Logica

LECTURE 1

Logic • The *systematic* study of *valid* reasoning.
Propositional logic • logic of 'not','and','or',etc.
Predicate logic • logic of 'for all', 'for some', etc.
Law of Excluded Middle *LEM* • every statement is either true or false.
Law of Non-Contradiction *LNC* • no statement is both true and false
Classical logic • both *LEM* and *LNC* hold.
Proposition • a statement that is either true or false.

LECTURE 2

Set • an abstract collection of things, its *members* or *elements*. We can list its elements ($\mathbb{N} = \{1, 2, 3, ..\}$) or give *defining properties* ($\{x \in A : P(x)\}$)
Axiom of Extensionality • For all sets X, Y : X = Y iff for all objects x, $x \in X iff x \in Y$ (unordered: {a,b}={b,a}, multiplicity: {a,a,b}={a,b})
Axiom of Separation • $\{x : x \in X \ \&\ P(x)\}$ is a set. Restricting to sets already known to exist when defining new sets is safe.
Union • $X \cup Y = \{x : x \in X \text{ or } x \in Y\}$.
Intersection • $X \cap Y = \{x : x \in X \text{ and } x \in Y\}$
Difference • $X \setminus Y = \{x \in X : x \notin Y\}$ (fe $\{a, b\} \setminus \{b\} = \{a\}$)
Subset • $X \subseteq Y \Leftrightarrow (x \in X \Rightarrow x \in Y)$
Power set • $\wp(X) = \{Y : Y \subseteq X\}$ fe $\wp$({1,2}) = {$\emptyset$, {1}, {2}, {1,2} }. Axiom : for alls sets X, $\wp$(X) is a set.
Proof strategies *(i)* • If you want to show a *universal claim* holds (for all...), then show that it holds for an *arbitrary* object.
Proof strategies *(ii)* • If you want to show a *conditional claim* (if...then...) then show that you can get the "then"-part if you assume the "if"-part.
Recursion rule • Determines what's the n-th place based on the places before and the starting point.
Recursive definition of a set • Give (a) the starting point of the set (b) rules to determine new members based on the old.
Proof by induction • We can easily prove things *about* recursively defined sets. Proof that the statement hold for the first element. Then prove, using recursive rules, it must be true for the next element as well.

LECTURE 3

Formal language • given by (a) basic symbols (b) rules for forming wffs.
PropLog *Alphabet* • (i)letters: $p_0, p_1, ..$ (ii) constant: $\perp$ (iii)connectives: $\neg, \wedge, \vee, \leftrightarrow, \rightarrow$ (iv) auxiliary: ()
PropLog *FOR set* • (i) $p_i \in FOR$ (ii) $\perp \in FOR$ (iii) $A \in FOR \Rightarrow \neg A \in FOR$ (iv) $A, B \in FOR \Rightarrow (A \circ B)$
PropLog *Parsing Trees* • for every $A \in FOR$ there is a unique parsing tree T(A)
Unique Readability Theorem • every formula has *one* correct way of reading it.
Order • From strong to weak: $\{\neg, \wedge \text{ or } \vee, \rightarrow, \leftrightarrow\}$
Occurence • An *occurence* of σ in A is a node in T(A) labelled with σ.
Function • A *function* $f : X \rightarrow Y$ is a mapping that assings to every element $x \in X$ exactly one element $f(x) \in Y$. *Domain*: set of all possible input. *Range*: set of all possible output. *Image*: The image of x under f : if $x \in X$ then $f(x) = Y$.
Recursive definition of a function *RD* • Define a function by (a)startingpoint and (b)recursive rules.
Depth • The depth of a formula (aka *logical complexity*) measures the longest branch in T(A), starting at 0.
Reasons for RD • (i)Infinite sets of value by finite statements (ii) Use recursively defined values on FOR to prove things about FOR.

LECTURE 4-5

Semantics • Aim is to characterixe the notion of a true sentence (under a given interpretation) and of entailment(implicaties)
Truth functions • Truth-values: {0,1}=**2**. Every p_i has a $V(p_i) \in \mathbf{2}$. The connectives express *truth-functions.*
Interpretation • V : FORM $\rightarrow$**2** is an interpretation: $V(p_i) = 1$ iff p_i is true; $V(p_i) = 0$ iff p_i is false.
Principle of Compositionality • The meaning of a sentence is a function of the meaning of its parts.
Principle of Truth-Functionality • The truth-value of a sentence is a function of truth-values of its parts.
Consequence • $A \in FORM$ is a *semantic consequence* of a set $\Gamma \in FORM$ iff it is impossible that all members of Γ are true, without A being true as well: $\Gamma \models A$.
Logical truth • For $A \in FORM$ iff it's impossible for A to be false: $\models A$.

Therefore • Premisses support the conclusion: $\{A_n\} \therefore B$. This is valid iff $\{A_n\} \models B$.
Functional complete • A set is functionally complete if every truth-function can be expressed as a combination of $(A \circ B)$. There are smaller sets that are functionally complete (fe 'Scheffer Stroke' : $\{f|\}$)
Truth tables *constructing* • (i)write down all p_i and the formula.(ii) Make 2^{p_i} rows. Divide rows and fill with 1/0; repeat.(ii) Work from bottom to top in the parsing tree.
Truth tables *Semantics checking* • (i)*Logical Status* Tautology: all 1. Contingency: mixed. Contradiction: all 0. (ii)*Equivalence* If formulas have the same truth-value, they are locigaly equivalent.(iii)*Validity* $\{A_n\} \therefore B$ iff $\{A_n\} \models B$ iff ($\{\wedge A_n\} \rightarrow B$ = tautology).
Interpretation by recursion • (i)V($\perp$)=0 (ii)V($\neg$A)=1-V(A) (iii)V(A$\wedge$B)=min(V(A),V(B)) (iv)V(A$\vee$B)=max(V(A),V(B)) (v)V(A$\rightarrow$B)=max(1-V(A),V(B)) (viV(A$\leftrightarrow$B)=1 if V(A)=V(B), 0 otherwise.

LECTURE 6-7

Proof theory • $\Gamma \vdash A$ means there is a proof from Γ to A. Obtain a proof system, such that $\Gamma \vdash A$ iff $\Gamma \models A$.
Why Proof theory? • (i)It is faster than large truth-tables. (ii) Truth tables don't work for more advanced logics.
Tableaus *theorem* • $\Gamma \models A$ iff there is no interpretation V, such that $V \models \Gamma \cup \{\neg A\}$.
Tableaus *constructing* • (i)Write down all members of the set (root) (ii)Apply tableaux rules, first non-branching (iii)Close if there is a branch tohrough both A and $\neg$A or $\perp$.(iv) Continue (ii) and (iii) till done. (v)*closed* iff all branches closed, *open* otherwise.
Tableaus *interpretation* • Pick an open branch $\mathfrak{B}$ and set $V(p_i) = 1$ if $p_i \in \mathfrak{B}$, $V(p_i) = 0$ otherwise.
Tableaus *proof* • A closed tableaux for $\Gamma \cup \{\neg A\}$ is proof for $\Gamma \vdash A$.
Tableaus *meta-theory* • If $\Gamma \vdash A$ then $\Gamma \models A$ (soundness) and if $\Gamma \models A$ then $\Gamma \vdash A$ (completeness).

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