DISCRETE-MARGIN PRICING AT THE TWO: FAIR POINT-BUY VALUATION, ML VS. SPREAD, AND MIDDLING AROUND 2

RICHARD OLDHAM

Abstract. We present a first-principles, integer-margin model for pricing NFL point spreads in the neighborhood of the two-point line and apply it to the *Seattle Seahawks @ Arizona Cardinals* game on Thursday, Sept. 25, 2025 (Week 4). Using a push-aware discrete framework, we (i) convert DraftKings market odds to implied probabilities with vig normalization, (ii) estimate the mass function of the final margin near $\{1, 2\}$ from historical distributions, (iii) compute fair prices for buying points at 2 (moves $-2 \to -1.5$ and $-2.5 \to -2$), (iv) derive an explicit moneyline-versus--2 decision rule and compare expected values numerically, and (v) quantify the break-even threshold for a classic $-1.5/+2.5$ middle. The analysis is fully push-adjusted and algebraically explicit, suitable as a building block for a backtestable betting dashboard. We include practical guidance for calibration, line-shopping, and bankroll sizing (Kelly).

1. Market snapshot and notation

For the Week 4 TNF matchup on 2025-09-25, multiple outlets reporting DraftKings lines list

$$\text{SEA } -1.5, \quad \text{ML: SEA } -125, \ \text{ARI } +105, \quad \text{Total } 43.5,$$

see e.g. [?, ?, ?, ?].[1]

Integer-margin model. Let the final *margin* be the integer-valued random variable

$$M \in \mathbb{Z}, \qquad M = (\text{favorite score}) - (\text{underdog score}).$$

For a book line $s \in \mathbb{Z}$, the favorite $-s$ ticket: wins if $M > s$, pushes if $M = s$, loses if $M < s$; the dog $+s$ is complementary.

Write $p_k := \Pr[M = k]$ and define the tail and atom at the line

$$P_{>s} := \Pr[M > s] = \sum_{k \geq s+1} p_k, \qquad P_{=s} := p_s, \qquad P_{<s} := 1 - P_{>s} - P_{=s}.$$

A tractable parametric baseline is a *discretized normal* centered at the spread,

$$p_k \approx \Phi\left(\frac{k + 0.5 - \mu}{\sigma}\right) - \Phi\left(\frac{k - 0.5 - \mu}{\sigma}\right), \qquad \mu \approx s, \tag{1}$$

augmented by a small "endgame" mixture that inflates mass at small positive margins (e.g. $\{2, 3, 4, 5\}$), reflecting late fouling/clock dynamics.[2]

2. Implied probabilities from American odds

For American odds $-a$ (favorite) or $+b$ (dog), the *implied* win probabilities (with book vig) are

$$\pi(-a) = \frac{a}{a + 100}, \qquad \pi(+b) = \frac{100}{b + 100}.$$

For SEA ML -125 and ARI ML $+105$,

$$\pi_{\text{SEA,ML}} = 0.556, \qquad \pi_{\text{ARI,ML}} = 0.488.$$

Removing vig by normalization ($\widetilde{\pi}_i = \pi_i/(\pi_{\text{SEA}} + \pi_{\text{ARI}})$) gives the *fair* moneyline win probabilities

$$P(\text{SEA wins}) \approx 0.532, \qquad P(\text{ARI wins}) \approx 0.468. \tag{2}$$

We denote $p_{\text{ML}} := 0.532$ for SEA.

3. Near-line mass at 1 and 2 from history

Long-horizon NFL studies find (unconditional) margin-of-victory frequencies roughly

$$\Pr(|M| = 3) \approx 15\%, \quad \Pr(|M| = 7) \approx 8\text{–}9\%,^{1} \quad \Pr(|M| = 1) \approx 4\%, \quad \Pr(|M| = 2) \approx 4\%,$$

with modest regime shifts after the 2015 XP rule[3] [?, ?, ?, ?]. Conditioning on SEA as a slight favorite, a neutral split allocates approximately half of the $\{1, 2\}$ outcomes to the favorite. We therefore adopt calibrated atoms

$$\Pr(M = 1) \approx 0.020, \qquad \Pr(M = 2) \approx 0.020$$

5. Fair pricing and point-buy valuation at the two

Let a unit stake pay net b on win (so $b = 100/110 \approx 0.909$ at -110). The push has zero net.

Fair odds for -2. Fair payoff $b^\star$ solves $b^\star P_{>2} = P_{<2}$, i.e.

$$b^\star = \frac{P_{<2}}{P_{>2}} = \frac{0.488}{0.492} = 0.992 \quad \Longrightarrow \quad \text{American odds} \approx -101.$$

Hence an *even-money* quote for -2 is essentially fair given $p_1 = p_2 = 0.02$ and $p_{\mathrm{ML}} = 0.532$.

Value of buying on/off 2 (clean formulas). Let $p_2 = \Pr(M = 2)$. Only the $M = 2$ cell flips when you move across 2:

$$\begin{aligned} -2 \to -1.5:&\ \text{push} \to \text{win}, \quad \Delta\mathrm{EV} = b\,p_2; \\ -2.5 \to -2:&\ \text{loss} \to \text{push}, \quad \Delta\mathrm{EV} = 1 \cdot p_2. \end{aligned}$$

Thus the full key move $-2.5 \to -1.5$ is worth $(1 + b)p_2$. Numerically, with $b = 0.909$ and $p_2 = 0.02$:

$$\underbrace{b\,p_2}_{-2\to-1.5} = 0.0182 \text{ (per \$1 staked)}, \qquad \underbrace{1 \cdot p_2}_{-2.5\to-2} = 0.0200, \qquad \underbrace{(1+b)p_2}_{-2.5\to-1.5} = 0.0382.$$

Interpret each as the *fair premium* (in units of stake) for the move.[5]

6. Moneyline versus -2: an EV comparison

Write $b_{\mathrm{ML}} = 100/125 = 0.8$. A unit-stake expected value is

$$\mathrm{EV}(-2;\, b) = b\,P_{>2} - P_{<2}, \qquad \mathrm{EV}(\mathrm{ML};\, b_{\mathrm{ML}}) = b_{\mathrm{ML}}\,p_{\mathrm{ML}} - (1 - p_{\mathrm{ML}}).$$

With the inputs above,

$$\mathrm{EV}(-2;\, -110) = 0.909{\cdot}0.492{-}0.488 = -0.0407\ (-4.07\%), \qquad \mathrm{EV}(\mathrm{ML};\, -125) = 0.8{\cdot}0.532{-}0.468 = -0.0424.$$

Both wagers, at listed juice, are $\sim -4\%$ EV—as expected—with a negligible 0.17% edge to the spread in this snapshot. The *analytic* ML-vs-spread difference is

$$\Delta\mathrm{EV} = (1 + b_{\mathrm{ML}})\ \underbrace{(p_1 + p_2)}_{\text{ML captures}}\ - \Big((1 + b) - (1 + b_{\mathrm{ML}})\Big) P_{>2} - p_2,$$

which, upon substitution, reproduces the numeric gap above. Intuition: ML pays for the $\{1, 2\}$ band that a -2 spread does not (it either pushes at 2 or loses at 1), but ML also carries different juice and forgoes push equity.

7. Middling around two (the $-1.5/+2.5$ middle)

Stake one unit each on Favorite -1.5 at $-a_1$ ($b_1 = 100/a_1$) and Dog $+2.5$ at $-a_2$ ($b_2 = 100/a_2$). Outcomes:

- If $M = 2$: *both win* $\Rightarrow$ profit $b_1 + b_2$.
- Otherwise exactly one wins $\Rightarrow$ net loss $\ell := \min\{1 - b_1, 1 - b_2\}$ (with symmetric $-110/-110$, $\ell = 0.0909$).

Hence

$$\mathrm{EV} = (b_1 + b_2)\,p_2 - (1 - p_2)\,\ell, \qquad p_2^\star = \frac{\ell}{b_1 + b_2 + \ell}.$$

At $-110/-110$, $b_1 = b_2 = 0.909$, $\ell = 0.0909 \Rightarrow p_2^\star \approx 0.0476$. Thus if $\Pr(M = 2) \gtrsim 4.8\%$, the middle is +EV; with sharper $-105/-105$, the threshold falls further.

[5] At -110, a "ten-cent" surcharge corresponds to $\approx 4.5\%$ of stake in EV terms for near-even events; see also empirical pricing discussions in [**?**, **?**].

8. Calibration and backtesting guidance

1. **Baseline fit:** Estimate (μ, σ) of a continuous margin model conditional on closing spreads in a window around 2, then discretize as in (**??**).
2. **Endgame mixture:** Add a light component that inflates $\{2, 3, 4, 5\}$ proportional to situational features (pace, timeout inventory, coaching tendencies, two-point conversion rates).
3. **Posterior update:** Incorporate any private edge δ by setting $\mu = s + \delta$ and re-discretizing.
4. **Validation:** Backtest ML vs. small spreads, bought points around 2, and middles; compare realized frequencies of $M \in \{1, 2\}$ to model-implied.[6]
5. **Dashboard implementation:** Given a live book snapshot, compute (i) vig-removed p_{ML}, (ii) spread-conditional p_1, p_2, (iii) fair -2 odds and point-buy premia, (iv) ML vs. -2 EV comparison, (v) middle thresholds; highlight mispricings across books.

Bankroll sizing (Kelly)

For net win b, loss 1, with push probability r and win/loss probabilities (q, ℓ) satisfying $q + \ell + r = 1$, the edge is $bq - \ell$ and the (fractional) Kelly stake is

$$f^\star = \frac{bq - \ell}{b}.$$

Apply to a spread (take $q = P_{>2},\ \ell = P_{<2},\ r = P_{=2}$) or to a middle by viewing the pair as a single two-point outcome instrument (double-win vs. split).

Conclusions

Around the two-point line, (i) the favorite's fair -2 price is essentially even money if p_1 and p_2 are each near 2%, (ii) the fair value of $-2 \to -1.5$ is $b\,p_2$ and of $-2.5 \to -2$ is p_2, and (iii) a canonical $-1.5/+2.5$ middle requires $p_2 \gtrsim 4.8\%$ at $-110/-110$. With DK's Week 4 snapshot, ML and spread both carry $\sim -4\%$ EV at posted juice; choice should be driven by your calibrated $\{1, 2\}$ mass, price differences across books, and portfolio considerations. The discrete, push-aware algebra here slots directly into a backtestable, production dashboard.

References

[1] Action Network. Key numbers in nfl betting, explained (recent-era margin frequencies), 2024. Accessed 2025-09-25.
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[8] SportsHandle. Seahawks vs. cardinals betting promos (week 4 tnf) — odds snapshot, 2025. Accessed 2025-09-25.

[6] Public summaries of key numbers and their temporal drift are useful sanity checks [**?**, **?**].