BASIC CONCEPTS: Units, Treatments, and Potential Outcomes

- **Unit**: A physical object at a particular point in time
- **Treatment**: An intervention, whose effects we wish to assess relative to no intervention (i.e., the "control")
- **Potential Outcomes**: The yet-to-be-observed values of a unit's measurements of interest after (a) application of the treatment *and* (b) non-application of the treatment (i.e., under control)
- **Causal Effect**: For each unit, the comparison of the potential outcome under treatment and the potential outcome under control
- **The Fundamental Problem of Causal Inference**: We can observe at most one of the potential outcomes for each unit
- Specific example clarifies

Potential Outcomes and Causal Effect with One Unit - Simple Difference

Hypothetical example:

- The unit is you at a particular point in time with a headache
- *Y* is your assessment of your headache pain two hours after taking an aspirin (Asp) or not taking aspirin (Not)

Unit	Initial Headache	Potential Outcomes		Causal Effect on *Y*
	X	*Y*(Asp)	*Y*(Not)	*Y*(Asp)-*Y*(Not)
you	80	25	75	-50

- No hidden versions of treatments: Asp+ and Asp-

Note: we do not use the column "*X*" yet.

Potential Outcomes and Causal Effect with One Unit – Gain Scores

Hypothetical example:

- The unit is you at a particular point in time with a headache
- *Y* is your assessment of your headache pain two hours after taking an aspirin (Asp) or not taking aspirin (Not)
- Outcome is reduction in headache pain, $Y - X$, where X is your assessment of the pain of your initial headache

Unit	Initial Headache	Potential Outcomes		Causal Effect on $Y - X$
	X	Y(Asp) - X	Y(Not) - X	Y(Asp)-Y(Not)
you	80	-55	-5	-50

Common Sense Use in the Arts

- It's a Wonderful Life
  - George Bailey, Mean Mr. Potter, Clarence the wingless angel
  - $Y(0)$ = Actual world events already seen
  - $Y(1)$ = Counterfactual world if George Bailey had never been born
- Dickens – A Christmas Carol
  - Ebenezer Scrooge, Ghost of Christmas Future
  - $Y(0)$ = Potential future if Scrooge continues his ways
  - Scrooge rejects this, and we see $Y(1)$ = future with changed Scrooge
- Many other examples in movies, fiction
- Differences between potential outcomes and counterfactuals

Legal Contexts

- The Federal Judicial Center's "Reference Manual on Scientific Evidence" (1994, Chapter 3, p. 481) states:

  The first step in a damages study is the translation of the legal theory of the harmful event into an analysis of the economic impact of that event. In most cases, the analysis considers the difference between the plaintiff's economic position if the harmful event had not occurred and the plaintiff's actual economic position. The damages study restates the plaintiff's position "but for" the harmful event; this part is often called the *but-for analysis*. Damages are the difference between the but-for value and the actual value.

- But-for world = counterfactual world
- To be plausible, generally need detail and logical steps supported by related data

Tobacco Litigation – Consequences of Misconduct of the Industry

In the September 22, 1999 news conference held to announce the United States' filing of its lawsuit against the tobacco Industry, Assistant Attorney General David Ogden stated:

> The number that's in the complaint is not a number that reflects a Particular demand for payment. What we've alleged is that each year the federal government expends in excess of $20 billion on *tobacco* related medical costs. What we would actually recover would be our Portion of that annual toll that is the result of the illegal conduct that we allege occurred, and it simply will be a matter of proof for the court, which will be developed through the course of discovery, what that amount will be. So, we have not put out a specific figure and we'll simply have to develop that as the case goes forward.

LEARNING ABOUT CAUSAL EFFECTS

- **Replication**:
  - At least one unit receives treatment and at least one unit receives control
  - Personal replication in time versus many physical objects (FDA)

- **Stable Unit-Treatment-Value Assumption ("SUTVA")**: Two parts:
  a) For each unit there is only one form of the active treatment and one form of the control treatment (i.e., no hidden versions), and
  b) There is no interference among units, i.e., each unit's potential outcomes remain the same no matter what treatments the other units receive

- **Assignment Mechanism**: The process governing which units receive treatment and which receive control

- Stability – No hidden versions of treatments
  - Two aspirin tables available for first unit
  - If not “equivalent” for first unit
    - Asp+ = active
    - Asp- = dud
  - Then three treatments, not just two.
  - All versions need to be represented with their potential outcomes
  - SUTVA can depend on choice of Y
    - Y = actual intensity from 0 to 100 – detailed
    - Y = mild versus severe – coarse

Potential Outcomes with Two Units and Interference Between Units (Part (b) of SUTVA Does Not Hold)

Potential Outcomes and Values in Example

You take: I take:	Asp Asp	Not Asp	Asp Not	Not Not
<u>Unit</u>				
1 = you	$Y_1([\text{Asp, Asp}]) = 0$	$Y_1([\text{Not, Asp}]) = 100$	$Y_1([\text{Asp, Not}]) = 50$	$Y_1([\text{Not, Not}]) = 100$
2 = me	$Y_2([\text{Asp, Asp}]) = 0$	$Y_2([\text{Not, Asp}]) = 0$	$Y_2([\text{Asp, Not}]) = 100$	$Y_2([\text{Not, Not}]) = 100$

Potential Outcomes in Aspirin Example for N Units Under the Stability Assumption

Unit	X	Y(Asp)	Y(Not)	Unit Level Causal Effect
1	X_1	Y_1(Asp)	Y_1(Not)	Y_1(Asp) - Y_1(Not)
2	X_2	Y_2(Asp)	Y_2(Not)	Y_2(Asp) – Y_2(Not)
⋮	⋮	⋮	⋮	⋮
i	X_i	Y_i(Asp)	Y_i(Not)	Y_i(Asp) - Y_i(Not)
⋮	⋮	⋮	⋮	⋮
N	X_N	Y_N(Asp)	Y_N(Not)	Y_N(Asp) - Y_N(Not)

This array of values of X, Y(1), and Y(0) represents the science, about which we want to learn

Various Population Level Causal Effects

- Comparison of Y_i(Asp) and Y_i(Not) on a common subset of units
- Average causal effect of "Asp" vs. "Not" = Ave[Y_i(Asp) – Y_i(Not)]

$$\text{Ave}[Y_i(\text{Asp}) - Y_i(\text{Not}) = \sum_{i=1}^{N} [Y_i(\text{Asp}) - Y_i(\text{Not})]/N$$

- Median causal effect of "Asp" vs. "Not" = Median{Y_i(Asp) – Y_i(Not)}
- Difference of median potential outcomes = Median {Y_i(Asp)} – Median{Y_i(Not)}
- If X_i includes male/female of each unit:
  Average causal effect of "Asp" vs. "Not" for males =
  $\text{Ave}_{X_i \rightarrow \text{male}}\{Y_i(\text{Asp}) - Y_i(\text{Not})\}$

Hypothetical Interventions OK if SUTVA OK

- Sun and planets' orbits
- Age discrimination?
- Race, sex?
- Examples of interventions for immutable characteristics
- Need for no versions of treatments of SUTVA
- All versions → same potential outcomes
- Generally, can be a distribution, but this is rarely needed
- Legal example – effect of "race" on application of capital punishment – Baldus, or on juror striking - NC

Need to Consider the Assignment Mechanism Example: Perfect Doctor

Potential Outcomes

Unit	Control $Y(0)$	New $Y(1)$
1	13	14
2	6	0
3	4	1
4	5	2
5	6	3
6	6	1
7	8	10
8	8	9
True Averages	7	5
Variances	7.7	27.4

The average causal effect $\overline{Y}(1) - \overline{Y}(0) = -2$, where $\overline{Y}$ denotes average.

Observed Data for the Perfect Doctor

The perfect doctor chooses the better treatment for each patient, i.e., the treatment under which the patient will live longer

W indicates which treatment each unit received.

Observed $\overline{y}_1 - \overline{y}_0 = 5.6 \neq -2$

		Potential Outcomes	
Unit	*W*	*Y*(0)	*Y*(1)
1	1	?	14
2	0	6	?
3	0	4	?
4	0	5	?
5	0	6	?
6	0	6	?
7	1	?	10
8	1	?	9
Observed Averages		5.4	11

Imperfect Conclusions using Data from the Perfect Doctor Example

- Drawing an inference based on the observed difference in sample means → New treatment on average, adds over 5 years
  - But new treatment, on average, subtracts 2 years from life
- If everyone received the new treatment, sample mean → people would live on average 11 years
  - But if everyone received the new treatment, people would live an average of 5 years
- Looking at all the observed values, years lived under the new are much greater than the years lived under the old
  - But this is wrong across all 8 units
- What is wrong with what we did? Where exactly was our mistake? Bad doctor? Hope for in real life!!

Imputation-Based Thinking

- Need to understand Assignment Mechanism to know how to impute missing potential outcomes, the “?”
- Impute and then compare the eight values of $Y(1)$ with the eight values of $Y(0)$
- Comparing observed values effectively randomly imputes observed values for missing values in each column
- But, for the Perfect Doctor, the observed value in a row $\geq$ the **missing** value in that row

Imputation of Missing Potential Outcomes Under the Perfect Doctor's Assignment Mechanism

The perfect doctor chooses the better treatment for each patient, i.e., the treatment under which the patient will live longer

		Potential Outcomes	
Unit	*W*	*Y*(0)	*Y*(1)
1	1	**$\leq$ 14**	14
2	0	6	**$\leq$ 6**
3	0	4	**$\leq$ 4**
4	0	5	**$\leq$ 5**
5	0	6	**$\leq$ 6**
6	0	6	**$\leq$ 6**
7	1	**$\leq$ 10**	10
8	1	**$\leq$ 9**	9
Column Averages		**$\leq$ 7.5**	**$\leq$ 7.5**

And 5/8 of the patients do at least as well under the old operation, according to the doctor.

Example: Lord's Paradox

From Holland and Rubin, "On Lord's Paradox," 1983.

"A large university is interested in investigating the effects on the students of the diet provided in the university dining halls and any sex differences in these effects. Various types of data are gathered. In particular, the weight of each student at the time of his [or her] arrival in September and his [or her] weight the following June are recorded."

The average weight for Males was 180 lbs. in both September and June. Thus, the average weight gain for Males was zero.

The average weight for Females was 130 lbs. in both September and June. Thus, the average weight gain for Females was zero.

For both males and females, correlation = 0.7

Two Statistician's Views

Question: What is the differential causal effect of the diet on male weights and on female weights?

Statistician 1: Look at gain scores: No effect of diet on weight for either males or females, and no evidence of a differential effect between the two sexes, because neither group shows any systematic change.

Statistician 2: Compare June weight for males and females with the same weight in September: On average, for a specified September weight, men weigh 15 pounds more in June than women. Thus the new diet leads to more weight gain for men. "Regression" answer: $50 – (0.7 \times 50) = 15$

Is Statistician 1 correct? Statistician 2? Neither? Both?

Unraveling Lord's Paradox

- Units = students, X = September weight and male/female
- Treatments:
  - Active = university dining hall diet, and
  - Control = diet the students would have had otherwise
- The assignment mechanism: all units receive active treatment with probability one
- The assignment mechanism is unconfounded and individualistic, but worthless for empirical causal inference because no observations under control treatment
- Either statistician could be correct if underlying assumptions were made explicit