CSE 527 –Introduction to Computer Vision

Homework 04

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Question 1

Files: q1.m

a) The matrix of intrinsic parameters is given by :

```
A = [f*sx   0      u/2;
      0     f*sy   v/2 ;
      0     0       1]
```

b) Using Quaternions to represent the rotation

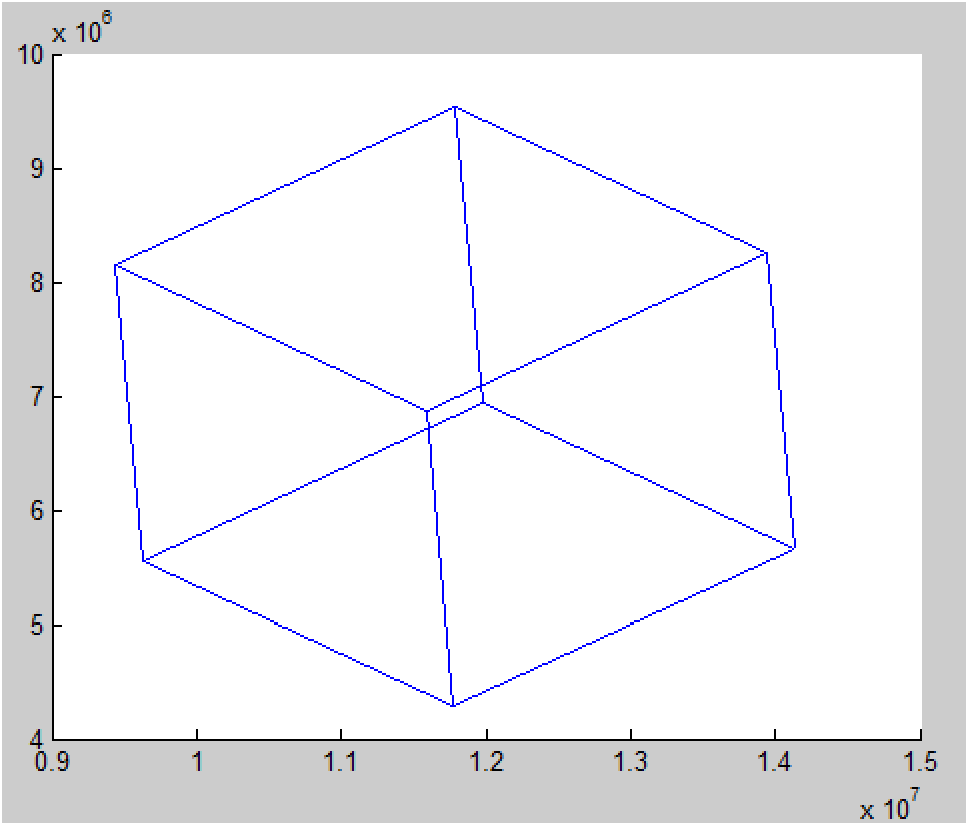
```
A  =75 % angle is 60 degrees
C = cosd(60)
S = sind (60)
c = 1-C
rx = r(:,1);
ry = r(:,2);
rz = r(:,3);

 R = [(c*(rx)^2)+C c*rx*ry-S*rz c*rx*rz+S*ry;
       c*rx*ry+S*rz (c*(ry)^2)+C c*ry*rz-S*rx;
       c*rx*rz-S*ry c*ry*rz+S*rx (c*(rz)^2)+C]
```

Transformation matrix (Rotation and translation)

T1 =

1.0e+03 *

0.0007	0.0001	-0.0007	7.3706
0.0004	0.0008	0.0004	-4.3260
0.0006	-0.0006	0.0005	-5.1923
0	0	0	0.0010



N = Matrix of vertices of the cube after transformation.

V1 = 1.0e+07 * (1.1786 0.9530 0.0000)

V2 = 1.0e+07 *(0.9430 0.8149 0.0000)

V3 = 1.0e+07 * (0.9622 0.5560 0.0000)

V4 = 1.0e+07 *(1.1979 0.6942 0.0000)

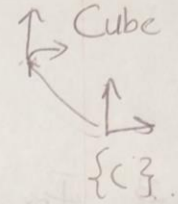
V5 = 1.0e+07 * (1.3943 0.8252 0.0000)

V6 = 1.0e+07 *(1.1586 0.6870 0.00000

V7 = 1.0e+07 *(1.1779 0.4282 0.0000)

V8 = 1.0e+07 * (1.4135 0.5663 0.0000)

If we consider the points at infinity in the cubes new frame, the points would be at u,v as shown in image below



Q1 d) To find out the points at infinity corr x, y, z dir^n of cube w.r.t frame.

We know from our code that the rotation matrix for Camera Frame to Cube Frame is

$$R = \begin{bmatrix} 0.6731 & 0.0609 & -0.7371 \\ 0.4006 & -0.8087 & 0.4326 \\ 0.6217 & -0.58 & 0.5192 \end{bmatrix}$$

$$p = K \begin{bmatrix} R & t \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

We take zero to consider infinity

$$p = K R \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K \begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix}$$

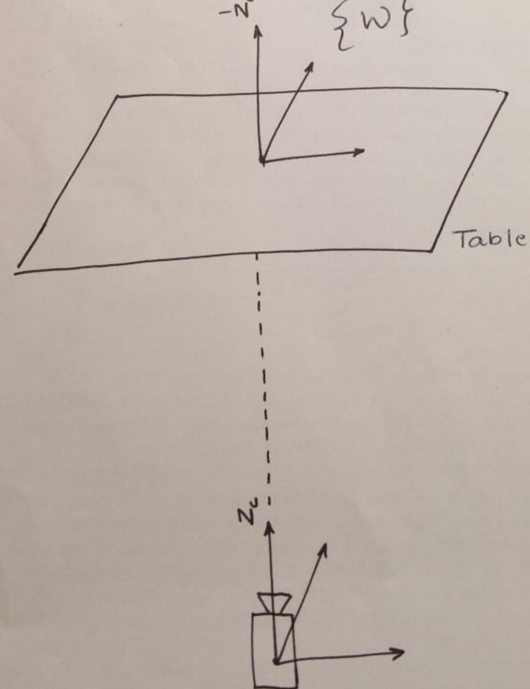
$$u = \frac{f(x_R)}{z_R} + u_0$$

$$v = \frac{f(y_R)}{z_R} + v_0$$

Question 2

Main difference between the perspective and projective models is that perspective model is dependent only on intrinsic parameters while the projective model depends on the intrinsic and extrinsic parameters.

We consider the camera as shown in fig. below.
We consider the world frame on the table at the center of the cross pattern. So for all cases, X = Y = Z = 0.
When the ht of table increases tz increases.



{C} = Camera Frame
.

Q.2 Perspective projection camera model is

a)

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & S_k & u/2 & 0 \\ 0 & S_y & v/2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \cdots (1)$$

S_x = Scale along X S_y = Scale along Y

S_k = Skew

$\frac{u}{2}$ = offset along x axis

$\frac{v}{2}$ = offset along Y axis

Projective camera model.

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underset{3\times3}{\begin{bmatrix} S_x & S_k & u/2 \\ 0 & S_y & v/2 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3\times4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & t_z \end{bmatrix}}$$

$$\times \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \cdots (2)$$

b) In the above defined projective camera model we have 6 degrees of freedom in order to calibrate the system. They are S_x, S_y, S_k, u, v, t_z

c) Multiplying the matrices in eq(2)

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & S_k & u/2 & t_z(u/2) \\ 0 & S_y & v/2 & t_z(v/2) \\ 0 & 0 & 1 & t_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$S_x X + S_k Y + \frac{u}{2} Z + t_z (u/2) - u = 0$$

$$S_y Y + v/2 \, Z + t_z (v/2) - v = 0$$

$\cancel{Y = t_z}$

From this we get the linear system of eq^n

$$\begin{bmatrix} X_1 & Y_1 & Z_1 + tz_1 & 0 & 0 & -u_1 \\ 0 & 0 & 0 & Y_1 & Z_1 + tz_1 & -V_1 \\ X_2 & Y_2 & Z_2 + tz_2 & 0 & 0 & -u_2 \\ 0 & 0 & 0 & Y_2 & Z_2 + tz_2 & -V_2 \\ X_3 & Y_3 & Z_3 + tz_3 & 0 & 0 & -u_3 \\ 0 & 0 & 0 & Y_3 & Z_3 + tz_3 & -V_3 \end{bmatrix} \begin{bmatrix} S_x \\ S_k \\ u/2 \\ S_y \\ V/2 \\ 1 \end{bmatrix} = 0$$

Performing computations to solve ~~Ax=b~~ $Ax = 0$

Using SVD.

Substituting the values x, Y, Z, t_z we get

$$\vec{X} = \begin{bmatrix} 0 \\ 0 \\ -0.37 \\ 0 \\ -0.87 \\ -0.31 \end{bmatrix}$$

But we know last term is 1 so scaling we get, $\vec{X} = \begin{bmatrix} 0 \\ 0 \\ 1.19 \\ 0 \\ 2.76 \\ 1 \end{bmatrix}$

Thus for our camera model we obtain

$S_x = 0$
$S_k = 0$
$u/2 = 1.19$
$S_y = 0$
$v/2 = 2.76$

d) From this we know the height is u/S_x and v/S_y

For case 1, $h_x = \frac{u}{S_x} = \frac{130}{1.19} = 109$

$h_y = \frac{V}{S_y} = \frac{310}{2.76} = 112$

Taking avg $h = 111$

Case 2

$$h_x = \frac{170}{1.19} = 142$$

$$h_y = \frac{380}{2.76} = 137$$

Avg h = 140

Case 3

$$h_x = \frac{190}{1.19} = 108$$

$$h_y = \frac{300}{2.76} = 159$$

Avg h = 132.

Question 3

Files: q3_script.m

ComputeWarpMapping.m

We first choose four images .

1. We align them:

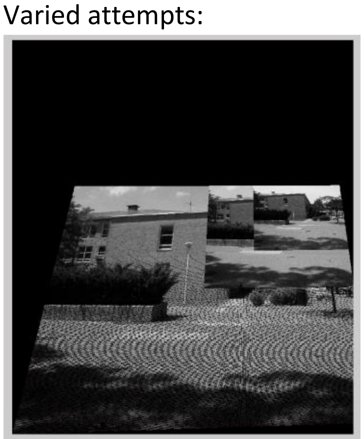
2. We manually select five overlapping features in the first two images.
   We need minimum 4 points and hence we select five points .

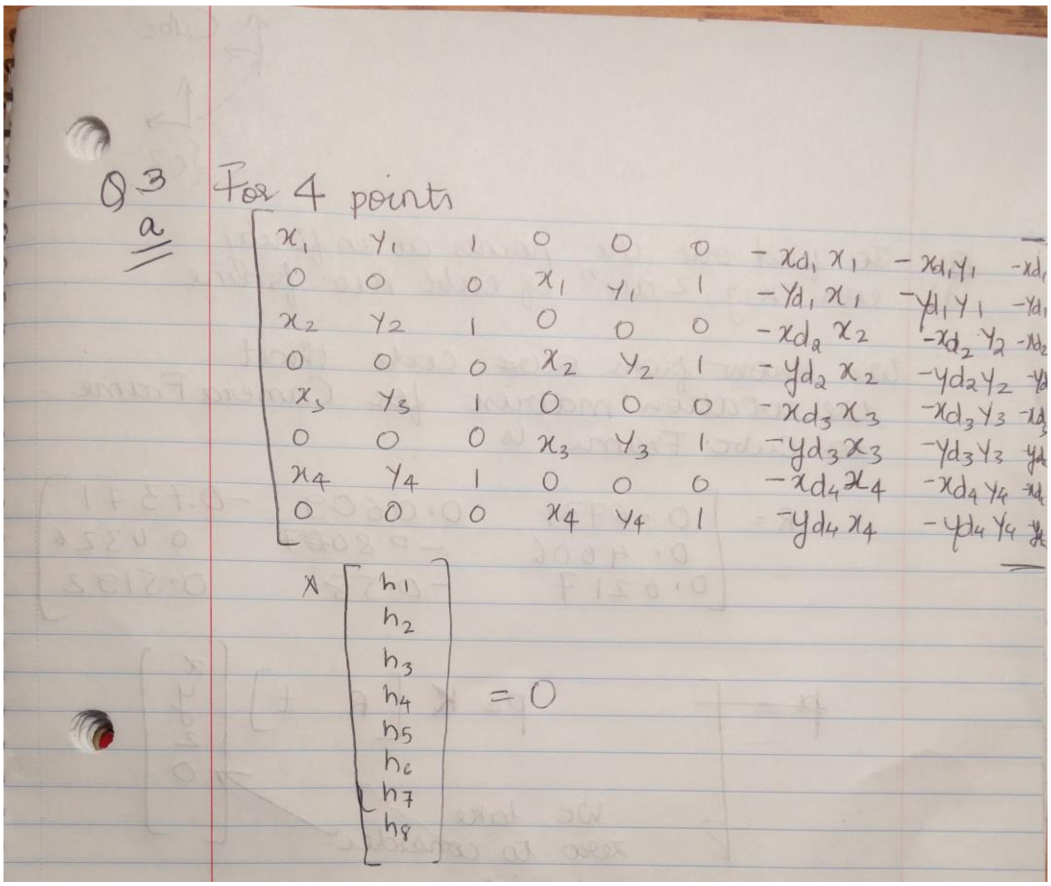
3) Next we compute the projective transformation – we call it homography matrix

H can be obtained using Singular Value decomposition.

4.) We now find the corresponding co-ordinates by multiplying the locations of the second image by the H matrix.

We can visualize the warped image.
5.) Now we finally put together the images.
Varied attempts:





Q3 a

For 4 points

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_{d_1}x_1 & -x_{d_1}y_1 & -x_{d_1} \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_{d_1}x_1 & -y_{d_1}y_1 & -y_{d_1} \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_{d_2}x_2 & -x_{d_2}y_2 & -x_{d_2} \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -y_{d_2}x_2 & -y_{d_2}y_2 & -y_{d_2} \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_{d_3}x_3 & -x_{d_3}y_3 & -x_{d_3} \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -y_{d_3}x_3 & -y_{d_3}y_3 & -y_{d_3} \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_{d_4}x_4 & -x_{d_4}y_4 & -x_{d_4} \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -y_{d_4}x_4 & -y_{d_4}y_4 & -y_{d_4} \end{bmatrix} \times \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{bmatrix} = 0$$

Question 4

Files: q4_script.m

ComputeWarpMapping.m

In this section we have used SIFT instead of manual feature selection.