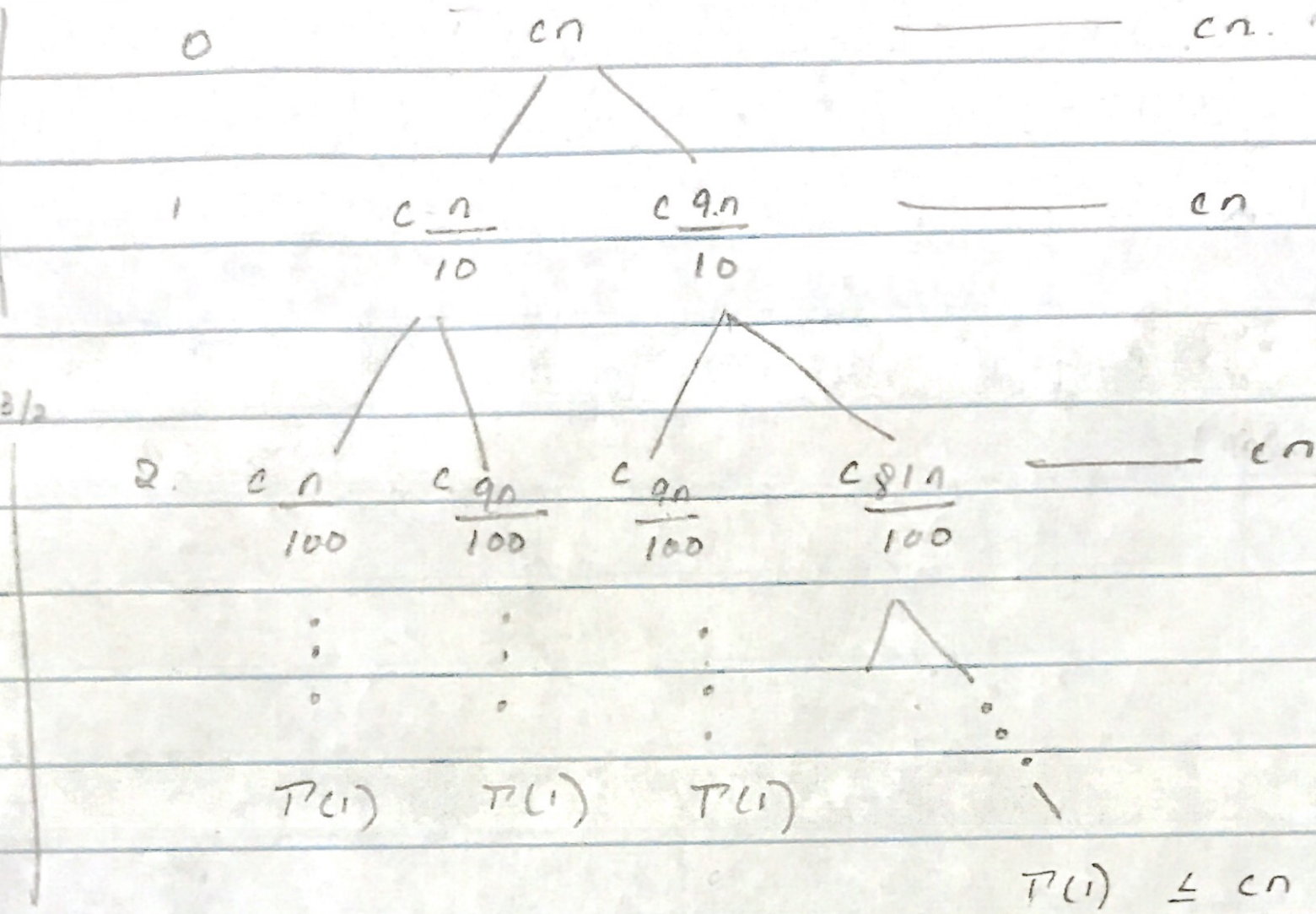
$\frac{9n}{1}$ $\quad\quad$ $\frac{\frac{9n}{10}}{\frac{10}{10}}$ $\quad\quad$ $\frac{9n}{10} \times \frac{1}{10}$

Q.3 $\quad T(n) = T(n/10) + T(9n/10) + n$



By assuming longest path: Total $O(n \lg n)$

Guess : $O(n \lg n)$.

Hypothesis : $T(n) \leq dn \lg(n)$

$T(n) \leq T(n/10) + T(9n/10) + cn$

$\leq d(n/10) \lg(n/10) + d(9n/10) \lg(9n/10) + cn$

$$= \left[a(n/10) \{ \lg n - \lg 10 \} \right] + \left[a(9n/10) \{ \lg n - \lg 9/10 \} \right]$$

$$= a(n/10) \lg n - a(n/10) \lg 10 + a(9n/10) \lg n - a(9n/10) \lg 9/10 + cn$$

$$= an \lg(n) - a \left[(n/10) \lg 10 + (9n/10) \lg (9/10) \right]$$

$$= an \lg(n) - a \left[(n/10) \lg 10 + \left(\frac{9n}{10}\right) \lg 9 - (9n/10) \lg 10 \right]$$

$$= an \lg(n) - an (\lg 10 - 9/10) + cn$$

$$\leq an \lg n$$

This inequality holds for any: $a \geq \epsilon / (\lg 10 - (9/10))$