

Advanced Financial Models

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0 Introduction

www.staslab.cam.ac.uk/ mike/AFM/ for course material. However lecture notes only come after lectures, so taking notes is still necessary..

m.tehranchi@statslab.cam.ac.uk

Assumptions for this course:

No dividends, zero tick size (continuous), no transaction costs, no short-selling constraints, infinitely divisible assets, no bid-ask spread, infinite market depth, agents have preferences for expected utility.

1 Discrete time models

We'll assume there are n assets with price P_t^i at time t for asset i . Apparently P_t^i is a random variable on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

We'll use the notation $P = (P_t^1, \dots, P_t^n)_{t \geq 0}$ which is a n -dimensional stochastic process.

Information available at time t is modelled by a σ -algebra $\mathcal{F}_t \subseteq \mathcal{F}$.

The assumption will be $\mathcal{F}_s \subseteq \mathcal{F}_t$ for $s \leq t$ (in other words, \mathcal{F} is a filtration):

Definition. (Filtration)

A *Filtration* is a collection of σ -algebra $(\mathcal{F}_t)_{t \geq 0}$ such that $\mathcal{F}_s \subseteq \mathcal{F}_t$ for $s \leq t$.

We'll assume that \mathcal{F}_0 is trivial, i.e. if A is \mathcal{F}_0 measure then $\mathbb{P} = 0$ or 1 . As a result P_0^1, \dots, P_0^n are constants.

We assume that $(P_t)_{t \geq 0}$ is adapted to the filtration, i.e. P_t is \mathcal{F}_t -measurable for all $t \geq 0$. Usually we assume the filtration is generated by P itself (so all the information is in the price).

Let c_t be the amount consumed at time t which will be a \mathcal{F}_t -measurable scalar, $H_t = (H_t^1, \dots, H_t^n)$ be the vector of portfolio weights, so H_t^i is the number of shares held at $(t-1, t]$ (remember we are in discrete time), which will be \mathcal{F}_{t-1} -measurable. $(c_t)_{t \geq 0}$ is adapted (\mathcal{F}_t -measurable), $(H_t)_{t \geq 1}$ is predictable/previsible (\mathcal{F}_{t-1} -measurable, n -dimensional).

Definition. The process (c, H) is self-financing if $H_t \cdot P_t = c_t + H_{t+1} \cdot P_t$ for all $t \geq 1$.

For example, X_0 be the initial wealth, $X_0 - c_0$ is the post-consumption wealth ($= H_1 \cdot P_0$), $X_1 = H_1 \cdot P_1$ is the pre-consumption wealth at time 1, $X_1 - c_1$ is the post consumption wealth (which is $H_2 \cdot P_1$). In other words, at any specific time, consumption takes place and then price updates.

Background assumption on behaviour of an agent:

Say c^2 is preferred to c^1 iff $\mathbb{E}U(c_0^1, c_1^1, \dots) < \mathbb{E}U(c_0^2, c_1^2, \dots)$ where U is some investor utility function, which is increasing in all c_i , concave (so risk-avert)

Definition. An arbitrage is a self-financing investment consumption strategy (c, H) such that there exists a non-random time $T > 0$ s.t. $c_0 = -H_1 \cdot P_0$, $c_t = (H_t - H_{t+1}) \cdot P_t$ for $1 \leq t \leq T-1$, and $c_T = H_T \cdot P_T$ (in words, with initial wealth $X_0 = 0$ and post-consumption wealth at T $X_T - C_T = 0$), that $\mathbb{P}(c_t \geq 0)$ for all $0 \leq t \leq T) = 1$, and $\mathbb{P}(c_t > 0 \text{ for some } 0 \leq t \leq T) > 0$.