

# Advanced Financial Models

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## 0 Introduction

*www.staslab.cam.ac.uk/ mike/AFM/* for course material. However lecture notes only come after lectures, so taking notes is still necessary..

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Assumptions for this course:

No dividends, zero tick size (continuous), no transaction costs, no short-selling constraints, infinitely divisible assets, no bid-ask spread, infinite market depth, agents have preferences for expected utility.

## 1 Discrete time models

We'll assume there are  $n$  assets with price  $P_t^i$  at time  $t$  for asset  $i$ . Apparently  $P_t^i$  is a random variable on some probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ .

We'll use the notation  $P = (P_t^1, \dots, P_t^n)_{t \geq 0}$  which is a  $n$ -dimensional stochastic process.

Information available at time  $t$  is modelled by a  $\sigma$ -algebra  $\mathcal{F}_t \subseteq \mathcal{F}$ .

The assumption will be  $\mathcal{F}_s \subseteq \mathcal{F}_t$  for  $s \leq t$  (in other words,  $\mathcal{F}$  is a filtration):

**Definition.** (Filtration)

A *Filtration* is a collection of  $\sigma$ -algebra  $(\mathcal{F}_t)_{t \geq 0}$  such that  $\mathcal{F}_s \subseteq \mathcal{F}_t$  for  $s \leq t$ .

We'll assume that  $\mathcal{F}_0$  is trivial, i.e. if  $A$  is  $\mathcal{F}_0$  measure then  $\mathbb{P} = 0$  or  $1$ .