MSEN660 HW1

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1 Assignment1

1.1 (a)

The mean and covariance matrices are

$$\Sigma_0 = \Sigma_1 = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

$$\mu_0 = \begin{pmatrix} 0 & 0 \end{pmatrix}$$
$$\mu_1 = \begin{pmatrix} 1 & 1 \end{pmatrix}$$

The prior probabilities P(Y=0) and P(Y=1) =.5. The optimal classifier in the Gaussian equal variance case is a hyperplane whose error is given by:

$$\epsilon^* = \phi(-\frac{1}{2} * \delta) \tag{1}$$

Where ϕ is the cdf of the Gaussian and δ is the Mahlanobis distance between the classes:

$$\delta^2 = (\mu_1 - \mu_0)^T \Sigma^{-1} (\mu_1 - \mu_0) \tag{2}$$

We get:

$$\Sigma^{-1} = \frac{1}{\sigma^2(1-\sigma^2)} \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix} \tag{3}$$

from here we use (2) and (3) to get:

$$\delta^2 = \begin{pmatrix} 1 & 1 \end{pmatrix} \frac{1}{\sigma^2(1-\rho^2)} \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\delta^2 = \begin{pmatrix} 1 & 1 \end{pmatrix} \frac{}{\sigma^2(1-\rho)(1+\rho)} \begin{pmatrix} 1-\rho \\ 1-\rho \end{pmatrix}$$

$$\delta^2 = \begin{pmatrix} 1 & 1 \end{pmatrix} \frac{1}{\sigma^2(1+\rho)} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\delta^2 = \frac{2}{\sigma^2(1+\rho)}$$

And so we have the optimal classification error by (1) :

$$\epsilon^* = \phi\Big(-\frac{1}{\sqrt{2}\sigma\sqrt{1+\rho}}\Big) \tag{4}$$

1.2 (b)

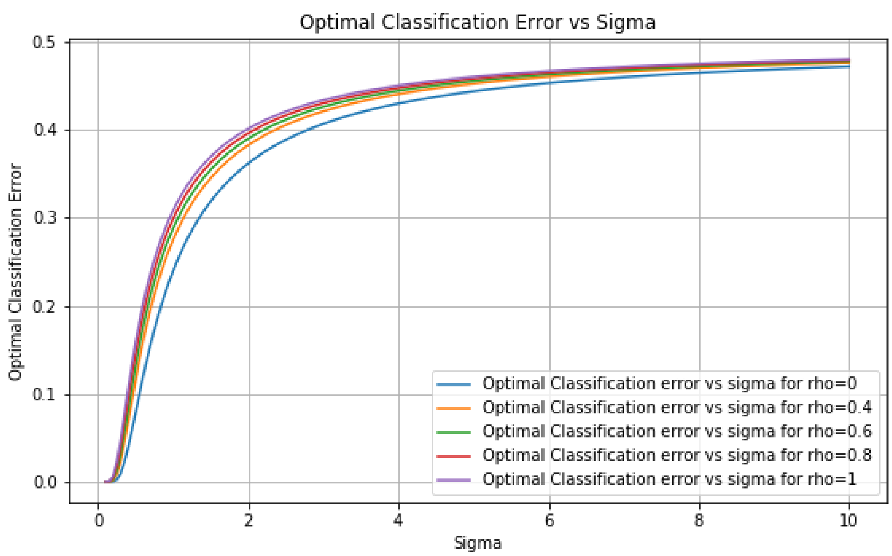


Figure 1: Optimal Classification Error for different ρ and σ

We see that the optimal error quickly increases with σ. This can be intuitively understood as the two classes becoming more spread out about their means. This would imply increased difficulty in separating them.

1.3 (c)

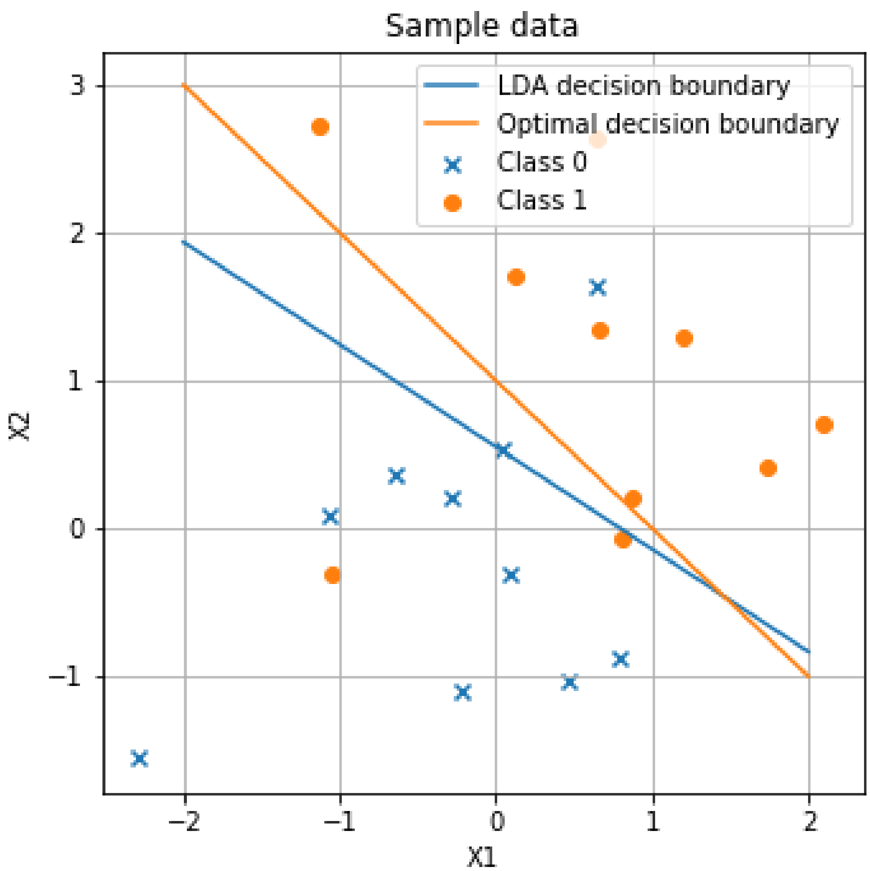


Figure 2: LDA decision boundary separating the two generated normal distributions for the classes

We see that the Optimal decision boundary separates the classes better, as to be expected. We expect that as the training size increases the LDA decision boundary should approach that of the optimal classifier. This is because for the optimal classifier we know the ground truth (optimal classifier) and as we get more and more samples we can better understand the ground truth(LDA).

1.4 (d)

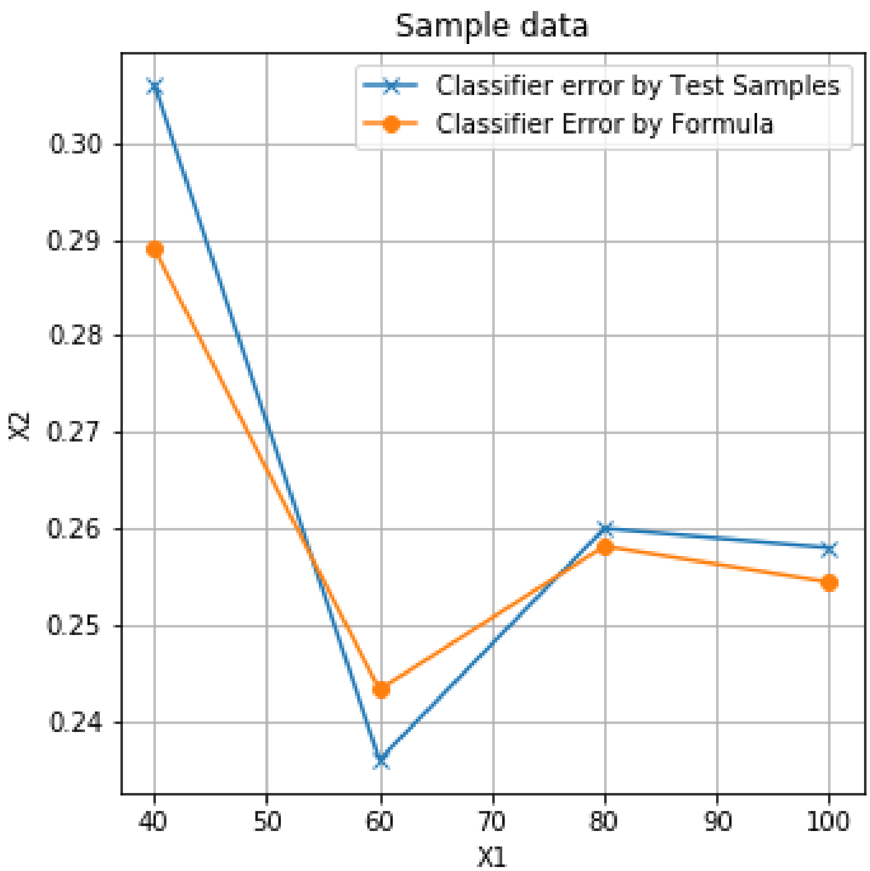


Figure 3: Classification error (y axis) vs training size (x axis)

Here we see that the classification error on the test set decreases as the training set size increases. This is to be expected. But it should be noted that the plot is very jumpy and erratic. This may have to do with the smaller training size.

2 Assignment2

2.1 (a)

	C	N	Ni	Fe	Mn	Si	Cr	SFE
0	0.04100	0.0540	8.10	70.87500	1.7100	0.3300	18.20	0.0
1	0.00400	0.0030	15.60	64.31700	0.0300	0.0200	17.50	1.0
2	0.07000	0.5400	16.13	54.67800	9.6400	0.4500	18.48	1.0
3	0.40000	0.0660	16.30	54.66000	9.6400	0.4500	18.48	0.0
4	0.00004	0.0001	12.00	71.99952	0.0001	0.0002	16.00	0.0

Figure 4: Sample of the training set after pre processing

2.2 (b)

```
[Ttest_indResult(statistic=-3.6652244046944773, pvalue=0.0013486369196097205), 'Ni']
[Ttest_indResult(statistic=2.359906793878512, pvalue=0.027438300016957032), 'Fe']
[Ttest_indResult(statistic=2.1848241668704795, pvalue=0.04179478729923675), 'Si']
[Ttest_indResult(statistic=1.4943828075999175, pvalue=0.15926508688174165), 'C']
[Ttest_indResult(statistic=-1.0278205792662476, pvalue=0.31477403802244214), 'Cr']
[Ttest_indResult(statistic=0.80913241119481, pvalue=0.4267305733623522), 'Mn']
[Ttest_indResult(statistic=0.2829267844867566, pvalue=0.780037720226723 2), 'N']
```

Figure 5: Sorted table of predictors with t statistics and p values after Welch's two sample t test

2.3 (c)

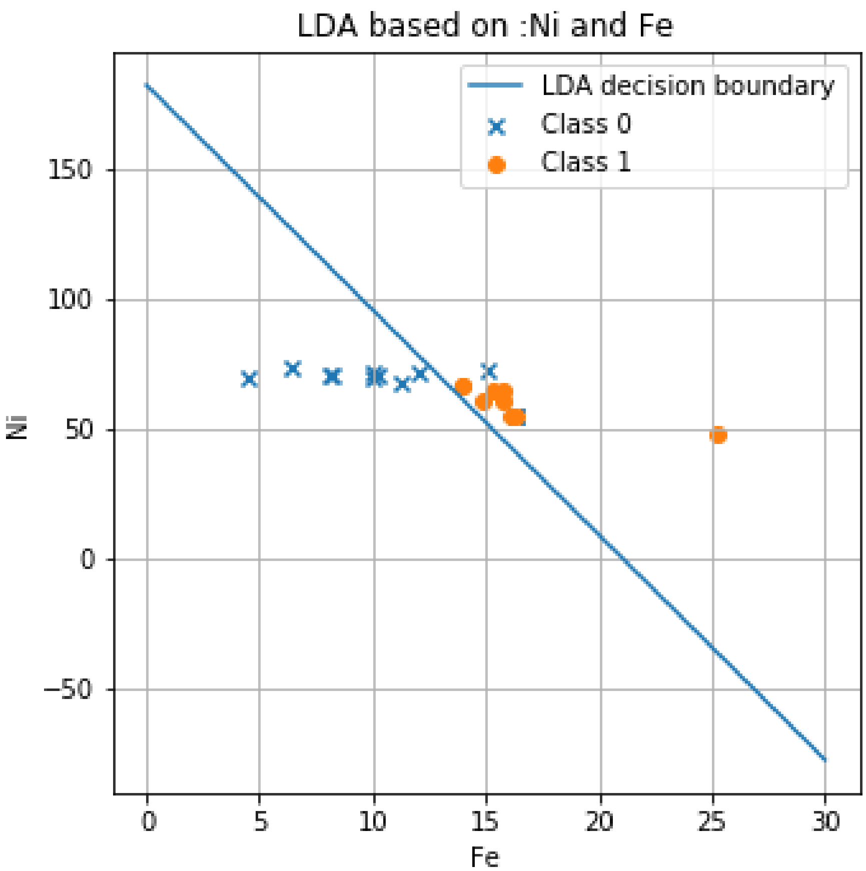


Figure 6: LDA decision boundary on two classes created using top two predictors

2.3.1 Estimated LDA error rate

The error for this LDA classifier is estimated by predicting on the test data. We get an error rate of 0.0792.

2.4 (d)

We now repeat this procedure for the top three,four and five predictors and estimate the classification error on the test set.

2.4.1 Three Predictors Ni,Fe and Si

We obtain a classification error of 0.4257

2.4.2 Four Predictors Ni,Fe,Si and C

We obtain a classification error of 0.4653

2.4.3 Four Predictors Ni,Fe,Si,C and Cr

We obtain a classification error of 0.4752

3 Code

3.1 Assignment 1 Code

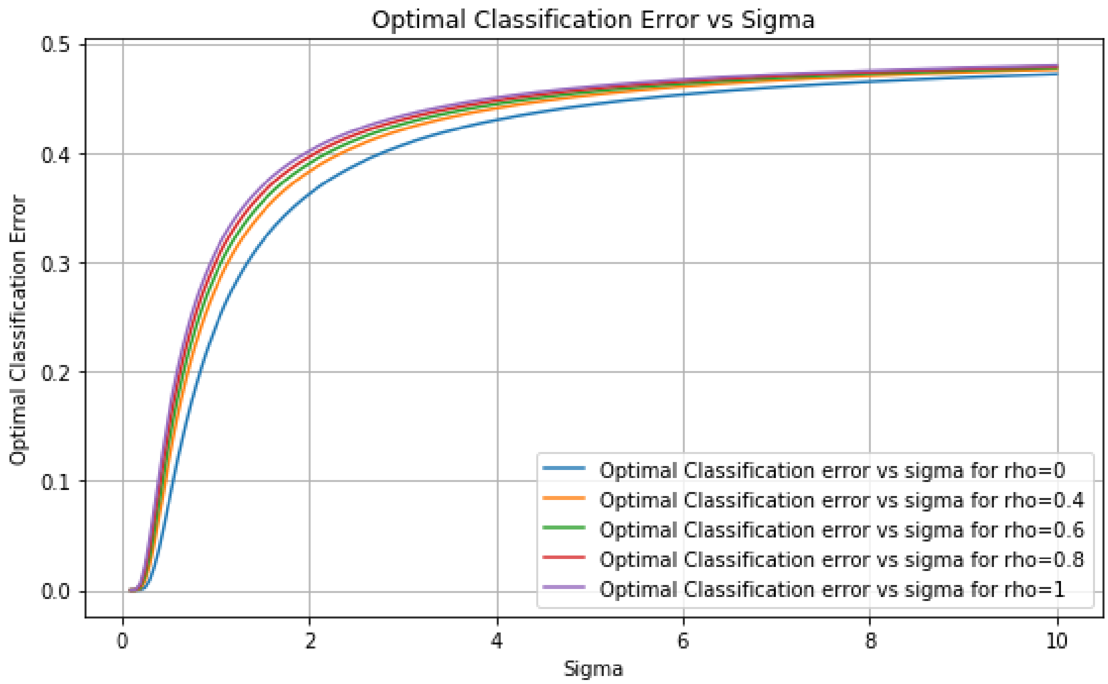
HW1_1

October 16, 2018

In [34]:
```
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
fig, ax = plt.subplots(figsize=[8,5])

x=np.linspace(stats.norm.cdf(0.01),stats.norm.cdf(0.99),100)
print(x)

rhoarray=list([0,0.4,0.6,0.8,1])
sigmarr=np.linspace(.1,10,num=200)
err=np.zeros((len(rhoarray),len(sigmarr)))
#import pdb; pdb.set_trace()
for i in range(0,len(rhoarray)):
    for j in range(0,len(sigmarr)):
        err[i,j]=stats.norm.cdf(-1/(np.sqrt(2)*sigmarr[j]*np.sqrt(1+rhoarray[i])))
    ax.plot(sigmarr,err[i,:],label='Optimal Classification error vs sigma for rho='+s
    ax.legend()
plt.title('Optimal Classification Error vs Sigma')
plt.ylabel('Optimal Classification Error')
plt.xlabel('Sigma')
fig.tight_layout()
ax.legend()
plt.grid(True)
plt.show
fig.savefig('hw1b.png')
```

```
[0.50398936 0.50737242 0.51075549 0.51413856 0.51752162 0.52090469
 0.52428776 0.52767082 0.53105389 0.53443695 0.53782002 0.54120309
 0.54458615 0.54796922 0.55135229 0.55473535 0.55811842 0.56150149
 0.56488455 0.56826762 0.57165069 0.57503375 0.57841682 0.58179989
 0.58518295 0.58856602 0.59194909 0.59533215 0.59871522 0.60209829
 0.60548135 0.60886442 0.61224748 0.61563055 0.61901362 0.62239668
 0.62577975 0.62916282 0.63254588 0.63592895 0.63931202 0.64269508
 0.64607815 0.64946122 0.65284428 0.65622735 0.65961042 0.66299348
 0.66637655 0.66975962 0.67314268 0.67652575 0.67990881 0.68329188
 0.68667495 0.69005801 0.69344108 0.69682415 0.70020721 0.70359028
 0.70697335 0.71035641 0.71373948 0.71712255 0.72050561 0.72388868
 0.72727175 0.73065481 0.73403788 0.73742095 0.74080401 0.74418708
```

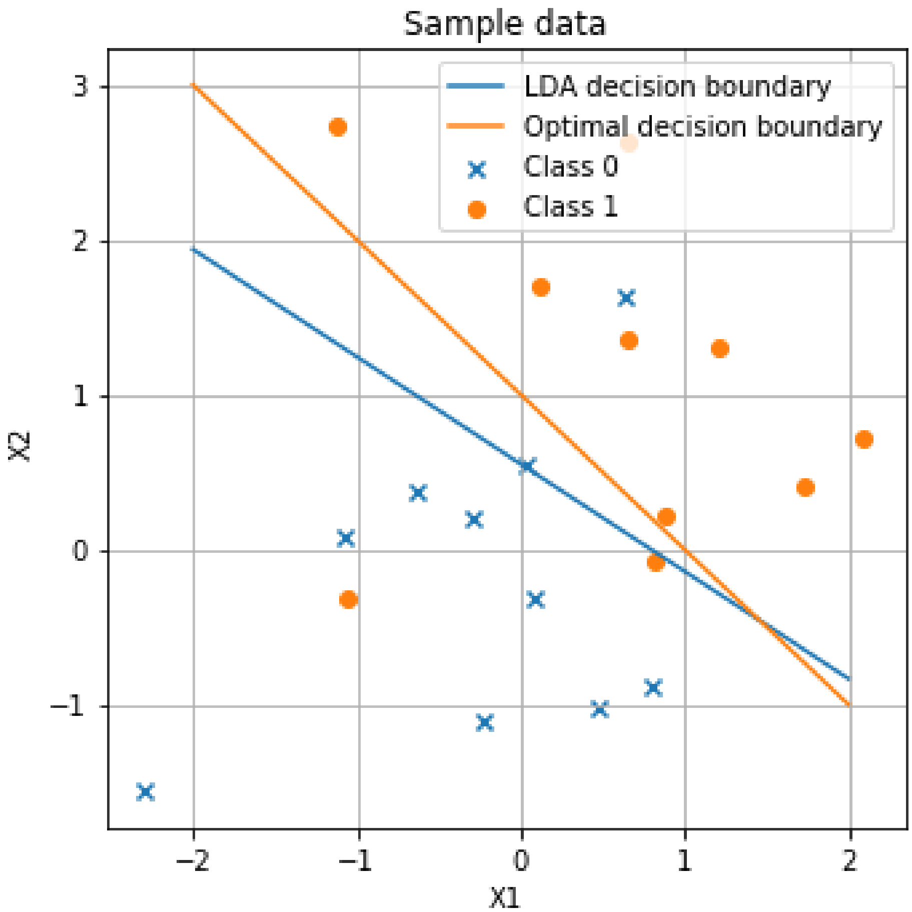
```
0.74757014 0.75095321 0.75433628 0.75771934 0.76110241 0.76448548
0.76786854 0.77125161 0.77463468 0.77801774 0.78140081 0.78478388
0.78816694 0.79155001 0.79493308 0.79831614 0.80169921 0.80508228
0.80846534 0.81184841 0.81523147 0.81861454 0.82199761 0.82538067
0.82876374 0.83214681 0.83552987 0.83891294]
```



In [4]:
```
sigma=1
rho=0.2
u1=np.array([0,0])
u2=np.array([1,1])
cov=np.array([[sigma**2,rho*sigma**2],[rho*sigma**2,sigma**2]])
print(cov)

#create sample two guassian distributions for each mean
x1=np.random.multivariate_normal(u1,cov,10)
x2=np.random.multivariate_normal(u2,cov,10)
xs=np.concatenate((x1,x2),axis=0)

#designed lda classifier
smean1=np.mean(x1,axis=0)
smean2=np.mean(x2,axis=0)
scov1=np.cov(x1.T,rowvar=True)
scov2=np.cov(x2.T,rowvar=True)
scov=(scov1+scov2)/2
```

```python
scov_inv=np.linalg.inv(scov)
a_lda=scov_inv.dot((smean2-smean1))
b_lda=-0.5*((smean2-smean1).dot(scov_inv)).dot((smean1+smean2))
x1_lda=np.linspace(-2,2,num=20)
y_lda=-a_lda[0]/a_lda[1]*x1_lda-b_lda/a_lda[1]

#optimal classifier
cov_inv=np.linalg.inv(cov)
a_opt=cov_inv.dot((u2-u1))
b_opt=-0.5*((u2-u1).dot(cov_inv)).dot((u1+u2))
x1_opt=np.linspace(-2,2,num=20)
y_opt=-a_opt[0]/a_opt[1]*x1_opt-b_opt/a_opt[1]

#import pdb; pdb.set_trace()
fig, ax = plt.subplots(figsize=[5,5])
plt.scatter(x1[:,0],x1[:,1],marker='x',label='Class 0')
plt.scatter(x2[:,0],x2[:,1],marker='o',label='Class 1')
plt.plot(x1_lda,y_lda,label='LDA decision boundary')
plt.plot(x1_opt,y_opt,label='Optimal decision boundary')
plt.title('Sample data')
plt.ylabel('X2')
plt.xlabel('X1')
fig.tight_layout()
ax.legend()
plt.grid(True)
plt.show
fig.savefig('hw1c.png')
```

```
[[1.  0.2]
 [0.2 1. ]]
```

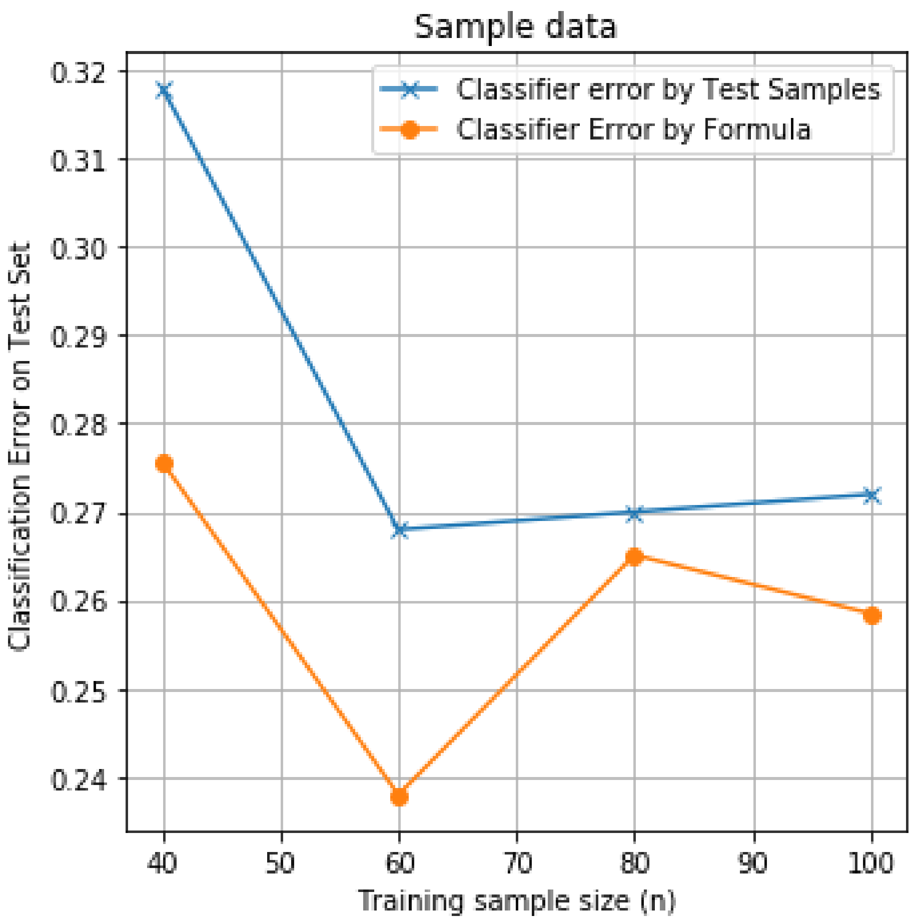


In [36]:
```
#part d

sigma=1
rho=0.2
u1=np.array([0,0])
u2=np.array([1,1])
cov=np.array([[sigma**2,rho*sigma**2],[rho*sigma**2,sigma**2]])
print(cov)

nlist=np.linspace(40,100,num=4)

test_error=np.zeros(len(nlist))
err_lda=np.zeros(len(nlist))
fig, ax = plt.subplots(figsize=[5,5])

# generate test set
x1_test=np.random.multivariate_normal(u1,cov,250)
x2_test=np.random.multivariate_normal(u2,cov,250)
```

```
for t in range(0,len(nlist)) :
    #import pdb; pdb.set_trace()
    #create sample two guassian distributions for each mean
    x1=np.random.multivariate_normal(u1,cov,int(nlist[t]/2))
    x2=np.random.multivariate_normal(u2,cov,int(nlist[t]/2))


    #train on training set
    smean1=np.mean(x1,axis=0)
    smean2=np.mean(x2,axis=0)
    scov1=np.cov(x1.T,rowvar=True)
    scov2=np.cov(x2.T,rowvar=True)
    scov=(scov1+scov2)/2
    scov_inv=np.linalg.inv(scov)
    a_lda=scov_inv.dot((smean2-smean1))
    b_lda=-0.5*((smean2-smean1).dot(scov_inv)).dot((smean1+smean2))
    err1=0

    #estimate error on test set
    for i in range(0,len(x1_test)):
        g1=a_lda.dot(x1_test[i])+b_lda
        if (g1>0):
            err1+=1
    err2=0
    for i in range(0,len(x2_test)):
        g2=a_lda.dot(x2_test[i])+b_lda
        if (g2<=0):
            err2+=1

    test_error[t]=(err1+err2)/500

    #obtaining error by formula for LDA
    err_lda[t]=0.5*(stats.norm.cdf((a_lda.dot(u1)+b_lda)/np.sqrt(a_lda.dot(scov.dot(a

   # print('Test error : ',test_error)

    #import pdb; pdb.set_trace()

plt.plot(nlist,test_error,marker='x',label='Classifier error by Test Samples')
plt.plot(nlist,err_lda,marker='o',label='Classifier Error by Formula')
plt.hold(True)
plt.title('Sample data')
plt.ylabel('Classification Error on Test Set')
plt.xlabel('Training sample size (n)')
fig.tight_layout()
ax.legend()
plt.grid(True)
```

```
plt.show
fig.savefig('hw1d.png')
```

```
[[1.  0.2]
 [0.2 1. ]]


C:\Users\aksha\Anaconda3\lib\site-packages\ipykernel_launcher.py:60: MatplotlibDeprecationWarn
    Future behavior will be consistent with the long-time default:
    plot commands add elements without first clearing the
    Axes and/or Figure.
C:\Users\aksha\Anaconda3\lib\site-packages\matplotlib\__init__.py:911: MatplotlibDeprecationWa
  mplDeprecation)
C:\Users\aksha\Anaconda3\lib\site-packages\matplotlib\rcsetup.py:156: MatplotlibDeprecationWar
  mplDeprecation)
```



3.2 Assignment 2 Code

Hw1_2

October 15, 2018

In [24]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

#import excel table
SFE_data=pd.read_excel('SFE_Dataset.xlsx')
col=SFE_data.columns
print(col)

#preprocessing

#remove columns with zero elements

nonzero_ratio=np.zeros(len(col))
counter=0
for col_i in col:
    n_zeros=0
    #print('column : ',col_i)
    #import pdb; pdb.set_trace()
    for j in range(0,len(SFE_data[col_i])):
        if(SFE_data[col_i][j]==0):
            n_zeros+=1
    nonzero_ratio[counter]=1-n_zeros/len(SFE_data[col_i])
    counter+=1
print('non zero ratio : ',nonzero_ratio)


drop_cols=[]
for i in range(0,len(SFE_data.columns)-1):
    if (nonzero_ratio[i]<.6):
        #print('non zero ratio',nonzero_ratio[i])
        drop_cols.append(col[i])

SFE_data_cleancols=SFE_data.drop(drop_cols,axis=1)
new_col=SFE_data_cleancols.columns
print(len(new_col),len(col))
#import pdb; pdb.set_trace()
```

```
Index(['C', 'N', 'P', 'S', 'V', 'Ni', 'Nb', 'Al', 'Ti', 'Fe', 'Hf', 'Mo', 'Mn',
       'Co', 'Si', 'Cr', 'Cu', 'SFE'],
      dtype='object')
non zero ratio :  [0.89006342 0.60887949 0.36997886 0.40169133 0.00422833 0.75052854
 0.01268499 0.09513742 0.01479915 1.         0.00211416 0.39957717
 0.80338266 0.00634249 0.60887949 0.8372093  0.05496829 1.        ]
8 18
```

```
In [25]: #SFE into high and low, remove rows

         #high and low SFE
         SFE=SFE_data_cleancols['SFE']
         non_SFE=[]
         for i in range(0,len(SFE)):
             if(SFE[i]<=35):
                 SFE_data_cleancols['SFE'][i]=0
             elif(SFE[i]>=45):
                 SFE_data_cleancols['SFE'][i]=1
             elif(SFE[i]<45 and SFE[i]>35):
                 non_SFE.append(i)
         SFE_data_cleanrows=SFE_data_cleancols.drop(non_SFE)
         SFE=SFE_data_cleanrows['SFE']
         #print(SFE)
         #SFE_data_cleanrows.head
         #SFE_data_cleancols.head
         #import pdb; pdb.set_trace()

         # for resetting the index after removing the columns or rows
         SFE_data_cleanrows=SFE_data_cleanrows.reset_index(drop=True)

         [m,n]=SFE_data_cleanrows.shape
         col=SFE_data_cleanrows.columns

         #clean rows that have any zeros
         zer=[]
         ncol=len(col)
         for i in range(0,m):
             #import pdb; pdb.set_trace()
             n_zeros=0
             for j in range (0,ncol-1):
                 if(SFE_data_cleanrows.at[i, col[j]]==0):
                     n_zeros+=1
             if(n_zeros>0):
                 zer.append(i)


         #import pdb; pdb.set_trace()
```

```
In [26]: SFE_data_cleanrows2=SFE_data_cleanrows.drop(zer)
         # for resetting the index after removing the columns or rows
         SFE_data_cleanrows2=SFE_data_cleanrows2.reset_index(drop=True)
```

```
In [27]: # randomly sample into training data and then reject data with over 55% of any one
         [m,n]=SFE_data_cleanrows2.shape
         import random
         import time
         random.seed(time.time())
         okay_sample_flag=0
         while(okay_sample_flag==0):
             train_set=SFE_data_cleanrows2.sample(frac=0.2,random_state=random.randint(1,100))
             test_set=SFE_data_cleanrows2.drop(train_set.index)

             train_set=train_set.reset_index(drop=True)
             test_set=test_set.reset_index(drop=True)

             #remove samples for more than 55%
             n_ones=0
             for i in range(0,len(train_set)):
                 if (train_set.at[i,'SFE']==1):
                     n_ones+=1
             if(n_ones/len(train_set)>.45 and n_ones/len(train_set)<.55):
                 okay_sample_flag+=1
         #    import pdb; pdb.set_trace()
```

```
In [36]: train_set.head()
```

Out[36]:

	C	N	Ni	Fe	Mn	Si	Cr	SFE
0	0.04100	0.0540	8.10	70.87500	1.7100	0.3300	18.20	0.0
1	0.00400	0.0030	15.60	64.31700	0.0300	0.0200	17.50	1.0
2	0.07000	0.5400	16.13	54.67800	9.6400	0.4500	18.48	1.0
3	0.40000	0.0660	16.30	54.66000	9.6400	0.4500	18.48	0.0
4	0.00004	0.0001	12.00	71.99952	0.0001	0.0002	16.00	0.0

```
In [28]: from scipy import stats

         col=train_set.columns

         ttest=[]
         for i in range (0,len(col)-1):
             ttest.append([stats.ttest_ind(train_set[train_set['SFE']==0][col[i]], train_set[t
```

```
In [29]: ttest_sorted=sorted(ttest,key=lambda x:x[:][0][1])
         print(ttest_sorted[0][1])
         col1=ttest_sorted[0][1]
         col2=ttest_sorted[1][1]
         col3=ttest_sorted[2][1]
         col4=ttest_sorted[3][1]
         col5=ttest_sorted[4][1]
```

```
Ni
```

```
In [50]: for i in range (0,len(ttest_sorted)):
             print(ttest_sorted[i][0:3])
```

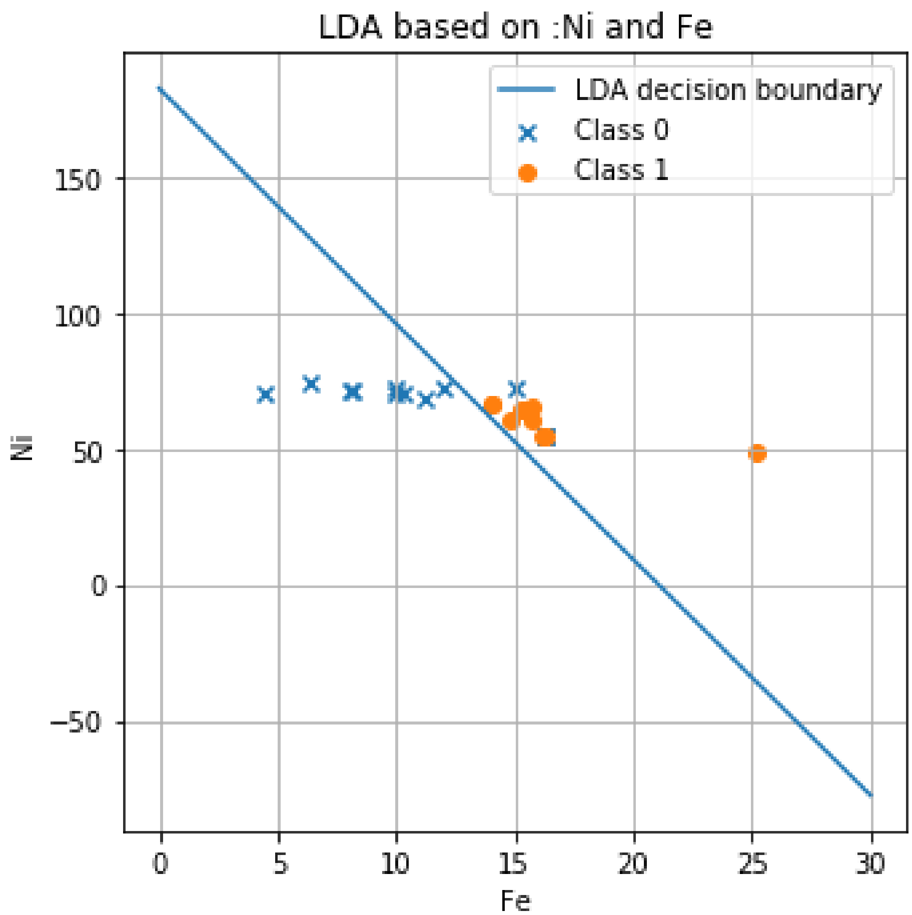
```
[Ttest_indResult(statistic=-3.6652244046944773, pvalue=0.0013486369196097205), 'Ni']
[Ttest_indResult(statistic=2.359906793878512, pvalue=0.027438300016957032), 'Fe']
[Ttest_indResult(statistic=2.1848241668704795, pvalue=0.04179478729923675), 'Si']
[Ttest_indResult(statistic=1.494382807599175, pvalue=0.15926508688174165), 'C']
[Ttest_indResult(statistic=-1.0278205792662476, pvalue=0.31477403802244214), 'Cr']
[Ttest_indResult(statistic=0.80913241119481, pvalue=0.4267305733623522), 'Mn']
[Ttest_indResult(statistic=0.2829267844867566, pvalue=0.7800377202267232), 'N']
```

```
In [51]: train_set_0=train_set[train_set['SFE']==0]
         train_set_1=train_set[train_set['SFE']==1]

         train_set_0=train_set_0.reset_index(drop=True)
         train_set_1=train_set_1.reset_index(drop=True)

         smean1=np.array([float(train_set_0.mean(0)[col1]),float(train_set_0.mean(0)[col2])])
         smean2=np.array([float(train_set_1.mean(0)[col1]),float(train_set_1.mean(0)[col2])])
         cov=train_set.cov()
         import pdb; pdb.set_trace()

         scov=np.zeros((2,2))
         scov=[[float(cov[col1][col1]),float(cov[col1][col2])],[float(cov[col2][col1]),float(c
         scov_inv=np.linalg.inv(scov)
         a_lda=scov_inv.dot((smean2-smean1))
         b_lda=-0.5*((smean2-smean1).dot(scov_inv)).dot((smean1+smean2))
         x1_lda=np.linspace(0,30,num=20)
         y_lda=-a_lda[0]/a_lda[1]*x1_lda-b_lda/a_lda[1]
```

```
--Return--
> <ipython-input-51-09331e3c5c42>(10)<module>()->None
-> import pdb; pdb.set_trace()
(Pdb) continue
```

```
In [52]: fig, ax = plt.subplots(figsize=[5,5])
         plt.scatter(train_set_0[col1],train_set_0[col2],marker='x',label='Class 0')
         plt.scatter(train_set_1[col1],train_set_1[col2],marker='o',label='Class 1')
         plt.plot(x1_lda,y_lda,label='LDA decision boundary')
         plt.title('LDA based on :'+ col1 +' and '+ col2)
         plt.ylabel(col1)
         plt.xlabel(col2)
         fig.tight_layout()
         ax.legend()
```

```
plt.grid(True)
plt.show
fig.savefig('hw2c.png')
```




```
In [32]: #estimating error on test set
         test_set_0=test_set[test_set['SFE']==0]
         test_set_1=test_set[test_set['SFE']==1]

         test_set_0=test_set_0.reset_index(drop=True)
         test_set_1=test_set_1.reset_index(drop=True)

         errtest_set_0=np.zeros((len(test_set_0),2))
         errtest_set_1=np.zeros((len(test_set_1),2))
         for i in range (0,len(test_set_0)):
             errtest_set_0[i][0]=test_set_0[col1][i]
             errtest_set_0[i][1]=test_set_0[col2][i]
         for i in range (0,len(test_set_1)):
             errtest_set_1[i][0]=test_set_0[col1][i]
```

```
        errtest_set_1[i][1]=test_set_0[col2][i]
        #[test_set[test_set['SFE']==0][col1],test_set[test_set['SFE']==0][col2]]
        #test_set_1[i]=[test_set[test_set['SFE']==1][col1],test_set[test_set['SFE']==1][c
    import pdb; pdb.set_trace()


    err1=0
    g1=0
    g2=0
    for i in range(0,len(errtest_set_0)):
        g1=a_lda.dot(errtest_set_0[i])+b_lda
        if (g1>0):
            err1+=1
    err2=0
    for i in range(0,len(errtest_set_1)):
        g2=a_lda.dot(errtest_set_1[i])+b_lda
        if (g2>0):
            err2+=1
    err=(err1+err2)/(len(errtest_set_0)+len(errtest_set_1))
```

```
--Return--
> <ipython-input-32-33d17c0aef56>(18)<module>()->None
-> import pdb; pdb.set_trace()
(Pdb) continue
```

In [33]:
```
err
```

Out[33]: 0.07920792079207921

In [53]:
```
#now repeat for the top 3,4 and 5 predictors
#first we do top 3 predictors

train_set_0=train_set[train_set['SFE']==0]
train_set_1=train_set[train_set['SFE']==1]

train_set_0=train_set_0.reset_index(drop=True)
train_set_1=train_set_1.reset_index(drop=True)

#clean train set to only contain columns of our interest
train_set_clean=train_set[[col1,col2,col3]]
train_set_0_clean=train_set_0[[col1,col2,col3]]
train_set_1_clean=train_set_1[[col1,col2,col3]]


smean1=np.array(train_set_0_clean.mean(0))
smean2=np.array(train_set_1_clean.mean(0))
cov=train_set_clean.cov()
import pdb; pdb.set_trace()
```

```
        cov_inv=np.linalg.inv(cov)
        a_lda=cov_inv.dot((smean2-smean1))
        b_lda=-0.5*((smean2-smean1).dot(cov_inv)).dot((smean1+smean2))
        x1_lda=np.linspace(0,40,num=20)
        y_lda=-a_lda[0]/a_lda[1]*x1_lda-b_lda/a_lda[1]
```

```
--Return--
> <ipython-input-53-607b6e9b46a5>(19)<module>()->None
-> import pdb; pdb.set_trace()
(Pdb) continue
```

```
In [54]: #estimating error on test set
        test_set_0=test_set[test_set['SFE']==0]
        test_set_1=test_set[test_set['SFE']==1]

        test_set_0=test_set_0.reset_index(drop=True)
        test_set_1=test_set_1.reset_index(drop=True)

        #consider only predictors of our importance
        test_set_clean=test_set[[col1,col2,col3]]
        test_set_0_clean=test_set_0[[col1,col2,col3]]
        test_set_1_clean=test_set_1[[col1,col2,col3]]

        errtest_set_0=np.array(test_set_0_clean)
        errtest_set_1=np.array(test_set_1_clean)



        import pdb; pdb.set_trace()
        err1=0
        g1=0
        g2=0

        for i in range(0,len(errtest_set_0)):
            g1=a_lda.dot(errtest_set_0[i])+b_lda
            if (g1>0):
                err1+=1
        err2=0

        for i in range(0,len(errtest_set_1)):
            g2=a_lda.dot(errtest_set_1[i])+b_lda
            if (g2>0):
                err2+=1
        print(err1,err2)
        err=(err1+err2)/(len(errtest_set_0)+len(errtest_set_1))
        print(err)
```

```
--Return--
```

```
> <ipython-input-54-b71cb07b1d07>(18)<module>()->None
-> import pdb; pdb.set_trace()
(Pdb) continue
3 40
0.42574257425742573
```

In [55]:
```python
#now we do for top 4 predictors

train_set_0=train_set[train_set['SFE']==0]
train_set_1=train_set[train_set['SFE']==1]

train_set_0=train_set_0.reset_index(drop=True)
train_set_1=train_set_1.reset_index(drop=True)

#clean train set to only contain columns of our interest
train_set_clean=train_set[[col1,col2,col3,col4]]
train_set_0_clean=train_set_0[[col1,col2,col3,col4]]
train_set_1_clean=train_set_1[[col1,col2,col3,col4]]


smean1=np.array(train_set_0_clean.mean(0))
smean2=np.array(train_set_1_clean.mean(0))
cov=train_set_clean.cov()
import pdb; pdb.set_trace()
cov_inv=np.linalg.inv(cov)
a_lda=cov_inv.dot((smean2-smean1))
b_lda=-0.5*((smean2-smean1).dot(cov_inv)).dot((smean1+smean2))
x1_lda=np.linspace(0,40,num=20)
y_lda=-a_lda[0]/a_lda[1]*x1_lda-b_lda/a_lda[1]
```

```
--Return--
> <ipython-input-55-6d83bec34c21>(18)<module>()->None
-> import pdb; pdb.set_trace()
(Pdb) continue
```

In [56]:
```python
#estimating error on test set
test_set_0=test_set[test_set['SFE']==0]
test_set_1=test_set[test_set['SFE']==1]

test_set_0=test_set_0.reset_index(drop=True)
test_set_1=test_set_1.reset_index(drop=True)

#consider only predictors of our importance
test_set_clean=test_set[[col1,col2,col3,col4]]
test_set_0_clean=test_set_0[[col1,col2,col3,col4]]
test_set_1_clean=test_set_1[[col1,col2,col3,col4]]
```

```
errtest_set_0=np.array(test_set_0_clean)
errtest_set_1=np.array(test_set_1_clean)



import pdb; pdb.set_trace()
err1=0
g1=0
g2=0

for i in range(0,len(errtest_set_0)):
    g1=a_lda.dot(errtest_set_0[i])+b_lda
    if (g1>0):
        err1+=1
err2=0

for i in range(0,len(errtest_set_1)):
    g2=a_lda.dot(errtest_set_1[i])+b_lda
    if (g2>0):
        err2+=1
print(err1,err2)
err=(err1+err2)/(len(errtest_set_0)+len(errtest_set_1))
print(err)
```

```
--Return--
> <ipython-input-56-de58f5100028>(18)<module>()->None
-> import pdb; pdb.set_trace()
(Pdb) continue
3 44
0.46534653465346537
```

In [57]:
```
#now we do for top 5 predictors

train_set_0=train_set[train_set['SFE']==0]
train_set_1=train_set[train_set['SFE']==1]

train_set_0=train_set_0.reset_index(drop=True)
train_set_1=train_set_1.reset_index(drop=True)

#clean train set to only contain columns of our interest
train_set_clean=train_set[[col1,col2,col3,col4,col5]]
train_set_0_clean=train_set_0[[col1,col2,col3,col4,col5]]
train_set_1_clean=train_set_1[[col1,col2,col3,col4,col5]]


smean1=np.array(train_set_0_clean.mean(0))
```

```
smean2=np.array(train_set_1_clean.mean(0))
cov=train_set_clean.cov()
import pdb; pdb.set_trace()
cov_inv=np.linalg.inv(cov)
a_lda=cov_inv.dot((smean2-smean1))
b_lda=-0.5*((smean2-smean1).dot(cov_inv)).dot((smean1+smean2))
x1_lda=np.linspace(0,40,num=20)
y_lda=-a_lda[0]/a_lda[1]*x1_lda-b_lda/a_lda[1]
```

```
--Return--
> <ipython-input-57-f049cac95ddd>(18)<module>()->None
-> import pdb; pdb.set_trace()
(Pdb) continue
```

In [58]:
```
#estimating error on test set
test_set_0=test_set[test_set['SFE']==0]
test_set_1=test_set[test_set['SFE']==1]

test_set_0=test_set_0.reset_index(drop=True)
test_set_1=test_set_1.reset_index(drop=True)

#consider only predictors of our importance
test_set_clean=test_set[[col1,col2,col3,col4,col5]]
test_set_0_clean=test_set_0[[col1,col2,col3,col4,col5]]
test_set_1_clean=test_set_1[[col1,col2,col3,col4,col5]]

errtest_set_0=np.array(test_set_0_clean)
errtest_set_1=np.array(test_set_1_clean)



import pdb; pdb.set_trace()
err1=0
g1=0
g2=0

for i in range(0,len(errtest_set_0)):
    g1=a_lda.dot(errtest_set_0[i])+b_lda
    if (g1>0):
        err1+=1
err2=0

for i in range(0,len(errtest_set_1)):
    g2=a_lda.dot(errtest_set_1[i])+b_lda
    if (g2>0):
        err2+=1
print(err1,err2)
```

```python
        err=(err1+err2)/(len(errtest_set_0)+len(errtest_set_1))
        print(err)
```

```
--Return--
> <ipython-input-58-1bb7c4f9af88>(18)<module>()->None
-> import pdb; pdb.set_trace()
(Pdb) continue
3 45
0.4752475247524752
```