Chapter 1

Literature Review and Methodological Foundations

1.1 Paired-comparison and Dynamic Rating Models

This chapter grounds our hybrid modeling + RL design in rigorous prior work.

1.1.1 Bradley–Terry / Thurstone Foundations

Let each team i have latent skill θ_i. The Bradley–Terry model gives

$$P(i \text{ beats } j) = \frac{\exp(\theta_i)}{\exp(\theta_i) + \exp(\theta_j)} = \operatorname{logit}^{-1}(\theta_i - \theta_j).$$

While elegant, this static model cannot capture temporal dynamics or strength drift.

1.1.2 Harville's Linear Mixed Models for NFL

Harville (1980) models score margin Y_{ij} as

$$Y_{ij} = \alpha_i - \alpha_j + \delta\,\mathrm{HFA}_{ij} + \mathbf{x}_{ij}^{\top}\beta + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim N(0, \sigma^2),$$

with α as random effects and shrinkage (BLUP) estimation. (**?**) This approach is interpretable and computationally efficient, but lacks temporal adaptability.

1.1.3 Glickman–Stern State-Space Model

Glickman & Stern (1998) model latent strengths $\theta_{i,t}$ evolving via

$$\theta_{i,t} = \gamma\,\theta_{i,t-1} + \eta_{i,t}, \quad \eta_{i,t} \sim N(0, \tau^2),$$

and margins:

$$Y_{ij,t} = (\theta_{i,t} - \theta_{j,t}) + \delta\,\mathrm{HFA}_{ij,t} + \varepsilon_{ij,t}, \quad \varepsilon_{ij,t} \sim N(0, \sigma^2).$$

They link spread to win probability via the normal approximation $P(\text{win}) = \Phi(p/\sigma)$. (**??**) Strengths: credible intervals, smooth drift, extensions for covariates. Weaknesses: Gaussian residuals may misfit extreme margins, MCMC scaling, linear covariate assumptions.

Stochastic Volatility Extensions. Some works allow σ_t to vary via

$$\log \sigma_t = \phi \log \sigma_{t-1} + \zeta_t,$$

which captures heteroskedasticity across games or seasons.

1.2 Score / Margin Distributions

1.2.1 Independent Poisson + Dixon-Coles Adjustment

Model goals/scores as independent Poissons with rates λ, μ. Dixon & Coles (1997) introduce an adjustment $\tau(x, y; \rho)$ to correct low-score dependence:

$$P(X = x, Y = y) \propto \tau(x, y; \rho)\,\text{Poisson}(x; \lambda)\,\text{Poisson}(y; \mu).$$

This improves fit in tightly contested, low-score games. (**?**)

1.2.2 Bivariate Poisson via Shared Component

Karlis & Ntzoufras model

$$X = U + Z,\ Y = V + Z,$$

with U, V, Z independent Poissons. The shared Z introduces positive covariance: $\text{Cov}(X, Y) = \mathbb{E}[Z]$. (**?**) This structure better captures correlated scoring (e.g. fast score pace, momentum).

1.2.3 Dynamic Bivariate Poisson Models

Koopman, Lit & Lucas (2015) evolve scoring intensities over time in a state-space framework, using simulation-based filtering to handle the non-Gaussian likelihood. (**?**) This allows adaptation of attack/defense parameters mid-season.

1.2.4 Skellam Margins and Key-Number Reweighting

For margins $D = X - Y$, where X, Y are Poisson, D follows a Skellam distribution:

$$P(D = d) = e^{-(\lambda+\mu)} \left(\frac{\lambda}{\mu}\right)^{d/2} I_{|d|}(2\sqrt{\lambda\mu}).$$

In the NFL, margins concentrate on key numbers (3,6,7,10). We reweight the Skellam/Poisson PMF to match empirical key-number frequencies before feeding into teaser/middle simulations (see Chapter **??**).

1.3 Calibration, Scoring & Uncertainty

1.3.1 Scoring Rules

We evaluate models by:

- **Brier score:** $\frac{1}{N}\sum(p_i - y_i)^2$
- **Log-loss:** $-\frac{1}{N}\sum[y_i \log p_i + (1 - y_i)\log(1 - p_i)]$
- **Reliability diagrams, ECE:** partition probabilities into bins and check empirical frequency

1.3.2 Uncertainty Quantification

Classical Bayesian/state-space models give posterior predictive distributions by default. For ML models, we will estimate predictive intervals via:

- Bootstrapping over training subsets
- Quantile regression layers
- Ensemble variance

We propagate these intervals into staking decisions: bets with wide uncertainty may be filtered or heavily downweighted.

1.4 Machine Learning Models in NFL Prediction

1.4.1 Feature Sets and Interactions

Key feature families include:

- Efficiency metrics: EPA/play, success rate (offense, defense, by down/distance)
- Play-calling: PROE (pass rate over expected), pace (sec/play), pass vs run splits
- Trench indicators: pressure allowed/created, stuff rate, line yards proxies
- Roster & injuries: QB status, adjusted games lost (AGL), starters out
- Environmental: weather (wind, rain, temp), turf/grass, altitude
- Market microstructure: implied probability, hold, line-move delta, cross-book spreads (CBV)

We use ML (e.g. gradient boosting, neural nets) to capture nonlinear interactions among these features, stacking with classical model outputs as base features.

1.4.2 Regularization, Calibration & Robustness

We guard against overfitting via:

- Time-based cross-validation (rolling windows)
- Strong regularization (ridge, lasso, elastic net)
- Probability calibration (Platt scaling, isotonic regression) on held-out data
- Ensemble bootstraps and variance reduction

1.5 Reinforcement Learning for Betting

1.5.1 MDP Formulation for Betting

We treat each potential bet (pre-game or intra-game) as a step in an MDP:

$$s_t = (\text{model predictions}, \text{market state}, \text{bankroll}, \text{time}), \quad a_t \in \{\text{no bet}, \text{stake bucket}\}, \quad r_t = \text{PnL (or utility}$$

Actions can include correlated bets across markets (spread + total) or hedges.

1.5.2 RL Algorithms and Offline Training

We experiment with:

- **DQN / Q-learning**: discretized stake buckets, value iteration + experience replay
- **PPO / Actor–Critic**: continuous or stochastic stake policies, clipped updates, entropy regularization
- **Uncertainty-aware gating**: suppress stakes when posterior CI is wide (e.g. if variance too high)

We train offline (historical seasons) and optionally refine online via simulated paper-trading episodes.

1.6 Betting Market Theory & Microstructure

1.6.1 Economics of Wagering Markets

Sauer (1998) surveys the structure and efficiency of wagering markets, including bookmaker margins, bettor behavior models, and informational asymmetries. (**?**) Levitt (2004) argues bookmakers sometimes exploit bettor biases (e.g. overbetting favorites) rather than purely balancing books. (**?**)

1.6.2 Market as Signal and Benchmark

We treat the market (closing lines) as both:

- A performance benchmark: our models must outperform or capture CLV (closing line value) edge
- A feature: cross-book spreads, line velocity, implied vs model delta, push rules

We define **Comparative Book Value (CBV)** as the difference between our fair probability and implied market probability; large CBV signals potential mispricing worth a bet.

1.7 Design Synthesis and Implications

Table **??** (placeholder) will compare model families on uncertainty, scalability, interpretability, and deployment fit.

From the literature, our design principles are:

1. Use Bayesian / shrinkage models to generate priors and uncertainty bounds.
2. Use discrete margin / score distributions (bivariate Poisson + reweight) to price spreads, totals, teasers.
3. Use ML meta-models to absorb nonlinear interactions among features.
4. Use RL to convert edges into action sequences under risk constraints.
5. Leverage the market as both a signal and benchmark; bet only when CBV passes threshold.

Insert Table **??**.

1.8 Canonical Works Integrated

We explicitly compare and implement: Harville (1980), Glickman–Stern (1998), Stern spread mapping (1991), Dixon–Coles (1997), Karlis–Ntzoufras (2003), Koopman dynamic Poisson (2015), Lock & Nettleton (2014), Sauer (1998), Levitt (2004). Implementation, ablation, and critique will occur in Chapter **??**.

Add figure: empirical vs model margin PMF reweighted at key integers (3,6,7,10).

Add paper-by-paper annotated reading list.