

①

$$X = \begin{bmatrix} -2 \\ -5 \\ -3 \\ 0 \\ -8 \\ -2 \\ 1 \\ 5 \\ -1 \\ 6 \end{bmatrix} \quad Y = \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

$$\theta = (X^T X)^{-1} X^T Y$$

$$y = \theta_0 x_0 + \theta_1 x_1$$

\downarrow
 1

Solution

$$X = \begin{bmatrix} x_0 & x_1 \\ 1 & -2 \\ 1 & -5 \\ 1 & -3 \\ 1 & 0 \\ 1 & -8 \\ 1 & -2 \\ 1 & 1 \\ 1 & 5 \\ 1 & -1 \\ 1 & 6 \end{bmatrix} \quad \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}_{10 \times 1}$$

$$X^T Y = \begin{bmatrix} 14 \\ -79 \end{bmatrix}_{2 \times 1}$$

$$(X^T X) \theta = X^T Y$$

$$\begin{bmatrix} 10 & -9 \\ -9 & 169 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 14 \\ -79 \end{bmatrix}$$

$$10\theta_0 - 9\theta_1 = 14$$

$$-9\theta_0 + 169\theta_1 = -79$$

Given θ ,

$$X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -5 & -3 & 0 & -8 & -2 & 1 & 5 & -1 & 6 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -2 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

$$\theta_0 = \frac{1655}{1609} \quad \theta_1 = \frac{-664}{1609}$$

$$X^T X = \begin{bmatrix} 10 & -9 \\ -9 & 169 \end{bmatrix}_{2 \times 2}$$

$$(2a) J = (x_1 + x_2 - 2)^2$$

$$\frac{dJ}{dx_1} = 2(x_1 + x_2 - 2) \quad (1)$$

$$\frac{dJ}{dx_1} = 2x_1 + 2x_2 - 4$$

$$\frac{dJ}{dx_2} = 2(x_1 + x_2 - 2) \quad (1)$$

$$\frac{dJ}{dx_2} = 2x_1 + 2x_2 - 4$$

(2b) Done in Jupyter notebook

(2c)

J for $x_2 = 0, x_1 = 2$ is 0

J for $x_2 = 1, x_1 = 1$ is 0

J for $x_2 = 2, x_1 = 0$ is 0

Part 2

- Done in Jupyter notebook

Part 3

- Done in Jupyter notebook

Part 4 - Done in Jupyter notebook