# Combinatorics

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# 0 Introduction

In this course we'll discuss three main aspects:
• Set systems;
• Isoperimetric Inequalities;
• Projections (combinatorics in continuous settings).

References:
*Combinatorics*, Bocabas, Cambridge University Press, 1986 (chapter 1,2);
*Combinatorics and finite sets*, Anderson, Oxford University Press, 1987 (chapter 1).

# 1 Set Systems

Let $X$ be a set. A *set system* on $X$ (or family of subsets of $X$) is a family $\mathcal{A} \subset \mathbb{P}(X)$.
For example, we define $X^{(r)} = \{A \subset X : |A| = r\}$.

Unless otherwise stated, $X = [n] = \{1, 2, ..., n\}$. For example, $|X^{(r)}| = \binom{n}{r}$ (assume finiteness). So $[4]^{(2)} = \{12, 13, 14, 23, 24, 34\}$.

We often make $\mathbb{P}(x)$ into a graph, called $Q_n$, by joining $A$ to $B$ if $|A \triangle B| = 1$ (symmetric difference).

(examples of $Q_3, Q_n$)

If we identify a set $A \subset X$ with a 0-1 sequence of length $n$ via $A \leftrightarrow 1_A$ (characteristic function), then $Q_3$ acn be thought of as a cube. In general, $Q_n$ is an $n$-dimensional cube (hypercube/discretecube/$n$-cube/...).

## 1.1 Chains and antichains

A family $\mathcal{A} \subset \mathbb{P}(X)$ is a *chain* if $\forall A, B \in \mathcal{A}, A \subset B$ or $B \subset A$. It is an antichain if $\forall A \neq B \in \mathcal{A}$, $A \not\subset B$.

Obviously the maximum size of a chain in $X$ is $n + 1$.

For antichains, we can take $X^{\lfloor \frac{n}{2} \rfloor}$, which has size $\binom{n}{\lfloor n/2 \rfloor}$. The result is that wee can't beat this, but the proof is not trivial.