

(i) $x^{90} + \frac{1}{x^{90}}$ (ii) $x^{96} + \frac{1}{x^{96}}$

Sol. $x + \frac{1}{x} = \sqrt{3}$

Then $x^6 = -1$

(i) $x^{90} + \frac{1}{x^{90}}$

$(x^6)^{15} + \frac{1}{\left(x^6\right)^{15}}$

$=(-1)^{15} + \frac{1}{(-1)^{15}}$

$= -1 - 1 = -2$

(ii) $x^{96} + \frac{1}{x^{96}}$

$(x^6)^{16} + \frac{1}{\left(x^6\right)^{16}}$

$=(-1)^{16} + \frac{1}{(-1)^{16}}$

$= 1 + 1 = 2$

Ex.90 If $x + \frac{1}{x} = \sqrt{3}$ find the value of

(i) $x^{92} + \frac{1}{x^{92}}$ (ii) $x^{93} + \frac{1}{x^{93}}$

Sol. (i) $x^{92} + \frac{1}{x^{92}}$

Break the power multiple of 6

$x^{90} \times x^2 + \frac{1}{x^{90} \times x^2}$

$= (-1)^{15} \times x^2 + \frac{1}{(-1)^{15} \times x^2}$

$= -\left(x^2 + \frac{1}{x^2}\right)$

$x + \frac{1}{x} = \sqrt{3}$ **(As Given)**

Then,

$x^2 + \frac{1}{x^2} = \left(\sqrt{3}\right)^2 - 2$

$x^2 + \frac{1}{x^2} = 3 - 2 = 1$

So,

$x^{92} + \frac{1}{x^{92}} = -1$

(ii) $x^{93} + \frac{1}{x^{93}}$

Break the power multiple of 6

$x^{90} \times x^3 + \frac{1}{x^{90} \times x^3}$

$= (-1)^{15} \times x^3 + \frac{1}{(-1)^{15} \times x^3}$

$= -\left(x^3 + \frac{1}{x^3}\right)$

$x + \frac{1}{x} = \sqrt{3}$ **(As Given)**

Then,

$x^3 + \frac{1}{x^3} = \left(\sqrt{3}\right)^3 - 3 \times \sqrt{3}$

$x^3 + \frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3} = 0$

So,

$x^{93} + \frac{1}{x^{93}} = 0$

Ex.91 If $x + \frac{1}{x} = \sqrt{3}$ find the value of

(i) $x^6 + 5$

(ii) $x^{102} + x^{96} + x^{101} + x^{95} + x^{100} + x^{94}$,

Sol. $x + \frac{1}{x} = \sqrt{3}$

Then,

$x^6 + 1 = 0$

(i) $x^6 + 5$

$\underbrace{x^6 + 1}_{0} + 4 = 4$

(ii) $x^{102} + x^{96} + x^{101} + x^{95} + x^{100} + x^{94}$,

$x^{96}(x^6 + 1) + x^{95}(x^6 + 1) + x^{94}(x^6 + 1)$

$= x^{96} \times 0 + x^{95} \times 0 + x^{94} \times 0 = 0$

Note: When difference of the power is 6 then the value of sum of both of terms value is 0.

TYPE – VIII

If x+y = 0 Then x = –y

or

x = 0, y = 0

If $x^2 + y^2 = 0$

Then $x^2 = 0$, $x = 0$

And $y^2 = 0$, y = 0

If $(x-1)^2 + (y-2)^2 = 0$ then we can say x = 1 and y = 2

Ex.92 If $(x + 3)^2 + (y - 5)^2 + (z + 2)^2 = 0$ find the value $\sqrt{x + y + z}$

Sol. $(x + 3)^2 + (y - 5)^2 + (z + 2)^2 = 0$

$\downarrow$ 0, $x = -3$; $\downarrow$ 0, $y = 5$; $\downarrow$ 0, $z = -2$

$\sqrt{x + y + z} = \sqrt{-3 + 5 - 2}$

= 0

Ex.93 If $(a - 4)^2 + (b - 5)^2 + (c - 3)^2 = 0$ find the value $\frac{a + b}{c}$

Sol. $(a-4)^2 + (b-5)^2 + (c-3)^2 = 0$

$$\downarrow \quad \downarrow \quad \downarrow$$
$$0 \quad 0 \quad 0$$
$$a = 4 \quad b = 5 \quad c = 3$$

$$\frac{a+b}{c} = \frac{4+5}{3} = \mathbf{3}$$

Ex.94 If $x^2 + y^2 + z^2 + 4x + 2y + 5 = 0$ find the value $x^2 + y^3 + z^4$

Sol. $x^2 + y^2 + z^2 + 4x + 2y + 5 = 0$

$x^2 + 4x + 4 + y^2 + 2y + 1 + z^2 = 0$

$(x+2)^2 + (y+1)^2 + z^2 = 0$

$$\downarrow \quad \downarrow \quad \downarrow$$
$$0 \quad 0 \quad 0$$
$$x = -2 \quad y = -1 \quad z = 0$$

$x^2 + y^3 + z^4 = (-2)^2 + (-1)^3 + (0)^4$

$= 4 - 1 + 0 = 3$

Ex.95 If $a^2 + b^2 + c^2 = 2(a - 2b - 2c) - 9$ find the value of $a^3 + b^4 - c^2$

Sol. $a^2 + b^2 + c^2 = 2(a - 2b - 2c) - 9$

$a^2 + b^2 + c^2 - 2a + 4b - 4c + 9 = 0$

$a^2 - 2a + 1 + b^2 + 4b + 4 + c^2 - 4c + 4 = 0$

$(a-1)^2 + (b+2)^2 + (c-2)^2 = 0$

$$\downarrow \quad \downarrow \quad \downarrow$$
$$0 \quad 0 \quad 0$$
$$a = 1 \quad b = -2 \quad c = 2$$

$a^3 + b^4 - c^2 = (1)^3 + (-2)^4 - (2)^2$

$= 1 + 16 - 4 = \mathbf{13}$

Ex.96 If $5x^2 + 4xy + y^2 + 1 + 2x = 0$ find the value of x & y

Sol. $5x^2 + 4xy + y^2 + 1 + 2x = 0$

$4x^2 + 4xy + y^2 + x^2 + 2x + 1 = 0$

$(2x + y)^2 + (x+1)^2 = 0$

$$\downarrow \quad \downarrow$$
$$0 \quad 0$$

$2x = -y \quad x = -1$

$\boldsymbol{x = -1},$

$2x = -y$

$2 \times -1 = -y$

$\mathbf{y = 2}$

Ex.97 If $(x + y - z - 1)^2 + (y + z - x - 5)^2 + (z + x - y - 3)^2 = 0$

find $\sqrt{x+y+z}$

Sol. $(x+y-z-1)^2 + (y+z-x-5)^2 + (z+x-y-3)^2 = 0$

$$\downarrow \quad \downarrow \quad \downarrow$$
$$0 \quad 0 \quad 0$$
$$x+y-z=1 \quad y+z-x=5 \quad z+x-y=3$$

Adding all three eqations

$x + y - z + y + z - x + z + x - y = 1 + 5 + 3$

$x + y + z = 9$

Then $\sqrt{x+y+z} = \sqrt{9} = \mathbf{3}$

TYPE – IX

Rationalising factor of the surd x = $\sqrt{a} \pm \sqrt{b}$ and

$$\frac{\mathbf{1}}{\mathbf{x}} = \sqrt{a} \mp \sqrt{b}.$$

Ex.98 If $x = \dfrac{1}{5+2\sqrt{6}}$, $y = \dfrac{1}{5-2\sqrt{6}}$

Find the value of $\dfrac{1}{x+1} + \dfrac{1}{y+1}$

Sol. $x = \dfrac{1}{5+2\sqrt{6}}$

Now, $\dfrac{1}{x} = \dfrac{1}{5-2\sqrt{6}} = y$

$$\frac{\mathbf{1}}{\boldsymbol{x}} = \mathbf{y}$$

Put the value of y in equation

$$= \frac{1}{x+1} + \frac{1}{y+1}$$

$$= \frac{1}{x+1} + \frac{1}{\frac{1}{x}+1}$$

$$\frac{1}{x+1} + \frac{1 \times x}{1+x} = \frac{1+x}{1+x} = 1$$

Note:

If xy = 1 then $\dfrac{1}{1+\boldsymbol{x}} + \dfrac{1}{1+\boldsymbol{y}} = 1$

Ex.99 $x = \left(\sqrt{3}+\sqrt{2}\right)^{-3}$,

$y = \left(\sqrt{3}-\sqrt{2}\right)^{-3}$

Find the value of $(x+1)^{-1} + (y+1)^{-1}$

Sol: $x = \left(\sqrt{3}+\sqrt{2}\right)^{-3}$

Then

$$\frac{1}{x} = \left(\sqrt{3}-\sqrt{2}\right)^{-3} = y$$

$$\frac{1}{x} = y$$

Now,

$(x+1)^{-1} + (y+1)^{-1}$

$$= \frac{1}{x+1} + \frac{1}{y+1}$$

$$= \frac{1}{x+1} + \frac{1}{\frac{1}{x}+1}$$

$$= \frac{1}{x+1} + \frac{1 \times x}{1+x} = \frac{1+x}{1+x} = 1$$

Ex.100. If $x = 2+\sqrt{3}$, find the value of $\dfrac{x^6+x^4+x^2+1}{x^3}$

Sol. $x = 2+\sqrt{3}$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$$

$$x + \frac{1}{x} = 4$$

Then,

$$x^3 + \frac{1}{x^3} = (4)^3 - 3 \times 4$$

$$= x^3 + \frac{1}{x^3} = 52$$

Now,

$$= \frac{x^6+x^4+x^2+1}{x^3}$$

$$= x^3 + x + \frac{1}{x} + \frac{1}{x^3}$$

$$= x^3 + \frac{1}{x^3} + x + \frac{1}{x}$$

Put the value

$52 + 4 = 56$

Ex101. If $x = 5+2\sqrt{6}$ Find the value of $\dfrac{x^4+\frac{1}{x^2}}{x^2-3x+1}$

Sol. $x = 5+2\sqrt{6}$

$$\frac{1}{x} = 5-2\sqrt{6}$$

$$x + \frac{1}{x} = 10$$

Then $x^3 + \dfrac{1}{x^3} = (10)^3 - 3 \times 10$

$$x^3 + \frac{1}{x^3} = \mathbf{970}$$

Now,

x divide or nominator and denominator

$$\frac{x^4+\frac{1}{x^2}}{x^2-3x+1}$$

$$= \frac{\frac{x^4}{x} + \frac{1}{x^2 \times x}}{\frac{x^2}{x} - \frac{3x}{x} + \frac{1}{x}}$$

$$= \frac{x^3 + \frac{1}{x^3}}{x + \frac{1}{x} - 3}$$

$$= \frac{970}{10-3} = \frac{970}{7}$$

Ex102. If $x = 7 + 4\sqrt{3}$, find the value of $\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$

Sol. $x = 7 + 4\sqrt{3}$

$\frac{1}{x} = 7 - 4\sqrt{3}$

$x + \frac{1}{x} = 14$

Then,

$x^2 + \frac{1}{x^2} = (14)^2 - 2$

$x^2 + \frac{1}{x^2} = \mathbf{194}$

x^2 divide or nominator and denominator

$$= \frac{\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^2}}{\frac{x^4 + 1}{x^2}}$$

$$= \frac{\frac{x^4}{x^2} + \frac{3x^3}{x^2} + \frac{5x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{x^4}{x^2} + \frac{1}{x^2}}$$

$$= \frac{x^2 + 3x + 5 + \frac{3}{x} + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}}$$

$$= \frac{x^2 + \frac{1}{x^2} + 3\left(x + \frac{1}{x}\right) + 5}{x^2 + \frac{1}{x^2}}$$

$$= \frac{194 + 3 \times 14 + 5}{194} = \frac{241}{194}$$

Ex.103 If $x = 3 - 2\sqrt{2}$, $y = 3 + 2\sqrt{2}$

Find the value of $\frac{x^2}{y} + \frac{y^2}{x}$

Sol. $x = 3 - 2\sqrt{2}$

$\frac{1}{x} = 3 + 2\sqrt{2}$

Then, $\frac{1}{x} = y$

$\frac{x^2}{y} + \frac{y^2}{x}$

Put the value of y = $\frac{1}{x}$

$$\frac{x^2}{\frac{1}{x}} + \left(\frac{1}{x}\right)^2 \times \frac{1}{x}$$

$x^3 + \frac{1}{x^3}$

Now,

$x + \frac{1}{x} = 6$

$x^3 + \frac{1}{x^3} = (6)^3 - 3 \times 6$

$x^3 + \frac{1}{x^3} = 198$

Ex.104. If $x = 5 + 2\sqrt{6}$ *and* $xy = 1$,

Find the value of $\frac{x^2 + y^2 + 2xy}{x^3 + y^3 + 3xy}$

Sol. $x = 5 + 2\sqrt{6}$

$\frac{1}{x} = 5 - 2\sqrt{6}$

$xy = 1$ (As given)

Then,

$y = \frac{1}{x} = 5 - 2\sqrt{6}$

$\frac{x^2 + y^2 + 2xy}{x^3 + y^3 + 3xy}$

Put the value of $y = \frac{1}{x}$

$$\frac{x^2 + \frac{1}{x^2} + 2}{x^3 + \frac{1}{x^3 + 3}}$$

Now,

$x + \frac{1}{x} = 10$

Then

$x^2 + \frac{1}{x^2} = (10)^2 - 2$

$x^2 + \frac{1}{x^2} = 98$

And,

$x^3 + \frac{1}{x^3} = (10)^3 - 3 \times 10$

$x^3 + \frac{1}{x^3} = 970$

Put the value in equation,

$\frac{98 + 2}{970 + 3} = \frac{100}{973}$

Ex.105. If $x + \frac{1}{x} = 10$, Find the value of $\frac{7x}{x^2 - 5x + 1}$

Sol. $\frac{7x}{x^2 - 5x + 1}$

x divide or nominator and denominator

$$\frac{\frac{7x}{x}}{\frac{x^2 - 5x + 1}{x}} = \frac{7}{x - 5 + \frac{1}{x}}$$

$$= \frac{7}{x + \frac{1}{x} - 5}$$

$x + \frac{1}{x} = 10$ (As Given)

$\frac{7}{10 - 5} = \frac{7}{5}$

Ex.106. $\frac{2p}{p^2 - 2p + 1} = \frac{1}{4}$, Find the value of $p + \frac{1}{p}$

Sol. $\frac{2p}{p^2 - 2p + 1} = \frac{1}{4}$

8p = p² – 2p + 1

10p = p² + 1

divide by p both sides,

$p + \frac{1}{p} = 10$

Ex.107 If $x + \frac{a}{x} = 1$, find the value of $\frac{x^2 + x + a}{x^3 - x^2}$

Sol. $x+\frac{a}{x}=1$

$x^2 + a = x$ ....(i)

$x^2 - x = -a$ ....(ii)

$\frac{x^2+x+a}{x^3-x^2}$

From equation (i)

$\frac{x+x}{x^3-x^2}$

$=\frac{2}{x^2-x}$

From equation (ii)

$\frac{x^2+x+a}{x^3-x^2}=\frac{-2}{a}$

Type – X

Ex.108 If $\frac{a^2-bc}{a^2+bc}+\frac{b^2-ac}{b^2+ac}+\frac{c^2-ab}{c^2+ab}$ = 1 Find $\frac{2a^2}{a^2+bc}+\frac{2b^2}{b^2+ac}+\frac{2c^2}{c^2+ab}$

Sol. $\frac{a^2-bc}{a^2+bc}+\frac{b^2-ac}{b^2+ac}+\frac{c^2-ab}{c^2+ab}=1$

Added 1 every terms

$\Rightarrow \frac{a^2-bc}{a^2+bc}+1+\frac{b^2-ac}{b^2+ac}+1+\frac{c^2-ab}{c^2+ab}+1=1+3$

$\Rightarrow \frac{a^2-bc+a^2+bc}{a^2+bc}+\frac{b^2-ac+b^2+ac}{b^2+ac}+\frac{c^2-ab+c^2+ab}{c^2+ab}=4$

$\Rightarrow \frac{2a^2}{a^2+bc}+\frac{2b^2}{b^2+ac}+\frac{2c^2}{c^2+ab}=4$

Note: In this type of question when based is same we add or substract 1

Ex.109 If $\frac{1}{a-1}+\frac{2}{b-2}+\frac{3}{c-3}=10$, Find the value of $\frac{a}{a-1}+\frac{b}{b-2}+\frac{c}{c-3}$

Sol. $\frac{1}{a-1}+\frac{2}{b-2}+\frac{3}{c-3}=10$

$\frac{1}{a-1}+1+\frac{2}{b-2}+1+\frac{3}{c-3}+1$

= 10+1+1+1

$\frac{a}{a-1}+\frac{b}{b-2}+\frac{c}{c-3}=13$

Ex.110 If $\frac{1}{x+1}+\frac{2}{x+2}+\frac{3}{x+3}$ ...... $\frac{1007}{x+1007}=1249$

find $\frac{x}{x+1}+\frac{x}{x+2}$ ...... $\frac{x}{x+1007}$

Sol. $\frac{1}{x+1}+\frac{2}{x+2}+\frac{3}{x+3}$ ...... $\frac{1007}{x+1007}=1249$

$\Rightarrow \frac{1}{x+1}-1+\frac{2}{x+2}-1+\frac{3}{x+3}-1$....... $\frac{x}{x+1007}-1=1249-1007$

$\Rightarrow \frac{x}{x+1}+\frac{x}{x+2}$ ...... $\frac{x}{x+1007}$ = –242

Ex.111 If x = 101 find the value of $x\left(x^2-3x+3\right)$

Sol: $x\left(x^2-3x+3\right)$

$x^3 - 3x^2 + 3x$

We add and substract 1 then,

$x^3-3x^2+3x-1+1$

$= (x-1)^3+1$

Put the value x

$= (101-1)^3 + 1$

$= 100^3+1$ = 1000001

Ex.112 If x = 102 find the value of $x(x^2+3x+3)$

Sol: $x(x^2+3x+3)$

$x^3 + 3x^2 + 3x$

We add and substract 1 then,

$x^3+3x^2+3x+1-1$

$= (x+1)^3-1$

Put the value x

$= (102 + 1)^3 + 1 = 103^3+1$

= 1092727+ 1 = 1092728

Ex.113 : If $\frac{1}{\sqrt[3]{25}-\sqrt[3]{5}+1}$ $= A\sqrt[3]{25} + B\sqrt[3]{5} + C$

Find the value of A + B – C

Sol. $\frac{1}{\sqrt[3]{25}-\sqrt[3]{5}+1}$

$= A\sqrt[3]{25} + B\sqrt[3]{5} + C$

$\Rightarrow \frac{1}{5^{\frac{2}{3}}-5^{\frac{1}{3}}+1}=A(5)^{\frac{2}{3}}+B(5)^{\frac{1}{3}}+C$

$\Rightarrow \left(5^{\frac{1}{3}}+1\right)$ multiply and divided

$\Rightarrow \frac{1\times\left(5^{\frac{1}{3}}+1\right)}{\left(5^{\frac{1}{3}}+1\right)\left[\left(5^{\frac{1}{3}}\right)^2-5^{\frac{1}{3}}\times1+(1)^2\right]}$

$(a^3+b^3)=(a+b)(a^2-ab+b^2)$

$\Rightarrow \frac{5^{\frac{1}{3}}+1}{\left(5^{\frac{1}{3}}\right)^3+1^3}$

$\Rightarrow \frac{5^{\frac{1}{3}}+1}{5+1}=\frac{5^{\frac{1}{3}}+1}{6}=A(5)^{\frac{2}{3}}+B\left(5^{\frac{1}{3}}\right)+c$

$\Rightarrow \frac{5^{\frac{1}{3}}}{6}+\frac{1}{6}=A(5)^{\frac{2}{3}}+B\left(5^{\frac{1}{3}}\right)+c$

Comparison of the terms

A = 0, B = $\frac{1}{6}$ and C = $\frac{1}{6}$

Then, A+ B – C = 0 + $\frac{1}{6}-\frac{1}{6}$ = 0

Ex.114 : If $\frac{1}{\sqrt[3]{16}+\sqrt[3]{4}+1}$ $=A\sqrt[3]{16}+B\sqrt[3]{4}+C$

Find the value of A + B + C

Sol. $\frac{1}{\sqrt[3]{16}+\sqrt[3]{4}+1}=A\sqrt[3]{16}+B\sqrt[3]{4}+C$

$\Rightarrow \frac{1}{4^{\frac{2}{3}}-4^{\frac{1}{3}}+1}=A(4)^{\frac{2}{3}}+B(4)^{\frac{1}{3}}+C$

$\Rightarrow \left(4^{\frac{1}{3}}-1\right)$ multiply and divided

$\frac{1\times\left(4^{\frac{1}{3}}-1\right)}{\left(4^{\frac{1}{3}}-1\right)\left[\left(4^{\frac{1}{3}}\right)^2+4^{\frac{1}{3}}\times1+(1)^2\right]}$

$$\frac{4^{\frac{1}{3}}-1}{\left(4^{\frac{1}{3}}\right)^3-1^3}$$

$$\Rightarrow \frac{4^{\frac{1}{3}}-1}{4-1} = \frac{4^{\frac{1}{3}}-1}{3}$$

$$= A(4)^{\frac{2}{3}}+B\left(4^{\frac{1}{3}}\right)+c$$

$$\Rightarrow \frac{4^{\frac{1}{3}}}{3}-\frac{1}{3} = A(4)^{\frac{2}{3}}+B\left(4^{\frac{1}{3}}\right)+c$$

Comparison of the terms

A = 0, B = $\frac{1}{3}$ and C = $-\frac{1}{3}$

Then, A+ B + C = 0 + $\frac{1}{3}-\frac{1}{3}$ = 0

TYPE – XI

Ex.115 If x = 11, Find the value of $x^5 - 12x^4 + 13x^3 - 12x^2 + 12x - 5$

Sol. $x^5 - 12x^4 + 13x^3 - 12x^2 + 12x - 5$

Expandable from the equation

Then,

$x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 5\ x^3$

$= \cancel{x^5} - \cancel{11x^4} - \cancel{x^4} + \cancel{11x^3} + x^3 - \cancel{11x^2} - \cancel{x^2} + \cancel{11x} + x - 5 + x^3$

Put the value of x = 11

= 11 – 5 + (11)³ = 11 – 5 + 1331

= 1337

Ex.116 x = 8 find $x^4 - 9x^3 + 9x^2 - 9x$

Sol. $x^4 - 9x^3 + 9x^2 - 9x$

Expandable from the equation

Then,

$\cancel{x^4} - \cancel{8x^3} - \cancel{x^3} + \cancel{8x^2} - \cancel{8x} - x$

Put the value of x = 8

= – 8

Ex.117 If x = 12 find $x^4 - 13x^3 + 13x^2 - 12x + 11$

Sol. $x^4 - 13x^3 + 13x^2 - 12x + 11$

Expandable from the equation

Then,

$\underbrace{x^4 - 12x^3}_{0} \underbrace{- x^3 + 12x^2}_{0} \underbrace{+ x^2 - 12x}_{0} + 11$

= 11

Ex.118 If x = 12 find $x^4 - 14x^3 + 13x^2 \div 12x + 11$

Sol. $x^4 - 14x^3 + 13x^2 \div 12x + 11$

Expandable form the equation

Then,

$\underbrace{x^4 - 12x^3}_{0} \underbrace{- x^3 + 12x^2}_{0} \underbrace{+ x^2 - 12x}_{0} + 11 - x^3$

Put the value of x = 12

Then,

= 11 – 12³

= 11 – 1728 = – 1717

Ex.119 find the value of $\frac{1+7231\times7233}{7232}$

Sol. Let 7232 = x

Then,

$$\frac{1+(x-1)\times(x+1)}{x}$$

$$= \frac{1+x^2-1}{x} = x$$

So,

x = 7232

Ex. 120 find the value of $\frac{4+673\times677}{675}$

Sol. Let 675 = x

$$\Rightarrow \frac{4+(x-2)(x+2)}{x}$$

$$\Rightarrow \frac{4+x^2-4}{x} = x$$

Then,

x = 675

Ex.121 Find the value of $\frac{16+9748\times9756}{9752}$

Sol. Let 9752 = x

$$= \frac{16+(x-4)(x+4)}{x}$$

$$= \frac{16+x^2-16}{x} = x$$

So,

***x* = 9752**

Ex.122 What will be added in the prodcut of numbers (30 × 36) That resulted will be perfect square number

Sol. (30 × 36)

Difference of the number between = 36 – 30

= 6

Divided by 2 in difference = $\frac{6}{2}$

= 3

Square of 3 = 3² = 9

(30 × 36) + 9 = 1080 + 9

= 1089

1089 is perfect square of 33

So,

9 will be added the product of number

Note:- $\left(\frac{\mathbf{D}}{\mathbf{2}}\right)^2$ **always added the prodcut of number we will get perfect square integer number**

Here D = Difference of numbers

Ex.123 What will be added in the prodcut of numbers (174 × 182) That resulted will be perfect number

Sol. D = 8

$$\frac{D}{2} = \frac{8}{2} = 4$$

$$\left(\frac{D}{2}\right)^2 = 4^2 = 16$$

So,

16 will be added the product of number

Maximum or minimum value of quadratic equation

Quadratic equation in general form

1. When a>0 (In the equation ax^2+bx+c.) The expression gives minimum value

$$y = \frac{4ac-b^2}{4a}$$

Max = ∞

2. When **a**<0 (In the equation ax^2+bx+c) The expression gives maximum value

$$= \frac{4ac-b^2}{4a}$$

Min = – ∞

Ex.124 Expression $5x^2-8x+14$ what will be minimum value ?

Sol. $5x^2-8x+14$

a> 0

Then, min value = $\frac{4ac-b^2}{4a}$

here,

a = 5, b = –8, c = 14

Now, min = $\frac{4\times5\times14-(-8)^2}{4\times5}$

$$= \frac{280-64}{20} = \frac{216}{20}$$

Min value = $\mathbf{\frac{54}{5}}$

Ex.125 Find the minimum value of $(x-2)(x-9)$

Sol. $(x-2)(x-9)$
$= x^2-2x-9x+18$
$= x^2-11x+18$
Coefficient of x^2 is 1 which 1>0
Then min value

$= \frac{4\times1\times18-(-11)^2}{4\times1} = \frac{72-121}{4}$

min value $= \mathbf{\frac{-49}{4}}$

Ex.126 Find the maximum value of $12-7x-x^2$

Sol. $12-7x-x^2$ or $-x^2-7x+12$
Here a<0
Cofficient of x^2 is –1 which less Than 0
So,

Max value $= \frac{4ac-b^2}{4a}$

$= a=-1,\ b=-7$ and $c=12$

$= \frac{4\times-1\times12-(-7)^2}{4\times(-1)}$

$= \frac{-48-49}{-4} = \frac{-97}{-4} = \mathbf{\frac{97}{4}}$

Ex.127 Find the maximum value of $5-12x-3x^2$

Sol. $5-12x-3x^2$
or
$-3x^2-12x5$
Here coefficient of x^2 is –3 which less than 0,
Now,
$a=-3,\ b=-12,\ c=5$

So, max value $= \frac{4ac-b^2}{4a}$

$= \frac{4\times-3\times5-(-12)^2}{4\times-3}$

$= \frac{-60-144}{-12}$

$= \frac{-204}{-12} = \mathbf{17}$

(ii) a and b are two numbers
Then,

$A,M = \frac{a+b}{2}$

and $G.M = \sqrt{ab}$

Thus when a, b and c are three numbers
Then,
$A.M = a+b+c$
$G.M = \sqrt[3]{abc}$

Relation between A.M & G.M
A.M $\geq$ G.M
When a,b,c are +ve Real number

m and $\frac{1}{m}$ are two number

$A.M = \frac{M+\frac{1}{m}}{2}$, $G.M = \sqrt{M\times\frac{1}{m}}$

A.M $\geq$ G.M

$\frac{m+\frac{1}{m}}{2} \geq \sqrt[2]{m\times\frac{1}{m}}$

$\frac{m+\frac{1}{m}}{2} \geq 1$

$\frac{m+\frac{1}{m}}{2} \geq 2$

When m +ve Real number.

Minimum value of $m+\frac{1}{m}=2$

And
We can say minimum value

$m^n+\frac{1}{m^n}=2$

When m is + ve Real number

Ex.128 If $x>1$, find the minimum value of $f(x) = x^2+\frac{1}{x^2-1}-3$

Sol. $x^2+\frac{1}{x^2-1}-3$

Add and substract 1,
Now,

$(x^2-1)+\frac{1}{(x^2-1)}-3+1$

Let $x^2-1 = m$

$m+\frac{1}{m}-2$

Minimum value of $m+\frac{1}{m}=2$

Then,
$2-2 = 0$
So, minimum value $F(x) = 0$

Ex.129 Find the minimum value of $x^2 + \frac{1}{x^2+1}-4$

Sol. Add and substract

Now, $(x^2+1)+\frac{1}{(x^2+1)}-4-1$

Let $x^2+1 = m$

Then, $m+\frac{1}{m}-5$

Minimum value of $m+\frac{1}{m}=2$

$= 2-5 = -3$

So, minimum value of $F(x) = -3$

Ex.130 Find the minimum value of $x^2+\frac{1}{x^2-2}+5$, here $x>\sqrt{2}$

Sol. $x^2+\frac{1}{x^2-2}+5$

$(x^2-2)+\frac{1}{(x^2-2)}+5+2$

Let $x^2-2 = m$

Now, $m+\frac{1}{m}+7$

Minimum value $= 7+2 = 9$

Ex.131 Find the maximum value of $\frac{x^4}{x^8+1}$, where x is Real number

Sol. $\frac{x^4}{x^8+1}$

componendo and dividendo divide by x^4

$\frac{\frac{x^4}{x^4}}{\frac{x^8+1}{x^4}} = \frac{1}{x^4+\frac{1}{x^4}}$

Min value $= x^4+\frac{1}{x^4} = 2$

Then, max value = –

(iii) If $x+y$ will be given then xy will be maximum When $x=y$

Ex.132 Find maximum value xy if $x+y = 20$

Sol. $x+y = 20$
For maximum $x = y$
Then $x = y = 10$
Max value $xy = 10\times10 = 100$

Ex.133 Find the maximum value xy, if $x+y = 25$

Sol. $x+y = 25$
For maximum $x = y$

Then $x=y=\frac{25}{2}$

Max value = $xy\ \dfrac{25}{2} \times \dfrac{25}{2}$

$= \dfrac{\mathbf{625}}{\mathbf{4}}$

(iv) If x y will be given then x + y will be minimum when x = y here x & y (+ve) Real number

Ex.134 Find the minimum value of x + y if xy = 16

Sol. xy = 16

For minimum value x = y = z

x = y = 4

Min value = x + y = 4 + 4 = 8

Ex.135 Find the minimum value of x + y + z if x y z = 216

Sol. xyz = 216

Min value x = y = z

$x \times x \times x = x^3 = 216$

x = 6

x = y = z = 6

Minimum value = x + y + z

= 6+6+6 = 18

Ex.136 If x+y +z = 18, Find the Maximum value of (x–1) (y–2)(z–3)

Sol. For maximum

(x–1)= (y–2)=(z–3) = m

x = m+1

y = m+2

z = m+3

∴ x+y+z = 18 (As given)

Put the value x, y, and z

m+1 + m+2 + m+3 = 18

3m = 12

***m* = 4**

So, max value of

(x–1) (y–2)(z–3) = $m \times m \times m$

= m^3

Put the value m

Then

Max value = $(4)^3$ = 64

Ex.137 If x+y+z = 24, maximum value of

(x–1) (y–2)(z+3)

Sol. For max value (x–1) = (y–2) = (z–3) = m

x = m +1

y = m +2

z = m –3

∴ x+y+z = 24

Put the value x, y and z

m +1 + m+2 + m – 3 = 24

3m = 24

m = 8

Max value = (x+1) (y+2)(z –3)

= m×m×m = m^3

So, max value = $(8)^3$ = 512

EXERCISE

1. The value of

$\left(1+\dfrac{1}{x}\right)\left(1+\dfrac{1}{x+1}\right)\left(1+\dfrac{1}{x+2}\right)\left(1+\dfrac{1}{x+3}\right)$ is:

(a) $1+\dfrac{1}{x+4}$ (b) $x+4$

(c) $\dfrac{1}{x}$ (d) $\dfrac{x+4}{x}$

2. If $x = 7-4\sqrt{3}$, then the value of $\left(x+\dfrac{1}{x}\right)$ is :

(a) $3\sqrt{3}$ (b) $8\sqrt{3}$

(c) $14+8\sqrt{3}$ (d) 14

3. If $\dfrac{2a+b}{a+4b} = 3$, then find the value of $\dfrac{a+b}{a+2b}$

(a) $\dfrac{5}{9}$ (b) $\dfrac{2}{7}$ (c) $\dfrac{10}{9}$ (d) $\dfrac{10}{7}$

4. If A : B = $\dfrac{1}{2} : \dfrac{3}{8}$ B : C = $\dfrac{1}{3} : \dfrac{5}{9}$ and C : D = $\dfrac{5}{6} : \dfrac{3}{4}$ then find the ratio of A : B : C : D

(a) 6 : 4 : 8 : 10 (b) 6 : 8 : 9 : 10

(c) 8 : 6 : 10 : 9 (d) 4 : 6 : 8 : 10

5. If $\dfrac{a}{3} = \dfrac{b}{4} = \dfrac{c}{7}$ then $\dfrac{a+b+c}{c}$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

6. If $\dfrac{144}{0.144} = \dfrac{14.4}{x}$, then the value of x is

(a) 144 (b) 14.4

(c) 1.44 (d) 0.0144

7. If 1 < x < 2, then the value of $\sqrt{(x-1)^2} + \sqrt{(x-3)^2}$ is

(a) 1 (b) –2

(c) 3 (d) 2x – 4

8. Given that $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$, then the value of z is close to

(a) 1.45 (b) 1.88 (c) 2.9 (d) 3.7

9. If 47.2506 = 4A + 7B + 2C + $\dfrac{5}{D}$ + 6E, then the value of 5A + 3B + 6C + D + 3E is

(a) 53.6003 (b) 53.603

(c) 153.6003 (d) 213.0003

10. If 3^{x+3}+7= 250, then x is equal to

(a) 5 (b) 3 (c) 2 (d) 1

11. If $\dfrac{1}{4} \times \dfrac{2}{6} \times \dfrac{3}{8} \times \dfrac{4}{10} \times \dfrac{5}{12} \times ..... \times \dfrac{31}{64} = \dfrac{1}{2^x}$ the value of x is

(a) 31 (b) 32 (c) 36 (d) 37

12. If $x = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ & $y = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$, then value of $x^2 + y^2$ is :

(a) 14 (b) 13 (c) 15 (d) 10

13. If $x = 3+2\sqrt{2}$, then the value of $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)$ is :

(a) 1 (b) 2

(c) $2\sqrt{2}$ (d) $3\sqrt{3}$

14. If p = 999, then the value of $\sqrt[3]{p\left(p^2+3p+3\right)+1}$ is

(a) 1000 (b) 999

(c) 998 (d) 1002

15. If x : y = 7 : 3 then the value of $\dfrac{xy+y^2}{x^2-y^2}$ is

(a) $\dfrac{3}{4}$ (b) $\dfrac{4}{3}$ (c) $\dfrac{3}{7}$ (d) $\dfrac{7}{3}$

16. If $x^{x\sqrt{x}} = \left(x\sqrt{x}\right)^x$, then x equals

(a) $\dfrac{4}{9}$ (b) $\dfrac{2}{3}$ (c) $\dfrac{9}{4}$ (d) $\dfrac{3}{2}$

17. If a = 7, b = 5 and c = 3, then the value of $a^2 + b^2 + c^2 - ab - bc - ca$ is

(a) 12 (b) –12 (c) 0 (d) 8

18. $\dfrac{\frac{1}{3}.\frac{1}{3}.\frac{1}{3}+\frac{1}{4}.\frac{1}{4}.\frac{1}{4}-3\frac{1}{3}.\frac{1}{4}.\frac{1}{5}+\frac{1}{5}.\frac{1}{5}.\frac{1}{5}}{\frac{1}{3}.\frac{1}{3}+\frac{1}{4}.\frac{1}{4}+\frac{1}{5}.\frac{1}{5}-\left(\frac{1}{3}.\frac{1}{4}+\frac{1}{4}.\frac{1}{5}+\frac{1}{5}.\frac{1}{3}\right)}$ is equal to

(a) $\dfrac{2}{3}$ (b) $\dfrac{3}{4}$ (c) $\dfrac{47}{60}$ (d) $\dfrac{49}{60}$

19. If x = $7-4\sqrt{3}$, then $\sqrt{x}+\frac{1}{\sqrt{x}}$ is equal to:

(a) 1 (b) 2 (c) 3 (d) 4

20. If $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}$ & $b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of

$\frac{a^2+ab+b^2}{a^2+-ab+b^2}$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

21. If a = 4.36, b = 2.39 and c = 1.97, then the value of $a^3-b^3-c^3-3abc$ is

(a) 3.94 (b) 2.39 (c) 0 (d) 1

22. If $\frac{3a+5b}{3a-5b}=5$, then a : b is equal to

(a) 2:1 (b) 2:3 (c) 1:3 (d) 5:2

23. If $p:q=r:s=t:u=2:3$, then $(mp+nr+ot):(mq+ns+ou)$ equals :

(a) 3:2 (b) 2:3 (c) 1:3 (d) 1:2

24. If $x:y=3:4$, then $(7x+3y):(7x-3y)$ is equal to :

(a) 5 : 2 (b) 4 : 3
(c) 11 : 3 (d) 37 : 19

25. For what value(s) of a is $x+\frac{1}{4}\sqrt{x}+a^2$ a perfect square ?

(a) $\pm\frac{1}{18}$ (b) $\frac{1}{8}$ (c) $-\frac{1}{5}$ (d) $\frac{1}{4}$

26. If a$\neq$b,then which of the following statements is true?

(a) $\frac{a+b}{2}=\sqrt{ab}$ (b) $\frac{a+b}{2}<\sqrt{ab}$

(c) $\frac{a+b}{2}>\sqrt{ab}$ (d) All of the above

27. If x, y are two positive real number and $x^{1/3}=y^{1/4}$, then which of the following relations is true?

(a) $x^3=y^4$ (b) $x^3=y$
(c) $x=y^4$ (d) $x^{20}=y^{15}$

28. If $x=\frac{\sqrt{3}}{2}$, then $\frac{\sqrt{1+x}}{1+\sqrt{1+x}}+\frac{\sqrt{1-x}}{1-\sqrt{1-x}}$ is equal to

(a) 1 (b) $2/\sqrt{3}$
(c) $2-\sqrt{3}$ (d) 2

29. If for non-zero, x, $x^2-4x-1=0$, the value of $x^2+\frac{1}{x^2}$ is

(a) 4 (b) 10 (c) 12 (d) 18

30. $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)\left(x^2+\frac{1}{x^2}+1\right)$ is equal to

(a) $x^6+\frac{1}{x^6}$ (b) $x^8+\frac{1}{x^8}$

(c) $x^8-\frac{1}{x^8}$ (d) $x^6-\frac{1}{x^6}$

31. If $\frac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b$, then the value of a is

(a) $\frac{11}{3}$ (b) $-\frac{4}{3}$

(c) $\frac{4}{3}$ (d) $\frac{-4\sqrt{7}}{3}$

32. If $a+\frac{1}{b}=1$ and $b+\frac{1}{c}=1$ then $c+\frac{1}{a}$ is equal to

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

33. If $x=\sqrt{3}+\sqrt{2}$, then the value of $\left(x^3+\frac{1}{x^3}\right)$ is

(a) $6\sqrt{3}$ (b) $12\sqrt{3}$
(c) $18\sqrt{3}$ (d) $24\sqrt{3}$

34. If $x+y=7$, then the value of x^3+y^3+21xy is

(a) 243 (b) 143 (c) 343 (d) 443

35. If $x^{\frac{1}{3}}+y^{\frac{1}{3}}=z^{\frac{1}{3}}$, then {(x + y – z)^3 +27 xyz} equals :

(a) –1 (b) 1 (c) 0 (d) 27

36. If $\frac{a}{b}+\frac{b}{a}=1$, $a\neq 0, b\neq 0$ the value of a^3+b^3 is

(a) 0 (b) 1 (c) – 1 (d) 2

37. If p = 99, then value of p(p^2 +3p +3) is

(a) 999 (b) 9999
(c) 99999 (d) 999999

38. If $5^{\sqrt{x}}+12^{\sqrt{x}}=13^{\sqrt{x}}$, then x is equal to

(a) $\frac{25}{4}$ (b) 4 (c) 9 (d) 16

39. If x, y and z are real number such that $(x-3)^2+(y-4)^2+(z-5)^2=0$ then $(x+y+z)$ is equal to

(a) – 12 (b) 0 (c) 8 (d) 12

40. If $x=3+\sqrt{8}$, then $x^2+\frac{1}{x^2}$ is equal to

(a) 38 (b) 36 (c) 34 (d) 30

41. If $x-\frac{1}{x}=4$, then $\left(x+\frac{1}{x}\right)$ is equal to

(a) $5\sqrt{2}$ (b) $2\sqrt{5}$ (c) $4\sqrt{2}$ (d) $4\sqrt{5}$

42. If $4b^2+\frac{1}{b^2}=2$, then the value of $8b^3+\frac{1}{b^3}$ is

(a) 0 (b) 1 (c) 2 (d) 5

43. If $\left(\frac{3}{5}\right)^3\left(\frac{3}{5}\right)^{-6}=\left(\frac{3}{5}\right)^{2x-1}$, then x is equal to

(a) –2 (b) 2 (c) – 1 (d) 1

44. $\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}-\sqrt{3-x}}=2$ then x is equal to

(a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{5}{7}$ (d) $\frac{7}{5}$

45. If $x=\frac{\sqrt{3}}{2}$, then the value of $\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right)$ is

(a) $-\sqrt{3}$ (b) – 1 (c) 1 (d) $\sqrt{3}$

46. If $\frac{\sqrt{x+4}+\sqrt{x-4}}{\sqrt{x+4}-\sqrt{x-4}}=2$ then x is equal to

(a) 2.4 (b) 3.2 (c) 4 (d) 5

47. If $x=\left(\sqrt{2}+1\right)^{\frac{1}{3}}$, the value of $\left(x^3-\frac{1}{x^3}\right)$ is

(a) 0 (b) $-\sqrt{2}$ (c) + 2 (d) $3\sqrt{2}$

48. If $\frac{x^2-x+1}{x^2+x+1}=\frac{3}{2}$, then the value of $\left(x+\frac{1}{x}\right)$ is

(a) 4 (b) – 5 (c) 6 (d) 8

49. If $x=3+\sqrt{8}$, then $x^2+\frac{1}{x^2}$ is equal to

(a) 38 (b) 36 (c) 34 (d) 30

50. If $x=5+2\sqrt{6}$, then the value of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ is.

(a) $2\sqrt{2}$ (b) $3\sqrt{2}$
(c) $2\sqrt{3}$ (d) $3\sqrt{3}$

51. If $x=\sqrt{3}+\sqrt{2}$, then the value of $\left(x^2+\frac{1}{x^2}\right)$ is :

(a) 4 (b) 6 (c) 9 (d) 10

52. If $x+\frac{9}{x}=6$, then the value of $\left(x^2+\frac{9}{x^2}\right)$ is

(a) 8 (b) 9 (c) 10 (d) 12

53. If $2p+\frac{1}{p}=4$, then value of $p^3+\frac{1}{8p^3}$ is

(a) 4 (b) 5 (c) 8 (d) 15

54. If $a^4+b^4=a^2b^2$, then (a^6+b^6) equals

(a) 0 (b) 1
(c) a^2+b^2 (d) $a^2b^4+a^4b^2$

55. If $x+\frac{1}{x}=3$, then the value of $\frac{x^3+\frac{1}{x}}{x^2-x+1}$ is :

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{11}{2}$

56. If $a+\frac{1}{a}+1=0(a\neq 0)$ then the value of (a^4-a) is:

(a) 0 (b) 1 (c) 2 (d) – 1

57. If $x=a+\frac{1}{a}$ and $y=a-\frac{1}{a}$, then the value of $x^4+y^4-2x^2y^2$ is

(a) 24 (b) 18 (c) 16 (d) 12

58. If a = 11 and b = 9, then the value of $\left(\frac{a^2+b^2+ab}{a^3-b^3}\right)$ is

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{20}$ (d) 20

59. If p = 101, then the value of $\sqrt[3]{p\left(p^2-3p+3\right)-1}$ is

(a) 100 (b) 101
(c) 102 (d) 1000

60. If x = 19 and y = 18, then the value of $\frac{x^2+y^2+xy}{x^3-y^3}$ is

(a) 1 (b) 37
(c) 324 (d) 361

61. If 50% of $(p-q)$ = 30% of $(p+q)$, then $p : q$ is equal to

(a) 5 : 3 (b) 4 : 1
(c) 3 : 5 (d) 1 : 4

62. If $\frac{a}{3}=\frac{b}{2}$, then value of $\frac{2a+3b}{3a-2b}$ is

(a) $\frac{12}{5}$ (b) $\frac{5}{12}$ (c) 1 (d) $\frac{12}{7}$

63. If $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$

(a) 1 (b) 2 (c) 3 (d) 4

64. If $\frac{2x-y}{x+2y}=\frac{1}{2}$, then value of $\frac{3x-y}{3x+y}$ is :

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (b) 1

65. If $x+\frac{1}{x}=5$, then $\frac{2x}{3x^2-5x+3}$ is equal to

(a) 5 (b) $\frac{1}{5}$ (c) 3 (d) $\frac{1}{3}$

66. If $\sqrt{1-\frac{x^3}{100}}=\frac{3}{5}$, then x equals

(a) 2 (b) 4
(c) 16 (d) $(136)^{1/3}$

67. If $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}=a+b\sqrt{6}$, then the values of a and b are respectively

(a) $\frac{9}{15},-\frac{4}{15}$ (b) $\frac{3}{11},\frac{4}{33}$

(c) $\frac{9}{10},\frac{2}{5}$ (d) $\frac{3}{5},\frac{4}{15}$

68. If $\sqrt{1+\frac{x}{961}}=\frac{32}{31}$, then the value of x is

(a) 63 (b) 61 (c) 65 (d) 64

69. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=3$

then $\frac{2a^2+3c^2+4e^2}{2b^2+3d^2+4f^2}=?$

(a) 2 (b) 3 (c) 4 (d) 9

70. If $2x+\frac{1}{3x}=5$. Find the value of $\frac{5x}{6x^2+20x+1}$.

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{1}{7}$

71. If x varies inversely as (y^2-1) and x is equal to 24 when $y = 10$, then the value of x when $y=5$ is

(a) 99 (b) 12 (c) 24 (d) 100

72. If $x^2+y^2+2x+1=0$, then the value of $x^{31}+y^{35}$ is

(a) – 1 (b) 0 (c) 1 (d) 2

73. If $\frac{x}{2x^2+5x+2}=\frac{1}{6}$, then value of $\left(x+\frac{1}{x}\right)$ is:

(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) – 2

74. If a, b, c are real and $a^2+b^2+c^2=2(a-b-c)-3$ then the value of $2a-3b+4c$ is

(a) – 1 (b) 0 (c) 1 (d) 2

75. If $(3a+1)^2+(b-1)^2+(2c-3)^2=0$, then the value of $(3a+b+2c)$ is equal to;

(a) 3 (b) – 1 (c) 2 (d) 5

76. The value of the expression $\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(a-b)(c-a)}+\frac{(c-a)^2}{(a-b)(b-c)}$

(a) 0 (b) 3 (c) $\frac{1}{3}$ (d) 2

77. If $(a-3)^2+(b-4)^2+(c-9)^2=0$, then the value of $\sqrt{a+b+c}$ is :

(a) –4 (b) 4 (c) ± 4 (d) ± 2

78. If $1.5x = 0.04y$, then the value of $\frac{y^2-x^2}{y^2+2xy+x^2}$ is

(a) $\frac{730}{77}$ (b) $\frac{73}{77}$ (c) $\frac{73}{770}$ (d) $\frac{74}{77}$

79. If $a^{\frac{1}{3}}=11$, then the value of a^2-331a is

(a) 1331331 (b) 1331000
(c) 1334331 (d) 1330030

80. If $x^2+y^2+\frac{1}{x^2}+\frac{1}{y^2}=4$, then the value of x^2+y^2 is

(a) 2 (b) 4 (c) 8 (d) 16

81. If $x^2=y+z$, $y^2=z+x$, $z^2=x+y$, then the value of $\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$ is

(a) – 1 (b) 1 (c) 2 (d) 4

82. If $a^2+b^2=2$ and $c^2+d^2=1$ then the value of $(ad-bc)^2+(ac+bd)^2$ is

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

83. If $x=\frac{4ab}{a+b}$ $a\neq b$, the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$ is

(a) a (b) b (c) 2 ab (d) 2

84. If $m+\frac{1}{m-2}=4$,

find the value of $(m-2)^2+\frac{1}{(m-2)^2}$

(a) – 2 (b) 0 (c) 2 (d) 4

85. If $a^2+b^2+2b+4a+5=0$, then the value of $\frac{a-b}{a+b}$ is

(a) 3 (b) – 3 (c) $\frac{1}{3}$ (b) $-\frac{1}{3}$

86. If $x-y=\frac{x+y}{7}=\frac{xy}{4}$, the numerical value of xy is

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

87. If $x+y+z=0$, then $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=?$

(a) $(xyz)2$ (b) $x^2+y^2+z^2$
(c) 9 (d) 3

88. If a +b+c = 0, then the value of

$\frac{1}{(a+b)(b+c)}+\frac{1}{(a+c)(b+a)}+\frac{1}{(c+a)(c+b)}$

(a) 1 (b) 0 (c) – 1 (d) – 2

89. If $a+b+c=0$, then the value of $\frac{a^2+b^2+c^2}{a^2-bc}$ is

(a) 0 (b) 1 (c) 2 (d) 3

90. If $x^2+y^2-4x-4y+8=0$, then the value of $x-y$ is

(a) 4 (b) – 4 (c) 0 (d) 8

91. If $x=b+c-2a$, $y=c+a-2b$, $z=a+b-2c$, then the value of $x^2+y^2-z^2+2xy$ is

(a) 0 (b) a +b +c
(c) a – b + c (d) a + b – c

92. For real a, b, c if $a^2+b^2+c^2=ab+bc+ca$, then value of $\frac{a+c}{b}$ is:

(a) 1 (b) 2 (c) 3 (d) 0

93. If $x+\frac{1}{x}=\sqrt{3}$ then the value of $x^{18}+x^{12}+x^6+1$ is

(a) 0 (b) 1 (c) 2 (d) 3

94. If for two real constants a and b the expression ax^3+3x^2-8x+b is exactly divisible by $(x+2)$ and $(x-2)$

(a) $a=2$, $b=12$ (b) $a=12$, $b=2$
(c) $a=2$, $b=-12$ (d) $a=-2$, $b=12$

95. If $x^2-3x+1=0$, then the value of $x^3+\frac{1}{x^3}$ is

(a) 9 (b) 18 (c) 27 (d) 1

96. If $x+\frac{1}{4x}=\frac{3}{2}$, find the value of $8x^3+\frac{1}{8x^3}$.

(a) 18 (b) 36 (c) 24 (d) 16

97. If $\frac{1}{x+y}=\frac{1}{x}+\frac{1}{y}$ $(x\neq 0, y\neq 0, x\neq y)$ then the value of x^3-y^3 is

(a) 0 (b) 1 (c) – 1 (d) 2

98. If $x=a(b-c)$, $y=b(c-a)$ and $z=c(a-b)$, then $\left(\frac{x}{a}\right)^3+\left(\frac{y}{b}\right)^3+\left(\frac{z}{c}\right)^3=?$

(a) $\frac{xyz}{3abc}$ (b) $3\,xyzabc$

(c) $\frac{3xyz}{abc}$ (d) $\frac{xyz}{abc}$

99. If xy(x +y) = 1, then the value of $\frac{1}{x^3y^3}-x^3-y^3$ is:

(a) 0 (b) 1 (c) 3 (d) –2

100. If $x^4+\frac{1}{x^4}=119$ and $x>1$, then the value of $x^3+\frac{1}{x^3}$ is

(a) $6\sqrt{13}$ (b) $8\sqrt{13}$

(c) $13\sqrt{13}$ (d) $10\sqrt{13}$

101. If $3x+\frac{1}{2x}=5$, then the value of $8x^3+\frac{1}{27x^3}$ is:

(a) $118\frac{1}{2}$ (b) $30\frac{10}{27}$

(c) 0 (d) 1

102. If $x+y=z$, then the expression $x^3+y^3-z^3+3xyz$ wil be equal to :

(a) 0 (b) $3xyz$
(c) $-3xyz$ (d) z3

103. If the sum of $\frac{a}{b}$ and its reciprocal is 1 and $a\neq 0$, $b\neq 0$, then the value of a^3+b^3 is

(a) 2 (b) –1 (c) 0 (d) 1

104. If $x=2-2^{1/3}+2^{2/3}$ then the value of $x^3-6x^2+18x+18$ is

(a) 22 (b) 33 (c) 40 (d) 45

105. If $a^3-b^3-c^3-3abc=0$, then

(a) a = b = c (b) $a+b+c=0$
(c) $a+c=b$ (d) $a=b+c$

106. If a = 2.361, b = 3.263 and c =5.624, then the value of $a^3+b^3-c^3+3abc$ is

(a) $(p-q)(q-r)^3+(r-p)^3$
(b) $3(p-q)(q-r)(r-p)$
(c) 0 (d) 1

107. If p = 124, $\sqrt[3]{p\left(p^2+3p+3\right)+1}=?$

(a) 5 (b) 7 (c) 123 (d) 125

108. If $x+\frac{1}{x}=2$ and x is real , then the value of $x^{17}+\frac{1}{x^{19}}$ is

(a) 1 (b) 0 (c) 2 (d) –2

109. If x : y = 3 : 4, then the value of $\frac{5x-2y}{7x+2y}=?$

(a) $\frac{7}{25}$ (b) $\frac{7}{23}$ (c) $\frac{7}{29}$ (d) $\frac{7}{17}$

110. If x +y = 2z then the value of $\frac{x}{x-z}+\frac{z}{y-z}$ is

(a) 1 (b) 3 (c) $\frac{1}{2}$ (d) 2

111. If $a^3b=abc=180$, a, b, c are positive integers, then the value of b is

(a) 110 (b) 180 (c) 4 (d) 25

112. If a, b are rational number and $(a-1)\sqrt{2}+3=b\sqrt{2}+a$, the value of $(a+b)$ is

(a) – 5 (b) 3 (c) – 3 (d) 5

113. If $ax^2+bx+c=a(x-p)^2$, then the relation among a, b, c would be

(a) $abc=1$ (b) $b^2=ac$
(c) $b^2=4ac$ (d) $2b=a+c$

114. If $a+b+c+d=1$, then the maximum value of $(1+a)(1+b)(1+c)(1+d)$ is

(a) 1 (b) $\left(\frac{1}{2}\right)^3$ (c) $\left(\frac{3}{4}\right)^3$ (d) $\left(\frac{5}{4}\right)^4$

115. If $a^2+b^2+c^2+3=2(a+b+c)$ then the value of $(a+b+c)$ is

(a) 2 (b) 3 (c) 4 (d) 5

116. If $x-\frac{1}{x}=5$, then $x^2+\frac{1}{x^2}$ is :

(a) 5 (b) 25 (c) 27 (d) 23

117. If x = $3+2\sqrt{2}$, then the value of $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)$ is:

(a) 1 (b) 2
(c) $2\sqrt{2}$ (d) $3\sqrt{3}$

118. If a +b +c = 0, then the value of $\frac{a^2+b^2+c^2}{a^2-bc}$ is
(a) 0 (b) 1 (c) 2 (d) 3

119. If n = $7+4\sqrt{3}$, then the value of $\left(\sqrt{n}+\frac{1}{\sqrt{n}}\right)$ is:
(a) $2\sqrt{3}$ (b) 4
(c) – 4 (d) $-2\sqrt{3}$

120. If $x=\sqrt{3}+\sqrt{2}$, then the value of $\left(x+\frac{1}{x}\right)$ is
(a) $2\sqrt{2}$ (b) $2\sqrt{3}$
(c) 2 (d) 3

121. If p + q =10 and pq = 5, then the numerical value of $\frac{p}{q}+\frac{q}{p}$ will be
(a) 16 (b) 20 (c) 22 (d) 18

122. If $x=3+2\sqrt{2}$ and xy = 1, then the value of $\frac{x^2+3xy+y^2}{x^2-3xy+y^2}$ is
(a) $\frac{30}{31}$ (b) $\frac{70}{31}$ (c) $\frac{35}{31}$ (d) $\frac{37}{31}$

123. If $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{a+b}$, then
यदि $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{a+b}$ है, तो :
(a) $\frac{x-y}{b-a}=\frac{y-z}{c-b}=\frac{z-x}{a-c}$
(b) $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$
(c) $\frac{x-y}{c}=\frac{y-z}{b}=\frac{z-x}{c}$
(d) None of the above is true

124. If $x-y$ = 2, xy = 24, then the value of (x^2+y^2) is
(a) 25 (b) 36 (c) 63 (d) 52

125. If the expression $\frac{x^2}{y^2}+tx+\frac{y^2}{4}$ is a perfect square, then the values of t is
(a) ±1 (b) ±2 (c) 0 (d) ±3

126. If $a = x+y$, $b = x-y$, $c = x+2y$, then $a^2+b^2+c^2-ab-bc-ca$ is
(a) $4y^2$ (b) $5y^2$ (c) $6y^2$ (d) $7y^2$

127. If $x+\frac{1}{x}=2, x\neq0$ then value of $x^2+\frac{1}{x^3}$ is equal to
(a) 1 (b) 2 (c) 3 (d) 4

128. If $\frac{a}{b}+\frac{b}{a}=1$, $a\neq0, b\neq0$ the value of a^3+b^3 is
(a) 0 (b) 1 (c) – 1 (d) 2

129. If $\left(x+\frac{1}{x}\right)^2=3$ then the value of $(x^{72}+x^{66}+x^{54}+x^{24}+x^6+1)$
(a) 0 (b) 1 (c) 84 (d) 206

130. If $a+\frac{1}{a}=\sqrt{3}$, then the value of $a^6-\frac{1}{a^6}+2$ will be
(a) 1 (b) 2 (c) $3\sqrt{3}$ (d) 5

131. If x^3+y^3 = 35 and $x+y$ = 5, then the value of $\frac{1}{x}+\frac{1}{y}$ will be :
(a) $\frac{1}{3}$ (b) $\frac{5}{6}$ (c) 6 (d) $\frac{2}{3}$

132. If a^3-b^3 = 56 and $a-b$ =2 then value of a^2+b^2 will be :
(a) 48 (b) 20 (c) 22 (d) 5

133. If $(a^2+b^2)^3=(a^3+b^3)^2$ then $\frac{a}{b}+\frac{b}{a}=?$
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$

134. If $x+\frac{1}{x}=5$, then the value of $\frac{x^4+3x^3+5x^2+3x+1}{x^4+1}$
(a) $\frac{43}{23}$ (b) $\frac{47}{21}$ (c) $\frac{41}{23}$ (d) $\frac{45}{21}$

135. If x is real, $x+\frac{1}{x}\neq0$ and $x^3+\frac{1}{x^3}$ = 0, then the value of $\left(x+\frac{1}{x}\right)^4$ is
(a) 4 (b) 9 (c) 16 (d) 25

136. If $x+\frac{1}{x}=3$, then the value of $\left(x^5+\frac{1}{x^5}\right)$ is
(a) 322 (b) 126 (c) 123 (d) 113

137. If $m^4+\frac{1}{m^4}$ = 119, then $m-\frac{1}{m}$ = ?
(a) ±3 (b) 4 (c) ±2 (d) ±1

138. If $x+y+z$ =6, then the value of $(x-1)^3+(y-2)^3+(z-3)^3$ is
(a) $3(x-1)(y+2)(z-3)$
(b) $3(x+1)(y-2)(z-3)$
(c) $3(x-1)(y-2)(z+3)$
(d) $3(x-1)(y-2)(z-3)$

139. If $a+b+c$ =6, $a^2+b^2+c^2$ = 14 and $a^3+b^3+c^3$ = 36, then the value of abc is
(a) 3 (b) 6 (c) 9 (d) 12

140. If $a+b$ = 1 and $a^3+b^3+3ab=k$, then the value of k is
(a) 1 (b) 3 (c) 5 (d) 7

141. If a = 34, $b=c$ = 33, then the value of $a^3+b^3+c^3-3abc$ is
(a) 0 (b) 111 (c) 50 (d) 100

142. If $(2^x)(2^y)$ =8 and $(9^x)(3^y)$ = 81, then (x, y) is:
(a) (1,2) (b) (2,1) (c) (1,1) (d) (2,2)

143. The expression x^4-2x^2+k will be a perfect square when the value of k is
(a) 2 (b) 1 (c) – 1 (d) – 2

144. $\frac{\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}-3\frac{1}{3}\cdot\frac{1}{4}\cdot\frac{1}{5}+\frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{5}}{\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{5}\cdot\frac{1}{5}-\left(\frac{1}{3}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{5}+\frac{1}{5}\cdot\frac{1}{3}\right)}$
(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{47}{60}$ (d) $\frac{49}{60}$

145. If $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is
(a) 1 (b) 2 (c) 3 (d) 4

146. If a, b, c are real numbers and $a^2+b^2+c^2=2(a-b-c)-3$ then the value of $2a-3b+4c$ is
(a) – 1 (b) 9 (c) 1 (d) 2

147. The value of the expression $\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(a-b)(c-a)}+\frac{(c-a)^2}{(a-b)(b-a)}$
(a) 0 (b) 3 (c) $\frac{1}{3}$ (d) 2

148. If $(x-3)^2+(y-5)^2+(z-4)^2$ =0 then the value of $\frac{x^2}{9}+\frac{y^2}{25}+\frac{z^2}{16}$ is
(a) 12 (b) 9 (c) 3 (d) 1

149. x varies inversely as square of y. Given that y = 2 for x = 1, the value of x for y = 6 will be equal to
(a) 3 (b) 9 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$

150. If x^2-y^2 = 80 and $x-y$ = 8, then the average of x and y is
(a) 2 (b) 3 (c) 4 (d) 5

151. The third proportional to $\left(\frac{x}{y}+\frac{y}{x}\right)$ and $\sqrt{x^2+y^2}$ is
(a) xy (b) $\sqrt{xy}$
(c) $\sqrt[3]{xy}$ (d) $\sqrt[4]{xy}$

152. The value of $\frac{4+3\sqrt{3}}{7+4\sqrt{3}}$ is

(a) $5\sqrt{3}-8$ (b) $5\sqrt{3}+8$

(c) $8\sqrt{3}+5$ (d) $8\sqrt{3}-5$

153. If $x=\frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$, the value of $\frac{x+\sqrt{20}}{x-\sqrt{20}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}$ is

(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{5}$

154. If x = 5 − $\sqrt{21}$, then the value of $\frac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}}$ is

(a) $\frac{1}{\sqrt{2}}\left(\sqrt{3}-\sqrt{7}\right)$ (b) $\frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$

(b) $\frac{1}{\sqrt{2}}\left(\sqrt{7}+\sqrt{3}\right)$ (d) $\frac{1}{\sqrt{2}}\left(7+\sqrt{3}\right)$

155. The value of $\left(x^{a+b}\right)^{a-b}\left(x^{b+c}\right)^{b-c}\left(x^{c+a}\right)^{c-a}$

(a) 1 (b) 2 (c) − 1 (d) 0

156. If $\frac{x}{a}=\frac{1}{a}-\frac{1}{x}$, then the value of $x-x^2$ is :

(a) $-a$ (b) $\frac{1}{a}$ (c) a (d) $-\frac{1}{a}$

157. If $x+\frac{1}{x}=99$, find the value of $\frac{100x}{2x^2+102x+2}$

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

158. If $\frac{4x-3}{x}+\frac{4y-3}{y}+\frac{4z-3}{z}=0$ then the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is

(a) 9 (b) 3 (c) 4 (d) 6

159. If $\frac{xy}{x+y}=a, \frac{xz}{x+z}=$ b and $\frac{yz}{y+z}=$c, where a, b, c are all non -zero numbers, then x equals to

(a) $\frac{2abc}{ab+bc-ac}$ (b) $\frac{2abc}{ab+ac-bc}$

(c) $\frac{2abc}{ac+bc-ab}$ (d) $\frac{2abc}{ab+bc-ac}$

160. If x and y are positive real numbers and xy =8, then the minimum value of $2x+y$ is

(a) 9 (b) 17 (c) 10 (d) 8

161. If the expression x^2+x+1 is written in the form $\left(x+\frac{1}{2}\right)^2+q^2$, then the possible values of q are

(a) $\pm\frac{1}{3}$ (b) $\pm\frac{\sqrt{3}}{2}$

(c) $\pm\frac{2}{\sqrt{3}}$ (d) $\pm\frac{1}{2}$

162. If $a^2-4a-1=0$, then value of $a^2+\frac{1}{a^2}+3a-\frac{3}{a}$ is

(a) 25 (b) 30 (c) 35 (d) 40

163. One of the factors of the expression $4\sqrt{3}x^2+5x-2\sqrt{3}$ is :

(a) $4x+\sqrt{3}$ (b) $4x+3$

(c) $4x-3$ (d) $4x-\sqrt{3}$

164. If $\sqrt{x}=\sqrt{3}-\sqrt{5}$, then the value of $x^2-16x+6$ is

(a) 0 (b) − 2 (c) 2 (d) 4

165. If $x+y+z=0$, then $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=?$

(a) $(xyz)^2$ (b) $x^2+y^2\ z^2$

(c) 9 (d) 3

166. If यदि a +b +c = 0, then the value of $\left(\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}\right)\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)$

(a) 8 (b) − 3 (c) 9 (d) 0

167. If a, b, c are non - zero $a+\frac{1}{b}=1$ and $b+\frac{1}{c}=1$, then the value of abc is :

(a) − 1 (b) 3 (c) − 3 (d) 1

168. If $a+b+c=2s$, then $\frac{(s-a)^2+(s-b)^2+(s-c)^2+s^2}{a^2+b^2+c^2}$

(a) $a^2+b^2+c^2$ (b) 0

(c) 1 (d) 2

169. If x = $3+2\sqrt{2}$, the value of $x^2+\frac{1}{x^2}$ is

(a) 36 (b) 30 (c) 32 (d) 34

170. If $x\left(3-\frac{2}{x}\right)=\frac{3}{x}$, then the value of $x^2+\frac{1}{x^2}$ is

(a) $2\frac{1}{9}$ (b) $2\frac{4}{9}$ (c) $3\frac{1}{9}$ (d) $3\frac{4}{9}$

171. If $x^2-3x+1=0$, then the value of $x^2+x+\frac{1}{x}+\frac{1}{x^2}$ is

(a) 10 (b) 2 (c) 6 (d) 8

172. If $a^2+b^2=5ab$, the value of $\left(\frac{a^2}{b^2}+\frac{b^2}{a^2}\right)$ is :

(a) 32 (b) 16 (c) 23 (d) − 23

173. If यदि $xy+yz+zx=0$, then तो $\left(\frac{1}{x^2-yz}+\frac{1}{y^2-zx}+\frac{1}{z^2-xy}\right)$ $(x,y,z\neq0)$

(a) 3 (b) 1

(c) $x+y+z$ (d) 0

174. If a +b +c = 9 (where a, b, c are real numbers), then the minimum value of $a^2+b^2+c^2$ is

(a) 100 (b) 9 (c) 27 (d) 81

175. If $a^2+b^2+4c^2=2(a+b-2c)-3$ and a, b, c are real, then the value of $(a^2+b^2+c^2)$ is

(a) 3 (b) $3\frac{1}{4}$ (c) 2 (d) $2\frac{1}{4}$

176. Number of solutions of the two equations $4x-y=2$ and $2x-8y+4=0$ is

(a) zero (b) one

(c) two

(d) infinitely many

177. If $\frac{a}{b}=\frac{4}{5}$ and $\frac{b}{c}=\frac{15}{16}$, then $\frac{18c^2-7a^2}{45c^2+20a^2}$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

178. If $x\neq0, y\neq0$ and $z\neq0$ and $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}$ then the relation among x, y, z is

(a) $x+y+z=0$

(b) $x+y=z$

(c) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

(d) $x=y=z$

179. If $x=3t$, $y=\frac{1}{2}(t+1)$, then the value of t for which $x=2y$ is

(a) 1 (b) $\frac{1}{2}$ (c) – 1 (d) $\frac{2}{3}$

180. If $x^2+\frac{1}{5}x+a^2$ is a perfect square, then a is

(a) $\frac{1}{100}$ (b) $\pm\frac{1}{10}$ (c) $\frac{1}{10}$ (d) $-\frac{1}{10}$

181. Find the value of x for which the expression $2-3x-4x^2$ has the greatest value.

(a) $-\frac{41}{16}$ (b) $\frac{3}{8}$ (c) $-\frac{3}{8}$ (d) $\frac{41}{16}$

182. The expression x^4-2x^2+k will be a perfect square if the value of k is

(a) 1 (b) 0 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

183. If $\frac{5x}{2x^2+5x+1}=\frac{1}{3}$, then the value of $\left(x+\frac{1}{2x}\right)$

(a) 15 (b) 10 (c) 20 (d) 5

184. If xy (x +y) = 1, then the value of $\frac{1}{x^3y^3}-x^3-y^3$ is:

(a) 0 (b) 1 (c) 3 (d) – 2

185. If $x>1$ and $x^2+\frac{1}{x^2}=83$ then $x^3-\frac{1}{x^3}$ is

(a) 764 (b) 750
(c) 756 (d) 760

186. If $\left(a+\frac{1}{a}\right)^2=3$, then $a^3+\frac{1}{a^3}=?$

(a) $2\sqrt{3}$ (b) 2
(c) $3\sqrt{3}$ (d) 0

187. If $\frac{x}{x^2-2x+1}=\frac{1}{3}$, then the value of $x^3+\frac{1}{x^3}$ is :

(a) 64 (b) 110 (c) 81 (d) 124

188. If $\left(x+\frac{1}{x}\right)=4$, then the value of $x^4+\frac{1}{x^4}$ is :

(a) 64 (b) 194 (c) 81 (d) 124

189. If $x+y+z=6$ and $x^2+y^2+z^2=20$ then the value of $x^3+y^3+z^3-3xyz$ is
(a) 64 (b) 70 (c) 72 (d) 76

190. If $x=1-\sqrt{2}$, the value of $\left(x-\frac{1}{x}\right)^3$ is:

(a) – 8 (b) 8 (c) $2\sqrt{2}$ (b) 1

191. If $x=a-b$, $y=b-c$, $z=c-a$, then the numerical value of the algebraic expression $x^3+y^3+z^3-3xyz$ will be
(a) $a+b+c$ (b) 0
(c) $4(a+b+c)$ (d) $3abc$

192. If $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then the value of x^3+y^3 is :
(a) 950 (b) 730 (c) 650 (d) 970

193. If $2x+\frac{2}{x}=3$, then the value of $x^3+\frac{1}{x^3}+2$ is

(a) $-\frac{9}{8}$ (b) $-\frac{25}{8}$ (c) $\frac{7}{8}$ (d) 11

194. If $a+b+c=15$ and $a^2+b^2+c^2=83$ then the value of $a^3+b^3+c^3-3abc$
(a) 200 (b) 180 (c) 190 (d) 210

195. If $x+\frac{1}{x+1}=1$, then $(x+1)^5+\frac{1}{(x+1)^5}$ equals
(a) 1 (b) 2 (c) 4 (d) 8

196. If $\frac{1}{a}+\frac{1}{b}=\frac{1}{a+b}$, then the value of a^3-b^3 is
(a) 0 (b) – 1 (c) 1 (d) 2

197. If $a+b+c=0$, then $a^3+b^3+c^3$ is equal to
(a) $a+b+c$ (d) abc
(c) $2abc$ (d) $3abc$

198. If $x=y=333$ and $z=334$, then the value of $x^3+y^3+z^3-3xyz$ is
(a) 0 (b) 667
(c) 1000 (d) 2334

199. Out of the given responses one of the factors of $(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3$ is
(a) (a + b)(a–b) (b) (a + b)(a+b)
(c) (a – b)(a–b) (d) (b – c)(b–c)

200. If $a=\frac{b^2}{b-a}$ then the value of a^3+b^3 is

(a) $6ab$ (b) 0 (c) 1 (d) 2

201. If $p-2q=4$, then the value of $p^3-8q^3-24pq-64$ is
(a) 2 (b) 0 (c) 3 (d) – 1

202. If $x=-1$, then the value of $\frac{1}{x^{99}}+\frac{1}{x^{98}}+\frac{1}{x^{97}}+\frac{1}{x^{96}}+\frac{1}{x^{95}}+\frac{1}{x^{94}}+\frac{1}{x}-1$

(a) 1 (b) 0 (c) – 2 (d) – 1

203. If $\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1}=a\sqrt[3]{4}+b\sqrt[3]{2}+c$ and a, b, c are rational numbers then a + b + c is equal to

(a) 0 (b) 1 (c) 2 (d) 3

204. If $x=\sqrt[3]{2+\sqrt{3}}$, then the value of $x^3+\frac{1}{x^3}$ is

(a) 8 (b) 9 (c) 2 (d) 4

205. If $x=\sqrt[3]{5}+2$, then the value of $x^3-6x^2+12x-13$
(a) – 1 (b) 1 (c) 2 (d) 0

206. The simplest form of the expression $\frac{p^2-p}{2p^3+p^2}+\frac{p^2-1}{p^2+3p}+\frac{p^2}{p+1}$

(a) $2p^2$ (b) $\frac{1}{2p^2}$

(c) p +3 (d) $\frac{1}{p+3}$

207. If $x+\frac{1}{x}=2$, then the value of $\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)$ is

(a) 20 (b) 4 (c) 8 (d) 16

208. If a,b,c be all positive integers then the least positive value of $a^3+b^3+c^3-3abc$ is.
(a) 0 (b) 2 (c) 4 (d) 3

209. When $f(x)=12x^3-13x^2-5x+7$ is divided by $(3x+2)$, then the remainder is
(a) 2 (b) 0 (c) – 1 (d) 1

210. If the equation $2x^2-7x+12=0$ has two roots α and β, then the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ is

(a) $\frac{7}{2}$ (b) $\frac{1}{24}$ (c) $\frac{7}{24}$ (d) $\frac{97}{24}$

211. If $x^3+\frac{3}{x}=4(a^3+b^3)$ and $3x+\frac{1}{x^3}=4(a^3+b^3)$, then a^2-b^2 is equal to

(a) 4 (b) 0 (c) 1 (d) 2

212. The term to be added to $121a^2+64b^2$ to make a perfect square is
(a) $176ab$ (b) $276a^2b$
(c) $178ab$ (d) $188b^2a$

213. If $a=2+\sqrt{3}$, then the value of $\left(a^2+\frac{1}{a^2}\right)$

(a) 12 (b) 14 (c) 16 (d) 10

214. For what value(s) of k the expression $p+\frac{1}{4}+\sqrt{p}+k^2$ is perfect square ?

(a) 0 (b) $\pm\frac{1}{4}$ (c) $\pm\frac{1}{8}$ (d) $\pm\frac{1}{2}$

215. The reciprocal of $x+\frac{1}{x}$ is

(a) $\frac{x}{x^2+1}$ (b) $\frac{x}{x+1}$

(c) $x-\frac{1}{x}$ (d) $\frac{1}{x}+x$

216. If a, b, c are positive and a+b+c =1, then the least value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ is

(a) 9 (b) 5 (c) 3 (d) 1

217. If $a\left(2+\sqrt{3}\right)=b\left(2-\sqrt{3}\right)=1$, then the value of

$\frac{1}{a^2+1}+\frac{1}{b^2+1}$

(a) – 1 (b) 1 (c) 4 (d) 9

218. If $\left(2+\sqrt{3}\right)a=\left(2-\sqrt{3}\right)b=1$ then the value of $\frac{1}{a}+\frac{1}{b}$ is

(a) 1 (b) 2 (c) $2\sqrt{3}$ (d) 4

219. If $a+\frac{1}{b}=b+\frac{1}{c}=c+\frac{1}{a}$ (a ≠ b ≠ c), then the value of abc is

(a) ± 1 (b) ± 2 (c) 0 (d) $\pm\frac{1}{2}$

220. If $(x-2)$ is a factor of $x^2+3Qx-2Q$, then the value of Q is
(a) 2 (b) – 2 (c) 1 (d) – 1

221. If $a+b=12$, $ab=22$, then (a^2+b^2) is equal to

(a) 188 (b) 144 (c) 34 (d) 100

222. If $x=\sqrt{3}-\frac{1}{\sqrt{3}}$ and $y=\sqrt{3}+\frac{1}{\sqrt{3}}$, then the value of $\frac{x^2}{y}+\frac{y^2}{x}$ is

(a) $\sqrt{3}$ (b) $3\sqrt{3}$

(c) $16\sqrt{3}$ (d) $2\sqrt{3}$

223. If x^2+ax+b is a perfect square, then which one of the following relations between a and b is true
(a) $a^2=b$ (b) $a^2=4b$
(c) $b=4a$ (d) $b^2=a$

224. If a + b + c +d =4, then find the value of $\frac{1}{(1-a)(1-b)(1-c)}+\frac{1}{(1-b)(1-c)(1-d)}+\frac{1}{(1-c)(1-d)(1-a)}+\frac{1}{(1-d)(1-a)(1-b)}$

(a) 0 (b) 5 (c) 1 (d) 4

225. If $x-\frac{1}{x}=1$, then the value of

$\frac{x^4-\frac{1}{x^2}}{3x^2+5x-3}$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 0

226. If $x+y=15$, then $(x-10)^3+(y-5)^3$ is
(a) 25 (b) 125
(c) 625 (d) 0

227. If $x^2+\frac{1}{x^2}=66$, then the value of

$\frac{x^2-1+2x}{x}=?$

(a) ±8 (b) 10, – 6
(c) 6, – 10 (d) ±4

228. If $a^2+a+1=0$, then the value of a^9 is
(a) 2 (b) 3 (c) 1 (d) 0

229. If $x+\frac{2}{x}=1$, then the value of

$\frac{x^2+x+2}{x^2(1-x)}$ is

(a) 1 (b) – 1 (d) 2 (d) – 2

230. If $x=-2k$ and $y=1-3k$, then for what value of k, will be $x=y$?
(a) 0 (b) 1 (c) – 1 (d) 2

231. Find the value of
$\sqrt{(x^2+y^2+z)(x+y-3z)}+\sqrt[3]{xy^3z^2}$
when $x=+1$, $y=-3$, $z=-1$,

(a) 1 (b) 0 (c) – 1 (d) $\frac{1}{2}$

232. If $x+\frac{1}{x}=2$, then the value of

$\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)$ is

(a) 20 (b) 4 (c) 8 (d) 16

233. If $x+\frac{1}{x}=5$, then $x^6+\frac{1}{x^6}$ is

(a) 12098 (b) 12048
(c) 14062 (d) 12092

234. If $x^2-3x+1=0$, then the value of

$\frac{x^6+x^4+x^2+1}{x^3}$ will be

(a) 18 (b) 15 (c) 21 (d) 30

235. If x is a rational number and $\frac{(x+1)^3-(x-1)^3}{(x+1)^2-(x-1)^2}=2$, then the sum of numerator and denominator of x is:
(a) 3 (b) 4 (c) 5 (d) 7

236. If $x=\sqrt{5}+2$, then the value $\frac{2x^2-3x-2}{3x^2-4x-3}$ is equal to

(a) 0.1785 (b) 0.525
(c) 0.625 (d) 0.785

237. If a = 2.234, b= 3.121 and c = –5.355, then the value of $a^3+b^3+c^3-3abc$ is
(a)– 1 (b) 0 (c) 1 (d) 2

238. If $x^2+y^2+1=2x$, then the value of x^3+y^5 is
(a) 2 (b) 0 (c) – 1 (d) 1

239. If $3(a^2+b^2+c^2)=(a+b+c)^2$ then the relation between a, b and c is
(a) a = b = c (b) a = b ≠ c
(c) a < b < c (d) a > b > c

240. If $x(x-3)=-1$, then the value of $x^3(x^3-18)$ is
(a) – 1 (b) 2 (c) 1 (d) 0

241. The factors of $(a^2+4b^2+4b-4ab-2a-8)$ are
(a) (a –2b–4) (a –2b+2)
(b) (a –b–2) (a + 2b +2)
(c) (a +2b–4) (a +2b+2)
(d) (a +2b–4) (a –2b+2)

242. The value of

$\frac{1}{a^2+ax+x^2}-\frac{1}{a^2-ax+x^2}+\frac{2ax}{a^4+a^2x^2+x^4}$

is
(a) 2 (b) 1 (c) – 1 (d) 0

243. If $x=11$, then the value of $x^5-12x^4+12x^3-12x^2+12x-1$ is
(a) 5 (b) 10 (c) 15 (d) 20

244. If $p=99$, then the value of p (p^2+3p+3) is
(a) 10000000 (b) 999000
(c) 99999 (d) 990000

245. Which one is not an example of an equality relation of two expressions in x:
(a) $(x+3)^2=x^2+6x+9$
(b) $(x+2y)^3=x^3+8y^3+6xy(x+2y)$
(c) $(x+2)^2=x^2+2x+4$
(d) $(x+3)(x-3)=x^2-9$

246. If $\left(a+\frac{1}{a}\right)^2=3$, then the value of $a^3+\frac{1}{a^3}$ is
(a) 0 (b) 1 (c) 2 (d) 6

247. If $a+\frac{1}{a}=\sqrt{3}$, then the value of $a^{18}+a^{12}+a^6+1$ is
(a) 0 (b) 1 (c) 2 (d) 6

248. If x = 997, y = 998 and z = 999 then the value of $x^2+y^2+z^2-xy-yz-zx$ is
(a) 0 (b) 1 (c) – 1 (d) 3

249. If $x+\frac{1}{x}=3$, then the value of $\frac{3x^2-4x+3}{x^2-x+1}$ is
(a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{5}{3}$

250. If $x=3+2\sqrt{2}$, then $\frac{x^6+x^4+x^2+1}{x^3}$ is equal to
(a) 216 (b) 192
(c) 198 (d) 204

251. If a + b + c = 0, then the value of $(a+b-c)^2+(b+c-a)^2+(c+a-b)^2$ is
(a) 0 (b) 8abc
(c) $4(a^2+b^2+c^2)$ (d) $4(ab+bc+ca)$

252. If $p^3+3p^2+3p=7$, then the value of p^2+2p is
(a) 4 (b) 3 (c) 5 (d) 6

253. If x = 2015, y = 2014 and z = 2013, then value of $x^2+y^2+z^2-xy-yz-zx$ is
(a) 3 (b) 4 (c) 6 (d) 2

254. If $3a^2=b^2\neq 0$, then the value of $\frac{(a+b)^3-(a-b)^3}{(a+b)^2+(a-b)^2}$ is
(a) $\frac{3b}{2}$ (b) b (c) $\frac{b}{2}$ (d) $\frac{2b}{3}$

255. If $x>1$ and $x+\frac{1}{x}=2\frac{1}{12}$, then the value of $x^4-\frac{1}{x^4}$ is
(a) $\frac{58975}{20736}$ (b) $\frac{59825}{20736}$
(c) $\frac{57985}{20736}$ (d) $\frac{57895}{20736}$

256. The value of $\frac{4x^3-x}{(2x+1)(6x-3)}$ when x = 9999 is
(a) 1111 (b) 2222
(c) 3333 (d) 6666

257. If $a^3+b^3=9$ and $a+b=3$, then the value of $\frac{1}{a}+\frac{1}{b}$ is
(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) – 1

258. If $t^2-4t+1=0$, then the value of $t^3+\frac{1}{t^3}$ is:
(a) 44 (b) 48 (c) 52 (d) 64

259. If $\sqrt[3]{a}+\sqrt[3]{b}=\sqrt[3]{c}$, then the simplest value of $(a+b-c)^3+27abc$ is
(a) – 1 (b) 3 (c) – 3 (d) 0

260. If 4x+5y = 83 and 3x : 2y = 21 : 22, then $(y-x)$ equals
(a) 3 (b) 4 (c) 7 (d) 11

261. If $x=\sqrt[3]{a+\sqrt{a^2+b^3}}+\sqrt[3]{a-\sqrt{a^2+b^3}}$, then x^3+3bx is equal to
(a) 0 (b) a (c) 2a (d) 1

262. If $\frac{x^{24}+1}{x^{12}}=7$ then the value of $\frac{x^{72}+1}{x^{36}}$
(a) 343 (b) 433 (c) 432 (d) 322

263. If P = 99 then the value of $P(P^2+3P+3)$
(a) 989898 (b) 998889
(c) 988899 (d) 999999

264. If x = 2 then the value of $x^3+27x^2+243x+631$
(a) 1321 (b) 1233
(c) 1231 (d) 1211

265. If $x^2+y^2+z^2=2(x+z-1)$, then the value of; $x^3+y^3+z^3$ = ?
(a) – 1 (b) 2 (c) 0 (d) 1

266. If $x+\frac{1}{x}=1$, then the value of $\frac{2}{x^2-x+2}$ = ?
(a) 2/3 (b) 2 (c) 1 (d) 4

267. If $x=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ and $y=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, then the value of $\frac{x^2+xy+y^2}{x^2-xy+y^2}$ = ?
(a) $\frac{65}{63}$ (b) $\frac{67}{65}$ (c) $\frac{69}{67}$ (d) $\frac{63}{61}$

268. If $4a-\frac{4}{a}+3=0$, then the value of $a^3-\frac{1}{a^3}+3$ = ?
(a) $\frac{7}{16}$ (b) $\frac{3}{16}$ (c) $\frac{21}{64}$ (d) $\frac{21}{16}$

269. If $x=z=225$ and $y=226$ then the value of: $x^2+y^3+z^3-3xyz$
(a) 765 (b) 676
(c) 674 (d) 576

270. If $x^2+x=5$ then the value of: $(x+3)^3+\frac{1}{(x+3)^3}$
(a) 140 (b) 110 (c) 130 (d) 120

271. If m = – 4, n = –2, then the value of $m^3-3m^2+3m+3n+3n^2+n^3$ is
(a) 124 (b) –124 (c) 126 (d) – 126

272. 2x – ky + 7 = 0 and 6x – 12y + 15 = 0 has no solution for:
(a) k = – 4 (b) k = 4
(c) k = 1 (d) k = –1

273. If x = 332, y = 333, z = 335, then the value of $x^3+y^3+z^3-3xyz$ is
(a) 7000 (b) 8000
(c) 9000 (d) 10000

274. If $2+x\sqrt{3}=\frac{1}{2+\sqrt{3}}$, then the simplest value of x is:
(a) 1 (b) –2 (c) 2 (d) –1

275. If m – 5n = 2, then the value of (m^3-125n^3-30mn) is :
(a) 6 (b) 7 (c) 8 (d) 9

276. If $x=\sqrt{a\sqrt[3]{b\sqrt{a\sqrt[3]{b\ldots\ldots\infty}}}}$, then the value of x is:
(a) $\sqrt[5]{ab^3}$ (b) $\sqrt[3]{a^5b}$
(c) $\sqrt[3]{a^3b}$ (d) $\sqrt[5]{a^3b}$

277. If $x+\frac{1}{x}=2$, then the value of $x^{12}-\frac{1}{x^{12}}$ is:
(a) – 4 (b) 4 (c) 2 (d 0

278. If $x+\frac{1}{x}=1$, then the value of $\frac{x^2+3x+1}{x^2+7x+1}$ is:
(a) 1/2 (b) 3/7 (c) 2 (d) 3

279. If x + (1/x) = 2, then the value of $x^7+(1/x^5)$ is:
(a) 2^5 (b) 2^{12} (c) 2 (d) 2^7

280. The term, that should be added to$(4x^2+8x)$ so that resulting expression be a perfect square, is:
(a) 2x (b) 2 (c) 1 (d) 4

281. If 999x + 888y = 1332 and 888x + 999y = 555
Then the value of x + y is?
(a) 888 (b) 1 (c) 555 (d) 999

282. If $x = \frac{1}{2+\sqrt{3}}$, $y = \frac{1}{2-\sqrt{3}}$, then the value of $8xy(x^2 + y^2)$ is
(a) 112 (b) 194 (c) 290 (d) 196

283. If $a = \frac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}$, then the value of $a^2 - ax$ is
(a) 2 (b) 1 (c) 0 (d) – 1

284. If a + b = 1, find the value of $a^3 + b^3 - ab - (a^2 - b^2)^2$
(a) 0 (b) 1 (c) – 1 (d) 2

285. If $a - \frac{1}{a-3} = 5$, then the value of $(a-3)^3 - \frac{1}{(a-3)^3}$
(a) 7 (b) 14 (c) 2 (d) 5

286. $(3x-2y) : (2x+3y) = 5 : 6$, then one of the value of $\left(\frac{\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt[3]{x}-\sqrt[3]{y}}\right)^2$ is
(a) $\frac{1}{25}$ (b) 5 (c) $\frac{1}{5}$ (d) 25

287. If $x-\sqrt{3}-\sqrt{2}=0$ and $y-\sqrt{3}+\sqrt{2}=0$, then value of $\left(x^3-20\sqrt{2}\right)-\left(y^3+2\sqrt{2}\right)$
(a) 2 (b) 3 (c) 1 (d) 0

288. $3(a^2+b^2+c^2) = (a+b+c)^2$, then the relation between a, b and c is
(a) $a = b \neq c$ (b) $a \neq b \neq c$
(c) $a \neq b = c$ (d) $a = b = c$

289. $x = a^{\frac{1}{2}} + a^{-\frac{1}{2}}$, $y = a^{\frac{1}{2}} - a^{-\frac{1}{2}}$, then value of $(x^4 - x^2y^2 - 1) + (y^4 - x^2y^2 + 1)$
(a) 13 (b) 12 (c) 14 (d) 16

290. If $m = \sqrt{5+\sqrt{5+\sqrt{5......}}}$
$n = \sqrt{5-\sqrt{5-\sqrt{5-......}}}$
then among the following the relation between m & n holds is
(a) m–n+1=0 (b) m+n+1=0
(c) m+n–1=0 (d) m–n–1=0

291. If $\frac{3-5x}{2x} + \frac{3-5y}{2y} + \frac{3-5z}{2z} = 0$, then the value of $\frac{2}{x} + \frac{2}{y} + \frac{2}{x}$ is
(a) 20 (b) 10 (c) 5 (d) 15

292. If 2s = a + b + c, then the value of s(s – c) + (s – a) (s – b) is
(a) ab (b) 0
(c) abc (d) $\frac{a+b+c}{2}$

293. If p + m = 6 and $p^3 + m^3 = 72$, then the value of pm is
(a) 6 (b) 9 (c) 12 (d) 8

294. When x^m is multiplied by x^n, product is 1. The relation between m and n is
(a) mn = 1 (b) m + n = 1
(c) m = n (d) m = – n

295. If $\frac{2p}{P^2 - 2P + 1} = \frac{1}{4}$, then the value of $\left(p + \frac{1}{p}\right)$ is
(a) 7 (b) 1 (c) $\frac{2}{5}$ (d) 10

296. If $x = 2$, y = 1 and z = – 3, then $x^3 + y^3 + z^3 - 3xyz$ is equal to
(a) 6 (b) 0 (c) 2 (d) 8

297. $(x^3 + y^6)(x^3 - y^6)$ is equal to
(a) $x^6 - y^{12}$ (b) $x^9 - y^{16}$
(c) $x^6 + y^{12}$ (d) $x^9 + y^{36}$

298. The sum of $\frac{1}{x+y}$ and $\frac{1}{x-y}$ is
(a) $\frac{2y}{x^2-y^2}$ (b) $\frac{2x}{x^2-y^2}$
(c) $\frac{-2y}{x^2-y^2}$ (d) $\frac{2x}{y^2-x^2}$

299. If x + y = 2a, then the value of $\frac{a}{x-a} + \frac{a}{y-a}$ is
(a) 0 (b) – 1 (c) 1 (d) 2

300. For real a, b, c if $a^2 + b^2 + c^2 = ab + bc + ca$, the value of $\frac{a+c}{b}$ is :
(a) 2 (b) 1 (c) 0 (d) 3

301. If $p^3 - q^3 = (p - q)\{(p - q)^2 - xpq\}$, then find the value of x is:
(a) – 1 (b) 3 (c) 1 (d) – 3

302. If $x + y + z = 6$ and $xy + yz + zx = 10$, then the value of $x^3 + y^3 + z^3 - 3xyz$ is:
(a) 36 (b) 40 (c) 42 (d) 48

303. If $\frac{x+1}{x-1} = \frac{a}{b}$ and $\frac{1-y}{1+y} = \frac{b}{a}$, then the value of $\frac{x-y}{1+xy}$ is:
(a) $\frac{a^2-b^2}{ab}$ (b) $\frac{a^2+b^2}{2ab}$
(c) $\frac{a^2-b^2}{2ab}$ (d) $\frac{2ab}{a^2-b^2}$

304. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$ then a : b : c is:
(a) 1 : 2 : 1 (b) 2 : 1 : 1
(c) 1 : 1 : 2 (d) 1 : 1 : 1

305. If $x = a(b - c)$, $y = b(c - a)$, $z = c(a-b)$ then the value of $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$ is:
(a) $\frac{xyz}{abc}$ (b) 0
(c) $\frac{3xyz}{abc}$ (d) $\frac{2xyz}{abc}$

306. If $\frac{x}{y} = \frac{a+2}{a-2}$, then the value of $\frac{x^2-y^2}{x^2+y^2}$
(a) $\frac{2a}{a^2+2}$ (b) $\frac{4a}{a^2+4}$
(c) $\frac{2a}{a^2+4}$ (d) $\frac{4a}{a^2+2}$

307 If $\frac{a}{b} + \frac{b}{a} = 2$, then the value of a – b is:
(a) 2 (b) –1 (c) 0 (d) 1

308. If x(x+y+z)=20, y(x+y+z)=30, & z(x+y+z)=50, then the value of 2(x+y+z) is:
(a) 20 (b) 10 (c) 15 (d) 18

309. If x+y=4, $x^2+y^2=14$ and x > y. Then the correct value of x and y is:
(a) $2-\sqrt{2}$, $\sqrt{3}$ (b) 3, 1
(c) $2+\sqrt{3}$, $2-\sqrt{3}$ (d) $2+\sqrt{3}$, $2\sqrt{2}$

310. If for non-zero x, $x^2 - 4x - 1 = 0$ the value of is $x^2 + \frac{1}{x^2}$:
(a) 4 (b) 10 (c) 12 (d) 18

311. If $a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then $\frac{a^2}{b} + \frac{b^2}{a}$ value of :
(a) 1030 (b) 970
(c) 1025 (d) 930

312. If $(2a - 1)^2 + (4b - 3)^2 + (4c + 5)^2 = 0$ then the value of $\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2}$ is:
(a) $3\frac{3}{8}$ (b) $2\frac{3}{8}$ (c) 0 (d) $1\frac{3}{8}$

313. If $\left(a + \frac{1}{a}\right)^2 = 3$, then find the value of $a^{30} + a^{24} + a^{18} + a^{12} + a^6 + 1$
(a) 0 (b) 27 (c) 1 (d) –1

314. If $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$, then the value of $a^3 - b^3$ is:
(a) 3 (b) 2 (c) 1 (d) 0

ANSWER KEY

1. (d)	33. (c)	65. (b)	97. (a)	129.(a)	161.(b)	193.(c)	225.(b)	257.(b)	286.(d)
2. (d)	34. (c)	66. (b)	98. (c)	130.(b)	162.(b)	194.(b)	226.(d)	258.(c)	287.(d)
3. (c)	35. (c)	67. (d)	99. (c)	131.(b)	163.(d)	195.(b)	227.(b)	259.(d)	288.(d)
4. (c)	36. (a)	68. (a)	100.(d)	132.(b)	164.(c)	196.(a)	228.(c)	260.(b)	289.(d)
5. (c)	37. (d)	69. (d)	101.(b)	133.(b)	165.(d)	197.(d)	229.(a)	261.(c)	290.(d)
6. (d)	38. (b)	70. (d)	102.(a)	134.(a)	166.(c)	198.(c)	230.(b)	262.(d)	291.(b)
7. (d)	39. (d)	71. (a)	103.(c)	135.(b)	167.(a)	199.(a)	231.(b)	263.(d)	292.(a)
8. (a)	40. (c)	72. (a)	104.(c)	136.(c)	168.(c)	200.(b)	232.(b)	264.(b)	293.(d)
9. (c)	41. (b)	73. (b)	105.(d)	137.(a)	169.(d)	201.(a)	233.(a)	265.(b)	294.(d)
10. (c)	42. (a)	74. (c)	106.(c)	138.(d)	170.(b)	202.(c)	234.(c)	266.(b)	295.(d)
11. (c)	43. (c)	75. (a)	107.(d)	139.(b)	171.(a)	203.(a)	235.(b)	267.(d)	296.(b)
12. (a)	44. (b)	76. (b)	108.(c)	140.(a)	172.(c)	204.(d)	236.(c)	268.(c)	297.(a)
13. (b)	45. (d)	77. (c)	109.(c)	141.(d)	173.(d)	205.(d)	237.(b)	269.(b)	298.(b)
14. (a)	46. (d)	78. (b)	110.(a)	142.(a)	174.(c)	206.(b)	238.(d)	270.(b)	299.(a)
15. (a)	47. (c)	79. (b)	111.(b)	143.(b)	175.(d)	207.(b)	239.(a)	271.(d)	300.(a)
16. (c)	48. (b)	80. (a)	112.(d)	144.(c)	176.(b)	208.(a)	240.(a)	272.(b)	301.(d)
17. (a)	49. (c)	81. (b)	113.(c)	145.(d)	177.(d)	209.(d)	241.(a)	273.(a)	302.(a)
18. (c)	50. (c)	82. (d)	114.(d)	146.(c)	178.(d)	210.(b)	242.(d)	274.(d)	303.(d)
19. (d)	51. (d)	83. (d)	115.(b)	147.(b)	179.(b)	211.(c)	243.(b)	275.(c)	304.(d)
20. (b)	52. (c)	84. (c)	116.(c)	148.(c)	180.(c)	212.(a)	244.(c)	276.(d)	305.(c)
21. (c)	53. (b)	85. (c)	117.(b)	149.(d)	181.(d)	213.(b)	245.(c)	277.(d)	306.(b)
22. (d)	54. (a)	86. (a)	118.(c)	150.(d)	182.(a)	214.(a)	246.(a)	278.(a)	307.(c)
23. (b)	55. (c)	87. (d)	119.(b)	151.(a)	183.(d)	215.(a)	247.(a)	279.(c)	308.(a)
24. (c)	56. (a)	88. (b)	120.(b)	152.(a)	184.(c)	216.(a)	248.(d)	280.(d)	309.(c)
25. (b)	57. (c)	89. (c)	121.(d)	153.(b)	185.(c)	217.(b)	249.(c)	281.(b)	310.(d)
26. (c)	58. (a)	90. (c)	122.(d)	154.(b)	186.(d)	218.(d)	250.(d)	282.(a)	311.(b)
27. (d)	59. (a)	91. (a)	123.(a)	155.(a)	187.(b)	219.(a)	251.(c)	283.(d)	312.(c)
28. (b)	60. (a)	92. (b)	124.(a)	156.(c)	188.(b)	220.(d)	252.(b)	284.(a)	313.(a)
29. (d)	61. (b)	93. (a)	125.(a)	157.(c)	189.(c)	221.(d)	253.(a)	285.(b)	314.(d)
30. (d)	62. (a)	94. (c)	126.(d)	158.(c)	190.(d)	222.(b)	254.(a)		
31. (b)	63. (d)	95. (b)	127.(b)	159.(c)	191.(b)	223.(b)	255.(a)		
32. (c)	64. (b)	96. (a)	128.(a)	160.(d)	192.(d)	224.(a)	256.(c)		

SOLUTION

1.(d) $\left(\frac{1}{x}+1\right)\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)\left(1+\frac{1}{x+3}\right)$

Taking L.C.M of each term.

$\Rightarrow \left(\frac{x+1}{x}\right)\left(\frac{x+1+1}{x+1}\right)\left(\frac{x+2+1}{x+2}\right)\left(\frac{x+3+1}{x+3}\right)$

$\Rightarrow \frac{1}{x}\times(x+4) \Rightarrow \frac{x+4}{x}$

2. (d) $x = 7-4\sqrt{3}$

$\frac{1}{x} \Rightarrow \frac{1}{7-4\sqrt{3}}$

By rationalisation

$\frac{1}{x} = \frac{1}{7-4\sqrt{3}}\times\frac{7+4\sqrt{3}}{7+4\sqrt{3}}$

$= \frac{7+4\sqrt{3}}{49-48} = 7+4\sqrt{3}$

$\therefore\ x+\frac{1}{x} = 7-4\sqrt{3}+7+4\sqrt{3} = 14$

3. (c) $\frac{2a+b}{a+4b} = 3$ (given)

$2a + b = 3(a+4b)$

$2a + b = 3a + 12b$

$\Rightarrow -a = 11b$

$a = -11b$

$\therefore\ \frac{a+b}{a+2b}$

$\Rightarrow \frac{-11b+b}{-11b+2b}$

$= \frac{-10b}{-9b} = \frac{10}{9}$

4. (c) $A : B = \frac{1}{2} : \frac{3}{8}$

$\Rightarrow 8 : 6$

$\Rightarrow 4 : 3$

$\Rightarrow B : C \Rightarrow \frac{1}{3} : \frac{5}{9}$

$\Rightarrow 9 : 15 \Rightarrow 3 : 5$

$\Rightarrow C : D \Rightarrow \frac{5}{6} : \frac{3}{4}$

$\Rightarrow 20 : 18 \Rightarrow 10 : 9$

A	:	B	:	C	:	D
4	:	3				
		3	:	5		
				10	:	9
8	:	6	:	10	:	9

5. (c) $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$

$\therefore$ a = 3k

b = 4k

c = 7k

$\therefore \dfrac{a+b+c}{c} = \dfrac{3k+4k+7k}{7k} = 2$

6. (d) $\dfrac{144}{0.144} = \dfrac{14.4}{x}$

$\Rightarrow \dfrac{144 \times 1000}{144} = \dfrac{144}{x \times 10}$

$\Rightarrow 1000 = \dfrac{144}{10x}$

$\Rightarrow x = \dfrac{144}{1000 \times 10}$

$x = \dfrac{144}{10000} = 0.0144$

7. (d) $1 < x < 2$

$\sqrt{(x-1)^2} + \sqrt{(x-3)^2}$

(square root cancel with square)

$\therefore x - 1 + x - 3 = 2x - 4$

8. (c) $10^{0.48} = x$

$10^{0.70} = y$

and $x^z = y^2$

$\therefore \left(10^{0.48}\right)^z = \left(10^{0.70}\right)^2$

$\Rightarrow 10^{0.48z} = 10^{1.40}$ (If $a^x = a^y$, if base equal power are equal: $(x = y)$

$\therefore 0.48z = 1.40$

$z = \dfrac{140}{48} = \dfrac{35}{12} = 2.9$

9. (c) $47.2506 = 4A + 7B + 2C + \dfrac{5}{D} + 6E$

$47.2506 = 4 \times 10 + 7 \times 1 + 2 \times 0.1000 + 5 \times 0.0100 + 0 + 6 \times 0.0001$

$\therefore A = 10 \quad B = 1 \quad C = 0.1000$

$D = \dfrac{1}{\frac{1}{100}} = 100, \quad E = 0.0001$

$\therefore 5A + 3B + 6C + D + 3E$

$= 5 \times 10 + 3 \times 1 + 6 \times 0.1 + 100 + 3 \times 0.0001$

$= 50 + 3 + 0.6 + 100 + 0.0003$

$= 153.6003$

10. (c) $3^{x+3} + 7 = 250$

$3^{x+3} = 250 - 7$

$3^{x+3} = 243$

$3^{x+3} = 3^5$

$x + 3 = 5$

$x = 2$

11.(c) $\dfrac{1}{4} \times \dfrac{2}{6} \times \dfrac{3}{8} \times \dfrac{4}{10} \times \dfrac{5}{12} \times \ldots\ldots. \times \dfrac{31}{64}$

$= \dfrac{1}{2^x}$

$\Rightarrow \left(\dfrac{1}{2}\right)^{30} \times \left(\dfrac{1}{2}\right)^{5}$

$= \left(\dfrac{1}{2}\right)^{30+6} = \dfrac{1}{2^x}$

or $\dfrac{1}{2^{36}} = \dfrac{1}{2^x}$

$\therefore x = 36$

12. (a) $x = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$\Rightarrow \therefore x = \dfrac{1}{y}$

$x = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{\left(\sqrt{3}+1\right)^2}{3-1}$

$= \dfrac{3+1+2\sqrt{3}}{2} = \dfrac{4+2\sqrt{3}}{2}$

$= \left(2+\sqrt{3}\right)$

$x^2 = \left(2+\sqrt{3}\right)^2 = 4 + 3 + 4\sqrt{3}$

$= 7 + 4\sqrt{3}$

$y^2 = \dfrac{1}{7+4\sqrt{3}} \times \dfrac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$y^2 = \dfrac{7-4\sqrt{3}}{49-48} = \dfrac{7-4\sqrt{3}}{1}$

$= 7 - 4\sqrt{3}$

$\therefore x^2 + y^2 = 7 + 4\sqrt{3} + 7 - 4\sqrt{3}$

$= 14$

Alternate:-

$x^2 + y^2 = x^2 + \dfrac{1}{x^2}$

$= \left(x + \dfrac{1}{x}\right)^2 - 2 \quad \left(\therefore x = \dfrac{1}{y}\right)$

$= \left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1} + \dfrac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2 - 2$

$= \left[\dfrac{\left(\sqrt{3}+1\right)^2 + \left(\sqrt{3}-1\right)^2}{3-1}\right]^2 - 2$

$\therefore (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

$= \left[\dfrac{2\left(\left(\sqrt{3}\right)^2 + (1)^2\right)}{2}\right]^2 - 2$

$= (3 + 1)^2 - 2 = 16 - 2 = 14$

13. (b) $x = 3 + 2\sqrt{2}$

$x = 2 + 1 + 2\sqrt{2} = \left(\sqrt{2}+1\right)^2$

$\sqrt{x} = \sqrt{2} + 1$

$\dfrac{1}{\sqrt{x}} = \dfrac{1}{\sqrt{2}+1}$

$\Rightarrow \dfrac{1}{\sqrt{x}} = \dfrac{1}{\sqrt{2}+1} \times \dfrac{\sqrt{2}-1}{\sqrt{2}-1}$

(हर का परिमेयकरण करने पर)

$= \dfrac{\sqrt{2}-1}{1} = \sqrt{2} - 1$

$\therefore \sqrt{x} - \dfrac{1}{\sqrt{x}}$

$= \sqrt{2} + 1 - \left(\sqrt{2} - 1\right)$

$= \sqrt{2} + 1 - \sqrt{2} + 1 = 2$

14. (a) $P = 999$

$\sqrt[3]{p\left(p^2 + 3p + 3\right) + 1}$

$\Rightarrow \sqrt[3]{p^3 + 3p^2 + 3p + 1}$

$\therefore \sqrt[3]{(p+1)^3}$

$= \sqrt[3]{(999+1)^3}$

$= \sqrt[3]{(1000)^3} = 1000$

15. (a) $x : y$

$7 : 3$

$\therefore \dfrac{xy + y^2}{x^2 - y^2} = \dfrac{21+9}{49-9} = \dfrac{30}{40} = \dfrac{3}{4}$

16. (c) $x^{x\sqrt{x}} = \left(x\sqrt{x}\right)^x$

$x^{x\sqrt{x}} = \left(x^{\frac{3}{2}}\right)^x$

$x^{x\sqrt{x}} = x^{\frac{3}{2}x}$

(If bases are same then their power is also same)

$\therefore x\sqrt{x} = \dfrac{3}{2}x$ or $\sqrt{x} = \dfrac{3}{2}$

$x = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

17. (a) $a^2 + b^2 + c^2 - ab - bc - ca$

$= \dfrac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

$= \dfrac{1}{2}\left[(7-5)^2 + (5-3)^2 + (3-7)^2\right]$

$= \dfrac{1}{2}(4+4+16) = \dfrac{24}{2} = 12$

18. (c)

$$\frac{\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}-3\times\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{5}-\left(\frac{1}{3}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{3}\right)}$$

$A^3 + B^3 + C^3 - 3ABC = (A + B + C)$
$(A^2 + B^2 + C^2 - AB - BC - CA)$

$$\therefore \frac{\left(\frac{1}{3}\right)^3+\left(\frac{1}{4}\right)^3-3\times\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}+\left(\frac{1}{5}\right)^3}{\left(\frac{1}{3}\right)^2+\left(\frac{1}{4}\right)^2+\left(\frac{1}{5}\right)^2-\frac{1}{3}\times\frac{1}{4}-\frac{1}{4}\times\frac{1}{5}-\frac{1}{5}\times\frac{1}{3}}$$

$$= \frac{\left(\frac{1}{3}+\frac{1}{5}+\frac{1}{4}\right)\left[\left(\frac{1}{3}\right)^2+\left(\frac{1}{4}\right)^2+\left(\frac{1}{5}\right)^2-\frac{1}{3}\times\frac{1}{4}-\frac{1}{4}\times\frac{1}{5}-\frac{1}{5}\times\frac{1}{3}\right]}{\left[\left(\frac{1}{3}\right)^2+\left(\frac{1}{4}\right)^2+\left(\frac{1}{5}\right)^2-\frac{1}{3}\times\frac{1}{4}-\frac{1}{4}\times\frac{1}{5}-\frac{1}{5}\times\frac{1}{3}\right]}$$

$$= \frac{20+15+12}{60} = \frac{47}{60}$$

19. (d) $x = 7 - 4\sqrt{3}$

$= 4 + 3 - 4\sqrt{3}$

$= (2)^2 + \left(\sqrt{3}\right)^2 - 2 \times 2\sqrt{3}$

$= \left(2-\sqrt{3}\right)^2$

$\therefore$ $[(a^2 + b^2 - 2ab = (a-b)^2]$

$\Rightarrow x = \left(2-\sqrt{3}\right)^2$

$\sqrt{x} = 2-\sqrt{3}$

$$\frac{1}{\sqrt{x}} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$= 2+\sqrt{3}$

$$\therefore \sqrt{x} + \frac{1}{\sqrt{x}}$$

$= 2 - \sqrt{3} + 2 + \sqrt{3} = \mathbf{4}$

20. (b) $a = \dfrac{\sqrt{5}+1}{\sqrt{5}-1}$ $b = \dfrac{\sqrt{5}-1}{\sqrt{5}+1}$

$$\therefore a = \frac{1}{b}$$

$$a + b = a + \frac{1}{a}$$

$$\Rightarrow \frac{\sqrt{5}+1}{\sqrt{5}-1} + \frac{\sqrt{5}-1}{\sqrt{5}+1}$$

$$\Rightarrow \frac{5+1+2\sqrt{5}+5+1-2\sqrt{5}}{\left(\sqrt{5}\right)^2-(1)^2}$$

$$\Rightarrow \frac{6+2\sqrt{5}+6-2\sqrt{5}}{5-1} = \frac{12}{4} = 3$$

$$\therefore \frac{a^2+ab+b^2}{a^2-ab+b^2}$$

$$= \frac{a^2+\frac{1}{a^2}+ab}{a^2+\frac{1}{a^2}-ab}$$

$$\Rightarrow a + \frac{1}{a} = 3$$

$a^2 + \dfrac{1}{a^2} = 9 - 2 = 7$ $(ab = 1)$

$$\therefore \frac{a^2+\frac{1}{a^2}+ab}{a^2+\frac{1}{a^2}-ab} = \frac{7+1}{7-1} = \frac{8}{6} = \frac{4}{3}$$

21. (c) $a = 4.36$ $b = 2.39$ $c = 1.97$
$a - b - c$
$= 4.36 - 2.39 - 1.97$
$= 0$
$a^3 - b^3 - c^3 - 3abc$

$= \dfrac{1}{2}(a-b-c)[(a-b)^2+(b-c)^2+(c-a)^2]$

$= 0$

22. (d) $\dfrac{3a+5b}{3a-5b} = 5$

$\Rightarrow 3a + 5b = 15a - 25b$

$\Rightarrow 12a = 30b$

$\Rightarrow 2a = 5b$

a : b
5 : 2

23. (b) p : q = r : s = t : u = 2 : 3

$$\therefore \frac{mp+nr+ot}{mq+ns+ou}$$

$$\Rightarrow \frac{m\times 2x+n\times 2x+o\times 2x}{m\times 3x+n\times 3x+o\times 3x}$$

$$\Rightarrow \frac{2x(m+n+o)}{3x(m+n+o)} = \frac{2}{3}$$

$\therefore$ mp + nr + ot : mq + ns + ou

$\boxed{2:3}$

24. (c) $x : y = 3 : 4$

$$\frac{7x+3y}{7x-3y} = \frac{y}{y}\left(\frac{7\frac{x}{y}+3}{7\frac{x}{y}-3}\right) = \frac{7\times\frac{3}{4}+3}{7\times\frac{3}{4}-3}$$

$$= \frac{\frac{21}{4}+3}{\frac{21}{4}-3} = \frac{\frac{21+12}{4}}{\frac{21-12}{4}} = \frac{11}{3}$$

25. (b) $x + \dfrac{1}{4}\sqrt{x} + a^2$

$$= \left(\sqrt{x}\right)^2 + 2\times\frac{1}{8}\times\sqrt{x}+a^2$$

$[(A^2 + 2BA + B^2) = (A + B)^2]$

Here, $A = \sqrt{x}$ and $B = a$

$B = \dfrac{1}{8}$ $\therefore a = \dfrac{1}{8}$

26. (c) Given that $a \neq b$
Let a = 16, b = 4
$\therefore$ by options

So, $\dfrac{a+b}{2} = \dfrac{16+4}{2} = 10$

and $\sqrt{ab} = \sqrt{16\times 4} = 8$

$\therefore \dfrac{a+b}{2} > \sqrt{ab}$

$\therefore$ option (c) is correct.

27. (d) $x^{1/3} = y^{1/4}$
$\Rightarrow$ LCM of 3, 4 = 12

$\therefore \left(x^{1/3}\right)^{12} \Rightarrow \left(y^{1/4}\right)^{12}$

$x^4 = y^3$
take power '5' on both sides

$\Rightarrow \left(x^4\right)^5 = \left(y^3\right)^5$

$\Rightarrow x^{20} = y^{15}$

28. (b) $x = \dfrac{\sqrt{3}}{2}$

or $1 + x = 1 + \dfrac{\sqrt{3}}{2} = \dfrac{2+\sqrt{3}}{2}$

$$= \frac{2\left(2+\sqrt{3}\right)}{2\times 2}$$

(divides and multiply by 2)

$$\Rightarrow 1 + x = \frac{4+2\sqrt{3}}{4}$$

$$= \frac{1+3+2\sqrt{3}}{4}$$

$$= \frac{(1)^2+\left(\sqrt{3}\right)^2+2\times 1\times\sqrt{3}}{4}$$

$$1 + x \Rightarrow \frac{\left(1+\sqrt{3}\right)^2}{4}$$

$$\therefore \sqrt{1+x} = \frac{1+\sqrt{3}}{2}$$

Similarly,

$$\sqrt{1-x} \Rightarrow \frac{\sqrt{3}-1}{2}$$

$$\therefore \frac{\sqrt{1+x}}{1+\sqrt{1+x}} + \frac{\sqrt{1-x}}{1-\sqrt{1-x}}$$

$$= \frac{\frac{1+\sqrt{3}}{2}}{1+\frac{1+\sqrt{3}}{2}} + \frac{\frac{\sqrt{3}-1}{2}}{1-\frac{\sqrt{3}-1}{2}}$$

$$= \frac{1+\sqrt{3}}{3+\sqrt{3}} + \frac{\sqrt{3}-1}{3-\sqrt{3}}$$

$$= \frac{1+\sqrt{3}}{\sqrt{3}\left(\sqrt{3}+1\right)} + \frac{\sqrt{3}-1}{\sqrt{3}\left(\sqrt{3}-1\right)}$$

$= \frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

29. (d) $x^2 - 4x - 1 = 0$
$x^2 - 1 = 4x$
(divide x both sides)

$x - \frac{1}{x} = 4$

$x^2 + \frac{1}{x^2} - 2 = 16$

$x^2 + \frac{1}{x^2} = 18$

30. (d) $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)\left(x^2+\frac{1}{x^2}+1\right)$

$=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)$

$\because$ $\boxed{\begin{array}{l}(A+B)(A^2-AB+B^2)=A^3+B^3\\(A-B)(A^2+AB+B^2)=A^3-B^3\end{array}}$

$=\left(x^3+\frac{1}{x^3}\right)\left(x^3-\frac{1}{x^3}\right) = \boxed{x^6-\frac{1}{x^6}}$

31. (b) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$

L.H.S $= \frac{\sqrt{7}-2}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2}$
(Rationalisation)

$= \frac{(\sqrt{7}-2)^2}{(\sqrt{7})^2-(4)} = \frac{7+4-4\sqrt{7}}{7-4}$

$= \frac{11-4\sqrt{7}}{3}$

$\frac{11}{3}-\frac{4}{3}\sqrt{7} = -\frac{4}{3}\sqrt{7} + \frac{11}{3}$

$= a\sqrt{7}+b$ = R.H.S

Compare the cofficients of $\sqrt{7}$ and constant term

$a = -\frac{4}{3}$

$b = \frac{11}{3}$

32. (c) $a+\frac{1}{b}=1,\ b+\frac{1}{c}=1,\ c+\frac{1}{a}=\ ?$

Put values,

$a = \frac{1}{2}$ $b = 2$ $c = -1$

$c+\frac{1}{a} = -1 + \frac{1}{\left(\frac{1}{2}\right)} = -1 + 2 = 1$

Alternate:

$\Rightarrow a + \frac{1}{b} = 1$..... (i)

$\Rightarrow a = 1 - \frac{1}{b} = \boxed{\frac{b-1}{b}}$

$\frac{1}{a} = \frac{b}{b-1} \Rightarrow b + \frac{1}{c} = 1$

$\frac{1}{c} = 1 - b,\ \boxed{c=\frac{1}{1-b}}$

$\therefore c + \frac{1}{a} = \frac{1}{1-b}+\frac{b}{b-1}$

$= \frac{1}{1-b}-\frac{b}{1-b} = \frac{1-b}{1-b} = 1$

33. (c) $x = \sqrt{3}+\sqrt{2}$

$\therefore \frac{1}{x} = \sqrt{3}-\sqrt{2}$

$x^3 + \frac{1}{x^3}$

$\therefore x^3 = \left(\sqrt{3}+\sqrt{2}\right)^3$

$= \left(\sqrt{3}\right)^3+\left(\sqrt{2}\right)^3+3\times\sqrt{3}\times\sqrt{2}\left(\sqrt{3}+\sqrt{2}\right)$

$= 3\sqrt{3}+2\sqrt{2}+3\sqrt{6}\left(\sqrt{3}+\sqrt{2}\right)$

$= 3\sqrt{3}+2\sqrt{2}+9\sqrt{2}+6\sqrt{3}$

$x^3 = 9\sqrt{3}+11\sqrt{2}$

$\frac{1}{x^3} = 9\sqrt{3}-11\sqrt{2}$

$x^3 + \frac{1}{x^3} = 9\sqrt{3}+11\sqrt{2}+9\sqrt{3}-11\sqrt{2}$

$= 18\sqrt{3}$

Alternate:

$x = \sqrt{3}+\sqrt{2}$

$\frac{1}{x} = \sqrt{3}-\sqrt{2}$

and $x + \frac{1}{x} = \sqrt{3}+\sqrt{2} + \sqrt{3}-\sqrt{2}$

$= 2\sqrt{3}$

$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$= \left(2\sqrt{3}\right)^3 - 3\left(2\sqrt{3}\right)$

$= 24\sqrt{3} - 6\sqrt{3} = 18\sqrt{3}$

34. (c) $x + y = 7$ (cubing both sides)
$(x+y)^3 = (7)^3$
$x^3 + y^3 + 3(x+y)xy = 343$
$x^3 + y^3 + 21xy = 343$

35. (c) $x^{1/3} + y^{1/3} = z^{1/3}$
(cubing both sides)

$\left(x^{1/3}+y^{1/3}\right)^3 = \left(z^{1/3}\right)^3$

$\Rightarrow x + y + 3x^{1/3}.y^{1/3}.(x^{1/3}+y^{1/3}) = z$

$\Rightarrow x + y - z + 3x^{1/3}.y^{1/3}.z^{1/3} = 0$

$\Rightarrow x + y - z = -3x^{1/3}\,y^{1/3}\,z^{1/3}$
(cubing again both sides)

$(x + y - z)^3 = -27xyz$
$(x + y - z)^3 + 27xyz = 0$

36. (a) $\frac{a}{b}+\frac{b}{a}=1$
$a \neq 0,\quad b \neq 0$
$a^2 + b^2 = ab$
$a^2 + b^2 - ab = 0$
$(a + b)(a^2 + b^2 - ab) = (a + b) \times 0$
[(multiply both sides by (a + b)]
$a^3 + b^3 = 0$

37. (d) $p = 99$
$p(p^2 + 3p + 3)$
$= p^3 + 3p^2 + 3p + 1 - 1$
$= (p + 1)^3 - 1$
$= (100)^3 - 1 = 1000000 - 1$
$= 999999$

38. (b) $5^{\sqrt{x}}+12^{\sqrt{x}}=13^{\sqrt{x}}$
By option put $x = 4$
$\Rightarrow 5^{\sqrt{4}}+12^{\sqrt{4}}=13^{\sqrt{4}}$
$\Rightarrow 5^2 + 12^2 = 13^2$
$\Rightarrow 169 = 169$
hence, $x = 4$

39. (d) $(x-3)^2 + (y-4)^2 + (z-5)^2 = 0$
$\therefore$ $(x-3)^2 = 0$ $x = 3$
$(y-4)^2 = 0$ $y = 4$
$(z-5)^2 = 0$ $z = 5$
$(x + y + z) \Rightarrow 4 + 3 + 5 \Rightarrow 12$

40. (c) $x = 3 + \sqrt{8}$

$\frac{1}{x} = \frac{1}{3+\sqrt{8}}$

$\frac{1}{x} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}}$

$= \frac{3-\sqrt{8}}{9-8} = 3 - \sqrt{8}$

$x + \frac{1}{x} = 3 + \sqrt{8} + 3 - \sqrt{8} = 6$

$x + \frac{1}{x} = 6$

squaring both sides

$x^2 + \frac{1}{x^2} + 2 = 36$

$x^2 + \frac{1}{x^2} = 34$

41. (b) $x - \frac{1}{x} = 4$

$x^2 + \frac{1}{x^2} - 2 = 16$ (On Squaring)

$\Rightarrow x^2 + \frac{1}{x^2} = 18$

$x^2 + \frac{1}{x^2} + 2 - 2 = 18$

$x^2 + \frac{1}{x^2} + 2 = 20$

$\left(x + \frac{1}{x}\right)^2 = 20$

$x + \frac{1}{x} = \sqrt{20}$

$= \sqrt{4 \times 5} = 2\sqrt{5}$

42. (a) $4b^2 + \frac{1}{b^2} = 2$

$(2b)^2 + \left(\frac{1}{b}\right)^2 + 4 - 4 = 2$

$\left(2b + \frac{1}{b}\right)^2 - 4 = 2$

$\left(2b + \frac{1}{b}\right)^2 = 6$

$2b + \frac{1}{b} = \sqrt{6}$

Take cube both sides

$\left(2b + \frac{1}{b}\right)^3 = (\sqrt{6})^3$

$8b^3 + \frac{1}{b^3} + 3 \times 2b \times \frac{1}{b}\left(2b + \frac{1}{b}\right) = 6\sqrt{6}$

$8b^3 + \frac{1}{b^3} + 6\sqrt{6} = 6\sqrt{6}$

$8b^3 + \frac{1}{b^3} = 0$

43. (c) $\left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$

$\Rightarrow \left(\frac{3}{5}\right)^{+3-6} = \left(\frac{3}{5}\right)^{2x-1}$

$\Rightarrow -3 = 2x - 1$

$\Rightarrow 2x = -2$

$\Rightarrow x = -1$

44. (b) $\frac{\sqrt{3+x} + \sqrt{3-x}}{\sqrt{3+x} - \sqrt{3-x}} = \frac{2}{1}$

(by c–d rule)

$\Rightarrow \frac{\sqrt{3+x}}{\sqrt{3-x}} = \frac{2+1}{2-1} = \frac{3}{1}$

$\left(\begin{array}{l} \frac{A}{B} = \frac{C}{D} \\ \frac{A+B}{A-B} = \frac{C+D}{C-D} \end{array}\right)$

$\frac{\sqrt{3+x}}{\sqrt{3-x}} = 3$

Squaring both sides

$\frac{3+x}{3-x} = 9$

$3 + x = 27 - 9x$

$10x = 24$

$x = \frac{24}{10} = \frac{12}{5}$

45. (d) $x = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}} \times \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$

$= \frac{\left(\sqrt{1+x} + \sqrt{1-x}\right)^2}{\left(\sqrt{1+x}\right)^2 - \left(\sqrt{1-x}\right)^2}$

$= \frac{1 + x + 1 - x + 2\sqrt{1-x^2}}{1 + x - 1 + x}$

$= \frac{2 + 2\sqrt{1-x^2}}{2x} = \frac{1 + \sqrt{1-x^2}}{x}$

$= \frac{1 + \sqrt{1 - \frac{3}{4}}}{\sqrt{3}} \times 2$

$= \frac{\left(1 + \frac{1}{2}\right)}{\sqrt{3}} \times 2 = \frac{\frac{3}{2} \times 2}{\sqrt{3}} = \sqrt{3}$

46. (d) $\frac{\sqrt{x+4} + \sqrt{x-4}}{\sqrt{x+4} - \sqrt{x-4}} = \frac{2}{1}$

by C – D rule

$\Rightarrow \frac{\sqrt{x+4}}{\sqrt{x-4}} = \frac{2+1}{2-1} = \frac{3}{1}$

$\Rightarrow \left(\frac{\sqrt{x+4}}{\sqrt{x-4}}\right)^2 = \left(\frac{3}{1}\right)^2$

$\Rightarrow \frac{x+4}{x-4} = 9$ again C & D rule

$\Rightarrow \frac{x}{4} = \frac{9+1}{9-1}$

$\Rightarrow \frac{x}{4} = \frac{10}{8}$

$\Rightarrow x = \frac{10}{8} \times 4 = 5$

47. (c) $x = \left(\sqrt{2} + 1\right)^{1/3}$

Take cube on both sides

$\Rightarrow x^3 = \sqrt{2} + 1$

$\Rightarrow \frac{1}{x^3} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \frac{\sqrt{2}-1}{1}$

$\frac{1}{x^3} = \sqrt{2} - 1$

$\Rightarrow x^3 - \frac{1}{x^3} = \sqrt{2} + 1 - \sqrt{2} + 1 = 2$

48. (b) $\frac{x^2 - x + 1}{x^2 + x + 1} = \frac{3}{2}$ Given

$\Rightarrow \frac{x\left\{\left(x + \frac{1}{x}\right) - 1\right\}}{x\left\{\left(x + \frac{1}{x}\right) + 1\right\}} = \frac{3}{2}$

Let $\left(x + \frac{1}{x} = y\right)$

$\Rightarrow \frac{\left(x + \frac{1}{x}\right) - 1}{\left(x + \frac{1}{x}\right) + 1} = \frac{3}{2}$

$\frac{y-1}{y+1} = \frac{3}{2}$

$\Rightarrow 2(y - 1) = 3(y + 1)$

$2y - 2 = 3y + 3$

$y = -2 - 3 = -5$

$\therefore x + \frac{1}{x} = -5$

49. (c) $x = 3 + \sqrt{8}$

$x^2 = 9 + 8 + 2 \times 3\sqrt{8}$

$x^2 = 17 + 6\sqrt{8}$

$\frac{1}{x^2} = 17 - 6\sqrt{8}$

$x^2 + \frac{1}{x^2} = 17 + 6\sqrt{8} + 17 - 6\sqrt{8} = 34$

50. (c) $x = 5 + 2\sqrt{6}$

$x = 3 + 2 + 2\sqrt{3} \times \sqrt{2}$

$x = \left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2 + 2\sqrt{3} \times \sqrt{2}$

$x = \left(\sqrt{3} + \sqrt{2}\right)^2$

$\sqrt{x} = \sqrt{3} + \sqrt{2}$

Similarly

$\Rightarrow \frac{1}{\sqrt{x}} = \sqrt{3} - \sqrt{2}$

$\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}$

$= 2\sqrt{3}$

51. (d) $x = \sqrt{3} + \sqrt{2}$

$x^2 = 3 + 2 + 2\sqrt{6}$

$x^2 = 5 + 2\sqrt{6}$

$\frac{1}{x^2} = 5 - 2\sqrt{6}$

$x^2 + \frac{1}{x^2}$

$= 5 + 2\sqrt{6} + 5 - 2\sqrt{6}$

$= 10$

52. (c) $x + \frac{9}{x} = 6$

Take values of x

Let $x = 3$

$3 + \frac{9}{3} = 6$

Prove So, $x = 3$

$\therefore x^2 + \frac{9}{x^2} = 9 + \frac{9}{9} = 10$

Alternate:

$x + \frac{9}{x} = 6$

On squaring

$\left(x + \frac{9}{x}\right)^2 = 36$

$x^2 + \frac{81}{x^2} + 2 \times \frac{9}{x} \times x = 36$

$x^2 + \frac{81}{x^2} - 18 = 0$

$\left(x - \frac{9}{x}\right)^2 = 0$

$x = \frac{9}{x}$

$x^2 = 9$

Hence $x^2 + \frac{9}{x^2} = 9 + \frac{9}{9} = 10$

53. (b) $2p + \frac{1}{p} = 4$

Divide by 2

$\frac{2p}{2} + \frac{1}{2p} = \frac{4}{2}$

$p + \frac{1}{2p} = 2$

Take cube on both sides

$\Rightarrow \left(p + \frac{1}{2p}\right)^3 = (2)^3$

$p^3 + \frac{1}{8p^3} + 3 \times p \times \frac{1}{2p}\left(p + \frac{1}{2p}\right) = 8$

$p^3 + \frac{1}{8p^3} + \frac{3}{2} \times 2 = 8$

$p^3 + \frac{1}{8p^3} = 8 - 3 = 5$

54. (a) $a^6 + b^6 = (a^2)^3 + (b^2)^3$

$= (a^2 + b^2)(a^4 - a^2b^2 + b^4)$

$\therefore a^6 + b^6 = (a^2 + b^2) \times 0 = 0$

55. (c) $x + \frac{1}{x} = 3$ (Given)

$\frac{x^3 + \frac{1}{x}}{x^2 - x + 1}$ (Divide by x)

$\Rightarrow \frac{\frac{x^3}{x} + \frac{1}{x^2}}{\frac{x^2}{x} - \frac{x}{x} + \frac{1}{x}} = \frac{x^2 + \frac{1}{x^2}}{x - 1 + \frac{1}{x}}$

$= \frac{x^2 + \frac{1}{x^2}}{x + \frac{1}{x} - 1}$

$\therefore x + \frac{1}{x} = 3$

$\therefore x^2 + \frac{1}{x^2} = 9 - 2 = 7$

$\therefore \frac{x^2 + \frac{1}{x^2}}{x + \frac{1}{x} - 1} = \frac{7}{3 - 1} = \frac{7}{2}$

56. (a) $a + \frac{1}{a} + 1 = 0$

$a + \frac{1}{a} = -1$

Squaring both sides

$\Rightarrow a^2 + \frac{1}{a^2} + 2 = 1$

$\Rightarrow a^2 + \frac{1}{a^2} = -1$

$\Rightarrow a^2 + 1 = \frac{-1}{a^2}$ .....(i)

$\Rightarrow a + \frac{1}{a} = -1$ (Given)

$\therefore a^2 + 1 = -a$ .....(ii)

$\Rightarrow -a = \frac{-1}{a^2}$ from equation (i) and (ii)

$a^3 = 1$

$\therefore a^3 - 1 = 0$

$\Rightarrow a^4 - a = 0 \times a = 0$

(Multiply a both sides)

57. (c) $x = a + \frac{1}{a}$

$y = a - \frac{1}{a}$

$\therefore (x + y) = a + \frac{1}{a} + a - \frac{1}{a} = 2a$

$\therefore (x - y) = a + \frac{1}{a} - a + \frac{1}{a} = \frac{2}{a}$

$\therefore x^4 + y^4 - 2x^2y^2 = (x^2 - y^2)^2$

$\Rightarrow ((x + y)(x - y))^2$

$\Rightarrow \left(2a \times \frac{2}{a}\right)^2$

$= (4)^2 = 16$

58. (a) $a = 11$

$b = 9$

$\Rightarrow \frac{a^2 + b^2 + ab}{a^3 - b^3}$

$\Rightarrow (a^3 - b^3) = ((a - b)(a^2 + ab + b^2))$

$\therefore \frac{a^2 + b^2 + ab}{(a - b)(a^2 + ab + b^2)}$

$= \frac{1}{a - b} = \frac{1}{11 - 9} = \frac{1}{2}$

59. (a) $p = 101$

$= \sqrt[3]{p(p^2 - 3p + 3) - 1}$

$= \sqrt[3]{p^3 - 3p^2 + 3p - 1}$

$\therefore$ $[(p-1)^3 = p^3 - (1)^3 - 3p(p-1)]$

$= \sqrt[3]{(p-1)^3}$

$= p - 1 = 101 - 1 = 100$

60. (a) $x = 19$ $y = 18$

$$\frac{x^2 + y^2 + xy}{x^3 - y^3}$$

$$= \frac{x^2 + y^2 + xy}{(x-y)(x^2 + y^2 + xy)}$$

$$= \frac{1}{x-y} = \frac{1}{19-18} = 1$$

61. (b) $50\% (p - q) = 30\% (p + q)$

$$\frac{p-q}{2} = \frac{3}{10} (p + q)$$

$$\left[50\% = \frac{1}{2}\right]$$

$\Rightarrow$ $5(p - q) = 3(p + q)$

$\Rightarrow$ $5p - 5q = 3p + 3q$

$\Rightarrow$ $2p = 8q$

$\Rightarrow$ $1p = 4q$

$\Rightarrow$ $p : q$

$\Rightarrow$ $4 : 1$

62. (a) $\frac{a}{3} = \frac{b}{2} \Rightarrow \frac{a}{b} = \frac{3}{2}$

$$\frac{2a + 3b}{3a - 2b} = \frac{2 \times 3 + 3 \times 2}{3 \times 3 - 2 \times 2}$$

$$= \frac{6+6}{9-4} = \frac{12}{5}$$

63. (d) $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$

Add 3 both sides

$$\Rightarrow \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} + 3 = 1 + 3$$

$$\Rightarrow \left(\frac{a}{1-a} + 1\right) + \left(\frac{b}{1-b} + 1\right) + \left(\frac{c}{1-c} + 1\right) = 4$$

$$\Rightarrow \left(\frac{a+1-a}{1-a}\right) + \left(\frac{b+1-b}{1-b}\right) + \left(\frac{c+1-c}{1-c}\right) = 4$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 4$$

64. (b)

$\frac{2x-y}{x+2y} \quad \frac{1}{2}$ (Cross Multiply)

$\Rightarrow$ $4x - 2y = x + 2y$

$3x = 4y$

$x : y = 4 : 3$

$$\Rightarrow \frac{3x - y}{3x + y} = \frac{3 \times 4 - 3}{3 \times 4 + 3}$$

$$= \frac{12-3}{12+3} = \frac{9}{15} = \frac{3}{5}$$

65. (b) $x + \frac{1}{x} = 5$

$$\therefore \frac{2x}{3x^2 - 5x + 3}$$

(Divide by x)

$$= \frac{\frac{2x}{x}}{\frac{3x^2}{x} - \frac{5x}{x} + \frac{3}{x}} = \frac{2}{3x + \frac{3}{x} - 5}$$

$$= \frac{2}{3\left(x + \frac{1}{x}\right) - 5} = \frac{2}{3 \times 5 - 5}$$

$$= \frac{2}{10} = \frac{1}{5}$$

66. (b) $\sqrt{1 - \frac{x^3}{100}} = \frac{3}{5}$

$$\Rightarrow 1 - \frac{x^3}{100} = \left(\frac{3}{5}\right)^2$$

$$\Rightarrow 1 - \frac{9}{25} = \frac{x^3}{100}$$

$$\Rightarrow \frac{16}{25} = \frac{x^3}{100}$$

$$\Rightarrow \frac{16 \times 100}{25} = x^3$$

$\Rightarrow$ $16 \times 4 = x^3$

$\Rightarrow$ $4 = x$

67. (d) $\frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}} = a + b\sqrt{6}$

$$\Rightarrow \frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{16 \times 3} + \sqrt{9 \times 2}}$$

$$= \frac{4\sqrt{3} + 5\sqrt{2}}{4\sqrt{3} + 3\sqrt{2}}$$

$$\Rightarrow \frac{4\sqrt{3} + 5\sqrt{2}}{4\sqrt{3} + 3\sqrt{2}} \times \frac{4\sqrt{3} - 3\sqrt{2}}{4\sqrt{3} - 3\sqrt{2}}$$

$$= \frac{(4\sqrt{3} + 5\sqrt{2})(4\sqrt{3} - 3\sqrt{2})}{48 - 18}$$

$$= \frac{8\sqrt{6} + 18}{30} = \frac{8\sqrt{6}}{30} + \frac{18}{30}$$

$$= \frac{4}{15}\sqrt{6} + \frac{3}{5}$$

$$= \frac{3}{5} + \frac{4}{15}\sqrt{6}$$

$$\therefore \frac{3}{5} + \frac{4}{15}\sqrt{6} = a + b\sqrt{6}$$

By comparing cofficients of rational and irrational parts.

$\Rightarrow$ $a = \frac{3}{5}$ $b = \frac{4}{15}$

$$\left(\frac{3}{5}, \frac{4}{15}\right)$$

68. (a) $\sqrt{1 + \frac{x}{961}} = \frac{32}{31}$

(Squaring both sides)

$$\Rightarrow 1 + \frac{x}{961} = \frac{1024}{961}$$

$$\Rightarrow \frac{961 + x}{961} = \frac{1024}{961}$$

$x = 1024 - 961 = 63$

69. (d) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{3}{1}$

$$\Rightarrow \frac{2 \times 9 + 3 \times 9 + 4 \times 9}{2 \times 1 + 3 \times 1 + 4 \times 1}$$

$$= \frac{18 + 27 + 36}{2 + 3 + 4} = \frac{81}{9} = \mathbf{9}$$

70. (d) $2x + \frac{1}{3x} = 5$

$\Rightarrow$ $6x^2 + 1 = 15x$

$$\therefore \frac{5x}{6x^2 + 20x + 1}$$

$$= \frac{5x}{15x + 20x} = \frac{5x}{35x} = \frac{1}{7}$$

71. (a) $x \propto \frac{1}{y^2 - 1}$ (Given)

$$x = k \times \frac{1}{y^2 - 1}$$

(k is constant)

Now x = 24 when y = 10 given

$\Rightarrow$ $24 = k \times \dfrac{1}{(10)^2 - 1}$

$\Rightarrow$ $24 = \dfrac{k}{99}$

$k = 24 \times 99$

$\Rightarrow$ x = ?

y = 5

$x = 24 \times 99 \times \dfrac{1}{25-1}$

$= 24 \times 99 \times \dfrac{1}{24}$ $x = 99$

72. (a) $x^2 + y^2 + 2x + 1 = 0$

$\Rightarrow$ $x^2 + 2x + 1 + y^2 = 0$

$(x+1)^2 + y^2 = 0$

Hence both terms are squares and there addition is zero so, it can be possible only when both terms are zeros.

$\therefore$ $x + 1 = 0$

$\Rightarrow x = -1$ $y = 0$

$\therefore$ $x^{31} + y^{35}$

$= (-1)^{31} + (0)^{35} = -1$

73. (b) $\dfrac{x}{2x^2 + 5x + 2} = \dfrac{1}{6}$

$\dfrac{\frac{x}{x}}{2\frac{x^2}{x} + \frac{5x}{x} + \frac{2}{x}} = \dfrac{1}{6}$

$\Rightarrow$ $\dfrac{1}{2x + \frac{2}{x} + 5} = \dfrac{1}{6}$

$\Rightarrow$ $2\left(x + \dfrac{1}{x}\right) + 5 = 6$

$\Rightarrow$ $6 - 5 = 2\left(x + \dfrac{1}{x}\right)$

$\Rightarrow$ $1 = 2\left(x + \dfrac{1}{x}\right)$

$\Rightarrow$ $x + \dfrac{1}{x} = \dfrac{1}{2}$

74. (c) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$

$\Rightarrow$ $a^2 + b^2 + c^2 = 2a - 2b - 2c - 3$

$\Rightarrow$ $a^2 + b^2 + c^2 - 2a + 2b + 2c + 1 + 1 + 1 = 0$

$\Rightarrow$ $(a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$

$\Rightarrow$ $(a-1)^2 + (b+1)^2 + (c+1)^2 = 0$

$a = 1$

$b = -1$

$c = -1$

$\therefore$ $2a - 3b + 4c$

$= 2 \times 1 - 3 \times (-1) + 4 \times (-1)$

$= 2 + 3 - 4 = 1$

75. (a) $(3a+1)^2 + (b-1)^2 + (2c-3)^2 = 0$

$\Rightarrow$ $(3a+1)^2 = 0$

$\Rightarrow$ $3a = -1$

$\Rightarrow$ $a = \dfrac{-1}{3}$

$(b-1)^2 = 0 \Rightarrow b - 1 = 0$

$\Rightarrow$ $b = 1$

$(2c - 3) = 0$

$\Rightarrow$ $c = \dfrac{3}{2}$

$\therefore$ $3a + b + 2c = 3 \times \dfrac{-1}{3} + 1 + \dfrac{3}{2} \times 2$

$= -1 + 1 + 3 = \mathbf{3}$

76. (b) $\dfrac{(a-b)^2}{(b-c)(c-a)} + \dfrac{(b-c)^2}{(a-b)(c-a)} + \dfrac{(c-a)^2}{(a-b)(b-c)}$

Now

$\Rightarrow$ $\dfrac{(a-b)^2}{(b-c)(c-a)} \times \dfrac{(a-b)}{a-b}$

Multiply divide by (a – b) in Ist term

$\Rightarrow$ (b – c) in IInd term

$\Rightarrow$ (c – a) in IIIrd term

$\Rightarrow$ $\dfrac{(a-b)^2(a-b)}{(b-c)(c-a)(a-b)} + \dfrac{(b-c)^2(b-c)}{(a-b)(b-c)(c-a)} + \dfrac{(c-a)^2(c-a)}{(a-b)(b-c)(c-a)}$

$\Rightarrow$ Let $a - b = x$

$b - c = y$

$c - a = z$

$\therefore$ $x + y + z = 0$

$\therefore$ $x^3 + y^3 + z^3 = 3xyz$

$\therefore$ $(a-b)^3 + (b-c)^3 + (c-a)^3$

$= 3(a-b)(b-c)(c-a)$

$\therefore$ $\dfrac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$

77. (c) $(a-3)^2 + (b-4)^2 + (c-9)^2 = 0$

$a - 3 = 0$ $a = 3$

$b - 4 = 0$ $b = 4$

$c - 9 = 0$ $c = 9$

$\therefore$ $\sqrt{a+b+c} = \sqrt{3+4+9}$

$= \sqrt{16} = \pm\mathbf{4}$

78. (b) $1.5x = 0.04y$

$\Rightarrow$ $\dfrac{x}{y} = \dfrac{0.04}{1.5} = \dfrac{4}{100} \times \dfrac{10}{15} = \dfrac{2}{75}$

$\therefore$ $\dfrac{y^2 - x^2}{y^2 + x^2 + 2xy} = \dfrac{(y-x)(y+x)}{(y+x)^2}$

$\Rightarrow$ $\dfrac{y-x}{y+x} = \dfrac{75-2}{75+2} = \dfrac{73}{77}$

79. (b) $a^{1/3} = 11$, $a = 11^3 = 1331$

$a^2 - 331a = a(a - 331)$

$= 1331\ (1331 - 331)$

$= 1331 \times 1000 = 1331000$

80. (a) $x^2 + y^2 + \dfrac{1}{x^2} + \dfrac{1}{y^2} = 4$

Take $x = y = 1$

$1 + 1 + \dfrac{1}{1} + \dfrac{1}{1} = 4$

Hence

$x^2 + y^2 = 1 + 1 = 2$

81. (b) $x^2 = y + z$ ....(i)

$x^2 + x = y + z + x$

add x on both sides

$x(x+1) = x + y + z$

$\Rightarrow$ $y^2 = x + z$ ....(ii)

$y^2 + y = x + y + z$

add y on both sides

$y(y+1) = (x + y + z)$

$\Rightarrow$ $z^2 = y + x$ ....(ii)

$z^2 + z = x + z + y$

add z on both sides

$z(z+1) = x + y + z$

$\therefore$ $x(x+1) = x + y + z$

$\dfrac{x}{x+y+z} = \dfrac{1}{x+1}$

$\dfrac{y}{x+y+z} = \dfrac{1}{y+1}$

$\dfrac{z}{x+y+z} = \dfrac{1}{z+1}$

By adding them

$= \dfrac{x}{x+y+z} + \dfrac{y}{x+y+z} + \dfrac{z}{x+y+z} = \dfrac{1}{x+1} + \dfrac{1}{y+1} + \dfrac{1}{z+1}$

$= \dfrac{x+y+z}{x+y+z} = \dfrac{1}{x+1} + \dfrac{1}{y+1} + \dfrac{1}{z+1} = 1$

Alternate:-

$x = y = z = 2$

$\therefore$ $\dfrac{1}{2+1} + \dfrac{1}{2+1} + \dfrac{1}{2+1} = \dfrac{1}{3} + \dfrac{1}{3} + \dfrac{1}{3} = 1$

82. (d) $a^2 + b^2 = 2$
$c^2 + d^2 = 1$
Put values of a, b, c, d
Take a = b = 1
c = 1
d = 0
$\Rightarrow (ad - bc)^2 + (ac + bd)^2$
$\Rightarrow (0-1)^2 + (1+0)^2$
$\Rightarrow (-1)^2 + (1)^2 = \mathbf{2}$

83. (d) $x = \frac{4ab}{a+b}$

$\Rightarrow \frac{x}{2a} = \frac{2b}{a+b}$

$\frac{x+2a}{x-2a} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a}$
(By C – D rule)
$\Rightarrow$ again

$\frac{x}{2b} = \frac{2a}{a+b}$

$\frac{x+2b}{x-2b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b}$

$\Rightarrow \frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$

$\Rightarrow \frac{3b+a-3a-b}{b-a}$

$\Rightarrow \frac{2b-2a}{b-a} = \frac{2(b-a)}{(b-a)} = 2$

84. (c) $m + \frac{1}{m-2} = 4$

$\Rightarrow (m-2) + \frac{1}{(m-2)} = 2$

$\Rightarrow$ Squaring both sides

$(m-2)^2 + \frac{1}{(m-2)^2} + 2 \times (m-2) \times \frac{1}{(m-2)} = 4$

$(m-2)^2 + \frac{1}{(m-2)^2} = 2$

85. (c) $a^2 + b^2 + 2b + 4a + 5 = 0$
$a^2 + b^2 + 2b + 4a + 4 + 1 = 0$
$a^2 + 4a + 4 + b^2 + 2b + 1 = 0$
$(a+2)^2 + (b+1)^2 = 0$
a + 2 = 0 a = –2
b + 1 = 0 b = – 1

$\frac{a-b}{a+b} \Rightarrow \frac{-2+1}{-2-1}$

$\Rightarrow \frac{-1}{-3} = \frac{1}{3}$

86. (a) $x - y = \frac{x+y}{7} = \frac{xy}{4} = k$ (let)

$x - y = k$ ....(i)
$x + y = 7k$ ....(ii)
$xy = 4k$ ....(iii)

$\therefore$ $x - y = k$ .....(i)
$x + y = 7k$ ...(ii)
$x = 4k$
$y = 3k$

$\therefore xy \Rightarrow 4k \times 3k = 12k^2$
$12k^2 = 4k$

$k = \frac{1}{3}$ $\therefore xy = 4k = \frac{4}{3}$

87. (d) $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = 0$

$= \frac{x^2}{yz} \times \frac{x}{x} + \frac{y^2 \times y}{zx \times y} + \frac{z^2}{xy} \times \frac{z}{z} = 0$

$= \frac{x^3 + y^3 + z^3}{xyz} = 0$

$\therefore$ If $x + y + z = 0$
then, $x^3 + y^3 + z^3$
$\Rightarrow 3xyz)$

$\therefore \frac{3xyz}{xyz} = 3$

88. (b) a + b + c = 0
$\frac{1}{(a+b)(b+c)} + \frac{1}{(a+c)(b+a)} + \frac{1}{(c+a)(c+b)}$

$\Rightarrow \frac{(a+c)+(b+c)+(a+b)}{(a+b)(a+c)(b+c)}$

$\Rightarrow \frac{2(a+b+c)}{(a+b)(a+c)(b+c)} = 0$

$(\therefore a + b + c = 0)$

89. (c) a + b + c = 0
Assume values a = 2 b = –2 c = 0
a+ b + c = 2 – 2 + 0 = 0(satisfy)

$\therefore \frac{a^2+b^2+c^2}{a^2-bc}$

$\Rightarrow \frac{4+4+0}{4-0} \Rightarrow \frac{8}{4} = 2$

Alternate:-
a + b + c = 0
b + c = – a
Squaring both sides
$(b + c)^2 = a^2$
$b^2 + c^2 + 2bc = a^2$
$b^2 + c^2 = a^2 - 2bc$

$\therefore \frac{a^2+b^2+c^2}{a^2-bc}$

$\Rightarrow \frac{a^2+a^2-2bc}{a^2-bc}$

$\Rightarrow \frac{2a^2-2bc}{a^2-bc} \Rightarrow \frac{2(a^2-bc)}{a^2-bc} = 2$

90. (c) $x^2 + y^2 - 4x - 4y + 8 = 0$
$x^2 + 4 - 4x + y^2 + 4 - 4y = 0$
$(x-2)^2 + (y-2)^2 = 0$
$x - 2 = 0$, $y - 2 = 0$
$x = 2$, $y = 2$
$\therefore x - y = 2 - 2 = 0$

91. (a) $x = b + c - 2a$
$y = c + a - 2b$
$z = a + b - 2c$
$\Rightarrow x + y + z = (b + c - 2a) + (c + a - 2b) + (a + b - 2c)$
$\therefore$ Now
$= x^2 + y^2 + 2xy - z^2$
$= (x+y)^2 - z^2$ $(A^2 - B^2 = (A+B)(A-B)$
$(x + y - z)(x + y + z)$
= As we know $(x + y + z) = 0$
$\therefore x^2 + y^2 - z^2 + 2xy = 0 \times (x + y - z) = 0$

92. (b) $a^2 + b^2 + c^2 = ab + bc + ca$
take value a = b = c = 2

$\Rightarrow \frac{a+c}{b} = \frac{2+2}{2} = 2$

93. (a) $x + \frac{1}{x} = \sqrt{3}$ (take cube on both sides)

$\left(x + \frac{1}{x}\right)^3 = \left(\sqrt{3}\right)^3$

$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 3\sqrt{3}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(\sqrt{3}\right) = 3\sqrt{3}$

$x^3 + \frac{1}{x^3} = 0$

$\therefore x^6 = -1$
$\therefore x^{18} + x^{12} + x^6 + 1$
$= (-1)^3 + (-1)^2 + (-1) + 1$
$= -1 + 1 - 1 + 1 = 0$

94. (c) $ax^3 + 3x^2 - 8x + b$ is divisible by $(x+2)$ and $(x-2)$
$\therefore$ $(x + 2)$ and $(x - 2)$ are factors
$\therefore x + 2 = 0 \Rightarrow x = -2$
$x - 2 = 0 \Rightarrow x = 2$
Put $x = -2$
$\therefore a(-2)^3 + 3(-2)^2 - 8(-2) + b = 0$
$= -8a + 12 + 16 + b = 0$
$-8a + b + 28 = 0$
$-8a + b = -28$............ (I)

and

Put $x = 2$

$\Rightarrow a(2)^3 + 3(2)^2 - 8 \times 2 + b = 0$

$\Rightarrow 8a + 12 - 16 + b = 0$

$8a + b - 4 = 0$

$8a + b = 4$ ......(II)

From equation (I) & (II)

$\therefore -8a + b = -28$

$8a + b = 4$

$2b = -24$

$b = -12$

$a = 2$

95. (b) $x^2 - 3x + 1 = 0$

$x^2 + 1 = 3x$

Divide by x

$= \frac{x^2}{x} + \frac{1}{x}$

$\Rightarrow \frac{3x}{x}$

$x + \frac{1}{x} = 3$

Cubing both sides

$\Rightarrow x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 27$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$

$x^3 + \frac{1}{x^3} = 18$

96. (a) $x + \frac{1}{4x} = \frac{3}{2}$

Multiply by 2 both sides

$\therefore 2x + \frac{1}{2x} = 3$

Take cube both sides

$= \left(2x + \frac{1}{2x}\right)^3 = (3)^3$

$= 8x^3 + \frac{1}{8x^3} + 3 \times 2x \times \frac{1}{2x}\left(2x + \frac{1}{2x}\right)$

$= 27$

$= 8x^3 + \frac{1}{8x^3} + 3 \times 3 = 27$

$8x^3 + \frac{1}{8x^3} = 27 - 9 = 18$

97. (a) $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$

$\Rightarrow \frac{1}{x+y}$

$= \frac{x+y}{xy}$

$xy = (x+y)^2$

$x^2 + y^2 + 2xy = xy$

$x^2 + y^2 + xy = 0$

$\therefore x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$

$(x^3 - y^3) = (x - y) \times 0 = 0$

98. (c) $x = a(b - c)$

$y = b(c - a)$

$z = c(a - b)$

Let $\frac{x}{a} = b - c$ $\quad \frac{x}{a} = A$

$\frac{y}{b} = c - a$ $\quad \frac{y}{b} = B$

$\frac{z}{c} = a - b$ $\quad \frac{z}{c} = C$

$\therefore A + B + C = b - c + c - a + a - b = 0$

$\therefore A^3 + B^3 + C^3 = 3ABC$

$\therefore \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$

$= 3 \times \frac{x}{a} \times \frac{y}{b} \times \frac{z}{c}$

$= \frac{3xyz}{abc}$

99. (c) $xy(x + y) = 1$

$x + y = \frac{1}{xy}$

Cubing both sides

$\Rightarrow (x+y)^3 = \frac{1}{x^3y^3}$

$x^3 + y^3 + 3xy(x+y) = \frac{1}{x^3y^3}$

$x^3 + y^3 + 3 = \frac{1}{x^3y^3}\left(x + y = \frac{1}{xy}\right)$

$\frac{1}{x^3y^3} - x^3 - y^3 = 3$

100. (d) $x^4 + \frac{1}{x^4} = 119$ $\quad x > 1$

$\therefore x^4 + \frac{1}{x^4} + 2 = 119 + 2 = 121$

$\left(x^2 + \frac{1}{x^2}\right)^2 = (11)^2$

$x^2 + \frac{1}{x^2} = 11$

$x^2 + \frac{1}{x^2} + 2 = 11 + 2$

$\left(x + \frac{1}{x}\right)^2 = 13$

$x + \frac{1}{x} = \sqrt{13}$

$\Rightarrow$ Taking cube both sides

$x^3 + \frac{1}{x^3} + 3\sqrt{13} = \left(\sqrt{13}\right)^3$

$x^3 + \frac{1}{x^3} + 3\sqrt{13} = 13\sqrt{13}$

$x^3 + \frac{1}{x^3} = 10\sqrt{13}$

101. (b) $3x + \frac{1}{2x} = 5$

$\Rightarrow$ Multiply both sides by $\frac{2}{3}$

$\therefore 3x \times \frac{2}{3} + \frac{1}{2x} \times \frac{2}{3} = 5 \times \frac{2}{3}$

$2x + \frac{1}{3x} = \frac{10}{3}$

$\therefore$ Taking cube on both side

$8x^3 + \frac{1}{27x^3} + 3 \times 2x \times \frac{1}{3x}\left(2x + \frac{1}{3x}\right)$

$= \left(\frac{10}{3}\right)^3$

$8x^3 + \frac{1}{27x^3} + 2 \times \frac{10}{3} = \frac{1000}{27}$

$8x^3 + \frac{1}{27x^3} = \frac{1000}{27} - \frac{20}{3}$

$= \frac{1000 - 180}{27} = \frac{820}{27} = 30\frac{10}{27}$

102. (a) $x + y = z$

$x + y - z = 0$

If $a + b + c = 0$

then $a^3 + b^3 + c^3 - 3abc = 0$

$\therefore x^3 + y^3 - z^3 = -3xyz$

$\therefore x^3 + y^3 - z^3 + 3xyz = 0$

$3xyz - 3xyz = 0$

103. (c) $\frac{a}{b} + \frac{b}{a} = 1$

$\therefore a^2 + b^2 = ab$

$\therefore a^2 + b^2 - ab = 0$

$\therefore a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (a + b) \times 0 = 0$

104. (c) $x = 2 - 2^{1/3} + 2^{2/3}$
$x - 2 = 2^{2/3} - 2^{1/3}$ .... (I)
Take cube both sides
$(x-2)^3 = (2^{2/3} - 2^{1/3})^3$
$x^3 - 8 - 6x(x-2) = (2^{2/3})^3 - (2^{1/3})^3 - 3\times 2^{2/3}.2^{1/3}(2^{2/3} - 2^{1/3})$
$x^3 - 8 - 6x^2 + 12x = 2^2 - 2 - 3\times 2^{\frac{2+1}{3}}(x-2)$
From equation (I)
$\therefore\ x^3 - 8 - 6x^2 + 12x = 4 - 2 - 3\times 2(x-2)$
$x^3 - 8 - 6x^2 + 12x = 2 - 6x + 12$
$x^3 + 18x - 6x^2 - 8 - 14 = 0$
$x^3 + 18x - 6x^2 - 22 = 0$
$\therefore x^3 - 6x^2 + 18x + 18 = 22 + 18 = 40$

105. (d) $a^3 - b^3 - c^3 - 3abc = 0$
$\therefore$ a – b – c = 0
a = b + c

106. (c)a = 2.361
b = 3.263
c = 5.624
a + b – c = 0
2.361 + 3.263 – 5.624 = 0
$\therefore a^3 + b^3 - c^3 + 3abc \Rightarrow 0$

107. (d) p = 124
$\sqrt[3]{p\left(p^2+3p+3\right)+1}$
$= \sqrt[3]{p^3+3p^2+3p+1}$
$= \sqrt[3]{(p+1)^3} = \sqrt[3]{(125)^3} = 125$

108. (c) $x + \frac{1}{x} = 2$
(assume $x = 1$, so,1 + 1 =2)
$x^{17} + \frac{1}{x^{19}} = (1)^{17} + \frac{1}{(1)^{19}}$
= 1 + 1 = 2

109. (c) $x : y = 3 : 4$
$\therefore\ \frac{5x-2y}{7x+2y} = \frac{5\times3-2\times4}{7\times3+2\times4}$
$= \frac{15-8}{21+8} = \frac{7}{29}$

110. (a) $x + y = 2z$
$x - z = z - y$
$x - z \Rightarrow -(y-z)$ ....(i)
$\frac{x}{x-z} + \frac{z}{y-z} = \frac{x}{x-z} - \frac{z}{x-z}$
$= \frac{x-z}{x-z} = 1$

111. (b) $a^3b = abc = 180$
or a =1, b = 180
then c = 1
$\therefore$ b = 180

112. (d) $(a-1)\sqrt{2} + 3 = b\sqrt{2} + a$
Comparing Cofficient of $\sqrt{2}$ & constant terms.
$\therefore$ a = 3
$\therefore$ a – 1 = b
3 – 1 = b
b = 2
a + b = 3 + 2 = 5

113. (c) $ax^2 + bx + c = a(x-p)^2$
$ax^2 + bx + c = a(x^2 + p^2 - 2px)$
$\Rightarrow ax^2 + bx + c = ax^2 + ap^2 - 2apx$
Comparing cofficients of x^2 and x
$\Rightarrow$ b = – 2ap
$\Rightarrow p = \frac{-b}{2a}$ ....(i)
and $c = ap^2$
$\Rightarrow c = a \times \frac{b^2}{4a^2}$ (From (i))
$\boxed{4ac = b^2}$

114. (d) a + b + c + d = 1
(1 + a) (1 + b) (1 + c) (1 + d)
$\Rightarrow$ For maximum value a, b, c, d
$a = b = c = d = \frac{1}{4}$
$= \left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right) = \left(\frac{5}{4}\right)^4$

115. (b) $a^2 + b^2 + c^2 + 3 = 2(a + b + c)$
$a^2 + b^2 + c^2 + 3 = 2a + 2b + 2c$
$a^2 - 2a + 1 + b^2 - 2b + 1 + c^2 - 2c + 1 = 0$
$(a-1)^2 + (b-1)^2 + (c-1)^2 = 0$
a = 1
b = 1
c = 1
(a + b + c) $\Rightarrow$ 1 + 1 +1 = 3

116. (c) $x - \frac{1}{x} = 5$
$x^2 + \frac{1}{x^2} - 2 = 25$
$\Rightarrow x^2 + \frac{1}{x^2} = 27$

117. (b) $x = 3 + 2\sqrt{2}$
$x = 2 + 1 + 2\sqrt{2}$
$\left(\sqrt{2}\right)^2 + (1)^2 + 2\times 1\times\sqrt{2}$
$x = \left(\sqrt{2}+1\right)^2$
$\sqrt{x} = \sqrt{\left(\sqrt{2}+1\right)^2}$
$\sqrt{x} = \sqrt{2}+1$
$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1} = \sqrt{2}-1$
$\therefore\ \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{2}+1-\sqrt{2}+1 = 2$

118. (c) Put a = b = 1 and c = – 2
we get a + b + c = 1 + 1 – 2 = 0
0 = 0 (satisfy)
$\therefore\ \frac{a^2+b^2+c^2}{a^2-bc}$
$= \frac{(1)^2+(1)^2+(-2)^2}{(1)^2-(1)(-2)} = \frac{6}{3} = 2$

119. (b) $n = 7 + 4\sqrt{3}$
$n = 4 + 3 + 4\sqrt{3}$
$n = (2)^2 + \left(\sqrt{3}\right)^2 + 2\times2\times\sqrt{3}$
$n = \left(2+\sqrt{3}\right)^2$
$\sqrt{n} = 2 + \sqrt{3}$
$\therefore\ \frac{1}{\sqrt{n}} = 2-\sqrt{3}$
$\sqrt{n} + \frac{1}{\sqrt{n}} = 2+\sqrt{3}+2-\sqrt{3} = 4$

120. (b) $x = \sqrt{3}+\sqrt{2}$
$\frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \sqrt{3}-\sqrt{2}$
$x + \frac{1}{x} = \sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2} = 2\sqrt{3}$

121. (d) p + q = 10 .... (i)
and pq = 5
Squaring both sides of equation (i)
$(p+q)^2 = (10)^2$
$p^2 + q^2 + 2pq = 100$
$p^2 + q^2 + 2\times 5 = 100$
$p^2 + q^2 = 90$
Now,
$\therefore\ \frac{p}{q}+\frac{q}{p} = \frac{p^2+q^2}{pq} = \frac{90}{5} = 18$

122. (d) $x = 3 + 2\sqrt{2}$, $xy = 1$, $y^2 = \frac{1}{x^2}$
$y = \frac{1}{x} = \frac{1}{3+2\sqrt{2}} = 3-2\sqrt{2}$
$\therefore\ x + \frac{1}{x} = 3+2\sqrt{2}+3-2\sqrt{2} = 6$
$\therefore\ x^2 + \frac{1}{x^2} = 36 - 2 = 34$
$= \frac{x^2+3xy+y^2}{x^2-3xy+y^2} = \frac{x^2+\frac{1}{x^2}+3}{x^2+\frac{1}{x^2}-3}$
$= \frac{34+3}{34-3} = \frac{37}{31}$

123. (a) $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$ (given)

$\therefore \frac{x}{b+c} = \frac{y}{c+a}$

$\Rightarrow \frac{x}{b+c} = \frac{x-y}{(b+c)-(c+a)} = \frac{x-y}{b-a}$

Similarly

$\frac{y}{c+a} = \frac{z}{a+b}$

$\therefore \frac{y}{c+a} = \frac{y-z}{(c+a)-(a+b)} = \frac{y-z}{c-b}$ again

$\Rightarrow \frac{z}{a+b} = \frac{x}{b+c}$

$\Rightarrow \frac{z}{a+b} = \frac{z-x}{(a+b)-(b+c)} = \frac{z-x}{a-c}$

$\therefore \frac{x-y}{b-a} = \frac{y-z}{c-b} = \frac{z-x}{a-c}$

124. (d) $x - y = 2,\ xy = 24$ (given)

$x^2 + y^2 - 2xy = 4$

$x^2 + y^2 - 2 \times 24 = 4$

$x^2 + y^2 = 4 + 48 = 52$

125. (a) $\frac{x^2}{y^2} + tx + \frac{y^2}{4}$ (given)

To make it a perfect square it should be in the form

$A^2 + 2AB + B^2 = (A + B)^2$

$= \left(\frac{x}{y}\right)^2 + tx + \left(\frac{y}{2}\right)^2$

$= A^2 + 2AB + B^2$

$A = \frac{x}{y},\ B = \frac{y}{2}$ & $2AB = tx$

So, $tx = 2 \times \frac{x}{y} \times \frac{y}{2}$

$tx = x$

$t = 1$

126. (d) $a = x + y$

$b = x - y$

$c = x + 2y$

$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}((a - b)^2 + (b - c)^2 + (c - a)^2)$

$= \frac{1}{2}((x + y - x + y)^2 + (x - y - x - 2y)^2 + (x + 2y - x - y)^2)$

$= \frac{1}{2}((2y)^2 + (-3y)^2 + y^2)$

$= \frac{1}{2}(4y^2 + 9y^2 + y^2)$

$= \frac{1}{2}(14y^2) = 7y^2$

127. (b) $x + \frac{1}{x} = 2, \quad x \neq 0$

put $x = 1$

$1 + 1 = 2$

$\therefore x^2 + \frac{1}{x^3} = 1 + 1 = 2$

128. (a) $\frac{a}{b} + \frac{b}{a} = 1$

$\frac{a^2+b^2}{ab} = 1$

$a^2 + b^2 - ab = 0$

$\therefore a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$= (a + b) \times 0 = 0$

129. (a) $\left(x + \frac{1}{x}\right)^2 = 3$

$x + \frac{1}{x} = \sqrt{3}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow x^3 + \frac{1}{x^3} = 0$

$\Rightarrow x^6 + 1 = 0$

$\Rightarrow x^6 = -1$

$\Rightarrow x^{72} + x^{66} + x^{54} + x^{24} + x^6 + 1$

$\Rightarrow (x^6)^{12} + (x^6)^{11} + (x^6)^9 + (x^6)^4 + x^6 + 1$

$\Rightarrow (-1)^{12} + (-1)^{11} + (-1)^9 + (-1)^4 + -1 + 1$

$\Rightarrow 1 - 1 - 1 + 1 - 1 + 1 = 0$

130. (b) $a + \frac{1}{a} = \sqrt{3}$

$a^6 = -1$

$\therefore a^6 - \frac{1}{a^6} + 2 = -1 - \frac{1}{(-1)} + 2$

$= -1 + 1 + 2 = 2$

131. (b) $x^3 + y^3 = 35$

$\Rightarrow x + y = 5$

Take cube on both sides,

$(x + y)^3 = (5)^3$

$x^3 + y^3 + 3xy(x + y) = 125$

$35 + 3xy(5) = 125$

$15xy = 125 - 35$

$15xy = 90$

$xy = 6$

$\therefore \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{5}{6}$

132. (b) $a^3 - b^3 = 56$

$\Rightarrow a - b = 2$

$\Rightarrow a^3 - b^3 - 3ab(a - b) = (2)^3$

(By cubing)

$56 - 3ab \times 2 = 8$

$-6ab = 8 - 56$

$6ab = 48$

$\Rightarrow ab = 8$

$(a - b) = 2$

$(a - b)^2 = a^2 + b^2 - 2ab = 4$

$= a^2 + b^2 = 4 + 2ab$

$a^2 + b^2 = 4 + 2 \times 8 = 20$

133. (b) $(a^2 + b^2)^3 = (a^3 + b^3)^2$

$\Rightarrow a^6 + b^6 + 3a^2b^2(a^2 + b^2) = a^6 + b^6 + 2a^3b^3$

$\Rightarrow a^6 + b^6 + 3a^4b^2 + 3a^2b^4 = a^6 + b^6 + 2a^3b^3$

$\Rightarrow 3a^4b^2 + 3a^2b^4 = 2a^3b^3$

$\Rightarrow a^2b^2(3a^2 + 3b^2) = 2a^3b^3$

$3a^2 + 3b^2 = 2ab$

$3(a^2 + b^2) = 2ab$

$\Rightarrow \frac{a^2+b^2}{ab} = \frac{2}{3}$

$\Rightarrow \frac{a}{b} + \frac{b}{a} = \frac{2}{3}$

134. (a) $x + \frac{1}{x} = 5 \qquad x^2 + \frac{1}{x^2} + 2 = 25$

$x^2 + \frac{1}{x^2} = 23$

now, $\frac{x^4+3x^3+5x^2+3x+1}{x^4+1}$

divided by x^2,

$= \frac{\frac{x^4}{x^2} + \frac{3x^3}{x^2} + \frac{5x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{x^4}{x^2} + \frac{1}{x^2}}$

$= \frac{x^2 + 3x + 5 + \frac{3}{x} + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}}$

$= \frac{x^2 + \frac{1}{x^2} + 3\left(x + \frac{1}{x}\right) + 5}{x^2 + \frac{1}{x^2}}$

$= \frac{23+3(5)+5}{23} = \frac{43}{23}$

135. (b) $x^3 + \frac{1}{x^3} = 0$

$\left(x + \frac{1}{x}\right)^3 - 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 0$

$\left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = 0$

$\left(x + \frac{1}{x}\right)^3 = 3\left(x + \frac{1}{x}\right)$

$\left(x + \frac{1}{x}\right)^2 = 3$

$\left(\left(x + \frac{1}{x}\right)^2\right)^2 = (3)^2$

$\left(x + \frac{1}{x}\right)^4 = 9$

136. (c) $x + \frac{1}{x} = 3$

(Squaring both sides)

$x^2 + \frac{1}{x^2} = 7$

On cubing both sides

$$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$

$$x^3 + \frac{1}{x^3} = 18$$

$$\therefore \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) = 18 \times 7$$

$$\left(x^5 + \frac{1}{x^5}\right) + \left(x + \frac{1}{x}\right) = 126$$

$$\left(x^5 + \frac{1}{x^5}\right) + 3 = 126$$

$$\left(x^5 + \frac{1}{x^5}\right) = 123$$

137. (a) $m^4 + \frac{1}{m^4} = 119$

$$m^4 + \frac{1}{m^4} + 2 = 119 + 2$$

$$\left(m^2 + \frac{1}{m^2}\right)^2 = 121$$

$$m^2 + \frac{1}{m^2} = 11$$

$$m^2 + \frac{1}{m^2} - 2 = 11 - 2$$

$$\left(m - \frac{1}{m}\right)^2 = 9$$

$$m - \frac{1}{m} = \pm 3$$

138. (d) $x + y + z = 6$

$(x - 1)^3 + (y - 2)^3 + (z - 3)^3$

$\therefore$ as $x + y + z = 6$

Take values

$x = 1, \ y = 2, \ z = 3$

$(1 + 2 + 3) = 6$

$\therefore (1 - 1)^3 + (2 - 2)^3 + (3 - 3)^2 = 0$

Now assume values in options. option 'd' satisfies the given relation.

Hence 'd' is correct.

139. (b) $a + b + c = 6$

$= a^2 + b^2 + c^2 = 14$

$a^3 + b^3 + c^3 = 36$

Put values as

$a = 1, \quad b = 2, \quad c = 3$

$1 + 2 + 3 = 6$

$1 + 4 + 9 = 14$

$1 + 8 + 27 = 36$

$\therefore \quad abc = 1 \times 2 \times 3 = 6$

Alternate:

$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$36 = 14 + 2(ab + bc + ca)$

$(ab + bc + ca) = 11$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow 36 - 3abc = 6\ (14 - 11)$

$\Rightarrow 36 - 3abc = 6 \times 3$

$-3abc = 18 - 36$

$3abc = 18$

$abc = 6$

140. (a) $a + b = 1$

By cubing

$a^3+b^3 +3ab\ (a + b) = 1^3$

$a^3 + b^3 + 3ab = 1\ (a + b=1)$

$a^3 + b^3 + 3ab = k$

From above both equations

$k = 1$

141. (d) $a = 34, \quad b = 33, \quad c = 33$

$\because a^3 + b^3 + c^3 - 3abc = (a + b + c) \times$

$$\frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$$

$$= (34 + 33 + 33) \times \frac{1}{2}$$

$$\left[(34-33)^2 + (33-33)^2 + (33-34)^2\right]$$

$$= 100 \times \frac{1}{2}(1 + 0 + 1)$$

$= 100 \times 1 = 100$

142. (a) $2^x.2^y = 8 \Rightarrow 2^{x+y} = 2^3$

$x + y = 3$ .....(i)

$9^x.3^y = 81$

$3^{2x}.3^y = 3^4$

$\Rightarrow 2x + y = 4$

.....(ii)

from equation (i) and (ii),

$$\begin{array}{r} x + y = 3 \\ \underline{2x + y = 4} \\ -x = -1 \end{array}$$

$x = 1$

$y = 2$

$\therefore (x, y) = (1, 2)$

143. (b) $x^4 - 2x^2 + k$

$\Rightarrow (A + B)^2 = A^2 + 2AB + B^2$

$(A - B)^2 = A^2 - 2AB + B^2$

$\Rightarrow (x^2)^2 - 2 \times x^2 + k$

$(A)^2 - 2 \times AB + B^2$

$\therefore A = x^2, B = -1$

$B^2 = K$

$(-1)^2 = K$

$K = 1$

144. (c) $$\frac{\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}-3\times\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{5}-\left(\frac{1}{3}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{3}\right)}$$

$$= \frac{\left(\frac{1}{3}\right)^3 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{5}\right)^3 - 3\times\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}}{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{5}\right)^2 - \left(\frac{1}{3}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{3}\right)}$$

$\therefore A^3 + B^3 + C^3 - 3ABC = (A^2 + B^2 + C^2 - AB - BC - CA)(A + B + C)$

Let $\frac{1}{3} = A$

$\frac{1}{4} = B$

$\frac{1}{5} = C$

$$\Rightarrow \frac{(A+B+C)(A^2+B^2+C^2-AB-BC-CA)}{(A^2+B^2+C^2-AB-BC-CA)}$$

$= A + B + C$

$$\therefore A + B + C = \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$

$$= \frac{20 + 15 + 12}{60} = \frac{47}{60}$$

145. (d) $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$

Adding 3 on both sides

$$\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} + 3 = 1 + 3$$

$$\frac{a}{1-a} + 1 + \frac{b}{1-b} + 1 + \frac{c}{1-c} + 1 = 4$$

$$\Rightarrow \frac{a+1-a}{1-a} + \frac{b+1-b}{1-b} + \frac{c+1-c}{1-c} = 4$$

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 4$$

146. (c) $a^2 + b^2 + c^2 = 2\ (a - b - c) - 3$

$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 1 + 1 + 1 = 0$

$\Rightarrow a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$

$\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

$a = 1, b = -1, c = -1$

$\therefore 2a - 3b + 4c = 2 \times 1 - 3 \times -1 + 4 \times -1$

$= 2 + 3 - 4 = 1$

147. (b) $$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$$

$$\Rightarrow \frac{(a-b)^2}{(b-c)(c-a)} \times \frac{(a-b)}{(a-b)} + \frac{(b-c)^2}{(a-b)(c-a)} \times \frac{(b-c)}{(b-c)} + \frac{(c-a)^2}{(a-b)(b-a)} \times \frac{(c-a)}{(c-a)}$$

$$\Rightarrow \frac{(a-b)^3}{(b-c)(c-a)(a-b)} + \frac{(b-c)^3}{(a-b)(c-a)(b-c)} + \frac{(c-a)^3}{(a-b)(b-a)(c-a)}$$

$$= \frac{(a-b)^3+(b-c)^3+(c-a)^3}{(b-c)(c-a)(a-b)}$$

(If $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xy$)

$$= \frac{3(a-b)(b-c)(c-a)}{(b-c)(c-a)(a-b)} = 3$$

148. (c) $(x-3)^2 + (y-5)^2 + (z-4)^2 = 0$

$\therefore$ $(x-3)^2 = 0$ $\quad x = 3$

$(y-5)^2 = 0$ $\quad y = 5$

$(z-4)^2 = 0$ $\quad z = 4$

$$\therefore \frac{x^2}{9} + \frac{y^2}{25} + \frac{c^2}{16}$$

$$\Rightarrow \frac{9}{9} + \frac{25}{25} + \frac{16}{16} = 3$$

149. (d) $x \propto \frac{1}{y^2}$ (Inversely proportional)

$$x = \frac{k}{y^2}$$

($y = 2$) for ($x = 1$) (Given)

$$\therefore 1 = \frac{k}{(2)^2} \Rightarrow 1 = \frac{k}{4}$$

$k = 4$

$\therefore$ For $y = 6$

$$x = \frac{4}{(6)^2} = \frac{1}{9} = \frac{1}{9}$$

150. (d) $x^2 - y^2 = 80$

$\Rightarrow$ $(x - y)(x + y) = 80$

$x - y = 8$ ......(I)

$\therefore$ $(x + y) \times 8 = 80$

$(x + y) = 10$ ......(II)

Now average of x and y

$$= \frac{x+y}{2} = \frac{10}{2}$$

$= 5$

151. (a) Third proportional of a and b $= \frac{b^2}{a}$

Third proportion of $\left(\frac{x}{y}+\frac{y}{x}\right)$ and $\sqrt{x^2+y^2}$

$$\frac{\left(\sqrt{x^2+y^2}\right)^2}{\frac{x}{y}+\frac{y}{x}} = \frac{x^2+y^2}{\frac{x^2+y^2}{xy}} = xy$$

152. (a) $\frac{4+3\sqrt{3}}{7+4\sqrt{3}}$

(By Rationalization of denominator)

$$\frac{4+3\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{(4+3\sqrt{3})(7-4\sqrt{3})}{49-48}$$

$$\begin{array}{r} 4+3\sqrt{3} \\ \times\ 7-4\sqrt{3} \\ \hline -16\sqrt{3}-12\times 3 \\ 28+21\sqrt{3} \\ \hline 28-36+5\sqrt{3} = 5\sqrt{3}-8 \\ \hline \end{array}$$

153. (b) $x = \frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$

$$\frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}} = ?$$

$$\therefore x = \frac{4\times\sqrt{5}\times\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2\times\sqrt{5}\times\sqrt{12}}{\sqrt{5}+\sqrt{3}}$$

or $\frac{2\times\sqrt{20}\times\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$\frac{x}{\sqrt{12}} = \frac{2\sqrt{5}}{\sqrt{5}+\sqrt{3}}$$

by C – D rule

$$\frac{x+\sqrt{12}}{x-\sqrt{12}} = \frac{2\sqrt{5}+\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{5}-\sqrt{3}}$$

$$= \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$\frac{x}{\sqrt{20}} = \frac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$\frac{x+\sqrt{20}}{x-\sqrt{20}} = \frac{2\sqrt{3}+\sqrt{5}+\sqrt{3}}{2\sqrt{3}-\sqrt{5}-\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$

$$\therefore \frac{x+\sqrt{12}}{x-\sqrt{12}} + \frac{x+\sqrt{20}}{x-\sqrt{20}}$$

$$= \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$

$$= \frac{3\sqrt{5}+\sqrt{3}-3\sqrt{3}-\sqrt{5}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{2\sqrt{5}-2\sqrt{3}}{\sqrt{5}-\sqrt{3}} = 2$$

154. (b) $x = 5 - \sqrt{21}$

$2x = 10 - 2\sqrt{21}$ ......(i)

$$\Rightarrow 2x = \left(\sqrt{7}\right)^2 + \left(\sqrt{3}\right)^2 - 2\left(\sqrt{7}\right)\left(\sqrt{3}\right)$$

$$\Rightarrow 2x = \left(\sqrt{7}-\sqrt{3}\right)^2$$

$$\Rightarrow x = \frac{1}{2}\left(\sqrt{7}-\sqrt{3}\right)^2$$

$$\Rightarrow \sqrt{x} = \frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{7}-\sqrt{3}\right)^2}$$

$$= \frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$$

$$\therefore \frac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}\left(\sqrt{32-\left(10-2\sqrt{21}\right)}-\sqrt{21}\right)}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}\left(\sqrt{22+2\sqrt{21}}-\sqrt{21}\right)}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}\left(\sqrt{\left(\sqrt{21}+1\right)^2}-\sqrt{21}\right)}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}\left(\sqrt{21}+1-\sqrt{21}\right)}$$

$$= \frac{\left(\sqrt{7}-\sqrt{3}\right)}{\sqrt{2}}$$

155. (a) $(x^{b+c})^{b-c}\,(x^{c+a})^{c-a}\,(x^{a+b})^{a-b}$ $(x \neq 0)$

$$= x^{b^2-c^2} \cdot x^{c^2-a^2} \cdot x^{a^2-b^2}$$

$$x^{b^2-c^2+c^2-a^2+a^2-b^2} = x^0 = 1$$

156. (c) $\frac{x}{a} = \frac{1}{a} - \frac{1}{x}$

$$\frac{x}{a} - \frac{1}{a} = -\frac{1}{x}$$

$$\left(\frac{x-1}{a}\right) = -\frac{1}{x}$$

$$\frac{1-x}{a} = \frac{1}{x}$$

$x(1-x) = a$

$x - x^2 = a$

157. (c) $x + \frac{1}{x} = 99$

$\therefore$ $x^2 + 1 = 99x$

$2(x^2 + 1) = 2 \times 99x$

$2x^2 + 2 = 198x$

$$= \frac{100x}{2x^2 + 2 + 102x}$$

$$= \frac{100x}{198x + 102x} = \frac{100x}{300x} = \frac{1}{3}$$

158. (c) $\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$

$$\Rightarrow \frac{4x}{x} - \frac{3}{x} + \frac{4y}{y} - \frac{3}{y} + \frac{4z}{z} - \frac{3}{z} = 0$$

$$= 4 - \frac{3}{x} + 4 - \frac{3}{y} + 4 - \frac{3}{z} = 0$$

$$12 - 3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$$

$$-3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = -12$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 4$$

159. (c) $\frac{xy}{x+y} = a$, $\frac{xz}{x+z} = b$, $\frac{yz}{y+z} = c$

Now

$$\Rightarrow \frac{x+y}{xy} = \frac{1}{a}, \frac{x+z}{xz} = \frac{1}{b}, \frac{y+z}{yz} = \frac{1}{c}$$

$$\Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{1}{a}, \frac{1}{z} + \frac{1}{x} = \frac{1}{b}, \frac{1}{z} + \frac{1}{y} = \frac{1}{c}$$

Now we have to find the value of x

$$\therefore \frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{1}{y} + \frac{1}{x} + \frac{1}{z} + \frac{1}{x} - \frac{1}{y} - \frac{1}{z}$$

$$\therefore \frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{2}{x}$$

$$\frac{bc + ac - ab}{abc} = \frac{2}{x}$$

$$x = \frac{2abc}{bc + ac - ab}$$

160. (d) $xy = 8$ Given

So, $(x, y) = (1, 8)$

We have to Questions the options and check them

(8, 1)

(2, 4)

(4, 2)

$\therefore$ $2x + y = 2 \times 1 + 8 = 10$

$2 \times 8 + 1 = 11$

$2 \times 2 + 4 = 8$ minimum

$2 \times 4 + 2 = 10$

Hence in this question we have all the options. So, take all positive factor otherwise we should have to take –ve values also.

$(x, y) = (1, 8)$

(8, 1)

(2, 4)

(4, 2)

(–1, –8)

(–8, –1)

(–2, –4)

(–4, –2)

161. (b) $x^2 + x + 1 = 0$ .....(i)

$$\left(x + \frac{1}{2}\right)^2 + q^2 = 0$$

$$\Rightarrow x^2 + \frac{1}{4} + 2 \times \frac{1}{2} \times x + q^2$$

$$= x^2 + x + \left(q^2 + \frac{1}{4}\right) = 0 \quad \text{......(ii)}$$

Comparing constant term of equation (i) and (ii)

$$\therefore q^2 + \frac{1}{4} = 1$$

$$= q^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$q = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$$

162. (b) $a^2 - 4a - 1 = 0$

$a^2 - 1 = 4a$

$$a - \frac{1}{a} = 4$$

Squaring both sides

$$a^2 + \frac{1}{a^2} - 2 = 16$$

$$a^2 + \frac{1}{a^2} = 18$$

$$\therefore a^2 + \frac{1}{a^2} + 3\left(a - \frac{1}{a}\right)$$

$= 18 + 3 \times 4 = 18 + 12 = 30$

163. (d) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$4x\left(\sqrt{3}x + 2\right) - \sqrt{3}\left(\sqrt{3}x + 2\right) = 0$

$\left(4x - \sqrt{3}\right)\left(\sqrt{3}x + 2\right) = 0$

164. (c) $\sqrt{x} = \sqrt{3} - \sqrt{5}$

$x = 3 + 5 - 2\sqrt{3}\sqrt{5}$

$x = 8 - 2\sqrt{5} \times \sqrt{3}$

$x - 8 = -2\sqrt{15}$

(Squaring both sides)

$x^2 + 64 - 16x = 60$

$x^2 + 4 - 16x = 0$

$x^2 + 6 - 16x = 2$

165. (d) $x + y + z = 0$

$$\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = ?$$

$$\frac{x^3}{xyz} + \frac{y^3}{yxz} + \frac{z^3}{zxy}$$

$$\Rightarrow \frac{x^3 + y^3 + z^3}{zxy}$$

$x + y + z = 0$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0$

$x^3 + y^3 + z^3 = 3xyz$

$$\therefore \frac{3xyz}{zxy} = 3$$

166. (c) $a + b + c = 0$

Have values

$a = 1$, $b = 2$, $c = -3$

$$\Rightarrow \left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right)\left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$$

$$\Rightarrow \left(\frac{1+2}{-3} + \frac{2-3}{1} + \frac{-3+1}{2}\right)\left(\frac{1}{2-3} + \frac{2}{-3+1} + \frac{-3}{1+2}\right)$$

$\Rightarrow (-1 - 1 - 1)(-1 - 1 - 1)$

$\Rightarrow -3 \times -3 = 9$

167. (a) $a + \frac{1}{b} = 1$, $b + \frac{1}{c} = 1$

Values of a, b, c assume

$a = \frac{1}{2}$

$b = 2$

$c = -1$

$\therefore$ $abc = \frac{1}{2} \times 2 \times -1 = -1$

168. (c) $a + b + c = 2s$

$\therefore$ let

$a = 2$

$b = 1$

$c = 1$

$s = 2$

$$\therefore \frac{(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2}{a^2 + b^2 + c^2}$$

$$\Rightarrow \frac{(2-2)^2 + (2-1)^2 + (2-1)^2 + 2^2}{2^2 + 1^2 + 1^2}$$

$$\Rightarrow \frac{0+1+1+4}{4+1+1} = \frac{6}{6} = 1$$

169. (d) $x = 3 + 2\sqrt{2}$

$x^2 = (3 + 2\sqrt{2})^2$

(Squaring both sides)

$= 9 + 8 + 12\sqrt{2}$

$= 17 + 12\sqrt{2}$

$\frac{1}{x^2} = \frac{1}{17+12\sqrt{2}} \times \frac{17-12\sqrt{2}}{17-12\sqrt{2}}$

$= 17 - 12\sqrt{2}$

$\therefore \frac{1}{x^2} + x^2 = 17+12\sqrt{2}+17-12\sqrt{2} = 34$

170. (b) $x\left(3 - \frac{2}{x}\right) = \frac{3}{x}$

$\Rightarrow 3x - 2 = \frac{3}{x}$

$\Rightarrow 3x - \frac{3}{x} = 2$

$\Rightarrow 3\left(x - \frac{1}{x}\right) = 2$

$\Rightarrow x - \frac{1}{x} = \frac{2}{3}$

Squaring both sides

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = \frac{4}{9}$

$\Rightarrow x^2 + \frac{1}{x^2} = \frac{4}{9} + 2 = 2\frac{4}{9}$

171. (a) $x^2 - 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x$

$x + \frac{1}{x} = 3$

Squaring both sides

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 9$

$x^2 + \frac{1}{x^2} = 7$

$\therefore x^2 + \frac{1}{x^2} + x + \frac{1}{x}$

$= 7 + 3 = 10$

172. (c) $a^2 + b^2 = 5ab$

$\Rightarrow \frac{a^2}{ab} + \frac{b^2}{ab} = 5$

$\Rightarrow \frac{a}{b} + \frac{b}{a} = 5$

Squaring both sides

$\left(\frac{a}{b}\right)^2 + \left(\frac{b}{a}\right)^2 + 2 \times \frac{a}{b} \times \frac{b}{a} = 25$

$\frac{a^2}{b^2} + \frac{b^2}{a^2} = 25 - 2 = \mathbf{23}$

173. (d) $xy + yz + zx = 0$

$\therefore xy + zx = -yz$

$\Rightarrow xy + yz = -zx$

$\Rightarrow yz + zx = -xy$

$\therefore \frac{1}{x^2 - yz} + \frac{1}{y^2 - zx} + \frac{1}{z^2 - xy}$

Puting values of $-yz$, $-zx$, $-xy$ from above

$\Rightarrow \frac{1}{x^2 + (xy + zx)} + \frac{1}{y^2 + (xy + yx)}$

$+ \frac{1}{z^2 + (yz + zx)}$

$\Rightarrow \frac{1}{x(x+y+z)} + \frac{1}{y(x+y+z)}$

$+ \frac{1}{z(x+y+z)}$

$\Rightarrow \frac{1}{(x+y+z)}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$

$\Rightarrow \frac{1}{(x+y+z)}\left(\frac{zy + xz + xy}{xyz}\right)$

$\Rightarrow \frac{1}{x+y+z} \times 0 = 0$

174. (c) $a + b + c = 9$

For minimum value $a = b = c$

$\Rightarrow 3a = 9$

$a = \frac{9}{3} = 3$

For minimum value $a = b = c = 3$

$a^2 + b^2 + c^2 = 3^2 + 3^2 + 3^2$

$\Rightarrow 9+9+9 \Rightarrow 27$

175. (d) $a^2 + b^2 + 4c^2 = 2(a + b - 2c) - 3$

$\Rightarrow a^2 + b^2 + 4c^2 - 2a - 2b + 4c + 3 = 0$

$\Rightarrow a^2 - 2a + 1 + b^2 - 2b + 1 + 4c^2 + 4c + 1 = 0$

$(a - 1)^2 + (b - 1)^2 + (2c + 1)^2 = 0$

$\therefore a - 1 = 0 \qquad a = 1$

$b - 1 = 0 \qquad b = 1$

$2c + 1 = 0 \qquad c = \frac{-1}{2}$

$\therefore a^2 + b^2 + c^2 = 1 + 1 + \frac{1}{4} = 2 + \frac{1}{4} = \frac{9}{4} = 2\frac{1}{4}$

176. (b) $4x - y = 2$

$2x - 8y + 4 = 0$

Note:-

For two linear equaltions

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where x and y are variable.

(i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then there will be unique solution.

(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then infinite soultion

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then no solution.

$\therefore \frac{4}{2} \neq \frac{-1}{-8}$

So, the equations have only one solution

177. (d) $\frac{a}{b} = \frac{4}{5}$ and $\frac{b}{c} = \frac{15}{16}$

$\Rightarrow \frac{a}{b} \times \frac{b}{c} = \frac{4}{5} \times \frac{15}{16} = \frac{3}{4}$

$\therefore \frac{a}{c} = \frac{3}{4}$

$\therefore \frac{18c^2 - 7a^2}{45c^2 + 20a^2} = \frac{c^2\left(18 - 7\frac{a^2}{c^2}\right)}{c^2\left(45 + 20\frac{a^2}{c^2}\right)}$

$= \frac{18 - 7\left(\frac{a}{c}\right)^2}{45 + 20\left(\frac{a}{c}\right)^2} = \frac{18 - 7 \times \frac{9}{16}}{45 + 20 \times \frac{9}{16}}$

$= \frac{18 - \frac{63}{16}}{45 + \frac{45}{4}} = \frac{225 \times 4}{16 \times 225} = \frac{1}{4}$

178. (d) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$

Go through options 'd'

take $x = y = z$

$\frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2} = \frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2}$

$\therefore$ Option d is right

179. (b) $x = 3t$, (I)

$y = \frac{1}{2}(t + 1)$

$x = 2y$

$\Rightarrow x = 2 \times \frac{1}{2}(t + 1)$

$x = t + 1$ (II)

∴ 3t = t + 1(from equation (i) and (ii)

2t = 1

$t = \frac{1}{2}$

180. (c) $x^2 + \frac{1}{5}x + a^2$

$A^2 + 2 \times AB + B^2 = (A + B)^2$

$x^2+2\times\frac{1}{10}\times x+a^2=\left(x+\frac{1}{10}\right)^2$

$A = x$

$B = \frac{1}{10}$

$B = a = \frac{1}{10}$

181. (d) $2 - 3x - 4x^2 = 0$

$-4x^2 - 3x + 2 = 0$

$ax^2 + bx + c = 0$

In quadratic equation

(i) When a > 0

Minimum value $= \frac{4ac - b^2}{4a}$

(ii) When a < 0

Maximum value $= \frac{4ac - b^2}{4a}$

∴ In $-4x^2 - 3x + 2$

$a < 0$

∴ Maximum value

= Maximum value

$\frac{4\times -4\times 2-(-3)^2}{4\times -4}$

$= \frac{-32-9}{-16} = \frac{41}{16}$

182. (a) $x^4 - 2x^2 + k$

$\Rightarrow (x^2)^2 - 2 \times x^2 \times 1 + (\sqrt{k})^2$

$\Rightarrow A^2 - 2 \times A \times B + (\sqrt{k})^2$

$\Rightarrow \sqrt{k} = \pm 1$

$k = 1$

183. (d) $\frac{5x}{2x^2+5x+1} = \frac{1}{3}$

$\frac{5}{\frac{2x^2}{x}+\frac{5x}{x}+\frac{1}{x}} = \frac{1}{3}$

$\frac{5}{2x+\frac{1}{x}+5} = \frac{1}{3}$

$\left(2x+\frac{1}{x}+5\right) = 15$

$2x + \frac{1}{x} = 10$

Divide by 2 both sides

$x + \frac{1}{2x} = \frac{10}{2} = 5$

184. (c) $xy\,(x + y) = 1$

$x + y = \frac{1}{xy}$

Cubing both sides

$\Rightarrow (x + y)^3 = \frac{1}{x^3y^3}$

$x^3 + y^3 + 3xy\,(x + y) = \frac{1}{x^3y^3}$

$x^3 + y^3 + 3 = \frac{1}{x^3y^3}\left(x+y=\frac{1}{xy}\right)$

$\frac{1}{x^3y^3} - x^3 - y^3 = 3$

185. (c) $x^2 + \frac{1}{x^2} = 83$

Subtracting 2 from both sides

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 83 - 2$

$\Rightarrow \left(x-\frac{1}{x}\right)^2 = 81$

$\Rightarrow x - \frac{1}{x} = 9$

Take cube on both sides

$x^3 - \frac{1}{x^3} - 3\left(x-\frac{1}{x}\right) = 729$

$x^3 - \frac{1}{x^3} - 3 \times 9 = 729$

$x^3 - \frac{1}{x^3} = 729 + 27 = 756$

186. (d) $\left(a+\frac{1}{a}\right)^2 = 3$

$a + \frac{1}{a} = \sqrt{3}$

Take cube on both sides

$\left(a+\frac{1}{a}\right)^3 = \left(\sqrt{3}\right)^3$

$= a^3 + \frac{1}{a^3} + 3a \times \frac{1}{a}\left(a+\frac{1}{a}\right) = 3\sqrt{3}$

$= a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$

$= a^3 + \frac{1}{a^3} = 0$

187. (b) $\frac{x}{x^2-2x+1} = \frac{1}{3}$

$\Rightarrow \frac{1}{\frac{x^2}{x}-\frac{2x}{x}+\frac{1}{x}} = \frac{1}{3}$

$\Rightarrow \frac{1}{x+\frac{1}{x}-2} = \frac{1}{3}$

$x + \frac{1}{x} - 2 = 3$

$\Rightarrow x + \frac{1}{x} = 3 + 2 = 5$

Taking cube on both sides

$\Rightarrow \left(x+\frac{1}{x}\right)^3 = (5)^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 5 = 125$

$\Rightarrow x^3 + \frac{1}{x^3} = 110$

188. (b) $x + \frac{1}{x} = 4$

Squaring both sides

$x^2 + \frac{1}{x^2} + 2 = 16$

$x^2 + \frac{1}{x^2} = 14$

Squaring again

$x^4 + \frac{1}{x^4} = 196 - 2 = 194$

189. (c) $x + y + z = 6$

$x^2 + y^2 + z^2 = 20$

$\Rightarrow (x + y + z)^2 = (6)^2$

$\Rightarrow x^2 + y^2 + z^2 + 2\,(xy + yz + zx) = 36$

$\Rightarrow 20 + 2\,(xy + yz + zx) = 36$

$\Rightarrow 2(xy + yz + zx) = 16$

$xy + yz + zx = 8$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)\,(x^2 + y^2 + z^2 - xy - zx - yz)$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 6\,(20 - 8)$

$= 6 \times 12 = 72$

190. (b) $x = 1 - \sqrt{2}$

$\frac{1}{x} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$

$= \frac{1+\sqrt{2}}{1-2} = \frac{1+\sqrt{2}}{-1} = -\left(\sqrt{2}+1\right)$

$\therefore\ x - \frac{1}{x} = 1 - \sqrt{2} + \sqrt{2} + 1 = 2$

Take cube

$\left(x - \frac{1}{x}\right)^3 = (2)^3 \Rightarrow \left(x - \frac{1}{x}\right)^3 = 8$

191. (b) $x = a - b$
$y = b - c$
$z = c - a$
$x + y + z = a - b + b - c + c - a = 0$
$\therefore\ x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = 0$

192. (d) $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$\therefore\ x = \frac{1}{y}$

$y = \frac{1}{x} \Rightarrow xy = 1$

$\therefore\ x+y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$\Rightarrow \frac{\left(\sqrt{3}-\sqrt{2}\right)^2 + \left(\sqrt{3}+\sqrt{2}\right)^2}{1}$

$\Rightarrow \frac{3+2-2\sqrt{6}+3+2+2\sqrt{6}}{1} = 10$

$\Rightarrow (x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow (10)^3 = x^3 + y^3 + 3 \times 1(10)$

$\Rightarrow x^3 + y^3 = 1000 - 30 = 970$

193. (c) $2x + \frac{2}{x} = 3$

$\Rightarrow x + \frac{1}{x} = \frac{3}{2}$

Taking cube on both sides

$= \left(x + \frac{1}{x}\right)^3 = \left(\frac{3}{2}\right)^3$

$x + \frac{1}{x} = a$

$x^3 + \frac{1}{x^3} = a^3 - 3a$

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times \frac{3}{2} = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} = \frac{27}{8} - \frac{9}{2}$

$= \frac{27-36}{8} = \frac{-9}{8}$

$\therefore\ x^3 + \frac{1}{x^3} + 2 = \frac{-9}{8} + 2$

$= x^3 + \frac{1}{x^3} + 2 = \frac{-9+16}{8} = \frac{7}{8}$

194. (b) $a + b + c = 15$
$a^2 + b^2 + c^2 = 83$
$\therefore\ (a + b + c)^2 = (15)^2$

$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 225$

$\Rightarrow 83 + 2(ab + bc + ca) = 225$

$\Rightarrow 2(ab + bc + ca) = 225 - 83 = 142$

$ab + bc + ca = 71$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = 15(83 - 71) = 15 \times 12 = 180$

195. (b) $x + \frac{1}{x+1} = 1$

adding (1) both sides

$\therefore\ x + 1 + \frac{1}{x+1} = 1 + 1$

$\Rightarrow (x+1) + \left(\frac{1}{x+1}\right) = 2$

Put $x + 1 = 1$

and $\frac{1}{x+1} = 1$

$\therefore\ (x + 1)^5 + \frac{1}{(x+1)^5}$

$= 1 + 1 = 2$

196. (a) $\frac{1}{a} + \frac{1}{b} = \frac{1}{a+b}$

$\Rightarrow \frac{a+b}{ab} = \frac{1}{a+b}$

$\Rightarrow (a + b)^2 = ab$

$\Rightarrow a^2 + b^2 + 2ab = ab$

$a^2 + b^2 + ab = 0$

$a^3 - b^3 = (a - b)(a^2 - ab + b^2)$

$a^3 - b^3 = 0$

197. (d) If $a + b + c = 0$
then, $a^3 + b^3 + c^3 - 3abc = 0$
$a^3 + b^3 + c^3 = 3abc$

198. (c) $x = y = 333,\ z = 334$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz$

$= \frac{1}{2}(x+y+z)((x-y)^2 + (y-z)^2) + (z-x)^2$

$\Rightarrow \frac{1}{2}(333 + 333 + 334)[(333 + 333)^2 + (333 - 334)^2 + (334 - 333)^2]$

$\Rightarrow \frac{1}{2}(1000)(0 + 1 + 1) = 1000$

199. (a) $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$
Let $a^2 - b^2 = A$
$b^2 - c^2 = B$
$c^2 - a^2 = C$
$A + B + C = a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$
$\therefore\ A^3 + B^3 + C^3 - 3ABC$

$= \frac{1}{2}(A + B + C)[(A - B)^2 + (B - C)^2 + (C - A)^2]$

$A^3 + B^3 + C^3 - 3ABC = 0$
$A^3 + B^3 + C^3 = 3ABC$
Where $A = a^2 - b^2$ etc.
$\therefore\ A^3 + B^3 + C^3 = 3 \times (a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$
Hence $(a + b)(a - b)$ is a factor

200. (b) $a = \frac{b^2}{b-a}$

$\Rightarrow a(b - a) = b^2$
$ab - a^2 = b^2$
$a^2 + b^2 - ab = 0$
$\Rightarrow a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
$\therefore\ a^3 + b^3 = 0$

201. (b) $p - 2q = 4$
Take cube on both sides
$(p - 2q)^3 = (4)^3$
$p^3 - 8q^3 - 3p \times 2q(p - 2q) = 64$
$p^3 - 8q^3 - 6pq \times 4 = 64$
$p^3 - 8q^3 - 24pq = 64$
$p^3 - 8q^3 - 24pq - 64 = 0$

202. (c) $x = -1$

$\frac{1}{x^{99}} + \frac{1}{x^{98}} + \frac{1}{x^{97}} + \frac{1}{x^{96}} + \frac{1}{x^{95}} + \frac{1}{x^{94}} + \frac{1}{x} - 1$

$= \frac{1}{(-1)^{99}} + \frac{1}{(-1)^{98}} + \frac{1}{(-1)^{97}} + \frac{1}{(-1)^{96}} + \frac{1}{(-1)^{95}} + \frac{1}{(-1)^{94}} + \frac{1}{(-1)} - 1$

$= -1 + 1 - 1 + 1 - 1 + 1 + \frac{1}{-1} - 1 = -2$

203. (a) $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$

$\Rightarrow \frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = \frac{1}{\left(2^{\frac{1}{3}}\right)^2 + 2^{\frac{1}{3}} + (1)^2}$

$\Rightarrow (\therefore A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$\therefore$ Put, $A = 2^{\frac{1}{3}}$, $B = 1$

$= \frac{\left(2^{\frac{1}{3}} - 1\right)}{\left(2^{\frac{1}{3}} - 1\right)\left(\left(2^{\frac{1}{3}}\right)^2 + 2^{\frac{1}{3}} + (1)^2\right)}$

$= \frac{\left(2^{\frac{1}{3}} - 1\right)}{\left(2^{\frac{1}{3}}\right)^3 - (1)^3} = \left(2^{\frac{1}{3}} - 1\right)$

$2^{\frac{1}{3}} - 1 = a\left(2^{\frac{2}{3}}\right) + b\,(2)^{\frac{1}{3}} + c$

(Comparing the terms)

a = 0

b = 1

c = –1

∴ a + b + c = 0 + 1 – = 0

204. (d) $x = \sqrt[3]{2+\sqrt{3}}$

$x^3 = 2 + \sqrt{3}$

$\frac{1}{x^3} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

⇒ $\frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

∴ $x^3 + \frac{1}{x^3} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

205. (d) $x = \sqrt[3]{5} + 2$

⇒ $x - 2 = \sqrt[3]{5}$

Take cube on both sides

⇒ $(x-2)^3 = \left(\sqrt[3]{5}\right)^3$

⇒ $x^3 - 8 - 3 \times 2 \times x\,[x-2] = 5$

⇒ $x^3 - 8 - 6x^2 + 12x = 5$

∴ $x^3 - 6x^2 + 12x - 13 = 0$

206. (b) $\frac{p^2-p}{2p^3+p^2} + \frac{p^2-1}{p^2+3p} + \frac{p^2}{p+1}$

In such type of question a ssume values of p.

∴ Let p = 1

∴ $\frac{1-1}{2+1} + \frac{1-1}{1+3} + \frac{1}{1+1}$

$= 0 + 0 + \frac{1}{2} = \frac{1}{2}$

Now check options (b)

$\frac{1}{2p^2} = \frac{1}{2}$

Hence option (b) is Answer.

207. (b) $x + \frac{1}{x} = 2$

⇒ Put x = 1

∴ $1 + \frac{1}{(1)} = 2$

2 = 2 (satisfy)

$\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)$

= (1 + 1) (1 + 1)

= 2 × 2 = 4

208. (a) a, b, c, are +ve integers

So, minimum value is $a=b=c=1$

∴ Putting the value of x in equation

$a^3 + b^3 + c^3 - 3abc$

= 1 + 1 + 1 – 3 × 1× 1× 1 =0

Hence minimum value is 0.

209. (d) $f(x) = 12x^3 - 13x^2 - 5x + 7$

If we divide $f(x)$ by

⇒ $(3x + 2)$ then

$3x + 2 = 0$

$x = -\frac{2}{3}$

∴ $f\left(\frac{-2}{3}\right) = 12\left(\frac{-2}{3}\right)^3 - 13\left(\frac{-2}{3}\right)^2$

$- 5\left(\frac{-2}{3}\right) + 7$

$= -12 \times \frac{8}{27} - \frac{52}{9} + \frac{10}{3} + 7$

$= \frac{-96-156+90+189}{27}$

$= \frac{-252+279}{27} = \frac{27}{27} = 1$

210. (b) $2x^2 - 7x + 12 = 0$

roots are α, β

∴ $\alpha\beta = +\frac{c}{a},\ \alpha + \beta = \frac{-b}{a}$

∴ $\alpha+\beta = +\frac{7}{2},\ \alpha\beta = \frac{12}{2} = 6$

∴ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{7}{2}\right)^2 - 2\times 6}{6} = \frac{\frac{49}{4}-12}{6}$

$= \frac{49-48}{6\times 4} = \frac{1}{24}$

211. (c) $x^3 + \frac{3}{x} = 4(a^3 + b^3)$

and $3x + \frac{1}{x^3} = 4(a^3 + b^3)$

∴ $x^3 + \frac{3}{x} = 3x + \frac{1}{x^3}$

⇒ $x^3 - \frac{1}{x^3} = 3x - \frac{3}{x}$

$\left(x^3 - \frac{1}{x^3}\right) = 3\left(x - \frac{1}{x}\right)$

⇒ $\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)$

$= 3\left(x-\frac{1}{x}\right)$

$x^2 + 1 + \frac{1}{x^2} = 3$

$x^2 + \frac{1}{x^2} = 2$ $\boxed{x=1}$

$4(a^3 + b^3) = 1 + 3$

⇒ $a^3 + b^3 = 1$

Let a = 1 b = 0

∴ $a^2 - b^2 = 1 - 0 = 1$

or you can directly put the value of x also,

212. (a) $121a^2 + 64b^2$

$= (11a)^2 + (8b)^2 + 2 \times 11a \times 8b$

$= (11a + 8b)^2$

∴ So term added to make perfect square = 176 ab

213. (b) $a = 2 + \sqrt{3}$

⇒ $a^2 = \left(2+\sqrt{3}\right)^2$

$= 4 + 3 + 4\sqrt{3}$

$= 7 + 4\sqrt{3}$

⇒ $\frac{1}{a^2} = \frac{1}{7+4\sqrt{3}}$

$= \frac{7-4\sqrt{3}}{\left(7+4\sqrt{3}\right)\left(7-4\sqrt{3}\right)}$

$= \frac{7-4\sqrt{3}}{1}$

∴ $a^2 + \frac{1}{a^2} = 7 + 4\sqrt{3} + 7 - 4\sqrt{3}$

= 14

214. (a) $p + \frac{1}{4} + \sqrt{p} + k^2$

⇒ $p + \sqrt{p} + \left(k^2+\frac{1}{4}\right)$

⇒ $\left(\sqrt{p}\right)^2 + 2\times\frac{1}{2}\times\sqrt{p} + \left(k^2+\frac{1}{4}\right)$

⇒ $A^2 + 2 \times A \times B + B^2$

⇒ $A = \sqrt{p}\quad B^2 = \left(k^2+\frac{1}{4}\right)$

$B = \frac{1}{2}$

∴ $k^2 + \frac{1}{4} = \left(\frac{1}{2}\right)^2$

$k^2 + \frac{1}{4} = \frac{1}{4}$

$k^2 = 0$

k = 0

215. (a) Reciprocal of $\left(x+\frac{1}{x}\right)$

$= \frac{1}{\left(x+\frac{1}{x}\right)} = \frac{x}{x^2+1}$

216. (a) For minimum value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

a = b = c

a + b + c = 1 (given)

$\therefore$ a = b = c = $\frac{1}{3}$

$\frac{1}{a} = \frac{1}{b} = \frac{1}{c} = 3$

$\therefore$ Minimum value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

= 3 + 3 + 3 = 9

217. (b) $a\left(2+\sqrt{3}\right) = b\left(2-\sqrt{3}\right) = 1$

$a = \frac{1}{2+\sqrt{3}}$ $\quad b = \frac{1}{2-\sqrt{3}}$

$\Rightarrow a = \frac{1}{b}$

$\Rightarrow \frac{1}{a^2+1} + \frac{1}{b^2+1}$

$\Rightarrow \frac{1}{\frac{1}{b^2}+1} + \frac{1}{b^2+1}$

$\Rightarrow \frac{1}{\frac{1+b^2}{b^2}} + \frac{1}{b^2+1}$

$\Rightarrow \frac{b^2}{b^2+1} + \frac{1}{b^2+1}$

$\Rightarrow \frac{b^2+1}{b^2+1} = 1$

218. (d) $\left(2+\sqrt{3}\right)a = \left(2-\sqrt{3}\right)b = 1$

$\Rightarrow \frac{1}{a} = \left(2+\sqrt{3}\right)$

By rationals

$\Rightarrow \frac{1}{b} = \left(2-\sqrt{3}\right)$

$\Rightarrow \frac{1}{a}+\frac{1}{b} = 2-\sqrt{3}+2+\sqrt{3} = 4$

219. (a) $a+\frac{1}{b} = b+\frac{1}{c} = c+\frac{1}{a}$

To save your time assume values of a, b, c according to equation.

Let a = 2, b = – 1 & c = $\frac{1}{2}$

$2+\frac{1}{-1} = -1+\frac{1}{1/2} = \frac{1}{2} + \frac{1}{2}$

= 1 = 1 = 1

$\therefore abc = 2 \times -1 \times \frac{1}{2} = -1$

220. (d) $(x-2)$ is a factor of $x^2+3Qx-2Q$

for $(x-2) = 0$

$x^2 + 3Qx - 2Q = 0$

$\Rightarrow 4 + 3 \times Q \times 2 - 2 \times Q = 0$

$\Rightarrow 4 + 6Q - 2Q = 0$

$4Q = -4$

$Q = -1$

221. (d) a + b = 12 .... (I)

ab = 22 ...... (II)

Squaring both sides of equation (I)

$\Rightarrow a^2 + b^2 + 2ab = 144$

$a^2 + b^2 + 2 \times 22 = 144$

$a^2 + b^2 = 144 - 44 = 100$

222. (b) $x = \sqrt{3} - \frac{1}{\sqrt{3}}$ & $y = \sqrt{3} + \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{x^2}{y} + \frac{y^2}{x} = \frac{x^3+y^3}{xy}$

$= \frac{(x+y)(x^2-xy+y^2)}{xy}$

$\therefore x+y = \sqrt{3} - \frac{1}{\sqrt{3}} + \sqrt{3} + \frac{1}{\sqrt{3}} = 2\sqrt{3}$

$\therefore xy = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$

$= 3 - \frac{1}{3} = \frac{8}{3}$

$\Rightarrow \frac{(x+y)(x^2+y^2+2xy-2xy-xy)}{xy}$

$\Rightarrow \frac{(x+y)\left((x+y)^2-3xy\right)}{xy}$

$\Rightarrow \frac{2\sqrt{3}\left(\left(2\sqrt{3}\right)^2 - 3 \times \frac{8}{3}\right)}{\frac{8}{3}}$

$\Rightarrow \frac{2\sqrt{3}(12-8)}{\frac{8}{3}} \Rightarrow \frac{2 \times 3\sqrt{3}(4)}{8} = 3\sqrt{3}$

223. (b) $x^2 + ax + b$

$\Rightarrow x^2 + 2 \times \frac{1}{2} a \times x + \left(\sqrt{b}\right)^2$

$\Rightarrow A^2 + 2 \times A \times B + B^2 = (A + B)^2$

$\therefore A = x,\ B = \frac{1}{2}a,\ B^2$

$= \left(\sqrt{b}\right)^2,\ B = \sqrt{b}$

$\Rightarrow \left(x+\frac{1}{2}a\right)^2$ be perfect square

at $\sqrt{b} = \frac{1}{2}a$ $\quad b = \frac{1}{4}a^2$

$\therefore a^2 = 4b$

224. (a) a + b + c + d = 4

$\frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-a)(1-b)(1-c)}$

$+ \frac{1}{(1-c)(1-d)(1-a)} + \frac{1}{(1-b)(1-c)(1-d)}$

Put a = 0, b = 0 and c = 2 and d = 2

a + b + c + d = 0 + 0 + 2 + 2 = 4 = 4 (satisfy)

$\frac{1}{(1-0)(1-0)(1-2)} + \frac{1}{(1-0)(1-2)(1-2)}$

$+ \frac{1}{(1-2)(1-2)(1-0)} + \frac{1}{(1-2)(1-0)(1-0)}$

$\Rightarrow \frac{1}{-1} + \left(\frac{1}{+1}\right) + \frac{1}{-1\times-1} + \frac{1}{-1}$

$\Rightarrow -1 + 1 + 1 - 1 = 0$

225. (b) $x - \frac{1}{x} = 1$

$\Rightarrow \frac{x^4 - \frac{1}{x^2}}{3x^2+5x-3}$

divide and multiply by x

$\Rightarrow \frac{\frac{x^4}{x} - \frac{1}{x^3}}{\frac{3x^2}{x} + \frac{5x}{x} - \frac{3}{x}}$

$= \frac{x^3 - \frac{1}{x^3}}{3x - \frac{3}{x} + 5} \Rightarrow \frac{x^3 - \frac{1}{x^3}}{3\left(x-\frac{1}{x}\right)+5}$

$\Rightarrow x - \frac{1}{x} = 1$

Take cube on both sides

$\left(x-\frac{1}{x}\right)^3 = (1)^3$

$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x-\frac{1}{x}\right) = 1$

$x^3 - \frac{1}{x^3} - 3 = 1$

$\Rightarrow x^3 - \frac{1}{x^3} = 4$

$= \frac{x^3 - \frac{1}{x^3}}{3\left(x-\frac{1}{x}\right)+5} = \frac{4}{3\times1+5} = \frac{4}{8} = \frac{1}{2}$

226. (d) $x + y = 15$

$\Rightarrow x - 10 = 5 - y$

$x - 10 = -(y - 5)$

Take cube on both sides

$\Rightarrow (x - 10)^3 = (-(y - 5))^3$

$\Rightarrow (x - 10)^3 + (y - 5)^3 = 0$

227. (b) $x^2 + \frac{1}{x^2} = 66$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 66 - 2$

$\left(x - \frac{1}{x}\right)^2 = 64$

$\left(x - \frac{1}{x}\right)^2 = (8)^2$

$\Rightarrow x - \frac{1}{x} = \pm 8$

$\Rightarrow \frac{x^2 - 1 + 2x}{x} = \frac{\frac{x^2}{x} - \frac{1}{x} + \frac{2x}{x}}{\frac{x}{x}}$

$\Rightarrow \frac{\left(x - \frac{1}{x}\right) + 2}{1}$

When $x - \frac{1}{x} = +8$

Then $\left(x - \frac{1}{x}\right) + 2 = 8 + 2 = 10$

When $x - \frac{1}{x} = -8$

$-8 + 2 = -6$

$\therefore$ (10, – 6)

228. (c) $a^2 + a + 1 = 0$

$\left[\begin{array}{l} a^3 + 1^3 = (a+1)(a^2 - a + 1) \\ a^3 - 1 = (a-1)(a^2 + 1 + a) \end{array}\right]$

$\therefore (a^3 - 1) = (a - 1) \times 0$

$a^3 - 1 = 0$

$a^3 = 1$

$(a^3)^3 = 1^3$

$a^9 = 1$

229. (a) $x + \frac{2}{x} = 1$

$x^2 + 2 = x$

$x^2 - x = -2$

$x - x^2 = 2$

$\therefore \frac{x^2 + x + 2}{x^2(1 - x)}$ = divide & mulitply by x

$\Rightarrow \frac{\frac{x^2}{x} + \frac{x}{x} + \frac{2}{x}}{\frac{x^2}{x}(1 - x)} = \frac{x + \frac{2}{x} + 1}{x(1 - x)}$

$\Rightarrow \frac{x + \frac{2}{x} + 1}{x - x^2} = \frac{1 + 1}{2} = 1$

230. (b) $y = 1 - 3k$ and $x = -2k$ (given)

$\therefore$ For $x = y$

$-2k = 1 - 3k$

$k = 1$

231. (b) $\sqrt{(x^2 + y^2 + z)(x + y - 3z)} + \sqrt[3]{xy^3z^2}$

$x = 1$ $y = -3$ $z = -1$ (given)

$\Rightarrow \sqrt{(1 + 9 - 1)(1 - 3 + 3)} + \sqrt[3]{1 \times (-3)^3 \times 1}$

$= 3 + (-3) = 0$

232. (b) $x + \frac{1}{x} = 2$ ....(I)

Squaring both sides

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4$

$\Rightarrow x^2 + \frac{1}{x^2} = +2$

Cubing equation (I)

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8$

$x^3 + \frac{1}{x^3} + 6 = 8$

$x^3 + \frac{1}{x^3} = 2$

$\therefore \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) = 2 \times 2 = 4$

233. (a) $x + \frac{1}{x} = 5$

$\therefore$ Take cube on both sides

$\Rightarrow \left(x + \frac{1}{x}\right)^3 = (5)^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 5 = 125$

$\Rightarrow x^3 + \frac{1}{x^3} = 110$

$\therefore$ Squaring both sides

$\left(x^3 + \frac{1}{x^3}\right)^2 = (110)^2$

$\Rightarrow x^6 + \frac{1}{x^6} + 2 = 12100$

$x^6 + \frac{1}{x^6} = 12100 - 2 = 12098$

234. (c) $x^2 - 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x$

$x + \frac{1}{x} = 3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$

$x^3 + \frac{1}{x^3} = 18$

$\Rightarrow \frac{x^6 + x^4 + x^2 + 1}{x^3}$

$\Rightarrow \frac{x^6}{x^3} + \frac{x^4}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3}$

$\Rightarrow x^3 + \frac{1}{x^3} + \frac{1}{x} + x$

$\Rightarrow 18 + 3 = 21$

235. (b) $\frac{(x+1)^3 - (x-1)^3}{(x+1)^2 - (x-1)^2} = 2$

$\Rightarrow A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$A^2 - B^2 = (A - B)(A + B)$

$\Rightarrow \frac{(x+1-x+1)\left((x+1)^2 + (x-1)(x+1) + (x-1)^2\right)}{(x+1-x+1)(x+1+x-1)}$

$= 2$

$\Rightarrow \frac{(x^2 + 1 + 2x + x^2 - 1 + x^2 + 1 - 2x)}{(2x)} = 2$

$\Rightarrow \frac{3x^2 + 1}{2x} = 2$

$\Rightarrow 3x^2 + 1 = 4x$

$3x^2 - 4x + 1 = 0$

$3x^2 - 3x - x + 1 = 0$

$3x(x - 1) - 1(x - 1) = 0$

$(3x - 1)(x - 1) = 0$

$\Rightarrow 3x - 1 = 0$

$x = \frac{1}{3}$

$\Rightarrow x - 1 = 0$

For $x = 1 = \frac{1}{1}$

By adding numinator and denominator

$1 + 1 = 2$

No option is satisfied

$\therefore x = \frac{1}{3}$

$1 + 3 = 4$

236. (c) $x = \sqrt{5} + 2$

$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{5} + 2} \times \frac{\sqrt{5} - 2}{\sqrt{5} - 2}$

$\Rightarrow \frac{\sqrt{5} - 2}{5 - 4} = \sqrt{5} - 2$

$\Rightarrow x - \frac{1}{x} = \sqrt{5} + 2 - \sqrt{5} + 2 = 4$

$\therefore \frac{2x^2 - 3x - 2}{3x^2 - 4x - 3}$

$$= \frac{\frac{2x^2}{x} - \frac{2}{x} - \frac{3x}{x}}{\frac{3x^2}{x} - \frac{3}{x} - \frac{4x}{x}} \Rightarrow \frac{2x - \frac{2}{x} - 3}{3x - \frac{3}{x} - 4}$$

$$= \frac{2\left(x - \frac{1}{x}\right) - 3}{3\left(x - \frac{1}{x}\right) - 4} \Rightarrow \frac{2 \times 4 - 3}{3 \times 4 - 4}$$

$$= \frac{8-3}{12-4} = \frac{5}{8} = 0.625$$

237. (b) a = 2.234
b = 3.121
c = –5.355
∴ a + b + c = 0
$a^3 + b^3 + c^3 - 3abc = (a + b + c)$
$(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

238. (d) $x^2 + y^2 + 1 = 2x$
$x^2 - 2x + 1 + y^2 = 0$
$(x - 1)^2 + y^2 = 0$
If $A^2 + B^2 = 0$

[As powers are even it can possible only when A = 0 & B = 0]

∴ $x - 1 = 0$
$x = 1$
$y = 0$
∴ $x^3 + y^5 = 1 + 0 = 1$

239. (a) $3(a^2 + b^2 + c^2) = (a + b + c)^2$
by options a = b = c
$3(a^2 + a^2 + a^2) = 9a^2$
$\Rightarrow 9a^2 = 9a^2$

240. (a) $x(x-3) = -1$

$\Rightarrow (x - 3) = \frac{-1}{x}$

Taking cube on both sides

$\Rightarrow (x - 3)^3 = \left(\frac{-1}{x}\right)^3$

$\Rightarrow x^3 - 27 - 9x(x - 3) = \frac{-1}{x^3}$

$\Rightarrow x^3 - 27 - 9 \times -1 = \frac{-1}{x^3}$

$\Rightarrow x^3 - 27 + 9 = \frac{-1}{x^3}$

$\Rightarrow x^3 - 18 = \frac{-1}{x^3}$

$\Rightarrow x^3(x^3 - 18) = -1$

241. (a) $a^2 + 4b^2 + 4b - 4ab - 2a - 8$
$= a^2 - 4ab + 4b^2 - 2a + 4b - 8$
$= (a - 2b)^2 - 2(a - 2b) - 8$
Put t = a – 2b
$= t^2 - 2t - 8$
$= t^2 - 4t + 2t - 8$
$= t(t - 4) + 2(t - 4)$
$= (t + 2)(t - 4)$
$= (a - 2b - 4)(a - 2b + 2)$
(Put the value of assume t)

242. (d) $\frac{1}{a^2 + ax + x^2} - \frac{1}{a^2 - ax + x^2} + \frac{2ax}{a^4 + a^2x^2 + x^4}$

$= \frac{a^2 - ax + x^2 - a^2 - ax - x^2}{(a^2 + x^2 + ax)(a^2 + x^2 - ax)} + \frac{2ax}{a^4 + a^2x^2 + x^4}$

$= \frac{-2ax}{(a^2 + x^2)^2 - (ax)^2} + \frac{2ax}{a^4 + x^4 + a^2x^2}$

$= \frac{-2ax}{a^4 + x^4 + 2x^2a^2 - a^2x^2} + \frac{2ax}{a^4 + x^4 + a^2x^2}$

$= \frac{-2ax}{a^4 + x^4 + x^2a^2} + \frac{2ax}{a^4 + x^4 + a^2x^2} = 0$

243. (b) $x = 11$
$x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$
$= x^5 - 11x^4 - x^4 + 11x^3 + x^3 - 11x^2 - x^2 + 11x + x - 1$
$= (11)^5 - 11 \times (11)^4 - (11)^4 + 11 \times (11)^3 + 11^3 - 11 \times (11)^2 - (11 \times 11) + (11 \times 11) + (11) - 1$
$= 0 - 0 + 0 + 0 + 11 - 1 = 10$

244. (c) p = 99
$p(p^2 + 3p + 3)$
$\Rightarrow p^3 + 3p^2 + 3p + 1 - 1$
$(p + 1)^3 - 1$
$\Rightarrow (99 + 1)^3 - 1$
$\Rightarrow (100)^3 - 1$
$\Rightarrow 1000000 - 1 = 99999$

245. (c) From option (c) LHS $(x + 2)^2 = x^2 + 4x + 4$
RHS = $x^2 + 2x + 4$
∴ LHS ≠ RHS

246. (a) $\left(a + \frac{1}{a}\right)^2 = 3$

$\Rightarrow a + \frac{1}{a} = \sqrt{3}$

Cube on both sides

$\left(a + \frac{1}{a}\right)^3 = \left(\sqrt{3}\right)^3$

$\Rightarrow a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} = 0$

247. (a) $a + \frac{1}{a} = \sqrt{3}$

Take cube on both sides

$\Rightarrow a^3 + \frac{1}{a^3} + 3a \times \frac{1}{a}\left(a + \frac{1}{a}\right) = \left(\sqrt{3}\right)^3$

$\Rightarrow a^3 + \frac{1}{a^3} + 3 \times \sqrt{3} \Rightarrow 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} = 0$

$\Rightarrow a^6 + 1 = 0$
$\Rightarrow a^6 = -1$
$= a^{18} + a^{12} + a^6 + 1$
$= (-1)^3 + (-1)^2 - 1 + 1$
$= -1 + 1 - 1 + 1 = 0$

248. (d) $x^2 + y^2 + z^2 - xy - xz - yz = \frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}[(997 - 998)^2 + (998 - 999)^2 + (999 - 997)^2]$

$= \frac{1}{2}(1 + 1 + 4) = 3$

249. (c) $x + \frac{1}{x} = 3$

$\frac{3x^2 + 3 - 4x}{x^2 + 1 - x} = \frac{\frac{3x^2}{x} + \frac{3}{x} - 4}{\frac{x^2}{x} + \frac{1}{x} - \frac{x}{x}} = \frac{3\left(x + \frac{1}{x}\right) - 4}{\left(x + \frac{1}{x}\right) - 1}$

$= \frac{3 \times 3 - 4}{3 - 1} = \frac{9 - 4}{3 - 1} = \frac{5}{2}$

250. (d) $x = 3 + 2\sqrt{2}$

∴ $\frac{1}{x} = 3 - 2\sqrt{2}$

$x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2}$

$x + \frac{1}{x} = 6$

$\left(x + \frac{1}{x}\right)^3 = (6)^3$

$x^3 + \frac{1}{x^3} + 3 \times 6 = 216$

$x^3 + \frac{1}{x^3} = 216 - 18 = 198$

∴ $\frac{x^6 + x^4 + x^2 + 1}{x^3}$

$= x^3 + x + \frac{1}{x} + \frac{1}{x^3}$

$= x^3 + \frac{1}{x^3} + x + \frac{1}{x}$

$= 198 + 6 = 204$

251. (c) $(a + b - c)^2 + (b + c - a)^2 + (c + a - b)^2 = ?$
$\Rightarrow a + b + c = 0$ (given)
$\Rightarrow a + b = -c$
$\Rightarrow b + c = -a$

$\Rightarrow$ a + c = –b

$\Rightarrow$ $(a + b - c)^2 + (b + c - a)^2 + (c + a - b)^2$

$\Rightarrow$ $(-c - c)^2 + (-a - a)^2 + (-b - b)^2$

$\Rightarrow$ $(-2c)^2 + (-2a)^2 + (-2b)^2$

$\Rightarrow$ $4c^2 + 4a^2 + 4b^2$

$\Rightarrow$ $4(a^2 + b^2 + c^2)$

252. (b) $p^3 + 3p^2 + 3p - 7 = 0$

$p^3 + 3p^2 + 3p + 1 = 7 + 1$

= $(p + 1)^3 = (2)^3$

= p + 1 = 2

$\Rightarrow$ p = 1

$\therefore$ $p^2 + 2p = 1 + 2 = 3$

253. (a) $x = 2015$

y = 2014

z = 2013

= $x^2 + y^2 + z^2 - xy - yz - zx = \frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$

= $\frac{1}{2}[(2015 - 2014)^2 + (2014 - 2013)^2 + (2013 - 2015)^2]$

= $\frac{1}{2}(1 + 1 + 4) = 3$

254. (a) $3a^2 = b^2$ (given)

$$\frac{(a+b)^3 - (a-b)^3}{(a+b)^2 + (a-b)^2}$$

$$\Rightarrow \frac{a^3 + b^3 + 3ab(a+b) - (a^3 - b^3 - 3ab(a-b))}{a^2 + b^2 + 2ab + a^2 + b^2 - 2ab}$$

$$\Rightarrow \frac{2b^3 + 6a^2b}{2a^2 + 2b^2} \Rightarrow \frac{b^3 + 3a^2b}{a^2 + b^2}$$

$$\Rightarrow \frac{b^3 + b^3}{\frac{b^2}{3} + b^2} = \frac{2b^3}{b^2\left(\frac{1}{3} + 1\right)}$$

$$\Rightarrow \frac{2b}{\frac{4}{3}} = \frac{3b}{2}$$

255. (a) $x + \frac{1}{x} = 2\frac{1}{12} = \frac{25}{12}$

$$x^2 + \frac{1}{x^2} + 2 = \frac{675}{144}$$

$$x^2 + \frac{1}{x^2} = \frac{625}{144} - 2$$

$$x^2 + \frac{1}{x^2} = \frac{625 - 288}{144}$$

$$x^2 + \frac{1}{x^2} = \frac{337}{144}$$

$$x^2 + \frac{1}{x^2} - 2 = \frac{337}{144} - 2$$

$$\left(x - \frac{1}{x}\right)^2 = \frac{337 - 288}{144} = \frac{49}{144}$$

$$x - \frac{1}{x} = \frac{7}{12}$$

$$\therefore \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = \frac{25}{12} \times \frac{7}{12} = \frac{175}{144}$$

$$\therefore \left(x^2 - \frac{1}{x^2}\right) = \frac{175}{144}$$

$$\therefore x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right) =$$

$$\frac{175}{144} \times \frac{337}{144} = \frac{58975}{20736}$$

256. (c) $x = 9999$

$$\frac{4x^3 - x}{(2x+1)(6x-3)} = \frac{x(4x^2 - 1)}{3(2x+1)(2x-1)}$$

$$= \frac{x(4x^2 - 1)}{3(4x^2 - 1)} = \frac{x}{3}$$

$$\therefore \frac{9999}{3} = 3333$$

257. (b) $a^3 + b^3 = 9$

a + b = 3

Assume values, a = 2, b = 1

$\therefore$ $(2)^3 + 1 = 9$

2 + 1 = 3

$$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2} + 1 = \frac{3}{2}$$

258. (c) $t^2 - 4t + 1 = 0$

$t^2 + 1 = 4t$

$$\frac{t^2 + 1}{t} = \frac{4t}{t}$$

$$t + \frac{1}{t} = 4$$

[take cube both sides]

$$t + \frac{1}{t} + 3t\,\frac{1}{t}\left(t + \frac{1}{t}\right) = 64$$

$$t^3 + \frac{1}{t^3} = 64 - 12 = 52$$

$$t^3 + \frac{1}{t^3} = 52$$

259. (d) $\sqrt[3]{a} + \sqrt[3]{b} = \sqrt[3]{c}$

Take cube both sides

$$\left(\sqrt[3]{a} + \sqrt[3]{b}\right)^3 = \left(\sqrt[3]{c}\right)^3$$

$$\Rightarrow a + b + 3a^{\frac{1}{3}}b^{\frac{1}{3}}\left(\sqrt[3]{a} + \sqrt[3]{b}\right) = c$$

$$\Rightarrow a + b + 3\,a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}} = c$$

$$\Rightarrow a + b - c = -3\,a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}$$

Again take cube both sides

$\Rightarrow$ $(a + b - c)^3 = -27\,abc$

$\Rightarrow$ $(a + b - c)^3 + 27abc = 0$

260. (b) 4x + 5y = 83

3x : 2y = 21 : 22

x: y = 7 : 11

let x = 7 and y = 11

y – x = 11 – 7

= 4

261. (c) $x = \sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - \sqrt{a^3 + b^3}}$

Take cube on both sides

$x^3 = (a + \sqrt{a^2 + b^3}) + (a - \sqrt{a^2 + b^3}) + 3\left(\sqrt{a + \sqrt{a^2 + b^3}}\right)^{\frac{1}{3}}\left(\sqrt{a - \sqrt{a^2 + b^3}}\right)^{1/3}$

$\left(\sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - \sqrt{a^3 + b^3}}\right)$

$$x^3 = 2a + 3\left(\sqrt{a^2 - (a^2 + b^3)}\right)^{\frac{2}{3}}(x)$$

$$x^3 = 2a + 3\left(\sqrt{-b^3}\right)^{\frac{2}{3}}(x)$$

$$x^3 = 2a + 3\left(-b^{\frac{3}{2}}\right)^{\frac{2}{3}}(x)$$

$x^3 = 2a - 3bx$

$x^3 + 3bx = 2a$

262. (d) Given

$$\frac{x^{24} + 1}{x^{12}} = 7$$

$$\frac{x^{24} + 1}{x^{12}} \Rightarrow \frac{x^{24}}{x^{12}} + \frac{1}{x^{12}}$$

$$\Rightarrow x^{12} + \frac{1}{x^{12}} = 7$$

$\Rightarrow$ Cubing both sides

$$\Rightarrow \left(x^{12} + \frac{1}{x^{12}}\right)^3 = 7^3$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} + \frac{3 \times x^{12} \times 1}{x^{12}}\left(x^{12} + \frac{1}{x^{12}}\right)$$

= 343

$$\Rightarrow x^{36} + \frac{1}{x^{36}} + 3 \times 7 = 343$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} = 343 - 21$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} = \frac{x^{72} + 1}{x^{36}}$$

= 322

263. (d) Given P = 99
find $P(P^2 + 3P + 3) = ?$
to put value in equation
$\Rightarrow$ $99\ ((99)^2 + (3 \times 99) + 3)$
$\Rightarrow$ $(100 - 1)\ [(100 - 1)^2 + [3\times(100 - 1] + 3)$
$\Rightarrow$ $(100 - 1)\ [10000 + 1 - 200 + 300 - 3 + 3]$
$\Rightarrow$ $(100 - 1)\ (10000 + 100 + 1)$
$\Rightarrow$ $(100 - 1)\ (10101)$
$\Rightarrow$ 99×10101
$\Rightarrow$ **99 99 99**

264. (b) Given, $x = 2$
Find $x^3 + 27x^2 + 243x + 631$
$\Rightarrow$ to put value $x = 2$
$\Rightarrow$ $2^3 + 27\times2^2 + (243 \times 2) + 631$
$\Rightarrow$ $8 + 108 + 486 + 631$
$\Rightarrow$ **1233**

265. (b) Given, $x^2+y^2+z^2 = 2\ (x+z-1)$
Find- $x^3+y^3+z^3 = ?$
$\Rightarrow$ $x^2+y^2+z^2 = 2\ (x+z-1)$
$\Rightarrow$ $x^2+y^2+z^2 = 2x+2z-2$
$\Rightarrow$ $x^2+y^2+z^2 = 2x+2z-1-1$
$\Rightarrow$ $(x^2+1-2x) + y^2 + (z^2+1-2z) = 0$
$\Rightarrow$ $(x-1)^2 + y^2 + (z-1)^2 = 0$
$\Rightarrow$ $(x-1)^2 = 0$
$\Rightarrow$ $x = 1$
$\Rightarrow$ $y^2 = 0$
$\Rightarrow$ $y = 0$
$\Rightarrow$ $(z - 1)^2 = 0$
$\Rightarrow$ $z = 1$
Value substituted in question,
$\Rightarrow$ $x^3+y^3+z^3$
$\Rightarrow$ $1^3 + 0 + 1^3$
$\Rightarrow$ 2

266. (b) Given, $x+\frac{1}{x}=1$

Find: $\frac{2}{x^2-x+2}=?$

$x+\frac{1}{x}=1$
$x^2+1 = x$
$(x^2-x) = -1$
Putting value in,
$\frac{2}{(x^2-x)+2} = 2$

267. (d) Given,
$x=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}},\ y=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Find: $\frac{x^2+y^2+xy}{x^2+y^2-xy}=?$

$\Rightarrow$ $\frac{x^2+y^2+2xy-xy}{x^2+y^2-2xy+xy}$

$\Rightarrow$ $\frac{(x+y)^2-xy}{(x-y)^2+xy}=?$

Now, $x+y=\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})}+\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$

$\Rightarrow$ $x+y=\frac{(\sqrt{5}-\sqrt{3})^2+(\sqrt{5}+\sqrt{3})^2}{\sqrt{5}^2-\sqrt{3}^2}$

$\Rightarrow$ $x+y=\frac{2(\sqrt{5}^2+\sqrt{3}^2)}{2}$

$\Rightarrow$ $x + y = 8$ ....(i)

Again, $x-y=\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})}-\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$

$=\frac{4\times\sqrt{5}\times\sqrt{3}}{2}$

$\Rightarrow$ $(x-y)=2\sqrt{15}$ ....(ii)

And, $xy=\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})}\times\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$

$xy = 1$
Substitutes values in the question.

$\Rightarrow$ $\frac{(x+y)^2-xy}{(x-y)^2+xy}$

$\Rightarrow$ $\frac{8^2-1}{(2\sqrt{15})^2+1}$ $\Rightarrow$ $\frac{63}{61}$

268. (c) Given, $4a-\frac{4}{a}+3=0$

Find: $a^3-\frac{1}{a^3}+3=?$

$\Rightarrow$ $4a-\frac{4}{a}=-3$

$\Rightarrow$ $a-\frac{1}{a}=\frac{-3}{4}$

$\Rightarrow$ $\left(a-\frac{1}{a}\right)^3 = \left(\frac{-3}{4}\right)^3$

[Cubing both sides]

$\Rightarrow$ $a^3-\frac{1}{a^3}-3a\times\frac{1}{a}\left(a-\frac{1}{a}\right)=\frac{-27}{64}$

$\Rightarrow$ $a^3-\frac{1}{a^3}-3\times\left(-\frac{3}{4}\right)=\frac{-27}{64}$

$\Rightarrow$ $a^3-\frac{1}{a^3}=\frac{-27}{64}-\frac{9}{4}$

$\Rightarrow$ $a^3-\frac{1}{a^3}+3=\frac{-27}{64}-\frac{9}{4}+3$

$\Rightarrow$ $\frac{192-171}{64}$ $\Rightarrow$ $a^3-\frac{1}{a^3}+3=\frac{21}{64}$

269. (b) According to the question,
$\because$ $x = z = 225$
$y = 226$
$\Rightarrow$ $x^3+y^3+z^3-3xyz = ?$
As we know,
$x^3+y^3+z^3-3xyz = \frac{1}{2}(x+y+z)$
$[(x-y)^2+(y-z)^2+(z-x)^2]$

$=\frac{1}{2}[225+225+226][(225226)^2 + (+226-225)^2+(225-225)^2]$

$=\frac{676}{2}\times[1+1+0] = 676$

270. (b) $x^2 + x = 5$ then $(x + 3)^3 + \frac{1}{(x+3)^3}$

Let $x + 3 = m$
$x = m - 3$
then $(m - 3)^2 + (m - 3) = 5$
$m^2 + 9 - 6m + m - 3 = 5$
$m^2 - 5m + 6 = 5$
$m^2 - 5m = -1$

$m - 5 = \frac{-1}{m}$

$m + \frac{1}{m} = 5$

then $m^3 + \frac{1}{m^3} = 125 - 15$

$m^3 + \frac{1}{m^3} = 110$
Here $m = x + 3$ then

$(x+3)^3 + \frac{1}{(x+3)^3} = 110$

271. (d) Given, $m = -4$, $n = -2$
Find $m^3-3m^2+3m+3n+3n^2+n^3$
Putting value of m and n
$\Rightarrow$ $(-4)^3 - 3(-4)^2 + 3(-4) + 3\times(-2) + 3(-2)^2 + (-2)^3$
$\Rightarrow$ $-64 - 48 - 12 - 6 + 12 - 8$
$\Rightarrow$ $-64 - 60 - 2$
$\Rightarrow$ -126

272. (b) $2x - ky + 7 = 0$ ......(i)
$6x - 12y + 15 = 0$ .....(ii)
There has no solution for

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow$ $\frac{2}{6} = \frac{-k}{-12}$

$\frac{1}{3} = \frac{k}{12}$ $\Rightarrow$ K = 4

273. (a) Here, $x = 332$,
$y = 333$, $z = 335$
Find $x^3 + y^3 + z^3 - 3xyz$

$=\frac{1}{2}(a+b+c)$
$\left[(a-b)^2+(b-c)^2+(c-a)\right]$

$=\left(\frac{332+333+335}{2}\right)\left[(333-332)^2+(335-333)^2\right.$

$\left.+(335-332)^2\right] = \frac{1000}{2}(1^2+2^2+3^2)$

$=\frac{1000}{2}(14) = 7000$

274. (d) If $2 + x\sqrt{3} = \frac{1}{2+\sqrt{3}}$

Find x = ?

$\Rightarrow \quad 2+\sqrt{3}\ x = \frac{1}{2+\sqrt{3}}$

$\Rightarrow \quad 2+\sqrt{3} \times x = \frac{2-\sqrt{3}}{1}$

$\Rightarrow \quad 2 + x\sqrt{3} = 2 - \sqrt{3}$

$\Rightarrow \quad x = -1$

275. (c) Given $m-5n = 2$
find $m^3 - 125n^3 - 30mn$

$\Rightarrow \quad m - 5n = 2$

$\Rightarrow \quad (m-5n)^3 = 2^3$ (cubing both sides)

$\Rightarrow \quad m^3 - 125n^3 - 3m\times 5n\,(m-5n) = 8$

$\Rightarrow \quad m^3 - 125n^3 - 15mn\times 2 = 8$

$\Rightarrow \quad m^3 - 125n^3 - 30mn = 8$

276. (d) Given

$x = \sqrt{a\sqrt[3]{b\sqrt{a\sqrt[3]{b}}}}\ .....\infty$ ......(i)

on squaring both side

$\Rightarrow \quad x^2 = a\sqrt[3]{b\sqrt{a\sqrt[3]{b}}}\ ....\infty$

On cubing both sides

$\Rightarrow \quad x^6 = a^3b\ \sqrt{a\sqrt[3]{b\sqrt{a\sqrt[3]{b}}}}\ .....\infty$

$\Rightarrow \quad x^6 = a^3b\ x$ from equation (i)

$\Rightarrow$ On dividing above eq. by x we get

$\Rightarrow \quad \frac{x^6}{x} = \frac{a^3bx}{x}$

$\Rightarrow \quad x^5 = a^3b$

$\Rightarrow \quad x = \sqrt[5]{a^3b}$

277. (d) **Given:** $x + \frac{1}{x} = 2$ ....(i)

The value of $x^{12} - \frac{1}{x^{12}}$ =?

$\Rightarrow$ if $x = 1 \Rightarrow x + \frac{1}{x} = 2$

$1+1=2$

Then, $x^{12} - \frac{1}{x^{12}}$

$\Rightarrow \quad 1^{12} - \frac{1}{1^{12}}$

$\Rightarrow \quad 1 - 1 = 0$

278. (a) Given: $x + \frac{1}{x} = 1$ ....(i)

Find $\frac{x^2+3x+1}{x^2+7x+1}$ =?

From equation (i)

$\Rightarrow \quad x + \frac{1}{x} = 1$

$\Rightarrow \quad x^2 + 1 = x$

$\Rightarrow \quad \frac{(x^2+1)+3x}{(x^2+1)+7x}$

$\Rightarrow \quad \frac{x+3x}{x+7x} \Rightarrow \frac{4x}{8x} = \frac{1}{2}$

279. (c) $x + \frac{1}{x} = 2$

Find $x^7 + \frac{1}{x^5} = 2$

$\Rightarrow x + \frac{1}{x} = 2 \Rightarrow$ Let $x = 1$

↓ ↓

1 1

To, put value in question,

$\Rightarrow x^7 + \frac{1}{x^5} \Rightarrow 1^7 + \frac{1}{1^5}$

$\Rightarrow 1+1 = 2$

280. (d) Given expression,

$\Rightarrow 4x^2 + 8x$

Let P should be added,

$\Rightarrow 4x^2 + 8x + p$

$\Rightarrow (2x)^2 + 2 \times (2x) \times 2$

$[(a+b)^2 = a^2+b^2+2ab]$

Term that should be added $= 2^2 = 4$

P = 4

281. (b)

$999x + 888y = 1332$

$888x + 999y = 555$

$1887\ (x+y) = 1887$

$x + y = 1$

282. (a) According to the question,

$\Rightarrow x = \frac{1}{2+\sqrt{3}},\quad y = \frac{1}{2-\sqrt{3}}$

$\Rightarrow x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}},$

$y = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$\Rightarrow \quad x = 2-\sqrt{3},\ y = 2+\sqrt{3}$

$8xy\ (x^2+y^2)$

$= 8\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)\left[\left(2-\sqrt{3}\right)^2 + \left(2+\sqrt{3}\right)^2\right]$

$\Rightarrow \quad 8\times 1\left[7 - 2\sqrt{3} + 7 + 2\sqrt{3}\right] = 112$

283. (d) According to the question,

$a = \frac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}$

Put $x = 2$

$a = \frac{\sqrt{2+2}+\sqrt{2-2}}{\sqrt{2+2}-\sqrt{2-2}}$

$a = \frac{\sqrt{4}}{\sqrt{4}} = 1$

$a^2 - ax = 1^2 - 1\times 2 = 1 - 2 = -1$

284. (a) Let $a = 0$
$b = 1$

$\Rightarrow a^3 + b^3 - ab - (a^2 - b^2)^2$

$\Rightarrow 0 + 1 - 0 - (0-1)^2$

$\Rightarrow 1 - 1 = 0$

285. (b) $a - \frac{1}{(a-3)} = 5$

$a - 3 - \frac{1}{a-3} = 5 - 3$

$(a-3) - \frac{1}{(a-3)} = 2$

Cubing both sides

$\left[(a-3) - \frac{1}{(a-3)}\right]^3 = 2^3$

$(a-3)^3 - \frac{1}{(a-3)^3} - 3.(a-3)\times\frac{1}{(a-3)}$

$\left[(a-3) - \frac{1}{(a-3)}\right] = 8$

$(a-3)^3 - \frac{1}{(a-3)^3} - 3\ [\ 2\ ] = 8$

$(a-3)^3 - \frac{1}{(a-3)^3} = 8 + 6 = 14$ Ans.

286. (d) $\frac{3x-2y}{2x+3y} = \frac{5}{6}$

$18x - 12y = 10x + 15y$

$8x = 27y$

$\frac{x}{y} = \frac{27}{8}$

$\left[\frac{\sqrt[3]{x} + \sqrt[3]{y}}{\sqrt[3]{x} - \sqrt[3]{y}}\right]^2$

$\left(\frac{\sqrt[3]{27} + \sqrt[3]{8}}{\sqrt[3]{27} - \sqrt[3]{8}}\right)^2$

$\Rightarrow \left(\frac{3+2}{3-1}\right)^2 = (5)^2 = 25$

287. (d) According to the Question

$x = \sqrt{3} + \sqrt{2}$

$y = \sqrt{3} - \sqrt{2}$

$(x^3 - 20\sqrt{2}) - (y^3 + 2\sqrt{2})$

$= [(\sqrt{3}+\sqrt{2})^3 - 20\sqrt{2} - (\sqrt{3}-\sqrt{2})^3 - 2\sqrt{2}]$

$= 3\sqrt{3} + 2\sqrt{2} + 9\sqrt{2} + 6\sqrt{3} - 20\sqrt{2} - 3\sqrt{3} + 2\sqrt{2} + 9\sqrt{2} - 6\sqrt{3} - 2\sqrt{2}$

$= 9\sqrt{3} - 9\sqrt{2} - 9\sqrt{3} + 9\sqrt{2} = 0$

288. (d) **SHORTCUT METHOD**
Always do these types of question with the help of
Put a = b = c = 1
$3(1^2 + 1^2 + 1^2) = (a + b + c)^2$
3 = 3 satified
So this is answer → a = b = c

289. (d) According to the question,

$x = \sqrt{a} + \frac{1}{\sqrt{a}}$ & $y = \sqrt{a} - \frac{1}{\sqrt{a}}$

∴ $x^4 - x^2y^2 - 1 + y^4 - x^2y^2 + 1$
$= x^4 - 2x^2y^2 + y^4$
$= [x^2 - y^2]^2$

$$\left[\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)^2 - \left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)^2\right]^2$$

$$= \left[a + \frac{1}{a} + 2 - a - \frac{1}{a} + 2\right]^2$$

$= [4]^2 = 16$

290. (d) Let m = $\sqrt{5+\sqrt{5+\sqrt{5}}}$
Factor = (a)×(a+1)
Here m = a + 1
or m – 1 = a ......(i)
Let n = $\sqrt{5-\sqrt{5-\sqrt{5}}}$
Factor = (a)×(a+1)
Here n = a ......(ii)
From (i) & (ii)
m – 1 = n
or m – n – 1 = 0

291. (b) $\frac{3-5x}{2x} + \frac{3-5y}{2y} + \frac{3-5z}{2z} = 0$

or $\frac{3}{2x} - \frac{5}{2} + \frac{3}{2y} - \frac{5}{2} + \frac{3}{2z} - \frac{5}{2} = 0$

or $\frac{3}{2x} + \frac{3}{2y} + \frac{3}{2z} = \frac{3\times5}{2}$

or $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3\times5\times2}{2\times3}$

or $\frac{2}{x} + \frac{2}{y} + \frac{2}{z} = \frac{3\times5\times2\times2}{2\times3}$
$= 10$

292. (a) According to the question
If 2S = a + b + c

$S = \frac{a+b+c}{2}$

Let a = 10, b = 10, c = 10

∵ $S = \frac{10+10+10}{2}$

$S = \frac{30}{2} = 15$

∵ S(S – C) + (S – a) (S – b)
15(15 – 10) + (15 – 10) (15 – 10) = 75 + 25 = 100
Now check from option.
Option (a) ab = 10×10 = 100 (Satisfied)

293. (d) p + m = 6 .....(i)
$p^3 + m^3 = 72$
$(p + m)(p^2 + m^2 - pm) = 72$
$(p + m)[(p + m)^2 - 3pm] = 72$
[∵ $p^2 + m^2 = (p + m)^2 - 2pm$]
$6[(6)^2 - 3pm] = 72$ from (i)
36 – 3pm = 12
pm = 8

294. (d) $x^m \times x^n = 1$
$x^{m+n} = x^o$ (∵ $x^o = 1$)
m + n = 0
m = –n

295. (d) $\frac{2p}{p^2-2p+1} = \frac{1}{4}$

$\frac{2}{p-2+\frac{1}{p}} = \frac{1}{4}$

(Divide p both in nu. & de.)

$p + \frac{1}{p} - 2 = 8$

$p + \frac{1}{p} = 10$

296. (b) According to the question,
x = 2, y = 1, z = – 3
∴ $x^3 + y^3 + y^3 - 3xyz = ?$
As we know that
a + b + c = 0 then $a^3 + b^3 + c^3 - 3abc = 0$
∴ 2 + 1 – 3 = 0
$x^3 + y^3 + 3^3 - 3xyz = 0$

297. (a) According to the question
⇒ $(x^3 + y^6)(x^3 - y^6)$
⇒ $x^6 + x^3y^6 - x^3y^6 - y^{12}$
⇒ $x^6 - y^{12}$

298. (b) According to the question,

⇒ $\frac{1}{x+y} + \frac{1}{x-y}$

⇒ $\frac{x-y+x+y}{x^2-y^2}$ ⇒ $\frac{2x}{x^2-y^2}$

299. (a) Given, x + y = 2a to

Find $\frac{a}{(x-a)} + \frac{a}{(y-a)} = ?$

⇒ $x + y = 2a$
↓ ↓ ↓
3 1 2
⇒ Let x = 3, y = 1, a = 2

⇒ $\frac{a}{(x-a)} + \frac{a}{y-a}$

⇒ $\frac{2}{(3-2)} + \frac{2}{(1-2)}$

⇒ $\frac{2}{1} + \frac{2}{-1} = 0$

300. (a) $a^2 + b^2 + c^2 = ab + bc + ca$
Let a = b = c = 1
⇒ $a^2 + b^2 + c^2 = ab + bc + ca$
⇒ $1^2+1^2+1^2 = 1\times1+1\times1+1\times1$
⇒ 3 = 3

⇒ to find $\frac{a+c}{b} = ?$

⇒ $\frac{1+1}{1} = 2$

301. (d) $p^3 - q^3 = (p - q)\{(p - q)^2 - xpq\}$
⇒ $p^3 - q^3 = (p - q)[p^2 + q^2 - 2pq - xpq]$
⇒ $p^3 - q^3 = (p - q)[p^2 + q^2 - 2pq - (-3)pq]$
⇒ $p^3 - q^3 = (p - q)(p^2 + q^2 + pq)$
⇒ So, x = –3
⇒ because $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

302. (a) Given
⇒ x + y + z = 6
⇒ xy + yz + zx = 10
To find $x^3 + y^3 + z^3 - 3xyz = ?$
⇒ Using formula.
⇒ $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$
⇒ $6^2 = x^2 + y^2 + z^2 + 2 \times 10$
⇒ $36 = x^2 + y^2 + z^2 + 20$
⇒ $x^2 + y^2 + z^2 = 16$
⇒ $x^3 + y^3 + z^3 - 3xyz =$

$(x+y+z)[x^2+y^2+z^2-xy-yz-zx]$

$= 6[16-(xy+yz+zx)]$

= 6 [16 – 10]
= 6 × 6 = 36

303. (d) Given:

⇒ $\frac{x+1}{x-1} = \frac{a}{b}$

⇒ $\frac{x}{1} = \frac{a+b}{a-b}$

(using componendo & dividendo)

⇒ $x = \frac{a+b}{a-b}$ ....(i)

Again,

$\Rightarrow \frac{1-y}{1+y} = \frac{b}{a}$

$\Rightarrow \frac{1+y}{1-y} = \frac{a}{b}$

$\Rightarrow \frac{1}{y} = \frac{a+b}{a-b}$

$\Rightarrow y = \frac{a-b}{a+b}$ ....(i)

$\Rightarrow$ From question, $\frac{x-y}{1+xy}$

$$\Rightarrow \frac{\frac{a+b}{a-b}-\frac{(a-b)}{(a+b)}}{1+\left(\frac{a+b}{a-b}\right)\left(\frac{a-b}{a+b}\right)}$$

$$\Rightarrow \frac{(a+b)^2-(a-b)^2}{(a^2-b^2)(1+1)}$$

$$\Rightarrow \frac{4ab}{2(a^2-b^2)} \Rightarrow \frac{2ab}{a^2-b^2}$$

304. (d) Given

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

to find $a : b : c = ?$

$\Rightarrow$ According to the question,

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow$ Let $a = b = c = 1$

$\Rightarrow 1^2+1^2+1^2=1\times1+(1\times1)+(1\times1)$

$\Rightarrow 3 = 3$

$\Rightarrow$ So ratio of $a : b : c = 1 : 1 : 1$

305. (c) $\frac{x}{a} = b - c$; $\frac{y}{b} = c - a$; $\frac{z}{c} = a - b$

$$\left(\frac{x}{a}\right)^3+\left(\frac{y}{b}\right)^3+\left(\frac{z}{c}\right)^3 = 3\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(\frac{z}{c}\right)$$

$$\left(\because \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0\right) = \frac{3x\,yz}{abc}$$

306. (b) $\frac{x}{y} = \frac{a+2}{a-2}$

$$\frac{x^2}{y^2} = \frac{(a+2)^2}{(a-2)^2}$$

Applying Componendo & Dividendo

$$\frac{x^2-y^2}{x^2+y^2} = \frac{(a+2)^2-(a-2)^2}{(a+2)^2+(a-2)^2} = \frac{8a}{2a^2+8} = \frac{4a}{a^2+4}$$

307. (c) $\frac{a}{b} + \frac{b}{a} = 2$

$a^2 + b^2 = 2ab$

$(a - b)^2 = 0$

$a - b = 0$

308. (a) Put $(x + y + z) = 10$

$x = 2$

$y = 3$

$z = 5$

$x(x + y + z) = 20$

$2(10) = 20$

Similarly other will satisfied

so value of $2(x + y + z)$

$\Rightarrow 2(10) = 20$

309. (c) $x^2 + y^2 = 14$

$x + y = 4$ ...... (i)

squaring both sides

$x^2 + y^2 + 2xy = 16$

$14 + 2xy = 16$

$2xy = 2$

$xy = 1$

$x^2 + y^2 = 14$

subtrance (2xy)from both sides.

$x^2 + y^2 - 2xy = 14 - 2xy$

$(x - y)^2 = 14 - 2 \times 1$

$x - y = \sqrt{12}$

$x - y = 2\sqrt{3}$ ....(2)

solve equation (1) and (2)

$y = 2 - \sqrt{3}$

$x = 2 + \sqrt{3}$

310. (d) $x^2 - 4x - 1 = 0$

$\Rightarrow x - \frac{1}{x} = 4$

$\Rightarrow \left(x-\frac{1}{x}\right)^2 = 4^2$

(squaring both sides)

$\Rightarrow x^2 + \frac{1}{x^2} = 18$

311. (b) **Given**

$\because a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}, b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

to find $\frac{a^2}{b} + \frac{b^2}{a} = ?$

$\Rightarrow \frac{a^3+b^3}{ab} = ?$

$$\Rightarrow \frac{(a+b)^3-3ab(a+b)}{ab} = ?$$

$$\Rightarrow a + b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$\Rightarrow \frac{\left(\sqrt{3}-\sqrt{2}\right)^2+\left(\sqrt{3}+\sqrt{2}\right)^2}{\sqrt{3}^2-\sqrt{2}^2}$$

$$\Rightarrow \frac{2\left(\sqrt{3}^2+\sqrt{2}^2\right)}{1}$$

$\Rightarrow 2 \times (5) \Rightarrow a + b = 10$

Again

$$\Rightarrow a \times b = \frac{\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)}\times\frac{\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)}$$

$\Rightarrow ab = 1$

$$\Rightarrow \frac{(a+b)^3-3ab(a+b)}{ab}$$

$$\Rightarrow \frac{10^3-3\times1\times10}{1}$$

$\Rightarrow 1000 - 30 = \mathbf{970}$

312. (c) $(2a-1)^2+(4b-3)^2+(4c+5)^2 = 0$

to find

$$\Rightarrow \frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2}$$

$$\Rightarrow \frac{\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]}{a^2+b^2+c^2}$$

$\Rightarrow$ then, $2a - 1 = 0$

$a = 1/2$

$\Rightarrow 4b - 3 = 0$

$\Rightarrow b = \frac{3}{4}$

$\Rightarrow 4c + 5 = 0$

$\Rightarrow c = \frac{-5}{4}$

$$\Rightarrow \frac{\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]}{a^2+b^2+c^2}$$

$$\Rightarrow \frac{\frac{1}{2}\left(\frac{1}{2}+\frac{3}{4}-\frac{5}{4}\right)\left[\left(\frac{1}{2}-\frac{3}{4}\right)^2+\left(\frac{3}{4}+\frac{5}{4}\right)^2+\left(\frac{-5}{4}-\frac{1}{2}\right)^2\right]}{\left(\frac{1}{2}\right)^2+\left(\frac{3}{4}\right)^2+\left(\frac{-5}{4}\right)^2}$$

$$\Rightarrow \frac{\frac{1}{2}\left(\frac{2+3-5}{4}\right)\left[\left(\frac{2-3}{4}\right)^2+\left(\frac{8}{4}\right)^2+\left(\frac{-5-2}{4}\right)^2\right]}{\frac{1}{4}+\frac{9}{16}+\frac{25}{16}}$$

$\Rightarrow \frac{1}{2}\times 0 = 0$ $(a + b + c = 0)$

313. (a) $\left(a+\frac{1}{a}\right)^2 = 3$

$a + \frac{1}{a} = \sqrt{3}$

cube both sides

$$a^3 + \frac{1}{a^3}+3\times a\times\frac{1}{a}\left(a+\frac{1}{a}\right) = \left(\sqrt{3}\right)^3$$

$a^3 + \frac{1}{a^3}+3\sqrt{3}$

$= 3\sqrt{3}$

$a^3 + \frac{1}{a^3} = 0$

$a^6 + 1 = 0$

$\Rightarrow a^{30} + a^{24} + a^{18} + a^{12} + a^6 + 1$

$= a^{24}(a^6+1)+a^{12}(a^6+1)+a^6+1$

$= a^{24}(0)+a^{12}(0)+0 = 0$

314. (d) $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$

$\frac{1}{a+b} = \frac{b+a}{ab}$

$ab = a^2 + b^2 + 2ab$

$a^2 + b^2 + ab = 0$

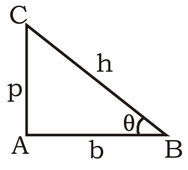
$a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$= (a-b)(0) = 0$

CHAPTER 12

TRIGONOMETRY

1. **Trigonometric Ratio:**



To study different trigonometric ratio functions we will consider a right angled triangle. Suppose ABC is a right angled triangle with $\angle A = 90°$.

We can obtain six different trigonometric ratio from the sides of these triangle. They are respectively $\frac{AC}{BC}, \frac{AB}{BC}, \frac{AC}{AB}, \frac{AB}{AC}, \frac{BC}{AB}$ and $\frac{BC}{AB}$. If $\angle B = \theta$ then these ratio are respectively called $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$ $\sec\theta$ and $\operatorname{cosec}\theta$. Clearly for the given angle θ, AC (p) is perpendicular, AB (b) is base and BC (h) is hypotenuse. Hence six different trigonometric ratios are follows (see the given figure)

Trigonometric Ratios:-

$$\sin\theta = \frac{AC}{BC} = \frac{p}{h} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{AB}{BC} = \frac{b}{h} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{AC}{AB} = \frac{p}{b} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\operatorname{cosec}\theta = \frac{BC}{AC} = \frac{h}{p} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

$$\sec\theta = \frac{BC}{AB} = \frac{h}{b} = \frac{\text{Hypotenuse}}{\text{Base}}$$

$$\cot\theta = \frac{AB}{AC} = \frac{b}{p} = \frac{\text{Base}}{\text{Perpendicular}}$$

Clearly $\sin\theta$ and $\operatorname{cosec}\theta$ are reciprocals to each other. Similarly $\cos\theta$ and $\sec\theta$ are reciprocals to each other while $\tan\theta$ and $\cot\theta$ are reciprocals to each other.

Relations between Trigonometric Ratios :-

(i) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

or $\operatorname{cosec}\theta \times \sin\theta = 1$

(ii) $\sec\theta = \frac{1}{\cos\theta}$

or $\sec\theta \times \cos\theta = 1$

(iii) $\cot\theta = \frac{1}{\tan\theta}$

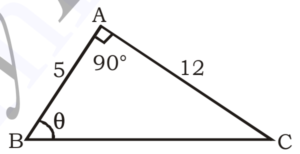
or $\cot\theta \times \tan\theta = 1$

(iv) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

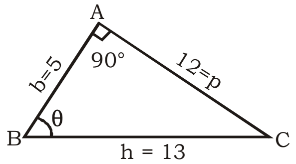
(v) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

TYPE - 1

Ex.1 Write all the six t-ratios value in the given figure:



Sol. In ΔABC is, a right angle triangle with $\angle A = 90°$,



Let AC = 12 = p and AB = 5 = b

Then from Pythagoras theorem,

$BC = \sqrt{AB^2 + AC^2} = \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144} = \sqrt{169} = 13$

Here side opposite to θ is AC which is p.

Side adjacent to θ is AB, which is b.

Side opposite to right angle is BC, which is hypotenuse h.

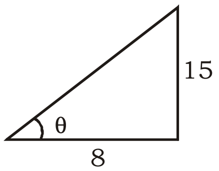
$\therefore \sin\theta = \frac{p}{h} = \frac{12}{13}$, $\operatorname{cosec}\theta = \frac{h}{p} = \frac{13}{12}$

$\cos\theta = \frac{b}{h} = \frac{5}{13}$, $\sec\theta = \frac{h}{b} = \frac{13}{5}$

$\tan\theta = \frac{p}{b} = \frac{12}{5}$, $\cot\theta = \frac{b}{p} = \frac{5}{12}$

Ex.2 If $15\cot\theta = 8$ then calculate the remaining trigonometric ratio.

Sol.



$\cot\theta = \frac{8}{15} = \frac{b}{p}$

Let b = 8k p = 15 k

From pythagoras theorem, $h^2 = p^2 + b^2 = (15k)^2 + (8k)^2$

or, $h^2 = 225k^2 + 64k^2 = 289k^2$

or, $h = \sqrt{289k^2} = 17k$

Hence, $\sin\theta = \frac{p}{h} = \frac{15k}{17k} = \frac{15}{17}$

$\cos\theta = \frac{b}{h} = \frac{8k}{17k} = \frac{8}{17}$

$\tan\theta = \frac{p}{b} = \frac{15k}{8k} = \frac{15}{8}$

$\sec\theta = \frac{h}{b} = \frac{17k}{8k} = \frac{17}{8}$

$\operatorname{cosec}\theta = \frac{h}{p} = \frac{17k}{15k} = \frac{17}{15}$

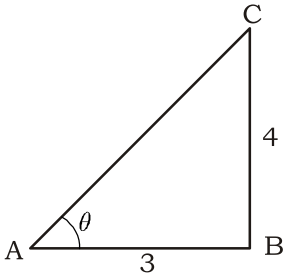
Ex.3 If $\tan\theta = \frac{4}{3}$, then $\cos\theta = ?$

(a) $4/5$ (b) $3/5$

(c) $3/4$ (d) $1/5$

Sol.(b) $\tan\theta = \frac{BC}{AB} = \frac{4}{3}$

$\therefore AC = \sqrt{(4)^2 + (3)^2} = 5$



$\therefore \cos\theta = \frac{AB}{AC} = \frac{3}{5}$

Ex.4 If $\tan\theta = \frac{4}{3}$, the value of $\frac{1-\sin\theta}{1+\sin\theta}$ is:-

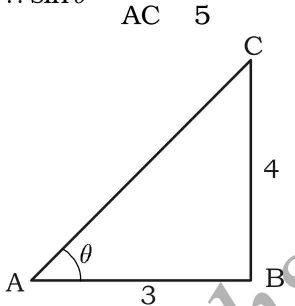
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{9}$ (d) $\frac{1}{13}$

Sol.(c) $\tan\theta = \frac{4}{3} = \frac{BC}{AB}$

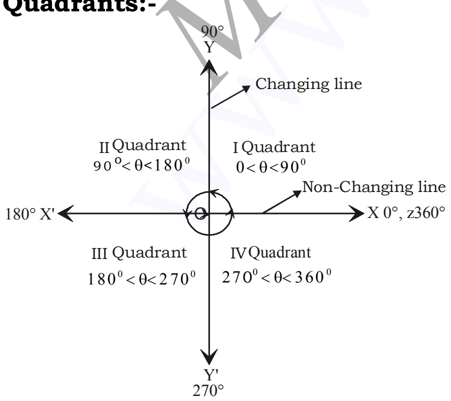
and $AC = \sqrt{(3)^2 + (4)^2} = 5$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{4}{5}$



$\therefore \frac{1-\sin\theta}{1+\sin\theta} = \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{1}{9}$

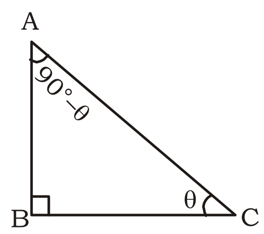
Quadrants:-



Let XOX' and YOY' be two mutually perpendicular lines. These lines divide the plane into four parts and each one of them is called a quadrant.

Complementary Angle.

For a given angle θ its complementary angle is (90°−θ).



From definition,

$\sin\theta = \frac{\text{side opposite angle }\theta}{\text{hypotenuse}} = \frac{AB}{AC}$

and cos (90° − θ)

$= \frac{\text{side along with angle }(90° - \theta)}{\text{hypotenuse}} = \frac{AB}{AC}$

$\therefore \boxed{\sin\theta = \cos(90° - \theta)}$

Similarly, we can prove that

$\therefore \boxed{\cos\theta = \sin(90° - \theta)}$

⇒ 90°,270°.....(odd multiple of 90°) will be changed

⇒ 0°,180°,360°.........(multiple of 180°) will not be changed

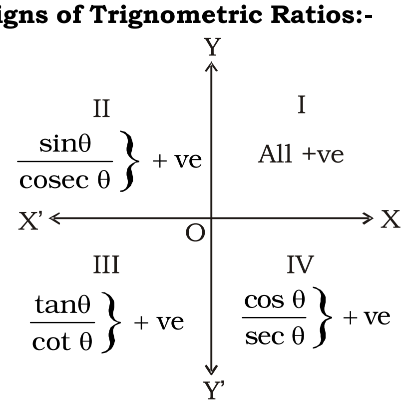
Change will be in following manner:

sinθ → cosθ & cosθ → sinθ

tanθ → cotθ & cotθ → tanθ

secθ→ cosecθ & cosec θ → secθ

Signs of Trignometric Ratios:-



Trigonometric Ratios of Allied Angles

(A) T-ratios of $(-\theta)$ in terms of those of θ :-

1. $\sin(-\theta) = -\sin\theta$
2. $\cos(-\theta) = \cos\theta$
3. $\tan(-\theta) = -\tan\theta$
4. $\cot(-\theta) = -\cot\theta$
5. $\sec(-\theta) = \sec\theta$
6. $\text{cosec}\,(-\theta) = -\text{cosec}\,\theta$

(B) T-ratios of $(90^0 - \theta)$ in terms of those of θ :-

1. $\sin(90^0 - \theta) = \cos\theta$
2. $\cos(90^0 - \theta) = \sin\theta$
3. $\tan(90^0 - \theta) = \cot\theta$
4. $\cot(90^0 - \theta) = \tan\theta$
5. $\sec(90^0 - \theta) = \text{cosec}\theta$
6. $\text{cosec}(90^0 - \theta) = \sec\theta$

(C) T-ratios of $(90^0 + \theta)$ in terms of those of θ :-

1. $\sin(90^0 + \theta) = \cos\theta$
2. $\cos(90^0 + \theta) = -\sin\theta$
3. $\tan(90^0 + \theta) = -\cot\theta$
4. $\cot(90^0 + \theta) = -\tan\theta$
5. $\sec(90^0 + \theta) = -\text{cosec}\theta$
6. $\text{cosec}\,(90^0 + \theta) = \sec\theta$

(D) T-ratios of $(180^0 - \theta)$ in terms of those of θ :-

1. $\sin(180^0 - \theta) = \sin\theta$
2. $\cos(180^0 - \theta) = -\cos\theta$
3. $\tan(180^0 - \theta) = -\tan\theta$
4. $\cot(180^0 - \theta) = -\cot\theta$
5. $\sec(180^0 - \theta) = -\sec\theta$
6. $\text{cosec}\,(180^0 - \theta) = \text{cosec}\,\theta$

(E) T-ratios of $(180^0 + \theta)$ in terms of those of θ :-

1. $\sin(180^0 + \theta) = -\sin\theta$
2. $\cos(180^0 + \theta) = -\cos\theta$
3. $\tan(180^0 + \theta) = \tan\theta$
4. $\cot(180^0 + \theta) = \cot\theta$
5. $\sec(180^0 + \theta) = -\sec\theta$
6. $\text{cosec}\,(180^0 + \theta) = -\text{cosec}\,\theta$

(F) T-ratios of $(270^\circ - \theta)$ in terms of those of θ :-

1. $\sin(270^0 - \theta) = -\cos\theta$
2. $\cos(270^0 - \theta) = -\sin\theta$
3. $\tan(270^0 - \theta) = \cot\theta$
4. $\cot(270^0 - \theta) = \tan\theta$
5. $\sec(270^0 - \theta) = -\text{cosec}\,\theta$
6. $\text{cosec}(270^0 - \theta) = -\sec\theta$

(G) T-ratios of $(270^\circ + \theta)$ in terms of those of θ :-

1. $\sin(270^0 + \theta) = -\cos\theta$
2. $\cos(270^0 + \theta) = \sin\theta$
3. $\tan(270^0 + \theta) = -\cot\theta$
4. $\cot(270^0 + \theta) = -\tan\theta$
5. $\sec(270^0 + \theta) = \text{cosec}\,\theta$
6. $\text{cosec}\,(270^0 + \theta) = -\sec\theta$

(H) T-ratios of $(360^\circ - \theta)$ in terms of those of θ :-

1. $\sin(360^0 - \theta) = -\sin\theta$
2. $\cos(360^0 - \theta) = \cos\theta$
3. $\tan(360^0 - \theta) = -\tan\theta$
4. $\cot(360^0 - \theta) = -\cot\theta$
5. $\sec(360^0 - \theta) = \sec\theta$
6. $\text{cosec}\,(360^0 - \theta) = -\text{cosec}\,\theta$

(I) T-ratios of $(360^\circ + \theta)$ in terms of those of θ :-

1. $\sin(360^0 + \theta) = \sin\theta$
2. $\cos(360^0 + \theta) = \cos\theta$
3. $\tan(360^0 + \theta) = \tan\theta$
4. $\cot(360^0 + \theta) = \cot\theta$
5. $\sec(360^0 + \theta) = \sec\theta$
6. $\text{cosec}\,(360^0 + \theta) = \text{cosec}\,\theta$

(J) T-ratios of $(n \times 360^\circ + \theta)$ in terms of those of θ :-

1. $\sin(n \times 360^0 + \theta) = \sin\theta$
2. $\cos(n \times 360^0 + \theta) = \cos\theta$
3. $\tan(n \times 360^0 + \theta) = \tan\theta$
4. $\cot(n \times 360^0 + \theta) = \cot\theta$
5. $\sec(n \times 360^0 + \theta) = \sec\theta$
6. $\text{cosec}\,(n \times 360^0 + \theta) = \text{cosec}\,\theta$

Value of some specific angle of trigonometrical (t)-ratio function.
We must learn the following table to solve the question based on trigonometrical (t)-ratio angle 0°, 30°, 45°, 60°, 90°

θ	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

TYPE - II

Ex.5 find the value of following
(i) sin 120° (ii) cos 210° (iii) Tan570° (iv) cot 780° (v) sin960° (vi) cos1020° (vii) sec 1500°

Sol. (i) sin 120°
= sin (90 + 30)°
$(\sin(90+\theta) = \cos\theta)$
= cos 30° = $\frac{\sqrt{3}}{2}$

Sol.(ii) cos210°
= cos (180 + 30)°
$(\cos(180+\theta) = -\cos\theta)$
= – cos30° = $\frac{-\sqrt{3}}{2}$

Sol.(iii) Tan 570°
= Tan (540 + 30)°
(540° multiple of 180°, Then no change Tan $(540+\theta)$ = Tan θ)
= Tan 30° = $\frac{1}{\sqrt{3}}$

Sol.(iv) cot 780°
= cot (720 + 60)°
∴ cot (n × 360 + θ) = cot θ
= cot (2 × 360 + 60)°
= cot 60° = $\frac{1}{\sqrt{3}}$

Sol.(v) sin 960°
= sin(900 + 60)°
∴ 900° multiple of 180°, so no change of Trignometry function.
= sin(2 × 360 + 180 + 60) = sin (180 + 60) = –sin60°
= $\frac{-\sqrt{3}}{2}$ $\begin{pmatrix}\sin(180+\theta)\\ = -\sin\theta\end{pmatrix}$

Sol.(vi) cos(1020)°
= cos (1080– 60)°
1080 multiple of 180°, so no change In Trignometry function.
= cos (3 × 360 – 60)°
= cos 60° = $\frac{1}{2}$

Sol.(vii) sec (1500°)
= sec (1440 + 60)°
= sec (4 × 360 + 60)°
$\begin{pmatrix}\sec(n \times 360+\theta)\\ = \sec\theta\end{pmatrix}$
= sec 60° = 2

Ex.6
$\frac{\cos(90^0+A).\sec(360^0-A).\tan(180^0-A)}{\sec(A-720)^\circ.\sin(A+540^0).\cot(A-90^0)} = ?$
(a) 0 (b) 1 (c) -1 (d) None of these

Sol. (b)
$\frac{\cos(90^0+A).\sec(360^0-A).\tan(180^0-A)}{\sec(A-720^0).\sin(A+540^0).\cot(A-90^0)}$
$= \frac{(-\sin A).(\sec A)(-\tan A)}{\sec(2\times360-A).\sin(3\times180^0+A).[-\cot(90^0-A)]}$
$(\because \sec(-\theta) = \sec\theta$
and $\cot(-\theta) = -\cot\theta]$
$= \frac{\sin A.\sec A.\tan A}{\sec A(-\sin A)(-\tan A)}$
$= \frac{\sin A.\sec A.\tan A}{\sin A.\sec A.\tan A} = 1$

Ex.7 sin720° – cot 270° – sin 150° cos120° is equal to:–
(a) $1/2$ (b) $1/3$
(c) $1/5$ (d) $1/4$

Sol.(d). sin720° – cot270° – sin150°.cos120°
= sin (2×360°+0°) –cot(360°–90°) – sin $(90^0+60^0).\cos(90^0+30^0)$

$= \sin 0^0 + \cot 90^0 + \cos 60^0 . \sin 30^0$

$= 0 + 0 + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Ex.8. Find the value of :- $\frac{\sin 37^0}{\cos 53^0}$

(a) 1 (b) -1 (c) 0 (d) 0

Sol. (a)

$$\frac{\sin 37^0}{\cos 53^0} = \frac{\sin 37^0}{\cos(90^0 - 37^0)} = \frac{\sin 37^0}{\sin 37^0} = 1$$

Ex.9. Evaluate :- $\sin^2 60° + \cos^2 30° + \cot^2 45°$ $+ \sec^2 60° - \operatorname{cosec}^2 30° + \cos^2 0°$:-

(a) $3/2$ (b) $5/2$

(c) $7/2$ (d) 2

Sol. (c) We know that $\sin 60^0 = \frac{\sqrt3}{2}$

$\cos 30^0 = \frac{\sqrt3}{2}, \cot 45^0 = 1$

$\sec 60° = 2$

$\therefore \sin^2 60^0 + \cos^2 30^0 + \cot^2 45^0 + \sec^2 60^0$
$- \operatorname{cosec}^2 30^0 + \cos^2 0^0$

$$= \left[\left(\frac{\sqrt3}{2}\right)^2 + \left(\frac{\sqrt3}{2}\right)^2 + 1^2 + 2^2 - 2^2 + 1^2\right] = \frac{7}{2}$$

Ex.10 If $\frac{x \operatorname{cosec}^2 30^0 . \sec^2 45^0}{8 \cos^2 45^0 . \sin^2 60^0}$ $= \tan^2 60° - \tan^2 30°$, then the value of x is :-

(a) -1 (b) 0 (c) 1 (d) 2

Sol. (c)

$$\frac{x \times (2)^2 \times (\sqrt2)^2}{8 \times \left(\frac{1}{\sqrt2}\right)^2 \times \left(\frac{\sqrt3}{2}\right)^2} = (\sqrt3)^2 - \left(\frac{1}{\sqrt3}\right)^2$$

$$\Rightarrow 8x = \left(\frac{8}{3} \times 8 \times \frac{1}{2} \times \frac{3}{4}\right) \Rightarrow x = 1$$

Ex.11. If $\sec\theta = \frac{13}{5}$ and θ lies in the fourth quadrant, then the value of $\sin\theta$ is :-

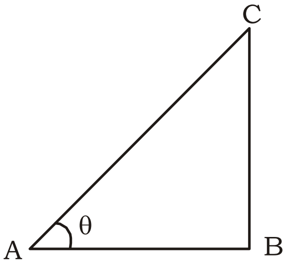
(a) $\frac{12}{13}$ (b) $-\frac{12}{13}$

(c) $-\frac{5}{13}$ (d) $\frac{5}{13}$

Sol.(b) $\sec\theta = \frac{13}{5} = \frac{AC}{AB}$

and $BC = \sqrt{13^2 - 5^2} = 12$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{12}{13}$



since θ lies in the fourth quadrant

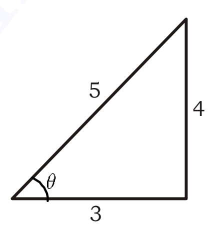
$\therefore \sin\theta = -\frac{12}{13}$

Ex.12. If $3\tan\theta + 4 = 0$, where $\frac{\pi}{2} < \theta < \pi$, then the value of $2\cot\theta - 5\cos\theta + \sin\theta$ is :-

(a) $-\frac{53}{10}$ (b) $\frac{7}{10}$

(c) $\frac{23}{10}$ (d) $\frac{37}{10}$

Sol. (c) $3\tan\theta + 4 = 0 \Rightarrow \tan\theta = -\frac{4}{3}$



$\Rightarrow \sin\theta = \frac{4}{5}, \cos\theta = -\frac{3}{5}, \cot\theta = -\frac{3}{4}$

[$\because \sin\theta$ is positive and $\cos\theta$ is negative in II quadrant].

Ex.13 If $\sin 17° = \frac{x}{y}$, then the value of $\sec 17° - \sin 73°$ is:

(a) $\frac{y^2 - x^2}{xy}$ (b) $\frac{x^2}{\sqrt{y^2 - x^2}}$

(c) $\frac{x^2}{y\sqrt{y^2 + x^2}}$ (d) $\frac{x^2}{y\sqrt{y^2 - x^2}}$

Sol.(d) $\sec 17° - \sin 73°$
$= \sec 17° - \sin(90° - 17°)$
$= \sec 17° - \cos 17°$

$= \frac{1}{\cos 17°} - \cos 17°$

$$= \frac{1 - \cos^2 17°}{\cos 17°} = \frac{\sin^2 17}{\cos 17°} = \frac{\frac{x^2}{y^2}}{\sqrt{1 - \frac{x^2}{y^2}}}$$

$$= \frac{\frac{x^2}{y^2}}{\sqrt{\frac{y^2 - x^2}{y^2}}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$$

Ex. 14. If $\operatorname{cosec} 39° = x$, the value of $\frac{1}{\operatorname{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51° \sec^2 39°}$ is:

(a) $\sqrt{x^2 - 1}$ (b) $\sqrt{1 - x^2}$

(c) $x^2 - 1$ (d) $1 - x^2$

Sol.(c) $\frac{1}{\operatorname{cosec}^2 51°} + \sin^2 39° + \tan^2 51°$

$- \frac{1}{\sin^2 51° . \sec^2 39°}$

$= \sin^2 51° + \sin^2 39° + \tan^2(90° - 39°)$

$- \frac{1}{\sin^2(90° - 39°).\sec^2 39°}$

$= \cos^2 39° + \sin^2 39° + \cot^2 39°$

$- \frac{1}{\cos^2 39° . \sec^2 39°}$

[$\because \sin(90° - \theta) = \cos\theta$), $\tan(90° - \theta) = \cot\theta$]

$= 1 + \cot^2 39° - 1 = \operatorname{cosec}^2 39° - 1$
$= x^2 - 1$

Ex. 15 Find the value of $\cos(180° + A) + \cos(180° + B) + \cos(180° + C) + \cos(180° + D)$ Where A, B, C and D are the vertices of a cyclic quadrilateral ?

(a) 0 (b) 1

(c) 2 (d) 2 cos A

Sol.(a)

$\cos(180^0 + A) + \cos(180^0 + B) + \cos$
$(180^0 + C) + \cos(180^0 + D)$

$= -\cos A - \cos B - \cos C - \cos D$

$= -\cos(180^0 - C) - \cos(180^0 - D)$

$-\cos C - \cos D$

[$\because A + C = B + D = 180^0$ cyclic quadrilateral]

$= \cos C + \cos D - \cos C - \cos D = 0$

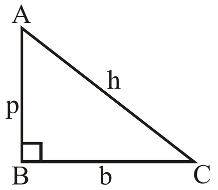
Some Useful formula

(i) $\sin^2\theta + \cos^2\theta = 1$
or $\sin^2\theta = 1 - \cos^2\theta$
or $\cos^2\theta = 1 - \sin^2\theta$

(ii) $1 + \tan^2 = \sec^2\theta$
or $\sec^2\theta - 1 = \tan^2\theta$
or $\sec^2\theta - \tan^2\theta = 1$

(iii) $1 + \cot^2\theta = \text{cosec}^2\theta$
or $\text{cosec}^2\theta - 1 = \cot^2\theta$
or $\text{cosec}^2\theta - \cot^2\theta = 1$

Proof we know,



$\sin\theta = \frac{p}{h}$ $\qquad \cos\theta = \frac{b}{h}$

Now,

$\sin^2\theta + \cos^2\theta$

$= \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = \frac{p^2}{h^2} + \frac{b^2}{h^2} = \frac{p^2 + b^2}{h^2}$

$\because$ In right angle ΔABC
$p^2 + b^2 = h^2$

then $\sin^2\theta + \cos^2\theta$

$= \frac{h^2}{h^2} = 1$

★ Same as we can proof all remaining results same this process

TYPE - III

If A + B = 90°,
Results

(i) sin A. secB = 1
or sinA = cosB
(ii) cos A. cosecB = 1
or secA = cosecB
(iii) tanA. tanB = 1
or tanA = cotB
(iv)cotA. cotB = 1
(v)$\sin^2 A + \sin^2 B = 1$
(vi)$\cos^2 A + \cos^2 B = 1$

Proof

(i) **sin A. secB = 1**
A + B = 90° (given)
Then, B = 90 – A
Now, sinA.sec.(90 – A)
$\Rightarrow$ sinA.cosec.A

$\Rightarrow \sin A \times \frac{1}{\sin A} = 1$

★ Same as we can proof all remaining results same this process

★ **And their vice-versa are also true.**
when sinA. secB = 1,
then we can say A + B = 90°

Ex. 16 The value of (sin25°. sec65°) is equal to:-

Sol. 25° + 65° = 90°

$\begin{pmatrix} \text{If } A + B = 90° \\ \text{sinA. secB} = 1 \end{pmatrix}$

So, sin25°. sec65° = 1

Ex.17 The value of $(\tan 23^0 \times \tan 67^0)$ is equal to :-

Sol. 23° + 67° = 90°

$\begin{pmatrix} \text{If } A + B = 90° \\ \text{tanA. tanB} = 1 \end{pmatrix}$

So, $\tan 23^0 . \tan 67^0 = 1$

Ex.18 The value of
tan10°. tan25°. tan 65°. tan 80° is

Sol.

tan10° tan25° tan65° tan80°

= 1

Ex.19 If sin(3x– 6) = cos (6x – 3) find the value of *x*.

Sol. 3x– 6 + 6x – 3 = 90°
9x = 99°
x = 11

$\begin{pmatrix} \text{If A+ B = 90,} \\ \text{then sinA = cosB} \end{pmatrix}$

Ex.20 The value of cos40°.cosec50°

Sol. 40° + 50° = 90°

$\begin{pmatrix} \text{If } A + B = 90° \\ \text{cosA. cosecB} = 1 \end{pmatrix}$

So, cos40°. cosec50° = 1

Ex.21 If tan 2θ tan3θ = 1
find the value of θ

Sol. $2\theta + 3\theta = 90°$
$5\theta = 90°$
$\theta = 18°$

$\begin{pmatrix} \text{If } A + B = 90° \\ \text{tanA. tanB} = 1 \end{pmatrix}$

Ex.22 If cot2θ cot3θ = 1
find the value of

$\sin\frac{5\theta}{2} . \cos\frac{5\theta}{2}$

Sol. $2\theta + 3\theta = 90°$
$5\theta = 90°$

$\begin{pmatrix} \text{If } A + B = 90° \\ \text{tanA. tanB} = 1 \end{pmatrix}$

Now, $\sin\frac{5\theta}{2} . \cos\frac{5\theta}{2}$

put the value of 5θ

$= \sin\frac{90}{2} . \cos\frac{90}{2}$

= sin45°. cos45°

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

Ex.23 If sin (x + 4)° sec (x – 4)° = 1

find the value of $\tan\frac{2x}{3}$

Sol. sin(x + 4)° sec (x – 4)° = 1
x + 4 + x – 4 = 90°
2x = 90°
x = 45
Now,

$\tan\frac{2x}{3}$

put of value of x

$= \tan\frac{2 \times 45}{3} = \tan\frac{90°}{3}$

$= \tan 30° = \frac{1}{\sqrt{3}}$

Ex.24 If cos(90 – θ) = sin (3θ – 5)
find the value of θ

Sol. cos(90 – θ) = sin(3θ – 50)
cos(90 – θ) cosec (3θ – 50) = 1
90 – θ + 3 θ – 50 = 90°

$\begin{pmatrix} \text{if cosA.cosecB} = 1 \\ A + B = 90° \end{pmatrix}$

$2\theta = 50°$
$\theta = 25°$

Ex.25 If cot (x – 50) = tan (80 – 2x) find the value of tanx + sinx

Sol. cot (x – 50) = tan (80 – 2x)

cot (x – 50) cot (80 – 2x) = 1

$\left(\begin{array}{l}\text{if } \cot A.\cot B = 1 \\ \text{then } A + B = 90^\circ\end{array}\right)$

x – 50 + 80 – 2x = 90

– x + 30 = 90

x = – 60°

Now, tanx + sinx

= tan(–60)° + sin (–60)°

= – tan60° – sin60°

$\left(\begin{array}{l}\tan(-\theta) = -\tan\theta \\ \sin(-\theta) = -\sin\theta\end{array}\right)$

= – (tan60° + sin60°)

$= -\left(\sqrt{3} + \frac{\sqrt{3}}{2}\right) = \frac{-3\sqrt{3}}{2}$

TYPE–IV

Sum and Difference Formula

(i) sin (A+B) = sin A. cos B+cos A sin B

(ii) sin (A – B) = sin A. cos B – cos A sin B

(iii) cos (A+B) = cos A. cos B – sin A sin B

(iv) cos (A – B) = cos A. cos B+sin A sin B

(v) 2 sin A. cos B = sin (A+B)+sin (A-B)

(vi) 2 cos A. sin B = sin (A+B)-sin (A-B)

(vii) 2 sin A. sin B = cos (A-B)-cos (A+B)

(viii) 2 cos A. cos B = cos (A+B)+cos (A-B)

(ix) $\sin^2 A - \sin^2 B$ = sin (A+B) . sin (A-B)

(x) $\cos^2 A - \cos^2 B$ = cos (A+B) . cos (A-B)

Tangent Formulae

(i) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$

(ii) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A.\tan B}$

(iii) $\cot(A+B) = \frac{\cot A\cdot\cot B - 1}{\cot A + \cot B}$

(iv) $\cot(A-B) = \frac{\cot B\cdot\cot A + 1}{\cot B - \cot A}$

(v) $\tan(45+\theta) = \frac{1+\tan\theta}{1-\tan\theta}$

$= \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

(vi) $\tan(45-\theta) = \frac{1-\tan\theta}{1+\tan\theta}$

$= \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

Ex.26 Find the value of the following

(i) sin75° (ii) cos75° (iii) tan15° (iv) tan75°

Sol.(i) sin75°

sin(45° + 30°)

= sin45° cos30° + cos45°sin30°

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

★ $\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}} = \cos 15^\circ$

(ii) cos75°

cos(45° + 30°)

= cos45°cos30° –sin45°sin30°

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

★ $\cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}} = \sin 15^\circ$

(iii) tan15°

tan(45 – 30)

$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ.\tan 30^\circ}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

★ $\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \cot 75^\circ$

(iv) tan75°

tan(45 + 30)

$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

★ $\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \cot 15^\circ$

Trignometric Ratios of Specific Angles

(i) $\sin 18^0 = \left(\frac{\sqrt{5}-1}{4}\right)$

(ii) $\cos 18^0 = \frac{\sqrt{10+2\sqrt{5}}}{4}$

(iii) $\cos 36^0 = \left(\frac{\sqrt{5}+1}{4}\right)$

(iv) $\sin 36^0 = \frac{\sqrt{10-2\sqrt{5}}}{4}$

(vii) $\sin 22\frac{1^0}{2} = \frac{\sqrt{2-\sqrt{2}}}{2}$

(viii) $\cos 22\frac{1^0}{2} = \frac{\sqrt{2+\sqrt{2}}}{2}$

Ex.27 The value of $\frac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ}$ is

Sol. $\frac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ}$

= tan (45 – 15)

$= \tan 30^\circ = \frac{1}{\sqrt{3}}$

Ex.28 The value of tan40° + 2tan10° is equal to

(a) tan40° (b) cot40°

(c) sin40° (d) cos40°

Sol. We know,

40° + 10° = 50°

both sides take tan

tan(40° + 10°) = tan50°

$\Rightarrow \frac{\tan 40^\circ + \tan 10^\circ}{1 - \tan 40^\circ.\tan 10^\circ} = \tan 50^\circ$

$\Rightarrow$ tan40° + tan10°

= tan50° –

$\underbrace{\tan 50^\circ.\tan 40^\circ}_{\Leftrightarrow\ 1}.\tan 10$

$\left(\begin{array}{l}\therefore \tan A.\tan B = 1 \\ \text{if } A + B = 90\end{array}\right)$

$\Rightarrow$ tan40° + tan10°

= tan50° – tan10°

$\Rightarrow$ tan40° + 2tan10° = tan50°

Now, tan50° = tan.(90° – 40°)

= cot40°

Ex.29 The value of

$\frac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ}$ is equal to

(a) tan33°.cot53°

(b) tan53°.cot37°

(c) tan33°.cot57°

(d) tan57°.cot37°

Sol. $\frac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ}$

$\Rightarrow \frac{\tan 57^\circ + \cot 37^\circ}{\tan(90-57) + \cot 53^\circ}$

$$\Rightarrow \quad \frac{\tan 57^\circ + \frac{1}{\tan 37^\circ}}{\cot 57^\circ + \cot(90-37)^\circ}$$

$$\Rightarrow \quad \frac{\tan 57^\circ + \frac{1}{\tan 37^\circ}}{\frac{1}{\tan 57^\circ} + \tan 37^\circ}$$

$$\Rightarrow \quad \frac{\frac{(\tan 57^\circ \tan 37^\circ + 1)}{\tan 37^\circ}}{\frac{(\tan 57^\circ \tan 37^\circ + 1)}{\tan 57^\circ}}$$

$$\Rightarrow \quad \frac{1}{\tan 37^\circ} \times \tan 57^\circ$$

$\Rightarrow$ $\tan 57^\circ . \cot 37^\circ$

TYPE-V

Use of componendo and dividendo-

If $\frac{x}{y} = \frac{a}{b}$, Then $\frac{x}{y} = \frac{a}{b}$

$$\frac{x+y}{x-y} = \frac{a+b}{a-b}$$

Proof $\frac{x}{y} = \frac{a}{b}$

Add 1 in both side.

$$\frac{x}{y} + 1 = \frac{a}{b} + 1$$

$$\frac{x+y}{y} = \frac{a+b}{b} \quad \text{..........(i)}$$

subtract 1 in both side.

$$\frac{x}{y} - 1 = \frac{a}{b} - 1$$

$$\frac{x-y}{y} = \frac{a-b}{b} \quad \text{.........(ii)}$$

(i) / (ii)

$$\frac{x+y}{x-y} = \frac{a+b}{a-b}$$

Ex.30 If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 9$ find the value $\tan\theta$ and $\cos\theta$

Sol. $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{9}{1}$

Apply C & D

$$\Rightarrow \quad \frac{(\sin\theta + \cos\theta) + (\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta) - (\sin\theta - \cos\theta)}$$

$$= \frac{9+1}{9-1}$$

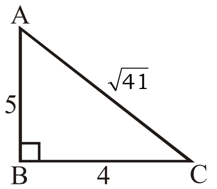
$$\Rightarrow \quad \frac{2\sin\theta}{2\cos\theta} = \frac{10}{8}$$

$$\tan = \frac{5}{4}$$

Now, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

Hypotenuse $= \sqrt{(5)^2 + (4)^2}$

$= \sqrt{41}$



Then, $\cos\theta = \frac{b}{h} = \frac{4}{\sqrt{41}}$

Ex.31 If $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$, then find The value of $\sin\theta$

Sol. $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$

Apply C & D

$$\Rightarrow \quad \frac{(\sec\theta + \tan\theta) + (\sec\theta - \tan\theta)}{(\sec\theta + \tan\theta) - (\sec\theta - \tan\theta)}$$

$$= \frac{5+3}{5-3}$$

$$\Rightarrow \quad \frac{2\sec\theta}{2\tan\theta} = \frac{8}{2}$$

$$\Rightarrow \quad \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = 4 \quad \Rightarrow \quad \frac{1}{\sin\theta} = 4$$

So, $\sin\theta = \frac{1}{4}$

Ex. 32 If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$, find the value of $\frac{\tan^2\theta + 1}{\tan^2\theta + 1}$

Sol. $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$,

Apply C and D

$\tan\theta = 9$

Now, $\frac{\tan^2\theta + 1}{\tan^2\theta + 1} = \frac{(9)^2 + 1}{(9)^2 - 1}$

$$= \frac{82}{80} = \frac{41}{40}$$

Ex. 33 If $\tan\theta = \frac{4}{3}$, then the value of $\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$ is

Sol. $\tan\theta = \frac{4}{3}$

$$\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{2 \times 2}{3}$$

$$\frac{3\sin\theta}{2\cos\theta} = \frac{2}{1}$$

Apply C and D

$$\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta} = \frac{2+1}{2-1}$$

$$\boxed{\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta} = 3}$$

Alternate:-

$$\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$$

divide all terms by $\cos\theta$

$$\frac{3\tan\theta + 2}{3\tan\theta - 2}$$

$\therefore$ $\tan\theta = \frac{4}{3}$ (given)

$$= \frac{3 \times \frac{4}{3} + 2}{3 \times \frac{4}{3} - 2} = \frac{4+2}{4-2} = \frac{6}{2} = 3$$

Ex.34 If $2\cot\theta = 3$, Then find the value of $\frac{2\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

Sol. $2\cot\theta = 3$

$$\cot\theta = \frac{3}{2}$$

$$\frac{\cos\theta}{\sin\theta} = \frac{3}{2}$$

$$\frac{2\cos\theta}{\sin\theta} = \frac{3}{1}$$

Apply C and D

$$\frac{2\cos\theta+\sin\theta}{2\cos\theta-\sin\theta} = \frac{3+1}{3-1}$$

$$\frac{2\cos\theta+\sin\theta}{2\cos\theta-\sin\theta} = \frac{4}{2} = 2$$

So, $$\boxed{\frac{2\cos\theta-\sin\theta}{2\cos\theta+\sin\theta} = \frac{1}{2}}$$

Alternate:-

$$\frac{2\cos\theta-\sin\theta}{2\cos\theta+\sin\theta}$$

Divide all terms (in numerator and in denominator) by sin. θ

$$\frac{2\cot\theta-1}{2\cot\theta+1}$$

$\because$ cot θ = $\frac{3}{2}$ (given)

$$\frac{2\times\frac{3}{2}-1}{2\times\frac{3}{2}+1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

TYPE - VI

Some pythagorean natural number will help in solving the problem on trigonometric ratio angle.

pythagorean theorem

$(\text{Base})^2$ + $(\text{Perpendicular})^2$ = $(\text{Hypotenuse})^2$

$3^2 + 4^2 = 5^2$, $6^2 + 8^2 = 10^2$,
$5^2 + 12^2 = 13^2$, $10^2 + 24^2 = 26^2$,
$8^2 + 15^2 = 17^2$, $7^2 + 24^2 = 25^2$,
$20^2 + 21^2 = 29^2$, $9^2 + 40^2 = 41^2$, etc.

Ex.35 If $\sin\theta + \cos\theta = \frac{17}{13}$ find the value of $\sin\theta .\cos\theta$

Sol. $\sin\theta + \cos\theta = \frac{17}{13}$

squaring of both side

$\Rightarrow$ $(\sin\theta + \cos\theta)^2 = \left(\frac{17}{13}\right)^2$

$\Rightarrow$ $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{289}{169}$

$\Rightarrow$ $1 + 2\sin\theta\cos\theta = \frac{289}{169}$

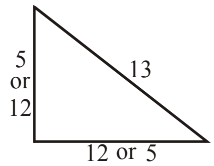
$\Rightarrow$ $2\sin\theta\ \cos\theta = \frac{289}{169} - 1$

$\Rightarrow$ $2\sin\theta\cos\theta = \frac{289-169}{169}$

$\Rightarrow$ $2\sin\theta\cos\theta = \frac{120}{169}$

$\Rightarrow$ $\sin\theta\cos\theta = \frac{60}{169}$

Alternate:-



$$\sin\theta + \cos\theta = \frac{17}{13} \rightarrow \frac{p+b}{h}$$

$$\downarrow \quad \downarrow$$

$$\frac{p}{h} \quad \frac{b}{h}$$

Apply pythagorean here hypotenuse is 13, Then other sides of right angle triangle will be 5 and 12.

Now, Check

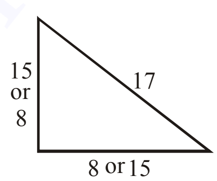
$$\frac{5}{13}+\frac{12}{13}=\frac{17}{13}$$

But we cannot find exact value of base and perpendicular, here no affect of value of $\sin\theta$ and $\cos\theta$. This question because both are product.

Hence, $\sin\theta\ \cos\theta = \frac{5}{13}\times\frac{12}{13}=\frac{60}{169}$

Ex.36 If $\sin\theta + \cos\theta = \frac{23}{17}$, find the value of $\sin\theta .\cos\theta$

Sol.



$$\sin\theta + \cos\theta = \frac{23}{17}$$

$$\downarrow \quad \downarrow$$

$$\frac{p}{h} + \frac{b}{h} = \frac{p+b}{h}$$

so, h $\rightarrow$ 17

Apply pythagorean sides here hypotenuse is 17, then other sides 8 and 15

Now, Check $\frac{8}{17}+\frac{15}{17}=\frac{23}{17}$

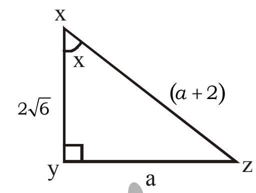
Hence, $\sin\theta .\cos\theta = \frac{8}{17}\times\frac{15}{17}=\frac{120}{289}$

Ex. 37 In a Δxyz, $\angle$y = 90°

xy = $2\sqrt{6}$ and xz – yz = 2

Find the value of secx + tanx = ?

Sol.



Let yz = a,

Then xz = 2 + yz = 2 + a

Apply pythagorean theorem

$\Rightarrow$ $(2\sqrt{6})^2 + (a)^2 = (a + 2)^2$

$\Rightarrow$ $(2\sqrt{6})^2 = (a + 2)^2 - (a)^2$

$\Rightarrow$ 24 = (a + 2 – a) (a + 2 + a)

$\Rightarrow$ 24 = 2 (2a + 2)

$\Rightarrow$ 24 = 4(a + 1)

$\Rightarrow$ 6 = a + 1

$\Rightarrow$ a = 5

Hence, xz = (5 + 2) = 7

yz = 5

$\Rightarrow$ $\text{secx} + \text{tanx} = \frac{7}{2\sqrt{6}} + \frac{5}{2\sqrt{6}}$

$$= \frac{12}{2\sqrt{6}} = \frac{6}{\sqrt{6}} = \sqrt{6}$$

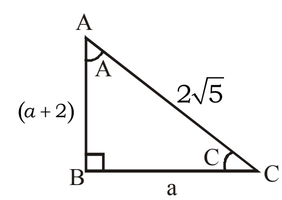
Ex. 38 In a ΔABC, $\angle$B = 90°

AB – BC = 2, and AC = $2\sqrt{5}$

find the value of

$\cos^2A - \cos^2C$ = ?

Sol.



Let the triangle of side BC = a

Then AB = BC + 2 = a + 2

Now,

$\Rightarrow$ $(a + 2)^2 + (a)^2 = (2\sqrt{5})^2$

$\Rightarrow$ $a^2 + 4 + 4a + a^2 = 20$

$\Rightarrow$ $2a^2 + 4a = 16$

$\Rightarrow$ $a^2 + 2a = 8$

$\Rightarrow$ $a^2 + 2a - 8 = 0$

$\Rightarrow$ $a^2 + 4a - 2a - 8 = 0$

$\Rightarrow$ (a + 4) (a – 2) = 0

a = –4, a = 2

side of Δ is always positive hence,

We take a = 2

Now,

$\Rightarrow$ AB = 4, BC = 2

$\Rightarrow$ $\cos^2A - \cos^2C$

$\Rightarrow$ $= \left(\frac{4}{2\sqrt{5}}\right)^2 - \left(\frac{2}{2\sqrt{5}}\right)^2$

$\Rightarrow$ $\frac{16}{20} - \frac{4}{20} = \frac{12}{20} = \frac{3}{5}$

Ex. 39 If $2\sin\alpha + 15\cos^2\alpha = 7$, $(0° < \alpha < 90°)$ find the value of $\cot\alpha$

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Sol. $2\sin\alpha + 15\cos^2\alpha = 7$,

$\Rightarrow$ $2\sin\alpha + 15(1 - \sin^2\alpha) = 7$

$\Rightarrow$ $2\sin\alpha + 15 - 15\sin^2\alpha = 7$

$\Rightarrow$ $15\sin^2\alpha - 2\sin\alpha - 8 = 0$

$\Rightarrow$ $15\sin^2\alpha - 12\sin\alpha + 10\sin\alpha - 8 = 0$

$\Rightarrow$ $(3\sin\alpha + 2)(5\sin\alpha - 4) = 0$

$\Rightarrow$ $3\sin\alpha + 2 = 0$ or $5\sin\alpha - 4 = 0$

$\Rightarrow$ $\sin\alpha = \frac{-2}{3}$ $\qquad \sin\alpha = \frac{4}{5}$

Value of α between 0° and 90° so $\sin\alpha$ is positive. Then we

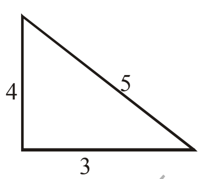
take $\sin\alpha = \frac{4}{5}$

$\sin\alpha = \frac{4}{5} = \frac{p}{h}$

$b = \sqrt{(5)^2 - (4)^2} = 3$

Hence, $\cot\alpha = \frac{b}{p} = \frac{3}{4}$

Alternate:-



We take option $\frac{3}{4}$

$\cot\alpha = \frac{3}{4} = \frac{b}{p}$

$h = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

Now, check

$2\sin\alpha + 15\cos^2\alpha = 7$

$2 \times \frac{4}{5} + 15 \times \left(\frac{3}{5}\right)^2$

$= \frac{8}{5} + 15 \times \frac{9}{25}$

$= \frac{8}{5} + \frac{27}{5} = \frac{35}{5} = 7$

L.H.S = R.H.S

So, $\cot\alpha = \frac{3}{4}$

Ex. 40 If $2 - \cos^2\theta = 3\sin\theta\cos\theta$, find the value of $\tan\theta$

(a) $\frac{1}{2}$ (b) 0 (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

Sol. $2 - \cos^2\theta = 3\sin\theta\cos\theta$

divide by $\cos^2\theta$ both sides

$\Rightarrow$ $\frac{2 - \cos^2\theta}{\cos^2\theta} = \frac{3\sin\theta\cos\theta}{\cos^2\theta}$

$\Rightarrow$ $2\sec^2\theta - 1 = 3\tan\theta$

$\Rightarrow$ $2(1 + \tan^2\theta) - 1 = 3\tan\theta$

$\Rightarrow$ $2 + 2\tan^2\theta - 1 = 3\tan\theta$

$\Rightarrow$ $2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow$ $(2\tan\theta - 1)(\tan\theta - 1) = 0$

$\Rightarrow$ $2\tan\theta - 1 = 0$, $\tan\theta - 1 = 0$

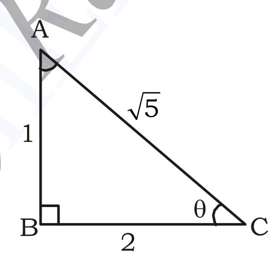
$\Rightarrow$ $\tan\theta = \frac{1}{2}$ or 1

So, option (a) is correct

$\tan\theta = \frac{1}{2}$

Alternate:-

Take options (a)



So, $\tan\theta = \frac{1}{2} = \frac{p}{b}$

then, $h = \sqrt{(2)^2 + (1)^2}$

$h = \sqrt{5}$

Now,

$2 - \cos^2\theta = 3\sin\theta\cos\theta$

$2 - \left(\frac{2}{\sqrt{5}}\right)^2 = 3 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$

$\frac{6}{5} = \frac{6}{5}$

L.H.S = R.H.S

$\therefore \tan\theta = \frac{1}{2}$

TYPE -VII

Function and Inverse function →

(a) **If $\sin\theta + \operatorname{cosec}\theta = 2$ then $\sin\theta = \operatorname{cosec}\theta = 1$**

$\because$ **$\sin^n\theta + \operatorname{cosec}^n\theta = 2$**
n → natural no.

Ex.41 If $\sin\theta + \operatorname{cosec}\theta = 2$ find the value of $\sin^{100}\theta + \operatorname{cosec}^{100}\theta$

Sol. $\sin\theta + \operatorname{cosec}\theta = 2$

Then, $\sin\theta = \operatorname{cosec}\theta = 1$

so, $\sin^{100}\theta + \operatorname{cosec}^{100}\theta = 1$
$= (1)^{100} + (1)^{100} = 2$

(b) **If $\cos\theta + \sec\theta = 2$ then $\cos\theta = \sec\theta = 1$**

$\because$ **$\cos^n\theta + \sec^n\theta = 2$**

Ex.42 If $\cos\theta + \sec\theta = 2$, find the value of $\cos^{10}\theta + \sec^{10}\theta = ?$

Sol. $\cos\theta + \sec\theta = 2$

$\cos\theta = \sec\theta = 1$

Then, $\cos^{10}\theta + \sec^{10}\theta$
$= (1)^{10} + (1)^{10} = 1+1 = 2$

(c) **If $\tan\theta + \cot\theta = 2$ so $\tan\theta = \cot\theta = 1$**
$\tan^n\theta + \cot^n\theta = 2$

Ex.43 If $\tan\theta + \cot\theta = 2$ find the value of $\tan^{50}\theta + \cot^{60}\theta$

Sol. $\tan\theta + \cot\theta = 2$

$\tan\theta = \cot\theta = 1$

$\tan^{50}\theta + \cot^{60}\theta = (1)^{50} + (1)^{60} = 1+1 = 2$

(d) **If sinA + cosB = 2 Then A = 90° B = 0°**

Ex.44 If sinA + cosB = 2, then find the value of $\tan\frac{(A+B)}{2}$

Sol.
$$\begin{array}{ccc} \text{SinA} & + & \text{cosB}=2 \\ \downarrow & & \downarrow \\ 1 & & 1 \\ A & = & 90° \\ B & = & 0° \end{array}$$

Now, $\tan\frac{(A+B)}{2}$

$= \tan\frac{(90+0)}{2} = \tan 45° = 1$

(e) **If sinA + cosB = 0 then A = 0°, B = 90°**

Ex.45 If sinA + cosB + sinC = 3, then find the value of $\cot\frac{(A+B+C)}{3}$

Sol. sinA + cosB + sinC=3

$$\underset{1}{\downarrow}\quad \underset{1}{\downarrow}\quad \underset{1}{\downarrow}$$

∵ A = 90°
B = 0°
C = 90°

$$\cot\frac{(90+0+90)}{3} = \cot 60° = \frac{1}{\sqrt{3}}$$

Ex.46 If θ is acute and $\tan\theta + \cot\theta = 2$, then $\tan^7\theta + \cot^9\theta$ is equal to:

(a) $\sqrt{3}$ (b) 3 (c) 2 (d) 4

Sol. $\tan\theta + \cot\theta = 2$

$\tan\theta = \cot\theta = 1$

So, $\therefore \tan^7\theta + \cot^9\theta = 1 + 1 = 2$

Ex.47 If $\tan(x+y) = \sqrt{3}$ and $\cot(x-y) = \sqrt{3}$, then what are the smallest positive value of x and y respectively?

(a) 45^0, 30^0

(b) 15^0, 60^0

(c) 45^0, 15^0

(d) 30^0, 45^0

Sol. $\tan(x + y) = \sqrt{3}$

then $x + y = 60°$ .....(i)

$\cot(x - y) = \sqrt{3}$

Then $x - y = 30°$ .....(ii)

from (i) and (ii)

$x = 45°$ & $y = 15°$

Ex.48 If $2\cos 3\theta_1 = 1$ and $2\sin 2\theta_2 = \sqrt{3}$, then what will be the value of θ_1 and θ_2

(a) $30^0, 20^0$ (b) $60^0, 40^0$
(c) $20^0, 30^0$ (d) $45^0, 45^0$

Sol. $2\cos 3\theta_1 = 1 \Rightarrow \cos 3\theta_1 = \frac{1}{2} = \cos 60°$

$\Rightarrow 3\theta_1 = 60° \Rightarrow \theta_1 = 20°$

$$2\sin 2\theta_2 = \sqrt{3} \Rightarrow \sin 2\theta_2 = \frac{\sqrt{3}}{2} = \sin 60^0$$

$$\Rightarrow 2\theta_2 = 60^0 \Rightarrow \theta_2 = 30^0$$

Ex. 49 If $\sin\alpha + \cos\beta = 2 (0° \le \beta \le \alpha \le 90°)$, then $\sin\left(\frac{2\alpha + 2\beta}{3}\right) =$

(a) $\sin\frac{\alpha}{2}$ (b) $\cos\frac{\alpha}{3}$

(c) $\sin\frac{\alpha}{3}$ (d) $\cos\frac{2\alpha}{3}$

Sol. $\sin\alpha + \cos\beta = 2$ $\sin\alpha \le 1$; $\cos\beta \le 1$

$\Rightarrow \alpha = 90°$; $\beta = 0°$

$$\therefore \sin\frac{(2\alpha + \beta)}{3} = \sin\left(\frac{180°}{3}\right)$$

$$= \sin 60° = \sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\alpha}{3} = \cos 30° = \frac{\sqrt{3}}{2}$$

Ex.50 If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = ?$

(a) 0 (b) 1 (c) 2 (d) 3

Sol. $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

$\Rightarrow \sin\theta_1 = \sin\theta_2 = \sin\theta_3$

$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \frac{\pi}{2}$

$\therefore \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

Ex.51 If $\cos^2\alpha + \cos^2\beta = 2$, then the value of $\tan^3\alpha + \sin^5\beta$ is :

(a) – 1 (b) 0 (c) 1 (d) $\frac{1}{\sqrt{3}}$

Sol. $\cos^2\alpha + \cos^2\beta = 2$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta = 2$

$\Rightarrow \sin^2\alpha + \sin^2\beta = 0$

$\Rightarrow \sin\alpha = \sin\beta = 0$

$\Rightarrow \alpha = \beta = 0$

$\therefore \tan^3\alpha + \sin^5\beta = 0$

TYPE - VIII

Series Base ⟶

Ex. 52 The value of cos1°, cos2°,cos3°...... $\cos 179^0$ is:-

(a) 1 (b) -1 (c) 2 (d) 0

Sol. $\because \cos 90^0 = 0$

$\therefore \cos 1^0.\cos 2^0................\cos 179^0 = 0$

Ex. 53 The value of tan1° tan2°tan3° ....tan 89° is :

(a) 1 (b) 0 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

Sol. tan 1°.tan 2°. tan3°......tan 45° .....tan88°tan89°

= (tan 1°.tan 89°) (tan2°. tan88°) .... tan 45°

= (tan 1°.cot 1°). (tan 2°. cot2°) .... tan 45° = 1

[∵ tan (90° – θ) = cot θ ,tan θ .cot θ = 1]

Ex.54 The value of : cos20° + cos40° + cos° + ... + cos160° + cos180° is:-

(a) 1 (b) -1

(c) 0 (d) ½

Sol. $\cos(180^0 - \theta) = -\cos\theta$

$\therefore \cos 160^0 = \cos(180^0 - 20^0) = -\cos 20^0$

similarly

$\Rightarrow$ cos140° = – cos40°,
cos120° = – cos60°
cos100° = – cos80°

Now,

$$\Rightarrow \left(\cos 20^0 + \cos 160^0\right) + \left(\cos 40^0 + \cos 140^0\right) + \left(\cos 60^0 + \cos 120^0\right) + \left(\cos 80^0 + \cos 100^0\right) + \cos 180^0$$

$$= \left(\cos 20^0 - \cos 20^0\right) + \left(\cos 40^0 - \cos 40^0\right) + \left(\cos 60^0 - \cos 60^0\right) + \left(\cos 80^0 - \cos 80^0\right) + \cos 180^0$$

$\Rightarrow \cos 180^0 = -1$

Ex. 55 sin²5° + sin²6° + ...sin²84° + sin²85° = ?

(a) $39\frac{1}{2}$ (b) $40\frac{1}{2}$

(c) 40 (d) $39\frac{1}{\sqrt{2}}$

Sol. Let the number of terms be n, then By $t_n = a + (n - 1)d$

Here,

$\Rightarrow$ a = 5, d = 1

$\Rightarrow$ 85 = 5 + (n – 1) 1

$\Rightarrow$ n – 1 = 85 – 5 = 80

$\Rightarrow$ n = 81

$\therefore$ $\sin^2 5° + \sin^2 6° + ... + \sin^2 45° + ... + \sin^2 84° + \sin^2 85°$

$= (\sin^2 5° + \sin^2 85°) + (\sin^2 6° + \sin^2 84°) + ...$ + to 40 terms + $\sin^2 45°$

$= (\sin^2 5° + \cos^2 5°) + (\sin^2 6° + \cos^2 6) + ...$ + to 40 terms + $\sin^2 45°$

$$= 40 + \frac{1}{2} = 40\frac{1}{2}$$

$$\begin{bmatrix} \sin(90° - \theta) = \cos\theta \\ \sin^2\theta + \cos^2\theta = 1 \end{bmatrix}$$

Ex.56 The value of sin10° + sin20° + ... + sin340° + sin350°

Sol. sin10° + sin20° + ... + sin340° + sin350°

$\Rightarrow$ sin(360° – 350°) + sin(360° – 340°) + ...sin180° ....sin340° + sin350°

$\Rightarrow$ – sin350° – sin340°....+sin180° +...sin340° + sin350° = 0

[sin{360° – θ} = – sinθ, sin180° = 0])

Ex.57 The value of $\cos^2 1° + \cos^2 3° + ... + \cos^2 89° + \cos^2 90°$

Sol. $\cos^2 1° + \cos^2 3° + \cos^2 5° ...... + \cos^2 89° + \cos^2 90°$

$$n = \frac{89 - 1}{2} + 1 = 45$$

$$\text{sum} = \frac{45}{2}$$

$$\therefore \frac{45}{2} + \underbrace{\cos^2 90°}_{0} = \frac{45}{2}$$

Ex.58 The value of

$$\sin^2\frac{\pi}{40} + \sin^2\frac{2\pi}{40} + \sin^2\frac{3\pi}{40} + ... \sin^2\frac{20\pi}{40}$$

Sol. $$\sin^2\frac{\pi}{40} + \sin^2\frac{2\pi}{40} + \sin^2\frac{3\pi}{40} + ... + \sin^2\frac{19\pi}{40} + \sin^2\frac{20\pi}{40}$$

$$\Rightarrow \sin^2\frac{\pi}{40} + \sin^2\frac{2\pi}{40} ...... \sin^2\frac{18\pi}{40} + \sin^2\frac{19\pi}{40} + \sin^2\frac{20\pi}{40}$$

$$\Rightarrow \left(\sin^2\frac{\pi}{40} + \sin^2\frac{19\pi}{40}\right) + \left(\sin^2\frac{2\pi}{40} + \sin^2\frac{18\pi}{40}\right) ..... + \sin^2\frac{20\pi}{40}$$

Here,

$$\frac{\pi}{40} + \frac{19\pi}{40} = \frac{20\pi}{40} = \frac{\pi}{2}$$

So, sum of 2 terms = $\sin^2\frac{\pi}{2} = 1$

n = 19

$$\text{sum} = \frac{19}{2} + \sin^2\frac{20\pi}{2}$$

$$= \frac{19}{2} + \sin^2 90°$$

$$\Rightarrow \frac{19}{2} + 1 = \frac{21}{2}$$

Ex.59 If $1 + \sin x + \sin^2 x + \sin^3 x + ... \infty = 4 + 2\sqrt{3}$, find the value of x

Sol. It is a G.P Series

then, $S_\infty = \frac{a}{1 - r}$

a = 1, r = sin$x$

$$\Rightarrow \frac{1}{1 - \sin x} = 4 + 2\sqrt{3} \times \frac{(4 - 2\sqrt{3})}{4 - 2\sqrt{3}}$$

$$\Rightarrow \frac{1}{1 - \sin x} = \frac{4}{4 - 2\sqrt{3}}$$

$$\Rightarrow \frac{1}{1 - \sin x} = \frac{\frac{4}{4}}{\frac{4}{4} - \frac{2\sqrt{3}}{4}}$$

(divide by 4 all terms)

$$\Rightarrow \frac{1}{1 - \sin x} = \frac{1}{1 - \frac{\sqrt{3}}{2}}$$

comparing both equation

$$\sin x = \frac{\sqrt{3}}{2}$$

sin x = sin60°

x = 60°

TYPE- IX

(A) $$\boxed{\begin{array}{c} \sin^2\theta + \cos^2\theta = 1 \\ \text{or} \\ \sin^2\theta = 1 - \cos^2\theta \\ \text{or} \\ \cos^2\theta = 1 - \sin^2\theta \end{array}}$$

Ex. 60 What is the value of $\sin^2 1000° + \cos^2 1000°$?

(a) 1000 (b) 100 (c) 10 (d) 1

Sol.(d) $\sin^2 1000° + \cos^2 1000° = 1$

for every value of θ in $\sin^2\theta + \cos^2\theta$ will be 1

Ex. 61 If $\sin^2 60° + \cos^2(3x - 9°) = 1$ Then value of x is

Sol. $\sin^2 60° + \cos^2(3x - 9)° = 1$

This is similar to $\sin^2\theta + \cos^2\theta$

$\Rightarrow$ So,

60° = 3x – 9

69° = 3x

x = 23°

Ex.62 If $3\sin^2\alpha + 7\cos^2\alpha = 4$, then the value of tan α is (where $0 < \alpha < 90°$):

(a) $\sqrt{2}$ (b) $\sqrt{5}$ (c) $\sqrt{3}$ (d) $\sqrt{6}$

Sol.(a) $3\sin^2\alpha + 7(1 - \sin^2\alpha) = 4$

$\Rightarrow 3\sin^2\alpha + 7 - 7\sin^2\alpha = 4$

$\Rightarrow 7 - 4\sin^2\alpha = 4$

$$\Rightarrow 4\sin^2\alpha = 3 \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$$

So, α = 60°

$\tan\alpha = \tan 60° = \sqrt{3}$

Ex.63 If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $2\cos^2\theta - 1$ is :

(a) 0 (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

Sol.(c) $$\cos^4\theta - \sin^4\theta = \frac{2}{3}$$

$$\Rightarrow (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$$

$$\Rightarrow \cos^2\theta - \sin^2\theta = \frac{2}{3}$$

$$\Rightarrow \cos^2\theta - (1 - \cos^2\theta) = \frac{2}{3}$$

$$\Rightarrow 2\cos^2\theta - 1 = \frac{2}{3}$$

Ex.64 $\sin\theta + \sin^2\theta = 1$

Find the value of $\cos^2\theta + \cos^4\theta$

Sol. $\sin\theta + \sin^2\theta = 1$

$\sin\theta = 1 - \sin^2\theta$

$\sin\theta = \cos^2\theta$

Now, $\cos^2\theta + \cos^4\theta$

Put the value $\cos^2\theta$

$\cos^2\theta + (\cos^2\theta)^2$

$\sin\theta + \sin^2\theta = 1$ (given)

Ex.65 If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x - 1$ is equal to :

(a) 2 (b) 1 (c) 0 (d) –1

Sol.(c) $\sin x + \sin^2 x = 1$

$\Rightarrow \sin x = 1 - \sin^2 x = \cos^2 x$

$\therefore \sin^2 x = \cos^4 x$

Now, $\cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x) - 1$

$= (\cos^4 x + \cos^2 x)^3 - 1 = (\sin^2 x + \sin x)^3 - 1$

$= 1^3 - 1 = 0$

Ex.66 If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, Find the value of $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = ?$

Sol. $\sin\theta + \sin^2\theta + \sin^3\theta = 1$

$\Rightarrow \sin\theta + \sin^3\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta(1 + \sin^2\theta) = \cos^2\theta$

$\Rightarrow \sin\theta(1 + 1 - \cos^2\theta) = \cos^2\theta$

$\Rightarrow \sin\theta\ (2 - \cos^2\theta) = \cos^2\theta$

Squaring. (for making $\sin\theta$ into $\cos\theta$)

$\Rightarrow \sin^2\theta(2 - \cos^2\theta)^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)\ [4 + \cos^4\theta - 4\cos^2\theta] = \cos^4\theta$

$\Rightarrow 4 + \cos^4\theta - 4\cos^2\theta - 4\cos^2\theta - \cos^6\theta + 4\cos^4\theta = \cos^4\theta$

$\Rightarrow -\cos^6\theta + 4\cos^4\theta - 8\cos^2\theta = -4$

Hence,

$\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

Ex.67 If $\cos\theta + \cos^2\theta = 1$, Find the value of $\sin^8\theta + 2\sin^6\theta + \sin^4\theta$

Sol. $\cos\theta = 1 - \cos^2\theta$

$\cos\theta = \sin^2\theta$

squaring both side

$\cos^2\theta = \sin^4\theta$ ......(i)

Now, $\sin^8\theta + 2\sin^6\theta + \sin^4\theta$

$\underbrace{\sin^8\theta}_{a^2} + \underbrace{2\sin^6\theta}_{2ab} + \underbrace{\sin^4\theta}_{b^2}$

$(\sin^4\theta + \sin^2\theta)^2$

From equation (i),

So, $(\cos^2\theta + \sin^2\theta)^2 = 1$

Ex.68 If $x = (1 + \sin\theta)(1 + \sin\alpha)(1 + \sin\beta) = (1 - \sin\theta)(1 - \sin\alpha)(1 - \sin\beta)$ Then, Find the value of x

(a) $\pm\cos\theta.\cos\alpha.\cos\beta$

(b) $\pm\cos^2\theta.\cos^2\alpha.\cos^2\beta$

(c) $\pm\sec\theta.\sec\alpha.\sec\beta$

(d) $\pm\sin\theta.\sin\alpha.\sin\beta$

Sol. $x = (1 + \sin\theta)(1 + \sin\alpha)(1 + \sin\beta)$ .....(i)

$x = (1 - \sin\theta)(1 - \sin\alpha)(1 - \sin\beta)$ .....(ii)

(i) × (ii)

$x^2 = \cos^2\theta.\cos^2\alpha.\cos^2\beta$

$x = \pm\cos\theta\cos\alpha\cos\beta$

Ex. 69 If $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$, Find the value of $(m^2 + n^2)\cos^2\beta$

Sol. $\dfrac{\cos\alpha}{\cos\beta} = m$

$\cos\alpha = m\cos\beta$ **......(i)**

$\dfrac{\cos\alpha}{\sin\beta} = n$

$\cos\alpha = n\sin\beta$ **......(ii)**

from (i) and (ii) squaring both sides

$\Rightarrow m^2\cos^2\beta = n^2\sin^2\beta$

$\Rightarrow m^2\cos^2\beta = n^2(1 - \cos^2\beta)$

$\Rightarrow m^2\cos^2\beta = n^2 - n^2\cos^2\beta$

$\Rightarrow m^2\cos^2\beta + n^2\cos^2\beta = n^2$

$\Rightarrow (m^2 + n^2)\cos^2\beta = n^2$

Ex. 70 If $r\sin\theta = 1$, and $r\cos\theta = \sqrt{3}$ Then the value of $(\sqrt{3}\tan\theta + 1)$ And r

Sol. $r\sin\theta = 1$ ....(i)

$r\cos\theta = \sqrt{3}$ ....(ii)

$(i)^2 + (ii)^2$

$(r\sin\theta)^2 + (r\cos\theta)^2 = (1) + (\sqrt{3})^2$

$r^2\sin^2\theta + r^2\cos^2\theta = 1 + 3$

$r^2(\sin^2\theta + \cos^2\theta) = 4$

$r^2 = 4$

$r = 2$

(i)/(ii)

$\dfrac{r\sin\theta}{r\cos\theta} = \dfrac{1}{\sqrt{3}}$

$\tan\theta = \dfrac{1}{\sqrt{3}}$

Now, $(\sqrt{3}\tan\theta + 1)$

Put the value of $\tan\theta$

$= \sqrt{3} \times \dfrac{1}{\sqrt{3}} + 1 = 2$

Ex.71 $\dfrac{\sin\theta}{x} = \dfrac{\cos\theta}{y}$, then $\sin\theta - \cos\theta$ is equal to

(a) $x - y$

(b) $x + y$

(c) $\dfrac{x-y}{\sqrt{x^2+y^2}}$

(d) $\dfrac{y-x}{\sqrt{x^2+y^2}}$

Sol. $\dfrac{\sin\theta}{x} = \dfrac{\cos\theta}{y} = k$ (Let)

$\sin\theta = kx$ .... (i)

$\cos\theta = ky$ ....(ii)

$(i)^2 + (ii)^2$

$\sin^2\theta + \cos^2\theta = (kx)^2 + (ky)^2$

$1 = k^2(x^2 + y^2)$

$k = \dfrac{1}{\sqrt{x^2+y^2}}$

Now, $\sin\theta - \cos\theta = kx - ky = k(x - y)$

Put the value of k

$\therefore \sin\theta - \cos\theta = \dfrac{x-y}{\sqrt{x^2+y^2}}$

Ex.72 If $3\sin\theta + 4\cos\theta = 5$ find the value of $\tan\theta$.

Sol. $3\sin\theta + 4\cos\theta = 5$

$\dfrac{3}{5}\sin\theta + \dfrac{4}{5}\cos\theta = 1$ ....(i)

We know, $\sin^2\theta + \cos^2\theta = 1$ or

$\sin\theta.\sin\theta + \cos\theta.\cos\theta = 1$ .... (ii)

comparison of (i) and (ii)

Then, $\sin\theta = \dfrac{3}{5}$

$\cos\theta = \dfrac{4}{5}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{3/5}{4/5} = \dfrac{3}{4}$

Alternative:-

$\underbrace{3}_{p}\sin\theta + \underbrace{4}_{b}\cos\theta = 5$

$\tan\theta = \dfrac{p}{b} = \dfrac{3}{4}$

Ex.73 If $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$ then find the value of $\tan\theta$

Sol. $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$

$\downarrow \qquad\qquad \downarrow$

$p \qquad\qquad b$

$\tan\theta = \frac{p}{b} = \frac{a^2-b^2}{2ab}$

Ex.74 If $x\cos\theta - y\sin\theta = \sqrt{x^2+y^2}$,

and $\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{x^2+y^2}$

which statement of true

(i) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(iii) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

(iv) $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$

Sol. $x\cos\theta - y\sin\theta = \sqrt{x^2+y^2}$

$\frac{x\cos\theta}{\sqrt{x^2+y^2}} + \frac{(-y)\sin\theta}{\sqrt{x^2+y^2}} = 1$...(i)

We know,

$\cos^2\theta + \sin^2\theta = 1$ .....(ii)

comparison of (i) and (ii)

$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$

$\sin\theta = \frac{-y}{\sqrt{x^2+y^2}}$

$\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{x^2+y^2}$ ....(iii)

Put the value of $\cos\theta$ & $\sin\theta$ in (iii)

$\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2 \times \frac{1}{a^2} +$

$\left(\frac{-y}{\sqrt{x^2+y^2}}\right)^2 \times \frac{1}{b^2} = \frac{1}{x^2+y^2}$

$\frac{x^2}{(x^2+y^2)a^2} + \frac{y^2}{(x^2+y^2)b^2}$

$= \frac{1}{x^2+y^2}$

Hence, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Ex.75 If $29\tan\theta = 31$, find the value of $\frac{1+2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta}$

Sol. $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1+2\sin\theta\cos\theta$ and $(\sin\theta - \cos\theta)^2 = 1- 2\sin\theta\cos\theta$.

Now, $= \frac{1+2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta}$

$= \frac{(\sin\theta + \cos\theta)^2}{(\sin\theta - \cos\theta)^2}$

Divide $\cos\theta$ all terms

$\left(\frac{\tan\theta+1}{\tan\theta-1}\right)^2$

$\therefore \tan\theta = \frac{31}{29}$ (given)

Put the value of $\tan\theta$

$= \left(\frac{\frac{31}{29}+1}{\frac{31}{29}-1}\right)^2 = \left(\frac{60/29}{2/29}\right)^2$

$= (30)^2 = 900$

Ex.76 If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$

Find the value of $\cos^2\alpha$

Sol. $\tan\alpha = n\tan\beta$

$\frac{\sin\alpha}{\cos\alpha} = \frac{n\sin\beta}{\cos\beta}$

put $(\sin\alpha = m\sin\beta)$

$m\cos\beta = n\cos\alpha$ .....(i)

$m\sin\beta = \sin\alpha$ ....(ii)

adding after squaring (i) and (ii)

$m^2\cos^2\beta + m^2\sin^2\beta = n^2\cos^2\alpha + \sin^2\alpha$

$m^2(\cos^2\beta + \sin^2\beta) = n^2\cos^2\alpha + (1-\cos^2\alpha)$

$m^2 = n^2\cos^2\alpha + 1 - \cos^2\alpha$

$m^2 - 1 = \cos^2\alpha(n^2-1)$

$\cos^2\alpha = \frac{m^2-1}{n^2-1}$

Ex.77 If $x\sin^3\theta + y\cos^3\theta = \sin\theta.\cos\theta$. and $x\sin\theta - y\cos\theta = 0$ find the value of $x^2 + y^2$

Sol. $x\sin\theta = y\cos\theta$ .....(i) (given)

$\Rightarrow x\sin^3\theta + y\cos^3\theta = \sin\theta.\cos\theta$.

$\Rightarrow x\sin\theta.\sin^2\theta + y\cos^3\theta = \sin\theta.\cos\theta$

$\Rightarrow y\cos\theta.\sin^2\theta + y\cos^3\theta = \sin\theta.\cos\theta$

$\Rightarrow y\cos\theta(\sin^2\theta + \cos^2\theta) = \sin\theta.\cos\theta$

$\Rightarrow y\cos\theta = \sin\theta.\cos\theta$

$\Rightarrow y = \sin\theta$

Thus $x = \cos\theta$

Now, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

Ex.78 $10\sin^4\alpha + 15\cos^4\alpha = 6$, find the value of $27\operatorname{cosec}^6\alpha + 8\sec^6\alpha$.

Sol. $10\sin^4\alpha + 15\cos^4\alpha = 6 \times 1$

$\Rightarrow 10\sin^4\alpha + 15\cos^4\alpha = 6\times(\sin^2\theta + \cos^2\theta)^2$

$\Rightarrow 10\sin^4\alpha + 15\cos^4\alpha = 6(\sin^4\alpha + \cos^4\alpha + 2\sin^2\alpha\cos^2\alpha)$

divide by $\cos^4\alpha$ both sides

$\Rightarrow 10\tan^4\alpha + 15 = 6(\tan^4\alpha + 1 + 2\tan^2\alpha)$

$\Rightarrow 10\tan^4\alpha + 15 = 6\tan^4\alpha + 6 + 12\tan^2\alpha$

$\Rightarrow 4\tan^4\alpha - 12\tan^2\alpha + 9 = 0$

$\Rightarrow (2\tan^2\alpha - 3) = 0$

$\Rightarrow 2\tan^2\alpha = 3$

$\Rightarrow \tan^2\alpha = \frac{3}{2}$

Now, $27\operatorname{cosec}^6\alpha + 8\sec^6\alpha$.

$= 27(\operatorname{cosec}^2\alpha)^3 + 8(\sec^2\alpha)^3$

$= 27(1+\cot^2\alpha)^3 + 8(1+\tan^2\alpha)^3$

Put $\tan^2\alpha = \frac{3}{2}$ then $\cot^2\alpha = \frac{2}{3}$

$= 27\left(1+\frac{2}{3}\right)^3 + 8\left(1+\frac{3}{2}\right)^3 = 27$

$\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3$

$= 27 \times \frac{125}{27} + 8 \times \frac{125}{8} = 125 + 125 = 250$

Alternate:-

$10\sin^4\alpha + 15\cos^4\alpha = 6$

$\frac{10}{6}\sin^2\alpha.\sin^2\alpha + \frac{15}{6}\cos^2\alpha.\cos^2\alpha = 1$ .....(i)

$\sin^2\alpha + \cos^2\alpha = 1$ ....(ii) comparison (i) and (ii)

$$\frac{10}{6}\sin^2\alpha = 1 \text{ and } \frac{15}{6}\cos^2\alpha = 1$$

$$\sin^2\alpha = \frac{3}{5} \text{ and } \cos^2\alpha = \frac{2}{5}$$

Now, $27(\csec^2\alpha)^3 + 8(\sec^2\alpha)^3$

$$= 27\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3 = 27 \times \frac{125}{27} + 8 \times \frac{125}{8}$$

$= 125 + 125 = 250$

Ex.79 If $T_n = \sin^n\theta + \cos^n\theta$, then $\frac{T_3 - T_5}{T_1} = ?$

(a) $\sin\theta.\cos\theta$

(b) $\sin^2\theta.\cos\theta$

(c) $\sin^2\theta.\cos^2\theta$

(d) $\sin\theta.\cos^2\theta$

Sol.(c) $\frac{T_3 - T_5}{T_1}$

$$= \frac{(\sin^3\theta + \cos^3\theta) - (\sin^5\theta + \cos^5\theta)}{\sin\theta + \cos\theta}$$

$$= \frac{(\sin^3\theta - \sin^5\theta) + (\cos^3\theta - \cos^5\theta)}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta(1 - \sin^2\theta) + \cos^3\theta(1 - \cos^2\theta)}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta.\cos^2\theta + \cos^3\theta.\sin^2\theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^2\theta.\cos^2\theta(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)}$$

$$= \sin^2\theta.\cos^2\theta$$

Alternative:-

Let $\theta = 45°$, then

$$T_n = \left(\frac{1}{\sqrt{2}}\right)^n + \left(\frac{1}{\sqrt{2}}\right)^n = 2\left(\frac{1}{\sqrt{2}}\right)^n = 2^{\left(\frac{2-n}{2}\right)}$$

$$\therefore \frac{T_3 - T_5}{T_1} = \frac{2^{-1/2} - 2^{-3/2}}{2^{1/2}} = \frac{\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}}{\sqrt{2}}$$

$$= \frac{\frac{1}{2\sqrt{2}}}{\sqrt{2}} = \frac{1}{4}$$

i.e. $\frac{T_3 - T_5}{T_1} = \sin^2\theta.\cos^2\theta$

Ex.80 If $u_n = \cos^n\alpha + \sin^n\alpha$, then the value of $2u_6 - 3u_4 + 1$ is :

(a) 1 (b) 4 (c) 6 (d) 0

Sol. $u_n = \cos^n\alpha + \sin^n\alpha$

$\therefore$ $u_6 = \cos^6\alpha + \sin^6\alpha$

$(\cos^2\alpha)^3 + (\sin^2\alpha)^3$

$\Rightarrow$ $(\cos^2\alpha + \sin^2\alpha)^3 - 3\cos^2\alpha.\sin^2\alpha(\sin^2\alpha + \cos^2\alpha)$

$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$

$\Rightarrow$ $1 - 3\cos^2\alpha.\sin^2\alpha$

$u_6 = 1 - 3\cos^2\alpha.\sin^2\alpha$

$u_4 = \cos^4\alpha + \sin^4\alpha$

$\Rightarrow$ $(\cos^2\alpha + \sin^2\alpha)^2 - 2\cos^2\alpha.\sin^2\alpha$

$\Rightarrow$ $1 - 2\cos^2\alpha.\sin^2\alpha$

$\therefore$ $2u_6 - 3u_4 + 1$

$\Rightarrow$ $2(1 - 3\sin^2\alpha\cos^2\alpha) - 3(1 - 2\sin^2\alpha\cos^2\alpha) + 1$

$\Rightarrow$ $2 - 3 + 1 = 0$

Alternate:-

$u_n = \cos^n\alpha + \sin^n\alpha$

$u_6 = \cos^6\alpha + \sin^6\alpha$

$u_4 = \cos^4\alpha + \sin^4\alpha$

Let $\alpha = 0°$

Then, $u_6 = 1$ & $u_4 = 1$

Now, $2u_6 - 3u_4 + 1$

$\Rightarrow$ $2 \times 1 - 3 \times 1 + 1 = 0$

(B). If **$a\sin\theta + b\cos\theta = m$.....(i)**

$b\sin\theta - a\cos\theta = n$ ....(ii)

Then $a^2+b^2 = m^2+n^2$

Proof $(i)^2 + (ii)^2$

$\Rightarrow$ $(a\sin\theta + b\cos\theta)^2 + (b\sin\theta - a\cos\theta)^2 = m^2 + n^2$

$\Rightarrow$ $(a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta) + (b^2\sin^2\theta + a^2\cos^2\theta - 2ab\sin\theta\cos\theta) = m^2 + n^2$

$\Rightarrow$ $a^2(\sin^2\theta + \cos^2\theta) + b^2(\cos^2\theta + \sin^2\theta) = m^2 + n^2$.

$a^2 + b^2 = m^2 + n^2$

Ex.81 If $\sin\theta + \cos\theta = x$, find the value of $\sin\theta - \cos\theta$

Sol. $1 \times \sin\theta + 1 \times \cos\theta = x$

Let $1\times\sin\theta - 1\times\cos\theta = n$

use property $a^2+b^2 = m^2+n^2$

Here, $a = b = 1$, $m = x$, n(let)

$(1)^2 + (1)^2 = (x)^2 + (n)^2$

$2 - x^2 = n^2$

$n = \pm\sqrt{2 - x^2}$

So, $\sin\theta - \cos\theta = \pm\sqrt{2 - x^2}$

Ex.82 If $a\cos\theta + b\sin\theta = c$, Then find $(a\sin\theta - b\cos\theta) = ?$

Sol. $a\cos\theta + b\sin\theta = c$ ....(i)

$a\sin\theta - b\cos\theta = x$ (let) ...(ii)

$(i)^2 + (ii)^2$

$a^2 + b^2 = c^2 + x^2$

$x^2 = a^2 + b^2 - c^2$

So, $(a\sin\theta - b\cos\theta)^2 = a^2 + b^2 - c^2$

Ex.83 If $\sin\theta + \cos\theta = \frac{7}{5}$, find the value of $\cos\theta - \sin\theta = ?$

Sol. $\sin\theta + \cos\theta = \frac{7}{5}$

$\cos\theta - \sin\theta = x$ (let)

$\Rightarrow$ $(1)^2 + (1)^2 = \left(\frac{7}{5}\right)^2 + (x)^2$

$\Rightarrow$ $2 - \frac{49}{25} = x^2$

$\Rightarrow$ $x^2 = 2 - \frac{49}{25} = \frac{1}{25}$

So, $\boldsymbol{x} = \pm\frac{1}{5}$

Ex.84 If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then find the value of $\sin\theta - \cos\theta$.

Sol. $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\sin\theta - \cos\theta = x$ (let)

$\Rightarrow$ $(1)^2 + (1)^2 = (\sqrt{2}\cos\theta)^2 + (x)^2$

$\Rightarrow$ $2 = 2\cos^2\theta + x^2$

$\Rightarrow$ $x^2 = 2 - 2\cos^2\theta$

$\Rightarrow$ $x^2 = 2 - 2\cos^2\theta$

$\Rightarrow$ $x^2 = 2(1 - \cos^2\theta)$

$\Rightarrow$ $x^2 = 2\sin^2\theta$.

$\boldsymbol{x = \pm\sqrt{2}\sin\theta}$

Ex.85 If $\frac{x}{a}\sin\theta + \frac{y}{b}\cos\theta = 1$, then.

Find the value of $\frac{y}{b}\sin\theta - \frac{x}{a}\cos\theta = ?$

Sol. $\frac{x}{a}\sin\theta + \frac{y}{b}\cos\theta = 1$

$\frac{y}{b}\sin\theta - \frac{x}{a}\cos\theta = P$ (let)

$\Rightarrow$ $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = (1)^2 + (P)^2$

$\Rightarrow$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + P^2$

$P = \pm\sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1}$

Ex.86 If $\sin\theta + \cos\theta = \frac{1}{2}$, Then find the value of $\sin\theta - \cos\theta$ =?

Sol. $\sin\theta + \cos\theta = \frac{1}{2}$

$\sin\theta - \cos\theta = x$ (let)

$(1)^2 + (1)^2 = \left(\frac{1}{2}\right)^2 + (x)^2$

$x^2 = 2 - \frac{1}{4}$

$x^2 = \frac{7}{4}$

$x = \pm\frac{\sqrt{7}}{2}$

Ex.87 If $3\sin\theta + 4\cos\theta = 5$, find the value of $4\sin\theta - 3\cos\theta$?

Sol. $3\sin\theta + 4\cos\theta = 5$

$4\sin\theta - 3\cos\theta = x$ (let)

$(3)^2 + (4)^2 = (5)^2 + x^2$

$x = 0$

$\therefore$ So, $4\sin\theta - 3\cos\theta = 0$

TYPE - X

(A) $\mathbf{1 + \tan^2\theta = \sec^2\theta}$

$\mathbf{\sec^2\theta - 1 = \tan^2\theta}$

$\mathbf{\sec^2\theta - \tan^2\theta = 1}$

Ex.88 If $\sec^2\theta + \tan^2\theta = 9$ find the value of $\sin\theta$ $(0° < \theta < 90°)$

Sol. $\sec^2\theta + \tan^2\theta = 9$

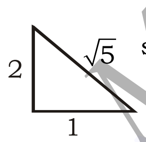
$1 + \tan^2\theta + \tan^2\theta = 9$

$2\tan^2\theta = 8$

$\tan^2\theta = 4$

$\tan\theta = 2, = \frac{p}{b}$

Now,



$\sin\theta = \frac{p}{h} = \frac{2}{\sqrt{5}}$

Ex.89 If $\sec^2\theta + \tan^2\theta = 11$, find the value of $\operatorname{cosec}\theta$ $(\therefore 0° < \theta < 90°)$

Sol. $\sec^2\theta + \tan^2\theta = 11$

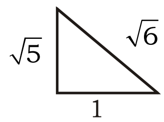
$1 + \tan^2\theta + \tan^2\theta = 11$

$2\tan^2\theta = 10$

$\tan^2\theta = 5$

$\tan\theta = \sqrt{5}$

Now,



$\operatorname{cosec}\theta = \frac{\sqrt{6}}{\sqrt{5}}$

Ex.90 If $\sec^2\theta + \tan^2\theta = \frac{5}{3}$ find the value of θ. while $(0° \le \theta \le 90°)$

Sol. $\sec^2\theta + \tan^2\theta = \frac{5}{3}$

$1 + \tan^2\theta + \tan^2\theta = \frac{5}{3}$

$2\tan^2\theta = \frac{5}{3} - 1$

$2\tan^2\theta = \frac{2}{3}$

$\tan\theta = \frac{1}{\sqrt{3}}$

so, $\theta = 30°$

Ex.91 If $\tan^2\alpha = 1 + 2\tan^2\beta$, find the value of $\sqrt{2}\cos\alpha - \cos\beta$ =?

Sol. $\tan^2\alpha = 1 + 2\tan^2\beta$ (Using identity)

$\Rightarrow$ $\sec^2\alpha - 1 = 1 + 2(\sec^2\beta - 1)$

$\Rightarrow$ $\sec^2\alpha - 1 = 1 + 2\sec^2\beta - 2$

$\Rightarrow$ $\sec^2\alpha - 1 = 2\sec^2\beta - 1$

$\Rightarrow$ $\sec^2\alpha = 2\sec^2\beta$

$\Rightarrow$ $\sec\alpha = \sqrt{2}\sec\beta$

$\Rightarrow \frac{1}{\cos\alpha} = \sqrt{2}\left(\frac{1}{\cos\beta}\right)$

$\Rightarrow$ $\cos\beta = \sqrt{2}\cos\alpha$

$\Rightarrow$ $\sqrt{2}\cos\alpha - \cos\beta = 0$

Alternative:-

$\alpha = 45°$ & $\beta = 0°$ satisfies

$\tan^2\alpha = 1 + 2\tan^2\beta$

put $\alpha = 45°$ & $\beta = 0°$ in $\sqrt{2}\cos\alpha - \cos\beta$

$= \sqrt{2}\cos 45° - \cos 0° = 1 - 1 = 0$

(B). $\mathbf{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1}$

$\mathbf{\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}}$

$\mathbf{\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}}$

if $\mathbf{\sec\theta + \tan\theta = x}$

then, $\mathbf{\sec\theta - \tan\theta = \frac{1}{x}}$

Ex.92 If $\sec\theta + \tan\theta = 3$, find the value of $\cos\theta$

Sol. $\sec\theta + \tan\theta = 3$ ....(i)

then $\sec\theta - \tan\theta = \frac{1}{3}$ ....(ii)

adding (i) + (ii)

$2\sec\theta = 3 + \frac{1}{3}$

$2\sec\theta = \frac{10}{3}$

$\sec\theta = \frac{5}{3}$

Hence, $\cos\theta = \frac{3}{5}$

Ex.93 If $\sec\theta + \tan\theta = x$, find the value of $\tan\theta$

Sol. $\sec\theta + \tan\theta = x$ .... (i)

$\sec\theta - \tan\theta = \frac{1}{x}$ ......(ii)

(i) and (ii)

$2\tan\theta = x - \frac{1}{x} = \frac{x^2 - 1}{x}$

$\tan\theta = \frac{x^2 - 1}{2x}$

Ex.94 If $\sec\theta + \tan\theta = 5$, find the value of $\sin\theta$.

Sol. $\sec\theta + \tan\theta = 5$ ....(i)

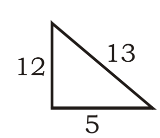
then $\sec\theta - \tan\theta = \frac{1}{5}$ ....(ii)

Adding (i) and (ii)

$2\sec\theta = 5 + \frac{1}{5} = \frac{26}{5}$

$\sec\theta = \frac{13}{5} = \frac{h}{b}$

Now,



$\sin\theta = \frac{12}{13}$

Ex.95 If $\sec\theta + \tan\theta = 5$, Then the value of $\frac{\tan\theta+1}{\tan\theta-1}$ is

Sol. $\sec\theta + \tan\theta = 5$ ....(i)

$\sec\theta - \tan\theta = \frac{1}{5}$ ....(ii)

(i) – (ii)

$2\tan\theta = 5 - \frac{1}{5}$

$2\tan\theta = \frac{24}{5}$

$\tan\theta = \frac{12}{5}$

Now, $\frac{\tan\theta+1}{\tan\theta-1}$

Put the value $\tan\theta$

$$\frac{\frac{12}{5}+1}{\frac{12}{5}-1} = \frac{17/5}{7/5} = \frac{17}{7}$$

Ex.96 If $\sec\theta = x + \frac{1}{4x}$ $(0° \ \theta < 90°)$, then $\sec\theta + \tan\theta$ is equal to

(a) $\frac{x}{2}$ (b) $2x$ (c) x (d) $\frac{1}{2x}$

Sol. $\sec\theta = x + \frac{1}{4x}$ (given) .....(i)

Let $\sec\theta + \tan\theta = a$ .....(ii)

Then $\sec\theta - \tan\theta = \frac{1}{a}$ ....(iii)

(ii) + (iii)

$2\sec\theta = a + \frac{1}{a}$

$\sec\theta = \frac{a}{2} + \frac{1}{2a}$ .....(iv)

Compare (i) and (iv)

Hence, $x = \frac{a}{2}$, $a = 2x$

so, $\sec\theta + \tan\theta = 2x$

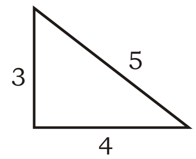
Alternate:-

$\sec\theta = x + \frac{1}{4x}$

Let $x = 1$

Then, $\sec\theta = 1 + \frac{1}{4\times1} = \frac{5}{4}$

Now,



$\sec\theta + \tan\theta$

$\frac{5}{4} + \frac{3}{4} = \frac{8}{4} = 2$

Take option (b) $2x = 2\times1 = 2$

Ex.97 If $\tan^2\theta = 1 - e^2$, then $\sec\theta + \tan^3\theta . \operatorname{cosec}\theta$ = ?

(a) $\left(2-e^2\right)^{3/2}$

(b) $(2 - e^2)^{2/3}$

(c) $\left(2-e^2\right)^{1/2}$

(d) None of these

Sol. We have, $\sec\theta + \tan^3\theta .\operatorname{cosec}\theta$

$= \sec\theta \left(1 + \tan^3\theta \frac{\operatorname{cosec}\theta}{\sec\theta}\right)$

$= \sec\theta \ (1 + \tan^2\theta)$

$= \sec\theta .\sec^2\theta$

$= \sec^3\theta = (\sec^2\theta)^{3/2} = (1 + \tan^2\theta)^{3/2}$

$= (1 + 1 - e^2)^{3/2}$

$\because \tan^2\theta = 1 - e^2 = (2 - e^2)^{3/2}$

Alternate:-

Let $\theta = 45°$

Then, $\tan^2\theta = 1 - e^2$

$\tan^2 45 = 1 - e^2$

$1 = 1 - e^2$

$\mathbf{e^2 = 0}$

Now, $\sec\theta + \tan^3\theta .\operatorname{cosec}\theta$

Put $\theta = 45°$

$= \sec45° + \tan^2 45°.\operatorname{cosec}45°$

$= \sqrt{2} + 1\times\sqrt{2}$

$= 2\sqrt{2} = (2)^{3/2}$

Now, we take option (a)

$(2-e^2)^{\frac{3}{2}}$

Put $e^2 = 0$

Then $= (2)^{\frac{3}{2}}$

So, (a) option is correct

Ex.98 Find the value of

$$\frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1}$$

Sol. $\frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1}$

$\Rightarrow \frac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$(\because \sec^2\theta - \tan^2\theta = 1)$

$\Rightarrow \frac{(\sec\theta+\tan\theta) - (\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{\tan\theta - \sec\theta + 1}$

$\Rightarrow \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{(\tan\theta - \sec\theta + 1)}$

$\Rightarrow \sec\theta + \tan\theta$

$\Rightarrow \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$

$\Rightarrow \frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)} = \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$

$\Rightarrow$ So, $\frac{\cos\theta}{1-\sin\theta}$

(C). **$a\sec\theta - b\tan\theta = c$ .....(i)**

$b\sec\theta - a\tan\theta = d$ .....(ii)

or

$a\tan\theta - b\sec\theta = d$

Then, $(a^2 - b^2 = c^2 - d^2)$

or $a\sec\theta + b\tan\theta = c$ .....(i)

$b\sec\theta + a\tan\theta = d$ .....(ii)

then, $(a^2 - b^2 = c^2 - d^2)$

Proof $(i)^2 - (ii)^2$

$\Rightarrow$ $(a\sec\theta - b\tan\theta)^2 - (b\sec\theta - a\tan\theta)^2 = c^2 - d^2$.

$\Rightarrow$ $(a^2\sec^2\theta + b^2\tan^2\theta - 2ab\sec\theta\tan\theta)$ $-(b^2\sec^2\theta + a^2\tan^2\theta - 2ab\sec\theta\tan\theta) = c^2 - d^2$

$\Rightarrow$ $a^2\sec^2\theta + b^2\tan^2\theta - b^2\sec^2\theta - a^2\tan^2\theta = c^2 - d^2$

$\Rightarrow$ $a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta) = c^2 - d^2$

$\mathbf{a^2 - b^2 = c^2 - d^2}$

Ex.99 If $a\sec\theta - b\tan\theta = 10$ and $b\sec\theta - a\tan\theta = 5$ find the value of $a^2 - b^2$

Sol. $a\sec\theta - b\tan\theta = 10$

$b\sec\theta - a\tan\theta = 5$

(use property)

$a^2 - b^2 = (10)^2 - (5)^2$

$a^2 - b^2 = 100 - 25 = 75$

Ex.100 If $5\sec\theta - 3\tan\theta = 7$ find the value of

$3\sec\theta - 5\tan\theta$ =?

Sol. $5\sec\theta - 3\tan\theta = 7$

$3\sec\theta - 5\tan\theta = m$. (let)

$(5)^2 - (3)^2 = (7)^2 - (m)^2$

$25 - 9 = 49 - m^2$

$m^2 = 33$

$\mathbf{m = \pm\sqrt{33}}$

TYPE- XI

(A) $\mathbf{1+\cot^2\theta = cosec^2\theta}$

$\mathbf{cosec^2\theta - 1 = \cot^2\theta}$

$\mathbf{cosec^2\theta - \cot^2\theta = 1}$

Ex.101 $cosec^2\theta + 2\cot^2\theta = 10$, then find the value of $\sin\theta$ + $\cos\theta$ when $0° < \theta < 90°$

Sol. $cosec^2\theta + 2\cot^2\theta = 10$

$1 + \cot^2\theta + 2\cot^2\theta = 10$

$3\cot^2\theta = 9$

$\cot\theta = \sqrt{3}$

So, $\theta = 30°$

Now, $\sin\theta + \cos\theta$

$= \sin 30° + \cos 30°$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

Ex.102 If $cosec^2\theta + \cot^2\theta = 3$, find the value of $\cos\theta$. when $(0° < \theta < 90°)$

Sol. $cosec^2\theta + \cot^2\theta = 3$

$1 + \cot^2\theta + \cot^2\theta = 3$

$2\cot^2\theta = 2$

$\cot^2\theta = 1$

$\cot\theta = 1$

So, $\theta = 45°$

Now, $\cos\theta$

$= \cos 45° = \frac{1}{\sqrt{2}}$

(B) $\mathbf{cosec^2\theta - \cot^2\theta = 1}$

$\mathbf{(cosec\theta - \cot\theta)(cosec\theta + \cot\theta) = 1}$

$\mathbf{cosec\theta - \cot\theta}$

$\mathbf{= \frac{1}{cosec\theta + \cot\theta}}$

If $\mathbf{cosec\theta - \cot\theta = x}$,

then , $\mathbf{cosec\theta + \cot\theta = \frac{1}{x}}$

Ex.103 If $cosec\theta - \cot\theta = 4$, find the value of $\cos\theta$

Sol. $cosec\theta - \cot\theta = 4$ .....(i)

then, $cosec\theta + \cot\theta = \frac{1}{4}$ .....(ii)

(i) and (ii)

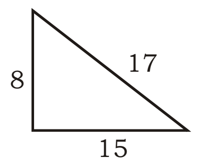
$2\,cosec\theta = 4 + \frac{1}{4}$

$2\,cosec\theta = \frac{17}{4}$

$cosec\theta = \frac{17}{8} = \frac{h}{p}$

$b = \sqrt{(17)^2 - (8)^2} = 15$

Now,



$\cos\theta = \frac{b}{h} = \frac{15}{17}$

Ex.104 If $cosec\theta + \cot\theta = \sqrt{5} + 2$, then find the value of $\sin\theta$.

Sol. $cosec\theta + \cot\theta = \sqrt{5} + 2$ .....(i)

Then, $cosec\,\theta - \cot\theta = \frac{1}{\sqrt{5}+2} = \sqrt{5} - 2$ .....(ii)

(i) + (ii)

$2\,cosec\theta = 2\sqrt{5}$

$cosec\theta = \sqrt{5}$

So, $\sin\theta = \frac{1}{\sqrt{5}}$

(c) $\mathbf{a\,cosec\theta - b\cot\theta = c}$

$\mathbf{b\,cosec\theta - a\cot\theta = d}$

or

$\mathbf{a\cot\theta - b\,cosec\theta = d}$

$\mathbf{(i)^2 - (ii)^2}$

Then, $\mathbf{(a^2 - b^2 = c^2 - d^2)}$

or

$\mathbf{a\,cosec\theta + b\cot\theta = c}$ **....... (i)**

$\mathbf{b\,cosec\theta + a\cot\theta = d}$ **.......(ii)**

$\mathbf{(i)^2 - (ii)^2}$

$\mathbf{(a^2 - b^2 = c^2 - d^2)}$

Ex.105 If $4\,cosec\theta + 5\cot\theta = 7$, then find the value of $5\,cosec\theta + 4\cot\theta$ =?

Sol. $4\,cosec\theta + 5\cot\theta = 7$ (given)

$5\,cosec\theta + 4\cot\theta = m$ (let)

Using identity

$(4)^2 - (5)^2 = (7)^2 - (m)^2$

$16 - 25 = 49 - m^2$

$m^2 = 49 + 9$

$m = \pm\sqrt{58}$

TYPE – XII

(A) **If A+B = 45° or 225° then,**

(i) **(1+tanA) (1+tanB) = 2**

and

(ii) **(1–cotA) (1–cotB) = 2**

Proof

(i) A+B = 45°

Both side take tan.

tan (A+B)= tan45°

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$

$\Rightarrow$ tanA + tanB = 1– tanA tanB

$\Rightarrow$ tanA + tanB + tanA tanB = 1

Adding 1 both side.

$\Rightarrow$ 1+ tanA + tanB + tanA tanB = 1+1

$\Rightarrow$ 1(1+tanA) + tanB (1+tanA) = 2

Hence, (1+tanB) (1+tanA) = 2

(ii) A+B = 45

Both side take cot.

cot (A+B)= cot45°

$\Rightarrow \frac{\cot A \cdot \cot B - 1}{\cot A + \cot B} = \frac{1}{1}$

$\Rightarrow$ cotA cotB –1 = cotA + cot B

$\Rightarrow$ cotA [cotB –1] –1– cotB +1 –1=0

$\Rightarrow$ cotA [cotB –1] –1 [cotB –1] = 2

$\Rightarrow$ (cotA–1)(cotB –1) = **2**

Ex.106 Find the value of (1+tan5°) (1+tan40°)

Sol. A+B = 5° + 40° = 45°

then, (1+tan5°) (1+tan40°) = 2

Ex.107 Find the value of (1+tan1°) (1+tan2°) (1+tan3°) (1+tan44°) (1+tan43°) (1+tan42°)

Sol. (1+tan1°) (1+tan44°)

(1+tan2°) (1+tan43°)

(1+tan3°) (1+tan42°)

1°+44° = 2°+43°

= 3°+42° = 45°

so, 3 pair of such term

= 2×2×2 = 8

Ex.108 If A+B = 45°, then find

$\frac{\tan A}{1 - \tan A} \cdot \frac{\tan B}{1 - \tan B} = ?$

Sol. $\frac{\tan A}{1 - \tan A} \cdot \frac{\tan B}{1 - \tan B}$

$$= \frac{\frac{1}{\cot A}}{1-\frac{1}{\cot A}} \cdot \frac{\frac{1}{\cot B}}{1-\frac{1}{\cot B}}$$

$$= \frac{1}{\cot A - 1} \cdot \frac{1}{\cot B - 1} =$$

$$\frac{1}{(\cot A - 1)} \cdot \frac{1}{(\cot B - 1)} = \frac{1}{2}$$

(B) **If A+B+C = 180°**
then,

(i) **tanA + tanB + tan C = tanA.tanB.tanC**

(ii) **cotA.cotB + cotB.cotC + cotC. cotA=1**

(iii) **sin2A+sin2B+sin2C = 4sinA.sinB.sinC**

Proof,

A+B+C = 180°
A+B = 180° – C
Both side take tan
tan (A+B) = tan (180°–C)

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{-\tan C}{1}$$

⇒ tanA + tanB = – tanC + tanA tanB TanC

⇒ **so, tanA+tanB+tanC = tanA.tanB.tanC**

(tanA.tanB.tanC) divided by both side

then,

$$\frac{1}{\tan B.\tan C} + \frac{1}{\tan A.\tan C} + \frac{1}{\tan A.\tan B} = 1$$

⇒ **cotA cotB + cotB cotC + cotC.cotA =1**

Ex.109 The value of tan37° + tan73° + tan70° is equal to

Sol. Sum of angle 37° + 73° +70° = 180°
so, tan37° + tan73° + tan70° = tan37°.tan73°.tan70°

TYPE - XIII

Morri's law

If 4θ < 60°

(i) $\sin\theta.\sin 2\theta.\sin 4\theta = \frac{1}{4}\sin 3\theta$

(ii) $\cos\theta.\cos 2\theta.\cos 4\theta = \frac{1}{4}\cos 3\theta$

(iii) $\tan\theta.\tan 2\theta.\tan 4\theta = \tan 3\theta$

For all value of θ

(i) $\sin(60-\theta)\sin\theta.\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

(ii) $\cos(60-\theta)\cos\theta.\cos(60+\theta) = \frac{1}{4}\cos 3\theta$

(iii) $\tan(60-\theta)\tan\theta.\tan(60+\theta) = \tan 3\theta$

Ex.110 The value of tan10° tan20° tan40° =?

Sol. Here θ = 10°

∴ tan(10°)tan(2×10°)tan(4×10°) = tan(3×10°)

⇒ $\tan 30° = \frac{1}{\sqrt{3}}$

Ex.111 The value of sin20°.sin40°.sin80° =?

Sol. sin20°.sin40°.sin80°

$$\underset{\theta}{\downarrow} \quad \underset{60-\theta}{\downarrow} \quad \underset{60+\theta}{\downarrow}$$

Here, θ = 20°

$$= \frac{1}{4}\sin 3\theta$$

$$= \frac{1}{4}\sin(3\times 20°)$$

$$= \frac{1}{4}\sin 60°$$

$$\frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$$

Ex.112 The value of

$\sin\frac{\pi}{9}.\sin\frac{5\pi}{9}.\sin\frac{7\pi}{9}.\sin\frac{3\pi}{9}$. is equal to

Sol. $\sin\frac{\pi}{9}.\sin\frac{5\pi}{9}.\sin\frac{7\pi}{9}.\sin\frac{3\pi}{9}$.

Put value of π = 180°

⇒ sin20°. sin100°. sin140°. sin60°

⇒ sin20°. sin(180°–80°). sin(180°–40°). $\frac{\sqrt{3}}{2}$

⇒ sin20°. sin40°. sin80°. $\frac{\sqrt{3}}{2}$

[∴ sin(180° – θ)= sinθ]

⇒ $\frac{1}{4}\sin 60° \times \frac{\sqrt{3}}{2}$

⇒ $\frac{1}{4} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \Rightarrow \frac{3}{16}$

Ex.113 The value of sin12°.sin48°.sin54° =?

Sol. sin12°.sin48°.sin54°
(sin72°divide and multiply)

⇒ sin12°.sin48°.sin72° × $\frac{1}{\sin 72°}$.sin54°

here θ = 12°

⇒ $\frac{1}{4}\sin 3\times 12° \times \frac{1}{\sin 72°}.\sin(90°-36°)$

⇒ $\frac{1}{4}\sin 36°.\cos 36° \times \frac{1}{\sin 72°}$

⇒ $\frac{1}{4\times 2}.2\sin 36°.\cos 36° \times \frac{1}{\sin 72°}$

⇒ $\frac{1}{8}\sin 2\times 36° \times \frac{1}{\sin 72°}$

⇒ $\frac{1}{8}\sin 72° \times \frac{1}{\sin 72°} = \frac{1}{8}$

Ex.114 The value of 1–sin10° sin50° sin70° = ?

Sol. 1–sin10°sin50°sin70°

here θ = 10°

$$= 1 - \left(\frac{1}{4}\sin 3\times 10°\right)$$

$$= 1 - \frac{1}{4}\times \sin 30° = 1 - \frac{1}{4}\times\frac{1}{2} = \frac{7}{8}$$

TYPE - XIV

T-radius of Multiple Angles :-

(i) $\sin 2A = 2\sin A.\cos A = \frac{2\tan A}{1+\tan^2 A}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$

$$= 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

(iii) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

(iv) $\sin 3A = 3\sin A - 4\sin^3 A$

(v) $\cos 3A = 4\cos^3 A - 3\cos A$

(vi) $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

(vii) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

(viii) sin C – sin D

$= 2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

(ix) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

(x) $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

Ex.115. If $\sin 2x = \frac{1}{5}$, the value of $(\sin x + \cos x)$ is :-

(a) $\sqrt{\frac{7}{5}}$ (b) $\sqrt{\frac{4}{5}}$

(c) $\sqrt{\frac{6}{5}}$ (d) $\sqrt{\frac{2}{5}}$

Sol. $\sin 2x = \frac{1}{5}$

add 1 both side

$1 + \sin 2x = 1 + \frac{1}{5} = \frac{6}{5}$

$\therefore \sin^2 x + \cos^2 x + 2\sin x.\cos x = \frac{6}{5}$

$[\because \sin^2 x + \cos^2 x = 1$ and $\sin 2x = 2\sin x.\cos x\,]$

$\Rightarrow (\sin x + \cos x)^2 = \frac{6}{5}$

$\Rightarrow \sin x + \cos x = \sqrt{\frac{6}{5}}$

Ex.116 The value of

$\cos 15°.\ \cos 7\frac{1}{2}^{0}.\sin 7\frac{1}{2}^{0}$. =?

Sol. $\cos 15°.\ \cos 7\frac{1}{2}^{0}.\sin 7\frac{1}{2}^{0}$

Multiply and divide by 2

$\Rightarrow \cos 15°.\frac{1}{2}\left[2\cos 7\frac{1}{2}^{0}.\sin 7\frac{1}{2}^{0}\right]$

$\Rightarrow \frac{1}{2}\cos 15°\times\sin 2.\frac{15}{2}$

$\Rightarrow \frac{1}{2}\cos 15°.\sin 15°$

Again multiply and divide by 2

$\Rightarrow \frac{1}{2\times 2}\times 2\sin 15.\cos 15°$

$\Rightarrow \frac{1}{4}.\sin 30° \Rightarrow \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$

Ex.117. The value of

$\frac{\sin 2x}{\sin\frac{x}{4}}$

Sol. $\frac{\sin 2x}{\sin\frac{x}{4}}$

First we solve sin2x

$\Rightarrow \sin 2x = 2\sin x.\cos x$

$\Rightarrow 2\sin 2\left(\frac{x}{2}\right).\cos x$

$= 4\sin\frac{x}{2}.\cos\frac{x}{2}.\cos x$

$= 4\sin 2\left(\frac{x}{2\times 2}\right).\cos\frac{x}{2}.\cos x$

$= 4\times 2\sin\frac{x}{4}.\cos\frac{x}{4}.\cos\frac{x}{2}.\cos x$

now,

$\Rightarrow \frac{4\times 2\sin\frac{x}{4}.\cos\frac{x}{4}.\cos\frac{x}{2}.\cos x}{\sin\frac{x}{4}}$

$\Rightarrow 8\cos\frac{x}{4}.\cos\frac{x}{2}.\cos x$

Ex.118. Find the value of $\frac{1-\tan^2 22\frac{1}{2}}{1+\tan^2 22\frac{1}{2}}$

Sol. $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

$\therefore \cos 2\times 22\frac{1}{2} = \cos 45°$

$= \frac{1}{\sqrt{2}}$

Ex.119 Find the value $\sin 22°\frac{1}{2}$ =?

Sol. [cos2A = 2 cos²A–1 = 1–2sin²A]

$22\frac{1}{2} = A$

45° = 2A

$\Rightarrow \cos 45° = 1 - 2\sin^2 22\frac{1}{2}$

$\left(\sin^2 22\frac{1}{2} = x\,(\text{let})\right)$

$\Rightarrow \frac{1}{\sqrt{2}} = 1 - 2x^2$

$\Rightarrow 2x^2 = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow x^2 = \frac{\sqrt{2}-1}{2\sqrt{2}}$

$x = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}}$

$= \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

Ex.120 The value of

3 sin20° – 4sin³20°

Sol. 3sin20° – 4sin³20° = sin3×20°

= sin60°

$= \frac{\sqrt{3}}{2}$

Ex.121 Find the value of

$\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°}$

Sol. $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°}$

$= \frac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10°\cos 10°}$

$= \frac{2\left(\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°\right)}{\sin 10°\cos 10°}$

We can write $\frac{1}{2}$ = sin30° & $\frac{\sqrt{3}}{2}$ = cos30°

$= \frac{2[\sin 30°\cos 10° - \cos 30°\sin 10°]}{\sin 10°\cos 10°}$

multiply and divide by 2

$= \frac{2.2\sin(30°-10°)}{2\sin 10°\cos 10°}$

$= \frac{4\sin 20°}{\sin 2\times 10°} = 4$

Ex.122. If $\cos x = \frac{2\cos y - 1}{2 - \cos y}$ then find the value of

$\tan\frac{x}{2}\cot\frac{y}{2}$

Sol. $\frac{\cos x}{1} = \frac{2\cos y - 1}{2 - \cos y}$

$\Rightarrow$ Applying Componendo-Dividendo rule

$\Rightarrow \frac{\cos x + 1}{\cos x - 1}$

$= \frac{2\cos y - 1 + 2 - \cos y}{2\cos y - 1 - 2 + \cos y}$

$\Rightarrow \frac{\cos x + 1}{\cos x - 1} = \frac{1 + \cos y}{3(\cos y - 1)}$

$\Rightarrow \frac{2\cos^2\frac{x}{2} - 1 + 1}{1 - 2\sin^2\frac{x}{2} - 1}$

$= \frac{1 + 2\cos^2\frac{y}{2} - 1}{3\left(1 - 2\sin^2\frac{y}{2} - 1\right)}$

$\Rightarrow \frac{2\cos^2\frac{x}{2}}{-2\sin^2\frac{x}{2}} = \frac{2\cos^2\frac{y}{2}}{3\left(-2\sin^2\frac{y}{2}\right)}$

$\Rightarrow \cot^2\frac{x}{2} = \frac{1}{3}\cot^2\frac{y}{2}$

$\Rightarrow 3 = \tan^2\frac{x}{2}.\cot^2\frac{y}{2}$

Hence, $\tan\frac{x}{2}.\cot\frac{y}{2} = \sqrt{3}$

Ex.123 Find the value of $\frac{1}{2}\operatorname{cosec}10° - 2\sin 70°$

Sol. $\frac{1}{2}\operatorname{cosec}10° - 2\sin 70°$

$= \frac{1}{2\sin 10°} - 2\sin 70°$

$= \frac{1 - 2.2\sin 70°\sin 10°}{2\sin 10°}$

$= \frac{1 - 2[\cos 60° - \cos 80°]}{2\sin 10°}$

(Using formula; 2sin A.sinB = cos(A – B) – cos(A + B)

$= \frac{1 - 2\left(\frac{1}{2} - \cos 80°\right)}{2\sin(90° - 80°)}$

$= \frac{1 - 1 + 2\cos 80°}{2\cos 80°}$

$= \frac{2\cos 80°}{2\cos 80°} = 1$

TYPE- XV

Trigonometry expression is independent of angle so we can put any value of θ except result should not indetermine

$\left(\infty, \frac{\infty}{\infty}, \frac{0}{0}, \frac{1}{0}\right)$

Note:- $\frac{1}{\infty} = 0$, $\frac{1}{\infty + 1} = \frac{1}{\infty} = 0$

$\Rightarrow$ It is better to put $\theta = 0°$ if expression does not contain $\operatorname{cosec}\theta$ or $\cot\theta$ otherwise $\theta = 45°$

(i) If $\sin\theta$, $\cos\theta$ in equation, Try to put $\theta = 0°$ or $90°$

(ii) If $\sin\theta$, $\cos\theta$, $\tan\theta$, $\sec\theta$, $\operatorname{cosec}\theta$, $\cot\theta$ try to put $\theta = 45°$

Ex.124 If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then find the value of $m^2 - n^2 = ?$

(a) $4\sqrt{mn}$ (b) mn
(c) m^2n^2 (d) m^3n^3

Sol. $m^2 - n^2 = (\tan\theta - \sin)^2 - (\tan\theta - \sin\theta)^2$

$= 4\tan\theta\ .\ \sin\theta$

$[\because (a+b)^2 - (a-b)^2 = 4ab]$

$= 4\tan\theta\ .\sin\theta$

$= 4\sqrt{\tan^2\theta.\sin^2\theta}$

$= 4\sqrt{\tan^2\theta(1 - \cos^2\theta)}$

$= 4\sqrt{\tan^2\theta - \tan^2\theta.\cos^2\theta}$

$= 4\sqrt{\tan^2\theta - \frac{\sin^2\theta}{\cos^2\theta}.\cos^2\theta}$

$= 4\sqrt{\tan^2\theta - \sin^2\theta}$

Now, $mn = (\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$

$mn = \tan^2\theta - \sin^2\theta$ ....(i)

From equation (i)

$= 4\sqrt{mn}$

Alternative:-

Let $\theta = 45°$

$m = \tan\theta + \sin\theta = 1 + \frac{1}{\sqrt{2}}$

$n = \tan\theta - \sin\theta = 1 - \frac{1}{\sqrt{2}}$

Now, $m^2 - n^2 = (m + n)(m - n)$

$= (2) \times \left(2 \times \frac{1}{\sqrt{2}}\right) = 4 \times \frac{1}{\sqrt{2}}$

take option (a) $4\sqrt{mn}$

$= mn = \left(1 + \frac{1}{\sqrt{2}}\right)\left(1 - \frac{1}{\sqrt{2}}\right)$

$= (1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$

So, $4\sqrt{mn} = 4 \times \sqrt{\frac{1}{2}}$

So option (a) is correct.

Ex.125 If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then what is the value of 2p:

(a) $p(q^2 - 1)$
(b) $p(1 - q^2)$
(c) $q(1 - p^2)$
(d) $q(p^2 - 1)$

Sol. Let $\theta = 45°$

then $p = \sin 45° + \cos 45° = \sqrt{2}$ and

$q = \sec 45° + \operatorname{cosec}45° = 2\sqrt{2}$

Now, $2p = 2\sqrt{2}$

Take option (d)

$\therefore q(p^2 - 1) = 2\sqrt{2}\left[\left(\sqrt{2}\right)^2 - 1\right] = 2\sqrt{2} = 2p$

Hence, option (d) is correct.

Ex.126 Find the value of :

$1 + 2\sec^2 A.\tan^2 A - \sec^4 - \tan^4 A$

(a) 0 (b) 1
(c) $\sec^2A.\tan^2A$
(d) None of these

Sol. $1 - \left(\sec^4 A + \tan^4 A - 2\sec^2 A.\tan^2 A\right)$

$= 1 - \left(\sec^2 A - \tan^2 A\right)^2$

$= 1 - 1 = 0$

Alternative:-

Take A = 45°, then

Given Exp.

$= 1 + 2\sec^2 45°.\tan^2 45° - \sec^4 45° - \tan^4 45°$

$= 1 + 2\left(\sqrt{2}\right)^2 \times 1 - \left(\sqrt{2}\right)^4 - 1$

$= 0$

Ex.127 If $x=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$, then $\frac{2x}{1-x^2}=?$

(a) $\sec\theta$ (b) $\tan\theta$ (c) $\cot\theta$ (d) $\cos\theta$

Sol. Let $\theta = 60°$

$\therefore \; x=\sqrt{\dfrac{1-\frac{1}{2}}{1+\frac{1}{2}}}=\dfrac{1}{\sqrt{3}}$

$\therefore \; \dfrac{2x}{1-x^2}=\dfrac{2/\sqrt{3}}{1-\frac{1}{3}}=\sqrt{3}$

and option (B). $\tan\theta = \tan 60°$

$= \sqrt{3}$

& option (B) is correct.

Note:- at $\theta = 0°$ $\cot\theta = \infty$

$\therefore$ θ can't be 0° & at $\theta = 45°$

option (B) and (C) contradicts.

Ex.128 If $x = \text{cosec}\,\theta - \sin\theta$ and $y = \sec\theta - \cos\theta$, then the value of $x^2y^2(x^2 + y^2 + 3)$ is :

(a) 0 (b) 1 (c) 2 (d) 3

Sol. take $\theta = 45°$

$\therefore \; x=\sqrt{2}-\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$ & $y=\sqrt{2}-\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

$\therefore \; x^2y^2\left(x^2+y^2+3\right)=\frac{1}{2}\times\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}+3\right)$

$=\frac{1}{4}(4)=1$

Ex.129 If $m = \text{cosec}\,\theta - \sin\theta$ and $n = \sec\theta - \cos\theta$, then find the value of $m^{2/3}+n^{2/3}=?$

(a) $(mn)^{2/3}$ (b) $(mn)^{-2/3}$

(c) $(mn)^{-1/3}$ (d) $(mn)^{1/3}$

Sol. $m = \text{cosec}\,\theta - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$

$= \dfrac{\cos^2\theta}{\sin\theta}$ and $n = \sec\theta - \cos\theta$

$= \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$

$\therefore \; mn = \dfrac{\cos^2\theta}{\sin\theta}\cdot\dfrac{\sin^2\theta}{\cos\theta} = \sin\theta.\cos\theta$

and $\quad m^{2/3}+n^{2/3}=\left(\dfrac{\cos^2\theta}{\sin\theta}\right)^{2/3}+\left(\dfrac{\sin^2\theta}{\cos\theta}\right)^{2/3}$

$= \dfrac{\cos^{4/3}\theta}{\sin^{2/3}\theta}+\dfrac{\sin^{4/3}\theta}{\cos^{2/3}\theta}$

$= \dfrac{\cos^2\theta+\sin^2\theta}{(\sin\theta.\cos\theta)^{2/3}}=\dfrac{1}{(mn)^{2/3}}$

$= (mn)^{-2/3}$

Alternative:-

Let $\theta = 45°$, then

$m = \dfrac{1}{\sqrt{2}}$ and $n = \dfrac{1}{\sqrt{2}}$

$\therefore \; m^{2/3}+n^{2/3}=\left(\dfrac{1}{\sqrt{2}}\right)^{2/3}+\left(\dfrac{1}{\sqrt{2}}\right)^{2/3}$

$\Rightarrow 2.\left(\dfrac{1}{\sqrt{2}}\right)^{2/3}=2.\left(\dfrac{1}{2}\right)^{1/3}$

$\Rightarrow 2.2^{-1/3} = 2^{1-\frac{1}{3}} = 2^{2/3}$

& $(mn)^{-2/3}=\left(\dfrac{1}{2}\right)^{-2/3}=2^{2/3}$

$\Rightarrow m^{2/3}+n^{2/3}=(mn)^{-2/3}$

Ex.130 If $\text{cosec}\,\theta - \sin\theta = l^3$ and $\sec\theta - \text{cosec}\,\theta = m^3$, then the value of $l^2m^2(l^2 + m^2)$ is :

(a) −1 (b) 0 (c) 1 (d) 2

Sol. take $\theta = 45°$

$l^3 = \sqrt{2}-\dfrac{1}{\sqrt{2}}=\dfrac{1}{\sqrt{2}}$

$m^3 = \sqrt{2}-\dfrac{1}{\sqrt{2}}=\dfrac{1}{\sqrt{2}}$

$\Rightarrow l^3 = m^3$

So, $l = m$

$\Rightarrow l^2m^2(l^2 + m^2)$

Put $(l = m)$

$\Rightarrow l^2 \times l^2(l^2 + l^2)$

$\Rightarrow 2l^6 = 2(l^3)^2 = 2 \times \left(\dfrac{1}{\sqrt{2}}\right)^2 = 1$

Ex.131 If $a\cos^3\theta + 3a\cos\theta.\sin^2\theta = m$ and $a\sin^3\theta + 3a\sin\theta.\cos^2\theta = n$, then the find value of $(m+n)^{2/3}+(m-n)^{2/3}=?$

(a) $2a^{1/3}$ (b) $2a^{2/3}$

(c) $a^{2/3}$ (d) $a^{1/3}$

Sol. $m + n = a\cos^3\theta + 3a\cos\theta.\sin^2\theta + a\sin^3\theta + 3a\sin\theta.\cos^2\theta$

$= a(\cos\theta + \sin\theta)^3$

Similarly,

$\Rightarrow m - n = a(\cos\theta - \sin\theta)^3$

$\Rightarrow \therefore \cos\theta+\sin\theta=\left(\dfrac{m+n}{a}\right)^{1/3}$ and

$\Rightarrow \cos\theta-\sin\theta=\left(\dfrac{m-n}{a}\right)^{1/3}$

$\Rightarrow \therefore \left(\dfrac{m+n}{a}\right)^{2/3}+\left(\dfrac{m-n}{a}\right)^{2/3}$

$\Rightarrow (\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2$

$\Rightarrow 2(\cos^2\theta + \sin^2\theta) = 2$

$\Rightarrow (m + n)^{2/3} + (m - n)^{2/3} = 2a^{2/3}$

Alternate:-

Let $\theta = 0°$, then

$m = a$ and $n = 0$

$(m + n)^{2/3} + (m - n)^{2/3} = a^{2/3} + a^{2/3} = 2a^{2/3}$

EXERCISE

1. $\frac{\tan\theta+\cot\theta}{\tan\theta-\cot\theta}=2, (0\le\theta\le 90°)$, then the value of $\sin\theta$ is
(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) 1

2. The value of cot 10°. cot 20°. cot 60°. cot 70°.cot 80° is
(a) 1 (b) –1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

3. The value of cot18°
$\left(\cot 72°\cos^2 22° + \frac{1}{\tan 72°\sec^2 68°}\right)$ is
(a) 1 (b) $\sqrt{2}$ (c) 3 (d) $\frac{1}{\sqrt{3}}$

4. If tan 15° = $2-\sqrt{3}$, the value of tan 15° cot 75°+ tan 75°cot15°is
(a) 14 (b) 12 (c) 10 (d) 8

5. If x, y are acute angles, $0 < x + y < 90°$ and sin $(2x-20°)$ = cos $(2y+20°)$, then the value of tan $(x + y)$ is :
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) 1

6. $\sin^2 5°+\sin^2 10°+\sin^2 15°+.........+\sin^2 85°+\sin^2 90°$ is equal to
(a) $7\frac{1}{2}$ (b) $8\frac{1}{2}$ (c) 9 (d) $9\frac{1}{2}$

7. The value of $\frac{\sin 39°}{\cos 51°}$ + 2 tan 11° tan 31° tan 45° tan 59° tan 79° – $3(\sin^2 21°+\sin^2 69°)$ is ;
(a) 2 (b) – 1 (c) 1 (d) 0

8. If $\frac{\cos^2\theta}{\cot^2\theta-\cos^2\theta}=3$ and $0° < \theta < 90°$, then value of θ is:
(a) 30° (b) 45°
(c) 60° (d) None of these

9. If A = tan 11° tan 29°, B = 2 cot 61° cot 79°, then ;
(a) A = 2B (b) A = – 2B
(c) 2A = B (d) 2A= – B

10. If $\sin\alpha+\cos\beta=2; (0° \le \beta < \alpha \le 90°)$, then $\sin\left(\frac{2\alpha+\beta}{3}\right)$ =
(a) $\sin\frac{\alpha}{2}$ (b) $\cos\frac{\alpha}{3}$
(c) $\sin\frac{\alpha}{3}$ (d) $\cos\frac{2\alpha}{3}$

11. If $\sin\alpha\sec(30°+\alpha)=1$, $(0<\alpha<60°)$, then the value of $\sin\alpha+\cos 2\alpha$ is
(a) 1 (b) $\frac{2+\sqrt{3}}{2\sqrt{3}}$
(c) 0 (d) $\sqrt{2}$

12. If tanθ = 1, then the value of $\frac{8\sin\theta+5\cos\theta}{\sin^3\theta-2\cos^3\theta+7\cos\theta}$ is
(a) 2 (b) $2\frac{1}{2}$ (c) 3 (d) $\frac{4}{5}$

13. If 'θ' be a positive acute angle satisfying $\cos^2\theta+\cos^4\theta=1$, then the value of $\tan^2\theta+\tan^4\theta$ is
(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) 0

14. If $\tan\theta=\frac{4}{3}$, then the value of $\frac{3\sin\theta+2\cos\theta}{3\sin\theta-2\cos\theta}$ is
(a) 0.5 (b) –0.5 (c) 3.0 (d) –3.0

15. The simplified value of $(\sec A-\cos A)^2+(\operatorname{cosec}A-\sin A)^2-(\cot A-\tan A)^2$ is
(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

16. If 'θ' be an acute angle and $\tan\theta+\cot\theta=2$, then the value of $\tan^5\theta+\cot^{10}\theta$ is
(a) 1 (b) 2 (c) 3 (d) 4

17. If $\sin\theta-\cos\theta=\frac{7}{13}$ and $0° < \theta < 90°$, then the value of sinθ + cosθ is
(a) $\frac{17}{13}$ (b) $\frac{13}{17}$ (c) $\frac{1}{13}$ (d) $\frac{1}{17}$

18. If $2\cos\theta-\sin\theta=\frac{1}{\sqrt{2}}$, $(0°<\theta<90°)$ the value of $2\sin\theta+\cos\theta$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

19. If $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=3$ then the value of $\sin^4\theta-\cos^4\theta$ is
(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{4}{5}$

20. If $\sec^2\theta+\tan^2\theta=7$, then the value of θ when $0°\le\theta\le 90°$, is
(a) 60° (b) 30° (c) 0° (d) 90°

21. The simplified value of $(\sec x\sec y+\tan x\tan y)^2-(\sec x\tan y+\tan x\sec y)^2$ is :
(a) –1 (b) 0 (c) $\sec^2 x$ (d) 1

22. If A = $\sin^2\theta+\cos^4\theta$ for any value of θ, then the value of A is
(a) $1\le A\le 1$ (b) $\frac{3}{4}\le A\le 1$
(c) $\frac{13}{16}\le A\le 1$ (d) $\frac{3}{4}\le A\le\frac{13}{16}$

23. If $\sin\theta+\operatorname{cosec}\theta=2$, then the value of $\sin^5\theta+\operatorname{cosec}^5\theta$, when $0°\le\theta\le 90°$ is
(a) 0 (b) 1 (c) 10 (d) 2

24. If tan2θ. tan4θ = 1, then the value of tan 3θ is
(a) $\sqrt{3}$ (b) 10 (c) 1 (d) $\frac{1}{\sqrt{3}}$

25. If $\cos^2\alpha+\cos^2\beta=2$, then the value of $\tan^3\alpha+\sin^5\beta$ is ;
(a) – 1 (b) 0 (c) 1 (d) $\frac{1}{\sqrt{3}}$

26. In a triangle ABC, $\angle ABC$ = 75° and $\angle ACB = \frac{\pi^c}{4}$, The circular measure of $\angle BAC$ is
(a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian
(c) $\frac{\pi}{6}$ radian (d) $\frac{\pi}{2}$ radian

27. The angles of a triangle are $(x+5)$°, $(2x-3)$°and $(3x+4)$°. The value of x is
(a) 30° (b) 31° (c) 29° (d) 28°

28. If $\angle A$ and $\angle B$ are complementary to each other, then the value of $Sec^2A+Sec^2B-Sec^2A.Sec^2B$ is
(a) 1 (b) –1 (c) 2 (d) 0

29. If sin 17°$=\frac{x}{y}$, then the value of (sec17°–sin73°) is
(a) $\frac{y^2}{x\sqrt{y^2-x^2}}$ (b) $\frac{x^2}{y\sqrt{y^2-x^2}}$
(c) $\frac{x^2}{y\sqrt{x^2-y^2}}$ (d) $\frac{y^2}{x\sqrt{x^2-y^2}}$

30. The value of $\frac{\cot 30° - \cot 75°}{\tan 15° - \tan 60°}$ is
(a) 0 (b) 1
(c) $\sqrt{3}-1$ (d) –1

31. The value of
$\cot\theta.\tan(90°-\theta)-\sec(90°-\theta)$
$\text{cosec}\theta+(\sin^2 25+\sin^2 65°)$
$+\sqrt{3}(\tan 5°\tan 15°.\tan 30°.\tan 75°.\tan 85°)$
is
(a) 1 (b) –1 (c) 2 (d) 0

32. If sin(3x–20°) = cos(3y+20°) then the value of $(x+y)$ is
(a) 20° (b) 30° (c) 40° (d) 45°

33. If cosθ cosec23°=1, the value of θ is
(a) 23° (b) 37° (c) 63° (d) 67°

34. If $2(\cos^2\theta - \sin^2\theta) = 1$, θ is a positive acute angle, then the value of θ is
(a) 60° (b) 30°
(c) 45° (d) $22\frac{1}{2}$

35. If sec(7θ+28°)= cosec (30° – 3θ), then the value of θ is
(a) 8° (b) 5° (c) 60° (d) 9°

36. If $\tan\left(\frac{\pi}{2}-\frac{\theta}{2}\right)=\sqrt{3}$, the value of cosθ is:
(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) 1

37. The value of cos25° – sin25°
(a) positive but less than 1
(b) positive but greater than 1
(c) negative
(d) 0

38. In a right angled Δ ABC, right angle at B, if cos A = $\frac{4}{5}$, then what is sin C is equal to?
(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{2}{5}$

39. If α and β are complementary angles, then what is $\sqrt{\cos\alpha\,\text{cosec}\beta-\cos\alpha\sin\beta}$ equal to?
(a) secβ (b) cos α
(c) sin α (d) –tanβ

40. If 2 cotθ = 3, then what is $\frac{2\cos\theta-\sin\theta}{2\cos\theta+\sin\theta}$ equal to?
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

41. If sinθ cosθ = $\frac{1}{2}$, then what is $\sin^6\theta + \cos^6\theta$ equal to?
(a) 1 (b) 2 (c) 3 (d) $\frac{1}{4}$

42. If secθ + tanθ = 2, then what is the value of secθ ?
(a) $\frac{3}{2}$ (b) $\sqrt{2}$ (c) $\frac{5}{2}$ (d) $\frac{5}{4}$

43. What is cosec (75°+θ) – sec (15°–θ) –tan(55°+θ) + cot (35°–θ) equal to?
(a) –1 (b) 0 (c) 1 (d) $\frac{3}{2}$

44. If sinθ + 2 cosθ = 1, where 0 < θ < π/2, what is 2sinθ – cosθ equal to?
(a) –1 (b) 1/2 (c) 2 (d) 1

45. If tan 8θ = cot2θ, where 0 < 8θ < $\frac{\pi}{2}$, then what is the value of tan 5θ ?
(a) $\frac{1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) 0

46. If sin(A + B) = 1, where 0 < B < 45°, then what is cos(A – B) equal to?
(a) sin2B (b) sinB
(c) cos 2B (d) cosB

47. If 5sinθ + 12cosθ = 13, then what is 5cosθ – 12sinθ equal to?
(a) –2 (b) –1 (c) 0 (d) 1

48. If 4tan = 3, then what is $\frac{4\sin\theta-\cos\theta}{4\sin\theta+9\cos\theta}$ equal to?
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

49. If sinθ – cosθ = 0, then what is $\sin^4\theta + \cos^4\theta$ equal to?
(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

50. What is $\frac{(\sin\theta+\cos\theta)(\tan\theta+\cot\theta)}{\sec\theta+\text{cosec}\theta}$ equal to
(a) 1 (b) 2
(c) sinθ (d) cosθ

51. What is $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$ equal to?
(a) 0 (b) 1 (c) 2 (d) 4

52. What is $\frac{(1+\sec\theta-\tan\theta)\cos\theta}{(1+\sec\theta+\tan\theta)(1-\sin\theta)}$ equal to?
(a) 1 (b) 2
(c) tanθ (d) cotθ

53. If sinθ + cosθ = $\sqrt{3}$, then what is tanθ + cotθ equal to?
(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\sqrt{3}$

54. If tanθ + secθ = m, then what is secθ equal to?
(a) $\frac{m^2-1}{2m}$ (b) $\frac{m^2+1}{2m}$
(c) $\frac{m+1}{m}$ (d) $\frac{m^2+1}{m}$

55. What is cosec (75° + θ) – sec(15° – θ) equal to?
(a) 0 (b) 1
(c) 2sinθ (d) 2cosθ

56. If ΔABC is right angled at C, then what is cos(A + B) + sin(A + B) equal to?
(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

57. If α, β and γ are acute angles such that $\sin\alpha = \frac{\sqrt{3}}{2}$, $\cos\beta = \frac{\sqrt{3}}{2}$ and $\tan\gamma = 1$, then what is $\alpha + \beta + \gamma$ equal to?
(a) 105° (b) 120° (c) 135° (d) 150°

58. If $\cos A + \cos^2 A = 1$, then what is the value of $2(\sin^2 A + \sin^4 A)$?
(a) 4 (b) 2 (c) 1 (d) 1/2

59. $(1 - \tan A)^2 + (1 + \tan A)^2 + (1 - \cot A)^2 + (1 + \cot A)^2$ is equal to
(a) $\sin^2 A.\cos^2 A$
(b) $\sec^2 A.\text{cosec}^2 A$
(c) $2\sec^2 A.\text{cosec}^2 A$
(d) None of these

60. What is the value of $\frac{\tan A - \sin A}{\sin^3 A}$?
(a) $\frac{\sec A}{1-\cos A}$ (b) $\frac{\sec A}{1+\cos^2 A}$
(c) $\frac{\sec A}{1+\cos A}$ (d) None of these

61. If $\sin\theta = \frac{x^2 - y^2}{x^2 + y^2}$, the which one of the following is correct?
(a) $\cos\theta = \frac{2xy}{x^2 - y^2}$
(b) $\cos\theta = \frac{2xy}{x^2 + y^2}$
(c) $\cos\theta = \frac{x-y}{x^2 + y^2}$
(d) $\cos\theta = \frac{xy(x-y)}{x^2 + y^2}$

62. If $a^2 = \frac{1 + 2\sin\theta\cos\theta}{1 - 2\sin\theta\cos\theta}$, then what is the value of $\frac{a+1}{a-1}$?
(a) $\sec\theta$ (b) 1
(c) 0 (d) $\tan\theta$

63. If the angle θ is in the first quadrant and $\tan\theta = 3$, then what is the value of $(\sin\theta + \cos\theta)$?
(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{2}{\sqrt{10}}$
(c) $\frac{3}{\sqrt{10}}$ (d) $\frac{4}{\sqrt{10}}$

64. If $0° < \theta < 90°$, then all the trigonometric ratios can be obtained when
(a) Only $\sin\theta$ is given
(b) Only $\cos\theta$ is given
(c) Only $\tan\theta$ is given
(d) any one of the six ratios is given

65. What is the value of sin A cos A tanA + cos A sin A cot A?
(a) $\sin^2 A + \cos A$
(b) $\sin^2 A + \tan^2 A$
(c) $\sin^2 A + \cot^2 A$
(d) $\text{cosec}^2 A - \cot^2 A$

66. What is the value of $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$?
(a) $2\text{cosec}\,\theta$ (b) $2\sec\theta$
(c) $\sec\theta$ (d) $\text{cosec}\,\theta$

67. If $\sin\theta\cos\theta = \sqrt{3}/4$, then $\sin^4\theta + \cos^4\theta$ is equal to
(a) 7/8 (b) 5/8
(c) 3/8 (d) 1/8

68. Consider the following
(i) $\sin^2 1° + \cos^2 1° = 1$
(ii) $\sec^2 33° - \cot^2 57° = \text{cosec}^2 37° - \tan^2 53°$
Which of the above statement is/are correct?
(a) Only I (b) Only II
(c) Both I and II
(d) Neither I nor II

69. If p = a sin$x$ + b cos$x$ and q = a cos$x$ – b sin x, then what is the value of $p^2 + q^2$?
(a) a + b (b) ab
(c) $a^2 + b^2$ (d) $a^2 - b^2$

70. The expression $\sin^2 x + \cos^2 x - 1 = 0$ is satisfied by how many values of x?
(a) Only one value of x
(b) Two values of x
(c) Infinite values of x
(d) No value of x

71. Consider the following statements
I. There is only one value of x in the first quadrant that satisfies $\sin x + \cos x = 2$
II. There is only one values of x in the first quadrant that satisfies $\sin x - \cos x = 0$.
Which of the statements above is/are correct?
(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

72. If $7\sin^2\theta + 3\cos^2\theta = 4, (0° \le \theta \le 90°)$, then value of θ is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

73. If $\tan(2\theta + 45°) = \cot 3\theta$, where $(2\theta + 45°)$ and 3θ are acute angles, then the value of θ is
(a) 5° (b) 9° (c) 12° (d) 15°

74. If θ be acute angle and $\cos\theta = \frac{15}{17}$, then the value of $\cot(90° - \theta)$ is
(a) $\frac{2\sqrt{8}}{15}$ (b) $\frac{8}{15}$
(c) $\frac{\sqrt{2}}{17}$ (d) $\frac{8\sqrt{2}}{17}$

75. If $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, then $\sec^4\theta - \tan^4\theta =$
(a) $\frac{7}{12}$ (b) $\frac{1}{2}$ (c) $\frac{5}{12}$ (d) 1

76. If $0 < x < \frac{\pi}{2}$ and sec$x$= cosec$y$ then the value of sin(x+y) is ;
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{3}}$

77. If A, B and C be the angles of a triangle, the incorrect relation is;
(a) $\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$
(b) $\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$
(c) $\tan\left(\frac{A+B}{2}\right) = \sec\frac{C}{2}$
(d) $\cot\left(\frac{A+B}{2}\right) = \tan\frac{C}{2}$

78. If θ is a positive acute angle and $\tan 2\theta.\tan 3\theta = 1$, then the value of $\left(2\cos^2\frac{5\theta}{2} - 1\right)$ is

(a) $-\frac{1}{2}$ (b) 1 (c) 0 (d) $\frac{1}{2}$

79. If $2\sin\left(\frac{\pi x}{2}\right) = x^2 + \frac{1}{x^2}$, then the value of $\left(x - \frac{1}{x}\right)$ is

(a) –1 (b) 2 (c) 1 (d) 0

80. If $\cos\theta + \sec\theta = 2$, the value of $\cos^6\theta + \sec^6\theta$ is

(a) 4 (b) 8 (c) 1 (d) 2

81. The numerical value of $\frac{5}{\sec^2\theta} + \frac{2}{1+\cot^2\theta} + 3\sin^2\theta$ is;

(a) 5 (b) 2 (c) 3 (d) 4

82. The numerical value of $\left(\frac{1}{\cos\theta} + \frac{1}{\cot\theta}\right)\left(\frac{1}{\cos\theta} - \frac{1}{\cot\theta}\right)$ is

(a) 0 (b) – 1 (c) +1 (d) 2

83. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$, the value of $\frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ is

(a) $\frac{25}{16}$ (b) $\frac{41}{9}$ (c) $\frac{41}{40}$ (d) $\frac{40}{41}$

84. If $\tan 7\theta \tan 2\theta = 1$, then the value of $\tan 3\theta$ is

(a) $\sqrt{3}$ (b) $-\frac{1}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $-\sqrt{3}$

85. The value of $\left(2\cos^2\theta - 1\right)\left(\frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}\right)$ is

(a) 4 (b) 1 (c) 3 (d) 2

86. If $\sec\theta + \tan\theta = 2$, then the value of $\sec\theta$ is

(a) $\frac{4}{5}$ (b) 5 (c) $\frac{5}{4}$ (d) $\sqrt{2}$

87. If $\sec\theta = x + \frac{1}{4x}, (0° < \theta < 90°)$ then $\sec\theta + \tan\theta$ is equal to

(a) $\frac{x}{2}$ (b) $2x$ (c) x (d) $\frac{1}{2x}$

88. The value of $\left(\sin^2 25° + \sin^2 65°\right)$ is

(a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) 0 (d) $\frac{2}{\sqrt{3}}$

89. If $\sec\theta + \tan\theta = \sqrt{3}\ (0° \le \theta \le 90°)$, then the value of $\tan 3\theta$ = ?

(a) undefined (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{3}$

90. If $\sin(60° - \theta) = \cos(\psi - 30°)$, then the value of $\tan(\psi - \theta)$ is (assume that θ and ψ are both positive acute angles with $\theta < 60°$ and $\psi > 30°$).

(a) $\frac{1}{\sqrt{3}}$ (b) 0 (c) $\sqrt{3}$ (d) 1

91. If $a\sin\theta + b\cos\theta = c$ then the value of $a\cos\theta - b\sin\theta$ is ;

(a) $\pm\sqrt{-a^2 + b^2 + c^2}$

(b) $\pm\sqrt{a^2 + b^2 - c^2}$

(c) $\pm\sqrt{a^2 - b^2 - c^2}$

(d) $\pm\sqrt{a^2 - b^2 + c^2}$

92. If sin(A – B) = $\frac{1}{2}$ and cos(A + B) = $\frac{1}{2}$ where A > B > 0 and A+ B is an acute angle, then the value B is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

93. The value of 152 ($\sin 30° + 2\cos^2 45° + 3\sin 30° + 4\cos^2 45°$ + ...+ $17\sin 30° + 18\cos^2 45°$) is

(a) an integer but not perfect square

(b) a rational number but not an integer

(c) a perfect square of an integer

(d) irrational

94. Evaluate : 3 cos 80° cosec 10° + 2 cos 59° cosec 31°

(a) 1 (b) 3 (c) 2 (d) 5

95. $\sin^2\theta - 3\sin\theta + 2 = 0$ will be true if

(a) $0 \le \theta < 90$ (b) $0 < \theta < 90$

(c) $\theta = 0°$ (d) $\theta = 90°$

96. If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$, then $\cos^2\alpha$ is

(a) $\frac{m^2}{n^2+1}$ (b) $\frac{m^2}{n^2}$

(c) $\frac{m^2-1}{n^2-1}$ (d) $\frac{m^2+1}{n^2+1}$

97. If $\operatorname{cosec}\theta - \cot\theta = \frac{7}{2}$, then value of $\operatorname{cosec}\theta$ is;

(a) $\frac{47}{28}$ (b) $\frac{51}{28}$ (c) $\frac{53}{28}$ (d) $\frac{49}{28}$

98. If x sin45°= y cosec30°, then $\frac{x^4}{y^4}$ is equal to

(a) 4^3 (b) 6^3 (c) 2^3 (d) 8^3

99. If 5 tanθ= 4, then $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

100. 2 $\operatorname{cosec}^2 23° \cot^2 67° - \sin^2 23° - \sin^2 67° - \cot^2 67°$ is equal to

(a) 1 (b) $\sec^2 23°$

(c) $\tan^2 23°$ (d) 0

101. The equation $\cos^2\theta = \frac{(x+y)^2}{4xy}$ is only possible when

(a) $x = -y$ (b) $x > y$

(c) $x = y$ (d) $x < y$

102. If $\alpha + \beta = 90°$, then the value of $\left(1 - \sin^2\alpha\right)\left(1 - \cos^2\alpha\right) \times \left(1 + \cot^2\beta\right)\left(1 + \tan^2\beta\right)$ is

(a) 1 (b) – 1 (c) 0 (d) 2

103. $\frac{2\sin 68°}{\cos 22°} - \frac{2\cot 15°}{5\tan 75°} - \frac{3\tan 45°.\tan 20°.\tan 40°.\tan 50°.\tan 70°}{5}$ is equal to
(a) –1 (b) 0 (c) 1 (d) 2

104. If $\sin 7x = \cos 11x$, then the value of $\tan 9x + \cot 9x$ is
(a) 1 (b) 2 (c) 3 (d) 4

105. If $\tan^2\alpha = 1 + 2\tan^2\beta$ (here, α, β are positive acute angles), then $\sqrt{2}\cos\alpha - \cos\beta$ is equal to
(a) 0 (b) $\sqrt{2}$ (c) 1 (d) – 1

106. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^{100}\theta + \cot^{100}\theta$ is
(a) 2 (b) 0 (c) 1 (d) $\sqrt{3}$

107. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$ is equal to
(a) $1 - \tan\theta - \cot\theta$
(b) $1 + \tan\theta - \cot\theta$
(c) $1 - \tan\theta + \cot\theta$
(d) $1 + \tan\theta + \cot\theta$

108. If $\sec\theta + \tan\theta = 2 + \sqrt{5}$, then the value of $\sin\theta + \cos\theta$ is ;
(a) $\frac{3}{\sqrt{5}}$ (b) $\sqrt{5}$ (c) $\frac{7}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{5}}$

109. The value of $(1+\cot\theta - \text{cosec}\theta)(1+\tan\theta + \sec\theta)$ is equal to
(a) 1 (b) 2 (d) 0 (d) – 1

110. If $x = a\sec\theta\cos\phi$, $y = b\sec\theta\sin\phi$, $z = c\tan\theta$, then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ is ;
(a) 1 (b) 4 (c) 9 (d) 0

111. If $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$, then $\sin\theta$ is equal to ;
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

112. If $(1+\sin\alpha)(1+\sin\beta)(1+\sin\gamma) = (1-\sin\alpha)(1-\sin\beta)(1-\sin\gamma)$, then third side is equal to
(a) $\pm\cos\alpha\cos\beta\cos\gamma$
(b) $\pm\sin\alpha\sin\beta\sin\gamma$
(c) $\pm\sin\alpha\cos\beta\sec\gamma$
(d) $\pm\sin\alpha\sin\beta\cos\gamma$

113. The numerical value of $\frac{1}{1+\cot^2\theta} + \frac{3}{1+\tan^2\theta} + 2\sin^2\theta$ will be
(a) 2 (b) 5 (c) 6 (d) 3

114. The value of $\frac{4}{1+\tan^2\alpha} + \frac{1}{1+\cot^2\alpha} + 3\sin^2\alpha$ is
(a) 4 (b) –1 (b) 2 (d) 3

115. The value of 3 $(\sin x + \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$ is
(a) 14 (b) 11 (c) 12 (d) 13

116. The value of
$$\sec\theta\left(\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}\right) - 2\tan^2\theta$$ is
(a) 4 (b) 1 (c) 2 (d) 0

117. The elimination of θ from $x\cos\theta - y\sin\theta = 2$ and $x\sin\theta + y\cos\theta = 4$ will give
(a) $x^2+y^2 = 20$ (b) $3x^2+y^2 = 20$
(c) $x^2-y^2 = 20$ (d) $3x^2-y^2 = 10$

118. The value of
$$\left[\frac{\cos^2 A(\sin A + \cos A)}{\text{cosec}^2 A(\sin A - \cos A)} + \frac{\sin^2 A(\sin A - \cos A)}{\sec^2 A(\sin A + \cos A)}\right] \times (\sec^2 A - \text{cosec}^2 A)$$
(a) 1 (b) 3 (c) 2 d) 4

119. The value of
$$\frac{1}{\text{cosec}\,\theta - \cot\theta} - \frac{1}{\sin\theta}$$ is
(a) 1 (b) $\cot\theta$
(b) $\text{cosec}\,\theta$ (d) $\tan\theta$

120. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta$ is
(a) $\sqrt{2}\tan\theta$ (b) $-\sqrt{2}\cos\theta$
(c) $-\sqrt{2}\sin\theta$ (d) $\sqrt{2}\sin\theta$

121. If $\sin\theta - \cos\theta = \frac{1}{2}$ then value of $\sin\theta + \cos\theta$ is;
(a) –2 (b) ±2
(c) $\frac{\sqrt{7}}{2}$ (d) 2

122. The value of $\frac{\sin A}{1+\cos A} + \frac{\sin A}{1-\cos A}$ is (0°<A<90°)
(a) 2 cosec A (b) 2 sec A
(c) 2 sin A (d) 2 cos A

123. If $\tan\theta - \cot\theta = 0$, find the value of $\sin\theta + \cos\theta$,
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2

124. If $3\sin\theta + 5\cos\theta = 5$, then $5\sin\theta - 3\cos\theta$ is equal to
(a) ±3 (b) ±5 (c) 1 (d) 0

125. If x sin 60°. tan30° = sec60°. cot45°, then the value of x is
(a) 2 (b) $2\sqrt{3}$
(c) 4 (d) $4\sqrt{3}$

126. If $\theta = 60°$, then $\frac{1}{2}\sqrt{1+\sin\theta} + \frac{1}{2}\sqrt{1-\sin\theta}$ is equal to
(a) $\cot\frac{\theta}{2}$ (b) $\sec\frac{\theta}{2}$
(c) $\sin\frac{\theta}{2}$ (d) $\cos\frac{\theta}{2}$

127. If $\frac{2\tan^2 30°}{1-\tan^2 30°} + \sec^2 45° - \sec^2 0° = x\sec 60°$, then the value of x is
(a) 2 (b) 1 (c) 0 (d) – 1

128. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then $\sin\alpha + \cos\alpha$ is
(a) $\pm\sqrt{2}\sin\theta$ (b) $\pm\sqrt{2}\cos\theta$
(c) $\pm\frac{1}{\sqrt{2}}\sin\theta$ (d) $\pm\frac{1}{\sqrt{2}}\cos\theta$

129. If $7\sin^2\theta + 3\cos^2\theta = 4$, $(0° < \theta < 90°)$, then the value of $\tan\theta$ is
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{3}$

130. If tan9° = $\frac{p}{q}$, then the value of $\frac{\sec^2 81°}{1+\cot^2 81°}$ is

(a) $\frac{q}{p}$ (b) 1 (c) $\frac{p^2}{q^2}$ (d) $\frac{q^2}{p^2}$

131. If $\sec\theta + \tan\theta = 5$ then the value of $\frac{\tan\theta+1}{\tan\theta-1}$ is

(a) $\frac{11}{7}$ (d) $\frac{13}{7}$ (c) $\frac{15}{7}$ (d) $\frac{17}{7}$

132. If $\frac{\cos\alpha}{\cos\beta} = a$ and $\frac{\sin\alpha}{\sin\beta} = b$, then the value of $\sin^2\beta$ in terms of a and b is

(a) $\frac{a^2+1}{a^2-b^2}$ (b) $\frac{a^2-b^2}{a^2+b^2}$

(c) $\frac{a^2-1}{a^2-b^2}$ (d) $\frac{a^2-1}{a^2+b^2}$

133. The value of $\frac{\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$ is

(a) $\frac{64}{\sqrt{3}}$ (b) $\frac{55}{12}$ (c) $\frac{67}{12}$ (d) $\frac{67}{10}$

134. If $\cos \pi x = x^2 - x + \frac{5}{4}$, the value of x will be

(a) 0 (b) 1 (d) –1

(d) None of the above

135. The numerical value of $1+\frac{1}{\cot^2 63°} - \sec^2 27° + \frac{1}{\sin^2 63°} - \cos ec^2 27°$ is

(a) 1 (b) 2 (c) – 1 (d) 0

136. If $x = \frac{\cos\theta}{1-\sin\theta}$, then $\frac{\cos\theta}{1+\sin\theta}$ is equal to

(a) $x-1$ (b) $\frac{1}{x}$

(b) $\frac{1}{x+1}$ (d) $\frac{1}{1-x}$

137. In ΔABC, $\angle B = 90°$ and AB : BC = 2 : 1, Then value of (SinA + CotC)

(a) $3+\sqrt{5}$ (b) $\frac{2+\sqrt{5}}{2\sqrt{5}}$

(c) $2+\sqrt{5}$ (d) $3\sqrt{5}$

138. If $\sin\frac{\pi x}{2} = x^2-2x+2$, then the value of x is

(a) 0 (b) 1

(c) –1 (d) None of these

139. The value of $\frac{\sin 43°}{\cos 47°} + \frac{\cos 19°}{\sin 71°} - 8\cos^2 60°$

(a) 0 (b) 1 (c) 2 (d) – 1

140. The value of $\left(\sin^2 7\frac{1}{2}° + \sin^2 82\frac{1}{2}°\right)$ is

(a) 1 (b) 2 (c) 0 (d) 4

141. If 3sin$x$ + 5cos$x$ = 5, then what is the value of (3cos$x$ – 5sin$x$)?

(a) 0 (b) 2 (c) 3 (d) 5

142. If α and β are complementary angles, then what is $\sqrt{(\text{cosec}\,\alpha.\text{cosec}\,\beta)}\left(\frac{\sin\alpha}{\sin\beta}+\frac{\cos\alpha}{\cos\beta}\right)^{-\frac{1}{2}}$ equal to?

(a) 0 (b) 1

(c) 2 (d) None of these

143. If A, B, C and D are the successive angles of a cyclic quadrilateral, then what is cosA + cosB + cosC + cosD are equal to?

(a) 4 (b) 2 (c) 1 (d) 0

144. How many degrees are there in an angle which equals two-third of its complement?

(a) 36° (b) 45° (c) 48° (d) 60°

145. If $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2$ with $0 < \theta < 90°$, then what is θ equal to?

(a) 30° (b) 45° (c) 60° (d) 75°

146. If $\sin 3\theta = \cos(\theta - 2°)$, where 3θ and $(\theta - 2°)$ are acute angles, what is the value of θ?

(a) 22° (b) 23° (c) 24° (d) 25°

147. What is $\frac{\sin^6\theta - \cos^6\theta}{\sin^2\theta - \cos^2\theta}$ equal to?

(a) $\sin^4\theta - \cos^4\theta$

(b) $1-\sin^2\theta\cos^2\theta$

(c) $1+\sin^2\theta\cos^2\theta$

(d) $1-3\sin^2\theta\cos^2\theta$

148. If tan A = $\frac{1-\cos B}{\sin B}$, then what is $\frac{2\tan A}{1-\tan^2 A}$ equal to?

(a) $\frac{\tan B}{2}$ (b) 2tanB

(c) tanB (d) 4tanB

149. Assume the Earth to be a sphere of radius R. What is the radius of the circle of latitude 40°S?

(a) R cos40° (b) R sin80°

(c) R sin 40° (d) R tan 40°

150. If $\cos\theta \geq \frac{1}{2}$ in the first quadrant, then which one of the following is correct?

(a) $\theta \leq \frac{\pi}{3}$ (b) $\theta \geq \frac{\pi}{3}$

(c) $\theta \leq \frac{\pi}{6}$ (d) $\theta \geq \frac{\pi}{6}$

151. If $\sin\theta + \cos\theta = 1$, then what is the value of $\sin\theta.\cos\theta$?

(a) 2 (b) 0

(c) 1 (d) $\frac{1}{2}$

152. What is $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ equal to?

(a) $\sec\theta - \tan\theta$

(b) $\sec\theta + \tan\theta$

(c) $\text{cosec}\,\theta + \cot\theta$

(d) $\text{cosec}\,\theta - \cot\theta$

153. Two sides of an acute angle triangle are 6 cm and 2 cm, respectively. Which one of the following represents the correct range of the third side in cm?

(a) (4, 8) (b) $(4, 2\sqrt{10})$

(c) $(4\sqrt{2}, 8)$ (d) $(4\sqrt{2}, 2\sqrt{10})$

154. If cos1° = p and cos89° = q, then which one of the following is correct?

(a) p is close to 0 and q is close to 1

(b) p < q
(c) p = q
(d) p is close to 1 and q is close to 0

155. If $7\cos^2\theta + 3\sin^2\theta = 4$ and $0 < \theta < \frac{\pi}{2}$, then what is the value of $\tan\theta$?
(a) $\sqrt{7}$ (b) $\frac{7}{3}$ (c) 3 (d) $\sqrt{3}$

156. What is the value of $[(1-\sin^2\theta)\sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$ when $0° < \theta < 90°$?
(a) 2 (b) >2 (c) ≥2 (d) <2

157. If $0 \le \theta < \frac{\pi}{2}$ and $p = \sec^2\theta$, then which one of the following is correct?
(a) p < 1 (b) p = 1
(c) p > 1 (d) p ≥ 1

158. In a ΔABC, $\angle ABC = 90°$, $\angle ACB = 30°$, AB = 5 cm. What is the length of AC?
(a) 10 cm (b) 5 c m
(c) $5\sqrt{2}$ cm (d) $5\sqrt{3}$ cm

159. If $0 \le \theta \le \frac{\pi}{2}$ and $\cos\theta + \sqrt{3}\sin\theta = 2$, then what is the value of θ?
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

160. If ABC is a right angled triangle at C and having u units, v units and w units as the length of its sides opposite to be vertices A,B and C respectively, then what is tan A + tan B equal to?
(a) $\frac{u^2}{vw}$ (b) 1
(c) $u + v$ (d) $\frac{w^2}{uv}$

161. ABC is a right triangle with right angle at A. If the value of $\tan B = \frac{1}{\sqrt{3}}$, then for any real k the length of the hypotenuse is of the form.
(a) $3k$ (b) $2k$ (c) $5k$ (d) $9k$

162. If α is an acute angle and $\sin\alpha = \sqrt{\frac{x-1}{2x}}$, then what is $\tan\alpha$ equal to?
(a) $\sqrt{\frac{x-1}{x+1}}$ (b) $\sqrt{\frac{x+1}{x-1}}$
(c) $\sqrt{x^2-1}$ (d) $\sqrt{x^2+1}$

163. $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2$ is satisfied by which one of the following values of θ?
(a) $\pi/2$ (b) $\pi/3$
(c) $\pi/4$ (d) $\pi/6$

164. If $0° < x < 45°$ and $45° < y < 90°$, then which one of the following is correct?
(a) $\sin x = \sin y$
(b) $\sin x < \sin y$
(c) $\sin x > \sin y$
(d) $\sin x \le \sin y$

165. What is the value of $\sin^3 60° \cot 30° - 2\sec^2 45° + 3\cos 60° \tan^2 45° - \tan^2 60°$?
(a) 35/8 (b) –35/8
(c) –11/8 (d) 11/8

166. If $\tan\theta = \frac{p}{q}$, then what is $\frac{p\sec\theta - q\operatorname{cosec}\theta}{p\sec\theta + q\operatorname{cosec}\theta}$ equal to?
(a) $\frac{p-q}{p+q}$ (b) $\frac{q^2-p^2}{q^2+p^2}$
(c) $\frac{p^2-q^2}{q^2+p^2}$ (d) 1

167. The value of $\operatorname{cosec}^2\theta - 2 + \sin^2\theta$ is always
(a) less then zero
(b) non-negative
(c) zero
(d) 1

168. Find the value of $1 - 2\sin^2\theta + \sin^4\theta$,
(a) $\sin^4\theta$ (b) $\cos^4\theta$
(c) $\operatorname{cosec}^4\theta$ (d) $\sec^4\theta$

169. $\sin\theta = 0.7$, then $\cos\theta$, $0° \le \theta < 90°$ is
(a) 0.3 (b) $\sqrt{0.49}$
(c) $\sqrt{0.51}$ (d) $\sqrt{0.9}$

170. The value of $\sin^2 65° + \sin^2 25° + \cos^2 35° + \cos^2 55°$ is
(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

171. If $\cos^2\theta - \sin^2\theta = \frac{1}{3}$, where $0 \le \theta < \frac{\pi}{2}$, then the value of $\cos^4\theta - \sin^4\theta$ is
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$

172. If $\tan\theta = \frac{1}{\sqrt{11}}$ and $0° < \theta < \frac{\pi}{2}$, then the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$ is
(a) $\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{5}{6}$ (d) $\frac{6}{7}$

173. If $\sin\theta = \frac{3}{5}$, then the value of $\frac{\tan\theta + \cos\theta}{\cot\theta + \operatorname{cosec}\theta}$ is equal to
(a) $\frac{29}{60}$ (b) $\frac{31}{60}$ (c) $\frac{34}{60}$ (d) $\frac{37}{60}$

174. If $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$, then the value of k is
(a) 1 (b) 7 (c) 3 (d) 5

175. If $\sin 21° = \frac{x}{y}$, then $\sec 21° - \sin 69°$ is equal to
(a) $\frac{x^2}{y\sqrt{y^2-x^2}}$ (b) $\frac{y^2}{x\sqrt{y^2-x^2}}$
(c) $\frac{x^2}{y\sqrt{x^2-y^2}}$ (d) $\frac{y^2}{x\sqrt{x^2-y^2}}$

176. If $\sec\alpha + \tan\alpha = 2$, then the value of $\sin\alpha$ is (assume that $0° < \alpha < 90°$)
(a) 0.4 (b) 0.5
(c) 0.6 (d) 0.8

177. If $7\sin\alpha = 24\cos\alpha$; $0 < \alpha < \frac{\pi}{2}$, then the value of $14\tan\alpha - 75\cos\alpha - 7\sec\alpha$ is equal to
(a) 3 (b) 4 (c) 1 (d) 2

178. ABCD is a rectangle of which AC is a diagonal. The value of $(\tan^2 \angle CAD + 1)\sin^2 \angle BAC$ is

(a) 2 (b) $\frac{1}{4}$ (c) 1 (d) 0

179. For any real value of $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}}$ = ?

(a) $\cot\theta - \text{cosec}\theta$
(b) $\sec\theta - \tan\theta$
(c) $\text{cosec}\theta - \cot\theta$
(d) $\tan\theta - \sec\theta$

180. In a ΔABC, $\angle B = \frac{\pi}{3}$ $\angle C = \frac{\pi}{4}$ and D divides BC internally in the ratio 1 : 3 then, $\frac{\sin\angle BAD}{\sin\angle CAD}$ is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{6}}$ (d) $\sqrt{6}$

181. If sin 3A = cos(A – 26°), where 3A is an acute angle then the value of A is

(a) 29° (b) 26° (c) 23° (d) 28°

182. The value of $\sec^2\theta$ – $\frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta}$ is

(a) 1 (b) 2 (c) – 1 (d) 0

183. If x = a $(\sin\theta + \cos\theta)$, y = b $(\sin\theta - \cos\theta)$ then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ is

(a) 0 (b) 1 (c) 2 (d) –2

184. If $\sin 5\theta = \cos 20°$ $(0° < \theta < 90°)$ then the value of θ is

(a) 4° (b) 22° (c) 10° (d) 14°

185. If $0° < \theta < 90°$ and $2\sec\theta = 3\,\text{cosec}^2\theta$, then θ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{5}$

186. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ is equal to

(a) $2\cos\theta$ (b) $2\sin\theta$
(c) $2\cot\theta$ (d) $2\sec\theta$

187. If α and β are positive acute angles, $\sin(4\alpha - \beta) = 1$ and $\cos(2\alpha + \beta) = \frac{1}{2}$, then the value of $\sin(\alpha + 2\beta)$ is

(a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

188. If θ is a positive acute angle and $\text{cosec}\,\theta = \sqrt{3}$, then the value of $\cot\theta - \text{cosec}\theta$ is

(a) $\frac{3\sqrt{2} - \sqrt{3}}{3}$ (b) $\frac{\sqrt{2}(3 + \sqrt{3})}{3}$

(c) $(\sqrt{2} - \sqrt{3})$ (d) $\frac{3\sqrt{2} + \sqrt{3}}{3}$

189. If $(r\cos\theta - \sqrt{3})^2 + (r\sin\theta - 1)^2 = 0$, then the value of $\frac{r\tan\theta + \sec\theta}{r\sec\theta + \tan\theta}$ is equal to

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{\sqrt{5}}{4}$

190. If $\frac{\cos\alpha}{\cos\beta} = a$, $\frac{\sin\alpha}{\sin\beta} = b$, then $\sin^2\beta$ is equal to

(a) $\frac{a^2 - 1}{a^2 + b^2}$ (b) $\frac{a^2 + 1}{a^2 - b^2}$

(c) $\frac{a^2 - 1}{a^2 - b^2}$ (d) $\frac{a^2 + 1}{a^2 + b^2}$

191. If $\sqrt{3}\tan\theta = 3\sin\theta$, then the value of $(\sin^2\theta - \cos^2\theta)$ is

(a) 1 (b) 3

(c) $\frac{1}{3}$ (d) None of these

192. ABC is a right angle triangle and right angle at B and $\angle A$ = 60° and AB = 20cm, then the ratio of sides BC and CA is

(a) $\sqrt{3} : 1$ (b) $1 : \sqrt{3}$
(c) $\sqrt{3} : \sqrt{2}$ (d) $\sqrt{3} : 2$

193. If tan(A + B) = $\sqrt{3}$ and tan (A – B) = $\frac{1}{\sqrt{3}}$, $(\angle A + \angle B) < 90°$, $A \geq B$, then $\angle A$ is

(a) 90° (b) 30° (c) 45° (d) 60°

194. The value of $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$ is equal to

(a) $\sin\theta$ (b) $\cos\theta$
(c) $\tan\theta$ (d) $\cot\theta$

195. If $\cot\theta = \frac{2xy}{x^2 - y^2}$, then what is $\cos\theta$ equal to?

(a) $\frac{x^2 - y^2}{x^2 + y^2}$ (b) $\frac{x^2 + y^2}{x^2 - y^2}$

(c) $\frac{2xy}{x^2 + y^2}$ (d) $\frac{2xy}{\sqrt{x^2 + y^2}}$

196. For what value of θ is $(\sin\theta + \text{cosec}\,\theta) = 2.5$, where $0° < \theta < 90°$?

(a) 30° (b) 45° (c) 60° (d) 90°

197. If x cos60° + y cos0° = 3 and 4x sin30° – y cot 45° = 2, then what is the value of x?

(a) –1 (b) 0 (c) 1 (d) 2

198. What is log (tan 1°) + log (tan 2°) + log (tan 3°) + ….+ log (tan 89°) equal to?

(a) 0 (b) 1 (c) 2 (d) –1

199. If sin x cos x = 1/2, then what is the value of sin$x$ – cos$x$?

(a) 2 (b) 1 (c) 0 (d) –1

200. If $\tan^2 y\,\text{cosec}^2 x - 1 = \tan^2 y$, then which one of the following is correct?

(a) x – y = 0 (b) x = 2y
(c) y = 2x (d) x – y = 1

201. If $\frac{\cos x}{1 + \text{cosec}\,x} + \frac{\cos x}{\text{cosec}\,x - 1} = 2$, then which one of the following is one of the values of x?

(a) $\pi/2$ (b) $\pi/3$
(c) $\pi/4$ (d) $\pi/6$

202. If x + y = 90° and sin x : sin y = $\sqrt{3}$: 1, then what is x : y equal to?

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 3 : 2

203. If $\frac{\cos x}{\cos y}$ = n and $\frac{\sin x}{\sin y}$ = m, then $(m^2 - n^2)\sin^2 y$ is equal to

(a) $1 - n^2$ (b) $1 + n^2$
(c) m^2 (d) n^2

204. If p = $\tan^2 x + \cot^2 x$, then which one of the following is correct?

(a) p ≤ 2 (b) p ≥ 2
(c) p < 2 (d) p > 2

205. What is the value of

$$\frac{5\sin 75^\circ \sin 77^\circ + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ} - \frac{7\sin 81^\circ}{\cos 9^\circ}$$

(a) –1 (b) 0 (c) 1 (d) 2

206. If sin$x$ + sin y = a and cos$x$ + cos y = b, what is sin x.sin y + cos$x$. cos y equal to?
(a) a + b – ab (b) a + b + ab
(c) $a^2 + b^2 - 2$ (d) $\left(\frac{a^2+b^2-2}{2}\right)$

207. If α is the angle of first quadrant such that $\text{cosec}^4\alpha = 17 + \cot^4\alpha$, then what is the value of $\sin\alpha$?
(a) 1/3 (b) 1/4
(c) 1/9 (d) 1/16

208. If $x + \left(\frac{1}{x}\right) = 2\cos\alpha$, then what is the value $x^2 + \left(\frac{1}{x^2}\right)$?
(a) $4\cos^2\alpha$
(b) $4\cos^2\alpha - 1$
(c) $2\cos^2\alpha - 2\sin^2\alpha$
(d) $\cos^2\alpha - \sin^2\alpha$

209. What is the value $\cot^2\theta - \frac{1}{\sin^2\theta}$?
(a) 1/2 (b) –1
(c) –1/2 (d) 3/2

210. If sin x = cos y and angle x and angle y are acute then what is the relation between x and y?
(a) $x - y = \pi/2$
(b) $x + y = 3\pi/2$
(c) $x + y = \pi/2$
(d) $x + y = \pi/4$

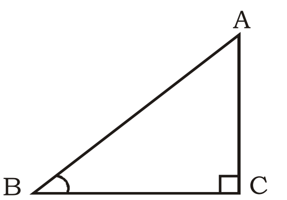
211. If $\sin\theta = \frac{m^2 - n^2}{m^2 + n^2}$, then what is the value of $\tan\theta$?
(a) $\frac{m^2+n^2}{m^2-n^2}$ (b) $\frac{2mn}{m^2+n^2}$
(c) $\frac{m^2-n^2}{2mn}$ (d) $\frac{m^2+n^2}{2mn}$

212. If sin$(x - y)$ = 1/2 and cos $(x + y)$ = 1/2, then what is the valueof x?
(a) 15° (b) 30° (c) 45° (d) 60°

213. If $1 + \tan\theta = \sqrt{2}$, then what is the value of $\cot\theta - 1$?
(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 0 (d) 1

214. If sin $(x + 54°)$ = cos x, where 0 < x, x + 54° < 90°, then what is the value of x?
(a) 54° (b) 36° (c) 27° (d) 18°

215. In the given figure, BC = 15 cm and sin B = 4/5. What is the value of AB?



(a) 25 cm (b) 20 cm
(c) 5 cm (d) 4 cm

216. The smallest side of a right angled triangle has length 2 cm. The tangent of one acute angle is $\frac{3}{4}$. What is the hypotenuse of the triangle?
(a) 5 cm (b) 2.5 cm
(c) 1.25 cm (d) $\frac{10}{3}$ cm

217. If sin x – cos x = 0, then what is the value of $\sin^4 x + \cos^4 x$?
(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

218. What is the expression $\frac{\tan x}{1+\sec x} - \frac{\tan x}{1-\sec x}$ equal to?
(a) cosec$x$ (b) 2 cosec$x$
(c) 2 sin$x$ (d) 2 cos$x$

219. What is the expression $(\sin^4 x - \cos^4 x + 1)\,\text{cosec}^2 x$ equal to?
(a) 1 (b) 2 (c) 0 (d) –1

220. If x + y = 90°, then what is $\sqrt{\cos x.\text{cosec}y - \cos x.\sin y}$ equal to?
(a) cos$x$ (b) sin$x$
(c) $\sqrt{\cos x}$ (d) $\sqrt{\sin x}$

221. If p = $\sin^{10} x$, then which one of the following correct for any value x?
(a) $p \geq 1$ (b) $0 \leq p \leq 1$
(c) $1 \leq p \leq 2$ (d) None of these

222. What is the value of the expression $\cos^2\frac{\pi}{8} + 4\cos^2\frac{\pi}{4} - \sec\frac{\pi}{3} + 5\tan^2\frac{\pi}{3} + \sin^2\frac{\pi}{8}$?
(a) 8 (b) 10 (c) 16 (d) 18

223. If cosecθ = p/q and θ is acute, then what is the value of $\left(\sqrt{p^2 - q^2}\right)\tan\theta$?
(a) p (b) q
(c) pq (d) $\sqrt{p^2 + q^2}$

224. If $2x^2\cos 60° - 4\cot^2 45° - 2\tan 60° = 0$, then what is the value of x?
(a) 2 (b) 3
(c) $\sqrt{3} - 1$ (d) $\sqrt{3} + 1$

225. Which one of the following statements is true in respect of the expression sin31° + sin32°?
(a) Its value is 0
(b) Its value is 1
(c) Its value is less than 1
(d) Its value is greater than 1

226. Which one of the following is correct?
(a) sin 35° > cos 55°
(b) $\cos 61° > \frac{1}{2}$
(c) $\sin 32° > \frac{1}{2}$
(d) tan 44° > 1

227. If $\sin\theta + \text{cosec}\theta = 2$, then what is the value of $\sin^4\theta + \cos^4\theta$?
(a) 2 (b) 2^2 (c) 2^3 (d) 1

228. If $\theta \in R$ be such that $\sec\theta > 0$ and $2\sec^2\theta + \sec\theta - 6 = 0$
Then, what is the value of cosecθ?
(a) $\sqrt{5}$ (b) $\sqrt{3}/2$
(c) $3/\sqrt{5}$ (d) $2/\sqrt{3}$

229. Under which one of the following conditions is the trigonometrical identity.
sin$x$/(1 + cos$x$) = (1 – cos$x$)/sin$x$ true?
(a) x is not a multiple of 360°
(b) x is not an odd multiple of 180°
(c) x is not a multiple of 180°
(d) None of the above

230. If $3\sin\theta + 4\cos\theta = 5$, then what is $3\cos\theta - 4\sin\theta$ equal to?
(a) 0 (b) 3 (c) 4 (d) 5

231. If $\sec\theta = 13/5$, then what is value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$?
(a) 1 (b) 2 (c) 3 (d) 4

232. If $r \sin\theta = \frac{7}{2}$ and $r\cos\theta = \frac{7\sqrt{3}}{2}$ then value of r is
(a) 4 (b) 3 (c) 5 (d) 7

233. If $\theta + \phi = \frac{\pi}{2}$ and $\sin\theta = \frac{1}{2}$, then the value of $\sin\phi$ is
(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

234. If sec θ + tan θ = P, (P $\neq$ 0) the sec θ is equal to:
(a) $\frac{1}{3}\left(P\frac{1}{P}\right)$, P $\neq 0$
(b) $\frac{1}{2}\left(P+\frac{1}{P}\right)$, P $\neq 0$
(c) $2\left(P+\frac{1}{P}\right)$, P $\neq 0$
(d) $\left(P-\frac{1}{P}\right)$, P $\neq 0$

235. The value of $\sin^2 22° + \sin^2 68° + \cot^2 30°$ is:
(a) 5/4 (b) 3/4
(c) 3 (d) 4

236. If $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} = 2\frac{51}{79}$ then the value of sinθ is
(a) $\frac{91}{144}$ (b) $\frac{39}{72}$ (c) $\frac{65}{144}$ (d) $\frac{35}{72}$

237. If $1 + \cos^2\theta = 3\sin\theta\cos\theta$, then the integral value of cotθ is $\left(0 < \theta < \frac{\pi}{2}\right)$
(a) 2 (b) 1 (c) 3 (d) 0

238. The value of following is $\cos 2\,4° + \cos 55° + \cos 125° + \cos 204° + \cos 300°$
(a) – 1/2 (b) 1/2
(c) 2 (d) 1

239. If sinA + cosecA = 3, then find the value of $\frac{\sin^4 A + 1}{\sin^2 A}$.
(a) 1 (b) 0 (c) 7 (d) 0

240. cos7° cos23° cos45° cosec83° cosec67° = ?
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

241. If $\frac{\tan\theta+\cot\theta}{\tan\theta-\cot\theta} = 2$, (0° ≤ θ ≤ 90°), then the value of sinθ is
(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) 1

242. If sin x + cos x = c, then $\sin^6 x + \cos^6 x$ is equal to.
(a) $\frac{1+6c^2-3c^4}{16}$
(b) $\frac{1+6c^2-3c^4}{4}$
(c) $\frac{1+6c^2+3c^4}{16}$
(d) $\frac{1+6c^2+3c^4}{4}$

243. $\frac{1-\sin A\cos A}{\cos A(\sec A - \operatorname{cosec} A)} \cdot \frac{\sin^2 A - \cos^2 A}{\sin^3 A + \cos^3 A} = ?$
(a) sinA (b) cosA
(c) tanA (d) cosecA

244. If cotθ + cosθ = m and cotθ – cosθ = n, then find the value of $m^2 - n^2$.
(a) $\sqrt{mn}$ (b) $2\sqrt{mn}$
(c) $3\sqrt{mn}$ (d) $4\sqrt{mn}$

245. If tanθ + sinθ = m and tanθ – sinθ = n, then find the value of $\sqrt{mn}$.
(a) $\frac{1}{2}(m^2-n^2)$ (b) $2(m^2-n^2)$
(c) $\frac{1}{4}(m^2+n^2)$ (d) $\frac{1}{4}(m^2-n^2)$

246. If cotθ + tanθ = x and secθ – cosθ = y, then $(x^2y)^{2/3} - (xy^2)^{2/3}$ =?
(a) 4 (b) 3 (c) 2 (d) 1

247. $\frac{\sin^8\theta - \cos^8\theta}{\cos 2\theta(1+\cos^2 2\theta)} = ?$
(a) 1 (b) $-\frac{1}{2}$ (c) – 1 (d) 2

248. If (secα + tanα) (secβ + tanβ) (secγ + tanγ) = (secα – tanα) (secβ – tanβ) (secγ – tanγ), then each of the side is equal to
(a) ±1 (b) – 1 (c) +1 (d) 4

249. If a secθ + b tanθ + c = 0 and p secθ + q tanθ + r = 0, then $(br - qc)^2 - (pc - ar)^2$ = ?
(a) $(aq - bp)^2$ (b) $(ap - bq)^2$
(c) $(aq + bp)^2$ (d) $(aq - bp)^3$

250. If $P = a\cos^3 x + 3a\cos x.\sin^2 x$ and $Q = a\sin^3 x + 3a\cos^2 x.\sin x$, then $(P+Q)^{2/3} + (P-Q)^{2/3}$ = ?
(a) $2a^{2/3}$ (b) $a^{1/3}$
(c) $2a^{1/3}$ (d) $a^{1/3}$

251. If $8\cos^2\theta + 8\sec^2\theta = 65$ and $0° < \theta < \frac{\pi}{2}$, then $4\cos 2\theta$ is equal to
(a) $-\frac{23}{8}$ (b) $-\frac{31}{8}$
(c) $-\frac{31}{32}$ (d) $-\frac{33}{32}$

252. If cos(θ – A) = a, cos(θ – B) = b, then $\sin^2(A-B) + 2ab\cos(A-B)$ is equal to
(a) $a^2 - b^2$ (b) $a^2 + b^2$
(c) $b^2 - a^2$ (d) 2ab

253. $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$ = ?
(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) 0

254. If sinθ + cosθ = a and secθ + cosecθ = b, then the value of b $(a^2 - 1)$ is equal to
(a) 2a (b) 3a (c) 0 (d) 2ab

255. $\cos 15° \cos 7\frac{1}{2}°. \cos 82\frac{1}{2}°$ = ?
(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

256. 3tanθ tanϕ =1, then $\frac{\cos(\theta-\phi)}{\cos(\theta+\phi)}$ =?
(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) 3

257. tan20° + tan40° + $\sqrt{3}$ tan20°. tan40° = ?
(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) 1

258. If x cosq – sinq = 1, then $x^2 + (1+x^2)$ sinq equals
(a) 1 (b) – 1 (c) 0 (d) 2

259. The simplified form of the given expression sinA cosA(tanA – cotA) is (where 0° <A < 90°):
(a) 1 (b) 1 – $\cos^2$A
(c) 1 – 2 $\sin^2$A (d) 2 $\sin^2$ A – 1

260. $\frac{\text{Cos}\,\alpha}{\text{Sin}\,\beta}$ = n and $\frac{\text{Cos}\,\alpha}{\text{Cos}\,\beta}$ = m, then the value of $\cos^2\beta$ is:
(a) $\frac{m^2-1}{n^2-1}$ (b) $\frac{m^2-3}{n^2-4}$
(c) $\frac{m^2+3}{n^2+3}$ (d) $\frac{n^2}{m^2+n^2}$

261. Provided, sin(A – B) = sinA cosB – cosA sin B, then sin15° will be
(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2\sqrt{2}}$
(c) $\frac{\sqrt{3}-1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

262. If α is an acute angle and 2sinα + 15 $\cos^2\alpha$ = 7, then the value of cotα is
(a) 4/3 (b) 5/4
(c) 4/5 (d) 3/4

263. If $\sin^4\theta + \cos^4\theta = 2\sin^2\theta\cos^2\theta$. θ is an acute angle, then value of tanθ is
(a) $\sqrt{2}$ (b) 1 (c) 3/5 (d) 0

264. If A is an acute angle and cotA +cosec A = 3, then the value of sin A is
(a) 1 (b) 4/5
(c) 3/5 (d) 0

265. The value of. cot41°.cot42°.cot43°.cot44°.cot45°.cot46°.cot47°.cot48°.cot49°.
(a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) 0 (d) $\frac{1}{\sqrt{2}}$

266. If 5 cosθ +12 sinθ = 13, θ < θ < 90°, then the value of sinθ is
(a) $\frac{6}{13}$ (b) $-\frac{12}{13}$
(c) $\frac{5}{13}$ (d) $\frac{12}{13}$

267. If tanθ – cotθ = 0 and θ is positive acute angle, then the value of $\frac{\tan(\theta+15^\circ)}{\tan(\theta-15^\circ)}$ is
(a) 3 (b) $\frac{1}{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

268. If secθ – tanθ = $\frac{1}{\sqrt{3}}$, the value of secθ .tanθ is:
(a) $\frac{4}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{\sqrt{3}}$

269. The value of (cosec a – sin a) (sec a – cos a) (tan a + cot a)
(a) 4 (b) 6 (c) 2 (d) 1

270. If tan A = n tan B and sin A = m sin B, then the value of $\cos^2$ A is
(a) $\frac{m^2+1}{n^2+1}$ (b) $\frac{m^2+1}{n^2-1}$
(c) $\frac{m^2-1}{n^2-1}$ (d) $\frac{m^2-1}{n^2+1}$

271. If $x\cos^2 30^\circ.\sin 60^\circ = \frac{\tan^2 45^\circ.\sec 60^\circ}{\text{cosec}\,60^\circ}$ then the value of x
(a) $\frac{1}{\sqrt{3}}$ (b) $2\frac{2}{3}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

272. If $\sin(\theta+30^\circ) = \frac{3}{\sqrt{12}}$, then find $\cos^2\theta$
(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

273. If $0 \le \theta \le 90^\circ$ and $4\cos^2\theta - 4\sqrt{3}\cos\theta + 3 = 0$, then the value of θ is
(a) 30° (b) 90° (c) 45° (d) 60°

274. The value of $\sec^4 A(1-\sin^4 A) - 2\tan^2 A$ is
(a) ½ (b) 0 (c) 2 (d) 1

275. If A, B, C are the angles of a Δ ABC then following is equal to:
$\text{Sin}\left(\frac{B+C}{2}\right)$
(a) $\sec\frac{B}{2}$ (b) $\sec\frac{A}{2}$
(c) $\text{cosec}\frac{A}{2}$ (d) $\cos\frac{A}{2}$

276. The numerical value of
$\frac{9}{\text{cosec}^2\theta} + 4\cos^2\theta + \frac{5}{1+\tan^2\theta}$:
(a) 7 (b) 9 (c) 4 (d) 5

277. If $\cos\theta = \frac{p}{\sqrt{p^2+q^2}}$, then the value of tanθ is:
(a) $\frac{q}{p}$ (b) $\frac{p}{p^2+q^2}$
(c) $\frac{q}{\sqrt{p^2+q^2}}$ (d) $\frac{q}{\sqrt{p^2-q^2}}$

278. If secθ +tanθ = 3, θ being acute, the value of 5 sinθ is:
(a) $\sqrt{3/5}$ (b) $\sqrt{5/3}$
(c) 4 (d) $\frac{5}{2}$

279. If $\frac{x - x\tan^2 30^\circ}{1+\tan^2 30^\circ} = \sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ$ Then value of x is:
(a) $\frac{1}{4}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$

280. If cosθ +sinθ =m and Secθ + cosecθ =n then the value of $n(m^2-1)$ is equal to:
(a) mn (b) 4mn
(c) 2n (d) 2m

281. If α + β = 90° then the expression $\frac{\tan\alpha}{\tan\beta} + \sin^2\alpha + \sin^2\beta$ is equal to :
(a) $\tan^2\alpha$ (b) $\tan^2\beta$
(c) $\sin^2\beta$ (d) $\sec^2\alpha$

282. The value of x in the equation
$\tan^2\frac{\pi}{4} - \cos^2\frac{\pi}{3} = x\sin\frac{\pi}{4}\cos\frac{\pi}{4}\tan\frac{\pi}{3}$ is :
(a) $\frac{3\sqrt{3}}{4}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

283. If $\sin 2\theta = \frac{\sqrt{3}}{2}$ then the value of $\sin 3\theta$ is equal to : (Take $0° \le \theta \le 90°$)

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 0 (d) 1

284. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$ then the value of $\sin^4\theta$ is :

(a) $\frac{16}{25}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{3}{5}$

285. If cos 20° = m and cos 70° = n, then the value of $m^2 + n^2$ is

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) $\frac{1}{\sqrt{2}}$

286. If $\sin A - \cos A = \frac{\sqrt{3}-1}{2}$ then the value of $\sin A \cdot \cos A$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{4}$ (d) $\frac{\sqrt{3}}{4}$

287. If $\sin\theta = \frac{a}{b}$, then the value of $\sec\theta - \cos\theta$ is (where $0° < \theta < 90°$)

(a) $\frac{a}{b\sqrt{b^2-a^2}}$ (b) $\frac{b^2}{a\sqrt{b^2-a^2}}$

(c) $\frac{a^2}{b\sqrt{b^2-a^2}}$ (d) $\frac{\sqrt{b^2+a^2}}{\sqrt{b^2-a^2}}$

288. The expression

$\frac{\sqrt{1+\sin\theta}}{\sqrt{1-\sin\theta}} + \frac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}}$ is equal to:

(a) 2 sinθ (b) 2 tanθ

(c) 2 secθ (d) 2 cosec θ

289. If $\frac{\sec^2 70° - \cot^2 20°}{2(\text{cosec}^2 59° - \tan^2 31°)} = \frac{2}{m}$, then m. is equal to:

(a) 2 (b) 3 (c) 4 (d) 1

290. If $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$ α, β lie between 0 and $\frac{\pi}{4}$, then $\tan 2\alpha$ = ?

(a) $\frac{56}{33}$ (b) $\frac{36}{33}$ (c) $\frac{33}{56}$ (d) $\frac{49}{36}$

291. $\cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \ldots + \ldots \cos\frac{6\pi}{7}$ = ?

(a) 0 (b) 1 (c) –1 (d) 2

292. tan9° – tan27° – tan63° + tan81° = ?

(a) 1 (b) 2 (c) 3 (d) 4

293. If 2 cos$x$ + sin$x$ = 1, then find 7 cos$x$ + 6 sin$x$.

(a) 6 (b) 2 (c) 7 (d) 1

294. $\cos^2 48° - \sin^2 12°$ = ?

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{8}$

(c) $\frac{\sqrt{3}-1}{4}$ (d) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

295. $\sin\left(\frac{\pi}{10}\right)\sin\left(\frac{3\pi}{10}\right)$ = ?

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 1

296. $\tan\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}-\theta\right)$ = ?

(a) 2 tan2θ (b) 2 cot2θ

(c) tan2θ (d) cot2θ

297. If $\sin(\alpha-\beta) = \frac{1}{2}$ and $\cos(\alpha+\beta) = \frac{1}{2}$, where α, β are positive acute angles, then α & β are_

(a) 45° and 15° (b) 60° and 15°

(c) 15° and 45° (d) 45° and 60°

298. If $\alpha + \beta - \gamma = \pi$, then $\sin^2\alpha + \sin^2\beta - \sin^2\gamma$ = ?

(a) $2\sin\alpha \sin\beta \cos\gamma$

(b) $2\cos\alpha \cos\beta \cos\gamma$

(c) $2\sin\alpha \sin\beta \sin\gamma$

(d) $2\cos\alpha \sin\beta \sin\gamma$

299. $1 + \cos 2x + \cos 4x + \cos 6x$ = ?

(a) $2\cos x \cos 2x \cos 3x$

(b) $4\sin x \cos 2x \cos 3x$

(c) $4\cos x \cos 2x \cos 3x$

(d) $\cos x \cos 2x \cos 3x$

300. sin12° sin48° sin54° = ?

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{32}$

301. $\frac{1}{4}\left[\sqrt{3}\cos 23° - \sin 23°\right]$ = ?

(a) cos43° (b) cos70°

(c) cos53° (d) $\frac{1}{2}$cos53°

302. $2\cos x - \cos 3x - \cos 5x$ = ?

(a) $16\cos^3 x \sin^2 x$

(b) $\sin^3 x \cos^2 x$

(c) $4\cos^3 x \sin^2 x$

(d) $4\sin^3 x \cos^2 x$

303. $\sqrt{3}$ cosec20° – sec20° = ?

(a) 2 (b) $\frac{2\sin 20°}{\sin 40°}$

(c) 4 (d) $\frac{4\sin 20°}{\sin 40°}$

304. $\frac{\cos 12° - \sin 12°}{\cos 12° + \sin 12°} + \frac{\sin 147°}{\cos 147°}$ = ?

(a) 1 (b) – 1 (c) 0 (d) $\frac{1}{2}$

305. cos24° + cos55° + cos125° + cos204° + cos300° = ?

(a) 2 (b) $\frac{-1}{2}$

(c) 0 (d) $\frac{1}{2}$

306. If m tan (θ – 30°) = n tan (θ + 120°), then $\frac{m+n}{m-n}$ = ?

(a) 2cos2θ (b) cos2θ

(c) 2sin2θ (d) sin2θ

307. If $\tan\theta = \frac{x\sin\phi}{1-x\cos\phi}$ & $\tan\phi = \frac{y\sin\theta}{1-y\cos\theta}$, then $\frac{x}{y}$ = ?

(a) $\frac{\sin\theta}{\sin\phi}$ (b) $\frac{\sin\phi}{\sin\theta}$

(c) $\frac{\sin\phi}{1-\cos\theta}$ (c) $\frac{\sin\theta}{1-\cos\phi}$

308. If $\tan x = \frac{b}{a}$, then $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$ = ?

(a) $\frac{2\sin x}{\sqrt{\sin 2x}}$ (b) $\frac{2\cos x}{\sqrt{\cos 4x}}$

(c) $\frac{2\cos x}{\sqrt{\sin 2x}}$ (d) $\frac{2\cos x}{\sqrt{\cos 2x}}$

309. $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$ = ?
(a) $\tan\alpha$ (b) $\tan 2\alpha$
(c) $\cot\alpha$ (d) $\cot 2\alpha$

310. If sinA = n sin B, then $\left(\frac{n-1}{n+1}\right)$ $\tan\left(\frac{A+B}{2}\right)$ = ?
(a) $\sin\left(\frac{A-B}{2}\right)$ (b) $\tan\left(\frac{A-B}{2}\right)$
(c) $\cot\left(\frac{A-B}{2}\right)$ (d) $\tan\left(\frac{A+B}{2}\right)$

311. $2\sin A\cos^3 A - 2\sin^3 A\cos A$ = ?
(a) sin4A (b) $\frac{1}{2}\sin 4A$
(c) $\frac{1}{4}\sin 4A$ (d) $\frac{1}{8}\sin 4A$

312. tanA + tan (180° + A)+ cot (90° + A)+ cot (360° – A) = ?
(a) 0 (b) 2 tan A
(c) 2 cot A (d) tan A – cot A

313. If $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$ = y, then $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ = ?
(a) y (b) 1/y
(c) 1+y (d) $\frac{1}{1+y}$

314. $\frac{\sin 70° + \cos 40°}{\cos 70° + \sin 40°}$ = ?
(a) 1 (b) $\frac{1}{\sqrt{3}}$
(c) $\sqrt{3}$ (d) $-\sqrt{3}$

315. If $\frac{\sin^4 A}{a} + \frac{\cos^4 A}{b} = \frac{1}{a+b}$, then $\frac{\sin^8 A}{a^3} + \frac{\cos^8 A}{b^3}$ = ?
(a) $\frac{1}{(a+b)^3}$ (b) $\frac{a^2b^2}{(a+b)^2}$
(c) $\frac{a^3b^3}{(a+b)^2}$ (d) $\frac{ab}{a+b}$

316. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then $x^2 + 4y^2$ = ?
(a) 4 (b) – 4 (c) ± 4 (d) 0

317. If $\tan\theta - \cot\theta = a$ and $\cos\theta + \sin\theta = b$, then $(b^2 - 1)^2 (a^2 + 4)$ =?
(a) 2 (b) – 4 (c) ± 4 (d) 4

318. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then $\sin\alpha + \cos\alpha$ and $\sin\alpha - \cos\alpha$ are equal to
(a) $\sqrt{2}\cos\theta$, $\sqrt{2}\sin\theta$
(b) $\sqrt{2}\sin\theta$, $\sqrt{2}\cos\theta$
(c) $\sqrt{2}\sin\theta$, $\sqrt{2}\sin\theta$
(d) $\sqrt{2}\cos\theta$, $\sqrt{2}\cos\theta$

319. If $\sin\theta + \sin\phi = a$ and $\cos\theta + \cos\phi = b$, then $\tan\left(\frac{\theta-\phi}{2}\right)$ = ?
(a) $\sqrt{\frac{a^2+b^2}{4-a^2-b^2}}$ (b) $\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$
(c) $\sqrt{\frac{a^2+b^2}{4+a^2+b^2}}$ (d) $\sqrt{\frac{4+a^2+b^2}{a^2+b^2}}$

320. $\cos^2\alpha + \cos^2(\alpha + 120°) + \cos^2(\alpha - 120°)$ = ?
(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) 0

321. If $\tan A = \frac{1-\cos B}{\sin B}$, express tan 2A in terms of tan B
(a) tan2A = tanB
(b) $\tan 2A = \tan^2 B$
(c) $\tan 2A = \tan^2 A + \tan^2 B$
(d) $\tan 2A = \tan^2 A - \tan^2 B$

322. If tan (A + B) = p and tan (A – B) = q, then the value of tan 2A is
(a) $\frac{p+q}{p-q}$ (b) $\frac{p-q}{1+pq}$
(c) $\frac{p+q}{1-pq}$ (d) $\frac{1+pq}{p-q}$

323. $\frac{\sec 8A - 1}{\sec 4A - 1}$ = ?
(a) $\frac{\tan 2A}{\tan 8A}$ (b) $\frac{\tan 8A}{\tan 2A}$
(c) $\frac{\cot 8A}{\cot 2A}$ (d) $\frac{\cot 2A}{\cot 8A}$

324. In a ΔABC, $\angle$C = 90°, then the equation whose roots are tan A & tan B is
(a) $abx^2 + c^2 + ab = 0$
(b) $abx^2 + c^2x - ab = 0$
(c) $abx^2 + c^2x - ab = 0$
(d) $abx^2 - c^2x + ab = 0$

325. If $\cos(A - B) = \frac{3}{5}$ and tan A tan B = 2, then
(a) cosA cosB = $\frac{2}{5}$
(b) sinA sinB = $\frac{2}{5}$
(c) cos A cos B = $\frac{1}{5}$
(d) Both (b) and (c)

326. $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$ = ?
(a) tan54° (b) tan36°
(c) tan18° (d) cot18°

327. If $\tan\alpha = \frac{1}{7}$ and $\tan\beta = \frac{1}{3}$, the $\cos 2\alpha$ = ?
(a) $\sin 2\beta$ (b) $\sin 4\beta$
(c) $\sin 3\beta$ (d) $\cos 3\beta$

328. If tan A = $\frac{1}{2}$, tan B = $\frac{1}{3}$, then cos2A = ?
(a) sinB (b) sin2B
(c) sin3B (d) cos3B

329. $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos^2(\alpha + \beta)$= ?
(a) $\sin^2\alpha$ (b) $\cos^2\beta$
(c) $\cos^2\alpha$ (d) $\sin^2\beta$

330. sin12° sin24° sin 48° sin 84° = ?
(a) cos20° cos40° cos60° cos80°
(b) sin20° sin40° sin60° sin80°
(c) $\frac{3}{16}$ (d) $\frac{1}{16}$

331. $\tan 5x \tan 3x \tan 2x$ = ?
(a) $\tan 5x - \tan 3x - \tan 2x$
(b) 0
(c) $\frac{\sin 6x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$
(d) $\tan 9x$

332. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then $\cos 2\alpha + \cos 2\beta$ = ?
(a) $-2\sin(\alpha + \beta)$
(b) $2\cos(\alpha + \beta)$
(c) $2\sin(\alpha + \beta)$
(d) $-2\cos(\alpha + \beta)$

333. If cosA = a cos B and sin A = b sin B, then $(b^2 - a^2)\sin^2 B$ = ?
(a) $1 + a^2$ (b) $2 + a^2$
(c) $1 - a^2$ (d) $2 - a^2$

334. If A + B + C = π, then cos2A + cos2B + cos2C = ?
(a) 1 + 4 cosA cosB cos C
(b) – 1 + 4 sinA sinB cos C
(c) – 1 – 4 cosA cosB cosC
(d) 1 + 4 sin A sin B sin C

335. If A, B C are angles of a triangle, then $\sin^2 A + \sin^2 B + \sin^2 C - 2\cos A\cos B\cos C$ = ?
(a) 1 (b) 2 (c) 3 (d) 4

336. If A + B + C = 180°, then sin2A + sin2B + sin2C = ?
(a) 4 sinA sinB cosC
(b) 4 cosA cosB cosC
(c) 4 sinA sinB sinC
(d) 8 sinA sin B sin C

337. tan10° – tan50° + tan70° = ?
(a) 0 (b) 1 (c) $\sqrt{3}$ (d) 3

338. For what value of α does the equation $4\cos\alpha + 3\cos 2\alpha - 2\sin 3\alpha + \cos 4\alpha = 2\sqrt{3} - 1$?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

339. If $(\sin A + \sin B + \sin C)^2 = \sin^2 A + \sin^2 B + \sin^2 C$, then which one is true?
(a) sinA+ sinB + sinC = 0
(b) cosA + cosB + cosC = 0
(c) $\frac{1}{\sin A} + \frac{1}{\sin B} + \frac{1}{\sin C} = 0$
(d) None of these

340. If $\frac{\sin x}{\sin y} = p$ and $\frac{\cos x}{\cos y} = q$, then $\tan x$ =?
(a) $\frac{p}{q}\sqrt{\frac{q^2-2}{1-p^2}}$ (b) $\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$
(c) $\frac{p}{q}\sqrt{\frac{1-q^2}{1-p^2}}$ (d) $\frac{q}{p}\sqrt{\frac{q^2-1}{1-p^2}}$

341. If $y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$ and $z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$, then
(a) y + z = a + c
(b) y + z = a + b
(c) y + a = x + b
(d) None of these

342. If A + B + C = π, then $\frac{\cos A}{\sin B\sin C} + \frac{\cos B}{\sin C\sin A} + \frac{\cos C}{\sin A\sin B}$ = ?
(a) 0 (b) 1 (c) 2 (d) 3

343. If sinA, cosA and tan A are in GP, then $\cos^3 A + \cos^2 A$ = ?
(a) 1 (b) 2 (c) 3 (d) 4

344. If A+ B = C and tan A = k tan B, and A – B = ϕ, then sinC = ?
(a) 0 (b) 1
(c) $\frac{k+1}{k-1}$ (d) $\frac{k+1}{k-1}\sin\phi$

345. If $\tan\alpha$, $\tan\beta$ are the roots of $x^2 + px + q = 0$ ($p \neq q$) then $\tan(\alpha + \beta)$ = ?
(a) p – 1 (b) $\frac{p}{q-1}$
(c) 2q + p (d) None of these

346. If A + B + C = $\frac{3\pi}{2}$, then cos2A + cos2B + cos2C = ?
(a) 1 – 4 sinA sinB sinC
(b) 1 – sinA sinB sinC
(c) 1– 2sinA sinB sinC
(d) 1 – 3 sinA sinB sinC

347. $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then $\frac{\tan x}{\tan y}$ =?
(a) a (b) b
(c) a/b (d) b/a

348. $\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}$ = ?
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{3}$

349. If A + B + C = 180°, then $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2}$ = ?
(a) 1 (b) 3 (c) 2 (d) 0

350. $\frac{\cot^2 15° - 1}{\cot^2 15° + 1}$ = ?
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$
(c) $-\frac{2}{\sqrt{3}}$ (d) $\sqrt{\frac{2}{3}}$

ANSWER KEY

1. (b)	36. (c)	71. (b)	106. (a)	141. (c)	176. (c)	211. (c)	246. (d)	281. (d)	316. (a)
2. (d)	37. (a)	72. (c)	107. (d)	142. (b)	177. (d)	212. (c)	247. (b)	282. (d)	317. (d)
3. (a)	38. (b)	73. (b)	108. (a)	143. (d)	178. (c)	213. (b)	248. (a)	283. (d)	318. (a)
4. (a)	39. (c)	74. (b)	109. (b)	144. (a)	179. (c)	214. (d)	249. (a)	284. (a)	319. (b)
5. (d)	40. (c)	75. (a)	110. (a)	145. (b)	180. (c)	215. (a)	250. (a)	285. (b)	320. (a)
6. (d)	41. (d)	76. (b)	111. (a)	146. (b)	181. (a)	216. (d)	251. (b)	286. (d)	321. (a)
7. (d)	42. (d)	77. (c)	112. (a)	147. (b)	182. (a)	217. (c)	252. (b)	287. (c)	322. (c)
8. (c)	43. (b)	78. (c)	113. (d)	148. (c)	183. (c)	218. (b)	253. (a)	288. (c)	323. (b)
9. (c)	44. (c)	79. (d)	114. (a)	149. (a)	184. (d)	219. (b)	254. (a)	289. (a)	324. (d)
10. (b)	45. (b)	80. (d)	115. (d)	150. (a)	185. (c)	220. (b)	255. (b)	290. (a)	325. (d)
11. (a)	46. (a)	81. (a)	116. (c)	151. (b)	186. (d)	221. (b)	256. (b)	291. (a)	326. (a)
12. (a)	47. (c)	82. (c)	117. (a)	152. (b)	187. (d)	222. (c)	257. (c)	292. (d)	327. (b)
13. (b)	48. (d)	83. (c)	118. (c)	153. (b)	188. (c)	223. (b)	258. (a)	293. (a)	328. (b)
14. (c)	49. (c)	84. (c)	119. (b)	154. (d)	189. (a)	224. (d)	259. (d)	294. (b)	329. (c)
15. (c)	50. (a)	85. (d)	120. (d)	155. (d)	190. (c)	225. (d)	260. (d)	295. (c)	330. (d)
16. (b)	51. (b)	86. (c)	121. (c)	156. (b)	191. (c)	226. (c)	261. (a)	296. (a)	331. (a)
17. (a)	52. (a)	87. (b)	122. (a)	157. (d)	192. (d)	227. (d)	262. (d)	297. (a)	332. (d)
18. (c)	53. (a)	88. (b)	123. (c)	158. (a)	193. (c)	228. (c)	263. (b)	298. (a)	333. (c)
19. (b)	54. (b)	89. (a)	124. (a)	159. (a)	194. (c)	229. (c)	264. (c)	299. (c)	334. (c)
20. (a)	55. (a)	90. (c)	125. (c)	160. (d)	195. (c)	230. (a)	265. (b)	300. (b)	335. (b)
21. (d)	56. (c)	91. (b)	126. (d)	161. (b)	196. (a)	231. (c)	266. (d)	301. (d)	336. (c)
22. (b)	57. (c)	92. (b)	127. (b)	162. (a)	197. (d)	232. (d)	267. (a)	302. (a)	337. (c)
23. (d)	58. (b)	93. (c)	128. (b)	163. (c)	198. (a)	233. (d)	268. (c)	303. (c)	338. (a)
24. (c)	59. (c)	94. (d)	129. (a)	164. (b)	199. (c)	234. (b)	269. (d)	304. (c)	339. (a)
25. (b)	60. (c)	95. (d)	130. (d)	165. (b)	200. (a)	235. (d)	270. (c)	305. (d)	340. (b)
26. (b)	61. (b)	96. (c)	131. (d)	166. (c)	201. (c)	236. (c)	271. (b)	306. (a)	341. (a)
27. (c)	62. (d)	97. (c)	132. (c)	167. (b)	202. (c)	237. (b)	272. (b)	307. (a)	342. (c)
28. (d)	63. (d)	98. (a)	133. (b)	168. (b)	203. (a)	238. (b)	273. (a)	308. (d)	343. (a)
29. (b)	64. (d)	99. (c)	134. (d)	169. (c)	204. (b)	239. (c)	274. (d)	309. (c)	344. (d)
30. (d)	65. (d)	100. (b)	135. (d)	170. (c)	205. (b)	240. (d)	275. (d)	310. (b)	345. (b)
31. (a)	66. (a)	101. (c)	136. (b)	171. (a)	206. (d)	241. (b)	276. (b)	311. (b)	346. (a)
32. (b)	67. (b)	102. (a)	137. (b)	172. (c)	207. (a)	242. (b)	277. (a)	312. (d)	347. (c)
33. (d)	68. (a)	103. (c)	138. (b)	173. (b)	208. (c)	243. (a)	278. (c)	313. (a)	348. (d)
34. (b)	69. (c)	104. (b)	139. (a)	174. (b)	209. (b)	244. (d)	279. (c)	314. (c)	349. (a)
35. (a)	70. (c)	105. (a)	140. (a)	175. (a)	210. (c)	245. (d)	280. (d)	315. (a)	350. (a)

SOLUTION

1. (b) $\frac{\tan\theta+\cot\theta}{\tan\theta-\cot\theta}=2$

By componendo and dividendo rule,

$\frac{\tan\theta}{\cot\theta}=\frac{3}{1}$

$\frac{\sin\theta}{\cos\theta}\times\frac{\sin\theta}{\cos\theta}=3$

$\sin^2\theta = 3\cos^2\theta$

$\sin^2\theta = 3(1-\sin^2\theta)$

$4\sin^2\theta = 3$

$\sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

Alternate:-

$\frac{\tan\theta+\cot\theta}{\tan\theta-\cot\theta}=2$

By C and D rule,

$\frac{\tan\theta}{\cot\theta}=\frac{3}{1}$

$\tan^2\theta = 3$

$\tan\theta = \sqrt{3} = \tan 60°$

$\theta = 60°$

$\therefore \sin\theta = \sin 60° = \frac{\sqrt{3}}{2}$

2. (d) cot10°. cot20° .cot60° .cot70° . cot80·

[In cotA.cotB If A + B = 90° then, cotA.cotB = 1]

$\underbrace{\cot 10°.\cot 80°}.\underbrace{\cot 20°.\cot 70°}.\underbrace{\cot 60°}_{\frac{1}{\sqrt{3}}}$

$= 1 \times 1 \times \frac{1}{\sqrt{3}}$

3. (a)

$\cot 18^0\left[\cot 72^0.\cos^2 22^0+\frac{1}{\tan 72^\circ.\sec^2 68^0}\right]$

$\underbrace{\cot 18^0.\cot 72^0}_{1}.\cos^2 22^0+\frac{\cot 18^0}{\tan 72^\circ.\sec^2 68^0}$

(If A + B = 90°, then cotA.cotB = 1)

$=\cos^2 22^0+\frac{\cot 18^0}{\tan 72^\circ.\sec^2 68^\circ}$

($\because$ tan72° = cot18°)

= cos²22° +cos²68°

=cos²22° +sin²22° =1

($\because$ sin²θ + cos²θ = 1)

4. (a) tan15° cot75° + tan75° cot15°

= tan15°.cot(90–15°)+ tan(90–15).cot15°

= tan15°.tan15° + cot15°. cot15°

= tan²15° + cot²15° ....(i)

[Formula]

cot(90° – θ) = tan θ

tan(90° – θ) = cotθ]

Put value of tan 15^0 ,

$$\left[\begin{array}{l}\cot 15^0=\frac{1}{\tan 15^0}\\ \cot 15^0=\frac{1}{(2-\sqrt{3})}\Rightarrow\frac{1}{(2-\sqrt{3})}\times\frac{(2+\sqrt{3})}{(2+\sqrt{3})}\end{array}\right]$$

$\boxed{\cot 15^0 = 2+\sqrt{3}}$

Now put value in eq (i)

$\tan^2 15^0+\cot^2 15^0$

= $(2-\sqrt{3})^2 + (2+\sqrt{3})^2$

= $4 + 3 - 4\sqrt{3} + 4 + 3 + 4\sqrt{3}$

= 14

5. (d) sin(2x–20°) = cos(2y+20°)

(2x–20°) + (2y + 20°) = 90°

$$\left[\begin{array}{l}\text{if } \sin A=\cos B\\ \text{then}\\ A+B=90^0\end{array}\right]$$

$2(x+y)=90^0$

$x+y=45^0$

$\tan(x+y)=\tan 45^0=\mathbf{1}$

Alternate:-

sin(2x – 20°) = cos(2y + 20°)

⇒ sin(2x – 20°) = sin [90° – (2y + 20°)]

⇒ 2x – 20° = 90° – (2y + 20°)

⇒ 2x – 20° = 90° – 2y – 20°

⇒ 2x + 2y = 90°

⇒ 2 (x + y) = 90°

⇒ x + y = 45°

$\therefore$ tan (x + y) = tan45° = 1

6. (d)

$(\sin^2 5^0+\sin^2 10^0+......+\sin^2 85^0)+\sin^2 90^0$

No. of terms = $\left\{\left(\frac{85-5}{5}\right)+1\right\}$ = 17

Sum of series= $\frac{17}{2}$ + 1 = $9\frac{1}{2}$

[use this approach in exam]

7. (d) $\frac{\sin 39°}{\cos 51°}$+2tan11°tan31° tan45°tan59° tan79° – 3(sin²21° + sin²69°)

$=\frac{\sin 39^0}{\cos(90^0-39^0)}+2\tan 11^0.\tan(90^0-11^0).\tan 31^0$

$.\tan(90^0-31^0).1-3(\sin^2 21^0+\sin^2(90^0-21^0))$

$=\frac{\sin 39^0}{\sin 39^0}+2\tan 11^0.\cot 11^0.\tan 31^0.$

$\cot 31^0-3(\sin^2 21^0+\cos^2 21^0)$

$=1+2-3=\mathbf{0}$

8. (c) $\frac{\cos^2\theta}{\cot^2\theta-\cos^2\theta}=3$

$\Rightarrow \cos^2\theta=3(\cot^2\theta-\cos^2\theta)$

$\Rightarrow \cos^2\theta=3\cot^2\theta-3\cos^2\theta$

$\Rightarrow 4\cos^2\theta=3\cot^2\theta$

$\Rightarrow 4\cos^2\theta=3\frac{\cos^2\theta}{\sin^2\theta}$

$\Rightarrow 4\sin^2\theta=3$

$\sin\theta=\frac{\sqrt{3}}{2}=\sin 60°$

$\therefore \theta = 60°$

Alternate method:-

Put $\theta = 60°$

$\frac{\cos^2 60°}{\cot^2 60°-\cos^2 60°}=3$

$\Rightarrow \text{LHS} = \frac{\frac{1}{4}}{\frac{1}{3}-\frac{1}{4}}=\frac{\frac{1}{4}}{\frac{1}{12}}=3$

$\therefore$ LHS = RHS = 3

Here,

3 = 3

So, $\theta = 60°$

9. (c) $\frac{A}{B}=\frac{\tan 11°\tan 29°}{2\cot 61°\cot 79°}$

$\frac{A}{B}=\frac{\tan 11°\tan 29°}{2[\cot(90°-29°)\cot(90°-11°)]}$

$\frac{A}{B}=\frac{\tan 11°\tan 29°}{2\tan 11°\tan 29°}$

$\frac{A}{B}=\frac{1}{2}$

2A=B

10. (b) $\sin\alpha + \cos\beta = 2$

put, $\alpha = 90°$, $\beta = 0°$

$\Rightarrow \sin 90° + \cos 0° = 2$

$\Rightarrow 1 + 1 = 2$

2 = 2 matched

So, $\alpha = 90°$, $\beta = 0°$

$\Rightarrow \sin\left(\frac{2\alpha+\beta}{3}\right)^o$

$= \sin\left(\frac{2\times 90+0}{3}\right)^o = \sin\left(\frac{180}{3}\right)^o$

$= \sin 60° = \cos 30° = \frac{\sqrt{3}}{2}$

Take option 'b'

$\cos\frac{\alpha}{3} = \cos\frac{90°}{3} = \cos 30°$

So, this is answer.

11. (a) $\sin\alpha\sec(30°+\alpha)=1$

put value of α between 0°to 60°,

if α =30°

$\sin 30°\sec(30°+30°)=1$

$\sin 30°\sec 60°=1$

$\Rightarrow \frac{1}{2}\times 2=1$

$\Rightarrow$ 1=1 (satisfy)

So, α =30°

$\sin\alpha + \cos 2\alpha$

$=\sin 30°+\cos 2\times 30°$

$= \sin 30°+\cos 60° = \frac{1}{2}+\frac{1}{2}=1$

Alternate:-

If, $\sin\alpha \sec\beta = 1$

then, $\alpha + \beta = 90°$

$\sin\alpha\sec(30° + \alpha) = 1$

$\alpha + 30° + \alpha = 90°$

$2\alpha = 60°$

$\alpha = 30°$

$\sin\alpha + \cos 2\alpha$

$\sin 30° + \cos 2\times 30° = 1$

12. (a) If, $\tan\theta=1$

It means, θ=45°

$\frac{8\sin\theta+5\cos\theta}{\sin^3\theta-2\cos^3\theta+7\cos\theta}$

$=\frac{8\sin 45°+5\cos 45°}{\sin^3 45°-2\cos^3 45°+7\cos 45°}$

$=\frac{8\times\frac{1}{\sqrt{2}}+5\times\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3-2\left(\frac{1}{\sqrt{2}}\right)^3+7\left(\frac{1}{\sqrt{2}}\right)} = 2$

13. (b) $\cos^2\theta + \cos^4\theta = 1$

$\cos^4\theta = 1 - \cos^2\theta$

$\cos^4\theta = \sin^2\theta$

$\cos^2\theta.\cos^2\theta = \sin^2\theta$

$\cos^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

$\cos^2\theta = \tan^2\theta$

$\cos^2\theta + \cos^4\theta = \tan^2\theta + \tan^4\theta$

$\because \cos^2\theta + \cos^4\theta = 1$

Here, $\tan^2\theta + \tan^4\theta = 1$

14. (c) $\frac{3\sin\theta+2\cos\theta}{3\sin\theta-2\cos\theta}$

divide numerator & de nominator by $\cos\theta$

$=\frac{3\frac{\sin\theta}{\cos\theta}+\frac{2\cos\theta}{\cos\theta}}{\frac{3\sin\theta}{\cos\theta}-\frac{2\cos\theta}{\cos\theta}}$ $\left[\because \frac{\sin\theta}{\cos\theta}=\tan\theta\right]$

$=\frac{3\tan\theta+2}{3\tan\theta-2}$

put value of $\tan\theta$

$=\frac{3\times\frac{4}{3}+2}{3\times\frac{4}{3}-2}=\frac{6}{2}=3$

15. (c) $(\sec A - \cos A)^2 + (\operatorname{cosec}A - \sin A)^2 - (\cot A - \tan A)^2$

$= (\sec^2A + \cos^2A - 2\sec A\cos A) + (\operatorname{cosec}^2A + \sin^2A - 2\operatorname{cosec}A\sin A) - (\cot^2A + \tan^2A - 2\cot A\tan A)$

$= \sec^2A - \tan^2A + \cos^2A + \sin^2A + \operatorname{cosec}^2A - \cot^2A - 2$

$= 3 - 2 = 1$

Alternate:-

$(\sec A - \cos A)^2 + (\operatorname{cosec}A - \sin A)^2 - (\cot A - \tan A)^2$

put θ = 45°,

$= (\sec 45° - \cos 45°)^2 + (\operatorname{cosec}45° - \sin 45°)^2 - (\cot 45° - \tan 45°)^2$

$= \left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)^2 - (1-1)^2$

$= \frac{1}{2}+\frac{1}{2}-0=1$

16. (b) $\tan\theta+\cot\theta=2$

If we put θ=45°,

tan45°+cot45°=2

1+1=2 $\Rightarrow$ 2=2

So, θ=45°

$\tan^5\theta+\cot^{10}\theta$

$= \tan^5 45°+\cot^{10}45°$

$= (1)^5+(1)^{10}= 1+1=2$

17. (a) $\sin\theta - \cos\theta = \frac{7}{13}$

$\sin\theta + \cos\theta = \sqrt{2-x^2}$

$= \sqrt{2-\left(\frac{7}{13}\right)^2} = \sqrt{2-\left(\frac{49}{169}\right)}$

$= \sqrt{\frac{289}{169}} = \frac{17}{13}$

18. (c) $2\cos\theta-\sin\theta=\frac{1}{\sqrt{2}}$

put value of θ=45°,

$2\cos 45°-\sin 45°=\frac{1}{\sqrt{2}}$

$2\times\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

So, it satisfies with question.

Now,

$2\sin\theta+\cos\theta = 2\sin 45°+\cos 45°$

$= 2\times\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{3}{\sqrt{2}}$

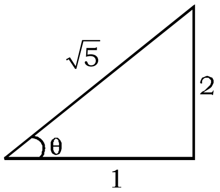
19. (b) $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=3$

$\sin\theta+\cos\theta=3\sin\theta-3\cos\theta$

$2\sin\theta=4\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta}=\frac{2}{1}$

$\Rightarrow \tan\theta = \frac{2}{1} = \frac{P}{B}$

$\left[\therefore \sin\theta = \frac{P}{H} = \frac{2}{\sqrt{5}} \text{ and } \cos\theta = \frac{B}{H} = \frac{1}{\sqrt{5}}\right]$



$\Rightarrow \sin^4\theta - \cos^4\theta$

$\Rightarrow (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$\Rightarrow 1(\sin^2\theta - \cos^2\theta)$

$\Rightarrow \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

20. (a) $\sec^2\theta + \tan^2\theta = 7$

$\Rightarrow 1 + \tan^2\theta + \tan^2\theta = 7$

$\Rightarrow 2\tan^2\theta = 6$

$\Rightarrow \tan^2\theta = 3$

$\Rightarrow \tan\theta = \sqrt{3} = \tan 60^\circ$

$\Rightarrow \theta = 60^\circ$

Alternate:-

take help from option

put $\theta = 60^\circ$,

$\sec^2 60^\circ + \tan^2 60^\circ = 7$

$(2)^2 + (\sqrt{3})^2 = 7$

7= 7 (matched)

So, $\theta = 60^\circ$

21. **(d)** $(\sec x .\sec y + \tan x .\tan y)^2 - (\sec x . \tan y + \tan x .\sec y)^2$

put value of $x=y=45^\circ$,

$= (\sec 45^\circ \sec 45^\circ + \tan 45^\circ \tan 45^\circ)^2 - (\sec 45^\circ \tan 45^\circ + \tan 45^\circ \sec 45^\circ)^2$

$= (\sqrt{2}\times\sqrt{2} + 1\times 1)^2 - (\sqrt{2}\times 1 + 1\times\sqrt{2})^2$

$= (2+1)^2 - (2\sqrt{2})^2 = 9 - 8 = 1$

22. **(b)** According to question,

$A = \sin^2\theta + \cos^4\theta$

Put $\theta = 90^\circ$ for maximum value of A

$A = \sin^2 90^\circ + \cos^4 90^\circ$

$A = 1 + 0$

$A = 1$

Put $\theta = 45^\circ$

for minimum value of A

$A = \sin^2 45^\circ + \cos^4 45^\circ$

$A = \frac{1}{2} + \frac{1}{4}$

$A = \frac{3}{4}$ ∴ A lies in $\frac{3}{4} \le A \le 1$

23. **(d)** $\sin\theta + \operatorname{cosec}\theta = 2$

put $\theta = 90^\circ$

$\Rightarrow \sin 90^\circ + \operatorname{cosec} 90^\circ = 2$

$\Rightarrow 1+1=2$

$\Rightarrow 2 = 2$

It, satisfies the question

$\sin^5\theta + \operatorname{cosec}^5\theta$

$= \sin^5 90^\circ + \operatorname{cosec}^5 90^\circ$

$= (1)^5 + (1)^5 = 1+1=2$

24. **(c)** $\tan 2\theta . \tan 4\theta = 1$

[If tanA. tanB = 1, then A + B = 90°]

$2\theta + 4\theta = 90^\circ$

$\Rightarrow 6\theta = 90^\circ$

$\Rightarrow 3\theta = 45^\circ$

$\therefore \tan 3\theta = \tan 45^\circ = \mathbf{1}$

25. **(b)** $\cos^2\alpha + \cos^2\beta = 2$

put value of $\alpha = \beta = 0^\circ$

$\Rightarrow \cos^2 0^\circ + \cos^2 0^\circ = 2$

$\Rightarrow (1)^2 + (1)^2 = 2$

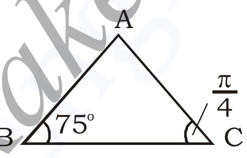
$\Rightarrow 2 = 2$

[If satifies the question]

$\tan^3\alpha + \sin^5\beta$

$= \tan^3 0^\circ + \sin^5 0^\circ = 0 + 0 = 0$

26. **(b)**



$\frac{\pi^c}{4} = \frac{180^\circ}{4} = 45^\circ$

$\angle BAC = 180^\circ - 75^\circ - 45^\circ = 60^\circ$

$180^\circ \to \pi^c$

$1^\circ \to \frac{\pi^c}{180^\circ}$

$60^\circ \to \frac{\pi^c}{180^\circ} \times 60^\circ = \frac{\pi^c}{3}$ radian

27. **(c)** $(x+5)^\circ + (2x-3)^\circ + (3x+4)^\circ = 180^\circ$

(Sum of all angles in triangle is 180°)

$6x + 6^\circ = 180^\circ$

$(x + 1) = 30^\circ$

$\therefore x = 29^\circ$

28. **(d)** A + B = 90°

(Complementary angle)

We can put A = B = 45°

Or A = 30° , B = 60°

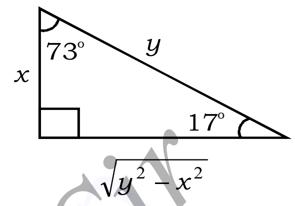
$= \sec^2 A + \sec^2 B - \sec^2 A.\sec^2 B$

$= \sec^2 45^\circ + \sec^2 45^\circ - \sec^2 45^\circ.\sec^2 45^\circ$

$= (\sqrt{2})^2 + (\sqrt{2})^2 - (\sqrt{2})^2 . (\sqrt{2})^2$

$= 4 - 4 = 0$

29. **(b)** $\sin 17^\circ = \frac{x}{y} \Rightarrow \frac{P}{H}$



$\Rightarrow \sec 17^\circ - \sin 73^\circ$

$= \frac{y}{\sqrt{y^2 - x^2}} - \frac{\sqrt{y^2 - x^2}}{y}$

$= \frac{y^2 - (y^2 - x^2)}{(y)(\sqrt{y^2 - x^2})}$

$= \frac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$

30. **(d)** $\frac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ}$

$= \frac{\tan 60^\circ - \tan 15^\circ}{\tan 15^\circ - \tan 60^\circ}$

$= \frac{-(\tan 15^\circ - \tan 60^\circ)}{\tan 15^\circ - \tan 60^\circ} = -1$

Alternate:-

$\frac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ} = \frac{\cot 30^\circ - \cot 75^\circ}{\cot 75^\circ - \cot 30^\circ}$

$= \frac{\cot 30^\circ - \cot 75^\circ}{-(\cot 30^\circ - \cot 75^\circ)} = -1$

31. **(a)** $\cot\theta . \tan(90^\circ - \theta) - \sec(90^\circ - \theta)\operatorname{cosec}\theta + (\sin^2 25^\circ + \sin^2 65^\circ) + \sqrt{3}(\tan 5^\circ . \tan 15^\circ . \tan 30^\circ . \tan 75^\circ \tan 85^\circ)$

$= \cot\theta . \cot\theta - \operatorname{cosec}\theta . \operatorname{cosec}\theta + (\sin^2 25^\circ + \cos^2 25^\circ) + \sqrt{3}[(\tan 5^\circ . \tan 85^\circ).(\tan 15^\circ . \tan 75^\circ) . \tan 30^\circ]$

$= (\cot^2\theta - \operatorname{cosec}^2\theta) + 1 + \sqrt{3}\,(1 . 1 . \frac{1}{\sqrt{3}})$ $\left[\begin{array}{l}\tan A.\tan B = 1 \\ \text{If } A+B = 90^\circ\end{array}\right]$

$= (-1) + 1 + \sqrt{3} \times \frac{1}{\sqrt{3}}$

$= -1 + 1 + 1 = \mathbf{1}$

32. (b) sin $(3x - 20^o)$ = cos $(3y + 20^o)$
[If, sinA = cosB then, A + B = 90°]
$\Rightarrow (3x - 20^o) + (3y + 20^o) = 90°$
$\Rightarrow 3x + 3y = 90^o$
$\Rightarrow x + y = 30^o$

Alternate:-
$\sin(3x - 20°) = \cos(3y + 20°)$
$\sin(3x - 20°) = \sin[90° - (3y + 20°)]$
$3x - 20° = 90° - 3y - 20°$
$(\because \sin(90° - \theta) = \cos\theta)$
$3x + 3y = 90°$
$\Rightarrow \quad x + y = 30°$

33. (d) $\cos\theta \cdot \text{cosec}23^o = 1$

$$\cos\theta \cdot \frac{1}{\sin 23^o} = 1$$

$\Rightarrow \cos\theta = \sin 23°$
(If sinA = cosB then A + B = 90°)

$\frac{1}{\text{cosecA}}$ = cosB
sinA = cosB
$A + B = 90^o$
$23^o + B = 90^o$
$B = 90^o - 23^o = 67^o$
$\cos B = \cos\theta$
$\Rightarrow B = \theta = 67°$
$\therefore \theta = 67°$

Alternate:-
$\cos\theta \cdot \text{cosec}23° = 1$

$\cos\theta \cdot \frac{1}{\sin 23^o} = 1$

$\cos\theta = \sin 23°$
$\cos\theta = \cos(90° - 23°)$
$\theta = 90° - 23°$
$\therefore \theta = 67°$

34. (b) $2(\cos^2\theta - \sin^2\theta) = 1$

$\Rightarrow \cos^2\theta - \sin^2\theta = \frac{1}{2}$

$\Rightarrow 2\sin^2\theta = 1 - \frac{1}{2}$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{1}{2} = \sin 30^o$

$\theta = 30^o$

35. (a) $\sec(7\theta + 28)^o = \text{cosec}(30^o - 3\theta)$
[If secA. sinB = 1, then A + B = 90°]
$\Rightarrow (7\theta + 28^o) + (30^o - 3\theta) = 90^o$
$\Rightarrow 4\theta + 58^o = 90^o$
$\Rightarrow 4\theta = 32^o \Rightarrow \theta = \mathbf{8^o}$

36. (c) $\tan\left[\frac{\pi}{2} - \frac{\theta}{2}\right] = \sqrt{3}$

$\Rightarrow \tan\left[90 - \frac{\theta}{2}\right] = \sqrt{3}$ $[\pi = 180^o]$

$\Rightarrow \cot\frac{\theta}{2} = \sqrt{3} = \cot 30^o$

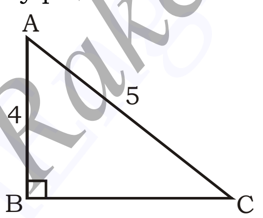
$\Rightarrow \frac{\theta}{2} = 30^o$

$\Rightarrow \theta = 60^o$

$\therefore \cos 60^o = \frac{1}{2}$

37. (a) Since, value of $\cos\theta$ decreases, from 0° to 90° at 45° it is equal to the value of $\sin\theta$.
Similarly,
Value of $\sin\theta$ increases from 0 to 90° and at 45° it is equal to the value of $\cos\theta$
For $0° < \theta < 45°$, $\cos\theta > \sin\theta$
So, value of cos 25° – sin25° is alway positive but less than1.

38. (b)



ΔABC,

$\cos A = \frac{4}{5}$ i.e.,

AB = 4 and AC = 5

$\sin C = \frac{AB}{AC} = \frac{4}{5}$

39. (c) Since, α and β are complementary angle.
$\therefore \alpha = 90 - \beta$
Now,
$\sqrt{\cos\alpha\,\text{cosec}\beta - \cos\alpha\sin\beta}$

$= \sqrt{\frac{\cos\alpha}{\sin\beta} - \cos\alpha\sin\beta}$

$= \sqrt{\frac{\cos\alpha}{\cos(90-\beta)} - \cos\alpha.\cos(90-\beta)}$

$= \sqrt{\frac{\cos\alpha}{\cos\alpha} - \cos\alpha.\cos\alpha}$

$= \sqrt{1 - \cos^2\alpha} = \sqrt{\sin^2\alpha} = \sin\alpha$

40. (c) $\because 2\cot\theta = 3$

$\Rightarrow \cot\theta = \frac{3}{2}$

divide numerator and denominator by $\sin\theta$,

$\therefore \frac{2\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{2\cot\theta - 1}{2\cot\theta + 1}$

$= \frac{2\times\frac{3}{2} - 1}{2\times\frac{3}{2} + 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$

41. (d) $\sin^6\theta + \cos^6\theta$
$= (\sin^2\theta)^3 + (\cos^2\theta)^3$
$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$
$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)$
$(\because \sin^2\theta + \cos^2\theta = 1)$
$= (1 - 3\sin^2\theta\cos^2\theta)$

$= 1 - 3 \times \frac{1}{4} = 1 - \frac{3}{4} = \frac{1}{4}$

42. (d) By trigonometric identity,
$\sec^2\theta - \tan^2\theta = 1$
$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{2}$ ....(i)

and given, $\sec\theta + \tan\theta = 2$
On adding Eqs. (i) and (ii) we get,

$2\sec\theta = \frac{1}{2} + 2$ ....(ii)

$\sec\theta = \frac{5}{4}$

Alternate:-
$\sec\theta + \tan\theta = 2$

$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 2$

$\frac{1+\sin\theta}{\cos\theta} = 2$

Squaring both sides,

$\frac{(1+\sin\theta)^2}{\cos^2\theta} = 4$

$\frac{(1+\sin\theta)^2}{1-\sin^2\theta} = 4$

$\frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)} = 4$

$\frac{1+\sin\theta}{1-\sin\theta} = 4$

$1 + \sin\theta = 4 - 4\sin\theta$

$5\sin\theta = 3$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \sqrt{1-\sin^2\theta}$

$\cos\theta = \sqrt{1-\left(\frac{3}{5}\right)^2}$

$\cos\theta = \sqrt{1-\frac{9}{25}}$

$\cos\theta = \frac{4}{5}$

$\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$

43. (b) $\text{cosec}(75°+\theta) - \sec(15°-\theta) - \tan(55°+\theta) + \cot(35°-\theta)$

$\Rightarrow \text{cosec}(75°+\theta) - \text{cosec}[90° - (15° - \theta)] - \tan(55° + \theta) + \tan[90° - (35° - \theta)]$

$\Rightarrow \text{cosec}(75° + \theta) - \text{cosec}(75° + \theta) - \tan(55°+\theta) + \tan(55° + \theta) = 0$

44. (c) $\sin\theta + 2\cos\theta = 1$

On squaring both sides, we get

$(\sin\theta + 2\cos\theta)^2 = 1$

$\Rightarrow \sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$

$\Rightarrow (1 - \cos^2\theta) + 4(1 - \sin^2\theta) + 4\sin\theta\cos\theta = 1$

$\Rightarrow -(\cos^2\theta + 4\sin^2\theta) + 4\sin\theta\cos\theta = 1 - 5$

$\Rightarrow \cos^2\theta + 4\sin^2\theta - 4\sin\theta\cos\theta = 4$

$\Rightarrow (2\sin\theta - \cos\theta)^2 = 4$

$\Rightarrow 2\sin\theta - \cos\theta = 2$

45. (b) $\tan 8\theta = \cot 2\theta$

$\Rightarrow \tan 8\theta = \tan(90 - 2\theta)$

$\Rightarrow 8\theta = 90 - 2\theta$

$\Rightarrow \theta = 9°$

$\therefore \tan 5\theta$

$\Rightarrow \tan 45° = 1$

46. (a) $\sin(A + B) = 1$

$A + B = \sin^{-1}1$

$\Rightarrow (A + B) = 90°$

$(\because \sin^{-1}1 = 90°)$

$\therefore B = 90° - A \Rightarrow A = 90° - B$

Now,

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

$= \cos(90 - B)\cos B + \sin(90 - B)\sin B$

$= \sin B\cos B + \cos B\sin B$

$= 2\sin B\cos B = \sin 2B$

47. (c) $\therefore 5\sin\theta + 12\cos\theta = 13$

On squaring both sides, we get

$25\sin^2\theta + 144\cos^2\theta + 120\sin\theta\cos\theta = 169$

$\Rightarrow 25(1 - \cos^2\theta) + 144(1 - \sin^2\theta) + 120\sin\theta\cos\theta = 169$

$\Rightarrow 25 - 25\cos^2\theta + 144 - 144\sin^2\theta + 120\sin\theta\cos\theta = 169$

$\Rightarrow 25\cos^2\theta + 144\sin^2\theta - 120\sin\theta\cos\theta = 169 - 169$

$\Rightarrow (5\cos\theta - 12\sin\theta)^2 = 0$

$\therefore 5\cos\theta - 12\sin\theta = 0$

48. (d) $\frac{4\sin\theta - \cos\theta}{4\sin\theta + 9\cos\theta}$

On dividing both numerator and denominator by $\cos\theta$, we get

$= \frac{\frac{4\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{\frac{4\sin\theta}{\cos\theta} + \frac{9\cos\theta}{\cos\theta}} = \frac{4\tan\theta - 1}{4\tan\theta + 9}$

$= \frac{3-1}{3+9} = \frac{2}{12} = \frac{1}{6}$

49. (c) $\because \sin\theta - \cos\theta = 0$

$\therefore \sin\theta = \cos\theta$

Since, $\sin\theta$ and $\cos\theta$ are equal for $\theta = 45°$

$\therefore \sin^4\theta + \cos^4\theta = (\sin 45°)^4 + (\cos 45°)^4$

$\left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

$= \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$

50. (a) $\frac{(\sin\theta + \cos\theta)(\tan\theta + \cot\theta)}{\sec\theta + \text{cosec}\theta}$

$= \frac{(\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)}{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}$

$= \frac{(\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)}{\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{(\sin\theta + \cos\theta)\left(\frac{1}{\sin\theta\cos\theta}\right)}{\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}}$

$= \frac{\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}}{\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}} = 1$

Alternate:-

$\frac{(\sin\theta + \cos\theta)(\tan\theta + \cot\theta)}{\sec\theta + \text{cosec}\theta}$

Put $\theta = 45°$

$\Rightarrow \frac{\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)(1+1)}{\sqrt{2} + \sqrt{2}}$

$\Rightarrow \frac{\frac{2}{\sqrt{2}} \times 2}{2\sqrt{2}} \Rightarrow \frac{4}{\sqrt{2}} \times \frac{1}{2\sqrt{2}} = 1$

51. (b) $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$[\because (a + b)^3 = a^3 + b^3 + 3ab(a + b)]$

Here,

$a = \sin^2\theta$ and $b = \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)^3$

$= (1)^3 = 1 \ [\because \sin^2\theta + \cos^2\theta = 1]$

52. (a) $\frac{(1 + \sec\theta - \tan\theta)\cos\theta}{(1 + \sec + \tan\theta)(1 - \sin\theta)}$

$= \frac{\left(1 + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)\cos\theta}{\left(1 + \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)}$

$= \frac{\left(\frac{\cos\theta + 1 - \sin\theta}{\cos\theta}\right)\cos\theta}{\frac{(\cos\theta + 1 + \sin\theta)(1 - \sin\theta)}{\cos\theta}}$

$= \frac{\cos\theta + 1 - \sin\theta}{\frac{\cos\theta + 1 + \sin\theta - \sin\theta\cos\theta - \sin\theta - \sin^2\theta}{\cos\theta}}$

$= \frac{\cos\theta + 1 - \sin\theta}{\frac{\cos\theta + 1 - \sin^2\theta - \sin\theta\cos\theta}{\cos\theta}}$

$= \frac{\cos\theta + 1 - \sin\theta}{\frac{\cos\theta + \cos^2\theta - \sin\theta\cos\theta}{\cos\theta}}$

$[\because 1 - \sin^2\theta = \cos^2\theta]$

$= \frac{\cos\theta + 1 - \sin\theta}{\frac{\cos\theta(\cos\theta + 1 - \sin\theta)}{\cos\theta}} = 1$

53. (a) $\sin\theta + \cos\theta = \sqrt{3}$

On squaring both sides, we get,

$(\sin\theta + \cos\theta)^2 = \left(\sqrt{3}\right)^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \sin\theta\cos\theta = \dfrac{3-1}{2} = \dfrac{2}{2} = 1$

Now,

$\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta} = \dfrac{1}{\sin\theta.\cos\theta}$

$\therefore \tan\theta + \cot\theta = \dfrac{1}{1} = 1$

54. (b) $\tan\theta + \sec\theta = m$

$\Rightarrow \sec\theta = m - \tan\theta$

On squaring both sides, we get

$(\sec\theta)^2 = (m - \tan\theta)^2$

$\Rightarrow \sec^2\theta = m^2 + \tan^2\theta - 2m\tan\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = m^2 - 2m\tan\theta$

$\Rightarrow 1 = m^2 - 2m\tan\theta$

$(\because \sec^2\theta - \tan^2\theta = 1)$

$\Rightarrow \tan\theta = \dfrac{m^2-1}{2m}$

On putting the value of $\tan\theta$ in initial equation, we get

$\dfrac{m^2-1}{2m} + \sec\theta = m$

$\Rightarrow \sec\theta = m - \dfrac{m^2-1}{2m}$

$\therefore \sec\theta = \dfrac{2m^2 - m^2 + 1}{2m} = \dfrac{m^2+1}{2m}$

Alternate:-

$\tan\theta + \sec\theta = m$

Put $\theta = 0°$

$\Rightarrow \tan 0° + \sec 0° = m$

$\Rightarrow 0 + 1 = m$

$\Rightarrow m = 1$

$\therefore \sec\theta = \sec 0° = 1$

Now check from option

Option:- (b) : $\dfrac{m^2+1}{2m} = \dfrac{1+1}{2\times 1}$

$= \dfrac{2}{2} = 1$ (Satisfy)

55. (a) $\operatorname{cosec}(75° + \theta) - \sec(15° - \theta)$

$= \operatorname{cosec}(75° + \theta) - \sec[90 - (75° + \theta)]$

$= \operatorname{cosec}(75° + \theta) - \operatorname{cosec}(75° + \theta)$

$= 0$

56. (c) In ΔABC, if $\angle C$ is 90°, then

$\angle A + \angle B = 180° - 90° = 90°$

Now,

$\cos(A + B) + \sin(A + B)$

$= 0 + 1 = 1$

57. (c) $\sin\alpha = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \alpha = 60° \left(\because \sin 60° = \dfrac{\sqrt{3}}{2}\right)$

Now,

$\cos\beta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \beta = 30° \left(\because \sin 30° = \dfrac{\sqrt{3}}{2}\right)$

and $\tan\gamma = 1$

$\Rightarrow \gamma = 45° \ (\because \tan 45° = 1)$

$\therefore \alpha + \beta + \gamma = 60° + 30° + 45° = 135°$

58. (b) Given that, $\cos A + \cos^2 A = 1$

$\Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$. (i)

Now,

$2(\sin^2 A + \sin^4 A)$

$= 2(\sin^2 A + \cos^2 A)$ [from Eq. (i)]

$= 2.(1)\ [\because \sin^2\theta + \cos^2\theta = 1) = 2$

59. (c) $(1 - \tan A)^2 + (1 + \tan A)^2 + (1 - \cot A)^2 + (1 + \cot A)^2$

$= 1 + \tan^2 A - 2\tan A + 1 + \tan^2 A + 2\tan A + 1 + \cot^2 A - 2\cot A + 1 + \cot^2 A + 2\cot A$

$= 4 + 2(\tan^2 A + \cot^2 A)$

$= (2 + 2\tan^2 A) + (2 + 2\cot^2 A)$

$= 2\sec^2 A + 2\operatorname{cosec}^2 A$

$= 2\left(\dfrac{1}{\cos^2 A} + \dfrac{1}{\sin^2 A}\right)$

$= 2\left(\dfrac{\sin^2 A + \cos^2 A}{\sin^2 A.\cos^2 A}\right)$

$= \dfrac{2.(1)}{\sin^2 A.\cos^2 A}$

$= 2\sec^2 A.\operatorname{cosec}^2 A$

Alternate:-

$(1 - \tan A)^2 + (1 + \tan A)^2 + (1 - \cot A)^2 + (1 + \cot A)^2$

Put A = 45°

$(1-1)^2 + (1+1)^2 + (1-1)^2 + (1+1)^2$

$4 + 4 = 8$

Now check from option

Option:- (c)

$2\sec^2 A \operatorname{cosec}^2 A$

$= 2\times\sec^2 45° \operatorname{cosec}^2 45°$

$= 2 \times 2 \times 2 = 8$ (satisfy)

60. (c) $\dfrac{\tan A - \sin A}{\sin^3 A} = \dfrac{\dfrac{\sin A}{\cos A} - \sin A}{\sin^3 A}$

$= \dfrac{\sin A\left(\dfrac{1}{\cos A} - 1\right)}{\sin^3 A} = \dfrac{1 - \cos A}{\cos A \sin^2 A}$

Rationalising above equation

$= \dfrac{(1 - \cos A)}{\cos A.\sin^2 A} \times \dfrac{(1 + \cos A)}{(1 + \cos A)}$

$= \dfrac{(1 - \cos^2 A)}{\cos A.\sin^2 A(1 + \cos A)}$

$= \dfrac{\sin^2 A}{\cos A.\sin^2 A(1 + \cos A)}$

$= \dfrac{1}{\cos A}.\dfrac{1}{1 + \cos A} = \dfrac{\sec A}{1 + \cos A}$

Alternate:-

$\dfrac{\tan A - \sin A}{\sin^3 A}$

Put A = 45°

$= \dfrac{\tan 45° - \sin 45°}{\sin^3 45°} = \dfrac{1 - \dfrac{1}{\sqrt{2}}}{\left(\dfrac{1}{\sqrt{2}}\right)^3}$

$= \dfrac{\dfrac{\sqrt{2}-1}{\sqrt{2}}}{\dfrac{1}{2\sqrt{2}}} = \dfrac{\sqrt{2}-1}{\sqrt{2}} \times 2\sqrt{2}$

$= 2\left(\sqrt{2} - 1\right)$

Now check from options

Option:- $\dfrac{\sec A}{1 + \cos A}$

$= \dfrac{\sec 45°}{1 + \cos 45°} = \dfrac{\sqrt{2}}{1 + \dfrac{1}{\sqrt{2}}}$

$= \dfrac{\sqrt{2} \times \sqrt{2}}{\sqrt{2} + 1} = \dfrac{2}{\sqrt{2} + 1}$

Rationalise above equation

$= \dfrac{2}{\sqrt{2} + 1} \times \dfrac{\sqrt{2} - 1}{\sqrt{2} - 1}$

$= \dfrac{2\left(\sqrt{2} - 1\right)}{2 - 1} = 2\left(\sqrt{2} - 1\right)$ (Satisfy)

61. (b) Given that, $\sin\theta = \dfrac{x^2 - y^2}{x^2 + y^2}$

$\because \cos^2\theta = 1 - \sin^2\theta$

$= 1 - \left(\dfrac{x^2 - y^2}{x^2 + y^2}\right)^2$

$= \dfrac{(x^2 + y^2)^2 - (x^2 - y^2)^2}{(x^2 + y^2)^2}$

$= \dfrac{2x^2.2y^2}{(x^2 + y^2)^2}$

$= \dfrac{4x^2.y^2}{(x^2 + y^2)^2} = \left(\dfrac{2xy}{x^2 + y^2}\right)^2$

$\therefore \cos\theta = 2xy/x^2 + y^2$

Alternate:-

$\sin\theta = \dfrac{x^2 - y^2}{x^2 + y^2}$

Put $\theta = 30°$

$x = \sqrt{3}$

$y = 1$

$\sin 30° = \dfrac{(\sqrt{3})^2 - (1)^2}{(\sqrt{3})^2 + (1)^2}$

$\dfrac{1}{2} = \dfrac{3-1}{3+1}$

$\dfrac{1}{2} = \dfrac{2}{4}$

$\dfrac{1}{2} = \dfrac{1}{2}$ (Satisfy)

62. (d) Given that,

$\Rightarrow a^2 = \dfrac{1 + 2\sin\theta.\cos\theta}{1 - 2\sin\theta.\cos\theta}$

$\Rightarrow a^2 = \dfrac{(\sin^2\theta + \cos^2\theta) + 2\sin\theta.\sin\theta}{(\sin^2\theta + \cos^2\theta) - 2\sin\theta.\cos\theta}$

$\Rightarrow a^2 = \dfrac{(\sin\theta + \cos\theta)^2}{(\sin\theta - \cos\theta)^2}$

$\Rightarrow \dfrac{a}{1} = \dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

(applying componendo dividendo formula)

$\Rightarrow \dfrac{a+1}{a-1} = \dfrac{(\sin\theta + \cos\theta) + (\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta) - (\sin\theta - \cos\theta)}$

$\Rightarrow \dfrac{a+1}{a-1} = \dfrac{2\sin\theta}{2\cos\theta} = \tan\theta$

63. (d) Given that, θ lies is in first quadrant and $\tan\theta = 3$

$\therefore \tan^2\theta = 9 \Rightarrow 1 + \tan^2\theta = 10$

$\Rightarrow \sec^2\theta = 10 \Rightarrow \sec\theta = \sqrt{10}$

$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{10}}$ ....(i)

$\because \sin^2\theta = 1 - \cos^2\theta$

$= 1 - \dfrac{1}{10} = \dfrac{9}{10}$

$\Rightarrow \sin\theta = \dfrac{3}{\sqrt{10}}$ ....(ii)

Now,

$\sin\theta + \cos\theta = \dfrac{3}{\sqrt{10}} + \dfrac{1}{\sqrt{10}} = \dfrac{4}{\sqrt{10}}$

(since, θ lies in first quadrant)

64. (d) If $0° < \theta < 90°$, then all the trigonometric ratios can be obtained when any one of the six ratios is given.

Since, We use any of the following identity to get any trigonometric ratios

$\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \text{cosec}^2\theta$

65. (d) sinA.cosA.tanA + cosA.sinA . cotA

$= \sin A.\cos A.\dfrac{\sin A}{\cos A} + \cos A . \sin A.\dfrac{\cos A}{\sin A} = \sin^2A + \cos^2A = 1$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \text{cosec}^2A - \cot^2A$

$[\because \text{cosec}^2A - \cot^2A = 1]$

Alternate:-

sinA cosA tanA + cosA sinA cotA

Put A = 45°

$\Rightarrow \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} \times 1 + \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} \times 1$

$\Rightarrow \dfrac{1}{2} + \dfrac{1}{2} = 1$

Now check from option,

$\text{cosec}^2 A - \cot^2 A$

Put A = 45°

$(\sqrt{2})^2 - 1$

$2 - 1 = 1$ (Satisfy)

66. (a) Let

$f(\theta) = \dfrac{\sin\theta}{1 + \cos\theta} + \dfrac{1 + \cos\theta}{\sin\theta}$

$= \dfrac{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{1 + 2\cos^2\frac{\theta}{2} - 1} + \dfrac{1 + 2\cos^2\frac{\theta}{2} - 1}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}$

$= \dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} + \dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$

$= \dfrac{2}{2}.\dfrac{\left(\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right)}{\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}$

$= \dfrac{2}{\sin\theta} = 2\text{cosec}\,\theta$

Alternate:-

$\dfrac{\sin\theta}{1 + \cos\theta} + \dfrac{1 + \cos\theta}{\sin\theta}$

Put $\theta = 90°$

$\dfrac{\sin 90°}{1 + \cos 90°} + \dfrac{1 + \cos 90°}{\sin 90°}$

$\dfrac{1}{1} + \dfrac{1}{1} = 2$

Now check the option,

Option: (a)

$2\text{cosec}\,\theta$

$= 2\text{cosec}\,90°$

$= 2 \times 1 = 2$ (Satisfy)

67. (b) Given that, $\sin\theta.\cos\theta = \dfrac{\sqrt{3}}{4}$

$\therefore \sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= (1)^2 - 2(\sin\theta.\cos\theta)^2$

$= 1 - 2\left(\dfrac{\sqrt{3}}{4}\right)^2 = 1 - 2.\dfrac{3}{16} = 1 - \dfrac{3}{8} = \dfrac{5}{8}$

Alternate:-

$\sin\theta\cos\theta = \dfrac{\sqrt{3}}{4}$

Put $\theta = 30°$

$\sin 30° \cos 30° = \dfrac{\sqrt{3}}{4}$

$\Rightarrow \dfrac{1}{2} \times \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4}$

$\Rightarrow \dfrac{\sqrt{3}}{4} = \dfrac{\sqrt{3}}{4}$ (Satisfy)

$\therefore$ $\sin^4\theta + \cos^4\theta$

$\Rightarrow$ $\sin^4 30° + \cos^4 30°$

$\Rightarrow \left(\frac{1}{2}\right)^4 + \left(\frac{\sqrt{3}}{2}\right)^4$

$\Rightarrow \frac{1}{16} + \frac{9}{16}$

$\Rightarrow \frac{1+9}{16} = \frac{10}{16}$

$\Rightarrow \frac{5}{8}$

68. (a) We know that, $\sin^2\theta + \cos^2\theta = 1$ is true

I. $\sin^2 1° + \cos^2 1° = 1$ which is also true.

II. $\sec^2 33° - \cot^2 57° = \text{cosec}^2 37° - \tan^2 53°$

Now,

$\sec^2(90-57)° = \text{cosec}^2 57°$ and

$\cot^2 57° = \cot^2(90–33)° = \tan^2 33°$

$\therefore \sec^2 33° - \cot^2 57° = \text{cosec}^2 57° - \tan^2 33°$

Hence,

Statement II is not true.

69. (c) Given,

$p = a\sin x + b\cos x$

$q = a\cos x - b\sin x$

$\Rightarrow p^2 = a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x$

and $q^2 = a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x$

$\therefore p^2 + q^2 = a^2(\sin^2 x + \cos^2 x) + b^2(\cos^2 x + \sin^2 x) = a^2 + b^2$

70. (c) Given that, $\sin^2 x + \cos^2 x - 1 = 0$

$\sin^2 x + \cos^2 x = 1$

which is an identity of trigonometric ratio and always true for every real value of x.

So, the equation have an infinite solution.

71. (b) I. Given that, $\sin x + \cos x = 2$

On Squaring both sides, we get

$\Rightarrow (\sin x + \cos x)^2 = 4$

$\Rightarrow (\sin^2 x + \cos^2 x) + 2\sin x.\cos x = 4$

$\Rightarrow 1 + \sin 2x = 4$

$\Rightarrow \sin 2x = 3 \Rightarrow \sin 2x \neq 3$

Hence, there is no value of x in the first quadrant that satisfies $\sin x + \cos x = 2$

II. $\sin x - \cos x = 0$

$\Rightarrow \tan x = 1 = \tan\frac{\pi}{4} \Rightarrow x = \frac{\pi}{4}$

Also, there is only one value of x in the first quadrant that satisfies in $\sin x - \cos x = 0$

72. (c) $7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow 7\sin^2\theta + 3(1-\sin^2\theta) = 4$

$\Rightarrow 7\sin^2\theta + 3 - 3\sin^2\theta = 4$

$\Rightarrow 4\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{1}{2} = \sin 30°$

$\theta = 30° = \frac{\pi}{6}$ $\quad [\because \pi^c = 180°]$

73. (b) $\tan(2\theta + 45^0) = \cot 3\theta$

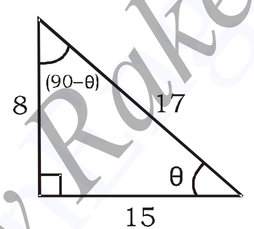
[if tanA = cotB then A + B = 90°]

$(2\theta + 45°) + 3\theta = 90°$

$5\theta + 45^0 = 90^0$

$\theta = \frac{45}{5} = 9^0$

74. (b) $\cos\theta = \frac{15 \to \text{Base}}{17 \to \text{Hypo.}}$



perpendicular = 8

$= \cot(90° - \theta) = \tan\theta$

$= \frac{8}{15}$ $\left[\because \tan\theta = \frac{P}{B}\right]$

75. (a) $(\sec^4\theta - \tan^4\theta)$

$\Rightarrow (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$

$\Rightarrow 1 \times (\sec^2\theta + \tan^2\theta)$

$(\because \sec^2\theta - \tan^2\theta = 1)$

$\Rightarrow 1 \times \frac{7}{12} = \mathbf{\frac{7}{12}}$

76. (b) $\sec x = \text{cosec}\, y$

$\left[\begin{matrix} \text{if } \sec A = \text{cosec} B \\ \text{then } A + B = 90^0 \end{matrix}\right]$

$x + y = 90^0$

$\sin(x + y) = \sin 90° = 1$

77. (c) $A + B + C = \pi = 180^0$

$\Rightarrow \frac{A+B}{2} = \frac{180}{2} - \frac{C}{2}$

$\Rightarrow \sin\left(\frac{A+B}{2}\right)$

$= \sin\left(\frac{\pi}{2} - \frac{c}{2}\right) = \cos\frac{c}{2}$

Similarly,

$\cos\left(\frac{A+B}{2}\right) = \sin\frac{c}{2}$

$\cot\left(\frac{A+B}{2}\right) = \tan\frac{c}{2}$

$\tan\left(\frac{A+B}{2}\right) = \cot\frac{c}{2}$

So, option (C) is incorrect

78. (c) $\tan 2\theta \tan 3\theta = 1$

$(2\theta + 3\theta) = 90^0$

$5\theta = 90^0$

(If tanA . tanB = 1 then A + B = 90°)

$= 2\cos^2\frac{5\theta}{2} - 1 = 2\cos^2 45° - 1$

$= 2 \times \frac{1}{2} - 1 = \mathbf{0}$

79. (d) $2\sin\left[\frac{\pi x}{2}\right] = x^2 + \frac{1}{x^2}$

Let $x = 1$

$2\sin 90.\ 1 = 1^2 + \frac{1}{1^2}$

$2 \times 1 = 1 + 1$

$2 = 2$ matched, so $x = 1$

so, $x - \frac{1}{x}$

$\Rightarrow 1 - \frac{1}{1} = \mathbf{0}$

80. (d) $\cos\theta + \sec\theta = 2$

Put $\theta = 0^0$

$\cos 0^0 + \sec 0^0 = 2$

$\Rightarrow 1 + 1 = 2 \Rightarrow 2 = 2$

(matched, so $\theta = 0°$)

$= \cos^6\theta + \sec^6\theta$

$= \cos^6 0^0 + \sec^6 0^0$

$= (1)^6 + (1)^6 = 1 + 1 = 2$

81. (a) $\dfrac{5}{\sec^2\theta}+\dfrac{2}{1+\cot^2\theta}+3\sin^2\theta$

$= 5\cos^2\theta+\dfrac{2}{\operatorname{cosec}^2\theta}+3\sin^2\theta$

$= 5\cos^2\theta+2\sin^2\theta+3\sin^2\theta$

$= 5(\cos^2\theta+\sin^2\theta)$

$(\because \sin^2\theta+\cos^2\theta=1)$

$= 5$

82. (c) $\left[\dfrac{1}{\cos\theta}+\dfrac{1}{\cot\theta}\right]\left[\dfrac{1}{\cos\theta}-\dfrac{1}{\cot\theta}\right]$

$= (\sec\theta+\tan\theta)\ (\sec\theta-\tan\theta)$

$= \sec^2\theta-\tan^2\theta$

$[\because 1+\tan^2\theta=\sec^2\theta] = 1$

83. (c) $\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} = \dfrac{5}{4}$

$\Rightarrow 4\sin\theta+4\cos\theta = 5\sin\theta-5\cos\theta$

$\Rightarrow \sin\theta = 9\cos\theta$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = 9$

$\Rightarrow \tan\theta = 9$

$\dfrac{\tan^2\theta+1}{\tan^2\theta-1} = \dfrac{9^2+1}{9^2-1} = \dfrac{82}{80} = \dfrac{41}{40}$

84. (c) $\tan 7\theta\ .\ \tan 2\theta = 1$

[If, tanA.tanB= 1 then, A + B = 90°]

$\Rightarrow 7\theta+2\theta = 90°$

$\Rightarrow 9\theta = 90°$

$\Rightarrow \theta = 10^0$

$\therefore \tan 3\theta = \tan 30^0 = \dfrac{1}{\sqrt{3}}$

85. (d)

$(2\cos^2\theta-1)\left[\dfrac{1+\tan\theta}{1-\tan\theta}+\dfrac{1-\tan\theta}{1+\tan\theta}\right]$

Put $\theta=60^0$

$\Rightarrow (2\cos^2 60^0-1)\left[\dfrac{1+\tan 60^0}{1-\tan 60^0}+\dfrac{1-\tan 60^0}{1+\tan 60^0}\right]$

$= \left[2(\tfrac{1}{2})^2-1\right]\left[\dfrac{1+\sqrt{3}}{1-\sqrt{3}}+\dfrac{1-\sqrt{3}}{1+\sqrt{3}}\right]$

$= \left[\dfrac{1}{2}-1\right]\left[\dfrac{(1+\sqrt{3})^2+(1-\sqrt{3})^2}{(1-\sqrt{3})(1+\sqrt{3})}\right]$

$= \left(-\dfrac{1}{2}\right)\left[\dfrac{(1+3+2\sqrt{3})+(1+3-2\sqrt{3})}{1-3}\right]$

$= \left(-\dfrac{1}{2}\right)\left[\dfrac{4+2\sqrt{3}+4-2\sqrt{3}}{-2}\right]$

$= -\dfrac{1}{2}\left[-\dfrac{8}{2}\right] = \dfrac{1}{2}\times 4 = 2$

Alternate (I):-

$(2\cos^2\theta-1)\times\left(\dfrac{1+\tan\theta}{1-\tan\theta}+\dfrac{1-\tan\theta}{1+\tan\theta}\right)$

$= (2\cos^2\theta-(\sin^2\theta+\cos^2\theta))$

$\times\left(\dfrac{(1+\tan\theta)^2+(1-\tan\theta)^2}{1-\tan^2\theta}\right)$

$= (\cos^2\theta-\sin^2\theta)$

$\times\left(\dfrac{1+\tan^2\theta+2\tan\theta+1+\tan^2\theta-2\tan\theta}{1-\dfrac{\sin^2\theta}{\cos^2\theta}}\right)$

$= (\cos^2\theta-\sin^2\theta)$

$\left(\dfrac{2+2\tan^2\theta}{\cos^2\theta-\sin^2\theta}\right)\times\cos^2\theta$

$= 2(1+\tan^2\theta)\times\cos^2\theta$

$= 2\sec^2\theta.\ \cos^2\theta = 2$

Alternate(II):-

$(2\cos^2\theta-1)\left(\dfrac{1+\tan\theta}{1-\tan\theta}+\dfrac{1-\tan\theta}{1+\tan\theta}\right)$

put $\theta = 0°$

$=(2\cos^2 0°-1)\times\left(\dfrac{1+\tan 0°}{1-\tan 0°}+\dfrac{1-\tan 0°}{1+\tan 0°}\right)$

$= (2\times 1-1)\left(\dfrac{1+0}{1-0}+\dfrac{1-0}{1+0}\right)$

$= (2-1)(1+1) = 2$

86. (c) $\sec\theta+\tan\theta = 2$ ....(i)

$\because \sec^2\theta-\tan^2\theta = 1$

$\Rightarrow (\sec\theta-\tan\theta)(\sec\theta+\tan\theta) = 1$

$\Rightarrow (\sec\theta-\tan\theta) = \dfrac{1}{\sec\theta+\tan\theta}$

$(\sec\theta-\tan\theta) = \dfrac{1}{2}$ ......(ii)

adding equation (i) and (ii)

$\sec\theta+\tan\theta+\sec\theta-\tan\theta = 2+\dfrac{1}{2}$

$\Rightarrow 2\sec\theta = \dfrac{5}{2} \Rightarrow \sec\theta = \dfrac{5}{4}$

87. (b) $\sec\theta = \dfrac{4x^2+1}{4x}$

$\tan\theta = \sqrt{\sec^2\theta-1}$

$= \sqrt{\left[\dfrac{4x^2+1}{4x}\right]^2-1}$

$= \sqrt{\dfrac{(4x^2+1)^2-(4x)^2}{(4x)^2}}$

$= \sqrt{\dfrac{16x^4+1+8x^2-16x^2}{(4x)^2}}$

$= \sqrt{\dfrac{16x^4+1-8x^2}{(4x)^2}}$

$= \sqrt{\dfrac{(4x^2-1)^2}{(4x)^2}}$

$= \dfrac{4x^2-1}{4x}$

$\therefore \sec\theta+\tan\theta$

$= \dfrac{4x^2+1}{4x}+\dfrac{4x^2-1}{4x}$

$= \dfrac{4x^2+1+4x^2-1}{4x}$

$= \dfrac{8x^2}{4x} = 2x$

Alternate:-

$\sec\theta = x+\dfrac{1}{4x}$

put $x = 1$

$\sec\theta = 1+\dfrac{1}{4} = \dfrac{5}{4} = \dfrac{H}{B}$

$\tan\theta = \dfrac{P}{B} = \dfrac{3}{4}$

Now,

$\sec\theta+\tan\theta$

$= \dfrac{5}{4}+\dfrac{3}{4} = \dfrac{5+3}{4} = \dfrac{8}{4} = 2$

by option (b),

$2x = 2\times 1 = 2$

88. (b) $\sin^2 25^0 + \sin^2 65^0$

$= \sin^2 25^0 + \sin^2(90 - 25^0)$

$= \sin^2 25^0 + \cos^2 25^0 = 1$

Alternate:-

If A + B = 90°

then $\sin^2 A + \sin^2 B = 1$

So, $\sin^2 25^0 + \sin^2 65^0 = 1$

89. (a) $\sec\theta + \tan\theta = \sqrt{3}$ ....(i)

$\because \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$ ......(ii)

Subtract equation (i) from (ii)

$\Rightarrow 2\tan\theta = \sqrt{3} - \frac{1}{\sqrt{3}}$

$\Rightarrow 2\tan\theta = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$

$\Rightarrow \theta = 30° \left[\because \tan 30^0 = \frac{1}{\sqrt{3}}\right]$

$\Rightarrow \tan 3\theta = \tan 90° = \text{undefined}$

90. (c) $\sin(60^0 - \theta) = \cos(\Psi - 30^0)$

$(60^0 - \theta) + (\Psi - 30^0) = 90^0$

[*if* $\sin A = \cos B$

then $A + B = 90^0$]

$(\Psi - \theta) = 90^0 - 30^0$

$(\Psi - \theta) = 60^0$

$\tan(\Psi - \theta) = \tan 60^0$

$\tan 60° = \sqrt{3}$

91. (b) $a \sin\theta + b\cos\theta = c$....(i)

let

$a \cos\theta - b \sin\theta = x$ ......(ii)

Squaring and adding equation (i)and (ii)

$= a^2 + b^2 = c^2 + x^2$

$= a^2 + b^2 - c^2 = x^2$

$= x = \pm\sqrt{a^2 + b^2 - c^2}$

92. (b) $\sin(A - B) = \frac{1}{2} = \sin 30°$

$\therefore$ A – B = 30°

$\cos(A+B) = \frac{1}{2} = \cos 60°$

$\therefore$ A + B = 60°

Adding both side

$\Rightarrow (A - B) + (A + B) = 30^0 + 60^0$

$\Rightarrow 2A = 90^0$

$\Rightarrow A = 45^0$

$\therefore A - B = 30^0$

$B = A - 30^0 = 45^0 - 30^0 = 15^0$

$\Rightarrow \frac{15° \times \pi}{180°} = \frac{\pi}{12}$ radian

93. (c) 152 (sin30° + 2cos²45° + 3 sin30°..........+17sin30° + 18cos²45°)

$\Rightarrow 152$

$\left(\frac{1}{2} + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 3 \times \frac{1}{2} .....17 \times \frac{1}{2} + 18\left(\frac{1}{\sqrt{2}}\right)^2\right)$

$\Rightarrow 152\left[\frac{1}{2} + 1 + 1\frac{1}{2} + ... + 8\frac{1}{2} + 9\right]$

$\Rightarrow$ This is in A.P. where,

$a = \frac{1}{2}$, $d = \frac{1}{2}$, $n = 18$

$\Rightarrow S_{152} = 152\left[\frac{18}{2}\left(2 \times \frac{1}{2} + (18-1)\frac{1}{2}\right)\right]$

$\Rightarrow 152\left[\frac{18}{2}\left(1 + \frac{17}{2}\right)\right]$

$\Rightarrow 152 \times 9 \times \frac{19}{2} \Rightarrow 12996 \Rightarrow$

$\sqrt{12996}$ = **114**

94. (d) 3cos80° cosec10°+ 2cos59° cosec31°

$\Rightarrow 3\cos 80° \frac{1}{\sin 10°} + 2\cos 59° \frac{1}{\sin 31°}$

$\Rightarrow 3\cos 80° \frac{1}{\sin(90° - 80°)} + 2$

$\cos 59° \frac{1}{\sin(90° - 59°)}$

$\Rightarrow 3 + 2 =$ **5**

95. (d) $\sin^2\theta - 3\sin\theta + 2 = 0$

$\Rightarrow \sin^2\theta - 2\sin\theta - \sin\theta + 2 = 0$

$\Rightarrow \sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) = 0$

$\Rightarrow (\sin\theta - 1)(\sin\theta - 2) = 0$

$[\because \sin\theta \neq 2]$

$\therefore \sin\theta = 1 = \sin 90°$

$\Rightarrow \theta = 90°$

Alternate:-

Put value of θ = 90° [take help from options]

$\sin^2\theta - 3\sin\theta + 2 = 0$

$\sin^2 90° - 3\sin 90° + 2 = 0$

$1 - 3 \times 1 + 2 = 0$

$[\because \sin 90° = 1]$

$0 = 0$

[matched]

So, this is answer.

96. (c) $\sin\alpha = m\sin\beta$

Squaring both sides

$\sin^2\alpha = m^2\sin^2\beta$ ...(i)

$\tan\alpha = n\tan\beta$

Squaring both sides

$\tan^2\alpha = n^2\tan^2\beta$

$\frac{\sin^2\alpha}{\cos^2\alpha} = n^2\frac{\sin^2\beta}{\cos^2\beta}$

(value put in $\sin^2\beta$)

$\frac{\sin^2\alpha}{\cos^2\alpha} = \frac{n^2\sin^2\alpha}{m^2(1 - \sin^2\beta)}$

(from equation (i))

$\frac{1}{\cos^2\alpha} = \frac{n^2}{m^2\left(1 - \frac{\sin^2\alpha}{m^2}\right)}$

$(m^2 - \sin^2\alpha) = n^2\cos^2\alpha$

$m^2 - (1 - \cos^2\alpha) = n^2\cos^2\alpha$

$m^2 - 1 + \cos^2\alpha = n^2\cos^2\alpha$

$m^2 - 1 = \cos^2\alpha(n^2 - 1)$

$\cos^2\alpha = \frac{m^2 - 1}{n^2 - 1}$

Alternate:-

According to question,

$\tan\alpha = n\tan\beta$

$\sin\alpha = m\sin\beta$

$n = \frac{\tan\alpha}{\tan\beta}$, $m = \frac{\sin\alpha}{\sin\beta}$

Put $\alpha = 30°$ and $\beta = 60°$

$n = \frac{\tan 30°}{\tan 60°} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$

$m = \frac{\sin 30°}{\sin 60°} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

Then, $\cos^2\alpha = \cos^2 30° = \frac{3}{4}$

by option (c),

$\frac{m^2-1}{n^2-1}$

Put the value of m and n,

$$\frac{\left(\frac{1}{\sqrt{3}}\right)^2-1}{\left(\frac{1}{3}\right)^2-1}=\frac{\frac{1}{3}-1}{\frac{1}{9}-1}$$

$= \frac{\mathbf{3}}{\mathbf{4}}$ **(satisfied)**

97. (c) $\operatorname{cosec}\theta-\cot\theta=\frac{7}{2}$ .....(i)

$\because\ \operatorname{cosec}^2\theta-\cot^2\theta=1$

$\Rightarrow [\operatorname{cosec}\theta-\cot\theta](\operatorname{cosec}\theta+\cot\theta)=1$

$\Rightarrow (\operatorname{cosec}\theta+\cot\theta)$

$=\frac{1}{(\operatorname{cosec}\theta-\cot\theta)}$

$\Rightarrow \operatorname{cosec}\theta+\cot\theta=\frac{2}{7}$ .. (ii)

Adding both equations,

$\Rightarrow 2\operatorname{cosec}\theta=\frac{7}{2}+\frac{2}{7}$

$\Rightarrow \frac{49+4}{14}=\frac{53}{14}$

$\Rightarrow \operatorname{cosec}\theta=\mathbf{\frac{53}{28}}$

98. (a) $x\sin45^\circ=y\operatorname{cosec}30^\circ$

$\Rightarrow \frac{x}{y}=\frac{\operatorname{cosec}30^\circ}{\sin45^\circ}$

$\Rightarrow \frac{x}{y}=\frac{2}{1/\sqrt{2}}=\frac{2\sqrt{2}}{1}$

$\Rightarrow \frac{x^4}{y^4}=\left(\frac{2\sqrt{2}}{1}\right)^4=\frac{64}{1}$

$=\mathbf{4^3}$

99. (c) $\because 5\tan\theta=4$

$\Rightarrow \tan\theta=\frac{4}{5}$

$\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}$

Divide numerator and denominator by $\cos\theta$

$$=\frac{5\frac{\sin\theta}{\cos\theta}-3\frac{\cos\theta}{\cos\theta}}{5\frac{\sin\theta}{\cos\theta}+2\frac{\cos\theta}{\cos\theta}}=\frac{5\tan\theta-3}{5\tan\theta+2}$$

$$=\frac{\left(5\times\frac{4}{5}\right)-3}{\left(5\times\frac{4}{5}\right)+2}=\mathbf{\frac{1}{6}}$$

100. (b) According to question,

$2\operatorname{cosec}^2 23^\circ\ \cot^2 67^\circ-\sin^2 23^\circ-\sin^2 67^\circ-\cot^2 67^\circ$

$=2\operatorname{cosec}^2 23^\circ\ \cot^2(90^\circ-23^\circ)-\sin^2 23^\circ-\sin^2(90^\circ-23^\circ)-\cot^2 67^\circ$

$=2\operatorname{cosec}^2 23^\circ\ \tan^2 23^\circ-(\sin^2 23^\circ+\cos^2 23^\circ)-\cot^2 67^\circ$

$=\frac{2}{\cos^2 23^\circ}-1-\cot^2 67^\circ$

$=2\sec^2 23^\circ-1-\cot^2(90-23^\circ)$

$=2\sec^2 23^\circ-1-\tan^2 23^\circ$

$=2\sec^2 23^\circ-(1+\tan^2 23^\circ)$

$=2\sec^2 23^\circ-\sec^2 23^\circ=\sec^2 23^\circ$

101. (c) $\cos^2\theta=\frac{(x+y)^2}{4xy}$

$\because$ max. value of $\cos^2\theta=1$

$\therefore 1=\frac{(x+y)^2}{4xy}$

$\Rightarrow 4xy=(x+y)^2$

$\Rightarrow 4xy=x^2+y^2+2xy$

$\Rightarrow 0=x^2+y^2-2xy$

$\Rightarrow 0=(x-y)^2$

$\Rightarrow 0=x-y \Rightarrow x=y$

102. (a) $(1-\sin^2\alpha)(1-\cos^2\alpha)\times(1+\cot^2\beta)(1+\tan^2\beta)$

$=(\cos^2\alpha)(\sin^2\alpha)(\operatorname{cosec}^2\beta)(\sec^2\beta)$

put $\alpha=\beta=45^\circ$,

$=\cos^2 45^\circ.\sin^2 45^\circ.\operatorname{cosec}^2 45^\circ.\sec^2 45^\circ$

$=\frac{1}{2}.\frac{1}{2}.2.2=\mathbf{1}$

Alternate:-

$\alpha+\beta=90^\circ$ or $\alpha=90-\beta$

$(1-\sin^2\alpha)(1-\cos^2\alpha)\times$

$(1+\cot^2\beta)(1+\tan^2\beta)$

$=(\cos^2\alpha)(\sin^2\alpha)(\operatorname{cosec}^2\beta)$

$(\sec^2\beta)$

$=\cos^2(90-\beta).\sin^2\alpha.$

$\operatorname{cosec}^2\beta.\sec^2(90-\alpha)$

$=\sin^2\beta.\operatorname{cosec}^2\beta.\sin^2\alpha.\operatorname{cosec}^2\alpha=1$

103. (c) $\frac{2\sin68^\circ}{\cos22^\circ}-\frac{2\cot15^\circ}{5\tan75^\circ}-\frac{3\tan45^\circ.\tan20^\circ.\tan40^\circ.\tan50^\circ.\tan70^\circ}{5}$

$=\frac{2\sin68^\circ}{\cos(90^\circ-68^\circ)}-\frac{2\cot15^\circ}{5\tan(90^\circ-15^\circ)}-\frac{3\times1.(\tan20^\circ.\tan70^\circ)(\tan40^\circ.\tan50^\circ)}{5}$

$=\frac{2\sin68^\circ}{\sin68^\circ}-\frac{2\cot15^\circ}{5\cot15^\circ}-\frac{3\times1\times1\times1}{5}$

($\tan A.\tan B=1$ if $A+B=90^\circ$)

$=2-\frac{2}{5}-\frac{3}{5}=\mathbf{1}$

104. (b) $\sin7x=\cos11x$

$\Rightarrow \sin7x=\sin(90^\circ-11x)$

$\Rightarrow 7x=90^\circ-11x$

$\Rightarrow 7x+11x=90^\circ$

$\Rightarrow 18x=90^\circ$

$\Rightarrow x=5^\circ$

$=\tan9x+\cot9x$

$=\tan45^\circ+\cot45^\circ$

$=1+1=\mathbf{2}$

105. (a) $\tan^2\alpha=1+2\tan^2\beta$

$\Rightarrow \sec^2\alpha-1=1+2(\sec^2\beta-1)$

$\Rightarrow \sec^2\alpha-1=2\sec^2\beta-1$

$\Rightarrow \frac{1}{\cos^2\alpha}=\frac{2}{\cos^2\beta}$

$\Rightarrow 2\cos^2\alpha=\cos^2\beta$

$\Rightarrow \sqrt{2}\cos\alpha=\cos\beta$

$\therefore \sqrt{2}\cos\alpha-\cos\beta=0$

Alternate:-

$\tan^2\alpha=1+2\tan^2\beta$

Put $\beta=45^\circ$

$\Rightarrow \tan^2\alpha=1+2.\tan^2 45^\circ$

$\Rightarrow \tan^2\alpha = 3$

$\Rightarrow \tan\alpha = \sqrt{3}$

$\Rightarrow \alpha = 60°$

Put $\alpha = 60°$, $\beta = 45°$

$\sqrt{2}\cos\alpha - \cos\beta$

$= \sqrt{2}\cos 60° - \cos 45°$

$= \sqrt{2} \times \frac{1}{2} - \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

106. (a) $\tan\theta + \cot\theta = 2$

Put $\theta = 45°$

$1 + 1 = 2$ (matched)

So, $\theta = 45°$

$\Rightarrow \tan^{100}45° + \cot^{100}45°$

$\Rightarrow 1^{100} + 1^{100} = \mathbf{2}$

107. (d) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$

$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{\tan\theta(1-\tan\theta)}$

$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)}$

$= \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$

$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$

[$\because a^3 - b^3 = (a-b)(a^2+b^2+ab)$]

$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$

$= \tan\theta + \cot\theta + 1$

108. (a)

$\sec\theta + \tan\theta = 2 + \sqrt{5}$ ..(i)

$\because \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{2+\sqrt{5}}$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{5}+2}$

$\Rightarrow \sec\theta - \tan\theta = \sqrt{5} - 2$ ....(ii)

$\left[\because \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2\right]$

add eq (i) + (ii)

$2\sec\theta = 2 + \sqrt{5} + \sqrt{5} - 2$

$\Rightarrow 2\sec\theta = 2\sqrt{5}$

$\Rightarrow \sec\theta = \sqrt{5}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{5}}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \sin^2\theta = 1 - \left(\frac{1}{\sqrt{5}}\right)^2$

$\Rightarrow \sin^2\theta = \frac{4}{5}$

$\Rightarrow \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore \sin\theta + \cos\theta$

$= \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \mathbf{\frac{3}{\sqrt{5}}}$

109. (b) $(1+\cot\theta - \mathrm{cosec}\,\theta)(1 + \tan\theta + \sec\theta)$

Put, $\theta = 45°$

$=(1 + \cot 45° - \mathrm{cosec}\,45°)(1 + \tan 45° + \sec 45°)$

$= (1 + 1 - \sqrt{2})(1 + 1 + \sqrt{2})$

$= (2 - \sqrt{2})(2 + \sqrt{2})$

$=[2^2 - (\sqrt{2})^2]$

[$\because (a-b)(a+b) = a^2 - b^2$]

$= 4 - 2 = 2$

110. (a) $x = a\sec\theta\cos\phi$

$y = b\sec\theta\sin\phi$

$z = c\tan\theta$

$\Rightarrow \frac{x}{a} = \sec\theta.\cos\phi$

$\Rightarrow \frac{y}{b} = \sec\theta.\sin\phi$

$\Rightarrow \frac{z}{c} = \tan\theta$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

$\sec^2\theta.\cos^2\phi + \sec^2\theta.\sin^2\phi - \tan^2\theta$

$=\sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta$

$(\because \sin^2\phi + \cos^2\phi = 1)$

$= \sec^2\theta - \tan^2\theta = \mathbf{1}$

111. (a) $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$

$\Rightarrow 3\sec\theta + 3\tan\theta = 5\sec\theta - 5\tan\theta$

$\Rightarrow 8\tan\theta = 2\sec\theta \Rightarrow \frac{\sec\theta}{\tan\theta} = 4$

$\Rightarrow \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = 4 \Rightarrow \sin\theta = \frac{1}{4}$

112. (a) Let,

$(1+\sin\alpha)(1+\sin\beta)(1+\sin\gamma) = (1-\sin\alpha)(1-\sin\beta)(1-\sin\gamma) = x$

$\therefore x.x = (1+\sin\alpha)(1-\sin\alpha)(1+\sin\beta)(1-\sin\beta)(1+\sin\gamma)(1-\sin\gamma)$

$x^2 = (1-\sin^2\alpha)(1-\sin^2\beta)(1-\sin^2\gamma)$

$x^2 = \cos^2\alpha . \cos^2\beta . \cos^2\gamma$

$x = \pm \cos\alpha.\cos\beta.\cos\gamma$

113. (d)

$\frac{1}{1+\cot^2\theta} + \frac{3}{1+\tan^2\theta} + 2\sin^2\theta$

$= \frac{1}{\mathrm{cosec}^2\theta} + \frac{3}{\sec^2\theta} + 2\sin^2\theta$

$=\sin^2\theta + 3\cos^2\theta + 2\sin^2\theta$

$=3(\sin^2\theta + \cos^2\theta)$

$=3 \times (1) = \mathbf{3}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

Alternate:-

$\frac{1}{1+\cot^2\theta} + \frac{3}{1+\tan^2\theta} + 2\sin^2\theta$

Put $\theta = 45°$

$\Rightarrow \frac{1}{1+1} + \frac{3}{1+1} + 2 \times \frac{1}{2}$

$\Rightarrow \frac{1}{2} + \frac{3}{2} + 1$

$\Rightarrow \frac{1+3+2}{2} = \frac{6}{2} = 3$

114. (a)

$\frac{4}{1+\tan^2\alpha} + \frac{1}{1+\cot^2\alpha} + 3\sin^2\alpha$

$= \dfrac{4}{\sec^2\alpha} + \dfrac{1}{\text{cosec}^2\alpha} + 3\sin^2\alpha$

$= 4\cos^2 + \sin^2\alpha + 3\sin^2\alpha$

$= 4(\cos^2\alpha + \sin^2\alpha)$

$= 4\times(1) = 4$

$(\because \sin^2\alpha + \cos^2\alpha = 1)$

Alternate:-

$\dfrac{4}{1+\tan^2\alpha} + \dfrac{1}{1+\cot^2\alpha} + 3\sin^2\alpha$

Put $\alpha = 45°$

$\Rightarrow \dfrac{4}{1+1} + \dfrac{1}{1+1} + 3 \times \dfrac{1}{2}$

$\Rightarrow 2 + \dfrac{1}{2} + \dfrac{3}{2}$

$\Rightarrow \dfrac{4+1+3}{2} = \dfrac{8}{2} = 4$

115. **(d)** $3(\sin x + \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

Put, $x = 90°$

$3(\sin 90° + \cos 90°)^4 + 6(\sin 90° + \cos 90°)^2 + 4(\sin^6 90° + \cos^6 90°)$

$= 3(1+0)^4 + 6(1+0)^2 + 4(1^6 + 0)$

$= 3 + 6 + 4 = 13$

116. **(c)**

$\sec\theta\left(\dfrac{1+\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta}\right) - 2\tan^2\theta$

Take, $\theta = 0°$

$= \sec 0° \times \left(\dfrac{1+\sin 0°}{\cos 0°} + \dfrac{\cos 0°}{1+\sin 0°}\right) - 2\tan^2 0°$

$= 1\times\left(\dfrac{1+0}{1} + \dfrac{1}{1+0}\right) - 2\times 0 = 2$

117. **(a)** $x\sin\theta + y\cos\theta = 4$

Squaring both sides,

$x^2\sin^2\theta + y^2\cos^2\theta + 2xy\sin\theta.\cos\theta = 16$ ....(i)

$x\cos\theta - y\sin\theta = 2$

again squaring both sides,

$x^2\cos^2\theta + y^2\sin^2\theta - 2xy\sin\theta.\cos\theta = 4$ .....(ii)

on adding eqn. (i) and (ii),

$(x^2 + y^2)(\sin^2\theta + \cos^2\theta) = 16 + 4$

$x^2 + y^2 = 20$

118. **(c)** $\left[\dfrac{\cos^2 A.\sin^2 A(\sin A + \cos A)}{(\sin A - \cos A)} + \dfrac{\sin^2 A.\cos^2 A(\sin A - \cos A)}{(\sin A + \cos A)}\right] \times \left[\dfrac{1}{\cos^2 A} - \dfrac{1}{\sin^2 A}\right]$

$= \left[\dfrac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}\right]$

$\times\left[\sin^2 A - \cos^2 A\right]$

$= 2(\sin^2 A + \cos^2 A) = \mathbf{2}$

119. **(b)** $\dfrac{1}{\text{cosec}\theta - \cot\theta} - \dfrac{1}{\sin\theta}$

$= \dfrac{\text{cosec}^2\theta - \cot^2\theta}{\text{cosec}\theta - \cot\theta} - \text{cosec}\theta$

$[\because \text{cosec}^2\theta - \cot^2\theta = 1]$

$= \text{cosec}\theta + \cot\theta - \text{cosec}\theta$

$= \boldsymbol{\cot\theta}$

120. **(d)** $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Squaring both sides,

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow 2\cos^2\theta - \cos^2\theta - \sin^2\theta = 2\cos\theta\sin\theta$

$\Rightarrow \cos^2\theta - \sin^2\theta = 2\sin\theta.\cos\theta$

$\Rightarrow (\cos\theta - \sin\theta)(\cos\theta + \sin\theta) = 2\sin\theta.\cos\theta$

$\Rightarrow (\cos\theta - \sin\theta)(\sqrt{2}\cos\theta) = 2\sin\theta.\cos\theta$

$\Rightarrow \cos\theta - \sin\theta = \dfrac{2\sin\theta.\cos\theta}{\sqrt{2}\cos\theta}$

$= \sqrt{2}\sin\theta$

Alternate:-

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\cos\theta - \sin\theta = x$

$2(\sin^2\theta + \cos^2\theta) = (\sqrt{2})^2\cos^2\theta + x^2$

$2 - 2\cos^2\theta = x^2$

$2(1 - \cos^2\theta) = x^2$

$2\sin^2\theta = x^2$

$x = \sqrt{2}\sin\theta$

121. **(c)** $\sin\theta - \cos\theta = \dfrac{1}{2}$ ....(i)

$\sin\theta + \cos\theta = m$ .....(ii)

on squaring and adding both sides,

$2(\sin^2\theta + \cos^2\theta) = \left(\dfrac{1}{2}\right)^2 + m^2$

$\Rightarrow 2 = \dfrac{1}{4} + m^2$

$\Rightarrow m^2 = 2 - \dfrac{1}{4} = \dfrac{7}{4}$

$\Rightarrow m = \dfrac{\sqrt{7}}{2}$

$\therefore \sin\theta + \cos\theta = \dfrac{\sqrt{7}}{2}$

122. **(a)** $\dfrac{\sin A}{1+\cos A} + \dfrac{\sin A}{1-\cos A}$

$= \dfrac{\sin A(1-\cos A) + \sin A(1+\cos A)}{(1+\cos A)(1-\cos A)}$

$= \dfrac{\sin A - \sin A\cos A + \sin A + \sin A\cos A}{1-\cos^2 A}$

$= \dfrac{2\sin A}{\sin^2 A} = 2\,\text{cosec}A$

123. **(c)** $\tan\theta - \cot\theta = 0$

Put $\theta = 45°$

$\tan 45° - \cot 45° = 0$

$1 - 1 = 0$

$0 = 0$ (matched)

So, $\theta = 45°$

$\sin\theta + \cos\theta$

$= \sin 45° + \cos 45°$

$= \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \sqrt{2}$

124. **(a)** $3\sin\theta + 5\cos\theta = 5$

$5\sin\theta - 3\cos\theta = x$

and squaring and adding both sides,

$5^2(\sin^2\theta + \cos^2\theta) + 3^2(\sin^2\theta + \cos^2\theta) = x^2 + 5^2$

$\Rightarrow 5^2 + 3^2 = x^2 + 5^2$

$\Rightarrow x^2 = 9$

$\therefore x = \pm 3$

125. **(c)** $x\sin 60°.\tan 30° = \sec 60°.\cot 45°$

$\Rightarrow x.\dfrac{\sqrt{3}}{2}.\dfrac{1}{\sqrt{3}} = 2.\,1$

$\Rightarrow \dfrac{x}{2} = 2$

$\Rightarrow \boldsymbol{x = 4}$

126. **(d)** $\theta = 60^\circ$

$= \frac{1}{2}\sqrt{1+\sin\theta} + \frac{1}{2}\sqrt{1-\sin\theta}$

$= \frac{1}{2}\sqrt{1+\sin 60^\circ} + \frac{1}{2}\sqrt{1-\sin 60^\circ}$

$= \frac{1}{2}\sqrt{1+\left(\frac{\sqrt{3}}{2}\right)} + \frac{1}{2}\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)}$

$= \frac{1}{2\sqrt{2}}\left(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}\right)$

$= \frac{1}{2\sqrt{2}} \times \frac{1}{\sqrt{2}}\left(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}}\right)$

$= \frac{1}{4}\left(\sqrt{(\sqrt{3}+1)^2} + \sqrt{(\sqrt{3}-1)^2}\right)$

$= \frac{1}{4}\left(\sqrt{3}+1+\sqrt{3}-1\right)$

$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = \cos 30^\circ = \cos\frac{\theta}{2}$

127.(b) $\frac{2\tan^2 30^\circ}{1-\tan^2 30^\circ} + \sec^2 45^\circ - \sec^2 0^\circ$

$= x\sec 60^\circ$

$\Rightarrow \frac{2\times\left(\frac{1}{\sqrt{3}}\right)^2}{1-\left(\frac{1}{\sqrt{3}}\right)^2} + \left(\sqrt{2}\right)^2 - 1$

$= x \times 2$

$\Rightarrow \frac{2\times\frac{1}{3}}{1-\frac{1}{3}} + 2 - 1 = 2x$

$\Rightarrow \left(\frac{2}{3}\times\frac{3}{2}\right) + 2 - 1 = 2x$

$\Rightarrow 2 = x \times 2$

$\therefore$ $\boldsymbol{x = 1}$

128. **(b)** $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$

Squaring both sides and after that adding '1' both sides,

$\Rightarrow 1 + \tan^2\theta = 1 + \frac{(\sin\alpha - \cos\alpha)^2}{(\sin\alpha+\cos\alpha)^2}$

$\Rightarrow \sec^2\theta$

$= \frac{(\sin\alpha+\cos\alpha)^2 + (\sin\alpha-\cos\alpha)^2}{(\sin\alpha+\cos\alpha)^2}$

$(\because 1 + \tan^2\theta = \sec^2\theta)$

$\Rightarrow \sec^2\theta = \frac{2(\sin^2\alpha + \cos^2\alpha)}{(\sin\alpha+\cos\alpha)^2}$

$\Rightarrow \frac{1}{\cos^2\theta} = \frac{2}{(\sin\alpha+\cos\alpha)^2}$

$\Rightarrow \frac{1}{\cos\theta} = \frac{\pm\sqrt{2}}{\sin\alpha+\cos\alpha}$

$\Rightarrow \sin\alpha + \cos\alpha = \pm\sqrt{2}\cos\theta$

129. **(a)** $7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow 7\sin^2\theta + 3(1-\sin^2\theta) = 4$

$\Rightarrow 7\sin^2\theta + 3 - 3\sin^2\theta = 4$

$\Rightarrow 4\sin^2\theta = 4 - 3$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ$

$\therefore \tan\theta = \tan 30^\circ = \boldsymbol{\frac{1}{\sqrt{3}}}$

130. **(d)** $\tan 9^\circ = \frac{p}{q}$

$\frac{\sec^2 81^\circ}{1+\cot^2 81^\circ} = \frac{\sec^2 81^\circ}{\text{cosec}^2 81^\circ}$

$(\because 1 + \cot^2\theta = \text{cosec}^2\theta)$

$= \frac{1}{\cos^2 81^\circ} \times \sin^2 81^\circ$

$= \tan^2 81^\circ = \tan^2(90^\circ - 9^\circ)$

$= \cot^2 9^\circ = \frac{1}{\tan^2 9^\circ} = \frac{q^2}{p^2}$

131. **(d)** If $\sec\theta + \tan\theta = 5$ .....(i)

$\because \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta+\tan\theta)(\sec\theta-\tan\theta) = 1$

$(\sec\theta - \tan\theta) = \frac{1}{5}$ ....(ii)

subtracting eq. (ii) from (i)

$\Rightarrow (\sec\theta + \tan\theta) - (\sec\theta - \tan\theta)$

$= 5 - \frac{1}{5}$

$\Rightarrow 2\tan\theta = \frac{25-1}{5} = \frac{24}{5}$

$\tan\theta = \frac{12}{5}$

$\Rightarrow \frac{\tan\theta + 1}{\tan\theta - 1} = \frac{\frac{12}{5}+1}{\frac{12}{5}-1} = \frac{12+5}{12-5} = \boldsymbol{\frac{17}{7}}$

132. **(c)** $\frac{\cos\alpha}{\cos\beta} = a$

$= \cos\alpha = a\cos\beta$

On squaring both sides,

$\Rightarrow \cos^2\alpha = a^2\cos^2\beta$

$\Rightarrow 1 - \sin^2\alpha = a^2(1 - \sin^2\beta)$ ....(i)

Again, $\sin\alpha = b\sin\beta$

Squaring both sides

$\Rightarrow \sin^2\alpha = b^2\sin^2\beta$

put the value of $\sin^2\alpha$ in equation (i)

$\Rightarrow 1 - b^2\sin^2\beta = a^2(1 - \sin^2\beta)$

$\Rightarrow 1 - b^2\sin^2\beta = a^2 - a^2\sin^2\beta$

$\Rightarrow a^2\sin^2\beta - b^2\sin^2\beta = a^2 - 1$

$\Rightarrow \sin^2\beta\,(a^2 - b^2) = a^2 - 1$

$\Rightarrow \sin^2\beta = \frac{a^2-1}{a^2-b^2}$

133. **(b)**

$\frac{\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$= \frac{\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1^2}{1}$

$(\because \sin^2 A + \cos^2 A = 1)$

$= \frac{1}{4} + \frac{4\times 4}{3} - 1 = \frac{1}{4} + \frac{16}{3} - 1$

$= \frac{3+64-12}{12} = \boldsymbol{\frac{55}{12}}$

134. **(d)** $\cos\pi x = x^2 - x + \frac{5}{4}$

$= x^2 - 2\times x\times\frac{1}{2} + \frac{1}{4} - \frac{1}{4} + \frac{5}{4}$

$= \left(x - \frac{1}{2}\right)^2 + 1 > 1$

$= -1 \le \cos x \le 1$

$\therefore$ so value of x is none of the above

135. **(d)** $1 + \frac{1}{\cot^2 63°} - \sec^2 27°$

$+ \frac{1}{\sin^2 63°} - \text{cosec}^2 27°$

$= 1 + \tan^2 63° - \sec^2 27° + \text{cosec}^2 63° - \text{cosec}^2 27°$

$= 1 + \cot^2 27° - \sec^2 27° + \sec^2 27° - \text{cosec}^2 27°$

$= 1 + \cot^2 27° - \text{cosec}^2 27°$

$= 1 - 1 = 0$

136. **(b)** $x = \frac{\cos\theta}{1 - \sin\theta}$

$= \frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$

$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$

$= \frac{1+\sin\theta}{\cos\theta}$

$\frac{\cos\theta}{1+\sin\theta} = \frac{1}{x}$

Alternate:-

$\therefore x = \frac{\cos\theta}{1-\sin\theta}$

put $\theta = 0°$

$x = \frac{\cos 0°}{1-\sin 0°} = \frac{1}{1-0}$

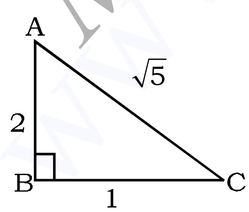
$\Rightarrow x = 1$

$= \frac{\cos\theta}{1+\sin\theta} = \frac{\cos 0°}{1+\sin 0°} = \frac{1}{1+0}$

$= 1$

now check option by putting $x = 1$ only option (b) satisfying.

137. **(b)**



$AC = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\sin A + \cot C$

$\frac{BC}{AC} + \frac{BC}{AB}$

$\frac{1}{\sqrt{5}} + \frac{1}{2} \Rightarrow \frac{2+\sqrt{5}}{2\sqrt{5}}$

138.(b) $\sin\frac{\pi x}{2} = x^2 - 2x + 2$

put value of x from options

$x = 1$

$\sin\frac{\pi}{2} \times 1 = 1^2 - 2\times 1 + 2$

$\sin 90° = 1 - 2 + 2$

$1 = 1$ (satisfied)

139. **(a)** $\frac{\sin 43°}{\cos 47°} + \frac{\cos 19°}{\sin 71°} - 8\cos^2 60°$

$= 1 + 1 - \left(8 \times \left(\frac{1}{2}\right)^2\right)$

(If A + B = 90°, then sinA = cosB)

$\frac{SinA}{CosB} = 1$ or $\frac{\cos B}{\sin A} = 1$

$= 2 - 2 = 0$

140. **(a)** $\text{Sin}^2\ 7\frac{1}{2}° + \sin^2 82\frac{1}{2}°$

$= \sin^2 7\frac{1}{2}° + \sin^2\left(90° - 7\frac{1}{2}°\right)$

$= \sin^2 7\frac{1}{2}° + \cos^2\ 7\frac{1}{2}°$

$= 1$ ($\because \sin^2\theta + \cos^2\theta = 1$)

141. **(c)** Given that, $3\sin x + 5\cos x = 5$

On squaring both sides, we get

$9\sin^2 x + 25\cos^2 x + 30\sin x\cos x = 25$

$\Rightarrow$ $9(1 - \cos^2 x) + 25(1 - \sin^2 x) + 30\sin x\cos x = 25$

$\Rightarrow$ $9 + 25 - (9\cos^2 x + 25\sin^2 x - 30\sin x \cos x) = 25$

$\Rightarrow$ $9 = (3\cos x - 5\sin x)^2$

$\Rightarrow$ $3\cos x - 5\sin x = 3$

142. **(a)** If $\alpha + \beta = 90°$

Put $\alpha = 45°$

$\beta = 45°$

$\sqrt{(\text{cosec}\,\alpha\ \text{cosec}\,\beta)} \times \left(\frac{\sin\alpha}{\sin\beta} + \frac{\cos\alpha}{\cos\beta}\right)^{-1/2}$

$\Rightarrow \sqrt{\sqrt{2}\times\sqrt{2}} \times \left(\frac{1/\sqrt{2}}{1/\sqrt{2}} + \frac{1/\sqrt{2}}{1/\sqrt{2}}\right)^{-1/2}$

$\Rightarrow \sqrt{2} \times (2)^{-\frac{1}{2}}$

$\Rightarrow \sqrt{2} \times \frac{1}{\sqrt{2}} = 1$

143. **(d)** We know that, in a cyclic quadrilateral sum of opposite angle is 180°

$\therefore$ A + C = 180° ....(i)

and B + D = 180° ....(ii)

$\therefore$ cosA + cosB + cosC + cosD

= cosA + cosB + cos(180° − A) + cos(180° − B)

From Eqs. (i) and (ii),

= cosA + cosB − cosA − cosB = 0

144. **(a)** Given, $\alpha + \beta = 90°$ ....(i)

By given condition,

$\beta = \frac{2}{3}\alpha$

$\therefore$ $\beta = \frac{2}{3}\alpha = \frac{2}{3}(90° - \beta)$

{from Eq. (i)}

$\Rightarrow$ $\beta = 60° - \frac{2}{3}\beta \Rightarrow \beta = 36°$

145. **(b)** Given, $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2$

$\therefore$ $\sin^2\theta + \cos^2\theta = 2\sin\theta\cos\theta$

$\Rightarrow \sin 2\theta = 1 = \sin 90°$

$\Rightarrow 2\theta = 90° \Rightarrow \theta = 45°$

146. **(b)** Given, $\sin 3\theta = \cos(\theta - 2°)$

$\Rightarrow \sin 3\theta = \sin[90° - (\theta - 2°)]$

$\Rightarrow 3\theta = 90° - \theta + 2°$

$\Rightarrow 4\theta = 92° \Rightarrow \theta = \frac{92°}{4} = 23°$

147. **(b)** $\frac{\sin^6\theta - \cos^6\theta}{\sin^2\theta - \cos^2\theta}$

$= \frac{(\sin^2\theta)^3 - (\cos^2\theta)^3}{\sin^2\theta - \cos^2\theta}$

$= \frac{(\sin^2\theta - \cos^2\theta)(\sin^4\theta + \cos^4\theta + \sin^2\theta\cos^2\theta)}{\sin^2\theta - \cos^2\theta}$

$= \sin^4\theta + \cos^4\theta + 2\sin^2\theta.\cos^2\theta - \sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta$

$= 1 - \sin^2\theta\cos^2\theta$

148. **(c)** $\tan A = \frac{1 - \cos B}{\sin B}$

Put A = 30°, B = 60°

$$\tan 30° = \frac{1-\cos 60°}{\sin 60°}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ (Satisfy)}$$

$$\therefore \frac{2\tan A}{1-\tan^2 A} = \frac{2\tan 30°}{1-\tan^2 30°}$$

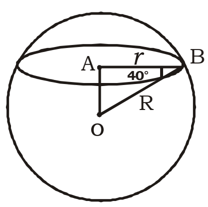
$$= \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

Now check the option,

Option (c):- tanB

$\tan 60° = \sqrt{3}$

149. (a)



In Δ OAB,

$$\cos 40° = \frac{AB}{OB} \Rightarrow \cos 40° = \frac{r}{R}$$

$\Rightarrow r = R\cos 40°$

So, the radius of the circle of latitude 40° S is R cos 40°.

150. (a) We know that, If value of $\cos\theta$ increases, then the value of θ decreases.

$$\because \cos\theta \geq \frac{1}{2}$$

$$\therefore \cos\theta \geq \cos 60° \Rightarrow \theta \leq \frac{\pi}{3}$$

151. (b) Given, $\sin\theta + \cos\theta = 1$

On squaring both sides, we get

$(\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta = 1$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 1$

$\Rightarrow \sin\theta.\cos\theta = 0$

152. (b) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}}$

$$= \sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$= \sec\theta + \tan\theta$

153. (b) $\because \cos\theta = \dfrac{a^2+b^2-C^2}{2ab}$

(By cosine rule)

$$= \frac{6^2+2^2-C^2}{2\times6\times2} = \frac{40-c^2}{24}$$

For acute angle,

$$\cos\theta > 0 \Rightarrow \frac{40-c^2}{24} > 0 \Rightarrow C^2 < 40$$

$\Rightarrow 0 < c < 2\sqrt{10}$

(since, C cannot be negative) ....(i)

Also, $b + c > ac > 6 - 2 \Rightarrow c > 4$

From Eqs. (i) and (ii), $c \in (4, 2\sqrt{10})$

154. (d) We know that, the value of cos θ is decreasing from 0° to 90°.

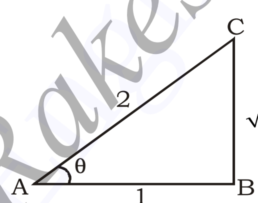
$\therefore$ $\cos 1° > \cos 89°$

$\Rightarrow$ $p > q$

Also, cos 1° is close to 1 and cos 89° is close to 0.

Hence, option (d) is correct.

155. (d)



Given, $7\cos^2\theta + 3\sin^2\theta = 4$

$\Rightarrow 7(1-\sin^2\theta) + 3(\sin^2\theta) = 4$

$\Rightarrow 7 - 4\sin^2\theta = 4$

$\Rightarrow 4\sin^2\theta = 3$

$$\Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$$

$$\text{For, } 0 < \theta < \frac{\pi}{2}$$

$$\sin\theta = \frac{\sqrt{3}}{2}, \theta = 60°$$

$$\therefore \tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$$

156. (b) $[(1-\sin^2\theta)\sec^2\theta + \tan^2\theta]$ $(\cos^2\theta + 1)$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= [\cos^2\theta.\sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$

$(\because \cos^2\theta.\sec^2\theta = 1)$

$= (1 + \tan^2\theta)(\cos^2 + 1)$

$(\because \sec^2\theta - \tan^2\theta = 1)$

$= \sec^2\theta(\cos^2\theta + 1)$

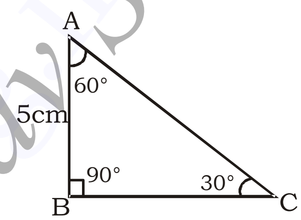
$= \sec^2\theta.\cos^2\theta + \sec^2\theta$

$= 1 + \sec^2\theta > 1 + 1 > 2$

$(\because \sec^2\theta > 1 \text{ for } 0 < \theta < 90°)$

157. (d) We know in the interval $\theta \in \left[0, \frac{\pi}{2}\right]$, $\sec^2\theta$ is increasing from 1 to ∞ $\therefore p \geq 1$

158. (a) In ΔABC,



$$\cos 60° = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{5}{AC}$$

$\therefore$ AC = 10cm

159. (a) Given, $\cos\theta + \sqrt{3}\sin\theta = 2$

$$\Rightarrow \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = 1$$

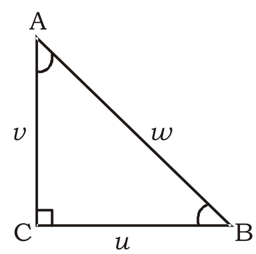
$\Rightarrow \sin 30°\cos\theta + \cos 30°\sin\theta = 1$

$\Rightarrow \sin(30°+\theta) = \sin 90°$

$\Rightarrow 30° + \theta = 90°$

$$\therefore \theta = 60° = \frac{\pi}{3}$$

160. (d) In ΔABC,



$$\tan A = \frac{BC}{AC} = \frac{u}{v}$$

$$\text{and} \tan B = \frac{v}{u}$$

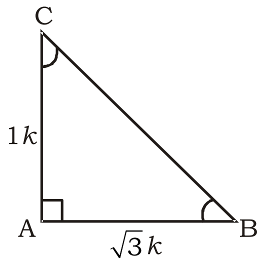
Also, $u^2 + v^2 = w^2$ .....(i)

(by Pythagoras theorem)

$$\therefore \tan A + \tan B = \frac{u}{v} + \frac{v}{u} = \frac{u^2+v^2}{uv}$$

$$= \frac{w^2}{uv} \quad \text{[from Eq. (i)]}$$

161. (b) Given,



$\tan B = \dfrac{k}{\sqrt{3}k}$

In ΔABC,

$AB^2 + AC^2 = BC^2$ ......(i)

(by Pythagoras theorem)

$\Rightarrow (\sqrt{3}k)^2 + (1k)^2 = BC^2$

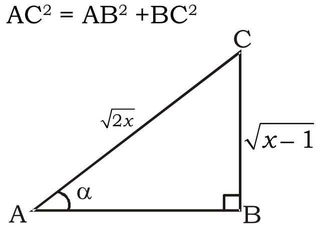
$\Rightarrow BC^2 = 4k^2 \Rightarrow BC = 2k$

162. (a) Given,

$\sin\alpha = \sqrt{\dfrac{x-1}{2x}}$

In ABC, using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$



$\Rightarrow 2x = AB^2 + (x-1)$

$\Rightarrow AB^2 = x + 1 \Rightarrow AB = \sqrt{x+1}$

$\therefore \tan\alpha = \dfrac{BC}{AB} = \dfrac{\sqrt{x-1}}{\sqrt{x+1}} = \sqrt{\dfrac{x-1}{x+1}}$

163. (c) Given,

$\dfrac{\cos\theta}{1-\sin\theta} - \dfrac{\cos\theta}{1+\sin\theta} = 2$

$\Rightarrow \dfrac{\cos\theta + \sin\theta\cos\theta - \cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta} = 2$

$(\because 1 - \sin^2\theta = \cos^2\theta)$

$\Rightarrow 2\sin\theta\cos\theta = 2\cos^2\theta$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = \dfrac{\pi}{4}$

164. (b) As we know that, $\sin x$ is increasing from 0° to 90°.

$\therefore$ $\sin y > \sin x$.

165. (b) $\sin^3 60^\circ . \cot 30^\circ - 2\sec^2 45^\circ + 3\cos 60^\circ . \tan^2 45^\circ - \tan^2 60^\circ$

$= \left(\dfrac{\sqrt{3}}{2}\right)^3 \times \sqrt{3} - 2\left(\sqrt{2}\right)^2 + 3 \times \left(\dfrac{1}{2}\right) \times (1)^2 - \left(\sqrt{3}\right)^2$

$= \dfrac{9}{8} - 4 + \dfrac{3}{2} - 3 = \dfrac{-35}{8}$

166. (c) $\tan\theta = \dfrac{p}{q}$

$= \dfrac{p\sec\theta - q\,\text{cosec}\theta}{p\sec\theta + q\,\text{cosec}\theta}$

$= \dfrac{\text{cosec}\theta\left(\dfrac{p\sec\theta}{\text{cosec}\theta} - q\right)}{\text{cosec}\theta\left(\dfrac{p\sec\theta}{\text{cosec}} + q\right)}$

$= \dfrac{p\tan\theta - q}{p\tan\theta + q}$

$= \dfrac{p \times \dfrac{p}{q} - q}{p \times \dfrac{p}{q} + q}$

$= \dfrac{p^2 - q^2}{p^2 + q^2}$

167. (b) $\text{cosec}^2\theta - 2 + \sin^2\theta$

$= (\sin\theta - \text{cosec}\theta)^2$

Hence, it is always non-negative.

168. (b) $1 - 2\sin^2\theta + \sin^4\theta$

$\Rightarrow (1 - \sin^2\theta)^2$

$\Rightarrow (\cos^2\theta)^2$

$\Rightarrow \cos^4\theta$

169. (c) $\text{Sin}\theta = 0.7$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow (0.7)^2 + \cos^2\theta = 1$

$\Rightarrow 0.49 + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - 0.49$

$\Rightarrow \cos\theta = \sqrt{0.51}$

170. (c) $\text{Sin}^2 65^\circ + \sin^2 25^\circ + \cos^2 35^\circ + \cos^2 55^\circ$

$= \sin^2 65^\circ + \sin^2(90^\circ - 65^\circ) + [\cos^2 35^\circ + \cos^2(90^\circ - 35^\circ)]$

$= (\sin^2 65^\circ + \cos^2 65^\circ) + (\cos^2 35^\circ + \sin^2 35^\circ)$

$= 1 + 1 = 2$

171. (a) $\cos^2\theta - \sin^2\theta = \dfrac{1}{3}$ (given)

$\cos^2\theta + \sin^2\theta = 1$ (property)

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= \dfrac{1}{3} \times 1$

$\cos^4\theta - \sin^4\theta = \dfrac{1}{3}$

$(\because (a^2 - b^2)(a^2 + b^2) = a^4 - b^4)$

172. (c) $\tan\theta = \dfrac{1}{\sqrt{11}}$

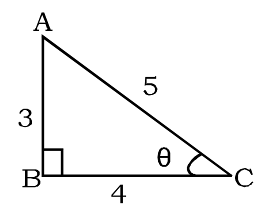
$\dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$

$= \dfrac{\dfrac{1}{\sin^2\theta} - \dfrac{1}{\cos^2\theta}}{\dfrac{1}{\sin^2\theta} + \dfrac{1}{\cos^2\theta}} \Rightarrow \dfrac{\dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta.\cos^2\theta}}{\dfrac{\cos^2\theta + \sin^2\theta}{\sin^2\theta.\cos^2\theta}}$

$\Rightarrow \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} \Rightarrow \dfrac{\cos^2\theta\left(1 - \dfrac{\sin^2\theta}{\cos^2\theta}\right)}{\cos^2\theta\left(1 + \dfrac{\sin^2\theta}{\cos^2\theta}\right)}$

$\Rightarrow \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta} \Rightarrow \dfrac{1 - \dfrac{1}{11}}{1 + \dfrac{1}{11}} = \dfrac{5}{6}$

173. (b) $\sin\theta = \dfrac{3}{5}$



$\sin\theta = \dfrac{3}{5} = \dfrac{P}{H}$

So, B = 4

P = 3

H = 5

$\Rightarrow \dfrac{\tan\theta + \cos\theta}{\cot\theta + \text{cosec}\theta} = \dfrac{\dfrac{P}{B} + \dfrac{B}{H}}{\dfrac{B}{P} + \dfrac{H}{P}}$

$\Rightarrow \dfrac{\dfrac{3}{4} + \dfrac{4}{5}}{\dfrac{4}{3} + \dfrac{5}{3}} = \dfrac{\dfrac{15+16}{20}}{\dfrac{4+5}{3}} = \dfrac{\dfrac{31}{20}}{\dfrac{9}{3}}$

$= \dfrac{31}{60}$

174. (b) $(\sin\alpha + \text{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$

put $\alpha = 45^\circ$,

$\Rightarrow$ $(\sin 45° + \text{cosec}45°)^2 + (\cos 45° + \sec 45°)^2 = K + \tan^2 45° + \cot^2 45°$

$\Rightarrow$ $\left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2$

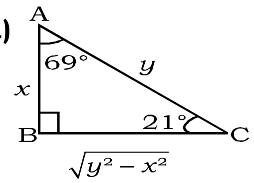
$= k + 1 + 1$

$\Rightarrow$ $\frac{1}{2} + 2 + \left(2\sqrt{2} \times \frac{1}{\sqrt{2}}\right) + \frac{1}{2} + 2 + \left(2\sqrt{2} \times \frac{1}{\sqrt{2}}\right)$

$= k + 2$

$\Rightarrow$ $4\frac{1}{2} + 4\frac{1}{2} = k + 2 \Rightarrow k = 7$

175. **(a)**



In ΔABC, $\sin 21° = \frac{x}{y}$

AB = x
AC = y
$BC = \sqrt{y^2 - x^2}$

$\Rightarrow$ $\sec 21° - \sin 69°$

$\Rightarrow$ $\frac{AC}{BC} - \frac{BC}{AC}$

$\Rightarrow$ $\frac{(AC)^2 - (BC)^2}{(BC)(AC)} = \frac{y^2 - \left(\sqrt{(y^2 - x^2)}\right)^2}{y\sqrt{y^2 - x^2}}$

$\Rightarrow$ $\frac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$

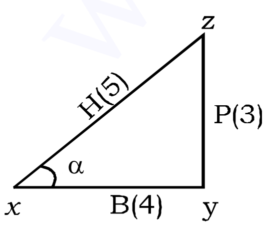
176. **(c)** $\sec\alpha + \tan\alpha = 2$ ....(i)

$\sec\alpha - \tan\alpha = \frac{1}{2}$ ....(ii)

On adding equation (i) and (ii)

$2\sec\alpha = 2 + \frac{1}{2}$

$\sec\alpha = \frac{5 \to H}{4 \to B}$

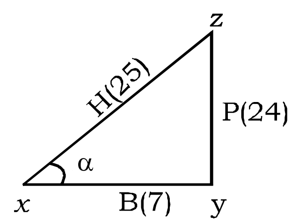


$\sin\alpha = \frac{3}{5} = 0.6$

177. **(d)** $7\sin\alpha = 24\cos\alpha$

$\frac{\sin\alpha}{\cos\alpha} = \frac{24}{7}$

$\Rightarrow$ $\tan\alpha = \frac{24 \to P}{7 \to B}$

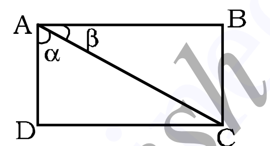


$14\tan\alpha - 75\cos\alpha - 7\sec\alpha$

$= 14 \times \frac{24}{7} - 75 \times \frac{7}{25} - 7 \times \frac{25}{7}$

$= 48 - 21 - 25 = 2$

178. **(c)**



$= (\tan^2\alpha + 1)\sin^2\beta$

$= (\tan^2 45° + 1)\sin^2 45°$

$= (1 + 1)\left(\frac{1}{\sqrt{2}}\right)^2 = 2 \times \frac{1}{2} = 1$

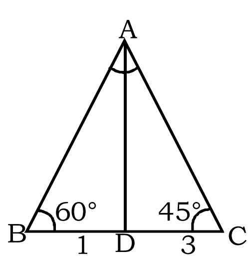
179. **(c)** $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} = \sqrt{\frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}}$

$= \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \sqrt{\frac{(1 - \cos\theta)^2}{\sin^2\theta}}$

$= \frac{1 - \cos\theta}{\sin\theta} = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$= \text{cosec}\theta - \cot\theta$

180. **(c)**



$\angle B = \frac{\pi}{3} = 60°,\ \angle C = \frac{\pi}{4} = 45°$

In ΔABD,

$\Rightarrow$ $\frac{BD}{\sin\angle BAD} = \frac{AD}{\sin 60°}$ (By sin rule)

$\Rightarrow$ $\frac{BD}{\sin\angle BAD} = \frac{AD}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ $AD = \frac{\sqrt{3}}{2} \cdot \frac{BD}{\sin\angle BAD}$ ....(i)

from ΔADC

$\frac{CD}{\sin\angle DAC} = \frac{AD}{\sin 45°} \Rightarrow AD$

$= \frac{1}{\sqrt{2}} \cdot \frac{CD}{\sin\angle DAC}$ ....(ii)

from equation (i) and (ii)

$\frac{\sqrt{3}}{2} \cdot \frac{BD}{\sin\angle BAD} = \frac{1}{\sqrt{2}} \cdot \frac{CD}{\sin\angle DAC}$

$\Rightarrow$ $\frac{\sin\angle BAD}{\sin\angle DAC} = \frac{\sqrt{3}}{2} \times \sqrt{2} \times \frac{1}{3}$

$= \frac{1}{\sqrt{3} \times \sqrt{2}} = \frac{1}{\sqrt{6}}$

181. **(a)** $\sin 3A = \cos(A - 26°)$

$\Rightarrow \sin 3A = \sin[90° - (A - 26°)]$

$\Rightarrow 3A = 90° - A + 26°$

$\Rightarrow 4A = 116°$

$\Rightarrow A = 29°$

182. **(a)** $\sec^2\theta - \frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta}$

$= \sec^2\theta - \frac{\sin^2\theta(1 - 2\sin^2\theta)}{\cos^2\theta(2\cos^2\theta - 1)}$

$[\because \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta]$

$= \sec^2\theta - \tan^2\theta = 1$

Alternate:-

$\sec^2\theta - \frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta}$

Put $\theta = 0°$

$\sec^2 0° - \frac{\sin^2 0° - 2\sin^4 0°}{2\cos^4 0° - \cos^2 0°}$

$= 1 - \frac{0 - 0}{2 - 1}$

$= 1 - 0 = 1$

183. **(c)** $x = a(\sin\theta + \cos\theta)$
$y = b(\sin\theta - \cos\theta)$

$\Rightarrow \frac{x}{a} = (\sin\theta + \cos\theta)$

$\Rightarrow \frac{x^2}{a^2} = (\sin\theta + \cos\theta)^2$ ....(i)

$\Rightarrow \frac{y}{b} = (\sin\theta - \cos\theta)$

$\Rightarrow \frac{y^2}{b^2} = (\sin\theta - \cos\theta)^2$ ....(ii)

On adding equation (i) and (ii)

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta.\cos\theta$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2(\sin^2\theta + \cos^2\theta)$

$= 2 \times 1 = 2$

184. **(d)** $\sin 5\theta = \cos 20°$

$\Rightarrow 5\theta + 20° = 90°$

(If $\sin A = \cos B$ then $A + B = 90°$)

$\Rightarrow 5\theta = 70°$

$\Rightarrow \theta = 14°$

185. **(c)** $2\sec\theta = 3\,\text{cosec}^2\theta$

$\frac{2}{\cos\theta} = \frac{3}{\sin^2\theta} = \frac{3}{1-\cos^2\theta}$

$\Rightarrow 2 - 2\cos^2\theta = 3\cos\theta$

$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\Rightarrow 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$\Rightarrow 2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$

$\Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$

$\therefore 2\cos\theta - 1 = 0$ or $\cos\theta + 2 \neq 0$

$\Rightarrow \cos\theta = \frac{1}{2} = \cos 60°$ or $\cos\frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{3}$

186. **(d)** $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \frac{\left(\sqrt{1+\sin\theta}\right)^2 + \left(\sqrt{1-\sin\theta}\right)^2}{\sqrt{1-\sin^2\theta}}$

$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$

$= \frac{2}{\cos\theta} = 2\sec\theta$

Alternate:-

put $\theta = 30°$,

$= \sqrt{\frac{1+\sin 30°}{1-\sin 30°}} + \sqrt{\frac{1-\sin 30°}{1+\sin 30°}}$

$= \sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}} + \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}}$

$= \sqrt{\frac{3}{1}} + \sqrt{\frac{1}{3}} = \frac{4}{\sqrt{3}}$

Now check with option by putting $\theta = 30°$,

$2\sec 30° = \frac{2\times 2}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

187. **(d)** $\sin(4\alpha - \beta) = 1 = \sin 90°$

$\cos(2\alpha + \beta) = \frac{1}{2} = \cos 60°$

$4\alpha - \beta = 90°$

$2\alpha + \beta = 60°$

adding $6\alpha = 150°$

$\alpha = 25°$

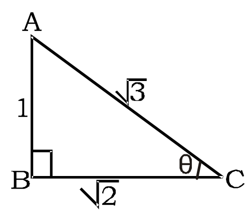
$\Rightarrow \beta = 10°$

$\Rightarrow \sin(\alpha + 2\beta)$

$\Rightarrow \sin(25° + 2\times 10°)$

$\Rightarrow \sin 45° = \frac{1}{\sqrt{2}}$

188. **(c)**



$\text{cosec}\theta = \sqrt{3}$

$\sin\theta = \frac{1}{\sqrt{3}} \begin{matrix}\to P \\ \to H\end{matrix}$

$\Rightarrow \cot\theta - \text{cosec}\theta$

$\Rightarrow \frac{\sqrt{2}}{1} - \frac{\sqrt{3}}{1}$

$\Rightarrow \sqrt{2} - \sqrt{3}$

189. **(a)** $(r\cos\theta - \sqrt{3})^2 + (r\sin\theta - 1)^2 = 0$

$(r\cos\theta - \sqrt{3})^2 = 0$, $(r\sin\theta - 1)^2 =$

Or $\cos\theta = \sqrt{3}$ ......(i)

$r\sin\theta = 1$ ......(ii)

squaring and adding equation (i) and (ii)

$r^2\cos^2\theta + r^2\sin^2\theta = 3 + 1$

$r^2(\cos^2\theta + \sin^2\theta) = 4$

$r^2 = 4$

$r = 2$

$\tan\theta = \frac{r\sin\theta}{r\cos\theta} = \frac{1}{\sqrt{3}}$ and $r\cos\theta = \sqrt{3}$

$\cos\theta = \frac{\sqrt{3}}{r}$ $\sec\theta = \frac{r}{\sqrt{3}}$

$\frac{r\tan\theta + \sec\theta}{r\sec\theta + \tan\theta} = \frac{\frac{r}{\sqrt{3}} + \frac{r}{\sqrt{3}}}{\frac{r^2}{\sqrt{3}} + \frac{1}{\sqrt{3}}}$

$= \frac{r\left(\frac{2}{\sqrt{3}}\right)}{\frac{r^2+1}{\sqrt{3}}} = \frac{2r}{r^2+1} = \frac{2\times 2}{2^2+1} = \mathbf{\frac{4}{5}}$

Alternate:-

$r = 2$

$\tan\theta = \frac{r\sin\theta}{r\cos\theta} = \frac{1}{\sqrt{3}}$

$\theta = 30°$

$= \frac{2\tan 30° + \sec 30°}{2\sec 30° + \tan 30°}$

$= \frac{2\times\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}}{2\times\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{4}{5}$

190. **(c)** $\frac{\sin\alpha}{\sin\beta} = b \Rightarrow \sin\alpha = b\sin\beta$

$\frac{\cos\alpha}{\cos\beta} = a \Rightarrow \frac{\cos^2\alpha}{\cos^2\beta} = a^2$

$\Rightarrow \frac{1-\sin^2\alpha}{1-\sin^2\beta} = a^2$

$\Rightarrow 1 - \sin^2\alpha = a^2(1 - \sin^2\beta)$

$\Rightarrow 1 - b^2\sin^2\beta = a^2 - a^2\sin^2\beta$

[value put in $\sin\alpha$]

$\Rightarrow 1 - a^2 = b^2 \sin^2\beta - a^2 \sin^2\beta$

$\Rightarrow 1 - a^2 = (b^2 - a^2) \sin^2\beta$

$\Rightarrow \sin^2\beta = \dfrac{1-a^2}{b^2-a^2} = \dfrac{a^2-1}{a^2-b^2}$

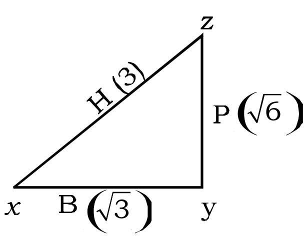
191. (c) $\sqrt{3}\tan\theta = 3\sin\theta$

$\sqrt{3}\dfrac{\sin\theta}{\cos\theta} = 3\sin\theta$

$\dfrac{\sqrt{3}}{\cos\theta} = 3$

$\cos\theta = \dfrac{\sqrt{3}}{3}$

then perpendicular = $\sqrt{6}$



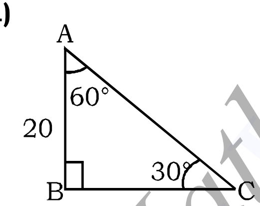
$(\sin^2\theta - \cos^2\theta)$

$\Rightarrow \left(\dfrac{P}{H}\right)^2 - \left(\dfrac{B}{H}\right)^2$

$\Rightarrow \left(\dfrac{\sqrt{6}}{3}\right)^2 - \left(\dfrac{\sqrt{3}}{3}\right)^2$

$\Rightarrow \dfrac{6}{9} - \dfrac{3}{9} = \dfrac{1}{3}$

192. (d)



AB = 20 cm

BC : CA = ?

$\Rightarrow \dfrac{BC}{CA} = \cos C$

$\Rightarrow \dfrac{BC}{CA} = \cos 30°$

$(\angle C = 180° - 90° - 60° \Rightarrow 30°)$

$\Rightarrow \dfrac{BC}{CA} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \sqrt{3} : 2$

193. (c) $\tan(A + B) = \sqrt{3} = \tan 60°$

$\tan(A - B) = \dfrac{1}{\sqrt{3}} = \tan 30°$

$\Rightarrow (A + B) = 60°$ ......(i)

$(A - B) = 30°$ .....(ii)

Adding both equation

$2A = 90°$

$A = \dfrac{90°}{2}$

$A = 45°$

$B = 15°$

194. (c) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$\Rightarrow \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} \times \dfrac{(1-2(1-\cos^2\theta))}{2\cos^2\theta - 1}$

$\Rightarrow \dfrac{\tan\theta(1+2\cos^2\theta-2)}{2\cos^2\theta-1}$

$\Rightarrow \dfrac{\tan\theta(2\cos^2\theta-1)}{(2\cos^2\theta-1)}$

$\Rightarrow \tan\theta$

Alternate:-

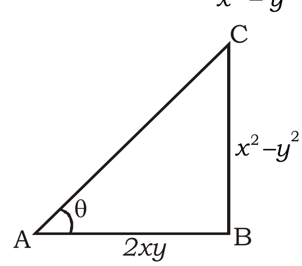
$\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$\Rightarrow \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$

$(1 - 2\sin^2\theta = 2\cos^2\theta - 1 = \cos 2\theta = \cos^2\theta - \sin^2\theta)$

$\Rightarrow \tan\theta$

195. (c) Given, $\cot\theta = \dfrac{2xy}{x^2-y^2}$



In ΔABC,

$AC^2 = (x^2 - y^2) + (2xy)^2$

$\Rightarrow AC^2 = (x^2 + y^2)^2 \Rightarrow AC = x^2 + y^2$

$\therefore \cos\theta = \dfrac{AB}{AC} = \dfrac{2xy}{x^2+y^2}$

196. (a) Given, $(\sin\theta + \operatorname{cosec}\theta) = 2.5$

$\Rightarrow \left(\sin\theta + \dfrac{1}{\sin\theta}\right) = \dfrac{5}{2}$

$\Rightarrow 2\sin^2\theta - 5\sin\theta + 2 = 0$

$\Rightarrow 2\sin^2\theta - 4\sin\theta - \sin\theta + 2 = 0$

$\Rightarrow 2\sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = \dfrac{1}{2}$ $(\because \sin\theta \neq 2)$

$\therefore \theta = 30°$

Alternate:-.

$\sin\theta + \operatorname{cosec}\theta = 2.5$

Put $\theta = 30°$

$\sin 30° + \operatorname{cosec} 30° = 2.5$

$\Rightarrow \dfrac{1}{2} + 2 = 2.5$

$\Rightarrow 2.5 = 2.5$ (Satisfy)

$\therefore \theta = 30°$

197. (d) Given, $x\cos 60° + y\cos 0° = 3$

$\Rightarrow \dfrac{x}{2} + y = 3$

$\Rightarrow x + 2y = 6$ ......(i)

$\Rightarrow$ and $4x\sin 30° - y\cot 45° = 2$

$\Rightarrow 4x \times \dfrac{1}{2} - y.1 = 2$

$\Rightarrow 2x - y = 2$ ......(ii)

On solving Eqs. (i) and (ii),

we get $x = y = 2$

198. (a) $\log(\tan 1°) + \log(\tan 2°) + ...$

$...+ \log(\tan 89°)$

$= \log(\tan 1° \tan 2° ...\tan 45° ..\tan 88° \tan 89°)$

$[\therefore \tan 89° = \tan(90° - 1) = \cot 1°]$

$= \log[\tan 1° \cot 1°)(\tan 2° \cot 2°) ....\tan ....\tan 45°]$

$= \log(1°.1°...1°) = 0$

199. (c) Now, $(\sin x - \cos x)^2 = (\sin^2 x + \cos^2 x) - 2\sin x\cos x$

$= 1 - 2\left(\dfrac{1}{2}\right)$

$\left(\because \sin x\cos x = \dfrac{1}{2}, \text{given}\right) = 0$

Alternate:-

$\sin x\cos x = \dfrac{1}{2}$

Put $x = 45°$

$\sin 45° \cos 45° = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

$\Rightarrow \frac{1}{2} = \frac{1}{2}$ (Satisfy)

$\therefore$ $\sin x - \cos x$

$\sin 45° - \cos 45°$

$\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

200. (a) $\tan^2 y \operatorname{cosec}^2 x - 1 = \tan^2 y$

Put $x = y = 45°$

$\tan^2 45° \operatorname{cosec}^2 45° - 1 = \tan^2 45°$

$\Rightarrow 1 \times (\sqrt{2})^2 - 1 = (1)^2$

$\Rightarrow 2 - 1 = 1$

$\Rightarrow 1 = 1$ (Satisfy)

$\therefore x = y$

$x - y = 0$

201. (c) Given,

$\frac{\cos x}{1+\operatorname{cosec} x} + \frac{\cos x}{\operatorname{cosec} x - 1} = 2$

$\Rightarrow \frac{2\cos x \operatorname{cosec} x}{\operatorname{cosec}^2 x - 1} = 2$

$\Rightarrow \frac{\cos x \operatorname{cosec} x}{\cot^2 x} = 1$

$\Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

202. (c) Given, $\sin x : \sin y = \sqrt{3} : 1$

$= \frac{\sqrt{3}}{2} : \frac{1}{2} = \sin 60° : \sin 30°$

$\therefore x : y = 60 : 30$

$\Rightarrow x : y = 2 : 1$

203. (a) Given, $\frac{\cos x}{\cos y} = n$ ......(i)

$\frac{\sin x}{\sin y} = m$ ...... (ii)

Now, $(m^2 - n^2)\sin^2 y$

$= \left(\frac{\sin^2 x}{\sin^2 y} - \frac{\cos^2 x}{\cos^2 y}\right)\sin^2 y$

$= \frac{(1-\cos^2 x)\cos^2 y - \cos^2 x(1-\cos^2 y)}{\cos^2 y}$

$= \frac{\cos^2 y - \cos^2 x}{\cos^2 y} = 1 - n^2$ [from Eq. (i)]

Alternate:-

$\frac{\cos x}{\cos y} = n$ and $\frac{\sin x}{\sin y} = m$

Put x = y = 45°

$\frac{\cos 45°}{\cos 45°} = n$ and $\frac{\sin 45°}{\sin 45°} = m$

$n = 1$ $\quad m = 1$

$\therefore (m^2 - n^2)\sin^2 y = (1^2 - 1^2)\sin^2 45° = 0$

Now check from options

Option (a):- $1 - n^2$

$\Rightarrow 1 - (1)^2$

$\Rightarrow 0$ (Satisfy)

204. (b) Given, $p = \tan^2 x + \cot^2 x$

$= (\tan x + \cot x)^2 - 2$

$= \left(\frac{\sin^2 x + \cos^2 x}{\sin x . \cos x}\right)^2 - 2$

$= \left(\frac{2}{\sin 2x}\right)^2 - 2 = \frac{4}{\sin^2 2x} - 2$

Since, the maximum value of $\sin 2x$ is 1.

$\therefore p_{min} = \frac{4}{1} - 2 = 2$

$\therefore p \geq 2$

Hence, $p \geq 2$

Alternate:-

$P = \tan^2 x + \cot^2 x$

Put $x = 45°$

$P = \tan^2 45° + \cot^2 45°$

$P = 1 + 1$

$P = 2$

Put $x = 30°$

$P = \tan^2 30° + \cot^2 30°$

$P = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$

$P = 0.33 + 2.99$

$P = 3.32$

$\therefore P \geq 2$

205. (b)

$\frac{5\sin 75° \sin 77° + 2\cos 13° \cos 15°}{\cos 15° \sin 77°}$

$- \frac{7\sin 81°}{\cos 9°}$

$= \frac{5\cos 15° \sin 77° + 2\sin 77° \cos 15°}{\cos 15° \sin 77°}$

$- \frac{7\cos 9°}{\cos 9°}$

$= \frac{7\cos 15° . \sin 77°}{\cos 15° . \sin 77°} - \frac{7\cos 9°}{\cos 9°}$

$= 7 - 7 = 0$

206. (d) Given, $\sin x + \sin y = a$ and $\cos x + \cos y = b$

on squaring both sides, we get

$\sin^2 x + \sin^2 y + 2\sin x \sin y = a^2$...... (i)

$\cos^2 x + \cos^2 y + 2\cos x \cos y = b^2$...(ii)

On adding Eqs. (i) and (ii), we get

$(\sin^2 x + \cos^2 x) + (\sin^2 y + \cos^2 y) + 2(\sin x \sin y + \cos x \cos y) = a^2 + b^2$

$\Rightarrow$ $(\sin x \sin y + \cos x \cos y)$

$= \frac{a^2 + b^2 - 2}{2}$

Alternate:-

$\sin x + \sin y = a$

$\cos x + \cos y = b$

Put x = y = 45°

$\sin 45° + \sin 45° = a$

$a = \sqrt{2}$

$b = \sqrt{2}$

$\therefore \sin x \sin y + \cos x \cos y = \frac{1}{2} + \frac{1}{2} = 1$

Now check from option,

Option: (d) $\frac{a^2 + b^2 - 2}{2}$

$\Rightarrow \frac{(\sqrt{2})^2 + (\sqrt{2})^2 - 2}{2}$

$\Rightarrow \frac{2+2-2}{2}$

$\Rightarrow \frac{2}{2} = 1$ (Satisfy)

207. (a) Given, $\operatorname{cosec}^4 \alpha - \cot^4 \alpha = 17$

$\Rightarrow (\operatorname{cosec}^2 \alpha - \cot^2 \alpha)(\operatorname{cosec}^2 \alpha + \cot^2 \alpha) = 17$

$\Rightarrow 1\left(\frac{1+\cos^2 \alpha}{\sin^2 \alpha}\right) = 17$

$(\because \operatorname{cosec}^2 \alpha - \cot^2 \alpha = 1)$

$\Rightarrow 2 - \sin^2 \alpha = 17\sin^2 \alpha$

$\Rightarrow 18\sin^2 \alpha = 2 \Rightarrow \sin^2 \alpha = \frac{1}{9}$

$\therefore \sin \alpha = \frac{1}{3}$

(since, α lie in first quadrant)

208. (c) Given, $x + \left(\frac{1}{x}\right) = 2\cos \alpha$

On squaring both sides, we get

$x^2 + \frac{1}{x^2} + 2 = 4\cos^2 \alpha$

$\Rightarrow x^2+\frac{1}{x^2} = 2\,(2\cos^2\alpha - 1)$

$= 2(2\cos^2\alpha - \sin^2\alpha - \cos^2\alpha)$

$= 2\cos^2\alpha\ - 2\sin^2\alpha$

209. **(b)** $\cot^2\theta - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta} = \frac{\cos^2\theta - 1}{\sin^2\theta}$

$= \frac{-(1-\cos^2\theta)}{\sin^2\theta} = \frac{-\sin^2\theta}{\sin^2\theta} = -1$

$(\because\ 1-\cos^2\theta\ = \sin^2\theta)$

Alternate:-

$\cot^2\theta - \frac{1}{\sin^2\theta}$

Put $\theta = 45°$

$\cot^2 45° - \frac{1}{\sin^2 45°}$

$\Rightarrow\ 1 - \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}$

$\Rightarrow\ 1 - 2 = -1$

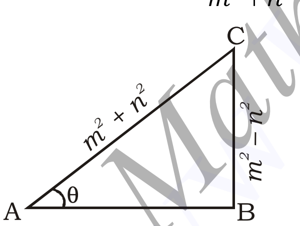
210. (c) Since, $\sin x = \cos y$

As x and y are acute angles, then

$x = y = \frac{\pi}{4}$

$\therefore\quad x + y = \frac{\pi}{2}$

211. (c) Given, $\sin\theta\ = \frac{m^2-n^2}{m^2+n^2}$



In ΔABC,

$AB = \sqrt{(AC)^2-(BC)^2}$

$= \sqrt{m^4+n^4+2m^2n^2-(m^4+n^4-2m^2n^2)}$

$= \sqrt{4m^2n^2} = 2mn$

$\therefore\ \tan\theta\ = \frac{m^2-n^2}{2mn}$

212. (c) Given, $\sin(x-y) = \frac{1}{2}$ and

$\cos\,(x+y) = \frac{1}{2}$

$\Rightarrow \sin(x-y) = \sin 30°$

and $\cos(x+y) = \cos 60°$

$\Rightarrow x - y = 30°$ and $x + y = 60°$

$\therefore x = 45°$ and $y = 15°$

213. (b) Given, $1 + \tan\theta\ = \sqrt{2}$

$\Rightarrow\ \tan\theta\ = \sqrt{2} - 1$

$\therefore\qquad \cot\theta - 1 = \frac{1}{\sqrt{2}-1} - 1$

$= \frac{1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1} - 1$

$= \frac{\sqrt{2}+1}{2-1}\ -1 = \sqrt{2}$

214. (d) Given, $\sin\,(x + 54°) = \cos x$

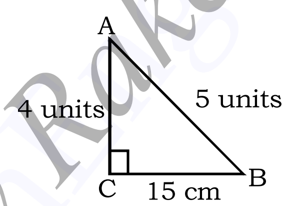
$\Rightarrow \sin(x + 54°) = \sin(90° - x)$

$(\because 0° < x < 90°)$

$\Rightarrow x + 54° = 90° - x$

$\Rightarrow 2x = 36°\ \Rightarrow x = 18°$

215. **(a)** By using Pythagoras theorem



$AB^2 = AC^2+BC^2$

$(5)^2 = (4)^2+BC^2$

BC = 3 units

3 units = 15 cm

1 unit = $\frac{15}{3}$

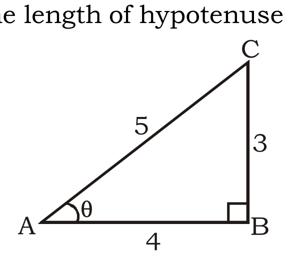
5 units = $\frac{15}{3}\times 5$ = 25 cm

$\therefore$ AB = 25 cm

216. (d) Since, $\tan\theta\ = \frac{3}{4} = \frac{P}{B}$

$H = \sqrt{P^2+B^2}\ = \sqrt{9+16}\ = \sqrt{25}\ = 5$

Let the length of hypotenuse = x cm



$\therefore\qquad \sin\theta\ = \frac{2}{x} = \frac{3}{5}$

$\Rightarrow \frac{2\times 5}{3} = \frac{10}{3}$ cm

217. (c) Given, $\sin x - \cos x = 0$

$\Rightarrow \sin x = \cos x \Rightarrow \tan x = 1$

$\Rightarrow \tan x = \tan\frac{\pi}{4} \Rightarrow\ x = \frac{\pi}{4}$

$\therefore \sin^4 x + \cos^4 x = \sin^4\frac{\pi}{4}\ + \cos^4\frac{\pi}{4}$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4\ = \frac{1}{4}+\frac{1}{4} = \frac{1}{2}$

218. (b) $= \frac{\tan x}{1+\sec x} - \frac{\tan x}{1-\sec x}$

$= \frac{\tan x(1-\sec x-1-\sec x)}{1-\sec^2 x}$

$= \frac{-2\tan x\sec x}{-\tan^2 x}$

$= \frac{2\ \frac{\cos x}{\sin x}}{\cos x} = 2\operatorname{cosec} x$

Alternate:-

$\frac{\tan x}{1+\sec x} - \frac{\tan x}{1-\sec x}$

Put x = 45°

$\frac{\tan 45°}{1+\sec 45°}\ -\ \frac{\tan 45°}{1-\sec 45°}$

$\Rightarrow\ \frac{1}{1+\sqrt{2}}\ -\ \frac{1}{1-\sqrt{2}}$

$\Rightarrow\ \frac{1-\sqrt{2}-1-\sqrt{2}}{1-\left(\sqrt{2}\right)^2}$

$\Rightarrow\ \frac{-2\sqrt{2}}{-1}$

$2\sqrt{2}$

Now check from option.

Option: (b)

2 cosec x = 2 × cosec45°

= 2 × $\sqrt{2}$ (Satisfy)

219. (b) $(\sin^4 x - \cos^4 x + 1)\operatorname{cosec}^2 x$

$= \{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) + 1\}\operatorname{cosec}^2 x$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$(\because 1-\cos^2 x = \sin^2 x)$

$= \{\sin^2 x - \cos^2 x + 1)\,\text{cosec}^2 x$

$(\because 1-\cos^2 x = \sin^2 x)$

$= 2\sin^2 x.\ \dfrac{1}{\sin^2 x} = 2$

Alternate:-

$(\sin^4 x - \cos^4 x + 1)\,\text{cosec}^2 x$

Put x = 90°

$(\sin^4 90° - \cos^4 90° + 1)\,\text{cosec}^2 90°$

$\Rightarrow (1-0+1)\times 1$

$\Rightarrow 2$

220. (b) $\sqrt{\cos x\,\text{cosec}y - \cos x \sin y}$

$(\because x + y = 90°,\text{ given})$

$= \sqrt{\cos x\,\text{cosec}(90° - x) - \cos x.\sin(90° - x)}$

$= \sqrt{\cos x.\sec x - \cos^2 x}$

$= \sqrt{1-\cos^2 x} = \sqrt{\sin^2 x} = \sin x$

221. (b) We know that, $0 \le \sin^2 x \le 1$

$\Rightarrow 0 \le \sin^{10} x \le 1$

$\Rightarrow 0 \le p \le 1$

$(\because p = \sin^{10} x)$

222. (c) $\left(\cos^2\dfrac{\pi}{8} + \sin^2\dfrac{\pi}{8}\right) + 4\cos^2\dfrac{\pi}{4}$

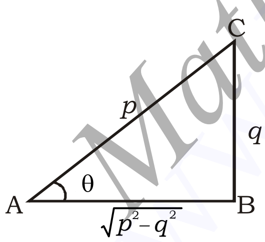
$- \sec\dfrac{\pi}{3} + 5\tan^2\dfrac{\pi}{3}$

$= 1 + 4\left(\dfrac{1}{\sqrt{2}}\right)^2 - 2 + 5\left(\sqrt{3}\right)^2$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1 + 2 - 2 + 15 = 16$

223. (b) Given, $\text{cosec}\,\theta = \dfrac{p}{q}$



In Δ ABC, $\tan\theta = \dfrac{q}{\sqrt{p^2 - q^2}}$

$\Rightarrow \sqrt{p^2 - q^2}.\ \tan\theta = q$

Alternate:-

$\text{cosec}\,\theta = \dfrac{p}{q}$

$\theta = 30°$

$\text{cosec}30° = \dfrac{p}{q}$

$2 = \dfrac{p}{q}$

$p = 2q$

$\therefore \sqrt{p^2 - q^2} \times \tan\theta$

$= \sqrt{4q^2 - q^2} \times \tan 30°$

$= \sqrt{3q^2} \times \dfrac{1}{\sqrt{3}} = q$

224. (d) Given, $2x^2\cos 60° - 4\cot^2 45° - 2\tan 60° = 0$

$\Rightarrow 2x^2 \times \dfrac{1}{2} - 4(1)^2 - 2\times\sqrt{3} = 0$

$\Rightarrow x^2 - 4 - 2\sqrt{3} = 0$

$\Rightarrow x^2 = 4 + 2\sqrt{3}$

$\Rightarrow x^2 = 3 + 1 + 2\sqrt{3}$

$\Rightarrow x^2 = \left(\sqrt{3}\right)^2 + (1)^2 + 2\sqrt{3}.1$

$\Rightarrow x^2 = \left(\sqrt{3}+1\right)^2$

$\Rightarrow x = \sqrt{3} + 1$

225. (d) We know that, $\sin 30° = \dfrac{1}{2}$

Value of sin increases 0° to 90°

$\therefore \sin 31° > \sin 30°$ and $\sin 32° > \sin 30°$

$\Rightarrow \sin 31° > \dfrac{1}{2}$ and $\sin 32° > \dfrac{1}{2}$

On adding both sides, we get

$\sin 31° + \sin 32° > \dfrac{1}{2} + \dfrac{1}{2}$

$\Rightarrow \sin 31° + \sin 32° > 1$

226. (c) We know that, $\sin\theta$ is increasing in 0° to 90°.

$\because \sin 30° = \dfrac{1}{2}$

$\therefore \sin 32° > \dfrac{1}{2}$

227. (d) Given, $\sin\theta + \text{cosec}\,\theta = 2$

$\Rightarrow \sin\theta + \dfrac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$

$\Rightarrow \sin\theta = \sin 90° \Rightarrow \theta = 90°$

$\Rightarrow \sin^4\theta + \cos^4\theta = \sin^4 90° + \cos^4 90°$

$\therefore = 1 + 0 = 1$

228. (c) Given, $2\sec^2\theta + \sec\theta - 6 = 0$

$\Rightarrow 2\sec^2\theta + 4\sec\theta - 3\sec\theta - 6 = 0$

$\Rightarrow 2\sec\theta(\sec\theta + 2) - 3(\sec\theta + 2) = 0$

$\Rightarrow (2\sec\theta - 3)(\sec\theta + 2) = 0$

$(\sec\theta \ne -2,\ \sec\theta > 0)$

$\Rightarrow \sec\theta = \dfrac{3}{2}$

$\Rightarrow \cos\theta = \dfrac{2}{3}$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\dfrac{4}{9}} = \dfrac{\sqrt{5}}{3}$

$\therefore \text{cosec}\,\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{\sqrt{5}}{3}} = \dfrac{3}{\sqrt{5}}$

229. (c) $\dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$

$\Rightarrow \sin^2 x = (1-\cos x)(1+\cos x)$

$\Rightarrow (1-\cos^2 x) = (1-\cos^2 x)$

which is possible for all values of x except multiples of 180°. Since for $x = 180°$, $\sin x = 0$ and $1 + \cos x = 0$

230. (a) $3\sin\theta + 4\cos\theta = 5$

$3\cos\theta - 4\sin\theta = x$ (Let)

Using identity,

$3^2 + 4^2 = 5^2 + x^2$

x = 0

So,

$3\cos\theta - 4\sin\theta = 0$

231. (c) Given, $\sec\theta = \dfrac{13}{5}$

$\Rightarrow \sec^2\theta = \dfrac{169}{25}$

$\Rightarrow 1 + \tan^2\theta = \dfrac{169}{25}$

$\Rightarrow \tan^2\theta = \dfrac{169}{25} - 1$

$\Rightarrow \tan^2\theta = \dfrac{144}{25} \Rightarrow \tan\theta = \dfrac{12}{5}$

$\therefore \dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \dfrac{2\frac{\sin\theta}{\cos\theta} - 3}{4\frac{\sin\theta}{\cos\theta} - 9}$

$$= \frac{2\tan\theta - 3}{4\tan\theta - 9} = \frac{2\left(\frac{12}{5}\right) - 3}{4\left(\frac{12}{5}\right) - 9}$$

[From Eq. (i)]

$$= \frac{24 - 15}{48 - 45} = \frac{9}{3} = 3$$

232. **(d)** r $\sin\theta = \frac{7}{2}$ ......(i)

$r\cos\theta = \frac{7\sqrt{3}}{2}$ .....(ii)

On squaring and adding both equation

$$r^2\sin^2\theta + r^2\cos^2\theta = \left(\frac{7}{2}\right)^2 + \left(\frac{7\sqrt{3}}{2}\right)^2$$

$$r^2(\sin^2\theta + \cos^2\theta) = \frac{49}{4} + \frac{147}{4}$$

$$r^2 = \frac{196}{4} = 49$$

$$r = \sqrt{49} = 7$$

233. **(d)** $\theta + \phi = \frac{\pi}{2}$

$\theta + \phi = 90°$......(i)

$\sin\theta = \frac{1}{2}$

$\sin\theta = \sin30° = \frac{1}{2}$ ......(ii)

put $\theta = 30°$ in equation(i)

$30° + \phi = 90°$

$\phi = 60°$

$\sin\phi = \sin60° = \frac{\sqrt{3}}{2}$

234. **(b)** Given

$\Rightarrow \sec\theta + \tan\theta = p$.....(i)

Then,

$\sec\theta - \tan\theta = \frac{1}{p}$.....(ii)

From equation (i) + (ii)

$\Rightarrow 2\sec\theta = p + \frac{1}{p}$

$\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$

235. **(d)** Given

$\Rightarrow \sin^2 22° + \sin^2 68° + \cot^2 30°$

$\Rightarrow \cos^2 68° + \sin^2 68° + \cot^2 30°$

$$\begin{Bmatrix}\sin(90-\theta) = \cos\theta \\ \cos(90-\theta) = \sin\theta\end{Bmatrix}$$

$\Rightarrow 1 + \cot^2 30°$

$\Rightarrow 1 + (\sqrt{3})^2$

$[\because \cot30° = \sqrt{3}]$

$\Rightarrow 4$

236. **(c)**Given,

$$\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = 2\frac{51}{79}$$

$$\Rightarrow \frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{209}{79}$$

$$\left[\begin{matrix}\text{by componendo-dividendo} \\ \frac{a}{b} = \frac{c}{d}, \ \frac{a+b}{a-b} = \frac{c+d}{c-d}\end{matrix}\right]$$

$$\Rightarrow \frac{\sec\theta}{\tan\theta} = \frac{288}{130}$$

$$\Rightarrow \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{288}{130}$$

$$\Rightarrow \frac{1}{\sin\theta} = \frac{288}{130}$$

$\Rightarrow$Therefore, $\sin\theta = \frac{130}{288}$

$\Rightarrow \sin\theta = \frac{65}{144}$

237. **(b)**Given, $1+\cos^2\theta = 3\sin\theta \cdot \cos\theta$ $[0<\theta<\pi/2]$

$1+\cos^2\theta = 3\sin\theta \cdot \cos\theta$

Find $\cot\theta$ = ?

By dividing $\sin^2\theta$ both sides

$$\Rightarrow \frac{1+\cos^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$$

$\Rightarrow \operatorname{cosec}^2\theta + \cot^2\theta = 3\cot\theta$

$\Rightarrow 1+\cot^2\theta + \cot^2\theta = 3\cot\theta$

$[1+\cot^2\theta = \operatorname{cosec}^2\theta]$

$\Rightarrow 1+2\cot^2\theta = 3\cot\theta$

$\Rightarrow 2\cot^2\theta = 3\cot\theta - 1$

Let $\theta = 45°$

$\because \cot45° = 1$

$2\cot^2 45° - 3\cot 45° + 1 = 0$

$2 - 3 + 1 = 0$

$0 = 0$

Therefore $\cot\theta = \cot45° = 1$

238. **(b)**The value of $\cos24° + \cos55° + \cos125° + \cos204° + \cos300°$

We know that, $\cos(180° \pm \theta) = -\cos\theta$

$\Rightarrow \cos24° + \cos55° + \cos(180°-55°) + \cos(180° + 24°) + \cos(360° - 60°)$

$\Rightarrow \cos24° + \cos55° - \cos55° - \cos24° + \cos60°$

$= \cos60° = \frac{1}{2}$

239. **(c)** $\sin A + \operatorname{cosec}A = 3$

$\sin A + \frac{1}{\sin A} = 3$

Squaring both sides

$\sin^2 A + \frac{1}{\sin^2 A} + 2 = 9$

$\frac{\sin^4 A + 1}{\sin^2 A} = 9 - 2 = 7$

240. **(d)** $\cos\alpha\ \operatorname{cosec}\beta = 1$ if $\alpha + \beta = 90°$

$\cos7°\ \cos23°\ \cos45°\ \operatorname{cosec}83°\ \operatorname{cosec}67°$

$= (\cos7°\ \operatorname{cosec}83°)\ (\cos23°\ \operatorname{cosec}67°)\ \cos45°$

$= 1 \times 1 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

241. **(b)** $\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = 2$

$\tan\theta + \cot\theta = 2\tan\theta - 2\cot\theta$

$3\cot\theta = \tan\theta$

$\frac{3}{\tan\theta} = \tan\theta$

$\tan^2\theta = 3$

$\Rightarrow \tan\theta = \sqrt{3}$ or $\theta = 60°$

$\sin\theta = \sin60° = \frac{\sqrt{3}}{2}$

242. **(b)**$\sin x + \cos x = c$

squaring both sides

$\sin^2 x + \cos^2 x + 2\sin x\cos x = c^2$

$\sin x\cos x = \frac{c^2 - 1}{2}$

we know that,

$\sin^6 x + \cos^6 x = 1 - 3\sin^2 x\cos^2 x$

$= 1 - 3\left(\frac{c^2 - 1}{2}\right)^2$

$= 1 - 3\left(\frac{c^4+1-2c^2}{4}\right)$

$= \frac{1+6c^2-3c^4}{4}$

Alternate:-

Put, $x = 0$

then c = 1

put, c = 1 in all option

option (a) = $\frac{1}{4}$, option (b) = 1

option (c) = $\frac{5}{8}$, option (d) = $\frac{5}{2}$

Hence, $\sin^6 x + \cos^6 x = 1$ = option (b)

243. (a)

$\frac{1-\sin A.\cos A}{\cos A(\sec A-\operatorname{cosec}A)}.\frac{\sin^2 A-\cos^2 A}{\sin^3 A+\cos^3 A}$

$= \frac{(1-\sin A\cos A)}{\cos A\left(\frac{\sin A-\cos A}{\sin A\cos A}\right)}.$

$\frac{(\sin A+\cos A)(\sin A-\cos A)}{(\sin A+\cos A)(\sin^2 A+\cos^2 A-\sin A\cos A)}$

$= \frac{\sin A(1-\sin A.\cos A)}{(1-\sin A.\cos A)} = \sin A$

244. (d) $mn = \cot^2\theta - \cos^2\theta$

$= \frac{\cos^2\theta(1-\sin^2\theta)}{\sin^2\theta}$

$mn = \cot^2\theta . \cos^2\theta$

$\Rightarrow \sqrt{mn} = \cot\theta.\cos\theta$

$m^2 - n^2 = (m+n)(m-n) = 2\cot\theta . 2\cos\theta$

$= 4\cot\theta\cos\theta = 4\sqrt{mn}$

Alternate:-

$\cot\theta + \cos\theta = m$

$\cot\theta - \cos\theta = n$

Put $\theta = 45°$

Cot 45° + cos45° = m

$\Rightarrow 1 + \frac{1}{\sqrt{2}} = m$

$\Rightarrow m = 1 + \frac{1}{\sqrt{2}}$

$\Rightarrow n = 1 - \frac{1}{\sqrt{2}}$

$m^2 - n^2 = \left(1+\frac{1}{\sqrt{2}}\right)^2 - \left(1-\frac{1}{\sqrt{2}}\right)^2$

$= 1 + \frac{1}{2}+\frac{2}{\sqrt{2}} - 1 - \frac{1}{2}+\frac{2}{\sqrt{2}}$

$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$

Now check from option

Option:- (d) $4\sqrt{mn}$

$\Rightarrow \Rightarrow 4\sqrt{\left(1+\frac{1}{\sqrt{2}}\right)\left(1-\frac{1}{\sqrt{2}}\right)}$

$\Rightarrow 4\sqrt{1^2-\frac{1}{(\sqrt{2})^2}}$

$\Rightarrow 4\sqrt{1-\frac{1}{2}}$

$\Rightarrow 4\sqrt{\frac{1}{2}} \Rightarrow 2\sqrt{\frac{4}{2}}$

$= 2\sqrt{2}$ (satisfy)

245. (d) $\tan\theta + \sin\theta = m$

$\tan\theta - \sin\theta = n$

Put $\theta = 45°$

tan45° + sin45° = m

$1 + \frac{1}{\sqrt{2}} = m$

$1 - \frac{1}{\sqrt{2}} = n$

$\therefore \sqrt{mn} = \sqrt{\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)}$

$= \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}$

Now check from option,

Option (d):- $\frac{1}{4}(m^2-n^2)$

$= \frac{1}{4}\left[\left(1+\frac{1}{\sqrt{2}}\right)^2-\left(1-\frac{1}{\sqrt{2}}\right)^2\right]$

$= \frac{1}{4}\left[1+\frac{1}{2}+\frac{2}{\sqrt{2}}-1-\frac{1}{2}+\frac{2}{\sqrt{2}}\right]$

$= \frac{1}{4}\left[\frac{4}{\sqrt{2}}\right] = \frac{1}{\sqrt{2}}$ (Satisfy)

246. (d) $x = \cot\theta + \tan\theta$

$= \frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}$

$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$

$y = \sec\theta - \cos\theta = \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$

$(x^2 y)^{2/3} = \left(\frac{1}{\sin^2\theta.\cos^2\theta}.\frac{\sin^2\theta}{\cos\theta}\right)^{2/3}$

$= \frac{1}{\cos^2\theta}$

$(xy^2)^{2/3} = \left(\frac{1}{\sin\theta.\cos\theta}.\frac{\sin^4\theta}{\cos^2\theta}\right)^{2/3}$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$(x^2 y)^{2/3} - (xy^2)^{2/3} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta} = 1$

Alternate:-

Put $\theta = 45°$

x = cot 45° + tan45° = 2

y = sec45° – cos45° = $\frac{1}{\sqrt{2}}$

$(x^2y)^{2/3}-(xy^2)^{2/3} = \left(\frac{4}{\sqrt{2}}\right)^{2/3} - \left(\frac{2}{2}\right)^{2/3}$

= 2 – 1 = 1

247. (b) $\frac{\sin^8\theta-\cos^8\theta}{\cos 2\theta(1+\cos^2 2\theta)}$

$= \frac{(\sin^4\theta+\cos^4\theta)(\sin^2\theta-\cos^2\theta)(\sin^2\theta+\cos^2\theta)}{(\cos^2\theta-\sin^2\theta)\left(1+(\cos^2\theta-\sin^2\theta)^2\right)}$

$= -\frac{(\sin^4\theta+\cos^4\theta)}{(1+\cos^4\theta+\sin^4\theta-2\sin^2\theta.\cos^2\theta)}$

We can say

$\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= -\frac{(1-2\sin^2\theta.\cos^2\theta)}{(1+1-2\sin^2\theta.\cos^2\theta-2\sin^2\theta.\cos^2\theta)}$

$= -\frac{1}{2}$

Alternate:-

Answer is independent of angle θ, so put $\theta = 90°$

$\frac{\sin^8 90°-\cos^8 90°}{\cos 180°(1+\cos^2 180°)}$

$= \frac{1-0}{-1(1+1)} = -\frac{1}{2}$

248. **(a)** $k = (\sec\alpha + \tan\alpha)(\sec\beta + \tan\beta)(\sec\gamma + \tan\gamma)$....(i)

$k = (\sec\alpha - \tan\alpha)(\sec\beta - \tan\beta)(\sec\gamma - \tan\gamma)$....(ii)

Multiplying equation (i)&(ii) $k^2 = (\sec^2\alpha - \tan^2\alpha)(\sec^2\beta - \tan^2\beta)(\sec^2\gamma - \tan^2\gamma)$

$k^2 = 1 \Rightarrow k = \pm 1$

249. **(a)** $a\sec\theta + b\tan\theta + c = 0$

$p\sec\theta + q\tan\theta + r = 0$

Apply cross multiplication method to solve these eq^n

$$\frac{\sec\theta}{br - qc} = \frac{\tan\theta}{pc - ar} = \frac{1}{aq - bp}$$

$$\sec\theta = \frac{br - qc}{aq - bp} \;\&\; \tan\theta = \frac{pc - ar}{aq - bp}$$

$\sec^2\theta - \tan^2\theta = 1$

$$\left(\frac{br - qc}{aq - bp}\right)^2 - \left(\frac{pc - ar}{aq - bp}\right)^2 = 1$$

Then, $(br - qc)^2 - (pc - ar)^2 = (aq - bp)^2$

250. **(a)** $P = a\cos^3 x + 3a\cos x \sin^2 x$

$Q = a\sin^3 x + 3a\cos^2 x \sin x$

Put $x = 45°$

$$P = \frac{a}{2\sqrt{2}} + \frac{3a}{2\sqrt{2}},$$

$$Q = \frac{a}{2\sqrt{2}} + \frac{3a}{2\sqrt{2}}$$

$$P = \frac{4a}{2\sqrt{2}} = \sqrt{2}a,\ Q = \sqrt{2}a$$

$(P + Q)^{2/3} + (P - Q)^{2/3}$

$$= \left(\sqrt{2}a + \sqrt{2}a\right)^{2/3} + 0$$

$$= \left(2\sqrt{2}a\right)^{2/3} = 2a^{2/3}$$

251. **(b)** $8\cos^2\theta + 8\sec^2\theta = 65$

$$0° < \theta < \frac{\pi}{2}$$

$$\Rightarrow 8\cos^2\theta + \frac{8}{\cos^2\theta} = 65$$

$\Rightarrow 8\cos^4\theta + 8 = 65\cos^2\theta$

$\Rightarrow 8\cos^4\theta - 64\cos^2\theta - \cos^2\theta + 8 = 0$

$\Rightarrow 8\cos^2\theta\,[\cos^2\theta - 8] - 1\,(\cos^2\theta - 8) = 0$

$$\Rightarrow \cos^2\theta = \frac{1}{8},$$

$\Rightarrow \cos^2\theta = 8$ (not possible)

$(\because \cos 2\theta = 2\cos^2\theta - 1)$

$$\Rightarrow \cos 2\theta = 2\times\frac{1}{64} - 1$$

$$= \frac{2 - 64}{64} = \frac{-62}{64} = -\frac{31}{32}$$

$$\Rightarrow 4\cos 2\theta = -\frac{31}{8}$$

252. **(b)** $\cos(\theta - A) = a$, $\cos(\theta - B) = b$

Let $\theta = 90°$

$a = \cos(90° - A) = \sin A$,

$b = \cos(90° - B) = \sin B$

$\cos A = \sqrt{1 - a^2}$, $\cos B = \sqrt{1 - b^2}$

$\Rightarrow \sin^2(A - B) + 2ab\cos(A - B)$

$= (\sin A\cos B - \cos A\sin B)^2 + 2ab(\cos A\cos B + \sin A\sin B)$

$$= \left(a\sqrt{1 - b^2} - b\sqrt{1 - a^2}\right)^2$$

$$+ 2ab\left(\sqrt{1 - a^2}.\sqrt{1 - b^2} + ab\right)$$

$= a^2(1 - b^2) + b^2(1 - a^2) - 2ab\sqrt{1 - a^2}.\sqrt{1 - b^2}$

$+ 2ab\sqrt{1 - a^2}.\sqrt{1 - b^2} + 2a^2b^2$

$= a^2 - a^2b^2 + b^2 - a^2b^2 + 2a^2b^2$

$= a^2 + b^2$

Alternate:-

$\cos(\theta - A) = a$

$\cos(\theta - B) = b$

Put $\theta = 90°$

$\cos(90° - A) = a$

$a = \sin A$

$b = \sin B$

Put $A = 60°$

$B = 30°$

$$a = \frac{\sqrt{3}}{2}$$

$$b = \frac{1}{2}$$

$\therefore\ \sin^2(A - B) + 2ab\cos(A - B)$

$$\sin^2(60° - 30°) + 2\times\frac{\sqrt{3}}{2}\times\frac{1}{2}\cos(60° - 30°)$$

$$\Rightarrow \frac{1}{4} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{1}{4} + \frac{3}{4} = 1$$

Now check the option

Option (b):- $a^2 + b^2$

$$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \quad \text{(Satisfy)}$$

253. **(a)** $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$

$$= \sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\left(\pi - \frac{3\pi}{8}\right) + \sin^4\left(\pi - \frac{\pi}{8}\right)$$

$$= 2\left(\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8}\right)$$

$$= 2\left(\sin^4\frac{\pi}{8} + \sin^4\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right)$$

$$= 2\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}\right)$$

$$\left(\because \sin\frac{3\pi}{8} = \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \cos\frac{\pi}{8}\right)$$

$$= 2\left(\left(\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right)^2 - 2\sin^2\frac{\pi}{8}.\cos^2\frac{\pi}{8}\right)$$

$$= 2\left(1 - \frac{1}{2}\left(\sin\frac{\pi}{4}\right)^2\right) = 2\left(1 - \frac{1}{4}\right) = \frac{3}{2}$$

254. **(a)** $\sin\theta + \cos\theta = a$,

$\sec\theta + \operatorname{cosec}\theta = b$

$$b(a^2 - 1) = \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)(\sin^2 + \cos^2\theta + 2\sin\theta\cos\theta - 1)$$

$$= \frac{(\sin\theta + \cos\theta)}{\sin\theta\cos\theta}\times 2\sin\theta\cos\theta$$

$= 2a$

255. **(b)** $\frac{1}{2}\left(\cos 15°.\cos 7\frac{1}{2}°.\cos 82\frac{1}{2}°\right)\times 2$

$$\Rightarrow \frac{1}{2}\left(\cos 15°.2\times\cos 7\frac{1}{2}^{0}.\sin 7\frac{1}{2}^{0}\right)$$

$$\Rightarrow \frac{1}{2}\cos 15°.\sin 15°$$

Multiply and divide by 2

$$\Rightarrow \frac{1}{2}\times\frac{1}{2}\times 2\cos 15°\sin 15°$$

$$\Rightarrow \frac{1}{2}\times\frac{1}{2}\times\sin 2\times 15°$$

$$\Rightarrow \frac{1}{2}\times\frac{1}{2}\times\sin 30°$$

$$\Rightarrow \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

256. (b) $3\tan\theta\tan\phi = 1$

$\theta = \phi = 30°$

$$\frac{\cos(30°-30°)}{\cos(30°+30°)} = \frac{\cos 0°}{\cos 60°} = 2$$

257. **(c)** $\tan 60°$

$$= \frac{\tan 20° + \tan 40°}{1-\tan 20°\tan 40°}$$

$\tan 20° + \tan 40° = \sqrt{3} - \sqrt{3}\tan 20°.\tan 40°$

$\tan 20° + \tan 40° + \sqrt{3}\tan 20°.\tan 40° = \sqrt{3}$

258. (a) Given let $x = 1$ and $\theta = 0°$

$\Rightarrow x\cos\theta - \sin\theta = 1$ ......(i)

$\Rightarrow 1\times 1 - (0) = 1$

$\Rightarrow 1 = 1$

putting value $x = 1$ and $\theta = 0°$ in equation (i)

$\Rightarrow x^2 + (1 + x^2)\sin\theta$

$\Rightarrow 1^2 + (1 + 1^2) \times \sin 0° = 1$

259. **(d)** Simplest form of

$\Rightarrow \sin A.\cos A(\tan A - \cot A)$

$$\Rightarrow \sin A.\cos A\left(\frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}\right)$$

$\Rightarrow \sin^2 A - \cos^2 A$

$\{\because \cos^2 A = 1 - \sin^2 A\}$

$\Rightarrow 2\sin^2 A - 1$

260. **(d)** Given:-

$$n = \frac{\cos\propto}{\sin\beta}, \quad m = \frac{\cos\propto}{\cos\beta}$$

$\Rightarrow \cos\propto = n\sin\beta$, and $\cos\propto = m\cos\beta$

$\cos^2\propto = n^2\sin^2\beta$ ....(i)

$\cos^2\propto = m^2\cos^2\beta$ ....(ii)

equation (i) = (ii)

$\Rightarrow n^2\sin^2\beta = m^2\cos^2\beta$

$\Rightarrow n^2(1-\cos^2\beta) = m^2\cos^2\beta$

$$\Rightarrow \cos^2\beta = \frac{n^2}{m^2+n^2}$$

261. **(a)** Given, $\sin(A-B) = \sin A.\cos B - \cos A.\sin B$

Let $A = 45°$, $B = 30°$

$\sin(45-30)° = \sin 45°.\cos 30° - \cos 45°.\sin 30$

$$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2}$$

$$\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

262. (d) α is an acute angle

$\Rightarrow 2\sin\alpha + 15\cos^2\alpha = 7$

$\Rightarrow 2\sin\alpha + 15(1-\sin^2\alpha) = 7$

$\Rightarrow 2\sin\alpha + 15 - 15\sin^2\alpha = 7$

$\Rightarrow 15\sin^2\alpha - 2\sin\alpha - 8 = 0$

Let $\sin\alpha = x$

$\Rightarrow 15x^2 - 2x - 8 = 0$

$\Rightarrow 15x^2 - (12-10)x - 8 = 0$

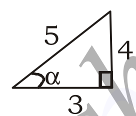
$\Rightarrow 15x^2 - 12x + 10x - 8 = 0$

$\Rightarrow 3x(5x-4) + 2(5x-4) = 0$

$\Rightarrow (3x+2)(5x-4) = 0$

$\Rightarrow x = -\frac{2}{3}$ (Rejected)

$\Rightarrow x = \frac{4}{5}$



$$\Rightarrow \sin\theta = \frac{4}{5} = \frac{\text{Perp.}}{\text{Hypo.}}$$

Base = 3

(Using triplets 3, 4, 5)

$$\Rightarrow \text{Therefore, } \cot\alpha = \frac{\text{Base}}{\text{Perp.}}$$

$$\cot\alpha = \frac{3}{4}$$

263. **(b)** According to the question,

$\sin^4\theta + \cos^4\theta = 2\sin^2\cos^2\theta$

Put $\theta = 45°$

$\sin^4 45° + \cos^4 45° = 2\sin^2\cos^2\theta$

$$\frac{1}{4}+\frac{1}{4} = 2\times\frac{1}{2}\times\frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} \text{ [Satisfy]}$$

$\therefore \tan 45° = 1$

264. **(c)** According to the question,

$\text{cosec}A + \cot A = 3$

$\text{cosec}A - \cot A = \frac{1}{3}$

$2\text{cosec}A = \frac{10}{3}$

$$\text{cosec}A = \frac{10}{6}$$

$$\sin A = \frac{6}{10} = \frac{3}{5}$$

265. **(b)** $\cot 41°.\cot 42°.\cot 43°.\cot 44°.\cot 45°.\cot 46°\cot 47°.\cot 48°.\cot 49°$

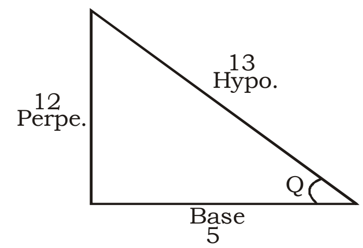
$\Rightarrow (\cot 41°.\cot 49°).(\cot 42°.\cot 48°)(\cot 43°.\cot 47°)(\cot 44°.\cot 46°)\cot 45°$ $[\cot A.\cot B = 1$ [*If* $A + B = 90°$]$] = 1$

266. **(d)** $5\cos\theta + 12\sin\theta = 13$

$$\frac{5}{13}\cos\theta + \frac{12}{13}\sin\theta = \frac{13}{13}$$

[Divide whole question by R.H.S. value]

$$\frac{5}{13}\,\frac{\text{Base}}{\text{Hypo}} + \frac{12}{13}\,\frac{\text{Perpendicular}}{\text{Hypo}} = 1$$



$$\sin\theta = \frac{P}{H} = \frac{12}{13}$$

267. **(a)** $\tan\theta - \cot\theta = 0$

Put $\theta = 45°$

$\tan 45° - \cot 45° = 0$

$1 - 1 = 0$

$0 = 0$ (Satisfied)

So, $\theta = 45°$

$$\text{Now, } \frac{\tan(\theta+15°)}{\tan(\theta-15°)}$$

$$\Rightarrow \frac{\tan(45°+15°)}{\tan(45°-15°)}$$

$$\Rightarrow \frac{\tan 60°}{\tan 30°} \Rightarrow \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = \sqrt{3}\times\sqrt{3} = 3$$

268. **(c) SHORTCUT METHOD**

Put $\theta = 30°$

$$\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$$

$$\sec 30° - \tan 30° = \frac{1}{\sqrt{3}}$$

$$\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ (satisfy)}$$

$\sec\theta = 30°$

$\Rightarrow \sec\theta \,.\, \tan\theta$

$\Rightarrow \sec 30°.\ \tan 30°$

$$\Rightarrow \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2}{3}$$

269. **(d)** (cosec a – sin a) (sec a – cos a) (tan a + cot a)

Put a = 45°

(cosec 45° – sin 45°) (sec 45° – cos 45°) (tan 45° + cot 45°)

$$\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)(1+1)$$

$$\frac{1}{2} \times 2 = 1$$

270. **(c)** According to the question tanA = ntanB and sinA = msinB

$$n = \frac{\tan A}{\tan B} \qquad m = \frac{\sin A}{\sin B}$$

Put A = 30° and B = 60°

$$n = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} \qquad m = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$n = \frac{1}{3} \qquad m = \frac{1}{\sqrt{3}}$$

$$\therefore \cos^2 A = \cos^2 30° = \frac{3}{4}$$

Now check from option to save your valuable time

Option (c) : $\frac{m^2-1}{n^2-1}$

$$= \frac{\left(\frac{1}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{3}\right)^2 - 1} = \frac{\frac{1}{3}-1}{\frac{1}{9}-1} = \frac{-\frac{2}{3}}{-\frac{8}{9}}$$

$$= \frac{3}{4} \text{ (satisfy)}$$

271.. **(b)** $x \cos^2 30°.\ \sin 60°$

$$= \frac{\tan^2 45°.\sec 60°}{\text{cosec} 60°}$$

$$x\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{\sqrt{3}}{2}\right) = \frac{(1)^2(2)}{\left(\frac{2}{\sqrt{3}}\right)}$$

$$x = 2\frac{2}{3}$$

272.(b) $\sin(\theta + 30°) = \frac{3}{\sqrt{12}}$

$$= \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$\sin(\theta + 30°) = \sin 60°$

$\therefore \theta = 30°$

$$\cos^2\theta = \cos^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

273. **(a)** $4\cos^2\theta - 4\sqrt{3}\cos\theta + 3 = 0$

Hit & trial method

Put $\theta = 30°$ option (a)

$4\cos^2 30° - 4\sqrt{3}\cos 30° + 3 = 0$

$$4\left(\frac{3}{4}\right) - 4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) + 3 = 0$$

$0 = 0$

274. **(d)** According to the question ,

$\Rightarrow \sec^4 A\,(1 - \sin^4 A) - 2\tan^2 A$

Put A = 45°

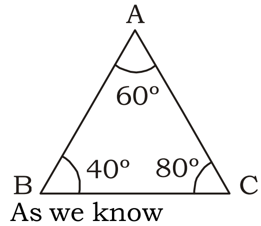
$\Rightarrow \sec^4 45°\,(1 - \sin^4 45°) - 2\tan^2 45°$

$$\Rightarrow 4\left(1 - \frac{1}{4}\right) - 2$$

$$\Rightarrow 4 \times \frac{3}{4} - 2$$

$\Rightarrow 3 - 2 = 1$

275. **(d)**



$\Rightarrow$ As we know

$\Rightarrow \sin(90 - \theta) = \cos\theta$

$$\Rightarrow \sin\left[\frac{(B+C)}{2}\right] = \cos\frac{A}{2}$$

Note:- Let ∠A = 60°, ∠B = 40°, ∠C = 80°

$$\Rightarrow \sin\left[\frac{B+C}{2}\right] = \cos\frac{A}{2}$$

$$\Rightarrow \sin\frac{(40° + 80°)}{2}$$

$$= \cos\frac{60°}{2}$$

$$\Rightarrow \sin\left(\frac{120°}{2}\right) = \cos\frac{60°}{2}$$

$\Rightarrow \sin 60° = \cos 30°$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ Therefore

$$\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$$

276. **(b)**

$$= \frac{9}{\text{cosec}^2\theta} + 4\cos^2\theta + \frac{5}{1+\tan^2\theta}$$

$$= 9\sin^2\theta + 4\cos^2\theta + \frac{5}{1+\tan^2\theta}$$

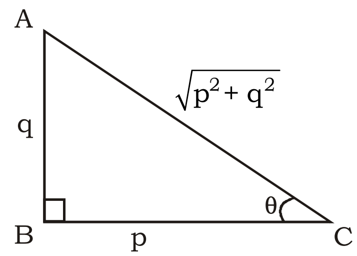
$\because (1+\tan^2\theta = \sec^2\theta)$

$= 9\sin^2\theta + 4\cos^2\theta + 5\cos^2\theta$

$= 9(\sin^2\theta + \cos^2\theta) = 9$

277. **(a)**

$$\cos\theta = \frac{\text{Base}}{\text{Hyp}} = \frac{p}{\sqrt{p^2+q^2}}$$



$(\text{Hyp})^2 = \text{Base}^2 + \text{perp}^2$.

Perpendicular = q

$$\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{q}{p}$$

278.(c) $\tan\theta + \sec\theta = 3$

$$\sec\theta - \tan\theta = \frac{1}{3}$$

$$2\sec\theta = \frac{10}{3}$$

$$\cos\theta = \frac{3}{5};\ \sin\theta = \frac{4}{5}$$

$5\sin\theta = 4$

279. (c) $\frac{x - x\tan^2 30°}{1+\tan^2 30°}$

$= \sin^2 30° + 4\cot^2 45° - \sec^2 60°$

$$\frac{x(1-\tan^2 30°)}{1+\tan^2 30°} = \left(\frac{1}{2}\right)^2 + 4\times 1 - (2)^2$$

$$\frac{x\left[1-\left(\frac{1}{\sqrt{3}}\right)^2\right]}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1}{4} + 4 - 4$$

$$\frac{x\left[1-\frac{1}{3}\right]}{1+\frac{1}{3}} = \frac{1}{4}$$

$$x \times \frac{2}{3} = \frac{1}{4} \times \frac{4}{3}$$

$$x = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$$

280.(d) According to question

$\cos\theta + \sin\theta = m$

$\sec\theta + \operatorname{cosec}\theta = n$

put $\theta = 45°$

$\cos45° + \sin45° = m$,

$\sec45° + \operatorname{cosec}45° = n$

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = m, \sqrt{2} + \sqrt{2} = n$$

$m = \sqrt{2}, n = 2\sqrt{2}$

$\therefore \quad n(m^2 - 1) = 2\sqrt{2}\left(\left(\sqrt{2}\right)^2 - 1\right)$

$= 2\sqrt{2}\,(2 - 1) = 2\sqrt{2}$

Now check from option only one option satisfy

$2m = 2 \times \sqrt{2} = 2\sqrt{2}$

281. (d) **Given:-**

$\propto + \beta = 90°$

$\because \quad \propto + \beta = 90°$

$\Rightarrow \quad \propto = 90 - \beta$

$\Rightarrow$ $\propto$, β are complementary angles

$$\Rightarrow \frac{\tan\propto}{\tan(90-\propto)} + \sin^2\propto + \sin^2(90-\propto)$$

$$\Rightarrow \frac{\tan\propto}{\cot\propto} + \sin^2\propto + \cos^2\propto$$

$\Rightarrow \quad \tan^2\propto + 1$

$(\because \sin^2\propto + \cos^2\propto = 1)$

$\Rightarrow \quad \sec^2\propto$

$(\because 1 + \tan^2\propto = \sec^2\propto)$

282. (d)

$$\tan^2\frac{\pi}{4} - \cos^2\frac{\pi}{3} = x\sin\frac{\pi}{4}\cos\frac{\pi}{4}\tan\frac{\pi}{3}$$

$\Rightarrow \tan^2 45° - \cos^2 60° = x \sin45° \cos45° \tan60°$

$$\Rightarrow 1 - \frac{1}{4} = x \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sqrt{3}$$

$$\Rightarrow \frac{3}{4} = \frac{x\times\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{\sqrt{3}}{2}$$

283. (d) $\sin2\theta = \frac{\sqrt{3}}{2}$

$$\Rightarrow \sin2\theta = \frac{\sqrt{3}}{2}$$

$\Rightarrow \sin2\theta = \sin60°$

$\Rightarrow 2\theta° = 60°$

$$\Rightarrow \theta° = \frac{60}{2} = 30°$$

$\Rightarrow$ So $\sin3\theta = \sin 3 \times 30° = \sin90° = 1$

284. (a) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{3}{1}$

to find $\sin^4\theta$ = ?

$\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{3}{1}$ (by componendo dividendo)

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{3+1}{3-1}$$

$\Rightarrow \tan\theta = 2$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{2}{1}$$

$$\Rightarrow \sin^4\theta = \left(\frac{2}{\sqrt{5}}\right)^4 = 16/25$$

285. (b) $\cos20° = m$

$\cos70° = n$

So, $m^2 + n^2 = \cos^2 20° + \cos^2 70°$

$= 1$

$$\left[\begin{array}{l}\cos^2 A + \cos^2 B = 1 \\ \text{if } A + B = 90°\end{array}\right]$$

286. (d) $\sin A - \cos A = \frac{\sqrt{3}-1}{2}$

Shortcut Method:-

Put $\theta = 60°$

$$\Rightarrow \sin60° - \cos60° = \frac{\sqrt{3}-1}{2}$$

$$\Rightarrow \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$$

$$\Rightarrow \frac{\sqrt{3}-1}{2} = \frac{\sqrt{3}-1}{2}$$

(Matched)

Hence, $\sin A . \cos A$

$\Rightarrow \sin60° . \cos60°$

$$\Rightarrow \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$$

Alternate:-

$$\sin A - \cos A = \frac{\sqrt{3}-1}{2}$$

Squaring both side,

$\Rightarrow \sin^2 A + \cos^2 A - 2\sin A\cos A$

$$= \left(\frac{\sqrt{3}-1}{2}\right)^2$$

$\Rightarrow 1 - 2\sin A\cos A$

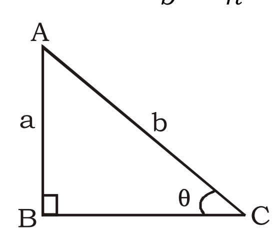
$$= \frac{3 + 1 - 2\sqrt{3}}{4}$$

$$\Rightarrow 2\sin A\cos A = 1 - \frac{2\left(2-\sqrt{3}\right)}{4}$$

$$\Rightarrow 2\sin A . \cos A = \frac{2 - 2 + \sqrt{3}}{2}$$

$$\sin A. \cos A = \frac{\sqrt{3}}{4}$$

287. (c) $\sin\theta = \frac{a}{b} = \frac{p}{h}$



$BC = \sqrt{b^2 - a^2}$

[using pythagoras theorem]

$\therefore \sec\theta - \cos\theta$

$$= \frac{H}{B} - \frac{B}{H} = \frac{AC}{BC} - \frac{BC}{AC}$$

$$= \frac{b}{\sqrt{b^2 - a^2}} - \frac{\sqrt{b^2 - a^2}}{b}$$

$$= \frac{b^2 - \left(\sqrt{b^2 - a^2}\right)^2}{b\sqrt{b^2 - a^2}}$$

$$= \frac{b^2 - b^2 + a^2}{b\sqrt{b^2 - a^2}} = \frac{a^2}{b\sqrt{b^2 - a^2}}$$

288. (c) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$$= \frac{\left(\sqrt{1+\sin\theta}\right)^2 + \left(\sqrt{1-\sin\theta}\right)^2}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{1+\sin\theta+1-\sin\theta}{\sqrt{\cos^2\theta}}$$

$$= \frac{2}{\cos\theta} = 2\sec\theta$$

Alternate:-

Put $\theta = 30°$

$$\Rightarrow \sqrt{\frac{1+\sin 30°}{1-\sin 30°}} + \sqrt{\frac{1-\sin 30°}{1+\sin 30°}}$$

$$\Rightarrow \sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}} + \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}}$$

$$\Rightarrow \sqrt{\frac{\frac{3}{2}}{\frac{1}{2}}} + \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}}$$

$$\Rightarrow \sqrt{3} + \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

Now check from option to save your valuable time, option 'c'

$2\sec\theta = 2\sec 30° = 2 \times \frac{2}{\sqrt{3}}$

$= \frac{4}{\sqrt{3}}$ (Satisfied)

289. (c) $\frac{\sec^2 70° - \cot^2 20°}{2\left(\mathrm{cosec}^2 59° - \tan^2 31°\right)}$

$$= \frac{2}{m}$$

$$\frac{\sec^2 70° - \cot^2(90°-70°)}{2\left(\mathrm{cosec}^2 59° - \tan^2(90°-59°)\right)}$$

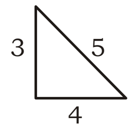
$$= \frac{2}{m}$$

$$\frac{\sec^2 70° - \tan^2 70°}{2(\mathrm{cosec}^2 59° - \cot^2 59°)} = \frac{2}{m}$$

$$\frac{1}{2} = \frac{2}{m} \quad \begin{bmatrix} \sec^2\theta - \tan^2\theta = 1 \\ \mathrm{cosec}^2\theta - \cot^2\theta = 1 \end{bmatrix}$$

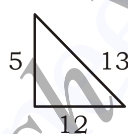
$m = 2 \times 2 = 4$

290. (a) $\cos(\alpha+\beta) = \frac{4}{5}$



$$\Rightarrow \tan(\alpha+\beta) = \frac{3}{4}$$

$\sin(\alpha-\beta) = \frac{5}{13}$



$$\Rightarrow \tan(\alpha-\beta) = \frac{5}{12}$$

$$\Rightarrow \tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta)$$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}} = \frac{14}{12} \times \frac{48}{48-15} = \frac{56}{33}$$

291. (a) $\cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7}$

$$= \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\left(\pi - \frac{3\pi}{7}\right) + \cos\left(\pi - \frac{2\pi}{7}\right) + \cos\left(\pi - \frac{\pi}{7}\right)$$

$$= \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{2\pi}{7}$$

$$- \cos\frac{\pi}{7} - \cos\frac{3\pi}{7} = 0$$

292. (d) $\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$\tan 9° + \cot 9° - (\tan 27° + \cot 27°)$

$$= \frac{\sin 9°}{\cos 9°} + \frac{\cos 9°}{\sin 9°} - \left(\frac{\sin 27°}{\cos 27°} + \frac{\cos 27°}{\sin 27°}\right)$$

$$= \frac{1}{\sin 9°\cos 9°} - \frac{1}{\sin 27°\cos 27°}$$

$$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°} = \frac{2(\sin 54° - \sin 18°)}{\sin 18°\sin 54°}$$

Using SinC and sinD formula

$$= \frac{2(2\sin 18°\cos 36°)}{\sin 18°\cos 36°} = 4$$

293. (a) $\sin x + 2\cos x = 1$ ...(i)

Put $x = 90°$

$\sin 90° + 2\cos 90° = 1$

$1 + 0 = 1$

$1 = 1$ (Satisfy)

Now,

$7\cos x + 6\sin x$

$= 7\cos 90° + 6\sin 90° = 6$

294. (b) $\cos^2 48° - \sin^2 12°$

[$\because \cos^2 A - \sin^2 B = \cos(A+B).\cos(A-B)$]

$$\cos 60°.\cos 36° = \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right) = \frac{\sqrt{5}+1}{8}$$

[Using $\cos 36° = \frac{\sqrt{5}+1}{4}$]

295. (c) $\sin\left(\frac{\pi}{10}\right)\sin\left(\frac{3\pi}{10}\right)$

$$\frac{1}{2}\left[2\sin\left(\frac{\pi}{10}\right)\sin\frac{3\pi}{10}\right]$$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$$= \frac{1}{2}\left(\cos\frac{2\pi}{10} - \cos\frac{4\pi}{10}\right)$$

$$= \frac{1}{2}(\cos 36° - \cos 72°)$$

$$= \frac{1}{2}(\cos 36° - \sin 18°)$$

(By putting the value of cos 36° and sin18°)

$$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}\right) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

296. (a) $\tan\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}-\theta\right)$

$$= \frac{1+\tan\theta}{1-\tan\theta} - \frac{1-\tan\theta}{1+\tan\theta}$$

$= \frac{1+\tan^2\theta+2\tan\theta-1-\tan^2\theta+2\tan\theta}{1-\tan^2\theta}$

$= \frac{2.2\tan\theta}{1-\tan^2\theta} = 2\tan 2\theta$

Alternate:-

$\tan\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}-\theta\right)$

Put $\theta = 15°$

$\tan(60°) - \tan(30°)$

$\Rightarrow \left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \frac{2}{\sqrt{3}}$

Now check from option,

Option (a):-

$2\tan 2\theta$

$\Rightarrow 2\tan 2 \times 15°$

$\Rightarrow 2\tan 30° = \frac{2}{\sqrt{3}}$ (Satisfy)

297. **(a)** $\sin(\alpha-\beta) = \cos(\alpha+\beta) = \sin(90° - \alpha - \beta)$

$\alpha - \beta = 90 - \alpha - \beta$

$\alpha = 45°, \beta = 15°$

Alternate:-

$\sin(\alpha-\beta) = \frac{1}{2}$

$\Rightarrow \sin(\alpha-\beta) = \sin 30°$

$\alpha - \beta = 30°$ ...(i)

$\Rightarrow \cos(\alpha+\beta) = \frac{1}{2}$

$\Rightarrow \cos(\alpha+\beta) = \cos 60°$

$\alpha+\beta = 60°$ ...(ii)

Solve eq(i) and (ii) we get

$\alpha = 45°$ and $\beta = 15°$

298. **(a)** $\alpha + \beta = \pi + \gamma$

$\Rightarrow \sin(\alpha+\beta) = \sin(\pi+\gamma)$

$\Rightarrow \sin\alpha\cos\beta + \cos\alpha\sin\beta = -\sin\gamma$

squaring both the sides

$\Rightarrow \sin^2\alpha(1-\sin^2\beta) + \sin^2\beta(1-\sin^2\alpha) + 2\sin\alpha.\cos\beta.\cos\alpha\sin\beta = \sin^2\gamma$

$\Rightarrow \sin^2\alpha - \sin^2\alpha\sin^2\beta + \sin^2\beta - \sin^2\alpha\sin^2\beta + 2\sin\alpha\cos\beta.\cos\alpha\sin\beta = \sin^2\gamma$

$\Rightarrow \sin^2\alpha + \sin^2\beta - \sin^2\gamma$

$= 2\sin\alpha\sin\beta(\sin\alpha\sin\beta - \cos\alpha\cos\beta)$

$= -2\sin\alpha\sin\beta\cos(\alpha+\beta)$

$= -2\sin\alpha\sin\beta\cos(180°+\gamma)$

$= 2\sin\alpha\sin\beta\cos\gamma$

Alternate:-

$\alpha+\beta-\gamma = \pi$

$\therefore \sin^2\alpha + \sin^2\beta - \sin^2\gamma$

Put $\alpha = 90°$

$\beta = 90°$

$\gamma = 0°$

$\therefore \sin^2 90° + \sin^2 90° - \sin^2 0°$

$\Rightarrow 1 + 1 - 0$

$\Rightarrow 2$

Now check from option,

Option (a):- $2\sin\alpha\sin\beta\cos\gamma$

$\Rightarrow 2\sin 90°\sin 90°\cos 0°$

$\Rightarrow 2 \times 1 \times 1 \times 1$

$\Rightarrow 2$ (Satisfy)

299. **(c)** $1 + \cos 2x + \cos 4x + \cos 6x$

$1 + 2\cos^2 x - 1 + 2\cos\left(\frac{4x+6x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)$

$= 2\cos^2 x + 2\cos 5x.\cos x$

$= 2\cos x(\cos x + \cos 5x)$

$= 2\cos x\left[2\cos\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right)\right]$

$= 2\cos x.\ 2\cos 3x.\cos 2x$

$= 4\cos x\cos 2x\cos 3x$

Alternate :

We can put $x = 0°$ in question and all option

$1 + \cos 2x + \cos 4x + \cos 6x$

$= 1 + \cos 0° + \cos 0° + \cos 0° = 4$

Option

(a) $2\cos x\cos 2x\cos 3x = 2$

(b) $4\sin x\cos 2x\cos 3x = 0$

(c) $4\cos x\cos 2x\cos 3x = 4$

(d) $\cos x\cos 2x\cos 3x = 1$

Hence, option (c) is correct

300. **(b)**

divide and multiply by $\sin 72°$,

$\frac{\sin 12°.\sin 48°.\sin 72°.\sin 54°}{\sin 72°}$

Use $\frac{1}{4}\sin 3\theta$

$= \sin(60°-\theta).\sin\theta.\sin(60°+\theta)$

Here, $\theta = 12°$

$= \frac{1}{4}.\frac{\sin 36°.\sin 54°}{2\sin 36°\cos 36°}$

$= \frac{1}{8}.\frac{\sin 54°}{\sin 54°} = \frac{1}{8}$

301. **(d)** $\frac{1}{4}\left(\sqrt{3}\cos 23° - \sin 23°\right)$

$\frac{1}{2}\left(\frac{\sqrt{3}}{2}\cos 23° - \frac{1}{2}\sin 23°\right)$

$= \frac{1}{2}(\cos 30°.\cos 23° - \sin 30°\sin 23°)$

$[\cos A\cos B - \sin A\sin B = \cos(A+B)]$

$= \frac{1}{2}(\cos 53°)$

302. (a) $2\cos x - \cos 3x - \cos 5x$

$= 2\cos x - (\cos 3x + \cos 5x)$

$= 2\cos x - \left[2\cos\left(\frac{3x+5x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

$= 2\cos x - 2\cos 4x\cos x$

$= 2\cos x(1-\cos 4x)$

$= 2\cos x(1 - 1 + 2\sin^2 2x)$

$= 4\cos x\sin^2 2x$

$= 4\cos x(2\sin x.\cos x)^2$

$= 16\cos^3 x.\sin^2 x$

Alternate:-

Put $x = 45°$ in question and all option

$2\cos 45° - \cos 135° - \cos 225°$

$= \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}}$

When $x = 45°$, $(\sin x = \cos x)$

Option (a) $16\cos^5 x = 16\left(\frac{1}{\sqrt{2}}\right)^5$

$= \frac{16}{4\sqrt{2}} = \frac{4}{\sqrt{2}}$

Option (b) $\sin^5 x = \left(\frac{1}{\sqrt{2}}\right)^5 = \frac{1}{4\sqrt{2}}$

Option (c) $4\cos^5 x = 4\left(\frac{1}{\sqrt{2}}\right)^5 = \frac{4}{4\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

Option (d) $4\cos^5 x = 4\left(\frac{1}{\sqrt{2}}\right)^5 = \frac{4}{4\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

Hence, Option (a) is correct.

303. **(c)** $\frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$

$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°.\cos 20°}$

$= \frac{2\left[\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right]}{\sin 20°.\cos 20°}$

$= \frac{2.2[\sin 60°\cos 20° - \cos 60°\sin 20°]}{2.\sin 20°.\cos 20°}$

$= \frac{4\sin 40°}{\sin 40°} = 4$

304.(c) $\frac{\cos 12° - \sin 12°}{\cos 12° + \sin 12°} + \frac{\sin 147°}{\cos 147°}$

$= \frac{\sin 78° - \sin 12°}{\sin 78° + \sin 12°}$

$+ \frac{\sin 147°}{\cos 147°}$

$\left[\begin{array}{l}\sin(180° - \theta) = \sin\theta \\ \cos(180° - \theta) = -\cos\theta\end{array}\right]$

$= \frac{2\cos 45°.\sin 33°}{2\sin 45°.\cos 33°} - \frac{\sin 33°}{\cos 33°} = 0$

305. **(d)** cos24° + cos55° + cos125° + cos204° + cos300°

= cos24° + cos(90° – 35)+ cos(90° + 35°)+ cos(180° + 24°)+ cos(360° – 60°)

=cos24° + sin35°– sin35° – cos24°+ cos60°= $\frac{1}{2}$

306. **(a)** mtan(θ–30°) = ntan(θ+120°)

m tan(θ–30°) = – n cot (θ+30°)

tan (θ–30°).tan(θ + 30°) = $\frac{-n}{m}$

$\frac{\sqrt{3}\tan\theta - 1}{\sqrt{3} + \tan\theta}.\frac{\sqrt{3}\tan\theta + 1}{\sqrt{3} - \tan\theta} = \frac{-n}{m}$

$\left[\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}\right]$

$\frac{3\tan^2\theta - 1}{3 - \tan^2\theta} = \frac{-n}{m}$

$\frac{m}{n} = \frac{3 - \tan^2\theta}{1 - 3\tan^2\theta}$

Use Componendo and Dividendo

$\frac{m+n}{m-n} = \frac{4 - 4\tan^2\theta}{2 + 2\tan^2\theta}$

$= \frac{4(1 - \tan^2\theta)}{2(1 + \tan^2\theta)} = 2\cos 2\theta$

Alternate:-

$\frac{n}{m} = \frac{\tan(\theta - 30°)}{\tan(\theta + 120°)}$

Put θ = 90°

$\Rightarrow \frac{n}{m} = \frac{\tan(90° - 30°)}{\tan(90° + 120°)}$

$\Rightarrow \frac{n}{m} = \frac{\tan 60°}{\tan 210°}$

$\Rightarrow \frac{n}{m} = \frac{\tan 60°}{\tan(180° + 30)°}$

$\Rightarrow \frac{n}{m} = \frac{\tan 60°}{\tan 30°}$

$\Rightarrow \frac{n}{m} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}}$

$\Rightarrow \frac{n}{m} = 3$

$\Rightarrow \frac{m}{n} = \frac{1}{3}$

$\therefore \frac{m+n}{m-n} = \frac{1+3}{1-3} = \frac{4}{-2} = -2$

Now check from option,

Option (a):- 2cos2θ

Put θ = 90°

= 2 cos180°

= 2cos(90° + 90°)

= –2sin90°

= –2 (Satisfy)

307. **(a)** $\tan\theta = \frac{x\sin\phi}{1 - x\cos\phi}$

$\Rightarrow \frac{1}{\tan\theta} = \frac{1}{x\sin\phi} - \cot\phi$

$\frac{1}{\tan\theta} + \cot\phi = \frac{1}{x\sin\phi}$

$\Rightarrow \cot\theta + \cot\phi = \frac{1}{x\sin\phi}$ ....(i)

$\tan\phi = \frac{y\sin\theta}{1 - y\cos\theta}$

$\Rightarrow \frac{1}{\tan\phi} = \frac{1}{y\sin\theta} - \cot\theta$

$\frac{1}{\tan\phi} + \cot\theta = \frac{1}{y\sin\theta}$

$\Rightarrow \cot\phi + \cot\theta = \frac{1}{y\sin\theta}$ ...(ii)

from equation (i) and (ii)

$\frac{1}{x\sin\phi} = \frac{1}{y\sin\theta}$

$\Rightarrow \frac{x}{y} = \frac{\sin\theta}{\sin\phi}$

308. **(d)** $\tan x = \frac{b}{a}$

$= \sqrt{\frac{1 + \frac{b}{a}}{1 - \frac{b}{a}}} + \sqrt{\frac{1 - \frac{b}{a}}{1 + \frac{b}{a}}}$

$= \sqrt{\frac{1 + \tan x}{1 - \tan x}} + \sqrt{\frac{1 - \tan x}{1 + \tan x}}$

Rationalising above equation

$= \sqrt{\frac{1 + \tan x}{1 - \tan x} \times \frac{1 + \tan x}{1 + \tan x}}$

$+ \sqrt{\frac{1 - \tan x}{1 + \tan x} \times \frac{1 - \tan x}{1 - \tan x}}$

$= \sqrt{\frac{(1 + \tan x)^2}{1 - \tan^2 x}} + \sqrt{\frac{(1 - \tan x)^2}{1 - \tan^2 x}}$

$= \frac{1 + \tan x}{\sqrt{1 - \tan^2 x}} + \frac{1 - \tan x}{\sqrt{1 - \tan^2 x}}$

$= \frac{(1 + \tan x) + (1 - \tan x)}{\sqrt{1 - \tan^2 x}} = \frac{2}{\sqrt{1 - \frac{\sin^2 x}{\cos^2 x}}}$

$= \frac{2}{\sqrt{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}} = \frac{2\cos x}{\sqrt{\cos 2x}}$

309. **(c)** $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + \frac{8(1 - \tan^2 4\alpha)}{2\tan 4\alpha}$

$\left(\because \cot 2\theta = \frac{1 - \tan^2\theta}{2\tan\theta}\right)$

$= \tan\alpha + 2\tan 2\alpha + \frac{8\tan^2 4\alpha + 8 - 8\tan^2 4\alpha}{2\tan 4\alpha}$

$= \tan\alpha + 2\tan 2\alpha + \frac{4}{\tan 4\alpha}$

$= \tan\alpha + 2\tan2\alpha + \dfrac{4(1-\tan^2 2\alpha)}{2\tan2\alpha}$

$= \tan\alpha + \dfrac{4\tan^2 2\alpha + 4 - 4\tan^2 2\alpha}{2\tan2\alpha}$

$= \tan\alpha + \dfrac{2}{\tan2\alpha}$

$= \tan\alpha + \dfrac{2(1-\tan^2\alpha)}{2\tan\alpha}$

$= \dfrac{2\tan^2\alpha + 2 - 2\tan^2\alpha}{2\tan\alpha}$

$\dfrac{1}{\tan\alpha} = \cot\alpha$

Alternate:-

$.\tan\alpha + 2\tan2\alpha + 4\tan4\alpha + 8\cot8\alpha$

Put $\alpha = 30^\circ$

$= \tan30^\circ + 2\tan60^\circ + 4\tan120^\circ + 8\cot240^\circ$

$= \tan30^\circ + 2\tan60^\circ - 4\cot30^\circ + 8\cot60^\circ$

$= \dfrac{1}{\sqrt3} + 2\sqrt3 - 4\sqrt3 + \dfrac{8}{\sqrt3}$

$= \dfrac{1}{\sqrt3} - 2\sqrt3 + \dfrac{8}{\sqrt3}$

$= \dfrac{9}{\sqrt3} - 2\sqrt3$

$= 3\sqrt3 - 2\sqrt3 = \sqrt3$

Now check from options

Option (c):- $\cot\alpha = \cot30^\circ$

$= \sqrt3$ (Satisfy)

310. **(b)** $\sin A = n \sin B$

$\dfrac{\sin A}{\sin B} = \dfrac{n}{1}$

applying componendo dividendo rule

$\Rightarrow \dfrac{\sin A + \sin B}{\sin A - \sin B} = \dfrac{n+1}{n-1}$

$\Rightarrow \dfrac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$

$= \dfrac{n+1}{n-1}$

$\Rightarrow \dfrac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)} = \dfrac{n+1}{n-1}$

$\left(\dfrac{n-1}{n+1}\right)\tan\left(\dfrac{A+B}{2}\right) = \tan\left(\dfrac{A-B}{2}\right)$

Alternate:-

$\sin A = n \sin B$

Put, $A = 90^\circ$ & $B = 30^\circ$

then, $n = 2$

$\left(\dfrac{n-1}{n+1}\right)\tan60^\circ = \dfrac{1}{3}\times\sqrt3 = \dfrac{1}{\sqrt3}$

Put these value in options

Option (a) $\sin\left(\dfrac{A-B}{2}\right) = \sin30^\circ = \dfrac{1}{2}$

Option (b) $\tan\left(\dfrac{A-B}{2}\right) = \tan30^\circ = \dfrac{1}{\sqrt3}$

Option (c) $\cot\left(\dfrac{A-B}{2}\right) = \cot30^\circ = \sqrt3$

Option (d) $\tan\left(\dfrac{A+B}{2}\right) = \tan60^\circ = \sqrt3$

Hence, option (b) is correct.

311. **(b)** $2\sin A\cos^3 A - 2\sin^3 A\cos A$

$2\sin A\cos A(\cos^2 A - \sin^2 A)$

$\dfrac{1}{2}(2\sin2A.\cos2A) = \dfrac{1}{2}\sin4A$

Alternate:-

$.2\sin A\cos^3 A - 2\sin^3 A\cos A$

Put $A = 30^\circ$

$2\sin30^\circ\cos^3 30^\circ - 2\sin^3 30^\circ\cos30^\circ$

$\dfrac{3\sqrt3}{8} - 2\times\dfrac{1}{8}\times\dfrac{\sqrt3}{2}$

$\dfrac{3\sqrt3}{8} - \dfrac{\sqrt3}{8} = \dfrac{\sqrt3}{4}$

Now check from options

Option (b):- $\dfrac{1}{2}\sin4A$

Put $A = 30^\circ$

$\Rightarrow \dfrac{1}{2}\times\sin120^\circ$

$\Rightarrow \dfrac{1}{2}\times\sin(90+30^\circ)$

$\Rightarrow \dfrac{1}{2}\times\cos30^\circ$

$\Rightarrow \dfrac{1}{2}\times\dfrac{\sqrt3}{2} = \dfrac{\sqrt3}{4}$ (Satisfy)

312. **(d)** $\tan A + \tan A - \tan A - \cot A$

$= \tan A - \cot A$

313. **(a)** $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$

Put $\alpha = 45^\circ$

$= \dfrac{\frac{2}{\sqrt2}}{1+\frac{1}{\sqrt2}+\frac{1}{\sqrt2}} = y$

$= y = \dfrac{2}{2+\sqrt2} = 2 - \sqrt2$

$= \dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$

$= \dfrac{1-\frac{1}{\sqrt2}+\frac{1}{\sqrt2}}{1+\frac{1}{\sqrt2}} = \dfrac{\sqrt2}{\sqrt2+1}$

$= \dfrac{\sqrt2(\sqrt2-1)}{1} = 2 - \sqrt2 = y$

314. **(c)** $\dfrac{\sin70^\circ+\cos40^\circ}{\cos70^\circ+\sin40^\circ}$

$= \dfrac{\cos20^\circ+\cos40^\circ}{\sin20^\circ+\sin40^\circ}$

$= \dfrac{2\cos30^\circ.\cos10^\circ}{2\sin30^\circ.\cos10^\circ}$

$= \cot30^\circ = \sqrt3$

315. **(a)** $\dfrac{\sin^4 A}{a} + \dfrac{\cos^4 A}{b} = \dfrac{1}{a+b}$

$\Rightarrow \left(\dfrac{a+b}{a}\right)\sin^4 A + \left(\dfrac{a+b}{b}\right)\cos^4 A = 1$

$\Rightarrow \left(\dfrac{a+b}{a}\sin^2 A\right)\sin^2 A +$

$\left(\dfrac{a+b}{b}\cos^2 A\right)\cos^2 A = 1$

$\left(\because \dfrac{a+b}{a}\sin^2 A = 1 \,\&\, \dfrac{a+b}{b}\cos^2 A = 1\right)$

$\Rightarrow \sin^2 A = \dfrac{a}{a+b}$ & $\cos^2 A = \dfrac{b}{a+b}$

Now,

$\Rightarrow \dfrac{\sin^8 A}{a^3} + \dfrac{\cos^8 A}{b^3}$

$= \dfrac{a^4}{(a+b)^4 a^3} + \dfrac{b^4}{(a+b)^4 b^3}$

$= \dfrac{a+b}{(a+b)^4} = \dfrac{1}{(a+b)^3}$

Alternate:-

Put A = 45° and a = 1, b = 1

$$\Rightarrow \frac{\sin^4 45°}{1} + \frac{\cos^4 45°}{1} = \frac{1}{1+1}$$

$$\Rightarrow \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{2} \quad \text{(Satisfy)}$$

$$\therefore \frac{\sin^8 A}{a^3} + \frac{\cos^8 A}{b^3}$$

$$\frac{\sin^8 45°}{1^3} + \frac{\cos^8 45°}{1^3}$$

$$\frac{1}{16} + \frac{1}{16} = \frac{1}{8}$$

Now check from option

Option (a):- $\frac{1}{(a+b)^3}$

$$\Rightarrow \frac{1}{(1+1)^3} \Rightarrow \frac{1}{8} \text{ (Satisfy)}$$

316. (a) $2y\cos\theta = x\sin\theta$

$= 2y\text{cosec}\,\theta = x\sec\theta$

Put value of $x\sec\theta$ as following

$2x\sec\theta - y\text{cosec}\,\theta = 3$

$2.2y\,\text{cosec}\,\theta - y\text{cosec}\,\theta = 3$

$4y\,\text{cosec}\,\theta - y\text{cosec}\,\theta = 3$

$3y\,\text{cosec}\,\theta = 3$

$= y\,\text{cosec}\,\theta = 1$

or $\sin\theta = y$

then, $2y\cos\theta = xy$

$\Rightarrow 2\cos\theta = x$

Hence,

$x^2 + 4y^2 = 4\cos^2\theta + 4\sin^2\theta = 4$

Alternate:-

Put $\theta = 45°$

$2y\cos 45° = x\sin 45° \Rightarrow x = 2y$

$2x.\sec 45° - y\,\text{cosec}\,45° = 3$

$\sqrt{2}(2x - y) = 3 \Rightarrow \sqrt{2}(4y - y) = 3$

$$y = \frac{1}{\sqrt{2}}$$

$$x = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$x^2 + 4y^2 = 2 + 4\times\frac{1}{2} = 4$$

317. (d) $\tan\theta - \cot\theta$

$= a \,\&\, \cos\theta + \sin\theta = b$

Put, $\theta = 45°$

$\tan 45° - \cot 45° = a,$

$\cos 45° + \sin 45° = b,$

$$a = 0, \ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = b$$

$$\frac{1+1}{\sqrt{2}} = b \Rightarrow b = \sqrt{2}$$

$(b^2 - 1)^2 (a^2 + 4)$

$$= \left[\left(\sqrt{2}\right)^2 - 1\right]^2 (0 + 4)$$

$= (2-1)^2 (4) = 4$

318. (a) $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$

$$\frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$$

let, $\sin\alpha - \cos\alpha = k\sin\theta$

$\sin\alpha + \cos\alpha = k\cos\theta$

Adding after squaring,

$\sin^2\alpha + \cos^2\alpha - 2\sin\alpha\cos\alpha +$
$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha$

$= k^2(\sin^2\theta + \cos^2\theta)$

$2 = k^2$

$k = \sqrt{2}$

Hence, $\sin\alpha - \cos\alpha = \sqrt{2}\sin\theta$

$\sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$

319. (b) $\sin\theta + \sin\phi = a,$

$\cos\theta + \cos\phi = b$

Put, $\theta = 90° \,\&\, \phi = 30°$

$$a = 1 + \frac{1}{2} = \frac{3}{2}, \quad b = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{\theta - \phi}{2}\right) = \tan 30° = \frac{1}{\sqrt{3}}$$

option (b) $\Rightarrow \sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$

$$= \sqrt{\frac{4 - \frac{9}{4} - \frac{3}{4}}{\frac{9}{4} + \frac{3}{4}}} = \frac{1}{\sqrt{3}}$$

Hence, option (b) is correct

320. (a) Answer is independent of α so put $\alpha = 0°$

$\cos^2\alpha + \cos^2(\alpha + 120°) + \cos^2(\alpha - 120°)$

$= \cos^2 0° + \cos^2 120° + \cos^2(-120°)$

$$= 1 + \frac{1}{4} + \frac{1}{4} = \frac{3}{2}$$

321.(a) $\tan A = \frac{1 - \cos B}{\sin B}$

$$= \frac{1 - \cos 2\times\frac{B}{2}}{\sin 2\times\frac{B}{2}} = \frac{1 - 1 + 2\sin^2\frac{B}{2}}{2\sin\frac{B}{2}\cos\frac{B}{2}}$$

$$\tan A = \frac{\sin\frac{B}{2}}{\cos\frac{B}{2}} = \tan\frac{B}{2}$$

$$A = \frac{B}{2} \Rightarrow B = 2A$$

$\tan 2A = \tan B$

322. (c) $\tan 2A = \tan(A + B + A - B)$

$$= \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B).\tan(A-B)} = \frac{p+q}{1-pq}$$

323. (b) $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\frac{1}{\cos 8A} - 1}{\frac{1}{\cos 4A} - 1}$

$$= \frac{1 - \cos 8A}{1 - \cos 4A} \times \frac{\cos 4A}{\cos 8A}$$

$$= \frac{2\sin^2 4A}{2\sin^2 2A} \times \frac{\cos 4A}{\cos 8A}$$

$$= \frac{2\sin 4A\cos 4A.\sin 4A}{\cos 8A \times 2\sin^2 2A}$$

$$= \tan 8A.\frac{\cos 2A}{\sin 2A} = \frac{\tan 8A}{\tan 2A}$$

Alternate:-

$$\frac{\sec 8A - 1}{\sec 4A - 1}$$

Put A = 15°

$$\Rightarrow \frac{\sec 120° - 1}{\sec 60° - 1}$$

$$\Rightarrow \frac{\frac{1}{\cos 120°} - 1}{\frac{1}{\cos 60°} - 1}$$

$$\Rightarrow \frac{-\frac{1}{\sin 30°} - 1}{\frac{1}{\cos 60°} - 1} = \frac{-2-1}{2-1} = -3$$

Now check from options

Option (b):- $\frac{\tan 8A}{\tan 2A}$

Put A = 15°

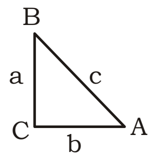
$\frac{\tan 120°}{\tan 30°} = \frac{-\cot 30°}{\tan 30°}$

$= -\sqrt{3} \times \sqrt{3} = -3$ (Satisfy)

324. (d) $\alpha + \beta = \tan A + \tan B = \frac{a}{b} + \frac{b}{a}$

$= \frac{a^2 + b^2}{ab}$

$\alpha\beta = \tan A . \tan B = \frac{a}{b} . \frac{b}{a} = 1$



then equation

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - \left(\frac{a^2+b^2}{ab}\right)x + 1 = 0$

$abx^2 - (a^2 + b^2)x + ab = 0$

$abx^2 - c^2x + ab = 0$

$(\because a^2 + b^2 = c^2)$

325. (d) $\cos(A - B) = \frac{3}{5}$ and

$\tan A \tan B = \frac{\sin A \sin B}{\cos A \cos B} = 2$

$\sin A \sin B = 2\cos A\cos B$.. (i)

$\cos A \cos B + \sin A \sin B = \frac{3}{5}$

From eq (i)

$3\cos A \cos B = \frac{3}{5}$

$\cos A \cos B = \frac{1}{5}$

$\sin A \sin B = \frac{2}{5}$

326. (a)

$\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = \frac{\sin 81° + \sin 9°}{\sin 81° - \sin 9°}$

$= \frac{2\sin 45° . \cos 36°}{2\cos 45° . \sin 36°} = \cot 36°$

$= \tan 54°$

327. (b) $\tan\alpha = \frac{1}{7}$, $\tan\beta = \frac{1}{3}$

$\cos 2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$

$= \frac{1 - \frac{1}{49}}{1 + \frac{1}{49}} = \frac{48}{50} = \frac{24}{25}$

$\sin 2\beta = \frac{2\tan\beta}{1 + \tan^2\beta} = \frac{\frac{2}{3}}{1 + \frac{1}{9}}$

$= \frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$

$\cos 2\beta = \frac{4}{5}$

$\sin 4\beta = 2\sin 2\beta \cos 2\beta$

$= 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

hence, $\cos 2\alpha = \sin 4\beta$

328. (b) $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \frac{1}{4}}{1 + \frac{1}{4}} = \frac{3}{5}$

$\sin 2B = \frac{2\tan B}{1 + \tan^2 B} = \frac{\frac{2}{3}}{1 + \frac{1}{9}}$

$= \frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$

Hence, $\cos 2A = \sin 2B$

329. (c) Put $\beta = 0°$ such that no option are same

$2\sin^2\beta + 4\cos(\alpha + \beta)$.

$\sin\alpha . \sin\beta + \cos^2(\alpha + \beta)$

$= 0 + 0 + \cos^2\alpha$

$= \cos^2\alpha$ option (c) (We can also put $\alpha = 0°$)

330. (d)

$\frac{(\sin 12° . \sin 48° . \sin 72°).(\sin 24° . \sin 36° . \sin 84)}{\sin 72° . \sin 36°}$

Multiply and divide by sin36° & sin72°

$\sin\theta . \sin(60 - \theta) . \sin(60 + \theta) = \frac{1}{4}\sin 3\theta$

$= \frac{1}{4}\sin 36° \times \frac{1}{4}\sin 72° \times \frac{1}{\sin 72° \sin 36°} = \frac{1}{16}$

331. (a) $\tan 5x = \frac{\tan 3x + \tan 2x}{1 - \tan 3x \tan 2x}$

$\Rightarrow \tan 5x - \tan 5x \tan 3x \tan 2x = \tan 3x + \tan 2x$

$\Rightarrow \tan 5x \tan 3x \tan 2x = \tan 5x - \tan 3x - \tan 2x$

332. (d) $\cos\alpha + \cos\beta = \sin\alpha + \sin\beta$

Squaring both sides

$\Rightarrow \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = \sin^2\alpha + \sin^2\beta + 2\sin\alpha . \sin\beta$

$\Rightarrow (\cos^2\alpha - \sin^2\alpha) + (\cos^2\beta - \sin^2\beta) + 2(\cos\alpha . \cos\beta - \sin\alpha \sin\beta) = 0$

$\Rightarrow \cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta) = 0$

$\Rightarrow \cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$

333. (c) $\cos A = a\cos B \Rightarrow \cos^2 A = a^2\cos^2 B = a^2(1 - \sin^2 B)$ .....(i)

$\sin A = b\sin B \Rightarrow \sin^2 A = b\sin^2 B$ ....(ii)

Adding both equation

$\cos^2 A + \sin^2 A = a^2 - a^2\sin^2 B + b^2\sin^2 B$

$1 = a^2 + \sin^2 B(b^2 - a^2)$

$(b^2 - a^2)\sin^2 B = 1 - a^2$

Alternate:-

$a = \frac{\cos A}{\cos B}$, $b = \frac{\sin A}{\sin B}$

Put A = 0°, B = 60°

a = 2, b = 0

$\therefore (b^2 - a^2)\sin^2 B = (0^2 - 2^2) \times \sin^2 60°$

$= -4 \times \frac{3}{4} = -3$

Now check from option

Option (c):- $1 - a^2$

Put a = 2

$1 - (2)^2 = 1 - 4 = -3$ (Satisfy)

334. (c) Put A = 90°, B = 60°, C = 30°

$(\because A + B + C = \pi)$

then, cos 2A + cos2B + cos2C

= cos 180° + cos120° + cos60°

$= -1 - \frac{1}{2} + \frac{1}{2} = -1$

Put these value in all option

(a) = 1+4cos A. cos B. cos C = 1

(b) = −1+ 4 sin A sin B. cosC

$= -1 + 4 \times 1 \times \frac{\sqrt{3}}{2} . \frac{\sqrt{3}}{2}$

$= -1 + 3 = 2$

(c) $= -1 - 4\cos A \cos B . \cos C$
$= -1 - 0 = -1$

(d) $= 1 + 4 \sin A \sin B . \sin C$

$$= 1 + 4 \times 1 \times \frac{\sqrt{3}}{2} . \frac{1}{2} = 1 + \sqrt{3}$$

Hence, option (c) is correct

335. (b) Let A = B = 45° and C = 90°
$\sin^2 A + \sin^2 B + \sin^2 C - 2\cos A . \cos B . \cos C$

$$= \frac{1}{2} + \frac{1}{2} + 1 - 2 \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times 0 = 2$$

336. (c) $\sin 2A + \sin 2B + \sin 2C$
$= 2\sin (A+B).\cos (A-B) + 2 \sin C . \cos C$
$= 2\sin (180° - C).\cos (A-B) + 2\sin C . \cos C$
$= 2\sin C [\cos(A-B) + \cos C]$
$= 2 \sin C [\cos(A-B) + \cos(180° - (A+B)]$
$= 2 \sin C [\cos(A - B) - \cos (A+B)]$
$= 2 \sin C (2\sin A . \sin B) = 4 \sin A \sin B \sin C$

Alternate:-
A+B+C=180°
Put A = B= 45° and C = 90° in equation and option too.
$\sin 2A + \sin 2B + \sin 2C = \sin 90° + \sin 90° + \sin 180°$
$= 1+1+0=2$
Now check for option as

(a) =0 (b) =0

(c) $= 4 \times \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} .1 = 2$

(d) $= 8 \times \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} .1 = 4$

Option (c) is correct.

337. (c) $\tan 10° - \tan 50° + \tan 70°$

$\Rightarrow \tan 10° + \tan (60°+10°) - \tan (60° - 10°)$

$$\Rightarrow \tan 10° + \frac{\tan 60° + \tan 10°}{1 - \tan 60° . \tan 10°} - \frac{\tan 60° - \tan 10°}{1 + \tan 60° . \tan 10°}$$

$$\Rightarrow \tan 10° + \frac{\sqrt{3} + \tan 10°}{1 - \sqrt{3}\tan 10°} - \frac{\sqrt{3} - \tan 10°}{1 + \sqrt{3}\tan 10°}$$

$$\Rightarrow \tan 10° + \frac{8\tan 10°}{1 - 3\tan^2 10}$$

$$\Rightarrow \frac{9\tan 10° - 3\tan^3 10°}{1 - 3\tan^2 10°}$$

$$= \frac{3(3\tan 10° - \tan^3 10°)}{1 - 3\tan^2 10°}$$

$$\left[\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right]$$

$= 3 \tan 30° = \sqrt{3}$

338. (a) Solve it by option

put, $\alpha = \frac{\pi}{6}$

$$4 \cos \frac{\pi}{6} + 3 \cos 2\frac{\pi}{6} - 2 \sin 3\frac{\pi}{6} + \cos 4\frac{2\pi}{6}$$

$$= 4.\frac{\sqrt{3}}{2} + \frac{3}{2} - 2 - \frac{1}{2}$$

$$= \frac{4\sqrt{3}}{2} - 1 = 2\sqrt{3} - 1$$

hence, option (a) is correct.

339. (a) $(\sin A + \sin B + \sin C)^2 = \sin^2 A + \sin^2 B + \sin^2 C$
As we know that
$(a+ b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\therefore$ 2(sinA sin B + sin B sin C + sin C sin A) =0
$\Rightarrow$ sin A sin B + sin B sin C + sin C sinA = 0
above condition will be true only when
A = B = C =0
$\therefore$ sin A + sin B + sin C =0

340. (b) $\frac{\sin x}{\sin y} = p$ & $\frac{\cos x}{\cos y} = q$

$$\frac{\sin x}{p} = \sin y$$

$$\Rightarrow \frac{\sin^2 x}{p^2} = \sin^2 y$$

$$\frac{\cos x}{q} = \cos y \Rightarrow \frac{\cos^2 x}{q^2} = \cos^2 y$$

Adding both eqation

$$\frac{\sin^2 x}{p^2} + \frac{\cos^2 x}{q^2} = \sin^2 y + \cos^2 y = 1$$

divide by $\cos^2 x$

$$\frac{\tan^2 x}{p^2} + \frac{1}{q^2} = \frac{1}{\cos^2 x}$$

$= \sec^2 x = 1 + \tan^2 x$

$$\tan^2 x \left(\frac{1}{p^2} - 1\right) = 1 - \frac{1}{q^2}$$

$$\tan^2 x = \frac{\frac{q^2-1}{q^2}}{\frac{1-p^2}{p^2}}$$

$$\tan x = \frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$$

Alternate:-

$$p = \frac{\sin x}{\sin y}$$

$$q = \frac{\cos x}{\cos y}$$

Put x = 30° and y = 60°

$$p = \frac{\sin 30°}{\sin 60°} = \frac{1}{\sqrt{3}}$$

$$q = \frac{\cos 30°}{\cos 60°} = \sqrt{3}$$

$$\therefore \tan x = \tan 30° = \frac{1}{\sqrt{3}}$$

Now check from options

Option (b):- $\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$

$$\Rightarrow \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}}\sqrt{\frac{(\sqrt{3})^2 - 1}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}}$$

$$\Rightarrow \frac{1}{3}\sqrt{\frac{3-1}{\frac{3-1}{3}}}$$

$$\Rightarrow \frac{1}{3}\sqrt{\frac{2}{2} \times 3}$$

$$\Rightarrow \frac{1}{\sqrt{3}}$$ (Satisfy)

341. (a) $y = a \cos^2 x + 2b \sin x \cos x + c \sin^2 x$
$z = a \sin^2 x - 2b \sin x \cos x + c \cos^2 x$
$y + z = a (\sin^2 x + \cos^2 x) + c (\sin^2 x + \cos^2 x)$
$y + z = a + c$

342.(c) $A + B + C = \pi$
answer is independent of A, B & C
So put A = B = C = 60°

$$= \frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B}$$

$= \frac{\cos 60^\circ}{\sin 60^\circ.\sin 60^\circ} + \frac{\cos 60^\circ}{\sin 60^\circ.\sin 60^\circ} + \frac{\cos 60^\circ}{\sin 60^\circ.\cos 60^\circ}$

$= 3 \times \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}} = 3.\frac{\frac{1}{2}}{\frac{3}{4}} = 2$

343. (a) sinA. cosA & tanA are in G.P.
then, $\cos^2 A = \sin A.\tan A$

$\cos^2 A = \frac{\sin A.\sin A}{\cos A}$

$\cos^3 A = \sin^2 A = 1 - \cos^2 A$

$\cos^3 A + \cos^2 A = 1$

344. (d) A + B = C, tan A = k tan B and A – B = ϕ

$\frac{\sin A}{\cos A} = \frac{k \sin B}{\cos B}$

$\frac{\sin A \cos B}{\cos A \sin B} = \frac{k}{1}$

Using componendo & dividendo rule

$\frac{\sin A \cos B + \cos A \sin B}{\cos B \sin A - \cos A \sin B} = \frac{k+1}{k-1}$

$\frac{\sin(A+B)}{\sin(A-B)} = \frac{k+1}{k-1}$

$\frac{\sin C}{\sin \phi} = \frac{k+1}{k-1}$

$\sin C = \frac{k+1}{k-1} \sin \phi$

345.(b) $\tan\alpha$, $\tan\beta$ are roots of $x^2 + px + q = 0$

$\tan\alpha + \tan\beta = -p$

$\tan\alpha.\tan\beta = q$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$

$= \frac{-p}{1-q} = \frac{p}{q-1}$

Alternate:-

$x^2 + px + q = 0$

Put $\alpha = 45^\circ$

$\beta = 0^\circ$

$\tan 45^\circ = 1$

$\tan 0^\circ = 0$

$\therefore\ 1^2 + p \times 1 + q = 0$

$p + q = -1$ ...(i)

$0^2 + p \times 0 + q = 0$

$q = 0$ ...(ii)

From (i) and (ii)

$p = -1$ and $q = 0$

$\therefore\ \tan(\alpha+\beta) = \tan 45^\circ = 1$

Now check from options,

Option (b):- $\frac{p}{q-1} = \frac{-1}{0-1}$

$= 1$ (Satisfy)

346.(a)If A + B + C = $\frac{3\pi}{2}$

Put A = B = C = $\frac{\pi}{2}$ in question and all options.

$\cos 2A + \cos 2B + \cos 2C = \cos\pi + \cos\pi + \cos\pi = -3$

option (a)

$= 1 - 4\sin\frac{\pi}{2}.\sin\frac{\pi}{2}.\sin\frac{\pi}{2} = 1 - 4$

$= -3$

347. (c) $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

$\frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y} = \frac{a+b}{a-b}$

Using componendo & dividendo rule

$\frac{2\sin x.\cos y}{2\cos x.\sin y} = \frac{2a}{2b}$

$= \frac{\tan x}{\tan y} = \frac{a}{b}$

348.(d) $\tan\frac{\pi}{3} = \tan\left(\frac{2\pi}{5} - \frac{\pi}{15}\right)$

$\tan\frac{\pi}{3} = \frac{\tan\frac{2\pi}{5} - \tan\frac{\pi}{15}}{1 + \tan\frac{2\pi}{5}\tan\frac{\pi}{15}}$

$\Rightarrow \tan\frac{\pi}{3} + \tan\frac{\pi}{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}$

$= \tan\frac{2\pi}{5} - \tan\frac{\pi}{15}$

$\Rightarrow \tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}$

$\tan\frac{\pi}{15} = \sqrt{3}$

349. (a) Answer is independent of A, B, C and A + B + C = 180°
So put, A = B = C = 60°

$\tan^2\frac{A}{2} + \tan^2\frac{B}{2}\tan^2\frac{C}{2} = \tan^2 30^\circ + \tan^2 30^\circ + \tan^2 30^\circ$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

350. (a) $\frac{\cot^2 15^\circ - 1}{\cot^2 15^\circ + 1}$

$= \frac{\cos^2 15^\circ - \sin^2 15^\circ}{\cos^2 15^\circ + \sin^2 15^\circ}$

divide by $\cos^2 15^\circ$ all terms

$= \frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} = \cos 2 \times 15^\circ$

$= \cos 30^\circ = \frac{\sqrt{3}}{2}$

CHAPTER 13

MAXIMUM AND MINMUM VALUE OF TRIGONOMETRIC FUNCTIONS

Limit of the values of Trignometric Functions:

	Min	Max
$\sin\theta$ & $\cos\theta$ (odd power)	-1	$+1$
$\sin^2\theta$ & $\cos^2\theta$ (even power)	0	$+1$
$\tan\theta$ & $\cot\theta$ (odd power)	$-\infty$	$+\infty$
$\tan^2\theta$ & $\cot^2\theta$ (even power)	0	∞
$\sec\theta$ & $\text{cosec}\,\theta$ (odd power)	$-\infty$	$+\infty$
$\sec^2\theta$ & $\text{cosec}^2\theta$ (even power)	1	∞

Note:- The value of **$\sec\theta$ & $\text{cosec}\,\theta$** can be anything $-\infty$ to $+\infty$ but value of $\sec\theta$ & $\text{cosec}\,\theta$ can't be between **– 1 & 1** but it can be – 1 & 1.

Increasing and Decreasing functions :

(i) The value of $\sin\theta$ increases from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$. As θ increases in this interval, then $\sin\theta$ also increases.

(ii) **Ist quadrant :** $\sin\theta$ increases from 0 to 1; $\cos\theta$ decreases from 1 to 0 and $\tan\theta$ increases from 0 to ∞.

(iii) **2nd quadrant :** $\sin\theta$ decreases from 1 to 0; $\cos\theta$ decreases from 0 to -1 and $\tan\theta$ decreases, from ∞ to 0.

(iii) **3rd quadrant :** $\sin\theta$ decreases from 0 to -1; $\cos\theta$ increases from -1 to 0; $\tan\theta$ increases from 0 to ∞.

(iv) **4th quadrant :** $\sin\theta$ increases from -1 to 0; $\cos\theta$ increases from 0 to 1; $\tan\theta$ decreases from ∞ to 0.

TYPE-I

Ex.1 Choose the correct statements among following.

(a) cos40°> cos70°
(b) sin35°> sin65°
(c) tan45°<tan46°
(d) cot40°< cot39°
(e) sec20°>sec40°
(f) cosec20°<cosec30°

Sol. When $0<\theta<90°$,

(a) Value of $\cos\theta$ decreases as θ increases, hence, 40° <70° ⇒ cos40° >cos70°. The statement is true.

(b) As θ increases, value of $\sin\theta$ also increase, hence 65°> 35°
⇒ sin65°>sin35°. Hence, the statement is false.

(c) As θ increases value of $\tan\theta$ increases, hence 45°<46°
⇒ tan45° <tan46°. Hence, given statement is true.

(d) As θ increases, value of $\cot\theta$ decreases, hence 40°>39°
⇒ cot40°<cot39°. Hence, the statement is true.

(e) As θ increases, value of $\sec\theta$ increases hence 20° <40°
⇒ sec20° <sec40°. Hence, the statement is false.

(f) As θ increases, value of $\text{cosec}\,\theta$ decreases, Hence 20°<30°
⇒ cosec20°>cosec30°. Hence, given statement is false.

Ex.2 Which statement is incorrect

(a) $\sin\theta = \frac{3}{4}$ (b) $\cos\theta = \frac{1}{3}$

(c) $\sec\theta = 4$ (d) $\text{cosec}\,\theta = \frac{2}{3}$

Sol. option (d) is incorrect because $\text{cosec}\,\theta = \frac{2}{3}$,

it means $\sin\theta = \frac{3}{2}$, it is not possible

therefore $\sin\theta$ maximum value is 1

Ex.3 Find minimum & maximum value of $15+\sin^2\theta$

Sol. $\sin^2\theta$

min. 0 max. +1

min. value = 15+0 = 15
max. value = 15+1 = 16

Ex.4 Find minimum & maximum value of $15-\sin^2\theta$

Sol. $\sin^2\theta$

min. 0 max. +1

max. value = 15–0 = 15
min. value = 15–1 = 14

Ex.5 Find min & max value of $10+\sec^2\theta$

Sol. $\sec^2\theta$

min. +1 max. ∞

min value = 10+1 =11
max. value = $10+\infty = \infty$

* maximum value will not be asked & equal to ∞

Ex.6 Find minimum & maximum value of $10\sin\theta - 1$

Sol. $\sin\theta$

min. –1 max. +1

max. value = 10 ×(1) –1 = 9
min. value = 10×(–1) –1 = –11

Ex.7 Find minimum & maximum value of $12\sin^2\theta - 3$?

Sol. $12\sin^2\theta - 3$

max. value = $12(1) - 3 = 9$

min value = $12 \times (0) - 3 = -3$

Ex.8 Find minimum & maximum value of $11 + \cos^2\theta$

Sol.

$\cos^2\theta$

max. +1   min. 0

max. = $11 + 1 = 12$

min. = $11 + 0 = 11$

TYPE – II

(i) In the expression of $a\sin^2\theta + b\cos^2\theta$.

if a > b	if b > a
max = a	max = b
min = b	min = a

Ex.9 find the maximum and minimum value of $15\sin^2\theta + 10\cos^2\theta$.

Sol. $15\sin^2\theta + 10\cos^2\theta$

$= 15\sin^2\theta + 10(1 - \sin^2\theta)$

$= 15\sin^2\theta + 10 - 10\sin^2\theta$

$= 5\sin^2\theta + 10$

$= 10 + 5\sin^2\theta$

$(0 < \sin^2\theta < 1)$

max. = $10 + 5 \times 1 = 15$

min. = $10 + 5 \times 0 = 10$

Alternate:-

$15\sin^2\theta + 10\cos^2\theta$

max. = 15 (use above identity)

min. = 10

Ex.10 find the maximum and minimum value of $4\sin^2\theta + 7\cos^2\theta + 5$

Sol. $4\sin^2\theta + 7\cos^2\theta + 5$

min = $4 + 5 = 9$

max = $7 + 5 = 12$

Ex.11 Find the maximum and minimum value of

$12\sin^2\theta - 17\cos^2\theta$

Sol. $12\sin^2\theta + (-17)\cos^2\theta$

max. = 12

min = –17

Ex.12 Find the maximum and minimum value of

$-14\sin^2\theta - 21\cos^2\theta$

Sol. max = –14

min = – 21

(ii) when angle is different & Independent then.

$\sin^2\alpha$	$\cos^2\beta$
max = +1	max = +1
min = 0	min = 0

Ex.13 Find the maximum and minimum value of $7\sin^2\alpha + 20\cos^2\beta$

Sol. $7\sin^2\alpha + 20\cos^2\beta$

Now,

max. = $7 \times 1 + 20 \times 1 = 27$

min. = $7 \times 0 + 20 \times 0 = 0$

Ex.14 Find the maximum and minimum value of $10\tan^2\theta + 15\sec^2\theta$

Sol. $10\tan^2\theta + 15\sec^2\theta$

$= 10\tan^2\theta + 15(1 + \tan^2\theta)$

$= 10\tan^2\theta + 15 + 15\tan^2\theta$

$= 25\tan^2\theta + 15$

min. value = $25 \times 0 + 15$

$= 15$ ($\because \tan^2\theta$ min = 0)

* When $a\sec^2\theta + b\tan^2\theta$ is given then coefficient of $\sec^2\theta$ is value minimum is equal to a.

(iii) If expression with $a\sin\theta \pm b\cos\theta$

max. value = $\sqrt{a^2 + b^2}$

min. value = $-\sqrt{a^2 + b^2}$

Ex.15 Find the maximum and minimum value of $5\sin\theta + 12\cos\theta$

Sol. $5\sin\theta + 12\cos\theta$

maxi. value = $\sqrt{(5)^2 + (12)^2} = \sqrt{25 + 144} = 13$

mini. value = $-\sqrt{(5)^2 + (12)^2} = -13$

Ex.16 Find the maximum and minimum value of $5\cos\theta + 3\cos(\theta + 60^\circ) + 5$

Sol. $5\cos\theta + 3\cos(\theta + 60^\circ) + 5$

$\Rightarrow 5\cos\theta + 3(\cos\theta\cos 60^\circ - \sin\theta\sin 60^\circ) + 5$

$\Rightarrow 5\cos\theta + 3\left(\cos\theta \times \frac{1}{2} - \sin\theta\frac{\sqrt{3}}{2}\right) + 5$

$\Rightarrow 5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 5$

$\Rightarrow \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 5$

max. value

$= \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} + 5$

$= \sqrt{\frac{169}{4} + \frac{27}{4}} + 5 = \frac{14}{2} + 5 = 12$

min. value $= -\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}$

$+5 = -7 + 5 = -2$

Ex.17 Find the maximum and minimum value of

$16^{\sin\theta} \cdot 4^{\sin\theta} \cdot 2^{\cos\theta} \cdot 128^{\cos\theta}$

Sol. $16^{\sin\theta} \cdot 4^{\sin\theta} \cdot 2^{\cos\theta} \cdot 128^{\cos\theta}$

$\Rightarrow (2^4)^{\sin\theta} \cdot (2^2)^{\sin\theta} \cdot (2)^{\cos\theta} \cdot (2^7)^{\cos\theta}$

$\Rightarrow 2^{4\sin\theta} \cdot 2^{2\sin\theta} \cdot 2^{\cos\theta} \cdot 2^{7\cos\theta}$

$\Rightarrow 2^{6\sin\theta + 8\cos\theta}$

Max. Value $6\sin\theta + 8\cos\theta$.

$= \sqrt{6^2 + 8^2} = 10$

Now,

max. value = 2^{10}

min. value = 2^{-10}

Ex.18 Find the maximum and minimum value of $27^{\sin x} \times 81^{\cos x}$

Sol. $27^{\sin x} \times 81^{\cos x}$

$\Rightarrow (3^3)^{\sin x} \times (3^4)^{\cos x}$

$\Rightarrow 3^{3\sin x} \times 3^{4\cos x}$

$\Rightarrow 3^{3\sin x + 4\cos x}$

max. value = $3\sin x + 4\cos x$

$= \sqrt{(3)^2 + (4)^2} = 5$

so, max. value = 3^5

min. value = 3^{-5}

Ex.19 find maximum & minimum value of $5\sin^2 x - 12\sin x\cos x + 10\cos^2 x$

Sol. $5\sin^2 x - 12\sin x\cos x + 10\cos^2 x$

$\Rightarrow (4\sin^2 x - 12\sin x\cos x + 9\cos^2 x) + \cos^2 x + \sin^2 x$

$\Rightarrow (2\sin x - 3\cos x)^2 + 1$

for minimum value of $2\sin x - 3\cos x = 0$

then,

minimum value = 1

for maximum value

$2\sin x - 3\cos x = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$

maximum value = $1 + \left(\sqrt{13}\right)^2$

$= 1 + 13 = 14$

Ex.20 Find the minimum & maximum value of $10 \sin\theta . \cos\theta + 1 - 2\sin^2\theta$

Sol. $10 \sin\theta . \cos\theta + 1 - 2\sin^2\theta$

$\Rightarrow$ $5 \times 2 \sin\theta \cos\theta + 1 - 2\sin^2\theta$

$\Rightarrow$ $5\sin 2\theta + 1\cos 2\theta$

max. $= +\sqrt{5^2+1^2} = +\sqrt{26}$

min. $= -\sqrt{26}$

TYPE-III

In expression of $\sin^{2n}\theta$ + $\cos^{2n}\theta$.
maximum value = 1
minimum value = put θ = 45°

Ex.21 Find the minimum & maximum value of $\sin^4\theta + \cos^4\theta$°

Sol. $\sin^4\theta + \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta \cos^2\theta$

$= 1 - 2\sin^2\theta \cos^2\theta$

multiply & divide by 2

$= 1 - \dfrac{2\times 2}{2} \sin^2\theta \cos^2\theta$

$= 1 - \dfrac{1}{2} (2 \sin\theta \cos\theta)^2$

$= 1 - \dfrac{1}{2} \sin^2 2\theta .$

maxi. value $= 1 - \dfrac{1}{2} \times (0) = 1$

mini. value $= 1 - \dfrac{1}{2} (1) = \dfrac{1}{2}$

Alternate:-

$\sin^4\theta + \cos^4\theta$

maxi. value = 1

for minimum value = θ = 45°

mini. value = $\sin^4 45° + \cos^4 45°$

$= \left(\dfrac{1}{\sqrt{2}}\right)^4 + \left(\dfrac{1}{\sqrt{2}}\right)^4 = \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$

Ex.22 Find the minimun & maximum value of $\sin^6\theta + \cos^6\theta$.

Sol. $\sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta \cos^2\theta (\sin^2\theta + \cos^2\theta)$

$= 1 - 3\sin^2\theta \cos^2\theta$

multiply & divide by 4

$= 1 - \dfrac{3}{4} \times 4 \sin^2\theta \cos^2\theta$

$= 1 - \dfrac{3}{4} (2\sin\theta \cos\theta)^2$

$= 1 - \dfrac{3}{4} \sin^2 2\theta .$

max. value $= 1 - \dfrac{3}{4} \times (0) = 1$

min. value $= 1 - \dfrac{3}{4} \times (1) = \dfrac{1}{4}$

Alternate:-

$\sin^6\theta + \cos^6\theta$.

maxi. value = 1

mini. value $= \left(\dfrac{1}{\sqrt{2}}\right)^6 + \left(\dfrac{1}{\sqrt{2}}\right)^6$

$= \dfrac{1}{8} + \dfrac{1}{8} = \dfrac{1}{4}$

Ex.23 Find the minimum & maximum value of $\sin^2\theta + \cos^4\theta$.

Sol. $\sin^2\theta + \cos^4\theta$

max. value = 1

min. value put θ = 45°

$= \left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^4$

$= \dfrac{1}{2} + \dfrac{1}{4} = \dfrac{3}{4}$

TYPE-IV

In the expression of $\sin^n\theta . \cos^n\theta$.

maxi. value = $\dfrac{1}{2^n}$

min. value =

(i) When n is even then min. value 0

(ii) When n is odd

then min value $\left(\dfrac{-1}{2^n}\right)$

Ex.24 Find the minimum & maximum value of $\sin^4\theta . \cos^4\theta$

Sol. $\sin^4\theta . \cos^4\theta$.

$= \dfrac{1\times 2^4}{2^4} \sin^4\theta . \cos^4\theta$

$= \dfrac{1}{2^4} (2\sin\theta . \cos\theta)^4$

$= \dfrac{1}{2^4} \sin^4 2\theta$

maxi. $= \dfrac{1}{2^4} \times (1) = \dfrac{1}{2^4}$

mini. $= \dfrac{1}{2^4} \times 0 = 0$

($\sin^n\theta$ when n even max.=1, min.=0)

Ex.25 Find the minimum & maximum value of

$\sin^3\theta \cos^3\theta$

Sol. max $= \dfrac{1}{2^3} = \dfrac{1}{8}$

min. $= \dfrac{-1}{2^3} = \dfrac{-1}{8}$ (3 - odd power)

Ex.26 Find the minimum & maximum value of $\sin^{110}\theta . \cos^{110}\theta$.

Sol. max $= \dfrac{1}{2^{110}}$

min = 0 (even power)

TYPE-V

(A) a $\tan^2\theta$ + b $\cot^2\theta$
or

a $\tan^2\theta$ + $\dfrac{\mathbf{b}}{\mathbf{\tan^2\theta}}$

min. value = 2$\sqrt{ab}$

max. value = ∞

Proof

a $\tan^2\theta$ + b $\cot^2\theta$

$= (\sqrt{a} \tan\theta - \sqrt{b} \cot\theta)^2 + 2 \sqrt{a} \sqrt{b} \tan\theta \cot\theta$

$= (\sqrt{a} \tan\theta - \sqrt{b} \cot\theta)^2 + 2\sqrt{ab}$

But $(\sqrt{a} \tan\theta - \sqrt{b} \cot\theta)^2$ is either 0 or greater than zero.

$\therefore$ $a \tan^2\theta + b \cot^2\theta \geq 0 + 2\sqrt{ab}$

or, $a \tan^2\theta + b\cot^2\theta \geq 2\sqrt{ab}$

Since value of $a \tan^2\theta + b \cot^2\theta$ is greater than or equal to $2\sqrt{ab}$,

its minimum value is $2\sqrt{ab}$

Note- Do not write the given expression as $(\sqrt{a} \tan\theta + \sqrt{b} \cot\theta)^2 - 2\sqrt{ab} \tan\theta \cot\theta$.

In this situation minimum value of, $\sqrt{a} \tan\theta + \sqrt{b} \cot\theta$ cannot be zero.

Ex.27 Find the minimum value of $16 \tan^2\theta + 9 \cot^2\theta$.

Sol. $16 \tan^2\theta + 9 \cot^2\theta$

min. value $= 2\sqrt{ab} = 2 \sqrt{16\times 9}$

$= 2\times 12 = 24$

Ex.28 Find the minimum value of $4\tan^2\theta + 25\cot^2\theta$

Sol. $4\tan^2\theta + 25\cot^2\theta$

min. value $= 2\sqrt{4\times25}$

$= 2\times10 = 20$

Ex.29 Find the minimum value of $4\sec^2\theta + 25\operatorname{cosec}^2\theta$

Sol. $4\sec^2\theta + 25\operatorname{cosec}^2\theta$

$= 4(1+\tan^2\theta) + 25(1+\cot^2\theta)$

$= 4 + 4\tan^2\theta + 25 + 25\cot^2\theta$

$= 29 + 4\tan^2\theta + 25\cot^2\theta$

min. value $= 4\tan^2\theta + 25\cot^2\theta$

$= 2\sqrt{4\times25} = 20$

min. value $= 29+20 = 49$

Ex.30 Find the minimum value of $\tan^2\theta + \cot^2\theta$.

Sol. $\tan^2\theta + \cot^2\theta$

min. value $= 2\sqrt{ab}$

$= 2\sqrt{1\times1} = 2$

(B) in expression of

$a\sin^2\theta + b\operatorname{cosec}^2\theta$

(i) if $a \le b$

min. value = $a+b$

(ii) if $a \ge b$

min. value = $2\sqrt{ab}$

Ex.31 Find the minimum value of $4\sin^2\theta + 25\operatorname{cosec}^2\theta$

Sol. minimum value $= 4+25 = 29$

($\because$ $a<b$ min value $= a+b$)

Ex.32 Find the minimum value of $16\sin^2\theta + 9\operatorname{cosec}^2\theta$

Sol. $16\sin^2\theta + 9\operatorname{cosec}^2\theta$

min. value $= 2\sqrt{16\times9}$

$= 2\times12 = 24$

($\because$ $a>b$ min value $= 2\sqrt{ab}$)

Ex.33 Find the minimum value of $25\operatorname{cosec}^2\theta + 25\sin^2\theta$.

Sol. $25\operatorname{cosec}^2\theta + 25\sin^2\theta$

mini. value $= 2\sqrt{ab}$

$2\sqrt{25\times25}$

$2\times25 = 50$

or

mini. value $= a+b = 25+25 = 50$

Ex.34 Find the minimum value of $\sin^2\theta + \operatorname{cosec}^2\theta$

Sol. $\sin^2\theta + \operatorname{cosec}^2\theta$

min. value $= 2\sqrt{1\times1} = 2$

or

min. value $1+1 = 2$

(C) In expression of

$a\cos^2\theta + b\sec^2\theta$

(i) If $a \le b$

minimum value = $a+b$

(ii) If $a \ge b$

minimum value = $2\sqrt{ab}$

Ex.35 Find the minimum value of $10\cos^2\theta + 15\sec^2\theta$

Sol. $10\cos^2\theta + 15\sec^2\theta$

minimum value $= 10+15 = 25$

($\because$ $a<b$ min. value $= a+b$)

Ex.36 Find the minimum value of $\cos^2\theta + \sec^2\theta$

Sol. $\cos^2\theta + \sec^2\theta$

min. value $= 1+1 = 2$

(D) In the expression of

$a\sec^2\theta + b\operatorname{cosec}^2\theta$

min. value = $\left(\sqrt{a}+\sqrt{b}\right)^2$

maxi. value = ∞

Ex.37 Find the value of $4\sec^2\theta + 9\operatorname{cosec}^2\theta$.

Sol. mini. value $= \left(\sqrt{a}+\sqrt{b}\right)^2 = \left(\sqrt{4}+\sqrt{9}\right)^2$

$= (2+3)^2 = 25$

Ex.38 Find the minimum value of $\sin^2\theta + \operatorname{cosec}^2\theta + \cos^2\theta + \sec^2\theta + \tan^2\theta + \cot^2\theta$.

Sol. $\sin^2\theta + \operatorname{cosec}^2\theta + \cos^2\theta + \sec^2\theta + \tan^2\theta + \cot^2\theta$

$= \sin^2\theta + 1 + \cot^2\theta + \cos^2\theta + 1 + \tan^2\theta + \tan^2\theta + \cot^2\theta$

$\Rightarrow 2 + \sin^2\theta + \cos^2\theta + 2\tan^2\theta + 2\cot^2\theta$

$= 2+1+2\tan^2\theta + 2\cot^2\theta$

$= 3 + 2\tan^2\theta + 2\cot^2\theta$

min. value of $2\tan^2\theta + 2\cot^2\theta$

$= 2\sqrt{ab}$

$= 2\sqrt{2\times2} = 4$

min. value $= 3+4 = 7$

EXERCISE

1. Find the maximum value of $\sin x + \cos x$.

 (a) $\sqrt{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 2 (d) 3

2. For $0<\theta<\frac{\pi}{2}$ the inequality which holds is :

 (a) $\theta < \sin\theta < \tan\theta$
 (b) $\sin\theta < \theta < \tan\theta$
 (c) $\theta < \tan\theta < \sin\theta$
 (d) $\tan\theta < \theta < \sin\theta$

3. Which is smaller ?

 (a) $\sin 76°$ (b) $\cos 76°$
 (c) both are equal
 (d) None of these

4. If the sine of an angle is 1/3, then cosine of that angle is :

 (a) equal of 1/3
 (b) less than 1/3
 (c) greater than 1/3
 (d) not known

5. The maximum value of $24\sin\theta + 7\cos\theta$ is

 (a) 7 (b) 17 (c) 24 (d) 25

6. Which of the following is correct?

 (a) $\sin 1° < \sin 1$
 (B) $\sin 1° = \sin 1$
 (c) $\sin 1° > \sin 1$
 (d) $\sin 1° = \frac{\pi}{180}\sin 1$

7. The greatest value of $\cos(xe^{|x|} + 7x^2 - 3x)$, $x \in [-1, \infty)$ is

 (a) 0 (b) 1 (c) –1
 (d) None of these

8. The greatest value of $\sin^4\theta + \cos^4\theta$ is :

 (a) 1 (b) $\frac{1}{2}$ (c) 0 (d) $\frac{\sqrt{3}}{2}$

9. The least value of $2\sin^2\theta + 3\cos^2\theta$ is:

 (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

10. What is the minimum value of $\sin^2\theta + \cos^4\theta$:

 (a) 0 (b) $\frac{3}{4}$ (c) 2 (d) $\frac{1}{4}$

11. Find the maximum value of $\sin^{113}\theta \cdot \cos^{113}\theta$:

(a) $\left(\frac{3}{2}\right)^{113}$ (b) 1

(c) $\left(\frac{1}{4}\right)^{113}$ (d) $\left(\frac{1}{2}\right)^{113}$

12. Find the minimum value of $16\text{cosec}^2\theta + 25\sec^2\theta$:
(a) 81 (b) 41 (c) 82 (d) 90

13. The least value of $\sin^2\theta + \text{cosec}^2\theta + \cos^2\theta + \sec^2\theta$ is:
(a) 3 (b) 4 (c) 5 (d) 6

14. If $0 \le \theta < \frac{\pi}{2}$, then which of the following trigonometric ratios can have the value 1.1?
(a) $\sin\theta$ (b) $\cos\theta$
(c) $2\sec\theta$ (d) $2\tan\theta$

15. $\cos 3\theta + \sin 3\theta$ is maximum when θ is:
(a) 15° (b) 30° (c) 45° (d) 60°

16. The value of x, for maximum value of ($\sin x + \cos x$) is :
(a) 30° (b) 45° (c) 60° (d) 90°

17. The maximum value of $\sin x . \cos x$ is:
(a) 2 (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) 1

18. The maximum and minimum value of ($1 + \cos 2x$) are:
(a) -1 and 1 (b) 1 and 2
(c) $-\frac{1}{2}$ and $\frac{1}{2}$ (d) 0 and 2

19. If P = $\sin^2 20° + \sin^2 40° + \sin^2 50° + \sin^2 70°$, then.
(a) 0 < P < 1 (b) 1 < P < 1.5
(c) 1.5 < P < 2 (d) P = 2

20. If $0 \le \theta \le \frac{\pi}{2}$, then which of the following is true?
(a) $(\tan^2\theta + \cot^2\theta) \ge 2$
(b) $(\tan^2\theta + \cot^2\theta) \le 2$
(c) $(\tan^2\theta + \cot^2) \le 1$
(d) None of these

21. If $0 < \theta < \frac{\pi}{2}$, which of the following is true?
(a) $\sin^2\theta + \frac{1}{\sin^2\theta} < 2$
(b) $\sin^2\theta + \frac{1}{\sin^2\theta} = 2$
(c) $\sin^2\theta + \frac{1}{\sin^2\theta} > 2$
(d) None of these

22. The least value of $(4\sec^2\theta + 9\text{cosec}^2\theta)$ is
(a) 1 (b) 19 (c) 25 (d) 7

23. If $\alpha + \beta = 90°$, then the maximum value of $\sin\alpha . \sin\beta$ is:
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$
(d) None of these

24. $\sin x + \sqrt{3}\cos x$ is maximum when
(a) x = 30° (b) x = 0°
(c) x = 45° (d) x = 60°

25. The least value of $\tan^2\theta + \cot^2\theta$ is:
(a) 2 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

26. What is the minimum value of $\sin^6\theta + \cos^6\theta$?
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1
(d) None of these

27. The greatest value of $81^{\sin x} . 27^{\cos x}$ is :
(a) 3^5 (b) 3^4 (c) 3 (d) 3^3

28. If $0° < A < 90°$ and cosA-sinA > 0 then cosA + sinA can not be greater than :
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

29. In a ΔABC, if angle C is obtuse, then
(a) tanA.tanB<1
(b) tanA.tanB $\le$ 1
(c) tanA.tanB>1
(d) None of these

30. The ratio of the greatest value of $2 - \cos x + \sin^2 x$ to its least value is :
(a) $\frac{1}{4}$ (b) $\frac{9}{4}$ (c) $\frac{13}{4}$ (d) $\frac{7}{4}$

ANSWER KEY

1.(a)	4.(c)	7.(b)	10.(b)	13.(c)	16.(b)	19.(d)	22.(c)	25.(a)	28.(d)
2.(b)	5.(d)	8.(a)	11.(d)	14.(d)	17.(c)	20.(a)	23.(b)	26.(b)	29.(a)
3.(b)	6.(a)	9.(c)	12.(a)	15.(a)	18.(d)	21.(c)	24. (a)	27.(a)	30.(c)

SOLUTION

1.(a) Maximum Value of :

$a\sin x + b\cos x = \sqrt{a^2+b^2}$

here a = b = 1

Maximum Value of :

$\sin x + \cos x = \sqrt{1^2+1^2} = \sqrt{2}$

Alternate

$\sin x + \cos x = \left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)\sqrt{2}$

$=(\sin x.\cos 45° + \cos x.\sin 45°)\sqrt{2}$

$= \sqrt{2}\sin(x+45°)$

Now, $-1 \le \sin(x+45°) \le 1$

$-1\times\sqrt{2} \le \sqrt{2}\sin(x+45°) \le 1\times\sqrt{2}$

$\Rightarrow -\sqrt{2} \le (\sin x + \cos x) \le \sqrt{2}$

2.(b) In Ist quadrant, $\sin\theta < \theta < \tan\theta$ is true.

3.(b) In Ist quadrant as θ increases, the value of $\sin\theta$ increases.

Now, cos 76° = cos (90°–14°) = sin 14°

out of sin 76° and sin 14°, the smaller one is sin14°

Hence, cos 76° is smaller.

4.(c)

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} > \sqrt{\frac{1}{9}}$

So, $\cos\theta > \frac{1}{3}$

5.(d) Max. value of $a\sin\theta + b\cos\theta$

$= \sqrt{a^2+b^2}$

Max. value of 24 $\sin\theta$ + 7$\cos\theta$

$=\sqrt{(24)^2+(7)^2}=25$

6.(a) Since 1 radian = 57°16'22'' approx,

and sin 57°16'22'' >sin1°,

sin 1°<sin1

7.(b) since $\cos\theta \le 1$ for all real θ. the given expression ≤ 1 for all x

Hence, the greatest value=1

8.(a)

$\sin^4\theta + \cos^4\theta = (\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta.\cos^2\theta - 2\sin^2\theta.\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta.\cos^2\theta$

$= 1 - 2\sin^2\theta.\cos^2\theta \le 1$

Greatest value of $\sin^4\theta + \cos^4\theta = 1$

9.(c) $2\sin^2\theta + 3\cos^2\theta =$

$2(\sin^2\theta + \cos^2\theta) + \cos^2\theta$

$= 2 + \cos^2\theta \ge 2$

Least value of $2\sin^2\theta$ + $3\cos^2\theta = 2$

10.(b) $x = \sin^2\theta + \cos^4\theta$

$= 1 - \cos^2\theta + \cos^4\theta$

$x = 1 - \cos^2\theta(1-\cos^2\theta)$

$x = 1 - \cos^2\theta.\sin^2\theta$

$x = 1 - \frac{1}{4}(2\sin\theta.\cos\theta)^2$

$x = 1 - \frac{1}{4}\sin^2 2\theta \quad 0 \le \sin^2 2\theta \le 1$

when $\sin^2 2\theta = 0 \Rightarrow x = 1 - \frac{1}{4}(0) = 1$

and when

$\sin^2 2\theta = 1, \Rightarrow x = 1 - \frac{1}{4}(1) = \frac{3}{4}$

i.e. $\frac{3}{4} \le x \le 1$

i.e the least value of

$\sin^2\theta + \cos^4\theta = \frac{3}{4}$

11.(d) $\sin^{113}\theta.\cos^{113}\theta$

$= \frac{1}{2^{113}}(2\sin\theta.\cos\theta)^{113}$

$= \left(\frac{1}{2}\right)^{113}(\sin 2\theta)^{113} \le \left(\frac{1}{2}\right)^{113}$

$(\because -1 \le \sin 2\theta \le 1)$

Hence, the greatest value of

$\sin^{113}\theta.\cos^{113}\theta = \left(\frac{1}{2}\right)^{113}$

Short Cut :

The maximum value of

$\sin^n\theta.\cos^n\theta = \left(\frac{1}{2}\right)^n$

The maximum value of

$\sin^{113}\theta.\cos^{113}\theta = \left(\frac{1}{2}\right)^{113}$

12.(a) $16\text{cosec}^2\theta + 25\sec^2\theta$

$= 16(1+\cot^2\theta) + 25(1+\tan^2\theta)$

$= 41 + 16\cot^2\theta + 25\tan^2\theta$

(i)

Now,

$\because$ Minimum value of

$ax^2 + \frac{b}{x^2} = 2\sqrt{ab}$

Minimum value of :

$16\cot^2\theta + 25\tan^2\theta = 2\sqrt{16\times 25} = 40$

From (i)

Minimum value of

$16\text{cosec}^2\theta + 25\sec^2\theta = 41 + 40 = 81$

Short Cut :

Minimum value of

$(a\text{cosec}^2\theta + b\sec^2\theta)$

$= (\sqrt{a}+\sqrt{b})^2$

$\therefore$ Minimum value of

$(16\text{cosec}^2\theta + 25\sec^2\theta)$

$= (\sqrt{16}+\sqrt{25})^2 = (9)^2 = 81$

13.(c)

$(\sin^2\theta + \cos^2\theta) + (\text{cosec}^2\theta + \sec^2\theta)$

$= 1 + (\text{cosec}^2\theta + \sec^2\theta)$

$\because$ Minimum value of

$(a\,\text{cosec}^2\theta + b\sec^2\theta) = (\sqrt{a}+\sqrt{b})^2$

$\therefore$ Minimum value of

$(\text{cosec}^2\theta + \sec^2\theta) = (\sqrt{1}+\sqrt{1})^2 = 4$

$\therefore$ Minimum value of the given Exp. = 1 + 4 = 5

14.(d) Clearly, $0 \le \sin\theta < 1$ and $0 \le \cos\theta < 1$

for $0 \le \theta < \frac{\pi}{2}$

Also $\sec\theta \ge 1$ so $2\sec\theta \ge 2$

Now, tan θ may have the value 0.55

$\therefore$ 2 tan θ may have the value 1.1.

15.(a) $\cos 3\theta + \sin 3\theta$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos 3\theta + \frac{1}{\sqrt{2}}\sin 3\theta\right)$

$= \sqrt{2}(\sin 45° \cos 3\theta$

$+ \cos 45°. \sin 3\theta)$

$= \sqrt{2}\sin(45° + 3\theta)$

The maximum value occurs when

$\sin(45° + 3\theta) = 1$

i.e. $3\theta = 45° \Rightarrow \theta = 15°$

16.(b) $\sin x + \cos x$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)$

$= \sqrt{2}(\sin x.\cos 45° + \cos x \sin 45°)$

$= \sqrt{2}\sin(x + 45°)$

$= \sqrt{2}$, when $x = 45°$

$\because$ max.

$(\sin x + \cos x) = \sqrt{2}$

17.(c) $\sin x . \cos x = \frac{1}{2}(2\sin x.\cos x)$

$= \frac{1}{2}\sin 2x \le \frac{1}{2}$

$[\because -1 \le \sin 2x \le 1]$

Hence max. value of $\sin x. \cos x$ is $\frac{1}{2}$.

18.(d) We know that

$-1 \le \cos\theta \le 1$

$\therefore -1 \le \cos 2x \le 1$ OR

$-1 + 1 \le 1 + \cos 2x \le 1 + 1$

$\Rightarrow 0 \le 1 + \cos 2x \le 2$

19.(d) $P = (\sin^2 20° + \sin^2 70°) +$

$(\sin^2 40° + \sin^2 50°)$

$= (\sin^2 20° + \cos^2 20°) +$

$(\sin^2 40° + \cos^2 40°)$

$= 1+1 = 2$

20.(a) $\tan^2\theta + \cot^2\theta =$

$\tan^2\theta + \cot^2\theta -$

$2\tan\theta.\cot\theta + 2\tan\theta.\cot\theta$

$= (\tan\theta - \cot\theta)^2 + 2 \ge 2$

21.(c) $\sin^2\theta + \frac{1}{\sin^2\theta}$

$= \sin^2\theta + \frac{1}{\sin^2\theta} - 2 + 2$

$= \left(\sin\theta - \frac{1}{\sin\theta}\right)^2 + 2 > 2$

$\left[\because 0 < \theta < \frac{\pi}{2} \Rightarrow \sin\theta \ne 1\right]$

22.(c) least value of

$(a\sec^2\theta + b\,\mathrm{cosec}^2\theta) = (\sqrt{a} + \sqrt{b})^2$

$\therefore$ least value of

$(4\sec^2\theta + 9\,\mathrm{cosec}^2\theta)$

$= (\sqrt{4} + \sqrt{9})^2 = (5)^2 = 25$

23.(b) Let $y = \sin\alpha.\sin\beta$

$= \frac{1}{2}(2\sin\alpha.\sin\beta)$

$= \frac{1}{2}[2\sin\alpha.\sin(90° - \alpha)]$

$= \frac{1}{2}(2\sin\alpha.\cos\alpha)$

OR

$y = \frac{1}{2}(\sin 2\alpha)$

Since, Maximum value of $\sin 2\alpha$ is 1.

$\therefore$ Maximum value of $y = \frac{1}{2}$

24. (a) Let $y = \sin x + \sqrt{3}\cos x$

$= 2\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right)$

$= 2\sin(x + 60°)$

y will be maximum, when $\sin(x + 60°)$ maximum

i.e. $\sin(x + 60°) = 1 = \sin 90° \Rightarrow x + 60° = 90°$

$\Rightarrow x = 30°$

25.(a) $\tan^2\theta + \cot^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{\sin^4\theta + \cos^4\theta}{\sin^2\theta.\cos^2\theta}$

$= \frac{(\sin^2\theta - \cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta}{\sin^2\theta.\cos^2\theta}$

$= \left(\frac{(\sin^2\theta - \cos^2\theta)}{\sin\theta.\cos\theta}\right)^2 + 2 \ge 2$

Least value of $\tan^2\theta + \cot^2\theta = 2$

Alternate:-

$\tan^2\theta + \cot^2\theta = \tan^2\theta + \frac{1}{\tan^2\theta}$

It is in the form of $ax^2 + \frac{b}{x^2}$

Where a = b = 1

$\therefore$ minimum value

$= 2\sqrt{ab} = 2\sqrt{1 \times 1} = 2$

26.(b) $x = \sin^6\theta + \cos^6\theta$

$\Rightarrow x = (\sin^2\theta)^3 + (\cos^2\theta)^3$

$\Rightarrow x = (\sin^2\theta + \cos^2\theta)$

$(\sin^4\theta + \cos^4\theta - \sin^2\theta.\cos^2\theta)$

$x = 1 \times [(\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta.\cos^2\theta]$

$x = 1 - 3\sin^2\theta.\cos^2\theta$

$x = 1 - \frac{3}{4}(2\sin\theta.\cos\theta)^2$

$x = 1 - \frac{3}{4}(\sin 2\theta)^2$

$= 1 - \frac{3}{4}\sin^2 2\theta$

$\because 0 \le \sin^2 2\theta \le 1$

$\therefore$ at $\sin^2 2\theta = 0$

$x = 1 - \frac{3}{4}(0) = 1$

and at $\sin^2 2\theta = 1$

$x = 1 - \frac{3}{4}(1) = \frac{1}{4}$

i.e. $\frac{1}{4} \le x \le 1$

i.e least value of

$\sin^6\theta + \cos^6\theta = \frac{1}{4}$

27.(a)

$81^{\sin x}.27^{\cos x} = 3^{4\sin x}.3^{3\cos x}$

$= 3^{4\sin x + 3\cos x}$

For maximum value,

4 sin$x$ + 3cos x must be maximum and maximum value of :

$4\sin x + 3\cos x$

$= \sqrt{4^2 + 3^2} = 5$

$\therefore$ Greatest value of

$81^{\sin x}.27^{\cos x} = 3^5$

28.(d) $\cos A - \sin A > 0$

$\sin A < \cos A$

$\tan A < 1 \Rightarrow A < 45°$

$\therefore$ sin A + cos A < sin 45° + cos45°

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

i.e. sin A + cosA = $\sqrt{2}$

29.(a) since A+B+C= π

$\therefore$ A+B = $\pi - C$

$\Rightarrow \tan(A+B) = \tan(\pi - C)$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

$\because$ angle C is obtuse

$\therefore$ tan C<0

$\therefore \frac{\tan A + \tan B}{1 - \tan A + \tan B} > 0$ ______(i)

But since C is obtuse angle, so A and B will

both be less than $\pi/2$

$\therefore$ Both tanA and tanB are positive.

Hence, from

(i) 1–tan A . tan B > 0

$\Rightarrow$ tan A .tan B <1

30.(c) $2 - \cos x + \sin^2 x$

$= 2 - \cos x + 1 - \cos^2 x$

$= -(\cos^2 x + \cos x) + 3$

$= -\left[\left(\cos x + \frac{1}{2}\right)^2 - \frac{1}{4}\right] + 3$

$= \frac{13}{4} - \left(\cos x + \frac{1}{2}\right)^2$

$\therefore$ Max. value occurs at

$\cos x = -\frac{1}{2}$

and it is $\frac{13}{4}$

and Min. value occurs at $\cos x = 1$ and it is 1

$\therefore$ The required ratio is $\frac{13}{4}$.

CHAPTER 14

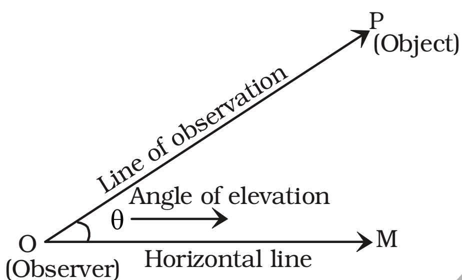
HEIGHT & DISTANCE

INTRODUCTION

One of the important application of trigonometry is in finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

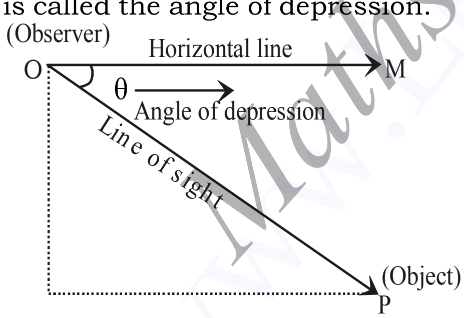
Angle of elevation

Let O and P be two point where P is at a higher level than O. Let O be at the position of the observer and P be the position of the object. Draw a horizontal line OM through the point O. OP is called the line of observation or line of sight, Then $\angle POM = \theta$ is called of elevation of P as observed from O.



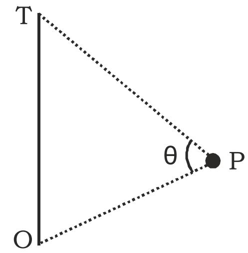
Angle of Depression

In the above figure, if P be at a lower level than O, then $\angle MOP = \theta$ is called the angle of depression.



Angle subtended by a line at a point

In the adjacent figure suppose OT is a tower, where O is the foot and *T* is the top of a tower. Suppose P is a point any where in the space (including ground). Join O – P and T – P, then $\angle OPT = \theta$ is the angle subtened by tower OT at point P.

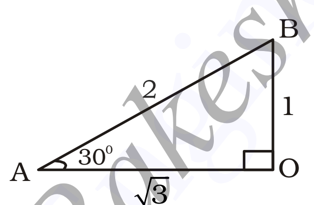


Some useful point

In this chapter we solve all the questions with the help of ratio. Some important ratios are as following :-

TYPE -I

(A)



Angle Ratio of 30° =

Base : Height : Hypotenuse
$\sqrt{3}$: 1 : 2

Proof:

In $\triangle$ OBA

$$\text{Tan}30° = \frac{OB}{OA}$$

$$\frac{1}{\sqrt{3}} = \frac{OB}{OA}$$

OB : OA

$1 : \sqrt{3}$

$$AB^2 = (OB)^2 + (OA)^2 = \left(\sqrt{3}\right)^2 + (1)^2$$

$$AB = \sqrt{3+1} = 2.$$

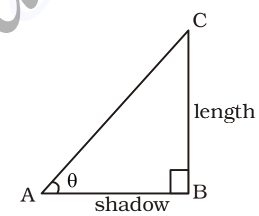
$$\boxed{\begin{array}{c}\mathbf{OB:OA:AB}\\ \mathbf{1:\sqrt{3}:2}\end{array}}$$

Ex.1 If the ratio of the length of a pen to its shadow is $1:\sqrt{3}$, the angle of elevation of the source of light is :-

(a) 40^0 (b) 30^0 (c) 60^0 (d) 90^0

Sol. $\tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{3}} = \tan 30^0$

$\Rightarrow \theta = 30^0$



Ex.2 The angle of elevation of the top of a tower at a distance of 500 m from its foot is 30^0. The height of the tower is :-

(a) $\frac{500\left(\sqrt{3}-1\right)}{3}$ m

(b) 500 m

(c) $\frac{500\sqrt{3}}{3}$ m

(d) $\frac{500\left(\sqrt{3}+1\right)}{3}$

Sol. $\tan 30^0 = \frac{BC}{AB} = \frac{h}{500}$

$\Rightarrow h = 500 \times \frac{1}{\sqrt{3}} = \frac{500\sqrt{3}}{3}$

Alternative

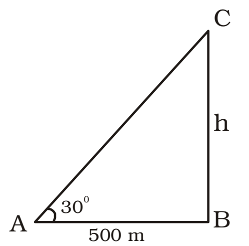
In 30° :- height : Base

1 : $\sqrt{3}$

but Base = 500 (given)

$\therefore \sqrt{3} \rightarrow 500$

$\therefore 1 \rightarrow \frac{500}{\sqrt{3}} = \frac{500.\sqrt{3}}{3}$



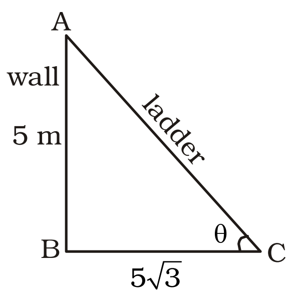
Ex.3 The foot of a ladder leaning against a wall of length 5 metre rest on a level ground $5\sqrt{3}$ metre from the base of the wall. The angle of inclination of the ladder with the ground is :-

(a) 60^0 (b) 50^0 (c) 40^0 (d) 30^0

Sol. In right angled ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^0$$

$$\Rightarrow \theta = 30^0$$

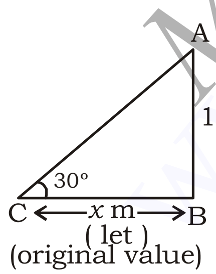


Ex.4 A ladder is resting against a wall at a height of 10m. If the ladder is inclined at an angle of 30° with the ground the distance of the ladder from the wall is:

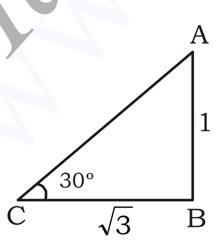
(a) $\frac{10}{\sqrt{3}}$ m (b) $\frac{20}{\sqrt{3}}$ m

(c) $10\sqrt{3}$ m (d) $20\sqrt{3}$ m

Sol.



(original value)



(ratio value)

Ratio value
Original value

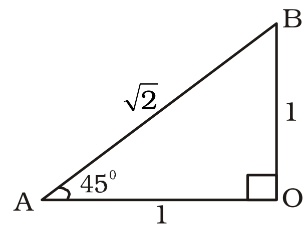
$AB \to 1 \quad \to \quad 10$ m

$\therefore BC \to \sqrt{3} \quad \to \quad 10\sqrt{3}$ m

i.e. Required distance (BC ratio value = $\sqrt{3}$)

$= x = 10\sqrt{3}$ m

(B)



Angle Ratio of 45°

= Base : Height : Hypotenuse
1 : 1 : $\sqrt{2}$

Proof:

In Δ OBA

$$\tan 45° = \frac{OB}{OA}$$

$$1 = \frac{OB}{OA}$$

$\Rightarrow$ OB : OA
1 : 1

$\Rightarrow AB^2 = (1)^2 + (1)^2$

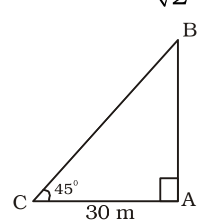
$\Rightarrow AB = \sqrt{2}$

$$\boxed{\begin{array}{c}\textbf{OB:OA:AB}\\ \mathbf{1:1:\sqrt{2}}\end{array}}$$

Ex.5 The angle of elevation of the top of a tower at a distance of 30 m from its foot is 45^0. The height of the tower is:-

(a) 20 m (b) 30 m (c) $15\sqrt{2}$ m (d) $\frac{15}{\sqrt{2}}$ m

Sol.



Base : height =1:1

$\therefore$ height = Base = 30 m

Alternate:-

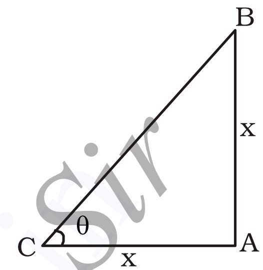
In $\Delta ABC, \tan 45^0 = \frac{AB}{AC}$

$$\Rightarrow 1 = \frac{AB}{30} \Rightarrow AB = 30 \text{ m}$$

Ex.6 The angle of elevation of a moon when the length of the shadow of a pole is equal to its height is :-

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

Sol.



Let $AB = x$, then AC=x

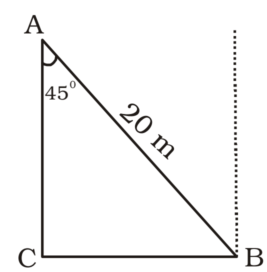
$$\therefore \tan\theta = \frac{AB}{AC} = \frac{x}{x} = 1$$

$$\Rightarrow \theta = 45^0$$

Ex.7 The banks of a river are parallel. A swimmer starts from a point on one of the banks and swims in a straight line inclined to the bank at 45^0 and reaches the opposite bank at a point 20 m from the point opposite to the starting point. The breadth of the river is :-

(a) 20 m (b) 28.28m (c) 14.14 m (d) 40 m

Sol.

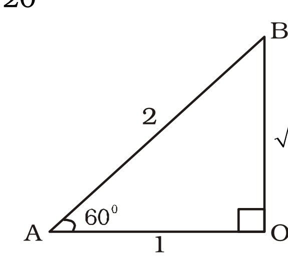


Let A be the starting point and B, the end point of the swimmer. Then AB=20 m & $\angle BAC = 45^0$

Now, $\sin 45^0 = \frac{BC}{AB}$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{20} \Rightarrow BC = 10\sqrt{2} = 14.14 \text{ m}$$

(C)



Angle Ratio of 60° =

Base : Height : Hypotenuse
1 : $\sqrt{3}$: 2

Proof:

In Δ OBA

$$\tan 60° = \frac{OB}{OA}$$

$$\sqrt{3} = \frac{OB}{OA}$$

$\Rightarrow$ OB : OA

$\sqrt{3}$: 1

$\Rightarrow AB^2 = (OB)^2 + (OA)^2$

$= \left(\sqrt{3}\right)^2 + (1)^2$

$\Rightarrow AB = \sqrt{3+1} = 2.$

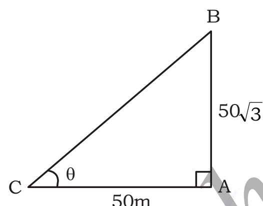
$$\boxed{\begin{matrix}\textbf{OB:OA:AB} \\ \sqrt{3} : 1 : 2\end{matrix}}$$

Ex.8 A tower is $50\sqrt{3}$ meters high. Find the angle of elevation of its top from a point 50 meters away from its foot:-

(a) $\theta = 60^0$ (b) $\theta = 45^0$

(c) $\theta = 30^0$ (d) $\theta = 22\frac{1}{2}^0$

Sol.



Height of tower (AB) = $50\sqrt{3}$

$$\therefore \tan\theta = \frac{AB}{AC} = \frac{50\sqrt{3}}{50} = \sqrt{3} = \tan 60^0$$

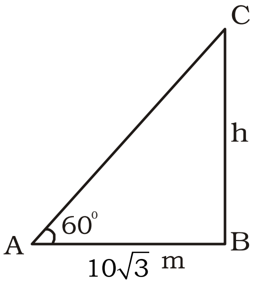
$\Rightarrow \theta = 60^0$

Ex.9 If the angle of elevation of the top of a building from a point $10\sqrt{3}$ m away from its base is 60^0, the height of the building is:-

(a) 10 m (b) 20 m

(c) $\frac{10}{\sqrt{3}}$ m (d) 30 m

Sol.



$$\tan 60° = \frac{h}{10\sqrt{3}}$$

$$\sqrt{3} = \frac{h}{10\sqrt{3}}$$

h = 30 m.

Ex.10 One flies a kite with a thread 150 metre long. If the thread of the kite makes an angle of 60° with the horizontol line, then the height of the kite from the ground (assuming the thread to be in a straight line) is:-

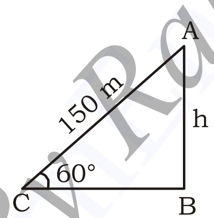
(a) 50 metre

(b) $75\sqrt{3}$ m

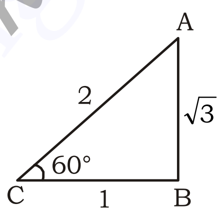
(c) $25\sqrt{3}$ metre

(d) 80 metre

Sol.



(original value)



(ratio value)

AC = length of thread, let height = h m

Ratio value		Original value
AC $\longrightarrow$ 2	$\longrightarrow$	150
$\therefore$ 1	$\longrightarrow$	$\frac{150}{2} = 75$
$\therefore$ $\sqrt{3}$	$\longrightarrow$	$75\sqrt{3}$

i.e. the height of the kite

= AB = h = $75\sqrt{3}$ m

Ex.11 The length of a string between a kite and a point on the ground is 50 m. The string makes an angle of 60^0 with the level ground. If there is no slack in the string, the height of the kite is:-

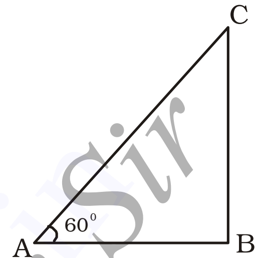
(a) $50\sqrt{3}$ m

(b) $25\sqrt{3}$ m

(c) 25 m

(d) $25/\sqrt{3}$ m

Sol.



Let C be the position of the kite and AC be the string.

$\therefore$ AC = 50 m and $\angle BAC = 60^0$

$$\therefore \frac{BC}{AC} = \sin 60^0 \Rightarrow \frac{BC}{50} = \frac{\sqrt{3}}{2}$$

$\Rightarrow BC = 25\sqrt{3}$ m

Hence, Height of the kite

= $25\sqrt{3}$ m

Alternate :-

In 60° AC : BC

$= 2 : \sqrt{3}$

$\times 25) \quad \times 25)$

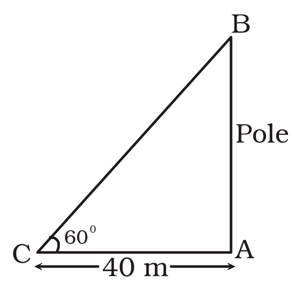
$50 \quad 25\sqrt{3}$

Ex.12 The angle of the sun at any distance is 60^0. The height of the vertical pole that will cast a shadow of 40 m is :-

(a) 20 m (b) $\frac{40}{\sqrt{3}}$ m

(c) $40\sqrt{3}$ m (d) $20\sqrt{3}$ m

Sol.



Let height of pole AB = h m

$$\therefore \tan 60^0 = \frac{h}{40}$$

$\Rightarrow h = 40\sqrt{3}$ m

Alternative :

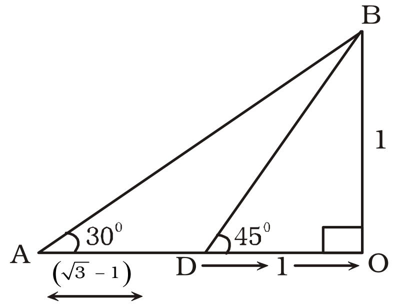
In 60° = Base : height

$$\begin{array}{ccc} 1 & : & \sqrt{3} \\ \times 40) & & \times 40) \\ 40 & & 40\sqrt{3} \end{array}$$

$\therefore$ height $= 40\sqrt{3}$

TYPE -II

(A)



Proof: In Δ AOB

$\tan 30° = \frac{OB}{OA}$

$\frac{1}{\sqrt{3}} = \frac{OB}{OA}$

$\Rightarrow$ **OB : OA**

1 : $\sqrt{3}$ .... **(i)**

ΔOBD

$\tan 45° = \frac{OB}{OD}$

$1 = \frac{OB}{OD}$

OB : OD

1 : 1 .... **(ii)**

From (i) and (ii)

To make equal ratio

OB : OA

OB : OD

$$\begin{array}{ccccc} 1 & : & \sqrt{3} & & \\ & & 1 & : & 1 \\ OA & : & OB & : & OD \\ \sqrt{3} & : & 1 & & \\ & & 1 & : & 1 \\ \sqrt{3} & : & 1 & : & 1 \end{array}$$

OB : OA : OD

1 : $\sqrt{3}$: 1

then,

AD = OA – OD

AD = $\sqrt{3}-1$

Ex.13 The shadow of an electric pole standing on a ground is 40 m less when the angle of elevation changes from 30^0 to 45^0. The length of the pole is:-

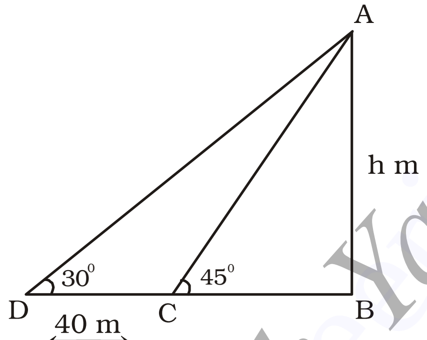
(a) $20\left(\sqrt{3}+1\right)$ m

(b) $20\left(\sqrt{3}-1\right)$ m

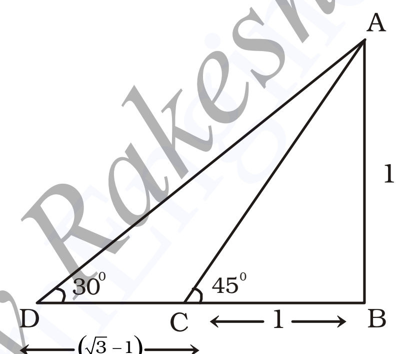
(c) 20 m

(d) $20\sqrt{3}$ m

Sol. (By ratio)



(original value)



(ratio value)

$\therefore \left(\sqrt{3}-1\right) \rightarrow 40$ m

$$\therefore 1 \rightarrow \frac{40}{\sqrt{3}-1} \times \frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)} = 20\left(\sqrt{3}+1\right)$$

$\therefore$ length of pole $= 20(\sqrt{3}+1)$

Ex.14 If the angle of elevation of the Sun changes from 30° to 45°, the length of the shadow of a pillar decreases by 20 metres. The height of the pillar is:

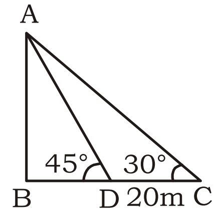
(a) $20\left(\sqrt{3}-1\right)$m

(b) $20\left(\sqrt{3}+1\right)$m

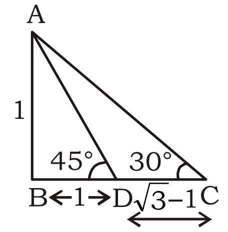
(c) $10\left(\sqrt{3}-1\right)$m

(d) $10\left(\sqrt{3}+1\right)$m

Sol.



(original value)



(ratio value)

Let AB be a pillar of height h metre.

Ratio value   original value

CD $\rightarrow \sqrt{3}-1 \rightarrow 20$

$$\therefore \quad 1 \rightarrow \frac{20}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

height of the pillar $= 10$

$\left(\sqrt{3}+1\right)$ metre

Ex.15. The length of the shadow of a vertical tower on level ground increases by 10 metres when the altitude of the sun changes from 45° to 30°. Then the height of the tower is

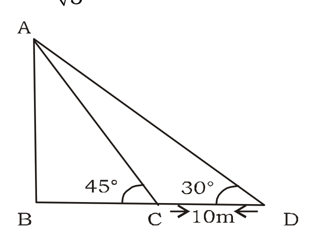
(a) $5\left(\sqrt{3}+1\right)$ metres

(b) $5\left(\sqrt{3}-1\right)$ metres

(c) $5\sqrt{3}$ metres

(d) $\frac{5}{\sqrt{3}}$ metres

Sol.



AB = height of tower

In ΔABC

$\tan 45° = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{AB}{BC}$ = AB : BC = 1 : 1 .....(i)

In ΔABD

$\tan 30° = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BD}$

$\Rightarrow$ AB : BD = 1 : $\sqrt{3}$ ....(ii)

Now,

BC	:	AB	:	BD
1	:	1		
		1	:	$\sqrt{3}$
1	:	1	:	$\sqrt{3}$

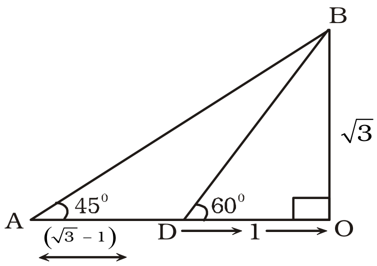
CD = BD − BC

$= \left(\sqrt{3}-1\right)$ units = 10m

$= 1$ unit $= \dfrac{10}{\sqrt{3}-1}$

AB = 1unit = $5\left(\sqrt{3}+1\right)$ metres

(B)



ΔOAB

$\tan 45° = \dfrac{OB}{OA} = \dfrac{1}{1}$

OB : OA ....(i)

1 : 1

ΔOBD

$\tan 60° = \dfrac{OB}{OD}$

$\sqrt{3} = \dfrac{OB}{OD}$

OB : OD .....(i)

$\sqrt{3} : 1$

From (i) and (ii)

To make equal ratio

OB : OA **OB : OD**

1 : 1 **$\sqrt{3}$: 1**

OB : OA : OD

$\sqrt{3}$: $\sqrt{3}$: 1

AD = OA − OD = $\sqrt{3}-1$

Ex.16 The angle of elevation of the top of a tower at a point on level ground is 45°. When moved 20 m towards the tower, the angle of elevation becomes 60°. What is the height of the tower?

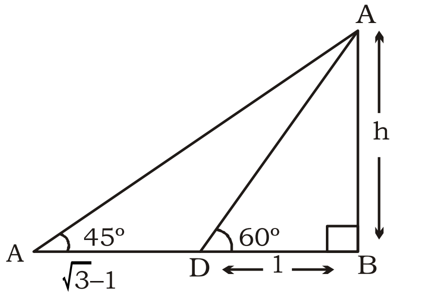
(a) $10\left(\sqrt{3}-1\right)$ m

(b) $10\left(\sqrt{3}+1\right)$ m

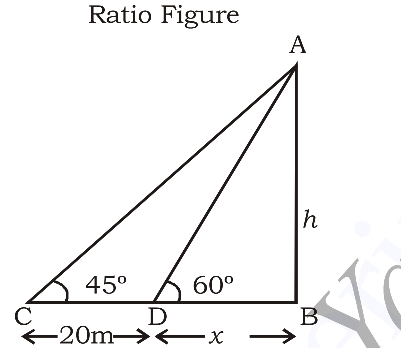
(c) $10\left(3-\sqrt{3}\right)$ m

(d) $10\left(3+\sqrt{3}\right)$ m

Sol. (d) Let the height of tower be h m and BD = xm



Ratio Figure



Both of the figures are in comparison ratio

$\left(\sqrt{3}-1\right)R = 20$

$R = \dfrac{20}{\sqrt{3}-1}$

$x = 1 \quad R = \dfrac{20}{\sqrt{3}-1}$

$= \dfrac{20}{\sqrt{3}-1} \times \dfrac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)} = \dfrac{20\left(\sqrt{3}+1\right)}{2}$

$= 10\ (\sqrt{3}+1)$ m.

Height of tower

$= AB = \sqrt{3} \times 10\left(\sqrt{3}+1\right)$

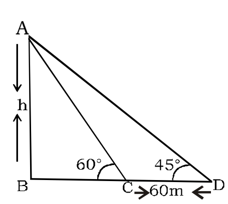
$= 10\ \left(3+\sqrt{3}\right)$ m.

Ex.17 The elevation of the top of a tower from a point on the ground is 45°. On travelling 60 m from the point towards the tower the elevation of the top becomes 60°. The height of the tower (in metres) is

(a) 30 (b) $30\left(3-\sqrt{3}\right)$

(c) $30\left(3+\sqrt{3}\right)$ (d) $30\sqrt{3}$

Sol.



AB height of tower

In ΔABC

$\tan 60° = \dfrac{AB}{BC}$

$\sqrt{3} = \dfrac{AB}{BC} \Rightarrow AB : BC = \sqrt{3} : 1$ ....(i)

In ΔABD

$\tan 45° = \dfrac{AB}{BD}$

$1 = \dfrac{AB}{BD}$

$\Rightarrow$ AB : BD = 1 : 1 ....(ii)

Now

BD	:	AB	:	BC
1	:	1		
		$\sqrt{3}$	:	1
$\sqrt{3}$	:	$\sqrt{3}$	:	1

CD = BD − BC = $\left(\sqrt{3}-1\right)$

$\left(\sqrt{3}-1\right)$ = 60 metre

1 unit $= \dfrac{60}{\sqrt{3}-1}$

AB = $\sqrt{3}$ units $= \dfrac{60}{\sqrt{3}-1} \times \sqrt{3}$

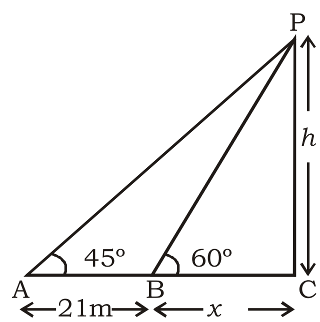
$= 30\left(3+\sqrt{3}\right)$ m

Ex.18 The angle of elevation of the tip of a tower from a point on the ground is 45°. Moving 21 m directly towards the base of the tower, the angle of elevation changes to 60°. What is the height of the tower, to the nearest meter?

(a) 48 m (b) 49 m

(c) 50 m (d) 51m

Sol. In Δ PBC,



$\tan 60° = \dfrac{h}{x} \Rightarrow \dfrac{h}{x} = \sqrt{3}$

$\Rightarrow x = \dfrac{h}{\sqrt{3}}$ ....(i)

In Δ PAC, $\tan 45° = \dfrac{h}{21+x} = 1$

$\Rightarrow$ h = 21 + x

$\Rightarrow$ h = 21 + $\frac{h}{\sqrt{3}}$ [from Eq. (i)]

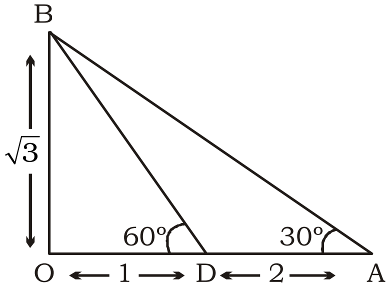
$$\Rightarrow h\left(1-\frac{1}{\sqrt{3}}\right)=21$$

$$\therefore \quad h=\frac{21\sqrt{3}}{\left(\sqrt{3}-1\right)}\cdot\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$

$$=\frac{21\sqrt{3}\left(\sqrt{3}+1\right)}{2}$$

= 49.68 $\approx$ 50m

(C)



Proof

$\triangle$ OAB

$$\tan 30^\circ=\frac{OB}{OA}$$

$$\frac{1}{\sqrt{3}}=\frac{OB}{OA}$$

OB : OA .....(i)

1 : $\sqrt{3}$

$\triangle$ OBD

$$\tan 60^\circ=\frac{OB}{OD}$$

$$\sqrt{3}=\frac{OB}{OD}$$

OB : OD ....(ii)

$\sqrt{3}$: 1

From (i) & (ii)

to make equal ratio

OB:OA OB : OD

1 :$\sqrt{3}$ $\sqrt{3}$:1

OB : OA : OD

$\sqrt{3}$: 3 : 1

AD = OA – OD = 3 –1 = 2

Ex.19 A man from the top a 50 m high tower, sees a car moving towards the tower at an angle of depression of 30°. After some time, the angle of depression becomes 60°. The distance (in m) travelled by the car during this time is:-

(a) $50\sqrt{3}$ (b) $\frac{50\sqrt{3}}{3}$

(c) $\frac{100\sqrt{3}}{3}$ (d) $100\sqrt{3}$

Sol. In $\triangle$ ADC

$$\Rightarrow \tan 30^\circ=\frac{DC}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{50}{AC}$$

AC = $50\sqrt{3}$ ....**(i)**

In $\triangle$ DBC

$$\Rightarrow \tan 60^\circ=\frac{50}{BC}$$

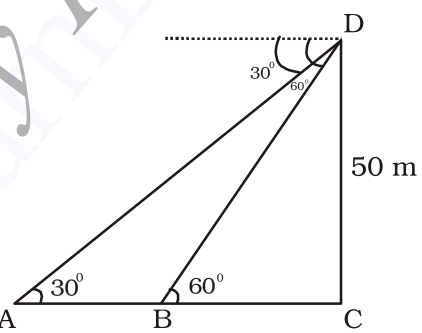
$$BC=\frac{50}{\sqrt{3}}$$ ....**(ii)**

(i) – (ii)

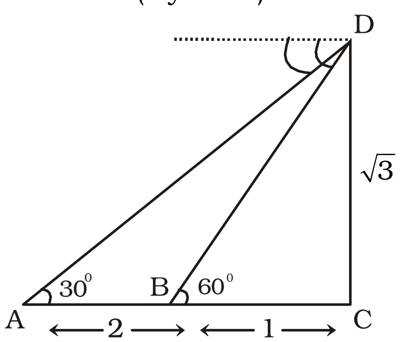
AB = AC – BC

$$=50\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)$$

$$=\frac{100}{\sqrt{3}}=\frac{100\sqrt{3}}{3}$$



Alternate :- (by ratio)



CD=50 m (given)

but CD = $\sqrt{3}$ (according to ratio)

Ratio value original value

CD $\sqrt{3}$ ⟶ 50

$\therefore$ 1 ⟶ $\frac{50}{\sqrt{3}}$

$\therefore$ 2 ⟶ $\frac{100}{\sqrt{3}}$

$$\therefore AB\left(\text{ratio value}\right)=\frac{100}{\sqrt{3}}=\frac{100\sqrt{3}}{3}$$

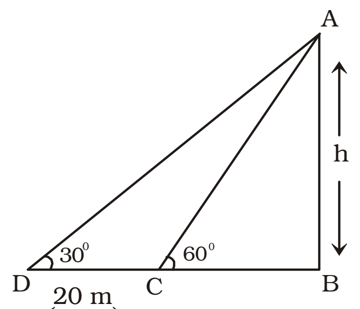
Ex.20 The angle of elevation of top of a tree on the bank of a river from its other bank is 60° and from a point 20 m further away from this is 30°. The width of the river is:-

(a) $10\sqrt{3}$ m (b) 10 m

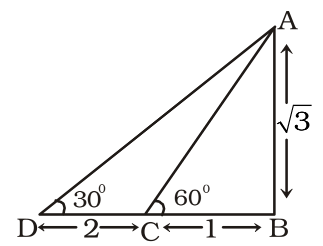
(c) 20 m (d) $20\sqrt{3}$ m

Sol. (By ratio)

BC = width of river



(original value)



(Ratio value)

Ratio value original value

2 ⟶ 20 m

$\therefore$ 1 ⟶ 10 m

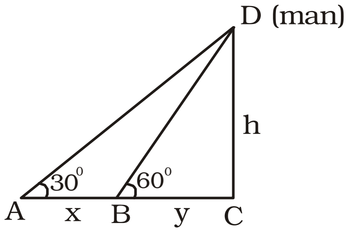
Hence, width of the river=BC=10 m

Ex.21 A man on the top of a rock rising on a sea-shore observes a boat coming towards it. If it takes 20 minute for the angle of depression to change from 30° to 60°, how soon will the boat reach the shore ?

(a) 20 minute

(b) 30 minute
(c) 10 minute
(d) 15 minute

Sol.



From figure, $\tan 60^0 = \frac{h}{y}$

$\Rightarrow h = y\sqrt{3}$............(i)

and $\tan 30^0 = \frac{h}{x+y}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y\sqrt{3}}{x+y}$ [using (i)]

$\therefore 3y = x + y$

$\Rightarrow 2y = x \Rightarrow y = \frac{x}{2}$

$\because$ Time taken to cover a distance from A to B = 20 minute

$\therefore$ time taken to cover a unit distance

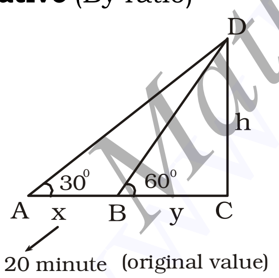
$= \frac{20}{x}$ minute

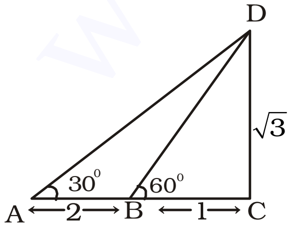
$\therefore$ For distance 'y' time taken

$= \frac{20}{x} \times y$

$= \frac{20}{x} \times \frac{x}{2} = 10$ minute

Alternative (By ratio)





(Real value)

Ratio value		original value
2	$\longrightarrow$	20 minute
$\therefore 1$	$\longrightarrow$	10 minute

$\therefore$ For distance y (or ratio value = 1) time taken =10minute

Ex.22 From the top of a 60 m high tower the angle of depression of the top and bottom of a building are observed to be 30^0 and 60^0 respectively. The height of the building is:-

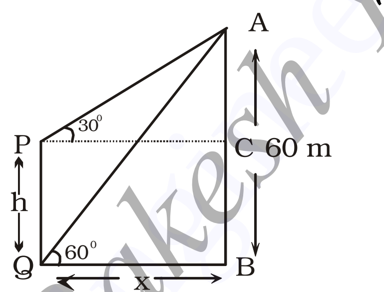
(a) $60\sqrt{3}$ m (b) $40\sqrt{3}$ m
(c) 40 m (d) 20 m

Sol. Let AB be the tower and PQ the building.

From right angled ΔABQ,

$\tan 60^0 = \frac{AB}{BQ} \Rightarrow \sqrt{3} = \frac{60}{x}$

$\Rightarrow x = 20\sqrt{3}$ ....**(i)**



and From right angled ΔAPC,

$\tan 30^0 = \frac{AC}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AC}{x}$

$AC = \frac{x}{\sqrt{3}}$

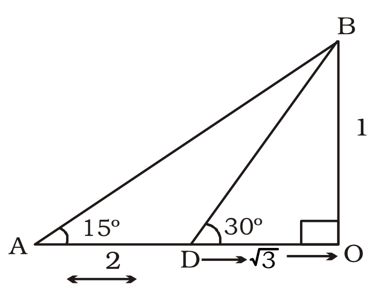
put the value of x in equation

(i) $AC = \frac{20\sqrt{3}}{\sqrt{3}} = 20$

$\therefore PQ = BC = AB - AC = 60 - 20 = 40$

$\therefore$ height of the building $= h = 40$ m

(D)



In Δ OAB

$\tan 15° = \frac{OB}{OA}$

$(2 - \sqrt{3}) = \frac{OB}{OA}$

$\left(\tan 15° = 2 - \sqrt{3} = \frac{1}{2+\sqrt{3}}\right)$

$\frac{1}{2+\sqrt{3}} = \frac{OB}{OA}$

OB : OA .....(i)

$1 : 2+\sqrt{3}$

In Δ BOD

$\tan 30° = \frac{OB}{OD}$

$\frac{1}{\sqrt{3}} = \frac{OB}{OD}$

OB : OD ....(ii)

$1 : \sqrt{3}$

From (i)& (ii)

to make same ratio

OB: OA	OB : OD
$1 : 2+\sqrt{3}$	$1 : \sqrt{3}$

OB : OA : OD

$1 : 2+\sqrt{3} : \sqrt{3}$

then,

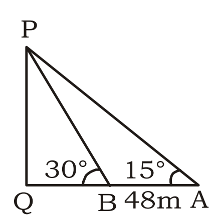
AD = OA – OD

$= (2+\sqrt{3}) - (\sqrt{3}) = 2$

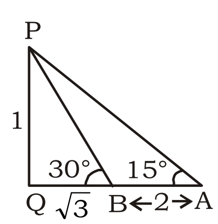
Ex.23 The angles of elevation of the top of a tower from two points A and B lying on the horizontal through the foot of the tower are respectively 15° and 30°. If A and B are on the same side of the tower and AB = 48 metre, then the height of the tower is:

(a) $24\sqrt{3}$ metre
(b) 24 metre
(c) $24\sqrt{2}$ metre
(d) 96 metre

Sol.



(original value)



(ratio value)

PQ = Tower = h metre

Ratio value		Original value
AB $\rightarrow$ 2	$\longrightarrow$	48
$\therefore$ 1	$\longrightarrow$	$\frac{48}{2}$

= 24 metre

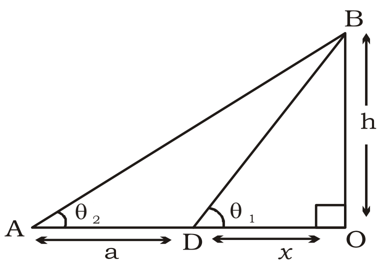
i.e. height of the building
= PQ (ratio value = 1) = h
= 24 metre.

Note:- you also can make other ratio for different angle.

Alternate:-

We can use formula for this type

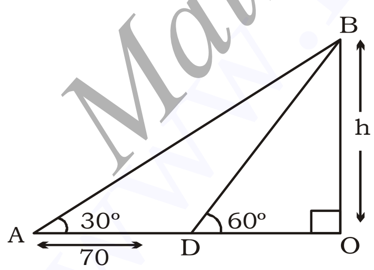
* The angle of elevation of the top of a tower from a point on the horizontal is θ_2 and moving 'a' distance towards the tower it becomes θ_1, the height of the tower is-



$$\boxed{h = \frac{a}{\cot\theta_2 - \cot\theta_1}}$$

Ex.24 The angle of elevation of the top of a tower from a point on the gound is 30° and moving 70 metres towards the tower it becomes 60°. The height of the tower is:

Sol.

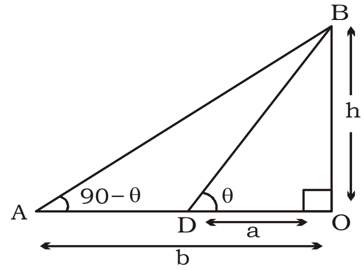


$$h = \frac{70}{\cot 30° - \cot 60°} = \frac{70}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$= \frac{70\sqrt{3}}{2} = 35\sqrt{3}$$

Type - III

* If the angles of a tower from two points distant a and b from its foot and in the same straight line from it is complementary each other then height of tower is.



$$\boxed{h = \sqrt{ab}}$$

Ex.25 The angle of elevation of the top of a tower at two points which are at a distance a and b from the foot in the same horizontal line and on the same side of the tower, are complementary. The height of the tower is :-

(a) ab (b) $\sqrt{ab}$

(c) $\sqrt{a/b}$ (d) $\sqrt{b/a}$

Sol. Let PQ be the given tower of height h. If A, B be given points then suppose.

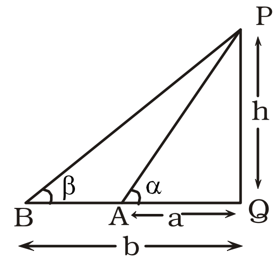
$\angle PAQ = \alpha$ and $\angle PBQ = \beta$

$\therefore \alpha + \beta = 90^0$

Now in ΔPAQ,

$$\tan\alpha = \frac{h}{a} \text{.............(i)}$$

in ΔPBQ,



$$\tan\beta = \frac{h}{b} \Rightarrow \tan\left(90^0 - \alpha\right) = \frac{h}{b}$$

$$\Rightarrow \cot\alpha = \frac{h}{b} \Rightarrow \tan\alpha = \frac{b}{h}$$

$$\Rightarrow \frac{h}{a} = \frac{b}{h}$$

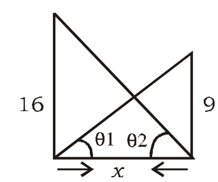
[from (i)- $\tan\alpha = \frac{h}{a}$]

$$\Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$$

Ex.26 The distance between two pillars of length 16 metres and 9 metres is x metres. If two angles of elevation of their respective top from the bottom of the other are complementary to each other then the value of x (in metres) is

(a) 15 (b) 16 (c) 12 (d) 9

Sol.



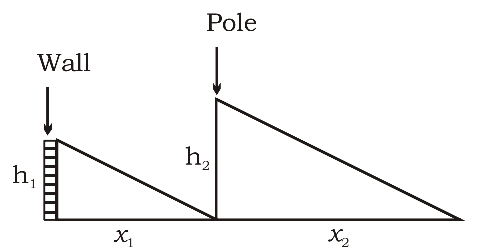
If $\theta_1 + \theta_2$ = 90° then $x = \sqrt{h_1 \times h_2}$

(h= height of towers)

$x = \sqrt{16 \times 9} = \sqrt{144}$ = **12 mtr**

TYPE - IV

* At a particular time for all object ratio of height shadow are same



$$\boxed{\tan\theta = \frac{h_1}{x_1} = \frac{h_2}{x_2}}$$

Ex.27 A vertical stick 12 cm long casts a shadow 8 cm long on the ground. At the same time, a tower casts a shadow 40 m long on the ground. The height of the tower is

(a) 72 m (b) 60 m
(c) 65 m (d) 70 m

Sol. $\frac{12}{8} = \frac{h_2}{40}$

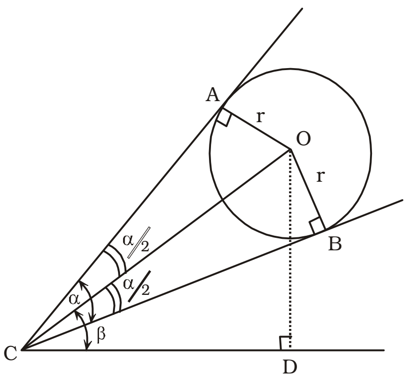
$h_2 = \frac{40 \times 12}{8}$

h_2 = 60 m.

TYPE - V

A round balloon of radius r subtends an angle α at the eye of an observer while the angle of elevation of its centre is β. Then the height of its centre from horizontal is

h $= r\sin\beta.\text{cosec}\,{}^{\alpha}\!/_{2}$



Proof....

Let O be the centre of balloon of radius r. The observer's eye is at C, $\angle ACB = \alpha$ and $\angle OCD = \beta$ clearly, CA and CB are tangents to the circle.

so $\angle ACO = \angle BCO = {}^{\alpha}\!/_{2}$

In right angled ΔOBC,

$\sin\frac{\alpha}{2} = \frac{OB}{OC} \Rightarrow OC = \frac{OB}{\sin{}^{\alpha}\!/_{2}} = r\,\text{cosec}\,{}^{\alpha}\!/_{2}$

In right angled ΔOCD,

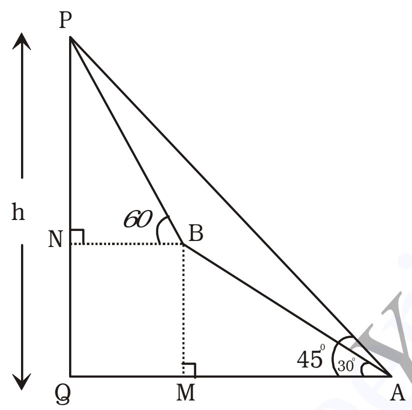
$\sin\beta = \frac{OD}{OC} \Rightarrow OD = OC\sin\beta = r\,\text{cosec}\,{}^{\alpha}\!/_{2}.\sin\beta$

$\therefore$ Height of the centre of the balloon is

$r\sin\beta.\text{cosec}\,{}^{\alpha}\!/_{2}$

TYPE – VI

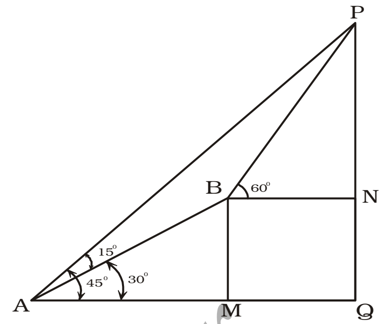
At the foot of the mountain the elevation of its summit is 45^0; after ascending AB km towards the mountain up a slope of 30^0 inclination, the elevation is found to be 60^0, Then the height of the mountain.



$$\boxed{h = AB\left(\frac{\sqrt{3}+1}{2}\right)}$$

Ex.28 At the foot of the mountain the elevation of its summit is 45^0; after ascending 4km towards the mountain up a slope of 30^0 inclination, the elevation is found to be 60^0, Then find the height of the mountain:-

Sol. Suppose P be the summit of the mountain and Q be the foot.



Here, $BN \perp PQ$ and $BM \perp AQ$,

$AB = 4km, \angle MAB = 30^0$,

$\angle MAP = 45^0$

$\angle NBP = 60^0 \quad \therefore \angle BAP = 15^0$

and $\angle APQ = 45^0$

and $\angle BPN = 30^0$

$\therefore \angle APB = 15^0$

$\therefore \Delta ABP$ is isosceles and

$AB = BP = 4km$

In ΔPBN,

$PN = BP\sin 60^0$

In ΔABM,

$BM = AB\sin 30^0$

$\therefore PQ = PN + NQ = PN + BM$

$= BP\sin 60^0 + AB\sin 30^0$

$= 4\frac{\sqrt{3}}{2} + 4\frac{1}{2} = 4\left(\frac{\sqrt{3}+1}{2}\right)$

$= 2\left(\sqrt{3}+1\right)$ km

$\therefore$ Height of the mountain is

$= 2\left(\sqrt{3}+1\right)$ km

Alternae:-

$PQ = AB\left(\frac{\sqrt{3}+1}{2}\right) = 4\left(\frac{\sqrt{3}+1}{2}\right)$

$= 2\left(\sqrt{3}+1\right)$ km

EXERCISE

1. At a point on a horizontal line through the base of a monument the angle of elevation of the top of the monument is found to be such that its tangent is $\frac{1}{5}$. On walking 138 metres towards the monument the secant of the angle of elevation is found to be $\frac{\sqrt{193}}{12}$. The height of the monument (in metre) is
(a) 42 (b) 49 (c) 35 (d) 56

2. The angle of elevation of the top of a building from the top and bottom of a tree are x and y respectively. If the height of the tree is h metre, then (in metre) the height of the building is
(a) $\frac{h\cot x}{\cot x+\cot y}$ (b) $\frac{h\cot y}{\cot x+\cot y}$
(c) $\frac{h\cot x}{\cot x-\cot y}$ (d) $\frac{h\cot y}{\cot x-\cot y}$

3. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metres towards the foot of the tower to a point B, the angle of elevation increases to 60°. The height of the tower is
(a) $\sqrt{3}$ m (b) $5\sqrt{3}$ m
(c) $10\sqrt{3}$ m (d) $20\sqrt{3}$ m

4. Two poles of equal height are standing opposite to each other on either side of a road which is 100m wide. From a point between them on road, angle of elevation of their tops are 30° and 60°. The height of each pole (in meter) is
(a) $25\sqrt{3}$ (b) $20\sqrt{3}$
(c) $28\sqrt{3}$ (d) $30\sqrt{3}$ m

5. The angle of elevation of the top of a chimney and roof of the building from a point on the ground are 45° and x^o respectively. The height of building is h metre. Then the height of the chimney,(in metre) is
(a) h cot x+ h (b) h cot x – h
(c) h tan x – h (d) h tan x +h

6. There are two vertical posts, one on each side of a road, just opposite to each other. One post is 108 metre high. From the top of this post the angle of depression of the top and foot of the other post are 30° and 60° respectively. The height of the other post (in metre) is
(a) 36 (b) 72 (c) 108 (d) 110

7. Two posts are x metres apart and the height of one is double that of the other. If from the mid-point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height (in metres) of the shorter post is
(a) $\frac{x}{2\sqrt{2}}$ (b) $\frac{x}{4}$
(c) $x\sqrt{2}$ (d) $\frac{x}{2}$

8. An aeroplane when flying at a height of 5000m from the ground passes vertically above another aeroplane at an instant, when the angles of elevation of the two aeroplanes from the same point on the ground are 60°and 45° respectively. The vertical distance between the aeroplanes at that instant is
(a) $5000\left(\sqrt{3}-1\right)$ m
(b) $5000\left(3-\sqrt{3}\right)$ m
(c) $5000\left(1-\frac{1}{\sqrt{3}}\right)$ m
(d) 4500 m

9. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30°. The man walks some distance towards the tower and then his angle of elevation of the top of the tower is 60°. If the height of tower is 30m, then the distance he moves is
(a) 22 m (b) $22\sqrt{3}$ m
(c) 20 m (d) $20\sqrt{3}$ m

10. An aeroplane when flying at a height of 3125m from the ground passes vertically below another plane at an instant when the angle of elevation of the two planes from the same point on the ground an 30° and 60° respectively. The distance between the two planes at that instant is
(a) 6520 m (b) 6000 m
(c) 5000 m (d) 6250 m

11. The shadow of the tower becomes 60 meters longer when the altitude of the sun changes from 45° to 30°. Then the height of the tower is
(a) $20\left(\sqrt{3}+1\right)$ m (b) $24\left(\sqrt{3}+1\right)$ m
(c) $30\left(\sqrt{3}+1\right)$ m (d) $30\left(\sqrt{3}-1\right)$ m

12. A vertical post 15 ft. high is broken at a certain height and its upper part, not completely separated meets the ground at an angle of 30°. Find the height at which the post is broken
(a) 10ft (b) 5ft
(c) $15\sqrt{3}\left(2-\sqrt{3}\right)$ ft
(d) $5\sqrt{3}$ ft

13. The shadow of a tower is $\sqrt{3}$ times its height . Then the angle of elevation of the top of the tower is
(a) 45° (b) 30° (c) 60° (d) 90°

14. A man 6ft tall casts a shadow 4ft long. At the same time when a flag pole casts a shadow 50 ft long. The height of the flag pole is
(a) 80ft (b) 75ft (c) 60ft (d) 70ft

15. The angle of elevation of an aeroplane from a point on the ground is 60°. After 15 seconds flight,the elevation changes to 30°, If the aeroplane is flying at a height of $1500\sqrt{3}$ m, find the speed of the plane.
(a) 300 m/sec (b) 200m/sec
(c) 100m/sec (d) 150m/sec

16. There are two temples, one on each bank of a river just opposite to each other. One temple is 54m high. From the top of this temple, the angles of

depression of the top and the foot of the other temple are 30° and 60° respectively. The length of the temple is;
(a) 18 m (b) 36 m
(c) $36\sqrt{3}$ m (d) $18\sqrt{3}$ m

17. The angle of elevation of the top of a tower from the point P and Q at distance of 'a' and 'b' respectively from the base of the tower and in the same straight line with it are complementary. The height of the tower is
(a) $\sqrt{ab}$ (b) $\frac{a}{b}$
(c) ab (d) a^2b^2

18. The angle of elevation of a tower from a distance 100 m from its foot is 30°. Height of the tower is
(a) $\frac{100}{\sqrt{3}}$ m (b) $50\sqrt{3}$ m
(c) $\frac{200}{\sqrt{3}}$ m (d) $100\sqrt{3}$ m

19. A pole stands vertically inside a scalene triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in ΔABC, the foot of the pole is at the
(a) centroid
(b) circumcentre
(c) incentre
(d) orthocentre

20. If the angle of elevation of a balloon from two consecutive kilometre-stones along a road are 30° and 60° respectively, then the height of the balloon above the ground will be
(a) $\frac{\sqrt{3}}{2}$ km (b) $\frac{1}{2}$ km
(c) $\frac{2}{\sqrt{3}}$ km (d) $3\sqrt{3}$ km

21. A tower standing on a horizontal plane subtends a certain angle at a point 160 m apart from the foot of the tower. On advancing 100 m towards it, the tower is found to subtend and angle twice as before. The height of the tower is
(a) 80 m (b) 100 m
(c) 160 m (d) 200 m

22. The angle of elevation of a tower from a distance 50 m from its foot is 30°. The height of the tower is
(a) $50\sqrt{3}$ m (b) $\frac{50}{\sqrt{3}}$ m
(c) $75\sqrt{3}$ m (d) $\frac{75}{\sqrt{3}}$ m

23. From two points on the ground lying on a straight line through the foot of a pillar, the two angles of elevation of the top of the pillar are complementary to each other. If the distance of the two points from the foot of the pillar are 9 metres and 16 metres and the two points lie on the same side of the pillar. Then the height of the pillar is
(a) 5m (b) 10m
(c) 9m (d) 12m

24. The top of two poles of height 24m and 36 m are connected by a wire. If the wire makes an angle of 60° with the horizontal, then the length of the wire is
(a) 6m (b) $8\sqrt{3}$ m
(c) 8 m (d) $6\sqrt{3}$ m

25. From the top of a hill 200 m high the angle of depression of the top and the bottom of a tower are observed to be 30° and 60°. The height of the tower is (in m);
(a) $\frac{400\sqrt{3}}{3}$ m (b) $166\frac{2}{3}$ m
(c) $133\frac{1}{3}$ m (d) $200\sqrt{3}$ m

26. From a tower 125 metres high the angle of depression of two objects, which are in horizontal line through the base of the tower are 45° and 30° and they are on the same side of the tower. The distance (in metres) between the objects is
(a) $125\sqrt{3}$ m (b) $125\left(\sqrt{3}-1\right)$ m
(c) $125/\left(\sqrt{3}-1\right)$ m (d) $125\left(\sqrt{3}+1\right)$ m

27. From a point P on the ground the angle of elevation of the top of a 10m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff (Take $\sqrt{3}=1.732$)
(a) $10\left(\sqrt{30}+2\right)$ m (b) $10\left(\sqrt{30}+1\right)$ m
(c) $10\sqrt{3}$ m (d) 7.32 m

28. The angle of elevation of the top of a vertical tower situated perpendicularly on a plane is observed as 60° from a point P on the same plane. From another point Q, 10m vertically above the point P, the angle of depression of the foot of the tower is 30°. The height of the tower is
(a) 15 m (b) 30 m
(c) 20 m (d) 25 m

29. From a point 20 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°. The height of the tower is
(a) $10\sqrt{3}$ m (b) $20\sqrt{3}$ m
(c) $\frac{10}{\sqrt{3}}$ m (d) $\frac{20}{\sqrt{3}}$ m

30. The angle of elevation of ladder leaning against a house is 60° and the foot of the ladder is 6.5 metres from the house. The length of the ladder is
(a) $\frac{13}{\sqrt{3}}$ (b) 13 meters
(c) 15 meters (d) 3.25 metres

31. The angle of elevation of sun changes form 30°to 45°, the length of the shadow of a pole decreases by 4 metres, the height of the pole is (Assume $\sqrt{3}=1.732$)
(a) 1.464m (b) 9.464 m
(c) 3.648 cm (d) 5.464 m

32. A vertical pole and a vertical tower are standing on the same level ground. Height of the pole is 10 metres. From the top of the pole the angle of elevation of the top of the tower and angle of depression of the foot of the tower are 60° and 30° respectively. The height of the tower is
(a) 20 m (b) 30 m
(c) 40 m (d) 50 m

33. If a pole of 12 m height caste a shadow of $4\sqrt{3}$ m long on the ground then the sun's angle of elevation at that instant is
(a) 30° (b) 60° (c) 45° (d) 90°

34. The angle of elevation of the top of a tower from a point on the ground is 30° and moving 70 meters towards the tower it becomes 60°. The height of the tower is

(a) 10 meter (b) $\frac{10}{\sqrt{3}}$ metre

(c) $10\sqrt{3}$ metre (d) $35\sqrt{3}$ metre

35. From the top of a tower of height 180m the angles of depression of two objects on both sides of the tower are 30° and 45°. Then the distance between the objects are

(a) $180(3+\sqrt{3})$ m

(b) $180(3-\sqrt{3})$ m

(c) $180(\sqrt{3}-1)$ m

(d) $180(\sqrt{3}+1)$ m

36. From the peak of a hill which is 300m high , the angle of depression of two sides of a bridge lying on a ground are 45° and 30°(both ends of the bridge are on the same side of the hill). Then the length of the bridge is

(a) $300(\sqrt{3}-1)$ m (b) $300(\sqrt{3}+1)$

(c) $300\sqrt{3}$ m (d) $\frac{300}{\sqrt{3}}$ m

37. From an aeroplane just over a river, trees on the opposite bank of the river are found to be at an angle of 60° and 30° respectively. If the breadth of the river is 400 metres, then the height of the aeroplane above the river at that instant is (Assume $\sqrt{3} = 1.732$)

(a) 173.2 metres
(b) 346.4 metres
(c) 519.6 metres
(d) 692.8 metres

38. The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30°, then when it is 60°. What is the height of the tower?

(a) 25 m (b) $25\sqrt{3}$ m

(c) $\frac{25}{\sqrt{3}}$ m (d) 30 m

39. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. The distance between the two towers is m times the height of the shorter tower. What is m equal to?

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

40. A spherical balloon of radius r subtends angle 60° at the eye of an observer. If the angle of elevation of its centre is 60° and h is height of the centre of the balloon, then which one of the following is correct?

(a) h = r (b) h = $\sqrt{2}r$

(c) h = $\sqrt{3}$r (d) h = 2r

41. What is the angle of elevation of the Sun, when the shadow of a pole of height x m is $\frac{x}{\sqrt{3}}$ m?

(a) 30° (b) 45° (c) 60° (d) 75°

42. On walking 120 m towards a chimney in a horizontal line through its base the angle of elevation of tip of the chimney changes from 30° to 45°. The height of the chimney is

(a) 120 m (b) $60(\sqrt{3}-1)$m

(c) $60(\sqrt{3}+1)$m

(d) None of these

43. A ladder 20 m long is placed against a wall, so that the foot of the ladder is 10 m from the wall. The angle of inclination of the ladder to the horizontal will be

(a) 30° (b) 45° (c) 60° (d) 75°

44. The angles of elevation of the top of a tower from two points which are at distance of 10 m and 5 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is

(a) 5 m (b) 15m

(c) $\sqrt{50}$ m (d) $\sqrt{75}$ m

45. The angles of elevation of the top of an inaccessible tower from two points on the same straight line from the base of the tower are 30° and 60°, respectively. If the point is are separated at a distance of 100 m, then the height of the tower is close to

(a) 86.6 m (b) 84.6 m
(c) 82.6 m (d) 80.6 m

46. Two poles of heights 6m and 11m stand on a plane ground. If the distance between their feet is 12 m, what is the distance their tops?

(a) 13 m (b) 17 m
(c) 18 m (d) 23 m

47. From the top of a cliff 200 m high, the angles of depression of the top and bottom of a tower of observed to be 30° and 45°, respectively. What is the height of the tower?

(a) 400 m (b) $400\sqrt{3}$

(c) $400\sqrt{3}$ m

(d) None of these

48. The angle of elevation the tip of a tower from a point on the ground is 45°. Moving 21 m directly towards the base of the tower, the angle of elevation changes to 60°. What is the height of the tower, to the nearest meter?

(a) 48 m (b) 49 m
(c) 50 m (d) 51m

49. What is the angle of elevation of the Sun when the shadow of a pole is $\sqrt{3}$ times the length of the pole?

(a) 30° (b) 45° (c) 60°
(d) None of these

50. The angles of elevation of the top of a tower from two points situated at distances 36 m and 64 m from its base and in the same straight line with it are complementary. What is the height of the tower?

(a) 50 m (b) 48 m
(c) 25 m (d) 24 m

51. The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 100 m from its base is 45°. If the angle of elevation of the top of complete pillar at the same point is to be 60°, then the height of the incomplete pillar is to be increased by

(a) $50\sqrt{2}$ m (b) 100 m

(c) 100 $\left(\sqrt{3}-1\right)$ m

(d) 100 $\left(\sqrt{3}+1\right)$ m

52. The length of shadow of a tree is 16 m when the angle of elevation of the Sun is 60°. What is the height of the tree?
(a) 8 m (b) 16 m
(c) $16\sqrt{3}$ m (d) $\frac{16}{\sqrt{3}}$ m

53. From a lighthouse the angles of depression of two ships on oppsite sides of the lighthouse are observed to 30° and 45°. If the height of lighthouse is h, what is the distance between the ships?
(a) $\left(\sqrt{3}+1\right)$ h (b) $\left(\sqrt{3}-1\right)$ h
(c) $\sqrt{3}$ h (d) $\left(1+\frac{1}{\sqrt{3}}\right)$

54. A telegraph post gets broken at a point against a storm and its top touches the ground at a distance 20 m from the base of the post making an angle 30° with the ground. What is the height of the post?
(a) $\frac{40}{\sqrt{3}}$ m (b) $20\sqrt{3}$ m
(c) $40\sqrt{3}$ m (d) 30 m

55. The angle of elevation of the top of a tower from the bottom of a building is twice that from its, top. What is the height of the building, if the height of the tower is 75 m and the angle of elevation of the top of the tower from the bottom of the building is 60°?
(a) 25 m (b) 37.5 m
(c) 50 m (d) 60 m

56. The shadow of a tower is 15 m when the Sun's altitude is 30°. What is the length of the shadow when the Sun's altitude is 60°?
(a) 3m (b) 4m (c) 5m (d) 6m

57. A man is watching from the top to tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at distance of 60 m from the bottom of tower. After 5 s, the angle of depression becomes 30°. What is the approximate speed of the boat assuming that it is running in still water?
(a) 31.5 km/h (b) 36.5 km/h
(c) 38.5 km/h (d) 40.5 km/h

58. Suppose the angle of elevation of the top of a tree at a point E due East of the tree is 60° and that at a point F due West of the tree is 30°. If the distance between the points E and F is 160 ft, then what is the height of the tree?
(a) $40\sqrt{3}$ ft (b) 60 ft
(c) $\frac{40}{\sqrt{3}}$ ft (d) 23 ft

59. The length of the shadow of a person s cm tall when the angle of elevation of the Sun is α is p cm. It is q cm when the angle of elevation of the Sun is β. Which one of the following is correct when $\beta = 3\alpha$?
(a) $p - q = s\left(\frac{\tan\alpha - \tan 3\alpha}{\tan 3\alpha \tan\alpha}\right)$
(b) $p - q = s\left(\frac{\tan 3\alpha - \tan\alpha}{3\tan\alpha \tan\alpha}\right)$
(c) $p - q = s\left(\frac{\tan 3\alpha - \tan\alpha}{\tan 3\alpha \tan\alpha}\right)$
(d) $p - q = s\left(\frac{\tan 2\alpha}{\tan 3\alpha \tan\alpha}\right)$

60. A radio transmitter antenna of height 100 m stands at the top of a tall building. At a point on the ground, the angle of elevation of bottom of the antenna is 45° and that of top of antenna is 60°. What is the height of the building?
(a) 100 m (b) 50 m
(c) 50 $\left(\sqrt{3}+1\right)$ m
(d) 50 $\left(\sqrt{3}-1\right)$ m

61. The angle of elevation of the top of an unfinished pillar at a point 150 m from its base is 30°. If the angle of elevation at the same point is to be 45°, then the pillar has to be raised to a height of how many metres?
(a) 59.4 m (b) 61.4 m
(c) 62.4 m (d) 63.4 m

62. A round balloon of unit radius subtends an angle of 90° at the eye of an observer standing at a point, say A. What is the distance of the centre of the balloon from the point A?
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{2}$

63. Person standing at the end of the shadow is $\frac{1}{\sqrt{3}}$ times the length of the pole. At what angle of elevation will the man see the Sun?
(a) 60° (b) 30° (c) 45° (d) 15°

64. The angle of depression of vertex of a regular hexagon lying in a horizontal plane, from the top of tower of height 75 m located at the centre of the regular hexagon is 60°. What is the length of each side of the hexagon?
(a) $50\sqrt{3}$ m (b) 75 m
(c) $25\sqrt{3}$ m (d) 25 m

65. Two houses are collinear with the base of a tower and are at distance 3 m and 12 m from the base of the tower. The angles of elevation from these two houses of the top of the tower are complementary.

What is the height of the tower?
(a) 4 m (b) 6 m
(c) 7.5 m (d) 36 m

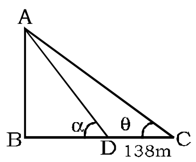
66. The angle of elevation from the bank of a river of the top of a tree standing on the opposite bank is 60°. The angle of elevation becomes 30° when observed from a point 40 m backwards in a direction perpendicular to the length of the river. What is the width of the river?
(a) 10 m (b) 20 m
(c) 30 m (d) 40 m

ANSWER KEY

1. (a)	8. (c)	15. (b)	22. (b)	29. (d)	36. (a)	43. (c)	49. (a)	55. (c)	61. (d)
2. (c)	9. (d)	16. (b)	23. (d)	30. (b)	37. (a)	44. (c)	50. (b)	56. (c)	62. (b)
3. (c)	10. (d)	17. (a)	24. (b)	31. (d)	38. (b)	45. (a)	51. (c)	57. (a)	63. (a)
4. (a)	11. (c)	18. (a)	25. (c)	32. (c)	39. (b)	46. (a)	52. (c)	58. (a)	64. (c)
5. (b)	12. (b)	19. (b)	26. (b)	33. (b)	40. (c)	47. (d)	53. (a)	59. (c)	65. (b)
6. (b)	13. (b)	20. (a)	27. (d)	34. (d)	41. (c)	48. (c)	54. (b)	60. (c)	66. (b)
7. (a)	14. (b)	21. (a)	28. (b)	35. (d)	42. (c)				

SOLUTION

1. (a)



Shortcut approach

I[st] Case:

$\tan\theta = \frac{AB}{BC} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{1}{5}$

II[st] Case:

$Sec\alpha = \frac{AD}{BD} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\sqrt{193}}{12}$

In ΔABD

Hypotenuse = $\sqrt{193}$

Base = 12

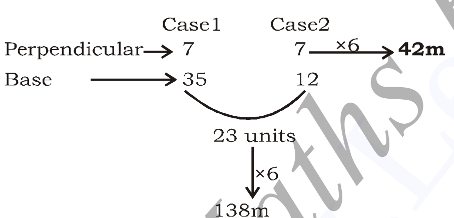
Then perpendicular = 7

(By pythagoras theorem

In Case I Perpendicular is 1.

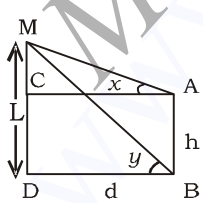
So equal this

$\tan\theta = \frac{1\times7}{5\times7} = \frac{7 \leftarrow \text{Perpen.}}{35 \leftarrow \text{Base}}$



AB = 7 × 6 = **42m**

2. (c)



AB = tree 'h'

MD = Building 'L'

DB = CA = 'd'

In ΔMCA ,

$\tan x = \frac{MC}{AC} = \frac{L-h}{d}$

$\Rightarrow d = \frac{L-h}{\tan x}$

$\Rightarrow d = (L - h) \cot x$ .....(i)

In ΔMDB ,

$\tan y = \frac{L}{d} = \frac{MD}{DB}$

$d = L \cot y$ .....(ii)

from equation (i) and (ii)

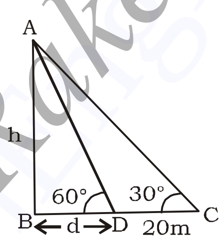
$(L - h) \cot x = L \cot y$

$L \cot x - h \cot x = L \cot y$

$h \cot x = L (\cot x - \cot y)$

$L = \frac{h\cot x}{\cot x - \cot y}$

3. (c)



AB ='h' metre

In ΔABC

$\tan 30° = \frac{AB}{BC}$

$\Rightarrow \frac{h}{(d+20)}$

$\frac{1}{\sqrt{3}} = \frac{h}{(d+20)}$

$\sqrt{3}h = d + 20$ ......(i)

In ΔABD

$\tan 60° = \frac{AB}{BD} = \frac{h}{d}$

$\sqrt{3} = \frac{h}{d}$

$h = \sqrt{3}d$

$d = \frac{h}{\sqrt{3}}$ ......(ii)

Put the value of d in equation (i)

$\sqrt{3}h = \frac{h}{\sqrt{3}} + 20$

$\sqrt{3}h = \frac{h+20\sqrt{3}}{\sqrt{3}}$

$3h = h + 20\sqrt{3}$

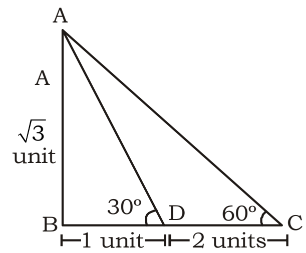
$2h = 20\sqrt{3}$

$h = \frac{20\sqrt{3}}{2}$

$h = 10\sqrt{3}$ metre

Alternate:-

As we know the ratio of



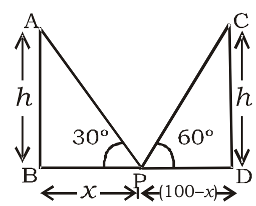
2 unit = CD = 20 metres

1 unit = 10 metres

$\sqrt{3}$ units = AB = $\sqrt{3}\times10$

= $10\sqrt{3}$ metres

4. (a)



BD = 100

AB = CD = 'h' metre (Height of pole)

In ΔABP ,

$\Rightarrow \tan 30° = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow \sqrt{3}h = x$ ....(i)

In ΔCDP

$\tan 60° = \dfrac{h}{(100 - x)}$

$\Rightarrow \sqrt{3}\,(100 - x) = h$

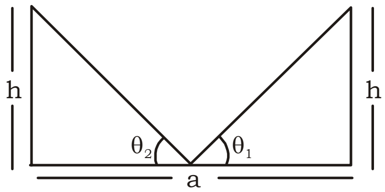
$\Rightarrow \sqrt{3}\left(100 - \sqrt{3}h\right) = h$

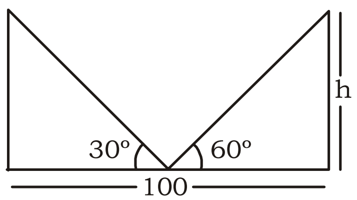
(Put the value of x from equation (i)

$\Rightarrow 100\sqrt{3} - 3h = h \Rightarrow 4h = 100\sqrt{3}$

$h = 25\sqrt{3}$ metre

Alternate:-



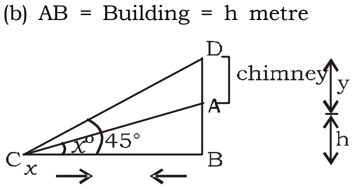


$h = \dfrac{a}{\cot\theta_2 + \cot\theta_1}$

$h = \dfrac{100}{\sqrt{3} + \dfrac{1}{\sqrt{3}}}$

$\Rightarrow 25\sqrt{3}$ metre

5. (b) AB = Building = h metre



AD = chimney = 'y' meter

In ΔDCB

$\tan 45° = \dfrac{DB}{BC} \Rightarrow 1 = \dfrac{h+y}{BC}$

$\Rightarrow BC = h + y$ ....(i)

In ΔACB

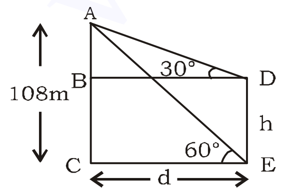
$\tan x° = \dfrac{AB}{BC} \Rightarrow \tan x = \dfrac{h}{BC} \Rightarrow BC = h\cot x$....(ii)

from equation (i) and (ii)

$\Rightarrow h + y = h\cot x$

$\Rightarrow y = (h\cot x - h)$ metre

6. (b)



In ΔACE

$\tan 60° = \dfrac{AC}{CE}$

$\dfrac{\sqrt{3}}{1} = \dfrac{AC}{CE} = AC : CE = \sqrt{3} : 1$......(i)

In ΔABD

$\tan 30° = \dfrac{AB}{BD}$

$\dfrac{1}{\sqrt{3}} = \dfrac{AB}{BD} \Rightarrow AB : BD = 1 : \sqrt{3}$ ..(ii)

Since BD = CE

$\therefore$ AC : CE : AB

equation(I)$\rightarrow \sqrt{3}$: 1

equation(II)$\rightarrow$ $\sqrt{3}$: 1

3 : $\sqrt{3}$: 1

$\downarrow \times 36$ $\qquad \downarrow \times 36$

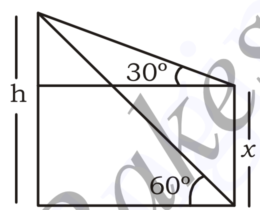
acutal height $\rightarrow$ 108 $\qquad$ 36

$\Rightarrow DE = AC - AB$

$= 108 - 36 = 72$ metre

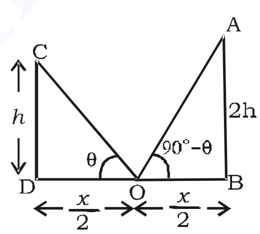
Alternate:

In such question when angles are 30° and 60°



$x = \dfrac{2h}{3} = \dfrac{2 \times 108}{3} = 72$cm

7. (a)



From figure,

$OB = OD = \dfrac{x}{2}$

In ΔOCD,

$\tan\theta = \dfrac{h}{\frac{x}{2}} \Rightarrow \dfrac{2h}{x}$ ....(i)

In ΔAOB

$\tan(90° - \theta) = \dfrac{AB}{OB}$

$\Rightarrow \cot\theta = \dfrac{2h}{\frac{x}{2}} = \dfrac{4h}{x}$ ....(ii)

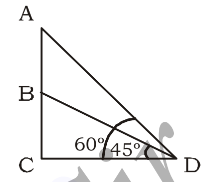
Multiplying both equations,

$\tan\theta.\cot\theta = \dfrac{2h}{x} \times \dfrac{4h}{x}$

$\Rightarrow x^2 = 8h^2$

$\Rightarrow h^2 = \dfrac{x^2}{8} \Rightarrow h = \dfrac{x}{2\sqrt{2}}$ metre

8. (c)



AC = 5000 m (given)

In ΔACD,

$\tan 60° = \dfrac{AC}{CD}$

$\sqrt{3} = \dfrac{AC}{CD} \Rightarrow AC : CD = \sqrt{3} : 1$

In ΔBCD,

$\tan 45° = \dfrac{BC}{CD}$

$1 = \dfrac{BC}{CD} = BC : CD = 1 : 1$ ....(ii)

Now,

BC	:	CD	:	AC
1	:	1	:	
		1	:	$\sqrt{3}$
1	:	1	:	$\sqrt{3}$

$AB = AC - BC$

$= \left(\sqrt{3} - 1\right)$ units

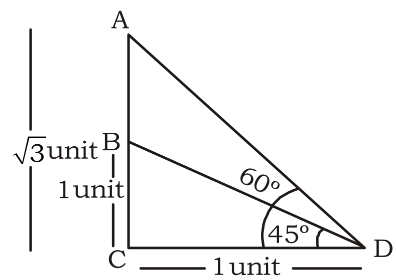
$\sqrt{3}$ units = 5000 m

1 unit = $\dfrac{5000}{\sqrt{3}}$ m

$AB = \left(\sqrt{3} - 1\right) units = \dfrac{5000}{\sqrt{3}}\left(\sqrt{3} - 1\right)$

$= 5000\left[1 - \dfrac{1}{\sqrt{3}}\right]$ m

Alternate:



AC = $\sqrt{3}$ unit

BC = 1 unit

AB = $(\sqrt{3} - 1)$ unit

According to question,

AC = $\sqrt{3}$ unit = 5000 m

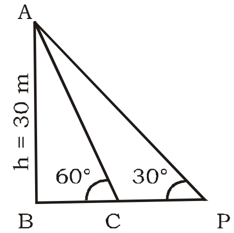
$1 \text{ unit} = \dfrac{5000}{\sqrt{3}}$ m

So, the difference between the aeroplane = AB = $(\sqrt{3}-1)$ unit

$(\sqrt{3}-1)\text{unit} = \dfrac{5000}{\sqrt{3}}(\sqrt{3}-1)\text{m}$

$AB = 5000\left(1-\dfrac{1}{\sqrt{3}}\right)$ m

9. (d)



h = 30 m

PC = ?

In ΔABP

$\tan 30° = \dfrac{AB}{BP}$

$\dfrac{1}{\sqrt{3}} = \dfrac{AB}{BP} \Rightarrow AB : BP = 1 : \sqrt{3}$ .....(i)

In ΔABC

$\tan 60° = \dfrac{AB}{BC}$

$\dfrac{\sqrt{3}}{1} = \dfrac{AB}{BC} \Rightarrow AB : BC = \sqrt{3} : 1$......(ii)

BP	:	AB	:	BC
$\sqrt{3}$	:	1		
		$\sqrt{3}$	:	1
3	:	$\sqrt{3}$	:	1

Now

$AB = \sqrt{3}$ units = 30 metre

$1 \text{ unit} = \dfrac{30}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

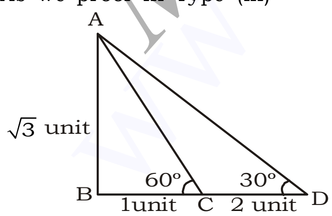
$= 10\sqrt{3}$ metre

PC = 3 – 1 = 2 units

$= 10\sqrt{3} \times 2 = 20\sqrt{3}$ metre

Alternate:

As we proof in Type (iii)



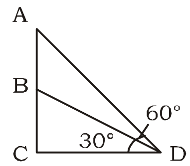
AB = 30m = $\sqrt{3}$ units

$1 \text{ unit} = \dfrac{30}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$ metre

So, PC = 2 units

$\Rightarrow 2 \times 10\sqrt{3} = 20\sqrt{3}$ metre

10. (d) BC = 3125 m



In ΔACD

$\tan 60° = \dfrac{AC}{DC}$

$\dfrac{\sqrt{3}}{1} = \dfrac{AC}{DC}$

$AC : DC = \sqrt{3} : 1$ ....(i)

In ΔDCB

$\tan 30° = \dfrac{BC}{DC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{BC}{DC}$

$BC : DC = 1 : \sqrt{3}$ ......(ii)

Now,

AC	:	DC	:	BC
$\sqrt{3}$	:	1		
		$\sqrt{3}$	:	1
3	:	$\sqrt{3}$	:	1

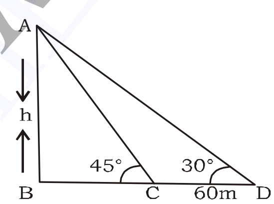
BC = 1 unit = 3125 m

AB = AC – BC

⇒ 3 – 1 = 2 units

AB = 2 × 3125 = **6250 m**

11. (c)



h = height

In ΔABC

$\tan 45° = \dfrac{AB}{BC}$

$\dfrac{1}{1} = \dfrac{AB}{BC} = AB : BC = 1 : 1$ ....(i)

In ΔABD

$\tan 30° = \dfrac{AB}{BD}$

$\dfrac{1}{\sqrt{3}} = \dfrac{AB}{BD}$

$\Rightarrow AB : BD = 1 : \sqrt{3}$ ......(ii)

Now,

BD	:	AB	:	BC
		1	:	1
$\sqrt{3}$	:	1		
$\sqrt{3}$	:	1	:	1

CD = BD – BC

$CD = \sqrt{3} - 1$

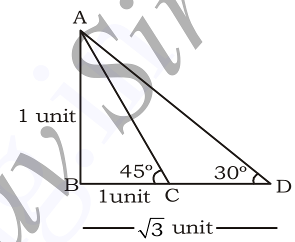
$(\sqrt{3} - 1)$ units = 60

$1 \text{ unit} = \dfrac{60}{\sqrt{3}-1}$

$h = \dfrac{60}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$h = 30\left(\sqrt{3}+1\right)$ m

Alternate:



AB = 1 unit

BC = 1 unit

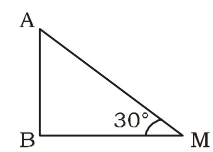
$CD = (\sqrt{3}-1)$ unit

$CD = (\sqrt{3}-1)$ unit = 60 m

$\text{Height (AB)} = \dfrac{60}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= 30(\sqrt{3}+1)$m

12. (b)



MAB was straight earlier

AB + AM = 15 ft

In ΔABM

$\tan 30° = \dfrac{AB}{BM}$

$\dfrac{1}{\sqrt{3}} = \dfrac{AB}{BM}$

If AB = 1

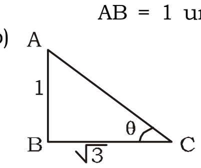
and BM = $\sqrt{3}$

then AM = 2(By pythagoras theorem)

AB + AM = 2 +1 ⇒ 3 units = 15 ft

AB = 1 unit = 5ft

13. (b)



In ΔABC

$\tan\theta = \dfrac{1}{\sqrt{3}}$

$\tan\theta = \tan 30° \Rightarrow \theta = 30°$

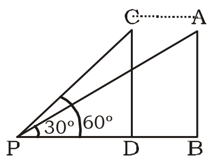
14. (b)

Height		Shadow
6ft		4ft
3	:	2

So height of pole will be in same ratio.

$= 50 \times \dfrac{3}{2} = $ **75ft**

15. (b)



AB = CD = $1500\sqrt{3}$ m
(height of aeroplane)
In ΔPDC

$\tan 60° = \frac{CD}{PD}$

$\sqrt{3} = \frac{CD}{PD} \Rightarrow CD:PD = \sqrt{3}:1 \ldots$(i)

In ΔPBA

$\tan 30° = \frac{AB}{PB}$

$\frac{1}{\sqrt{3}} = \frac{AB}{PB} \Rightarrow AB : PB = 1:\sqrt{3}$ .....(ii)

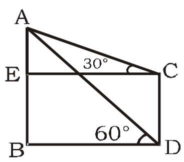
AC = BD and AB = CD

PD : AB : PB
1 : $\sqrt{3}$
 1 : $\sqrt{3}$
1 : $\sqrt{3}$: 3

DB = PB − PD
= 3 − 1 = 2 units
AB = $\sqrt{3}$ units = $1500\sqrt{3}$ m
$\Rightarrow$ 1 unit = 1500 m
CA = DB $\Rightarrow$ 2 units = 3000 metre

Speed = $\frac{\text{Distance}}{\text{Time}} = \frac{3000}{15}$ =200 m/s

16. (b)



AB and CD are temples
BD = width of river
AB = 54 m
In ΔAEC

$\tan 30° = \frac{AE}{EC} = \frac{1}{\sqrt{3}}$

$\Rightarrow AE : EC = 1 : \sqrt{3}$

In ΔABD

$\tan 60° = \frac{AB}{BD}$

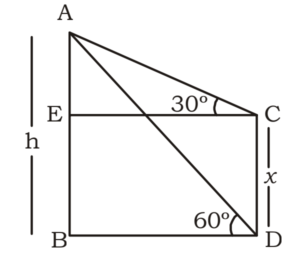
$\sqrt{3} = \frac{AB}{BD} \Rightarrow AB : BD = \sqrt{3}:1$ ......(ii)

EB = CD and EC = BD
Now,

AB : BD : AE
 $\sqrt{3}$: 1
$\sqrt{3}$: 1
3 : $\sqrt{3}$: 1

CD = AB − AE
= 3 − 1 = 2 units
AB = 3 units × 18 = 54 m
CD =2 units × 18 = 36 m

Alternate:



AB = h = 54m
CD = x

$x = \frac{2h}{3} = \frac{2\times 54}{3}$ = 36m

Alternate:-

$H = \frac{h\cot\theta}{\cot\theta - \cot\alpha}$

Where, θ = smaller angle
α = greater angle

$54 = \frac{h\cot 30°}{\cot 30° - \cot 60°}$

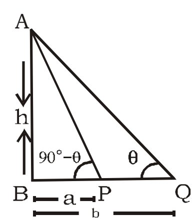
$54 = \frac{h\sqrt{3}}{\sqrt{3} - \frac{1}{\sqrt{3}}}$

$54 = \frac{h\sqrt{3}}{\frac{3-1}{\sqrt{3}}}$

$54 = \frac{3h}{2}$

$h = \frac{54\times 2}{3}$ = 36 m

17. (a)



AB is tower
$\therefore \angle AQB = \theta \ \& \ \angle APB = 90° - \theta$
PB = a, BQ = b
In ΔAQB

$\tan\theta = \frac{AB}{BQ}$

$\tan\theta = \frac{h}{b}$ .....(i)

In ΔAPB

$\tan(90° - \theta) = \frac{h}{PB}$

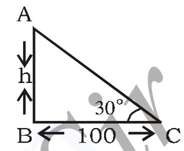
$\Rightarrow \cot\theta = \frac{h}{a}$ ....(ii)

By multiplying both equation

$\tan\theta . \cot\theta = \frac{h}{b}\times\frac{h}{a}$

$h^2 = ab \Rightarrow h = \sqrt{ab}$

18. (a)



In ΔABC

$\tan 30° = \frac{AB}{BC}$

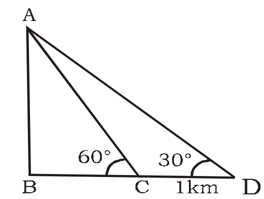
$\frac{1}{\sqrt{3}} = \frac{AB}{BC}$

AB : BC = 1 : $\sqrt{3}$
↓ ↓
$\frac{100}{\sqrt{3}}$m 100m

$\therefore$ Height of tower = $\frac{100}{\sqrt{3}}$ m

19. (b) It should be on circumcentre.

20. (a)



AB = height of balloon
In ΔABC

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{BC} \Rightarrow AB : BC = \sqrt{3}:1$

In ΔABD

$\tan 30° = \frac{AB}{BD} \Rightarrow AB : BD = 1:\sqrt{3}$

Now,

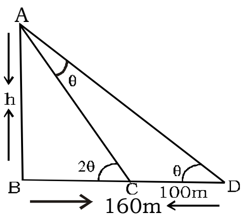
BC : AB : BD
1 : $\sqrt{3}$
 1 : $\sqrt{3}$
1 : $\sqrt{3}$: 3

CD = BD − BC
= 3 − 1 = 2 units
2 units = 1 km

1 unit = $\frac{1}{2}$

AB = $\sqrt{3}$ unit = $\frac{1}{2}\times\sqrt{3} = \frac{\sqrt{3}}{2}$ km

21. (a)



BD = 160m (given)

In ΔACD

exterior $\angle ACB = \angle CAD + \angle ADC$

$2\theta = \angle CAD + \theta$

$\angle CAD = \theta$

$\therefore$ AC = CD

AC = 100 m

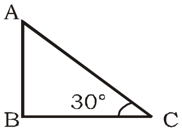
In ΔABC

AC = 100 m

BC = 160 − 100 = 60 m

Then AB = 80 m (By pythagoras theorem)

22. (b)



AB = Tower

BC = 50 m (Given)

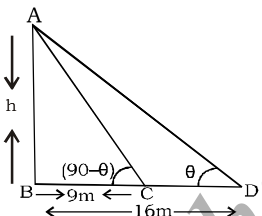
In ΔABC

$\tan\theta = \frac{AB}{BC}$

$\tan 30° = \frac{AB}{50}$

AB = $\frac{50}{\sqrt{3}}$ m

23. (d)



AB = Pillar

BC = 9 metre

BD = 16 metre

$\angle ADB = \theta$

In ΔABC

$\tan(90° - \theta) = \frac{AB}{BC}$

$\cot\theta = \frac{AB}{BC} = \frac{h}{9}$ ...(i)

In ΔABD

$\tan\theta = \frac{h}{16}$ ...(ii)

By multiplying equation (i) and (ii)

$\tan\theta \cdot \cot\theta = \frac{h}{9} \times \frac{h}{16}$

$\Rightarrow \frac{h^2}{144} = 1$

$\Rightarrow h^2 = 144$

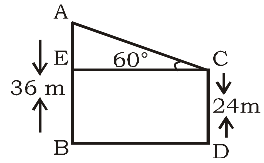
h = $\sqrt{144}$

h = 12 metre

Alternate:-

h = $\sqrt{16 \times 9}$ = 12 m.

24. (b)



AC = wire

AB and CD are two poles

In ΔAEC

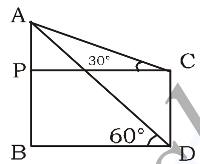
$\sin 60° = \frac{AE}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{12}{AC}$

(AE = AB − CD = 36 − 24 = 12 m)

AC = $\frac{24}{\sqrt{3}}$ = $8\sqrt{3}$ m

25. (c)



AB = hill =200 metre

CD = tower

In ΔAPC

$\tan 30° = \frac{AP}{PC}$

$\frac{1}{\sqrt{3}} = \frac{AP}{PC}$ = AP : PC = $1 : \sqrt{3}$ ....(i)

In ΔABD

$\tan 60° = \frac{AB}{BD}$

$\sqrt{3} = \frac{AB}{BD}$ = AB : BD = $\sqrt{3} : 1$ ......(ii)

PB = CD and PC = BD

Now

AB	:	BD	:	AP
$\sqrt{3}$	:	1		
		$\sqrt{3}$	:	1
3	:	$\sqrt{3}$	:	1

CD = PB $\Rightarrow$ AB − AP

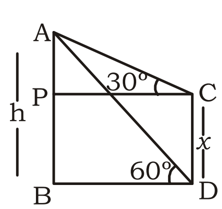
CD = 3 − 1 = 2 units

AB = 3 units = 200 metre

CD = 2 units = $\frac{200}{3} \times 2$

= $133\frac{1}{3}$ metre

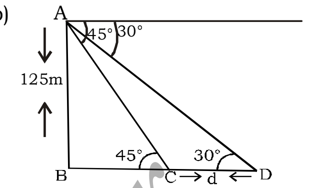
Alternate:-



AB = h = 200z

CD = x = $\frac{2h}{3}$ = $\frac{2 \times 200}{3}$ = $133\frac{1}{3}$ m

26. (b)



AB = Tower

In ΔABC $\tan 45° = \frac{AB}{BC}$

$1 = \frac{AB}{BC}$ $\Rightarrow$ AB : BC = 1 : 1 ....(i)

In ΔABD,

$\tan 30° = \frac{AB}{BD}$ = AB : BD = $1 : \sqrt{3}$ ....(ii)

Now,

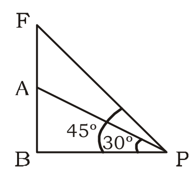
BC	:	AB	:	BD
1	:	1		
		1	:	$\sqrt{3}$
1	:	1	:	$\sqrt{3}$

CD = BD − BC = $\left(\sqrt{3} - 1\right)$ units

AB = 1unit = 125 metre

CD = $\left(\sqrt{3}-1\right)$ units = $125\left(\sqrt{3}-1\right)$ metre

27. (d)



AB − building = 10 m

In ΔABP

$\tan 30° = \frac{AB}{BP}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BP}$ = AB : B P

= 1 : $\sqrt{3}$ ....(i)

In ΔFBP

$\tan 45° = \frac{FB}{BP}$

$1 = \frac{FB}{BP}$ = FB : BP

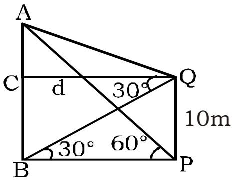
= 1 : 1 .....(ii)

Now,

AB	:	BP	:	FB
1	:	$\sqrt{3}$		
		$\frac{1}{\sqrt{3}}$	:	$\frac{1}{\sqrt{3}}$
1	:			
$\downarrow \times 10$				$\downarrow \times 10$
10m				$10\sqrt{3}$ m

FB = 17.32 m
FA = FB – AB
= 17.32 – 10
= 7.32 metre

28. (b)



AB = Tower
QP = 10 metre

In ΔQBP, $\tan 30° = \frac{QP}{PB}$

$\frac{1}{\sqrt{3}} = \frac{QP}{PB} \Rightarrow QP : PB = 1 : \sqrt{3}$ ....(i)

In ΔABP

$\tan 60° = \frac{AB}{BP}$

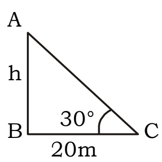
$\sqrt{3} = \frac{AB}{BP} \Rightarrow AB : BP = \sqrt{3} : 1$ .....(ii)

CB = QP and CQ = BP

Now,

AB : BP : CB
$\sqrt{3}$: 1
$\sqrt{3}$: 1
3 : $\sqrt{3}$: 1
$\times 10$ … $\times 10$
30 metre … 10 metre

29. (d)

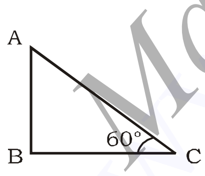


In ΔABC

$\frac{AB}{BC} = \tan 30° \Rightarrow \frac{h}{20} = \frac{1}{\sqrt{3}}$

$\Rightarrow h = \frac{20}{\sqrt{3}}$ m

30. (b)



AC = Ladder
BC = 6.5 metres

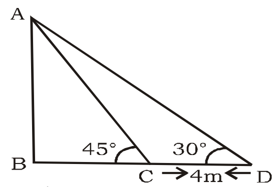
In ΔABC

$\cos 60° = \frac{BC}{AC}$

$\frac{1}{2} = \frac{6.5}{AC}$ m

AC = 13 m

31. (d)



AB = pole

In ΔABC

$\tan 45° = \frac{AB}{BC}$

$1 = \frac{AB}{BC}$ = AB : BC = 1 : 1 ......(i)

In ΔABD

$\tan 30° = \frac{AB}{BD}$ = AB : BD = $1 : \sqrt{3}$ ....(ii)

BC : AB : BD
1 : 1
1 : $\sqrt{3}$
1 : 1 : $\sqrt{3}$

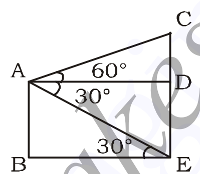
CD = BD – BC

$= \sqrt{3} - 1 = \sqrt{3} - 1$ units = 4 m

AB = 1unit = $\frac{4}{\sqrt{3}-1}$

$= 2\left(\sqrt{3}+1\right) = 5.464$ m

32. (c)



AB = pole CE = tower
AB = 10 metre

In ΔABE

$\tan 30° = \frac{AB}{BE}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BE}$ = AB : BE = $1 : \sqrt{3}$ ......(i)

In ΔACD

$\tan 60° = \frac{CD}{AD}$

$\frac{\sqrt{3}}{1} = \frac{CD}{AD}$ = CD : AD = $\sqrt{3}$: 1....(ii)

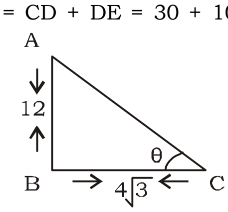
AD = BE and AB = DE

Now,

AB : BE : CD
1 : $\sqrt{3}$
1 : $\sqrt{3}$
1 : $\sqrt{3}$: 3
$\times 10$ … $\times 10$
10 metre … 30 metre

CE = CD + DE = 30 + 10 = 40 metre

33. (b)



In ΔABC

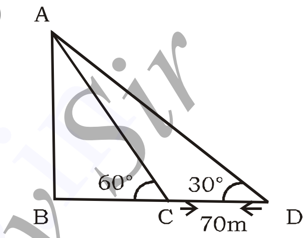
$\tan\theta = \frac{AB}{BC} = \frac{12}{4\sqrt{3}}$

$\tan\theta = \frac{3}{\sqrt{3}}$

$\tan\theta = \sqrt{3} = \tan 60°$

$\theta = 60°$

34 (d)



In ΔACD

$\angle ACB = \angle CAD + \angle ADC$

$60° = \angle CAD + 30°$

$\angle CAD = 30°$

So,

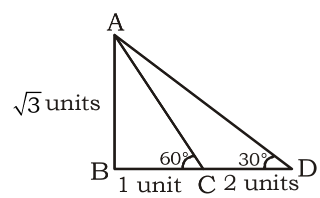
AC = CD
AC = 70m

$\text{cosec } 60° = \frac{AC}{AB}$

$\frac{2}{\sqrt{3}} = \frac{70}{AB}$

$AB = 35\sqrt{3}$ m

Alternate:-

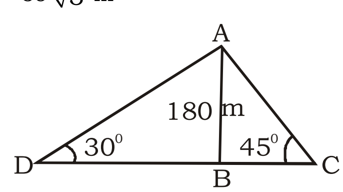


CD = 2 unit = 70 m

$\Rightarrow$ 1 unit = 35 m

$AB = 35\sqrt{3}$ m

35. (d)



AB = 180m

In ΔABC

$\tan 45° = \frac{AB}{BC}$

$1 = \frac{AB}{BC} \Rightarrow$ AB : BC = 1:1 .....(1)

In ΔABD

$\tan 30° = \frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BD}$

AB : BD = $1:\sqrt{3}$ .....(ii)

Now,

BC : AB : BD

1 : 1

1 : $\sqrt{3}$

to make equal ratio

BC : AB : BD

1 : 1 : $\sqrt{3}$

CD = BD + BC

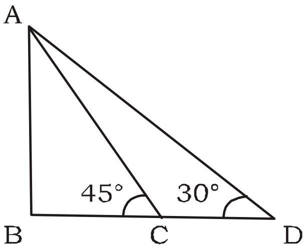
CD = $\left(\sqrt{3}+1\right)$ units

AB = 1 unit = 180 m

CD = $\left(\sqrt{3}+1\right)$ units

= 180 $\left(\sqrt{3}+1\right)$ m

36. (a)



AB = height of peak = 300 m

CD = length of Bridge

In ΔABC

$\tan 45° = \dfrac{AB}{BC}$

$1 = \dfrac{AB}{BC}$ = AB : BC = 1 : 1

In ΔABD

$\tan 30° = \dfrac{AB}{BD}$

$\dfrac{1}{\sqrt{3}} = \dfrac{AB}{BD} \Rightarrow$ AB : BD = 1 : $\sqrt{3}$

Now,

BC : AB : BD

1 : 1

1 : $\sqrt{3}$

1 : 1 : $\sqrt{3}$

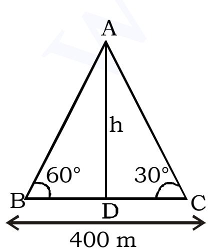
CD = BD – BC

CD = ($\sqrt{3}$ – 1) unit

AB = 1 unit = 300 metre

$\left(\sqrt{3}-1\right)$ units = 300 $\left(\sqrt{3}-1\right)$ metre

37. (a)



BC = 400 metres

In ΔABD

$\tan 60° = \dfrac{AD}{BD}$

$\dfrac{\sqrt{3}}{1} = \dfrac{AD}{BD} \Rightarrow$ AD : BD = $\sqrt{3}$: 1....(i)

In ΔADC

$\tan 30° = \dfrac{AD}{DC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{AD}{DC} \Rightarrow$ AD : DC = 1 : $\sqrt{3}$ .....(ii)

Now,

BD : AD : DC

1 : $\sqrt{3}$

1 : $\sqrt{3}$

1 : $\sqrt{3}$: 3

BC = BD + DC

= 1 + 3 = 4 units.

4 units = 400m

1 unit = 100m

AD = $\sqrt{3}$ unit

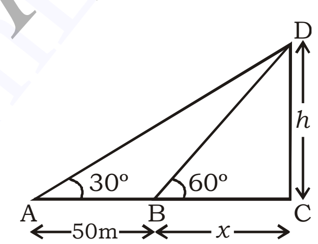
= $100\sqrt{3}$ = 100 × 1.732 = 173.2m

38. (b) Let h be the height of the tower and BC be x m.

In Δ BCD, $\tan 60° = \dfrac{h}{x}$

$\Rightarrow \sqrt{3} = \dfrac{h}{x}$

$\Rightarrow h = x\sqrt{3}$ ...(i)



Now in Δ ACD, $\tan 30°$

$= \dfrac{h}{50+x}$

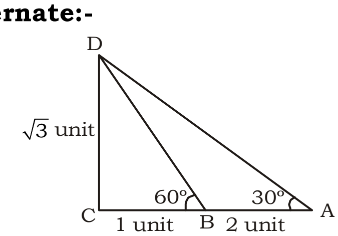
$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{x\sqrt{3}}{50+x}$

$\Rightarrow$ 50 + x = 3x

$\Rightarrow$ x = 25m

$\therefore$ h = $25\sqrt{3}$ [from eq. (i)]

Alternate:-

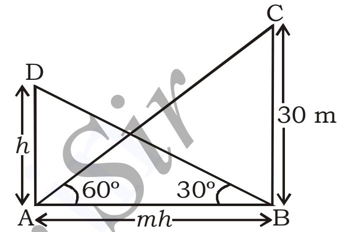


2 units = 50m

1 unit = 25m

CD = $\sqrt{3}$ = $25\sqrt{3}$ m.

39. (b) Let h be the height of shorter tower, then the distance between the two towers is mh m.

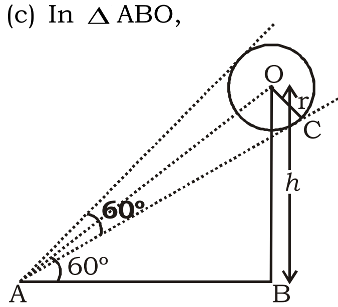


In Δ ABD, $\tan 30° = \dfrac{h}{mh}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{1}{m}$

$\therefore$ m = $\sqrt{3}$

40. (c) In Δ ABO,



$\sin 60° = \dfrac{OB}{AO}$

$\Rightarrow AO = \dfrac{OB}{\sin 60°}$ ....(i)

Now, In Δ AOC,

$\sin\dfrac{60°}{2} = \dfrac{OC}{AO}$

$\Rightarrow AO = \dfrac{OC}{\sin 30°}$ ....(ii)

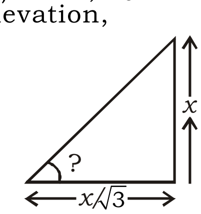
From Eqs. (i) and (ii),

$\dfrac{OB}{\sin 60°} = \dfrac{OC}{\sin 30°}$

$\dfrac{h}{\frac{\sqrt{3}}{2}} = \dfrac{r}{\frac{1}{2}}$

$\therefore$ h = $\sqrt{3}r$

41. (c) Here, θ is the angle of elevation,



$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{x}{\frac{x}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}x}{x} = \sqrt{3}$$

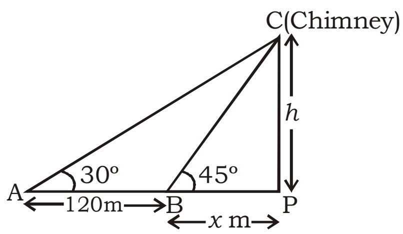
Here, $\tan\theta = \sqrt{3}$

$\therefore\ \theta = 60^\circ \qquad (\because \tan 60^\circ = \sqrt{3})$

42. (c) Let h be the height of the chimney.

In $\triangle BPC$,

$\tan 45^\circ = \frac{h}{x} = 1 \Rightarrow h = x \quad \ldots(i)$



Now, In $\triangle APC$,

$$\tan 30^\circ = \frac{h}{120+x} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \frac{h}{120+h} = \frac{1}{\sqrt{3}}$ [from Eq. (i)]

$\Rightarrow \sqrt{3}\,h = 120 + h$

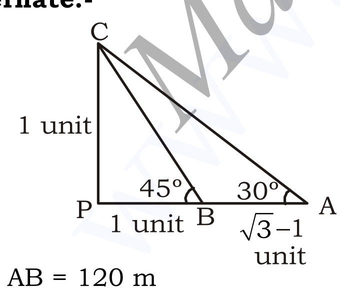
$\Rightarrow \sqrt{3}\,h - h = 120$

$\Rightarrow h\left(\sqrt{3}-1\right) = 120$

$\therefore\ h = \frac{120}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{120\left(\sqrt{3}+1\right)}{2}$

$\therefore$ Required height of the chimney(h) $= 60\left(\sqrt{3}+1\right)$ m

Alternate:-



AB = 120 m

$= \left(\sqrt{3}-1\right)$unit = 120 m

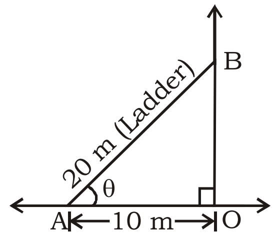
$1 \text{ unit} = \frac{120}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= 60\left(\sqrt{3}+1\right)$m

So, the height of PC

$= 60(\sqrt{3}+1)$ metres

43. (c) Let θ be the inclination of the ladder to the horizontal.



Now, In $\triangle AOB$,

$$\cos\theta = \frac{AO}{AB} = \frac{10}{20} = \frac{1}{2}$$

$\Rightarrow \cos\theta = \cos 60^\circ$

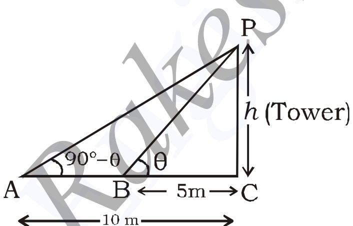
$\therefore\ \theta = 60^\circ$

44. (c) Given that, angles are complementary.

Let $\angle PBC = \theta$

$\therefore\ \angle PAC = 90 - \theta$

Let h be the height of the tower



Now, in $\triangle PBC$

$\tan\theta = \frac{h}{5} \quad \ldots\ldots (i)$

and in $\triangle PAC$

$\tan(90^\circ - \theta) = \frac{h}{10}$

$\Rightarrow \cot\theta = \frac{h}{10} \quad \ldots\ldots (ii)$

On multiplying Eqs. (i) and (ii), we get

$\tan\theta . \cot\theta = \frac{h}{5} . \frac{h}{10}$

$\Rightarrow \frac{h^2}{50} = 1 \Rightarrow h = \sqrt{50}$ m

Alternate:-

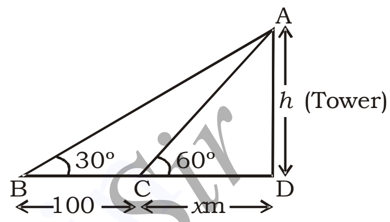
$h = \sqrt{5 \times 10} = \sqrt{50}$ m

45. (a) Let h be the height of inaccessible tower.

Now, in $\triangle ACD$

$\tan 60^\circ = \frac{h}{x} = \sqrt{3}$

$\Rightarrow x = \frac{h}{\sqrt{3}} \quad \ldots..(i)$



and in $\triangle ABD$

$$\tan 30^\circ = \frac{h}{100+x} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \sqrt{3}\,h = 100 + x = 100 + \frac{h}{\sqrt{3}}$

[From Equation (i)]

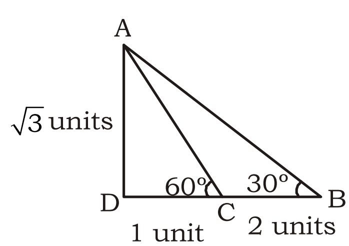
$\Rightarrow \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) h = 100$

$\Rightarrow \frac{2}{\sqrt{3}} h = 100 \Rightarrow h = 50\sqrt{3}$

$\therefore\ h = 50 \times 1.732 = 86.6$m

So, the required height is 86.6m

Alternate:-



BC = 2 units = 100 m

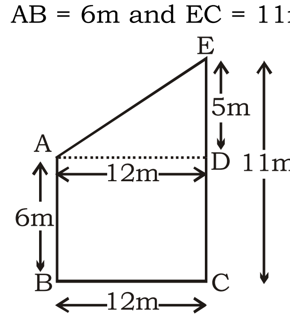
1unit = 50m

Height = AD = $\sqrt{3} \times 50$

$= 50\sqrt{3} = 50 \times 1.732 = 86.6$ m

46. (a) Given,

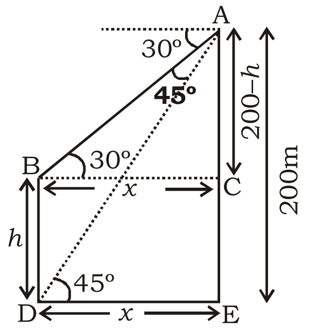
AB = 6m and EC = 11m



$\Rightarrow$ BC = 12m

$\because$ BC = AD = 12m
and ED = EC – CD = EC – AB
($\because$ AB = CD)
= 11 – 6 = 5m
In $\triangle$ AED,
$(AE)^2 = (AD)^2 + (ED)^2$
(by Pythagoras theorem)
$= (12)^2 + (5)^2 = 144 + 25$
$= 169 = (13)^2$
$\therefore$ AE = 13m
So, the distance between their tops is 13 m.

47. (d) Let AE = 200 m be the height of the cliff and BD = h m be the height of the tower.



In $\triangle$ ABC,

$$\tan 30° = \frac{200-h}{x} = \frac{1}{\sqrt{3}} = \frac{200-h}{x}$$

$\Rightarrow x = (200 - h)\sqrt{3}$

and In $\triangle$ ADE, $\tan 45° = \dfrac{200}{x}$

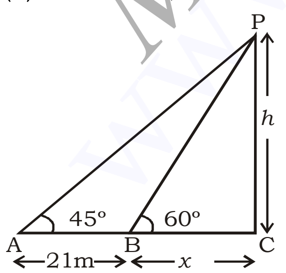
$\Rightarrow 1 = \dfrac{200}{x} \Rightarrow x = 200$ m

From Eq. (i),

$200 = (200 - h)\sqrt{3}$

$$\Rightarrow h = 200\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)\text{m}$$

48. (c) In $\triangle$ PBC,



$\tan 60° = \dfrac{h}{x} \Rightarrow \dfrac{h}{x} = \sqrt{3}$

$\Rightarrow x = \dfrac{h}{\sqrt{3}}$ ....(i)

In $\triangle$ PAC, $\tan 45° = \dfrac{h}{21+x} = 1$

$\Rightarrow h = 21 + x$

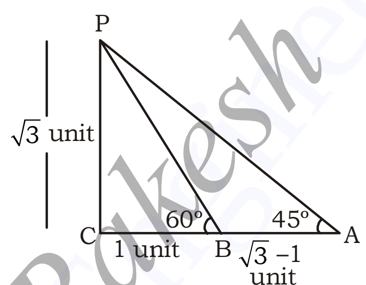
$\Rightarrow h = 21 + \dfrac{h}{\sqrt{3}}$ [from Eq. (i)]

$$\Rightarrow h\left(1 - \frac{1}{\sqrt{3}}\right) = 21$$

$$\therefore h = \frac{21\sqrt{3}}{(\sqrt{3}-1)}\cdot\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{21\sqrt{3}(\sqrt{3}+1)}{2} = 49.68 \approx 50\text{m}$$

Alternate:-



AB = $(\sqrt{3}-1)$ unit = 21m

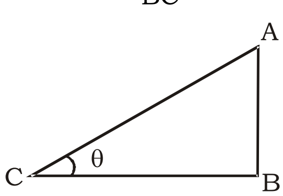
$$1\text{ unit} = \frac{21}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$1\text{ unit} = \frac{21(\sqrt{3}+1)}{2}\text{m}$$

$$PC = \sqrt{3} \times \frac{21(\sqrt{3}+1)}{2}$$

= 49.68 = 50m

49. (a) $\therefore \tan\theta = \dfrac{AB}{BC}$



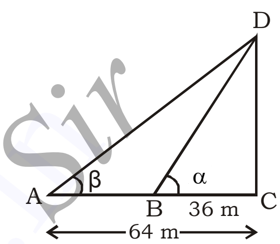
BC = $\sqrt{3}$AB

$\Rightarrow \dfrac{AB}{BC} = \dfrac{1}{\sqrt{3}}$

$\therefore \tan\theta = \dfrac{1}{\sqrt{3}}$

$$\Rightarrow \theta = 30° \left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$$

50. (b) Let CD = h be height of the tower.



In $\triangle$ BCD, $\tan\alpha = \dfrac{h}{36}$

and In $\triangle$ ACD,

$\tan\beta = \dfrac{h}{64}$

But $\alpha + \beta = \dfrac{\pi}{2}$

$\Rightarrow \tan(\alpha + \beta) = \tan\dfrac{\pi}{2}$

$$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{1}{0}$$

$\Rightarrow 1 - \tan\alpha\tan\beta = 0$

$\Rightarrow \tan\alpha\ \tan\beta = 1$

$\Rightarrow \dfrac{h}{36} \times \dfrac{h}{64} = 1$

$\Rightarrow h^2 = 36 \times 64 \Rightarrow h = 6 \times 8$

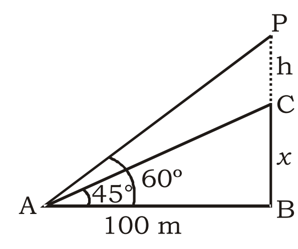
$\Rightarrow$ h = 48 m

Alternate:-

h = $\sqrt{36 \times 64}$

= 6×8 = 48 m.

51. (c) Let the height of the incomplete pillar be x m and the increaseing height be PC = h m



In $\triangle$ ABC,

$\tan 45° = \dfrac{x}{100} \Rightarrow x = 100$ m

and In $\triangle$ APB,

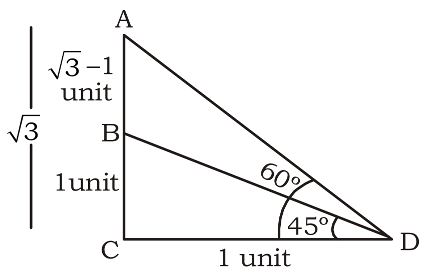
$\tan 60° = \frac{x+h}{100}$

$x + h = 100\sqrt{3}$

$\Rightarrow h = 100\sqrt{3} - x = 100\sqrt{3} - 100$

$\therefore h = 100\left(\sqrt{3}-1\right) m$

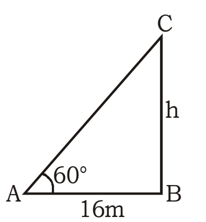
Alternate:-



CD = 1 unit = 100 m

AB = 100 ($\sqrt{3}$ −1)m

52. (c) Let the height of the tree be h m.



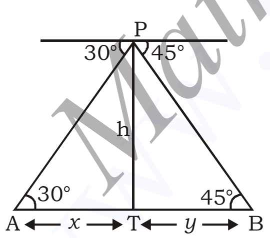
In $\triangle$ABC, $\tan 60° = \frac{h}{16}$

$\Rightarrow h = 16\sqrt{3}$ m

53. (a) In $\triangle$PBT

$\tan 45° = \frac{h}{y} = 1$

$\therefore y = h$



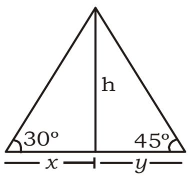
and in $\triangle$PTA

$\tan 30° = \frac{h}{x} \Rightarrow x = \sqrt{3}h$ ...(ii)

$\therefore$ Required distance,

$x + y = \sqrt{3}h + h = h\left(\sqrt{3}+1\right) m$

Alternate:-

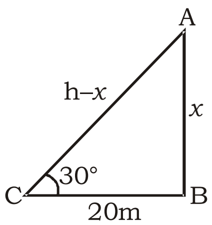


$x + y = h (\cot 30° + \cot 45°)$

$\Rightarrow h (\sqrt{3} + 1)$

$x + y \Rightarrow (\sqrt{3} + 1)h$

54. (b) Let the height of post be x m.



In $\triangle$ABC, $\tan 30° = \frac{x}{20} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{20}{\sqrt{3}}$ m ....(i)

and $\cos 30° = \frac{20}{h-x}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{20}{h-x}$

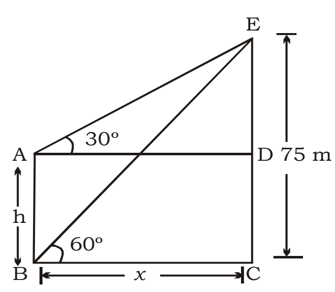
$\Rightarrow h - x = \frac{40}{\sqrt{3}}$

From equation (i),

$h = \frac{40}{\sqrt{3}} + \frac{20}{\sqrt{3}} = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 20\sqrt{3}$ m

55. (c) Let height of the building be h m and distance between building and tower be x m.



In $\triangle$ADE,

$\tan 30° = \frac{ED}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75-h}{x}$

$\Rightarrow x = 75\sqrt{3} - h\sqrt{3}$

and In $\triangle$BCE,

$\tan 60° = \frac{CE}{BC}$

$\Rightarrow \sqrt{3} = \frac{75}{x}$

$\Rightarrow x\sqrt{3} = 75$ (from eq.(i))

$\Rightarrow \left(75\sqrt{3} - h\sqrt{3}\right)\sqrt{3} = 75$

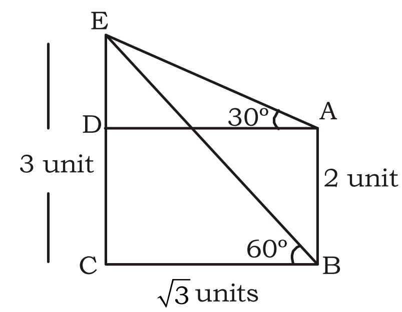
$\Rightarrow 75 \times 3 - 3h = 75$

$\Rightarrow 3h = 75 \times 3 - 75$

$\Rightarrow h = \frac{75 \times 2}{3}$

$\therefore h = 50$ m

Alternate:-



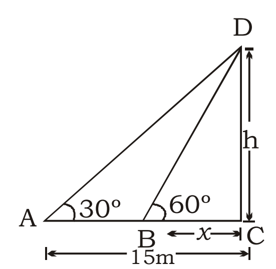
$AB = \frac{2EC}{3} = \frac{2 \times 75}{3} = 50$ m.

56. (c) In $\triangle$ACD,

$\tan 30° = \frac{CD}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{15}$

$\Rightarrow h = \frac{15}{\sqrt{3}}$



and in $\triangle$BCD,

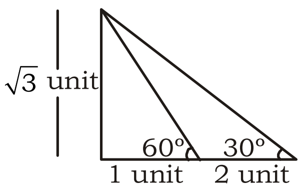
$\tan 60° = \frac{CD}{BC}$

$\Rightarrow \sqrt{3} = \dfrac{h}{x}$

$\Rightarrow \dfrac{h}{\sqrt{3}} = x$

$\therefore\ x = \dfrac{15}{3} = 5\text{m}$ (from Eq. (i))

Alternate:-

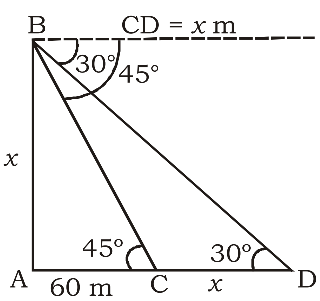


3 units = 15 m

1 unit = $\dfrac{15}{3}$ = 5m

So, the shadow will be 5m when angle becomes 60°

57. (a) Let AB be a height of tower,



In Δ ACB,

$\tan 45° = \dfrac{AB}{AC}$

$\Rightarrow 1 = \dfrac{AB}{60}$

$\Rightarrow$ AB = 60 m

Now, In Δ ADB,

$\Rightarrow \tan 30° = \dfrac{60}{60+x}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{60}{60+x}$

$\Rightarrow 60 + x = 60\sqrt{3}$

$\Rightarrow\ x = 60\left(\sqrt{3}-1\right) = 60\ (1.73 - 1)$

$= 60 \times 0.73 = 43.8$ m

$\therefore$ Speed of boat $= \dfrac{43.8}{5} \times \dfrac{18}{5}$

$= \dfrac{788.4}{25} = 31.5$ km/h

58. (a) Let AC = h (Height of a tower)

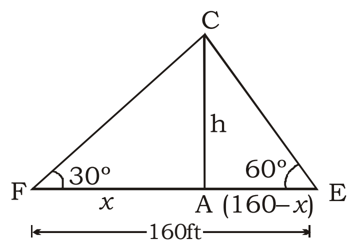
x = Distance between A and F

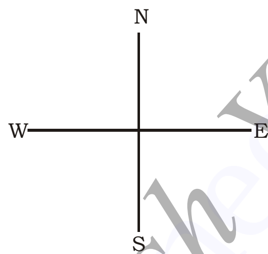
$\therefore$ AE = 160 – x

In Δ AFC,

$\tan 30° = \dfrac{h}{x}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{x} \Rightarrow x = \sqrt{3}\,h$ ...(i)





and In Δ AEC,

$\tan 60° = \dfrac{h}{160-x}$

$\Rightarrow \sqrt{3}\,(160 - x) = h$

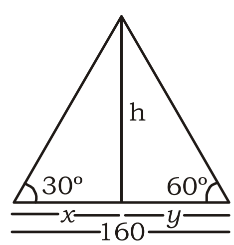
$\Rightarrow \sqrt{3}\,(160 - \sqrt{3}\,h) = h$ [from Eq. (i)]

$\Rightarrow 160\sqrt{3} - 3h) = h$

$\Rightarrow 4h = 160\sqrt{3}$

$\therefore$ h = $40\sqrt{3}$ ft

Alternate:-



x + y = h (cotB + cotA)

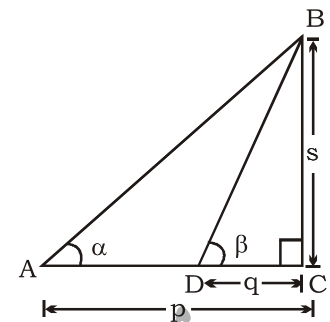
160 = h (cot30° + cot60°)

$160 = h\left(\sqrt{3} + \dfrac{1}{\sqrt{3}}\right) = h\left(\dfrac{4}{\sqrt{3}}\right)$

$h = \dfrac{160 \times \sqrt{3}}{4} = 40\sqrt{3}$ ft.

59. (c) In Δ ABC,

$\tan\alpha = \dfrac{s}{p} \Rightarrow p = \dfrac{s}{\tan\alpha}$ . . . (i)



In Δ BDC,

$\tan\beta = \dfrac{s}{q} \Rightarrow q = \dfrac{s}{\tan 3\alpha}$

($\because \beta = 3\alpha$, given) .....(i)

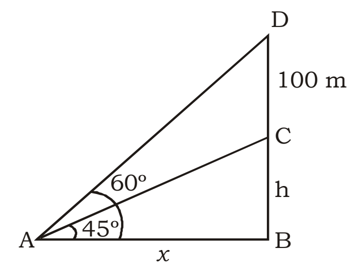
On subtracting Eq. (ii) from Eq. (i), we get

$p - q = \dfrac{s}{\tan\alpha} - \dfrac{s}{\tan 3\alpha}$

$= s\left(\dfrac{\tan 3\alpha - \tan\alpha}{\tan\alpha \tan 3\alpha}\right)$

60. (c) Let BC be a building of height h m and CD = 100 m be a height of antenna.

x = Distance between A and B



In Δ ABC,

$\tan 45° = \dfrac{h}{x} \Rightarrow x = h$ . . . .(i)

and in Δ ABD,

$\tan 60° = \dfrac{100+h}{x}$

$\Rightarrow \sqrt{3} = \dfrac{100+h}{h}$ [from Eq. (i)]

$\Rightarrow \sqrt{3}\ h = 100 + h$

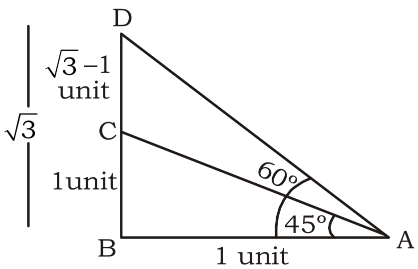
$\Rightarrow \left(\sqrt{3}-1\right) h = 100$

$\Rightarrow h = \dfrac{100}{\sqrt{3}-1}$

$\Rightarrow h = \dfrac{100}{\left(\sqrt{3}-1\right)} \times \dfrac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$

$= 50\ (\sqrt{3} + 1)$m

Alternate:-



CD = DB – BC

= $(\sqrt{3}-1)$ unit

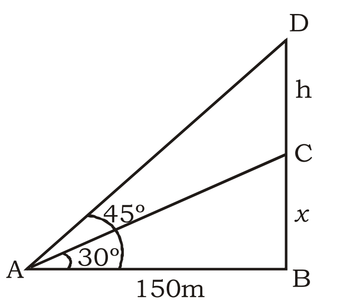
$(\sqrt{3}-1)$ unit = 100 m

1 unit = $\frac{100}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

= $50(\sqrt{3}+1)$ metre

BC=1 unit = $50(\sqrt{3}+1)$ metres

61. (d) Let BC = x m height of unfinished pillar and CD= h m = Raised height of pillar



In $\triangle$ ABC,

$\tan 30° = \frac{x}{150} \Rightarrow x = \frac{150}{\sqrt{3}}$ ...(i)

and in $\triangle$ ABD,

$\tan 45° = \frac{h+x}{150} \Rightarrow 1 = \frac{h+x}{150}$

$\Rightarrow 150 = h + \frac{150}{\sqrt{3}}$ [from Eq. (i)]

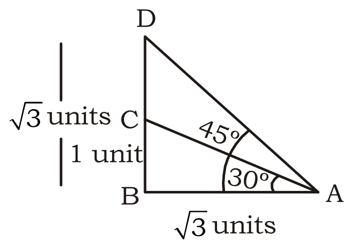
$\Rightarrow h = \frac{150(\sqrt{3}-1)}{\sqrt{3}}$

$\Rightarrow h = 150 \times \frac{(1.732-1)}{1.732}$

$= \frac{150 \times 0.732}{1.732} = 63.39$

= 63.4 m (approx)

Alternate:-



AB = $\sqrt{3}$ units = 150 m

1 unit = $\frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

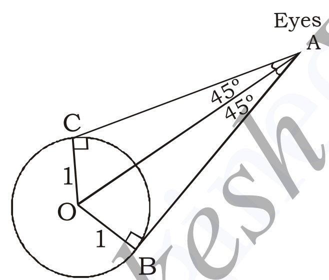
= $50\sqrt{3}$ metre

So, CD = $50\sqrt{3}(\sqrt{3}-1)$

= 150 – 86.60 = 63.4 metres

62. (b) Let O = Centre of the ballon

OB = OC = Radii of the balloon

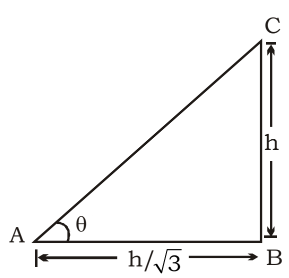


In $\triangle$ OAB,

$\sin 45° = \frac{OB}{OA}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{OA} \Rightarrow OA = \sqrt{2}$

63. (a) Let h be the height of pole, then its shadow = $h\sqrt{3}$ (given) θ (suppose) be the angle of elevation.



Then, $\tan\theta = \frac{h}{h/\sqrt{3}}$

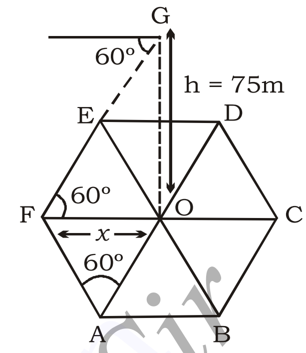
$\Rightarrow \tan\theta = \sqrt{3} = \tan 60°$

$\therefore \theta = 60°$

64. (c) Let OG be a height of the tower,

Angle of elevation = Angle of depression

x = Distance between O and F.



In $\triangle$ FOG,

$\tan 60° = \frac{75}{x}$

$\Rightarrow x = \frac{75}{\sqrt{3}} = 25\sqrt{3}$ m

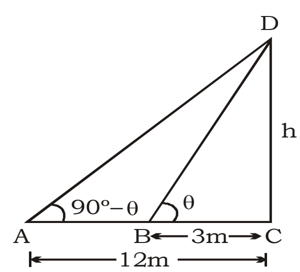
In regular hexagon $\triangle$ OEF, $\triangle$ OED,...are equilateral triangles.

$\therefore$ OF = OE = OA = OD = OC = OB = OA = side of hexagon

$\therefore$ Length of hexagon = $25\sqrt{3}$ m

65. (b) Let h be the height of the tower and $\angle$CBD = θ

$\angle$DAC = 90° – θ



$\therefore$ In $\triangle$ BCD,

$\tan\theta = \frac{CD}{BC} \Rightarrow \tan\theta = \frac{h}{3}$ . . (i)

and in $\triangle$ ACD

$\tan(90° - \theta) = \frac{CD}{AC}$

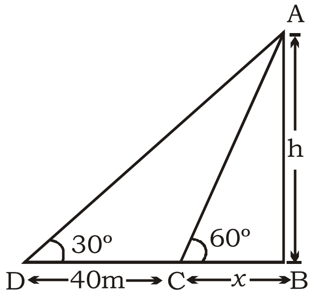
$\Rightarrow \cot\theta = \frac{h}{12}$ . . . .(i)

On multiplying Eqs. (i) & (ii), we get

$\tan\theta \cdot \cot\theta = \frac{h}{3} \cdot \frac{h}{12}$

$\Rightarrow \frac{h^2}{36} = 1 \Rightarrow h^2 = 36 \Rightarrow h = 6m$

66. (b) Let AB = h m be the height of the tree and BC= x m be the width of the river



In $\triangle ABC$,

$\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}x$ . . . (i)

and in $\triangle$ ADB, tan 30°

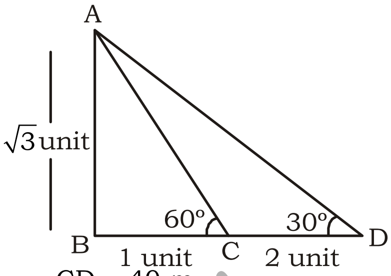
$= \frac{h}{40+x} \Rightarrow h = \frac{40+x}{\sqrt{3}}$ . . . (ii)

From Eqs. (i) & (ii),

$\sqrt{3}x = \frac{40+x}{\sqrt{3}} \Rightarrow 2x = 40 \Rightarrow x$

=20 m

Alternate:-

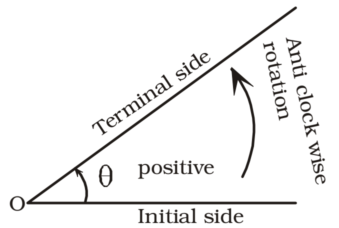


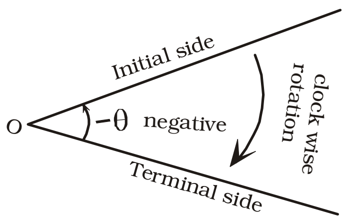
CD = 40 m
2 units = 40m
1 unit = 20 m
So, the width of river = 20 m

CHAPTER
15

TRIGONOMETRY CIRCULAR MEASURE OF ANGLES

Angles: When two rays (initial and terminal) meet at a point after rotation in a plane then they are said to have described an angle.





Unit of Angle:

1. Degree (°) 2. Radian (^{c})
3. Grade (^{g})

Systems of Measurement of Angles:

1. Hexagonal System

The angle between two perpendicular lines is called a right angle. A right angle is equal to 90 degree, it is written as 90°.

Thus, if a right angle is divided into 90 equal parts then one part is called one degree. It is written as1°.

If 1° is divided into 60 equal parts, each part is called 1 minute. It is denote by 1'

$\frac{1}{60}$ th part of 1' is called one second. It is written as 1".

1 degree $=1^0= \frac{1}{90}$ right angle

1 minute $=1' = \frac{1}{60}$ degree

1 second $=1'' = \frac{1}{60}$ minute

In other words:- 90^0 = 1 right angle

60' = 1^0

60'' = 1'

Hence 1 right angle = 90°

= 90 × 60 = 5400

= 5400 minutes

= 90 × 60 × 60 = 324000"

= 324000 seconds

Again, $\boxed{1° = 60' = 60 \times 60'' = 3600''}$

2. Centesimal or French System:-

1 right angle = 100 grades (= 100^g)

1 grade = 60 minutes (= 60')

1 minute = 60 seconds (= 60")

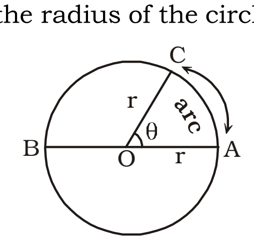
3. Circular System:-

In this system, the unit of measurement is "radian".

Angle (in radian)

$$\theta = \frac{\text{arc}\,AC}{\text{radius}} = \frac{\overset{\frown}{AC}}{r}$$

1 radian or 1^C is the angle subtended by an arc at the centre of a circle whose length is equal to the radius of the circle.



i.e. If arc=radius = r, then

$$\theta = \frac{r}{r} = 1, \text{ radian} = 1^c$$

when arc ACB = πr

4. Relation between degree measure and radian measure:

$\therefore\ \pi$ rad = 180°

$\therefore \boxed{x° = \frac{\pi x}{180} rad}$

and $\boxed{x\ rad = \frac{180}{\pi} x°}$

$\boxed{1\ rad = \frac{180}{3.14} = 57°16'22''}$

Thus to change degree into radian, multiply by $\frac{\pi}{180}$ and to change radian into degree multiply by $\frac{180}{\pi}$.

If mentioned , take $\pi = \frac{22}{7}$ or 3.14.

$\left\{\text{and } 1° = \frac{\pi}{180} \text{ radian} = \left(\frac{22}{7 \times 180}\right) \text{rad}\right\}$

$\boxed{\text{or } 1° = 0.01746 \text{ radian}}$

For Ex:

$\sin 1^c$	$\sin 1°$	$\cos 1^c$	$\cos 1°$
$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
$\approx 57°$	$\approx 0°$	$\approx 57°$	$\approx 0°$
$\approx \sin 60°$	$\approx \sin 0°$	$\approx \cos 60°$	$\approx \cos 0°$

then sin 60°> sin 0° then cos 60°< cos 0°

$\therefore \boxed{\sin 1^c > \sin 1°}$ $\boxed{\cos 1^c < \cos 1°}$

Degree	Radian	Degree	Radian
30°	$\pi/6$	135°	$3\pi/4$
45°	$\pi/4$	150°	$5\pi/6$
60°	$\pi/3$	180°	π
90°	$\pi/2$	270°	$3\pi/2$
120°	$2\pi/3$	360°	2π

Ex. 1 Convert the following degree measures in the radian measure.

(i) 42°30' (ii) –520°
(iii) 72° 45' (iv) 115° 40'
(v) 15° 12' 30"

Sol. We know that $x° = \frac{\pi x}{180}$ rad

$\therefore$ **(i)** $42°30' = 42\frac{1°}{2} = \frac{85°}{2}$

$= \frac{\pi}{180} \times \frac{85}{2}$

$= \frac{\pi \times 17 \times 5}{5 \times 36 \times 2} = \frac{17\pi}{72}$

(ii) $-520° = -520 \times \frac{\pi}{180} = \frac{-26\pi}{9}$

(iii) 72° 45'

1° = 60'

1' $= \left(\frac{1}{60}\right)^{\circ}$

45' $= \frac{3°}{4}$

So, $= 72\frac{3°}{4} = \frac{291°}{4}$

$= \frac{291}{4} \times \frac{\pi}{180}$

$= \frac{3 \times 97 \times \pi}{4 \times 3 \times 60} = \frac{97\pi}{240}$

(iv) $115° 40' = 115\frac{2°}{3}$

$\left(\therefore 40' = \frac{2}{3}\right) = \frac{347°}{3}$

$= \frac{347°}{3} \times \frac{\pi}{180} = \frac{347\pi}{540}$

(v) 15° 12' 30"

$30" = \left(\frac{30}{60}\right)' = \left(\frac{1}{2}\right)'$

$12' 30" = \left(12\frac{1}{2}\right)' = \left(\frac{25}{2}\right)'$

$= \left(\frac{25}{2} \times \frac{1}{60}\right)^{\circ} = \left(\frac{5}{24}\right)^{\circ}$

$\therefore 15° 12' 30" = \left(15\frac{5}{24}\right)^{\circ}$

$= \left(\frac{365}{24}\right)^{\circ} = \frac{365}{24} \times \frac{\pi}{180} = \frac{73\pi}{864}$

Ex. 2 Convert the following radian measure in degree measures

(i) 4^c (ii) $\frac{-5\pi^c}{3}$ (iii) $\frac{5\pi^c}{24}$

(iv) $\frac{\pi^c}{16}$ (v) $\frac{5\pi^C}{7}$

Sol. π radian = 180°

1radian $= \frac{180}{\pi} = 180° \times \frac{7}{22}$

(i) 4 radian $= \frac{180° \times 7 \times 4}{22}$

$= \frac{90° \times 7 \times 4}{11} = \frac{2520°}{11}$

$= 229\frac{1}{11}$ degree

(ii) $-\frac{5\pi}{3} = -\frac{5}{3} \times 180°$

$= -5 \times 60° = -300°$

(iii) $\frac{5\pi}{24} = \frac{5\pi}{24} \times \frac{180°}{\pi}$

$= \frac{75°}{2} = 37\frac{1°}{2}$

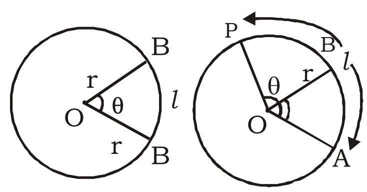
(iv) $\frac{\pi}{16} = \frac{\pi}{16} \times \frac{180°}{\pi}$

$= \frac{45°}{4} = 11\frac{1°}{4}$

(v) $\frac{5\pi}{7} \times \frac{180°}{\pi} = \frac{5 \times 180°}{7} = \frac{900°}{7}$

$= \boxed{128°57'14"}$

5. Relation between length of arc (l), radius (r) and angle (θ):



If an arc of length l of a circle subtends θ at its centre and radius of the circle is r then

$\theta = \frac{l}{r}$.

Hence,

(i) when $\theta = \frac{l}{r}$ and r is constant then $\theta \propto l$

i.e., $\theta_1 : \theta_2 = l_1 : l_2$

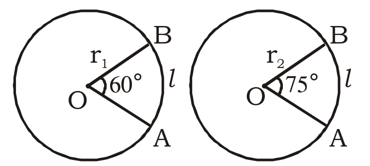
(ii) when $\theta = \frac{l}{r}$ and θ is constant then $l \propto r$ i.e., $l_1 : l_2 = r_1 : r_2$

(iii) when $\theta = \frac{l}{r}$ and l is constant then $\theta \propto \frac{l}{r}$

or $r \propto \frac{l}{\theta}$

i.e, $\theta_1 : \theta_2 = r_2 : r_1$ (reverse order)

Ex.3 If in two circles, arcs of the same length subtend angle 60° and 75° at the centre, find the ratio of their radii.



Sol. Let the radii of two circles be r_1 and r_2 respectively.
According to the question, arc AB = l (say) in the two circle.
Given that $\theta_1 = 60°$

$= 60 \times \frac{\pi}{180} = \frac{60\pi}{180}$ radian

And $\theta_2 = 75° = \frac{75\pi}{180}$ radian

$\because \theta = \frac{l}{r} \therefore \theta_1 = \frac{l}{r_1}$ and $\theta_2 = \frac{l}{r_2}$

or, $l = r_1\theta_1 = r_2\theta_2$

or, $\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\frac{75\pi}{180}}{\frac{60\pi}{180}}$

$= \frac{75}{60} = \frac{5}{4}$

Alternate: -
since l is constant,
therefore $r_1 : r_2$
$= \theta_2 : \theta_1 = 75° : 60° = 5 : 4$

Ex.4 If the arc of same length in two circles subtends angle 75° and 120° at their respective centres, then ratio of their diameter is

(a) 8 : 5 (b) 5 : 8
(c) 3 : 5 (d) 5 : 3

Sol. (a) Since l is constant therefore $r_1 : r_2 = \theta_2 : \theta_1$

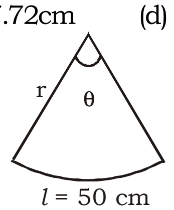
$= 120° : 75° = 8 : 5$

$\therefore d_1 : d_2 = 8 : 5$

Ex.5 The tip a pendulum swings. It covers an arc of 50 cm and subtends 60° at the fixed point. The length of pendulum is

(a) 43.72cm (b) 45.72cm
(c) 47.72cm (d) 45.27 cm

Sol. (c)



$\theta = 60° = \frac{\pi}{3}$ and $l = 50$ cm

$\therefore$ using $\theta = \frac{l}{r}$

$r = \frac{l}{\theta} = \frac{50\text{cm}}{\frac{\pi}{3}} = \frac{150}{\pi}\text{cm} \frac{150}{\frac{22}{7}}$

$= \frac{150 \times 7}{22} = 47.72$ cm

Ex.6 The minutes hand of a watch is 5 cm. How far does the tip move in 20 minutes?

(a) 10 cm (b) 9.53 cm
(c) 11 cm (d) 10.47cm

Sol. (d) In 20 minute, hand covers

$\frac{20}{60} \times 2\pi$

$= \frac{2\pi}{3}$ rad distance.

From $\theta = \frac{1}{r'}$, $l = \theta r$

$= \frac{2\pi}{3} \times 5 = \frac{10\pi}{3} = \frac{10}{3} \times \frac{22}{7}$

$= \frac{220}{21} = 10.47$ cm

Ex. 7 When a pendulum of length 50 cm oscillates, it produces an arc of 16 cm. The angle so formed in degree measure is (approx) :

(a) 18°25' (b) 18°35'
(c) 18°20' (d) 18°08'

Sol. (c) $l = 16$ cm

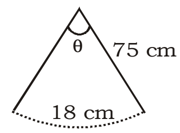
r = 50 cm

$\therefore \theta = \frac{l}{r} = \frac{16}{50} = \frac{8}{25}$ radian

$= \frac{8}{25} \times \frac{180°}{\pi} = \frac{8}{25} \times \frac{180}{22} \times 7 = \frac{1008}{55}$

$= 18\frac{18°}{55} = 18°\left(\frac{18}{55} \times 60\right) \approx 18°20'$

Ex.8 Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc length 18 cm.

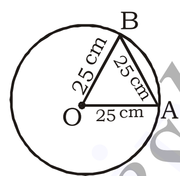


Sol. Suppose the pendulum swings through an angle of radian. then $\theta = \frac{l}{r} = \frac{18}{75}$ rad

(see figure)

$= \frac{6}{25}$ rad

Ex.9 In a circle of diameter 50 cm, the length of a chord is 25 cm. Find the length of minor arc and major arc of the chord.



Sol. See the figure

Given that radius of the circle

$= \frac{50cm}{2} = 25$ cm and

chord AB of the circle = 25 cm

Clearly ΔOAB is an equilateral triangle, therefore

$\angle$AOB = 60°

$= \frac{\pi}{3} = \theta$ (say)

In minor arc AB = l then from

$\theta = \frac{l}{r}$

$l = r\theta = \frac{25\pi}{3}$

Here major angle = 360° – 60°

$= 300° = 300 \times \frac{\pi}{180} = \frac{5\pi}{3}$

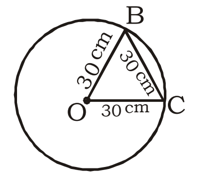
Major arc $= 25 \times \frac{5\pi}{3} = \frac{125\pi}{3}$

Ex.10 The diameter of a circle is 60 cm. The length of minor arc created by a chord of 30 cm is

(a) $31\frac{3}{7}$ cm (b) 34 cm

(c) $32\frac{2}{7}$ cm (d) $32\frac{4}{7}$ cm

Sol. (a)



BOC is an equilateral triangle since all sides are equal.

$\theta = 60° = \frac{\pi}{3}$ rad

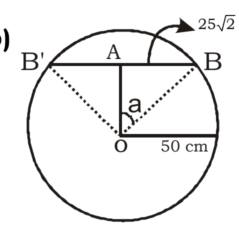
$\theta = \frac{l}{r} \Rightarrow l = \theta r = \frac{\pi}{3} \times 30$

$= 10 \times \frac{22}{7} = 31\frac{3}{7}$ cm

Ex.11 In a circle of radius 50 cm the length of a chord is $50\sqrt{2}$ cm. The length of major arc of the chord is

(a) 245.5 cm (b) 235.5 cm
(c) 255.5 cm
(d) None of these

Sol. (b)



$\sin\alpha = \frac{25\sqrt{2}}{50} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45°$

$\therefore 2\alpha = 90°$

Hence major arc of chord BB' subtends angle

$= 360° - 90° = 270° = \frac{3\pi}{2}$ at centre.

$\therefore$ using $\theta = \frac{l}{r}$

major arc $l = \theta \times r = \frac{3\pi}{2} \times 50$

$= 75\pi = 75 \times 3.14$ cm

= 235.50 cm

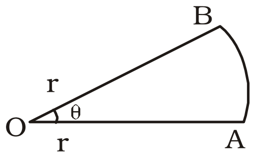
6. If θ is in radian and is very small then $\sin\theta = \theta = \tan\theta$ (approximate)

7. **Area of a sector (or Sectorial area) :-**

The area of the sector

$OAB = \frac{1}{2}r^2\theta$

Here θ is in radian.



Note :- Radian is a constant angle.

Some Useful Points :-

The angle between two consecutive digits in a clock is

$$30^0 \left(= \frac{\pi}{6}\text{radians}\right)$$

- The hour hand rotates through an angle of 30^0 in one hour i.e. $\left(\frac{1}{2}\right)^0$ in one minute.
- The minute hand rotates through an angle of 6^0 in one minute.

Ex. 12 The angle between the hands of a clock at 4 hour 45 minute is

(a) $112\frac{1}{2}^\circ$ (b) $122\frac{1}{2}^\circ$

(c) 125° (d) $127\frac{1}{2}^\circ$

Sol. (d) Using $\frac{11}{2}$ M = 30 H $\pm$ A

here, H = 4, M = 45

$\because \frac{11}{2} \times 45 = 30 \times 4 \pm A$

$247.5 = 120^\circ \pm A$

$\therefore A = 247.5^\circ - 120^\circ = 127.5^\circ$

Ex. 13 The angle between the hands of a clock at half past one is

(a) $\frac{3\pi}{4}$ rad (b) $\frac{2\pi}{3}$ rad

(c) $\frac{5\pi}{12}$ rad (d) $\frac{5\pi}{6}$ rad

Sol. (a) Using $\frac{11}{2}$ M = 30 H $\pm$ A

$\frac{11}{2} \times 30 = 30 \times 1 \pm A$

$\Rightarrow A = 11 \times 15 - 30 = 135^\circ$

$= \frac{135}{180} \times \pi \text{ rad} = \frac{3\pi}{4}$ rad

Ex.14 Find the angle between the minute hand of a clock and the hour hand when the time is 5:20 AM.

(a) 50^0 (b) 30^0 (c) 40^0 (d) 45^0

Sol. (c) $\theta = \left|\frac{11}{2}M - 30H\right|$

Where θ = angle

M = minute

H = hour

$$\theta = \left|\frac{11}{2} \times 20 - 30 \times 5\right|$$

$= |110^\circ - 150^\circ| = 40^\circ$

Ex. 15 A wheel makes 180 revolutions in one minute. Through how many radians does it turn in one second? Also find its degree measure.

Sol. Wheel makes 180 revolutions in 1 minute.

$\therefore$ Wheel makes $\frac{180}{60} = 3$ revolutions in 1 second.

Now, $\because$ One complete revolution measure 2π radian.

$\because$ Three complete revolutions measure $2\pi \times 3 = 6\pi$ radian

Again π rad = 180°

$\therefore 6\pi$ rad = 6 × 180° = 1080°

Ex.16 A wheel makes 240 revolutions per minute. Through how many radians does it turn in 1 second ?

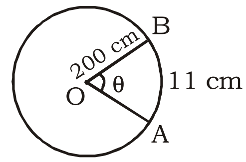
(a) 8π (b) 6π (c) 4π (d) 16π

Sol. (a) Number of revolutions per second $= \frac{240}{60} = 4$ complete circles.

A circle subtends an angle of $2\pi^c$ at its centre it 1 revolution

$\therefore$ Number of radians in 4 revolution $= 4 \times 2\pi = 8\pi^c$

Ex. 17 Find the degree and radian measure of the angle subtended at the centre of a circle of radius 200 cm by an arc of length 11 cm.



Sol. Given r = 200 cm,

l = Arc AB = 11cm

Suppose angle subtended at the centre of circle be θ radian

then, $\theta = \frac{l}{r} = \frac{11}{200}$ rad

$\because \pi$ rad = 180°

$\therefore l \text{ rad} = \frac{180^\circ}{\pi}$

or, $l \text{ rad} = \frac{7}{22} \times 180^\circ$

$\therefore \frac{11}{200} \text{ rad} = \frac{11}{200} \times \frac{7}{22} \times 180^\circ$

$= \frac{7 \times 180^\circ}{200 \times 2} = \frac{7 \times 45^\circ}{100} = \frac{7 \times 9^\circ}{20}$

$= \frac{63^\circ}{20} = 3\frac{3}{20} = 3$ degree

$\frac{3}{20} \times 60$ minutes = 3°9'

Ex.18 The moon's distance from the earth is 360000 km and its diameter subtends an angle of 30' at the eye of the observer. Find the diameter of the moon.

(a) 100π km

(b) 1000π km

(c) 1500π km

(d) 2000π km

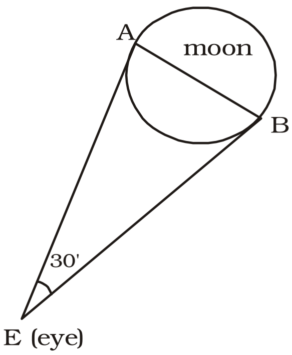
Sol.(b) Diameter d = Arc AB as the distance between moon and the earth is very large

$\theta = 30' = \left(\frac{30}{60} \times \frac{\pi}{180}\right)^c$

$= \left(\frac{\pi}{360}\right)^c$

$\therefore \theta = \frac{\text{arc}}{\text{radius}}$

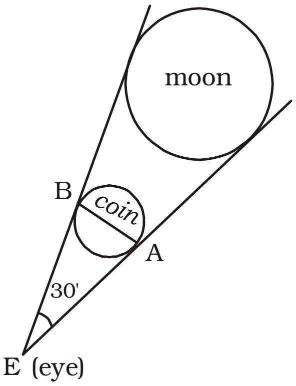
$$\Rightarrow \frac{\pi}{360} = \frac{d}{360000} \Rightarrow d = 1000\pi \text{ km}$$



Ex.19 If the angular diameter of the moon be 30' , how far from the eye a coin of diameter 4.4 cm be kept to hide the moon ?

(a) 252cm (b) 504cm
(c) 300cm (d) 500 cm

Sol.(b) arc AB = diameter AB =4.4 cm



$$\theta = 30' = \left(\frac{30}{60}\right)^0 = \left(\frac{1}{2}\right)^0$$

$$= \left(\frac{1}{2} \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{360}\right)^c$$

$$\theta = \frac{\text{arc}}{\text{radius}} \Rightarrow \frac{\pi}{360} = \frac{4.4}{r}$$

$$\Rightarrow r = \frac{4.4 \times 360}{\pi}\text{cm} = \frac{4.4 \times 360}{22} \times 7$$

$$\Rightarrow r = 504 \text{ cm}$$

EXERCISE

1. In radian measure 120° equals

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) $\frac{4\pi}{3}$

2. $37\frac{1}{2}^\circ$ is equal to which of the following radian measure?

(a) $\frac{5\pi}{12}$ (b) $\frac{7\pi}{12}$ (c) $\frac{5\pi}{24}$ (d) $\frac{7\pi}{24}$

3. $11\frac{1}{4}^\circ$ is equivalent to the radian measure

(a) $\frac{\pi}{8}$ rad (b) $\frac{3\pi}{8}$ rad

(c) $\frac{3\pi}{16}$ (d) $\frac{\pi}{16}$ rad

4. $\frac{5}{6}$ right angle in radian equals

(a) $\frac{5\pi}{24}$ (b) $\frac{5\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{\pi}{12}$

5. 1 radian is equal to:-

(a) 100° (b) $\left(\frac{\pi}{180}\right)^0$

(c) $\left(\frac{180}{\pi}\right)^0$ (d) 90^0

6. Find the degree measure corresponding to $\left(\frac{1}{6}\right)^c$:-

(a) $9^0 32'$ (b) $9^0 32' 43.6''$
(c) 10^0 (d) None of these

7. Three interior angles of a quadrilateral are 60°, 120°, 90°. The remaining angle in circular measure is given by :

(a) $\frac{\pi^c}{3}$ (b) $\frac{\pi^c}{2}$ (c) $\frac{\pi^c}{4}$ (d) $\frac{3\pi^c}{4}$

8. In Δ ABC, $\angle$A = 30°, $\angle$B = 60°. Find $\angle$C in circular measure :

(a) $\frac{2\pi^c}{3}$ (b) $\frac{3\pi^c}{4}$ (c) $\frac{\pi^c}{4}$ (d) $\frac{\pi^c}{2}$

9. Radian value of 560°20' is

(a) $\frac{1481\pi}{540}$ rad (b) $\frac{1481\pi}{360}$ rad

(c) $\frac{1681\pi}{540}$ rad (d) $\frac{1681\pi}{360}$ rad

10. Radian measure of 72°40' is

(a) $\frac{109\pi}{270}$ rad (b) $\frac{109\pi}{180}$ rad

(c) $\frac{219\pi}{540}$ rad (d) $\frac{219\pi}{360}$ rad

11. Measure of 6 rad is
(a) 343°18'11" (b) 341°18'11"
(c) 341°38'11" (d) 343°38'11"

12. If 1 rad = 57° 16' 21" then 10 rad equals
(a) 570°16'21" (b) 573°43'10"
(c) 571°43'40" (d) 572°43'30"

13. If one unit of an angle is 29° 46'55" then five units of the angle equals
(a) 148°54'35" (b) 146°54'35"
(c) 149°34'25" (d) 147°44'35"

14. If one unit of an angle is 15°49'50" then measure of 100 units of the angle equals
(a) 1580°30'20" (b) 1582°3'20"
(c) 1583°3'20" (d) 1581°30'20"

15. A wheel make 90 revolutions in half hour. Through how many degree does it turn in one minute?
(a) 120° (b) 720°
(c) 1080° (d) 540°

16. The angle in degree through which a pendulum of length 100 cm savings and the tip describes an arc length of 10 cm is
(a) 5°43'38" (b) 7°43'38°
(c) 5°34'18" (d) 7°34'18"

17.. Find in degrees the angle through which a pendulum swings if its length is 90cm and its tip describes an arc of length 22cm .
(a) 14^0 (b) $13^0 16'$
(c) $14^0 8'$ (d) 13^0

18. A rail road curve is to be laid out on a circle. What radius should be used if the track is to change direction by 25° in a distance of 40 metres ?
(a) 91.64 metres
(b) 90.46 metres
(c) 89.64 metres
(d) 93.64 metres

19. By decreasing 15° of each angle of a triangle, the ratios of their angles are 2 : 3 : 5. The radian measure of greatest angle is :

(a) $\frac{11\pi}{24}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{24}$ (d) $\frac{5\pi}{24}$

20. In a triangle ABC, $\angle ABC = 75°$ and $\angle ACB = \frac{\pi^{\circ}}{4}$. The circular measure of $\angle BAC$ is :

(a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian

(c) $\frac{\pi}{24}$ (d) $\frac{5\pi}{24}$

21. The minute hand of a big wall-clock is 35 cm long. Taking $\pi = \frac{22}{7}$, length of the arc, its extremity moves in 18 seconds is :

(a) 11 cm (b) 1.1 cm
(c) 6.6 cm (d) 6 cm

22. Two angles of a triangle are $\frac{3}{2}$ rad and $\frac{4}{3}$ rad. The triangle

(a) is an acute angled triangle
(b) is an obtuse angled triangle
(c) is a right angled triangle
(d) does not form

23. If two angle of a triangle are 2 rad and $\frac{1}{2}$ rad then its third angle in degree is

(a) $105\frac{3°}{7}$ (b) $15\frac{3°}{7}$

(c) $105\frac{5°}{7}$ (d) $36\frac{9°}{11}$

24. A wheel revolves 24 times in 10 seconds. How many time does it take in revolving an angle of 110 rad?

(a) 5 sec (b) 7.3 sec
(c) 10 sec (d) None of these

25. Radian measure of 40°20'50" is

(a) $\frac{481}{1196}\pi$ rad (b) $\frac{681}{1296}\pi$ rad

(c) $\frac{581}{2592}\pi$ rad (d) $\frac{581}{1296}\pi$ rad

26. A pendulum of length 60 cm swings and creates an arc of 18 cm. The angle at the fixed point of the pendulum is

(a) 15° (b) $17\frac{1°}{2}$

(c) 20° (d) $22\frac{1°}{2}$

27. Radius of a circle is 54 cm. If an arc of circle subtends an angle of 20° at centre then length of the arc is

(a) $19\frac{1}{7}$ (b) $17\frac{4}{7}$ cm

(c) $18\frac{6}{7}$ cm (d) None of these

28. An arc of length 40 cm subtends $22\frac{1°}{2}$ at the centre of the circle. Radius of the circle is

(a) 92 cm (b) 102 cm
(c) 96 cm (d) 108 cm

29. The minute hand of a watch is 3cm long. How far does its tip move in 50 minute ?

(a) 10.32 cm (b) 17.67 cm
(c) 15.71 cm (d) 18.23 cm

30. Find the angle between the hour hand and the minute hand at half past four .

(a) $\frac{\pi}{4}$ radian (b) $\frac{\pi}{6}$ radian

(c) $\frac{2\pi}{3}$ radian (d) $\frac{\pi}{3}$ radian

31. In a circle of diameter 30cm , the length of the chord is 15cm. Find the length of the minor arc corresponding to the chord .

(a) $\frac{5\pi}{3}$ cm (b) 5π cm

(c) $\frac{5\pi}{2}$ cm (d) None of these

32. The angles of a triangle are in Arithmetic Progression. The ratio of the least angle in degrees to the number of radians in the greatest angle is 60 : π. The angles in degrees are:

(a) 30°, 60°, 90°
(b) 35°, 55°, 90°
(c) 40°, 50°, 90°
(d) 40°, 55°, 85°

33. Ananta's (A) and Shailvia's (S) house are situated at a circular road and subtends 90° at a fixed point. If fixed point is at a distance of 100 metre from each house, the distance travelled between the both house on the road is

(a) 628 metres (b) 314 metres
(c) 157 metres (d) 235.5 metres

34. The angle covered by minute hand of a watch during 1hour 15 minutes noon to half past three noon is

(a) 4.5 π (b) 5 π
(c) 4.25 π (d) None of these

35. The angle covered by hour hand of a clock from half past six in the morning to three O'clock in the noon is

(a) 270° (b) 245°
(c) 255° (d) 265°

36. Assuming that the Moon's diameter subtends and angle $\left(\frac{1}{2}\right)^{\circ}$ at the eye of an observer, find how far from the eye of a coin of 10 cm diameter must be held so as just to hide Moon?

(a) $112\frac{5}{11}$ cm (b) $110\frac{6}{11}$ cm

(c) $116\frac{5}{11}$ cm (d) $114\frac{6}{11}$ cm

37. The earth revolves in its axis in 24 hours. How much angle does it move in 4 hours and 12 minutes?

(a) 63° (b) 64°
(c) 65° (d) 70°

38. The angle formed by the hour-hand and the minute-hand of a clock at 2 : 15 p.m. is

(a) $22\frac{1}{2}^{\circ}$ (b) 30° (c) $27\frac{1}{2}^{\circ}$ (d) 45°

39. Two angles of triangle are $\frac{1}{2}$ and $\frac{1}{3}$ radian. The measure of the third angle in degree $\left(\text{taking } \pi = \frac{22}{7}\right)$ is

(a) $132\frac{1°}{11}$ (b) $132\frac{2°}{11}$

(c) $132\frac{3°}{11}$ (d) 132°

40. A wheel rotates 3.5 times in one second. What time (in second) does the wheel take to rotate 55 radian of angle?
(a) 1.5 (b) 2.5 (c) 3.5 (d) 4.5

41. The radian measure of 63°14'51" is
(a) $\left(\frac{2811}{8000}\right)^c$ (b) $\left(\frac{3811\pi}{8000}\right)^c$
(c) $\left(\frac{4811\pi}{8000}\right)^c$ (d) $\left(\frac{5811\pi}{8000}\right)^c$

42. When a pendulum of length 50 cm oscillates, it produce an arc of 16 cm. The angle so formed in degree measure is (approx)
(a) $22\frac{1}{2}^\circ$ (b) 18°35'
(c) $27\frac{1}{2}^\circ$ (d) 45°

43. A rail road curve is to be laid out on a circle. What radius should be used if the track is to change direction by 25° in a distance of 40 metres.
(a) 91.64 m (b) 81.23 m
(c) 95.67 m (d) 98.72 m

44. An arc of a circle of radius 42 cm subtends an angle of 15° at the centre. Taking $\pi = \frac{22}{7}$' the length of the arc is:
(a) $\frac{88}{5}$ cm (b) 11 cm
(c) 12 cm (d) $\frac{14}{5}$ cm

45. If the sum and difference of two angles are 135° and $\frac{\pi}{12}$ respectively, then the value of the angles in degree measure are
(a) 70°, 65° (b) 75°, 60°
(c) 45°, 90° (d) 80°, 55°

46. The degree measure of 1 radian $\left(\text{taking } \pi = \frac{22}{7}\right)$
(a) 50°16'22" (approx.)
(b) 57°16'22" (approx.)
(c) 57°22 '16"(approx.)
(d) 57°32'16" (approx.)

47. In a triangle ABC, $\angle ABC = 75°$ and $\angle ACB = \frac{\pi^c}{4}$. The circular measure of $\angle BAC$ is
(a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian
(c) $\frac{5\pi}{6}$ radian (d) $\frac{5\pi}{2}$ radian

48. At what point of time after 3 O' clock hour hand and the minute hand of a clock occur at right angles for the first time?
(a) 9O'clock
(b) 4th $37\frac{1}{6}$ min
(c) 3h $30\frac{8}{11}$ min
(d) 3h $32\frac{8}{11}$ min

49. Consider the following
I. The angular measure in radian of a circular arc of fixed length subtending at its centre decreases, if the radius of the arc is increases
II. 1800° is equal to 5 radian.
Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II
(d) Neither I nor II

50. How many degrees are there in an angles which equals two-third of its complement?
(a) 36° (b) 45° (c) 48° (d) 60°

51. The earth takes 24 h to rotate about its own axis. Through what angle will it turn in 4 h and 12 min?
(a) 64° (b) 63° (c) 65° (d) 70°

52. If $\cos\theta \geq \frac{1}{2}$ in the first quadrant, then which one of the following is correct?
(a) $\theta \leq \frac{\pi}{3}$ (b) $\theta \geq \frac{\pi}{3}$
(c) $\theta \leq \frac{\pi}{6}$ (d) $\theta \geq \frac{\pi}{6}$

53. What is the angle (in radian) included between the hands of a clock, when the time is 10 min past 5?
(a) $17\pi/36$ (b) $19\pi/36$
(c) $5\pi/9$ (d) $7\pi/12$

54. If clock started at noon, then what is the angle turned by hour hands at 3 : 45 pm?
(a) 67.5° (b) 97.5°
(c) 112.5° (d) 142.5°

ANSWER KEY

1. (b)	7. (b)	13. (a)	19. (a)	25. (c)	30. (a)	35. (c)	40. (b)	45. (b)	50. (a)
2. (c)	8. (d)	14. (c)	20. (b)	26. (b)	31. (b)	36. (d)	41. (a)	46. (b)	51. (b)
3. (d)	9. (c)	15. (c)	21. (b)	27. (c)	32. (a)	37. (a)	42. (b)	47. (b)	52. (a)
4. (b)	10. (a)	16. (a)	22. (a)	28. (b)	33. (c)	38. (a)	43. (a)	48. (d)	53. (b)
5. (c)	11. (d)	17. (a)	23. (d)	29. (c)	34. (a)	39. (c)	44. (b)	49. (a)	54. (c)
6. (b)	12. (d)	18. (a)	24. (b)						

SOLUTION

1. (b) $120° = \frac{120}{180} \times \pi \text{ rad}$

$= \frac{2\pi}{3} \text{ rad}$

2. (c) $37\frac{1°}{2} = \left(\frac{75}{2}\right)^°$

$= \frac{75}{2 \times 180} \times \pi = \frac{5\pi}{2 \times 12} = \frac{5\pi}{24} \text{ rad}$

3. (d) $11\frac{1°}{4} = \left(\frac{45}{4}\right)^°$

$= \frac{\left(\frac{45}{4}\right)}{180} \times \pi \text{ rad} = \frac{\pi}{16} \text{ rad}$

4. (b) $\frac{5}{6}$ right angle

$= \frac{5}{6} \times \frac{\pi}{2} \text{ rad} = \frac{5\pi}{12}$

5. (c) π rad = 180^0

$\therefore 1 \text{ rad} = \left(\frac{180}{\pi}\right)^°$

6. (b) $\left(\frac{1}{6}\right)^c = \left(\frac{1}{6} \times \frac{180}{\pi}\right)^0 = \left(\frac{1}{6} \times \frac{180}{22} \times 7\right)^0$

$= \left(\frac{105}{11}\right)^0$

$= \left(9\frac{6}{11}\right)^° = 9°\left(\frac{6}{11} \times 60\right)' = 9°\left(32\frac{8}{11}\right)'$

$= 9°32'\left(\frac{8}{11} \times 60\right)'' = 9°32'43.6''$

7.(b) Fourth angle of quadrilateral
= 360° – (60° + 120° + 90°)
= 90°
$\because$ $180° = \pi^c$

$90° = \frac{\pi^c}{2}$

8. (d) $\angle C = 180° - 30° - 60° = 90°$
$\because$ $180° = \pi$ radian

$\therefore 90° = \frac{\pi}{180} \times 90 = \frac{\pi}{2} \text{ radian}$

9. (c) $560°20' = \left(560\frac{20}{60}\right)^°$

$= \left(560\frac{1}{3}\right)^° = \left(\frac{1681}{3}\right)^°$

$= \frac{1681}{3} \times \frac{\pi}{180} = \frac{1681}{540} \pi \text{ rad}$

10. (a) $72°40' = \left(72\frac{40}{60}\right)^° = \left(72\frac{2}{3}\right)^°$

$= \left(\frac{218}{3}\right)^° = \left(\frac{218}{3} \times \frac{\pi}{180}\right)$

$= \left(\frac{109\pi}{3 \times 90}\right) = \frac{109\pi}{270} \text{ rad}$

11. (d) π rad = 180°

$\therefore$ 1 rad = $\frac{180°}{\pi}$

6 rad = $\frac{6 \times 180°}{\pi} = \frac{6 \times 180° \times 7}{22}$

$= \frac{21 \times 180°}{11} = \frac{3780°}{11}$

$\left(343\frac{7}{11}\right)^° = 343°\left(\frac{420}{11}\right)'$

$= 343°\left(38\frac{2}{11}\right)'$

$= 343°38'\left(\frac{120}{11}\right)''$

= 343°38'11" (approximate)
Second Method,
$\because$ 1 rad = 57°16'22"
6 rad = 57° × 6 + 16' × 6 + 22" × 6
= 342° + 96' + 132''
= 342° + (1° + 36') + (2' + 12")
= ($\because$ 1° = 60' and 1' = 60")
= 343° 38'12" (approximate)

12. (d) 10 rad = (57°16'21") × 10
= 570° + 160' + 210"
= 570° + 2° + 40' + 3' + 30"
= 572°43'30"

13. (a) 5 units = (29°46'55") × 5
= 145° 230' 275"
= 145° + 3°+ 50' + 4' + 35"
= 148° 54' 35"

14. (c) 100 units
= (15° 49' 50") × 100
= 1500° 4900' 5000"
= 1500° + (81° + 40') + 83' + 20"

$\left(\because \frac{4900}{60} = 81\frac{40}{60}, \frac{5000}{60} = 83\frac{20}{60}\right)$

= 1500° + 81° + 40' + 1° + 23' + 20"
= 1582°63'20" = 1583°3'20"

15. (c) Wheel revolves = $\frac{90}{30}$
= 3 turn in one minute
$\because$ 1 turn = 360°
$\therefore$ 3 turn = 1080°

16. (a) From $\theta = \frac{l}{r}$, $\theta = \frac{10}{100}$

$= \frac{1}{10} \text{ rad} = \frac{180°}{\pi \times 10} = \left(\frac{18 \times 7}{22}\right)^°$

$= \left(\frac{63}{11}\right)^° = \left(5\frac{8}{11}\right)^°$

$= 5°\left(\frac{8}{11} \times 60'\right) = 5°\left(\frac{480'}{11}\right)$

$= 5°\left(43\frac{7}{11}\right)' = 5°43'\left(\frac{7}{11} \times 60''\right)$

= 5°43'38" (approximate)

17. (a) r = 90 cm and
arc (s) = 22 cm

$\therefore \theta = \left(\frac{S}{r}\right)^c = \left(\frac{22}{90}\right) = \left(\frac{11}{45}\right)^c$

$\Rightarrow \theta = \left(\frac{11}{45} \times \frac{180}{\pi}\right)^0 = \left(\frac{11}{45} \times \frac{180}{22} \times 7\right)^0 = 14^0$

18. (a) $\theta = 25°$

$= \frac{25 \times \pi}{180} \text{ radians}$

$= \frac{5\pi}{36} \text{ radians}$

$\theta = \frac{s}{r}$

$\Rightarrow r = \frac{s}{\theta} = \frac{40}{\frac{5\pi}{36}} = \frac{40 \times 36}{5\pi} = \frac{40 \times 36 \times 7}{5 \times 22} \text{ metre}$

$= 91.64$ metre

19. (a) $2x + 3x + 5x = 180° - 45° = 135°$

$\Rightarrow 10x = 135°$

$\Rightarrow x = \frac{135}{10} = \frac{27}{2}$

$\therefore$ Largest angle

$= 5x + 15° = \left(5 \times \frac{27}{2}\right)° + 15°$

$= \frac{135 + 30}{2} = \frac{165°}{2}$

$\because 180° = \pi$ radian

$\therefore \frac{165°}{2} = \frac{\pi}{180} \times \frac{165}{2} = \frac{11\pi}{24}$ radian

20. (b) $\angle ABC = 75°$

$\because 180° = \pi$ radian

$\therefore 75° = \frac{\pi}{180} \times 75 = \frac{5\pi}{12}$ radian

$\angle BAC = \pi - \frac{\pi}{4} - \frac{5\pi}{12}$

$= \frac{12\pi - 3\pi - 5\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$ radian

21. (b) Traced arc length by minute hand in 60×60 seconds = $2\pi r$

$\therefore$ Length of arc made in 18 seconds $= \frac{2\pi r}{60 \times 60} \times 18$

$= 2 \times \frac{22}{7} \times \frac{35 \times 18}{60 \times 60} = 1.1$ cm

22. (a) 1 right angle = $\frac{\pi}{2}$ rad

= 1.57 rad (approximate)

$\frac{3}{2}$ = 1.5 rad, which is an acute angle.

$\frac{4}{3}$ = 1.33 rad, which is an acute angle

Third angle = $(\pi - 1.5 - 1.33)$

and = (3.14 – 1.5 – 1.33)

= 0.31 rad which is also an acute angle.

23. (d) Third angle

$= \pi \text{ rad} - \frac{1}{2} \text{ rad} - 2\text{rad}$

$= \left(\frac{22}{7} - \frac{5}{2}\right)\text{rad} = \frac{9}{14}$ rad

$= \frac{9}{14} \times \frac{180°}{\pi} = \frac{9}{14} \times \frac{180}{22} \times 7$

$= \frac{45 \times 9}{11} = \frac{405}{11} = 36\frac{9°}{11}$

24. (b) The wheel makes $\frac{24}{10}$ rotation in one second

It covers $\frac{24}{10} \times 2\pi$ rad angle in 1 second.

Hence in covering 110 rad, wheel takes

$\frac{110}{\frac{24}{10} \times 2\pi} = \frac{110 \times 10 \times 7}{24 \times 2 \times 22}$

= 7.3 second

25. (c) $40°20'50'' = 40°\left(20\frac{50}{60}\right)'$

$= 40°\left(\frac{125}{6}\right)' = 40°\left(\frac{125}{6 \times 60}\right)°$

$= \left(40\frac{25}{72}\right)° = \left(\frac{2905}{72}\right)°$

$= \frac{2905}{72} \times \frac{\pi}{180}$ rad

$= \frac{581\pi}{72 \times 36} = \frac{581\pi}{2592}$ rad

26. (b) $\theta = \frac{l}{r}$, $\theta = \frac{18}{60} = \frac{3}{10}$ rad

= 0.3 rad

1 rad = 57°16'22" (approximate)

0.3 rad = 5.7° × 3 = more than 17° and less than 18°

$\therefore$ From options $= 17\frac{1°}{2}$

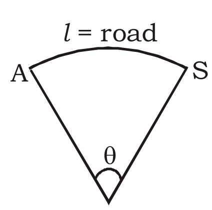
27. (c) $20° = \frac{20°}{180°}\pi = \frac{\pi}{9}$ rad

From, $\theta = \frac{l}{r}$ $l = \theta r = \frac{\pi}{9} \times 54$

$= \frac{22}{7 \times 9} \times 54$ cm

$= \frac{22 \times 6}{7} = \frac{132}{7} = 18\frac{6}{7}$ cm

28. (b)



$22\frac{1°}{2} = \frac{\left(\frac{45°}{2}\right)}{180} \times \pi \text{ rad} = \frac{\pi}{8}$ rad

$\because \theta = \frac{l}{r} \Rightarrow r = \frac{l}{\theta} = \frac{(40)}{\left(\frac{\pi}{8}\right)}$

$= \frac{320}{\pi} = \frac{320 \times 7}{22} = 101.8$ cm

29. (c) The minute hand complete one revolution in 60 minute.

$\therefore$ In 50 minute it will cover $\frac{50}{60} = \frac{5}{6}$ of the revolution.

$\because$ 1 revolution $= 2\pi$ radian.

$\therefore \frac{5}{6}$ revolution $= 2\pi \times \frac{5}{6} = \frac{5\pi}{3}$ radian

$\therefore$ Distance moved by tip

$= 3 \times \frac{5\pi}{3}$ cm $= 5\pi$ cm

$= 5 \times \frac{22}{7}$ cm $= 15.71$ cm

30. (a) Angle traced by the hour hand in 12 hours = 360^0

$\therefore$ Angle traced by the hour hand in 4 hrs 30 min.

$\left(= \frac{9}{2}\text{hrs}\right) = \frac{360}{12} \times \frac{9}{2} = 135^0$

Angle traced by the minute hand in 60 min. $= 360^0$

$\therefore$ Angle traced by the minute hand in 30 min

$= \left(\frac{360}{60} \times 30\right)^0 = 180^0$

Thus, the angle between two hands = 180° – 135° = 45°

$= \frac{\pi}{4}$ radian

Alternate:-

$\theta = \left|\frac{11}{2}M - 30H\right|$

Where θ = angle

M = minute

H = hour

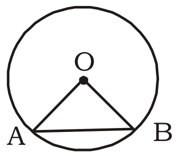
$$\theta = \left|\frac{11}{2}\times 30 - 30\times 4\right|$$

$= |165° - 120°|$

$= 45° = \dfrac{\pi}{4}$ radian

31. (b) OA = OB = 15 cm (radius)
and chord AB = 15 cm

$\therefore$ ΔOAB is an equilateral triangle.



$$\therefore \angle AOB = 60^0 = \left(\frac{\pi}{3}\right)^c$$

$$\therefore \theta = \frac{arc}{radius} \Rightarrow arc = \theta\times r = \frac{\pi}{3}\times 15 = 5\pi \text{ cm}$$

32. (a) Angles of triangle
(a – d)°, a°, (a + d)°
a – d + a + a + d = 180°

$\Rightarrow 3a = 180° \Rightarrow a = 60°$

$$\therefore \frac{a-d}{a+d} = \frac{60}{\pi} = \frac{60}{180} = \frac{1}{3}$$

$$\Rightarrow \frac{60-d}{60+d} = \frac{1}{3}$$

$\Rightarrow$ 180 – 3d = 60 + d

$\Rightarrow$ 4d = 120° $\Rightarrow$ d = 30°

$\therefore$ Angles of triangle :
a – d = 60° – 30° = 30°
a = 60°
a + d = 60 + 30 = 90°

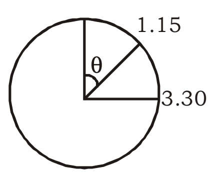
33. (c) $90° = \dfrac{\pi}{2}$

$\theta = \dfrac{l}{r}$ or $l = \theta r$

$= \dfrac{\pi}{2}\times 100$ metres

$= \dfrac{3.14}{2}\times 100 = 157$ metres

34. (a)



From 1 hour 15 minutes to half past three, minute hand covers 2 hours 15 minutes i.e.,

$2\frac{1}{4}$ rotations.

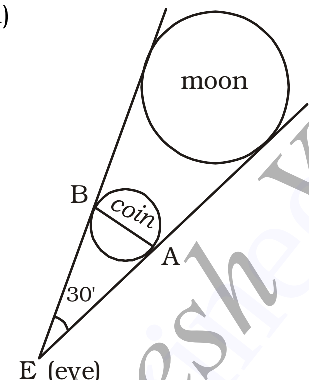
$\therefore$ It covers $2\frac{1}{4}\times 2\pi$

= $(4.5)\pi$ rad distance.

35. (c) From half past six in the morning to 3 o'clock at noon, time elapsed is 8 hours 30 minutes. Since hour hand covers 30° in 5 minute therefore it covers

$30°\times 8\frac{1}{2} = 255°$ in $8\frac{1}{2}$ hours.

36. (d)



arc AB = diameter AB = 10cm

$$\theta = \left(\frac{1}{2}\right)^{\circ} = \frac{1}{2}\times\frac{\pi}{180} = \left(\frac{\pi}{360}\right)^c$$

$$\theta = \frac{l}{r} \Rightarrow \frac{\pi}{360} = \frac{10}{r}$$

$$\Rightarrow r = \frac{360\times 10\times 7}{22} = 114\frac{6}{11}\text{ cm}$$

37. (a) Revolution in 24 hours = 360°

$\therefore$ Revolution in 1 hours

$= \dfrac{360°}{24} = 15°$

Revolution in 4 hours
= 15° × 4 = 60°

$\because$ Revolution in 60 minutes = 15°

Revolution in 12 minutes

$= \dfrac{15°\times 12°}{60°} = 3°$

$\therefore$ Revolution in 4 hours 12 minutes = 60° + 3° = 63°

38. (a) From Trick,

using, $= \dfrac{11}{2}M = 30H \pm A$

Here, M = 15, H = 2

Hence, $\dfrac{11}{2}\times 15 = 30\times 2 \pm A$

or, $A = \dfrac{165}{2} - 60 = 82\frac{1}{2} - 60$

$= 22\frac{1}{2}^{\circ}$

Therefore, angle will be $22\frac{1}{2}^{\circ}$

39. (c) Sum of two angle be

$= \dfrac{1}{2}+\dfrac{1}{3} = \dfrac{5}{6}$ rad

$\because \dfrac{22}{7}$ rad $= 180° \left(\because \pi = \dfrac{22}{7}\right)$

$\therefore \dfrac{5}{6}$ rad $= \dfrac{180°}{22}\times 7\times\dfrac{5}{6}$

$= \dfrac{30°\times 35}{22} = \dfrac{15°\times 35}{11}$

Remaining angle

$= 180° - \dfrac{15°\times 35}{11} = 180° - \dfrac{525°}{11}$

$= 180° - 47\dfrac{8°}{11} = 132\dfrac{3°}{11}$

40. (b) 1 rotation = 2π radian
3.5 rotation = 3.5 × 2π radian

$= 3.5\times 2\times\dfrac{22}{7} = 22$ radian

$\because$ Wheel rotation in one second is 22 radian

$\therefore$ Wheel rotation in 55 radian

$\dfrac{55}{22} = 2.5$ second.

41. (a) $63°14'51'' = 63\left(14\dfrac{51}{60}\right)'$

$= 63\left(14\dfrac{17}{20}\right)'$

$= 63\left(\dfrac{297}{20}\right)' = \left(63\dfrac{297}{20\times 60}\right)^{\circ}$

$= \left(63\dfrac{99}{20\times 20}\right)^{\circ} = \left(63\dfrac{99}{400}\right)^{\circ}$

$= \left(\dfrac{25299}{400}\right)^{\circ}$

$= \left(\dfrac{25299}{400}\times\dfrac{\pi}{180}\right)^{\circ}$ rad

$= \left(\dfrac{2811\pi}{400\times 20}\right)^{\circ} = \left(\dfrac{2811\pi}{8000}\right)$

Trick, Value of 63°14'51" is 60° (approximate)
$\therefore$ Value of 63°14'51" should be more than $\frac{\pi}{3} = 0.33\pi$

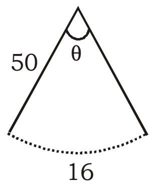
From option (a),

$$\left(\frac{2811\pi}{8000}\right) = \left(\frac{2800\pi}{8000}\right) \text{ (approx)}$$

$$= \left(\frac{28}{80}\pi\right) = 0.35\pi \text{ (approx)}$$

So, option (a) is correct.

42. (b)



$$\theta = \frac{l}{r} \Rightarrow \theta = \frac{16}{50} \text{ rad}$$

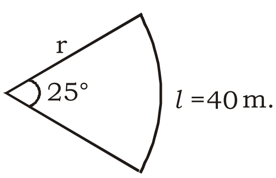
$$= \left(\frac{16}{50} \times \frac{180}{\pi}\right)^\circ$$

$$= \left(\frac{16 \times 18}{5} \times \frac{7}{22}\right)^\circ$$

$$\left(\frac{16 \times 9 \times 7}{5 \times 11}\right)^\circ = \left(\frac{1008}{55}\right)^\circ$$

$$\left(18\frac{18}{55}\right)^\circ = 18°35' \quad \text{(approx)}$$

43. (a)



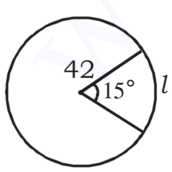
$$25° = 25 \times \frac{\pi}{180} \text{ rad} = \frac{5\pi}{36} \text{ rad}$$

From $\theta = \frac{l}{r}$, $\frac{5\pi}{36} = \frac{40}{r}$

$$\therefore r = \frac{40 \times 36}{5\pi}$$

$$= \frac{1008}{11} = 91.64 \text{ meter}$$

44. (b)



$$15° = \frac{15}{180} \times \pi = \frac{\pi}{12}$$

From $\theta = \frac{l}{r}$, $l = \theta r$

or, $l = \frac{\pi}{12} \times 42 = \frac{22}{7} \times \frac{1}{12} \times 42$

= 11 cm

45. (b) A + B = 135° ....(i)

$$A - B = \frac{\pi}{12} \times \frac{180°}{\pi} = 15° \quad ...(ii)$$

(i) + (ii) 2A = 150
A = 75°
B = 135° – 75° = 60°

46.(b) 1 radian = $1 \times \frac{180}{\pi}$ degrees

$$= \frac{180}{22} \times 7 = \frac{630°}{11} = \left(57\frac{3}{11}\right)^\circ$$

$$= 57° + \left(\frac{3}{11} \times 60\right)'$$

$$= 57° + \left(\frac{180}{11}\right)' = 57° + \left(16\frac{4}{11}\right)'$$

$$= 57° + 16' + \left(\frac{4}{11} \times 60\right)''$$

= 57°16'22" (approx.)

47. (b) Given, $\angle ABC = 75°$,

$$\angle ACB = \frac{\pi}{4} \text{ rad} = \left(\frac{\pi}{4} \times \frac{180}{\pi}\right)^\circ = 45°$$

$$\angle BAC = 180° - (75° + 45°) = 60°$$

$$= \left(60 \times \frac{\pi}{180°}\right) \text{ radian}$$

$$= \frac{\pi}{3} \text{ radian}$$

48. (d) In 60 minutes, minute hand rotate 360°
In x minutes, minute hand rotate $\frac{360}{60}x = 6x$

In 60 minutes, hours hand rotate 30°
In x minutes, hour hand rotate

$$\frac{30}{60}x = \frac{x}{2}$$

Initial angle of hour hand at 30' clock is 90° and minutes hand is 0°
Angle after x minutes

$$= 6x - \left(90 + \frac{x}{2}\right) = 90$$

$$\Rightarrow \frac{11x}{2} = 180°$$

$$\Rightarrow x = \frac{360}{11} = 3h\ 32\frac{8}{11} \text{ min}$$

49. (a) I. Arc length $l = r\theta$
& for constant l, $\theta = \frac{l}{r}$ so if the radius increases, θ decreases

II. $1800° = 1800 \times \frac{\pi}{180} = 10\pi$

50. (a) Given, $\theta + \phi = 90°$ (complementary angles)

$$\& \ \theta = \frac{2}{3}\phi$$

$$\text{or } \theta = \frac{2}{3}(90° - \theta)$$

$$\Rightarrow \theta = 60° - \frac{2\theta}{3}$$

$$\Rightarrow \frac{5\theta}{3} = 60° \Rightarrow \theta = 36°$$

51. (b) In 24h, Earth rotate 360°

$\therefore$ In 1 hr, Earth rotate $\frac{360°}{24} = 15°$

In 4h 12 min = $\left(4 + \frac{12}{60}\right)$hr

$$= \frac{21}{5} \text{hr},$$

Earth rotate = $15 \times \frac{21}{5} = 63°$

52. (a) If θ decreases then $\cos\theta$ increases

$$\cos\theta \geq \frac{1}{2}$$

$$\cos\theta \geq \cos\frac{\pi}{3} = \theta \leq \frac{\pi}{3}$$

53. (b) At 5 'o' clock, hour hand rotate = $5 \times \frac{\pi}{6} = \frac{5\pi}{6}$

In 60 min hour hand rotate

$$= 30° = \frac{\pi}{6}$$

10 min hour hand rotate

$$= \frac{10}{60} \times \frac{\pi}{6} = \frac{\pi}{36}$$

Total angle of hour hand

$= \frac{5\pi}{6} + \frac{\pi}{36} = \frac{30\pi + \pi}{36} = \frac{31\pi}{36}$

Angle rotate by minute hand in 10 minute

$= 60° = \frac{\pi}{3}$

Angle between two hands

$= \frac{31\pi}{36} - \frac{\pi}{3} = \frac{31\pi - 12\pi}{36} = \frac{19\pi}{36}$

54. (c) In 12h, hour hand rotate = 360°

In 1h, hour hand rotate = 30°

In 3h, hour hand rotate = 90°

In 45 minute, hour hand rotate

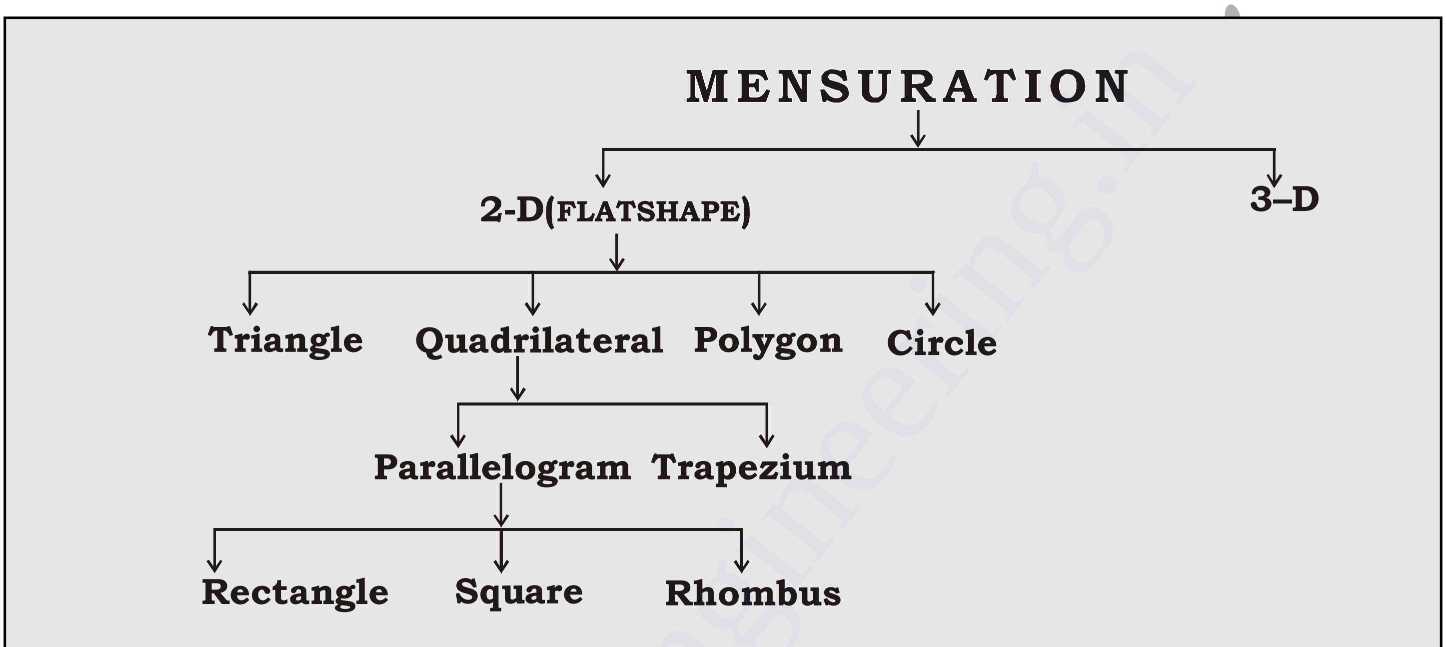
$= \frac{45}{60} \times 30° = 22.5°$

Total angle turned by hour hands at 3 : 45 pm = 90° + 22.5° = 112.5°

■■■

CHAPTER 16

MENSURATION
2-D (TWO DIMENSIONAL)



Area:- The area of any plane figure is the amount of surface enclosed within its bounding lines. It is always expressed in square units e.g. square metres, square inches etc.

Perimeter : The perimeter of a geometrical figure is the total length of the sides enclosing the figure.

Note:-

1 hectare = 10,000 m^2

100 hectare = 1,000,000 m^2

Weight (mass) = Volume $\times$ density.

What is Quadrilateral?

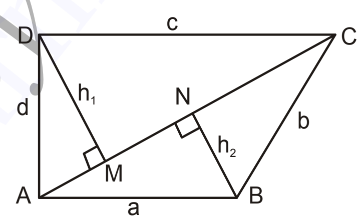
A quadrilateral is a four-sided polygon with four angles. There are many kinds of quadrilaterals. The sum of the angle of quadrilateral is 360°.

Types of quadrilaterals

(1) Rectangle **(2)** Square **(3)** Parallelogram **(4)** Rhombus **(5)** Trapezium

Quadrilateral : A closed figure bounded by four sides.

(i) $\angle A + \angle B + \angle C + \angle D = 360°$



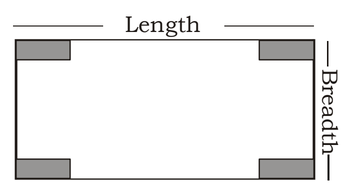
(ii) area of quadrilateral = $\frac{1}{2}\times$ one diagonal × (sum of perpendicular to it from opposite vertices)

= $\frac{1}{2}(AC)(h_1 \quad h_2)$

(iii) P = a + b + c + d

Rectangle

A rectangle is a four sided flat shape where every angle is a right angle (90°) also opposite sides are parallel and of equal length. A rectangle has two diagonals, they are equal in length and interesect in the middle.



(1) Its diagonals are equal & bisect each other.

(2) Area = Length × Breadth

(3) Perimeter = 2 (Length + Breadth)

(4) Diagonal (d) = $\sqrt{l^2 + b^2}$

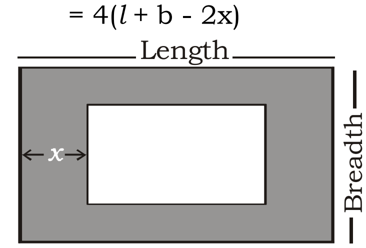
(5) (i) Area of a path inside a rectangular field:-

Area of path = $2x(l + b - 2x)$

(ii) Perimeter (P) = inner P + Outer P

= $2(l + b) + 2(l + b - 4x)$

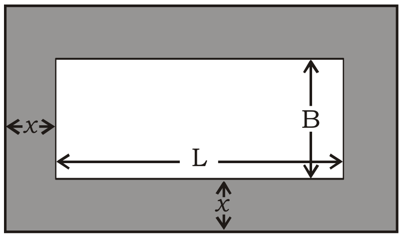
= $4(l + b - 2x)$



(6) (i) Area of path outside a rectangular field:-

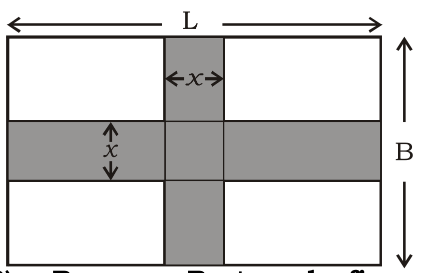
Area of path outside = $2x(l + b + 2x)$

(ii) Perimeter (P) = inner Perimeter + outer Perimeter
$= 2(l + b) + 2(l + b + 4x)$
$= 4(l + b + 2x)$



(7) (i) Area of path midway = $x(l + b - x)$

(ii) Perimeter of Path (P) = $2(l + b) - 4x = 2(l + b - 2x)$



(8) Room as a Rectangular figure:-
Area of four walls of a room = Perimeter × Height = 2 × (L + B) × H

(9) Area of Roof and 4 walls
= 2H (L+B)+LB
(this formula can be use when we have to paint a whole room.)

EXAMPLES

1. Area of a rectangular field of breadth 15 cm is 180 sq. cm. Find the length and perimeter of a rectangle

Sol. Area = Length × Breadth
$180 = x \times 15$
$x = \frac{180}{15} = 12$ cm = length
Perimeter $\Rightarrow$ 2 (length + Breadth)
$\Rightarrow$ 2 (15+12) = 54 cm

2. Area of a rectangular field is 560 sq. metre. Ratio of their length & Breadth is 5:7. Find the diagonal of a rectangle?

Sol. Area = Length × Breadth
$560 = 5x \times 7x$
$560 = 35x^2$
$x^2 = 16$
$x = 4$
Length = $5x = 5\times4 = 20$,
Breadth = $7x = 7\times4 = 28$
Diagonal = $\sqrt{20^2 + 28^2}$
$= \sqrt{400+784} = \sqrt{1184}$ m.

3. The ratio between the length and width of the rectangular field is 3 : 2. If only length is increased by 5m. The new area of the field is 2600m². What is the width of the rectangular field?
(a) 60 (b) 50 (c) 40 (d) 65

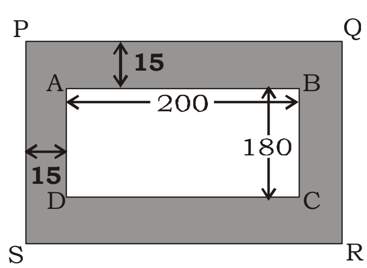
Sol. Let length = $3x$, then width = $2x$
$\therefore$ $(3x + 5)2x = 2600$
$\Rightarrow (3x + 5)x = 1300$
we, go through the option
option (c) $2x = 40 \Rightarrow x = 20$
which satisfy the above equation
$\therefore$ width = $2x$ = 40m
Note : you can also solve the above equation.

4. The length of rectangle, which is 24cm is equal to the length of a square and the area of the rectangle is 176cm less than the area of the square. What is the breadth of the rectangle?

Sol. Area of square = (side)² = (24)² = 576cm²
Area of rectangle = length × breadth = 576 –176 = 24 × x
$\Rightarrow$ Breadth of rectangle
$= \frac{400}{24} = \frac{50}{3} = 16\frac{2}{3}$ cm

5. A street of width 15 metres surrounds from outside a rectangular garden whose measurment is 200 m × 180 m. The area of the path?

Sol.



Area of Rectangle ABCD = L × B
$\Rightarrow$ 200×180=36,000 m²
Area of Rectangle PQRS
= (200+30)×(180+30)
$\Rightarrow$ 230×210 = 48,300 m²
Area of Path outside ABCD
= Area of PQRS – Area of ABCD
$\Rightarrow$ 48,300 – 36,000
$\Rightarrow$ 12,300 m²

Alternate:-
Area of Path outside
= $2x(l+b+2x)$
$\Rightarrow$ 2×15 (200+180+30)
$\Rightarrow$ 30×410 $\Rightarrow$ 12,300 m²

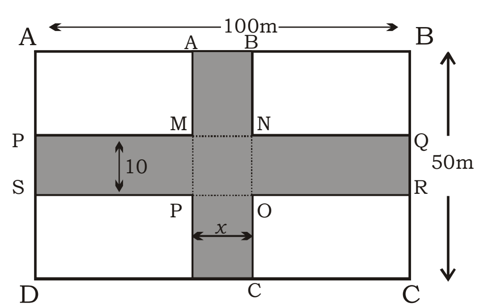
6. A path of uniform width runs mid-way of the Rectangle field having lenght 100m & Breadth 50m. If the path occupies 1400m². then the width of the path is?
(a) 5 (b) 10 (c) 12 (d) 8

Sol. Area of path midway = $x(l + b - x)$
$1400 = x(100+50-x)$
$1400 = 150x - x^2$
$x^2 - 150x + 1400 = 0$
$\Rightarrow x^2 - (140+10)x + 1400 = 0$
$\Rightarrow x^2 - 140x - 10x + 1400$
$\Rightarrow x(x-140) - 10(x-140)$
$= (x-140)(x-10)$
$\Rightarrow x \neq 140$ (is not possible because breadth is less than width)
$x = 10$

Alternate:-
In these question we take option (b)
= 10(100 + 50 – 10)
= 10 × 140 = 1400
So, (b) is correct.

7. Find the perimeter of a path in the sixth question?

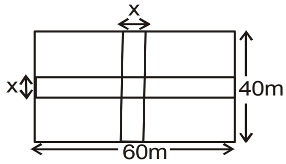
Sol.



L = 100m
B = 50 m
x = width of path = 10m.
Perimetor of path = permeter of ABCD + Peremeter of PQRS – Peremeter of MNOP
$= 2(B + x) + 2(L + x) - 2(x + x)$
$= 2L + 2B + 4x - 4x$
$= 2(L + B) = 2(100 + 50) = 300$

8. A rectangular park $60 \times 40m^2$ has two cross roads running in the middle of the park and the rest park has been lawn. If the area of the lawn is $2109m^2$. What is the width of the road?

Sol.



Total area of park = 60 × 40
= $2400m^2$

and area of lawn = $2109m^2$ (given)

area of the cross roads = 2400-2109 = $291m^2$

$\Rightarrow x(60+40-x) = 291$

$\Rightarrow x^2 - 100x + 291 = 0$

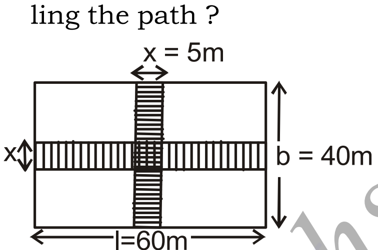
$\Rightarrow (x-97)(x-3) = 0$

$\Rightarrow x = 3$ or 97

$\Rightarrow x = 3$

[$\therefore x = 97$ is not possible]

9. A rectangular lawn $60 \times 40m^2$ has two roads each 5m wide running between the park. One is parallel to length and other is parallel to width. Cost of gravelling is 60 paise/m^2. Find the total cost of gravelling the path ?



Sol. Area of path = x (l + b - x)

= 5(60 + 40 - 5)

= 5 × 95 = $475m^2$

$\therefore$ Total cost = $475 \times \frac{60}{100}$

= ₹ 285

10. A room 8 meters long, 6 metres broad and 3 metres high. Find the area of Room?

Sol. As room consist of floor, roof & 4 walls, then the area of room will be = Total surface area of room

$\Rightarrow$ 2 (l + b) × h + 2lb

$\Rightarrow$ 2[(l + b)h + lb]

$\Rightarrow$ 2[3(14)+48]

$\Rightarrow$ 2[42+48] = 180 m²

11. A room 8m long, 6m broad and 3m high has two windows $1\frac{1}{2}$ m × 1m and a door 2m× $1\frac{1}{2}$ m. Find the cost of papering the walls with paper 50 cm wide at 25 paise per meter:

Sol. Area of walls = 2(length + breadth) × height = 2(8 + 6) × 3 = $84m^2$

Area of two windows and a door

$= 2\left(1\frac{1}{2}\times 1\right)+\left(2\times 1\frac{1}{2}\right) = 6m^2$

$\therefore$ Area to be covered = 84 – 6 = $78m^2$

$\therefore$ Area of paper = Area to be covered = 78 m²

$\Rightarrow$ (length × breadth) of paper = 78

$\Rightarrow$ length of paper = $\frac{78}{50} \times 100m$ = 156cm

$\therefore$ cost $= \frac{156 \times 25}{100}$ = ₹ 39

12. The dimensions of a room are 12.5 metres by 9 metres by 7 metres. There are 2 doors and 4 windows in the room, each door measures (2.5×1.2) metres and each window (1.5×1) metres. Find the cost of painting the whole room at ₹ 3.50 per square metre.

Sol. Area of 2 doors & 4 windows

= 2 (2.5×1.2) + 4 (1.5×1)

$\Rightarrow$ 6+6 = 12 m²

Area of roof and 4 walls

= 2H (L + B)+ LB

= 2×7 (12.5+9) +12.5×9

= 14×21.5 + 112.5

= 413.5 m²

Area of painting = 413.5 –12

= 401.5m²

Total Expense of painting

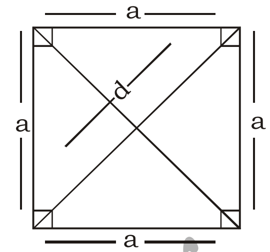
= 401.5 ×3.5

= ₹ 1405.25

Square

A square is a four-Sided flat shape where every angle is 90° and all the four sides are equal also the diagonals are equal and bisect each other at 90°.

* Every Square is a Rhombus but every Rhombus is not a square.



1. **Area** = a² = (side)²

= $\frac{1}{2}$ × (diagonal)²

2. **Perimeter** = 4a

3. **Diagonal** (d) = $\sqrt{2}$ a

4. **Area of Path Inside Square**

= 4d (x–d)

$\therefore$ d = length of Path

x = length of Square

5. **Area of Path outside Square**

= 4d (x+d)

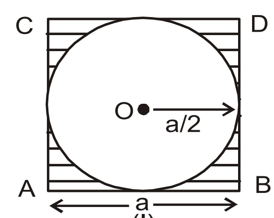
6. **Area of Path midway Square**

= d (2x–d)

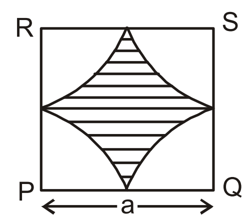
7. **In circle Radius** = $\frac{\text{Side}}{2}$

8. **Circumcircle Radius** = $\frac{\text{Side}}{\sqrt{2}}$

Some-useful Results :



(I)



(ii)

9. In figure (i) ABCD is a square of side 'a'

(a) O is the centre of the incircle.

In figure (ii) PQRS is a square of side 'a'

(b) P,Q,R and S are the centres of four quad -rant of radius a/2 each.

In both case– Area of shaded region $= \frac{3}{14}a^2$

10. If the additional of square increases by x times, then the area of the square becomes x^2 times.

11. If the area of the square is a cm^2, then the area of the circle formed by the same perimeter is = $\frac{4a}{\pi}cm^2$.

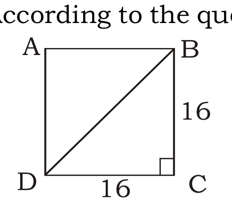
EXAMPLES

1. A square field has an arm of length 125 cm. Find the area and Perimeter of a square?

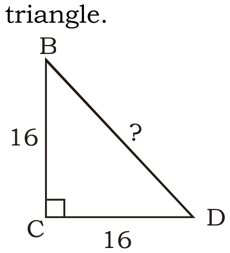
Sol. Side (a) = 125
Area = $(side)^2 = (125)^2$
= 15625 cm^2
Perimeter = 4×side
= 4×125 = 500 cm.

2. A square park has a side of 16 cm. A person cross it across diagonally. Find the distance he covered?

Sol. According to the question,



Here Δ BCD is an right angle triangle.



In Square diagonal is Angle Bisector.
$\angle D = \angle B = 45^\circ$

$\sin D = \frac{P}{H} = \sin 45^\circ$

$= \frac{16}{x} = \frac{1}{\sqrt{2}} = \frac{16}{x}$

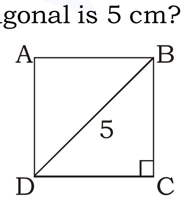
$\Rightarrow \sqrt{2} \times 16 = 16\sqrt{2}$ cm

Alterate:-

$d = \sqrt{2} \times a = 16\sqrt{2}$

3. Find the area of a Square whose diagonal is 5 cm?

Sol.



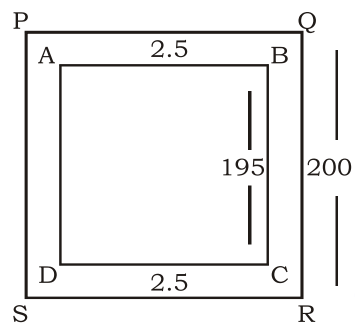
BD = 5 cm

Area of Square = $\frac{1}{2}\times(\text{diagonal})^2$

$= \frac{1}{2} \times (5)^2 = 12.5 \text{ cm}^2$

4. A square plot is 200 m long. It has a path 2.5 m wide all round it inside. Find the area of path?

Sol.



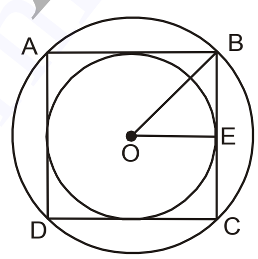
Area of Path Inside = Area of Square PQRS – Area of Square ABCD
$\Rightarrow (200)^2 - (195)^2$
$\Rightarrow 40,000 - 38025 \Rightarrow 1975 \text{ m}^2$

Alternate:-

Area of path inside = $4d(x - d)$
Square = $4 \times 2.5\,(200 - 2.5)$
= 1975 m^2

5. The length of the side of a square is 14cm. Find out the ratio of the radii of the inscribed and circumscribed circle of the square.

Sol.



Radius of incircle = OE = $\frac{AB}{2}$

= 7cm

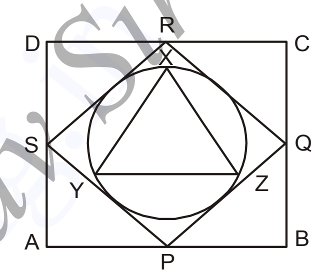
Radius of circum-circle = OB

$= \frac{\text{Diagonal BD}}{2}$

$= \frac{\sqrt{2} \times 14}{2} = 7\sqrt{2}cm$

$\therefore \text{Required ratio} = 7 : 7\sqrt{2} = 1 : \sqrt{2}$

6. In the given figure ABCD is a square and PQRS is also a square made by joining the mid-points of the sides of the larger square ABCD. There is inscribed a circle, In $\square$PQRS and an equilateral ΔXYZ inscribed in the circle.
Find the ratio of the side of the square ABCD to the side of the equilateral triangle XYZ.



Sol. Let side of $\square$ABCD = 2a

$\therefore$ side of $\square$PQRS =

$= \sqrt{AP^2 + AS^2} = \sqrt{a^2 + a^2} = \sqrt{2}a$

$\therefore$ radius of circle $= \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

Let side of ΔXYZ = b

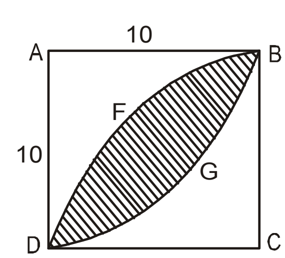
$\therefore$ radius of circumcircle of

$\Delta XYZ = \frac{b}{\sqrt{3}}$

$\therefore \quad \frac{b}{\sqrt{3}} = \frac{a}{\sqrt{2}} \Rightarrow \frac{a}{b} = \frac{\sqrt{2}}{\sqrt{3}}$

$\therefore \quad \frac{2a}{b} = \frac{2\sqrt{2}}{\sqrt{3}}$

7. In the figure, ABCD is a square with side 10 cm. BFD is an arc of a circle with centre C. BGD is and arc of a circle with centre A. What is the area of the shaded region :



Sol. Area of portion DFBGD = Area of portion DFBC – area of $\triangle$ BCD

$\frac{1}{4}\pi r^2 - \frac{1}{2}ab\sin\theta$

$= \frac{1}{4}\pi(10)^2 - \frac{1}{2}\times 10\times 10$

$= 25\pi - 50$

= Area of portion DFBGD

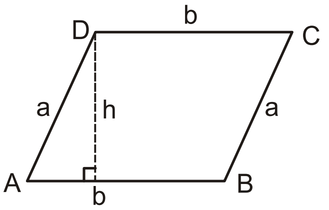
$\therefore$ Area of portion DFBGD

$= (25\pi - 50) + (25\pi - 50)$

$= 50\pi - 100$

Parallelogram

If opposite sides of a quadrilateral are parallel, it is called parallelogram. Its opposite sides are also equal in length.



(i) **Area** = base $\times$ height = bh

(ii) **Perimeter** = 2(a + b)

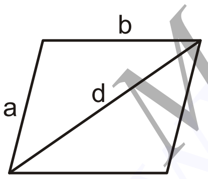
(iii) $d_1^2 + d_2^2 = 2(a^2 + b^2)$ (d_1, d_2 = length of diagonals)

(iv) Area = A

$= 2\sqrt{s(s-a)(s-b)(s-d)}$

where a & b are adjacent sides, d is the length of diagonal connecting the ends of the two sides and,

$$s = \frac{a+b+d}{2}$$



EXAMPLES

1. The area of a Parallelogram 486 cm². If its altitude is 66.66% of its base then find the base and altitude?

Sol. $66.66\% = 66\frac{2}{3}\% = \frac{2}{3}$,

$\therefore$ Let Base = x ; Altitude = $\frac{2}{3}x$

Area of Parallelogram = Base × Height

$486 = x \times \frac{2}{3}x$

$x^2 = \frac{486\times 3}{2} = 729$

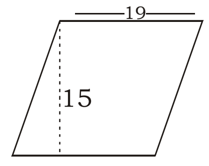
$x = 27$

then Base = 27 cm

Altitude = $27 \times \frac{2}{3}$ = 18 cm

2. One side of a Parallelogram is 19 cm. Its distance from the opposite side is 15 cm. Then area of the parallelogram will be:

Sol.



Area of Parallelogram = Base × Height

$\Rightarrow 19\times 15 \Rightarrow 285$ cm²

3. In a parallelogram, the lengths of adjacent sides are 12 cm and 14 cm respectively. If the length one diagonal is 16 cm, find the length of other diagonal?

Sol. $d_1^2 + d_2^2 = 2(a^2+b^2)$

$d_1^2 + (16)^2 = 2(12^2+14^2)$

$d_1^2 + 256 = 2(144+196)$

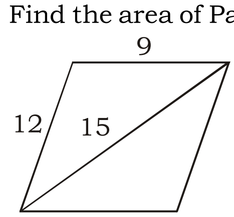
$d_1^2 + 256 = 2\times 340 = 680$

$d_1^2 = 680 - 256 = 424$

$d_1 = 20.6$ cm

4. The two adjacent sides of a parallelogram are 12 and 9 cm and the length of diagonal is 15 cm. Find the area of Parallelogram?

Sol.



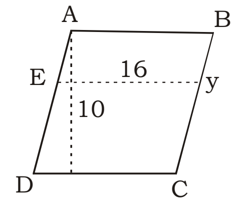
$S = \frac{12+9+15}{2} = 18$

Area = $2\sqrt{18\times 6\times 9\times 3} \Rightarrow 2\times 54$

= 108 cm²

5. A parallelogram has an area of 160 cm². If the distance between its opposite sides are 10 cm and 16 cm. Find the sides of the Parallelogram.

Sol.



Area of Parallelogram

ABCD = B×H

$160 = x \times 10$

x = 16 cm

Area of Parallelogram ABCD = B×H

$160 = y \times 16$

y = 10 cm

Length of the parallelogram = 16 cm

Breadth of the Paralleogram = 10 cm

6. In a ||gm, the lengths of adjacent sides are 11 and 13cm. If the length of one diagonal is 16cm, find the length of other diagonal.

Sol. In a ||gm, $d_1^2 + d_2^2 = 2(a^2 + b^2)$

$\Rightarrow (16)^2 + d_2^2 = \left[(11)^2 + (13)^2\right]\times 2$

$\Rightarrow d_2^2 = 2(290) - 256 \Rightarrow d^2 = 324$

$\Rightarrow$ d = 18cm

7. Sides of a parallelogram are in the ratio 5 : 4. Its area is 1000sq.units. Altitude on the greater side is 20units, Altittude on the smaller side is:

Sol. Let the side of parallelogram be $5x$ and $4x$

Area of parallelogram = Base $\times$ height

$1000 = 5x \times 20$

$\Rightarrow x = \frac{1000}{5\times 20} = 10$

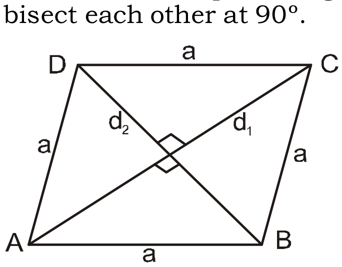
$\therefore$ Sides = 50 and 40units

$\therefore$ $40\times$ h = 1000

$\Rightarrow$ h = $\frac{1000}{40}$ = 25 units

Rhombus

It is a Quadrilateral whose all four sides are equal. Diagonal bisect each other at 90°.



1. Diagonals bisect each other at 90°

2. $A = \frac{1}{2}(d_1 \times d_2)$

3. $a = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

4. P = 4a

5. $d_1^2 + d_2^2 = 4a^2$

EXAMPLES

1. The area of Rhombus is 24 cm² has one of its diagonal is 6cm. Find the other diagonal?

Sol. Area = $\frac{1}{2} \times d_1 \times d_2$

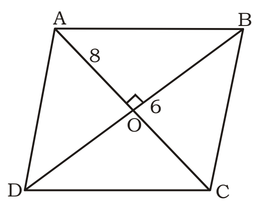
$24 = \frac{1}{2} \times 6 \times x$

$\frac{24 \times 2}{6} = x$

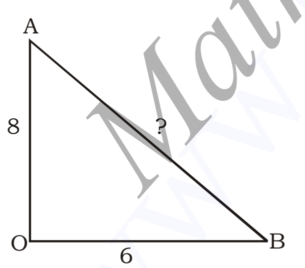
$x = 8$cm

2. Find the side of a Rhombus whose diagonals are12 cm and 16 cm.

Sol. As diagonals of Rhombus bisect each other at 90^0



In Δ AOB,



$(AB)^2 = (AO)^2 + (OB)^2$

$(AB)^2 = 64 + 36$

AB = 10 cm

Alternate:-

Side, $a = \frac{1}{2}\sqrt{d_1^2 + d_2^2} = \frac{1}{2}\sqrt{16^2 + 12^2}$

$\Rightarrow \frac{1}{2} \times 20 = 10$ cm

3. In a Rhombus, the length of two diagonal are 120 cm & 90 cm respectively, find its perimeter.

Sol. $D_1 = 120$ cm

$D_2 = 90$ cm

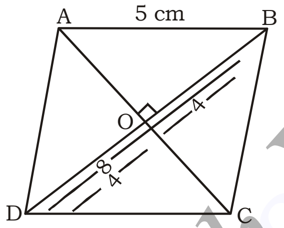
Side (a) = $\frac{1}{2}\sqrt{D_1^2 + D_2^2} = \frac{1}{2}\sqrt{(120)^2 + (90)^2}$

$= \frac{1}{2} \times 150 = 75$ cm

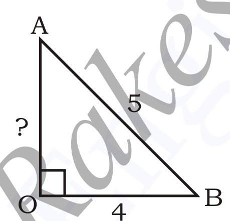
Perimeter = 4 × 75 = 300 cm

4. One of the diagonals of a Rhombus of side 5 cm measures 8 cm. Find the area of the Rhombus.

Sol.



In Δ AOB



By phythagoras theorem,

$(AB)^2 = (AO)^2 + (OB)^2$

$25 = (AO)^2 + 16$

$AO^2 = 9$

AO = 3cm

AS AO = 3cm, then AC = 2 × AO

= 2 × 3 = 6 cm

$D_1 = 8$ cm, $D_2 = 6$ cm

Area = $\frac{1}{2} \times D_1 \times D_2$

$= \frac{1}{2} \times 8 \times 6 = 24cm^2$

Alternate : $d_1^2 + d_2^2 = 4a^2$

$8^2 + d_2^2 = 4 \times 5^2$

$d_2^2 = 36$

$d_2 = 6$

area = $\frac{1}{2} d_1 \times d_2$

$= \frac{1}{2} \times 8 \times 6 = 24$ cm²

5. Area of Rhombus is 256 cm². One of the diagonal is half of the other . Find the perimeter and sum of the diagonals?

Sol. Let the one of the diagonals is $(d_1) = x$

other diagonal (d_2) is $= 2x$

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

$256 = \frac{1}{2} \times x \times 2x$

$x^2 = 256$

$x = 16$

$d_1 = 16$ cm, $d_2 = 32$cm

Sum of the diagonals = (16 + 32) = 48 cm

Perimeter = $2\left(\sqrt{d_1^2 + d_2^2}\right)$

$= 2\left(\sqrt{256 + 1024}\right)$

$= 2\sqrt{1280} = 71.55$ cm.

6. Perimeter of a Rhombus is 2p unit and sum of length of diagonal is m unit, then area of the Rhombus is

Sol. Side of Rhombus = $\frac{2P}{4} = \frac{P}{2}$

$d_1 + d_2 = m$ (Given)

$\left(\frac{P}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$

$\frac{P^2}{4} = \frac{d_1^2}{4} + \frac{d_2^2}{4}$

$\mathbf{P^2 = d_1^2 + d_2^2}$

$\therefore (d_1 + d_2)^2 = d_1^2 + d_2^2 + 2d_1 d_2$

$\Rightarrow m^2 = p^2 + 2 d_1 d_2$

$\Rightarrow d_1 d_2 = \frac{m^2 - p^2}{2}$

Area of Rhombus = $\frac{1}{2} d_1 d_2$

$= \frac{m^2 - p^2}{4}$

7. The perimeter of a rhombus is 146 cm and one of its diagonals is 55cm. The other diagonal is :

Sol. Perimeter $= 4 \times Side$

$= 4 \times \frac{1}{2} \cdot \sqrt{d_1^2 + d_2^2} = 2\sqrt{d_1^2 + d_2^2}$

$\Rightarrow 146 = 2\sqrt{55^2 + d_2^{\,2}}$

$\Rightarrow 73 = \sqrt{55^2 + d_2^{\,2}}$

$\Rightarrow 73^2 = 55^2 + d_2^{\,2}$

$\Rightarrow 73^2 - 55^2 = d_2^{\,2}$

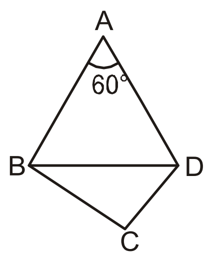
$\Rightarrow d_2^{\,2} = (73+55)\ (73-55)$

$= 128 \times 18$

$\Rightarrow d_2 = 48cm$

8. The perimeter of a rhombus is 40cm and the measure of an angle is 60°, then the area of it is:

Sol.



Side $= \dfrac{40}{4} = 10cm$

AB = AD = 10cm

$\angle$ABD = $\angle$ADB = 60°

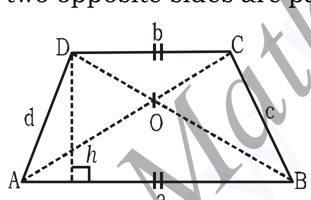
$\therefore$ Area of the rhombus

$= 2 \times \dfrac{\sqrt{3}}{4} \times (AB)^2$

$= 2 \times \dfrac{\sqrt{3}}{4} \times 10 \times 10 = 50\sqrt{3}cm^2$

Trapezium

It is a quadrilateral, whose any two opposite sides are parallel.



(i) **Perimeter** = a + b + c + d

(ii) **Area** = $\frac{1}{2}$(Sum of Parallel sides) × Distance b/w them

$= \frac{1}{2}$ (a + b) × h

(iii) $\mathbf{d_1^2 + d_2^2}$ = $c^2 + d^2 + 2(ab)$

(sum of squares of non-parallel sides) + 2 (product of parallel sides)

(iv) **If diagonals intersect at O, than**

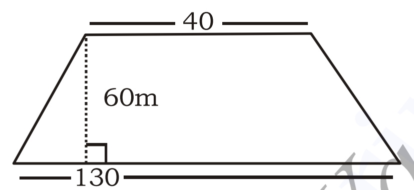
$\dfrac{AO}{OC} = \dfrac{BO}{OD} = \dfrac{AB}{DC}$

$[\because \Delta AOB \sim \Delta COD]$

EXAMPLES

1. A trapezium has the perpendicular distance between the two parallel sides 60m. If the lengths of the parallel sides be 40m and 130 m, than find the area of the trapezium.

Sol.



Area = $\dfrac{1}{2}(a+b) \times h$

$= \dfrac{1}{2}(130+40)60$ = 85 × 60

= 5100 m²

2. The cross section of a canal is a trapezium in shape. If the canal is 10 m wide at the top and 6 m wide at the bottom and the area of cross section is 640m². Find the length of canal?

Sol. Let the length of a canal = h m

Area of trapezium canal

$= \dfrac{1}{2}$(a+b)×h

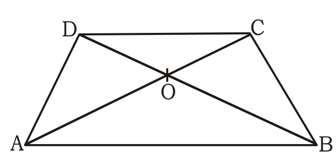
$640 = \dfrac{1}{2}(10+6)h$

1280 = 16 h

h = 80m.

3. ABCD is a trapezium with AB||CD whose diagonal meet at O. If AB = 2 CD and length of CD = 2.5 cm. Find the area of Δ AOB and Δ COD ?

Sol.



AB = 5 cm

CD = 2.5 cm

$\dfrac{\text{Area of }\Delta AOB}{\text{Area of }\Delta COD} = \dfrac{(AB)^2}{CD^2} = \dfrac{(2CD)^2}{CD^2} = \dfrac{4CD^2}{CD^2}$

$\Rightarrow \dfrac{4 \times 6.25}{6.25} = \dfrac{25}{6.25}$

Area of Δ AOB = 25 cm²

Area of Δ COD = 6.25 cm²

4. A wall is the form of a trapezium with height 8 m and parallel sides being 6 m and 10 m. What is the cost of painting the wall, if the rate of painting is Rs. 25 per sq. m?

Sol. Area of trapezium = $\dfrac{1}{2}(a+b) \times h$

$= \dfrac{1}{2}$(6+10)×8 = 64m²

Total cost of painting

= Rs 25 × 64 = Rs. 1600

5. Area of the trapezium formed by x-axis; y-axis and the lines $3x + 4y = 12$ and $6x + 8y = 60$ is:

Sol. For $3x + 4y = 12$

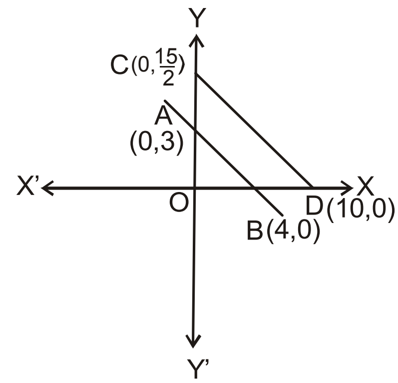
By putting $x = 0$, $y = 3$

By puttting, $y = 0$, $x = 4$

For $6x + 8y = 60$,

By putting $x = 0$, $y = \dfrac{15}{2}$

By putting $y = 0$, $x = 10$



$\therefore$ Area of Δ OCD $= \dfrac{1}{2} \times OD \times OC$

$= \dfrac{1}{2} \times 10 \times \dfrac{15}{2} = \dfrac{75}{2}$

$\therefore$ Area of ΔOAB

$= \dfrac{1}{2} \times OB \times OA = \dfrac{1}{2} \times 4 \times 3 = 6$

$\therefore$ Area of trapezium $= \dfrac{75}{2} - 6$

$= \dfrac{75-12}{2} = \dfrac{63}{2}$

$= 31.5 sq.units$

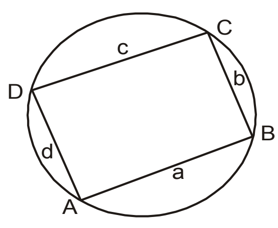
6. Find the distance between the two parallel sides of a trapezium if the area of the trapezium is 250sq.m and the two parallel sides are equal to 15m and 10m respectively.

Sol. Area= $\frac{1}{2}$ (sum of parallel sides) × (distance between them)

$\Rightarrow\ 250 = \frac{1}{2}(15+10)\times h$

$\Rightarrow$ h = 20m

Cyclic Quadrilateral: A quadrilateral whose vertices lie on the circumference of the circle.



(i) **Area (A)**

$= \sqrt{s(s-a)(s-b)(s-c)(s-d)}$

where, s $= \dfrac{a+b+c+d}{2}$

(ii) $\angle A + \angle B + \angle C + \angle D = 2\pi$

(iii) $\angle A + \angle C = \angle B + \angle D = 180° = \pi$

Triangle

A triangle is a polygon with three edges and three vertices. It is one of the basic shapes in geometry. It denoted by ΔABC.

Type of triangles:-

(1) **Scalene Triangle:-**

A scalene triangle has all its sides of different lengths. Equivalently, it has all angles of different

(i) Area = $\frac{1}{2}$×Base×height

(ii) Area = $\sqrt{s(s-a)(s-b)(s-c)}$

(Hero's Formula)

(iii) Semi-perimeter (S)

$= \dfrac{(a+b+c)}{2}$

(iv) If lengths of three medians of Δ ABC are x, y and z units, then :

$A = \frac{4}{3}\sqrt{Sm(Sm-x)(Sm-y)(Sm-z)}$

where, $Sm = \dfrac{x+y+z}{2}$

(v) **In circle Radius**

$= \dfrac{\text{Area of }\Delta}{\text{Semi–Perimeter}}$

r $= \dfrac{\Delta}{s}$

(vi) **Circum-circle Radius (R)**

$= \dfrac{abc}{4\times\text{area of }\Delta}$

(a, b, c = length of sides of Triangle)

EXAMPLES

1. The area of a triangle whose sides are 15m, 16m and 17m is :

Sol. $\dfrac{a+b+c}{2} = \dfrac{15+16+17}{2} = 24$m

Aera (A) = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-15)(24-16)(24-17)}$

$= \sqrt{24\times9\times8\times7}$

9 = $24\sqrt{21}\text{m}^2$

2. If the area of a triangle is 250m² and base : height is 4 : 5, find its height :

Sol. Let base = 4x, then height = 5x

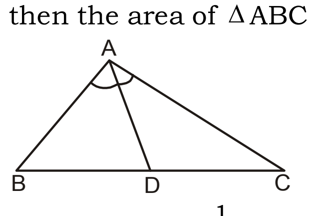
$\therefore\ \frac{1}{2}\times4x\times5x = 250$

$\Rightarrow x^2 = 25$

$\Rightarrow x = 5$

$\therefore$ height = $5x$ = 5 × 5 = 25m

3. In the figure given below, AD bisects $\angle$BAC. If the area of ΔABD = 40cm² and AC = 3AB, then the area of ΔABC :

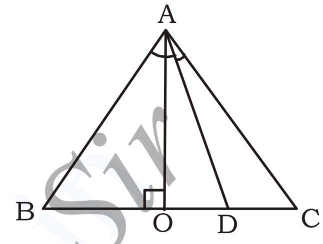


Sol. $\dfrac{\text{Area of }\Delta\text{ ABD}}{\text{Area of }\Delta\text{ ADC}} = \dfrac{\frac{1}{2}\times(BD)\times(OA)}{\frac{1}{2}\times(DC)\times(OA)}$

$\dfrac{BD}{DC} = \dfrac{AB}{AC} = \dfrac{1}{3}$

($\therefore$ AD is the angle bisector of $\angle$BAC)

$\Rightarrow$ Area of ΔADC = 3 × 40 = 120cm²



$\therefore$ Area of ΔABC = 120 + 40 = 160cm²

4. What is the area of a triangle having perimeter 32cm, one side 11cm and difference of other two sides 5cm?

Sol. Let the sides of triangle be a, b and c respectively,

$\therefore$ 2s = a + b + c = 32

$\Rightarrow$ 11 + b + c = 32

$\Rightarrow$ b + c = 32 - 11 = 21 ....(i)

and b - c = 5 ...(ii)

By equations (i) and (ii)

2b = 26 $\Rightarrow$ b = 13

$\therefore$ c = 13 - 5 = 8

$\therefore$ 2s = 32 $\Rightarrow$ s = 16

a = 11, b = 13, c = 8

$\therefore$ Area of triangle

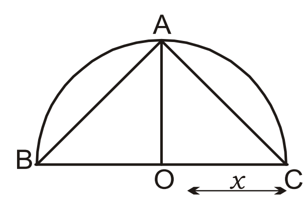
$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-11)(16-13)(16-8)}$

$= \sqrt{16\times5\times3\times8} = 8\sqrt{30}$ sq. cm

5. The area of the largest triangle that can be inscribed in a semi circle of radius x in square unit is:

Sol.



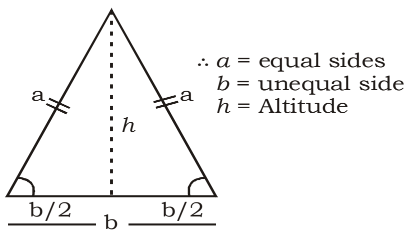
$OA = \frac{1}{2}BC = \text{radius}$

Area of the largest triangle

$= \frac{1}{2}\times BC\times OA = \frac{1}{2}\times2x\times x = x^2$

2. Isosceles Triangle

An Isosceles triangle is a triangle with two equal sides also their opposite angles are equal.

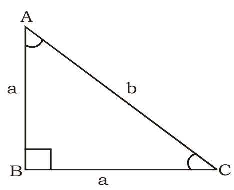


(i) Area = $\frac{b}{4}\sqrt{4a^2-b^2}$

(ii) Perimeter = 2a + b

(iii) *h* (Altitude)= $\frac{\sqrt{4a^2-b^2}}{2}$

If an isosceles triangle is right angle triangle than



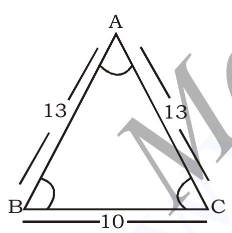
(iv) Area $=\frac{1}{2}a^2$

(v) *h* (hypotenuse) = $a\sqrt{2}$

EXAMPLES

1. The base and the same sides of an isosceles triangle is 10 cm and 13 cm respectively. Find its area and Perimeters ?

Sol.



a = 13 cm
b = 10

Area = $\frac{b}{4}\sqrt{4a^2-b^2}$

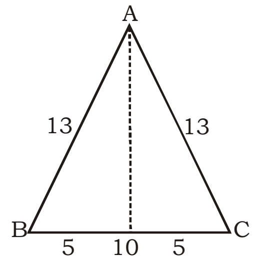
$=\frac{10}{4}\sqrt{4\times(13)^2-(10)^2}$

$=\frac{10}{4}\times 24=60\,cm^2$

Perimeter = 2a + b
= 2×13+10 = 36 cm.

2. Find the altitude in the last question?

Sol.



h (altitude) = $\frac{\sqrt{4a^2-b^2}}{2}$

$=\frac{\sqrt{4\times169-100}}{2}$

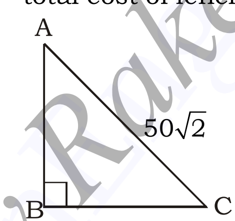
$=\frac{24}{2}$ = 12 cm.

Alternate:-

h = $\sqrt{(13)^2-(5)^2}$ = 12

3. A plot of land is in the shape of a right angled Isosceles triangle. The length of hypotenuse is $50\sqrt{2}$ m. The cost of fencing is Rs. 3 per square meter. Find the total cost of fencing the plot ?

Sol.



$\angle B = 90°$

$\angle A = \angle C = 45°$

In 45° triangle,

$\sqrt{2}$ a = $50\sqrt{2}$

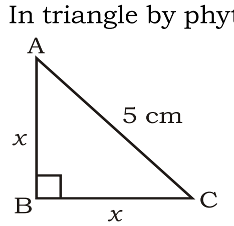
a = 50
AB = 50 cm
BC = 50 cm

Area = $\frac{1}{2}a^2=\frac{1}{2}\times(50)^2$ = 1250 cm²

Total cost of fencing the land
= 1250 × 3 = Rs 3750

4. The hypotenuse of a right angle Isosceles triangle is 5 cm. Its area will be?

Sol. In triangle by phythagoras



∴ $x^2 + x^2 = 5^2$

$2x^2 = 25$

$x^2 = \frac{25}{2}$

Area of triangle

$=\frac{1}{2}\times x^2=\frac{1}{2}\times\frac{25}{2}$ = 6.25 cm²

5. Two sides of a triangular field are 85 metres and 154 metres respectively and its perimeter is 324 metres. Find the cost of leveling the field at the rate of Rs 5 per sq. m.

Sol. P = Sum of all three sides

Third side of triangle
= 324 – (154 + 85)
= 85 metres

Area of the field

$=\sqrt{s(s-a)(s-b)(s-c)}$

$S=\frac{a+b+c}{2}=\frac{p}{2}=\frac{324}{2}=162$

Area = $\sqrt{162\times8\times77\times77}$
= 2772m²

Cost of leveling = 5×2772
= Rs. 13,860

6. The perimeter of an isosceles, right-angled triangle is 2p unit. The area of the same triangle is:

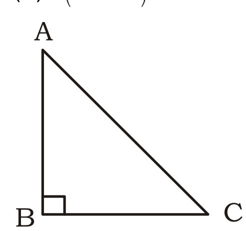
(a) $\left(3-2\sqrt{2}\right)p^2 sq.unit$

(b) $\left(2+\sqrt{2}\right)p^2 sq.unit$

(c) $\left(2-\sqrt{2}\right)p^2 sq.unit$

(d) $\left(3-\sqrt{2}\right)p^2 sq.unit$

Sol.



AB = BC = x

$\therefore AC=\sqrt{AB^2+BC^2}=\sqrt{x^2+x^2}$

$=\sqrt{2}x$ units

$\therefore 2x+\sqrt{2}x=2p$

$\Rightarrow x\left(2+\sqrt{2}\right)=2p$

$\Rightarrow x=\frac{2p}{2+\sqrt{2}}=\frac{2p\left(2-\sqrt{2}\right)}{\left(2+\sqrt{2}\right)\left(2-\sqrt{2}\right)}$

$$=\frac{2\left(2-\sqrt{2}\right)p}{4-2}=\left(2-\sqrt{2}\right)p$$

$$\therefore \text{Area of triangle} = \frac{1}{2}x^2$$

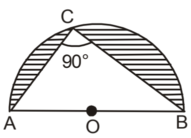
$$=\frac{1}{2}\times\left(2-\sqrt{2}\right)^2 p^2$$

$$=\frac{4+2-4\sqrt{2}}{2}p^2$$

$$=\left(3-2\sqrt{2}\right)p^2 \, sq.units$$

7. A right angled isosceles triangle is inscribed ina semicircle of radius 7cm. The area enclosed by the semi-circle but exterior to the triangle is :

Sol.



$\angle ACB = 90°$

AC = CB = X cm

AB = 14 cm

From $\triangle ABC$

$AC^2 + BC^2 = AB^2$

$\Rightarrow x^2 + x^2 = 14^2$

$\Rightarrow 2x^2 = 14 \times 14$

$\Rightarrow x^2 = 14 \times 7$

$\Rightarrow x = \sqrt{14\times 7} = 7\sqrt{2}cm$

$\therefore \text{Area of } \Delta ABC = \frac{1}{2}\times AC\times BC$

$=\frac{1}{2}\times 7\sqrt{2}\times 7\sqrt{2} = 49 sq.cm$

area of semicircle = $\frac{\pi r^2}{2}$

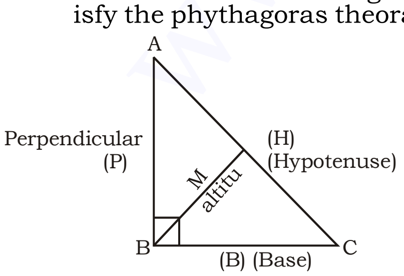
$= \frac{22}{7}\times\frac{7\times 7}{2} = 77 \text{ cm}^2$

Required area

$= 77 - 49 = 28 \text{ cm}^2$

3. Right angle triangle:-

It is a triangle with an angle of 90° $(\pi/2)$ radians. The sides a, b and c of such a triangle satisfy the phythagoras theoram.



(i) Area = $\frac{1}{2}$ × Base × Height

(ii) Perimeter = P + B + H

(iii) Altitude (M) = $\frac{P\times B}{H}$

(iv) $H^2 = P^2 + B^2$

(v) In radius (r) = $\frac{P+B-H}{2}$ **or**

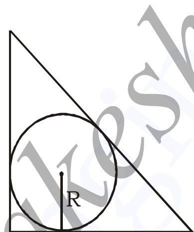
$\frac{PB}{P+B+H}$

(vi) Circum radius (R) = $\frac{H}{2}$

EXAMPLES

1. What is the radius of the incircle of a triangle with sides 18, 24 and 30 cm?

Sol. As we know 18, 24 and 30 are triplets, than the triangle will be right angle triangle .



Incircle Radius

$= \frac{\text{Area of triangle}}{\text{Semi–perimeter}}$

$S = \frac{a+b+c}{2} = \frac{18+24+30}{2} = 36cm$

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{36\times 18\times 12\times 6} = 216$

Incirle Radius $= \frac{\text{Area of }\Delta}{S}$

$= \frac{216}{36} = 6cm.$

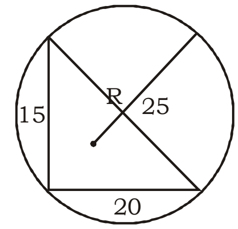
Alternate:-

$r = \frac{P+B-H}{2}$

$= \frac{18+24-30}{2} = 6 \text{ cm}$

2. What is the radius of circle drawn outside the triangle whose length of sides are 15, 20 and 25 cm?

Sol.



Circumcircle Radius

$= \frac{abc}{4\times\text{area of triangle}}$

Area of triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$S = \frac{15+20+25}{2} = 30$

Area of triangle

$= \sqrt{30\times 15\times 10\times 5} = 150 \text{ cm}^2$

Circumcircle Radius

$= \frac{abc}{4\times\text{area of triangle}}$

$= \frac{15\times 20\times 25}{4\times 150} = 12.5 \text{ cm}$

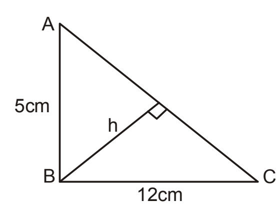
Alternate:-

R $= \frac{\text{Hypotenus}}{2} = \frac{H}{2}$

$= \frac{25}{2} = 12.5 \text{ cm}$

3. The base and altitude of a right angled triangle are 12cm and 5cm respectively. the perpendicular distance of its hypotenuse from the opposite vertex is:

Sol.



$AC = \sqrt{(12)^2 + (5)^2} = 13cm$

Area of Δ ABC $= \frac{1}{2}\times 12\times 5$

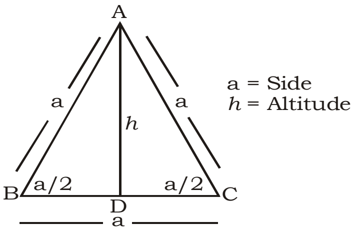
$= 30 \text{cm}^2$

also area of Δ ABC

$= \frac{1}{2}\times (AC)\times h = \frac{13}{2}h \text{ cm}^2$

$\therefore \frac{13}{2}h = 30 \Rightarrow h = \frac{60}{13} = 4\frac{8}{13}cm$

4. Equilateral triangle:-
It is a triangle whose all sides and angle are equal.



(i) Area = $\frac{\sqrt{3}}{4}a^2$

(ii) Perimeter (2s) = 3a

(iii) Altitude (h) = $\frac{\sqrt{3}}{2}a$

(iv) Incircle Radius (r) = $\frac{a}{2\sqrt{3}}$

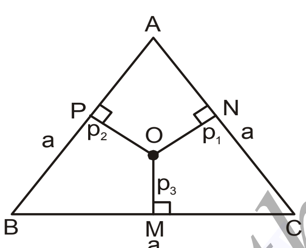
(v) Circumcircle Radius (R) = $\frac{a}{\sqrt{3}}$

We can say "r" = $\frac{R}{2}$

(vi) $\angle A = \angle B = \angle C = 60°$

$$A = \frac{\sqrt{3}}{4}a^2 = \frac{h^2}{\sqrt{3}}$$

(vii) If P_1, P_2 and P_3 are perpendicular lengths from any interior point (O) of an equilateral $\triangle$ ABC to all its three sides respectively, then:-



$$P_1 + P_2 + P_3 = \frac{\sqrt{3}}{2}a = h$$

$$\Rightarrow a = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$$

EXAMPLES

1. The area of an equilateral triangle is $400\sqrt{3}$ sq. m. Its perimeter will be:-

Sol. Area = $\frac{\sqrt{3}}{4}a^2$ = $400\sqrt{3}$

a^2 = $\frac{400\sqrt{3} \times 4}{\sqrt{3}}$ = 1600

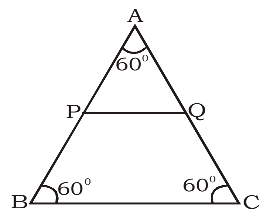
a = 40

Perimeter = 3a

= 3 × 40 = 120 m.

2. ABC is an equilateral triangle. P and Q are two point on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{PQ} \parallel \overline{BC}$. If $\overline{PQ}$ = 5cm then the area of $\triangle$ APQ ?

Sol.



$\therefore \overline{PQ} \parallel \overline{BC}$

$\angle APQ = \angle ABC = 60°$
(corresponding anlge)

$\angle AQP = \angle ACB = 60°$
(corresponding anlge)

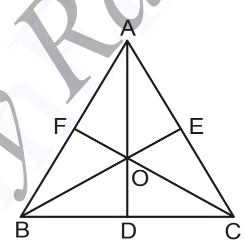
$\triangle$ APQ is an equilateral triangle

Area of $\triangle$ APQ = $\frac{\sqrt{3}}{2}a^2$

$= \frac{\sqrt{3}}{4} \times 25 = \frac{25\sqrt{3}}{4}$ cm

3. If the circumradius of an equilateral triangle be 10cm, then the measure of its in-radius is:

Sol.



Let AB = x cm

$$\therefore BD = \frac{X}{2}$$

$$AD = \sqrt{x^2 - \frac{x^2}{4}} = \frac{\sqrt{3}}{2}x\ cm.$$

$$\therefore OD = \frac{1}{3} \times \frac{\sqrt{3}}{2}x = \frac{x}{2\sqrt{3}}cm.$$

$$OB = \sqrt{BD^2 + OD^2}$$

$$= \sqrt{\frac{x^2}{4} + \frac{x^2}{12}} = \sqrt{\frac{4x^2}{12}} = \frac{x}{\sqrt{3}}cm.$$

$$\therefore \frac{x}{\sqrt{3}} = 10 \Rightarrow x = 10\sqrt{3}cm.$$

$$\therefore OD = \frac{x}{2\sqrt{3}} = \frac{10\sqrt{3}}{2\sqrt{3}} = 5cm.$$

Alternatively:-

Circumradius(R) = $\frac{a}{\sqrt{3}}$

$\Rightarrow$ 10 = $\frac{a}{\sqrt{3}}$

$\Rightarrow$ a = $10\sqrt{3}$

$\therefore$ Inradius = $\frac{a}{2\sqrt{3}}$

= $\frac{10\sqrt{3}}{2\sqrt{3}}$ = 5 cm

Alternate:-

r = $\frac{R}{2}$ = $\frac{10}{2}$ = 5

4. If the area of square is $3\sqrt{3}$ times the area of an equilateral triangle, then the ratio of the sides of the square to the side of the equilateral triangle is equal to :

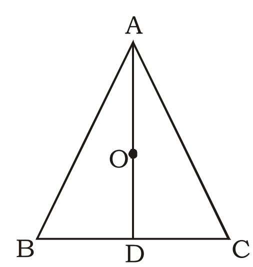
Sol. Let the side of square = x and the side of equilateral triangle = y

$$\therefore x^2 = 3\sqrt{3} \times \left(\frac{\sqrt{3}}{4}y^2\right) \Rightarrow x^2 = \frac{9}{4}y^2$$

$$\Rightarrow \frac{x}{y} = \frac{3}{2} \Rightarrow x : y = 3 : 2$$

5. The in-radius of an equilateral triangle is of length 3 cm. Then the length of each medians is.

Sol.



AO : OD = 2:1

AO = Circum radius

OD = Inradius = 3 cm

AD = 2 + 1 = 3

AD = 3 × 3 = 9 cm

6. The circum-circle radius of an equilateral triangle is 8 cm. The inradius of the triangle is-

Sol. Circum circle radius = $\frac{a}{\sqrt{3}}$

$8 = \frac{a}{\sqrt{3}} \Rightarrow a = 8\sqrt{3}$

In radius = $\frac{a}{2\sqrt{3}} = \frac{8\sqrt{3}}{2\sqrt{3}}$ = 4cm.

Alternate:-

r = $\frac{R}{2} = \frac{8}{2}$ = 4 cm

7. The area of an equilateral triangle inscribed in a circle is $9\sqrt{3}cm^2$. The area of the circle is :

Sol. Area of equilateral Δ

$= \frac{\sqrt{3}}{4}(\text{side})^2 = 9\sqrt{3}$

$\Rightarrow$ side = 6 cm

$\therefore$ circum-radius of equilateral Δ

$= \frac{side}{\sqrt{3}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ So, Area of circle = $\pi \times (2\sqrt{3})^2 = 12\pi\ cm^2$

8. The area of an equilateral triangle is $4\sqrt{3}$ cm². the length of each side of triangle is

Sol. Area = $\frac{\sqrt{3}}{4}a^2 \Rightarrow 4\sqrt{3} = \frac{\sqrt{3}}{4}a^2$

$a^2 = \frac{16\sqrt{3}}{\sqrt{3}} = 16 \Rightarrow a = 4$ cm

Each side of triangle is 4 cm.

9. From a point in the interior of an equilateral triangle, the perpendicular distance of the sides are $\sqrt{3}m, 2\sqrt{3}m$ and $5\sqrt{3}$ m respectively. The perimeter of the triangle is :

Sol. $P_1 = \sqrt{3}$,

$P_2 = 2\sqrt{3},\ P_3 = 5\sqrt{3}$

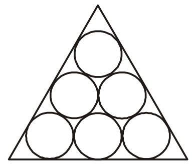
$\therefore \quad P = \frac{\sqrt{3}}{2}a$

$= P_1 + P_2 + P_3 = 8\sqrt{3}$

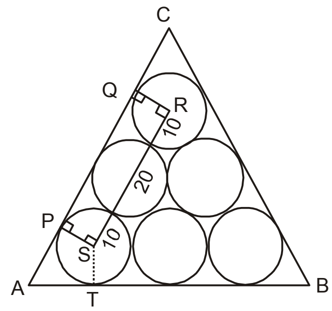
$\Rightarrow$ a = 16

$\therefore$ Perimeter = 3a = 48m

10. An equilateral triangle circumscribes all the cricles, each with radius 10cm. What is the perimeter of the equilatral triangle?



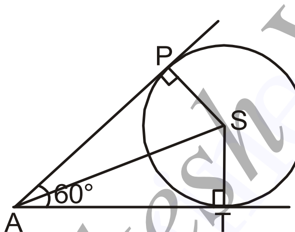
Sol.



PQRS is a rectangle.

$\therefore$ PQ = 10 + 20 + 10 = 40cm

for AP,



$\angle A = 60°,\ \angle PST = 120°$

$\therefore \angle PSA = \angle AST = \frac{120°}{2} = 60°$

$and\ \angle PAS = \angle SAT = 30°$

$\therefore$ In Δ PSA , tan 30°

$= \frac{PS}{AP} = \frac{10}{AP}$

$\Rightarrow$ AP = $\frac{10}{\tan 30°}$

$\Rightarrow$ AP = $10\sqrt{3}$

Similarly;

QC = $10\sqrt{3}$

$\therefore$ AC = PQ + AP + QC

= 40 + $10\sqrt{3}$ + $10\sqrt{3}$

= $20(2+\sqrt{3})$cm

$\therefore$ AB = BC = AC

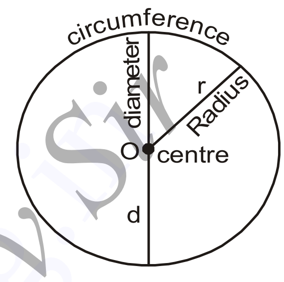
= $20(2+\sqrt{3})$cm

$\therefore$ Perimeter of ΔABC

= $60(2+\sqrt{3})$cm

Circle

A circle is a set of points on a plane which lie at a fixed distance from a fixed-point. The fixed point is known as 'centre' and the fixed disatance is called the 'radius'.



(i) Circumference or Perimeter of circle

(P) = $2\pi r = \pi d$ (d = diameter)

(ii) Area = A = $\pi r^2 = \frac{\pi d^2}{4}$

$\therefore$ Diameter of the circle = d

$= \sqrt{\frac{4A}{\pi}}$

EXAMPLES

1. The radii of two circle 7 cm and 24 cm. the area of third circle is equal to the sum of the area of the two circles. The radius of the third circle is.

Sol. Area of the C_1 = πr_1^2 = 49π cm²

Area of the C_2 = πr_2^2 = 576π cm²

Area of C_3 = Area of C_1 + Area of C_2

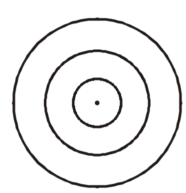
$\pi r_3^2 = 49\pi + 576\pi = 625\pi$

$r_3^2 = 625$

r_3 = 25 cm.

2. The area of circle whose radius is 6 cm is trisected by two concentric circles. The radius of the smalest circle is

Sol.



Area of C_1 = πr^2

= $\pi (6)^2 = 36\pi$

After Trisected, the area of

Smallest circle = $\frac{1}{3}\times 36\pi = 12\pi$

Area of smallest circle,

$12\pi = \pi r^2$

$r^2 = 12$

$r = 2\sqrt{3}$ cm

3. The area of circle is increased by 22 cm² when its radius is increased by 1 cm. The original radius of the circle is.

Sol. $\pi(r+1)^2 - \pi r^2 = 22$

$\pi(r^2 + 1 + 2r - r^2) = 22$

$\pi(2r + 1) = 22$

$\frac{22}{7}(2r+1) = 22$

$2r = 7 - 1$

$r = 3$ cm

4. The area of a circle is halved when its radius is decreased by n. Find its radius :

Sol. By the question, we have

$\pi r^2 - \pi(r-n)^2 = \frac{\pi r^2}{2}$

$\Rightarrow\ -(r-n)^2 = \frac{-r^2}{2}$

$\Rightarrow\ r^2 - \left(\sqrt{2}(r-n)\right)^2 = 0$

$\Rightarrow\ \left[r-\sqrt{2}(r-n)\right]\left[r+\sqrt{2}(r-n)\right] = 0$

$\Rightarrow\ r = \sqrt{2}(r-n)$ or $r = -\sqrt{2}(r-n)$

$\Rightarrow\ r = \sqrt{2}r - \sqrt{2}n$

$\Rightarrow\ r = \frac{\sqrt{2}n}{\sqrt{2}-1}$

5. The area of a circular field is equal to the area of a rectangular field. The ratio of the length and the breadth of the rectangular field is 14 : 11 respectively and perimeter is 100 meters. What is the diameter of circular field ?

Sol. Let length = $14x$, then breadth = $11x$

$\therefore\ 2(14x + 11x) = 100$

$\Rightarrow\ 50x = 100 \Rightarrow x = 2$

$\therefore$ Area of rectangular field $= 28 \times 22\text{m}^2$

$\therefore$ Area of circular field $= 28\times 22\text{m}^2$

$\Rightarrow\ \pi r^2 = 28\times 22\text{m}^2$

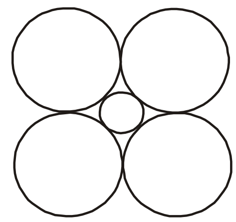
$r^2 = \frac{28\times 22}{22}\times 7$

$= 28\times 7 = 7\times 7\times 4$

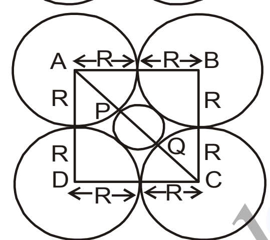
$\Rightarrow\ r = 7\times 2 = 14$m

$\therefore\ d = 2r = 28$m

6. In the given figure, when the outer circles all have radii 'R' then the radius of the inner circle will be :



Sol.



let radius of inner circle = r

A, B, C, D are the centres of the four outer circles

$\therefore$ ABCD is a square of side 2R

$\therefore\ AC = \sqrt{2}(side) = \sqrt{2}(2R) = 2\sqrt{2}R$

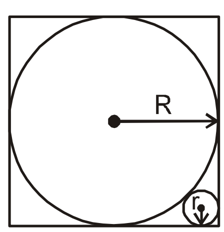
$\therefore$ PQ = AC - AP - QC

$= 2\sqrt{2}R - R - R = 2R(\sqrt{2}-1)$

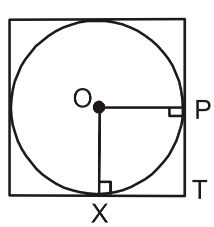
$\Rightarrow 2r = 2R(\sqrt{2}-1)$

$\Rightarrow r = R(\sqrt{2}-1)$

7. In the given figure, find the radius of smaller circle (r) :



Sol.



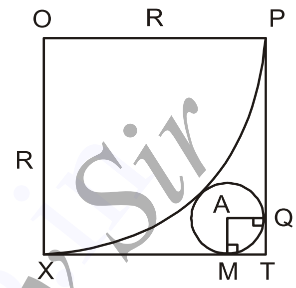
in □OPTX, $\angle P = \angle X = \angle T = 90°$

$\therefore\ \angle O = 90°$ and OP = OX = R

$\Rightarrow$ □OPTX, is a square of side R

$\therefore\ OT = \sqrt{2}R$

Similarly, AQTM is a square of side r.



$\therefore\ AT = \sqrt{2}\,r$

$\therefore\ OT = OA + AT$

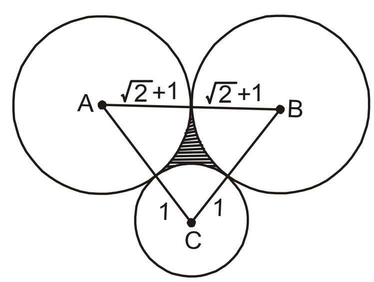
$\Rightarrow\ \sqrt{2}R = (R+r)+\sqrt{2}\,r$

$\Rightarrow\ \left(\sqrt{2}-1\right)R = \left(1+\sqrt{2}\right)r$

$\Rightarrow\ r = \left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)R = \left(3-2\sqrt{2}\right)R$

8. Three circles of radius $\sqrt{2}+1, \sqrt{2}+1$ and 1unit, touch each other externally, then find the perimeter of the surrounded part by three circles.

Sol.



$AB = 2\left(\sqrt{2}+1\right)$ and AC = BC

$= 2 + \sqrt{2}$

$\therefore\ AB^2 = 4\left(\sqrt{2}+1\right)^2$

and $AC^2 = BC^2 = \left(2+\sqrt{2}\right)^2$

So, it is clear

$AB^2 = AC^2 + BC^2$

i.e. ABC is an isosceles right angled triangle.

$\therefore\ \angle ACB = 90°$ and $\angle CAB = \angle ABC = 45°$

$\therefore$ required perimeter

$$= 2\pi\left(\sqrt{2}+1\right)\frac{45°}{360°}$$

$$+\ 2\pi\left(\sqrt{2}+1\right)\frac{45°}{360°}\ +$$

$$2\pi\times1\times\frac{90°}{360°}$$

$$= 2\times2\pi\left(\sqrt{2}+1\right)\frac{1}{8}+2\pi\frac{1}{4}$$

$$= \frac{2\pi}{4}\left(\sqrt{2}+1+1\right)$$

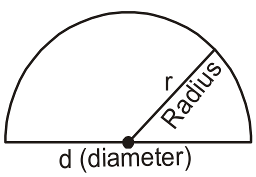
$$= \frac{\pi}{2}\left(\sqrt{2}+2\right)$$

Semi Circle

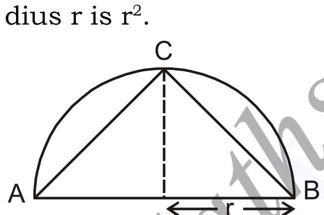
It is a figure enclosed by a diameter and the part of the circumference cut off by it.

(i) Circumference (Perimeter) $= \pi r + 2r \quad = \pi r + d$

(ii) Area(A)= $\dfrac{\pi r^2}{2}$.



(iii) The area of largest triangle inscribed in a semi-circle of radius r is r^2.



Area of the ΔACB = r^2

$$\left[\frac{1}{2}\times r\times 2r\right]$$

EXAMPLES

1. If the perimeter of a Semi-circular field 36m. Find its radius?

Sol. Circumference = $\pi r + 2r$

π r + 2r = 36

r (π +2) = 36

$$r\left(\frac{22}{7}+2\right) = 36$$

$$r = \frac{36}{36}\times 7 = 7 \text{ cm.}$$

2. The perimeter of a Semi-circle is numerically equal to its area. The lenth of the diameter is:-

Sol. π r + 2r = $\frac{1}{2}\pi r^2$

r (π + 2) = $\frac{1}{2}\pi r^2$

2 π + 4 = π r

$$r = \frac{4}{\pi}+2$$

$$\Rightarrow r\ \frac{4\times7}{22}+2 = \frac{72}{22}$$

$$\text{Diameter} = 2\times\frac{72}{22}$$

$$= 6\frac{6}{11} \text{ meters.}$$

3. A semi-circular shaped window has diameter 98 cm, its perimeter equals:-

Sol. Radius = $\frac{\text{Dia}}{2} = \frac{98}{2}$ = 49 cm

Perimeter = π r + 2r

$= \frac{22}{7}\times49+2\times49$ = 252 cm

4. The perimeter of semi-circular area is 18cm, then the the radius is : (using $\pi = \frac{22}{7}$)

Sol. Perimeter of semi-circular region = 18cm

$\Rightarrow \quad \pi r + 2r = 18$

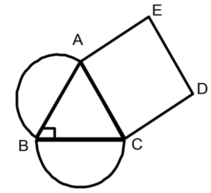
$\Rightarrow \quad r(\pi+2) = 18$

$$\Rightarrow \quad r\left(\frac{22}{7}+2\right) = 18$$

$$\Rightarrow \quad r\left(\frac{36}{7}\right) = 18$$

$$\Rightarrow \quad r = \frac{18\times7}{36} = \frac{7}{2} = 3\frac{1}{2}cm$$

5. The area of the square on AC as a side is 128cm. What is the sum of the areas of semicircles drawn on AB and AC as diameters, given ABC is an isoscles right angled triangle and AC is its hypotenuse.



Sol. Let AB = BC = x,

then $AC = \sqrt{2}\,x$

But AC = $\sqrt{128} = 8\sqrt{2}cm$

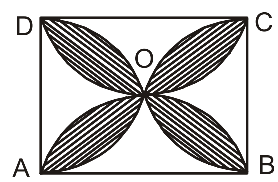
$\sqrt{2}x = 8\sqrt{2} \Rightarrow x$ = 8cm

Area of semicircls

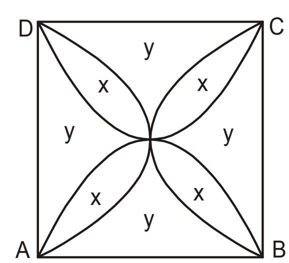
$$= \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

$$= \frac{1}{2}\pi\ [2\times16] = 16\ \pi\ \text{cm}^2$$

6. In the given figure ABCD is a square. Four equal semicircles are drawn in such a way that they meet each other at 'O'. Sides AB, BC, CD and DA are the respective diameters of the four semicircles. Each of the side of the square is 8cm. Find the area of the shaded region.



Sol.



Let area of each shaded portion = x

and area of each unshaded portion = y

total area of square = $(8)^2$

= 64 cm^2

$\therefore \quad 4(x + y) = 64$

$\Rightarrow \quad x + y = 16$ .....(i)

Again in a semicircle,

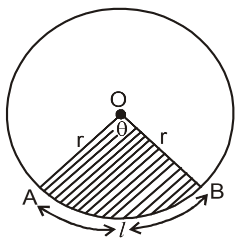
$$\text{AOB} = x + y + x = \frac{1}{2}\pi\times(4)^2$$

$\Rightarrow \quad 2x + y = 8\pi$ ......(ii)

From (i) & (ii) we get.
$x = 8\pi - 16 = 8(\pi - 2)$
∴ Total area of shaded region
$= 32(\pi - 2)cm^2$

Sector

A sector is a figure enclosed by two radii and an arc lying between them.
For sector AOB,



(i) l = Arc AB $= (2\pi r)\dfrac{\theta}{360°}$

(ii) Area of sector ACBO

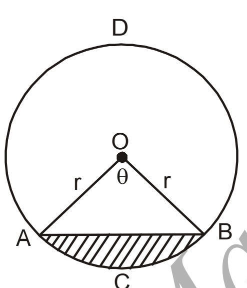
$= \dfrac{1}{2}$(arc AB)× radius

$= (\pi r^2)\dfrac{\theta}{360°}$

(iii) Perimeter (P) = Arc AB + 2r
= l + 2r

Segment of a circle

A figure enclosed by a chord and an arc which it cuts off.



(i) Area of segment ACB. (minor segment) = area of sector ACBO – area of △ OAB.

(ii) Area of segment ADB (major segment) = area of circle – area of segment ACB.

(iii) Perimeter (P) = arc AB + $\theta.r$

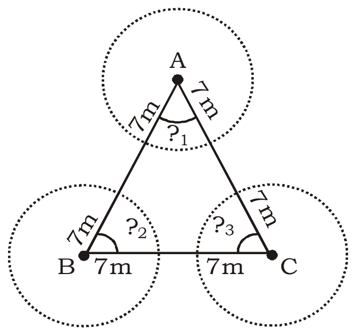
$$= 2r\left[\frac{\pi\theta}{360°} + \sin\left(\frac{\theta}{2}\right)\right]$$

(iv) Arc = Angle × Radius

EXAMPLES

1. At each corner of a triangles field of sides 26m, 28m, and 30m, a cow is tethered by a rope of length 7m. The area ungrazed by the the cows is.

Sol.



Area grazed by all cows

Area of sector = $\dfrac{\pi r^2 \theta}{360°}$

Now, $= \dfrac{\pi r^2 \theta_1}{360°} + \dfrac{\pi r^2 \theta_2}{360°} + \dfrac{\pi r^2 \theta_3}{360°}$

$= \left(\dfrac{\pi r^2}{360°}\right)(\theta_1 + \theta_2 + \theta_3)$

Here, $\theta_1 + \theta_2 + \theta_3 = 180°$
r = 7

$= \dfrac{\pi r^2}{360°} \times 180° = \dfrac{1}{2}\pi r^2$

$= \dfrac{1}{2} \times \dfrac{22}{7} \times 7^2 = 77$ cm

Now, Area of △ ABC,

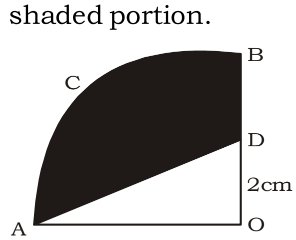
Semi-perimeter = $\dfrac{26 + 28 + 30}{2} = 42$

Area of △ ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42 \times 16 \times 14 \times 12} = 336$

Ungrazed area = 336 – 77
= 259 m²

2. In the adjoining figure, AOBCA represents a quadrant of a circle of radius 4cm with centre O. Calculate the area of the shaded portion.



Sol. Area of shaded region = Area of quadrant – Area of △AOD

$= \dfrac{\pi r^2}{4} - \dfrac{1}{2} \times 4 \times 2$

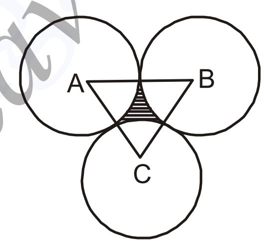
$= \dfrac{\pi \times 4 \times 4}{4} - 4 = 4\pi - 4$

$= 4(\pi - 1)cm^2$

= 4(3.14-1) = 4 × 2.14

= 8.56 cm²

3. Find the area of the shaded region if the radius of each of the circle is 1cm.



Sol. ABC is an equilateral triangle with sides = 2cm

∴ Area of shaded region
= Area of △ABC - Area of 3 quadrants.

$\dfrac{\sqrt{3}}{4}(2)^2 - 3\left(\pi r^2 \dfrac{\theta}{360°}\right),$

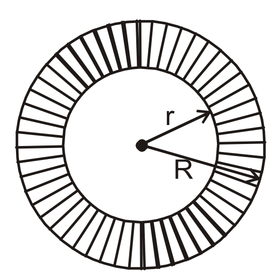
$[\theta = 60° \because \Delta ABC \text{ is an equilateral triangle}]$

$= \dfrac{\sqrt{3}}{4} \times 4 - 3\left(\pi \times 1 \times \dfrac{1}{6}\right)$

$= \sqrt{3} - \dfrac{\pi}{2}$

Ring or Circular Path :

R → outer radius
r → inner radius

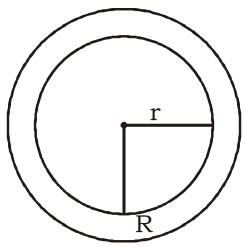


(i) Area (A) = π $(R^2 - r^2)$
(ii) Perimeter = $2\pi(R + r)$

EXAMPLES

1. The area of ring between two concentric circles, whose circumferences are 44 cm and 66 cm.

Sol.



C_1 (Smaller Circle);
C_2 (Bigger Circle)

Circumference of $C_1 = 2\pi r_1$

$2\pi r_1 = 44$

Circumt of $C_2 = 2\pi r_2$

$2\pi r_2 = 66$

$r_1 = \frac{44}{44} \times 7 = 7$ cm

$r_2 = \frac{66}{44} \times 7 = 10.5$ cm

Area of ring between circles

$\Rightarrow \pi\left(R_2^2 - R_1^2\right)$

$\Rightarrow \pi\left((10.5)^2 - (7)^2\right)$

$\Rightarrow \pi\left((10.5+7)(10.5-7)\right)$

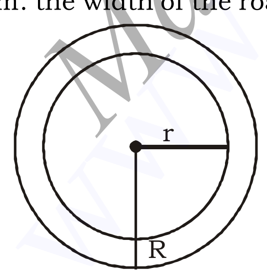
$\Rightarrow \pi(17.5 \times 3.5)$

$\Rightarrow \frac{22}{7} \times 17.5 \times 3.5$

$\Rightarrow$ 192.5 cm²

2. A circular road runs ground a circular ground. If the difference between the circumference of the outer and inner circle is 99m. the width of the road is:-

Sol.



Width of the Road $= 2\pi R - 2\pi r$

$\Rightarrow 2\pi$ (R – r) = 66

$\Rightarrow \text{R} - \text{r} = \frac{66}{22} \times \frac{7}{2} = \frac{21}{2} = 10.5$ cm

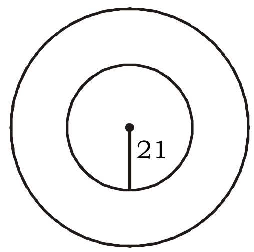
Width of the road = 10.5 cm.

3. A circular Swimming pool with a diameter of 42 ft has a deck of uniform width built around it. If the area of deck is 43π sq. ft. Find its width.

Sol. Area of swimming pool

$= \pi r^2 = \pi(21)^2$

$= 441\pi$ sq. ft.



Area of deck = 43π Sq.ft

$\Rightarrow \pi(R^2 - (21)^2) = 43\pi$

$\Rightarrow R^2 - 441 = \frac{43\pi}{\pi}$

$R^2 = 441 + 43$

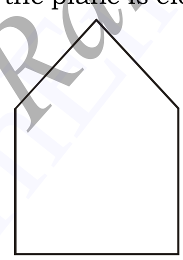
$R^2 = 484 \Rightarrow R = 22$

Width of the deck = 22 – 21

= 1 feet.

Polygon

It is a 2– Dimensional shapes. They are made of three or more than three straight lines, and the plane is closed.



Polygon
n = n: of sides

(i) Sum of Exterior angles

$= 2\pi = 360°$

(ii) Sum of Interior angles

= (n – 2) × 180°

(iii) Interior angle + exterior angle = 180°

(iv) Each Interior angle

$= \left(\frac{n-2}{n}\right) \times 180^0$

(v) Each Exterior angle $= \frac{360}{n}$

(vi) No of diagonals in polygon

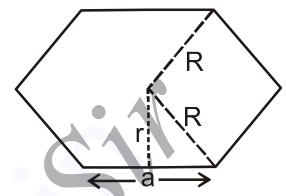
$= \frac{n(n-3)}{2}$

(vii) Perimeter (P) = n × a

(where n = no. of sides, and, a = length of each side)

(viii) Area (A)

$= \frac{1}{2} \times p \times r = \frac{1}{2} \times n \times a \times r$



where, r = radius of inscribed circle

$\Rightarrow A = \frac{1}{2} \times n \times a \times \sqrt{R^2 - \left(\frac{a}{2}\right)^2}$

R = radius of circumscribed circle.

$or\ A = \frac{na^2}{4} \cot \frac{\pi}{n}$

EXAMPLES

1. An exterior angle of a regular polygon measures 36°. How many sides does the polygon have?

Sol. No of sides of polygon = n

Each exterior angle $= \frac{360°}{n}$

$36° = \frac{360^0}{n}$

$n = \frac{360°}{36°} = 10$

Sides of a polygon is 10.

2. How many sides does a polygon have if the sum of its interior angles is 1260°?

Sol. Sum of interior angles

= (n – 2) × 180

1260 = 180n – 360

1260 + 360 = 180n

$n = \frac{1620}{180} = 9$

Sides of a polygon is 9.

3. The ratio of internal and external angle is 7:1. Find the number of sides?

Sol. Ratio of I : E = 7:1

$7x + x = 180°$
$8x = 180°$
$x = 22.5°$

So, exterior angles is = 22.2°

Each exterior angle $= \frac{360°}{n}$

$22.5 = \frac{360°}{n}$

$n = \frac{360}{22.5} = 16$

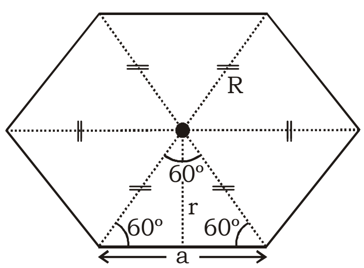
No. of sides is 16.

There are several other parts of polygon such as

No. of Sides	Name	Internal angle	External angle
3	Triangle	60°	120°
4	Quadrilateral	90°	90°
5	Pentagon	108°	72°
6.	Hexagon	120°	60°
7.	Heptagon	128.57°	51.43
8.	Octogon	135°	45°
9.	Nonagon	140°	40°
10.	Decagon	144°	36°

Hexagon

A polygon with 6 sides is known as Hexagon.



(i) Area = 6 × (Area of Equilateral triangle of side a)

$= 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$ (a = side)

(ii) Perimeter = 6a

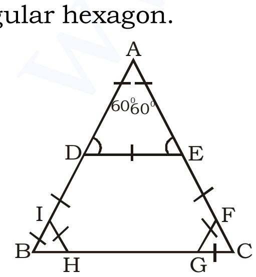
(iii) Incircle radius (r) $= \frac{\sqrt{3}}{2} a$

(iv) Circum Radius = a

EXAMPLES

1. An equilateral triangle of side 6 cm has its corner cut off form a regular hexagon. Area of this regular hexagon.

Sol.



DE || BC

$\angle ADE = \angle AED = 60°$

$\therefore AD = DE = AE$

$\Rightarrow$ Side of regular hexagon

$\Rightarrow \frac{1}{3} \times 6 = 2$ cm

Area $= \frac{3\sqrt{3}}{2} \times 4 = 6\sqrt{3}$ cm²

2. If area of a regular hexagon is $216\sqrt{3}$ sq. cm, then its perimeter.

Sol. Area $= \frac{3\sqrt{3}}{2} a^2$

$216\sqrt{3} = \frac{3\sqrt{3}}{2} a^2$

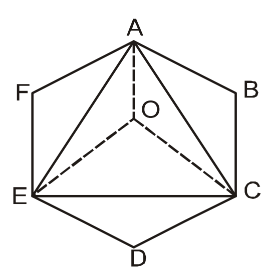
$\frac{432\sqrt{3}}{3\sqrt{3}} = a^2$

$a^2 = 144 \Rightarrow a = 12$

Perimeter = 6 × 12 = 72 cm.

3. Let ABCDEF be a regular hexagon. What is the ratio of the area of the triangle ACE to that of the hexagon ABCDEF ?

Sol.



ABCDEF is a regular hexagon. Joining the centre O with vertices A,C and E, we get,

$\Delta AFE = \Delta AOE$

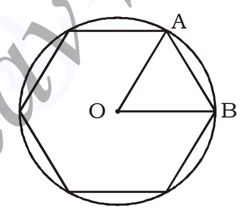
similarly, $\Delta OAC = \Delta BAC$

also, $\Delta OEC = \Delta DEC$

$\therefore \Delta ACE = \frac{1}{2}$ the area of regular hexagon.

4. A 6 sided regular polygon is inscribed in a circle of radius 10 cm, find the length of one side of the hexagon?

Sol.



$\angle AOB = \frac{360°}{6} = 60°$

Since OA = OB = 10 cm, triangle OAB is isosceles which gives

$\angle OAB = \angle OBA$

As all angle of the triangle are equal, therefore it is equilateral triangle

Hence AB = OA = OB = 10 cm

5. The area of a square is 2304 cm². Calculate the area of a regular hexagon that has the same perimeter as this square.

Sol. Perimeter of square = 4a

Perimeter of hexagon = 6a

Area of square = a² = 2304

a = 48

4a = 192 cm

= Perimeter of square

6a = 192

a = 32

Area of hexagon $= \frac{3\sqrt{3}a^2}{2}$

$= \frac{3\sqrt{3}}{2} \times 1024 = 1536\sqrt{3}$ cm²

6. A regular hexagon with an area of $150\sqrt{3}$ cm² is inscribed in a circle. Find the area not covered by hexagon?

Sol. Area of hexagon $= \frac{3\sqrt{3}}{2} a^2$

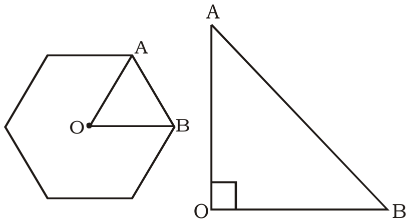
$\frac{3\sqrt{3}}{2} a^2 = 150\sqrt{3}$

$a^2 = \dfrac{300\sqrt{3}}{3\sqrt{3}} = 100$

a = 10

Covered = πr^2

As hexagon consist of 6 equilateral triangles



Side of hexagon = Radius of circle = 10 cm.

Area not couered = $\pi (10)^2 - 150\sqrt{3}$

$\Rightarrow 100\pi - 150\sqrt{3} \Rightarrow 54.35$ cm^2

Octagon

A polygon with 8 sides is known as Octagon.

(i) Perimeter = 8 × side

(ii) Area of regular octagon

= $2\left(\sqrt{2}+1\right)(\text{side})^2$

EXAMPLES

1. The side of a regular octagon is 5 cm. Its area is ?

Sol. Area of regular octagon

= $2\left(\sqrt{2}+1\right)(\text{side})^2$

= $2\left(\sqrt{2}+1\right)(5)^2 = 50\left(\sqrt{2}+1\right)$

2. If the perimeter of a regular octagon is 80 cm. Its area is ?

Sol. Perimeter of regular octagon is = 80

8 × side = 80

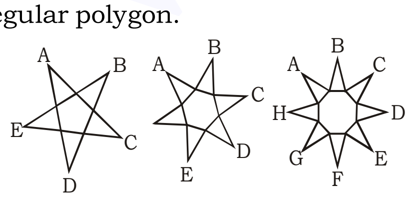
side = 10 cm

Area of regular octagon

= $2\left(\sqrt{2}+1\right)(\text{side})^2$

= $2\left(\sqrt{2}+1\right)(10)^2 = 200\left(\sqrt{2}+1\right)$

Star:- Sum of outer angles of a star Star forms by extending sides of a regular polygon.



Let outer triangles are 'n' then sum of outer angles = n × 180 – two times sum of exterior angles

= (n × 180 – 2 × 360)

= 180 (n – 4)

If n = 5 then $\angle A + \angle B + \dots + \angle E$ =180 (5 – 4) = 180°

If n = 6 then $\angle A + \angle B + \dots + \angle F$ =180 (6 – 4) = 360°

If n = 8 then $\angle A + \angle B + \dots + \angle H$ =180 (8 – 4) = 720°

Miscellaneous Problem

Some Useful Results:

1. If each of the defining dimensions or sides of any 2-D figures are increased (or decreased) by x%, its Perimeter also increases (or decreases) by x%.

2. If all the sides of a quadrilateral are increased (or decreased by x%, its diagonals also increases (or decreasses) by x%.

3. The number of revolutions made by a circular wheel of radius r in travelling distance 'd' is given by-

(no. of revolutions) n = $\dfrac{d}{2\pi r}$

4. If the length and breadth of a rectangle are increased by x% and y% respectively, then the area of rectangle will increases by:

$\left(x + y + \dfrac{xy}{100}\right)\%$

5. If the length and breadth of a rectangle are dicreased by x% and y% respectively, then the area of rectangle will decreases by :

$\left(x + y - \dfrac{xy}{100}\right)\%$

6. If the length of a rectangle is increased by x%, then its breadth will have to be decreased by $\left(\dfrac{100x}{100+x}\right)\%$ in order to maintain the same area of the rectangle.

7. If each of the degining dimensions or sides of any 2D figure (triangle, rectangle, square, circle, quadrilateral, pentagon, hexagon etc.) is changed by x%, its area changes by

$x\left(2 + \dfrac{x}{100}\right)\%$

$x \rightarrow +\text{ve}$ if increases

$x \rightarrow -ve$ if decreases

EXAMPLES

1. The length of a rectangle is increased by 50%. By what % should the width be decreased to maintain the same area ?

Sol. % decrease in breadth

= $\left(\dfrac{100x}{100+x}\right)\%$

= $\dfrac{100 \times 50}{150} = \dfrac{100}{3} = 33\dfrac{1}{3}\%$

Alternatively :-

Let length = x and breadth = y

New length = $x\left(\dfrac{150}{100}\right) = \dfrac{3x}{2}$

As the area remains the same, the new breadth of the rectangle - so,

$\dfrac{3x}{2} \times \text{New breadth} = xy$

$\Rightarrow \text{New breadth} = \dfrac{2y}{3}$

$\therefore$ decrease in breadth

$= y - \dfrac{2y}{3} = \dfrac{y}{3}$

$\therefore$ % decrease in breadth

= $\dfrac{y/3}{y} \times 100 = \dfrac{100}{3} = 33\dfrac{1}{3}\%$

2. The length of a rectangle is increased by 60%. By what per-cent would the width be decreased so as to maintain the same area ?

Sol. Let length = width = 100m

If length = 160m, then let width = x m

s.t. 160x = 10000

$\Rightarrow x = \dfrac{10000}{160} = \dfrac{1000}{16} = 62\dfrac{1}{2}$

$\therefore$ width is reduced to 37 $\dfrac{1}{2}$%

3. A wire is in the form of an equilateral triangle with area $4\sqrt{3}$ m^2. If it is change into a square, the side of a square will be.

Sol. Area of an equilateral trianle

$= \frac{\sqrt{3}}{4}a^2$

$4\sqrt{3} = \frac{\sqrt{3}}{4}a^2$

$a^2 = \frac{16\sqrt{3}}{\sqrt{3}} \Rightarrow a = 4$ m

Perimeter of equilateral triangle

$= 3a \Rightarrow 3 \times 4 = 12$ cm

Perimeter of square = 4a

4a = 12, a = 3m

Side of a square = 3m.

4. If the ratio of areas of two squares is 225:256 than the ratio of their perimeter is:

Sol. Area of first square, $S_1 = a^2$

= 225

a = 15

Area of second square,

$S_2 = a^2 = 256$

a = 16

Perimeter of square, S_1 = 4a

$= 4 \times 15 = 60$, $S_2 = 4a = 4 \times 64$

= 64

Ratio of their perimeter

= 15 : 16

Alternate:-

$$\frac{A_1}{A_2} = \left(\frac{P_1}{P_2}\right)^2$$

$$\frac{S_1}{S_2} \times \left(\frac{P_1}{P_2}\right)^2 \Rightarrow \frac{P_1}{P_2}$$

$$= \frac{\sqrt{225}}{\sqrt{256}} = \frac{15}{16}$$

5. A circular wire of diameter 210 cm is folded in the shape of a rectangle whose sides are in the ratio of 6:5. Find the area enclosed by the rectangle.

Sol. Perimeter of Circular wire

= Perimeter of Rectangle

$2\pi r = 2\ (l + b)$

$\Rightarrow 2\pi r = 2 \times \frac{22}{7} \times 105$

= 660 cm

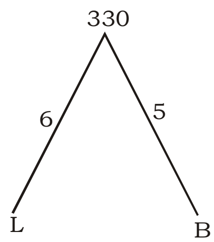
In Rectangle

$2\ (l + b) = 660$

$l + b = 330$

Length and breadth are in the ratio of 6:5

$\Rightarrow$



Area of rectangle = $l \times b$

$= 180 \times 150$

= 27,000 cm^2

6. The diameter of a wheel is 0.14 m. How many revolutions did it makes in moving 440 m.

Sol. Circumference of a wheel

= distance in 1 revolution

$2\pi r = 2 \times \frac{22}{7} \times 0.07 = \frac{44}{100}$ m

In 1 revolution wheel covers

$\frac{44}{100}$ m

In covering 440 m distacne, wheel makes

$\Rightarrow \frac{440}{44} \times 100$

= 1000 revolution.

Alternate:-

$n = \frac{d}{2\pi r}$

($\therefore$ n = n : of revolution d

$= \text{distance}) = \frac{440}{2 \times 22 \times 0.07}$

= 1000 revolution.

7. A metal wire when bent in the form of a square encloses an area 484cm^2. If the same wire is bent in the form of a circle, then (talking $\pi = \frac{22}{7}$) its area is:

Sol. Side of square

$= \sqrt{484} = 22$ cm

$\therefore$ length of wire $= 22 \times 4$

= 88cm

$\therefore 2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88$

$\Rightarrow r = \frac{88 \times 7}{2 \times 22} = 14cm$

$\therefore$ Area = πr^2

$= \frac{22}{7} \times 14 \times 14 = 616 sq.cm.$

8. The radius of a circular wheel is 1.75m. The number of revolutions that it will make in covering 11km is:

Sol. Total no. of revolution

$= \frac{d}{\text{Perimeter}} = \frac{d}{2\pi r}$

$= \frac{11 \times 1000 \times 7}{2 \times 22 \times 1.75} = 1000$

9. The wheel of a motor car makes 1000 revolutions in moving 110m. The diameter of the wheel is:-

Sol. As wheel makes 1000 revolutions in moving 110 m

$1000 \rightarrow 110$m

$1 \rightarrow \frac{110}{1000}$ = Circumference

$2\pi r = \frac{110}{1000}$

$r = \frac{110 \times 7}{44 \times 1000}$

$r = \frac{7}{400}$

Diameter = 2r = $2 \times \frac{7}{400}$

$= \frac{7}{200} = 0.035$ m

10. The cost of cultivating a square field at the rate of Rs. 190 per hectare is Rs. 1710. The cost of putting a fence around it at the Rate of 50 paise per metre.

Sol. Area of square field = $\frac{1710}{190}$

= 9 hectare

1 hectare = 10,000 m^2

9 hectare = 90,000 m^2

Side of square

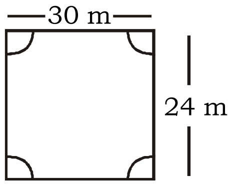
$= \sqrt{a} = \sqrt{90,000} = 300$

Cost of fencing $= \frac{50}{100} \times 4a$

$= \frac{1}{2} \times 300 \times 4 =$ Rs. 600

11. A rectangular piece is 30m long and 24 m wide. From its four corrers, quadrants of radii 2.1 metres have been cut. The area of the remaining part is.

Sol.



Area of Remaning part is

= Area of Rectangle – 4 × area of quarter circle

$= L \times B - 4 \times \frac{1}{4} \times \pi r^2$

$= 720 - \frac{22}{7} \times 2.1 \times 2.1$

$= 720 - 13.86 = 706.14$

12. Area of circle is equal to the area of a rectangle having perimeter of 100cms and length is more than the breadth by 6cms. What is the diameter of the circle?

Sol. Let breadth = x,

then length = $(x + 6)$

$\therefore$ $2(x + x + 6) = 100$

$\Rightarrow$ $2x + 6 = 50$

$\Rightarrow$ $x = 22$cm

$\therefore$ breadth = x

= 22cm & length

= 22 + 6 = 28cm

$\therefore$ Area of circle

= Area of rectangle

$\Rightarrow \pi r^2 = 22 \times 28$

$\Rightarrow r^2 = \frac{22 \times 28}{22} \times 7 = 7 \times 4 \times 7$

$\Rightarrow r = 7 \times 2 = 14cm$

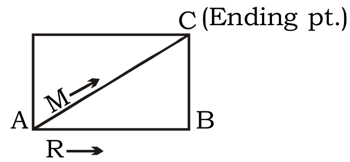
$\therefore$ Diameter = $2r$ = 28cm

13. In between the race of two friends Mohit and Rakesh in a rectangular field Mohit cheated Rakesh by taking diagonal path rather than a side path. Mohit took 15 secs to reach ending point at the rate of 100 m/min whereas Rakesh took same time to. Reach the ending point at the rate of 124 m/min. Find the area of field.

Sol. Distance Covered by Mohit in 15 sec $= 100 \times \frac{15}{60} = 25$ m

Distance covered by Rakesh in 15 Sec.

$= 124 \times \frac{15}{60} = 31$m



(Starting pt.)

Sum of length and Breadth = 31 metre.

We know that hypotenuse in a right angle triangle 25 m then length and breadth may be 24 and 7.

$(l + b) = 31$ area $= 24 \times 7 = 168$ m².

14. A person observed that he required 40 seconds less time to cross a circular ground along its diameter than to cover it once along a boundary. If his speed 40 m/min, then the radius of the circular ground is:

Sol. Along boundary he covers perimeter = $2\pi r$

Along diameter = 2r

Time distance

$= \frac{2\pi r}{\text{Speed}} - \frac{2r}{\text{Speed}}$

$\frac{40}{60}$ min $= \frac{2r}{40}(\pi - 1)$

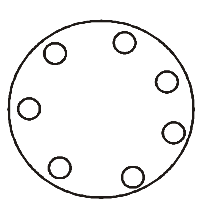
$\frac{2}{3} = \frac{2r}{40}\left(\frac{22}{7} - 1\right)$

$\frac{2 \times 40 \times 7}{3 \times 15} = 2r$

r = 6.2 m (approx.)

15. In a circular park with a radius of 25 m there are 7 lamps whose base are circles with a radius of 1 m. The entire area of the park has grass with the exception of the base for the lamps. Calculate the total lawn area?

Sol.



Area of the circular par $= \pi r^2$

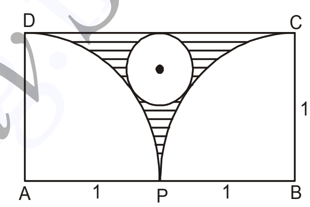
$\Rightarrow \pi (25)^2 = 625\pi$ m²

Area of 7 lamps $= 7 \times \pi (1)^2 = 7\pi$ m²

Total lawn area $= 625\pi - 7\pi$

$= 618\pi$ m²

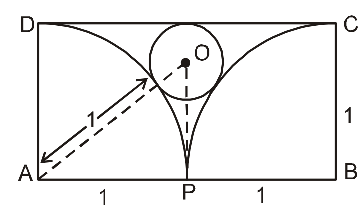
16. In the following figure ABCD is a rectangle with AD and DC equal to 1 and 2 units respectively. Two quarter circles are drawn with centres at B and A respectively. Now a circle is drawn touching both the quarter circles and done of the sides of the rectangle. Find the area of the shaded region :



Sol. Let radius fo the circle is 'r' units

OP = (1- r), OA = (1+ r) and AP = 1

In ΔAOP; OA² = AP² + OP²



$\Rightarrow (1 + r)^2 = 1^2 + (1 - r)^2$

$\Rightarrow r = \frac{1}{4}$ units

$\therefore$ Area of smaller circle $= \pi\left(\frac{1}{4}\right)^2$

$= \frac{\pi}{16}$ square units

Sum of the area of the quarter circles

$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ square units

Area of shaded region

$= 2 - \left(\frac{\pi}{16} + \frac{\pi}{2}\right)$

$= 2 - \frac{9}{16}\pi \cong 0.23$ square units

Among the option choices, option (b) is closest.

EXERCISE

1. If the length of the diagonal AC of a square ABCD is 5.2 cm, then the area of the square is :
(a) 15.12 sq. cm
(b) 13.52 sq. cm
(c) 12.62 sq. cm
(d) 10.00 sq . cm

2. The length of the diagonal of a square is 'a' cm. Which of the following represents the area of the square (in sq. cm) ?
(a) 2a (b) $\frac{a}{\sqrt{2}}$
(c) $a^2/2$ (d) $a^2/4$

3. The breadth of a rectangular hall is three-fourth of its length. If the area of the floor is 768 sq. m. , then the difference between the length and breadth of the hall is:
(a) 8 metres (b) 12 metres
(c) 24 metres (d) 32 metres

4. Find the length of the largest rod that can be placed in a room 16m long, 12m broad and $10\frac{2}{3}m$ high,
(a) 123 m (b) 68 m
(c) $22\frac{2}{3}m$ (d) $22\frac{1}{3}m$

5. Between a square of perimeter 44 cm and a circle of circumference 44 cm, which figure has larger area and by how much ?
(a) Square, 33cm^2
(b) Circle, 33 cm^2
(c) Both have equal area.
(d) square, 495 cm^2

6. The perimeter of a square and a circular field are the same. If the area of the circular field is 3850 sq meter. What is the area (in m^2) of the square?
(a) 4225 (b) 3025
(c) 2500 (d) 2025

7. The perimeter of the top of a rectangular table is 28m., whereas its area is 48m^2. What is the length of its diagonal?
(a) 5 m (b) 10 m
(c) 12 m (d) 12.5 m

8. The breadth of a rectangular hall is three- fourth of its length. If the area of the floor is 192 sq. m., then the difference between the length and breadth of the hall is:
(a) 8 meters (b) 12 meters
(c) 4 meters (d) 32 meters

9. The diagonal of a square is $4\sqrt{2}$ cm. The diagonal of another square whose area is double that of the first square is:
(a) $8\sqrt{2}$ cm (b) 16 cm
(c) $\sqrt{32}$ cm (d) 8 cm

10. The diagonal of a square A is (a +b). The diagoanal of a square whose area is twice the area of square A is
(a) 2(a +b) (b) 2(a+b)^2
(c) $\sqrt{2}$(a–b) (d) $\sqrt{2}(a+b)$

11. The length of a rectangular garden is 12 metres and its breadth is 5 metres. Find the length of the diagonal of a square garden having the same area as that of the rectangular garden:
(a) $2\sqrt{30}$ m (b) $\sqrt{13}$ m
(c) 13 m (d) $8\sqrt{15}$ m

12. The areas of a square and a rectangle are equal. The length of the rectangle is greater than the length of any side of the square by 5 cm and the breadth is less by 3 cm. Find the perimeter of the rectangle.
(a) 17 cm (b) 26 cm
(c) 30 cm (d) 34 cm

13. The perimeter of a rectangle is 160 meter and the difference of two sides is 48 meter. Find the side of a square whose area is equal to the area of this rectangle.
(a) 32m (b) 8m (c) 4m (d) 16m

14. The perimeter of two squares are 24 cm and 32 cm. The perimeter (in cm) of a third square equal in area to the sum of the areas of these squares is :
(a) 45 (b) 40 (c) 32 (d) 48

15. A wire when bent in the form of a square encloses an area of 484 sq. cm. What will be the enclosed area when the same wire is bent into the form of a circle? (Take $\pi=\frac{22}{7}$)
(a) 125 cm^2 (b) 230 cm^2
(c) 550 cm^2 (d) 616 cm^2

16. Find the length of the longest rod that can be placed in a hall of 10 m length, 6 m breadth and 4 m height,
(a) $2\sqrt{38}$ m (b) $4\sqrt{38}$ m
(c) $2\sqrt{19}$ m (d) $\sqrt{154}$ m

17. The difference of the areas of two squares drawn on two line segments of different lengths is 32sq. cm, Find the length of the greater line segment if one is longer than the other by 2 cm.
(a) 7 cm (b) 9 cm
(c) 11 cm (d) 16 cm

18. A took 15 sec. to cross a rectangular field diagonally walking at the rate of 52m/min and B took the same time to cross the same field along its sides walking at the rate of 68 m/ min. The area of the field is:
(a) 30 m^2 (b) 40 m^2
(c) 50 m^2 (d) 60 m^2

19. The difference between the length and breadth of a rectangle is 23 m. If its perimeter is 206 m, then its area is
(a) 1520 m^2 (b) 2420 m^2
(c) 2480 m^2 (d) 2520 m^2

20. The area (in m^2) of the square which has the same perimeter as a rectangle whose length is 48 m and is 3 times its breadth is:
(a) 1000 (b) 1024
(c) 1600 (d) 1042

21. The perimeter of two squares are 40 cm and 32 cm. The perimeter of a third square whose area is the difference of the area of the two squares is
(a) 24 cm (b) 42 cm
(c) 40 cm (d) 20 cm

22. The perimeter of five squares are 24 cm, 32 cm,40 cm,76cm and 80 cm respectively. The perimeter of another square equal in area to sum of the areas of these squares is:
(a) 31 cm (b) 62 cm
(c) 124 cm (d) 961 cm

23. There is a rectangular tank of length 180 m and breadth 120 m in a circular field, If the area of the land portion of the field is 40000 m^2, what is the radius of the field ? (Take $\pi=\frac{22}{7}$)
(a) 130 m (b) 135 m
(c) 140 m (d) 145 m

24. The length of a rectangular hall is 5m more than its breadth. The area of the hall is 750m². The length of the hall is :
(a) 15 m (b) 22.5 m
(c) 25 m (d) 30 m

25. A cistern 6 m long and 4 m wide contains water up to a depth of 1 m 25 cm. The total area of the wet surface is
(a) 55 m² (b) 53.5 m²
(c) 50 m² (d) 49 m²

26. If the length and breadth of a rectangle are in the ratio 3 : 2 and its perimeter is 20 cm, then the area of the rectangle (in cm²) is
(a) 24 cm² (b) 36 cm²
(c) 48 cm² (d) 12 cm²

27. The perimeter of a rectangle and a square are 160 m each. The area of the rectangle is less than that of the square by 100 sq m. The length of the rectangle is
(a) 30m (b) 60m (c) 40m (d) 50m

28. A path of uniform width runs round the inside of a rectangular field 38 m long and 32 m wide, If the path occupies 600m², then the width of the path is
(a) 30 m (b) 5 m
(c) 18.75 m (d) 10 m

29. The perimeter of the floor of a room is 18 m. What is the area of the walls of the room, If the height of the room is 3 m ?
(a) 21 m² (b) 42 m²
(c) 54 m² (d) 108 m²

30. A copper wire is bent in the shape of a square of area 81 cm². If the same wire is bent in the form of a semicircle, the radius (in cm) of the semicircle is (take $\pi=\frac{22}{7}$)
(a) 126 (b) 14 (c) 10 (d) 7

31. A copper wire is bent in the form of square with an area of 121 cm². If the same wire is bent in the form of a circle, the radius (in cm) of the circle is (Take $\pi=\frac{22}{7}$)
(a) 7 (b) 14 (c) 8 (d) 12

32. Water flows into a tank which is 200 m long and 150 m wide through a pipe of cross- section 0.3m × 0.2m at 20 km/hour. Then the time (in hours) for the water level in the tank to reach 8 m is
(a) 50 (b) 120 (c) 150 (d) 200

33. A street of width 10 metres surrounds from outside a rectangular garden whose measurement is 200 m × 180 m. The area of the path (in square metres) is
(a) 8000 (b) 7000
(c) 7500 (d) 8200

34. The area of the square inscribed in a circle of radius 8 cm is
(a) 256 sq. cm (b) 250 sq. cm
(c) 128 sq. cm (d) 125 sq. cm

35. Area of square with diagonal $8\sqrt{2}$ cm is
(a) 64 cm² (b) 29 cm²
(c) 56 cm² (d) 128 cm²

36. If the area of a rectangle be $(x^2+7x+10)$ sq. cm, then one of the possible perimeter of it is
(a) $(4x+14)$ cm (b) $(2x+14)$ cm
(c) $(x+14)$ cm (d) $(2x+7)$ cm

37. If the perimeter of a square and a rectangle are the same. then the area P and Q enclosed by them would satisfy the condition
(a) $P < Q$ (b) $P \le Q$
(c) $P > Q$ (d) $P = Q$

38. A cube of edge 6 cm is painted on all sides and then cut into unit cubes. The number of unit cubes with no sides painted is
(a) 0 (b) 64 (c) 186 (d) 108

39. The length of diagonal of a square is $15\sqrt{2}$ cm. Its area is
(a) 112.5 cm² (b) 450 cm²
(c) $\frac{255\sqrt{2}}{2}$ cm² (d) 225 cm²

40. A kite in the shape of a square with a diagonal 32 cm attached to an equilateral triangle of the base 8 cm. Approximately how much paper has been used to make it? (Use $\sqrt{3}$ = 1.732)
(a) 539.712 cm²
(b) 538.721 cm²
(c) 540.712 cm²
(d) 539.217 cm²

41. A lawn is in the form of a rectangle having its breadth and length in the ratio 3 : 4. The area of the lawn is $\frac{1}{12}$ hectare. The breadth of the lawn is
(a) 25 metres (b) 50 metres
(c) 75 metres (d) 100 metres

42. The area of a rectangle is thrice that of a square. The length of the rectangle is 20 cm and the breadth of the rectangle is $\frac{3}{2}$ times that of the side of the square. The side of the square, (in cm) is
(a) 10 (b) 20 (c) 30 (d) 60

43. The length and breadth of a rectangular field are in the ratio 7 : 4. A path 4 m wide running all around outside has an area of 416 m². The breadth (in m) of the field is
(a) 28 (b) 14 (c) 15 (d) 16

44. How many tiles, each 4 decimeter square. will be required to cover the floor of a room 8 m long and 6 m broad?
(a) 200 (b) 260 (c) 280 (d) 300

45. A godown is 15 m long and 12 m broad. The sum of the area of the floor and the ceiling is equal to the sum of areas of the four walls. The volume (in m³) of the godown is:
(a) 900 (b) 1200
(c) 1800 (d) 720

46. Length of a side of a square inscribed in a circle is $a\sqrt{2}$ units. The circumference of the circle is
(a) $2\pi a$ units (b) πa units
(c) $4\pi a$ units (d) $\frac{2a}{\pi}$ units

47. The perimeter and length of a rectangle are 40 m and 12 m respectively. Its breadth will be
(a) 10m (b) 8m (c) 6m (d) 3m

48. If each edge of a square be doubled. then the increase percentage in its area is
(a) 200% (b) 250%
(c) 280% (d) 300%

49. An elephant of length 4 m is at one corner of a rectangular cage of size (16 m × 30 m) and faces towards the diagonally opposite corner. If the elephant starts moving towards the diagonally opposite conrner it takes 15 seconds to reach this corner. Find the speed of the elephant
(a) 1 m/sec (b) 2 m/sec
(c) 1.87 m/sec (d) 1.5 m/sec

50. A circle is inscribed in a square of side 35 cm. The area of the remaining portion of the square which is not enclosed by the circle is
(a) 962.5 cm^2 (b) 262.5 cm^2
(c) 762.5 cm^2 (d) 562.4 cm^2

51. If the side of a square is $\frac{1}{2}(x+1)$ units and its diagonal is $\frac{3-x}{\sqrt{2}}$ units, then the length of the side of the square would be
(a) $\frac{4}{3}$ units (b) 1 unit
(c) $\frac{1}{2}$ units (d) 2 units

52. A rectangular carpet has an area of 120 m^2 and a perimeter of 46 metre. The length of its diagonal is:
(a) 17 metres (b) 21 metres
(c) 13 metres (d) 23 metres

53. If the length of a diagonal of a square is $6\sqrt{2}$ cm, then its area will be
(a) $24\sqrt{2}$ cm^2 (b) 24 cm^2
(c) 36 cm^2 (d) 72 cm^2

54. The length of a room is 3m more than its breadth. If the area of a floor of the room is 70 m^2, then the perimeter of the floor will be-
(a) 14 metres (b) 28 metres
(c) 34 metres (d) 17 metres

55. The length of a rectangle is twice the breadth. If area of the rectangle be 417.605 sq. m., then length is-
(a) 29.08 metres
(b) 29.80 metres
(c) 29.09 metres
(d) 28.90 metres

56. The area of a sector of a circle of radius 5 cm, formed by an arc of length 3.5 cm is :
(a) 8.5 cm^2 (b) 8.75 cm^2
(c) 7.75 cm^2 (d) 7.50 cm^2

57. The radius of a circular wheel is 1.75 m. The number of revolutions it will make in travelling 11 km is $\left(use\ \pi = \frac{22}{7}\right)$:
(a) 800 (b) 900
(c) 1000 (d) 1200

58. The radius of a wheel is 21 cm, How many revolutions will it make in travelling 924 metres?
$\left(use\ \pi = \frac{22}{7}\right)$
(a) 7 (b) 11 (c) 200 (d) 700

59. The area (in sq. cm) of the largest circle that can be drawn inside a square of side 28 cm is :
(a) 1724 (b) 784
(c) 8624 (d) 616

60. The area of the ring between two concentric circles, whose circumference are 88 cm and 132 cm, is
(a) 78 cm^2 (b) 770 cm^2
(c) 715 cm^2 (d) 660 cm^2

61. The diameter of a toy wheel is 14 cm, What is the distance travelled by it in 15 revolutions?
(a) 880 cm (b) 660 cm
(c) 600 cm (d) 560 cm

62. A can go round a circular path 8 times in 40 minutes. If the diameter of the circle is increased to 10 times the original diameter, the time required by A to go round the new path once travelling at the same speed as before is :
(a) 25 min (b) 20 min
(c) 50 min (d) 100 min

63. The base of a triangle is 15 cm and height is 12 cm. The height of another triangle of double the area having the base 20 cm is
(a) 9cm (b) 18 cm
(c) 8 cm (d) 12.5 cm

64. If a wire is bent into the shape of a square, the area of the square is 81 sq. cm, When the wire is bent into a semicircular shape, the area of the semicircle (taking $\pi=\frac{22}{7}$) is :
(a) 154 cm^2 (b) 77 cm^2
(c) 44 cm^2 (d) 22 cm^2

65. If the area of a triangle with base 12 cm is equal to the area of square with side 12 cm. the altitude of the triangle will be
(a) 12 cm (b) 24 cm
(c) 18 cm (d) 36 cm

66. The sides of a triangle are 3cm, 4 cm and 5 cm. The area (in cm^2) of the traingle formed by joining the mid points of this triangle is:
(a) 6 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

67. Three circles of radius 3.5 cm each are placed in such a way that each touches the other two. The area of the portion enclosed by the circles is
(a) 1.975 cm^2 (b) 1.967 cm^2
(c) 19.68 cm^2 (d) 21.22 cm^2

68. The area of a circular garden is 2464 sq. m. how much distance will have to be covered if you like to cross the garden along its diameter? (use $\pi=\frac{22}{7}$)
(a) 56m (b) 48m (c) 28m (d) 24m

69. Four equal circles each of radius 'a' units touch one another. The area enclosed between them ($\pi = \frac{22}{7}$). In square units, is
(a) $3a^2$ (b) $\frac{6a^2}{7}$
(c) $\frac{41a^2}{7}$ (d) $\frac{a^2}{7}$

70. The area of the greatest circle inscribed inside a square of side 21 cm is (Take $\pi=\frac{22}{7}$)
(a) $351\frac{1}{2}$ cm^2 (b) $350\frac{1}{2}$ cm^2
(c) $346\frac{1}{2}$ cm^2 (d) $347\frac{1}{2}$ cm^2

71. The area of an equilateral triangle is $400\sqrt{3}$ sq. m. Its perimeter is :
(a) 120 m (b) 150 m
(c) 90 m (d) 135 m

72. From a point in the interior of an equilateral triangle, the perpendicular distance of the sides are $\sqrt{3}$ cm $2\sqrt{3}$ cm and $5\sqrt{3}$ cm. The perimeter (in cm) of the triangle is
(a) 64 (b) 32 (c) 48 (d) 24

73. The perimeter of a triangle is 30 cm and its area is 30 cm^2. If the largest side measures 13 cm, What is the length of the smallest side of the triangle?
(a) 3cm (b) 4cm
(c) 5cm (d) 6cm

74. Diameter of a wheel is 3 m. The wheel revolves 28 times in a minute. To cover 5.280 km distance, the wheel will take

(Take $\pi = \frac{22}{7}$):

(a) 10 minutes (b) 20 minutes
(c) 30 minutes (d) 40 minutes

75. Find the diameter of a wheel that makes 113 revolutions to go 2 km 26 decameters.

(Take $\pi = \frac{22}{7}$)

(a) $4\frac{4}{13}$ m (b) $6\frac{4}{11}$ m

(c) $12\frac{4}{11}$ m (d) $12\frac{8}{11}$ m

76. The radius of a circular wheel is 1.75m. The number of revolutions that it will make in travelling 11 km, is
(a) 1000 (b) 10,000
(c) 100 (d) 10

77. The circumference of a circle is 100 cm. The side of a square inscribed in the circle is

(a) $\frac{100\sqrt{2}}{\pi}$ cm (b) $\frac{50\sqrt{2}}{\pi}$ cm

(c) $\frac{100}{\pi}$ cm (b) $50\sqrt{2}$ cm

78. A path of uniform width surrounds a circular park, The difference of internal and external circumference of this circular path is 132 metres. Its width is:

(Take $\pi = \frac{22}{7}$)

(a) 22m (b) 20m (c) 21m (d) 4m

79. Four equal sized maximum circular plates are cut off from a square paper sheet of area 784 sq. cm. The circumference of each plate is

(Take $\pi = \frac{22}{7}$)

(a) 22 cm (b) 44 cm
(c) 66 cm (d) 88 cm

80. The circum-radius of an equilateral triangle is 8 cm. The in- radius of the triangle is
(a) 3.25 cm (b) 3.50 cm
(c) 4 cm (d) 4.25 cm

81. Three coins of the same size (radius 1 cm) are placed on a table such that each of them touches the other two. The area enclosed by the coins is

(a) $\left(\frac{\pi}{2} - \sqrt{3}\right)$ cm²

(b) $\left(\sqrt{3} - \frac{\pi}{2}\right)$ cm²

(c) $\left(2\sqrt{3} - \frac{\pi}{2}\right)$ cm²

(d) $\left(3\sqrt{3} - \frac{\pi}{2}\right)$ cm²

82. The area of the largest triangle that can be inscribed in a semicircle of radius r cm, is
(a) 2r cm² (b) r² cm²

(c) 2 cm² (d) $\frac{1}{2}r^2$ cm²

83. The area of the greatest circle, which can be inscribed in a square whose perimeter is 120 cm, is :

(a) $\frac{22}{7} \times 15^2$ cm²

(b) $\frac{22}{7} \times \left(\frac{7}{2}\right)^2$ cm²

(c) $\frac{22}{7} \times \left(\frac{15}{2}\right)^2$ cm²

(d) $\frac{22}{7} \times \left(\frac{9}{2}\right)^2$ cm²

84. The area of the incircle of an equilateral triangle of side 42 cm is (Take $\pi = \frac{22}{7}$):

(a) 231 cm² (b) 462 cm²
(c) $22\sqrt{3}$ cm² (d) 924 cm²

85. The number of revolutions a wheel of diameter 40 cm makes in travelling a distance of 176 m, is (Take $\pi = \frac{22}{7}$):

(a) 140 (b) 150
(c) 160 (d) 166

86. The length of the perpendiculars drawn from any point in the interior of an equailateral triangle to the respective sides are p_1,p_2 and p_3. The length of each side of the triangle is

(a) $\frac{2}{\sqrt{3}}(p_1 + p_2 + p_3)$

(b) $\frac{1}{3}(p_1 + p_2 + p_3)$

(c) $\frac{1}{\sqrt{3}}(p_1 + p_2 + p_3)$

(d) $\frac{4}{\sqrt{3}}(p_1 + p_2 + p_3)$

87. A circle is inscribed in a square, An equilateral triangle of side $4\sqrt{3}$ cm is inscribed in that circle. The length of the diagonal of the square (in cm) is

(a) $4\sqrt{2}$ (b) 8 (c) $8\sqrt{2}$ (d) 16

88. The hypotenuse of a right angle isosceles triangle is 5 cm. Its area will be
(a) 5 sq. cm (b) 6.25 sq. cm
(c) 6.50 sq. cm (d) 12.5 sq. cm

89. From a point within an equilateral triangle, perpendiculars drawn to the three sides are 6 cm, 7 cm and 8 cm respectively, the length of the side of the triangle is :
(a) 7 cm (b) 10. 5 cm

(c) $14\sqrt{3}$ cm (b) $\frac{14\sqrt{3}}{3}$ cm

90. In an isosceles triangle, the measure of each of equal sides is 10 cm and the angle between them is 45°, then area of the triangle is

(a) 25 cm² (b) $\frac{25}{2}\sqrt{2}$ cm²

(c) $25\sqrt{2}$ cm² (d) $2\sqrt{3}$ cm²

91. The area of circle whose radius is 6 cm is trisected by two concentric circles. The radius of the smallest circle is

(a) $2\sqrt{3}$ cm (b) $2\sqrt{6}$ cm
(c) 2 cm (d) 3 cm

92. The area of an equilateral triangle inscribed in a circle is $4\sqrt{3}$ cm². The area of the circle is

(a) $\frac{16}{3}\pi$ cm² (b) $\frac{22}{3}\pi$ cm²

(c) $\frac{28}{3}\pi$ cm² (d) $\frac{32}{3}\pi$ cm²

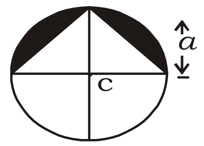
93. If the difference between the circumference and diameter of a circle is 30 cm, then the radius of the circle must be
(a) 6cm (b) 7cm
(c) 5cm (d) 8cm

94. The base and altitude of a right angled triangle are 12 cm and 5 cm respectively. The perpendicular distance of its hypotenuse from the opposite vertex is

(a) $4\frac{4}{13}$ cm (b) $4\frac{8}{13}$ cm
(c) 5 (d) 7 cm

95. From a point in the interior of an equilateral triangle , the length of the perpendiculars to the three sides are 6 cm, 8 cm and 10 cm respectively. The area of the triangle is
(a) 48 cm² (c) $16\sqrt{3}$ cm²
(c) $192\sqrt{3}$ cm² (d) 192 cm²

96. The area of the shaded region in the figure given below is



(a) $\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$ sq. units
(b) $a^2(\pi-1)$ sq. units
(c) $a^2\left(\frac{\pi}{2}-1\right)$ sq. units
(d) $\frac{a^2}{b^2}(\pi-1)$ sq. units

97. The area of a circle is increased by 22 cm, if its radius is increased by 1 cm. The original radius of the circle is
(a) 6 cm (b) 3.2 cm
(c) 3 cm (d) 3.5 cm

98. The area of the largest circle, that can be drawn inside a rectangle with sides 148 cm. by 14 cm is
(a) 49 cm² (b) 154 cm²
(c) 378 cm² (d) 1078 cm²

99. A circle is inscribed in an equilateral triangle of side 8 cm. The area of the portion between the triangle and the circle is
(a) 11 cm² (b) 10.95 cm²
(c) 10 cm² (d) 10.50 cm²

100. In a triangular field having sides 30m, 72m and 78m, the length of the altitude to the side measuring 72m is :
(a) 25 m (b) 28 m
(c) 30 m (d) 35 m

101. If the perimeter of a right-angled isosceles triangle is $4\sqrt{2}+4$ cm, the length of the hypotenuse is;
(a) 4 cm (b) 6 cm
(c) 8 cm (d) 10 cm

102. A right triangle with sides 3 cm,4 cm and 5 cm is rotated about the side 3 cm to from a cone. The volume of the cone so formed is
(a) 16π cm³ (b) 12π cm³
(c) 15π cm³ (d) 20π cm³

103. ABC is an equilateral triangle of side 2 cm. With A, B, C as centre and radius 1 cm three arcs are drawn. The area of the region within the triangle bounded by the three arcs is
(a) $\left(3\sqrt{3}-\frac{\pi}{2}\right)$ cm²
(b) $\left(\sqrt{3}-\frac{3\pi}{2}\right)$ cm²
(c) $\left(\sqrt{3}-\frac{\pi}{2}\right)$ cm²
(d) $\left(\frac{\pi}{2}-\sqrt{3}\right)$ cm²

104. The circumference of a circle is 11 cm and the angle of a sector of the circle is 60°. The area of the sector is (use $\pi=\frac{22}{7}$)
(a) $1\frac{29}{48}$ cm² (b) $2\frac{29}{48}$ cm²
(c) $1\frac{27}{48}$ cm² (d) $2\frac{27}{48}$ cm²

105. If the difference between areas of the circumcircle and the incircle of an equilateral triangle is 44 cm², then the area of the triangle is (Take $\pi=\frac{22}{7}$)
(a) 28 cm² (b) $7\sqrt{3}$ cm²
(c) $14\sqrt{3}$ cm² (d) 21 cm²

106. If the area of a circle inscribed in a square is 9π cm², then the area of the square is
(a) 24 cm² (b) 30 cm²
(c) 36 cm² (d) 81 cm²

107. The sides of a triangle are 6 cm, 8 cm and 10 cm. The area of the greatest square that can be inscribed in it, is
(a) 18 cm² (b) 15 cm²
(c) $\frac{2304}{49}$ cm² (d) $\frac{576}{49}$ cm²

108. The length of a side of an equilateral triangle is 8 cm. the area of the region lying between the circumcircle and the incircle of the triangle is (use $\pi=\frac{22}{7}$)
(a) $50\frac{1}{7}$ cm² (b) $50\frac{2}{7}$ cm²
(c) $75\frac{1}{7}$ cm² (d) $75\frac{2}{7}$ cm²

109. A wire, when bent in the form of a square, encloses a region having area 121 cm². If the same wire is bent into the form of a circle, then the area of the circle is (Take $\pi=\frac{22}{7}$)
(a) 144 cm² (b) 180 cm²
(c) 154 cm² (d) 176 cm²

110. If the perimeter of a semicircular field is 36 m. Find its radius (use $\pi=\frac{22}{7}$)
(a) 7 m (b) 8 m
(c) 14 m (d) 16 m

111. The perimeter (in metres) of a semicircle is numerically equal to its area (in m²). The length of its diameter is (Take $\pi=\frac{22}{7}$)
(a) $3\frac{3}{11}$ metres (b) $5\frac{6}{11}$ metres
(c) $6\frac{6}{11}$ metres (d) $6\frac{2}{11}$ metres

112. One acute angle of a right angled triangle is double the other. If the length of its hypotenuse is 10 cm, then its area is
(a) $\frac{25}{2}\sqrt{3}$ cm² (b) 25 cm²
(c) $25\sqrt{3}$ cm² (d) $\frac{75}{2}$ cm²

113. If a triangle with base 8 cm has the same area as a circle with radius 8 cm, then the corresponding altitude (in cm) of the triangle is
(a) 12π (b) 20π
(c) 16π (d) 32π

114. The measures (in cm) of sides of a right angled triangle are given by consecutive integers. its area (in cm^2) is
(a) 9 (b) 8 (c) 5 (d) 6

115. The area of a right-angled isosceles triangle having hypotenuse $16\sqrt{2}$ cm is
(a) 144 cm^2 (b) 128 cm^2
(c) 112 cm^2 (d) 110 cm^2

116. The area of an equilateral triangle is $4\sqrt{3}$ cm^2. The length of each side of the triangle is:
(a) 3 cm (b) $2\sqrt{2}$ cm
(c) $2\sqrt{3}$ cm (d) 4 cm

117. An equilateral triangle of side 6 cm has its corners cut off to form a regular hexagon. Area (in cm^2) of this regular hexagon will be
(a) $3\sqrt{3}$ (b) $3\sqrt{6}$
(c) $6\sqrt{3}$ (d) $\frac{5\sqrt{3}}{2}$

118. A 7 m wide road runs outside around a circular park, whose circumference is 176 m. the area of the road is : (use $\pi=\frac{22}{7}$)
(a) 1386 m^2 (b) 1472 m^2
(c) 1512 m^2 (d) 1760 m^2

119. The length (in cm) of a chord of a circle of radius 13 cm at a distance of 12 cm from its centre is
(a) 5 (b) 8 (c) 10 (d) 12

120. The four equal circles of radius 4 cm drawn on the four corners of a square touch each other externally. Then the area of the portion between the square and the four sectors is
(a) 9(π -4)sq. cm
(b) $16(4-\pi)$ sq. cm
(c) 99 $(\pi-4)$ sq. cm
(d) 169 $(\pi-4)$ sq. cm

121. If the four equal circles of radius 3 cm touch each other externally, then the area of the region bounded by the four circles is
(a) 4 ($9-\pi$) sq. cm
(b) $9(4-\pi)$ sq. cm
(c) $5(6-\pi)$ sq. cm
(d) $6(5-\pi)$ sq. cm

122. The length of each side of an equilateral triangle is $14\sqrt{3}$ cm. The area of the incircle (in cm^2) is
(a) 450 (b) 308
(c) 154 (d) 77

123. Area of the incircle of an equilateral triangle with side 6 cm is
(a) $\frac{\pi}{2}$ sq. cm (b) $\sqrt{3}\pi$ sq. cm
(c) 6π sq. cm (d) 3π sq. cm

124. A copper wire is bent in the form of an equilateral triangle and has area $121\sqrt{3}$ cm^2. If the same wire is bent into the form of a circle. the area (in cm^2) enclosed by the wire is (take $\pi=\frac{22}{7}$)
(a) 364.5 (b) 693.5
(c) 346.5 (d) 639.5

125. At each corner of a triangular field of sides 26 m, 28 m and 30 m, a cow is tethered by a rope of length 7m, the area (in m) ungrazed by the cows is
(a) 336 (b) 259
(c) 154 (d) 77

126. In an equilateral triangle ABC, P&Q are mid point of sides AB & AC respectively such that PQ || BC. If PQ = 5 cm then find the length of BC.
(a) 5 cm (b) 10 cm
(c) 15 cm (d) 12 cm

127. ABC is an equilateral triangle, P and Q are two points on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{PQ} \parallel \overline{BC}$. If $\overline{PQ}$ =5 cm, then area of $\triangle APQ$ is :
(a) $\frac{25}{4}$ sq. cm (b) $\frac{25}{\sqrt{3}}$ sq. cm
(c) $\frac{25\sqrt{3}}{4}$ sq.cm (d) $25\sqrt{3}$ sq. cm

128. The area of a circle with circumference 22cm is
(a) 38.5 cm² (b) 39 cm²
(c) 36.5 cm² (d) 40 cm²

129. In $\triangle ABC$, O is the centroid and AD, BE, CF are three medians and the area of $\triangle AOE$ = 15 cm^2 then area of quadrilateral BDOF is
(a) 20 cm^2 (b) 30 cm^2
(c) 40 cm^2 (d) 25 cm^2

130. A straight line parallel to the base BC of the triangle ABC intersects AB and AC at the points D and E respectively. If the area of the $\triangle ABE$ be 36 sq. cm. then the area of the $\triangle ACD$ is
(a) 18 sq.cm (b) 36 sq. cm
(c) 120 sq. cm (d) 54 sq. cm

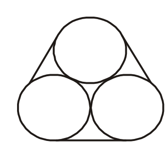
131. The length of two sides of an isosceles triangle are 15 and 22 respectively. What are the possible values of perimeter ?
(a) 52 or 59 (b) 52 or 60
(c) 15 or 37 (d) 37 or 29

132. The diameter of a wheel is 98 cm. The number of revolutions in which it will have to cover a distance of 1540 m is
(a) 500 (b) 600
(c) 700 (d) 800

133. The wheel of a motor car makes 1000 revolutions in moving 440 m. The diameter (in metre) of the wheel is
(a) 0.44 (b) 0.14
(c) 0.24 (d) 0.34

134. A bicycle wheel makes 5000 revolutions in moving 11 km . Then the radius of the wheel (in cm) is (take $\pi=\frac{22}{7}$)
(a) 70 (b) 35 (c) 17.5 (d) 140

135. Three circles of diameter 10 cm each are bound together by a rubber band as shown in the figure.



the length of the rubber band (in cm) if it is stretched is
(a) 30 (b) 30 +10 π
(c) 10 π (d) 60+20 π

136. If chord of length 16 cm is at a distance of 15 cm from the centre of the circle, then the length of the chord of the same circle which is at a distance of 8 cm from the centre is equal to
(a) 10 cm (b) 20 cm
(c) 30 cm (d) 40 cm

137. A semicircular shaped window has diameter of 63 cm, its perimeter equals ($\pi=\frac{22}{7}$)
(a) 126 cm (b) 162 cm
(c) 198 cm (d) 251 cm

138. In an equilateral triangle ABC of side 10 cm, the side BC is trisected at D & E. Then the length (in cm) of AD is
(a) $3\sqrt{7}$ (b) $7\sqrt{3}$
(c) $\frac{10\sqrt{7}}{3}$ (d) $\frac{7\sqrt{10}}{3}$

139. The perimeter of a triangle is 40cm and its area is 60 cm^2. If the largest side measures 17 cm, then the length (in cm) of the smallest side of the triangle is
(a) 4 (b) 6 (c) 8 (d) 15

140. From four corners of a square sheet of side 4 cm four pieces each in the shape of arc of a circle with radius 2 cm are cut out. The area of the remaining portion is :
(a) $(8 - \pi)$ sq. cm
(b) $(16 - 4\pi)$ sq. cm
(c) $(16 - 8\pi)$ sq. cm
(d) $(4 - 2\pi)$ sq. cm

141. If the numerical value of the perimeter of an equilateral triangle is $\sqrt{3}$ times the area of it, then the length of each side of the triangle is
(a) 2 units (b) 3 units
(c) 4 units (d) 6 units

142. Each side of an equilateral triangle is 6 cm. Find its area
(a) $9\sqrt{3}$ sq. cm (b) $6\sqrt{3}$ sq. cm
(c) $4\sqrt{3}$ sq. cm (d) $8\sqrt{3}$ sq. cm

143. The length of three medians of a triangle are 9 cm, 12 cm and 15 cm. The area (in sq. cm) of the triangle is
(a) 24 (b) 72
(c) 48 (d) 144

144. The area of the triangle formed by the straight line $3x + 2y = 6$ and the co-ordinate axes is
(a) 3 square units
(b) 6 square units
(c) 4 square units
(d) 8 square units

145. If the length of each side of an equilateral triangle is increased by 2 unit, the area is found to be increased by $3 + \sqrt{3}$ square unit. The length of each side of the triangle is
(a) $\sqrt{3}$ units (b) 3 units
(c) $3\sqrt{3}$ units (d) $3\sqrt{2}$ units

146. What is the area of the triangle whose sides are 9cm, 10cm and 11 cm ?
(a) 30 cm^2 (b) 60 cm^2
(c) $30\sqrt{2}$ cm^2 (d) $60\sqrt{2}$ cm^2

147. The area of an isosceles triangle is 4 square units, If the length of the unequal side is 2 unit, the length of each equal side is
(a) 4 units (b) $2\sqrt{3}$ units
(c) $\sqrt{17}$ units (d) $3\sqrt{2}$ units

148. What is the area of a triangle having perimeter 32 cm, one side 11 cm and difference of other two sides 5 cm?
(a) $8\sqrt{30}$ cm^2 (b) $5\sqrt{35}$ cm^2
(c) $6\sqrt{30}$ cm^2 (d) $8\sqrt{2}$ cm^2

149. Area of equilateral triangle having side 2cm is
(a) 4 cm^2 (b) $\sqrt{3}$ cm^2
(c) 3 cm^2 (d) $\sqrt{6}$ cm^2

150. The area of a circle is increased by 22 cm^2 when its radius is increased by 1 cm. The original radius of the circle is
(a) 3 cm (b) 5 cm
(c) 7cm (d) 9 cm

151. The radii of two circles are 5 cm and 12 cm. The area of a third circle is equal to the sum of the area of the two circles. The radius of the third circle is :
(a) 13 cm (b) 21 cm
(c) 30 cm (d) 17 cm

152. The perimeter of a semicircular path is 36 m. Find the area of this semicircular path.
(a) 42 sq. m (b) 54 sq. m
(a) 63 sq. m (d) 77 sq. m

153. The area of a circle inscribed in a square of area 2m^2 is
(a) $\frac{\pi}{4}$ m^2 (b) $\frac{\pi}{2}$ m^2
(c) π m^2 (d) 2π m^2

154. Three circles of radii 4 cm, 6 cm and 8 cm touch each other pair wise externally. The area of the triangle formed by the line-segments joining- the centres of the three circles is
(a) $144\sqrt{13}$ sq. cm
(b) $12\sqrt{105}$ sq. cm
(c) $4\sqrt{3}$ sq. cm
(d) $24\sqrt{6}$ sq. cm

155. Two circles with centre A and B and radius 2 units touch each other externally at 'C', A third circle with centre 'C' and radius '2' units meets other two at D and E. Then the area of the quadrilateral ABDE is
(a) $2\sqrt{2}$ sq. units
(b) $3\sqrt{3}$ sq. units
(c) $3\sqrt{2}$ sq. units
(d) $2\sqrt{3}$ sq. units

156. If the perimeter of a right angled triangle is 56 cm and area of the triangle is 84 sq. cm, then the length of the hypotenuse is (in cm)
(a) 25 (b) 50 (c) 7 (d) 24

157. If the length of each median of an equilateral triangle is $6\sqrt{3}$ cm, the perimeter of the triangle is
(a) 24 cm (b) 32 cm
(c) 36 cm (d) 42 cm

158. The area of an equilateral triangle is $4\sqrt{3}$ sq. cm. Its perimeter is
(a) 12 cm (b) 6 cm
(c) 8 cm (d) $3\sqrt{3}$ cm

159. A gear 12 cm in diameter is turning a gear 18 cm in diameter. When the smaller gear has 42 revolutions. how many has the larger one made?
(a) 28 (b) 20 (c) 15 (d) 24

160. The perimeter of a semicircle is 18 cm, then the radius is:
(using $\pi = \frac{22}{7}$)
(a) $5\frac{1}{3}$ cm (b) $3\frac{1}{2}$ cm
(c) 6 cm (d) 4 cm

161. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. The area of the circle is
(Take $\pi = \frac{22}{7}$)
(a) 125 cm^2 (b) 230 cm^2
(c) 550 cm^2 (d) 616 cm^2

162. The area of a circle is 38.5 sq. cm. Its circumference (in cm) is
$\left(use\ \pi = \frac{22}{7}\right)$
(a) 22 (b) 24 (c) 26 (d) 32

163. A circle is inscribed in a square whose length of the diagonal is $12\sqrt{2}$ cm. An equilateral triangle is inscribed in that circle. The length of the side of the triangle is
(a) $4\sqrt{3}$ cm (b) $8\sqrt{3}$ cm
(c) $6\sqrt{3}$ cm (d) $11\sqrt{3}$ cm

164. The area (in sq. unit) of the triangle formed in the first quadrant by the line $3x+4y=12$ is
(a) 8 (b) 12 (c) 6 (d) 4

165. The height of an equilateral triangle is 15 cm. the area of the triangle is
(a) $50\sqrt{3}$ sq. cm
(b) $70\sqrt{3}$ sq. cm
(c) $75\sqrt{3}$ sq. cm
(d) $150\sqrt{3}$ sq. cm

166. The area of an equilateral triangle is $9\sqrt{3}$ m^2. The length (in m) of the median is
(a) $2\sqrt{3}$ (b) $3\sqrt{3}$
(c) $3\sqrt{2}$ (d) $2\sqrt{2}$

167. The sides of a triangle are 16 cm, 12 cm and 20 cm. Find the area,
(a) 64 cm^2 (b) 112 cm^2
(c) 96 cm^2 (d) 81 cm^2

168. 360 sq. cm and 250 sq. cm are the area of two similar triangles. If the length of one of the sides of the first triangle be 8 cm , then the length of the corresponding side of the second triangle is
(a) $6\frac{1}{5}$ cm (b) $6\frac{1}{3}$ cm
(c) $6\frac{2}{3}$ cm (d) 6 cm

169. The perimeter of an isosceles triangle is 544 cm and each of the equal sides is $\frac{5}{6}$ times the base . What is the area (in cm^2) of the triangle ?
(a) 38172 (b) 18372
(c) 31872 (d) 13872

170. The altitude drawn to the base of an isosceles triangle is 8 cm and its perimeter is 64 cm. The area (in cm^2) of the triangle is
(a) 240 (b) 180
(c) 360 (d) 120

171. Three circles of radius a, b, c touch each other externally. The area of the triangle formed by joining their centre is
(a) $\sqrt{(a+b+c)abc}$
(b) $(a+b+c)\sqrt{ab+bc+ca}$
(c) $ab + bc + ca$
(d) None of the above

172. The radii of two circles are 10 cm and 24 cm. The radius of a circle whose area is the sum of the area of these two circles is
(a) 36 cm (b) 17 cm
(c) 34 cm (d) 26 cm

173. A circle is inscribed in an equilateral triangle and a square is inscribed in that circle. The ratio of the areas of the triangle and the square is
(a) $\sqrt{3}:4$ (b) $\sqrt{3}:8$
(c) $3\sqrt{3}:2$ (d) $3\sqrt{3}:1$

174. If area of an equilateral triangle is a and height b, then value of $\frac{b^2}{a}$ is:
(a) 3 (b) $\frac{1}{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

175. If $\triangle ABC$ is similar to $\triangle DEF$ such that BC = 3 cm, EF = 4 cm and area of $\triangle ABC$ = 54 cm^2, then the area of $\triangle DEF$ is :
(a) 66 cm^2 (b) 78 cm^2
(c) 96 cm^2 (d) 54 cm^2

176. The area of two similar triangles ABC and DEF are 20cm^2 and 45 cm^2 respectively. If AB =5 cm, then DE is equal to
(a) 6.5 cm (b) 7.5 cm
(c) 8.5 cm (d) 5.5 cm

177. C_1 and C_2 are two concentric circles with centre at O, Their radii are 12 cm and 3 cm, respectively. B and C are the point of contact of two tangents drawn to C_2 from a point A lying on the circle C_1. Then, the area of the quadrilateral ABOC is
(a) $\frac{9\sqrt{15}}{2}$ sq . cm
(b) $12\sqrt{15}$ sq. cm
(c) $9\sqrt{15}$ sq. cm
(d) $6\sqrt{15}$ sq. cm

178. From a point P which is at a distance of 13 cm from centre O of a circle of radius 5 cm in the same plane, a pair of tangents PQ and PR are drawn to the circle Area of quadrilateral PQOR is
(a) 65 cm^2 (b) 60 cm^2
(c) 30 cm^2 (d) 90 cm^2

179. A circular road runs around a circular ground. If the difference between the circumference of the outer circle and the inner circle is 66 meters, the width of the road is: (Take $\pi = \frac{22}{7}$)
(a) 10.5 metres (b) 7 metres
(c) 5.25 metres (d) 21 metres

180. A person observed that he required 30 seconds less time to cross a circular ground along its diameter than to cover it once along the boundary. If his speed was 30 m/ minutes. then the radius of the circular ground is (Take $\pi = \frac{22}{7}$):
(a) 5.5 m (b) 7.5 m
(c) 10.5 m (d) 3.5 m

181. The difference of perimeter and diameter of a circle is X unit. The diameter of the circle is
(a) $\frac{X}{\pi-1}$ unit (b) $\frac{X}{\pi+1}$ unit
(c) $\frac{X}{\pi}$ unit (d) $\left(\frac{X}{\pi}-1\right)$ unit

182. The area of the circumcircle of an equilateral triangle is 3π sq. cm .The perimeter of the triangle is
(a) $3\sqrt{3}$ cm (b) 9 cm
(c) 18 cm (d) 3 cm

183. A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope streched and describes 88 metres when it has traced out 72° at the centre, the length of the rope is (Take $\pi = \frac{22}{7}$)
(a) 70 m (b) 75 m
(c) 80 m (d) 65 m

184. Three circles of radii 3.5 cm, 4.5 cm and 5.5 cm touch each other externally. Then the perimeter of the triangle formed by joining the centres of the circles, in cm is

(a) 27
(b) π [$(3.5)^2 + (4.5)^2 + (5.5)^2$]
(c) 27π
(d) 13.5

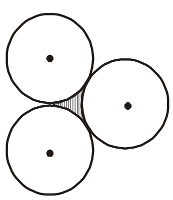
185. Three sides of a triangular field are of length 15 m, 20m and 25 m long respectively. Find the cost of sowing seeds in the field at the rate of 5 rupees per sq. m
(a) Rs.300 (b) Rs.600
(c) Rs.750 (d) Rs.150

186. A chord of length 30 cm is at a distance of 8 cm from the centre of a circle. The radius of the circle is:
(a) 17 cm (b) 23 cm
(c) 21 cm (d) 19 cm

187. The radius of the incircle of a triangle whose sides are 9 cm, 12 cm and 15 cm is
(a) 9 cm (b) 13 cm
(c) 3 cm (d) 6 cm

188. The ratio of inradius and circumradius of a square is :
(a) $1:\sqrt{2}$ (b) $\sqrt{2}:\sqrt{3}$
(c) 1 : 3 (d) 1 : 2

189. Three circles of equal radius 'a' cm touch each other. The area of the shaded region is :



(a) $\left(\frac{\sqrt{3}+\pi}{2}\right)$ a²sq. cm
(b) $\left(\frac{6\sqrt{3}-\pi}{2}\right)$ a²sq. cm
(c) $\left(\sqrt{3}-\pi\right)$ a²sq. cm
(d) $\left(\frac{2\sqrt{3}-\pi}{2}\right)$ a²sq. cm

190. ABC is a right angled triangle. B being the right angle Mid- points of BC and AC are respectively B' and A'. Area of $\triangle$A'B'C is
(a) $\frac{1}{2}\times$ area of $\triangle ABC$
(b) $\frac{2}{3}\times$ area of $\triangle ABC$
(c) $\frac{1}{4}\times$ area of $\triangle ABC$
(d) $\frac{1}{8}\times$ area of $\triangle ABC$

191. A wire of length 44 cm is first bent to form a circle and then rebent to form a square. The difference of the two enclosed areas is
(a) 44 cm² (b) 33 cm²
(c) 55 cm² (d) 66 cm²

192. $\angle ACB$ is an angle in the semicircle of diameter AB = 5 cm and AC : BC = 3: 4. The area of the triangle ABC is
(a) $6\sqrt{2}$ sq. cm (b) 4 sq. cm
(c) 12 sq. cm (d) 6 sq. cm

193. If the lengths of the sides AB, BC and CA of a triangle ABC are 10 cm, 8 cm and 6 cm respectively and If M is the mid-point of BC and $MN \parallel AB$ to cut AC at N, then area of the trapezium ABMN is equal to
(a) 18 sq. cm (b) 20 sq. cm
(c) 12 sq. cm (d) 16 sq. cm

194. In an equilateral triangle of side 24 cm, a circle is inscribed touching its sides. The area of the remaining portion of the triangle is
($\sqrt{3}$ =1.732)
(a) 98.55 sq. cm (b) 100 sq. cm
(c) 101 sq. cm (d) 95 sq. cm

195. Two sides of a plot measuring 32 m and 24 m and the angle between them is a perfect right angle. The other two sides measure 25 m each and the other three angles are not right angles. The area of the plot in m² is
(a) 768 (b) 534
(c) 696.5 (d) 684

196. A and b are two sides adjacent to the right angle of a right angled triangle and p is the perpendicular drawn to the hypotenuse from the opposite vertex. Then p² is equal to
(a) a² +b² (b) $\frac{1}{a^2}+\frac{1}{b^2}$
(c) $\frac{a^2b^2}{a^2+b^2}$ (d) a² – b²

197. A is the centre of circle whose radius is 8 and B is the centre of a circle whose diameter is 8. If these two circles touch externally, then the area of the circle with diamter AB is
(a) 36π (b) 64π
(c) 144π (d) 256π

198. If the numerical value of the height and the area of an equilateral triangle be same. then the length of each side of the triangle is
(a) 2 units (b) 4 units
(c) 5 units (d) 8 units

199. If the length of a side of the square is equal to that of the diameter of a circle, then the ratio of the area of the square and that of the circle ($\pi = \frac{22}{7}$)
(a) 14 : 11 (b) 7 : 11
(c) 11 : 14 (d) 11 : 7

200. The median of an equilateral triangle is $6\sqrt{3}$ cm. The area (in cm²) of the triangle is
(a) 72 (b) 108
(c) $72\sqrt{3}$ (d) $36\sqrt{3}$

201. If the numerical value of the circumference and area of a circle is same, then the area is
(a) 6π sq. units
(b) 4π sq. units
(c) 8π sq. units
(d) 12π sq. units

202. The area of an equilateral triangle is 48 sq. cm. The length of the side is
(a) $4\sqrt{8}$ cm (b) $4\sqrt{3}$ cm
(c) 8 cm (d) $8\sqrt[4]{3}$ cm

203. The external fencing of a circular path around a circular plot of land is 33m more than its interior fencing. The width of the path around the plot is
(a) 5.52 m (b) 5.25 m
(c) 2.55 m (d) 2.25 m

204. The perimeter of a triangle is 54 m and its sides are in the ratio 5 : 6 : 7. The area of the triangle is
(a) 18 m² (b) $54\sqrt{6}$ m²
(c) $27\sqrt{2}$ m² (d) 25 m²

205. A circular wire of diameter 112 cm is cut and bent in the form of a rectangle whose sides are in the ratio of 9 : 7. The smaller side of the rectangle is
(a) 77 cm (b) 97 cm
(c) 67 cm (d) 84 cm

206. If the perimeter of an equilateral triangle be 18 cm, then the length of each median is

(a) $3\sqrt{2}$ cm (b) $2\sqrt{3}$ cm

(c) $3\sqrt{3}$ cm (d) $2\sqrt{2}$ cm

207. Two equal maximum sized circular plates are cut off from a circular paper sheet of circumference 352 cm. Then the circumference of each circular plate is

(a) 176 cm (b) 150 cm
(c) 165 cm (d) 180 cm

208. The inradius of an equilateral triangle is $\sqrt{3}$ cm, then the perimeter of that triangle is

(a) 18 cm (b) 15 cm
(c) 12 cm (d) 6 cm

209. The difference between the circumference and diameter of a circle is 150 m. The radius of that circle is (Take $\pi = \frac{22}{7}$)

(a) 25 metre (b) 35 metre
(c) 30 metre (d) 40 metre

210. The perimeters of a circle, a square and an equilateral triangle are same and their areas are C, S and T respectively. Which of the following statement is true ?

(a) C = S = T (b) C > S > T
(c) C < S < T (d) S < C < T

211. A horse takes $2\frac{1}{2}$ seconds to complete a round around a circular field. If the speed of the horse was 66 m/sec, then the radius of the field is, [Given $\pi = \frac{22}{7}$]

(a) 25.62 m (b) 26.52 m
(c) 25.26 m (d) 26.25 m

212. The diameter of the front wheel of an engine is $2x$ cm and that of rear wheel is $2y$ cm to cover the same distance, find the number of times the rear wheel will revolve when the front wheel revolves 'n' times,

(a) $\frac{n}{xy}$ times (b) $\frac{yn}{x}$ times

(c) $\frac{nx}{y}$ times (d) $\frac{xy}{n}$ times

213. A bicycle wheel has a diameter (including the tyre) of 56 cm. The number of times the wheel will rotate to cover a distance of 2.2 km is (Assume $\pi = \frac{22}{7}$)

(a) 625 (b) 1250
(c) 1875 (d) 2500

214. If the altitude of an equilateral triangle is $12\sqrt{3}$ cm, then its area would be;

(a) $36\sqrt{3}$ cm² (b) $144\sqrt{3}$ cm²
(c) 72 cm² (d) 12 cm²

215. Let C_1 and C_2 be the inscribed and circumscribed circles of a triangle with sides 3 cm, 4 cm and 5 cm, then $\frac{\text{area of } C_1}{\text{area of } C_2}$ is

(a) $\frac{9}{25}$ (b) $\frac{16}{25}$

(c) $\frac{9}{16}$ (d) $\frac{4}{25}$

216. A circular swimming pool is surrounded by a concrete wall 4m wide. If the area of the concrete wall surrounding the pool is $\frac{11}{25}$ that of the pool, then the radius(in m) of the pool :

(a) 8 (b) 16 (c) 30 (d) 20

217. If the area of a circle is A, radius of the circle is r and circumference of it is c, then

(a) rC = 2A (b) $\frac{C}{A} = \frac{r}{2}$

(c) $AC = \frac{r^2}{4}$ (d) $\frac{A}{r} = C$

218. The sides of a triangle having area 7776 sq. cm are in the ratio 3 : 4 : 5. The perimeter of the triangle is:

(a) 400 cm (b) 412 cm
(c) 424 cm (d) 432 cm

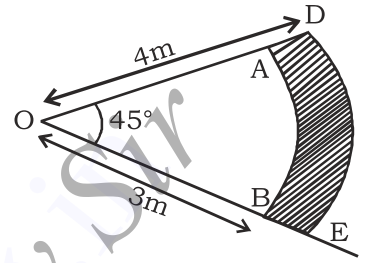
219. The perimeter of a sheet of paper in the shape of a quadrant of a circle is 75 cm. Its area would be ($\pi = \frac{22}{7}$)

(a) 512.25 cm² (b) 346.5 cm²
(c) 100 cm² (d) 693 cm²

220. A circle is inscribed in an equilateral triangle of side 8m. The approximate area of the unoccupied space inside the triangle is

(a) 21 m² (b) 11 m²
(c) 20 m² (d) 22 m²

221. In the figure, OED and OBA are sectors of a circle with centre O. The area of the shaded portion.



(a) $\frac{11}{16}$ m² (b) $\frac{11}{8}$ m²

(c) $\frac{11}{2}$ m² (d) $\frac{11}{4}$ m²

222. If the circumference of a circle is $\frac{30}{\pi}$, then the diameter of the circle is

(a) 30 (b) $\frac{15}{\pi}$ (c) 60π (d) $\frac{30}{\pi^2}$

223. The outer and inner diameter of a circular path be 728 cm and 700 cm respectively. The breadth of the path is

(a) 7 cm (b) 14 cm
(c) 28 cm (d) 20 cm

224. A piece of wire when bent to form a circle will have a radius of 84 cm. If the wire is bent to form a square, the length of a side of the square is

(a) 152 cm (b) 168 cm
(c) 132 cm (d) 225 cm

225. The area of a circle is 324 π sq.cm. The length of its longest chord (in cm.) is

(a) 36 (b) 38 (c) 28 (d) 32

226. The circumference of a triangle is 24 cm and the circumference of its in-circle is 44 cm. Then the area of the triangle is (taking $\pi = \frac{22}{7}$)

(a) 56 square cm
(b) 48 square cm
(c) 84 square cm
(d) 68 square cm

227. If the length of each of two equal sides of an isosceles triangle is 10 cm. and the adjacent angle is 45°, then the area of the triangle is
(a) $20\sqrt{2}$ square cm
(b) $25\sqrt{2}$ square cm
(c) $12\sqrt{2}$ square cm
(d) $15\sqrt{2}$ square cm

228. The inner-radius of a triangle is 6 cm, and the sum of the lengths of its sides is 50 cm. The area of the triangle (in sq. cm.) is
(a) 150 (b) 300 (c) 50 (d) 56

229. One of the angles of a right-angled triangle is 15°, and the hypotenuse is 1 m. The area of the triangle (in sq. cm.) is
(a) 1220 (b) 1250
(c) 1200 (d) 1215

230. If an isosceles triangle the length of each equal side is 'a' units and that of the third side is 'b' units, then its area will be
(a) $\frac{a}{4}\sqrt{4a^2-a^2}$ sq. units
(b) $\frac{b}{4}\sqrt{4a^2-b^2}$ sq. units
(c) $\frac{a}{2}\sqrt{2a^2-b^2}$ sq. units
(d) $\frac{b}{2}\sqrt{a^2-2b^2}$ sq. units

231. What is the position of the circumcentre of an obtuse-angles triangle?
(a) It is the vertex opposite to the largest side.
(b) It is the mid point of the largest side.
(c) It lies outside the triangles.
(d) It lies inside the triangles.

232. The ratio of cirumference and diameter of a circle is 22 : 7. If the circumference be $1\frac{4}{7}$ m, then the radius of the circle is:
(a) $\frac{1}{4}$ m (b) $\frac{1}{3}$ m
(c) $\frac{1}{2}$ m (d) 1 m

233. The area of a circle whose radius is the diagonal of a square whose area is 4 is:
(a) 4π (b) 8π
(c) 6π (d) 16π

234. The diagonals of a rhombus are 32 cm and 24 cm respectively. The perimeter of the rhombus is:
(a) 80 cm (b)72 cm
(c) 68 cm (d) 64 cm

235. The diagonals of a rhombus are 24 cm and 10 cm. The perimeter of the rhombus (in cm) is :
(a) 68 (b) 65 (c) 54 (d) 52

236. The perimeter of a rhombus is 40 cm, If one of the diagonals be 12 cm long, what is the length of the other diagonal ?
(a) 12 cm (b) $\sqrt{136}$ cm,
(c) 16 cm (d) $\sqrt{44}$ cm

237. The perimeter of a rhombus is 40 m and its height is 5m its area is:
(a) 60 m^2 (b) 50 m^2
(c) 45 m^2 (d) 55 m^2

238. The perimeter of a rhombus is 40 cm. If the length of one of its diagonals be 16 cm, the length of the other diagonal is
(a) 14 cm (b) 15 cm
(c) 16 cm (d) 12 cm

239. The area of a rhombus is 150 cm^2. The length of one of its diagonals is 10 cm. The length of the other diagonal is :
(a) 25 cm (b) 30 cm
(c) 35 cm (d) 40 cm

240. The area of a regular hexagon of side $2\sqrt{3}$ cm is :
(a) $18\sqrt{3}$ cm^2 (b) $12\sqrt{3}$ cm^2
(c) $36\sqrt{3}$ cm^2 (d) $27\sqrt{3}$ cm^2

241. Each side of a regular hexagon is 1 cm. The area of the hexagon is
(a) $\frac{3\sqrt{3}}{2}$ cm^2 (b) $\frac{3\sqrt{3}}{4}$ cm^2
(c) $4\sqrt{3}$ cm^2 (d) $3\sqrt{2}$ cm^2

242. The length of one side of a rhombus is 6.5 cm and its altitude is 10 cm. If the length of its diagonal be 26 cm, the length of the other diagonal will be:
(a) 5 cm (b) 10 cm
(c) 6.5 cm (d) 26 cm

243. The measure of each of two opposite angles of a rhombus is 60° and the measure of one of its sides is 10 cm. The length of its smaller diagonal is :
(a) 10cm (b) $10\sqrt{3}$ cm
(c) $10\sqrt{2}$ cm (d) $\frac{5}{2}\sqrt{2}$ cm

244. The perimeter of a rhombus is 100 cm, If one of its diagonals is 14 cm.Then the area of the rhombus is
(a) 144 cm^2 (b) 225 cm^2
(c) 336 cm^2 (d) 400 cm^2

245. The ratio of the length of the parallel sides of a trapezium is 3 : 2. The shortest distance between them is 15 cm. If the area of the trapezium is 450 cm^2 the sum of the length of the parallel sides is
(a) 15 cm (b) 36 cm
(c) 42 cm (d) 60 cm

246. A parallelogram has sides 15 cm and 7 cm long. The length of one of the diagonals is 20 cm. The area of the parallelogram is
(a) 42 cm^2 (b) 60 cm^2
(c) 84 cm^2 (d) 96 cm^2

247. Sides of a parallelogram are in the ratio 5 : 4. Its area is 1000 sq. units. Altitude on the greater side is 20 units. Altitude on the smaller side is
(a) 20 units (b) 25 units
(c) 10 units (d) 15 units

248. The perimeter of a rhombus is 40 cm and the measure of an angle is 60°, then the area of it is:
(a) $100\sqrt{3}$ cm^2 (b) $50\sqrt{3}$ cm^2
(c) $160\sqrt{3}$ cm^2 (d) 100 cm^2

249. Two adjacent sides of a parallelogram are of length 15 cm and 18 cm, If the distance between two smaller sides is 12 cm, then the distance between two bigger sides is
(a) 8 cm (b) 10 cm
(c) 12 cm (d) 15 cm

250. A parallelogram ABCD has sides AB = 24 cm and AD = 16 cm. The distance between the sides AB and DC is 10 cm. Find the distance between the sides AD and BC.
(a) 15 cm (b) 18 cm
(c) 16 cm (d) 9 cm

251. The adjacent sides of a paralleogram are 36 cm and 27 cm in length, If the distance between the shorter sides is 12 cm, then the distance between the longer sides is
(a) 10 cm (b) 12 cm
(c) 16 cm (d) 9 cm

252. If the diagonals of a rhombus are 8 cm and 6 cm, then the area of square having same side as that of rhombus is
(a) 25 (b) 55 (c) 64 (d) 36

253. Two circles with centres A and B and radius 2 units touch each other externally at 'C' A third circle with centre 'C' and radius '2' units meets other two at D and E. Then the area of the quadrilateral ABDE is
(a) $2\sqrt{2}$ sq. units
(b) $3\sqrt{3}$ sq. units
(c) $3\sqrt{2}$ sq. units
(d) $2\sqrt{3}$ sq. units

254. The perimeter of a non-square rhombus is 20 cm. One of its diagonal is 8 cm. The area of the rhombus is
(a) 28 sq. cm (b) 20 sq. cm
(c) 22 sq. cm (d) 24 sq. cm

255. The perimeter of a rhombus is 100 cm and one of its diagonals is 40 cm. Its area (in cm^2) is
(a) 1200 (b) 1000
(c) 600 (d) 500

256. In ΔABC, D and E are the points of sides AB and BC respectively such that $DE \parallel AC$ and AD : BD = 3 : 2. The ratio of area of trapezium ACED to that of ΔBED is
(a) 4 : 15 (b) 15 : 4
(c) 4 : 21 (d) 21 : 4

257. ABCD is a trapezium in which $AB \parallel DC$ and AB = 2 CD. The diagonals AC and BD meet at O. The ratio of area of triangles AOB and COD is
(a) 1 : 1 (b) $1:\sqrt{2}$
(c) 4 : 1 (d) 1 : 4

258. The length of each side of a rhombus is equal to the length of the side of a square whose diagonal is $40\sqrt{2}$ cm. If the length of the diagonals of the rhombus are in the ratio 3 : 4, then its area (in cm^2) is
(a) 1550 (b) 1600
(c) 1535 (d) 1536

259. ABCD is a parallelogram BC is produced to Q such that BC = CQ. Then
(a) area (ΔABC) = area (ΔDCQ)
(b) area (ΔABC) > area (ΔDCQ)
(c) area (ΔABC) < area (ΔDCQ)
(d) area $(\Delta ABC) \neq$ area (ΔDCQ)

260. ABCD is a parallelogram. P and Q are the mid- points of sides BC and CD respectively. If the area of ΔABC is 12 cm^2, then the area of ΔAPQ is
(a) 12 cm^2 (b) 8 cm^2
(c) 9 cm^2 (d) 10 cm^2

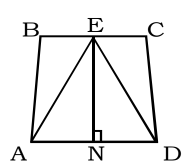
261. The area of a rhombus is 216 cm^2 and the length of its one diagonal is 24 cm. The perimeter (in cm) of the rhombus is
(a) 52 (b) 60
(c) 120 (d) 100

262. One of the four angles of a rhombus is 60°. If the length of each side of the rhombus is 8 cm, then the length of the longer diagonal is
(a) $8\sqrt{3}$ cm (b) 8 cm
(c) $4\sqrt{3}$ cm (d) $\frac{8}{\sqrt{3}}$ cm

263. The diagonals of a rhombus are 12 cm and 16 cm respectively. The length of one side is
(a) 8 cm (b) 6 cm
(c) 10 cm (d) 12 cm

264. A parallelogram has sides 60 m and 40 m and one of its diagonals is 80 m long. Its area is
(a) $500\sqrt{15}$ m^2 (b) $600\sqrt{15}$ m^2
(c) $400\sqrt{15}$ m^2 (d) $450\sqrt{15}$ m^2

265. ABCD is a trapezium with AD and BC parallel sides. The ratio of the area of ABCD to that of Δ AED is



(a) $\frac{\overline{AD}}{\overline{BC}}$ (b) $\frac{\overline{BE}}{\overline{EC}}$
(c) $\frac{\overline{AD}+\overline{BE}}{\overline{AD}+\overline{CE}}$ (d) $\frac{\overline{AD}+\overline{BC}}{\overline{AD}}$

266. perimeter of a rhombus is 2p unit and sum of length of diagonals is m unit, then area of the rhombus is
(a) $\frac{1}{4}m^2p$ sq unit
(b) $\frac{1}{4}mp^2$ sq unit
(c) $\frac{1}{4}\left(m^2 - p^2\right)$ sq unit
(d) $\frac{1}{4}\left(m^2 + p^2\right)$ sq unit

267. Area of regular hexagon with side 'a' is
(a) $\frac{3\sqrt{3}}{4}$ a^2 sq. unit
(b) $\frac{12}{2\sqrt{3}}$ a^2 sq. unit
(c) $\frac{9}{2\sqrt{3}}$ a^2 sq. unit
(d) $\frac{6}{\sqrt{2}}$ a^2 sq. unit

268. In ΔABC, D and E are two points on the sides AB and AC respectively so that $DE \parallel BC$ and $\frac{AD}{BD} = \frac{2}{3}$. Then $\frac{\text{the area of trapezium } DECB}{\text{the area of } \Delta ABC}$ is equal to
(a) $\frac{5}{9}$ (b) $\frac{21}{25}$
(c) $1\frac{4}{5}$ (d) $5\frac{1}{4}$

269. The sides of a rhombus are 10 cm each and a diagonal measures 16 cm. Area of the rhombus is
(a) 96 sq. cm (b) 160 sq. cm
(c) 100 sq. cm (d) 40 sq. cm

270. The lengths of two parallel sides of a trapezium are 6 cm and 8 cm. If the height of the trapezium be 4 cm, then its area is
(a) 28 cm² (b) 56 cm²
(c) 30 cm² (d) 36 cm²

271. If diagonals of a rhombus are 24 cm and 32 cm, then perimeter of that rhombus is
(a) 80 cm (b) 84 cm
(c) 76 cm (d) 72 cm

272. The area of an isosceles trapezium is 176 cm² and the height is 2/11th of the sum of its parallel sides. If the ratio of the length of the parallel sides is 4 : 7, then the length of a diagonal (in cm) is
(a) $2\sqrt{137}$ (b) 24
(c) $\sqrt{137}$ (d) 28

273. The perimeter of a rhombus is 60 cm and one of its diagonal is 24 cm. The area of the rhombus is
(a) 432 sq.cm (b) 216 sq.cm
(c) 108 sq.cm (d) 206 sq.cm

274. The area of the parallelogram whose length is 30 cm, width is 20 cm and one diagonal is 40 cm is
(a) $200\sqrt{15}$ cm²
(b) $300\sqrt{15}$ cm²
(c) $100\sqrt{15}$ cm²
(d) $150\sqrt{15}$ cm²

275. The area of a rhombus is 256 sq.cm. and one of its diagonal is twice the other in length.Then length of its larger diagonal is
(a) 32 cm (b) 48 cm
(c) 36 cm (d) 24 cm

276. The length of two parallel sides of a trapezium are 15 cm and 20 cm. If its area is 175 sq.cm, then its height is:
(a) 25 cm (b) 10 cm
(c) 20 cm (d) 15 cm

277. The cost of carpenting a room is Rs. 120 . If the width had been 4 metres less, the cost of the Carpet would have been Rs. 20 less. The width of the room is :
(a) 24 m (b) 20 m
(c) 25 m (d) 18.4 m

278. The floor of a corridor is 100 m long and 3 m wide. Cost of covering the floor with carpet 50 cm wide at the ratio of Rs. 15 per m is
(a) Rs. 4500 (b) Rs. 9000
(c) Rs. 7500 (d) Rs. 1900

279. A playground is in the shape of a rectangle. A sum of Rs. 1,000 was spent to make the ground usable at the rate of 25 paise per sq. m. The breadth of the ground is 50 m. If the length of the ground is increased by 20 m. what will be the expenditure (in rupees) at the same rate per sq. m.?
(a) 1,250 (b) 1,000
(c) 1,500 (d) 2,250

280. A hall 25 metres long and 15 metres broad is surrounded by a verandah of uniform width of 3.5 metres. The cost of flooring the varandah, at Rs. 27.50 per square metre is
(a) Rs. 9149.50 (b) Rs. 8146.50
(c) Rs. 9047.50 (d) Rs. 4186.50

281. The outer circumference of a circular race-track is 528 metre. The track is everywhere 14 metre wide. Cost of levelling the track at the rate of Rs.10 per sq. metre is :
(a) Rs. 77660 (b) Rs. 67760
(c) Rs. 66760 (d) Rs. 76760

282. The length and breadth of a rectangular field are in the ratio of 3 : 2. If the perimeter of the field is 80m, its breadth (in metres) is :
(a) 18 (b) 16 (c) 10 (d) 24

283. The sides of a rectangular plot are in the ratio 5 : 4 and its area is equal to 500 sq.m The perimeter of the plot is :
(a) 80 m (b) 100 m
(c) 90 m (d) 95 m

284. ABC is a triangle with base AB, D is a point on AB such that AB = 5 and DB =3. What is the ratio of the area of $\triangle ADC$ to the area of $\triangle ABC$?
(a) 2/5 (b) 2/3
(c) 9/25 (d) 4/25

285. If the area of a triangle is 1176 cm² and the ratio of base and corresponding altitude is 3 : 4, then the altitude of the triangle is:
(a) 42 cm (b) 52 cm
(c) 54 cm (d) 56 cm

286. The sides of a triangle are in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. If the perimeter of the triangle is 52 cm, the length of the smallest side is :
(a) 24 cm (b) 10 cm
(c) 12 cm (d) 9 cm

287. If the diagonals of two squares are in the ratio of 2 : 5. Their area will be in the ratio of
(a) $\sqrt{2}:\sqrt{5}$ (b) 2 : 5
(c) 4 : 25 (d) 4 : 5

288. The ratio of base of two triangles is $x : y$ and that of their areas is $a : b$. Then the ratio of their corresponding altitudes will be:
(a) $\frac{a}{y}:\frac{b}{x}$ (b) $1 : 1$
(c) $ay : bx$ (d) $\frac{x}{a}:\frac{b}{y}$

289. The area of a field in the shape of a trapezium measures 1440m². The perpendicular distance between its parallel sides is 24m. If the ratio of the parallel sides is 5 : 3, the length of the longer parallel side is :
(a) 75 m (b) 45 m
(c) 120 m (d) 60 m

290. If the ratio of areas of two squares is 225 : 256, then the ratio of their perimeter is :
(a) 225 : 256 (b) 256 : 225
(c) 15 : 16 (d) 16 : 15

291. The area of a triangle is 216 cm² and its sides are in the ratio 3 : 4 : 5. The perimeter of the triangle is:
(a) 6 cm (b) 12 cm
(c) 36 cm (d) 72 cm

292. A circular wire of radius 42 cm is bent in the form of a rectangle whose sides are in the ratio of 6 : 5. The smaller side of the rectangle is (Take $\pi=\frac{22}{7}$):
(a) 60 cm (b) 30 cm
(c) 25 cm (d) 36 cm

293. The ratio of the outer and the inner perimeter of a circular path is 23 : 22, If the path is 5 meters wide the diameter of the inner circle is:
(a) 110 m (b) 55 m
(c) 220 m (d) 230 m

294. The angles of a triangle are in the ratio 3 : 4 : 5. The measure of the largest angle of the triangle is
(a) 60° (b) 75°
(c) 120° (d) 150°

295. The ratio of the area of a square to that of the square drawn on its diagonal is:
(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

296. A square and an equilateral triangle are drawn on the same base. The ratio of their area is
(a) 2 :1 (b) 1 : 1
(c) $\sqrt{30}:\sqrt{4}$ (d) $4:\sqrt{3}$

297. If the area of a circle and a square are equal, then the ratio of their perimeter is
(a) 1 : 1 (b) 2 : π
(c) π : 2 (d) $\sqrt{\pi}$: 2

298. The area of two equilateral triangles are in the ratio 25 : 36. Their altitudes will be in the ratio:
(a) 36 : 25 (b) 25 : 36
(c) 5 : 6 (d) $\sqrt{5}:\sqrt{6}$

299. If the length and the perimeter of a rectangle are in the ratio 5 : 16. then its length and breadth will be in the ratio
(a) 5 : 11 (b) 5 : 8
(c) 5 : 4 (d) 5 :3

300. Through each vertex of a triangle, a line parallel to the opposite side is drawn. the ratio of the perimeter of the new triangle. thus formed, with that of the original triangle is
(a) 3 : 2 (b) 1 : 2
(c) 2 : 1 (d) 2 : 3

301. The ratio of the number giving the measure of the circumference and the area of a circle of radius 3 cm is
(a) 1 : 3 (b) 2 : 3
(c) 2 : 9 (d) 3 : 2

302. The height of an equilateral triangle is $4\sqrt{3}$ cm. The ratio of the area of its circumcircle to that of its in-circle is
(a) 2 : 1 (b) 4 : 1
(c) 4 : 3 (d) 3 : 2

303. The radius of circle A is twice that of circle B and the radius of circle B is twice that of circle C. Their area will be in the ratio
(a) 16 : 4 : 1 (b) 4 : 2 : 1
(c) 1 : 2 : 4 (d) 1 : 4 : 16

304. A circle and a square have equal areas. the ratio of a side of the square and the radius of the circle is
(a) $1:\sqrt{\pi}$ (b) $\sqrt{\pi}:1$
(c) $1:\pi$ (d) $\pi:1$

305. The sides of a triangle are in the ratio $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$ and its perimeter is 94cm. The length of the smallest side of the triangle is:
(a) 18 cm (b) 22.5 cm
(c) 24 cm (d) 27 m

306. The sides of a quadrilateral are in the ratio 3 : 4 : 5 : 6 and its perimeter is 72 cm. The length of its greatest side (in cm) is
(a) 24 (b) 27 (c) 30 (d) 36

307. The ratio of the radii of two wheels is 3: 4. The ratio of their circumference is
(a) 4 : 3 (b) 3 : 4
(c) 2 : 2 (d) 3 : 2

308. The sides of a triangle are in the ratio 2 : 3 : 4. the perimeter of the triangle is 18 cm. The area (in cm^2) of the triangle is
(a) 9 (b) 36
(c) $\sqrt{42}$ (d) $3\sqrt{15}$

309. The ratio of the areas of the circumcircle and the incirle of an equilateral triangle is
(a) 2 : 1 (b) 4 : 1
(c) 8 : 1 (d) 3 :2

310. In ΔABC, the medians CD and BE intersect each other at O, then the ratio of the areas of ΔODE and ΔOBC is
(a) 1 : 4 (b) 6 : 1
(c) 1 : 12 (d) 12 : 1

311. The ratio of the area of two isosceles triangles having the same vertical angle (i.e. angle between equal sides) is 1 : 4. The ratio of their heights is
(a) 1 : 4 (b) 2 : 5
(c) 1 : 2 (d) 3 : 4

312. The ratio of length of each equal side and the third side of an isosceles triangle is 3 : 4. If the area is $8\sqrt{5}$ units^2. the third side is
(a) 3 units
(b) $2\sqrt{5}$ square units units
(c) $8\sqrt{2}$ units
(d) 12 units

313. The ratio of sides of a triangle is 3 : 4 : 5. If area of the triangle is 72 square unit then the length of the smallest side is :
(a) $4\sqrt{3}$ unit (b) $5\sqrt{3}$ unit
(c) $6\sqrt{3}$ unit (d) $3\sqrt{3}$ unit

314. The ratio of sides of a triangle is 3 : 4 : 5 and area of the triangle is 72 square units. Then the area of an equilateral triangle whose perimeter is same as that of the previous triangle is
(a) $32\sqrt{3}$ square units
(b) $48\sqrt{3}$ square units
(c) 96 square units
(d) $60\sqrt{3}$ square units

315. The parallel sides of a trapezium are in a ratio 2 : 3 and their shortest distance is 12 cm.If the area of the trapezium is 480 sq. cm., the longer of the parallel sides is of length :
(a) 56 cm (b) 36 cm
(c) 42 cm (d) 48 cm

316. An equilateral triangle is drawn on the diagonal of a square . The ratio of the area of the triangle to that of the square is
(a) $\sqrt{3}:2$ (b) $1:\sqrt{3}$
(c) $2:\sqrt{3}$ (d) $4:\sqrt{3}$

317. Two triangles ABC and DEF are similar to each other in which AB = 10 cm, DE = 8 cm. Then the ratio of the area of triangles ABC and DEF is
(a) 4 : 5 (b) 25 : 16
(c) 64 : 125 (d) 4 : 7

318. The ratio between the area of two circles is 4 : 7. What will be the ratio of their radii ?
(a) $2:\sqrt{7}$ (b) 4 : 7
(c) 16 : 49 (d) 4 : $\sqrt{7}$

319. The area of a circle is proportional to the square of its radius. A small circle of radius 3 cm is drawn within a larger circle of radius 5 cm. Find the ratio of the area of the annular zone to the area of the larger circle (Area of the annular zone is the difference between the area of the larger circle and that of the smaller circle)
(a) 9 : 16 (b) 9 : 25
(c) 16 : 25 (d) 16 : 27

320. The diameter of two circles are the side of a square and the diagonal of the square. The ratio of the area of the smaller circle and the larger circle is
(a) 1 : 2 (b) 1 : 4
(c) $\sqrt{2}:\sqrt{3}$ (d) $1:\sqrt{2}$

321. The ratio of the area of an equilateral triangle and that of its circumcircle is
(a) $2\sqrt{3}:2\pi$ (b) $4:\pi$
(c) $3\sqrt{3}:4\pi$ (d) $7\sqrt{2}:2\pi$

322. If the perimeters of a rectangle and a square are equal and the ratio of two adjacent sides of the rectangle is 1 : 2 then the ratio of area of the rectangle and that of the square is
(a) 1 : 1 (b) 1 : 2
(c) 2 : 3 (d) 8 : 9

323. The perimeter of a rectangle and an equilateral triangle are same. Also, one of the sides of the rectangle is equal to the side of the triangle. The ratio of the area of the rectangle and the triangle is
(a) $\sqrt{3}:1$ (b) $1:\sqrt{3}$
(c) $2:\sqrt{3}$ (d) $4:\sqrt{3}$

324. The radius of a circle is a side of a square. The ratio of the area of the circle and the square is
(a) $1 : \pi$ (b) $\pi : 1$
(c) $\pi : 2$ (d) $2 : \pi$

325. ABC is an isosceles right angled triangle with $\angle B$ = 90°, On the sides AC and AB, two equilateral triangles ACD and ABE have been constructed. The ratio of area of ΔABE and ΔACD is
(a) 1 : 3 (b) 2 : 3
(c) 1 : 2 (d) $1:\sqrt{2}$

326. Two triangles ABC and DEF are similar to each other in which AB= 10 cm, DE=8 cm. Then the ratio of the area of triangles ABC and DEF is
(a) 4 : 5 (b) 25 : 16
(c) 64 : 125 (d) 4 : 7

327. ABC is a right angled triangle, B being the right angle. Mid-points of BC and AC are respectively B' and A'. The ratio of the area of the quadrilateral AA'B'B to the area of the triangle ABC is
(a) 1 : 2 (b) 2 : 3
(c) 3 : 4
(d) None of the above

328. The sides of a triangle are in the ratio $\frac{1}{4}:\frac{1}{6}:\frac{1}{8}$ and its perimeter is 91 cm. The difference of the length of longest side and that of shortest side is
(a) 19 cm (b) 20 cm
(c) 28 cm (d) 21 cm

329. If the arcs of unit length in two circles subtend angles of 60° and 75° at their centres, the ratio of their radii is
(a) 3 : 4 (b) 4 : 5
(c) 5 : 4 (d) 3 : 5

330. ABCD is a parallelogram in which diagonals AC and BD intersect at O. If E, F, G and H are the mid-points of AO, DO, CO and BO respectively, then the ratio of the perimeter of the quadrilateral EFGH to the perimeter of parallelogram ABCD is
(a) 1 : 4 (b) 2 : 3
(c) 1 : 2 (d) 1 : 3

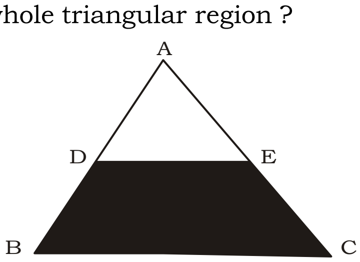
331. If the circumference of a circle increases from 4π to 8π, what change occurs in its area ?
(a) It doubles (b) It triples
(c) It quadruples (d) It is halved

332. If the length of a rectangle is increased by 25% and the width is decreased by 20%, then the area of the rectangle :
(a) Increases by 5%
(b) decreases by 5%
(c) remains unchanged
(d) increases by 10%

333. The area of a circle of radius 5 is numerically what percent of its circumfernce ?
(a) 200% (b) 255%
(c) 240% (d) 250%

334. If the circumference and area of a circle are numerically equal, then the diameter is equal to :
(a) area of the circle
(b) $\frac{\pi}{2}$ (c) 2π (d) 4

335. If D and E are the mid-points of the side AB and AC respectively of the ΔABC in the given figure here, the shaded region of the triangle is what per cent of the whole triangular region ?



(a) 50% (b) 25%
(c) 75% (d) 60%

336. The length of a rectangle is decreased by 10% and its breadth is increased by 10%, By what percent is its area changed?
(a) 0% (b) 1%
(c) 5% (d) 100%

337. The percentage increase in the area of a rectangle. If each of its sides is increased by 20%, is:
(a) 40% (b) 42%
(c) 44% (d) 46%

338. If the circumference of a circle is reduced by 50%, its area will be reduced by
(a) 12.5% (b) 25%
(c) 50% (d) 75%

339. If the side of a square is increased by 25%, then its area is increased by:
(a) 25% (b) 55%
(c) 40.5% (d) 56.25%

340. If the radius of a circle is increased by 50% . its area is increased by :
(a) 125% (b) 100%
(c) 75% (d) 50%

341. If the length of a rectangle is increased by 20% and its breadth is decreased by 20%, then its area
(a) increases by 4%
(b) decreases by 4%
(c) decreases by 1%
(d) None of these

342. If each side of a rectangle is increased by 50%, its area will be increased by
(a) 50% (b) 125%
(c) 100% (d) 250%

343. If the altitude of a triangle is increased by 10% while its area remains same, its corresponding base will have to be decreased by
(a) 10% (b) 9%
(c) $9\frac{1}{11}\%$ (d) $11\frac{1}{9}\%$

344. If the circumference of a circle is increased by 50% then the area will be increased by
(a) 50% (b) 75%
(c) 100% (d) 125%

345. The length and breadth of a rectangle are increased by 12% and 15% respectively. Its area will be increased by :
(a) $27\frac{1}{5}\%$ (b) $28\frac{4}{5}\%$
(c) 27% (d) 28%

346. If the sides of an equilateral triangle are increased by 20%, 30% and 50% respectively to form a new triangle the increase in the perimeter of the equilateral triangle is
(a) 25% (b) $33\frac{1}{3}$%
(c) 75% (d) 100%

347. Each side of a rectangular field is diminished by 40%. By how much percent is the area of the field diminished ?
(a) 32% (b) 64%
(c) 25% (d) 16%

348. The length of rectangle is increased by 60%. By what percent would the breadth to be decreased to maintain the same area?
(a) $37\frac{1}{2}$% (b) 60%
(c) 75% (d) 120%

349. The length and breadth of rectangle are increased by 20% and 25% respectively. The increase in the area of the resulting rectangle will be :
(a) 60% (b) 50%
(c) 40% (d) 30%

350. If each side of a square is increased by 10%. its area will be increased by
(a) 10% (b) 21%
(c) 44% (d) 100%

351. If the length of a rectangular plot of land is increased by 5% and the breadth is decreased by 10%. How much will its area increase or decrease?
(a) 6.5% increase
(b) 5.5% decrease
(c) 5.5% increase
(d) 6.5% decrease

352. The radius of circle is increased by 1%. How much does the area of the circle increase ?
(a) 1% (b) 1.1%
(c) 2% (d) 2.01%

353. The length of a room floor exceeds its breadth by 20m . The area of the floor remains unaltered when the length is decreased by 10 m but the breadth is increased by 5 m. The area of the floor (in square meters) is:
(a) 280 (b) 325
(c) 300 (d) 420

354. In measuring the sides of a rectangle, there is an excess of 5% on one side and 2% deficeit on the other. Then the error percent in the area is
(a) 3.3% (b) 3.0 %
(c) 2.9% (d) 2.7%

355. The length and breadth of a rectangle are increased by 30% and 20% respectively. The area of the rectangle so formed exceeds the area of the square by
(a) 46% (b) 66%
(c) 42% (d) 56%

356. If side of a square is increased by 40%, the percentage increase in its surface area is
(a) 40% (b) 60%
(c) 80% (d) 96%

357. If the diameter of a circle is increased by 8%, then its area is increased by :
(a) 16.64% (b) 6.64%
(c) 165 (d) 16.46%

358. One side of a rectangle is increased by 30%. To maintain the same area, the other side will have to be decreased by
(a) $23\frac{1}{13}$% (b) $76\frac{12}{13}$%
(c) 30% (d) 15%

359. The length and breadth of a rectangle are doubled. Percentage increase in area is
(a) 150% (b) 200%
(c) 300% (d) 400%

360. The length of a rectangle is increased by 10% and breadth decreased by 10%. The area of the new rectangle is
(a) neither increased nor decreased
(b) increased by 1%
(c) decreased by 2%
(d) decreased by 1%

361. How many circular plates of diameter d be taken out of a square plate of side 2d with minimum loss of material?
(a) 8 (b) 6 (c) 4 (d) 2

362. What is the total area of three equilateral triangles inscribed in a semi-circle of radius 2 cm?
(a) 12 cm² (b) $\frac{3\sqrt{3}}{4}$ cm²
(c) $\frac{9\sqrt{3}}{4}$ cm² (d) $3\sqrt{3}$ cm²

363. The area of a sector of a circle of radius 36 cm is 72π cm². The length of the corresponding arc of the sector is
(a) π cm (b) 2π cm
(c) 3π cm (d) 4π cm

364. A square is inscribed in a circle of diameter $2a$ and another square is circumscribing circle. The difference between the areas of outer and inner squares is
(a) a² (b) 2a² (c) 3a² (d) 4a²

365. ABC is a triangle right angled at A. AB = 6 cm and AC = 8 cm. Semi-circles drawn (outside the triangle) on AB, AC and BC as diameters which enclose areas x, y and z square units, respectively. What is x+y–z equal to?
(a) 48 cm² (b) 32 cm²
(c) 0 (d) None of these

366. Consider an equilateral triangle of a side of one unit length. A new equilateral triangle is formed by joining the mid-points of one, then a third equilateral triangle is formed by joining the mid-points of second. The process is continued. The perimeter of all triangles, thus formed is
(a) 4 (b) 5
(c) 6 (d) 7

367. What is the area of the larger segment of a circle formed by a chord of length 5 cm subtending an angle of 90° at the centre?
(a) $\frac{25}{4}\left(\frac{\pi}{2}+1\right)$ cm²
(b) $\frac{25}{4}\left(\frac{\pi}{2}-1\right)$ cm²
(c) $\frac{25}{4}\left(\frac{3\pi}{2}+1\right)$ cm²
(d) None of these

368. A rectangle of maximum area is drawn inside a circle of diameter 5 cm. What is the maximum area of such a rectangle?
(a) 25 cm² (b) 12.5 cm²
(c) 12 cm² (d) None of these

369. If AB and CD are two diameters of a cricle of radius r and they are mutually perpendicular, then

what is the ratio of the area of the circle to the area of the $\triangle ACD$?

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 2π

370. What is the area of a circle whose area is equal to that of a triangle with sides 7 cm, 24 cm and 25 cm?
(a) 80 cm^2 (b) 84 cm^2
(c) 88 cm^2 (d) 90 cm^2

371. If the area of an equilateral triangle is x and its perimeter is y, then which one of the following is correct?
(a) $y^4 = 432x^2$ (b) $y^4 = 216x^2$
(c) $y^2 = 432x^2$ (d) None of these

372. A rectangular field is 22 m long and 10 m wide. Two hemispherical pitholes of radius 2 m are dug from two places and the mud is spread over the remaining part of the field. The rise in the level of the field is

(a) $\frac{8}{93}$ m (b) $\frac{13}{93}$ m

(c) $\frac{16}{93}$ m (d) $\frac{23}{93}$ m

373. The area of an isosceles $\triangle ABC$ with AB = AC is 12 sq cm and altitude AD = 3 cm. What is its perimeter?
(a) 18 cm (b) 16 cm
(c) 14 cm (d) 12 cm

374. A hospital room is to accommodate 56 patients. It should be done in such a way that every patient gets 2.2 m^2 of floor and 8.8 m^3 of space. If the length of the room is 14m, then breadth and the height of the room are respectively
(a) 8.8 m,4m (b) 8.4 m,4.2m
(c) 8 m, 4 m (d) 7.8 m,4.2m

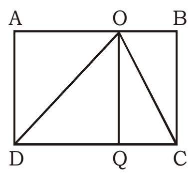
375. How many 200 mm lengths can be cut from 10 m of ribbon?
(a) 50 (b) 40 (c) 30 (d) 20

376. What is the area between a square of side 10 cm and two inverted semi-circular, cross-sections each of radius 5 cm inscribed in the square?
(a) 17.5 cm^2 (b) 18.5 cm^2
(c) 20.5 cm^2 (d) 21.5 cm^2

377. The perimeter of a rectangle having area equal to 144 cm^2 and sides in the ratio 4:9 is
(a) 52 cm (b) 56 cm
(c) 60 cm (d) 64 cm

378. One side of a parallelogram is 8.06 cm and its perpendicular distance from opposite side is 2.08 cm. What is the approximate area of the parallelogram?
(a) 12.56 cm^2 (b) 14.56 cm^2
(c) 16.76 cm^2 (d) 22.56 cm^2

379. In the figure given below, the area of rectangle ABCD is 100 sq cm, O is any point on AB and CD=20cm. Then, the area of $\triangle$ COD is



(a) 40 sq cm (b) 45 sq cm
(c) 50 sq cm (d) 80 sq cm

380. If an isosceles right angled triangle has area 1sq unit, then what is its perimeter?
(a) 3 units
(b) $2\sqrt{2}+1$ units
(c) $\left(\sqrt{2}+1\right)$ units
(d) $2\left(\sqrt{2}+1\right)$ units

381. A circular water fountain 6.6 m in diameter is surrounded by a path of width 1.5 m. The area of this path (in sq m) is
(a) 13.62π (b) 13.15π
(c) 12.15π (d) None of these

382. The area of a rectangular filed is 4500 sq m. If its length and breadth are in the ratio 9:5, then its perimeter is
(a) 90 m (b) 150 m
(c) 280 m (d) 360 m

383. The area of a square inscribed in a circle of radius 8 cm is
(a) 32 sq cm (b) 64 sq cm
(c) 128 sq cm (d) 256 sq cm

384. The short and long hands of a clock are 4 cm and 6 cm long, respectively. Then, the ratio of distances travelled by tips of short hand in 2 days and long hand in 3 days is
(a) 4 : 9 (b) 2 : 9
(c) 2 : 3 (d) 1 : 27

385. The arc AB of the circle with centre at O and radius 10 cm has length 16 cm. What is the area of the sector bounded by the radii OA, OB and the arc AB?
(a) 40 π sq cm (b) 40 sq cm
(c) 80 sq cm (d) 20π sq cm

386. The length of a room floor exceeds its breadth by 20m. The area of the floor remains unaltered when the length is decreased by 10m but the breadth is increased by 5m. The area of the floor (in square metres) is:
(a) 280 (b) 325
(c) 300 (d) 420

387. Find the perimeter of a square which is symmetrically inscribed in semicircle of radius 10 cm.
(a) $\sqrt{80}$cm (b) 80 cm
(c) $8\sqrt{24}$ cm (d) $16\sqrt{5}$ cm

388. Consider the following statement
I. Area of a segment of a circle is less than area of its corresponding sector.
II. Distance travelled by a circular wheel of diameter 2d cm in one revolution is greater than 6d cm.

Which of the above statements is/are correct?
(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

389. The Perimeter of a rectangle is 82 m and its area is 400 sq m. What is the breadth of the rectangle?
(a) 18 m (b) 16 m
(c) 14 m (d) 12 m

390. The area enclosed between the circumference of two concentric circles is 16π sq cm and their radii are in the ratio 5:3. What is the area of the outer circle?
(a) 9π sq cm (b) 16π sq cm
(c) 25π sq cm (d) 36π sq cm

391. If the circumference of a circle is equal to the perimeter of square, then which one of the following is correct?
(a) Area of circle = Area of square
(b) Area of circle $\geq$ Area of square
(c) Area of circle > Area of square
(d) Area of circle< Area of square

392. If the circumference of two circle are in the ratio 2:3, then what is the ratio of their areas?
(a) 2:3 (b) 4:9
(c) 1:3 (d) 8:27

393. If the area of a circle inscribed in an equilateral triangle is 154 sq cm, then what is the perimeter of the triangle?
(a) 21 cm (b) $42\sqrt{3}$ cm
(c) $21\sqrt{3}$ cm (d) 42 cm

394. In the ΔABC, the base BC is trisected at D and E. The line through D, Parallel to AB, meets AC at F and the line through E parallel to AC meets AB at G. If EG and DF intersect at H, then what is the ratio of the sum of the area of parallelogram AGHF and the area of the Δ DHE to the area of the ΔABC?
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

395. If the area of a circle is equal to the area of a square with side $2\sqrt{\pi}$ units, then what is the diameter of the circle?
(a) 1 unit (b) 2 units
(c) 4 units (d) 8 units

396. A square, a cricle and an equilateral triangle have same perimeter.
Consider the following statements
I. The area of square is greater than the area of the triangle.
II. The area of circle is less than the area of triangle.
Which of the above statement is/ are correct?
(a) Only I (b) Only II
(c) Both I and II
(d) Neither I nor II

397. If the area of a rectangle whose length is 5 more than twice its width is 75 sq units. What is the perimeter of the rectangle?
(a) 40 units (b) 30 units
(c) 24 units (d) 20 units

398. If the altitude of an equilateral triangle is $\sqrt{3}$ cm, then what is its perimeter?
(a) 3 cm (b) $3\sqrt{3}$ cm
(c) 6 cm (d) $6\sqrt{3}$ cm

399. The area of a rectangle, whose one side is a is $2a^2$. What is the area of a square having one of the diagonal of the rectangle as side?
(a) $2a^2$ (b) $3a^2$ (c) $4a^2$ (d) $5a^2$

400. If the outer and inner diameters of a stone parapet around a well are 112 cm and 70 cm respectively. Then, what is the area of the parapet?
(a) 264 sq cm (b) 3003 sq cm
(c) 6006 sq cm (d) 24024 sq cm

401. If the area of a ΔABC is equal to area of square of side length 6 cm,then what is the length of the altitude to AB, where AB = 9 cm?
(a) 18 cm (b) 14 cm
(c) 12 cm (d) 8 cm

402. What is the area of an equilateral triangle having altitude equal to $2\sqrt{3}$ cm?
(a) $\sqrt{3}$ sq cm (b) $2\sqrt{3}$ sq cm
(c) $3\sqrt{3}$ sq cm (d) $4\sqrt{3}$ sq cm

403. If a lawn 30 m long and 16 m wide is surrounded by a path 2 m wide, then what is the area of the path?
(a) 200 m^2 (b) 280 m^2
(c) 300 m^2 (d) 320 m^2

404. If a circle circumscribes a rectangle with side 16 cm and 12 cm, then what is the area of the circle?
(a) 48π sq cm (b) 50π sq cm
(c) 100π sq cm (d) 200π sq cm

405. The lengths of two sides of a right angled triangle which contain the right angle are a and b, respectively. Three squares are drawn on the three sides of the triangle on the outer side.What is the total area of the triangle and the three squares?
(a) $2(a^2+b^2)+ab$
(b) $2(a^2+b^2)+2.5ab$
(c) $2(a^2+b^2)+0.5ab$
(d) $2.5(a^2+b^2)$

406. A wall is of the form of a trapezium with height 4 m and parallel sides being 3 m and 5m. What is the cost of painting the wall, if the rate of painting is Rs.25 per sq m?
(a) Rs. 240 (b) Rs. 400
(c) Rs. 480 (d) Rs. 800

407. A grassy field has the shape of an equilateral triangle of side 6 m. If a horse with 4.2 m long rope tied at a vertex. The percentage of the total area of the field which is available for grazing is best approximated by
(a) 50% (b) 55%
(c) 59% (d) 62%

408. The areas of two circles are in the ratio 1:2. If the two circles are bent in the form of squares, then what is the ratio of their areas?
(a) 1 : 2 (b) 1 : 3
(c) $1:\sqrt{4}$ (d) 1 : 4

409. If the four equal circles of radius 3cm touch each other externally, then the area of the region bounded by the four circles is:
(a) $4(9-\pi)sq.cm$
(b) $9(4-\pi)sq.cm$
(c) $5(6-\pi)sq.cm$
(d) $6(5-\pi)sq.cm$

410. If the diameter of a circle circumscribing a square is $15\sqrt{2}$ cm, then what is the length of the side of the square?
(a) 15 cm (b) 12 cm
(c) 10 cm (d) 7.5 cm

411. Three congruent circles each of radius 4 cm touch one another. What is the area (in cm^2) of the portion included between them?
(a) 8π (b) $16\sqrt{3}-8\pi$
(c) $16\sqrt{3}-4\pi$ (d) $16\sqrt{3}-2\pi$

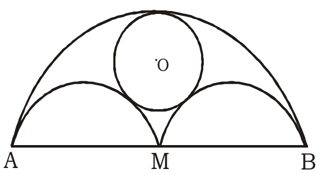
412. The two diagonals of a rhombus are of lengths 55 cm and 48 cm. If P is the perpendicular height of the rhombus, then which one of the following is correct?
(a) 36 cm < p < 37 cm
(b) 35 cm < p < 36 cm
(c) 34 cm < p < 35 cm
(d) 33 cm < p < 34 cm

413. The Perimeter of a triangular field is 240 m. If two of its sides are 78 m and 50 m, then what is the length of the perpendicular on the side of length 50 m from the opposite vertex?
(a) 43 m (b) 52.2 m
(c) 67.2 m (d) 70 m

414. A piece of wire 78 cm long is bent in the form of and isosceles triangle. If the ratio of one of the equal sides to the base is 5:3, then what is the length of the base?
(a) 16 cm (b) 18 cm
(c) 20 cm (d) 30 cm

415. The length of a minute hand of a wall clock is 9 cm. What is the area swept (in cm²) by the minute hand in 20 min? (take π = 3.14)
(a) 88.78 (b) 84.78
(c) 67.74 (d) 57.78

416. In the figure given below, AB is a line of length 2a, with M as mid-point. Semi-circles are drawn on one side with AM, and AB as diameters. A circle with centre O and radius r is drawn such that this circle touches all the three semi-circles. What is the value of r?



(a) $\frac{2a}{3}$ (b) $\frac{a}{2}$ (c) $\frac{a}{3}$ (d) $\frac{a}{4}$

417. A circle and a square have the same perimeter.Which one of the following is correct?
(a) The area of the circle is equal to that of square
(b) The area of the circle is larger than that of square
(c) The area of the circle is less than that of square
(d) No conclusion can be drawn

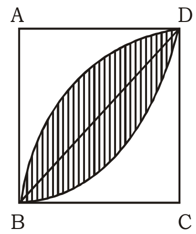
418. What is the radius of the circle inscribed in a triangle having side lengths 35 cm, 44 cm and 75 cm?
(a) 3 cm (b) 4 cm
(c) 5 cm (d) 6 cm

419. A rectangle area of 6 sq m is to be painted on a 3m × 4m board leaving a border of uniform width on all sides. What should be the width of the border?
(a) 0.25 m (b) 0.5 m
(c) 1 m (d) 3 m

420. A wheel of a bicycle has inner diameter 50 cm and thickness 10 cm. What is the speed of the bicycle, If it makes 10 revolutions is 5 s?
(a) 5.5 m/s (b) 4.4 m/s
(c) 3.3 m/s (d) 2.2 m/s

421. If a wire of length 36 cm is bent in the form of a semi-circle, then what is the radius of the semi-circle?
(a) 9 cm (b) 8 cm
(c) 7 cm (d) 6 cm

422. In the given figure, the side of square ABCD is 7 cm. What is the area of the shaded portion, formed by the arcs BD or the circles with centre at C and A?



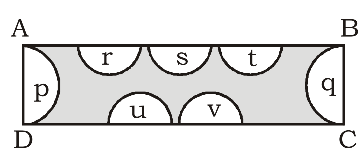
(a) 7cm² (b) 28 cm²
(c) 14 cm² (d) 21 cm²

423. What is the maximum area of a rectangle, the perimeter of which is 18 cm?
(a) 20.25 cm² (b) 20.00 cm²
(c) 19.75 cm (d) 19.60 cm²

424. Three circular laminas of the same radius are cut out from a larger circular lamina. When the radius of each lamina cut out is the largest possible, then what is the ratio (approximate) of the area of the residual piece of the original lamina to its original total area?
(a) 0.30 (b) 0.35
(c) 0.40 (d) 0.45

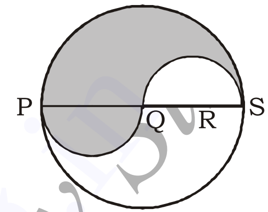
425. A wire is in the form of a radius 42 cm. If it is bent into a square, then what is the side of the square?
(a) 66 cm (b) 42 cm
(c) 36 cm (d) 33 cm

426. Seven semi- circluar areas are removed from the rectangle ABCD as shown in the figure below, in which AB = 2 cm and AD = 0.5 cm. The radius of each semi-circle, r,s,t, u and v is half of that of semi-circle p or q. What is the area of the remaining portion?



(a) $(128 - 13\pi)/128$ cm²
(b) $(125 - 13\pi)/125$ cm²
(c) $(128 - 15\pi)/128$ cm²
(d) None of these

427. PQRS is a diameter of a circle of radius 6 cm as shown in the figure above. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters. What is the perimeters of the shaded region?



(a) 12 π cm (b) 14π cm
(c) 16 π cm (d) 18π cm

428. A person rides a bicycle round a circular path of radius 50 m. The radius of the wheel of the bicycle is 50 cm. The cycle comes to the starting point for the first time in 1 h. What is the number of revolutions of the wheel in 15 min?
(a) 20 (b) 25 (c) 30 (d) 35

429. If a man walking at the rate 3 km/h crosses a square field diagonally in 1 min, then what is the area of the field?
(a) 1000 m² (b) 1250 m²
(c) 2500 m² (d) 5000 m²

430. The difference between the area of a square and that of an equilateral triangle on the same base is 1/4 cm². What is the length of side of triangle?
(a) $(4 - \sqrt{3})^{1/2}$ cm
(b) $(4 + \sqrt{3})^{1/2}$ cm
(c) $(4 - \sqrt{3})^{-1/2}$ cm
(d) $(4 + \sqrt{3})^{-1/2}$ cm

431. A horse is tied to a pole fixed at one corner of a 50 m × 50 m square field of grass by means of a 20 m long rope. What is the area of that part of the field which the horse can graze?
(a) 1256 m² (b) 942 m²
(c) 628 m² (d) 314 m²

432. From a rectangular metal sheet of sides 25 cm and 20 cm; a circular sheet as large as possible is cut-off. What is the area of the remaining sheet?
(a) 186 cm² (b) 144 cm²
(c) 93 cm² (d) 72 cm²

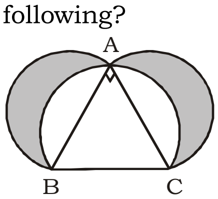
433. What is the area of a right angled isosceles triangle whose hypotenuse is $6\sqrt{2}$ cm?
(a) 12 cm² (b) 18 cm²
(c) 24 cm² (d) 36 cm²

434. If A is the area of a triangle in cm², whose sides are 9 cm, 10 cm and 11 cm, then which one of the following is correct?
(a) A < 40 cm²
(b) 40 cm² < A < 45 cm²
(c) 45 **cm² < A < 50 cm²**
(d) A > 50 cm²

435. If x and y are respectively the areas of a square and a rhombus of sides of same length, then what is x : y?
(a) 1 : 1 (b) $2 : \sqrt{3}$
(c) $4 : \sqrt{3}$ (d) 3 : 2

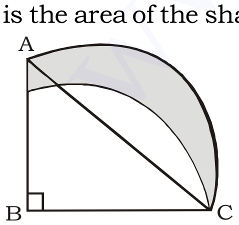
436. If the area of a circle, inscribed in an equilateral triangle is 4π cm², then what is the area of the triangle?
(a) $12\sqrt{3}$ cm² (b) $9\sqrt{3}$ cm²
(c) $8\sqrt{3}$ cm² (d) 18 cm²

437. In the given figure, $\triangle$ABC is a right angled triangle, right angled at A. Semi-circles are drawn on the sides AB, BC and AC. Then the area of shaded portion is equal to which one of the following?



(a) Area of $\triangle$ABC
(b) 2 times the area of $\triangle$ABC
(c) Area of semi-circle ABC
(d) None of the above

438. In the given figure, ABC is a right angled triangle, right angled at B. BC = 21 cm and AB = 28 cm. Width AC as diameter of a semi-circle and width BC as radius a quarter circle are drawn. What is the area of the shaded portion?



(a) 425 cm² (b) 425.75 cm²
(c) 428 cm² (d) 428.75 cm²

439. The Perimeter of a square S_1 is 12 m more than perimeter of the square S_2. If the area of S_1 equals three times, the area of S_2 minus 11, then what is the perimeter of S_1?
(a) 24 m (b) 32 m
(c) 36 m (d) 40 m

440. From a rectangular sheet of cardboard of size 5 cm×2 cm, the greatest possible circle is cut-off. What is the area of the remainting part?
(a) $(25-\pi)$ cm²
(b) $(10-\pi)$ cm²
(c) $(4-\pi)$ cm²
(d) $(10-2\pi)$ cm²

441. A chord AB of a cricle of radius 20 cm makes a right angle at the centre of the circle. What is the area of the minor segment in cm²? (take π = 3.14)
(a) 31.4 cm² (b) 57 cm²
(c) 62.8 cm² (d) 114 cm²

442. The minute hand of a clock is 14 cm long. How much distance does the end of the minute hand travel in 15 min? (take $\pi = \frac{27}{7}$)
(a) 11 cm (b) 22 cm
(c) 33 cm (d) 44 cm

443. A square of side x is taken. A rectangle is cut out from this square such that on side of the rectangle is half that of the square and the other is $\frac{1}{3}$rd of the first side of the rectangle. What is the area of the remaining portion?
(a) $\left(\frac{3}{4}\right)x^2$ (b) $\left(\frac{7}{8}\right)x^2$
(c) $\left(\frac{11}{12}\right)x^2$ (d) $\left(\frac{15}{16}\right)x^2$

444. A rectangle cardboard is 18 cm×10 cm. From the four corners of the rectangle, quarter circles of radius 4 cm are cut. What is the perimeter (approximate) of the remaining portion?
(a) 47.1 cm (b) 49.1 cm
(c) 51.0 cm (d) 53.0 cm

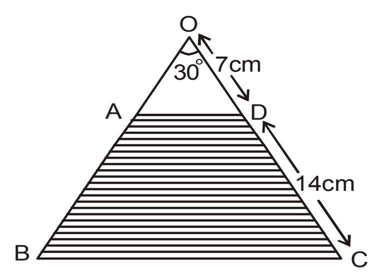
445. A cycle wheel makes 1000 revolution is moving 440 m. What is the diameter of the wheel?
(a) 7 cm (b) 14 cm
(c) 28 cm (d) 21 cm

446. A circle is inscribed in a equilateral triangle of side a. What is the area of any square inscribed in this circle?
(a) $\frac{a^2}{3}$ (b) $\frac{a^2}{4}$ (c) $\frac{a^2}{6}$ (d) $\frac{a^2}{8}$

447. Consider a circle C of radius 6 cm with centre at O. What is the difference in the area of the circle C and the area of the sector of C subtending an angle of 80° at O?
(a) 26π cm² (b) 16π cm²
(c) 28π cm² (d) 30π cm²

448. The ratio of the areas of the in-circle and the circum-circle of a square is:
(a) 1 : 2 (b) $\sqrt{2}:1$
(c) $1:\sqrt{2}$ (d) 2 : 1

449. The diagram represents the area swept by the wiper of a car. With the dimensions given in the figure, calculate the shaded area swept by the wiper.



(a) 102.67cm² (b) 205.34cm²
(b) 51.33cm² (d) 208.16cm²

450. If the length of a chord of a circle at a distance of 12cm from the centre is 10cm, then the diameter of the circle is :
(a) 13 cm (b) 15 cm
(c) 26 cm (d) 30 cm

451. Area of the incircle of an equilateral triangle with side 6cm is :
(a) $\frac{\pi}{2}$ sq.cm (b) $\sqrt{3}\pi$ sq.cm
(c) 6π sq.cm (d) 3π sq.cm

452. The adjacent sides of a parellelogram are 36 cm and 27 cm in length. If the distance between the shorter sides is 12cm, then the distance between the longer sides is :
(a) 10 cm (b) 12 cm
(c) 16 cm (d) 9 cm

453. A circle and a rectangle have the same perimeter. The sides of the rectangle area 18cm and 26cm. The area of the circle is :
(a) 125 cm^2 (b) 230 cm^2
(c) 550 cm^2 (d) 616 cm^2

454. The perimeter of a semicircular path is 36m. Find the area of this semicircular path.
(a) 42sq.m (b) 54sq.m
(c) 63sq.m (d) 77sq.m

455. Find the area of a rectangle whose area is equal to the area of a circle with perimter equal to 24 π :
(a) 144 (b) 144 π
(c) 154 π (d) none of these

456. A circle is inscribed in an equilateral triangle of side 8cm. The area of the portion between the triangle and the circle is :
(a) 11cm^2 (b) 10.95cm^2
(c) 10cm^2 (d) 10.50cm^2

457. Find the ratio of the diameter of the circles inscribed in and circumscribed an equilateral triangle to its height.
(a) 1 : 2 : 3 (b) 2 : 4 : 3
(c) 1 : 3 : 4 (d) 3 : 2 : 1

458. Find the area of the largest (or maximum sized) square that can be made inside a right angle triangle having sides 6cm, 8cm & 10cm when one of vertices of the square coincide with the vertex of right angle of the triangle?
(a) $\frac{576}{49}cm^2$ (b) 24cm^2
(c) $\frac{24}{7}cm^2$ (d) None of these

459. Area of the trapezium formed by x-axis; y-axis and the lines $3x+4y=12$ and $6x+8y=60$ is:
(a) 37.5sq.unit (b) 31.5sq.unit
(c) 48sq.unit (d) 36.5sq.unit

460. A square having area 200sq.m, is formed in such a way that the length of its diagonal is $\sqrt{2}$ times of the diagonal of the given square. Then the area of the new square formed is:
(a) $200\sqrt{2}sq.m$ (b) $400\sqrt{2}sq.m$
(c) 400sq.m (d) 800sq.m

ANSWER KEY

1. (b)	47. (b)	93. (b)	139. (c)	185. (c)	231. (c)	277. (a)	323. (c)	369. (b)	415. (b)
2. (c)	48. (d)	94. (b)	140. (b)	186. (a)	232. (a)	278. (b)	324. (b)	370. (b)	416. (c)
3. (a)	49. (b)	95. (c)	141. (c)	187. (c)	233. (b)	279. (a)	325. (c)	371. (a)	417. (b)
4. (c)	50. (b)	96. (c)	142. (a)	188. (a)	234. (a)	280. (d)	326. (b)	372. (c)	418. (d)
5. (b)	51. (b)	97. (c)	143. (b)	189. (d)	235. (d)	281. (b)	327. (c)	373. (a)	419. (b)
6. (b)	52. (a)	98. (b)	144. (a)	190. (c)	236. (c)	282. (b)	328. (d)	374. (a)	420. (b)
7. (b)	53. (c)	99. (b)	145. (a)	191. (b)	237. (b)	283. (c)	329. (c)	375. (a)	421. (c)
8. (c)	54. (c)	100. (c)	146. (c)	192. (d)	238. (d)	284. (d)	330. (c)	376. (d)	422. (c)
9. (d)	55. (d)	101. (a)	147. (c)	193. (a)	239. (b)	285. (d)	331. (c)	377. (a)	423. (a)
10. (d)	56. (b)	102. (a)	148. (a)	194. (a)	240. (a)	286. (c)	332. (c)	378. (c)	424. (b)
11. (a)	57. (c)	103. (c)	149. (b)	195. (d)	241. (a)	287. (c)	333. (d)	379. (c)	425. (a)
12. (d)	58. (d)	104. (a)	150. (a)	196. (c)	242. (a)	288. (c)	334. (d)	380. (d)	426. (a)
13. (a)	59. (d)	105. (c)	151. (a)	197. (a)	243. (a)	289. (a)	335. (c)	381. (c)	427. (a)
14. (b)	60. (b)	106. (c)	152. (d)	198. (a)	244. (c)	290. (c)	336. (b)	382. (c)	428. (b)
15. (d)	61. (b)	107. (d)	153. (b)	199. (a)	245. (d)	291. (d)	337. (c)	383. (c)	429. (b)
16. (a)	62. (c)	108. (b)	154. (d)	200. (d)	246. (c)	292. (a)	338. (d)	384. (d)	430. (c)
17. (b)	63. (b)	109. (c)	155. (b)	201. (b)	247. (b)	293. (c)	339. (d)	385. (c)	431. (d)
18. (d)	64. (b)	110. (a)	156. (a)	202. (d)	248. (b)	294. (b)	340. (a)	386. (c)	432. (a)
19. (d)	65. (b)	111. (c)	157. (c)	203. (b)	249. (b)	295. (b)	341. (b)	387. (d)	433. (b)
20. (b)	66. (c)	112. (a)	158. (a)	204. (b)	250. (a)	296. (d)	342. (b)	388. (c)	434. (b)
21. (a)	67. (b)	113. (c)	159. (a)	205. (a)	251. (d)	297. (d)	343. (c)	389. (b)	435. (a)
22. (c)	68. (a)	114. (d)	160. (b)	206. (c)	252. (a)	298. (c)	344. (d)	390. (c)	436. (a)
23. (c)	69. (b)	115. (b)	161. (d)	207. (a)	253. (b)	299. (d)	345. (b)	391. (c)	437. (a)
24. (d)	70. (c)	116. (d)	162. (a)	208. (a)	254. (d)	300. (c)	346. (b)	392. (b)	438. (d)
25. (d)	71. (a)	117. (c)	163. (c)	209. (b)	255. (c)	301. (b)	347. (b)	393. (b)	439. (b)
26. (a)	72. (c)	118. (a)	164. (c)	210. (b)	256. (d)	302. (b)	348. (a)	394. (b)	440. (b)
27. (d)	73. (c)	119. (c)	165. (c)	211. (d)	257. (c)	303. (a)	349. (b)	395. (c)	441. (d)
28. (b)	74. (b)	120. (b)	166. (b)	212. (c)	258. (d)	304. (b)	350. (b)	396. (a)	442. (b)
29. (c)	75. (b)	121. (b)	167. (c)	213. (b)	259. (a)	305. (c)	351. (b)	397. (a)	443. (c)
30. (d)	76. (a)	122. (c)	168. (c)	214. (b)	260. (c)	306. (a)	352. (d)	398. (c)	444. (b)
31. (a)	77. (b)	123. (d)	169. (d)	215. (d)	261. (b)	307. (b)	353. (c)	399. (d)	445. (b)
32. (d)	78. (c)	124. (c)	170. (d)	216. (d)	262. (a)	308. (d)	354. (c)	400. (c)	446. (c)
33. (a)	79. (b)	125. (b)	171. (a)	217. (a)	263. (c)	309. (b)	355. (d)	401. (d)	447. (c)
34. (c)	80. (c)	126. (b)	172. (d)	218. (d)	264. (b)	310. (a)	356. (d)	402. (d)	448. (a)
35. (a)	81. (b)	127. (c)	173. (c)	219. (b)	265. (d)	311. (c)	357. (a)	403. (a)	449. (a)
36. (a)	82. (b)	128. (a)	174. (c)	220. (b)	266. (c)	312. (a)	358. (a)	404. (c)	450. (c)
37. (c)	83. (a)	129. (b)	175. (c)	221. (d)	267. (c)	313. (c)	359. (c)	405. (c)	451. (d)
38. (b)	84. (b)	130. (b)	176. (b)	222. (d)	268. (b)	314. (b)	360. (d)	406. (b)	452. (d)
39. (d)	85. (a)	131. (a)	177. (c)	223. (b)	269. (a)	315. (d)	361. (c)	407. (c)	453. (d)
40. (a)	86. (a)	132. (a)	178. (b)	224. (c)	270. (a)	316. (a)	362. (d)	408. (a)	454. (d)
41. (a)	87. (c)	133. (b)	179. (a)	225. (a)	271. (a)	317. (b)	363. (d)	409. (b)	455. (b)
42. (a)	88. (b)	134. (b)	180. (d)	226. (c)	272. (a)	318. (a)	364. (b)	410. (a)	456. (b)
43. (d)	89. (c)	135. (b)	181. (a)	227. (b)	273. (b)	319. (c)	365. (c)	411. (b)	457. (b)
44. (d)	90. (c)	136. (c)	182. (b)	228. (a)	274. (d)	320. (a)	366. (c)	412. (a)	458. (a)
45. (b)	91. (a)	137. (b)	183. (a)	229. (b)	275. (a)	321. (c)	367. (c)	413. (c)	459. (b)
46. (a)	92. (a)	138. (c)	184. (a)	230. (b)	276. (b)	322. (d)	368. (c)	414. (b)	460. (c)

SOLUTION

1. (b) Side of a square = $\frac{\text{Diagonal}}{\sqrt{2}}$

Area of square = $\left(\frac{\text{Diagonal}}{\sqrt{2}}\right)^2$

$= \frac{(5.2)^2}{2} = \frac{5.2 \times 5.2}{2}$

$= 2.6 \times 5.2 = 13.52\ \text{cm}^2$

2. (c) Area of square

$= \frac{\text{Diagonal}^2}{2} = \frac{a^2}{2}$

3. (a) Let the length of rectangular hall = x

∴ Breadth of rectangular hall

$= \frac{3}{4}x$

According to question,

Area = 768 m²

$x \times \frac{3}{4}x = 768$

$\frac{3}{4}x^2 = 768$

$x^2 = \frac{768 \times 4}{3} = 256 \times 4$

$x^2 = \sqrt{256 \times 4} = 32$ m.

Difference of length and

breadth $= x - \frac{3}{4}x = \frac{x}{4} = \frac{32}{4} = 8$ m

4. (c) Since the room is in cuboid shape

Length of largest rod = Diagonal of cuboid

$= \sqrt{l^2+b^2+h^2} = \sqrt{16^2+12^2+\frac{32^2}{3^2}}$

$= \sqrt{256+144+\frac{1024}{9}}$

$= \sqrt{\frac{2304+1296+1024}{9}}$

$= \sqrt{\frac{4624}{9}} = \frac{68}{3} = 22\frac{2}{3}$ m

5. (b) Perimeter of square = 44 cm

4 × side = 44

side = 11 cm

area of square = (side)² = (11)²

= 121cm²

Circumference of circle = 44cm

2 π (radius) = 44

radius $= \frac{44 \times 7}{2 \times 22} = 7$ cm

area of circle $= \frac{22}{7} \times 7 \times 7$

= 154 cm²

Option (b) is the answer. (circle, 33 cm²)

6. (b) Let the side of square = a and the radius of circle = r

perimeter of square

= circumference of circle

4a = 2 π r

$r = \frac{4a}{2\pi}$

area of circle = 3850 m²

$\pi \times \frac{4a}{2\pi} \times \frac{4a}{2\pi} = 3850$

$16a^2 = \frac{3850 \times 2 \times 2 \times 22}{7}$

a² = 3025 m²

7. (b) 2(l + b) = 28

l + b = 14

and l × b = 48

(l + b)² = l² + b² + 2lb

(14)² = l² + b² + 48 × 2

196 - 96 = l² + b²

l² + b² = 100

$\sqrt{l^2+b^2} = 10$

Diagonal = 10 m

8. (c) Let the length of rectangular hall = x

∴ Breadth of rectangular hall

$= \frac{3}{4}x$

According to question,

Area = 192 m²

$x \times \frac{3}{4}x = 192$

$\frac{3}{4}x^2 = 192$

$x^2 = \frac{192 \times 4}{3} = 64 \times 4$

$x = \sqrt{64 \times 4} = 16$ cm

difference of length and

breadth $= x - \frac{3}{4}x = \frac{x}{4}$

$= \frac{16}{4} = 4$ cm

9. (d) Side of the square

$= \frac{\text{Diagonal}}{\sqrt{2}} = \frac{4\sqrt{2}}{\sqrt{2}} = 4$

area of the square = 16

area of new square = 32

side of new square = $\sqrt{32}$

$= 4\sqrt{2}$

Diagonal of new square

$= 4\sqrt{2} \times \sqrt{2} = 8$ cm

10. (d) Diagonal of square A = (a + b)

side of square

$= \frac{\text{Diagonal}}{\sqrt{2}} = \frac{a+b}{\sqrt{2}}$

area of square A $= \left(\frac{a+b}{\sqrt{2}}\right)^2$

$= \frac{(a+b)^2}{2}$

area of square

B = 2 × area of square A

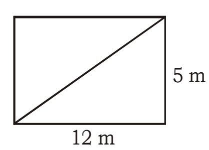
$= 2 \times \frac{(a+b)^2}{2} = (a+b)^2$

side of square B $= \sqrt{(a+b)^2}$

= (a + b)

diagonal of square B = $\sqrt{2}(a+b)$

11. (a)



area of the rectangular garden

= 12 × 5 = 60 m²

∴ area of square = 60

(side)² = 60

side = $\sqrt{60}$

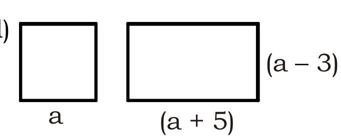
diagonal of the square

$= \sqrt{2}$ side

$= \sqrt{2} \times \sqrt{60} = \sqrt{120}$

$= 2\sqrt{30}$ m

12. (d)



According to question,
$a^2 = (a-3)(a+5)$
$a^2 = a^2 + 5a - 3a - 15$
$2a = 15$
$a = \frac{15}{2}$

Length $= a + 5 = \frac{15}{2} + 5 = \frac{25}{2}$

breadth $= a - 3 = \frac{15}{2} - 3$
$= \frac{15-6}{2} = \frac{9}{2}$

perimeter of the rectangle
$= 2(l + b)$
$= 2\left(\frac{25}{2} + \frac{9}{2}\right) = 34$ cm

13. (a) According to question,
$2(l + b) = 160$
$l + b = 80$ ..... (i)
$l - b = 48$ ....(ii)
on solving (i) and (ii)
$l = 64$, $b = 16$
area of square = area of rectangle
$(side)^2 = 64 \times 16$
side $= \sqrt{64 \times 16} = 32$ m

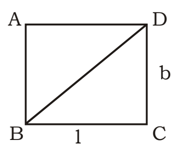
14. (b) Side of square, whose perimeter is 24 cm
$= \frac{24}{4} = 6$cm
So, area of square $= 6^2$
$= 36$ cm²
Again, side of square, whose perimeter is 32 cm $= \frac{32}{4} = 8$ cm
So, area of this square
$= 8^2 = 64$ cm²
According to question,
Area of new square
$= 64 + 36 = 100$ cm²
∴ side of the new square
$= \sqrt{100} = 10$ cm
Hence perimeter of new square
$= 10 \times 4 = 40$ cm

15. (d) $(side)^2 = 484$ cm²
side $= 22$ cm
perimeter of square $= 4 \times 22$
$= 88$ cm
According to question,
$2\pi r = 88$ cm
$r = \frac{88 \times 7}{2 \times 22} = 14$ cm

area of circle $= \pi r^2 = \frac{22}{7} \times 14 \times 14$
$= 616$ cm²

16. (a) $l = 10$ m, $b = 6$ m, $h = 4$m
length of diagonal (longest rod)
$= \sqrt{100+36+16} = \sqrt{152}$ m
$= 2\sqrt{38}$

17. (b) Let the length of smaller line segment $= x$ cm
The length of larger line segment $= (x + 2)$ cm
According to question,
$(x + 2)^2 - x^2 = 32$
$x^2 + 4x + 4 - x^2 = 32$
$x = \frac{28}{4} = 7$
The required length $= x + 2$
$= 7 + 2 = 9$ cm

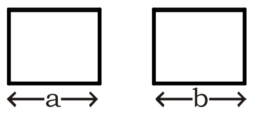
18. (d)


BD = length of diagonal
= speed × time
$= \frac{52}{60} \times 15 = 13$ m
BD $= \sqrt{l^2+b^2}$
$\Rightarrow l^2 + b^2 = 13^2 = 169$
Again, $l + b = \frac{68}{60} \times 15 = 17$
$(l + b) = l^2 + b^2 + 2lb$
$17^2 = 169 + 2lb$
$lb = \frac{120}{2} = 60$ m²

19. (d) Let the breadth be $= x$ m
∴ length $= (23 + x)$ m
$\Rightarrow 2(x + 23 + x) = 206$
$4x = 206 - 46$
$x = \frac{160}{4} = 40$ m
∴ length $= 40 + 23 = 63$ m
∴ Required area
$= 63 \times 40 = 2520$ m²

20. (b) Length of rectangle = 48 m
Breadth of rectangle =16 m
According to question,
Perimeter of square
= Perimeter of rectangle
$= 2(48 + 16)$
$4 \times$ side $= 2 \times 64$

side $= \frac{2 \times 64}{4} = 32$ m
∴ Area of the square
$= (side)^2 = (32)^2 = 1024$

21. (a)


$4a = 40$ | $4b = 32$
$a = 10$ cm | $b = 8$ cm
area of third square
$= a^2 - b^2 = 10^2 - 8^2$
$= 100 - 64 = 36$cm²
side of third square $= \sqrt{36} = 6$ cm
perimeter of third square
$= 4 \times 6 = 24$ cm

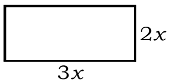
22. (c) side of the square
$= \frac{perimeter}{4}$
∴ Sides of all five squares are
$= \frac{24}{4}, \frac{32}{4}, \frac{40}{4}, \frac{76}{4}, \frac{80}{4}$
$= 6, 8, 10, 19, 20$
ATQ
area of another square
$= 6^2 + 8^2 + 10^2 + 19^2 + 20^2$
$(side)^2 = 36 + 64 + 100 + 361 + 400$
side $= \sqrt{961} = 31$
perimeter of square
$= 31 \times 4 = 124$

23. (c) Area of the tank
= length × breadth
$= 180 \times 120 = 21600$ m²
Total area of the circular plot
$= 40000 + 21600 = 61600$ m²
∴ area of circle = 61600
$\pi (radius)^2 = 61600$
$(radius)^2 = \frac{61600 \times 7}{22}$
radius $= \sqrt{2800 \times 7}$
$= \sqrt{7 \times 7 \times 400} = 7 \times 20$
$= 140$m

24. (d) Let the breadth of rectangle
$= x$ m
∴ length $= (x + 5)$ m
∴ Area of hall = length × breadth
$750 = (x + 5)x$
$750 = 30 \times 25$
(clearly $750 = 30 \times 25$)
∴ $x = 25$, breadth = 25m
length $= 25 + 5$
$= 30$ m

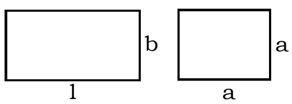
25. (d) Required total area
= Area of four walls + area of base
= 2 × 1.25(6 + 4) + 6 × 4
= 49 m^2

26. (a)



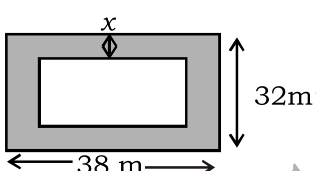
Ratio of length and breadth
= 3 : 2
2(l + b) = 20 cm
2(3x + 2x) = 20 cm
2 ×5x = 20 cm
10x = 20
x = 2
∴ length = 3 × 2 = 6 cm,
breadth = 2 × 2 = 4 cm
area = length × breadth
= 6 × 4 =24 cm^2

27. (d)



2(l +b) = 160m   4a = 160
l + b = 80m   ......(i)
a = 40 m
ATQ
a^2 – lb = 100
$(40)^2$ –lb = 100
1600 – lb = 100
lb = 1500   ....(ii)
Clearly, 50 + 30 = 80
and 50 × 30 = 1500
length = 50 m

28. (b)



area of path= 600 m^2
(l + b – 2x) 2x = 600
(38 + 32 – 2x)2x = 600
(70 – 2x)2x = 600

$(70 - 2x)x = \frac{600}{2} = 300$

$70x - 2x^2 = 300$
$2x^2 - 70x + 300 = 0$
$x^2 - 35x + 150 = 0$
$x^2 - 30x - 5x + 150 = 0$
x(x – 30) –5 (x – 30) = 0
(x – 30) (x – 5) =0
x = 30 not possible
x = 5 (right)

Alternate
(l + b –2x) 2x = area of path = 600
take help from options to save your valuable time take option(b) x = 5m
(38 + 32 – 2 × 5) 2 × 5 = (70 – 10) × 10 = 60 × 10 = 600

29. (c) Area of walls = Perimeter of base × height
= 18 × 3 = 54 m^2

30. (d) $a^2 = 81$, a = 9
⇒ Perimeter of square
= 9 × 4 = 36 cm
⇒ 2r + πr = 36
r(2 + π) = 36

$r = \frac{36}{2+\frac{22}{7}} = 7$ cm

31. (a) $a^2 = 121$,a = 11
⇒ Perimeter of square
= 11 × 4= 44 cm
⇒ Circumference of circle
= 44
2πr = 44

$2 \times \frac{22}{7} \times r = 44$

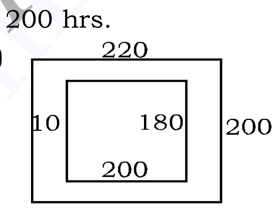
⇒ r = 7 cm

32. (d) Let the no. of hours be x
⇒ (0.3 × 0.2 × 20000) × x = 200 × 150 × 8

$\Rightarrow x = \frac{200 \times 150 \times 8}{3 \times 2 \times 200}$

= 200 hrs.

33. (a)



Area of path
= 200 × 220 – 200 × 180
= 44000 – 36000 = 8000 m^2

34. (c) Diagonal of square = diameter of circle = 8 × 2 = 16 cm

∴ side of square $= \frac{16}{\sqrt{2}} = 8\sqrt{2}$ cm

⇒ area of square $= \left(8\sqrt{2}\right)^2$
= 128 cm^2

35. (a) Side of square $= \frac{8\sqrt{2}}{\sqrt{2}} = 8$ cm
∴ Area of square = 8 × 8 = 64 cm^2

36. (a) $x^2 + 7x + 10 = x^2 + 5x + 2x + 10$
= x(x + 5) + 2(x + 5)
= (x+ 2) (x +5)
∴ Two sides of rectangle
= (x+ 2) (x +5)
∴ Perimeter = 2(x + 2 + x + 5)
= 2(2x + 7) = 4x + 14

37. (c) Let the sides of rectangle be 6 cm and 2 cm (or any other number)
⇒ Area of rectangle (Q) = 6 × 2
= 12 cm^2
∴ Side of square = 4 cm
⇒ Area of square(P)
= 4 × 4 = 16 cm^2
⇒ P > Q

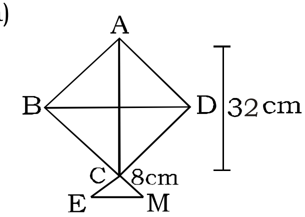
38. (b) No. of cubes with no side is painted = $(n-2)^3$
Where n is the side of the bigger cube
Required number
= $(6-2)^3$ = 64

39. (d) Side of square $= \frac{Diagonal}{\sqrt{2}}$

$= \frac{15\sqrt{2}}{\sqrt{2}} = 15$ cm

area of square = $(side)^2$
= $(15)^2$ = 225 cm^2

40. (a)



area of square $= \frac{1}{2}(Diagonal)^2$

$= \frac{1}{2}(32)^2 = \frac{1}{2} \times 32 \times 32 = 16 \times 32$

= 512 cm^2
area of triangle

$= \frac{\sqrt{3}}{4} 8^2 = \frac{1.732 \times 8 \times 8}{4}$

= 1.732×2×8 =27.712 cm^2
Required are =(512+27.712)cm^2
= 539.712 cm^2

41. (a) Area of the lawn

$= \frac{1}{12}$ hectare

length × breadth $= \frac{1}{12} \times 10000 m^2$

$4x \times 3x = \frac{10000}{12} m^2$

$$12x^2 = \frac{10000}{12}$$

$$x^2 = \frac{10000}{12\times12}$$

$$x = \frac{100}{12}$$

Breadth = 3x = $3 \times \frac{100}{12}$

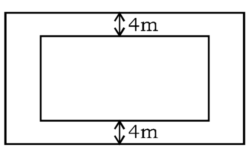
$= \frac{100}{4} = 25$ m

42. (a) Let the side of square = a cm
ATQ
l × b = 3a²

$20 \times \frac{3}{2}a = 3a^2$

a = 10 cm

43. (d)



Area of path = (l + b + 2x)2x
∴ where x = thickness of path
Let l = 7p , b = 4p
{7p + 4p + 2(4)}2(4)= 416
(11p + 8)8 = 416
11p + 8 = 52
11p = 44

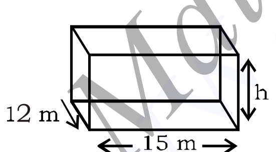
$p = \frac{44}{11} = 4$, p = 4

∴ breadth = 4 × 4 = 16 m

44. (d) Area of the floor= 8 × 6 = 48 m²
= 4800 dm² (1m =10 dm)
Area of square tile
= 4 × 4 = 16 dm²

No. of tiles = $\frac{4800}{16} = 300$

45. (b)



Shape of godown is cuboidcal
length = 15 m , breadth = 12 m,
height = h m
Area of four walls = 2(l + b)× h
area of floor = l × b
area of ceiling = l × b
ATQ
l × b + l × b = 2(l + b)× h
2(l × b) = 2(l + b)× h
2(15 × 12) = 2(15+12)× h
2 × 180 = 2 × 27 × h

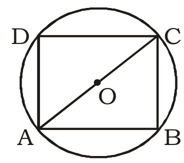
$h = \frac{180}{27} = \frac{\mathbf{20}}{\mathbf{3}}$ m

Volume of the cuboid = l × b × h

$= 15 \times 12 \times \frac{20}{3}$

= 60 × 20 = 1200 m³

46. (a)



side of a square = AB
= $\sqrt{2}$ a units

∴ AC = Diagonal = $\sqrt{2}\times\sqrt{2}$ a

∴ Diameter = 2 a units
Circumference= π × diameter
= π × 2a = 2πa units.

47. (b) Perimeter of rectangle = 40 m
Length = 12 metre
∴ 2(l + b) = 40
2 (12 + b) = 40

12 + b = $\frac{40}{2}$ = 20

b = 20 – 12 = 8 m

48. (d) Percentage increase in

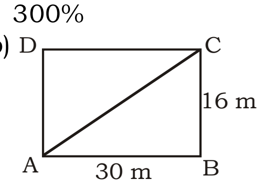
area = $\left(x + y + \frac{xy}{100}\right)\%$

Here, x = 100%, y = 100%

$= \left(100 + 100 + \frac{100\times100}{100}\right)\%$

= 300%

49. (b)



AC = $\sqrt{AB^2 + BC^2}$

= $\sqrt{30^2 + 16^2}$

= $\sqrt{900+256}$

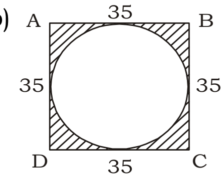
= $\sqrt{1156}$ = 34 metre.

Distance travelled by elephant
= 34 – 4 = 30 metre

speed of elephant = $\frac{30}{15}$

= 2 m/s

50. (b)



According to the question,

Radius of circle = $\frac{35}{2}$

Required area of shaded portion

$= (35)^2 - \frac{22}{7}\times\frac{35}{2}\times\frac{35}{2}$

= 1225 – 962.5 = 262.5 m²

51. (b) Diagonal of square = $\sqrt{2}$ side of square

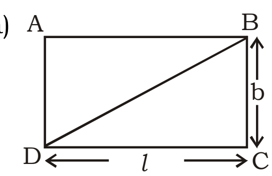
Here a = $\frac{1}{2}$ (x + 1) and d = $\frac{3-x}{\sqrt{2}}$

∵ d = $\sqrt{2}a$

⇒ $\frac{3-x}{\sqrt{2}} = \sqrt{2}\left[\frac{1}{2}(x+1)\right]$

∵ x = 1 unit

52. (a)



Let ABCD is a rectangular carpet having length l metre and breadth b metre and BD is a diagonal
⇒ As we know
⇒ Area = l × b = 120 ....(i)
⇒ Perimeter
= 2(l + b) = 46
Using formula
⇒ (l + b)² = l² + b² + 2l b
⇒ (23)² = l² + b² + 2 × 120
⇒ 529 = l² + b² + 240
⇒ l² + b² = 529 – 240
⇒ l² + b² = 289

⇒ $\sqrt{l^2 + b^2} = \sqrt{289}$

diagonal = 17
diagonal of carpet is17 metres

53. (c) Diagonal of a square
= $6\sqrt{2}$ cm

Side of a square = $\frac{6\sqrt{2}}{\sqrt{2}}$ = 6 cm

Area of a square = 6 × 6 = 36 cm²

54. (c) Let the breadth of floor = x m
Then the length of floor = (x+3) m
A.T.Q,
$x \times (x + 3) = 70$
$x^2 + 3x - 70 = 0$
$x^2 + 10x - 7x - 70 = 0$
$(x + 10)(x - 7) = 0$
$x = 7, x = -10$
Breadth = 7m
Length = 10m
Perimeter of floor = 2 (L + B)
= 2 (10 + 7) = 34 m

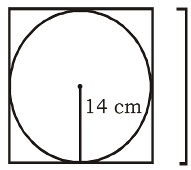
55. (d) Let the breadth of rectangle = x m
then the length of rectangle = 2x m
A.T.Q,
$x \times 2x = 417.605$
$2x^2 = 417.605$
$x^2 = \frac{4170.605}{2}$
$x = \sqrt{\frac{83521}{400}}$
$x = \frac{289}{20}$ cm
Breadth = $\frac{289}{20}$ m
Length = $\frac{289}{20} \times 2 = 28.90$ m

56. (b) Radius of circle = 5 cm
Length of arc = 3.5 cm
$\therefore$ Area of sector = $\frac{1}{2} lr$
$\frac{1}{2} \times 3.5 \times 5 = 8.75 \text{ cm}^2$

57. (c) Radius of circular wheel = 1.75 m
Circumference of circular wheel = $2\pi r = 2 \times \frac{22}{7} \times 1.75$ m
No. of revolutions
$= \frac{\text{Distance to be covered}}{\text{Circumference of circle}}$
$= \frac{11000\text{m}}{2 \times \frac{22}{7} \times 1.75 \text{ m}}$
$= \frac{11000}{11} = 1000$

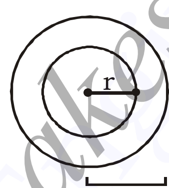
58. (d) Circumference of wheel
$= 2\pi r$
$= 2 \times \frac{22}{7} \times 21 \text{ cm} = 132 \text{ cm}$
No. of revolutions
$= \frac{\text{Distance to be covered}}{\text{Circumference of circle}}$
$= \frac{924 \times 100}{132} = 700$

59. (d)



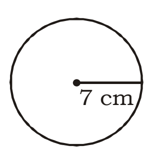
Radius of the largest circle
$= \frac{1}{2} \times (\text{side of square})$
$= \frac{1}{2} \times 28 = 14$ cm
area of the circle = $\pi (\text{radius})^2$
$= \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2$

60. (b)



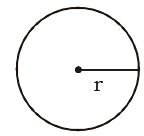
$\because 2\pi r = 88$
$r = \frac{88 \times 7}{2 \times 22} = 14$cm
$2\pi R = 132$ cm
$R = \frac{132 \times 7}{2 \times 22} = 21$ cm
The area between two circles
$= \pi(21)^2 - \pi(14)^2$
$= \pi\{21^2 - 14^2\}$
$= \pi\ (21+14)(21-14)$
$= \frac{22}{7} \times 35 \times 7 = 770 \text{ cm}^2$

61. (b)



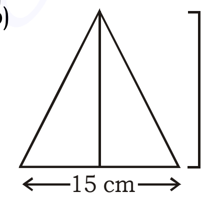
circumference of wheel
$= 2\pi r$
$= 2 \times \frac{22}{7} \times 7$
= 44cm
$\therefore$ Total distance travelled by wheel in 15 revolutions
$= 15 \times 44 \text{ cm} = 660$ cm

62. (c)



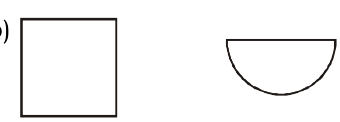
Circumference = $2\pi r$
Distance covered in 1 min
$= 2 \times \frac{8}{40} \times \pi r$
New circumference
$= 2 \times \pi \times r \times 10$
Time taken = $\frac{2\pi r \times 10 \times 40}{2\pi r \times 8}$
= 50 min

63. (b)



area of the triangle
$= \frac{1}{2} \times \text{base} \times \text{height}$
$= \frac{1}{2} \times 15 \times 12$
$= 90 \text{ cm}^2$
area of another triangle = 2×90
$= 180 \text{ cm}^2$
$= \frac{1}{2} \times \text{base} \times \text{height} = 180$
$= \frac{1}{2} \times 20 \times \text{height} = 180$
height = $\frac{180 \times 2}{20} = 18$ cm

64. (b)



area of the square = 81 cm²
side of the square = $\sqrt{81}$ = 9 cm
perimeter of the square = 4×9
= 36 cm
Now, According to question,
$\pi r + 2r = 36$
$r(\pi + 2) = 36$
$r = \frac{36}{\frac{22}{7} + 2} = \frac{36 \times 7}{22 + 14}$

$= \frac{36 \times 7}{36} = 7$

area of the semi circle

$= \frac{\pi r^2}{2} = \frac{22}{7} \times \frac{7^2}{2} = 77 \text{ cm}^2$

65. (b) Area of square $= (12)^2$
$= 144 \text{ cm}^2$

Area of triangle

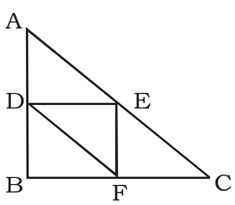
$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 12 \times \text{height}$

$= \frac{1}{2} \times 12 \times \text{height} = 144$

height $= \frac{144 \times 2}{12} = 24$ cm

66. (c)



$\because 3^2 + 4^2 = 5^2$

Δ ABC is a right angled triangle

ar (ABC) $= \frac{1}{2} \times AB \times BC$

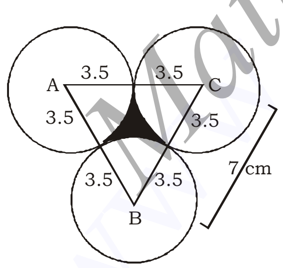
$= \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$

$\therefore$ Required Area of Δ DEF

$= \frac{1}{4} \times 6$

$= \frac{3}{2} \text{ cm}^2$

67. (b)



AB = BC = AC = 7 cm

Area enclosed

= Area of equilateral

$\Delta ABC - \frac{1}{2}(\text{area of 1 circle})$

$= \frac{\sqrt{3}}{4} \times (7)^2 - \frac{1}{2}\left[\frac{22}{7} \times (3.5)^2\right]$

$= 1.967 \text{ cm}^2$

68. (a) $\pi r^2 = 2464 \text{ cm}^2$

$\Rightarrow r = \sqrt{\frac{2464 \times 7}{22}}$

$= \sqrt{784} = 28$m

$\therefore$ diameter $= 2r = 2 \times 28$
$= 56$ cm

69. (b) Required area = Area of square − Area of circle

$= (2a)^2 - \pi (a)^2 = 4a^2 - \frac{22}{7} a^2$

$= \frac{28a^2 - 22a^2}{7} = \frac{6a^2}{7}$

70. (c) Diameter of the circle
= Side of square
$2r = 21$

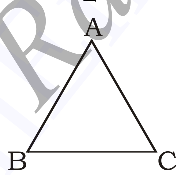
$r = \frac{21}{2}$ m

Area $= \pi r^2 = \pi \left(\frac{21}{2}\right)^2$

$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2} \text{ cm}^2$

$= 346\frac{1}{2} \text{ cm}^2$

71. (a)



$\because$ Area of an equilateral triangle $= 400\sqrt{3}$

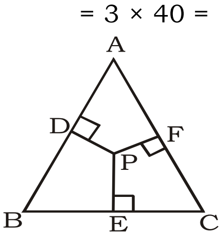
$\frac{\sqrt{3}}{4}(\text{side})^2 = 400\sqrt{3}$

$(\text{side})^2 = \frac{400\sqrt{3} \times 4}{\sqrt{3}} = 1600$

side = 40 m

perimeter = 3 × side
= 3 × 40 = 120 m

72. (c)



Let P be the point inside the equilateral Δ ABC

$PD = \sqrt{3}$, $PE = 2\sqrt{3}$, $PF = 5\sqrt{3}$

and AB = BC = AC = x

Ar (ABC)

$= \frac{1}{2} \times x \times \sqrt{3} + \frac{1}{2} \times x \times 2\sqrt{3} + \frac{1}{2} \times x \times 5\sqrt{3}$

$\frac{\sqrt{3}}{4} x^2 = \frac{1}{2} \times x \times \sqrt{3} + \frac{1}{2} \times x \times 2\sqrt{3} + \frac{1}{2} \times x \times 5\sqrt{3}$

$\frac{\sqrt{3}}{4} x^2 = \frac{1}{2} \times x \times 8\sqrt{3}$

$x = 16$

$\therefore$ perimeter of triangle
$= 3x = 3 \times 16 = 48$ cm

Alternative:-

side of equilateral $\Delta = \frac{2}{\sqrt{3}}$ (sum of the altitudes draw from internal point)

side $= \frac{2}{\sqrt{3}}\left(\sqrt{3} + 2\sqrt{3} + 5\sqrt{3}\right)$

$= \frac{2}{\sqrt{3}} \times 8\sqrt{3} = 16$ cm

perimeter = 3 × sides
= 3 × 16 = 48 cm

73. (c) Perimeter of Δ = 30 cm
Area = 30 cm²

Check the triplet

$\{(5, 12, 13), (3, 4, 5)\}$

whose largest side is 13.

Also, $5^2 + 12^2 = 13^2$

And perimeter = 5 + 12 + 13
= 30 cm

Smallest side = 5 cm

74. (b) Diameter of the wheel = 3 m

Circumference = $\pi \times$ diameter

$= \frac{22}{7} \times 3 = \frac{66}{7}$

Since a wheel covers a distance equal to its circumference in one revolution therefore distance covered in 28 revolutions

$= 28 \times \frac{66}{7} = 264$ m

264 metres covered = 1 minutes

1 metre covered $= \frac{1}{264}$ minute

5280 metres covered $= \frac{5280}{264}$

= 20 minutes

75. (b) distance covered = 2 km 26 decameters

$= (2 \times 1000 + 26 \times 10)$
(1 decameter = 10 meter)
$= 2260$ m

Distance covered in 1 revolution

$$= \frac{\text{Total distance}}{\text{Number of revolutions}}$$

$$= \frac{2260}{113} = 20 \text{ m}$$

Now, $\pi \times$ diameter = 20

$$\text{diameter} = \frac{20 \times 7}{22}$$

$$= \frac{70}{11} = 6\frac{4}{11} \text{ m}$$

76. (a) Distance covered in 1 revolution
= circumference of wheel

$$= 2 \times \frac{22}{7} \times 1.75 \text{ m}$$

$\therefore$ Number of revolution

$$= \frac{11 \times 1000}{2 \times \frac{22}{7} \times 1.75} = 1000$$

77. (b) Radius of circle

$$= \frac{\text{circumference}}{2\pi} = \frac{100}{2\pi}$$

When a square is inscribed in the circle, diagonal of the square is equal to diameter of the circle
$\therefore$ Diagonal of square

$$= 2 \times \frac{100}{2\pi} = \frac{100}{\pi}$$

$\therefore$ side of square

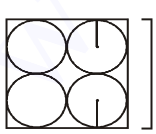
$$= \frac{\text{Diagonal}}{\sqrt{2}} = \frac{100}{\sqrt{2}\pi} = \frac{50\sqrt{2}}{\pi}$$

78. (c) Let outer Radius = R and inner Radius = r

$$2\pi R - 2\pi r = 132$$
$$2\pi (R - r) = 132$$
$$R - r = \frac{132 \times 7}{2 \times 22} = 21$$

Hence, width of path = 21 metres.

79. (b)



side of square papersheet

$= \sqrt{784} = 28$ cm

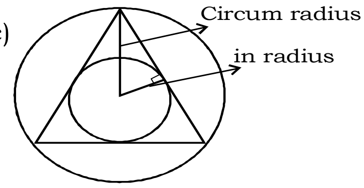
radius of each circle

$$= \frac{28}{4} = 7 \text{ cm}$$

$\therefore$ circumference of each circular plate $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$$

80. (c)



Circum radius of equilateral triangle $= \frac{(side)}{\sqrt{3}} = 8$

side $= 8\sqrt{3}$

In radius of equilateral triangle

$$= \frac{side}{2\sqrt{3}} = \frac{8\sqrt{3}}{2\sqrt{3}} = 4 \text{ cm}$$

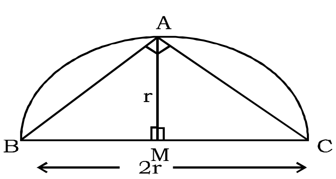
81. (b) radius of each circle = 1 cm with all the three centres an equilateral triangle of side 1 cm is formed .
area enclosed by coins = (area of equilateral triangle) – 3× (area of sector of angle 60°)

$$= \frac{\sqrt{3}}{4}(2)^2 - 3 \times \frac{60}{360} \times \pi (1)^2$$

$$= \frac{\sqrt{3}}{4} \times 4 - 3 \times \frac{1}{6} \times \pi$$

$$= \left(\sqrt{3} - \frac{\pi}{2}\right) \text{cm}^2$$

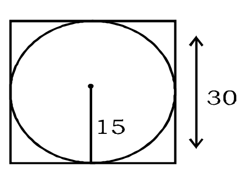
82. (b)



M is the centre, BM = CM = r
AM $\perp$ BC, (AM = r)

area of $\triangle$ ABC $= \frac{1}{2} r \times 2r = r^2$

83. (a)



side of the square

$$= \frac{\text{Perimeter}}{4} = \frac{120}{4} = 30 \text{ cm}$$

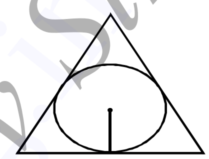
radius of the circle

$$= \frac{\text{side}}{2} = \frac{30}{2} = 15 \text{ cm}$$

area of the circle

$$= \frac{22}{7} \times (\text{radius})^2 = \frac{22}{7} \times (15)^2$$

84. (b)



radius of in circle

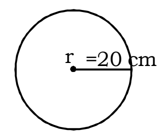
$$= \frac{\text{side}}{2\sqrt{3}} = \frac{42}{2\sqrt{3}} = \frac{21}{\sqrt{3}} \text{ cm}$$

area of incircle

$$= \frac{22}{7} \times \left(\frac{21}{\sqrt{3}}\right)^2 = \frac{22}{7} \times \frac{21 \times 21}{3}$$

$= 22 \times 21 = 462$ cm²

85. (a)

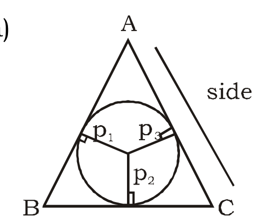


wheel of radius 20 cm
no. of revolutions

$$= \frac{\text{distance to cover}}{\text{circumference of wheel}}$$

$$= \frac{17600 \times 7}{2 \times 22 \times 20} = 140$$

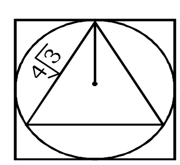
86. (a)



In an equilateral triangle

$$\text{side} = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$$

87. (c)



side of equilateral triangle

$= 4\sqrt{3}$

circumradius of triangle

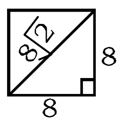
$= \frac{\text{side}}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}} = 4$

see the figure

side of square

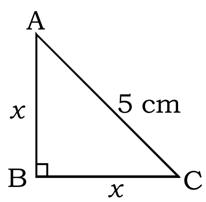
$= 2 \times$ circum radius

$= 2 \times 4 = 8$



Diagonal of square = $8\sqrt{2}$ cm

88. (b)



isosceles right triangle

$\therefore x^2 + x^2 = 5^2 = 25$

$2x^2 = 25$

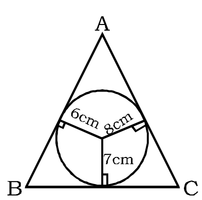
$x^2 = \frac{25}{2}$

Area of triangle

$= \frac{1}{2} \times x^2 = \frac{1}{2} \times \frac{25}{2}$

$= 6.25$ cm²

89. (c)



length of side $= \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$

$= \frac{2}{\sqrt{3}}(6 + 7 + 8) = \frac{2}{\sqrt{3}} \times 21$

$= \frac{42}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{42\sqrt{3}}{3} = 14\sqrt{3}$ cm

90. (c) Area of isosceles triangle

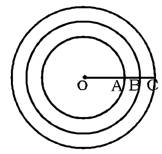
$= \frac{1}{2}a^2 \sin\theta$ (θ is angle between equal sides)

$= \frac{1}{2}(10)^2 \times \sin 45°$

$= \frac{100}{2} \times \frac{1}{\sqrt{2}} = \frac{50}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= 25\sqrt{2}$ cm²

91. (a)



Radius of circle = 6 cm

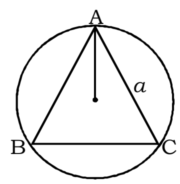
Area of smallest circle

$= \frac{6^2 \times \pi}{3} = 12\pi$

Radius of smallest circle

$= \sqrt{\frac{12\pi}{\pi}} = 2\sqrt{3}$ cm

92. (a)



$\frac{\sqrt{3}}{4}a^2 = 4\sqrt{3}$

$a^2 = 4 \times 4$

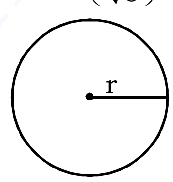
$a = 4$ cm

Circum radius $= \frac{a}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

area of circle $= \pi r^2$

$= \pi\left(\frac{4}{\sqrt{3}}\right)^2 = \frac{16}{3}\pi\ cm^2$

93. (b)



Circumference – diameter = 30 cm

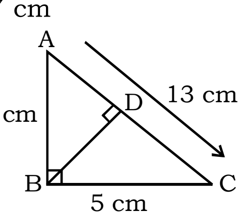
$2\pi r - 2r = 30$

$2r(\pi - 1) = 30$

$r = \frac{30}{2\left(\frac{22}{7} - 1\right)} = \frac{30 \times 7}{2 \times 15}$

$= 7$ cm

94. (b)



AC $= \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$

$= \sqrt{169} = 13$ cm

length of perpendicular,

$BD = \frac{AB \times BC}{AC}$

$\therefore$ length of perpendicular to hypotenuse to

$= \frac{\text{perpendicular} \times \text{Base}}{\text{Hypotenuse}}$

$= \frac{12 \times 5}{13} = \frac{60}{13} = 4\frac{8}{13}$ cm

95. (c) Side of equilateral triangle

$= \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$

$= \frac{2}{\sqrt{3}}(6 + 8 + 10)$

$= \frac{2}{\sqrt{3}} \times 24 = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

Side $= 16\sqrt{3}$ cm

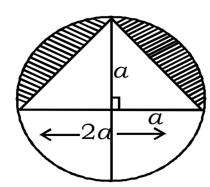
area of triangle

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times \left(16\sqrt{3}\right)^2$

$= \frac{\sqrt{3}}{4} \times 3 \times 16 \times 16$

$= 192\sqrt{3}$ cm²

96. (c)



area of shaded region = area of semicircle – area of triangle

$= \frac{\pi a^2}{2} - \frac{1}{2} \times a \times 2a$

$= \frac{\pi a^2}{2} - a^2 = a^2\left(\frac{\pi}{2} - 1\right)$ sq units

97. (c) According to question

$\pi(R + 1)^2 - \pi R^2 = 22$

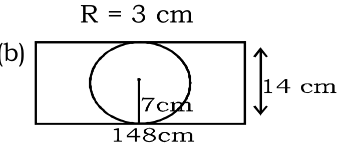
$\pi\{(R+1)^2 - R^2\} = 22$

$(R + 1 + R)(R + 1 - R) = \frac{22 \times 7}{22} = 7$

$2R + 1 = 7$

$R = 3$ cm

98. (b)

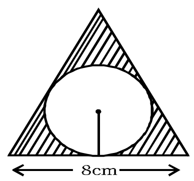


radius of largest circle

$= \frac{\text{breadth}}{2} = \frac{14}{2} = 7$ cm

area $= \frac{22}{7} \times 7 \times 7 = 154$ cm²

99. (b)



in-radius of circle (r) $= \frac{\text{side}}{2\sqrt{3}}$

$= \frac{8}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$

area of circle $= \pi\left(\frac{4}{\sqrt{3}}\right)^2$

$= \frac{22}{7} \times \frac{4 \times 4}{3} = \frac{22 \times 16}{21} = 16.76$

Required area

$= \frac{\sqrt{3}}{4}(8)^2 - \frac{22 \times 16}{21}$

$= \frac{\sqrt{3}}{4} \times 64 - 16.76$

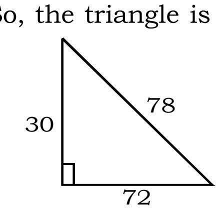
$= 16\sqrt{3} - 16.76$

$= 27.71 - 16.76$

$= 10.95$ cm²

100. (c) $\begin{matrix} 30 & : & 72 & : & 78 \\ 5 & : & 12 & : & 13 \end{matrix}$

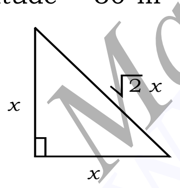
So, the triangle is right triangle



$\frac{1}{2} \times 30 \times 72 = \frac{1}{2} \times \text{altitude} \times 72$

altitude = 30 m

101. (a)



perimeter of triangle $= 4\sqrt{2} + 4$

$x + x + \sqrt{2}x = 4\sqrt{2} + 4$

$2x + \sqrt{2}x = 4\sqrt{2} + 4$

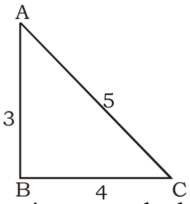
$x(2 + \sqrt{2}) = 4(\sqrt{2} + 1)$

$x = \frac{4}{\sqrt{2}}$

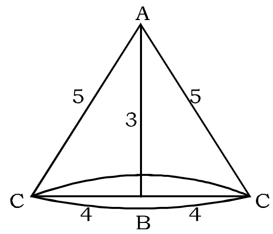
Hypotenuse $= \sqrt{2}x$

$= \sqrt{2} \times \frac{4}{\sqrt{2}} = 4$ cm

102. (a)



cone is rotated about 3 cm



The cone so formed after rotating about Side AB.

So, slant height of cone = 5 cm

$\therefore$ Volume of cone $= \frac{1}{3} \times \pi r^2 h$

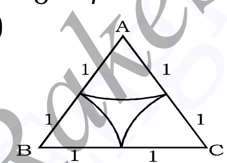
r = radius

h = height

$\therefore$ Volume of cone

$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3 = 16\pi$ cm³

103. (c)



Area of bounded region

$= \frac{\sqrt{3}}{4} \times 2^2 - \frac{1}{2}\pi(1)^2$

$= \left(\sqrt{3} - \frac{\pi}{2}\right) cm^2$

104. (a) $2\pi r = 11$

$\Rightarrow r = \frac{11 \times 7}{22 \times 2} = \frac{7}{4}$

Area of sector

$= \frac{\theta}{360} \times \pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}$

$= \frac{77}{48} = 1\frac{29}{48}$ cm²

105. (c) Let the side of the triangle be 'a' cm

$\Rightarrow$ Circumradius $= \frac{a}{\sqrt{3}}$

and Inradius $= \frac{a}{2\sqrt{3}}$

$\Rightarrow \pi\left(\frac{a}{\sqrt{3}}\right)^2 - \pi\left(\frac{a}{2\sqrt{3}}\right)^2 = 44$

$\Rightarrow \pi\left(\frac{a^2}{3} - \frac{a^2}{12}\right)^2 = 44$

$\Rightarrow \frac{4a^2 - a^2}{12} = \frac{44 \times 7}{22} = 14$

$\Rightarrow \frac{3a^2}{12} = 14$

$\Rightarrow a^2 = 56$

$\Rightarrow a = 2\sqrt{14}$

$\Rightarrow$ area $= \frac{\sqrt{3}}{4} \times 2\sqrt{14} \times 2\sqrt{14}$

$= 14\sqrt{3}$ cm²

106. (c) Side of square = diameter of the circle

area of circle $= \pi r^2 = 9\pi$

$\Rightarrow$ r = 3 cm

$\Rightarrow$ Side of square = 3 × 2 = 6 cm

$\Rightarrow$ Area = 6 × 6

= 36 cm²

107. (d) The given triangle is a right angled triangle

$\Rightarrow$ side of the square

$= \frac{P \times b}{P + b} = \frac{8 \times 6}{8 + 6} = \frac{24}{7}$

$\Rightarrow$ Area of square $= \left(\frac{24}{7}\right)^2$

$= \frac{576}{49}$ cm²

108. (b) Radius of circumcircle

$= \frac{8}{\sqrt{3}}$ cm

Radius of incircle

$= \frac{8}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$ cm

$\Rightarrow$ Required area $= \pi(R^2 - r^2)$

$= \frac{22}{7}\left(\left(\frac{8}{\sqrt{3}}\right)^2 - \left(\frac{4}{\sqrt{3}}\right)^2\right)$

$= \frac{22}{7}\left(\frac{64}{3} - \frac{16}{3}\right)$

$= \frac{22}{7} \times 16 = 50\frac{2}{7}$ cm²

109. (c) Side of square = $\sqrt{121}$ = 11 cm

Perimeter of square = Circumference of circle = 44 cm

$\Rightarrow 2\pi r = 44$

$\Rightarrow r = \frac{44 \times 7}{22 \times 2} = 7$ cm

$\Rightarrow$ Area = πr^2

$= \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$

110. (a) $2r + \pi r = 36$

$\Rightarrow r(2 + \pi) = 36$

$\Rightarrow r(2 + \frac{22}{7}) = 36$

$\Rightarrow r = \frac{36 \times 7}{36} = 7$ m

111. (c) $2r + \pi r = \frac{1}{2}\pi r^2$

$\Rightarrow r(2 + \pi) = \frac{1}{2}\pi r^2$

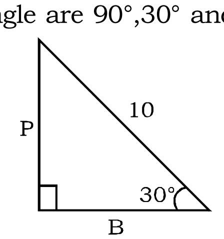
$\Rightarrow 4 + 2\pi = \pi r$

$\Rightarrow r = \frac{4}{\pi} + 2$

$\Rightarrow$ Diameter $= 2\left(\frac{4}{\pi} + 2\right)$

$= 6\frac{6}{11}$ m

112. (a) The angles of the given triangle are 90°, 30° and 60°



$P = \frac{10}{2} = 5$

$B = 5\sqrt{3}$

$\Rightarrow$ Area $= \frac{1}{2} \times 5\sqrt{3} \times 5$

$= \frac{25\sqrt{3}}{2} \text{ cm}^2$

113. (c) Let the altitude = x cm

$\Rightarrow \frac{1}{2} \times x \times 8 = \pi \times 8^2$

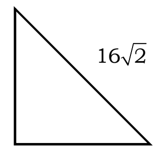
$\Rightarrow x = \frac{\pi \times 64}{4}$

$\Rightarrow x = 16\pi$

114. (d) The sides of the given triangle are 3,4 and 5 cm

area $= \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$

115. (b)



Other sides $= \frac{16\sqrt{2}}{\sqrt{2}} = 16$ cm (as the isosceles $\triangle$)

$\Rightarrow$ Area $= \frac{1}{2} \times 16 \times 16 = 128 \text{ cm}^2$

116. (d) $\frac{\sqrt{3}}{4}a^2 = 4\sqrt{3}$

$\Rightarrow a^2 = 16$

$\Rightarrow a = 4$ cm

117. (c) Side of hexagon

$= \frac{\text{Side of equilateral triangle}}{3}$

= 2 cm

Area of hexagon

$= \frac{3\sqrt{3}}{2}a^2 = \frac{3\sqrt{3}}{2} \times 4$

$= 6\sqrt{3} \text{ cm}^2$

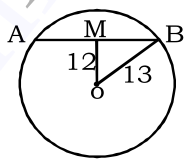
118. (a) The radius of park $= \frac{176}{2\pi}$

= 28 m

$\Rightarrow$ Area of road $= \pi(28 + 7)^2 - \pi(28)^2 = \pi(35 + 28)(35 - 28)$

$= \frac{22}{7} \times 7 \times 63 = 1386 \text{ m}^2$

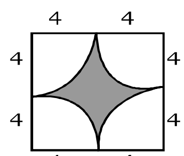
119. (c)



In $\triangle$ OMB MB $= \sqrt{13^2 - 12^2} = 5$

$\Rightarrow$ AB = 5 × 2 = 10 cm

120. (b)



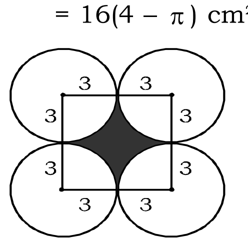
Area of shaded portion

$= 8 \times 8 - \pi \times 4^2$

$= 64 - 16\pi$

$= 16(4 - \pi) \text{ cm}^2$

121. (b)



Area of the shaded portion

$= 6 \times 6 - \pi(3)^2$

$= 36 - 9\pi$

$= 9(4 - \pi) \text{ cm}^2$

122. (c) Radius of incircle

$= \frac{14\sqrt{3}}{2\sqrt{3}} = 7$ cm

$\Rightarrow$ Area $= \pi r^2 = \frac{22}{7} \times 7 \times 7$

$= 154 \text{ cm}^2$

123. (d) Radius of incircle

$= \frac{6}{2\sqrt{3}} = \sqrt{3}$ cm

Area $= \pi r^2$

$= 3\pi \text{ cm}^2$

124. (c) $\frac{\sqrt{3}}{4}a^2 = 121\sqrt{3}$

$\Rightarrow a = 22$ cm

$\Rightarrow 3a = 66$ cm

Circumference of circle = 66 cm

$2\pi r = 66$

$r = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$

Area $= \pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$

$= 346.5 \text{ cm}^2$

125. (b) Area grazed by the cow

$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 77 \text{ m}^2$

$S = \frac{26 + 30 + 28}{2} = 42$

Area of field

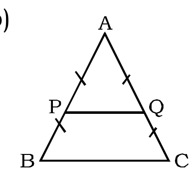
$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42 \times 16 \times 14 \times 12} = 336 \text{ m}^2$

$\Rightarrow$ Remaining area = 336 – 77

$= 259 \text{ m}^2$

126. (b)



As P and Q are mid-point and PQ||BC

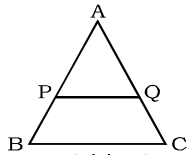
$\Rightarrow \triangle APQ \sim \triangle ABC$

$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} = \frac{1}{2}$

$\Rightarrow PQ = \frac{BC}{2}$

$\Rightarrow BC = 2PQ = 2 \times 5 = 10$ cm

127. (c)



As PQ||BC

$\Rightarrow \triangle APQ \sim \triangle ABC$

$\Rightarrow \triangle APQ$ is also an equilateral $\triangle$

$\Rightarrow \triangle APQ = \frac{\sqrt{3}}{4}(5)^2 = \frac{25\sqrt{3}}{4}$ cm²

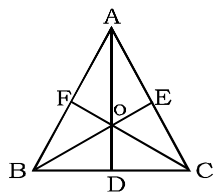
128. (a) $2\pi r = 22$

$\Rightarrow r = \frac{22 \times 7}{22 \times 2} = \frac{7}{2}$

$\Rightarrow \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2}$

$= 38.5$ cm²

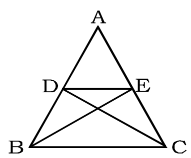
129. (b)



ar ($\triangle$ AOE) = 15 cm²

ar$\square$BDOF = 2 × ar $\triangle$ AOE = 30 cm²

130. (b)



ar ($\triangle$ABE) = ar($\triangle$ACD) = 36 cm²

131. (a) The third side will be either 15 or 22

$\Rightarrow$ Possible perimeter

$= 15 \times 2 + 22 = 52$

and $22 \times 2 + 15 = 59$

132. (a) No. of revolutions

$= \frac{\text{Distance}}{\text{Circumference}}$

$= \frac{1540 \times 100}{2 \times \frac{22}{7} \times \frac{98}{2}} = 500$

133. (b) $2\pi r = \frac{440}{1000}$

$\Rightarrow r = \frac{22 \times 7}{50 \times 22 \times 2} = .07$

$\Rightarrow$ Diameter = .14 m

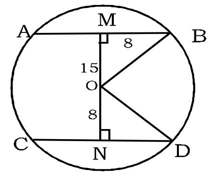
134. (b) $2\pi r = \frac{11000 \times 100}{5000}$

$\Rightarrow r = \frac{11 \times 100 \times 7}{5 \times 2 \times 22} = 35$ cm

135. (b) Length of rubber band

$= 3d + 2\pi r = 30 + 10\pi$

136. (c)



In $\triangle$ OMB

OB = $\sqrt{15^2+8^2}$ = 17 cm

OB = OD = radius

In $\triangle$ OND

ND = $\sqrt{17^2 - 8^2}$ = 15 cm

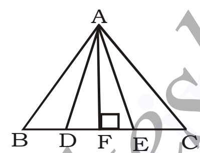
CD = 15 × 2 = 30 cm

137. (b) Perimeter = $2r + \pi r$

$= 63 + \frac{22}{7} \times \frac{63}{2}$

$= 63 + 99 = 162$ cm

138. (c)



In $\triangle$ AFB

AF $\perp$ BC

AF² = AB² – FB² = 100 – 25

AF = $5\sqrt{3}$

In $\triangle$ ADF

AD² = AF² + DF²

AD² = $75 + \left(5 - \frac{10}{3}\right)^2$

AD = $\frac{10\sqrt{7}}{3}$

139. (c) Let sides of triangle are a,b and c respectively

$\therefore$ largest side given =17 cm

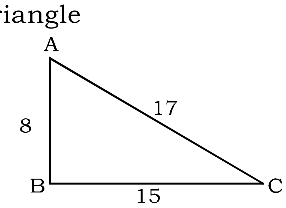
Perimeter = a + b + c

= 40 cm (given)

area = 60 cm² (given)

In such questions take the help of triplets which form right angle triangle

3,4,5
6,8,10
8,15,17
etc



So, here we have a side 17 cm

$\Rightarrow$ by triplet we get sides 8 and 15

$\Rightarrow$ check the sides perimeter

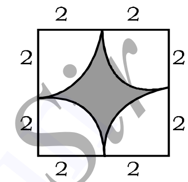
= 8+15+ 17 = 40

area = $\frac{1}{2} \times 8 \times 15 \Rightarrow 60$

Hence sides are 15,8.

smaller side = 8 cm.

140. (b)



Area of shaded region

= (4)² – π (2)² = (16 – 4π) cm²

141. (c) Let the side of the triangle be a

$\Rightarrow$ Perimeter = 3a

$3a = \left(\frac{\sqrt{3}}{4}a^2\right)\sqrt{3}$

$3 = \frac{3}{4}a$

a = 4 units

142. (a) Area of $\triangle$ = $\frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4} \times 36 = 9\sqrt{3}$ cm²

143. (b) Area of $\triangle$ = $\frac{4}{3}$(Areaof $\triangle$ formed by median as side)

$= \frac{4}{3}\left(\frac{1}{2} \times 9 \times 12\right)$

($\because$ 9,12,15 from triplet)

$= \frac{4}{3} \times 54 = 72$ cm²

144. (a) 3x + 2y = 6

$\frac{x}{2} + \frac{y}{3} = 1$

(Make R. H.S. equal to one)

$\Rightarrow$ Coordinates of $\triangle$

= (0,3),(2,0),(0,0)

$\Rightarrow$ Area of $\triangle$ = $\frac{1}{2} \times 3 \times 2$

= 3 square units

145. (a) Let each side of the triangle be a units

$\Rightarrow \frac{\sqrt{3}}{4}((a+2)^2 - a^2) = 3+\sqrt{3}$

$\frac{1}{4}(a^2 + 4 + 4a - a^2) = 1+\sqrt{3}$

$\frac{1}{4}(4 + 4a) = 1+\sqrt{3}$

$1 + a = 1+\sqrt{3}$

$a = \sqrt{3}$ units

146. (c) $S = \frac{9 + 10 + 11}{2} = 15$

using hero's formula

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15\times6\times5\times4}$

$= \sqrt{1800} = 30\sqrt{2}$ cm²

147. (c) Let the length of each equal side be a unit

$\Rightarrow \frac{2}{4}\sqrt{4a^2-4} = 4$

$\sqrt{4a^2-4} = 8$

$4a^2 - 4 = 64$

$a^2 - 1 = 16$

$a^2 = 17$

$a = \sqrt{17}$ units

148. (a) Sum of other two sides

$(a + b) = 32 - 11 = 21$

and $a - b = 5$

$\Rightarrow a = \frac{21+5}{2} = 13$ cm

$b = \frac{21-5}{2} = 8$ cm

Sides of the Δ = 11,8,13 cm

$S = \frac{13+8+11}{2} = 16$

$\Rightarrow$ area

$= \sqrt{16(16-13)(16-8)(16-11)}$

$= \sqrt{16 \times 3 \times 8 \times 5} = 8\sqrt{30}$ cm²

149. (b) Area of $\Delta = \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4} \times (2)^2 = \sqrt{3}$ cm²

150. (a) Let the original radius = r cm

$\Rightarrow \pi((r+1)^2 - r^2) = 22$

$r^2 + 1 + 2r - r^2 = \frac{22 \times 7}{22} = 7$

$2r + 1 = 7$

$r = 3$ cm

151. (a) Area of two circles

$= \pi(5^2 + 12^2) = 169\pi$ cm²

$\Rightarrow \pi r^2 = 169\pi$

$r^2 = 169$

$r = 13$ cm

$\therefore$ Radius of third circle = 13 cm

152. (d) Let the radius of the semi-circle be = r

$\Rightarrow 2r + \pi r = 36$

$r(2 + \pi) = 36$

$r\left(2 + \frac{22}{7}\right) = 36$

$r\left(\frac{36}{7}\right) = 36$

$r = 7$ m

$\Rightarrow$ Area $= \frac{\pi \times 7^2}{2}$

$= \frac{22 \times 7 \times 7}{7 \times 2} = 77$ m²

153. (b) side of square $= \sqrt{\text{area}}$

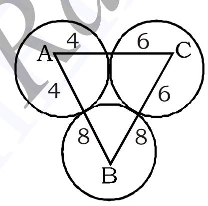
$= \sqrt{2}$ m = Diameter of circle

$\Rightarrow$ Radius of circle

$= \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ m

$\therefore$ Area $= \frac{22}{7} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{\pi}{2}$ m²

154. (d)



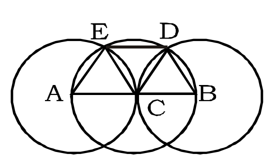
Side of ΔABC = 10, 14, 12

$S = \frac{10+14+12}{2} = 18$

area $= \sqrt{s(s-a).(s-b)(s-c)}$

$= \sqrt{18\times8\times4\times6} = 24\sqrt{6}cm^2$

155. (b)



Area (□ABDE) = 3 × ar(ΔAEC)

$= 3 \times \frac{\sqrt{3}}{4}(2)^2$ (ADC is equilateral triangle)

$= 3\sqrt{3}$ square units

156. (a) Check tripletes

3,4,5

6,8,10

7,24,25

$\Rightarrow$ 7, 24, 25 fulfill the given conditions

area $= \frac{1}{2} \times 24 \times 7 = 84$ cm²

Perimeter = 7 + 24 + 25 = 56 cm

$\Rightarrow$ Hypotenuse = 25 cm

157. (c) Length of median

$= \frac{\sqrt{3}}{2}a = 6\sqrt{3}$

$a = 12$ cm

$\therefore$ Perimeter = 12 × 3 = 36 cm

158. (a) Area of equilateral Δ

$\frac{\sqrt{3}}{4}a^2 = 4\sqrt{3}$

$a^2 = 16$

$a = 4$

$\therefore$ Perimeter = 4 × 3 = 12 cm

159. (a) Distance covered by small gear

$= 2\pi r \times 42$

$= 84\pi \times \frac{12}{2} = 504\pi$

No. of revolution by big gear

$\frac{504\pi}{2\pi \times 9} = 28$

160. (b) Perimeter of semi-circle

$= 2r + \pi r = r(2 + \pi)$

$\Rightarrow r(2 + \pi) = 18$

$r = \frac{18}{2+\frac{22}{7}} = \frac{18\times7}{36}$

$= \frac{7}{2}$ cm $= 3\frac{1}{2}$ cm

161. (d) Perimeter of circle = $2\pi r$

= 2(18 + 26) = 88 cm

$\pi r = 44$ cm

$r = 14$ cm

$\therefore$ Area of circle $= \frac{22}{7} \times 14 \times 14$

= 616 cm²

162. (a) Area of a circle = 38.5 cm²

$\pi r^2 = 38.5$

$r^2 = \frac{38.5 \times 7}{22}$

$r = \frac{7}{2}$ cm

Circumference of a circle = $2\pi r$

$= 2 \times \frac{22}{7} \times \frac{7}{2} = 22$cm

163. (c) Diameter of circle

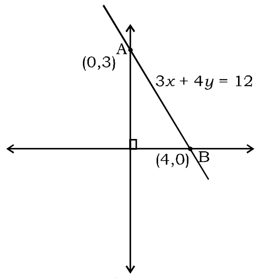
$= \dfrac{\text{Diagonal}}{\sqrt{2}} = \dfrac{12\sqrt{2}}{\sqrt{2}} = 12\text{cm}$

Radius of circle = $\dfrac{12}{2}$ = 6 cm

Radius of circumcircle of equilateral Δ = $\dfrac{a}{\sqrt{3}}$

$\Rightarrow$ a = Radius × $\sqrt{3}$ = $6\sqrt{3}$ cm

164. (c)



3x + 4y = 12

$\dfrac{3x}{12} + \dfrac{4y}{12} = 1$

∴ Divide by 12 on both sides make R.H.S = 1

$\dfrac{x}{4} + \dfrac{y}{3} = 1$

∴ Coordinates of point A = (0,3)
point B = (4,0)

area of ΔOAB = $\dfrac{1}{2} \times 4 \times 3$

= 6 sq units

165. (c) Height of equilateral Δ = 15 cm

$\dfrac{\sqrt{3}}{2}$ (side) = 15

side = $\dfrac{15\times2}{\sqrt{3}}$

area = $\dfrac{\sqrt{3}}{4}$ (side)^2

$= \dfrac{\sqrt{3}}{4}\left(\dfrac{15\times2}{\sqrt{3}}\right)^2 = \dfrac{\sqrt{3}}{4} \times \dfrac{225\times4}{3}$

= $\mathbf{75\sqrt{3}}$ cm^2

166. (b) $\dfrac{\sqrt{3}}{4}(side)^2 = 9\sqrt{3}$

(side)^2 = 9 × 4 = 36

side = $\sqrt{36}$ = 6 cm

length of median = $\dfrac{\sqrt{3}}{2}(side)$

$= \dfrac{\sqrt{3}}{2} \times 6 = \mathbf{3\sqrt{3}}$ cm

Note: In an equilateral triangle, length of median, angle bisector, altitude is equal to $\dfrac{\sqrt{3}}{2}$ sides

167. (c) clearly, 12 cm , 16 cm and 20 cm from a triplet

3 4 5 → triplet
×4 ×4 ×4
12 16 20 → triplet

They from a right triangle,

area of triangle = $\dfrac{1}{2}$ × 16 × 12

= 96 cm^2

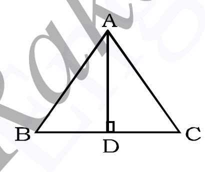
168. (c) $\dfrac{(8)^2}{(x)^2} = \dfrac{360}{250} = \dfrac{36}{25}$

$\dfrac{8}{x} = \sqrt{\dfrac{36}{25}} = \dfrac{6}{5}$

$x = \dfrac{40}{6} = \dfrac{20}{3} = \mathbf{6\frac{2}{3}}$ cm

Note: The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides

169. (d)



AB = AC = $\dfrac{5}{6}$ BC

AB + BC + AC = 544

$\dfrac{5}{6}$ BC + BC + $\dfrac{5}{6}$ BC = 544

$\dfrac{5BC+6BC+5BC}{6} = 544$

$\dfrac{16BC}{6} = 544$

BC = $\dfrac{544\times6}{16}$ = 204

$\Rightarrow$ AB = AC = $\dfrac{5}{6}\times204$ = 170 cm

Area of ΔABC = $\dfrac{b}{4}\sqrt{4a^2 - b^2}$

∴ where a = equal side
b = base

$= \dfrac{204}{4}\sqrt{4(170)^2 - (204)^2}$

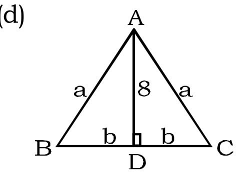
$= 51\sqrt{11560 - 41616}$

$= 51 \times \sqrt{73984}$

= 51 × 272

= 13872 cm^2

170. (d)



Let AB = AC = a cm

BD = DC = b cm

∴ Altitude of isosceles triangle is also median

In right ΔADC

AD^2 = a^2 – b^2

64 = a^2 – b^2 ....(i)

Perimeter = 64

a + a + 2b = 64

2a+ 2b = 64

a + b = 32 ....(ii)

On dividing $\dfrac{a^2 - b^2}{a+b} = \dfrac{64}{32} = 2$

[a^2 – b^2 = (a + b) (a–b)]

a – b = 2

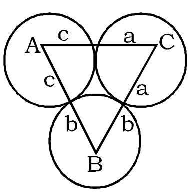
∴ a + b = 32

On solving a = 17 , b = 15

area of ΔABC = $\dfrac{1}{2}$ × AD × BC

= $\dfrac{1}{2}$ × 8 × 30 = 120 cm^2

171. (a)



x = AB = b + c

y = BC = a + b

z = AC = a + c

∴ semi-perimeter(s)

$= \dfrac{AB+BC+AC}{2} = \dfrac{2a+2b+2c}{2}$

= a + b + c

area of

$\Delta ABC = \sqrt{s(s-x)(s-y)(s-z)}$

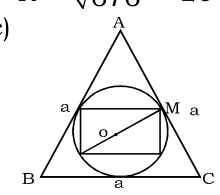
$= \sqrt{\mathbf{(a+b+c)abc}}$

172. (d) $\pi R^2 = \pi(10)^2 + \pi(24)^2$

R^2 = 10^2 + 24^2 = 100 + 576

R = $\sqrt{676}$ = 26 cm

173. (c)



Let the side of an equilateral triangle = 'a'
and the side of square = 'b'
In-circle radius of equilateral

$\Delta = \dfrac{a}{2\sqrt{3}}$

$\therefore$ Diagonal of square

$= 2\times\dfrac{a}{2\sqrt{3}} = \dfrac{a}{\sqrt{3}}$

Now,

$b = \dfrac{Diagonal}{\sqrt{2}} = \dfrac{\frac{a}{\sqrt{3}}}{\sqrt{2}} = \dfrac{a}{\sqrt{6}}$

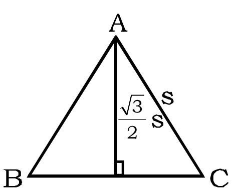
Required ratio

$= \dfrac{\frac{\sqrt{3}}{4}a^2}{\left(\frac{a}{\sqrt{6}}\right)^2} = \dfrac{\sqrt{3}}{4}a^2 \times \dfrac{6}{a^2}$

$= \dfrac{3\sqrt{3}}{2} \Rightarrow$ **$3\sqrt{3} : 2$**

174. (c) Let the side of equilateral triangle = s

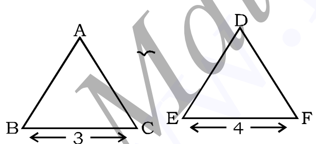
area of equilateral $\Delta = \dfrac{\sqrt{3}}{4}s^2$



height of equilateral triangle = $\dfrac{\sqrt{3}}{2}s$

$\dfrac{b^2}{a} = \dfrac{\left(\frac{\sqrt{3}}{2}s\right)^2}{\frac{\sqrt{3}}{4}s^2} = \dfrac{\frac{3}{4}s^2}{\frac{\sqrt{3}}{4}s^2} = \dfrac{3}{\sqrt{3}} = \sqrt{3}$

175. (c)



$\Delta ABC \sim \Delta DEF$

$\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{3^2}{4^2}$

$\dfrac{54}{ar(\Delta DEF)} = \dfrac{9}{16}$

$ar(\Delta DEF) = \dfrac{16\times 54}{9} = 96\text{ cm}^2$

176. (b) $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AB^2}{DE^2}$

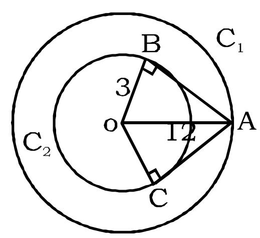
$\dfrac{20}{45} = \dfrac{25}{DE^2}$

$DE^2 = \dfrac{45\times 25}{20} = \dfrac{225}{4}$

$DE = \sqrt{\dfrac{225}{4}}$

$= \dfrac{15}{2} = 7.5$ cm

177. (c)



AB = AC tangents drawn from the same point equal
OB = OC = 3 cm
OA = 12 cm
$\angle ABO = \angle ACO = 90°$
In Right ΔABO

$AB = \sqrt{12^2 - 3^2} = \sqrt{135}$

$= \sqrt{15\times 9} = 3\sqrt{15}$

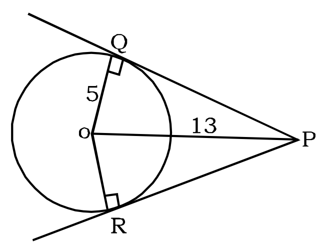
ar (ABOC) = 2× ar(ABO)

$= 2\times \dfrac{1}{2}\times$ AB × OB

$= 3\sqrt{15}\times 3$

$=$ **$9\sqrt{15}$** cm²

178. (b)



$\angle OQP = \angle ORP = 90°$
(radius is $\perp$ tangent)
and PQ = PR (tangent drawn from same point are equal)

$PQ = \sqrt{OP^2 - OQ^2} = \sqrt{13^2 - 5^2}$

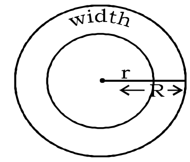
= 12

ar (PQOR) = 2 × ar(PQO)

$= 2\times\dfrac{1}{2}\times$ PQ × OQ

= 12 × 5 = 60 cm²

179. (a)



Let radius of outer circle = R
and radius of inner circle = r
ATQ $2\pi R - 2\pi r = 66$
$2\pi(R - r) = 66$

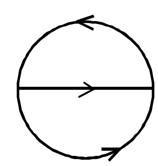
$R - r = \dfrac{66}{2\pi} = \dfrac{66\times 7}{2\times 22} = \dfrac{21}{2}$

width = 10.5 m

180. (d) Distance covered in 30 seconds

$= 30\text{ m/min}\times\dfrac{30}{60} = 15$ m

This is the difference of distance of the boundary and the diameter
Let 'R' be the radius



$2\pi R - 2R = 15$

$2R(\pi - 1) = 15$

$2R = \dfrac{15}{\pi - 1} = \dfrac{15}{\frac{22}{7} - 1} = \dfrac{15\times 7}{15} = 7$

$R = \dfrac{7}{2} = 3.5$ m

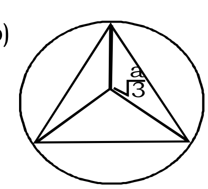
181. (a) Perimeter of the circle = circumference of circle
Let 'R' be the radius
ATQ $2\pi R - 2R = X$
$2R(\pi - 1) = X$

$2R = \dfrac{X}{\pi - 1}$

Diameter= $\dfrac{\mathbf{X}}{\boldsymbol{\pi}\mathbf{-1}}$

$\therefore$ 2R = diameter of the circle

182. (b)



Let the side of an equilateral triangle = 'a'

$\therefore$ Circumcircle radius = $\frac{a}{\sqrt{3}}$

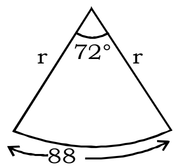
area of circumcircle = $\pi\left(\frac{a}{\sqrt{3}}\right)^2$

$\frac{\pi a^2}{3} = 3\pi$

$a^2 = 9, \quad a = 3$

perimeter = 3 × a = 3 × 3 = 9 cm

183. (a)

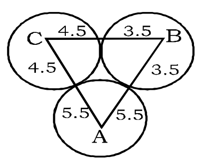


Length of arc = $\frac{\theta}{360}\times 2\pi r$

$\frac{72}{360}\times 2\times\frac{22}{7}\times r = 88$

$r = \frac{88\times 7\times 360}{72\times 2\times 22} = 70$ m

184. (a)

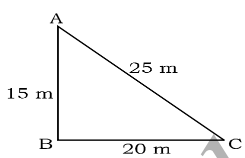


In $\triangle ABC$

perimeter of $\triangle ABC$ = (AB + BC + AC) = 2(3.5 + 4.5 + 5.5)

= (13.5)×2 = 27

185. (c)



$\therefore$ 15,20,25 form a triplet

Clearly, $25^2 = 15^2 + 20^2$

ABC is a right triangle

Area of Right $\triangle ABC$

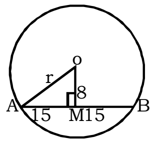
$= \frac{1}{2}\times AB\times BC$

$= \frac{1}{2}\times 15\times 20 = 150$

Cost of sowing seeds

= 150 × ₹ 5 = ₹ 750

186. (a)



AB = 30 cm

OM ⊥ AB and OM = 8

$\therefore$ AM = BM = 15 cm

In Right $\triangle OMA$

$OA^2 = OM^2 + AM^2$

$OA^2 = 15^2 + 8^2$

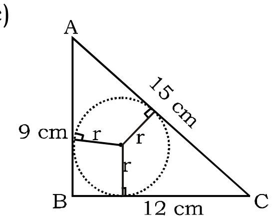
$OA^2 = 289$

$OA = \sqrt{289}$

OA = 17 cm

Radius of circle = 17 cm

187. (c)



Since, 9,12,15 forms a triplet

area of $\triangle ABC = \frac{1}{2}\times 9\times 12$

= 54 cm²

In-circle radius of triangle

$= \frac{\text{area of triangle}}{\text{semiperimeter of triangle}}$

$= \frac{54}{\frac{9+15+12}{2}} = \frac{54\times 2}{36} = 3$ cm

Alternate:

In a right triangle, with P, B and H incircle radius

$= \frac{P+B-H}{2}$

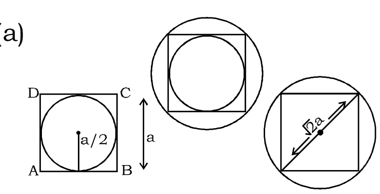
Hence, $r = \frac{9+12-15}{2}$

$= \frac{6}{2} = 3$ cm

Also Circumcircle radius

$= \frac{H}{2} = \frac{15}{2} = 7.5$ cm

188. (a)



Let the side of square = a

In circle radius of square = $\frac{a}{2}$

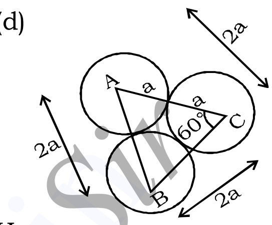
Circumcircle radius of square

$= \frac{Diagonal}{2} = \frac{a\sqrt{2}}{2}$

$\therefore \frac{\text{Incircle radius}}{\text{Circumcircle radius}} = \frac{\frac{a}{2}}{\frac{a\sqrt{2}}{2}}$

$= \frac{1}{\sqrt{2}} = 1:\sqrt{2}$

189. (d)



Hence

ABC is an equilateral triangle

AB = BC = AC = '2a' cm

area of $\triangle ABC = \frac{\sqrt{3}}{4}(2a)^2$

$= \frac{\sqrt{3}}{4}\times 4a^2 = \sqrt{3}\,a^2$

area of 3 sectors (θ = 60°)

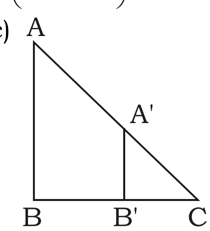
$= 3\times\frac{60°}{360°}\times\pi a^2 = \frac{\pi a^2}{2}$

area of shaded region = area of $\triangle ABC$ – area of 3 sector

$= \sqrt{3}a^2 - \frac{\pi a^2}{2}$

$= \left(\frac{2\sqrt{3}-\pi}{2}\right)a^2$ cm²

190. (c)



In ΔABC and ΔA'B'C

$\angle$C = $\angle$C (common)

$\angle$B' = $\angle$B ($\therefore$ AB | | A'B')

$\Rightarrow \Delta ABC \sim \Delta A'B'C$

$\Rightarrow \frac{area\ of\ \Delta A'B'C}{area\ of\ \Delta ABC} = \left(\frac{B'C}{BC}\right)^2$

$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow$ ar ΔA'B'C = $\frac{1}{4}$(area ΔABC)

191. (b) Perimeter of square = 44 cm

Area of square = $\left(\frac{44}{4}\right)^2$

= 121 cm²

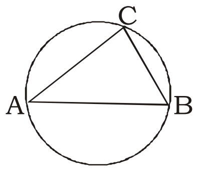
Circumference of circle
$= 2\pi r = 44$ cm

$r = \frac{22 \times 7}{22} = 7$ cm

$\Rightarrow$ area of circle $= \pi r^2 = \frac{22}{7} \times (7)^2$
$= 154$ cm²

Required difference
$= 154 - 121 = 33$ cm²

192. (d)



$\angle ACB = 90°$ (angle in semi-circle)

AC : BC = 3 : 4

$AB^2 = \sqrt{AC^2 + BC^2} = \sqrt{3^2 + 4^2}$

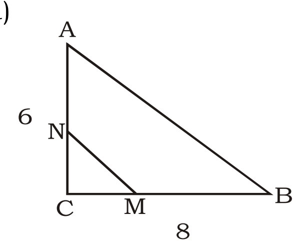
= 5 units

5 units = 5 cm

$\therefore$ ar $\Delta ABC = \frac{1}{2} \times 3 \times 4$

$= 6$ cm²

193. (a)



$Ar(\Delta ABC) = \frac{1}{2} \times 6 \times 8 = 24$ cm²

$\Delta ABC \sim \Delta MCN$

$\angle C = \angle C$

$\angle M = \angle B$ $(\because MN \| AB)$

$\therefore \frac{ar(\Delta CMN)}{ar(\Delta ABC)} = \left(\frac{CM}{BC}\right)^2$

$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

ar (□MNAB)

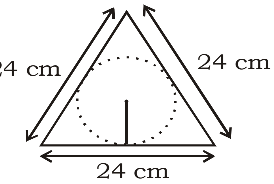
= ar(ΔABC) – ar(ΔCMN)

= 4 – 1 = 3

$\therefore$ ar (□MNAB) $= \frac{24}{4} \times 3$

$= 18$ cm²

194. (a)



Area of equilateral triangle

$= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times 24 \times 24$

$= \sqrt{3} \times 6 \times 24$

$= 144\sqrt{3}$ cm²

$= 144 \times 1.732$

$= 249.408$ cm²

Inradius of an equilateral triangle $= \left(\frac{\text{side}}{2\sqrt{3}}\right)$

$= \frac{24}{2\sqrt{3}} = 4\sqrt{3}$ cm

Now, Area of incircle

$= \frac{22}{7} \times (\text{Inradius})^2$

$= \frac{22}{7} \times 4\sqrt{3} \times 4\sqrt{3}$

$= \frac{22 \times 16 \times 3}{7} = \frac{1056}{7}$

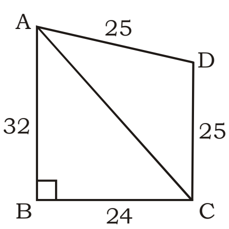
$= 150.86$ cm²

Area of remaining part = area of Δ – area of incircle

$= 249.408 - 150.86$

$= 98.548$ cm²

195. (d)



$\angle ABC = 90°$

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{32^2 + 24^2}$

$= \sqrt{1024 + 576}$

$= \sqrt{1600} = 40$ m

Now, area of Δ ABC

$= \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 32 \times 24 = 384$cm²

Now, In ΔADC,

$s = \frac{25+25+40}{2} = 45$ m

area of Δ ADC

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{45(45-25)(45-25)(45-40)}$

$= \sqrt{45 \times 20 \times 20 \times 5}$

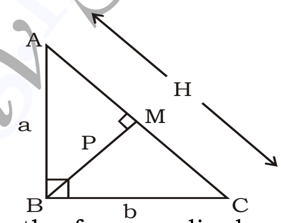
$= 20 \times 3 \times 5$

$= 300$ m²

Area of the plot

$= 384 + 300 = 684$ m²

196. (c)



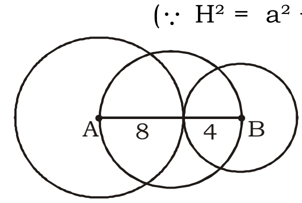
Length of perpendicular drawn from the right angle hypotenuse, $P = \frac{a \times b}{H}$

$P^2 = \frac{a^2 b^2}{H^2}$

$P^2 = \frac{a^2 b^2}{a^2 + b^2}$

$(\because H^2 = a^2 + b^2)$

197. (a)



Diameter of the circle AB
$= 8 + 4 = 12$ units

Radius $= \frac{12}{2} = 6$ units

$\therefore$ Area of circle $= \pi r^2 = \pi \times (6)^2$
$= 36\pi$ sq. units

198. (a) $\frac{\sqrt{3}}{2}(\text{side}) = \frac{\sqrt{3}}{4}(\text{side})^2$

side = 2 units

199. (a) Let the length of side of square = a
then the diameter of circle = d
According to question,
a = d

$\therefore \frac{\text{area of square}}{\text{area of circle}} = \frac{a^2}{\pi\left(\frac{d^2}{4}\right)}$

$= \frac{a^2 \times 4}{\pi d^2} = \frac{a^2 \times 4}{\pi a^2}$

$= \frac{4}{\pi} = \frac{4\times7}{22} = \frac{14}{11}$

$\Rightarrow$ 14 : 11

200. (d) Length of median of an equilateral triangle = $\frac{\sqrt{3}}{2}$(side)

Length of median, altitude, and angle bisector is

$= \frac{\sqrt{3}}{2}$(side)

$\therefore \frac{\sqrt{3}}{2}a = 6\sqrt{3}$

$a = \frac{6\sqrt{3}\times2}{\sqrt{3}} = 12$ cm

$\therefore$ area of Δ ABC = $\frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4}\times12\times12$

$= \sqrt{3}\times3\times12 = 36\sqrt{3}$ cm²

201. (b) $\pi r^2 = 2\pi r$

r = 2 units

$\therefore$ Area of circle = $\pi(2)^2$

$= 4\pi$ sq. units

202. (d) Area of equilateral triangle

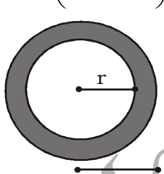
$= \frac{\sqrt{3}}{4}(\text{side})^2$

$\frac{\sqrt{3}}{4}(\text{side})^2 = 48$

$(\text{side})^2 = \frac{48\times4}{\sqrt{3}} = 64\sqrt{3}$

side = $\left(64\,(3)^{\frac{1}{2}}\right)^{\frac{1}{2}} = 8\,(3)^{\frac{1}{4}}$

203. (b)



$2\pi R - 2\pi r = 33$

$(R - r) = \frac{33}{2\pi} = \frac{33\times7}{2\times22}$

$= \frac{3\times7}{2\times2} = \frac{21}{4}$

thickness = 5.25 m

204. (b) Ratio = 5 : 6 : 7

sum of sides = perimeter = 18

sides,

$\frac{5}{18}\times54 = 15$ $\quad \frac{6}{18}\times54 = 18$

$\frac{7}{18}\times54 = 21$ metres

$S = \frac{15+18+21}{2} = 27$

$\therefore$ Area of Δ

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{27\times12\times9\times6}$

$= 54\sqrt{6}$ m²

205. (a) circumference of circle

= $\pi\times$ diameter

$= \frac{22}{7}\times112 = 352$ cm

$\therefore$ perimeter of rectangle = 352

2 (l + b) = 352

$l + b = \frac{352}{2} = 176$

$\therefore$ smaller side = $\frac{7}{16}\times176$

= 77 cm

206. (c) perimeter of equilateral triangle = 18 cm

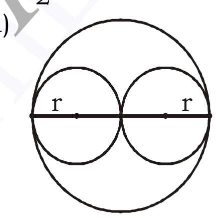
3 × side = 18 cm

side = $\frac{18}{3}$ = 6cm

length of median = $\frac{\sqrt{3}}{2}$ side

$= \frac{\sqrt{3}}{2}\times6 = \mathbf{3\sqrt{3}}$ cm

207. (a)



Circumference of paper sheet = 352

$2\pi R = 352$

$R = \frac{352}{2\pi} = \frac{352\times7}{2\times22}$

= 56 cm

$r = \frac{R}{2} = \frac{56}{2} = 28$ cm

$\therefore$ circumference of circular plate $= 2\pi r$

$= 2\times\frac{22}{7}\times28$

= 176 cm

208. (a) Inradius of equilateral triangle = $\frac{\text{side}}{2\sqrt{3}}$

$\sqrt{3} = \frac{\text{side}}{2\sqrt{3}}$

side = 6 cm

perimeter of equilateral triangle = 3 × 6 = 18 cm

209. (b) Circumference of circle = πd

$\therefore \pi d - d = 150$

$d(\pi - 1) = 150$

$d\left(\frac{22}{7} - 1\right) = 150$

$d\times\frac{15}{7} = 150$

$d = \frac{150\times7}{15} = 70$

Radius $= \frac{d}{2} = \frac{70}{2}$

= 35m

210. (b) Let radius of circle = r

Side of square = a

Side of equilateral Δ = b

According to question,

$2\pi R = 4a = 3b$

$\therefore a = \frac{\pi R}{2} \quad b = \frac{2}{3}\pi R$

Ratio of their areas:

$\pi R^2 : a^2 : \frac{\sqrt{3}}{4}b^2$

$\pi R^2 : \left(\frac{\pi R}{2}\right)^2 : \frac{\sqrt{3}}{4}\left(\frac{2}{3}\pi R\right)^2$

$1 : \frac{\pi}{4} : \frac{\sqrt{3}}{9}\pi$

C : S : T

Here, we can see that C > S > T

Quicker Approach : When perimeter of two or more figures are same then the figure who has more vertex is greater in the area. Since, circle has infinite vertex.

Therefore, C > S > T

211. (d) Distance covered in 1 revolution = Circumference of circular field = $2\pi r$

Distance = speed × time

$= 66$ m/s $\times \frac{5}{2}$s $= 165$ m

$\therefore 2\pi r = 165$

$2\times\frac{22}{7}\times r = 165$

$r = \frac{165\times7}{2\times22}$

= 26.25 m.

212. (c) Circumference of front wheel × no. of its revolutions = circumference of rear wheel × no. of its revolutions
$2\pi x \times n = 2\pi y \times m$ (let 'm' is the revolution of rear wheel)

$$m = \frac{nx}{y}$$

213. (b) Distance to be covered in one revolution = Circumference of wheel
$= \pi \times$ diameter

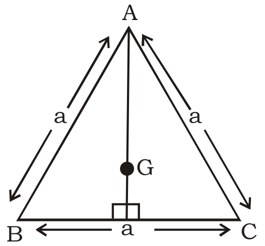
$= \frac{22}{7} \times 56 = 176$ cm

Total distance = 2.2 km
$= (2.2 \times 1000 \times 100)$ cm
$= 22,0000$ cm

$\therefore$ Number of revolutions

$= \frac{220000}{176} = 1250$

214. (b)



We know that in an equilateral triangle a median also be a altitude

$\Rightarrow$ Altitude of an equilateral triangle $= \frac{\sqrt{3}}{2}a$

$\Rightarrow \frac{\sqrt{3}}{2}a = 12\sqrt{3}$ (given)

$\Rightarrow$ a = 24 cm

$\Rightarrow$ Then area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4} \times 24 \times 24$

$= 144\sqrt{3}$ cm^2

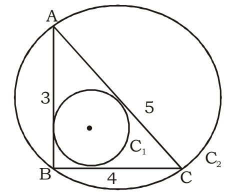
215. (d) Let a triangle ABC has sides of measurement 3 cm, 4cm and 5 cm
using triplets (3, 4, 5)

$\Rightarrow$ Δ ABC will be a right angled triangle

$\Rightarrow$ Inner radius of circle C_1

$= \frac{AB + BC - CA}{2} = \frac{4+3-5}{2}$

r = 1cm



$\Rightarrow$ Circum-radius of circle C_2

$R = \frac{\text{Hypotenuse}}{2}$

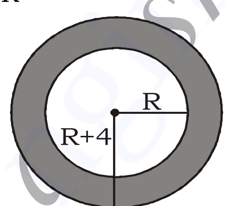
In a right angled triangle half of hypotenuse is circum radius

$R = \frac{5}{2} = 2.5$ cm

$\Rightarrow \frac{\text{Area of } C_1}{\text{Area of } C_2} = \frac{\pi r^2}{\pi R^2}$

$= \frac{1^2}{\left(\frac{5}{2}\right)^2} = \frac{4}{25}$

216. (d) Let the radius of Swimming Pool = R



Outer radius of Pool with concrete wall = (R + 4)
According to question

$\pi R^2 \times \frac{11}{25} = \pi (R+4)^2 - \pi R^2$

$R^2 \times \frac{11}{25} = R^2 + 16 + 8R - R^2$

$\frac{11}{25} R^2 = 16 + 8R$

$11R^2 - 200R - 400 = 0$

By option (d), (In such type of equation go through the option to save your valuable time)
R = 20
$11 \times (20)^2 - 200 \times 20 - 400 = 0$
$4400 - 4000 - 400 = 0$
0 = 0 (satisfy)
Radius of pool R = 20 cm

217. (a) Area of circle = A
Radius of circle = r
Circumference of circle = c

$\pi r^2 = A$ (i)

$2\pi r = c$ (ii)

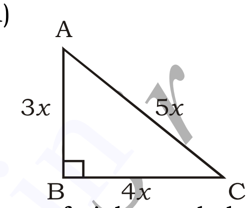
From (i) ÷ (ii)

$\frac{\pi r^2}{2\pi r} = \frac{A}{C}$

$\frac{r}{2} = \frac{A}{C}$

rc = 2A

218. (d)



Area of right angled triangle = 7776

$\Rightarrow \frac{1}{2} \times 4x \times 3x = 7776$

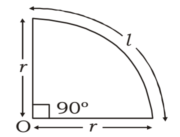
$\Rightarrow 6x^2 = 7776$

$\Rightarrow x^2 = 1296$

$\Rightarrow x = 36$

$\Rightarrow$ Perimeter of triangle
$= 3x + 4x + 5x = 12x$
$= 12 \times 36 = 432$ cm

219. (b)



According to the figure,
$\Rightarrow$ Perimeter = r + r + l
$\Rightarrow$ 75 cm = 2r + length of arc

$\Rightarrow 75 \text{ cm} = 2r + \frac{2\pi r}{4}$

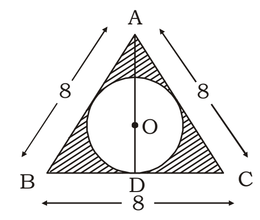
$\Rightarrow 75 \text{ cm} = 2r + \frac{22 \times r}{7 \times 2}$

$\Rightarrow$ r = 21 cm.
$\Rightarrow$ Its area

$= \frac{1}{4}\left[\frac{22}{7} \times 21 \times 21\right]$

$= 346.5$ cm^2

220. (b)



According to the question,
Here OD = radius,

$\therefore r = \frac{a}{2\sqrt{3}} = \frac{8}{2\sqrt{3}}$

$r = \frac{4}{\sqrt{3}}$

Required area of shaded portion

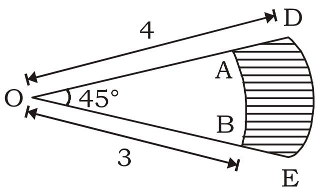
$= \frac{\sqrt{3}}{4} \times (8)^2 - \pi\left(\frac{4}{\sqrt{3}}\right)^2$

$= \frac{\sqrt{3}}{4} \times 64 - \pi \times \frac{16}{3}$

$= \sqrt{3} \times 16 - \frac{22}{7} \times \frac{16}{3}$

$= 10.95 \text{ m}^2 = 11 \text{ m}^2$

221. (d)



According to the question,

Area of sector OED $= \pi r^2 \times \frac{\theta}{360}$

$= \pi \times 4 \times 4 \times \frac{45}{360} = 2\pi \text{ m}^2$

Area of the sector OAB

$= \pi r^2 \times \frac{\theta}{360}$

$= \pi \times 3 \times 3 \times \frac{45}{360}$

$= \frac{9}{8}\pi \text{ m}^2$

So, Area of shaded portion = Area of OED – Area of OAB

$= 2\pi - \frac{9}{8}\pi = \frac{16\pi - 9\pi}{8}$

$= \frac{7}{8}\pi = \frac{7}{8} \times \frac{22}{7} = \frac{11}{4} \text{m}^2$

222. (d) According to the question,

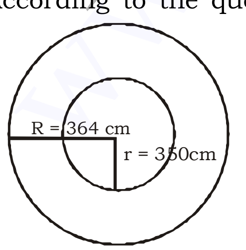
Circumference of a circle $= 2\pi r$

$2\pi r = \frac{30}{\pi}$

$r = \frac{15}{\pi^2}$

$D = 2r = \frac{30}{\pi^2}$

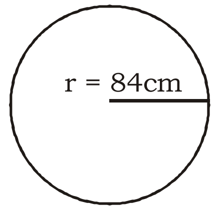
223. (b) According to the question



The breadth of the path = (R – r)

= (364– 350)cm = 14 cm

224. (c) According to the questions,



=

a = ?

Let the length of side of the square be a cm

(circumference of circle = perimeter square)

$2\pi r = 4a$

$2 \times \frac{22}{7} \times 84 = 4a$

132 cm = a

225. (a) Area of circle $= 324\pi \text{ cm}^2$

$\pi r^2 = 324\pi$

r = 18cm

Longest chord = diameter = 2r

= 2×18 = 36 cm

226. (c) Circumference of a Δ = 24 cm

a + b + c = 24 cm

or $S = \frac{a+b+c}{2} = 12$ cm

Circumference of incircle

$2\pi r$ (inner) = 44 cm

r (inner) = 7 cm

Area of Δ = S × r (inner)

= 12 × 7 = 84 cm²

227. (b) Area of $\Delta = \frac{1}{2}$ ab $\sin\theta$

$= \frac{1}{2} \times 10 \times 10 \times \sin 45°$

$= 25\sqrt{2}$ cm²

228. (a) According to the question

$r = \frac{\Delta}{S}$

semiperimeter $= \frac{50}{2} = 25$

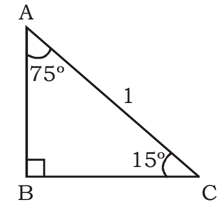
inner radius

$= \frac{\text{Area}}{\text{Semi-perimeter}}$

$6 = \frac{\text{Area}}{25}$

Area = 150 cm²

229. (b) According to the question,



$\sin 15° = \frac{P}{H} = \frac{AB}{1}$

AB = sin15°

$\cos 15° = \frac{B}{H} = \frac{BC}{1}$

BC = Cos15°

Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times \sin 15° \cos 15°$

$= \frac{1}{4} \times \sin 2 \times 15$

$[\because \sin 2\theta = 2\sin\theta\cos\theta]$

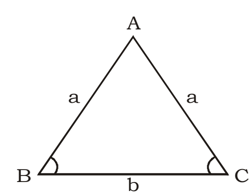
$= \frac{1}{4} \times \sin 30°$

$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \text{m}^2$

$= \frac{1}{8} \times 100 \times 100$

= 1250 cm²

230. (b) According to the question,



Let AB = AC = a

BC = b.

$\therefore S = \frac{a+a+b}{2}$

$S = a + \frac{b}{2}$

Area $= \sqrt{S(S-a)(S-b)(S-c)}$

Area =

$\sqrt{\left(a+\frac{b}{2}\right)\left(a+\frac{b}{2}-a\right)\left(a+\frac{b}{2}-a\right)\left(a+\frac{b}{2}-b\right)}$

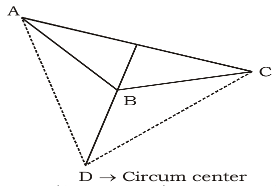
Area $= \sqrt{\left(a+\frac{b}{2}\right)\left(\frac{b}{2}\right)\left(\frac{b}{2}\right)\left(a-\frac{b}{2}\right)}$

Area $= \frac{b}{2}\sqrt{a^2 - \frac{b^2}{4}}$

Area $= \frac{b}{4}\sqrt{4a^2 - b^2}$ sq. units.

231. (c) As we know circumcentre always made by the intersection of half altitude

$\Rightarrow$ In obtuse angle it will always be out.



232. (a) According to the question,

$2\pi r \rightarrow$ circumference

$2r \rightarrow$ Diameter

$\Rightarrow \frac{2\pi r}{2r} = \frac{22}{7}$

$\Rightarrow \frac{1\frac{4}{7}}{2r} = \frac{22}{7}$

$\Rightarrow \frac{11}{7 \times 2r} = \frac{22}{7}$

$\Rightarrow \frac{1}{2r} = \frac{2}{1}$

$\Rightarrow R = \frac{1}{4}$ m

233. (b) Given:

$\Rightarrow$ Area of square = 4

side² = 4

side = 2

$\Rightarrow$ Diagonal of square = radius of circle

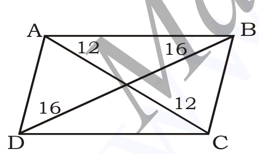
$\sqrt{2}\ side = r$

$\Rightarrow r = 2\sqrt{2}$

$\Rightarrow$ Area of circle = πr^2

$\Rightarrow \pi \times (2\sqrt{2})^2 = 8\pi$ cm²

234. (a) We know that rhombus is parallelogram whose all four sides are equal and its diagonals bisect each other at 90°.



$\therefore AB = \sqrt{16^2 + 12^2}$

$= \sqrt{400} = 20$cm

= side of rhombus

$\therefore$ perimeter of the rhombus = 20 × 4 = 80cm

235. (d) If d_1 and d_2 are the lengths of diagonals of rhombus.

Then,

Perimeter = $2\sqrt{d_1^2+d_2^2}$

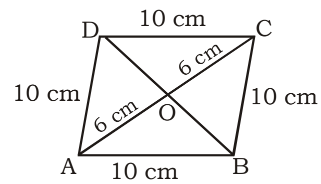
$= 2\sqrt{24^2+10^2}$

$= 2\sqrt{676}$

$= 2 \times 26 = 52$ cm

236. (c) 4 × side = 40 cm(given)

$\Rightarrow$ side = $\frac{40}{4} = 10$ cm



In Δ AOB,

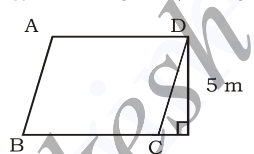
$OB = \sqrt{10^2 - 6^2}$

$= \sqrt{100 - 36}$

$= \sqrt{64} = 8$cm

Diagonal BD = 8 × 2 = 16 cm.

237. (b)



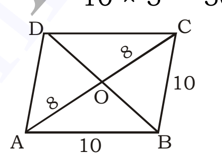
4 × side of rhombus = 40 m

side of rhombus = 10 m

Since rhombus is also a parallelogram therefore its area = base × height

= 10 × 5 = 50 m²

238. (d)



Perimeter of Rhombus = 40 cm

4 × side = 40

side = 10 cm

We know that diagonals of rhombus bisect each other at right angle,

Therefore In right ΔOAB

OB² = AB² – OA²

= 10² – 8² = 100 – 64 = 36

OB = $\sqrt{36}$ = 6 cm

Diagonal BD= 2 × OB = 2 × 6

= 12 cm

Alternative

Side of rhombus = $\frac{1}{2}\sqrt{d_1^2 + d_2^2}$

$10 = \frac{1}{2}\sqrt{16^2+d^2}$

$20 = \sqrt{256+d_2^2}$

$256 + d_2^2 = 400$

$d_2^2 = 400 - 256 = 144$

$d_2 = \sqrt{144} = 12$ cm

239. (b) Diagonal (d_1) = 10 cm

area of Rhombus = 150 cm²

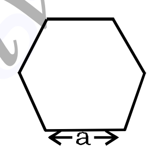
$\frac{1}{2} \times d_1 \times d_2 = 150$

$\frac{1}{2} \times 10 \times d_2 = 150$

$d_2 = \frac{150 \times 2}{10}$

= 30 cm

240. (a)



A regular hexagon consists of 6 equilateral triangle area of regular hexagon

$= 6 \times \frac{\sqrt{3}}{4}(\text{side})^2$

$= 6 \times \frac{\sqrt{3}}{4}a^2$

$= 6 \times \frac{\sqrt{3}}{4} \times \left(2\sqrt{3}\right)^2$

$= 6 \times \frac{\sqrt{3}}{4} \times 12 = 18\sqrt{3}$ cm²

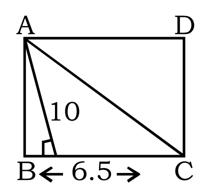
241. (a) area of hexagon

$= 6 \times \frac{\sqrt{3}}{4}(\text{side})^2$

$= 6 \times \frac{\sqrt{3}}{4}(1)^2$

$= 6 \times \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$ cm²

242. (a)



($\therefore$ Rhombus is a ||gm

$\therefore$ area of Rhombus = base × height)

area of Rhombus

= base × height

= 6.5 × 10 = 65 cm²

Also area of Rhombus

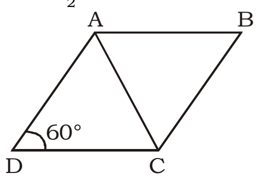
$= \frac{1}{2} \times d_1 \times d_2$

$\frac{1}{2} \times 26 \times d_2 = 65$

$13 \times d_2 = 65$

$d_2 = 5$cm

243. (a)



In the above figure $\triangle ADC$ is an equilateral triangle (as AC is angle bisector)

$\Rightarrow$ AC = 10 cm (smaller diagonal)

244. (c) Side of rhombus

$= \frac{100}{4} = 25$ cm

we know that in a rhombus $4a^2 = d_1^2 + d_2^2$

$\Rightarrow d_2^2 = 4\times(25)^2 - (14)^2 = 2500 - 196 = 2304$

$\Rightarrow d_2 = \sqrt{2304} = 48$ cm

$\Rightarrow$ Area $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 14 \times 48 = 336$ cm²

245. (d) Let the parallel sides be 3x and 2x

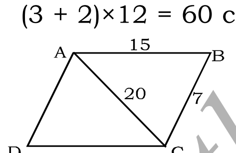
$\Rightarrow \frac{1}{2}(3x + 2x) \times 15 = 450$

$\Rightarrow 5x = 60$

$x = 12$

$\Rightarrow$ Sum of length of parallel sides $= (3 + 2)\times 12 = 60$ cm

246. (c)



Using Hero's formula

$S = \frac{15+7+20}{2} = 21$ cm

Area of $\triangle$ABC

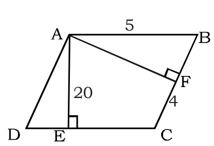
$= \sqrt{21(21-20)(21-7)(21-15)}$

$= \sqrt{21\times1\times14\times6} = 42$ cm²

$\Rightarrow$ Area of $\square$ABCD

$= 42 \times 2 = 84$ cm²

247. (b)



Area of parallelogram $= AB\times AE$

$5x \times 20 = 1000$

$x = 10$

Area of parallelogram $= AD \times AF$

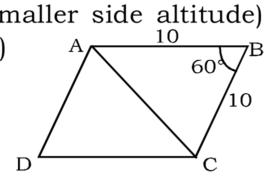
$1000 = 4x \times AF$

$1000 = 4 \times 10 \times AF$

AF = 25 cm

(smaller side altitude)

248. (b)



as $\square$ ABCD is a rhombus

$\therefore \triangle ABC$ is an equilateral $\triangle$

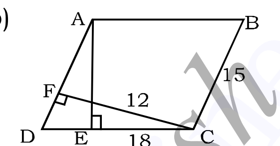
$\Rightarrow$ ar $(\triangle ABC) = \frac{\sqrt{3}}{4} \times 10 \times 10$

$= 25\sqrt{3}$ cm²

$\Rightarrow$ ar $(\square ABCD) = 25\sqrt{3} \times 2$

$= 50\sqrt{3}$ cm²

249. (b)



Area of parallelogram

= AD × FC = 15 × 12

= 180 cm²

Area of parallelogram

= DC × AE = 180

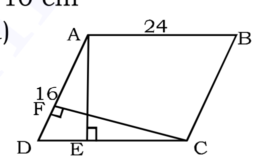
18 × AE = 180

AE = 10 cm

$\therefore$ Distance between bigger sides

= 10 cm

250. (a)



AB = 24 cm

AD = 16 cm

AE = 10 cm (Given)

Area of Parallelogram

= AE × DC = 10 × 24

= 240 cm²

also, area of Parallelogram

= FC × AD = 240

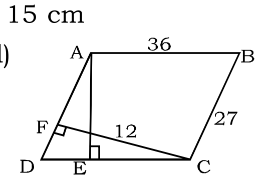
FC × 16 = 240

FC = 15

$\therefore$ Distance between AD and BC

= 15 cm

251. (d)



Area of parallelogram

= AE × DC = CF × AD

AE × 36 = 12 × 27

= AE = 9 cm

$\therefore$ Distance between bigger sides

= 9 cm

252. (a) In a rhombus

$4a^2 = d_1^2 + d_2^2$

$4a^2 = 8^2 + 6^2$

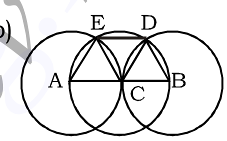
$a^2 = \frac{100}{4} = 25$

$a = 5$

$\Rightarrow$ Side of square = 5 cm

$\therefore$ Area of square = 25 cm²

253. (b)

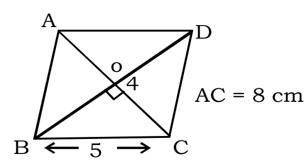


Area ($\square$ ABDE) = 3 × ar($\triangle$ ADC)

(ADC is an equilateral triangle)

$= 3\times\frac{\sqrt{3}}{4}\times2^2 = 3\sqrt{3}$ unit²

254. (d) side of rhombus $= \frac{20}{4} = 5$ cm



OC = 4 cm

In Right $\triangle OBC$

$OB^2 = BC^2 - OC^2$

$= 5^2 - 4^2 = 9$

$OB = \sqrt{9} = 3$ cm

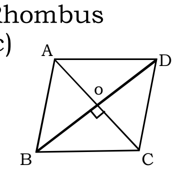
BD = 2 × OB = 2 × 3 = 6 cm

area of Rhombus

$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 8 \times 6 = 24$ cm²

Note: In the question do not get confused with the words non–square its simply to clear that it is Rhombus

255. (c)



side of Rhombus $= \frac{100}{4} = 25$ cm

BD = 40 cm

OB = 20 cm

In right $\triangle OBC$

$OC^2 = BC^2 - OB^2$

OC = $\sqrt{25^2 - 20^2}$ = 15 cm

$\therefore$ AC = 2 × OC

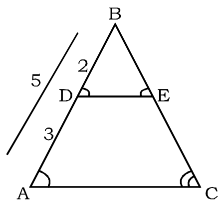
= 2 × 15 = 30 cm

Area = $\frac{1}{2}$ × BD × AC

= $\frac{1}{2}$ × 40 × 30

= 600 cm²

256. (d)



$\therefore$ DE || AC

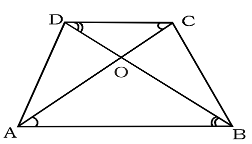
$\therefore$ $\Delta BDE \sim \Delta BAC$

$\frac{ar(\Delta BDE)}{ar(\Delta BAC)} = \frac{2^2}{5^2} = \frac{4}{25}$

ar(trap.ACED) = ar(BAC) – ar(ΔBDE) = 25 – 4 = 21

$\therefore \frac{ar(ACED)}{ar(\Delta BDE)} = \frac{21}{4}$ = 21 : 4

257. (c)



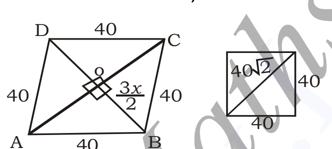
$\therefore$ AB = 2 CD

$\frac{AB}{CD} = \frac{2}{1}$

$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$ = 4:1

$(\Delta AOB \sim \Delta COD)$

258. (d)



Let AC = 4x and BD = 3x

$\therefore$ OA = 2x and OB = $\frac{3x}{2}$

In Right ΔOAB

$\sqrt{(2x)^2 + \left(\frac{3x}{2}\right)^2}$ = 40

4x² + $\frac{9x^2}{4}$ = 40² = 1600

16x² + 9x² = 1600 × 4

25x² = 6400

x² = $\frac{6400}{25}$

x = $\sqrt{\frac{6400}{25}} = \frac{80}{5}$ = 16

$\therefore$ AC = 4x = 4 × 16 = 64

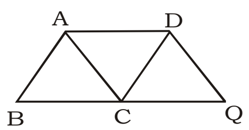
BD = 3x = 3 × 16 = 48

area = $\frac{1}{2}$ × AC × BD

= $\frac{1}{2}$ × 64 × 48

= 1536 cm²

259. (a)



In ΔABC & ΔDCQ

$\angle ABC = \angle DCQ$

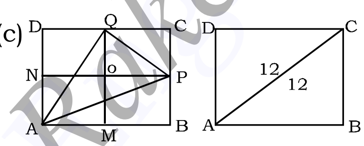
$\angle ACB = \angle DQC$

BC = CQ

$\Delta ABC \cong \Delta DCQ$

$ar(\Delta ABC) = ar(\Delta DCQ)$

260. (c)



area of ABCD = 24

Draw QM and PN and intersect them at O

ar($\square POQC$) = $\frac{1}{4}$ × 24 = 6

$\therefore$ area (ΔPQC) = $\frac{1}{2}$ × 6 = 3

ar (ΔPQC) = 3

ar (QMAD) = $\frac{1}{2}$ × 24 = 12

ar (ΔQAD) = $\frac{1}{2}$ × 12 = 6

ar (ΔABP) = 6

ar(ΔPQC) + ar(ΔQAD) +ar(ΔABP) = 15

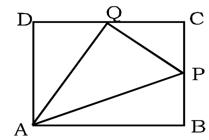
ar(ΔAPQ) = 24 – 15 = 9 cm²

also

$\frac{ar(\Delta APQ)}{ar(ABCD)} = \frac{9}{24} = \frac{3}{8}$

$\therefore$ always it will be 3 : 8

Alternate:-



In this question

ar (ΔAPQ) = $\frac{3}{8}(ABCD)$

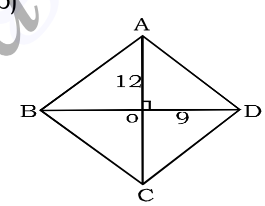
$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{3}{8}$

$\therefore$ ar (ΔABC) = 12

$\therefore$ ABCD = 2 × 12 = 24

ar (ΔAPQ) = $\frac{3}{8}$ × 24 = 9 cm²

261. (b)



d_1 = 24 cm

area of Rhombus = 216

$\frac{1}{2} \times d_1 \times d_2 = 216$

$\frac{1}{2} \times 24 \times d_2$ = 216

$d_2 = \frac{216 \times 2}{24}$ = 18 cm

OA = $\frac{1}{2} \times d_1 = \frac{1}{2}$ × 24 = 12 cm

$\therefore$ Diagonals of Rhombus bisect each other at right angle

OD = $\frac{1}{2} \times d_2 = \frac{1}{2}$ × 18 = 9 cm

Now,

In Right ΔAOD

AD² = AO² + OD²

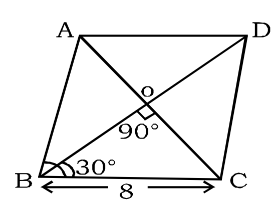
= 12² + 9² = 144 + 81 = 225

AD = $\sqrt{225}$ = 15 cm

$\therefore$ Perimeter of Rhombus

= 4 × AD = 4 × 15 = 60 cm

262. (a)



Let $\angle ABC$ = 60°

$\angle OBC$ = 30°

$\therefore$ Diagonals of Rhombus are the angle bisectors

In right ΔBOC

$$\frac{OB}{BC} = \text{Cos } 30°$$

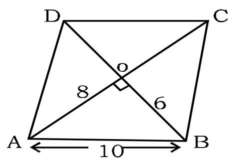
$$\frac{OB}{8} = \frac{\sqrt{3}}{2}$$

$$OB = 4\sqrt{3}$$

$\therefore$ $BD = 2 \times OB$

$$= 2 \times 4\sqrt{3} = 8\sqrt{3} \text{ cm}$$

263. (c)



AC = 16, BD = 12 cm

$\therefore$ OA = 8 cm, OB = 6 cm

$\therefore$ Diagonals of rhombus bisect each other at 90°

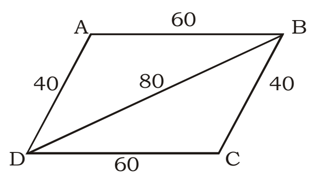
In Right ΔOAB

$$AB^2 = OA^2 + OB^2$$

$$= 8^2 + 6^2 = 100$$

$$AB = \sqrt{100} = 10 \text{ cm}$$

264. (b)



$$S(\Delta ABD) = \frac{60+80+40}{2} = 90$$

ar(ΔABD)

$$= \sqrt{90(90-80)(90-60)(90-40)}$$

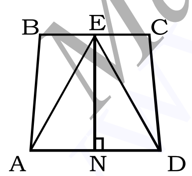
$$= \sqrt{90 \times 10 \times 30 \times 50}$$

$$= 300\sqrt{15} \text{ m}^2$$

ar ($\square$ABCD) = 2 × ar(ΔABD)

$$= 600\sqrt{15} \text{ m}^2$$

265. (d)



Let EN $\perp$ AD

$$\text{area of } \Delta AED = \frac{1}{2} \times EN \times AD$$

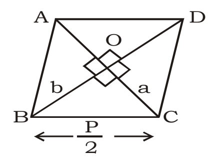
area of trapezium ABCD

$$= \frac{1}{2}(AD+BC) \times EN$$

$$\frac{ar(ABCD)}{ar(AED)} = \frac{\frac{1}{2}(AD+BC) \times EN}{\frac{1}{2} \times EN \times AD}$$

$$= \frac{AD+BC}{AD}$$

266. (c)



side of Rhombus

$$= \frac{\text{perimeter}}{4} = \frac{2P}{4} = \frac{P}{2}$$

Let, AC = 2a

$\therefore$ OA = OC = a

BD = 2b

OB = OD = b

In Right Δ OBC,

$$a^2 + b^2 = \frac{P^2}{4}$$

$4a^2 + 4b^2 = P^2$ .....(i)

Also, $2a + 2b = m$

on squaring,

$4a^2 + 4b^2 + 8ab = m^2$

$4a^2 + 4b^2 = m^2 - 8ab$ ......(ii)

from (i) and (ii)

$m^2 - 8ab = P^2$

$8ab = m^2 - P^2$

$4 \times (2ab) = m^2 - P^2$

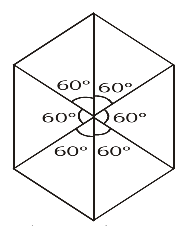
$$2ab = \frac{1}{4}(m^2 - P^2)$$

area of Rhombus

$$= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 2a \times 2b$$

$$= 2ab = \frac{1}{4}(m^2 - P^2)$$

267. (c)



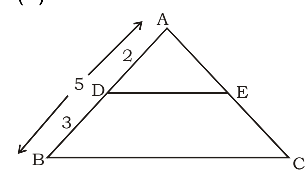
Regular hexagon has 6 equilateral triangle

$\therefore$ Area of Regular hexagon = 6×area of equilateral triangle

$$= 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$$

$$= \frac{\mathbf{9}}{\mathbf{2\sqrt{3}}} \mathbf{a^2}$$

268. (b)



$\therefore$ DE || BC

$\therefore$ $\angle$ADE = $\angle$ABC and

$\angle$AED = $\angle$ACB

$\therefore$ Δ ADE ~ ΔABC

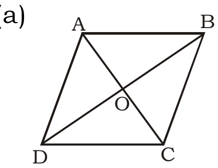
$$\frac{ar(ADE)}{ar(ABC)} = \frac{(2)^2}{(5)^2} = \frac{4}{25}$$

$\therefore$ area ($\square$DECB) = area (ΔABC) – area (ΔADE)

= 25 – 4 = 21

$$\therefore \frac{(ar\Delta \text{ DECB})}{(ar\Delta \text{ ABC})} = \frac{21}{25}$$

269. (a)



AB = BC = CD = DA = 10 cm

BD = 16 cm

In Δ ODC,

OD = 8, CD = 10, $\angle$DOC = 90°

$\therefore$ OC

$$= \sqrt{CD^2 - OD^2} = \sqrt{10^2 - 8^2}$$

= 6 cm

$\therefore$ AC = 2 × OC = 2 × 6 = 12 cm

Now, Area of Rhombus ABCD

$$= \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 16 \times 12 = 96 \text{ cm}^2$$

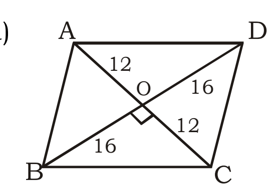
270. (a) Area of trapezium

$$= \frac{1}{2}(\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2}(6+8) \times 4 = \frac{1}{2} \times 14 \times 4$$

$$= 28 \text{ cm}^2$$

271. (a)



AC = 24, BD = 32

$\therefore$ OB = OD = 16 and

OA = OC = 12

(Diagonals of Rhombus bisect each other at 90°

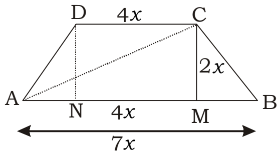
In Δ OBC,

$BC^2 = OB^2 + OC^2 = 16^2 + 12^2 = 400$

BC = $\sqrt{400}$ = 20 cm

perimeter = 20 × 4 = 80 cm

272. (a)



area = $\frac{1}{2}$ (sum of parallel sides)× distance between them

$\frac{1}{2}(7x+4x)\times 2x = 176$

$11x^2 = 176 \Rightarrow x^2 = 16$

$\Rightarrow \quad x = 4$

$AB = 7 \times 4 = 28$ cm

$CD = 4 \times 4 = 16$ cm

$CM = 2 \times 4 = 8$ cm

AM=AN+NM

= AN+16

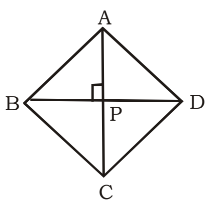
= 6+16=22 (AN=BM=$\frac{12}{2}$=6)

$AC^2 = CM^2 + AM^2$

$AC^2 = 8^2 + 22^2$

$AC = \sqrt{64+484} \Rightarrow \sqrt{548} \Rightarrow 2\sqrt{137}$

273. (b)



ABCD is a rhombus

AB=$\frac{60}{4}$=15cm

(Perimeter = 60 cm)

AC = 24, AP = 12

[Diagonals of rhombus bisect perpendicularly]

In Δ APB

AB = 15, AP = 12

$\therefore$ BP = 9

(By pythagoras theorem)

$BD = 9 \times 2 = 18$

Area of rhombus

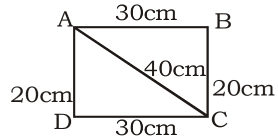
$= \frac{1}{2} \times \text{diagonal}_1 \times \text{diagonal}_2$

$\Rightarrow \frac{1}{2} \times 18 \times 24 = 216$ sq cm

274. (d) Let ABCD is a || gm

area of ▭ABCD = 2 ×area of Δ ADC

For area of (ΔADC)



a = 20cm, b = 30cm, c = 40cm

$S = \frac{a+b+c}{2} = \frac{20+30+40}{2}$

= 45 cm

area (ΔADC)

$= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{45(45-20)(45-30)(45-40)}$

$= \sqrt{45 \times 25 \times 15 \times 5}$

$= 75\sqrt{15}$ cm²

ar (▭ABCD) = $2 \times 75\sqrt{15}$

$= 150\sqrt{15}$ cm²

275. (a) Let the diagonal of rhombus $d_1 = x$ & $d_2 = 2x$

Area of rhombus = $\frac{1}{2} d_1 \, d_2$

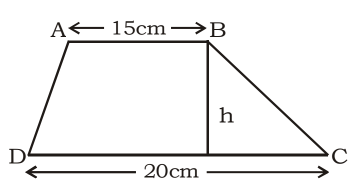
$256 = \frac{1}{2}(x)(2x)$

16 = x

Longer diagonal = 2x = 2 (16)

= 32 cm

276. (b)



As we know

$\Rightarrow$ Area of trapezium

$= \frac{1}{2}(\text{sum of parallel sides}) \times \text{height}$

$\Rightarrow 175 = \frac{1}{2}(20+15) \times h$

$\Rightarrow$ height = 10 cm

277. (a) let the rate of carpenting = Rs x/m²

$\therefore$ length × breadth × x = Rs 120 ...(i)

length × (breadth – 4) × x = Rs 100 ...(ii)

$\frac{\text{breadth}}{\text{breadth}-4} = \frac{120}{100} = \frac{6}{5}$

breadth = 24m

278. (b) Area of corridor = 100 × 3

= 300 m²

Carpet length

$= \frac{300 \times 100}{50} = 600$ cm

Cost of Carpet = ₹ 15 × 600

= 9000

279. (a) Old expenditure = ₹1000

increase in area = 50 × 20 m²

Increase in expenditure

= 50 × 20 × .25 = ₹250

$\Rightarrow$ New expenditure

= 1000 + 250 = ₹1250

280. (d) Area of verandah

= (25+3.5)×(15+3.5)–25×15

= 527.25–375 = 152.25 m²

cost of flooring= 152.25×27.5

= Rs. 4186.50 (app.)

281. (b) $2\pi R_1 = 528$

$\Rightarrow \quad 2 \times \frac{22}{7} \times R_1 = 528$

$\Rightarrow \quad R_1 = 84$ cm

$\Rightarrow$ New Radius = $R_1 - 14 = R_2$

$\Rightarrow \quad R_2 = 84 - 14$

$\Rightarrow \quad R_2 = 70$

New Radius R_2 = 84 – 14 = 70

Area of Road = $\pi(R_1^2 - R_2^2)$

$\Rightarrow$ = $\pi \times 14 \times 154$

$\Rightarrow$ Total expenditure

$= \frac{22}{7} \times 14 \times 154 \times 10$

= Rs. 67760

282. (b) Since the ratio of length and breadth = 3 : 2

Let length of rectangular field = 3x

Breadth of rectangular field = 2x

Perimeter of the field = 80 m

2 (l + b) = 80

2 (2x + 3x) = 80

2 × 5x = 80

$x = \frac{80}{10} = 8$

then breadth = 2x

= 2× 8 = 16 cm

283. (c) The sides of a rectangular plot are in the ratio = 5 : 4

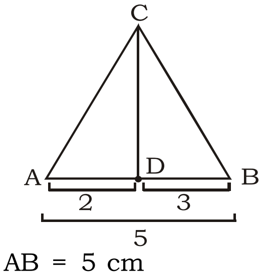
Let the length of rectangular

field = 5x
and the breadth of rectangular fieled = 4x
According to question,
Area = 500 m²
$5x \times 4x = 500$ m²
$20x^2 = 500$ m²

$x^2 = \frac{500}{20} = 25$

x = 5
Length = 5x = 5 × 5 = 25 m
Breadth = 4x = 4 × 5 = 20 m
Perimeter of the rectangle
= 2(25 + 20)
= 2 × 45 = 90m

284. (d)



AB = 5 cm
DB = 3 cm
∴ AD = 2 cm

$\frac{ar(\Delta \text{ ADC})}{ar(\Delta \text{ ABC})} = \left(\frac{\text{AD}}{\text{AB}}\right)^2$

$= \left(\frac{2}{5}\right)^2 = \frac{4}{25}$

285. (d) Base : Corresponding altitude = 3 : 4
Let the base = 3x
altitude = 4x
∵ area of triangle = 1176

$\frac{1}{2} \times 3x \times 4x = 1176$

$x^2 = \frac{1176 \times 2}{3 \times 4} = 196$

x = 14
∴ altitude = 4 × 14 = 56 cm

286. (c) According to question,
Ratio of sides of triangle are

$= \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

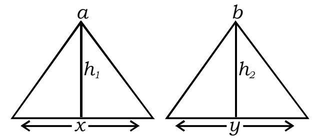
(Take L.C.M of 2, 3, and 4 which is 12)
= 6 : 4 : 3
Now, 6x + 4x + 3x = 52
13x = 52
x = 4
∴ length of smallest side = 3x
= 3 × 4 = 12 cm

287. (c) Let diagonals be 2x and 5x

$\frac{A_1}{A_2} = \frac{\frac{1}{2} \times (2x)^2}{\frac{1}{2} \times (5x)^2} = \frac{4}{25}$

⇒ 4 : 25

288. (c)



$\frac{\frac{1}{2} \times h_1 \times x}{\frac{1}{2} \times h_2 \times y} = \frac{a}{b}$

$\frac{h_1}{h_2} \times \frac{x}{y} = \frac{a}{b}, \quad \frac{h_1}{h_2} = \frac{ay}{bx}$

ay : bx

289. (a) Ratio of parallel sides
= 5 : 3
Let sides are 5x and 3x

$\frac{1}{2}$(sum of parallel sides)× perpendicular distance= 1440 m²

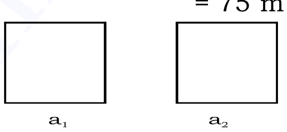
$\frac{1}{2}(5x + 3x) \times 24 = 1440$

4x × 24 = 1440

$x = \frac{1440}{4 \times 24} = 15$m

∴ length of longer side = 5x
= 5 × 15
= 75 m

290. (c)



ATQ, $\frac{a_1^2}{a_2^2} = \frac{225}{256}$

$\frac{a_1}{a_2} = \sqrt{\frac{225}{256}} = \frac{15}{16}$

Ratio of their perimeters

$= \frac{4a_1}{4a_2} = \frac{a_1}{a_2} = \frac{15}{16}$

⇒ 15 : 16

291. (d) Clearly, 3,4 and 5 form a triplet therefore, consider the triangle, a right triangle
Let the sides are 3x,4x,and 5x
perimeter = 3x + 4x + 5x = 12x

area of triangle = $\frac{1}{2} \times 3x \times 4x$

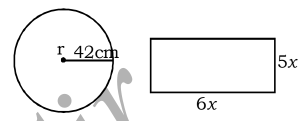
$\frac{1}{2} \times 3x \times 4x = 216$

$x^2 = \frac{216 \times 2}{3 \times 4} = 36$

$x = \sqrt{36} = 6$

∴ Perimeter =12 × 6 = 72 cm

292. (a)

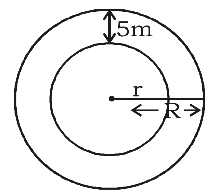


perimeter of rectangle = circumference of circular wire

$2(6x + 5x) = 2 \times \frac{22}{7} \times 42$

22x = 2 × 22 × 6
x = 12
clearly,
smaller side of rectangle
= 5 × 12 = 60 cm

293. (c)



$\frac{2\pi R}{2\pi r} = \frac{23}{22}$

$\frac{R}{r} = \frac{23}{22}$

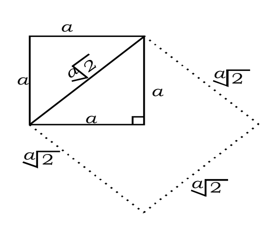
Let R = 23x , r = 22x
∴ R – r = 5
23x – 22x = 5
x = 5
⇒ r = 22 × 5 = 110
diameter of inner circle = 2r
= 2 × 110 = 220 m

294. (b) Ratio of angles = 3 : 4 : 5
3 + 4+ 5 =180°
12 = 180°

$1 = \frac{180°}{12} = 15°$

3 : 4 : 5
×15 ×15 ×15
45 60 75→largest angle

295. (b)



Let the side of square = a

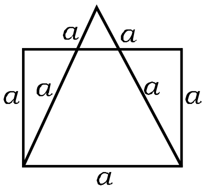
$\therefore$ Diagonal = $a\sqrt{2}$

$\left\{\sqrt{a^2+a^2} = a\sqrt{2}\right\}$

$\frac{\text{Area of square}}{\text{Area of square on diagonal}}$

$= \frac{a^2}{\left(a\sqrt{2}\right)^2} = \frac{a^2}{a^2 \times 2} = \frac{1}{2} = 1:2$

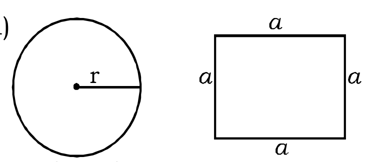
296. (d)



$= \frac{\text{area of square}}{\text{area of equilateral triangle}}$

$= \frac{a^2}{\frac{\sqrt{3}}{4}a^2} = \frac{4}{\sqrt{3}} = 4:\sqrt{3}$

297. (d)



$\pi r^2 = a^2$

$r^2 = \frac{a^2}{\pi}$

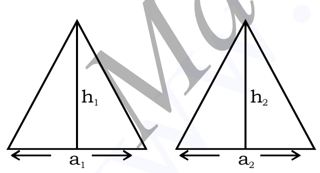
$r = \frac{a}{\sqrt{\pi}}$

Ratio of perimeter = $\frac{2\pi r}{4a}$

$= \frac{\pi r}{2a}$

$= \frac{\pi \times \frac{a}{\sqrt{\pi}}}{2a} = \frac{\sqrt{\pi}}{2} = \sqrt{\pi}:2$

298. (c)



$\frac{\frac{\sqrt{3}}{4}(a_1)^2}{\frac{\sqrt{3}}{4}(a_2)^2} = \frac{25}{36}$

$\frac{a_1}{a_2} = \sqrt{\frac{25}{36}} = \frac{5}{6}$

Ratio of altitudes = $\frac{\frac{\sqrt{3}}{2}a_1}{\frac{\sqrt{3}}{2}a_2}$

$= \frac{a_1}{a_2} = \frac{5}{6} = 5:6$

299. (d) Let length = 5x

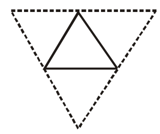
$\frac{l}{2(l+b)} = \frac{5}{16}$

$8l = 5l + 5b$

$3l = 5b$

$\frac{l}{b} = \frac{5}{3} = 5:3$

300. (c)



When we draw such figures as mentioned in the question the vertex of the old triangle are the mid points of the sides of new triangle and the sides of the old triangle are half of the opposite side.

$\therefore$ required ratio = 2 : 1

301. (b) $\frac{\text{Circumference}}{\text{Area}} = \frac{2\pi r}{\pi r^2}$

$= \frac{2}{r} = \frac{2}{3}$

302. (b) Ratio of area= (Ratio of radius)^2 = $\left(\frac{\frac{a}{\sqrt{3}}}{\frac{a}{2\sqrt{3}}}\right)^2$

= 4 : 1

303. (a) Ratio of area= (Ratio of radius)^2

	A		B		C
Radius	4	:	2	:	1
Area	16	:	4	:	1

304. (b) $\pi r^2 = a^2$

$\frac{a^2}{r^2} = \frac{\pi}{1}$

$\frac{a}{r} = \sqrt{\pi} : 1$

305. (c) Ratio of sides = $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$

= 20 : 15 : 12

20 + 15 + 12 = 47

$\Rightarrow$ 47 $\rightarrow$ 94

1 $\rightarrow$ 2

$\Rightarrow$ Smallest side = 12 × 2

= 24 cm

306. (a) Let the sides be 3x,4x,5x and 6x

$\Rightarrow$ 18x $\rightarrow$ 72, x $\rightarrow$ 4

$\Rightarrow$ Greatest side = 6 × 4

= 24 cm

307. (b) Ratio of circumference = Ratio of radius = 3 : 4

308. (d) Let the sides be 2x, 3x and 4x

$\Rightarrow$ 9x = 18 $\Rightarrow$ x = 2

$\Rightarrow$ Sides are 4,6 and 8 cm respectively

Using hero's formula

$S = \frac{4+6+8}{2} = 9$ cm

$\Rightarrow$ area = $\sqrt{s(s-a)(s-b)(s-c)}$

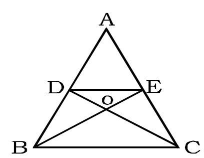
$= \sqrt{9 \times 5 \times 3 \times 1}$

$= 3\sqrt{15}$ cm^2

309. (b) Ratio of area = (Ratio of radius)^2

$= \left(\frac{a}{\sqrt{3}} : \frac{a}{2\sqrt{3}}\right)^2 = 4:1$

310. (a)



As D and E are mid-points of AB and AC

$\Rightarrow$ DE| |BC

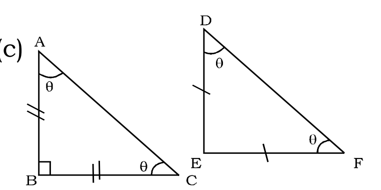
$\Rightarrow \triangle ODE \sim \triangle BOC$

and also $\frac{DE}{BC} = \frac{1}{2}$

(as D and E are mid-points)

$\Rightarrow \frac{ar(\Delta ODE)}{ar(\Delta OBC)} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

311. (c)



The given angle is same

let vertical angle = θ

($\therefore \triangle$ABC and $\triangle$DEF are isoceles triangles)

$\Rightarrow$ when two angles are equal then third angle is also equal

$\therefore \triangle ABC \sim \triangle DEF$

$\triangle$ABC is similar to $\triangle$DEF

$\therefore \frac{\text{area of } \triangle \text{ ABC}}{\text{area of } \triangle \text{ DEF}}$

$= \left(\dfrac{\text{side of } \triangle \text{ ABC}}{\text{side of } \triangle \text{ DEF}}\right)^2$

$\Rightarrow \sqrt{\dfrac{1}{4}} = \dfrac{\text{side of } \triangle \text{ ABC}}{\text{side of } \triangle\text{DEF}}$

$= \dfrac{\text{side of } \triangle \text{ ABC}}{\text{side of } \triangle\text{DEF}} = \dfrac{1}{2}$

312. (a) Let the sides be 3x ,3x and 4x

$\Rightarrow$ Area $= \dfrac{(4x)^2}{4}\sqrt{4(3x)^2-(4x)^2}$

$= 4x^2\sqrt{36x^2-16x^2} = 4x^2\sqrt{20x^2}$

$= 8x^3\sqrt{5} = 8\sqrt{5} = x^3 = 1$

$= x = 1$

$\therefore$ 3rd side = 3 × 1 = 3 units

313. (c) 3, 4 and 5 from triplet
Let the sides be 3x,4x and 5x

$\Rightarrow \dfrac{1}{2} \times 3x \times 4x = 72$

$\Rightarrow 6x^2 = 72$

$\Rightarrow x^2 = 12$

$\Rightarrow x = 2\sqrt{3}$

$\therefore$ Smallest side $= 3 \times 2\sqrt{3}$

$= 6\sqrt{3}$

314. (b) Let the sides be 3x, 4x and 5x

$\Rightarrow$ area $= \dfrac{1}{2} \times 3x \times 4x = 72$

$\Rightarrow 6x^2 = 72$

$x^2 = 12$

$x = 2\sqrt{3}$

$\Rightarrow$ Perimeter of equilateral $\triangle = 12 \times 2\sqrt{3} = 24\sqrt{3}$ units

Side of $\triangle = \dfrac{24\sqrt{3}}{3} = 8\sqrt{3}$ units

area of $\triangle = \dfrac{\sqrt{3}}{4} \times \left(8\sqrt{3}\right)^2$

$= \dfrac{\sqrt{3}}{4} \times 64 \times 3 = 48\sqrt{3}$ unit²

315. (d) Let the parallel sides be 2x and 3x

$\Rightarrow$ area $= \dfrac{1}{2}(2x + 3x) \times 12 = 480$

$5x = 80$

$x = 16$

$\Rightarrow$ Longer parallel side

$= 16 \times 3 = 48$ cm

316. (a) Let the side of square = a

$\therefore$ Side of equilateral $\triangle = \sqrt{2}$ a

Required ratio $= \dfrac{\dfrac{\sqrt{3}}{4}\left(\sqrt{2}a\right)^2}{a^2}$

$= \dfrac{\sqrt{3}}{4} \times 2 = \sqrt{3} : 2$

317. (b) Ratio of area = (Ratio of side)²

$\dfrac{ar(\triangle ABC)}{ar(\triangle DEF)} = \left(\dfrac{10}{8}\right)^2 = 25 : 16$

318. (a) $\dfrac{\pi r_1^2}{\pi r_2^2} = \dfrac{4}{7}$

$\dfrac{r_1^2}{r_2^2} = \dfrac{4}{7}$

$\dfrac{r_1}{r_2} = \sqrt{\dfrac{4}{7}} = \dfrac{2}{\sqrt{7}} = \mathbf{2 : \sqrt{7}}$

319. (c) Required ratio =

$\dfrac{\pi(5)^2 - \pi(3)^2}{\pi(5)^2} = \dfrac{(5)^2-(3)^2}{(5)^2} = \dfrac{16}{25}$

$\Rightarrow$ 16 : 25

320. (a) Let side of square = a

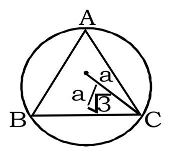
radius of smaller circle $= \dfrac{a}{2}$

radius of larger circle $= \dfrac{\sqrt{2}a}{2}$

Required ratio $= \dfrac{\pi\left(\dfrac{a}{2}\right)^2}{\pi\left(\dfrac{\sqrt{2}a}{2}\right)^2}$

$= \dfrac{\dfrac{a^2}{4}}{\dfrac{2a^2}{4}} = \dfrac{1}{2} \Rightarrow 1 : 2$

321. (c)



Circumradius $= \dfrac{side}{\sqrt{3}} = \dfrac{a}{\sqrt{3}}$

Equilateral $\triangle$

Required ratio $= \dfrac{\dfrac{\sqrt{3}}{4}a^2}{\pi\left(\dfrac{a}{\sqrt{3}}\right)^2} = \dfrac{3\sqrt{3}}{4\pi}$

$= \mathbf{3\sqrt{3} : 4\pi}$

322. (d) 2(l + b) = 4a
(a = side of square)

2(2 + 1) = 4a

2 × 3 = 4a

$a = \dfrac{3}{2}$

Required ratio

$= \dfrac{l \times b}{a^2} = \dfrac{1 \times 2}{\left(\dfrac{3}{2}\right)^2} = \dfrac{2 \times 4}{9} = \dfrac{8}{9} = 8 : 9$

323. (c) 2(l + b) = 3a

$\therefore$ (a = side of equilateral triangle)

Let (b = a)

$\Rightarrow$ 2(l + a) = 3a

2(l + a) = 3a

2l + 2a = 3a

2l = a

Required Ratio

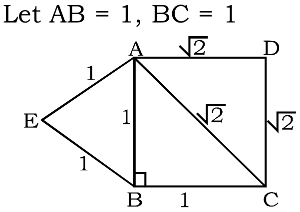
$= \dfrac{l \times b}{\dfrac{\sqrt{3}}{4}a^2} = \dfrac{\dfrac{a}{2} \times a}{\dfrac{\sqrt{3}}{4}a^2} = \dfrac{a^2}{2} \times \dfrac{4}{\sqrt{3}a^2}$

$= \dfrac{2}{\sqrt{3}} = \mathbf{2 : \sqrt{3}}$

324. (b) Required ratio $= \dfrac{\pi r^2}{r^2} = \dfrac{\pi}{1}$

$= \mathbf{\pi : 1}$

325. (c) Let AB = 1, BC = 1



$\therefore$ AC $= \sqrt{1^2+1^2} = \sqrt{2}$

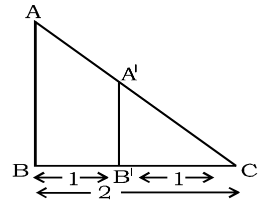
(using pythagoras)

$= \dfrac{ar(\triangle ABE)}{ar(\triangle ACD)} = \dfrac{\dfrac{\sqrt{3}}{4}(1)^2}{\dfrac{\sqrt{3}}{4}\left(\sqrt{2}\right)^2} = \dfrac{1}{2}$

= 1 : 2

326. (b) $\dfrac{ar\triangle ABC}{ar\triangle DEF} = \left(\dfrac{AB}{DE}\right)^2 = \left(\dfrac{10}{8}\right)^2 = \dfrac{25}{16}$

327. (c)



A'B' || AB

$\therefore$ A' and B' are the mid-point. By mid point theorem

$\therefore$ $\Delta A'B'C \sim \Delta ABC$

Let BB' = B'C = 1

$\therefore$ BC = 2

(B' is the mid-point of BC)

$$\frac{ar(\Delta A'B'C)}{ar(\Delta ABC)} = \left(\frac{B'C}{BC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

ar(AA'B'B) = ar(Δ ABC) – ar(Δ A'B'C)

$$\frac{ar(AA'B'B)}{ar(\Delta ABC)} = \frac{3}{4} = 3 : 4$$

328. (d) Ratio of sides = $\frac{1}{4} : \frac{1}{6} : \frac{1}{8}$

$= \frac{1}{4} \times 24 : \frac{1}{6} \times 24 : \frac{1}{8} \times 24$

= 6 : 4 : 3

(Take L.C.M = 24)

ATQ perimeter = 91

6 + 4 + 3 = 91

13 units = 91

1 unit = $\frac{91}{13}$ = 7

Diff. between longer and shorter side = 6 - 3 = 3 units

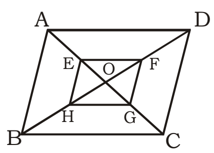
$\Rightarrow$ 3 units = 7 × 3 = 21 cm

329. (c) By using result,

$R_1\theta_1 = R_2\theta_2$

$$\frac{R_1}{R_2} = \frac{\theta_2}{\theta_1} = \frac{75°}{60°} = \frac{5}{4} = 5 : 4$$

330. (c)



In Δ OBC,

H and G are the midpoints of OB and OC

$\therefore$ HG = $\frac{1}{2}$BC

similarly, FG = $\frac{1}{2}$ CD

and EF = $\frac{1}{2}$ AD,

HE = $\frac{1}{2}$ AB

on adding,

HE + HG + FG + EF

= $\frac{1}{2}$(AB + BC + CD + AD)

perimeter of EFGH.

= $\frac{1}{2}$ × perimeter of ABCD

$$\frac{\text{perimeter of EFGH}}{\text{perimeter of ABCD}} = \frac{1}{2}$$

331. (c) Old circumference = 4π

2πr = 4π

r = $\frac{4\pi}{2\pi}$ = 2cm

Old area = π(2)² = 4π cm²

New circumference = 8π

2πR = 8π

R = $\frac{8\pi}{2\pi}$ = 4cm

New area = 16π cm²

Option (c) is the answer

($\because$ area is quadruples)

332. (c) Length 4 → 5

Breadth 5 → 4

area 20 → 20

area remains unchanged

333. (d) Area of circle = π(5)² = 25π

Circumference of circle

= 2π(5) = 10π

= $\frac{25\pi}{10\pi}$ × 100 = 250%

334. (d) According to question,

Circumference of a circle = area of circle

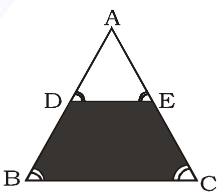
2πr = πr²

r = 2

$\therefore$ diameter of circle = 2r

= 2 × 2 = 4

335. (c)



$\therefore$ D and E are the mid points of sides AB and AC

$\therefore$ DE||BC (By mid point theorem)

also DE = $\frac{1}{2}$ BC

ΔADE ~ ΔABC

$$\begin{Bmatrix} \angle ADE = \angle ABC \\ \angle AED = \angle ACB \end{Bmatrix}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \left(\frac{DE}{BC}\right)^2$$

$$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\therefore \frac{ar(DECB)}{ar(\Delta ABC)} = \frac{3}{4}$$

Percentage of ar (DECB)

= $\frac{3}{4}$ × 100 = 75%

336. (b) Increment in breadth = 10%

= $\frac{10}{100} = \frac{1 \rightarrow \text{Increment}}{10 \rightarrow \text{breadth}}$

Decrement in length = 10%

= $\frac{10}{100} = \frac{1 \rightarrow \text{decrement}}{10 \rightarrow \text{length}}$

	length	breadth	Area
Original	10	10	100
New	9	11	99) –1

% change = $\frac{-1}{100}$ = 1%

Alternate:-

using x = 10% (breadth),

y = –10% (length)

% change = x + y $\frac{xy}{100}$

= 10 – 10 + $\frac{10 \times (-10)}{100}$ = –1%

337. (c) % increase = x + y + $\frac{xy}{100}$

= 20 + 20 + $\frac{20 \times 20}{100}$ = 44%

338. (d) If circumference of circle is reduced by 50% then radius is reduced by 50%

50% = $\frac{1 \rightarrow \text{decrement}}{2 \rightarrow \text{original}}$

	radius	Area
Original	2	4
New	1	1) –3

(π is constant)

Reduction in area

= $\frac{3}{4}$ × 100 = 75%

339. (d) Increase in area

= 25 + 25 + $\frac{25 + 25}{100}$

use formula : $(x + y + \frac{xy}{100})$

= 50 + 6.25

= 56.25%

340. (a) Increase in area

= 50 + 50 + $\frac{50+50}{100}$

= 100 + 25 = 125%

341. (b) using $x + y + \frac{xy}{100}$

$= 20 - 20 + \frac{20 \times (-20)}{100}$

$= -4\%$

(decrease by 4%)

342. (b) Increase in area

$= 50 + 50 + \frac{50 \times 50}{100}$

$= 100 + 25 = 125\%$

343. (c) Increase in altitude = 10%

$= \frac{1}{10} \begin{array}{l} \rightarrow \text{Increment} \\ \rightarrow \text{original} \end{array}$

altitude	base
10	11
11	10

$\left.\right) -1$

Area no change

decrease in base

$= \frac{1}{11} \times 100 = 9\frac{1}{11}\%$

344. (d) Increase in circumference = Increase in radius

$= 50\% = \frac{1}{2} \begin{array}{l} \rightarrow \text{Increment} \\ \rightarrow \text{Original} \end{array}$

Radius	area
2	4
3	9

$\left.\right) +5$

Increase % $= \frac{5}{4} \times 100 = 125\%$

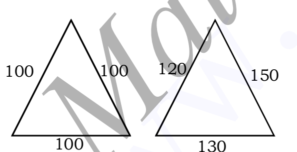
345. (b) use $x + y + \frac{xy}{100}$

percentage change

$= 12 + 15 + \frac{12 \times 15}{100} = 27 + \frac{9}{5}$

$= 27 + 1 + \frac{4}{5} = 28\frac{4}{5}\%$

346. (b)



Perimeter of equilateral triangle = 100 +100 +100 = 300

Perimeter of New triangle = 120 + 150 +130 = 400

% increase $= \frac{100}{300} \times 100 = 33\frac{1}{3}\%$

347. (b)

Length	5 →	3
breadth	5 →	3
Area	25 →	9

% decrease $= \frac{25-9}{25} \times 100 = 64\%$

348. (a)

Length	5 →	8
Breadth	8 →	5
Area	40 →	40

$\Rightarrow$ % Decrease $= \frac{8-5}{8} \times 100$

$= 37\frac{1}{2}\%$

349. (b)

Length	5 →	6
Breadth	4 →	5
Area	20 →	30

% Increase $= \frac{30-20}{20} \times 100 = 50\%$

350. (b) Side 10 → 11

Area 100 → 121

% Increase $= \frac{121-100}{100} \times 100$

$= 21\%$

351. (b)

Length	20 →	21
Breadth	10 →	9
Area	200 →	189

% Decrease $= \frac{200-189}{200} \times 100 = 5.5\%$

352. (d) Radius 100 → 101

Area $10000\pi \rightarrow 10201\pi$

%Increase $= \frac{201}{10000} \times 100 = 2.01\%$

353. (c) Let the breadth = x cm

$\Rightarrow$ length = (x + 20) cm

According to the question,

$x(x + 20) = (x + 10)(x + 5)$

$\Rightarrow x^2 + 20x = x^2 + 15x + 50$

$\Rightarrow 5x = 50$

$\Rightarrow x = 10$

$\Rightarrow$ Area = 10(10 + 20) = 300 m^2

354. (c)

Length	20 →	21
Breadth	50 →	49
Area	1000 →	1029

%error $= \frac{1029-1000}{1000} \times 100 = 2.9\%$

355. (d)

Length	10 →	13
Breadth	10 →	12
Area	100 →	156

% increase in area

$= \frac{156-100}{100} \times 100 = 56\%$

356. (d) $40\% = \frac{4}{10} = \frac{2}{5}$

	Side	Surface area
40%	5	$(5)^2 = 25$
	7	$(7)^2 = 49$

$\left.\right) 24$

% increase $= \frac{24}{25} \times 100 = 96\%$

Alternate

Percentage increase in surface area

$= 40 + 40 + \frac{40 \times 40}{100}\%$

$= 80 + 16 = 96\%$

[% effect using $x + y + \frac{xy}{100}$]

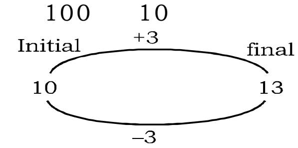
357. (a) percentage increase in area

$= \left(8 + 8 + \frac{8 \times 8}{100}\right)$

$= 16 + 0.64 = 16.64\%$

358. (a) Side of square is increased by 30%

$= \frac{30}{100} = \frac{3}{10}$



Other side will have to be decreased by

$= \frac{3}{13} \times 100 = \mathbf{23\frac{1}{13}}\%$

359. (c) Percentage increase in area

$= 100 + 100 + \frac{100 \times 100}{100} = 300\%$

Alternate

L	B	Area
1	1	1
2	2	4

$\left.\right) +3$

Percentage increase

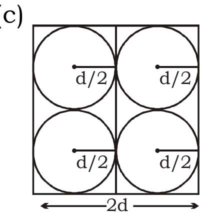
$= \frac{3}{1} \times 100 = 300\%$

360. (d) $x + y + \frac{xy}{100}$

$= 10 - 10 + \frac{10 \times (-10)}{100} = -1\%$

(Negative sign shows decrease)

361. (c)

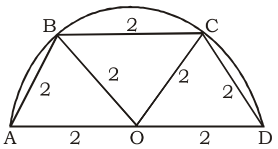


From the figure it is clear that, 4 circular plates of diameter d can be made of a. Square plate of side 2d with minimum loss of material.

362. (d) $\triangle$ AOB, $\triangle$ BOC and $\triangle$ COD are equilateral $\triangle$.

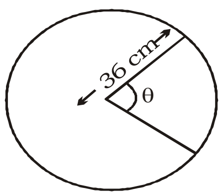
$\therefore$ Side = 2 cm

Now, total area $= 3 \times \frac{\sqrt{3}}{4}(\text{Side})^2$



$= 3 \times \frac{\sqrt{3}}{4} \times 4 = 3\sqrt{3}$ cm²

363. (d) Area of sector = 72π cm²



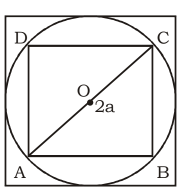
$\Rightarrow \frac{\pi r^2 \theta}{360^\circ} = 72\pi$

$\therefore \theta = \frac{72 \times 360}{36 \times 36} = 20^\circ$

Now, length of arc $= \frac{\pi r \theta}{180^\circ}$

$= \frac{\pi \times 36 \times 20}{180} = 4\pi$ cm

364. (b)



For inscribed circle,
Diameter of circle = Diagonal of square
Since, sides of square are equal.
Now, In $\triangle$ABC by Pythagoras theorem
AB² + BC² = AC²

$\Rightarrow$ 2 AB² = 4a²

AB² = 2a² $\Rightarrow$ AB = $\sqrt{2}$a

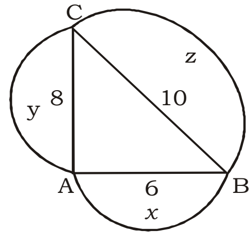
$\therefore$ Area of inner square = AB² = $(\sqrt{2}a)^2$ = 2a²

For circumscribed square,
Diameter of circle = Side of square

$\therefore$ Area of circumscribed square = (2a)² = 4a²

$\therefore$ Difference between areas of outer and inner squares
= 4a² – 2a² = 2a²

365. (c) In $\triangle$ ABC, by Pythagoras



BC² = AB² + AC² = 36 + 64 = 10 cm

Now, area of semi-circle = x

$= \frac{\pi(3)^2}{2} = \frac{9\pi}{2}$ cm²

Area of semi-circle = y

$= \frac{16\pi}{2}$ cm²

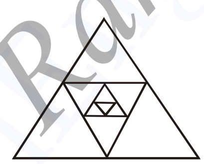
Area of semi-circle = z

$= \frac{25\pi}{2}$ cm²

Now, value of x+y–z

$= \left(\frac{9\pi}{2} + \frac{16\pi}{2}\right) - \frac{25\pi}{2} = 0$

366. (c) Perimeters of triangles,

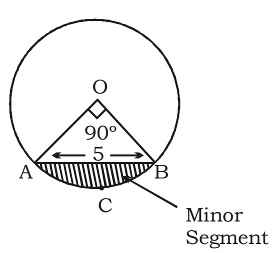


= (3×1) + (3×0.5) + (3×.025) + (3×0.125)+ ...

$\Rightarrow$ 3 + 1.5 + 0.75 + 0.375

$S_n = \frac{a}{1-n} = \frac{3}{1-\frac{1}{2}} = \frac{3 \times 2}{2-1} = 6$ units

367. (c) In $\triangle$ AOB,
AO = OB = r (radius of circle)



Using Pythagoras theorem,
AB² = OA² + OB² $\Rightarrow$ (5)² = r² + r²

$\therefore r^2 = \frac{25}{2}$ cm

How, area of sector AOB

$= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times \pi \times \frac{25}{2} = \frac{25\pi}{8}$ cm²

Now, area of minor segment
= Area of sector – Area of triangle

$= \frac{25\pi}{8} - \frac{r^2}{2} = \frac{25\pi}{8} - \frac{25}{4}$

$= \left(\frac{25\pi - 50}{8}\right)$

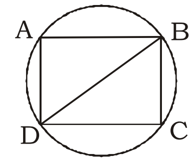
Area of major segment = Area of circle – Area of minor segment

$= \pi r^2 - \left(\frac{25\pi - 50}{8}\right)$

$= \frac{100\pi - 25\pi + 50}{8} = \frac{75\pi + 50}{8}$

$= \frac{25}{8}(3\pi + 2) = \frac{25}{4}\left(\frac{3\pi}{2} + 1\right)$ cm²

368. (c) ABCD be the rectangle inscribed in the circle of diameter 5 cm.



$\therefore$ Diameter = Diagonal of rectangle

Now, let x and y be the length and breadth of rectangle are respectively,

Now In $\triangle$ ABD, AB² + AD² = (5)²

$\Rightarrow$ x² + y² = 25

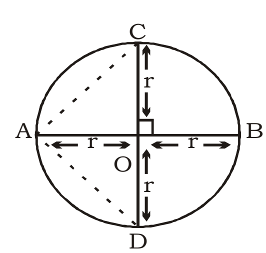
Since, they form Pythagoras triplet,

$\therefore$ x = 4 and y = 3

So, area of rectangle = 3×4
= 12cm²

369. (b) Required ratio

$= \frac{\text{Area of circle}}{\text{Area of } \Delta ACD}$



$= \dfrac{\pi r^2}{\frac{1}{2}\times 2r\times r} = \pi$

370. (b) Semi-perimeter of triangle

$= \dfrac{a+b+c}{2}$

$= \dfrac{7+24+25}{2} = \dfrac{56}{2} = 28$ cm

Area of circle = Area of triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{28(28-7)(28-24)(28-25)}$

$= \sqrt{28\times 21\times 4\times 3}$

$= \sqrt{7056} = 84$ cm²

371. (a) Area of equilateral triangle

$= \dfrac{\sqrt{3}a^2}{4} = x$ .....(i)

and perimeter = 3a = y

$\Rightarrow$ $a = \dfrac{y}{3}$ ......(ii)

Now, puting the value of a from Eq. (ii) in Eq. (i), we get

$\dfrac{\sqrt{3}\left(\frac{y}{3}\right)^2}{4} = x \Rightarrow x = \dfrac{\sqrt{3}\times y^2}{9\times 4}$

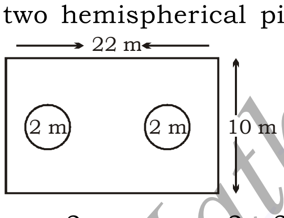
$\Rightarrow$ $x = \dfrac{y^2}{3\sqrt{3}\times 4} \Rightarrow x = \dfrac{y^2}{12\sqrt{3}}$

$\Rightarrow$ $12\sqrt{3}\,x = y^2$

On squaring both sides, we get

$y^4 = 432\,x^2$

372. (c) Volume of mud dug out in two hemispherical pitholes



$= 2\times\dfrac{2}{3}\pi r^3 = 2\times\dfrac{2}{3}\times\dfrac{22}{7}\times 2^3$

$= \dfrac{2\times 2\times 22\times 8}{21} = \dfrac{704}{21}$ m³

Area on which the muds is spread over

= Area of fieled – Area of pitholes

$= l\times b - 2\times\pi r^2$

$= 22\times 10 - 2\times\dfrac{22}{7}\times 2^2$

$= 220 - \dfrac{176}{7} = \dfrac{1540-176}{7}$

$= \dfrac{1364}{7}$ m²

Now, let the rise in level by h m, then

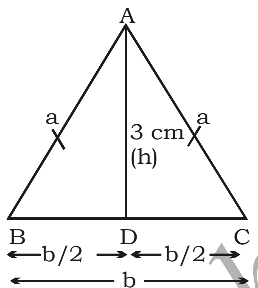
Area of remaining field × h = Volume of mud dugged out

$\Rightarrow$ $\dfrac{1364}{7}\times h = \dfrac{704}{21}$

$\therefore$ $h = \dfrac{704\times 7}{1364\times 21} = \dfrac{16}{93}$ m

373. (a) Area of the $\triangle$ABC

$= \dfrac{1}{2}\times b\times h$



$\Rightarrow$ $12 = \dfrac{1}{2}\times b\times 3$

$\therefore$ $b = \dfrac{12\times 2}{3} = 8$ cm

Here, $BD = CD = \dfrac{b}{2} = \dfrac{8}{2}$

= 4cm

In right angled $\triangle$ABD, by pythagoras theorem,

$AB = \sqrt{BD^2+AD^2}$

$\Rightarrow a = \sqrt{4^2+3^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$ cm

Now, perimeter of an isosceles triangle

$= 2a+b = 2\times 5+8 = 10+8 = 18$ cm

374. (a) Let the breadth and height of room be b and h m, respectively.

Then, according to the question,

$\Rightarrow$ $l\times b$ = n Area occupied by one patient

$\Rightarrow$ $14\times b = 56\times 2.2$

$\Rightarrow$ $b = \dfrac{56\times 2.2}{14} = 8.8$ m

Now, total volume of the room is equal to total patients multiplied by volume occupied by each patient.

Then, $14\times 8.8\times h = 8.8\times 56$

$\therefore$ $h = \dfrac{8.8\times 56}{14\times 8.8} = 4$m

375. (a) 1 m = 1000 mm

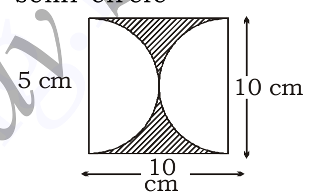
$\therefore$ 10 m = 10000 mm

Number of 200 mm lengths that can be cut from 10 m of ribbon

$= \dfrac{10000}{200} = 50$

376. (d) Area between square and semi-circles

= Area of square – 2 Area of semi-circle



$= (10)^2 - 2\times\dfrac{22}{7}\times(5)^2$

$= 100 - 78.5 = 21.5$ cm²

377. (a) Let l = 4x and b = 9x

$\therefore$ Area of rectangle = $l\times b$

$144 = 4x\times 9x$

$\Rightarrow$ $x^2 = \dfrac{144}{36}$

$\Rightarrow x^2 = 4 \Rightarrow x = 2$

Now, $l = 4\times 2 = 8$ cm and

$b = 9\times 2 = 18$ cm

$\therefore$ Perimeter of rectangle

$= 2(l+b) = 2(8+18)$

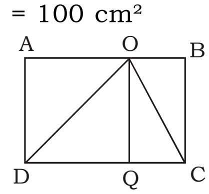
$= 2\times 26 = 52$ cm

378. (c) Area of parallelogram

= Base × Height

= 8.06 × 2.08 = 16.76 cm²

379. (c) Given that, CD = 20 cm and area of rectangle ABCD = 100 cm²



$\Rightarrow$ AD × CD = 100 cm²

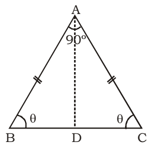
$\Rightarrow$ AD ×20 = 100

$\therefore$ AD = 5 cm [in rectangle AB = CD = 20 cm and AD = BC = OQ = 5 cm]

$\therefore$ Area of $\triangle$ODC $= \dfrac{1}{2}\times PQ\times CD = \dfrac{1}{2}\times 5\times 20$

$= 5\times 10 = 50$ cm²

380. (d) Let AB = AC = a
$\therefore$ $BC^2 = AB^2 + AC^2$
(by Pythagoras theorem)



In Δ ABC,

$a^2+a^2 = 2a^2 \Rightarrow BC = a\sqrt{2}$

$90^\circ + \theta + \theta = 180^\circ$
(since, sum of all interior angles of any triangle is 180°)

$\Rightarrow$ $2\theta = 90^\circ$

$\therefore$ $\theta = 45^\circ$

Now, In Δ ABD,

$\sin 45^\circ = \frac{AD}{a} \Rightarrow AD = \frac{a}{\sqrt{2}}$

$\therefore$ Area of Δ ABC $= \frac{1}{2} \times AD \times BC$

$= \frac{1}{2} \times a\sqrt{2} \times \frac{a}{\sqrt{2}}$
= 1sq unit (given)

$\Rightarrow \frac{a^2}{2} = 1$

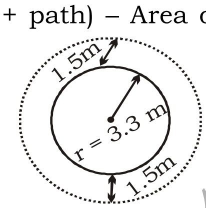
$\therefore$ $a = \sqrt{2}$

$\therefore$ Perimeter of Δ ABC

$= 2a + \sqrt{2}a = 2\sqrt{2} + \sqrt{2}.\sqrt{2}$

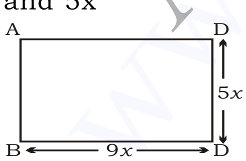
$= 2(1+\sqrt{2})$ units

381. (c) Area of path = Area of (fountain + path) – Area of fountain



$= r\,(3.3 + 1.5)^2 - \pi\,(3.3)^2$
$= [(4.8)^2 - (3.3)^2]\,\pi$
$= (23.04 - 10.89)\,\pi = 12.15\,\pi\ m^2$

382. (c) Let length and breadth of a rectangular field are 9x and 5x

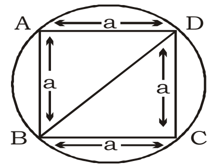


$\therefore$ Area of a rectangular field
$= 4500\ m^2$
$\Rightarrow 9x \times 5x = 4500$
$\Rightarrow x^2 = 100 = (10)^2$
$\therefore$ x = 10

So, the length and breadth of a rectangular field are 90 m and 50 m.

$\therefore$ Perimeter of rectangular field
= 2 (Length + Breadth)
= 2 (90+50)=2×140 = 280 m

383. (c) Given that, radius of a circle = 8 cm
and diameter of a circle = 16 cm



$\therefore$ Length of a diameter of a square = Diameter of a cricle

$\Rightarrow$ $a\sqrt{2} = 16$

$\therefore$ $a = 8\sqrt{2}$ cm

$\therefore$ Area of square ABCD = a^2

$= \left(8\sqrt{2}\right)^2 = 64 \times 2$

= 128 sq cm

384. (d) Given that, length of hour hand = 4 cm
and length of minute hand = 6 cm

$\therefore$ Hour hand rotating in 1 day = $2 \times 360^\circ = 720^\circ$

$\therefore$ Hour hand rotating in 2 days =

$2 \times 720^\circ = 1440 \times \frac{\pi}{180}$ radius

Similarly,
Minute hand rotating in 1 day $= 24 \times 360^\circ$

$\therefore$ Minute hand rotating in 3 days

$= 72 \times 360^\circ \times \frac{\pi}{180}$ radius

$\therefore$ Distance travelled by hour

hand $= 4 \times 1440^\circ \times \frac{\pi}{180^\circ} = 32\pi$

and distance travelled by minute hand

$= 6 \times 72^\circ \times 360^\circ \times \frac{\pi}{180}$

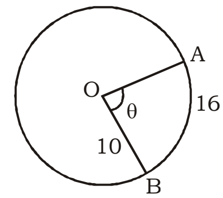
$= 6 \times 144\pi$

Required ratio

$= \frac{32\pi}{6 \times 144\pi}$

$= \frac{1}{27}$

385. (c) Arc of length $= 2\pi r.\frac{\theta}{360^\circ}$



$\Rightarrow$ $16 = 2\pi r.\frac{\theta}{360^\circ}$

$\Rightarrow$ $\frac{\theta}{360^\circ} = \frac{16}{2\pi r}$

Now, area of sector OAB

$= \pi r^2.\frac{\theta}{360^\circ}$

$= \pi r^2.\frac{16}{2\pi r} = 8r = 8 \times 10$

= 80 sq cm

386. (c) Let the breadth of floor be x metre.

$\therefore$ Length = (x + 20)metre

$\therefore$ Area of the floor = (x + 20)x sq.metre
In case II,
(x + 10)(x + 5) = x(x + 20)

$\Rightarrow$ $x^2 + 15x + 50 = x^2 + 20x$

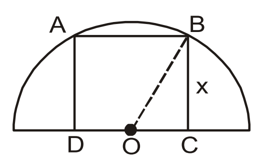
$\Rightarrow$ 20x = 15x + 50

$\Rightarrow$ 5x = 50

$\Rightarrow$ x = 10 metre

Area of the floor
= x (x + 20)
= 10(10 + 20)
= 300

387. (d)



Here ABCD is a square of side x.

$\therefore$ $OC = \frac{x}{2}$, BC = x, and
OB = radius of circle = 10cm
In ΔOCB,
$OB^2 = OC^2 + BC^2$

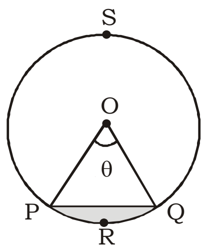
$\Rightarrow$ $\left(\frac{x}{2}\right)^2 + x^2 = (10)^2$

$\Rightarrow$ $\frac{5x^2}{4} = 100 \Rightarrow x^2 = 80$

$\Rightarrow$ $x = 4\sqrt{5}$cm

Hence, perimeter of the square ABCD = 4x = $16\sqrt{5}$

388. (c) 1. We know that, Area of segment (PRQP)
= Area of sector (OPRQO) – Area of $\triangle$ OPQ = $\frac{\pi r^2\theta}{360} - \frac{1}{2} r^2 \sin\theta$



So, the area of a segment of a circle is always less than area of its corresponding sector.

II. Distance travelled by a circular wheel of diameter 2 d cm in one revolution

$= 2\pi \frac{(2d)}{2} = 2\times3.14\times d = 6.28d$

which is greater than 6d cm.

389. (b) Given that, perimeter of a rectangle = 82 m

$\therefore$ 2 (Length + Breadth) = 82 m

$\Rightarrow$ Length + Breadth = 41 m

$\Rightarrow$ l+b = 41 m. . . .(i)

Also, its area = 400 m²

$\Rightarrow$ l.b = 400m²

Now, $(l-b)^2 = (l+b)^2 - 4lb$

$=(41)^2 - 4\ (400)$

= 1681 – 1600 = 81

$\therefore$ l – b = 9 .....(iii)

From Eqs. (i) and (iii),

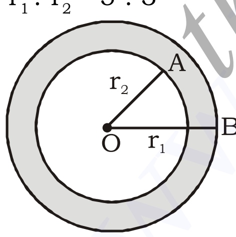
2l = 50 $\Rightarrow$ l = 25 m and

b = 16 m

$\therefore$ Required breadth (b) = 16 m

390. (c) Given that, ratio of their radii = 5:3

i.e., $r_1 : r_2 = 5 : 3$



$\Rightarrow \frac{r_1}{r_2} = \frac{5}{3}$ .....(i)

Let $r_1 = 5x$ and $r_2 = 3x$

Also, given that, area enclosed between the circumferences of two concentric circles

$= 16\pi$ cm²

$\therefore \pi (r_1^2 - r_2^2) = 16\pi$

$\Rightarrow (5x)^2 - (3x)^2 = 16$

$\Rightarrow 25x^2 - 9x^2 = 16$

$\Rightarrow 16x^2 = 16$

$\Rightarrow x^2 = 1 \Rightarrow x = 1$

$\therefore r_1 = 5$ and $r_2 = 3$

$\therefore$ Area of the outer circle

$= \pi r_1^2 = \pi (5)^2 = 25\pi$ cm²

391. (c) Let the radius of a circle is r and a be the length of the side of a square.

Given, circumference of a circle = Perimeter of a square

$\Rightarrow 2\pi r = 4a$

$\Rightarrow a = \frac{\pi}{2} r = 1.57r$

Now, area of the circle (A_C)

$= \pi r^2 = 3.14r^2$

and area of the square (A_s)

$= a^2 = 2.4649r^2$

$\therefore$ Area of circle > Area of square

392. (b) Let the radii of two circles are r_1 and r_2 respectively.

Given,

$$\frac{\text{Circumference of 1st circle}}{\text{Circumference of IInd circle}} = \frac{2}{3}$$

$\Rightarrow \frac{2\pi r_1}{2\pi r_2} = \frac{2}{3} \Rightarrow \frac{r_1}{r_2} = \frac{2}{3}$

$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$ ...(i)

$\therefore \frac{\text{Area of 1st circle}}{\text{Area of IInd circle}} = \frac{\pi r_1^2}{\pi r_2^2}$

$= \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$

393. (b) We know that the radius of a circle inscribed in a equilateral triangle $= \frac{a}{2\sqrt{3}}$

where, a be the length of the side of an equilateral triangle. Given that, area of a circle inscribed in an equilateral triangle = 154 cm²

$\therefore \pi\left(\frac{a}{2\sqrt{3}}\right)^2 = 154$

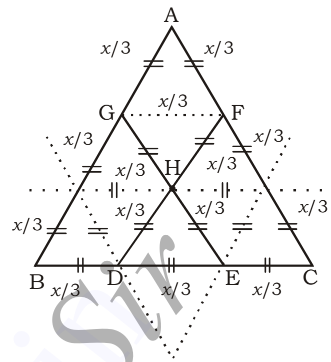
$\Rightarrow \left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{154\times7}{22} = (7)^2$

$\Rightarrow a = 14\sqrt{3}$ cm

$\therefore$ Perimeter of an equilateral triangle = 3a = $3(14\sqrt{3})$

$= 42\sqrt{3}$ cm

394. (b) $\triangle$ ABC forms an equilateral triangle.



where, AGHF form a rhombus and $\triangle$ HDE is also an equilateral triangle.

$\therefore$ Area of rhombus = (Area of $\triangle$ AGF + Area of $\triangle$ GFH)

$= \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 + \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2$

$= 2.\frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2$

Now, area of $\triangle$ HDE $= \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2$

and area of $\triangle$ ABC $= \frac{\sqrt{3}}{4} x^2$

By given condition,

$$\frac{\text{Area of rhombus AGHF + Area of }\Delta\text{HDE}}{\text{Area of }\Delta\text{ABC}}$$

$$= \frac{3\times\frac{\sqrt{3}}{4}\times\left(\frac{x}{3}\right)^2}{\frac{\sqrt{3}}{4}x^2} = \frac{1}{3}$$

395. (c) Given that,

Area of the circle = Area of the square = (Side)²

$\pi r^2 = (2\sqrt{\pi})^2 \Rightarrow \pi r^2 = 4\pi \Rightarrow$

$r^2 = \frac{4\pi}{\pi} = 4$

$\therefore r = \sqrt{4} = 2$ units

$\therefore$ Diameter of circle (d)

= 2.r = 2.2 = 4 units

396. (a) Let the radius of circle is r and the side of a square is a, then by given condition,

$2\pi r = 4a \Rightarrow a = \frac{\pi r}{2}$

$\therefore$ Area of square $= \left(\frac{\pi r}{2}\right)^2$

$= \frac{\pi^2 r^2}{4} = \frac{9.86r^2}{4} = 2.46r^2$

and area of circle = πr^2 = 3.14r^2

and let the side of equilateral triangle is x.

Then, by given condition,

$$3x = 2\pi r \Rightarrow x = \frac{2\pi r}{3}$$

$\therefore$ Area of equilateral triangle = $\frac{\sqrt{3}}{4}$

$$x^2 = \frac{\sqrt{3}}{4} \times \frac{4\pi^2 r^2}{9}$$

$$= \frac{\pi^2}{3\sqrt{3}} r^2 = 1.89r^2$$

Hence, Area of circle

> Area of square

> Area of equilateral triangle

397. (a) Let the width of the rectangle = x unit

$\because$ Length = (2 x + 5) unit

According to the question,

Area = x (2x+5)

$\Rightarrow$ 75 = $2x^2$ +5x

$\Rightarrow 2x^2 + 5x - 75 = 0$

$\Rightarrow 2x^2 + 15x - 10x - 75 = 0$

$\Rightarrow$ x (2x +15) – 5 (2x +15) = 0

$\Rightarrow$ (2x +15) (x–5)=0

$\Rightarrow$ x =5 and $\frac{-15}{2}$

Since, width cannot be negative.

$\therefore$ Width = 5 units and

length = 2x +5

= 2×5+5 = 15 units

$\therefore$ Perimeter of the rectangle = 2 (15+5) = 40 units

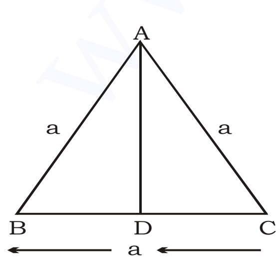
398. (c) Height of equilateral triangle,

$$(AD) = \frac{\sqrt{3}}{2} \times \text{Side}$$

$$\Rightarrow \sqrt{3} = \frac{\sqrt{3}\times\text{side}}{2}$$

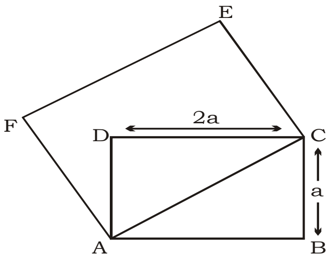
$$\Rightarrow 2\sqrt{3} = \sqrt{3}\times\text{side}$$

$\therefore$ Side = $\frac{2\sqrt{3}}{\sqrt{3}}$ = 2 cm



$\therefore$ Perimeter of an equilateral triangle = 3a = 3×2 = 6 cm

399. (d) Given that,



Area of rectangle = $2a^2$ = l×b

$\Rightarrow$ l×b = $2a^2$ = l×a $\Rightarrow$ l=2a

Now, In $\triangle$ ACD,

$AC^2 = AD^2 + CD^2$

$a^2 + 4a^2 = 5a^2$

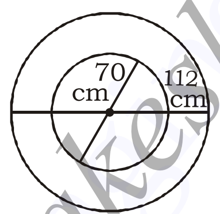
$\therefore$ Side of square, AC

= a$\sqrt{5}$ unit

Hence, area of square

= $(a\sqrt{5})^2$= $5a^2$ sq units

400. (c) Outer diameter = 112 cm and inner diameter = 70 cm



$\therefore$ Required area

$$= \frac{1}{4}\pi\ (112^2 - 70)^2$$

$$= \frac{1}{4}(12544 - 4900)\pi$$

$$= \frac{1}{4} \times 7644 \times \frac{22}{7}$$

$$= \frac{1}{4} \times 24024 = 6006 \text{ cm}^2$$

401. (d) Let the length of altitude AB =l

By given condition,

Area of $\triangle$ ABC = Area of square

$$\therefore \frac{1}{2}\times\text{Base}\times\text{Altitude} = (\text{Side})^2$$

$$\Rightarrow \frac{1}{2}\times 9\times l = 36 \Rightarrow l = \frac{36\times 2}{9}$$

$\therefore$ l = 8 cm

402. (d) Area of equilateral triangle

$$= \frac{\sqrt{3}}{4}a^2$$

But altitude = $\frac{\sqrt{3}}{2}$a

$$\Rightarrow 2\sqrt{3} = \frac{\sqrt{3}}{2}a$$

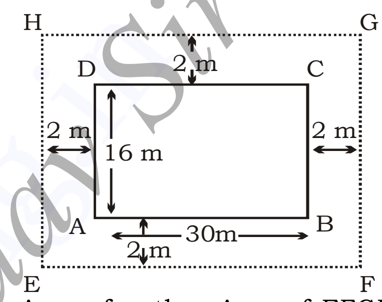
$\Rightarrow$ a = 4 cm

$\therefore$ Area of equilateral triangle =

$$\frac{\sqrt{3}}{4}(4)^2 = 4\sqrt{3}\ \text{cm}^2$$

403. (a) Required area of the path

EF = 30+4 = 34m, GF = 16+4 =20 m

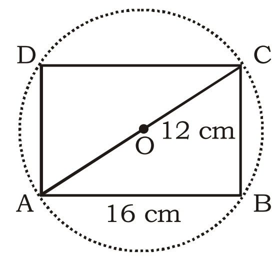


$\therefore$ Area of path = Area of EFGH - Area of ABCD

= 34 × 20 – 30 × 16

= 680 – 480 = 200 m^2

404. (c) In right $\triangle$ ABC,



$AC^2 = AB^2 + BC^2$

$AC^2 = (16)^2 + (12)^2$

= 256 + 144 = 400

$\Rightarrow$ AC = 20 cm

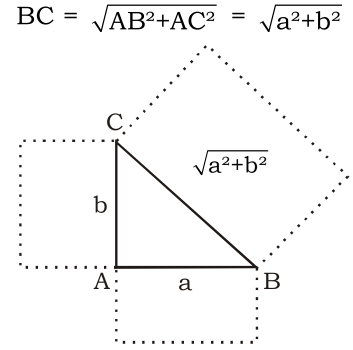
$\therefore$ AO = 10 cm (radius)

and area of circumcircle = πr^2

= $\pi \times (10)^2 = 100\pi$ cm^2

405. (c) In $\triangle$ ABC,

$$BC = \sqrt{AB^2+AC^2} = \sqrt{a^2+b^2}$$



$\therefore$ Required total area = $a^2 + b^2$

$$+ \left(\sqrt{a^2+b^2}\right)^2 + \frac{1}{2}ab$$

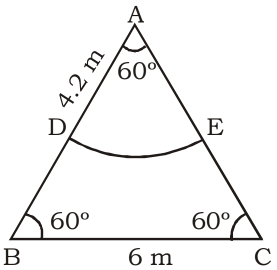
= $2(a^2+b^2)$+0.5ab

406. (b) Area of trapezium

$= \frac{1}{2}(3+5) \times 4 = 16\ m^2$

∴ Total cost of painting Rs. 25 per sq m = 16×25 = Rs. 400

407. (c) Suppose a horse is tied at vertex A. Then, area available grazing field is ADE.



Now, area of curve ADE

$= \frac{\pi r^2 \theta}{360°}$

$= \frac{22 \times (4.2)^2 \times 60°}{7 \times 360°} = 9.24\ m^2$

and area of equilateral △ABC

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times (6)^2 = 15.57$

∴ Required percentage

$= \frac{9.24}{15.57} \times 100$

= 59.34% = 59% (approx)

408. (a) By given condition,

$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{1}{2}$

$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{2}$ .....(i)

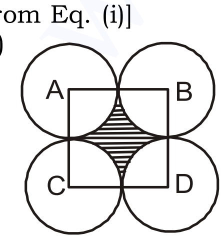
If circle or bent in the form of square, then $2\pi r_1 = 4a_1$

$\Rightarrow a_1 = \frac{\pi r_1}{2}$ and $a^2 = \frac{\pi r_2}{2}$

∴ $\frac{A_1}{A_2} = \frac{a_1^2}{a_2^2} = \frac{\left(\frac{\pi r_1}{2}\right)^2}{\left(\frac{\pi r_2}{2}\right)^2} = \frac{r_1^2}{r_2^2} = \frac{1}{2}$

[from Eq. (i)]

409. (b)



Area of the shaded region

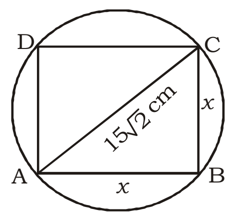
= Area of square of side 6cm - 4× a right angled sector

$= 36 - 4 \times \frac{\pi \times 3^2}{4}$

$= 36 - 9\pi = 9(4-\pi)$ sq. cm.

410. (a) Let the sides of a square be x cm,

In △ABC, AC² = AB² + BC²



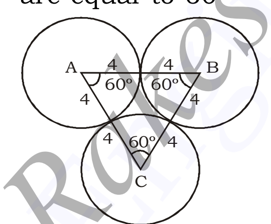
AC² = x² + x²

$\Rightarrow (15\sqrt{2})^2 = 2x^2 \Rightarrow 2x^2 = 225 \times 2$

$\Rightarrow x^2 = 225 \Rightarrow x = 15$ cm

Hence, length of the side of the square be 15 cm.

411. (b) Since, all sides of a △ABC are equal, so their all angles are equal to 60°



Area of Portion include between circles

= Area of triangle – Area of 3 sectors

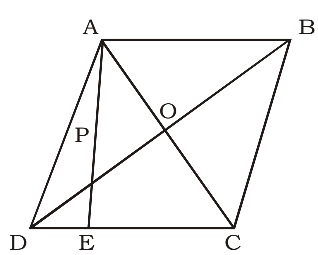
$= \frac{\sqrt{3}}{4}(8)^2 - 3 \times \frac{60°}{360°} \times \pi(4)^2$

$= (16\sqrt{3} - 8\pi)\ cm^2$

412. (a) Area of rhombus

$= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 55 \times 48$

= 1320 cm



∵ Area of rhombus

= Base × Height = DC × AE

⇒ DC×AE = 1320

$\Rightarrow p \times \sqrt{OD^2 + OC^2} = 1320$

$\Rightarrow p \times \sqrt{\left(\frac{55}{2}\right)^2 + \left(\frac{48}{2}\right)^2} = 1320$

$\left(\because OD = \frac{1}{2}BD \text{ and } OC = \frac{1}{2}AC\right)$

$\Rightarrow p \times \sqrt{\frac{5329}{4}} = 1320$

$\Rightarrow p = \frac{1320}{36.5} = 36.16$

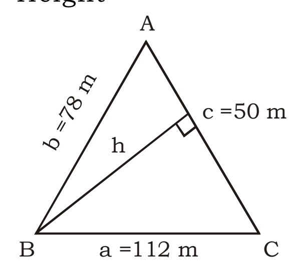
36 cm < p < 37 cm

413. (c) Perimeter = a + b + c

240 = a + 78 + 50

a = 112

∴ Area of triangle $= \frac{1}{2} \times$ Base × Height



and also, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

∴ Area of $= \sqrt{120(120-112)(120-78)(120-50)}$

$= \sqrt{120 \times 8 \times 42 \times 70}$

= 1680 m²

∵ Area of triangle $= \frac{1}{2} \times$ Base × Height

$\Rightarrow 1680 = \frac{1}{2} \times 50 \times 1$

∴ $h = \frac{2 \times 1680}{50} = 67.2$ m

414. (b) Let the sides of isosceles triangle be 5x, 5x and 3x cm, respectively.

By given condition,

Perimeter of isosceles triangle = Length of wire

5x + 5x +3x = 78 ⇒ 13x = 78

⇒ x = 6 cm

∴ Length of base = 3×6 = 18 cm

415. (b) The angle made by the minute hand in 20 min = 120°,

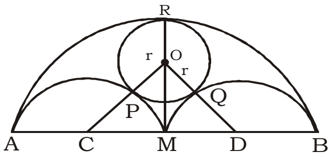
(1 minute = 6°)

∴ The area swept by the minute hand in 20 min

$= \frac{\theta}{360°} \times \pi r^2 = \frac{120°}{360°} \times 3.14 \times 9 \times 9$

= 84.78 cm²

416. (c) Given, AB = 2a



$\Rightarrow$ AM = a

and AC = CM = MD = BD = $\frac{a}{2}$

Now, OC = OP+PC = OP+CM

$= r+\frac{a}{2}$ and

OD = OQ+QD = OQ+MD = $r + \frac{a}{2}$

So, $\triangle$OCD is an isosceles triangle. ($\because$ OC = OD)

$\Rightarrow \angle OMC = 90°$

In $\triangle$OMC,

$OC^2 = OM^2 + CM^2$

$\Rightarrow \left(r+\frac{a}{2}\right)^2 = (a-r)^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow r^2 + \frac{a^2}{4} + ar$

$= a^2 + r^2 - 2ar + \frac{a^2}{4}$

$\therefore r = \frac{a}{3}$

417. (b) Let r be the radius of circle and a be the side of square.

By given condition, $2\pi r = 4a$

$\Rightarrow a = \frac{\pi r}{2}$

$\therefore$ Area of square = $\left(\frac{\pi r}{2}\right)^2$

$= \frac{\pi^2 r^2}{4} = \frac{9.86r^2}{4} = 2.46\ r^2$

and area of circle $= \pi r^2$

$= 3.14r^2$

Hence, area of the circle is larger than that of square.

418. (d) Let a = 35 cm, b = 44 cm and c = 75cm

$\therefore s = \frac{a+b+c}{2}$

$= \frac{35+44+75}{2} = 77$

Now, Area of Δ

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{77\times42\times33\times2}$

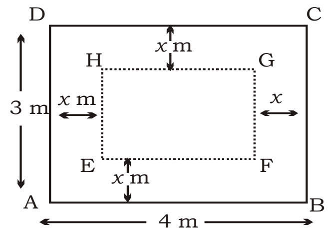
$= \sqrt{7\times11\times2\times3\times7\times3\times11\times2}$

$= 7\times11\times2\times3 = 462\ cm^2$

Hence, radius of incircle

$= \frac{\Delta}{s} = \frac{462}{77} = 6$ cm

419. (b) Width of the border = x m



Given, area of EFGH = $6m^2$

$\Rightarrow (4-2x)(3-2x) = 6$

$\Rightarrow 12-8x-6x+4x^2 = 6$

$\Rightarrow 4x^2-14x+12 = 6$

$\Rightarrow 4x^2-14x+6=0$

$\Rightarrow 2x^2-7x+3=0$

$\Rightarrow 2x^2-6x-x+3=0$

$\Rightarrow 2x(x-3)-1(x-3)=0$

$\Rightarrow (x-3)(2x-1) = 0$

$\therefore x = 3, \frac{1}{2}$

$\therefore x = \frac{1}{2} = 0.5\ (\because x \neq 3)$

420. (b) Inner radius, r_1 = 25 cm and external radius, r_2 = 25+10 = 35 cm

Distance covered in 1 revolution $= 2\pi\times35 = 70\times\frac{22}{7}$

= 220 cm.

and distance covered in 10 revolutions = 2200 cm

$\therefore$ Speed of bicycle

$= \frac{\text{Covered distance}}{\text{Time}}$

$= \frac{2200}{5}$ cm/s $= \frac{22}{5}$ m/s

= 4.4 m/s

421. (c) Length of wire = 36 cm

$\therefore$ Perimeter of semi-circle

$= \pi r+2r$

$\Rightarrow 36 = r\left(\frac{22}{7}+2\right)$

{$\because$ Perimeter of semi-circle = Length of wire}

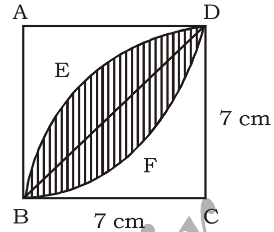
$\Rightarrow r = \frac{36\times7}{36} = 7$ cm

Hence, radius of semi-circle=7 cm

422. (c) Area of curve BCDE =

$\frac{1}{4}\pi(7)^2 = \frac{22}{7\times4}\times7\times7 = \frac{77}{2}\ cm^2$

Area of $\triangle$BCD $= \frac{1}{2}\times7\times7 = \frac{49}{2}\ cm^2$



$\therefore$ Required area of shaded region = 2 Area of curve BEDF

$= 2\left(\frac{77}{2}-\frac{49}{2}\right) = 2\left(\frac{28}{2}\right) = 28\ cm^2$

423. (a) Let sides of a rectangle be l and b.

Then, 2(l+b) = 18 $\Rightarrow$ l+b = 9

$\therefore$ Area of rectangle = l×b

For maximum, area of rectangle, l=b

$\therefore$ 2l = 9 $\Rightarrow$ l = 4.5

$\therefore$ Maximum area of rectangle = l×b = $(4.5)^2$ = 20.25 cm^2

424. (b) Let r = Radius of 3 smaller laminas

In $\triangle$ ADC, $(2r)^2 = r^2 + DC^2$

$\Rightarrow DC = \sqrt{3}r\ \ \because OC = \frac{2}{3}DC$

$= \frac{2}{3}\times\sqrt{3}r = \frac{2r}{\sqrt{3}}$

Radius of larger circular lamina = OE

$OC + CE = \frac{2r}{\sqrt{3}} + r = \frac{(2+\sqrt{3})r}{\sqrt{3}}$

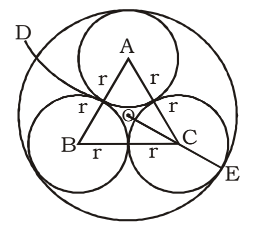
Area of 3 laminas = $3\pi r^2$

Area of larger lamina

$= \pi\left[\frac{(2+\sqrt{3})}{\sqrt{3}}r\right]^2$

$= \pi\ \frac{(4+3+4\sqrt{3})}{3}r^2$

$= \frac{(7+4\sqrt{3})\pi r^2}{3}$



Residual area

$= \left(\frac{7+4\sqrt{3}}{3}-3\right)\pi r^2$

$= \dfrac{(4\sqrt{3}-2)}{3}\pi r^2$

$\therefore$ Required ratio

$= \dfrac{\left(\dfrac{4\sqrt{3}-2}{3}\right)\pi r^2}{\dfrac{7+4\sqrt{3}}{3}\pi r^2}$

$= \dfrac{4\sqrt{3}-2}{7+4\sqrt{3}} \times \dfrac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$= \dfrac{28\sqrt{3}-48-14+8\sqrt{3}}{49-48}$

$= 36\sqrt{3} - 62 = 36 \times 1.732 - 62$

$= 62.352 - 62 = 0.35$

425. (a) Circumference of circle $= 2\pi \times 42$

$= 2 \times \dfrac{22}{7} \times 42 = 264$ cm

Perimeter of square = 4x

$\Rightarrow$ 264 = 4x

$\therefore$ x = 66 cm

426. (a) Area of 2 bigger semi-circles $= 2 \times \dfrac{\pi r^2}{2}$

$= 2\pi\left(\dfrac{0.5}{2}\right)^2 \times \dfrac{1}{2} = \dfrac{0.25\pi}{4}$ cm²

and area of 5 smaller semi – circles $= \dfrac{5\pi r^2}{2} = 5 \times \pi \times \dfrac{1}{2} \times \left(\dfrac{0.5}{4}\right)^2$

$= \dfrac{5\pi}{2} \times \dfrac{0.25}{16} = \dfrac{1.25}{32}$ cm²

Area of rectangle ABCD

= 2×0.5 = 1 cm²

Area of remaining portion

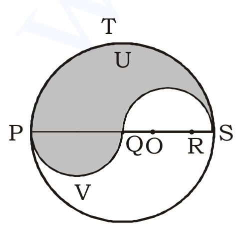
$= 1 - \dfrac{0.25\pi}{4} - \dfrac{1.25\pi}{32}$

$= 1 - \dfrac{\pi}{16} - \dfrac{5\pi}{128}$

$= \dfrac{128 - 8\pi - 5\pi}{128}$

$= \dfrac{128 - 13\pi}{128}$ cm²

427. (a) Given, OS = 6 cm



$\therefore$ PQ = QR = RS = 4 cm

$\therefore$ Perimeter of shaded region

= Perimeter of semi-circle PTS

+ Perimeter of semi-circle QUS

+ Perimeter of semi-circle PVQ

$= \pi(6) + \pi(4) + \pi(2) = 12\pi$ cm

428. (b) Circumference of circular path $= 2\pi \times 50$ m

$= 10000\pi$ cm

and circumference of wheel

$= 2\pi \times 50 = 100\pi$ cm

$\therefore$ Distance covered in 60 min = 10000π cm

Distance covered in 15 min

$= \dfrac{10000}{60}\pi \times 15 = 2500\pi$ cm

$\therefore$ Number of revolutions

$= \dfrac{2500\pi}{100\pi} = 25$

429. (b) The distance covered by a man diagonally is

$d = \dfrac{3 \times 1000}{60} \times 1 = 50$m

$\therefore$ Area of field $= \dfrac{1}{2}d^2$

$= \dfrac{1}{2} \times (50)^2 = 1250$ m²

430. (c) Let the side of an square be a cm.

By given condition,

Area of square – Area of an equilateral triangle $= \dfrac{1}{4}$

$\Rightarrow a^2 - \dfrac{\sqrt{3}}{4}a^2 = \dfrac{1}{4}$

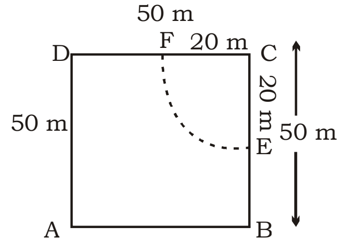
$\Rightarrow a^2\left(1 - \dfrac{\sqrt{3}}{4}\right) = \dfrac{1}{4}$

$\Rightarrow a^2(4 - \sqrt{3}) = 1 \Rightarrow a^2 = \dfrac{1}{4-\sqrt{3}}$

$\therefore$ $a = (4 - \sqrt{3})^{-1/2}$ cm

431. (d) Suppose a pole is fixed at a point C.

$\therefore$ Area of field in which the horse can graze = Area of field in which the horse can graze = Area of curve CFE



$= \dfrac{1}{4}(\pi r^2) = \dfrac{3.14 \times 20 \times 20}{4}$

= 314 m²

432. (a) Here width of sheet is 20 cm, which is the maximum diameter of the circular sheet.

$\therefore$ Remaining area of sheet

= Area of rectangle sheet

– Area of circular sheet

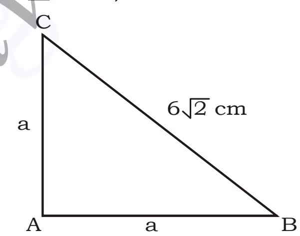
$= 25 \times 20 - \pi(10)^2$

= 500 – 314

= 186 cm²

433. (b) Let the other sides of a right isosceles triangle be a cm.

In $\triangle$ ABC,



$\sqrt{a^2+a^2} = 6\sqrt{2}$

$\Rightarrow a\sqrt{2} = 6\sqrt{2}$

$\Rightarrow$ a = 6 cm

$\therefore$ Area of $\triangle$ ABC $= \dfrac{1}{2} \times a^2$

$= \dfrac{1}{2} \times 6 \times 6 = 18$ cm²

434. (b) a = 9cm, b = 10 cm and c = 11 cm

$\because s = \dfrac{9+10+11}{2} = 15$ cm

$\because A = \sqrt{s(s-a)(s-b)(s-c)}$

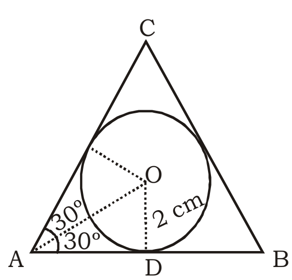
$= \sqrt{15(15-9)(15-10)(15-11)}$

$= \sqrt{15 \times 6 \times 5 \times 4} = 30\sqrt{2} = 42.3$ cm²

435. (a) As we know that, if the length of square and rhombus are same, then the area should be same.

436. (a) Area of circle $= 4\pi$ cm² (given)

$\Rightarrow \pi r^2 = 4\pi \Rightarrow r = 2$ cm



In $\triangle$ OAD, tan 30° $= \frac{OD}{AD}$

$\Rightarrow$ AD = $2\sqrt{3}$ cm

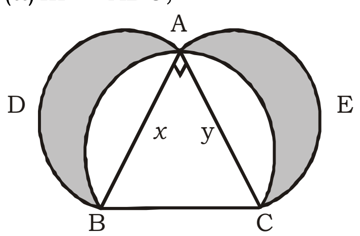
Now, AB = 2 AD = $4\sqrt{3}$ cm

$\therefore$ Area of equilateral $\triangle$ ABC

$= \frac{\sqrt{3}}{4}(AB)^2 = \frac{\sqrt{3}}{4}(4\sqrt{3})^2$

$= 12\sqrt{3}$ cm²

437. (a) In $\triangle$ ABC,



BC = $\sqrt{x^2+y^2}$

$\therefore$ Area of $\triangle$ ABC

$= \frac{1}{2} \times x \times y = \frac{1}{2}xy$

Area of semi-circle BACB

$= \frac{\pi(x^2+y^2)}{4}$

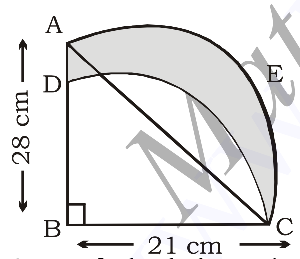
$\therefore$ Area of shaded portion
= Semi-circle ABDA
+ Area of semi-circle AECA-
(Area of semi-circle BACB –
Area of $\triangle$ ABC)

$= \frac{\pi x^2}{4} + \frac{\pi y^2}{4} - \pi\left(\frac{x^2+y^2}{4}\right)$ + Area of $\triangle$ ABC = Area of $\triangle$ ABC

438. (d) In $\triangle$ ABC,

AC² = $\sqrt{28^2+21^2} = \sqrt{784+441}$

$= \sqrt{1225} \Rightarrow$ AC = 35 cm



Area of shaded portion = Area of Semi - circle ACE
+ Area of $\triangle$ ABC – Area of quadrant circle BCD

$= \frac{\pi r^2}{2} + \frac{1}{2} \times BC \times BA - \frac{\pi}{4} \times r_1^2$

$= \frac{22}{7} \times \frac{1}{2} \times \frac{35}{2} \times \frac{35}{2} + \frac{1}{2} \times 21 \times 28 - \frac{22}{7\times4} \times 21 \times 21$

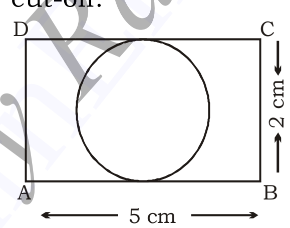
$= \frac{5\times11\times35}{4} + \frac{1}{2}(21\times28-33\times21)$

$= \frac{1925}{4} + \frac{1}{2}(-105)$

= 481.25 – 52.50 = 428.75 cm²

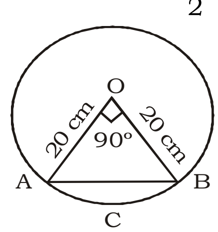
439. (b) Let the sides of squares S_1 and S_2 are a and b, respectively. So, perimeters of square S_1 and S_2 are 4a and 4b, respectively.
By given condition,
4a = 4b + 12
$\Rightarrow$ a = b + 3 ....(i)
and a² = 3 (b)²–11
$\Rightarrow$ (b+3)² = 3 b²–11
$\Rightarrow$ b²+6b+9 = 3b² – 11
$\Rightarrow$ 2b² – 6b – 20 = 0
$\Rightarrow$ 2b² – 10b + 4b – 20 = 0
$\Rightarrow$ 2b (b–5)+4(b–5) = 0
$\Rightarrow$ (b–5)(2b+4) = 0
$\Rightarrow$ b = 5m ($\because b \neq -2$)
On putting the value of b in Eq. (i), we get
a = 5+3 = 8
$\therefore$ Perimeter of S_1 = 4×8 = 32 m

440. (b) From a rectangular sheet of cardboard of size 5×2 cm², a circle of radius 1 cm, can be cut-off.



Area of rectangular sheet
= 5×2 = 10 cm²
and area of circle
= π (1)² = π cm²
$\therefore$ Required area = Area of sheet – Area of circle
= (10 – π) cm²

441. (d) Area of $\triangle$ AOB = $\frac{1}{2}$ × OA × OB



$= \frac{1}{2} \times 20 \times 20$ = 200 cm²
and area of sector OACBO

$= \frac{\pi r^2\theta}{360°} = \frac{3.14\times20\times20\times90°}{360°}$

$= \frac{3.14\times400}{4}$ = 314 cm²

$\therefore$ Area of minor segment
= Area of sector OACBO – Area of $\triangle$ AOB = 314 – 200 = 114 cm²

442. (b) Angle made in 60 min by minute hand of a clock = 360° and angle made in 15 min by minute hand of a clock

$= \frac{360°}{60°} \times 15° = 90°$

$\therefore$ Required distance

$= \frac{2\pi(14)90°}{360°} = \frac{22}{7} \times \frac{14\times2}{4}$

= 22 cm.

443. (c) Let the length of rectangle $= \frac{x}{2}$

and breadth of rectangle = $\frac{x}{6}$

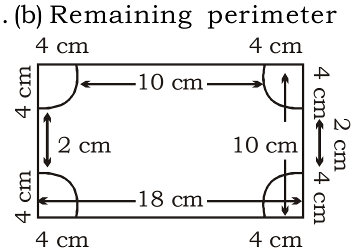
$\therefore$ Area of rectangle

$= \frac{x}{2} \times \frac{x}{6} = \frac{x^2}{12}$

and area of square = x²
Hence, area of remaining

portion = $x^2 - \frac{x^2}{12} = \frac{11x^2}{12}$

444. (b) Remaining perimeter



$= \left(\frac{2\pi r}{4}\right)$4+10+2+10+2
= 2 × 3.14 × 4 + 24
= 25.12 +24 = 49.12 cm
= 49.1 cm (approx)

445. (b) Distance travel in 1 revolution = $\frac{440}{1000}$ m

and circumference = π ×d

$= \frac{44000}{1000}$ cm

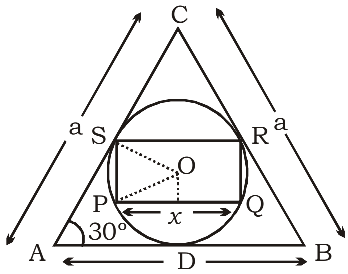
$\therefore$ d = $\frac{44000\times7}{1000\times22}$ = 14 cm

446. (c) Side of an equilateral triangle is a

then the altitude of equilateral triangle is $\frac{\sqrt{3}}{2}a$

$\therefore$ Radius of incircle

$= \frac{a\sqrt{3}}{2} \times \frac{1}{3} = \frac{a}{2\sqrt{3}}$



$\therefore$ Diameter of incircle

$= 2\left(\frac{a}{2\sqrt{3}}\right) = \frac{a}{\sqrt{3}}$

Let side of a square be x.

$\therefore \left(\frac{a}{\sqrt{3}}\right)^2 = x^2 + x^2$

$\Rightarrow \frac{a^2}{3} = 2x^2$

$\Rightarrow x^2 = \frac{a^2}{6}$ = Area of square

447. (c) Radius of circle, r = 6 cm

$\therefore$ Area of circle $= \pi r^2 = \pi \times 6^2$

$= 36\pi \text{ cm}^2$

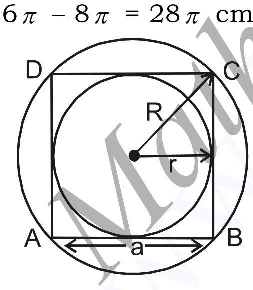
and area of sector subtending an angle of 80° at O

$\frac{\pi r^2 \theta}{360°} = \frac{\pi \times 6^2 \times 80°}{360°} = 8\pi \text{ cm}^2$

$\therefore$ Required difference

$= 36\pi - 8\pi = 28\pi \text{ cm}^2$

448. (a)



Radius of incircle $= r = \frac{a}{2}$

and Radius of circum-circle

$= \frac{a}{\sqrt{2}} = R$

$\therefore$ Ratio of area $= \frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2}$

$= \frac{a^2/4}{a^2/2} = \frac{2}{4} = \frac{1}{2} = 1:2$

449. (a) Larger Radius (R) = 14 + 7 = 21cm

Smaller Radius (S) = 7cm

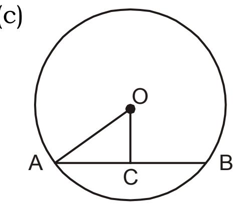
$\therefore$ Area of shaded portion

$= \pi R^2 \frac{\theta}{360°} - \pi r^2 \frac{\theta}{360°}$

$= \pi \frac{30°}{360°}(21 \times 21 - 7 \times 7)$

$= \frac{22}{7} \times \frac{1}{12} \times 28 \times 14 = 102.67\text{cm}^2$

450. (c)



OC = 12cm AC = CB = 5cm

$\therefore$ Radius 'OA' $= \sqrt{OC^2 + AC^2}$

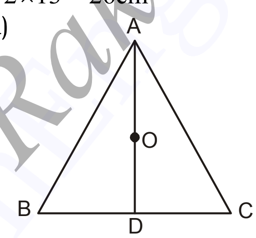
$= \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$

$= \sqrt{169} = 13cm$

$\therefore$ *Diameter* of circle

$= 2 \times 13 = 26$cm

451. (d)



DB = DC = 3cm.

$AD = \sqrt{AB^2 - BD^2} = \sqrt{6^2 - 3^2}$

$= \sqrt{36 - 9} = \sqrt{27} = 3\sqrt{3}cm.$

$\therefore$ OD = In-radius

$= \frac{1}{3} \times 3\sqrt{3} = \sqrt{3}cm.$

$\therefore$ *Area* of circle $= \pi r^2$

$= \pi \times \sqrt{3} \times \sqrt{3} = 3\pi$ sq.cm

Alternatively :

Inradius(r)$= \frac{a}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} = \sqrt{3}$

$\therefore$ Area $= \pi r^2 = \pi(\sqrt{3})^2 = 3\pi$

452. (d) Area of parallelogram

= base $\times$ height

= 27 $\times$ 12 = 324sq. cm.

Again,

324 = 36 $\times$ h

$\Rightarrow h = \frac{324}{36} = 9cm$

453. (d) $2\pi r = 2(18 + 26)$

$2 \times \frac{22}{7} \times r = 44 \times 2 \quad \Rightarrow r = 14cm$

$\therefore$ *Area* of circle $= \pi r^2$

$= \frac{22}{7} \times 14 \times 14 = 616 sq.cm.$

454. (d) $\pi r + 2r = 36$

$\Rightarrow r\left(\frac{22}{7} + 2\right) = 36$

$\Rightarrow r\left(\frac{22 + 14}{7}\right) = 36$

$\Rightarrow r = \frac{36 \times 7}{36} = 7\, metre$

$Area = \frac{\pi r^2}{2} = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 77 sq.metre$

455. (b) Let the radius of circle be 'r'

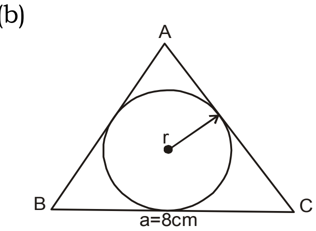
$\Rightarrow 2\pi r = 24\pi \Rightarrow r = 12$

$\therefore$ Area of circle

$= \pi(12)^2 = 144\pi$

$\therefore$ Area of the rectangle = area of circle = 144π

456. (b)



Area of triangle $= \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4} \times 8 \times 8 = 16\sqrt{3}cm^2$

radius of incircle (r)

$= \frac{a}{2\sqrt{3}} = \frac{8}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$

Area of inscribed circle $= \pi r^2$

$\pi\left(\frac{4}{\sqrt{3}}\right)^2 = \frac{22}{7} \times \frac{16}{3}$

$\therefore$ Required area

$= \left(16\sqrt{3} - \frac{22 \times 16}{21}\right)$

$= \dfrac{16}{21}(21 \times 1.732 - 22)$

$= \dfrac{16}{21}(14.372) = 10.95cm^2$

457.(b) height = h

$= \dfrac{\sqrt{3}}{2}a$ (a side of)

inradius = $\dfrac{a}{2\sqrt{3}}$

incirle's diameter (d)

$= \dfrac{2a}{2\sqrt{3}} = \dfrac{a}{\sqrt{3}}$

circum-radius = $\dfrac{a}{\sqrt{3}}$

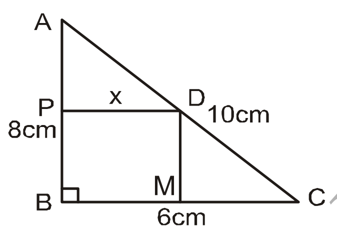
$\Rightarrow$ circumcircle's diameter

(D) $= \dfrac{2a}{\sqrt{3}}$

$\therefore$ d : D : H

$= \dfrac{a}{\sqrt{3}} : \dfrac{2a}{\sqrt{3}} : \dfrac{\sqrt{3}}{2}a = 2:4:3$

458. (a)



Side of maximum sized square

$\dfrac{AB \times BC}{AB + BC}$

$\dfrac{8 \times 6}{8+6} = \dfrac{24}{7}$

$\therefore$ area of square

$= \left(\dfrac{24}{7}\right)^2 = \dfrac{576}{49}cm^2$

Alternatively,

let side of largest square = x

$\therefore$ AP = (8 - x)cm and

MC = (6 - x)cm

In $\triangle$ ABC and $\triangle$ DMC,

$\angle B = \angle M = 90°$

$\angle C = \angle C$ = (common)

$\triangle ABC \sim \triangle DMC$,

$\Rightarrow \dfrac{BC}{MC} = \dfrac{AB}{DM} \Rightarrow \dfrac{6}{6-x} = \dfrac{8}{x}$

$\Rightarrow 3x = 24 - 4x \Rightarrow x = \dfrac{24}{7}cm$

$\therefore$ area of square

$= x^2 = \dfrac{576}{49}cm^2$

459.(b) For 3x + 4y = 12

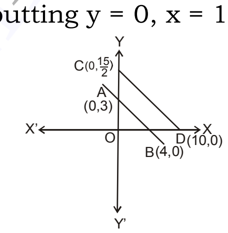
By putting x = 0, y = 3

By puttting, y = 0, x = 4

For 6x + 8y = 60,

By putting x = 0, $y = \dfrac{15}{2}$

By putting y = 0, x = 10



$\therefore$ Area of $\Delta OCD = \dfrac{1}{2} \times OD \times OC$

$= \dfrac{1}{2} \times 10 \times \dfrac{15}{2} = \dfrac{75}{2}$

$\therefore$ Area of ΔOAB

$= \dfrac{1}{2} \times OB \times OA = \dfrac{1}{2} \times 4 \times 3 = 6$

$\therefore$ Area of trapezium $= \dfrac{75}{2} - 6$

$= \dfrac{75-12}{2} = \dfrac{63}{2} = 31.5 sq.units$

460. (c) Side of the first square

$= \sqrt{Area}$

$= \sqrt{200} = 10\sqrt{2} metre$

Its diagonal $= \sqrt{2} \times side$

$= 10\sqrt{2} \times \sqrt{2}$

$= 20 metre$

$\therefore$ Diagonal of new square

$= \sqrt{2} \times 20 = 20\sqrt{2}$ metre

$\therefore$ Its area $= \dfrac{1}{2} \times (diagonal)^2$

$= \dfrac{1}{2} \times 20\sqrt{2} \times 20\sqrt{2}$

$= 400 sq.metre$

CHAPTER 17

MENSURATION 3–D (THREE DIMENSIONAL)

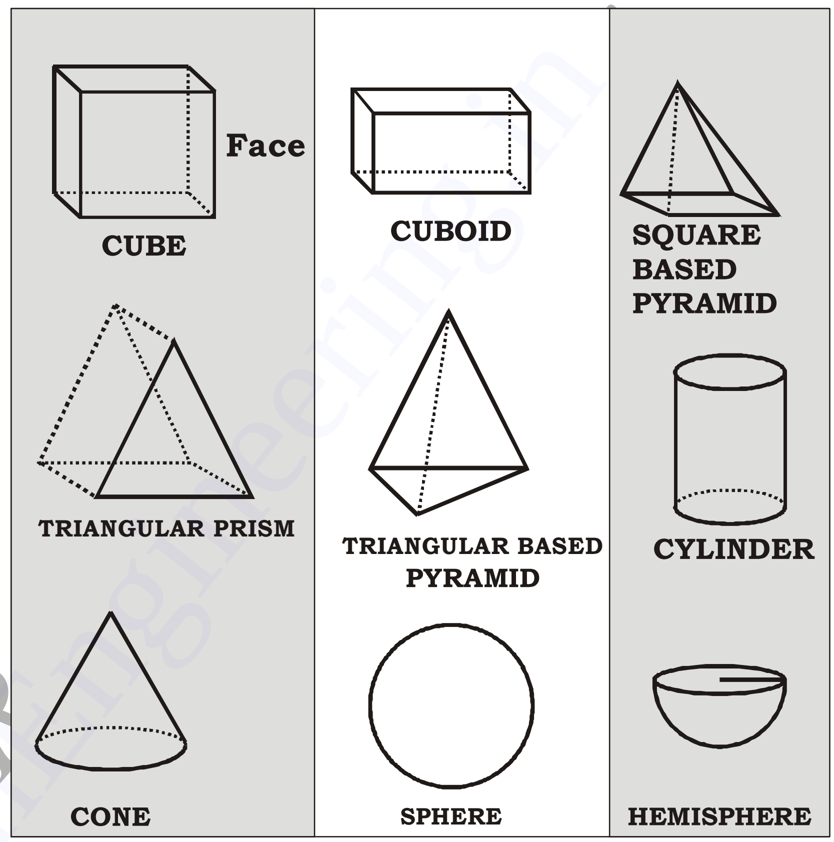
Mensuration is the branch of mathematics which deals with the study of different geometrical shapes, their areas and Volume. In the broadest sense, it is all about the process of measurement. It is based on the use of algebric equations and geometric calculations to provide measurement data regarding the width, depth and volume of a given object or group of objects. while the measurement results obtained by the use of mensuration are estimates rather than actual physical measurements, the caluclations are usually considered very accurate.

There are two types of geometric shapes:-

1. 2D **2. 3D**

3D shapes: They have surface area and volume.

(1) Cube
(2) Rectangular Prism (Cuboid)
(3) Cylinder **(4)** Cone
(5) Sphere and Hemisphere
(6) Prism **(7)** Pyramid



What is 3D....?

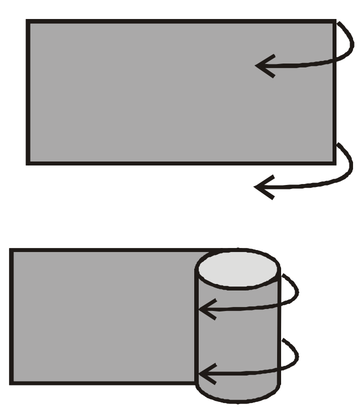
A **three-dimensional shape** is a solid shape that has height and depth. For example, a sphere and a cube are three-dimensional, but a circle and a square are not.

What is Volume.....?

Volume is the measure of the amount of space inside of a solid figure, like a cube, ball, cylinder or pyramid. It's units are always "cubic", that is the number of little element cubes that fit inside the figure.

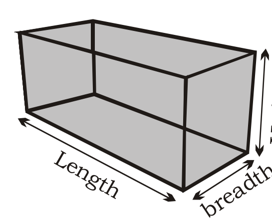
Difference between Curved surface area and Total surface area

The area of all the curved surfaces of any solid. The Curved surface area is the circumference of the base of the solid and the face parallel to it. The Total Surface area is the sum of both the curved surface area and the area of the base and top.



Cuboid (Parallelopiped)

A cuboid is a 3 dimensional shape.
It is a solid figure which has 6-regular faces.
12 edges, **8** vertices and **4** diagonals.



Look at this shape.
There are 3 different measurements:
Length, **Breadth**, **Height**

Formula

(i) The volume is found using the formula:

Volume = Length × Breadth × Height

Which is usually shortened to:

V = l × b × h

Or more simply:

V = **lbh**

(a) $V = l \times b \times h$ cubic units

(b) $V = \sqrt{A_1 \times A_2 \times A_3}$ cubic units

Where,

A_1 = area of base or top = lb sq. units

A_2 = area of one side face = bh sq. units

A_3 = area of other side face = hl sq. units

(ii) **Lateral surface Area /Curved surface area/ Area of four walls**

= Perimeter of Base × height

= $2(l + b) \times h$ sq.units

(iii) **Total surface Area** = $2(lb + bh + hl)$ sq. units

(iv) **Diagonal of cuboid**

$= \sqrt{l^2 + b^2 + h^2}$ units

(v) To find total surface area of a cuboid if the sum of all three sides and diagonal are given.

Total surface area = (Sum of all three sides)² – (Diagonal)²

Note : For painting the surface area of a box or to know how much tin sheet is required for making a box, we use formulae **(iii)** i.e. Total surface Area.

To find the length of the longest pole to be placed in a room, we use formulae **(iv)** i.e. Diagonal.

EXAMPLES

1. The dimensions of a cuboid are 16 cm, 18 cm and 24 cm. Find:

(a) Volume

(b) Surface area (c) Diagonal

Sol. (a) Volume = $l \times b \times h = 16 \times 18 \times 24 = 6912$ cm³

(b) Surface area = $2(lb + bh + hl) = 2(16\times18 + 18\times24 + 24\times16) = 2208$ cm²

(c) Diagonal

$= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{16^2 + 18^2 + 24^2}$

$= \sqrt{1156} =$ **34 cm**

2. Find the length of the longest pole that can be placed in a room 30 m long, 24 m broad and 18 m high.

Sol. $d = \sqrt{l^2 + b^2 + h^2}$

$d = \sqrt{900 + 576 + 324}$

$= \sqrt{1800}$

$d = \mathbf{30\sqrt{2}}$ **m**

3. A brick measures 20 cm × 10 cm × 7.5 cm. How many bricks will be required for a wall 20 m × 2 m × 0.75 m ?

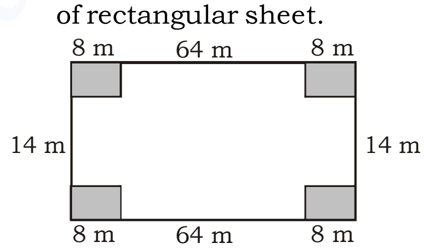
Sol. Number of bricks = $\dfrac{\text{Total Volume of wall}}{\text{Volume of one brick}}$

$= \dfrac{20 \times 2 \times 0.75 \times 100 \times 100 \times 100}{20 \times 10 \times 7.5}$

= **20,000**

4. A rectangular sheet of metal is 80 m by 30 m. Equal squares of side 8 m are cut off at the corners and the remainder is folded up to form an open rectangular box. Find: **(i)** Volume **(ii)** Total surface area **(iii)** Surface area of box.

Sol. **(i)** When four square of 8 cm are removed from four corners of rectangular sheet.



Length and Breadth of remaining rectangular sheet will 64 cm and 14 cm & height of sheet will be 8 cm.

Volume of open rectangular box

= Length × Breadth × Height

= 64 × 14 × 8 = **7168 m³**

(ii) Surface area = 2(Length + Breadth) × Height

= 2(64 + 14)×8 = 2 × 78 × 8

= **1248 m²**

(iii) Total surface area = Surface area + Base area

= 1248 + Length × Breadth

= 1248 + 64 × 14 = 1248 + 896

= **2144 m²**

5. In swimming pool measuring 90 m by 40 m, how much water will be displaced by 150 men, if the displacement of water by one man is 8 cm³, what will be the rise in water level?

Sol. Volume of water displaced by 150 men = Volume of water came out

(Let the height raised in water = h)

⇒ 8 × 150 = 90 × 40 × h

⇒ $h = \dfrac{1}{3}$ m = **33.33 cm**

6. A rectangular water reservoir is 15 m × 12 m at the base. Water flows into it through a pipe whose cross section is 5 cm by 3 cm at the rate of 16 m/s. Find the height to which water will rise in the reservoir in 25 minutes.

Sol. Volume of water comes out from pipe in 1 sec

$= \dfrac{5}{100} \times \dfrac{3}{100} \times 16\,\text{m}^3 = 0.0240\,\text{m}^3$

Volume of water comes out from pipe in 25 min = 0.0240 × 25 × 60 = 36 m³

⇒ Volume of water pour into tank = Volume of water comes out from pipe.

∴ 15 × 12 × h = 36

(∴ h = rise in level of water)

⇒ h = **0.2 m**

7. The sum of length, breadth and height of a cuboid is 25 cm and its diagonal is 15 cm long. Find the total surface area of the cuboid.

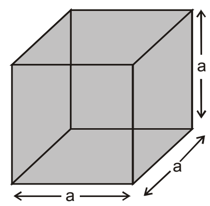
Sol. We have the total surface area,

= (25)² – (15)² = 625 – 225

= **400 sq. cm.**

CUBE

A cube whose length, breadth and height are all equal is called a cube. A cube has **6** equal faces, **12** equal edges, **8** vertices and **4** equal diagonals.



Consider a cube of edge a units.
It is a special type of cuboid in which l = b = h = a units i.e. each face is a square.

(i) Volume = a^3 cubic units

(ii) Lateral surface Area
= $4a^2$sq.units

(iii) Total surface Area
= $6a^2$sq.units

(iv) Diagonal of cube (d)
= $\sqrt{3}$ a units

(v) Face diagonal of cube
= $\sqrt{2}$ a units

(vi) Volume of cube

$$=\left(\sqrt{\frac{\text{surface area}}{6}}\right)^3 \text{ cubic units}$$

EXAMPLES

1. Edge of a cube is 5 cm. Find:
(a) Volume
(b) Surface area
(c) Diagonal

Sol. Volume = $a^3 = (5)^3 = 125$ cm³
Surface area = $6a^2 = 6 \times (5)^2$
= 150 cm²
Diagonal = $a\sqrt{3}$ = $5\sqrt{3}$
= 8.660 = **8.66 cm**

2. Three cubes of volume 1 cm³, 216 cm³ and 512 cm³ are melted to form a new cube. What is the diagonal of the new cube ?

Sol. Volume of new cube = 1 + 216 + 512 = 729 cm³
$\therefore$ Edge of new cube
= $\sqrt[3]{729}$ = 9 cm
$\therefore$ Surface area = $6a^2$
= $6 \times (9)^2$ = 486 cm²
$\therefore$ Diagonal of the new cube
= $a\sqrt{3}$ = $9\sqrt{3}$
= **15.6 cm (approx)**

3. The surface area of a cube is 864 cm². Find the volume.

Sol. $6a^2 = 864 \Rightarrow a^2 = 144$
$\Rightarrow$ a = 12 cm
$a^3 = (12)^3$ = **1728 cm³**

4. The Cost of painting the whole surface area of a cube at the rate of 13 paise per sq. cm is Rs. 343.98. Then the volume of the cube is:

Sol. Cost of painting the whole surface are = Rs. 343.98 = 34398 paise

$$\text{Total surface area} = \frac{34398}{13}$$

= 2646 cm²
$6a^2 = 2646$
$a^2 = 441$
a = 21 cm
$\therefore$ Volume of cube = a^3
= $(21)^3$ = **9261 cm³**

5. A solid cube with an edge of 10 cm is melted to form two equal cubes. The edge of smaller cube to the bigger cube is.

Sol. Volume of larger cube = summation of volume of smaller cubes
$\therefore$ Let the edge of smaller cubes be 'a'
$\Rightarrow (10)^3 = (a)^3 + (a)^3$
$\Rightarrow (10)^3 = 2(a^3)$

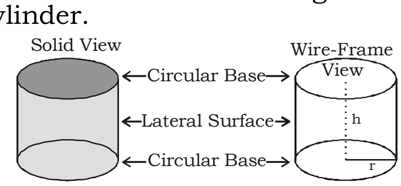
$$\Rightarrow a = \frac{10}{(2)^{1/3}}$$

$$\Rightarrow \frac{\text{edge of smaller cube}}{\text{edge of bigger cube}} = \frac{\frac{10}{\sqrt[3]{2}}}{10}$$

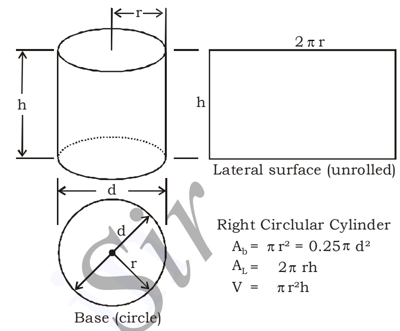
$$= \frac{1}{\sqrt[3]{2}}$$

Right Circular Cylinder

A **Right circular cylinder** is a three-dimensional object with two congruent circles as parallel bases and a lateral surface consisting of a rectangle. Volume and surface area of a **Right Circular Cylinder.** if 'r' is the radius of a circular base of the cylinder and 'h' is the height of the cylinder.



A right circular cylinder is a cylinder whose base is a circle and whose elements are perpendicular to its base.



Properties of a Right Circular Cylinder

1. The axis of a right circular cylinder is the line joining the centers of the bases.

2. For any oblique or non-oblique sections which do not pass any one base, the center of which is at the axis.

3. A right circular cylinder can be formed by revolving a rectangle about one side as axis of revolution.

4. Every section of a right circular cylinder made by a cutting plane containing two elements and parallel to the axis is a rectangle.

Fromulae for Right Circular Cylinder

Area of the base, A_b

$$A_b = \pi r^2$$
$$A_b = \frac{\pi}{4} d^2$$

Lateral surface Area, A_L

$$A_L = 2\pi rh$$
$$A_L = \pi dh$$

Volume, V

$$V = A_b h$$
$$V = \pi r^2 h$$
$$V = \frac{\pi}{4} d^2 h$$

Total surface Area, A_T

Total surface area (open both ends), A_L = A
Total surface Area (open one end), A = $A_b + A_L$
Total surface Area (closed both ends), A = $2A_b + A_L$

EXAMPLES

1. Find the volume of an iron rod which is 7 cm long and whose diameter is 1 cm.

Sol. Diameter = 1 cm.

Radius = $\frac{1}{2}$ cm

Height = 7 cm

Volume of Cylinder = $\pi r^2 h$

$= \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 7 = \frac{11}{2}$

= **5.5 cubic cm**

2. Water flows at 10 km per hour through a pipe with cross section a circle of radius 35 cm, into a cistern of dimensions 25 m by 12 m by 10 m. By how much will the water level rise in the cisten in 24 minutes ?

Sol. Volume flown in 24 minutes

$= \left(\frac{22}{7} \times \frac{35}{100} \times \frac{35}{100} \times \frac{10000}{60} \times 24\right)$

= 1540 cubic m

Rise in level $= \left(\frac{1540}{25 \times 12}\right) =$ **5.13 m**

3. A powder tin has a square base with side 8 cm and height 13 cm. Another is cylindrical with radius of its base 7 cm and height 15 cm. Find the difference in their capacities.

Sol. Difference in capacities

$= \frac{22}{7} \times 7 \times 7 \times 15 - 8 \times 8 \times 13$

= 2310 – 832 = **1478 cubic m**

4. A metallic sphere of radius 21 cm is dropped into a cylinderical vessel, which is partially filled with water. The diameter of the vessel is 1.68 metres. If the sphere is completely submerged, find by how much the surface of water will rise.

Sol. Volume of sphere $= \frac{4}{3}\pi r^3$

$= \left(\frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21\right)$

= 38808 cubic cm.

$\therefore \frac{22}{7} \times 84 \times 84 \times h = 38808$

$\therefore$ **h = 1.75 cm**

5. The radius of a wire is decreased to one third. If volume remains same, length will increase:

Sol. Let, radius = R and length = h,

Volume = $\pi R^2 h$

New radius = $\frac{1}{3}R$

Let, new length = H

Volume = $\pi \times \left(\frac{1}{3}R\right)^2 \times H$

$= \frac{\pi R^2 H}{9}$

$\therefore \pi R^2 h = \frac{\pi R^2 H}{9}$ or H = 9h

6. A cylindrical iron rod is 70 cm long, and the diameter of its end is 2 cm. What is its weight, reckoning a cubic cm of iron to weigh 10 grams ?

Sol. Volume of the iron rod = $\pi r^2 h$

$= \frac{22}{7} \times 1 \times 1 \times 70 = 220 \text{ cm}^3$

$\therefore$ weight of the cylinder

$= \frac{220 \times 10}{1000}$ = **2.2 kg.**

7. A cylinderical vessel, whose base is 14 dm in diameter holds 2310 litres of water. Taking a litre of water to occupy 1000 cubic cm, what is the height of the vessel in dm?

Sol. $\frac{22}{7} \times 70 \times 70 \times h = 2310 \times 1000$

$[\because 1\text{dm} = 10\text{cm}]$

$\therefore h = \frac{2310 \times 1000 \times 7}{22 \times 70 \times 70}$

= 150 cm = **15 dm.**

8. Find how many pieces of money $\frac{3}{4}$ cm in diameter and $\frac{1}{8}$ cm thick must be melted down to form a cube whose edge is 3 cm long ?

Sol. Volume of one piece of money

$= \pi r^2 h = \frac{22}{7} \times \frac{3}{8} \times \frac{3}{8} \times \frac{1}{8}$

$\therefore \frac{22}{7} \times \frac{3}{8} \times \frac{3}{8} \times \frac{1}{8} \times n = 3 \times 3 \times 3$

$n = \frac{3 \times 3 \times 3 \times 7 \times 8 \times 8 \times 8}{22 \times 3 \times 3 \times 1}$

n = **488.72**

9. The diameter of a cylindrical tank is 24.5 metres and depth 32 metres. How many metric tons of water will it hold ? (One cubic metre of water weighs 1000 kg.)

Sol. Volume of the cylinder

$= \frac{22}{7} \times \frac{24.5 \times 24.5}{2 \times 2} \times 32$

= 15092 m^3

Since 1 cubic metre = 1000 kg.

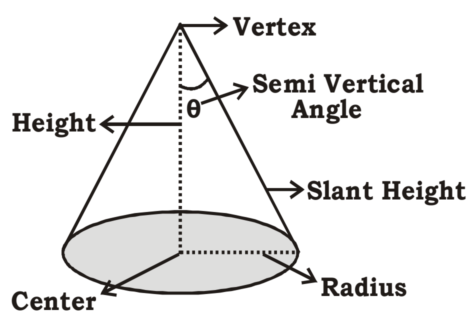
$\therefore$ 1 cubic metre = 1 metric ton

$(\because 1000\text{kg} = 1 \text{metric ton})$

$\therefore$ Volume of cylinder = **15092 metric tones**

Right Circular Cone

Cone is a three dimensional geometric shape. If one end of a line is twisted about a second set line while keeping the lines other end fixed, we get a cone. The point about which the line is curved is known as the vertex and the base of the cone is a circle. The vertex is directly above the centre of the bottom.



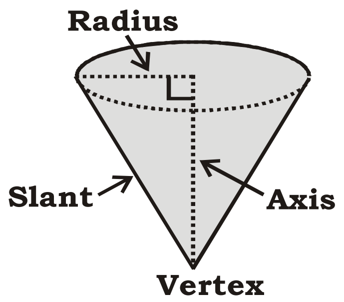
Properties of a Cone

There are number of properties of a cone. Some of them are as follows:

- Volume of a Cone
- Lateral surface Area of a Cone
- Total Surface Area of a Cone

Definition of Right Circular Cone

A right circular cone is one whose axis is perpendicular to the plane of the base. We can generate a right cone by revolving a right triangle about one of its legs.



Formulae of Right Circular Cone

For a right circular cone of radius r, height h and slant height l, we have

Lateral surface area of a right circular cone = πrl

Total surface area of a right circular cone = $\pi(r + l)r$

Volume of a right circular cone= $\frac{1}{3}\pi r^2h$

Note:-

Area is measured in square units and volume is measured in cubic units.

Surface Area of a Right Circular Cone:-

The surface area of a right circular cone is the sum of area of base and lateral surface area of a cone. The surface area is measured in terms of square units.

Surface area of a cone = Base Area + Lateral surface Area of a cone

$= \pi r^2 + \pi r l = \pi r(r+l)$

Surface Area of a Right Circular Cone can be calculated by the following formula.

Area of a right circular cone $= \pi r(r+l)$

Here, $l = \sqrt{r^2+h^2}$

Where, r = Radius

h = Height and

l = Slant height of cone

Volume of a Right Circular Cone

The volume of a cone is one third of the product of the area of base and the height of the cone. the volume of a right circular cone is measured in terms of cubic units.

Volume of a right circular cone can be calculated by the following formula.

Volume of a right circular cone

$= \frac{1}{3} \times \text{Base area} \times \text{height}$

Where, Base area = πr^2

Volume of a Circular cone = $\frac{1}{3} \times \pi \times \text{Radius}^2 \times \text{Height}$

EXAMPLES

1. Find the volume of a cone whose diameter of the base is 21 cm and the slant height is 37.5 cm.

Sol. $h = \sqrt{(37.5)^2 - (10.5)^2} = 36$ cm

$\therefore$ volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 36$

= **4158 cm³**

2. The radius and height of a right circular cone are in the ratio 5 : 12 and its volume is 2512 cm³. Find the slant height, radius and curved surface area of the cone. (Take π = 3.14)

Sol. Let radius = $5x$, height = $12x$

$\therefore$ volume $= \frac{1}{3}\pi r^2 h$

$\frac{1}{3} \times 3.14 \times (25x^2) \times (12x) = 2512$

$x = 2$

$\therefore$ Radius = $5x$ = 10 cm &

Height = $12x$ = 24 cm

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{(10)^2 + (24)^2}$

= 26 cm

$\therefore$ Curved surface area of cone

$= \pi r l = 3.14 \times 10 \times 26 = 31.4 \times 26 =$ **816.4 cm²**

3. If a right circular cone of vertical height 24 cm has a volume of 1232 cm³, then the area of its curved surface in cm² is:

Sol. $\because$ Volume $= \frac{1}{3}\pi r^2 h$

$\therefore \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24$

= 1232 = r² = 49 = r = 7

Slant height $= \sqrt{(24)^2 + (7)^2}$

= 25 cm

$\therefore$ Curved surface

$= \left(\frac{22}{7} \times 7 \times 25\right) =$ **550 cm²**

4. A right cylindrical vessel is full with water. How many right cones having same diameter and height as those of right cylinder will be needed to store that water ?

Sol. Volume of 1 cylinder = $\pi r^2 h$

Volume of 1 cone = $\frac{1}{3}\pi r^2 h$

Number of cones $= \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} =$ **3**

5. A cylindrical piece of metal of radius 2 cm and height 6 cm is shaped into a cone of same radius. The height of cone is:

Sol. Volume of Cone $= \frac{1}{3}\pi r^2 h =$

$\frac{1}{3}\pi(2)^2 h$

Volume of cylinder = $\pi \times (2)^2 \times 6$

$\because$ Volume of cone = Volume of cylinder

$\frac{1}{3}\pi \times 2^2 \times h = \pi \times 2^2 \times 6$

$\therefore$ **h = 18 cm**

6. From a solid right circular with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and base is removed. Find the volume of the remaining solid.

Sol. Volume of remaining solid

$= \pi(r)^2 h - \frac{1}{3}\pi(r)^2 h$

$= \frac{22}{7} \times 6 \times 6 \times 10 - \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 10$

$= \frac{22}{7} \times 6 \times 6 \times 10 \times \frac{2}{3}$

$= \frac{5280}{7} = 754\frac{2}{7}$ cm³

7. The slant height of a conical tomb is $17\frac{1}{2}$ metres. If its diameter be 28 metres, find the cost of constructing it at Rs. 135 per cubic metre and also find the cost of white-washing its slant surface at Rs. 3.30 per square metre.

Sol. Height of the cone

$= \sqrt{\left(\frac{35}{2}\right)^2 - 14^2} = \frac{21}{2}$ m

Volume of the cone

$= \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times \frac{21}{2} = 2156 \text{ m}^3$

Cost of constructing the conical tomb = 2156 × 135

= Rs. 291060

Curved surface area of the conical tomb $= \pi r l$

$= \frac{22}{7} \times 14 \times \frac{35}{2} = 770 \text{ m}^3$

$\therefore$ Cost of white washing = 770 × 3.30 = Rs. 2541

8. Radius of the base of a right circular cone is 3 cm and the height of the cone is 4 cm. Find the total surface area of the cone.

Sol. Applying to the question,

Total surface area $= \pi \times r\,(l + r)$

$= \frac{22}{7} \times 3\left(\sqrt{4^2 + 3^2} + 3\right)$

$\left(\because l = \sqrt{h^2 + r^2}\right)$

$= \frac{22 \times 3 \times 8}{7} = \frac{528}{7} = 75\frac{3}{7}$ sq. cm

9. If the heights of two cones are in the ratio 1 : 4 and their diameters in the ratio 4 : 5 what is the ratio of their volumes ?

Sol. We have,

Ratio of Volumes = (4 : 5)² × (1 : 4) $= \frac{16}{25} \times \frac{1}{4}$

= 4 : 25

[$\because$ ratio of diameters = ratio of radii]

10. If the volumes of the two cones are in the ratio 4 : 1 and their heights in the ratio 4 : 9, what is the ratio of their radii ?

Sol. Ratio of radii $= \sqrt{(4:1) \times \left(\frac{1}{4} : \frac{1}{9}\right)}$

$= \sqrt{(4:1) \times (9:4)} = 3 : 1$

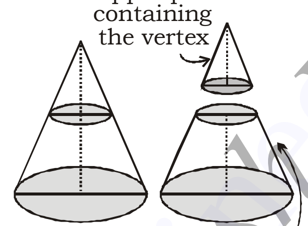
11. If the heights and the curved surface areas of two circular cylinder are in the ratio 1 : 3 and 4 : 5 respectively. Find the ratio of their radii.

Sol. Required ratio $= (4:5) \times \left(1 : \frac{1}{3}\right)$

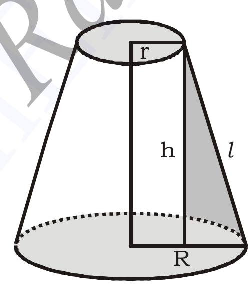
= (4 : 5) × (3 : 1) = **12 : 5**

Frustum of a right circular cone

A frustum of a cone or truncated cone is the result of cutting a cone by a plane parallel to the base and removing the part containing the apex.



The height is the line segment that joins the two bases perpendicularly.



The radii are of their bases are ‘r’ and ‘R’.

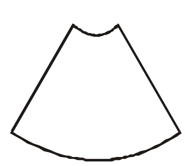
The slant height is the shortest possible distance between the edges of the two bases.

The **slant height** of the truncated cone is obtained by applying the Pythagoras theorem for the shaded triangle:

$l^2 = h^2 + (R - r)^2$

$l = \sqrt{h^2 + (R - r)^2}$

Unfold of a Truncated Cone



Lateral Area of a Truncated Cone

$A_L = \pi\ (R + r)\ l$

Surface Area of a Truncated Cone

$A_T = \pi\ [\ l\,(R + r) + R^2 + r^2\ ]$

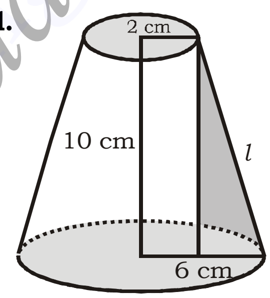
Volume of a Truncated Cone

$V = \frac{1}{3}.\pi.h\left(R^2 + r^2 + R.r\right)$

EXAMPLES

1. Calculate the lateral surface area, surface area and volume of a truncated cone of radii 2 and 6 cm and height of 10 cm.

Sol.



$l^2 = 10^2 + (6 - 2)^2$

$l = \sqrt{10^2 + (6-2)^2} = 10.77$ cm

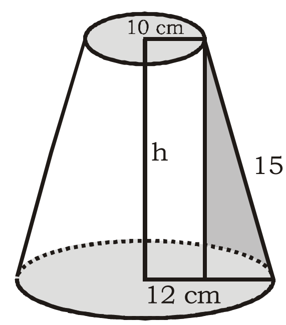
$A_L = \pi\ (6 + 2) \times 10.77 = 270.78 \text{ cm}^2$

$A_T = 270.78 + \pi \times 6^2 + \pi \times 2^2 = 396.35 \text{ cm}^2$

$V = \frac{1}{3}\pi \times 10 \times \left(6^2 + 2^2 + 6 \times 2\right) = \mathbf{544.54\ cm^3}$

2. Calculate the lateral surface area, surface area and volume of a truncated cone of radii 10 and 12 cm and a slant height of 15 cm.

Sol.



$\mathbf{A_L} = \pi\ (R + r)\ l$

$= \pi\,(12 + 10) \times 15 = 1{,}036.73 \text{ cm}^2$

$\mathbf{A_T} = 1036.72 + \pi \times 12^2 + \pi \times 10^2$

$= 1803.27 \text{ cm}^2$

$\because\ l^2 = h^2 + (R - r)^2$

$15^2 = h^2 + (12 - 10)^2$

$\mathbf{h} = \sqrt{15^2 - 2^2} = 14.866$ cm

$\mathbf{V} =$

$\frac{1}{3}\pi \times 14.866(12^2 + 10^2 + 12 \times 10)$

$= \mathbf{5{,}666.65\ cm^3}$

Sphere

A sphere is a solid bounded by a closed surface every point of which is equidistant from a fixed point called the centre. Most familiar examples of a sphere are baseball, tennis ball, bowl, and so forth.

Terms such as radius, diameter, chord, and so forth, as applied to the sphere are defined in the same sense as for the circle.

Thus, a radius of a sphere is a straight line segment connecting its centre with any point on the sphere. Obviously, all radii of the same sphere are equal.

Diameter of the sphere is a straight line drawn from the surface and after passing through the centre ending at the surface.

The sphere may also be considered as generated by the complete rotation of a semicircle about a diameter.

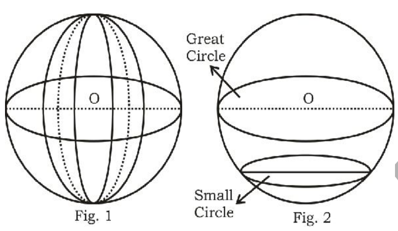


Fig. 1 Fig. 2

Great and Small Circles:

Every section made by a plane passed through a sphere is a circle. If the plane passes through the centre of a sphere, the plane section is a great circle; otherwise, the section is a small circle (Fig. 2). Clearly any plane through the centre of the sphere contains a diameter. Hence, all great circles of a sphere are equals have for their common centre, the centre of the sphere and have for their radius, the radius of the sphere.

Surface Area and Volume of a Sphere:

If 'r' is the radius and 'd' is the diameter of a great circle, then

(i) Surface area of a sphere = 4 times the area of its great circle = $\mathbf{4\pi r^2} = \pi\,\mathbf{d^2}$

(ii) Volume of a sphere = $\mathbf{\frac{4}{3}\pi r^3}$

$= \mathbf{\frac{\pi}{6} d^3}$

(iii) For a spherical shell if R and r are outer and inner radii respectively, then the volume of a shell is = $\mathbf{\frac{4}{3}\pi(R^3 - r^3)}$

$= \mathbf{\frac{\pi}{6}(D^3 - d^3)}$

EXAMPLES

1. The diameter of a sphere is 13.5m. Find its surface area and volume.

Sol. Here, d = 13.5 m

Surface area = $4\pi r^2 = \pi d^2 =$

$\pi(13.5)^2 = 572.56$ sq. m.

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{\pi}{6}d^3$

$= \frac{\pi}{6}(13.5)^3 = \mathbf{1288.25\ cu.\ m.}$

2. Two spheres each of 10m diameter are melted down and recast into a cone with a height equal to the radius of its base, Find the height of the cone.

Sol. Here, d = 10m

$\because$ Radius of cone = height of the cone (Given)

$\therefore$ r = h

Volume of sphere = $\frac{\pi}{6}(d)^3$

$= \frac{\pi}{6} \times 10^3 = \frac{\pi}{6} \times 1000$

= 523.599 cu. m.

Volume of two sphere

= 1047.2 cu. m.

Volume of the cone

= Volume of two spheres

$\Rightarrow \frac{1}{3}\pi r^2 h = 1047.2$

$\Rightarrow \frac{1}{3}\pi h^3 = 1047.2$

$\Rightarrow h^3 = \frac{3 \times 1047.2}{\pi}$

$\Rightarrow h^3 = 1000$

$\Rightarrow h = \mathbf{10m}$

3. How many leaden ball of a $\frac{1}{4}$ cm. in diameter can be cast out of metal of a ball 3 cm in diameter supposing no waste.

Sol. Here, diameter of leaden ball

$= \frac{1}{4}$ cm. $= d_1$

Diameter of metal ball

= 3cm = d_2

Volume of leaden ball

$= \frac{4}{3}\pi\left(\frac{d_1}{2}\right)^3 = \frac{\pi}{6}d_1^3 = \frac{\pi}{6}\left(\frac{1}{4}\right)^3$

= 0.0082 cu. m.

Volume of metal ball = $\frac{\pi}{6}d_2^3$

$= \frac{\pi}{6}(3)^3 = 14.137$ cu. m.

$\therefore$ Number of leaden ball =

$\frac{14.137}{0.0082} = \mathbf{1728.00}$

4. A metal sphere of diameter 14 cm is dropped into a cylindrical vessel, which is partly filled with water. The diameter of the vessel is 1.68 metres. If the sphere is completely submerged, find by how much the surface of water will rise.

Sol. Radius of the sphere = 7 cm

Volume of sphere

$= \left(\frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\right)$

$= 1437\frac{1}{3}$ cu. m

Volume of water displaced by sphere = $1437\frac{1}{3}$ cu cm.

Let the water rise by h cm,

Then, $\frac{22}{7} \times 28 \times 28 \times h$

$= 1437\frac{1}{3}$

or $h = \frac{4312 \times 7}{22 \times 28 \times 28 \times 3} = \frac{7}{12}$

= 0.58 cm

5. Find the weight of an iron shell, the external and internal diameters of which are 13 cm and 10 cm respectively, if 1 cubic cm of iron weighs 8 gms.

Sol. Volume of iron shell

$$= \frac{4}{3}\pi\left\{\left(\frac{13}{2}\right)^3 - (5)^3\right\}$$

$$= \frac{4}{3}\times\frac{22}{7}\times\frac{1197}{8} = 627 \text{ cu cm.}$$

Weight of iron shell = 627 × 8 = 5016 gms. = 5.016 kg.

6. Find the surface area of a sphere whose volume is 310464 cu cm.

Sol. $\frac{4}{3}\pi r^3 = 310464$

$$\text{or } r^3 = \frac{310464\times3\times7}{4\times22} = 74088$$

$\therefore$ r = 42 cm

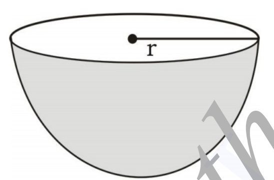
$\therefore$ surface area = $4\pi r^2$

$$= 4\times\frac{22}{7}\times42\times42$$

= **22176 sq. cm.**

Hemisphere

A plane through the center of the sphere cuts it into two equal parts. Each part is called hemisphere.



1. Volume of hemispher = $\frac{2}{3}\pi r^3$

2. Curved surface area or Surface area of hemisphere = $2\pi r^2$

3. Total surface area of solid hemisphere

$$= 2\pi r^2 + \pi r^2 = 3\pi r^2$$

EXAMPLES

1. Find the volume of a hemisphere of radius 21 cm.

Sol. According to the question, We have

r = 21 cm

Volume of hemisphere = $\frac{2}{3}\pi r^3$

Volume of hemisphere

$$= \left(\frac{2}{3}\times\frac{22}{7}\times21\times21\times21\right)$$

= 19404 cm³

2. Find the curved surface area of a hemisphere of radius 21 cm.

Sol. According to the question, We have

Curved surface area = $2\pi r^2$

$$= \left(2\times\frac{22}{7}\times21\times21\right)\text{cm}^2$$

= **2772 cm²**

3. A hemisphere of lead of radius 7 cm is cast into a right circular cone of height 49 cm. Find the radius of the base.

Sol. Volume of hemisphere

$$= \frac{1}{2}\left(\frac{4}{3}\times\frac{22}{7}\times7\times7\times7\right)$$

$$= \frac{2156}{3} \text{ cubic cm}$$

Let, the radius of the base of cone be 'r' cm.

$$\text{Then, } \frac{1}{3}\times\frac{22}{7}\times r^2\times49 = \frac{2156}{3}$$

$$\text{or } r^2 = \frac{2156\times7\times3}{3\times22\times49} = 14$$

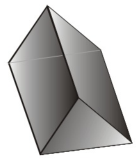
$\therefore$ Radius (r) = $\sqrt{14}$ = 3.74 cm

Right Prism

A prism is a solid object with:

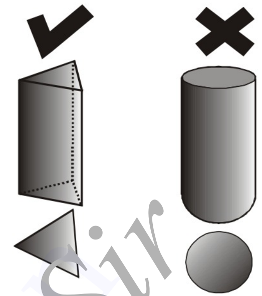
- Identical ends
- Flat faces
- Same cross section all along its length

The cross section of this object is a triangle ..... it has the same cross section all along its length ... so it's a triangular prism.



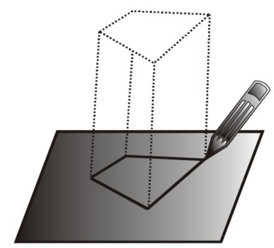
No Curves :

A prism is a polyhedron, which means all faces are flat.



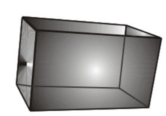
For example, a **cylinder** is **not prism**, because it has curved sides.

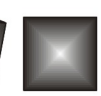
Try drawing a shape on a piece of paper (using straight lines) Then imagine it extending up from the sheet of paper ...... that's a prism !



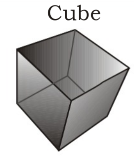
There are all Prism:

Square Prism | Cross-Section



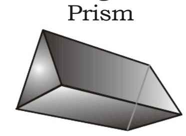


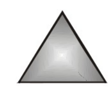
Cube | Cross-Section



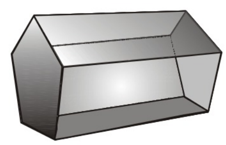
(yes, a cube is a prism, because it is a square all along its length)

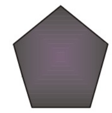
Triangular Prism | Cross-Section





Pentagonal Prism | Cross-Section



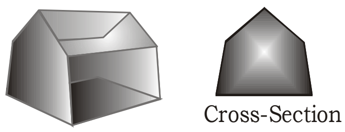


Regular and Irregular Prisms

All the previous examples are of **Regular** Prisms, because the cross section is regular (in other words it is a shape with equal edge lengths, and equal angles.)

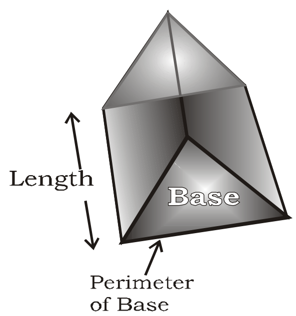
Here is an example of an **Irregular Prism**:

Irregular Pentagonal Prism:



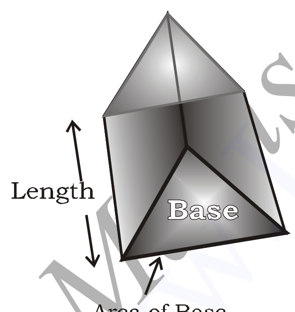
It is "irregular" because the cross-section is not "regular" in shape.

Surface Area of Prism



Surface Area = 2 × Base Area + Base Perimeter × Length

Volume of Prism



The Volume of a prism is the product of its base area and length.

Volume = Base Area × Length

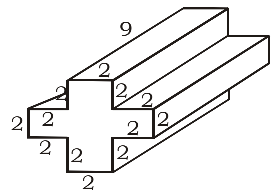
Volume = Base Area × Length

Curved Surface Area = Base perimeter × Height

Total Surface Area = CSA + 2 Base Area

EXAMPLES

1. What is the volume of this prism ?



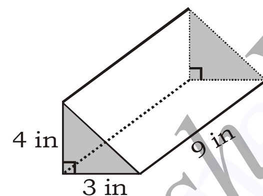
Sol. Volume = Base area × height

There are 5 squares

∴ Base area = 5 × 2 × 2 = 20

Volume = 20 × 9 = **180 units.**

2. The diagram show a prism whose cross-section is a right triangle. What is the volume of the prism ?



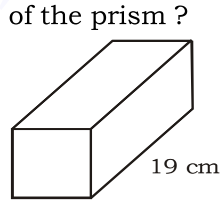
Sol. Volume = base area × height.

Base area = $\frac{1}{2} \times 4 \times 3$ = 6 in²

Volume = 6 × 9 = **54 in³**

3. The diagram shows a prism whose cross-section is a square. The length of the base of the prism is 19 cm and its volume is 1,539 cm³.

What is the total surface area of the prism ?



Sol. Volume = Base area × Height

1539 = Base area × 19

Base area = $\frac{1539}{19}$ = 81

Base area = a² = 81

a = 9

C.S.A = Base perimeter × Height

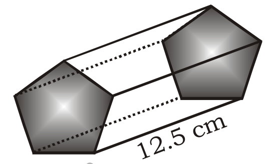
= 4 × 9 × 19 = 684 cm²

T.S.A = C.S.A + 2 Base area

= 684 + 2 × 81

= 684 + 162 = **846 cm²**

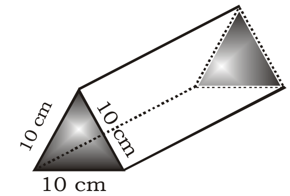
4. The diagram shows a prism with a cross section that is a regular pentagon. If the area of the cross-section is 30 cm², then what is the volume of the prism ?



Sol. Volume = Base area × Height

= 30 × 12.5 = **375 cm³**

5. The diagram shows a prism whose corss-section is an equilateral triangle of lengths 10 cm. Given that its volume is 866 cm², what is the total surface area of the prism ?



Sol. Volume = Base area × Height

Base area = $\frac{\sqrt{3}}{4} \times (10)^2$ = 43.3 cm²

Height = Volume ÷ Base area

= $\frac{866}{43.3}$ = 20

C.S.A = Base perimeter × height

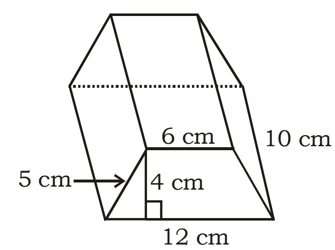
C.S.A = 3 × 10 × 20 = 600 cm²

T.S.A = C.S.A + 2 base area

T.S.A = 600 + 2 × 43.3

= 600 + 86.6 = **686.6 cm²**

6. There is a 10 cm long prism whose cross-section is an isosceles trapezoid:



What is the total surface area of the prism ?

Sol. Base area = $\frac{1}{2}$ × Sum of || sides × height

$= \frac{1}{2} \times (6 + 12) \times 4 = 36 \text{ cm}^2$

Base perimeter = 5 + 6 + 5 + 12 = 28 cm

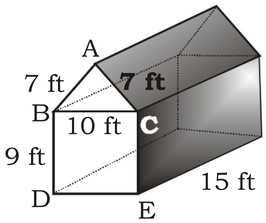
C.S.A. = Base perimeter × h

C.S.A. = 28 × 10 = 280 cm²

T.S.A. = C.S.A. + 2 Base area

T.S.A. = 280 + 2 × 36 = **352 cm²**

7.



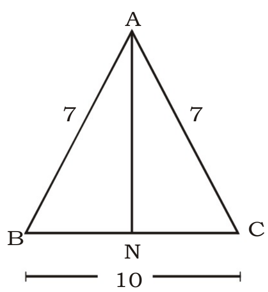
The diagram shows a barn. What is the volume of the barn?

(The length of the hypotenuse in the right triangle is rounded to the nearest foot.)

Sol. Volume = Base area × height

Total base area = Area of □ BCED + Area of Δ ABC

Area of BCDE = 10 × 9 = 90 ft²



In Δ ABC,

$AB^2 = AN^2 + BN^2$

$AN^2 = 49 - 25$

$AN^2 = 24$

$AN = 2\sqrt{6}$ ft

∴ Area of Δ ABC

$= \frac{1}{2} \times BC \times AN$

$= \frac{1}{2} \times 10 \times 2\sqrt{6}$

$= 10\sqrt{6}$ ft²

∴ Total base area

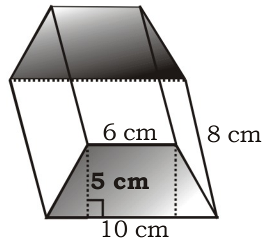
$= \left[10\sqrt{6} + 90\right] \text{ft}^2$

Volume $= \left(10\sqrt{6} + 90\right) \times 15$

$= 1350 + 150\sqrt{6}$

$= 1350 + 150 \times 2.45 =$ **1717.5 ft³**

8. The diagram shows a prism whose cross-section is an isosceles trapezoid.



What is the volume of the prism ?

Sol. Volume = Base area × Height

Base area $= \frac{1}{2}$ (Sum of || sides) × height

$= \frac{1}{2} \times (10 + 6) \times 5$

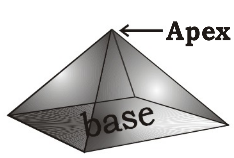
$= \frac{1}{2} \times 16 \times 5 = 40 \text{ cm}^2$

Volume = 40 × 8 = 320 cm³

Pyramids

When we think of pyramids we think of the Great Pyramids of Egypt. They are actually Square Pyramids, because their base is a Square.

Parts of Pyramid

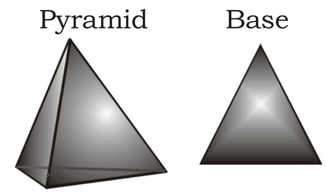


A pyramid is made by connecting a base to an apex.

Types of Pyramid

There are many types of Pyramids, and they are named after the shape of their base.

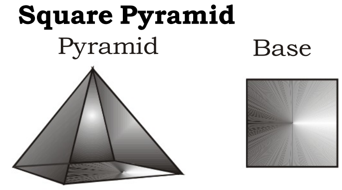
1. Triangular Pyramid



Notice these interesting things:

- It has 4 Faces
- The 3 Side Faces are Triangles
- The Base is also a Triangle
- It has 4 Vertices (corner points)
- It has 6 Edges
- It is also a Tetrahedron (if all triangles are equilateral triangles)

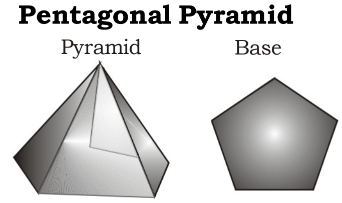
2. Square Pyramid



Notice these interesting things:

- It has 5 Faces
- The 4 Side Faces are Triangles
- The Base is a Square
- It has 5 Vertices (corner points)
- It has 8 Edges

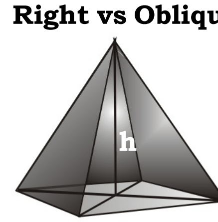
3. Pentagonal Pyramid

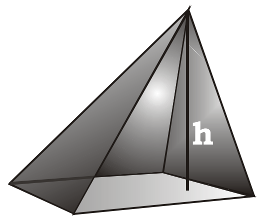


Notice these interesting things:

- It has 6 Faces
- The 5 Side Faces are Triangles
- The Base is a Pentagon
- It has 6 Vertices (corner points)
- It has 10 Edges

Right vs Oblique Pyramid:-

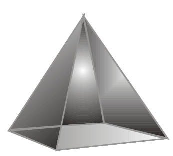




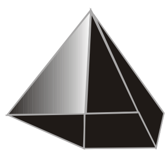
This tells us where the top (apex) of the pyramid is. When the apex is directly above the center of the base it is a **Right Pyramid**, otherwise it is an **Oblique Pyramid**.

Regular vs Irregular Pyramid:-

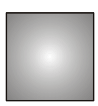
This tells us about the **shape of the base**. When the base is a regular polygon it is a **Regular Pyramid**, otherwise it is an **Irregular Pyramid**.



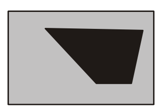
Regular Pyramid



Irregular Pyramid



Base is Regular



Base is Irregular

FORMULAE

(i) Volume of pyramid

$$= \frac{1}{3} \times \text{(area of base)} \times \text{height}$$

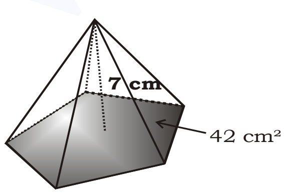
(ii) Curved surface area

$$= \frac{1}{2} \times \text{(perimeter of base)} \times \text{slant height}$$

(iii) Total surface area = curved surface area + area of the base.

EXAMPLES

1. The diagram shows a pyramid whose base is a regular pentagon of area 42 cm² and whose height is 7 cm. What is the volume of the pyramid?

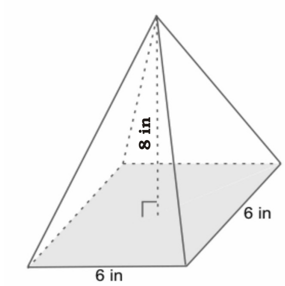


Sol. Volume $= \frac{1}{3} \times$ Area of base $\times$ height

$$\text{Volume} = \frac{1}{3} \times 42 \times 7$$

Volume = 98 cm³

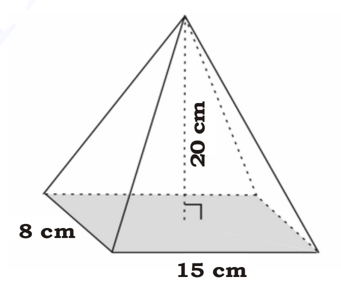
2. The diagram shows a square-based pyramid with base lengths 6 in and height 8 in. What is the volume of the pyramid?



Sol. Volume $\frac{1}{3} \times$ Area of base $\times$ height

$$\text{Volume} = \frac{1}{3} \times 6 \times 6 \times 8 = \mathbf{96\ in^3}$$

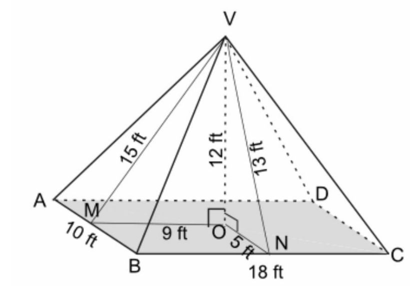
3. The diagram shows a rectangular-based pyramid with base length 15 cm and width 8 cm. The height of the pyramid is 20 cm. What is the volume of pyramid ?



Sol. Volume $= \frac{1}{3} \times$ Area of base $\times$ height $= \frac{1}{3} \times 15 \times 8 \times 20$

Volume = **800 cm³**.

4. The diagram shows a pyramid with vertex V and a rectangular base ABCD. M is the midpoint of AB, N is the midpoint of BC and O is the point at the center of the base.



AB = 10 ft BC = 18 ft
VO = 12 ft VM = 15 ft
VN = 13 ft

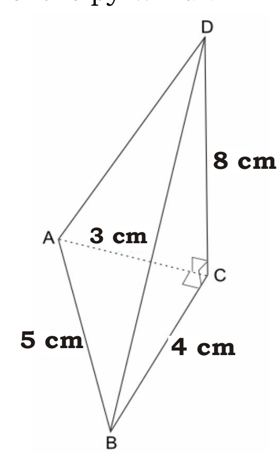
What is the total surface area of the pyramid ?

Sol. Total surface area = 2 × area ΔABV + 2 × area ΔVBC + Area of ABCD

$$= 2 \times \frac{1}{2} \times 10 \times 15 + 2 \times \frac{1}{2} \times 18 \times 13 + 10 \times 18$$

= 150 +234 +180 = 564 ft²

5. The diagram shows a pyramid with a triangular base ABC. The point D is vertically above the point C. What is the volume of the pyramid ?



Sol. Volume

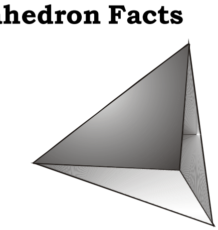
$$= \frac{1}{3} \times \text{Area of base} \times \text{height}$$

$$= \frac{1}{3} \times \frac{1}{2} \times 3 \times 4 \times 8$$

Volume = 16 cm³

Tetrahedron

Tetrahedron Facts



Notice these interesting things:

- It has 4 Faces
- Each face is an Equilateral Triangle
- It has 6 Edges
- It has 4 Vertices (corner points) and at each vertex 3 edges meet
- It is one of the Platonic Solids

The tetrahedron also has a beautiful and unique property ... all four vertices are the same distance from each other!

And it is the only Platonic Solid with no parallel faces.

When we say "tetrahedron" we often mean "regular tetrahedron" (in other words all faces are the same size and shape)

Surface Area = $\sqrt{3} \times (\text{Edge Length})^2$

Volume = $\dfrac{\sqrt{2}}{12} \times (\text{Edge Length})^3$

EXAMPLES

1. The length of one edge of a regular tetrahedron is 9 units. What is its surface area?

Sol. Surface area

$= \sqrt{3} \times (\text{Edge length})^2$

$= \sqrt{3} \times (9)^2 = 81 \times 1.732$

= **140.30**

Alternate:-

Calculate the area of a side, which is an equilateral triangle.

The base is 9, the height is

$= \dfrac{\sqrt{3}}{2} \times 9 = \dfrac{9\sqrt{3}}{2}$

Area of triangle $= \dfrac{1}{2} \times b \times h$

$= \dfrac{1}{2} \times 9 \times 9 \times \dfrac{\sqrt{3}}{2}$

$= \dfrac{81\sqrt{3}}{4}$

Area of 4 triangles

$= 4 \times \dfrac{81 \times \sqrt{3}}{4} = 81 \times \sqrt{3}$

= **140.30**

2. The length of one edge of a regular tetrahedron is 9 units. What is its volume?

Sol. Volume $= \dfrac{\sqrt{2}}{12} \times (\text{Edge length})^3$

$= \dfrac{\sqrt{2}}{12} \times (9)^3 = \dfrac{\sqrt{2}}{12} \times 729$

= **85.91 units³**

3. The total length of the edges of a tetrahedron is 24 cm. What is its surface area ?

Sol. **Note:-** In tetrahedron has 6 edges.

One edge length = 24 ÷ 6

= 4 cm

Surface area

$= \sqrt{3} \times (\text{Edge length})^2$

$= \sqrt{3} \times (4)^2 = 16 \times 1.732$

= **27.71 cm²**

4. The total length of the edges of a tetrahedron is 36cm. What is its volume?

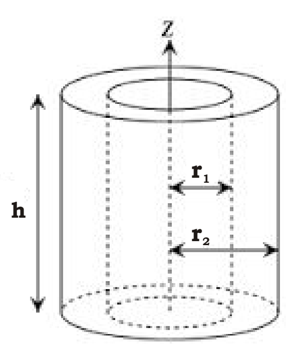
Sol. A tetrahedron has 6 edges

So one Edge Length = 36cm ÷ 6 = 6cm

Volume $= \dfrac{\sqrt{2}}{12} \times (\text{Edge length})^3$

$= \dfrac{\sqrt{2}}{12} \times (6)^3 = \dfrac{\sqrt{2}}{12} \times 216 = 25.46$ cm³

Hollow Cylinder



(i) Volume (V) = $\pi(r_2^2 - r_1^2)h$

(ii) Curved surface area = $2\pi(r_1 + r_2)h$

(iii) Total surface area = inner curved surface area + Outer curved surface area + area of Base and Top = $2\pi(r_1 + r_2)[h + (r_2 - r_1)]$

(iv) Thickness (t) = $(r_2 - r_1)$

EXAMPLES

1. A hollow garden roller 63cm wide with a girth of 440cm is made of iron 4cm thick. The volume of iron is:

Sol. Circumference = 440 cm

$\Rightarrow 2\pi r = 440$

$r = \left(\dfrac{440}{2 \times 22} \times 7\right) = 70$ cm

Inner radius = 70 – 4 = 66 cm

Volume of Iron

$= \pi\left[(70)^2 - (66)^2\right] \times 63$

$= \dfrac{22}{7} \times 136 \times 4 \times 63 = 58752$ cm³

2. A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the external diameter be 50 cm and the length of tube be 140 cm, find the volume of iron in it.

Sol. External diameter = 50 cm

$\therefore$ External radius $= \dfrac{50}{2} = 25$ cm

Volume $= \dfrac{22}{7} \times 140\,[(25)^2 - (23)^2]$

$= \dfrac{22}{7} \times 140 \times 48 \times 2 = 42240$ cm³

EXERCISE

1. If diagonal of a cube is $\sqrt{12}$ cm, then its volume in cm^3 is :
(a) 8 (b) 12 (c) 24 (d) $\sqrt[3]{2}$

2. How many cubes, each of edge 3 cm, can be cut from a cube of edge 15 cm ?
(a) 25 (b) 27 (c) 125 (d) 144

3. What is the volume of a cube (in cubic cm) whose diagonal measures $4\sqrt{3}$ cm?
(a) 16 (b) 27 (c) 64 (d) 8

4. A cuboidal water tank has 216 litres of water. Its depth is $\frac{1}{3}$ of its length and breadth is $\frac{1}{2}$ of $\frac{1}{3}$ of the difference of length and breadth. The length of the tank is
(a) 72 dm (b) 18 dm
(d) 6 dm (d) 2 dm

5. The volume of cuboid is twice the volume of a cube. If the dimensions of the cuboid are 9 cm, 8 cm and 6 cm, the total surface area of the cube is:
(a) 72 cm^2 (b) 216 cm^2
(c) 432 cm^2 (d) 108 cm^2

6. The length, breadth and height of a room is 5m, 4 m and 3m respectively. Find the length of the largest bamboo that can be kept inside the room.
(a) 5 m (b) 60 m
(c) 7 m (d) $5\sqrt{2}$ m

7. A wooden box measures 20 cm by 12 cm by 10 cm . Thickness of wood is 1 cm. Volume of wood to make the box (in cubic cm) is
(a) 960 (b) 519
(c) 2400 (d) 1120

8. A cuboidal block of 6 cm × 9 cm × 12 cm is cut up into exact number of equal cube. The least posible number of cubes will be
(a) 6 (b) 9 (c) 24 (d) 30

9. A cistern of capacity 8000 litres measures externally 3.3 m by 2.6 m by 1.1 m and its walls are 5 cm thick. The thickness of the bottom is:
(a) 1 m (b) 13.5 m
(c) 1 dm (d) 90 cm

10. The area of three adjacent faces of a cuboid are x, y, z square units respectively. If the volume of the cuboid be v cube units. then the correct relation between v,x, y, z is
(a) $v^2 = xyz$ (b) $v^3 = xyz$
(c) $v^2 = x^3y^3z^3$ (d) $v^3 = x^2y^2z^2$

11. The largest sphere is carved out of a cube of side 7 cm. The volume of the sphere (in cm^3) will be
(a) 718.66 (b) 543.72
(c) 481.34 (d) 179.67

12. The length (in meters) of the longest rod that can be put in a room of dimensions 10 m× 10 m × 5 m is
(a) $15\sqrt{3}$ (b) 15
(c) $10\sqrt{2}$ (d) $5\sqrt{3}$

13. A rectangular sheet of metal is 40 cm by 15 cm . equal squares of side 4cm are cut off at the corners and the remainder is folded up to form an open rectangular box The volume of the box is
(a) 896 cm^3 (b) 986 cm^3
(c) 600 cm^3 (d) 916 cm^3

14. The areas of three consecutive faces of a cuboid are 12 cm^2, then the volume (in cm^3) of the cuboid is
(a) 3600 (b) 100
(c) 80 (d) $24\sqrt{3}$

15. The length of the longest rod that can be placed in a room which is 12 m long, 9 m broad and 8 m high is
(a) 27m (b) 19m
(c) 17m (d) 13m

16. The floor of a room is of size 4 m × 3 m and its height is 3 m. The walls and celling of the room require painting. The area to be painted is
(a) 66 m^2 (b) 54 m^2
(c) 42 m^2 (d) 33 m^2

17. If the sum of three dimensions and the total surface area of a rectangular box are 12 cm and 94 cm^2 respectively, then the maximum length of a stick that can be placed inside the box is
(a) $5\sqrt{2}$ cm (b) 5 cm
(c) 6 cm (d) $2\sqrt{5}$ cm

18. The area of the four walls of a room is 660 m^2 and its length is twice of its breadth. If the height of the room is 11 m, then area of its floor (in m^2) is
(a) 120 (b) 150
(c) 200 (d) 330

19. If the length of the diagonal of a cube is $8\sqrt{3}$ cm, then its total surface area is
(a) 192 cm^2 (b) 512 cm^2
(c) 768 cm^2 (d) 384 cm^2

20. The maximum length of a pencil that can be kept in a rectangular box of dimensions 8cm × 6cm × 2cm is
(a) $2\sqrt{13}$ cm (b) $2\sqrt{14}$ cm
(c) $2\sqrt{26}$ cm (d) $10\sqrt{2}$ cm

21. The volume of a cubical box is 3.375 cubic metres. The length of edge of the box is
(a) 75 m (b) 1.5 m
(c) 1.125 m (d) 2.5 m

22. Two cubes of sides 6 cm each are kept side by side to form a rectangular parallelopiped. The area (in sq. cm) of the whole surface of the rectangular parallelopiped is
(a) 432 (b) 360
(c) 396 (d) 340

23. 2 cm of rain has fallen on a square km of land. Assuming that 50% of the raindrops could have been collected and contained in a pool having a 100 m× 10 m base, by what level would the water level in the pool have increased ?
(a) 1 km (b) 10 m
(c) 10 cm (d) 1 m

24. A parallelopiped whose sides are in ratio 2 : 4 :8 have the same volume as a cube. The ratio of their surface area is:
(a) 7 : 5 (b) 4 : 3
(c) 8 : 5 (d) 7 :6

25. If two adjacent sides of a rectangular parallelopiped are 1 cm and 2 cm and the total surface area of the parallelopiped is 22 square cm,

then the diagonal of the parallelopiped is

(a) $\sqrt{10}$ cm (b) $2\sqrt{3}$ cm
(c) $\sqrt{14}$ cm (d) 4 cm

26. If the sum of the length, Breadth and height of a rectangular parallelopiped is 24 cm and the length of its diagonal is 15 cm, then its total surface area is
(a) 256 cm^2 (b) 265 cm^2
(c) 315 cm^2 (d) 351 cm^2

27. If the total surface area of a cube is 96 cm^2, its volume is
(a) 56 cm^3 (b) 16 cm^3
(c) 64 cm^3 (d) 36 cm^3

28. The length of the largest possible rod that can be placed in a cubical room is $35\sqrt{3}$ m. The surface area of the largest possible sphere that fit within the cubical room (asuming $\pi = \frac{22}{7}$) (in sq. m) is
(a) 3,500 (b) 3,850
(c) 2,450 (d) 4,250

29. The volume of air in a room is 204 m^3. The height of the room is 6 m. What is the floor area of the room?
(a) 32 m^2 (b) 46 m^2
(c) 44 m^2 (d) 34 m^2

30. A square of side 3 cm is cut off from each corner of a rectangular sheet of length 24 cm and breadth 18 cm and the remaining sheet is folded to form an open rectangular box. The surface area of the box is
(a) 468 cm^2 (b) 396 cm^2
(c) 615 cm^2 (d) 423 cm^2

31. Three solid iron cubes of edges 4 cm, 5 cm and 6 cm are melted together to make a new cube. 62 cm^3 of the melted material is lost due to improper handing. The area (in cm^2) of the whole surface of the newly formed cube is
(a) 294 (b) 343 (c) 125 (d) 216

32. Area of the floor of a cubical room is 48 sq. m. The length of the longest rod that can be kept in that room is
(a) 9 metre (b) 12 metre
(c) 18 metre (d) 6 metre

33. Three cubes of sides 6 cm, 8 cm and 1 cm are melted to form a new cube. The surface area of the new cube is
(a) 486 cm^2 (b) 496 cm^2
(c) 586 cm^2 (d) 658 cm^2

34. Some bricks are arranged in an area measuring 20 cu.m. If the length, breadth and height of each brick is 25 cm, 12.5 cm and 8 cm respectively, then the number of bricks are (suppose there is no gap in between two bricks)
(a) 6,000 (b) 8,000
(c) 4,000 (d) 10,000

35. The whole surface of a cube is 150 sq. cm. Then the volume of the cube is
(a) 125 cm^3 (b) 216 cm^3
(c) 343 cm^3 (d) 512 cm^3

36. The ratio of the length and breadth of a rectangular parallelopiped is 5 : 3 and its height is 6 cm. If the total surface area of the parallelopiped be 558 sq. cm, then its length in dm is
(a) 9 (b) 1.5 (c) 10 (d) 15

37. If the sum of the dimensions of a rectangular parallelopiped is 24 cm and the length of the diagonal is 15 cm, then the total surface area of it is
(a) 420 cm^2 (b) 275 cm^2
(c) 351 cm^2 (d) 378 cm^2

38. The length, breadth and height of a cuboid are in the ratio 3 : 4 : 6 and its volume is 576 cm^3. The whole surface area of the cuboid is
(a) 216 cm^2 (b) 324 cm^2
(c) 432 cm^2 (d) 460 cm^2

39. If the number of vertices, edges and faces of a rectangular parallelopiped are denoted by v, e and f respectively, the value of (v – e + f) is
(a) 4 (b) 1 (c) 0 (d) 2

40. A low land, 48 m long and 31.5m broad is raised to 6.5 dm. For this, earth is removed from a cuboidal hole, 27 m long and 18.2 m broad, dug by the side of the land. The depth of the hole will be.
(a) 3 m (b) 2 m
(c) 2.2 m (d) 2.5 m

41. A cuboidal shaped water tank, 2.1 m long and 1.5 m broad is half filled with water. If 630 litres more water is poured into tank, the water level will rise
(a) 2 cm (b) 0.15 cm
(c) 0.20 m (d) 0.18 cm

42. A solid cuboid of dimensions 8 cm × 4 cm × 2 cm is melted and cast into identical cubes of edge 2 cm. Number of such identical cubes is.
(a) 16 (b) 4 (c) 10 (d) 8

43. A metallic hemisphere is melted and recast in the shape of cone with the same base radius (R) as that of the hemisphere. If H is the height of the cone, then:
(a) H = 2 R (b) H = $\frac{2}{3}$ R
(c) H = $\sqrt{3R}$ (d) B = 3R

44. If the radius of a sphere is increased by 2 cm, its surface area increased by 352 cm^2. The radius of sphere before change is :
(a) 3 cm (b) 4 cm
(c) 5 cm (d) 6 cm

45. The height of a conical tank is 60 cm and the diameter of its base is 64cm. The cost of painting it from outside at the rate of Rs. 35 per sq. m. is :
(a) Rs. 52.00 approx,
(b) Rs. 39.20 approx,
(c) Rs. 35.20 approx,
(d) Rs. 23.94 approx,

46. A solid metallic cone of height 10 cm, radius of base 20 cm is melted to make spherical balls each of 4 cm diameter. How many such balls can be made?
(a) 25 (b) 75 (c) 50 (d) 125

47. A cylindrical tank of diameter 35 cm is full of water. If 11 litres of water is drawn off, the water level in the tank will drop by :
(a) $10\frac{1}{2}$ cm (b) $12\frac{6}{7}$ cm
(c) 14 cm (d) $11\frac{3}{7}$ cm

48. The volume of a right circular cylinder whose height is 40cm, and circumference of its base is 66 cm is:
(a) 55440 cm^3 (b) 3465 cm^3
(c) 7720 cm^3 (d) 13860 cm^3

49. The circumference of the base of a circular cylinder is 6π cm. The height of the cylinder is equal to the diameter of the base. How many litres of water can it hold ?
(a) 54π cc (b) 36π cc
(c) 0.054π cc (d) 0.54π cc

50. The volume of a right circular cylinder is equal to the volume of that right circular cone whose height is 108 cm and diameter of base is 30 cm. If the height of the cylinder is 9 cm, the diameter of its base is
(a) 30 cm (b) 60 cm
(c) 50 cm (d) 40 cm

51. Three solid metallic spheres of diameter 6 cm, 8 cm and 10 cm are melted and recast into a new soild sphere. The diameter of the new sphere is :
(a) 4 cm (b) 6 cm
(c) 8 cm (d) 12 cm

52. Find the volume of a prism which is based on a regular Hexagon & of height 10cm. If total S.A. is $156\sqrt{3}$ cm².
(a) $60\sqrt{3}$ (b) $180\sqrt{3}$
(c) $120\sqrt{3}$ (d) $240\sqrt{3}$

53. Three solid spheres of a metal whose radii are 1 cm, 6 cm and 8 cm are melted to form an other solid sphere. The radius of this new sphere is
(a) 10.5 cm (d) 9.5 cm
(c) 10 cm (d) 9 cm

54. The slant height of a conical mountain is 2.5 km and the area of its base is 1.54 km². then find the height of conical mountain.
(a) 2.2 km (b) 2.4 km
(c) 3 km (d) 3.11 km

55. The base of a conical tent is 19.2 metres in diameter and the height is 2.8 metres . The area of the canvas required to put up such a tent (in square meters) (taking $\pi = \frac{22}{7}$) is nearly.
(a) 3017.1 (b) 3170
(c) 301.7 (d) 30.17

56. A hollow cylinderical tube 20 cm long. is made of iron and its external and internal diameters are 8 cm and 6 cm respectively. The volume of iron used in making the tube is ($\pi = \frac{22}{7}$)
(a) 1760 cu.cm (b) 880 cu.cm.
(c) 440 cu.cm (d) 220 cu.cm

57. A sphere of radius 2 cm is put into water contained in a cylinder of base- radius 4 cm. If the sphere is completely immersed in the water, the water level in the cylinder rise by
(a) $\frac{1}{3}$ cm (b) $\frac{1}{2}$ cm
(c) $\frac{2}{3}$ cm (d) 2 cm

58. A solid metallic spherical ball of diameter 6 cm is melted and recasted into a cone with diameter of the base as 12 cm. The height of the cone is
(a) 6 cm (b) 2 cm
(c) 4 cm (d) 3 cm

59. The volume of a right circular cone is 1232 cm³ and its vertical height is 24 cm . Its curved surface area is
(a) 154 cm² (b) 550 cm²
(c) 604 cm² (d) 704 cm²

60. The volume of a sphere is $\frac{88}{21}\times(14)^3$ cm³ The curved surface area of the sphere is (Take $\pi = \frac{22}{7}$)
(a) 2424 cm² (b) 2446 cm²
(c) 2484 cm² (d) 2464 cm²

61. The surface area of a sphere is 64π cm² Its diameter is equal to
(a) 16 cm (b) 8 cm
(c) 4 cm (d) 2 cm

62. The diameter of the base of a cylinderical drum is 35 dm. and the height is 24 dm. It is full of kerosene. How many tins each of size 25 cm × 22 cm × 35 cm can be filled with kerosene from the drum ?
(use $\pi = \frac{22}{7}$)
(a) 1200 (b) 1020
(c) 600 (d) 120

63. A hollow iron pipe is 21 cm long and its exterior diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weights 8 g/cm³, then the weight of the pipe is (Take $\pi = \frac{22}{7}$):
(a) 3.696 kg (b) 3.6 kg
(c) 36 kg (d) 36.9 kg

64. The volume of a right circular cylinder, 14 cm in height, is equal to that of a cube whose edge is 11 cm Take $\pi = \frac{22}{7}$ the radius of the base of the cylinder is
(a) 5.2 cm (b) 5.5 cm
(c) 11.0 cm (d) 22.0 cm

65. If the volume of a right circular cylinder is $9\pi h$ m³, where h is its height (in metres) then the diameter of the base of the cylinder is equal to
(a) 3 m (b) 6 m (c) 9 m (d) 12 m

66. Each of the measure of the radius of base of a cone and that of a sphere is 8 cm. Also, the volume of these two solids are equal. the slant height of the cone is
(a) $8\sqrt{17}$ cm (b) $4\sqrt{17}$ cm
(c) $34\sqrt{2}$ cm (d) 34 cm

67. A well 20 m in diameter is dug 14 m deep and the earth taken out is spread all around it to a width of 5 m to form an embankment. The height of the embankment is:
(a) 10 m (b) 11 m
(c) 11.2 m (d) 11.5 m

68. The diameter of the iron ball used for the shot-put game is 14 cm. It is melted and then a solid cylinder of height $2\frac{1}{3}$ cm is made. What will be the diameter of the base of the cylinder?
(a) 14 cm (b) 28 cm
(c) $\frac{14}{3}$ cm (d) $\frac{28}{3}$ cm

69. The sum of radii of two spheres is 10 cm and the sum of their volume is 880 cm³. What will be the product of their radii?
(a) 21 (b) $26\frac{1}{3}$
(c) $33\frac{1}{3}$ (d) 70

70. A rectangular paper sheet of dimensions 22 cm × 12 cm is folded in the form of a cylinder along its length. What will be the volume of this cylinder?
(Take $\pi = \frac{22}{7}$)
(a) 460 cm^3 (b) 462 cm^3
(c) 624 cm^3 (d) 400 cm^3

71. A copper rod of 1 cm diameter and 8 cm length is drawn into a wire of uniform diameter and 18 m length. The radius (in cm) of the wire is
(a) $\frac{1}{15}$ (b) $\frac{1}{30}$ (c) $\frac{2}{15}$ (d) 15

72. 12 spheres of the same size are made by melting a solid cylinder of 16 cm diameter and 2 cm height. The diameter of each sphere is :
(a) 2 cm (b) 4 cm
(c) 3 cm (d) $\sqrt{3}$ cm

73. When the circumference of a toy ballon is increased from 20 cm to 25 cm its radius (in cm) is increased by :
(a) 5 (b) $\frac{5}{\pi}$ (c) $\frac{5}{2\pi}$ (d) $\frac{\pi}{5}$

74. If the volume and surface area of a sphere are numerically the same, then its radius is
(a) 1 unit (b) 2 units
(c) 3 units (d) 4 units

75. In a right circular cone, the radius of its base is 7 cm and its height 24 cm. A cross- section is made through the midpoint of the height parallel to the base. The volume of the upper portion is
(a) 169 cm^3 (b) 154 cm^3
(c) 1078 cm^3 (d) 800 cm^3

76. Some solid metallic right circular cones. each with radius of the base 3 cm and height 4 cm, are melted to form a solid sphere of radius 6 cm. The number of right circular cones is
(a) 12 (b) 24 (c) 48 (d) 6

77. A right circular cylinder of height 16 cm is covered by a rectangular tin foil of size 16 cm × 22 cm, The volume of the cylinder is
(a) 352 cm^3 (b) 308 cm^3
(c) 616 cm^3 (d) 176 cm^3

78. If the area of the base of a cone is 770 cm^2 and the area of its curved surface is 814 cm^2. then find its volume.
(a) $213\sqrt{5}$ cm^3 (b) $392\sqrt{5}$ cm^3
(c) $550\sqrt{5}$ cm^3 (d) $616\sqrt{5}$ cm^3

79. The size of a rectangular piece of paper is 100 cm × 44 cm. A cylinder is formed by rolling the paper along its breadth. The volume of the cylinder is
(Use $\pi = \frac{22}{7}$)
(a) 4400 cm^3 (b) 15400 cm^3
(c) 35000 cm^3 (d) 144 cm^3

80. The radius of the base and height of a metallic soild cylinder are r cm and 6 cm respectively. It is melted and recast into a solid cone of the same radius of base. The height of the cone is:
(a) 54 cm (b) 27 cm
(c) 18 cm (d) 9 cm

81. The total surface area of a metallic hemisphere is 1848 cm^2. The hemisphere is melted to form a solid right circular cone. If the radius of the base of the cone is the same as the radius of the hemisphere its height is
(a) 42 cm (b) 26 cm
(c) 28 cm (d) 30 cm

82. A right circular cylinder is formed by rolling a rectangular paper 12 cm long and 3 cm wide along its length. The radius of the base of the cylinder will be
(a) $\frac{3}{2\pi}$ cm (b) $\frac{6}{\pi}$ cm
(c) $\frac{9}{2\pi}$ cm (d) 2π cm

83. What part of a ditch, 48 metres long. 16.5 metres broad and 4 metres deep can be filled by the earth got by digging a cylinderical tunnel of diameter 4 metres and length 56 metres? (Use $\pi = \frac{22}{7}$)
(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{7}{9}$ (d) $\frac{8}{9}$

84. The volume of the metal of cylinderical pipe is 748 cm^3. The length of the pipe is 14 cm and its external radius is 9 cm. its thickness is
(Take $\pi = \frac{22}{7}$)
(a) 1 cm (b) 5.2 cm
(c) 2.3 cm (d) 3.7 cm

85. Two iron sphere each of diameter 6 cm are immersed in the water contained in a cylinerical vessel of radius 6 cm. The level of the water in the vessel will be raised by
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 6 cm

86. The height of the cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume is $\frac{1}{27}$ of the volume of the cone. at what height above the base, is the section made ?
(a) 6 cm (b) 8 cm
(c) 10 cm (d) 20 cm

87. The total surface area of a solid hemisphere is 108π cm^2. The volume of the hemisphere is
(a) 72π cm^3 (b) 144π cm^3
(c) $108\sqrt{6}$ cm^3 (d) $54\sqrt{6}$ cm^3

88. A solid metallic sphere of radius 3 decimetres is melted to form a circular sheet of 1 milimetre thickness. The diameter of the sheet so formed is
(a) 26 metres (b) 24 metres
(c) 12 metres (d) 6 metres

89. Water flows through a cylinderical pipe. whose radius is 7 cm, at 5 metre per second. The time, it takes to fill an empty water tank with height 1.54 metres and area of the base (3 × 5) square metres, is
(take $\pi = \frac{22}{7}$)
(a) 6 minutes (b) 5 minutes
(c) 10 minutes (d) 9 minutes

90. If S denotes the area of the curved surface of a right circular cone of height h and semivertical angle α then S equals
(a) $\pi h^2 \tan^2\alpha$
(b) $\frac{1}{3}\pi h^2 \tan^2\alpha$
(c) $\pi h^2 \sec\alpha \tan\alpha$
(d) $\frac{1}{3}\pi h^2 \sec\alpha \tan\alpha$

91. The height and the radius of the base of a right circular cone are 12 cm and 6 cm respectively. The radius of the circular

cross-section of the cone cut by a plane parallel to its base at a distance of 3 cm from the base is
(a) 4 cm (b) 5.5 cm
(c) 4.5 cm (d) 3.5 cm

92. If S_1 and S_2 be the surface areas of a sphere and the curved surface area of the circumscribed cylinder respectively, then S_1 is equal to
(a) $\frac{3}{4}S_2$ (b) $\frac{1}{2}S_2$
(c) $\frac{2}{3}S_2$ (d) S_2

93. The volume of a right circular cylinder and that of a sphere are equal and their radii are also equal. If the height of the cylinder be h and the diameter of the sphere d. then which of the following relation is correct?
(a) h =d (b) 2h =d
(c) 2h = 3d (d) 3h =2d

94. Water is being pumped out through a circular pipe whose internal diameter is 7cm. If the flow of water is 12cm per second, how many litres of water is being pumped out in one hour?
(a) 1663.2 (b) 1500
(c) 1747.6 (d) 2000

95. The lateral surface area of a cylinder is 1056 cm² and its height is 16cm. Find its volume.
(a) 4545 cm³ (b) 4455 cm³
(c) 5445 cm³ (d) 5544 cm³

96. The radius of the base and height of a right circular cone are in the ratio 5 : 12. If the volume of the cone is $314\frac{2}{7}$ cm³, the slant height (in cm) of the cone will be
(a) 12 (b) 13 (c) 15 (d) 17

97. A solid metallic cone is melted and recast into a solid cylinder of the same base as that of the cone. If the height of the cylinder is 7 cm, the height of the cone was
(a) 20 cm (b) 21 cm
(c) 28 cm (d) 24 cm

98. A copper wire of length 36m and diameter 2mm is melted to form a sphere. The radius of the sphere (in cm) is
(a) 2.5 (b) 3 (c) 3.5 (d) 4

99. The diameter of the base of a right circular cone is 4 cm and its height $2\sqrt{3}$ cm. The slant height of the cone is
(a) 5 cm (b) 4 cm
(c) $2\sqrt{3}$ (d) 3 cm

100. The rain water from a roof 22 m × 20 m drains into a cylinderical vessel having a diameter of 2 m and height 3.5 m, If the vessel is just full, then the rainfall (in cm) is :
(a) 2 (b) 2.5 (c) 3 (d) 4.5

101. From a solid cylinder of height 10 cm and radius of the base 6 cm, a cone of same height and same base is removed. The volume of the remaining solid is :
(a) 240π cu. cm
(b) 5280 cu. cm
(c) 620 π cu. cm
(d) 360π cu. cm

102. Two solid right cones of equal height and of radii r_1 and r_2 are melted and made to form a solid sphere of radius R. Then the height of the cone is
(a) $\frac{4R^2}{r_1^2 r_2^2}$ (b) $\frac{R^3}{r_1^2 r_2^2}$
(c) $\frac{4R^3}{r_1^2+r_2^2}$ (d) $\frac{R^2}{r_1^2 r_2^2}$

103. The ratio of height and the diameter of a right circular cone is 3 : 2 and its volume is 1078 cc, then (taking $\pi = \frac{22}{7}$) its height is :
(a) 7 cm (b) 14 cm
(c) 21 cm (d) 28 cm

104. From a right circular cylinder of radius 10 cm and height 21 cm. a right circular cone of same base radius is removed. If the volume of the remaining portion is 4400 cm³, then the height of the removed cone (take $\pi = \frac{22}{7}$) is :
(a) 15 cm (b) 18 cm
(c) 21 cm (d) 24 cm

105. A child reshapes a cone made up of clay of height 24 cm and radius 6 cm into a sphere. The radius (in cm) of the sphere is
(a) 6 (b) 12 (c) 24 (d) 48

106. A solid cylinder has total surface area of 462 sq. cm. Its curved surface area is one third of the total surface area. Then the radius of the cylinder is
(a) 7 cm (b) 3.5 cm
(c) 9 cm (d) 11 cm

107. The diameter of a cylinder is 7 cm and its height is 16 cm. Using the value of $\pi = \frac{22}{7}$, the lateral surface area of the cylinder is
(a) 352 cm² (b) 350 cm²
(c) 355 cm² (d) 348 cm²

108. The height of a solid right circular cylinder is 6 metres and three times the sum of the area of its two end faces is twice the area of its curved surface, The radius of its base (in meter) is
(a) 4 (b) 2 (c) 8 (d) 10

109. A semi-circular sheet of metal of diameter 28 cm is bent into an open conical cup. The depth of the cup is approximately
(a) 11 cm (b) 12 cm
(c) 13 cm (d) 14 cm

110. A right angled sector of radius r cm is rolled up into a cone in such a way that the two binding radii are joined together . then the curved surface area of the cone is
(a) πr^2 cm² (b) $\frac{\pi r^2}{4}$ cm²
(c) $\frac{\pi r^2}{2}$ cm² (d) $2\pi r^2$ cm²

111. The radius of the base of a conical tent is 16 metre. If $427\frac{3}{7}$ sq. metre canvas is required to construct the tent, then the slant height of the tent is : (take $\pi = \frac{22}{7}$)
(a) 17 metre (b) 15 metre
(c) 19 metre (d) 8.5 metre

112. A circus tent is cylinderical up to a height of 3 m and conical above it. If its diameter is 105m and the slant height of the conical part is 63 m, then the total area of the canvas required to make the tent is (take $\pi = \frac{22}{7}$)
(a) 11385 m² (b) 10395 m²
(c) 9900 m² (d) 990 m²

113. A toy is in the form of a cone mounted on a hemisphere. The radius of the hemisphere and that of the cone is 3 cm and height of the cone is 4 cm. The total surface area of the toy (takeing $\pi = \frac{22}{7}$) is
(a) 75.43 sq. cm,
(b) 103.71 sq. cm,
(c) 85.35 sq. cm,
(d) 120.71 sq. cm,

114. Marbles of diameter 1.4 cm are dropped into a cylinderical beaker containing some water and fully submerged. The diameter of the beaker is 7 cm. Find how many marbles have been dropped in it if the water rises by 5.6 cm ?
(a) 50 (b) 150
(c) 250 (d) 350

115. A cylindrical rod of iron whose height is eight times its radius is melted and cast into spherical balls each of half the radius of the cylinder. The number of such spherical balls is
(a) 12 (b) 16 (c) 24 (d) 48

116. A cylinder has 'r' as the radius of the base and 'h' as the height. The radius of base of another cylinder, having double the volume but the same height as that of the first cylinder must be equal to
(a) $\frac{r}{2}$ (b) $2r$ (c) $r\sqrt{2}$ (d) $\sqrt{2r}$

117. From a solid cylinder whose height is 12 cm and diameter 10 cm. a conical cavity of same height and same diameter of the base is hollowed out. The volume of the remanining solid is approximately ($\pi = \frac{22}{7}$)
(a) 942.86 cm^3 (b) 314.29 cm^3
(c) 628.57 cm^3 (d) 450.76 cm^3

118. The radius of a cylinder is 10 cm and height is 4 cm. The number of centimetres that may be added either to the radius or to the height to get the same increase in the volume of the cylinder is
(a) 5 cm (b) 4 cm
(c) 25 cm (d) 16 cm

119. The radius of the base of a right circular cone is doubled keeping its height fixed. The volume of the cone will be :
(a) Three times of the previous volume
(b) four times of the previous volume
(c) $\sqrt{2}$ times of the previous volume
(d) double of the pervious volume

120. The base of a right circular cone has the radius 'a' which is same as that of a sphere. Both the sphere and the cone have the same volume. Height of the cone is
(a) $3a$ (b) $4a$
(c) $\frac{7}{4}a$ (d) $\frac{7}{3}a$

121. The circumference of the base of a 16 cm high solid cone is 33 cm What is the volume of the cone in cm^3 ?
(a) 1028 (b) 616
(c) 462 (d) 828

122. A solid sphere of 6 cm diameter is melted and recast into 8 solid spheres of equal volume. The radius (in cm) of each small sphere is
(a) 1.5 (b) 3 (c) 2 (d) 2.5

123. In a cylinderical vessel of diameter 24 cm filled up with sufficient quantity of water, a solid spherical ball of radius 6 cm is completely immersed. Then the increase in height of water level is :
(a) 1.5 cm (b) 2 cm
(c) 3 cm (d) 4.2 cm

124. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm find the the volume of wooden toy (nearly).
(a) 104 cm^3 (b) 162 cm^3
(c) 421 cm^3 (d) 266 cm^3

125. If a solid cone of volume 27π cm^3 is kept inside a hollow cylinder whose radius and height are that of the cone, then the volume of water needed to fill the empty space is
(a) 3π cm^3 (b) 18π cm^3
(c) 54π cm^3 (d) 81π cm^3

126. A cylindrical can whose base is horizontal and is of internal radius 3.5 cm contains sufficient water so that when a solid sphere is placed inside, water just covers the sphere. The sphere fits in the can exactly. The depth of water in the can before the sphere was put, is
(a) $\frac{35}{3}$ cm (b) $\frac{17}{3}$ cm
(c) $\frac{7}{3}$ cm (d) $\frac{14}{3}$ cm

127. The radius and height of a cylinder are in the ratio 5 : 7 and its volume is 550 cm^3. Calculate it curved surface area in sq. cm.
(a) 110 (b) 444
(c) 220 (d) 616

128. The area of the curved surface and the area of the base of a right circular cylinder are a square cm and b square cm respectively. The height of the cylinder is
(a) $\frac{2a}{\sqrt{\pi b}}$ cm (b) $\frac{a\sqrt{b}}{2\sqrt{\pi}}$ cm
(c) $\frac{a}{2\sqrt{\pi b}}$ cm (d) $\frac{a\sqrt{\pi}}{2\sqrt{b}}$ cm

129. The volume of a solid hemisphere is 19404 cm^3. Its total surface area is
(a) 4158 cm^2 (b) 2858 cm^2
(c) 1738 cm^2 (d) 2038 cm^2

130. A solid hemisphere is of radius 11 cm. the curved surface area in sq. cm is
(a) 1140.85 (b) 1386.00
(c) 760.57 (d) 860.57

131. The base of a cone and a cylinder have the same radius 6 cm. They have also the same height 8 cm. The ratio of the curved surface area of the cylinder to that of the cone is
(a) 8 : 5 (b) 8 : 3
(c) 4 : 3 (d) 5 : 3

132. A right cylindrical vessel is full with water. How many right

cones having the same diameter and height as that of the right cylinder will be needed to store that water ?

(take $\pi = \frac{22}{7}$)

(a) 4 (b) 2 (c) 3 (d) 5

133. A spherical lead ball of radius 10 cm is melted and small lead balls of radius 5mm are made The total number of possible small lead balls is (Take $\pi = \frac{22}{7}$)

(a) 8000 (b) 400
(c) 800 (d) 125

134. The number of spherical bullets that can be made out of solid cube of lead whose edge measures 44 cm each bullet being of 4 cm diameter, is (Take $\pi = \frac{22}{7}$)

(a) 2541 (b) 2451
(c) 2514 (d) 2415

135. The radius of a metallic cylinder is 3 cm and its height is 5 cm. It is melted and moulded into small cones, each of height 1 cm and base radius 1 mm. The number of such cones formed is

(a) 450 (b) 1350
(c) 8500 (d) 13500

136. A sector is formed by opening out a cone of base radius 8 cm and height 6 cm. Then the radius of the sector is (in cm)

(a) 4 (b) 8 (c) 10 (d) 6

137. A solid cone of height 9 cm with diameter of its base 18 cm is cut out from a wooden solid sphere of radius 9 cm. The percentage of wood wasted is :

(a) 25% (b) 30%
(c) 50% (d) 75%

138. The perimeter of the base of a right circular cylinder is 'a' unit . If the volume of the cylinder is V cubic unit, then the height of the cylinder is

(a) $\frac{4a^2V}{\pi}$ unit (b) $\frac{4\pi a^2}{V}$ unit
(c) $\frac{\pi a^2 V}{4}$ unit (d) $\frac{4\pi V}{a^2}$ unit

139. What is the height of a cylinder that has the same volume and radius as a sphere of diameter 12 cm ?

(a) 7 cm (b) 10 cm
(c) 9 cm (d) 8 cm

140. The perimeter of the base of a right circular cone is 8 cm. If the height of the cone is 21 cm, then is volume is :

(a) 108 π cm³ (b) $\frac{112}{\pi}$ cm³
(c) 112π cm³ (d) $\frac{108}{\pi}$ cm³

141. If the volume of two right circular cones are in the ratio 4 : 1 and their diameter are in the ratio 5 : 4, then the ratio of their height is :

(a) 25 : 16 (b) 25 : 64
(c) 64 : 25 (d) 16 : 25

142. The volume of a conical tent is 1232 cu. m and the area of its base is 154 sq. m. Find the length of the canvas required to build the tent, if the canvas is 2m in width.

(Take $\pi = \frac{22}{7}$)

(a) 270 m (b) 272 m
(c) 276 m (d) 275 m

143. If the ratio of the diameters of two right circular cones of equal height be 3: 4, then the ratio of their volume will be

(a) 3 : 4 (b) 9 : 16
(c) 16 : 9 (d) 27 : 64

144. The surface area of two spheres are in the ratio 4 : 9. Their volumes will be in the ratio

(a) 2 : 3 (b) 4 : 9
(c) 8 : 27 (d) 64 : 729

145. The total surface area of a sphere is 8π square unit. The volume of the sphere is

(a) $\frac{8\sqrt{2}}{3}\pi$ cubic unit

(b) $\frac{8}{3}\pi$ cubic unit

(c) $8\sqrt{3}\pi$ cubic unit

(d) $\frac{8\sqrt{3}}{5}\pi$ cubic unit

146. A semicircular sheet of metal of diameter 28 cm is bent into an open conical cup. The capacity of the cup (Take $\pi = \frac{22}{7}$) is

(a) 624.26 cm³ (b) 622.36 cm³
(c) 622.56 cm³ (d) 623.20 cm³

147. A conical flask is full of water. The flask has base radius r and height h. This water is poured into a cylindrical flask of base radius m, height of cylindrical flask is

(a) $\frac{m}{2h}$ (b) $\frac{h}{2}m^2$
(c) $\frac{2h}{m}$ (d) $\frac{r^2h}{3m^2}$

148. A solid spherical copper ball whose diameter is 14 cm is melted and converted into a wire having diameter equal to 14 cm. The length of the wire is

(a) 27 cm (b) $\frac{16}{3}$ cm
(c) 15 cm (d) $\frac{28}{3}$ cm

149. A sphere of diameter 6 cm is droped in a right circular cylinderical vessel partly filled with water. The diameter of the cylinderical vessel is 12 cm. If the sphere is just completely submerged in water, then the rise of water level in the cylindrical vessel is

(a) 2 cm (b) 1 cm
(c) 3 cm (d) 4 cm

150. A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. The length of the wire in metre is :

(a) 2.43 m (b) 243 m
(c) 2430 m (d) 24.3 m

151. A rectangular block of metal has dimensions 21 cm, 77 cm and 24 cm. The block has been melted into a sphere. The radius of the sphere is (Take $\pi = \frac{22}{7}$)

(a) 21 cm (b) 7 cm
(c) 14 cm (d) 28 cm

152. The radius of cross-section of a solid cylindrical rod of iron is 50 cm. the cylinder is melted down and formed into 6 solid spherical balls of the same

radius as that of the cylinder. The length of the rod (in metres) is
(a) 0.8 (b) 2 (c) 3 (d) 4

153. Two right circular cones of equal height and radii of there respective base 3 cm and 4 cm are melted together and made to a solid sphere of radius 5 cm. The height of a cone is
(a) 10 cm (b) 20 cm
(c) 30 cm (d) 40 cm

154. The radius of the base and the height of a right circular cone are doubled. The volume of the cone will be
(a) 8 times of the previous volume
(b) three times of the previous volume
(c) $3\sqrt{2}$ times of the preivous volume
(d) 6 times of the previous volume

155. If h, c, v are respecitvely the height, curved surface area and volume of a right circular cone then the value of $3\pi vh^3 - c^2h^2 + 9v^2$ is
(a) 2 (b) – 1 (c) 1 (d) 0

156. The total number of spherical bullets, each of diameter 5 decimeter, that can be made by utilizing the maximum of a rectangular block of lead with 11 metre length, 10 metre breadth and 5 metre width is (assume that $\pi = 3$)
(a) equal to 8800
(b) less than 8800
(c) equal to 8400
(d) greater than 9000

157. If a metallic cone of radius 30 cm and height 45 cm is melted and recast into metallic spheres of radius 5 cm, find the number of spheres ,
(a) 81 (b) 41 (c) 80 (d) 40

158. A metallic sphere of radius 10.5 cm is melted and then recast into small cones each of radius 3.5 cm and height 3 cm. The number of cones thus formed is
(a) 140 (b) 132 (c) 112 (d) 126

159. A right circular cone is 3.6 cm high and radius of its base is 1.6 cm. It is melted and recast into a right circular cone with radius of its base as 1.2 cm. Then the height of the cone (in cm) is
(a) 3.6 cm (b) 4.8 cm
(c) 6.4 cm (d) 7.2 cm

160. If surface area and volume of a sphere are S and V respectively, then value of $\frac{S^3}{V^2}$ is
(a) 36π units (b) 9π units
(c) 18π units (d) 27π units

161. Assume that a drop of water is spherical and its diameter is one-tenth of a cm. A conical glass has a height equal to the diameter of its rim. If 32,000 drops of water fill the glass completely. Then the height of the glass (in cm) is
(a) 1 (b) 2 (c) 3 (d) 4

162. A tank 40 m long, 30 m broad and 12 m deep is dug in a field 1000 m long and 30 m wide. By how much will the level of the field rise if the earth dug out of the tank is evenly spread over the field ?
(a) 2 metre (b) 1.2 metre
(c) 0.5 metre (d) 5 metre

163. A sphere is cut into two hemispheres. One of them is used as bowl. It takes 8 bowlfuls of this to fill a conical vessel of height 12 cm and radius 6 cm. The radius of the sphere(in centimetre) will be
(a) 3 (b) 2 (c) 4 (d) 6

164. A ball of lead 4 cm in diameter is covered with gold. If the volume of the gold and lead are equal then the thickness of gold [given $\sqrt[3]{2}$ = 1.259) is approximately
(a) 5.038 cm (b) 5.190 cm
(c) 1.038 cm (d) 0.518 cm

165. A conical cup is filled with icecream. The ice-cream forms a hemispherical shape on its open top. The height of the hemispherical part is 7 cm. the radius of the hemispherical part equals to the height of the cone. Then the volume of the ice-cream is $\left[\pi = \frac{22}{7}\right]$
(a) 1078 cubic cm
(b) 1708 cubic cm
(c) 7108 cubic cm
(d) 7180 cubic cm

166. A hollow sphere of internal and external diameter 6 cm and 10 cm respectively is melted into a right circular cone of diameter 8 cm. The height of the cone is
(a) 22.5 cm (b) 23.5 cm
(c) 24.5 cm (d) 25.5 cm

167. A flask in the shape of a right circular cone of height 24 cm is filed with water. The water is poured in right cirular cylindrical flask whose radius is $\frac{1}{3}$ rd of radius of the base of the circular cone. Then the height of the water in the cylindrical flask is
(a) 32 cm (b) 24 cm
(c) 48 cm (d) 72 cm

168. A solid metallic spherical ball of diameter 6 cm is melted and recast into a cone with diameter of the base as 12 cm. The height of the cone is
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 6 cm

169. A hemispherical bowl of internal radius 15 cm contains a liquid. The liquid is to be filled into cylindrical shaped bottles of diameter 5 cm and height 6 cm. The number of bottles required to empty the bowl is
(a) 30 (b) 40 (c) 50 (d) 60

170. If V_1, V_2 and V_3 be the volumes of a right circular cone. a sphere and a right circular cylinder having the same radius and same height then
(a) $V_1 = \frac{V_2}{4} = \frac{V_3}{3}$ (b) $\frac{V_1}{2} = \frac{V_2}{3} = V_3$
(c) $\frac{V_1}{3} = \frac{V_2}{2} = V_3$ (d) $\frac{V_1}{3} = V_2 = \frac{V_3}{2}$

171. If the surface area of a sphere is 346.5 cm², then its radius [taking $\pi = \frac{22}{7}$]
(a) 7 cm (b) 3.25 cm
(c) 5.25 cm (d) 9 cm

172. Deepali makes a model of a cylindrical kaleidoscope for her science project. She uses a chart paper to make it. If the length of the kaleidoscope is 25 cm and radius 35 cm, the area of the paper she used, in sq. cm, is $\left(\pi = \frac{22}{7}\right)$

(a) 1100 (b) 5500
(c) 500 (d) 450

173. If the volume of a sphere is numerically equal to its surface area then its diameter is;

(a) 4 cm (b) 6 cm
(c) 3 cm (d) 2 cm

174. 5 persons live in a tent. If each person requires 16 m^2 of floor area and 100 m^3 space for air then the height of the cone of smallest size to accomodate these persons would be?

(a) 16 m (b) 18.75 m
(c) 10.25 m (d) 20 m

175. The numerical values of the volume and the area of the lateral surface of a right circular cone are equal. If the height of the cone be h and radius be r, the value of $\frac{1}{h^2}+\frac{1}{r^2}$ is

(a) $\frac{9}{1}$ (b) $\frac{3}{1}$ (c) $\frac{1}{3}$ (d) $\frac{1}{9}$

176. There is wooden sphere of radius $6\sqrt{3}$ cm. The surface area of the largest possible cube cut out from the sphere will be

(a) $464\sqrt{3}$ cm² (b) $646\sqrt{3}$ cm²
(c) 864 cm² (d) 462 cm²

177. If a hemisphere is melted and four spheres of equal volume are made, the radius of each sphere will be equal to

(a) 1/4th of the hemisphere
(b) radius of the hemisphere
(c) 1/2 of the radius of the hemisphere
(d) 1/6th of the radius of the hemisphere

178. The portion of a ditch 48 m long. 16.5 m wide and 4 m deep that can be filled with stones and earth available during excavation of a tunnel, cylindrical in shape, of diameter 4 m and length 56 m is $\left(\text{Take } \pi = \frac{22}{7}\right)$

(a) $\frac{1}{9}$ Part (b) $\frac{1}{2}$ Part
(c) $\frac{1}{4}$ Part (d) $\frac{2}{9}$ Part

179. From a solid right circular cylinder of length 4 cm and diameter 6 cm, a conical cavity of the same height and base is hollowed out. The whole surface area of the remaining solid (in square cm.) is

(a) 48π (b) 63π
(c) 15π (d) 24π

180. A spherical ball of radius 1 cm is dropped into a conical vessel of radius 3 cm and slant height 6 cm. The volume of water (in cm³), that can just immerse the ball, is

(a) $\frac{5\pi}{3}$ (b) 3π (c) $\frac{\pi}{3}$ (d) $\frac{4\pi}{3}$

181. If the height of a cylinder is 4 times its circumference, the volume of the cylinder in terms of its circumference, c is

(a) $\frac{2c^3}{\pi}$ (b) $\frac{c^3}{\pi}$
(c) $4\pi c^3$ (d) $2\pi c^3$

182. The radii of a sphere and a right circular cylinder are 3 cm each. If their volumes are equal, then curved surface area of the cylinder is $\left(\text{Assume } \pi \ \frac{22}{7}\right)$

(a) $75\frac{3}{7}$ cm² (b) $65\frac{3}{7}$ cm²
(c) $74\frac{3}{7}$ cm² (d) $72\frac{3}{7}$ cm²

183. The radius of a hemispherical bowl is 6 cm. The capacity of the bowl is: $\left(\text{Take } \Pi = 22/7\right)$

(a) 452.57 cm³ (b) 452 cm³
(c) 345.53 cm³ (d) 495.51 cm³

184. The total surface area of a right circular cylinder with radius of the base 7 cm and heigtht 20 cm is:

(a) 140 cm² (b) 1000 cm²
(c) 900 cm² (d) 1188 cm²

185. The radius of base and curved surface area of a right cylinder are 'r' units and $4\pi rh$ square units respectively. The height of the cylinder is:

(a) 4h units (b) $\frac{h}{2}$ units
(c) h units (d) 2h units

186. A hemi-spherical bowl has 3.5 cm radius. It is to be painted inside as well as outside. The cost of painting it at the rate of Rs. 5 per 10sq. cm. will be:

(a) Rs. 77 (b) Rs. 175
(c) Rs. 50 (d) Rs. 100

187. The volume of a right circular cone which is obtained from a wooden cube of edge 4.2 dm wasting minimum amount of wood is:

(a) 194.04 cu. dm
(b) 19.404 cu. dm
(c) 1940.4 cu. dm
(d) 1940.4 cu. dm

188. If the radius of a sphere is increased by 2 cm, then its surface area increases by 352cm². The radius of the sphere initially was: $\left(\text{use}\,\pi = \frac{22}{7}\right)$

(a) 3cm (b) 5cm
(c) 4cm (d) 6cm

189. A right triangle with sides 9 cm, 12 cm and 15 cm is rotated about the side of 9 cm to form a cone. The volume of the cone so formed is:

(a) 432 π cm³ (b) 327 π cm³
(c) 334 π cm³ (d) 324 π cm³

190. The volume of the largest right circular cone that can be cut out of a cube of edge 7 cm ? $\left(\text{use}\,\pi = \frac{22}{7}\right)$

(a) 13.6 cm³ (b) 147.68 cm³
(c) 89.9 cm³ (d) 121 cm³

191. By melting two solid metallic spheres of radii 1 cm and 6 cm,

a hollow sphere of thickness 1 cm is made. The external radius of the hollow sphere will be
(a) 8 cm (b) 9 cm
(c) 6 cm (d) 7 cm

192. Water is flowing at the rate of 5 km/h through a pipe of diameter 14 cm into a rectangular tank which is 50 m long 44m wide, The time taken (in hours) for the rise in the level of water in the tank to 7 cm is
(a) 2 (b) $1\frac{1}{2}$ (c) 3 (d) $2\frac{1}{2}$

193. The volume (in m^3) of rain water that can be collected from 1.5 hectares of ground in a rainfall of 5 cm is
(a) 75 (b) 750
(c) 7500 (d) 75000

194. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour, How much water (in-litres) will fall into sea in a minute ?
(a) 4,00,000 m^3 (b) 40,00,000 m^3
(c) 40,000 m^3 (d) 4,000 m^3

195. Water is flowing at the rate of 3 km/hr through a circular pipe of 20 cm internal diameter into a circular cistern of diameter 10 m and depth 2m. In how much time will the cistern be filled ?
(a) 1 hour
(b) 1 hour 40 minutes
(c) 1 hour 20 minutes
(d) 2 hours 40 minutes

196. Water flows at the rate of 10 metres per minute from cylindrcial pipe 5 mm in diameter, How long it will take to fill up a conical vessel whose diameter at the base is 30 cm and depth 24 cm?
(a) 28 mintues 48 seconds
(b) 51 minutes 12 seconds
(c) 51 minutes 24 seconds
(d) 28 mintues 36 seconds

197. The radius of the base of conical tent is 12 m. The tent is 9 m high. Find the cost of canvas required to make the tent, if one square metre of canvas costs Rs.120 (Take π = 3.14)
(a) Rs. 67,830 (b) Rs. 67,800
(c) Rs. 67,820 (d) Rs. 67,824

198. A plate of square base made of brass is of length x cm and thickness 1 mm.The plate weighs 4725 gm. If 1 cubic cm of brass weighs 8.4 gram, then the value of x is:
(a) 76 (b) 72 (c) 74 (d) 75

199. The diameter of a 120 cm long roller is 84 cm. It takes 500 complete revolutions of the roller to level a ground. The cost of levelling the ground at Rs. 1.50 sq. m. is:
(a) Rs. 5750 (b) Rs. 6000
(c) Rs. 3760 (d) Rs. 2376

200. Two right circular cylinders of equal volume have their heights in the ratio 1 : 2. The ratio of their radii is :
(a) $\sqrt{2}:1$ (b) 2 : 1
(c) 1 : 2 (d) 1 : 4

201. If the volume of two cubes are in the ratio 27 : 1, the ratio of their edge is :
(a) 3 : 1 (b) 27 : 1
(c) 1 : 3 (d) 1 : 27

202. The edges of a cuboid are in the ratio 1 : 2 : 3 and its surface area is 88 cm^2. The volume of the cuboid is :
(a) 48 cm^3 (b) 64 cm^3
(c) 16 cm^3 (d) 100 cm^3

203. The volume of two spheres are in the ratio 8 : 27. The ratio of their surface area is:
(a) 4 : 9 (b) 2 : 3
(c) 4 : 5 (d) 5 : 6

204. The base radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is :
(a) 27 : 20 (b) 20 : 27
(c) 9 : 4 (d) 4 : 9

205. The curved surface area of a cylinderical pillar is 264 m^2 and its volume is 924 m^3 (Taking $\pi = \frac{22}{7}$). find the ratio of its diameter to its height .
(a) 7 : 6 (b) 6 : 7
(c) 3 : 7 (d) 7 : 3

206. The ratio of the volume of two cones is 2 : 3 and the ratio of radii of their base is 1 : 2. The ratio of their height is
(a) 3 : 8 (b) 8 : 3
(c) 4 : 3 (d) 3 : 4

207. If the volume of two cubes are in the ratio 27 : 64, then the ratio of their total surface area is:
(a) 27 : 64 (b) 3 : 4
(c) 9 : 16 (d) 3 : 8

208. A hemisphere and a cone have equal base . If their heights are also equal, the ratio of their curved surface will be :
(a) $1:\sqrt{2}$ (b) $\sqrt{2}:1$
(c) 1 : 2 (d) 2 : 1

209. If the height of a given cone be doubled and radius of the base remains the same the ratio of the volume of the given cone to that of the second cone will be
(a) 2 : 1 (b) 1 : 8
(c) 1 : 2 (d) 8 : 1

210. Spheres A and B have their radii 40 cm and 10 cm respectively. Ratio of surface area of A to the surface area of B is :
(a) 1 : 16 (b) 4 : 1
(c) 1 : 4 (d) 16 : 1

211. If the radius of the base of a cone be doubled and height is left unchanged, then ratio of the volume of new cone to that of the original cone will be:
(a) 1 : 4 (b) 2 : 1
(c) 1 : 2 (d) 4 : 1

212. A cube of edge 5 cm is cut into cubes each of edge of 1 cm. The ratio of the total surface area of one of the small cubes to that of the large cube is equal to:
(a) 1 : 125 (b) 1 : 5
(c) 1 : 625 (d) 1 : 25

213. The diameter of two hollow spheres made from the same metal sheet are 21 cm and 17.5 cm respectively. The ratio of the area of metal sheets required for making the two spheres is
(a) 6 : 5 (b) 36 : 25
(c) 3 : 2 (d) 18 : 25

214. By melting a solid lead sphere of diameter 12 cm, three small spheres are made whose diameters are in the ratio 3 : 4 : 5. The radius (in cm) of the smallest sphere is
(a) 3 (b) 6 (c) 1.5 (d) 4

215. A cone is cut at mid point of its height by a frustum parallel to its base. The ratio between the volumes of two parts of cone would be
(a) 1 : 1 (b) 1 : 8
(c) 1 : 4 (d) 1 : 7

216. The ratio of the area of the in-circle and the circum-circle of a square is
(a) 1 : 2 (b) $\sqrt{2}:1$
(c) $1:\sqrt{2}$ (d) 2 : 1
(d) remains unchanged

217. The ratio of the surface area of a sphere and the curved surface area of the cylinder circumscribing the sphere is
(a) 1 : 2 (b) 1 : 1
(c) 2 : 1 (d) 2 : 3

218. The radii of two spheres are in the ratio 3 : 2. Their volume will be in the ratio :
(a) 9 : 4 (b) 3 : 2
(c) 8 :27 (d) 27 : 8

219. The volume of a sphere and a right circular cylinder having the same radius are equal. The ratio of the diameter of the sphere to the height of the cylinder is
(a) 3 : 2 (b) 2 : 3
(c) 1 :2 (d) 2 : 1

220. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their respective volume is
(a) 1 : 2 : 3 (b) 2 : 1 : 3
(c) 1 : 3 : 2 (d) 3 : 1 : 2

221. The radii of the base of two cylinders are in the ratio 3 : 5 and their heights in the ratio 2 : 3. The ratio of their curved surface will be :
(a) 2 : 5 (b) 2 : 3
(c) 3 : 5 (d) 5 :3

222. If the radii of two spheres are in the ratio 1 : 4, then their surface area are in the ratio :
(a) 1 : 2 (b) 1 : 4
(c) 1 : 8 (d) 1 :16

223. The radii of the base of two cylinders A and B are in the ratio 3 : 2 and their height in the ratio x : 1. If the volume of cylinder A is 3 times that of cylinder B, the value of x is
(a) $\frac{4}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{3}{2}$

224. A solid metallic sphere of radius 8 cm is melted to form 64 equal small solid spheres. The ratio of the surface area of this sphere to that of a small sphere is
(a) 4 : 1 (b) 1 :16
(c) 16 :1 (d) 1 : 4

225. The diameter of two cylinders, whose volumes are equal, are in the ratio 3 : 2, Their heights will be in the ratio.
(a) 4 : 9 (b) 5 : 6
(c) 5 : 8 (d) 8 : 9

226. The radius of base and slant height of a cone are in the ratio 4 : 7. If slant height is 14 cm then the radius (in cm) of its base is (use $\pi=\frac{22}{7}$)
(a) 8 (b) 12 (c) 14 (d) 16

227. A right circular cylinder just encloses a sphere of radius r. The ratio of the surface area of the sphere and the curved surface area of the cylinder is
(a) 2 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 1

228. The ratio of radii of two cone is 3 : 4 and the ratio of their height is 4 : 3. Then the ratio of their volume will be
(a) 3 : 4 (b) 4 : 3
(c) 9 : 16 (d) 16 : 9

229. If a right circular cone is separated into solids of volumes V_1, V_2, V_3 by two planes parallel to the base which also trisect the altitude, then $V_1 : V_2 : V_3$ is
(a) 1 : 2 : 3 (b) 1 : 4 : 6
(c) 1 : 6 : 9 (d) 1 : 7 : 19

230. The total surface area of a solid right circular cylinder is twice that of a solid sphere. If they have the same radii, the ratio of the volume of the cylinder to that of the sphere is given by
(a) 9 : 4 (b) 2 : 1
(c) 3 : 4 (d) 4 :9

231. The respective height and volume of a hemisphere and a right circular cylinder are equal, then the ratio of their radii is
(a) $\sqrt{2}:\sqrt{3}$ (b) $\sqrt{3}:1$
(c) $\sqrt{3}:2$ (d) $2:\sqrt{3}$

232. The ratio of the volume of a cube and of a solid sphere is 363 : 49. The ratio of an edge of the cube and the radius of the sphere is (take $\pi=\frac{22}{7}$)
(a) 7 : 11 (b) 22 : 7
(c) 11 :7 (d) 7 : 22

233. The radius and the height of a cone are in the ratio 4 : 3. The ratio of the curved surface area and total surface area of the cone is
(a) 5 : 9 (b) 3 : 7
(c) 5 : 4 (d) 16 : 9

234. A sphere and a cylinder have equal volume and equal radius. The ratio of the curved surface area of the cylinder to that of the sphere is
(a) 4 : 3 (b) 2 : 3
(c) 3 : 2 (d) 3 : 4

235. A right circular cylinder and a cone have equal base radius and equal height. If their curved surfaces are in the ratio 8 : 5, then the radius of the base to the height are in the ratio:
(a) 2 : 3 (b) 4 : 3
(c) 3 : 4 (d) 3 : 2

236. A right prism with trapezium base of parallel side 8 cm & 14 cm. Height of prism is 12 cm & its volume is 1056 cm³ then. Find the distance two parallel lines.
(a) 8 (b) 10 (c) 16 (d) 6

237. The radii of the base of cylinder and a cone are in the ratio $\sqrt{3}:\sqrt{2}$ and their heights are in the ratio $\sqrt{2}:\sqrt{3}$. Their volumes are in the ratio of
(a) $\sqrt{3}:\sqrt{2}$ (b) $3\sqrt{3}:\sqrt{2}$
(c) $\sqrt{3}:2\sqrt{2}$ (d) $\sqrt{2}:\sqrt{6}$

238. The heights of two cones are in the ratio 1 : 3 and the diameters of their base are in the ratio 3 : 5, The ratio of their volume is
(a) 3 : 25 (b) 4 : 25
(c) 6 : 25 (d) 7 : 25

239. A sphere and a hemisphere have the same volume . The ratio of their radii is
(a) 1 : 2 (b) 1 : 8
(c) 1 : $\sqrt{2}$ (d) $1:\sqrt[3]{2}$

240. The diameter of the moon is assumed to be one fourth of the diameter of the earth. Then the ratio of the volume of the earth to that of the moon is
(a) 64 : 1 (b) 1 : 64
(c) 60 : 7 (d) 7 : 60

241. If A denotes the volume of a right circular cylinder of same height as its diameter and B is the vol-

ume of a sphere of same radius then $\frac{A}{B}$ is:

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

242. Diagonal of a cube is $6\sqrt{3}$ cm. Ratio of its total surface area and volume (numerically) is
(a) 2 : 1 (b) 1 : 6
(c) 1 : 1 (d) 1 : 2

243. A sphere and a hemisphere have the same volume. The ratio of their curved surface area is :
(a) $2^{\frac{3}{2}}:1$ (b) $2^{\frac{2}{3}}:1$
(c) $4^{\frac{2}{3}}:1$ (d) $2^{\frac{1}{3}}:1$

244. The volume of a cylinder and a cone are in the ratio 3 : 1. Find their diameters and then compare them when their heights are equal.
(a) Diameter of cylinder = 2 times of diameter of cone
(b) Diameter of cylinder = Diameter of cone
(c) Diameter of cylinder > Diameter of cone
(d) Diameter of cylinder < Diaameter of cone

245. A solid sphere is melted and recast into a right circular cone with a base radius equal to the radius of sphere. What is the ratio of the height and radius of the cone so formed ?
(a) 4 : 3 (b) 2 : 3
(c) 3 : 4 (d) 4 : 1

246. Find the total surface area of a prism which is based on Δ of perimeter 45 cm & incircle radius 9cm, if its volume is 810 cm^3.
(a) 405 (b) 585
(c) 616 (d) 468

247. The ratio of weights of two spheres of different materials is 8 : 17 and the ratio of weights per 1 cc of materials of each is 289 : 64. The ratio of radii of the two spheres is
(a) 8 : 17 (b) 4 : 17
(c) 17 : 4 (d) 17 : 8

248. If the ratio of volumes of two cones is 2 : 3 and the ratio of the radii of their bases is 1 : 2, then the ratio of their heights will be
(a) 8 : 3 (b) 3 : 8
(c) 4 : 3 (d) 3 : 4

249. The volumes of a right circular cylinder and a sphere are equal. The radius of the cylinder and the diameter of the sphere are equal. The ratio of height and radius of the cylinder is
(a) 3 : 1 (b) 1 : 3
(c) 6 : 1 (d) 1 : 6

250. A large solid sphere is melted and moulded to form identical right circular cones with base radius and height same as the radius of the sphere. One of these cones is melted and moulded to form a smaller solid sphere. Then the ratio of the surface area of the smaller to the surface area of the larger sphere is
(a) $1:3^{\frac{4}{3}}$ (b) $1:2^{\frac{3}{2}}$
(c) $1:2^{\frac{2}{3}}$ (d) $1:2^{\frac{4}{3}}$

251. A plane divides a right circular cone into two parts of equal volume. If the plane is parallel to the base, then the ratio, in which the height of the cone is divided, is
(a) $1:\sqrt{2}$ (b) $1:\sqrt[3]{2}$
(c) $1:\sqrt[3]{2}-1$ (d) $1:\sqrt[3]{2}+1$

252. A rectangle based pyramid, length and width of the base is 18cm and 10cm respectively. Find the total surface area, if its height is 12cm :
(a) 267cm^2 (b) 564cm^2
(c) 516cm^2
(d) None of these

253. A cone of height 7 cm and base radius 1 cm is carved from a cuboidal block of wood 10 cm × 5 cm × 2 cm .[Assuming $\pi=\frac{22}{7}$] The percentage of wood wasted in the process is :
(a) $92\frac{2}{3}\%$ (b) $46\frac{1}{3}\%$
(c) $42\frac{1}{3}\%$ (d) $41\frac{1}{3}\%$

254. If the radius of a cylinder is decreased by 50% and the height is increased by 50% to form a new cylinder, the volume will be decreased by
(a) 0% (b) 25%
(c) 62.5% (d) 75%

255. Each of the height and base radius of a cone is increased by 100%. The percentage increase in the volume of the cone is
(a) 700% (b) 400%
(c) 300% (d) 100%

256. If both the radius and height of a right circular cone are increased by 20%, its volume will be increased by
(a) 20% (b) 40%
(c) 60% (d) 72.8%

257. A cone of height 15 cm and base diameter 30 cm is carved out of a wooden sphere of radius 15 cm. The percentage of used wood is :
(a) 75% (b) 50%
(c) 40% (d) 25%

258. If the height of a right circular cone is increased by 200% and the radius of the base is reduced by 50%, the volume of the cone
(a) increases by 25%
(b) increases by 50%
(c) remains unaltered
(d) decreases by 25%

259. If the height and the radius of the base of a cone are each increased by 100%, then the volume of the cone becomes
(a) double that of the original
(b) three times that of the original
(c) six times that of the original
(d) eight times that of the original

260. If the height of a cylinder is increased by 15 per cent and the radius of its base is decreased by 10 percent then by what percent will its curved surface area change ?
(a) 3.5 percent decrease
(b) 3.5 percent increase
(c) 5 percent increase
(d) 5 percent decrease

261. If the radius of a sphere is doubled, its volume becomes
(a) double (b) four times
(c) six times (d) eight times

262. If the radius of a right circular cylinder is decreased by 50% and its height is increased by 60% its volume will be decreased by
(a) 10% (b) 60%
(c) 40% (d) 20%

263. The length, breadth and height of a cuboid are in the ratio 1 : 2 : 3. If they are increased by 100%, 200% and 200% respectively. Then compared to the original volume the increase in the volume of the cuboid will be
(a) 5 times (b) 18 times
(c) 12 times (d) 17 times

264. Each of the radius of the base and the height of a right circular cylinder is increased by 10%. The volume of the cylinder is increased by
(a) 3.31% (b) 14.5%
(c) 33.1% (d) 19.5%

265. If the height of a cone is increase by 100% then its volume is increased by :
(a) 100% (d) 200%
(d) 300% (d) 400%

266. A hemispherical cup of radius 4 cm is filled to the brim with coffee. The coffee is then poured into a vertical cone of radius 8 cm and height 16 cm. The percentage of the volume of the cone that remains empty is :
(a) 87.5% (b) 80.5%
(c) 81.6% (d) 88.2%

267. The height of a circular cylinder is increased six times and the base area is decreased to one ninth of its value. The factor by which the lateral surface of the cylinder increases is
(a) 2 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

268. If the radius of a sphere be doubled. the area of its surface will become
(a) Double
(b) Three times
(c) Four times
(d) None of the mentioned

269. If each edge of a cube is increased by 50%, the percentage increase in its surface area is
(a) 150% (b) 75%
(c) 100% (d) 125%

270. If the radius of a sphere be doubled, then the percentage increase in volume is
(a) 500% (b) 700%
(c) 600% (d) 800%

271. Find the radius of maximum size sphere which can be inscribed or put in a cone whose base radius and height are 6cm and 8cm respectively?
(a) 4cm (b) 5cm
(c) 3cm
(d) None of these

272. If the length of each side of a regular tetrahedron is 12 cm, then the volume of the tetrahedron is
(a) $144\sqrt{2}$ cu. cm,
(b) $72\sqrt{2}$ cu. cm,
(c) $8\sqrt{2}$ cu. cm,
(d) $12\sqrt{2}$ cu. cm,

273. If the radii of the circular ends of a truncated conical bucket which is 45cm high be 28 cm and 7 cm then the capacity of the bucket in cubic centimetre is (use $\pi=\frac{22}{7}$)
(a) 48510 (b) 45810
(c) 48150 (d) 48051

274. There is a pyramid on a base which is a regular hexagon of side 2a cm. If every slant edge of this pyramid is of length $\frac{5a}{2}$ cm, then the volume of this pyramid is
(a) $3a^3$ cm^3 (b) $3\sqrt{2}\,a^2$ cm^3
(c) $3\sqrt{3}\,a^3$ cm^3 (d) $6a^3$ cm^3

275. The base of a right pyramid is a square of side 40 cm long. If the volume of the pyramid is 8000 cm^3, then its height is :
(a) 5 cm (b) 10 cm
(c) 15 cm (d) 20 cm

276. The base of a right prism is a trapezium. The length of the parallel sides are 8 cm and 14 cm and the distance between the parallel sides is 8 cm, If the volume of the prism is 1056 cm^3 , then the height of the prism is
(a) 44 cm (b) 16.5 cm
(c) 12 cm (d) 10.56 cm

277. Each edge of a regular tetrahedron is 3 cm, then its volume is
(a) $\frac{9\sqrt{2}}{4}$ c.c. (b) $27\sqrt{3}$ c.c.
(c) $\frac{4\sqrt{2}}{9}$ c.c. (d) $9\sqrt{3}$ c.c.

278. The perimeter of the triangular base of a right prism is 15 cm and radius of the incircle of the triangular base is 3 cm. If the volume of the prism be 270 cm^3 then the height of the prism is
(a) 6 cm (b) 7.5 cm
(c) 10 cm (d) 12 cm

279. The base of a solid right prism is a triangle whose sides are 9 cm, 12 cm and 15 cm, The height of the prism is 5 cm. The the total surface area of the prism is
(a) 180 cm^2 (b) 234 cm^2
(c) 288 cm^2 (d) 270 cm^2

280. The base of a right prism is an equilateral triangle of area 173 cm^2 and the volume of the prism is 10380 cm^3. The area of the lateral surface of the prism is
(use $\sqrt{3}$ =1.73)
(a) 1200 cm^2 (b) 2400 cm^2
(c) 3600 cm^2 (d) 4380 cm^2

281. The base of a right pyamid is a square of side 16 cm long . If its height be 15 cm, then the area of the lateral surface in square cm is :
(a) 136 (b) 544
(c) 800 (d) 1280

282. Area of the base of a pyramid is 57 sq. cm. and height is 10 cm, then its volume (in cm^3), is
(a) 570 (b) 390
(c) 190 (d) 590

283. The height of a right prism with a square base is 15 cm. If the area of the total surface of the prism is 608 sq. cm, its volume is
(a) 910 cm^3 (b) 920 cm^3
(c) 960 cm^3 (d) 980 cm3

284. The base of a right prism is an equilateral triangle of side 8 cm and height of the prism is 10 cm. Then the volume of the prism is
(a) $320\sqrt{3}$ cubic cm
(b) $160\sqrt{3}$ cubic cm
(c) $150\sqrt{3}$ cubic cm
(d) $300\sqrt{3}$ cubic cm

285. A prism has as the base a right angled triangle whose sides adjacent to the right angles are 10 cm and 12 cm long. The height of the prism is 20 cm . the density of the material of the prism is 6gm/cubic cm. the weight of the prism is
(a) 6.4 kg (b) 7.2 kg
(c) 3.4 kg (d) 4.8 kg

286. If the slant height of a right pyramid with square base is 4 metre and the total slant surface of the pyramid is 12 square metre, then the ratio of total slant surface and area of the base is :
(a) 16 : 3 (b) 24 : 5
(c) 32 : 9 (d) 12 : 3

287. The length of each edge of a regular tetrahedron is 12 cm. The area (in sq. cm) of the total surface of the tetrahedron is
(a) $288\sqrt{3}$ (b) $144\sqrt{2}$
(c) $108\sqrt{3}$ (d) $144\sqrt{3}$

288. The base of right prism is a triangle whose perimeter is 28 cm and the inradius of the triangle is 4 cm. If the volume of the prism is 366 cc, then its height is
(a) 6 cm (b) 8 cm
(c) 4 cm
(d) None of these

289. If the base of a right pyramid s triangle of sides 5 cm, 12 cm and 13 cm and its volume is 330 cm, then its height (in cm) will be
(a) 33 (b) 32 (c) 11 (d) 22

290. The base of a right pyramid is equilateral triangle of side $10\sqrt{3}$ cm. If the total surface are of the pyramid is $270\sqrt{3}$ sq. cm. its height is
(a) $12\sqrt{3}$ cm (b) 10 cm
(c) $10\sqrt{3}$ cm (d) 12 cm

291. A right prism stands on a base of 6 cm side equilateral triangle and its volume is $81\sqrt{3}$ cm³. the height (in cm) of the prism is
(a) 9 (b) 10 (c) 12 (d) 15

292. A right pyramid stands on a square base of diagonal $10\sqrt{2}$ cm. If the height of the pyramid is 12 cm, the area (in cm²) of its slant surface is
(a) 520 (b) 420 (c) 360 (d) 260

293. If the altitude of a right prism is 10 cm and its base is an equilateral triangle of side 12 cm, then its total surface area (in cm²) is
(a) $\left(5+3\sqrt{3}\right)$ (b) $36\sqrt{3}$
(c) 360 (d) $72\left(5+\sqrt{3}\right)$

294. A right pyramid stands on a square base of side 16 cm and its height is 15 cm. The area (in cm²) of its slant surface is
(a) 514 (b) 544 (c) 344 (d) 444

295. The base of a right prism is a right angled triangle whose sides are 5 cm, 12 cm and 13 cm. If the total surface area of the prism is 360 cm², then its height (in cm) is
(a) 10 (b) 12 (c) 9 (d) 11

296. A right pyramid 6 m high has a square base of which the diagonal is $\sqrt{1152}$ m. Volume of the pyramid is
(a) 144 m³ (b) 288 m³
(c) 576 m³ (d) 1152 m³

297. The height of the right pyramid whose area of the base is 30 m² and volume is 500 m² is
(a) 50 m (b) 60 m
(c) 40 m (d) 20 m

298. The base of a right. prism is an equilateral triangle. If the lateral surface area and volume is 120 cm², $40\sqrt{3}$ cm³ respectively then the side of base of the prism is
(a) 4 cm (b) 5 cm
(c) 7 cm (d) 40 cm

299. Each edge of a regular tetrahedron is 4 cm. its volume (in cubic cm) is
(a) $\frac{16\sqrt{3}}{3}$ (b) $16\sqrt{3}$
(c) $\frac{16\sqrt{2}}{3}$ (d) $16\sqrt{2}$

300. The base of a prism is a right angled triangle with two sides meeting at right angle are 5 cm and 12 cm. The height of the prism is 10 cm. The total surface area of the prism is
(a) 360 sq. cm (b) 300 sq. cm
(c) 330 sq. cm (d) 325 sq. cm

301. The radius of a cylinder is 10 cm and height is 4 cm. The number of centimetres that may be added either to the radius or to the height to get the same increase in the volume of the cylinder is :
(a) 25 (b) 4 (c) 5 (d) 16

302. If the area of the base, height and volume of a right prism be $\left(\frac{3\sqrt{3}}{2}\right)$ p² cm², $10\sqrt{3}$ cm and 7200 cm³ respectively, then the value of P (in cm) will be?
(a) 4 (b) $\frac{2}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{3}{2}$

303. If the base of right prism remains same and the lateral edges are halved, then its volume will be reduced by
(a) 33.33% (b) 50%
(c) 25% (d) 66%

304. The total surface area of a regular triangular pyramid with each edges of length 1 cm is?
(a) $\frac{4}{2}\sqrt{2}$ cm² (b) $\sqrt{3}$ cm²
(c) 4 cm² (d) $4\sqrt{3}$ cm²

305. Base of a right pyramid is a square of side 10 cm. If the height of the pyramid is 12 cm, then its total surface area is
(a) 360 cm² (b) 400 cm²
(c) 460 cm² (d) 260 cm²

306. A right prism has a triangular base whose sides are 13 cm, 20 cm and 21 cm, If the altitude of the prism is 9 cm, then its volume is
(a) 1143 cm³ (b) 1314 cm³
(c) 1413 cm³ (d) 1134 cm³

307. Base of a prism of height 10 cm is square. Total surface area of the prism is 192 sq. cm. The volume of the prism is
(a) 120 cm³ (b) 640 cm³
(c) 90 cm³ (d) 160 cm³

308. A right prism has triangular base. If v be the number of vertices, e be the number of edges and f be the number of faces of the prism. The value of $\frac{v+e-f}{2}$ is
(a) 2 (b) 4 (c) 5 (d) 10

309. The base of a right prism is a trapezium whose lengths of two parallel sides are 10 cm and 6 cm and distance between them is 5 cm. If the height of the prism is 8 cm, its volume is:
(a) 300 cm^3 (b) 300.5 cm^3
(c) 320 cm^3 (d) 310 cm^3

310. Base of a right prism is a rectangle, the ratio of whose length and breadth is 3 : 2. If the height of the prism is 12 cm and total surface area is 288 sq. cm, the volume of the prism is:
(a) 288 cm (b) 290 cm^3
(c) 286 cm^3 (d) 291 cm^3

311. Height of a prism-shaped part of a machine is 8 cm and its base is an isosceles triangle, whose each of the equal sides is 5 cm and remaining side is 6 cm. The volume of the part is
(a) 90 cm^3 (b) 96 cm^3
(c) 120 cm^3 (d) 86 cm^3

312. Find the total surface area of pyramid which is based on a equal Δ of side $18\sqrt{3}$ & the height of pyramid is 12 cm.
(a) $124\sqrt{3}$ (b) $624\sqrt{3}$
(c) $648\sqrt{3}$ (d) $405\sqrt{3}$

313. The base of a right prism is a $\square$ABCD. If the volume of prism is 2070. Then find the lateral surface area. AB = 9, BC = 14, CD = 13, DA = 12 DAB = 90°.
(a) 720 (b) 540
(c) 920 (d) 960

314. Find the volue of pyramid which is based on a equilateral traingle of side 4 cm & height of pyramid is $20\sqrt{3}$ cm.
(a) 100 (b) 160
(c) 80 (d) 40

315. Find the total surface area of pyramid of 4 cm height which is based on a square of side 6cm.
(a) 48 (b) 72 (c) 96 (d) 120

316. Find the volue of a pyramid which is based on a square of side 10cm & lateral edge of pyramid is 12 cm.
(a) $\frac{100}{3}$ (b) $\frac{100\sqrt{119}}{3}$
(c) $\frac{100\sqrt{119}}{9}$ (d) $100\sqrt{119}$

317. A rectangular water tank is open at the top. Its capacity is 24 m^3. Its length and breadth are 4 m and 3 m respectively. Ignoring the thickness of the material used for building the tank, the total cost of painting the inner and outer surfaces of the tank at the rate of Rs. 10 per m^2 is:
(a) Rs. 400 (b) Rs. 500
(c) Rs. 600 (d) Rs. 800

318. If V be the volume and S the surface area of a cuboid of dimensions a, b and c then $\frac{1}{V}$ is equal to:
(a) $\frac{S}{2}(a+b+c)$
(b) $\frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$
(c) $\frac{2S}{a+b+c}$
(d) 2S(a + b + c)

319. An open box is made of wood 3 cm thick. Its external length is 1.46 m, breadth 1.16 m and height 8.3 dm. The cost of painting the inner surface of the box at 50 paise per 100 cm^2 is:
(a) Rs. 138.50 (b) Rs. 277
(c) Rs. 415.50 (d) Rs. 554

320. The areas of three adjacent faces of a cuboid are x, y & z square units respectively. If the volume of the cuboid be v cubic units, then the correct relation between v, x, y, z is:
(a) $v^2 = xyz$ (b) $v^3 = xyz$
(c) $v^2 = x^3y^3z^3$ (d) $v^3 = x^2y^2z^2$

321. 1 m^3 piece of copper is melted and recast into a square cross section bar 36 m long. An exact cube is cut off from this bar. If 1 m^3 of copper cost Rs. 108, then the cost of the cube is.
(a) 50 paise (b) 25 paise
(c) 75 paise (d) 1 paise

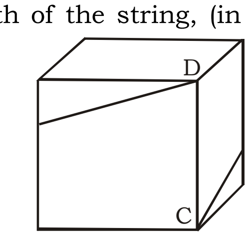
322. The volume of a rectangular block of stone is 10368 dm^2, its dimensions are in the ratio of 3:2:1, If its entire surface is polished at 2 paise per dm^2, then what is the total cost?
(a) Rs. 31.68 (b) Rs. 31.50
(c) Rs. 63 (d) Rs. 63.36

323. A rectangular water tank measure 15m × 6m at top and is 10m deep. It is full of water. If water is drawn out lowering the level by 1 meter how much of water has been drawn out?
(a) 90,000 litres
(b) 45,000 litres
(c) 80,000 litres
(d) 40,000 litre

324. A rectangular tank is 45 m long and 26 m broad. Water flows into it through a pipe whose cross section is 13 cm^2, at the rate of 9 km/hour. How much will the level of the water rise in the tank in 15 min?
(a) 0.0016m (b) 0.0020m
(c) 0.0025m (d) 0.0018

325. The diagonals of the three faces of a cuboid are x, y and z respectively. Find the volume of cuboid?
(a) $\frac{xyz}{2\sqrt{2}}$
(b) $\frac{\sqrt{(y^2+z^2-x^2)(z^2+x^2-y^2)(x^2+y^2-z^2)}}{2\sqrt{2}}$
(c) $\sqrt{\frac{(y^2+z^2(z^2+x^2)(x^2+y2))}{2\sqrt{2}}}$
(d) None of these

326. The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can given exactly one turn. The length of the string, (in cm) is.



(a) $\sqrt{2}n$ (b) $\sqrt{17}n$
(c) n (d) $\sqrt{13}n$

327. A reservoir is supplied water by a pipe 6 cm in diameter. How many pipes of 1.5 cm diameter would discharge the same quantity, supposing the velocity of water is same?
(a) 8 (b) 12 (c) 16 (d) 20

328. Given a solid cylinder of radius 10 cm and length 1000 cm, a cylindrical hole is made into it to obtain a cylindrical shell of uniform thickness and having volume equal to one-fourth of the original cylinder. The thickness of the cylindrical shell is:
(a) $5\left(\sqrt{5}-2\right)$cm
(b) $5(2-\sqrt{3})$ cm
(c) 5 cm (d) $5\sqrt{2}$ cm

329. A well of radius 'r' is dug 20 m deep and the earth taken out is spread all around it to a width of 1 m to form an embankment. The height of the embankment is 5 m then find the value of 'r':
(a) $\frac{1+\sqrt{5}}{2}$ (b) $\frac{1+\sqrt{5}}{4}$
(c) $\frac{\sqrt{5}-1}{2}$ (d) $\frac{\sqrt{5}-1}{4}$

330. A cylinder is filled to 4/5th of volume. If is then tilted so that level of water coincides with one edge of its bottom and top edge of the opposite side. In the process, 30 litre of the water is spilled. What is the value of the cylinder?
(a) 75 litre (b) 96 litre
(c) Data insufficient
(d) 100 litre

331. A monument has 50 cylindrical pillars each of diameter 50 cm and height 4 m. what will be the labour charges for getting these pillars cleared at the rate of 50 paise per m^2 (Use π = 3.14).
(a) Rs. 237 (b) Rs. 157
(c) Rs. 257 (d) Rs. 353

332. A right circular cyindrical tank has the storage capacity 38808 ml. If the radius of the base of the cylinder is three fourth of the height what is the radius of base?
(a) 28 cm (b) 56 cm
(c) 21 cm (d) 42 cm

333. A rectangular piece of iron sheet measuring 50 cm and 100 cm is rolled into cylinder of height 50 cm. If the cost of painting the cylinder is Rs. 50 per square meter, then what will be the cost of painting the surface of the cylinder?
(a) Rs. 25.00 (b) Rs. 37.50
(c) Rs 75.00 (d) Rs. 87.50

334. Sixteen cylindrical cans, each with a radius of 1 unit, are placed inside a cardboard box four in a row. If the cans touch the adjacent cans and or the walls of the box, then which of the following could be the interior area of the bottom of the box in square units?
(a) 16 (b) 32 (c) 64 (d) 128

335. Find the volume of a right circular cone formed by joining the edges of a sector of a circle of radius 4cm where the angle of the sector is 90°.
(a) $\frac{2\sqrt{3}}{\pi}$ cm^3 (b) $\frac{2\sqrt{2}\pi}{3}$ cm^3
(c) $\frac{\pi\sqrt{5}}{\sqrt{3}}$ cm^3 (d) $\frac{\sqrt{3}}{\pi}$ cm^3

336. A sector of circle of radius 3cm has an angle of 120°. if it is modulated into a cone, find the volume of the cone.
(a) $\frac{\pi}{\sqrt{3}}$ cm^3 (b) $\frac{2\sqrt{2}\pi}{3}$ cm^3
(c) $\frac{2\sqrt{3}}{\pi}$ cm^3 (d) $\frac{\sqrt{3}}{\pi}$ cm^3

337. If the slant height and the radius of the base of a right circular cone are H and r respectively then the ratio of the areas of the lateral surface and the base is:
(a) 2H :r (b) H : r
(c) H : 2r (d) H^2 : r^2

338. A sector of a circle of radius 15cm has the angle 120.It is rolled up so that two bounding radii are joined toghether to form a cone.the volume of the cone is.
(a) $\left(250\sqrt{2}\right)\pi$ cm^3
(b) $\left(100\sqrt{2}\right)\pi$ cm^3
(c) $\left[\left(250\sqrt{2}\right)\pi/3\right]$cm^3
(d) $\left[\left(100\sqrt{2}\right)\pi/3\right]$cm^3

339. The base radius and height of a cone is 5cm and 25cm respectively.if the cone is cut parallel to its base at a height of h from the base. If the volume of this frustrum is 110 cm^3 find the radius of smaller cone?
(a) $(104)^{1/3}$ cm (b) $(104)^{1/2}$ cm
(c) 5 cm
(d) None of these

340. A hemisphereical bowl is 176cm round the brim. supposing it to be half full, how many persons may be swerved from it in hemispherical glasses 4 cm in diameter at the top?
(a) 1372 (b) 1272
(c) 1172 (d) 1472

341. A sphere of radius 3 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vesasel is 6 cm.If the sphere is submerged completely, then the surface of the water is raised by
(a) $\frac{1}{4}$ cm (b) $\frac{1}{2}$ cm
(c) 1 cm (d) 2cm

342. Let A and B be two solid spheres area of B is 300% higher than surface area of A.
The volume of A is found to be k% lower than the volume of B.The value of k must be
(a) 85.5 (b) 92.5
(c) 90.5 (d) 87.5

343. The base of a prism is a regular hexagon. If every edge of the prism measures 1 metre and height is 1 metre, than volume of the prism is
(a) $\frac{3\sqrt{2}}{2}$ cu m (b) $\frac{3\sqrt{3}}{2}$ cu m
(c) $\frac{6\sqrt{2}}{5}$ cu m (d) $\frac{5\sqrt{3}}{2}$ cu m

344. The base of a right prism is a pentagon whose sides are in the ratio $1:\sqrt{2}:\sqrt{2}:1:2$ and its height is 10 cm. If the longest side of the base be 6 cm, the volume of the prism is
(a) 270 cm^3 (b) 360 cm^3
(c) 540 cm^3
(d) None of these

345. There are two prism, one has equilateral triangle as a base and the other regular hexagon. If both of the prisms have equal heights and volumes, then find the ratio between the length of each side at their bases.
(a) $1:\sqrt{6}$ (b) $\sqrt{6}:1$
(c) $\sqrt{3}:2$ (d) $2:\sqrt{3}$

346. The base of a right prism is a trapezium. The lengths of the parallel sides are 8 cm and 14 cm and the distance between the parallel sides is 8 cm. If the volume of the prism is 1056 cm^3, then the height of the prism is
(a) 44 cm (b) 16.5 cm
(c) 12 cm (d) 10.56 cm

347. If the base of a right rectangular prism is left unchanged and the measure of the lateral edges are doubled, then its volume will be
(a) unchanged (b) tripled
(c) doubled (d) quadrupled

348. Prism has as the base a right angled triangle whose sides adjacent to the right angles are 10 cm and 12 cm long. The height of the prism is 20 cm. The density of the material of the prism is 6 gm. cubic cm. The weight of the prism is.
(a) 6.4 kg (b) 7.2 kg
(c) 3.4 kg (d) 4.8 kg

349. The perimeter of the triangular base of a right prism is 15 cm and radius of the incircle of the triangular base is 3 cm. If the volume of the prism be 270 cm^3, then the height of the prism is-
(a) 6 cm (b) 7.5 cm
(c) 10 cm (d) 12 cm

350. The base of a right prism is an equilateral tirangle. If its height is one-fourth and each side of the base is tripled, then the ratio of the volumes of the old to the new prism is-
(a) 4 : 3 (b) 1 : 4
(c) 1 : 2 (d) 4 : 9

351. A right pyramid is on a regular hexagonal base. Each side of the base is 10 m and the height is 30 m. The volume of the pyramid is
(a) 2500 m^3 (b) 2550 m^3
(c) 2598 m^3 (d) 5196 m^3

352. There is a pyramid on a base which is a regular hexagon of side 2a. If every slant edge of this pyramid is of length $\frac{5a}{2}$, then the volume of this pyramid is.
(a) $3a^3$ (b) $3a^2\sqrt{2}$
(c) $3a^3\sqrt{3}$ (d) $6a^3$

353. If the area of the base of a regular hexagonal pyramid is $96\sqrt{3}$ m^2 and the area of one of its side faces is $32\sqrt{3}$ m^3, then the volume of the pyramid is:
(a) $380\sqrt{3}$ m^3 (b) $382\sqrt{2}$ m^3
(c) $384\sqrt{3}$ m^3 (d) $386\sqrt{3}$ m^3

354. What part of a ditch, 48 metres long 16.5 metres broad and 4 metres deep can be filled by the sand got by digging a cylindrical tunnel of diameter 4 metres and length 56 metres?
$\left[\text{use } \pi = \frac{22}{7}\right]$
(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{7}{9}$ (d) $\frac{8}{9}$

355. A cylindrical rod of iron whose height is eight times its radius is melted and cast into spherical balls each of half the radius of the cylinder. The number of such pherical balls is
(a) 12 (b) 16 (c) 24 (d) 48

356. Water flows at the rate of 10 meters per minute from a cylindrical pipe 5 mm in diameter. How long it take to fill up a conical vessel whose diameter at the base is 30 cm and depth 24 cm?
(a) 28 minutes 48 seconds
(b) 51 minutes 12 seconds
(c) 51 minutes 24 seconds
(d) 28 minutes 36 second

357. A semi-circular sheet of metal of diameter 28 cm is bent into an open conical cup. The depth of the cup is aproximately :
(a) 11 cm (b) 12 cm
(c) 13 cm (d) 14 cm

358. The height of a right prism with a square base 15 cm. If the total S.A. of prism of 608 cm^2. The find its volume.
(a) 480 (b) 460
(c) 1500 (d) 960

359. A slab of ice 8 inches in length 11 inches in breadth, and 2 inches thick was melted and resolidified in the form of a rod of 8 inches diameter. The length of such a rod, in inches, is nearest to.
(a) 3 (b) 3.5 (c) 4 (d) 4.5

360. A storage tank consists of a circular cylinder with a hemisphere adjoined on either side. If the external diameter of the cylinder be 14 m and its length be 50 m, then what will be the cost of painting it at the rate of Rs. 10 per sq m?
(a) Rs. 38160 (b) Rs. 28160
(c) Rs. 39160 (d) None of these

361. The diameter of the iron ball used for the shotput game is 14 cm. It is melted and then a solid cylinder of height $2\frac{1}{3}$ cm is made. What will be the diameter of the base of the cylinder?
(a) 14 cm (b) 28 cm
(c) $\frac{14}{3}$ cm (d) $\frac{28}{3}$ cm

362. If the area of the circular shell having inner and outer radii of 8 cm and 12 cm respectively is equal to the total surface area of cylinder of radius R_1 and height h, then h, in terms or R_1 will be.
(a) $\frac{3R_1^2-30}{7R_1}$ (b) $\frac{R_1^2-40}{R_1^2}$
(c) $\frac{30-R_1}{R_1^2}$ (d) $\frac{40-R_1^2}{R_1}$

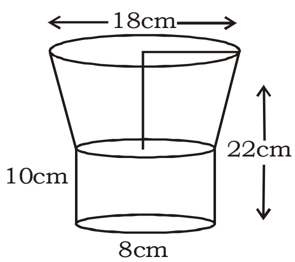
363. Two solid right cones of equal heights are of radii r_1 and r_2 are melted and made to form a solid sphere of radius R. Then the height of the cone is:
(a) $\frac{4R^2}{r_1^2+r_2^2}$ (b) $\frac{4R}{r_1+r_2}$
(c) $\frac{4R^3}{r_1^2+r_2^2}$ (d) $\frac{R^2}{r_1^2+r_2^2}$

364. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the

volume of the wooden by (nearly).
(a) 104 cm^3 (b) 162 cm^3
(c) 427 cm^3 (d) 266 cm^3

365. The volume of a cylinder and a cone are in the ratio 3 : 1. Find their diameters and then compare them when their heights are equal.
(a) Diameter of cylinder = 2 times diameter of cone
(b) Diameter of cylinder = Diameter of cone
(c) Diameter of cylinder > Diameter of cone
(d) Diameter of cylinder < Diameter of cone

366. A oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attatched to 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make a funnel.



(a) 728.57 cm^3 (b) 782.57 cm^3
(c) 872.57 cm^3 (d) 827.57 cm^3

367. A conical flask has radius a cm and height h cm. It was completely filled with milk. The milk is poured into a cylindrical therefore flask whose base radius is p cm. What will be the height of the soultion level in the flask?
(a) $\frac{a^2h}{3p^2}$ cm (b) $\frac{3hp^2}{a^2}$ cm
(c) $\frac{p^2}{3h^2}$ cm (d) $\frac{3a^2}{hp^2}$ cm

368. The perimeter of an equilateral triangle is $72\sqrt{3}$ cm. Find its height.
(a) 63 metres (b) 24 metres
(c) 18 metres (d) 36 metres

369. A pit 7.5 metre long, 6 metre wide and 1.5 metre deep is dug in a field. Find the volume of soil removed in cubic metres.
(a) 135 m^3 (b) 101.25 m^3
(c) 50.625 m^3 (d) 67.5 m^3

370. In a shower, 10 cm of rain falls. What will be the volume of water that falls on 1 hectare area of ground?
(a) 500 m^3 (b) 650 m^3
(c) 1000 m^3 (d) 750 m^3

371. Seven equal cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.
(a) 750 cm^2 (b) 1500 cm^2
(c) 2250 cm^2 (d) 700 cm^2

372. In a swimming pool measuring 90 m by 40m, 150 men take a dip. If the average displacement of water by a man is 8 cubic metres, what will be rise in water level?
(a) 30 cm (b) 50 cm
(c) 20 cm (d) 33.33 cm

373. A conical tent is to accommodate 10 persons. Each person must have 6 m^2 space to sit and 30 m^3 of air to breath. What will be the height of the cone ?
(a) 37.5 m (b) 150 m
(c) 75 m
(d) None of these

374. A hollow spherical shell is made of a metal of density 4.9 g/cm^3. If its internal and external radii are 10 cm and 12 cm respectively, find the weight of the shell.
(Take = 3.1416)
(a) 5016 gm (b) 1416.8 gm
(c) 14942.28gm (d) 5667.1 gm

375. A spherical cannon ball, 28 cm in diameter, is melted and cast into a right circular conical mould the base of which is 35 cm in diameter. Find the height of the cone correct up to two places of decimals.
(a) 8.96 cm (b) 35.84 cm
(c) 5.97 cm (d) 17.9 cm

376. A rope is wound round the outside of a circular drum whose diameter is 70 cm and a bucket is tied to the other end of the rope. Find the number of revolutions made by the drum if the bucket is raised by 11 m.
(a) 10 (b) 2.5 (c) 5 (d) 5.5

377. A cube whose edge is 20 cm long has circle on each of its faces painted black. What is the total area of the unpainted surface of the cube if the circles are of the largest area possible?
(a) 85.71 cm^2 (b) 257.14 cm^2
(c) 514.28 cm^2 (d) 331.33 cm^2

378. The areas of three adjacent faces of a cuboid are x, y, z. If the volume is V, then V^2 will be equal to
(a) xy/z (b) yz/x^2
(c) x^2y^2/z^2 (d) xyz

379. The dimensions of a field are 20 m by 9m. A pit 10 m long, 4.5 m wide and 3m deep is dug in one corner of the field and the soil removed has been evenly spread over the remaining area of the field. What will be the rise in the height of field as a result of this operation?
(a) 1m (b) 2m (c) 3m (d) 4m

380. A vessel is in the form of a hollow cylinder mounted on a hemispherical bowl. The diameter of the sphere is 14 cm and the total height of the vessel is 13cm. Find the capacity of the vessel.
(Take = 22/7)
(a) 321.33 cm (b) 1642.67 cm^3
(c) 1232 cm^3 (d) 1632.33 cm^3

381. A circular tent is cylindrical to a height of 3 metres and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 6 m wide to make the required tent.
(a) 3894 m (b) 973.5 m
(c) 1947 m (d) 1800 m

382. A steel sphere of radius 4 cm is drawn into a wire of diameter 4 mm. Find the length of wire.
(a) 10,665 mm
(b) 42.660 mm
(c) 21,333 mm
(d) 14,220 mm

383. A cylinder and a cone having equal diameter of their bases are placed in the Qutab Minar one on the other, with the cylinder placed in the bottom. If their curved surface area are in the ratio of 8 : 5, find the ratio of their heights. Assume the height of the cylinder to be equal to the radius of Qutab Minar. (Assume Qutab Minar to

be having same radius throughout).
(a) 1 : 4 (b) 3 : 4
(c) 4 : 3 (d) 2 : 3

384. If the curved surface area of a cone is thrice that of another cone and slant height of the second cone is thrice that of the first, find the ratio of the area of their base.
(a) 81 : 1 (b) 9 : 1
(c) 3 : 1 (d) 27 : 1

385. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If external radius of the base of the cylinder is 5 cm and its height is 32 cm, find the uniform thickness of the cylinder.
(a) 2 cm (b) 3 cm
(c) 1 cm (d) 3.5 cm

386. A hollow sphere of external and internal radius 6 cm and 4 cm respectively is melted into a cone of base diameter 8cm. Find the height of the cone
(a) 25 cm (b) 35 cm
(c) 30 cm (d) 38 cm

387. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.
(a) 7 : 9 (b) 49 : 81
(c) 9 : 7 (d) 27 : 23

388. If V be the volume of a cuboid of dimension x, y, z and A is its surface, then A/V will be equal to
(a) $x^2y^2z^2$
(b) 1/2(1/xy+1/xz+1/yz)
(c) None of these
(d) 1/xyz

389. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if the height of conical part is 12 cm.
(a) 1440 cm² (b) 385 cm²
(c) 1580 cm² (d) 770 cm²

390. A solid wooden toy is in the form of a cone mounted on a hemisphere. If the radii of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of wood used in the toy.
(a) 353.72 cm³ (b) 266.11 cm³
(c) 532.22 cm³ (d) 133.55 cm³

391. A cylindrical container whose diameter is12 cm and height is 15 cm, is filled with ice cream. The whole ice-cream is distributed to 10 children in equal cones having hemispherical tops. If the height of the conical portion is twice the diameter of the cylindrical container its base, find the diameter of the ice-cream cone.
(a) 6 cm (b) 13 cm
(c) 3 cm (d) 18 cm

392. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the total surface area of the solid. (Use = 22/7).
(a) 398.75 cm² (b) 418 cm²
(c) 444 cm² (d) 412 cm²

393. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. What is the ratio of their volumes?
(a) 2 : 1 : 3 (b) 2.5 : 1 : 3
(c) 1 : 2 : 3 (d) 1.5 : 2 : 3

394. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost of painting 1 cm² of the surface is ` 0.05. Find the total cost of painting the vessel all over. (Take π = 22/7)
(a) ₹ 97.65 (b) ₹ 86.4
(c) ₹ 184 (d) ₹ 96.28

395. A cyclindrical cane whose base is horizontal is of internal radius 3.5 cm contain sufficient water so that when a solid sphere of max. size is placed, water just immersed it. Calculate the depth of water in the cane before the sphere was put.
(a) $\frac{5}{2}$ (b) $\frac{7}{3}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$

396. A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. Their common diameter is 3.5 cm and the heights of conical and cylindrical portion are respectively 6 cm and 10 cm. Find the volume of the solid. (Use = 3.14)
(a) 117 cm² (b) 234 cm²
(c) 58.5 cm²
(d) None of these

397. A right elliptical cylinder full of petrol has its widest elliptical side 2.4m and the shortest 1.6m. Its height is 7m. Find the time required to empty half the tank through a hose of diameter 4cm if the rate of flow of petrol is 120 m/min
(a) 60 min (b) 90 min
(c) 75 min (d) 70 min

398. The radius of a right circular cylinder is increased by 50%. Find the percentage increase in volume
(a) 120% (b) 75%
(c) 150% (d) 125%

399. Water flows out at the rate of 10m/min from a cylindrical pipe of diameter 5 mm. Find the time taken to fill a conical tank whose diameter at the surface is 40 cm and depth 24 cm.
(a) 50 min (b) 102.4 min
(c) 51.2 min (d) 25.6 min

400. The section of a solid right circular cone by a plane containing vertex and perpendicular to base is an equilateral triangle of side 12 cm. Find the volume of the cone.
(a) 72 cm³ (b) 144 cm³
(c) 74 cm³ (d) $72\sqrt{3}\pi$ cm³

401. Iron weight 8 times the weight of oak. Find the diameter of an iron ball whose weight is equal to that of a ball of oak 18 cm in diameter.
(a) 4.5 cm (b) 9 cm
(c) 12 cm (d) 15 cm

402. A piece of squared timber is 7 metres long and 0.1 metre both in width and thickness. What is its weight at the rate of 950 kg per cubic metres?
(a) 66 kg (b) 67 kg
(c) 66.5 kg (d) 68.5 kg

403. How many cubic metres of masonry are there in a wall 81

metres long, 4 metres high and 0.2 metre thick.
(a) 64.8 cub m (b) 69 cub m
(c) 68 cub m (d) 68.9 cub m

404. A river 10 metres deep and 200 metres wide is flowing at the rate of $4\frac{1}{2}$ km/hr. Find how many cubic m of water run into the sea per second.
(a) 2500 cub metres
(b) 2000 cub metres
(c) 2200 cub metres
(d) None of these

405. A cistern is constructed to hold 200 litres, and the base of the cistern is a square metre. what is the depth of the cistern? A cubic metre is 1000 litres.
(a) 50 cm (b) 20 cm
(c) 25 cm (d) 40 cm

406. A field is 500 metres long and 30 metres broad and a tank 50 metres long, 20 metres broad and 14 metres deep is dug in the field, and the earth taken out of it is spread evenly over the field, How much is the level of the field raised?
(a) 0.5 m (b) 1.5 m
(c) 1 m (d) 2 m

407. Find the volume and surface area of a cube, whose each edge measures 25 cm.
(a) 15265 cu cm, 3750 sq cm
(b) 15625 cu cm, 2500 sq cm
(c) 15625 cu cm, 3850 sq cm
(d) Data inadequate

408. The three co-terminus edges of a rectangular solid are 36, 75 and 80 cm respectively. Find the edge of a cube which will be of the same capacity?
(a) 70 cm (b) 36 cm
(c) 60 cm
(d) Data inadequate

409. A cube of metal each edge of which measures 5 cm, weighs 0.625 kg. What is the length of each edge of a cube of the same metal which weighs 40 kg?
(a) 20 cm (b) 25 cm
(c) 15 cm (d) 30 cm

410. The sum of the radius of the base and the height of a solid cylinder is 37 m. If the total surface area of the cylinder be 1628 sq m, find the volume.
(a) 4620 cu m (b) 4630 cu m
(c) 4520 cu m (d) 4830 cu m

411. If the diameter of the base of a closed right circular cylinder is equal to its height h, then its whole surface area is:
(a) $2\pi h^2$ (b) $\frac{4}{3}\pi h^2$
(c) $\frac{3}{2}\pi h^2$ (d) πh^2

412. How many bullets can be made out of a cube of lead whose edge measures 22 cm, each bullet being 2 cm in diameter?
(a) 5324 (b) 2662
(c) 1347 (d) 2541

413. A cylindrical vessel 60 cm in diameter is partially filled with water. A sphere, 30 cm in radius is gently dropped into the vessle. To what further height will water in the cylinder rise?
(a) 15 cm
(b) 30 cm
(c) 40 cm
(d) Can't be determined

414. The difference between the outside and inside surface of a cylindrical metallic pipe, 14cm long, is $44cm^2$. If the pipe is made of $99cm^3$ of metal. Find the outer radii of the pipe?
(a) 2cm (b) 2.5cm
(c) 4cm (d) 5cm

415. How many bullets can be made out of a cube of lead whose edge measures 22 cm, each bullet being 2 cm in deameter?
(a) 2341 (b) 2641
(c) 2541 (d) 2451

416. A right cylindrical vessel is full with water. How many right cones having same diameter and height as those of right cylinder will be needed to store that water?
(a) 2 (b) 3 (c) 4 (d) 5

417. An open rectgangular cistern when measured from out side is 1 m 35 cm long; 1 m 8 cm broad and 90 cm deep, and is made of iron 2.5 cm thick. Find (i) the capacity of the cistern, (ii) the volume of the iron used.
(a) 1171625 cu cm, 140575 cu cm
(b) 1711625 cu cm, 104575 cu cm
(c) 1171625 cu cm, 145075 cu cm
(d) None of these

418. Find the weight of a lead pipe 3.5 metres long, if the external diameter of the pipe is 2.4 cm and the thickness of the lead is 2 mm and 1 cc of lead weight 11.4 gm.
(a) 5.5 kg (b) 5 kg
(c) 8 kg (d) 10 kg

419. A closed rectangular box has inner dimensions 24 cm by 12 cm by 10 cm. Calculate its capacity and the area of tin foil needed to line its inner surface.
(a) 2680 cu cm, 1296 sq cm
(b) 2880 cu cm, 1396 sq cm
(c) 2880 cu cm, 1296 sq cm
(d) 2860 cu cm, 1296 sq cm

420. The dimension of an open box are 52 cm, 40 cm and 29 cm. Its thickness is 2 cm. If 1 cm^3 of metal used in the box weight 0.5 gm, the weight of the box is:
(a) 8.56 kg (b) 7.76 kg
(c) 7.756 kg (d) 6.832 kg

421. Half cubic metre of gold sheet is extended by hammering so as to cover an area of 1 hectare. Find the thickness of the gold.
(a) 0.05 cm (b) 0.5 cm
(c) 0.005 cm (d) 0.0005 cm

422. Two cubic metres of gold are extended by hammering so as to cover an area of twelve hectares. Find the thickness of gold.
(a) 0.017 cm (b) 0.0017 cm
(c) 1.7 cm (d) 0.17 cm

423. A cub of silver is drawn into a wire $\frac{1}{10}$ mm in diameter, find the length of the wire.
(π = 3.1416)
(a) 128 metres (b) 127.3 metres
(c) 129.3 metres (d) 128.3 metres

424. A hollow cylindrical tube open at both ends ins made of iron 4 cm thick. If the internal diameter be 40 cm and the length of the tube be 144 cm, find the volume of iron in it.
(a) 25344π (b) 23544π
(c) 26344π (d) None of these

425. A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the internal diameter be 33 cm and the length of the tube be 70 cm, find the volume of iron in it.
(a) 12400 cu cm
(b) 15400 cu cm
(c) 13800 cu cm
(d) 16400 cu cm

426. One cm of rain has fallen on 2 square km of land. Assuming that 25% of the raindrops could have been collected and contained in a pool having a 50 m × 5 m base, by what level would the water level in the pool have in creased?
(a) 20 m (b) 40 m
(c) 25 m
(d) Data inadequate

427. Two cm of rain has fallen on a square km of land. Assuming that 40% of the raindrops could have been collected and contained in a pool having a 200 m × 20 m base, by what level would the water level in the pool have incrased?
(a) 2 m (b) 1 m
(c) 4 m (d) 1.5 m

428. The length of a tank is thrice that of breadth, which is 256 cm deep and holds 3000 L water. What is the base area of the tank? (1000 L = 1 cubic metre)
(a) 111775 m^2 (b) 1171.875 m^2
(c) 1.171875 m^2
(d) None of these

429. A lid of rectangular box of sides 39.5 cm by 9.35 cm by 9.35 cm is sealed all around with tape such that there is an ovrelapping of 3.75 cm of the tape. What is the length of the tape used?
(a) 111.54 cm (b) 101.45 cm
(c) 110.45 cm (d) None of these

430. A cistern from inside is 12.5 m long, 8.5 m broad and 4m high and is open at top. Find the cost of cementing the inside of a cistern at Rs. 24 per sq. m:
(a) Rs. 6582 (b) Rs. 8256
(c) Rs. 7752 (d) Rs. 8752

431. 250 men took a dip in a water tank at a time, which is 80m × 50m. What is the rise in the water level if the average displacement of 1 man is 4 m^3?
(a) 22 cm (b) 25 cm
(c) 18 cm (d) 30 cm

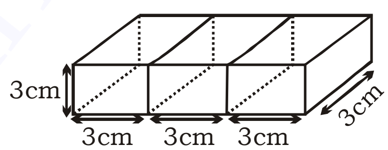
432. The edge of a cube is increased by 100% the surface area of the cube is increased by:
(a) 100% (b) 200%
(c) 300% (d) 400%

433. The external dimensions of a wooden box closed at both ends are 24 cm, 16 cm and 10 cm respectively and thickness of the wood is 5 mm. If the empty box weighs 7.35 kg, find the weight of 1 cubic cm of wood:
(a) 10 g (b) 12.5 g
(c) 27 g (d) 15 g

434. The internal dimensions of a tank are 12 dm, 8 dm and 5 dm. How many cubes each of edge 7 cm can be placed in the tank with faces parallel to the sides of the tank. Find also, how much space is left unoccupied?
(a) 35 ; 113 dm^3
(b) 1313 ; 31.13 dm^3
(c) 35 ; 31.013 dm^3
(d) 1309 ; 13.31 dm^3

435. The length, breadth and height of box are 2 m, 1.5 m and 80 cm respectively. What would be the cost of canvas to cover it up fully, if one square metre of canvas costs Rs. 25.00?
(a) Rs. 260 (b) Rs. 290
(c) Rs. 285
(d) None of these

436. Three cubes each of edge 3 cm long are placed together as shown in the adjoining digure. Find the surface area of the cuboid so formed:



(a) 182 sq. cm (b) 162 sq. cm
(c) 126 sq. cm
(d) None of these

437. A room is 36 m long, 12 m wide and 10 m high. It has 6 window, each 3 m × 2.5 m; one door 9.5 m × 6 m and one fire chimney 4 m × 4.5 m. Find the expenditure of papering its walls at the rate of 70 paise per metre, if the width of the paper is 1.2 m:
(a) Rs. 490 (b) Rs. 690
(c) Rs. 1000
(d) None of these

438. When each side of a cube is increased by 2 cm, the volume is increased by 1016 cm^3. Find the side of the cube. If each side of it is decreased by 2 cm, by how much will the volume decrease?
(a) 12 cm, 729 cm^3
(b) 8 cm, 512 cm^3
(c) 9 cm, 729 cm^3
(d) 12 cm, 728 cm^3

439. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the resulting cuboid to that of the total surface areas of the three cubes:
(a) 5 : 7 (b) 7 : 9
(c) 9 : 7 (d) None of these

440. A hollow square shaped tube open at both ends is made of iron. The internal square is of 5 cm side and the length of the tube is 8 cm. There are 192 cm^3 of iron in the tube. Find the thickness:
(a) 2 cm (b) 0.5 cm
(c) 1 cm
(d) can't be determined

441. A cube of 11 cm edge is immersed completely in a rectangular vessel containing water. If the dimesions of base are 15 cm and 12 cm. Find the rise in water level in the vessel:
(a) 6.85 cm (b) 7 cm
(c) 7.31 cm (d) 7.39 cm

442. A rectanular tank 25 cm long and 20 cm wide contains water to a depth of 5 cm. A metal cube of side 10 cm is placed in the tank so that one face of the cube rests on the bottom of the tank. Find how many litres of water must be poured into the tank so as to just cover the cube?
(a) 1 L (b) 1.5 L
(c) 2 L (d) 2.5 L

443. A rectangular block has length 10 cm, breadth 8 cm and height 2 cm. From this block, a cubical hole of side 2 cm is drilled out. Find the volume and the surface area of the remaining solid:
(a) 152 cm^2, 512 cm^2
(b) 125 cm^2, 215 cm^2
(c) 152 cm^2, 240cm^2
(d) 125 cm^2, 512 cm^2

444. How many bricks (number near to next hundred) will be required to build a wall 30 m long, 30 cm thick and 5m high with a provision of 2doors, each 4 m × 2.5 m and each brick being 20 cm × 16 cm × 8 cm when one-ninth of the wall is filled with lime?
(a) 13500 bricks
(b) 13600 bricks
(c) 20050 bricks
(d) 18500 bricks

445. A rectangular water reservoir is 15 m by 12 m at the base. Water flows into it through a pipe whose cross-section is 5 cm by 3 cm at the rate of 16 m/s second. Find the height to which the water will rise in the reservoir in 25 minutes:
(a) 0.2 m (b) 2 cm
(c) 0.5 m
(d) None of these

446. The volume of a wall, 3 times as high as it is broad and 8 times as long as it is high, is 36.864 m^3. The height of the wall is:
(a) 1.8 m (b) 2.4 m
(c) 4.2 m
(d) None of these

447. If the areas of 3 adjacent sides of a cuboid are x, y, z respectively, then the voulum of the cuboid is:
(a) xyz (b) $\sqrt{xyz}$
(c) $3xyz$
(d) None of these

448. A cylindrical cistern whose diameter is 21 cm is partly filled with water. If a rectangular block of iron 14 cm in length, 10.5 cm in breadth and 11 cm in thickness is wholly immersed in water, by how many centimetres will the water level rise?
(a) 14 cm (b) 20 cm
(c) $\frac{14}{3}$ cm (d) 12 cm

449. A right circular cylindrical tunnel of diameter 4 m and length 10 m is to be constructed from a sheet of iron. The area of the iron sheet required.
(a) $\frac{280}{\pi}$ (b) 40π
(c) 80π (d) None of these

450. A conical vessel has a capacity of 15 L of milk. Its height is 50 cm and base radius is 25 cm. How much milk can be contained in a vessel in cylindrical form having the same dimensions as that cone?
(a) 15 L (b) 30 L
(c) 45 L (d) None of these

451. The height of a metric cylinder is 14cm & the different of its in curved S.A. is 44 cm^2. If the cylinder is made up of 99 cm^3 metal the find the inner & outer radius of cylinder.
(a) 464 (b) 564
(c) 660 (d) 366

452. If the base radius and the height of a right circular cone are increased by 40% then the percentage increase in volume (approx) is:
(a) 175% (b) 120%
(c) 64% (d) 540%

453. From a circular sheet of paper of radius 25 cm, a sector area 4% is removed. If the remaining part is used to make a conical surface, then the ratio of the radius and height of the cone is:
(a) 16 : 25 (b) 9 : 25
(c) 7 : 12 (d) 24 : 7

454. A conical tent has 60° angle at the vertex. The ratio of its radius and slant height is:
(a) 3 : 2 (b) 1 : 2
(c) 1 : 3
(d) can't be determined

455. Water flows at the rate of 5 m per min from a cylindrical pipe 8 mm in radius. How long will it take to fill up a conical vessel whose radius is 12 cm and depth 35 cm?
(a) 315 s (b) 365 s
(c) 5 min (d) None of these

456. A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across at the bottom. It is 6 m deep its capacity is:
(a) 224 m^3 (b) 176 m^3
(c) 225 m^3 (d) None of these

457. A conical vessel whose internal radius is 10 cm and height 72 cm is full of water. If this water is poured into a cylindrical vessel with internal radius 30 cm, the height of the water level rises in it is:
(a) $2\frac{2}{3}$ cm (b) $3\frac{2}{3}$ cm
(c) $5\frac{2}{3}$ cm (d) None of these

458. If h, c, v are respectively the height, the curved surface area and the volume of a cone then the value of $3\pi vh^3 - c^2h^2 + 9v^2$ is equal to :
(a) 1 (b) 2
(c) 0
(d) None of these

459. If P is the height of a tetra hedron & each side is of 2cm the find the value of $3p^2$.
(a) $6a^2$ (b) $8a^2$ (c) $5a^2$ (d) $7a^2$

460. If 'h' be the height of a pyramid standing on a base which is an equilateral trinagle of side 'a' units, then the slant edge is:
(a) $\sqrt{h^2+a^2/4}$ (b) $\sqrt{h^2+a^2/8}$
(c) $\sqrt{h^2+a^2/3}$ (d) $\sqrt{h^2+a^2}$

461. Find the volume of a tetrahedren whose height is $4\sqrt{3}$ cm.
(a) 72 (b) 108 (c) 54 (d) 36

462. In a shower 10 cm of rain fall the volume of water that falls on 1.5 hectares of ground is:
(a) 1500 m^3 (b) 1400 m^3
(c) 1200 m^3 (d) 1000 m^3

463. Find the total surface area of pyramid of heights 12 which is based on a rectangle of length 18 and & breadth 10cm.
(a) 117 (b) 564 (c) 120 (d) 456

464. If from a circular sheet of paper of radius 15 cm, a sector of 144° is removed and the remaining is used to make a conical surface, then the angle at the vertex will be:
(a) $\sin^{-1}\left(\frac{3}{10}\right)$ (b) $\sin^{-1}\left(\frac{6}{5}\right)$
(c) $2\sin^{-1}\left(\frac{3}{5}\right)$ (d) $2\sin^{-1}\left(\frac{4}{5}\right)$

465. Find the length of the string bound on a cylindrical tank whose base diameter and height are $5\frac{1}{11}$ cm and 48cm

respectively. The string makes exactly four complete turns round the cylinder, while its two ends touch the tank's top and bottom :
(a) 75cm (b) 70cm
(c) 60cm (d) 80cm

466. A cone, a hemisphere and a cylinder stand on equal bases of radius R and have equal heights H. Their whole surfaces are in the ratio:
(a) $(\sqrt{3}+1):3:4$
(b) $(\sqrt{2}+1):7:8$
(c) $(\sqrt{2}+1):3:4$
(d) None of these

467. If l, b, p be the length, breadth and perimeter of a rectangle and b, l, p are in GP (in order), then l/b is:
(a) 2 : 1
(b) $(\sqrt{3}-1):1$
(c) $(\sqrt{3}+1):1$
(d) can't be determined

468. The height of a circular cylinder is increased by 6 times and base area is decreased by 1/9th times. By what factor its lateral surface area is increased ?
(a) 2 (b) 3 (c) 6 (d) 1.5

469. A pyramid with an equal Δ based of each side 4 cm while its slant height is twice the height of pyramid. Find its volume-
(a) $8\sqrt{3}$ (b) $\frac{8}{3\sqrt{3}}$
(c) $4\sqrt{3}$ (d) $3\sqrt{3}$

470. A cube and a sphere have equal surface areas. The ratio of their volume is :
(a) p : 3 (b) $\sqrt{\pi}:\sqrt{6}$
(c) $\sqrt{6}:\sqrt{\pi}$ (d) $6:\pi$

471. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their respective volumes is :
(a) 1 : 2 : 3 (b) 2 : 1 : 3
(c) 1 : 3 : 2 (d) 3 : 1 : 2

472. A cube of sides 3cm is melted and smaller cubes of sides 1cm each are formed. How many such cubes are possible?
(a) 21 (b) 23 (c) 25 (d) 27

473. A cuboidal block of $6cm \times 9cm \times 12cm$ is cut into exact number of equal cubes. The least possible number of cubes will be :
(a) 6 (b) 9 (c) 24 (d) 30

474. If the cone is cut along its axis from the middle, the new shape we obtain after opening the paper is :
(a) isosceles triangle
(b) right angle triangle
(d) equilateral triangle
(d) None of these

475. What is the height of the cone which is formed by joining the two ends of a sector of circle with radius r and angle 60° :
(a) $\frac{\sqrt{35}}{6}r$ (b) $\frac{\sqrt{25}}{6}r$
(c) $\frac{r^2}{\sqrt{3}}$ (d) $\frac{35}{6}r$

476. A sphere of 20cm radius is dropped into a cylindrical vessel of 60cm diameter, which is partly filled with water, then its level rises by x cm. Find x:
(a) $11\frac{21}{27}$cm (b) 12cm
(c) 22.5cm (d) $11\frac{23}{27}$cm

477. A right circular cone resting on its base is cut at $\frac{4}{5}th$ its height along a parallel to the circular base. The height of original cone is 75cm and base diameter is 42cm. What is the base radius of cut out (top portion) cone ?
(a) 4.2cm (b) 8.4cm
(c) 2.8cm (d) 3.5cm

478. A solid sphere is melted and recast into a right circular cone with a base radius equal to the radius of the sphere. What is the ratio of the height and radius of cone so formed ?
(a) 5 : 2 (b) 4 : 3
(c) 4 : 1 (d) 3 : 2

479. 125 identical cubes are cut from a big cube and all the smaller cubes are arranged in a row to form a long cuboid. What is the percentage increase in the total surface area of the cuboid over the total surface area of the cube ?
(a) $234\frac{1}{3}\%$ (b) $234\frac{2}{3}\%$
(c) 117% (d) None of these

480. What is the total surface area of the identical cubes of largest possible size that are cut from a cuboid of size 75cm × 15cm × 4.5cm ?
(a) 20, 250cm^2 (b) 20, 520cm^2
(c) 22, 250cm^2 (d) None of these

481. If the volume of a sphere, a cube, a tetrahedron and a octahedron be same then which of the following has maximum surface area ?
(a) sphere (b) cube
(c) tetrahedron (d) octahedron

482. A spherical ball of lead 6cm in radius is melted and recast into three spherical balls. The radii of two of these balls are 3cm and 4cm. What is the radius of the third sphere ?
(a) 6cm (b) 6.5cm
(c) 5.5cm (d) 5cm

483. The base of a right prism is a triangle whose perimeter is 45cm and the radius of incircle is 9cm. If the volume of the prism is 810cm^3. Find its height :
(a) 5cm (b) 4cm
(c) 6cm (d) 4.5cm

484. What is the semi-vertical angle of a cone whose lateral surface area is double the base area ?
(a) 30° (b) 45°
(c) 60° (d) None of these

485. What is the number of cones of semi-vertex angle α and having r as the radius of the midsection which can be moduled out of a cylinder of base radius r and height 2r cotα :
(a) 5 (b) 7 (c) 6 (d) 4

486. A water tank is 30cm long, 20cm wide and 12m deep. It is made of iron sheet which is 3m wide. The tank is open at the top. If the cost of iron sheet is ₹ 10per meter. Find the total cost of iron required to build the tank ?
(a) ₹ 6000 (b) ₹ 5000
(c) ₹ 5500 (d) ₹ 5800

487. A trapezium based prism with two parallel sides 8cm and 14cm respectively and distance between two parallel sides is 8cm. Find the height of the prism if the volume of the prism is 1056cm^3 ?
(a) 11cm (b) 10cm
(c) 9cm (d) 12cm

488. From a circular sheet of paper, radius 10cm, A sector of area 40% of the sheet is removed. If the remaining part is used to makde a conical surface. Then the ratio of radius and height will be :
(a) 4 : 3 (b) 3 : 4
(c) 2 : 3 (d) 2 : 1

489. If the volume of circular cell having inner and outer radius 8cm and 12cm respectively is equal to the total surface area of cylinder of radius R_1 and height h, then h in terms of R_1 will be :
(a) $\frac{40-R_1}{R_1}$ (b) $\frac{40-R_1}{R_1{}^2}$
(c) $\frac{40-R_1{}^2}{R_1}$ (d) None of these

490. An iron pipe 20cm long has exterior diameter 25cm. If the thickness of the pipe is 1cm, then the whole surface area of the pipe ?
(a) 3168cm^2 (b) 3186cm^2
(c) 3200cm^2 (d) 3150cm^2

491. The capacity of two hemispherical bowls are 64 litre and 216 litre respectively. Then the ratio of their internal curved surface area will be:
(a) 2 : 3 (b) 1 : 3
(c) 16 : 81 (d) 4 : 9

492. If the length of a rectangular parallel pipe is three times of its breadth and five times of its height. If its volume is 14400cm^3, then the total surface area will be :
(a) 4230cm^2 (b) 4320cm^2
(c) 4203cm^2
(d) None of these

493. A right angled triangle with its sides 5cm, 12cm and 13cm is revolved about the side 12cm. Find the volume of the solid formed ?
(a) 942cm^3 (b) 298cm^3
(c) 314cm^3 (d) 302cm^3

494. A hemisphere bowl V_1 and a hollow right circular cylinder V_2 (having length equal to its radius) have the same diameter equal to the length of a side of a hollow cubical box V_3. Water is filled in all these vessels upto the same level and such that hemispherical bowl is full of water and the volumes of filled water are v_1, v_2 and v_3 respectively in V_1, V_2 and V_3then :
(a) $V_1<V_2<V_3$ (b) $V_2<V_3<V_1$
(c) $V_3<V_2<V_1$ (d) $V_3<V_1<V_2$

495. A vertical cone of volume V with vertex downward is filled with water up to half of its height. The volume of the water is :
(a) V/16 (b) V/8
(c) V/4 (d) V/2

496. The heights of a cone, cylinder and hemisphere are equal. If their radii are in the ratio 2 : 3 : 1, then the ratio of their volumes is :
(a) 2 : 9 : 2 (b) 4 : 9 : 1
(c) 4 : 27 : 2 (d) 2 : 3 : 1

497. The height of a right circular cone and the radius of its circular base are respectively 9 cm and 3 cm. The cone is cut by a plane parallel to its base so as to divide it into two parts. The volume of the frustum (i.e., the lower part) of the cone is 44 cubic cm. The radius of the upper circular surface of the frustum $\left(take\ \pi=\frac{22}{7}\right)$ is :
(a) $\sqrt[3]{12}$ cm (b) $\sqrt[3]{13}$ cm
(c) $\sqrt[3]{13}$ cm (d) $\sqrt[3]{20}$ cm

498. A solid cylinder has total surface area of 462 sq. cm. Curved surface area is $\frac{1}{3}$rd of its total surface area. The volume of the cylinder is :
(a) 530 cm^3 (b) 536 cm^3
(c) 539 cm^3 (d) 545 cm^3

499. A solid is hemispherical at the bottom and conical above. If the surface areas of the two parts are equal, then the ratio of radius and height of its conical part is :
(a) 1 : 3 (b) 1 : 1
(c) $\sqrt{3}$: 1 (d) 1 : $\sqrt{3}$

500. The radius and the height of a cone are in the ratio 4 :3 . The ratio of the curved surface area and total surface area of the cone is :
(a) 5 : 9 (b) 3 : 7
(c) 5 : 4 (d) 16 : 9

501. From a right circular cylinder of radius 10 cm and height 21 cm, a right circular cone of same base- radius is removed. If the volume of the remaining portion is 4400 cm^3 then the height of the removed cone is: $\left(take\ \pi=\frac{22}{7}\right)$
(a) 15 cm (b) 18 cm
(c) 21 cm (d) 24 cm

502. A right circular cylinder and a cone have equal base radius and equal heights. If their curved surfaces are in the ratio 8 : 5, then the radius of the base to the height are in the ratio :
(a) 2 : 3 (b) 4 : 3
(c) 3 : 4 (d) 3 : 2

503. The curved surface area of a cylindrical pillar is 264 sq.m. and its volume is 924 cu.m. The ratio of its diameter to height is :
(a) 3: 7 (b) 7 : 3
(b) 6 : 7 (d) 7 : 6

504. A cube of edge 6 cm is painted on all sides and then cut into unit cubes. The number of unit cubes with no sides painted is:
(a) 0 (b) 64 (c) 186 (d) 108

505. There is a pyramid on a base which is a regular hexagon of side 2a cm. If every slant edge of this pyramid is of length $\frac{5a}{2}$ cm, then the volume of this pyramid is :
(a) $3a^3cm^3$ (b) $3\sqrt{2}a^3cm^3$
(c) $3\sqrt{3}a^3cm^3$ (d) $6a^3cm^3$

506. The base of a right prism is an equilateral triangle of area 173 cm^2 and the volume of the prism is 10380 cm^3. The area of the lateral surface of the prism is $use\left(\sqrt{3}=1.73\right)$
(a) 1200 cm^2 (b) 2400 cm^2
(c) 3600 cm^2 (d) 4380 cm^2

507. Three spherical balls of radii 1 cm, 2 cm and 3 cm are melted to form a single spherical ball. In the process, the loss of material is 25 %. The radius of the new ball is :
(a) 6 cm (b) 5 cm
(c) 3 cm (d) 2 cm

508. The height of a cone is 40cm. The cone is cut parllel to its base such that the volume of the small cone is $\frac{1}{64}$ of the cone. Find at which height from base the cone is cut ?
(a) 20cm (b) 30cm
(c) 25cm (d) 22.5cm

509. A cube of side 8 metre is reduced 3 times in the ratio 2 : 1. The area of one face of the reduced cube to that of the original cube is in the ratio :
(a) 1 : 4 (b) 1 : 8
(c) 1 : 16 (d) 1 : 64

510. The volume of the largest cylinder formed, when a rectangular sheet of paper of size 22 cm $\times$ 15cm is rolled along its larger side, is $\left(use\ \pi = \frac{22}{7}\right)$:
(a) 288.75 cm^3 (b) 577.50 cm^3
(c) 866.25 cm^3 (d) 1155.00 cm^3

511. Each of the height and radius of the base of a right circular cone is increased by 100 %. The volume of the cone will be increased by :
(a) 700 % (b) 500 %
(c) 300 % (d) 100 %

512. The height of a right prism with a square base is 15cm. If the area of the total surfaces of the prism is 608 sq.cm, its volume is :
(a) 910 cm^3 (b) 920 cm^3
(c) 960 cm^3 (d) 980 cm

513. The internal radius and thickness of a hollow metallic pipe are 24 cm and 1 cm respectively. It is melted and recast into a solid cylinder of equal length. The diameter of the solid cylinder will be :
(a) 7 cm (b) 14 cm
(c) 960 cm^3 (d) 980 cm

514. The radius of the base of a right circular cone is doubled. To keep the volume fixed, the height of the cone will be
(a) One-fourth of the previous height
(b) $\frac{1}{\sqrt{2}}$ times of the prevoious height
(c) half of the previous height
(d) one - third of the previous height

515. If a cube maximum possible volume is cut off from a solid sphere of diameter d, then the volume of the remaining (waste) material of the sphere would be equal to :
(a) $\frac{d^3}{3}\left(\pi - \frac{d}{2}\right)$ (b) $\frac{d^3}{3}\left(\frac{\pi}{2} - \frac{1}{\sqrt{3}}\right)$
(c) $\frac{d^2}{4}\left(\sqrt{2} - \pi\right)$ (d) None of these

516. A big cube of side 9cm is formed by rearranging together 27 small but identical cubes each of side 3cm. further, if the corner cubes in the top most layer of the big cube are removed, what is the change in total surface area of the big cube?
(a) 18cm^2, decreases
(b) 54cm^2, decreases
(c) 36cm^2, decreases
(d) remains the same

517. The base radius and height of a cone is 5cm and 25cm respectively. If the cone is cut parallel to its base at a height of h from the base. If the volume of this frustum is 110cm^3. Find the radius of smaller cone ?®
(a) $(104)^{1/3}\,cm$ (d) $(104)^{1/2}\,cm$
(c) 5cm (d) None of these

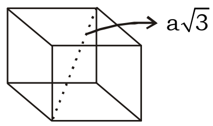
518. A spherical steel ball was silver polished then it was cut into 4 similar pieces. What is the ratio of the polished area to the non-polished area :
(a) 1 : 1 (b) 1 : 2
(c) 2 : 1
(d) None of these

ANSWER KEY

1. (a)	53. (d)	105.(a)	157.(a)	209.(c)	261.(d)	313.(a)	365.(b)	417.(a)	469.(b)
2. (c)	54. (b)	106.(a)	158.(d)	210.(d)	262.(b)	314.(c)	366.(b)	418.(a)	470.(b)
3. (c)	55. (c)	107.(a)	159.(c)	211.(d)	263.(d)	315.(c)	367.(a)	419.(c)	471.(a)
4. (b)	56. (c)	108.(a)	160.(a)	212.(d)	264.(c)	316.(b)	368.(d)	420.(d)	472.(d)
5. (b)	57. (c)	109.(b)	161.(d)	213.(b)	265.(a)	317.(d)	369.(d)	421.(c)	473.(c)
6. (d)	58. (d)	110.(b)	162.(c)	214.(a)	266.(a)	318.(b)	370.(c)	422.(b)	474.(b)
7. (a)	59. (b)	111.(d)	163.(a)	215.(d)	267.(a)	319.(b)	371.(a)	423.(b)	475.(a)
8. (c)	60. (d)	112.(a)	164.(d)	216.(a)	268.(c)	320.(a)	372.(d)	424.(a)	476.(d)
9. (c)	61. (b)	113.(b)	165.(a)	217.(b)	269.(d)	321.(a)	373.(d)	425.(b)	477.(a)
10. (a)	62. (a)	114.(b)	166.(c)	218.(d)	270.(b)	322.(d)	374.(c)	426.(a)	478.(c)
11. (d)	63. (a)	115.(d)	167.(d)	219.(a)	271.(c)	323.(a)	375.(b)	427.(a)	479.(b)
12. (b)	64. (b)	116.(c)	168.(b)	220.(a)	272.(a)	324.(c)	376.(c)	428.(c)	480.(a)
13. (a)	65. (b)	117.(c)	169.(d)	221.(a)	273.(a)	325.(b)	377.(c)	429.(b)	481.(c)
14. (d)	66. (a)	118.(a)	170.(a)	222.(d)	274.(c)	326.(b)	378.(d)	430.(a)	482.(d)
15. (c)	67. (c)	119.(b)	171.(c)	223.(a)	275.(c)	327.(c)	379.(a)	431.(b)	483.(b)
16. (b)	68. (b)	120.(b)	172.(b)	224.(c)	276.(c)	328.(b)	380.(b)	432.(c)	484.(a)
17. (a)	69. (b)	121.(c)	173.(b)	225.(a)	277.(a)	329.(b)	381.(c)	433.(a)	485.(c)
18. (c)	70. (b)	122.(a)	174.(b)	226.(a)	278.(d)	330.(d)	382.(c)	434.(c)	486.(a)
19. (d)	71. (b)	123.(b)	175.(d)	227.(d)	279.(c)	331.(b)	383.(b)	435.(b)	487.(d)
20. (c)	72. (b)	124.(c)	176.(d)	228.(a)	280.(c)	332.(c)	384.(a)	436.(c)	488.(b)
21. (b)	73. (c)	125.(c)	177.(c)	229.(d)	281.(b)	333.(a)	385.(c)	437.(a)	489.(c)
22. (b)	74. (c)	126.(c)	178.(d)	230.(c)	282.(c)	334.(c)	386.(d)	438.(d)	490.(a)
23. (b)	75. (b)	127.(c)	179.(a)	231.(c)	283.(c)	335.(c)	387.(a)	439.(b)	491.(d)
24. (d)	76. (b)	128.(c)	180.(a)	232.(b)	284.(b)	336.(b)	388.(c)	440.(c)	492.(b)
25. (c)	77. (c)	129.(a)	181.(b)	233.(a)	285.(b)	337.(b)	389.(d)	441.(d)	493.(c)
26. (d)	78. (d)	130.(c)	182.(a)	234.(b)	286.(a)	338.(c)	390.(b)	442.(b)	494.(a)
27. (c)	79. (b)	131.(a)	183.(a)	235.(c)	287.(d)	339.(a)	391.(a)	443.(c)	495.(b)
28. (b)	80. (c)	132.(c)	184.(d)	236.(a)	288.(d)	340.(a)	392.(b)	444.(b)	496.(c)
29. (d)	81. (c)	133.(a)	185.(d)	237.(b)	289.(a)	341.(c)	393.(c)	445.(a)	497.(b)
30. (b)	82. (b)	134.(a)	186.(a)	238.(a)	290.(d)	342.(d)	394.(d)	446.(b)	498.(c)
31. (a)	83. (b)	135.(d)	187.(b)	239.(d)	291.(a)	343.(b)	395.(b)	447.(b)	499.(d)
32. (b)	84. (a)	136.(c)	188.(d)	240.(a)	292.(d)	344.(a)	396.(d)	448.(c)	500.(a)
33. (a)	85. (b)	137.(d)	189.(a)	241.(b)	293.(d)	345.(b)	397.(d)	449.(b)	501.(c)
34. (b)	86. (d)	138.(d)	190.(c)	242.(c)	294.(b)	346.(c)	398.(d)	450.(c)	502.(c)
35. (a)	87. (b)	139.(d)	191.(b)	243.(d)	295.(a)	347.(c)	399.(c)	451.(b)	503.(b)
36. (b)	88. (d)	140.(b)	192.(a)	244.(b)	296.(d)	348.(b)	400.(d)	452.(a)	504.(b)
37. (c)	89. (b)	141.(c)	193.(b)	245.(d)	297.(a)	349.(d)	401.(b)	453.(d)	505.(c)
38. (c)	90. (c)	142.(d)	194.(d)	246.(b)	298.(a)	350.(d)	402.(c)	454.(b)	506.(c)
39. (d)	91. (c)	143.(b)	195.(b)	247.(a)	299.(c)	351.(c)	403.(a)	455.(a)	507.(c)
40. (b)	92. (d)	144.(c)	196.(a)	248.(a)	300.(a)	352.(c)	404.(a)	456.(d)	508.(b)
41. (c)	93. (d)	145.(a)	197.(d)	249.(d)	301.(c)	353.(c)	405.(b)	457.(a)	509.(d)
42. (d)	94. (a)	146.(b)	198.(d)	250.(d)	302.(a)	354.(b)	406.(c)	458.(c)	510.(b)
43. (a)	95. (d)	147.(d)	199.(d)	251.(c)	303.(b)	355.(d)	407.(b)	459.(b)	511.(a)
44. (d)	96. (b)	148.(d)	200.(a)	252.(b)	304.(b)	356.(a)	408.(c)	460.(c)	512.(c)
45. (d)	97. (b)	149.(b)	201.(a)	253.(a)	305.(a)	357.(b)	409.(a)	461.(a)	513.(b)
46. (d)	98. (b)	150.(b)	202.(a)	254.(c)	306.(d)	358.(d)	410.(a)	462.(a)	514.(a)
47. (d)	99. (b)	151.(a)	203.(a)	255.(a)	307.(d)	359.(b)	411.(c)	463.(b)	515.(b)
48. (d)	100.(b)	152.(d)	204.(b)	256.(d)	308.(c)	360.(d)	412.(d)	464.(c)	516.(d)
49. (a)	101.(a)	153.(b)	205.(d)	257.(d)	309.(c)	361.(b)	413.(c)	465.(d)	517.(a)
50. (b)	102.(c)	154.(a)	206.(b)	258.(d)	310.(a)	362.(d)	414.(b)	466.(c)	518.(a)
51. (d)	103.(c)	155.(d)	207.(c)	259.(d)	311.(b)	363.(c)	415.(c)	467.(c)	
52. (b)	104.(c)	156.(a)	208.(b)	260.(b)	312.(c)	364.(d)	416.(b)	468.(a)	

SOLUTION

1. (a) Let the side of cube = a cm



Diagonal of cube $= a\sqrt{3}$ cm

$a\sqrt{3} = \sqrt{12}$

on squaring , $a^2(3) = 12$

$a^2 = 4$

$a = 2$ cm

volume of cube = $a^3 = 2^3 = 8$ cm

2. (c) Number of cubes = $\frac{(15)^3}{(3)^3}$ = **125**

3. (c) Side of the cube = $\frac{\text{Diagonal}}{\sqrt{3}}$

$= \frac{4\sqrt{3}}{\sqrt{3}} = 4$cm

Volume of the cube = (side)^3

$\Rightarrow (4)^3 = 4\times4\times4 =$ **64 cm³**

4. (b) Let $l = 9x$, $h = 3x$, $b = x$

$l \times b \times h = 216 \times 1000$

(1 litre = 1000 cm³)

$9x \times x \times 3x = 216000$

$27x^3 = 216000$

$x^3 = 8000$

$x = 20$

$l = 180$ cm = 18 dm

5. (b) Volume of Cuboid = 2 × volume Cube

$l \times b \times h = 2 \times (\text{side})^3$

$\frac{9\times8\times6}{2} = (\text{side})^3 = 216$

side = $\sqrt[3]{216}$ = 6 cm

Total surface area of cube

9. (c) Volume of the cistern

$\Rightarrow (330-10)\times(260-10)\times(110-x)$

$= 8000 \times 1000$

(where x = thickness of bottom)

$x = 110 - 100 = 10$ cm = 1 dm

10. (a) Let the length, breadth and height be l, b, h respectively

$\Rightarrow lb = x$

$bh = y$

$lh = z$

$\Rightarrow l^2b^2h^2 = xyz$

$(lbh)^2 = xyz$

$\Rightarrow v^2 = xyz$

11. (d) The diameter of sphere = side of cube = 7 cm

Radius (r)= $\frac{7}{2}$ cm

volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2}$

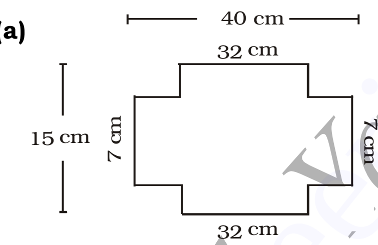
= 179.67 cm³

12. (b) Length of rod

$= \sqrt{10^2+10^2+5^2} = \sqrt{225}$

= 15cm

13. (a)



Volume of the box = $l \times b \times h$

= (40 – 8) × (15 – 8)× 4

= 32 × 7 × 4 = 896 cm³

14. (d) Let the three sides of the cuboid be l, b and h

$\Rightarrow lb = bh = hl = 12$

$\Rightarrow l^2b^2h^2 = 12 \times 12 \times 12$

$= 1728$

$\Rightarrow lbh = \sqrt{1728} = 12\sqrt{12}$

$= 24\sqrt{3}$ cm³

15. (c) dimensions of room

length(l) = 12 cm

breadth(b) = 9 cm

height(h) = 8 cm

∴ diagonal of cube = $\sqrt{l^2+b^2+h^2}$

∴ length of longest rod

⇒ length of diagonal of cuboid

$\Rightarrow \sqrt{12^2+9^2+8^2}$

$\Rightarrow \sqrt{144+81+64}$

$\Rightarrow \sqrt{289} = 17$ cm

16. (b) area of floor ⇒ 3 × 4 = 12 m²

height ⇒ 3 m

∴ area of walls of room

⇒ (Perimeter × height of of floor)

$\Rightarrow 2(l + b) \times h$

$\Rightarrow l$ = length = 4 m

b = breadth = 3 m

h = height = 3 m

∴ Area of walls ⇒ 2

(4 + 3) × 3 = 42 m²

Area of painted part

= 42 m² + 12m² = 54 m²

17. (a) Let

length = l,

breadth = b

height = h

given that (l + b + h) = 12 cm

= total surface area of box

= 2(lb + bh + hl) =94 m² (given)

$(l + b + h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + hl)$

$(12)^2 = l^2 + b^2 + h^2 + 94$

$144 - 94 \Rightarrow l^2 + b^2 + h^2$

$50 = l^2 + b^2 + h^2$

diagonal of box = $\sqrt{l^2+b^2+h^2}$

∴ length of longest rod that can be put inside the box

$= \sqrt{l^2+b^2+h^2} = 5\sqrt{2}$ cm

18. (c) Let breadth = b m

∴ length of room = 2b m

(l = 2b)

height = 11 m

Area of four walls of room

= 660 m² (given)

2(l + b) × h = 660

2(2b +b) × 11 = 660

3b × 22 = 660

b = 10

∴ Breadth = 10 m

Length = 20 m

area of floor = l × b

length × breadth =

20 × 10 = 200 m²

19. (d) Side of cube (a) = $\frac{8\sqrt{3}}{\sqrt{3}}$ = 8 cm

⇒ Total surface area

= 6(a)² = 6×8² = 384 cm²

20. (c) Length of pencil

$= = \sqrt{a^2+b^2+c^2}$

$= \sqrt{8^2+6^2+2^2} = \sqrt{64+36+4}$

$= \sqrt{104} = 2\sqrt{26}$ cm

21. (b) Edge of box

$= \sqrt[3]{3.375} = 1.5$ m

22. (b) Whole surface area of cuboid

= 2(whole surface area of cube)

– 2×area of one face

(∵ two faces of the two cubes are not visible now)

⇒ Required area = 12a² – 2a²

= 10a² = 10×6² = 360 cm²

23. (b) Let the increase in level = x m

$$\Rightarrow \left(1000\times1000\times\frac{2}{100}\right)\times\frac{1}{2}$$

$= 100 \times 10 \times x$

$\Rightarrow \quad x = 10$m

24. (d) Sides of parallelopiped are in ratio = 2 : 4 : 8

Let length = 2 units

breadth = 4 units

Height = 8 units

Let the side of cube = a unit

According to question,

volume of cube = volume of parallelopiped

$a^3 = 2 \times 4 \times 8$

$a^3 = 64$

$a = \sqrt[3]{64} = 4$ units

$$\frac{\text{Suface area of parallelopiped}}{\text{Surface area of cube}}$$

$$= \frac{2(lb+bh+hl)}{6a^2}$$

$$= \frac{2(8+32+16)}{6(4)^2} = \frac{7}{6} = 7 : 6$$

25. (c) Let

length = 1 cm

breadth = 2 cm,

height = h cm

2(lb + bh + hl) = 22

2(2 + 2h + h) = 22

2 + 3h = 11

3h = 9

h = 3 cm

Diognal $= \sqrt{l^2+b^2+h^2}$

$\sqrt{1^2+2^2+3^2} = \sqrt{1+4+9} = \sqrt{14}$

26. (d) $\sqrt{l^2+b^2+h^2} = 15$

$l^2 + b^2 + h^2 = 225$...............(i)

$\therefore \quad l + b + h = 24$

$(1 + b + h)^2 = 576$

$\Rightarrow \quad l^2 + b^2 + h^2 + 2(lb + bh + hl) = 576$

$= 225 + 2(lb + bh + hl) = 576$

$2(lb + bh + hl) =$ **351 cm ²**

27. (c) Total surface area of cube

$= 6(\text{side})^2$

$6(\text{side})^2 = 96$

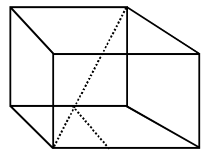
$(\text{side})^2 = \frac{96}{6} = 16$

side $= \sqrt{16} = 4$ cm

Volume of the cube = (side)³

$= (4)^3 =$ **64 cm³**

28.(b)



Diagonal $= 35\sqrt{3}$

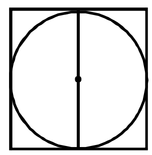
$\therefore$ The length of largest rod

= Diagonal = side $\sqrt{3}$

Side $\sqrt{3} = 35\sqrt{3}$

side $= \frac{35\sqrt{3}}{\sqrt{3}} = 35$

side of cube = 35



Diameter of the sphere

= side of the cube

= 2 × radius = side

radius $= \frac{35}{2}$ cm

Surface area of the sphere

$= 4\pi r^2$

$= 4\times\frac{22}{7}\times\frac{35}{2}\times\frac{35}{2}$

= **3850 m²**

29. (d) volume of air in room = 204 m³

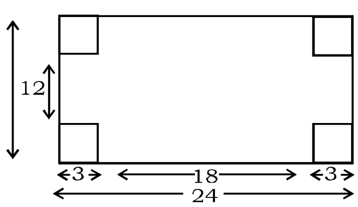
(area of floor) × height = 204

$\therefore$ volume = area of base × height

(area of floor) × 6 = 204

area of floor $= \frac{204}{6} = 34$ m²

30. (b)



The box will be of cuboid shape

Length of the box, $l = 24 - (2 \times 3) = 24 - 6 = 18$ cm

breadth of the box, b

$= 18 - (2 \times 3) = 18 - 6 = 12$ cm

Height of the box ,h = 3 cm

surface area of the box

$= 2(l + b)\times h + l \times b$

$= 2(18 + 12)\times 3 + 18 \times 12$

$= 2 \times 30 \times 3 + 18 \times 12$

= 180 + 216 = **396 cm²**

31. (a) volume of all three cube

$= 4^3 + 5^3 + 6^3$

= 64 + 125 + 216 cm³

= 405 cm ³

Now, 62 cm³ is lost

$\therefore$ Volume of new cube = 405 – 62

= 343

(side of new cube)³ = 343

side of new cube $= \sqrt[3]{343}$

= 7

Total surface area of new cube

$= 6\ (\text{side})^2$

$= 6 \times (7)^2 = 6 \times 49 =$ **294 cm²**

32. (b) Area of cubical floor = 48

Side² = 48

side $= \sqrt{48} = 4\sqrt{3}$

Diagonal of cube = side $\sqrt{3}$

$= 4\sqrt{3} \times \sqrt{3} = 12$ m

Length of longest rod = 12 m

33. (a) Let side of new cube = a

According to question,

$a^3 = 6^3 + 8^3 + 1^3$

= 216 + 512 + 1

= 729

$a = \sqrt[3]{729} = 9$

then surface area = 6(a)² = 6 × 9²

= 6 × 81 = **486 cm²**

34. (b) Volume = 20 m³ = 20 × (100)³ cm³

Volume of one brick = (25 × 12.5 × 8) cm³

$\therefore$ Required number of bricks

$$= \frac{20\times100\times100\times100}{25\times12.5\times8} = 8000$$

35. (a) The total surface area of cube

6 (side)² = 150cm²

6 (side)² = 150 cm²

$(\text{side})^2 = \frac{150}{6} = 25$

side $= \sqrt{25} = 5$ cm

$\therefore$ volume of cube = (side)³

$= (5)^3 =$ **125 cm³**

36. (b) Let Ratio of length : breadth

$= 5x : 3x$

Total surface area of parallelopiped

= 558 cm²

2 (lb + bh + hl) = 558

$2(5x \times 3x + 3x \times 6 + 6 \times 5x) = 558$

$2\ (15x^2 + 18x + 30x) = 558$

$15x^2 + 48x = 279$

$15x^2 + 48x - 279 = 0$

On solving, $x = 3$

$\therefore$ length = 5 × 3 = 15 cm $= \frac{15}{10}$

= **1.5 dm**

37. (c) $l + b + h = 24$ cm
Length of diagonal = 15 cm

$$\sqrt{l^2 + b^2 + h^2} = 15$$
$$l^2 + b^2 + h^2 = 225 \text{ cm}$$
$$(l + b + h)^2 - 2(lb + bh + hl) = 225$$
$$(24)^2 - 2(lb + bh + hl) = 225$$
$$576 - 225 = 2(lb + bh + hl)$$
$$351 = 2(lb + bh + hl)$$
$\therefore$ Total surface area = 351 cm^2

38. (c) Let length = $3x$,
breadth = $4x$
height = $6x$
$$3x \times 4x \times 6x = 576$$
$$x^3 = \frac{576}{3 \times 4 \times 6} = 8$$
$$x = \sqrt[3]{8} = 2 \text{ cm}$$
$\therefore$ length = 3 × 2 = 6cm
breadth = 4 × 2 = 8 cm,
height= 6 × 2 = 12 cm
Total surface area= 2 (lb + bh + hl)
= 2 (6 × 8 + 8 × 12 + 12 × 6)
= 2 (48 + 96 + 72)
= 2 × 216 = **432 cm²**

39. (d) As we know that
A parallelopiped has vertices (v) = 8
edge (e) = 12
face (f) = 6
Put into equation (v – e + f)
$\Rightarrow$ 8 – 12 + 6 $\Rightarrow$ 2

40. (b) According to the question.
$$1 \text{ dm} = \frac{1}{10} \text{m}$$
Let depth of the hole = d
$$\therefore \quad 48 \text{ m} \times 31.5 \times \frac{6.5}{10} \text{m}$$
= 27 × 18.2 × d
d = 2m

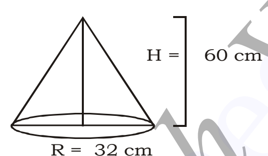
41. (c)
$$2.1 \text{ m} \times 1.5 \text{ m} \times h = 630 \text{ lt}$$
$$\frac{21}{10}\text{m} \times \frac{15}{10}\text{m} \times h = \frac{630}{1000} \text{ m}^3$$
$$\begin{bmatrix} \because 1 \text{ m}^3 = 1000\text{lt} \\ 1000 \text{ cm}^3 = 1\text{lt} \end{bmatrix}$$
$$h = \frac{1}{5}\text{m} = 0.20 \text{ metre}$$

42. (d) Number of cubes = $\frac{8 \times 4 \times 2}{2 \times 2 \times 2}$
= 8

43. (a) when we change shape of a solid figure, volume remains constant ,
$$\therefore \quad \begin{matrix}\text{Volume} \\ \text{Hamisphere}\end{matrix} = \begin{matrix}\text{Volume} \\ \text{of cone}\end{matrix}$$
$$\frac{2}{3} \pi R^3 = \frac{1}{3} \pi R^2 h$$
$$\therefore \quad 2R = h$$

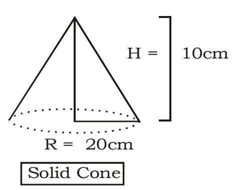
44. (d) According to question
Let the radius of sphere = r cm
$$4\pi (r + 2)^2 - 4\pi r^2 = 352$$
$$4\pi \left\{(r+2)^2 - r^2\right\} = 352$$
$$4\pi \left\{r^2 + 4 + 4r - r^2\right\} = 352$$
$$\pi (1 + r) = \frac{352}{16} = 22$$
$$\frac{22}{7}(1 + r) = 22$$
$$1 + r = 7$$
$$r = 6 \text{ cm}$$

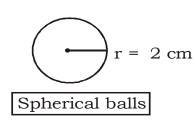
45(d)



We have to find the slant height
Take ratio of H and R
$$= \cancel{60} : \cancel{32}$$
$$15 : 8$$
$$L = \sqrt{15^2 + 8^2} = 17$$
$$= 17 \times 4 = 68 \text{ cm}$$
Cost of painting = Surface area of cone × 35
$$= \pi R L \times 35$$
$$= \frac{22}{7} \times \frac{32 \times 68}{10000} \times 35$$
= **Rs. 23.94 (approx)**

46. (d)

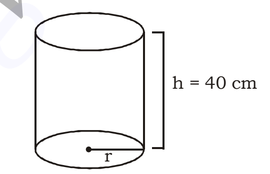




Let the spherical balls made = 'x'
According to question,
Volume of cone = x × volume of sphere
$$\frac{1}{3}\pi R^2 H = x \times \frac{4}{3}\pi r^3$$
$$(20)^2 \times 10 = x \times 4 \times (2)^3$$
$$x = \mathbf{125}$$

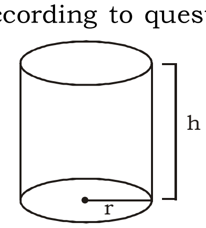
47. (d) Radius of tank , r = $\frac{35}{2}$ cm
Let initial height = H
Final height = h
According to question ,
$$\pi\left(\frac{35}{2}\right)^2 \times H - \pi\left(\frac{35}{2}\right)^2 h = 11000 \text{ cm}^3$$
$$\pi\left(\frac{35}{2}\right)^2 \times (H - h) = 11000$$
$$H - h = \frac{11000 \times 2 \times 2 \times 7}{35 \times 35 \times 22}$$
$$= \frac{80}{7} = 11\frac{3}{7} \text{ cm}$$

48. (d)



$\because$ circumference of its base = 66 cm
$$2\pi r = 66$$
$$r = \frac{66}{2\pi} = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm}$$
$$\therefore \quad \text{volume} = \pi r^2 h$$
$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 40$$
= **13860 cm³**

49. (a) According to question,



$$2\pi r = 6\pi$$
$$r = 3 \text{ cm}$$
height of cylinder = diameter
= 2×r = 2 × 3 = 6cm
volume of water
= $\pi r^2 h = \pi (3)^2 \times 6$
= **54π cm³**

50. (b) Volume of the cone
$$= \frac{1}{3}\pi (15)^2 \times 108 \text{ cm}^3$$
Volume of the cylinder
= $\pi \times r^2 \times 9$ cm³
According to question,
$$\pi \times r^2 \times 9 = \frac{1}{3}\pi \times 15 \times 15 \times 108$$
$$r^2 = \frac{5 \times 15 \times 108}{9} = 900$$

$r = \sqrt{900} = 30$

Diameter of base = 2r

= 2 × 30 = 60 cm

51. (d) Volume of new solid sphere

$= \frac{4}{3}\pi\left(\frac{6}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{8}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{10}{2}\right)^3$

$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left[(3)^3+(4)^3+(5)^3\right]$

$r^3 = 216, \quad r = 6$ cm

∴ Diameter of the new sphere

= 2 × 6 = 12 cm

52. (b) Total surface area of prism (regular hexagon)

= Surface area + (base + top) area

$156\sqrt{3}$ = Perimeter of base × height + 2 × Base area

$156\sqrt{3} = 6 \times a \times 10 + 2 \times 6 \times \frac{\sqrt{3}}{4} \times a^2$

$156\sqrt{3} = 60a + 3\sqrt{3}a^2$

$\sqrt{3}a^2 + 20a = 52\sqrt{3}$

$a\left(\sqrt{3}a + 20\right) = 52\sqrt{3}$

$a = 2\sqrt{3}$

Volume of prism = Base area × Height

$= 6 \times \frac{\sqrt{3}}{4} \times \left(2\sqrt{3}\right)^2 \times 10$

$= 18\sqrt{3} \times 10 = 180\sqrt{3}$ cm³

53. (d) Volume of the new sphere

$\Rightarrow \frac{4}{3}\pi[r_1^3 + r_2^3 + r_3^3] = \frac{4}{3}\pi R^3$

$R^3 = r_1^3 + r_2^3 + r_3^3$

$R^3 = 1^3 + 6^3 + 8^3$

$= 1 + 216 + 512 = 729$

$R = 729 = 9$ cm

54. (b) $l = 2.5$ km

area of base = 1.54 km²

$\pi r^2 = 1.54$

$r^2 = \frac{1.54 \times 7}{22}$

$r = \sqrt{\frac{1.54 \times 7}{22}} = 0.7$ km

We know that, $l^2 = r^2 + h^2$

$h^2 = \sqrt{l^2 - r^2} = \sqrt{2.5^2 - 0.7^2}$

$= \sqrt{5.76} = 2.4$ km

55. (c) radius $= \frac{\text{diameter}}{2}$

$= \frac{19.2}{2} = 9.6$ m

height = 2.8

$l^2 = r^2 + h^2 = 9.6^2 + 2.8^2$

$= 92.16 + 7.84$

$= 100$

$l = \sqrt{100} = 10$ m

area of the canvas $= \pi r l$

$= \frac{22}{7} \times 9.6 \times 10 =$ **301.7**

56. (c) External radius R = 4 cm

Internal radius r = 3cm

volume of iron used

$= \pi R^2 h - \pi r^2 h$

$= \pi h (R^2 - r^2)$

$= \pi h (R + r)(R - r)$

$= \frac{22}{7} \times 20 \times (4+3) \times (4-3)$

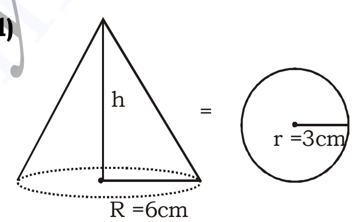
$= \frac{22}{7} \times 20 \times 7 \times 1 = 440$ cu. cm

57. (c) Volume of sphere = Volume of displaced water

$\frac{4}{3}\pi \times 2 \times 2 \times 2 = \pi \times 4 \times 4 \times h$

$h =$ **$\frac{2}{3}$ cm**

58.(d)



Volume of cone = volume of sphere

$\frac{1}{3}\pi R^2 h = \frac{4}{3}\pi r^3$

$\frac{1}{3}\pi \times 6 \times 6 \times h = \frac{4}{3}\pi \times 3 \times 3 \times 3$

h = 3 cm

59. (b) Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi r^2 \, 24 = 1232$ cm ²

$r^2 = \frac{1232 \times 3 \times 7}{24 \times 22}$

$r^2 = 7 \times 7$

$r = \sqrt{7 \times 7} = 7$ cm

$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{625}$

= 25

curved surface area $= \pi r l$

$= \frac{22}{7} \times 7 \times 25 = 550$ cm²

60. (d) Volume of a sphere

$= \frac{88}{21} \times (14)^3$

$= \frac{4}{3} \times \frac{22}{7} \times (14)^3 \left\{\frac{4}{3}\pi r^3\right\}$

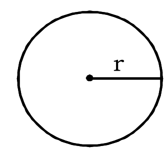
Radius = 14

Curved surface area of sphere

$= 4\pi(\text{radius})^2$

$= 4 \times \frac{22}{7} \times 14 \times 14 = 2464$ cm²

61.(b)



surface area of sphere

$= 64\pi$

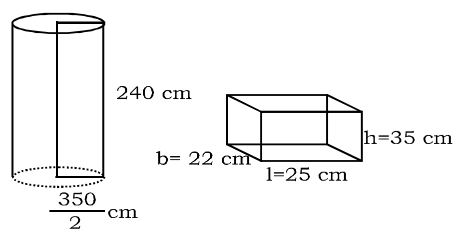
$4\pi(\text{Radius})^2 = 64\pi$

$(\text{radius})^2 = \frac{64}{4} = 16$

radius $= \sqrt{16} = 4$ cm

diameter = 8 cm

62. (a) 1 dm = 10 cm



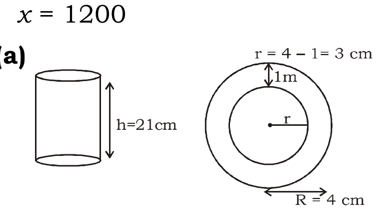
x × volume of 1 tin = volume of cylinder

$\Rightarrow x \times (25 \times 22 \times 35)$

$= \frac{22}{7} \times \frac{350}{2} \times \frac{350}{2} \times 240$

$x = 1200$

63. (a)



Volume of hollow iron pipe

$= \pi\{R^2 - r^2\} \times h$

$= \pi\{4^2 - 3^2\} \times 21$

$= \frac{22}{7} \times 7 \times 21 = 22 \times 21$

$= 462\ cm^3$

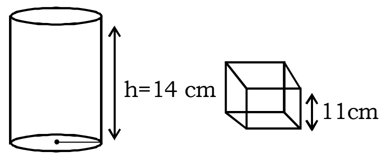
Now ,

$1\ cm^3 = 8g$

$462\ cm^3 = 8 \times 462\ g$

$= 3696\ g = 3.696kg$

64. (b)



Volume of the cylinder = volume of cube

$\pi r^2 h = (side)^3$

$\frac{22}{7} \times r^2 \times 14 = 11 \times 11 \times 11$

$r^2 = \frac{11 \times 11 \times 11}{22 \times 2}$

$= \frac{121}{4}$

$r = \frac{11}{2}\ cm = 5.5\ cm$

65. (b) Let the radius = r

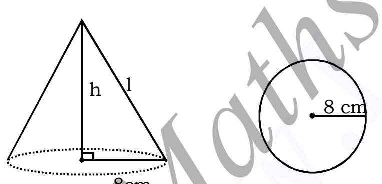
$\pi r^2 h = 9\pi h$

$r^2 = 9$

$r = \sqrt{9} = 3\ m$

diameter = 3×2 = 6 m

66. (a)



volume of cone = volume of sphere

$\frac{1}{3}\pi(8)^2 \times h = \frac{4}{3}\pi(8)^3$

$8 \times 8 \times h = 4 \times 8 \times 8 \times 8$

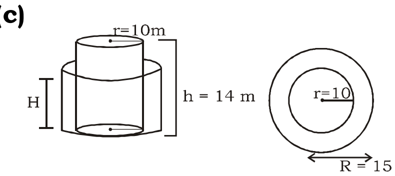
$h = 32\ cm$

slant height $= (l) = \sqrt{r^2 + h^2}$

$= \sqrt{8^2 + 32^2}$

$= \sqrt{64 + 1024} = 8\sqrt{17}$

67.(c)

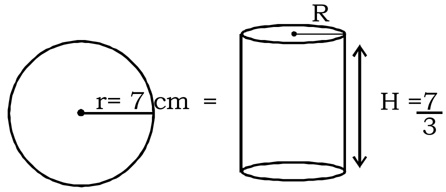


volume of well = volume of embankment

$\pi(10)^2 \times 14 = \pi\{15^2 - 10^2\} \times H$

$H = \frac{100 \times 14}{125} = 11.2\ m$

68. (b)



Volume of sphere = volume of cylinder

$\frac{4}{3}\pi 7^3 = \pi R^2 \times \frac{7}{3}$

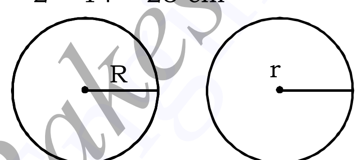
$R^2 = 4 \times 7 \times 7 = 2 \times 2 \times 7 \times 7$

$R = \sqrt{2 \times 2 \times 7 \times 7} = 2 \times 7 = 14\ cm$

diameter of base of cylinder = 2R

= 2 × 14 = 28 cm

69. (b)



ATQ,

$R + r = 10$

$(R + r)^2 = 100$

$R^2 + r^2 + 2Rr = 100$

$R^2 + r^2 = 100 - 2Rr$ ...(i)

$\frac{4}{3}\pi R^3 + \frac{4}{3}\pi r^3 = 880$

$\frac{4}{3}\pi R^3 + r^3 = 880$

$R^3 + r^3 = \frac{880 \times 3}{\pi \times 4} = \frac{880 \times 3 \times 7}{22 \times 4}$

$(R + r)(R^2 + r^2 - Rr) = 210$

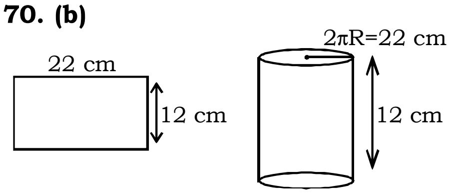
$10 \times (100 - 2Rr - Rr) = 210$

$100 - 3Rr = 21$

$3Rr = 100 - 21 = 79$

$Rr = \frac{79}{3} = 26\frac{1}{3}$

70. (b)



∴ Cylinder is folded along the length of rectangle

∴ $2\pi R = 22$

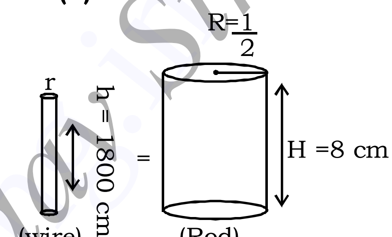
$R = \frac{22}{2\pi} = \frac{22 \times 7}{2 \times 22} = \frac{7}{2}\ cm$

Volume of the cylinder $= \pi R^2 H$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12$

$= 22 \times 7 \times 3 = 462\ cm^3$

71. (b)



Volume of wire = volume of Rod

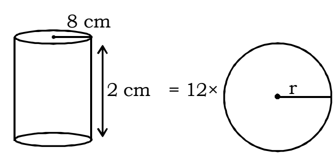
$\pi r^2 h = \pi R^2 h$

$r^2 \times 1800 = \frac{1}{4} \times 8 = 2$

$r^2 = \frac{2}{1800} = \frac{1}{900}$

$r = \sqrt{\frac{1}{900}} = \frac{1}{30}$

72. (b)



Volume of cylinder = 12 × volume of sphere

$\pi(8)^2 \times 2 = 12 \times \frac{4}{3}\pi r^3$

$r^3 = \frac{8 \times 8 \times 2 \times 3}{12 \times 4}$

$r = \sqrt{2 \times 2 \times 2} = 2\ cm$

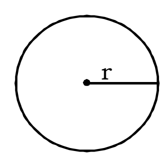
r = 2 cm

d = 4cm

73. (c) $2\pi R - 2\pi r = 5$

$(R - r) = \frac{5}{2\pi}$

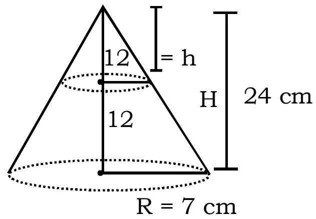
74. (c)



$\frac{4}{3}\pi r^3 = 4\pi r^2$

radius (r) = 3 units

75. (b)



Volume of bigger cone

$= \frac{1}{3} \times \pi \times (7)^2 \times 24$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$

$= 22 \times 7 \times 8 = 1232 \text{ cm}^3$

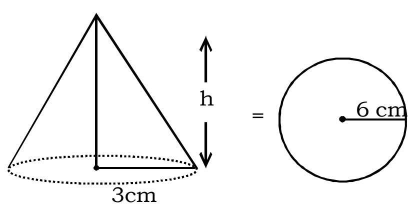
$\frac{\text{volume of smaller cone}}{\text{volume of bigger cone}} = \frac{h^3}{H^3}$

$\frac{\text{Volume of smaller cone}}{1232} = \frac{12^3}{24^3}$

volume of smaller cone= 154 cm³

∴ When the cone is cut in between then the ratio of volume of smaller cone to the bigger one is always equal to the ratio of the cubes of their heights

76. (b)



$n = \frac{\text{Volume of sphere}}{\text{volume of cone}}$

$= \frac{\frac{4}{3}\pi 6^3}{\frac{1}{3}\pi 3^2 \times 4} = 24$

77. (c) Height of cylinder = Breadth of tin foil

⇒ Circumference of the base of cylinder = Length of the foil = 22 cm

⇒ $2\pi r = 22$

$r = \frac{22 \times 7}{22 \times 2} = \frac{7}{2}$ cm

Volume $= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 16 = 616 \text{cm}^3$

78. (d) $\pi r^2 = 770$

$\Rightarrow r^2 = \frac{770 \times 7}{22}$

$\Rightarrow r = 7\sqrt{5}$ cm

and $\pi r l = 814$

$\Rightarrow l = \frac{814 \times 7}{22 \times 7\sqrt{5}} = \frac{37}{\sqrt{5}}$

$l^2 = h^2 + r^2$

$\Rightarrow \frac{37 \times 37}{5} = h^2 + 245$

$\Rightarrow h^2 = \frac{1369}{5} - 245 = \frac{144}{5}$

$\Rightarrow h = \frac{12}{\sqrt{5}}$

volume $= \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7\sqrt{5} \times 7\sqrt{5} \times \frac{12}{\sqrt{5}}$

$= 616\sqrt{5} \text{ cm}^3$

79. (b) In this case the breadth becomes the circumference of the base of the cylinder

$\Rightarrow 2\pi r = 44$

$\Rightarrow r = \frac{44 \times 7}{22 \times 2} = 7$ cm

New volume $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 100$

$= 15400 \text{ cm}^3$

80. (c) $\pi r^2 H = \frac{1}{3} \pi r^2 h$

$\Rightarrow H = \frac{1}{3} h$

$\Rightarrow h = 3H = 3 \times 6 = 18$ cm

81. (c) $3\pi r^2 = 1848$

$r^2 = \frac{1848 \times 7}{3 \times 22} = 196$

$\Rightarrow r = 14$ cm

According to the question

$\frac{2}{3} \pi r^3 = \frac{1}{3} \pi r^2 h$

$\Rightarrow 2r = h$

$\Rightarrow h = 2 \times 14 = 28$ cm

82. (b) The length of the paper becomes the circumference of the base of cylinder when it is rolled along its length

$\Rightarrow 2\pi r = 12$

$\Rightarrow r = \frac{12}{2\pi} = \frac{6}{\pi}$ cm

83. (b) Volume of tunnel $= \pi \times r^2 \times H$

$= \frac{22}{7} \times \frac{4}{2} \times \frac{4}{2} \times 56 = 704 \text{ m}^3$

Volume of ditch = $48 \times 16.5 \times 4$

$= 3168 \text{ m}^3$

Required part $= \frac{704}{3168} = \frac{2}{9}$

84. (a) According to the question

$\pi h(R^2 - r^2) = 748$

$R^2 - r^2 = \frac{748 \times 7}{22 \times 14}$

$9^2 - r^2 = 17$

$\Rightarrow 9^2 - r^2 = 17$

$\Rightarrow r^2 = 81 - 17 = 64$

$\Rightarrow r = 8$

⇒ Thickness = 9 − 8 = 1 cm

85. (b) $2 \times \left(\frac{4}{3} \times \pi \times r^3\right) = \pi R^2 h$

$\Rightarrow 2 \times \frac{4}{3} \times \pi \times 27 = \pi \times 36 \times h$

$h = \frac{27 \times 4 \times 2}{36 \times 3}$

$\Rightarrow h = \frac{8 \times 27}{3 \times 36} = 2$ cm

86. (d)

Ratio of height $= \sqrt[3]{\text{Ratio of volume}}$

$= \sqrt[3]{\frac{1}{27}}$

$\Rightarrow \frac{h}{H} = \frac{1}{3}$

3 units → 30

2 units → 20

⇒ The cut is made 20 cm above the base

87. (b) $3\pi r^2 = 108\pi$

$\Rightarrow r^2 = 36$

$\Rightarrow r = 6$ cm

Volume $= \frac{2}{3} \pi r^3 = \frac{2}{3} \times 216 \times \pi$

$= 144\pi$

88. (d) Radius = 3 Decimeters

= 30 cm

Height of circular sheet = 1 mm

= 0.1 cm

$\Rightarrow \frac{4}{3}\pi \times (30)^3 = \pi r^2 \times \frac{1}{10}$

$\Rightarrow r = \sqrt{10000 \times 9 \times 4}$

$\Rightarrow r = 600$ cm = 6 meters

89. (b) Let no. of seconds required to fill the tank = x

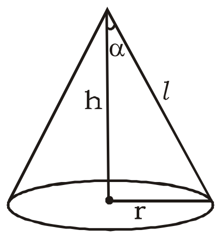
$\Rightarrow (\pi r^2 h) \times x = 3 \times 5 \times 1.54$

$\Rightarrow x = \frac{3 \times 5 \times 1.54 \times 7 \times 100 \times 100}{22 \times 7 \times 7 \times 5}$

= 300 seconds

$\Rightarrow$ Time required = 5 minutes

90. (c)



$\frac{r}{h} = \tan\alpha$

$\Rightarrow r = h\tan\alpha$

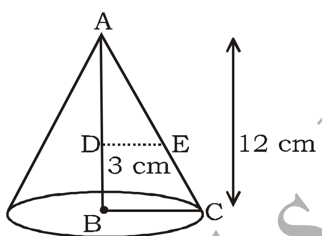
and $\frac{l}{h} = \sec\alpha$

$\Rightarrow l = h\sec\alpha$

$\Rightarrow S = \pi \times h\tan\alpha \times h\sec\alpha$

$= \pi h^2(\tan\alpha \times \sec\alpha)$

91. (c)



As DE||BC, $\triangle ADE \sim \triangle ABC$

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{12-3}{12} = \frac{DE}{6} \Rightarrow \frac{9 \times 6}{12} = DE$

$\Rightarrow DE = 4.5$ cm

92. (d) Height of cylinder = Diameter of sphere

$\Rightarrow \frac{S_1}{S_2} = \frac{4\pi r^2}{2\pi r \times h} = \frac{4r^2}{4r^2} = \frac{1}{1}$

$\Rightarrow S_1 = S_2$ (h = 2r)

93. (d) $\frac{\pi r^2 h}{\frac{4}{3}\pi r^3} = 1$

$\frac{h}{r} = \frac{4}{3}$

$\frac{h}{\frac{d}{2}} = \frac{4}{3}$

$h = \frac{4d}{6}$, $3h = 2d$

94. (a) Volume of water pumped out in one hour

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 \times 3600$

$= 1663200$ cm³

= 1663.2 ltr.

95. (d) $2\pi rh = 1056$

$r = \frac{1056 \times 7}{2 \times 22 \times 16} = \frac{21}{2}$

Volume = $\pi r^2 h = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 16$

= 5544 cm³

96. (b) Let the radius and height be $5x$ and $12x$

$\Rightarrow \frac{1}{3} \times \pi \times 25x^2 \times 12x = \frac{2200}{7}$

$\Rightarrow x^3 = \frac{2200 \times 7 \times 3}{7 \times 22 \times 25 \times 12} = 1$

$\Rightarrow x = 1$

$\Rightarrow$ slant height = $\sqrt{5^2+12^2}$

= 13 cm

97. (b) $\pi r^2 H = \frac{1}{3}\pi r^2 h$

$H = \frac{r^2 h}{r^2 \times 3} = \frac{h}{3}$

$\Rightarrow h = 3H = 3 \times 7$

= 3 × 7 = 21 cm

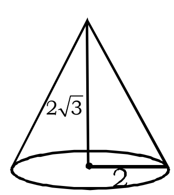
98. (b) $\pi \times r^2 \times H = \frac{4}{3}\pi r^3$

$\pi \times \frac{1}{10} \times \frac{1}{10} \times 36 \times 100 = \frac{4}{3}\pi r^3$

$r^3 = 27$

r = 3 cm

99. (b)



Slant height = $\left(\sqrt{\left(2\sqrt{3}\right)^2 + 2^2}\right)$

$= \sqrt{12+4} = 4$ cm

100. (b) Volume of vessel

= Volume of roof

$\pi \times r^2 \times h = 22 \times 20 \times x$

(where x is rainfall in cm)

$\Rightarrow \frac{22}{7} \times \frac{100 \times 100 \times 350}{22 \times 20 \times 100 \times 100} = x$

$\Rightarrow x = 2.5$ cm

101. (a) Volume of remaining solid = Volume of cylinder − Volume of cone

$\pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$

$\frac{2}{3} \times \frac{22}{7} \times 36 \times 10 = 240\pi$ cm³

102. (c) Let the height be H

$\Rightarrow \frac{1}{3}\pi r_1^2 H + \frac{1}{3}\pi r_2^2 H$

$= \frac{4}{3}\pi R^3$

$\Rightarrow \frac{1}{3}\pi H(r_1^2 + r_2^2) = \frac{4}{3}\pi R^3$

$\Rightarrow H = \frac{4R^3}{r_1^2 + r_2^2}$

103. (c) Let height and diameter be $3x$ and $2x$

$\Rightarrow \frac{1}{3}\pi x^2 \times 3x = 1078$

$\Rightarrow x^3 = \frac{1078 \times 7}{22} = 49 \times 7$

$\Rightarrow x = 7$

$\Rightarrow$ height = 7 × 3 = 21 cm

104. (c) Radius of cylinder r = 10 cm

height of cylinder h = 21 cm

volume of cylinder = $\pi r^2 h$

radius of cone $\Rightarrow$ radius of cylinder = 10 cm

Let height of cone = h_1

$\therefore$ volume of cone = $\frac{1}{3}\pi r^2 h_1$

$\therefore$ volume of remaining portion

$\Rightarrow$ 4400 cm³(given)

(after removing cone)

$\pi r^2 h - \frac{1}{3}\pi r^2 h_1 = 4400$

$\pi r^2\left(h - \frac{h_1}{3}\right) = 4400$

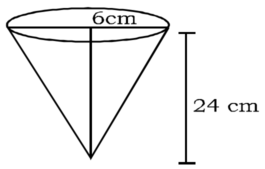
$$\frac{22}{7} \times 10 \times 10 \left(21 - \frac{h_1}{3}\right) = 4400$$

$$\Rightarrow \quad 21 - \frac{h_1}{3} = 14$$

$$21 - 14 = \frac{h_1}{3}$$

$$h_1 = 21$$

105. (a)



radius of cone = 6 cm
height of cone = 24 cm

$\therefore$ volume of cone = $\frac{1}{3}\pi 6^2 \times 24$ cm³

cone is converted to sphere
Let radius of sphere = r

$\therefore$ Volume of sphere $\Rightarrow \frac{4}{3}\pi r^3$

volume of sphere = volume of cone

$$\therefore \quad \frac{4}{3}\pi r^3 = \frac{1}{3} \times \pi \times 6 \times 6 \times 24$$

$$\Rightarrow r^3 \Rightarrow \frac{1}{3} \times \frac{6 \times 6 \times 24}{4} \times 3$$

$$\Rightarrow r^3 = 3 \times 3 \times 24$$

$$= 3 \times 3 \times 3 \times 8$$

$$r^3 = (3)^3 \times (2)^3$$

$$r = 3 \times 2 = 6 \text{ cm}$$

$\therefore$ radius of sphere = 6 cm

106. (a) total surface area of cylinder

$\Rightarrow$ 462 (given)

$\Rightarrow$ $(2\pi rh + 2\pi r^2) = 462$ cm²

r = radius, h = height

$$2\pi r^2 = 462 - 154 = 308$$

$$\pi r^2 = 154$$

$$r^2 = \frac{154}{22} \times 7 = 49$$

r = 7 cm

107. (a) Diameter of cylinder = 7 cm

radius $= \frac{7}{2}$ cm, height = 16 cm

$\therefore$ lateral or curved surface area

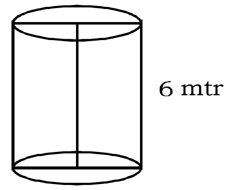
$\Rightarrow$ $2\pi rh$

$\Rightarrow$ r = radius

h = height

$$\therefore \quad 2 \times \frac{22}{7} \times \frac{7}{2} \times 16 \Rightarrow 352 \text{ cm}^2$$

108. (a)

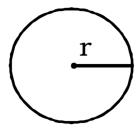


height of cylinder h = 6 mtr
Let radius of cylinder = r mtr

$\therefore$ curved surface area = $2\pi rh$

area of end face = πr^2

$\Rightarrow$ total area of two end faces = $2\pi r^2$

$\Rightarrow$ 

$\Rightarrow$ given that

$$3 \times 2\pi r^2 = 2 \times 2\pi rh$$

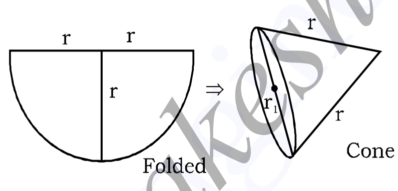
$$3r = 2h$$

$$3 \times r = 2h$$

$$r = 4 \text{ mtr}$$

$\therefore$ radius of base = 4 mtr

109. (b)



radius of semi-circular sheet

$$= r \Rightarrow \frac{28}{2}$$

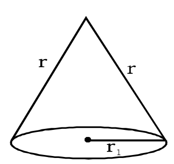
r = 14 cm

Circumference of sheet = πr

= 14π cm

Sheet is folded to form a cone

Let radius of cone = r_1



$\therefore$ The circumference of base of cone = circumference of sheet

$$\therefore \quad 2\pi r_1 = 14\pi$$

$$r_1 = 7 \text{ cm}$$

$\therefore$ radius of cone = 7 cm

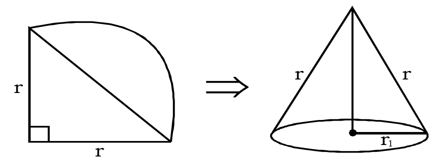
slant height = radius of semi-circular sheet

l = 14 cm

$$\therefore \text{ height} = \sqrt{14^2 - 7^2}$$

$$= \sqrt{147} = 12 \text{ cm } \textbf{(approx)}$$

110.(b)



$\Rightarrow$ Circumference of sectors $= \frac{\pi r}{2}$

$\Rightarrow$ Circumference of base of cone of radius $r_1 = 2\pi r_1$

$$\frac{\pi r}{2} = 2\pi r_1$$

$$r_1 = \frac{r}{4}$$

$\therefore$ radius of cone $= \frac{r}{4}$

curved surface area of cone $= \pi r_1 l$

l = slant height

l = r

$\therefore$ surface area of cone $= \pi \times \frac{r}{4} \times r$

$$\Rightarrow \frac{\pi r^2}{4}$$

111.(d) radius of cone = r = 16 meter (given)

Let slant height = l meter

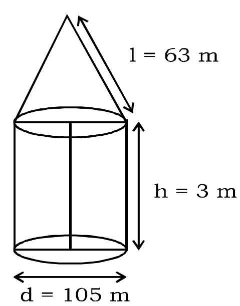
curved surface area = πrl

$$= 427\frac{3}{7} m^2 \text{ (given)}$$

$$\Rightarrow \frac{22}{7} \times 16 \times l = \frac{2992}{7}$$

$$l = \frac{2992}{22 \times 16} = 8.5 \text{ meter}$$

112. (a)



$\therefore$ radius of cone $= \frac{105}{2}$ m

slant height of cone = 63 m

$\Rightarrow$ curved surface area of cone = (πrl)

$$= \frac{22}{7} \times \frac{105}{2} \times 63$$

= 10395 m²

$\Rightarrow$ radius of cylinder = $\frac{105}{2}$ m

height = 3 m (given)

$\therefore$ curved surface area of cylinder $= 2\pi rh$

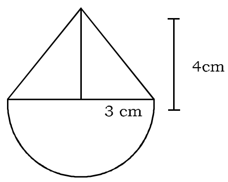
$= 2 \times \frac{22}{7} \times \frac{105}{2} \times 3 = 990\ m^2$

$\therefore$ Total curved area of structure

$\Rightarrow$ curved area of cone + curved area of cylinder = 10395 + 990 = 11385 m^2

$\therefore$ Total area of canvas = 11385 m^2

113. (b)



Surface area of hemishpere

$= 2\pi r^2 = 2 \times \frac{22}{7} \times 9 = 56.57\ cm^2$

height of cone = 4 cm

radius = 3 cm

$\therefore$ slant height = $\sqrt{16+9}$ = 5 cm

$\therefore$ surface area of cone = πrl

$= \frac{22}{7} \times 3 \times 5$

$\Rightarrow$ 47.14 cm^2

$\therefore$ total surface area of the toy area of cone + area of hemisphere

$\Rightarrow$ 47.14 + 56.57 $\Rightarrow$ 103.71 cm^2

114. (b) diameter of beaker = 7 cm

radius = $\frac{7}{2}$ cm

level of water rises = 5.6 cm

diameter of a marble = 1.4 cm

$\therefore$ radius = $\frac{1.4}{2}$ = 0.7 cm

Let n marbles are dropped so, Volume of n marbles

$= n \times \frac{4}{3}\pi \times (0.7)^3$

$\Rightarrow n \times \frac{4}{3}\pi \times (0.7)^3 = \pi \times \frac{7}{2} \times \frac{7}{2} \times 5.6$

$\Rightarrow n \times \frac{4}{3} \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$

$= \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10}$

$\Rightarrow$ n = 150

115. (d) Let radius of iron rod = r

$\therefore$ height = 8r

$\therefore$ volume of iron rod

$= \pi \times (r)^2 \times 8r \Rightarrow 8\pi r^3$

$\Rightarrow$ radius of spherical ball = $\frac{r}{2}$

volume of spherical ball

$= \frac{4}{3}\pi \times \left(\frac{r}{2}\right)^3$

Let n balls are cast

$\therefore\ n \times \frac{4}{3}\pi\left(\frac{r^3}{8}\right) = 8\pi r^3$

$\Rightarrow \frac{n}{6} \Rightarrow 8 \Rightarrow n = 48$

116. (c) Let the radius of base of second cylinder = R

$\Rightarrow 2(\pi r^2 h) = \pi R^2 h$

$\Rightarrow 2r^2 = R^2$

$\Rightarrow R = r\sqrt{2}$

117. (c) Volume of remaining solid = Volume of cylinder – Volume of cone

$\pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$

$= \frac{2}{3} \times \frac{22}{7} \times 5 \times 5 \times 12$

= 628.57 cm^3

118. (a) Let the required increase = x cm

$\Rightarrow \pi(10+x)^2 \times 4 = \pi \times 10^2 \times (4+x)$

$100 + x^2 + 20x = 25(4+x)$

$x^2 + 20x + 100 = 100 + 25x$

$x^2 - 5x = 0$

$x - 5 = 0$

$x = 5$

$\therefore$ Required increase = 5 cm

119. (b) Let the old volume = $\frac{1}{3}\pi r^2 h$

$\Rightarrow$ New volume $= \frac{1}{3}\pi(2r)^2 h$

$= \frac{4}{3}\pi r^2 h$

$\Rightarrow$ New volume is four times the old volume

120. (b) Let the height of cone be 'h' cm

$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$

$a^2 h = 4a^3$

$h = 4a$

121. (c) Radius of base = $\frac{33}{2\pi}$

$= \frac{33 \times 7}{2 \times 22} = \frac{21}{4}$ cm

$\therefore$ Volume of cone = $\frac{1}{3} \times \pi \times r^2 \times h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{21}{4} \times \frac{21}{4} \times 16$

= 462 cm^3

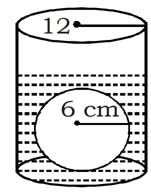
122. (a) Let the radius of small spheres be r cm

$\Rightarrow \left(\frac{4}{3}\pi r^3\right) \times 8 = \frac{4}{3}\pi \times (3)^3$

$\Rightarrow 8r^3 = 3^3$

$\Rightarrow r = \frac{3}{2} = 1.5$ cm

123. (b)



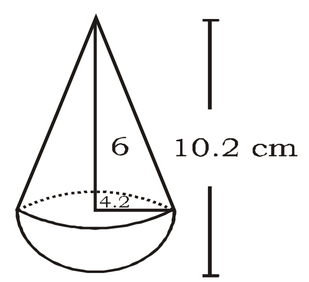
Let the increase in height = h cm

$\Rightarrow \pi R^2 h = \frac{4}{3}\pi r^3$

$(12)^2 \times h = \frac{4}{3} \times 6^3$

$h = \frac{4}{3} \times \frac{216}{144} = 2$ cm

124. (c) Height of the cone = 10.2 – 4.2 = 6 cm



$\Rightarrow$ Volume of the toy

$= \frac{1}{3}\pi r^2 h + \frac{4}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 (h + 4r)$

$= \frac{1}{3} \times \frac{22}{7} \times (4.2)^2 (4 \times 4.2 + 6)$

$= \frac{1}{3} \times \frac{22}{7} \times (4.2)^2 \times 22.8$

= 421 cm^3 **(approx.)**

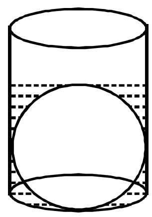
125. (c) Volume of water
= Volume of cylinder – volume of cone

$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$

$= 2\left(\frac{1}{3}\pi r^2 h\right)$

$= 2 \times 27\pi = 54\pi$ cm³

126. (c)



Height of water after ball is immersed = 3.5 × 2 = 7 cm

⇒ Volume of water = $\pi r^2 h - \frac{4}{3}\pi r^3$

⇒ $\pi r^2\left(h - \frac{4}{3} r\right)$

⇒ $\frac{22}{7}\times 3.5\times 3.5\left(7-\frac{4}{3}\times 3.5\right)$ =

Volume of water before ball was immersed

$\frac{22}{7}\times 3.5\times 3.5\left(7-\frac{4}{3}\times 3.5\right)$

$= \pi(3.5)^2 \times h$

$= h = \frac{7}{3}$ cm

127. (c) Let height and radius be = 7x and 5x respectively

⇒ $\pi r^2 h = 550$

$\pi(5x)^2 \times 7x = 550$

$\frac{22}{7}\times 25x^2\times 7x = 550$

$x^3 = 1$

$x = 1$

∴ height = 7 cm
radius = 5 cm

⇒ Curved surface are = $2\pi rh$

$= 2\times\frac{22}{7}\times 5\times 7 = 220$ cm²

128. (c) Let the height of the cylinder be 'h' cm and the radius be r cm

⇒ $\pi r^2 = b$

⇒ $r = \sqrt{\frac{b}{\pi}}$

also, $2\pi rh = a$

$2\pi\sqrt{\frac{b}{\pi}}\times h = a$

$h = \frac{a}{2\sqrt{\pi b}}$ cm

129. (a) $\frac{2}{3}\pi r^3 = 19404$

$r^3 = \frac{19404\times 7\times 3}{22\times 2}$

⇒ r = 21 cm

⇒ Total surface area = $3\pi r^2$

$= 3\times\frac{22}{7}\times 21\times 21 = 4158$ cm²

130. (c) Curved surface area = $2\pi r^2$

$= 2\times\frac{22}{7}\times 11\times 11 = 760.57$ cm²

131. (a) Slant height of the cone (l)

$= \sqrt{6^2+8^2} = 10$ cm

⇒ Required ratio = $\frac{2\pi rh}{\pi rl} = \frac{2h}{l}$

$= \frac{2\times 8}{10} = 8 : 5$

132. (c) The volume of cone having same height & diameter as that of a cylinder

$n \times \frac{1}{3}\pi r^2 h = \pi r^2 h$

$= \frac{1}{3}\times$ volume of cylinder

No. of cones required = 3

133. (a) Let the no. of small balls = x

⇒ $\frac{4}{3}\pi\times(10)^3 = x\times\frac{4}{3}\times\pi\times\left(\frac{1}{2}\right)^3$

⇒ $1000 = x\times\frac{1}{8}$

⇒ $x = 8000$

134. (a) Let the no. of balls = x

⇒ $44\times 44\times 44 = x\times\frac{4}{3}\times\pi\left(\frac{4}{2}\right)^3$

⇒ $\frac{44\times 44\times 44\times 7\times 3}{22\times 4\times 8} = x$

⇒ x = 2541

135. (d) Let the no. of cones = x

⇒ $\pi\times 3^2\times 5 = x\times\frac{1}{3}\times\pi\times\left(\frac{1}{10}\right)^2\times 1$

⇒ x = 9×5×3×100 = 13500

136. (c) Slant height of cone

$= \sqrt{8^2+6^2} = 10$ cm

Slant height of cone = radius of sector = 10 cm

137. (d) Volume of sphere

$= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\pi\times(9)^3$

$= 972\pi$ cm³

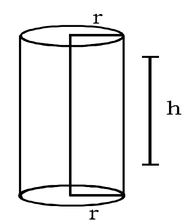
volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\ 9^2\times 9$

$= 243\pi$ cm³

⇒ % of wasted wood

$= \frac{(972-243)\pi}{972\pi}\times 100 = 75\%$

138. (d)



$2\pi r = a, r = \frac{a}{2\pi}$

Volume of cylinder = V

$\pi r^2 h = V$

$\pi\left(\frac{a}{2\pi}\right)^2\times h = V$

$4\pi\frac{a^2}{4\pi^2} = V$

$h = \frac{V\times 4\pi}{a^2} = \frac{4\pi V}{a^2}$

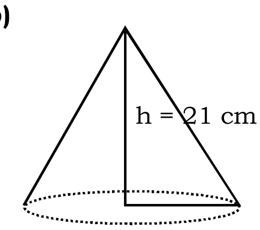
139. (d) Radius of sphere = $\frac{12}{2} = 6$ cm

Let the height of the cylinder = h
According to questions,
Volume and radius are same

$\pi(6)^2\times h = \frac{4}{3}\pi(6)^3$

$h = \frac{4\times 6}{3}$ = **8 cm**

140. (b)



Perimeter of base = 8 cm

$2\pi r = 8$

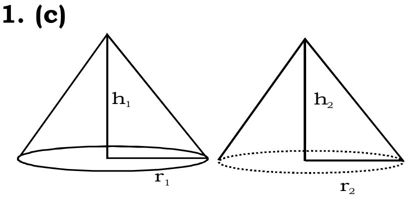
$$r = \frac{8}{2\pi}$$

h = 21 cm

Volume of cone

$$= \frac{1}{3} \times \pi \times \frac{4}{\pi} \times \frac{4}{\pi} \times 21 = \mathbf{\frac{112}{\pi}\ cm^3}$$

141. (c)



$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1{}^2 h_1}{\frac{1}{3}\pi r_2{}^2 h2} = \frac{r_1{}^2 h_1}{r_2{}^2 h_2} = \frac{4}{1}$$

$$\left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right) = \frac{4}{1}$$

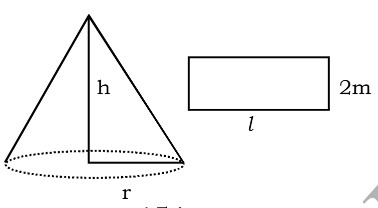
$$\therefore \frac{2r_1}{2r_2} = \frac{5}{4} \therefore \frac{r_1}{r_2} = \frac{5}{4}$$

$$\left(\frac{5}{4}\right)^2 \times \frac{h_1}{h_2} = \frac{4}{1}$$

$$\frac{25}{16} \times \frac{h_1}{h_2} = \frac{4}{1}$$

$$\frac{h_1}{h_2} = \mathbf{\frac{64}{25}}$$

142. (d)



πr^2 = 154

$$r^2 = \frac{154 \times 7}{22} = 49$$

$r = \sqrt{49}$ = 7 m

also, volume = 1232

$$\frac{1}{3}\pi r^2 \times h = 1232$$

$$h = \frac{1232 \times 3}{\pi r^2} = \frac{1232 \times 3}{154} = 8$$

h = 8 m

Area of canvas required = πrl

$$= \pi r\sqrt{r^2 + h^2}$$

$$= \frac{22}{7} \times 7 \times \sqrt{24^2 + 7^2}$$

$$= \frac{22}{7} \times 7 \times 25 = 550\ m^2$$

length × 2 = 550 m²

$$\text{length}(l) = \frac{550}{2} = \mathbf{275\ m}$$

143. (b) Ratio of the volume of cones

$$= \frac{\frac{1}{3}\pi r_1{}^2 h}{\frac{1}{3}\pi r_2{}^2 h} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

= **9 : 16**

144. (c) Ratio of surface area of sphere

$$\frac{4\pi r_1{}^2}{4\pi r_2{}^2} = \frac{4}{9}$$

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$$

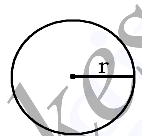
$$\frac{r_1}{r_2} = \frac{2}{3}$$

Ratio of their volume

$$= \frac{\frac{4}{3}\pi r_1{}^3}{\frac{4}{3}\pi r_2{}^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

= **8 : 27**

145. (a)



Total surface area of sphere

= 8π squares

4πr² = 8π

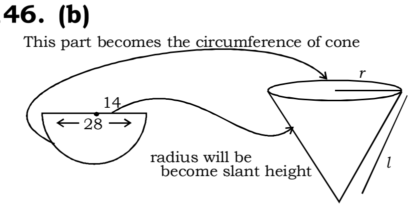
r² = 2

r = $\sqrt{2}$ units

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times \left(\sqrt{2}\right)^3 = \frac{8\sqrt{2}\pi}{3} \text{ units}$$

146. (b)



In this question just cut the semicircular paper and told it to form cone

Circumference of base of cone= $2\pi r$

$$\therefore \frac{2\pi r}{2} = \text{circumference of semi circlar Sheet}$$

$2\pi r = \pi \times 14$

r = 7 cm

Slant height,(l) of = radius of semicirular plate

l = 14 cm

h² = l² – r²

= 14² – 7²

= 196 – 49

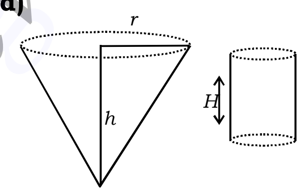
= 147

h = $\sqrt{147}$ = $7\sqrt{3}$

$$\text{Volume of cone} = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\sqrt{3}$$

= **622.36 cm³**

147. (d)



Volume of water in conical flask

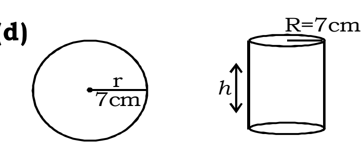
$$= \frac{1}{3}\pi r^2 h$$

If the height of water level in cylindrical flask = H units

$$\therefore \pi m^2 H = \frac{1}{3}\pi r^2 h$$

$$H = \frac{1}{3} \times \frac{\pi r^2 h}{\pi m^2} = \frac{hr^2}{3m^2}$$

148.(d)



$$\text{volume of the solid sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi \times 7 \times 7 \times 7\ cm^3$$

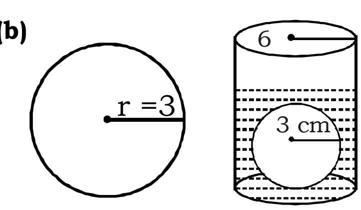
Let the length of wire = h cm

$$\pi R^2 h = \frac{4}{3}\pi \times 7 \times 7 \times 7$$

$$7 \times 7 \times h = \frac{4}{3} \times 7 \times 7 \times 7$$

$$\mathbf{h = \frac{28}{3}\ cm}$$

149. (b)



Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 3 \times 3 \times 3 = 36\pi \text{ cm}^3$

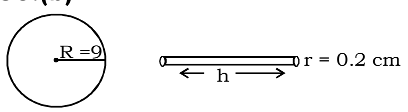
If the water level rises by H cm

$\pi R^2H = 36\pi$

$6 \times 6 \times h = 36$

$h = 1$ cm

150.(b)



Volume of sphere $= \frac{4}{3}\pi R^3$

$972\pi \text{ cm}^3$

Let the length of wire $= h$ cm

$\pi(0.2)^2 \times h = 972\pi$

$h = \frac{972}{0.2 \times 0.2} = 24300$ cm

$=$ **243 metres**

151. (a) Volume of sphere = volume of rectangular block

$\frac{4}{3}\pi \text{ (radius)}^3$

$=$ length $\times$ breadth $\times$ height

$\frac{4}{3}\pi \text{ (radius)}^3 = 21 \times 77 \times 24$

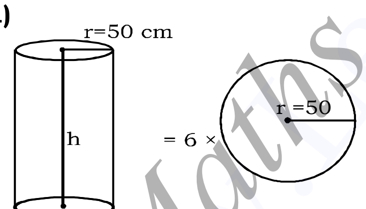
$\text{(radius)}^3 = \frac{21 \times 77 \times 24 \times 3 \times 7}{4 \times 22}$

(radius) $=$

$\sqrt[3]{7 \times 7 \times 7 \times 3 \times 3 \times 3}$

radius $= 7 \times 3 = 21$ cm

152. (d)



Volume of cylinder

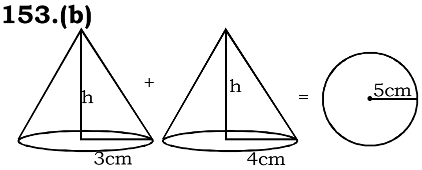
$= 6 \times$ volume of a sphere

$\pi 50^2 h = 6 \times \frac{4}{3}\pi(50)^3$

$h = 6 \times \frac{4}{3} \times 50$

$= 400$ cm $= 4$ m

153.(b)



volume of both the cones will be equal to the volume of sphere

$\frac{1}{3}\pi(3)^2 h + \frac{1}{3}\pi(4)^2 h = \frac{4}{3}\pi(5)^3$

$\frac{1}{3}h(3^2+4^2) = \frac{4}{3} \times 5 \times 5 \times 5$

$\frac{1}{3} \times h \times 25 = \frac{4}{3} \times 5 \times 5 \times 5$

$h = \frac{20}{3} \times 3 = 20$ cm

154. (a) Volume of cone $= \frac{1}{3}\pi r^2 h$

Now, $r_1 = 2r$, $h_1 = 2h$

$\therefore$ Volume of second cone

$= \frac{1}{3}\pi r_1^2 h_1$

$= \frac{1}{3}\pi(2r)^2 2h = \frac{1}{3}\pi r^2 h \times 8$

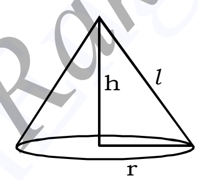
$= 8$ times of the previous volume

Alternate:

In the formula of volume of cone, there is power 2 on radius and power 1 on height

$\therefore$ $(2)^2 \times 2 = 8$ times

155. (d)



$C = \pi rl$

$C^2 = \pi^2 r^2 l^2$

$V = \frac{1}{3}\pi r^2 h$

$V^2 = \frac{1}{9}\pi^2 r^4 h^2$

$3\pi vh^3 - c^2h^2 + 9v^2$

$3\pi \times \frac{1}{3}\pi r^2 h \times h^3 - \pi^2 r^2 l^2 h^2 + 9 \times \frac{1}{9}\pi^2 r^4 h^2$

$= \pi^2 r^2 h^4 - \pi^2 r^2 h^2 (r^2 + h^2) + \pi^2 r^4 h^2$

$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2$

$=$ **0**

156. (a) volume of rectangular block

$= 11 \times 10 \times 5 = 550 \text{ m}^3$

$= 550000 \text{ dm}^3$ (1 m = 10 dm)

Volume of a sphere

$= \frac{4}{3}\pi \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \text{ dm}^3$

$= \frac{500}{8} \text{ dm}^3$

According to question,

$n \times \frac{500}{8} = 550000$

$n = \frac{550000 \times 8}{500} =$ **8800**

157. (a) Required number of spheres

$= \frac{\text{volume of metallic cone}}{\text{volume of a sphere}}$

$= \frac{\frac{1}{3}\pi \times 30 \times 30 \times 45}{\frac{4}{3}\pi \times 5 \times 5 \times 5} = 81$

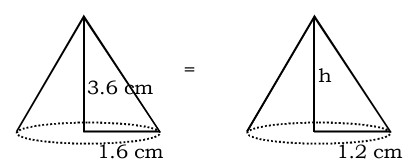
158. (d) Number of cones

$= \frac{\text{Volume of sphere}}{\text{Volume of cone}}$

$= \frac{\frac{4}{3}\pi(10.5)^3}{\frac{1}{3}\pi(3.5)^2 \times 3}$

$= \frac{4 \times 10.5 \times 10.5 \times 10.5}{3.5 \times 3.5 \times 3} =$ **126**

159.(c)



According to question

$\frac{1}{3} \times \pi \times 1.6 \times 1.6 \times 3.6$

$= \frac{1}{3} \times \pi \times 1.2 \times 1.2 \times h$

$h = \frac{1.6 \times 1.6 \times 3.6}{1.2 \times 1.2} = \frac{16 \times 16 \times 36}{12 \times 12 \times 10}$

$= \frac{64}{10} =$ **6.4 cm**

160.(a) $\frac{S^3}{V^2} = \frac{(4\pi r^2)^3}{\left(\frac{4}{3}\pi r^3\right)^2} = \frac{4^3 \times \pi^3 \times r^6}{4^2 \times \pi^2 \times r^6} \times 3^2$

$= 4 \times \pi \times 9 = \frac{36\pi}{1} = 36\pi$ units

161. (d) Radius of sphere $= \frac{1}{20}$ cm

volume of a sphere

$= \frac{4}{3}\pi \times \frac{1}{20} \times \frac{1}{20} \times \frac{1}{20}$

Let the radius of cone $= R$

height $= 2R$

According tio question,

$$\frac{1}{3}\pi \times R\times R\times 2R$$

$$= \frac{4}{3}\pi \times \frac{1}{20}\times\frac{1}{20}\times\frac{1}{20}\times 32000$$

$$R^3 = \frac{2\times 32000}{20\times 20\times 20} = \frac{64000}{20\times 20\times 20}$$

$$R^3 = \frac{40\times 40\times 40}{20\times 20\times 20}$$

$$R = \frac{40}{20} = 2$$

height of glass = 2R = 2 × 2 = 4 cm

162. (c) Volume of earth taken out

= 40 × 30 × 12 = 14400 m^3

Area of rectangular field

= 1000×30 = 30000 m^2

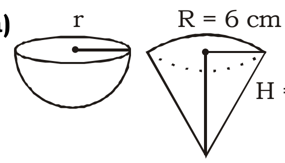
Area of region of tank = 40 × 30 = 1200 m^2

Remaining area = 30000 – 1200

= 28800 m^2

Increase in height $= \frac{14400}{28800}$ = **0.5 m**

163.(a)



According to question,

$$8 \times \frac{2}{3}\pi r^3 = \frac{1}{3}\pi (6)^2 \times 12$$

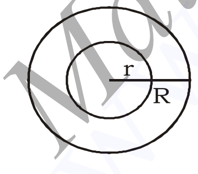
$$r^3 = \frac{6\times 6\times 12}{8\times 2}$$

$= 3 \times 3 \times 3$

$r = \sqrt[3]{3\times 3\times 3}$

= **3 cm**

164. (d)



Volume of lead $= \frac{4}{3}\pi r^3$

Volume of Gold $= \frac{4}{3}\Pi R^3 - \frac{4}{3}\Pi r^3$

According to question,

$$\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi R^3 = \frac{8}{3}\pi r^3$$

$R^3 = 2r^3$

$R^3 = 2(2)^3$

$R = \sqrt[3]{2}\times 2 = 1.259 \times 2$

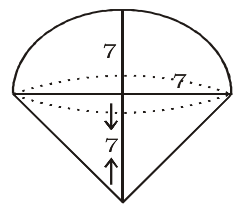
$= 2.518$

$\therefore$ **Thickness** = R – r

= 2.518 – 2

= 0.518 cm

165. (a)



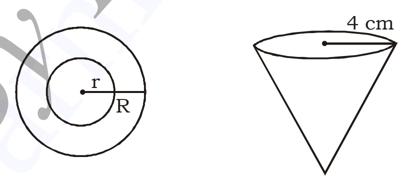
In the question, Radius of hemisphere = Radius of cone = height of cone = 7 cm

($\because$ height of hemisphere = radius of hemisphere) volume of ice cream = volume of hemisphere part + volume of conical part

$$= \frac{2}{3}\times\frac{22}{7}\times(7)^3 + \frac{1}{3}\times\frac{22}{7}\times 7^3$$

$$= \frac{22}{7}\times 7^3 = 22 \times 7^2 = 1078\ cm^3$$

166. (c)



Volume of material of hollow sphere = Volume of cone

$$\frac{4}{3}\Pi(5^3 - 3^3) = \frac{1}{3}\Pi(4)^2 \times h$$

$98 = 4h$

$h = \frac{98}{4}$ = **24.5 cm**

167.(d) Radius of the base of conical shape = r cm

$\therefore$ Radius of base of cylinder $= \frac{r}{3}$ cm

Volume of water = volume of cone

$$= \frac{1}{3}\Pi r^2 h = \frac{1}{3}\Pi r^2 \times 24$$

$= 8\pi r^2\ cm^3$

volume of cylinder = volume of water

$$\pi\left(\frac{r}{3}\right)^2 \times H = 8\pi r^2$$

H = 9 × 8 = **72 cm**

168. (b) volume of metallic sphere = volume of cone

$$\Rightarrow \frac{4}{3}\pi \times 3\times 3\times 3$$

$$= \frac{1}{3}\pi R^2 h$$

$$\Rightarrow \frac{4}{3}\pi \times 3\times 3\times 3$$

$$= \frac{1}{3}\times\pi\times 6\times 6\times h$$

$h = \frac{108}{6\times 6}$ = **3 cm**

169. (d) Number of bottle

$$= \frac{\text{volume of hemispherical bowl}}{\text{volume of cylindrical bottle}}$$

$$= \frac{\frac{2}{3}\times\pi\times 15\times 15\times 15}{\pi\times\frac{5}{2}\times\frac{5}{2}\times 6} = \mathbf{60}$$

170. (a) volume of cone V_1

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi}{3} r^3 \quad (\because h = r)$$

Volume of sphere , V_2

$$= \frac{4}{3}\pi r^3$$

Volume of cylinder V_3

$= \pi r^2 h = \pi r^3$

$\therefore\ V_1 : V_2 : V_3 = \frac{1}{3} : \frac{4}{3} : 1$

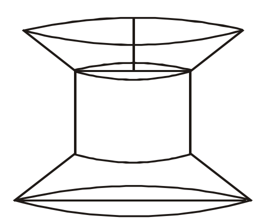
= 1 : 4 : 3

$$V_1 = \frac{V_2}{4} = \frac{V_3}{3}$$

171. (c) $4\pi\ \text{radius}^2 = 346.5$

$\text{radius}^2 = \frac{346.5\times 7}{4\times 22}$ = **5.25 cm**

172. (b)



Height of kaleidoscope = 25 cm

Radius of kaleidoscope = 35 cm
paper used = curved surface area
of cylinder $= 2 \times \frac{22}{7} \times 35 \times 25$
$= 2 \times 22 \times 5 \times 25$
$= 5500 \text{ cm}^2$

173. (b) According to the question,

$\Rightarrow$ Volume of sphere = Surface area of sphere

$\Rightarrow \frac{4}{3}\pi r^3 = 4\pi r^2 \Rightarrow \frac{r}{3} = 1$

$\Rightarrow$ r = 3 cm

$\Rightarrow$ then diameter of sphere will be = 2r
$= 2 \times 3 = 6$ cm

174. (b) Let the height of cone h metre)

$\Rightarrow$ Total area of ground will be required
$= 5 \times 16\text{m}^2 = 80 \text{ m}^2$

$\Rightarrow$ Total volume of air is needed
$= 100 \times 5 \text{ m}^3 = 500 \text{ m}^3$

According to the question

$\Rightarrow$ volume of cone = 500 m³

$\Rightarrow \frac{1}{3}$ area of ground × height = 500

$\Rightarrow \frac{1}{3} \times \pi r^2 \times h = 500$

$= \frac{1}{3} \times 80 \times h = 500$

$\Rightarrow$ height $= \frac{500 \times 3}{80}$

$\Rightarrow$ height of cone = 18.75 metres

175. (d) Volume of cone = Lateral surface Area

$\frac{1}{3}\pi r^2 h = \pi r l \quad [l = \sqrt{h^2 + r^2}]$

$\frac{rh}{3} = \sqrt{h^2 + r^2}$

Squaring both sides

$\frac{1}{9} = \frac{h^2 + r^2}{r^2 h^2}$

$\frac{1}{9} = \frac{h^2}{r^2 h^2} + \frac{r^2}{r^2 h^2}$

$\frac{1}{9} = \frac{1}{r^2} + \frac{1}{h^2}$

176. (c) Diagonal of cube will be equal to diameter of sphere

$\sqrt{3}a = 2 \times r$

$\sqrt{3}a = 2 \times 6\sqrt{3}$

a = 12

Surface area $= 6a^2 = 6 \times 12 \times 12$
$\Rightarrow 864 \text{ cm}^2$

177. (c) Let hemisphere radius be = R
& Sphere radius be = r
ATQ,

$\frac{2}{3}\pi R^3 = 4 \times \frac{4}{3}\pi r^3$

$2R^3 = 16r^3$

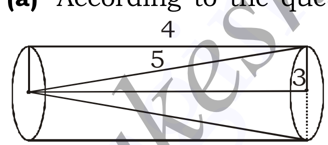
$\frac{R^3}{r^3} = \frac{8}{1} = \frac{R}{r} = \frac{2}{1}$

178. (d) Let part filled be 'x'
ATQ,
$x \times (48\text{m} \times 16.5\text{m} \times 4\text{m})$
$= \pi (2)^2 \times 56$

$x = \frac{22 \times 4 \times 56}{7 \times 48 \times 16.5 \times 4}$

$x = \frac{2}{9}$

179. (a) According to the question,



Whole surface of remaining Solid
$= \pi r l + 2\pi r h + \pi r^2$

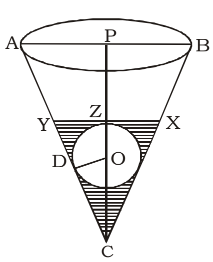
Hence $l = \sqrt{h^2 + r^2}$
$l = \sqrt{4^2 + 3^2}$
$l = 5$

$\therefore \quad = \pi r[l + 2h + r]$

$= \frac{22}{7} \times 3 \; [5 + 2 \times 4 + 3]$

$= \frac{22}{7} \times 3 \times 16 = 48\pi$

180. (a)



ΔABC = equilateral Δ

$\therefore \angle ACB = 60^0 \;\&\; \angle BCP = 30^0$

$\Delta CDO, \angle CDO = 90^0$
and tangent is 90
OD = 1P = 1 cm
OC = 2P = 2(1) = 2 cm
then, CZ = OC + OZ
= 2+1 = 3 cm

$\Delta CZY, \angle CZY = 90^0$

$CZ = \sqrt{3}Q = 3\text{cm}$

$YZ = 1Q = \sqrt{3}\text{cm}$

Now, In cone XYC

$r = ZY = \sqrt{3}\text{cm}$
h = CZ = 3 cm
Vol.of cone

$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(\sqrt{3})^2(3)$

$= 3\pi \text{ cm}^2$

Vol. of sphere $= \frac{4}{3}\pi r_s^3$

$(\therefore r_S = 1 \text{ cm})$

$= \frac{4}{3}\pi \text{ cm}^3$

Vol. of water that can immerse the ball

$= \left(3\pi - \frac{4\pi}{3}\right)\text{cm}^3 = \frac{5\pi}{3} \text{ cm}^3$

181. (b) Here h = 4c,
Volume of cylinder $= \pi r^2 h$

$= \frac{4\pi \times \pi r^2 h}{4\pi}$

(Multiply 4π both in Numerator & denominator)

$= \frac{(2\pi r)^2 \times (4c)}{4\pi} = \frac{c^3}{\pi}$

182. (a) According to the question,

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of cyclinder $= \pi r^2 h$

$\frac{4}{3}\pi r^3 = \pi r^2 h$

$= \frac{4}{3}r = h = h = \frac{4}{3} \times 3 = 4\text{cm}$

C.S.A of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 3 \times 4$

$= \frac{44 \times 12}{7} = \frac{528}{7} = 75\frac{3}{7} \text{ cm}^2$

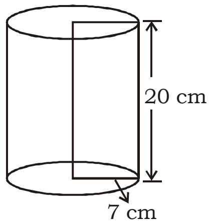
183. (a) According to question,
R = 6 cm.
$\Rightarrow$ The capacity of the

hemispheri-cal bowl

$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 6^3$

$= 452.57 \text{ cm}^3$

184. (d)



According to the question,

$\Rightarrow$ r = 7 cm

$\Rightarrow$ h = 20 cm

$\Rightarrow$ Total surface Area of cylinder = curved surface Area + 2 × area of base

$= 2\pi rh + 2\pi r^2$

$= 2\pi r [r + h]$

$= 2 \times \frac{22}{7} \times 7\ (7 + 20) = 44 \times 27$

$\Rightarrow$ TSA of cylinder = 1188 cm²

185. (d) According to question,
Given:

$\Rightarrow$ Radius of cylinder = r

$\Rightarrow$ CSA of cylinder = $4\pi r h$

$\Rightarrow$ As we know

$\Rightarrow$ Curved surface area of cylinder $= 2\pi RH$

$4\pi r h = 2\pi \times r \times \text{Height}$

$\Rightarrow$ Height = 2h unit

186. (a) According to the question,
Radius = 3.5 cm

$\Rightarrow$ In the question it is given that A hemi-shperical bowl is to be painted inside as well as outside

Total area that is to be painted = Inside area of bowl + outside area of bowl

$= 2\pi r^2 + 2\pi r^2$

$= 4\pi r^2$

$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

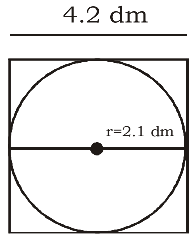
$\Rightarrow$ painting Rate = 10cm² in 5 Rs.

1 cm² will be painted $= \frac{5}{10}$ = Rs. $\frac{1}{2}$

so total cost will be painted in

$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{1}{2}$ = Rs. 77

187. (b)



r = 2.1 dm
h = 4.2 dm
(for Max.)

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4.2$

$= 19.404 \text{ dm}^3$

188. (d) Let the intial radius = r
According to the question.

$4\pi (r + 2)^2 - 4\pi r^2 = 352$

$4\pi [(r + 2)^2 - r^2] = 352$

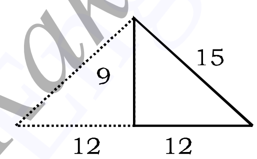
$r^2 + 4 + 4r - r^2 = \frac{352 \times 7}{22 \times 4}$

$4r + 4 = 28$

$4r = 24$

$r = 6$

189. (a)



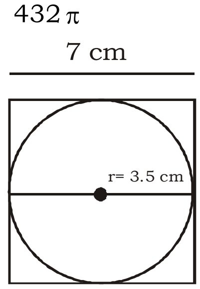
Volume $\Rightarrow \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{1}{3}\pi\ 12 \times 12 \times 9$

$\Rightarrow 144 \times 3\pi$

$\Rightarrow 432\pi$

190. (c)



According to the question,

Volume of cone $= \frac{1}{3}\pi r^2 h$

Height = 7 cm

Radius $= \frac{7}{2}$ cm

$\therefore$ Volume of cone

$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7 =$ **89.8 cm³**

191. (b) Radius of Ist solid metallic spheres = R = 6 cm

Radius of IInd solid metallic spheres = r = 1 cm

Internal Radius of hollow sphere = x

External Radius of hollow sphere $= x + 1$

So, $\frac{4}{3}\pi\left(R^3 + r^3\right)$

$= \frac{4}{3}\pi\left[(x+1)^3 - x^3\right]$

$216 + 1 = x^3 + 1 + 3x(x+1) - x^3$

$216 = 3x(x + 1)$

$72 = x^2 + x$

$\Rightarrow x^2 + x - 72 = 0$

After solving,

$x = 8$ cm

so, the external radius of the hollow sphere

$= x + 1 = 8 + 1 = 9$ cm

192. (a) Let the time taken to fill the tank = x hrs

$\Rightarrow (\pi r^2 h) \times x = 50 \times 44 \times \frac{7}{100}$

$\Rightarrow x = \frac{50 \times 44 \times 7 \times 7 \times 100 \times 100}{22 \times 7 \times 7 \times 100 \times 5000}$

= 2 hrs

193. (b) The area of ground

$\Rightarrow$ 1.5 hectares = 1.5 × 10000 m²

$\Rightarrow$ 15000 m²

($\therefore$ 1 hectare = 10000 m²)

$\Rightarrow$ level of rainfall = height of water level

$= 5 \text{ cm} = \frac{5}{100}$ m

$\therefore$ volume of collected water

$\Rightarrow 15000 \times \frac{5}{100} = 750 \text{ m}^3$

194. (d) Required quantity of water

$= \frac{3 \times 40 \times 2000}{60} = 4{,}000 \text{ m}^3$

195. (b) Let r = Radius of pipe
Let R = Radius of cistern
Let H = Height of cistern
Let the no. of hours be 'x'

$x\left(\pi R^2 H\right) = \pi r^2 h$

$\Rightarrow 3000 \times \pi \times \frac{10}{100} \times \frac{10}{100} \times x$

$= \pi \times \frac{10}{2} \times \frac{10}{2} \times 2$

$\Rightarrow \frac{6}{10} \times x = 1$

$x = \frac{10}{6}$ = 1 hour 40 minutes

196. (a) Diameter = 5 mm = 0.5 cm
radius = 0.25 cm
volume of water flowing from the pipe in 1 minute
$= \pi \times 0.25 \times 0.25 \times 1000$ cm³
volume of conical vessel

$= \frac{1}{3}\pi \times 15 \times 15 \times 24$ cm³

$= \frac{1}{3}\pi \times 15 \times 15 \times 24$ cm³

$\therefore$ Time $= \dfrac{\frac{1}{3} \times \pi \times 15 \times 15 \times 24}{\pi \times 0.25 \times 0.25 \times 1000}$

$= 28\frac{4}{5}$

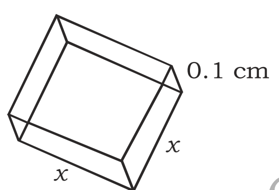
= 28 minutes 48 second

197. (d) r = 12 m, h = 9 m

$l = \sqrt{r^2 + h^2} = \sqrt{12^2 + 9^2} = 15$ m

cost of canvas = curved surface area × cost of 1 m²
$= \pi r l \times 120$
= 3.14 × 12× 15 ×120
= Rs. 67824

198. (d)



8.4 gm = 1 cm³

4725 gm $= \frac{4725}{8.4}$ cm³

volume $= x \times x \times 0.1 = \frac{4725}{8.4}$ cm³

$= x^2 \times 0.1 = \frac{4725}{8.4}$ cm³

x = 75 cm

199. (d) According to the question.
diameter = 84 cm
radius = 42 cm = 0.42 m
height = 120 cm = 1.2 m
$\therefore$ Curved surface area of cylinder

$= 2\pi rh = \frac{2 \times 22 \times 0.42 \times 1.2}{7}$

Cost of levelling the ground
$= 2\pi rh \times 500 \times 1.5$

$= \frac{2 \times 22 \times 0.42 \times 1.2 \times 1.5 \times 500}{7}$

= 2376

200. (a) since the volume of the two cylinders is same

$\therefore \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = 1$

$\frac{h_1}{h_2} = \frac{1}{2}$

$\left(\frac{r_1}{r_2}\right)^2 = \frac{h_2}{h_1} = \frac{2}{1}$

$\frac{r_1}{r_2} = \sqrt{\frac{2}{1}} = \frac{\sqrt{2}}{1}$

$\therefore$ Ratio of their radii $= \sqrt{2} : 1$

201. (a) We are given that volume of two cube are in the ratio = 27 :1

$\left(\frac{a_1}{a_2}\right)^3 = \frac{27}{1}$

$\frac{a_1}{a_2} = \sqrt[3]{\frac{27}{1}} = \frac{3}{1} =$ **3 : 1**

202. (a) Ratio of edges of cuboid = 1 : 2 : 3
Let,
$l = x,\ b = 2x, h = 3x$
Surface area = 88 cm²
$2(lb + bh + hl) = 88$
$2(2x^2 + 6x^2 + 3x^2) = 88$
$11x^2 = 44$
$\Rightarrow x^2 = 4,\ x = 2$
$\therefore l = 2$ cm, $b = 4$ cm, $h = 6$cm
$\therefore$ Volume $= l\,b\,h$
= 2 × 4 × 6
= 48 cm³

203. (a) $\frac{V_1}{V_2} = \frac{8}{27}$

$\dfrac{\frac{4}{3}\pi (r_1)^3}{\frac{4}{3}\pi (r_2)^3} = \frac{8}{27}$

$\left(\frac{r_1}{r^2}\right)^3 = \frac{8}{27}$

$\left(\frac{(r_1)^2}{(r_2)^2}\right) = \left(\frac{2}{3}\right)^2$

Ratio of surface area $= \frac{4\pi r_1^2}{4\pi r_2^2}$

$= \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

204. (b) $\frac{R_1}{R_2} = \frac{2}{3},\ \frac{H_1}{H_2} = \frac{5}{3}$

Ratio of volumes $= \frac{V_1}{V_2}$

$= \frac{\pi R_1^2 H_1}{\pi R_2^2 H_2} = \left(\frac{R_1}{R_2}\right)^2 \times \left(\frac{H_1}{H_2}\right)$

$= \left(\frac{2}{3}\right)^2 \times \left(\frac{5}{3}\right) = \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$

205. (d) $2\pi rh = 264$ .... (i)
$\pi r^2 h = 924$ ....(ii)

on dividing: $\frac{2\pi rh}{\pi r^2 h} = \frac{264}{924}$

$\frac{2}{r} = \frac{264}{924}$

$r = \frac{924 \times 2}{264} = 7$ cm

diametre = 2r = 2 × 7 = 14 cm
putting, r = 7 in (i)
$2\pi rh = 264$

$h = \frac{264 \times 7}{2 \times 22 \times 7}$
= 6 cm

Required ratio $= \frac{2r}{h} = \frac{14}{6} = \frac{7}{3}$

206. (b) $\dfrac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h^2} = \frac{2}{3}$

$\left(\frac{1}{2^2}\right)\frac{h_1}{h_2} = \frac{2}{3}$

$\frac{h_1}{h_2} = \frac{8}{3}$

207. (c) $\frac{(a_1)^3}{(a_2)^3} = \frac{27}{64}$

$\frac{a_1}{a_2} = \frac{3}{4}$

Ratio of their total surface area

$$= \frac{6a_1^2}{6a_2^2} = \left(\frac{a_1}{a_2}\right)^2$$

$$= \left(\frac{3}{4}\right)^2 = \frac{9}{16} = \mathbf{9 : 16}$$

208. (b) Radius of both hemisphere and cone = R

Also height of hemisphere is equal to its Radius = R

$\therefore$ height of both hemisphere and cone = R

Now, In cone

slant height, $l = \sqrt{R^2 + R^2}$

$= \sqrt{2}R$

$$\frac{\text{C.S.A of hemisphere}}{\text{C.S.A of cone}}$$

$$= \frac{2\pi R^2}{\pi R \times \sqrt{2} R} = \frac{\sqrt{2}}{1}$$

$= \sqrt{2} : 1$

209. (c) Let height of cone = h

radius of cone = r

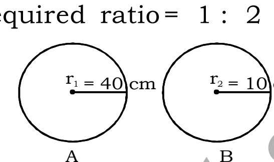
volume of cone = $\frac{1}{3}\pi r^2 h$

Now height is doubled , volume of new cone

$$= \frac{1}{3}\pi r^2 (2h) = \frac{2}{3}\pi r^2 h$$

Required ratio = 1 : 2

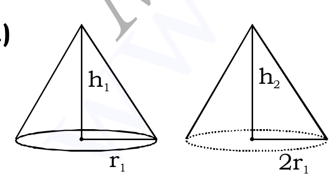
210. (d)



$$\frac{\text{surface area of A}}{\text{Surface area of B}} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{40}{10}\right)^2 = \frac{16}{1} \Rightarrow 16 : 1$$

211.(d)



$r_2 = 2r_1$

$h_2 = h_1$

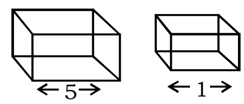
$$\frac{\text{volume of new cone}}{\text{volume of old cone}}$$

$$= \frac{\frac{1}{3}\pi r_2^2 \times h_2}{\frac{1}{3}\pi r_1^2 \times h_1} = \frac{r_2^2 \times h_2}{r_1^2 \times h_1}$$

$$= \frac{(2r_1)^2 \times h_1}{r_1^2 \times h_1} = \frac{4}{1}$$

$\Rightarrow$ **4 : 1**

212. (d)



Ratio of total surface area $= \frac{6(1)^2}{6(5)^2}$

$$\Rightarrow = \frac{1}{25} = \mathbf{1 : 25}$$

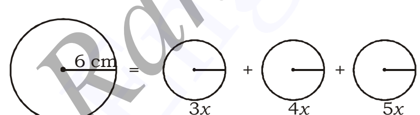
213. (b) Let $r_1 = \frac{21}{2}$ cm

$r_2 = \frac{17.5}{2}$ cm

$\therefore$ Required ratio $= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2}$

$$= \frac{21 \times 21}{17.5 \times 17.5} = \frac{36}{25} = \mathbf{36 : 25}$$

214. (a)



$$\frac{4}{3}\pi \{(3x)^3 + (4x)^3 + (5x)^3\}$$

$$= \frac{4}{3}\pi (6)^3$$

$x^3 (27 + 64 + 125) = 216$

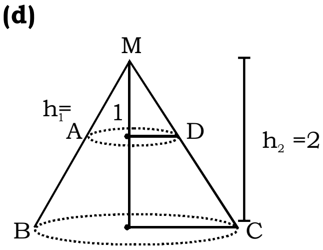
$x^3 \times 216 = 216$

$$x^3 = \frac{216}{216} = 1$$

$x = \sqrt[3]{1} = 1$

Radius of smallest sphere = $3x$ = 3 × 1 = 3 cm

215. (d)



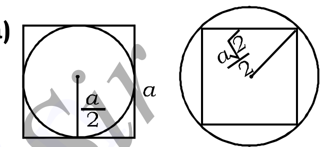
$$\frac{\text{ratio of smaller cone}}{\text{ratio of larger cone}} = \frac{h_1^3}{h_2^3}$$

$$= \frac{1^3}{2^3} = \frac{1}{8}$$

area of part (ABCD) (i.e frustum) = 8 – 1 = 7

$\therefore$ Required ratio = 1 : 7

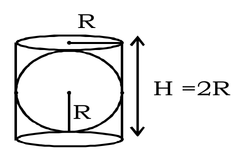
216. (a)



$$\frac{\text{area of incircle}}{\text{area of circum circle}}$$

$$= \frac{\pi\left(\frac{a}{2}\right)^2}{\pi\left(\frac{a\sqrt{2}}{2}\right)^2} = \frac{1}{2} \Rightarrow \mathbf{1 : 2}$$

217. (b)



(height of cylinder = 2 ×R)

$$\frac{\text{Surface area of sphere}}{\text{C.S.A of cylinder}}$$

$$\frac{4\pi R^2}{2\pi R \times H} = \frac{4\pi R^2}{2\pi R(2R)}$$

$$= \frac{4\pi R^2}{4\pi R^2} = \frac{1}{1} = 1 : 1$$

218. (d) Ratio of volume = (Ratio of radius)³

$$= \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3$$

$$= \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

219. (a) $\frac{4}{3}\pi r^3 = \pi r^2 h$

$\Rightarrow h = \frac{4}{3} r$

$\Rightarrow r = \frac{3}{4} h$

$\Rightarrow$ 2r(diameter) $= \frac{3}{4} \times 2h = \frac{3}{2} h$

$$\Rightarrow \frac{\text{Diameter}}{\text{Height}} = \frac{3}{2}$$

220. (a) In this case height of cylinder and cone is equal to the radius of hemisphere

$\Rightarrow h = r$ Ratio of volumes

Cone : hemisphere : cylinder

$= \frac{1}{3}\pi r^2 \times r : \frac{2}{3}\pi r^3 : \pi r^2 \times r$

$= 1 : 2 : 3$

221.(a) Ratio of curved surface area = Ratio of product of height and radius

$\Rightarrow$ Required ratio

$\frac{C_1}{C_2} = \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{r_1 h_1}{r_2 h_2}$

$= \frac{3\times 2}{5\times 3} = \frac{2}{5}$

222. (d) Ratio of surface area = (Ratio of radius)²

$= \frac{C_1}{C_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$

$= \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

223. (a) $\frac{\pi R^2 H}{\pi r^2 h} = \frac{3}{1}$

$\Rightarrow \frac{3\times 3\times H}{2\times 2\times h} = \frac{3}{1}$

$\Rightarrow \frac{H}{h} = \frac{4}{3}$

$\Rightarrow \frac{x}{1} = \frac{4}{3}$

$\Rightarrow x = \frac{4}{3}$

224. (c) $\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$

$\frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = 64$

$\left(\frac{8}{r}\right)^3 = (4)^3$

$\Rightarrow r = 2$ cm

$\frac{C_1}{C_2} = \frac{4\pi R^2}{4\pi r^2} = \left(\frac{R}{r}\right)^2$

Ratio of area = (Ratio of radius)²

= (8 : 2)²

= 16 : 1

225. (a) $\frac{\pi R_1^2 H}{\pi R_2^2 h} = 1$

$\frac{3^2 \times H}{2^2 \times h} = 1$

$\Rightarrow \frac{H}{h} = \frac{4}{9}$

226.(a) Let the radius and slant height be $4x$ and $7x$

$\Rightarrow 7x = 14$ cm

$x = 2$ cm

$\Rightarrow$ Radius $= 4 \times 2 = 8$ cm

227. (d) Height of cylinder = Diameter of sphere

$\Rightarrow 4\pi r^2 : 2\pi rh$

$\Rightarrow 4\pi r^2 : 4\pi r^2$ (h=2r)

$\Rightarrow$ Required ratio = **1 : 1**

228. (a) $\frac{V_1}{V_2} = \frac{r^2 h}{R^2 H} = \frac{3^2 \times 4}{4^2 \times 3} = \frac{3}{4}$

229. (d) Ratio of volume of bigger cone and smaller cones

= (Ratio of altitude)³

= (1 : 2 : 3)³

= (1 : 8 : 27)

∴ Ratio of parts = 1 : 8–1 : 27–8

= 1 : 7 : 19

230. (c) Let radii of cylinder and sphere be r

∴ Volume of cylinder of height (h)

$= \pi r^2 h$

Total surface area of cylinder

$= 2\pi rh + 2\pi r^2$

Total surface area of sphere

$= 4\pi r^2$

∴ given that

$4\pi r^2 = 2\pi rh + 2\pi r^2$

$4\pi r^2 = 2\pi r(h + r)$

$\Rightarrow 2r = h + r$

$r = h$

∴ radius of sphere or cylinder's equal to height of cylinder

∴ Ratio of volume of cylinder and sphere

$= \pi r^2 \times r : \frac{4}{3}\pi r^3 =$ **3 : 4**

231.(c) $\frac{4}{3}\pi R^3 = \pi r^2 H$

$\frac{4}{3}R^3 = r^2 H$

$\frac{R^2}{r^2} = \frac{3}{4}$ $(\because H = R)$

$R : r = \sqrt{3} : \sqrt{4} = \sqrt{3} : 2$

232. (b) $\frac{a^3}{\frac{4}{3}\pi r^3} = \frac{363}{49}$

$\frac{a^3}{r^3} = \frac{363\times 22\times 4}{49\times 7\times 3}$

$\frac{a^3}{r^3} = \left(\frac{22}{7}\right)^3$

$\frac{a}{r} = \frac{22}{7}$

233. (a) cone $\Rightarrow$ radius : height

4 : 3

Let 4x : 3x

∴ curved surface area of cone $= \pi r l$

r = radius

l = slant height

$l = \sqrt{h^2 + r^2}$

$= \sqrt{(4x)^2 + (3x)^2} = 5x$

∴ curved surface area

$\Rightarrow \pi \times 4x \times 5x$

$\Rightarrow 20\pi x^2$

∴ total surface area $\Rightarrow \pi r l + \pi r^2$

$\Rightarrow \pi r(l + r)$

$\Rightarrow \pi \times 4x(5x + 4x)$

$\Rightarrow \pi \times 4x \times 9x$

$\Rightarrow 36\pi x^2$

∴ Curved area : total area

$20\pi x^2 : 36\pi x^2$

5 : 9

234. (b) Let radius of sphere = r

radius of cylinder = r

∴ let height of cylinder = h

∴ given that volume of sphere = volume of cylinder

$\Rightarrow \frac{4}{3}\pi r^3 = \pi r^2 h$

$\Rightarrow \frac{4}{3}r = h$

∴ curved surface area

cylinder : sphere

$2\times \pi \times r \times \frac{4r}{3} : 4\pi r^2$

$\Rightarrow \frac{8}{3} : 4$

2 : 3

235. (c) radius of cone = radius of cylinder = r height of cone = height of cylinder = h

$\frac{\text{curved surface area of cylinder}}{\text{curved surface area of cone}}$

$\Rightarrow \frac{2\pi rh}{\pi rl} = \frac{8}{5}$

$\Rightarrow \frac{h}{1} = \frac{4}{5}$

$\Rightarrow \frac{h^2}{l^2} = \frac{16}{25}$

$\Rightarrow l^2 = h^2 + r^2$

$\Rightarrow h^2 = 16$

$25 = 16 + r^2$

$r^2 = 9$

$r = 3$

$\therefore$ radius : height

3 : 4

236. (a) Volume of prism = Base Area × Height

$$1056 = \frac{1}{2}(8 + 14) \times h \times 12$$

$$h = \frac{1056 \times 2}{22 \times 12} = 8 \text{ cm}$$

237. (b) Ratio of volume =

$$\frac{\pi\left(\sqrt{3}\right)^2 \times \sqrt{2}}{\frac{1}{3}\pi\left(\sqrt{2}\right)^2 \times \sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{2}}$$

$$= 3\sqrt{3} : \sqrt{2}$$

238. (a) $$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$

$$\text{Ratio of volume} = \frac{(3)^2 \times 1}{(5)^2 \times 3} = 3$$

: 25

239. (d) Let the radius of sphere and hemisphere be = R and r

$\Rightarrow \frac{4}{3}\pi R^3 = \frac{2}{3}\pi r^3$

$2R^3 = r^3$

$\frac{R^3}{r^3} = \frac{1}{2}$

$\Rightarrow R : r = 1 : \sqrt[3]{2}$

240. (a) Ratio of radius of earth and moon = 4 : 1

$\Rightarrow$ Ratio of volume $= 4^3 : 1^3$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = 64 : 1$$

241. (b) Let the radius of cylinder and sphere be = r cm

$\Rightarrow$ height of cylinder = 2r cm

$\Rightarrow A = \pi r^2 \times 2r = 2\pi r^3$

$B = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{A}{B} = \frac{2\pi r^3}{\frac{4}{3}\pi r^3} = 3 : 2$$

242. (c) Side of cube $= \frac{6\sqrt{3}}{\sqrt{3}}$

= 6cm

$$\text{Required rate} = \frac{6 \times 6^2}{6^3} = 1 : 1$$

243. (d) Let the radius of hemi-sphere and sphere be 'r' and 'R'

$\Rightarrow \frac{4}{3}\pi R^3 = \frac{2}{3}\pi r^3$

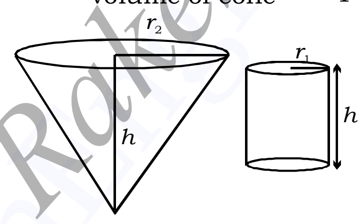
$\frac{R^3}{r^3} = \frac{1}{2}$

$\frac{R}{r} = \frac{1}{\sqrt[3]{2}}$

$\Rightarrow$ Ratio of curved surface area

$$= \frac{4\pi R^2}{2\pi r^2} = \frac{2R^2}{r^2} = \frac{2 \times 1}{\left(\sqrt[3]{2}\right)^2} = 2^{\frac{1}{3}} : 1$$

244. (b) $$\frac{\text{Volume of cylinder}}{\text{volume of cone}} = \frac{3}{1}$$

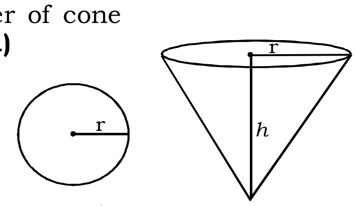


$$\frac{\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h} = \frac{3}{1}$$

$\Rightarrow r_1 = r_2$

Diameter of cylinder = Diameter of cone

245.(d)



Volume remains same

volume of sphere = volume of cone

$$\frac{4}{3}\pi r^3 = \frac{1}{3} \times \pi \times r^2 \times h$$

4r = h

$$\frac{h}{r} = \frac{4}{1} = \mathbf{4 : 1}$$

246. (b) Area of Δ (Base Area) = Semiperimeter × in radius

$$= \left(\frac{45}{2} \times 9\right) = \text{cm}$$

Volume of prism = Base area × Height

$$810 = \frac{45}{2} \times 9 \times \text{Height}$$

$$\text{Height} = \frac{810 \times 2}{45 \times 9} = 4 \text{ cm}$$

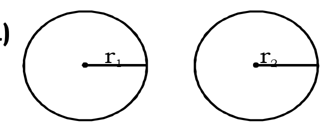
Total surface area = Lateral Surface Area + (Bottom + Top) Area

= (Perimeter of Base × Height) + (2 × Base area)

$$= (45 \times 4) + 2 \times \left(9 \times \frac{45}{2}\right)$$

$= 180 + 405 = 585$ cm²

247. (a)



Ratio of volume of sphere × ratio of weight per 1 cc. of material of each = Ratio of weight of two sphere

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} \times \frac{289}{64} = \frac{8}{17}$$

$$\frac{r_1^3}{r_2^3} = \frac{8 \times 64}{17 \times 289} = \frac{8 \times 8 \times 8}{17 \times 17 \times 17}$$

$$\frac{r_1}{r_2} = \frac{8}{17} \Rightarrow \mathbf{8 : 17}$$

248. (a) $$\frac{R_1}{R_2} = \frac{1}{2}$$

$$\frac{V_1}{V_2} = \frac{2}{3}$$

$$\frac{\frac{1}{3}\pi R_1^2 H_1}{\frac{1}{3}\pi R_2^2 H_2} = \frac{2}{3}$$

$$\left(\frac{R_1}{R_2}\right)^2 \times \left(\frac{H_1}{H_2}\right) = \frac{2}{3}$$

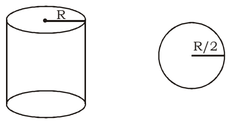
$$\left(\frac{1}{2}\right)^2 \times \frac{H_1}{H_2} = \frac{2}{3}$$

$$\frac{1}{4}\times\frac{H_1}{H_2} = \frac{2}{3}$$

$$\frac{H_1}{H_2} = \frac{2}{3}\times\frac{4}{1}$$

$$= \frac{8}{3} \Rightarrow 8 : 3$$

249. (d)



Let the Radius of cylinder = R

⇒ Therefore, Radius of sphere

$= \frac{R}{2}$

Volume of Right circular cylinder

$= \pi R^2 H$

$$\text{Volume of sphere} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3$$

$$= \frac{4}{3}\pi\frac{R^3}{8} = \frac{\pi R^3}{6}$$

According to question,

Volume of Cylinder = Volume of sphere

$$\pi R^2 H = \frac{\pi R^3}{6}$$

$$\frac{\pi R^2 H \times 6}{\pi R^3} = 1$$

$$\frac{H}{R} = \frac{1}{6} \Rightarrow 1 : 6$$

250. (d) Radius of larger sphere = R units

$$\text{Its volume} = \frac{4}{3}\pi R^3$$

Now cones are formed with base radius and height same as the radius of larger sphere

$$\therefore \text{Volume of smaller cone} = \frac{1}{3}\pi R^3$$

and one of the cone is converted into smaller sphere

Therefore volume of smaller sphere

$$= \frac{1}{3}\pi R^3$$

$$\therefore \quad \frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^3$$

$$\frac{r^3}{R^3} = \frac{1}{4}$$

$$\frac{r}{R} = \frac{1}{\sqrt[3]{4}}$$

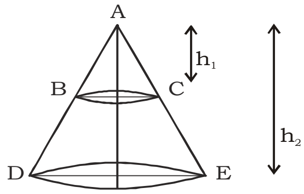
$$\therefore \frac{\text{Surface area of smaller shpere}}{\text{Surface area of larger sphere}}$$

$$\therefore = \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2}$$

$$\Rightarrow \frac{(1)^2}{\left((4)^{\frac{1}{3}}\right)^2} = \frac{(1)^2}{\left((2)^{\frac{2}{3}}\right)^2} = \frac{1}{2^{\frac{4}{3}}}$$

$$\Rightarrow 1 : 2^{\frac{4}{3}}$$

251. (c)



$$\frac{\text{Volume of Cone ABC}}{\text{Volume of BCED}} = \frac{1}{1}$$

$$\frac{\text{Volume of Cone ABC}}{\text{Volume of Cone ADE}} = \frac{1}{1+1}$$

$$= \frac{1}{2}$$

If a cone is cut in any parts parallel to its base then the ratio of volume of smaller cone to the volume of larger cone is equal to the ratio of the cubes of their corresponding heights/ radii/slant height(it is proved by similarity)

$$= \left(\frac{\text{height of Cone}(h_1)}{\text{height of Cone}(h_2)}\right)^3 = \frac{1}{2}$$

$$\frac{h_1}{h_2} = \frac{1}{\sqrt[3]{2}}$$

$$\Rightarrow h_1 : h_2 - h_1$$

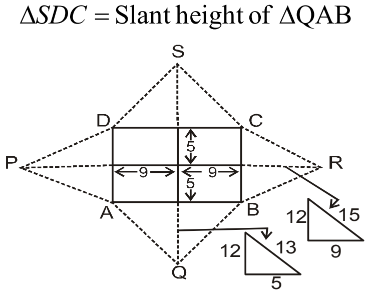
$$= 1 : \left(\left(\sqrt[3]{2} - 1\right)\right)$$

$$1 : \left(\sqrt[3]{2} - 1\right)$$

252. (b) Height (h) = 12cm (given)

∴ Slant height of

ΔSDC = Slant height of ΔQAB



$$= \sqrt{(12)^2 + (5)^2} = 13\text{cm}$$

and slant height of ΔPDA = Slant height of ΔRCB

$$= \sqrt{(12)^2 + (9)^2} = 15cm$$

∴ Area of ΔABQ

= Area of ΔSDC

$$= \frac{1}{2}\times 18\times 13 = 117cm^2$$

Area of Δ PDA = Area of ΔRCB

$$\frac{1}{2}\times 10\times 15 = 75cm^2$$

& Area of □ABCD (Base area)

= 10 × 18

= 180 cm²

Total surface area = Base area + area of (ΔABQ + ΔSDC + ΔPDA + ΔRCB)

$= 180 + 2\times 117 + 2\times 75 = 564\text{cm}^2$

253. (a) volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times 1^2 \times 7 = \frac{22}{3}\ \text{cm}^3$$

Volume of cubical block

= 10×5×2 cm³ =100 cm³

Wastage of wood

$$= \left(100 - \frac{22}{3}\right)\text{cm}^3$$

$$= \left(\frac{300-22}{3}\right) = \frac{278}{3}\ \text{cm}^3$$

$$\%\ \text{wastage} = \frac{\frac{278}{3}}{100}\times 100$$

$$= \frac{278}{3} = \mathbf{92\frac{2}{3}\%}$$

254. (c) Decrease in radius = $50\% = \frac{1}{2}$

increase in height = 50%

$$= \frac{1 \rightarrow \text{Increment}}{2 \rightarrow \text{Original}}$$

(Let)

	Radius	Height	Volume
Original	2	2	$(2)^2\times(2) = 8$
	↓ 50% decrease	↓ 50% increase	↓ −5
New	1	3	$(1)^2\times(3) = 3$

Reduction in volume

$$= \frac{5}{8} \times 100 = \mathbf{62\frac{1}{2}}\%$$

255. (a) Increase in radius = 100%

$$= \frac{1}{1}$$

Increase in height = 100% $= \frac{1}{1}$

	Radius	Height	Volume
Original	1	1	$(1)^2 \times (1) = 1$
	↓	↓	↓ 7
New	2	2	$(2)^2 \times (2) = 8$

% increase $= \frac{7}{1} \times 100 = \mathbf{700\%}$

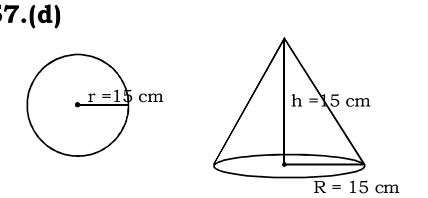
256. (d) $20\% = \frac{1 \rightarrow \text{Increment}}{5 \rightarrow \text{Original}}$

Radius	Height	volume
5	5	$(5)^2 5 = 125$
6	6	$(6)^2(6) = 216$

(Difference: 91)

$$= \frac{91}{125} \times 100 = 72.8\%$$

257.(d)



volume of cone $= \frac{1}{3} \times \pi (15)^2 \times 15$

$$= \frac{1}{3}\pi(15)^3$$

Volume of sphere $= \frac{4}{3}\pi(15)^3$

Required Percentage

$$= \frac{\text{volume of cone}}{\text{volume of sphere}} \times 100$$

$$= \frac{\frac{1}{3} \times \pi \times (15)^3}{\frac{4}{3} \times \pi (15)^3} \times 100$$

$$= \frac{1}{4} \times 100 = 25\%$$

258. (d)

	Radius	Height	Volume
Original	2	1	$(2)^2 \times (1) = 4$
	↓	↓	↓ 1
New	1	3	$(1)^2 \times (3) = 3$

% decrease $= \frac{4-3}{4} \times 100 =$ 25%

259. (d) height = 100% Radius = 100%

$\frac{1 \rightarrow \text{Increment}}{1 \rightarrow \text{Original}}$ $\frac{1 \rightarrow \text{Increment}}{1 \rightarrow \text{Original}}$

	Height	Radius	Volume
Original	1	1	$(1)^2 1 = 1$
New	2	2	$(2)^2 2 = 8$

$\left[\frac{1}{3}\pi \text{ is Constant}\right]$

= eight times that of original

260. (b) use $x + y + \frac{xy}{100}$

percentage change in area

$$= 15 - 10 + \frac{15 \times (-10)}{100}$$

$$= 5 - 1.5 = 3.5\%$$

(3.5 % increase)

Remember : when change in area is asked in the question, then use this formula to save your valuable time.

261. (d) Let old radius = r

$\Rightarrow$ volume $= \frac{4}{3}\pi r^3$

New radius = 2r

$\Rightarrow$ New volume $= \frac{4}{3}\pi(2r)^3$

$$= \frac{4}{3}\pi \times 8r^3$$

$\Rightarrow$ Volume becomes eight times

262. (b)

	Radius	Height	Volume
Original	2	5	$(2)^2 \times (5) = 20$
	↓	↓	↓ 12
New	1	8	$(1)^2 \times (8) = 8$

$\Rightarrow$ Volume decreases

% decrease (% कमी) $= \frac{20-8}{20} \times 100$

= 60%

263. (d) Length $1 \rightarrow 2$

Breadth $2 \rightarrow 6$

Height $3 \rightarrow 9$

Volume $6 \rightarrow 108$

$\Rightarrow$ New volume = 18 times the original volume

$\Rightarrow$ Increase in volume

= 18 – 1 = 17 times

264. (c)

	Radius	Height	Volume
Original	10	10	$(10)^2 \times (10) = 1000$
	↓	↓	↓ 331
New	11	11	$(11)^2 \times (11) = 1331$

$\Rightarrow$ % Increase $= \frac{1331 - 1000}{1000} \times 100$

= 33.1%

265. (a) % Change in height = % change in volume = 100%

266. (a) Volume of coffee

$$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (4)^3$$

$$= \frac{128}{3}\pi \,\text{cm}^3$$

Volume of cone $= \frac{1}{3}\pi r^2 \times h$

$$= \frac{1}{3}\pi(8)^2 \times 16 = \frac{1024}{3}\pi$$

∴ Required percentage

$$= \frac{\frac{1024}{3} - \frac{128}{3}}{\frac{1024}{3}} \times 100 = 87.5\%$$

267. (a) Decrease in base radius = (Decrease in base area)$^{1/2}$

$$= \left(\frac{1}{9}\right)^{\frac{1}{2}} = \frac{1}{3}$$

Let initial radius and height be $3r$ and h

∴ New radius and height are r and 6 h

old lateral surface area

$= 2 \times \pi \times 3r \times h$

$= 6\pi rh$

New lateral surface area

$= 2 \times \pi \times r \times 6h$

$= 12\pi rh$

Required factor $= \frac{12\pi rh}{6\pi rh} = 2$

268. (c) Let the original radius be 'r'

$\Rightarrow$ Area $= 4\pi r^2$

New area $= 4\pi(2r^2) = 16\pi r^2$

$\Rightarrow$ New area is 4 times the old area

269. (d) Edge is increased by 50% =

$$\frac{50}{100} = \frac{1 \rightarrow \text{Increased}}{2 \rightarrow \text{Original}}$$

Let original edge = 2

$\therefore$ increased edge = 3

edge	surface area
2	4
3	9

$\left.\begin{matrix}4\\9\end{matrix}\right) + [5,\ 6 \text{ is constant}]$

% increase = $\frac{5}{4}\times 100$ = **125%**

270. (b) Let the inital radius = 1 unit
New radius = 2 unit (radius is doubled)

Radius	:	Volume
1		$(1)^3$
2		$(2)^3$

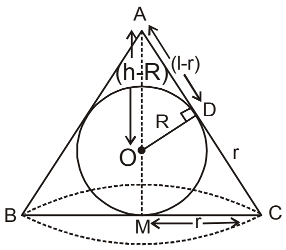
$\left(\frac{4}{3}\pi \text{ is constant}\right)$

1	1
2	8

$\left.\begin{matrix}1\\8\end{matrix}\right)7$

Percentage increase = $\frac{7}{1}\times 100$
= **700%**

271. (c)



Let the radius of cone = r
and height = h, slant height = l
and radius of sphere = R

$\therefore R = \frac{hr}{l+r} = \frac{8\times 6}{10+6} = 3cm$

Detailed method

CD and CM are tangents at c,
$\therefore$ CD = CM
Now in ΔADO,
$(h - R)^2 = (R)^2 + (l - r)^2$
$\Rightarrow h^2 + R^2 - 2Rh = R^2 + l^2 + r^2 - 2lr$
$\Rightarrow h^2 - 2rh = h^2 + r^2 + r^2 - 2lr$
$[\because l^2 = h^2 + r^2]$

$\Rightarrow R = \frac{r(l-r)}{h} = \frac{rh(l-r)}{h^2}$

$\Rightarrow R = \frac{rh(l-r)}{(l^2-r^2)} = \frac{rh(l-r)}{(l-r)(l+r)} = \frac{rh}{l+r}$

$= \frac{8\times 6}{10+6} = 3cm$

272. (a) Volume of tetrahedron

$= \frac{a^3}{6\sqrt{2}} = \frac{12^3}{6\sqrt{2}}$

$= \frac{1728}{6\sqrt{2}} = 144\sqrt{2}\ \text{cm}^3$

273. (a) Volume of bucket

$= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$

$= \frac{1}{3}\times\frac{22}{7}\times 45\,(28^2 + 7^2 + 28\times 7)$

$= \frac{22}{7}\times 15\times 1029 = 48510\ \text{cm}^3$

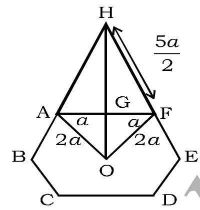
274. (c) Side of regular hexagon = 2a cm

area of hexagon = $6\times\frac{\sqrt{3}}{4}\times(2a)^2$

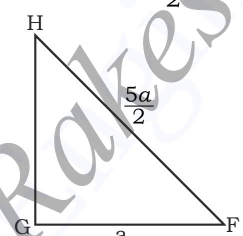
$\Rightarrow 6\sqrt{3}a^2\ \text{cm}^2$

slant edge of pyramid = $\frac{5a}{2}$ cm

$\therefore$



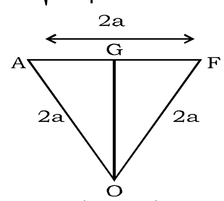
slant edge $\Rightarrow \frac{5a}{2}$ (Given)



$\Rightarrow$ HF = $\frac{5a}{2}$ (slant edge)
$\Rightarrow$ HG = slant height (l)
$\Rightarrow$ GF = base
$\Rightarrow$ (a) (given)

slant height $\Rightarrow \sqrt{\left(\frac{5a}{2}\right)^2 - (a)^2}$

$= \sqrt{\frac{25a^2}{4} - a^2} = \frac{\sqrt{21}a}{2}$



AOF is equilateral triangle of side 2a

$\therefore$ Altitude GO $= \frac{\sqrt{3}}{2}\times 2a$
$= \sqrt{3}\ a$

$\therefore$ Slant height $\Rightarrow \frac{\sqrt{21}}{2}$a

altitude $= \sqrt{3}$ a

$\therefore$ height of the pyramid =h

$\Rightarrow \sqrt{\left(\frac{\sqrt{21}a}{2}\right)^2 - \left(\sqrt{3}a\right)^2}$

$= \sqrt{\frac{21}{4}a^2 - 3a^2} = \sqrt{\frac{9a^2}{4}}$

$= \frac{3}{2}a$

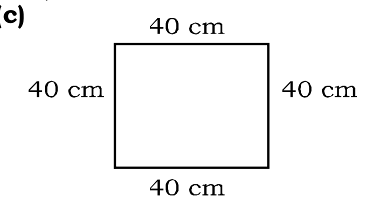
$\therefore$ Volume of pyramide

$= \frac{1}{3}$ area of base × height

$= \frac{1}{3}\times 6\sqrt{3}a^2\times\frac{3}{2}$a

$= 3\sqrt{3}a^3\ \text{cm}^3$

275. (c)



$\Rightarrow$ area of base = 40 × 40
= 1600 cm²

Let height of pyramid= h

$\therefore$ Volume = $\frac{1}{3}$ × h × area of base

$= \frac{1}{3}$ × h × 1600

$\Rightarrow$ 8000 (given)
= h = 15 cm

276. (c) area of trapizium

$= \frac{1}{2}$ × h (AB + CD)

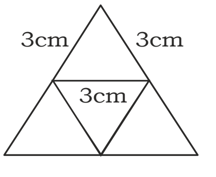
$= \frac{1}{2}$ ×8 × (8 + 14)

= 4 × 22 = 88 cm²
= volume of prism
=Height of prism × area of base
$\Rightarrow$ height × 88 = 1056 (given)

$\Rightarrow$ height = $\frac{1056}{88}$

$\Rightarrow$ 12 cm

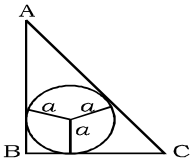
277. (a) Edge of regular tetra hadron = 3 cm
$\therefore$ a = 3 cm



$\therefore$ volume $\Rightarrow \frac{\sqrt{2}}{12}a^3$ cm³

$\Rightarrow \frac{\sqrt{2}}{12} \times (3)^3 = \frac{9}{4}\sqrt{2}$ cm³

278. (d)



r - inradius of incircle of triangle
Perimeter = 15 cm (given)

$\therefore$ Semiperimeter (S) = $\frac{15}{2}$ cm

Inradius of any triangle

$r \Rightarrow \frac{\Delta}{s}$

$r = \frac{\text{area}}{\text{semiperimeter}}$

where Δ is the area of triangle
$\therefore$ r = 3 cm given

$3 \Rightarrow \frac{\text{area of triangle}}{\frac{15}{2}}$

$3 \times \frac{15}{2}$ = area of traingle

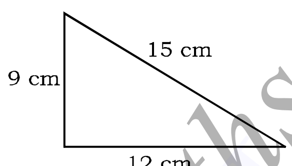
$\Rightarrow \frac{45}{2}$ cm = area of triangle

$\therefore$ volume of prism $\Rightarrow$ 270 cm³ (given)

$\therefore 270 = h \times \frac{45}{2}$

$\Rightarrow$ h = 12 cm

279. (c)

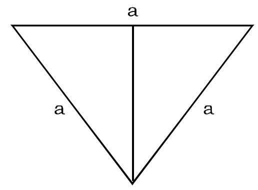


9, 12, 15 is a triplet which forms a right angle triangle
$\therefore$ area of base of prism

$\Rightarrow \frac{1}{2} \times 9 \times 12 =$ 54 cm²

Perimeter of triangle = 9 + 12 + 15 = 36 cm
$\therefore$ total surface area of prism
= perimeter base × height + 2× area of base
$\Rightarrow$ height of prism = 5 cm given
$\therefore$ total surface area
= (36 × 5) + (2 × 54)
$\Rightarrow$ 180 + 108 = 288 cm²

280. (c)



Let side equilateral triangle be = a

$\therefore$ area $= \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4}a^2 = 173 \text{ cm}^2$

$\Rightarrow a^2 = \frac{173}{\sqrt{3}} \times 4$

$(\sqrt{3} = 1.73)$

$\therefore a^2 = \frac{173}{1.73} \times 4$

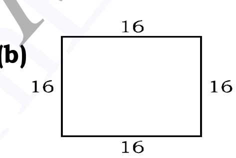
$= \frac{173}{173} \times 4 \times 100$

a² = 400
a = 20 cm.
Perimeter of base = 20 × 3 = 60 cm
$\therefore$ Volume of prism = 10380 cm³ (given)
area of base × height = 10380

height $= \frac{10380}{173} = 60$

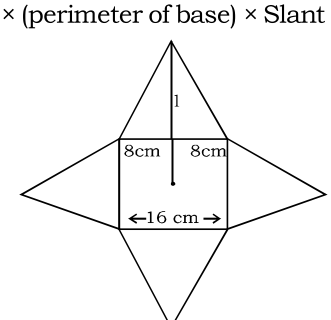
LSA = 60 × 60 = 3600 cm²

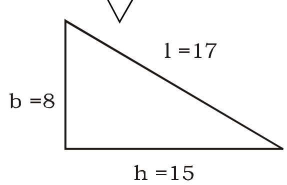
281. (b)



Perimeter of the base = 4 × 16
= 64 cm
Curved or lateral surface area of pyramid

$= \frac{1}{2}$ × (perimeter of base) × Slant height





$\Rightarrow$ height of pyramid $\Rightarrow$ 15 cm
$\Rightarrow$ base = 8 cm
$\Rightarrow$ slant height of pyramid

$l = \sqrt{(15)^2 + (8)^2} \Rightarrow$ 17 cm

$\Rightarrow$ Curved surface area of pyramid

$\Rightarrow \frac{1}{2} \times 64 \times 17 \Rightarrow$ 544 cm²

282. (c) Volume of pyramid

$= \frac{1}{3}$ × Area of base × height

$= \frac{1}{3} \times 57 \times 10 =$ 190 cm³

283. (c) Let the side of square base = a cm
$\Rightarrow$ 2a² + 4a × h = 608
$\Rightarrow$ 2a² + 4a × 15 = 608
$\Rightarrow$ a² + 30a = 304
$\Rightarrow$ a² + 38a – 8a – 304 = 0
$\Rightarrow$ a(a + 38) – 8 (a + 38) = 0
$\Rightarrow$ a = – 38,8
$\Rightarrow$ a = 8 cm
$\therefore$ Volume of prism = 8 × 8 × 15
= 960 cm³

284. (b) Volume of prism $= \frac{\sqrt{3}}{4}a^2 \times h$

$= \frac{\sqrt{3}}{4} \times (8)^2 \times 10 = 160\sqrt{3}$ cm³

285. (b) Volume of prism

$= \frac{1}{2} \times 10 \times 12 \times 20 = 1200 \text{ cm}^3$

$\Rightarrow$ Weight of prism = 1200 × 6
= 7200 gm = 7.2 kg

286. (a) Total slant surface area

$= 4 \times \frac{1}{2} \times 4 \times a = 12$

(where a is the side of the square base)

$\Rightarrow a = \frac{12}{8} = \frac{3}{2}$ cm

$\Rightarrow$ area of base $= \frac{9}{4}\text{cm}^2$

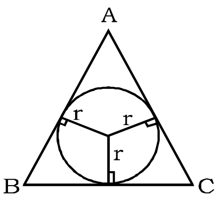
$\therefore$ Required ratio $= \frac{12}{\frac{9}{4}}$

= 16 : 3

287 (d) Total surface area of tetrahedron

$= \sqrt{3}a^2 = \sqrt{3} \times 12^2 = 144\sqrt{3}$ cm²

288. (d)



In radius of triangle

$= \dfrac{\text{area of triangle}}{\text{semiperimeter}}$

$\therefore ar(\Delta ABC) = \text{Inradius} \times \text{semiperimeter}$

$= 4 \times \dfrac{28}{2} = 4 \times 14 = 56$ cm

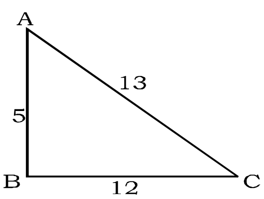
Volume of the prism =366 cm³

(area of base)× height = 366 cm³

56 × height = 366 cm

height = $\dfrac{366}{56}$ = **6.535 cm**

289. (a)



Clearly the base triangle is the right triangle

∴ area of triangle ABC

$= \dfrac{1}{2} \times 5 \times 12 = 30$ cm²

Volume of the pyramid

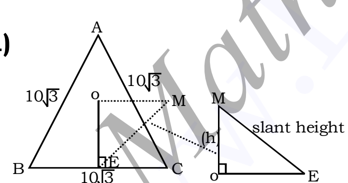
$= \dfrac{1}{3} \times (\textit{base area}) \times$ height

$\dfrac{1}{3} \times$ Base area × height = 330

$\dfrac{1}{3} \times 30 \times$ height = 330

height = $\dfrac{330 \times 3}{30}$ = **33 cm**

290. (d)



Base is equilateral triangle

In radius of equilateral triangle

$= OE = \dfrac{\text{side of equilateral } \Delta}{2\sqrt{3}}$

$= \dfrac{10\sqrt{3}}{2\sqrt{3}} = 5$ cm

slant length, $l = \sqrt{h^2 + OE^2}$

$= \sqrt{h^2 + 25}$

Total surface area = $270\sqrt{3}$

$\dfrac{1}{2}$(perimeter of base×slant height) + Base area = $270\sqrt{3}$

$\dfrac{1}{2}\left\{30\sqrt{3} \times \sqrt{(h^2+25)}\right\} + \dfrac{\sqrt{3}}{4}$

$\left(10\sqrt{3}\right)^2 = 270\sqrt{3}$

$15\sqrt{3}\sqrt{h^2+25} + 75\sqrt{3} = 270\sqrt{3}$

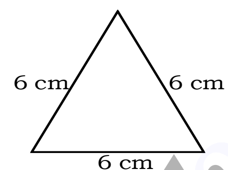
$\sqrt{h^2+25} = 13$

h² +25 = 169

h² = 169 – 25 = 144

h = $\sqrt{144}$ = **12 cm**

291. (a)



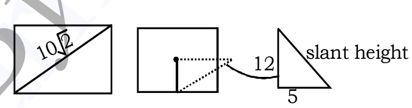
Volume of prism = area of base × height

$= \dfrac{\sqrt{3}}{4}(6)^2 \times$ height

$\dfrac{\sqrt{3}}{4} \times 6 \times 6 \times$ height = $81\sqrt{3}$

height = $\dfrac{81\sqrt{3} \times 4}{\sqrt{3} \times 6 \times 6}$ = **9 cm**

292. (d)



Side of square = $\dfrac{1}{\sqrt{2}} \times 10\sqrt{2}$ = 10 cm

slant height = $\sqrt{5^2 + 12^2}$ = 13 cm

lateral surface area = $\dfrac{1}{2} \times$ perimeter of base × slant height

$= \dfrac{1}{2} \times 40 \times 13 = 260$ cm ²

293. (d) Total surface area of prism

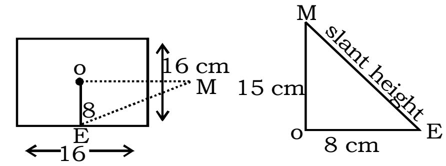
= (perimeter of base × height + 2 × base area)

$= (3 \times 12 \times 10) + 2 \times \dfrac{\sqrt{3}}{4} \times (12)^2$

$= 360 + 72\sqrt{3}$

$= 72\left(5+\sqrt{3}\right)$ cm²

294. (b)



Slant height of pyramid

$= \sqrt{8^2 + 15^2} = 17$

(8,15,17) is triplet)

lateral surface area

$= \dfrac{1}{2} \times$ perimeter of base × slant height

$= \dfrac{1}{2} \times 64 \times 17$

= 32 × 17 = **544 cm²**

295. (a) Perimeter of right Δ

= (5 + 12 + 13) = 30

total surface area = lateral surface area + 2 × area of base

= (perimeter of base × height)+ 2 × area of base

$= (30 \times \text{ height}) + 2 \times \dfrac{1}{2} \times 5 \times 12$

= (30 × height) + 60

ATQ,

30 × height + 60 = 360

30 × height = 360 – 60 = 300

height = **10 cm**

296. (d) Height of pyramid = 6 m

Diagonal of square base = $24\sqrt{2}$ m

Side of square = 24 m

Area of square= (24)² = 576 m²

Volume of the pyramid

$= \dfrac{1}{3} \times$ area of base × height

$= \dfrac{1}{3} \times 576 \times 6$

= 576 × 2 = **1152 m³**

297. (a) Volume of pyramid

$= \dfrac{1}{3} \times$ area of base × height

$500 = \dfrac{1}{3} \times 30 \times$ height

height= $\dfrac{500 \times 3}{30}$ = 50 m

298. (a) Lateral surface area of prism = 120

base perimeter × height = 120

L.S.A of prism = (Base perimeter × height)

3 × (side) × height = 120

(perimeter of eq. Δ = 3× side)

side × height = $\frac{120}{3}$ = 40 .... (i)

volume of prism = $40\sqrt{3}$

area of base × height = $40\sqrt{3}$

$\frac{\sqrt{3}}{4}(\text{side})^2 \times \text{height} = 40\sqrt{3}$

$(\text{side})^2$ × height

$= \frac{40\sqrt{3}\times 4}{\sqrt{3}} = 160$ .. (ii)

Dividing (ii) by (i)

$\frac{(\text{side})^2 \times \text{height}}{\text{side} \times \text{height}} = \frac{160}{40}$

side = 4 cm

299.(c) Volume of tetrahedron

$= \frac{\sqrt{2}}{12}(\text{side})^3 = \frac{\sqrt{2}}{12}(4)^3$

$\Rightarrow \frac{\sqrt{2}\times 4\times 4\times 4}{12} = \mathbf{\frac{16\sqrt{2}}{3}}$ **cm³**

300. (a) Area of the base of prism (a right triangle)

$= \frac{1}{2}\times 5\times 12 = 30$ cm²

Third side of the triangle

$= \sqrt{12^2+5^2} = 13$ cm

Perimeter of the triangle

= 5 + 12 + 13 = 30 cm

Total surface area

= lateral surface + 2 × (base area)

= (perimeter of base × height) + 2× (base area)

= (30 × 10) + 2 × 30

= 300 + 60 = **360 cm²**

301. (c) Let radius be increased by x cm.

Volume of cylinder

$= \pi\times 10^2(4+x)$

$\therefore \pi(10+x)^2\times 4 = \pi(10)^2(4+x)$

$\Rightarrow (10+x)^2 = 25(4+x)$

$\Rightarrow 100+20x+x^2 = 100+25x$

$\Rightarrow x^2-5x=0$

$\Rightarrow x(x-5)=0$

$\Rightarrow x=5$ cm

302. (a) As we know,

Volume of Right Prism = Area of the base × Height

$\Rightarrow 7200 = \frac{3\sqrt{3}}{2}P^2\times 100\sqrt{3}$

$\Rightarrow$ 72×2 = 9P²

$\Rightarrow$ P² = 16

$\Rightarrow$ P = 4

303. (b) Half of its lateral edges

$\Rightarrow$ Half of its edges

$\Rightarrow$ Half of its volume

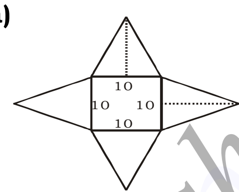
Then, volume reduced by = 50%

304. (b) Total surface area

$= 4\times\left[\frac{\sqrt{3}}{4}\times 1^2\right]$

$= \sqrt{3}$ cm²

305. (a)



Area of base = 10 × 10 = 100 cm²

Area of 4 Phase

$= (\frac{1}{2}$ ×Base× slant height) × 4

$\Rightarrow (\frac{1}{2}$ × 10 × 13) × 4

= 65 × 4 = 260

[slant height = $\sqrt{12^2+5^2} = \sqrt{169} = 13$]

Total Surface area

$\Rightarrow$ 260 + 100

$\Rightarrow$ 360 m²

306.(d) Volume of prism = (area of base × height

Area of base (i.e area of triangle)

$\Rightarrow$ Area of base

$= \sqrt{s(s-a)(s-b)(s-c)}$

= (By Hero's formula)

So, $S = \frac{13+20+21}{2} = \frac{54}{2} = 27$

$\Rightarrow \sqrt{27(27-13)(27-20)(27-21)}$

$\Rightarrow \sqrt{27\times 14\times 7\times 6}$

$\Rightarrow \sqrt{9\times 3\times 2\times 7\times 7\times 2\times 3}$

$\Rightarrow \sqrt{9\times 9\times 7\times 7\times 2\times 2}$

$\Rightarrow 9\times 7\times 2$

Volume of Prism

$= (9\times 7\times 2)\times 9 = 1134 cm^3$

307. (d) Let the side of the square = a cm

ATQ

T.S.A = C.S.A + 2 base area

C.S.A = base perimeter × h

Volume = base area × h

$\therefore$ T.SA = (base perimeter × h) + (2 base area)

192= 4a × 10 +2a²

2a² + 40a – 192 = 0

a² + 20a – 96 = 0

a² + 24a– 4a – 96 = 0

a (a+ 24)–4 (a+24) = 0

(a+24) (a–4) = 0

$\therefore$ a = 4, (–24)

$\therefore$ a = 4 (Side can never be in – ve)

Volume = base area × h

Volume = 16×10

Volume = 160 cm³

308. (c) According to the question,

V = number of vertices of prism = 6

e = edges of prism = 9

f= faces of the prism = 5

ATQ,

$\frac{v+e-f}{2} = \frac{6+9-5}{2}$

$= \frac{10}{2} = 5$

309. (c) ATQ

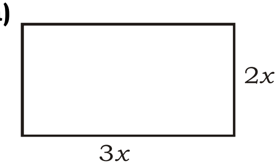
Volume of prism = Area of base × height

= trapezium area × height

$= \frac{1}{2}(10+6)\times 5\times 8$

= 16 × 5 × 4 = 320 cm³

310. (a)



Base of prism

$\Rightarrow$ length : breadth

$3x : 2x$

Perimeter of base

$= 2(3x + 2x) = 10x$

area of base

$\Rightarrow\ 2x \times 3x = 6x^2$

height of Prism = 12 cm (given)

total surface area of prism

= (Perimeter of base × height) + (2 × area of base)

$288 = 10x \times 12 + 12x^2$

$12x^2 + 120x - 288 = 0$

$x^2 + 10x - 24 = 0$

$\boldsymbol{x = 2}$

$\therefore$ area of base $\Rightarrow 6 \times 4$

$\Rightarrow$ 24 cm²

$\therefore$ volume of prism $\Rightarrow 24 \times 12$

$\Rightarrow$ 288 cm³

311. (b) Volume of the part (prism) = Area of base × height

Area of base (Isoscales Δ)

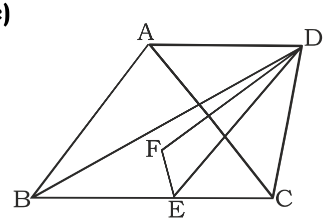
$= \frac{b}{4}\sqrt{4a^2 - b^2}$

$= \frac{6}{4}\sqrt{4(5)^2 - (6)^2} = 12 \text{ cm}^2$

Volume of prism = 12 × 8

= 96 cm³

312.c)



In circle radius (EF) = $\frac{a}{2\sqrt{3}}$

$= \frac{18\sqrt{3}}{2\sqrt{3}} = 9 \text{ cm}$

Slant height (DE) = $\sqrt{(12)^2 + 9^2}$

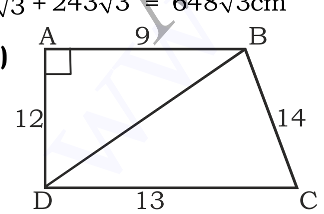
= 15 cm

Total surface area

$= 3 \times \left(\frac{1}{2} \times 18\sqrt{3} \times 15\right) + \frac{\sqrt{3}}{4}(18\sqrt{3})^2$

$= 405\sqrt{3} + 243\sqrt{3} = 648\sqrt{3}\text{cm}^2$

313.(a)



In ΔDAB

$BD^2 = AD^2 + AB^2$

$BD^2 = (12)^2 + 9^2$

BD = 15cm

Area of ΔDAB = $\frac{1}{2}$ × AB × AD

$= \frac{1}{2} \times 9 \times 12 = 54 \text{ cm}^2$

In ΔBDC

Semi perimeter = $\frac{14 + 13 + 15}{2}$

= 21 cm

Area of Δ

$= \sqrt{21 \times (21-14) \times (21-13) \times (21-15)}$

$= \sqrt{21 \times 7 \times 8 \times 6} = 84 \text{ cm}$

Volume of prism = Base area × Height

2070 = (54 + 84) × Height

Height = $\frac{2070}{138}$ = 15 cm

Lateral surface area = Perimeter of base × Height

= (9 + 14 + 13 + 12) × 15

= 48 × 15 = 720 cm²

314.(c) Volume of pyramid = $\frac{1}{3}$ ×Area of base × Heights

$= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times 4 \times 4 \times 20\sqrt{3}$

= 80cm³

315.(c) Slant height of pyramid

$= \sqrt{4^2 + \left(\frac{6}{2}\right)^2} = 5 \text{ cm}$

Total surface area = Lateral Surface area + Base area

$= \frac{1}{2}$ × Perimeter of base × Slant height + Base area

$= \left(\frac{1}{2} \times 24 \times 5\right) + (6 \times 6)$

= 60 + 36 = 96 cm²

316.(b) slant height = $\sqrt{(13)^2 - \left(\frac{10}{2}\right)^2}$

Height of pyramid

$= \sqrt{(12)^2 - 5^2} = \sqrt{119}$ cm

Volume of pyramid = $\frac{1}{3}$ × Area of base × Height

$= \frac{1}{3} \times 10 \times 10 \times \sqrt{119}$

$= \frac{100}{3}\sqrt{119} \text{ cm}^3$

317(d) Length of tank (l) = 4m

Breadth of tank (b) = 3m

Depth of tank (d) = h

$\therefore$ Volume = lbh

$24 = 4 \times 3 \times h$

$\Rightarrow\ h = 2$m

(As tank is open from upper sides remaining five sides of the tank will be painted)

Area of 5 sides = 2(hb + lh) + lb

= 2(2 × 3 + 2 × 4) + 4 × 3

= 28 + 12 = 40m²

$\therefore$ Total area to be painted inner and outer side.

40 m² + 40 m² = 80 m²

= 80m² cost of painting = 80 × 10

= 800 Rs.

318. (b) V = abc

$\therefore$ S = 2 (ab + bc + ca)

$\Rightarrow$ S = 2abc $\left(\frac{1}{c} + \frac{1}{a} + \frac{1}{b}\right)$

$\Rightarrow$ S = 2V $\left(\frac{1}{c} + \frac{1}{a} + \frac{1}{b}\right)$

$\Rightarrow \frac{1}{V} = \frac{2}{S}\left(\frac{1}{c} + \frac{1}{a} + \frac{1}{b}\right)$

319. (b) External length (L) = 1.46 = 146 cm

External breadth (B) = 1.16 m = 116 cm

External Height (H) = 8.3 dm = 83 cm

$\Rightarrow$ Internal length (l) = 146 – 6 = 140 cm

$\Rightarrow$ Internal breadth (b) = 116 – 6 = 110 cm

$\Rightarrow$ Internal Height (h) = 83 – 3 = 80 cm

Total surface area of inner walls

= $2h\,(l + b) + lb$

= 2 × 80 (140 + 110) + 140 × 110

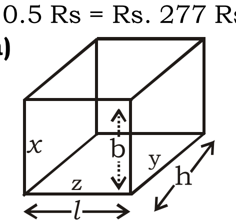
= 40,000 + 1540

= 55400 cm²

Cost of painting 100 cm² = 50 paise = 0.5 Rs.

cost of painting 55400 cm² = 554 × 0.5 Rs = Rs. 277 Rs.

320. (a)



Volume = l × b × h

x = hb

y = lh

z = l b
xyz = $(l\,b\,h)^2$
$V^2 = xyz$

321. (a) Volume of bar = Volume of piece of copper
area of cross section of square × length = $1m^3$
$a^2 \times 36 = 1$

$$\Rightarrow a = \frac{1}{6} m$$

then volume of cube

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216} m^3$$

Cost of cube = Volume × cost of $1m^3$

$$= \frac{1}{216} \times 108 = \text{Rs. 50 paise}$$

322. (d) Let the length, breadth and height of block be $3x$, $2x$ and x
∴ Volume of block = lbh
$3x \times 2x \times x = 10368$
$x = 12$ dm
Now, length breadth and height of block will be 36 dm, 24 dm and 12 dm respectively.
∴ Total surface area of block
$= 2\,(lb + bh + hl)$
$= 3168\ dm^2$
⇒ cost of polishing entire surface = 3168 × 0.02 = 63.36 Rs.

323.(a) Volume of water drawn out = (Volume of water initially in tank) – (Volume of water remained in tank)
$= 15 \times 6 \times 10 - 15 \times 6 \times 9\ m^3$
= 90,000 litre ($1m^3$ = 1000 litre)

324. (c) Volume of water will come out of pipe in 15 min
= Cross section area × Length per 15 min

$$= \frac{13}{10000} \times \frac{9000}{60} \times 15 = 2.925 m^3.$$

∴ Volume of water filled in tank in 15 min
= Volume of water came out of pipe in 15 min
$\Rightarrow 45 \times 26 \times h = 2.925$
(∴ h = level of water rise in tank)
$\Rightarrow h = 0.0025$m

325. (b) Let the length, breadth and height of cuboid be l, b and h
$\Rightarrow x^2 = l^2 + b^2$ ....(i)
$y^2 = b^2 + h^2$ ....(ii)
$z^2 + l^2 + h^2$ ....(iii)
adding (i), (ii), (iii) we get,
$x^2 + y^2 + z^2 = 2(l^2+b^2+h^2)$

$$\Rightarrow l^2 + b^2 + h^2 = \frac{x^2 + y^2 + z^2}{2} \ ...(iv)$$

subtracting (i) from (iv) we get,

$$h^2 = \frac{y^2 + z^2 - x^2}{2}$$

$$\Rightarrow h = \sqrt{\frac{y^2 + z^2 - x^2}{2}}$$

Subtracting (ii) from (iv) we get,

$$l^2 = \frac{x^2 + z^2 - y^2}{2} \Rightarrow l = \sqrt{\frac{x^2 + z^2 - y^2}{2}}$$

subtracting (iii) from (iv) we get

$$\therefore b^2 = \frac{x^2 + y^2 - z^2}{2}$$

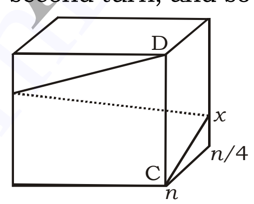
$$\Rightarrow b = \sqrt{\frac{x^2 + y^2 - z^2}{2}}$$

∴ Volume = lbh

$$= \sqrt{\frac{(x^2 + y^2 - z^2)(y^2 + z^2 - x^2)(x^2 + z^2 - y^2)}{8}}$$

$$= \frac{\sqrt{(x^2 + y^2 - z^2)(y^2 + z^2 - x^2)(x^2 + z^2 - y^2)}}{2\sqrt{2}}$$

326. (b) The string of minimum length, if starting from C, touches next corner at height $n/4$ on the completion of one turn, starting from height $n/4$ touches next corner at height $n/2$ in the second turn, and so on.



$$CX = \sqrt{n^2 + \left(\frac{n}{4}\right)^2} = \frac{\sqrt{17} \times n}{4}$$

⇒ Length of string

$$= 4 \times \frac{\sqrt{17} \times n}{4} = \sqrt{17}n.$$

Alternate:-
Opening up the four vertical sides of the cubes of side n,

$$\text{Length of string} = \sqrt{(4n)^2 + (n^2)}$$

$$= \sqrt{17} \times n$$

327. (c) Volume of supplied water by diameter of 6 cm = n × volume of water supplied by diameter 1.5 cm

$$\Rightarrow \pi \times \left(\frac{6}{2}\right)^2 = n \times \pi \times \left(\frac{1.5}{2}\right)^2$$

(n = no. of pipes)
$\Rightarrow n = 16$

328. (b) Volume of cylindrical shell

$$= \frac{1}{4} \times \text{volume of solid cylinder}$$

$$\pi\,(R^2 - r^2) \times 1000 = \frac{1}{4}\,\pi R^2 \times 1000$$

$$= R^2 - r^2 = \frac{R^2}{4}$$

$$R^2 - \frac{R^2}{4} = r^2$$

$3R^2 = 4r^2$

$$r^2 = \frac{3 \times 10^2}{4}$$

$r^2 = 75$
$r = 5\sqrt{3}$
thickness of cylinder = radius of solid cylinder – Inner radius of cylinderical hole = $10 - 5\sqrt{3}$

$$= 5 \times \left(2 - \sqrt{3}\right) \text{ cm}$$

329. (b) Let the radius of well = r
Volume of embankment (hollow cylinder)
= volume of earth taken out

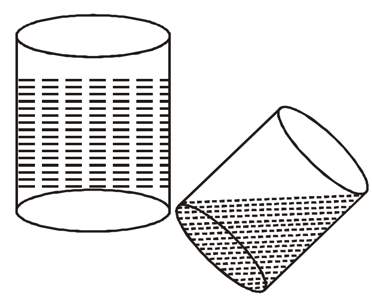
$$\Rightarrow \pi\left((r+1)^2 - r^2\right) \times 5 = \pi r^2 \times 20$$

$$\Rightarrow \left[(r+1+r)(r+1-r)\right] = r^2 \times 4$$

$\Rightarrow (2r + 1) \times 1 = 4r^2$
$\Rightarrow 4r^2 - 2r - 1 = 0$

$$r = \frac{2 \pm \sqrt{4+16}}{8} = \frac{\sqrt{5}+1}{4} m$$

330. (d)



Let total volume of cylinder = $5x$ litre

$$\text{Volume of water} = \frac{4}{5} \times 5x = 4x \text{ litre}$$

After tilting, there will be water half of total volume $4x - \frac{5x}{2}$

$\Rightarrow \quad 30 = \dfrac{3x}{2}$

$\Rightarrow \quad x = 20$ litre

Hence, volume of cylinder

$= 5x = 100$ litre.

331. (b) Curved surface area $2\pi rh$

Curved surface area of 50 pillars $= 50 \times 2\pi rh$

$= 50 \times 2 \times \dfrac{22}{7} \times \dfrac{50}{2 \times 100} \times 4 = 314 \text{m}^2$

Labout charge for getting these pillars cleared = 314 × 0.5

= Rs. 157

332. (c) $r = \dfrac{3}{4}h$ ....(given)

$\Rightarrow \quad h = \dfrac{4r}{3}$

$\therefore$ Volume $= \pi r^2 h = 38808$ ml

$= 38.808\ l$

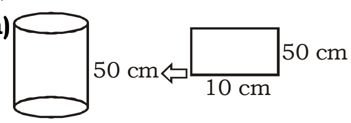
$= 38.808 \times 1000 \text{ cm}^3$

$\Rightarrow \quad \dfrac{22}{7} \times r^2 \times \left(\dfrac{4r}{3}\right) = 38808$

$\Rightarrow \quad r^3 = 9261$

$\Rightarrow \quad r = 21$ cm

333. (a)



$2\pi r = 100$ cm

Then, curved surface area

$= 2\pi rh$

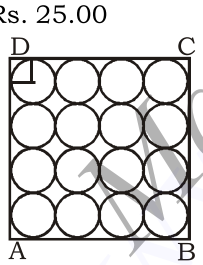
$100 \times 50 = 5000 \text{ cm}^2$

Cost of painting

$= 5000 \times \dfrac{50}{100 \times 100}$

= Rs. 25.00

334. (c)



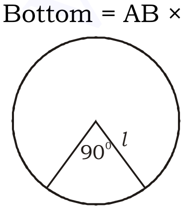
Diameter of each circle = 2cm

$\therefore$ AB = 2 × 4 = 8

Similarly, CD = 8

Area of Bottom = AB × CD = 64 cm²

335. (c)



Slant height (l) = radius of circle

= 4 cm

r = radius of cone

$\therefore$ Perimeter of base of cone

$= 2r\pi$

$\Rightarrow \quad 2r\pi = \dfrac{90^\circ}{360^\circ} \times$ (perimeter of circle)

$\Rightarrow \quad 2r\pi = \dfrac{1}{4} \times 2\pi \times 4$

$\Rightarrow \quad r = 1$ cm

height of cone (h) $= \sqrt{l^2 - r^2}$

$= \sqrt{16 - 1} = \sqrt{15}$

$\therefore$ Volume of cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times (1)^2 \times \sqrt{15}$

$= \dfrac{\pi\sqrt{5}}{\sqrt{3}} \text{ cm}^3$

336. (b) Arc of Circle $= \dfrac{\theta}{180} \times \pi \times r$

$= \dfrac{120}{180} \times \pi \times 3 = \dfrac{2}{3} \times \pi \times 3 = 2\pi$

Base of cone = Arc of Circle

$2\pi r = 2\pi$

$r = 1$ cm

$\Rightarrow \quad h^2 = l^2 - b^2$

$= (3)^2 - (1)^2 = 8$

$h = 2\sqrt{2}$ cm

Volume of Cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \pi \times 1 \times h$

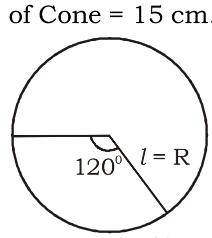
$= \dfrac{1}{3} \times \pi \times 1 \times 2\sqrt{2} = \dfrac{2\sqrt{2}\,\pi}{3}$

337. (b) Slant height = H, and radius = r

$\Rightarrow \quad \dfrac{\text{curved surface area}}{\text{area of base}} = \dfrac{\pi rl}{\pi r^2}$

$= \dfrac{\pi rH}{\pi r^2} = \dfrac{H}{r} = H : r$

338.(c) Radius of cone = Slant height of Cone = 15 cm.



$\therefore$ Perimeter of base of sector = perimeter of base of cone

$\Rightarrow \quad \dfrac{120^\circ}{360^\circ} \times 2\pi(15) = 2\pi r$

(where, r is radius. of cone

$\Rightarrow \quad r = 5$cm

$h = \sqrt{l^2 - r^2} = \sqrt{(15)^2 - (5)^2}$

$= \sqrt{200} = 10\sqrt{2}$

$\therefore$ Volume $= \dfrac{1}{3}\pi r^2 h$

$\dfrac{1}{3} \times \pi \times 5 \times 5 \times 10\sqrt{2}$

$= [(250\sqrt{2})\pi/3] \text{ cm}^3$

339. (a) Volume of cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 5 \times 5 \times 25 = 644.761$

Volume of smaller cone

$= 654.76 - 110 = 544.761$

$\therefore \quad \dfrac{\text{Volume of smaller cone}}{\text{Volume of larger cone}}$

$= \left(\dfrac{\text{radius of smaller cone}}{\text{radius of large cone}}\right)$

$\Rightarrow \quad \dfrac{544.761}{654.761} = \left(\dfrac{r}{5}\right)^3$

$\Rightarrow \quad r = (104)^{1/3}$ cm

340. (a) Circumference of hesmispherical bowl = 176

$\Rightarrow \quad 2\pi r = 176$

$\Rightarrow \quad r = 28$ cm

When bowl is half full

$= \dfrac{2}{3}\pi r^3 \times \dfrac{1}{2} = \dfrac{1}{3}\pi(28)^3$

Volume of hemispherical glass

$= \dfrac{2}{3}\pi(2)^3$

No. of persons may be served =

$\dfrac{\frac{1}{3}\pi(28)^3}{\frac{2}{3}\pi(2)^3} = 1372$

341. (c) Volume of cylindrical vessel = Volume of sphere

$\pi \times (6)^2 \times h = \dfrac{4}{3} \times \pi \times (3)^3$

$\therefore$ Surface of water rise (h) = 1 cm

342. **(d)** Surface area of B = surface area of A + 300% of surface of A
= 4 (Surface of area of A)
Let the radius of A be a and radius of B be b.
$\therefore$ $4\pi b^2 = 4 \times 4\pi a^2$
$\Rightarrow$ b = 2a

Volume of A $= \frac{4}{3}\pi a^3$

Volume of B $= \frac{4}{3}\pi b^3 = \frac{4}{3}\pi(2a)^3$

$= \frac{4}{3}\pi 8a^3$

% of volume of A lower than B = K%

$$= \frac{\frac{4}{3}\pi 8a^3 - \frac{4}{3}\pi a^3}{\frac{4}{3}\pi 8a^3}$$

$= \frac{7}{8} \times 100 = 87.5\%$

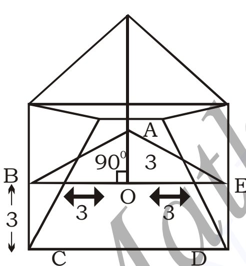
$\therefore$ k = 87.5

343. **(b)** Given prism is a solid with regular hexagonal base
$\therefore$ Its volume = Area of the base × Height

$= \frac{3\sqrt{3}}{2} \times 1 = = \frac{3\sqrt{3}}{2}$ cu m

$\left[\text{Since the area of regular hexagon with side l m} = \frac{3\sqrt{3}}{2}\text{m}^2\right]$

344. **(a)**



AB = $3\sqrt{2}$

AE = $3\sqrt{2}$

$\angle$ABE = 45°

$\angle$AEB = 45°

Area of $\square$ BCDE = 18cm²

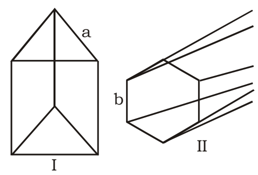
Area of $\triangle$ OAB = $\frac{1}{2} \times 3 \times 3$

= 4.5 cm²

$\therefore$ Area of $\triangle$ ABE = 9 cm²

$\therefore$ Area of ABCDE (base) = (18 + 9) = 27 cm²
Volume of prism = Area of the base × Height
= (27 × 10)
= 270 cm³

345. **(b)** Let the height of each prism be h units and the length of each side of equilateral triangle at the base of first prism be a units and that the second prism having regular hexagon as base be b units.
(See the figures given below)



According to the question,
Volume of first prism = Volume of second prism

$\frac{\sqrt{3}}{4}a^2 \times h = 6\frac{\sqrt{3}}{4}b^2 \times h$

$\Rightarrow \frac{1}{4}a^2 = \frac{3}{2}b^2 \Rightarrow a^2 = 6b^2$

$a = \sqrt{6}b \Rightarrow \frac{a}{b} = \frac{\sqrt{6}}{1}$

$\therefore$ a : b = $\sqrt{6}$: 1

346. **(c)** Area of trapezium $= \frac{1}{2}$ × height × (sum of parallel sides)

$= \frac{1}{2}$ × 8 (8 + 14) = 88 cm²

$\therefore$ Volume = area of traperium × height 1056 = 88 × h
h = 12 cm

347. **(c)** The base of the prism is rectangular and we are not changing the base so length & breadth will be remain same. If we double the lateral edges it means we are doing double its height so Volume of the prism will be doubled.

348. **(b)** Area of base $= \frac{1}{2}$ × base × height

$= \frac{1}{2}$ × 10 × 12 = 60 cm²

$\therefore$ Volume of prism = Area of base × height
= 60 × 20 = 1200 cm³
Material used for 1 cubic cm.
= 6 gm
Material used for 1200 cm³
= 1200 × 6 = 7200 gm = 7.2 kg

349. **(d)** Perimeter of triangle = 15 cm

semiperimeters $= \frac{15}{2}$ cm

inradius r = 3 cm

$\triangle$ = r.s $= \frac{15}{2} \times 3 = \frac{45}{2}$ cm

Volume of prism = area of base × height

$270 = \frac{45}{2} \times h$

h = 12 cm

350. **(d)** $\frac{V_1}{V_2} = \frac{\frac{\sqrt{3}}{4}a_1^2.h_1}{\frac{\sqrt{3}}{4}a_1^2.h^2} = = \left(\frac{a_1}{a_2}\right)^2 \cdot \frac{h_1}{h_2}$

$= \left(\frac{a_1}{3a_2}\right)^2 \cdot \left(\frac{h_1}{h_2/4}\right) = \frac{4}{9}$

$(\because a_2 = 3a_1 \ \& \ 4h_1 = h_2)$

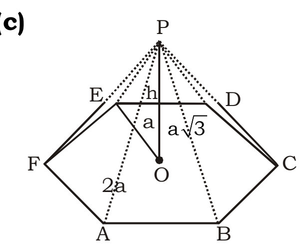
351. **(c)** Volume of a pyramid

$= \frac{1}{3}$ × Area of the base × Height

$= \frac{1}{3} \times \frac{3\sqrt{3}}{2}$ × (10)²× 30 = 2598m³

[$\because$ Area of the regular hexagon of side $a = \frac{3\sqrt{3}}{2}a^2$ sq units]

352. **(c)**



AB = BC = CD = EF = FA = 2a

PE $= \frac{5a}{2}$ and OE = 2a

$\therefore h = \text{OP} = \sqrt{\left(\frac{5a}{2}\right)^2 - 4a^2} = \frac{3a}{2}$

Volume of the pyramid $= \frac{1}{3}$ ×

Area of the base × Height

$= \frac{1}{3} \times 6 \times \frac{1}{2} \times 2az \times a\sqrt{3} \times \frac{3a}{2}$

$= 3a^3\sqrt{3}$

353. (c) Area of regular hexagon of side $a = \frac{3\sqrt{3}}{2}a^2$

$\Rightarrow \frac{3\sqrt{3}}{2}a^2 = 96\sqrt{3} \Rightarrow a = 8$ m

Let h be the height of the pyramid. Then area of one side face of the pyramid $= \frac{1}{2}a \times l$, where l is the slant height of the face.

$\Rightarrow \frac{1}{2}a \times l = 32\sqrt{3} \Rightarrow l = 8\sqrt{3}$

$\Rightarrow \frac{3a^2}{4} + h^2 = l^2$

$\Rightarrow \frac{3 \times 64}{4} + h^2 = 64 \times 3$

$\Rightarrow h^2 = 64 \times 3 \left[1 - \frac{1}{4}\right] = 144$

$\Rightarrow$ h = 12 m

∴ Volume of the pyramid

$= \frac{1}{3} \times$ Area of the base $\times h$

$= \frac{1}{3} \times 96\sqrt{3} \times 12 = 384\sqrt{3}\text{m}^3$

354. (b) Volume of tunnel $= \pi r^2 h$

$= \frac{22}{7} \times 2 \times 2 \times 56 = 704\text{m}^3$

Volume of the ditch = 48 × 16.5 × 4 = 3168 m^3

Part of the ditch filled $= \frac{704}{3168} = \frac{2}{9}$

355. (d) Height of cylindrical rod (h) = $8r$

Radius of spherical ball $= \frac{r}{2}$

Number of spherical balls

$= \frac{\pi r^2 h}{\frac{4}{3}\pi\left(\frac{r}{2}\right)^3}$

$= \frac{\pi \times r^2 \times 8r}{\frac{4}{3} \times \pi \times \left(\frac{r}{2}\right)^3}$

= 16 × 3 = 48

356. (a) Radius of pipe (r) $= \frac{1}{4}$ cm

Radius of cone (R) = 15 cm

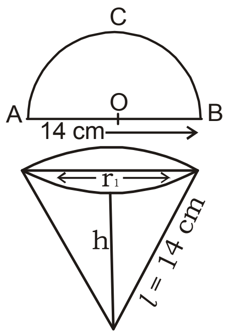
vol. passed by pipe = vol of cone

$\pi r^2 \times 1000 \times T = \frac{1}{3}\pi \times 15 \times 15 \times 24$

[∵ speed of water = 1000 cm/ minute]

$T = \frac{28800}{1000}$ minute = 28 minute 48 sec

357. (b)



Length (ACB) of semi-circular sheet = π r

$= \frac{22}{7} \times 14 = 44$ cm.

Slant height of the cone = 14cm.

Circumference of the base of the cone

$= 2\pi r_1 = \frac{44}{7} r_1$

$\Rightarrow 44 = \frac{44}{7} r_1 \Rightarrow r_1 = 7\text{cm}$

$\therefore h = \sqrt{14^2 - 7^2} = 7\sqrt{3}$ cm

$= 7 \times 1.732 = 12$ cm.

358. (d) Total surface area of prism = Surface area + (base + top) area

608 = Perimeter of base × height + 2 × base area

608 = 4 × a × 15 + 2 × a × a

608 = 60 a + 2a^2

a^2 + 30 a = 304

a(a + 30) = 304

a = 8 cm

Volume of prism = Base area × Height

= 8 × 8 × 15 = 960 cm^3

359. (b) A.T.Q.

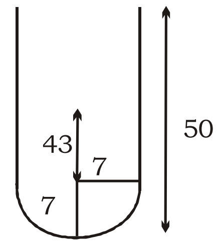
$l \times b \times h = \pi r^2 h_1$

$8 \times 11 \times 2 = \frac{22}{7} \times 4 \times 4 \times h_1$

$h_1 = \frac{7}{2}$

$h_1 = 3.5$

360. (d) C.S.A. of storage tank = C.S.A. of cylinder + C.S.A. of hemisphere



$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r) = 2\pi r(43 + 7)$

$= 2 \times \frac{22}{7} \times 7 \times 50 = 2200 \text{ m}^2$

361. (b) Diameter of ball = 14 cm

Radius = 7 cm

Height of solid cylinder $= \frac{7}{3}$ cm

A.T.Q.

Volume of ball = Volume of cylinder

$\Rightarrow \frac{4}{3}\pi r^3 = \pi r^2 h$

$\Rightarrow \frac{4}{3} \times \pi \times 7 \times 7 \times 7 = \pi \times r^2 \times \frac{7}{3}$

$\Rightarrow r^2 = 49 \times 4$

$\Rightarrow r = 7 \times 2 = 14$ cm

Diameter of the base of the cylinder

$\Rightarrow D = 2r = 2 \times 14 = 28$ cm

362. (d) Inner radius r = 8 cm

Outer Radius R = 8 cm

Area of Circular shell

$= \pi(R^2 - r^2) = \pi(144 - 64) = 80\pi$

A.T.Q.

Total surface area of cylinder = area of shell

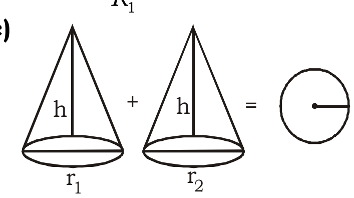
$\Rightarrow 2\pi R_1(h + R_1) = 80\pi$

$\Rightarrow h + R_1 = \frac{40}{R_1}$

$\Rightarrow h = \frac{40}{R_1} - R_1$

$\Rightarrow h = \frac{40 - R_1^2}{R_1}$

363. (c)



$\frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h = \frac{4}{3}\pi R^3$

$\frac{1}{3}\pi h(r_1^2 + r_2^2) = \frac{4}{3}\pi R^3$

$h = \frac{4R^3}{r_1^2 + r_2^2}$

364. (d) height of cone
$10.2 - 4.2 = 6$
Volume of woods

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{\pi r^2}{3}(h+2r)$

$= \frac{\pi r^2}{3}(6 + 8.4)$

$\Rightarrow \frac{22}{7} \times \frac{4.2 \times 4.2}{3} \times 14.4$

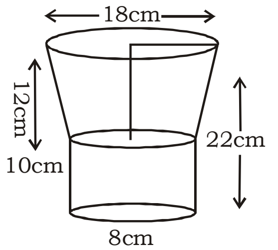
$\Rightarrow 22 \times 0.6 \times 1.4 \times 14.4$
$\Rightarrow 266.112 \Rightarrow 266$ (nearly)

365. (b) $h_1 = h_2$

$\frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3}{1}$

$r_1^2 = r_2^2$
$r_1 = r_2$
$D_1 = D_2$
(b) Diameter of cylindar = diameter of cone

366. (b)



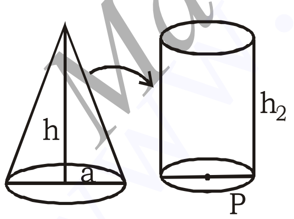
R = 9
r = 4

$l = \sqrt{(12)^2 + (5)^2}$

$l = 13$
Area of funnel
$= 2\pi rh \times \pi (R + r)l$
$= \pi [2 \times 4 \times 10 + (9 + 4) \times 13]$
$= 249\pi$

$= 249 \times \frac{22}{7} = 782.57$ cm³

367. (a)

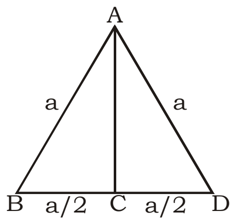


$\pi r^2 h_2 = \frac{1}{3}\pi r^2 h$

$\pi p^2 h_1 = \frac{1}{3}\pi a^2 h$

$h_1 = \frac{a^2 h}{3P^2}$

368.(d)



Let one side of Δ be = a
Perimeter of equilateral triangle = $3a$

$\therefore \quad 3a = 72\sqrt{3} = a = 24\sqrt{3}$ cm

Height = AC; by Pythagoras theorem

$AC^2 = a^2 - \left(\frac{a}{2}\right)^2$

AC = 36 cm

369. (d) Volume of soil removed
$= l \times b \times h$
$= 7.5 \times 6 \times 1.5 = 67.5$ m³

370. (c) I hectare = 10000 m²

Height = 10 cm = $\frac{1}{10}$ m

Volume = $10000 \times \frac{1}{10} = 1000$ m³

371. (a) Total surface are of 7 cubes
$\Rightarrow 7 \times 6a^2 = 1050$
But on joining end to end, 12 sides will be covered.
So there area = $12 \times a^2$
$\Rightarrow 12 \times 25 = 300$
So the surface area of the resulting figure = 1050 – 300 = 750

372. (d) Let the rise in height be = h
Then, as per the question, the volume of water should be equal in both the cases.
Now, $90 \times 40 \times h = 150 \times 8$

$h = \frac{150 \times 8}{90 \times 40} = \frac{1}{3}$ m $= \frac{100}{3}$ cm

= 33.33 cm

373. (d) Area of base = $6 \times 10 = 60$m²
Volume of tent = $30 \times 10 = 300$m³
Let the radius be = r, height = h, slant height= l

$\pi r^2 = 60 \Rightarrow r = \frac{60}{\pi}$

$300 = \frac{\pi r^2 h}{3} \Rightarrow 900 = \pi \cdot \frac{60}{\pi}$.

h $\Rightarrow$ h = 15 m

374. (c) Volume of metal used

$= \frac{4\pi R^3}{3} - \frac{4\pi r^3}{3}$

$= \frac{4\pi}{3}(12^3 - 10^3) = 3047.89$ cm³

Weight = volume × density
$\Rightarrow 4.9 \times 3047.89$
$\Rightarrow$ 14942.28 gm

375. (b) The volume in both the cases will be equal. Let the height of cone be = h

$\frac{4}{3} \times \frac{22}{7} \times (14)^3 = \frac{22}{7} \times \left(\frac{35}{2}\right)^2 \times \frac{h}{3}$

$\Rightarrow 4(14)^3 = h\left(\frac{35}{2}\right)^2 = h$

$= \frac{4 \times 14 \times 14 \times 14 \times 2 \times 2}{35 \times 35}$

= h = 35.84 cm

376. (c) Circumference of the circular face of the cylinder = $2\pi r$

$\Rightarrow 2 \times \frac{22}{7} \times \frac{35}{100} = 2.2$ m

Number of revolutions required

to lift the bucket by = $\frac{11}{2.2} = 5$

377. (c) Surface area of the cube $6a^2$
$= 6 \times (20)^2 = 2400$
Area of 6 circles of radius 10 cm
$= 6\pi r^2$
$= 6 \times \pi \times 100 = 1885.71$
Remaining area = 2400 – 1884
= 514.28

378. (d) $x \cdot y \cdot z = lb \times bh \times lh$
$= (lbh)^2$
(V) Volume of a cuboid = lbh
So $V^2 = (lbh)^2 = xyz$

379. (a) Volume of mud dug out
$= 10 \times 4.5 \times 3 = 135$ m³
Let the remaining ground rise by = h m
Then $\{(20 \times 9) - (10 \times 4.5)\}$ h = 135
135h = 135 $\Rightarrow$ h = 1 m

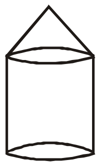
380. (b) Height of the cylinder
= 13 – 7 = 6 cm
Radius of the cylinder and the hemisphere = 7 cm
Volume of the vessel = volume of cylinder + volume of hemisphere

$\Rightarrow \pi r^2 h + \frac{4\pi r^3}{3 \times 2}$

$\Rightarrow 3.14 \times (7)^2 \times 6 + \frac{4 \times 3.14 \times (7)^2}{3 \times 2}$

$\Rightarrow$ 1642.6 cm²

381. (c)



Radius = $\frac{105}{2}$ =52.5 cm

Area of the entire canvas, used for the tent

= Area of cylinder = area of cone

= $2\pi rh + \pi rl$

= $\pi r(2h + l)$ = 3.14 × 52.5 $(2\sqrt{53^2 - 52.5^2} + 53)$

= 5 × l (because area of canvas = l × b also)

= $l \Rightarrow$ 1947 m

382. (c) The volume in both the cases would be the same.

Therefore = $\frac{4\pi r^3}{3} = \pi r^2 h$

$\frac{4 \times 3.14 \times (4 \times 10)^3}{3}$ = 3.14 × 2²×h

$\Rightarrow h = \frac{64000}{3}$ = 21333.33 mm

383. (b) As the cylinder and cone have equal diameters. So they have equal area. Let cone's height be h_2 and as per question, cylinder's height be h_1.

$$\frac{2\pi rh_1}{\pi r\sqrt{h_2^2 + r^2}} = \frac{8}{5}$$

On solving we get the desired ratio as 3 : 4

384. (a) Let the slant height of 1st cone = L

Then the slant height of 2nd cone = 3L

Let the radius of 1st cone = r_1

And let the radius of 2nd cone = r_2

Then, $\pi r_1 L = 3 \times \pi r_2 \times 3L$

$\Rightarrow \pi r_1 L = 9\pi r_2 L \Rightarrow r_1 = 9r_2$

Ratio of area of the base 81 : 1

385. (c) Let the internal radius of the cylinder = r

Then, the volume of sphere = Volume of sphere cylinder

$\Rightarrow \frac{4\pi.6^3}{3} = \pi h(5^2 - r^2)$

$\Rightarrow \frac{864\pi}{3} = 32\pi(25 - r^2)$

$\Rightarrow r^2 = 16 = r = 4$ cm

So thickness of the cylinder = 5 – 4 = 1 cm

386. (d) The volume in both the cases would be the same.

Let the height of the cone = h

Then, external radius = 6cm

Internal radius = 4cm

$\Rightarrow \frac{4\pi.(6^3 - 4^3)}{3} = \frac{\pi.4^2.h}{3}$

$\Rightarrow h = \frac{6^3 - 4^3}{4}$

$\Rightarrow h = \frac{216 - 64}{4}$ = 38 cm

387. (a) Let arc side of the cube be = a units

Total surface area of 3 cubes

= 3 × 6a² = 18 a²

Total surface area of cuboid

= 18a² – 4a² = 14a²

Ratio = $\frac{14a^2}{18a^2}$ 7 : 9

388. (c) A = 2 $(xy + yz + zx)$

V = xyz

A/V = $\frac{2(xy + yz + zx)}{xyz}$

= $\frac{2}{z} + \frac{2}{x} + \frac{2}{y} \Rightarrow 2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$

389. (d) Radius of cylinder, hemisphere and cone = 5cm

Height of cone = 12 cm

Surface area of toy = $2\pi rh + \frac{4\pi r^2}{2} + \pi rL$

L = $\sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2}$ =13

Then ⇒ (2×3.14×5×13) + (2×3.14×25) + (3.14×5×13)

⇒ 770 cm²

390. (b) Height of cone = 10.2 – 4.2 = 6cm

Volume of wood = $\frac{\pi r^2 h}{3} + \frac{4\pi r^3}{3 \times 2}$

$\Rightarrow \frac{3.14 \times (4.2)^2 \times 6}{3} + \frac{4 \times 3.14 \times (4.2)^3}{3 \times 2}$

⇒ 266 cm²

391. (a) Volume of ice cream = $\pi r^2 h$

= 3.14 × (6)² × 15

= 1695.6 cm²

Volume of 1 cone = $\frac{\pi r^2 \times 24}{3}$

Then ⇒ 10×3.14 × r² × 8 = 1695.6

⇒ r = 3cm (approx.)

So diameter = 2 × 3 = 6 cm

392. (b) Radius of cylinder and hemispheres = $\frac{7}{2}$ =3.5cm

Height of cylinder = 19 – (3.5 × 2) = 12 cm

Total surface area of solid= $2\pi rh + 4\pi r^2$

⇒ 2 × 3.14 × 3.5 × 12 + 4 × 3.14 × (3.5)²

⇒ 418 cm²

393 (c) As they stand on the same base so their radius is also same.

Then; volume of cone = $\frac{\pi r^2 h}{3}$

Volume of hemisphere = $\frac{2\pi r^2}{3}$

Volume of cylinder = $\pi r^2 h$

Ratio = $\frac{\pi r^2 h}{3} : \frac{2\pi r^3}{3} : \pi r^2 h$

$\Rightarrow \frac{h}{3} : \frac{2r}{3} : h \Rightarrow h : 2r : 3h$

Radius of a hemisphere = Its height

So h : 2h : 3h ⇒ 1 : 2 : 3

394. (d) Total surface to be painted = external surface area + internal surface + surface area of right area

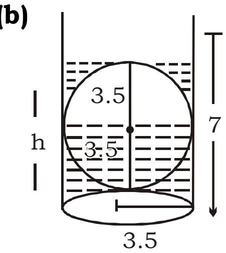
= $2\pi(R^2 + r^2) + 2\pi(R^2 - r^2)$

Cost of painting

$\Rightarrow 2 \times 3.14 \times \left\{\left(\frac{25}{2}\right)^2 + \left(\frac{24}{2}\right)^2 + \left(\frac{25}{2}\right)^2 - \left(\frac{24}{2}\right)^2\right\} \times 0.05$

⇒ 2 × 6.28 × (12.5)² × 0.05 × ₹ 96.28

395. (b)



$\pi\,(3.5)^2\times 7-\frac{4}{3}\,\pi\,(3.5)^3$

$=\pi\,(3.5)^2\times h$

$7-\frac{4}{3}\times\frac{35}{10}=h$

$7-\frac{14}{3}=h=\frac{7}{3}$

396. (d) Radius $=\frac{3.5}{2}=1.75$ cm

Volume of solid $=\pi r^2h+\frac{\pi r^2h}{3}+\frac{2\pi r^3}{3}$

$\Rightarrow 3.14\times(1.75)^2\times(10+\frac{6}{3}+\frac{2(1.75)}{3})\Rightarrow 121\text{ cm}^3$

397. (d) Volume of the elliptical cylinder

$=\pi\times\frac{2.4}{2}\times\frac{1.6}{2}\times 7$

$=3.14\times1.2\times0.8\times7\Rightarrow 9\text{ m}^3$

Amount of water emptied per minute $=120\times3.14\times\left(\frac{2}{100}\right)^2$

Time required to empty half the tank $=\frac{4.5}{120\times3.14\times(0.02)^2}$

$=70$ min

398. (d) Let initial radius = r

Then volume $=\pi r^2h$

New radius $=r+\frac{r}{2}r\Rightarrow\frac{3r}{2}$

New volume $=\pi\frac{9}{4}r^2h$

Increased volume $=\frac{5}{4}\pi r^2h$

Percent increase $=\frac{5\pi r^2h}{4\pi r^2h}\times100$

$=125\%$

399. (c) Volume of the cone

$=\frac{\pi r^2h}{3}\Rightarrow\frac{3.14\times20\times20\times24}{3}$

$\Rightarrow 10048\text{ cm}^3$

Diameter of pipe = 5m

Volume of water flowing out of the pipe per minute

$\Rightarrow 10\times(2.5)^2\times3.14\Rightarrow 196.25\text{ cm}^3$

Time taken to fill the tank

$=\frac{10048}{196.25}=51.2$ mins

400. (d) One side of the equilateral triangle = diameter of cone.

Therefore radius of cone $=\frac{12}{2}=6$

Height of cone = Height of equilateral triangle be

$\because$ Height of cone $\frac{\sqrt{3}a}{2}=6\sqrt{3}$

Volume of cone $=\frac{\pi r^2h}{3}$

$\Rightarrow\frac{\pi\times6^2\times6\sqrt{3}}{3}=72\sqrt{3}\pi\text{ cm}^3$

401. (b) Let the radius of iron ball $=r_1$

Let the radius of ball $=r_0$

Then, as iron weights 8 times oak

$\because\frac{4\pi r_0^3}{3}=\frac{8\times4\pi r_1^3}{3}=\frac{r_0}{r_1}=2\Rightarrow r_0=2r_1$

So diameter of iron $=\frac{1}{2}$ diameter of oak

$\Rightarrow\frac{1}{2}\times18=9$ cm

402. (c) Volume of the timber $=7\times0.1\times0.1=0.07$ cu m

$\therefore$ Weight of the timber $=0.07\times950=66.5$ kg

403. (a) Volume of masonary = Length × Breadth × Height

$=81\times0.2\times4=64.8\text{ m}^3$

404. (a) Speed of the river $=\frac{9}{2}$ km/hr

$=\frac{9}{2}\times\frac{5}{18}=\frac{5}{4}$ m/sec.

$\therefore$ required answer

$=10\times200\times\frac{5}{4}=2500$ cub m.

405. (b) 1 sq m × depth $=\frac{200}{1000}$

$\therefore$ depth $=\frac{1}{5}\text{m}=\frac{1}{5}\times100=20$ cm.

406. (c) Area of field $=500\times30$

$=15000\text{ m}^2$

Area of field after constrauction of tank $=15000-(50\times20)$

$=1400\text{ m}^2$

Volume of tank = Area of field after constraction of tank × h

$50\times20\times14=14000\times h$

$\Rightarrow h=1$m

407. (b) Volume of cube $=a^3=25^3$

$=15625\text{ cm}^3$

Surface $=4a^2$

$=4\times25\times25$

$=2500\text{ cm}^2$

408. (c) Volume of the rectangular solid

$=36\times75\times80=21600$ cu cm

$\therefore$ Edge of the cube $=\sqrt[3]{216000}$

$=60$ cm

409. (a) Volume of the cube $=5\times5\times5=125$ cu cm

0.625 kg ≡ 125 cu cm

$\therefore$ 40 kg $\equiv\frac{125}{0.625}\times40$

$=8000$ cu cm

$\therefore$ edge $=\sqrt{8000}=20$ cm.

410. (a) $r+h=37$ and $2\pi r(r+h)=1628$

or, $\pi r=\frac{1628}{74}=22$

$\therefore r=7$ cm and $h=37-7=30$ cm

$\therefore$ Volume $=\pi r^2h=\frac{22}{7}\times7\times7\times30=4620$ cu cm.

411. (c) Radius $=\frac{1}{2}h$ and height = h

$\therefore$ Whole surface

$=2\pi\times\left(\frac{1}{2}h\right)\times h+2\pi\times\left(\frac{1}{2}h\right)^2$

$=\frac{3}{2}\pi h^2$

412. (d) Volume of cube

$=(22\times22\times22)\text{ cm}^3$

Volume of 1 bullet

$=\left(\frac{4}{3}\times\frac{22}{7}\times1\times1\times1\right)\text{ cm}^3$

$\therefore$ Number of bullets

$=\left(\frac{22\times22\times22\times3\times7}{4\times22}\right)=2541$

413. (c) Let h and H be the heights of water level before and after dropping the sphere into it.

Then, $[\pi\times(30)^2\times H]-[\pi\times(30)^2\times h]=\frac{4}{3}\pi\times(30)3$

or $\pi\times900\times(H-h)=\frac{4}{3}\pi\times27000$

or, $(H-h)=40$ cm.

414.(b) Let outer radii = R_1 and inner radii = R_2

$\therefore$ $2\pi R_1h - 2\pi R_2h = 44$

[Where, h = height of pipe]

$$\Rightarrow 2\times\frac{22}{7}\times14[R_1-R_2]=44$$

$$\Rightarrow R_1-R_2=\frac{1}{2}=0.5----(i)$$

and $\pi(R_1^2-R_2^2)\times h=99$ (given)

$$\Rightarrow\frac{22}{7}(R_1+R_2)(R_1-R_2)\times14=99$$

$$\Rightarrow 4\times0.5(R_1+R_2)=9$$

$R_1 + R_2 = 4.5$ ----------(ii)

On adding (i) and (ii) :-

$2R_1 = 5 \Rightarrow R_1 = 2.5$cm

415. (c) Required answer

$$=\frac{22\times22\times22\times3\times7}{4\times22\times1\times1\times1}=2541$$

416. (b) Volume of 1 cylinder = πr^2h

Volume of 1 cone = $\frac{1}{3}\pi r^2h$

Number of cones = $\dfrac{\pi r^2h}{\frac{1}{3}\pi r^2h}$ = 3

417. (a) (i) Capacity = (135 – 5) (108 – 5) (90 – 2.5) [Since cistern is open] = 1171625

(ii) Volume of iron = [(135 × 108 × 90) – (1171625)]

= 140575 cu cm

418. (a) External radius of the pipe = 1.2 cm

Internal radius of the pipe = (1.2 – 0.2) = 1 cm

External volume

$$=\left(\frac{22}{7}\times1.2\times1.2\times3.5\times100\right)$$

= 1584 cu cm

Internal voulme

$$=\left(\frac{22}{7}\times1\times1\times3.5\times100\right)$$

= 1100 cu cm

Volume of lead = (External Volume) – (Internal Volume)

= (1584 – 1100) = 484 cu cm

Weight of the pipe

$$=\left(\frac{484\times11.4}{1000}\right)=5.5176\text{ kg.}$$

419. (c) Capacity = Volume = (24 × 12 × 10) = 2880 cu cm

Area of tin foil neede = Total surface area = 2(lb + bh + hl)

= 2 (288 +120 +240)

= 1296 cm^2

420. (d) Volume of metal

= (52 × 40 × 29 – 48 × 36 × 27) = 13664 cm^3

$\therefore$ Weight of metal

$$=\left(13664\times0.5\times\frac{1}{1000}\right)$$

= 6.832 kg.

421. (c) Volume of Sheet

$= \frac{1}{2}$ cu m

$=\left(\frac{1}{2}\times100\times100\times100\right)$ cu cm

Area of sheet = 1 hectare

= 10000 sqr. metres

= (10000 × 100 × 100) sq cm.

Thickness = $\frac{\text{Volume}}{\text{Area}}$

$$=\frac{1\times100\times100\times100}{2\times10000\times100\times100}=\frac{1}{200}$$

= 0.005 cm

422. (b) Volume of sheet = 2 cm^3 = 2 × 100 × 100 × 100 cm^3

Area of sheet = 12 hectare

= 12 × 10000

= 120000 m^2 = 120000 × 100 × 100 cm^2

Thickness = $\frac{\text{Volume}}{\text{Area}}$

$$=\frac{2\times100\times100\times100}{120000\times100\times100}$$

$$=\frac{1}{600}=0.0017\text{ cm}$$

423. (b) $3.1416\times\frac{0.01}{2}\times\frac{0.01}{2}\times h=1$

$$\therefore h=\frac{4}{3.1416\times0.01\times0.01}$$

= 12732.365 cm = 127.3 m

424. (a) Volume of hollow cylindrical tube = $\pi(R^2 - r^2)h$

$= \frac{22}{7}((24)^2 - (20)^2) \times 144$

$= \frac{22}{7} \times 44 \times 4 \times 144$

= 25344π

425. (b) Volume of Hollow cylindrical Tube

= $\pi(R^2 - r^2)h$

$$=\frac{22}{7}\left(\left(\frac{37}{2}\right)^2-\left(\frac{33}{2}\right)^2\right)\times70$$

$$=22\times10\times\left[\frac{37}{2}+\frac{33}{2}\right]\times\left[\frac{37}{2}-\frac{33}{2}\right]$$

= 220 × 35 × 2

= 15400 cm^3

426. (a) Volume of collected Rain = Volume of swimming pool

$$\frac{25}{100}\left(\frac{1}{100}\times2\times1000\times1000\right)$$

= 50 × 5 × h

$$h=\frac{25\times2\times1000\times1000}{100\times100\times50\times5}$$

h = 20 m

427. (a) Volume of collected Rain = Volume of swimming pool

$$\frac{40}{100}\left(\frac{2}{100}\times1\times1000\times1000\right)$$

= 200 × 20 × h

$$h=\frac{40\times2\times1000\times1000}{100\times100\times200\times20}$$

h = 2 m

428. (c) Volume of the tank = 3 m^3

$\therefore$ Base area × height = 3 m^3

$\Rightarrow$ Base area = $\frac{3}{2.56}$

= 1.171875 m^2

[$\therefore$ Volume of cuboid = (l × b) × h = (base area) × height]

429. (b) Total length of tape

= 2(l + b) + 3.75

= 2(39.5 + 9.35) + 3.75

= 101.45 cm

430. (a) Area of surface to be cemented = 2 × (l + b) × h + (l × b)

i.e, area of four walls + area of floor

= 2 × (21) × 4 + (106.25)

= 274.25 m^2

$\therefore$ Cost of cementing = 24 × 274.25 = Rs. 6582

431. (b) Total volume of water displaced by 250 men

= 250 × 4 = 1000 m^3

$\therefore$ Rise in water level (h)

$$=\frac{\text{Volume}}{\text{Base area}}$$

$$=\frac{1000}{80\times50}=25\text{ cm}$$

432. (c) Let each edge of smaller cube = 1 m

$\therefore$ Each edge of larger cube = 2m and surface area of smaller cube = $6 \times (1)^2 = 6m^2$

$\therefore$ Surface area of larger cube = $6 \times (2)^2 = 24\ m^2$

$\therefore$ % increase in surface area = $\frac{24-6}{6} \times 100 = 300\%$

Alternatively:

$$\frac{S_2}{S_1} = \left(\frac{e_2}{e_1}\right)^2 \Rightarrow \frac{S_2}{S_1} = \frac{4}{1}$$

$\therefore$ percnetage increase in surface area = $\frac{4-1}{1} \times 100 = 300\%$

where S = surface area, e = edge of cube.

433. (a) External volume of the bos = $24 \times 16 \times 10 = 3484\ cm^3$

Thickness of the wood = 5mm = 0.5 cm

$\therefore$ Internal breadth of box = $24 - 2 \times 0.5 = 23$ cm

Internal breadth of box = $16 - 2 \times 0.5 = 9$ cm

$\therefore$ Internal volume of the box = $23 \times 15 \times 9 = 3105\ cm^3$

$\therefore$ Volume of the wood = 3840 – 3105 = 735 cm^3

Now, total weight of wood = Volume × weight of 1 cm^3 wood

7350 = 735 × weight of 1 cm^3 wood

$\therefore$ Weight of 1 cm^3 wood = 10 gm

434. (c) Length of tank = 120 m

But since, $\frac{120}{7} = 17\frac{1}{7}$, hence 17 cubes can be placed along length and breadth of tank = 80 cm

But since, $\frac{80}{7} = 11\frac{3}{7}$, hence 11 cubes can be placed along breadth and height of tank = 50 cm.

But since, $\frac{50}{7} = 7\frac{1}{7}$, hence only 7 cubes can be placed along height of the tank.

$\therefore$ Total volume occupied by these cubes = $(7)^3 \times 17 \times 11 \times 7$ = 448987 cm^3

Total volume of the tank = $120 \times 80 \times 50 = 480000\ cm^3$

$\therefore$ Area of unoccuppied space = 480000 – 448987 = 31013 cm^3 = 31.013 dm^3

435. (b) Surface area of the cuboid = $2(lb + bh + hl) = 11.6\ m^3$

$\therefore$ Cost of canvas = 11.6 × 25 = Rs. 290

436. (c) Required surface area = $2(9 \times 3 + 3 \times 3 + 3 \times 9) = 126\ cm^2$

437. (a) Area of 4 walls = $2(36 + 12) \times 10 = 960\ m^2$

Total area of (windows + door + chimney) = 120 m^2

$\therefore$ Net area for papering = 960 – 120 = 840 m^2

$\therefore$ Length of required paper = $\frac{840}{1.2} = 700$ m

Hence, cost of papering = 700 × 0.7 = Rs. 490

438. (d) $(x + 2)^3 - x^3 = 1016$

$\Rightarrow x = 12$ cm

and $x^3 - (x - 2)^3 = (12)^2 - (10)^2$ = 728

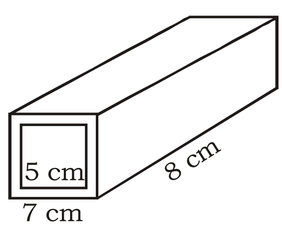
439. (b) Now let us consider that surface area of each face of the cube 1 cm^3.

$\therefore$ Total surface area of the cuboid = 14 cm^2

and Total surface area of the 3 cubes = 18 cm^2

Hence, required ratio = 14 : 18 = 7 : 9

440. (c) Iron used in the tube = Difference in external and internal volumes of the tube



$\therefore 192 = 8x^2 - 8(5)^2$

$\Rightarrow x = 7$ cm

Hence, the thickness of the tube = $\frac{7-5}{2}$ = 1 cm

441. (d) Base area of vessel × rise in water level = Volume of cube

$15 \times 12 \times h = 11 \times 11 \times 11$

$\Rightarrow$ h = 7.39 cm

442. (b) (Initial volume of water + required volume of water + volume of cube)

= Base area of vessel × 10

$\therefore 25 \times 20 \times 5$ + required volume of water + 1000 = $25 \times 20 \times 10$

$\Rightarrow$ Required volume of water = 1500 cm^3 = 1.5 litre

443. (c) Net volume = $(10 \times 8 \times 2) - (2 \times 2 \times 2) = 152\ cm^3$

Net surface area = $2(10 \times 8 + 8 \times 2 + 2 \times 10) + 4(2 \times 2) - 2(2 \times 2)$ = 240 cm^2

444. (b) Net volume of the wall = Total volume – Volume taken away due to doors

= $(30 \times 0.3 \times 5) - 2(4 \times 2.5 \times 0.3)$ = 39 m^3

Volume of the bricks = $39 \times \frac{8}{9}$

(Since $\frac{1}{9}$ part is lime in the wall)

$\therefore$ Number of bricks

$= \frac{39 \times 8}{9 \times 0.2 \times 016 \times 0.08}$

= 13541.66 = 13600

445. (a) Volume of water which flow in 25 minutes

= $25 \times 60 \times 0.05 \times 0.03 \times 16$

= 36 m^3

$\therefore$ Rise in water level = $\frac{36}{15 \times 12}$

= $\frac{1}{5}$ m = 0.2 m

446. (b) h : b = 3 : 1 and l : h = 8 : 1

$\Rightarrow l$: h : b = 24 : 3 : 1

$\therefore 24x \times 3x \times x = 36.846$

$\Rightarrow x^3 = 0.512 \Rightarrow x = 0.8$

$\therefore$ h = 3x = 2.4 m

447. (b) $lb = x$, $bh = y$, $hl = z$

$\therefore lb \times bh \times hl = xyz$

$\Rightarrow (lbh)^2 = xyz \Rightarrow lbh = \sqrt{xyz}$

448. (c) Volume of the block = $14 \times 10.5 \times 11\ cm^3$

Radius of the tank = $\frac{21}{2}$ = 10.5 cm

Volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times h$

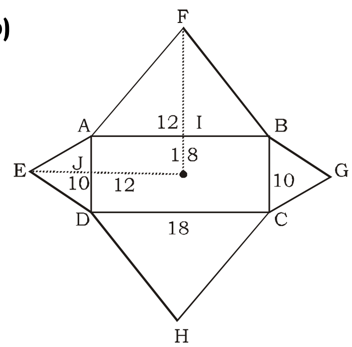
$\therefore \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times h = 14 \times 10.5 \times 11$

$h = \frac{14}{3} = 4\frac{2}{3}$ cm

449. (b) $2\pi rh = 2 \times \pi \times 2 \times 10 = 40\pi\ m^2$

450. (c) Sicne radius and height of the cylinder are same as that of cone. Therefore cylinder can contain 15 × 3 = 45 litre of milk.

451.(b)



Height of $\triangle$ AFB (FI)

$= \sqrt{(12)^2 + 5^2} = 13$ cm

Height of $\triangle$ AED (EJ)

$= \sqrt{(12)^2 + 9^2} = 15$cm

Total surface Area = area of ($\triangle$ AFB + $\triangle$ BGC + $\triangle$ CDH + $\triangle$ AED + ($\square$ ABCD)

$= 2 \times \frac{1}{4} \times 13 \times 18 + 2 \times \frac{1}{4} \times 15 \times 10 + 18 \times 10$

= 234 + 150 + 180 = 564

452. (a) $\frac{V_2}{V_1} = \frac{(1.4)^3}{(1)^3} = \frac{2.744}{1}$

$\therefore$ % increase in volume

$= \left[\frac{2.744 - 1}{1}\right] \times 100 = 174.4\%$

453. (d) Area of circular sheet $= 625\pi$

Since length of arc and area of sector are directly proportional to the central angle.

Therefore, length of remaining arc $= \frac{96}{100} \times 2 \times \pi \times 25 = 48\pi$

But the remaining arc is equal to the circumference of the base of circular cone.

$\therefore$ $2\pi R = 48\pi \Rightarrow R = 24$ cm

Now, since the slant height of cone is equal to the radius of the original circular sheet.

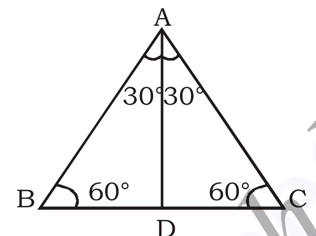
Hence, $l = 25$ cm

$h = 7$ cm $(\therefore l = \sqrt{r^2 + h^2})$

$\therefore \frac{\text{Radius}}{\text{Height}} = \frac{24}{7}$

454. (b) $\frac{BD}{AB} = \cos 60°$

$\frac{BD}{AB} = \frac{1}{2}$



BD = CD , are the radii of the base and AB = AC are the slant heights of the cone. A is vertex and BC is the base.

455. (a) Volume of cone $= \frac{1}{3}\pi \times 144 \times 35$

Volume of water flowing per second $= \pi \times (0.8)^2 \times \frac{500}{60}$

$\therefore$ Required time $= \frac{\left(\frac{\pi}{3}\right) \times 144 \times 35}{\pi \times 0.64 \times \frac{500}{60}} =$

315 seconds

456. (d) Volume of frustum

$= \frac{1}{3}\pi h (R^2 + Rr + r^2)$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times (16 + 8 + 4)$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 28 = \frac{528}{3}$

$= 176\ cm^3$

457. (a) $\frac{1}{3}\pi \times (10)^2 \times 72 = \pi \times (30)^2 \times h$

$\Rightarrow h = \frac{8}{3}$

$= 2\frac{2}{3}$ cm

458.(c) Let slant height = l and radius = r

$\therefore v = \frac{1}{3}\pi r^2 h \Rightarrow 3v = \pi r^2 h$

$\Rightarrow 9v^2 = \pi^2 r^4 h^2$

and C = πrl

$\Rightarrow C^2 = \pi^2 r^2 l^2 = \pi^2 r^2 (h^2 + r^2)$

$[\because l^2 = h^2 + r^2]$

$\Rightarrow C^2 = \pi^2 r^2 h^2 + \pi^2 r^4$

$\therefore 3\pi v h^3 - c^2 h^2 + 9v^2$

$= (\pi r^2 h)\pi h^3 - (\pi^2 r^2 h^2 + \pi^2 r^4) h^2 + \pi^2 r^4 h^2$

$= \pi^2 r^2 h^4 - \pi^2 r^2 h^4 - \pi^2 r^4 h^2 + \pi^2 r^4 h^2 = 0$

459. (b) Side of Tetrahedren $= \frac{\sqrt{3}}{\sqrt{2}}$ H

$2a = \frac{\sqrt{3}}{\sqrt{2}}$ P

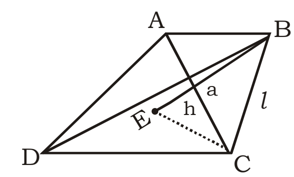
$P = \frac{2\sqrt{2}}{\sqrt{3}}$ a

$P^2 = \frac{8}{3}a^2$

$3P^2 = \frac{8}{3}a^2 \times 3$

$3P^2 = 8a^2$

460. (c)



BE = height of pyramid = hm

BC = slant edge of pyramid = l

EC = base $= \frac{a}{\sqrt{3}}$

In $\triangle$ BEC,

$BC^2 = EC^2 + BE^2$

$l^2 = \left(\frac{a}{\sqrt{3}}\right)^2 + h^2$

$l = \sqrt{\frac{a^2}{3} + h^2}$

461.(a) Side of tetrahydren $= \frac{\sqrt{3}}{\sqrt{2}}$ H

$= \frac{\sqrt{3}}{\sqrt{2}} \times 4\sqrt{3} = 6\sqrt{2}$

Volume of terahydren

$= \frac{\sqrt{2}}{12} \times (6\sqrt{2})^3$

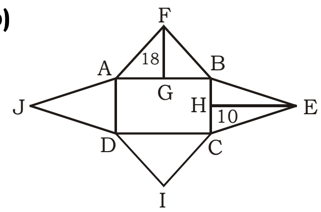
$= \frac{\sqrt{2}}{12} \times 6\sqrt{2} \times 6\sqrt{2} \times 6\sqrt{2}$

$= 72cm^3$

462. (a) Volume of rain = 1.5 hectare×10cm

$= 1.5\times100\times100\times\frac{1}{10} = 1500\ \text{cm}^3$

463. (b)



Area of base (ABCD) = 18 × 10
= 180

In $\triangle$ AFB

$GF^2 = (12)^2 + 5^2$

GF = 13 cm

Area of $\triangle$ AFB = $\frac{1}{2}\times13\times18$
= 117 cm²

In $\triangle$ BEC

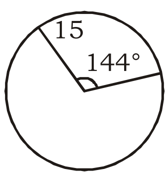
$EH^2 = (12)^2 + 9^2$

EH = 15 cm

Area of $\triangle$ BEC = $\frac{1}{2}\times15\times10$
= 75 cm²

Total surface area of Pyramid
= 180 + 2 × 117 + 2 × 75
= 180 + 234 + 150 = 564 cm²

464. (c)



Remaining portion of circle of

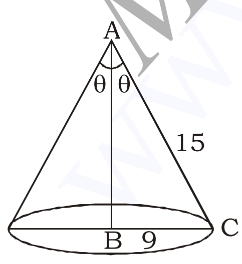
circumference = $\frac{\theta}{360^\circ}\times2\pi r$

$= \frac{216}{360}\times2\times\frac{22}{7}\times15 = 18\pi$

Circumference of base of cone
= 18π

$2\pi r = 18\pi$

r = 9



In $\triangle$ ABC,

$AB^2 + BC^2 = AC^2$

$AB^2 + 9^2 = 15^2$

$AB^2 = 225 - 81$

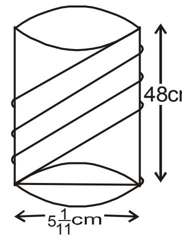
AB = 12 cm

$\sin\theta = \frac{BC}{AC} = \frac{9}{15}$

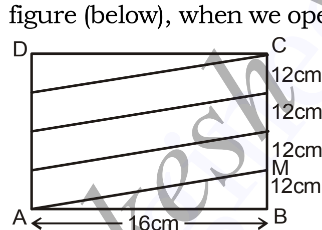
$\theta = \sin^{-1} = \frac{3}{5}$

$2\theta = 2\sin^{-1}\frac{3}{5}$

465. (d) According to the given information, the string will be bounded on a cylindrical tank as shown in the figure,



The above figure, will look like the figure (below), when we open it.



The base circumference

$= 2\pi r = 2\times\frac{22}{7}\times\frac{56}{11}\times\frac{1}{2} = 16\text{cm}$

$\therefore$ AM = length of one complete turn $= \sqrt{16^2+12^2}$
= 20cm

$\therefore$ Total length = 4×20 = 80cm

466.(c) Whole surface area = cone : hemisphere : cylinder

$= \pi r(l + r) : 3\pi r^2 : 2\pi r(r + h)$

$= \pi r\left(\sqrt{2}r+r\right) : 3\pi r^2 : 2\pi r(r + r)$

$= (\sqrt{2} + 1)r : 3r : 4r$

$= (\sqrt{2} + 1) : 3 : 4$

467. (c) Since, b l and $2(l + b)$ are in GP, therefore

$\frac{l}{b} = \frac{2(l+b)}{l}$

Suppose $\frac{l}{b} = x$,

then $x = 2\left(1+\frac{1}{x}\right)$

$\Rightarrow x^2 - 2x - 2 = 0$

$\Rightarrow x = \sqrt{3} + 1$

468. (a) Lateral surface area c ylinder (A) = $2\pi rh$

$\because$ Base area is decreased by 1/9th times

$\therefore$ side (radius) will decrease by 1/3 time

[$\because$ area d(side)²]

$\therefore$ new radius = $r' = r/3$

and new height = $h' = 6h$

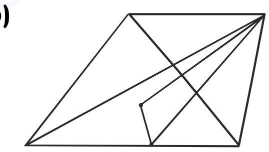
$\therefore$ new lateral surface area of cylinder

$\Rightarrow A' = 2\pi\left(\frac{r}{3}\right)\times(6h) = 4\pi rh$

$\Rightarrow A' = 2(2\pi rh) = 2A$

i.e. $A' = 2$times of A.

469. (b)



Inradius = $\frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$

l = 2h

$(2h)^2 = h^2 + \left(\frac{2}{\sqrt{3}}\right)^2$

$4h^2 = h^2 + \frac{4}{3}$

$3h^2 = \frac{4}{3}$

$h^2 = \frac{4}{9}$

$h = \frac{2}{3}$

Volume of pyramid = $\frac{1}{3}$ × Area of base × Height

$\frac{1}{3}\times\frac{\sqrt{3}}{4}\times4\times4\times\frac{2}{3} = \frac{8}{3\sqrt{3}}\ \text{cm}^3$

470. (b) Let the side of the cube and radius of the sphere be a and r respectively

$\because 6a^2 = 4\pi r^2 \Rightarrow a = r\left(\frac{2}{3}\pi\right)^{\frac{1}{2}}$

$\frac{V_1}{V_2} = \frac{a^3}{\frac{4}{3}\pi r^3} = \frac{r^3\left(\frac{2}{3}\pi\right)^{\frac{3}{2}}}{\frac{4}{3}\pi r^3}$

$= \frac{2\sqrt{2}\pi^{3/2}}{3\sqrt{3}}\times\frac{3}{4\pi} = \sqrt{\pi} : \sqrt{6}$

471. (a) V(cone) : V(hemisphere) : V(cylinder)

$=\frac{1}{3}\pi r^2.r:\frac{2}{3}\pi r^3:\pi r^2.r$

$=1:2:3$

472. (d) Volume of original cube = n $\times$ volume of smaller cubes

$\Rightarrow n=\frac{3\times3\times3}{1\times1\times1}=27$

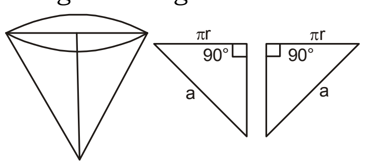
473. (c) $\because$ H.C.F of 6,9 and 12 = 3

$\therefore$ Least possible number of

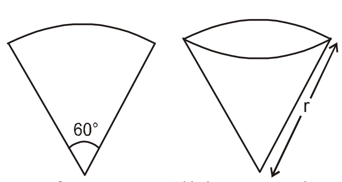
cubes $=\frac{6\times9\times12}{3\times3\times3}=24$

[Note :- The cuboid is cut into smaller cubes of equal size i.e. size should be maximum]

474. (b) It will be in the form of a right angled triangle.



475. (a) Arc of sector $=2\pi r\frac{60}{360}=\frac{2\pi r}{6}$



This arc of sector will be equal to the perimeter of cone. Let the radius of cone be R, then:

$2\pi R=\frac{2\pi r}{6}\Rightarrow R=\frac{r}{6}$, and slant height of cone (l) = radius of sector = r

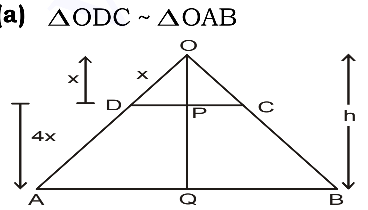
$\therefore h=\sqrt{l^2-R^2}=\sqrt{r^2-\frac{r^2}{36}}=\frac{\sqrt{35}}{6}r$

476. (d) Volume of water displaced = volume of sphere

$\pi(30)^2\times x=\frac{4}{3}\pi(20)^3$

$\Rightarrow x=\frac{320}{27}=11\frac{23}{27}cm$

477. (a) ΔODC ~ ΔOAB



$\therefore\frac{OP}{OQ}=\frac{1}{5}=\frac{PC}{BQ}$

Since, the ratio in radii of the two cones is 1 : 5.

There fore the radius of smaller cone ODC is $\frac{21}{5}$ = 4.2 cm.

478. (c) $\frac{4}{3}\pi r^3=\frac{1}{3}\pi r^2h$

(since radii of sphere and cone are same)

$\Rightarrow$ 4r = h

$\therefore\frac{h}{r}=\frac{4}{1}\Rightarrow h:r=4:1$

479. (b) Area of large cube = $6(5)^2$ $=150$ (unit)^2

Area of cuboid = 2(1 $\times$ 1 + 1 $\times$ 125 + 125 $\times$ 1) = 502sq. units

$\therefore$ % increase in surface area=

$\frac{502-150}{150}\times100=234\frac{2}{3}\%$

480. (a) H.C.F of 75, 15, 4.5 = 1.5

$\therefore$ No. of cubes

$=\frac{75\times15\times4.5}{1.5\times1.5\times1.5}=1500$

Area of each cube = $6\,(1.5)^2$

Area of all the 1500 cubes

$=1500\times6\times(1.5)^2=20{,}250cm^2$

481. (c) The solid with the least number of sides will have maximum surface area. So, tetrahedron will have maximum surface area. Notice that in a sphere there are infinite number of sides with least possible length. So, the surface area of the sphere will be least.

482. (d) $\frac{4}{3}\pi(r_1{}^3+r_2{}^3+r_3{}^3)=\frac{4}{3}\pi(6)^3$

$\Rightarrow 27+64+r_3{}^3=216$

$\Rightarrow r_3{}^3=125\Rightarrow r=5$ cm

483. (b) In radius (r) = 9cm

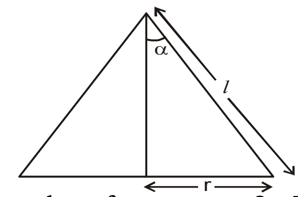
$=\frac{\text{Area (A)}}{\text{semiperimeter (s)}}$

$\Rightarrow$ A= $9\times\frac{45}{2}cm^2$

V= Area of base (A) $\times$ h

$\Rightarrow$ h = $\frac{810}{9\times45}\times2=4cm$

484. (a)



Lateral surface area = 2 $\times$ Base area

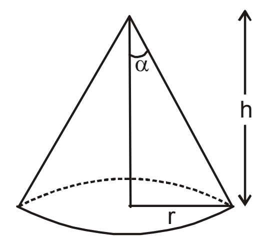
$\Rightarrow\pi rl=2\pi r^2\Rightarrow l=2r$

$Sin\alpha=\frac{r}{l}=\frac{r}{2r}=\frac{1}{2}$

$\Rightarrow\alpha=30°$

485. (c) $\cot\alpha=\frac{h}{r}\Rightarrow h=r\cot\alpha.$

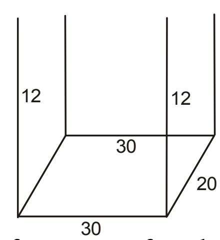
let no. of cones = n



$\therefore$ n $\left(\frac{1}{3}\pi r^2\times r\cot\alpha\right)=\pi r^2(2r\cot\alpha)$

$\Rightarrow n=6$

486. (a)



Total surface area of tank

TS A = 30 $\times$ 20+2(12 $\times$ 20) + 2(30 $\times$ 12) =1800

$\therefore$ area of iron sheet = T.S.A

$\Rightarrow$ Length $\times$ width = 1800

$\Rightarrow$ Length = $\frac{1800}{3}=600\ m$

$\therefore$ Cost = 600 $\times$ 10 = ₹ 6000

487.(d) Volume of prism = (Base area) $\times$ height.

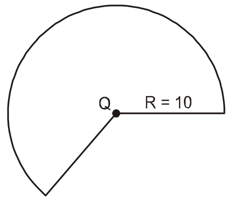
Base Area = Area of trapezium

$=\frac{1}{2}(8+14)\times8=88$

$\therefore$ 1056 = 88 × height

height = $\frac{1056}{88}$ = 12cm

488. (b)



Let radius of conical surface = r

$\therefore 2\pi r = 60\% \; of \; 2\pi R$

$\Rightarrow r = \frac{3}{5} \times 10 = 6cm$

and slant height of cone = l = R =10cm

$\therefore height(h) = \sqrt{l^2 - r^2} = \sqrt{(10)^2 - (6)^2} = 8cm$

$\therefore r : h = 6 : 8 = 3 : 4$

489. (c) we have ,

$2\pi R_1(R_1 + h) = \pi(12^2 - 8^2)$

$\Rightarrow R_1 + h = \frac{80}{2R_1} = \frac{40}{R_1}$

$\Rightarrow h = \frac{40}{R_1} - R_1 = \frac{40 - R_1^2}{R_1}$

490. (a) Total surface area of pipe (hollow cylinder)

$= 2\pi(R + r)[h + (R - r)]$

$here - R - r = thickness = 1cm, h = 20cm$

$R = \frac{25}{2} = 12.5cm$

$\therefore \quad r = 12.5 - 1 = 11.5$

$Area = 2 \times \frac{22}{7}(12.5 + 11.5)(20 + 1)$

= 44 × 72 = 3168 cm²

491. (d) $v_1 = 64$ ltr $v_2 = 216$ ltr

$\therefore \frac{r_1}{r_2} = \left(\frac{64}{216}\right)^{1/3} = \frac{4}{6} = \frac{2}{3}$

$\left[\because v \propto r^3\right]$

$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$

$\left[\because v \propto r^2\right]$

492. (b) Let lenght (l) = 15R

$\therefore$ breadth (b) = 5R

and height (h) = 3R

$\therefore$ volume (V) = lbh

= 15R × 5R × 3R

$\Rightarrow$ 14400 = 225R³

$\Rightarrow$ R³ = 64

$\Rightarrow$ R = 4

$\therefore$ l = 60cm, b = 20cm, h = 12cm

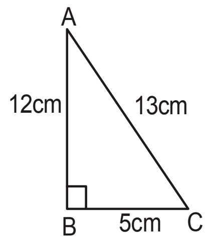
$\therefore$ Total surface area

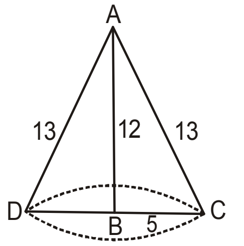
= 2(lb + bh + hl)

= 2(1200 + 240 + 720)

= 4320cm²

493. (c)





i.e. after revolution, a cone of radius 5cm & height 12cm is formed

$\therefore$ volume of the cone

$= \frac{1}{3}\pi(5)^2 \times 12 \quad = 100\pi$

$= 100 \times 3.14 = 314cm^3$

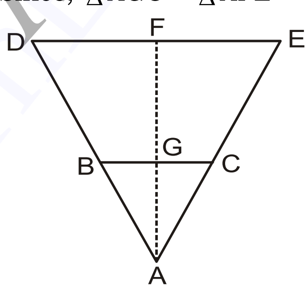
494. (a) $V_1 = \frac{2}{3}\pi r^3, V_2 = \pi r^2.r = \pi r^3$

$V_3 = (2r)^3 = 8r^3$

$\Rightarrow V_1 < V_2 < V_3$

495. (b) Let r be the radius and h be the height of the cone.

Since, $\triangle AGC \sim \triangle AFE$



$\therefore \frac{AG}{AF} = \frac{GC}{FE} \Rightarrow GC = \frac{h/2}{h} \times r = \frac{r}{2}$

$\therefore$ volume of water filled in cone

$= \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \times \frac{h}{2} = \frac{1}{8}\left(\frac{1}{3}\pi r^2 h\right) = \frac{V}{8}$

496. (c) Height of cone = height of cylinder = radius of hemisphere = r units

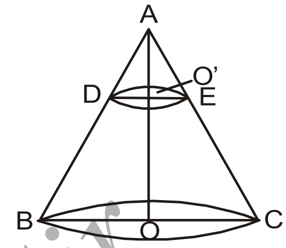
$\therefore$ Ratio of the volumes of cone, cylinder and hemisphere

$= \frac{1}{3}\pi r_1^2 h : \pi r_2^2 h : \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi 2^2 r^3 : \pi 3^2 r^3 : \frac{2}{3}\pi r^3$

$= \frac{4}{3} : 9 : \frac{2}{3} = 4 : 27 : 2$

497. (b)



Let DO` = r cm and OO` = h cm, From similar triangles ADO` and ABO,

$= \frac{AO'}{AO} = \frac{DO'}{BO} \Rightarrow \frac{9-h}{9} = \frac{r}{3}$

$\Rightarrow 9 - h = 3r \Rightarrow h = 9 - 3r$

volume of frustum $= \frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r^2)$

$\Rightarrow 44 = \frac{1}{3} \times \frac{22}{7}(9 - 3r)(9 + r^2 + 3r)$

$\Rightarrow 44 = \frac{22}{7}(3 - r)(3^2 + 3r + r^2)$

$\Rightarrow \frac{44 \times 7}{22} = 3^3 - r^3$

$\Rightarrow 14 = 27 - r^3 \Rightarrow r^3 = 27 - 14 = 13$

$\therefore r = \sqrt[3]{13}$ cm

498. (c) Let the height of cylinder be h cm and radius of base= r cm

$\therefore \; 2\pi r^2 + 2\pi rh = 462$ ______(i)

Area of curved surface = 2π rh

$= \frac{1}{3} \times 462 = 154$

$\therefore 2\pi r^2 + 154 = 462$

$\Rightarrow 2\pi r^2 = 462 - 154 = 308$

$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 308$

$\Rightarrow r^2 = \frac{308 \times 7}{2 \times 22} = 49 \Rightarrow r = 7cm$

$\therefore 2\pi rh = 154 \Rightarrow 2 \times \frac{22}{7} \times 7 \times h = 154$

$\Rightarrow h = \frac{154}{2 \times 22} = \frac{7}{2}cm$

$\therefore$ Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times \frac{7}{2} = 539$ cm³

499. (d) $\therefore 2\pi r^2 = \pi r\sqrt{r^2+h^2}$

$\Rightarrow 2r = \sqrt{r^2+h^2} \Rightarrow 4r^2 = r^2+h^2$

$\Rightarrow 3r^2 = h^2 \Rightarrow \sqrt{3}r = h \Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}}$

500. (a) $\frac{r}{h} = \frac{4}{3} \Rightarrow \frac{r}{4} = \frac{h}{3} = k$

$\Rightarrow r = 4k; h = 3k$

$\therefore l = \sqrt{r_2 + h_2} = \sqrt{16k^2+9k^2}$

$= \sqrt{25k^2} = 5k$

$\therefore \frac{\text{Curved surface area}}{\text{Total surface area}}$

$= \frac{\pi rl}{\pi r(r+l)} = \frac{l}{r+l} = \frac{5k}{4k+5k} = \frac{5}{9}$

501. (c) Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 10 \times 10 \times 21 = 6600$ cu.cm

Volume of the cone $= 6600 - 4400 = 2200$ cu.cm.

$\therefore 2200 = \frac{1}{3}\pi \times 10^2 \times h$

$\Rightarrow 2200 = \frac{2200}{21} \times h \Rightarrow 21$ cm.

502. (c) Radius of the base = r units and height = h units

$\Rightarrow \frac{\text{Curved surface of cylinder}}{\text{Curved surface of cone}} = \frac{2\pi rh}{\pi rl}$

$\Rightarrow \frac{8}{5} = \frac{2h}{l} \Rightarrow \frac{4}{5} = \frac{h}{\sqrt{h^2+r^2}}$

$\Rightarrow \frac{16}{25} = \frac{h^2}{h^2+r^2} \Rightarrow \frac{h^2+r^2}{h^2} = \frac{25}{16}$

$\Rightarrow 1 + \frac{r^2}{h^2} = \frac{25}{16} \Rightarrow \frac{r^2}{h^2} = \frac{25}{16} - 1 = \frac{9}{16}$

$\Rightarrow \frac{r}{h} = \frac{3}{4}$

503. (b) Curved surface area of cylinder = $2\pi rh$ and volume = $\pi r^2 h$

$\therefore \frac{\pi r^2 h}{2\pi rh} = \frac{924}{264} \Rightarrow \frac{r}{2} = \frac{924}{264}$

$\Rightarrow r = \frac{924 \times 2}{264} = 7$metre

$\therefore 2\pi rh = 264$

$\Rightarrow 2 \times \frac{22}{7} \times 7 \times h = 264$

$\therefore h = \frac{264}{2 \times 22} = 6$

$\therefore \frac{\text{Diameter}}{\text{Height}} = \frac{2 \times 7}{6} = \frac{7}{3}$

504. (b) Volume of bigger cube
$= 6 \times 6 \times 6 = 216$ cu. cm
Volume of unit cube $= 1 \times 1 \times 1$ $= 1$ cu.cm
Number of uncoloured cubes
$= 4 \times 4 \times 4$
= 64, because edge of uncoloured cube = 4 cm

505. (c) Area of the base $= 6 \times \frac{\sqrt{3}}{4} \times (2a)^2$

$= 6 \times \frac{\sqrt{3}}{4} \times 4a^2 = 6\sqrt{3}a^2$ sq.cm

Height $= \sqrt{\left(\frac{5a}{2}\right)^2 - (2a)^2}$

$= \sqrt{\frac{25}{4}a^2 - 4a^2} = \sqrt{\frac{9a^2}{4}} = \frac{3}{2}a$ cm.

$\therefore$ volume of pyramid

$= \frac{1}{3} \times$ area of base $\times$ height

$= \frac{1}{3} \times 6\sqrt{3}a^2 \times \frac{3}{2}a = 3\sqrt{3}a^3 cm^3$

506. (c) Volume of right prism = Area of the base $\times$ height
$\Rightarrow 10380 = 173 \times h$

$\Rightarrow h = \frac{10380}{173} = 60$ cm

Now, Area of triangle $= \frac{\sqrt{3}}{4} \times (\text{Side})^2$

$\Rightarrow 173 = \frac{\sqrt{3}}{4} \times (\text{Side})^2$

$\therefore$ Side $= \sqrt{\frac{173 \times 4}{\sqrt{3}}} = \sqrt{\frac{173 \times 4}{1.73}} = 20$ cm

$\therefore$ Perimeter $= 3 \times 20 = 60$ cm

$\therefore$ Area of the lateral surface

$=$ Perimeter base $\times$ height

$= 60 \times 60 = 3600$ sq.cm.

507. (c) Volume of the new ball

$= \frac{3}{4} \times \frac{4}{3}\pi\left(r_1^3 + r_2^3 + r_3^3\right)$

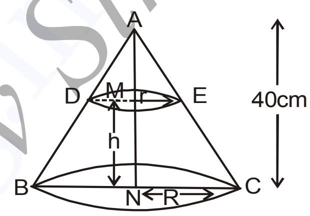
$= \pi\left(1^3 + 2^3 + 3^3\right)$

$= \pi(1+8+27) = 36\pi$ cubic cm

$\therefore \frac{4}{3}\pi r^3 = 36\pi \Rightarrow r^3 = \frac{36 \times 3}{4} = 27$

$\therefore r = \sqrt[3]{27} = 3\,cm$

508. (b)



Let the radius of small cone = r and height = h from the base,

$\therefore \frac{1}{64} \times \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi r^2(40-h)$

$\Rightarrow \frac{R^2}{r^2} = (40-h) \times \frac{8}{5}$ ----(i)

$\Delta AME \sim \Delta ANC$

$\therefore \frac{40-h}{40} = \frac{r}{R}$ ------(ii)

$\therefore$ from (i) and (ii)

$\therefore \left(\frac{40}{40-h}\right)^2 = (40-h) \times \frac{8}{5}$

$\Rightarrow (40-h)^3 = 25 \times 40 = 125 \times 8$

$\Rightarrow 40 - h = 5 \times 2 = 10$

$\Rightarrow h = 40 - 10 = 30cm$

Short Trick:-

Bigger Smaller conecone

Ratio of vol. 64 1

Ratio of (height $\sqrt[3]{64} = 4$ $\sqrt[3]{1} = 1$

/radius / slant height)

i.e. 4 represent = 40

$\Rightarrow$ 4 $\cong$ 40 cm

$\therefore$ 1 $\cong \frac{40}{4}$ = 10cm

$\therefore$ Required height = h
= 40 – 10 = 30 cm

509. (d) Let Edge reduced from 2 $\rightarrow$ 1

2 $\rightarrow$ 1

2 $\rightarrow$ 1

$2 \to 1$

$2^3 \to 1$

$8 \to 1$

Edge of the smaller cube = 1 metre

$\therefore$ Required ratio = 1 : 64

510. (b) $2\pi r = 22$

$$\Rightarrow 2\times\frac{22}{7}\times r = 22 \Rightarrow r = \frac{22\times 7}{2\times 22} = \frac{7}{2}cm$$

$h = 15\ cm$

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 15 = 577.5cm^3$$

511. (a) Volume of the original cone

$$= \frac{1}{3}\pi r^2 h$$

Volume of the new cone

$$= \frac{1}{3}\pi 4r^2 h \times 2h = 8\left(\frac{1}{3}\pi r^2 h\right)$$

$\therefore$ Percentage increase

$$= \frac{7\left(\frac{1}{3}\pi r^2 h\right)}{\frac{1}{3}\pi r^2 h}\times 100 = 700\%$$

512. (c) Total surface area of prism = Curved surface area + 2 × Area of base

$\Rightarrow$ 608 = Perimeter of base × height + 2 × Area of base

$\Rightarrow$ $608 = 4x \times 15 + 2x^2$

(Where x = side of square)

$\Rightarrow$ $x^2 + 30x - 304 = 0$

$\Rightarrow$ $x^2 + 38x - 8x - 304 = 0$

$\Rightarrow$ $x(x + 38) - 8(x + 38) = 0$

$\Rightarrow$ $(x - 8)(x + 38) = 0$

$\Rightarrow$ $x = 8$

Volume = Base area × Height

$= 8^2 \times 15 = 960\ cm^3$

513. (b) If the length of the pipe be h cm, then

Its volume $= \pi r_1^2 h - \pi r_2^2 h$

$= \pi h(r_1^2 - r_2^2) = \pi h(25^2 - 24^2)$

$= 49\pi h$ cu.cm.

$\Rightarrow$ Volume of new cylinder

$\Rightarrow \pi r^2 h = 49\pi h$

$\Rightarrow r^2 = 49$

$\Rightarrow r = \sqrt{49} = 7$cm

$\Rightarrow$ Didameter = 14cm

514. (a) New volume $= \frac{1}{3}\pi r^2 h$

$$\therefore \frac{1}{3}\pi(2r)^2 \times h' = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow 4h' = h \Rightarrow h' = \frac{h}{4}$$

515. (b) The diagonal of cube will be equal to the diameter of the sphere.

$$\therefore \text{volume of sphere} = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{\pi d^3}{6}$$

and each side of cube = a = $\frac{d}{\sqrt{3}}$

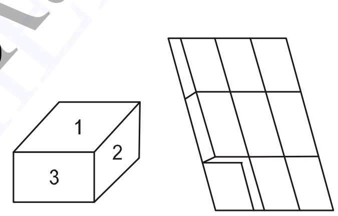
$$\therefore \text{volume of cube} = a^3 = \frac{d^3}{3\sqrt{3}}$$

$\therefore$ Remaining volume

$$= \frac{\pi\ d^3}{6} - \frac{d^3}{3\sqrt{3}}$$

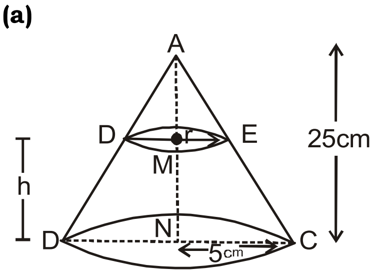
$$= \frac{d^3}{3}\left(\frac{\pi}{2} - \frac{1}{\sqrt{3}}\right)$$

516. (d)



Since, there are three faces which are visible in a corner cube. When the cube of corner is removed then the 3 faces of other cubes will be visible from outside. So, there will not be any change in the surface area of this solid figure.

517. (a)



$\Delta AME \sim \Delta ANC$

$$\therefore \frac{25-h}{25} = \frac{r}{5}$$

$\Rightarrow h = 25 - 5r$ ______(i)

Volume of frustum (V)

$$= \frac{1}{3}\pi\left[5^2 + r^2 + 5r\right]h$$

$$\Rightarrow 110 = \frac{1}{3}\pi\left[25 + r^2 + 5r\right](25 - 5r)$$

$$\Rightarrow \frac{5}{3}\pi\left[(5 - r)(5^2 + r^2 + 5r)\right] = 110$$

$$\Rightarrow \frac{5}{3}\pi\left[5^3 - r^3\right] = 110$$

$$\Rightarrow 5^3 - r^3 = \frac{110\times 3}{5\pi}$$

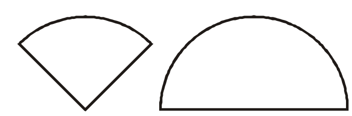
$$\Rightarrow 125 - r^3 = \frac{110\times 3}{5\times\frac{22}{7}}$$

$$\Rightarrow r = (104)^{1/3}\ cm$$

Short Trick :-

$$R^3 - r^3 = \frac{\text{Volume of frustrum}\times 3}{R\pi}$$

518. (a)



(Front view) (Side View)

Polished area = $4\pi (r)^2$

Non-polished area

$$= 4\times\left(2\times\frac{\pi r^2}{2}\right) = 4\pi r^2$$

In the adjoining figure one of the four parts of the sphere is shown.(To understand it properly, take an apple and cut it in the four parts one across horizontal and another cut make vertical to it then you will notice that in a piece there are 2 semicircles.)

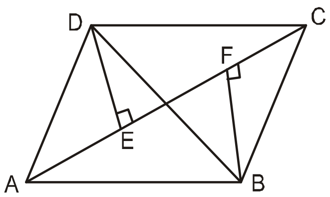
Therefore required ratio = 1 : 1

CHAPTER 18

QUADRILATERAL

QUADRILATERAL

A plane figure bounded by four line segments AB, BC, CD and DA is called a quadrilateral. It is denoted by symbol '⏢' i.e. ⏢ ABCD.



Pairs of consecutive (adjacent) angles:

$(\angle A, \angle B)$, $(\angle B, \angle C)$, $(\angle C, \angle D)$, $(\angle D, \angle A)$

Pairs of adjacent sides :

(AB,BC), (BC,CD), (CD,DA) and (DA,AB)

Properties

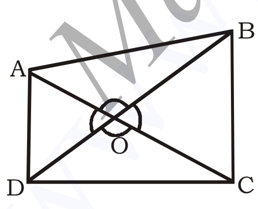
- Sum of four interior angles is 360°. *i.e.*

  $\angle A + \angle B + \angle C + \angle D = 360°$

  The figure formed by joining the mid-points of a quadrilateral is a parallelogram.

1. If the diagonals intersect at right angle then sum of square of opposite sides are equal

   $AB^2 + DC^2 = AD^2 + BC^2$



Proof:- Apply Pythagoras theorem in all right angled △

$OA^2 + OB^2 = AB^2$

$OB^2 + OC^2 = BC^2$

$OD^2 + OC^2 = DC^2$

$OA^2 + OD^2 = AD^2$

Add all

$2(OA^2 + OB^2 + OC^2 + OD^2)$

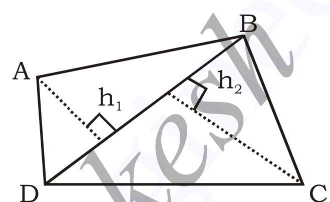
$= AB^2 + BC^2 + DC^2 + AD^2$

$2(AB^2 + DC^2) = AB^2 + BC^2 + DC^2 + AD^2$

$AB^2 + DC^2 = BC^2 + AD^2$

2. Area of a quadrilateral

   $= \frac{1}{2} \times$ **One diagonal** $\times$ sum of the perpendiculars drawn to the diagonals from the opposite vertices.



Area (□ ABCD) = Area (△ ABD) + Area (△ BDC)

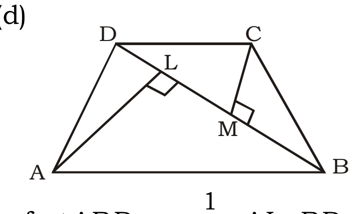
$= \frac{1}{2} \times BD \times h_1 + \frac{1}{2} \times BD \times h_2 = \frac{1}{2} \times BD \times (h_1 + h_2)$

EXAMPLES

1. ABCD is a quadrilateral in which diagonal BD = 64 cm. AL ⊥ BD and CM ⊥ BD, such that AL = 13.2 cm and CM = 16.8 cm. The area of the quadrilateral ABCD in square centimeters is:

(a) 422.4 (b) 690.0 (c) 537.6 (d) 960.0

Sol. (d)



Area of ΔABD $= \frac{1}{2} \times AL \times BD = \frac{1}{2} \times 13.2 \times 64 = 422.4$ cm²

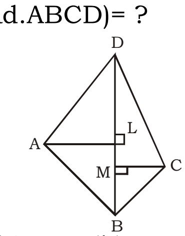
Area of ΔBCD $= \frac{1}{2} \times CM \times BD = \frac{1}{2} \times 16.8 \times 64 = 537.6$ m²

⇒ Area of quadrilateral ABCD = Area of △ ABD + Area of △ BCD = 422.4 + 537.6 = 960 cm²

Alternate

Area of □ABCD $= \frac{1}{2} \times BD \times (h_1 + h_2) = \frac{1}{2} \times 64\ (13.2 + 16.8) = 32 \times 30$ = 960 cm²

2. In a quadrilateral ABCD, it is given that BD = 16 cm. If AL ⊥ BD and CM ⊥ BD such that AL = 9 cm and CM = 7 cm, then ar (quad.ABCD)= ?



(a) 256 cm² (b) 128 cm² (c) 64 cm² (d) 96 cm²

Sol. (b) Area of quadrilateral ABCD

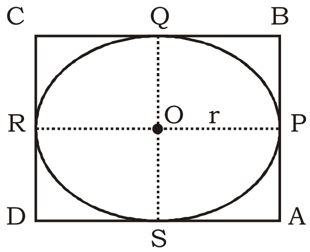
$= \frac{1}{2} \times BD(AL + CM)$

$= \frac{1}{2} \times 16 \times (9 + 7) = 128$ cm²

3. ABCD is a quadrilateral such that ∠D= 90°. A circle C (o,r) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm. Find r.

(a) 7 cm (b) 14 cm
(c) 13 cm (d) 11 cm

Sol. (b)



(Two tangents drawn from an external point to a circle are equal)

$\therefore$ BP = BQ = 27 cm

$\Rightarrow$ CQ = BC – BQ=38–27=11cm

$\Rightarrow$ CQ = CR =11cm

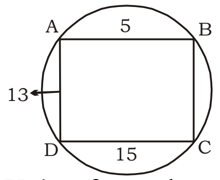
$\Rightarrow$ DR = CD – CR = 25–11=14 cm

$\therefore$ $OR \perp CD$ and $OS \perp AD$ (line drawn from centre to tangent will be perpendicular to tangent)

$\Rightarrow$ Radius (r) = DR = 14 cm

4. ABCD is a cyclic quadrilateral. If AB = 5 cm, CD = 15 cm, DA = 13 cm. If diagonals intersect at right angle. Find BC?

Sol.



Using formula

$\Rightarrow AB^2 + CD^2 = BC^2 + AD^2$

$5^2 + 15^2 = BC^2 + 13^2$

$25 + 225 = BC^2 + 169$

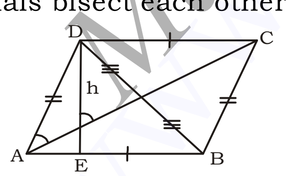
$BC^2 = 81$

BC = 9 cm

TYPES OF QUADRILATERAL

1. Parallelogram

If opposite sides of a quadrilateral are parallel, it is called a parallelogram. Its opposite sides are also equal in length and its diagonals bisect each other.



Properties

(i) The opposite sides are equal and parallel.

(ii) Opposite angles are equal. ($\angle A = \angle C$) and ($\angle B = \angle D$)

(iii) Sum of any two adjacent angles are 180°.

(iv) Diagonals bisect each-other.

(v) Diagonals need not be equal in length.

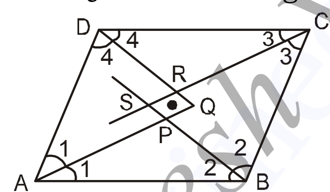
(vi) Diagonals need not bisect at right angle.

(vii) Each diagonal divides a ||gm into two congruent triangles.

i.e. $\Delta ABC \cong \Delta ADC$ and $\Delta ABD \cong \Delta BCD$.

(viii) Bisectors of the angles of a ||gm form a rectangle.

i.e. PQRS is a rectangle.



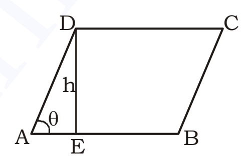
(ix) A ||gm inscribed in a circle is a rectangle.

(x) A ||gm circumscribed about a circle is a rhombus.

(xi) Area of ||gm ABCD

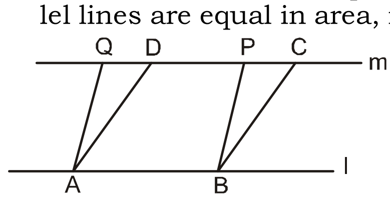
= Base $\times$ height = AB $\times$ h

= AB $\times$ AD Sinθ



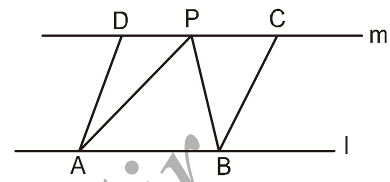
(xii) A ||gm is a rectangle if its diagonals are equal.

(xiii) ||gm that lie on the same base and between the same parallel lines are equal in area, i.e.



if l || m, then ar(□ABCD)= ar(□ABPQ)

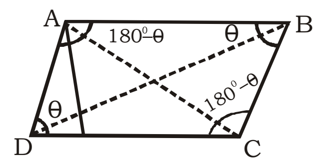
(xiv) if l || m, and ||gm ABCD and ΔAPB made on the same base AB then,



ar(ΔAPB) = $\frac{1}{2}ar$ (□ABCD)

(xv) Sum of square of diagonals is equal to sum of square of all sides

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2 = 2(AB^2 + BC^2)$



Proof :- Apply cosine formula

$$\cos(180-\theta) = \frac{AD^2 + AB^2 - BD^2}{2AD.AB} \quad ...(i)$$

$$\cos\theta = \frac{AD^2 + DC^2 - AC^2}{2AD.DC} \quad ...(ii)$$

From (i) & (ii)

$$-\frac{AD^2 + AB^2 - BD^2}{2AD.AB} = \frac{AD^2 + DC^2 - AC^2}{2AD.DC}$$

$\because$ AB = DC & AD = BC

$BD^2 - AD^2 - AB^2 = AD^2 + DC^2 - AC^2$

$BD^2 - BC^2 - DC^2 = BC^2 + DC^2 - AC^2$

$2DC^2 + 2BC^2 = BD^2 + AC^2$

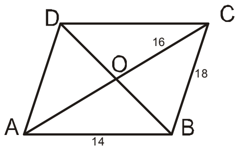
$AC^2 + BD^2 = DC^2 + BC^2 + AB^2 + AD^2$

EXAMPLES

1. ▭ABCD is a ||gm, AB = 14cm, BC = 18cm and AC = 16cm. Find the length of the other diagonal ?

(a) 30cm (b) 32cm
(c) 26cm (d) 28cm

Sol. (d)



$2(AB^2 + BC^2) = AC^2 + BD^2$

$\Rightarrow \quad BD^2 = [2(196+324)] - 256$

$BD^2 = 784 \Rightarrow BD = 28cm$

2. The perimeter of a ||gm is 22cm. If the longer side measures 6.5cm. What is the measure of the shorter side?

(a) 5.5cm (b) 4.5cm
(c) 6.0cm (d) 5.0cm

Sol. (b) Perimeter of ||gm = 22cm

$\Rightarrow$ 2(a + b) = 22cm $\Rightarrow$ a + b = 11

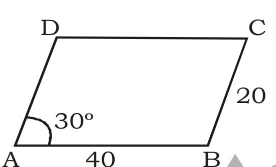
$\Rightarrow$ b = 11 - a = 11 - 6.5 = 4.5cm

$\therefore$ shorter side, b = 4.5cm

3. ABCD is a parallelogram, ∠DAB = 30°, BC = 20 cm and AB = 40 cm. Find the area of parallelogram:

(a) 150 cm² (b) 200 cm² (c) 400 cm² (d) 260 cm²

Sol. (c)



$\therefore$ AD || BC

$\Rightarrow$ ∠ABC = 180°–30° = 150°

$\therefore$ Area of parallelogram

$\Rightarrow$ AB.BC sin150°

$\Rightarrow$ 40×20×sin150°

$\Rightarrow$ 400 cm²

4. If an angle of a ||gm is two-third of its adjacent angle, then the largest angle of ||gm:

(a) 72° (b) 60°
(c) 108° (d) 120°

Sol. (c) Since, adjacent angles of a ||gm are supplementary.

$\therefore x + \frac{2}{3} \times x = 180° \Rightarrow \frac{5x}{3} = 180°$

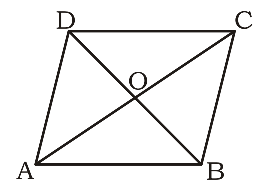
$\Rightarrow x = 108°$

$\therefore \frac{2}{3}x = \frac{2}{3} \times 108° = 72°$

$\therefore$ Angles are = 108°, 72°, 108°, 72°

$\therefore$ largest angle = 108°

5. In the given figure, ABCD is a ||gm in which diagonals AC and BD intersect at O. If ar(||gm ABCD) is 56cm², then the ar(△OAB) = ?



(a) 28cm² (b) 22cm²
(c) 42cm² (d) 14cm²

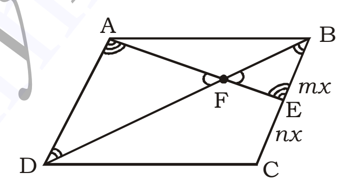
Sol. (d)

$$ar(\Delta OAB) = \frac{1}{4} ar(||gmABCD)$$

$$= \frac{1}{4} \times 56 = 14cm^2$$

6. ABCD is a parallelogram. E is a point on BC such that BE : EC = m : n. If AE and DB intersect at F, then what is the ratio of the area of △FEB to the area of △AFD?

Sol.



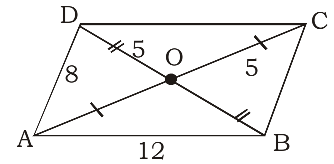
AFD~ EFB

$$\frac{Ar\ (\Delta EFB)}{Ar\ (\Delta AFD)} = \frac{EB^2}{AD^2}$$

$$= \frac{mx^2}{(mx+nx)^2} = \left[\frac{m}{m+n}\right]^2$$

7. The adjacent sides of a parallelogram are 12 cm. and 8 cm. and its one diagonal is 10 cm. then other diagonal is:

Sol.



Diagonals of parallelogram bisect each other

So In △ADB apply appollonious theorem.

$8^2 + 12^2 = 2\ (5^2 + AO^2)$

$AO^2 = 79 \Rightarrow AO = \sqrt{79}$

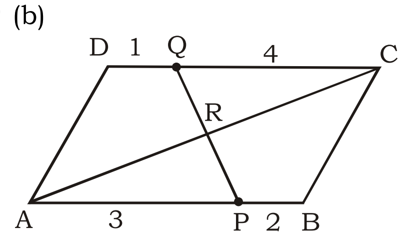
$\therefore$ AC = $2\sqrt{79}$

8. ABCD is a parallelogram AB is divided at P and CD at Q so that AP:PB = 3:2 and CQ:QD = 4:1 if PQ meets AC at R then AR = ?

(a) $\frac{2}{7}$AC (b) $\frac{3}{7}$AC

(c) $\frac{4}{7}$AC (d) $\frac{5}{7}$AC

Sol. (b)



AB || CD

In △ARP and △ QRC

$\Rightarrow$ ∠RAP = ∠RCQ (alternate interior angles)

$\Rightarrow$ ∠RPA = ∠RQC

△ARP ~ △RCQ (similar triangle)

$\Rightarrow \quad \frac{RC}{AR} = \frac{QC}{AP} = \frac{4}{3}$

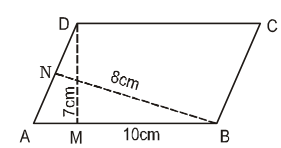
adding 1 both the sides

$\Rightarrow \quad \frac{RC}{AR} + 1 = \frac{4}{3} + 1$

$\Rightarrow \quad \frac{RC+AR}{AR} = \frac{7}{3}$

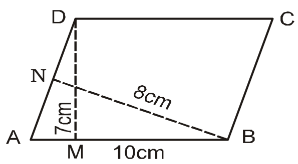
$\Rightarrow \quad AR = \frac{3}{7}\ AC$

9. In ||gm ABCD, AB = 10cm. The altitude corresponding to the sides AB and AD are 7cm and 8cm respectively. Find AD :



(a) 8.50cm (b) 8.25cm
(c) 8.75cm (d) 9.00cm

Sol. (c)



Area of ||gm = Base $\times$ Height

$\therefore$ ar(||gm ABCD) = AB $\times$ DM

= $(10 \times 7)cm^2$ **...(i)**

Also, ar(||gm ABCD) = AD $\times$ BN

= $(AD \times 8)cm^2$ **...(ii)**

From (i) and (ii), we have,

$10 \times 7 = AD \times 8$

$\Rightarrow AD = \frac{35}{4} = 8.75cm$

10. ABCD is a ||gm and $\angle DAB = 60°$. If the bisectors AP and BP of angles A and B respectively, meet at P on CD, then :

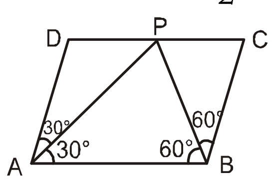
(a) CP = 2DP (b) $CP = \frac{1}{2}DP$

(c) $CP = \frac{1}{3}DP$ (d) CP = DP

Sol. (d) $\angle DAB = 60°$

$\Rightarrow \angle B = 120°$

$\therefore \angle ABP = PBC = \frac{120°}{2} = 60°$



$\angle DPA = \angle BAP = 30°$ [$\because$ AB || DC and AP intersects them]

Thus, In $\triangle ADP$,

$\angle DPA = \angle DAP = 30°$

$\Rightarrow$ AD = DP ______(i)

Similarly, $\angle BPC = \angle ABP = 60°$

$\therefore$ In $\triangle$ BPC, $\angle BPC = \angle PBC = 60°$

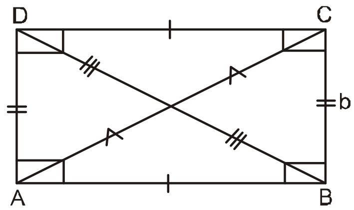
$\Rightarrow$ BC = CP = AD _____(ii) ($\because$ BC = AD)

$\therefore$ from (i) and (ii) CP = DP.

2. Rectangular

A parallelogram is called a rectangle if its all angles are 90°. Hence every rectangle is a parallelogram but every parallelogram is not a rectangle.

If diagonals of a parallelogram are equal (i.e. AC = BD) then it is a rectangle.



Properties :

(i) Diagonals are equal and bisect each other, but not necessarily bisects at right angles.

(ii) For the given perimeter of rectangles, a square has maximum area.

(iii) The figure formed by joining the mid-points of the adjacent sides of a rectangle is rhombus.

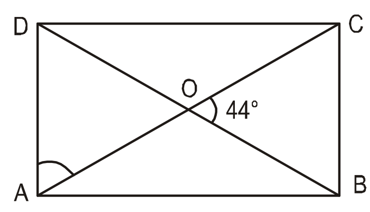
(iv) Area of rectangle ABCD = length $\times$ breadth = $l \times b$

(v) Diagonals of a rectangle = $\sqrt{l^2 + b^2}$

(vi) Bisectors of the angles of a rectangle form another rectangle.

EXAMPLES

1. The diagonals of rectangle ABCD meet at O. If $\angle BOC = 44°$, then $\angle OAD$ is equal to :



(a) 90° (b) 60° (c) 100° (d) 68°

Sol. (d) The diagonals of a rectangle bisect each other.

$\therefore$ OA = OD $\Rightarrow \angle ODA = \angle OAD$

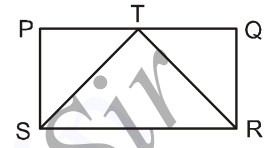
But, $\angle AOD = 44°$

(vertically opposite angle to $\angle BOC$)

$\therefore \angle OAD = \frac{1}{2}(180° - 44°)$

$= \frac{1}{2}(136°) = 68°$

2. In the given figure, what is the ratio of the area of the $\triangle$ STR to the area of the rectangle PQRS?



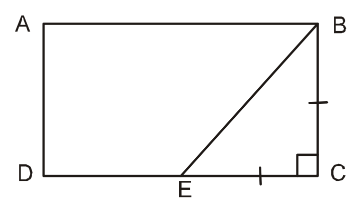
(a) 1 : 4 (b) 1 : 2 (c) 1 : 3 (d) 2 : 1

Sol. (b)

$\frac{\text{Area of } (\Delta STR)}{\text{Area of (quadrilateral PQRS)}} = \frac{\frac{1}{2}(SR \times PS)}{(SR \times PS)}$

$= \frac{1}{2} = 1 : 2$

3. The diagram below, ABCD is a rectangle. The area of isosceles right angle $\triangle$ BCE is $14cm^2$ and DE = 3EC. What is area of ABCD ?



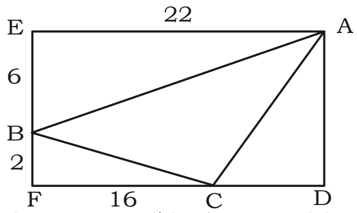
(a) 56 (b) 84 (c) 112 (d) $3\sqrt{28}$

Sol. (c) Area of ($\triangle$ BCE) = $\frac{1}{2} \times x \times x$ = 14 $\Rightarrow x^2 = 28$

$\therefore$ Area of ($\square$ ABCD) = (DE + EC) $\times x$ = 4EC $\times x$ = $4x.x$

$\Rightarrow 4x^2 = 4 \times 28 = 112$

4. In the given figure EADF is a rectangle and ABC is a triangle whose vertices lie on the sides of □EADF. AE = 22, BE = 6, CF = 16 and BF = 2. Find the length of the line joining the mid-points of the sides AB and BC.

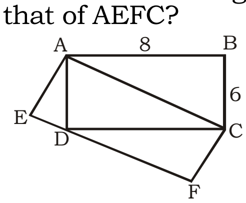


(a) 4 (b) 5 (c) 3.5

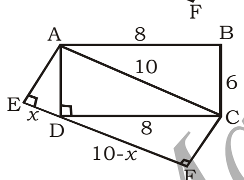
(d) $4\sqrt{2}$

Sol. (b) EF = AD = 8 (∵ EADF is a rectangle)
CD = 22 - 16 = 6
So, In right angled ΔADC,
AC = $\sqrt{8^2+6^2}=10$
∴ length of the line joining the mid-points of AB & BC = $\frac{1}{2}$(AC) = 5

5. ABCD is a rectangle of dimensions 8 units and 6 units. AEFC is a rectangle drawn in such way that diagonal AC of the first rectangle is one side and side opposite to it is touching the first rectangle at D as shown in the figure. What is the ratio of the area of rectangle ABCD to that of AEFC?



Sol.



AC = $\sqrt{(8)^2+(6)^2}$ = 10
AC = EF (AEFC is rectangle)
Let ED = x
then, DF = 10–x
In ΔAED, AE² = AD²–ED² ....(i)
= 36 – x^2
In ΔCFD, CF² = CD² –DF² ...(ii)
= 64 – (10 – x)²
Both (i)& (ii) is equal

$36 - x^2 = 64 - (100 + x^2 - 20x)$

$20x = 72 \Rightarrow x = \frac{18}{5}$

AE² = AD² – ED²

$= 6^2 - \left(\frac{18}{5}\right)^2 = \frac{24}{5}$

$\frac{\text{area of ▭ABCD}}{\text{area of ▭AEFC}} = \frac{8\times6}{10\times\frac{24}{5}} = \frac{1}{1}$

6. If the perimeter of a rectangle is P unit and its diagonal is d unit, then the difference between the length and width of the rectangle is-

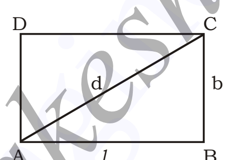
(a) $\sqrt{\frac{8d^2-p^2}{4}}$ unit

(b) $\sqrt{\frac{8d^2-p^2}{2}}$ unit

(c) $\sqrt{\frac{8d^2+p^2}{2}}$ unit

(d) $\sqrt{\frac{8d^2+p^2}{4}}$ unit

Sol. (a)



Perimeter (p) = 2 (l+b)

diagonal (d) = $\sqrt{l^2+b^2}$

$\Rightarrow$ d² = l^2 + b² **...(i)**

$\Rightarrow \frac{P^2}{4} = l^2 + b^2 + 2lb$ **...(ii)**

from eq. [(ii) – (i)] we get

$\frac{P^2}{4} - d^2 = 2lb$

Formula

$(l - b)^2 = l^2 + b^2 - 2lb$

Now, putting value of P and d

$\because (l - b)^2 = d^2 - \frac{P^2}{4} + d^2 = \frac{8d^2 - P^2}{4}$

$\Rightarrow l - b = \sqrt{\frac{8d^2 - P^2}{4}}$ unit

7. If l, b and p be the length, breadth and perimeter of a rectangle and b, l and p are in GP (in order) then $\frac{l}{b}$ is-

(a) 2:1 (b) ($\sqrt{3}$ –1):1

(c) ($\sqrt{3}$ +1):1 (d) 2: $\sqrt{3}$

Sol. (c) P = 2 (l+b)
∴ b, l and P are in G.P.
$\Rightarrow l^2 = bp$
$\Rightarrow l^2 = b \times 2(l + b)$

$\Rightarrow \frac{l^2}{b^2} = 2\left(\frac{l}{b}+1\right)$

$\Rightarrow \left(\frac{l}{b}\right)^2 - 2\left(\frac{l}{b}\right) - 2 = 0$

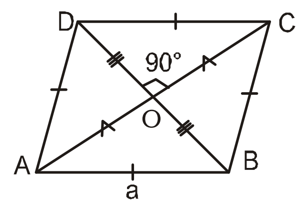
$\frac{l}{b} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2+2\sqrt{3}}{2}$

$= \left(\sqrt{3}+1\right):1$

(Negative factor can not consider other wise value of l/b will be negative which is not possible)

3. Rhombus

If all the sides of a parallelogram are equal it is rhombus. Diagonals of a rhombus bisect each other at right angle. i.e.



Properties

(i) AB = BC = CD = DA = a (say)

(ii) Diagonals bisect each other at right angle, but they are not necessarily equal.

(iii) A rhombus may or may not be a square but all squares are rhombus.

(iv) The figure formed by joining the mid-points of the adjacent sides of a rhombus is a rectangle.

(v) A ||gm is a rhombus if its diagonals are perpendicular to each other.

(vi) **(a)** Area of rhombus = $\frac{1}{2}\times$ product of diagonals
$= \frac{1}{2}\times d_1\times d_2$

(b) Area of rhombus = Product of adjacent sides $\times$ sine of the include angle.

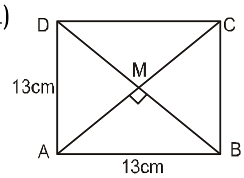
(vii) $AC = d_1$ and $BD = d_2$ (say)
then, $d_1^2 + d_2^2 = AB^2 + BC^2 + CD^2 + DA^2$
$\Rightarrow d_1^2 + d_2^2 = 4a^2$

(viii) A rhombus is a square if its diagonals are equal.
i.e. if $d_1 = d_2 \Rightarrow$ ABCD is a square.

EXAMPLES

1. The length of a side of a rhombus is 13cm and one of its diagonal is 24cm. The length of the other diagonal is:
(a) 14cm (b) 12cm
(c) 20cm (d) 10cm

Sol. (d)

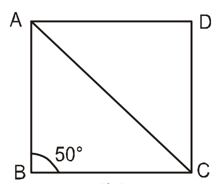


Let BD = 24cm, $\therefore$ BM = 12cm

$$\therefore AM = \sqrt{13^2 - 12^2} = 5cm$$

$\therefore$ AC= 2AM = 10cm

2. ABCD is a rhombus with $\angle ABC = 50°$, then $\angle ACD$ is :



(a) 50° (b) 90°
(c) 65° (d) 70°

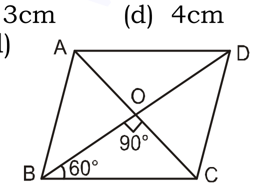
Sol. (c) AB = BC

$\therefore \angle BAC = \angle BCA = \frac{1}{2}(180° - 50°)$
$= 65°$

$\angle ACD = \angle ACB = 65°$

3. ABCD is a rhombus whose side AB = 4cm and $\angle ABC$ = 120°,then the length of diagonal BD is:
(a) 1cm (b) 2cm
(c) 3cm (d) 4cm

Sol. (d)



From $\triangle$ BOC

$$\cos 60° = \frac{BO}{4}$$

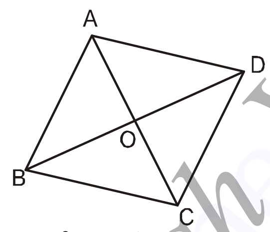
$$BO = \frac{1}{2} \times 4 = 2cm$$

$\therefore$ BD = 2 × 2 = 4 cm

4. If the diagonals of a rhombus are 8 and 6, then the square of its sides is:
(a) 25 (b) 55
(c) 64 (d) 36

Sol. (a) BO = 4 units, OC = 3units
$\angle BOC = 90°$

$$\therefore BC = \sqrt{4^2 + 3^2} = 5\,units$$



$\therefore$ BC^2 = 25 sq.units

5. If the perimeter of rhombus is 150 cm and length of one diagonal is 50 cm. Then find the length of second diagonal and area of rhombus.

Sol. Perimeter = 4a =150 $\Rightarrow$ 2a = 75 cm.

$4a^2 = d_1^2 + d_2^2 \Rightarrow (2a)^2 = (d_1)^2 + (d_2)^2$

$\Rightarrow (75)^2 = (50)^2 + d_2^2$

$\Rightarrow (75)^2 - (50)^2 = d_2^2$

$\Rightarrow d_2^2 = (75 + 50)(75 - 50)$
$= 125 \times 25 = 25 \times 25 \times 5$
$d_2 = 25\sqrt{5}$

$\Rightarrow$ Area $= \frac{1}{2} d_1 d_2 = \frac{1}{2} \times 50 \times 25\sqrt{5}$
$= 625\sqrt{5}$ cm²

6. Perimeter of a rhombus is 2p unit and sum of length of diagonals is m unit, then area of the rhombus is-

(a) $\frac{1}{4}m^2P$ sq. unit

(b) $\frac{1}{4}mp^2$sq. unit

(c) $\frac{1}{4}(m^2-p^2)$sq. unit

(d) $\frac{1}{4}(p^2-m^2)$sq. unit

Sol. (c) Sum of diagonal length = m . . . . (given)
Perimeter = 2p . . . .(given)
area of Rhombus = ?
$4a^2 = d_1^2 + d_2^2$
$\because 4a = 2P$ (perimeter)

$\Rightarrow a = \frac{P}{2}$

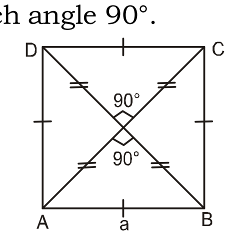
and $d_1 + d_2 = m$
Squaring both sides
$\Rightarrow d_1^2 + d_2^2 + 2d_1d_2 = m^2$
$\Rightarrow 4a^2 + 2d_1d_2 = m^2$
$\Rightarrow 2d_1d_2 = m^2 - 4a^2$

$\Rightarrow 2d_1d_2 = m^2 - 4\left(\frac{P}{2}\right)^2$ (Put a = P/2)

$\Rightarrow \frac{1}{2}d_1d_2 = \frac{m^2 - P^2}{4}$

4. Square

A square is a rectangle with adjacent sides of equal length or a rhombus with each angle 90°.



Properties

(i) AB = BC = CD = AD = a (say)
$\& \angle A = \angle B = \angle C = \angle D = 90°$

(ii) Diagonals are equal and bisect each other at right angle.

(iii) The figure formed by joining the mid-points of the adjacent sides of a square is a square.

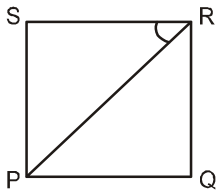
(iv) Every square is a rhombus but every rhombus is not a square.

(v) Perimeter of square = 4 × side of square = **4 × a**

(vi) Area = (side)² = $a^2 = \frac{d^2}{2}$, and diagonal(d) = $a\sqrt{2}$.

EXAMPLES

1. PQRS is a square. The $\angle SRP$ is equal to :



Sol. PQRS is a square, SP = SR and $\angle S = 90°$

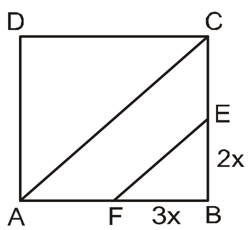
and $\angle SRP = \angle SPR = \frac{1}{2}(90°) = 45°$

Hence, $\angle SRP = 45°$

2. ABCD is a square, F is the mid-point of AB and E is a point on BC such that BE is one-third of BC. If area of $\triangle FBE = 147m^2$, then the length of AC is :

(a) $21\sqrt{2}$ m (b) 63m (c) $63\sqrt{2}$ m (d) $42\sqrt{2}$ m

Sol. (d)



Let the side of square be $6x$ then,

Area of $\triangle FBE$ = $\frac{1}{2} \times 3x \times 2x = 147$

$\Rightarrow$ $x^2 = 49 \Rightarrow x = 7$

$\therefore$ side of square = 42m

$\Rightarrow$ $AC^2 = (42)^2 + (42)^2 = 2(42)^2$

$\Rightarrow$ $AC = 42\sqrt{2}$ m

3. ABCD is a square. M is the mid-point of AB and N is the mid-point of BC. DM and AN are joined and they meet at O. Then which of the following is correct?

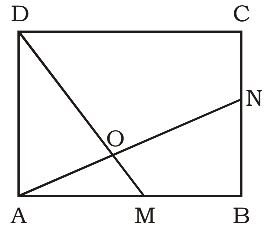
(a) OA : OM = 1:2

(b) AN = MD

(c) $\angle ADM = \angle ANB$

(d) $\angle AMD = \angle BAN$

Sol. (b)



ABCD is a square and M, N are mid Points of AB and BC respectively

If AB = $2x$

then, BN = x

$\therefore AN = \sqrt{AB^2 + BN^2}$

$= \sqrt{4x^2 + x^2} = \sqrt{5x^2} = \sqrt{5}\,x$

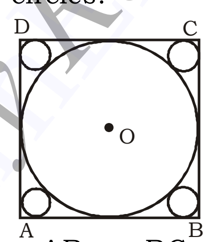
Similarly: $MD = \sqrt{AD^2 + AM^2}$

$= \sqrt{4x^2 + x^2} = \sqrt{5x^2} = \sqrt{5}\,x$

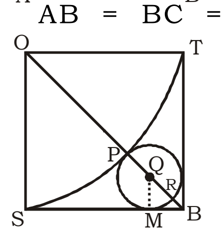
Hence, AN = MD

4. ABCD is square, in which a circle is inscribed touching all the sides of square. In the four corners of square 4 smaller circles of equal radii are drawn, containing maximum possible area.

What is the ratio of the area of larger circle to that of sum of the areas of four smaller circles?



Sol. AB = BC = CD = DA



Let OS = SB = R (radius of the larger circle) then $OB = R\sqrt{2}$

Similarly PQ = MQ = QR = r (radius of the smaller circle)

then $BQ = r\sqrt{2}$

$BP = r + r\sqrt{2}$

and $BP = OB - OP = R\sqrt{2} - R$

$\Rightarrow$ $R\sqrt{2} - R = r + r\sqrt{2}$

$\Rightarrow$ $R\left(\sqrt{2}-1\right) = r\left(\sqrt{2}+1\right)$

$\Rightarrow$ $r = R\left(\sqrt{2}-1\right)^2$

$\Rightarrow$ $r = R\left(3-2\sqrt{2}\right)$

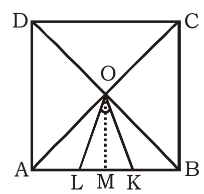
$\Rightarrow$ $\therefore$

$\frac{\text{Area of larger circle}}{\text{Area of 4 smaller circles}} = \frac{\pi R^2}{4\pi r^2}$

$= \frac{R^2}{4R^2\left(3-2\sqrt{2}\right)^2} = \frac{1}{4\left(17-12\sqrt{2}\right)}$

5. ABCD is a square. The diagonals AC and BD meet at O. Let K, L be the points on AB such that AO = AK and BO = BL. If $\theta = \angle LOK$, then what is the value of $\tan\theta$?

Sol.



Let sides of square be a

$OB = BL = \frac{a}{\sqrt{2}}$ (given)

$LM = BL - BM = \frac{a}{\sqrt{2}} - \frac{a}{2}$

$OM = \frac{a}{2}$

In $\triangle OML$

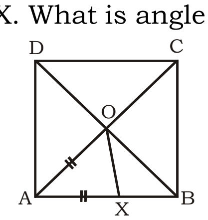
$\tan\frac{\theta}{2} = \frac{ML}{OM}$

$\tan\frac{\theta}{2} = \frac{\frac{a}{\sqrt{2}} - \frac{a}{2}}{\frac{a}{2}} = \frac{\frac{\sqrt{2}-1}{2}}{\frac{1}{2}} = \sqrt{2}-1$

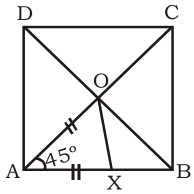
$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{2\left(\sqrt{2}-1\right)}{1-\left(2+1-2\sqrt{2}\right)}$

$= \frac{\left(2\sqrt{2}-2\right)}{2\sqrt{2}-2} = 1$

6. In the figure given below, ABCD is a square in which AO = AX. What is angle XOB?



Sol.



$\angle XAO = 45°$ (AC is diagonal angle bisect of square ABCD)
AO = AX (given)

$\angle AOX = \frac{180-45}{2} = \frac{135}{2}$

$\angle AOB = 90°$ (Diagonals bisect at right angle)

$\angle XOB = \angle AOB - \angle AOX$

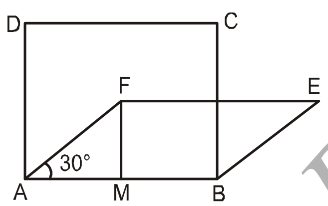
$\angle XOB = 90 - \frac{135}{2}$

$= \frac{180-135}{2} = \frac{45}{2} = 22.5°$

7. A square and a rhombus have the same base and the rhombus is inclined at 30°. What is the ratio of the area of the square to the area of the rhombus :

(a) $\sqrt{2}:1$ (b) 2 : 1 (c) 1 : 1 (d) $2:\sqrt{3}$

Sol. (b)



ABCD is a square and ABEF is a rhombus

$\sin 30° = \frac{FM}{AF} = \frac{1}{2}$

$\Rightarrow FM = \frac{AF}{2}, AF = AB = a$

Area of square = a^2 (AB = AD = a)
Area of rhombus = AB × FM

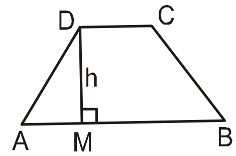
$= a \times \left(\frac{a}{2}\right) = \frac{a^2}{2}$

$\therefore \frac{\text{Area of square}}{\text{Area of rhombus}} = \left(\frac{a^2}{a^2/2}\right)$

$= \frac{2}{1}$

5. Trapezium

If two sides of a quadrilateral are parallel and other two sides are non parallel then it is called a trapezium.

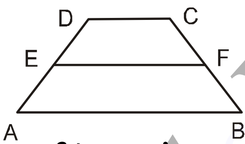


Properties

(i) $\angle A + \angle D = \angle B + \angle C = 180°$

(ii) If E and F are the mid-points of two non-parallel sides AD and BC respectively, then -

Median(EF) $= \frac{1}{2}(AB + DC)$



(iii) Area of trapezium

$= \frac{1}{2}$ (sum of parallel sides) × height

$= \frac{1}{2} \times (AB + CD) \times DM$

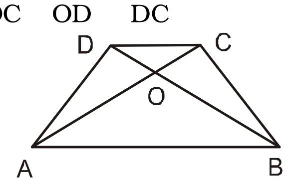
$= \frac{1}{2}(AB + CD) \times h$

(iv) Sum of square of diagonals = (sum of squares of non-parallel side) + 2(product of || sides)

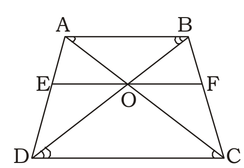
$AC^2 + BD^2 = BC^2 + AD^2 + 2AB.CD$

(v) By joining the mid-points of adjacent sides of a trapezium four similar triangles are obtained.

(vi) $\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$



(vii) Any line in a trapezium, parallel to parallel sides divides the non-parallel sides in equal ratio.



$\frac{AE}{ED} = \frac{BF}{FC}$

Proof: In $\triangle DBA$, OE||AB then

By Thales theorem $\frac{DE}{EA} = \frac{DO}{OB}$ ...(i)

& In $\triangle BDC$, OF||DC then

$\frac{DO}{OB} = \frac{CF}{FB}$ .... (ii)

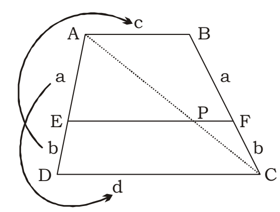
By (i) & (ii)

$\frac{DE}{EA} = \frac{CF}{FB}$

(viii) When Only one pair of sides is parallel

Case I

The length of parallel sides EF = $\frac{ad + bc}{a + b}$



Proof :- In $\triangle ABC$,

$\frac{PF}{c} = \frac{b}{a+b}$

($\triangle CFP \sim \triangle CBA$)

$\therefore PF = \frac{bc}{a+b}$

In $\triangle ADC$,

$\frac{EP}{d} = \frac{a}{a+b}$

($\triangle AEP \sim \triangle ADC$)

$\therefore EP = \frac{ad}{a+b}$

$\therefore EF = EP + PF$

$= \frac{ad}{a+b} + \frac{bc}{a+b}$

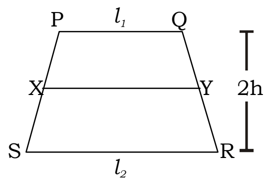
$= \frac{ad+bc}{a+b}$

Case II

If E and F are mid-points of AD and BC respectively then

$a = b \Rightarrow EF = \frac{d+c}{2}$

(ix) The line joining the mid-point of non-parallel sides divides the trapezium into two trapezium then ratio of their area.



$$\frac{\text{Area of Trapezium PQYX}}{\text{Area of Trapezium XYRS}} = \frac{3l_1+l_2}{3l_2+l_1}$$

Proof :-

$$\frac{\text{Area of PQYX}}{\text{Area of XYRS}} = \frac{\frac{1}{2}\left(l_1+\frac{l_1+l_2}{2}\right)\times h}{\frac{1}{2}\left(l_2+\frac{l_1+l_2}{2}\right)\times h} = \frac{3l_1+l_2}{3l_2+l_1}$$

The figure formed by joinnig the mid-point

- Quadrilateral is Parallelogram
- Rectangle is Rhombus
- Rhombus is Rectangle
- Square is Square
- Area becomes half of basic figure

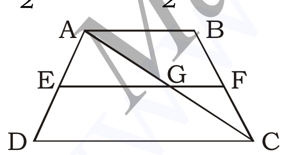
EXAMPLES

1. The parallel sides of a trapezium are p and q respectively. The line joining the mid points of its non-parallel sides will be

(a) $\sqrt{pq}$ (b) $\frac{2pq}{p+q}$

(c) $\frac{(p+q)}{2}$ (d) $\frac{1}{2}(p-q)$

Sol. (c)



In a trapezium ABCD

$\begin{bmatrix} DC=p \\ AB=q \end{bmatrix}$

AB || CD

E and F are mid points of its non parallel sides a line is drawn from A to C, which intersects EF at G

In $\triangle$ ADC

EF || DC

$EG=\frac{1}{2}DC$ **...(i)**

Similarly, In $\triangle$ ABC,

AB || GF

$GF=\frac{1}{2}AB$ **...(ii)**

Adding eq. (i) and (ii) we get,

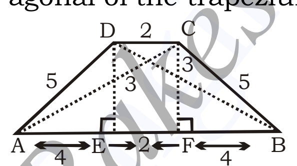
$EG+GF=\frac{1}{2}DC+\frac{1}{2}AB$

$EF = \frac{1}{2}(DC + AB)$

$EF = \frac{1}{2}(p + q)$

2. In a trapezium, the two non-parallel sides are equal in length, each being of 5cm. The parallel sides are at a distance of 3cm. If the smaller side of the parallel sides is of length 2cm., then the sum of the diagonal of the trapezium is:

Sol.



In $\triangle$ ACF

$AC=\sqrt{6^2+3^2}=\sqrt{45}$

$=\sqrt{9\times5}=3\sqrt{5}$

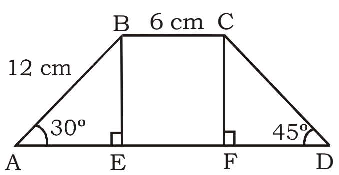
In $\triangle$ BDE ,

$DB = \sqrt{6^2+3^2}=3\sqrt{5}$

sum of diagonals

$=3\sqrt{5}+3\sqrt{5}=6\sqrt{5}$cm

3. In a trapezium ABCD, $\angle$BAE = 30°, $\angle$CDF = 45°, BC = 6 cm and AB = 12 cm. Find the area of ABCD.



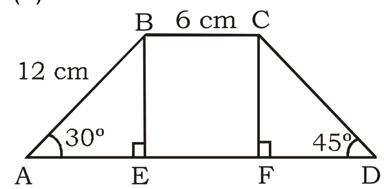
(a) 18 (3 + $\sqrt{3}$) cm²

(b) 36 $\sqrt{3}$ cm²

(c) 12 (3+2 $\sqrt{3}$) cm²

(d) None of these

Sol. (a)



Trapezium ABCD,

In $\triangle$ ABE,

$\frac{BE}{AB}=\sin30°$

$\therefore$ BE = 6 cm

BE = CF

$\cos30° = \frac{AE}{AB}$

$\Rightarrow \frac{\sqrt{3}}{2}=\frac{AE}{12}$

$\Rightarrow$ AE = 6 $\sqrt{3}$ cm.

In $\triangle$ CFD,

$\tan 45° = \frac{CF}{FD}$

$\Rightarrow 1=\frac{6}{FD}$

$\Rightarrow$ FD = 6 cm

$\because$ BC = EF = 6 cm.

Now the area of trapezium = $\frac{1}{2}\times$(Sum of parallel sides) $\times$height

$=\frac{1}{2}\times$ (BC + AD) × BE

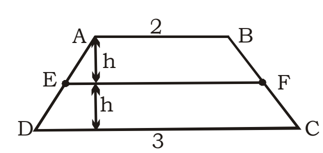
($\therefore$ AD =AE+EF+FD=6 $\sqrt{3}$ +6+6)

$=\frac{1}{2}$ (18 + 6 $\sqrt{3}$)×6

Area = 18 (3 + $\sqrt{3}$) cm²

4. ABCD is a trapezium with parallel sides AB = 2 cm and DC = 3 cm E and F are the mid-points of the non-parallel sides. The ratio of area of ABFE to the area of EFCD is:

Sol.



By property of trapezium

$EF = \frac{1}{2}(AB + CD) = \frac{1}{2}(2+3)=\frac{5}{2}$

$\dfrac{\text{Area of trapezium ABFE}}{\text{Area of trapezium EFCD}} =$

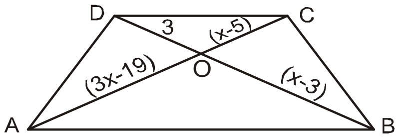
$$\frac{\frac{1}{2}\left(2+\frac{5}{2}\right)\times h}{\frac{1}{2}\left(3+\frac{5}{2}\right)\times h} = \frac{9}{11}$$

Alternate

$\dfrac{\text{Area of trapezium ABFE}}{\text{Area of trapezium EFCD}} =$

$$\frac{3a+b}{a+3b} = \frac{9}{11}$$

5. In the given figure, AB || CD, find the value of x:



(a) $x = 8$ (b) $x = 9$
(c) $x = 8$ or 9 (d) $x = 10$

Sol. (c) The diagonals of a trapezium divide each other proportionally,

$$\therefore \frac{AO}{CO} = \frac{BO}{OD} \Rightarrow \frac{3x-19}{x-5} = \frac{x-3}{3}$$

$\Rightarrow\ 9x - 57 = x^2 - 8x + 15$

$\Rightarrow\ x^2 - 17x + 72 = 0$

$\Rightarrow\ x = 8$ or $x = 9$

6. The parallel sides of a trapezium are in a ratio 2 : 3 and their shortest distance is 12cm. If the area of the trapezium is 480sq.cm.,the longer of the parallel sides is of length:
(a) 56cm (b) 36cm
(c) 42cm (d) 48cm

Sol. (d) Sides of the trapezium = $2x$ and $3x$ cm

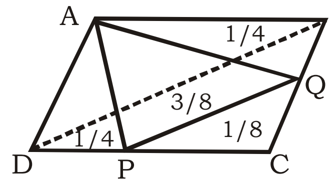
$$\therefore \frac{1}{2}(2x+3x)\times 12 = 480$$

$$\Rightarrow\ 5x = \frac{480}{6} = 80$$

$$\Rightarrow\ x = \frac{80}{5} = 16$$

Longer side = 16 × 3 = 48 cm

- If P and Q are mid-point of DC & BC respectively then Area of $\triangle$ APQ = $\mathbf{\frac{3}{8}}$ Area of ▫ ABCD



Proof

Area of $\triangle$ ABQ = $\frac{1}{4}$ Area of ▫ABCD

[$\triangle$ ABQ & ▫ABCD lie in between same || lines & QB = $\frac{BC}{2}$]

Similarly Area of $\triangle$ ADP = $\frac{1}{4}$ Area of ▫ABCD

Area of $\triangle$ PQC = $\frac{1}{4}$ Area of $\triangle$ BDC =

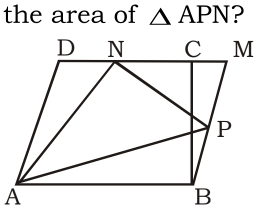
$\frac{1}{8}$ Area of ▫ABCD [mid-point theorem]

$\therefore$ Area of $\triangle$ APQ = ar (▫ABCD – $\triangle$ ADP – $\triangle$ PQC – $\triangle$ ABQ) =

▫ABCD – $\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{8}\right)$ ▫ABCD

$= \frac{3}{8}$ ▫ABCD

7. Parallelograms ABCD and ABMN are on the base AB, where AB$\parallel$DM. If the area of $\parallel$gm ABMN is 80 sq. unit, what will be the area of $\triangle$ APN?



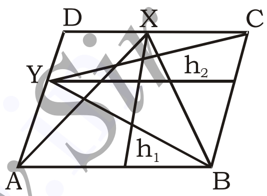
(a) 20 sq. unit (b) 30 sq. unit
(c) 40 sq. unit (d) 160 sq. unit

Sol. (c) Parallelogram ABCD and ABMN are on the base AB where,
So, area of ABCD = area of ABMN = 80
We know that

Area of $\triangle$ APN = $\frac{1}{2}$ ×area of parallelogram ABMN

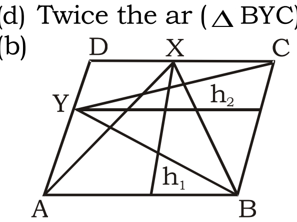
= $\frac{1}{2}$ × 80 = 40 sq. unit

8. Two points X and Y are one the sides DC and AD of the parallelogram ABCD. The ar ($\triangle$ ABX) is-



(a) $\frac{1}{2}$ of ar ($\triangle$ BYC)
(b) equal to ar ($\triangle$ BYC)
(c) $\frac{1}{3}$ of ar ($\triangle$ BYC)
(d) Twice the ar ($\triangle$ BYC)

Sol. (b)



Area of $\triangle$ AXB = $\frac{1}{2}$ × AB × h_1

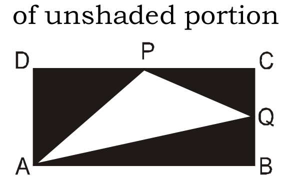
= $\frac{1}{2}$ ×(area of parallelogram ABCD)

Area of $\triangle$ BYC = $\frac{1}{2}$ × BC × h_2

= $\frac{1}{2}$ ×(area of parallelogram ABCD)

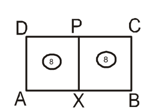
$\Rightarrow$ Area of $\triangle$ AXB = Area of $\triangle$ BYC

9. In the given figure, ABCD is a rectangle. P and Q are the mid-points of sides CD and BC respectively. Then the ratio of area of shaded portion : area of unshaded portion is :



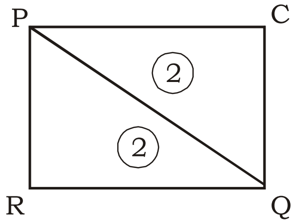
(a) 5 : 4 (b) 3 : 5
(c) 5 : 3 (d) 5 : 8

Sol. (c) Let total area of rectangle ABCD = 16 units

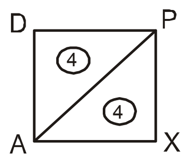


$\therefore$ area of ▭ AXPD =

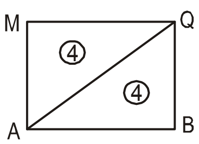
$\frac{16}{2}$ = 8 units



$\therefore$ area of ΔPCQ = 2 units



$\therefore$ area of ΔADP = 4 units



$\therefore$ area of ΔABQ = 4units

$\therefore$ total area of shaded portion = area of ΔADP + area of ΔABQ + area of ΔPCQ

= 4 + 2 + 4 = 10 units

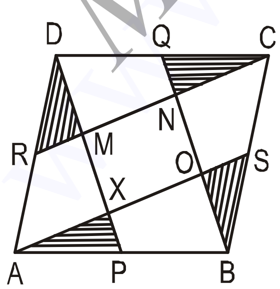
$\therefore$ area of unshaded portion = area of ΔAPQ

= 16 – Area of shaded portion

= 16 – 10 = 6 units

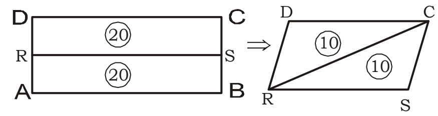
$\therefore$ Required ratio = 10 : 6 = 5 : 3

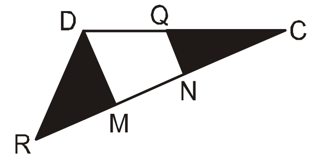
10. In the ||gm ABCD, P,Q,R and S are mid-points of sides AB, CD, DA and BC respectively. AS, BQ, CR and DP are joined. Find the ratio of the area of the shaded region to the area of the ||gm ABCD.



(a) 1/5 (b) 1/4
(c) 4/15 (d) 1/6

Sol. (a) Let total area of || ABCD = 40 units





In ΔDMC, Q is the mid point of DC and QN || DM

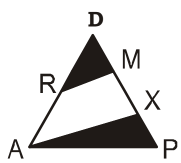
[$\because$ DMNQ is also a ||gm)

$\Rightarrow$ N is the mid-point of MC

$\therefore$ ar (ΔQCN) : ar(ΔDMC) = 1 : 4

Let ar (ΔQCN) = 1unit

$\Rightarrow$ ar (quadrilateral DMNQ) = 4 - 1 = 3 units



Similarly, In ΔDAX

ar (ΔDRM) : ar (ΔDAX) = 1 : 4

ar (Δ DRM) = 1 unit

$\Rightarrow$ ar (Δ DAX) = 4 units

$\Rightarrow$ ar ($\square$ RMXA) = 4 – 1 = 3 units

$\therefore$ ar(Δ DRM) + ar (Δ QNC)

= 1 +1 = 2units

and ar(Δ DRC) = 4 + 1 = 5units

but from (i) ar (Δ DRC) = 10 unit (2 times)

$\therefore$ ar (Δ DRM) + ar (Δ QNC)

= 2 $\times$ (2 times)

= 4 units

Similarly, ar (Δ APX) + ar (Δ BOS)

= 2 $\times$ 2

= 4units

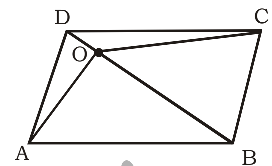
$\therefore$ total shaded area = 4 + 4

= 8 units

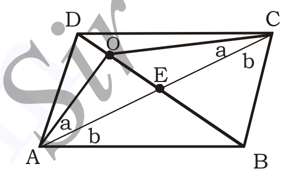
& area of ||gm ABCD = 40units

$\therefore$ Required ratio = $\frac{8}{40} = \frac{1}{5}$

11. In the below figure, ABCD is a parallelogram. If area of ΔOAB = 19 cm² then, find the area of ΔOBC.



Sol.



Line joining A and C cuts BD at E. AC diagonals of parallelogram bisects each other,

OE and BE will be median of ΔOAC and ΔABC respectively,

Let Area of ΔOAE = Area of ΔOEC = a

Then, Area of ΔAEB = Area of ΔEBC = b

Area of ΔOAB = Area of ΔOAE + Area of ΔAEB = (a + b)

Similarly,

Area of ΔOBC = Area of ΔOEC + Area of Δ EBC = (a +b)

Hence, if O is any point on diagonal BD in parallelogram ABCD then,,

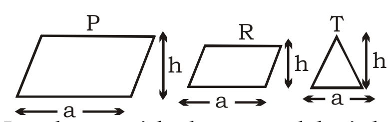
Area of ΔOAB = Area of ΔOBC

Area of ΔOBC = 19 cm²

12. If P, R, T are the area of a Parallelogram, a rhombus and a triangle standing on the same base and between the same parallels, Which of the following is ture?

(a) R < P < T (b) P > R > T
(c) R = P = T (d) R = P = 2T

Sol. (d)



Let base side be a and height be h

area of parallelogram (P) = ah . . . . . . (i)

area of rhombus (R) = ah . . . . (ii)

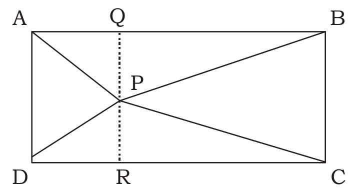
area of triangle (T) = $\frac{1}{2}$ ah

$\Rightarrow$ R = P = 2T

If ABCD is a square or rectangle

If 'P' is point inside a rectangle then

$AP^2 + PC^2 = BP^2 + PD^2$



(Subtract $AQ^2 - BQ^2 = AP^2 - BP^2$ .....(i))

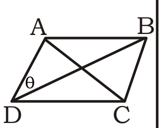
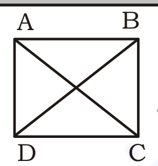
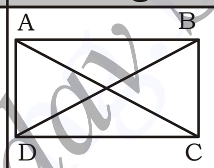
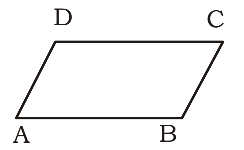
(Subtract $DR^2 - CR^2 = PD^2 - PC^2$ .....(ii))

$AQ = DR$ & $BQ = CR$

(1) = (2)

$AP^2 - BP^2 = PD^2 - PC^2$

$AP^2 + PC^2 = PD^2 + BP^2$

S.No.	Property	Rhombus	Square	Rectangle	Parallelogram
					
1.	Opposite sides are equal	✓	✓	✓	✓
2.	Opposite sides are parallel	✓	✓	✓	✓
3.	Opposite angles are equal	✓	✓	✓	✓
4.	All sides are equal	✓	✓	×	×
5.	All angles are equal and right angle	×	✓	✓	✓
6.	Diagonals bisect each other	✓	✓	✓	×
7.	Diagonals bisect each other at right angles	✓	✓	×	✓
8.	Diagonals bisect vertex angles	✓	✓	×	×
9.	Diagonals are equal	×	✓	✓	×
10.	Diagonals form four triangles of equal area	✓	✓	✓	✓
11.	Diagonals form four congruent triangles	✓	✓	×	×
12.	Area	$\frac{1}{2}d_1 \times d_2$	Side^2	$l \times b$	base × height
13.	The figure formed by joining the mid-points	Rectangle	Square	Rhombus	Parallelogram

EXERCISE

1. The side AB of a parallelogram ABCD is produced to E in such way that BE = AB, DE intersects BC at Q. The point Q divides BC in the ratio
(a) 1 : 2 (b) 1 : 1
(c) 2 : 3 (d) 2 : 1

2. ABCD is a rhombus. A straight line through C cuts AD produced at P and AB produced at Q. If DP = $\frac{1}{2}$ AB, then the ratio of the length of BQ and AB is
(a) 2:1 (b) 1:2
(c) 1:1 (d) 3:1

3. In a quadrilateral ABCD, with unequal sides if the diagonals AC and BD intersect at right angles then
(a) $AB^2 + BC^2 = CD^2 + DA^2$
(b) $AB^2 + CD^2 = BC^2 + DA^2$
(c) $AB^2 + AD^2 = BC^2 + CD^2$
(d) $AB^2 + BC^2 = 2(CD^2 + DA^2)$

4. The ratio of the angles $\angle A$ and $\angle B$ of a non-square rhombus ABCD is 4 : 5, then the value of $\angle C$ is :
(a) 50° (b) 45° (c) 80° (d) 95°

5. ABCD is a rhombus whose side AB = 4 cm and $\angle ABC = 120°$, then the length of diagonal BD is equal to :
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm

6. The tangents at two points A and B on the circle with centre O intersects at P. If in quadrilateral PAOB, $\angle AOB : \angle APB$ = 5 : 1, then measure of $\angle APB$ is :
(a) 30° (b) 60° (c) 45° (d) 15°

7. ABCD is a trapezium whose side AD is parallel to BC, Diagonals AC and BD intersect at O. If AO = 3, CO = x –3, BO = 3x –19 and DO = x – 5, the value(s) of x will be :
(a) 7, 6 (b) 12, 6
(c) 7, 10 (d) 8, 9

8. Q is a point in the interior of a rectangle ABCD, if QA = 3 cm, QB = 4 cm and QC = 5 cm then the length of QD (in cm) is
(a) $3\sqrt{2}$ (b) $5\sqrt{2}$
(c) $\sqrt{34}$ (d) $\sqrt{41}$

9. ABCD is a rectangle where the ratio of the length of AB and BC is 3 : 2 . If P is the mid- point of AB, then the value of sin $\angle CPB$ is
(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$

10. Inside a square ABCD , BEC is an equilateral triangle. If CE and BD intersect at O, then $\angle BOC$ is
(a) 60° (b) 75° (c) 90° (d) 120°

11. The diagonals AC and BD of a cyclic quadrilateral ABCD intersect each other at the point P. Then, it is always true that
(a) BP . AB = CD . CP
(b) AP. CP = BP . DP
(c) AP. BP = CP . DP
(c) AP. CD = AB . CP

12. If the opposite sides of a quadrilateral and also its diagonals are equal, then each of the angles of the quadrilateral is
(a) 90° (b) 120°
(c) 100° (d) 60°

13. In a rhombus ABCD, $\angle A$ = 60° and AB = 12 cm. Then the diagonal BD is:
(a) $2\sqrt{3}$ cm (b) 6 cm
(c) 12 cm (d) 10 cm

14. If PQRS is a rhombus and $\angle SPQ$ = 50°, then $\angle RSQ$ is:
(a) 75° (b) 45°
(c) 55° (d) 65°

15. The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. The area of the field is?
(a) 252 m² (b) 1152 m²
(c) 96 m² (d) 156 m²

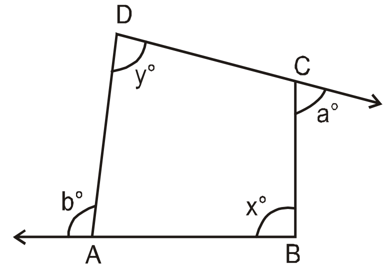
16. In trapezium ABCD, AB || CD and AB = 2 CD. Its diagonals intersect at O. If the area of Δ AOB = 84 cm², then the area of Δ COD is equal to
(a) 21 cm² (b) 72 cm²
(c) 42 cm² (d) 26 cm²

17. Quadrilateral ABCD is circumscribed about a circle. If the lengths of AB, BC, CD are 7 cm, 8.5 cm and 9.2 cm respectively, then the length (in cm) of DA is
(a) 16.2 (b) 7.7
(c) 10.2 (d) 7.2

18. If the ratio of the angles of a quadrilateral is 2 : 7 :2 : 7, then it is a
(a) trapezium (b) square
(c) parallelogram
(d) rhombus

19. The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4, the largest angle is :
(a) 120° (b) 134°
(c) 144° (d) 150°

20. The sides BA and DC of a quadrilateral ABCD are produced as shown in figure. Then the true statement is :



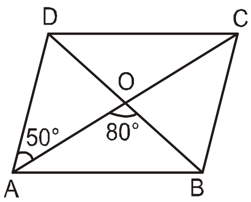
(a) $x° + y° = a° + b°$
(b) $x° + a° = y° + b°$
(c) $2x° + y° = a° + b°$
(d) $x + \frac{1}{2}y° = \frac{a° + b°}{2}$

21. If ABCD is a rhombus, then :
(a) $AC^2 + BD^2 = 4AB^2$
(b) $AC^2 + BD^2 = AB^2$
(d) $AC^2 + BD^2 = 2AB^2$
(d) $2(AC^2 + BD^2) = 3AB^2$

22. Two parallelograms stand on equal bases and between the same parallel lines. The ratio of their areas is :
(a) 1 : 1 (b) $\sqrt{2}:1$
(c) 1 : 3 (d) 1 : 2

23. The diagonals AC and BD of a ||gm ABCD intersect each other at the point O such that $\angle$ DAC = 50° and $\angle$ AOB =

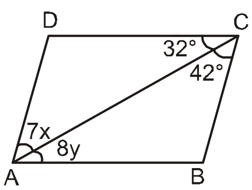
80°. Then $\angle$DBC = ?



(a) 50° (b) 40° (c) 45° (d) 30°

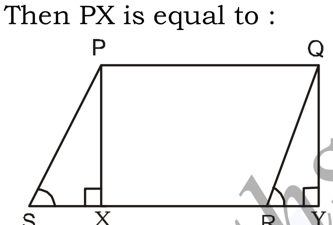
24. If ABCD is a ||gm with two adjacent angles A and B are equal to each other, then the ||gm is a :
(a) square (b) rhombus
(c) rectangle
(d) both (a) and (c)

25. In the adjoining figure, ABCD is a parellelogram, then the value of x and y are :



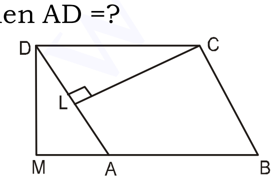
(a) 6, 4 (b) 5, 4
(c) 4, 5
(d) None of these

26. PQRS is a ||gm. PX and QY are respectively, the perpendicular from P and Q to SR and SR produced.
Then PX is equal to :



(a) QY (b) 2QY
(c) $\frac{1}{2}QY$ (d) XR

27. ABCD is a ||gm $CL \perp AD$ and $DM \perp BA$. If CD = 16 units, DM = 12 units and CL=15units, then AD =?



(a) 12.8 units (b) 13.6 units
(c) 11.1 units (d) 12.4 units

28. If a square and a rhombus stand on the same base and between two parallel lines then the ratio of the areas of the square and the rhombus is:
(a) 2 : 1 (b) 1 : 4
(c) 1 : 4 (d) 1 : 1

29. If area of a ||gm with sides a and b is A and that of a rectangle with sides a and b is B, then :
(a) A > B (b) A < B
(c) A = B
(d) none of these.

30. ABCD is a quadrilateral in which diagonal BD = 64cm, AL $\perp$ BD, such that AL = 13.2cm and CM = 16.8cm.The area of the quadrilateral ABCD in square centimetres is:
(a) 422.4 (b) 690.0
(c) 537.6 (d) 960.0

31. ABCD is cyclic trapezium whose sides AD and BC are parallel to each other. If $\angle$ABC = 72°, then the measure of the $\angle$BCD is :
(a) 162° (b) 18°
(c) 108° (d) 72°

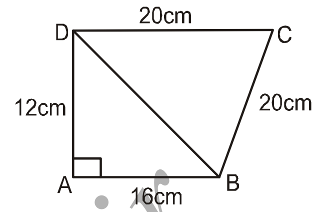
32. If an exterior angle of a cyclic quadrilateral be 50°, then the interior opposite angle is :
(a) 130° (b) 40° (c) 50° (d) 90°

33. In a quadrilateral ABCD, AO and BO are the bisectors of $\angle$A and $\angle$B respectively, then $\angle$AOB is equal to:
(a) $\angle$C + $\angle$D
(b) 2$\angle$C + 2$\angle$D
(c) $\frac{1}{2}(\angle C + \angle D)$
(d) $\frac{1}{2}(\angle C - \angle D)$

34. In a parallelogram ABCD, AO and BO are the bisectors of $\angle$A and $\angle$B respectively, then $\angle$AOB is equal to:
(a) 60° (b) 120°
(c) 100° (d) 90°

35. The measures of the angles of a quadrilateral taken in order are proportionates to :
(a) parallelogram
(b) trapezium
(c) rectangle
(d) rhombus

36. Find the area of □ABCD :

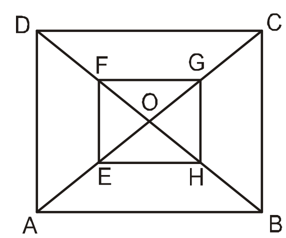


(a) $4(24+25\sqrt{3})cm^2$
(b) $4(25+24\sqrt{3})cm^2$
(c) $2(24+25\sqrt{3})cm^2$
(d) None of these

37. ABCD is a trapezium in which AB || DC and AB = 8cm, BC = 10cm, CD = 12cm, AD = 16cm, then AC² + BD² is equal to :
(a) 458cm² (b) 448cm²
(b) 546cm² (d) 548cm²

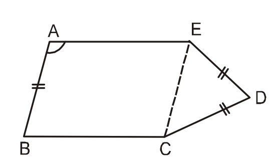
38. If O is a point within a rectangle ABCD then :
(a) OA² + OC² = OB² + OD²
(b) OA² + OB² = OC² + OD²
(c) OA + OC = OB + OD
(d) OA × OC = OB × OD

39. In the given figure, ABCD is a ||gm and E, F, G, H are the mid-points of AO, DO, CO and BO respectively, then
$\frac{EF+FG+GH+HE}{AD+DC+CB+BA} = ?$



(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

40. In the given figure AE = BC and AE || BC and the three sides AB, CD and ED are equal in length. If $\angle$A = 102°, find measure of $\angle$BCD :



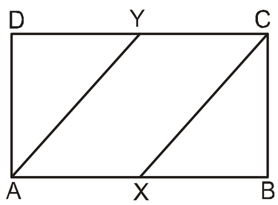
(a) 138° (b) 162°
(c) 88°
(d) None of these

41. If ABCD is a rectangle. P, Q are the mid-points of BC and AD respectively and R is any point on PQ, then ΔARB equals :

(a) $\frac{1}{6}$(□ABCD) (b) $\frac{1}{3}$(□ABCD)

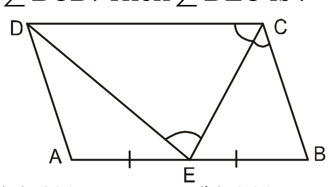
(c) $\frac{1}{4}$(□ABCD) (d) $\frac{1}{2}$(□ABCD)

42. ABCD is a ||gm and X, Y are the mid-points of sides AB and CD respectively.Then quadrilateral AXCY is :



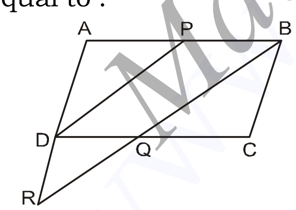
(a) parallelogram
(b) rhombus
(c) square
(d) rectangle

43. ABCD is a ||gm, E is the mid-point of AB and CE bisects ∠BCD. Then ∠DEC is :



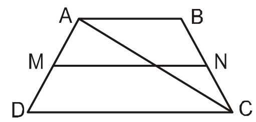
(a) 60° (b) 90°
(c) 100° (d) 120°

44. P is a mid-point of sideAB to a ||gm ABCD. A line through B parallel to PD meets DC at Q and AD produced at R. Then BR is equal to :



(a) BQ (b) $\frac{1}{2}$BQ
(c) 2BQ
(d) None of these

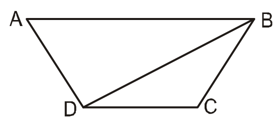
45. ABCD is a trapezium in which AB || CD. M and N are the mid-points of AD and BC respectively. If AB = 14cm and MN = 15cm, find CD.



(a) 16 cm (b) 18cm
(c) 8 cm (d) 10cm

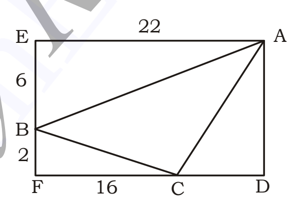
46. In a quadrilateral ABCD, ∠B = 90° , and $AD^2 = AB^2 + BC^2 + CD^2$. Then ∠ACD is equal to :
(a) 60° (b) 90° (c) 30° (d) 45°

47. In the quadrilateral ABCD



AB + BC + CD + DA is :
(a) greater than 2BD
(b) less than 2BD
(c) equal to 2BD
(d) none of these

48. In the given figure EADF is a rectangle and ABC is a triangle whose vertices lie on the sides of □EADF. AE = 22. BE = 6, CF = 16 and BF = 2. Find the length of the line joining the mid-points of the sides AB and BC.

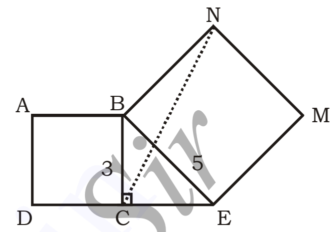


(a) 4 (b) 5
(c) 3.5 (d) $4\sqrt{2}$

49. If the length of the side PQ of the rhombus PQRS is 6 cm and ∠PQR = 120°, then the length of QS, in cm is
(a) 4 (b) 6 (c) 3 (d) 5

50. ABCD is a square. M is the mid-point of AB and N is the mid-point of BC. DM and AN are joined and they meet at O. Then which of the following is correct ?
(a) OA : OM = 1 : 2
(b) AN = MD
(c) ∠ADM = ∠ANB
(d) ∠AMD = ∠BAN

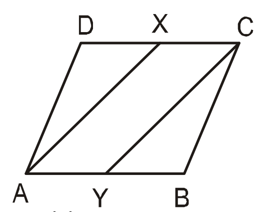
51. In the given figure, there is a square of 3 cm. If an another square of 5 cm with side BE is formed. In triangle BCE, C is right angle. Find the length of CN?



(a) $\sqrt{56}$ cm (b) $\sqrt{57}$ cm
(c) $\sqrt{58}$ cm (d) $\sqrt{59}$ cm

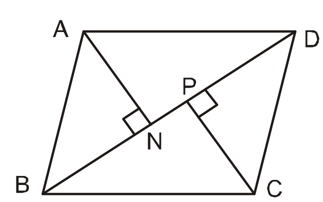
52. ABCD is a quadrilateral inscribed in a circle with centre O. If ∠COD = 120° and ∠BAC = 30°, then ∠BCD is:
(a) 75° (b) 90° (c) 120° (d) 60°

53. In the given figure, ABCD is a ||gm and line segments AX, CY bisect the angles A and C respectively, then which one is true:



(a) AX || CY
(b) AX || CY is a trapezium
(c) AX is not parallel to CY
(d) None of these.

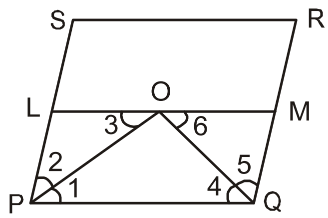
54. In the given figure, $AN \perp BD$ and $CP \perp BD$ and ABCD is a parallelogram, then :



(a) AN ≠ CP (b) AN = CP
(c) AN = $\frac{1}{2}$CP
(d) none of these.

55. In the given figure, PQRS is a ||gm, PO and QO are respectively, the angle bisectors of

$\angle$P and $\angle$Q. Line LOM is drawn parallel to PQ, then :



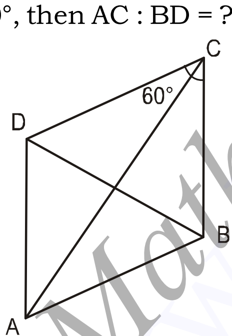
(a) LO = 2OM (b) LO = $\frac{1}{2}$ OM
(c) LO = OM
(d) None of these.

56. The diagonals of a | | gm ABCD intersect at O. A line through O intersects AB at X and DC at Y, then:
(a) OX = 2OY (b) OX = OY
(c) OY = 2OX
(d) None of these.

57. In a | | gm ABCD, the bisector of $\angle$A also bisects BC at X, then :
(a) AD = 2AB
(b) AD = AB
(c) AD = 3AB
(d) None of these

58. Diagonals of a | | gm are 8m and 6m respectively. If one of side is 5m, then the area of | | gm is:
(a) 18m^2 (b) 30m^2
(c) 24m^2 (d) 48m^2

59. ABCD is rhombus in which $\angle$C = 60°, then AC : BD = ?

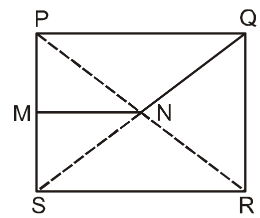


(a) $\sqrt{3}:1$ (b) $\sqrt{3}:2$
(c) 3 : 1 (d) 3 : 2

60. In a | | gm, the adjacent side are 36cm and 27cm in length. If the distance between the longer sides is 12cm, then the distance between the smaller sides is :
(a) 12 cm (b) 16 cm
(c) 14 cm (d) 15 cm

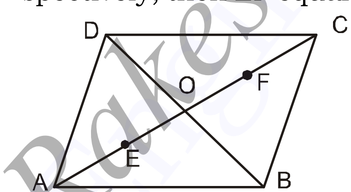
61. The length of the diagonal BD of the | | gm ABCD is 18cm. If P and Q are the centroid of $\triangle$ ADC and $\triangle$ ABC, then length of PQ is :
(a) 5.5 cm (b) 7cm
(c) 5 cm (d) 6cm

62. PQRS is a square, M is the mid-point of side PS and N is the intersecting point of its diagonals. Then the ratio Area ($\square$PQNM) : Area ($\square$PQRS) is :



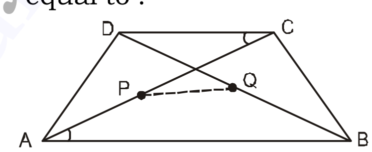
(a) 5 : 8 (b) 3 : 8
(c) 1 : 4
(d) none of these

63. In the adjoining figure ABCD is a | | gm and E,F are the centroids of Δ ABD and ΔBCD respectively, then EF equals :



(a) AE (b) BE (c) CE (d) DE

64. ABCD is a trapezium and P, Q are the mid-points of the diagonals AC and BD. Then PQ is equal to :

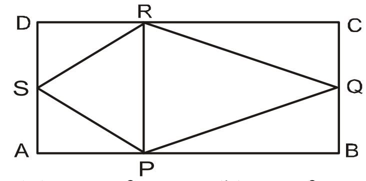


(a) $\frac{1}{2}(AB)$ (b) $\frac{1}{2}(CD)$

(c) $\frac{1}{2}(AB-CD)$

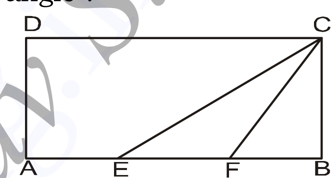
(d) $\frac{1}{2}(AB+CD)$

65. ABCD is a | | gm. P, Q, R and S are points on sides AB, BC, CD and DA respectively such that AP = DR. If the area of the | | gm ABCD is 20cm^2, then the area of quadrilateral PQRS is:



(a) 10cm^2 (b) 8cm^2
(c) 12cm^2 (d) 8.5cm^2

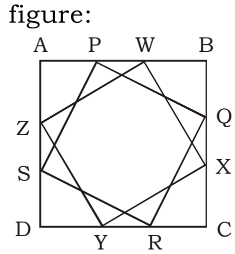
66. In the given figure, ABCD is a rectangle with AE = EF = FB. What is the ratio of the area of the $\triangle$CEF to that of the rectangle ?



(a) 2 : 5 (b) 2 : 3
(c) 1 : 4 (d) 1 : 6

67. Side AB of rectangle of ABCD is divided into four equal parts by points *x*, *y*, *z*. The ratio of the $\frac{\text{Area }(\Delta XYC)}{\text{Area(Rectangle ABCD)}}$ is :
(a) 1/7 (b) 1/6
(c) 1/9 (d) 1/8

68. In the adjoining figure ABCD, PQRS and WXYZ are three squares. Find number of triangles and quadrilaterals in the figure:



(a) 24 and 16 (b) 28 and 15
(c) 27 and 16
(d) none of the above

69. Two light rods AB = a+b, CD = a–b symmetrically lying on a horizontal AB. There are perpendicular distance between rods is a. The length of AC is given by
(a) a (b) b
(c) $\sqrt{a^2-b^2}$ (d) $\sqrt{a^2+b^2}$

70. If PQRS be a rectangle such PQ = $\sqrt{3}$ QR. Then, what is $\angle$PRS equal to?
(a) 60° (b) 45° (c) 30° (d) 15°

71. In a trapezium, the two non-parallel sides are equal in length, each being of 5 cm. The parallel sides are at a distance of 3 cm apart. If the smaller side of the parallel sides is of length 2 cm, then the sum of the diagonals of the trapezium is

(a) $10\sqrt{5}$ cm (b) $6\sqrt{5}$ cm

(c) $5\sqrt{5}$ cm (d) $3\sqrt{5}$ cm

72. The area of a rectangle lies between 40 cm² and 45 cm². If one of the sides is 5 cm, then its diagonal lies between

(a) 8 cm and 10 cm
(b) 9 cm and 11 cm
(c) 10 cm and 12 cm
(d) 11 cm and 13 cm

73. Let ABCD be a parallelogram. Let P, Q, R and S be the mid-points of sides AB, BC, CD and DA, respectively. Consider the following statements.

I. Area of triangle APS < Area of triangle DSR, if BD<AC.

II. Area of triangle ABC=4 (Area of triangle BPQ).

Select of correct answer using the codes given below.

(a) Only I (b) Only II
(c) Both I and II
(d) Neither I nor II

74. Consider the following statements

I. Let ABCD be a parallelogram which is not a rectangle. Then, $2(AB^2+BC^2) \neq AC^2+BD^2$

II. If ABCD is a rhombus with AB = 4 cm, then $AC^2+BD^2 = n^3$ for some positive integer n.

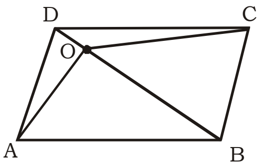
Which of the above statements is/are correct?

(a) Only I (b) Only II
(c) Both I and II
(d) Neither I nor II

75. ABCD is parallelogram. E is a point on BC such that BE:EC=m : n. AC and DB intersect at F, then what is the ratio of the area of $\triangle$ FEB and $\triangle$ AFD?

(a) m/n (b) $(m/n)^2$
(c) $(n/m)^2$ (d) $[m/(m+n)]^2$

76. In the below figure, ABCD is a parallelogram. If area of $\triangle$ OAB = 19 cm² then, find the area of $\triangle$ OBC.



(a) 19 (b) 20 (c) 38 (d) $\frac{19}{2}$

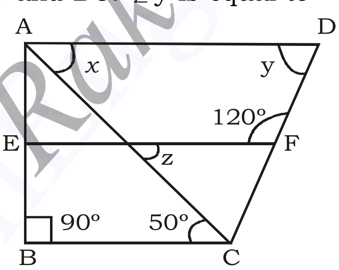
77. If the diagonals of a rhombus are 4.8 cm and 1.4 cm, then what is the perimeter of the rhombus?

(a) 5 cm (b) 10 cm
(c) 12 cm (d) 20 cm

78. ABCD is a trapezium with parallel sides AB=2cm and DC = 3 cm. E and F are the mid-points of the non-parallel sides. The ratio of area of ABFE to area of EFCD is

(a) 9:10 (b) 8:9
(c) 9:11 (d) 11:9

79. In the figure given above, ABCD is a trapezium. EF is parallel to AD and BC. $\angle y$ is equal to



(a) 30° (b) 45° (c) 60° (d) 65°

80. The locus of a point in rhombus ABCD which is equidistant from A and C is

(a) a fixed point on diagonal BD
(b) diagonal BD
(c) diagonal AC
(d) None of the above

81. If two parallel lines are cut by two distinct transversals, then the quadrilateral formed by the four lines is always a

(a) square
(b) parallelogram
(c) rhombus (d) trapezium

82. ABCD is a parallelogram. If the bisectors of the $\angle$A and $\angle$C meet the diagonal BD at points P and Q respectively, then which one of the following is correct?

(a) PCQA is a straight line
(b) $\triangle$ APQ is similar to $\triangle$ PCQ
(c) AP = CP (d) AP = AQ

83. The sides of a parallelogram are 12 cm and 8 cm long and one of the diagonals is 10 cm long. If d is the length of other diagonal, then which one of the following is correct?

(a) d<8 cm
(b) 8 cm< d<10 cm
(c) 10 cm<d<12 cm
(d) d>12 cm

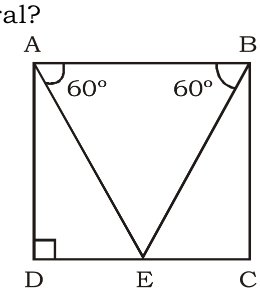
84. Let LMNP be a parallelogram and NR be perpendicular to LP. If the area of the parallelogram is six times the area of $\triangle$ RNP and RP = 6 cm what is LR equal to?

(a) 15 cm (b) 12 cm
(c) 9 cm (d) 8 cm

85. Two poles of heights 6 m and 11 m stand vertically upright on a place ground. If the distance between their feet is 12 m, what is the distance between their tops?

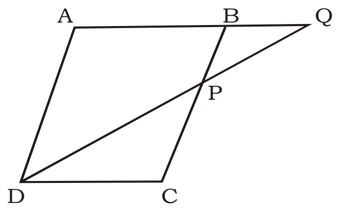
(a) 11 m (b) 12 m
(c) 13 m (d) 14m

86. In the given figure, ABCD is a quadrilateral with AB parallel to DC and AD parallel to BC, ADE is a right angle. If the perimeter of the $\triangle$ ABE is 6 units, what is the area of the quadrilateral?



(a) $2\sqrt{3}$ sq units
(b) 4 sq units
(c) 3 sq units
(d) $4\sqrt{3}$ sq units

87. In the figure given below, ABCD is a parallelogram. P is a point in BC such that PB:PC=1:2. DP produced meets AB produced at Q. If the area of the $\triangle$ BPQ is 20 sq units, what is the area of the $\triangle$ DCP?



(a) 20 sq units (b) 30 sq units
(c) 40 sq units
(d) None of the above

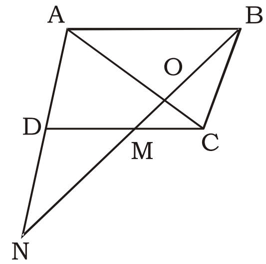
88. ABCD is a square, P, Q, R and S are points on the sides AB, BC, CD and DA respectively such that AP = BQ = CR = DS. What is $\angle$SPQ equal to?
(a) 30° (b) 45° (c) 60° (d) 90°

89. Two similar parallelograms have corresponding sides in the ratio 1:k. What is the ratio of their areas?
(a) $1:3k^2$ (b) $1:4k^2$
(c) $1:k^2$ (d) $1:2k^2$

90. ABC is a triangle in which AB = AC. Let BC be produced to D. From a point E on the line AC let EF be a straight line such that EF is parallel to AB. Consider the quadrilateral ECDF thus formed. If $\angle$ABC = 65° and $\angle$EFD =80°, then what is $\angle$D equal to?
(a) 43° (b) 41° (c) 37° (d) 35°

91. Two sides of a parallelogram are 10 cm and 15 cm. If the altitude corresponding to the side of length 15 cm is 5 cm, then what is the altitude to the side of length 10 cm?
(a) 5 cm (b) 7.5 cm
(c) 10 cm (d) 15 cm

92.

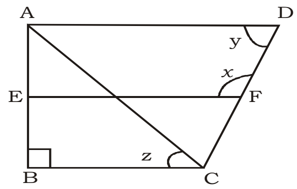


In the figure given above, M is the mid-point of the side CD of the parallelogram ABCD. What is ON:OB?
(a) 3:2 (b) 2:1
(c) 3:1 (d) 5:2

93. An equilateral triangle and a regular hexagon are inscribed in a given circle. If a and b are the lengths of their sides respectively, then which one of the following is correct?
(a) $a^2=2b^2$ (b) $b^2=3a^2$
(c) $b^2=2a^2$ (d) $a^2=3b^2$

94. In a cricket match, the first 5 batsmen of a team scored runs :30, 40, 50, 30 and 40. If these data represent a four sided figure with 50 as its one of the diagonals, then what does second diagonal represent?
(a) 30 runs (b) 40 runs
(c) 50 runs (d) 70 runs

95.



ABCD is a trapezium in which EF is parallel to BC. $\angle x$ = 120° and $\angle z$ =50°, then what is $\angle$y?
(a) 50° (b) 60° (c) 70° (d) 80°

96. An obtuse angle made by a side of a parallelogram PQRS with other pair of parallel sides is 150°. If the perpendicular distance between these parallel sides (PQ and SR) is 20 cm, what is the length of the sides RQ?
(a) 40 cm (b) 50 cm
(c) 60 cm (d) 70 cm

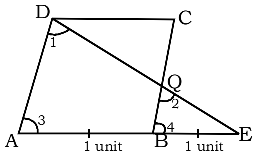
97. ABCD be a quadrilateral the diagonals AC and BD meet at O. the perpendicular drawn from A to CD, meet CD at E. Further, AO: OC = BO : OD, AB = 30 cm, CD = 40 cm, and the area of the quadrilateral ABCD is 1050 sq. cm.
(a). What is BE equal to?
(b). What is the area of the ΔADC is equal to?
(c). What is $\angle$AEB equal to?

ANSWER KEY

1. (b)	11. (b)	21. (a)	31. (d)	41. (c)	51. (c)	61. (d)	71. (b)	81. (d)	91. (b)
2. (a)	12. (a)	22. (a)	32. (c)	42. (a)	52. (b)	62. (b)	72. (b)	82. (b)	92. (b)
3. (b)	13. (c)	23. (d)	33. (c)	43. (b)	53. (a)	63. (a)	73. (b)	83. (d)	93. (d)
4. (c)	14. (d)	24. (d)	34. (d)	44. (c)	54. (b)	64. (c)	74. (b)	84. (b)	94. (c)
5. (d)	15. (a)	25. (a)	35. (b)	45. (a)	55. (c)	65. (a)	75. (d)	85. (c)	95. (b)
6. (a)	16. (a)	26. (a)	36. (a)	46. (b)	56. (b)	66. (d)	76. (a)	86. (a)	96. (a)
7. (d)	17. (b)	27. (a)	37. (d)	47. (a)	57. (a)	67. (d)	77. (b)	87. (d)	97. (*)
8. (a)	18. (c)	28. (d)	38. (a)	48. (b)	58. (c)	68. (c)	78. (c)	88. (d)	
9. (d)	19. (c)	29. (b)	39. (b)	49. (b)	59. (a)	69. (d)	79. (c)	89. (c)	
10. (b)	20. (a)	30. (d)	40. (b)	50. (b)	60. (b)	70. (c)	80. (a)	90. (d)	

SOLUTION

1. (b) According to questions



AD | | BC and AB | | DC

Point B is the midpoint of AE

$\angle 1 = \angle 2$ (Alternate Angle)

$\angle 3 = \angle 4$ (Alternate Angle)

$\therefore \quad \Delta EQB \sim \Delta EDA$

$\therefore \quad \frac{EB}{EA} = \frac{EQ}{ED} = \frac{QB}{AD}$

$\Rightarrow \quad \frac{1}{2} = \frac{QB}{AD}$

$\Rightarrow \quad \frac{QB}{AD} = \frac{1}{2}$

If AD = 2

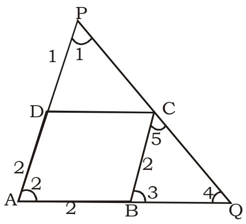
QB = 1

Then QC = 1

$\therefore$ Q divides BC in the ratio (1:1)

2. (a) According to question,

Given:



ABCD is a rhombus

AB = BC= CD =DA

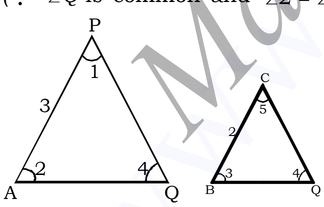
$\Rightarrow$ DP = $\frac{1}{2}$ AB

$\Rightarrow \quad \frac{DP}{AB} = \frac{1}{2}$

In a rhombus $\angle 2 = \angle 3$

$\therefore \quad \Delta APQ \sim \Delta BCQ$

($\because$ $\angle Q$ is common and $\angle 2 = \angle 3$



$\Rightarrow \quad \frac{AP}{BC} = \frac{AQ}{BQ} \qquad \frac{AQ}{BQ} = \frac{3}{2}$

$\Rightarrow \quad \frac{AB + BQ}{BQ} = \frac{3}{2}$ ($\therefore$ AQ = AB + BQ)

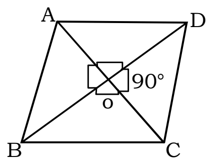
$\Rightarrow \quad \frac{AB}{BQ} + 1 = \frac{3}{2}$

$\Rightarrow \quad \frac{AB}{BQ} = \frac{3}{2} - 1$

$\Rightarrow \quad \frac{AB}{BQ} = \frac{1}{2}$

$\Rightarrow \quad \therefore \frac{BQ}{AB} = \frac{2}{1}$

3. (b) According to question



$OB^2 + OC^2 = BC^2$ **...(i)**

$OB^2 + OA^2 = AB^2$ **...(ii)**

$OA^2 + OD^2 = AD^2$ **...(iii)**

[By pythagoras theorem]

$OC^2 + OD^2 = CD^2$ **...(iv)**

Add equation (i),(ii),(iii) and (iv)

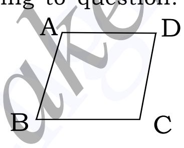
$\Rightarrow$ $2(OB^2 + OC^2 + OD^2 + OA^2) = BC^2 + AB^2 + AD^2 + CD^2$

$\Rightarrow$ $2BC^2 + 2AD^2 = BC^2 + AB^2 + AD^2 + CD^2$

$BC^2 + AD^2 = AB^2 + CD^2$

or $AB^2 + CD^2 = BC^2 + DA^2$

4. (c) According to question.



Given:

Ratio of $\angle A$ and $\angle B$ is 4 : 5

$\Rightarrow \quad \frac{\angle A}{\angle B} = \frac{4}{5}$

We know that $\angle A + \angle B = 180°$

9 units = 180°

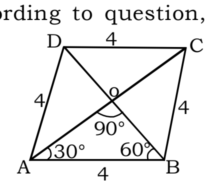
1 unit = 20°

$\angle A$ = 4 units = 4 × 20° = 80°

$\angle A = \angle C = 80°$

[Opposite $\angle$ of rhombus are equal

5. (d) According to question,



Given : $\angle B = 120°$

In a rhombus diagonal are angle bisector and diagonal cut at right triangle.

$\therefore \quad \text{Sin}30° = \frac{P}{H} = \frac{BO}{AB} = \frac{1}{2} = \frac{BO}{4}$

BO = 2 cm

$\therefore$ BD = 2 × BO

= 2 × 2 = 4 cm

Alternate

$\angle ABD = \frac{1}{2} \angle ABC$

$= \frac{1}{2} \times 120° = 60°$

$\therefore \quad \angle A = \angle ABD = \angle ADB = 60°$

$\therefore \quad \Delta ABD$ = equilateral triangle

So, AB = BD = 4 cm

6. (a)

According to question

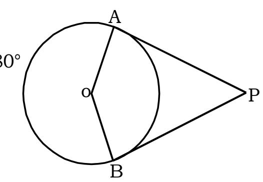
Given: PAOB is quadrilateral

$\because \quad \angle AOB : \angle APB$

$5x : 1x$

Note: In Quadrilateral Sum of opposite angle is 180°

$\therefore \quad \angle AOB + \angle APB = 180°$



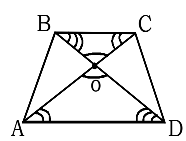
Then $5x + x = 180°$

$6x = 180°$

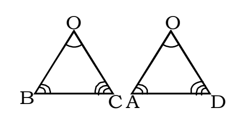
$x = 30°$

$\therefore \quad \angle APB = 30°$

7. (d) According to question,



$\Delta AOD \sim \Delta COB$



$\therefore \quad \frac{OB}{OD} = \frac{OC}{OA}$

$\Rightarrow \quad \frac{3x - 19}{x - 5} = \frac{x - 3}{3}$

$\Rightarrow \quad 9x - 57 = x^2 - 8x + 15$

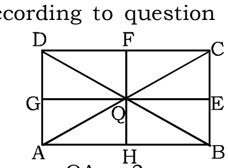
$\Rightarrow \quad x^2 - 17x + 72 = 0$

$\Rightarrow \quad x(x - 8) - 9(x - 8) = 0$

$\Rightarrow \quad (x - 8)(x - 9) = 0$

$\Rightarrow \quad x = 8$ or 9

8. (a) According to question



Given: QA = 3 cm

QB = 4 cm

QC = 5 cm

QD = ?

As we know that

$\Rightarrow$ $QD^2 + QB^2 = QA^2 + QC^2$

(By using Pythagoras theorem)

$\Rightarrow$ $QD^2 + (4)^2 = (3)^2 + (5)^2$

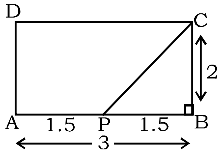
$\Rightarrow$ $QD^2 + 16 = 9 + 25$

$\Rightarrow$ $QD^2 = 34 - 16$

$\Rightarrow$ $QD^2 = 18$

$QD = \sqrt{18}$, $QD = 3\sqrt{2}$

9. (d) According to question



$\Rightarrow$ In ΔCBP

$\Rightarrow$ $CP^2 = BP^2 + BC^2$

$\Rightarrow$ $CP^2 = (1.5)^2 + (2)^2$

$\Rightarrow$ $CP^2 = 2.25 + 4$

$\Rightarrow$ $CP^2 = 6.25$

$\Rightarrow$ $CP = \sqrt{6.25}$

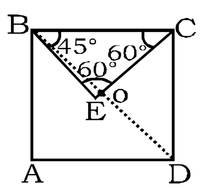
$\Rightarrow$ $CP = 2.5$

$\therefore \text{Sin } \angle CPB = \frac{BC}{CP}$

$\text{Sin } \angle CPB = \frac{2}{2.5}$

$\text{Sin } \angle CPB = \frac{4}{5}$

10. (b) According to question



ABCD is a square and BCE is an equilateral triangle

$\therefore \angle CEB = 60°$

If BD is a diagonal

$\therefore \angle CBD = 45°$

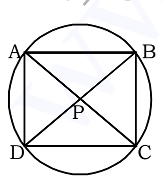
Then In ΔBOC

$\angle CBO + \angle BOC + \angle BCE = 180°$

$\angle BOC = 180° - 60° - 45°$

$\angle BOC = 75°$

11. (b) According to question



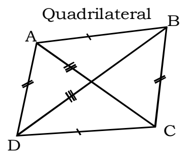
ABCD is a cyclic quadrilateral.

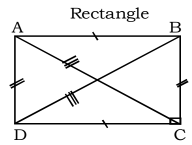
$\therefore$ AP × PC = DP × BP (theorem)

AC and BD are chords of circle

AP. CP = BP.DP

12. (a) According to question





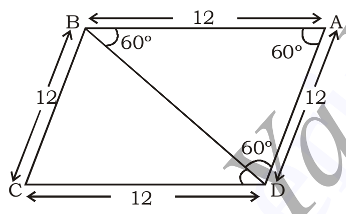
AB = CD	AB = CD
BC = AD	AD = BC
AC = BD	AC = BD

$\angle DCB = 90°$

Note: Only rectangle follows these condition

$\therefore$ angles of the quadrilateral is same as each angle of rectangle = 90°

13. (c) We know that in a Rhombus diagonal bisect the angle.



$\because \angle A = 60°$

then $\angle B = 180° - 60°$

$\angle ABC = 120°$

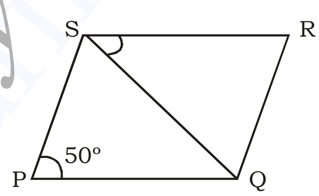
Now $\angle ABD = \angle CBD = \frac{120}{2} = 60°$

$\Rightarrow$ So $\angle ABD = \angle BDA = \angle BAD = 60°$

$\Rightarrow$ So ΔABD is an equilateral triangle then AD = AB = BD = 12 cm

$\Rightarrow$ So Diagonal BD = 12 cm

14. (d) According to the question,



$\angle P = 50°$, $\angle R = 50°$

then $\angle PSR = 180° - 50°$
$= 130°$

then $\angle RSQ = \frac{130°}{2} = 65°$

15. (a) Let ABCD is a quadrilateral and its BD diagonal = 24 metres

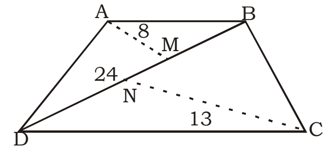
And, AM = 8 metres

CN = 13 metres

Area of □ABCD = ar(ΔABD)+ ar (ΔBCD)

$= \left(\frac{1}{2} BD \times AM\right) + \left(\frac{1}{2} BD \times CN\right)$

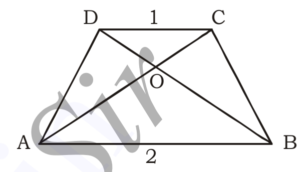
$= \frac{1}{2} \times BD\left[AM + CN\right]$



$= \frac{1}{2} \times 24\left[8 + 13\right] = 12 \times 21$

Area of □ABCD = 252 m^2

16. (a)



$\frac{\text{area of } \Delta COD}{\text{area of } \Delta AOB} = \frac{CD^2}{AB^2}$

$\frac{\text{area of } \Delta COD}{84} = \left(\frac{1}{2}\right)^2 \Rightarrow \frac{1}{4}$

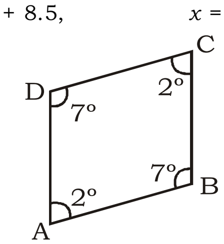
area of ΔCOD = 21 cm^2

17. (b) AB + CD = BC + DA (Property)

$\Rightarrow$ $7 + 9.2 = x + 8.5$

$\Rightarrow$ $16.2 = x + 8.5$, $x = 7.7$

18. (c) ATQ



As we know that in a parallelogram opposite angle are same.

$\therefore \angle A = \angle C$

$\angle B = \angle D$

Note:- Parallelogram is rhombus but rhombus is not a parallelogram

19. (c) Angles be x, $2x$, $3x$, $4x$

$\therefore x + 2x + 3x + 4x = 360°$

$\Rightarrow 10x = 360° \Rightarrow x = 36°$

$\therefore$ largest angle = $4x$ = 144°

20. (a) $\angle DAB = 180° - b$ and

$\angle BCD = 180° - a$

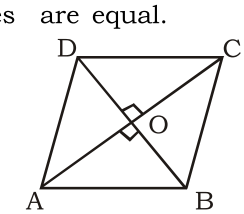
But,

$\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow$ $(180° - b°) + x° + (180° - a°) + y = 360°$

$\Rightarrow$ $x° + y° = a° + b°$

21. (a) Since diagonals bisect each other at right angles and all sides are equal.



$AO^2 + BO^2 = AB^2$

$\left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 = AB^2$

$AC^2 + BD^2 = 4AB^2$

22.(a) Since, their base and perpendicular heights are same.

23.(d) $\angle ACB = \angle DAC = 50°$ (Alternate interior $\angle$s)

$\angle BOC = 180° - 80° = 100°$

$\therefore$ Now in ΔBOC,

$\angle DBC = 180° - (100° + 50°)$

$= 30°$

24.(d) $\angle A + \angle B = 180°$ and $\angle A = \angle B$

$\Rightarrow \angle A = \angle B = 90°$

So, the given ||gm may be a square or a rectangle as in both the cases the adjacent angles are equal.

25.(a) $7x = 42 \Rightarrow x = 6$

and $8y = 32 \Rightarrow y = 4$

26.(a) In Δ PSX and Δ QRY

$\angle X = \angle Y = 90°$ and

SX = RY

[$\because$ SX = SY - XY and RY = SY - SR = SY - PQ = SY - XY]

and PS = QR (sides of a ||gm)

$\therefore \Delta PSX \cong \Delta QRY$

(R.H.S axion)

$\therefore$ PX = QY

27.(a) Area of ||gm ABCD

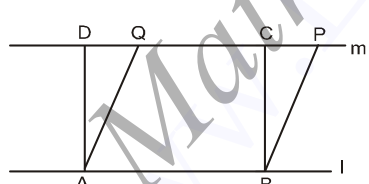
= Base $\times$ height

$\Rightarrow AB \times DM = AD \times CL$

$\Rightarrow 16 \times 12 = AD \times 15$

$\Rightarrow$ AD = 12.8 units

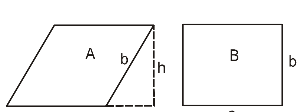
28.(d)



area of square ABCD = area of rhombus ABPQ

they lie on the same base AB and between two parallel lines (l || m).

29.(b)

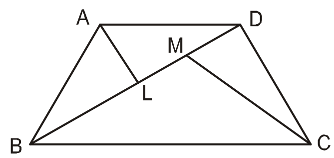


B = ab

A = ah $\Rightarrow$ A < ab [$\because$ h < b]

$\Rightarrow$ A < B

30.(d)



Area of quadrilateral ABCD = Area of ΔABD + Area of ΔBCD

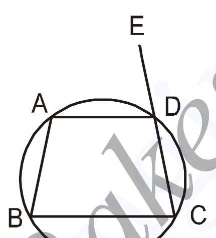
$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$

$= \frac{1}{2} \times BD(AL + CM)$

$= \frac{1}{2} \times 64(13.2 + 16.8)$

$= \frac{1}{2} \times 64 \times 30 = 960$sq.cm.

31.(d)



$\angle ABC + \angle CDA = 180°$

$\Rightarrow \angle CDA = 180° - 72° = 108°$

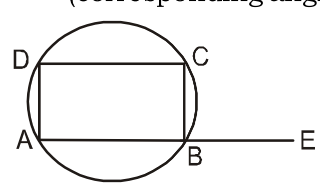
$\therefore \angle ADE = 180° - 108° = 72°$

AD || BC

$\angle BCD = \angle ADE = 72°$

(corresponding angles)

32.(c)



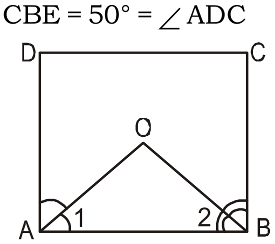
$\angle ABC + \angle ADC = 180°$

$\therefore \angle ABC = 180° - 50° = 130°$

$\therefore \angle ADC = 180° - 130° = 50°$

$\angle CBE = 50° = \angle ADC$

33.(c)



$\angle AOB = 180° - (\angle 1 + \angle 2)$

$= 180° - \left(\frac{1}{2}\angle A + \frac{1}{2}\angle B\right)$

$= 180° - \frac{1}{2}$ [360° - ($\angle C + \angle D$)]

[$\therefore \angle A + \angle B + \angle C + \angle D = 360°$]

$\therefore \angle AOB = 180° - 180° + \frac{1}{2}(\angle C + \angle D) = \frac{1}{2}(\angle C + \angle D)$

34.(d) $\angle AOB = \frac{1}{2}(\angle A + \angle B)$

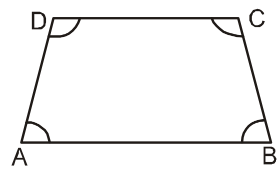
$= \frac{1}{2}(180°)$ [$\because \angle C + \angle D = \angle A + \angle B = 180°$] = 90°

35.(b) x + 2x + 3x + 4x = 360

$\Rightarrow$ x = 36

$\therefore$ The angles of quadrilateral (in order) are

36°,72°,108°,144°



Since, opposite angles are supplementary,

Therefore, AB || CD. Hence, it is a trapezium.

36.(a) BD = $\sqrt{12^2 + 16^2} = 20cm$

(Δ ABD is a right angle triangle)

$\therefore \Delta$ BCD is an equilateral triangle.

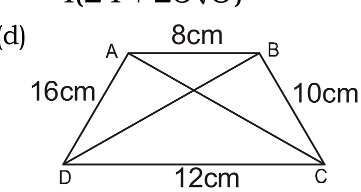
$\therefore$ Area of ☐ ABCD = Area of Δ ABD + Area of Δ BCD

$= \frac{1}{2} \times 16 \times 12 + \frac{\sqrt{3}}{4}(20)^2$

$= 96 + 100\sqrt{3}$

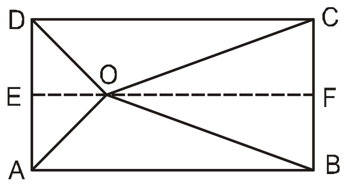
$= 4(24 + 25\sqrt{3})$ cm²

37.(d)



$AC^2 + BD^2 = (AD^2 + BC^2) + 2(AB \times CD)$
$= (256 + 100) + 2(8 \times 12)$
$= 356 + 192 = 548cm^2$

38.(a)



Draw EF || AB

In right angled $\triangle$ EOA and $\triangle$ OCF.

$OA^2 = OE^2 + AE^2$ and OC^2
$= OF^2 + CF^2$

$\therefore$ $OA^2 + OC^2$
$= OE^2 + AE^2 + OF^2 + CF^2$ **...(i)**

Similarly in the right angled $\triangle$ DEO and $\triangle$ OBF,

$OB^2 + OD^2 = OE^2 + OF^2 + DE^2 + BF^2$

$\Rightarrow$ $OB^2 + OD^2 = OE^2 + OF^2 + CF^2 + AE^2$ **...(ii)**

($\because$ DE = CF and BF = AE)

$\therefore$ from (i) and (ii)

$OA^2 + OC^2 = OB^2 + OD^2$

39. (b) By mid-point theorem

$$\frac{EF}{AD} = \frac{FG}{DC} = \frac{GH}{CB} = \frac{HE}{BA} = \frac{1}{2}$$

$$\therefore \quad \frac{EF + FG + GH + HE}{AD + DC + CB + BA}$$

$$\therefore \quad \frac{\frac{1}{2}(AD + DC + CB + BA)}{(AD + DC + CB + BA)} = \frac{1}{2}$$

40.(b) $\angle BCE = 102°$, AB = CD = ED (given)

$\therefore$ CD = ED = CE [$\because$ AB = CE]

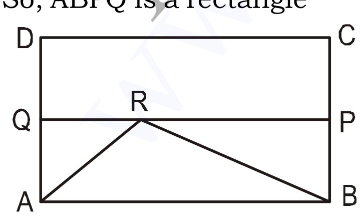
$\triangle$ ECD is an equilateral triangle.

$\therefore$ $\angle ECD = 60°$

$\angle BCD = 102° + 60°$
$= 162°$

41.(c) AB || PQ || CD.

So, ABPQ is a rectangle



$\therefore$ $\triangle ARB = \frac{1}{2}(\square ABPQ)$

$$= \frac{1}{2} \times (\frac{1}{2} \times \square ABCD)$$

$$= \frac{1}{4}(\square ABCD)$$

42.(a) $AX = \frac{1}{2}AB$ and $CY = \frac{1}{2}DC$

but, AB = DC

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow$ AX = CY

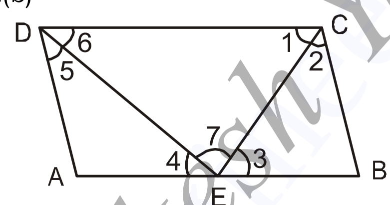
also, AB || DC[$\because$ ABCD is a ||gm]

Thus in quadrilateral AXCY,

$\Rightarrow$ AX || CY and AX = YC

Hence, quadrilateral AXCY is a ||gm.

43.(b)



AB || DC and EC cuts them

$\Rightarrow$ $\angle 3 = \angle 1$

$\Rightarrow$ $\angle 3 = \angle 2$ ($\because$ $\angle 1 = \angle 2$)

$\Rightarrow$ EB = BC

$\Rightarrow$ AE = AD

Now, AE = AD

$\Rightarrow$ $\angle 4 = \angle 5$ and $\angle 4 = \angle 6$

$\Rightarrow$ DE bisects $\angle ADC$,

Again, $\angle ADC + \angle BCD = 180°$ (Co.Int. Angles)

$$\Rightarrow \quad \frac{1}{2}\angle ADC + \frac{1}{2}\angle BCD = 90°$$

$\Rightarrow$ $\angle 6 + \angle 1 = 90°$

$\Rightarrow$ $\angle 1 + \angle 6 + \angle 7 = 180°$

$\Rightarrow$ $\angle 7 = 180° - 90°$

$\Rightarrow$ $\angle DEC = 90°$

44.(c) In $\triangle$ ARB, P is the mid-point of AB and PD || BR

$\Rightarrow$ D is the mid-point of AR.

$\because$ ABCD is a ||gm

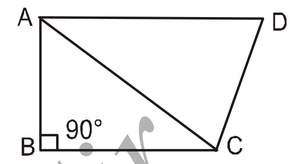
$\Rightarrow$ DC || AB $\Rightarrow$ DQ || AB

$\therefore$ Q is the mid-point of RB $\Rightarrow$ BR = 2BQ.

45.(a) $MN = \frac{1}{2}(AB + CD) \Rightarrow 2 \times 15 =$ 14 + CD

$\Rightarrow$ CD = 16cm.

46.(b)



$AD^2 = AB^2 + BC^2 + CD^2$

$\therefore$ In $\triangle$ ABC, $\angle B = 90°$

$\therefore$ $AC^2 = AB^2 + BC^2$

$\therefore$ $AD^2 = AC^2 + CD^2$

$\Rightarrow$ $\angle ACD = 90°$

47.(a) As we know that sum of two sides of a triangle is greater than the third side.

$\therefore$ In $\triangle$ ABD

AB + DA > BD___(i)

and In $\triangle$ BDC

BC + CD > BD__(ii)

$\therefore$ from (i) and (ii) AB + BC + CD + DA > 2BD.

48.(b) EF = AD = 8 ($\because$ EADF is a rectangle)

CD = 22 - 16 = 6

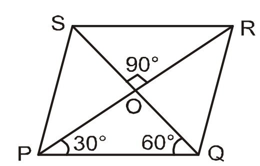
So, In right angled $\triangle$ ADC,

$AC = \sqrt{8^2 + 6^2} = 10$

$\therefore$ length of the line joining the mid-points of AB & BC = $\frac{1}{2}$(AC)

= 5

49.(b)



$\angle PQO = \frac{1}{2}\angle PQR = 60°$

From$\triangle$POQ,

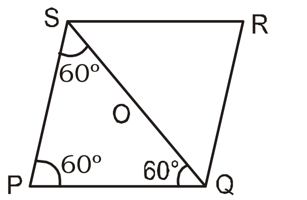
$\angle OPQ = 180° - 90° - 60°$
$= 30°$

$\sin\angle OPQ = \frac{OQ}{PQ}$

$\Rightarrow$ $OQ = PQ \sin 30° = 6 \times \frac{1}{2} = 3$

$\therefore$ $QS = 2 \times 3 = 6cm$

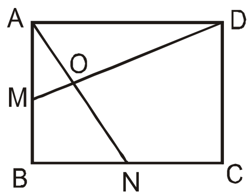
Alternatively



$\triangle$ SPQ is an equilateral triangle

$\therefore$ QS = 6 cm

50.(b)



If AB = $2x$, then BN = x

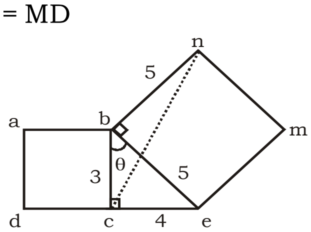
$\therefore$ AN = $\sqrt{4x^2 + x^2} = \sqrt{5}x$

Similarly

MD = $\sqrt{4x^2 + x^2} = \sqrt{5}x$

AN = MD

51.(c)



In ΔCBE

$$\text{Cos}(90 + \theta) = \frac{bc^2 + bn^2 - cn^2}{2bc \times bn}$$

$$-\text{Sin}\theta = \frac{(3^2 + 5^2 - cn^2)}{2 \times 3 \times 5}$$

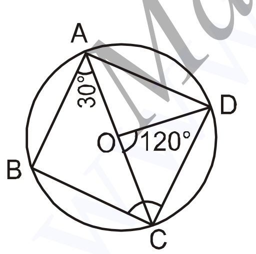
$$-\text{Sin}\theta = \frac{34 - cn^2}{30}$$

$$-\frac{4}{5} = \frac{34 - cn^2}{30}$$

$-24 - 34 = -cn^2$

cn = $\sqrt{58}$

52. (b)



$\angle$COD = 120°

$\angle$BAC = 30°

$\angle CAD = \frac{1}{2} \times 120° = 60°$

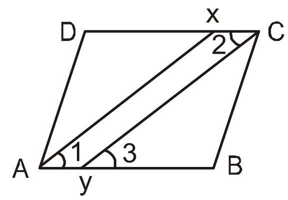
(angle made on other part of circle is half of angle made at centre by same arc)

$\therefore$ $\angle$BAD = 90°

$\therefore$ $\angle$BCD = 180° - 90°
= 90°

(cyclic quadrilateral)

53.(a)



ABCD is a ||gm (given)

$\therefore$ $\angle$A = $\angle$C

$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow$ $\angle$1 = $\angle$2 **...(i)**

Now, AB || DC and the transversal CY intersects then:

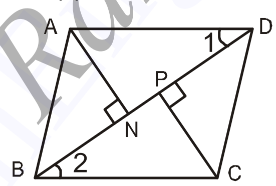
$\therefore$ $\angle$2 = $\angle$3 **....(ii)**

from (i) and (ii), we get $\angle$1 = $\angle$3

Thus, transversal AB intersects AX and YC at A and Y such that $\angle$1 = $\angle$3 i.e. corresponding angles are equal.

$\therefore$ AX || CY.

54.(b)



In $\triangle$ ADN and $\triangle$ CBP,

$\angle$1 = $\angle$2 [$\because$ AD || BC]

$\angle$AND = $\angle$CPB and

AD = BC ($\because$ opposite sides of a ||gm are equal)

So, by AAS criterion of congruence. $\therefore$ AN = CP

55.(c) PQRS is a ||gm.

$\therefore$ PS || QR

$\Rightarrow$ PL || QM and LM || PQ (given)

$\Rightarrow$ PQML is a ||gm

$\Rightarrow$ PL = QM (Opposite sides of a ||gm are equal)

$\angle$1 = $\angle$2 ...(i)

[OP is the bisector of $\angle$P]

and $\angle$1 = $\angle$3 ...(ii)

[$\because$ PQ || LM]

from (i) and (ii) $\angle$2 = $\angle$3

$\therefore$ In ΔOPL, $\angle$2 = $\angle$3

$\Rightarrow$ OL = PL ...(iii)

Similarly,

$\angle$4 = $\angle$5 and $\angle$4 = $\angle$6

$\therefore$ $\angle$5 = $\angle$6

$\therefore$ In ΔQOM, $\angle$5 = $\angle$6

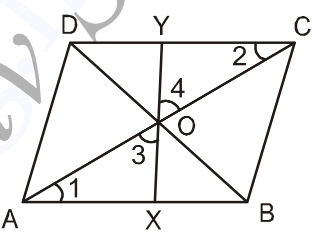
$\Rightarrow$ OM = QM

$\Rightarrow$ OM = PL ....(iv)

[$\because$ PL = QM]

$\therefore$ from (iii) and (iv), OL = OM

56.(b)



In $\triangle$ OAX and $\triangle$ OCY.

$\angle$1 = $\angle$2 ($\because$ AB || DC)

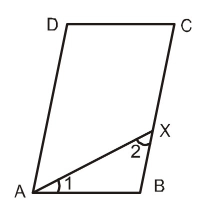
$\angle$3 = $\angle$4 (vertically opposite angles)

and OA = OC ($\because$ diagonals of a ||gm bisect each other)

$\therefore$ $\triangle$ OAX $\cong$ $\triangle$ OCY

$\Rightarrow$ OX = OY

57.(a)



$\angle$B = 180° - $\angle$A = 180°-2$\angle$1

In ΔABX,

$\angle$1 + $\angle$2 + $\angle$B = 180°

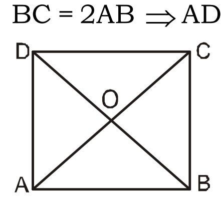
$\Rightarrow$ $\angle$1 + $\angle$2 +180° - 2$\angle$1 = 180°

$\Rightarrow$ $\angle$1 = $\angle$2

$\Rightarrow$ AB = BX $\Rightarrow$ 2BX = 2AB

$\Rightarrow$ BC = 2AB $\Rightarrow$ AD = 2AB

58.(c)



Let BD = 6cm and AC = 8m

$\therefore$ AO = 4m and BO = 3m

let AB = 5m $\therefore$ $\angle$AOB = 90°

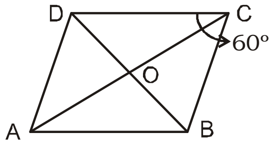
$\Rightarrow$ $\angle$BOC = $\angle$AOD = $\angle$DOC = 90°

Here, ABCD is a rhombus

$\therefore$ Area of rhombus ABCD

$= \dfrac{AC \times BD}{2} = \dfrac{6 \times 8}{2} = 24m^2$

59.(a)



In $\triangle$ BDC, BC = CD

$\Rightarrow$ Let, $\angle BDC = \angle DBC = x°$

$\therefore x + x + 60° = 180° \Rightarrow x = 60°$

$\therefore \triangle$ BDC is an equilateral triangle

$\therefore$ BD = BC = a

In $\triangle$AOB, $\angle AOB = 90°$

$\therefore AB^2 = OA^2 + OB^2$

$\Rightarrow OA^2 = AB^2 - OB^2$

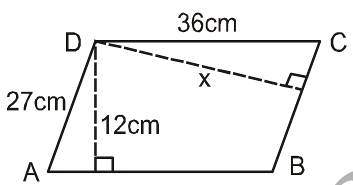
$\Rightarrow OA^2 = a^2 - \left(\dfrac{a}{2}\right)^2$

$\Rightarrow OA = \dfrac{\sqrt{3}a}{2}$

$\Rightarrow AC = 2(OA) = \sqrt{3}a$

$\therefore AC : BD = \sqrt{3}a : a = \sqrt{3} : 1$

60.(b)



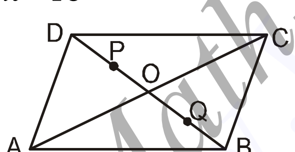
let distance = x

Area of | | gm = Base $\times$ Height

$36 \times 12 = x \times 27$

$\Rightarrow x = 16$

61.(d)



Since, diagonals of a | | gm bisect each other.

$\therefore BO = OD = \dfrac{18}{2} = 9cm$

P = centroid of ΔADC

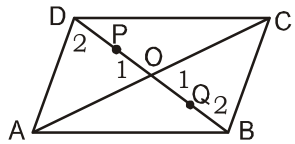
$\therefore OP = \dfrac{1}{3} OD = \dfrac{1}{3} \times 9 = 3cm$

Q = centroid of ΔABC

$\therefore OQ = \dfrac{1}{3} OB = \dfrac{1}{3} \times 9 = 3cm$

$\therefore$ PQ = OP + OQ = 3 + 3 = 6cm

Alternatively

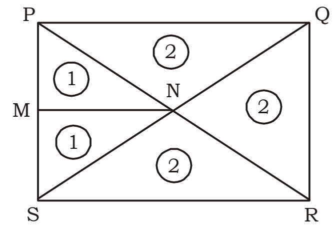


6 units = 18

1 unit = 3

$\therefore$ 2 unit = 6

62.(b) Let ar($\square$ PQRS) = 8 units



Area of PQNM = area of $\triangle$ PNQ + area of $\triangle$ PNM

= 2 + 1 = 3 units

$\therefore$ area ($\square$PQNM) : area ($\square$PQRS)

= 3 : 8

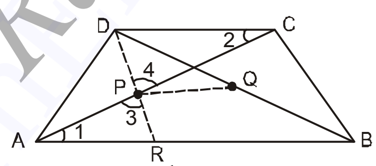
63.(a) AE : EO = 2 : 1 and CF : FO = 2 : 1

$\therefore OE = \dfrac{1}{3} AO$ and $OF = \dfrac{1}{3} OC$

$\therefore$ EF = OE + OF

$= \dfrac{1}{3}(AO + OC) = \dfrac{1}{3} AC = AE$

64.(c)



In $\triangle$ APR and $\triangle$ DPC,

$\angle 1 = \angle 2$ (alternate angles)

AP = CP ($\because$ P is mid-point of AC)

and $\angle 3 = \angle 4$ (vertically opposite angles)

So, $\triangle APR \cong \triangle DPC$ (ASA)

$\Rightarrow$ AR = DC and PR = DP

Again, P & Q are the mid-points of sides DR and DB respectively.

In $\triangle$ DRB,

$PQ = \dfrac{1}{2} BR$

$\therefore PQ = \dfrac{1}{2}(AB - AR)$

$\therefore PQ = \dfrac{1}{2}(AB - CD)$

($\because$ AR = DC)

65.(a) Area of ($\triangle$ PRS + $\triangle$ PQR) = $\dfrac{1}{2}$ (area of $\square$ APRD) $+ \dfrac{1}{2}$(area of $\square$BPRC)

$= \dfrac{1}{2}(AP \times AD) + \dfrac{1}{2}(PB \times BC)$

$= \dfrac{1}{2}(AP \times AD) + \dfrac{1}{2}(PB \times AD)$ ($\because$ BC = AD)

$= \dfrac{1}{2} AD\,(AP + PB)$

$= \dfrac{1}{2}(AD \times AB)$

$= \dfrac{1}{2}$(area of $\square$ ABCD)

$= \dfrac{1}{2} \times 20 = 10cm^2$

66.(d) Let BC = x and FB = EF = AE = y

$\therefore$ AB = CD = 3y

Now, Area of

$\triangle CBF = \dfrac{1}{2} xy$

and area of $\triangle$ CBE =

$\dfrac{1}{2} x \times 2y = xy$

$\therefore$ area of $\triangle$ CEF

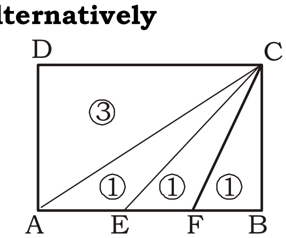
$= xy - \dfrac{1}{2} xy = \dfrac{1}{2} xy$

and area of rectangle ABCD = 3xy

$\therefore$ Required ratio = $\dfrac{1}{2} xy : 3xy$

= 1 : 6

Alternatively



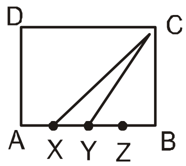
Let ar($\square$ ABCD) = 6 units

Base and height are same

ar(Δ CAE) = ar(Δ CEF) = ar(Δ CFB) = 1 unit

$\therefore$ required ratio = 1 : 6

67.(d)



Let, AB = 4x units and
BC = y units

$\therefore$ $\square$ ABCD = 4xy sq. units

In $\triangle$XYC

XY = x units

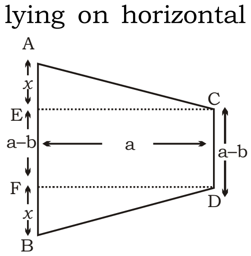
Height = y units

$\therefore$ Area of $\Delta XYC = \frac{1}{2}xy$

$\therefore \frac{\text{Ar}(\Delta \text{ XYC})}{\text{Ar (Rectangle ABCD)}} = \frac{\frac{1}{2}xy}{4xy} = \frac{1}{8}$

68.(c) Calculate them manually

69. (d) They are symmetrically lying on horizontal place.



$\therefore$ AC = BD

$\therefore$ AE = BF = x (say)

Now, AB = (a–b)+2x

i.e., a+b=a–b+2x $\Rightarrow$

2b =2x

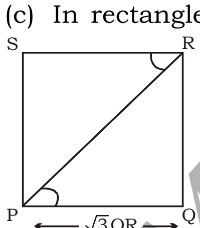
$\therefore$ x = b

Now In $\triangle$ ACE,

x^2+a^2 = AC²

$\Rightarrow$ AC² = b² +a² $\Rightarrow$ AC = $\sqrt{b^2+a^2}$

70. (c) In rectangle PQRS,



PQ || RS

$\therefore$ $\angle$RPQ = $\angle$PRS (alternate angles) ....(i)

Now In $\triangle$ PQR,

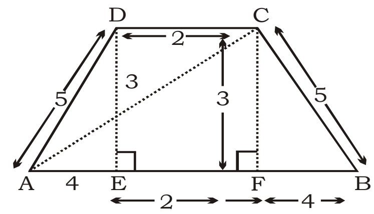
$\tan \angle QPR = \frac{RQ}{PQ}$

$\Rightarrow \tan \angle QPR = \frac{QR}{\sqrt{3}QR}$

$\Rightarrow$ $\angle$QPR = 30°

$\therefore$ $\angle$PRS = 30° [from Eq. (i)]

71. (b) In $\triangle$ BCF,



By Pythagoras theorem,

(5)² = (3)² +(BF)² $\Rightarrow$ BF = 4 cm

$\therefore$ AB = 2+4+4=10 cm

Now In $\triangle$ ACF,

AC² = CF²+FA²

$\Rightarrow$AC²=3²+6², AC = $\sqrt{45}$ cm

Similarly, BD = $\sqrt{45}$ cm

$\therefore$ Sum of diagonal = 2× $\sqrt{45}$ = 2 × 3$\sqrt{5}$ = 6$\sqrt{5}$ cm

72. (b) Area of rectangle lies between 40 cm² and 45 cm²

Now, one side = 5 cm

Since, area cannot be less than 40 cm².

$\therefore$ Other side cannot be less than

= $\frac{40}{5}$ = 8 cm

Since, area cannot be greater than 45 cm²

$\therefore$ Other side cannot be greater than = $\frac{45}{5}$ =9cm

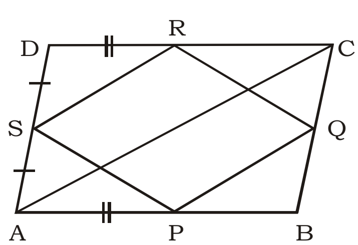
$\therefore$ Minimum value of diagonal = $\sqrt{8^2+5^2}$ = $\sqrt{89}$

Maximum value of diagonal

= $\sqrt{9^2+5^2}$ = $\sqrt{106}$ = 10.3 cm

So, diagonal lies between 9 cm and 11 cm.

73. (b) Area of $\triangle$ APS = Area of $\triangle$ DSR

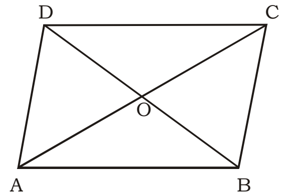


$\because$ AS = SD and AP = DR

$\therefore$ ar ($\triangle$ ABC) = 4 ar ($\triangle$ BPQ)

74. (b) I. ABCD is parallelogram, then

AC² +BD²=2(AB²+BC²)



II. ABCD is a rhombus and diagonals AC and BD bisect each other

$\therefore$ AO = OC

and OB = OD

In $\triangle$ AOB, AB² = AO² + OB²

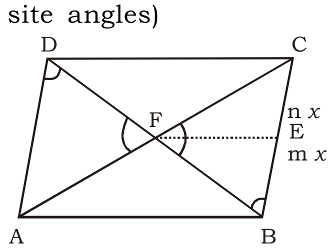
$(4)^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$

$\therefore$ AC²+BD²=64

= (4)³ i.e., n³

75. (d) In $\triangle$ AFD and $\triangle$ BFE,

$\angle$AFD = $\angle$BFE (vertically opposite angles)



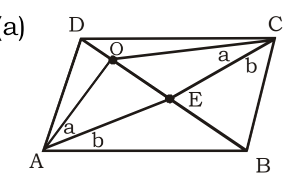
and $\angle$ADF = $\angle$FBE (alternate angles)

$\therefore$ $\triangle$ AFD ~ $\triangle$ BFE

So, $\frac{ar(\Delta FEB)}{ar(\Delta AFD)} = \frac{EB^2}{AD^2}$

$= \frac{m^2x^2}{(mx+nx)^2} = \frac{m^2}{(m+n)^2} = \left[\frac{m}{(m+n)}\right]^2$

76. (a)



Line joining A and C cuts BD at E.

AC diagonals of parallelogram bisects each other,

OE and BE will be median of $\triangle$ OAC and $\triangle$ ABC respectively,

Let Area of $\triangle$ OAE = Area of $\triangle$ OEC = a

Then,

Area of $\triangle$ AEB = Area of $\triangle$ EBC = b

Area of $\triangle$ OAB = Area of $\triangle$ OAE + Area of $\triangle$ AEB = (a + b)

Similarly,

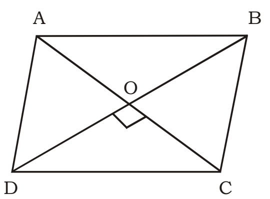
Area of $\triangle$ OBC = Area of $\triangle$ OEC + Area of $\triangle$ EBC = (a + b)

Hence, if O is any point on diagonal BD in parallelogram ABCD then,,

Area of ΔOAB = Area of $\triangle$ OBC

Area of OBC = 19 cm²

77. (b)



Here, OD = $\frac{BD}{2} = \frac{4.8}{2} = 2.4 \Rightarrow$ OC

$= \frac{AC}{2} = \frac{1.4}{2} = 0.7$

Since, In rhombus diagonal bisect at 90°. Then, In $\triangle$ ODC.

$OD^2+OC^2=CD^2$

$\Rightarrow CD = \sqrt{OD^2+OC^2} = \sqrt{(2.4)^2+(0.7)^2}$

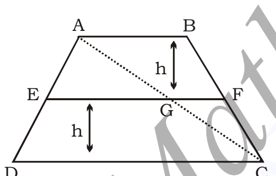
$CD = \sqrt{6.25} \Rightarrow CD = 2.5 = \frac{5}{2}$

$\therefore$ Perimeter of rhombus = 4a

$= 4\times\frac{5}{2} = 10$ cm

78. (c) Join AC.

In $\triangle$ ACD, EG $\parallel$ DC and E and G are mid-points of AD and AC, respectively.



$\therefore$ EG = $\frac{1}{2}$DC = $\frac{3}{2}$

Similarly, In $\triangle$ ABC

GF = $\frac{1}{2}$ AB = 1

EF = EG + GF = $1+\frac{3}{2} = \frac{5}{2}$

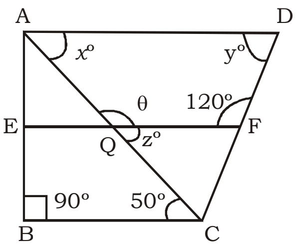
$\therefore$ Area of trapezium = $\frac{1}{2}$ (Sum of parallel sides×Height)

Now, Required ratio

$= \frac{\text{Area of ABFE}}{\text{Area of EFCD}} = \frac{\frac{1}{2}\left(2+\frac{5}{2}\right)\times h}{\frac{1}{2}\left(3+\frac{5}{2}\right)\times h}$

$= \frac{9}{11}$

79. (c) From figure,



BC $\parallel$ EF $\parallel$ AD

$\therefore x° = z°$ = 50°(corresponding interior angle)

$\therefore$ $\theta + z°$ = 180° (linear pair)

θ = 180° – 50° = 130°

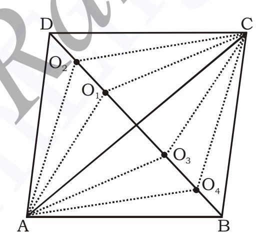
In quadrilateral AQFD,

$x°+y° + 120° + \theta$ = 360°

50° + y° + 120° + 130° = 360°

y = 360° – 300° = 60°

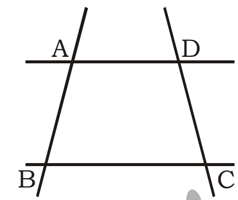
80. (a)



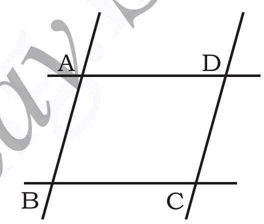
We know that, in a rhombus ABCD diagonals are bisect each other at point O. Which means that the distance of O from four vertices of rhombus i.e., A, B, C and D are equal. Even, If we take any fixed point on diagonal BD of rhombus ABCD and join with vertices A and C, we get which is equidistant from A and C. (by property of congruent triangle). Hence, the locus of a point in rhombus ABCD which is equidistant from A and C is a fixed point on diagonal BD.

81. (d) If two parallel lines are cut by two distinct transversals, then the quadrilateral formed by the four lines is always a Trapezium'.

Case I. If two distinct transversals (are not parallel), then always $\rightarrow$ (Trapezium)



Case II. It two distinct transversals are parallel, then always (Trapezium+Parallelogram)



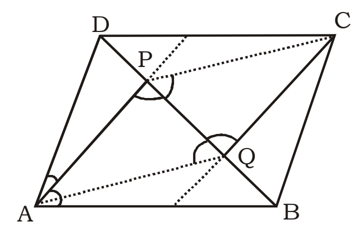
82. (b) Line segment

AP and CQ bisects the $\angle$A and $\angle$C, respectively.

Then, AP $\parallel$ CQ

Now, In $\triangle$ APQ and $\triangle$ CQP,

$\because$ AP $\parallel$ QC



$\because$ $\angle$APQ = $\angle$PQC (alternate angle)

PQ = PQ (common)

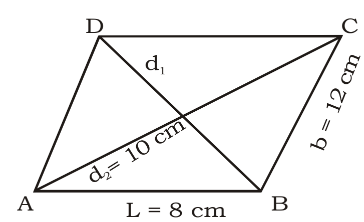
Also, PC $\parallel$ AQ

$\therefore$ $\angle$CPQ = $\angle$PQA (alternate angle)

$\therefore$ $\triangle$ APQ ~ $\triangle$ CQP (by ASA)

Hence, $\triangle$ APQ is similar to $\triangle$ PCQ.

83. (d) In parallelogram, $d_1^2+d_2^2 = 2(l^2+b^2)$



$\therefore$ $d^2+(10)^2 = 2\,(64+144)$

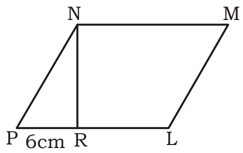
$\Rightarrow d^2 = 2 \times 208 - 100$

$\Rightarrow d^2 = 416 - 100 = 316 \Rightarrow d = \sqrt{316}$

$\Rightarrow d = 17.76$ cm

$\therefore$ $d > 12$

84. (b) By given condition,



Area of parallelgram = 6 × Area of $\triangle$ NPR

$\therefore$ NR × PL = 6 × $\frac{1}{2}$ × NR × PR

$\Rightarrow$ PL = 3PR (here, PL = PR + RL)

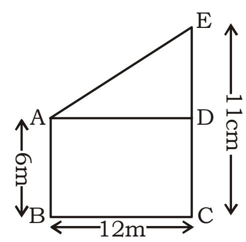
$\Rightarrow$ PR + RL = 3PR

$\Rightarrow$ RL = 2PR = 2 × 6 = 12cm

85. (c) AD = BC = 12m

and ED = 11 – 6 = 5cm

Since, AE is distance between top point of AB and CE.



In $\triangle$ ADE,

$AE^2 = AD^2 + ED^2 = 12^2 + 5^2$

= 144 + 25 = 169

$\therefore$ AE = 13m

Hence, the distance between their tops = 13m

86. (a) AB||DC and AD||BC

In $\triangle$ ABE,

$\angle$EAB = $\angle$ABE = 60°

$\Rightarrow$ $\angle$AEB = 60°

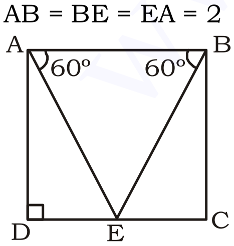
$\Rightarrow$ $\triangle$ ABE is an equilateral triangle.

Now, perimeter of $\triangle$ ABE = 6

$\Rightarrow$ AB + BE + EA = 6

$\Rightarrow$ AB = 2 units

AB = BE = EA = 2



and In $\triangle$ ADE, $AE^2 = AD^2 + ED^2$

$\Rightarrow$ $4 = AD^2 + 1$ (since, E is mid-point of CD)

$\Rightarrow$ AD = $\sqrt{3}$ units

Hence, area of quadrilateral ABCD = AB × AD

= 2 × $\sqrt{3}$ = 2× $\sqrt{3}$ sq units

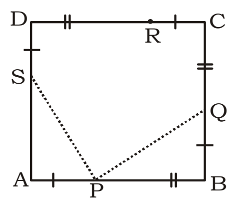
87. (d) We know that, ratio of the areas of two similar triangles is equal to the ratios of squares of their corresponding sides.

$$\therefore \frac{\text{Area}(\triangle BPQ)}{\text{Area}(\triangle DPC)} = \frac{PB^2}{PC^2}$$

$$\Rightarrow \frac{20}{\text{Area}(\triangle DPC)} = \frac{1}{4}$$

$\Rightarrow$ Area ($\triangle$ DPC) = 80 sq units

88. (d) In $\triangle$ APS and $\triangle$ PBQ,



PB = AS

AP = BQ (given)

and $\angle$A = $\angle$B = 90°

(since, ABCD is a square)

So, $\triangle$ APS and $\triangle$ PBQ are congruent.

$\therefore$ SP = PQ

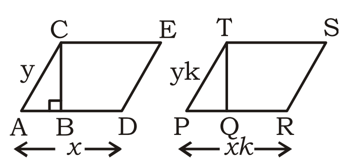
$\angle$SPA = $\angle$BQP and

$\angle$ASP = $\angle$BPQ

$\therefore$ $\angle$SPQ = 90° (by RHS rule)

$\triangle$ APS $\cong$ $\triangle$ PBQ

89. (c) Let the sides of a parallelogram are x, y and xk, yk.



Since, sides of two parallelogram are in 1 : k

$\therefore$ $\triangle$ ABC ~ $\triangle$ PQT

$$\Rightarrow \frac{AC}{PT} = \frac{BC}{QT} \Rightarrow \frac{BC}{QT} = \frac{y}{yk} = \frac{1}{k}$$

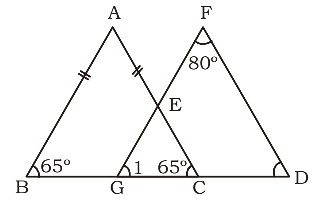
Let BC = z and QT = zk

$\therefore$ Ratio of areas of two similar parallelograms

$$= \frac{x \times z}{xk \times zk} = \frac{1}{k^2}$$

90. (d) $\angle$B = $\angle$C = 65°

Here, GF||AB, Which is intersects



Now $\angle$B = $\angle$G

$\angle$1 = $\angle$B = 65°

(corresponding angles)

In $\triangle$ FGD,

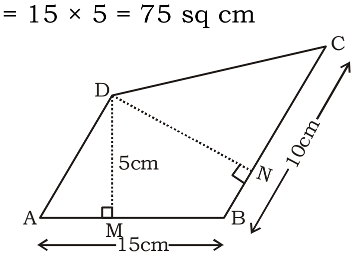
$\angle$1 + $\angle$F + $\angle$D = 180°

$\Rightarrow$ 65° + 80° + $\angle$D = 180°

$\Rightarrow$ $\angle$D = 35°

91. (b) Area of parallelogram = Base × Height

= 15 × 5 = 75 sq cm



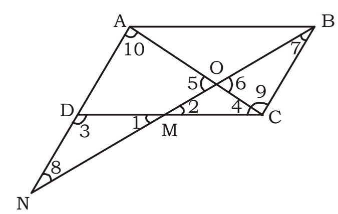
Area of parallelogram = Base × Height = 10 × DN

$\Rightarrow$ 10 × DN = 75

$\therefore$ DN = $\frac{75}{10}$ = 7.5 cm

92. (b) In $\triangle$ DMN and $\triangle$ BMC,

DM = MC (mid-point) (given)



$\angle$1 = $\angle$2

Since, BC||AD and intersects by CD.

$\triangle$ DMN $\cong$ $\triangle$ BMC

DN = BC = AD

So, AN = 2BC $\Rightarrow \frac{AN}{BC} = \frac{2}{1}$

In $\triangle$ OAN and $\triangle$ OBC,

$\angle$5 = $\angle$6

(vertically opposite angle)

$\angle$7 = $\angle$8

(alternate interior angle)

$\angle$9 = $\angle$10 (alternate interior angle)

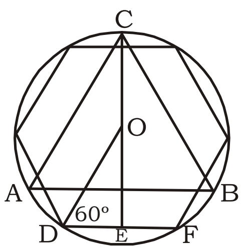
$\therefore$ $\triangle OAN \sim \triangle OBC$

So, the sides will be in same ratio

$\frac{AN}{BC} = \frac{ON}{OB}$

$\Rightarrow \frac{2}{1} = \frac{ON}{OB}$ [from Eq. (i)]

93. (d) We know altitude of equilateral $\triangle ABC$ is $\frac{\sqrt{3}}{2}$a.



$\therefore$ Length of OC = $\frac{\sqrt{3}}{2} a \times \frac{2}{3} = \frac{a}{\sqrt{3}}$ = radius

Also DF = b $\Rightarrow$ DE = $\frac{b}{2}$

In $\triangle ODE$, $\cos 60° = \frac{DE}{OD}$

$\frac{1}{2} = \frac{b/2}{a/\sqrt{3}}$

$\Rightarrow \frac{1}{2} = \frac{\sqrt{3}b}{2a} \Rightarrow a = \sqrt{3}b$

$\therefore a^2 = 3b^2$

Alternatively

Circum radius of equilateral triangle = $\frac{a}{\sqrt{3}}$

Circum radius of regular hexagon = side(b)

$\frac{a}{\sqrt{3}} = b$

$a = \sqrt{3}\,b \Rightarrow a^2 = 3b^2$

94. (c) $(50)^2 = (30)^2 + (40)^2$

$\Rightarrow 2500 = 900 + 1600$

$\Rightarrow 2500 = 2500$

It means given scores are the sides of a rectangle. So, other diagonal should be 50 runs.

95. (b) ABCD is a trapezium.

$\therefore$ AD||BC and EF||BC (given)

Hence, EF||AD

$\therefore \angle x + \angle y = 180°$

(interior angles)

$\therefore \angle y = 180° - 120° = 60°$

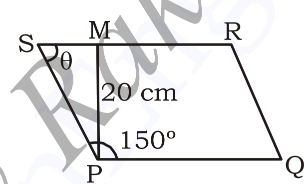
96. (a) Given that, $\angle SPQ = 150°$ and PM = 20 cm in parallelogram PQRS,

$\angle RSP + \angle SPQ = 180°$

(interior angles)

$\Rightarrow \angle RSP = 180° - 150° = 30°$

$\Rightarrow \theta = 30°$



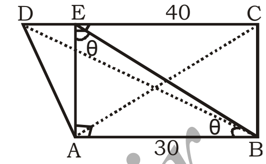
In $\triangle PSM$,

$\sin\theta = \sin 30° = \frac{PM}{SP}$

$\Rightarrow \frac{1}{2} = \frac{20}{SP} \Rightarrow SP = 40cm$

$\therefore$ RQ = SP = 40 cm

97.



In trapezium diagonal intersect proportionally

$\therefore$ ABCD is a trapezium where DC$\|$AB

Area of trapezium ABCD = $\frac{1}{2}$(AB +CD)×AE

$1050 = \frac{1}{2}(30+40) \times AE$

$\Rightarrow$ AE = 30cm.

(a) $\angle EAB = 90°$ (AB$\|$DC)

In right $\triangle EAB$,

$BE = \sqrt{AE^2 + AB^2} = \sqrt{30^2 + 30^2}$

$= 30\sqrt{2}cm$

(b). Area of $\triangle ADC = \frac{1}{2} \times CD \times AE$

$= \frac{1}{2} \times 40 \times 30 = 600cm^2$.

(c). In $\triangle AEB$, AE = AB

so $\angle AEB = \angle EBA$

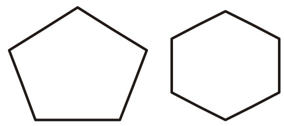
$2\theta = 90° \Rightarrow \theta = 45°$

CHAPTER 19

POLYGON

POLYGONS

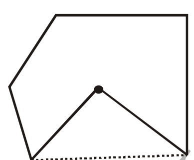
A closed-figure bounded by three or more than three straight lines.

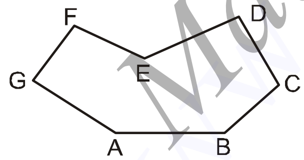


e.g.

No. of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octogon
9	Nonagon
10	Decagon

- **Convex Polygon:-** A polygon in which none of its interior angle is more than 180° and all line inside the figure, is known as a 'convex polygon'.
- **Concave Polygon:-** A polygon in which atleast one interior angle is more than 180° and at least one diagonal lies outside the figure , then it is said to be 'concave'.





- **Regular Polygon:-** A polygon in which all the sides are equal and also the interior angles are equal, is called a 'Regular polygon'.

  If **n** = total no. of sides of a regular polygon, then:-

1. Sum of interior angles $= (n-2) \times 180°$

2. Each exterior angle $= \left(\dfrac{360°}{n}\right)$

3. Sum of all exterior angle $= 360°$

4. Each interior angle = 180°- exterior angle

$\Rightarrow$ interior angle + exterior angle = 180°

5. Number of diagonals $= \dfrac{n(n-3)}{2}$

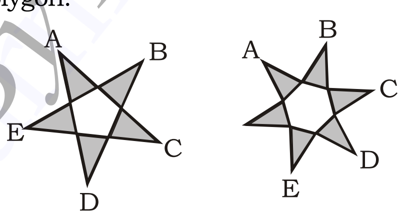
6. Perimeter = $\mathbf{n \times a}$

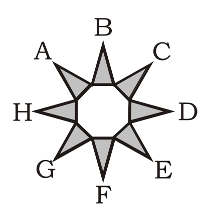
where a = length of side

7. Area of regular polygon = $\dfrac{na^2}{4} \times \cot\left(\dfrac{180}{n}\right)^0$

Star

Sum of outer angles of a star Star forms by extending sides of a regular polygon.





Let outer triangles are 'n' then sum of outer angles = n × 180 – two times sum of exterior angles = (n × 180 – 2 × 360) = 180 (n – 4)

If n = 5 then $\angle A + \angle B + \ldots + \angle E$ = 180 (5 – 4) = 180°

If n = 6 then $\angle A + \angle B + \ldots + \angle F$ = 180 (6 – 4) = 360°

If n = 8 then $\angle A + \angle B + \ldots + \angle H$ = 180 (8 – 4) = 720°

Some important points about Polygon.

- A convex polygon in which there is maximum number of sides, it has the greater enclosed area when the perimeter of the polygon is constant.

Note : Which of the following figures will have maximum area if the perimeter of all figures is same

Circle > Octagon> Hexagon> Square> Rhombus

Circle is a polygon with infinite sides of minimum possible length.

Example

1. Each interior angle of a regular polygon is 140°. The number of sides is:

Sol. Each interior angle = 140°

$\therefore$ Exterior angle = 180° – 140° = 40°

$\therefore$ Number of sides = $\dfrac{360°}{\text{Exterior angle}} = \dfrac{360°}{40°} = 9$

2. Each interior angle of a regular hexagon is:

Sol. Exterior angle of a regular hexagon = $\dfrac{360°}{6} = 60°$

Interior angle = 180° – 60° = 120°

3. If one of the interior angles of a regular polygon is equal to 5/6 times of one of the interior angles of a regular pentagon, then the number of sides of the polygon is:

Sol. Exterior angle of pentagon = $\dfrac{360°}{5} = 72°$

$\therefore$ Interior angle of pentagon = 180° – 72° = 108°

$\therefore$ Interior angle of required polygon $= \dfrac{5}{6} \times 108° = 90°$

$\therefore$ Each exterior angle of the required polygon = 180°– 90° = 90°

∴ Number of sides

$$= \frac{360^\circ}{Exterior\ angle} = \frac{360^\circ}{90^\circ} = 4$$

4. The sum of the interior angles of a polygon is 1260°. The number of sides of the polygon is:

Sol. Sum of all the interior angles = $(n-2)\ 180^\circ$

$\Rightarrow$ $(n-2)\ 180^\circ = 1260$

$\Rightarrow$ $n - 2 = 7$

$\Rightarrow$ $n = 9$ ($n \rightarrow$ number of sides)

5. If each interior angle of a regular polygon is 3 times its exterior angle, the number of sides of the polygon is:

Sol. Interior angle = 3 × exterior angle

Interior angle : exterior angle

3 : 1

Interior angle + exterior angle = 180°

$3x + x = 180^\circ$

$4x = 180^\circ$

$x = 45^\circ$

exterior angle = 45°

$$\text{Number of sides} = \frac{360^\circ}{\text{exterior angle}}$$

$$= \frac{360^\circ}{45}$$

Number of sides = 8

6. Difference between the interior and exterior angles of regular polygon is 60°. The number of sides in the polygon is:

(a) 5 (b) 6

(c) 8 (d) 9

Sol. Go through option

$$\text{Exterior angle} = \frac{360^\circ}{6} = 60^\circ$$

Interior angle = 180° – 60° = 120°

Difference between interior angle and exterior angle

= 120° – 60° = 60°

Alternate

$x + y = 180^\circ$........(i)

$x - y = 60^\circ$ ....(ii)

Solve equation (i) and (ii) we get

$x = 120^\circ$ and $y = 60^\circ$

$$\therefore \quad \text{Number of sides} = \frac{360^\circ}{60^\circ} = 6$$

7. A polygon has 54 diagonals. The number of sides in the polygon is:

(a) 7 (b) 9 (c) 12

(d) None of these

Sol. $\frac{n(n-3)}{2} = 54$

$\Rightarrow$ $n^2 - 3n = 108$

$\Rightarrow$ $n^2 - 3n - 108 = 0$

$\Rightarrow$ $n^2 - 12n + 9n - 108 = 0$

$\Rightarrow$ $n(n - 12) + 9(n-12) = 0$

$\Rightarrow$ $(n + 9)(n - 12) = 0$

$\Rightarrow$ $n = 12,\ n \neq -9$

No. of side always + ve

So, number of sides = 12

Alternate

$\frac{n(n-3)}{2} = 54$

$n(n - 3) = 108$

We take option (a) = 12

$12 \times (12 - 3) = 12 \times 9 = 108$

satisfy the equation

So, number of sides = 12

8. The ratio between the number of sides of two regular polygon 1 : 2 and the ratio between their interior angle is 3 : 4. The number of sides of these polygons are respectively:

(a) 3, 6 (b) 4, 8

(c) 6, 9 (d) 5, 10

Sol. Go through options. Let us consider the correct option (d).

Number of sides 5		:	10
Exterior angle = 72°		:	36°
Interior angle = 108°		:	144
	3	:	4

Hence, option (d) is correct.

Alternatively:-

Let side = n & 2n

$$\frac{\frac{(n-2)180}{n}}{\frac{(2n-2)180}{2n}} = \frac{3}{4}$$

$$\frac{2n-4}{2n-2} = \frac{3}{4}$$

$8n - 16 = 6n - 6$

$n = 5$

$2n = 10$

9. The sum of the all interior angles of a regular polygon is four times the sum of its exterior angles. The polygon is:

(a) hexagon (b) triangle

(c) decagon (d) nonagon

Sol. Sum of a exterior angles = 360°

∴ Sum of interior angles = 4 × 360° = 1440°

∴ $1440^\circ = (n-2)\ 180^\circ$

$\Rightarrow n = 10$ (no. of sides)

∴ The polygon is decagon

10. The ratio of the measure of an interior angle of a regular nonagon to the measure of its exterior angle is:

(a) 3 : 5 (b) 5 : 2

(c) 7 : 2 (d) 4 : 5

Sol. $\text{Exterior angle} = \frac{360^\circ}{9} = 40^\circ$

Interior angle 180° – 40° = 140°

Interior angle : exterior angle = 140° : 40° = 7 : 2

EXERCISE

1. Raju has drawn an angle of measure 45°27' when he was asked to draw an angle of 45°. The percentage error in his drawing is
(a) 0.5% (b) 1.0%
(c) 1.5% (d) 2.0%
2. In a regular polygon, the exterior and interior angles are in the ratio 1 : 4. The number of sides of the polygon is
(a) 5 (b)10 (c) 3 (d) 8
3. The difference between the interior and exterior angles at a vertex of a regular polygon is 150°. The number of sides of the polygon is
(a) 10 (b) 15 (c) 24 (d) 30
4. Each interior angle of a regular polygon is 144°. The number of sides of the polygon is
(a) 8 (b) 9 (c) 10 (d) 11
5. If the sum of the interior angles of a regular polygon be 1080°, the number of sides of the polygon is
(a) 6 (b) 8 (c) 10 (d) 12
6. The number of sides in two regular polygons are in the ratio of 5 : 4. The difference between their Interior angles of the polygon is 6°. Then the number of sides are
(a) 15, 12 (b) 5, 4
(c) 10, 8 (d) 20 , 16
7. Each internal angle of regular polygon is two times its external angle. Then the number of sides of the polygon is :
(a) 8 (b) 6 (c) 5 (d) 7
8. Ratio of the number of sides of two regular polygons is 5 : 6 and the ratio of their each interior angle is 24 : 25. Then the number of sides of these two polygons are
(a) 10,12 (b) 20,24
(c) 15,18 (d) 35,42
9. Measure of each interior angle of a regular polygon can never be:
(a) 150° (b) 105°
(c) 108° (d) 144°
10. Each interior angle of a regular polygon is three times its exterior angle, then the number of sides of the regular polygon is:
(a) 9 (b) 8 (c) 10 (d) 7
11. The sum of all interior angles of a regular polygon is twice the sum of all its exterior angles. The number of sides of the polygon is
(a) 10 (b) 8 (c) 12 (d) 6
12. The ratio between the number of sides of two regular polygons is 1 : 2 and the ratio between there interior angles is 2 : 3. The number of sides of these polygons is respectively
(a) 6, 12 (b) 5 ,10
(c) 4, 8 (d) 7, 14
13. Each internal angle of regular polygon is two times its external angle. Then the number of sides of the polygon is:
(a) 8 (b) 6 (c) 5 (d) 7
14. The sum of interior angles of a regular polygon is 1440°. The number of sides of the polygon is
(a) 10 (b) 12 (c) 6 (d) 8
15. An interior angle of a regular polygon is 5 times its exterior angle. Then the number of sides of the polygon is
(a) 14 (b) 16 (c) 12 (d) 18
16. In a regular polygon, if one of its internal angle is greater than the external angle by 132°, then the number of sides of the polygon is
(a) 14 (b) 12 (c) 15 (d) 16
17. If the ratio of an external angle and an internal angle of a regular polygon is 1 : 17, then the number of sides of the regular polygon is
(a) 20 (b) 18 (c) 36 (d) 12
18. The ratio of each interior angle to each exterior angle of a regular polygon is 3:1. The number of sides of the polygon is:
(a) 6 (b) 7 (c) 8 (d) 9
19. The interior angle of regular polygon exceeds its exterior angle by 108°. The number of sides of the polygon is
(a) 10 (b) 14 (c) 12 (d) 16
20. If the sum of all interior angles of a regular polygon is 14 right angles, then its number of sides is
(a) 6 (b) 8 (c) 9 (d) 7
21. How many diagonals are there in a octagon ?
(a) 10 (b) 14 (c)18 (d) 20
22. A polygons has 44 diagonals. The number of sides of the polygon is :
(a) 11 (b) 10 (c)13 (d) 12
23. The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4, the largest angle is :
(a) 120° (b) 134°
(c) 144° (d) 150°
24. Each interior angle of a regular polygon is 120°. The number of sides is :
(a) 7 (b) 6 (c) 5 (d) 8
25. Each interior angle of a regular octagon is :
(a) 120° (b) 90°
(c) 135° (d) None of these
26. The sum of the interior angles of polygon is 1440°. the number of sides of the polygon is :
(a) 9 (b) 10 (c)8 (d) 12
27. One angle of a pentagon is 140°. If the remaining angles are in the ratio 1 : 2 : 3 : 4, the size of the greatest angle is :
(a) 150° (b) 180°
(c) 160° (d) 170°
28. Each interior angle of a regular polygon is 144°. The number of sides of the polygon is :
(a) 8 (b) 12 (c)10 (d) 11
29. A regular polygon is inscribed in a circle. If a side subtends an angle of 36° at the centre, then the number of sides of the polygon is :
(a) 5 (b) 10 (c) 12 (d) 9
30. The difference between an exterior angle of (n - 1) sided regular polygon and an exterior angle of (n + 2) sided regular polygon is 6°, then the value of n is :
(a) 15 (b) 14 (c)12 (d) 13
31. If a regular polygon has each of its angles equal to $\frac{3}{5}$ times of two right angles, then the number of sides is
(a) 3 (b) 5 (c) 6 (d) 8

ANSWER KEY

1. (b)	4. (c)	7. (b)	10. (b)	13. (b)	16. (c)	19. (a)	22. (a)	25. (c)	28. (c)
2. (b)	5. (b)	8. (a)	11. (d)	14. (a)	17. (c)	20. (c)	23. (c)	26. (b)	29. (b)
3. (c)	6. (a)	9. (b)	12. (c)	15. (c)	18. (c)	21. (d)	24. (b)	27. (c)	30. (d)
									31. (b)

SOLUTION

1. (b) According to question

Angle of measure = 45° 27'

$= 45° + \frac{27}{60}$

Asked to draw an angle of = 45°

Error $= 45° + \frac{27}{60} - 45° = \frac{27}{60}$

Error % $= \frac{\left(\frac{27}{60}\right)}{45} \times 100$

$= \frac{27}{60 \times 45} \times 100 = 1.0\%$

2. (b) According to question

$\frac{\text{Exterior angle}}{\text{Interior angle}} = \frac{1}{4} = \frac{x}{4x}$

As we know that

Interior angle + Exterior angle = 180°

$\therefore\ x + 4x = 180°$

$5x = 180°$

$x = 36°$

$\therefore$ No. of sides

$= \frac{360°}{\text{Exterior angle (बाह्य कोण)}}$

No. of sides $= \frac{360^0}{36^0} = 10$

No. of sides = 10

3. (c) According to question

Given :

Interior angle – Exterior angle = 150° ....(i)

We know,

Interior angle + Exterior angle = 180° ....(ii)

Solve equation (i) and (ii)

Interior angle = 165°

Exterior angle = 15°

$\therefore$ No. of sides $= \frac{360°}{Exterior\ angle}$

$= \frac{360°}{15°} = 24$

4. (c) According to question

Given:

Interior angle = 144°

Exterior angle = 180° – 144°

= 36°

$\therefore$ no. of sides $= \frac{360°}{Exterior\ angle}$

$= \frac{360°}{36°} = 10$

5. (b) According to question

Sum of interior angle

= (n – 2) × 180°

Given:

Sum of interior angle = 1080°

(n – 2) × 180° = 1080°

$(n - 2) = \frac{1080°}{180}$

(n – 2) = 6

n = 6 + 2 = 8

No. of sides n = 8

6. (a) Let the no. of sides is $5x$ and $4x$

According to questions

$\left(180° - \frac{360°}{5x}\right) - \left(180° - \frac{360°}{4x}\right) = 6°$

$180° - \frac{360°}{5x} - 180° + \frac{360°}{4x} = 6°$

$\frac{360°}{4x} - \frac{360°}{5x} = 6°$

$360°\left(\frac{1}{4x} - \frac{1}{5x}\right) = 6°$

$\frac{1}{20x} = \frac{1}{60}, \quad x = 3$

No. of sides is $5x$ and $4x$ = 15,12

7. (b) According to questions

Given:

Internal Angle = 2 (External Angle)

As we know that

Internal Angle + External Angle = 180°

$\therefore$ 2 External Angle + External Angle = 180°

3 External Angle = 180°

External Angle $= \frac{180°}{3} = 60°$

No. of sides $= \frac{360°}{External\ angle}$

$= \frac{360°}{60°} = 6$ (no. of sides)

8. (a) Let the number of sides be $5x$ and $6x$

As we know that

Each interior angle

$= \frac{(2n - 4) \times 90°}{n}$

Given:

$\frac{n_1}{n_2} = \frac{5x}{6x}$

$\frac{\text{Interior angle}_1}{\text{Interior angle}_2} = \frac{24}{25}$

$\therefore$ Using Interior angle formula

$\frac{\frac{(n_1 - 2)180°}{n_1}}{\frac{(n_2 - 2)180°}{n_2}} = \frac{24}{25}$

$\frac{\frac{5x - 2}{5x}}{\frac{6x - 2}{6x}} = \frac{24}{25}$

$x = 2$

Then, No. of sides = 5 × 2 = 10,

6 × 2 = 12

= 10,12

9. (b) According to question

$n \rightarrow$ No. of sides

Interior angle

$= \frac{(2n - 4) \times 90}{n} = 180 - \frac{360}{n}$

(a) $150° = 180° - \frac{360°}{n}$

$$\frac{360°}{n} = 30°$$
n = 12

(b) $105° = 180° - \frac{360°}{n}$

$$\frac{360°}{n} = 75°$$

$$n = \frac{24}{5}$$

(c) $108° = 180° - \frac{360°}{n}$

$$\frac{360°}{n} = 72°$$
n = 5

(d) $144° = 180° - \frac{360°}{n}$

$$\frac{360°}{n} = 36°$$
n = 10

∴ Only 105° angle which can never be interior angle of regular polygon

10. (b) According to question.
Given:
Interior Angle = 3 × exterior angle Angle
As we know that
Interior Angle + Exterior Angle = 180°
3 Exterior Angle + Exterior Angle = 180°
4 exterior = 180°

Exterior angle = $\frac{180°}{4} = 45°$

∴ No. of Sides = $\frac{360°}{Exterior\ angle}$

No. of Sides = $\frac{360°}{45°} = 8$

11. (d) Sum of all interior angle of regular polygon = (n – 2)× 180
Sum of all exterior angle of polygon = 360
(n – 2)× 180 = 2×360
n = 6

12. (c) Let the sides be x and $2x$
According to question

$$\frac{180° - \frac{360°}{n_1}}{180° - \frac{360°}{n_2}} = \frac{2}{3}$$

$$\frac{180° - \frac{360°}{x}}{180° - \frac{360°}{2x}} = \frac{2}{3}$$

$$540° - \frac{1080°}{x} = 360° - \frac{360°}{x}$$

$$180° = \frac{720°}{x}$$

$x = 4$

∴ Sides be x and $2x$ = 4,8

Alternate
In this question go through option.
Option: (c) 4, 8

Given: $\frac{n_1}{n_2} = \frac{1}{2}$

(n = no. of sides)

$\frac{I_1}{I_2} = \frac{2}{3}$ (I = Interior Angle)

∴ Through option $n_1 = 4$
$n_2 = 8$

∴ $E_1 = \frac{360°}{n_1} = \frac{360°}{4} = 90°$

$E_2 = \frac{360°}{n_2} = \frac{360°}{8} = 45°$

As we know that
I + E = 180°
$I_1 + E_1 = I_1 + 90° = 180°$
$I_1 = 90°$
$I_2 + E_2 = I_2 + 45° = 180°$
$I_2 = 180° - 45° = 135°$

$\frac{I_1}{I_2} = \frac{90°}{135°} = \frac{2}{3}$ (satisfied)

13. (b) According to question
Given:
Internal angle = 2× External Angle
As we know that
Internal angle + Exterior Angle = 180°
2 × External angle + Exterior Angle = 180°
3 Exterior Angle = 180°
Exterior Angle = 60°

No. of sides = $\frac{360°}{Exterior\ Angle}$

No. of sides = $\frac{360°}{60°}$

No. of sides = 6

14. (a) If the number of sides of regular Polygon be = n Sum of the interior angle
= (n – 2)× 180°

∴ (n – 2) × 180° = 1440°

$n - 2 = \frac{1440°}{180°}$

n – 2 = 8 n = 10

15. (c) According to question
sum of interior angles = 5 × sum of exterior angles
As we know that
Exterior angle + Interior angle = 180°
Exterior angle + 5 Exterior angle = 180°
6 Exterior angle = 180°
Exterior angle = 30°

∴ no. of sides = $\frac{360°}{External\ angle}$

$= \frac{360°}{30°} = 12$

16. (c)According to question
Interior angle – Exterior angle = 132°
As we know that
Interior angle + Exterior angle = 180° ....(i)
Interior angle – Exterior angle = 132° ....(ii)
2 Interior angle = 312°
Interior angle = 156°
Put this value in equation (i) and (ii)
∴ Exterior angle = 180° – 156° = 24°
∴ no. of sides

$= \frac{360°}{External\ angle}$

no. of sides = $\frac{360°}{24°} = 15$

17. (c) According to question

$$\frac{External\ angle}{Internal\ angle} = \frac{1}{17}$$

As we know that
External angle + Internal angle = 180°
∴ 18 units = 180°

1 unit = $\frac{180°}{18}$ = 10°

∴ External angle = 10° × 1 = 10°

∴ no. of sides

$= \frac{360°}{External\ angle}$

no. of sides = $\frac{360°}{10°} = 36$

18. (c) Interior angle + exterior angle = 180°
$3x + x = 180°$
$4x = 180°$
$x = 45°$
each exterior angle = 45°

No. of sides = $\frac{360°}{\text{exterior angle}}$

$= \frac{360°}{45°} = 8$

19. (a) Let internal angle = x
External angle = y
$x-y = 108$ ....(i)
$x+y = 180$ .....(ii)
from equation (i) and (ii)
$y = 36°$

No. of sides = $\frac{360°}{\text{exterior angle}}$

$= \frac{360°}{36°}$

$n = 10$
Thus, side of polygon is 10

20. (c) Sum of Interior angles
$= (n-2)\times180$
Hence, $(n-2)\times 180$
= 14× Right angle triangle (90°)

$n-2 = \frac{14\times90}{180}$

$n = 7 + 2 = 9$

21. (d) no. of diagonals of a polygon of n sides

$=\frac{n(n-3)}{2}=\frac{8(8-3)}{2}=20$

22. (a) $=\frac{n(n-3)}{2} = 44 \Rightarrow n(n-3)$
$= 88 = 11 \times 8$
$\therefore$ n = 11

23. (c) angles be x, $2x$, $3x$, $4x$
$\therefore$ $x + 2x + 3x + 4x = 360° \Rightarrow 10x = 360° \Rightarrow x = 36°$
$\therefore$ largest angle = $4x$ = 144°

24. (b) Each interior angle = 120°
$\therefore$ Exterior angle
= 180° - 120° = 60°
$\therefore$ no. of sides

$= \frac{360°}{\text{exteriorangle}} = \frac{360}{60} = 6$

25. (c) Exterior angle of a regular octagon = $\frac{360}{8} = 45°$

$\therefore$ interior angle = 180° - 45°
= 135°

26. (b) Sum of all interior angles
$= (n-2)\times180°$

$\Rightarrow 1440 = (n-2)\times180$

$\Rightarrow n = 10 (n \to no. of sides)$

27. (c) Sum of interior angles of pentagon = $(n-2)\times180°$
$= (5-2)\times180° = 540°$
$\Rightarrow 140° + x + 2x + 3x + 4x = 540$
$\Rightarrow 10x = 400 \Rightarrow x = 40$
$\therefore$ largest angle = $4x$ = 4×40 = 160°

28. (c) If the number of sides of the polygon be n, then
Interior angle + exterior angle = 180°
Exterior angle = 180° – 144°
= 36°

Number of sides = $\frac{360°}{36°} = 10$

29. (b) Let no. of sides = n
each equal side subtends equal angle at the centre.

$\therefore$ $n\times36 = 360° \Rightarrow n = \frac{360}{36} = 10$

30. (d) $\frac{360°}{n-1} - \frac{360°}{n+2} = 6$

$\Rightarrow 60\left(\frac{1}{n-1} - \frac{1}{n+2}\right) = 1$

$\Rightarrow 60(n + 2 - n + 1) = (n - 1)(n + 2)$
$\Rightarrow 60 \times 3 = n^2 + n - 2$
$\Rightarrow n^2 + n - 182 = 0 \Rightarrow (n+14)(n-13) = 0$
$\Rightarrow n = -14$ or $n = 13$
$\Rightarrow n = 13$ ($\because$ n can not be negative)

31. (b) Each interior angle of a regular polygon

$= 180\times\frac{3}{5} = 108°$

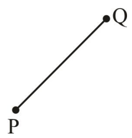
$\therefore$ Each exterior angle = 180° – 108° = 72

$\therefore$ No. of sides = $\frac{360}{72} = 5$

CHAPTER 20

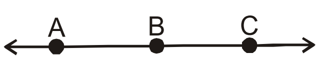
LINES & ANGLES

A line is made up of an infinite number of points and it has only length. i.e. A line has no end points on either side.
A **line segment** has two end points.
A **ray** has one end point.



Line segment line ray

- A line segment PQ is generally denoted by $\overline{PQ}$
- A line AB is denoted by $\overleftrightarrow{AB}$
- The ray OP is denoted by $\overrightarrow{OP}$
- **Collinear points :** Three or more than three points are said to be collinear if there is a line which contains them all.

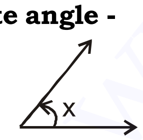


Here A,B and C are collinear points

- **Concurrent lines-** Three or more than three lines are said to be concurrent if there is a point which lies on all of them.
  AB, CD and EF are concurrent lines.

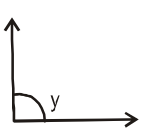
Related Angles

Acute angle -



$0° < x < 90°$

Right Angle -



$y = 90°$

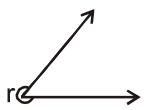
Obtuse Angle -

$90° < z < 180°$

Straight Angle -

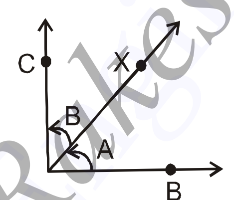
$t = 180°$

Reflex Angle



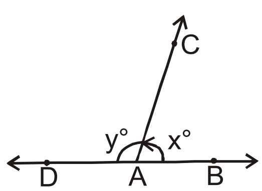
$180° < r < 360°$

Complementary Angles:- When the sum of the measures of the two angles is 90°, the angles are called "Complementary Angles".



i.e. $\angle A + \angle B = 90°$ or
$\angle A = \angle 90° - \angle B$ = complement of $\angle A$

Supplementary Angles - Sum of two angles which are supplementary is 180°



Sum of two angles which are supplementary is
$180°$. *i.e.* $\angle x + \angle y = 180°$

EXAMPLES

1. If one of angle is four times the complement of the other angle. Find the angles.

Sol. Let the one angle = θ
Other angle = $90° - \theta$
A.T.O
$\theta = 4\,(90° - \theta)$
$\theta = 360° - 4\theta$
$5\theta = 360°$
$\theta = 72°$
One angle = 72°
Other angle = $90° - 72° = 18°$

2. One angle of the complementary is 45°. Find the other angles.

Sol. Sum of the complementary angles = 90°
$\text{Angle}_1 + \text{Angle}_2 = 90°$
$45° + \text{Angle}_2 = 90°$
$\text{Angle}_2 = 90° - 45° = 45°$

3. If two supplementary angles are $(7x + 58)°$ and $(16x + 53)°$ then $5x$ is:

Sol. Sum of supplementary angles = 180°
$7x + 58° + 16x + 53° = 180°$
$23x + 111 = 180°$
$23x = 69$
$x = 3$
$5x = 5 \times 3 = 15°$

4. Ratio of two complementary angles are 11: 19 then both angles are:

Sol. Sum of two complementary angle = 90°
$11x + 19x = 90°$
$30x = 90°$
$x = 3$
Ist angle = $11x = 11 \times 3 = 33°$
IInd angle = $19x = 19 \times 3 = 57°$

5. Angle Q_1 & Q_2 are complemetary angles Q_1 is $\frac{9}{11}$ times of its supplementary angle, them Q_2 is

Sol. $Q_1 = (180° - Q_1)\frac{9}{11}$
$11 \times Q_1 = 180 \times 9 - 9\,Q_1$
$20Q_1 = 1620$
$Q_1 = 81°$
$Q_2 = 90° - 81° = 9°$

6. What is the complementary angle of 37° 49' 13"

Sol. Complementary angle = 90° – 37° 49'13"
= 89° 59'60" – 37° 49"13"
= 52° 10'47"

7. Angles a and b are linear pair. b is 24° more than one fifth of its supplementary angle. Find the complementary of half of a:

Sol. $b-\left(\frac{180^\circ-b}{5}\right)=24^\circ$

$5b-180^\circ+b=120^\circ$
$6b=300^\circ$
$b=50^\circ$
$a=180^\circ-50^\circ=130^\circ$

Complementary of $\left(\frac{a}{2}\right)=90^\circ-65^\circ=25^\circ$

8. Rakesh has drawn an angle of measure 45° 36', when he was asked to drawn an angle of 45°, the percentage error in his drawing:

Sol. Percentage error

$=\frac{45^\circ 36'-45^\circ}{45\times 60}\times 100$

$=\frac{36}{45\times 60}\times 100=\frac{4}{3}=1.33\%$

9. Ratio of supplementary angles are 14 : 22. Find the smaller angles.

Sol. Sum of supplementary angles = 180°
$14x+22x=180^\circ$
$36x=180^\circ$
$x=5$
Smaller angle = $14x$ = 14 × 5 = 70°

10. Angle Q_1 & Q_2 are supplementary angles Q_2 is $\frac{3}{7}$ times of its complementary angle, then Q_1

Sol. $Q_2=\frac{3}{7}(90^\circ-Q_2)$

$7Q_2=270^\circ-3Q_2$
$10Q_2=270^\circ$
$Q_2=27^\circ$
$Q_1+Q_2=180^\circ$
$Q_1=180^\circ-27^\circ=153^\circ$

11. Angle Q_1 & Q_2 are supplementary angles Q_2 is $46\frac{2^\circ}{3}$ greater than the complementary angle, then Q_1 is:

Sol. $Q_2-(90^\circ-Q_2)=46\frac{2^\circ}{3}$

$Q_2-90^\circ+Q_2=\frac{140^\circ}{3}$

$=Q_2=\frac{140^\circ+270^\circ}{2\times 3}$

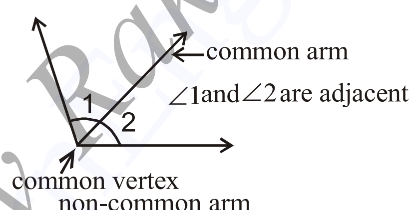
$Q_2=\frac{410}{2\times 3}$

$Q_2=\frac{205}{3}=68\frac{1^\circ}{3}$

$Q_1=180^\circ-\frac{205}{3}=\frac{335}{3}=111\frac{2^\circ}{3}$

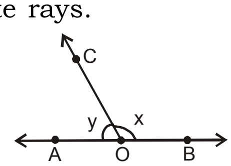
Adjacent Angles

Two angles are said to be adjacent if



1. They have a common vertex;
2. They have a common arm; and
3. The non-common arms are on either side of the common arm.

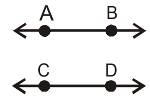
❑ **Linear pair-** A linear pair is a pair of adjacent angles whose non-common sides are opposite rays.



$\angle x$ and $\angle y$ are linear pair and $\angle x+\angle y=180^\circ$

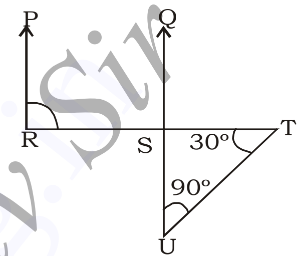
i.e. linear pair Angles are supplementary.

❑ **Parallel Lines-** If two lines have no point in common they are said to be Parellel lines.



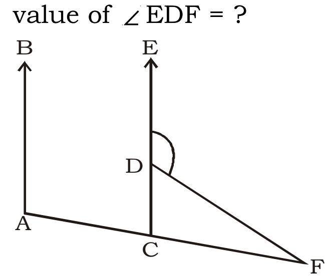
EXAMPLES

1. In this figure, PR||QU, ∠STU = 30° and ∠SUT = 90° then the value of ∠PRS = ?



Sol. In △ SUT
∠S+∠U+∠T=180°
[sum of angle of △ is 180°]
∠S + 90° + 30° = 180°
∠S = 60°
∠QST + ∠TSU = 180°
[Linear angle]
∠QST + 60 = 180°
∠QST = 120°
∠RSQ + ∠QST = 180°
∠RSQ + 120° = 180°
∠RSQ = 60°
∠PRS + ∠RSQ = 180°
∠PRS + 60° = 180°
∠PRS = 120°

2. In the figure, AB||CE, ∠BAF = 115°, ∠AFD = 30° then the value of ∠EDF = ?



Sol. ∠BAC + ∠ECA = 180°
[Linear angle]
115° + ∠ECA = 180°
∠ECA = 65°
∠ACE + ∠ECF = 180°
65° + ∠ECF = 180°
∠ECF = 115°
In △ DCF
∠D + ∠C + ∠F = 180°
∠D + 115° + 30° = 180°

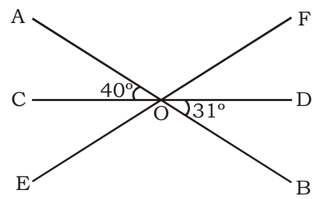
$\angle D = 35°$

$\angle EDF + \angle CDF = 180°$

$\angle EDF + 35° = 180°$

$\angle EDF = 145°$

3. In the following figure. Find the value of $\angle BOC$?



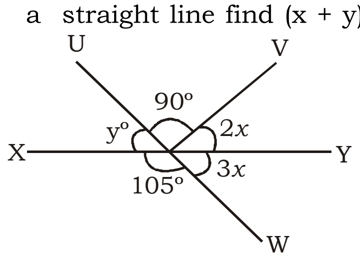
Sol. $\angle BOC + \angle BOD = 180°$

[CD is a straight line]

$\angle BOC + 31° = 180°$

$\angle BOC = 149°$

4. In the following figure U W is a straight line find (x + y)



Sol. $\angle UOV + \angle VOY + \angle YOW = 180°$ [UW is straight line]

$90° + 2x + 3x = 180°$

$5x = 90°$

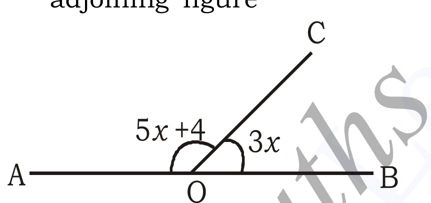
$x = 18°$

$\angle XOU + \angle XOW = 180°$

$y° + 105° = 180°$

$y = 75° \Rightarrow x + y = 18° + 75° = 93°$

5. What is the value of x in the adjoining figure



Sol. $\angle AOC + BOC = 180°$ [AB is a straight line]

$5x + 4 + 3x = 180$

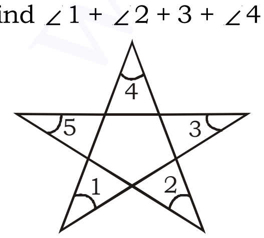
$8x = 180° - 4$

$8x = 176$

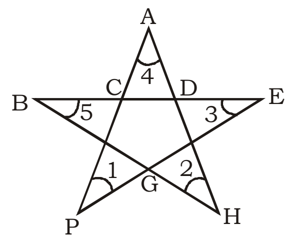
$x = 22°$

6. In the given figure.

Find $\angle 1 + \angle 2 + 3 + \angle 4 + \angle 5$ =?



Sol.



$\angle ADC = \angle 2 + \angle 5$

$\angle ACD = \angle 1 + \angle 3$

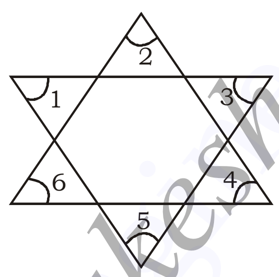
[Sum of two Interior angle is equal to opposite of exterior angle]

In $\triangle$ ACD

$\angle A + \angle C + \angle D = 180°$

$\angle 4 + \angle 1 + \angle 3 + \angle 2 + \angle 5 = 180°$

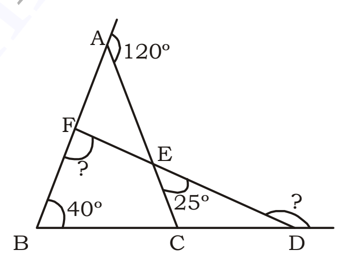
7. In the given figure. Find $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = ?$



Sol. Sum of two triangle = 180 + 180 = 360°

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$

8.



Sol. $\angle B + \angle BCA = \angle A$

[Sum of two interior angle is equal to opposite of exterior angle]

$40° + \angle BCA = 120°$

$\angle BCA = 80°$

$\angle BCE + \angle ECD = 180°$

[Linear angle]

$80° + \angle ECD = 180°$

$\angle ECD = 100°$

$\angle D = \angle E + \angle C$ [use above property exterior angle]

$\angle D = 25° + 100° = 125°$

$\angle EDC = 180° - 125° = 55°$

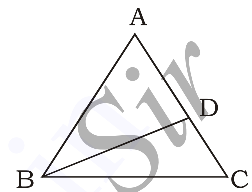
In $\triangle$ FBD

$\angle F + \angle B + \angle D = 180°$

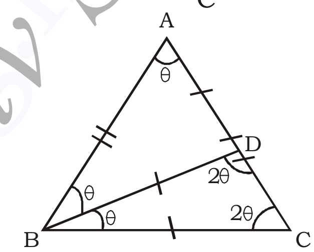
$\angle F + 40° + 55° = 180°$

$\angle F = 85°$

9. In the given figure, AB = AC, AD = BD = BC, find $\angle C$ = ?



Sol.



Let $\angle A = \theta$

$\angle A = \angle ABD = \theta$ [AD = BD]

$\angle BDC = \angle A + \angle ABD$ [Sum of two interior angle is equal to opposite of exterior angle]

$\angle C = \angle BDC = 2\theta$ [BD = BC]

$\therefore \angle DBC = \theta$

$B = \angle C = 2\theta$ [AB = AC]

In $\triangle$ ABC

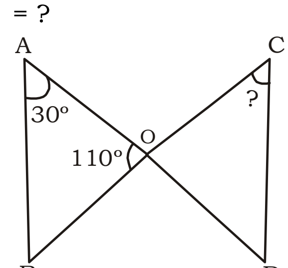
$\angle A + \angle B + \angle C = 180°$

$\theta + 2\theta + 2\theta = 180°$

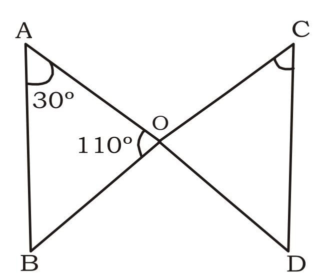
$\theta = 36°$

$\angle C = 2\theta = 2 \times 36° = 72°$

10. In the given figure AB||CD, $\angle A = 30°$, $\angle O = 110°$, find $\angle C$ = ?



Sol.



In ABO

$\angle A + \angle B + \angle O = 180°$

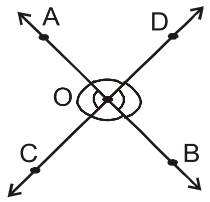
$30° + \angle B + 110° = 180°$

$\angle B = 40°$

$\angle B = \angle C = 40°$

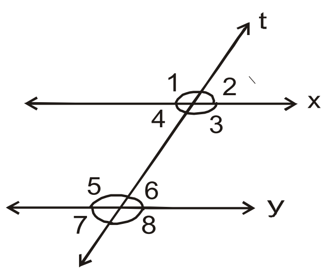
[Alternate Angle]

Vertically opposite Angles



$\left.\begin{array}{l}\angle AOD = \angle BOC \\ \angle AOC = \angle BOD\end{array}\right\}$ are vertically opposite angles

Angles made by a transversal line -



Interior Angles- $\angle 3, \angle 4, \angle 5, \angle 6$

Exterior Angles- $\angle 1, \angle 2, \angle 7, \angle 8$

Pairs of corresponding angles-

$\angle 1 = \angle 5,\ \angle 2 = \angle 6,$

$\angle 4 = \angle 7,\ \angle 3 = \angle 8$

Pairs of alternate interior angles-

$\angle 3 = \angle 5, \angle 4 = \angle 6$

Pairs of alternate exterior angles-

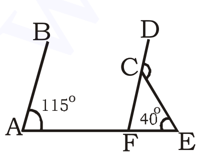
$\angle 1 = \angle 8, \angle 2 = \angle 7$

Pairs of interior angles on the same side of the transversal-

$\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$

EXAMPLES

1. In the figure, AB$\parallel$ CD and $\angle BAE = 115°$, $\angle AEC = 40°$, then the value of $\angle DCE$?



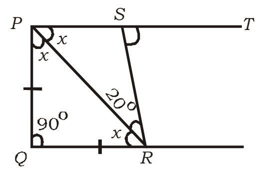
Sol. $\angle BAE = \angle CFE = 115°$ [Corresponding angles]

$\angle CFE + \angle CEF = \angle DCE$ [Sum of interior angle is equal to opposite of exterior angle]

$115° + 40° = \angle DCE$

$\angle DCE = 155°$

2. In this figure, PQRS is a quadrilateral. QR $\parallel$ PS. If PQ = QR then then value of $\angle TSR$?



Sol. In $\triangle$ PQR

$\angle P + \angle Q + \angle R = 180°$

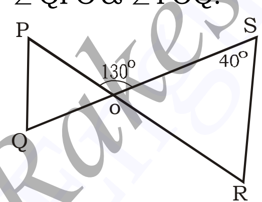
$x + 90° + x = 180°$

$x = 45°$

$\angle SRQ = 45° + 20° = 65°$

$\angle TSR = \angle QRS = 65°$ [Alternate angle]

3. Two line PR and QS intersect each other at O. $\angle POS = 130°$ & $\angle QSR = 40°$, PQ$\parallel$RS. Find $\angle QPO$ & $\angle POQ$.



Sol. $\angle ORS + \angle OSR = \angle POS$ [sum of interior angle is equal to opposite of exterior angle]

$\angle ORS + 40° = 130°$

$\angle ORS = 90°$

$\angle POS + \angle SOR = 180°$ [Linear angle]

$130° + \angle SOR = 180°$

$\angle SOR = 50°$

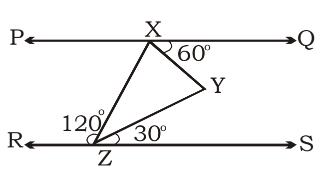
$\angle POQ = \angle SOR$ [vertically opposite angle]

PQ $\parallel$ RS

$\angle QPO = \angle SRO$ [Alternative angle]

$\angle QPO = 90°$

4. In this figure, PQ $\parallel$ RS, then $\angle XYZ$ =?



Sol. PQ $\parallel$ RS

$\angle QXZ = \angle XZR = 120°$ [Alternate angle]

$\angle YXZ = 120° - 60° = 60°$

$\angle RZX + \angle XZY + \angle YZS = 180°$ [Linear angle]

$120° + \angle XZY + 30° = 180°$

$\angle XZY = 30°$

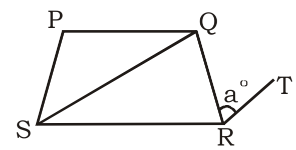
In $\triangle$ XYZ

$\angle X + \angle Y + \angle Z = 180°$ [Sum of angle of $\triangle$ is 180°]

$60° + \angle Y + 30° = 180°$

$\angle Y = 90°$

5. In the figure, SQ $\parallel$ RT and $\angle SPQ = 130°$, $\angle PQS = 40°$, $\angle PSR = 40°$, $\angle QRS = 90°$, then the value of a.



Sol. In $\triangle$ PQS

$\angle P + \angle Q + \angle S = 180°$

$130° + 40° + \angle S = 180°$

$\angle S = 10°$

$\angle QSR + \angle PSQ = \angle PSR$

$\angle QSR + 10° = 40°$

$\angle QSR = 30°$

In $\triangle$ QSR

$\angle Q + \angle S + \angle R = 180°$

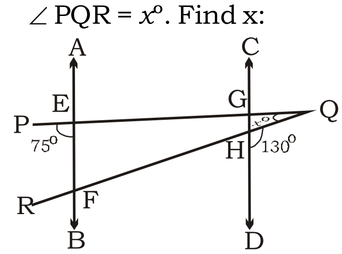
$\angle Q + 30° + 90° = 180°$

$\angle Q = 60°$

$\angle SQR = \angle QRT = 60°$ [Alternate angle]

a = 60°

6. In the adjoining figure AB$\parallel$CD and PQ, QR intersects AB and CD both at E, F and G, H respectively. Given that $\angle PEB = 75°$, $\angle QHD = 130°$ and $\angle PQR = x°$. Find x:



Sol. $\angle PEF = \angle EGH = 75°$ [Corresponding angle]

$\angle EGH + \angle HGQ = 180°$ [Linear angle]

$\angle HGQ = 180° - 75°$

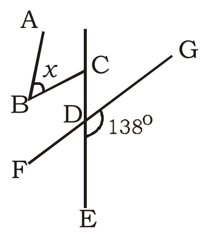
$\angle HGQ = 105°$

$\angle HGQ + \angle GQH = \angle DHQ$ [Sum of interior angle is equal to opposite of exterior angle]

$105° + x = 130°$

$x = 25°$

7. In this given figure AB∥CE and BC∥FG. Find the value of x°:



Sol. $\angle FDC = \angle GDE = 138°$ [Vertically opposite angle]

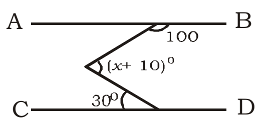
$\angle BCD + \angle FDC = 180°$

$\angle BCD + 138° = 180°$

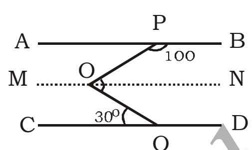
$\angle BCD = 42°$

$\angle ABC = \angle BCD = x = 42°$ [Alternate angle]

8. AB∥CD, shown in the figure. Find the value of x.



Sol.



$\angle OPB + \angle PON = 180°$ [Linear angle]

$\angle PON = 80°$

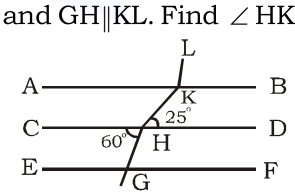
$\angle NOQ = \angle OQC = 30°$ [Alternate angle]

$\angle POQ = (x + 10)°$

$80° + 30° = x + 10°$

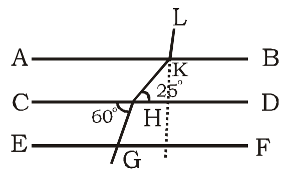
$x = 100°$

9. In the given figure AB∥CD∥EF and GH∥KL. Find $\angle HKL$



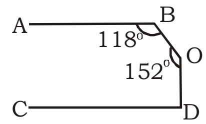
Sol. $\angle AKH = \angle KHD = 25°$ [Alternate angle]

$\angle EGH = \angle AKL = 180° - 60° = 120°$ [Corresponding angle]

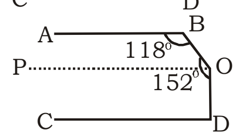


$\angle HKL = 120° + 25° = 145°$

10. AB∥CD. $\angle ABO = 118°$, $\angle BOD = 152°$, find $\angle CDO$:



Sol.



$\angle ABO + \angle BOP = 180°$ [Linear angle]

$118° + \angle BOP = 180°$

$\angle BOP = 62°$

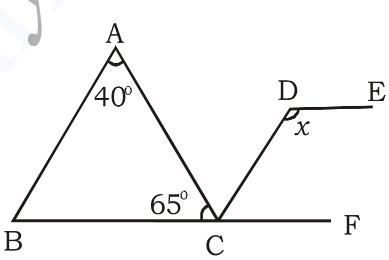
$\angle POD = 152° - \angle BOP$
$= 152° - 62° = 90°$

$\angle CDO + \angle POD = 180°$

$\angle CDO + 90° = 180°$

$\angle CDO = 90°$

11. In the figure AB∥DC and DE∥BF. Find the value of x & $\angle ACD$



Sol. In $\triangle ABC$

$\angle A + \angle B + \angle C = 180°$ [Sum of angle of $\triangle$ is 180°]

$40° + \angle B + 65° = 180°$

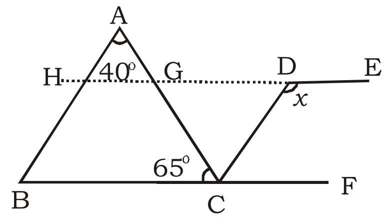
$\angle B = 75°$

$\angle ABC = \angle DCF = 75°$ [Corresponding angle]

$\angle EDC + \angle DCF = 180°$ [Interior angle on same side]

$\angle EDC + 75° = 180°$

$\angle EDC = 105°$



$\angle DGC = \angle BCG = 65°$ [Alternate angle]

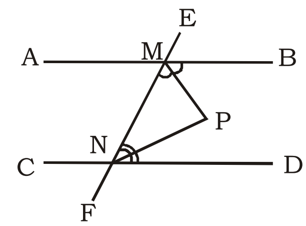
$\angle DGC + \angle GCD = \angle CDE$ [Sum of interior angle is equal to opposite of exterior angle]

$65° + \angle GCD = 105°$

$\angle GCD = 40°$

12. Two parallel lines AB and CD are intersected by a transversal EF at M & N respectively. The lines MP and NP are the bisectors of interior angles $\angle BMN$ and $\angle DNM$ on the same side of the transversal. Then $\angle MPN$ is equal to:

Sol.



$\angle BMN = 2a$, $\angle DNM = 2b$

$2a + 2b = 180°$

$a + b = 90°$

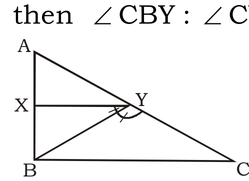
$\angle MPN = 180° - \angle NMP - \angle MNP$

$\angle MPN = 180° - (a + b)$

$\angle MPN = 180° - 90°$

$\angle MPN = 90°$

13. In a $\triangle ABC$, a line XY parallel to BC intersects AB at X and AC at Y. If BY bisects $\angle XYC$, then $\angle CBY : \angle CYB$ is

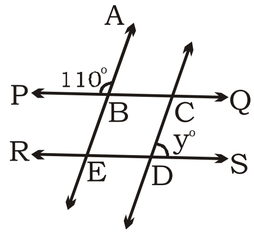


Sol. $\angle XYB = \angle YBC$ [Alternate angle]

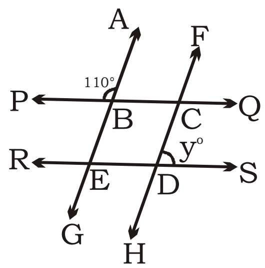
$\angle XYB = \angle BYC$ [Internal bisector]

$\angle CBY : \angle CYB = 1 : 1$

14. In the adjoining figure $AE \| CD$ and $BC \| ED$, then find Y.



Sol.



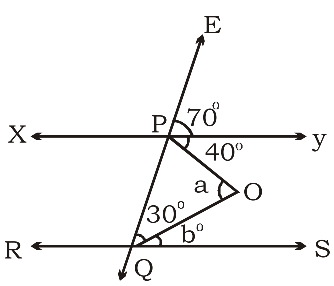
$\angle ABP + \angle ABC = 180°$

$110° + \angle ABC = 180°$

$\angle ABC = 70°$

$\angle ABC = \angle FCQ = \angle CDS = 70°$ [Corresponding angle]

15. In the given figure $XY \| RS$, Find the value of a & b.



Sol. $\angle EPY = \angle PQS = 70°$ [Corresponding angle]

$\angle PQO + OQS = \angle PQS$

$30° + b = 70°$

$b = 40°$

$\angle EPY + \angle YPO + \angle OPQ = 180°$

$70° + 40° + \angle OPQ = 180°$

$\angle OPQ = 70°$

In $\triangle OPQ$

$\angle P + \angle O + \angle Q = 180°$

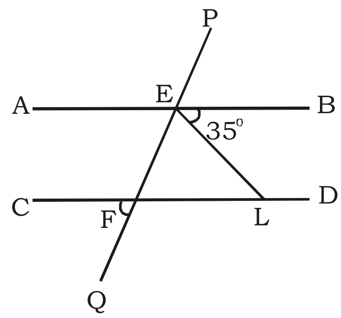
$70° + \angle a + 30° = 180°$

$\angle a = 80°$

$a = 80°, b = 40°$

16. AB and CD are two parallel lines. PQ cuts AB & CD at E & F respectively. EL is the bisector of $\angle BEF$. It $\angle LEB = 35°$ then $\angle CFQ$ will be:

Sol.



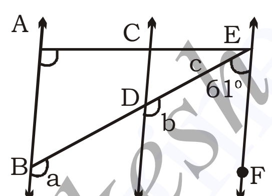
$\angle BEL = \angle LEF = 35°$ [Angle bisector]

$\angle BEF = \angle DFQ = 70°$ [Corresponding angle]

$\angle DFQ + \angle CFQ = 180°$

$\angle CFQ = 180° - 70° = 110°$

17. In the given figure. $AB \| CD \| EF$. Also $EA \perp AB$. If $\angle BEF = 61°$. Find the value of a, b & c.



Sol. $\angle A = \angle C = \angle E = 90°$

$\angle CED + \angle DEF = 90°$

$C + 61° = 90°$

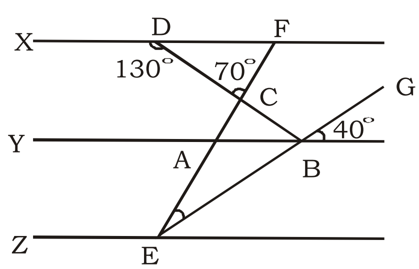
$C = 29°$

$61° + b = 180°$ [Linear angle]

$b = 119°$

$a = b = 119°$ [Corresponding angle]

18. Three straight lines x, y and z are parallel and the angles are shown in the figure above $\angle AEB$ equal to?



$\angle XDC = \angle DFC + \angle DCF$

[Sum of two interior angle is opposite to exterior angle]

$\angle DFC = 60°$

$\angle DFC = \angle CAB = 60°$ [Alternate angle]

$\angle B = \angle ABE = 40°$ [Vertically opposite angle]

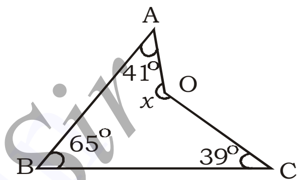
$\angle AEB + \angle ABE = \angle FAB$

[Sum of two interior angle is opposite to exterior angle]

$\angle AEB + 40° = 60°$

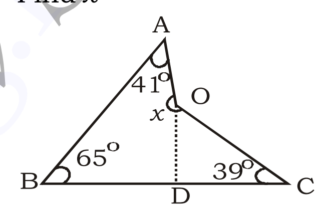
$\angle AEB = 20°$

19.



Find x

Sol.



$\angle ADC = \angle A + \angle B$ [Sum of two interior angle is equal to opposite of exterior angle]

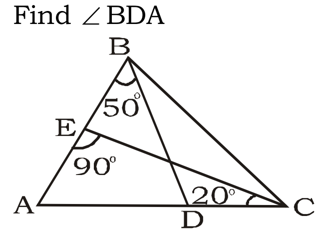
$\angle ADC = 41° + 65° = 106°$

$\angle AOC = \angle C + \angle D$

$= 39° + 106° = 145°$

Exterior angle of $AOC = 360° - 145° = 215°$

20. In the given figure. $CE \perp AB$, $\angle ACE = 20°$ and $\angle ABD = 50°$. Find $\angle BDA$



Sol. In $\triangle AEC$

$\angle A + \angle E + \angle C = 180°$

$\angle A + 90° + 20° = 180°$

$\angle A = 70°$

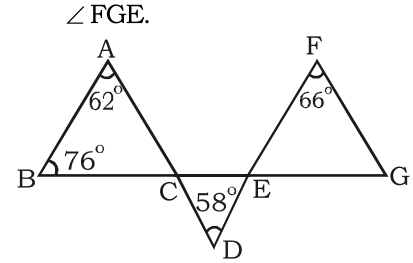
In $\triangle BAD$

$\angle B + \angle A + \angle D = 180°$

$50° + 70° + \angle D = 180°$

$\angle D = 60°$

21. In the adjoining figure $\angle CAB = 62°$, $\angle CBA = 76°$, $\angle ADE = 58°$ and $\angle DFG = 66°$, Find $\angle FGE$.



Sol. In $\triangle ABC$

$\angle A + \angle B + \angle C = 180°$

$62° + 76° + \angle C = 180°$

$\angle C = 42°$

$\angle C = \angle DCE = 42°$ [Vertically opposite angle]

In $\triangle$ CDE

$\angle C + \angle D + \angle E = 180°$

$42° + 58° + \angle E = 180°$

$\angle E = 80°$

$\angle E = \angle FEG = 80°$ [Vertically opposite angle]

In $\triangle$ FGE

$\angle F + \angle G + \angle E = 180°$

$66° + \angle G + 80° = 180°$

$\angle G = 34°$

$\angle FGE = 34°$

EXERCISE

1. Find the measures of an angle which is complement of itself.
(a) 40° (b) 30° (c) 45° (d) 50°

2. AB is a straight line and O is a point on AB, if line OC is drawn not conciding with OA or OB, then $\angle$AOC and $\angle$ BOC are :
(a) equal
(b) complementary
(c) supplementary
(d) together equal to 100

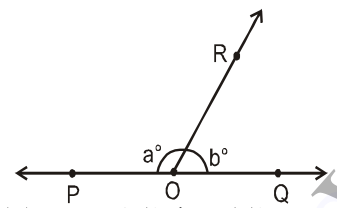
3. An angle is equal to one-third of its supplement. Find its measure :
(a) 45° (b) 50°
(c) 55°
(d) None of these

4. The ratio of two complementary angle is 1 : 5. What is the difference between the two angles?
(a) 60° (b) 90° (c) 120°
(d) Cannot be determined ith the given data

5. Which of the following statements are true :
(a) Angles forming a linear pair are supplementary.
(b) If two adjacent angles are equal, then measures of each angle will only be 90°.
(c) Angles forming a linear pair can both be acute angles.
(d) If angles forming a linear pair are equal,then each of these angles is of measure 90°.
(a) Only a (b) a, b and d
(c) c and d (d) a and d

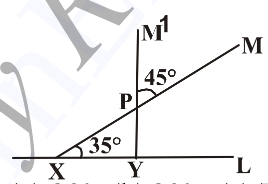
6. If one angle of a linear pair is acute, then its other angle will be :
(a) acute (b) obtuse
(c) right angle
(d) None of these

7. In figure $\angle$POR and $\angle$QOR form a linear pair. If a-b = 80°, find the value of 'a' and 'b':



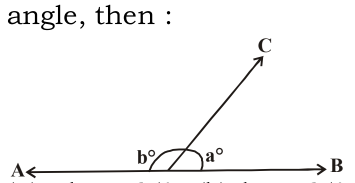
(a) a = 125°, b = 55°
(b) a = 110°, b = 70°
(c) a = 130°, b = 50°
(d) a = 75°, b = 105°

8. The angle between lines L and M measures 35°. If line M is rotated 45° counter clockwise about point P to line M[1] what is the angle in degrees between lines L and M[1] :



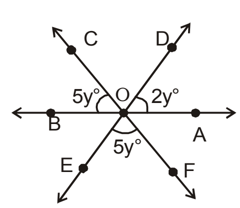
(a) 90° (b) 80° (c) 75° (d) 60°

9. In the given figure, $\angle$a is greater than one-sixth of right angle, then :



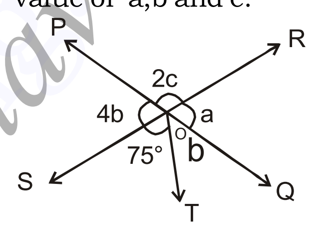
(a) b > 165° (b) b < 165°
(c) b $\leq$ 165° (d) b $\geq$ 165°

10. Determine the value of y.



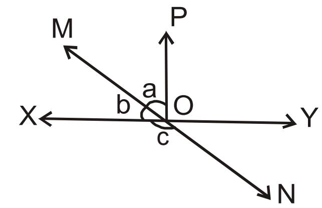
(a) 17° (b) 15° (c) 20° (d) 25°

11. In the given two straight lines PQ and RS intersect each other at 'O'. If $\angle$SOT = 75°,find the value of a,b and c:



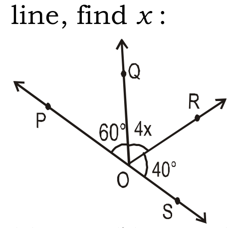
(a) a = 84°, b = 21°, c = 48°
(b) a = 48°, b = 20°, c = 50°
(c) a = 72°, b = 24°, c = 54°
(d) a = 64°, b = 28°, c = 45°

12. In the given figure XY and MN intersect at O. If POY = 90° and a:b = 2 : 3, then find c:



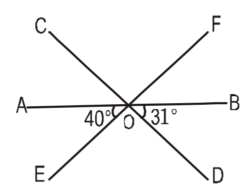
(a) 113° (b) 54°
(c) 126° (d) 48°

13. In the given figure POS is a line, find x :



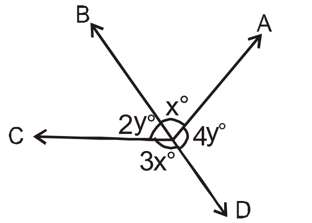
(a) 20° (b) 80° (c) 10° (d) 100°

14. In the following figure find the value of $\angle$BOC :



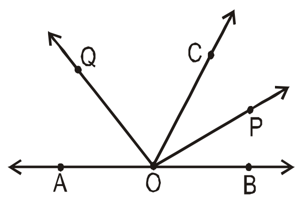
(a) 101° (b) 149°
(c) 71° (d) 140°

15. Find y, if $x° = 36°$, as per the given diagram :



(a) 36° (b) 16° (c) 12° (d) 42°

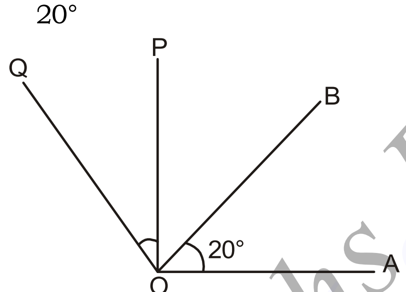
16. In figure OP bisects ∠BOC and OQ, ∠AOC. Find the value of ∠POQ :



(a) 60° (b) 75° (c) 90°
(d) None of these

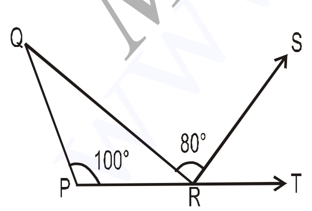
17. A,O,B, are three points on a line segment and C is a point not lying on AOB. If ∠AOC = 40° and OX, OY are the internal and external bisectors of ∠AOCand ∠BOC respectively, then ∠BOY is :
(a) 70° (b) 80° (c) 72° (d) 68°

18. In the figure, OP⊥OA and OQ⊥OB. Find ∠POQ if ∠AOB = 20°



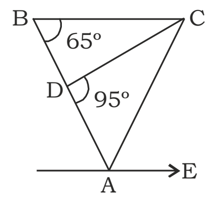
(a) 20° (b) 30° (c) 40°
(d) None of these

19. In the figure ∠PRQ = ∠SRT. If ∠QPR = 100° and ∠QRS = 80°, Find ∠PQR



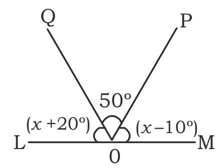
(a) 20° (b) 30° (c) 40° (d) 60°

20. In the figure given below, ABC is a triangle. BC is parallel to AE. If BC = AC, then what is the value of ∠CAE ?



(a) 20° (b) 30° (c) 40° (d) 50°

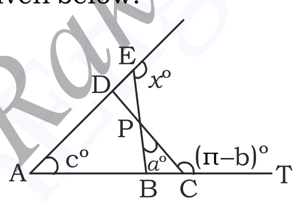
21. In the figure given below LOM is a straight line.



What is the valule of $x°$?
(a) 20° (b) 60° (c) 40° (d) 50°

22. In a ΔABC, $\frac{1}{2}\angle A + \frac{1}{3}\angle C + \frac{1}{2}\angle B = 80°$, then what is the value of ∠C ?
(a) 35° (b) 40° (c) 60° (d) 70°

23. The angles x°, a°, c° and (π –b)° are indicated in the figure given below.

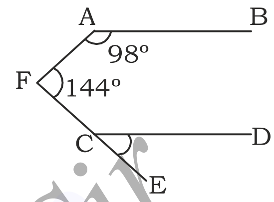


Which one of the following is correct ?
(a) $x = a + c - b$
(b) $x = b - a - c$
(c) $x = a + b + c$
(d) $x = a - b - c$

24. Two transversals S and T cut a set of distinct parallel lines. S cuts the parallel lines in points A, B, C, D and T cuts the parallel line in points E, F, G and H, respectively. If AB = 4, CD = 3 and EF = 12, then what is the length of GH?
(a) 4 (b) 6 (c) 8 (d) 9

25. AB is a straight line. C is a point whose distance from AB is 3 cm. What is the number of points which are at a distance of 1 cm from AB and 5 cm from C ?
(a) 1 (b) 2 (c) 3 (d) 4

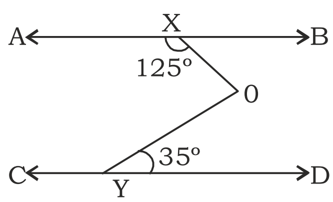
26. In the figure given below, AB is parallel to CD. If ∠BAF = 98° and ∠AFC = 144°, then what is ∠ECD equal to ?



(a) 62° (b) 64° (c) 82° (d) 84°

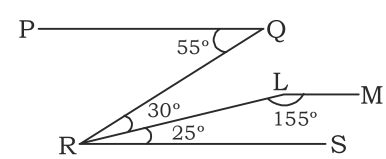
27. In a ΔABC, side AB is extended beyond B, side BC beyond C and side CA beyond A. What is the sum of the three exterior angles ?
(a) 270° (b) 305°
(c) 360° (d) 540°

28. In the figure given below, AB is parallel to CD, What is the value ∠XOY ?



(a) 80° (b) 90°
(c) 95° (d) 100°

29. In the figure given below, PQ is parallel to RS, What is the angle between the lines PQ and LM ?

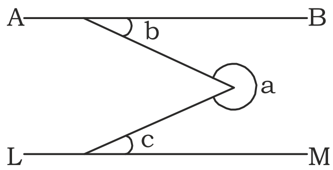


(a) 175° (b) 177°
(c) 179° (d) 180°

30. The line segments AB and CD intersect at O. OF is the internal bisector of obtuse ∠BOC and OE is the internal bisector of acute ∠AOC. If ∠BOC = 130°, what is the measure of ∠FOE ?
(a) 90° (b) 110°
(c) 115° (d) 120°

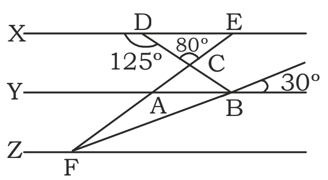
31. In the figure given below, AB is parallel to LM. What is the

angle a equal to ?



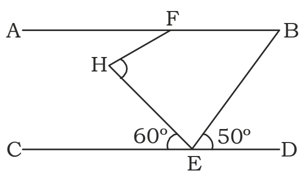
(a) $\pi + b + c$ (b) $2\pi - b + c$
(c) $2\pi - b - c$ (d) $2\pi + b - c$

32. Three straight lines X, Y and Z are parallel and the angles are as shown in the figure above. What is $\angle AFB$ equal to?



(a) 20° (b) 15° (c) 30° (d) 10°

33. In the figure given below, AB is parallel to CD and BE is parallel to FH. What is the $\angle FHE$ is equal to ?

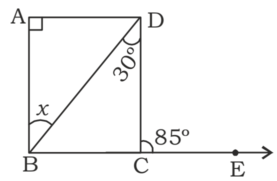


(a) 110° (b) 120°
(c) 125° (d) 130°

34. Let AB and AC be two rays intersecting at A. If D, E be the points lying on AB, AC respectively and P be the point such that P divides the line DE such that PD : PE = AD : AE. Then, what is the locus of the point P?
(a) The angle bisector of angle A
(b) The angle trisector of angle A
(c) The perpendicular bisector of angle A
(d) None of the above

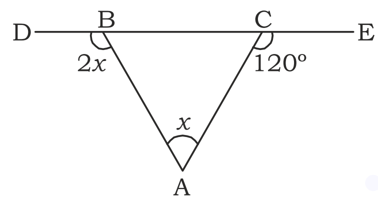
35. The length of a line segment AB is 2 unit. It is divided into two parts at the point C such that $AC^2 = AB \times CB$. What is the length of CB ?
(a) $3+\sqrt{5}$ units
(b) $3-\sqrt{5}$ units
(c) $2-\sqrt{5}$ units
(d) $\sqrt{3}$ units

36. In figure AD$\parallel$BE, $\angle DCE = 85°$ and $\angle BDC = 30°$, then what is the value of x ?



(a) 30° (b) 35° (c) 45° (d) 55°

37. In the figure given below, what is the value of x ?

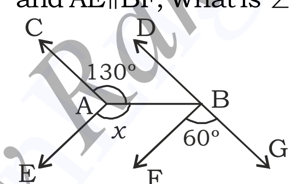


(a) 30° (b) 40° (c) 45° (d) 60°

38. LM is a straight line and O is a point on LM. Line ON is drawn not coinciding with OL or OM. If $\angle MON$ is one-third of $\angle LON$, then what is $\angle MON$ equal to ?

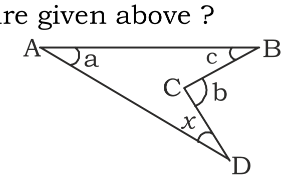
(a) 45° (b) 30° (c) 60° (d) 75°

39. In the figure given below, AC$\parallel$BD and AE$\parallel$BF, What is $\angle x$?



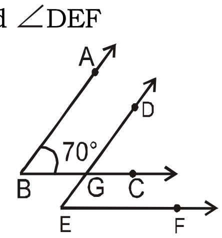
(a) 130° (b) 110° (c) 70° (d) 50°

40. What is the value of x in the figure given above ?



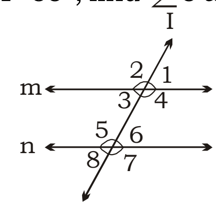
(a) $b - a - c$ (b) $b - a + c$
(c) $b + a - c$ (d) $\pi - (a + b - c)$

41. In the given figure, the arms of two angles are parallel. If $\angle ABC = 70°$, then find - $\angle DGC$ and $\angle DEF$



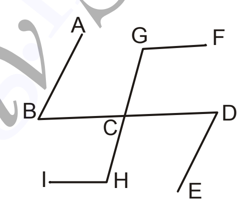
(a) 70°, 60° (b) 60°, 50°
(c) 70°,70° (d) None of these

42. In the given figure m||n and $\angle 1 = 65°$, find $\angle 5$ and $\angle 8$:-



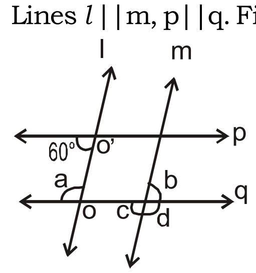
(a) 125°, 55° (b) 115°, 65°
(c) 105° , 75° (d) None of these

43. In the given diagram AB || GH ||DE and GF||BD||HI, $\angle FGC$ = 80°. Find the value of $\angle CHI$:



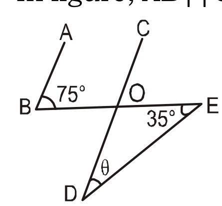
(a) 80° (b) 120°
(c) 100° (d) 160°

44. Lines l||m, p||q. Find a, b, c, d:



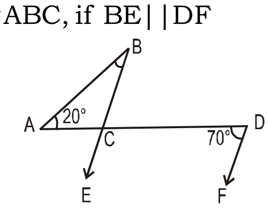
(a) a =120°, b = 60°, c = 60°, d = 120°
(b) a =120°, b = 120°, c = 60°, d = 120°
(c) a = 120°, b = 60°, c = 120°, d = 60°
(d) None of these

45. In figure, AB||CD. Find θ :



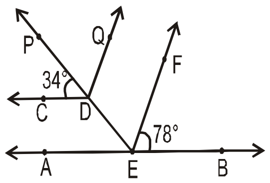
(a) 30° (b) 35°
(c) 40° (d) 45°

46. From the given figure, find $\angle ABC$, if BE||DF



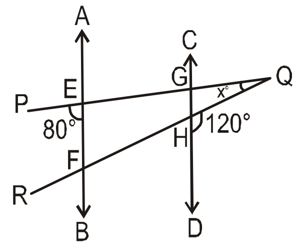
(a) 50° (b) 40° (c) 35°
(d) None of these

47. In the figure, AB||CD and EF||DQ, the value of $\angle$PDQ is:



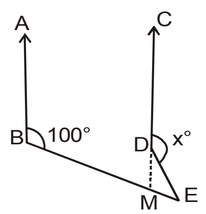
(a) 68° (b) 78° (c) 56°
(d) None of these

48. In the given figure AB||CD, given that $\angle$PEB = 80°, $\angle$QHD = 120° and $\angle$PQR = X°, find the value of x :



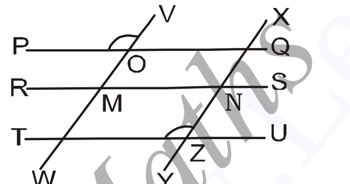
(a) 40° (b) 20° (c) 100° (d) 30°

49. In the given figure AB||CD, $\angle$ABE = 100°, $\angle$MED = 25°. Find $\angle$CDE :



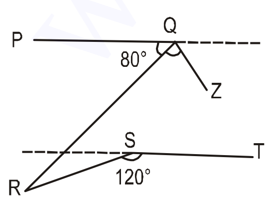
(a) 125° (b) 55° (c) 65° (d) 75°

50. In the given figure, $\angle$XZT = 130°, PQ, RS and TU are parallel. VW||XY. Find $\angle$VOP :



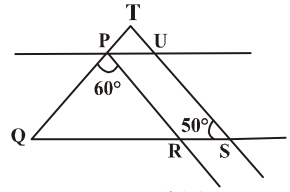
(a) 125° (b) 130°
(c) 120° (d) 135°

51. From the following figure, find $\angle$RQZ if $\angle$RQZ = 2$\angle$QRS and PQ||ST :



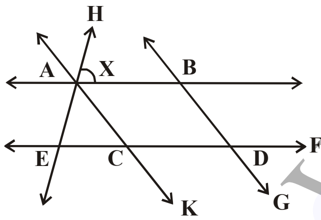
(a) 20° (b) 30° (c) 40° (d) 60°

52. In the given figure, PR||TS and PU||RS. Find $\angle$TPU :



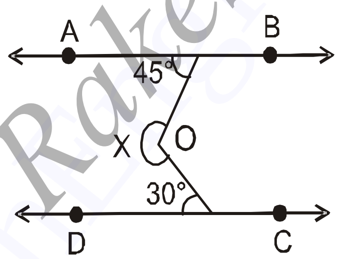
(a) 60° (b) 70°
(c) 80° (d) 100°

53. In the given figure, AB||CD and AC||BD. If $\angle$EAC = 40°, $\angle$FDG = 55°, $\angle$HAB = x, then the value of x is :



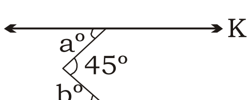
(a) 95° (b) 70°
(c) 35° (d) 85°

54. In the given figure, AB||CD, then X is equal to :



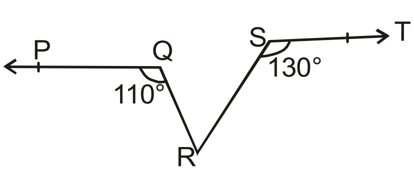
(a) 290° (b) 300°
(c) 280° (d) 285°

55. In the figure below, lines K and L are parallel . The value of a° + b° is :



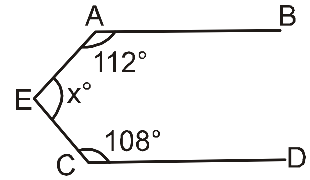
(a) 45° (b) 180°
(c) 180° (d) 360°

56. In the figure, if PQ||ST, $\angle$PQR = 110° and $\angle$RST = 130°, find $\angle$QRS.



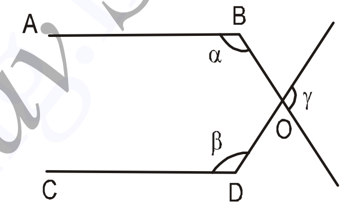
(a) 40° (b) 50° (c) 60° (d) 70°

57. In the figure, AB||CD, the value of x is :



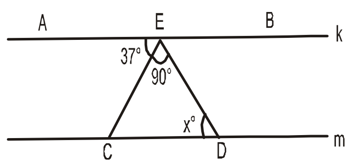
(a) 220° (b) 140°
(c) 150° (d) none of these

58. If AB || CD, then find the value of $\alpha + \beta + \gamma$:



(a) 180° (b) 270°
(c) 360° (d) 90°

59. In the figure below, if AB||CD and CE$\perp$ED, then the value of x is :



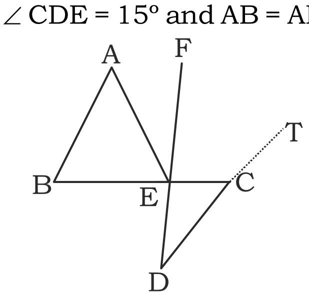
(a) 53° (b) 63° (c) 37° (d) 45°

60. If a straight line L makes an angle θ (θ > 90°) with the positive direction of x-axis, then the acute angle made by a straight line L_1, perpendicular to L, with the y-axis is:

(a) $\frac{\pi}{2}+\theta$ (b) $\frac{\pi}{2}-\theta$

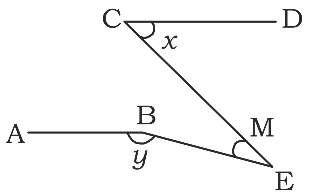
(c) $\pi+\theta$ (d) $\pi-\theta$

61. In the figure given below, AB is parallel to CD. $\angle$ABC = 65°, $\angle$CDE = 15° and AB = AE.



What is the value of $\angle$AEF ?
(a) 30° (b) 35° (c) 40° (d) 45°

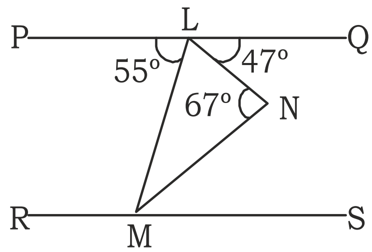
62. In the figure given below, AB is parallel to CD. If $\angle DCE = x$ and $\angle ABE = y$, then what is $\angle CEB$ equal to ?



(a) $y - x$ (b) $(x + y)/2$

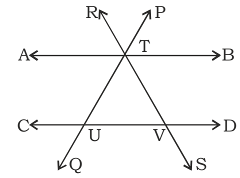
(c) $x + y - (\pi/2)$ (d) $x + y - \pi$

63. In the figure given below, PQ is parallel to RS. What is the $\angle NMS$ equal to ?



(a) 20° (b) 23° (c) 27° (d) 47°

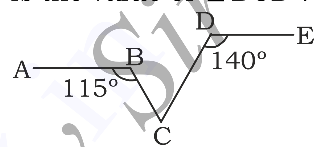
64. In the given figure, If AB||CD, $\angle PTB$ = 55° and $\angle DVS$ = 45°, then what is the sum of the measures of $\angle CUQ$ and $\angle RTP$?



(a) 180° (b) 135°

(c) 110° (d) 100°

65. In given that AB||DE. $\angle ABC$ = 115°, $\angle CDE$ = 140°. Then, what is the value of $\angle BCD$?



(a) 45° (b) 55° (c) 65° (d) 75°

ANSWER KEY

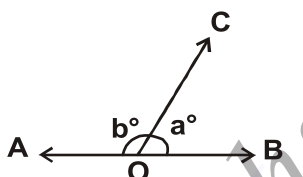
1. (c)	8. (b)	15. (a)	22. (c)	29. (d)	36. (b)	43. (a)	50. (b)	57. (b)	64. (b)
2. (c)	9. (b)	16. (c)	23. (c)	30. (a)	37. (d)	44. (a)	51. (c)	58. (c)	65. (d)
3. (a)	10. (b)	17. (a)	24. (d)	31. (c)	38. (a)	45. (c)	52. (b)	59. (a)	
4. (a)	11. (a)	18. (a)	25. (d)	32. (b)	39. (b)	46. (a)	53. (d)	60. (d)	
5. (d)	12. (c)	19. (b)	26. (a)	33. (a)	40. (a)	47. (a)	54. (d)	61. (b)	
6. (b)	13. (a)	20. (d)	27. (c)	34. (a)	41. (c)	48. (b)	55. (a)	62. (d)	
7. (c)	14. (b)	21. (b)	28. (b)	35. (b)	42. (b)	49. (a)	56. (c)	63. (a)	

SOLUTION

1. (c) Let the measure of angle = $x°$ measure of its complement= $x°$

$\therefore x° + x° = 90° \Rightarrow x° = 45°$

2. (c)



$\therefore a + b = 180°$

3. (a) Let the measure of angle = $x°$

$\therefore$ its supplement = $(180 - x)°$

$x = \frac{1}{3}(180 - x) \Rightarrow x = 45°$

4. (a) Given that,

$\frac{\alpha}{\beta} = \frac{1}{5} \Rightarrow \alpha = k$ and $\beta = 5k$

or complementary angles,

$\alpha = 90° - \beta \Rightarrow k = 90° - 5k$

$\Rightarrow k = 15°$

$\therefore$ a = 15° and β = 75°

$\therefore$ Difference between angles = 75° – 15° = 60°

5. (d)

6. (b) Let X < 90° and other angle = y

$\therefore$ X + Y = 180° (linear pair)

$\therefore$ Y = 180° - X , $\therefore$ Y > 90°

($\because$ X < 90°)

7. (c) $\angle POR$ = a° and $\angle QOR$ = b° form a linear pair

$\therefore$ a + b = 180° ______(1)

a – b = 80° ______(2) given

(1) + (2)

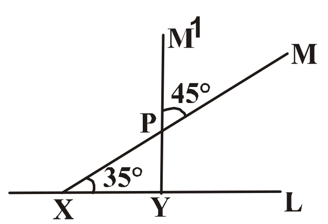
2a = 260° $\Rightarrow$ a = 130°

$\therefore$ Put in (1)

b = 180°- a = 180°–130° = 50°

$\therefore$ a = 130° and b = 50°

8. (b)



$\angle PYX = 180° - 35° - \angle XPY$
= 180°- 35°- 45° = 100°

$\therefore \angle PYL = 180° - \angle PYX$
= 80°

9. (b) $a > \frac{90°}{6} \Rightarrow a > 15°$

$a + b = 180° \Rightarrow b < 165° (\angle a > 15°)$

10. (b) $\angle COD = \angle EOF = 5Y°$ (vertically opposite angle)

$\therefore \angle AOD + \angle DOC + \angle COB = 180°$

$\Rightarrow 2y° + 5y° + 5y° = 180°$

$\Rightarrow 12y° = 180°$

$y = 15°$

11. (a) $\angle ROQ = \angle POS$ (vertically opposite angles)

$\therefore$ a = 4b

75° + b + a = 180°

$\Rightarrow$ b + 4b = 180°- 75° = 105°

$\Rightarrow$ b = 21°

$\therefore$ a = 4b = 4 × 21 = 84°

a + 2c = 180°

$\Rightarrow$ 2c = 180°- 84°

$\Rightarrow$ 2c = 96°

$\Rightarrow$ c = 48°

$\therefore$ a = 84°, b = 21°, c = 48°

12. (c) $\angle POX = 90°$
By line property:-
$\angle a + \angle b = 90°$
a : b = 2 : 3
So, $\angle a = 36°$ and $\angle b = 54°$
Here MN is a line so
$\Rightarrow 54° + \angle C = 180°$
$\Rightarrow \angle C = 126°$

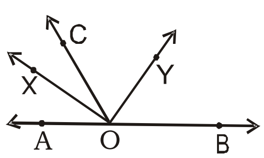
13. (a) by line property,
$60° + 4x + 40° = 180°$
$\Rightarrow 4x = 80°$
$\Rightarrow x = 20°$

14. (b) $\angle AOC = \angle BOD = 31°$ (vertically opposite)
$\therefore \angle BOC = 180° - \angle AOC = 149°$

15. (a) $2y + 3x = 180° \Rightarrow y = 36°$
$(\because x = 36°)$
or $x + 4y = 180° \Rightarrow y = 36°$
$(\because x = 36°)$

16. (c) $\angle BOC = 2\angle POC$
$\because$ OP bisects $\angle BOC$
$\angle AOC = 2\angle QOC$
$\because$ OQ bisects $\angle AOC$
Since, ray OC stands on line AB.
Therefore,
$\angle AOC + \angle BOC = 180°$
$\Rightarrow 2\angle QOC + 2\angle POC = 180°$
$\Rightarrow \angle QOC + \angle POC = 90°$
$\Rightarrow \angle POQ = 90°$

17. (a)

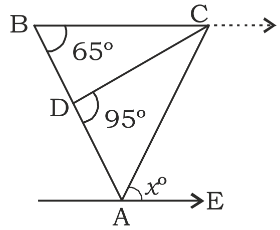


$\because \angle AOC = 40°$
$\therefore \angle BOC = 180° - 40° = 140°$
$\because$ OY is the bisector of $\angle BOC$
$\therefore \angle BOY = \frac{1}{2}\angle BOC$
$= \frac{1}{2} \times 140 = 70°$

18. (a) $\angle BOP = 90° - \angle AOB$
$= 90° - 20° = 70°$
$\therefore \angle POQ = 90° - \angle BOP$
$= 90° - 70° = 20°$

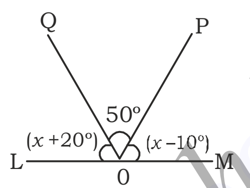
19. (b) $\angle PRQ + \angle QRS + \angle SRT = 180°$
$\therefore \angle PRQ + 80° + \angle PRQ = 180$
$(\therefore \angle PRQ = \angle SRT)$
$\Rightarrow \angle PRQ = 50°$
$\Rightarrow \angle PQR = 180° - \angle QPR - \angle PRQ$
$= 180° - 100° - 50° = 30°$

20. (d) Given that, BC || AE
$\angle CBA + \angle EAB = 180°$
$\Rightarrow \angle EAB = 180° - 65° = 115°$
$\because$ BC = AC
Hence, ΔABC is an isosceles triangle.



$\Rightarrow \angle CBA = \angle CAB = 65°$
Now, $\angle EAB = \angle EAC + \angle CAB$
$\Rightarrow 115° = x + 65° \Rightarrow x = 50°$

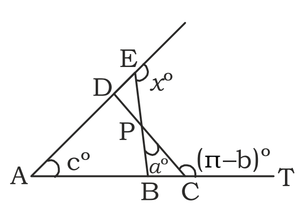
21. (b)



$\angle LOQ + \angle QOP + \angle POM = 180°$
$\therefore (x+20)° + 50° + (x - 10)° = 180°$
$\Rightarrow 2x + 60° = 180° \Rightarrow 2x = 120°$
$\therefore x = 60°$

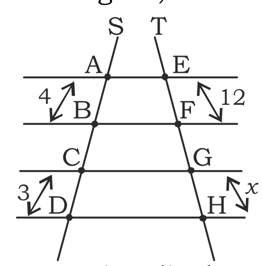
22. (c) Given that,
$\Rightarrow \frac{1}{2}\angle A + \frac{1}{3}\angle C + \frac{1}{2}\angle B = 80°$
$\Rightarrow 3\angle A + 2\angle C + 3\angle B = 480°$
$\Rightarrow 3(\angle A + \angle B) + 2\angle C = 480°$
Also In ΔABC,
$\angle A + \angle B + \angle C = 180°$
$\Rightarrow 3(\angle A + \angle B) + 3\angle C = 540°$ ....(ii)
On subtracting Eq. (i) from (ii), we get
$\angle C = 60°$

23. (c) $\angle PCT + \angle PCB = \pi$ (linear pair)
$\angle PCB = \pi - (\pi - b°) = b°$ ...(i)



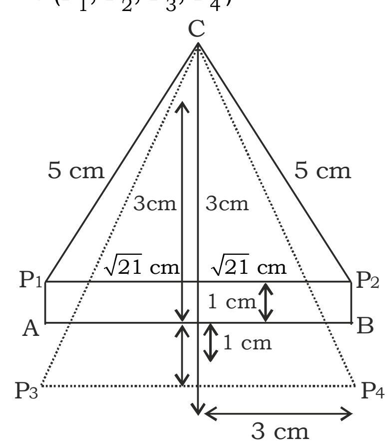
In ΔBPC,
$\angle PCB + \angle BPC + \angle PBC = \pi$
$\Rightarrow \angle PBC = \pi - \angle PCB - \angle BPC$
$= \pi - b° - a°$ ...(ii)
$\because \angle ABE + \angle EBC = \pi$
$(\because \angle PBC = \angle EBC)$
(Linear pair)
$\Rightarrow \angle ABE = \pi - \angle PBC = \pi - (\pi - b° - a°)$
$= a° + b°$ ...(iii)
Now is ΔABE,
Sum of two interior angles = Exterior angle
$\angle EAB + \angle ABE = \angle BES$
$\Rightarrow c° + b° + a° = x°$
$x° = a° + b° + c°$

24. (d) From figure,

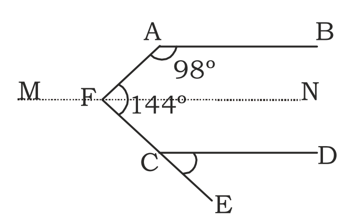


By proportionality law,
$$\frac{AB}{CD} = \frac{EF}{GH} \Rightarrow \frac{4}{3} = \frac{12}{x}$$
$x = 3 \times 3 = 9$

25. (d) Required number of points
$= 4\ (P_1, P_2, P_3, P_4)$



26. (a) Draw a line MN parallel to AB and CD.



$\therefore \angle AFN = 180° - 98° = 82°$

$\Rightarrow \angle CFN = 144° - 82° = 62°$

$\therefore \angle FCD = 180° - 62° = 118°$

$\Rightarrow \angle ECD = 180° - 118° = 62°$

27. (c) Sum of the three exterior angles

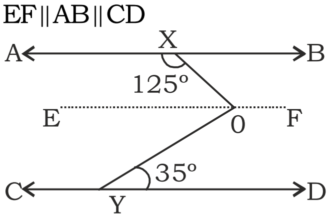
$=(\angle 1+\angle 2)+(\angle 2+\angle 3)+(\angle 3+\angle 1)$

$= 2(\angle 1+\angle 2+\angle 3)$

$= 2 \times 180° = 360°$

Sum of exterior angle of any polygon = 360°

28. (b) Draw a line EF such that EF || AB || CD



Now, AB || EF

$\therefore \angle AXO + \angle XOE = 180°$ (linear pair)

$\Rightarrow \angle XOE = 180° - 125° = 55°$

Also, EF || CD

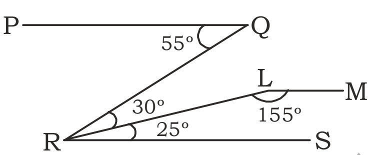
$\Rightarrow \angle EOY = \angle OYD = 35°$ (Alter. angle)

Hence,

$\angle XOY = \angle XOE + \angle EOY$

$= 55° + 35° = 90°$

29. (d) PQ || RS ...(i)



$\therefore \angle PQR = \angle QRS$ (Alter. angle)

and $\angle SRL + \angle RLM = 180°$

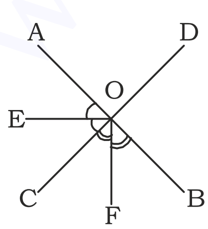
$\Rightarrow$ RS || LM ...(ii)

From Eqs. (i) and (ii)

Hence, angle between the line PQ and LM is 180°.

30. (a) Given, $\angle BOC = 130°$

$\because \angle BOC + \angle AOC = 180°$ (linear pair)



$\Rightarrow 130° + \angle AOC = 180°$

$\Rightarrow \angle AOC = 50°$

Now, $\angle BOC = 130°$

$\Rightarrow \angle BOF + \angle FOC = 130°$

$\Rightarrow \angle FOC + \angle FOC = 130°$

($\because$ OF is bisector of $\angle BOC$)

$\Rightarrow \angle FOC = 65°$

and $\angle AOC = 50°$

$\Rightarrow \angle AOE + \angle EOC = 50°$

$\Rightarrow \angle EOC + \angle EOC = 50°$

($\because$ OE is bisector of $\angle AOC$)

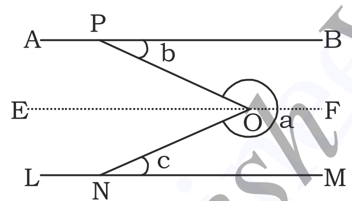
$\Rightarrow \angle EOC = 25°$

Hence,

$\angle EOF = \angle EOC + \angle FOC$

$= 25° + 65° = 90°$

31. (c) Let we draw a line parallel to AB which is EF.



$\therefore \angle EOP = \angle OPB$ (Alter. angle)

$\Rightarrow \angle EOP = b$

and

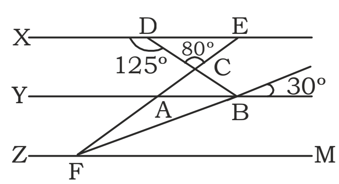
$\angle EON = \angle ONM$ (Alter. angle)

$\Rightarrow \angle EON = c$

$\therefore \angle PON = b + c$

$\because a = 2\pi - (b + c)$

32. (b) $\angle CDE = 180° - 125° = 55°$



In ΔDCE,

$\angle CED = 180° - 55° - 80° = 45°$

and $\angle ABF = 30°$ (vertically opposite angle)

Also, $\angle ABF = \angle BFM = 30°$ (Alternate angle)

and $\angle DEF = \angle EFM$ (Alternate angle)

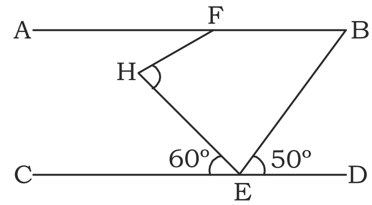
$\angle EFM = 45°$

$\Rightarrow \angle EFB + \angle BFM = 45°$

$\Rightarrow \angle EFB = 45° - 30°$

$\Rightarrow \angle AFB = 15°$

33. (a) $\angle HEB = 180° - 60° - 50° = 70°$



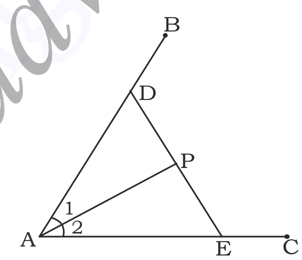
Since, HF || BE and HE is transversal line.

$\because \angle FHE + \angle HEB = 180°$

$\Rightarrow \angle FHE + 70° = 180°$

$\Rightarrow \angle FHE = 110°$

34. (a) $\frac{PD}{PE} = \frac{AD}{AE} = \frac{AP}{AP}$



ΔDAP and ΔAPE are similar

So, $\angle 1 = \angle 2$

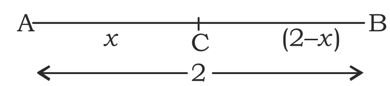
AP is bisector of $\angle A$

Hence, the locus of P is the angle bisector of angle A.

35. (b) Given, $AC^2 = AB \times CB$

$\Rightarrow x^2 = 2 \times (2 - x)$

$\Rightarrow x^2 = 4 - 2x$



$\Rightarrow x^2 + 2x - 4 = 0$

$\Rightarrow x = \frac{-2 \pm \sqrt{4+16}}{2 \times 1}$

$\Rightarrow x = -1 \pm \sqrt{5}$

Now, $BC = 2 - (-1 \pm \sqrt{5})$

$= 3 - \sqrt{5}$

(neglect $3+\sqrt{5} \Rightarrow 3+\sqrt{5} > 2$)

36. (b) AD || BE

$\therefore \angle ADC = \angle DCE$ (Alter. angle)

$\Rightarrow \angle ADB + 30° = 85°$

$\Rightarrow \angle ADB = 55°$

and $\angle BAD = 90°$

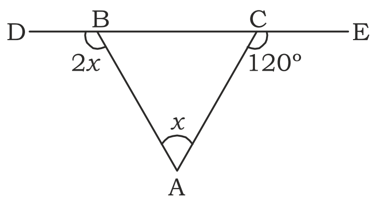
Now, In ΔABD,

$\angle ABD + \angle ADB + \angle BAD = 180°$

$\Rightarrow x + 55° + 90° = 180°$

$\Rightarrow x = 180° - 145° = 35°$

37. (d) $\angle ABC = 180° - \angle DBA$



$= 180° - 2x$

and $\angle ACB = 180° - \angle ACE$

$= 180° - 120° = 60°$

We know that,

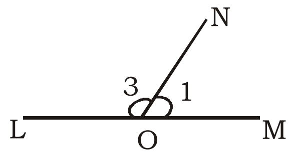
$\angle ABC + \angle ACB + \angle BAC = 180°$

$\Rightarrow 180° - 2x + 60° + x = 180°$

$\Rightarrow 240° - 180° = x$

$x = 60°$

38. (a)



$\angle MON = \frac{1}{3} \angle LON$

$\frac{\angle MON}{\angle LON} = \frac{1}{3}$

$\angle MON + \angle LON = 180°$

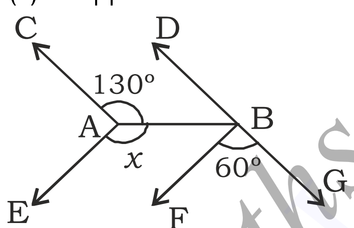
$x + 3x = 180°$

$4x = 180°$

$x = 45°$

$\angle MON = x = 45°$

39. (b) AC| |BD



$\therefore$ $\angle DBA = 180° - 130° = 50°$

Since, DBG is a straight line,

$\therefore$ $\angle DBA + \angle ABF + \angle FBG = 180°$

$\Rightarrow$ $50° + \angle ABF + 60° = 180°$

$\Rightarrow \angle ABF = 70°$

Since, AE$\parallel$BF

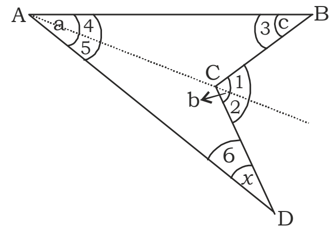
$\therefore$ $x = 180° - \angle ABF$

$x = 180° - 70° = 110°$

40. (a) $\angle 1 = \angle 3 + \angle 4$

$\angle 2 = \angle 5 + \angle 6$

(exterior angle is equal to sum of two opposite interior angles)



$\Rightarrow$ $\angle 1 + \angle 2 = \angle 3 + \angle 4 + \angle 5 + \angle 6$

$= b = c + a + x$

$x = b - c - a$

41. (c) $\angle DGC = \angle DEF = \angle ABC = 70°$ (corresponding angles)

42. (b) $\angle 1 = \angle 3$ (vertically opposite angles)

$\angle 3 = \angle 8$ (corresponding angles)

$\therefore$ $\angle 1 = \angle 8 \Rightarrow \angle 8 = 65°$

Now,

$\angle 5 + \angle 8 = 180° \Rightarrow \angle 5 = 115°$

Thus, $\angle 5 = 115°$ and $\angle 8 = 65°$

43. (a) $\because$ GF $\parallel$ HI

$\therefore$ CHI = $\angle FGC = 80°$

44. (a) $60° + a = 180° \Rightarrow a = 120°$

$\therefore$ $\angle loq = 180° - a = 60°$

$\therefore$ $\angle c = \angle loq = 60°$ (Alternate angles)

and $\angle b = \angle c = 60°$

(Vertically opposite angles)

and $\angle d = 180° - \angle c = 120°$

$\therefore$ a = 120°, b = 60°, c = 60°, d = 120°

45. (c) $\angle COE = \angle ABE = 75°$ (corresponding angles)

$\therefore$ $\angle DOE = 180° - \angle COE$

$= 180° - 75° = 105°$

In ΔDOE

$105° + 35° + \theta = 180°$

$\Rightarrow$ $\theta = 180° - 140° = 40°$

46. (a) BE | | DF

$\therefore$ $\angle ACE = \angle CDF = 70°$

(Corresponding angle)

$\therefore$ $\angle ACB = 180° - \angle ACE$

$= 110°$

$\therefore$ $\angle ABC = 180° - 20° - 110°$

$= 50°$

47. (a) CD | | AB

$\therefore$ $\angle AED = \angle PDC = 34°$

$\therefore$ $\angle DEF = 180° - 78° - 34°$

$= 68°$

$\because$ QD | | EF

$\therefore$ $\angle PDQ = \angle DEF = 68°$

(Corresponding angle)

48. (b) $\angle PEF = \angle PGH = 80°$

(Corresponding angle)

$\Rightarrow$ $\angle QGH = 100°$

$\angle QHD = 120°$

$\Rightarrow$ $\angle CHQ = 60°$

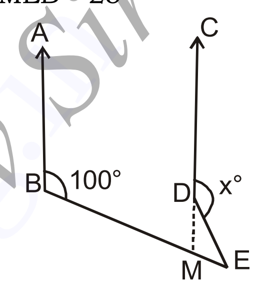
$\therefore$ $\angle x + 100° + 60° = 180°$

$\Rightarrow$ $x = 20°$

49. (a) Extend CD to M, then $\angle DME$

$= \angle ABE = 100°$

$\angle MED = 25°$



$\therefore$ $\angle MDE = 180° - (100° + 25°)$

$= 55°$

$\therefore$ $\angle CDE = 180° - 55° = 125°$

Alternate:

$\angle CDE = \angle DME + \angle MED = 125°$

50.(b) RS | | TU

$\therefore$ $\angle XNR = \angle XZT = 130°$ (corresponding angles)

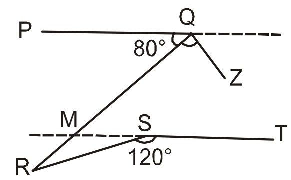
$\therefore$ VW | | XY

$\therefore$ $\angle OMR = \angle XNR = 130°$ (corresponding angles)

$\because$ PQ | | RS

$\therefore$ $\angle VOP = \angle OMR = 130°$ (corresponding angles)

51.(c)



$\angle MSR = 180° - \angle RST$

$= 180° - 120° = 60°$

$\because$ PQ | | ST

$\therefore$ $\angle QMS = \angle PQR = 80°$ (alternate angles)

$\therefore$ $\angle RMS = 180° - \angle QMS = 100°$

$\angle RMS + \angle MSR + \angle SRM = 180°$

$\Rightarrow \angle SRM = 180° - 100° - 60° = 20°$

$\because$ $\angle RQZ = 2\angle QRS = 2\angle SRM$

$\therefore$ $\angle RQZ = 2 \times 20° = 40°$

52. (b) $\because$ PR | | TS

$\therefore$ $\angle PRQ = \angle USR = 50°$

In ΔPQR :

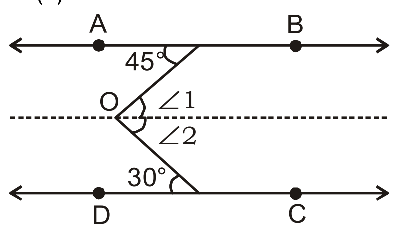
$\angle PQR = 180° - (50° + 60°) = 70°$

$\therefore$ $\angle TPU = \angle PQR = 70°$

[$\because$ PU | | RS | | QS]

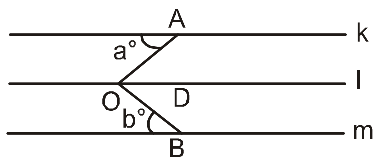
53. (d) $\angle DCK = \angle FDG = 55°$
(corresponding angle)
$\therefore$ $\angle ACE = \angle DCK = 55°$
(vertically opposite angle)
So, $\angle AEC = 180° - (40° + 55°) = 85°$
$\therefore$ $\angle HAB = \angle AEC = 85°$
(corresponding angle)
Hence, $x = 85°$

54. (d)



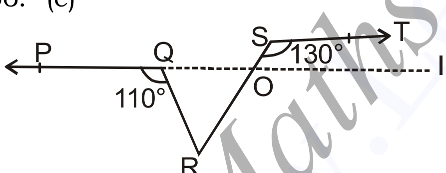
Through O, draw a line parallel to both AB & CD
Then, $\angle 1 = 45°$
(alternate angle)
and $\angle 2 = 30°$
(alternate angle)
$\therefore$ $\angle BOC = \angle 1 + \angle 2$
$= 45° + 30° = 75°$
So, $x = 360° - 75°$
$= 285°$
Hence, $x = 285°$

55. (a)



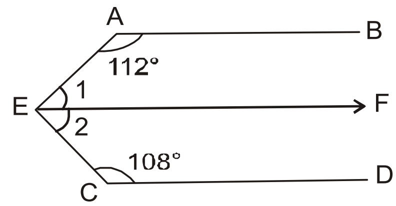
$\angle BOA = 45°$
$\because$ $k \,||\, l \,||\, m$
$\therefore$ $\angle DOB = b°$ and $\angle AOD = a°$
$\therefore$ $a° + b° = \angle AOB = 45°$

56. (c)



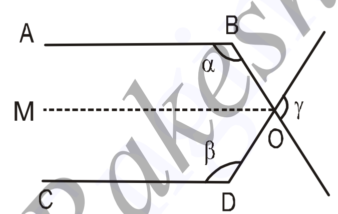
draw a line QI, then
$\angle RQO = 180° - 110° = 70°$
$\angle ROI = 130°$
(corresponding angle)
So, $\angle ROQ = 180° - 130° = 50°$
In ΔQRO
$\Rightarrow$ $\angle RQO + \angle ROQ + \angle QRO$
$= 180°$
$\Rightarrow$ $\angle QRO = 180° - (70° + 50°)$
$\Rightarrow \angle QRO = 60° = \angle QRS$

57. (b)



Draw EF parallel to both AB and CD
$\because$ AB || EF and AE transversal cuts them at A and E respectively.
$\therefore$ $\angle BAE + \angle FEA = 180°$
$\Rightarrow$ $112° + \angle 1 = 180°$
$\angle 1 = 68°$
Again EF || CD and transversal cuts them at C and E.
$\therefore$ $\angle FEC + \angle ECD = 180°$
$\Rightarrow$ $\angle 2 + 108° = 180°$
$\Rightarrow$ $\angle 2 = 72°$
Now, $x = \angle 1 + \angle 2$
$\Rightarrow$ $x = 72° + 68° = 140°$

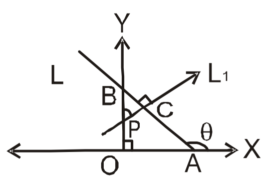
58. (c) Draw OM || AB || CD



$\because$ AB || OM
$\therefore$ $\angle BOM = 180° - \alpha$
Also,
$\because$ OM || CD
$\therefore$ $\angle DOM = 180° - \beta$
$\therefore$ $\gamma = \angle BOD$ (vertically opposite angle)
$\Rightarrow$ $\gamma = \angle BOM + \angle DOM$
$\Rightarrow$ $\gamma = 180° - \alpha + 180° - \beta$
$\Rightarrow$ $\alpha + \beta + \gamma = 360°$

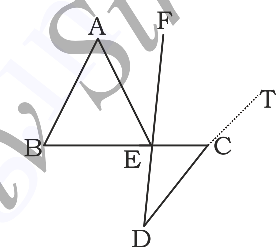
59. (a) $\angle AEC + \angle CED + \angle DEB = 180°$
$\Rightarrow$ $37° + 90° + \angle DEB = 180°$
$\Rightarrow$ $\angle DEB = 180° - 127°$
$= 53°$
$\because$ EB || CD
$\therefore$ $\angle DEB = \angle EDC = 53°$

60.(d)



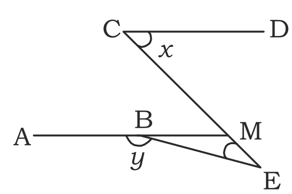
$\angle BCP = 180° - \angle BCL_1 = 90°$
In Δ BOA,
$\angle O = 90°$, $\angle A = 180° - \theta$
$\therefore$ $\angle OBA = 180° - (90° + 180° - \theta)$
$= \theta - 90$
In Δ BPC,
$\angle BPC = 180° - (90° + \theta - 90°)$
$= 180° - \theta$
$= \pi - \theta$

61. (b)



Given that,
$\angle ABC = 65°$ and $\angle CDE = 15°$
Here,
$\angle ABC + \angle TCB = 180°$
($\because AB \| CD$)
$\therefore$ $\angle TCB = 180° - 65° = 115°$
$\because$ $\angle TCB + \angle DCB = 180°$
(linear pair)
$\therefore$ $\angle DCB = 65°$
Now, In ΔCDE
$\angle CED = 180° - (\angle ECD + \angle EDC)$
$= 180° - (65° + 15°) = 100°$
$\because$ $\angle DEC + \angle FEC = 180°$
$\Rightarrow$ $\angle FEC = 180° - 100° = 80°$
Given that, AB = AE
i.e.,
ΔABE is an isosceles triangle.
$\therefore$ $\angle ABE = \angle AEB = 65°$
$\because$ $\angle AEB + \angle AEF + \angle FEC = 180°$
(Straight line)
$65° + x° + 80° = 180°$
$\therefore$ $x° = 180° - 145° = 35°$

62. (d) Here, we produced line AB to AM.



Since, AM is parallel to CD.
$\therefore$ $\angle DCM = \angle BMC = x$
(alternate angle)
Also, ABM is a straight line.
$\therefore$ $\angle EBM = \pi - y$

$\Rightarrow \angle MBE + \angle MEB = \angle CMB$

$\pi - y + \angle MEB = x$

$\angle MEB = x + y - \pi$

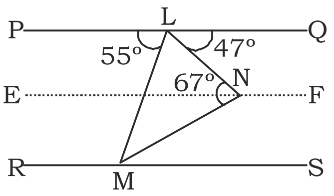
63. (a) $\angle PLM = \angle LMS$

$= 55°$ (Alter. angle)

Let draw EF line which is parallel to PQ and bisects by LN.

Then, $\angle QLN = \angle LNE = 47°$

$\because \angle ENL + \angle MNE = 67°$



$\Rightarrow 47° + \angle MNE = 67°$

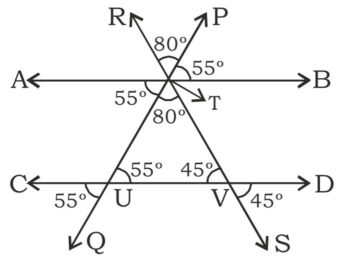
$\angle MNE = 67° - 47°$

$\angle MNE = 20°$

Similarly, EF | | RS,

Then $\angle ENM = \angle NMS = 20°$

64. (b) $\angle PTB = 55°$



Then, $\angle TUV = 55°$

(Corresponding anlge)

Also, $\angle PTB = \angle UTA = 55°$

(Vertically opposite angle)

$\angle CUQ = \angle UTA = 55°$ (Corresponding angle)

Also, given $\angle DVS = 45°$

Then, $\angle UVT = 45°$

(Vertically opposite angle)

In ΔUTV,

$\angle T = 180° - (55° + 45°) = 80°$

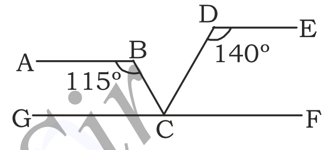
$\Rightarrow \angle T = \angle PTR = 80°$

(Vertically Opposite angle)

$\therefore \angle CUQ + \angle RTP = 55° + 80° = 135°$

65. (d) Draw a line GF through C parallel to AB and DE.

Now,



$\therefore \angle BCG = 180° - \angle ABC$

$= 180° - 115° = 65°$

and $\angle DCF = 180° - \angle CDE$

$= 180° - 140° = 40°$

Now, $\angle BCG + \angle BCD + \angle DCF = 180°$

$\Rightarrow 65° + \angle BCD + 40° = 180°$

$\Rightarrow \angle BCD = 180° - 105° = 75°$

CHAPTER 21

TRIANGLE

A **triangle** is closed figure with three sides or a closed plane figure having **three sides** and **three angles**.

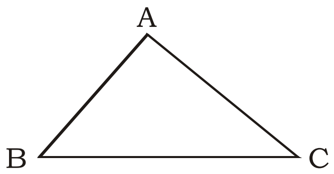
Classification of Triangles

There are many different kinds of triangles. The following table outlines some basic types of triangles.

On the basis of sides

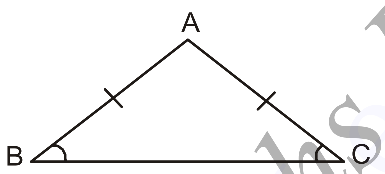
(i) **Scalene Triangle -** All three sides are of different lengths is called a Scalene Triangle.

$AB \neq BC \neq CA$



(ii) **Isosceles Triangle :-** Two sides are of equal length, is called a Isosceles Triangle.

AB = AC

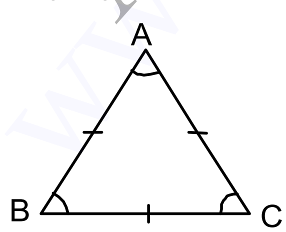


→ Angles opposite to equal sides are equal.

i.e. $\angle B = \angle C$

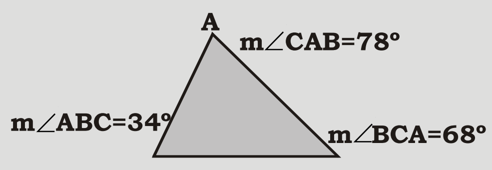
(iii) **Equilateral Triangle -** A triangle having all sides equal is called Equilateral triangle.

AB = BC = CA

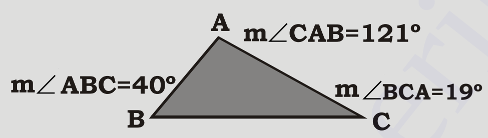


- All angles are equal and is equal to 60°

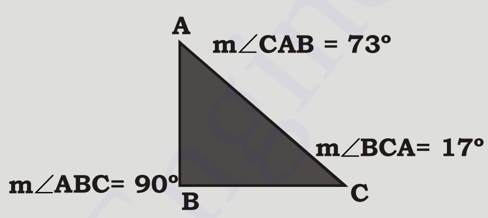
$\angle A = \angle B = \angle C = 60°$



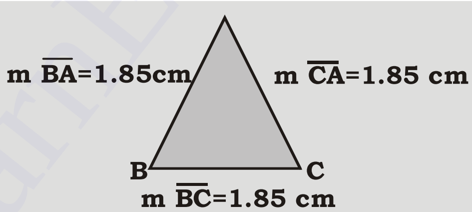
Acute triangles are triangles in which the measure of all three are less than 90° degrees.



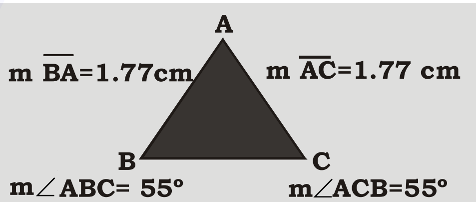
Obtuse triangles are triangles in which the measure of one angle is greater than 90° degrees.



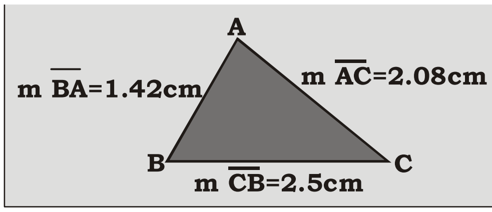
Right triangles are triangles in which the measure of one angle equals 90° degrees.



Equilateral triangles are triangles in which all three sides are the same length.



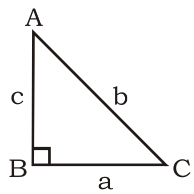
Isosceles triangles are triangles in which two of the sides are the same length.



Scalene triangles are triangles in which none of the same length.

On the basis of angles

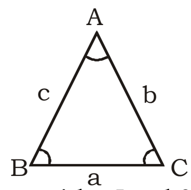
(i) **Right-angled Triangle -** In a triangle, in which one of the angles measures 90°. Rest two angles are complementary to each other.



Largest side, $b^2 = a^2 + c^2$

$\therefore \angle B = \angle A + \angle C$

(ii) Acute-angled Triangle - A triangle in which every angle is more than 0° and less than 90°.



Largest side, Let $b^2 < (a^2 + c^2)$

$\therefore$ According to cosine formula

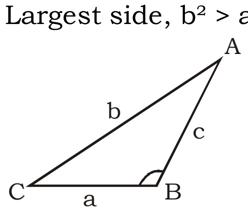
$\boxed{b^2 = a^2 + c^2 - 2ac \cos B}$

Here cos gives +ve value because $B < 90^\circ$

Each angle is less than the sum of the other two.

(iii) Obtuse-angled triangle - A triangle in which one of the angles is more than 90°.

Largest side, $b^2 > a^2 + c^2$



$\therefore$ According to cosine formula

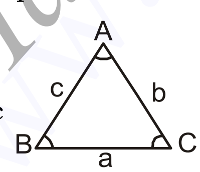
$\boxed{b^2 = a^2 + c^2 + 2ac \cos B}$

Here cos gives negative value So, –ve becomes +ve because $B > 90^\circ$

$\therefore \angle B > \angle A + \angle C$

Perimeter of a triangle (2s) : The sum of lengths of three sides of a triangle is called its perimeter.

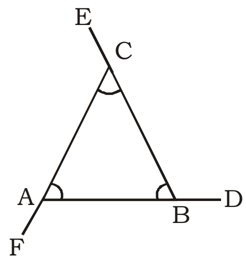
$2s = a + b + c$



Some basic theorem on the basis of Triangle

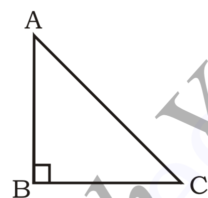
Theorem – 1

"A triangle is a closed figure that has three sides and three angles. The sum of the three angles of every triangle is always equals to **180°.** "



i.e., $\angle CAB + \angle ABC + \angle BCA = 180^\circ$

"A right triangle always has one angle equal to **90°** and two angles whose sum is **90°**, making a total of **180°**."



$\angle B = 90^\circ$

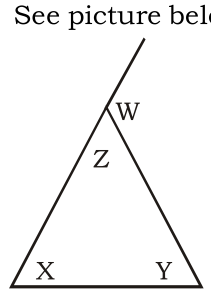
$\therefore \quad \angle A + \angle C = 90^\circ$

Then, $\angle A + \angle B + \angle C = 180^\circ$

Theorem – 2

Exterior Angle Theorem

At each vertex of a triangle, an exterior angle of the triangle may be formed by extending ONE SIDE of the triangle. See picture below.



Calculating the Angles

We can use equations to represent the measures of the angles described above. One equation might tell us the sum of the angles of the triangle. For example,

$x + y + z = 180^\circ$

We know this is true, because the sum of the angles inside a triangle is always 180°. What is w ? We don't know yet. But, we may observe that the measure of angle w plus the measure of angle z = 180°, because they are a pair of supplementary angles. Notice how Z and W together make a straight line ? That's 180°. So, we can make a new equation:

$w + z = 180$

Then, if we combine the two equations above, we can determine that the measure of angle $w = x + y$. Here's how to do that:

$x + y + z = 180^\circ$ ....(i)

$w + z = 180^\circ$ ....(ii)

Now, rewrite the second equation as $z = 180 - w$ and the substitute that for z in the first equation.

$x + y + (180 - w) = 180$

$x + y - w = 0$

$x + y = w$

The measure of the exterior angle equals the total of the other two interior angles. In fact, there is a theorem called the Exterior Angle Theorem.

Theorem – 3

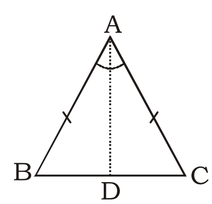
Angles Opposite to Equal Sides of a Triangles are Equal

Angles Opposite to equal sides of an isosceles triangle are equal.

Given : $\triangle ABC$ such that $AB = AC$

To Prove : $\angle ABC = \angle ACB$

Construction : Draw AD, the bisector of $\angle BAC$ to meet BC at D.



Proof

In $\triangle ABD$ and $\triangle ADC$,

$AB = AC$ **....(Given)**

$AD = AD$ **....(Common)**

$\angle BAD = \angle CAD$ **...(by construction)**

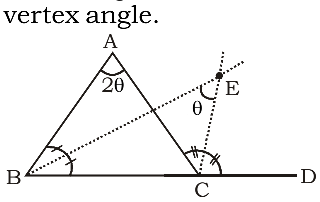
$\therefore \triangle ABD \cong \triangle ACD$**...(SAS Cong. Axiom)**

$\therefore \angle ABD = \angle ACD$ (CPCT)

$\Rightarrow \angle ABC = \angle ACB$

Theorem – 4

The angle between internal bisector of a base angle and external bisector of the other base angle is half of the remaining vertex angle.

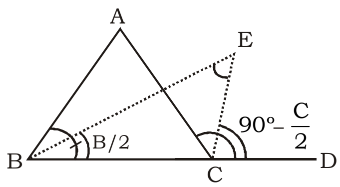


Proof

$\angle ACD = 180 - C$

$\angle ECD = \dfrac{180°-C}{2} = 90° - \dfrac{C}{2}$

In ΔBEC exterior angle,



$\angle ECD = \angle EBC + \angle BEC$

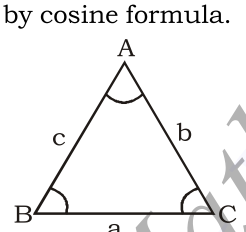
$90° - \dfrac{C}{2} = \dfrac{B}{2} + \angle BEC$

$\angle BEC = 90 - \left(\dfrac{B+C}{2}\right) = 90 - \left(\dfrac{180-A}{2}\right) = \dfrac{A}{2}$

Theorem – 5

Cosine Formula

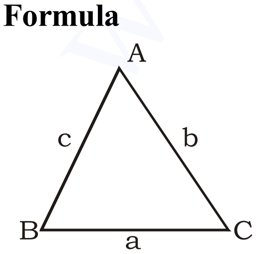
If two sides and angle between sides are given then we can find the opposite side by cosine formula.



$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$

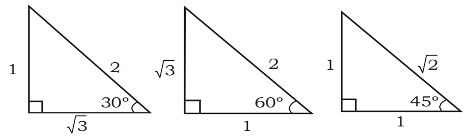
Theorem – 6

Sine Formula



$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

Some Important Facts About Right Angle Triangle



EXAMPLES

1. The sides of a triangle are 5cm, 6 cm and 4 cm respectively the triangle is:

Sol. $6^2 < 5^2 + 4^2$

$\therefore$ The triangle is Acute angled triangle

2. The sides of a triangle are 7cm, 2cm and 10cm respectively the triangle is:

Sol. 10^2 (largest side) $> 7^2 + 2^2$

$\therefore$ The triangle is Obtuse angled triangle

3. The length of the three sides of a right angled triangle are $(x - 2)$cm, x cm & $(x + 2)$ cm respectively. Then the value of x is:

Sol. $(x + 2)^2 = (x - 2)^2 + x^2$

$\Rightarrow x^2 + 4 + 4x = x^2 + 4 - 4x + x^2$

$\Rightarrow x^2 = 8x$

$\Rightarrow x = 8$

4. If two angles of a right angled triangle other than right angle are $(2x+21)°$ and $(3x+34)°$ then x is:

Sol. $2x + 21° + 3x + 34° = 90°$

$5x = 35°$ or $x = 7°$

5. In a ΔABC, $\angle A + \angle B = 113°$, $\angle B + \angle C = 105°$, then find $\angle B$.

Sol. $\angle A + \angle B + \angle B + \angle C = 113° + 105°$

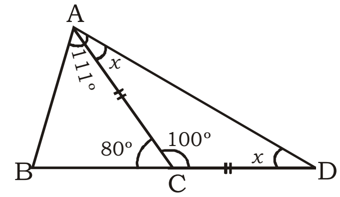
$180° + \angle B = 218°$

$\angle B = 218° - 180° = 38°$

6. In a triangle ABC, BC is produced to D so that CD = AC. If $\angle BAD = 111°$ and $\angle ACB = 80°$, then the measure of $\angle ABC$ is:

Sol. In ACB, $2x = 180° - 100° = 80°$

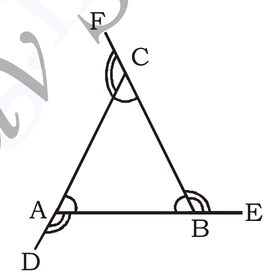
$\Rightarrow x = 40°$



In ΔABD,

$\angle ABC = 180° - (111° + 40°) = 29°$

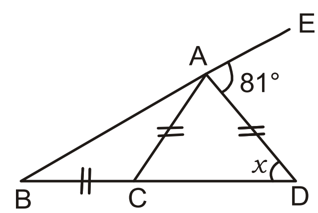
7. If the sides of a triangle are produced then the sum of the exterior angles i.e. $\angle DAB + \angle EBC + \angle FCA$ is equal to



Sol. Sum of exterior angles = 360°

$\therefore$ $\angle DAB + \angle EBC + \angle FCA = 360°$

8. In the given figure, BC = AC = AD, $\angle EAD = 81°$. Find the value of x :



Sol. In ΔACD,

$\because$ $\angle ACD = \angle ADC = x$

$\therefore$ $\angle CAD = (180° - 2x)$

In ΔABC,

$\angle ABC = \angle BAC = \dfrac{x}{2}$

$(\because \angle ABC + \angle BAC = \angle ACD = x)$

$\therefore$ $\angle BAC + \angle CAD + 81° = 180°$

$\dfrac{x}{2} + (180° - 2x) + 81° = 180°$

$\dfrac{3}{2}x = 81° \Rightarrow x = 54°$

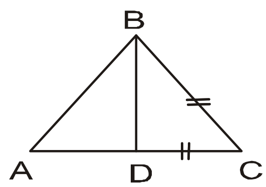
9. In the given triangle ABC, BC = CD and $(\angle ABC - \angle BAC) = 30°$. The measure of $\angle ABD$ is:

Sol. Given,

$\because$ BC = CD

$\therefore$ $\angle BDC = \angle DBC$

& $\angle ABC - \angle BAC = 30°$



$\angle ABD = \angle ABC - \angle DBC$

$= \angle ABC - \angle BDC$

$(\because \angle DBC = \angle BDC)$

$= \angle ABC - (\angle ABD + \angle BAD)$

$\Rightarrow 2(\angle ABD) = \angle ABC - \angle BAD$

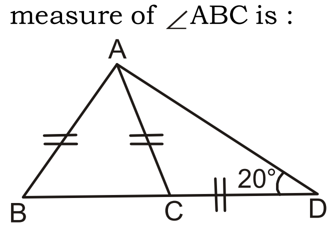
$= \angle ABC - \angle BAC$

$(\because \angle BAD = \angle BAC)$

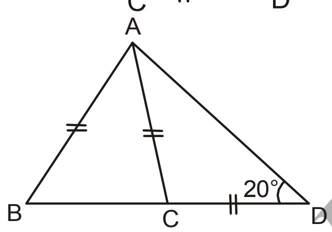
$= 30°$

$\Rightarrow \angle ABD = 15°$

10. Consider ΔABD such that $\angle ADB = 20°$ and C is a point on BD such that AB = AC and CD = CA. Then the measure of $\angle ABC$ is :



Sol.



$\angle ADB = 20°$

$\Rightarrow \angle ADB = \angle CAD = 20°$

$(\because AC = CD)$

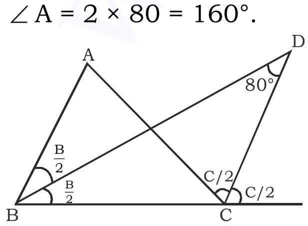
$\therefore$ $\angle ACB = 20° + 20° = 40°$ (exterior angle of ΔACD)

$\therefore$ $\angle ABC = \angle ACB = 40°$ $(\because AB = AC)$

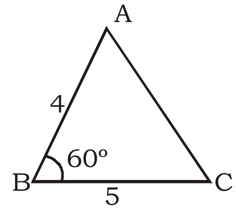
11. In ΔABC, the bisectors of the internal angle $\angle B$ & external angle $\angle C$ intersects at D. If $\angle BDC = 80°$, then $\angle A$ is :

Sol. $\angle A = 2\angle BDC$.

$\angle A = 2 \times 80 = 160°$.



12.



AC = ?

Sol. By Cosine formula,

$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cos B$

$AC^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \cos 60°$

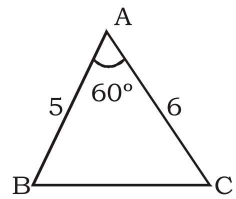
$AC^2 = 16 + 25 - 40 \times \frac{1}{2}$

$AC^2 = 41 - 20$

$AC^2 = 21$

$\therefore AC = \sqrt{21}$

13.



Find BC = ?

Sol. By using Cosine formula,

$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cos 60°$

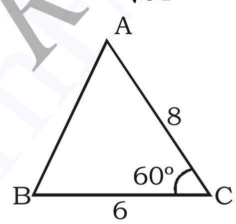
$BC^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \frac{1}{2}$

$BC^2 = 25 + 36 - 30$

$BC^2 = 31$

$\therefore BC = \sqrt{31}$

14.



Find AB = ?

Sol. By using Cosine formula,

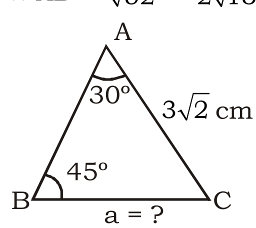
$AB^2 = AC^2 + BC^2 - 2AC \times BC \cos 60°$

$AB^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \times \frac{1}{2}$

$AB^2 = 64 + 36 - 48$

$\therefore AB = \sqrt{52} = 2\sqrt{13}$

15.



Sol. By using Sine formula,

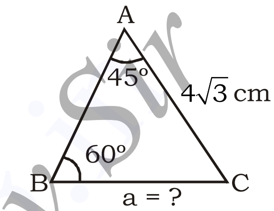
$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\Rightarrow \frac{a}{\sin 30°} = \frac{3\sqrt{2}}{\sin 45°}$

$\Rightarrow a = 3\sqrt{2} \times \sqrt{2} \times \frac{1}{2}$

$\therefore a = 3$ cm

16.



Sol. By using sine formula,

$\frac{a}{\text{Sin}A} = \frac{b}{\sin B}$

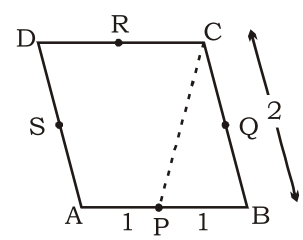
$\Rightarrow \frac{a}{\sin 45°} = \frac{4\sqrt{3}}{\sin 60°}$

$\Rightarrow a = \frac{4\sqrt{3}}{\sqrt{3}} \times 2 \times \frac{1}{\sqrt{2}}$

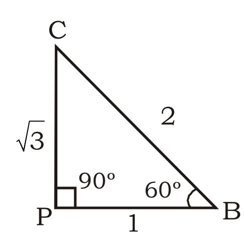
$\therefore a = 4\sqrt{2}$ cm

17. A, B, C, D are the vertex of a Rhombus and P, Q, R, S are the mid points of AB, BC, CD & DA. Find the largest angle of the rhombus ? & $CP \perp AB$

Sol. Let the side of rhombus = 2 units



As we know that,



$\therefore$ Largest angle = $\angle DAB$

$= 180° - 60° = 120°$

EXERCISE

1. In a triangle ABC, if AB, BC and AC are the three sides of the triangle, then which of the statement is necessarily true?
(a) AB + BC < AC
(b) AB + BC > AC
(c) AB + BC = AC
(b) AB – BC > AC

2. The sides of a triangle are 12 cm, 8 cm and 6 cm respectively, the triangle is :
(a) Acute Angled
(b) Obtuse Angled
(c) Right Angled
(d) Can't be determined

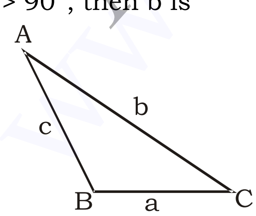
3. In a ΔABC, $\angle BAC > 90°$, then $\angle ABC$ and $\angle ACB$ must be:
(a) Acute (b) Obtuse
(c) One Acute and One Obtuse
(d) Can't be determined

4. In a ΔABC, $\angle A = x$, $\angle B = y$ and $\angle C = (y + 20)°$. If $4x - y = 10$, then the triangle is:
(a) Right-angled
(b) Obtuse-angled
(c) Equilateral
(d) None of these

5. If one angle of a triangle is equal to the sum of the other two, then the triangle is:
(a) Right-angled
(b) Obtuse-angled
(c) Acute-angled
(d) None of these

6. If each angle of a triangle is less than the sum of the other two, then the triangle is :
(a) Right-angled
(b) Acute-angled
(c) Obtuse-angled
(d) None of these

7. ABC is a triangle. It is given that a + c > 90°, then b is



(a) greater than 90°
(b) less than 90°
(c) equal to 90°
(d) can't be said

8. A man goes 150 m due east and then 200 m due north, How far is he from the starting point ?
(a) 200 m (b) 350 m
(c) 250 m (d) 175 m

9. In a ΔABC, $\angle A : \angle B : \angle C = 2:3:4$. Then find the smallest angle is :
(a) 45° (b) 40° (c) 60° (d) 65°

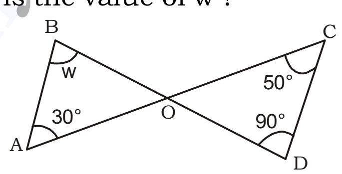
10. The sum of two angles of a triangle is equal to its third angle. Determine the measure of third angle :
(a) 100° (b) 80°
(c) 120° (d) 90°

11. Of the three angles of a triangle, one is twice the smallest and another is three times the smallest. Then find smallest angle :
(a) 60° (b) 90° (c) 30° (d) 45°

12. In a Δ ABC, If $2\angle A = 3\angle B = 6\angle C$, then $\angle A$ is equal to :
(a) 60° (b) 30°
(c) 90° (d) 120°

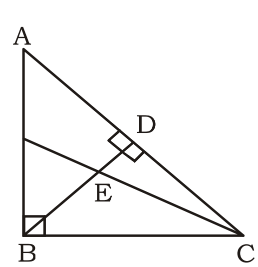
13. The degree measures each of the three angles of a triangle is an integer. Which of the following could not be the ratio of their measures ?
(a) 2 : 3 : 4 (b) 3 : 4 : 5
(c) 5 : 6 : 7 (d) 6 : 7 : 8

14. In the given figure below, what is the value of w ?



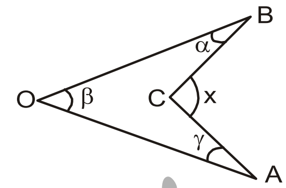
(a) 100° (b) 110°
(c) 120° (d) 130°

15. $AB \perp BC$, $BD \perp AC$ and CE bisects $\angle C$, $\angle A = 30°$. then, what is $\angle CED$?



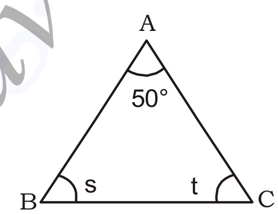
(a) 30° (b) 60° (c) 45° (d) 65°

16. In the given figure, x = ?



(a) $\alpha + \beta - \gamma$ (b) $\alpha - \beta + \gamma$
(c) $\alpha + \beta + \gamma$ (d) $\alpha + \gamma - \beta$

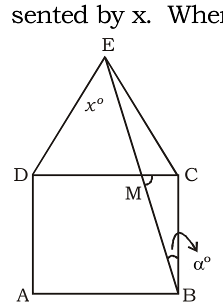
17. In the figure below, if s < 50°< t, then



(a) $t < 80$ (b) $s + t < 130$
(c) $50 < t < 80$
(d) $t > 80$

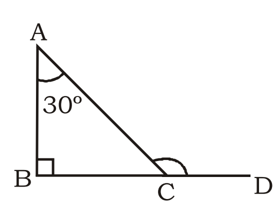
18. The sum of two angles of a triangle is 80° and their difference is 20°, then the smallest angle:
(a) 50° (b) 100° (c) 30°
(d) None of these

19. In the given diagram, equilateral triangle EDC surmounts square ABCD. Find the $\angle BED$ represented by x. Where, $\angle EBC = \alpha°$:



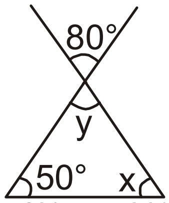
(a) 45° (b) 60° (c) 30°
(d) None of these

20. In the given figure, if $\angle ABC = 90°$, and $\angle A = 30°$, then $\angle ACD$ =



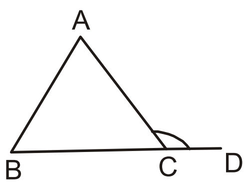
(a) 120° (b) 100° (c) 110°
(d) None of these

21. Value of x and y is :



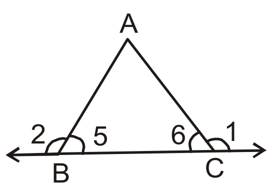
(a) $x = 60°$, $y = 80°$
(b) $x = 80°$, $y = 50°$
(c) $x = 50°$, $y = 80°$
(d) None of these

22. In the triangle ABC, side BC is produced to D, $\angle ACD = 100°$ if BC = AC, then find $\angle ABC$ is :



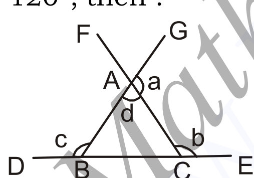
(a) 40° (b) 50° (c) 80°
(d) Can't be determined

23. In the given figure, the side BC of a ΔABC is produced on both sides, then $\angle 1 + \angle 2$ is equal to:



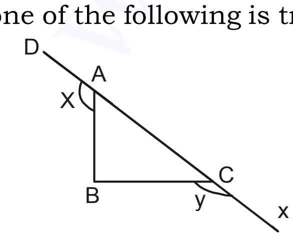
(a) $\angle A + 180°$
(b) $180° - \angle A$
(c) $\frac{1}{2}(\angle A + 180°)$
(d) $\angle A + 90°$

24. It is given that d = 70°, b = 120°, then :



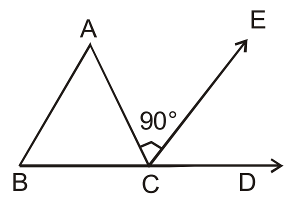
(a) c = 130° (b) a = 110°
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

25. It is given that $AB \perp BC$. Which one of the following is true :



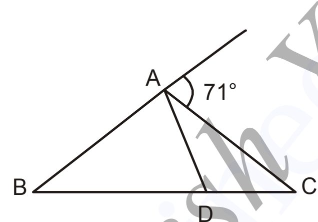
(a) $x + y = 180°$
(b) $x + y = 270°$
(c) $x + y = 300°$
(d) can not be said

26. In the given figure, $AC \perp CE$ and $\angle A : \angle B : \angle C = 3 : 2 : 1$, find the value of $\angle ECD$:



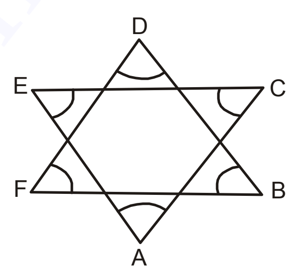
(a) 50° (b) 45° (c) 55° (d) 60°

27. In the given figure, if AD = BD = AC, then the value of $\angle C$ will be :



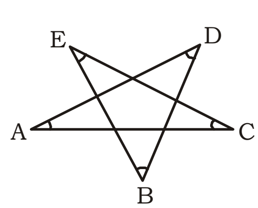
(a) $\frac{124°}{3}$ (b) $\frac{142°}{3}$
(c) 39°
(d) None of these

28. In the given figure, $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F =$



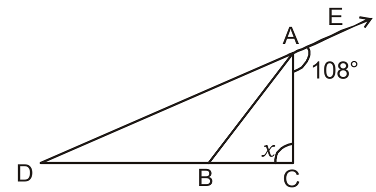
(a) 360° (b) 720°
(c) 180° (d) 300°

29. In the given figure, $\angle A + \angle B + \angle C + \angle D + \angle E =$



(a) 900° (b) 720°
(c) 180° (d) 540°

30. In the given figure, AB divides $\angle DAC$ in the ratio 1: 3 and AB = DB. The value of x :

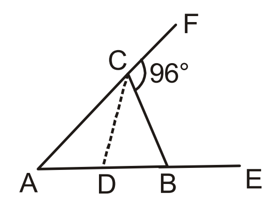


(a) 90° (b) 80°
(c) 100° (d) 70°

31. The side BC of ΔABC is produced to D. If $\angle ACD = 108°$ and $\angle B = \frac{1}{2}\angle A$ then $\angle A$ is :

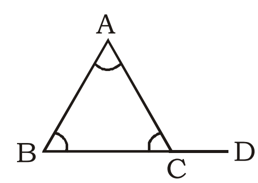
(a) 36° (b) 108°
(c) 59° (d) 72°

32. In the given figure below, if AD = CD = BC, and $\angle BCF = 96°$, How much is $\angle DBC$?



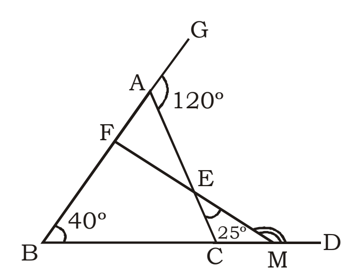
(a) 32° (b) 84° (c) 64°
(d) Can't be determined

33. In the given figure BC is produced to D and $\angle BAC = 40°$ and $\angle ABC = 70°$. Find the value of $\angle ACD$



(a) 30° (b) 40°
(c) 70° (d) 110°

34. In the given figure, $\angle CAG = 120°$, $\angle CEM = 25°$, $\angle ABC = 40°$, Find $\angle EMD$ = ?



(a) 125° (b) 140°
(c) 70° (d) 110°

35. In ΔABC, the line BC extend up to D. If $\angle$ACD = 170° and $\angle$B = $\frac{2}{3}\angle$A then $\angle$A is :

(a) 10° (b) 68°
(c) 102° (d) Not

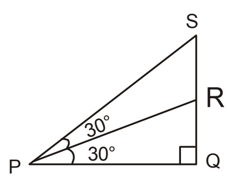
36. In ΔABC, value of exterior angle at B and C are 111° & 148° respectively then $\angle$A is :

(a) 32° (b) 69°
(c) 79° (d) 101°

37. In a ΔABC, $\angle$BAC = 75°, $\angle$ABC = 45° BC is produced to D. If $\angle$ACD = x, then $\frac{x}{3}$% of 60° is:

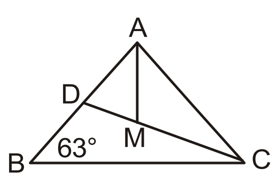
(a) 30° (b) 48° (c) 15° (d) 24°

38. In the given figure which of the following statements is true ?



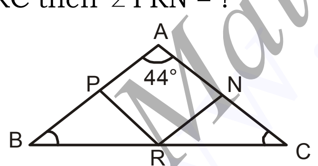
(a) PQ = QS (b) PR = RS
(c) QR = RS (d) PS < RS

39. In the given figure, AM = AD, B = 63° and CD is an angle bisector of $\angle$C, then $\angle$MAC = ?



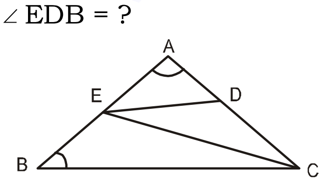
(a) 27° (b) 37° (c) 63°
(d) None of these

40. If $\angle$A = 44°, BP = BR and CN = RC then $\angle$PRN = ?



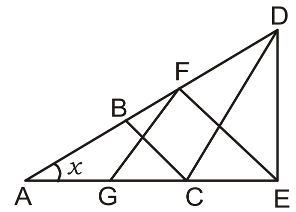
(a) 58° (b) 78° (c) 68° (d) 66°

41. In the given figure, if $\angle$B = $\angle$C = 78°, BC = EC, CD = BC and DE not parallel to BC, then $\angle$EDB = ?



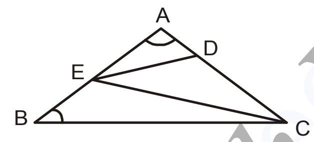
(a) 18° (b) 12° (c) 22°
(d) None of these

42. In the given figure, if AB = BC = CD = EF = DE = GA = FG, then x =?



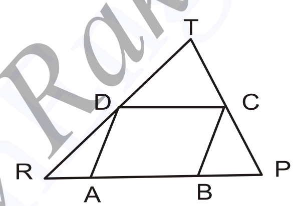
(a) $\frac{153°}{7}$ (b) 28°
(c) $\frac{180°}{7}$ (d) None of these

43. In the given figure, if AD = DE = EC = BC then $\angle$A : $\angle$B =



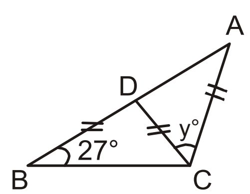
(a) 1 : 3 (b) 2 : 5
(c) 3 : 1 (d) 1 : 2

44. In the given figure, ABCD is a rhombus and AR = AB = BP, then the value of $\angle$RTP is :



(a) 60° (b) 90° (c) 120° (d) 75°

45. In the following figure ADBC, BD = CD = AC, $\angle$ABC = 27°, $\angle$ACD = Y. Find the value of y :

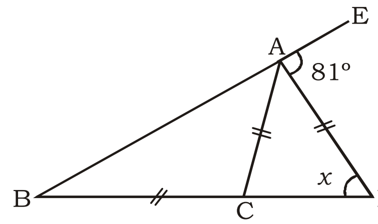


(a) 27° (b) 54° (c) 72° (d) 58°

46. ABC is an isosceles triangle with AB = AC. Side BA is produced to D such that AB = AD. Find $\angle$BCD :

(a) 60° (b) 90° (c) 120°
(d) can't be determined

47. In the given figure, BC = AC = AD and $\angle$EAD = 81°. Find the value of x :



(a) 45° (b) 54° (c) 63° (d) 36°

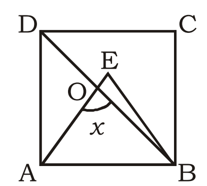
48. ABC is an equilateral triangle and CD is the internal bisector of $\angle$C. If DC is produced to E such that AC = CE then $\angle$CAE is equal to :

(a) 45° (b) 75° (c) 30° (d) 15°

49. In ΔABC, $\angle$ABC = 120° then relation between sides is :

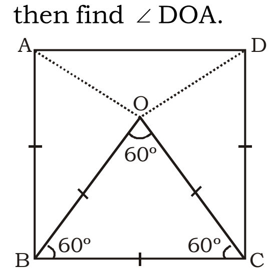
(a) $b^2 = a^2 + c^2 + ac$
(b) $b^2 = a^2 + c^2 - ac$
(c) $b^2 = a^2 + c^2 - 2ac$
(d) $b^2 = a^2 + c^2 + 2ac$

50. In the figure ΔABE is an equilateral triangle in a square ABCD. Find the value of angle x in degrees :



(a) 60° (b) 45° (c) 75° (d) 90°

51. ABCD is a square in which ΔOBC is an equilateral triangle then find $\angle$DOA.



(a) 60° (b) 45°
(c) 150° (d) 90°

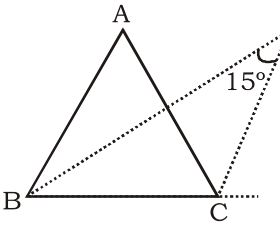
52. In ΔABC, $\angle$A = 110° and D, E are two points on BC is such that $\angle$BDA = 140°, $\angle$CEA = 120° and $\angle$EAC = 2 × $\angle$BAD. Find the angle $\angle$ABD

(a) 30° (b) 40° (c) 60° (d) 80°

53. In an obtuse angle ΔABC the external angle bisector of $\angle$A intersect the extended part of line CB at M and the external angle bisector of $\angle$C intersect the extended part of line AB at N. MA = AC = CN, then find $\angle B$ = ?

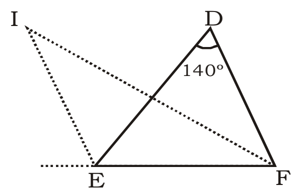
(a) 108° (b) 110°
(c) 112° (d) 114°

54. In ΔABC, the bisectors of the internal $\angle B$ and external $\angle C$ at D. If $\angle BDC$ = 15°, then $\angle A$ is:



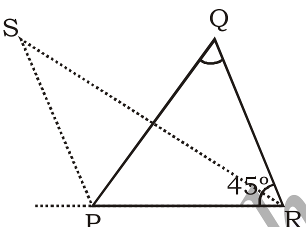
(a) 45° (b) 75° (c) 30° (d) 15°

55. In ΔDEF, the bisectors of the External $\angle E$ & internal $\angle F$ interect at I, If $\angle EDF$ = 140°, then $\angle I$ is:



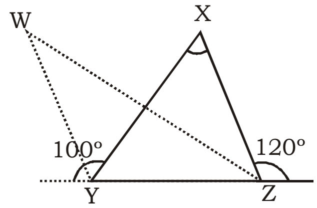
(a) 45° (b) 70° (c) 30° (d) 15°

56. In ΔPQR, the bisector of the external $\angle P$ & internal $\angle R$ intersect at S. If external angle of $\angle P$ = 100° & $\angle PRQ$ = 45° then $\angle PSR$:-



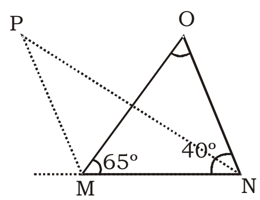
(a) 27.5° (b) 37.5°
(c) 30° (d) 15°

57. In Δxyz, the bisector of the external $\angle y$ & internal $\angle z$ meet at w. If external angle of $\angle y$ & $\angle z$ are 100° and 120° respectively then $\angle YWZ$ is :



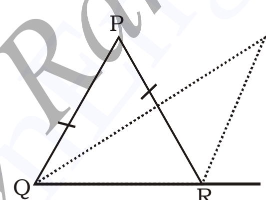
(a) 45° (b) 40° (c) 30° (d) 20°

58. In ΔMNO, the bisector of the external $\angle M$ & Internal $\angle N$ are 65° & 40° and meet at point P. Find the ratio of $\angle O : \angle P$ is:-



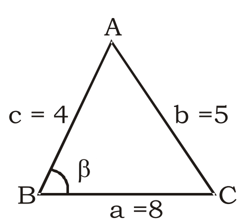
(a) 2 : 1 (b) 5 : 1
(c) 3 : 1 (d) 4 : 1

59. In ΔPQR, the bisector of internal $\angle Q$ & external $\angle R$ meet at O. If $\angle QOR = \frac{31°}{2}$, PQ = PR, then external angle of $\angle Q$ is:



(a) 105.5° (b) 106.5°
(c) 106° (d) 105°

60. In ΔABC, AB = 4 cm, BC = 8 cm & AC = 5 cm, then find $\cos \beta$?

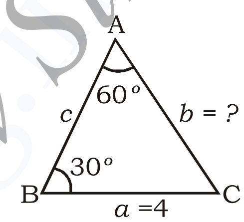


(a) $\frac{55}{64}$ (b) $\frac{64}{55}$
(c) 120 (d) 60

61. In ΔABC, AB = 5 cm, BC = 8 cm and $\angle ABC$ = 60°, then find AC?

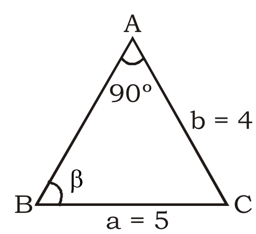
(a) 7 (b) $\sqrt{55}$
(c) 120 (d) $\sqrt{56}$

62. In ΔABC, BC = 4 cm and $\angle A$ = 60°, $\angle B$ = 30°, Find AC ?



(a) $\frac{4}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{3}{\sqrt{4}}$ (d) $\frac{\sqrt{3}}{2}$

63. In ΔABC, AC = 4 cm, BC=5 cm, $\angle A$ = 90°, Find $\sin\beta$.



(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{\sqrt{4}}$ (d) $\frac{2}{5}$

64. If the angles of triangle are 60°, 90° and 30°, then what is the ratio of the sides opposite to these angles ?

(a) $\sqrt{3}:\sqrt{2}:1$ (b) $1:\sqrt{2}:2$
(c) $2:\sqrt{3}:1$ (d) $\sqrt{3}:2:1$

65. Three sides of a triangle are 7 cm, $4\sqrt{3}$ cm and $\sqrt{13}$ cm then the smallest angle is :

(a) 15° (b) 30° (c) 45° (d) 60°

ANSWER KEY

1. (b)	8. (c)	15. (b)	22. (b)	29. (c)	36. (c)	43. (a)	50. (c)	57. (d)	64. (d)
2. (b)	9. (b)	16. (c)	23. (a)	30. (a)	37. (d)	44. (b)	51. (c)	58. (a)	65. (b)
3. (a)	10. (d)	17. (d)	24. (c)	31. (d)	38. (b)	45. (c)	52. (a)	59. (a)	
4. (a)	11. (c)	18. (c)	25. (b)	32. (c)	39. (c)	46. (b)	53. (a)	60. (a)	
5. (a)	12. (c)	19. (a)	26. (d)	33. (d)	40. (c)	47. (b)	54. (c)	61. (a)	
6. (b)	13. (d)	20. (a)	27. (b)	34. (a)	41. (b)	48. (d)	55. (b)	62. (a)	
7. (b)	14. (b)	21. (c)	28. (a)	35. (c)	42. (c)	49. (b)	56. (a)	63. (a)	

SOLUTION

1. (b) The sum of the smaller sides cannot be equal to or less than the largest side.

2. (b) $12^2 > 8^2 + 6^2$

$\therefore$ the triangle is Obtuse angled triangle.

3. (a) In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180°$

$\because \angle BAC > 90°$

$\therefore \angle ABC + \angle ACB < 90°$

4. (a) $x + y + (y + 20°) = 180°$

$\Rightarrow x + 2y = 160°$

$4x - y = 10° \Rightarrow y = 70°, x = 20°$

$\because$ The angles of the triangle are $20°, 70°, 90°$.

$\therefore$ The triangle is right angled.

5. (a) Let, ABC be a triangle.

$\angle A = \angle B + \angle C$ (Given)

$\because \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle A = 180°$

$\Rightarrow \angle A = 90°$

$\therefore$ The triangle is right angled.

6. (b) Let $\angle A < \angle B + \angle C$. Then

$\angle A < 180° - \angle A$ [$\therefore \angle A + \angle B + \angle C = 180°$]

$\Rightarrow 2\angle A < 180°$

$\Rightarrow \angle A < 90°$

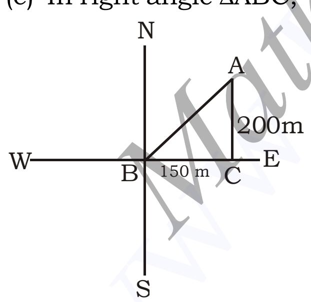
Similarly, $\angle B < 90°$ and $\angle C < 90°$

7. (b) If $a + b + c = 180°$

$\Rightarrow b = 180° - (a + c)$

$\therefore b < 90°$ ($\because a + c > 90°$)

8. (c) In right angle ΔABC,



By Pythagoras theorem,

$AB^2 = BC^2 + AC^2$

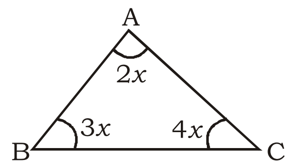
$\Rightarrow AB^2 = (150)^2 + (200)^2$

$\Rightarrow AB^2 = 22500 + 40000$

$\Rightarrow AB^2 = 62500$

$\Rightarrow AB = 250$ m

9. (b)



As we know that,

$\angle A + \angle B + \angle C = 180°$

$\therefore 2x + 3x + 4x = 180°$

$9x = 180°$

$x = 20°$

$\therefore$ Smallest angle $= \angle A = 2x$

$= 2 \times 20° = 40°$

10. (d) Let ABC be a triangle,

$\angle A + \angle B = \angle C$

We know that,

$(\angle A + \angle B + \angle C) = 180°$

$\Rightarrow \angle C + \angle C = 180°$

$\Rightarrow 2\angle C = 180°$

$\Rightarrow \angle C = 90°$

11 (c) Let the smallest angle $= x$ then, other two angles $= 2x$ and $3x$

$\therefore x + 2x + 3x = 180°$

$\Rightarrow 6x = 180°$

$\Rightarrow x = 30°$

Hence, Smallest angle = 30°

12 (c) Let, $2\angle A = 3\angle B = 6\angle C = K$

$\therefore \angle A = \frac{K}{2}, \angle B = \frac{K}{3}, \angle C = \frac{K}{6}$

$\therefore \angle A + \angle B + \angle C = 180°$

$$\frac{K}{2} + \frac{K}{3} + \frac{K}{6} = 180° \Rightarrow K = 180°$$

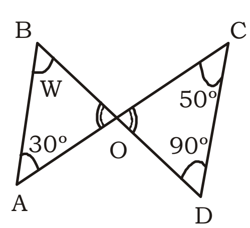
$$\therefore \angle A = \frac{180°}{2} = 90°$$

13. (d) By option (d)

$\because (6 + 7 + 8)x = 180°$

$$\Rightarrow x = \frac{180°}{21} \neq \text{integer}$$

14. (b)



In ΔOCD,

$\because \angle OCD + \angle CDO + \angle COD = 180°$

$\therefore 50° + 90° + \angle COD = 180°$

$\Rightarrow \angle COD = 180° - 140° = 40°$

$\angle COD = \angle BOA = 40°$

(Vertically opposite angle)

In ΔBOA,

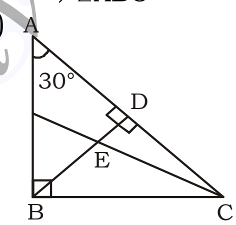
$\because \angle BOA + \angle OAB + \angle ABO = 180°$

$\therefore 40° + 30° + \angle ABO = 180°$

$\Rightarrow \angle ABO = 180° - 70° = 110°$

Hence, $\angle ABO = w = 110°$

15. (b)



In ΔABC,

$\angle B = 90°, \angle A = 30°$

$\because \angle A + \angle B + \angle C = 180°$

$\therefore 30° + 90° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 120° = 60°$

$\because$ CE Bisects $\angle BCD$ (given)

$\therefore \angle ECD = \frac{60°}{2} = 30°$

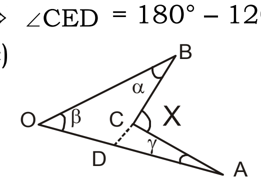
In ΔCDE,

$\because \angle CED + \angle EDC + \angle ECD = 180°$

$\therefore \angle CED + 90° + 30° = 180°$

$\Rightarrow \angle CED = 180° - 120° = 60°$

16. (c)



Produce BC to D,

$\therefore \angle CDA = \alpha + \beta$ (exterior angle)

In ΔADC, x is an exterior angle.

$\therefore x = \angle CDA + \angle CAD$

$\Rightarrow x = \alpha + \beta + \gamma$

17. (d) In ΔABC,

$\because s + t + 50° = 180°$

$\Rightarrow s + t = 130°$

$\Rightarrow t = 130° - s$

$\because s < 50°$

$\therefore t > 130° - 50°$

$\Rightarrow t > 80°$

18. (c) Let, ABC be a triangle

$\angle A + \angle B = 80°$ ...(i)

$\angle A - \angle B = 20°$ ...(ii)

On adding equation (i) & (ii),

$2\angle A = 100°$

$\Rightarrow \angle A = 50°$

$\therefore \angle B = 80° - \angle A$

$\angle B = 80° - 50°$

$\Rightarrow \angle B = 30°$

$\because \angle A + \angle B + \angle C = 180°$

$\therefore \angle C = 180° - (50° + 30°)$
$= 100°$

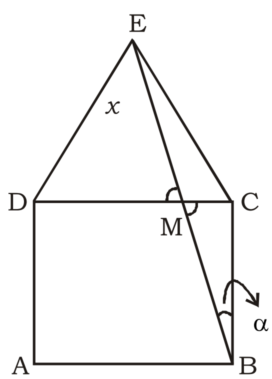
Hence, Smallest angle $= \angle B = 30°$

19. (a) According to the question,

$\angle BCD = 90°$ ($\because$ each angle of square is 90°)

$\angle ECD = 60°$ ($\because$ each angle of an equilateral triangle is 60°)

$\therefore \angle BCE = \angle BCD + \angle ECD$
$= 90° + 60° = 150°$



$\because \angle EBC + \angle BEC + \angle BCE = 180°$

$\therefore \angle EBC + \angle BEC = 180° - 150°$

$\Rightarrow \angle EBC + \angle BEC = 30°$

$\Rightarrow 2\angle BEC = 30°$ ($\because$ BC = EC)

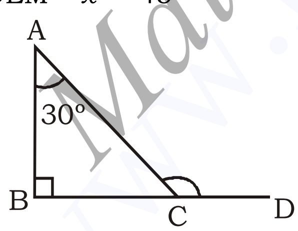
$\Rightarrow \angle BEC = 15°$

$\because \angle DEC = 60°$

$\therefore \angle DEM = 60° - 15°$

$\Rightarrow \angle DEM = x° = 45°$

20. (a)



$\because \angle B + \angle A = \angle ACD$ (exterior angle)

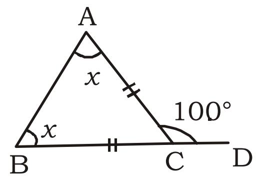
$\therefore 90° + 30° = \angle ACD$

$\Rightarrow \angle ACD = 120°$

21. (c) $y = 80°$ (vertically opposite angle)

$\therefore x = 180° - 50° - 80° = 50°$]

22. (b)



In ΔABC,

$\because$ AC = BC

$\therefore \angle CAB = \angle ABC = x$

$\angle ABC + \angle BAC = \angle ACD$ (exterior angle)

$\Rightarrow x + x = 100°$

$\Rightarrow x = 50°$

Hence, $\angle ABC = x = 50°$

23. (a) $\angle 1 = \angle A + \angle 5$ ..... (i)

and $\angle 2 = \angle A + \angle 6$ ..... (ii) (exterior angle)

$\Rightarrow \angle 1 + \angle 2 = \angle A + \angle 5 + \angle A + \angle 6$

$\Rightarrow \angle 1 + \angle 2 = \angle A + (\angle A + \angle 5 + \angle 6)$

$\Rightarrow \angle 1 + \angle 2 = \angle A + 180°$

($\because \angle A + \angle 5 + \angle 6 = 180°$)

24. (c) a = 180° - d = 180° - 70°
= 110°

and c = d + $\angle$ACB
= 70° + (180° - b)
= 70° + (180° – 120°)
= 70° + 60° = 130°

25. (b) $\because \angle BCA = 180° - Y$

$\therefore 90° + (180° - y) = x$ (exterior angle)

$\Rightarrow x + y = 270°$

26. (d) $\because \angle A : \angle B : \angle C = 3 : 2 : 1$

$\therefore 3x + 2x + x = 180°$

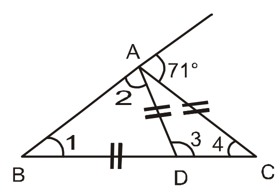
$\Rightarrow 6x = 180° \Rightarrow x = 30°$

$\therefore \angle C = x = 30°$

$\Rightarrow \angle ECD = 180° - \angle ACB - \angle ACE$

$\Rightarrow \angle ECD = 180° - 90° - 30° = 60°$

27. (b)



$\because$ AD = BD

$\therefore \angle 1 = \angle 2 = x$ (let)

$\therefore \angle 3 = \angle 1 + \angle 2 = 2x$ (external angle)

$\therefore \angle 4 = \angle 3 = 2x$ ($\because$ AD = AC)

In Δ ABC,

71° is an external angle.

$\therefore \angle 1 + \angle 4 = 71°$

$\Rightarrow x + 2x = 71° \Rightarrow x = \dfrac{71°}{3}$

$\therefore \angle C = \angle 4 = 2x = \dfrac{142°}{3}$

28. (a) In ΔAEC,

$\angle A + \angle E + \angle C = 180°$ ...(i)

In ΔBFD,

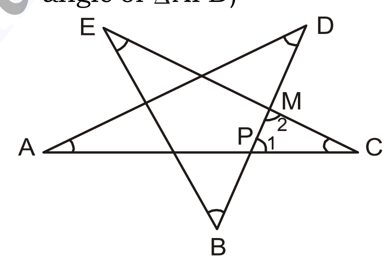
$\angle B + \angle F + \angle D = 180°$ ...(ii)

On Adding (i) and (ii), we get

$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$

29. (c) $\angle 2 = \angle E + \angle B$ (exterior angle of ΔMEB)

and $\angle 1 = \angle A + \angle D$ (exterior angle of ΔAPD)



In ΔPMC,

$\angle 1 + \angle 2 + \angle C = 180°$

$\therefore \angle A + \angle B + \angle C + \angle D + \angle E = 180°$

30. (a) $\angle DAC = 180° - 108° = 72°$

Let, $\angle DAB = y°$

$\therefore \angle BAC = 3y°$

$\therefore y + 3y = 72° \Rightarrow y = 18°$

Now, DB = AB

$\Rightarrow \angle ADB = \angle DAB = y° = 18°$

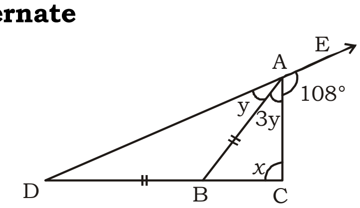
Now, in ΔACD, $\angle$CAE is exterior angle

$\therefore 108° = \angle ADC + x°$

$108° = 18° + x$

$x = 90°$

Alternate



$\because \angle DAC = 180° - 108° = 72°$

$\angle DAB : \angle BAC = 1 : 3$ (given)

$\therefore \angle DAB = y$ and $\angle BAC = 3y$

$\angle DAB + \angle BAC = 72°$

$\Rightarrow y + 3y = 72°$

$\Rightarrow$ $4y = 72°$

$\Rightarrow$ $y = 18°$

$\because$ DB = AB (given)

$\Rightarrow$ $\angle ADB = \angle DAB = y = 18°$

$\because$ $\angle BAC = 3y = 3 \times 18° = 54°$

and $\angle ABC = 2y = 2\times18° = 36°$

In ΔABC,

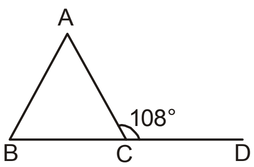
$\angle ABC + \angle BAC + \angle ACB = 180°$

$\Rightarrow$ $36° + 54° + \angle ACB = 180°$

$\Rightarrow$ $\angle ACB = 90°$

Hence, $x = 90°$

31. (d)



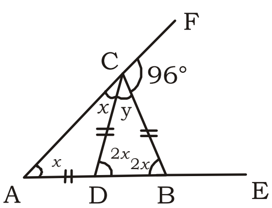
$\angle ACD = \angle ABC + \angle BAC$

$\Rightarrow$ $108° = \dfrac{\angle A}{2} + \angle A$

$\Rightarrow$ $\dfrac{3\angle A}{2} = 108°$

$\Rightarrow$ $\angle A = \dfrac{108\times 2}{3} = 72°$

32. (c)



$x + y = 180° - 96 = 84$ **...(i)**

Also for Δ CDB,

$4x + y = 180°$ **...(ii)**

Subtract eqn. (i) from eqn. (ii),

$3x = 96$ or $x = 32°$

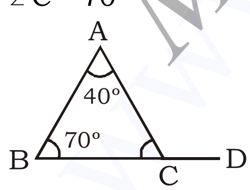
$\therefore$ $\angle DBC = 2x = 64°$

33. (d) As we know that,

$\Rightarrow$ $\angle A + \angle B + \angle C = 180$

$\Rightarrow$ $\angle C = 180° - 70° - 40°$

$\Rightarrow$ $\angle C = 70°$



$\therefore$ $\angle ACD = 180° - 70° = 110°$

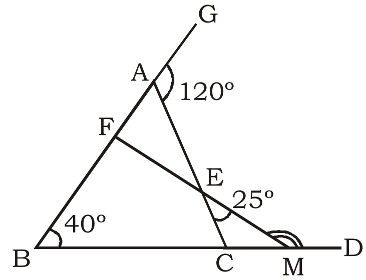
Alternate

As we know that

$\angle ACD = \angle A + \angle B$

$\therefore$ $\angle ACD = 40° + 70° = 110°$

34. (a) According to the question,



$\angle CAG = \angle ABC + \angle ACB = 120°$

($\because$ $\angle CAG$ is exterior angle)

$\Rightarrow$ $\angle ACB = 120° - 40°$

$\Rightarrow$ $\angle ACB = 80°$

$\because$ $\angle ECM = 180° - 80° = 100°$

$\angle EMD = \angle ECM + \angle CEM$

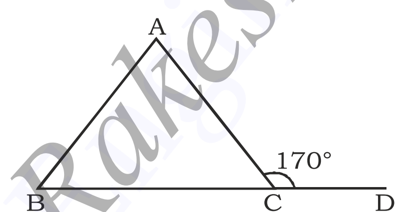
$\Rightarrow$ $\angle EMD = 100 + 25°$

$\therefore \angle EMD = 125°$

35. (c) According to the question,

$\angle ACD = 170°$

$\because$ $\dfrac{\angle B}{\angle A} = \dfrac{2}{3}$



Let, $\angle A = 3x$ & $= \angle B = 2x$

$\therefore$ $\angle ACD = \angle A + \angle B$

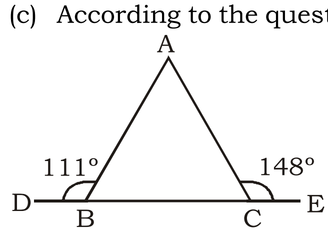
$3x + 2x = 170°$

$5x = 170°$

$x = 34°$

$\therefore$ $\angle A = 3x = 3 \times 34 = 102°$

36. (c) According to the question,



$\because$ $\angle ABD = 111°$

& $\angle ACE = 148°$

$\therefore$ $\angle ACB = 180° - 148° = 32°$

& $\angle ABC = 180° - 111° = 69°$

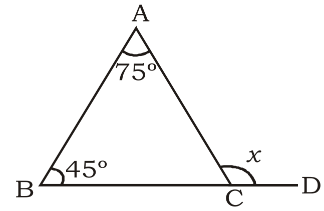
In ΔABC,

$\because$ $\angle A + \angle B + \angle C = 180°$

$\therefore$ $\angle A = 180° - 69° - 32°$

Hence, $\angle A = 79°$

37. (d) According to the question,



$\angle ACD = x = \angle A + \angle B$

($\because$ $\angle ACD$ is an exterior angle)

$= 75° + 45°$

$\angle ACD = 120°$

$\therefore$ $\dfrac{x}{3}\%$ of $60° = \dfrac{120}{3}\%$ of $60°$

$= 40\%$ of $60°$

$= \dfrac{40}{100}\times 60° = 24°$

38. (b) $\angle PSQ = 180° - (90° + 60°)$

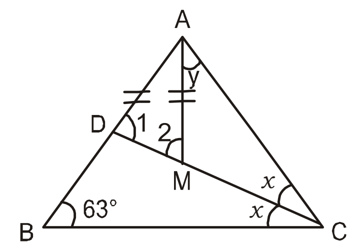
$= 30°$

In Δ PSR,

$\because \angle PSR = \angle RPS$

$\therefore$ PR = RS

39. (c)



In ΔBDC, $\angle 1$ is an external angle

$\therefore$ $\angle 1 = 63° + x$

and in ΔAMC, $\angle 2$ is an external angle

$\therefore$ $\angle 2 = x + y$

$\because$ AM = AD (given)

$\therefore$ $\angle 1 = \angle 2$

$\Rightarrow$ $63° + x = x + y$

$\Rightarrow$ $y = 63°$

$\therefore \angle MAC = 63°$

40. (c) $\because$ BP = BR

$\therefore$ $\angle BPR = \angle BRP = x$ (let)

and CN = RC

$\therefore \angle CRN = \angle RNC = y$ (let)

$\therefore$ $\angle PBR = 180° - 2x$ and $\angle NCR = 180° - 2y$

In Δ ABC,

$\angle A + \angle B + \angle C = 180°$

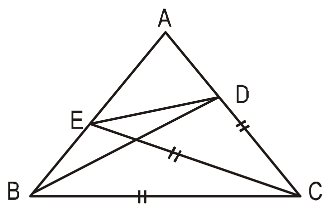
$\Rightarrow$ $44° + 180° - 2x + 180° - 2y = 180°$

$\Rightarrow$ $x + y = 112°$

Hence, $\angle PRN = 180° - (x + y)$

$= 180° - 112° = 68°$

41. (b)



$\because$ BC = EC

$\therefore$ $\angle$CEB = $\angle$EBC = 78°

In Δ BCD,

$\because$ CD = BC

$\therefore$ $\angle$DBC = $\angle$CDB

$\because$ $\angle$CDB + $\angle$DBC + $\angle$DCB = 180°

$\Rightarrow$ 2$\angle$CDB + 78° = 180°

$\Rightarrow$ $\angle$CDB = 51°

In ΔBEC,

$\because$ EC = BC

$\therefore$ $\angle$EBC = $\angle$BEC = 78°

$\angle$EBC + $\angle$BEC + $\angle$ECB = 180°

$\Rightarrow$ 78° + 78° + $\angle$ ECB = 180°

$\Rightarrow$ $\angle$ECB = 24°

$\because$ $\angle$ACB = 78°

$\therefore$ $\angle$DCE + $\angle$ECB = 78°

$\Rightarrow$ $\angle$DCE = 54°

In ΔEDC,

$\because$ DC = EC

$\therefore$ $\angle$EDC = $\angle$DEC

$\angle$DCE+$\angle$EDC+$\angle$DEC = 180°

2$\angle$EDC = 180° – 54° = 126°

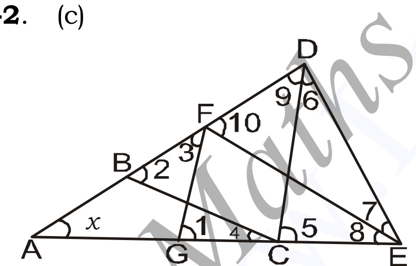
$\angle$EDC = 63°

$\because$ $\angle$EDC = $\angle$EDB + $\angle$CDB

$\Rightarrow$ 63° = $\angle$EDB +51°

Hence, $\angle$EDB = 12°

42. (c)



$\because$ AB = BC

$\therefore$ $\angle 4 = x$

$\therefore$ $\angle 2 = x + \angle 4 = 2x$

(exterior angle) [and BC = CD]

$\therefore$ $\angle 9 = \angle 2 = 2x$

$\therefore$ $\angle 3 = x$ ($\because$ FG = GA)

$\therefore$ $\angle 1 = x + \angle 3 = 2x$

(exterior angle)

$\because$ EF = FG $\angle 8 = \angle 1 = 2x$

$\angle 5 = \angle A + \angle 9 = x + 2x$

$= 3x$ (exterior angle)

$\because$ CD = DE

$\therefore$ $\angle 7 + \angle 8 = \angle 5$

$\Rightarrow$ $\angle 7 = 3x - 2x = x$

$\angle 10 = A + \angle 8 = 3x$

(exterior angle)

$\because$ DE = EF

$\therefore$ $\angle 9 + \angle 6 = \angle 10$

$\Rightarrow$ $\angle 6 = 3x - 2x = x$

Now in Δ ADE,

$\Rightarrow$ $\angle$A + $\angle$D + $\angle$E = 180°

$\Rightarrow$ $\angle x + 3x + 3x = 180°$

$\Rightarrow$ $x = \dfrac{180°}{7}$

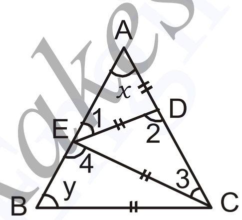
Alternate:-

If the figure is given in the form of zig-zag way then calculate how many sides are equal. In above figure seven sides are equal.

$\therefore \angle DAE = \dfrac{180°}{\text{No. of sides equal}}$

$= \dfrac{180°}{7}$

43. (a)



$\because$ AD = DE

$\therefore$ $\angle 1 = x$

$\therefore$ $\angle 2 = \angle 1 + x = 2x$

(exterior angle)

$\because$ DE = EC

$\because$ $\angle 3 = \angle 2 = 2x$

In Δ AEC, $\angle 4$ is an exterior angle.

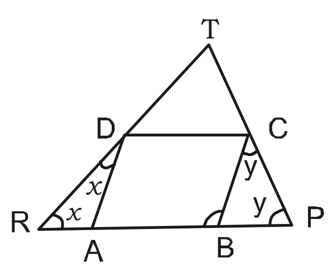
$\therefore$ $\angle 4 = x + \angle 3 = 3x$

$\therefore$ $y = \angle 4 = 3x$ ($\because$ EC = BC)

$\therefore$ $\angle A : \angle B = x : y : = x : 3x$

$= 1 : 3$

44. (b)



Let $\angle$ARD = x and $\angle$BPC = y

$\angle$ARD = $\angle$RDA ($\because$ AR = AD)

$\therefore$ $\angle$DAB = $\angle$ARD + $\angle$RDA

= $x + x = 2x$ (exterior angle)

Similarly,

$\angle$BPC = $\angle$BCP = y

$\therefore$ $\angle$ABC = $y + y = 2y$

(exterior angle)

$\therefore$ $2x + 2y = 180°$ ($\because$ ABCD is a rhombus)

$\Rightarrow$ $x + y = 90°$

Now in Δ RTP,

$\angle$RTP = 180° - ($x + y$)

= 180° - 90°

= 90°

45. (c) $\because$ BD = CD

$\therefore$ $\angle$DBC = $\angle$BCD = 27°

In ΔBDC,

$\because$ $\angle$BDC + $\angle$DBC + $\angle$BCD = 180°

$\Rightarrow$ $\angle$BDC + 27° + 27° = 180°

$\Rightarrow$ $\angle$BDC = 126°

$\angle$ADC = 180° – 126° = 54°

In ΔADC,

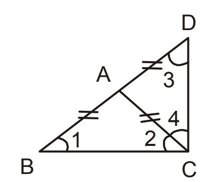
$\because$ CD = AC

$\therefore$ $\angle$ADC = $\angle$CAD

$\angle$ACD = y° = 180° – (54°+54°)

= 72°

46. (b)



$\angle 1 = \angle 2$ ($\because$ AB = AC)

and $\angle 3 = \angle 4$

($\because$ AB = AC = AD)

In Δ BDC,

$\angle$B + $\angle$C + $\angle$D = 180°

$\Rightarrow \angle 1 + \angle 2 + \angle 4 + \angle 3 = 180°$

$\Rightarrow \angle 2 + \angle 2 + \angle 4 + \angle 4 = 180°$

$\Rightarrow 2(\angle 2 + \angle 4) = 180°$

$\Rightarrow \angle 2 + \angle 4 = 90° \Rightarrow \angle C = 90°$

$\therefore$ $\angle$BCD = 90°

47. (b) According to the question,

BC = AC = AD

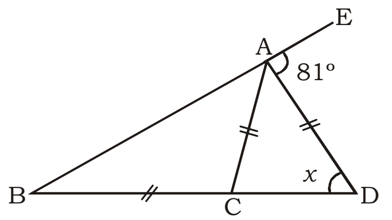
$\angle$EAD = 81°

$\angle$ACD = $\angle$ADC = x

$\angle$CAD = 180° – 2x

$\angle ABC = \angle BAC = \dfrac{x}{2}$

[$\therefore$ $\angle$ACD = $\angle$BAC + $\angle$ABC]



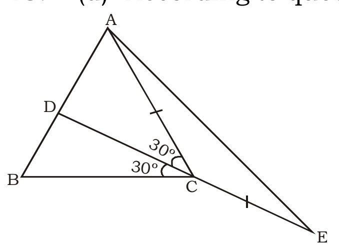
$\angle BAC + \angle CAD + 81° = 180°$

$\frac{x}{2} + 180° - 2x + 81 = 180°$

$-\frac{3x}{2} = -81$

$\therefore x = 54°$

48. (d) According to question,



$\because \angle A = \angle B = \angle C = 60°$

$\angle ACE = 180° - 30° = 150°$

$\angle CAE = \angle CEA$ ($\because$ CE = AC)

In ΔCAE,

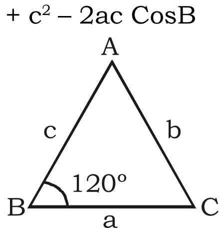
$\angle CAE + \angle CEA + \angle ACE = 180°$

$\Rightarrow 2\angle CAE = 180° - 150°$

$\Rightarrow \angle CAE = 15°$

49. (a) By using cosine formula,

$b^2 = a^2 + c^2 - 2ac \cos B$



$b^2 = a^2 + c^2 - 2ac \cos 120°$

$b^2 = a^2 + c^2 - 2ac \times \left(\frac{-1}{2}\right)$

$b^2 = a^2 + c^2 + ac$

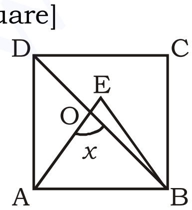
50. (c) According to the question,

ΔABE is an equilateral triangle

$\angle BAE = \angle ABE = \angle AEB = 60°$

As we know that,

$\angle ABD = 45°$ [$\because$ BD is diagonal of a square]



In ΔABO,

$\angle BAO + \angle ABO + \angle AOB = 180°$

$60 + x + 45° = 180$

$x = 75°$

51. (c) According to the question,

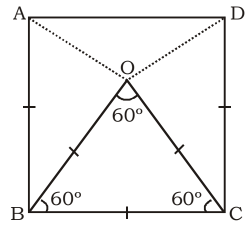
$\angle BCD = 90°$

$\angle OBA = \angle DCO = 90° - 60° = 30°$

In ΔOBA,

$\because$ OB = AB

$\therefore \angle BOA = \angle BAO = \frac{180°-30°}{2} = 75°$



Same in ΔCOD

$\angle COD = \angle ODC = 75°$

$\therefore \angle DOA = 360° - 75° - 75° - 60°$

$\Rightarrow \angle DOA = 150°$

52. (a) According to the question,

$\angle BDA = 140°$

$\angle CEA = 120°$

$\because \angle EAC = 2 \times \angle BAD$

$\Rightarrow \frac{\angle EAC}{\angle BAD} = \frac{2}{1}$

$\angle ADE = 180° - 140° = 40°$

$\angle AED = 180° - 120° = 60°$

$\angle DAE = 180° - 40° - 60°$

$\angle DAE = 80$

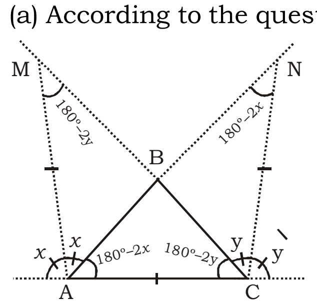
$\therefore \angle EAC = 20°$

$\angle BAD = 10°$

In ΔBDA,

$\angle ABD = 180° - 10° - 140° = 30°$

53. (a) According to the question,



In ΔMAC,

$\Rightarrow 180° - 2y + 180° - 2y + x + 180 - 2x = 180°$

$x + 4y = 360°$ ....(i)

In ΔNAC,

$180° - 2x + 180° - 2x + 180° - 2y + y = 180°$

$4x + y = 360°$ ...(ii)

Solve Eq. (i) and (ii), We get

$x + y = 144°$

In ΔABC,

$\because \angle A + \angle B + \angle C = 180°$

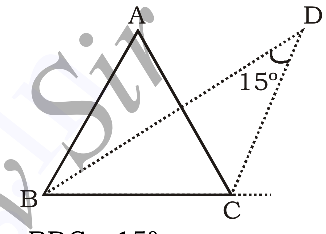
$\Rightarrow \angle B + 180° - 2x + 180° - 2y = 180°$

$\Rightarrow \angle B = 2(x + y) - 180°$

$\Rightarrow \angle B = 288° - 180°$

$\Rightarrow \angle B = 108°$

54. (c)

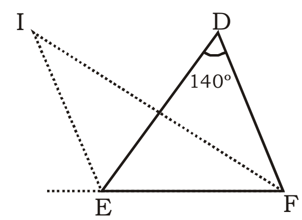


$\because \angle BDC = 15°$

$\therefore \angle BAC = 2 \times \angle BDC$

$= 2 \times 15° = 30°$

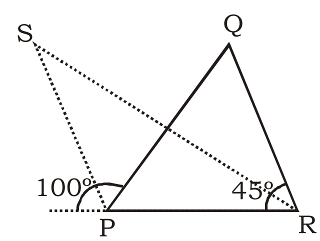
55. (b)



$\because \angle EDF = 140°$

$\therefore \angle EIF = \frac{\angle EDF}{2} = \frac{140°}{2} = 70°$

56. (a)



$\angle QPR = 180° - 100° = 80°$

In ΔPQR

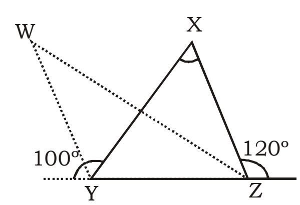
$\because \angle P + \angle Q + \angle R = 180°$

$\Rightarrow 80° + \angle Q + 45° = 180°$

$\Rightarrow \angle Q = 55°$

$\angle PSR = \frac{\angle Q}{2} = \frac{55}{2} = 27.5°$

57. (d)



$\angle YZX = 180° - 120° = 60°$

& $\angle XYZ = 180° - 100° = 80°$

In Δxyz,

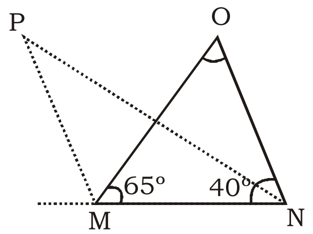
$\angle x + \angle y + \angle z = 180°$

$\angle x + 80° + 60° = 180°$

$\angle x = 40°$

$\therefore \angle YWZ = \dfrac{40}{2} = 20°$

58. (a)



In ΔMNO

$\angle M + \angle N + \angle O = 180°$

$65° + 40° + \angle O = 180°$

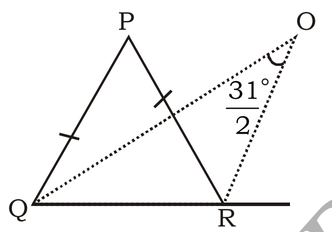
$\angle O = 75°$

$\angle P = \dfrac{\angle O}{2} = \dfrac{75°}{2}$

Required Ratio $= \dfrac{\angle O}{\angle P} = \dfrac{75}{75} \times 2 = \dfrac{2}{1}$

$\therefore \angle O : \angle P = 2 : 1$

59. (a)



$\angle P = 2 \times \angle O = 2 \times \dfrac{31}{2} = 31°$

In ΔPQR,

$\angle P + \angle Q + \angle R = 180°$

$\left[\begin{array}{l} \because PQ = PR \\ \therefore \angle Q = \angle R \end{array}\right]$

$31° + \angle Q + \angle Q = 180°$

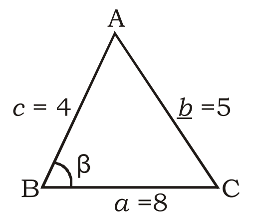
$\Rightarrow 2\angle Q = 149°$

$\Rightarrow \angle Q = 74.5°$

$\therefore$ External angle of $\angle Q$

$= 180° - 74.5° = 105.5°$

60. (a)



By using Cosine formula,

$\text{Cos}B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \text{Cos}\,\beta = \dfrac{64 + 16 - 25}{2 \times 8 \times 4}$

$\Rightarrow \text{Cos}\,\beta = \dfrac{55}{64}$

61. (a) By using Cosine formula,

$\text{Cos}B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \text{Cos}60° = \dfrac{5^2 + 8^2 - b^2}{2 \times 5 \times 8}$

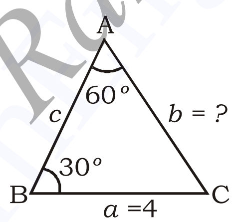
$\Rightarrow \dfrac{1}{2} = \dfrac{25 + 64 - b^2}{80}$

$\Rightarrow 40 = 89 - b^2$

$\Rightarrow b^2 = 49$

$\Rightarrow b = 7$

62. (a) Using sine formula,



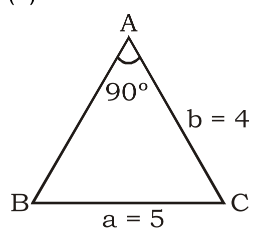
$\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$

$\Rightarrow \dfrac{4}{\sin 60°} = \dfrac{AC}{\sin 30°}$

$\Rightarrow \dfrac{4 \times 2}{\sqrt{3}} = \dfrac{AC \times 2}{1}$

$\Rightarrow AC = \dfrac{4}{\sqrt{3}}$

63. (a)



Using sine formula,

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\Rightarrow \dfrac{5}{\sin 90°} = \dfrac{4}{\sin B}$

$\Rightarrow \dfrac{5}{1} = \dfrac{4}{\sin\beta}$

$\Rightarrow \sin\beta = \dfrac{4}{5}$

64. (d) According to the sine formula,

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = k$ (Let)

$\dfrac{a}{\sin 60°} = \dfrac{b}{\sin 90°} = \dfrac{c}{\sin 30°} = k$

$a = k \sin 60° = k \times \dfrac{\sqrt{3}}{2}$

$b = k \sin 90° = k \times 1$

$c = k \sin 30° = k \times \dfrac{1}{2}$

a	**:**	**b**	**:**	**c**
$\dfrac{\sqrt{3}}{2}k$	:	K	:	$\dfrac{k}{2}$
$\sqrt{3}$	:	2	:	1

65. (b) By using cosine formula,

$\cos\theta = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \cos\theta = \dfrac{49 + 48 - 13}{2 \times 7 \times 4\sqrt{3}}$

$\Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30°$

CHAPTER 22

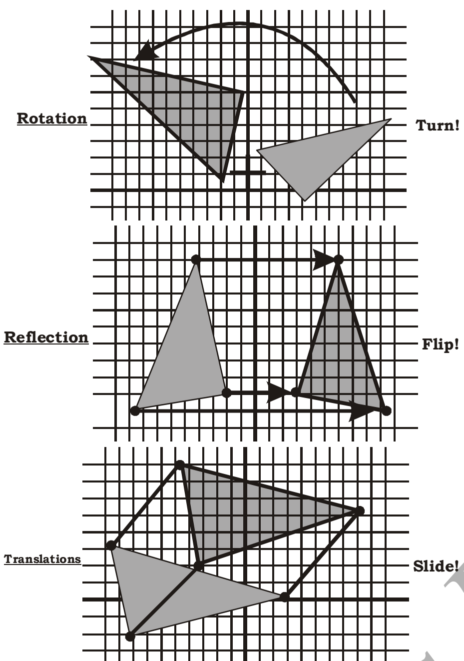
TRANGLES CONGRUENCE & SIMILARITY

Congruent Triangles

Triangles are congruent when they have exactly the **same three sides** and exactly the **same three angles.**

What is "Congruent" ..... ?

It means that one shape can become another using turns, flips and/or slides:

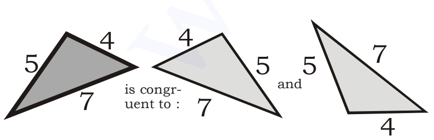


The equal sides and angles may not be in the same position (if there is a turn or a flip), but they are there.

Same sides

When the sides are the same then the triangles are congruent.

For example :

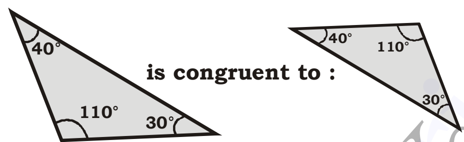


Because the two triangles do not have exactly the same sides.

Same Angles

Does this also work with angles? Not always!

Two triangles with the same angles **might be** congruent;



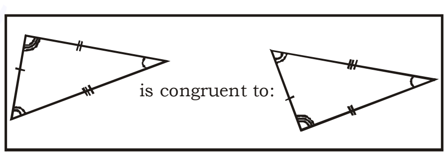
Only because they are the same size

*But they might **NOT** be congruent because of **different sizes:***

because, even though all angles match, **one is larger than the other**. *So just having the same angles is no guarantee they are congruent.*

Marking

When two triangles are congruent we often mark corresponding sides and angles like this;



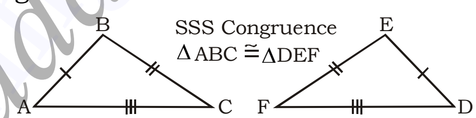
These sides marked one are equal in length. Similarly for the sides marked with two lines. Also for the sides marked with three lines.

The angles marked with one arc are equal in size. Similarly for the angles marked with two arcs. Also for the angles marked with three arcs.

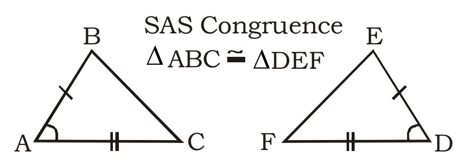
Congruence Tests for Triangles

SSS, SAS, ASA, AAS and HL. These tests describe combinations of congruent sides and/or angles that are used to determine if two triangles are congruent.

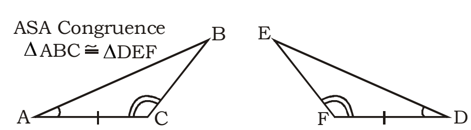
1. **SSS Congruence →** **Side-Side-Side** congruence. When two triangles have corresponding sides equal that are congruent as shown below, the triangles are congruent.



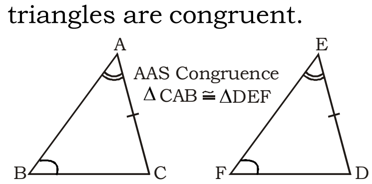
2. SAS Congruence → **Side-Angle-Side** congruence. When two triangles have corresponding angles and sides equal that are congruent as shown below, the triangles are congruent.



3. ASA Congruence → Angle-side-angle congruence. When two triangles have corresponding angles and sides equal that are congruent as shown below, the triangles themselves are congruent.

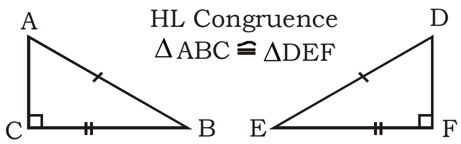


4. AAS Congruence OR SAA Congruence → **Angle-Angle-Side** congruence. When two triangles have corresponding angles and sides equal that are congruent as shown below, the triangles are congruent.



5. HL Congruence →

Hypotenuse-leg congruence. When two right triangles have corresponding sides equal that are congruent as shown below, the triangles are congruent.



Similarity of Triangles

Two triangles are **similar** if and only if the corresponding sides are in proportion and the corresponding angles are congruent.

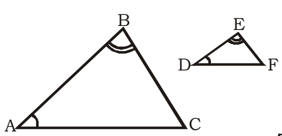
There are three accepted methods of proving triangles similar:

AA

To show two triangles are similar, it is sufficient to show that two angles of one triangle are congruent (equal) to two angles of the other triangle.

Theorem

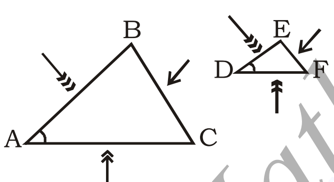
If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.



If $\angle A = \angle D$

$\angle B = \angle E$

Then: $\Delta ABC \sim \Delta DEF$



If $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF}$

Then: $\Delta ABC \sim \Delta DEF$

SSS for Similarity

BE CAREFUL!! SSS for similar triangles is NOT the same theorem as we used for congruent triangles. To show triangles are similar, it is sufficient to show that the three sets of corresponding sides are in proportion.

Theorem

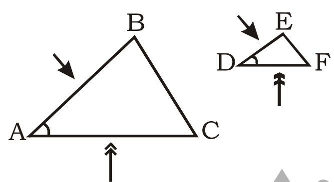
If the three sets of corresponding sides of two triangle are in proportion, the triangles are similar.

SAS for similarity

BE CAREFUL!! SAS for similar triangles is NOT the same theorem as we used for congruent triangles. To show triangles are similar, it is sufficient to show that two sets of corresponding sides are in proportion and the angles they include are congruent.

Theorem

If an angle of one triangle is congruent to the corresponding angle of another triangle and the lengths of the sides including these angles are in proportion, the triangles are similar.



If $\angle A = \angle D$

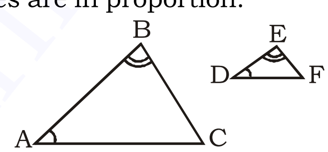
$\dfrac{AB}{DE} = \dfrac{AC}{DF}$

Then: $\Delta ABC \sim \Delta DEF$

Once the triangles are similar

Theorem :

The corresponding sides of similar triangles are in proportion.

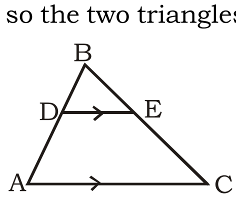


Then: $\Delta ABC \sim \Delta DEF$

Then: $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

Dealing with overlapping triangles :

Many problems involving similar triangles have one triangle ON TOP of (overlapping) another ___ triangle. Since DE is marked to be parallel to AC, we know that we have $\angle BDE$ congruent to $\angle DAC$ by corresponding angles. $\angle B$ is shared by both triangles, so the two triangles are similar by AA



There is an additional theorem that can be used when working with overlapping triangles.

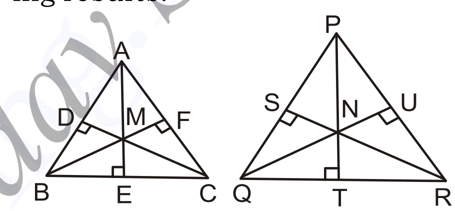
Additional Theorem :

Basic propprtaionality theorem (BPT) or Thales theorem

if a time is parallel to one side of a triangle and intersects the other two sides of the triangle, the line divides these two sides proportionality.

Properties of Similar triangles

If the two triangles are similar, then for the proportional/corresponding sides we have the following results.



1. $\begin{Bmatrix}\text{Ratio of} \\ \text{sides}\end{Bmatrix} = \begin{Bmatrix}\text{Ratio of height} \\ \text{(altitudes)}\end{Bmatrix}$

= Ratio of medians

= Ratio of angle bisectors

= Ratio of inradii

= Ratio of circumradii

2. Ratio of areas = Ratio of squares of corresponding sides.

i.e. If $\Delta ABC \sim \Delta PQR$,

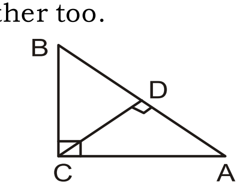
Then,

$$\frac{Area(\Delta ABC)}{Area(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

* Perimeter of triangle is:-

$$\frac{\text{Perimeter }(\Delta ABC)}{\text{Perimeter }(\Delta PQR)} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

3. In a right angled triangle, the triangles on each side of the altitude drawn from the vertex of the right angle to the hypotenuse are similar to the original triangle and to each other too.



i.e., $\Delta BCA \sim \Delta BDC \sim \Delta CDA$.

Some facts on Right angle triangle

Some facts on Right angle triangle

(A)	$CD^2 = BD \times DA$
(B)	$BC \times CA = BA \times CD$
(C)	$BC^2 = BD \times BA$
(D)	$AC^2 = AD \times BA$
(E)	$\frac{BD}{DA} = \frac{BC^2}{AC^2}$
(F)	$\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{CA^2}$

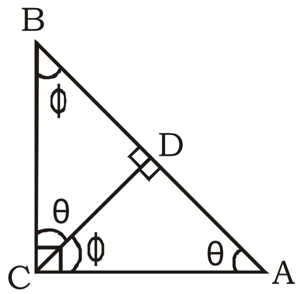
Explanation of these facts

$\angle DCB + \angle DCA = 90^\circ$

$\angle DCB + \angle CBD = 90^\circ$

$\therefore \angle DCA = \angle DBC$

$\therefore \Delta BCA \sim \Delta BDC \sim \Delta CDA$



A. If $\Delta BDC \sim \Delta CDA$

$$\frac{BD}{CD} = \frac{DC}{DA}$$

$$\mathbf{CD^2 = BD \times DA}$$

B. Area of ΔABC

$$\Rightarrow \frac{1}{2} \times BC \times AC = \frac{1}{2} \times AB \times CD$$

$$BC \times AC = AB \times CD$$

$$CD = \frac{BC \times AC}{AB}$$

C. $\Delta BCA \sim \Delta BDC$

$$\frac{BC}{BD} = \frac{BA}{CB}$$

$$BC^2 = BA \times BD$$

D. $\Delta BCA \sim \Delta CDA$

$$\frac{BA}{CA} = \frac{CA}{DA}$$

$$AC^2 = AB \times AD$$

E. In below equation C divided by equation D

$$\frac{BC^2}{AC^2} = \frac{BD \times BA}{AD \times BA}$$

$$\frac{BD}{DA} = \frac{BC^2}{AC^2}$$

F. In eq. (ii) put the value of AB = $\sqrt{BC^2 + AC^2}$

$$BC \times CA = CD\left(\sqrt{BC^2 + AC^2}\right)$$

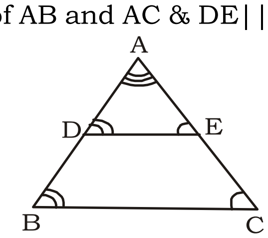
Squaring both sides,

$$BC^2 \times CA^2 = CD^2 \times (BC^2 + AC^2)$$

$$CD^2 = \frac{BC^2 \times CA^2}{BC^2 + AC^2}$$

$$\boxed{\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{AC^2}}$$

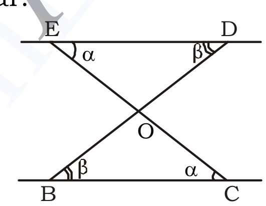
4. If D and E are the mid-points of AB and AC & DE| |BC



then, $DE = \frac{1}{2} BC$

Vice versa is also true i.e.
If a line divides any two sides in the same ratio $\left(i.e. \frac{AD}{DB} = \frac{AE}{EC}\right)$ then the line is parallel to third line
i.e. $DE \parallel BC$

5. Two triangles b/w the two parallel lines will always be similar.



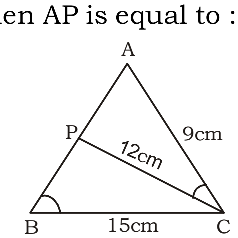
If ED | | BC

Then, $\Delta EOD \sim \Delta COB$

EXAMPLES

1. ABC is a triangle and P is any point on AB such that $\angle ACP = \angle ABC$, if AC = 9cm, CP = 12cm and BC = 15cm, then AP is equal to :

Sol.



In ΔAPC and ΔABC,

$\angle ACP = \angle ABC$ (Given)

$\angle A = \angle A$ (common)

$\therefore \Delta APC \sim \Delta ACB$

$$\therefore \frac{AP}{AC} = \frac{PC}{BC} \Rightarrow \frac{AP}{9} = \frac{12}{15}$$

$\Rightarrow$ AP = 7.2cm

2. If the three side of one triangle are equal to the corresponding sides of the other triangle then the triangle are :

Sol. Here, triangles are congruent. And congruent triangles are always similar.

3. If ΔABC and ΔDEF are so related that $\frac{AB}{FD} = \frac{BC}{DE} = \frac{CA}{EF}$, then which of the following is true?

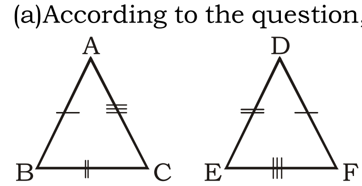
(a) $\angle A = \angle F$ and $\angle B = \angle D$

(b) $\angle C = \angle F$ and $\angle A = \angle D$

(c) $\angle B = \angle F$ and $\angle C = \angle D$

(d) $\angle A = \angle E$ and $\angle B = \angle D$

Sol. (a)According to the question,



$$\frac{AB}{FD} = \frac{BC}{DE} = \frac{CA}{EF}$$

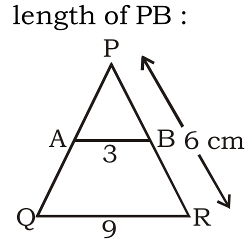
$\angle C = \angle E$

$\angle B = \angle D$

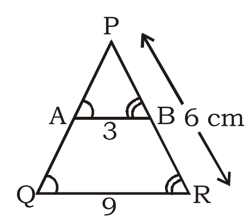
$\angle A = \angle F$

So, option (a) is true.

4. In the figure AB$\parallel$QR. Find the length of PB :



Sol. According to the question,



$\Delta PQR \sim \Delta PAB$

$$\frac{AB}{QR} = \frac{PB}{PR}$$

$$\frac{3}{9} = \frac{PB}{6}$$

PB = 2 cm

5. In Δ ABC and Δ DEF, If $\angle A$ = 50°, $\angle B$ = 70°, $\angle C$ = 60°, $\angle E$ = 70°, $\angle F$ = 50°, $\angle D$ = 60° then :

Sol. In Δ ABC,

$\angle A$ = 50°, $\angle B$ = 70°,

$\angle C$ = 60°

and In Δ DEF,

$\angle E$ = 70°, $\angle F$ = 50° and

$\angle D$ = 60°

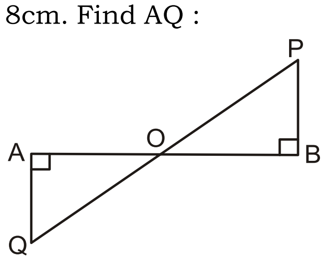
$\therefore$ In Δ FED,

$\angle F$ = 50°, $\angle E$ = 70° and

$\angle D$ = 60°

$\therefore$ $\Delta ABC \sim \Delta FED$.

6. In the given figure, QA and PB are perpendiculars to AB. If AO = 9cm, BO = 6cm and BP = 8cm. Find AQ :



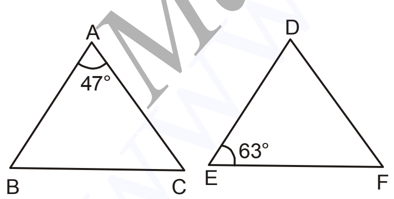
Sol. $\frac{AO}{BO} = \frac{AQ}{BP}$

$\frac{9}{6} = \frac{AQ}{8}$

AQ = 12 cm

7. If Δ ABC is similar to Δ DEF, such that $\angle A$ = 47° and $\angle E$ = 63°, then $\angle C$ is equal to :

Sol.



$\Delta ABC \sim \Delta DEF$

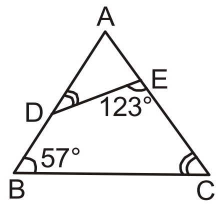
$\therefore$ $\angle A$ = 47° = $\angle D$

$\angle B$ = $\angle E$ = 63°

$\therefore$ $\angle C$ = 180° - 47° - 63°

= 70°

8. In the given figure, AD = 11cm, AB = 18 cm and AE = 9 cm. Find EC :



Sol. $\Delta ADE \sim \Delta ACB$ (A-A property)

$\because$ $\angle A = \angle A$, $\angle AED = \angle ABC$ Then

$\angle ADE = \angle ACB$

$\therefore$ $\frac{AE}{AB} = \frac{AD}{AC} \Rightarrow \frac{9}{18} = \frac{11}{AC}$

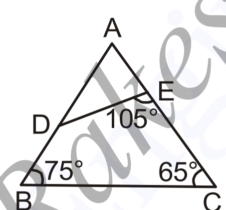
$\Rightarrow$ AC = 22cm

$\therefore$ EC = AC - AE

= 22 - 9

= 13cm

9. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and AE = 12. Find AB:



Sol. In Δ ABC and Δ ADE,

$\angle BAC = \angle DAE$

= 180°- (75° + 65°)

= 40°

$\angle AED$ = 75° = $\angle ABC$

$\therefore$ $\Delta AED \sim \Delta ABC$ (by AA)

$\therefore$ $\frac{DE}{BC} = \frac{AE}{AB} \Rightarrow \frac{2}{3} = \frac{12}{AB}$

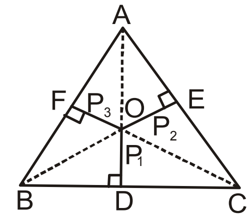
$\Rightarrow$ AB = 18cm

10. The corresponding sides of two similar triangles are in the ratio 1 : 3. Their altitude will be in the ratio:

Sol. Ratio of altitude = Ratio of corresponding sides = 1 : 3

11. The lengths of perpendiculars drawn from any point in the interior of an equilateral triangle to the respective side of the triangle are P_1, P_2 and P_3, then the side of triangle is :

Sol.



Let the side of Δ ABC be a . O is the point in the interior of Δ ABC.

OD,OE,OF are perpendiculars of BC,AC and AB

ar (Δ OAB) + ar (Δ OBC) + ar (Δ OAC) = ar (Δ ABC)

$\Rightarrow \frac{1}{2}a \times P_3 + \frac{1}{2}a \times P_1 + \frac{1}{2}a \times P_2 = \frac{\sqrt{3}}{4}a^2$

$\Rightarrow \frac{1}{2}a(P_1 + P_2 + P_3) = \frac{\sqrt{3}}{4}a^2$

$\Rightarrow a = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$

12. The lengths of perpendiculars drawn from any point in the interior of an equilateral triangle to the respective sides are 6cm, 8cm, and 10cm. The length of each side of the triangle is :

Sol. $a = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$

$= \frac{2}{\sqrt{3}}(6 + 8 + 10)$

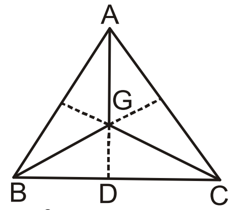
$= \frac{48}{\sqrt{3}} = 16\sqrt{3}$cm

13. Two triangles ABC and DEF are similar to each other in which AB = 10cm, DE = 8cm. Then the ratio of the areas of triangles ABC and DEF is :

Sol. $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{100}{64} = \frac{25}{16}$

14. If G be the centroid of Δ ABC and the area of Δ GBD is 6 sq.cm, where D is the mid-point of side BC, then the area of Δ ABC is :

Sol.



Area of Δ ABC

$= 6 \times ar(\Delta BGD)$

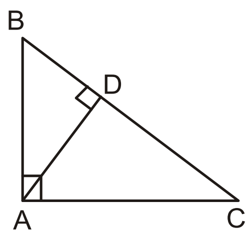
$= 6 \times 6 = 36$sq.cm

15. The perimeters of two similar triangles Δ ABC and Δ PQR are 36cm and 24cm respectively. If PQ = 10 cm, then AB is :

Sol. $\frac{AB}{PQ} = \frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta PQR} = \frac{36}{24}$

$\Rightarrow AB = \frac{36}{24} \times 10 = 15cm$

16. Which of the following is true in the given figure, where AD is the altitude to the hypotenuse of a right angle triangled ABC ?



(i) Δ ABD ~ Δ CAD

(ii) Δ ABD $\cong$ Δ CDA

(iii) Δ ADB ~ Δ CAB

Of these statements, the correct ones are :

Sol. In Δ ABD and Δ CAD

$\angle$ADB = $\angle$ADC = 90° each

$\angle$BAD = $\angle$ACD = 90° – $\angle$B

and

AD = AD (common)

$\therefore$ Δ ADB ~ Δ CAD and

Δ ABD $\cong$ Δ CAD

In Δ ADB and Δ CAB

$\angle$ADB = $\angle$BAC = 90° each

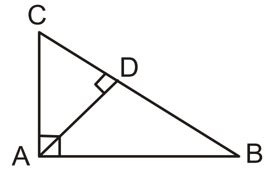
and $\angle$ABC = $\angle$ABD (common)

$\therefore$ Δ ADB ~ Δ CAB

Here, (i) and (iii) are correct statements.

17. In a triangle ABC, $\angle$BAC = 90° and AD is perpendicular to BC. If AD = 6cm and BD = 4cm, then the length of BC is:

Sol.



AB = $\sqrt{AD^2 + BD^2}$ = $\sqrt{36+16} = \sqrt{52}$cm

Δ ABC ~ Δ DBA

$\therefore \frac{AB}{BC} = \frac{BD}{AB}$

$\Rightarrow$ AB² = BC $\times$ BD

$\Rightarrow$ 52 = BC $\times$ 4

$\Rightarrow$ BC = 13cm

Alternate

AD² = BD × CD

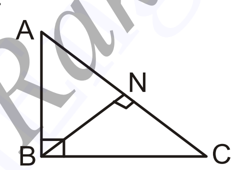
6² = 4 × CD

CD = 9 cm

$\therefore$ BC = 9 + 4 = 13 cm

18. In a right angled Δ ABC, $\angle$ABC = 90°; BN $\perp$ AC, AB = 6cm, AC = 10cm. Then AN : NC is :

Sol.



In Δ ABC & Δ BNC,

$\angle$ABC = $\angle$BNC = 90°

and $\angle$C = $\angle$C (common)

$\therefore$ Δ ABC ~ Δ BNC

and BC = $\sqrt{10^2 - 6^2}$ = 8cm

$\therefore \frac{AC}{BC} = \frac{BC}{NC}$

$\Rightarrow \frac{10}{8} = \frac{8}{NC}$

$\Rightarrow NC = 6.4$

$\therefore$ AN = 10 – 6.4 = 3.6

$\therefore$ AN : NC = 3.6 : 6.4 = 9:16

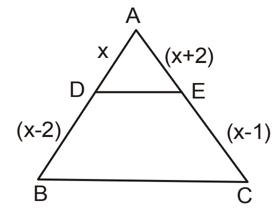
Alternate

$\frac{AN}{NC} = \frac{AB^2}{BC^2} = \frac{6^2}{8^2}$

$= \frac{36}{64} = \frac{9}{16}$

$\therefore$ AN : NC = 9 : 16

19. In the given figure, DE || BC then the value of x is :



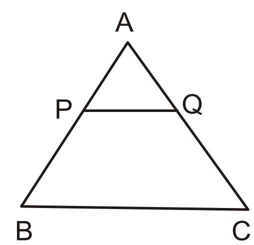
Sol. $\frac{AD}{DB} = \frac{AE}{EC}$ (by basic proportionality theorem $\because$ (DE || BC)

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow x^2 - x = x^2 - 4 \Rightarrow x = 4$

20. In Δ ABC, PQ || BC, If AP : PB = 1 : 2 and AQ = 3cm, AC is:

Sol.



$\frac{AP}{BP} = \frac{AQ}{QC} = \frac{1}{2}$

$\Rightarrow$ QC = 2AQ

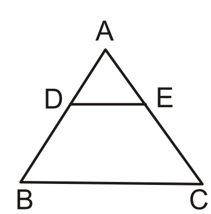
$\Rightarrow$ QC = 2 × 3 = 6

$\Rightarrow$ AC = AQ +QC

= 3 + 6 = 9 cm

21. The points D and E are taken on the sides AB and AC of Δ ABC such that AD = $\frac{1}{3}$AB, AE = $\frac{1}{3}$AC. If the length of BC is 15cm, then the length of DE is :

Sol.



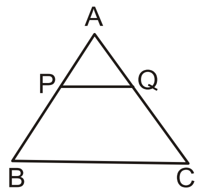
$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$

$\therefore \Delta ABC \sim \Delta ADE$

$\therefore \frac{DE}{BC} = \frac{1}{3} \Rightarrow DE = \frac{15}{3} = 5cm$

22. ABC is an equilateral triangle. P and Q are two points on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{PQ} || \overline{BC}$. If $\overline{PQ}$ = 5cm then the area of Δ APQ is:

Sol.



PQ || BC

$\therefore$ $\angle$APQ = $\angle$ABC = 60°

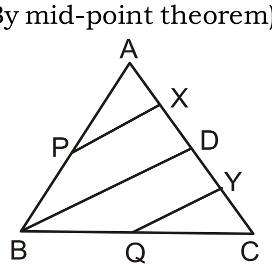
& $\angle$AQP = $\angle$ACB = 60°

$\therefore$ Area of Δ APQ = $\frac{\sqrt{3}}{4} \times (PQ)^2$

$= \frac{\sqrt{3}}{4} \times 25 = \frac{25\sqrt{3}}{4}\, sq.cm$

23. D is any point on side AC of Δ ABC. If P,Q,X & Y are the mid-points of AB, BC, AD and DC respectively, then the ratio of PX and QY is :

Sol. (By mid-point theorem)



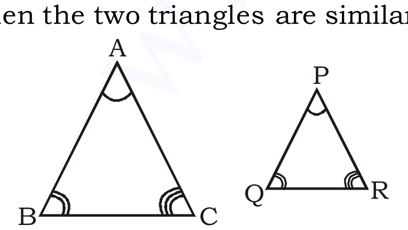
PX || BD and $PX = \frac{1}{2}BD$

QY || BD and $QY = \frac{1}{2}BD$

$\therefore$ PX : QY = 1 : 1

Area Based question when two triangles are similar →

Condition for two triangle to be similar angle of two triangles are equal then the two triangles are similar



If Δ ABC ~ Δ PQR Then

* The Ratio of their area is the square of their sides, Medians, Altitudes, Perimeters, Angle Bisectors.

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

$$= \left(\frac{BC}{QR}\right)^2 \left(\frac{\text{Angle Bisector}_1}{\text{Angle Bisector}_2}\right)^2$$

$$= \left(\frac{\text{Altitude}_1}{\text{Altitude}_2}\right)^2 = \left(\frac{\text{Median}_1}{\text{Median}_3}\right)^2$$

$$= \left(\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}\right)$$

24. The corresponding sides of two similar triangle in the ratio 5 : 6. Their area will be in the ratio.

Sol. When triangle similar,

Area of triangle = (side of triangle)²

$$\left(\frac{\text{Area of } \Delta_1}{\text{Area of } \Delta_2}\right) = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

25. Area of two similar triangle are respectively 196 cm² and 289 cm². If shortest side of the shorter triangle 7 cm. Then find the shorter side of the larger triangle.

Sol. In two similar triangle the ratio of their area is the square of the ratio of their sides.

26. Area of two similar triangles are respectively 81 cm² & 121 cm². If altitude of first triangle is 4.5 cm. Find the corresponding altitude of the second triangle.

Sol. $\frac{\text{Area of first triangle}}{\text{Area of second triangle}}$

$$= \left(\frac{\text{Altitude of first } \Delta}{\text{Altitude of second } \Delta}\right)^2$$

$$\left(\frac{81}{121}\right) = \left(\frac{4.5}{M}\right)^2$$

$$\frac{9}{11} = \frac{4.5}{M}$$

$$M = \frac{11}{2}$$

Here M = (Altitude of second triangle)

27. The corresponding medians of two similar triangle are respectively 12 cm & 15 cm. If area of first triangle is 288 cm². Find the corresponding area of second triangle.

Sol.

$$\left(\frac{\text{Area of first triangle}}{\text{Area of second triangle}}\right) = \left(\frac{\text{Median of first triangle}}{\text{Median of second triangle}}\right)^2$$

$$\frac{288}{A_2} = \left(\frac{12}{15}\right)^2$$

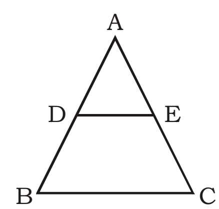
$$\frac{288}{A_2} = \frac{16}{25}$$

A_2 = 450 cm²

$\because$ (A_2 = Area of second triangle.)

28. In a Δ ABC a line DE is drawn parallel to BC. If $\frac{AD}{DB} = \frac{2}{3}$ then find the ratio of area of Δ ADE & Area of ▭ DECB.

Sol.



DE || BC (Given)

So Δ ADE ~ Δ ABC

AD : DB = 2 : 3

Then AB = AD + DB = 5

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \left(\frac{AD}{AB}\right)^2$$

$$= \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

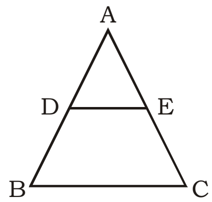
Area of □ DECB = Area of Δ ABC – Δ ADE

= 25 – 4 = 21

Area of Δ ADE : Area of □ DECB = 4 : 21

29. In a Δ ABC a line DE is Drawn parallal to BC. If $\frac{AD}{DB} = \frac{6}{7}$, Find the ratio of area of $\triangle$ ADE and Area of $\square$ DEBC

Sol.



DE $\parallel$ BC (Given)

$\triangle$ ADE ~ $\triangle$ ABC

$\frac{AD}{DB} = \frac{6}{7}$

AB = AD + DB = 6 + 7 = 13

$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \left(\frac{AB}{AD}\right)^2 =$

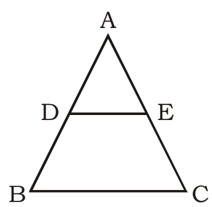
$\left(\frac{13}{6}\right)^2 = \frac{169}{36}$

Area of $\square$ DECB = 169 – 36 = 133

Area of $\triangle$ ADE : Area of $\square$ DEBC = 36 : 133

30. In a $\triangle$ ABC a line DE is drawn $\parallel$ to BC. If ratio of areas of $\triangle$ ADE & $\square$ DECB is 9 : 16 find the value of $\frac{AD}{DB}$

Sol.



DE $\parallel$ BC **(Given)**

So, $\triangle$ ADE ~ $\triangle$ ABC

Area of $\triangle$ ABC = Area of $\triangle$ ADE+Area of $\square$DECB

= 9 + 16 = 25

$\triangle$ ADE	:	$\triangle$ ABC
Area $\rightarrow$ 9	:	25
Side $\rightarrow$ 3	:	5

The ratio of their sides is the square root of the ratio of their area

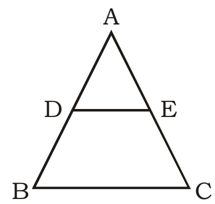
So, $\frac{AB}{AD} = \frac{5}{3}$

BD = AB – AD = 5– 3 = 2

Then, $\frac{AD}{DB} = \frac{3}{2}$

31. In a $\triangle$ ABC a line DE draw $\parallel$ to BC. If area of $\square$ DECB is 238 cm² and length of AD & AB respectively 5 cm & 12 cm find the area of $\triangle$ ABC.

Sol.



DE $\parallel$ BC **(Given)**

So, $\triangle$ ABC ~ $\triangle$ ADE

AD = 5 cm, AB = 12 cm

$\frac{\text{Area of } \Delta \text{ ADF}}{\text{Area of } \Delta \text{ ABC}} = \left(\frac{AD}{AB}\right)^2$

$= \left(\frac{5}{12}\right)^2 == \frac{25}{144}$

Area of $\square$ DECB = 144 – 25 = 119

119 units = 238

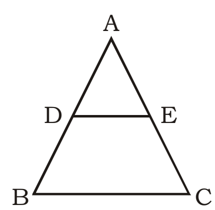
1 unit = 2

So, Area of $\triangle$ ABC = 144 units

= 144 × 2 = 288 cm²

32. In a $\triangle$ ABC a line DE is Drawn $\parallel$ to BC. Which Divide the triangle In equal Area. Then find $\frac{AD}{DB}$.

Sol.



Area of $\triangle$ ADE : Area of $\square$ DECB = 1 : 1

So, Area of $\triangle$ ABC = 1 + 1 = 2 units

$\triangle$ ADE	:	$\triangle$ ABC
Area $\rightarrow$ 1	:	2
Side $\rightarrow$ 1	:	$\sqrt{2}$

$\frac{AD}{AB} = \frac{1}{\sqrt{2}}$

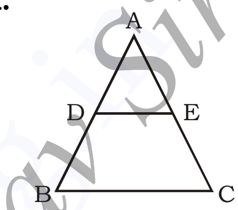
BD = AB – AD = $\sqrt{2} - 1$

Then,

$\frac{AD}{DB} = \frac{1}{\sqrt{2}-1} = 1 : \sqrt{2}-1$

33. In a $\triangle$ ABC, D and E are the mid point of AB and AC, find $\frac{\text{Ar. of } \Delta ADE}{\text{Ar. of } \square \text{ DECB}}$

Sol.



AD = DB = $\frac{AB}{2}$(D mid point of AB)

AE = EC = $\frac{AC}{2}$(E mid point of AC)

Let AB = 2 cm

Then AD = 1 cm

$\triangle$ ABC	:	$\triangle$ ADE
Side $\rightarrow$ 2	:	1
Area $\rightarrow$ 4	:	1

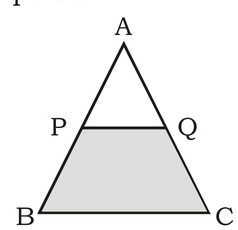
Area of $\square$DEBC = Area of $\triangle$ ABC– Area of $\triangle$ ADE

= 4 – 1 = 3 cm²

Now, $\frac{\text{Ar. of } \Delta \text{ ADE}}{\text{Ar. of } \square \text{ DECB}} = \frac{1}{3}$

34. In a $\triangle$ ABC a line PQ is Draw parallel to side BC, if AB = 9 cm and AP = 4 cm, find the ratio of shaded and unshaded portion

Sol.



PQ $\parallel$ BC **(Given)**

So $\triangle$ APQ ~ $\triangle$ ABC

AP = 4 cm, AB = 9 cm

$\frac{\text{Ar. of } \Delta APQ}{\text{Ar. of } \Delta ABC} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Ar. of $\square$ PQCB = Ar. of $\triangle$ ABC – Ar. of $\triangle$ APQ

= 81 – 16 = 65

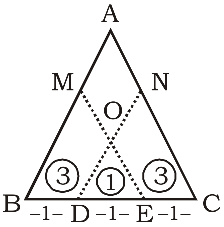
= Area of shaded part : Area of unshaded part

= 65 : 16

35. In a $\triangle ABC$, D and E are two points on the side BC such that they trisect the line BC. In M and N are the two point on line AB & AC. ME $\parallel$ AC and ND $\parallel$ AB. The line ME and ND intersect at O. Find

$$\frac{\text{Ar. of } \triangle DOE + \text{Ar. of} \square AMON}{\text{Ar. of } \triangle ABC}$$

Sol.



Let BC = 3 cm

Then BD = DE = EC = 1 cm

In $\triangle$ MEB

OD $\parallel$ BM **($\therefore$ DN $\parallel$ AB)**

$\triangle ODE \sim \triangle EMB$

$$\frac{\text{Ar. of } \triangle ODE}{\text{Ar. of } \triangle EMB} = \left(\frac{ED}{BE}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Ar. of $\square$ ODBM = Ar. of $\triangle$ EBM – Ar. of $\triangle$ ODE

= 4 – 1 = 3 cm²

Same as

In $\triangle$ DNC

$$\frac{\text{Ar. of } \triangle ODE}{\text{Ar. of } \triangle DNC} = \left(\frac{DE}{DC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Ar. of $\square$OECN = 4 – 1 = 3 cm²

Area of $\triangle$ ABC = (BC)² = (3)² = 9 cm²

Area of $\square$AMON = 9 – (3 + 1 + 3) = 2 cm²

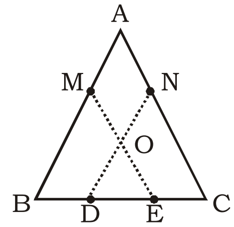
Now,

$$\frac{\text{Ar. of } \triangle ODE + \text{Ar. of } \square AMON}{\text{Ar. of } \triangle ABC} = \frac{1+2}{9} = \frac{1}{3}$$

36. In a $\triangle$ ABC, D and E are two points on the side BC. Such that $\frac{BD}{DE} = \frac{2}{3}$ and $\frac{DE}{EC} = \frac{6}{5}$. M and N are the two points on line AB & AC. ME $\parallel$ AC, ND $\parallel$ AB. The line ME and ND intersect at O Find

$$\frac{\text{Ar. of } \triangle DOE + \text{Ar. of } \square AMON}{\text{Ar. of } \triangle ABC}.$$

Sol.



$\frac{BD}{DE} = \frac{2}{3}$, $\frac{DE}{EC} = \frac{6}{5}$

BD : DE : EC

2 : 3

6 : 5

(To make equal ratio)

BD : DE : EC

4 : 6 : 5

BE = BD + DE = 4 + 6 = 10 units

DC = DE + EC = 6 + 5 = 11 units

BC = BD + DC = 4 + 11 = 15 units

In $\triangle$ BME , OD $\parallel$ MB ($\because$ AB $\parallel$ ND)

ODE ~ $\triangle$ BME

$$\frac{\text{Ar. of } \triangle ODE}{\text{Ar. of } \triangle BME} = \left(\frac{DE}{BE}\right)^2 = \left(\frac{6}{10}\right)^2 = \frac{36}{100}$$

Ar. of $\square$ODBM = Ar. of $\triangle$ BME– Ar. of $\triangle$ ODE

= 100 – 36 = 64 units

Same as In $\triangle$ DNC

$$\frac{\text{Ar. of } \triangle ODE}{\text{Ar. of } \triangle DNC} = \left(\frac{DE}{DC}\right)^2 = \left(\frac{6}{11}\right)^2 = \frac{36}{121}$$

Ar. of $\square$ONCE = Area of $\triangle$ DNC – Area of $\triangle$ DOE

= 121 – 36 = 85 units

Area of $\triangle$ ABC = (BC)² = (15)² = 225 units

Area of $\square$AMON = 225 – (64 + 36 + 85)

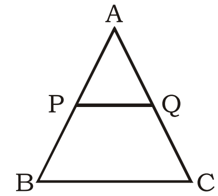
= 40 units

Now,

$$\frac{\text{Ar. of } \triangle ODE + \text{Ar. of } \square AMON}{\text{Ar. of } \triangle ABC} = \frac{36+40}{225} = \frac{76}{225}$$

37. In a $\triangle$ ABC Line PQ is draw parallel to side BC where P and Q are respectively lie on side AB and AC. If AB = 3AP what is percentage of Area $\square$PQCB in the respect of $\triangle$ ABC.

Sol.



PQ $\parallel$ BC

$\triangle APQ \sim \triangle ABC$

AB = 3AP

$\frac{AB}{3} : \frac{AP}{1}$

$$\frac{\text{Ar. of } \triangle APQ}{\text{Ar. of } \triangle ABC} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

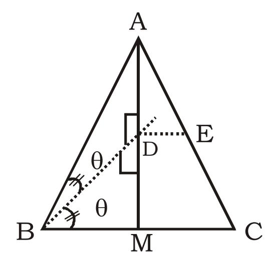
Area of $\square$PQCB = 9 – 1 = 8 cm²

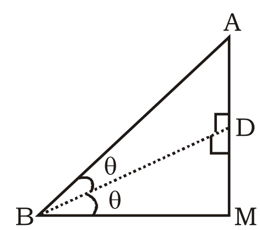
Percentage of $\square$PQCB in the respect of $\triangle$ ABC

$$= \frac{8}{9} \times 100 = 88\frac{8}{9}\%$$

38. In $\triangle$ ABC, AD is perpendicular to the angle bisector of angle B. line DE is drawn parallel to BC. E is a point on line AC. If AC = 12 cm. Find the length of AE.

Sol.





$\triangle$ ABD and $\triangle$ BDM

$\angle ADB = \angle BDM$ (90°)

BD = BD (Common)

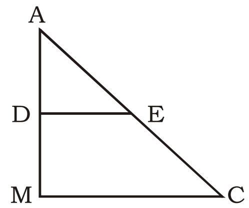
$\angle ABD = \angle DBM$ (BD Angle Bisector)

$\triangle ABD \cong \triangle BDM$

$AD = DM = \frac{AM}{2}$

We can say,

D is the mid point of AM



In $\Delta ADE \sim \Delta AMC$

$$\frac{AD}{AM} = \frac{AE}{AC}$$

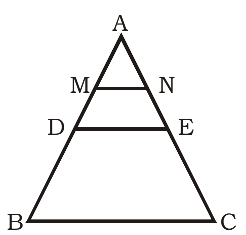
$$\frac{AM/2}{AM} = \frac{AE}{AC}$$

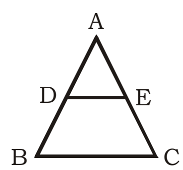
$$\frac{1}{2} = \frac{AE}{12}$$

AE = 6 cm

39. In the ΔABC, D and E are the mid points of line AB & AC. M and N are the mid point of AD & AE find the ratio of Area of ΔAMN : Area of ADE : Area of ΔABC.

Sol.



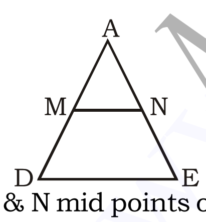


We know that when D & E mid point AB and AC then DE$\parallel$BC and

$$DE = \frac{1}{2}BC.$$

$$\text{Now, } \frac{DE}{BC} = \frac{1}{2} = \frac{AD}{AB} = \frac{AE}{AC}$$

$$\frac{\text{Ar. of } \Delta ADE}{\text{Ar. of } \Delta ABC} = \left(\frac{AD}{AB}\right)^2 = \frac{1}{4}$$



M & N mid points of AD & AE (Given)

Then, MN$\parallel$DE and $MN = \frac{1}{2}DE$

$$\frac{MN}{DE} = \frac{1}{2} = \frac{AM}{AD} = \frac{AN}{AE}$$

$$\frac{\text{Ar. of } \Delta AMN}{\text{Ar. of } \Delta ADE} = \left(\frac{AM}{AD}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

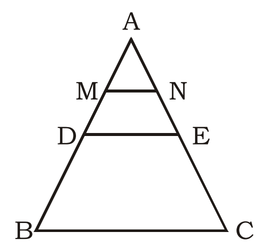
Ar. of ΔAMN	:	Ar. of ΔADE	:	Ar. of ΔABC
1	:	4	:	
		1	:	4

So,

Ar. of ΔAMN	:	Ar. of ΔADE	:	Ar. of ΔABC
1	:	4	:	16

(To make equal ratio)

Alternate



Let, AM = 1

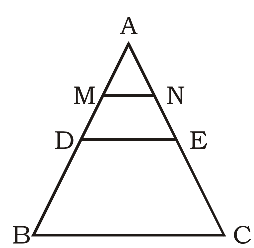
Then, AD = 2 AM = 2 × 1 = 2

and AB = 2 × AD = 2 × 2 = 4

	Ar. of ΔAMN	:	Ar. of ΔADE	:	Ar. of ΔABC
Side →	1	:	2	:	4
Area →	1	:	4	:	16

40. In a triangle ABC, D and E are the mid points of line AB & AC. M and N are the mid points of AD & AE. Then find the ratio of Area of ΔAMN, Area of ▭DENM & Area of ▭BCED.

Sol.



Ar. of ΔAMN	:	Ar. of ΔADE	:	Ar. of ΔABC
1	:	4	:	16

(Discuss in above question)

Area of ▭DENM = Area of ΔADE – Area of ΔAMN

= 4 – 1 = 3

Area of ▭BCED = Area of ΔABC – Area of ΔADE

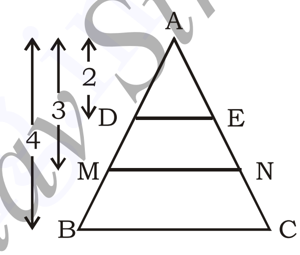
= 16 – 4 = 12

So,

Ar. of ΔAMN	:	Ar. of ▭DENM	:	Ar. of ▭BCED
1	:	3	:	12

41. In a ΔABC, D and E are the mid points of AB & AC. M and N are the mid point BD & EC. Then find the ratio of Ar. of ΔADE : Ar. of ΔAMN : Ar. of ΔABC

Sol.



Let, AD = 2

$$AD = BD = \frac{AB}{2} = 2$$

$\left(\because \text{ D is mid point AB}\right)$

Then, AB = 2 × 2 = 4

M is mid point BD

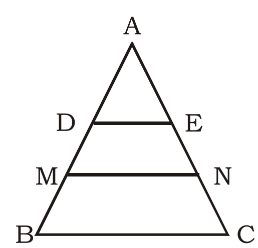
$$\text{Then, } BM = DM = \frac{BD}{2} = \frac{2}{2} = 1$$

AM = AD + DM = 2 + 1 = 3

	Ar. of ΔADE	:	Ar. of ΔAMN	:	Ar. of ΔABC
Side →	2	:	3	:	4
Area →	4	:	9	:	16

42. In a ΔABC, D and E are the mid point of AB & AC. M and N are the mid points of BD & EC then find the ratio of Ar. of ΔADE : Ar. of ▭DENM : Ar. of ▭MNCB

Sol.



Ar. of ΔADE	:	Ar. of ΔAMN	:	Ar. of ΔABC
4	:	9	:	16

(Discuss in above solution)

$\Rightarrow$ Area of ⏢DENM = Area of ΔAMN – Area of ΔADE

= 9 – 4 = 5 units

$\Rightarrow$ Area of ⏢MNCB = Area of ΔABC – Area of ΔAMN

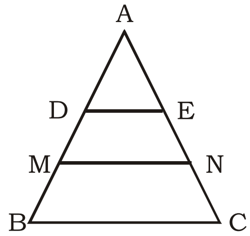
= 16 – 9 = 7 units

Now,

Ar. of ΔADE	:	Ar. of ⏢DENM	:	Ar. of ⏢MNCB
4	:	5	:	7

43. In a ΔABC, D and E are points lie on AB and AC. M and N are points lie on BD & EC. If $\frac{AD}{DM} = \frac{2}{3}$ and $\frac{DM}{MB} = \frac{4}{5}$ and Area of ΔAMN 800 cm². Find the area of ⏢MNCB.

Sol.



$\frac{AD}{DM} = \frac{2}{3}, \frac{DM}{MB} = \frac{4}{5}$

AD : DM : MB

2 : 3

4 : 5

(To make equal ratio)

AD : DM : MB

8 : 12 : 15

AM = AD + DM = 8 + 12 = 20

AB = AM + MB = 20 + 15 = 35

	Ar. of ΔADE	:	Ar. of ΔAMN	:	Ar. of ΔABC
Side→	8	:	20	:	35
Area→	64	:	400	:	1225

400 units = 800 cm²

1 unit = 2 cm²

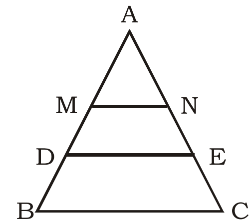
Area of ΔABC = 1225 × 2 = 2450

Area of ⏢MNCB = Area of ΔABC – Area of ΔAMN

= 2450 – 800 = 1650 cm²

44. In a ΔABC, D and E are point lies on AB & AC, and M and N are points lies on AD & AE. If $\frac{AM}{DM} = \frac{1}{3}$ and $\frac{DM}{DB} = \frac{6}{5}$ and Area of ⏢DECB is 210 cm² find the area of ⏢DENM.

Sol.



$\frac{AM}{MD} = \frac{1}{3}$ and $\frac{MD}{DB} = \frac{6}{5}$

AM : MD : DB

1 : 3

6 : 5

(to make equal ratio)

AM : MD : DB

2 : 6 : 5

AD = AM + MD = 2 + 6 = 8 AB = AD + DB = 8 + 5 = 13

	Ar. of ΔAMN	:	Ar. of ΔADE	:	Ar. of ΔABC
Side→	2	:	8	:	13
Area→	4	:	64	:	169

Area of ⏢DECB

= Area of ΔABC – Area of ΔADE

= 169 – 64 = 105

105 units = 210 cm²

1 unit = 2 cm²

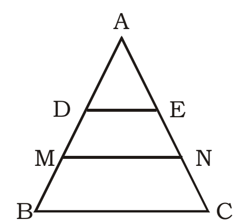
Area of ⏢MNED = Area of ΔADE – Area of ΔAMN

= 64 – 4 = 60 units

= 60 × 2 = 120 cm²

45. In a ΔABC, D and E are points lie on AB & AC. M and N are point lie on BD & EC. If $\frac{AD}{DM} = \frac{3}{2}, \frac{DM}{MB} = \frac{6}{7}$ and Area of ⏢DENM is 432 cm² Find the area of ⏢MNCB.

Sol.



$\frac{AD}{DM} = \frac{3}{2}, \frac{DM}{MB} = \frac{6}{7}$

AD : DM : MB

3 : 2

6 : 7

(To make equal ratio)

AD : DM : MB

9 : 6 : 7

AM = AD + DM = 9 + 6 = 15

AB = AM + MB = 15 + 7 = 22

	Ar. of ΔADE	:	Ar. of ΔAMN	:	Ar. of ΔABC
Side→	9	:	15	:	22
Area→	81	:	225	:	484

Area of ⏢DENM = Area of ΔAMN – Area of ΔADE

= 225 – 81 = 144 units

144 units = 432 cm²

1 unit = 3 cm²

Area of ⏢MNCB = Area of ΔABC – Area of ΔAMN

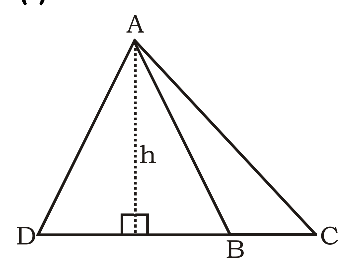
= 484 – 225 = 259 units

Now, Area of ⏢MNCB = 259 × 3 = 777 cm²

* (Area of triangles based on a same straight line.)

- If two or more than two triangles are based on a same straight line. Then the ratio of their area are same as the ratio of length of their base these triangles have common vertex.

(i)

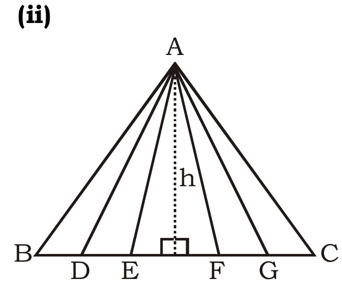


Proof:- Area of ΔABC : Area of ΔADB

$\frac{1}{2} \times BC \times h \quad : \quad \frac{1}{2} \times BD \times h$

BC : BD

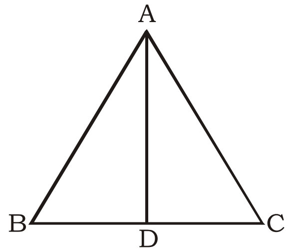
(ii)



Ar. of ΔABD	:	Ar. of ΔADE	:	Ar. of ΔAEF	:	Ar. of ΔAGC
BD	**:**	**DE**	**:**	**EF**	**:**	**GC**

46. In a triangle ABC, D is a point on BC. If area of ΔABD is 80 cm² and length of BD & DC are respectively 4 cm & 5 cm. Find the area of ΔABC.

Sol.



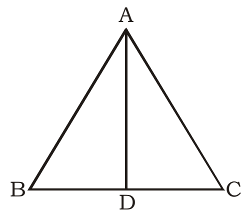
$$\frac{\text{Area of }\Delta ABD}{\text{Area of }\Delta ADC} = \frac{BD}{DC} = \frac{80}{A_2} = \frac{4}{5}$$

A_2 = 100 cm²

(here $A_2 \rightarrow$ Area of ΔABD)

47. In ΔABC, D is a point on BC Area of ΔABD 60 cm² and Area of ΔADC 75 cm². Find the ratio of BD and DC.

Sol.

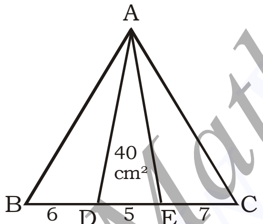


$$\frac{\text{Area of }\Delta ABD}{\text{Area of }\Delta ADC} = \frac{\text{Base of }\Delta ABD}{\text{Base of }\Delta ADC} = \frac{BD}{DC}$$

$$\frac{60}{75} = \frac{BD}{DC}$$

BD : DC = 4 : 5

48. In figure find the area of ΔABC ? If BD = 6 cm, DE = 5 cm, EC = 7 cm



Sol. In ΔADE,

5 units = 40 cm²

1 unit = 8 cm²

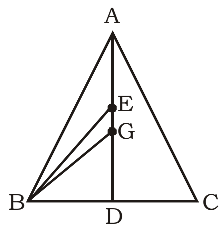
Area of ΔABC,

BC = BD + DE + EC = 5 + 6 + 7 = 18 units

18 units = 18 × 8 = 144 cm²

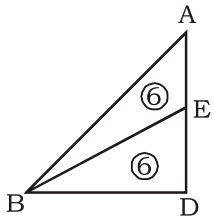
49. In ΔABC, AD is median, E is the mid-point of line AD and G is centroid find the ratio of area of ΔBEG and ΔABC.

Sol.

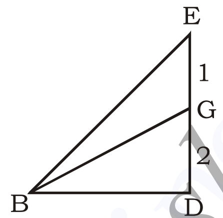


Let area of ΔABC = 24 units

then area of ΔABD = 12 units



Area of ΔABE = Area ΔBED = 6 units (E is mid point of AD)



Let AD = 6 units

$ED = \frac{AD}{2} = \frac{6}{2} = 3$ (E is mid point of AD)

$AG = \frac{2}{3}AD = \frac{2}{3} \times 6 = 4$ units

$GD = \frac{1}{3}AD = 6 \times \frac{1}{3} = 2$ units

EG = AG – AE = 4 – 3 = 1 unit

$$\frac{\text{Ar. of }\Delta BEG}{\text{Ar. of }\Delta BGD} = \frac{EG}{GD} = \frac{1}{2}$$

* (When two triangles are based on a same straight line then the ratio of their area are same as the ratio of there base)

Area of ΔBED = 6 units

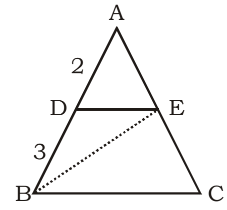
Then, Area of $\Delta BEG = 6 \times \frac{1}{3} =$ 2 units

$$\frac{\text{Ar. of }\Delta BEG}{\text{Ar. of }\Delta ABC} = \frac{2}{24} = \frac{1}{12}$$

Ar. of ΔBEG : Ar. of ΔABC = 1 : 12

50. In a ΔABC a line DE is draw parallel to BC. If area of ΔABC 150 cm² and AD : DB = 2 : 3 find the area of ΔDBE.

Sol.



DE || BC (Given)

$\Delta ADE \sim \Delta ABC$

AD : DB = 2 : 3

AB = AD + DB = 5 units

Area of ΔADE : Area of ΔABC

Side → 2 : 5

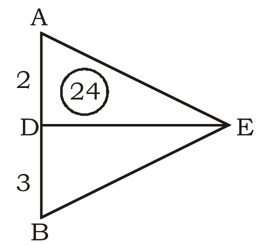
Area → 4 : 25

25 units = 150 cm²

1 unit = 6 cm²

Area of ΔADE = 4 units = 4 × 6 = 24 cm² In ΔAEB,

* When two triangle are based on a same straight line (AB). Then the ratio of their area are same as the ratio of length of their base have triangles have common vertex.



2 units = 24 cm²

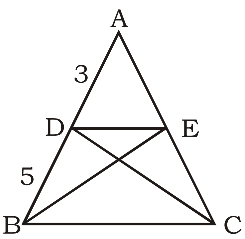
1 unit = 12 cm²

Area of ΔBDE = 3 units

= 3 × 12 = 36 cm²

51. In a ΔABC a line DE is parallel to BC. If Area of ΔABC 192 cm² and AD : DB = 3 : 5 find the area of ΔDEB.

Sol.



DE || BC (Given)

$\Delta ADE \sim \Delta ABC$

$$\frac{AD}{DB} = \frac{3}{5}$$

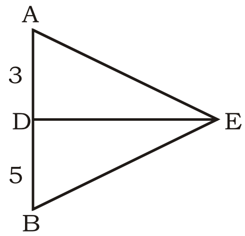
AB = 3 + 5 = 8 units

$$\frac{\text{Area of }\Delta ADE}{\text{Area of }\Delta ABC}=\left(\frac{AD}{AB}\right)^2=\left(\frac{3}{8}\right)^2=\frac{9}{64}$$

64 units = 192 cm²

1 unit = 3 cm²

Area of ΔADE = 9 × 3 = 27 cm²



$$\frac{\text{Area of }\Delta ADE}{\text{Area of }\Delta DEB}=\frac{AD}{DB}=\frac{3}{5}$$

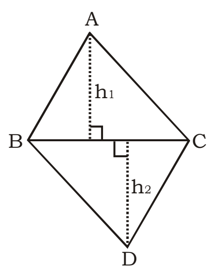
3 unit = 27

1 unit = 9

Area of ΔDBE = 5 unit = 5 × 9 = 45 cm²

* <u>Area of triangle which have common base</u>

If base of two or more triangles have common base then the ratio of their area are same as the ratio of length of altitude drawn on same base.



Proof

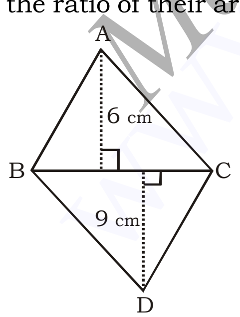
Area of ΔABC : Area of ΔBCD

$$\frac{1}{2}\times BC\times h_1 \quad : \quad \frac{1}{2}\times BC\times h_2$$

$$h_1 \quad : \quad h_2$$

52. ΔABC and ΔBCD have common base BC. If the lenght of their altitude 6 cm and 9 cm. Find the ratio of their areas.

Sol.



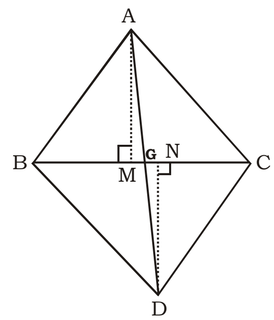
$$\frac{\text{Ar. of }\Delta ABC}{\text{Ar. of }\Delta BCD}=\frac{\text{Altitude of }\Delta ABC}{\text{Altitude of }\Delta BCD}=$$

$$\frac{6}{9}=\frac{2}{3}$$

Ar. of ΔABC : Area of ΔBCD = 2 : 3

53. ΔABC and ΔBCD have common base. If the length of their median 7 cm and 11 cm. Find the ratio of their areas.

Sol.



Draw AM & DN are altitude on BC.

AG and GD are medians

ΔAMG and ΔDGN

$\angle$AMG = $\angle$DNG (90°)

$\angle$DGN = $\angle$AGM (Vertically opposite)

ΔAMG ~ ΔDGN

$$\frac{AG}{GD}=\frac{AM}{DN}$$

So, $\frac{AM}{DN}=\frac{7}{11}$

We know that when two triangle have common base. Then the ratio of their area are same as the ratio of length of altitude.

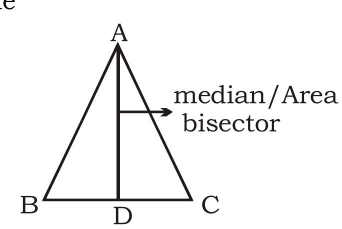
So,

Area of ΔABC : Area of ΔBDC = AM : DN

= 7 : 11

Median

A line from corner to the midpoint of the opposite side. It is called median. It is also called Area Bisector of triangle

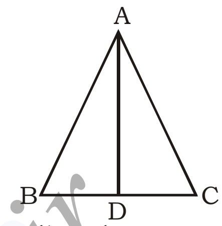


When AD is median, BD = DC

ar ΔABD = ar of ΔADC = $\frac{1}{2}$Ar. of Δ ABC

54. In a triangle ABC, AD is median. If area of ΔABC 120 cm² find the Area of ΔABD.

Sol. According to the Question



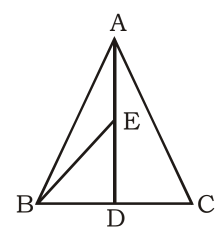
AD is median then ,

ar of ΔABD = $\frac{1}{2}$Ar. of ΔABC

= $\frac{1}{2}$ × 120 = 60 cm²

55. In a ΔABC, D is the midpoint of line BC and E is the mid point of AD. then find the ratio of area of ΔBEA and ΔABC.

Sol.



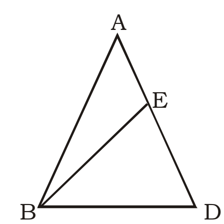
Let the area of ΔABC = 100 units

AD is median

Then,

Area of ΔABD = $\frac{1}{2}$ × area of ΔABC

= $\frac{1}{2}$ × 100 = 50 units



E is mid point of AD

So,

BE is median.

Ar of ΔBED = Ar of ΔBEA = $\frac{1}{2}$ × ABD

Ar of ΔBEA = $\frac{1}{2}$ × 50 = 25 units

Ar of ΔBEA : Ar of ΔABC = 25 : 100

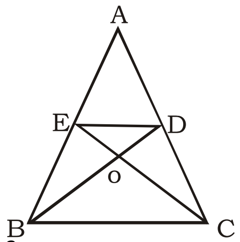
= 1 : 4

In a $\triangle$ ABC, BD and CE are two median which intersect each other at 'O'.$\Rightarrow$
Some Important results

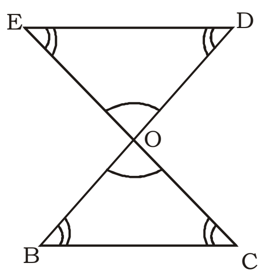
(i) Ar of $\triangle$ DOE : Ar of $\triangle$ ABC = 1 : 12

(ii) Ar of $\triangle$ DOE : Ar of $\triangle$ DOC = 1 : 2

(iii) Ar of $\triangle$ DOE : Ar of $\triangle$ BOC = 1 : 4



Proof



E and D mid point
Then,

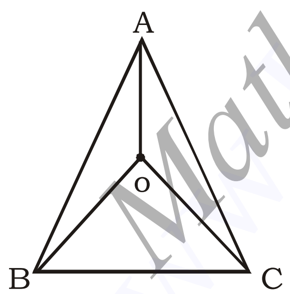
ED | | BC and ED = $\frac{1}{2}$ BC

So,

$$\frac{Ar\triangle OED}{Ar\triangle BOC} = \left(\frac{ED}{BC}\right)^2 = \left(\frac{BC/2}{BC}\right)^2 = \frac{1}{4}$$

Ar of $\triangle$ OED : Ar of $\triangle$ BOC = 1 : 4

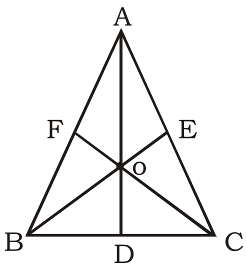
(iv). In a $\triangle$ ABC, O is centroid then



Ar. of $\triangle$ AOB = Ar. of BOC = Ar. of $\triangle$ AOC

$= \frac{1}{3}(Ar.of\,\triangle ABC)$

(v). In a $\triangle$ ABC, AD, BE and CF are medians and then all trianges have same Areas.

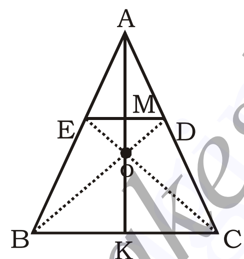


Ar. of $\triangle$ BOD = Ar. of $\triangle$ DOC = Ar. of $\triangle$ OEC = Ar. of $\triangle$ OAE = Ar. of $\triangle$ AOF = Ar. of $\triangle$ OBF.

$= \frac{1}{6}(Ar.of\,\triangle ABC)$

56. In a $\triangle$ ABC, BD and CE are two medians which intersects each other at 'O'. AO intersect the line ED at M. find the ratio of AM : MO

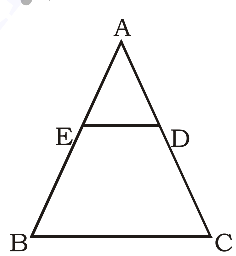
Sol.



Let AO = 4 unit (Large median)

then AK = $\frac{3}{2}$ AO

$= \frac{3}{2} \times 4 = 6$ units



E and D are mid points
(Because CE and BD are median)
Then ED | | BC

ED = $\frac{1}{2}$ BC

$$\frac{AE}{AB} = \frac{\text{median of } \triangle AED}{\text{median of } \triangle ABC} = \frac{AM}{AK}$$

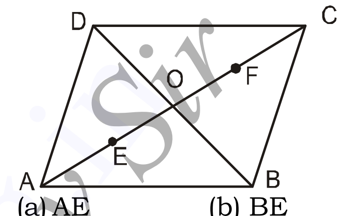
$$\frac{1}{2} = \frac{AM}{6}$$

AM = 3 units

OM = AO – AM
= 4 – 3 = 1

So,
AM : MO
3 : 1

57. In the adjoining figure ABCD is a | | gm and E,F are the centroids of Δ ABD and ΔBCD respectively, then EF equals :



(a) AE (b) BE
(c) CE (d) DE

Sol. AE : EO = 2 : 1 and CF : FO = 2 : 1

$\therefore$ OE = $\frac{1}{3}AO$ and OF = $\frac{1}{3}OC$

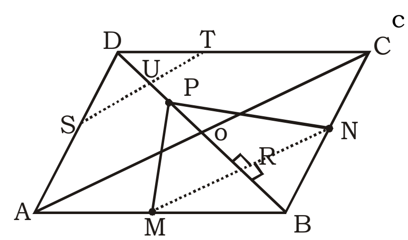
$\therefore$ EF = OE + OF

$= \frac{1}{3}(AO + OC) = \frac{1}{3}AC = AE$

58. A, B, C and D are the vertex of a Rhombous M and N are the mid-points of AB and BC. P is a point on diagonal BD In such a way BP $= \frac{3}{5}$ BD

Find $\frac{Ar \text{ of } \triangle BMN}{Ar \text{ of } \triangle MNP}$

Sol.



We know
BR : RO : OU : UD (By property)
1 : 1 : 1 : 1
(when M, N, S and T are mid-points)

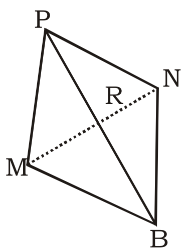
BP = $\frac{3}{5}$ BD (given)

BR= $\frac{1}{4}$ BD

Let BD =20 cm
then BR = OR = OU = UD = 5 cm

and BP = 20 × $\frac{3}{5}$ = 12 cm

RP = BP –BR = 12 – 5 = 7 cm



$\frac{\text{Ar of } \triangle BMN}{\text{Ar of } \triangle MNP} = \frac{BR}{RP} = \frac{5}{7}$

When two triangles have common base then the ratio of their Area is same as the ratio length of their Altitude.

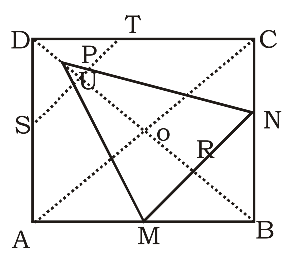
So,

Ar of $\triangle$ BMN: Ar of $\triangle$ MNP = 5 : 7

59. A,B,C and D are the vertex of a square M and N are the mid point of line AB and BC. P is a point on the Diagonal BD, if BP = $\frac{5}{6}$BD.

Find $\frac{\text{Ar. of} \triangle BMN}{\text{Ar. of} \triangle MNP}$

Sol.



BR : OR : OU : UD (When M,N, S, and T are mid points)
1 : 1 : 1 : 1

So,

BR = $\frac{1}{4}$ BD

BP = $\frac{5}{6}$ BD

L.C.M. of 4 and 6 = 12

Let BD = 12 cm.

BR = $\frac{1}{4}$ ×12 = 3

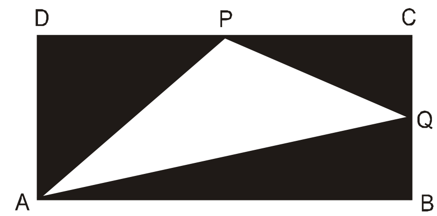
BP = $\frac{5}{6}$ × 12 = 10

RP = BP – BR = 10 – 3 = 7 cm

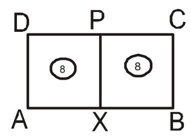
$\frac{\text{Ar of } \Delta BMN}{\text{Ar of } \Delta MNP} = \frac{BR}{RP} = \frac{3}{7}$

Ar. of Δ BMN : Ar. of Δ MNP = 3 : 7

60. In the given figure, ABCD is a rectangle. P and Q are the mid-points of sides CD and BC respectively. Then the ratio of area of shaded portion : area of unshaded portion is :

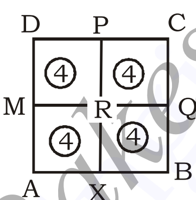


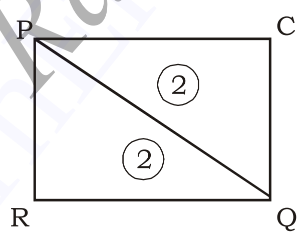
Sol. (c) Let total area of rectangle ABCD = 16 units



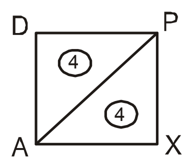
∴ area of □

AXPD = $\frac{16}{2}$ = 8 units

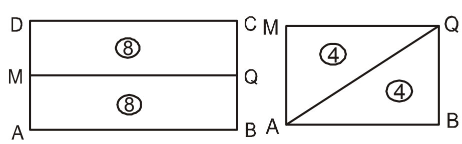




∴ area of Δ PCQ = 2 units



∴ area of Δ ADP = 4 units



∴ area of Δ ABQ = 4units

∴ total area of shaded portion

= area of Δ ADP + area of Δ ABQ + area of Δ PCQ

= 4 + 2 + 4 = 10 units

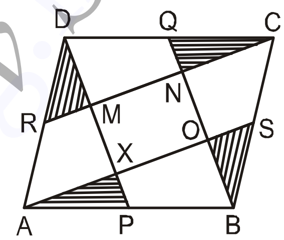
∴ area of unshaded portion = area of Δ APQ

= 16 – Area of shaded portion

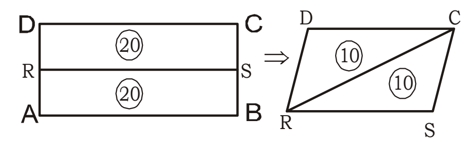
= 16 – 10 = 6 units

∴ Required ratio = 10 : 6 = 5 : 3

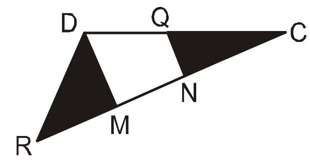
61. In the ||gm ABCD, P,Q,R and S are mid-points of sides AB, CD, DA and BC respectively. AS, BQ, CR and DP are joined. Find the ratio of the area of the shaded region to the area of the ||gm ABCD.



Sol. (a)



Let total area of || ABCD = 40 units



In Δ DMC, Q is the mid point of DC and QN || DM

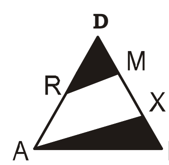
[∵ DMNQ is also a ||gm)

⇒ N is the mid-point of MC

∴ ar (Δ QCN) : ar(Δ DMC) = 1 : 4

Let ar (Δ QCN) = 1unit

⇒ ar (quadrilateral DMNQ) = 4 - 1 = 3 units



Similarly, In Δ DAX

ar (Δ DRM) : ar (Δ DAX) = 1 : 4

ar (Δ DRM) = 1 unit

$\Rightarrow$ ar ($\triangle$ DAX) = 4 units

$\Rightarrow$ ar ($\square$ RMXA) = 4 – 1 = 3 units

$\therefore$ ar($\triangle$ DRM) + ar ($\triangle$ QNC)
= 1 +1
= 2units

and ar($\triangle$ DRC) = 4 + 1 = 5units

but from (i) ar ($\triangle$ DRC) = 10 unit (2 times)

$\therefore$ ar ($\triangle$ DRM) + ar ($\triangle$ QNC)
= 2 $\times$ (2 times) = 4 units

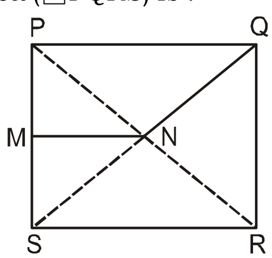
Similarly, ar ($\triangle$ APX) + ar ($\triangle$ BOS) = 2 $\times$ 2
= 4units

$\therefore$ total shaded area = 4 + 4
= 8 units

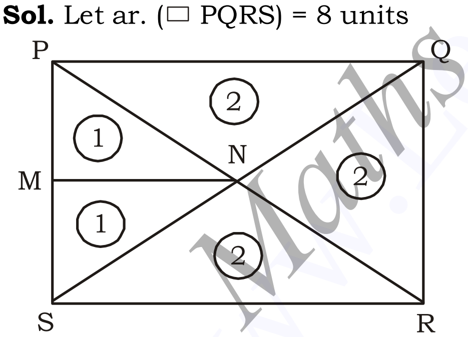
& area of ||gm ABCD = 40units

$\therefore$ Required ratio = $\frac{8}{40}=\frac{1}{5}$

62. PQRS is a square, M is the mid-point of side PS and N is the intersecting point of its diagonals. Then the ratio Area ($\square$PQNM) : Area ($\square$PQRS) is :



Sol. Let ar. ($\square$ PQRS) = 8 units

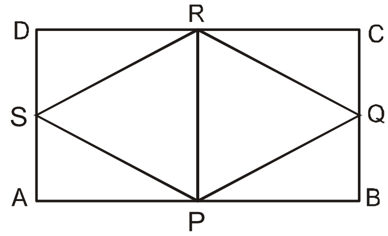


Area of PQNM = area of $\triangle$ PNQ + area of $\triangle$ PNM

= 2 + 1 = 3 units

$\therefore$ area ($\square$PQNM) : area ($\square$PQRS) = 3 : 8

63. ABCD is a ||gm. P, Q, R and S are points on sides AB, BC, CD and DA respectively such that AP = DR. If the area of the ||gm ABCD is 20cm², then the area of quadrilateral PQRS is :



Sol. Area of ($\triangle$ PRS + $\triangle$ PQR) = $\frac{1}{2}$ (area of $\square$ APRD)

$+\frac{1}{2}$(area of $\square$BPRC)

= $\frac{1}{2}$(AP $\times$ AD) + $\frac{1}{2}$(PB $\times$ BC)

= $\frac{1}{2}$(AP $\times$ AD) + $\frac{1}{2}$(PB $\times$ AD)

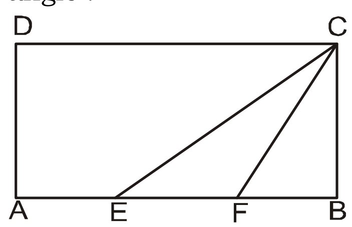
($\because$ BC = AD)

= $\frac{1}{2}$AD (AP + PB) = $\frac{1}{2}$(AD $\times$ AB)

= $\frac{1}{2}$(area of $\square$ ABCD)

= $\frac{1}{2}\times 20$ = 10cm²

64. In the given figure, ABCD is a rectangle with AE = EF = FB. What is the ratio of the area of the $\triangle$ CEF to that of the rectangle ?



Sol. Let BC = x and
FB = EF = AE = y
$\therefore$ AB = CD = 3y

Now, Area of $\triangle$ CBF = $\frac{1}{2}xy$

and area of $\triangle$ CBE = $\frac{1}{2}x\times 2y = xy$

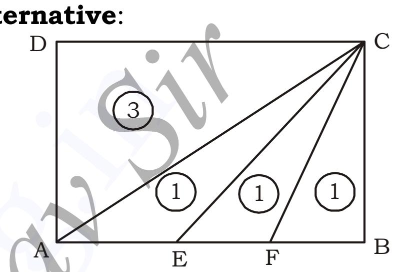
$\therefore$ area of $\triangle$ CEF

= $xy-\frac{1}{2}xy=\frac{1}{2}xy$

and area of rectangle ABCD = 3xy

$\therefore$ Required ratio = $\frac{1}{2}xy:3xy = 1:6$

Alternative:



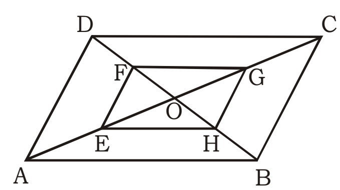
Let ar($\square$ ABCD) = 6 units
Base and height are same

ar($\triangle$ CAE) = ar($\triangle$ CEF) = ar($\triangle$ CFB) = 1 unit

$\therefore$ required ratio = 1 : 6

65. ABCD is a parallelogram in which diagonals AC and BD intersect at O. If E,F, G and H are the mid points of AO, DO, CO and BO respectively, then the ratio of the perimeter of the quadrilateral EFGH to the perimeter of parallelogram ABCD is:

Sol.



$\because$ E and H are mid points of OA and OB respectively

$\Rightarrow$ EH = $\frac{1}{2}$AB

similarly, GH = $\frac{1}{2}$ BC,

FG = $\frac{1}{2}$ CD, EF = $\frac{1}{2}$AD

Perimeter of parallelogram = (AB+BC+CD+DA)

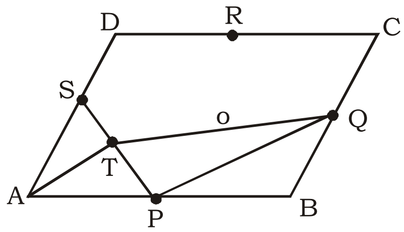
Perimeter of quadrilateral EFGH = EH + HG + FG + EF

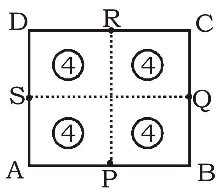
= $\frac{1}{2}$(AB+BC+CD+DA)

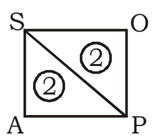
$\therefore$ $\frac{\text{Perimeter of EFGH}}{\text{Perimeter of ABCD}}$ = 1 : 2

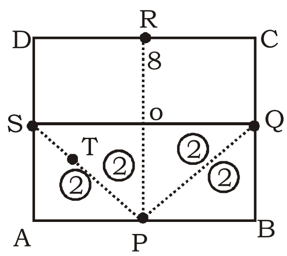
66. A,B,C and D are the vertex of a || gram. P,Q, R and S are the mid points of side AB,BC,CD and DA. T is the midpoint of line PS. then find the ratio of Areas of △ ATS and △ PTQ

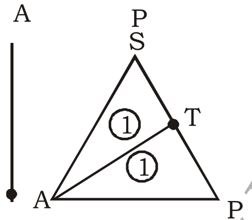
Sol. Let the area of parallelogram = 16 units











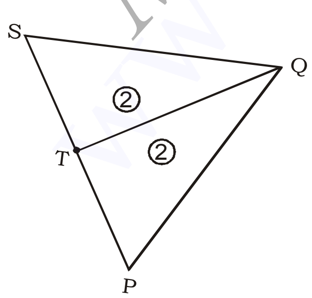
In △ ASP

Area of △ ASP = 2 units

T is the mid point of PS

The

Area of △ ATS = 1 units



Area of △ PSQ = 4 units

T is midpoint of SP

Then, Ar of △ PTQ = $\frac{1}{2} \times \text{Ar.of}\Delta PSQ$

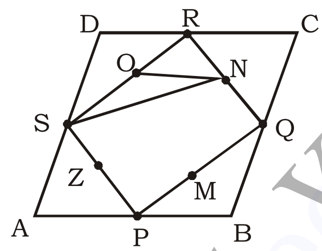
$= \frac{1}{2} \times 4 = 2$ units

Now,

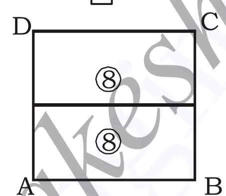
Ar of △ ATS : Ar of △ PTQ = 1 : 2

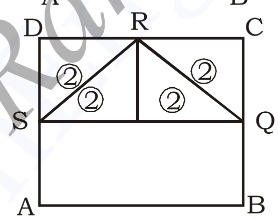
67. A,B,C and D are the vertex of || gram P,Q,R and S are the mid-point AB, BC, CD and DA. M, N, O and Z are the mid points PQ, QR, RS, and SP. find ratio of area of △ SON and □ ABCD.

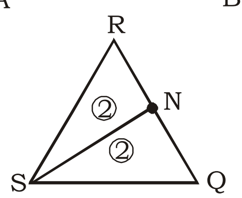
Sol.



Let Area of □ABCD = 16 units



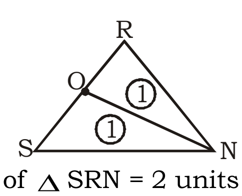




Ar of △ SRQ = 4 units

N is the mid point on RQ

Then



Ar of △ SRN = 2 units

O is midpoint on RS

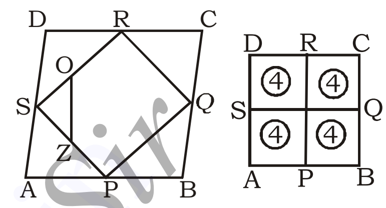
So,

Ar of △ SON = 1 unit

area of △ SON : area of □ ABCD = 1 : 16

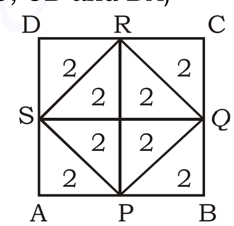
68. A, B, C and D are the vertex of || gram P, Q, R and S are the mid point AB, BC, CD and DA. O and Z are the mid points RS and SP. Find ratio of area of △ of SOZ and □ ABCD

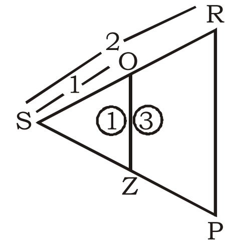
Sol.



Let the ar. of Parallelogram ABCD = 16 units.

(P, Q, R and S are the mid points of AB, BC, CD and DA)





Let SR = 2 cm

SO = 1 cm

O and Z are mid points SR and SP.

Then,

OZ || RP and OZ = $\frac{1}{2}$ RP

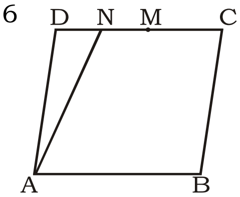
$$\frac{\text{Area of }\Delta SOZ}{\text{Area of } SRP} = \left(\frac{OS}{SR}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Now,

Area of △ SOZ : area of □ABCD = 1 : 16

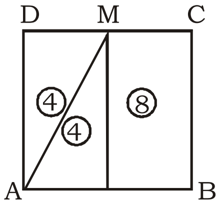
69. A, B, C and D are the vertex of a || gram. M and N are the mid points of CD and MD respectively. Find the ratio of area of △ ADN and □ ABCD

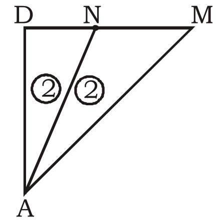
Sol. Let area of □ ABCD = 16



M is the mid point of CD

Ar. of $\triangle$ ADM = 4 units





N is mid point of DM

Then area of $\triangle$ ADN = 2 unit

So,

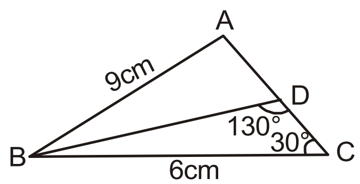
area of $\triangle$ ADN : area of $\square$ABCD

= 2 : 16

= 1 : 8

EXERCISE

1. In the given figure, AD : DC = 3 : 2, then $\angle$ABC :



(a) 30° (b) 40°
(c) 45° (d) 50°

2. Triangle ABC is such that AB = 9 cm, BC = 6 cm, AC = 7.5 cm. Triangle DEF is similar to $\triangle$ABC. If EF = 12 cm then DE is:
(a) 6 cm (b) 16 cm
(c) 18 cm (d) 15 cm

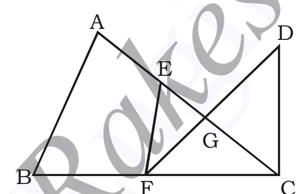
3. ABCD is a rhombus. A straight line through C cuts AD produced at P and AB produced at Q. If DP $= \frac{1}{2}$ AB, then the ratio of the length of BQ and AB is
(a) 2 : 1 (b) 1 : 2
(c) 1 : 1 (d) 3 : 1

4. D and E are two points on the sides AC and BC respectively of $\triangle ABC$ such that DE = 18 cm, CE = 5 cm and $\angle DEC$ = 90°. If tan $\angle ABC$ = 3.6, then AC : CD is
(a) BC : 2 CE (b) 2CE : BsC
(c) 2BC : CE (d) CE : 2BC

5. In $\triangle$ABC and $\triangle$DEF, AB = DE and BC = EF, then one can infer that $\triangle ABC \cong \triangle DEF$, when
(a) $\angle BAC = \angle EDF$
(b) $\angle ACB = \angle EDF$
(c) $\angle ACB = \angle DFE$
(d) $\angle ABC = \angle DEF$

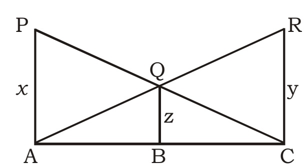
6. For a triangle ABC, D, E, F are the mid - points of its sides. if area of $\triangle$ABC = 24 sq. units, then area of $\triangle$DEF is
(a) 4 sq. units (b) 6 sq. units
(c) 8 sq. units (d) 12 sq. units

7. In the adjoining figure (not drawn to scale) AB, EF and CD are parallel lines. Given that EG = 5 cm, GC = 10 cm and DC = 18 cm. Calculate AC, if AB = 15 cm:



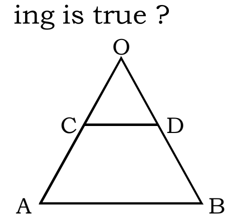
(a) 21 cm (b) 25 cm
(c) 18 cm (d) 28 cm

8. In the adjoining figure PA, QB and RC are each perpendicular to AC. Which one of the following is true :



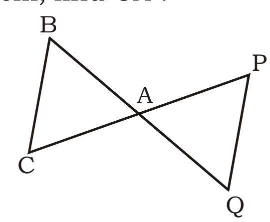
(a) x + y = z (b) xy = 2z
(c) $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$ (d) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

9. In the given diagram AB$\parallel$CD, then which one of the following is true ?



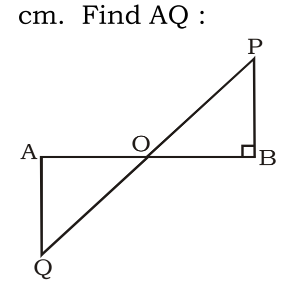
(a) $\frac{AB}{CD}=\frac{AO}{OC}$
(b) $\frac{AB}{CD}=\frac{BO}{OD}$
(c) $\triangle$AOB ~ $\triangle$COD
(d) All of these

10. In the figure $\triangle$ACB ~ $\triangle$APQ. If BC = 8 cm, PQ = 4 cm, AP = 2.8 cm, find CA :



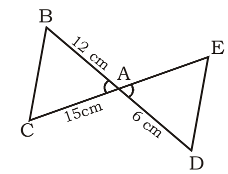
(a) 8 cm (b) 6.5 cm
(c) 5.6 cm
(d) None of these

11. In the figure QA and PB are perpendicular to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ :



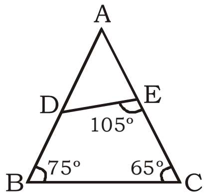
(a) 8 cm (b) 9 cm
(c) 15 cm (d) 12 cm

12. In the given figure AB = 12 cm, AC = 15 cm and AD = 6 cm. BC$\parallel$DE, Find the length of AE:



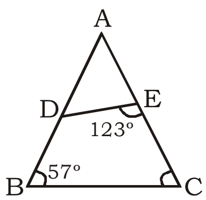
(a) 6 cm (b) 7.5 cm
(c) 9 cm (d) 10 cm

13. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and AE = 10 cm. Find AB:



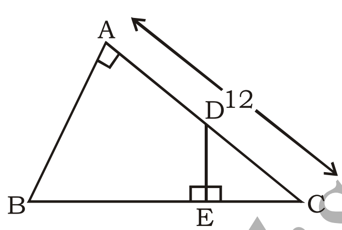
(a) 10cm (b) 15 cm
(c) 8 cm (d) 9 cm

14. In the figure AD = 12 cm, AB = 20 cm and AE = 10 cm. Find EC:



(a) 14 cm (b) 10 cm
(c) 8 cm (d) 15 cm

15. In a right angle triangle Δ BAC in which AB = BE = 5 cm and AC = 12 cm. DE is perpendicular on BC. Then find DE?

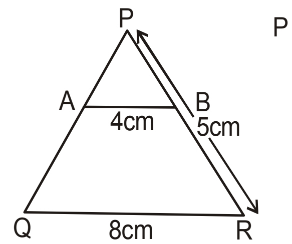


(a) 3.3 cm (b) 4.3 cm
(c) 5.3 cm (d) 6.3 cm

16. The side AB of a parallelogram ABCD is produced to E in such way that BE = AB, DE intersects BC at Q. The point Q divides BC in the ratio
(a) 1 : 2 (b) 1 : 1
(c) 2 : 3 (d) 2 : 1

17. ABCD is a trapezium whose side AD is parallel to BC, Diagonals AC and BD intersect at O. If AO = 3, CO = x–3, BO = 3x–19 and DO = x– 5, then the value(s) of x will be :
(a) 7, 6 (b) 12, 6
(c) 7, 10 (d) 8, 9

18. In the given figure, AB | | QR. Find the length of PB :



(a) 2.5 cm (b) 2 cm
(c) 3 cm (d) 3.5 cm

19. The areas of two similar triangles are in the ratio 9 : 16. Their corresponding sides will be in the ratio :
(a) 3 : 5 (b) 3 : 4
(b) 4 : 5 (d) 4 : 3

20. If G is centroid and AD, BE,CF are three medians of Δ ABC with area 72cm^2 , then the area of Δ BDG is :
(a) 12 cm^2 (b) 16 cm^2
(c) 24 cm^2 (d) 8 cm^2

21. The three medians AD, BE and CF of Δ ABC intersect at G. If the area of Δ ABC is 60sq.cm then the area of the quadrilateral BDGF is :
(a) 10 sq.cm (b) 15 sq.cm
(c) 20 sq.cm (d) 30 sq.cm

22. The areas of the similar triangles are in the ratio of 25 : 36. What is the ratio of their respective heights ?
(a) 5 : 6 (b) 6 : 5
(c) 1 : 11 (d) 2 : 3

23. Given that : $\Delta ABC \sim \Delta PQR$, if $\frac{\text{area}(\Delta PQR)}{\text{area}(\Delta ABC)} = \frac{256}{441}$ and PR = 12 cm, then AC is equal to?
(a) $12\sqrt{2}$ cm (b) 15.5 cm
(c) 16 cm (d) 15.75 cm

24. Two isosceles triangles have equal vertical angles and their areas are in the ratio 9 : 16. then the ratio of their corresponding heights is
(a) 4.5 : 8 (b) 3 : 4
(c) 4 : 3 (d) 8 : 4.5

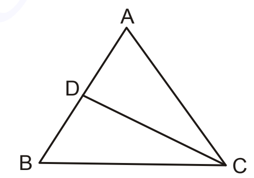
25. Given that the ratio of altitudes of two triangles is 4:5, ratio of their areas is 3 : 2, The ratio of their corresponding bases is
(a) 5 : 8 (b) 15 : 8
(c) 8 : 5 (d) 8 : 15

26. ABCD is a square. Draw a triangle QBC on side BC considering BC as base and draw a triangle PAC on AC as it base such that $\Delta QBC \sim \Delta PAC$ then,

$\frac{\text{Area of } \Delta\text{ QBC}}{\text{Area of } \Delta\text{ PAC}}$ is equal to :

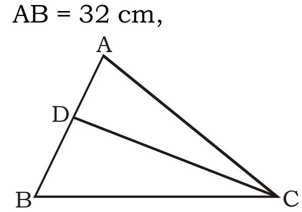
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{1}$

27. In the given figure, $\angle$BAC = $\angle$BCD, AB = 32cm and BD = 18cm, then the ratio of perimeter of Δ BCD and Δ ABC is :



(a) 4 : 3 (b) 8 : 5
(c) 5 : 8 (d) 3 : 4

28. In the given figure In ΔABC, $\angle$BAC = $\angle$BCD and AD = 14 cm, AB = 32 cm,



Find $\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta BDC}$?

(a) $\frac{4}{3}$ (b) $\frac{5}{3}$ (b) $\frac{6}{3}$ (d) $\frac{7}{3}$

29. ABC is a right-angled triangle. AD is perpendicular to the hypotenuse BC. If AC = 2AB, then the value of BD is :

(a) $\frac{BC}{2}$ (b) $\frac{BC}{3}$

(c) $\frac{BC}{4}$ (d) $\frac{BC}{5}$

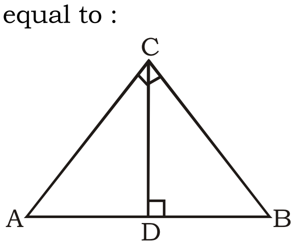
30. The difference between altitude and base of a right angled triangle is 17 cm and its hypotenuse is 25 cm. What is the sum of the base and altitude of the triangle is :
(a) 30 cm (b) 31 cm
(c) 32 cm
(d) con't be determined

31. One side other than the hypotenuse of right angle isosceles triangle is 6 cm. The length of the perpendicular on the hypotenuse from the opposite vertex is :
(a) 6 cm (b) $6\sqrt{2}$ cm
(c) 4 cm (d) $3\sqrt{2}$ cm

32. In right angled ΔABC, $\angle$B = 90°, If P and Q are points on the sides AB and BC respectively, then :
(a) $AQ^2 + CP^2 = PQ^2 + AC^2$
(b) $AB^2 + CP^2 = PQ^2 + AC^2$
(c) $2(AQ^2 + CP^2) = PQ^2 + AC^2$
(d) $AQ^2 + CP^2 = 2(PQ^2 + AC^2)$

33. If ABC is right angled triangle at B and M, N are the mid-points of AB and BC, then $4(AN^2 + CM^2)$ is equal to:
(a) $4AC^2$ (b) $6AC^2$
(c) $5AC^2$ (d) $\frac{5}{4}AC^2$

34. In a right angled ΔABC, $\angle$C = 90° and CD is the perpendicular on hypotenuse AB if BC = 15 cm and AC = 20 cm then CD is equal to :



(a) 18 cm (b) 12 cm
(c) 17.5 cm
(d) can't be determined

35. ABC is a right angle triangle at A and AD is perpendicular to the hypotenuse. Then $\frac{BD}{CD}$ is equal to :
(a) $\left(\frac{AB}{AC}\right)^2$ (b) $\left(\frac{AB}{AD}\right)^2$
(c) $\frac{AB}{AC}$ (d) $\frac{AB}{AD}$

36. Suppose ΔABC be a right-angled triangle where $\angle A = 90°$ and $AD \perp BC$. If ar(ΔABC) = 40cm², ar(ΔACD) = 10 cm² and AC = 9 cm, then the length of BC is
(a) 12 cm (b) 18 cm
(c) 4 cm (d) 6 cm

37. In a triangle ABC, $\angle BAC = 90°$ and AD is perpendicular to BC. If AD = 6 cm and BD = 4 cm then the length of BC is :
(a) 8 cm (b) 10 cm
(c) 9 cm (d) 13 cm

38. In a right angled ΔABC, $\angle ABC = 90°$, AB = 3, BC = 4, CA = 5; BN is perpendicular to AC, AN : NC is
(a) 3 : 4 (b) 9 : 16
(c) 3 : 16 (d) 1 : 4

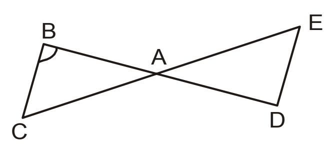
39. In ΔABC $\angle A = 90°$ and $AD \perp BC$ where D lies on BC. If BC = 8 cm , AC = 6 cm , then area of ΔABC : area of ΔACD = ?
(a) 4 : 3 (b) 25 : 16
(c) 16 : 9 (d) 25 : 9

40. A point D is taken from the side BC of a right-angled triangle ABC, where AB is hypotenuse. Then
(a) $AB^2 + CD^2 = BC^2 + AD^2$
(b) $CD^2 + BD^2 = 2AD^2$
(c) $AB^2 + AC^2 = 2AD^2$
(d) $AB^2 = AD^2 + BC^2$

41. BL and CM are medians of ΔABC right- angled at A and BC = 5 cm. If BL = $\frac{3\sqrt{5}}{2}$ cm, then the length of CM is
(a) $2\sqrt{5}$ cm (b) $5\sqrt{2}$ cm
(c) $10\sqrt{2}$ cm (d) $4\sqrt{5}$ cm

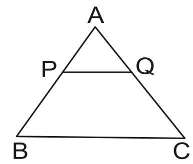
42. Q is a point in the interior of a rectangle ABCD, if QA = 3 cm, QB = 4 cm and QC = 5 cm then the length of QD (in cm) is
(a) $3\sqrt{2}$ (b) $5\sqrt{2}$
(c) $\sqrt{34}$ (d) $\sqrt{41}$

43. In the given figure, AB = 18cm, AC = 11cm and AD = 9 cm and BC || DE, find the length of AE



(a) 6 cm (b) 7 cm
(c) 5.5 cm (d) 7.5cm

44. In the given triangle ABC, BP = 3 AP, QC = 3AQ and BC = 36cm. Find the value of PQ ?



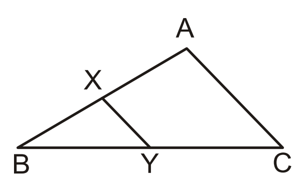
(a) 9cm (b) 8cm
(c) 6cm (d) 7cm

45. A straight line parallel to base BC of the triangle ABC intersects AB and AC at the points D and E respectively. If the area of the Δ ACD is 36sq cm, then the area of Δ ABE is :
(a) 36 sq.cm (b) 18 sq.cm
(c) 12 sq.cm (d) None of these

46. In Δ ABC, P and Q are the mid points of the sides AB and AC respectively. R is a point on the segment PQ such that PR : RQ = 1 : 2, If PR = 2cm, then BC =
(a) 4 cm (b) 2 cm
(c) 12 cm (d) 6 cm

47. Find the maximum area that can be enclosed in a triangle of perimeter 24cm :
(a) 32 cm² (b) $16\sqrt{3}$ cm²
(c) $16\sqrt{2}$ cm² (d) 27cm²

48. In the given figure, the line segment XY || AC and it divides the triangle into two parts of equal areas. Find ratio $\frac{AX}{AB}$:



(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{\sqrt{2}\quad 1}{\sqrt{2}}$ (d) $\frac{\sqrt{2}\quad 1}{\sqrt{2}}$

49. D and E are the mid-points of AB and AC of Δ ABC, BC is produced to any point P; DE, DP and EP are joined. then, area of :

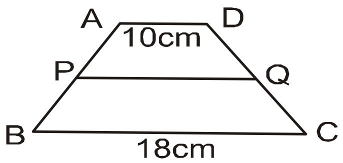
(a) $\Delta PED = \frac{1}{4} \Delta ABC$

(b) $\Delta PED = \Delta BEC$

(c) $\Delta ADE = \Delta BEC$

(d) $\Delta BDE = \Delta BEC$

50. In the given figure, ABCD is a trapezium such that AD || BC and P, Q are the points on AB and CD respectively such that PQ || AD and AP : PB = 5 : 3. Then PQ is :

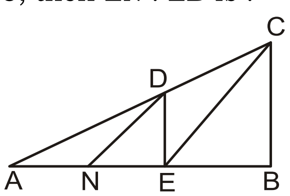


(a) 12.5cm (b) 15cm
(c) 17.5cm (d) 20cm

51. In a Δ ABC, D is the mid-point of BC and E is the mid-point of AD. The line BE is extended and it intersects AC at T. If AB = 18cm, BC = 17cm and AC = 15cm. Find TC ?

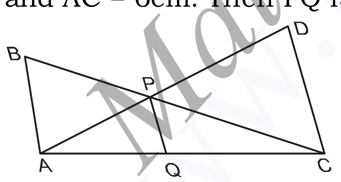
(a) 8cm (b) 9cm
(c) 10cm (d) 7cm

52. In the given figure, DE || BC and EC || ND, AE : EB = 4 : 5, then EN : EB is :



(a) 5 : 9 (b) 9 : 4
(c) 4 : 5 (d) 4 : 9

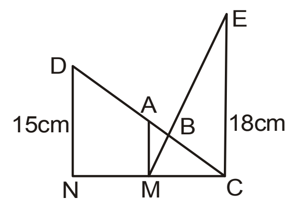
53. In the given figure, AB || CD || PQ, AB = 12cm, CD = 18cm and AC = 6cm. Then PQ is:



(a) $\frac{36}{5}cm$ (b) $\frac{18}{5}cm$

(c) 9cm (d) $\frac{14}{5}cm$

54. In the given figure, EC || AM || DN and AB = 5cm, BC = 10 cm. Find DC:



(a) 19 cm (b) 20cm
(c) 25 cm (d) 17.5cm

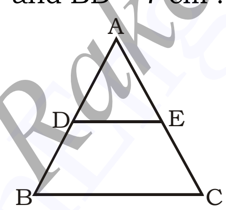
55. The triangle is formed by joining the mid-points of the sides AB, BC and CA of ΔABC and the area of ΔPQR is 6 cm², then the area of ΔABC is

(a) 36 cm² (b) 12 cm²
(c) 18 cm² (d) 24 cm²

56. ABC is a triangle and DE is drawn parallel to BC cutting the other side at D and E. If AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm, then AE is equal to:

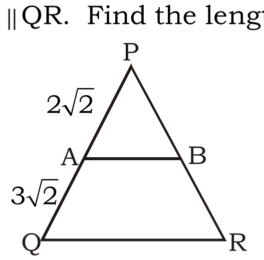
(a) 1.4 cm (b) 1.8 cm
(c) 1.2 cm (d) 1.05 cm

57. In ΔABC, AC = 5 cm. Calculate the length of AE where DE ∥BC. Given that AD = 3 cm and BD = 7 cm :



(a) 2 cm (b) 1 cm
(c) 1.5 cm (d) 2.5 cm

58. In ΔPQR, AP = $2\sqrt{2}$ cm, AQ = $3\sqrt{2}$ cm and PR = 10 cm, AB ∥QR. Find the length of BR :

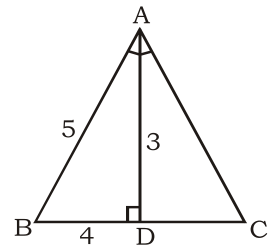


(a) $6\sqrt{2}$ cm (b) 6 cm

(c) $5\sqrt{2}$ cm

(d) None of these

59. In the adjoining figure the ∠BAC and ∠ADB are right angles. BA = 5 cm, AD = 3 cm and BD = 4 cm, what is the length of DC ?



(a) 2.5 (b) 3
(c) 2.25 (d) 2

60. ABC is a triangle in which ∠A = 90°, AN is perpendicular to BC, AC = 12 cm and AB = 5 cm. Find the ratio of the areas of ΔANB and ΔANC :

(a) 125 : 44 (b) 25 : 144
(c) 144 : 25 (d) 12 : 5

61. Area of ΔABC = 30 cm². D and E are the mid-points of BC and AB respectively. Find Ar (ΔBDE) :

(a) 10 cm (b) 7.5 cm
(c) 15 cm
(d) None of these

62. ABCD is a trapezium in which AB∥CD and E and F are the point on DA & BC, DC = 40 cm, AB = 105 cm. The ratio of $\frac{DE}{EA} = \frac{2}{3}$. Then find EF.

(a) 66 cm (b) 70 cm
(c) 72 cm (d) 73 cm

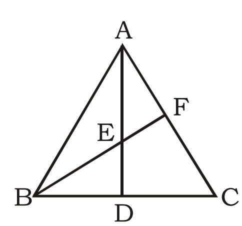
63. In ΔABC, D and E are mid-point of side AB and BC. P is point on line AC in such a way that R and S are the mid-point of AP & PC find DR : ES

(a) 1 : 1 (b) 2 : 1
(c) 4 : 1 (d) 3 : 1

64. In the given figure

$\frac{AE}{ED} = \frac{BD}{DC} = \frac{2}{3}$, AC = 760 cm

Then find AE = ?



(a) 80 cm (b) 90 cm
(c) 100 cm (d) 120 cm

65. The length of the diagonal BD of the parallelogram ABCD is 18 cm. If P and Q are the centroid of the $\triangle ABC$ and $\triangle ADC$ respectively then the length of the line segment PQ is
(a) 4 cm (b) 6 cm
(c) 9 cm (d) 12 cm

66. A straight line parallel to BC of $\triangle ABC$ intersects AB and AC at points P and Q respectively. AP = QC, PB =4 units and AQ = 9 units, then the length of AP is :
(a) 25 units (b) 3 units
(c) 6 units (d) 6.5 units

67. D is any point on side AC of $\triangle ABC$. If P, Q, X, Y are the mid-points of AB, BC, AD and DC respectively, then the ratio of PX and QY is
(a) 1 : 2 (b) 1 : 1
(c) 2 : 1 (d) 2 : 3

68. In $\triangle ABC$, PQ is parallel to BC. If AP : PB = 1 : 2 and AQ = 3 cm; AC is equal to
(a) 6 cm (b) 9 cm
(c) 12 cm (d) 8 cm

69. In $\triangle ABC$, D and E are points on AB and AC respectively such that $DE \parallel BC$ and DE divides the $\triangle ABC$ into two parts of equal areas. Then ratio of AD and BD is
(a) 1 : 1 (b) $1:\sqrt{2}-1$
(c) $1:\sqrt{2}$ (d) $1:\sqrt{2}+1$

70. ABCD is a rectangle where the ratio of the length of AB and BC is 3 : 2 . If P is the mid- point of AB, then the value of sin $\angle CPB$ is
(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$

71. Inside a triangle ABC, a straight line parallel to BC intersects AB and AC at the point P and Q respectively. If AB = 3 PB, then PQ : BC is
(a) 1 : 3 (b) 3 : 4
(c) 1 : 2 (d) 2 : 3

72. In $\triangle ABC$, $DE \parallel AC$, D and E are two points on AB and CB respectively. If AB = 10 cm and AD = 4 cm, then BE : CE is
(a) 2 : 3 (b) 2 : 5
(c) 5 : 2 (d) 3 : 2

73. For a triangle ABC, D and E are two points on AB and AC such that AD = $\frac{1}{4}$ AB, AE = $\frac{1}{4}$ AC. If BC = 12 cm, then DE is
(a) 5 cm (b) 4 cm
(c) 3 cm (d) 6 cm

74. In $\triangle PQR$, S and T are point on sides PR and PQ respectively such that $\angle PQR = \angle PST$, If PT = 5 cm, PS = 3 cm and TQ = 3 cm, then length of SR is
(a) 5 cm (b) 6 cm
(c) $\frac{31}{3}$ cm (d) $\frac{41}{3}$ cm

75. In $\triangle ABC$, two points D and E are taken on the lines AB and BC respectively in such a way that AC is parallel to DE. Then $\triangle ABC$ and $\triangle DBE$ are
(a) similar only If D lies outside the line segment AB
(b) congruent only If D lies out side the line segment AB
(c) always similar
(d) always congruent

76. In triangle ABC a straight line parallel to BC intersects AB and AC at D and E respectively. If AB = 2AD, then DE : BC is
(a) 2 : 3 (b) 2 : 1
(c) 1 : 2 (d) 1 : 3

77. In a $\triangle ABC$, D and E are two points on AB and AC respectively such that $DE \parallel BC$. DE bisects the $\triangle ABC$ in two equal areas. Then the ratio BD : AB is
(a) $1:\sqrt{2}$ (b) $1:2$
(c) $\left(\sqrt{2}-1\right):\sqrt{2}$ (d) $\sqrt{2}:1$

78. If in a triangle ABC, D and E are on the sides AB and AC, such that, DE is parallel to BC and $\frac{AD}{BD}=\frac{3}{5}$. If AC = 4 cm, then AE is
(a) 1.5 cm (b) 2.0 cm
(c) 1.8 cm (d) 2.4 cm

79. ABC is a triangle in which $DE \parallel BC$ and AD : DB = 5 : 4. Then DE : BC is
(a) 4 : 5 (b) 9 : 5
(c) 4 : 9 (d) 5 : 9

80. D and E are mid-points of sides AB and AC respectively of the $\triangle$ ABC. A line drawn from A meets BC at H and DE at K. AK : KH = ?
(a) 2 : 1 (b) 1 : 1
(c) 1 : 3 (d) 1 : 2

81. If ABC be an equilateral triangle and AD perpendicular to BC, Then
$AB^2 + BC^2 + CA^2$ = ?
(a) $3AD^2$ (b) $5AD^2$
(c) $2AD^2$ (d) $4AD^2$

ANSWER KEY

1. (b)	9. (d)	17. (d)	25. (b)	33. (c)	41. (a)	49. (a)	57. (c)	65. (b)	73. (c)
2. (c)	10. (c)	18. (b)	26. (c)	34. (b)	42. (a)	50. (b)	58. (b)	66. (c)	74. (c)
3. (a)	11. (c)	19. (b)	27. (d)	35. (a)	43. (c)	51. (c)	59. (c)	67. (b)	75. (c)
4. (a)	12. (b)	20. (a)	28. (a)	36. (b)	44. (a)	52. (d)	60. (b)	68. (b)	76. (c)
5. (d)	13. (b)	21. (c)	29. (d)	37. (d)	45. (a)	53. (a)	61. (b)	69. (b)	77. (c)
6. (b)	14. (a)	22. (a)	30. (b)	38. (b)	46. (c)	54. (c)	62. (a)	70. (d)	78. (a)
7. (b)	15. (a)	23. (d)	31. (d)	39. (c)	47. (b)	55. (d)	63. (a)	71. (d)	79. (d)
8. (c)	16. (b)	24. (b)	32. (a)	40. (a)	48. (d)	56. (a)	64. (a)	72. (d)	80. (b)
									81. (d)

SOLUTION

1. (b) $\frac{AD}{DC}=\frac{3}{2} and \frac{AB}{BC}=\frac{3}{2}$,

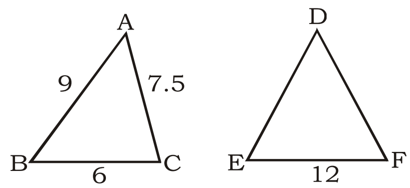
$\therefore \quad \frac{AD}{DC}=\frac{AB}{BC}$

$\therefore$ BD is the bisector of $\angle$B

Now, $\angle$CBD = 180° – (130° + 30°) = 20°

$\therefore \quad \angle B = 2(\angle CBD) = 40°$

2. (c) According to the question,



Δ ABC~ Δ DEF

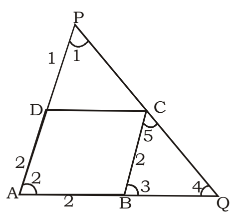
$\boxed{\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}}$

$\frac{9}{DE}=\frac{6}{12}$

DE = 18 cm

3. (a) According to question,

Given:



ABCD is a rhombus

AB = BC= CD =DA

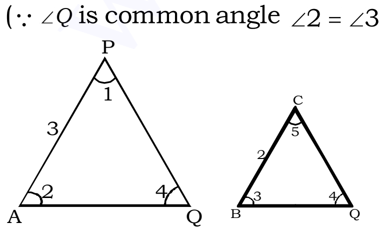
DP = $\frac{1}{2}$ AB

$\frac{DP}{AB}=\frac{1}{2}$

In a rhombus $\angle 2 = \angle 3$

$\therefore \quad \Delta APQ \sim \Delta BCQ$

($\because \angle Q$ is common angle $\angle 2 = \angle 3$)



$\frac{AP}{BC}=\frac{AQ}{BQ} \qquad \frac{AP}{BC}=\frac{3}{2}$

$\frac{AB+BQ}{BQ}=\frac{3}{2}$

($\therefore$ AQ = AB + BQ)

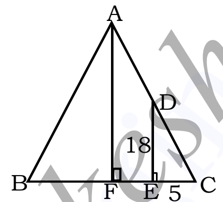
$\frac{AB}{BQ}+1=\frac{3}{2}$

$\frac{AB}{BQ}=\frac{3}{2}-1$

$\frac{AB}{BQ}=\frac{1}{2}$

$\therefore \quad \frac{BQ}{AB}=\frac{2}{1}$

4. (a) According to question



Given: DE = 18 cm

EC = 5 cm

$\tan\angle ABC = 3.6$

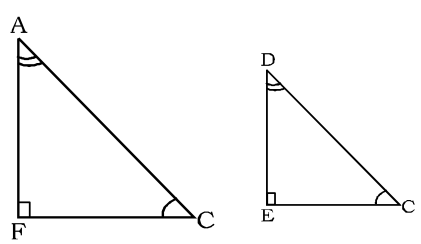
$\tan C = \frac{DE}{EC}$

$\tan C = \frac{18}{5} = 3.6$

$\therefore \tan\angle ABC = \tan\angle ACB$

Note:- In a isosceles triangle perpendicular bisects the opposite sides

$\therefore \quad \Delta AFC \sim \Delta DEC$



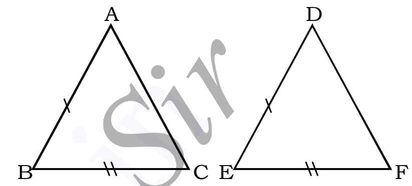
$\frac{AF}{DE}=\frac{AC}{DC}=\frac{FC}{EC}$

$\therefore \frac{AC}{CD}=\frac{FC}{EC}$ ($\therefore$ FC = $\frac{BC}{2}$)

$\frac{AC}{CD}=\frac{BC}{2EC}$

$\Rightarrow$ AC : CD = BC : 2EC

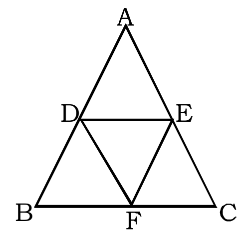
5. (d) According to question



$\angle ABC = \angle DEF$

Note: Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angles of the other triangle (SAS criterion).

6. (b) According to question



As we know that

Given: area of Δ ABC = 24 square units

D,E and F are the midpoints of AB, AC and BC

$\therefore$ Area of Δ ADE = area of Δ DBF = area of Δ DEF = area of Δ EFC

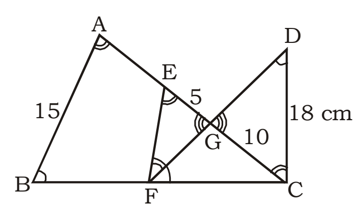
$\therefore$ Area of Δ DEF

$= \frac{1}{4}$ area of Δ ABC

Area of Δ DEF $= \frac{1}{4} \times 24$

= 6 sq. units

7. (b) According to the question,



$\angle$E = $\angle$C

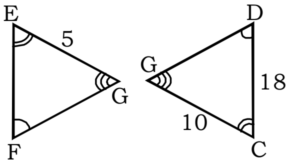
$\angle$EFG = $\angle$D

$\therefore$ Δ GEF ~ Δ GCD

$\frac{EF}{CD}=\frac{EG}{CG}$

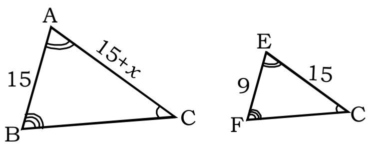
$$\frac{EF}{18} = \frac{5}{10}$$

EF = 9 cm



$\angle A = \angle E$

$\angle B = \angle F$



$\therefore$ Δ CAB ~ Δ CEF

$$\frac{AB}{EF} = \frac{AC}{EC}$$

$$\frac{15}{9} = \frac{15+x}{15}$$

$135 + 9x = 225$

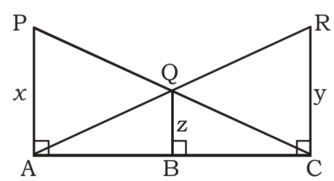
$9x = 90$

$x = 10$ cm

$\therefore$ AC = 15 + x

AC = 15 + 10 = 25 cm

8. (c)



In Δ APC & Δ BQC

$\angle PAC = \angle QBC$ (both 90°)

$\angle C = \angle C$ (common angle)

$\therefore$ $\Delta APC \sim \Delta BQC$ (AA similarity)

So,

$\frac{AC}{BC} = \frac{x}{z}$ (sides are proportional)

or $\frac{AB+BC}{BC} = \frac{x}{z}$

$$\frac{AB}{BC} + 1 = \frac{x}{z}$$

$$\frac{AB}{BC} = \frac{x}{z} - 1 = \frac{x-z}{z} \quad \text{...(i)}$$

Similarily,

Δ CRA ~ BQA

So, $\frac{AC}{AB} = \frac{y}{z}$

$$\frac{AB+BC}{AB} = \frac{y}{z}$$

$$1 + \frac{BC}{AB} = \frac{y}{z}$$

$$\frac{BC}{AB} = \frac{y}{z} - 1$$

$$\frac{BC}{AB} = \frac{y-z}{z}$$

or $\frac{AB}{BC} = \frac{z}{y-z}$ ......(ii)

From (i) & (ii)

$$\frac{x-z}{z} = \frac{z}{y-z}$$

$\therefore$ $z^2 = xy - xz - zy + z^2$

$xy = zx + zy$

(divide xyz both sides)

$$\boxed{\frac{1}{z} = \frac{1}{y} + \frac{1}{x}}$$

9. (d) According to the question,

$\angle DOC = \angle AOB$ (Common)

$\angle OCD = \angle OAB$

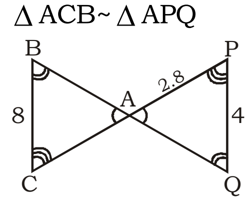
$\angle ODC = \angle OBA$

$\therefore$ Δ COD ~ Δ AOB

$$\frac{CD}{AB} = \frac{OC}{AO} = \frac{OD}{OB}$$

10. (c) According to the question,

$\Delta ACB \sim \Delta APQ$

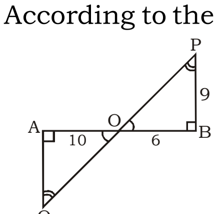


$$\frac{BC}{PQ} = \frac{CA}{AP}$$

$$\frac{8}{4} = \frac{CA}{2.8}$$

CA = 5.6 cm

11. (c) According to the question,



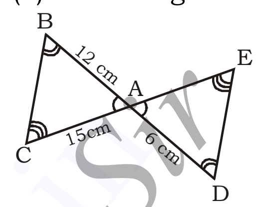
Δ OAQ ~ Δ OBP

$$\frac{AQ}{BP} = \frac{OA}{OB}$$

$$\frac{AQ}{9} = \frac{10}{6}$$

AQ = 15 cm

12. (b) According to the question,



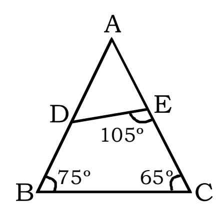
Δ ACB ~ Δ AED

$$\frac{AC}{AE} = \frac{AB}{AD}$$

$$\frac{15}{AE} = \frac{12}{6}$$

AE = 7.5 cm

13. (b) According to the question,



$\angle AED = 180° - 105° = 75°$

$\angle A = 180° - 75° - 65°$

$\angle A = 40°$

$\angle AED = \angle ABC$

$\angle ADE = \angle ACB$

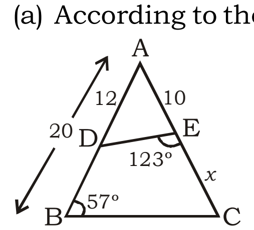
$\therefore$ Δ ABC ~ Δ ADE

$$\frac{BC}{DE} = \frac{AB}{AE}$$

$$\frac{3}{2} = \frac{AB}{10}$$

AB = 15 cm

14. (a) According to the question,



$\angle AED = 180° - 123° = 57°$

$\angle D = \angle C$

$\angle E = \angle B$

$\therefore$ Δ ABC ~ Δ AED

$$\frac{AB}{AE} = \frac{AC}{AD}$$

$$\frac{20}{10} = \frac{10+x}{12}$$

$24 = 10 + x$

$x = 14$ cm

15. (a) According to the question,

AB = BE = 5 cm

AC = 12 cm

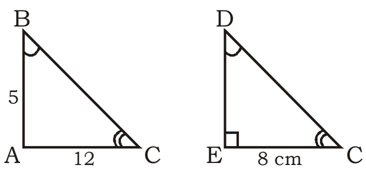
$\angle$BAC = $\angle$DEC [each 90°]

$\angle$ECD = $\angle$ACB [common]

$\therefore$ ΔABC ~ ΔEDC

In ΔABC **&** ΔEDC

By using pythagoras theorem.



$BC^2 = AB^2 + AC^2$

$BC^2 = (5)^2 + (12)^2$

$BC^2 = 25 + 144$

BC = 13 cm

$\therefore$ BC = BE + EC

EC = 13 – 5

EC = 8 cm

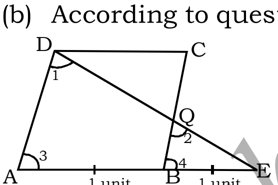
$$\therefore \frac{AB}{DE} = \frac{AC}{EC}$$

$$\frac{5}{DE} = \frac{12}{8}$$

$$DE = \frac{10}{3}$$

DE = 3.3 cm

16. (b) According to question



AD | | BC and AB | | DC

Point B is the midpoint of AE

$\angle 1 = \angle 2$

(Corresponding Angle)

$\angle 3 = \angle 4$

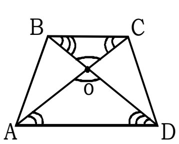
(Corresponding Angle)

$\therefore$ $\Delta EQB \sim \Delta EDA$

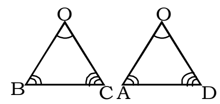
$$\therefore \frac{EB}{EA} = \frac{EQ}{ED} = \frac{QB}{AD} = \frac{1}{2}$$

QC : QB = 1 : 1

17. (d) According to question,



$\Delta AOD \sim \Delta COB$



$$\therefore \frac{OB}{OD} = \frac{OC}{OA}$$

$$\frac{3x-19}{x-5} = \frac{x-3}{3}$$

$9x - 57 = x^2 - 8x + 15$

$x^2 - 17x + 72 = 0$

$x(x-8) - 9(x-8) = 0$

$(x-8)(x-9) = 0$

$x = 8$ or 9

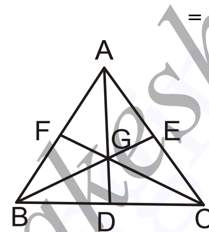
18. (a) $\dfrac{AB}{QR} = \dfrac{PB}{PR}$

$$\frac{4}{8} = \frac{PB}{5} \Rightarrow PB = 2.5cm$$

19. (b) Required ratio = $\sqrt{\dfrac{9}{16}} = \dfrac{3}{4}$

= 3 : 4

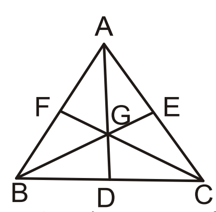
20. (a)



Area of $\Delta BDG = \dfrac{1}{6} \times$ Area of Δ ABC

$$= \frac{1}{6} \times 72 = 12cm^2$$

21. (c)

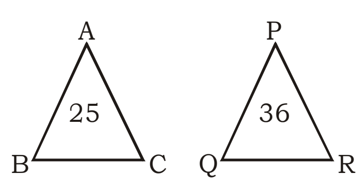


Required area of ▭ BDGF

$$= \frac{1}{3} \times 60 = 20 \text{ sq.cm.}$$

22. (a) According to the question,

As we know that

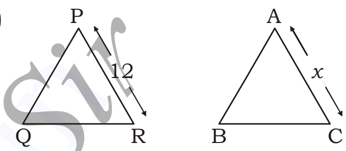


$$\frac{AB^2}{PQ^2} = \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR}$$

$$\frac{AB}{PQ} = \sqrt{\frac{25}{36}}$$

$$\frac{AB}{PQ} = \frac{5}{6}$$

23. (d)



$\because$ Δ PQR ~ Δ ABC

$\Rightarrow$ We know that in similar triangle

$$\Rightarrow \frac{\text{Area of triangle}_1}{\text{Area of triangle}_2} = \frac{(\text{Corresponding side}_1)^2}{(\text{Corresponding side}_2)^2}$$

$$\Rightarrow \frac{\Delta PQR}{\Delta ABC} = \frac{256}{441}$$

$$\Rightarrow \frac{\Delta PQR}{\Delta ABC} = \frac{(PR)^2}{(AC)^2}$$

$$\Rightarrow \frac{256}{441} = \frac{(12)^2}{(AC)^2}$$

$$\Rightarrow \left(\frac{16}{21}\right)^2 = \left(\frac{12}{AC}\right)^2$$

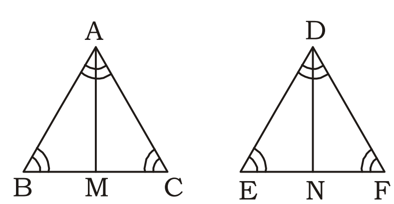
$$\Rightarrow \frac{16}{21} = \frac{12}{AC}$$

$$\Rightarrow \frac{4}{21} = \frac{3}{AC}$$

$$\Rightarrow AC = \frac{63}{4}$$

$\Rightarrow$ AC = 15.75 cm.

24. (b)



If two isosceles triangles have equal vertical angles then both triangle is similar

So, Δ ABC ~ Δ DEF

We know,

In similarity case

$\Rightarrow \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF}$

$= \dfrac{(AB)^2}{(DE)^2}$ corresponding sides square

$= \dfrac{(AM)^2}{(DN)^2}$ height

$\Rightarrow \dfrac{9}{16} = \dfrac{(AM)^2}{(DN)^2}$

$= \sqrt{\dfrac{9}{16}}$ = Ratio of their height

$\Rightarrow$ Ratio of height = 3:4

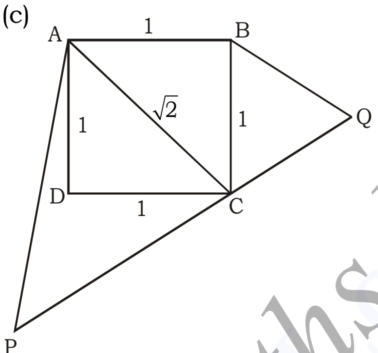
25. (b) $\dfrac{\text{area of triangle}_1}{\text{area of triangle}_2} = \dfrac{3}{2}$

$$\frac{\frac{1}{2} \times B_1 \times 4}{\frac{1}{2} \times B_2 \times 5} = \frac{3}{2}$$

$$\frac{B_1}{B_2} = \frac{3}{2} \times \frac{5}{4} = \frac{15}{8}$$

$B_1 : B_2 = 15 : 8$ Ans.

26. (c)



Given

$\because \quad \Delta QBC \sim \Delta PAC$

$\Rightarrow$ Let each side of square =1

$\Rightarrow$ then diagonal of square = $\sqrt{2}$

$\Rightarrow \because \Delta QBC \sim \Delta PAC$

$\Rightarrow \dfrac{\text{Area of } \Delta QBC}{\text{Area of } \Delta PAC} = \dfrac{(BC)^2}{(AC)^2}$

$= \dfrac{(QC)^2}{(PC)^2} = \dfrac{(QB)^2}{(PA)^2} = \dfrac{1^2}{(\sqrt{2})^2} = \dfrac{1}{2}$

27. (d) In Δ ABC and Δ BDC

$\angle BAC = \angle BCD$ (given)

and $\angle B = \angle B$ (common)

$\therefore \Delta ABC \sim \Delta CBD$

$\dfrac{AB}{BC} = \dfrac{BC}{BD} \Rightarrow \dfrac{32}{BC} = \dfrac{BC}{18}$

$\Rightarrow BC^2 = 18 \times 32$

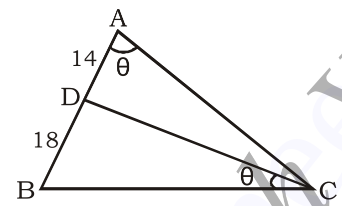
$\Rightarrow$ BC = 24cm

$\therefore$ perimeter of Δ BCD : perimeter of Δ ABC = BC : AB

= 24 : 32

= 3 : 4

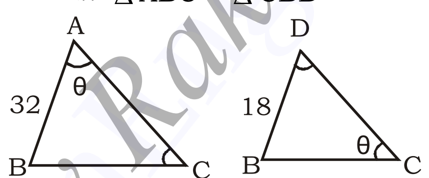
28. (a) According to the question,



Let $\angle BAC = \angle BCD = \theta$

$\angle B$ is common in both triangle

$\therefore \Delta ABC \sim \Delta CBD$



$\dfrac{BC}{AB} = \dfrac{BD}{BC}$

$BC^2 = AB \times BD$

$BC^2 = 32 \times 18$

$BC^2 = 576$

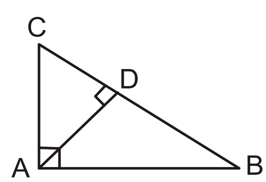
BC = 24 cm

$\Rightarrow \dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta BDC} = \dfrac{BC}{BD}$

$= \dfrac{AB}{BC} = \dfrac{AC}{CD}$

$\Rightarrow = \dfrac{24}{18} = \dfrac{4}{3}$

29. (d)



In Δ ABC

By using pythagoras theorem

$AC^2 + AB^2 = BC^2$

$(2AB)^2 + AB^2 = BC^2$

$5AB^2 = BC^2$

$BC = \sqrt{5}\,AB$

$AB = \dfrac{BC}{\sqrt{5}}$

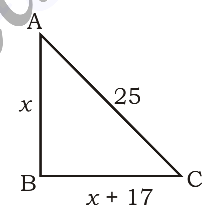
$AB^2 = BD \times BC$

$\left(\dfrac{BC}{\sqrt{5}}\right)^2 = BD \times BC$

$BD = \dfrac{BC}{5}$

30. (b) Let the altitude of a triangle = x

According to the question,



$AC^2 = AB^2 + BC^2$

$(25)^2 = x^2 + (x+17)^2$

$625 = x^2 + x^2 + 289 + 34x$

$2x^2 + 34x - 336 = 0$

$x^2 + 17x - 168 = 0$

$x^2 + 24x - 7x - 168 = 0$

$x(x + 24) - 7(x + 24) = 0$

$(x + 24)(x - 7) = 0$

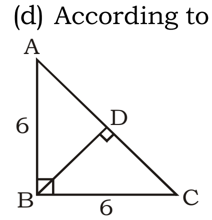
$x = 7$

$\therefore$ Sum of altitude and base

$= x + x + 17$

= 7 +7 +17 = 31 cm

31. (d) According to the question,



AB = BC = 6 cm

By using pythagoras theorem.

$AC^2 = BC^2 + AB^2$

$AC^2 = 6^2 + 6^2$

$AC^2 = 72$

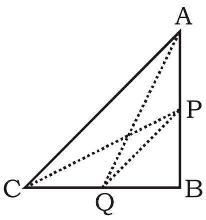
$AC = 6\sqrt{2}$

$\therefore BD = \dfrac{AB \times BC}{AC}$

$BD = \dfrac{6 \times 6}{6\sqrt{2}}$

$BD = 3\sqrt{2}$ cm

32. (a) According to the question,
By using Pythagoras theorem,



In Δ AQB,
$AQ^2 = BQ^2 + AB^2$ ...(i)

In Δ PCB
$PC^2 = BC^2 + BP^2$ ...(ii)

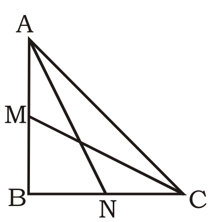
In Δ ACB
$AC^2 = BC^2 + AB^2$ ...(iii)

In Δ PQB
$PQ^2 = BQ^2 + BP^2$ ...(iv)

Adding eq. (i), (ii), (iii) & (iv)
$AQ^2 + PC^2 + AC^2 + PQ^2 = BQ^2 + AB^2 + BC^2 + BP^2 + BC^2 + AB^2 + BQ^2 + BP^2$
$AQ^2 + PC^2 + AC^2 + PQ^2 = 2(BQ^2 + AB^2 + BC^2 + BP^2)$
$AQ^2 + PC^2 + AC^2 + PQ^2 = 2(PQ^2 + AC^2)$

$\boxed{AQ^2 + CP^2 = PQ^2 + AC^2}$

33. (c) According to the question,
By using Pythagoras theorem,



In Δ ABN,
$AN^2 = AB^2 + BN^2$ ...(i)

In Δ MBC,
$MC^2 = BM^2 + BC^2$ ...(ii)

In Δ ABC,
$AC^2 = AB^2 + BC^2$ ...(iii)

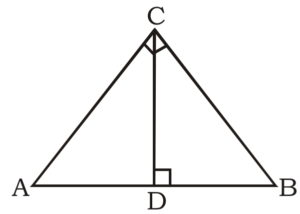
Adding eq (i) and (ii)
$AN^2 + CM^2 = AB^2 + BN^2 + BM^2 + BC^2$
$AN^2 + CM^2 = AC^2 + BN^2 + BM^2$

$$AN^2 + CM^2 = AC^2 + \frac{BC^2}{4} + \frac{AB^2}{4}$$

$$AN^2 + CM^2 = \frac{4AC^2 + AC^2}{4}$$

$\boxed{4\left(AN^2 + CM^2\right) = 5AC^2}$

34. (b) According to the question,



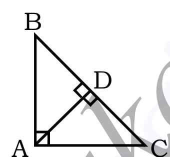
$\angle C = 90°$
BC = 15 cm
AC = 20 cm
By using pythagoras theorem,
$AB^2 = AC^2 + BC^2$
$AB^2 = (20)^2 + (15)^2$
$AB^2 = 400 + 225$
AB = 25 cm
As we know that,

$$CD = \frac{BC \times AC}{AB}$$

$$CD = \frac{20 \times 15}{25}$$

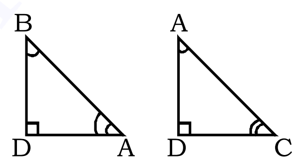
CD = 12 cm

35. (a) According to the question,



$\therefore \angle DAB = \angle DCA$
$\angle DBA = \angle DAC$
$\therefore \Delta ADB \sim \Delta CDA$

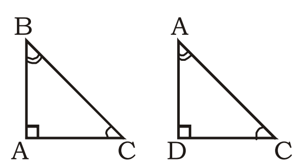
$$\frac{AD}{CD} = \frac{AB}{AC} = \frac{BD}{AD}$$



$AD^2 = BD \times CD$ ...(i)

$\Delta BAC \sim \Delta ADC$

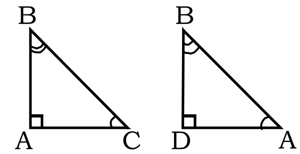
$$\frac{AB}{AD} = \frac{BC}{AC} = \frac{AC}{CD}$$



$AC^2 = BC \times CD$ ...(ii)

$\Delta BAC \sim \Delta BDA$

$$\frac{BA}{BD} = \frac{BC}{BA} = \frac{AC}{AD}$$



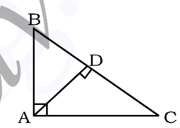
$AB^2 = BC \times BD$ ..(iii)

eq. (iii) divide by eq. (ii)

$$\frac{AB^2}{AC^2} = \frac{BC \times BD}{BC \times CD}$$

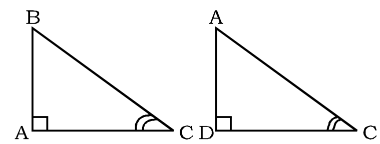
$$\boxed{\frac{BD}{CD} = \left(\frac{AB}{AC}\right)^2}$$

36. (b) According to Question
Given: AC = 9cm



area of ΔABC = 40 cm²
area of ΔADC = 10 cm²

$\Delta ABC \sim \Delta DAC$



$$\frac{area\ of\ \Delta ABC}{area\ of\ \Delta ADC} = \frac{AB^2}{AD^2} = \frac{BC^2}{AC^2} = \frac{AC^2}{DC^2}$$

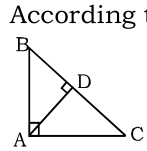
(In similar Δ ratio of their area is square of ratio of corresponding sides)

$$\frac{40}{10} = \frac{BC^2}{(9)^2}$$

$$\frac{40}{10} \times 81 = BC^2$$

BC = 18 cm

37. (d) According to Question



Given: BAC is a right angle triangled

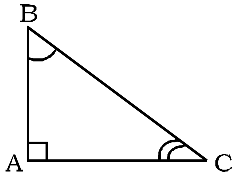
$AD \perp BC$
AD = 6 cm
BD =4 cm
BC = ?

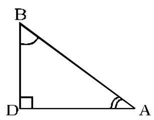
In ΔBAD

$$AB = \sqrt{BD^2 + AD^2}$$

$$AB = \sqrt{4^2 + 6^2} = \sqrt{52} \text{ cm}$$

$\Delta BAC \sim \Delta BDA$





$\therefore \frac{BC}{AB} = \frac{AB}{BD}$

$\therefore \frac{BC}{\sqrt{52}} = \frac{\sqrt{52}}{4}$

$BC = \frac{52}{4}$

BC = 13 cm

Alternate:-

$AB^2 = BD.BC$

$\left(\sqrt{BD^2+AD^2}\right)^2 = BD.BC$

$\left(\sqrt{4^2+6^2}\right)^2 = 4.BC$

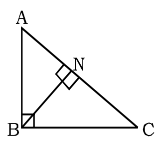
$\frac{52}{4} = BC,$

$\therefore BC = 13$ cm

38. (b) According to question

Given: $\angle ABC = 90°$

$\frac{AN}{NC} = ?$

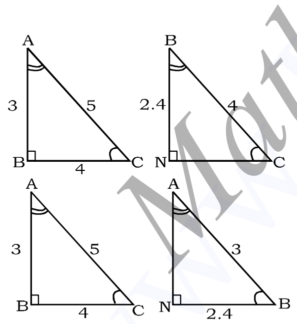


$\Delta ABC \sim \Delta BNC$

$\Delta ABC \sim \Delta ANB$

$\therefore \Delta ABC \sim \Delta BNC \sim \Delta ANB$

AB = 3, BC = 4, AC = 5



$BN = \frac{AB \times BC}{AC} = \frac{3 \times 4}{5} = 2.4$

$\frac{BC}{NC} = \frac{AB}{NB}$

$\frac{4}{NC} = \frac{3}{2.4}$

NC = 3.2

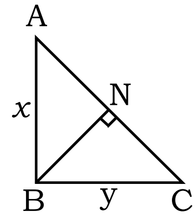
$\frac{AB}{AN} = \frac{BC}{NB} \qquad \frac{3}{AN} = \frac{4}{2.4}$

AN = 1.8

$\therefore \frac{AN}{NC} = \frac{1.8}{3.2} = \frac{9}{16}$

Alternate:-

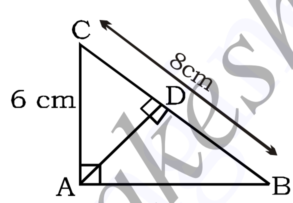
In such cases use the following method to save your valuable time.



$\frac{AN}{NC} = \frac{x^2}{y^2} = \frac{3^2}{4^2} = \frac{9}{16}$

$\Rightarrow$ AN : NC = 9 : 16

39. (c) According to question



$\Delta CAB \sim \Delta CDA$

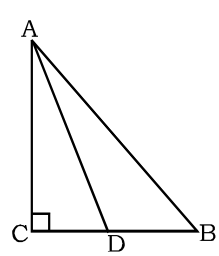
$\therefore \frac{area\ of\ \Delta CAB}{area\ of\ \Delta CDA} = \frac{BC^2}{AC^2}$

$\frac{area\ of\ \Delta CAB}{area\ of\ \Delta CDA} = \frac{8^2}{6^2}$

$\frac{area\ of\ \Delta CAB}{area\ of\ \Delta CDA} = \frac{64}{36}$

$\frac{area\ of\ \Delta CAB}{area\ of\ \Delta CDA} = \frac{16}{9}$

40. (a) According to question



In ΔABC

$AB^2 = AC^2 + BC^2$ ....(i)

In ΔACD

$AD^2 = AC^2 + CD^2$

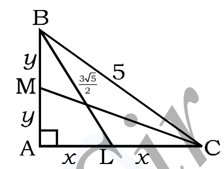
$AC^2 = AD^2 - CD^2$ ....(ii)

Put the value of AC^2 in equation (i)

$AB^2 = AD^2 - CD^2 + BC^2$

$AB^2 + CD^2 = AD^2 + BC^2$

41. (a) According to question



According to figure, when two medians intersect each other in a right angled triangle then we use, this equation.

$\Rightarrow 4\,(BL^2 + CM^2) = 5BC^2$

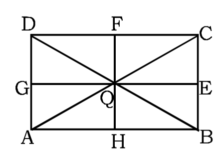
$\Rightarrow 4 \times \left(\frac{3\sqrt{5}}{2}\right)^2 + 4CM^2 = 5(5)^2$

$\Rightarrow 45 + 4CM^2 = 125$

$\Rightarrow CM^2 = \frac{125-45}{4} = 20$

$\Rightarrow CM = 2\sqrt{5}$ cm

42. (a) According to question



Given:

QA = 3 cm

QB = 4 cm

QC = 5 cm

QD = ?

As we know that

$QD^2 + QB^2 = QA^2 + QC^2$

(By using Pythagoras theorem)

$QD^2 + (4)^2 = (3)^2 + (5)^2$

$QD^2 + 16 = 9 + 25$

$QD^2 = 34 - 16$

$QD^2 = 18$

$QD = \sqrt{18}$,

$QD = 3\sqrt{2}$

43. (c) BC || DE

$\therefore \angle ABC = \angle ADE$

$\angle ACB = \angle AED$ and $\angle BAC = \angle EAD$

$\therefore \Delta ABC \sim \Delta AED$

$\frac{AB}{AD} = \frac{AC}{AE} \Rightarrow \frac{18}{9} = \frac{11}{AE}$

$\Rightarrow AE = 5.5$cm

44. (a) AB = AP + BP

$\Rightarrow$ AP + 3AP = 4AP

$\Rightarrow \frac{AB}{AP} = \frac{4}{1}$ &

AC = AQ + QC

= AQ + 3AQ

= 4AQ

$\Rightarrow \frac{AC}{AQ} = \frac{4}{1}$

In ΔABC and ΔAPQ

$\angle$BAC = $\angle$PAQ (common) and

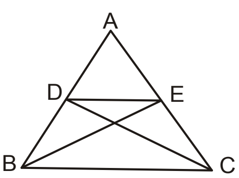
$\frac{AB}{AP} = \frac{AC}{AQ}$

$\therefore \Delta$ ABC ~ Δ APQ

$\therefore \frac{AB}{AP} = \frac{AC}{AQ} = \frac{BC}{PQ} = 4$

$\Rightarrow \frac{BC}{PQ} = \frac{4}{1} \Rightarrow \frac{36}{PQ} = 4 \Rightarrow PQ = 9cm$

45. (a)



ΔDBC & ΔEBC lie on the same base BC and between same parallel lines .

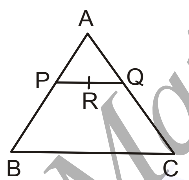
$\therefore$ ar (ΔDBC) = ar (ΔEBC)

$\Rightarrow$ ar (ΔABE) = ar (ΔACD)

$\Rightarrow$ ar (ΔACD) = ar (ΔABE)

= 36sq.cm

46. (c)



$\frac{PR}{RQ} = \frac{1}{2} \Rightarrow \frac{2}{RQ} = \frac{1}{2} \Rightarrow$ RQ = 4

$\therefore$ PQ = PR + RQ

= 2 + 4 = 6cm

The line joining the mid-points of two sides of a triangle is parallel to and half of the third side.

BC = 2PQ

= 2 × 6

= 12cm.

47. (b) For the given perimeter of a triangle the maximum area is enclosed by an equilateral triangle.

$\therefore$ 3a = 24 cm $\Rightarrow$ a = 8cm

$\therefore$ Area = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\times(8)^2$

$= 16\sqrt{3}cm^2$

48.(d) XY || AC (given)

$\therefore$ $\angle$BXY = $\angle$A and

$\angle$BYX = $\angle$C

$\therefore$ ΔABC ~ ΔXBY

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta XBY)} = \left(\frac{AB}{XB}\right)^2$ ....(i)

Also,

ar(ΔABC) = 2 ar(ΔXYB) (given)

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta XBY)} = 2$ ....(ii)

therefore from (i) & (ii)

$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1}$

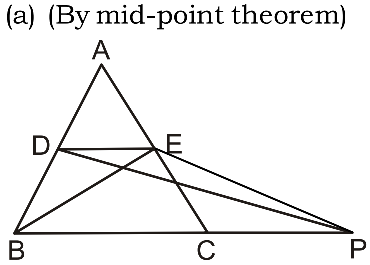
$\frac{AB}{XB} = \frac{\sqrt{2}}{1}$

or $\frac{XB}{AB} = \frac{1}{\sqrt{2}}$ or $1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{AB - XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

$\Rightarrow \frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

49. (a) (By mid-point theorem)



DE || BC

DE = $\frac{1}{2}BC$

$\Rightarrow$ ar (ΔBDE)

$= \frac{1}{4}\times ar(\Delta ABC)$

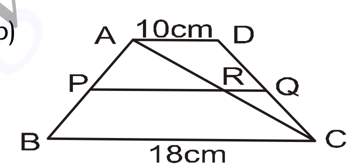
and ΔBDE = ΔPED

[$\because$ both triangles lie on the same base DE and between two parallel lines DE and BP]

$\therefore$ ar (ΔPED)

$= \frac{1}{4}\times ar(\Delta ABC)$

50. (b)



Now In ΔAPR and ΔABC

$\angle$APR = $\angle$ABC ($\because$ PQ || AD || BC)

and $\angle$ARP = $\angle$ACB ($\because$ PQ || BC)

$\therefore$ ΔAPR ~ ΔABC

$\therefore$

$\frac{AP}{AB} = \frac{PR}{BC} \Rightarrow PR = \frac{AP}{AB}\times BC$

$= \frac{AP}{AP+PB}\times BC$

$\Rightarrow PR = \frac{5}{8}\times 18 = \frac{45}{4}cm$

and $\frac{AP}{PB} = \frac{AR}{RC} = \frac{5}{3}$

similarly, ΔRCQ ~ ΔCAD

$\therefore \frac{RQ}{AD} = \frac{RC}{AC}$

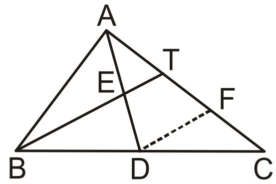
$\Rightarrow$ RQ $= \frac{RC}{AR+RC}\times AD$

$= \frac{3}{8}\times 10 = \frac{15}{4}$ cm

$\therefore$ PQ = PR + RQ $= \frac{45}{4} + \frac{15}{4}$

= 15cm

51. (c)



Draw DF | | ET

In ΔADF, E is the mid-point of AD and DF | | ET

$\therefore$ T will be the mid-point of AF.

i.e. AT = TF ....(i)

Now, In ΔBTC,

D is the mid-point of BC & DF | | ET | | BT

$\therefore$ F will be the mid-point of TC

i.e TF = FC ....(ii)

$\therefore$ from (i) and (ii)

AT = TF = FC

$\therefore$ $AC = 15cm, AT = TF = FC = \frac{AC}{3}$

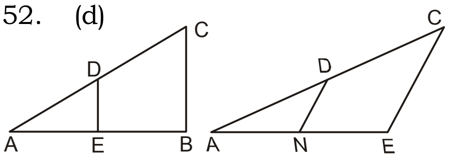
= 5cm

$\therefore$ TC = TF + FC

= 5 + 5

= 10cm

52. (d)



In ΔABC,

$\because$ DE | | BC

$\therefore \frac{AD}{DC} = \frac{AE}{EB} = \frac{4}{5} = 4:5$ ......(i)

In ΔAEC

EC | | ND

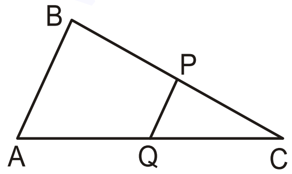
$\therefore \frac{AN}{NE} = \frac{AD}{DC} = \frac{4}{5} = 4:5$ .....(ii)

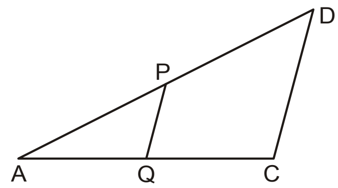
Let AE = 40 $\therefore$ EB = 50 and

$\therefore$ EN = $40 \times \frac{5}{9} = \frac{200}{9}$

EN : EB $= \frac{200}{9} : 50 = 4:9$

53. (a)





PQ | | AB

$\therefore$ $\Delta ABC \sim \Delta PQC$

$\frac{AB}{PQ} = \frac{AC}{QC}$

$\Rightarrow PQ = \frac{AB}{AC} \times QC$ ....(i)

$\because$ PQ| |CD

$\therefore$ $\Delta ACD \sim \Delta AQP$

$\therefore \frac{CD}{PQ} = \frac{AC}{AQ}$

$\Rightarrow PQ = \frac{AQ}{AC} \times CD$ ....(ii)

from (i) & (ii) -

$\frac{AQ}{AC} \times CD = \frac{AB}{AC} \times QC$

$\Rightarrow AQ \times CD = AB \times QC$

$\Rightarrow (AC - QC) \times 18 = 12 \times QC$

$\Rightarrow (6 - QC) \times 18 = 12 \times QC$

$\Rightarrow (6 - QC) \times 3 = 2QC$

$\Rightarrow 5QC = 18 \Rightarrow QC = \frac{18}{5}$

$\therefore$ from (i)

$$PQ = \frac{12}{6} \times \frac{18}{5} = \frac{36}{5} cm$$

Alternatively

Let QC = $x \Rightarrow$ AQ = $6 - x$

In ΔABC and triangle PQC

$\frac{PQ}{AB} = \frac{QC}{AC}$

$\frac{PQ}{12} = \frac{x}{6} \Rightarrow PQ = 2x$.......(i)

In ΔACD and Triangle AQP

$\frac{PQ}{CD} = \frac{AQ}{AC}$

$\frac{PQ}{18} = \frac{6 - x}{6}$

$\Rightarrow$ PQ = $18 - 3x$ ....(ii)

From (i) and (ii),

$2x = 18 - 3x$

$x = \frac{18}{5}$

$\therefore$ From (i), PQ = $2x = 2 \times \frac{18}{5} = \frac{36}{5} cm$

Alternate

When AB| | PQ | | CD

$\frac{1}{PQ} = \frac{1}{AB} + \frac{1}{CD}$

$= \frac{1}{12} + \frac{1}{18} = \frac{5}{36}$

$PQ = \frac{36}{5}$

54. (c) In ΔABM and ΔBEC

$\angle BAM = \angle BCE$

$\angle BMA = \angle BEC$

($\because$ AM || EC)

$\therefore$ $\Delta ABM \sim \Delta BEC$

$\therefore \frac{AB}{BC} = \frac{AM}{EC} \Rightarrow \frac{5}{10} = \frac{AM}{18}$

$\Rightarrow$ AM = 9 cm

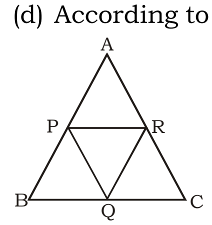
$\because$ AM || DN

$\therefore$ $\Delta AMC \sim \Delta DNC$

$\therefore \frac{DN}{AM} = \frac{DC}{AC} \Rightarrow \frac{15}{9} = \frac{DC}{15}$

$\Rightarrow$ $DC = \frac{15 \times 15}{9} = 25cm$

55. (d) According to the question,



Area of ΔPQR = 6 cm²

There are 4 congruent triangles.

$\Delta APR \sim \Delta BPQ \sim \Delta PQR \sim \Delta RQC$

Hence area of ΔABC = 4× area of ΔPQR = 4 × 6 =24 cm²

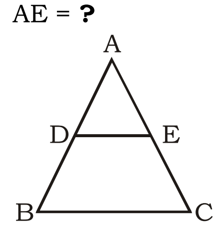
56. (a) According to the question,

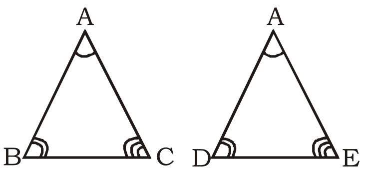
AB = 3.6 cm

AC = 2.4 cm

AD = 2.1 cm

AE = **?**





$\Delta ABC \sim \Delta ADE$

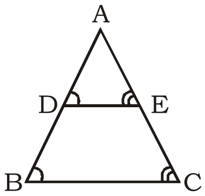
$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\frac{3.6}{2.1} = \frac{2.4}{AE}$$

$$AE = \frac{2.1 \times 2.4}{3.6}$$

AE = 1.4 cm

57. (c) According to the question,



AC = 5 cm
AD = 3 cm
BD = 7 cm
By using midpoint theorem,

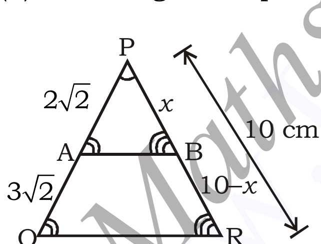
$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\frac{3}{10} = \frac{AE}{5}$$

AE = 1.5 cm

58. (b) According to the question,

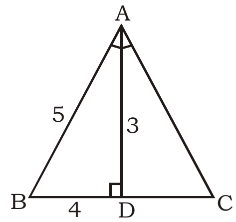


$\angle PAB = \angle PQR$
$\angle PBA = \angle PRQ$
$\therefore\ \Delta PAB \sim \Delta PQR$

$$\frac{x}{10} = \frac{2\sqrt{2}}{5\sqrt{2}}$$

$x = 4$ cm
$\therefore$ BR = $10 - x = 10 - 4 = 6$ cm

59. (c) According to the question,
As we know that,



$$\frac{1}{AD^2} = \frac{1}{AB^2} + \frac{1}{AC^2}$$

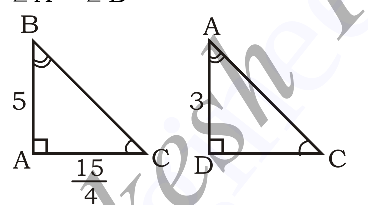
$$\frac{1}{(3)^2} = \frac{1}{(5)^2} + \frac{1}{(AC)^2}$$

$$\frac{1}{AC^2} = \frac{1}{9} - \frac{1}{25}$$

$$\frac{1}{AC^2} = \frac{25-9}{225}$$

$$AC = \frac{15}{4}$$

In ΔBAC and ΔADC
$\angle B = \angle A$
$\angle A = \angle D$



$\therefore\ \Delta CAB \sim \Delta CDA$

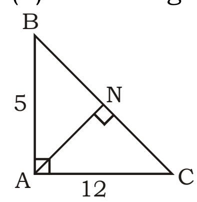
$$\frac{AC}{DC} = \frac{AB}{AD}$$

$$\frac{15}{4DC} = \frac{5}{3}$$

$$DC = \frac{9}{4}$$

DC = 2.25

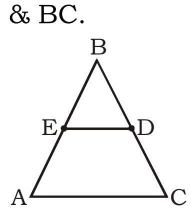
60. (b) According to the question,



$$\frac{\text{Area of } \Delta ANB}{\text{Area of } \Delta ANC} = \frac{AB^2}{AC^2} = \frac{(5)^2}{(12)^2}$$

$$= \frac{25}{144}$$

61. (b) According to the question,
E & D are the midpoint of AB & BC.

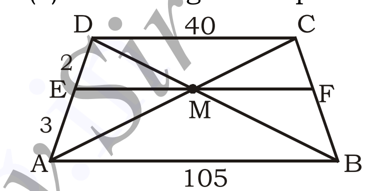


$$\therefore\ \frac{BE}{BA} = \frac{BD}{BC} = \frac{ED}{AC} = \frac{1}{2}$$

$$\therefore\ \frac{\text{Area of } \Delta BED}{\text{Area of } \Delta BAC} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

4 units = 30
1 unit = 7.5
$\therefore$ Area of $\Delta BED = 7.5\ cm^2$

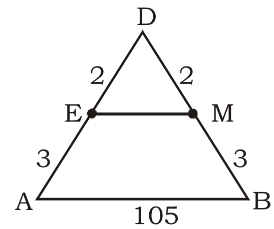
62. (a) According to the question,



AB = 105 cm
DC = 40 cm

$$\frac{DE}{EA} = \frac{2}{3}$$

In ΔADB, E & M are the mid points of AD & BD.
By using B.P.T. theorem.



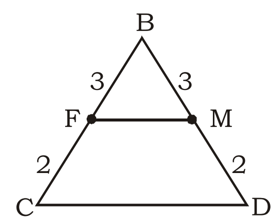
$$\frac{DE}{EA} = \frac{DM}{MB} = \frac{2}{3}$$

$$\frac{DE}{DA} = \frac{DM}{DB} = \frac{EM}{AB}$$

$$\frac{2}{5} = \frac{EM}{105}$$

EM = 42 cm

In ΔBDC, M & F are the mid points of BC & BD
By using B.P.T theorem



$$\frac{BF}{BC} = \frac{FM}{CD}$$

$$\frac{3}{5} = \frac{FM}{40}$$

FM = 24 cm

$\therefore$ EF = EM + FM

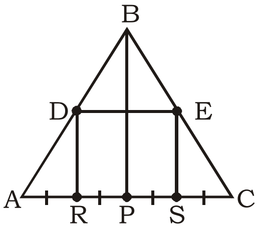
EF = 24 + 42 = 66 cm

63. (a) Let ES = x

According to the question,

In Δ BPC,

E is the midpoint of BC and S is the midpoint of PC.



$\therefore$ By using midpoint theorem.

$ES = \frac{1}{2}BP$

$BP = 2x$

In Δ BPA,

D is the midpoint of AB and R is the midpoint of AP

By using midpoint theorem,

$DR = \frac{1}{2} \times BP$

$DR = \frac{1}{2} \times 2x$

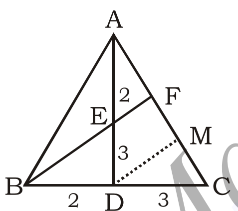
$DR = x$

$\therefore$ ES = DR

$\frac{ES}{DR} = \frac{x}{x} = \frac{1}{1}$

64. (a) According to the question,

Let draw a imaginary line DM which is parallel to EF.



In Δ ADM.

By using B.P.T.

$\frac{AE}{ED} = \frac{AF}{FM} = \frac{2}{3}$

In Δ CBF

By using B.P.T.

$\frac{CD}{DB} = \frac{CM}{FM} = \frac{3}{2}$

$\therefore$ to make FM same

AF	:	FM	:	MC
$2_{\times 2} = 4$		$3_{\times 2} = 6$		
		$2_{\times 3} = 6$		$3_{\times 3} = 9$
4	:	6	:	9

$\therefore$ Length of AC = AF + FM + MC

= 4 + 6 + 9 = 19 units

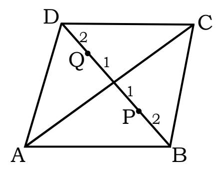
19 units = 760

$1 \text{ unit} = \frac{760}{19}$

$2 \text{ units} = \frac{760}{19} \times 2 = 80 \text{ cm}$

$\therefore$ AE = 2 units = 80 cm

65. (b) According to question,



Given: BD = 18 cm

Note: Centroid is the point where medians intersects and it divides median in 2 : 1

BD = 6 units, PQ = 2 units

6 units = 18 cm

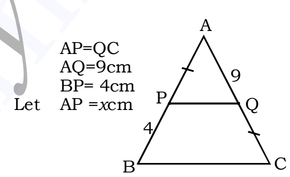
$1 \text{ unit} = \frac{18}{6} = 3$

2 units = 3 × 2 = 6

$\therefore$ PQ = 6 cm

66. (c) According to Question

Given:



$\Delta APQ \sim \Delta ABC$

To apply similarity property

$\frac{AP}{BP} = \frac{AQ}{QC}$

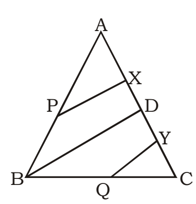
$\frac{x}{4} = \frac{9}{x}$

$x^2 = 36$

$x = 6$

$\therefore$ AP = 6 cm

67. (b) According to question



PX | | BD [mid point theorem]

$\therefore$ $PX = \frac{1}{2}BD$

Similarly,QY | | BD

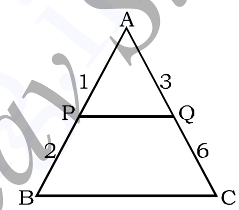
$\therefore$ $QY = \frac{1}{2}BD$

$\therefore$ $PX : QY = \frac{1}{2}BD : \frac{1}{2}BD$

PX : QY = 1 : 1

68. (b) According to question

Given:



$\frac{AP}{PB} = \frac{1}{2}$

AQ = 3

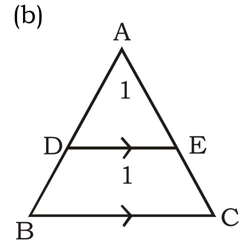
To apply similar triangle property.

$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$

$\frac{1}{3} = \frac{3}{AC}$

AC = 9 cm

69. (b)



ar (Δ ADE) = ar DECB

So, ar Δ ADE= 1 unit^2 and ar ABC = 2unit^2

$\frac{ar\Delta ADE}{ar\Delta ABC} = \frac{AD^2}{AB^2}$

$\frac{1}{2} = \left(\frac{AD}{AB}\right)^2$

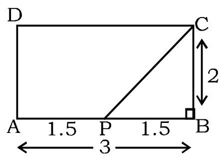
$\frac{1}{\sqrt{2}} = \frac{AD}{AB}$

$\therefore \frac{AD}{DB} = \frac{1}{\sqrt{2}-1}$

$\left(\therefore DB = AB - AD = \sqrt{2}-1\right)$

So, AD : BD = 1 : $\sqrt{2}$ - 1

70. (d) According to question



In $\triangle CBP$

$CP^2 = BP^2 + BC^2$

$CP^2 = (1.5)^2 + (2)^2$

$CP^2 = 2.25 + 4$

$CP^2 = 6.25$

$CP = \sqrt{6.25}$

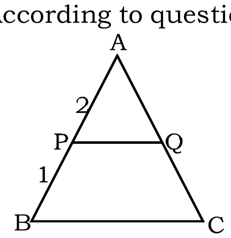
$CP = 2.5$

$\therefore \ \text{Sin} \angle CPB = \frac{BC}{CP}$

$\text{Sin} \angle CPB = \frac{2}{2.5}$

$\text{Sin} \angle CPB = \frac{4}{5}$

71. (d) According to question



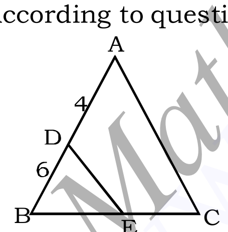
Given: $\frac{AB}{PB} = \frac{3}{1}$

To apply B.P.T

$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$

$\frac{PQ}{BC} = \frac{2}{3}$

72. (d) According to question



Given: AB = 10 cm

AD = 4 cm

DE||AC

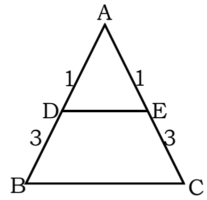
$\triangle ABC \sim \triangle DBE$

$\therefore \frac{BD}{AD} = \frac{BE}{CE}$

$\frac{BE}{CE} = \frac{6}{4}$

$\frac{BE}{CE} = \frac{3}{2}$

73. (c) According to question



By using B.P.T

$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

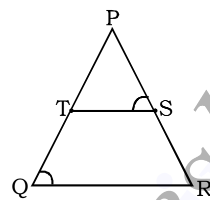
$\frac{AD}{AB} = \frac{DE}{BC}$,

$\frac{1}{4} = \frac{DE}{12}$,

DE = 3 cm

74. (c) According to question

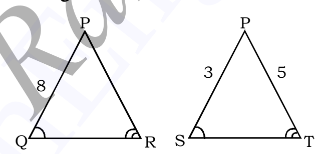
Given:



PT = 5 cm, PS = 3 cm

TQ = 3 cm, SR = ?

$\triangle PQR \sim \triangle PST$



$\frac{PR}{PT} = \frac{PQ}{PS} \Rightarrow \frac{PR}{5} = \frac{8}{3}$

$PR = \frac{40}{3}$

$\therefore$ SR = PR – PS

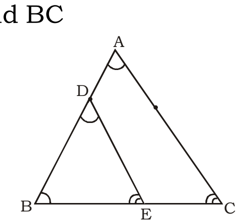
$SR = \frac{40}{3} - 3$

$SR = \frac{40-9}{3}$, $SR = \frac{31}{3}$ cm

75. (c) According to question

Given:

'D' and 'E' are the points on AB and BC



AC||DE

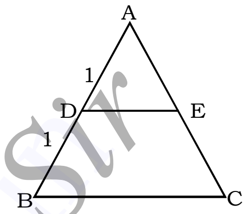
$\angle D = \angle A$

$\angle E = \angle C$

$\therefore \ \triangle BDE \sim \triangle BAC$

76. (c) According to question

Given:



AB = 2AD

$\frac{AB}{AD} = \frac{2}{1}$

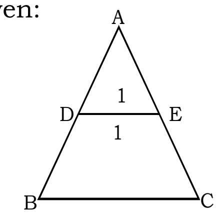
By applying B.P.T

$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$

$\frac{DE}{BC} = \frac{1}{2}$

77. (c) According to question

Given:



DE||BC

Area of $\triangle$ ABC = 2 (Area of $\triangle$ ADE)

$\therefore \ \angle E = \angle C$

$\angle D = \angle B$

$\therefore \ \triangle ABC \sim \triangle ADE$

$\therefore \ \frac{\text{area of } \triangle ABC}{\text{area of } \triangle ADE}$

$= \frac{AB^2}{AD^2} = \frac{AC^2}{AE^2} = \frac{BC^2}{DE^2}$

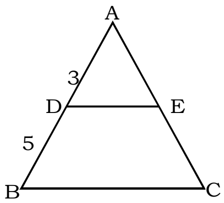
$\Rightarrow \frac{2}{1} = \frac{AB^2}{AD^2}$

$\therefore \ \frac{AB}{AD} = \frac{\sqrt{2}}{1}$

$\therefore$ BD = $\sqrt{2} - 1$

$\therefore \ \frac{BD}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

78. (a) According to question



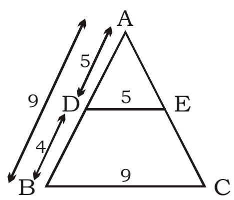
Given: AD = 3, BD = 5
AB = 8, AC = 4
AE = ?
By applying B.P.T

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\frac{3}{8} = \frac{AE}{4}$$

$$AE = \frac{3}{2} = 1.5 \text{ cm}$$

79. (d) In ΔABC, DE||BC



$$\frac{AE}{AC} = \frac{AD}{AB} = \frac{DE}{BC}$$

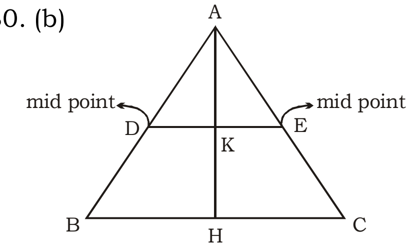
(Basic Prop. theorem)

Here, $\frac{AD}{DB} = \frac{5}{4}$

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC} = \frac{5}{9}$

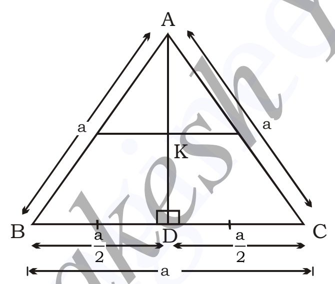
$\Rightarrow$ DE : BC = 5 : 9

80. (b)



$\because$ Point D and E are mid points of sides AB and AC respectively
Then DE will be parallel to BC [by thales theorem]
$\Rightarrow$ And DE, always cuts the two equal part
$\Rightarrow$ Therefore AK : KH 1 : 1

81. (d)



Let, the side of equilateral triangle is = a

Then AB = BC = CA = a

$$BD = DC = \frac{a}{2}$$

$AB^2 = AD^2 + BD^2$ ...(i)

$AC^2 = AD^2 + CD^2$ ...(ii)

Add eq. (i) & (ii)

$AB^2 + AC^2 = 2AD^2 + BD^2 + CD^2$

Add BC^2 both sides

$AB^2+AC^2+BC^2$

$= 2AD^2+BD^2+CD^2 + BC^2$

Put the value of side in R.H.S

$$2AD^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + a^2$$

$$2AD^2 + \frac{3}{2}a^2 = 4AD^2$$

$$\left(a^2 - \frac{a^2}{4} = AD^2 = \frac{3a^2}{4}\right)$$

CHAPTER 23

CENTRES OF TRIANGLES

In geometry, a triangle centre is a point in the plane that is in some sense a centre of a triangle and can be obtained by simple constructions. Centre of triangle always occupy the same position (relative to the vertices) under the operations of rotation, reflection and dilation.

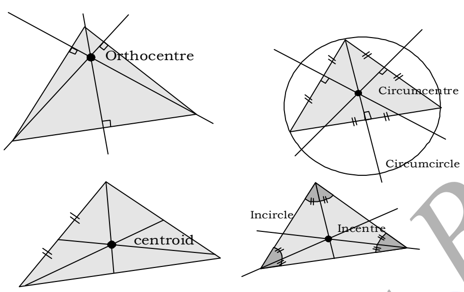
Ancient Greeks discovered the classic centres of triangle (centroid, circumcentre, incentre and orthocentre)

Triangle Centres

Where is the centre of triangle ?

There are actually thousands of centres !

Here are the 4 most popular ones:



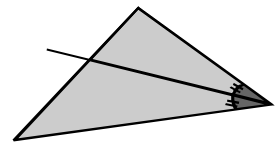
Centroid, Circumcentre, Incentre and Orthocentre

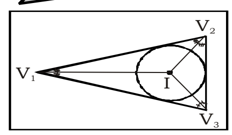
For each of those, the "centre" is where special lines cross, so **it all depends on those lines**!

INCENTRE

The incentre is the point of intersection of the three angle bisectors. The angle bisectors of a triangle are each one of the lines that divide an angle into two equal parts.

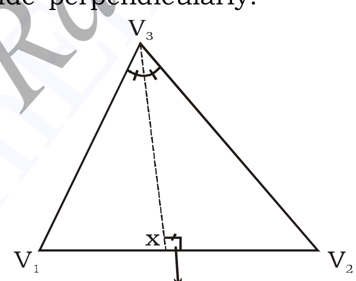
The Incentre is also the centre of the circle inscribed in the triangle.





Properties

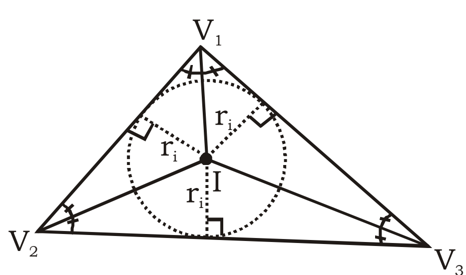
1. Incentre of any triangle lies inside that triangle.
2. Incentre is the only centre which is equiperpendicular distance from all the sides of a triangle.
3. Generally angle bisector doesn't intersect the opposite side perpendicularly.



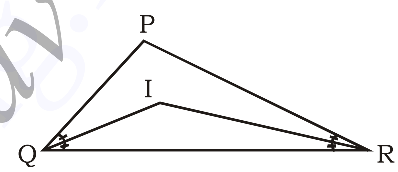
Not a right angle.

Note:- The angle x as given in the figure above is a right angle only in the case of a isosceles and equilateral triangle.

4. The radius of the circle inscribed in a triangle is known as inradius (r_i) of the triangle.



5. The angle betwen line segments drawn from the two vertices to the incentre is equal to the sum of a right angle (90°) and half of third vertex.

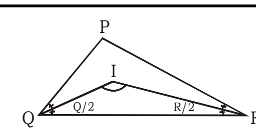


$\angle QIR = 90° + \frac{\angle P}{2}$

$\angle PIQ = 90° + \frac{\angle R}{2}$

$\angle PIR = 90° + \frac{\angle Q}{2}$

Proof



In∆ QIR,

$I + Q/2 + R/2 = 180°$

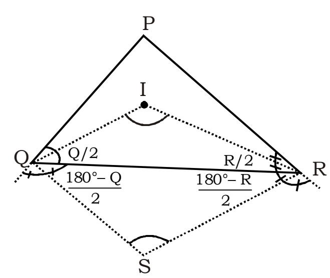
$I = 180° - \frac{Q}{2} - \frac{R}{2} = 180° - \left(\frac{Q+R}{2}\right)$

$I = 180° - \left(\frac{180° - P}{2}\right) = 180° - 90° + \frac{P}{2}$

$I = 90° + \frac{P}{2}$

6. The angle between the external bisectors of two angles of a triangle is difference between right angle and half of the third angle.

$\angle QSR = 90° - \frac{\angle P}{2}$



Proof

$\angle IQS = \frac{Q}{2} + \frac{180^\circ - Q}{2} = 90^\circ$

$\angle IRS = \frac{R}{2} + \frac{180^\circ - R}{2} = 90^\circ$

In Quadrilateral QIRS,

$\angle IRS + \angle IQS = 180^\circ$

Then $\angle QSR + \angle QIR = 180^\circ$

$\angle QSR = 180^\circ - \angle QIR$

$= 180^\circ - \left(90^\circ + \frac{\angle P}{2}\right)$

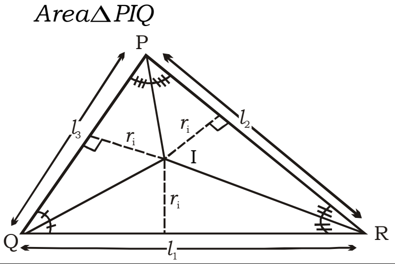
$= 90^\circ - \frac{\angle P}{2}$

7. The ratio of sides of a triangle is equal to the area of triangle formed by corresponding sides and incentre.

$l_1 : l_2 : l_3$ = Area$\triangle$ QIR : Area$\triangle$ PIR : Area$\triangle$ PIQ

$\Rightarrow \frac{l_1}{Area\triangle QIR} : \frac{l_2}{Area\triangle PIR} : \frac{l_3}{Area\triangle PIQ} = 1 : 1 : 1$

$\Rightarrow \frac{l_1}{Area\triangle QIR} = \frac{l_2}{Area\triangle PIR} = \frac{l_3}{Area\triangle PIQ}$



Proof Ar$\triangle$QIR : Ar$\triangle$PIR : Ar$\triangle$PIQ

$= \frac{1}{2} \times l_1 \times r_i : \frac{1}{2} l_2 \times r_i : \frac{1}{2} l_3 \times r_i$

$= l_1 : l_2 : l_3$

[Because altitude of all triangles is same and equal to inradius (r)]

8. Area of a triangle can be obtained by multiplying inradius (r_1) of triangle to the semiperimeter of that triangle.

Area$\triangle$ = $r_i \times$ s

Proof Area of triangle PQR

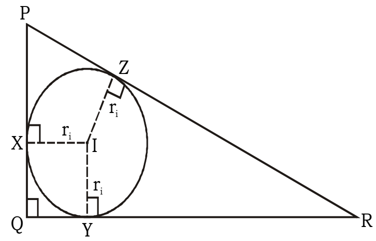
$\Rightarrow$ Ar$\triangle$ QIR + Ar$\triangle$ PIR + Ar$\triangle$ PIQ

$\Rightarrow \frac{1}{2} \times l_1 \times r_i + \frac{1}{2} \times l_2 \times r_i + \frac{1}{2} \times l_3 \times r_i$

$\Rightarrow r_i \times \left[\frac{1}{2}(l_1 + l_2 + l_3)\right]$

$\Rightarrow r_i \times S$

9. Inradius of a right angled triangle PQR.



$r_i = \frac{PQ + QR - PR}{2}$

Proof

PXQ is tangent at X and QYR is tangent at Y
So that IX $\perp$ PQ and IY $\perp$ QR

PX = PZ, QX = QY = r_i, RY = RZ
(Tangents from external points)

$\Rightarrow$ PQ + QR – PR = (PX + r_i) + (r_i + YR) – (PZ + ZR)

$= 2 \times r_i$

($\because$ PX = PZ, ZR = YR)

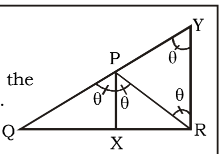
$r_i = \frac{PQ + QR - PR}{2}$

10. ANGLE BISECTOR THEOREM:-

The angle bisector theorem states that an angle bisector divides the opposite side of a triangle into two segments that are proportional to the triangle's other two sides.

In other words $\Rightarrow \frac{PQ}{PR} = \frac{QX}{XR}$

Proof Given that PX is the bisector of $\angle$QPR.



$\Rightarrow$ Draw a line parallel to PX from R. Which intersects extended QP at Y.

Now,

QPX = $\angle$XPR = θ $\Leftarrow$ bisected angles

$\angle$QPX = $\angle$PYR = θ $\Leftarrow$ corresponding angles

$\angle$XPR = $\angle$PRY = θ $\Leftarrow$ alternative interior angles

$\triangle$PRY is isosceles and PY = PR

(as$\angle$PRY = $\angle$PYR = θ)

Now as $\triangle$QPX ~ $\triangle$QYR

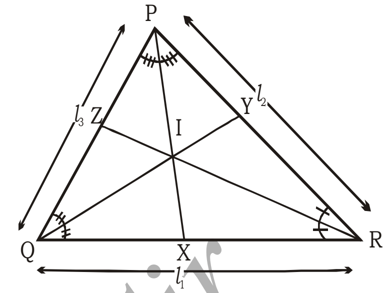
So,

$\frac{PQ}{PY} = \frac{QX}{XR}$

Since PY = PR, we have

$\frac{PQ}{PR} = \frac{QX}{XR}$

11. Each angle bisector divided by Incentre is divided in the ratio equal to the ratio of length of sum of two adjacent sides and opposite side.



PI : IX = $l_2 + l_3 : l_1$

QI : IY = $l_1 + l_3 : l_2$

RI : IZ = $l_1 + l_2 : l_3$

Proof By Interior angle bisector theorem if PX is angle bisector of $\angle$P, then

$\frac{QX}{XR} = \frac{l_3}{l_2} \Rightarrow \frac{QX}{XR} + 1 = \frac{l_3}{l_2} + 1$

$\Rightarrow \frac{QX + XR}{XR} = \frac{l_3 + l_2}{l_2} \Rightarrow \frac{QR}{XR} = \frac{l_3 + l_2}{l_2}$

$XR = \frac{l_2 \times QR}{l_2 + l_3} = \frac{l_1 \times l_2}{l_2 + l_3}$

In $\triangle$PXR

$\frac{PI}{IX} = \frac{PR}{XR} = \frac{l_2}{XR} = \frac{l_2 \times (l_2 + l_3)}{l_1 \times l_2}$

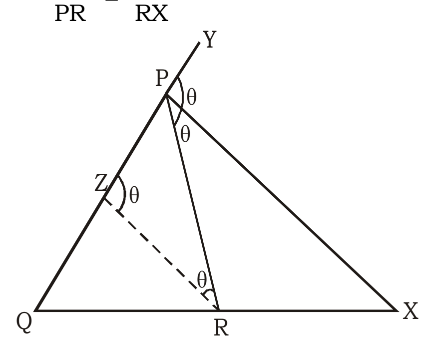
$\frac{l_2 + l_3}{l_1}$ $\left[\therefore XR = \frac{l_1 \times l_2}{l_2 + l_3}\right]$

12. EXTERNAL ANGLE BISECTOR THEOREM.

The external angle bisector theorem states that the external bisector of an angle of triangle divides the opposite side externally in the ratio of the sides containing the angle.

In other words.

$\frac{PQ}{PR} = \frac{QX}{RX}$



$\Rightarrow$ Given that PX is the external bisector of $\angle$RPY

$\Rightarrow$ Draw a line parallel to PX from R, which intersect PQ at Z.

Now

$\angle$RPX = $\angle$XPY = θ $\Leftarrow$ bisected angle

$\angle$ ZRP = $\angle$ RPX = θ $\Leftarrow$ alternative interior angles

$\angle$ RZP = $\angle$ XPY = θ = corresponding angles

$\triangle$ PRZ is isosceles and PZ = PR as $\angle$ PZR = $\angle$ PRZ

Now as $\triangle$ QPX ~ QZR

So,

$$\frac{PQ}{PZ} = \frac{QX}{RX}$$

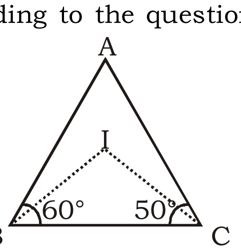
Since PZ = PR, we have

$$\frac{PQ}{PR} = \frac{QX}{RX}$$

Examples

1. I is the incentre of $\triangle ABC$, $\angle ABC$ = 60° and $\angle ACB$ = 50°. Then $\angle BIC$ is

Sol. According to the question



BI and CI are the angle bisector

$\therefore \angle CBI = 30°$

$\angle BCI = 25°$

In $\triangle$ BIC

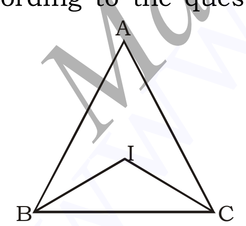
$\angle CBI + \angle BCI + \angle BIC = 180°$

30° +25° + $\angle BIC$ = 180°

$\angle BIC$ = 125°

2. I is the incentre of $\triangle ABC$, If $\angle ABC$ = 60°, $\angle BCA$ = 80°, then the $\angle BIC$ is

Sol. According to the question



Given: $\angle ABC$ =60°

$\angle BCA$ = 80°

$\angle BIC$ = ?

$\angle$ BAC = 40°

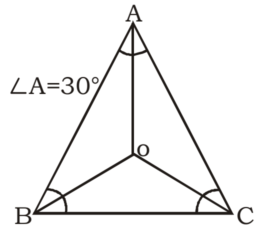
$\therefore \quad \angle BIC = 90° + \frac{1}{2} \times 40°$

$\angle BIC$ = 110°

3. O is the incentre of $\triangle ABC$ and $\angle A$ = 30°, then $\angle BOC$ is

Sol. According to the Question,

Given:



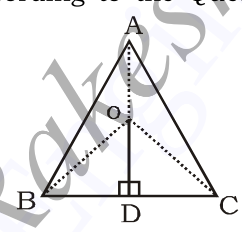
$\therefore \quad \angle BOC = 90° + \frac{1}{2}\angle A$

$= 90° + \frac{1}{2} \times 30° = 90° + 15°$

$\angle BOC$ = 105°

4. Let O be the in-centre of a triangle ABC and D be a point on the side BC of $\triangle ABC$, such that $OD \perp BC$. If $\angle$BOD = 15°, then $\angle ABC$ =

Sol. According to the Question,



Given: $\angle BOD = 15°$

In ΔBOD

$\therefore \angle BDO + \angle DOB + \angle DBO = 180°$

$\angle DBO$ = 180 – (90 + 15) = 75°

$\angle ABC$ = $2 \times \angle DBO$

$\angle ABC$ = 2 × 75°

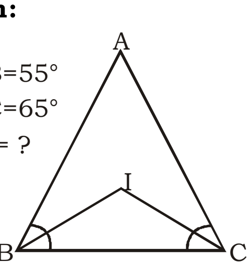
$\angle ABC$ = 150°

5. I is the incentre of a triangle ABC. If $\angle ACB$ = 55°, $\angle ABC$ = 65° then the value of $\angle BIC$ is

Sol. According to the question

Given:

$\angle$ACB=55°
$\angle$ABC=65°
$\angle$BIC= ?



$\therefore \quad \angle ACB + \angle ABC + \angle BAC$ = 180°

$\angle BAC$ = 180° – 55° – 65°

$\angle BAC$ = 60°

We know that

$\angle BIC = 90 + \frac{1}{2}\angle A$

$\angle BIC = 90 + \frac{1}{2} \times 60 = 90 + 30$

$\angle BIC$ = 120°

Alternate

In $\triangle$ BIC,

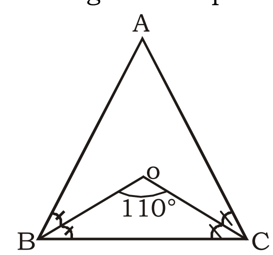
$\frac{1}{2}\angle B + \frac{1}{2}\angle C + \angle BIC = 180°$

$\frac{1}{2}(65° + 55°) + \angle BIC = 180°$

$\angle$ BIC = 180° – 60° = 120°

6. The internal bisectors of $\angle ABC$ and $\angle ACB$ of $\triangle ABC$ meet each other at O. If $\angle BOC$ = 110°, then $\angle BAC$ is equal to

Sol. According to the question



Given: $\angle BOC$ = 110°

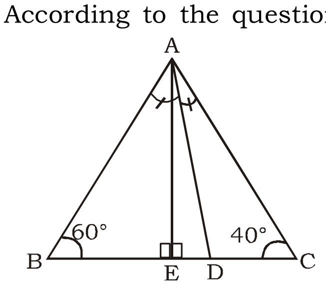
$\angle BOC = 90° + \frac{\angle A}{2}$

$110° = 90° + \frac{\angle A}{2}$

$20° = \frac{\angle A}{2}$, $\quad \angle A = 40°$

7. In $\triangle ABC$, $\angle B = 60°$ and $\angle C = 40°$. If AD and AE be respectively the internal bisector of $\angle A$ and perpendicular on BC, then the measure of $\angle DAE$ is

Sol. According to the question,



Given: $\angle B$ = 60°

$\angle C$ = 40°

As we know that

$\angle A + \angle B + \angle C = 180°$

$\angle A = 180° - 60° - 40°$

$\angle A = 80°$

$\therefore \angle BAD = \dfrac{80°}{2} = 40°$

In $\triangle AEB$

$\angle A + \angle B + \angle E = 180°$

$\angle A = 180° - 60° - 90°$

$\angle A = 30°$

Then,

$\angle DAE = \angle DAB - \angle EAB$

$= 40 - 30$

$\angle DAE = 10°$

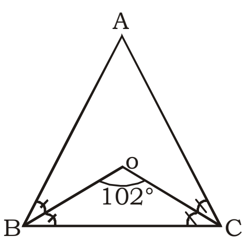
Alternate

$\angle DAE = \dfrac{\angle B - \angle C}{2} = \dfrac{60° - 40°}{2} = 10°$

8. Internal bisectors of $\angle B$ and $\angle C$ of $\triangle ABC$ intersect at O. If $\angle BOC = 102°$, then the value of $\angle BAC$ is

Sol. According to the Question,

Given:



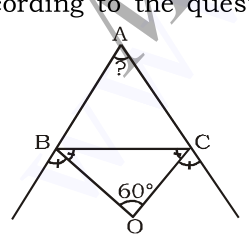
$\angle BOC = 102°$

$102° = 90° + \dfrac{1}{2}\angle A$

$\dfrac{\angle A}{2} = 12°, \angle A = 24°$

9. The angle between the external bisectors of two angles of a triangle is 60°. Then the third angle of the triangle is

Sol. According to the question



Given: $\angle BOC = 60°$

As we know that

$\therefore \angle O = 90 - \dfrac{1}{2}\angle A$

$\dfrac{1}{2}\angle A = 90° - 60°$

$\dfrac{1}{2}\angle A = 30°$

$\angle A = 60°$

10. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is :

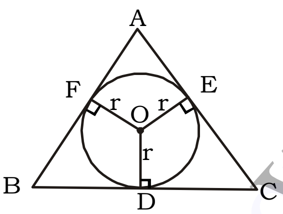
(a) circumcentre

(b) centroid

(c) incentre

(d) orthocentre

Sol.



Incentre is a point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle

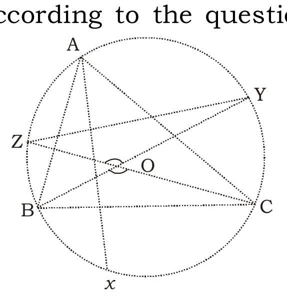
Here,

OD $\perp$ BC, OE $\perp$ AC, OF $\perp$ AB

and OD = OE = OF (Inradius)

11. In $\triangle$ABC, the internal bisector of the $\angle$A, $\angle$B and $\angle$C intersect the circumcircle at x, y and z respectively. If $\angle$A = 50°, $\angle$CZY = 30° then $\angle$BYZ will be

Sol. According to the question



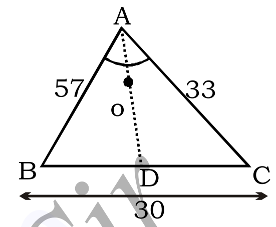
$\angle ZOY = \angle BOC = 90° + \dfrac{A}{2}$

$= 90° + \dfrac{50°}{2} = 115°$

In $\triangle$ZOY, $\angle$BYZ = 180 – (30 + 115) = 35°

12. Three sides of a $\triangle$ABC are, a =30 cm, b = 33cm, c =57 cm. The internal bisector of $\angle$A meets BC at D, and the bisector passes through incentre O. The ratio AO : OD is-

Sol. By 7 property of incentre

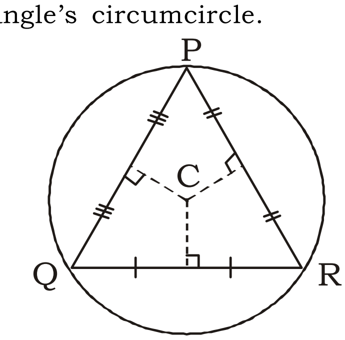


$\dfrac{AO}{OD} = \dfrac{33+57}{30} = \dfrac{90}{30} = \dfrac{3}{1}$

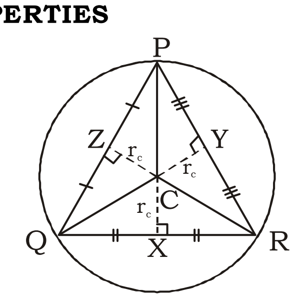
CIRCUMCENTRE

The circumcentre is the point where the perpendicular bisectors of all 3 sides of a triangle intersect.

It is also the centre of the triangle's circumcircle.



PROPERTIES



1. The length from all 3 vertices to the circumcentre is equal. and it is called circumradius.

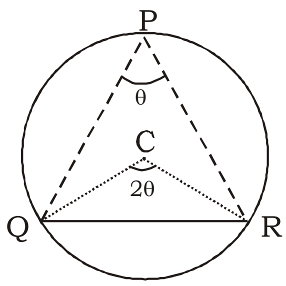
PC = QC = RC = R_c

2. If we draw a circle joining all 3 vertices of the triangle, it will be called the circumcircle of that triangle. And circumcentre of that triangle will also be the centre of its circumcircle.

3. The angle between line segments joining the circumcentre and two vertices is double the angle of the third vertex.

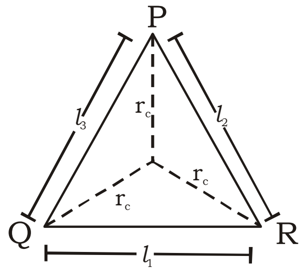
$\angle QCR = 2\angle P$
$\angle PCQ = 2\angle R$
$\angle PCR = 2\angle Q$

Proof:-



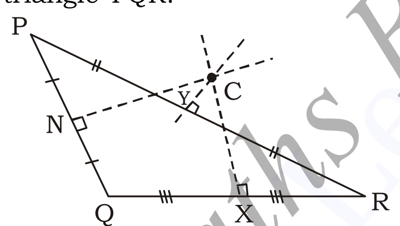
Because of the property of chord of a circle, an angle subtended by a chord at the centre is twice the angle subtended by the chord at any point on major arc of the circle.

4. Circumradius of a triangle (r_c) is

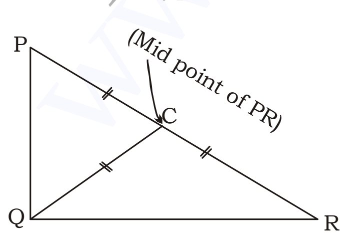


$$r_c = \frac{l_1 \times l_2 \times l_3}{4 \times (Area \Delta PQR)}$$

5. If Δ PQR is an obtuse angled triangle its circumcentre lies outside the triangle PQR.



6. If Δ PQR is a right angled triangle, then its circumcentre lies on the mid-point of its hypotenuse and circumradius (r_c) is half of its hypotenuse.



PC = QC = RC = r_c (radius of circumcentre)

= half of hypotenuse

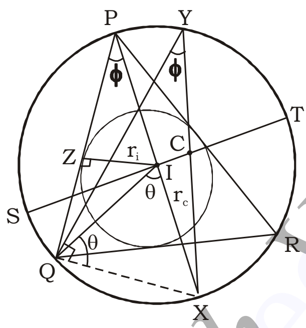
(**Note:-** QC is also the median of Δ PQR)

7. The distance (d) between the circumcentre (r_c) and Incentre (r_i) of a triangle can be expressed as

$$d = \sqrt{r_c^2 - 2r_c r_i}$$

or

$$\frac{1}{r_c + d} + \frac{1}{r_c - d} = \frac{1}{r_i}$$



Let C be the circumcentre of triangle PQR and I be the incentre

⇒ The extension of PI meets circumcircle at X, then X is the midpoint of arc QR

⇒ Join XC and extend it so that it intersects the circumcircle at Y.

⇒ From I construct a perpendicular to PQ and let Z be its foot. so IZ = r_i

Now Δ PZI ~ YQX

or

$\angle PZI = \angle YQX = 90°$

($\because$ $\angle YQX = 90°$, an angle in a semicircle)

$\angle ZPI = \angle QYX = \phi$

($\therefore$ angles made by a chord)

So $\frac{IZ}{XQ} = \frac{PI}{YX}$

⇒ IZ × YX = PI × XQ

PI × XQ = $2r_c r_i$ $\quad \left[\begin{array}{l} \therefore IZ = r_i \\ XY = 2 \times r_c \end{array}\right]$

Now join QI.

⇒ $\angle QIX = \angle QPI + \angle PQI$
$= \angle P/2 + \angle PQR/2$

⇒ $\angle IQX = \angle IQR + \angle RQX$
$= \angle PQR/2 + \angle P/2$

[$\therefore$ $\angle RQX = \angle P/2$ angles from same chord XR]

We have $\angle QIX = \angle IQX$

So IX = QX

Now extend IC, so that it intersects the circumcircle at S and T.

Now,

TI × SI = PI × IX

$(r_c + d) \times (r_c - d) = 2r_c r_i$

$r_c^2 - d^2 = 2r_c r_i$

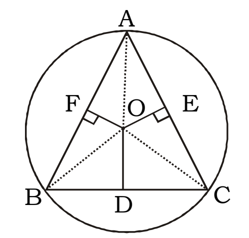
$d^2 = r_c^2 - 2r_c r_i$

$d = \sqrt{r_c^2 - 2r_c r_i}$

Examples

13. The equidistant point from the vertices of a triangle is called its :
(a) centroid
(b) incentre
(c) circumcentre
(d) orthocentre

Sol. The Perpendicular bisector of sides meet at a point called 'circumcentre'.

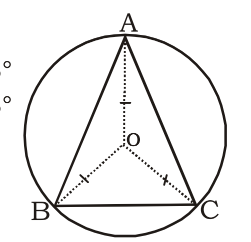


OA = OB = OC (Circumradius)

14. The circumcentre of a triangle ABC is O. If $\angle BAC = 85°$ and $\angle BCA = 75°$, then the value of $\angle OAC$ is

Sol. According to the Question Given:

$\angle BAC = 85°$
$\angle BCA = 75°$
$\angle OAC = ?$



In ΔABC

$\angle ABC + \angle BCA + \angle CAB = 180°$

$\angle ABC = 20°$

$\therefore \angle COA = 2 \times \angle ABC$

$\angle COA = 2 \times 20 = 40°$

In Δ AOC

We know OC = OA
(Circum Radius)

$\therefore\ \angle OAC = \angle OCA$

$\therefore\ \angle OAC + \angle OCA + \angle COA = 180°$

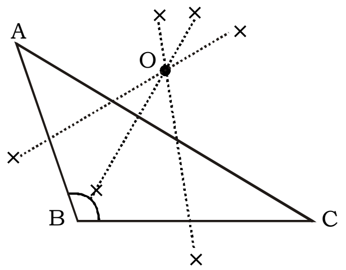
$2\angle OAC = 180° - 40°$

$2\angle OAC = 140°$

$\angle OAC = 70°$

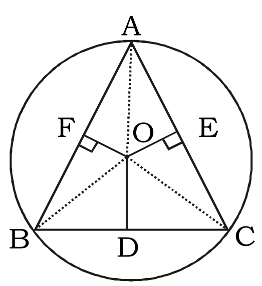
15. If the circumcentre of a triangle lies outside it, then the triangle is

Sol. Circumcentre of a triangle lies outside it, then triangle is obtuse angled triangle.



16. The equidistant point from the vertices of a triangle is called its:

Sol. The equidistant point from the vertices of a triangle is called circumcentre

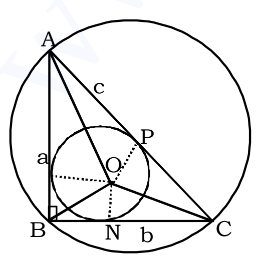


Here OA = OB = OC (Circum Radius)

17. The radius of the circumcircle of a right angled triangle is 15 cm and the radius of its incircle is 6 cm. Find the sides of the triangle.

(a) 30, 40, 41
(b) 18, 24, 30
(c) 30, 24, 25
(d) 24, 36, 20

Sol. According to question



Given:

PC = 15 cm = I_R(circumradius)

ON = 6 cm = I_r (Inradius)

As we know that

$$I_R = \frac{AC}{2},$$

AC = 2 × I_R = 2 × 15 = 30 cm

and $I_r = \dfrac{a+b-c}{2}$

a + b – c = 12

a + b = 12 + c

a + b = 12 + 30

a + b = 42 cm

Now check the option, any one option is satisfied

option:(b) Here a = 18

b = 24

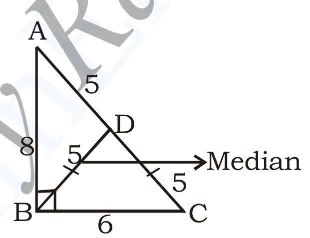
c = 30

a + b = 18 + 24 = 42 cm

18. If the length of the three sides of a triangle are 6 cm, 8 cm and 10 cm, then the length of the median to its greatest side is

Sol. According to question

Length of the three sides of a triangle are 6 cm, 8 cm and 10 cm, this is right angle triangle.



In right angle triangle median divides the hypotenuse in two equal parts

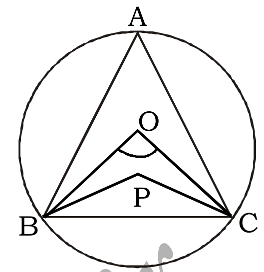
$$\therefore \quad BD = \frac{H}{2}$$

$$BD = \frac{10}{2}$$

BD = 5 cm

19. O is the circumcentre of a triangle ABC whose $\angle$A = 50°. If bisector of $\angle$OBC and $\angle$OCB intersect at P then what is the measure of $\angle$BPC?

Sol. Since angle subtended at the centre of the circle is double the angle subtended at circumference



$\therefore\ \angle BOC = 50° \times 2 = 100°$

$\therefore$ OB= OC

$\therefore\ \angle BOC = \angle OCB$

$= \dfrac{180° - 100°}{2} = 40°$

In $\triangle$ BPC,

$\angle BPC + \angle PBC + \angle PCB = 180°$

or, $\angle BPC + \dfrac{1}{2} \times 40° + \dfrac{1}{2} \times 40° = 180°$

$\therefore\ \angle BPC = 180° - 20° - 20° = 140°$

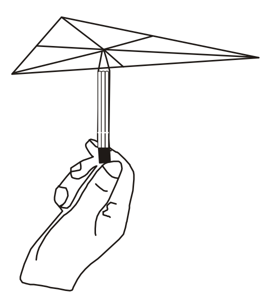
Alternate

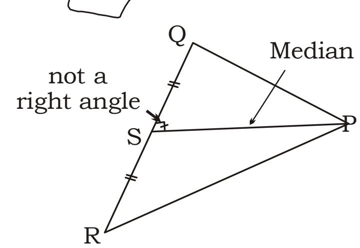
$\therefore\ \angle BPC = 90° + \dfrac{\angle BOC}{2} = 90° + A =$

90° + 50° = 140°

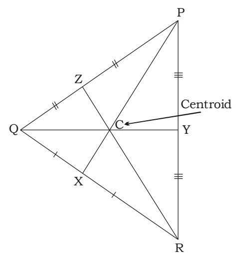
CENTROID

Draw a line (called a "median") from the vertices to the midpoint of the opposite sides. When all three medians intersect is called the centroids which is also the centre of mass.





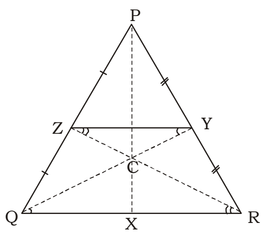
In the above figure the $\angle$PSR is 90° only in the case of an equilateral triangle or if P is the top vertex of isosceles triangle.



Properties

1. Each median is divided by the centroid in the ratio 2 : 1
   i.e. PC : CX = QC : CY = RC : CZ = 2 : 1

Proof



$\triangle CZY \sim \triangle CQR$

$\angle CZY = \angle CRQ$

$\angle CYZ = \angle CQR$

alternate interior angle.

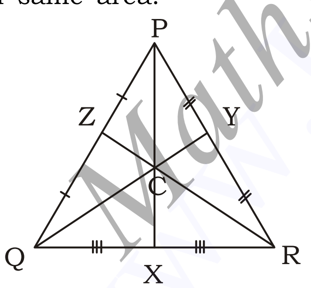
$$\frac{CY}{CQ} = \frac{ZY}{QR}$$

$$\left[\therefore \frac{ZY}{QR} = \frac{1}{2}(\text{by mid point theorem})\right]$$

So

$$\frac{CY}{CQ} = \frac{1}{2}$$

2. When we draw all 3 medians in a triangle. then it divides the triangle into six small triangles of same area.

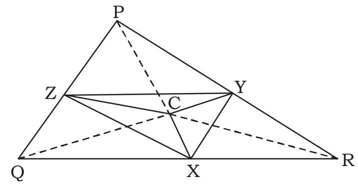


Area $\triangle PCZ$ = Area $\triangle ZCQ$ = Area $\triangle QCX$ = Area $\triangle XCR$ = Area $\triangle RCY$ = Area $\triangle CPY$ = $\frac{1}{6}$ Area $\triangle PQR$

3. Area of triangle formed by joining the centroid to mid points of 2 sides is $\frac{1}{12}$th of area of triangle

   Area $\triangle XYC$ = Area $\triangle YZC$ = Area $\triangle ZXC$

   $= \frac{1}{12}$ Area $\triangle PQR$



Proof:-

$\triangle CZY \sim \triangle CRQ$

[ZY||QR = Mid-point theorem]

So,

$\angle YZC = \angle CRQ$ and

$\angle ZYC = \angle CQR$

$$\therefore \frac{\text{Area}\triangle CZY}{\text{Area}\triangle CQR} = \frac{(ZY)^2}{(QR)^2}$$

$$= \frac{\left(\frac{1}{2}QR\right)^2}{(QR)^2} = \frac{1}{4}$$

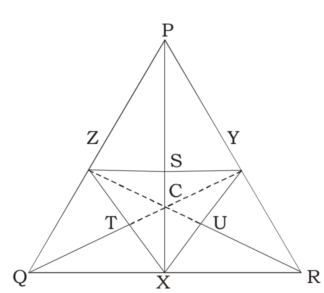
Area $\triangle CZY = \frac{1}{4} \times$ Area $\triangle CQR = \frac{1}{4} \times \left[\frac{1}{3}\text{Area PQR}\right]$

$$\left[\therefore \text{Area}\,\triangle CQR = \frac{1}{3}\text{Area}\,\triangle PQR\right]$$

So,

Area $\triangle CZY = \frac{1}{12}$ Area $\triangle PQR$

4. C is also the centroid of $\triangle XYZ$
5. The line segment drawn via joining the mid points of two sides divides the line joining from centroid to the common vertex of two sides in the ratio 1 : 3



CS : SP = CT : TQ = CU : UR = 1 : 3

PS : SC = QT : TC = RU : UC = 3 : 1

Proof

Let PX = 6n then PC = 4n and CX = 2n

$\triangle PZS \sim \triangle PQX$

$\Rightarrow \angle PZS = \angle PQX$

[$\therefore$ ZS||QX corresponding angle]

$\Rightarrow \angle ZPS = \angle QPX$, [$\therefore$ common angle]

$$\frac{PS}{PX} = \frac{ZS}{QX} = \frac{1}{2}$$

[S is the mid point of PX]

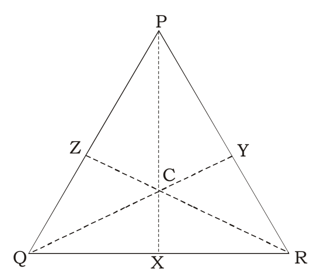
So, $PS = \frac{PX}{2} = \frac{6n}{2} = 3n$

then SC = PC – PS = 4n – 3n = n

$$\therefore \frac{PS}{SC} = \frac{3n}{n} = \frac{3}{1}$$

6. In a triangle the ratio of sum of medians to the perimeter is always smaller than $\frac{3}{2}$, or vice-versa $\frac{PX+QY+RZ}{PQ+QR+RP} < \frac{3}{2}$ or

   $$\frac{PQ+QR+RP}{PX+QY+RZ} > \frac{2}{3}$$



Proof:-

Apply triangle property in $\triangle PQX$

PQ + QX > PX

$PQ + \frac{QR}{2} > PX$ ...... (i)

Similarly in $\triangle QYR$

$QR + \frac{PR}{2} > QY$ ...... (ii)

Similarly in $\triangle RZP$

$PR + \frac{PQ}{2} > RZ$ ...... (iii)

adding (i), (ii) and (iii), we get

$$\frac{3}{2}(PQ + QR + RP) > (PX + QY + RZ)$$

$$\frac{(PX+QY+RZ)}{(PQ+QR+RP)} < \frac{3}{2}$$

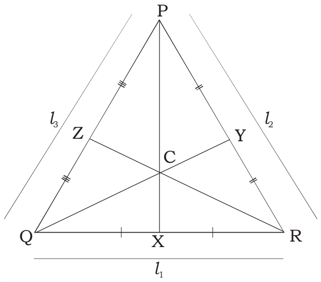
or $\frac{(PQ+QR+RP)}{(PX+QY+RZ)} > \frac{2}{3}$

7. The length of medians can be obtained from Apollonius theorem

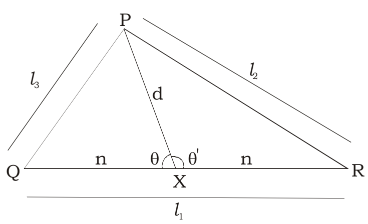
$PX = M_P = \frac{1}{2}\sqrt{2l_2^2 + 2l_3^2 - l_1^2}$

$QY = M_Q = \frac{1}{2}\sqrt{2l_1^2 + 2l_3^2 - l_2^2}$

$RZ = M_R = \frac{1}{2}\sqrt{2l_1^2 + 2l_2^2 - l_3^2}$



Proof:-



Let PX = d
and QX = XR = n
So

$n = \frac{l_1}{2}$

Now In $\triangle$ PQX

$l_3^2 = n^2 + d^2 - 2dn\cos\theta$ .... (i)

[$\therefore$ using cosine formula]

In $\triangle$ PXR,

$l_2^2 = n^2 + d^2 - 2dn\cos\theta'$

$= n^2 + d^2 - 2dn\cos(180^\circ - \theta)$

[$\theta + \theta' = 180^\circ$]

$= n^2 + d^2 + 2dn\cos\theta$ .... (ii)

[$\cos(180^\circ - \theta) = -\cos\theta$]

adding (i) and (ii)

$l_2^2 + l_3^2 = 2(n^2 + d^2)$ .... (iii)

Multiply (iii) by 2.

$2l_2^2 + 2l_3^2 = 4n^2 + 4d^2$

$2l_2^2 + 2l_3^2 = l_1^2 + 4d^2$

[$4n^2 = (2n)^2 = l_1^2$]

$4d^2 = 2l_2^2 + 2l_3^2 - l_1^2$

$2d = \sqrt{2l_2^2 + 2l_3^2 - l_1^2}$

$d = \frac{1}{2}\sqrt{2l_2^2 + 2l_3^2 - l_1^2}$

$PX = \frac{1}{2}\sqrt{2l_2^2 + 2l_3^2 - l_1^2}$

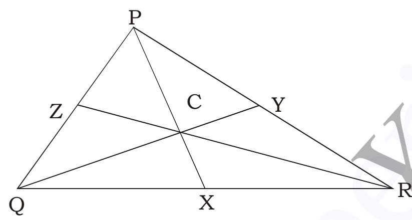
Note:- We can also find the length of sides if medians are given

$l_1 = \frac{2}{3}\sqrt{2M_Q^2 + 2M_R^2 - M_P^2}$

$l_2 = \frac{2}{3}\sqrt{2M_P^2 + 2M_R^2 - M_Q^2}$

$l_3 = \frac{2}{3}\sqrt{2M_P^2 + 2M_Q^2 - M_R^2}$

8. In a triangle four times of sum of squares of median is equal to three times the sum of squares of sides.



$4(PX^2 + QY^2 + RZ^2) = 3(PQ^2 + QR^2 + RP^2)$

Proof:

Apply appolonium theorem for all three medians

$\Rightarrow$ $4M_P^2 + 4M_Q^2 + 4M_R^2 = (2l_2^2 + 2l_3^2 - l_1^2) + (2l_1^2 + 2l_3^2 - l_2^2) + 2l_1^2 + 2l_2^2 - l_3^2$

$4(M_P^2 + M_Q^2 + M_R^2) = 3(l_1^2 + l_2^2 + l_3^2)$

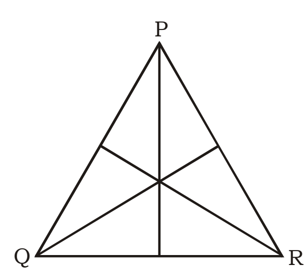
9. Determining the area of a triangle by Heron's formula for medians, providing medians of a triangles are given

let M_P, M_Q and M_R are the three medians of a triangle, and

$S_M = \frac{M_P + M_Q + M_R}{2}$

Area ΔPQR =

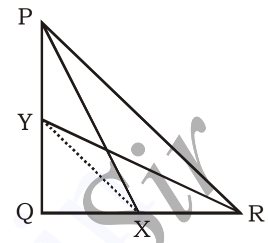
$\frac{4}{3}\sqrt{S_M(S_M - M_P)(S_M - M_Q)(S_M - M_R)}$



Note:- If $M_P^2 + M_Q^2 = M_R^2$ then area of triangle

$= \frac{4}{3} \times \left(\frac{1}{2} \times M_P \times M_Q\right) = \frac{2}{3} \times M_P \times M_Q$

10. In a right angle triangle five times square of hypotenuse is equal to four times of sum of square of two medians (not right angle vertex median)



$5(PR)^2 = 4(M_P^2 + M_R^2)$

$\Rightarrow$ Points To Remember:-

$M_P^2 + M_R^2 = (XY)^2 + (PR)^2$ ...(i)

$4(M_P^2 + M_R^2) = 5(PR)^2$ ...(ii)

$M_P^2 + M_R^2 = 5(XY)^2$ ...(iii)

$M_P^2 + M_R^2 = 5M_Q^2$ ...(iv)

Proof:-

$M_P^2 + M_R^2 = (PQ^2 + QX^2) + (YQ^2 + QR^2)$

$M_P^2 + M_R^2 = XY^2 + PR^2$

[$PQ^2 + QR^2 = PR^2$ & $YQ^2 + QX^2 = XY^2$]

According to mid-point theorem.

$XY = \frac{PR}{2}$ So, $XY^2 = \frac{PR^2}{4}$

$M_P^2 + M_R^2 = \frac{PR^2}{4} + PR^2$

$4(M_P^2 + M_R^2) = 5PR^2$

Now as PR = 2XY, $PR^2 = 4XY^2$

$4(M_P^2 + M_R^2) = 5 \times (4XY^2)$

$M_P^2 + M_R^2 = 5XY^2$

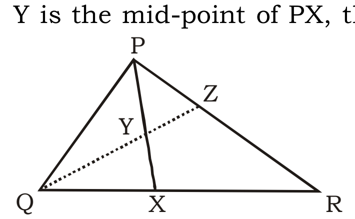
According to property of circumcentre of right angled triangle

$M_Q = \frac{PR}{2}$

$M_P^2 + M_R^2 = 5M_Q^2$

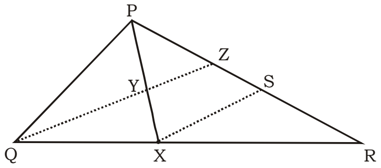
11. The line segment joining the mid-point of a median to vertex divides opposite side in the ratio 1 : 2.

Y is the mid-point of PX, then



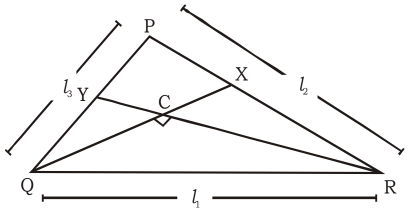
PZ : ZR = 1 : 2

Proof:-



In ΔPQR draw $XS||QZ$
as PX is a median then X is the mid-point of QR, then S is also the mid-point of ZR.
(According to mid point theorem)
So,
ZS : SR = 1 : 1
In ΔPXS,
$YZ||XS$ and Y is the mid point of PX, then Z is also the mid-point of PS.
PZ : ZS = 1 : 1
PZ : ZR = 1 : 2

12. If 2 medians are perpendicular to each other then five times the square of common side is equal to sum of square of other two sides



Proof

If $M_Q \perp M_R$ then
In ΔQCR,
$QC^2 + CR^2 = QR^2$

$$\left(\frac{2}{3}M_Q\right)^2 + \left(\frac{2}{3}M_R\right)^2 = l_1^2$$

$$M_Q^2 + M_R^2 = \frac{9}{4}l_1^2$$

by appolonius theorem

$$\frac{2l_1^2+2l_3^2-l_2^2}{4} + \frac{2l_1^2+2l_2^2-l_3^2}{4}$$

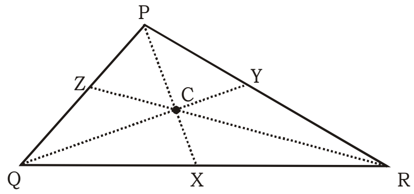
$$= \frac{9}{4}l_1^2$$

$$l_2^2 + l_3^2 = 5l_1^2$$

Note:- If median form pythagoras triplet with each other, i.e.,
$M_Q^2 + M_R^2 = M_P^2$
then also result will be the same
i.e.,
$l_2^2 + l_3^2 = 5l_1^2$

⇒ If two medians are perpendicular then all medians will form pythagorean triplets.

13. The sum of any two sides of a triangle is greater than twice the median drawn to the third side.



$PQ + PR > 2PX$ ...(i)
$PQ + QR > 2QY$ ...(ii)
$PR + QR > 2RZ$ ...(iii)

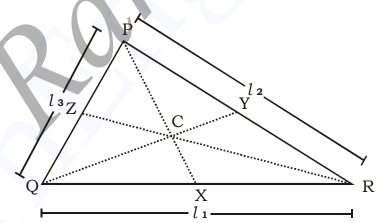
Adding (i), (ii) and (iii)
$2(PQ + QR + RP) > 2(PX + QY + RZ)$
$PQ + QR + RP > PX + QY + RZ$

14. Sum of three medians of a triangle is always greater than $\frac{3}{4}$ of perimeter

i.e.,

$$M_P + M_Q + M_R > \frac{3}{4}(l_1 + l_2 + l_3)$$

Proof:-



$PC = \frac{2}{3}M_P$ $[\because PX = M_P]$

$QC = \frac{2}{3}M_Q$ $[\because QY = M_Q]$

$RC = \frac{2}{3}M_R$ $[\because RZ = M_R]$

Apply inequality of triangle

In ΔPCQ

$\frac{2}{3}M_P + \frac{2}{3}M_Q > l_3$ ...(i)

In ΔPCR

$\frac{2}{3}M_P + \frac{2}{3}M_R > l_2$ ...(ii)

In ΔQCR

$\frac{2}{3}M_Q + \frac{2}{3}M_R > l_1$ ...(iii)

Adding (i), (ii) & (iii),
We get,

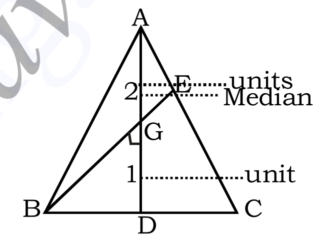
$$\frac{4}{3}(M_P + M_Q + M_R) > l_1 + l_2 + l_3$$

$$M_P + M_Q + M_R > \frac{3}{4}(l_1 + l_2 + l_3)$$

Examples

20. Two medians AD and BE of ΔABC intersect at G at right angles. If AD = 9 cm and BE = 6 cm, then the length of BD (in cm) is

Sol. According to question



G is the centroid which divides the median in 2 : 1
$\therefore$ AD = 3 units = 9 cm
3 units = 9 cm

1 unit = $\frac{9}{3}$ = 3 cm

$\therefore$ GD = 3 cm
BE = 3 units = 6 cm
3 units = 6 cm

1 unit = $\frac{6}{3}$

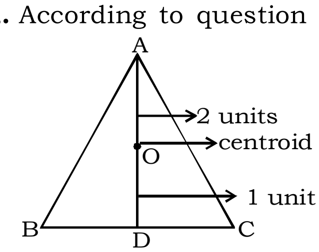
2 units = $\frac{6}{3}\times 2$ = 4 cm

$\therefore$ BG = 4 cm

Δ BGD is a right angle triangle
$BD^2 = BG^2 + GD^2$
$BD^2 = (4)^2 + (3)^2$
$BD^2 = 16 + 9$
$BD = \sqrt{25}$
BD = 5 cm

21. AD is the median of a triangle ABC and O is the centroid such that AO = 10 cm. The length of OD (in cm) is

Sol. According to question



AD is the median and 'O' is the centroid

$\therefore$ AO = 10 cm

2 units = 10

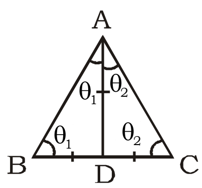
1 units = 5

$\therefore$ OD = 5 cm

22. If the median drawn on the base of a triangle is half its base the triangle will be

Sol. According to question

If the median drawn on the base of a triangle is half its base of the triangle will be right angled triangle.



In ΔABC, AD is median on base BC

According to the question,

$$AD = \frac{BC}{2}$$

AD = BD = DC (AD is median)

In ΔABD,

BD = DA

$\angle DBA = \angle DAB = \theta_1$

In ΔADC

DC = DA

$\angle DAC = \angle DCA = \theta_2$

Now,

In ΔABC

$\angle A + \angle B + \angle C = 180°$

$\theta_1 + \theta_2 + \theta_1 + \theta_2 = 180°$

$2(\theta_1 + \theta_2) = 90°$

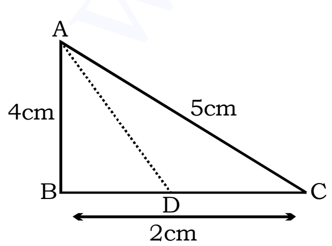
$\theta_1 + \theta_2 = 90°$

$\because$ $\angle A = \theta_1 + \theta_2 = 90°$

So, Triangle will be Right Angle triangle

24. In a ΔABC, three sides are 5cm, 4cm. and 2 cm. Find the length of median from smallest angle vertex.

Sol. Smallest angle is opposite to smallest side. By Apollonius theorem.



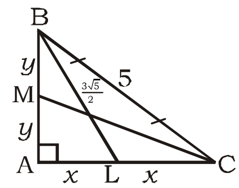
$$AD = \sqrt{\frac{2AB^2 + 2AC^2 - BC^2}{4}}$$

$$= \sqrt{\frac{32 + 50 - 4}{4}} = \sqrt{\frac{78}{4}}$$

$$AD = \sqrt{\frac{39}{2}}$$

25.. BL and CM are medians of ΔABC right- angled at A and BC = 5 cm. If BL = $\frac{3\sqrt{5}}{2}$ cm, then the length of CM is

Sol. According to question



According to figure, when two medians intersect each other in a right angled triangle then we use, this equation.

$\Rightarrow 4\,(BL^2 + CM^2) = 5BC^2$

$\Rightarrow 4 \times \left(\frac{3\sqrt{5}}{2}\right)^2 + 4CM^2 = 5 \times (5)^2$

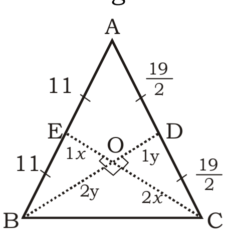
$\Rightarrow 45 + 4CM^2 = 125$

$\Rightarrow CM^2 = \frac{125 - 45}{4} = 20$

$\Rightarrow CM = 2\sqrt{5}$ cm

26. In a ΔABC, BD&CE are the two medians which intersect each other at right angle. AB = 22, AC = 19, find BC = ?

Sol. According to the question,



In right angle ΔCOD,

$OC^2 + OD^2 = CD^2$

$4x^2 + y^2 = \frac{361}{4}$ ...(i)

In right angle ΔBOE,

$OB^2 + OE^2 = BE^2$

$x^2 + 4y^2 = 121$ ...(ii)

Add Both Eq. (i) & (ii)

$5x^2 + 5y^2 = \frac{361}{4} + 121$

$x^2 + y^2 = \frac{169}{4}$ ....(iii)

In right angle ΔBOC,

$OB^2 + OC^2 = BC^2$

$4y^2 + 4x^2 = BC^2$

$4(y^2 + x^2) = BC^2$

From Eq. (iii)

$BC^2 = 169$

BC = 13

Alternate:

In this type of question we use direct formula.

$5BC^2 = AB^2 + AC^2$

$$BC = \sqrt{\frac{AB^2 + AC^2}{5}}$$

$$= \sqrt{\frac{(22)^2 + (19)^2}{5}} = \sqrt{\frac{484 + 361}{5}}$$

$$= \sqrt{\frac{845}{5}} = \sqrt{169} = 13 \text{ cm}$$

27. If AD, BE and CF are medians of ΔABC, then which one of the following statements is correct?

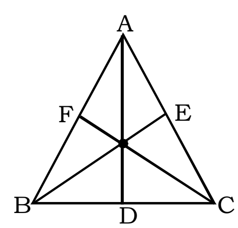
(a) AB + BC + CA < AD + BE + CF

(b) AB + BC + CA > AD + BE + CF

(c) AB + BC + CA = AD + BE + CF

(d) None of these

Sol. According to question



Points D, E & F are midpoint of BC,CA and AB.

$\therefore$ AB + AC > 2AD ....(i)

AB + BC > 2 BE ....(ii)

BC + CA> 2CF ......(iii)

Adding to equation (i),(ii) and (iii) we get

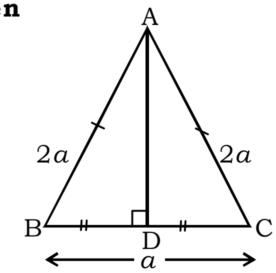
2(AB + BC + CA) > 2(AD + BE + CF)

$\therefore$ (AB + BC + CA) > (AD + BE + CF)

28. ΔABC is an isosceles triangle and $\overline{AB} = \overline{AC} = 2a$ unit, $\overline{BC} = a$ unit. Draw $\overline{AD} \perp \overline{BC}$, and find the length of $\overline{AD}$.

Sol. According to the question

Given



AB = AC = 2a

BC = a

$AD \perp BC$

In isosceles triangle perpendicular sides bisects the opposite side of the length

$\therefore \quad BD = \dfrac{BC}{2}$

$BD = \dfrac{a}{2}$

In $\triangle ADB$ using pythagoras theorem .

$AB^2 = BD^2 + AD^2$

$\Rightarrow \quad (2a)^2 = \left(\dfrac{a}{2}\right)^2 + AD^2$

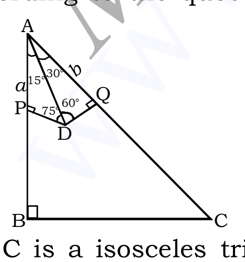
$\Rightarrow \quad 4a^2 = \dfrac{a^2}{4} + AD^2$

$\Rightarrow \quad AD^2 = 4a^2 - \dfrac{a^2}{4}$

$\Rightarrow \quad AD = \dfrac{\sqrt{15}a}{2}$ units

29. An isosceles triangle ABC is right-angled at B.D is a point inside the triangle ABC. P and Q are the feet of the perpendiculars drawn from D on the side AB and AC respectively of $\triangle ABC$. If AP = a cm , AQ = b cm and $\angle BAD = 15°$, Find the value of sin 75° ?

Sol. According to the question,



ΔABC is a isosceles triangle

So, $\angle B = 90°$, $\angle C = \angle A$ = 45°

In ΔAQD

$\angle DAC = \angle BAC - \angle BAD$ = 45 – 15 = 30°

and $\angle QDA$ = 180° – (90 + 30) = 60°

From $\triangle AQD$

$\text{Sin}60° = \dfrac{AQ}{AD}$

$\dfrac{\sqrt{3}}{2} = \dfrac{b}{AD}$

$AD = \dfrac{2b}{\sqrt{3}}$

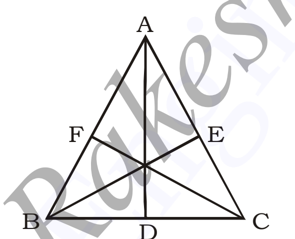
From $\triangle APD$

$\text{Sin}75° = \dfrac{AP}{AD}$

$\text{Sin}75° = \dfrac{a}{2b} \times \sqrt{3} = \dfrac{\sqrt{3}a}{2b}$

30. In $\triangle$ ABC, BE, AD, CF is a median on AC, BC and AB respectively. AD = 10, BE = 6 and CF = 8 cm. Then find the area of $\triangle$ ABC ?

Sol. According to the question,



By using this formula,

We can calculate the area of $\triangle$ ABC.

$\text{Area of } \triangle ABC = \dfrac{4}{3}\sqrt{S(S-a)(S-b)(S-c)}$

Here, a, b & c are the length of the median.

$\therefore \quad S = \dfrac{a+b+c}{2} = \dfrac{8+6+10}{2} = 12$

$\therefore \text{Area} = \dfrac{4}{3}\sqrt{12(12-6)(12-8)(12-10)}$

$= \dfrac{4}{3}\sqrt{12 \times 6 \times 4 \times 2}$

$= \dfrac{4}{3} \times 24 = 32 \text{ cm}^2$

Alternate:

When $m_1^2 + m_2^2 = m_3^2$

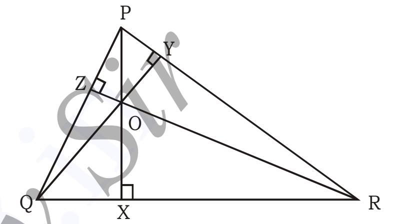
Then

$\text{Area of } \Delta = \dfrac{2}{3} \times m_1 \, m_2$

$= \dfrac{2}{3} \times 6 \times 8 = 32 \text{ cm}^2$

Orthocentre

A point where all 3 altitudes of a triangle meet is called the orthocentre.



Here,

PX, QY & RZ are the altitudes of $\triangle$ PQR & O is the orthocentre.

Properties

The sum of angle between line segments joining the orthocentre and two vertices and the third angle is always a supplementary angle.

$\angle QOR + \angle QPR = 180° = \angle ZOY + \angle QPR$

[$\angle QOR = \angle ZOY$, Opposite angle]

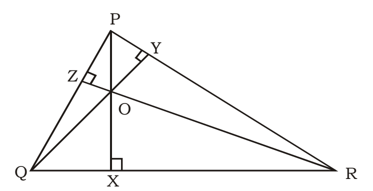
$\Rightarrow \angle POR + \angle PQR = 180° = \angle ZOX + \angle PQR$

[$\angle POR = \angle ZOX$, Opposite angles]

$\Rightarrow \angle POQ + \angle PRQ = 180° = \Rightarrow \angle YOX + \angle PRQ$

[$\angle POQ = \angle YOX$, Opposite angles]

Proof:-



In Quadrilateral PYOZ

$\angle Z = \angle Y = 90°$

So,

$\angle P + \angle O = 180°$

$\angle QPR + \angle ZOY = 180°$

$\angle QPR + \angle QOR = 180°$

2. Orthocentre of triangles

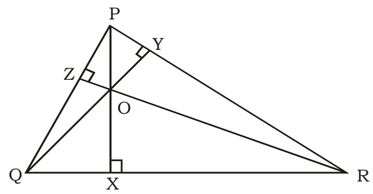
Orthocentre of $\triangle$ PQR = O

Orthocentre of $\triangle$ QOR = P

Orthocentre of $\triangle$ POQ = R

Orthocentre of $\triangle$ POR = Q

3. Pair of similar triangles when all three altitudes are drawn



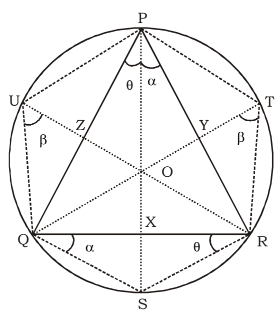
Δ OYR ~ Δ OZQ

Δ OXR ~ Δ OZP

Δ OXQ ~ Δ OYP

4. It must be noted that, In triangles PQR Δ OXQ, Δ OXR, Δ OYP, Δ OYR, Δ OZP & Δ OZQ are right angle triangles.

5. O is the orthocentre of Δ PQR. Draw a circumcircle to triangle PQR. Since angles in the same segments of a circle are equal.

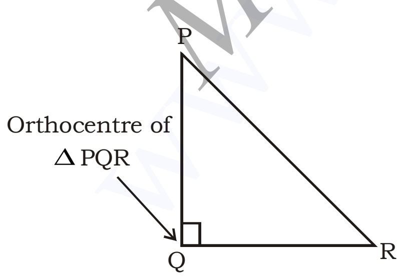


On Base, QS $\angle$QPS = $\angle$QRS

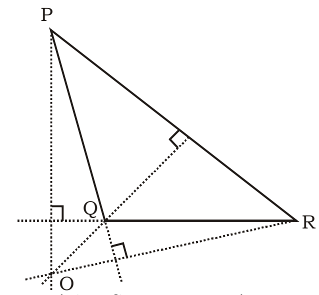
On Base, RS $\angle$RPS = $\angle$RQS

On Base, QR $\angle$QTR = $\angle$QUR etc.

6. The orthocentre of a right angled triangle is that point where triangle forms the right angle.

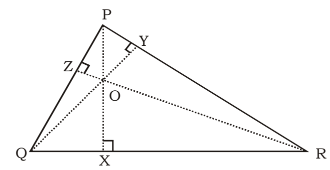


7. The orthocentre of an obtuse angled triangle lies outside the triangle.



In this figure Orthocentre (O) lies outside the triangle PQR.

8. Sum of three altitudes of a triangle is less than sum of three sides of the triangle.



In any right angled triangle Hypotenuse > side (altitude)

In PXQ, PQ > PX ...(i)

In Δ QYR, QR > QY ...(ii)

In Δ PRZ, RP > RZ ...(iii)

adding (i), (ii) & (iii)

We get,

PQ + QR + RP > PX + QY + RZ

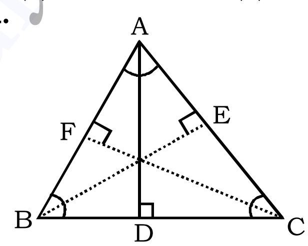
Examples

31. In Δ ABC, AD, BE and CF are the altitudes in the ratio 1:2 : 3 respectively, then the ratio of AB : BC : CA is:

(a) 3 : 2 : 1 (b) 1 : 2 : 3

(c) 1 : 4 : 9 (d) 2 : 6 : 3

Sol.



Area of Δ ABC = $\frac{1}{2}$ BC×AD

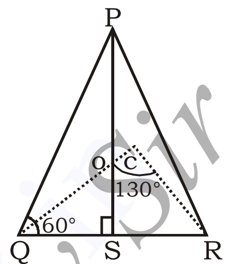
$= \frac{1}{2}$ AC×BE = $\frac{1}{2}$ AB×CF

$\therefore$ AB : BC : AC = $\frac{1}{CF} : \frac{1}{AD} : \frac{1}{BE}$

$= \frac{1}{3} : \frac{1}{1} : \frac{1}{2}$ = 2 : 6 :3

32. O and C are respectively the orthocentre and the circumcentre of an acute-angled triangle PQR. The points P and O are joined and produced to meet the side QR at S. If $\angle$PQS= 60° and $\angle QCR = 130°$, then $\angle RPS = ?$

Sol. According to question



Given $\angle PQS = 60°$

$\angle QCR = 130°$

$\therefore \angle QPR = \frac{1}{2} \angle QCR$

$\angle QPR = \frac{1}{2} \times 130° = 65°$

Now,

$\angle PQS + \angle PSQ + \angle QPS = 180°$

$60° + 90° + \angle QPS = 180°$

$\angle QPS = 30°$

$\angle RPS = \angle QPR - \angle QPS$

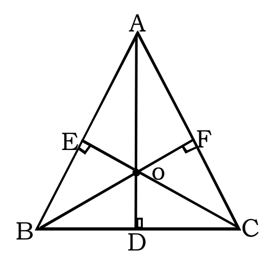
$= 65° - 30°$

$\angle RPS = 35°$

33. The perpendiculars drawn from the vertices to the opposite sides of a triangle, meet at the point whose name is

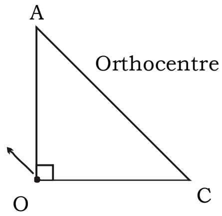
Sol. According to question

Draw a line (called the "altitude") at right angles to a side and going through the opposite corner. Where all three lines intersect is the "orthocentre" O is Orthocentre.



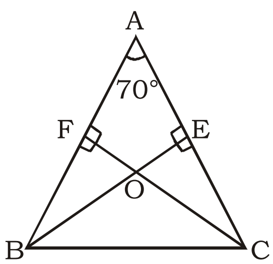
34. The orthocentre of a right angled triangle lies

Sol. The orthocentre of a right angled triangle lies at the right angular vertex



35. In $\triangle ABC$, draw $BE \perp AC$ and $CF \perp AB$ and the perpendicular BE and CF intersect at the point O. If $\angle BAC = 70°$, then the value of $\angle BOC$ is

Sol. According to question



Given: $\angle A = 70°$

AEOF is a quadrilateral

$\therefore$ In a quadrilateral sum of all angles are 360°

$\angle A + \angle F + \angle O + \angle E = 360°$

$70° + 90° + \angle O + 90° = 360°$

$\angle O = 360° - 250°$

$\angle O = 110°$

$\angle BOC = 110°$

(Vertically Opposite angle)

36. Let ABC be an equilateral triangle and AX, BY, CZ be the altitudes. Then the right statement out of the four given responses is

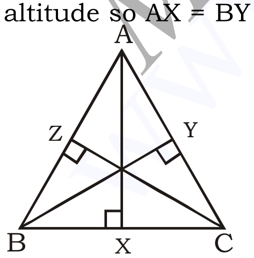
(a) AX = BY = CZ

(b) AX $\neq$ BY $\neq$ CZ

(c) AX = BY $\neq$ CZ

(d) None of these

Sol. ABC is an equilateral triangle and AX, BY and CZ be the altitude so AX = BY = CZ



Area of ΔABC = $\frac{1}{2}$ × Base × height

$= \frac{1}{2} \times AB \times CZ = \frac{1}{2} \times BC \times AX$

$= \frac{1}{2} \times AC \times BY$

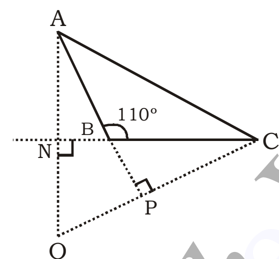
CZ = AX = BY

$\because$ AB = BC = CA (Side of equilateral)

so, we can say if three ltitudes are equal then the triangle is equilateral triangle

37. In obtuse angle Δ, obtuse angle is 110°. Find the angle made on its ortho centre:

Sol. According to the question,



$\angle ABC = 110° = \angle NBP$

(Vertically opposite)

In quadrilateral BNOP

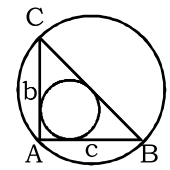
$\therefore$ $\angle N + \angle B + \angle P + \angle O = 360°$

$\angle B = 360° - 90° - 90° - 110°$

$\angle B = 70°$

Mixed properties of centres of a triangle

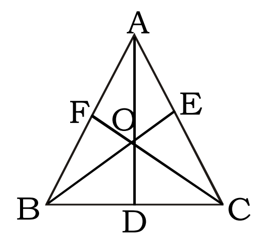
1. In an equilateral triangle all the four centres are coincident i.e., centroid, incentre, circumcentre and orthocentre of an equilateral triangle lie at the same point.
2. Centroid (G), orthocentre (P) and circumcentre (O) of a triangle are always collinear (i.e, lie in a straight line) and PG: GO = 2 : 1.
3. The orthocentre of a right angled triangle lies at the right angled vertex while its circumcentre is mid point of hypotenuse.
4. Circumcentre and orthocentre of an obtuse angled triangle always lie outside the triangle.
5. The sum of diameters of circumcircle and incircle of right angled triangle is equal to the sum of its perpendicular sides.



In the given figure ABC is a right angled triangle with $\angle A = 90°$. If radius of circumcircle and incircle of the triangle be respectively R and r then 2(R + r) = b + c

6. The distance between incentre and circumcentre of a triangle is $\sqrt{R^2 - 2rR}$ where R is circumradius and r is inradius.
7. In an equilateral triangle, length or radius of the circumcircle is equal to twice the radius of its incircle ie., if Δ ABC is equilateral then R = 2r
8. **Ceva Theorem:-** If O is any point inside the triangle ABC and AO, BO, CO meet sides BC, CA, AB respectively at point D, E, F then

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$$

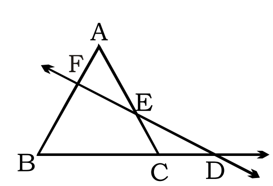


Since Ceva Theorem is true for any point inside the triangle, it is therefore also true for centroid, incentre, orthocentre and circumcentre of the triangle.

Converse of Ceva Theorem is also true.

9. **Menelaus Theorem:-** If a transverse cuts the sides BC, CA and AB (or its produced part) of a triangle at D, E, F then

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = -1$$



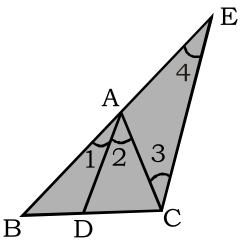
Converse of the Theorem is also true.

Interior Angle Bisector Theorem

Interior Angle Bisector Theorem : The angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

Given: In $\triangle ABC$ in which AD is the internal bisector of $\angle A$ and meets BC at D.

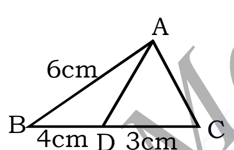
Prove that : BD /DC = AB/AC



Statements	Reasons
1) CE || DA	1) By Construction
2) $\angle 2 = \angle 3$	2) Alternate interior angles
3) $\angle 1 = \angle 4$	3) Corresponding angles
4) AD is the bisector	4) Given
5) $\angle 1 = \angle 2$	5) Definition of angle bisector
6) $\angle 3 = \angle 4$	6) From (2),(3) and (5)
7) AE = AC	7) InΔ ACE, side opposite to equal angles are equal
8) $\frac{BD}{DC} = \frac{BA}{AE}$	8) InΔBCE DA||CE and by BPT theorem
9) $\frac{BD}{DC} = \frac{AB}{AC}$	9) From (7)

Examples

38. In the given figure, AD is the Internal bisector of $\angle$A, If BD =4cm, DC = 3 cm and AB = 6 cm, find AC.



Sol. In $\triangle$ ABC, AD is the bisector of $\angle$A

$\therefore \quad \frac{BD}{DC} = \frac{AB}{AC}$ (Angle bisector theorem)

$\Rightarrow \quad \frac{4}{3} = \frac{6}{AC} \Rightarrow 4AC = 18$

$\Rightarrow \quad AC = \frac{18}{4} = 4.5$ cm.

Exterior Angle Bisector Theorem

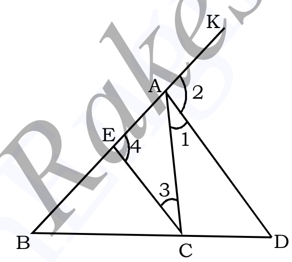
Exterior angle bisector theorem: The external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle.

Given

In $\triangle$ ABC, in which AD is the bisector of the exterior $\angle$A and intersects BC produced at D.

Prove that : **BD / CD = AB / AC**

Construction : Draw CE || DA meeting AB at E.

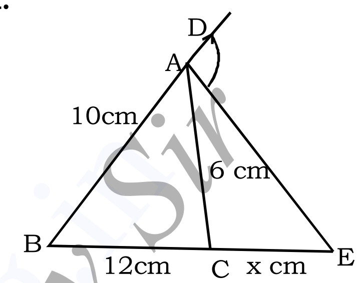


Statements	Reasons
1) CE||DA	1) By Construction
2) $\angle 1 = \angle 3$	2) Alternate interior angles
3) $\angle 2 = \angle 4$	3) Corresponding angles (CE ||DA and BK is a transversal)
4) AD is a bisector of A	4) Given
5) $\angle 1 = \angle 2$	5) Definition of angle bisector
6) $\angle 3 = \angle 4$	6) From (2),(3) and (5)
7) AE = AC	7) If angles are equal then side opposite to them are also equal
8) $\frac{BD}{CD} = \frac{BA}{EA}$	8) By Basic proportionality theorem (EC||AD)
9) $\frac{BD}{DC} = \frac{AB}{AE}$	9) BA =AB and EA =AE
10) $\frac{BD}{CD} = \frac{AB}{AC}$	10) AE = EC and from (7)

Examples

39. In the given figure, AE is the bisector of the exterior $\angle$CAD meeting BC produced in E. If AB = 10 cm, AC = 6 cm and BC = 12 cm, Find CE.

Sol.



Given : AB = 10 cm,AC = 6 cm and BC = 12 cm,

By exterior angle bisector theorem

CE = x (Let)

$\frac{BE}{CE} = \frac{AB}{AC}$

$\frac{12+x}{x} = \frac{10}{6}$

6(12 + x) = 10x [by cross multiplication]

72 + 6x = 10x

72 = 10x – 6x

72 = 4x

x = 18

CE = 18 cm

EXERCISE

1. Internal bisectors of angles $\angle B$ and $\angle C$ of a triangle ABC meet at O. If $\angle BAC$ = 80°, then the value of $\angle BOC$ is
(a) 120° (b) 140°
(c) 110° (d) 130°

2. The external bisector of $\angle B$ and $\angle C$ of ΔABC (where AB and AC extended to E and F respectively) meet at point P. If $\angle BAC = 100°$, then the measure of $\angle BPC$ is
(a) 50° (b) 80° (c) 40° (d) 52°

3. Internal bisectors of angles $\angle B$ and $\angle C$ of a triangle ABC meet at O. If $\angle BAC$ = 70°, then the value of $\angle BOC$ is
(a) 125° (b) 140°
(c) 110° (d) 130°

4. In ΔABC, the internal bisectors of $\angle$ABC and $\angle ACB$ meet at I and $\angle BAC$ = 50°. The measure of $\angle BIC$ is
(a) 105° (b) 115°
(c) 125° (d) 130°

5. If I be the incentre of Δ ABC and $\angle$ B=70^0 and $\angle$ C=50^0, then the magnitude of $\angle$ BIC is
(a) 130^0 (b) 60^0
(c) 120^0 (d) 105^0

6. ABC is an equilateral triangle and CD is the internal bisector of $\angle C$. If DC is produced to E such that AC = CE, then $\angle CAE$ is equal to
(a) 45° (b) 75° (c) 30° (d) 15°

7. The radius of the incircle of the equilateral triangle having each side 6 cm is
(a) $2\sqrt{3}$ cm (b) $\sqrt{3}$ cm
(c) $6\sqrt{3}$ cm (d) 2 cm

8. The internal bisectors of the angles B and C of a triangle ABC meet at I. If $\angle BIC = \frac{\angle A}{2}$ + X, then X is equal to
(a) 60° (b) 30° (c) 90° (d) 45°

9. Internal bisectors of $\angle Q$ and $\angle R$ of ΔPQR intersect at O. If $\angle ROQ$ = 96° then the value of $\angle RPQ$ is :
(a) 12° (b) 24° (c) 36° (d) 6°

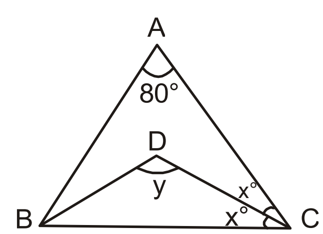
10. O is the incentre of ΔPQR and $\angle$QPR = 50°, then the measure of $\angle$QOR is:
(a) 125° (b) 100°
(c) 130° (d) 115°

11. AD is perpendicular to the internal bisector of $\angle$ ABC of Δ ABC. DE is drawn through D and parallel to BC to meet AC at E. If the length of AC is 12 cm, then the length of AE (in cm.) is
(a) 8 (b) 3 (c) 4 (d) 6

12. If any two sides of a triangle are produced beyond its base and the exterior angles thus obtained are bisected, then these bisectors will include :
(a) half the sum of the base angles
(b) sum of the base angles
(c) half the difference of the base angles
(d) difference of the base angles

13. If I is the in-centre of Δ ABC and $\angle$A = 60°, then the value of $\angle$BIC is:
(a) 100° (b) 120°
(c) 150° (d) 110°

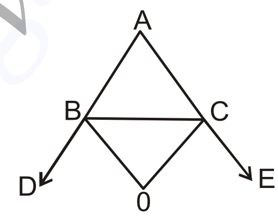
14. In the given figure, $\angle$A = 80°, $\angle$B = 60°, $\angle$C = 2x and $\angle$BDC = y°, BD and CD bisect angles B and C respectively. The value of x and y respectively are :



(a) 15° and 70° (b) 10° and 160°
(c) 20° and 130° (d) 20° and 125°

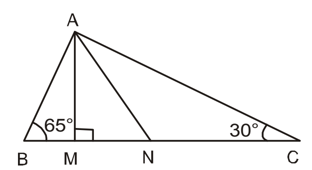
15. The sides of a right angle triangle containing the right angle measure 3 cm and 4 cm. The radius of the incircle of the triangle is :
(a) 3.5 cm (b) 1.75 cm
(c) 1 cm (d) 0.875 cm

16. In the given figure, BO and CO are the bisector of $\angle$CBD and $\angle$BCE respectively and $\angle$A = 40°, then $\angle$BOC is equal to :



(a) 60° (b) 65° (c) 75° (d) 70°

17. In the given figure, AM $\perp$ BC and AN is the bisector of $\angle$A. What is the measure of $\angle$MAN ?



(a) 17.5° (b) 15.5°
(c) 16° (d) 20°

18. If the bisector of an angle of Δ bisects the opposite side, then Δ is :
(a) Scalaene
(b) Isosceles
(c) Right triangle
(d) None of these.

19. Incentre of a triangle lies in the interior of :
(a) an isosceles triangle only
(b) an equilateral triangle only
(c) every triangle
(d) a Right-triangle only.

20. In Δ ABC, the bisectors of $\angle$B and $\angle$C intersect each-other at a point O, then $\angle$BOC is equal to :

(a) $90° - \frac{1}{2}\angle A$

(b) $90° + \frac{1}{2}\angle A$

(c) $120° + \frac{1}{2}\angle A$

(d) $120° - \frac{1}{2}\angle A$

21. In Δ ABC, the sides AB and AC are produced to P and Q respectively. The bisectors of $\angle$PBC and $\angle$QCB intersect at a point O, then $\angle$BOC is equal to :

(a) $90° - \frac{1}{2}\angle A$

(b) $90° + \frac{1}{2}\angle A$

(c) $120° + \frac{1}{2}\angle A$

(d) $120° - \frac{1}{2}\angle A$

22. O is the incentre of Δ ABC and $\angle$BOC = 130°. Find $\angle$BAC:
(a) 80° (b) 40° (c) 150° (d) 50°

23. The internal bisector of $\angle$ABC and $\angle$ACB of Δ ABC meet each-other at O. If $\angle$BOC = 120°, then $\angle$BAC is equal to :
(a) 80° (b) 50° (c) 60° (d) 90°

24. O is the incentre of Δ ABC and $\angle$A = 30°, then $\angle$BOC is :
(a) 100° (b) 105°
(c) 110° (d) 90°

25. In Δ ABC, AD is the internal bisector of $\angle$A, meeting the side BC at D. If BD = 5cm, BC = 7.5cm, then AB : AC is :
(a) 2 : 1 (b) 1 : 2
(c) 4 : 5 (d) 3 : 5

26. The external bisector of $\angle$B and $\angle$C of Δ ABC meet at point P. If $\angle$BAC = 80°, then $\angle$BPC is:
(a) 50° (b) 40°
(c) 80° (d) 100°

27. Internal bisectors of $\angle$B and $\angle$C of a Δ ABC intersect at O, if $\angle$BOC = 102°, then the value of $\angle$BAC is :
(a) 12° (b) 24° (c) 48° (d) 60°

28. The in-radius of an equilateral triangle is of length 3cm, Then the length of each of its medians is :
(a) 12 cm (b) $\frac{9}{2}$ cm
(c) 4cm (d) 9 cm

29. O is any point on the bisector of the acute angle $\angle$ ABC. From O a line parallel to CB meets AB at P. Then Δ BPO is:
(a) right angled isosceles triangle
(b) isosceles but not a right angled
(c) equilateral triangle
(d) None of these

30. If O be the circumcentre of a triangle PQR and $\angle QOR$ = 110°, $\angle OPR$ = 25° , then the measure of $\angle PRQ$ is
(a) 65° (b) 50° (c) 55° (d) 60°

31. For a triangle circumcentre lies on one of its sides. The triangle is
(a) right angled
(b) obtused angled
(c) isosceles
(d) equilateral

32. In ΔABC, $\angle ABC = 70°$, $\angle BCA$ = 40°, O is the point of intersection of the perpendicular bisectors of the sides, then the angle $\angle BOC$ is
(a) 100° (b) 120°
(c) 130° (d) 140°

33. ABC is an equilateral triangle and O is its circumcentre, then the $\angle BOC$ is
(a) 100° (b) 110°
(c) 120° (d) 130°

34. O is the circumcentre of ΔABC. If $\angle$BAC = 85°, $\angle$BCA = 75°, the $\angle OAC$ is equal to:
(a) 70° (b) 60° (c) 50° (d) 40°

35. If O is the circumcentre of a triangle ABC lying inside the triangle, the $\angle$ OBC + $\angle$ BAC is equal to
(a) 120° (b) 110°
(c) 90° (d) 60°

36. The circum-centre of a triangle is always the point of intersection of the:
(a) Medians
(b) bisectors
(c) Perpendiculars
(d) Perpendiculars bisector of the sides

37. The radius of circum-circle of an equilateral triangle of side 12cm is :
(a) $(4/3)\sqrt{3}$ (b) $4\sqrt{2}$
(c) $4\sqrt{3}$ (d) 4

38. In Δ ABC, $\angle$B is a right angle, AC = 6cm, and D is the mid-point of AC. The length of BD is:
(a) 4 cm (b) $\sqrt{6}$ cm
(c) 3 cm (d) 3.5 cm

39. If P and Q are the mid- points of the sides CA and CB respectively of a triangle ABC, right-angled at C, then the value of $4(AQ^2 + BP^2)$ is equal to :
(a) $4BC^2$ (b) $2AC^2$
(c) $2BC^2$ (d) $5AB^2$

40. The length of the two sides forming the right angle of a right angled triangle are 6cm and 8cm. The length of its circum-radius :
(a) 5 cm (b) 7 cm
(c) 6 cm (d) 10 cm

41. In Δ ABC, $\angle$ B = 90°, $\angle$ C = 45° and D is the mid-point of AC. If AC = $4\sqrt{2}$ units, then BD is:
(a) $2\sqrt{2}$ units (b) $4\sqrt{2}$
(c) $\frac{5}{2}$ units (d) 2 units

42. In a triangle ABC, median is AD and centroid is O, AO = 10 cm. The length of OD (in cm) is
(a) 6 (b) 4 (c) 5 (d) 3.3

43. G is the centroid of the equilateral ΔABC. If AB = 10 cm then length of AG is
(a) $\frac{5\sqrt{3}}{3}$ cm (b) $\frac{10\sqrt{3}}{3}$ cm
(c) $5\sqrt{3}$ cm (d) $10\sqrt{3}$ cm

44. If the three medians of a triangle are same, then the triangle is
(a) equilateral
(b) isosceles
(c) right- angled
(d) obtuse-angle

45. In $\triangle ABC$, D is the mid-point of BC. Length AD is 27 cm. N is a point in AD such that the length of DN is 12 cm. The distance of N from the centroid of $\triangle ABC$ is equal to
(a) 3 cm (b) 6 cm
(c) 9 cm (d) 15 cm

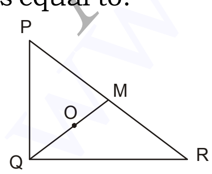
46. G is the centroid of $\triangle$ABC. The medians AD and BE intersect at right angles. If the lengths of AD and BE are 9 cm and 12 cm respectively; then the length of AB (in cm) is?
(a) 11 (b) 10
(c) 10.5 (d) 85

47. The centroid of a $\triangle$ ABC is G. The area of $\triangle$ ABC is 60 cm². The area of $\triangle$ GBC is
(a) 30 cm² (b) 40 cm²
(c) 10 cm² (d) 20 cm²

48. The medians CD and BE of a triangle ABC intersect each other at O. The ratio of $\triangle$ODE : $\triangle$ABC is equal to
(a) 1 : 12 (b) 12 : 1
(c) 4 : 3 (d) 3 : 4

49. In $\triangle$ ABC,the median BE intersects AC at E, if BG = 6cm, where G is the centroid, then BE is equal to :
(a) 8 cm (b) 10 cm
(c) 7 cm (d) 9 cm

50. If in the given figure, $\angle$PQR = 90°, O is the centroid of $\triangle$ PQR, PQ = 5cm and QR = 12cm, then OQ is equal to:



(a) $3\frac{1}{2}$ (b) $4\frac{1}{3}$
(c) $4\frac{1}{2}$ (d) $5\frac{1}{3}$

51. The medians AD, BE CF of a triangle ABC intersect in G. Which of the following istrue for any $\triangle$ ABC ?
(a) GB + GC = 3GA
(b) GB + GC < GA
(c) GB + GC > GA
(d) GB + GC = GA

52. If G is the centroid and AD be a median with length 12cm of $\triangle$ ABC, then the value of AG is:
(a) 4 cm (b) 6 cm
(c) 10 cm (d) 8 cm

53. Two medians AD and BE of $\triangle$ABC intersect at G at right angles. If AD = 9cm and BE = 6cm, then the length of BD, in cm is :
(a) 10 (b) 6 (c) 5 (d) 3

54. In $\triangle$ ABC, AD is the median and AD $=\frac{1}{2}$BC. If $\angle$BAD = 30° is, then $\angle$ACB is :
(a) 90° (b) 45° (c) 30° (d) 60°

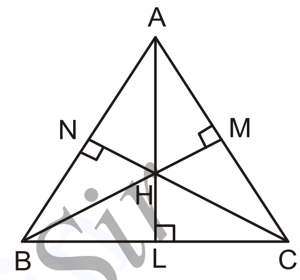
55. In $\triangle$ ABC, G is the centroid, AB = 15cm, BC = 18cm, and AC = 25cm. Find GD, where D is th mid-point of BC :
(a) $\frac{1}{2}\sqrt{86}$ (b) $\frac{1}{3}\sqrt{86}$
(c) $\frac{7}{3}\sqrt{86}$ cm (d) $\frac{2}{3}\sqrt{86}$ cm

56. If G is the centroid of $\triangle$ ABC and AG = BC, then $\angle$BGC is :
(a) 75° (b) 45° (c) 90° (d) 60°

57. I and O are respectively the in-centre and circumcentre of a triangle ABC. The line AI produced intersects the circumcircle of $\triangle ABC$ at the point D. If $\angle ABC = x°$, $\angle BID = y°$ and $\angle BOD = z°$, then $\frac{z+x}{y}$ =?
(a) 3 (b) 1 (c) 2 (d) 4

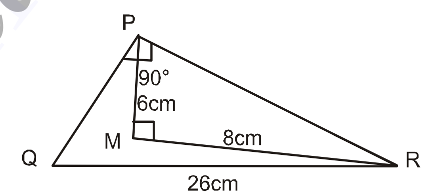
58. In an obtuse-angled triangle ABC, $\angle$A is the obtuse angle and O is the orthocentre. If $\angle$BOC = 54°, then $\angle$BAC is :
(a) 108° (b) 126°
(c) 136° (d) 116°

59. If H is the orthocentre of $\triangle$ ABC, then the orthocentre of $\triangle$ HBC is (figure given) :



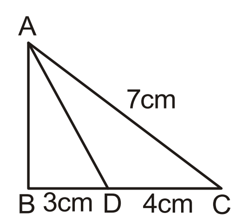
(a) N (b) A (c) L (d) M

60. In the given figure $\angle$QPR = 90°, QR = 26 cm, PM = 6 cm, MR = 8cm and $\angle$PMR = 90°, find the area of $\triangle$ PQR.



(a) 180 cm² (b) 240 cm²
(c) 120 cm² (d) 150 cm²

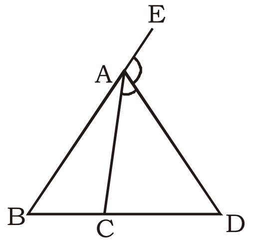
61. In the given figure, if AD is bi-sector of $\angle$BAC then AB is:



(a) 6cm (b) 5 cm
(c) 5.25cm (d) 5.75 cm

62. In $\triangle$ ABC, D is a point on BC such that $\frac{AB}{AC}=\frac{BD}{DC}$. If $\angle$B = 70°, $\angle$C = 50°, then the value of $\angle$BAD :
(a) 30° (b) 60°
(c) 40° (d) 50°

63. In the figure AD is the external bisector of $\angle$EAC, intersects BC produced to D. If AB =12 cm, AC = 8 cm and BC = 4 cm, find CD:



(a) 10cm (b) 6 cm
(c) 8 cm (d) 9 cm

64. In triangle ABC, DE || BC where D is a point on AB and E is point on AC. DE divides the area of Δ ABC into two equal parts. Then DB : AB is equal to

(a) $\sqrt{2}:(\sqrt{2}+1)$ (b) $(\sqrt{2}-1):\sqrt{2}$

(c) $\sqrt{2}:(\sqrt{2}-1)$ (d) $(\sqrt{2}+1):\sqrt{2}$

65. Let ABC be a triangle with AB = 3cm. and AC = 5 cm. If AD is a median drawn from the vertex A to the side BC, then which one of the following is correct?

(a) AD is always greater than 4cm. but less then 5 cm
(b) AD is always greater than 5 cm.
(c) AD is always less than 4 cm
(d) None of the Above

66. In ΔABC, the internal bisector of the $\angle$A, $\angle$B and $\angle$C intersect the circumcircle at x,y and z respectively. If $\angle$A= 50°, $\angle$czy = 30° then $\angle$Byz will be

(a) 10° (b) 35°
(c) 100° (d) 115°

67. In an isosceles Δ ABC,AB = AC and $\angle$A is two times of $\angle$B. If AB =3 cm. then ratio of inradius to the circumradius is-

(a) 1 : 2 (b) $\sqrt{2}-1:1$

(c) $2\sqrt{2}-1:1$ (d) $1:2\sqrt{2}-1$

68. If O be the orthocentre of ABC, OF $\perp$r AB and OE $\perp$r AC. If OE = 2cm and BE = 5 cm then find the value of OF × OC.

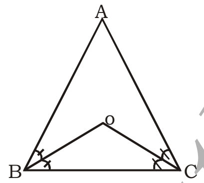
(a) 10 (b) 3 (c) 6 (d) 3

ANSWER KEY

1. (d)	8. (c)	15. (c)	22. (a)	29. (b)	36. (d)	43. (b)	50. (b)	57. (c)	64. (b)
2. (c)	9. (a)	16. (d)	23. (c)	30. (d)	37. (c)	44. (a)	51. (c)	58. (b)	65. (c)
3. (a)	10. (d)	17. (a)	24. (b)	31. (a)	38. (c)	45. (a)	52. (d)	59. (b)	66. (b)
4. (b)	11. (d)	18. (b)	25. (a)	32. (d)	39. (d)	46. (b)	53. (c)	60. (c)	67. (b)
5. (c)	12. (a)	19. (c)	26. (a)	33. (c)	40. (a)	47. (d)	54. (d)	61. (c)	68. (c)
6. (d)	13. (b)	20. (b)	27. (b)	34. (a)	41. (a)	48. (a)	55. (d)	62. (a)	
7. (b)	14. (c)	21. (a)	28. (d)	35. (c)	42. (c)	49. (d)	56. (c)	63. (c)	

SOLUTION

1. (d) According to question



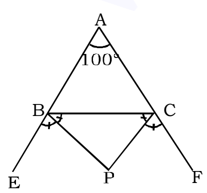
Given: $\angle BAC$ = 80°

$\angle BOC = 90° + \frac{1}{2}\angle A$

$\angle BOC = 90 + \frac{1}{2} \times 80$

= 130°

2. (c) According to question
Given:

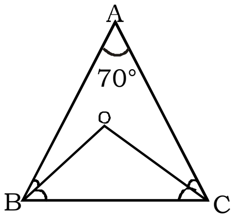


$\therefore \angle BPC = 90° - \frac{1}{2}\angle A$

$= 90° - \frac{1}{2} \times 100°$

$\angle BPC = 40°$

3. (a) According to question.



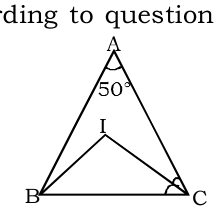
Given: $\angle BAC$ = 70°
$\angle BOC$ = ?

$\therefore \angle BOC = 90° + \frac{1}{2}\angle A$

$\angle BOC = 90° + \frac{1}{2} \times 70° = 90° + 35°$

$\angle BOC$ = 125°

4. (b) According to question



Given: $\angle A$ = 50°
I is the incentre

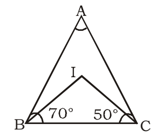
$\angle BIC = 90° + \frac{1}{2}\angle A$

$\angle BIC = 90° + \frac{1}{2} \times 50°$

$\angle BIC$ = 90° + 25°

$\angle BIC$ = 115°

5. (c) According to question



As we know that

$\angle A + \angle B + \angle C = 180°$

$\therefore \angle A = 180° - 70° - 50°$

$\angle A = 60°$

$\therefore \angle BIC = 90° + \frac{1}{2} \times 60°$

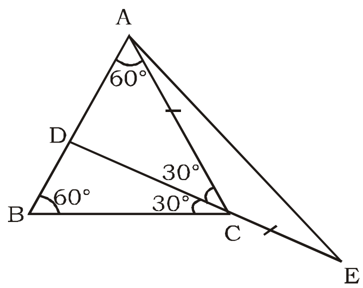
$\angle BIC = 120°$

6. (d) According to question

Given:

ABC is an equilateral triangle

CD is the angle bisector of $\angle C$



AC = CE

$\therefore \angle CAE = \angle CEA$

$\angle ACD = 30°$

$\therefore \angle ECA = 180° - 30°$

$= 150°$

In ΔCAE

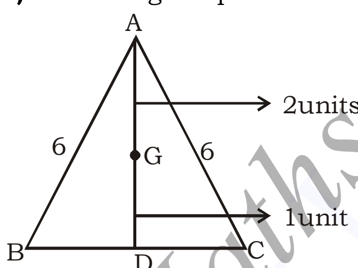
$\angle CAE + \angle CEA + \angle ECA = 180°$ (CE = CA)

$\therefore 2\angle CAE = 180° - 150°$

$2\angle CAE = 30°$

$\angle CAE = 15°$

7. (b) According to question



Given

AB = BC = CA = 6 cm

AG = I_R = Circumradius = 2 units

GD = I_r = Inradius = 1 unit

AD = height = 3 units

As we know that height of the equilateral triangle is h = $\frac{\sqrt{3}}{2}a$,

where'a' is the sides of a triangle

AD = $\frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3}$ cm

$\therefore$ 3 units = $3\sqrt{3}$

1 unit = $\frac{3\sqrt{3}}{3} = \sqrt{3}$

$\therefore$ GD = I_r = $\sqrt{\mathbf{3}}$

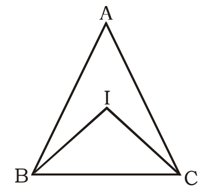
Alternate

$r_{in} = \frac{a}{2\sqrt{3}}$

$r_{in} = \frac{6}{2\sqrt{3}} = \sqrt{3}$ cm

8. (c) According to question

Given:



$\angle BIC = \frac{\angle A}{2}$ + X ......(i)

As we know that

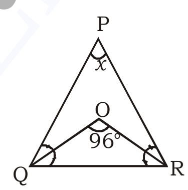
$\angle BIC = 90° + \frac{\angle A}{2}$ ....(ii)

Compare equation (i) and (ii)

X = 90°

9. (a) According to the question,

Value of $\angle$ROQ = 90 + $\frac{\angle P}{2}$



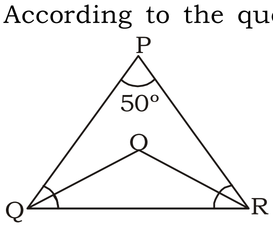
$\Rightarrow$ 96 = 90 + $\frac{\angle P}{2}$

$\Rightarrow$ 6 = $\frac{\angle P}{2}$

$\Rightarrow$ $\angle$P = 12°

Therefore $\angle$RPQ = 12°

10. (d) According to the question,

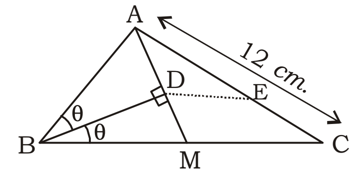


$\Rightarrow$ $\angle$QOR = 90°+$\frac{\angle A}{2}$

$\Rightarrow$ $\angle$QOR = 90° + $\frac{50°}{2}$

$\Rightarrow$ $\angle$QOR = 90° + 25° = 115°

11. (d) According to the question,



$\angle$ABD = $\angle$MBD = θ

(angle bisector)

$\therefore$ BD ⊥ AM

$\angle$BDA = $\angle$BDM = 90°

It happens only in equilateral and isosceles triangle

$\therefore$ AD = DM

i.e. AD = AM/2

Given DE || BC

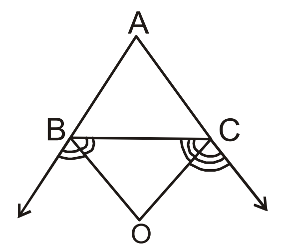
From thales theorem

E will be mid point of AC.

$\because$ AC = 12 cm.

So, AE = 6 cm.

12.(a) $\angle$BOC = 90° - $\frac{1}{2}$ $\angle$A ....(i)



$\angle$A + $\angle$B + $\angle$C = 180°

$\Rightarrow$ $\angle$A = 180° – ($\angle$B + $\angle$C)

multiply both sides by $\frac{1}{2}$

$\Rightarrow \frac{1}{2}\angle A = 90° - \frac{1}{2}(\angle B + \angle C)$

$\Rightarrow \frac{1}{2}(\angle B + \angle C) = 90° - \frac{1}{2}\angle A$ ...(ii)

From Eq. (i) = (ii)

$\therefore \angle BOC = \frac{1}{2}(\angle B + \angle C)$

13.(b) $\angle BIC = 90° + \frac{A}{2} = 90° + 30°$

= 120°

14.(c) $\angle$A + $\angle$B + $\angle$C = 180°

$\Rightarrow$ $\angle$C = 180° - ($\angle$A+$\angle$B)

$\Rightarrow$ $\angle$C = 180° - (80° + 60°) = 40°

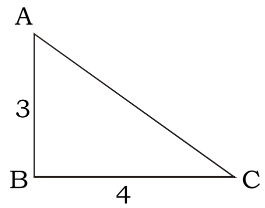
$\Rightarrow$ $2x = 40 \Rightarrow x = 20°$

$(\because C = 2x)$

$y = 90° + \frac{1}{2}\angle A = 90° + \frac{1}{2}(80)$

$= 130°$

15.(c) According to the question



$AC = \sqrt{3^2 + 4^2} = 5cm$

$A = \text{area of } \Delta ABC = \frac{1}{2} \times 4 \times 3$

$= 6cm$

$S = \frac{3+4+5}{2} = 6cm$

$\therefore \quad r = A/S = 6/6 = 1cm$

Alternate:-

For right angle triangle

$r = \frac{AB + BC - CA}{2}$

$r = \frac{3+4-5}{2} = 1$

16.(d) According to the question

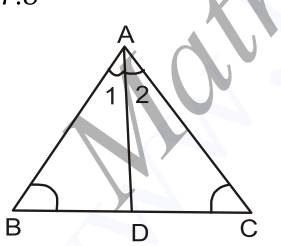
$\angle BOC = 90° - \frac{1}{2}\angle A = 90° - \frac{1}{2}(40) = 70°$

17.(a) $\angle MAN = \frac{1}{2}(\angle B - \angle C)$

$= \frac{1}{2}(65° - 30°) = \frac{1}{2}(35°)$

$= 17.5°$

18.(b)



Since $\angle 1 = \angle 2$

$\therefore \quad \frac{AB}{AC} = \frac{BD}{CD} \Rightarrow \frac{AB}{AC} = 1 \quad (\because BD = CD \text{ given})$

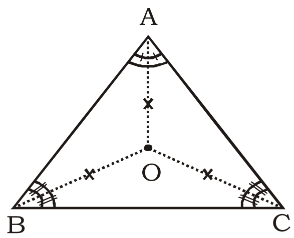
$\Rightarrow AB = AC$

$(\therefore$ the given Δ is isosceles)

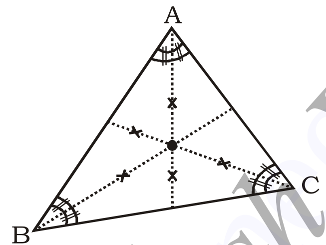
* This theorem apply for equilateral triangle because equilateral has also all sides equal (AB = BC = CA)

19.(c) Incentre:- meeting point of angle bisector.

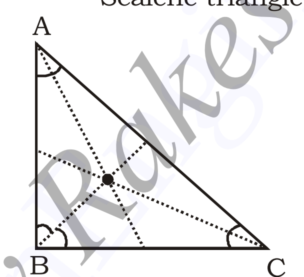
Every triangle:- lies incentre of a triangle in the interior



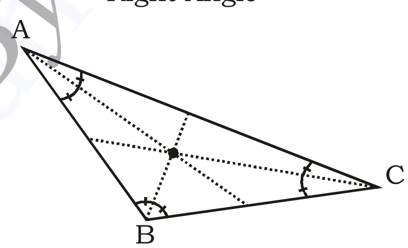
Equilareral triangle



Scalene triangle

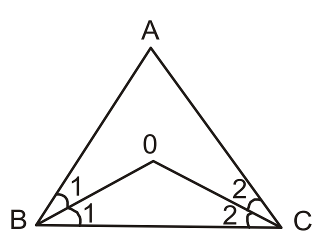


Right Angle



Obtuse angle triangle

20.(b)



$A + B + C = 180°$

$\Rightarrow \quad A + 2\angle 1 + 2\angle 2 = 180°$

$\Rightarrow \angle 1 + \angle 2 = \frac{1}{2}(180° - A)$

$= 90° - \frac{1}{2}A \quad \text{...(i)}$

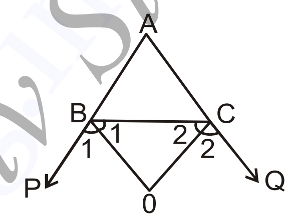
$\therefore \quad \angle BOC = 180° - (\angle 1 + \angle 2)$

From Eq. (i)

$= 180° - (90° - \frac{1}{2}\angle A)$

$= 90° + \frac{1}{2}\angle A$

21.(a)



$2\angle 1 + \angle B = 180°$

$\Rightarrow \angle 1 = 90° - \frac{1}{2}\angle B$ ______(i)

similarly,

$\angle 2 = 90° - \frac{1}{2}\angle C$ ______(ii)

Add both (i) and (ii)

$\angle 1 + \angle 2 = 180° - \frac{1}{2}(B + C)$

$\therefore \quad \angle BOC = 180° - (\angle 1 + \angle 2)$

$= \quad 180° - \left(180° - \frac{1}{2}(\angle B + \angle C)\right)$

$= \frac{1}{2}[\angle B + \angle C) = \frac{1}{2}(180° - \angle A]$

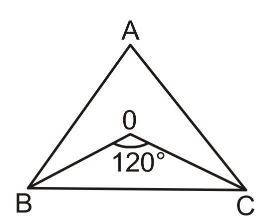
$= 90° - \frac{1}{2}\angle A$

22.(a) $\angle BOC = 90° + \frac{1}{2}(\angle BAC)$

$\Rightarrow \quad \angle BAC = (130 - 90) \times 2$

$= 80°$

23.(c)



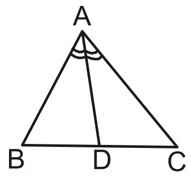
$\angle BOC = 90° + \frac{1}{2}(\angle BAC)$

$\Rightarrow \quad \angle BAC = (120° - 90°) \times 2 = 60°$

24.(b) $\angle BOC = 90° + \frac{1}{2}(\angle A)$

$= 90° + \frac{1}{2} \times 30° = 105°$

25.(a)



$\frac{AB}{AC} = \frac{BD}{DC}$

$DC = BC - BD$

$= 7.5 - 5 = 2.5$

$= \frac{AB}{AC} = \frac{5}{2.5}$

$= 2 : 1$

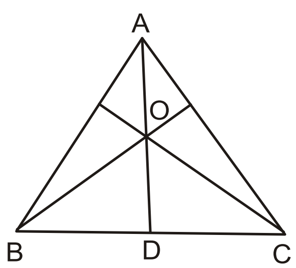
26.(a) $\angle BPC = 90° - \frac{A}{2}$

$= 90° - 40° = 50°$

27.(b) $\angle BOC = 90° + \frac{1}{2}(\angle BAC)$

$\Rightarrow \angle BAC = (102° - 90°) \times 2 = 24°$

28.(d)



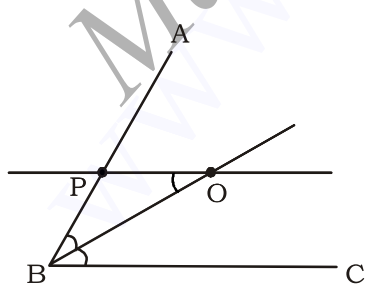
In equilateral triangle centroid, incentre, orthocentre coincide at the same point.

$\therefore$ in-radius = OD = 3 cm

$\therefore$ $\frac{AD}{3} = 3cm \Rightarrow AD = 9cm$

= median.

29.(b)



$\because OP \,||\, BC$

$\Rightarrow \angle POB = \angle OBC$ ....(i)

(Alternate angle)

$\Rightarrow \angle PBO = \angle POB$ $\because$ OB is the bisector of $\angle B$.

$\angle PBO = \angle OBC$ .....(ii)

From (i) and (ii)

$\therefore$ PB = PO

$\because$ $\angle ABC$ is an acute angle or $\angle ABC < 90°$

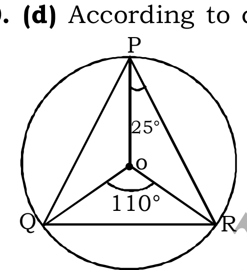
$\therefore$ $\frac{1}{2}\angle ABC < 45°$

$\Rightarrow \angle POB = \angle PBO < 45°$

$\therefore$ $\angle BPO > 90°$

Hence, Δ PBO is isosceles Δ but not a right-angled triangle.

30. (d) According to question



Given: $\angle QOR = 110°$

$\angle OPR = 25°$

'O' is the circumcentre then

OP = OR = OQ

$\because$ $\angle OPR = \angle ORP = 25°$

In ΔOQR

$\Rightarrow \angle OQR + \angle ORQ + \angle QOR = 180°$

$\Rightarrow 2\angle ORQ = 180° - 110°$ (OQ = OR)

$\Rightarrow 2\angle ORQ = 70°$

$\Rightarrow \angle ORQ = \frac{70°}{2}$

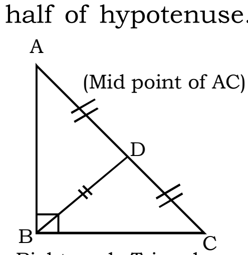
$\Rightarrow \angle ORQ = 35°$

$\therefore$ $\angle PRQ = \angle PRO + \angle ORQ$

$\angle PRQ = 60°$

31. (a) Only right angled triangle where circumcentre lies on hypotenuse (side)

For a right angled triangle, circumcentre is **midpoint of hypotenuse** and ircumradius is half of hypotenuse.



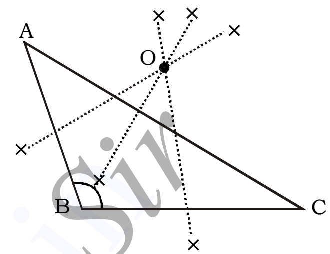
Right angle Triangle

BD = Median at hypotenuse

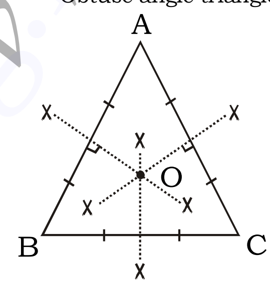
= Same as circumradius

= Half of hypotenuse

For an obtuse angled triangle, circumcentre lies outside the triangle



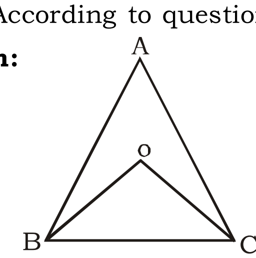
Obtuse angle triangle



Scalene Triangle

32. (d) According to question

Given:



$\angle C = 40°$ $\angle B = 70°$

$\therefore$ In ΔABC

$\angle A + \angle B + \angle C = 180°$

$\angle A = 180° - 40° - 70°$

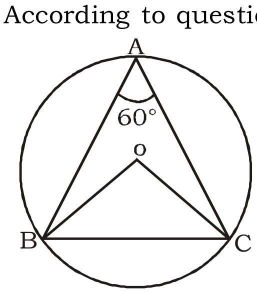
$\angle A = 70°$As we know that

$\therefore$ $\angle BOC = 2\angle A$

(O is a circumcentre)

$\angle BOC = 2 \times 70° = 140°$

33. (c) According to question



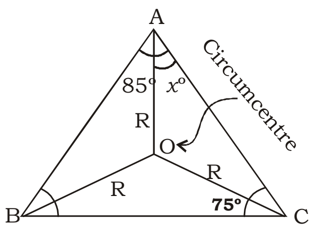
$\angle A = \angle B = \angle C = 60°$

$\angle BOC = 2\angle A$

$\angle BOC = 2 \times 60°$

$\angle BOC = 120°$

34. (a)



$\because$ O is circumcentre

So, OA = OB = OC = R (Radius)

$\because$ $\angle BAC = 85°$, $\angle BCA = 75°$

Then, $[\angle ABC = 180° - (\angle BAC + \angle BCA)]$

$\angle ABC = 180° - (85° + 75°)$

$\Rightarrow \angle ABC = 180° - 160°$

$\Rightarrow \angle ABC = 20°$

[Angle made by same chord at the centre is doubled than that of any other part of the circumference at same sector]

Then, $\angle AOC = 20°\times 2 = 40°$

$\because$ OA = OC = R

So, ΔAOC = Isosceles triangle

Then, $\angle OCA + \angle OAC + \angle AOC = 180°$

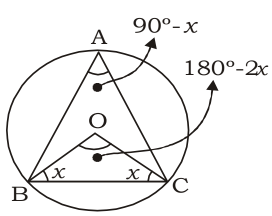
$x + x + \angle AOC = 180°$

$2x + 40° = 180°$

$x = 70°$

Therefore $\angle OAC = 70°$

35. (c)



In $\triangle$ BOC

OB = OC (circum radius)

$\angle OBC = \angle OCB = x$ (Let)

then,

$\Rightarrow \angle BOC = 180° - 2x°$

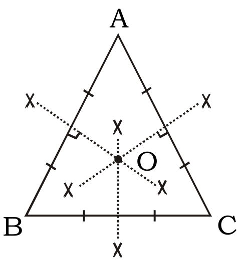
$\Rightarrow \angle BAC = \frac{\angle BOC}{2} = \frac{180-2x}{2}$

$\Rightarrow \angle BAC = 90 - x$

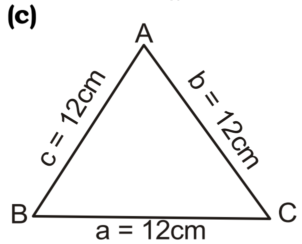
$\Rightarrow \angle OBC + \angle BAC = 90° - x + x = 90°$

36.(d) Perpendicular bisectors

Draw a line (called a "perpendicular bisector") at right angles to the midpoint of each side. Where all three lines intersect is the centre of a triangle's "circumcircle", called the "circumcentre":

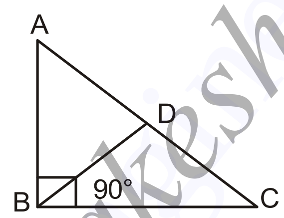


37. (c)



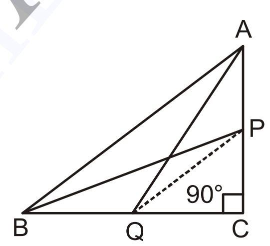
$$R = \frac{a}{\sqrt{3}} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

38.(c) In the right-angled triangle the length of median to the hypotenuse is half the length of the hypotenuse.



Hence, $BD = \frac{1}{2}AC = 3cm$

39.(d)



$\Rightarrow AQ^2 = AC^2 + QC^2$ ....(i)

$\Rightarrow BP^2 = BC^2 + CP^2$ ....(ii)

add (i) and (ii)

$\Rightarrow AQ^2 + BP^2 = (AC^2 + BC^2) + (QC^2 + CP^2)$

$$\Rightarrow AB^2 + \left(\frac{BC}{2}\right)^2 + \left(\frac{AC}{2}\right)^2$$

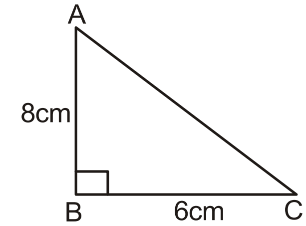
$$\Rightarrow AB^2 + \frac{1}{4}(BC^2 + AC^2)$$

$\Rightarrow (AQ^2 + BP^2)$

$$= AB^2 + \frac{1}{4}AB^2 = \frac{5}{4}AB^2$$

$\Rightarrow 4(AQ^2 + BP^2) = 5\,AB^2$

40.(a)

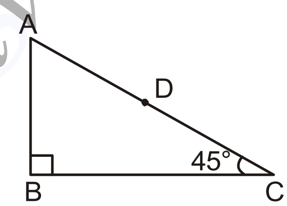


$$AC = \sqrt{6^2 + 8^2} = 10cm.$$

$\therefore$ circum radius $= \frac{10.}{2} = 5cm.$

i.e. mid-point of hypotenuse.

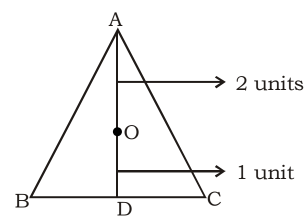
41.(a)



BD = AD = CD (mid-point of hypotenuse is circumcentre.)

$\therefore$ BD $= \frac{1}{2}(4\sqrt{2}) = 2\sqrt{2}$ units.

42. (c) According to question



Let AO = 2 units

OD = 1 unit

Given:-

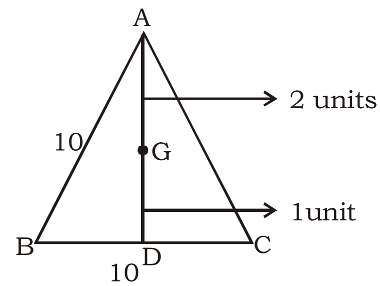
AO = 10 cm

$\therefore$ 2 units = 10 cm

1 unit = 5 cm

$\therefore$ OD = 5 cm

43. (b) According to question



Given:

AB = BC = CA = 10 cm

G = Centroid

AG = 2 units

GD = 1 unit

$\therefore$ AD = 3 units = Height

As we know that the height of the equilateral triangle is = $\frac{\sqrt{3}}{2}$ × 10 = $5\sqrt{3}$

$\therefore$ 3 units = $5\sqrt{3}$

1 unit = $\frac{5\sqrt{3}}{3}$

2 units = $\frac{5\sqrt{3}}{3} \times 2 = \frac{10\sqrt{3}}{3}$

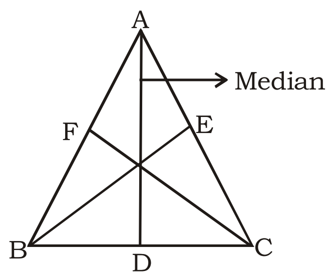
$\therefore$ AG = $\mathbf{\frac{10\sqrt{3}}{3}}$ cm

Alternate

AG (r_c) = $\frac{AB(a)}{\sqrt{3}}$

AG = $\frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$ cm

44. (a) According to question



Given: AD = BE = CF = median

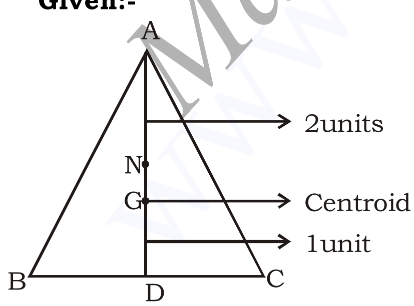
Then

AB = BC = CA

$\therefore$ The triangle is an equilateral triangle.

45. (a) According to question

Given:-



AD = 27 cm,

DN = 12 cm

As we know that

AG = 2 units,

GD = 1 unit

$\therefore$ AD = 3 units = 27 cm

3 units = 27 cm

1 unit = 9 cm

$\therefore$ GD = 9 cm

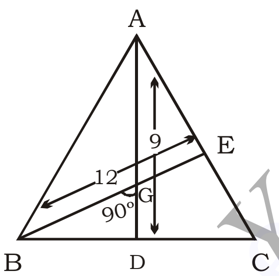
$\therefore$ GN = DN – GD

= 12 – 9 = 3 cm

46. (b) Medians AD and BE intersect at G on 90°

i.e $\angle AGB$ = 90° and ΔAGB will be a right angled triangle

We know, In a triangle centroid divides the medians in 2 : 1 Ratio



$\Rightarrow$ Then BG = $\frac{2}{3}$ × BE

BG = $\frac{2}{3}$ × 12

BG = 8

$\Rightarrow$ AG = $\frac{2}{3}$ × AD

$\Rightarrow$ AG = $\frac{2}{3}$ × 9

$\Rightarrow$ AG = 6 cm

In right angled triangle AB will be a hypotenuse

Using pythagoras theorem

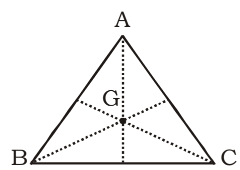
$\Rightarrow$ $(AB)^2 = (AG)^2 + (BG)^2$

$\Rightarrow$ $(AB)^2 = 6^2 + 8^2$

$\Rightarrow$ AB = 10 cm

Therefore, length of AB = 10 cm.

47. (d)



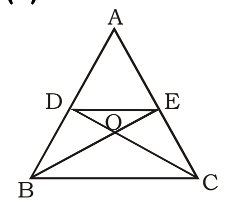
Area of ΔABC = 60 cm²

Area of ΔGBC = $2 \times \left(\frac{1}{6} \text{of } \Delta ABC\right)$

[A Median divides a triangle in two equal parts]

$\Rightarrow$ $2 \times \frac{1}{6} \times 60 = 20$ cm²

48. (a) In ΔODE & ΔBCO



$\frac{(OE)^2}{(OB)^2} = \frac{\text{Area of } \Delta ODE}{\text{Area of } \Delta BCO}$

$\frac{1}{4} = \frac{\text{Area of } \Delta ODE}{\text{Area of } \Delta BCO}$

Area of ΔBCO = $\frac{1}{3}$ Area of ΔABC

4 Area of ΔODE= $\frac{1}{3}$ of ΔABC

Area of ΔABC = 12 ×area of ΔODE

ΔODE :	ΔABC
1 :	12

49.(d) We know that the centroid of a triangle divides each median in the ratio of 2 : 1

$\therefore$ BG : BE = 2 : 3

$\Rightarrow$ BE = $\frac{3}{2}$BG = $\frac{3}{2} \times 6$

= 9 cm.

50.(b) By Pythagoras theorem,

PR = $\sqrt{PQ^2 + QR^2} = \sqrt{5^2 + 12^2} = 13cm$

$\because$ O is the centroid $\Rightarrow$ QM is median and M is the mid-point of PR .

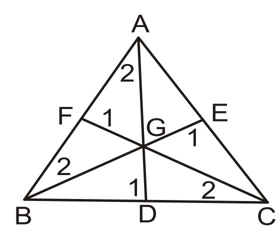
$\therefore$ QM = PM = $\frac{13}{2}$

$\because$ centroid divides median in ratio 2 : 1

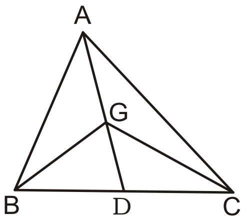
$\therefore$ OQ = $\frac{2}{3}$QM

$= \frac{2}{3} \times \frac{13}{2} = \frac{13}{3} = 4\frac{1}{3}cm$

51.(c)

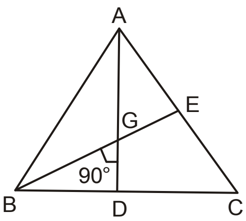


52.(d)



$AG = \frac{2}{3}\ AD = \frac{2}{3} \times 12 = 8cm.$

53.(c)



AD = 9cm

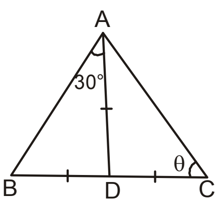
$\Rightarrow\ GD = \frac{1}{3} \times 9 = 3cm$

BE = 6cm

$\Rightarrow\ BG = \frac{2}{3} \times 6 = 4cm$

$\therefore\ BD = \sqrt{3^2 + 4^2} = 5cm.$

54.(d)



BD = CD = AD

$\because\ \angle BAD = 30°$

Now in $\triangle ABD$

$\angle ABD = 30°$ ($\because$ BD = AD)

$\because$ AD = DC

$\therefore\ \angle DAC = \angle DCA = \theta$ (let)

Now,

In $\triangle ABC$

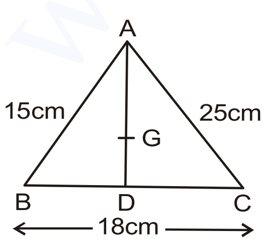
$\angle A + \angle B + \angle C = 180°$

$\Rightarrow\ 30° + \theta + 30° + \theta = 180°$

$\Rightarrow\ 2\theta = 120°$

$\Rightarrow\ \theta = \angle ACB = 60°$

55.(d)



$AB^2 + AC^2 = 2(AD^2 + BD^2)$

$\Rightarrow\ 225 + 625 = 2(AD^2 + 81)$

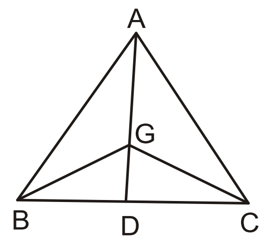
$\Rightarrow\ AD^2 = 344$

$AD = 2\sqrt{86}$ and

$GD = \frac{1}{3} AD$

$\Rightarrow\ GD = \frac{2}{3}\sqrt{86}cm$

56.(c)



AG = BC = $2x$ (let)

$\therefore$ GD = x ($\because$ centroid divides median in 2 : 1)

Now in $\triangle BDG$, BD = GD = x

$\therefore\ \angle DBG = \angle BGD = \theta_1$ (let)

Similarly in $\triangle DGC$, CD = GD = x

$\therefore\ \angle DCG = \angle DGC = \theta_2$ (let)

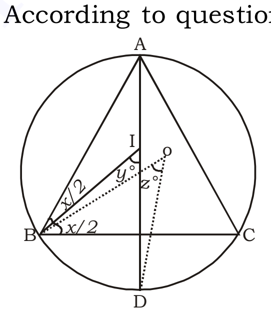
$\therefore\ \angle BGC = \theta_1 + \theta_2$

Now in $\triangle BGC$ = $\theta_1 + \theta_2 + (\theta_1 + \theta_2) = 180°$

$\Rightarrow\ \theta_1 + \theta_2 = 90°$

$\Rightarrow\ \angle BGC = 90°$

57. (c) According to question



Given: $\angle ABC = x°$

$\angle BID = y°$,

$\angle BOD = z°$

$\therefore$ 'I' is the incentre

$\therefore\ \angle ABI = \frac{1}{2}\angle ABC$

$\angle ABI = \frac{1}{2}x° = \frac{x°}{2}$

$\angle BAD = \frac{1}{2}\angle BOD$

$\therefore$ Angle subtended on the circumcircle is half the angle subtended on the centre of circle

$\angle BAD = \frac{1}{2}\angle BOD$

$\angle BAD = \frac{z°}{2}$

$\therefore\ y° = \frac{x°}{2} + \frac{z°}{2}$ (Exterior angle)

$\therefore\ y° = \frac{x° + z°}{2}$

$2y° = x° + z°$

Now,

$\frac{z° + x°}{y°} = \frac{2y°}{y°} = 2$

58.(b) $\angle BAC = 180° - \angle BOC$

$= 180° - 54° = 126°$

59.(b) In $\triangle ABC$, $HL \perp BC$ and $BN \perp HN$

Thus, the two altitudes HL and BN of $\triangle HBC$, intersect at A.

60.(c) $PR = \sqrt{PM^2 + MR^2}$

$= \sqrt{36 + 64} = 10cm$

$\Rightarrow\ PQ = \sqrt{QR^2 - PR^2}$

$= \sqrt{26^2 - 10^2} = 24cm$

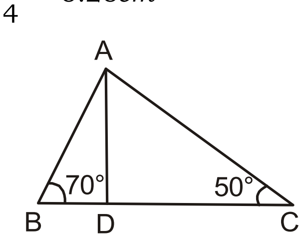
$\therefore$ ar ($\triangle PQR$) = $\frac{1}{2}$ (PR) (PQ)

$= \frac{1}{2} \times 10 \times 24 = 120cm^2$

61.(c) $\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow AB = \frac{3}{4} \times 7$

$= \frac{21}{4} = 5.25cm$

62.(a)



$\angle A + 70° + 50° = 180°$

$\Rightarrow\ \angle A = 60°$

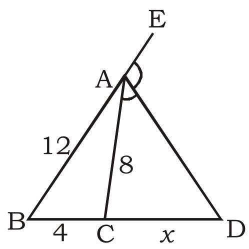
$\because \quad \dfrac{AB}{AC} = \dfrac{BD}{DC}$

$\therefore$ AD is the bisector of $\angle$BAC.

$\therefore \quad \angle$BAD

$= \dfrac{1}{2}\angle BAC = \dfrac{1}{2} \times 60^\circ = 30^\circ$

63. (c) According to the question, AD is the external bisector of $\angle$CAE By using external bisector theorem.



$\dfrac{AB}{AC} = \dfrac{BD}{CD}$

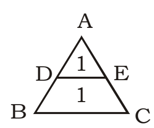
$\dfrac{12}{8} = \dfrac{4+x}{x}$

$12x = 32 + 8x$

$4x = 32$

$x = 8$

64. (b)



ar of ΔADE = 1,

ar of ΔABC= 2

$\dfrac{\text{ar of } \Delta \text{ ABC}}{\text{ar of } \Delta \text{ ADE}} = \dfrac{2}{1}$

$= \dfrac{\text{Side of } \Delta ABC\ (AB)^2}{\text{Side of } \Delta ADE\ (AD)^2}$

By square root $\dfrac{\sqrt{2}}{1} = \dfrac{AB}{AD}$

DB = AB – AD

$= \sqrt{2} - 1$

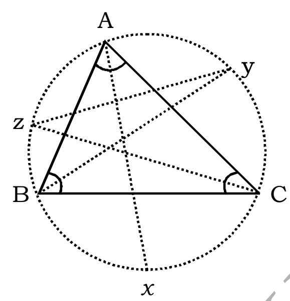
DB : AB = $\left(\sqrt{2} - 1\right) : \sqrt{2}$

65. (c) By property 12.

AB + AC> 2AD

3 + 5 >2AD $\Rightarrow$ AD<4

66. (b)



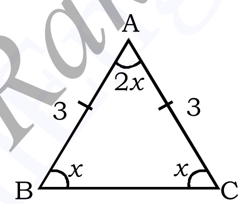
$\angle$ZOY = $\angle$BOC = 90 + $\dfrac{50}{2}$ = 115°

(Property 2)

In Δ ZOY,

$\angle$BYZ = 180– (30 + 115) = 35°

67. (b)



$x + x + 2x = 180^\circ$

$x = 45^\circ$

$\therefore \quad \angle A = 2 \times 45 = 90^\circ$

BC = $\sqrt{3^2 + 3^2} = 3\sqrt{2}$ cm.

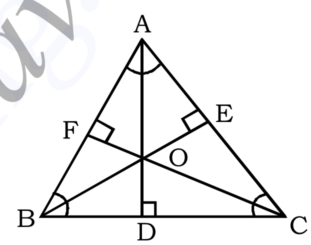
Inradius(r) = $\dfrac{3 + 3 - 3\sqrt{2}}{2}$

$= \dfrac{3\left(2 - \sqrt{2}\right)}{2}$

Circumradius(R) = $\dfrac{3\sqrt{2}}{2}$

$\dfrac{r}{R} = \dfrac{2 - \sqrt{2}}{\sqrt{2}} = \sqrt{2} - 1 : 1$

68. (c) In Δ OBF and Δ OCE,



$\angle$OFB = $\angle$OEC = 90°

and $\angle$BOF = $\angle$EOC

$\therefore$ Δ OBF ~ Δ OCE

Hence, $\dfrac{OB}{OC} = \dfrac{OF}{OE}$ or,

$\dfrac{OB}{OF} = \dfrac{OC}{OE}$

or, OB × OE = OF × OC

OF × OC = OB × OE

= (BE – OE) (OE)

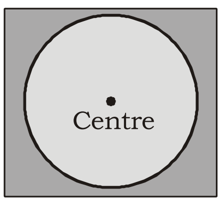
= (5 –2)×2 = 6

CHAPTER 24

CIRCLES, CHORDS & TANGENTS

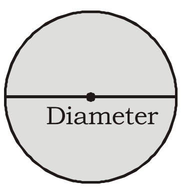
Circle

A circle is a shape with all points the same distance from its center. Some real world examples of a circle are a wheel, a dinner plate and the surface of a coin.



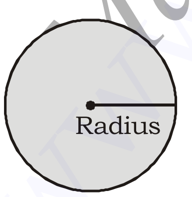
Diameter

The distance across a circle through the center is called the diameter. A real-world example of diameter is a 9-inch plate.

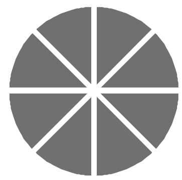


Radius

The distance between the centre and a point lies on the circumference of a circle is called radius. A real-world example of radius of a 9-inch plate.



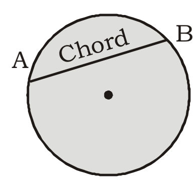
We can look at a pizza pie to find real-world examples of diameter and radius. Look at the pizza to the right which has been sliced into 8 equal parts through its center. A radius is formed by making a straight cut from the center to a point lies on the circumference of a circle.



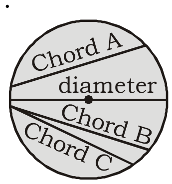
A straight cut made from a point on the circle, continuing through its center to another point on the circle, is a diameter. As you can see, a circle has many different radii and diameters, each passing through its center.

Chord

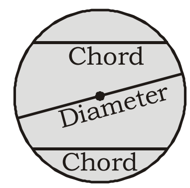
A chord is a line segment that joins two points on a curve. In geometry, a chord is often used to describe a line segment joining two endpoints that lie on a circle. The circle to the right contains chord AB. If this circle was a pizza pie, you could cut off a piece of pizza along chord AB. By cutting along chord AB, you are cutting off a segment of pizza that includes this chord.



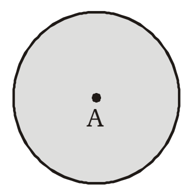
A circle has many different chords. Some chords pass through the center and some do not. A chord that passes through the center is called a diameter.



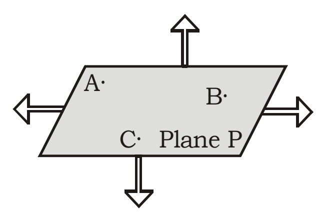
It turns out that a diameter of a circle is the longest chord of that circle since it passes through the center. A diameter satisfies the definition of a chord, however, a chord is not necessarily a diameter. This is because every diameter passes through the center of a circle, but some chords do not pass through the center. Thus, it can be stated, every diameter is a chord, but not every chord is a diameter.



Let's revisit the definition of a circle. A circle is the set of points that are equidistant from a special point in the plane. The special point is the center. In the circle to the right, the center is point A. Thus we have circle A.

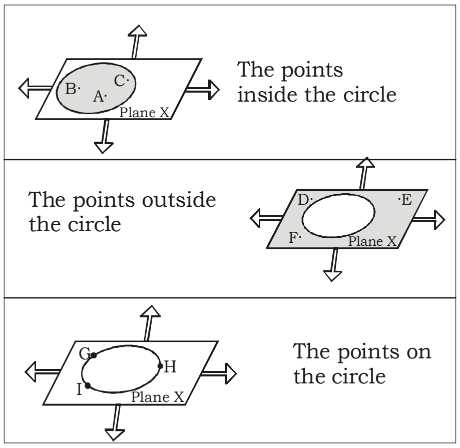


A plane is flat surface that extends without end in all directions. In the diagram to the right, Plane P contains points A, B and C.



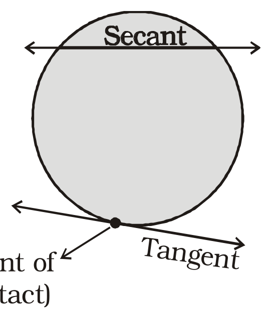
A circle divides the plane into three parts:

1. The points INSIDE the circle
2. The points OUTSIDE the circle
3. The points ON the circle



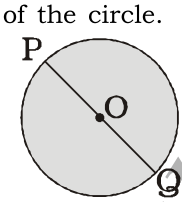
Secant

A line which intersects a circle in two distinct points is called a "Secant".

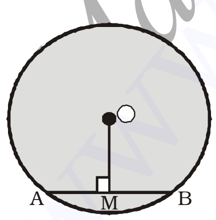


PROPERTIES OF CIRCLE

1. A chord which passes through the centre is called the diameter of the circle.It is a largest chord of the circle.

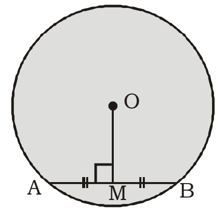


2. The perpendicular from the centre of a circle to a chord bisects the chord.



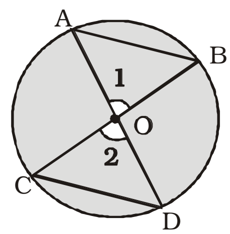
i.e. if OM $\perp$ AB, then AM = BM

3. **Converse of the above theorem:-** The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.



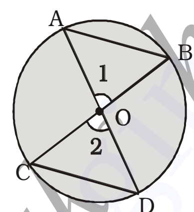
i.e. AM = MB, then OM $\perp$ AB.

4. Equal chords of a circle subtend equal angles at the centre.



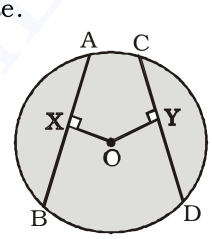
i.e. if AB = CD, then $\angle 1 = \angle 2$.

5. **Converse of the above theorem :** Angles subtended by two chords at the centre of a circle are equal then the chords are equal.



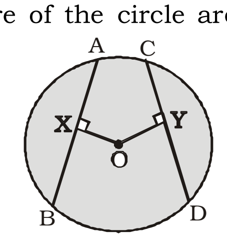
i.e. if $\angle 1 = \angle 2$, then AB = CD.

6. Equal chords of a circle are equidistance from the centre.



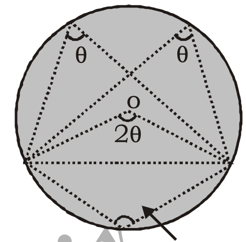
i.e. if AB = CD, OX $\perp$ AB and OY $\perp$ CD, then OX = OY.

7. **Converse of the above theorem :** chords equidistant from the centre of the circle are equal.



i.e, If OX $\perp$ AB, OY $\perp$ CD and OX = OY then AB = CD.

8. **Degree Measure Theorem:-** The angle subtended by an chord at the centre of a circle is twice the angle subtended by the chord at any point on the major arc.

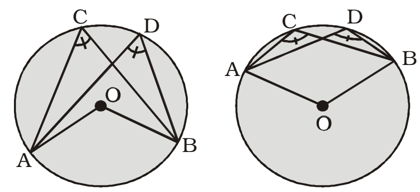


In a Minor Arc it is 180–θ

i.e $\angle x$ at the centre and $\angle y$ at the circumference made by the same arc AB, then

$$\boxed{\angle x = 2\angle y}$$

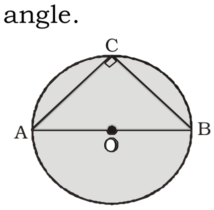
9. Angles in the same segment of a circle are equal.



i.e. $\angle$ACB = $\angle$ADB

(angles in same arc) or (angles in same segment)

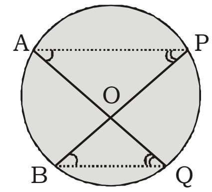
10. The angles in a semi circle is a right angle.



i.e $\angle$ACB = 90°.

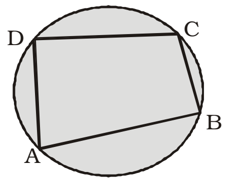
11. **Converse of the above theorem :** The circle, drawn with hypotenuse of a right triangle as diameter, passes through its opposite vertex.

12. If $\angle$APB = $\angle$AQB, and if P, Q are on the same side of AB, then A, B, Q, O & P are concylic i.e. lie on the same circle.



13. The sum of the either pair of the

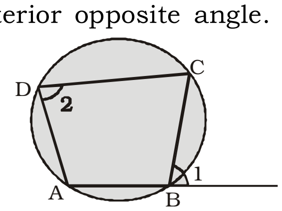
opposite angles of a cyclic quadrilateral is 180°.



i.e. $\angle A + \angle C = \angle B + \angle D = 180°$

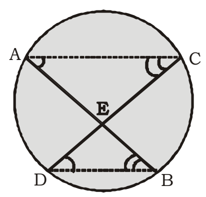
14. Converse of the above theorem : If the two angles of a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is 'cyclic'.

15. If a side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.



i.e. $\angle 1 = \angle 2$.

16. If two chords AB and CD intersect internally or externally at a point E, then



Then, AE×EB = CE ×ED

PROOF

In $\triangle$ ACE & $\triangle$ DBE,

$\angle$EAC = $\angle$EDB (Angle subtended by same chord)

$\angle$ECA = $\angle$EBD (Angle subtended by same chord)

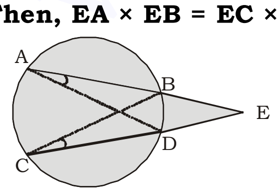
Then, $\Delta ACE \sim \Delta DBE$

$\therefore \dfrac{AE}{ED} = \dfrac{CE}{EB}$

$\Rightarrow AE \times EB = CE \times ED$

(b)

Then, EA × EB = EC × ED



PROOF

In $\triangle$ EAD & $\triangle$ ECB

$\angle$BCD = $\angle$BAD

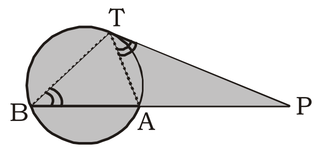
(Angle subtended by Same Chord)

$\angle$AED = $\angle$BED (common)

Then, $\triangle EAD \sim \triangle ECB$

$\therefore \dfrac{EA}{EC} = \dfrac{ED}{EB} \Rightarrow EA \times EB = CE \times ED$

17. If a secant and a tangent externally intersect each other.



Then, $PT^2 = PA \times PB$

PROOF

$\angle$ATP = $\angle$PBT (Alternate segment theorem)

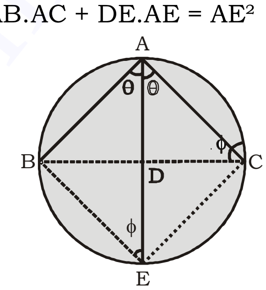
$\angle$BPT = $\angle$APT (common angle)

Then, $\triangle BTP \sim \triangle ATP$

$\therefore \dfrac{PT}{PA} = \dfrac{PB}{PT} \Rightarrow PT^2 = PA \times PB$

18. AE is angle bisector of $\angle$BAC then

AB.AC + DE.AE = AE^2



PROOF

$\angle AEB = \angle ACB = \phi$ (say)

[Angles by same chord]

$\angle BAD = \angle DAC = \theta$ (given)

$\therefore \triangle AEB \sim \triangle ACD$

$\Rightarrow \dfrac{AE}{AC} = \dfrac{AB}{AD} \Rightarrow AD = \dfrac{AB.AC}{AE}$

$\because$ AE = AD + DE, **AD = AE – DE......(i)**

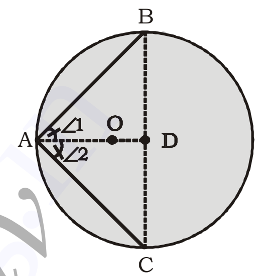
Put the value of AD

$AE - DE = \dfrac{AB.AC}{AE}$

$AE = \dfrac{AB.AC}{AE} + DE$

$\therefore AE^2 = AB.AC + DE.AE$

19. If two chords AB and AC of a circle are equal, then the bisector of $\angle$BAC passes through the centre O of the circle. $\angle 1 = \angle 2$



PROOF :-

Let, AD is angle bisector of $\angle$BAC then $\triangle BAD \cong \triangle CAD$

$$\left[\begin{array}{l} BA = CA \\ \angle BAD = \angle CAD \\ AD = AD \end{array}\right] SAS$$

$\therefore \angle BDA = \angle ADC = 90°$ (Angle at a straight line)

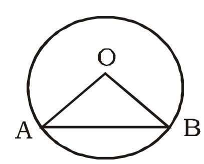
So, AD is the perpendicular bisector of the chord BC.

$\Rightarrow$ AD passes through the centre O.

TYPE I

Ex1. O is the centre of the circle. A chord is drawn equal to the radius. Find the centre angle.

Sol.

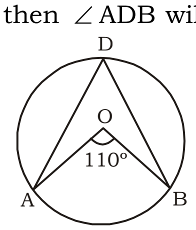


OA = OB = AB (given)

$\therefore$ $\triangle$ OAB is an equilateral triangle.

Hence, $\angle O = \angle A = \angle B = 60°$

Ex2. In the given figure, O is the centre of the circle and $\angle AOB = 110°$, then $\angle ADB$ will be:

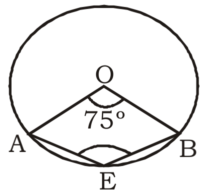


Sol. $\angle AOB = 2 \times \angle ADB$ [Centre angle of a circle is twice the angle of the major arc]

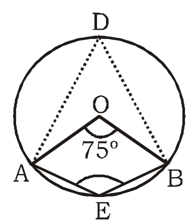
$110° = 2 \times \angle ADB$

$\therefore \angle ADB = 55°$

Ex3. In the given figure, O is the centre of the circle and $\angle AOB = 75°$, then $\angle AEB$ will be:



Sol.



$\angle AOB = 75°$

$\angle ADB = \dfrac{\angle AOB}{2}$ [Center angle of a circle is twice the angle of the major arc]

$= \dfrac{75°}{2} = 37.5°$

AEBD is a cyclic quadrilateral then,

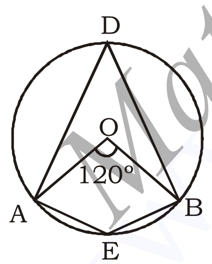
$\angle E + \angle D = 180°$

$\angle E + 37.5° = 180°$

$\therefore \angle E = 142.5°$

Ex4. In a circle, center angle is 120°. Find the ratio of major angle and minor angle.

Sol.



$\angle ADB = \dfrac{\angle O}{2} = \dfrac{120°}{2} = 60°$

[Center angle of a circle is twice the angle of the major arc]

AEBD is a cyclic quadrilateral then,

$\angle AEB + \angle ADB = 180°$

$\angle AEB = 180° - 60°$

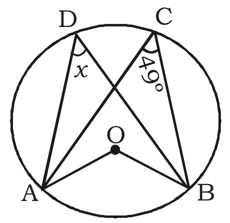
$\angle AEB = 120°$

Required ratio = Major angle : Minor angle

$= 120° : 60° = 2 : 1$

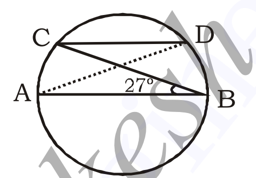
Ex5. In the given figure, O is the centre of the circle, then x is equal to:

Sol.



$\angle ADB = \angle ACB = 49°$ [Angle in the same segment of a circle are equal]

Ex6. AB is the diameter of the circle. AB is parallel to CD. If $\angle ABC = 27°$, then find $\angle BCD$ & $\angle CDA$?

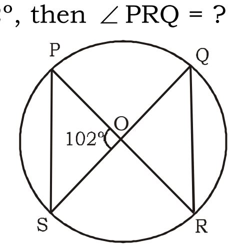


Sol. $\angle ABC = \angle BCD = 27°$ [Alternate angle]

$\angle CDA = \angle CBA = 27°$

[Angle in the same segment of a circle are equal]

Ex7. In the given figure O is the centre of the circle. If $\angle POS = 102°$, then $\angle PRQ$ = ?



Sol. In $\triangle OPS$,

$\angle O + \angle P + \angle S = 180°$

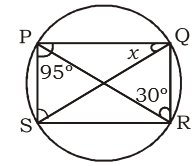
$102° + \angle S + \angle S = 180°$ [OP = OS = OQ = OR = Radius]

$2\angle S = 78°$

$\angle S = 39°$

$\angle PSQ = \angle PRQ = 39°$ [Angle is the same segment of a circle are equal]

Ex8. The value of x will be :



Sol. $\angle PRQ = \angle PSQ = 30°$ [Angle in the same segment of a circle are equal]

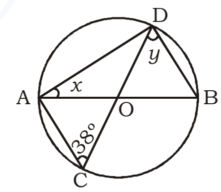
In $\triangle QPS$,

$\angle Q + \angle P + \angle S = 180°$

$\Rightarrow x + 95° + 30° = 180°$

$\therefore x = 55°$

Ex9. In the given figure, AB & CD is a diameter of circle, $\angle ACD = 38°$. Find x & y.



Sol. In $\triangle ACD$,

$\angle A + \angle C + \angle D = 180°$ [$\angle A = 90°$, It is a semicircle]

$90° + 38° + \angle D = 180°$

$\angle D = 52°$

$\angle ADB = 90°$ [Semicircle]

$\therefore y = 90° - 52° = 38°$

$\angle DBA = \angle ACD = 38°$ [Angle in the same segment of a circle are equal]

In $\triangle ADB$

$\angle A + \angle D + \angle B = 180°$

$x + 90° + 38° = 180°$

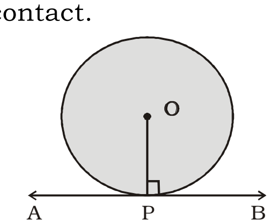
$\therefore x = 52°$

THEOREM ON TANGENTS

Tangent

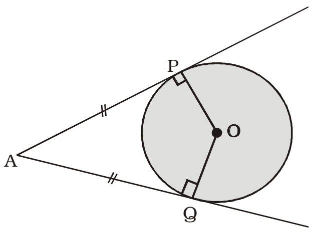
A line meeting a circle in only one point is called a tangent.

1. A tangent at any point of a circle is perpendicular to the radius through the point of contact.



i.e. If AB is a tangent at P, then OP $\perp$ AB. (converse of this theorem is also true)

2. The lengths of two tangents, drawn from an external point to a circle, are equal.



i.e. AP = AQ.

3. When tangents drawn from an external point to a circle

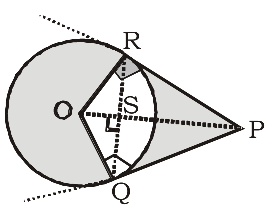
(a) Tangents are equal in length

(b) $\angle$QPO= $\angle$OPR

(c) $\angle$QOP= $\angle$POR

(d) $\frac{PQ}{QO} = \frac{QS}{SO}$

Proof:-



Δ OQP $\cong$ Δ ORP

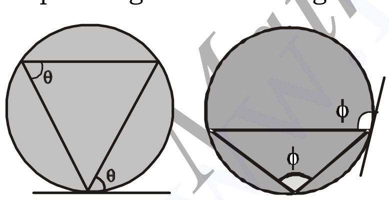
OQ = OR = Radius

OP = OP

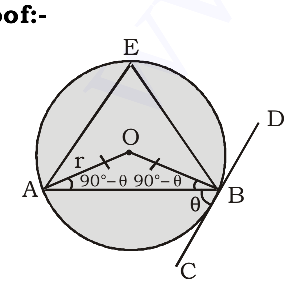
$\therefore$ $\angle$PQO = $\angle$ORP = 90°

4. Alternate Segment Theorem

The angle between a chord and a tangent drawn at end point of chord is equal respectively to the angle formed in the corresponding alternate segments.



Proof:-



$\angle$OBC = 90°

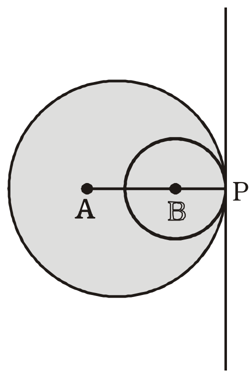
Let $\angle$ABC = θ then $\angle$OBA = 90° – θ

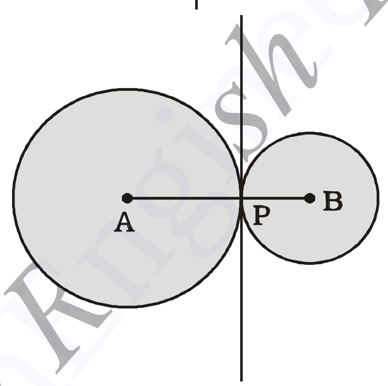
$\angle$OAB = $\angle$OBA = 90° – θ (OA = OB, Radius)

$\angle$AOB = 180° – (90°–θ +90°–θ)=2θ

$\therefore$ $\angle$AEB = θ

5. If two circles touch each other internally or externally the point of contact lies on the line joining their centres.





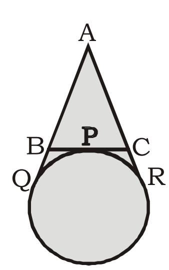
i.e. A, B and P are collinear.

Distance between their centres (d)

(i) When touch internally, d = AP – BP

(ii) When touch externally, d = AP + BP

6. A circle externally touch side BC of a ΔABC at P, AB produced at Q and AC produced at R. If QA = a cm, then **perimeter of ΔABC is = 2a**



PROOF:-.

Perimeter = 2×AQ = 2a

AQ = AR = a

AQ = AB + BQ = AB + BP

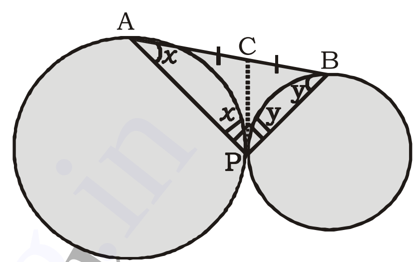
($\because$ BQ = BP)

AR = AC + CR = AC + CP

($\because$ CR = CP)

Perimeter = (AB+BP) + (AC+CP) = a + a = 2a

7.



Two circles externally touch each other at P. AB is direct common tangent (DCT) of the circles. If $\angle$BAP = x then find $\angle$ABP = ?

Remember, $\angle$APB is always right angle

$\angle$ABP = 90° – x

PROOF:-

CA = CP (tangent of circle).....(i)

CP = CB (tangent of circle)......(ii)

From (i) & (ii)

So, CA = CP = CB

In ΔACP,

$\angle$CAP = $\angle$CPA = x

In ΔCPB,

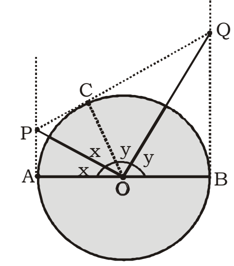
$\angle$CBP = $\angle$CPB = y

In ΔAPB

$\therefore$ x + (x + y) + y = 180°

$\Rightarrow$ x + y = 90°

8. AB is a diameter of a circle. Two tangents drawn at A & B Tangent drawn at any point C of the circle meet both tangents at P & Q. Find angle $\angle$POQ = ?



PROOF:-

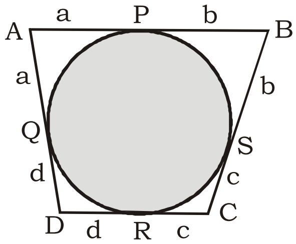
Remember, $\angle$POQ = 90°

ΔPOA $\cong$ ΔPOC

ΔQCO $\cong$ ΔQBO

$\therefore$ $x + x$ + y + y = 180° $\Rightarrow$ x + y = 90°

9. If a quadrilateral circumscribed a circle.(Not general case/only for some specific quadrilateral)



then, sum of opposite sides are equal

AB + CD = AD + BC

PROOF

Length of tangents drawn from an external point to a circle are equal.

AP = AQ = a

BP = BS = b

DQ = DR = d

CR = CS = c

L.H.S.

AB + DC = (a+b) + (d+c).....(i)

R.H.S.

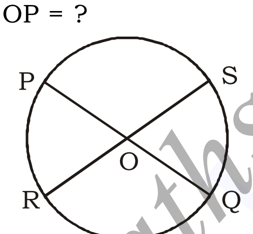
AD + BC = (a+d) + (b+c)......(ii)

$\therefore$ L.H.S. = R.H.S.

AB + CD = AD + BC

TYPE II

Ex1. In the given figure, PQ = 25 cm, OS = 12 cm, if OS = OR then OP = ?



Sol. Let, OP = x

$\therefore$ OQ = 25 – x

$\because$ OP × OQ = OR × OS

$\therefore$ $x \times (25 - x) = 12 \times 12$

$\Rightarrow$ $25x - x^2 = 144$

$\Rightarrow$ $x^2 - 16x - 9x + 144 = 0$

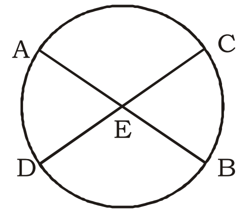
$\Rightarrow$ $x(x - 16) - 9(x - 16) = 0$

$\Rightarrow$ $(x - 16)(x - 9) = 0$

$\Rightarrow$ $x = 16, x = 9$

$\Rightarrow$ OP = 16, 9

Ex2. In the given figure, AE = 7, EB = 6 cm, EC = 14 cm. Find DE = ?

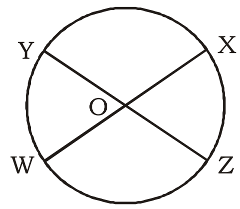


Sol. $\because$ AE × EB = DE × EC

$\therefore$ 7 × 6 = DE × 14

$\Rightarrow$ DE = 3 cm

Ex3. In the given figure, OY : OX = 5 : 4, OZ = 40 cm, YZ = 60 cm. Find WX?



Sol. OY = YZ – OZ = 60 – 40 = 20 cm

$\Rightarrow$ $\dfrac{OY}{OX} = \dfrac{5}{4} = \dfrac{20}{OX}$

$\Rightarrow$ OX = 16 cm

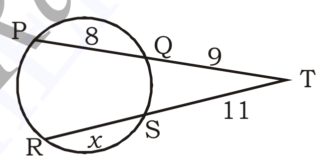
$\because$ OX × OW = OY × OZ

$\Rightarrow$ 16 × OW = 20 × 40

$\Rightarrow$ OW = 50 cm

$\therefore$ WX = OW + OX = 50 + 16 = 66 cm

Ex4. In the given figure, the value of x



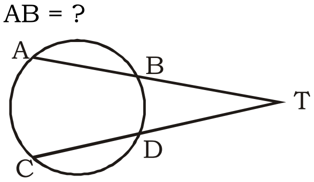
Sol. $\because$ QT × PT = ST × RT

$\therefore$ 9 × (9 +8) = 11 × (11 + x)

$\Rightarrow$ $\dfrac{9 \times 17}{11} = 11 + x$

$\Rightarrow$ $x = \dfrac{153 - 121}{11} = \dfrac{32}{11} = 2\dfrac{10}{11}$

Ex5. In the given figure, BT = CD, BT = 7 cm, DT = 21 cm, find AB = ?



$\because$ AT × BT = DT × CT

$\Rightarrow$ AT × 7 = 21 × 28 ($\because$ CT = CD + DT = 7 + 21 =28)

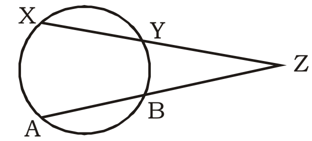
$\Rightarrow$ AT = 28 × 3 = 84 cm

$\Rightarrow$ AB = AT – BT

$\Rightarrow$ AB = 84 – 7

$\Rightarrow$ AB = 77 cm

Ex6. In the given figure, XY : YZ = 3 : 4, AB = 15 cm, BZ = 20 cm. Find XZ.



Sol. XZ × YZ = BZ × AZ

$\Rightarrow$ $7x \times 4x = 20 \times 35$

$\Rightarrow$ $x^2 = 25$

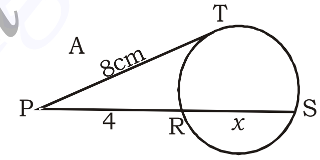
$\Rightarrow$ $x = 5$

$\Rightarrow$ XZ = $3x + 4x$

$\Rightarrow$ XZ = $7x$ = 7 × 5

$\Rightarrow$ XZ = 35

Ex7. The value of x is



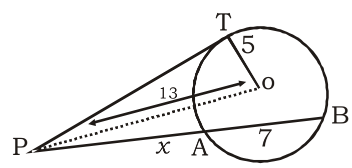
Sol. $PT^2 = PR \times PS$

$\Rightarrow$ $8^2 = 4 \times (x + 4)$

$\Rightarrow$ $x + 4 = 16$

$\Rightarrow$ $x = 12$ cm

Ex8. The value of x is



In $\triangle$PTO,

$\Rightarrow$ $PT^2 + OT^2 = OP^2$

$\Rightarrow$ $PT^2 = 169 - 25$

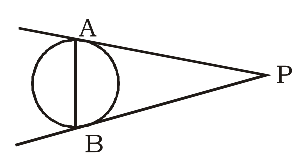
$\Rightarrow$ PT = 12 cm

$\Rightarrow$ $PT^2 = PA \times PB$

$\Rightarrow$ $144 = x(x + 7)$

$\Rightarrow$ $x = 9$

Ex9. In the given figure, PA and PB are tangents from a point P to a circle such that PA = 16 cm and $\angle$APB = 60°. What is the length of the chord AB?



Sol. AP = PB [Tangent of the same exterior point]

($\angle$PAB = $\angle$PBA)

In $\triangle$APB

$\angle A + \angle P + \angle B = 180°$

$\Rightarrow$ $\angle A + 60° + \angle A = 180°$

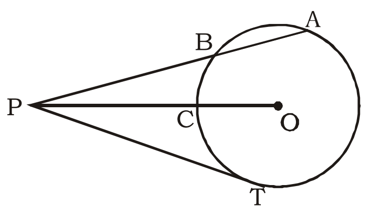
$\Rightarrow \quad 2\angle A = 120^\circ$

$\Rightarrow \quad \angle A = 60^\circ$

$\Rightarrow \quad \angle B = 60^\circ$

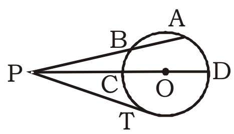
$\therefore \quad \Delta APB$ is an equilateral Δ.
Hence, AP = PB = AB = 16 cm.

Ex10.



In the figure, PT is a tangent to a circle of radius 6 cm. If P is at a distance of 10 cm from the centre O and PB = 5 cm, then what is the length of the chord BA?

Sol.



PD = PO + OD, 10 + 6 = 16 cm
PC = PO – OC, 10 – 6 = 4 cm
(OD = OC = 6 Radius)

$\because$ PB × PA = PC × PD

$\therefore$ 5 × PA = 4 × 16

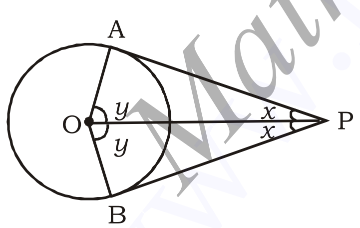
$\Rightarrow \quad PA = \dfrac{64}{5}$

$\because$ AB = PA – PB

$= \dfrac{64}{5} - 5$

$= \dfrac{39}{5} = 7.8cm$

Ex11. If x = 30°, then y = ?



Sol. $\angle A = \angle B = 90^\circ$ [Tangent of the circle]

In ΔAOP,

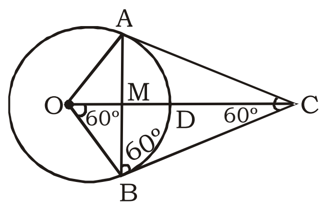
$\angle A + \angle O + \angle P = 180^\circ$

$\Rightarrow \quad 90^\circ + y + 30^\circ = 180^\circ$

$\therefore$ y = 60°

Ex12. O is the centre of the circle and OA and OB is radius. $\angle AOB = 120^\circ$. Two tangents A & B are drawn intersect each other at point C. If D is a point divided OC is equally, then the ratio of D divides OC?

Sol.



$\Delta OAM \cong \Delta OBM$

$\angle AOC = \angle BOC = \dfrac{120^\circ}{2} = 60^\circ$

$\angle A = \angle B = 90^\circ$ [tangents]

$\angle CAB = \angle CBA = 90^\circ - \angle OAM$

In Δ ABC,

$\angle ACB = 180^\circ - \angle AOB = 180^\circ - 120^\circ = 60^\circ$

$\Rightarrow$ CA = CB (tangent of circle)

$\Rightarrow \quad \angle CAB = \angle CBA$

Now, $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad \angle A + \angle A + \angle C = 180^\circ$

$\Rightarrow \quad 2\angle A = 180^\circ - 60^\circ$

$\Rightarrow \quad 2\angle A = 120^\circ$

$\angle A = 60^\circ$

So,

$\Rightarrow \quad \angle A = \angle B = \angle C = 60^\circ$

Δ ABC is an equilateral Δ.

In Δ AMC,

$\Rightarrow \quad \cos 30^\circ = \dfrac{CM}{CA}$

$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{CM}{\sqrt{3}r}$

$\Rightarrow \quad CM = \dfrac{3}{2}r$

In ΔOAM

$\Rightarrow \quad \cos 60^\circ = \dfrac{OM}{OA}$

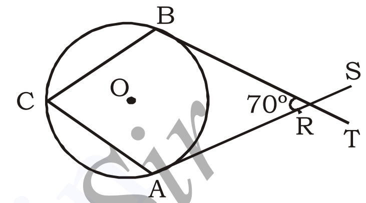
$\Rightarrow \quad \dfrac{1}{2} = \dfrac{OM}{r}$

$\Rightarrow \quad OM = \dfrac{r}{2}$

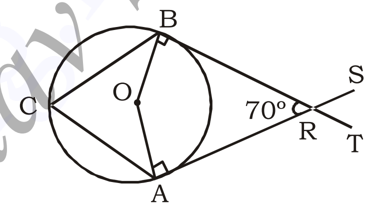
OC = OM + CM = 2r

$\therefore$ OD : DC = 1 : 1

Ex13. In the given figure, BT & AS are two tangents. Then $\angle C$ is:



Sol.



In ◻AOBR,

$\angle A + \angle O + \angle B + \angle R = 360^\circ$

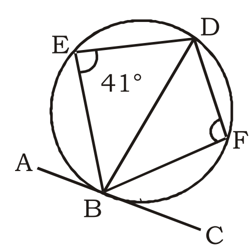
$90^\circ + \angle O + 90^\circ + 70^\circ = 360^\circ$

[$\because \angle A = \angle B = 90^\circ$ tangents]

$\angle O = 110^\circ$

$\therefore \angle BCA = \dfrac{\angle BOA}{2} = 55^\circ$

Ex14. Find $\angle ABD$



Sol. ◻DEBF is a cyclic quadrilateral,

$\because \angle E + \angle F = 180^\circ$

($\because$ Sum of opposite angles of cyclic quadrilateral is 180°)

$\Rightarrow \quad 41^\circ + \angle F = 180^\circ$

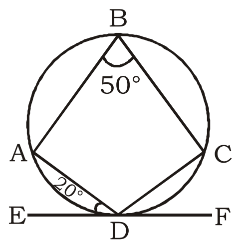
$\Rightarrow \quad \angle F = 139^\circ$

$\angle DFB = \angle ABD = 139^\circ$

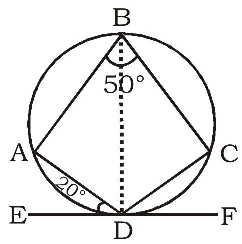
(alternate segment angle)

Ex15. In the given figure, $\angle ABC = 50^\circ$, $\angle ADE = 20^\circ$,

Find $\angle CDF$ = ?



Sol.

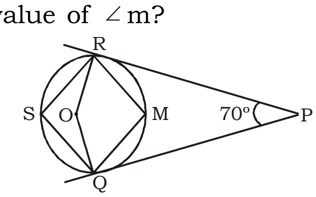


$\angle ABD = \angle ADE = 20°$ [Alternate segment]

$\angle DBC = 50° - 20° = 30°$

$\angle DBC = \angle CDF = 30°$ [Alternate segment]

Ex–16 In the given figure, Find the value of $\angle m$?



Sol. In ⏢ PQOR,

$\because \quad \angle O + \angle R + \angle P + \angle Q = 360°$

$\Rightarrow \quad \angle O + 90° + 70° + 90° = 360°$

$\Rightarrow \quad \angle O = 110°$

$\therefore \quad \angle QSR = \dfrac{\angle O}{2} = 55°$

$\because \quad \angle S + \angle M = 180°$

$\therefore \quad \angle M = 125°$

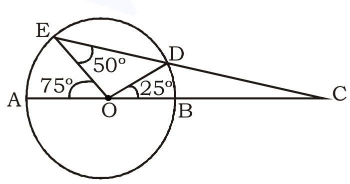
Alternate

$\angle m = 90° + \dfrac{\angle P}{2}$

$= 90° + \dfrac{70°}{2}$

$\angle m = 125°$

Ex–17 In the given circle, AB is a diameter. $\angle BOD = 25°$ and $\angle EOA = 75°$, then $\angle ECA = ?$



Sol. $\angle AOE + \angle EOD + \angle DOB = 180°$

[$\because$ AOC is a Straight Line]

$\Rightarrow \quad 75° + \angle EOD + 25° = 180°$

$\Rightarrow \quad \angle EOD = 80°$

$\because \angle E + \angle EOD = \angle ODC$

[Sum of two interior angle is equal to opposite of exterior angle]

$\therefore \; 50° + 80° = \angle ODC$

$\Rightarrow \quad \angle ODC = 130°$

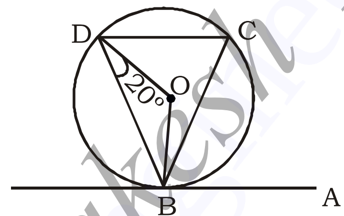
In $\triangle$ ODC,

$\angle O + \angle D + \angle C = 180°$

$\Rightarrow \quad 25° + 130° + \angle C = 180°$

$\Rightarrow \quad \angle C = 25°$

Ex–18 In the given figure, O is the centre of the circle and AB is tangent $\angle ODB = 20°$ and $\angle BDC$ and $\angle ABD$ are supplementary to each other then find $\angle OBC$



Sol. $\angle ODB = \angle OBD = 20°$ [$\because$ OD = OB = radius]

$\angle ABD = \angle ABO + \angle OBD$

$\Rightarrow \angle ABD = 90° + 20° = 110°$

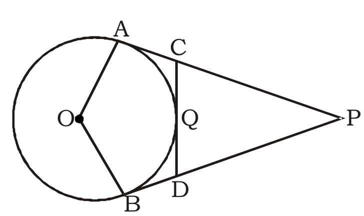
$\because \quad \angle BDC + \angle ABD = 180°$ (supplementary angle) (given)

$\Rightarrow \quad \angle BDC = 180° - 110° = 70°$

$\therefore \angle ABC = \angle BDC = 70°$ [Alternate segment]

$\therefore \angle OBC = 90° - 70° = 20°$

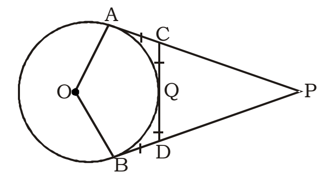
Ex–19



In the given, circle with centre O have two tangents PA, PB. CD is also tangent, then

(a) PC + PD = CD
(b) 3PB = PD + DC + PC
(c) 4PA = PD + CD + PC
(d) 2PB = PD + CD + PC

Sol.



As template question (4)

PA = PC + CA = PC + CQ

($\because$ CA = CQ)

PB = PD + DB = PD + DQ

($\because$ DB = DQ)

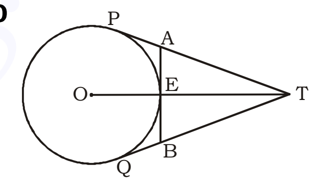
& PA = PB

$\therefore$ 2PB = (PC + CQ) + (PD + DQ)

= PC + CD + PD

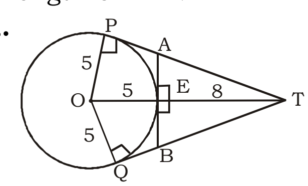
($\because$ CD = CQ + DQ)

Ex–20



In the figure given above, from a point T, 13cm. away from the centre O of a circle of radius 5cm., tangents PT and QT are drawn. What is the length of AB ?

Sol.



ET = OT – OE = 13 – 5 = 8 cm

In $\triangle$ OPT,

By pythagoras theorem,

$TP = \sqrt{OT^2 - OP^2} = \sqrt{13^2 - 5^2}$

=12cm

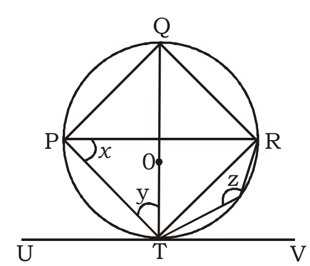
$\triangle TPO \sim \triangle TEA$

$\dfrac{TP}{TE} = \dfrac{PO}{EA}$

$\dfrac{12}{8} = \dfrac{5}{EA} \Rightarrow EA = \dfrac{10}{3}$

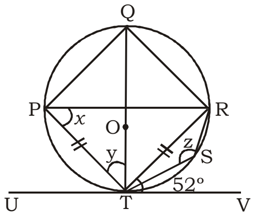
Hence, $AB = 2EA = \dfrac{20}{3}$ cm.

Ex–21



In the figure given above, O is the centre of the circle. The line UTV is a tangent to the circle at T, $\angle VTR = 52°$ and ΔPTR is an isosceles triangle such that TP = TR. What is $\angle x + \angle y + \angle z$ equal to?

Sol.



$x = \angle VTR = 52°$ (By alternate segment theroem)

$z = 180° - x = 180° - 52°$

$= 128°$ (Since PTSR is a cyclic Quadrilateral)

In ΔPTR,

PT = TR (given)

$\therefore$ $\angle PRT = x = 52°$

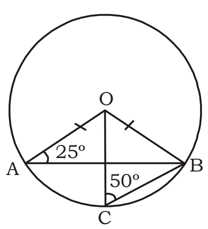
$\angle PTU = \angle PRT = 52°$ (By alternate Segment theorem)

$\therefore$ $y = 90° - \angle PTU$

$= 90° - 52° = 38°$

$\Rightarrow$ $x + y + z = 52° + 38° + 128° = 218°$

Ex–22 In the given figure, $\angle OCB = 50°$ and $\angle OAB = 25°$. Then the value of the $\angle AOC$ is :



Sol. $\angle OAB = \angle OBA = 25°$

[OA = OB= radius]

$\angle OCB = \angle OBC = 50°$

[OB = OC = radius]

In ΔCOB,

$\angle COB + \angle OCB + \angle OBC = 180°$

$\angle COB + 50° + 50° = 180°$

$\angle COB = 80°$

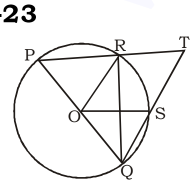
In ΔAOB,

$\angle AOB = 180° - 25° - 25° = 130°$

$\therefore$ $\angle AOC = \angle AOB - \angle COB$

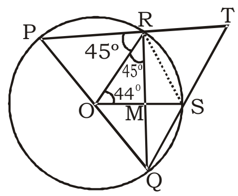
$= 130° - 80° = 50°$

Ex–23



In the figure given below, PQ is a diameter of the circle whose centre is at O. If $\angle ROS = 44°$ and OR a bisector of $\angle PRQ$ then what is the value of $\angle PTQ$?

Sol.



In ΔORS, OR = OS = radius

$\Rightarrow$ $\angle ORS = \angle OSR = \dfrac{180° - 44°}{2} = 68°$

$\angle PRS = 45° + 68° = 113°$

$\angle PQS = 180° - \angle PRS = 180° - 113° = 67°$ (By property of cyclic Quadrilateral)

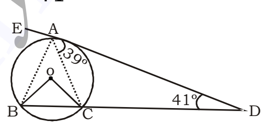
In ΔOPR, OP = OR $\Rightarrow$ $\angle PRO = 45°$

In ΔPQT, $\angle PTQ = 180° - \angle TPQ - \angle PQT$

$= 180° - 45° - 67° = 68°$

Ex–24 A, B & C are three points on a circle such that a tangent touches the circle at A and intersects the extended part of chord BC at D. Find the central angle made by chord BC. If $\angle CAD = 39°$, $\angle CDA = 41°$

Sol.



$\angle ACB = \angle CAD + \angle CDA$ [Sum of two Interior angle is equal to opposite of exterior angle]

$\angle ACB = 39° + 41° = 80°$

$\angle BAE = \angle BCA = 80°$

[Alternate segment]

$\angle EAB + \angle BAC + \angle CAD = 180°$

[Linear angle]

$80° + \angle BAC + 39° = 180°$

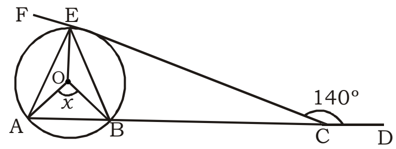
$\angle BAC = 61°$

$\therefore$ $\angle BOC = 2 \times \angle BAC$

[Center angle is twice the angle subtended by the major arc]

$= 2 \times 61° = 122°$

Ex–25 In the given figure, BE = BC, $\angle ECD = 140°$, final $\angle AOB$ = ?



Sol. $\angle BEC = \angle BCE$ [$\because$ BE = BC]

$\angle BCE = 180° - 140°$

$\angle BCE = 40°$

$\angle EBA = \angle BEC + \angle BCE$

$\angle EBA = 40° + 40°$

$\angle EBA = 80°$

$\angle OEB = 90° - \angle BEC$

$= 90° - 40°$

$= 50°$

$\angle OEB = \angle OBE = 50°$ [$\because$ OB = OE= radius]

$\angle OBA = \angle ABE - \angle OBE$

$\angle OBA = 80° - 50°$

$\angle OBA = 30° = \angle OAB$ [$\because$ OA = OB = radius]

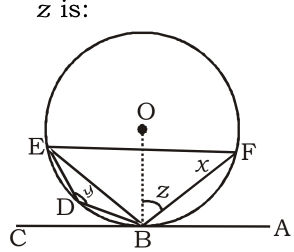
In ΔAOB,

$\angle A + \angle O + \angle B = 180°$

$30° + \angle O + 30° = 180°$

$\therefore$ $\angle O = 120°$

Ex–26. In the given figure, if BF = BE, $\angle EBC = 44°$, then $x + y + z$ is:



Sol. $\angle EBC = \angle EFB = x = 44°$ (alternate segment)

$\Rightarrow$ $\angle BEF = \angle BFE = 44°$ [$\because$ BF = BE]

In Δ BEF,

$\Rightarrow$ $\angle B = 180° - 44° - 44° = 92°$

$\Rightarrow$ $\angle CBF = \angle CBO + \angle OBF$

$\Rightarrow$ $= 90° + z$..................**(i)**

$\Rightarrow$ $\angle CBF = \angle CBE + \angle EBF$....**(ii)**

(i) = (ii)

$\Rightarrow$ $90° + z = \angle CBE + \angle EBF$

$\Rightarrow$ $\angle Z = \angle CBE + \angle EBF - 90°$

$\Rightarrow$ $\angle Z = 44° + 92° - 90°$

$\Rightarrow$ $\therefore$ $\angle Z = 46°$

In $\square$ BDEF,

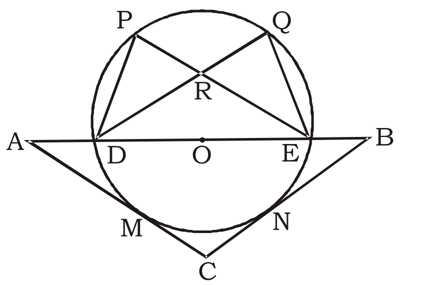
$\Rightarrow \quad \angle D + \angle F = 180°$

$\Rightarrow \quad \angle D = 180° - 44°$

$\Rightarrow \quad \angle D = 136° = y$

$\Rightarrow \quad \therefore x + y + z = 44° + 136° + 46° = 226°$

Ex– 27. ABC is an isosceles triangle and AC, BC are the tangents at M & N respectively. DE is the diameter of the circle. $\angle$ADP = $\angle$BEQ = 100°. What is the value of $\angle$PRD?



Sol. $\angle PDB = 180° - \angle PDA$ (linear angle)

$= 180° - 100° = 80°$ .....(i)

$\angle QEA = 180° - \angle QEB$

$= 180° - 100° = 80°$........(ii)

From (i) & (ii)

$\angle PDB = \angle QEA = 80°$

$(\because \angle DPE = \angle DQE = 90°)$

Angle in the semicircle

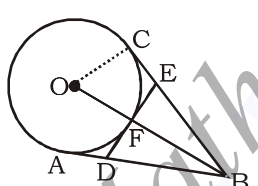
$\therefore \quad \angle PED = \angle QDE = 10°$

$\therefore \quad \angle DRE = 180° - (10° + 10°) = 160°$

$\therefore \quad \angle PRD = 180° - \angle DRE$ (Linear angle)

$= 180° - 160° = 20°$

Ex–28.



In the given figure, BE = 9 cm, BF = 6 cm, Find OB.

Sol. In ΔBEF,

$EF^2 = BE^2 - BF^2$

$\Rightarrow \quad EF^2 = 9^2 - 6^2$

$\Rightarrow \quad EF^2 = 45$

$\Rightarrow \quad EF = 3\sqrt{5}$

$CE = EF = 3\sqrt{5}$ [tangents of the circle]

$BC = BE + EC$

$\Rightarrow \quad BC = 9 + 3\sqrt{5}$

In ΔOBC,

$OB^2 = OC^2 + BC^2$

$(r + 6)^2 = r^2 + (9 + 3\sqrt{5})^2$

$r^2 + 36 + 12r = r^2 + 81 + 45 + 54\sqrt{5}$

$12r = 90 + 54\sqrt{5}$

$2r = 15 + 9\sqrt{5}$

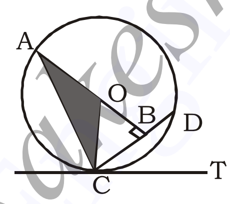
$r = \dfrac{15 + 9\sqrt{5}}{2}$

$\because OB = OF + BF$

$OB = \dfrac{15 + 9\sqrt{5}}{2} + 6$

$\Rightarrow \quad OB = \dfrac{27 + 9\sqrt{5}}{2}$ cm

Ex–29. In the given diagram CT is tangent at C, making an angle of $\dfrac{\pi}{4}$ with CD, O is the centre of the circle. CD = 10 cm. What is the perimeter of the shaded region (ΔAOC) approximately



Sol. $\angle OCT = 90°$ (tangent angle)

$\angle DCT = 45°$ (given)

$\therefore \angle OCB = \angle OCT - \angle DCT = 90° - 45° = 45°$

$\therefore \angle COB = 45°$ (BOC is a right angled triangle)

In ΔBOC isosceles right angle triangle

$\therefore \angle AOC = 180° - 45° = 135°$

Now, CD = 10 $\Rightarrow$ BC = 5 cm = OB

$OC^2 = OB^2 + BC^2 = (5)^2 + (5)^2 = 50$

$\Rightarrow \quad OC = 5\sqrt{2}\,cm = OA$ (radius)

Again, $AC^2 = OA^2 + OC^2 - 2OA.OC \cos 135°$

$= 2(OA)^2 - 2(OA)^2.\cos 135°$

$= 2\left(5\sqrt{2}\right)^2 - 2\left(5\sqrt{2}\right)^2 \times \left(-\dfrac{1}{\sqrt{2}}\right)$

$= 100 + \dfrac{100}{\sqrt{2}}$

$AC^2 \approx 170.70$

$\Rightarrow \quad AC \approx 13$cm

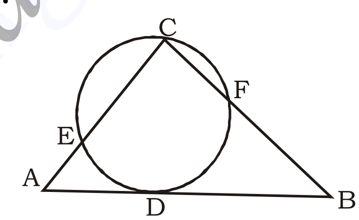
$\therefore \quad$ Perimeter of ΔOAC = OA + OC + AC

$= 5\sqrt{2} + 5\sqrt{2} + 13 = 10\sqrt{2} + 13$

$= 10 \times 1.414 + 13 \approx 27$

Ex–30. ABC is an isosceles triangle a circle is such that it passes through vertex C and AB acts as a tangent at D for the same circle. AC and BC intersects the circle at E and F respectively AC = BC = 4 cm and AB = 6 cm. Also, D is the mid-point of AB. What is the ratio of EC : (AE + AD)?

Sol.



Here, AC and BC are the secants of the circle and AB is tangent at D

$AD = DB = \dfrac{6}{2}$

$\therefore AE \times AC = AD^2$

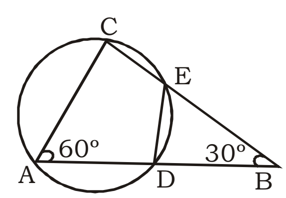
$AE \times 4 = (3)^2 \Rightarrow AE = \dfrac{9}{4}$

$\therefore \quad CE = 4 - \dfrac{9}{4} = \dfrac{7}{4}$

$\therefore CE : (AE + AD)$

$= \dfrac{7}{4} : \left(\dfrac{9}{4} + 3\right) = \dfrac{7}{4} : \dfrac{21}{4} = 1 : 3$

Ex–31. In the given figure ADEC is cyclic quadrilateral, CE and AD are extended to meet at B. $\angle$CAD = 60° and $\angle$CBA = 30°. BD = 6 cm and CE = $5\sqrt{3}cm$, What is the ratio of AC : AD?



Sol. $\angle CED = 120°$ [$\square$ ACED is cyclic]

In $\triangle BED$,

$\therefore$ $\angle BED = 180° - \angle CED = 180° - 120° = 60°$

$\therefore$ $\angle EDB = 180° - (30° + 60°) = 90°$

$\frac{BD}{BE} = \cos 30°$

$\frac{6}{BE} = \frac{\sqrt{3}}{2}$

$\Rightarrow$ $BE = 4\sqrt{3}cm$

$\therefore$ $BC = BE + EC$

$= 4\sqrt{3} + 5\sqrt{3} = 9\sqrt{3}cm$

Now, since AB and CB are the secants of the circle.

$\therefore$ $BD \times BA = BE \times BC$

$6 \times BA = 4\sqrt{3} \times 9\sqrt{3}$

$\Rightarrow$ BA = 18cm

In $\triangle ACB$,

$\angle ACB = 180° - (60° + 30°) = 90°$

Again, $\triangle ACB$ is a right angled triangle

$(\because \angle C = 90°)$

$\therefore$ AD = AB – BD = 12cm

$AC^2 = AB^2 - BC^2$

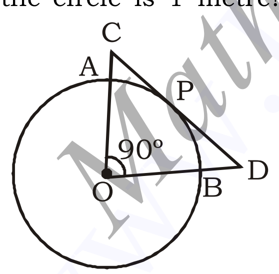
$= (18)^2 - (9\sqrt{3})^2$

$= 324 - 243$

$AC = \sqrt{81} = 9$

$\therefore$ $\frac{AC}{AD} = \frac{9}{12} = \frac{3}{4}$

Ex–32. In a circle O is the centre and COD is right angle. AC = BD and CD is the tangent at P. What is the value of AC + CP, if the radius of the circle is 1 metre?



Sol. OC = OA + AC ....(i)

OD = OB + BD ...(ii)

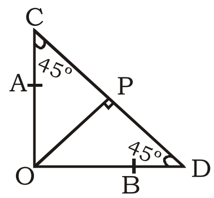
AC = BD (given)

OA = OB (radius)

then,

OC = OD and OA = OP = OB (radius)

In $\triangle OCP$,



$\angle OPC = 90°$ $(OP \perp CD)$

$\angle COP = 180° - (90° + 45°) = 45°$

OP = PC = 1

Now,

$OC^2 = OP^2 + PC^2 = (1)^2 + (1)^2 = 2$

$OC = \sqrt{2}\,m$

AC = OC – OA

$= \left(\sqrt{2} - 1\right)m$

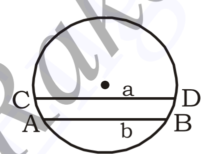
and $AC + CP = \left(\sqrt{2} - 1\right) + 1$

$= \sqrt{2}\,m$ = 1.414 m = 141.4 cm

TWO CIRCLE

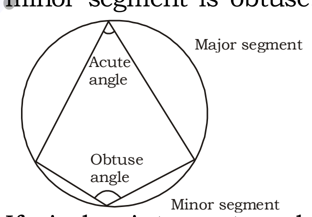
1. Two circles are congruent if and only if they have equal radii.

2. Of any two chords of a circle, the one which is greater is nearer to the centre.

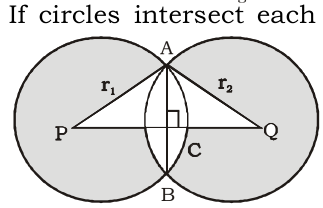


Here, CD > AB = a > b

3. Angle in a major segment of a circle is acute and angle in a minor segment is obtuse.



4. If circles intersect each other



AB is common chord of the circles,

Distance between centres, PQ=PC+CQ

$\Rightarrow$ $PQ = \sqrt{r_1^2 - \left(\frac{AB}{2}\right)^2} + \sqrt{r_2^2 - \left(\frac{AB}{2}\right)^2}$

Case - 1

If AQ is tangent of $circle_1$ or PA is tangent of $circle_2$ then ΔPAQ is a right angled triangle and $AC \perp PQ$

then $PA \times AQ = PQ \times AC$

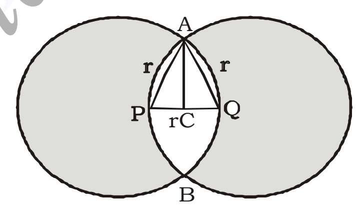
$\Rightarrow$ $AC = \frac{PA \times AQ}{PQ}$

Length of common chord AB = 2AC

$= \frac{2PA \times AQ}{PQ}$

Case - 2

If each circles passes through the centre of the other then length of common chord is $\sqrt{3}$ r.

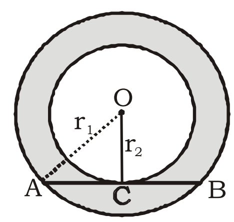


ΔPAQ is an equilateral triangle

& Height $AC = \frac{\sqrt{3}}{2}r$

So, **length of common chord** = **2AC** = $\sqrt{3}$ **r**

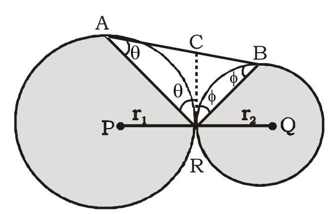
5. If circles are concentric



AB is chord of the greater circle be a tangent to smaller circle

Length of AB = 2AC = $2\sqrt{r_1^2 - r_2^2}$

6. If circles externally touch each other



Distance between centre = $r_1 + r_2$

R is common point of circles AB is direct common tangent (DCT)

then $\angle$**ARB = 90°**

Proof

Tangent at R meet AB at C. AC = CR & CB = CR (Tangents from external point)

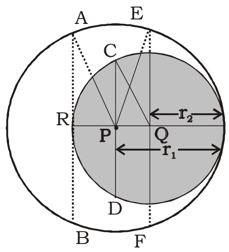
So, $\angle CAR = \angle CRA = \theta$

& $\angle CBR = \angle BRC = \phi$

Sum of three interior angle of $\Delta = 180^\circ$

$\therefore \quad \theta + (\theta + \phi) + \phi = 180^\circ \Rightarrow \theta + \phi = 90^\circ$

7. If circles internally touch each other



P & Q are centre of greater and smaller circle respectively. then **distance between centre = $r_1 - r_2$**

(a) AB - The biggest chord of the greater circle which is outside the inner circle

AB = $2\sqrt{AP^2 - RP^2}$

Where AP = r_1, RP = $(r_2 + r_2) - r_1 = 2r_2 - r_1$

AB = $2\sqrt{r_1^2 - (2r_2 - r_1)^2}$ = $2\sqrt{4r_2(r_1 - r_2)} = 4\sqrt{r_2(r_1 - r_2)}$

(b) CD - The smallest chord of the smaller circle which passes through the centre of greater circle.

CD =

$2\sqrt{CQ^2 - PQ^2} = 2\sqrt{r_2^2 - (r_1 - r_2)^2}$

CD = $2\sqrt{r_1(2r_2 - r_1)}$

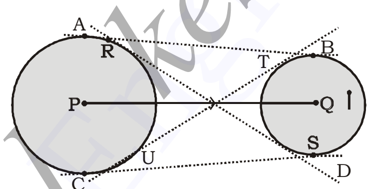
(c) **EF** - The smallest chord of greater circle which passes through centre smaller circle.

EF = $2\sqrt{PE^2 - PQ^2}$ =

$2\sqrt{r_1^2 - (r_1 - r_2)^2}$

EF = $2\sqrt{r_2(2r_2 - r_1)}$

8. If circles place at some distance

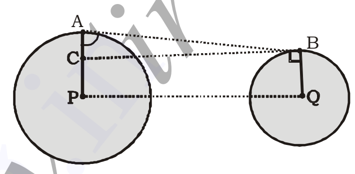


$\Rightarrow$ **Direct common tangent (DCT) AB and CD** =

$\sqrt{(\text{Distance between centres})^2 - (r_1 - r_2)^2}$

$\Rightarrow$ **Transverse common tangent (TCT) RS and TU**

$= \sqrt{\begin{pmatrix}\text{Distance between} \\ \text{centres}\end{pmatrix}^2 - (r_1 + r_2)^2}$



Proof,

$PA \perp AB$ and $QB \perp AB$ then

$PA \parallel BQ$

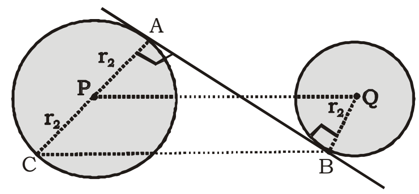
$\therefore$ AC = PA – PC = $r_1 - r_2$

In ΔCAB,

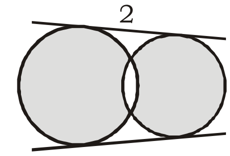
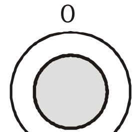
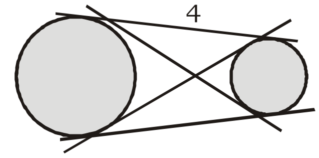
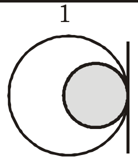
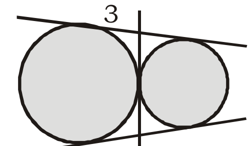
Length of AB = $\sqrt{CB^2 - AC^2}$

$\downarrow$

DCT= $\sqrt{\begin{pmatrix}\text{Distance between} \\ \text{centres} - (r_1 + r_2)^2\end{pmatrix}^2}$



Minimum and Maximum Possible Common Tangents in two Circles

Circle are	Minimum Common Tangent	Maximum Common Tangent
(1) Intersecting each other	2	2 
(2) Neither touch nor Intersecting	0 	4 
(3) Touch each other	1  (Internally)	3  (externally)

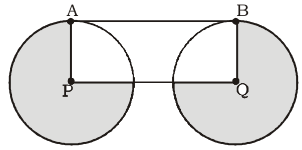
In ΔACB, $AB = \sqrt{CB^2 - AC^2}$

$$= \sqrt{\left(\text{Distance between centres}\right)^2 - \left(r_1 - r_2\right)^2}$$

Case : 1

If radius of circles are same

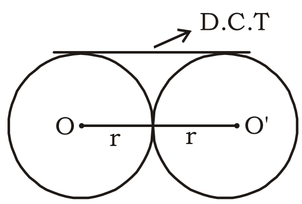
$r_1 = r_2 = r$



DCT, $AB = \sqrt{(PQ)^2 - (r - r)}$

AB = PQ i.e. length of direct common tangent equal to distance between circle.

Case : 2



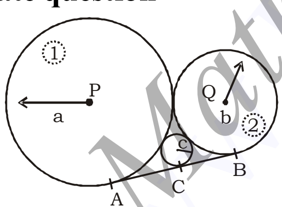
If circle externally touch each other i.e. distance between centre $= r_1 + r_2$

$\Rightarrow TCT = \sqrt{(r_1 + r_2)^2 - (r_1 + r_2)^2} = 0$

$\Rightarrow$ D C T

$= \sqrt{(r_1 + r_2)^2 - (r_1 - r_2)^2} = 2\sqrt{r_1 r_2}$

Template question



Two circles of radius a cm. & b cm. externally touch each other. AB is DCT of circles. A third circle with radius C cm. externally two circles and DCT. Then relation between a, b & c is:

According formula of Case 2

$AB = 2\sqrt{ab}$

$AC = 2\sqrt{ac}$ [It is DCT of (1) & (3) circles]

$CB = 2\sqrt{bc}$ [It is DCT of (2) & (3) circles]

AB = AC + CB

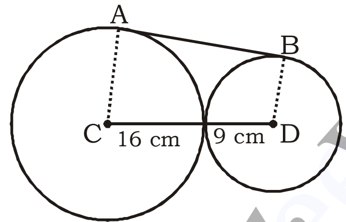
$2\sqrt{ab} = 2\sqrt{ac} + 2\sqrt{bc}$

$\Rightarrow \frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$

Type IV

Ex–1. Find the length of the common tangent of two externally touch circle with radius 16 cm & 9 cm respectively.

Sol.



Length of common tangent = (Distance between two circle)² – (Radius₁ – Radius₂)²

$AB^2 = CD^2 - (16 - 9)^2$

$AB^2 = (16 + 9)^2 - (7)^2$

$AB^2 = 625 - 49$

$AB^2 = 576$

$AB = 24cm$

Alternate $\rightarrow AB^2 = (R_1 + R_2)^2 - (R_1 - R_2)^2$

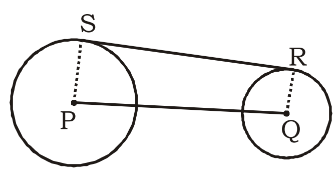
$AB = 2\sqrt{Rr} \qquad = 2\sqrt{16 \times 9}$

$= 2 \times 12 = 24cm$

Note: Use this formula, two circle intersect each other externally.

Ex–2. Find the length of the common tangent of two circle with radius 5 cm & 12 cm. Distance between two centre of the circle is 25 cm.

Sol.



$RS^2 = \sqrt{PQ^2 - (R_1 - R_2)^2}$

$RS^2 = \sqrt{(25)^2 - (12 - 5)^2}$

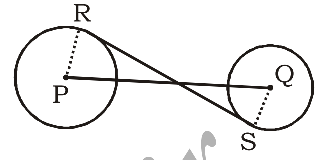
$RS^2 = \sqrt{625 - 49}$

$RS^2 = 576$

$RS = 24cm$

Ex–3. Find the length of common transversal tangent of two circle with radius 5 cm & 4 cm. Distance between two centre of the circle is 15 cm.

Sol.



$RS^2 = \sqrt{(PQ)^2 - (R_1 + R_2)^2}$

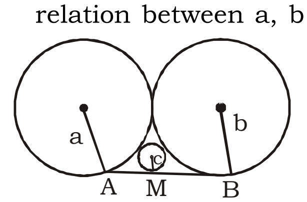
$RS^2 = \sqrt{(15)^2 - (5 + 4)^2}$

$RS^2 = \sqrt{225 - 81}$

$RS^2 = 144$

$RS = 12cm$

Ex–4. Two circle of radius a cm & b cm. Externally touch each other. AB is a direct common tangent of circles. A third circle with radius c cm externally touch two circle & direct common tangent. The relation between a, b & c is:



$\Rightarrow AB = 2\sqrt{ab}$

$\Rightarrow AM = 2\sqrt{ac}$

$\Rightarrow BM = 2\sqrt{bc}$

$\Rightarrow AB = AM + MB$

$\Rightarrow 2\sqrt{ab} = 2\sqrt{ac} + 2\sqrt{bc}$

$\Rightarrow \sqrt{ab} = \sqrt{ac} + \sqrt{bc}$

Divide by $\sqrt{abc}$

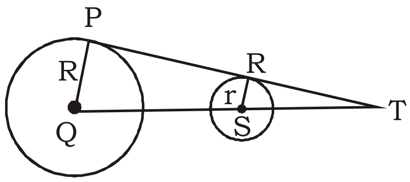
$$\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{a}}$$

Ex–5. If direct common tangent and line joining of centre of circle of extended and meet each other. Find the distance between external point & smaller circle & larger circle:

Sol. 'T' externally divides QS in ratio of radius two circles

$$\Rightarrow \frac{QT}{ST} = \frac{R}{r}$$

$\Rightarrow \dfrac{QS+ST}{ST}=\dfrac{R}{r}$



$\Rightarrow 1+\dfrac{QS}{ST}=\dfrac{R}{r}$

$\Rightarrow \dfrac{QS}{ST}=\dfrac{R-r}{r}$

$\Rightarrow QS = \left(\dfrac{R-r}{r}\right)ST$

$ST=\left(\dfrac{r}{R-r}\right)QS$ **[Smaller circle]**

Now,

$\Rightarrow \dfrac{QT}{ST}=\dfrac{R}{r}$

$\Rightarrow \dfrac{QT}{QT-QS}=\dfrac{R}{r}$

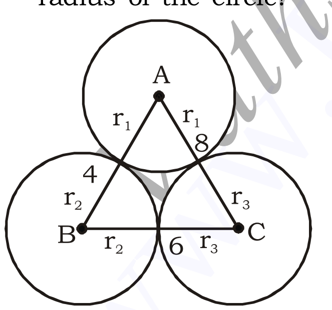
$\Rightarrow 1-\dfrac{QS}{QT}=\dfrac{r}{R}$

$\Rightarrow \dfrac{QS}{QT}=\left(\dfrac{R-r}{R}\right)$

$QT = \left(\dfrac{R}{R-r}\right)QS$ **[Larger circle]**

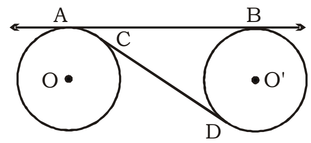
Ex–6. With A, B & C centres, three circles are drawn such that they touch each other externally. If the sides of the $\triangle ABC$ are 4 cm, 6 cm & 8cm, then what is the sum of the radius of the circle?

Sol.

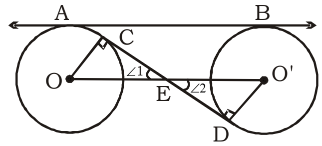


$r_1 + r_2 = 4$ .....(i)
$r_2 + r_3 = 6$ .....(ii)
$r_3 + r_1 = 8$ ....(iii)
(i) + (ii) + (iii)
$2(r_1 + r_2 + r_3) = 18$
$r_1 + r_2 + r_3 = 9$
Solve the equations
We get,
$r_1 = 3,\ r_2 = 1,\ r_3 = 5$

Ex–7. Two circle having same radius 12cm have two commmon tangent AB and CD. Which touch the circle on A, C & B, D respectively and if CD = 32 cm. Find the length of AB.

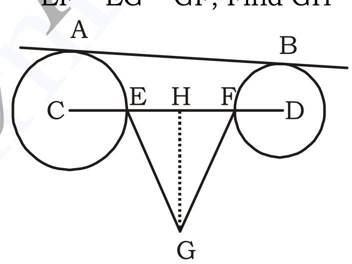


Sol.



OC = O'D = 12 cm (radius)
CD = 32 cm
$\triangle COE \sim \triangle EO'D$
($\because \angle C = \angle D = 90°$ & $\angle 1 = \angle 2$)
OE = O'E
CE = ED = 16cm
In $\triangle COE$,
$OE^2 = CE^2 + OC^2$
$OE^2 = (16)^2 + (12)^2 = 400$
OE = 20 cm
OO' = 2 × OE = 40 cm
$\therefore$ AB = OO' = 40

Ex–8. In the given figure, CE = 15 cm, FD = 8cm, AB = 24 cm, EF = EG = GF, Find GH = ?



Sol.

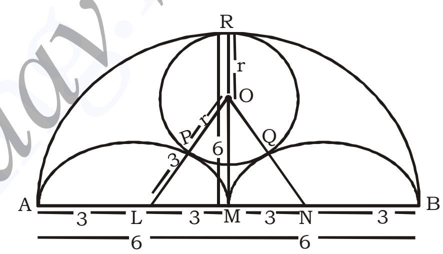
$$D.C.T = \sqrt{\text{Distance b/w centres}^2 - \text{diff. b/w radii}^2}$$

$AB^2=CD^2-[(CE)-(DF)]^2$
$(24)^2 = (CD)^2 - [(15) - (8)]^2$
$CD^2 = 576 + 49$
$\Rightarrow$ CD = 25 cm
EF = CD – CE – FD = 25 –15 – 8
= 2 cm
$\triangle EFG$ is an equilateral $\triangle$.
EF = GF = EG = 2 cm.

$GH = \dfrac{\sqrt{3}}{2}EF = \dfrac{\sqrt{3}}{2}\times 2 = \sqrt{3}$cm.

Ex–9. AB is a line segment of length 12 cm & M is the mid point semicircle are drawn with AM, MB & AB is diameter on the same side of the line AB. A circle with centre 'O' and radius 'r' is drawn so that it touches all the three semicircle. Find the value of 'r' if L & N are midpoint of AM & MB respectively & the point O, P, L & the point O, Q, N & the point R, O, M are collinear?

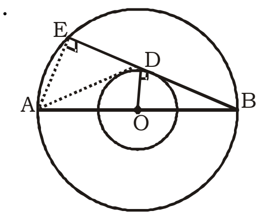
Sol.



In $\triangle$ OML,
$OL^2 = OM^2 + LM^2$
$(3 + r)^2 = (6 - r)^2 + 3^2$
$9 + r^2 + 6r = 36 + r^2 - 12r + 9$
$18r = 36$
$\therefore r = 2$

Ex–10. The radii of two concentric circle are 13 cm & 8 cm. AB is diameter of bigger circle. BD is tangent of smaller circle touching it at D. Find the length of AD?

Sol.



$\because$ AO = OB = 13cm [radius of bigger circle
& OD = 8 cm [radius of smaller circle]
DE = DB [tangents divide equally]
In $\triangle ODB$,
$OB^2 = OD^2 + DB^2$
$(13)^2 = 8^2 + DB^2$
$169 = 64 + DB^2$
$\Rightarrow DB = \sqrt{105} = DE$
In $\triangle$ BDO & $\triangle$ BEA,

$$\frac{OD}{AE} = \frac{BD}{BE}$$

$$\frac{8}{AE} = \frac{\sqrt{105}}{2\sqrt{105}}$$

$\Rightarrow$ AE = 16 cm

In $\triangle$ AED,

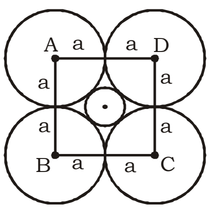
$AD^2 = AE^2 + ED^2$

$AD^2 = (16)^2 + \left(\sqrt{105}\right)^2$

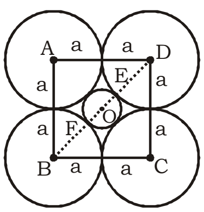
$AD^2 = 256 + 105$

$\therefore$ AD = 19 cm

Ex–11. In the given figure, radius of smaller circle is:



Sol.



In $\triangle$ BCD,

$BD^2 = CD^2 + BC^2$

$\Rightarrow$ BD = $2\sqrt{2}a$

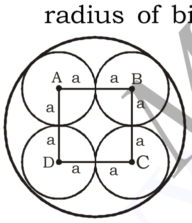
EF = $2\sqrt{2}$ a– a–a

EF = $2\left(\sqrt{2}a - a\right)$

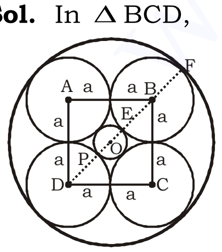
$\because$ OE = $\frac{1}{2} \times$ EF

$\therefore$ OE$=\frac{1}{2}\times\left[2a\left(\sqrt{2}-1\right)\right] = \left(\sqrt{2}-1\right)a$

Ex–12. In the given figure, find the radius of bigger circle is



Sol. In $\triangle$ BCD,



$BD^2 = BC^2 + CD^2$

$\Rightarrow$ BD = $2\sqrt{2}a$

$\because$ OB = $\frac{1}{2} \times$ BD

$\therefore$ OB = $\frac{1}{2} \times 2\sqrt{2}a = \sqrt{2}a$

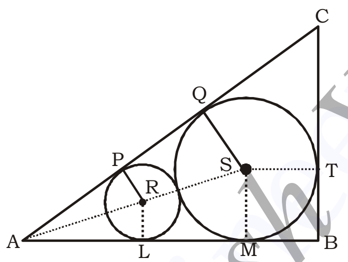
$\because$ OF = OB + BF

OF = $\sqrt{2}a + a$

$\therefore$ OF = $\left(\sqrt{2}+1\right)a$

Ex–13. Two circles inscribed in a right angle triangle radius is 12 cm & 3 cm. Find length of AP.

Sol.



PQ=$\sqrt{(RS)^2-(QS-PR)^2}$

PQ=$\sqrt{(15)^2-(12-3)^2}$

PQ = 12 cm = LM

$\because$ STBM is a square

MB = BT = ST = SM = 12 cm.

In $\triangle$APR & $\triangle$ASQ,

$$\frac{AP}{AQ} = \frac{PR}{SQ}$$

$$\frac{AP}{AP+PQ} = \frac{3}{12}$$

$$\frac{AP}{AP+12} = \frac{1}{4}$$

4AP = AP + 12

3AP = 12

$\therefore$ AP = 4cm.

Ex–14. Two circles with R & r cm, one circle is inscribed in another circle. If shortest distance between the circles is S cm. Then the distance between the centres is:

Sol. OA = R

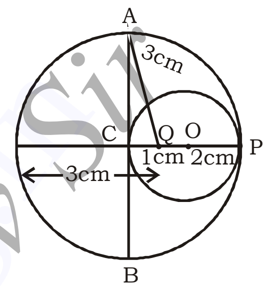
O'B = r

AB = S

OO' = OA – O'B – AB

$\therefore$ OO' = R – r – S

Ex–15. Two circles internally connect each other. It's radius are 2cm & 3 cm respectively. Find the maximum chord in the outer circle that Joint incircle.



Sol. In $\triangle$ ACQ,

$AQ^2 = AC^2 + CQ^2$

$3^2 = AC^2 + 1^2$

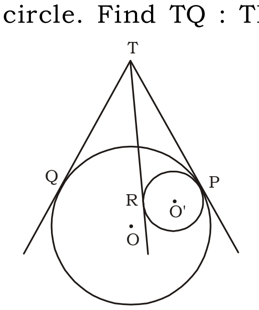
$AC^2 = 8$

AC = $2\sqrt{2}$

$\therefore$ AB = 2 × AC

$\Rightarrow$ AB = 2 × 2 $\sqrt{2}$ = $4\sqrt{2}cm$

Ex–16. In the given figure, there are two circles with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Find TQ : TR



Sol. TQ = TP ....(i) (tangent of larger circle)

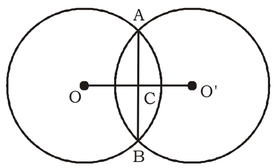
TR = TP.....(ii) (tangent of smaller circle)

From (i) and (ii)

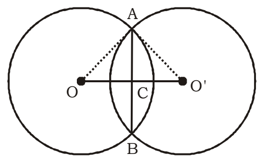
TQ = TP = TR [Tangents of the circle]

Required ratio = TQ : TR = 1 : 1

Ex–17. O and O' the centres of circle of radii 20 cm and 37 cm. AB = 24cm. What is the distance OO'?



Sol. $AC = \frac{AB}{2} = 12cm$



OA = 20 cm, O'A = 37 cm.

In Δ OAC,

$OA^2 = AC^2 + OC^2$

$\Rightarrow$ $(20)^2 = (12)^2 + OC^2$

$\Rightarrow$ OC = 16cm

In Δ ACO',

$(AO')^2 = (AC)^2 + (O'C)^2$

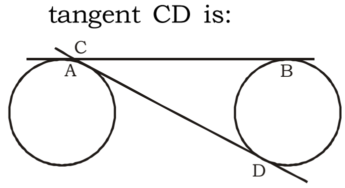
$\Rightarrow$ $(37)^2 = (12)^2 + (O'C)^2$

$\Rightarrow$ O'C = 35 cm

$\therefore$ OO' = OC + O'C

= 16 + 35 = 51 cm

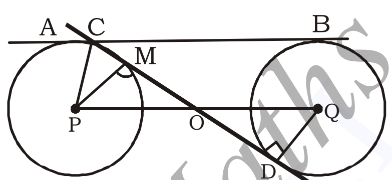
Ex–18. There are two circles each with radius 5 cm. Tangent AB is 26 cm. The length of tangent CD is:



Sol. AB = PQ = 26 cm

PO = OQ = 13 cm

$CO = \sqrt{(PO)^2 - (PC)^2}$



$CO = \sqrt{(13)^2 - (5)^2}$

CO = 12cm

$\therefore$ CD = 2 CO = 24 cm

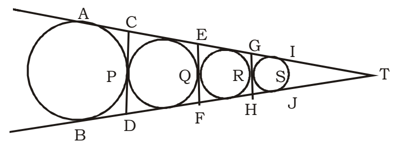
Ex–19. In the adjoining figure AT and BT are the two tangents at A and B respectively CD is also a tangent at P. There are some more circles touching each other and the tangents AT and BT also. Which one of the following is true?

(a) PC + CT = PD + DT

(b) AE + ET = BF + FT

(c) AQ + ET = BQ + FT

(d) FH + HT = EG + GT



Sol. (c) AT = BT

($\because$ Tangents on the same circle from a fixed point is equal)

AC = PC and BD = PD

$\therefore$ AT = BT

$\Rightarrow$ AC + CT = BD + DT

PC + CT = PD + DT

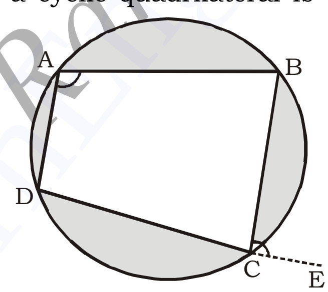
Similarly, all the relation (i), (ii) & (iv) are true.

Cyclic Quadrilateral

If all vertices of a quadrilateral lie on the circle, it is called a cyclic quadrilateral.

Properties

(i) The sum of opposite angles of a cyclic quadrilateral is 180°.



$\angle$**A+**$\angle$**C=**$\angle$**B+**$\angle$**D = 180°**

(ii) The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

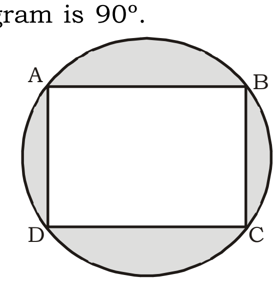
$\angle$**DAB =** $\angle$**BCE**

$\angle DAB + \angle BCD = 180°$

& $\angle BCD + \angle BCE = 180°$

$\therefore \angle DAB = BCE$

(iii) All angles of cyclic parallelogram is 90°.



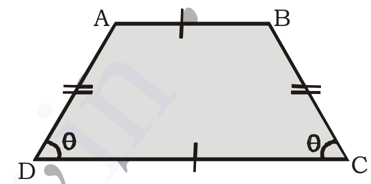
$\angle$**A+**$\angle$**C = 180°**

but according to parallelogram property $\angle A = \angle C = \theta$

$\therefore 2\theta = 180 \Rightarrow \theta = 90°$

then, $\angle D = \angle B = 90°$

(iv) If non parallel sides of a trapezium are equal then it will be a cyclic quadrilateral.



$\therefore$ AB || DC

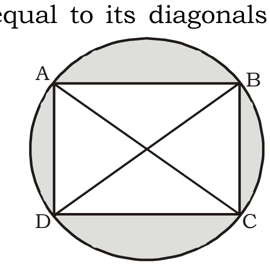
So, $\angle A = 180° - \theta$ & $\angle B = 180° - \theta$

If sum of opposite angles is 180°

($\angle A + \angle C = \angle B + \angle D = 180°$)

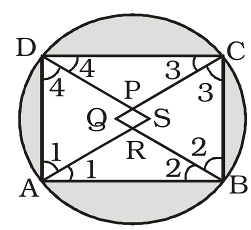
then it is cyclic quadrilateral.

v. **Ptolemy's theorem** : In a cyclic quadrilateral the sum of products of the measures of the pairs of opposite sides is equal to its diagonals.



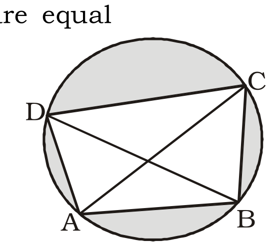
AC + BD = AB. DC + AD. BC

VI. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.



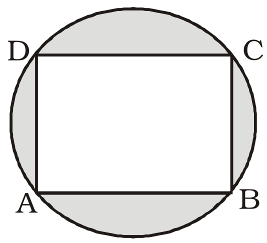
i.e. If ABCD is a cyclic quadrilateral, then $\square$ PQRS is also a cyclic.

VII. If A cyclic trapezium is isosceles and its diagonals are equal



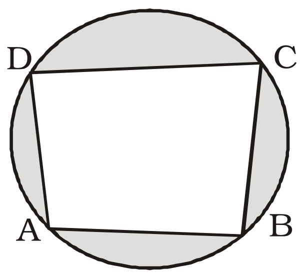
i.e. If ABCD is a cyclic trapezium s.t. AB||DC, then AD = BC and AC = BD.

VIII. If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel.



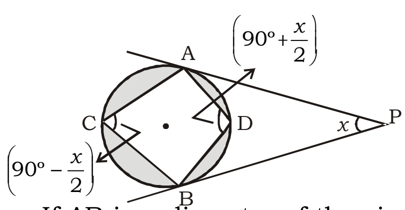
i.e. If AD = BC, then AB || CD.

IX. An isosceles trapezium is always cyclic.



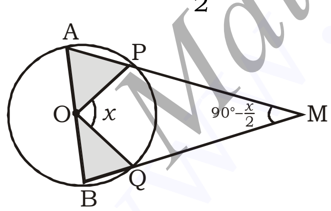
i.e. If AB || DC and AD = BC. Then, ABCD is a cyclic trapezium.

X.



XI. If AB is a diameter of the circle (centre O) and APM & BQM are its two secants. If $\angle POQ = x$ then

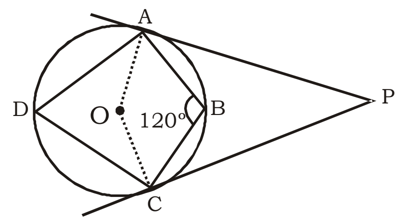
$\angle PMQ = 90° - \frac{x}{2}$



Type -VI

Ex–1. ABCD is a concyclic quadrilateral. The tangents at A & C intersect each other at P. If $\angle ABC = 120°$, then what is $\angle APC$?

Sol.



$\angle ABC + \angle ADC = 180°$ ($\because$ ABCD is a cyclic quadrilateral)

$\therefore \angle ADC = 180° - 120° = 60°$

$\angle AOC = 2 \times \angle ADC$ ($\because$ Centre angle is double at the angle of major arc)

$\angle AOC = 2 \times 60° = 120°$

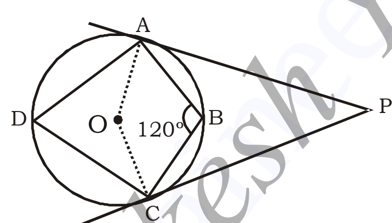
In $\square$ AOCP,

$\angle A + \angle O + \angle C + \angle P = 360°$

$90° + 120° + 90° + \angle P = 360°$

$\therefore \angle P = 60°$

Alternate



$\angle ADC = 180° - \angle ABC$

($\because$ ABCD is a cyclic quadrilateral)

$= 180 - 120° = 60°$

$\angle CAP = \angle ACP = \angle ADC = 60°$ [Alternate segments]

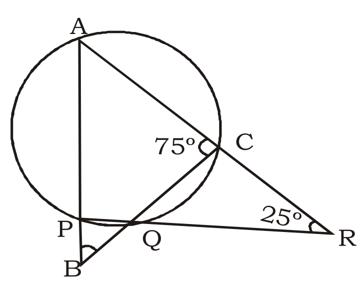
In $\triangle$ APC,

$\angle A + \angle P + \angle C = 180°$

$60° + \angle P + 60° = 180°$

$\angle P = 60°$

Ex–2.



In the given figure, find $\angle CBA$ = ?

Sol. $\angle APQ + \angle ACQ = 180°$ ($\because$ APQC is a cyclic quadrilateral)

$\angle APQ = 180° - 75°$

$\therefore \angle APQ = 105°$

$\angle BPQ = 180° - 105° = 75°$

$\angle ACQ = \angle CQR + \angle CRQ$ [Sum of Interior angle]

$75° = \angle CQR + 25°$

$\angle CQR = 50° = \angle PQB$ [Vertically opposite angle]

In $\triangle$ PBQ,

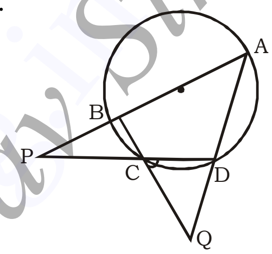
$\angle P + \angle B + \angle Q = 180°$

$75° + \angle B + 50° = 180°$

$\angle PBQ = 55°$

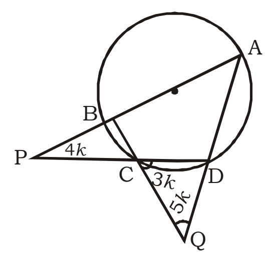
$\therefore$ CBA = 55°

Ex–3.



In the given figure, if $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, where $\angle DCQ = x$, $\angle BPC = y$ and $\angle DQC = z$, then what are the values of x, y and z respectively?

Sol.



$\frac{x}{3} = \frac{y}{4} = \frac{z}{5} = k$ (say)

$x = 3k$, $y = 4k$, $z = 5k$

In $\triangle$CDQ, $\angle CDQ = 180° - (3k + 5k)$

$= 180° - 8k$

ABC = CDQ (exterior angle of cyclic quadrilateral equal to opposite interior angle)

$= 180° - 8k$

In $\triangle$PBC,

$\angle BCP = \angle DCQ = 3k$ (vertically opposite angle)

$\angle ABC = \angle P + \angle C$

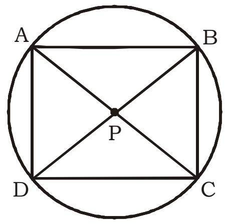
$180° - 8k = 4k + 3k \Rightarrow k$

$= \frac{180°}{15} = 12°$

$\therefore$ $x = 36°$, $y = 48°$, $z = 60°$

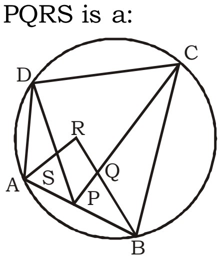
Ex–4. The diagonals AC and BD of a cyclic quadrilateral ABCD intersect each other at the point P.
AP = 3, CP = 6, DP = 9 and find BP = ?

Sol.



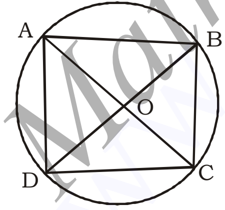
By property of chords of circle
Then, it is always true that:
AP×PC = BP×PD
3 × 6 = BP × 9
BP = 2

Ex–5. ABCD is a cyclic quadrilateral. The angle bisector of A, B, C and D intersect at P, Q, R and S as shown in the figure. These four points form a quadrilateral PQRS. Quadrilateral PQRS is a:



Sol. PQRS is the rectangle

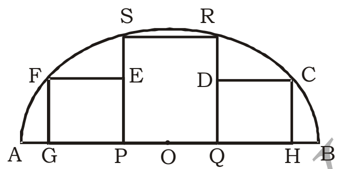
Ex–6. In the given figure ABCD is a cyclic quadrilateral DO = 8 cm and CO = 4 cm. AC is the angle bisector of BAD. The length of AD is equal to the length of AB. DB intersects diagonal AC at O, then what is the length of the diagonal AC?



Sol. $\frac{AD}{AB} = \frac{DO}{BO} = 1$

OB = OD = 8 cm
Also, $\triangle$ ADB is isosceles $\triangle$, and Ao is the angle bisector
$\therefore$ AO will also be the median
$\therefore$ ABCD is a cyclic quadrilateral
$\therefore$ DO × BO = CO × AO
8 × 8 = 4 × AO
$\Rightarrow$ AO = 16 cm
$\therefore$ AC = 16 + 4 = 20 cm

Ex–7. In the following figure there is semicircle with centre 'O' and diameter AB (= 2r). PQRS is a square of maximum possible area. P and Q lie one the diameter AB and R, S lie on the arc of semicircle, there are two more squares of maximum possible area EFGP and CDQH. What is the sum of lengths of RC and FS?



Sol. In $\triangle$ ORQ,
$\Rightarrow$ $OR^2 = (OQ)^2 + (RQ)^2$
$\Rightarrow$ $OR^2 = 5OQ^2$ $\quad$ ($\because$ RQ = 2(OQ))
radius (r) = $OQ\sqrt{5}$
Again,
In $\triangle$ OCH,
$OC^2 = OH^2 + HC^2$
$r^2 = (OQ + QH)^2 + (QH)^2$
($\because$ HQ = HC)

$$\Rightarrow r^2 = \left(\frac{r}{\sqrt{5}} + QH\right)^2 + (QH)^2$$

$$\Rightarrow QH = \frac{r}{\sqrt{5}}$$

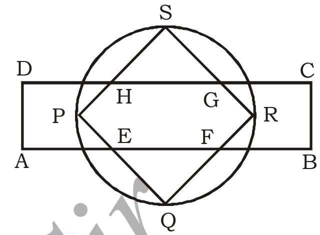
$$\therefore HC = \frac{r}{\sqrt{5}} = \frac{RQ}{2}$$

$$RC = \sqrt{(RD)^2 + (DC)^2}$$

$$= \sqrt{\left(\frac{r}{\sqrt{5}}\right)^2 + \left(\frac{r}{\sqrt{5}}\right)^2} = r\sqrt{\frac{2}{5}}$$

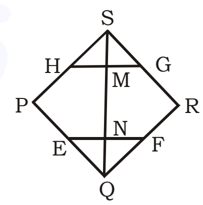
$$\therefore RC + FS = 2r\sqrt{\frac{2}{5}}$$

Ex–8. In the adjoining figure ABCD is a rectangle in which length is twice of breadth. H and G divide the line CD into three equal parts. Similarly points E and F trisect the line AB. A circle PQRS is circumscribed a square PQRS which passes through the points E, F, G and H. What is the ratio of areas of circle to that of rectangle?



Sol. Let AD = 3a and DC = 6a

$$DH = HG = GC = \frac{6a}{3} = 2a$$



$$HM = MG = \frac{2a}{2} = a = SM$$

NQ = a
SQ = SM + MN + NQ
= a + 3a + a = 5a
Since diagonal of square SQ = 5a
But, diameter of circle SQ = diagonal of square SQ

$$\text{Radius of the circle} = \frac{5a}{2}$$

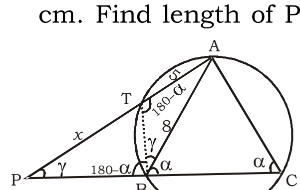
$$\text{Area of the circle} = \pi \times \left(\frac{5a}{2}\right)^2$$

Here ,

$$\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\frac{25}{4}(a^2\pi)}{3a \times 6a} = \frac{25\pi}{72}$$

Ex–9. There are two chords AB and AC of equal length 8 cm. CB is produced to P. AP cuts circle at T such that AT = 5 cm. Find length of PT.

Sol.

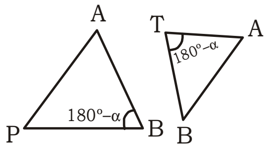


$\because$ AB = AC
$\Rightarrow \angle ABC = \angle ACB = \alpha$ (Let)
$\angle ABP = 180° - \angle ABC = 180° - \alpha$
$\therefore$ ATBC is a cyclic quadrilateral

$\Rightarrow \quad \angle ATB + \angle ACB = 180^\circ$

$\Rightarrow \quad \angle ATB = 180^\circ - \alpha$

Now,



In $\triangle ABP$ and $\triangle BTA$

$\angle ABP = \angle BTA = 180^\circ - \alpha$

$\angle BAP = \angle BAT$ (common)

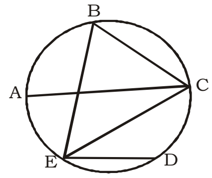
$\angle BPA = \angle TBA = \gamma$ (Let)

$\therefore \quad \triangle ABP \sim \triangle BTA$

$$\frac{AP}{AB} = \frac{AB}{AT} = \frac{x+5}{8} = \frac{8}{5} \quad = x = \frac{39}{5}$$

$$PT = \frac{39}{5}$$

Ex–10. In the figure, chord ED is paralled to the diameter AC of the circle. If angle CBE = 65°, then, what is the value of angle DEC.



Sol. $\angle ABC = 90^\circ$

$\angle ABE = \angle ABC - \angle CBE$

$\angle ABE = 90^\circ - 65^\circ = 25^\circ$,

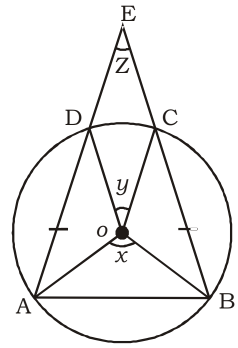
$\angle ABE = \angle ACE = 25^\circ$

$\angle ACE = \angle CED = 25^\circ$

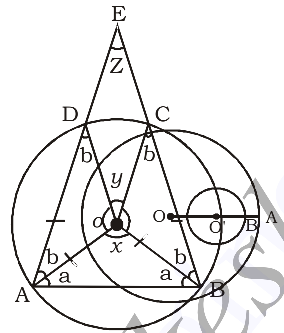
(alternate angles)

Ex–11. In the given figure, AD = CB,

find $\dfrac{x-y}{z}$



Sol.



$\angle OAB = \angle OBA = a$

[$\because$ OA = OB = radius]

In $\triangle ODA$ and $\triangle OCB$,

AD = CB (given)

OA = OB (radius)

$\triangle ODA \sim \triangle OCA$ [by S & S]

$\angle ADO = \angle DAO = \angle OCB = \angle OBC = b$ [by C.P.C.T]

$\angle DOA = \angle BOC = 180^\circ - 2b$

$\angle AOB = x = 180^\circ - 2a$

$\angle DOC = y = 360^\circ - [180^\circ - 2b + 180^\circ - 2b + 180^\circ - 2a]$

$y = 360^\circ - 180^\circ + 2b - 180^\circ + 2b - 180^\circ + 2a$

$y = 4b + 2a - 180^\circ$.....**(i)**

both sides substract by x

$x - y = 180^\circ - 2a - [4b + 2a - 180^\circ]$

$x - y = 180^\circ - 2a - 4b - 2a + 180^\circ$

$x - y = 360^\circ - 4a - 4b$

$\angle ODE = \angle OCE = 180^\circ - b$

In □CODE,

$\angle C + \angle O + \angle D + \angle E = 360^\circ$

$180^\circ - b + 4b + 2a - 180^\circ + 180^\circ - b + \angle E = 360^\circ$

$\angle E = Z = 180^\circ - 2a - 2b$

$$\therefore \quad \frac{x-y}{z} = \frac{360^\circ - 4a - 4b}{180^\circ - 2a - 2b} = \frac{2}{1}$$

EXERCISE

1. The length of tangent drawn from an external point P to a circle of radius 5 cm. is 12 cm. The distance of P from the centre of the circle is:
(a) 12 cm. (b) 9 cm.
(c) 7 cm. (d) 13 cm.

2. The centres of three circles of equal radii, touching each other externally form a/an
(a) Scalene triangle
(b) Equilateral triangle
(c) Isosceles triangle
(d) Right angle triangle

3. The length of two parallel chords of a circle of radius 5 cm are 6 cm and 8 cm in the same side of the centre. The distance between them is
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 1.5 cm

4. AB is a diameter of a circle having centre at O.P is a point on the circumference of the circle. If $\angle$POA = 120°, then measure of $\angle$PBO is
(a) 75° (b) 60° (c) 68° (d) 70°

5. AB is the diameter of a circle with centre O. P be a point on it. If $\angle POA = 120°$. Then, $\angle PBO = ?$
(a) 60° (b) 50° (c) 120° (d) 45

6. AB and AC are tangents to a circle with centre O. A is the external point of the circle. The line AO intersect the chord BC at D. The measure of the $\angle BDO$ is:
(a) 45° (b) 75° (c) 90° (d) 60°.

7. In a circle with centre at O and radius 5 cm, AB is a chord of length 8 cm. If OM is perpendicular to AB, then the length of OM is:
(a) 3 cm (b) 4 cm
(c) 1 cm (d) 2.5 cm

8. AB is a diameter of a circle having centre at O. PQ is a chord which does not intersect AB. Join AP and BQ. If $\angle$PAB = $\angle$ABQ, then ABQP is a:
(a) Cyclic rhombus
(b) Cyclic rectangle
(c) Cyclic trapezium
(d) Cyclic square

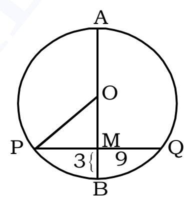
9. The distance between centres of two circles of radii 3 cm and 8 cm is 13 cm. If the points of contact of a direct common tangent the circles are P and Q, then the length of the line segment PQ is:
(a) 11.9 cm (b) 12 cm
(c) 11.5 cm (d) 11.58 cm

10. Two circles of radii 5 cm and 3 cm touch externally, then the ratio in which the direct common tangent to the circles divides externally the line joining the centres of the circles is:
(a) 5 : 3 (b) 3 : 5
(c) 1.5 : 2. 5 (d) 2.5 : 1.5

11. The diameter of a circle is 10 cm. If the distance of a chord from the centre of the circle be 4 cm, then the length of chord:
(a) 5 cm. (b) 6 cm.
(c) 4 cm. (d) 3 cm.

12. Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Then the distance between their centres is :
(a) 13.3 (b) 15 (c) 10 (d) 8

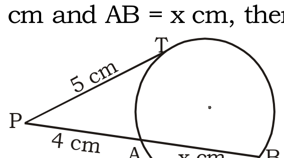
13. In a given circle, the chord PQ is of length 18 cm. AB is the perpendicular bisector of PQ at M. If MB = 3.find the length of AB



(a) 25 cm (b) 30 cm
(c) 28 cm (d) 27 cm

14. The chord of a circle is equal to its radius. The angle subtended by this chord at the minor arc of the circle is
(a) 150° (b) 60° (c) 75° (d) 120°

15. In the given figure, PAB is a secant and PT is a tangent to the circle from P. If PT = 5 cm, PA = 4 cm and AB = x cm, then x is



(a) 4/9 cm (b) 2/3 cm
(c) 9/4 cm (d) 5 cm

16. Two circles with their centres at O and P and radii 8 cm and 4 cm respectively touch each other externally. The length of their common tangent is
(a) 8 cm (b) 8.5 cm
(c) $8\sqrt{2}$ cm (d) $8\sqrt{3}$ cm

17. Two circles of diameters 10 cm and 6 cm have the same centre. A chord of the larger circle is a tangent of the smaller one. The length of the chord is
(a) 8 cm (b) 10 cm
(c) 6 cm (d) 4 cm

18. A and B are centres of two circles of radii 11 cm and 6 cm, respectively, PQ is a direct common tangent to the circle. If $\overline{AB} = 13$ cm, then length of $\overline{PQ}$ will be
(a) 12 cm (b) 13 cm
(c) 8.5 cm (d) 17 cm

19. The radii of two concentric circles are 17 cm and 25 cm. a straight line PQRS intersects the larger circle at the points P and S and intersects the smaller circle at the points Q and R. If QR = 16 cm, then the length (in cm.) of PS is
(a) 41 (b) 33 (c) 32 (d) 40

20. AB is a diameter of a circle with centre O. The tangents at C meets AB produced at Q. If $\angle CAB = 34°$, then measure of $\angle CBA$ is
(a) 56° (b) 68° (c) 34° (d) 124°

21. Let P and Q be two points on a circle with centre O. If two tangents of the circle through P and Q meet at A with $\angle PAQ = 48°$, then $\angle APQ$ is
(a) 96° (b) 66° (c) 48° (d) 60°

22. The radius of a circle is 6 cm. The distance of a point lying outside the circle from the centre is 10 cm. The length of the tangent drawn from the outside point to the circle is
(a) 5 cm (b) 6 cm
(c) 7 cm (d) 8 cm

23. DE is a tangent to the circumcircle of $\triangle ABC$ at the vertex A such that $DE \parallel BC$. If AB = 17

cm, then the length of AC is equal to
(a) 16.0 cm (b) 16.8 cm
(c) 17.3 cm (d) 17 cm

24. ST is a tangent to the circle at P and QR is a diameter of the circle. If $\angle RPT$ = 50°, then the value of $\angle SPQ$ is
(a) 40° (b) 60° (c) 80° (d)100°

25. If PA and PB are two tangents to a circle with centre O such that $\angle AOB$ = 110°, then $\angle APB$ is
(a) 90° (b) 70° (c) 60° (d) 55°

26. AC is a transverse common tangent to two circle with centres P and Q and radii 6 cm and 3 cm at the point A and C respectively. If AC cuts PQ at the point B and AB = 8 cm, then the length of PQ is:
(a) 12 cm (b) 15 cm
(c) 13 cm (d) 10 cm

27. AB and CD are two parallel chords of a circle lying on the opposite side of the centre and the distance between them is 17 cm. The length of AB and CD are 10 cm and 24 cm respectively. The radius(in cm) of the circle is:
(a) 13 (b) 18 (c) 9 (d) 15

28. A tangent is drawn to a circle of radius 6 cm from a point situated at a distance of 10 cm from the centre of the circle. The length of tangent will be (a) 4 cm (b) 5 cm
(c) 8 cm (d) 7 cm

29. Two chords of length a unit and b unit of a circle make angles 60° and 90° at the centre of a circle respectively, then the correct relation is:
(a) b = $\sqrt{2}$ a (b) b = 2a
(c) b = $\sqrt{3}$ a (d) b = 3/2a

30. Two circles touch externally. The sum of their areas is 130π sq cm and the distance between their centres is 14 cm. The radius of the smaller circle is:
(a) 2cm (b) 3 cm
(c) 4 cm (d) 5 cm

31. XY and XZ are tangents to a circle. ST is another tangent to the circle at the point R on the circle which intersects XY and XZ at S and T respectively, If XY = 9 cm and TX = 15 cm, then RT is :
(a) 4.5 cm (b) 3 cm
(c) 7.5 cm (d) 6 cm

32. A circle touches the four sides of a quadrilateral ABCD. The value of $\frac{(AB+CD)}{CB+DA}$ is equal to:
(a) $\frac{1}{3}$ (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

33. Two chords AB and CD of a circle with centre O, intersect each other at P. If $\angle AOD$ = 100° and $\angle BOC$ = 70°, then the value of $\angle APC$ is
(a) 80° (b) 75° (c) 85°(d) 95°

34. Chords AC and BD of a circle with centre O intersect at right angles at E. If $\angle OAB$ = 25°, then the value of $\angle EBC$ is
(a) 30° (b) 25° (c) 20°(d) 15°

35. Two circles touch externally at P. QR is a common tangent of the circles touching the circles at Q and R. Then measure of $\angle QPR$ is
(a) 120° (b) 60° (c) 90°(d) 45°

36. Two circles intersect each other at the points A and B. A straight line parallel to AB intersects the circles at C, D, E and F. If CD = 4.5 cm, then the measure of EF is
(a) 1.50 cm (b) 2.25 cm
(c) 4.50 cm (d) 9.00 cm

37. Two circles C_1 and C_2 touch each other internally at P. Two lines PCA and PDB meet the circles C_1 in C, D and C_2 in A, B respectively. If $\angle BDC = 120°$, then the value of $\angle ABP$ is equal to
(a) 60° (b) 80°
(c) 100° (d) 120°

38. Two circles having radii r units intersect each other in such a way that each of them passes through the centre of the other. Then the length of their common chord is
(a) $\sqrt{2r}$ units (b) $\sqrt{3r}$ units
(c) $\sqrt{5r}$ units (d) r units

39. Two circles with centres A and B of radii 5 cm and 3 cm respectively touch each other internally. If the perpendicular bisector of AB meets the bigger circle in P and Q, then the value of PQ is
(a) $\sqrt{6}$ cm (b) $2\sqrt{6}$ cm
(c) $3\sqrt{6}$ cm (d) $4\sqrt{6}$ cm

40. The length of a tangent from an external point to a circle is $5\sqrt{3}$ unit. If radius of the circle is 5 units, then the distance of the point from the circle is
(a) 5 units (b) 15 units
(c) – 5 units (d) – 15 units

41. Two circles are of radii 7 cm and 2 cm their centres being 13cm apart. Then the length of direct common tangent to the circles between the points of contact is
(a) 12 cm (b) 15 cm
(c) 10 cm (d) 5 cm

42. The length of chord of a circle is equal to the radius of the circle. The angle which this chord subtends in the major segment of the circle is equal to
(a) 30° (b) 45° (c) 60° (d) 90°

43. AB = 8 cm and CD = 6 cm are two parallel chords on the same side of the centre of a circle. The distance between them is 1 cm. The radius of the circle is
(a) 5 cm (b) 4 cm
(c) 3 cm (d) 2 cm

44. The length of two chords AB and AC of a circle are 8 cm and 6 cm and $\angle BAC$ = 90°, then the radius of circle is
(a) 25 cm (b) 20 cm
(c) 4 cm (d) 5 cm

45. The distance between two parallel chords of length 8 cm each in a circle of diameter 10 cm is
(a) 6 cm (b) 7cm
(c) 8 cm (d) 5.5 cm

46. The radius of two concentric circles are 9 cm and 15 cm. If the chord of the greater circle be a tangent to the smaller circle, then the length of that chord is
(a) 24 cm (b) 12 cm
(c) 30 cm (d) 18 cm

47. If chord of a circle of radius 5 cm is a tangent to another circle of radius 3 cm, both the circles being concentric, then the length of the chord is

(a) 10 cm (b) 12.5 cm
(c) 8 cm (d) 7 cm

48. The two tangents are drawn at the extermities of diameter AB of a circle with centre P. If a tangent to the circle at the point C intersects the other two tangents at Q and R, then the measure of the $\angle QPR$ is
(a) 45° (b) 60°
(c) 90° (d) 180°

49. AB is a chord to a circle and PAT is the tangent to the circle at A. If $\angle BAT$ = 75° and $\angle BAC$ = 45° and C being a point on the circle, then $\angle ABC$ is equal to
(a) 40° (b) 45° (c) 60° (d) 70°

50. Two circles touch each other externally at point A and PQ is a direct common tangent which touches the circles at P and Q respectively. Then $\angle PAQ$ = ?
(a) 45° (b) 90° (c) 80°(d) 100°

51. PR is tangent to a circle, with centre O and radius 4 cm, at point Q. If $\angle POR = 90°$, OR= 5 cm and OP = $\frac{20}{3}$ cm, then (in cm) the length of PR is :
(a) 3 (b) $\frac{16}{3}$ (c) $\frac{23}{3}$ (d) $\frac{25}{3}$

52. Two chords AB and CD of circle whose centre is O, meet at the point P and $\angle AOC$ = 50°, $\angle BOD = 40°$, Then the value of $\angle BPD$ is
(a) 60° (b) 40° (c) 45° (d) 75°

53. Two equal circles of radius 4 cm intersect each other such that each passes through the centre of the other. The length of the common chord is
(a) $2\sqrt{3}$ cm (b) $4\sqrt{3}$ cm
(c) $2\sqrt{2}$ cm (d) 8 cm

54. One chord of a circle is known to be 10.1 cm. The radius of this circle must be ;
(a) 5 cm
(b) greater than 5 cm
(c) greater than or equal to 5 cm
(d) less than 5 cm

55. The length of the chord of a circle is 8 cm and perpendicular distance between centre and the chord is 3 cm. Then the radius of the circle is equal to :
(a) 4 cm (b) 5 cm
(c) 6 cm (d) 8 cm

56. The length of the common chord of two intersecting circles is 24 cm. If the diameter of the circles are 30 cm and 26 cm, then the distance between the centre (in cm) is
(a) 13 (b) 14 (c) 15 (d) 16

57. In a circle of radius 21 cm an arc subtends an angle of 72° at the centre. The length of the arc is
(a) 21.6 cm (b) 26.4 cm
(d) 13.2 cm (d) 198.8 cm

58. A unique circle can always be drawn through x number of given non-collinear points, then x must be
(a) 2 (b) 3 (c) 4 (d) 1

59. Two parallel chords are drawn in a circle of diameter 30 cm. The length of one chord is 24 cm and the distance between the two chords is 21 cm. The length of the other chord is
(a) 10 cm (b) 18 cm
(c) 12 cm (d) 16 cm

60. If two equal circles whose centres are O and O' intersect each other at the point A and B , OO' = 12 cm and AB = 16 cm, then the radius of the circle is
(a) 10 cm (b) 8 cm
(c) 12 cm (d) 14 cm

61. Chords AB and CD of a circle intersect externally at P. If AB = 6 cm, CD= 3 cm and PD = 5 cm, then the length of PB is
(a) 5 cm (b) 7.35 cm
(c) 6 cm (d) 4 cm

62. AB and CD are two parallel chords on the opposite sides of the centre of the circle. If AB = 10 cm, CD = 24 cm and the radius of the circle is 13 cm, the distance between the chords is
(a) 17 cm (b) 15 cm
(c) 16 cm (d) 18 cm

63. Two circles touch each other externally at P. AB is a direct common tangent to the two circles, A and B are point of contact and $\angle PAB$ = 35°. Then $\angle ABP$ is
(a) 35° (b) 55° (c) 65° (d) 75°

64. If the radii of two circles be 6 cm and 3 cm and the length the transverse common tangent be 8 cm, then the distance between the two centres is
(a) $\sqrt{145}$ cm (b) $\sqrt{140}$ cm
(c) $\sqrt{150}$ cm (d) $\sqrt{135}$ cm

65. The distance between the centre of two equal circles each of radius 3 cm, is 10 cm. The length of a transverse common tangent is
(a) 8 cm (b) 10 cm
(c) 4 cm (d) 6 cm

66. The radii of two circles are 5 cm and 3 cm, the distance between their centre is 24 cm. Then the length of the transverse common tangent is
(a) 16 cm (b) $15\sqrt{2}$
(c) $16\sqrt{2}$ (d) 15 cm

67. P and Q are centre of two circles with radii 9 cm and 2 cm respectively, where PQ =17 cm. R is the centre of another circle of radius x cm, which touches each of the above two circles externally. If $\angle PRQ = 90°$, then the value of x is
(a) 4 cm (b) 6 cm
(c) 7 cm (d) 8 cm

68. Two chords AB and CD of a circle with centre O intersect each other at the point P. If $\angle AOD$ = 20° and $\angle BOC$ = 30°, then $\angle BPC$ is equal to :
(a) 50° (b) 20° (c) 25° (d) 30°

69. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm . If the chords are on the opposite sides of the centre and distance between them is 17 cm, then the radius of the circle is :
(a) 11 cm (b) 12 cm
(c) 13 cm (d) 10 cm

70. A chord AB of a circle C_1 of radius $(\sqrt{3}+1)$ cm touches a circle C_2 which is concentric to C_1. If the radius of C_2 is $(\sqrt{3}-1)$ cm. The length of AB is :
(a) $2\sqrt[4]{3}$ cm (b) $8\sqrt{3}$ cm
(c) $4\sqrt[4]{3}$ cm (d) $4\sqrt{3}$ cm

71. The length of the common chord of two circles of radii 30 cm and 40 cm whose centres are 50 cm apart is (in cm)
(a) 12 (b) 24 (c) 36 (d) 48

72. Chords AB and CD of a circle intersect at E and are perpendicular to each other. Segments AE, EB and ED are of lengths 2 cm, 6 cm and 3 cm respectively. Then the length of the diameter of the circle (in cm) is

(a) $\sqrt{65}$ (b) $\frac{1}{2}\sqrt{65}$

(c) 65 (d) $\frac{65}{2}$

73. Two circles of same radius 5 cm, intersect each other at A and B. If AB = 8 cm, then the distance between the centre is ;

(a) 6 cm (b) 8 cm
(c) 10 cm (d) 4 cm

74. AD is the chord of a circle with centre O and DOC is a line segment orginating from a point D on the circle and intersecting AB produced at C such that BC = OD. If $\angle BCD$ =20°, then $\angle AOD$ = ?

(a) 20° (b) 30° (c) 40° (d) 60°

75. In a circle of radius 17 cm, two parallel chords of length 30 cm and 16 cm are drawn. If both chords are on the same side of the centre. then the distance between the chords is

(a) 9 cm (b) 7 cm
(c) 23 cm (d) 11 cm

76. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the greater circle which is outside the inner circle is of length

(a) $2\sqrt{2}$ cm (b) $3\sqrt{2}$ cm

(c) $2\sqrt{3}$ cm (d) $4\sqrt{2}$ cm

77. Two circles touch each other externally. The distance between their centre is 7 cm. If the radius of one circle is 4 cm, then the radius of the other circle is

(a) 3.5 cm (b) 3 cm
(c) 4 cm (d) 2 cm

78. A, B and C are the three points on a circle such that the angles subtended by the chords AB and AC at the centre O are 90° and 110° respectively. $\angle BAC$ is equal to

(a) 70° (b) 80° (c) 90° (d)100°

79. N is the foot of the perpendicular from a point P of a circle with radius 7 cm, on a diameter AB of the circle. If the length of the chord PB is 12 cm , the distance of the point N from the point B is

(a) $6\frac{5}{7}$ cm (b) $12\frac{2}{7}$ cm

(c) $3\frac{5}{7}$ cm (d) $10\frac{2}{7}$ cm

80. A, B, C, D are four points on a circle, AC and BD intersect at a point E such that $\angle BEC$ = 130° and $\angle ECD$ = 20°. $\angle BAC$ is

(a) 120° (b) 90° (c) 100°(d) 110°

81. If two concentric circles are of radii 5 cm and 3 cm, then the length of the chord of the larger circle which touches the smaller circle is :

(a) 6 cm (b) 7 cm
(c) 10 cm (d) 8 cm

82. A chord 12 cm long is drawn in a circle of diameter 20 cm. The distance of the chord from the centre is

(a) 8 cm (b) 6 cm
(c) 10 cm (d) 16 cm

83. If the chord of a circle is equal to the radius of the circle, then the angle subtended by the chord on centre is

(a) 150° (b) 60° (c) 120°(d) 30°

84. In a right angled triangle, the circumcentre of the triangle lies

(a) inside the triangle
(b) outside the triangle
(c) on midpoint of the hypotenuse
(d) on one vertex

85. P and Q are two points on a circle with centre at O. R is a point on the minor arc of the circle, between the points P and Q. The tangents to the circle at the points P and Q meet each other at the point S. If $\angle PSQ$ = 20°, then $\angle PRQ$ = ?

(a) 80° (b) 200°
(c) 160° (d) 100°

86. Two circles intersect at A and B, P is a point on produced BA. PT and PQ are tangents to the circles. The relation of PT and PQ is

(a) PT = 2PQ (b) PT < PQ
(c) PT > PQ (d) PT = PQ

87. The length of the tangent drawn to a circle of radius 4 cm from a point 5 cm away from the centre of the circle is

(a) 3 cm (b) $4\sqrt{2}$ cm

(c) $5\sqrt{2}$ cm (d) $3\sqrt{2}$ cm

88. From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP is equal to diameter of the circle, then $\angle APB$ is

(a) 45° (b) 90° (c) 30° (d) 60°

89. The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle and BD is a tangent to the smaller circle touching it at D and the bigger circle at E. Point A is joined to D. The length of AD is

(a) 20 cm (b) 19 cm
(c) 18 cm (d) 17 cm

90. PQ is a chord of length 8 cm of a circle with centre O and radius 5 cm. The tangents at P and Q intersect at a point T. The length of TP is

(a) $\frac{20}{3}$ cm (b) $\frac{21}{4}$ cm

(c) $\frac{10}{3}$ cm (d) $\frac{15}{4}$ cm

91. The maximum number of common tangents drawn to two circles when both the circles touch each other externally is

(a) 1 (b) 2 (c) 3 (d) 0

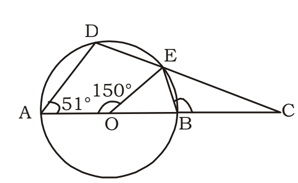
92. The radius of two concentric circles are 17 cm and 10 cm. A straight line ABCD intersects the larger circle at the point A and D and intersects the smaller circle at the points B and C. If BC = 12 cm, then the length of AD (in cm) is

(a) 20 (b) 24 (c) 30 (d) 34

93. Two chords AB , CD of a circle with centre O intersect each other at P. $\angle ADP$ = 23° and $\angle APC$ = 70°, then the $\angle BCD$ is

(a) 45° (b) 47° (c) 57° (d) 67°

94. In the following figure, AB is the diameter of a circle whose centre is O. If $\angle AOE$ = 150°, $\angle DAO$ = 51° then the measure of $\angle CBE$ is :



(a) 115° (b) 110°
(c) 105° (d) 120°

95. The angle in a semi-circle is
(a) a reflex angle
(b) an obtuse angle
(c) an acute angle
(d) a right angle

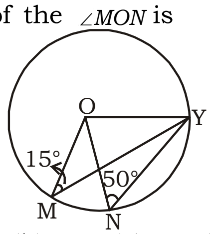
96. The angle subtended by a chord at its centre is 60°, then ratio between chord and radius is
(a) 1 : 2 (b) 1 : 1
(c) $\sqrt{2}:1$ (d) 2 : 1

97. Each of the circles of equal radii with centres and B pass through the centre of one another circle they cut at C and D then $\angle DBC$ is equal to
(a) 60° (b) 100° (c) 120° (d) 140°

98. The three equal circles touch each other externally. If the centres of these circles are A, B, C, then ABC is
(a) a right angle triangle
(b) an equilateral triangle
(c) an isosceles triangle
(d) a scalene triangle

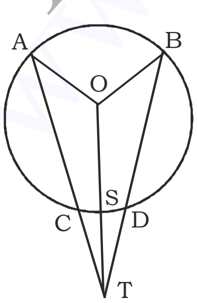
99. 'O' is the centre of the circle, AB is a chord of the circle, $OM \perp AB$. If AB = 20 cm, $OM = 2\sqrt{11}$ cm, then radius of the circle is
(a) 15 cm (b) 12 cm
(c) 10 cm (d) 11 cm

100. In the given figure, $\angle ONY$ = 50°and $\angle OMY$ =15°. Then the value of the $\angle MON$ is



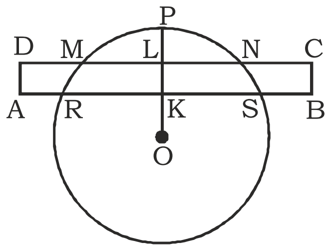
(a) 30° (b) 40° (c) 20° (d) 70°

101. In the adjoining figure 'O' is the centre of circle AC and BD are the two chords of circle which meeets at T outside the circle. OT bisects CD, OA = OB = 8cm and OT = 17cm. What isthe ratio of distance of AC and BD from the centre of the circle ?



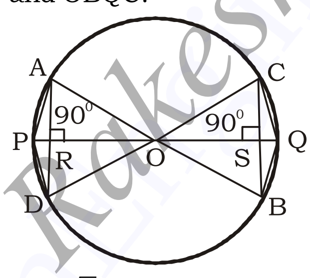
(a) 15 : 17 (b) 8 : 15
(c) 8 : 9
(d) none of these

102. In the adjoining figure O is the centre of the circle. The radius OP bisects a rectangle ABCD, at right angle. DM = NC = 2cm and AR = SB = 1cm and KS = 4cm and OP = 5cm. What is the area of the rectangle?



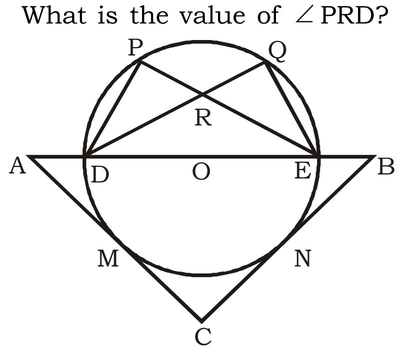
(a) 8 cm² (b) 10 cm²
(c) 12 cm²
(d) none of these

103. In the adjoining figure O isthe centre of the circle. $\angle$AOD = 120°. If the radius of the circle be 'r', then find the sum of the areas of quadrilaterals AODP and OBQC:



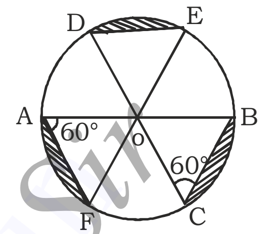
(a) $\frac{\sqrt{3}}{2}r^2$ (b) $3\sqrt{3}r^2$
(c) $\sqrt{3}r^2$ (d) none of these

104. ABC is an isosceles triangle and AC, BC are the tangents at M and N respectively. DE is the diameter of the circle. $\angle$ADP = $\angle$BEQ = 100°.
What is the value of $\angle$PRD?



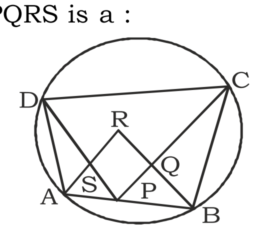
(a) 60° (b) 50°
(c) 20°
(d) Can't be determined.

105. In the adjoining figure O is the centre of the circle with radius 'r' AB, CD and EF are the diameters of the circle. $\angle$OAF = $\angle$OCB = 60°. What is the area of the shaded region?



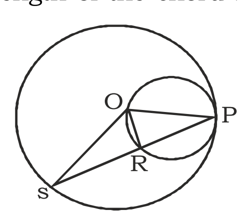
(a) $\frac{r^2}{2}\left(\pi-\frac{3\sqrt{3}}{2}\right)$
(b) $\frac{r^2}{2}\left(\pi-\frac{3\sqrt{3}}{4}\right)$
(c) $\frac{r^2}{3}\left(\pi-\frac{2\sqrt{3}}{4}\right)$
(d) data insufficient

106. ABCD is a cyclic quadrilateral. The angle bisector of $\angle$A, $\angle$B, $\angle$C and $\angle$D intersect at P, Q, R and S as shown in the figure. These four points form a quadrilateral PQRS. Quadrilateral PQRS is a :



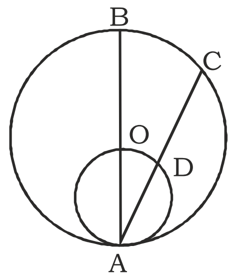
(a) Square
(b) rhombus
(c) rectangle
(d) cyclic quadrilateral

107. In the adjoining figure the diameter of the larger circle is 10 cm and the smaller circle touches internally the larger circle at P and passes through O, the centre of the larger circle at R and OR is equal to 4 cm. What is the length of the chord SP?



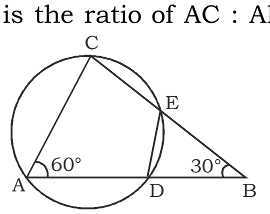
(a) 9cm (b) 12 cm
(c) 6cm (d) $8\sqrt{2}$ cm

108. A smaller circle touches internally to a larger circle at A and passes through the centre of the larger circle. O is the centre of the larger circle and BA, OA are of the diameters of the larger and smaller circles respectively. Chord AC intersects the smaller circle at a point D. If AC = 12 cm, then AD is :



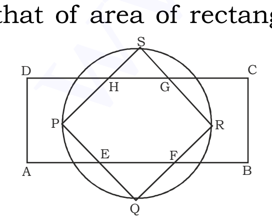
(a) 4 cm (b) 6 cm
(c) 5.6 cm
(d) data insufficient

109. In the given figure ADEC is a cyclic quadrilateral, CE and AD are extended to meet at B. $\angle CAD = 60°$ and $\angle CBA = 30°$. BD = 6cm and CE = $5\sqrt{3}$ cm, What is the ratio of AC : AD?



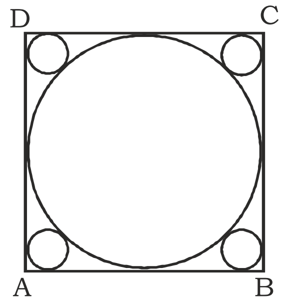
(a) $\frac{3}{4}$ (b) $\frac{4}{5}$
(c) $\frac{2\sqrt{3}}{5}$
(d) can't be determined

110. In the adjoining figure ABCD is a rectangle in which length is twice of breadth. H and G divide the line CD into three equal parts. Similarly points E and F trisect the line AB. A circle PQRS is circumscribed by a square PQRS which passes through the points E,F,G and H. What is the ratio of area of circle to that of area of rectangle?



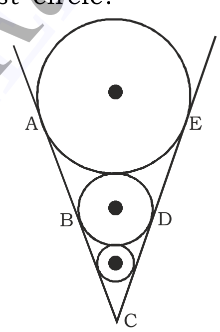
(a) $3\pi : 7$ (b) 3 : 4
(c) $25\pi : 72$ (d) $32\pi : 115$

111. ABCD is a square, in which a circle is inscribed touching all the sides of square. In the four corners of square 4 smaller circles of equal radii are drawn, containing maximum possible area.
What is the ratio of the area of larger circle to that of sum of the areas of four smaller circles?



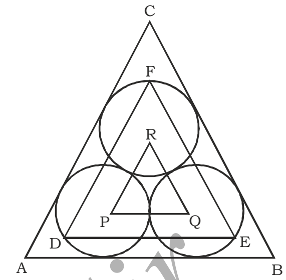
(a) $1 : (68 - 48\sqrt{2})$
(b) $1 : (17 - 12\sqrt{2})$
(c) $12 : 17\sqrt{2}$
(d) None of these

112. In the adjoining figure $\angle ACE$ is a right angle there are three circles which just touch each other and AC and EC are the tangents to all the three circles. What is the ratio of the smallest circle?



(a) $17 : 12\sqrt{2}$
(b) $1 : (17 - 12\sqrt{2})$
(c) $12 : 17\sqrt{2}$
(d) none of these

113. In the adjoining figure three congruent circles ar touching each other. Triangle ABC circumscribes all the three circles. Triangles PQR is formed by joining the centres of the circle. There is a third triangle DEF. Points A, D, P and B, E, Q and C, F, R lie in the same straight respectively.
What is the ratio of perimeters of $\Delta ABC : \Delta DEF : \Delta PQR$



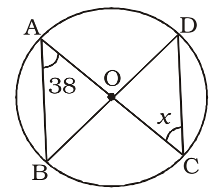
(a) $3\sqrt{2} : 2\sqrt{2} : 1$
(b) $2(4 + \sqrt{3}) : (2 + \sqrt{3}) : \sqrt{3}$
(c) $2(1 + \sqrt{3}) : (2 + \sqrt{3}) : 2$
(d) $2(1 + \sqrt{3}) : 2\sqrt{3} : \sqrt{3}$

114. Through any given set of four points A, B, C, D it is possible to draw:-
(a) atmost one circle
(b) exactly one circle
(c) exactly two circles
(d) exactly three circles

115. The number of common tangents that can be drawn to two given circles intersect each other is :-
(a) one (b) two
(c) three (d) four

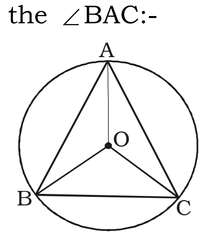
116. The radius of a circle is 13 cm and the length of one of its chords is 10 cm. Find the distance of chord from the centre.
(a) 8 cm (b) 10 cm
(c) 9 cm (d) 12 cm

117. In the given figure O is the centre of the circle. If $\angle BAC = 38°$, then $\angle OCD$ is



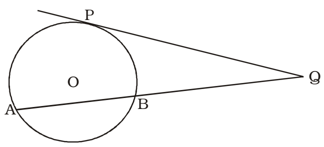
(a) 76° (b) 52°
(c) 38° (d) 19°

118. In the given figure, O is the centre of the circle. If $\angle OBC = 20°$, the $\angle BAC$:-



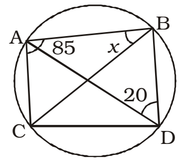
(a) 80° (b) 70°
(c) 100° (d) 140°

119. In the given figure PQ = 12 cm, BQ = 8cm, then the length of chord AB:-



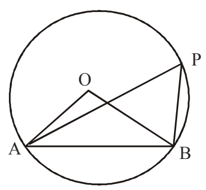
(a) 10 cm (b) $4\sqrt{5}$ cm
(c) 4 cm (d) 18 cm

120. The value of x will be:-



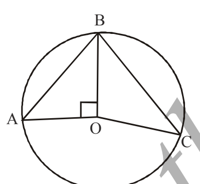
(a) 70° (b) 90°
(c) 60° (d) 75°

121. In the given figure, O is the centre of the circle and $\angle AOB = 90°$, then $\angle APB$ will be:-



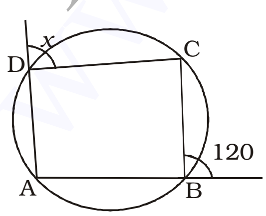
(a) 90° (b) 60°
(c) 45° (d) 30°

122. In the given figure, O is the centre of the circle $\angle AOB = 90°$, $\angle BOC = 110°$, then $\angle ABC$ is



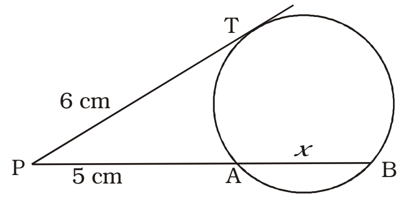
(a) 75° (b) 60°
(c) 80° (d) 70°

123. ABCD is a cyclic quadrilateral, then the value of x will be :-



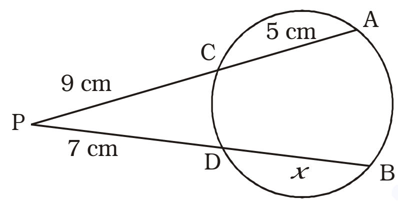
(a) 50° (b) 60°
(c) 120° (d) 70°

124. The value of x :-



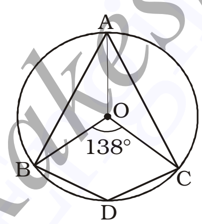
(a) 2.2 cm (b) 1.6 cm
(c) 3 cm (d) 2.6 cm

125. The value of x :-



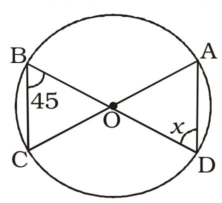
(a) 10 cm (b) 9 cm
(c) 7.5 cm (d) 11 cm

126. In the given figure ABCD is a cyclic quadrilateral and O is the centre of the circle. If $\angle BOC$ = 138°, then $\angle BDC$ will be



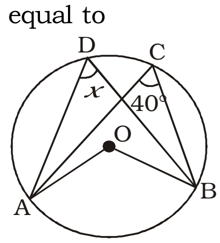
(a) 112° (b) 111°
(c) 109°
(d) None of these

127. In the given figure, O is the centre of the circle, then the value of x will be:-



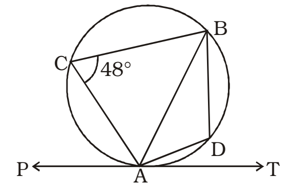
(a) 40° (b) 90°
(c) 45° (d) 30°

128. If O is centre of the circle, then x is equal to



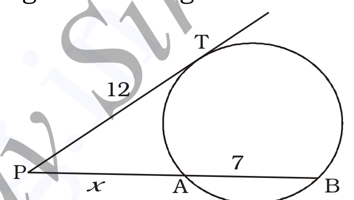
(a) 40° (b) 45°
(c) 39° (d) 35°

129. In the given figure, $\angle ADB$:-



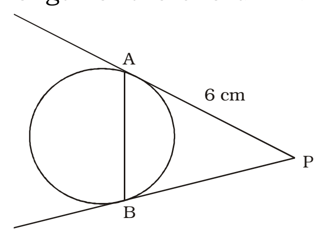
(a) 144° (b) 132°
(c) 48° (d) 96°

130. Find the value of x in the given figure:



(a) 16 cm (b) 9 cm
(c) 12 cm (d) 7 cm

131. In the given figure, PA and PB are tangents from a point P to a circle such that PA = 6 cm and $\angle APB = 60°$. What is the length of the chord AB?



(a) 12 cm (b) 8 cm
(c) 9 cm (d) 6 cm

132. ABC is a right angled triangle AB = 3 cm,BC = 5 cm and AC = 4 cm, then the inradius of the circle is
(a) 1 cm (b) 1.25 cm
(c) 1.5 cm
(d) None of these

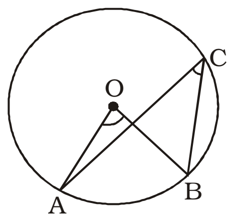
133. The number of common tangents that can be drawn to two given circles is at the most
(a) 1 (b) 2
(c) 3 (d) 4

134. Two circle of radii 12 cm and 7 cm touch each other internally. Find the distance between their centres.
(a) 6 cm (b) 13 cm
(c) 9 cm (d) 5 cm

135. Three circles touch each other externally.The distance between their centres is 5 cm, 6 cm and 7 cm. Find radii of the circles:-

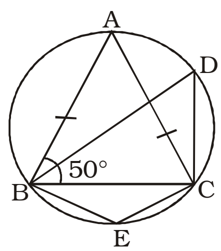
(a) 2 cm, 3 cm, 4 cm
(b) 3 cm, 4 cm, 1 cm
(d) 1 cm, 2 cm, 4 cm
(d) None of these

136. In the given figure, O is the centre of the circle and $\angle ACB = 30°$. Find $\angle AOB$.



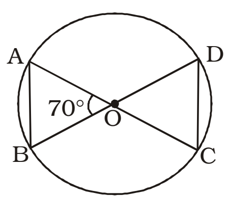
(a) 30° (b) 90°
(c) 60° (d) 50°

137. In the given figure, AB = AC and $\angle ABC = 50°$, Find $\angle BDC$:



(a) 60° (b) 80°
(c) 100° (d) 90°

138. In the given figure, O is the centre of the circle. $\angle AOB = 70°$, find $\angle OCD$.



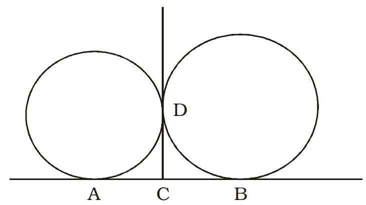
(a) 70° (b) 55°
(c) 65° (d) 110°

139. If the diagonals of a cyclic quadrilateral are equal, then the quadrilateral is
(a) rhombus (b) square
(c) rectangle
(d) None of these

140. The quadrilateral formed by angle bisec tors of cyclic quadrilateral is a
(a) rectangle
(b) square
(c) parallelogram
(d) cyclic quadrilateral

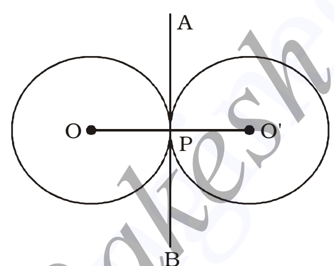
141. The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is
(a) 4 cm (b) 6 cm
(c) 8 cm (d) 10 cm

142. In the given figure, AB and CD are two common tangents to the two touching circle. If CD = 7 cm, then AB is equal to



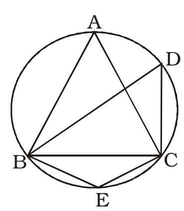
(a) 14 cm (b) 10.5 cm
(c) 12 cm
(d) None of these

143. O and O' are the centres of two circles which touch each other externally at P. If AB is a common tangent. Find $\angle APO$.



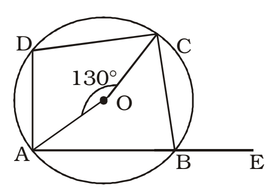
(a) 90° (b) 120°
(c) 60°
(d) data insufficient

144. In the given figure, ΔABC is an equilateral triangle. Find $\angle BEC$.



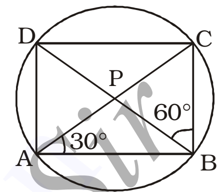
(a) 60° (b) 120°
(c) 80° (d) 90°

145. In the given figure, $\angle AOC = 130°$. Find $\angle CBE$, where O is the centre.



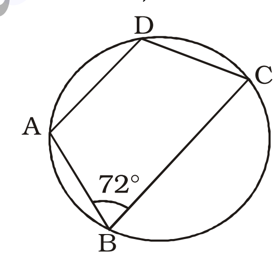
(a) 130° (b) 100°
(c) 115° (d) 105°

146. In the given figure, ABCD is a cyclic quadrilateral and diagonals bisect each other at P. If $\angle DBC = 60°$, and $\angle BAC = 30°$ then $\angle BCD$ is



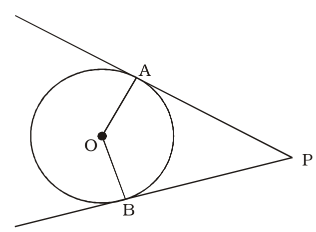
(a) 90° (b) 60°
(c) 80°
(d) None of these

147. In the given figure, AD||BC, if $\angle ABC = 72°$, then $\angle BCD$ = ?



(a) 108° (b) 36°
(c) 90° (d) 72°

148. In the given figure, O is the centre of the circle. PA and PB are tangents if $\angle AOB : \angle APB = 5 : 1$, then $\angle APB$



(a) 150° (b) 30°
(c) 60° (d) 90°

149. R and r are the radius of two circles (R>r). If the distance between the centre of the two circles be d, then length of common tangent of two circles is :

(a) $\sqrt{r^2-d^2}$

(b) $\sqrt{d^2-(R-r)^2}$

(c) $\sqrt{(R-r)^2-d^2}$

(d) $\sqrt{R^2-d^2}$

150. Two circles of radii 8cm and 2 cm respectively touch each other externally at the point A. PQ is the direct common tangent of those two circles of centres O_1 and O_2 respectively. Then length of QP is equal to :
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 8 cm

151. PQ is a direct common tangent of two circles of radii r_1 and r_2 touching each other externally at A. Then the value of PQ^2 is :
(a) r_1r_2 (b) $2r_1r_2$
(c) $3r_1r_2$ (d) $4r_1r_2$

152. Two circles with radii 5 cm and 8 cm touch each other externally at a point A. If a straight line through the point A cuts the circles at points P and Q respectively, then AP : AQ is :
(a) 8 : 5 (b) 5 : 8
(c) 3 : 4 (d) 4 : 5

153. The radius of a circle is 6 cm. An external point is at a distance of 10 cm from the centre. Then the length of the tangent drawn to the circle from the external point upto the point of contact is :
(a) 8 cm (b) 10 cm
(c) 6 cm (d) 12 cm

154. A triangle is inscribed in a circle and the diameter of the circle is its one side. Then the triangle will be :
(a) right-angled
(b) obtuse-angled
(c) equilateral
(d) a square

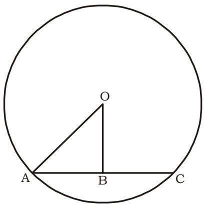
155. Two circles of radii 4 cm and 9 cm respectively touch each other externally at a point and a common tangent touches them at the points P and Q respectively. Then the area of square with one side PQ, is :
(a) 97 sq.cm (b) 194 sq.cm
(c) 72 sq.cm (d) 144 sq.cm

156. The length of the chord of a circle is 8 cm and perpendicular distance between centre and the chord is 3 cm. Then the radius of the circle is equal to :
(a) 4 cm (b) 5 cm
(c) 6 cm (d) 8 cm

157. The radius of a circle is 13 cm and XY is a chord which is at a distance of 12 cm from the centre. The length of the chord is :
(a) 15 cm (b) 12 cm
(c) 10 cm (d) 20 cm

158. SR is a direct common tangent to the circles of radii 8 cm and 3 cm respectively, their centres being 13 cm apart. If the points S and R are the respective points of intersect, then the length of SR is :
(a) 12 cm (b) 11 cm
(c) 17 cm (d) 10 cm

159. In the following figure, if OA = 10 and AC = 16, then OB must be :



(a) 5 (b) 6
(c) 3 (d) 4

160. One chord of a circle is known to be 10.1 cm. The radius of this circle must be:
(a) 5 cm
(b) greater than 5 cm
(c) greater than or equal to 5 cm
(d) less than 5 cm

161. The length of two chords AB and AC of a circle are 8 cm and 6 cm and $\angle BAC = 90°$, then the radius of circle is :
(a) 25 cm (b) 20 cm
(c) 30 cm (d) 5 cm

162. If a chord of length 16 cm is at a distance of 15 cm from the centre of the circle, then the length of the chord of the same circle which is at distance of 8 cm from the centre is equal to:
(a) 10 cm (b) 20 cm
(c) 30 cm (d) 40 cm

163. PR is tangent to circle, with centre O and radius 4 cm, at point Q. If $\angle POR = 90°$, OR = 5 cm and $OP = \frac{20}{3}$ cm, then, in cm, the length of PR is :
(a) 3 (b) $\frac{16}{3}$
(c) $\frac{23}{3}$ (d) $\frac{25}{3}$

164. Circumcentre of ΔABC is O. If $\angle BAC = 85°, \angle BCA = 80°$, then $\angle AOC$ is :
(a) 80° (b) 30°
(c) 60° (d) 75°

165. If O is the circumcentre of ΔABC and $\angle OBC = 35°$, then the $\angle BAC$ is equal to :
(a) 55° (b) 110°
(c) 70° (d) 35°

166. If I is the incentre of ΔABC and $\angle BIC = 135°$, then the ΔABC is :
(a) Acute angled
(b) equilateral
(c) right angled
(d) obtuse angled

167. If S is the circumcentre of ΔABC and $\angle A = 50°$, then the value of $\angle BCS$ is :
(a) 20° (b) 40°
(c) 60° (d) 80°

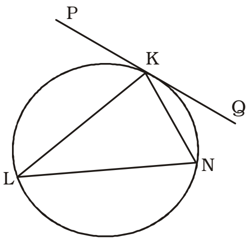
168. The distance between the centres of two equal circles, each of radius 3 cm, is 10 cm. The length of a transverse common tangent is :
(a) 8 cm (b) 10 cm
(c) 4 cm (d) 6 cm

169. A unique circle can always be drawn through x number of given non-collinear points, then x must be :
(a) 2 (b) 3 (c) 4 (d) 1

170. The length of radius of a circumcircle of a triangle having sides 3cm, 4cm and 5cm is:
(a) 2 cm (b) 2.5 cm
(c) 3 cm (d) 1.5 cm

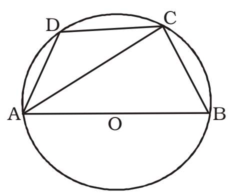
171. AB and CD are two parallel chords of a circle such that AB = 6cm and CD=8cm. If the chords lie on the same side of centre O and radius 5cm the distance between AB and CD is :
(a) 2 cm (b) 1 cm
(c) 2.5 cm (d) 3 cm

172. In the given figure PKQ is a tangent and LN is the diameter of the circles. If $\angle KLN = 30°$ then $\angle PKL$ will be :



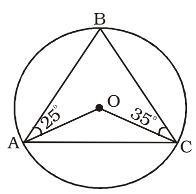
(a) 30° (b) 50°
(c) 60° (d) 70°

173. In the given figure ∠ ADC=120º and AOB is the diameter of the circle, then∠ BAC :



(a) 30° (b) 40°
(c) 50° (d) 60°

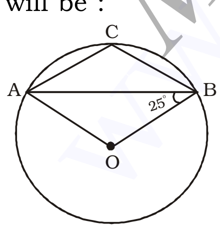
174. ∠OAB = 25°, ∠OCB = 35° then ∠AOC will be :



(a) 60° (b) 80°
(c) 100° (d) 120°

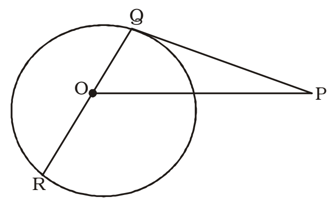
175. AB and CD are two parallel chords of a circle such that AB =10cm and CD = 24cm, If the chords are on the opposite sides of the centre and the distance between them is 17cm, then the radius of the circle is :
(a) 8 cm (b) 15 cm
(c) 11 cm (d) 13 cm

176. In the given figure, O is the centre of the circle then∠ACB will be :



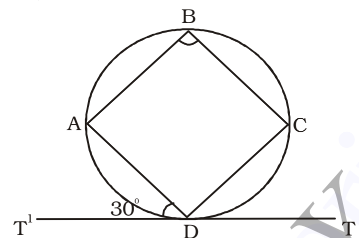
(a) 105° (b) 230°
(c) 115° (d) 100°

177. In the given figure, ROQ is the diameter of the circle. If ∠POR = 120^0 then ∠ QPO will be:



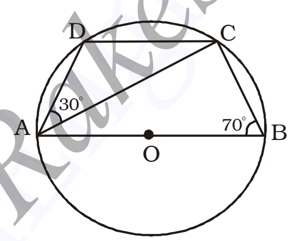
(a) 40° (b) 30°
(c) 60° (d) 50°

178. In the given figure ∠ ABC = 55°, the ∠ CDT is:



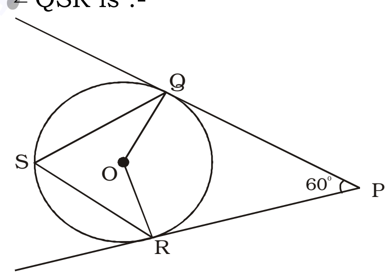
(c) 15° (b) 20°
(c) 25° (d) 30°

179. In the given figure if AB is the diameter of the circle, then ∠ ACD will be :



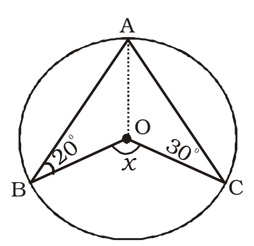
(a) 40° (b) 50° (c) 35° (d) 90°

180. ∠ QSR is :-



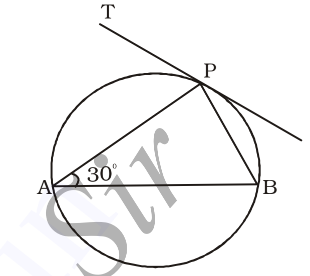
(a) 55^0 (b) 90^0
(c) 70^0 (d) 60^0

181. Find the value of ∠x in the given figure :-



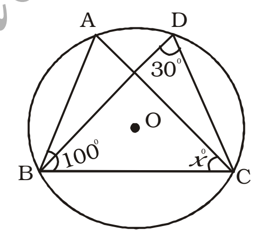
(a) 120^0 (b) 130^0
(c) 110^0 (d) 100^0

182. In the given figure AB is the diameter of the circle and ∠ PAB = 30°, Find ∠ TPA



(a) 30^0 (b) 60^0
(c) 50^0 (d) 70^0

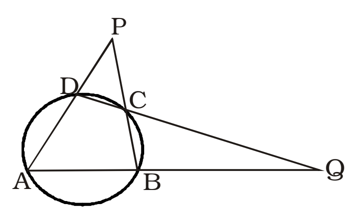
183. In the following figure, find the value of x



(a) 40^0 (b) 45^0
(c) 50^0 (d) 60^0

184. If a circle is provided with a measure of 19^0 on centre, is it possible to divide the circle into 360 equal parts ?
(a) Never
(b) Possible when one measure of 20^0 is given
(c) Always
(d) Possible when one measure of 21^0 is given

185. In the adjoining figure ∠ A=60°and ∠ ABC = 80°, ∠ BQC = ?



(a) 40^0 (b) 80^0 (c) 20^0 (d) 30^0

186. Two circles of radius 37cm and 20cm intersect each other at A and B.O and O' are the centres of the circles. If the length of AB is 24cm , then OO':-
(a) 50cm (b) 51cm
(c) 40cm (d) 57cm

187. Two circles of radius 4cm and 6cm touch each other internally. Find the longest chord of the bigger circle which is outside of the smaller circle?

(a) $8\sqrt{2}$ cm (b) $4\sqrt{2}$ cm

(c) $6\sqrt{2}$ cm (d) $3\sqrt{2}$ cm

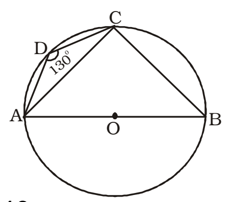
188. In a circle of radius 17cm two parallel chords are present on the opposite side of the diameter. If the distance between them is 23cm and the length of one chord is 16cm then the length of other chord is:-

(a) 15 cm (b) 20cm
(c) 18 cm (d) 30cm

189. AB is a chord of the circle (centre O). P is a point on the circle such that OP $\perp$ AB and OP intersect AB at point M. If AB = 8cm, MP = 2cm then radius (r):-

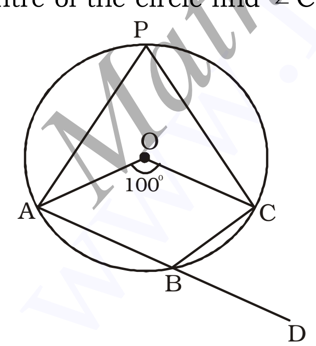
(a) 7 cm (b) 5cm
(c) 6 cm (d) 4cm

190. In the given figure, ABCD is a cyclic quadrilateral whose side AB is a diameter of the circle through A,B and C. If $\angle ADC = 130^0$ find $\angle CAB$.



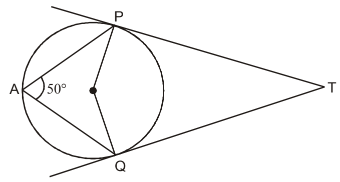
(a) 40^0 (b) 50^0
(c) 30^0 (d) 130^0

191. In the given figure, O is the centre of the circle find $\angle$CBD



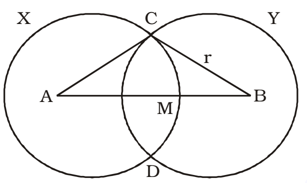
(a) 140^0 (b) 50^0
(c) 40^0 (d) 130^0

192. In the given figure, TP and TQ are tangents to the circle. If $\angle PAQ = 50°$, what is $\angle PTQ$?



(a) 80^0 (b) 70^0
(c) 100^0 (d) 90^0

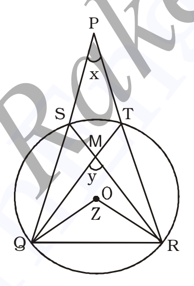
193. Two circles X and Y with centres A and B intersect at C and D. If Area of circle X is 4 time area of circle Y, then AB = ?



(a) 5 r (b) $\sqrt{5}$ r

(c) 3 r (d) $\frac{\sqrt{5}}{2}$ r

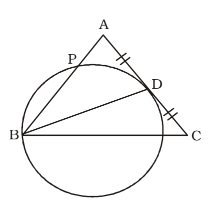
194. In the given figure, O is the centre of the circle. Then $\angle x + \angle y$ is equal to-



(a) 2 Z (b) $\frac{Z}{2}$

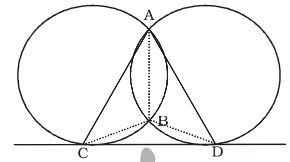
(c) Z
(d) None of these

195. In the figure, ABC is a triangle in which AB = AC. A circle through B touches AC at D and intersects AB at P. If D is the mid-point of AC, Find the value of AB :-



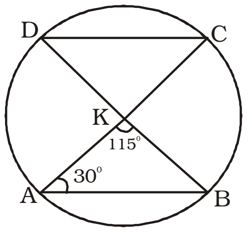
(a) 2AP (b) 3AP
(c) 4AP
(d) None of these

196. In the given figure, CD is a direct common tangent to two circles intersecting each other at A and B, then $\angle$CAD + CBD = ?



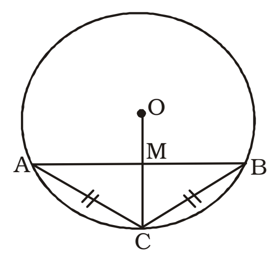
(a) 120° (b) 90°
(c) 360°
(d) None of these

197. In the given figure, $\angle$CAB = 30° and $\angle$AKB = 115°, find $\angle$KCD :-



(a) 65^0 (b) 35^0
(c) 40^0 (d) 72^0

198. In the given figure,the chords AC and BC are equal. The radius OC intersect AB at M then AM : BM :-



(a) 1:1 (b) $\sqrt{2}:3$

(c) $3:\sqrt{2}$

(d) None of these

199. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :-

(a) $\sqrt{3}:2$ (b) $\sqrt{3}:1$

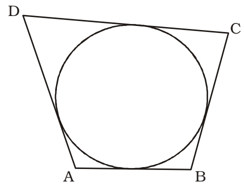
(c) $\sqrt{5}:1$

(d) None of these

200. ABCD is a cyclic trapezium whose sides AD and BC are parallel to each other; if $\angle ABC = 75°$ then the measure of $\angle BCD$ is:

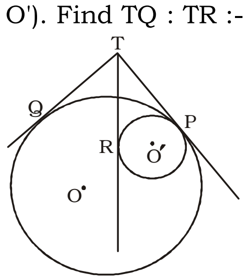
(a) 75° (b) 95° (c) 45° (d)105°

201. A circle touches a quadrilateral ABCD. Find the true statement :-



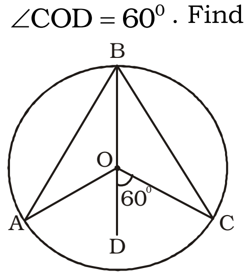
(a) BD = AC
(b) AB + BC = CD + AD
(c) AB + BC = AC
(d) AB + CD = BC + AD

202. In the given figure, Tangents TQ and TP are drawn to the larger circle centre O and tangents TP and TR are drawn to the smaller circle (centre O'). Find TQ : TR :-



(a) 1 : 1 (b) 5 : 4
(c) 8 : 7 (d) 7 : 8

203. 'O' is the centre of the circle, line segment $\angle$BOD is the angle bisector of $\angle AOC$, $\angle COD = 60^0$. Find $\angle ABC$:-



(a) 120^0 (b) 60^0
(c) 30^0 (d) 90^0

204. If O is the centre of the circle and PA and PB are two tangents drawn from a point P on the circumference of the circle. If $\angle APB = 68^0$, then $\angle POA$ = ?
(a) 68^0 (b) 34^0
(c) 56^0 (d) 90^0

205. In a circle, AB is the diameter of the circle, and CD is a chord such that CD||AB .P is any point on the circle such that $\angle BPC = 48^0$, then $\angle BCD$ = ?
(a) 48^0 (b) 42^0 (c) 24^0 (d) 96^0

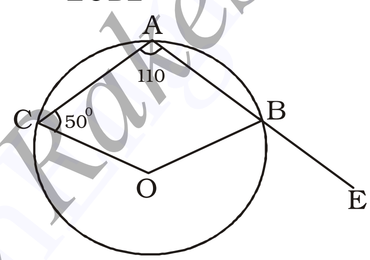
206. AB and CD are two chords of a circle intersect at a point P. If $\angle APC = 80^0$ and $\angle ADP = 30°$, then $\angle BCD$ = ?
(a) 30^0 (b) 80^0
(c) 100^0 (d) 50^0

207. ABCD is a cyclic quadrilateral. Side AB and DC when produced meet at P and side AD and BC when Produced meet at Q. If $\angle$APD = 40°, $\angle$ADC = 85°, then $\angle$AQB is equal to :-
(a) 30^0 (b) 40^0
(c) 50^0 (d) 55^0

208. AB is the diameter of the circle with centre O. DC is a chord such that DC||AB. If $\angle$BAC = 20°, then $\angle$ADC is equal to :-
(a) 100^0 (b) 90^0
(c) 110^0 (d) 120^0

209. In question above, find $\angle$COD?
(a) 50^0 (b) 100^0
(c) 25^0 (d) 90^0

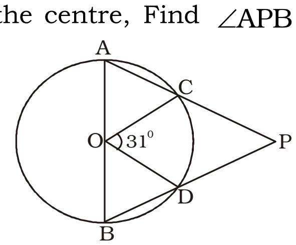
210. Find $\angle$OBE ?



(a) 120^0 (b) 100^0
(c) 115^0
(d) None of these

211. AB is the diameter of a circle whose center is O and CD is a chord in the circle and $CD = \frac{1}{2}AB$. AC and BD on producing meet at P. Find $\angle APB$?
(a) 30^0 (b) 40^0
(c) 50^0 (d) 60^0

212. In the given figure, AB is the diameter of the circle and O is the centre, Find $\angle APB$?

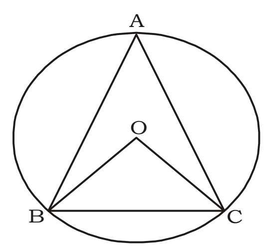


(a) 149^0 (b) 74.5^0
(c) 62^0
(d) None of these

213. O is the circum centre of the triangle ABC with circumradius 13 cm. Let BC = 24 cm and OD is perpendicular to BC. Then the length of OD is:
(a) 7 cm (b) 3 cm
(c) 4 cm (d) 5 cm

214. A, B, C are three points on a circle. The tangent at A meets BC produced at T, $\angle$BTA = 40° and $\angle$CAT = 44°. The angle subtended by BC at the centre of the circle is:
(a) 84° (b) 92°
(c) 96° (d) 104°

215. BC is the chord of a circle with centre O. A is a point on major arc BC as shown in the figure. What is the value of $\angle$BAC + $\angle$OBC?



(a) 120° (b) 60°
(c) 90° (d) 180°

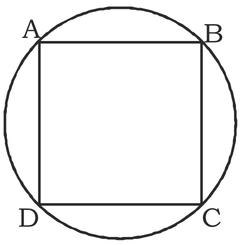
216. AB and CD are two parallel chords drawn on two opposite sides of their parallel diameter such that AB = 6 cm, CD = 8 cm. If the radius of the circle is 5 cm, the distance between the chords, in cm, is :
(a) 2 (b) 7 (c) 5 (d) 3

217. A chord AB of length $3\sqrt{2}$ unit subtends a right angle at the centre O of a circle. Area of the sector AOB (in sq. units) is:
(a) $\frac{9}{4}$ sq. units
(b) 5sq. units
(c) 9sq. units
(d) $\frac{9}{2}$ sq. units

218. AB and BC are two chords of a circle with centre O. If P and

Q are the mid-points of AB and BC respectively, then the quadrilateral OQBP must be:
(a) a rhombus (b) concyclic
(c) a rectangle (d) a square

219. If the area of the circle in the figure is 36 sq. cm, and ABCD is a square, then the area of ΔACD, in sq. cm, is :



(a) 12π (b) $\frac{36}{\pi}$
(c) 12 (d) 18

220. Two tangents are drawn from a point P to a circle at A and B. O is the centre of the circle. If $\angle$AOP = 60°, then $\angle$APB is :
(a) 120° (b) 90°
(c) 60° (d) 30°

221. If the length of a chord of a circle, which makes an angle 45° with the tangent drawn at one end point of the chord, is 6cm, then the radius of the circle is :
(a) $6\sqrt{2}$ cm (b) 5 cm
(c) $3\sqrt{2}$ cm (d) 6 cm

222. Two equal circles pass through each other's centre. If the radius of each circle is 5 cm, what is the length of the common chord?
(a) 5 (b) $5\sqrt{3}$
(c) $10\sqrt{3}$ (d) $\frac{5\sqrt{3}}{2}$

223. PA and PB are two tangents drawn from external point P to a circle with centre O where the points A and B are the points of contact. The quadrilateral OAPB must be:
(a) a rectangle(b) a rhombus
(c) a square (d) concyclic

224. The radius of two concentric circles are 9 cm and 15 cm. If the chord of the greater circle be a tangent to the smaller circle, then the length of that chord is :
(a) 24 cm (b) 12 cm
(c) 30 cm (d) 18 cm

225. The length of a chord of a circle is equal to the radius of the circle. The angle which this chord subtends in the major segment of the circle is equal to :
(a) 30° (b) 45° (c) 60° (d) 90°

226. The ratio of the areas of the circumcircle and the incircle of an equilateral triangle is:
(a) 2 : 1 (b) 4 : 1
(c) 8 : 1 (d) 3 : 2

227. AB = 8 cm and CD = 6 cm are two parallel chords on the same side of the centre of a circle. The distance between them is 1 cm. The radius of the circle is :
(a) 5 cm (b) 4 cm
(c) 3 cm (d) 2 cm

228. Two equal circles of radius 4 cm intersect each other such that each passes through the centre of the other. The length of the common chord is :
(a) $2\sqrt{2}$ cm (b) $4\sqrt{3}$ cm
(c) $2\sqrt{3}$ cm (d) 8 cm

229. ABCD is a cyclic parallelogram. The $\angle$B is equal to :
(a) 30° (b) 60° (c) 45° (d) 90°

230. From four corners of a square sheet of side 4 cm, four pieces, each in the shape of arc of a circle with radius 2 cm, are cut out. The area of the remaining portion is :
(a) $(8 - \pi)$sq.cm.
(b) $(16 - 4\pi)$sq.cm.
(c) $(16- 8\pi)$sq.cm.
(d) $(4 - 2\pi)$ sq.cm.

231. If a chord of a circle of radius 5 cm is a tangent to a circle of radius 3 cm, both the circles being concentric, then the length of the chord is :
(a) 10 cm (b) 12.5 cm
(c) 8 cm (d) 7 cm

232. Two circles touch each other externally at point A and PQ is a direct common tangent which touches at P and Q respectively. Then $\angle$PAQ = ?
(a) 45° (b) 90°
(c) 80° (d) 100°

233. AB is a chord to a circle and PAT is the tangent to the circle at A. If $\angle$BAT = 75° and $\angle$BAC = 45°, C being a point on the circle, then $\angle$ABC is equal to :
(a) 40° (b) 45° (c) 60° (d) 70°

234. O is the centre of a circle and arc ABC subtends an angle of 130° at O. AB is extended to P. Then $\angle$PBC is :
(a) 75° (b) 70° (c) 65° (d) 80°

235. The circumcentre of a triangle ABC is O. If $\angle$BAC = 85° and $\angle$BCA = 75°, then the value of $\angle$OAC is:
(a) 40° (b) 60° (c) 70° (d) 90°

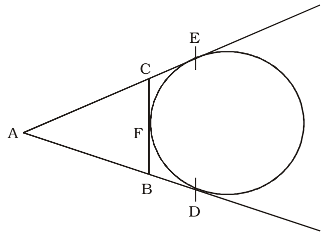
236. The length of each side of an equilateral triangle is $14\sqrt{3}$ cm. The area of the incircle, in cm^2, is :
(a) 450 (b) 308
(c) 154 (d) 77

237. If the radii of two circles be 6 cm and 3 cm and the length of the transverse common tangent be 8 cm, then the distance between the two centres is :
(a) $\sqrt{154}$ cm (b) $\sqrt{140}$ cm
(c) $\sqrt{145}$ cm (d) $\sqrt{135}$ cm

238. Two parallel chords are drawn in a circle of diameter 30 cm. The length of one chord is 24 cm and the distance between the two chords in 21 cm. The length of the other chord is :
(a) 10 cm (b) 18 cm
(c) 12 cm (d) 16 cm

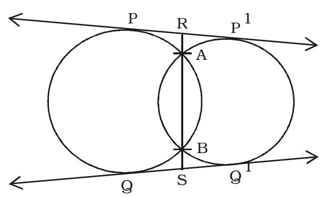
239. If two equal circles whose centres are O and O', intersect each other at the points A and B, OO'= 12 cm and AB = 16cm, then the radius of the circles is :
(a) 10 cm (b) 8 cm
(c) 12 cm (d) 14 cm

240. In the given figure, AD, AE and BC are tangents, then:-



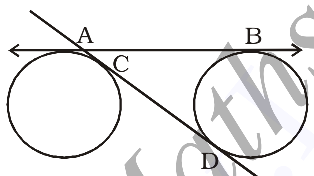
(a) AD = AB + BC + CA
(b) 2AD = AB + BC + CA
(c) 3AD = AB + BC + CA
(d) 4AD = AB + BC + CA

241. PP^1 and QQ^1 are two direct common tangents to two circles intersecting at points A and B. The common chord on produced intersects PP^1 in R and QQ^1 in S. Which of the following is true ?



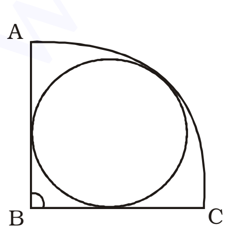
(a) $RA^2 + BS^2 = AB^2$
(b) $RS^2 = PP'^2 + AB^2$
(c) $RS^2 + PP'^2 = QQ'^2$
(d) $RS^2 = BS^2 + PP'^2$

242. If two equal circles of radius 5cm have two common tangent AB and CD which touch the circle on A,C, and B,D respectively and if CD=24cm, find the length of AB.



(a) 27cm (b) 25cm
(c) 26 cm (d) 30cm

243. If ABC is a Quarter Circle and a circle is inscribed in it and if AB=1cm, find radius of smaller circle.



(a) $\sqrt{2} - 1$ (b) $\frac{\sqrt{2} - 1}{2}$

(c) $\frac{\sqrt{2} + 1}{2}$ (d) $1 - 2\sqrt{2}$

244. Find the length of the common chord of two circles of radius 15cm and 20cm if their centres are 25cm apart?
(a) 12cm (b) 20cm
(c) 18cm (d) 24cm

245. AB and AC are two chords of a circle such that AB=AC=6cm. If radius of the circle is 5cm, then BC is:-
(a) 4.8cm (b) 9.6cm
(c) 2.4cm (d) 8.4cm

246. '2a' and '2b' are the length of two chords which intersect at right angle. If the distance between the centre of the circle and the intersecting point of the chords is 'C' then the radius of the circle is:-

(a) $\frac{\sqrt{a^2+b^2+c^2}}{2}$

(b) $\sqrt{a^2+b^2+c^2}$

(c) $\sqrt{\frac{a^2+b^2+c^2}{2}}$

(d) None of these

247. AB and CD are two chords of a circle which intersect at right angle at E. If AE = 2cm, EB = 6cm, ED = 3cm, then radius (r) is equal to:-

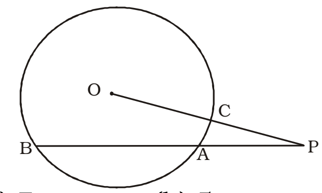
(a) $\frac{\sqrt{65}}{2}$ (b) $\sqrt{65}$

(c) $2\sqrt{65}$

(d) None of these

248. AB is a chord of a circle (centre O) and DOC is a line segment originating from a point D on the circle and intersecting AB on producing at C such that BC=OD. If $\angle$ BCD = 20°, then $\angle$ AOD :-
(a) 30° (b) 40°
(c) 100° (d) 60°

249. In the given figure, O is the centre of the circle. If BA = 7cm, OP = 13cm & AP = 9cm then radius (r):-



(a) 7cm (b) 5cm
(c) 4cm (d) 6cm

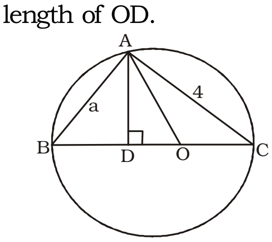
250. Two tangents PA and PB are drawn to the circle(centre O) from a point P. CD is another tangent on the circle which intersects PA and PB at C and D respectively. If $\angle APB = 34°$ then $\angle COD$:-
(a) 146° (b) 68°
(c) 73°
(d) None of these

251. Two tangents PA and PB are drawn from a point P to the circle. If the radius of the circle is 5 cm and AB=6cm and O is the centre of the circle. OP cuts AB at C and OC = 4cm, then OP:-

(a) $\frac{25}{4}$ cm (b) 25cm

(c) 13 cm
(d) None of these

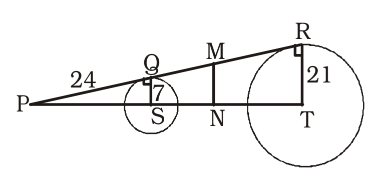
252. If in the given figure, AB=a, AC=4cm, while O is the centre of the circle and D is a point between O and B such that $AD \perp BC$. Find the length of OD.



(a) $\frac{4-a}{4}$ (b) $\frac{16-a^2}{2\sqrt{a^2+16}}$

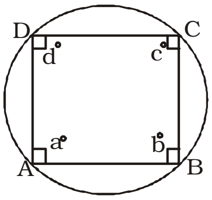
(c) $\frac{4a-16}{16a-a^2}$ (d) $\frac{2\sqrt{a^2-16}}{16+a^2}$

253. In the given figure, PQ = 24cm. M is the mid-point of QR. Also, $MN \perp PR$, QS = 7cm and TR = 21cm, then SN = ?



(a) 50 cm (b) 12.5cm
(c) 31 cm (d) 25 cm

254. In the given figure, AB || CD if a,b,c and d are integers, what is the number of possible value of (a+b–c–d)?



(a) 179 (b) 89
(c) 357 (d) 358

255. Three equal circle of unit radius touch each other. Then , the area of the circle circumscribing the three circle is :-

(a) $6\pi(2+\sqrt{3})^2$

(b) $\frac{\pi}{6}(2+\sqrt{3})^2$

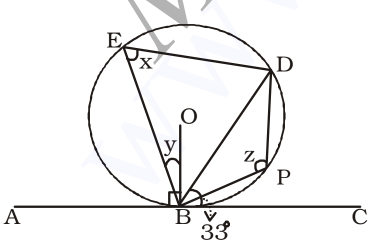
(c) $\frac{\pi}{3}(2+\sqrt{3})^2$

(d) $3\pi(2+\sqrt{3})^2$

256. In ΔABC, AB=4cm, BC=3.4cm and AC=2.2cm. Three circles are drawn with centre A, B and C in such a way that each circle touches the other two. Then the diameter of the bigger circle is:
(a) 5.2 cm (b) 2.6 cm
(c) 2.8 cm
(d) None of these

257. The angle bisectors of angle A, B and C of a ΔABC intersect the circumference of the circumcircle at X, Y and Z respectively. If $\angle A = 50°$, $\angle CZY = 42°$, then $\angle BYZ$ is equal to :-
(a) 46° (b) 42° (c) 23° (d) 21°

258. In the given figure, chord BE = BD, $\angle CBD = 33°$, & OB $\perp$ AC then $x+y+z$ is equal to



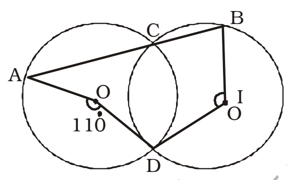
(a) 230° (b) 237°
(c) 337°
(d) None of these

259. ABC and MNC are two secants of a circle whose centre is O. AN is the diameter of the circle if $\angle BAN = 38°$ and $\angle ACM = 20°$ then $\angle MBN$:-
(a) 38° (b) 42°
(c) 28° (d) 32°

260. PT is a tangent of a circle at T and AB is a chord. If AB =18cm and PT = 2AP then find PT ?

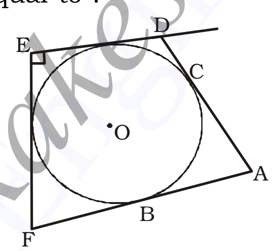
(a) 12cm (b) 18cm
(c) 6 cm (d) 9cm

261. Find $\angle$BO'D ?



(a) 220° (b) 110°
(c) 55° (d) 70°

262. In the given figure, AB = 27, AD = 38cm, ED = 24cm and $\angle E = 90°$, then radius of the circle is equal to :-



(a) 11 cm (b) 15 cm
(c) 13 cm (d) 17 cm

263. Two circles having radius 'a'cm and 'b'cm touch each other externally. another circle whose radius is 'c'cm, touches both the circles and also their common tangent. Then which statement will be true :-

(a) $\sqrt{a}+\sqrt{b}=\sqrt{c}$

(b) $\sqrt{a}=\sqrt{b}+\sqrt{c}$

(c) $\sqrt{ab}+\sqrt{bc}=\sqrt{ac}$

(d) $\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{c}}$

264. In a ΔABC, I and O are the in-centre and circum-centre respectively. The line AI is produced to a point D on the circumcircle. If $\angle BOD = Z$, $\angle BID = Y$ and $\angle ABC = X$, then $\frac{x+z}{3y}$ is equal to :-

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{4}{3}$ (d) 1

265. P and Q are the middle points of two chords (not diameters) AB and AC respectively of a circle with centre at a point O. The lines OP and OQ are produced to meet the circle respectively at the points R and S. T is any point on the major arc between the points R and S of the circle. If $\angle BAC = 32°$, $\angle RTS$ = ?
(a) 32° (b) 74°
(c) 106° (d) 64°

266. O and C are respectively the orthocentre and circumcentre of an acute-angled triangle PQR. The points P and O are joined and produced to meet the side QR at S. If $\angle PQS = 60°$ and $\angle QCR = 130°$, then $\angle RPS$ = ?
(a) 30° (b) 35° (c) 100°(d) 60°

267. Two chords AB and CD of circle whose centre is O, meet at the point P and $\angle AOC = 50°$, $\angle BOD = 40°$. Then the value of $\angle BPD$ is :
(a) 60° (b) 40° (c) 45° (d) 75°

268. The tangents are drawn at the extremities of a diameter AB of a circle with centre P. If a tangent to the circle at the point C intersects the other two tangents at Q and R, then the measure of the $\angle QPR$ is :
(a) 45° (b) 60° (c) 90° (d) 180°

269. Two circles of radii 9 cm and 2 cm respectively have centres X and Y and $\overline{XY}$ = 17cm. Circle of radius r cm with centre Z touches two given circles externally. If $\angle XZY = 90°$, find r :
(a) 18 cm (b) 3 cm
(c) 12 cm (d) 6 cm

270. A circle (with centre at O) is touching two intersecting lines AX and BY. The two points of contact A and B subtend an angle of 65° at any point C on the circumference of the

circle. If P is the point of intersection of the two lines, then the measure of $\angle$APO is:
(a) 25° (b) 65° (c) 90° (d) 40°

271. ABCD is a cyclic trapezium such that $AD \parallel BC$, if $\angle ABC = 70°$, then the value of $\angle BCD$ is :
(a) 60° (b) 70° (c) 40° (d) 80°

272. ABCD is a cyclic trapezium whose sides AD and BC are parallel to each other. If $\angle ABC$ = 72°, then the measure of the $\angle BCD$ is
(a) 162° (b) 18°
(c) 108° (d) 72°

273. If an exterior angle of a cyclic quadrilateral be 50°, then the interior opposite angle is :
(a) 130° (b) 40° (c) 50° (d) 90°

274. ABCD is cyclic parallelogram. The angle $\angle B$ is equal to :
(a) 30° (b) 60° (c) 45° (d) 90°

275. ABCD is a cyclic quadrilateral and O is the centre of the circle. If $\angle COD$ = 140° and $\angle BAC$ = 40°, then the value of $\angle BCD$ is equal to
(a) 70° (b) 90° (c) 60° (d) 80°

276. ABCD is a quadrilateral inscribed in a circle with centre O. If $\angle COD$ = 120° and $\angle BAC$ = 30°, then $\angle BCD$ is :
(a) 75° (b) 90° (c) 120°(d) 60°

277. ABCD is a cyclic trapezium with $AB \parallel DC$ and AB is a diameter of the circle. If $\angle CAB = 30°$, then $\angle ADC$ is
(a) 60° (b) 120° (c) 150°(d) 30°

278. ABCD is a cyclic quadrilateral. AB and DC are produced to meet at P. If $\angle ADC$ = 70° and $\angle DAB$ = 60°, then the $\angle PBC + \angle PCB$ is
(a) 130° (b) 150°
(c) 155° (d) 180°

279. A cyclic quadrilateral ABCD is such that AB = BC , AD = DC, $AC \perp BD$, $\angle CAD = \theta$, then the angle $\angle ABC$ = ?
(a) θ (b) $\frac{\theta}{2}$ (c) 2θ (d) 3θ

280. A quadrilateral ABCD circumscribes a circle and AB = 6 cm, CD = 5 cm and AD = 7 cm. The length of side BC is
(a) 4 cm (b) 5 cm
(c) 3 cm (d) 6 cm

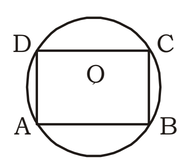
281. In a cyclic quadrilateral ABCD, $\angle A + \angle B + \angle C + \angle D$ = ?
(a) 90° (b) 360°
(c) 180° (d) 120°

282. A square ABCD is inscribed in a circle of 1 unit radius. Semicircles are inscribed on each side of the square. The area of the region bounded by the four semi-circles and the circle is
(a) 1 sq. unit (b) 2 sq. unit
(c) 1.5 sq. unit (d) 2.5 sq. unit

283. All sides of a quadrilateral ABCD touch a circle, If AB = 6 cm, BC = 7.5 cm, CD = 3 cm, then DA is
(a) 3.5 cm (b) 4.5 cm
(c) 2.5 cm (d) 1.5 cm

284. ABCD is a cyclic quadrilateral. The side AB is extended to E in such a way that BE = BC, If $\angle ADC$ = 70°, $\angle BAD$ = 95°, then $\angle DCE$ is equal to
(a) 140° (b) 120°
(c) 165° (d) 110°

285. In a cyclic quadrilateral $\angle A + \angle C = \angle B + \angle D$ = ?



(a) 270° (b) 360° (c) 90°(d) 180°

286. If ABCD be a cyclic quadrilateral in which $\angle A = 4x°$, $\angle B = 7x°$, $\angle C = 5y°$, $\angle D = y°$, then x : y is
(a) 3 : 4 (b) 4 : 3
(c) 5 : 4 (d) 4 : 5

287. ABCD is a cyclic quadrilateral and AD is a diameter. If $\angle DAC$ = 55°, then value of $\angle ABC$ is
(a) 55° (b) 35°
(c) 145° (d)125°

288. A square is inscribed in a quarter-circle in such a manner that two of its adjacent vertices lie on the two radii at an equal distance from the centre, while the other two vertices lie on the circular arc. If the square has sides of length x. then the radius of the circle is:
(a) $\frac{16x}{\pi + 4}$ (b) $\frac{2x}{\sqrt{x}}$
(c) $\frac{\sqrt{5}x}{\sqrt{2}}$ (d) $\sqrt{2}x$

289. ABC is a cyclic triangle and the bisectors of $\angle$BAC, $\angle$ABC and $\angle$BCA meet the circle at P, Q and R respectively. Then the angle $\angle$RQP is :
(a) $90° - \frac{B}{2}$ (b) $90° + \frac{C}{2}$
(c) $90° - \frac{A}{2}$ (d) $90 + \frac{B}{2}$

290. ABCD is a cyclic quadrilateral. AB and DC when produced meet at P, If PA = 8 cm, PB = 6, PC = 4 cm, then the length (in cm) of PD is
(a) 10 cm (b) 6 cm
(c) 12 cm (d) 8 cm

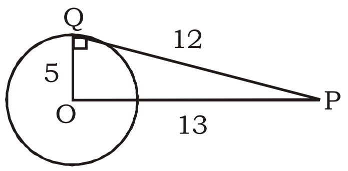
291. AB is a diameter of a circle with centre O. The tangents at C meets AB produced at Q. If $\angle$CAB = 34°, then measure of $\angle$CBA is
(a) 56° (b) 68° (c) 34°(d) 124°

ANSWER KEY

1. (d)	31. (d)	61. (d)	91. (c)	121.(c)	151.(d)	181.(d)	211.(d)	241.(b)	271.(b)
2. (b)	32. (b)	62. (a)	92. (c)	122.(c)	152.(b)	182.(b)	212.(b)	242.(c)	272.(d)
3. (a)	33. (d)	63. (b)	93. (b)	123.(b)	153.(a)	183.(c)	213.(d)	243.(a)	273.(c)
4. (b)	34. (b)	64. (a)	94. (c)	124.(a)	154.(a)	184.(c)	214.(d)	244.(d)	274.(d)
5. (a)	35. (c)	65. (a)	95. (d)	125.(d)	155.(d)	185.(d)	215.(c)	245.(b)	275.(a)
6. (c)	36. (c)	66. (c)	96. (b)	126.(b)	156.(b)	186.(b)	216.(b)	246.(c)	276.(b)
7. (a)	37. (a)	67. (b)	97. (c)	127.(c)	157.(c)	187.(a)	217.(a)	247.(a)	277.(b)
8. (c)	38. (b)	68. (c)	98. (b)	128.(a)	158.(a)	188.(d)	218.(b)	248.(d)	278.(a)
9. (b)	39. (d)	69. (c)	99. (b)	129.(b)	159.(b)	189.(b)	219.(b)	249.(b)	279.(c)
10. (a)	40. (a)	70. (c)	100. (d)	130.(b)	160.(b)	190.(a)	220.(c)	250.(c)	280.(a)
11. (b)	41. (a)	71. (d)	101.(d)	131.(d)	161.(d)	191.(b)	221.(c)	251.(a)	281.(b)
12. (a)	42. (a)	72. (a)	102.(b)	132.(a)	162.(c)	192.(a)	222.(b)	252.(b)	282.(b)
13. (b)	43. (a)	73. (a)	103.(c)	133.(d)	163.(d)	193.(b)	223.(d)	253.(d)	283.(d)
14. (a)	44. (d)	74. (d)	104.(c)	134.(d)	164.(b)	194.(c)	224.(a)	254.(a)	284.(a)
15. (c)	45. (a)	75. (b)	105.(a)	135.(a)	165.(a)	195.(c)	225.(a)	255.(c)	285.(d)
16. (c)	46. (a)	76. (d)	106.(d)	136.(c)	166.(c)	196.(d)	226.(a)	256.(a)	286.(b)
17. (a)	47. (c)	77. (b)	107.(c)	137.(b)	167.(b)	197.(b)	227.(a)	257.(c)	287.(c)
18. (a)	48. (c)	78. (b)	108.(b)	138.(b)	168.(a)	198.(a)	228.(b)	258.(b)	288.(c)
19. (d)	49. (c)	79. (d)	109.(a)	139.(c)	169.(b)	199.(b)	229.(d)	259.(d)	289.(a)
20. (a)	50. (b)	80. (d)	110.(c)	140.(d)	170.(b)	200.(a)	230.(b)	260.(a)	290.(c)
21. (b)	51. (d)	81. (d)	111.(a)	141.(c)	171.(b)	201.(d)	231.(c)	261.(b)	291.(a)
22. (d)	52. (c)	82. (a)	112.(*)	142.(a)	172.(c)	202.(a)	232.(b)	262.(c)	
23. (d)	53. (b)	83. (b)	113.(c)	143.(a)	173.(a)	203.(b)	233.(c)	263.(d)	
24. (a)	54. (b)	84. (c)	114.(a)	144.(b)	174.(d)	204.(c)	234.(c)	264.(a)	
25. (b)	55. (b)	85. (d)	115.(b)	145.(c)	175.(d)	205.(b)	235.(c)	265.(b)	
26. (b)	56. (b)	86. (d)	116.(d)	146.(a)	176.(c)	206.(d)	236.(c)	266.(b)	
27. (a)	57. (b)	87. (a)	117.(c)	147.(d)	177.(b)	207.(a)	237.(c)	267.(c)	
28. (c)	58. (b)	88. (d)	118.(b)	148.(b)	178.(c)	208.(c)	238.(b)	268.(c)	
29. (a)	59. (b)	89. (b)	119.(a)	149.(b)	179.(a)	209.(b)	239.(a)	269.(d)	
30. (b)	60. (a)	90. (a)	120.(d)	150.(d)	180.(d)	210.(a)	240.(b)	270.(a)	

SOLUTION

1. (d)



$PQ = 12cm$, $OQ = 5cm$

By using pythagoras theorem

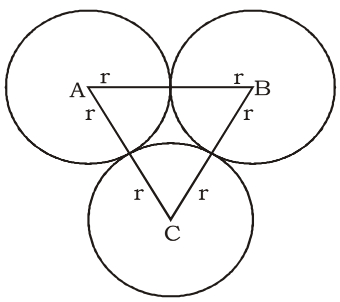
$OP^2 = OQ^2 + PQ^2$

$OP^2 = 5^2 + 12^2$

$OP^2 = 25 + 144$

$OP = \sqrt{169}$, $OP = 13cm$

2. (b)



$AB = BC = AC = 2r$

So, ABC is an equilateral Δ

3. (a) ATQ

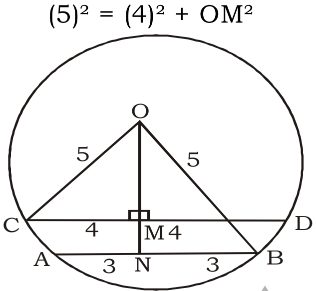
$CM = MD = 4$ cm

$AN = NB = 3$ cm

In ΔOMC

$OC^2 = MC^2 + OM^2$

$(5)^2 = (4)^2 + OM^2$



$OM = 3$ cm

In ΔONB

$OB^2 = ON^2 + NB^2$

$(5)^2 = ON^2 + (3)^2$

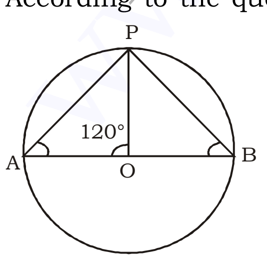
$ON = 4cm$

$\therefore MN = ON - OM$

$MN = 4 - 3$

$MN = 1$ cm.

4. (b) According to the question,



$\angle POA = 120°$

$\therefore \angle POB = 180 - 120°$

$\angle POB = 60°$

$OB = OP$

$\angle OBP = \angle OPB$

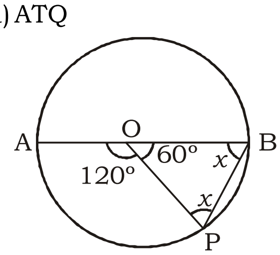
In ΔOPB,

$\angle POB + \angle OPB + \angle OBP = 180°$

$2\angle OBP = 180 - 60°$

$\angle PBO = 60°$

5. (a) ATQ



$\because \angle AOP = 120°$

$\Rightarrow \angle POB = 180° - 120° = 60°$

$\because OP = OB = R$

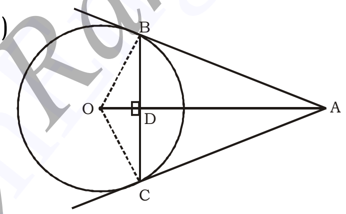
$\Rightarrow$ So Let $\angle PBO = x = \angle BPO$

$\Rightarrow x + x + 60° = 180°$

$\Rightarrow x = 60°$

$\Rightarrow \angle PBO = 60°$

6. (c)



$\Rightarrow$ According to figure

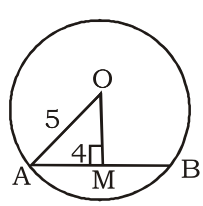
BC will be a chord of circle having centre 'O'

$\Rightarrow$ OD will be perpendicular on BC

And $BD = DC$

$\Rightarrow$ Therefore $\angle BDO = 90°$

7. (a)

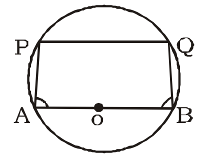


$AM = \frac{AB}{2}$ ($\perp$ bisect chord form centre)

$AO^2 = OM^2 + AM^2$

$OM^2 = 9$, $OM = 3$ cm

8. (c) ATQ



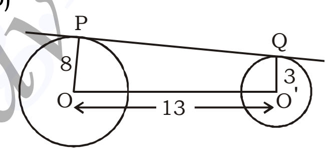
$\angle A = \angle B$

$AB \neq PQ$

$AB \| PQ$

$\therefore$ out of given option only cyclic trapezium follow the property.

9. (b)



$\Rightarrow$ Length of the direct common tangent PQ

$= \sqrt{(OO')^2 - (R - r)^2}$

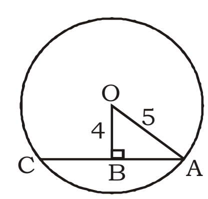
$= \sqrt{13^2 - (8-3)^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144} = 12$ cm

10. (a) Required ratio = 5 : 3

11. (b)



$\text{radius} = \frac{10}{2} = 5cm = OA$

$OB = 4$

By using pythagoras theorem

$OA^2 = OB^2 + AB^2$

$5^2 = 4^2 + AB^2$

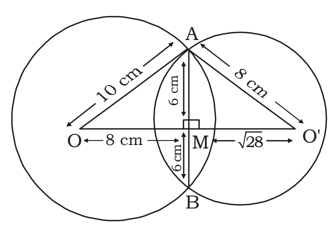
$AB^2 = 25 - 16$

$AB = \sqrt{9}$

$AB = 3cm$

$AC = 2 \times AB = 2 \times 3 = 6cm$

12. (a)



$O\,O' = 8 + \sqrt{28}$
$= 8 + 5.29$
$= 13.3$ cm

Note: $\because \Delta AMO$ = Right angled triangle

In, ΔAMO

$\Rightarrow$ AM = 6, AO = 10

then,

OM = 8

In, $\Delta AMO'$

$\Rightarrow$ AM = 6, AO' = 8

then,

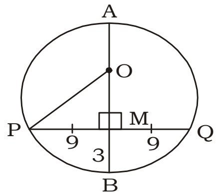
$O'M = \sqrt{28}$

$\Rightarrow O\,O' = OM + O'M$

$= 8 + \sqrt{28}$

$\Rightarrow$ 13.3 cm

13. (b)



According to the question

Let OA = x = OP

AB = $2x$

OM = $x - 3$

In ΔOMP,

$x^2 = (9)^2 + (x-3)^2$

$x^2 = 81 + x^2 + 9 - 6x$

$90 = 6x$

$x = 15$

$\therefore$ AB = 2×15 = 30 cm.

Alternate:-

PM × MQ = MB × AM

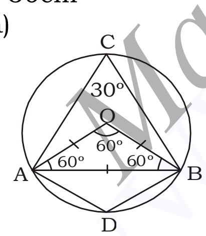
9 × 9 = 3 × x

x = 27

$\therefore$ AB = AM + BM= 27 + 3

= 30cm

14. (a)



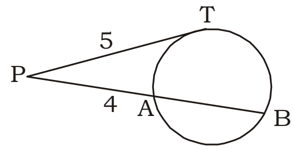
Angle subtended by an arc at the centre is twice the angle subtended by the arc on the circle.

$\therefore \angle C = 30°$

$\angle D = 150°$

[Because in a cyclic quadrilateral sum of opposite angle is 180°]

15. (c)



According to the question

PT = 5 cm.

PA = 4 cm.

PB = $(4+x)$ cm.

As we know that

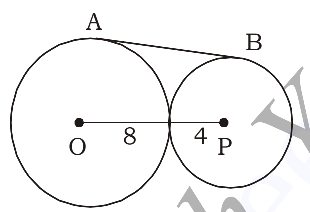
$PT^2 = PA \times PB$

$25 = 4\,(4 + x)$

$25 = 16 + 4x$

$x = \frac{9}{4}$ cm .

16. (c)



We know that,

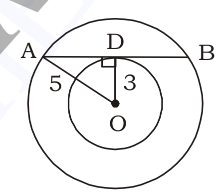
$AB = 2\sqrt{r_1 r_2}$

$AB = 2\sqrt{8 \times 4}$

$AB = 2 \times \sqrt{32}$

$AB = 8\sqrt{2}$ cm

17. (a)



According to the question

Let AD = DB = x

OA = 5

OD = 3

AB = $2x$

In ΔODA,

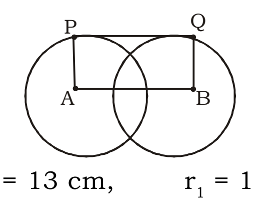
$OA^2 = OD^2 + AD^2$

$(5)^2 = (3)^2 + (x)^2$

$x = 4$

$\therefore$ AB = 2×4 = 8 cm.

18. (a)



AB = 13 cm, r_1 = 11 cm

r_2 = 6cm

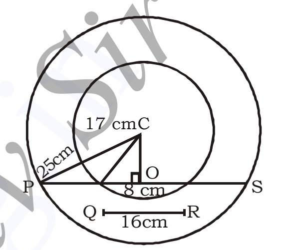
$PQ = \sqrt{(\text{Distance between centres})^2 - (\text{radius}_1 - \text{radius}_2)^2}$

$PQ = \sqrt{(13)^2 - (11-6)^2}$

$PQ = \sqrt{169 - 25}$

$PQ = \sqrt{144}$, PQ = 12cm

19. (d) In right ΔCOQ,



$QC^2 = OQ^2 + OC^2$ (By pt)

$17^2 = 8^2 + OC^2$

OC = 15 cm

In right ΔCOP

$CP^2 = OP^2 + CO^2$

$25^2 = OP^2 + 15^2$

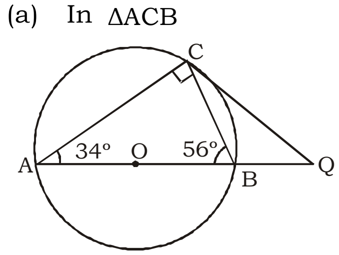
OP = 20cm

PS = 2× OP

= 2 × 20

= 40 cm

20. (a) In ΔACB



$\angle ACB = 90°$

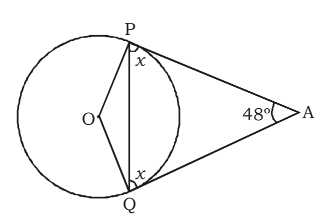
(Angle formed by semicircle is 90°)

$\angle ACB + \angle CAB + \angle CBA = 180°$

$90° + 34° + \angle CBA = 180°$

$\angle CBA = 56°$

21. (b) In ΔAPQ, $\angle P = \angle Q = x$



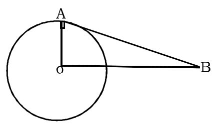
$x° + x° + 48 = 180°$

$2x = 132°$

$x = 66°$

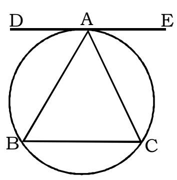
$\because \angle APQ = 66°$

22 (d) According to question
Given:



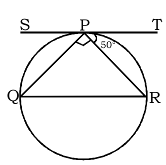
OB =10 cm, OA = radius=6 cm
AB = tangent
In right angle $\triangle OAB$
By using pythagoras theorem
$OB^2 = OA^2 + AB^2$
$(10)^2 = (6)^2 + (AB)^2$
$(AB)^2 = 100 - 36$
$(AB)^2 = 64$
AB = 8

23. (d) According to question
Given:



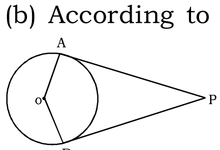
DE||BC, AB = 17 cm
AC = ?
$\angle DAB = \angle ACB$
(By alternate segment theorem
$\angle DAB = \angle ABC$ (Alternate angle)
$\therefore \angle ABC = \angle ACB$
AB = AC = 17 cm

24. (a) According to question
Given:



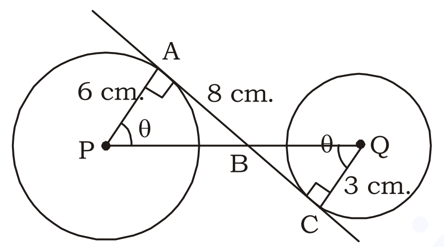
$\angle RPT = 50°$
$\angle QPR = 90°$ (Angle in Semicircle
$\therefore \angle PQR = 50°$
(Alternate segment theorem)
In $\triangle$ PQR
$\angle P + \angle Q + \angle R = 180°$
$\angle R = 180° - 90° - 50°$
$\angle R = 40°$
$\therefore \angle SPQ = 40°$
(Alternate segment theorem)

25. (b) According to question



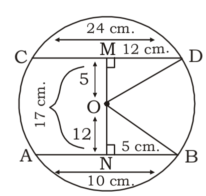
$\angle AOB = 110°$
$\angle APB = ?$
AOPB is a quadrilateral
$\angle O + \angle A + \angle P + \angle B = 360°$
$110° + 90° + 90° + \angle P = 360°$
$\angle P = 360° - 290°$
$\angle APB = 70°$
Therefore $\angle APB = 70°$

26. (b)



According to the question
$\Rightarrow$ AP = 6 cm (Radius$_1$)
$\Rightarrow$ QC = 3 cm (Radius$_2$)
As we know, any line draw from centre to the tangent be perpendicular
$\Rightarrow$ So, $\angle PAB = \angle QCB = 90°$
$\Rightarrow$ $\angle APB = \angle CQB = \theta$
[same alternative angle]
$\Rightarrow$ So, $\Delta APB \sim \Delta CQB$
$\Rightarrow$ $\frac{AP}{CQ} = \frac{AB}{CB}$
$\Rightarrow$ $\frac{6}{3} = \frac{8}{CB}$
$\Rightarrow$ CB = 4 cm.
$\Rightarrow$ In right angled triangle ΔPAB
$\Rightarrow$ $(PB)^2 = (PA)^2 + (AB)^2$
$\Rightarrow$ $(PB)^2 = 6^2 + 8^2$
$\Rightarrow$ PB = 10 cm.
$\Rightarrow$ Again, right angled triangle ΔCQB
$\Rightarrow$ $BQ^2 = (BC)^2 + (CQ)^2$
$\Rightarrow$ $(BQ)^2 = 3^2 + 4^2$
$\Rightarrow$ BQ = 5 cm.
$\Rightarrow$ Therefore PQ = PB + BQ
$\Rightarrow$ 10 + 5 = 15 cm.

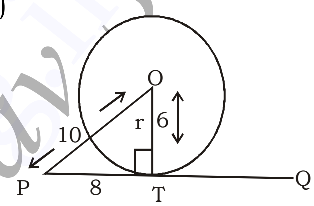
27. (a)



According to the question

$\because$ ΔOMD and ΔONB are right angled triangle
And, OD = OB = R
In ΔOMD, from triplets
5, 12, 13
OM = 5 cm, OD = 13 cm, MD = 12 cm.
Again, In ΔONB, from triplets
5, 12, 13
OB = 13 cm, ON = 12 cm
$\because$ OB = OD = 13 cm
Radius = 13 cm.

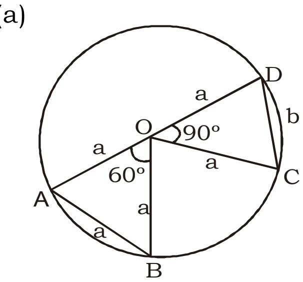
28. (c)



Let PTQ is the
Tangent of circle having centre O and Radius = r and point T, touches the circle
$\Rightarrow$ We know that any line draw on tangent's touching point from centre, always makes a perpendicular
So $\angle OTP = 90°$ length of PT = 8 cm

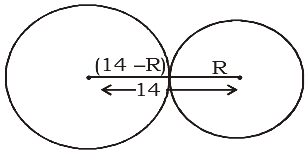
$$\left\{\begin{matrix} 3, & 4, & 5 \\ 2\times\downarrow & \times 2\downarrow & \downarrow\times 2 \\ 6 & 8 & 10 \end{matrix}\right\}$$

29. (a)



$\angle AOB = 60°$
$\angle COD = 90°$
$\Rightarrow$ length of chord AB = a
$\Rightarrow$ length of chord CD = b
$\Rightarrow$ AO = OB = AB = OD = OC = a
$\Rightarrow$ In ΔODC
$\Rightarrow$ $OD^2 + OC^2 = CD^2$
$\Rightarrow$ $a^2 + a^2 = b^2$
$\Rightarrow$ $b = \sqrt{2}a$

30. (b) Let smallest circle radius = R
Then biggest circle radius
= (14 – R)



⇒ According to the question

⇒ $\pi(14 - R)^2 + \pi R^2 = 130\pi$

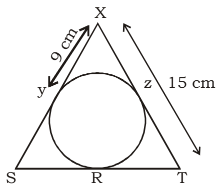
⇒ $(14 - R)^2 + R^2 = 130$

⇒ $196 + R^2 - 28R + R^2 = 130$

⇒ R = 3 cm

⇒ Radius of smallest circle R = 3 cm

31. (d)



xy = 9 cm, tx = 15 cm

⇒ We know

Length of tangents drawn from a point to the circle are equal

Therefore

xy = xz = 9cm, tz = rt

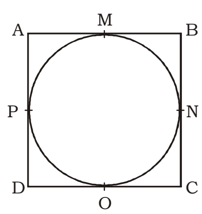
tx = 15 cm

xz + zt = 15

zt = 15 – 9 = 6

RT = ZT = 6 cm

32. (a) (b)



⇒ According to figure

⇒ PA = AM
(equal tangent drawn from a external point)

⇒ PD = OD

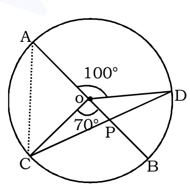
⇒ MB = BN

⇒ OC = CN

⇒ $\dfrac{(AB+CD)}{(CB+AD)}$

$= \dfrac{(AM+BM)+(OD+OC)}{(CN+NB)+(AP+DP)} = 1$

33. (d) According to question
Given :



$\angle AOD = 100°\ \angle BOC = 70°$

$\therefore\ \angle ACD = \angle ACP = \dfrac{100°}{2°} = 50°$

∴ The angle subtended at the centre is twice to that of angle at the circumferance by the same arc

$\angle BOC = 70°$

$\therefore \angle BDC = \angle BAC = \dfrac{70°}{2} = 35°$

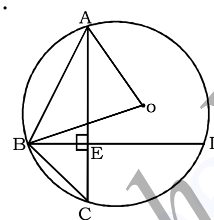
In △ APC,

$\angle PAC + \angle ACP + \angle APC = 180°$

$\angle APC = 180° - 50° - 35°$

$\angle APC = 95°$

34. (b)According to question
Given:



$\angle OAB = 25°$ OA = OB = r

$\therefore\ \angle OAB = \angle OBA = 25°$

$\therefore\ \angle AOB = 180° - 25° - 25°$

$\angle AOB = 130°$

$\therefore\ \angle ACB = \dfrac{1}{2}\angle AOB$

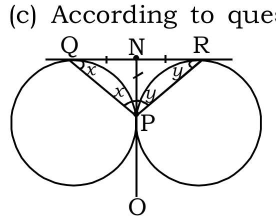
$= \dfrac{130°}{2} = 65°$

In right angle $\triangle BEC$

$\angle BEC + \angle CBE + \angle ECB = 180°$

$\angle CBE = 180° - 65° - 90°$
$= 25°$

35. (c) According to question



QR is the common tangent and NO is also the common tangent.

∴ QN = NP = NR

In $\triangle QPN$

$\angle NQP = \angle NPQ$

$\angle NRP = \angle NPR$

In $\triangle PQR$

$\angle P + \angle Q + \angle R = 180°$

$x + y + x + y = 180°$

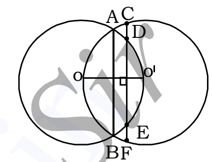
$2x + 2y = 180°$

$x + y = 90°$

As shown in the figure

$x + y = \angle P = 90°$

36. (c) According to question
Given :



CD= 4.5 cm

EF = ?

Let O and O' be the centre of a circle

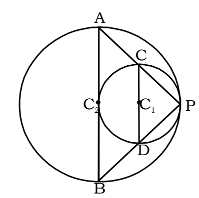
∴ OC = OF = radius of circle B

$\therefore\ OO' \perp O'E$

∴ O'D = O'E

Clearly CD = EF = 4.5 cm

37. (a) According to question
Given:



$\angle BDC = 120°\ \angle ABP = ?$

$\therefore\ \angle CDP = 180° - \angle BDC$

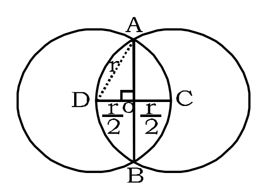
$\angle CDP = 180° - 120°$

$\angle CDP = 60°$

CD||AB

$\therefore\ \angle CDP = \angle ABP = 60°$

38. (b) According to question



Let the radius of the circle be = r

$\therefore$ DO = OC = $\dfrac{r}{2}$

In right angle $\triangle AOD$

By using pythagoras theorem

$AD^2 = OD^2 + AO^2$

$r^2 = \dfrac{r^2}{4} + AO^2$

$AO^2 = r^2 - \frac{r^2}{4}$

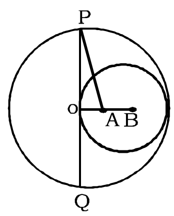
$AO^2 = \frac{3r^2}{4}$

$AO = \frac{\sqrt{3}r}{2}$ $\quad AB = 2 \times AO$

$AB = \frac{\sqrt{3}}{2} r \times 2,$

$AB = \sqrt{3}\mathbf{r}$ units

39. (d) According to question



AP = 5 cm, $\quad$ OB = 3 cm

PQ is $\perp$ bisector

$\therefore$ AO = 1, $\quad$ PO = OQ

In right angle $\triangle POA$

$AP^2 = OA^2 + OP^2$

$(5)^2 = (1)^2 + (OP)^2$

$(OP)^2 = 25 - 1$

$(OP)^2 = 24$

$(OP) = 2\sqrt{6}$ cm

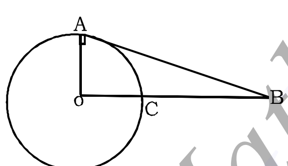
$\therefore PQ = 2 \times OP$

$PQ = 2 \times 2\sqrt{6}$

$PQ = 4\sqrt{6}$ cm

40. (a) According to question

Given:



OA = radius = 5 units

AB = $5\sqrt{3}$ units

BC = ?

In right angle $\triangle OAB$

$OB^2 = AB^2 + OA^2$

$OB^2 = \left(5\sqrt{3}\right)^2 + (5)^2$

$OB^2 = 75 + 25$

$OB^2 = 100$

OB = 10 units

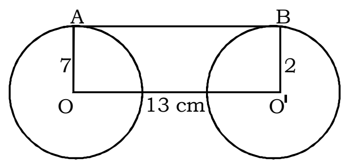
$\therefore$ BC = OB – OC

BC = 10 – 5

BC = 5 units

41. (a) According to question

Given:



OO' = 13 cm, $\quad$ OA = 7 cm

O'B = 2 cm

$\therefore$ Length of direct common tangent

$AB = \sqrt{(OO')^2 - (R-r)^2}$

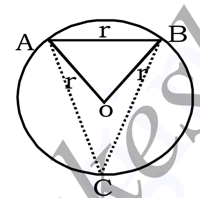
$AB = \sqrt{(13)^2 - (7-2)^2}$

$AB = \sqrt{169 - 25}$

$AB = \sqrt{144}$

AB = 12 cm

42. (a) According to question



Let AB is the chord and 'O' is the centre of circle

Given : The length of AB is equal to radius

$\therefore$ OA = OB = AB = r

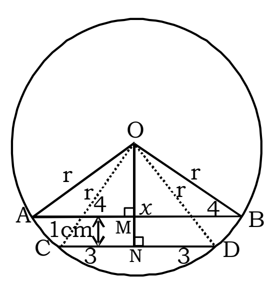
$\therefore$ ΔAOB is an equilateral triangle

$\angle AOB = 60°$

$\therefore$ $\angle ACB$ which chord subtends in the major segment is $= \frac{60°}{2} = 30°$

43. (a) According to question,

Given:



AB and CD are chord

AB = 8 cm

Let ON = x cm

$\therefore$ In, $\triangle OMA$

$OA^2 = OM^2 + AM^2$

$r^2 = (x-1)^2 + (4)^2$

$r^2 = (x-1)^2 + 16$ ....(i)

In $\triangle OND$

$OD^2 = ON^2 + ND^2$

$r^2 = x^2 + (3)^2$

$r^2 = x^2 + 9$ ....(ii)

Comparing equations (i) and (ii)

$(x-1)^2 + 16 = x^2 + 9$

$x^2 + 1 - 2x + 16 = x^2 + 9$

$17 - 2x = 9$

$2x = 8$

$x = 4$

Put the value of 'x' in equations(ii)

$r^2 = (4)^2 + 9$

$r^2 = 16 + 9$

$r^2 = 25$

r = 5 cm

Alternate:-

AM = 4 cm

CN = 3 cm

{3,4,5 } = formed a triplet

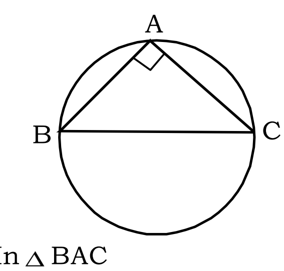
$\therefore$ Radius = 5 cm

44. (d) According to question,

AB and AC is a chord

AB = 8

AC = 6



In $\triangle$ BAC

$\angle A = 90°$

$\therefore BC^2 = AB^2 + AC^2$

$BC^2 = 8^2 + 6^2$

$BC^2 = 64 + 36$

$BC^2 = 100$

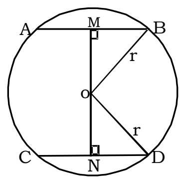
BC = 10 cm

Here BC is the diameter of a circle because only subtended on the arc of semi circle is 90°

$\therefore \frac{BC}{2}$ = radius= $\frac{10}{2}$ = 5 cm

45. (a) According to question

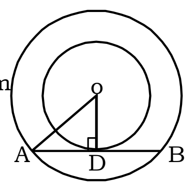
Given:



AB = CD = 8 cm
r = 5 cm
$\therefore$ In $\triangle OMB$
$OB^2 = OM^2 + MB^2$
$r^2 = OM^2 + (4)^2$
$(5)^2 = OM^2 + 16$
$25 - 16 = OM^2$
$OM^2 = 25 - 16$
$OM^2 = 9$
OM = 3
$\therefore$ MN = 2 × OM
MN = 2 × 3 = 6 cm

46. (a) According to questions,
Let 'O' be the centre of a circle and 'AB' is the chord of the biggest circle

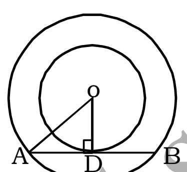
$\therefore$ AO= 15 cm
OD= 9cm



$\therefore$ In $\triangle ODA$
$OA^2 = OD^2 + AD^2$
$(15)^2 = (9)^2 + AD^2$
$AD^2 = 225 - 81$
$AD^2 = 144$
AD = 12 cm
$\therefore$ AB = 2 × AD
AB = 2 × 12 = 24 cm

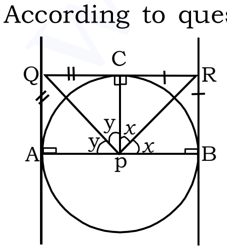
47. (c) According to question
Let'AB' is the chord of biggest circle and 'O' be the centre of a circle

OA= 5 cm
$\therefore$ OD= 3 cm



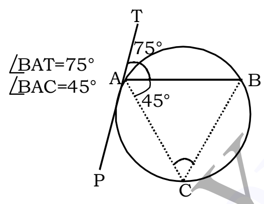
$\therefore$ In $\triangle$ ODA
$OA^2 = OD^2 + AD^2$
$(5)^2 = (3)^2 + AD^2$
$AD^2 = 25 - 9$
$AD^2 = 16$
AD = 4 cm
$\therefore$ AB = 2 × AD
= 2 × 4
AB = 8 cm

48. (c) According to question



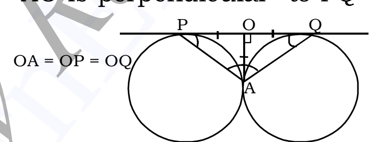
In $\triangle PCR$ and $\triangle RBP$
PC = PB (radius)
RC = RB
PR is common
$\therefore$ $\triangle PCR \cong \triangle RBP$
$\angle CPR = \angle RPB = x$
(CPCT)
Similarly,
$\angle APQ = \angle CPQ = y$ (CPCT)
$\therefore$ $2y + 2x = 180°$
$x + y = 90°$
$\therefore$ $\angle QPR = 90°$

49. (c) According to question,
Given:



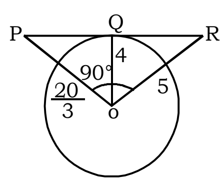
$\angle BAT = \angle BCA$
($\because$ Due to Alternate Segment theorem)
$\because$ $\angle BCA = 75°$
Then $\angle BAC + \angle BCA + \angle ABC = 180°$
$45° + 75° + \angle ABC = 180°$
$\angle ABC = 60°$

50. (b) According to Question,
AO is perpendicular to PQ



$\angle OPA = \angle OAP = \angle OQA = \angle OAQ$
$\angle OAQ = 45°$
$\therefore$ $\angle PAQ = 90°$

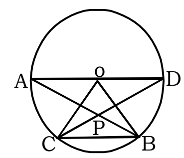
51. (d) According to Question,
Given:



$\angle POR = 90°$
OR =5 cm
OQ = 4 cm
$OP = \frac{20}{3}$ cm
$\therefore$ In $\triangle POR$
$PR^2 = PO^2 + OR^2$

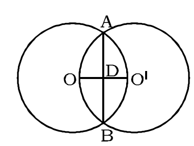
$$PR^2 = \left(\frac{20}{3}\right)^2 + (5)^2$$
$$PR^2 = \frac{400}{9} + 25$$
$$PR^2 = \frac{400 + 225}{9}$$
$$PR^2 = \frac{625}{9}$$
$$PR = \frac{\mathbf{25}}{\mathbf{3}} \text{ cm}$$

52. (c) According to Question



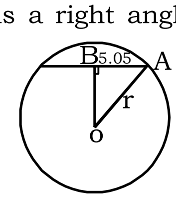
$\angle AOC + \angle BOD = 2\angle ABC + 2\angle BCD$
(Angle formed on major arc is half the angle formed on centre)
$= 2\angle ABC + 2\angle BCD$
$= 2\angle BPD$
[Exterior angles of triangles
$\angle AOC + \angle BOD = 2\angle BPD$
$2\angle BPD = 50° + 40°$
$\angle BPD = \frac{1}{2} \times 90° = 45°$

53. (b) According to question
AB is a common chord
O and 0' is the centre of the circle.



In $\triangle ODA$
$AO^2 = AD^2 + OD^2$
$(4)^2 = (AD)^2 + (2)^2$
$AD^2 = 12$
$AD = 2\sqrt{3}$
$\therefore$ AB = 2 × AD
$AB = 2 \times 2\sqrt{3} = \mathbf{4\sqrt{3}}$

54. (b) According to question
OBA is a right angle triangle

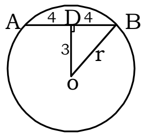


∴ OA is a hypotenuse

∴ Hypotenuse is always greater than other two sides

∴ Radius is always greater than 5 cm

55. (b) According to question.



In $\triangle BDO$, using pythagoras

$BO^2 = OD^2 + BD^2$

$r^2 = (4)^2 + (3)^2$

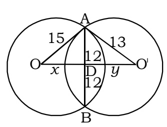
$r^2 = 16 + 9$

$r^2 = 25$

$r = 5$

56. (b) Let OD = x and DO' = y

According to question



In $\triangle ADO$

$AO^2 = OD^2 + AD^2$

$(15)^2 = x^2 + (12)^2$

$x^2 = 225 - 144$

$x^2 = 81$

$x = 9$

In $\triangle ADO'$

$(AO')^2 = AD^2 + DO'^2$

$(13)^2 = (12)^2 + y^2$

$169 = 144 + y^2$

$y^2 = 169 - 144$

$y^2 = 25$

$y = 5$

∴ $x + y = 9 + 5$

$= 14$

57. (b) According to question

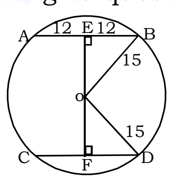
length of arc $= \frac{\theta}{360°} \times 2\pi r$

$= \frac{72°}{360°} \times 2 \times \frac{22}{7} \times 21$

$= 26.4$ cm

58. (b) one and only one circle can pass through 3 non- collinear points.

59. (b) According to question



AB = 24 cm

AE = EB = 12 cm

$OE = \sqrt{OB^2 - EB^2}$

$= \sqrt{15^2 - 12^2}$

$= \sqrt{225 - 144} = \sqrt{81}$

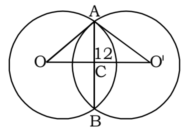
$= 9$ cm

∴ OF = 21 – 9 = 12 cm

also FD $= \sqrt{15^2 - 12^2} = 9$ cm

∴ CD = 2 × 9 = 18 cm

60. (a) According to Question



AB = 16

AC = BC = 8 cm

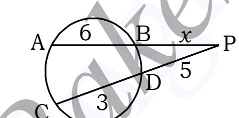
OC = OC' = 6 cm

$OA = \sqrt{OC^2 + CA^2}$

$OA = \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$

$OA = \sqrt{100} = 10$ cm

61. (d) According to Question



Given: AB = 6, CD = 3.

PD = 5

Let PB = x

Note: If chords AB and CD intersect externally at point, p then

PB × PA = PD × PC

$x \times (6 + x) = 5 \times 8$

$x^2 + 6x - 40 = 0$

$x^2 + 10x - 4x - 40 = 0$

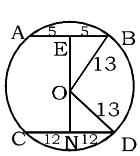
$x(x + 10) - 4(x + 10) = 0$

$(x + 10)(x - 4) = 0$

$x = 4, -10$ (–10 neglected)

∴ PB = 4 cm

62. (a) According to question.



AE = EB = 5 cm

CN = DN = 12 cm

$\triangle EOB$

$r = \sqrt{OE^2 + EB^2}$

$(13)^2 = (OE)^2 + (5)^2$

$169 = (OE)^2 + 25$

$OE^2 = 169 - 25$

$OE^2 = 144$

$OE = 12$ cm

In $\triangle OND$

$OD^2 = ON^2 + ND^2$

$(13)^2 = ON^2 + (12)^2$

$169 = ON^2 + 144$

$ON^2 = 169 - 144$

$ON^2 = 25$

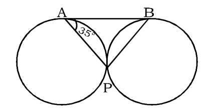
$ON = 5$

∴ EN = OE + ON

EN = 12 + 5

= 17 cm

63. (b) According to question



Given: $\angle PAB = 35°$

As we know that

$\angle APB = 90°$

Therefore,

∴ $\angle PAB + \angle APB + \angle ABP = 180°$

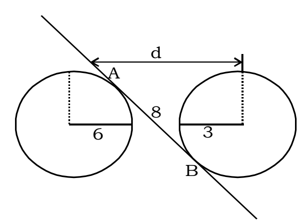
$\angle ABP = 180° - 90° - 35°$

$\angle ABP = 55°$

64. (a) According to question

Let length of transverse common tangent = AB = 8 cm

Distance between them = d



$AB = \sqrt{d^2 - (R_1 + R_2)^2}$

$AB^2 = d^2 - (R_1 + R_2)^2$

$(8)^2 = d^2 - (6 + 3)^2$

$64 = d^2 - 81$

$d^2 = 145$

$d = \sqrt{145}$

65. (a) According to Question

Let length of transverse common tangent be AB

Distance between them = 10 cm

$AB = \sqrt{d^2 - (R_1 + R_2)^2}$

$AB = \sqrt{(10)^2 - (3 + 3)^2}$

$AB = \sqrt{100-36}$

$AB = \sqrt{64}$

AB = 8 cm

66. (c) According to question

Length of the transverse common tangent

$AB = \sqrt{d^2 - (R_1 + R_2)^2}$

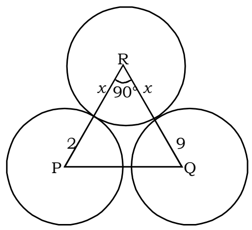
$AB = \sqrt{(24)^2 - (5+3)^2}$

$AB = \sqrt{576-64}$

$AB = \sqrt{512}$

$AB = 16\sqrt{2}$

67. (b) According to question



$\angle PRQ = 90°$

PR = 2 + x

PQ = 17

RQ = 9 + x

By using pythagoras theorem

$PQ^2 = PR^2 + RQ^2$

$(17)^2 = (2+x)^2 + (9+x)^2$

$289 = 4 + x^2 + 4x + 81 + x^2 + 18x$

$x^2 + 11x - 102 = 0$

$x^2 + 17x - 6x - 102 = 0$

$x(x+17) - 6(x+17) = 0$

$(x+17)(x-6) = 0$

$x = 6$ as $x \neq -17$

$\therefore$ $x = 6$ cm

Alternate

$\triangle$ PRQ = Right angle $\triangle$

PQ(H) ↓ 17cm

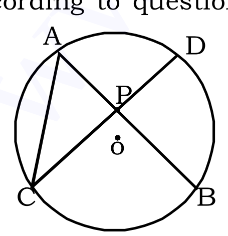
QR(B) ↓ (9 + x)cm

PR(P) ↓ (2 + x) cm

Triplet = (17,15,8)

$\therefore$ $x = 6$ cm

68. (c) According to question



$\angle BOC = 2\angle BAC$

$\therefore$ $\angle AOD = 2\angle ACD$

$\therefore$ $\angle BOC + \angle AOD = 2(\angle BAC + \angle ACD)$

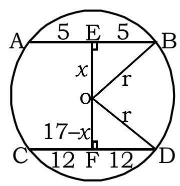
$= 2\angle BPC$

$30° + 20° = 2\angle BPC$

$\angle BPC = \frac{50°}{2}$

$\angle BPC = 25°$

69. (c) Accrding to question



AE = EB = 5 cm

CF = FD = 12 cm

BO = OD = r cm

$\therefore$ In $\triangle BOE$

$r^2 = x^2 + 5^2$ ....(i)

In $\triangle DOF$

$r^2 = (17-x)^2 + (12)^2$ ....(ii)

Compare equation (i) and (ii)

$x^2 + 25 = 289 + x^2 - 34x + 144$

$25 = 433 - 34x$

$34x = 408$

$x = 12$ ....(iii)

Put the value of x in equation (i)

$r^2 = (12)^2 + (5)^2$

$r^2 = 144 + 25$

$r^2 = 169$

r = 13 cm

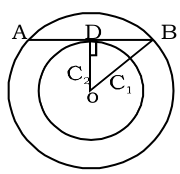
Alternate:-

Apply triplet

5,12,13

r = 13 cm

70. (c) According to question



AD = DB = x

$C_2 = (\sqrt{3}-1)$ cm

$C_1 = (\sqrt{3}+1)$ cm

In $\triangle BOD$

$C_1^2 = C_2^2 + BD^2$

$(\sqrt{3}+1)^2 = (\sqrt{3}-1)^2 + x^2$

$4 + 2\sqrt{3} = 4 - 2\sqrt{3} + x^2$

$x^2 = 4\sqrt{3}$

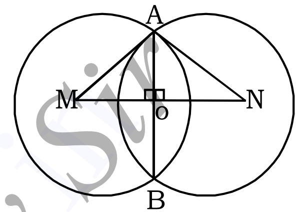
$x = (4\sqrt{3})^{1/2} = 2\sqrt[4]{3}$

$\therefore$ AB = 2 × BD

$AB = 2 \times 2\sqrt[4]{3}$

$AB = 4\sqrt[4]{3}$ cm

71. (d) According to question



Let AO = OB = x

MO = y

ON = 50 – y

AM = 30 cm

AN = 40 cm

In $\triangle AOM$

$AM^2 = OA^2 + OM^2$

$(30)^2 = OA^2 + y^2$

$x^2 = 900 - y^2$ ....(i)

In $\triangle AON$

$AN^2 = ON^2 + OA^2$

$(40)^2 = (50-y)^2 + x^2$

$(x)^2 = 1600 - (50-y)^2$......(ii)

Compare equation (i) and (ii)

$900 - y^2 = 1600 - (50-y)^2$

$900 - y^2 = 1600 - (2500 + y^2 - 100y)$

$900 - y^2 = 1600 - 2500 - y^2 + 100y$

$y = 18$ .... (iii)

put the value of y in equation (i)

$x^2 = 900 - 324$

$x^2 = 576$

$x = 24$ cm

OA = 24 cm

AB = 2 × 24

AB = 48 cm

Alternate:-

30,40,50 (triplet)

$\triangle$ AMN = Right triangle

$\angle$ MAN = 90°

$\triangle AMN = \frac{1}{2} \times b \times h$

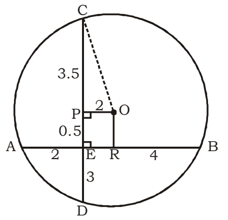
$\frac{1}{2} \times 30 \times 40 = \frac{1}{2} \times 50 \times AO$

AO = 24

AB = 2AO

$\therefore$ AB = 48 cm

72. (a) According to questions



Given:-

AE = 2 cm

EB = 6 cm

ED = 3 cm

As we know that

AE × EB = EC × ED

2 × 6 = EC × 3

EC = 4 cm

∴ In ΔOPC

$OC^2 = CP^2 + PO^2$

$r^2 = (2)^2 + \left(\frac{7}{2}\right)^2$

$r^2 = 4 + \frac{49}{4}$

$r^2 = \frac{65}{4}$

$r = \frac{\sqrt{65}}{2}$

∴ Diameter = 2r

$D = 2 \times \frac{\sqrt{65}}{2}$

$D = \sqrt{65}$

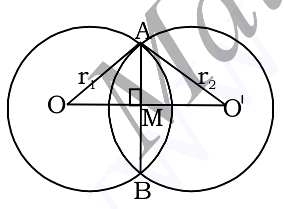
Alternate:-

Diameter = $\sqrt{AE^2 + EB^2 + EC^2 + ED^2}$

$= \sqrt{2^2 + 6^2 + 3^2 + 4^2} = \sqrt{65}$

radius is equal to $\frac{\sqrt{65}}{2}$

73. (a) According to question



$r_1 = r_2 = 5$ cm

AM = MB = 4 cm

∴ In Δ AMO

$OA^2 = OM^2 + AM^2$

$25 = OM^2 + 16$

$OM^2 = 25 - 16$

$OM^2 = 9$

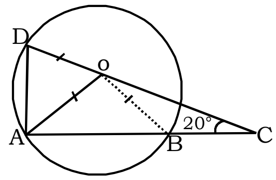
OM = 3 cm

∴ OO' = 2 × OM

OO' = 2 × 3

OO' = 6 cm

74. (d) According to question



BC = DO = OA = OB = r

In Δ OBC

$\angle$OCB = $\angle$COB = 20°

In Δ AOB

$\angle$OBA = 20°+ 20°

$\angle$OBA = 40°

$\angle$OBA = $\angle$OAB = 40°

In Δ AOB

$\angle$A + $\angle$O + $\angle$B = 180°

40° + $\angle$O + 40° = 180°

$\angle$O = 100°

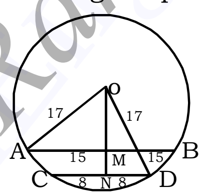
DOC is a line

$\angle$COB + $\angle$AOB + $\angle$DOA = 180°

20° + 100° + $\angle$DOA = 180°

$\angle$DOA = 60°

75. (b) According to question



OA = OD = 17 cm

AM = MB = 15 cm

CN = ND = 8 cm

In ΔOMA

$OA^2 = AM^2 + OM^2$

$(17)^2 = (15)^2 + OM^2$

$289 = 225 + OM^2$

$OM^2 = 289 - 225$

$OM^2 = 64$

OM = 8

In Δ OND

$OD^2 = ON^2 + ND^2$

$(17)^2 = (8)^2 + ON^2$

$289 = 64 + ON^2$

$ON^2 = 289 - 64$

$ON^2 = 225$

ON = 15

∴ MN = ON – OM

MN = 15 – 8

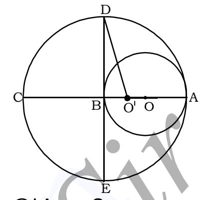
MN = 7 cm

Alternate:-

17,15,8 (triplet)

distance on same side between chords = (15 –8) = 7 cm

76. (d) According to question



O'A = 3 cm

OA = 2 cm

O'D = 3 cm

O'B = 1 cm

In Δ BDO

$O'D^2 = DB^2 + BO'^2$

$BD^2 = (3)^2 - (1)^2$

$BD^2 = 9 - 1$

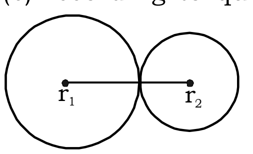
$BD^2 = 8$, BD = $2\sqrt{2}$

∴ DE = 2 × BD

DE = 2 × $2\sqrt{2}$

DE = $4\sqrt{2}$ cm

77. (b) According to question



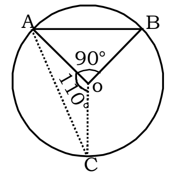
Given: $r_1 + r_2$ = 7 cm

r_1 = 4 cm

∴ r_2 = 7 – 4

r_2 = 3 cm

78. (b) According to question



OA = OB = OC

∴ In Δ OAB

$\angle OAB + \angle OBA + \angle AOB = 180°$

$2\angle OAB = 180° - 90$

$2\angle OAB = 90°$

$\angle OAB = 45°$

In Δ OAC

$\angle OAC + \angle OCA + \angle AOC = 180°$

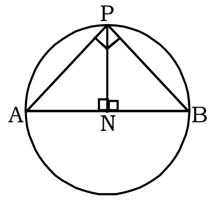
$2\angle OAC = 180° - 110°$

$\angle OAC = 35°$

∴ $\angle BAC = 45° + 35°$

$= 80°$

79. (d) According to question



AB = 2r = 14 cm
PB = 12 cm
$\angle APB$ = 90° (angle in the semicircle)
Let AN = x and NB = (14 – x)
$\therefore$ In $\triangle APB$
AB² = PB² + AP²
(14)² = (12)² + (AP)²
196 = 144 + (AP)²
(AP)² = 196 – 144
(AP)² = 52
AP = $\sqrt{52}$

In $\triangle$ APN
AP² = PN² + AN²
$\left(\sqrt{52}\right)^2 = x^2 + PN^2$
PN² = 52 – x^2 ....(i)
In $\triangle PNB$
PB² = PN² + NB²
(12)² = PN² + (14 – x)²
PN² = 144 – (14 – x)²..........(ii)
52 – x^2 = 144 – 196 – x^2 + 28x
28x = 104

$$x = \frac{104}{28}$$

$$x = \frac{26}{7}$$

NB = 14 – x

$$NB = 14 - \frac{26}{7}$$

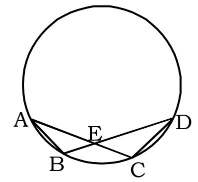
$$NB = \frac{72}{7}$$

$$NB = 10\frac{2}{7} \text{ cm}$$

Alternate:-
PB² = PN × AB
12² = 14 × BN
BN = 12 × 12 ÷ 14
BN = 72 ÷ 7

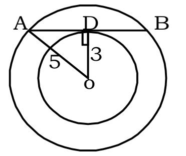
$$BN = 10\frac{2}{7}$$

80. (d) According to question



Given: $\angle BEC$ = 130°
$\Rightarrow$ $\angle DEC$ = 180° – 130° = 50°
$\therefore$ $\angle EDC$ = 180° – 50° – 20 = 110°
$\therefore$ $\angle BAC = \angle EDC$ = 110°
(Angle on the same arc are equal)

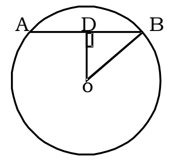
81. (d) According to question



Let AD = DB = x
OA = 5 cm
OD = 3 cm
In $\triangle$ ODA
OA² = OD² + AD²
(5)² = (3)² + (AD)²
25 = 9 + (AD)²
(AD)² = 25 – 9
(AD)² = 16
AD = 4
$\therefore$ AB = 2 × AD
AB = 2 × 4 = 8 cm

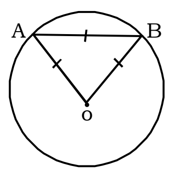
Alternate:-
In $\triangle$ AOD, 3,4,5 (triplet)
AD = 4 cm
AB = 2AD
= 2 × 4 = 8 cm

82. (a) According to question



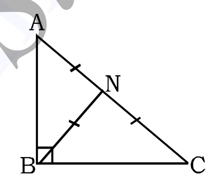
Let OD = x, AD = DB = 6 cm
OB = 10 cm
In $\triangle$ ODB
OB² = DB² + OD²
OD² = OB² – DB²
OD² = (10)² – (6)²
OD² = 100 – 36
OD² = 64
OD = 8 cm

83. (b) According to question



Let AB is the chord and 'O' is the centre of a circle
Given:-
OA = OB = AB
$\therefore$ All sides are equal then triangle is equilateral triangle.
$\therefore$ Then the angle subtended by the chord is 60°

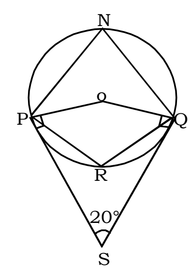
84. (c) In a right angled triangle the circumcentre of the triangle lies on mid point of the hypotenuse



BN = circumradius
N = circumcentre
$\therefore$ BN = AN = NC

$$BN = \frac{AC}{2} = \frac{H}{2}$$

85. (d) According to question,



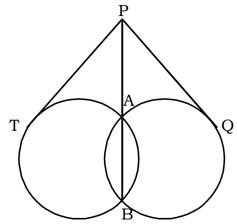
Given: $\angle$PSQ = 20°
$\angle$PRQ = ?
OPSQ is a quadrilateral
$\angle$OPS = $\angle$OQS = 90°
$\therefore$ $\angle OPS + \angle OQS + \angle POQ + \angle QSP = 360°$
$\angle OPS + \angle OQS + \angle POQ + \angle QSP = 360°$
$\angle POQ$ = 360° – 90° – 90°– 20°
$\angle POQ$ = 160°

$$\therefore \angle PNQ = \frac{1}{2}\angle POQ$$

$$\angle PNQ = \frac{1}{2} \times 160° = 80°$$

$\therefore$ NPRQ is a cyclic quadrilateral
$\therefore$ sum of opposite angles of cyclic quadrilateral is 180°
$\therefore$ $\angle PNQ + \angle PRQ$ = 180°
$\angle$PRQ = 180° – 80°
$\angle$PRQ = 100°

86. (d) According to question



As shown in the figure Tangent are equal

$\therefore$ PT = PQ

Alternate:-

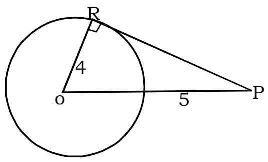
$PQ^2 = PA \times PB$ ..... (i)

$PT^2 = PA \times PB$ .....(ii)

From both equation,

$PT^2 = PQ^2$ PT = PQ

87. (a) According to question



Δ ORP is a right angle triangle

$\therefore$ By using pythagoras theorem.

$OP^2 = OR^2 + RP^2$

$(5)^2 = (4)^2 + (RP)^2$

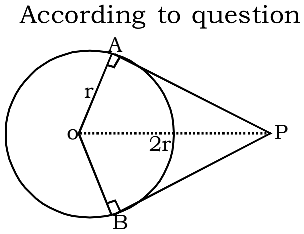
$25 = 16 + (RP)^2$

$(RP)^2 = 25 - 16$

$(RP)^2 = 9$

RP = 3 cm

88. (d) According to question



Given: OA = OB = r.(radius)

OP = 2r (diameter)

In Δ OAP

$OP^2 = OA^2 + AP^2$

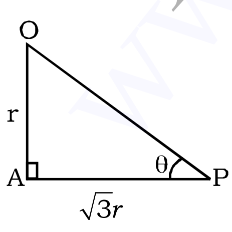
$(2r)^2 = r^2 + AP^2$

$AP^2 = 4r^2 - r^2$

$AP^2 = 3r^2$

$AP = \sqrt{3}r$

In Δ OAP



$\tan\theta = \frac{OA}{AP}$ $\tan\theta = \frac{r}{\sqrt{3}r}$

$\tan\theta = \frac{1}{\sqrt{3}}$ $\tan 30° = \frac{1}{\sqrt{3}}$

$\therefore$ $\angle OPA = 30°$

Similarly in $\angle$ OPB

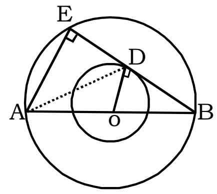
$\therefore$ $\angle OPB = 30°$

$\therefore$ $\angle APB = \angle OPA + \angle OPB$

$\angle APB = 30° + 30°$

$\angle APB = 60°$

89. (b) According to question



Given: OA = OB = 13 cm

OD = 8 cm

$\therefore$ AE = 2× OD

AE = 2 × 8 = 16 cm

In ΔODB

$OB^2 = OD^2 + BD^2$

$BD^2 = OB^2 - OD^2$

$BD^2 = (13)^2 - (8)^2$

$BD^2 = 169 - 64$

$BD^2 = 105$

$BD = \sqrt{105}$ cm

$\therefore$ DE = BD = $\sqrt{105}$ cm

$\therefore$ In ΔAED

$AD^2 = DE^2 + AE^2$

$AD^2 = \left(\sqrt{105}\right)^2 + (16)^2$

$AD^2 = 105 + 256$

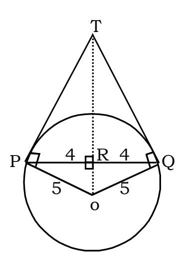
$AD^2 = 361$

AD = 19 cm

90. (a) According to question

OT is the perpendicular bisector of chord PQ.

let TR = y



In right angle ΔPRO

$PO^2 = PR^2 + RO^2$

$(5)^2 = (4)^2 + RO^2$

$(RO)^2 = 25 - 16$

$(RO)^2 = 9$, RO = 3 cm

Right angle ΔTPO and ΔTRP

$TO^2 = PT^2 + OP^2$ ....(i)

$PT^2 = TR^2 + PR^2$ ....(ii)

Put the value of PT^2 in equation (i)

$TO^2 = TR^2 + PR^2 + OP^2$

$(y + 3)^2 = y^2 + (4)^2 + (5)^2$

$y^2 + 9 + 6y = y^2 + 16 + 25$

$9 + 6y = 41,$ $6y = 32$

$y = \frac{32}{6} = \frac{16}{3}$ cm

In right angle ΔTRP

$PT^2 = TR^2 + PR^2$

$PT^2 = \left(\frac{16}{3}\right)^2 + (4)^2$

$PT^2 = \frac{256}{9} + 16$

$PT^2 = \frac{400}{9}$, $PT = \frac{20}{3}$ cm

Alternate:-

In Δ POR,

$OP^2 = OR^2 + PR^2$

$5^2 = OR^2 + 4^2$

$OR^2 = 25 - 16 = 9$

$\Rightarrow$ OR = 3cm

In Δ POR and Δ POT

$\angle PRO = \angle TOP$

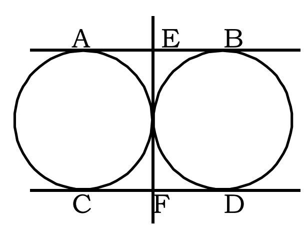
$\Rightarrow$ $\Delta POR \sim \Delta POT$

$\Rightarrow \frac{PR}{PT} = \frac{OR}{OP}$

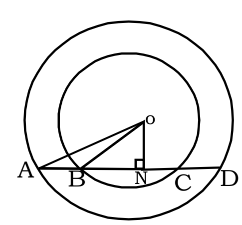
$\Rightarrow \frac{4}{PT} = \frac{3}{5}$

$\Rightarrow PT = \frac{20}{3}$ cm

91. (c) Maximum no. of tangent are



92. (c) According to question



Given: $BC = 12 cm, OA = 17$ cm

$OB = 10$ cm

$\therefore$ $BN = NC = 6$ cm

$\therefore$ In right angle $\triangle ONB$

$OB^2 = ON^2 + BN^2$

$(10)^2 = ON^2 + (6)^2$

$ON^2 = 100 - 36$

$ON^2 = 64$

$ON = 8$ cm

In right angle $\triangle ONA$

$OA^2 = ON^2 + AN^2$

$(17)^2 = (8)^2 + AN^2$

$AN^2 = 289 - 64$

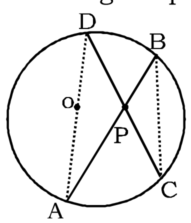
$AN^2 = 225$

$AN = 15$ cm

$AD = 2 \times AN$

$\therefore$ $AD = 15 \times 2 = 30$ cm

93. (b) According to question



$\angle ADP = \angle ABC = 23°$

$\angle APC = 70° = \angle DPB$

$\therefore$ $\angle APD = 180° - 70° = 110°$

$\angle APD = \angle BPC$ (Vertically opposite angle)

Also $\angle BCD = 180° - 23° - 110°$

$= 47°$

94. (c) According to figure

$\angle DAC = 51°$

$\angle EOB = 180° - 150° = 30°$

$OB = OE =$ radius

$\therefore$ $\angle OEB = \angle OBE$

then

$\angle OEB + \angle OBE + \angle EOB = 180°$

$2\angle OBE = 180° - 30°$

$\angle OBE = 75°$

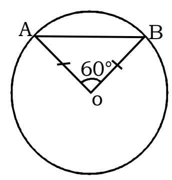
$\therefore$ $\angle CBE = 180° - 75°$

$\angle CBE = 105°$

95. (d) The angle in a semi-circle is a right angle

96. (b) According to question

Given :



$\angle BOA = 60°$

$OA = OB = r$

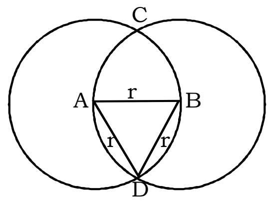
$\therefore$ $\angle OAB = \angle OBA = 60°$

$\therefore$ $\triangle OAB$ is a equilateral triangle

$\therefore$ $OA = OB = AB$

$\frac{AB}{OB} = \frac{1}{1}$

97. (c) According to question



$AB = AD = DB = r$

$\therefore$ $\triangle ADB$ is a equilateral triangle

$\angle DBA = 60°$

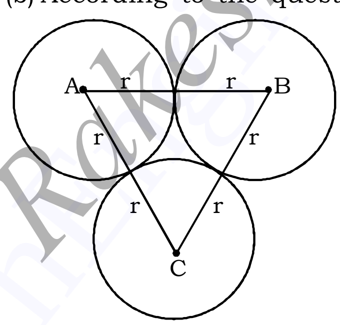
Similar In $\triangle ABC$

$\angle ABC = 60°$

$\angle DBC = 60° + 60°$

$\therefore$ $\angle DBC = 120°$

98. (b) According to the question



Let radius of the circle be $= r$

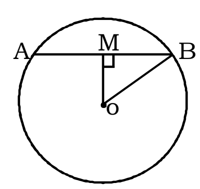
$\therefore$ $AB = 2r, BC = 2r, CA = 2r$

All these sides are equal

$\therefore$ Triangle ABC is an equilateral $\triangle$

99. (b) According to the question

Given:



$AB = 20$ cm

$AM = MB = 10$ cm

$OM = 2\sqrt{11}$ cm

$OM \perp AB$

$OB =$ radius

$\therefore$ In right angle $\triangle OMB$

By using pythagoras theorem

$OB^2 = OM^2 + MB^2$

$OB^2 = \left(2\sqrt{11}\right)^2 + (10)^2$

$OB^2 = 44 + 100$

$OB^2 = 144$

$OB = 12$ cm

100. (d) According to the figure.

$OM = OY = ON$

$\therefore$ In $\triangle OMY$

$\angle OMY = \angle OYM = 15°$

$\therefore$ $\angle MOY = 180° - 15° - 15°$

$\angle MOY = 150°$

In $\triangle ONY$

$\angle ONY = \angle OYN = 50°$

$\therefore$ $\angle NOY = 180° - 50° - 50°$

$\angle NOY = 80°$

$\therefore$ $\angle MON = 150° - 80°$

$\angle MON = 70°$

101. (d) Since $CS = SD$

The two chords must be equidistant from the centre 'O'.

Thus, the required ratio is 1 : 1

102. (b) $(OS)^2 = (OK)^2 + (KS)^2$

$25 = OK^2 + 16 \Rightarrow OK = 3$

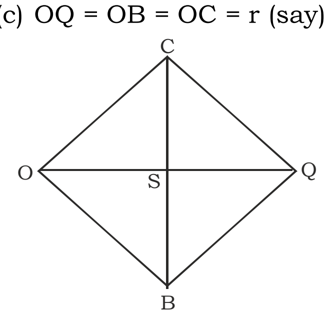
and $(OS)^2 = (OL)^2 + (LN)^2$

$\Rightarrow OL = 4$ cm

$KL = OL - OK = 1$cm

Area of rectangle= $1 \times 10 = 10cm^2$

103. (c) $OQ = OB = OC = r$ (say)



$\angle AOD = \angle BOC = 120°$

$\angle BOQ = \angle COQ = 60°$

$\therefore$ $\frac{SB}{OB} = \sin 60° = \frac{\sqrt{3}}{2}$

$\Rightarrow SB = \frac{r\sqrt{3}}{2}$

$BC = 2SB = r\sqrt{3}$

Area of quadrilateral BQCO

$= \frac{1}{2} \times BC \times OQ$

$= \frac{1}{2} \times r\sqrt{3} \times r = \frac{r^2\sqrt{3}}{2}$ cm²

Area of both the quadrilateral

$= 2\left(\frac{r^2\sqrt{3}}{2}\right) = r^2\sqrt{3}$ cm²

104. (c) $\angle PDB = \angle QEA = 80°$

$\angle PED = QDE = 10°$

($\therefore \angle DPE = \angle DQE = 90°$)

$\angle DRE = 180–(10+10) = 160°$

$\angle PRD = 180 - \angle DRE = 20°$

105. (a) Notice that all the given triangles are equilateral

Area of shaded region

$= 3\left[\pi r^2 \frac{60}{360} - \frac{\sqrt{3}}{4} \times r^2\right]$

$= \frac{r^2}{2}\left[\pi - \frac{3\sqrt{3}}{2}\right]$

106. (d)

107. (c) Note: $\angle ORP = 90°$ ($\because$ OP is a diameter ofsmaller circle)

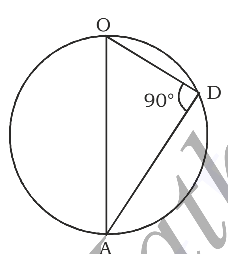
OS = 5cm and OR = 4cm

$\therefore SR = \sqrt{5^2 - 4^2} = 3$cm

$\therefore$ SP = 2(SR) = 6cm

(Since, OR passes through centre O and perpendicular to SP therefore OR bisects SP.)

108. (b) $\angle ADO$ is a right angle (angle of semicircle)



Again when OD is perpendicular on the chord AC and OD passes through the centre of circle ABC, then it must bisect the chord AC at D.

AD = CD = 6cm

109. (a) $\angle CED = 120°$(ACED is cyclic)

$\angle BED = 60°$

$\angle EDB = 90°$

$\frac{BD}{BE} = \cos 30°$

$\frac{6}{BE} = \frac{\sqrt{3}}{2}$

$BE = 4\sqrt{3}$ cm

BC = BE + CE

$= 4\sqrt{3} + 5\sqrt{3} = 9\sqrt{3}$

Now, since AB and CB are the secants of the circle.

BD × BA = BE × BC

$6 \times BA = 4\sqrt{3} \times 9\sqrt{3}$

BA = 18cm

Again Δ ACB is a right angle triangle ($\therefore \angle C = 90°$)

AC = AB sin30° (sin30° = 1/2)

AC = 9cm (alternatively apply Pythagorus theorem).

and AD = AB – BD = 12cm

$\frac{AC}{AD} = \frac{9}{12} = \frac{3}{4}$

110. (c) Let AD = 3a and DC = 6a

$DH = HG = GC = \frac{6a}{3} = 2a$

$HM = MG = \frac{2a}{2} = a = SM$

NQ = a

SQ = SM + MN + NQ

= a + 3a + a = 5a

Since diagonal of square SQ = 5a

But, diameter of circle SQ = diagonal of square SQ

$\therefore$ Radius of the circle $= \frac{5a}{2}$

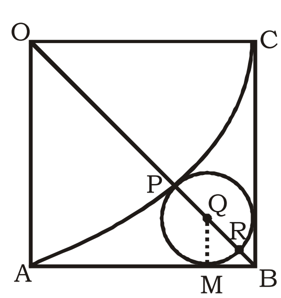
$\Rightarrow$ Area of the circle

$= \pi \times \left(\frac{5a}{2}\right)^2$

Here $\frac{\text{Area of circle}}{\text{Area of rectangle}}$

$= \frac{\frac{25}{4}(a^2\pi)}{3a \times 6a} = \frac{25\pi}{72}$

111. (a)



OA = AB = BC = OC = OP (radius)

Let OA = R (radius of the larger circle) then $OB = R\sqrt{2}$

Similarly PQ = MQ = QR = r (radius of the smaller circle) then $BQ = r\sqrt{2}$

and $BP = OB - OP = R\sqrt{2} - R$

$R\sqrt{2} - R = r + r\sqrt{2}$

$R(\sqrt{2} - 1) = r(\sqrt{2} + 1)$

$r = R(\sqrt{2} - 1)^2$

$r = R(3 - 2\sqrt{2})$

$\therefore \frac{\text{Area of larger circle}}{\text{Area of 4 smaller circle}}$

$= \frac{\pi R^2}{4\pi r^2} = \frac{R^2}{4(3-2\sqrt{2})^2 R^2}$

$= \frac{1}{4(3-2\sqrt{2})^2} = \frac{1}{4(17-12\sqrt{2})}$

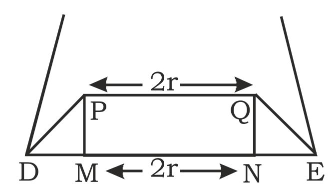
$= \frac{1}{68 - 48\sqrt{2}}$

112. Apply the same logic as in the previous problem.

113. (c) Let the radius of each circle be r unit then

PQ = QR = PR = 2r

$\angle PDM = \angle QEN = 30°$



$\frac{DM}{DP} = \cos 30°$

$DM = DP \times \frac{\sqrt{3}}{2}$ [DP = QE = (r)]

$DM = \frac{r\sqrt{3}}{2}$

DE = DM + MN + NE

$= \frac{r\sqrt{3}}{2} + 2r + \frac{r\sqrt{3}}{2} = (2 + \sqrt{3})r$

$DE = DF = EF = (2 + \sqrt{3})r$

$\angle PAM = \angle QBN = 30°$

Again,

$$\frac{PM}{AM} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\frac{r}{AM} = \frac{1}{\sqrt{3}}$$

$\Rightarrow AM = r\sqrt{3} = BN$

$AB = AM + MN + NB$

$= r\sqrt{3} + 2r + r\sqrt{3}$

$= 2r\,(1+\sqrt{3})$

$AB = BC = AC = 2r\,(1+\sqrt{3})$

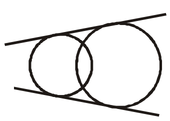
Ratio of perimeter of equilateral triangle = ratio of their sides

Ratio of perimeter of ΔABC : $\Delta DEF : \Delta PQR$

$= 2\,(1 + \sqrt{3}) : (2 + \sqrt{3}) : 2$

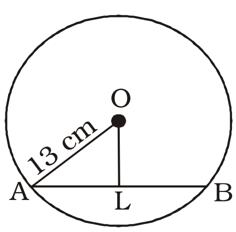
114. (a)

115. (b)



If two circle intersect each other then maximum two tangents will be drawn.

116. (d)



The perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore$ $AL = LB = \frac{1}{2} AB = 5$ cm

In ΔOAL

$\because$ $OA^2 = OL^2 + AL^2$

$\Rightarrow$ $13^2 = OL^2 + 5^2$

$\Rightarrow$ $OL^2 = 13^2 - 5^2$

$\Rightarrow$ $OL = 12$ cm

117.(c) $\angle ODC = \angle BAC = 38°$(Angle made by same arc BC) and OC = OD = Radius

$\therefore$ $\angle OCD = \angle ODC = 38°$

118.(b) OB = OC

$\Rightarrow$ $\angle OCB = \angle OBC = 20°$

$\therefore$ $\angle BOC = 180° - (20 + 20)$
$= 140°$

$\therefore$ $\angle BAC = \frac{1}{2} \angle BOC = 70°$

119. (a) $PQ^2 = BQ \times AQ$

$\Rightarrow (12)^2 = AQ \times 8$

$\Rightarrow$ $AQ = 18$ cm

$\therefore$ $AB = AQ - BQ$

$= 18 - 8 = 10$ cm

120. (d) $\angle ACB = \angle ADB = 20°$ (made by same arc AB)

$\therefore$ In ΔACB,

$\angle x° = 180° - 85° - 20° = 75°$

121.(c) $\angle APB = \frac{1}{2} \times \angle AOB$

$$\frac{1}{2} \times \angle AOB = \frac{1}{2} \times 90°$$

$= 45°$

122.(c) $\angle AOC = 360° - (90° + 110°)$
$= 160°$

$\therefore$ $\angle ABC = \frac{1}{2} \angle AOC = 80°$

123.(b) $\angle ADC$ = Opposite exterior angle = 120°

$\therefore$ $x° = 180° - 120° = 60°$

124..(a) $PT^2 = PA \times PB$

$\Rightarrow$ $36 = 5(5 + x)$

$\Rightarrow$ $5 + x = \frac{36}{5} = 7.2$

$\Rightarrow$ $x = 2.2$ cm

125. (d) $PA \times PC = PB \times PD$

$\Rightarrow$ $14 \times 9 = (7 + x) \times 7$

$\Rightarrow$ $18 = 7 + x \Rightarrow x = 11$ cm

126. (b) $\angle BAC = \frac{1}{2} \times 138° = 69°$

$\therefore$ $\angle BDC = 180° - 69° = 111°$

($\therefore$ ABCD is a cyclic quadrilateral)

127.(c) In ΔOBC

OB = OC

$\therefore$ $\angle B = \angle C = 45°$

$\therefore$ $\angle D = \angle C$

($\because$ made by same arc AB)

$\therefore$ $\angle D = x = 45°$

128.(a) $x = 40°$

($\because$ made by same arc AB)

129.(b) $\because$ ABCD is a cyclic quadrilateral.

$\therefore$ $\angle C + \angle D = 180°$

$\Rightarrow$ $\angle D = 180° - 48° = 132°$

130.(b) $(PT)^2 = PA \times PB$

$\Rightarrow$ $144 = x \times (7 + x)$

$\Rightarrow$ $x^2 + 7x - 144 = 0$

$\Rightarrow (x + 16)(x - 9) = 0$

$\Rightarrow$ $x = 9$ or -16

-16 cannot be the length, hence this value is discarded thus, $x = 9$ cm.

131.(d) PA = PB

$\therefore$ $\angle PAB = \angle PBA$

In ΔAPB

Also, $\angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow$ $\angle PAB + \angle PBA = 120°$

$\therefore$ $\angle PAB = \angle PBA = 60°$

i.e. ΔPAB is an equilateral triangle.

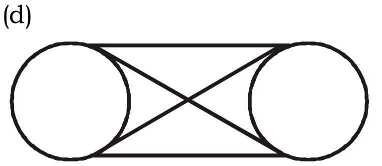
$\therefore$ AB = 6 cm

132.(a) A = Area of $\Delta ABC = \frac{1}{2} \times 3 \times 4 = 6$ cm^2

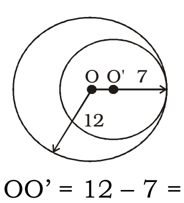
S = Semiperimeter of ΔABC = $\frac{3+5+4}{2} = 6$ cm

$\therefore$ inradius $= \frac{A}{S} = \frac{6}{6} = 1$ cm

133.(d)

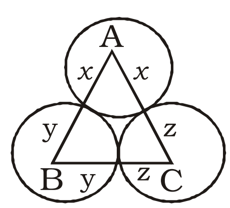


134.(d)



$\therefore$ OO' = 12 − 7 = 5 cm

135.(a)



AB = 5 cm = $x + y$

BC = 6 cm = $y + z$
AC = 7 cm = $z + x$
$\therefore$ $2(x + y + z) = 5 + 6 + 7 = 18$
$\Rightarrow$ $x + y + z = 9$
$\Rightarrow$ $5 + z = 9 \Rightarrow z = 4$ cm
$\therefore$ $x = 7 - z = 3$ cm and
$y = 6 - z = 2$ cm
$\therefore$ $x = 3$ cm, $y = 2$ cm, $z = 4$ cm

136.(c) $\angle AOB = 2\angle ACB = 2 \times 30° = 60°$

137.(b) AB = AC
$\Rightarrow$ $\angle ACB = \angle ABC = 50°$
$\therefore \angle BAC = 180° - (50 + 50) = 80°$
$\therefore$ $\angle BDC = \angle BAC = 80°$
(angle by same arc BC)

138.(b) $\angle DOC = \angle AOB = 70°$
$\because$ OD = OC = radius
$\therefore \angle OCD = \angle ODC = \frac{1}{2}(180° - 70°) = 55°$

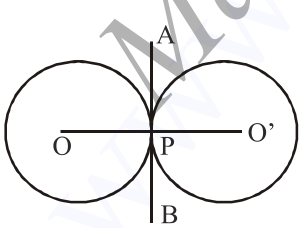
139.(c) It is necessarily a rectangle.

140.(d)

141.(c) Length of transverse tangent
$= \sqrt{d^2 - (R_1 + R_2)^2}$
here d = 10 cm, $R_1 = R_2 = 3$ cm
$\therefore$ length = $\sqrt{(10)^2 - (6)^2}$
= 8 cm

142.(a) CD = 7cm
$\therefore$ AC = 7 cm and BC = 7 cm
($\because$ AC = CD and BC = CD. Two tangents from the same point are always equal)
$\therefore$ AB = 7 + 7 = 14 cm

143.(a) Tangent is always perpendicular to the radius.



144.(b) $\angle BAC = 60°$
$\therefore \angle BEC = 180° - 60° = 120°$

145.(c) $\angle CBA = \frac{1}{2}\angle AOC = 65°$
$\therefore$ $\angle CBE = 180° - 65° = 115°$

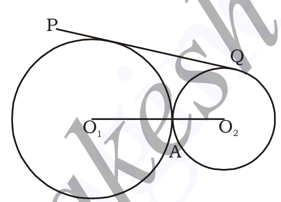
146.(a) $\angle BDC = \angle BAC = 30°$
In $\triangle$ BCD
$\therefore \angle BCD + \angle BDC + \angle DBC = 180°$
$\therefore$ $\angle BCD = 180° - (30° + 60°)$
$= 90°$

147.(d) $\angle D = 180° - 72° = 108°$
$\therefore$ $\angle BCD = 180° - 108°$
$= 72°$ ($\because$ AD||BC)

148.(b) $\angle OAP = \angle OBP = 90°$
In $\square$AOBP, $\angle O + \angle P = 180°$
$\Rightarrow$ $\angle AOB + \angle APB = 180°$
$\therefore$ $\angle APB = \frac{1}{6} \times 180° = 30°$
[$\angle AOB : \angle APB = 5 : 1$]

149.(b) Length of common tangent
$= \sqrt{d^2 - (R - r)^2}$

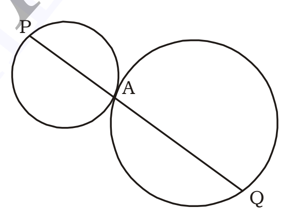
150.(d)



$\therefore$ PQ = $2\sqrt{r_1 r_2} = 2\sqrt{8 \times 2} = 8cm$

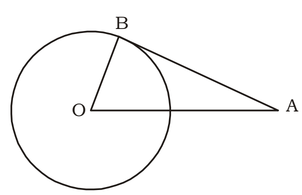
151.(d) $PQ^2 = (r_1 + r_2)^2 - (r_1 - r_2)^2 = 4r_1r_2$

152.(b)



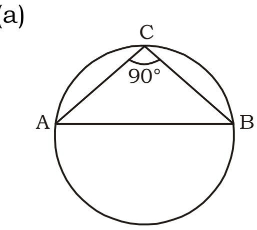
$\therefore$ AP : AQ = 5 : 8

153.(a)



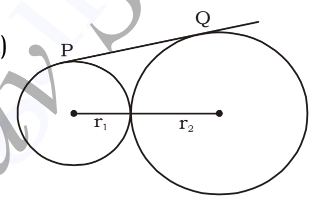
$\angle OBA = 90°$;
OA = 10 cm, OB = 6 cm
From Δ OAB,
$AB = \sqrt{OA^2 - OB^2}$
$= \sqrt{10^2 - 6^2}$
$= \sqrt{100 - 36} = \sqrt{64} =$ 8 cm

154.(a)



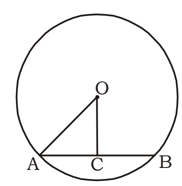
AB = diameter of circle.
Angle of a semi-circle is a right angle.
i.e $\angle ACB = 90°$
$\therefore$ ABC is a right angled triangle.

155.(d)



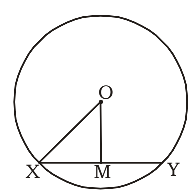
$PQ = 2\sqrt{r_1 r_2} = 2\sqrt{4 \times 9} = 12cm$
Required area (side PQ)
= $(12)^2$ = 144

156.(b)



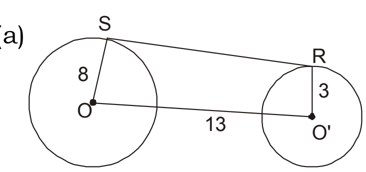
AC = CB = 4 cm
OC = 3 cm
$\therefore OA = \sqrt{OC^2 + CA^2}$
$= \sqrt{3^2 + 4^2}$
$= \sqrt{9 + 16} = \sqrt{25}$ = 5 cm

157.(c)



XM = MY
OM = 12 cm
OX = 13 cm
$\therefore XM = \sqrt{OX^2 - OM^2} = \sqrt{13^2 - 12^2}$
= 5
$\therefore$ XY = 2XM = 10 cm

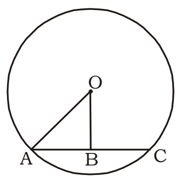
158.(a)



$SR = \sqrt{(OO')^2 - (R_1 - R_2)^2}$

$= \sqrt{(13)^2 - (5)^2} = \sqrt{18 \times 8} = 12$ cm

159.(b)



AB = BC = 8

OA = 10

In $\triangle$ OAB

$\therefore OB = \sqrt{OA^2 - AB^2}$

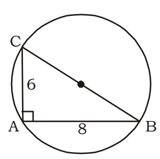
$= \sqrt{10^2 - 8^2} = \sqrt{36} = 6$

160.(b) The largest chord of circle is its diameter.

$R = \dfrac{10.1}{2} = 5.05$

(Greater than 5)

161.(d)



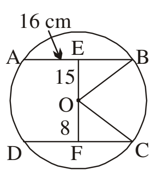
$\angle BAC = 90°$

$\therefore$ BC is the diameter of the circle.

$\therefore$ BC $= \sqrt{64+36} = \sqrt{100} = 10\ cm$

$\therefore$ Radius of the circle = 5cm

162.(c)



In $\triangle$ BEO

$OB^2 = BE^2 + OE^2$

$OB^2 = 15^2 + 8^2$

OB = 17 cm

In $\triangle$ OFC

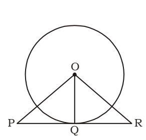
$OC^2 = OF^2 + FC^2$

$(17)^2 = 8^2 + FC^2$

FC = 15 cm

DC = 2 × FC = 30 cm.

163.(d)



OR = 5cm

$OP = \dfrac{20}{3}$

$\angle POR = 90°$

In triangle POR

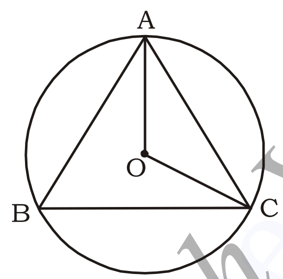
$PR^2 = PO^2 + OR^2$

$PR = \sqrt{\left(\dfrac{20}{3}\right)^2 + 5^2}$

$\sqrt{\dfrac{400}{9} + 25}$

$\sqrt{\dfrac{625}{9}} = \dfrac{25}{3}$

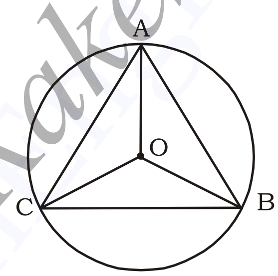
164.(b)



$\angle ABC = 180° - 80° - 85° = 15°$

$\Rightarrow \angle AOC = 2\angle ABC = 2 \times 15° = 30°$

165. (a)



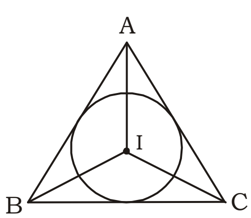
OB = OC = radius

$\therefore \angle OBC = \angle OCB = 35°$

$\angle BOC = 180° - 35° - 35° = 110°$

$\therefore \angle BAC = \dfrac{1}{2} \times \angle BOC = 55°$

166.(c)



$\angle BIC = 135°$

$\angle BIC = 90° + \dfrac{1}{2}\angle A$

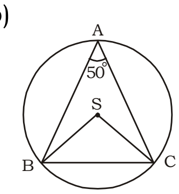
$\dfrac{1}{2}\angle A = 135 - 90$

$\angle A = 45 \times 2 = 90°$

$\angle B = \angle C$ = Incentre is angle bisector

So, $\triangle$ABC is a right angled $\triangle$

167.(b)



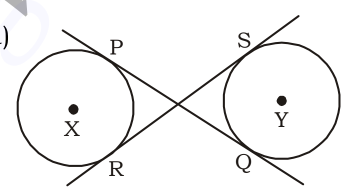
$\angle BAC = 50°$

$\therefore \angle BSC = 100°$

BS = SC = radius

$\therefore \angle BCS = \dfrac{1}{2}(180 - 100) = 40°$

168. (a)



Transverse common tangent

$= \sqrt{(\text{Distance between centres})^2 - (r_1 + r_2)^2}$

$= \sqrt{10^2 - 6^2} = \sqrt{16 \times 4} = 8cm$

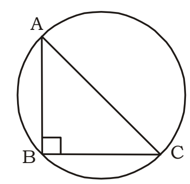
169.(b) One and only one circle can pass through three non-collinear points.

170.(b) $3^2 + 4^2 = 5^2$

ΔABC is a right angled triangle.

$\angle B = 90°$ = angle at the circumference

$\therefore$ Diameter of circle = 5 cm



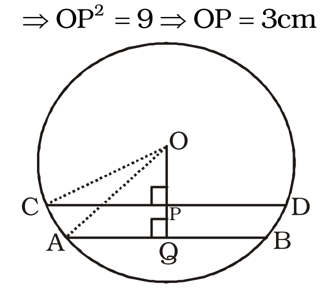
$\therefore$ Circum-radius = 2.5 cm

171.(b) In ΔOPC

$OC^2 = OP^2 + CP^2$

$\Rightarrow 5^2 = OP^2 + \left(\dfrac{8}{2}\right)^2 \Rightarrow OP^2 = 5^2 - 4^2$

$\Rightarrow OP^2 = 9 \Rightarrow OP = 3cm$



In ΔOQA

$OA^2 = OQ^2 + AQ^2$

$\Rightarrow 5^2 = OQ^2 + \left(\frac{6}{2}\right)^2 \Rightarrow OQ^2 = 5^2 - 3^2$

$\Rightarrow OQ = 4cm$

$\therefore$ Distance between chords AB and CD =

PQ = OQ - OP = 4-3 = 1cm

172.(c) $\angle LKN = 90^0$ (angle in semicircle)

$\therefore \angle LNK = 180^0 - (90 + 30) = 60^0$

$\therefore \angle PKL = \angle LNK = 60^0$

(angle in alternate segment)

173.(a) $\angle ADC = 120°$

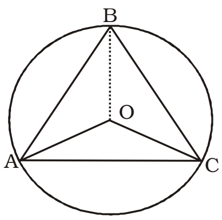
$\angle ABC = 180° - 120 = 60°$

and $\angle ACB = 90^0$ (angle in semicircle)

$\therefore \angle BAC = 180^0 - (90^0 + 60^0) = 30^0$

174.(d) In ΔAOB,

AO = OB (radius)



$\therefore \angle OBA = \angle OAB = 25^0$

similarly In ΔOBC,

$\therefore \angle OBC = \angle OCB = 35^0$

$\therefore \angle ABC = 25 + 35 = 60^0$

$\therefore \angle AOC = 2 \times \angle ABC$

$= 2 \times 60° = 120°$

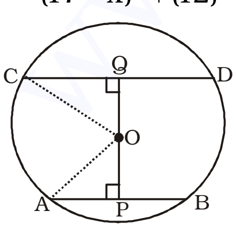
175.(d) Let OP = x cm

$\therefore$ OQ=$(17-x)$ cm and radius = r cm

$\therefore$ In ΔOQC,

$(OC)^2 = (OQ)^2 + (QC)^2$

$\Rightarrow r^2 = (17 - x)^2 + (12)^2$ ....(i)



and In ΔOAP,

$(OA)^2 = (OP)^2 + (AP)^2$

$\Rightarrow r^2 = x^2 + (5)^2$ ....(ii)

$\therefore (17 - x)^2 + (12)^2 = x^2 + 5^2$

$\Rightarrow 289 - 34x + x^2 + 144 = x^2 + 25$

$\Rightarrow 34x = 408 \Rightarrow x = 12cm$

$\therefore$ from(ii)

$r^2 = (12)^2 + (5)^2 = (13)^2$

$\Rightarrow r = 13$ cm

176. (C) OA = OB = radius

$\Rightarrow \angle OAB = \angle OBA = 25^0$

$\therefore \angle AOB = 180° - 50° = 130°$

$\therefore$ Major

$\angle AOB = 360^0 - 130^0 = 230^0$

$\Rightarrow \angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 230 = 115^0$

177.(b) $\angle QOP = 180^0 - 120^0 = 60^0$

and $\angle PQO = 90^0$

$\therefore \angle QPO = 180^0 - (90^0 + 60^0) = 30^0$

178.(c) $\angle ADC = 180^0 - 55 = 125^0$

$\therefore \angle CDT = 180^0 - (125 + 30^0) = 25^0$

179.(a) In cyclic $\square$ABCD,

$\angle ADC + \angle ABC = 180^0$

$\Rightarrow \angle ADC = 180^0 - 70^0 = 110^0$

now In ΔADC,

$\angle ACD = 180° - (30°+110°) = 40°$

180.(d) $\angle PQO = \angle PRO = 90^0$

($\because$ PQ and PR are tangents)

$\therefore$ In $\square$PQOR

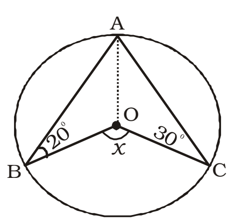
$\angle ROQ = 360^0 - (90^0 + 90^0 + 60^0) = 120^0$

$\therefore \angle QSR = \frac{1}{2}\angle QOR = \frac{1}{2} \times 120° = 60°$

181.(d) In ΔAOB

OA=OB=radius

$\therefore \angle OAB = \angle OBA = 20^0$



similarly In ΔAOC,

$\angle OAC = \angle OCA = 30°$

$\therefore \angle BAC = 20° + 30° = 50°$

$\therefore \angle BOC = 2 \times \angle BAC \Rightarrow \angle X = 100°$

182.(b) $\angle APB = 90^0$

(angle in a semicircle = 90^0)

$\therefore \angle PBA = 180° - (90° + 30°) = 60°$

$\therefore \angle TPA = \angle PBA = 60°$

(by alternate segment theorem)

183.(c) $\angle BAC = \angle BDC = 30^0$

($\because$ made by same arc BC)

In ΔABC,

$\angle x = 180^0 - (100 + 30^0) = 50^0$

184.(c) It will always be possible to divide a circle into 360 equal parts, because the sum of angle that can be subtended at the centre = 360^0

185.(d) ABCD is a cyclic quadrilateral. Therefore

$\angle DCB = 180° - \angle A$

$= 180° - 60° = 120°$

$\therefore \angle BCQ = 180° - 120°$

$= 60°$ $\angle ABC = 80$;

$\therefore \angle CBQ = 180° - 80° = 100°$

In ΔBCQ,

$\angle Q = 180° - (100 + 60°) = 20°$

186.(b) AB=24cm

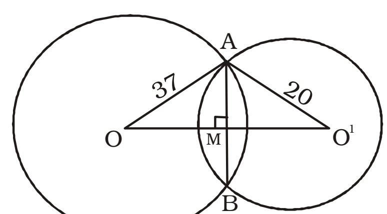
$\therefore$ AM=MB=12cm

In ΔAMO,

$(OM)^2 = (AO)^2 - (AM)^2$

$(OM)^2 = (37)^2 - (12)^2$

$\Rightarrow OM = 35cm$



In $\Delta AMO'$;

$(O'M)^2 = (AO')^2 - (AM)^2$

$(O'M)^2 = (20)^2 - (12)^2$

$\Rightarrow O'M = 16cm$

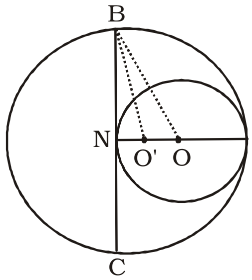
$\therefore OO' = OM + O'M = 35 + 16$

$= 51cm$

187. (a) OM = 4cm = radius of smaller circle and O'M = 6cm = radius of bigger circle

$\therefore$ O'N = 8–6 = 2cm

In $\Delta O'NB$,



$(O'B)^2=(O'N)^2+(BN)^2$

$\Rightarrow$ $(BN)^2 = 36-4 = 32$

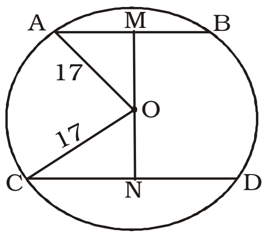
$\Rightarrow$ BN = $4\sqrt{2}$

$\therefore NC = BN = 4\sqrt{2}$

$\therefore BC = 4\sqrt{2}+4\sqrt{2} = 8\sqrt{2}cm$

188.(d) MN=23cm

AM = MB = $\dfrac{16}{2}$ = 8cm



$\therefore$ In ΔAMO,

$(OM)^2 = (17)^2 - (8)^2$

$\Rightarrow$ OM = 15cm

$\therefore ON = 23-15 = 8cm$

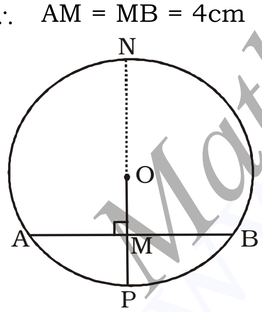
In ΔONC,

$(CN)^2 = (17)^2-(8)^2 \Rightarrow CN = 15cm$

$\therefore CD = 2CN = 30cm$

189.(b) AB = 8cm

$\therefore$ AM = MB = 4cm



$AM \times MB = PM \times MN$

$\Rightarrow 4\times 4 = 2\times(2r-2)$

$\Rightarrow 4 = r-1 \Rightarrow r = 5\,cm$

190.(a) ABCD is a cyclic quadrilateral

$\therefore \angle ADC + \angle ABC = 180^\circ$

$\Rightarrow \therefore \angle ABC = 180^\circ - 130^\circ = 50^\circ$

also, $\angle ACB = 90^\circ$

$\therefore$ In ΔABC,

$\angle CAB = 180^\circ - (90^\circ + 50^\circ) = 40^\circ$

191.(b) $\angle AOC = 2\angle APC$

$\Rightarrow \angle APC = 50^0$

Also, ABCP is a cyclic quadrilateral

$\therefore \angle ABC + \angle APC = 180^0$

$\Rightarrow \angle ABC = 180^0 - 50^0 = 130^0$

$\therefore \angle CBD = 180^0 - 130^0 = 50^0$

192.(a) $\angle TPQ = \angle PAQ = 50^0$

($\angle$s in the alternate segment)

$\because TP = TQ \Rightarrow \angle TQP = \angle TPQ = 50^0$

$\because \angle PTQ = 180^0 - 50^0 - 50^0 = 80^0$

193.(b) $\angle ACB = 90^0$ [angle at the point of intersection to the centres of the circles.]

BC = r

AC = 2r (as area of x=4 area of y)

$\therefore AB = \sqrt{r^2+4r^2} = \sqrt{5}r$

194.(c) $\angle QSR = \angle QTR = \dfrac{Z}{2}$

$\therefore \angle PSR = \angle PTQ = 180^0 - \dfrac{Z}{2}$

Also, $\angle QMR = \angle SMT = y$

$\therefore$ In quadrilateral PSMT

$180^0 - \dfrac{z}{2} + 180^0 - \dfrac{z}{2} + x + y = 360^0$

$\Rightarrow x + y = z$

195.(c) AB = AC and AD = CD

$\therefore$ AB = 2AD

Now, since AD is a tangent

$\therefore AD^2 = AP \times AB$

$\Rightarrow \left(\dfrac{AB}{2}\right)^2 = AP \times AB \Rightarrow AB = 4AP$

196. (d) $\angle CAB = \angle BCD$

and $\angle DAB = \angle BDC$

(alternate segment theorem)

$\therefore \angle CAD = \angle CAB + \angle DAB$

$= \angle BCD + \angle BDC$

$\therefore \angle CAD + \angle CBD = \angle BCD +$

$\angle BDC + \angle CBD = 180^0$

197.(b)

$\angle ABK = 180^0 - (115+30) = 35^0$

$\therefore \angle KCD = \angle ABK = 35^0$

($\angle$s made by same arc AD)

198.(a) $\angle AOC = \angle BOC$

$(\because AC = BC)$

$\therefore$ OC is the perpendicular bisector of AB

$\therefore$ AM = BM

IInd -Method

In ΔAOM and ΔBOM

OM = OM (common)

$\angle AOM = \angle BOM$ $(\because AC = BC)$

OA = OB = radius

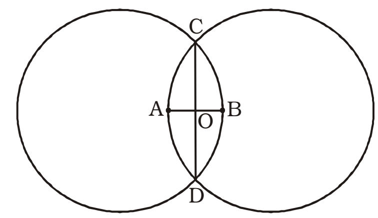
$\therefore \Delta AOM \cong \Delta BOM$

$\therefore AM = BM \Rightarrow AM : BM = 1:1$

199.(b) AB = r (say)

then AC = BC = r, also

$\therefore OA = OB = \dfrac{r}{2}$



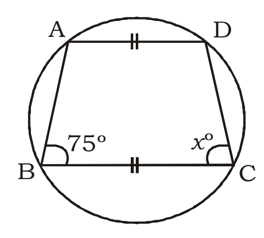
$\therefore OC = \sqrt{(AC)^2-(OA)^2} =$

$\sqrt{r^2-\left(\dfrac{r}{2}\right)^2} = \dfrac{\sqrt{3}}{2}r$

$\therefore CD = 2CO = \sqrt{3}r$

$\therefore \dfrac{CD}{AC} = \dfrac{\sqrt{3}.r}{r} = \dfrac{\sqrt{3}}{1}$

200. (a)



According to figure

$\Rightarrow$ AD || BC

$\Rightarrow \angle ABC = 75$

Then

$\Rightarrow \angle ABC + \angle ADC = 180^\circ$

$\Rightarrow$ 75° + $\angle$ADC = 180°

$\Rightarrow$ $\angle$ADC = 180° – 75°

$\Rightarrow$ $\angle$ADC = 105°

$\Rightarrow$ As we know in a cyclic quadrilateral

$\angle$ADC + $\angle$DCB = 180°

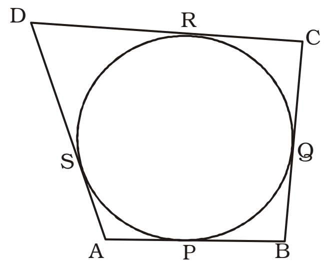
(AD || BC, corresponding angle)

$\Rightarrow$ 105 + $\angle$DCB = 180°

$\Rightarrow$ $\angle$DCB = 75°

201.(d) AP = AS, BP = BQ, CQ = CR and DR = DS

$\therefore$ AB = AP + BP = AS + BQ



CD = CR + DR = CQ + DS

$\therefore$ AB + CD = (AS + DS) + (BQ + CQ) = BC + AD

202.(a) TQ = TP and TP = TR

$\therefore$ TQ = TP = TR

$\Rightarrow TQ : TR = 1 : 1$

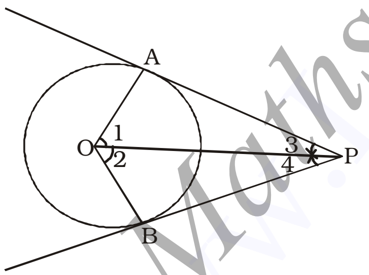
203.(b) $\angle AOC = 2 \times 60^0 = 120^0$

$$\angle ABC = \frac{120}{2} = 60^0$$

204.(c) $\angle APB = \angle 3 + \angle 4 = 68^0$

In ΔAOP and ΔBOP

PA = PB



OP = OP (common)

and OA=OB= radius

$\therefore \Delta$ AOP$\cong\Delta$BOP

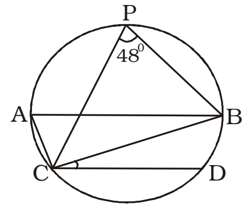
$\therefore \angle 1 = \angle 2$ and $\angle 3 = \angle 4$

$$\therefore \angle 3 = \frac{68}{2} = 34^0$$

$\therefore \angle 1 = \angle POA = 180^0 - (90^0 + 34^0) = 56^0$

205.(b) $\angle BAC = \angle BPC = 48^0$ (by same arc BC) and $\angle ACB = 90^0$

($\because$ AB is diameter)

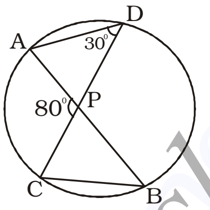


$\therefore \angle ABC = 180^0 - (90^0 + 48^0) = 42^0$

$\therefore \angle BCD = \angle ABC = 42^0$

($\because$ AB || CD)

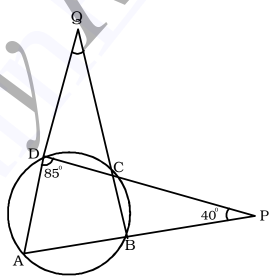
206. (d) $\angle APD = 180^0 - 80^0 = 100^0$



$\therefore \angle PAD = 180^0 - (100^0 + 30^0) = 50^0$

$\therefore \angle BCD = \angle BAD = 50^0$ [$\angle$s by same arc BD]

207.(a) $\angle ABC = 180^0 - 85^0 = 95^0$



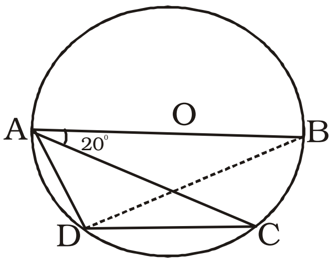
In ΔADP,

$\angle DAP = 180^0 - (85 + 40) = 55^0$

$\therefore$ In ΔAQB,

$\angle AQB = 180^0 - (95^0 + 55^0) = 30^0$

208. (c)



$\angle ADB = 90^0$ [semi circle]

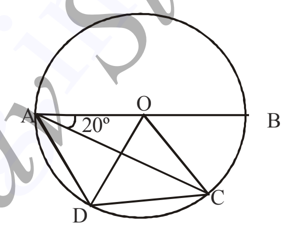
$\angle BDC = \angle BAC = 20^0$ [$\angle$s by same arc BC]

$\therefore \angle ADC = 90^0 + 20^0 = 110^0$

209. (b) $\angle ACD = \angle BAC = 20^0$

($\because$ AB || DC) and

$\angle ADC = 110^0$ (from Q above)

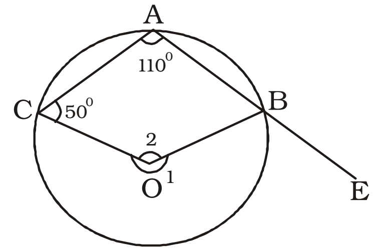


$\angle$DAC = 180°– (110° + 20°) = 50°

$\therefore \angle COD = 2 \times \angle DAC = 2 \times 50^0 = 100^0$

210. (a) $\angle 1 = 2\angle A = 220^0$

$\therefore \angle 2 = 360^0 - 220^0 = 140^0$

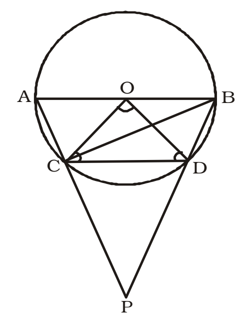


$\therefore \angle ABO = 360^0 - (110^0 + 50^0 + 140^0)$

$= 60^0$

$\therefore \angle OBE = 180^0 - 60^0 = 120^0$

211. (d) $CD = \frac{1}{2}AB = AO = OB =$ radius



$\therefore$ CD = OC = OD

$\therefore \Delta OCD$ is an equilateral triangle

$\therefore \angle COD = \angle OCD = \angle ODC = 60^0$

$\angle ACB = 90^0$ (angle in semicircle)

$\therefore \angle BCP = 90^0$

$\angle CBD = \frac{1}{2}\angle COD = 30^0$

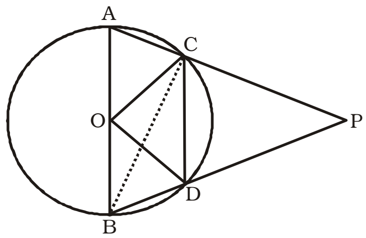
(made by same arc CD)

$\therefore$ In ΔBCP,

$\angle APB = 180^0 - 90^0 - 30^0 = 60^0$

212. (b) $\angle ACB = 90^0$

$\Rightarrow \angle BCP = 90^0$

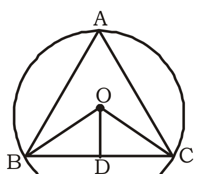


$\angle CBP = \frac{1}{2}\angle COD = 15.5^0$

$\therefore$ In ΔBPC,

$\angle APB = 180^0 - 90^0 - 15.5^0 = 74.5^0$

213. (d)



$BD = \frac{BC}{2} = 12$ cm

OB = 13 cm

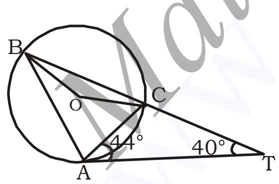
In ΔOBD,

OD = $\sqrt{OB^2 - BD^2}$

$= \sqrt{13^2 - 12^2} = \sqrt{169 - 144}$

$= \sqrt{25} = 5$ cm

214. (d)



$\angle$CAT = 44°

$\angle$BTA = 40°

$\angle$ACT = 180°- 44°- 40°

= 96°

$\angle$CAT = $\angle$CBA = 44°

$\angle$BCA = 180°- 96° = 84

$\therefore$ $\angle$BAC = 180°- 84° - 44°

= 52°

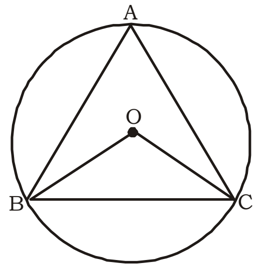
$\therefore$ Angle subtended by BC at centre ($\angle$BOC)

= 2 $\times$ 52° = 104°

215.(c) $\angle$BOC = 2$\angle$BAC

OB = OC

$\therefore$ $\angle$OBC = $\angle$OCB



In Δ BOC

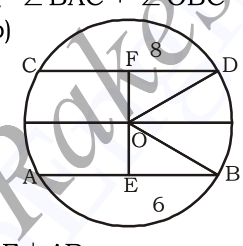
2$\angle$OBC = 180° – $\angle$BOC

$\therefore \angle OBC = 90° - \frac{\angle BOC}{2}$

$\angle$OBC = 90° - $\angle$BAC

$\therefore$ $\angle$BAC + $\angle$OBC = 90°

216. (b)



OE$\perp$AB

$\therefore$ BE = AE = 3 cm

and, OF$\perp$CD

$\therefore$ FD = CF = 4 cm

InΔODF

OF = $\sqrt{5^2 - 4^2}$ = 3 cm

InΔ OBE,

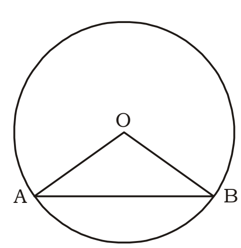
OE = $\sqrt{5^2 - 3^2}$ = 4 cm

$\therefore$ EF = OE + OF

= 4 + 3

= 7 cm

217.(a)



InΔ OAB,

$\angle$AOB = 90°

$OA^2 + OB^2 = AB^2$

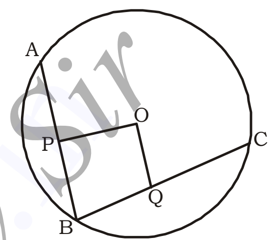
$\Rightarrow 2r^2 = \left(3\sqrt{2}\right)^2 = 18$

$\Rightarrow r^2 = 9 \Rightarrow r = 3$ units

$\therefore$ Area of the sector AOB

$= \frac{1}{4}\pi r^2 = \frac{1}{4}\pi \times 9 = \frac{9\pi}{4}$ sq.units

218.(b)



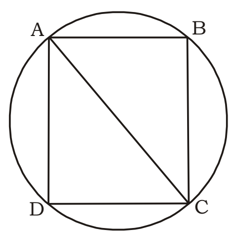
$\angle$OPB = $\angle$OQB = 90°

$\therefore$ $\angle$OPB + $\angle$OQB = 180°

and, $\angle$PBQ + $\angle$POQ = 180°

hence, OQBP must be concylic quadrilateral

219.(b)



Area of circle = $\pi r^2 = 36$

$\Rightarrow r^2 = \frac{36}{\pi}$

$r = \frac{6}{\sqrt{\pi}} cm$

$\therefore$ AC = Diameter = $\frac{12}{\sqrt{\pi}} cm$

Diagonal of square = $\frac{12}{\sqrt{\pi}}$ cm

$\therefore$ Side of square =

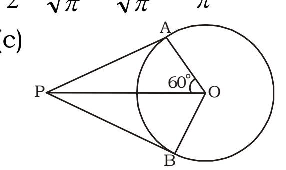
$S = \frac{1}{\sqrt{2}} \times$ Diagonal

$= \frac{1}{\sqrt{2}} \times \frac{12}{\sqrt{\pi}} = \frac{6\sqrt{2}}{\sqrt{\pi}}$ cm

$\therefore$ Area ofΔ ACD =

$\frac{1}{2} \times \frac{6\sqrt{2}}{\sqrt{\pi}} \times \frac{6\sqrt{2}}{\sqrt{\pi}} = \frac{36}{\pi}$ sq.cm

220.(c)



In right ΔOAP and ΔOBP,

AP = PB, OA = OB = radius

OP = OP

$\therefore\ \Delta OAP \cong \Delta OBP$

$\therefore\ \angle AOP = \angle POB$

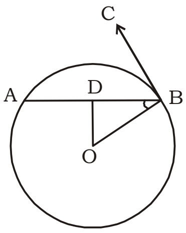
and $\angle APO = \angle OPB$

In ΔAOP,

$\angle APO = 180° - 90° - 60° = 30°$

$\therefore\ \angle APB = 2 \times 30 = 60°$

221.(c)



$\angle ABC = 45°$

$\Rightarrow \angle ABO = 45°\ (\because \angle OBC = 90°)$

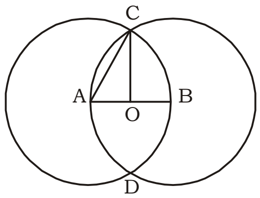
BD = 3 cm

$\therefore$ In ΔOBD

$$\cos 45° = \frac{3}{OB} \Rightarrow \frac{1}{\sqrt{2}} = \frac{3}{OB}$$

$\Rightarrow\ OB = 3\sqrt{2}$ cm

222.(b)



$$AO = OB = \frac{5}{2}$$

AC = 5

In ΔOAC

$$\therefore OC = \sqrt{5^2 - \left(\frac{5}{2}\right)^2} = \sqrt{25 - \frac{25}{4}}$$

$$= \sqrt{\frac{100-25}{4}} = \sqrt{\frac{75}{4}} = \frac{5\sqrt{3}}{2}$$

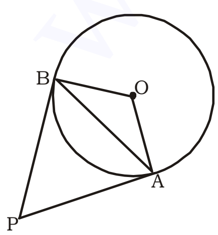
$$\therefore CD = 2 \times OC = 2 \times \frac{5\sqrt{3}}{2} = 5\sqrt{3} \text{ cm}$$

223.(d) $OA \perp AP$ and $OB \perp BP$

$\angle OAP = 90°$ and

$\angle OBP = 90°$

$\therefore\ \angle OAP + \angle OBP = 90° + 90° = 180°$



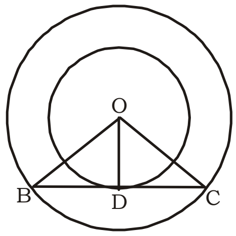
In quadrilateral OAPB,

$\angle AOB + \angle OAP + \angle APB + \angle OBP = 360°$

$\Rightarrow\ \angle APB + \angle AOB = 180°$

$\therefore$ The quadrilateral will be concyclic.

224.(a)



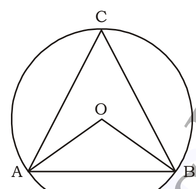
BO = OC = 15 cm.

OD = 9 cm

$$BD = \sqrt{15^2 - 9^2} = \sqrt{24 \times 6} = 12 \text{ cm}$$

$\therefore BC = 2 \times 12 = 24$ cm.

225.(a)



AO = OB = AB

$\therefore\ \angle AOB = 60°$

$\therefore\ \angle ACB = 30°$

226.(b) Side of the equilateral triangle a,

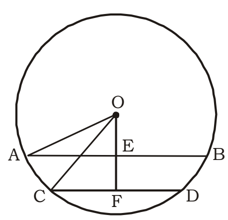
$$\text{In radius} = \frac{a}{2\sqrt{3}}$$

$$\text{Circum-radius} = \frac{a}{\sqrt{3}}$$

Required ratio

$$= \pi\left(\frac{a}{\sqrt{3}}\right)^2 : \pi\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{1}{3} : \frac{1}{12} = 4:1$$

227.(a)



Let OE = x cm

$\therefore$ OF = $(x + 1)$cm

OA = OC = r cm

AE = 4 cm, CF = 3 cm

In ΔOAE,

$OA^2 = AE^2 + OE^2$

$\Rightarrow\ r^2 = 16 + x^2$

$\Rightarrow\ x^2 = r^2 - 16$

In ΔOCF,

$(x + 1)^2 = r^2 - 9$

By equation (ii) - (i)

$(x + 1)^2 - x^2 = r^2 - 9 - r^2 + 16$

$\Rightarrow\ 2x + 1 = 7$

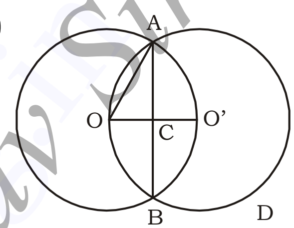
$\Rightarrow\ x = 3$ cm

$\therefore$ From equation (i),

$9 = r^2 - 16 \Rightarrow r^2 = 25$

$\Rightarrow\ r = 5$

228.(b)



OC = 2 cm

OA = 4 cm

$$\therefore AC = \sqrt{4^2 - 2^2} = \sqrt{16-4} = \sqrt{12} = 2\sqrt{3}$$

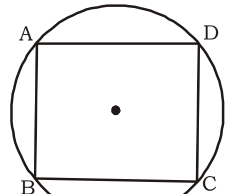
$\therefore AB = 4\sqrt{3}$ cm

229.(d) ABCD is cyclic parallelogram.

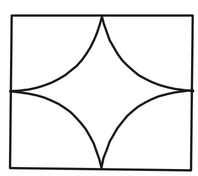
$\therefore\ \angle B + \angle D = 180°$

$\Rightarrow\ 2\angle B = 180°$

$\Rightarrow\ \angle B = 90°$



230.(b)



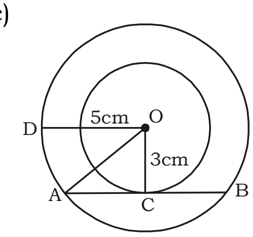
Area of sectors =

$\pi r^2 = 4\pi$ sq.cm.

Area of square = 4×4 = 16 cm.

Area of the remaining portion = $(16 - 4\pi)$sq.cm.

231.(c)

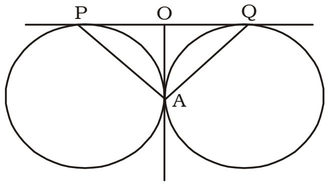


$$AC = \sqrt{AO^2 - OC^2} = \sqrt{5^2 - 3^2}$$

$=\sqrt{25-9}=\sqrt{16}=4$ cm

$\therefore$ AB = 2×4 = 8 cm

232.(b)



OA = OP

and OA = OQ

$\therefore$ OA = OP = OQ

Let $\angle OPA = \angle OAP = \alpha$

$\angle OQA = \angle OAQ = \beta$

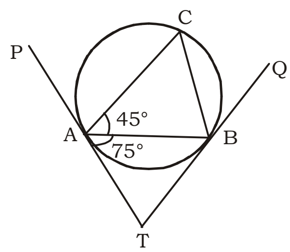
Now In $\triangle PAQ$,

$\beta+\alpha+(\alpha+\beta)=180°$

$\Rightarrow \alpha+\beta = 90°$

$\therefore \angle PAQ = 90°$

233. (c)

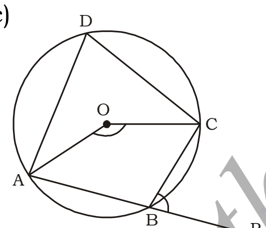


$\angle ACB = \angle BAT = 75°$ (angles in the alternate segment)

In $\triangle ABC$,

$\angle ABC = 180° - 45° - 75° = 60°$

234.(c)



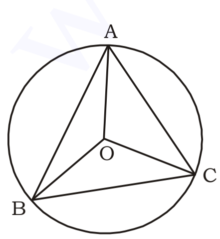
$\angle AOC = 130°$

$\angle ADC = \frac{1}{2}\times130° = 65°$

$\angle PBC = \angle ADC = 65°$

(exterior angle is equal to the opposite interior angle)

235.(c)



$\angle ABC = 180° - 85° - 75°$

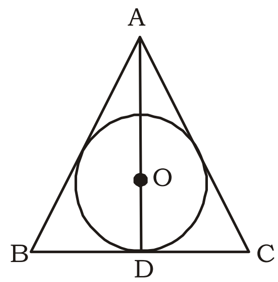
$= 20°$

$\therefore \angle AOC = 2\times20°$

$= 40°$

$\angle OAC = \frac{180^0 - 40^0}{2} = 70°$

236.(c)



BD = DC = $7\sqrt{3}$ cm

AD = $\sqrt{AB^2 - BD^2} = \sqrt{(14\sqrt{3})^2 - (7\sqrt{3})^2}$

$=\sqrt{(14\sqrt{3}+7\sqrt{3})(14\sqrt{3}-7\sqrt{3})}$

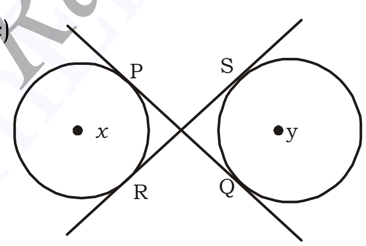
$=\sqrt{21\sqrt{3}\times7\sqrt{3}} = 21$ cm

$\therefore$ OD = $\frac{1}{3}\times21 = 7$ cm

$\therefore$ Area of circle = πr^2

$= \frac{22}{7}\times7\times7 = 154\ sq.cm$

237.(c)



Length of common transverse tangent

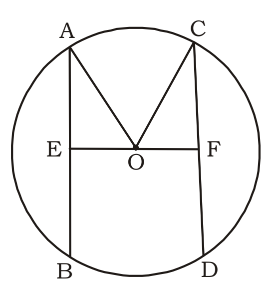
$=\sqrt{XY^2-(r_1+r_2)^2}$

$\Rightarrow 8=\sqrt{XY^2-9^2}$

$\Rightarrow XY^2=64+81=145$

$\Rightarrow XY=\sqrt{145}$

238.(b)



AO = OC $\Rightarrow$ 5 cm (radius)

AB = 24 cm

AE = EB = 12 cm

In $\triangle$ AEO

OE = $\sqrt{15^2-12^2}$

= $\sqrt{225-144}=\sqrt{81}=9$ cm

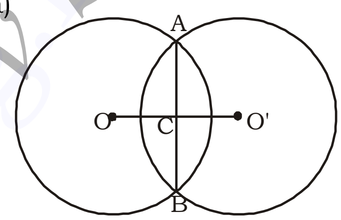
$\therefore$ OF = 21 - 9 = 12 cm

In $\triangle$ OCF

$\therefore$ CF = $\sqrt{15^2-12^2}=9$ cm

$\therefore$ CD = $2\times9 = 18$ cm

239. (a)



AB = 16, AC = BC = 8cm

OC = CO' = 6 cm

$\therefore OA = \sqrt{OC^2 + CA^2}$

$=\sqrt{6^2+8^2}=\sqrt{36+64}$

$=\sqrt{100}$ = 10 cm

240.(b) Tangents drawn from any external point are of same length

$\therefore$ AD = AE, BD = BF and CE = CF

AD = AB + BD = AB + BF

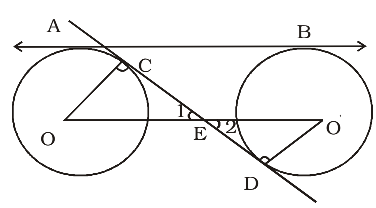
and AD = AE = AC + CE

= AC + CF

$\therefore$ 2AD = AB + AC + BF + CF 2AD = AB + BC + CA

241.(b)

242.(c)



OC = O' D = 5cm (radius)

CE = ED (CD = 24cm)

$\triangle COE \cong \triangle EO'D$

($\because \angle C = \angle D = 90°$ and $\angle 1 = \angle 2$)

$\therefore$ OE = O'E

and CE= ED=12cm

In $\triangle$ COE ;

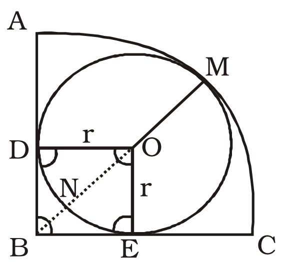
$OE^2 = CE^2 + OC^2$

$OE^2 = 12^2 + 5^2 = 169$

$\Rightarrow OE = 13$

$\therefore OO^1 = OE + EO' = 13 + 13$
$= 26cm = AB$

243.(a) Let radius of smaller circle=r



∵ AB and BC are tangents to smaller circle

∴ $OD \perp AB$ and $OE \perp BC$

∴ $\angle DOE = 90°$

& OD=OE=r= radius

BM=1cm

∴ OB=(1-r)cm

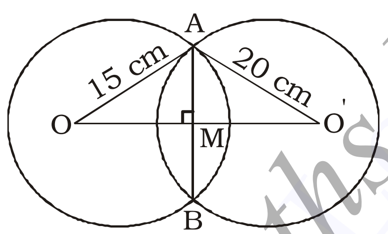
∵ ODBE is a square

∴ $OB = \sqrt{2}r$

∴ $\sqrt{2}r = 1 - r \Rightarrow r(\sqrt{2}+1) = 1$

$$\Rightarrow r = \frac{1}{\sqrt{2}+1} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = (\sqrt{2}-1)cm$$

244.(d)



OO'=25cm

∵ $(25)^2 = (15)^2 + (20)^2$

∴ $\Delta OAO'$ is a right angle triangle

AB is common chord

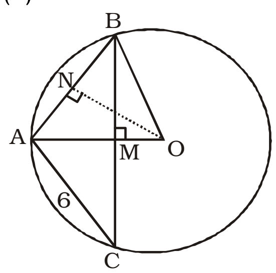
$\Delta OMA \cong \Delta OAO'$

{∵ $\angle OAO' = \angle AMO = 90°$ and $\angle O = \angle O$ (common)}

$$\therefore \frac{OA}{OO'} = \frac{AM}{O'A} \Rightarrow AM = \frac{15 \times 20}{25} = 12cm$$

∴ AB = 2AM = 24cm

245.(b) Let BM = x cm



$$\therefore \text{Area of } \Delta AOB = \frac{1}{2} \times OA \times BM$$

$$= \frac{1}{2} \times 5 \times x = \frac{5x}{2} cm$$

$ON \perp AB$

$$\therefore AN = BN = \frac{6}{2} = 3 \text{ cm}$$

In ΔANO,

$$ON = \sqrt{(5)^2 - (3)^2} = 4cm$$

∴ Again Area of ΔAOB

$$= \frac{1}{2} \times AB \times ON = \frac{1}{2} \times 6 \times 4 = 12cm$$

$$\therefore \frac{5x}{2} = 12 \Rightarrow x = \frac{24}{5} = 4.8cm$$

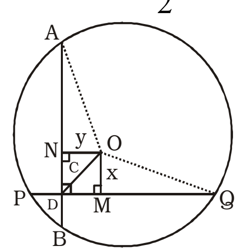
∴ BC = $2x$ = 9.6 cm

246.(c) Let AB = 2a and PQ = 2b & 'O' is the centre

$OM \perp PQ$ and $ON \perp AB$

$$\therefore PM = MQ = \frac{2b}{2} = b$$

$$\text{and } AN = NB = \frac{2a}{2} = a$$



In quadrilateral DMON,

$\angle D = \angle M = \angle N = 90^0$

∴ $\angle NOM = 90^0$

it mean DMON is a rectangle

∴ let $OM = DN = x$

then $ON = DM = y$

Let radius = OQ =r cm

In ΔOMQ, $r^2 = x^2 + b^2$ ____(i)

In ΔONA, $r^2 = y^2 + a^2$ ____(ii)

(i)+(ii)

$2r^2 = a^2 + b^2 + (x^2 + y^2)$

$2r^2 = a^2+b^2+c^2$

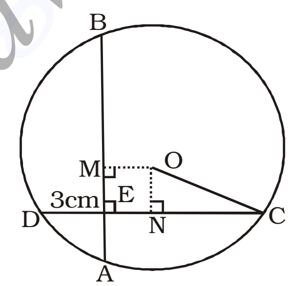
$$\Rightarrow r = \sqrt{\frac{a^2+b^2+c^2}{2}}$$

247.(a) AE × EB = DE ×CE

$$\Rightarrow CE = \frac{2 \times 6}{3} = 4cm$$

Let O is the centre

$OM \perp AB$



$$\therefore AM = BM = \frac{8}{2} = 4cm$$

∴ EM = 4-2 = 2cm

∴ ON = EM = 2cm

(∵ ONEM is a rectangle)

Now In ΔONC,

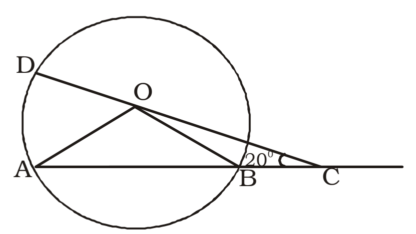
$(OC)^2 = (ON)^2 + (NC)^2$

$$\Rightarrow r^2 = (2)^2 + \left(\frac{7}{2}\right)^2 = 4 + \frac{49}{4} = \frac{65}{4}$$

$$\Rightarrow r = \frac{\sqrt{65}}{2} cm$$

248.(d) BC = OD (given)

∴ BC = OD = OB = BA = radius



In ΔBOC,

BC = OB

∴ $\angle BOC = \angle OCB = 20^0$

∴ $\angle ABO = 20^0 + 20^0 = 40^0$

In ΔOAB,

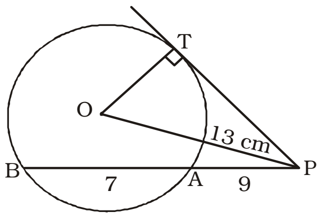
AO = OB

$\therefore \angle OAB = \angle ABO = 40^0$

$\therefore \angle AOB = 180^0 - (40^0 + 40^0) = 100^0$

$\therefore \angle AOD = 180^0 - (100^0 + 20^0) = 60^0$

249.(b) Draw a tangent (PT) from P-



$\therefore PT^2 = PA \times PB$

$\Rightarrow PT^2 = 9 \times 16 \Rightarrow PT = 12cm$

In ΔOTP, $\angle T = 90^0$

$\therefore (OT)^2 = (13)^2 - (12)^2 = 25$

$\Rightarrow OT = r = 5cm$

250.(c) In $\square AOBP$

$(\because \angle B = \angle A = 90^0)$

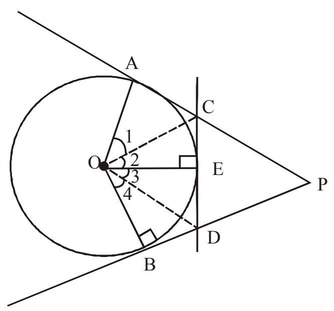
$\therefore \angle AOB = 180^0 - 34 = 146^0$

In ΔOAC and ΔOEC

OC = OC (common)

OA = OE = radius

and CA = CE



$\therefore \Delta OAC \cong \Delta OEC$

$\therefore \angle AOC = \angle COE \Rightarrow \angle 1 = \angle 2$

Similarly $\Delta OBD \cong \Delta OED$

$\therefore \angle 3 = \angle 4$

Δ In AOB

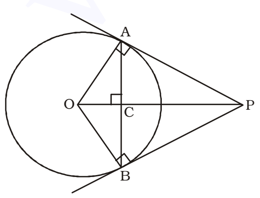
$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 146^0$

$\Rightarrow \angle 2 + \angle 2 + \angle 3 + \angle 3 = 146^0$

$\Rightarrow \angle 2 + \angle 3 = 73^0$

$\Rightarrow \angle COD = 73^0$

251.(a)



AB = 6cm

$\therefore AC = BC = \frac{6}{2} = 3cm$

In ΔOAP and ΔOCA

$\angle OAP = \angle OCA = 90^0$

$\angle AOP = \angle AOC$

$\therefore \Delta OAP \cong \Delta OCA$

$\frac{PO}{AO} = \frac{OA}{OC} \Rightarrow OP = \frac{OA^2}{OC} = \frac{(5)^2}{4}$

$\Rightarrow OP = \frac{25}{4} cm$

252.(b) $BC = 2(OB) = \sqrt{a^2 + 4^2}$

$= \sqrt{a^2 + 16}$

$(\because \angle A = 90^0)$

$\because \Delta ABD \cong \Delta CBA$

$\therefore \frac{BD}{AB} = \frac{AB}{BC} \Rightarrow BD.BC = a^2$

$\Rightarrow BD = \frac{a^2}{BC} = \frac{a^2}{\sqrt{a^2 + 16}}$

$\therefore OD = OB - BD = \frac{\sqrt{a^2+16}}{2} -$

$\frac{a^2}{\sqrt{a^2+16}} = \frac{16 - a^2}{2\sqrt{a^2+16}}$

253.(d) $\Delta PQS \cong \Delta PMN \cong \Delta PRT$

$\therefore$ N is the mid-point of ST

Also In ΔPQS, $PS^2 = (24)^2 + (7)^2$

$= 625$

$\Rightarrow PS = 25cm$

As $\Delta PQS \cong \Delta PRT$

$\Rightarrow \frac{QS}{RT} = \frac{PQ}{PR} = \frac{PS}{PT} = \frac{7}{21} = \frac{1}{3}$

$\therefore PR = 3 \times PQ = 72cm$

and $PT = 3 \times PS = 75cm$

$\therefore ST = PT - PS = 50cm$

$\therefore SN = 25cm$

254.(a)If a pair of sides of a cyclic quadrilateral are parallel, it become an isosceles trapezium.

Here, a + c = b + d = 180^0

(cyclic quadrilateral) a = b and c = d

(Isosceles trapezium)

$\therefore$ a + b - c - d = (a + b + c + d) - 2(c + d) = 360^0 - 4c $(\because$ c=d)

Since, no angle of the quadrilateral ABCD is reflex i,e, $\rangle 180^0$

$\therefore$ C can take any value 1 to 179

$\therefore$ (a + b - c -d) cm take one value for each value of C, i,e,179 values.

255.(c) AB = BC = AC = 2cm

$(\because$ radius of each circle=1cm)

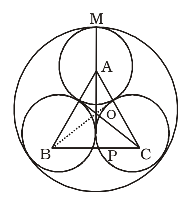
$\therefore AP = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}cm$

Let O is the centroid, then

$OA = \frac{2}{3} \times \sqrt{3} = \frac{2}{\sqrt{3}} cm$

$\therefore OM = OA + AM = \frac{2}{\sqrt{3}} + 1$

$= \frac{2+\sqrt{3}}{\sqrt{3}} cm$



OM is the radius of the larger circle.

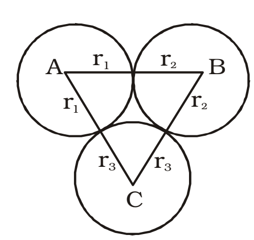
$\therefore$ Area of the circumscribing circle= πR^2

$= \pi \left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)^2$

$= \frac{\pi}{3}(2+\sqrt{3})^2$

256.(a) $\therefore r_1 + r_2 = 4$

$r_2 + r_3 = 3.4, r_1 + r_3 = 2.2$



$\Rightarrow 2(r_1+r_2+r_3)=4+3.4+2.2=9.6$

$\Rightarrow r_1+r_2+r_3=4.8$

$\therefore r_1=1.4cm,\ r_2=2.6cm$, and

$r_3=0.8cm$,

$\therefore$ diameters

$d_1=2r_1=2.8cm$

$d_2=2r_2=5.2cm$

$d_3=2r_3=1.6cm$

$\therefore$ d_2 is bigger circle &

d_2=5.2cm

257.(c) $\angle$CBY= $\angle$CZY=42^0

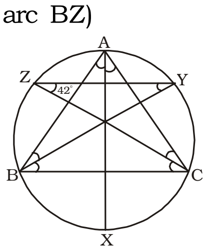
($\angle$S by same arc YC)

$\therefore$ $\angle$ABC = 42 + 42 = 84

$\therefore$ $\angle$C = 180^0 - (84 + 50^0) = 46^0

$\therefore$ $\angle$BCZ=$\frac{\angle C}{2}=23^0$

$\therefore$ $\angle$BYZ = $\angle$BCZ = 23^0 ($\angle$S by same arc BZ)



258.(b) $\angle x=\angle CBD=33^0$

(by alternate segment theorem)

$\therefore \angle Z=180^0-\angle x=147^0$

(BPDE is a cyclic quadrilateral)

and $\angle$BDE = $\angle$BED= $x=33^0$

{$\because$ BE=BD}

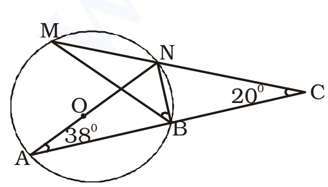
$\therefore$ $\angle$ABE= $\angle$BDE= 33^0

{by alternate segment theorem}

$\therefore$ $\angle$y=90^0-33^0=57^0

$x+y+z=33+57+147=237^0$

259.(d)



$\angle ABN$ = 90^o { angle in semicircle}

$\therefore$ $\angle CBN$ = 90^o

$\therefore$ In Δ CBN

$\therefore$ $\angle BNC$ = 180° – 90° – 20° = 70°

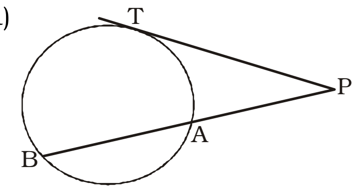
$\therefore$ $\angle$MNB = 180° – 70° = 110°

and $\angle BMN$ = $\therefore$ $\angle BAN$ = 38^o

$\therefore$ In Δ BMN

$\angle$MBN = 180° – 38° – 110° = 32°

260.(a)



$(PT)^2$ = $PA \times PB$

$\Rightarrow$ $(2AP)^2$ = $PA \times PB$

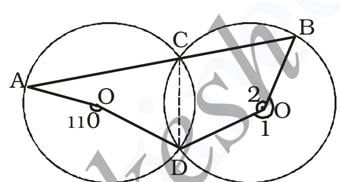
$\Rightarrow$ $4AP^2$ = AP × BP $\Rightarrow$ 4AP = BP

$\Rightarrow$ 4AP = (18 + AP) $\Rightarrow$ 3AP = 18

$\Rightarrow$ AP = 6cm

$\therefore$ PT = 2AP = 12 cm

261.(b)



$\angle$ACD = $\frac{1}{2}$ $\angle$AOD = 55°

$\therefore$ $\angle$BCD = 180° – 55°

$=125^0$

$\therefore$ $\angle$1 = 2$\angle$BCD = 250°

$\therefore$ $\angle$2 = $\angle$BO'D = 360°–250° = 110°

262.(c) AB = 27 cm

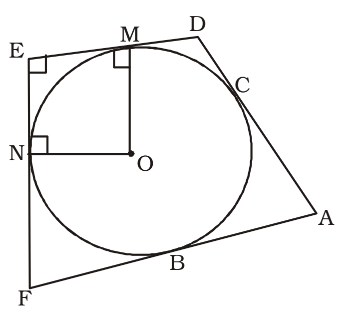
$\therefore$ AC = 27 cm

$\therefore$ CD = 38 – 27 = 11 cm

$\therefore$ DM = CD = 11 cm

$\therefore$ EM = 24 – 11= 13 cm

$\therefore$ EN = EM = 13 cm



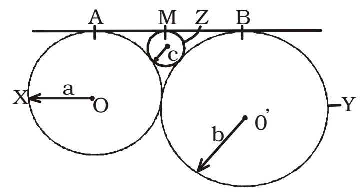
$OM \perp DE$ and $ON \perp EF$

$\therefore$ ONEM is a square

$(\therefore EM = EN)$

$\therefore$ $OM = ON = radius = 13cm$

263.(d)



AB is a common tangent of X and Y

$\therefore$ AB = $2\sqrt{ab}$

Similarly $AM = 2\sqrt{ac}$ and

$BM = 2\sqrt{bc}$

$\therefore$ $2\sqrt{ab}=2\sqrt{ac}+2\sqrt{bc}$

[$\because$ AB = AM + BM]

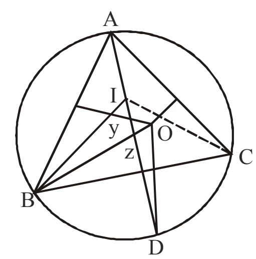
$\Rightarrow \sqrt{ab}=\sqrt{ac}+\sqrt{bc}$

on dividing both sides by $\sqrt{abc}$

$\frac{1}{\sqrt{c}}=\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{a}}$

$\Rightarrow \frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{c}}$

264.(a)



$\because$ $\angle$BOD = Z

$\therefore$ $\angle$BAI = $\frac{z}{2}$

[$\because$ angle made by arc are at the circumferences is half of the angle made by the arc at the centre.

$\angle$ABI = $\angle$IBC = $\frac{X}{2}$

[$\because$ I is the incentre]

Now, In ΔABI,

$\angle$BID = y (exterior angle)

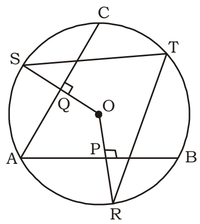
$\therefore$ y = $\angle$BAI + $\angle$ABI

$=\frac{Z}{2}+\frac{X}{2}=\frac{X+Z}{2}$

$\Rightarrow$ x + z = 2y

$\therefore$ $\frac{x+z}{3y}=\frac{2y}{3y}=\frac{2}{3}$

265.(b)



$\angle OQA = \angle OPA = 90°$

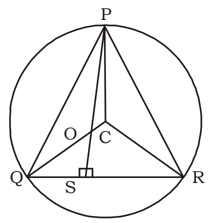
$\angle QOP + \angle QAP = 180°$

$\Rightarrow \angle QOP = 180° - 32° = 148°$

$\angle QOP = \angle SOR = 2\angle STR$

$\therefore \angle RTS = \frac{148}{2} = 74°$

266.(b)



$\angle PQS = 60°$

$\angle QCR = 130°$

$\therefore \angle QPR = \frac{1}{2} \times 130° = 65°$

In $\triangle QSP$,

$\angle PQS = 60°$

$\angle PSQ = 90°$

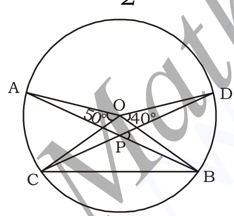
$\angle QPS = 180° - (60° + 90°) = 30°$

$\angle RPS = \angle QPR - \angle QPS = 65° - 30° = 35°$

267.(c) Join BC

$\therefore \angle BCP = \frac{1}{2}\angle BOD = 20°$

and $\angle CBP = \frac{1}{2}\angle AOC = 25°$

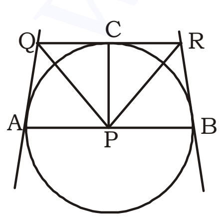


In $\triangle PBC$,

$\angle BPD = \angle BCP + \angle CBP$

$= 20° + 25° = 45°$

268.(c)



In ΔPCR and ΔRBP,

PC = PB (radii)

RC = RB

PR is common.

$\therefore \Delta PCR \cong \Delta RPB$

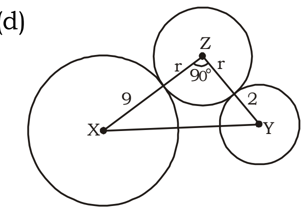
Similarly,

$\angle CPQ \cong \angle QPA$

$\therefore \angle QPR = 90°$

Because $\angle APB = 180°$

269.(d)



$\angle XZY = 90°$

XZ = (9 + r)cm,

YZ = (r + 2) cm

XY = 17 cm

$\therefore XY^2 = XZ^2 + ZY^2$

$\Rightarrow 17^2 = (9 + r)^2 + (r + 2)^2$

$\Rightarrow 289 = 81 + 18r + r^2 + r^2 + 4r + 4$

$\Rightarrow 2r^2 + 22r - 204 = 0$

$\Rightarrow r^2 + 11r - 102 = 0$

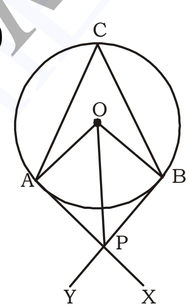
$\Rightarrow r^2 + 17r - 6r - 102 = 0$

$\Rightarrow r(r + 17) - 6(r + 17) = 0$

$\Rightarrow (r-6)(r + 17) = 0$

$\Rightarrow r = 6$ cm

270.(a)



$\angle ACB = 65°$

$\angle AOB = 2 \times 65° = 130°$

$\angle OAP = 90°$, $\angle AOP = 65°$

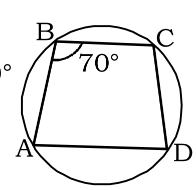
$\angle APO = 180° - 90° - 65° = 25°$

271.(b) According to question

Given:

$\angle ABC = 70°$

AD||BC



$\therefore \angle BAD = 180 - 70° = 110°$

($\therefore$ Corresponding angles between two parallel line sums to 180°)

$\angle BCD = 180 - 110° = 70°$

Note: In trapezium sum of opposite angles are 180°:

Alternate

In cyclic trapezium

$\angle A = \angle D$

and $\angle B = \angle C$

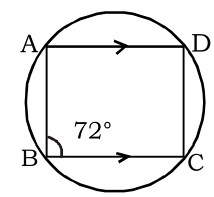
$\therefore \angle BCD = \angle ABC = 70°$

272.(d) According to question

Given:

AD||BC

$\angle ABC = 72°$



$\angle ABC + \angle CDA = 180°$

$\angle CDA = 180 - 72 = 108°$

AD||BC

$\therefore \angle ADC + \angle BCD = 180°$

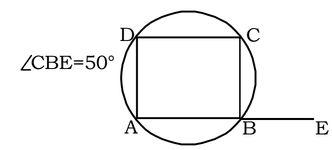
($\therefore$ Sum of corresponding angle of parallel line is 180°)

$\angle BCD = 180° - 108°$

$\angle BCD = 72°$

273.(c) According to question,

Given:



$\angle ABC + \angle CBE = 180°$

$\angle ABC = 180° - 50°$

$\angle ABC = 130°$

In cyclic quadrilateral, sum of opposite angles is 180°

$\angle CDA = 180° - 130°$

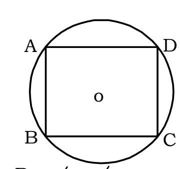
$\angle CDA = 50°$

274.(d) According to question

ABCD is a cyclic parallelogram

In a cyclic quadrilateral, sum of opposite angle is 180°

But In cyclic parallelogram opposite angle is same



But $\angle B + \angle D = 180°$

$\angle B + \angle D = 180°$

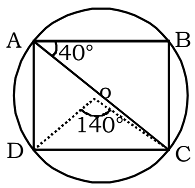
$2\angle B = 180°$

$\angle B = \dfrac{180°}{2}$

$\angle B = 90°$

275. (a) According to question

ABCD is a cyclic quadrilateral



$\angle CAD = \dfrac{1}{2}\angle COD$

(The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle)

$\angle CAD = \dfrac{1}{2} \times 140°$

$\angle CAD = 70°$

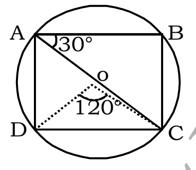
$\therefore\ \angle DAB = 70 + 40 = 110°$

In cyclic quadrilateral sum of opposite angles are 180°

$\angle A + \angle C = 180°$

$\angle C = 180° - 110° = 70°$

276. (b) According to question, ABCD is cyclic quadrilateral with centre'O'.



Given:

$\angle COD = 120°$

$\angle BAC = 30°$

$\angle BCD = ?$

$\angle CAD = \dfrac{1}{2}\angle COD$

$\angle CAD = \dfrac{1}{2} \times 120° = 60°$

$\therefore\ \angle BAD = 30° + 60° = 90°$

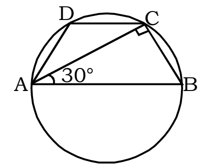
Note: In cyclic quadrilateral sum of opposite angle is 180°

$\angle BAD + \angle BCD = 180°$

$\angle BCD = 180° - 90°$

$\angle BCD = 90°$

277. (b) According to question



Given: AB is a diameter

$\angle CAB = 30°$

As we know that

$\angle ACB = 90°$

$\therefore\ \angle ACB + \angle CAB + \angle CBA = 180°$

$\angle CBA = 180° - 90° - 30°$

$\angle CBA = 60°$

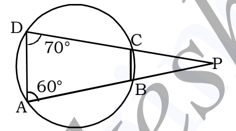
Note: In a cyclic trapezium sum of opposite angle is 180°

$\therefore\ \angle D + \angle B = 180°$

$\angle D = 180° - 60°$

$\angle D = 120°$

278. (a) According to question



$\angle ADC = 70°$

$\angle ABC = 180° - 70° = 110°$

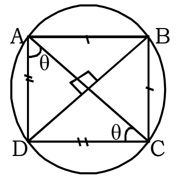
$\Rightarrow \angle PBC = 70°$

$\angle BCD = 180° - 60° = 120°$

$\Rightarrow\ \angle PCB = 60°$

$\therefore \angle PBC + \angle PCB = 70° + 60° = 130°$

279. (c) According to question



In $\triangle ADC$

$\angle A + \angle D + \angle C = 180°$

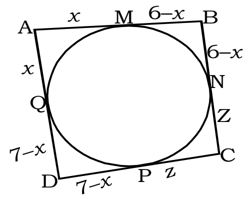
$\angle D = 180° - 2\theta$

$\angle B + \angle D = 180°$

$180° - 2\theta + \angle B = 180°$

$\angle B = 2\theta$

280. (a) According to question



We know tangents drawn to circle from same external point are equal

$\Rightarrow$ AM = AQ = x

$\therefore$ MB = BN = 6 – x

QD = DP = 7 – x

Let NC = PC = z

Now 7 – x + z = 5 (consider side DC)

–x + z = – 2 ....(i)

BC = 6 –x + z ....(ii)

Put the value of equation (i) in equation (ii)

BC = 6 – 2

BC = 4 cm

Alternate:-

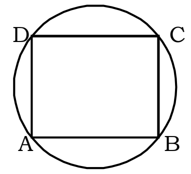
AB + CD = BC + AD

6 + 5 = BC + 7

11 – 7 = BC

4 cm = BC

281. (b) According to question



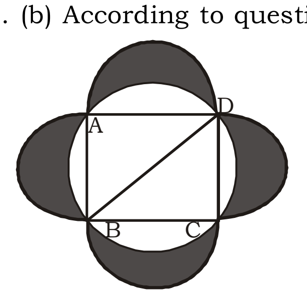
ABCD is a cyclic quadrilateral

$\therefore\ \angle A + \angle C = 180°$

$\angle B + \angle D = 180°$

$\angle A + \angle B + \angle C + \angle D = 360°$

282. (b) According to question



BD = 2 units

$\therefore$ AB = $\sqrt{2}$ units

Area of square = 2 units

Area of four semicircles

$= 4 \times \dfrac{\pi r^2}{2} = 2\pi r^2$

$= 2\pi\left(\dfrac{\sqrt{2}}{2}\right)^2 = 2\pi\left(\dfrac{2}{4}\right) = \pi$ units

Area of circle = πr^2

$= \pi\, 1^2 = \pi$

Required area of shaded portion = Area of square – Area of circle + area of semicircle

$= 2 + \pi - \pi$

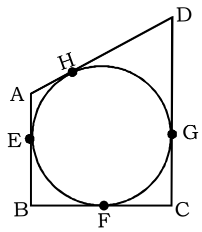
= 2 sq. units

Alternate :

In such type of questions the area of the shaded portion is equal to the area of the regular figure on which the semi-circles are made.

In this case the area of the square.

283. (d) According to questions



AE = AH ....(i)

BE = BF ....(ii)

DG = DH .....(iii)

GC = FC ......(iv)

Add equation (i) ,(ii),(iii) and (iV)

AE + BE + DG + GC = AH +BF + DH + FC

AB + CD = AD + BC

$\therefore$ 6 + 3 = AD + 7.5

AD = 9 – 7.5 = 1.5 cm

Alternate:-

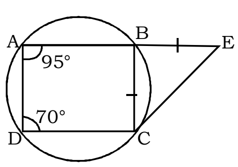
AB + CD = DA + BC

6 + 3 = 7.5 + DA

DA = 1.5 cm

284. (a) According to question

Given:



BE = BC

$\angle ADC = 70°$, $\angle BAD = 95°$

$\angle DCE = ?$

In cyclic quadrilateral sum of opposite angle is 180°

$\therefore \angle BCD = 180° - 95° = 85°$

$\angle ABC = 180° - 70° = 110°$

$\therefore \angle EBC = 180° - 110° = 70°$

BE = BC

$\therefore \angle BCE = \angle BEC$

In $\triangle BCE$

$\angle BCE + \angle BEC + \angle EBC = 180°$

$2\angle BCE = 180° - 70°$

$\angle BCE = 55°$

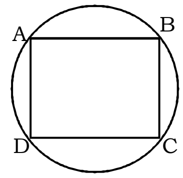
$\angle DCE = \angle BCE + \angle BCD$

$= 55° + 85° = 140°$

285. (d) In a cyclic quadrilateral sum of opposite angle is 180°

$\therefore \angle A + \angle C = \angle B + \angle D = 180°$

286. (b) According to question



$\angle A = 4x°$ $\angle B = 7x°$

$\angle C = 5y°$ $\angle D = y°$

As we know that in a cyclic quadrilateral sum of opposite angle is 180°

$\angle A + \angle C = 180°$

$4x° + 5y° = 180°$ ......(i)

$\angle B + \angle D = 180°$

$7x° + y° = 180°$ .... (ii)

From equation (i) and (ii)

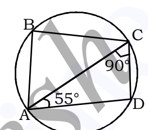
4x + 5y = 7x +y

4y = 3x

$$\frac{x}{y} = \frac{4}{3}$$

287. (c) According to question

Given :



$\angle DAC = 55°$

$\angle DCA = 90°$ [Semi-circle]

In $\triangle CAD$

$\angle DAC + \angle DCA + \angle CDA = 180°$

$\angle CDA = 180° - 90° - 55°$

$= 35°$

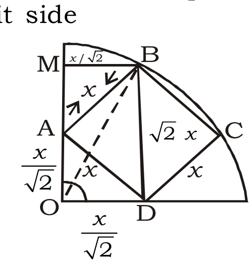
As we know that in a cyclic quadrilateral sum of opposite angle is 180°

$\therefore \angle D + \angle B = 180°$

$\angle B = 180° - 35°$

$\angle B = 145°$

288. (c)Let ABCD is a square of x unit side



Then $\angle AOD = 90°$

then $OD = \frac{x}{\sqrt{2}}$

diagonal of square ABCD

$= \sqrt{2}\,x$

Line MB inscribed || OD

i.e $OD = MB = \frac{x}{\sqrt{2}}$

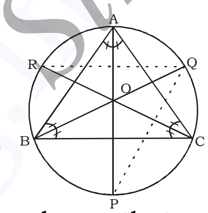
$\Rightarrow$ then MBDO will be a Rectangle become

$MB \parallel OD, MB = OD = x/\sqrt{2}$

$BD \parallel MO, MO = BD = \sqrt{2}\,x$

$$R = \sqrt{\left(\frac{x}{\sqrt{2}}\right)^2 + \left(\sqrt{2}\,x\right)^2} = \frac{\sqrt{5}x}{\sqrt{2}} \text{ Ans.}$$

289. (a)



As we know that

Angle subtended by the chord at the centre is double to the angle subtended by the same chord at the circle

$\therefore \angle BOP = 2\angle BAP$

$$2 \times \frac{\angle A}{2} = \angle A$$

Similarly $\angle BOR = 2\angle BCR$

$= 2\times\angle C/2$

$= \angle C$

Therefore, $\angle RQP = 1/2\ (\angle ROB + \angle BOP) = 1/2\ (\angle C + \angle A)$

And also we know that

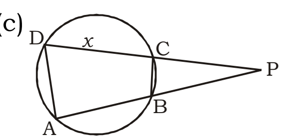
$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle C = 180° - \angle B$

$\angle RQP = 1/2\ (180 - \angle B)$

$$\angle RQP = 90 - \frac{\angle B}{2}$$

290. (c)



$\Rightarrow$ PA.PB = PC.PD

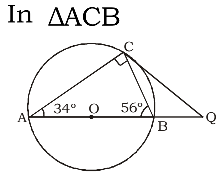
$\Rightarrow$ 8 × 6 = 4 × (PC + CD)

$\Rightarrow$ 48 = 4 (4 + x)

$\Rightarrow$ 12 = 4 + x

$x = 8$, PD = 12

291. (a) In ΔACB



$\angle ACB = 90°$

(Angle formed by semicircle is 90°)

$\angle ACB + \angle CAB + \angle CBA = 180°$

$90° + 34° + \angle CBA = 180°$

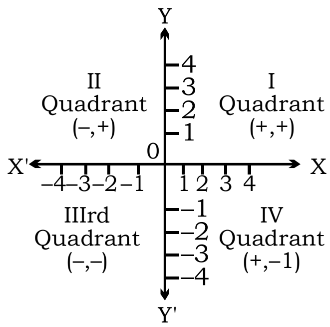
$\angle CBA = 56°$

CHAPTER 25

CO-ORDINATE GEOMETRY

From the SSC point of view, Co-Ordinate Gemoetry is an important Chapter. From which generally 3 to 4 questions are asked in every exam.

Co-ordinates are a set of values that show an exact position. On maps and graphs it is common to have a pair of numbers to show where a point is; the first number shows the distance along and the second number shows the distance up or down.



Region	Quad-rant	Nature of X & Y	Sings of co-ordin-ate
XOY	I	$x>0, y>0$	(+,+)
YOX'	II	$x<0, y>0$	(–,+)
X'OY'	III	$x<0, y<0$	(–,–)
Y'OX'	IV	$x>0, y<0$	(+,–)

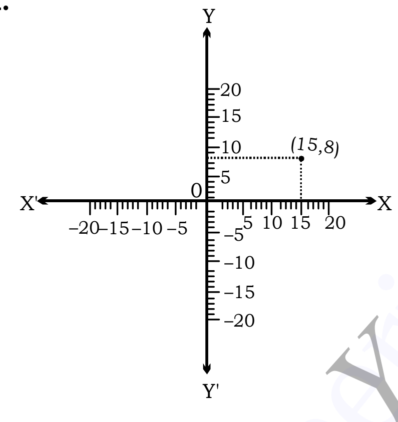
CARTESIAN CO-ORDINATE SYSTEM

• **Rectangular Co-ordinate System :** Let X'OX and Y'OY be two mutually perpendicular lines through any point O in the plane of the paper. Point O is known as the origin. The line X'OX is called the x-axis or axis of x; the line Y'OY is known as the y-axis or axis of y, and the two lines taken together are called the co-ordinates axes or the axes of co-ordinates.

Ex.1 The point (9,6) is 9 units along, 6 units up if it is (9 ,– 6) then – 6 unit down.

Ex.2 Mark the point (15,8) on the Graph?

Sol.



$\therefore$ (15,8) lies in Ist quadrant. In first quadrant x and y is always greater than'0'.

Ex.3 In which quadrants do the given points lie ?

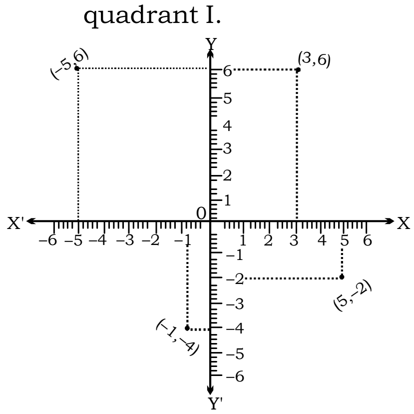
(a) (5,–2) (b) (–5,6)
(c) (–1,–4) (d) (3,6)

Sol. (a) Point of the type (+, –) lie in the 4th quadrant.
Hence, the point (5, – 2) lies in quadrant IV.

(b) Point of the type (–, +) lie in the 2nd quadrant.
Hence, the point (–5,6) lies in quadrant II. rant.

(c) Point of the type (–,–) lie in the 3rd quadrant.
Hence, the point (–1,–4) lies in quadrant III.

(d) Points of the type (+,+) lie in the 1st quadrant.
Hence, the point (3,6) lies in quadrant I.



Cartesian Co-ordinates

Using cartesian Co-ordinates you marks a point on a graph how for along and how for up it is.

X axis and Y Axis:-

→ The X axis runs horizontally through Zero

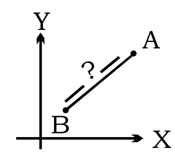
→ The Y axis runs vertically through zero

→ Axis is the reference line from which distance are measured.

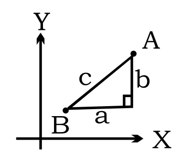
→ The horizontal "x" value in a pair at coordinates is known as Abscissa.

→ The vertical "Y" value in a pair of Co-ordinates is known as ordinate.

1. Distance Formula
Distance between 2 Points:-



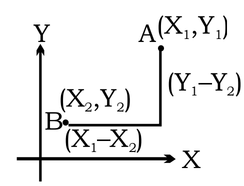
How to calculate the distance between A and B



We can extend Line down from A and along from B to make a right angle.

Now with the help from phythagoras theorem

$a^2 + b^2 = c^2$



X_1 means the X - Co-ordinate of Point A

Y_1 means the Y- Co-ordinate of point A

Horizontal distance a is $(X_1 - X_2)$

Vertical distance b is $(Y_1 - Y_2)$

Now we can solve for C the distance between the points is

$C^2 = a^2 + b^2$

$C^2 = (X_1 - X_2)^2 + (Y_1 - Y_2)^2$

$C = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Ex.4 Find the distance between the point (9,7) and (3,2) ?

$(X_1, Y_1) = (9,7)$

$(X_2, Y_2) = (3,2)$

$= \sqrt{(9-3)^2 + (7-2)^2}$

$\Rightarrow \sqrt{36+25} = \sqrt{61}$

Ex. 5 Find the distance between the point (5,2) and (3,4)

Sol. $C = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(5-3)^2 + (2-4)^2}$

$= \sqrt{4+4} = 2\sqrt{2}$

Ex.6 Find the distance between points (–2,5) and (6, –1)

Sol. $C = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(-2-6)^2 + (5-(-1))^2}$

$= \sqrt{100} = 10$

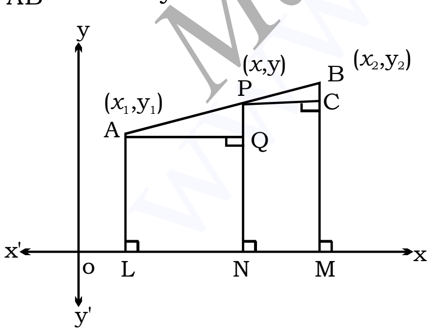
Sectional Formula:- This formula tells us the coordinate of a point $P(x,y)$ which divides a line Segment $\overline{AB}$ where $A=(x_1,y_1)$ and $B = (x_2,y_2)$ in the Ratio m : n internally

$$x = \frac{(mx_2+nx_1)}{(m+n)};$$

$$y = \frac{(my_2+ny_1)}{(m+n)}$$

Proof of Internal division:-

Consider Line Segment $\overline{AB}$ joining two points $A = (x_1,y_1)$ and $B = (x_2,y_2)$ and assume that point $P = (x,y)$ divide $\overline{AB}$ internally in the Ratio of m : n.



From the figure we get

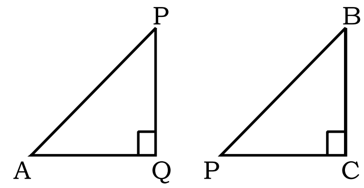
$\angle PAQ = \angle BPC$

$\angle AQP = \angle BCP = 90°$

$\angle APQ = \angle PBC$

So, $\triangle APQ \sim \triangle PBC$.

Since the triangle are Similar, the ratio of their sides are also equal



$$\frac{AQ}{PC} = \frac{AP}{PB}$$

$$\frac{x - x_1}{x_2 - x} = \frac{m}{n}$$

$nx - nx_1 = mx_2 - mx$

$nx + mx = mx_2 + nx_1$

$x(n+m) = nx_1 + mx_2$

$$x = \frac{mx_2 + nx_1}{m+n} \quad \text{......(i)}$$

Similary for y

$$\frac{PQ}{BC} = \frac{m}{n}$$

$$\frac{y-y_1}{y_2 - y} = \frac{m}{n}$$

$$y = \frac{my_2 + ny_1}{m+n} \quad \text{.....(ii)}$$

From (1) and (2)

$$P = (x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Ex. 7 Find the point which divides the line Segment joining (2,5) and (1,2) in the ratio 2 : 1 Internally

Sol. $P(x) = \frac{mx_2 + nx_1}{m+n} = \frac{2.1+1.2}{2+1} = \frac{4}{3}$

$P(y) = \frac{my_2 + ny_1}{m+n} = \frac{2.2+1.5}{2+1} = 3$

$P(x,y) = \left(\frac{4}{3}, 3\right)$

Ex. 8 Find the point which divides the line Segment joining (3,7) and (1,–2) in the ratio 3 : 2 Internally

Sol. $P(x) = \frac{mx_2 + nx_1}{m+n}$

$= \frac{3 \times 1 + 2 \times 3}{3+2}$

$= \frac{3+6}{5} = \frac{9}{5}$

$P(y) = \frac{my_2 + ny_1}{m+n}$

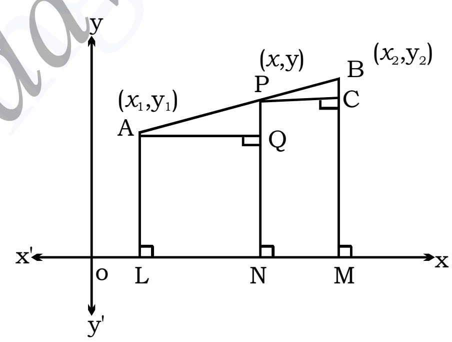
$= \frac{-3 \times 2 + 2 \times 7}{3+2} = \frac{8}{5}$

$P(x,y) = \left(\frac{9}{5}, \frac{8}{5}\right)$

Proof of External Division

Consider Line Segment $\overline{AB}$ joining two points $A = (x_1,y_1)$ and $B = (x_2,y_2)$ and assume that point $P = (x,y)$ divide $\overline{AB}$ internally in the Ratio of m : n.

$$(x, y) = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$



From the figure we get

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$\angle APQ = \angle PBC$

So,

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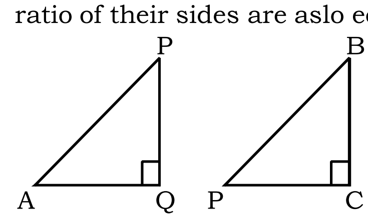
Since the triangle are Similar, the ratio of their sides are aslo equal



Since the $\triangle$ are Similar, the ratio of the Sides

$$\frac{AQ}{PC} = \frac{AP}{PB}$$

$$\frac{x - x_1}{x - x_2} = \frac{m}{n}$$

$nx - nx_1 = mx - mx_2$

$nx - mx = nx_1 - mx_2$

$x(m - n) = mx_2 - nx_1$

$$x = \frac{mx_2 - nx_1}{m-n} \quad \text{......(iii)}$$

Similary for y

$$\frac{PQ}{BC} = \frac{m}{n}; \quad \frac{y-y_1}{y-y_2} = \frac{m}{n}$$

$$y = \frac{my_2 - ny_1}{m-n} \qquad .....(iv)$$

From (iii) and (iv)

P = (x,y)

$$= \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

Ex9. Find the coordinate of the point P when it divides the line AB Externally in the ratio of 3:1 where A = (9, 4) and B = (5, 2)

Sol. $P(x) = \frac{mx_2 - nx_1}{m-n}$

$$= \frac{3\times5 - 1\times9}{3-1} = \frac{6}{2} = 3$$

$$P(y) = \frac{my_2 - ny_1}{m-n} = \frac{3\times2 - 1\times4}{3-1}$$

$$= \frac{2}{2} = 1$$

$P(x,y) = (3,1)$

Ex10. Find the co-ordinates of the point which divides the joint of the points (2,4) and (6,8) externally in the ratio 5 : 3.

Sol. $P(x) = \frac{5\times6 - 3\times2}{5-3} = \frac{24}{2} = 12$

$$P(y) = \frac{5\times8 - 3\times4}{5-3} = \frac{28}{2} = 14$$

Mid-Point:-The Co-ordinates of the mid-point of the Line Segment joining the two points A(x_1, y_1) and B(x_2,y_2) are given by

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

The midpoint is halfway between the two end points

Ex11. What is the mid-point of the points (–3,5), (8,–1)

Sol. $M = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

$$= \frac{(-3)+8}{2}, \frac{5+(-1)}{2}$$

$$\Rightarrow \frac{5}{2}, 2 \Rightarrow 2.5, 2$$

Ex12. What is the mid-point of the points (6,–8), (10,–4)

Sol. $M = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

$$= \frac{6+10}{2}, \frac{-8-4}{2}$$

$\Rightarrow 8, -6$

Division by Axes : If P (x_1, y_1) and Q (x_2, y_2), then PQ is divided by

(i) x - axis in the ratio = $\frac{-y_1}{y_2}$

(ii) y - axis in the ratio = $-\frac{x_1}{x_2}$

Division by a Line : A line $ax + by + c = 0$ divides PQ in the ratio

$$= -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$$

Area of a triangle : The area of a triangle ABC whose vertices are A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) is denoted by Δ.

$$\therefore \quad \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\begin{matrix} x_1(y_2 - y_3) + x_2(y_3 - y_1) \\ + x_3(y_1 - y_2) \end{matrix}\right]$$

Area of Polygon : The area of the polygon whose vertices are (x_1, y_1), (x_2, y_2),……(x_n, y_n) is -

$$= \frac{1}{2}\left[\begin{matrix} (x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + \ldots \\ \ldots\ldots + (x_ny_1 - x_1y_n) \end{matrix}\right]$$

Ex.13. The area of the triangle whose vertices are P (3, 5), Q(–2, 8) and R(4, 7), (in square units) is :

Sol.

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3 (y_1 - y_2)\right]$$

$$\frac{1}{2}\left[3(8-7) + (-2)(7-5) + 4(5-8)\right]$$

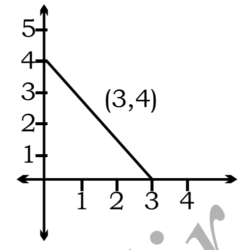
$$= \frac{1}{2}(-13) = \frac{13}{2} \text{ Square units.}$$

Ex14. Find the area of triangle which formed by $4x + 3y = 12$ on x and y Axis.

Sol. $4x + 3y = 12$

If $x = 0$; $y = 4$

$x = 3$; $y = 0$



Area of Triangle

$$= \frac{1}{2} \times 4 \times 3 = 6 \text{ Square units.}$$

- **Centroid :** If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of a triangle, then the co-ordinates of its centroid are -

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Ex15:- Find the centroid of a triangle whose vertices are (10,7), (9,5) and (5,6)

Sol. $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$$= \frac{10+9+5}{3}, \frac{7+5+6}{3} = 8, 6$$

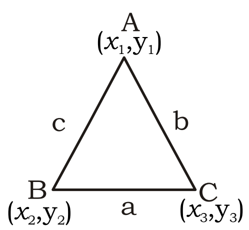
Ex16:- Find the centroid of a triangle whose verties are (5, 3), (4, 6) and (8, 2)

Sol. Centroid of Δ (x,y)

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \frac{5+4+8}{3}, \frac{3+6+2}{3} = \frac{17}{3}, \frac{11}{3}$$

- **Incentre :** If A (x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of a ΔABC, BC = a, CA = b and AB = c, then the co-ordinates of its incentre are

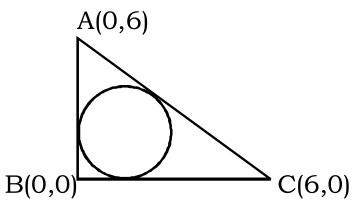


$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

Ex.17 Find the in-centre of the right angled isosceles triangle having one vertex at the origin and having the other two vertics at (6,0) and (0,6).

Sol. AS it is an isosceles triangle the length of the two sides AB and BC of the triangle is 6 units and the length of the third side is $(6^2 + 6^2)^{1/2}$.

Hence a = c = 6, b = $6\sqrt{2}$



In- centre will be at

$$\frac{(6.0+6\sqrt{2}.0+6.6)}{(6+6+6\sqrt{2})} \cdot \frac{(6.6+6\sqrt{2}.0+6.0)}{(6+6+6\sqrt{2})}$$

$$= \frac{36}{12+6\sqrt{2}} \cdot \frac{36}{12+6\sqrt{2}}$$

Note:-

- If the triangle is equilateral, then centroid, incentre, orthocentre, circumcentre coincides.
- Orthocentre, centroid and circumcentre are always collinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.
- In an isosceles triangle centroid, orthocentre, incentre, circumcentre lies on the same line.
- Incentre divides the angles bisectors in the ratio (b + c) : a, (c + a) : b, (a + b) : c.
- Area of the triangle formed by co- ordinate axes and the line a x + by + c = 0 is $\frac{c^2}{2ab}$

Straight Line : A straight line is a curve such that every point on the line segment joining any two points on it lies on it.

Different Forms of the Equations of a Straight Line :

(a) **General Form :** The general Form of the equation of a straight line is $ax + by + c = 0$

(First degree equation in x and y). Where a, b and c are real constants and a, b are not simultaneously equal to zero.

In this equation, slope of the line is given by $\left(-\frac{a}{b}\right)$

The general form is also given by $y = mx + c$; where m is the slope and c is the intercept on y-axis.

(b) **Line Parallel to the X-axis :** The equation of a straight line to the x -axis and at a distance b from it, is given by $y = b$

Obviously, the equation of the x-axis is $y = 0$

(c) **Line Parallel to Y-axis :** The equation of a straight line parallel to the y-axis and at a distance a from it given by x = a

obviously, the equation of y-axis is $x = 0$

(d) **Slope Intercept Form :** The equation of a straight line passing through the point A(x_1,y_1) and having a slope m is given by

$(y - y_1) = m\ (x - x_1)$

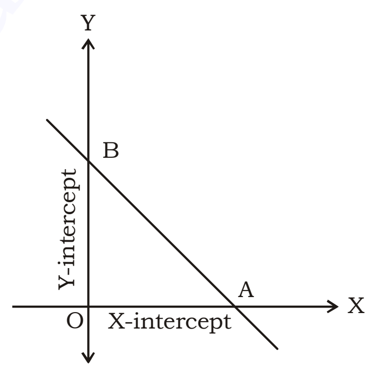
(e) **Two Points Form :** The equation of a straight line passing through two points A(x_1, y_1) and B(x_2, y_2) is given by

$$(y - y_1) = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$$

Its slope (m) = $\frac{y_2-y_1}{x_2-x_1}$

(f) **Intercept Form :** The equation of a straight line making intercepts a and b on the axes of x and y respectively is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$



Ex18. Find the point where the straight line $2x - 3y = 12$ cuts x–axis and y – axis. Also find the length intercepted by the line between the axis.

Sol. $2x - 3y = 12$

or $\frac{2x}{12} - \frac{3x}{12} = 1$

or, $\frac{x}{6} + \frac{y}{(-4)} = 1$

Thus straight line cuts x -axis at (6,0) and y-axis at (0,–4)

Length intercepted between axis

$= \sqrt{6^2+(-4)^2} = \sqrt{36+16}$

$= \sqrt{52} = 2\sqrt{13}$

Ex19. Find the equation of line whose ending points are (4,6) and (10,8)

Sol. $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$(x_1, y_1) = 4,6$

$x_2, y_2 = 10,8$

$y - 6 = \left(\frac{8-6}{10-4}\right)(x - 4)$

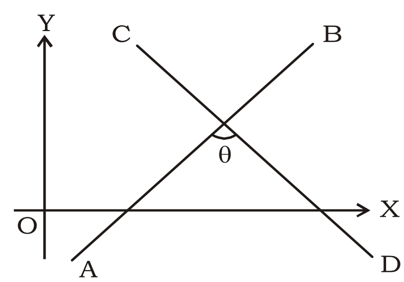
$y - 6 = \left(\frac{2}{6}\right)(x - 4)$

$6y - 36 = 2x - 8$

Equation of Given = $2x - 6y + 28 = 0$

Angle between two lines

$$\tan\theta = \pm\left(\frac{m_2 - m_1}{1+m_1m_2}\right)$$



- **Condition of Parallellism of lines:** If the slopes of two lines is m_1 and m_2 and if they are parallel, then,

$$\boxed{m_1 = m_2}$$

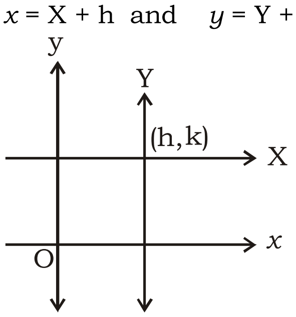
- **Length of Perpendicular it y or Distance of a Point from a Line:** The length of perpendicular from a given point (x_1, y_1) to a line $ax + by + c = 0$ is :

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

Note: The length of Perpendicular from the origin to the line $ax + by + c = 0$ is given by

$$\frac{|c|}{\sqrt{a^2+b^2}}$$

- **Distance between two Parallel Lines :** If two lines are parallel, the distance between them will always be the same.
  When two straight lines are parallel whose equations are $ax_1 + by_1 + c_1 = 0$ and $ax_2 + by_2 + c_2 = 0$, then the distance between them is given by $\frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$.
- **Changes of Axes :** If origin (0, 0) is shifted to (h, k) then the coordinates of the point (x, y) referred to the old axes and (X, Y) referred to the new axes can be related with the relation
  $x = X + h$ and $y = Y + k$



- **Point of Intersection of Two Lines:** Point of intersection of two lines can be obtained by solving the equations as simultaneous equations.
- If the given equations of straight line are
  $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then
  (i) The angle between the lines 'θ' is given by
  $$\tan\theta = \frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}$$
  (ii) If the lines are parallel, then
  $$a_2b_1 - a_1b_2 = 0 \text{ or } \frac{a_1}{a_2} = \frac{b_1}{b_2}$$
  (iii) If the lines are perpendicular, then
  $$a_1a_2 + b_1b_2 = 0$$
  (iv) Coincident : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- Angle between lines
  $x\cos\alpha + y\sin\alpha = P_1$ and
  $x\cos\beta + y\sin\beta = P_2$ is
  $|\alpha - \beta|$

Ex20. Find the perpendicular distance from a point (2,3) to the line $3x + 4y + 7 = 0$

Sol. $P = \left|\frac{3(2)+4(3)+7}{\sqrt{3^2+4^2}}\right|$

$= \left|\frac{6+12+7}{5}\right| = \frac{25}{5} = 5$

Ex21. Find the angle between two lines $x - 3y + 13 = 0$ and $x + 2y - 111 = 0$.

Sol. $3y = x + 13 \Rightarrow y = \frac{x}{3} + \frac{13}{3}$ then $m_1 = \frac{1}{3}$

$2y = -x + 111$

$\Rightarrow y = \frac{-1}{2}x + \frac{111}{2}$ then $m_2 = -\frac{1}{2}$

$$\tan\alpha = \left|\frac{\frac{1}{3} - \left(-\frac{1}{2}\right)}{1 + \frac{1}{3}\left(\frac{-1}{2}\right)}\right| = \left|\frac{\frac{5}{6}}{1 - \frac{1}{6}}\right| = 1$$

$\therefore \alpha = 45°$

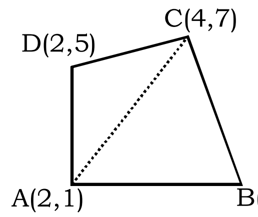
Quadrilateral

There is a quadrilateral whose vertices are A $(x_1\, y_1)$ B $(x_2 y_2)$, C$(x_3 y_3)$ and D$(x_4\, y_4)$ then quadrilateral will be Rhombus, Square, Rectangle and Parallelogram.

	Name of the figure	Conditions
1.	Square	Four sides are equal and the diagonals are also equal
2.	Rhombus	Four sides are equal.
3.	Rectangle	Opposite sides are equal and diagonals are also equal
4.	Parallelogram	Opposite sides are equal
5.	Parallelogram but not a rectangle	Opposite sides are equal but the diagonals are not equal
6.	Rhombus but not a square	All sides are equal but the diagonals are not equal

Ex.22 Find the area quadrilateral formed by joining points (2,1), (4,1), (2,5) and (4,7)

Sol.



Area of triangle ACD

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[2(7 - 5) + 4(5 - 1) + 2(1 - 7)] = 4$

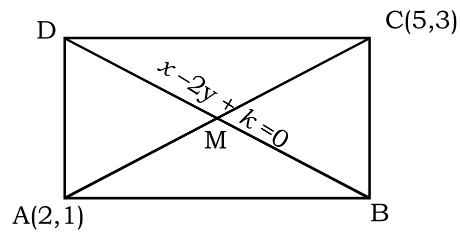
Area of triangle ABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[2(1 - 7) + 4(7 - 1) + 4(1 - 1)] = 6$

Hence, Area of quadrilateral ABCD = Area of triangle ADC + Area of triantgle ABC = 4 + 6 = 10 sq. unit

Ex.23. If two opposite vertex of a rectangle are (2,1) and (5,3) and the equation of other diagonal is $x - 2y + k = 0$ find k.



M is the mid-point of diagonal AC & BD.

Co-ordinate of M

$= \left(\frac{2+5}{2}, \frac{3+1}{2}\right) = (3.5,2)$

Point M is also situated on line $x - 2y + k = 0$

So, $3.5 - 2 \times 2 + k = 0$

$\Rightarrow k = \frac{1}{2}$

Conditions for Solvability

The system of equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ has :

(i) a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) an infinite number of solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iii) no solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Homogenous System of Equations:

The system of equations $a_1x + b_1y = 0$; $a_2x + b_2y = 0$ has

(i) only solution $x = 0$, $y = 0$, when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) an infinite number of solutions when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

- The graphs of $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + C_2 = 0$ will be :

(i) Parallel, if the system has no Solution;

(ii) Coincident, if the system has infinite number of solutions ;

(iii) Intersecting, if the system has a unique solution.

Ex25. If $9x + 4y = 15$ and $kx + 12y = 45$ are coincide lines find value of k.

Sol. $\frac{9}{k} = \frac{4}{12} = \frac{15}{45}$

$\Rightarrow \frac{1}{3} = \frac{9}{k}$

$\Rightarrow k = 27$

Ex26. If $2x + 3y = 122$ and $4x + ky = 119$ have unique solution.

Sol. For unique solution,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{2}{4} \neq \frac{3}{k}$

$\Rightarrow k \neq 6$

- **Ordered Pair :** A pair of numbers a and b listed in a specific order with a at the first place and b at the second place is called an ordered pair (a, b)

Note that (a, b) $\neq$ (b, a)

Thus (3, 5) is one ordered pair and (5, 3) is another ordered pair.

Example: Draw the lines XOX' and YOY' as axes on the plane of a paper and plot the points given below :

(i) A(3, 7)
(ii) B(–4, 3)
(iii) C(–5, –2)
(iv) D(3, –5)

Sol: Let XOX' and YOY' be the co-ordinate axes.

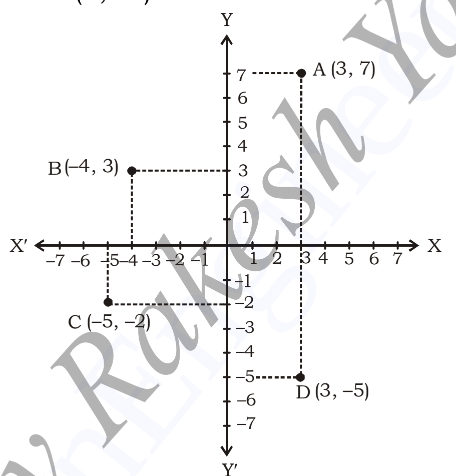
Fix a convenient unit of length and starting from O, mark equal distances on OX, OX', OY and OY'. Use the convention of signs.

(i) Starting from O, take +3 units on the x-axis and then +7 units on the y-axis to obtain the point A(3, 7).

(ii) Starting from O, take – 4 units on the x-axis and then +3 units on the y-axis to obtain the point B (–4, 3).

(iii) Starting from O, take – 5 units on the x-axis and then – 2 units on the y-axis to obtain the point C(–5, –2)

(iv) Starting from O, take + 3 units on the x-axis and then – 5 units on the y-axis to obtain the point C(3, –5)



♦ **Graph of $y = mx + c$**

Example: Draw the graph of the equation $y = 3x + 2$:

Sol. Construct a table and choose simple x values.

x	–2	–1	0	1	2
y					

In order to find the y - values for the table, substitute each x - values into the rule $y = 3x + 2$.

when $x = -2$, $y = 3(-2) + 2$
$= -6 + 2 = -4$

when $x = -1$, $y = 3(-1) + 2$
$= -3 + 2 = -1$

when $x = 0$, $y = 3(0) + 2$
$= 0 + 2 = 2$

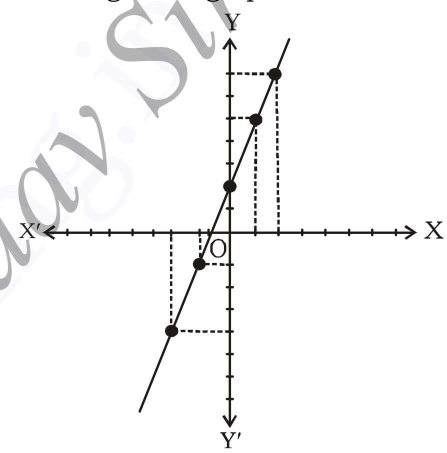
when $x = 1$, $y = 3 \times 1 + 2$
$= 3 + 2 = 5$

when $x = 2$, $y = 3 \times 2 + 2$
$= 6 + 2 = 8$

The table of values obtained after entering the values of y is as follows:

x	– 2	– 1	0	1	2
y	– 4	– 1	2	5	8

Now, Draw a Cartesian plane and plot the points. Then join the points with a ruler to obtain a straight line graph.



- **Solving Simultaneous Linear Equation (Graphical Method) :**

Let the given system of linear equations be

$a_1x + b_1y + c_1 = 0$ ...(i)

$a_2x + b_2y + c_2 = 0$ ...(ii)

On the same graph paper, we draw the graph of each one of the given linear equations.

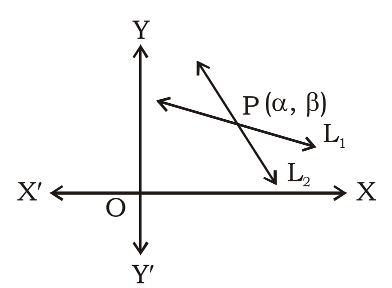
Each such graph is always a straight line.

Let the lines L_1 and L_2 represent the graph of (i) and (ii) respecitvely.

Now, the following cases arise:

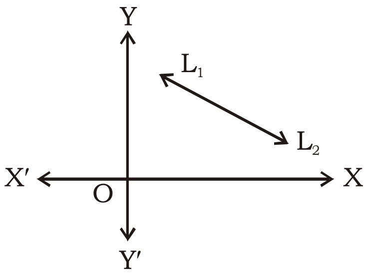
Case–1. When the lines L_1 and L_2 intersect at a point P(α ,β)

Then, $x = \alpha$, $y = \beta$ is the unique solution of the given system of equations.



Case-2 . When the lines L_1 and L_2 are coincident.

Then, given system of equations has infinitely many solutions.



Case-3. When the line L_1 ans L_2 are parallel. In this case, there is no common solution of the given system of equations.

Thus, in this case, the given system is inconsistent.

- **Point of Intersection of Two Lines :** Point of intersection of two lines can be obtained by solving the equations as simultaneous equations.

Inequations

Graph : Let us consider an inequation $ax + by \leq c$.

Step 1. Consider the equation $ax + by = c$. Draw the graph of this equation, which is a line.

In case of strict inequalities draw the line dotted, otherwise mark it thick.

Step 2. Choose a point [if possible (0, 0)], not lying on this line. Substitute its coordinates in the given in equations. If this point satisfies the given in equation, then shade the portion of the plane which contains the choosen point, otherwise shade the portion which does not contain this point.

The shaded portion represents the solution set. The dotted line is not a part of the solution set, while thick line is a part of it.

Example: Graph of the inequation $2x - y \geq 1$?

Sol : Consider the equation

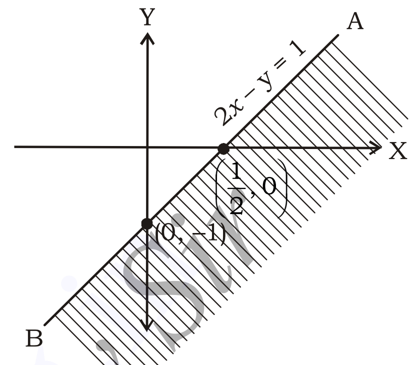
$2x - y = 1$

or $\dfrac{x}{1/2} + \dfrac{y}{-1} = 1$

i.e. x-intercept = 1/2 and y-intercept = –1

i.e. it meets x-axis at (1/2, 0) and y-axis at (0, – 1)

Join these two point with a thick line AB.



Now, consider (0, 0) put (0, 0) in the given inequation.

L.H.S = $2 \times 0 - 0 \neq$ R.H.S

i.e. it does not satisfy $2x - y \geq 1$.

So, shade the portion of the plane not containing (0, 0).

Shaded portion constitutes the solution set.

Note : The area bounded by $|x| + |y| = k$ is $2k^2$.

EXERCISE

1. Find the value of 'A' if the distance between the points (8,A) and (4,3) is 5.
   (a) 6 (b) 0
   (c) Both (a) and (b)
   (d) Not
2. Find the value of C if the distance between the point (C,4) and the origin is 5 units
   (a) 3 (b) –3
   (c) Both a & b (d) Not.
3. Point A(– 3, 2) and B(5,4) are the end points of a line segment, find the Co-Ordinates of the mid points of the line Segment.
   (a) $\left(\frac{3}{2}, 1\right)$ (b) $\left(\frac{2}{3}, 0\right)$
   (c) 1,3 (d) (2,3)
4. Find the area of quadrilateral formed by joining points (4,2),(8,2),(4,10) and (8,14).
   (a) 5 Sq. units (b) 10 Sq. units
   (c) 25 Sq. units (d) 40 sq. units
5. If two opposite vertex of a Rectangle are (4,2) and (10,6) and the equation of other diagonal is $x - 3y + k = 0$. Find the value of k?
   (a) $\frac{1}{2}$ (b) 3 (c) 5 (d) 4
6. Find the Intercepts made by the line $7x + 8y - 56 = 0$. On the axis
   (a) (5,6) (b) (8,7)
   (c) (7,8) (d) None of these
7. Find the slope of line passing through the points (2,8) and (6,9).
   (a) 5 (b) –5
   (c) .25 (d) –.25
8. Find the area of triangle formed by lines $8x - 3y = 24$, x axis and y – axis
   (a) 12 sq. units (b) 6 sq. units
   (c) 18 sq. units (d) 9 sq. units
9. Find the area of triangle formed by lines $-6x + 9y = 36$, x-axis and y-axix
   (a) 6 sq. units (b) 12 sq. units
   (c) 3 sq. units (d) 8 sq. units
10. Find the area of triangle formed by lines $2x - 2y = 10$, $2x + 3y = 10$ and x- axis
    (a) $\frac{75}{6}$ sq. units
    (b) $\frac{125}{6}$ sq. units
    (c) $\frac{-125}{6}$ sq. units
    (d) None of these
11. Find the area of $4x + 3y = 12$; $5x + 7y = 35$. On x-axis
    (a) $\frac{160}{13}$ sq.units
    (b) $\frac{80}{13}$ sq. units
    (c) $\frac{40}{13}$ sq.units
    (d) $\frac{150}{13}$ sq. units

12. Find the area of quadrilateral intercepted by straight lines $4x + 2y = 12$, $8x + 4y = 48$ between the axis.
(a) 36 sq. units (b) 27 sq. units
(c) 18 sq. units (d) 9 sq. units

13. Find the area of quadrilateral intercepted by straight lines $x + y = 2$, $3x + 4y = 24$ between the axis
(a) 22 sq. units (b) 20 sq. units
(c) 18 sq. unit (d) 16 sq. units

14. Find the area of quadrilateral formed by straight lines $x = 1$, $x = 3$, $y = 2$ and $x = y + 3$.
(a) 6 sq. units (b) 12 sq. units
(c) 3 sq. units (d) None of these

15. For what value of k given system of equations $5x + 20y = 11$ and $2x + ky = 17$ are intersecting lines ?
(a) $k \neq 8$ (b) $k \neq 4$
(c) $k \neq 14$ (d) $k \neq 2$

16. For what value of k the lines $4x + ky = 3$ and $3x + 2y = 7$ are perpendicular to each other ?
(a) 6 (b) ± 6
(c) –6 (d) 4

17. For what value of k the lines $(k+1)x + ky = 3$ and $5x - 2y = 7$ are perpendicular to each other ?
(a) $\frac{1}{3}$ (b) $\frac{-5}{3}$ (c) $-\frac{1}{3}$ (d) 5

18. If the points A (4,3) and B (x, 5) are on the circle with centre O (2,3), find the value of x.
(a) 1 (b) 2 (c) 2 (d) –2

19. Find a point on the y-axis which is equidistant from the points A (6, 5) and B (–4, 3)
(a) (0,9) (b) (9,0)
(c) (3,0) (d) (4,0)

20. Find the equation of the line which cuts off intercepts 2 and 3 on the axis.
(a) $9x - 7y = 6$
(b) $3x - 2y = 5$
(c) $4x - 3y = 7$
(d) $3x + 2y = 6$

21. Find the co-ordinates of the centroid of a triangle whose vertices are (0, 6) (8, 12) and (8, 0).
(a) $\left(\frac{16}{3}, 6\right)$ (b) $\left(6, \frac{16}{3}\right)$
(c) (6,5) (d) (6,3)

22. The vertices of a triangle are (3, –5) and (–7,4). If its centroid is (2, –12), find the third vertex.
(a) (10, –35) (b) (–2, 10)
(c) (10, 35) (d) (–3, 10)

23. Rhombus ABCD has A at (2,0) and B at (4,4). If one of the sides of the rhombus lies on the x– axis, what is the area of this rhombus?
(a) 40 sq. units
(b) $8\sqrt{6}$ sq. units
(c) $4\sqrt{5}$ sq. units
(d) $8\sqrt{5}$ sq. units

24. What are the co-ordinates of the incentre of triangle given by (3,4), (3,7) and (7,4) ?
(a) (5,5) (b) (5,4)
(c) (4,5) (d) (5,5)

25. The equation of a line passing through the point (4,4) and cutting off intercepts on the axis whose sum is 18?
(a) $x + 2y - 12 = 0$ but not $2x + y - 12 = 0$
(b) neither $x + 2y - 12 = 0$ nor $2x + y - 12 = 0$
(c) $2x + y - 12 = 0$ but not $x + 2y - 12 = 0$
(d) $x + 2y - 12 = 0$ or $2x + y - 12 = 0$

26. The co-ordinates of the point P which divides the join of A (5, –2) and B (9,6) in the ratio 3: 1 will be –
(a) (4,–7) (b) $\left(\frac{7}{2}, 4\right)$
(c) (8,4) (d) (12,8)

27. If line $x\cos\theta + y\sin\theta = 2$ is perpendicular to the line $x - y = 3$, then what is one of the value of θ?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

28. The foot of the perpendicular drawn from the point (2,–1)to a straight line L is (1,3). The equation of straight line L is–
(a) $x - 4y + 11 = 0$
(b) $x + 4y + 13 = 0$
(c) $4x - y - 1 = 0$
(d) $4x + y - 7 = 0$

29. Find the area of triangle of lines $\frac{x}{6} + \frac{y}{7} = 1$, $-\frac{x}{4} + \frac{y}{7} = 1$ and x-axis.
(a) 30 sq.units (b) 35 sq. units
(c) 70 sq. units (d) 60 sq. units

30. Let the vertices of a triangle ABC be (4,3),(7,–1), (9,3) then the triangle is :
(a) Scalene (b) Isosceles
(c) Equilateral (d) None of these

31. Let the vertices of a triangle ABC be (4,4),(3,5), (–1,–1), then the triangle is :
(a) scalene (b) equilateral
(c) right angled (d) None of these

32. A(–3,2) and B(5,4) are the end points of a line segment, find the coordinates of the mid points of the line segment:
(a) (1,3) (b) (2,3)
(c) (3,2) (d) (4,3)

33. Find the co-ordinates of the point which divides the join of the points (2,4) and (6,8) externally in the ratio 5 : 3.
(a) (5,6) (b) (12,14)
(c) (3,8) (d) (2,7)

34. Find the co-ordinates of the incentre of the triangle whose vertices are the points (4,–2), (5,5) and (–2,4).
(a) $\left(\frac{7}{3}, \frac{2}{3}\right)$ (b) $\left(\frac{5}{2}, \frac{5}{2}\right)$
(c) $\left(\frac{6}{5}, 5\right)$ (d) None of these

35. A(–2,–1), B(1,0),C(4,3) and D(1,2) are the four points of a quadrilateral. The quadilateral is a:
(a) Square
(b) Rhombus
(c) Parallelogram
(d) None of these

36. If A(–2,1),B(2,3) and C(–2,–4) are three points, find the angle between BA and BC.
(a) $\tan^{-1}\left(\frac{3}{5}\right)$ (b) $\tan^{-1}\left(\frac{2}{3}\right)$
(c) $\tan^{-1}\left(\frac{3}{2}\right)$ (d) None of these

37. Find the intercepts made by the line $3x + 4y - 12 = 0$ on the axes:
(a) 2 and3 (b) 4 and3
(c) 3 and5 (d) none of these

38. A firm produces 50 units of a good of ₹ 320 and 80 units for ₹ 380. Supposing that the cost curve is a straight line, estimate the cost of producing 110 units.
(a) ₹ 330 (b) ₹ 1665
(c) ₹ 440 (d) ₹ 365

39. Find the length of the perpendicular from the point (3, –2) to the straight line $12x - 5y + 6 = 0$.

(a) 5 (b) 4 (c) 6 (d) 8

40. Find the equation of the line through the intersection of the lines $3x + 4y = 7$ and $x - y + 2 = 0$ having slope 3.
(a) $4x - 3x + 4y = 7$
(b) $21x - 7y + 16 = 0$
(c) $8x + y + 8 = 0$
(d) none of these

41. A straight line intersects the x-axis at A and the y-axis at B. AB is divided internally at C(8,10) in the ratio 5 : 4. Find the equation of AB.
(a) $x + y = 18$ (b) $x + y + 2 = 0$
(c) $x + y - 2 = 0$ (d) None of these

42. Find the distance between the points (–5,3) and (3,1).
(a) $2\sqrt{7}$ (b) $3\sqrt{14}$
(c) $5\sqrt{17}$ (d) $2\sqrt{17}$

43. The point (6, –3) lies in the quadrant:
(a) First (b) Second
(c) Third (d) Fourth

44. If $x < 0$ and $y > 0$, then the point (x, y) lies in :
(a) quadrant I (b) quadrant II
(c) quadrant III (d) quadrant IV

45. Which of the following points lies on the line $y = 3x + 5$?
(a) (2, 11) (b) (3, 15)
(c) (4, 19) (d) (5, 15)

46. The co-ordinates of a point situated on x-axis at a distance of 7-units from y-axis is :
(a) (0, 7) (b) (7, 0)
(c) (7, 7) (d) (– 7, 7)

47. The co-ordinates of a point below x-axis at a distance of 8 units from x-axis but lying on y-axis is :
(a) (0, 8) (b) (– 8, 0)
(c) (0, – 8) (d) (8, – 8)

48. The point of intersection of the lines $2x + 7y = 1$ and $4x + 5y = 11$ is :
(a) (4, – 1) (b) (2, 3)
(c) (–1, 4) (d) (4, – 2)

49. Which of the following points does not lie on the line $y = 2x + 3$?
(a) (2, 7) (b) (3, 9)
(c) (4, 10) (d) (5, 13)

50. The area of ΔOAB with O(0, 0), A (8, 0) and B(0, 6) is :
(a) 12 sq. units (b) 48 sq. units
(c) 36 sq. units (d) 24 sq. units

51. The line $2x - 3y = 6$ meets y-axis at the point :
(a) (– 2, 0) (b) (0, – 2)
(c) (0, 3) (d) (3, 0)

52. The line $4x + 7y = 12$ meets x-axis at the point :
(a) (3, 1) (b) (0, 3)
(c) (3, 0) (d) (4, 0)

53. Graphical equation of the line $x = 0$ represents :
(a) y-axis (b) x-axis
(c) origin
(d) both x-axis and y-axis

54. The graph of the line $x = 5$ is parallel to :
(a) x-axis (b) y-axis
(c) both x-axis and y-axis
(d) none of these

55. The graph of the line $x = -3$ is parallel to :
(a) x-axis (b) y - axis
(c) both x - axis and y-axis
(d) none of these

56. when we draw the graph of $y = 2$, the line is parallel to :
(a) x - axis (b) y - axis
(c) both x- axis and y - axis
(d) none of these

57. If point $\left(3, \frac{11}{2}\right)$ lies on the graph of the equation $2y = ax + 5$, then a is equal to :
(a) – 2 (b) 4
(c) 2 (d) 3

58. If point $\left(3, \frac{5}{2}\right)$ lies on the graph of the equation $2y = ax - 5$, then a is equal to :
(a) $\frac{10}{3}$ (b) $-\frac{10}{3}$
(c) $\frac{7}{3}$ (d) 0

59. Find the distance between pair of points (8, 0) and (0, 6) :
(a) 9 (b) 8
(c) – 10 (d) 10

60. Distance of the point(7, 24) from the origin is :
(a) 24 (b) 25
(c) – 25 (d) 22

61. When we plot the points (4, 3), (0, 0) and (– 4, –3) on the graph paper, then the graph shows :
(a) straight line (b) curve
(c) zig-zag line (d) None of these

62. The equation of the line which passes through the origin is :
(a) $3x + 4y = 5$ (b) $3x + 4y = 7$
(c) $3y + 4x = 0$ (d) None of these

63. If we plot the points (4, 3), (–3, 3), (–3, –2), (– 4, –2) on the graph paper, the shape formed is:
(a) parallelogram
(b) rectangle
(c) square (d) rhombus

64. If the side of a figure are represented by $x = 2$, $x = -2$, $y = 2$, $y = -2$ then the graph shows :
(a) Rhombus
(b) Rectangle
(c) Parallelogram
(d) Sqaure

65. If a straight line $ax + by = c$ meets x-axis at P and y-axis at Q. Then area of the triangle OPQ where O is the point of intersection of co-ordinate axes is :
(a) $\frac{c^2}{ab}$ (b) $\frac{c^2}{2ab}$
(c) $\frac{c^2}{a^2b}$ (d) $\frac{c^2}{ab^2}$

66. The line passing through the points (–3, 7) and (4, 6) :
(a) cut x-axis only
(b) cuts y-axis only
(c) cuts both the axes
(d) does not cut any axes.

67. For what value of k will the equations $x + 2y + 6 = 0$ and $3x + ky + 18 = 0$ represent coincident lines ?
(a) $k = 8$ (b) $k = 4$
(c) $k = 5$ (d) $k = 6$

68. The value of k for which the system of equations $x + 3y + 7 = 0$, $4x + ky + 19 = 0$ has no solution is :
(a) $k = 6$ (b) $k = -12$
(c) $k = 12$ (d) $k = 8$

69. The pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ in two variables represents pair straight lines which are intersecting , if :
(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(d) $\frac{a_1}{a_2} \neq \frac{c_1}{c_2} = \frac{b_1}{b_2}$

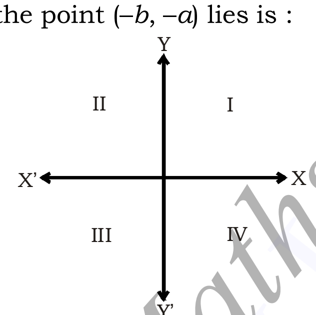
70. The point of the form (b, b) always lies on the straight lines :
(a) y = b (b) $x - y = 0$
(c) x = b (d) $x + y = 0$

71. The straight lines $2x + y = 4$ and $x - 3y = 9$ intersect at a point, the point of intersection is :
(a) (3, –2) (b) (–3, 2)
(c) (2, –3) (d) (–2, 3)

72. The value of k for which the system of equations $kx + y = 7$, $9x + 3y = 17$ has a unique solution :
(a) $k = 3$ (b) k = 0
(c) $k \neq 3$ (d) k $\neq$ 0

73. When we plot the points A (–1, 2), B(–1, –2) and C(2, –2) on a graph paper, the figure formed by these points is :
(a) right angled triagnle
(b) scalene triangle
(c) equilateral triangle
(d) isosceles triangle

74. If $ab > 0$ and the point (a, b) lies in the third quadrant in which the point ($-b$, $-a$) lies is :



(a) I (b) II
(c) III (d) IV

75. The area of the triangle formed by the lines $5x + 7y = 35$, $4x + 3y = 12$ x-axis is :

(a) $\frac{160}{13}$ sq.unit

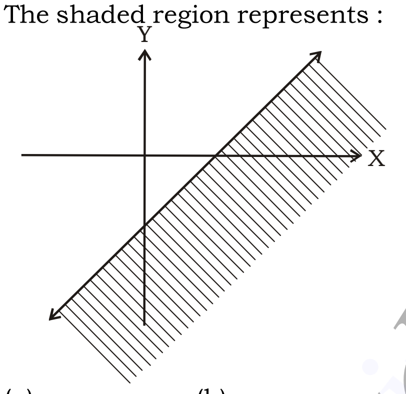
(b) $\frac{150}{13}$ sq.unit

(c) $\frac{140}{13}$ sq.unit

(d) 10 sq. unit

76. The area of triangle formed by the lines $3x - 6y = 12$, $3x - y = 3$ and x-axis is :
(a) 5.4 sq. units
(b) 2.7 sq. units
(c) 4.5 sq. units
(d) 3.6 sq. units

77. The shaded region represents :



(a) $y \geq x$ (b) $y \leq -x$
(c) $y \leq x$ (d) $y \geq -x$

78. The three vertices of ΔABC are A (2, 4), B(–3, 2), C(4, 2). The area is :
(a) 7 sq. units (b) 9 sq. units
(c) 14 sq. units (d) 10 sq. units

79. If we plot the point A(4, 11) and B (–2, 3) on graph, then the distance between them is :
(a) 15 (b) 9
(c) 12 (d) 10

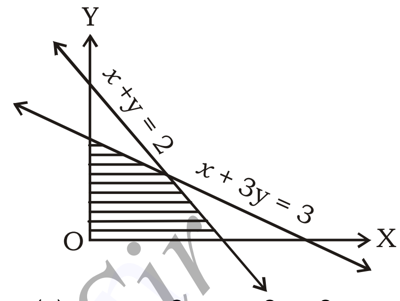
80. Find the area of the region bounded by lines $3x + 4y = 24$, $x + y = 2$ and the coordinate axes :
(a) 25 sq. units (b) 24 sq.units
(c) 22 sq. units (d) 20 sq. units

81. Find the area of the region bounded by lines $3x + 4y = 12$, $6x + 8y = 60$, $x = 0$ and $y = 0$:
(a) 37.5 sq. units
(b) 31.5 sq.units
(c) 25 sq. units
(d) 32 sq. units

82. The area of the region bounded by the lines $x - y = 0$, $x + 2y = 0$ and $y = 3$ is:
(a) 17 sq. units
(b) 6.75 sq.units
(c) 27 sq. units
(d) 13.5 sq. units

83. The region specified by $x \geq 0$, $x + y \geq 0$ includes :
(a) 1st quadrant
(b) 2nd quadrant
(c) 3rd quadrant
(d) 4th quadrant

84. The shaded region in the given figure is the solution set of the inequalities :



(a) $x + y \leq 2$, $x + 3y \geq 3$, $x \geq 0$, $y \geq 0$
(b) $x + y \geq 2$, $x + 3y \geq 3$, $x \geq 0$, $y \geq 0$
(c) $x + y \geq 2$, $x + 3y \leq 3$, $x \geq 0$, $y \geq 0$
(d) $x + y \leq 2$, $x + 3y \leq 3$, $x \geq 0$, $y \geq 0$

85. Area of the rectangular region $2 \leq x \leq 5$, $-1 \leq y \leq 3$ is :
(a) 9 sq. units (b) 12 sq. units
(c) 15 sq. unit (d) 20 sq. units

86. Graph of the inequation $2x - 5y \geq 5$ in cartesian plane is :
(a) above the line $2x - 5y = 5$
(b) below the line $2x - 5y = 5$
(c) on & below the line $2x - 5y = 5$
(d) on & above the line $2x - 5y = 5$

87. Find the area bounded by $|x| + |y| = 6$
(a) 72 sq. units (b) 48 sq. units
(c) 54 sq. units (d) 84 sq. units

88. The area of the region bounded by $y = |x| - 1$ and $y = 1 - |x|$
(a) 3 sq. units (b) 4 sq. units
(c) 2 sq. units (d) 1 sq. unit

89. The point (–5, 7) lies in the quadrant:
(a) First (b) Second
(c) Third (d) Fourth

90. The point (7, –5) lies in the quadrant:
(a) First (b) Second
(c) Third (d) Fourth

91. Find the distance between the points (–6,2) and (2 , 4) :
(a) $2\sqrt{17}$ (b) $4\sqrt{17}$
(c) $2\sqrt{5}$ (d) 10

92. The distance between the points A (b, 0) and B (0, a) is :
(a) $\sqrt{a^2 - b^2}$ (b) $\sqrt{a^2 + b^2}$
(c) $\sqrt{a + b}$ (d) a + b

93. The distance between the points A (7, 4) and B(3, 1) is :
(a) 6 units (b) 3 units
(c) 4 units (d) 5 units

94. The co-ordinates of point situated on x-axis at a distance of 5 units from y-axis is :
(a) (0, 5) (b) (5, 0)
(c) (5, 5) (d) (–5, 5)

95. The co-ordinates of a point situated on y-axis at a distance of 7 units from x-axis is :
(a) (0, 7) (b) (7, 0)
(c) (7, 7) (d) (–7, 7)

96. The co-ordinates of a point below x-axis at a distance of 6 units from x-axis but lying on y-axis is :
(a) (0, 6) (b) (–6, 0)
(c) (0, –6) (d) (6, – 6)

97. The distance of the point (6, –8) from the origin is :
(a) 2 units (b) 14 units
(c) 7 units (d) 10 units

98. The point of intersection of the lines $2x + 7y = 1$ and $4x + 5y = 11$ is :
(a) (4, –1) (b) (2, 3)
(c) (–1, 4) (d) (4, –2)

99. The line $4x + 7y = 12$ meets x-axis at the point :
(a) (3, 1) (b) (0, 3)
(c) (3, 0) (d) (4, 0)

100. The line $4x - 9y = 11$ meets y-axis at the point :
(a) $\left(-\frac{11}{9}, 0\right)$ (b) $\left(0, -\frac{11}{9}\right)$
(c) $\left(0, \frac{11}{4}\right)$ (d) $\left(0, -\frac{11}{4}\right)$

101. The slope of the line $3x + 7y + 8 = 0$ is :
(a) 3 (b) 7
(c) $-\frac{3}{7}$ (d) $\frac{3}{7}$

102. The slope of the line joining P(–4, 7) and Q(2, 3) is :
(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$
(c) $-\frac{3}{2}$ (d) $\frac{3}{2}$

103. The equation of a line parallel to x-axis at a distance of 6 units and above x-axis is :
(a) $x = 6$ (b) $y = 6x$
(c) $x = 6y$ (d) $y = 6$

104. The equation of a line parallel to y-axis at a distance of 5 units to the left of y-axis, is :
(a) $y = -5$ (b) $x = -5$
(c) $x + 5y = 0$ (d) $y + 5x = 0$

105. The equation of a line parallel to x-axis and at a distance of 7 units below x-axis is :
(a) $y = -7$ (b) $x = 7$
(c) $x = -7$ (d) $y = -7x$

106. The area of the triangle whose vertices are P (4, 5), Q(–3, 8) and R (3, –4), (in square units) is :
(a) 66 (b) $16\frac{1}{2}$
(c) 33 (d) 35

107. The points A(0, 0), B(0, 3) and C(4, 0) are the vertices of a triangle which is :
(a) Isosceles
(b) Right angled
(c) Equilateral
(d) None of these

108. The co-ordinates of the centroid of Δ PQR with vertices P(–2, 0), Q(9, –3) and R (8, 3) is :
(a) (1, 0) (b) $\left(\frac{19}{3}, 0\right)$
(c) (0, 5) (d) (5, 0)

109. The equation of a line passing through the points A (0, –3) and B (–5, 2) is :
(a) $x + y + 3 = 0$ (b) $x + y - 3 = 0$
(c) $x - y + 3 = 0$ (d) $x - y - 3 = 0$

110. The length of perpendicular from the origin to the line $12x + 5y + 7 = 0$ is :
(a) 2 units (b) 1 unit
(c) $\frac{7}{13}$ units (d) $\frac{7}{11}$ units

111. The angle which the line joining the points $\left(\sqrt{3}, 1\right)$ and $\left(\sqrt{15}, \sqrt{5}\right)$ makes with x–axis is :
(a) 30° (b) 45°
(c) 60° (d) 90°

112. The lines whose equations are $2x - 5y + 7 = 0$ and $8x - 20y + 28 = 0$ are:
(a) parallel
(b) perpendicualr
(c) coincident
(d) intersecting

113. If the distance of the point P(x, y) from A(a, 0) is a + x, then y^2 = ?
(a) 2 ax (b) 4ax
(c) 6ax (d) 8ax

114. If the point (x, y) is equidistant from the points (a + b, b – a) and (a–b, a + b) then bx = ?
(a) a^2y (b) ay^2
(c) ay (d) $a^2 y^2$

115. If the sum of the square of the distance of the point (x, y) from the point (a, 0) and (–a, 0) is $2b^2$, then :
(a) $x^2 + a^2 = b^2 + y^2$
(b) $x^2 + a^2 = 2b^2 - y^2$
(c) $x^2 - a^2 = b^2 + y^2$
(d) $x^2 + a^2 = b^2 - y^2$

116. P (– 4, a) and Q(2, a + 4) are two points and the co-ordinates of the middle point of PQ are (–1, 4). The value of a is :
(a) 0 (b) 2
(c) –2 (d) 3

117. If the points P(2, 3), Q(5, a) and R (6, 7) are collinear, the value of a is :
(a) 5/2 (b) – 4/3
(c) 6 (d) 5

118. The equation of a line parallel to x-axis and passing through (– 6 ,– 5) is :
(a) $y = -5$ (b) $x = -6$
(c) $y = -5x$ (d) $y = -6x - 5$

119. The equation of a line parallel to y-axis and passing through (2 ,– 5) is :
(a) $x = 2$ (b) $y = -5$
(c) $y = 2x$ (d) $x = -5y$

120. Two vertices of a triangle PQR are P(–1, 0) and Q(5, –2) and its centroid is (4, 0). The co-ordinates of R are :
(a) (8, –2) (b) (8, 2)
(c) (–8, 2) (d) (–8, –2)

121. The co-ordinates of the point of intersection of the medians of a triangle with vertices P(0, 6), Q(5, 3) and R(7, 3) are :
(a) (4, 5) (b) (3, 4)
(c) (4, 4) (d) (5, 4)

122. The ratio in which the line segment joining A(3, –5) and B(5, 4) is divided by x-axis is :
(a) 4 : 5 (b) 5 : 4
(c) 5 : 7 (d) 6 : 5

123. The ratio in which the line segment joining P(–3, 7) and Q (7, 5) is divided by y-axis is :
(a) 3 : 7 (b) 4 : 7
(c) 3 : 5 (d) 4 : 5

124. The ratio in which the point $P\left(1, \frac{10}{3}\right)$ divides the join of the point A(–3, 2) and B(3, 4) is :
(a) 2 : 3 (b) 1 : 2
(c) 2 : 1 (d) 3 : 1

125. The equation of a line with slope 5 and passing through the point (–4, 1) is :
(a) $y = 5x + 21$ (b) $y = 5x - 21$
(c) $5y = x + 21$ (d) $5y = x - 21$

126. The value of 'a' so that the lines $x + 3y - 8 = 0$ and $ax + 12y + 5 = 0$ are parallel is :
(a) 0 (b) 1
(c) 4 (d) – 4

127. The value of P for which the lines $3x + 8y + 9 = 0$ and $24x + py + 19 = 0$ are perpendicuIar is :
(a) –12 (b) – 9
(c) – 11 (d) 9

128. The value of a so that line joining P(–2, 5) and Q (0, –7) and the line joining A (–4, –2) and B(8, a) are perpendicular to each other is :
(a) –1 (b) 5
(c) 1 (d) 0

129. The angle between the lines represented by the equations $2y - \sqrt{12}x - 9 = 0$ and $\sqrt{3}y - x + 7 = 0$, is:
(a) 30° (b) 45°
(c) 60° (d) $22\frac{1}{2}^{\circ}$

130. If P(3, 5), Q (4, 5) and R(4, 6) be any three points, the angle between PQ and PR is :
(a) 30° (b) 45°
(c) 60° (d) 90°

131. Given a ΔPQR with vertices P (2, 3), Q (– 3, 7) and R (– 1, –3). The equation of median PM is :
(a) $x - y + 10 = 0$
(b) $x - 4y - 10 = 0$
(c) $x - 4y + 10 = 0$
(d) None of these

132. The co-ordinates of the point P which divides the join of A(3, –2) and $B\left(\frac{11}{2}, \frac{21}{2}\right)$ in the ratio 2 : 3 are :
(a) (4, 3) (b) (4, 5)
(c) $\left(4, \frac{5}{2}\right)$ (d) $\left(\frac{3}{2}, \frac{7}{2}\right)$

133. The length of the portion of the straight line $8x + 15y = 120$ intercepted between the axes is :
(a) 14 units (b) 15 units
(c) 16 units (d) 17 units

134. The equation of the line passing through the point (1, 1) and perpendicular to the line $3x + 4y - 5 = 0$, is :
(a) $3x + 4y - 7 = 0$
(b) $3x + 4y + k = 0$
(c) $3x - 4y - 1 = 0$
(d) $4x - 3y - 1 = 0$

135. The equation of a line passing through the point (5, 3) and parallel to the line $2x - 5y + 3 = 0$, is :
(a) $2x - 5y - 7 = 0$
(b) $2x - 5y + 5 = 0$
(c) $2x - 2y + 5 = 0$
(d) $2x - 5y = 0$

136. The sides PQ, QR, RS and SP of a quadrilateral have the equations $x + 2y = 3$, $x = 1$, $x - 3y = 4$, $5x + y + 12 = 0$ respectively, then the angle between the diagonals PR and QS is:
(a) 30° (b) 45°
(c) 60° (d) 90°

137. The equations of two equal sides of an isosceles triangle are $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point (1, –10). The equation of the third side is :
(a) $x - 3y - 31 = 0$ but not $x - 3y - 31 = 0$
(b) neither $3x + y + 7 = 0$ nor $x - 3y - 31 = 0$
(c) $3x + y + 7 = 0$ or $x - 3y - 31 = 0$
(d) $3x + y + 7 = 0$ but not $x - 3y - 31 = 0$

138. If P_1 and P_2 be perpendicular from the origin upon the straight lines $x\sec\theta + y\,\text{cosec}\,\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$ respectively, then the value of $4P_1^2 + P_2^2$ is :
(a) a^2 (b) $2a^2$
(c) $\sqrt{2}\,a^2$ (d) $3a^2$

139. Find the equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9 ?
(a) $x + 2y - 6 = 0$ but not $2x + y - 6 = 0$
(b) neither $x + 2y - 6 = 0$ nor $2x + y - 6 = 0$
(c) $2x + y - 6 = 0$ but not $x + 2y - 6 = 0$
(d) $x + 2y - 6 = 0$ or $2x + y - 6 = 0$

140. Romila went to a statomeru shop and purchased 2 pencils and 3 erasers for ₹ 9. Her friend Sonali saw the new variety of pencils and erasers with and she also bought 4 pencils and 6 erasers of the same kind for ₹ 18. representationis situation algebraicallyand graphically.

141. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were. Also, three years from now, I shall be three times as old as you will be." interesting?) Represent this situation algebebraically and graphically.

142. 5Pens and 7 Pencil together cost ₹ 50. Whereas 7 pens and 5 pencils together Cost ₹ 46. find the cost of one pencil and that of one pen.

ANSWER KEY

1. (c)	15. (a)	29. (b)	43. (d)	57. (c)	71. (a)	85. (b)	99. (c)	113.(b)	127.(b)
2. (c)	16. (c)	30. (b)	44. (b)	58. (a)	72. (c)	86. (c)	100.(b)	114.(c)	128.(d)
3. (c)	17. (b)	31. (c)	45. (a)	59. (d)	73. (a)	87. (a)	101.(c)	115.(d)	129.(a)
4. (d)	18. (c)	32. (a)	46. (b)	60. (b)	74. (c)	88. (c)	102.(c)	116.(b)	130.(b)
5. (c)	19. (a)	33. (b)	47. (c)	61. (a)	75. (a)	89. (b)	103.(d)	117.(c)	131.(c)
6. (b)	20. (b)	34. (b)	48. (a)	62. (c)	76. (b)	90. (d)	104.(b)	118.(a)	132.(a)
7. (c)	21. (a)	35. (c)	49. (c)	63. (b)	77. (c)	91. (a)	105.(a)	119.(a)	133.(d)
8. (a)	22. (a)	36. (b)	50. (d)	64. (d)	78. (a)	92. (b)	106.(c)	120.(b)	134.(d)
9. (b)	23. (d)	37. (b)	51. (b)	65. (b)	79. (d)	93. (d)	107.(b)	121.(c)	135.(b)
10. (b)	24. (c)	38. (c)	52. (c)	66. (b)	80. (c)	94. (b)	108.(d)	122.(b)	136.(d)
11. (a)	25. (d)	39. (b)	53. (a)	67. (d)	81. (b)	95. (a)	109.(a)	123.(a)	137.(c)
12. (b)	26. (c)	40. (b)	54. (b)	68. (c)	82. (d)	96. (c)	110.(c)	124.(c)	138.(a)
13. (a)	27. (b)	41. (a)	55. (b)	69. (a)	83. (a)	97. (d)	111.(a)	125.(a)	139.(d)
14. (a)	28. (a)	42. (d)	56. (a)	70. (b)	84. (d)	98. (a)	112.(c)	126.(c)	

EXERCISE

1. (c) Distance between the points

$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$5 = \sqrt{(8-4)^2 + (A-3)^2}$

$25 = (4)^2 + (A - 3)^2$

$25 = 16 + A^2 + 9 - 6A$

$9 - 9 = A^2 - 6A$

$\Rightarrow A^2 - 6A = 0$

$\Rightarrow A\,(A - 6) = 0$

A = (6,0)

2. (c) $(5)^2 = (C - 0)^2 + (4 - 0)^2$

$25 = C^2 + 16$

$C^2 = 9$

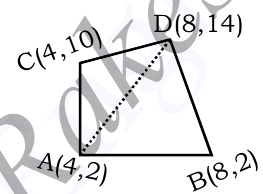
$C = \pm 3$

3. (c) Co-ordinates of Mid-points

of AB $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-3+5}{2}\right)\left(\frac{2+4}{2}\right) = 1,3$

4. (d) Area of Triangle ADC



$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x3(y_1 - y_2)\right]$

$= \frac{1}{2}\left[4(14-10) + 8(10-2) + 4(2-14)\right]$

$= \frac{1}{2}\left[4 \times 4 + 8 \times 8 - 4 \times 12\right]$

= 16 sq. units

Area of ΔABD

$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x3(y_1 - y_2)\right]$

$= \frac{1}{2}\left[4(2-14) + 8(14-2) + 8(2-2)\right]$

$= \frac{1}{2}\left[4 \times (-12) + 8 \times 12 + 8 \times 0\right]$

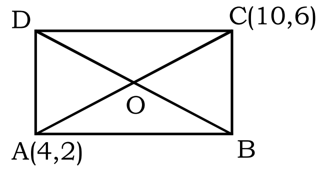
$= \frac{1}{2}\left[-48 + 96\right]$ = 24 sq. units

Hence Area of Qadrilateral ABCD

= Area of [ΔABD + ΔADC]

= 16 + 24 = 40 Sq. units

5. (c)



O is the mid point of diagonal AC and BD Co-ordinate

of O = $\left(\frac{4+10}{2}, \frac{2+6}{2}\right)$

= (7,4)

Point O is also situated on line

$x - 3y + k = 0$

$\Rightarrow$ $7 - 3 \times 4 + k = 0$

$\Rightarrow$ $7 - 12 + k = 0$

$\Rightarrow$ $k = 5$

6. (b) $7x + 8y - 56 = 0$

$7x + 8y = 56$

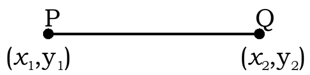
$\frac{x}{8} + \frac{y}{7} = 1$

Length of Intercepts made at

x axis at $x = 8$

y axis at y = 7

7. (c)



Slope of line $m = \frac{y_2 - y_1}{x_2 - x_1}$

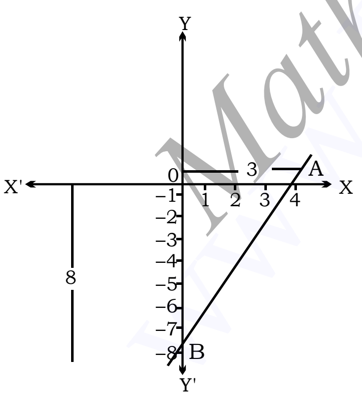
P(2,8) and Q(6,9)

Slope $m = \frac{9-8}{6-2} = \frac{1}{4} = .25$

8. (a) $8x - 3y = 24$

$\frac{x}{3} - \frac{y}{8} = 1$

$\frac{x}{3} + \frac{y}{-8} = 1$



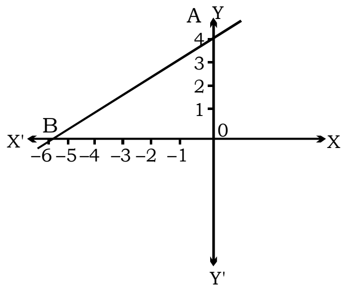
Area of Δ OAB

$= \frac{1}{2} \times 8 \times 3$

= 12 Sq. units

9. (b) $-6x + 9y = 36$

$\frac{x}{-6} + \frac{y}{4} = 1$



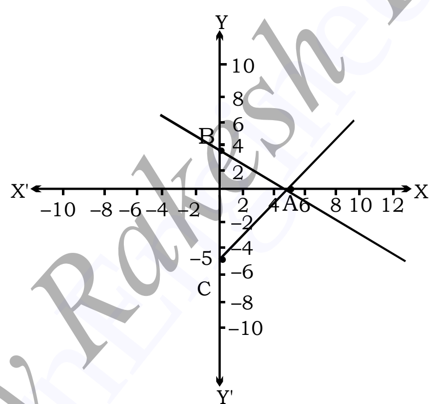
Area of Δ OAB

$= \frac{1}{2} \times 6 \times 4 = 12$ sq. units

10. (b) $2x - 2y = 10$

$\Rightarrow$ $\frac{x}{5} - \frac{y}{5} = 1$

$\Rightarrow$ $2x + 3y = 10 = \frac{x}{5} + \frac{3y}{10} = 1$



OA = 5

OB = 10/3

OC = –5 (But distance is always positive)

Area of Δ ABC = area of Δ OAB + area of Δ OAC

$= \frac{1}{2} \times 5 \times \frac{10}{3} + \frac{1}{2} \times 5 \times 5$

$= \frac{50}{6} + \left(\frac{.25}{2}\right) = \frac{25}{3} + \frac{25}{2}$

$= \frac{50+75}{6} = \frac{125}{6}$ sq.units.

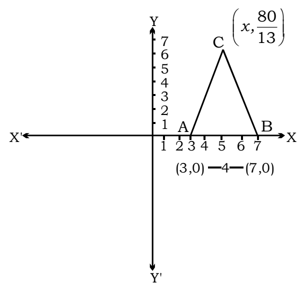
Sol.11(a) On x-axis; y = 0

$4x + 3y = 12$

$= x = (3,0)$ ....(i)

$5x + 7y = 35$

$\Rightarrow$ $x = (7,0)$ ......(ii)



$_{5\times}$ $4x + 3y = 12$ $_{\times 5}$

$_{4\times}$ $5x + 7y = 35$ $_{\times 4}$

$20x + 15y = 60$

$20x + 28y = 140$

$-13y = -80$

$\Rightarrow$ $y = \frac{80}{13}$

Area of Δ ABC

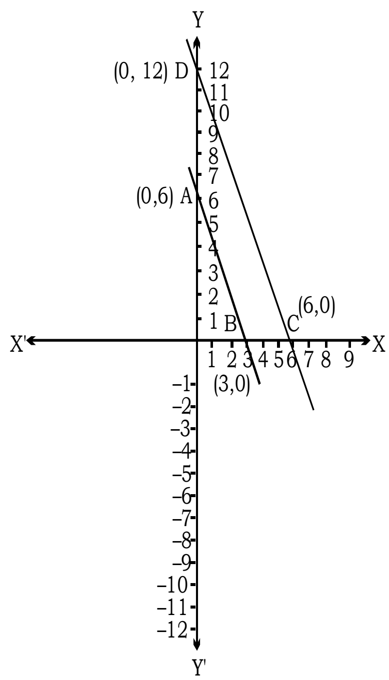
$= \frac{1}{2} \times 4 \times \frac{80}{13} = \frac{160}{13}$ sq. units

12. (b) $4x + 2y = 12$

$8x + 4y = 48$

$\Rightarrow$ $\frac{x}{3} + \frac{y}{6} = 1$

$\Rightarrow$ $\frac{x}{6} + \frac{y}{12} = 1$



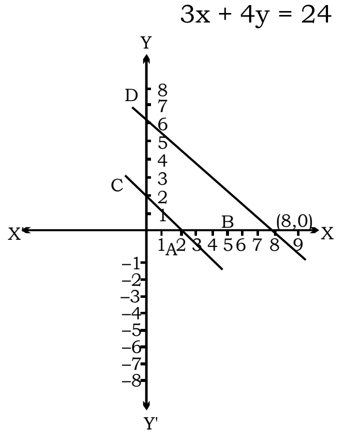
Area of quadrilateral = Area of Δ OCD – Area of Δ OAB

$= \frac{1}{2} \times 6 \times 12 - \frac{1}{2} \times 3 \times 6$

= 27 sq. units

13. (a) x + y = 2

3x + 4y = 24



$\Rightarrow \quad \frac{x}{2}+\frac{y}{2}=1$ ......(i)

$\Rightarrow \quad \frac{x}{8}+\frac{y}{6}=1$ .....(ii)

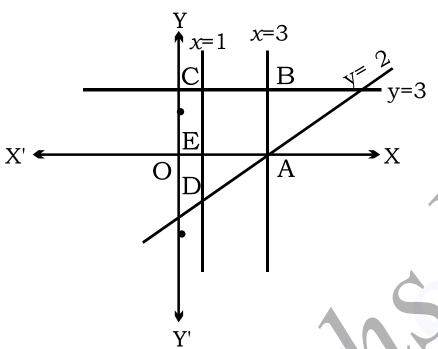
Area of quadrilateral = Area of Δ OBD – Area of Δ OAC

$= \frac{1}{2}\times 8\times 6-\frac{1}{2}\times 2\times 2$

= 22 Sq. units

14. (a) x = 1, x = 3, y = 2 and x = y + 3

$\Rightarrow \frac{x}{3}+\frac{y}{-3}=1$



Point D wil be (1, –2)

ABCD is a trapezium.

AB = 2, CD = CE + DE + 2 + 2 =4

height = AE = 3 – 1 = 2

Area of trapezium ABCD

$= \frac{1}{2}$ AE(AB + CD)

$= \frac{1}{2}\times 2\,(2+4)$ = 6 sq. unit.

15. (a) $5x + 20y = 11$ and $2x + ky = 17$ will be intersecting lines or having unique solution

$\Rightarrow \frac{5}{2}\neq\frac{20}{k}$

$\Rightarrow k\neq 8$

16. (c) $4x + ky = 3$ and $3x + 2y = 7$ are perpendicular to each other.

Condition for perpendicular :

$a_1a_2 + b_1b_2 = 0$

$\Rightarrow 4 \times 3 + k \times 2 = 0$

$\Rightarrow k = -6$

17. (b) $(k + 1)x + ky = 3$ and $5x - 2y = 7$ are perpendicular for perpendicular : $a_1a_2 + b_1b_2 = 0$

$\Rightarrow (k + 1) \times 5 + k \times (-2) = 0$

$\Rightarrow 5k + 5 - 2k = 0$

$\Rightarrow k = \frac{-5}{3}$

18. (c) OA = OB = radius

$= \sqrt{(4-2)^2+(3-3)^2}$

$= \sqrt{(x-2)^2+(5-3)^2}$

$\Rightarrow \quad (x-2)^2 = 0$

$\Rightarrow \quad x = 2$

19. (a)Let the required point be P (0,y).

$\therefore$ PA = PB

$= \sqrt{(0-6)^2+(y-5)^2}$

$= \sqrt{(0-4)^2+(y-3)^2}$

$\Rightarrow \quad 36 + (y-5)^2 = 16 + (y-3)^2$

$\Rightarrow \quad 36 + y^2 - 10y + 25$

$\Rightarrow \quad 16 + y^2 - 6y + 9$

$\Rightarrow \quad 4y = 36$

$\Rightarrow \quad y = 9$

$\therefore$ Required point is (0,9).

20. (d) a = 2 , b =3.

$\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \frac{x}{2}+\frac{y}{3}=1$

$\Rightarrow 3x + 2y = 6$

21. (a) Centroid a triangle=

$\equiv\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

$\equiv\left(\frac{0+8+8}{3},\frac{6+12+0}{3}\right)$

$\equiv\left(\frac{16}{3},6\right)$

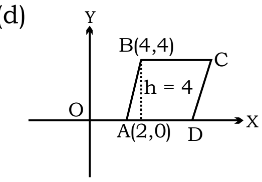
22. (a) Vetices of triangle are (3, – 5)(–7,4) and (h,k) and centroid is (2,–12)

$(2,-12)\equiv\left(\frac{3-7+h}{3},\frac{-5+4+k}{3}\right)$

On solving.

(h,k) $\equiv$ (10,–35)

23. (d)



$AB = \sqrt{(4-2)^2+(4-0)^2}$

$\sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

Area of Rhombus = Base × height

$2\sqrt{5}\times 4 = 8\sqrt{5}$ sq. units

24. (c)

$a = \sqrt{(7-3)^2+(4-7)^2} = 5$

$b = \sqrt{(7-3)^2+(4-4)^2} = 4$

$c = \sqrt{(3-3)^2+(7-4)^2} = 3$

Incentre I $\equiv$

$\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy3}{a+b+c}\right)$

$= \left(\frac{5\times3+4\times3+3\times7}{5+4+3},\frac{5\times4+4\times7+3\times4}{5+4+3}\right)$

= (4,5)

25. (d) Let a line $\frac{x}{a}+\frac{y}{b}=1$

Given, a + b = 18 ....(i)

Pasing throug (4,4)

It means $\frac{4}{a}+\frac{4}{b}=1$

$\Rightarrow$ ab = 4(a + b) =72

$(a - b)^2 = (a + b)^2 - 4ab$

$= (18)^2 - 4 \times 72$

= 324 – 288 = 36

a – b = $\pm$ 6 .....(ii)

On solving equation (i) and (ii)

(a,b) =(12,6) or (6,12)

Line will be $\frac{x}{12}+\frac{y}{6}=1$

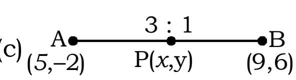
$\Rightarrow x + 2y = 12$

Or,

$\frac{x}{6}+\frac{y}{12}=1$

$\Rightarrow 2x + y = 12$

26. (c)



$(x,y)=\left(\frac{9\times3+1\times5}{3+1},\frac{6\times3+1\times-2}{3+1}\right)$

=(8,4)

27. (b) $x\cos\theta + y\sin\theta = 2$

$\Rightarrow y\sin\theta = -x\cos\theta + 2$

$\Rightarrow y = -x\cot\theta + 2\operatorname{cosec}\theta$

$m_1 = -\cot\theta$

$x - y = 3 \Rightarrow y = x - 3$

$m_2 = 1$

Both lines are perpendiculars to each other so $m_1.m_2 = -1$

$(-\cot\theta).(1) = -1$

$\Rightarrow \cot\theta = 1$

$\Rightarrow \theta = \dfrac{\pi}{4}$

28. (a) $m_1 = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{3+1}{1-2} = -4$

Slope of line L $= \dfrac{-1}{m_1} = \dfrac{-1}{-4} = \dfrac{1}{4}$

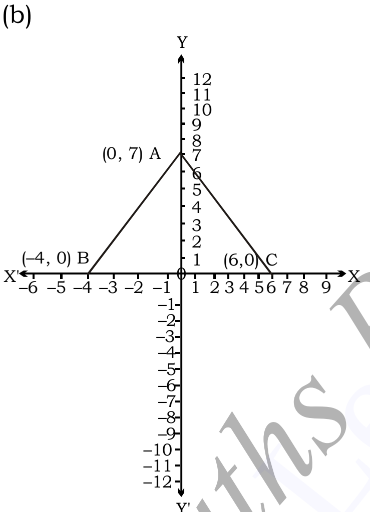
Equation of line L will be

$(y - 3) = \dfrac{1}{4}(x - 1)$

$4(y - 3) = x - 1$

$x - 4y + 11 = 0$

29. (b)



$\dfrac{x}{6} + \dfrac{y}{7} = 1$, $\dfrac{x}{-4} + \dfrac{y}{7} = 1$ and x-axis

BC = 4 + 6 = 10 unit

AO = 7 units

Area of triangle $= \dfrac{1}{2} \times BC \times OA$

$= \dfrac{1}{2} \times 10 \times 7 = 35$ units

30. (b) Let A = (4,3), B = (7, –1), C = (9,3)

$AB = \sqrt{(7-4)^2 + (-1-3)^2}$

$= \sqrt{25} = 5$

$BC = \sqrt{(9-7)^2 + (3+1)^2}$

$= \sqrt{20} = 2\sqrt{5}$

$CA = \sqrt{(4-9)^2 + (3+3)^2}$

$= \sqrt{25} = 5$

$\therefore$ AB = CA = 5

Hence ABC is an isosceles triangle

31. (c)Let A = (4,4), B = (3,5), C = (–1,–1)

Then

$AB = \sqrt{(3-4)^2 + (5-4)^2}$

$= \sqrt{2}$

$BC = \sqrt{(-1-3)^2 + (-1-5)^2}$

$= \sqrt{52}$

$AC = \sqrt{(-1-4)^2 + (-1-4)^2}$

$= \sqrt{50}$

$\therefore$ $AB^2 + AC^2 = BC^2$

Hence by the Pythagoras Theorem, ABC is a right angled triangle.

32. (a) The coordinates of the mid points of AB

$= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{-3+5}{2}, \dfrac{2+4}{2}\right) = (1,3)$

33. (b) The required coordinates of the point which divides AB in the ratio 5 : 3

$\dfrac{mx_2 - nx_1}{m-n}, \dfrac{my_2 - ny_1}{m-n}$

$= \dfrac{5\times6 - 3\times2}{5-3}, \dfrac{5\times8 - 3\times4}{5-3}$

= (12,14)

34. (b) Let A = (4, –2), B= (5,5) and C = (–2,4)

Then

$a = BC = \sqrt{(-2-5)^2 + (4-5)^2}$

$= 5\sqrt{2}$

$b = AC = \sqrt{(4+2)^2 + (-2-4)^2}$

$= 6\sqrt{2}$

$c = AB = \sqrt{(5-4)^2 + (5+2)^2}$

$= 5\sqrt{2}$.

and

$(x_1, y_1) = (4, -2)$, $(x_2, y_2) = (5,5)$.

$(x_3, y_3) = (-2,4)$

$\therefore$ The coordinates of the incentre of the $\triangle$ ABC are

$\left(\dfrac{ax_1 + bx_2 + cx_3}{a+b+c}, \dfrac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

$= \left(\dfrac{5}{2}, \dfrac{5}{2}\right)$

{substitute the value of (a,b,c} $\{x_1, x_2, x_3, y_1, y_2$ and $y_3\}$.

35. (c) Let A, B, C and D be the vertices of the quadrilateral whose coordinates are (–2,1) (1,0), (4,3) and (1,2) respectively.

Now, AB = $\sqrt{10}$,BC = $\sqrt{18}$,DC = $\sqrt{10}$,AD = $\sqrt{18}$

$\therefore$ AB = CD and BC = AD i.e., the opposite sides are equal.

Hence ABCD is a parallelogram.

36. (b) Let m1 and m2 be the slope of BA and BC respectively. Then

$m1 = \dfrac{3-1}{2-(-2)} = \dfrac{1}{2}$

and

$m2 = \dfrac{-4-3}{-2-2} = \dfrac{7}{4}$

Let θ be the angle between BA and BC. Then

$\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|$

$= \left|\dfrac{\dfrac{7}{4} - \dfrac{1}{2}}{1 + \dfrac{7}{4} \times \dfrac{1}{2}}\right| = \pm\dfrac{2}{3}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{2}{3}\right)$

37. (b) We have 3x + 4y – 12 = 0

$\Rightarrow 3x + 4y = 12$

$\Rightarrow \dfrac{3x}{12} + \dfrac{4y}{12} = 1$

$\Rightarrow \dfrac{x}{4} + \dfrac{y}{3} = 1$

Which is of the form $\dfrac{x}{a} + \dfrac{y}{b} = 1$

Then the required intercepts on the axes are 4 and 3.

38. (c)Let the equation of the cost curve as a straight line be

y = mx + c ....(i)

Where x = number of units of a good produced and y = cost of x units in rupees.

Given, when x = 50, y = 320 and when x = 80, y = 380

from (1) 320 = 50m + c ...(2)

380 = 80m + c ....(3)

Subtracting (2) from (3), we get m = 2

Substituting m = 2 in equation (2), we get c = 220

∴ From (1) y = 2x + 220

When x = 110, y = 2 × 110 + 220 = 440

Hence, the required cost of producing 110 units is ₹ 440.

39. (b) Length of the perpendicular from the point (3, – 2) to the straight line 12x – 5y + 6 = 0 is

$$\frac{12\times 3-5\times -2+6}{\sqrt{(12)^2+(-5)^2}}$$

$$= \frac{36+10+6}{\sqrt{169}} = 4 \text{ units.}$$

40. (b) The intersection point of the lines 3x + 4y – 7 = 0 and x – y + 2 = 0 is,

3x + 4y – 7 = 0 ...... (i)

x – y + 2 = 0 .... (ii)

x = y – 2 puting in equation (i)

3y – 6 + 4y – 7 = 0

$y = \frac{13}{7}$, $x = y - 2 = \frac{13}{7} - 2 = \frac{1}{7}$

equation of line

$y = mx + c$

m = slope of line

$y = 3x + c$ .... (iii)

equation (iii) passing the point

$$\left(-\frac{1}{7}, \frac{13}{7}\right)$$

$$\frac{13}{7} = \frac{-3}{7} + c$$

$c = \frac{16}{7}$ puting in equation (iii)

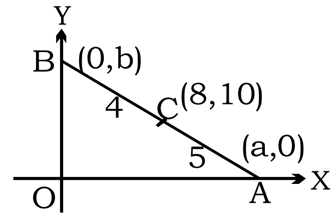
$$y = 3x + \frac{16}{7}$$

21x – 7y + 16 = 0

41. (a) Let the equation of the line AB be $\frac{x}{a}+\frac{y}{b}=1$ ....(i)

Then the coordinates of A and B are respectively (a,0) and (o,b).

since C (8, 10) divides AB in the ratio 5 : 4, we have



$$\frac{5\times 0+4\times a}{5+4} = 8$$

and $\frac{5\times b+4\times 0}{5+4} = 10$

or a = 18 and b = 18

Hence from (1), the required equation of the line AB is

$$\frac{x}{18}+\frac{y}{18}=1$$

or x + y = 18

42. (d) Distance between two points

$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Let (x_1,y_1) = (–5,3) and (x_2,y_2) = (3,1)

∴ Required distance

$$= \sqrt{(3+5)^2+(1-3)^2}$$

$$= \sqrt{64+4} = \sqrt{68} = 2\sqrt{17} \text{ units}$$

43. (d) The point (6, –3) lies in the fourth quadrant.

44. (b) If $x < 0$ & $y > 0$, (x, y) lies in quadrant II

45. (a)????? When $x = 2$, $y = 3\times 2 + 5 = 6 + 5 = 11$

So,

(2, 11) lies on $y = 3x + 5$

46. (b) Clearly, the point of x-axis has ordinate 7 and abscissa 0

So,

the point is (7, 0)

47. (c) Clearly, the point is (0, –8).

48. (a) $2x + 7y = 1$ ....(i)

$4x + 5y = 11$ ....(ii)

On solving (i) and (ii), we get x = 4 and y = –1

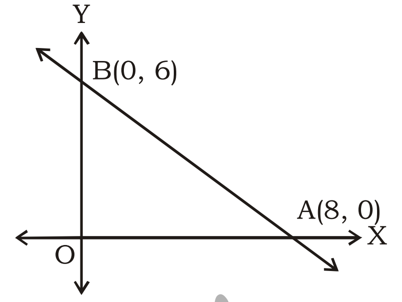
∴ Required point of intersection = (4, –1)

49. (c) When $x = 4 \Rightarrow y = 2\times 4+3 = 11$

So,

(4, 11) lies on y = 2x + 3 but (4, 10) does not lie on it.

50. (d)



Clearly, OA = 8 units and OB = 6 units

$$\therefore \text{ ar}(\Delta \text{OAB}) = \frac{1}{2}\times \text{OA}\times \text{OB}$$

$$= \frac{1}{2}\times 8\times 6 = 24 \text{ sq. units}$$

51. (b) at y - axis, $x = 0$

∴ $2\times 0 - 3y = 6 \Rightarrow y = -2$

∴ Required point (0, – 2)

52. (c) at x-axis, y = 0

∴ $4x + 7\times 0 = 12 \Rightarrow x = 3$

∴ Required point = (3, 0)

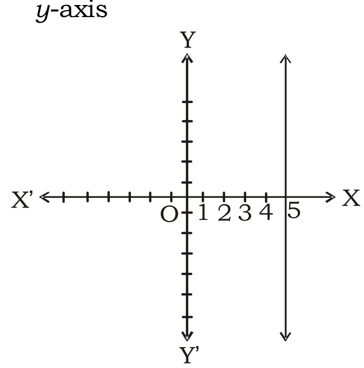
53. (a) The equation of y-axis is $x = 0$

54. (b) The given equation is $x = 5$.

Here, y-coordinate is 0.

So,

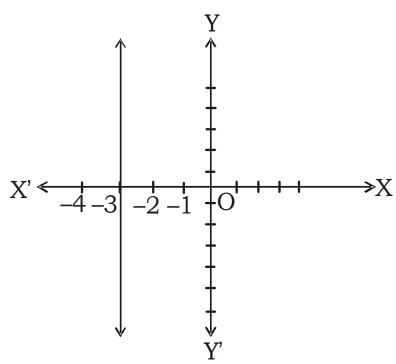
the given line is parallel to y-axis



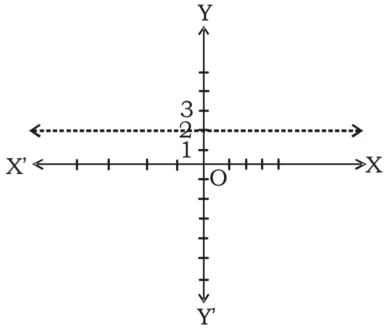
55. (b) The given equation is $x = -3$

Here y-coordinate is 0.

So,the given line is parallel to y-axis



56. (a) The given equation is $y = 2$, here x-coordinate is 0.
So,
the given line is parallel to x-axis.



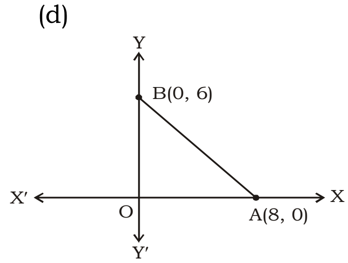
57. (c) Point $\left(3, \frac{11}{2}\right)$ will satisfy the given equation.

$$\therefore \quad 2 \times \frac{11}{2} = a \times 3 + 5 \Rightarrow 6 = 3a \text{ or } a = 2$$

58. (a) Point $\left(3, \frac{5}{2}\right)$ will satisfy the given equation.

$$\therefore \quad 2 \times \frac{5}{2} = a \times 3 - 5 \Rightarrow 3a = 10 \text{ or } a = \frac{10}{3}$$

59. (d)

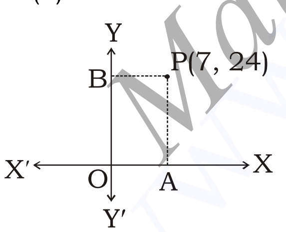


Required distance = AB

$$\sqrt{(OA)^2 + (OB)^2}$$

$$= \sqrt{8^2 + 6^2} = 10 \text{ units}$$

60. (b)

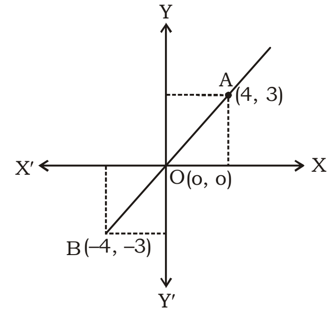


Clearly, OA = 7, AP = 24

$\therefore OP^2 = OA^2 + AP^2 = 7^2 + 24^2 = 625$

$$\therefore OP = \sqrt{625} = 25$$

61. (a) This graph shows AB is a straight line which passes through the origin.

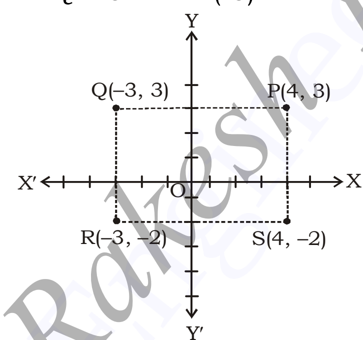


62. (c) $3y + 4x = 0$ or $y = -\frac{4}{3}x$

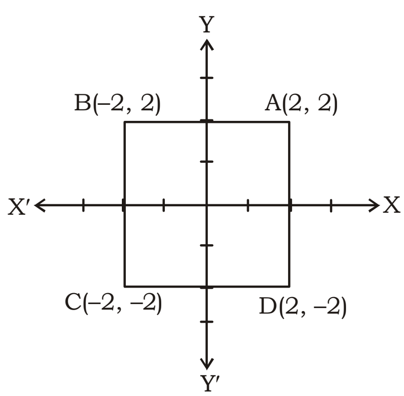
which passes through origin as(0, 0) satisfy it.

63. (b) Taking the points in the order given, it is easily seen that they are represented P, Q, R, S. Then the shape shows rectangle (PQRS) having sides.

PS = QR = 3 - (-2) = 5
PQ = SR = 4 - (-3) = 7



64. (d) The graph of $x = 2$ will be a line parallel to y-axis at a distance of 2 units to its right.
Similarly, the graph of $x = -2$ will be a line parallel to y-axis at a distance of 2 units to its left.
and $y = 2$ and $y = -2$ will be the lines parallel to x-axis at a distance of 2 units to its right and 2 units to its left respectively.
Thus ABCD is a square having each side of 4 units.

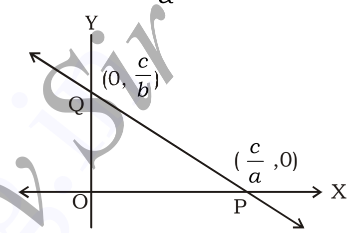


65. (b) The equation is - $ax + by = c$ (given)
When $x = 0 \Rightarrow by = c$
or

$$y = \frac{c}{b}$$

When $y = 0 \Rightarrow ax = c$

$$\text{or} \quad x = \frac{c}{a}$$

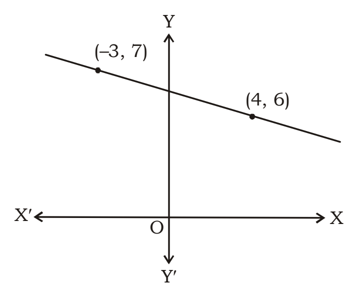


Clearly, $OP = \frac{c}{a}$, $OQ = \frac{c}{b}$

$$\therefore \quad ar(\Delta OPQ) = \frac{1}{2} \times OP \times OQ$$

$$= \frac{1}{2} \times \frac{c}{a} \times \frac{c}{b} = \frac{c^2}{2ab}$$

66. (b) (–3, 7) lies in second quadrant and (4, 6) lies in first quadrant.



67. (d) The given equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.These will represent coincident lines it they have infinitely many solutions.The condition for which is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{1}{3} = \frac{2}{k} = \frac{6}{18}$$

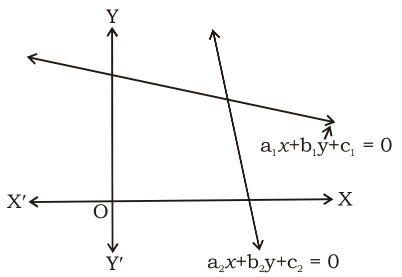
$$\Rightarrow k = 6$$

68. (c) $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ will have no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow \frac{1}{4} = \frac{3}{k} \Rightarrow k = 12$

69. (a)



Given that lines are intersecting i.e. they have unique solution.

$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}.$

70. (b) The point (b, b) always lies on the line $x - y = 0$

71. (a) $2x + y = 4$ ....(i)

$x - 3y = 9$ ....(ii)

$3\times$ (i) + (ii) we get

$\Rightarrow 7x = 21 \Rightarrow x = 21/7$

$\Rightarrow x = 3$

From (i),

$y = 4 - 6 \Rightarrow y = -2$

The straight lines intersect at (3, –2)

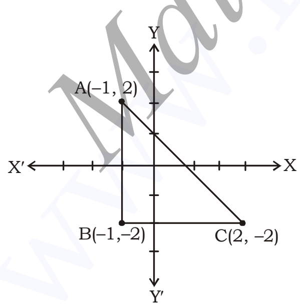
Alternatively:-

(a) Since the point (3, –2) satisfies both the equations. Put the values of x, y in the given equations. You get LHS = RHS

72. (c) $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution if

$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{9} \neq \frac{1}{3} \Rightarrow k \neq 3$

73. (a)



Here, AB = 4, BC = 3

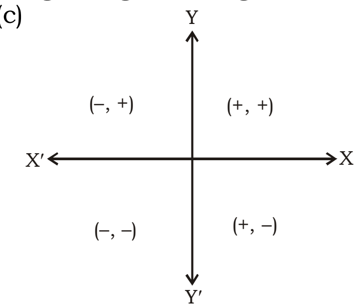
and AC $= \sqrt{(2+1)^2+(2+2)^2}$

$= \sqrt{9+16} = 5$

i.e. $AC^2 = AB^2 + BC^2$

hence, the figure represents right angled triangle.

74. (c)



ab > 0

Hence, a and b are both positive or negative.

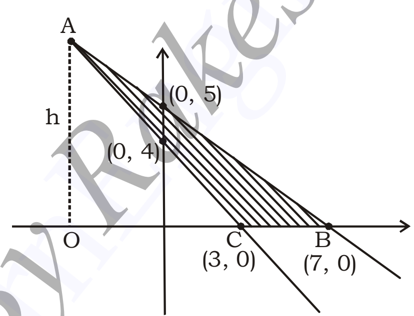
$\therefore$ If (a, b) lies in first quadrant, (–b, –a) will lie in third quadrant.

75. (a) $5x + 7y = 35$

or $\frac{x}{7} + \frac{y}{5} = 1$.........(i)

and $4x + 3y = 12$

or $\frac{x}{3} + \frac{y}{4} = 1$.........(ii)



$\therefore$ Base of ΔABC = BC = 7 – 3

= 4 unit

let height (OA) = h

from(i) and (ii) we get $y = \frac{80}{13}$

$\therefore ar(\Delta ABC)$

$= \frac{1}{2} \times 4 \times \frac{80}{13} = \frac{160}{13}$ sq.unit

76. (b) $3x - 6y = 12$ ....(i)

$3x - y = 3$ ....(ii)

(i) – (ii) we get, $-5y = 9$

or $y = -\frac{9}{5}$

$\Rightarrow$ Height of traingle $= \left|\frac{-9}{5}\right| = \frac{9}{5}$

put $y = 0$ in (i), we get $x = 4$

i.e. point of intersection of (i) on x-axis = (4,0)

put $y = 0$ in (ii), we get $x = 1$

i.e. point of intersection of (ii) on x-axis = (1, 0)

$\therefore$ Base = 4 – 1 = 3

area of $= \frac{1}{2} \times 3 \times \frac{9}{5}$

$= \frac{27}{10} = 2.7$ *sq.unit*

77. (c) Consider any point in the shaded portion

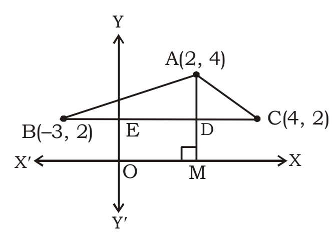
let (3, –3)

i.e. $x = 3, y = -3 \Rightarrow -3 < 3$ i.e. $y < x$

& when $x = 3, y = 3 \Rightarrow 3 = 3$ i.e. $y = x \quad \therefore y \leq x$

78. (a) Draw AM $\perp$ x-axis meeting BC at D.

Now, BC = BE + EC = 3 + 4 = 7 units

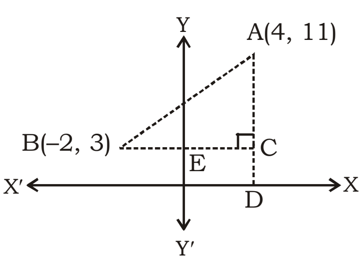


and AD = AM – DM = 4 – 2 = 2 units

$\therefore$ ar(Δ ABC)

$= \frac{1}{2} \times 2 \times 7 = 7$sq.unit

79.(d)



AC = AD – CD = 11 – 3 = 8 & BC = BE + EC = 2 + 4 = 6

$\therefore \quad AB = \sqrt{8^2 + 6^2} = 10$

Alternatively:-

Required distance =

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_1)^2}$

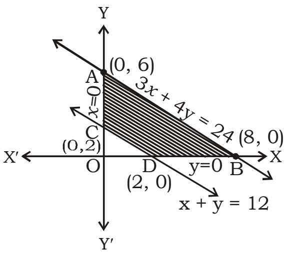
$\therefore$ AB = $\sqrt{(4+2)^2+(11-3)^2}$

$= \sqrt{6^2+8^2}$

or AB = $\sqrt{100} = 10$

80.(c) $3x + 4y = 24$ or $\frac{x}{8}+\frac{y}{6}=1$

i.e. it passes through (8, 0) and (0, 6)



Now, $x + y = 2$

put $x = 0$, $0 + y = 2$ or $y = 2$

again put $y = 0$, we get $x = 2$

i.e. it passes through (2, 0) & (0, 2)

$\therefore$ the required region is ABDC

Now, ar(ΔAOB)

$= \frac{1}{2}\times OB \times OA$

$= \frac{1}{2}\times 8\times 6 = 24$ sq.units

and ar(Δ OCD)

$= \frac{1}{2}\times OD \times OC$

$= \frac{1}{2}\times 2\times 2 = 2$ sq.units

$\therefore$ ar ($\square$ABDC) = 24 – 2 = 22 sq. units

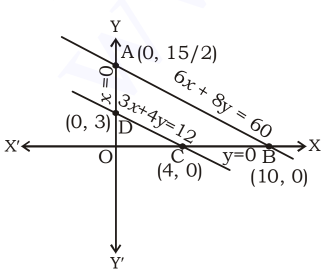
81.(b) $3x + 4y = 12$ or $\frac{x}{4}+\frac{y}{3}=1$

i.e. it passes through (4, 0) and (0, 3)

Similarly, $6x + 8y = 60$ or

$\frac{x}{10}+\frac{y}{15/2}=1$

i.e. it passes through (10, 0) and (0, 15/2)



$\therefore$ the required region is ABCD

Now,

ar (Δ OAB) $= \frac{1}{2}\times 10\times\frac{15}{2}$

= 37.5 sq. units

and ar(Δ ODC) $= \frac{1}{2}\times 4\times 3$

= 6 sq.units

$\therefore$ ar($\square$ABCD) = 37.5 – 6

= 31.5 units

82.(d) $x - y = 0$ or $x = y$

i.e. when $y = 3 \Rightarrow x = 3$

and $x + 2y = 0$

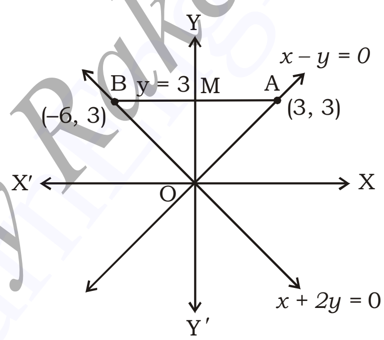
when $y = 3 \Rightarrow x = -6$

$\therefore$ the required region is a Δ OAB

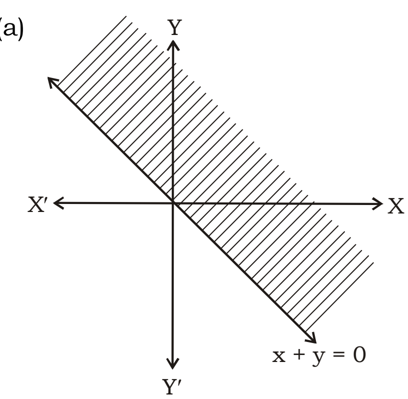
$\therefore$ ar(Δ OAB) $= \frac{1}{2}\times AB\times OM$

$= \frac{1}{2}\times 9\times 3$

$= \frac{27}{2} = 13.5$ sq.units



83.(a)



Consider the line $x + y = 0$

when, $x = 0$, $y = 0$ and

when, $x = 1$, $y = -1$

Now join the points (0, 0) & (1, –1) to get a line $x + y = 0$

Now, consider any point (1, 1)

Clearly (1, 1) satisfies the inequality, $x + y \geq 0$

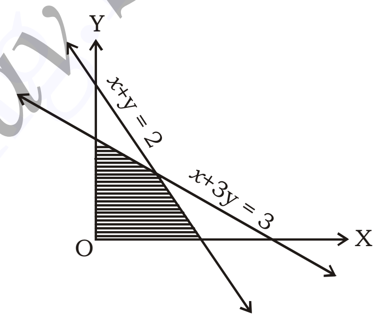
Shade the part of the plane containing (1, 1)

when $x \geq 0$, clearly, the first quadrant will be include as a whole.

84.(d) Clearly (0, 0) satisfies

$x + 3y \leq 3$

and (0, 0) satisfies $x + y \leq 2$



Clearly, shaded region is the portion common to the line x + 3y = 3 & below it and that of the line $x + y = 2$ and below it.

So shaded region is the solution set of (d)

85.(b) Clearly, the region is a rectangle bounded by the lines

$x = 2$, $x = 5$, $y = -1$ & y = 3

Clearly, the length of rectangle = 5 – 2 = 3 units

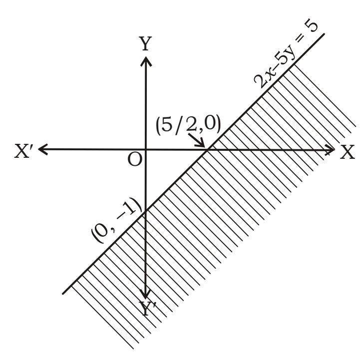
and the breadth of rectangle

= 3 – (–1) = 4 units

$\therefore$ Area of the rectangle = 3×4

= 12 sq.unit.

86.(c) $2x - 5y = 5$, or $\frac{x}{5/2}+\frac{y}{-1} = 1$



i.e. it passes through (5/2, 0) & (0, –1). Plot these points and join them with a thick line.

Clearly, (0, 0) does not satisfy $2x - 5y \geq 5$.

$\therefore$ Shade the portion of the plane not containing (0, 0)

So, required graph is on & below the line $2x - 5y = 5$

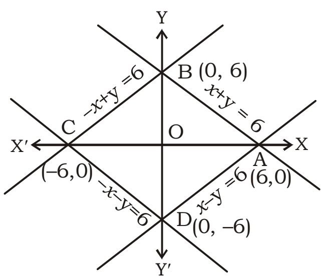
87.(a) As we know, area bounded by $|x| + |y| = k$ is $2k^2$

$\therefore$ area bounded by $|x| + |y| = 6$ is $2 \times 6^2 = 72$ sq. units

Alternatively :

$|x| + |y| = 6$, this represents four lines -

$x + y = 6$, $x - y = 6$, $-x + y = 6$ and $-x - y = 6$



Hence, the required region is ABCD.

Now ar(ΔOAB) = $\frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 6 \times 6$

$= 18$

$\therefore$ ar ($\square$ABCD) = $4 \times$ ar(ΔOAB)

$= 4 \times 18$

$= 72$ sq. units

88.(c) $y = |x| - 1$ represents two lines.

$y = x - 1$ ....(i)

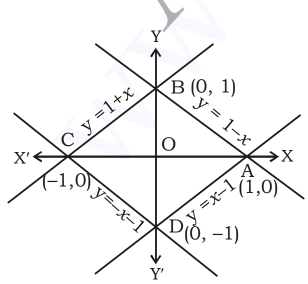
and $y = -x - 1$ ....(ii)

Similarly, $y = 1 - |x|$ represents two lines

$y = 1 - x$ ....(iii)

and $y = 1 + x$ ....(iv)

Now, draw the graph of (i) , (ii), (iii) & (iv)



$\therefore$ Required region is ABCD

Now ar(Δ OAB) $= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

$\therefore$ ar ($\square$ABCD) = $4 \times$ ar(Δ OAB)

$= 4 \times \frac{1}{2} = 2$ sq.units

89.(b) The point (–5, 7) lies in the second quadrant.

90.(d) The point (7, –5) lies in the fourth quadrant.

91.(a) Distance between two points

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

here $(x_1, y_1) = (-6, 2)$ and $(x_2, y_2) \equiv (2, 4)$

$\therefore$ Required distance

$= \sqrt{(-6-2)^2 + (2-4)^2}$

$= \sqrt{64+4} = \sqrt{68} = 2\sqrt{17}$unit

92.(b) $AB = \sqrt{(b-0)^2 + (0-a)^2}$

$= \sqrt{b^2 + a^2} = \sqrt{a^2 + b^2}$

93.(d) $AB^2 = (7-3)^2 + (4-1)^2$

$= 4^2 + 3^2$

$= 16 + 9 = 25$

$\Rightarrow AB = \sqrt{25} = 5$ units

94.(b) Clearly, the point of x-axis has ordinate 5 and abscissa 0.

So, the point is (5, 0)

95.(a) Clearly, the point on y-axis has abscissa 0.

So, the point is (0, 7)

96.(c) Clearly, the point is (0, – 6)

97.(d) Required distance

$= \sqrt{(6-0)^2 + (-8-0)^2}$

$= \sqrt{36+64} = \sqrt{100} = 10$ units

98.(a) $2x + 7y = 1$ .... (i)

$4x + 5y = 11$ ......(ii)

on solving (i) and (ii), we get $x = 4$ and $y = -1$

$\therefore$ Required point of intersection = (4, –1)

99.(c) Equation of x-axis is $y = 0$

put $y = 0$ in $4x + 7y = 12$ we get $x = 3$

$\therefore$ Required point = (3, 0)

100.(b) Equation of y-axis is $x = 0$

put $x = 0$ in $4x - 9y = 11$ we get $y = -\frac{11}{9}$

$\therefore$ Required point = $\left(0, -\frac{11}{9}\right)$

101.(c) $3x + 7y + 8 = 0 \Rightarrow 7y = -3x - 8$

$\Rightarrow y = \left(-\frac{3}{7}\right)x - \left(\frac{8}{7}\right)$

$\therefore$ Slope of the line is $= -\frac{3}{7}$

102.(c) Slope of PQ = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-7}{2-(-4)} = \frac{-4}{6} = \frac{-2}{3}$

103.(d) Clearly; the equation of the line is, $y = 6$

104.(b) Clearly, the equation of the line is, $x = -5$

105.(a) Clearly, the equation of the line is $y = -7$

106.(c)

$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

$\Delta = \frac{1}{2}\left|4(4+8) - 3(-4-5) + 3(5-8)\right|$

$= \frac{1}{2}|66| = 33$ sq. units

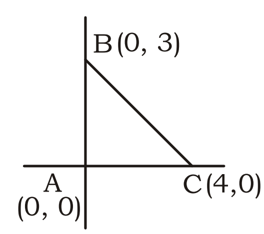
107.(b) $AB = \sqrt{(0+0)^2 + (0-3)^2} = 3$

$AC = \sqrt{(4-0)^2 + (0+0)^2} = 4$

and $BC = \sqrt{(4-0)^2 + (0-3)^2} = 5$

$\because$ $AB^2 + AC^2 = BC^2$

$\therefore$ ΔABC is a right angled triangle.



108.(d) The co-ordinates of the centroid of ΔPQR are -

$\left(\frac{-2+9+8}{3}, \frac{0-3+3}{3}\right) = (5, 0)$

109.(a) The required equation is

$$(y+3)=\frac{2+3}{-5-0}(x-0)$$

$\Rightarrow y+3=-x \Rightarrow x+y+3 = 0$

110.(c) Length of perpendicular =

$$=\frac{12\times0+5\times0+7}{\sqrt{12^2+5^2}}=\frac{7}{13} \text{ units}$$

111.(a) The slope of the line is

$$\frac{\sqrt{5}-1}{\sqrt{15}-\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$\therefore \tan\theta=\frac{1}{\sqrt{3}}$ or $\theta = 30°$

112.(c) Here,

$$\frac{a_1}{a_2}=\frac{2}{8}=\frac{1}{4},\ \frac{b_1}{b_2}=\frac{-5}{-20}=\frac{1}{4}$$

and $\frac{c_1}{c_2}=\frac{7}{28}=\frac{1}{4}$

$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$,

So the given lines are coincident.

113.(b) $\sqrt{(x-a)^2+(y-0)^2}=a+x$

$\Rightarrow (x-a)^2+y^2=(a+x)^2$

$\Rightarrow y^2=(x+a)^2-(x-a)^2$

$\Rightarrow y^2=4ax$

114.(c) Let P (x, y), Q$(a + b, b - a)$ and R$(a - b, a + b)$ are given points.

$\therefore$ PQ = PR.

$$\Rightarrow \sqrt{\{x-(a+b)\}^2+\{y-(b-a)\}^2}$$
$$=\sqrt{\{x-(a-b)\}^2+\{y-(a+b)\}^2}$$

$\Rightarrow x^2-2x(a+b)+(a+b)^2+y^2-2y(b-a)+(b-a)^2=x^2+(a-b)^2-2x(a-b)+y^2+(a+b)^2-2y(a+b)$

$\Rightarrow ax+bx+by-ay=ax-bx+ay+by$

$\Rightarrow 2bx=2ay$

$\Rightarrow bx=ay$

115.(d) Let A (x, y), P$(a, 0)$ and Q$(-a, 0)$, Then,

$\Rightarrow AP^2+AQ^2=2b^2$

$\Rightarrow [(x-a)^2+(y-0)^2]+[(x+a)^2+(y-0)^2]=2b^2$

$\Rightarrow x^2+a^2-2ax+y^2+x^2+a^2+2ax+y^2=2b^2$

$\Rightarrow 2(x^2+a^2+y^2)=2b^2$

$\Rightarrow x^2+a^2+y^2=b^2$

$\Rightarrow x^2+a^2=b^2-y^2$

116.(b) co-ordinates of middle point $=(-1, 4)$

$\therefore \frac{a+a+4}{2}=4$

$\Rightarrow 2a+4=8$

$\Rightarrow 2a=4$

$a=2$

117(c) Since, P,Q and R collinear

$\therefore$ slope of PQ = slope of PR

$\Rightarrow \frac{a-3}{5-2}=\frac{7-3}{6-2}\Rightarrow\frac{a-3}{3}=\frac{4}{4}$

$\Rightarrow a-3=3 \Rightarrow a=6$

118.(a) The equation of a line parallel to x-axis is $y=b$.

Since, it passes through $(-6, -5)$, so $b=-5$

$\therefore$ The required equation is, $y=-5$

119.(a) The equation of a line parallel to y-axis is, $x=a$.

Since, it passes through $(2, -5)$, so $a=2$

$\therefore$ The required equation is, $x=2$

120.(b) Let the co-ordinates of R be (x, y). Then,

$$\frac{-1+5+x}{3}=4 \text{ and } \frac{0-2+y}{3}=0$$

or $4+x=12$ and $-2+y=0$

or $x=8$ and $y=2$

$\therefore$ R $=(x, y)=(8, 2)$

121.(c) Since, point of intersection of median is "centroid".

$\therefore$ co-ordinates of centroid

$$=\left(\frac{0+5+7}{3},\frac{6+3+3}{3}\right)$$

$$=\left(\frac{12}{3},\frac{12}{3}\right)=(4,4)$$

122.(b) Let the ratio be k : 1

The ordinate of a point lying on x-axis must be zero

$\therefore \frac{4k-5\times1}{k+1}=0\Rightarrow 4k=5\Rightarrow k=\frac{5}{4}$

$\therefore$ Required ratio is $\frac{5}{4}:1=5:4$

123.(a) Let the ratio be k : 1

The abcissa of a point lying on y-axis must be zero

$\therefore \frac{7k-3\times1}{k+1}=0$

$\Rightarrow 7k-3=0$

$\Rightarrow k=\frac{3}{7}$

$\therefore$ Required ratio is $\frac{3}{7}:1=3:7$

124.(c) Let the ratio be k : 1

$\therefore \frac{3k-3\times1}{k+1}=1$

$\Rightarrow 3k-3=k+1$

$\Rightarrow 2k=4 \Rightarrow k=2$

$\therefore$ Required ratio is 2 : 1

125.(a) Let the equation be $y=5x+c$

Since it passes through $(-4, 1)$, we have $1=5(-4)+c$

$\therefore$ $c=21$ so, its equation is, $y=5x+21$

126.(c) Condition of parallelism

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}$$

$\therefore \frac{1}{a}=\frac{3}{12}\Rightarrow a=4$

Alternatively,

$x+3y-8+0$

$\Rightarrow y=\left(-\frac{1}{3}\right)x+\left(\frac{8}{3}\right)$

$\therefore m_1=-\frac{1}{3}$

$ax+12y+5+0=0$

$\Rightarrow y=\left(-\frac{a}{12}\right)x-\frac{5}{12}$

$\therefore m_2=\frac{a}{12}$

for parallelism, $m_1=m_2$

$\therefore -\frac{1}{3}=-\frac{a}{12}\Rightarrow a=4$

127.(b) Condition of perpend icularism,

$a_1a_2+b_1b_2=0$

$\therefore 3\times24+8\times p=0$

$\Rightarrow 8p=-3\times24\Rightarrow p=-9$

Alternatively

$3x+8y+9=0 \Rightarrow$

$y=\left(-\frac{3}{8}\right)x-\frac{9}{8}\ \therefore m_1=-\frac{3}{8}$

$24x + py + 19 = 0$

$\Rightarrow y = \left(-\frac{24}{p}\right)x - \frac{19}{p}$

$\therefore m_2 = \frac{24}{p}$

for perpendicularism, $m_1.m_2 = -1$

$\therefore \left(-\frac{3}{8}\right)\left(-\frac{24}{p}\right) = -1$

$\Rightarrow P = -9$

128.(d) m_1 = Slope of PQ

$= \frac{-7-5}{0+2} = \frac{-12}{2} = -6$

m_2 = Slope of AB $= \frac{a+2}{8+4} = \frac{a+2}{12}$

$\therefore m_1m_2 = -1 \Rightarrow -6 \times \frac{a+2}{12} = -1$

$\Rightarrow a + 2 = 2 \Rightarrow a = 0$

129.(a) $2y - \sqrt{12}.x - 9 = 0$

$\Rightarrow y = \frac{\sqrt{12}}{2}x + \frac{9}{2}$

$\Rightarrow y = \frac{\sqrt{12}}{2}x + \frac{9}{2}$

$\Rightarrow m_1 = \frac{\sqrt{12}}{2} = \sqrt{3}$

$\sqrt{3}y - x + 7 = 0$

$\Rightarrow m_2 = \frac{1}{\sqrt{3}}$

$\therefore \tan\theta = \frac{m_1 - m_2}{1+m_1.m_2} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+\sqrt{3}.\frac{1}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$

So, $\theta = 30°$

130. (b) Slope of PQ,

$m_1 = \frac{5-5}{4-3} = 0$

Slope of PR, $m_2 = \frac{6-5}{4-3} = 1$

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right| = \left|\frac{0-1}{1+0}\right| = 1$

So, $\theta = 45°$

131. (c) Clearly, M is the mid-point of QR.

$\therefore$ Co-ordinates of M are

$\left(\frac{-3-1}{2}, \frac{7-3}{2}\right)$

i.e.M (–2, 2)

Now, find the equation of the line joining P(2, 3) and M(–2, 2)

Required equation is , $(y - 3)$

$= \frac{2-3}{-2-2}(x-2)$

$\Rightarrow y - 3 = \frac{1}{4}(x-2)$

$\Rightarrow 4y - 12 = x - 2$

$\Rightarrow x - 4y + 10 = 0$

132. (a) Required point is :

$\left(\frac{3\times3+2\times\frac{11}{2}}{3+2}, \frac{3\ (-2)+2\times\frac{21}{2}}{3+2}\right)$

$= \left(\frac{20}{5}, \frac{15}{5}\right) = (4, 3)$

133. (d) Point of intersection at x-axis

$= (x, 0)$

$\therefore$ $8x + 15y = 120$

$\Rightarrow$ $8x + 15 \times 0 = 120 \Rightarrow x = 15$

$\Rightarrow$ point ot intersection = (15, 0)

Point of intersection at y-axis

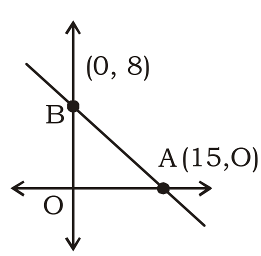
= (0, y)

$\therefore$ $8x + 15y = 120$

$\Rightarrow$ $0 + 15y = 120 \Rightarrow y = 8$

$\Rightarrow$ Point of intersection = (0, 8)

$\therefore$ Required length = AB



$= \sqrt{(15-0)^2 + (0-8)^2}$

$= \sqrt{225+64} = \sqrt{289}$

$= 17$ units

134.(d) Given line - $3x + 4y - 5 = 0$

$\Rightarrow y = \left(-\frac{3}{4}\right)x + \frac{5}{4}$

$\therefore$ its slope, $m_1 = -\frac{3}{4}$

Let m_2 be the slope of required line.

Then, $m_1m_2 = -1$ or $\left(-\frac{3}{4}\right)$ m2 = –1

$\Rightarrow m_2 = \frac{4}{3}$

Let the required equation be, $y = m_2x + c$

$\Rightarrow y = \frac{4}{3}x + c$

Since, it passes through (1, 1)

$\therefore 1 = \frac{4}{3} \times 1 + c \Rightarrow c = 1 - \frac{4}{3} = -\frac{1}{3}$

$\therefore$ The required equation is ,

$y = \frac{4}{3}x - \frac{1}{3}$

or $4x - 3y - 1 = 0$

135.(b) $2x - 5y + 3 = 0$

$\Rightarrow y = \left(\frac{2}{5}\right)x + \left(\frac{3}{5}\right)$

$\therefore$ its slope $m_1 = \frac{2}{5}$

Let the slope of line which is parallel to the given line is m_2

$\therefore m_2 = m_1 = \frac{2}{5}$

Let the required equation be,

$y = \frac{2}{5}x + c$

Since, it passes through (5, 3)

$\therefore 3 = \frac{2}{5} \times 5 + c \Rightarrow c = 1$

Required equation is,

$y = \frac{2}{5}x + 1$

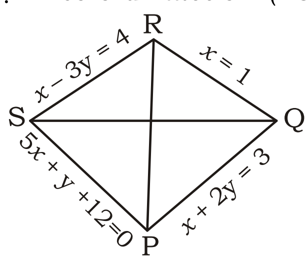
or $2x - 5y + 5 = 0$

136.(d) $x + 2y = 3$ .... (i)

$5x + y = -12$ ....(ii)

On solving (i) and (ii), we get $x = -3$, $y = 3$

$\therefore$ co-ordinates of P(– 3, 3)



Similarly, Q(1, 1), R(1, –1) and S(–2, 2)

Now, m_1 = slope of PR

$= \frac{-1-3}{1+3} = -1$

m_2 = slope of QS

$= \frac{-2-1}{-2-1} = 1$

$\because$ $m_1 m_2 = -1$

$\therefore$ the required angle is 90°

137.(c) $\because$ Third side passes through (1, –10)

so its equation $y + 10 = m(x - 1)$ ....(i)

This side makes equal angle with the given two sides.

let this angle be θ.

Now, slope of line $7x - y + 3 = 0$ is m_1,

$\therefore$ $m_1 = 7$

and slope of line $x + y - 3 = 0$ is m_2,

$\therefore$ $m_2 = -1$

angle between (i) and $7x - y + 3 = 0$ = angle between (i) and $x + y - 3 = 0$

$\therefore$ $\tan\theta = \frac{m-7}{1+7m} = \frac{m-(-1)}{1+m(-1)}$

$\Rightarrow m = -3$ or $1/3$

Hence possible equations of third side are $y + 10 = -3(x - 1)$

and $y + 10 = \frac{1}{3}(x-1)$

or $3x + y + 7 = 0$

and $x - 3y - 31 = 0$

138.(a) P_1 = length of perpendicular from (0, 0) on $x \sec\theta + y \operatorname{cosec}\theta = a$

$\therefore P_1 = \frac{a}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}$

$= \frac{a}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}}$

$= a\sin\theta.\cos\theta$

or $2P_1 = a(2\sin\theta.\cos\theta)$

$\Rightarrow$ $2P_1 = a\sin 2\theta$

Similarly,

$P_2 = \frac{a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} = a\cos 2\theta$

$\therefore 4P_1^2 + P_2^2 = a^2$

$\left(\sin^2 2\theta + \cos^2 2\theta\right) = a^2$

139.(d) Let a and b are the intercepts on x and y-axes respectively.

$\therefore$ a + b = 9 $\Rightarrow$ b = 9 – a ....(i)

and the equation of the line is

$\frac{x}{a} + \frac{y}{b} = 1$ ....(ii)

From (i) and (ii)

$\frac{x}{a} + \frac{y}{9-a} = 1$ ......(iii)

this line also passes through the point (2,2)

$\therefore$ from (iii) $\frac{2}{a} + \frac{2}{9-a} = 1$

On solving we get a = 6

or a = 3

If a = 6 then b = 9 – 6 = 3

$\therefore$ equation of the line is $\frac{x}{6} + \frac{y}{3} = 1$

or $x + 2y - 6 = 0$

If a = 3 then b = 9 – 3 = 6

$\therefore$ equation of the line is $\frac{x}{3} + \frac{y}{6} = 1$

or $2x + y - 6 = 0$

Hence, required equation is

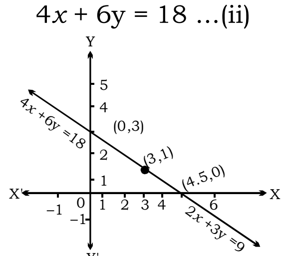
$x + 2y - 6 = 0$ or $2x + y - 6 = 0$

Note : Solve this type of question with the help of given options.

140. Let the C.P. of Pencil and Eraser be x and y

$2x + 3y = 9$ ......(i)

$4x + 6y = 18$ ...(ii)



After solving equation (i)

x	0	4.5
y	3	0

$2x + 3y = 9$

After solving equation(ii)

x	0	3
y	3	1

$4x + 6y = 18$

141. let the present age of Aftab and his daughter will by x and y Respectively.

	Aftab	Daughter
Present age	x	y

Before 7 yrs,

$x - 7 = 7(y - 7)$ ....(i)

Present age

After 3 yrs,

$x + 3 = 3(y + 3)$ ....(ii)

After solving eq. (i)

$= x - 7 = 7y - 49$

$x - 7y = -42$

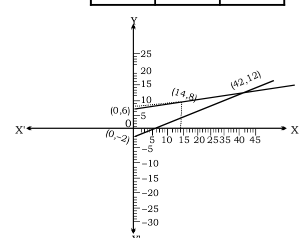
x	0	14
y	6	8

After Eq. (ii)

$= x + 3 = 3y + 9$

$x - 3y = 6$

x	0	6
y	–2	0



The two liner Intersect each other at (42,12) Present age of after and his daughter will be 42 yrs and 12 yrs respactively

142. Cost of 1 Pen be x and 1 Pencile be y $5x + 7y = 50$ ...(i)

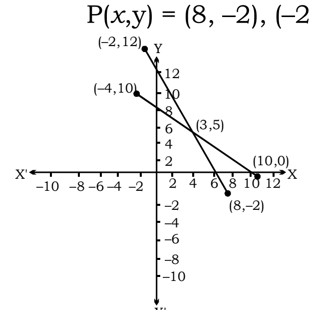
$7x + 5y = 46$ (ii)

From eq. (i) value of (x,y) after solving

$= P(x,y) = (10, 0), (-4,10)$

eq.(ii)

$P(x,y) = (8, -2), (-2,12)$



Cost of Pen = ₹ 3

Pencile = ₹ 5

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