(2) Formulation of HMM for Markov assumption length "2".

$$S^* = \text{argmax}_S \; P(S|O)$$

$\left\{ \begin{array}{l} S = \text{number of states} \\ O = \text{number of observations} \\ S^* = \text{Top path} \end{array} \right\}$

assumption

① number of states = 4
② ______ observations = 4

$$P(S|O) = P(\{S_1, S_2, S_3, S_4\} | \{O_1, O_2, O_3, O_4\})$$

$$= P(S_1|O) \times P(S_2|S_1, O) \dots \times P(S_4|S_3, S_2, S_1, O)$$

{Chain rule}

$$= P(S_1|O) \times P(S_2|S_1, O) \times P(S_3|S_2, S_1, O) \times P(S_4|S_3, S_2, O)$$

{ Markov assumption for length "2" }

→ using Bayes theorem

$$\text{Posterior} = P(A|B) = \frac{\overbrace{P(B|A)}^{\text{likelihood}} \cdot \overbrace{P(A)}^{\text{Prior}}}{\underbrace{P(B)}_{\text{Normalizing constant}}}$$

$$P(S|O) = \operatorname*{argmax}_{S} P(O|S)P(S)$$

→ $\text{Prior} = P(S_1) \times P(S_2|S_1) \times P(S_3|S_2, S_1) \times P(S_4|S_3, S_2)$

→ $\text{likelihood} = P(O_1|S_1) \cdot P(O_2|S_2) \cdot P(O_3|S_3) \cdot P(O_4|S_4)$

{ observation depends on current state only }

Prior comes from the transition probability matrix

likelihood comes from the emission probability matrix

$$\Rightarrow P(S|O) = P(O|S)\,P(S)$$

$$= P(S_1) \times P(S_2|S_1) \times P(S_3|S_2,S_1) \times P(S_4|S_3,S_2) \times$$

$$P(O_1|S_1) \times P(O_2|S_2) \times P(O_3|S_3) \times P(O_4|S_4)$$