Model Theory Example Sheet 2

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Question 3

For the first part, $\mathcal{N}$ is not connected, so it's not a random graph.

For the second part simply use $\psi(x, y) = \exists z(R(z, x) \wedge R(z, y))$ – it will be true if x and y are from the same random subgraph by the property of random graphs, and will be false otherwise since the two random subgraphs are disconnected.

Question 7

(ii) $\implies$ (i) is trivial as $|\mathcal{N}|$ is infinite by definition of saturation.
For (i) $\implies$ (ii), suppose otherwise, that $|\phi(\mathcal{N})| < |\mathcal{N}|$. Then let $\langle a_i : i < |\phi(\mathcal{N})|\rangle$ enumerate $\phi(\mathcal{N})$, and consider the formula

$$p(x) = \{\phi(x) \wedge (x \neq a_i) : i < |\phi(\mathcal{N})|\}$$

this is finitely satisfiable (as $\phi(\mathcal{N})$ is infinite), and has parameters $\phi(\mathcal{N})$ which, by assumption, has cardinality less than $\mathcal{N}$. Since $\mathcal{N}$ is saturated, $\mathcal{N}$ realizes $p(x)$. But this is impossible.