Project No. $2'$
Minimum Variance Portfolio Simulation with Eight Stocks

Portfolio Management

November 3, 2022

Submitted by Liteshwar Rao
MA Mathematics, Yeshiva University
Mathematics of Finance, Fall 2022, MAT-5640-1

1 Introduction

This project is in continuation of **Project 2** with the simulation of portfolio optimisation with eight different stocks.

2 Data and variables

We use data on the prices of eight securities from *Yahoo Finance* over last one year providing everyday adjusted closing price. Specifically, we use the data for the period $(11-03-2021)$ to $(11-02-2021)$. The name of the companies along with their ticker/symbol is presented below in the table.

Table 1: Ticker and Company Name

Sr. No.	Ticker	Company Name
1	AAL	American Airlines Group Inc.
2	DIS	The Walt Disney Company
3	GM	General Motors Company
4	JPM	JPMorgan Chase
5	PEP	PepsiCo. Inc.
6	AMD	Advanced Micro Devices Inc.
7	NFLX	Netflix Inc.
8	AAPL	Apple Inc.

List of variables:

- $P(t)$ = price of stock at time t, adjusted closing price of the previous day.
- $P(t+1)$ = price of stock at time $t+1$, adjusted closing price of the current day.
- R_i = daily return on stock i. It is defined as $R_i = (P(t+1) - P(t))/P(t)$.
- n = no. of days in consideration/total no. of returns of a stock in consideration
- μ_i = mean of daily return of stock i. It is equal to $\mu_i = \sum R_i/n$
- σ_i = standard deviation of daily returns of stock i. It is defined as $\sigma_i = \sqrt{(\sum(R_i - \mu_i)^2)/(n-1)}$
- i = 1,2,3, …, n securities.
- v = denotes portfolio of n securities.

- ω_i = weight/proportion of a stock i in portfolio v such that

$$\sum_{i=1}^{n} \omega_i = 1$$

- μ_v = expected return of the portfolio v.
- σ_v = risk of the portfolio v.
- C_{ij} = covariance of the stock i and j. It is defined as $C_{ij} = \sum((R_i - \mu_i).(R_j - \mu_j))/(n-1)$
- ρ_{ij} = correlation between stock i and j. It is defined as $\rho_{ij} = C_{ij}/\sigma_i.\sigma_j$. And $-1 \leq \rho_{ij} \leq 1$

Matrix-vector notation:

- $W = (w_1, w_2, \dots, w_n)$
- $U = (1, 1, \dots, 1)$
- $\mathbb{E} = (\mu_1, \mu_2, \dots, \mu_n)$
- $C_{ij} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nn} \end{bmatrix}$. Assuming $|C_{ij}| \neq 0 \implies C^{-1} exists.$
- $\rho_{ij} = \begin{bmatrix} \rho_{11} & \rho_{12} & \dots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \dots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \dots & \rho_{nn} \end{bmatrix}$
- $\sum\limits_{i=1}^{n} w_i = 1 = W.U^T = 1$
- $\mu_v = \sum\limits_{i=1}^{n} w_i.\mu_i = W.\mathbb{E}$
- $\sigma_v^2 = \sum\limits_{i,j=1}^{n} w_i.w_j.c_{ij} = W.C.W^T$

3 Methodology

The objective function is $\min\limits_{w_i} \sigma_v^2 = W.C.W^T$ s.t. $W.U^T = 1$. Setting up the Lagrangian function as below:

$$\mathcal{L}(W, \lambda) = W.C.W^T - \lambda(W.U^T - 1)$$

First order necessary conditions gives the following:

$$2W.C - \lambda U = 0$$
$$2W.C = \lambda.U$$
$$W = \frac{\lambda}{2} U.C^{-1}$$

Substituting W in the constraint $W.U^T = 1$ to get the following:

$$\frac{\lambda}{2} U.C^{-1}.U^T = 1$$
$$\frac{\lambda}{2} = \frac{1}{U.C^{-1}.U^T}$$

Substituting the value of $\frac{\lambda}{2}$ in the W we found above to get the following:

$$W = \frac{U.C^{-1}}{U.C^{-1}.U^T}$$

This gives the optimum portfolio weights minimizing the portfolio variance.

4 Results

The following table shows daily returns of securities. Due to excessive size of table, it only shows first and last five entries.

Table 1: Daily stock returns

Date	AAL	DIS	GM	JPM	PEP	AMD	NFLX	AAPL
2021-11-03	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2021-11-04	0.000	0.001	0.047	-0.013	0.000	0.053	-0.029	-0.003
2021-11-05	0.058	0.031	-0.002	-0.001	0.010	-0.008	-0.034	0.004
2021-11-08	0.020	0.007	0.018	0.006	-0.022	0.101	0.009	-0.006
2021-11-09	0.000	-0.010	-0.016	-0.008	0.007	-0.008	0.007	0.002
2022-10-27	-0.002	-0.002	0.008	0.004	-0.001	-0.019	-0.006	-0.030
2022-10-28	0.002	0.014	0.018	0.012	0.019	0.058	-0.004	0.076
2022-10-31	0.013	0.006	0.010	-0.002	-0.004	-0.031	-0.013	-0.015
2022-11-01	-0.005	-0.005	0.003	0.018	-0.005	-0.007	-0.018	-0.018
2022-11-02	-0.038	-0.039	-0.021	-0.009	-0.014	-0.017	-0.048	-0.037

The following table shows mean returns and standard deviation of securities.

Table 2: Mean and standard deviation of stock returns

Ticker	Mean μ_i	Std. dev σ_i
AAL	-0.00100	0.03640
DIS	-0.00182	0.02088
GM	-0.00108	0.02847
JPM	-0.00088	0.01872
PEP	0.00051	0.01245
AMD	-0.00244	0.03866
NFLX	-0.00268	0.04266
AAPL	0.00008	0.02144

The following table shows covariance between security i, j. Since, $\sigma_{ii} = \sigma_i^2$, the diagonal entries represent σ_i^2 for stock i.

Table 3: Covariance-Variance Matrix of stock returns

Ticker	AAL	DIS	GM	JPM	PEP	AMD	NFLX	AAPL
AAL	0.33396	0.12583	0.16815	0.09085	0.03374	0.19785	0.15221	0.10881
DIS	0.12583	0.10987	0.09560	0.05970	0.02332	0.11257	0.14544	0.07109
GM	0.16815	0.09560	0.20423	0.08584	0.03034	0.15896	0.13457	0.08894
JPM	0.09085	0.05970	0.08584	0.08827	0.02385	0.08298	0.07169	0.05182
PEP	0.03374	0.02332	0.03034	0.02385	0.03908	0.03371	0.02423	0.03469
AMD	0.19785	0.11257	0.15896	0.08298	0.03371	0.37658	0.19760	0.14322
NFLX	0.15221	0.14544	0.13457	0.07169	0.02423	0.19760	0.45867	0.10408
AAPL	0.10881	0.07109	0.08894	0.05182	0.03469	0.14322	0.10408	0.11584

The following table shows correlation between security i, j. Since, $\rho_{ii} = 1$, the diagonal entries represent $\rho_{ii} = 1$ for stock i.

Table 4: Correlation Matrix of stock returns

Ticker	AAL	DIS	GM	JPM	PEP	AMD	NFLX	AAPL
AAL	1.000	0.657	0.644	0.529	0.295	0.558	0.389	0.553
DIS	0.657	1.000	0.638	0.606	0.356	0.553	0.648	0.630
GM	0.644	0.638	1.000	0.639	0.340	0.573	0.440	0.578
JPM	0.529	0.606	0.639	1.000	0.406	0.455	0.356	0.512
PEP	0.295	0.356	0.340	0.406	1.000	0.278	0.181	0.516
AMD	0.558	0.553	0.573	0.455	0.278	1.000	0.475	0.686
NFLX	0.389	0.648	0.440	0.356	0.181	0.475	1.000	0.452
AAPL	0.553	0.630	0.578	0.512	0.516	0.686	0.452	1.000

The following table shows portfolio weights and if short selling is required or not in order to achieve minimum variance portfolio. The negative weights represent short-sale of the security.

Table 5: Portfolio Weights

Ticker	Weights	Short-selling
AAL	0.08621	No
DIS	-0.90709	Yes
GM	0.00878	No
JPM	0.35246	No
PEP	1.0000	No
AMD	-0.46911	Yes
NFLX	-0.07125	Yes
AAPL	1.0000	No
Total	**1.000**	

The following table shows portfolio parameters after solving for optimum weights minimizing the portfolio variance.

Table 6: Portfolio Parameters

Parameter	Percentage
Expected Return	59.4
Expected Risk	40.4