

# Observational Study Design: Selection on Observables

Northwestern Causal Inference Workshop

Brigham R. Frandsen

BYU

July 30, 2024







\$\$\$\$

\$15M



\$\$\$

\$15M



\$\$\$\$

\$15M

\$

\$500K



\$\$\$\$

\$

\$15M

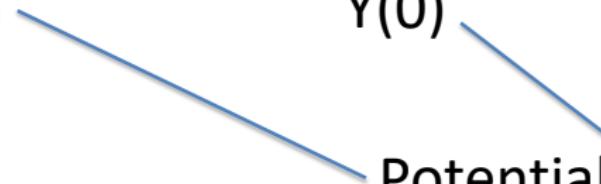
\$500K

D = 1

D = 0

Y(1)

Y(0)



Potential outcomes



\$\$\$

\$15M

\$

\$500K = \$14.5M

Y(1)

-

Y(0)

= Treatment effect



\$\$\$

\$

\$15M

\$700K = \$42.3M

Y(1)

= Treatment effect

-

Y(0)

counterfactual



\$\$\$\$

\$43M

\$

\$700K = \$42.3M

Y(1)

-

Y(0)

= Treatment effect

## What it would take to get individual-level treatment effects



## 1st Assumption: SUTVA



\$15M

\$12M



\$500k

\$85k

## 1st Assumption: SUTVA



\$15M

\$12M



\$500k

\$85k

## 1st Assumption: SUTVA



\$15M

\$12M



\$500k

\$85k



\$\$\$

\$

\$15M

\$700K = \$42.3M

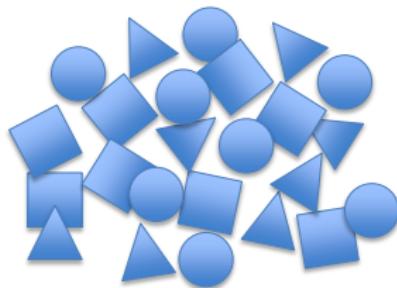
Y(1)

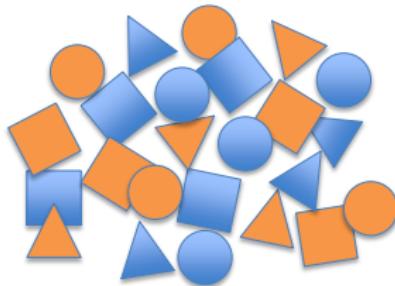
= Treatment effect

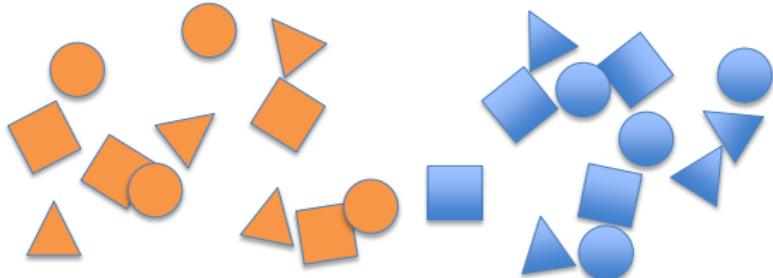
-

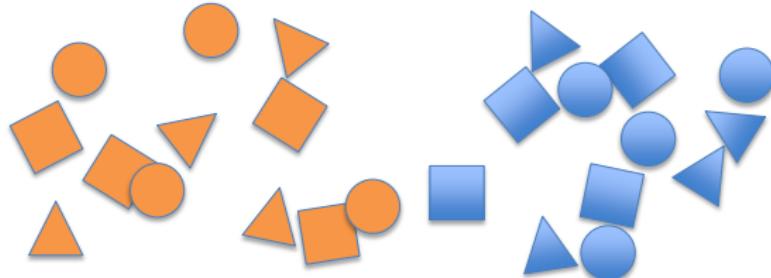
Y(0)

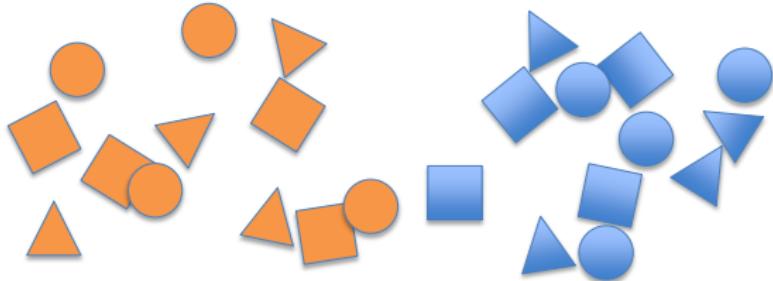
counterfactual





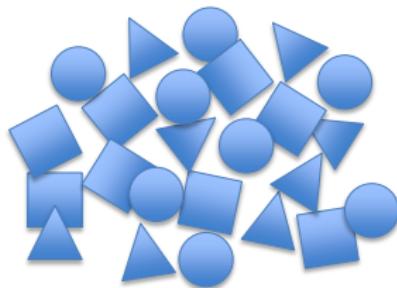


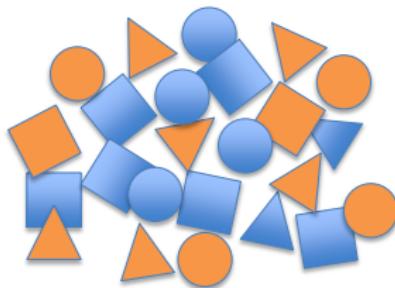
 $E[Y(1)]$  $E[Y(0)]$

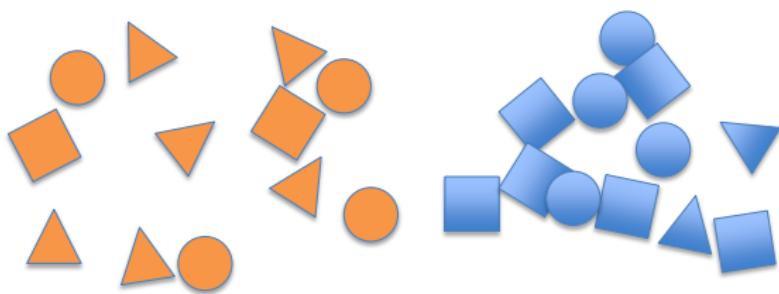


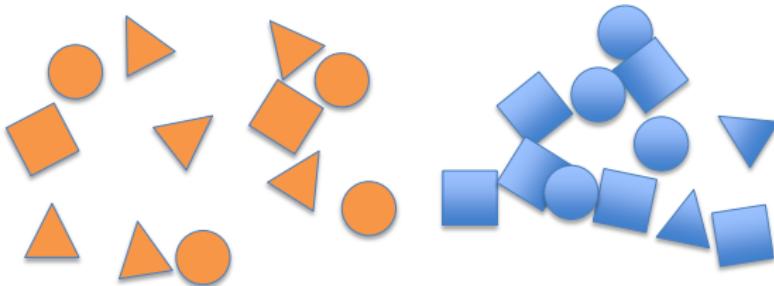
$$E[Y(1)] - E[Y(0)] = E[Y(1) - Y(0)]$$

ATE







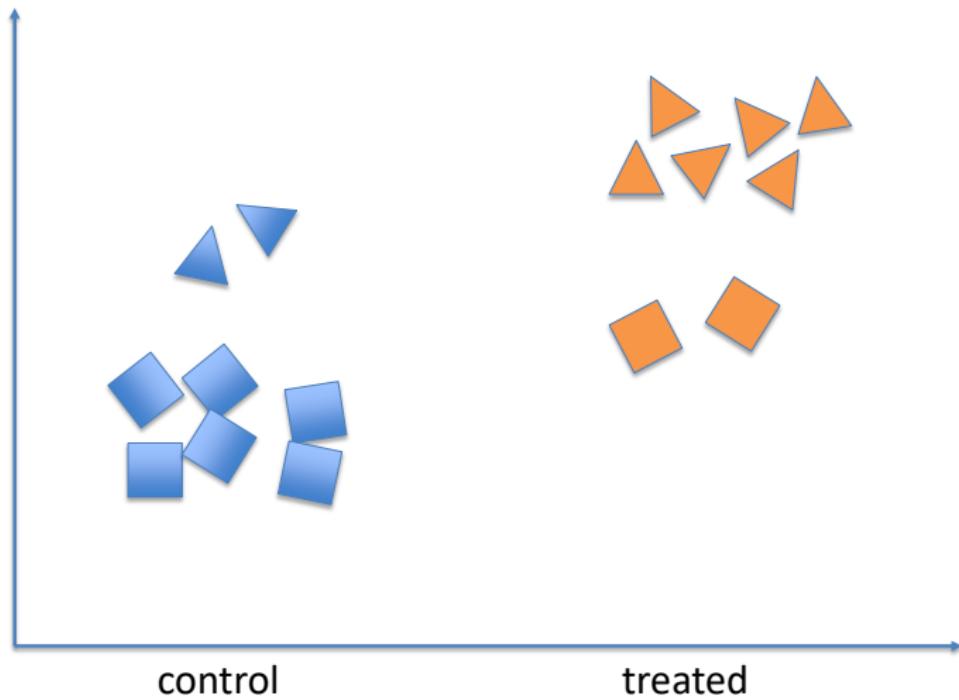


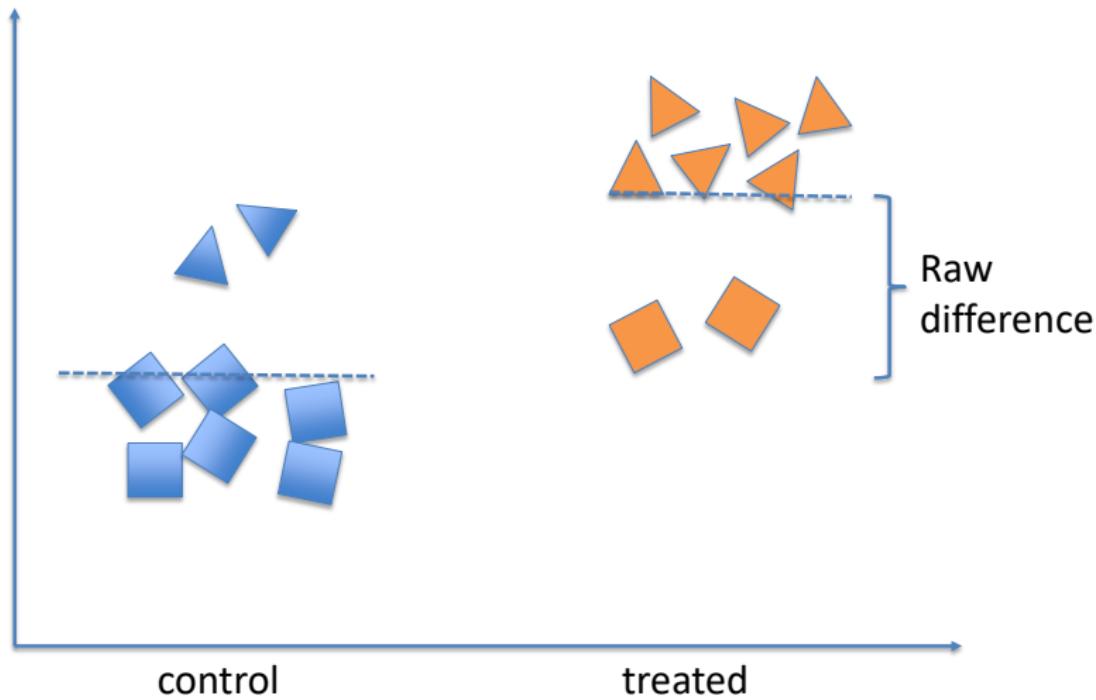
ATT

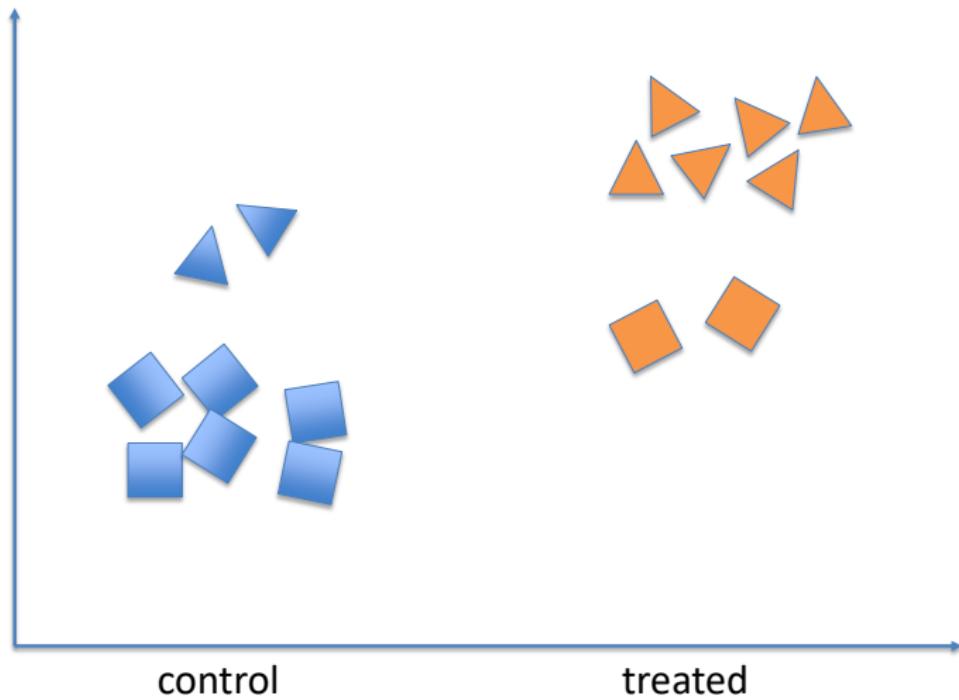
$$E[Y|D=1] - E[Y|D=0] = E[Y(1) - Y(0)|D=1]$$

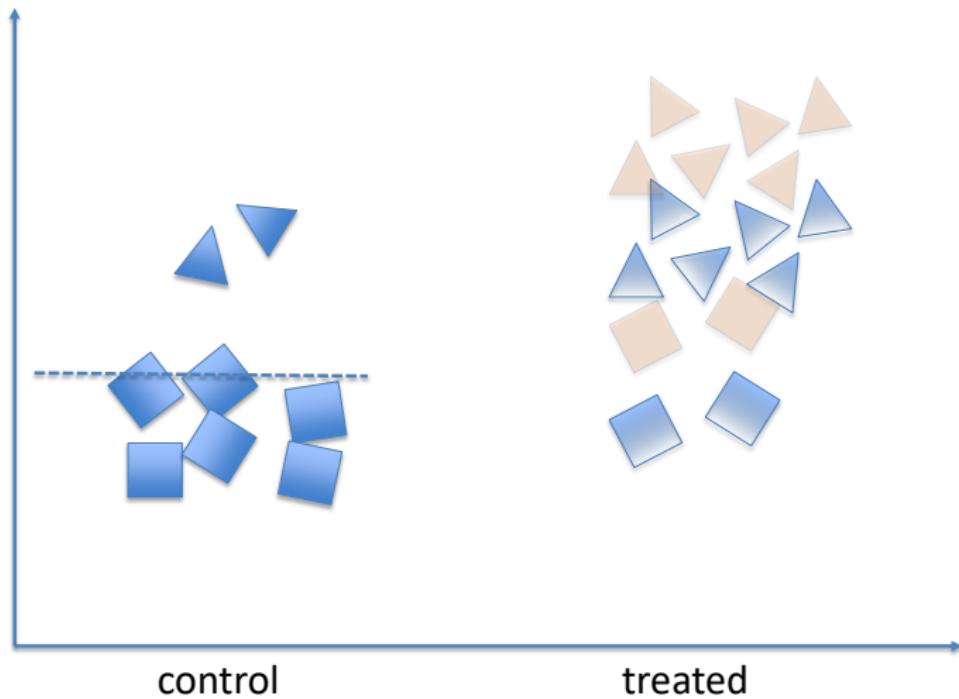
$$+ E[Y(0)|D=1] - E[Y(0)|D=0]$$

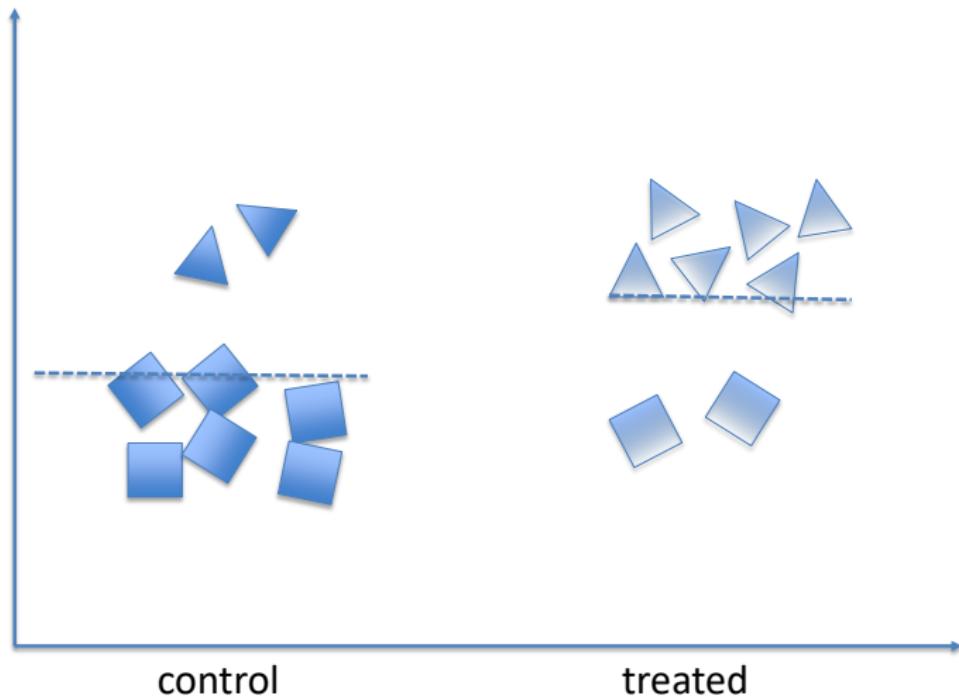
selection bias

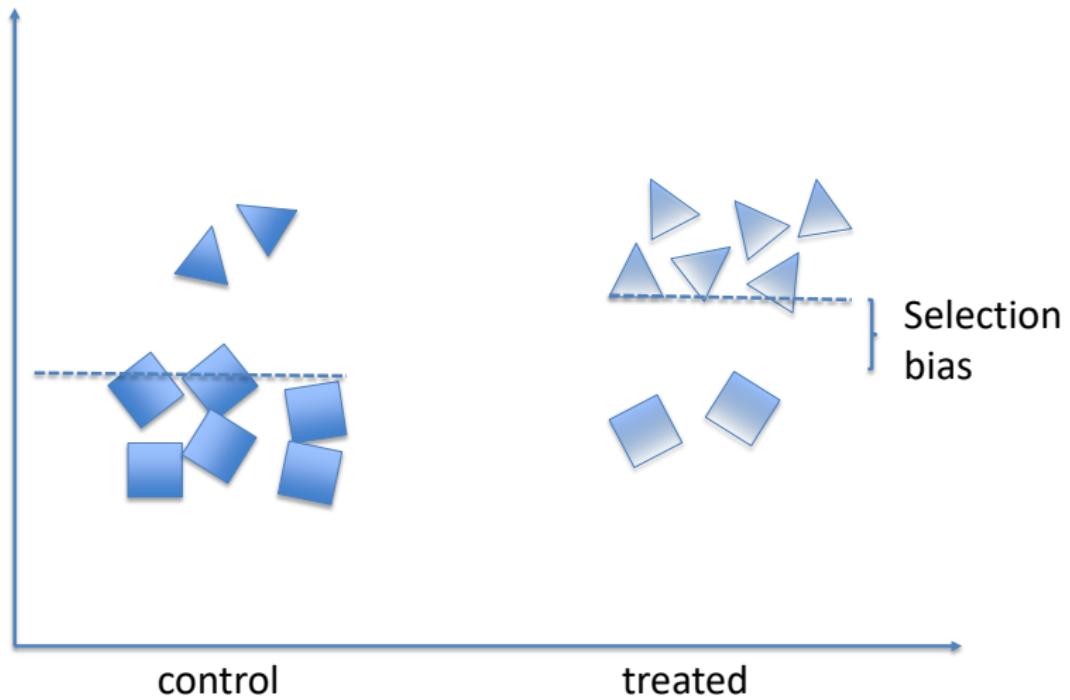


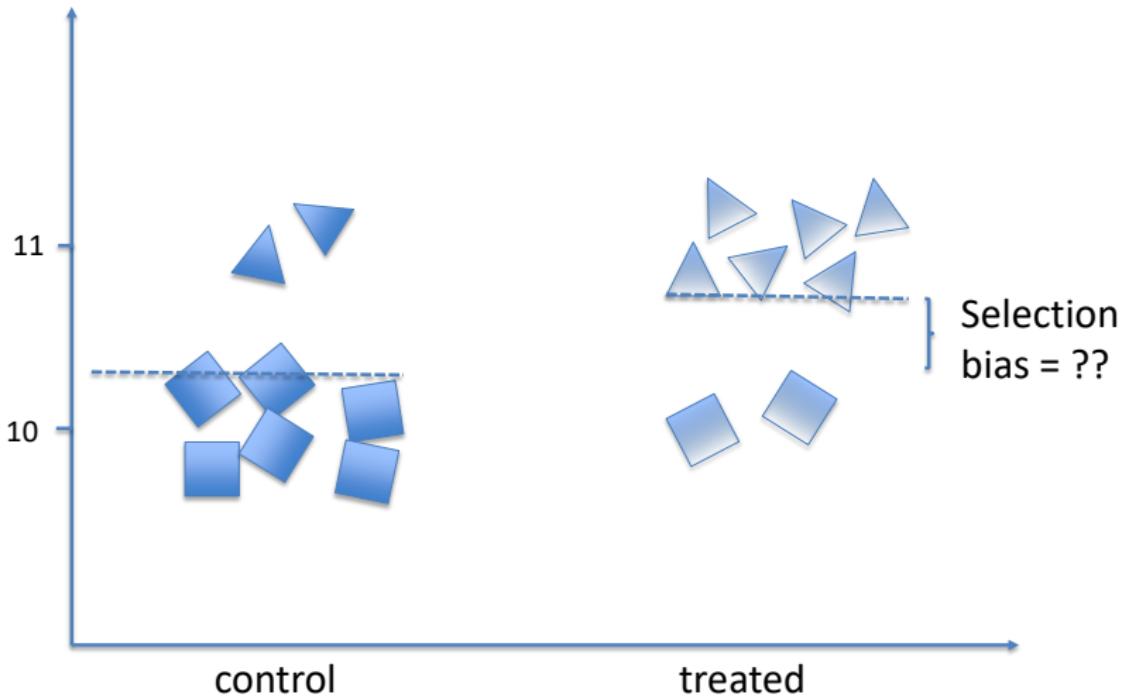


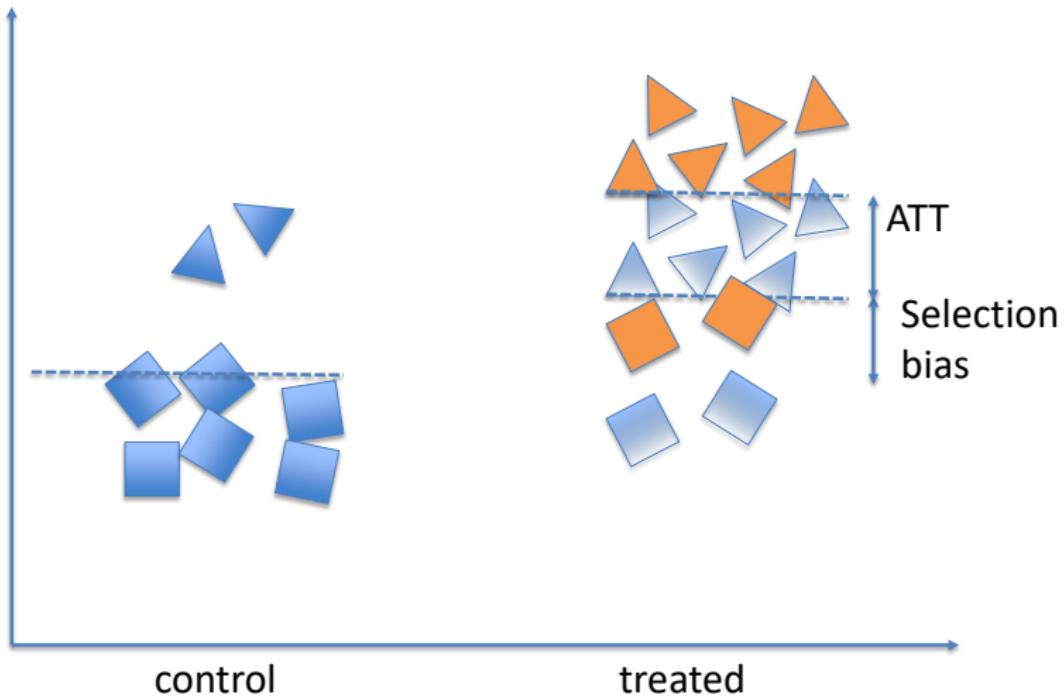




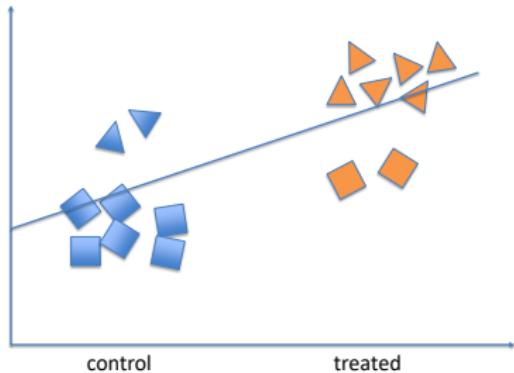








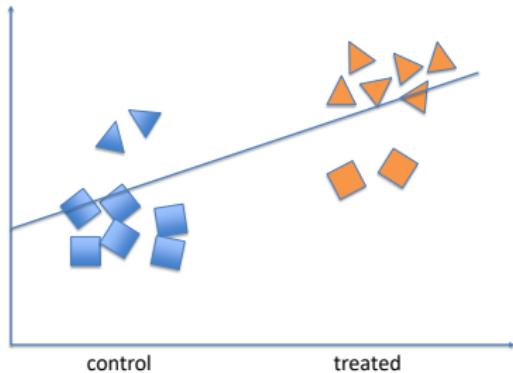
## Aside: Selection bias by another name



$$Y = \alpha + \beta D + \underbrace{\gamma S + \eta}_{\varepsilon}, \quad \text{Cov}(D, \eta) = 0$$

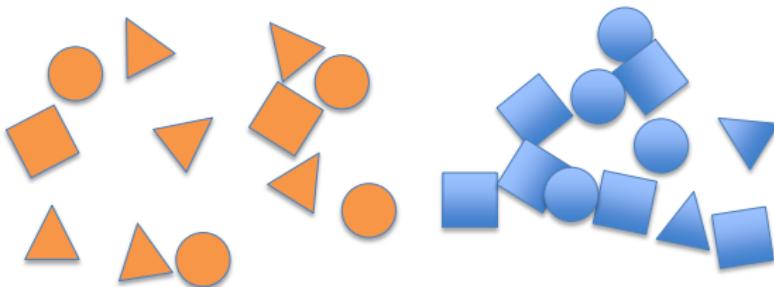
$$\hat{b}^{OLS} \rightarrow \beta + \underbrace{\gamma \frac{\text{Cov}(S, D)}{\text{Var}(D)}}_{OVB}$$

## Aside: Selection bias by another name



$$OVB = \gamma \times \frac{Cov(S, D)}{Var(D)}$$

$$\begin{aligned} \text{Selection Bias} &= E[Y(0)|D=1] - E[Y(0)|D=0] \\ &= \underbrace{(E[Y(0)|S=1] - E[Y(0)|S=0])}_{\gamma} \\ &\quad \times \underbrace{\Pr(S=1|D=1) - \Pr(S=1|D=0)}_{Cov(S,D)/Var(D)} \end{aligned}$$

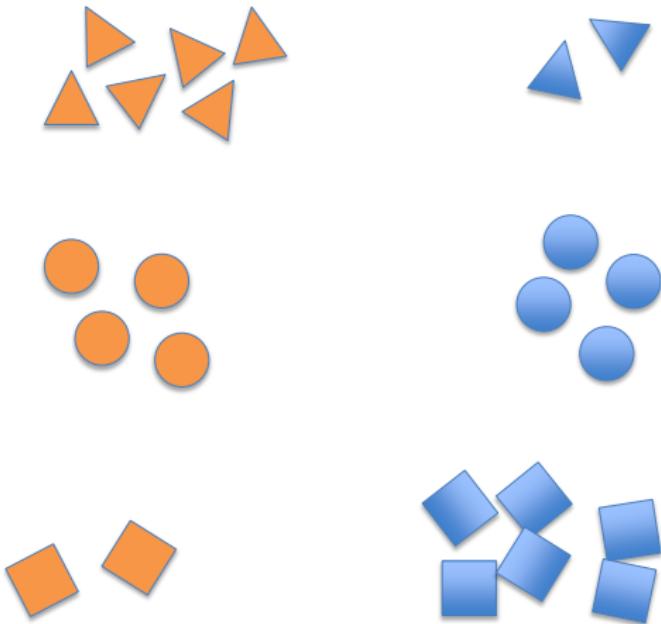


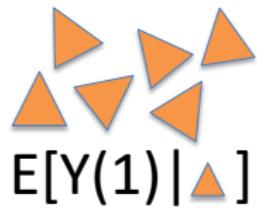
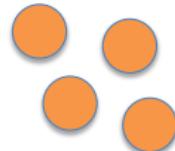
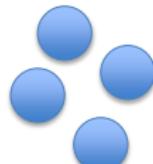
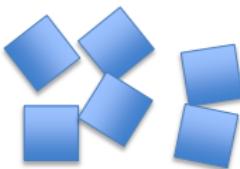
ATT

$$E[Y|D=1] - E[Y|D=0] = E[Y(1) - Y(0)|D=1]$$

$$+ E[Y(0)|D=1] - E[Y(0)|D=0]$$

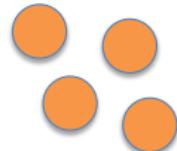
selection bias



 $E[Y(1) | \triangle]$  $E[Y(1) | \circ]$  $E[Y(1) | \square]$  $E[Y(0) | \triangle]$  $E[Y(0) | \circ]$  $E[Y(0) | \square]$



$E[Y(1) | \blacktriangle]$



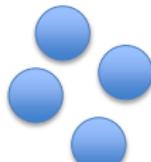
$E[Y(1) | \bullet]$



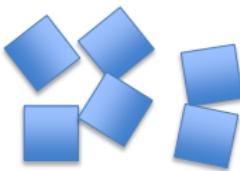
$E[Y(1) | \blacksquare]$



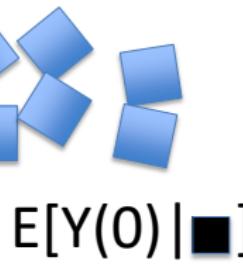
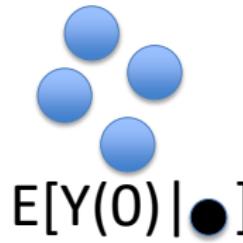
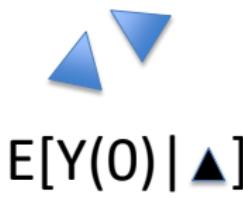
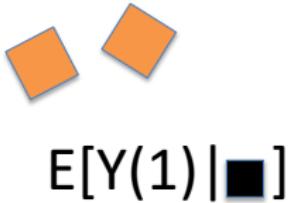
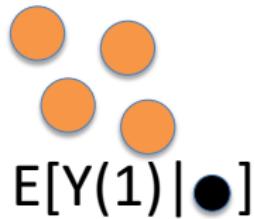
$E[Y(0) | \blacktriangle]$



$E[Y(0) | \bullet]$



$E[Y(0) | \blacksquare]$



As long as:

$$E[Y(1)|\triangle] = E[Y(1)|\blacktriangle]$$
$$E[Y(0)|\triangle] = E[Y(0)|\blacktriangle]$$

$$E[Y(1)|\circleddash] = E[Y(1)|\bullet]$$
$$E[Y(0)|\circleddash] = E[Y(0)|\bullet]$$

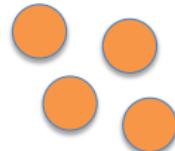
$$E[Y(1)|\square] = E[Y(1)|\blacksquare]$$
$$E[Y(0)|\square] = E[Y(0)|\blacksquare]$$



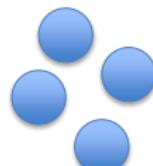
$$E[Y(1) | \blacktriangle] -$$



$$E[Y(0) | \blacktriangle] = E[Y(1) - Y(0) | \blacktriangle]$$



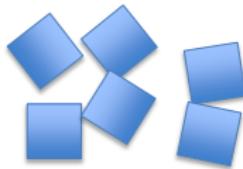
$$E[Y(1) | \bullet] -$$



$$E[Y(0) | \bullet] = E[Y(1) - Y(0) | \bullet]$$



$$E[Y(1) | \blacksquare] -$$



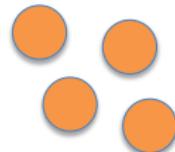
$$E[Y(0) | \blacksquare] = E[Y(1) - Y(0) | \blacksquare]$$



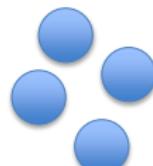
$$E[Y(1) | \blacktriangle] -$$



$$E[Y(0) | \blacktriangle] = E[Y(1) - Y(0) | \blacktriangle]$$



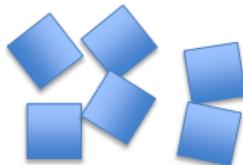
$$E[Y(1) | \bullet] -$$



$$E[Y(0) | \bullet] = E[Y(1) - Y(0) | \bullet]$$



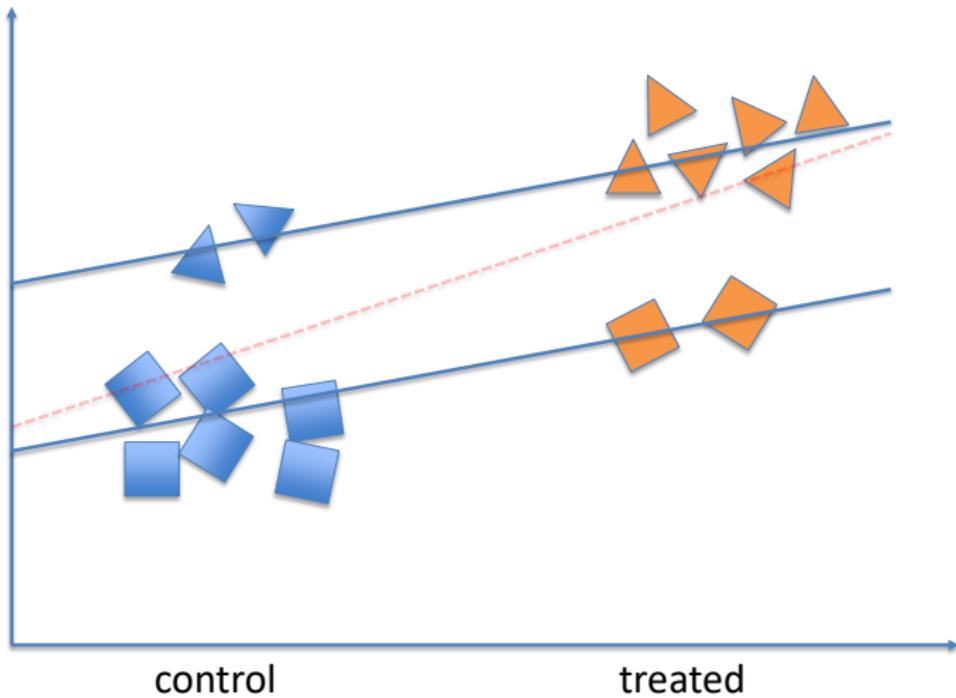
$$E[Y(1) | \blacksquare] -$$



$$E[Y(0) | \blacksquare] = E[Y(1) - Y(0) | \blacksquare]$$

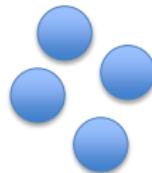
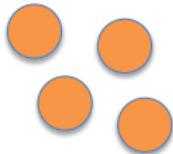
---

E[Y(1) - Y(0)]

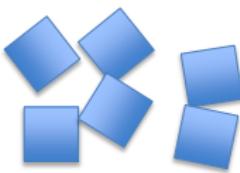




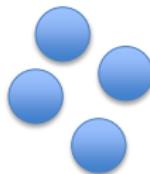
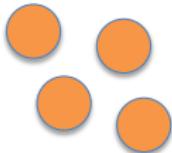
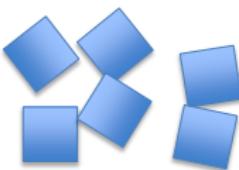
$Y(1), Y(0) \perp\!\!\!\perp D | \blacktriangle$



$Y(1), Y(0) \perp\!\!\!\perp D | \bullet$

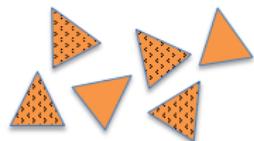


$Y(1), Y(0) \perp\!\!\!\perp D | \blacksquare$

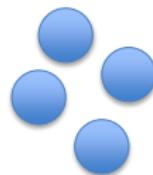
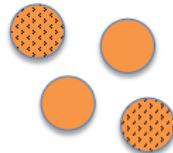

$$Y(1), Y(0) \perp\!\!\!\perp D | \blacktriangle$$

$$Y(1), Y(0) \perp\!\!\!\perp D | \bullet$$

$$Y(1), Y(0) \perp\!\!\!\perp D | \blacksquare$$

Selection on observables,  
“unconfoundedness”:

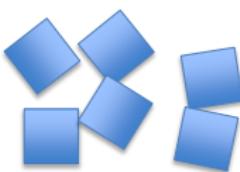
$$Y(1), Y(0) \perp\!\!\!\perp D | X$$



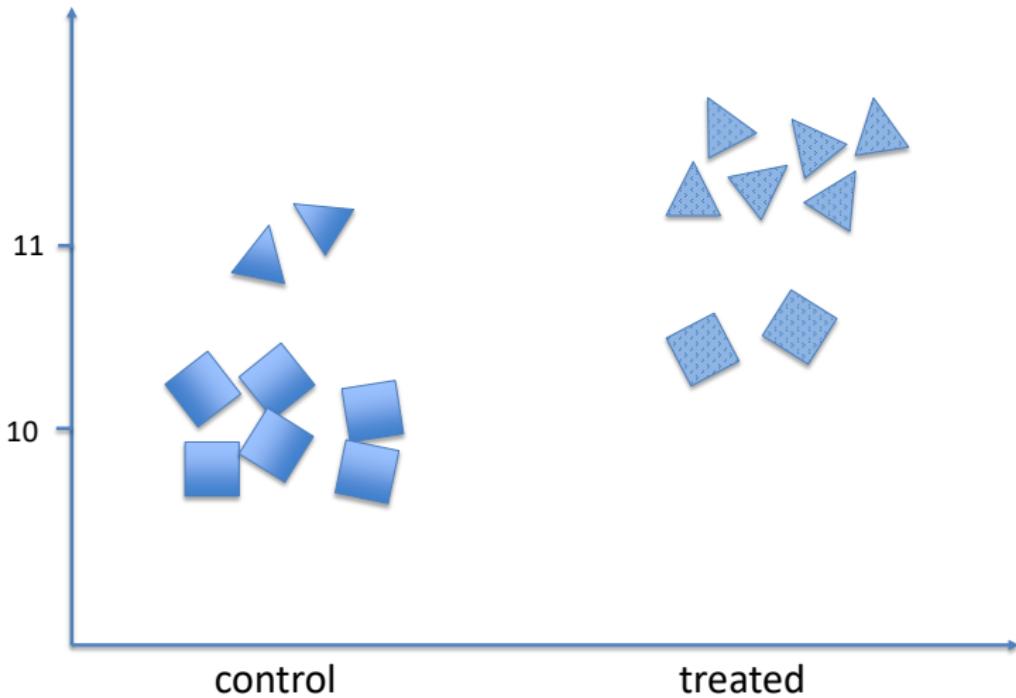
$Y(1), Y(0)$  ~~⊥~~ D | ▲

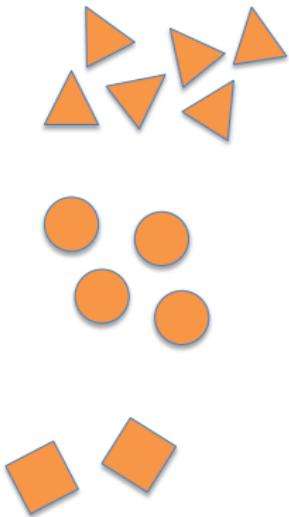


$Y(1), Y(0)$  ~~⊥~~ D | ●

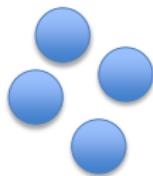
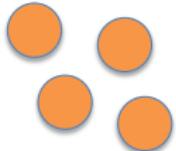


$Y(1), Y(0)$  ~~⊥~~ D | ■

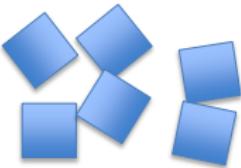




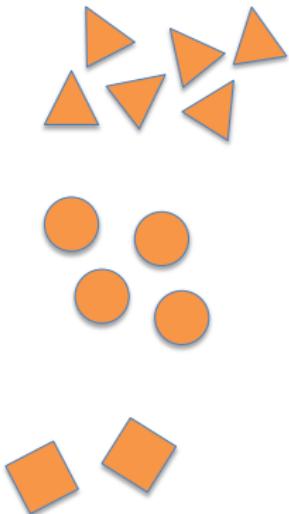
$$\Pr(D=1 | \blacktriangle) = .75$$



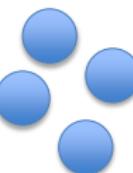
$$\Pr(D=1 | \bullet) = .5$$



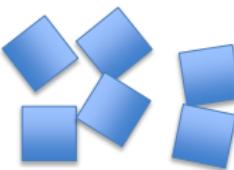
$$\Pr(D=1 | \blacksquare) = .25$$



$$\Pr(D=1 | \blacktriangle) = .75$$



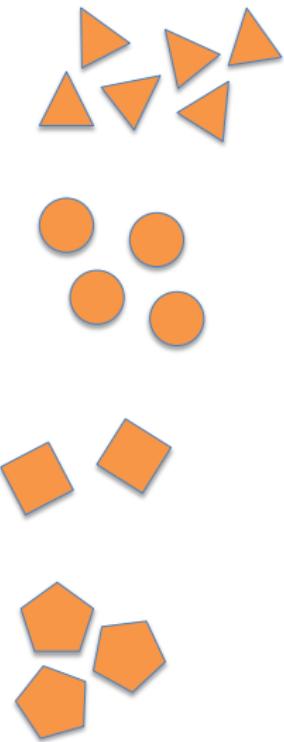
$$\Pr(D=1 | \bullet) = .5$$



$$\Pr(D=1 | \blacksquare) = .25$$

Common support,  
“overlap”:

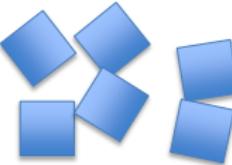
$$0 < \Pr(D=1 | X) < 1$$



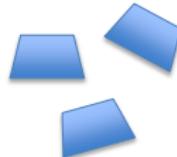
$$\Pr(D=1 | \blacktriangle) = .75$$

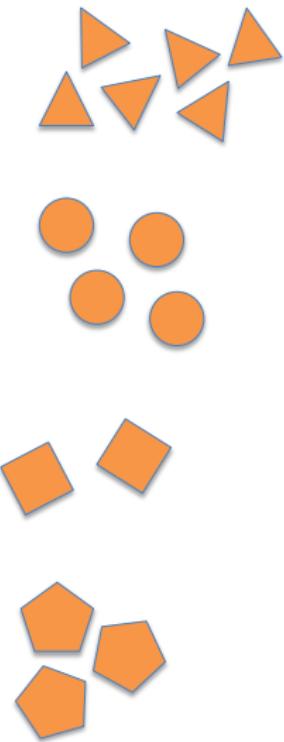


$$\Pr(D=1 | \bullet) = .5$$

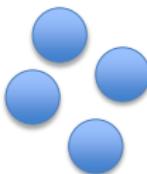


$$\Pr(D=1 | \blacksquare) = .25$$

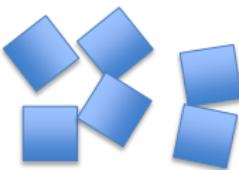




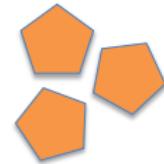
$$\Pr(D=1 | \blacktriangle) = .75$$



$$\Pr(D=1 | \bullet) = .5$$



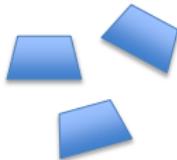
$$\Pr(D=1 | \blacksquare) = .25$$



!!

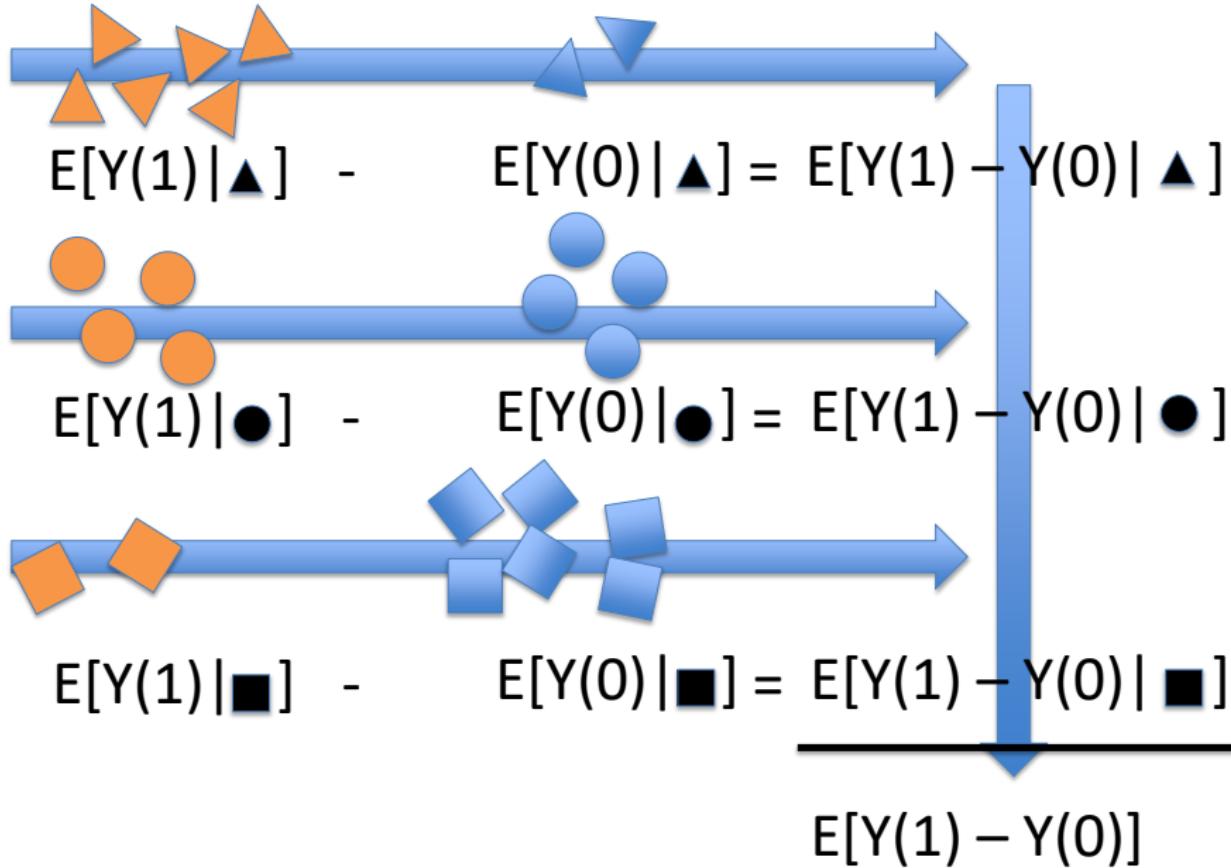
$$\Pr(D=1 | \blacklozenge) = 1$$

!!

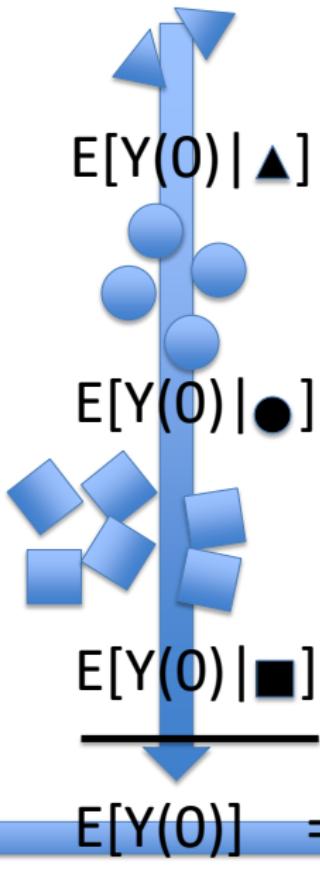
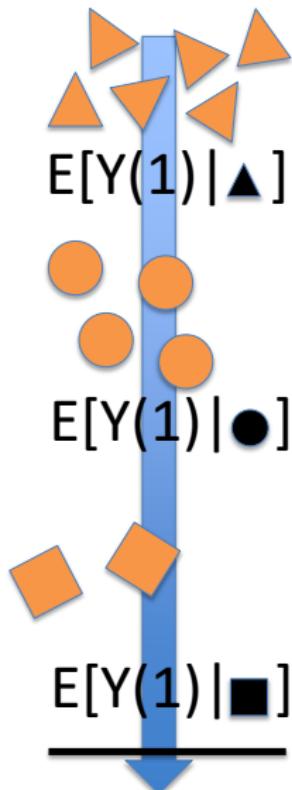


$$\Pr(D=1 | \blacksquare) = 0$$

## Matching, subclassification, p-score blocking

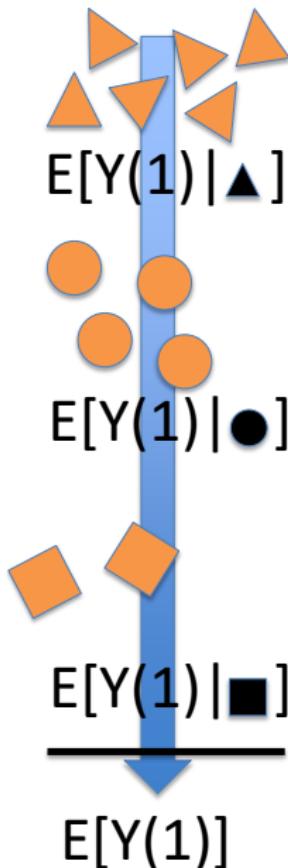


## Inverse p-score weighting, regression



$$E[Y(1)] - E[Y(0)] = E[Y(1) - Y(0)]$$

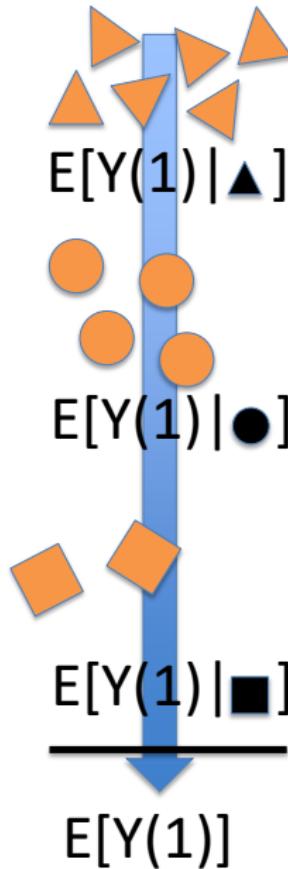
## Inverse p-score weighting, regression



Inverse propensity score weighting:

$$\begin{aligned}\hat{\mu}_1 &= \bar{Y}_{1\blacktriangle} \frac{n_{1\blacktriangle} + n_{0\blacktriangle}}{n} + \dots \\ &= \frac{1}{n} \left[ n_{1\blacktriangle} \bar{Y}_{1\blacktriangle} \frac{n_{1\blacktriangle} + n_{0\blacktriangle}}{n_{1\blacktriangle}} + \dots \right] \\ &= \frac{1}{n} \left[ \sum_{i:D_i=1, X_i=\blacktriangle} Y_i \frac{1}{\pi_\blacktriangle} + \dots \right] \\ &= \frac{1}{n} \sum_{i=1}^n \frac{Y_i D_i}{\pi_i}\end{aligned}$$

## Inverse p-score weighting, regression



Regression:

$$\begin{aligned}\hat{\mu}_1 &= \bar{Y}_{1\blacktriangle} \frac{n_{1\blacktriangle} + n_{0\blacktriangle}}{n} + \dots \\ &= \frac{1}{n} [\bar{Y}_{1\blacktriangle} (n_{1\blacktriangle} + n_{0\blacktriangle}) + \dots] \\ &= \frac{1}{n} \left[ \sum_{i:X_i=\blacktriangle}^n \bar{Y}_{1\blacktriangle} + \dots \right] \\ &= \frac{1}{n} \sum_{i=1}^n \hat{Y}_i(1)\end{aligned}$$

## Mathematical setup

$D_i$ : treatment indicator

$Y_i(0)$ : potential outcome if untreated

$Y_i(1)$ : potential outcome if treated

## Mathematical setup

$D_i$ : treatment indicator

$Y_i(0)$ : potential outcome if untreated

$Y_i(1)$ : potential outcome if treated

## Mathematical setup

$D_i$ : treatment indicator

$Y_i(0)$ : potential outcome if untreated

$Y_i(1)$ : potential outcome if treated

Parameter of Interest (Average Treatment Effect)

$$E[Y_i(1) - Y_i(0)]$$

# Mathematical setup

$D_i$ : treatment indicator

$Y_i(0)$ : potential outcome if untreated

$Y_i(1)$ : potential outcome if treated

Parameter of Interest (Average Treatment Effect)

$$E[Y_i(1) - Y_i(0)]$$

- ▶ Main challenge:

$$E[Y_i|D_i = 1] \neq E[Y_i(1)]$$

$$E[Y_i|D_i = 0] \neq E[Y_i(0)]$$

# Identification

## Assumption (Stable Unit Treatment Value (SUTVA))

Let  $d$  be an  $n \times 1$  vector of possible treatment assignments. For all  $d$  and  $d'$  such that  $d_i = d'_i$ ,  $Y_i(d) = Y_i(d') := Y_i(d_i)$ .

## Assumption (Selection on Observables)

$$Y_i(1), Y_i(0) \perp\!\!\!\perp D_i | X_i$$

## Assumption (Common Support)

$$0 < \Pr(D_i = 1 | X_i) < 1 \text{ almost surely}$$

## Theorem (ATE Identification)

$$E[Y_i(1) - Y_i(0)] = E[E[Y_i | D_i = 1, X_i] - E[Y_i | D_i = 0, X_i]]$$

(matching!)

# Identification

## Assumption (Stable Unit Treatment Value (SUTVA))

Let  $d$  be an  $n \times 1$  vector of possible treatment assignments. For all  $d$  and  $d'$  such that  $d_i = d'_i$ ,  $Y_i(d) = Y_i(d') := Y_i(d_i)$ .

## Assumption (Selection on Observables)

$$Y_i(1), Y_i(0) \perp\!\!\!\perp D_i | X_i$$

## Assumption (Common Support)

$$0 < \Pr(D_i = 1 | X_i) < 1 \text{ almost surely}$$

## Theorem (ATE Identification)

$$E[Y_i(1) - Y_i(0)] = E[E[Y_i | D_i = 1, X_i]] - E[E[Y_i | D_i = 0, X_i]]$$

(regression!)

# General guidelines for choosing covariates

Remember the formula for selection bias:

$$\begin{aligned}\text{Selection Bias} &= (E[Y(0)|X=1] - E[Y(0)|X=0]) \\ &\quad \times (\Pr(X=1|D=1) - \Pr(X=1|D=0))\end{aligned}$$

- ▶ for which treated and control groups differ
- ▶ which affect outcomes
- ▶ not affected by treatment (beware of bad control!) example
- ▶ trade off between controlling for all confounding factors and curse of dimensionality
- ▶ possible resolution to tradeoff: control for propensity score

## The role of the propensity score

Define the *propensity score*  $\pi(X_i) := \Pr(D_i = 1 | X_i)$

- ▶ A sufficient control variable (Rosenbaum and Rubin 1983)

Selection on Observables  $\implies Y_i(1), Y_i(0) \perp\!\!\!\perp D_i | \pi(X_i)$

# The role of the propensity score

Define the *propensity score*  $\pi(X_i) := \Pr(D_i = 1 | X_i)$

- ▶ A sufficient control variable (Rosenbaum and Rubin 1983)

Selection on Observables  $\implies Y_i(1), Y_i(0) \perp\!\!\!\perp D_i | \pi(X_i)$

- ▶ As a weighting strategy

$$E[Y_i(1)] = E\left[\frac{Y_i D_i}{\pi(X_i)}\right]$$

$$E[Y_i(0)] = E\left[\frac{Y_i (1 - D_i)}{1 - \pi(X_i)}\right]$$

## Empirical Example: Return to Ivy League Education

- ▶ Research question: what is the financial return to attending an Ivy League college, relative to a less-prestigious college?

## Empirical Example: Return to Ivy League Education

- ▶ Research question: what is the financial return to attending an Ivy League college, relative to a less-prestigious college?
- ▶ Data: College and Beyond Survey ( $N = 14,238$ , all attended a participating college)

## Empirical Example: Return to Ivy League Education

- ▶ Research question: what is the financial return to attending an Ivy League college, relative to a less-prestigious college?
- ▶ Data: College and Beyond Survey ( $N = 14,238$ , all attended a participating college)
- ▶  $D_i$ : binary indicator for attending Ivy-plus (Barnard, Columbia, Duke, Northwestern, Penn, Princeton, Stanford, Yale)

## Empirical Example: Return to Ivy League Education

- ▶ Research question: what is the financial return to attending an Ivy League college, relative to a less-prestigious college?
- ▶ Data: College and Beyond Survey ( $N = 14,238$ , all attended a participating college)
- ▶  $D_i$ : binary indicator for attending Ivy-plus (Barnard, Columbia, Duke, Northwestern, Penn, Princeton, Stanford, Yale)
- ▶  $Y_i$ : personal income mid-career

## Empirical Example: Return to Ivy League Education

- ▶ Research question: what is the financial return to attending an Ivy League college, relative to a less-prestigious college?
- ▶ Data: College and Beyond Survey ( $N = 14,238$ , all attended a participating college)
- ▶  $D_i$ : binary indicator for attending Ivy-plus (Barnard, Columbia, Duke, Northwestern, Penn, Princeton, Stanford, Yale)
- ▶  $Y_i$ : personal income mid-career
- ▶  $X_i$ : ???

## Empirical Example: Return to Ivy League Education

- ▶ Research question: what is the financial return to attending an Ivy League college, relative to a less-prestigious college?
- ▶ Data: College and Beyond Survey ( $N = 14,238$ , all attended a participating college)
- ▶  $D_i$ : binary indicator for attending Ivy-plus (Barnard, Columbia, Duke, Northwestern, Penn, Princeton, Stanford, Yale)
- ▶  $Y_i$ : personal income mid-career
- ▶  $X_i$ : ???
  - ▶ SAT score

## Empirical Example: Return to Ivy League Education

- ▶ Research question: what is the financial return to attending an Ivy League college, relative to a less-prestigious college?
- ▶ Data: College and Beyond Survey ( $N = 14,238$ , all attended a participating college)
- ▶  $D_i$ : binary indicator for attending Ivy-plus (Barnard, Columbia, Duke, Northwestern, Penn, Princeton, Stanford, Yale)
- ▶  $Y_i$ : personal income mid-career
- ▶  $X_i$ : ???
  - ▶ SAT score
  - ▶ parents' education

## Empirical Example: Return to Ivy League Education

- ▶ Research question: what is the financial return to attending an Ivy League college, relative to a less-prestigious college?
- ▶ Data: College and Beyond Survey ( $N = 14,238$ , all attended a participating college)
- ▶  $D_i$ : binary indicator for attending Ivy-plus (Barnard, Columbia, Duke, Northwestern, Penn, Princeton, Stanford, Yale)
- ▶  $Y_i$ : personal income mid-career
- ▶  $X_i$ : ???
  - ▶ SAT score
  - ▶ parents' education
  - ▶ . . .

## Estimation options

- ▶ Matching
- ▶ Weighting
- ▶ Regression
- ▶ Doubly-robust weighting + regression
- ▶ Propensity score subclassification + regression

## Matching details (1)

1. For each observation  $i$ , find the  $m$  “nearest” neighbors in the opposite treatment group,  $J_m(i)$
2. Impute  $\hat{Y}_i(0), \hat{Y}_i(1)$  for each observation:

$$\hat{Y}_i(0) = \begin{cases} Y_i & , D_i = 0 \\ \frac{1}{m} \sum_{j \in J_m(i)} Y_j & , D_i = 1 \end{cases}$$

$$\hat{Y}_i(1) = \begin{cases} Y_i & , D_i = 1 \\ \frac{1}{m} \sum_{j \in J_m(i)} Y_j & , D_i = 0 \end{cases}$$

3.  $\hat{\delta}^{\text{match}} = \frac{1}{n} \sum_{i=1}^n (\hat{Y}_i(1) - \hat{Y}_i(0))$

## Matching details (2): What do we mean by “nearest”?

- ▶ simple Euclidean distance:

$$\sum_{l=1}^k (X_{li} - X_{lj})^2$$

- ▶ scaled Euclidean distance:

$$\sum_{l=1}^k (X_{li} - X_{lj})^2 / \text{Var}(X_l)$$

- ▶ Mahalanobis distance:

$$(X_i - X_j)' \Sigma_X^{-1} (X_i - X_j)$$

## Weighting details

1. Estimate propensity score, e.g. logit  $D | X \rightarrow \hat{\pi}(X_i)$
2. Weight by inverse of propensity score to get average potential outcomes:

$$\hat{\mu}_1 = \frac{\sum_{i=1}^n Y_i D_i / \hat{\pi}(X_i)}{\sum_{i=1}^n D_i / \hat{\pi}(X_i)}$$

$$\hat{\mu}_0 = \frac{\sum_{i=1}^n Y_i (1 - D_i) / (1 - \hat{\pi}(X_i))}{\sum_{i=1}^n (1 - D_i) / (1 - \hat{\pi}(X_i))}$$

- 3.

$$\hat{\delta}^{\text{IPW}} = \hat{\mu}_1 - \hat{\mu}_0$$

## Regression details

1. Regress  $Y_i$  on  $X_i$  in  $D_i = 0$  sample  $\rightarrow \hat{\mu}_0(X_i)$
2. Regress  $Y_i$  on  $X_i$  in  $D_i = 1$  sample  $\rightarrow \hat{\mu}_1(X_i)$
3.  $\hat{\delta}^{\text{reg}} = \frac{1}{n} \sum_{i=1}^n (\hat{\mu}_1(X_i) - \hat{\mu}_0(X_i))$

Or, in one step:

$$Y_i = \delta D_i + X'_i \beta + D_i \times (X_i - \bar{X})' \gamma + \varepsilon_i$$

## Doubly-robust details

1. Estimate regression fits  $\hat{\mu}_1(X_i)$ ,  $\hat{\mu}_0(X_i)$  and propensity score  $\hat{\pi}(X_i)$  as above

2.

$$\hat{\mu}_1^{\text{DR}} = \frac{1}{n} \sum_{i=1}^n \left( \hat{\mu}_1(X_i) + \frac{D_i(Y_i - \hat{\mu}_1(X_i))}{\hat{\pi}(X_i)} \right)$$

$$\hat{\mu}_0^{\text{DR}} = \frac{1}{n} \sum_{i=1}^n \left( \hat{\mu}_0(X_i) + \frac{(1 - D_i)(Y_i - \hat{\mu}_0(X_i))}{1 - \hat{\pi}(X_i)} \right)$$

3.

$$\hat{\delta}^{\text{DR}} = \hat{\mu}_1^{\text{DR}} - \hat{\mu}_0^{\text{DR}}$$

- ▶ Advantage: consistent even if either propensity or regression functions misspecified
- ▶ But what if both are misspecified? (Kang and Schafer 2007)

# Machine Learning + Double Robust (Chernozhukov, et al. 2018)

Divide sample into  $K$  folds and for each fold  $k = 1, \dots, K$ :

1. Estimate regression fits  $\hat{\mu}_1^k(X_i)$ ,  $\hat{\mu}_0^k(X_i)$  and propensity score  $\hat{\pi}^k(X_i)$  using all data *except* fold  $k$  using favorite ML method (e.g. LASSO, random forest)
- 2.

$$\hat{\mu}_1^k = \frac{1}{n_k} \sum_{i \in \text{fold } k} \left( \hat{\mu}_1^k(X_i) + \frac{D_i(Y_i - \hat{\mu}_1^k(X_i))}{\hat{\pi}^k(X_i)} \right)$$

$$\hat{\mu}_0^k = \frac{1}{n_k} \sum_{i \in \text{fold } k} \left( \hat{\mu}_0^k(X_i) + \frac{(1 - D_i)(Y_i - \hat{\mu}_0^k(X_i))}{1 - \hat{\pi}^k(X_i)} \right)$$

- 3.

$$\hat{\delta}^{\text{DR+ML}} = \frac{1}{K} \sum_{k=1}^K \left( \hat{\mu}_1^k - \hat{\mu}_0^k \right)$$

## Propensity score subclassification + regression (Imbens and Wooldridge, 2007)

Poor man's double robustness (but works great!)

- ▶ Estimate propensity score, e.g. logit  $D | X \rightarrow \hat{\pi}(X_i)$

## Propensity score subclassification + regression (Imbens and Wooldridge, 2007)

Poor man's double robustness (but works great!)

- ▶ Estimate propensity score, e.g. logit  $D \mid X \rightarrow \hat{\pi}(X_i)$
- ▶ Divide sample into  $J$  (e.g.,  $J = 5$ ) equal-sized blocks, sorting on  $\hat{\pi}(X_i)$

## Propensity score subclassification + regression (Imbens and Wooldridge, 2007)

Poor man's double robustness (but works great!)

- ▶ Estimate propensity score, e.g. logit  $D | X \rightarrow \hat{\pi}(X_i)$
- ▶ Divide sample into  $J$  (e.g.,  $J = 5$ ) equal-sized blocks, sorting on  $\hat{\pi}(X_i)$
- ▶ For blocks  $j = 1, \dots, J$ , estimate

$$Y_i = \alpha_j + \delta_j D_i + X'_i \beta_j + \epsilon_i$$

## Propensity score subclassification + regression (Imbens and Wooldridge, 2007)

Poor man's double robustness (but works great!)

- ▶ Estimate propensity score, e.g. logit  $D | X \rightarrow \hat{\pi}(X_i)$
- ▶ Divide sample into  $J$  (e.g.,  $J = 5$ ) equal-sized blocks, sorting on  $\hat{\pi}(X_i)$
- ▶ For blocks  $j = 1, \dots, J$ , estimate

$$Y_i = \alpha_j + \delta_j D_i + X'_i \beta_j + \epsilon_i$$

- ▶ Final treatment effect:

$$\hat{\delta} = \frac{1}{J} \sum_{j=1}^J \hat{\delta}_j$$

# Connection to multiple linear regression

Remember the familiar model:

$$Y_i = \delta D_i + X'_i \beta + \varepsilon_i$$

How does this relate to matching, propensity score weighting, etc.? (suppose  $X_i$  is saturated)

- ▶ Analogous assumptions:  
 $E[\varepsilon_i | X_i] = 0 \sim Y_i(1), Y_i(0) \perp\!\!\!\perp D_i | X_i$
- ▶ Slightly different estimands:

$$\delta^{OLS} = \frac{E[\delta(X_i) \operatorname{Var}(D_i|X_i)]}{E[\operatorname{Var}(D_i|X_i)]}$$

$$\delta^{matching} = \delta^{IPW} = ATE$$

# Inference

- ▶ Under suitable conditions the estimators above are approximately normally distributed in large samples:

$$\sqrt{n} \left( \hat{\delta} - ATE \right) \xrightarrow{d} N(0, V)$$

with a variance  $V$  that can be estimated easily (formulas, bootstrap<sup>1</sup>)

- ▶ Confidence intervals and hypothesis tests are then straightforward:

$$CI_{.95}(ATE) = \hat{\delta} \pm 1.96 \times \sqrt{V/n}$$

---

<sup>1</sup>except matching in some cases

## Assessing the assumptions: Selection on observables

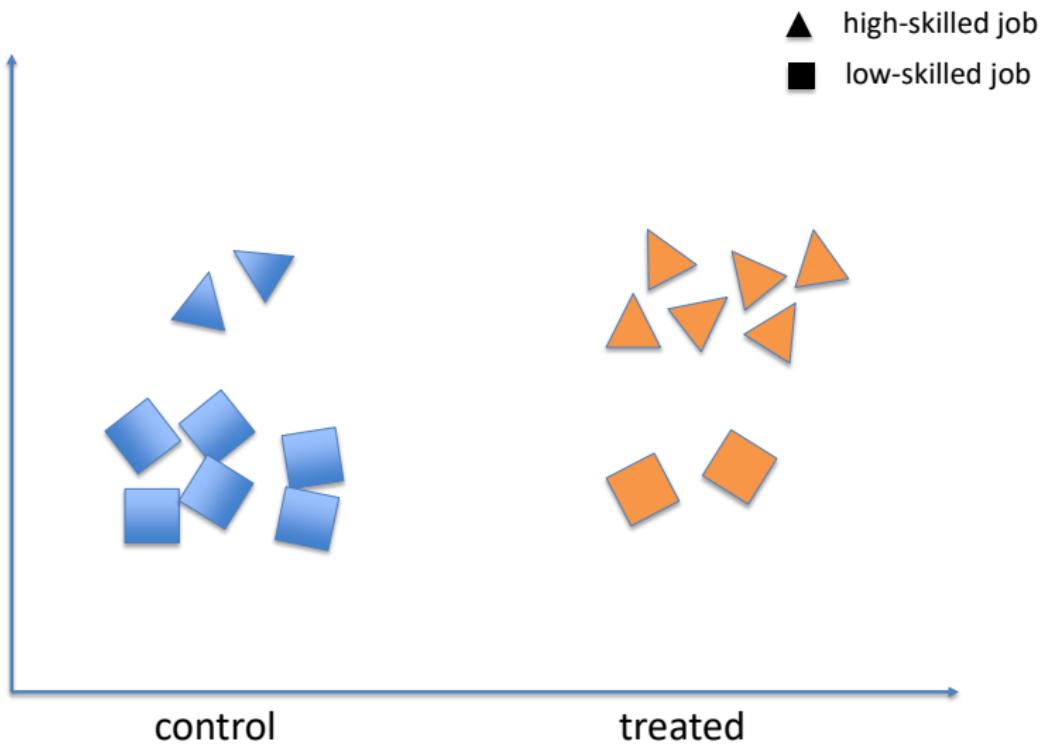
- ▶ Estimate “effects” on predetermined variables (besides those in  $X_i$ )
- ▶ Jointly test null of zero effect
- ▶ If reject, consider expanding  $X_i$

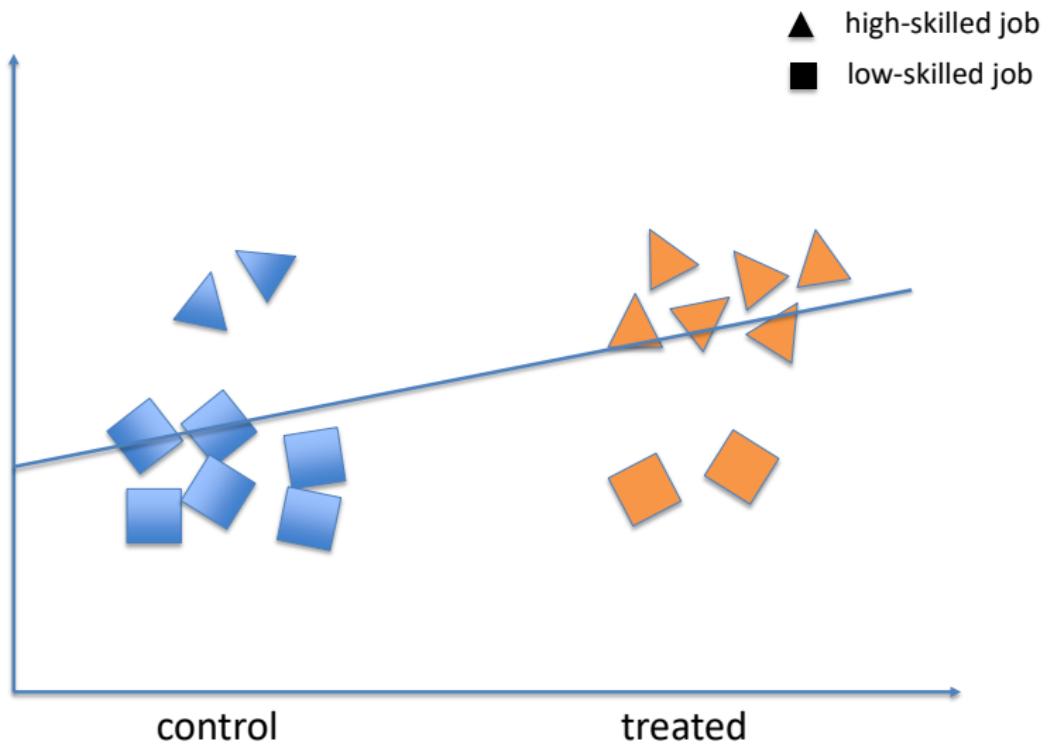
## Assessing the assumptions: Common support

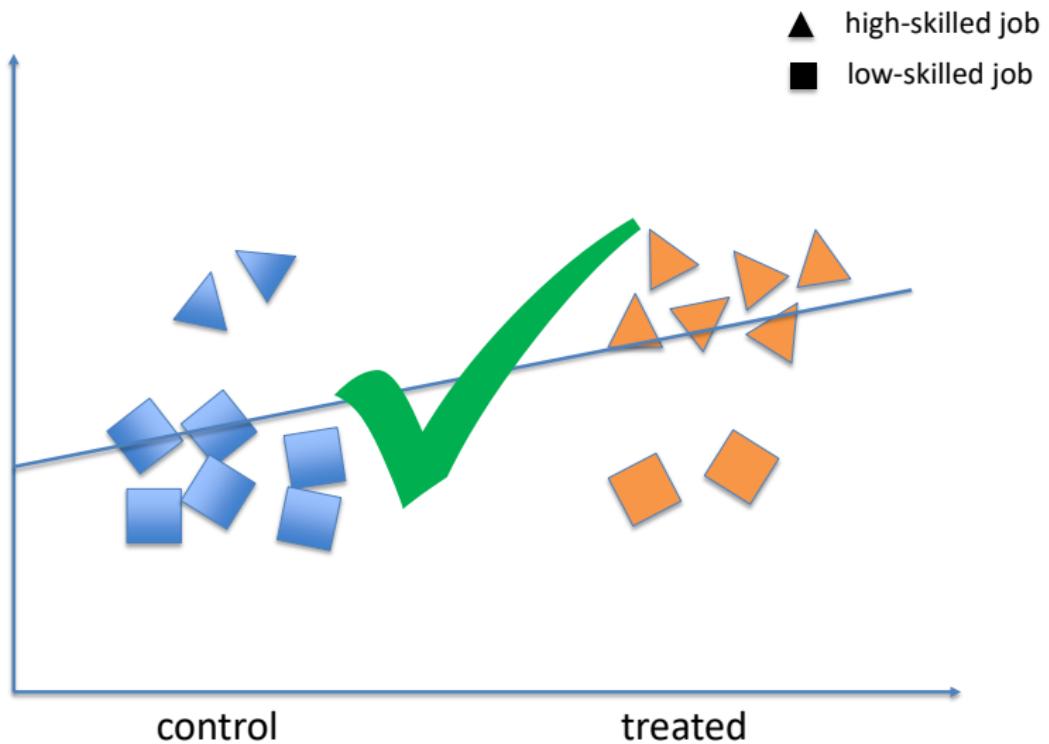
- ▶ Visually plot covariate distributions by treatment status
- ▶ Compute normalized imbalances (Imbens 2015):

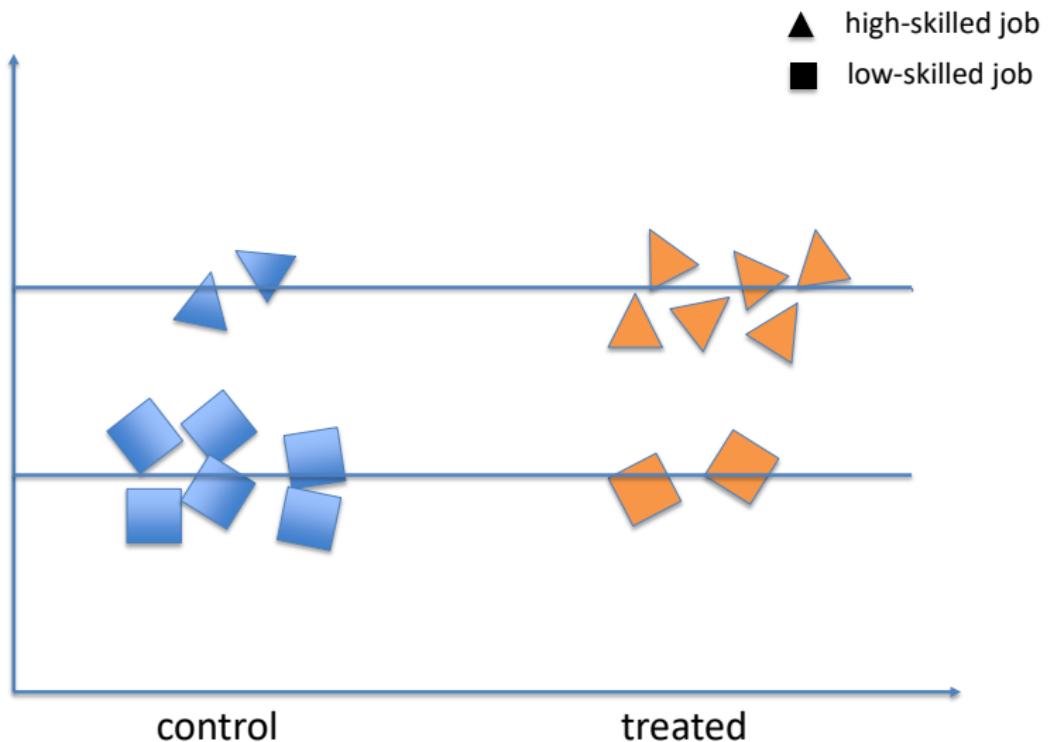
$$\frac{\bar{X}_1 - \bar{X}_0}{\sqrt{(\hat{S}_1^2 + \hat{S}_0^2)/2}}$$

- ▶ Examine estimated propensities
- ▶ Consider trimming observations with propensities very near zero or one









▲ high-skilled job  
■ low-skilled job

