1] (a)

$$X = \begin{array}{cc} x_1 & x_2 \\ -2 & 1 \\ -5 & -4 \\ -3 & 1 \\ 0 & 3 \\ -8 & 11 \\ -2 & 5 \\ 1 & 0 \\ 5 & -1 \\ -1 & -3 \\ 6 & 1 \end{array} \quad 10 \times 2$$

$$\mu_1 = \frac{-2-5-3-8-2+1+5-1+6}{10} = -0.9$$

$$\mu_2 = \frac{1-4+1+3+11+5+0-1-3+1}{10} = 1.4$$

$$\sigma_1 = 4.228212$$

$$\sigma_2 = 4.273952$$

new standardized data

$$X = \begin{array}{cc} x_1 & x_2 \\ -0.260157 & -0.093590 \\ -0.969677 & -1.263468 \\ -0.496664 & -0.093590 \\ 0.212856 & 0.374361 \\ -1.679197 & 2.246165 \\ -0.260157 & 0.842312 \\ 0.449363 & -0.327566 \\ 1.395389 & -0.561541 \\ -0.023651 & -1.029492 \\ 1.631895 & -0.093590 \end{array}$$

Now computing co-variance matrix,

$$\Sigma(X) = \frac{X^T X}{N-1}$$

$$= \begin{vmatrix} 1.00 & -0.408262 \\ -0.408262 & 1.00 \end{vmatrix}$$

Now for the eigen values.

$$|\Sigma - \lambda I| = 0$$

$$\begin{vmatrix} 1 & -0.408262 \\ -0.408262 & 1 \end{vmatrix} - \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & -0.408262 \\ -0.408262 & 1-\lambda \end{vmatrix} = 0$$

find the determinant:

$$(1-\lambda)(1-\lambda) - (-0.408262)^2 = 0$$

$$1-\lambda-\lambda+\lambda^2 - 0.166677 = 0$$

$$\lambda^2 - 2\lambda + 0.8333 = 0$$

Use $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$\frac{-(-2) \pm \sqrt{4 - 4(0.8333)}}{2a} = \lambda$$

$$\frac{2 \pm \sqrt{0.6668}}{2} = \lambda$$

$\lambda_1 = \frac{2 + \sqrt{0.6668}}{2}$, $\lambda_2 = \frac{2 - \sqrt{0.6668}}{2}$

$\lambda_1 = 1.4083$ $\lambda_2 = 0.5917$

for λ_1

$$\left[\begin{vmatrix} 1 & -0.4083 \\ -0.4083 & 1 \end{vmatrix} - \begin{vmatrix} 1.4083 & 0 \\ 0 & 1.4083 \end{vmatrix}\right] \cdot \begin{vmatrix} x \\ y \end{vmatrix} = 0$$

$$\begin{vmatrix} -0.4083 & -0.4083 \\ -0.4083 & -0.4083 \end{vmatrix} \begin{vmatrix} x \\ y \end{vmatrix} = 0$$

$-0.4083x - 0.4083y = 0$

$-0.4083x - 0.4083y = 0$

$x = -y$

for λ_2

$x = y$

Hence for λ_1 eigenvector $= \begin{vmatrix} x \\ -x \end{vmatrix}$, and for λ_2 eigenvector $= \begin{vmatrix} x \\ x \end{vmatrix}$

(1b)

projection $= \underset{10\times 2}{X}$ @ (eigenvector of x_1)$_{2\times 1}$

assume eigenvector of λ_1

$= \begin{vmatrix} 0.7071 \\ -0.7071 \end{vmatrix}$

projection $= \begin{vmatrix} -0.11778 \\ 0.2077 \\ -0.2850 \\ -0.1142 \\ -2.7757 \\ -0.7796 \\ 0.5494 \\ 1.3838 \\ 0.7112 \\ 1.2201 \end{vmatrix}$