Sheet 4 Q12
$Y_1, Y_2, ..., Y_n$ are independent normal random variables. Suppose $(a_1, a_2, ..., a_n)$, $(b_1, b_2, ..., b_n)$ s.t.

$$\sum_i^n a_i b_i = 0$$

Show that $\sum_i^n a_i Y_i$, $\sum_i^n b^i y^i$ are independent.
Let

$$A = \begin{pmatrix} a_1 & a_2 & ... & a_n \\ b_1 & b_2 & ... & b_n \\ ... & .. & ... & .. \end{pmatrix}$$

Choose another $n-2$ orthogonal rows. Then

$$\sum = AA^T$$

is diagonal, so $X = AY$ has independent components.
So the two sums are independent.