Inverse Problems in NeuroMANCER

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I. Introduction

Motivated by applications in the field of Computational Imaging (CI), such as Computational Microscopy, scene reconstruction with incomplete measurements is essential. A camera object captures a 3D scene in the real world, and maps it to a 2D Image. The camera system is a Linear-Spacial-Invariant (LSI) system, therefore the measurement (output) taken by the camera can be modeled by the following *forward model*:

$$\boldsymbol{y} = H\boldsymbol{x} \tag{1}$$

We are interested in recovering the scene $\boldsymbol{x}$, given measurement $\boldsymbol{y}$, with forward matrix H. In most CI applications, inverting the matrix H is not feasible because the number of columns in H are more than the number of rows, therefore the problem in (1) is highly underdetermined. As a consequence of this, it is possible to formulate the solution of (1) as a optimization problem, promoting a sparse solution:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} ||\boldsymbol{y} - H\boldsymbol{x}||_2^2 + \lambda||\Phi\boldsymbol{x}||_1 \tag{2}$$
$$s.t.\ \boldsymbol{x} \geq 0$$

Where $\lambda > 0$ is essentially promoting the sparsity constraint imposed by the non-smooth regularization term $||\Phi\boldsymbol{x}||_1$, which is a 3D total variation semi-norm (3D TV), Φ is simply a sparsity basis, and the constraint $\boldsymbol{x} \geq 0$ is promoting a scene with non-negative intensity, because the converse does not make sense physically.
This inverse problem is extremely large in scale, requiring millions of inputs and outputs. Projected Gradient Descent schemes are extremely slow due to the presence of the 3D TV proximal operator. Consider $H = DM$, with diagonal component D and convolution operator M. Now, both the forward operator (H) and the regularizer can be computed in 3D Fourier Space, thus recasting (2) in the following new optimization problem, via variable-splitting [1-3]:

$$\hat{\boldsymbol{x}} = \arg\min_{w\geq 0,u,v} \frac{1}{2}||\boldsymbol{y} - Dv||_2^2 + \lambda||u||_1 \tag{3}$$
$$s.t.\ v = M\boldsymbol{x},\ u = \Phi\boldsymbol{x},\ w = \boldsymbol{x}$$

Where u, v and w are auxiliary variables. The current literature solves this non-linear optimization problem via the use of the alternating direction method of multipliers (ADMM) algorithm, by splitting the problem in (3) into solvable and more manageable pieces, which in general performs quite well when the regularizer is non-smooth due to the presence of the l_1-norm.

II. Directions for improving NeuroMANCER

After carefully reviewing the NeuroMANCER library, we can start mentioning how methods for solving optimization problems of the form (3), such as ADMM can improve the overall versatility of the library. In a nutshell, NeuroMANCER can solve parametric constrained optimization problems for physics-informed system identification. Problem (3) is physics-informed, and thus, methods such as ADMM are robust and interpretable, but slow. By combining Deep Learning paradigms with the formulation in (3), we get the best of both worlds, thus leading to efficient parametrization. ADMM needs to be adapted to fit into NeuroMANCER's philospy, possibly through unrolling. This Phyisics-based learning approach can be compatible with NeuroMANCER's overall philosophy, which mainly deals with control theoretic problems, where non-smooth regularizers do not often appear, because of the presence of smooth regularizers, like quadratic costs. Therefore, the implementation of methods which deal with l_1-norm regularizers can improve the versatility of the library, and additionally attract users from other fields of engineering, such as CI or Signal Processing for example.
Inspired by CI, the following illustrates how ADMM's philosophy can possibly improve NeuroMANCER's Gradient-based approach:

A. *Enhanced Optimization Capabilities:*

1) *Handling non-smooth Objectives:*

- ADMM excels at solving problems with non-smooth regularization terms, such as the l_1-norm used in (3), thus sparse promoting.
- Gradient-based methods may struggle with non-smooth terms, leading to suboptimal convergence or requiring smoothing approximations.

2) *Performance Improvement:*

- **Efficiency**: ADMM may converge faster than gradient-based methods for certain classes of problems, reducing computational time.
- **Robustness**: Solutions and methods for solving (3) can provide more stable convergence in the presence of underdetermined problems or when dealing with high-dimensional data.

3) *Broadening NeuroMANCERR's Applicability:*

- **Diverse Problem Types**: Including, but not limited to, ADMM or the Fast Iterative Shrinkage-Thersholding Algorithm (FISTA) [4] would allow NeuroMANCER to handle a wider range of optimization problems, particularly those that are convex but non-smooth.

B. Considerations for Implementation

1) Integration with Differentiable Framework::

- **Differentiability**: For example, ADMM is not inherently differentiable because it involves iterative updates that may not be expressible as differentiable operations. However, techniques like unrolling, i.e. treating the iterations as layers in a neural network, can make ADMM compatible with backpropagation.

References

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