Problem Chosen	2025 MCM/ICM Summary Sheet	Team Control Number
B		2510151

To be or not to be: The Balancing Act for Sustainable Tourism

Summary

The prosperity of the tourism industry has brought significant economic benefits to residents, but it has also triggered a series of **environmental** and **social issues**. Quantifying the relationship between tourism **revenue** and its **hidden costs** is a key issue in current **sustainable tourism research**. Based on this, we have developed a **comprehensive model** for **sustainable tourism development**. This model integrates tourism-related problems into the relationship between tourism **revenue** and **tourist numbers**. It considers three dimensions: **economic income, nature conservation, and social maintenance**.

For Task 1, we established a **sustainable tourism model** tailored to the specific conditions of Juneau, Alaska. By analyzing relevant data from Juneau, we constructed a nonlinear **objective function** that relates tourism **revenue to tourist numbers**. Subsequently, by imposing constraints on tourist numbers and hidden costs, we used **iterative gradient descent methods** to find the optimal number of tourists and maximum tourism revenue. Additionally, we allocated **additional expenditures** to environmental protection and social maintenance to enhance the self-regulation capabilities of both the environment and society, thereby accommodating more tourists and increasing tourism revenue. Finally, we conducted **sensitivity and robustness analyses** of the model by iterating the growth coefficient of environmental carrying capacity, observing changes in the optimal number of tourists and maximum revenue, verifying the model's stability and robustness, and **discussing the critical factors** influencing the model.

For Task 2, we applied the model to **Lhasa** and **Iceland**, adjusting relevant parameters based on the actual conditions of each location. We then input these parameters into the model to calculate the optimal number of tourists and maximum revenue, comparing the results with actual conditions. The findings suggest that Lhasa's tourism development is excessive, recommending increased investment in environmental maintenance costs and limiting tourist numbers; conversely, Iceland's tourism resources are underdeveloped, necessitating accelerated resource development. These **results align with actual conditions**, validating the model's broad applicability and practicality. Users need only adjust model parameters according to the actual conditions of the study area to determine the optimal number of tourists, thereby **supporting sustainable tourism development**.

For Task 3, based on the model results, the optimal number of tourists for Juneau is **1.05 million**, with a **maximum revenue of $356 million**. We found that allocating additional income to social governance and environmental maintenance can significantly increase tourism revenue. Therefore, we offer the following **recommendations** to the Juneau Tourism Commission: **Implement a reservation system** for visitors to limit the number of daily admissions. During peak tourist seasons, the number of visitors will be strictly controlled to within 20,000 people per day.

Keywords: Number of Tourists, Tourism Revenue, Environmental protection costs, Sustainable Development, Nonlinear Programming

Contents

1 Introduction

1.1 Problem Background

"Let the tourists go home!" In Spain, thousands of residents have taken to the streets in cities like Barcelona and the Canary Islands, chanting slogans and staging protests. This phenomenon is emblematic of the broader "anti-tourism" movement sweeping across Europe in 2024. According to CNN, as the summer tourism season approaches, Europe has emerged as a focal point for demonstrations against "**over-tourism**."

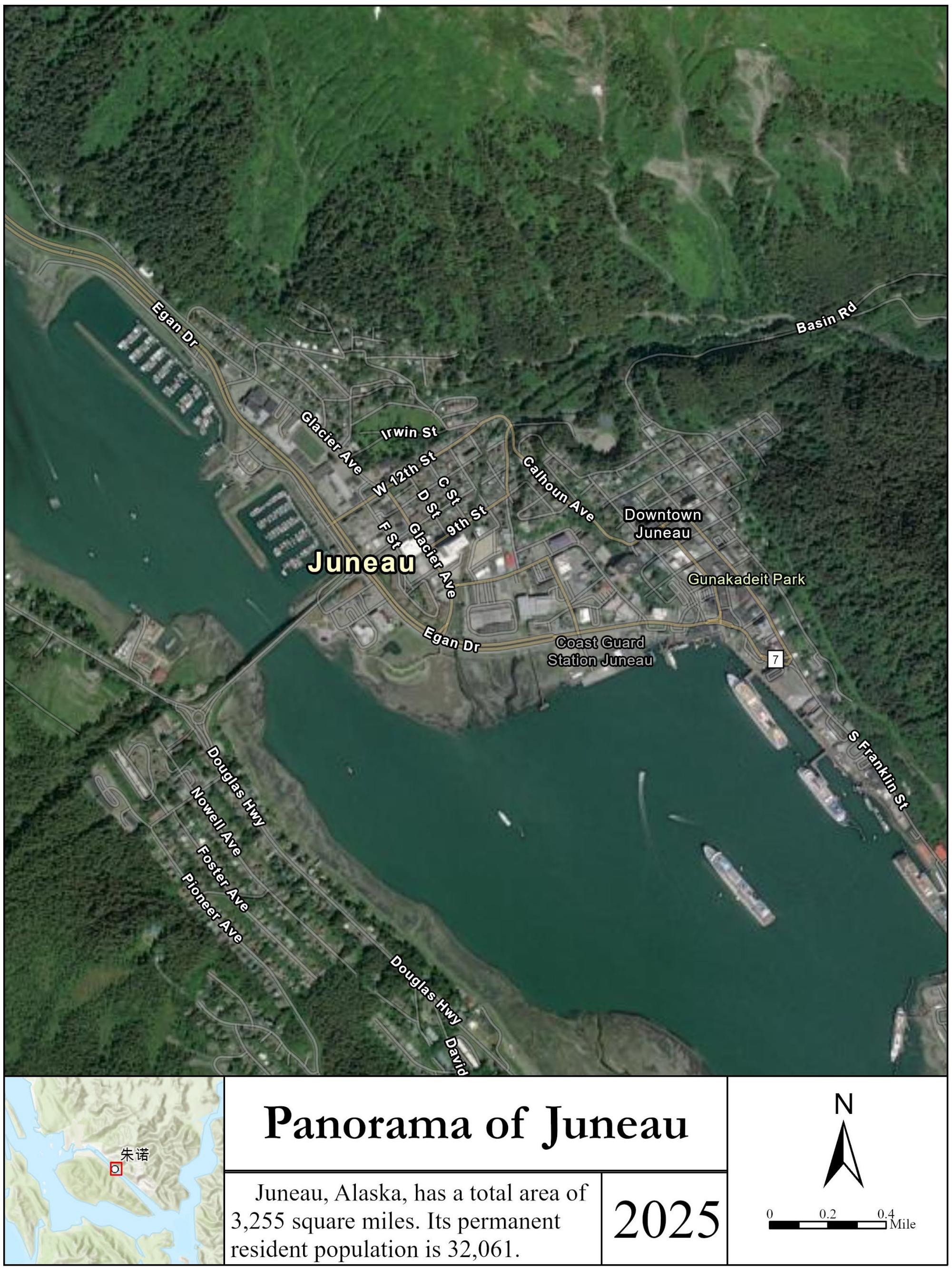


Figure 1: Panorama of Juneau

The core issue of "over-tourism" lies in the imbalance between tourism development and the carrying capacity of destinations, highlighting inadequacies in resource management and equitable spatial distribution. Implementing sustainable and high-quality development models is crucial for ensuring the harmonious and healthy growth of both the tourism industry and the economy.

Tourism serves multiple functions, including economic, social, and cultural roles. Sustainable tourism can address issues such as social employment and income inequality. To promote sustainable tourism development, it is essential to **balance the benefits for tourism professionals** with those for **residents**, thereby preventing a scenario where only a portion of the population profits while others bear the costs and sacrifices associated with tourism development [1]. Figure 2 presents some visualized data related to sustainable tourism.

The economic and social benefits of tourism can coexist. Environmental protection can also enhance residents' income. Promoting high-quality and sustainable tourism requires a comprehensive consideration of tourism's multifaceted impacts, correctly assessing the flow of people, information, technology, and capital brought by tourism, and striking a balance between **short-term gains** and **long-term interests** [2].

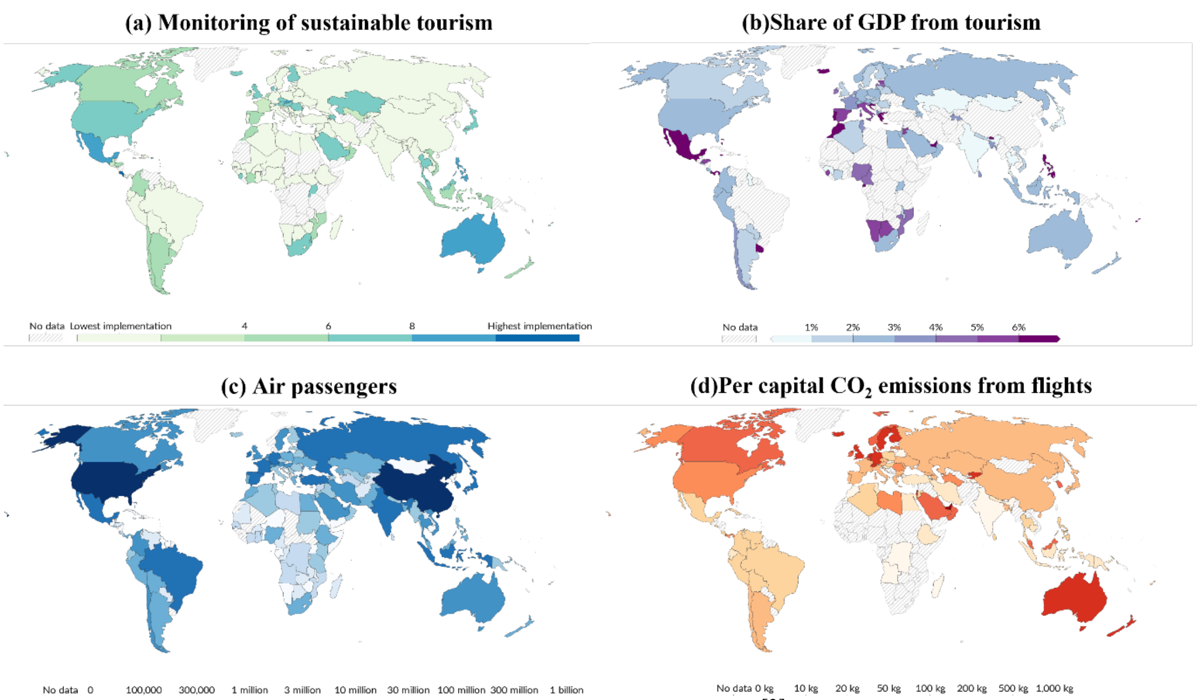


Figure 2: Tourism-related data [3]

1.2 Restatement of the Problem

Given the aforementioned background information and constraints, we need to address the following challenges：

Problem 1: In Juneau, Alaska, comprehensively consider multiple factors to develop a sustainable tourism model. The model should clearly outline optimization objectives and constraints, and include an expenditure plan for additional revenue. Evaluate the importance of each factor and conduct sensitivity analysis on the model.

Problem 2: Apply this model to another tourist destination affected by overtourism, discussing how changes in location impact these factors. Use the model to promote less-visited sites to achieve a more balanced tourism distribution.

Problem 3: Prepare a one-page memorandum for the Juneau Convention and Visitors Bureau, summarizing forecast results and the effectiveness of various measures, and providing specific recommendations for optimizing tourism.

1.3 Our Work

The work we have done in this problem is mainly shown in the following Figure 3.

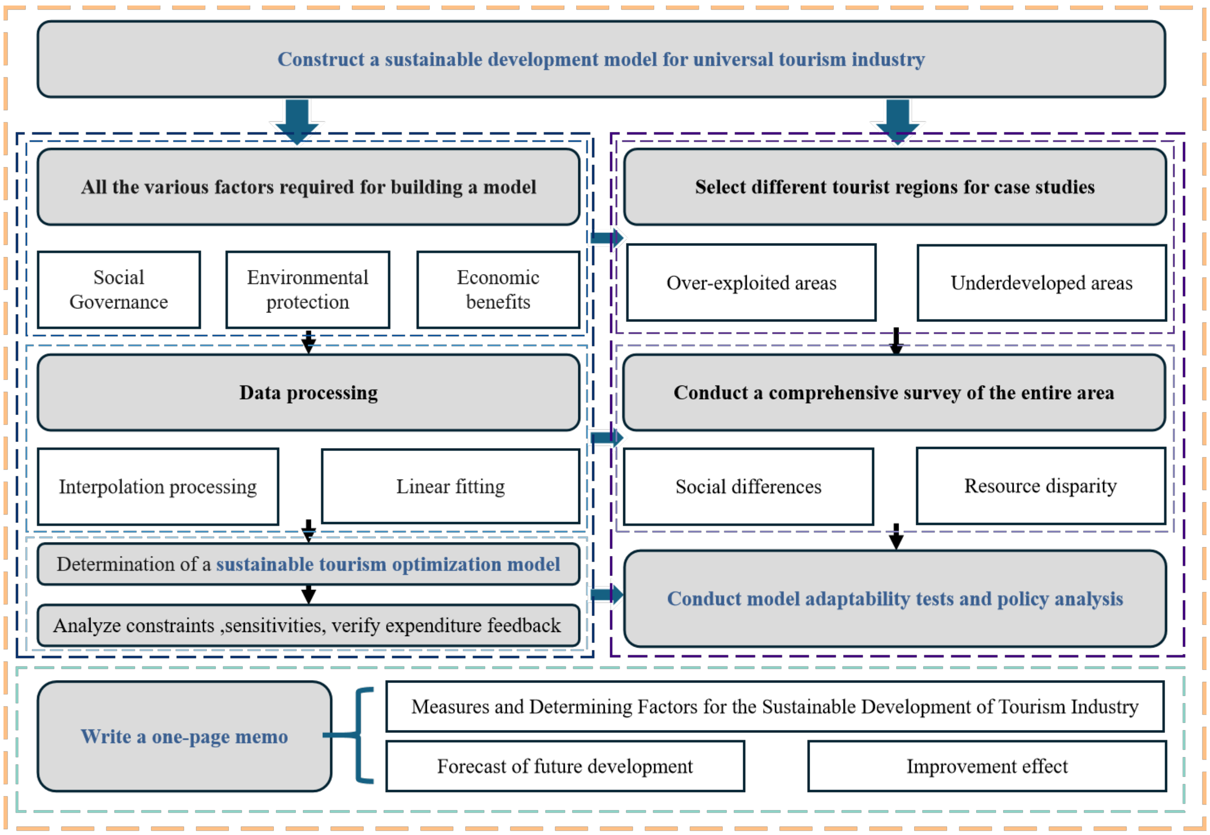


Figure 3: Our work

2 Assumptions and Justifications

- Considering those practical problems always contain many complex factors, first, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation:
- Explanations: The data in this article comes directly from the latest results of the major online official databases and published literature.

- The glaciers are disappearing at a certain rate.
- Explanations: Considering the actual situation, it is essential to incorporate natural factors when establishing the model. In recent years, the rate of glacier retreat has not exhibited significant sudden changes. Therefore, using historical data to determine the rate of glacier retreat is reasonable [4].

- It is assumed that the selected target area will not experience severe natural disasters or extraordinary events such as pandemics in the coming years.
- Explanations: The focus of this study is on optimizing sustainable tourism rather than addressing how tourism should respond under extremely abnormal circumstances. Natural disasters and pandemics are rare occurrences. Hence, this assumption is justified.

3 Notations

The process of building and using mathematical models inevitably involves the use of a considerable amount of mathematical notation. To avoid ambiguity and vagueness of meaning, the significance of the important symbols is listed in Table 1 below：

Table 1: Notations used in this paper

Symbol	Description	Unit
x	Tourist volume	million
$\bar{C}$	Per capita carbon emissions	
μ_t	Climate sensitivity parameter	
k_m	The sensitivity coefficient of glaciers to temperature changes	
μ_r	The cost of managing waste per unit volume	

*Some variables are not listed here and will be discussed in detail in each section.

4 Model Preparation

4.1 Parameter Determination

Tourism can be categorized into broad tourism and narrow tourism. Given that broad tourism intersects extensively with other sectors and involves a wide range of influencing factors, constructing a sustainable tourism optimization model requires considering the impacts of multiple industries. Therefore, it is more appropriate to choose broad tourism for this study. All references to tourism in the following text refer to **broad tourism** [5].

Tourism is interdependent with and mutually influential on numerous industries within the national economy. Its industrial linkages primarily manifest in two ways: backward linkages, which refer to the impact of tourism on industries that supply production factors to it; and forward linkages, which refer to the impact of tourism on industries that use its products or services as production factors. This model aims to establish a **sustainable tourism optimization framework**, thus unifying backward and forward linkages under the term "constraint factors."

In this study, we focus on three main aspects to construct a sustainable tourism model: **environmental protection, social governance**, and **economic benefits**. Each aspect is further divided into several sub-factors [6]. The specific indicators are shown in the following Figure 4:

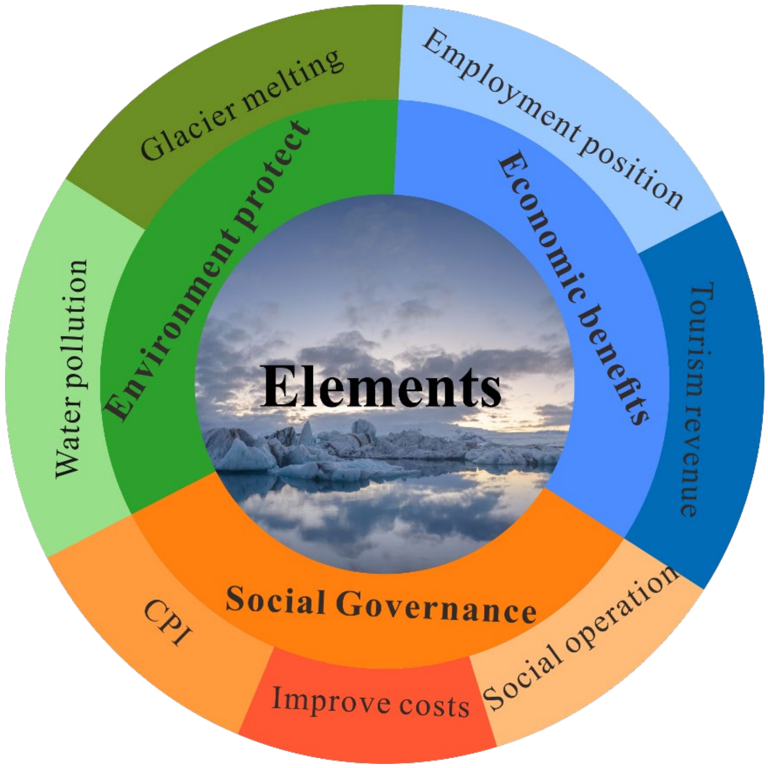


Figure 4: Elements to consider

4.2 Data Overview

The problem does not provide us with data directly, so we need to consider what data to collect when building the model and what data to collect during the process of building the model. Through the analysis of the problem, we collected the main data in Table 2.

Table 2: Main data description and source

Data Description	Data Source
Environmental protection costs Expenditure on social governance Tourism economic benefits	https://juneau.org/
The number of tourists over the years	https://www.alaskatia.org/
The rate of glacier melting	https://www.nature.com/ https://www.cambridge.org/
Juneau Demographics	https://zh.globalpopulations.com/
Carbon emissions in the city of Juno	https://www.ceicdata.com.cn/ https://dec.alaska.gov/air/

5 Tourism Revenue Model

This model mainly transforms the original problem into a **nonlinear programming problem**, converting all issues into the relationship between **tourism revenue** and **tourist visits**. It then constructs a nonlinear equation between tourism revenue and tourist visits and uses the basic situation of the study area to construct constraints, obtaining the domain of the nonlinear function. Within this domain, it finds the **maximum** value of **tourism revenue** and obtains the total number of **tourists** corresponding to the maximum tourism revenue, providing suggestions for tourism population. To better quantify and structure the model, this model mainly analyzes three aspects: tourism economic income, environmental maintenance costs of tourist destinations, and social maintenance costs of the tourism industry [7]. Figure 5 shows the specific analysis steps.

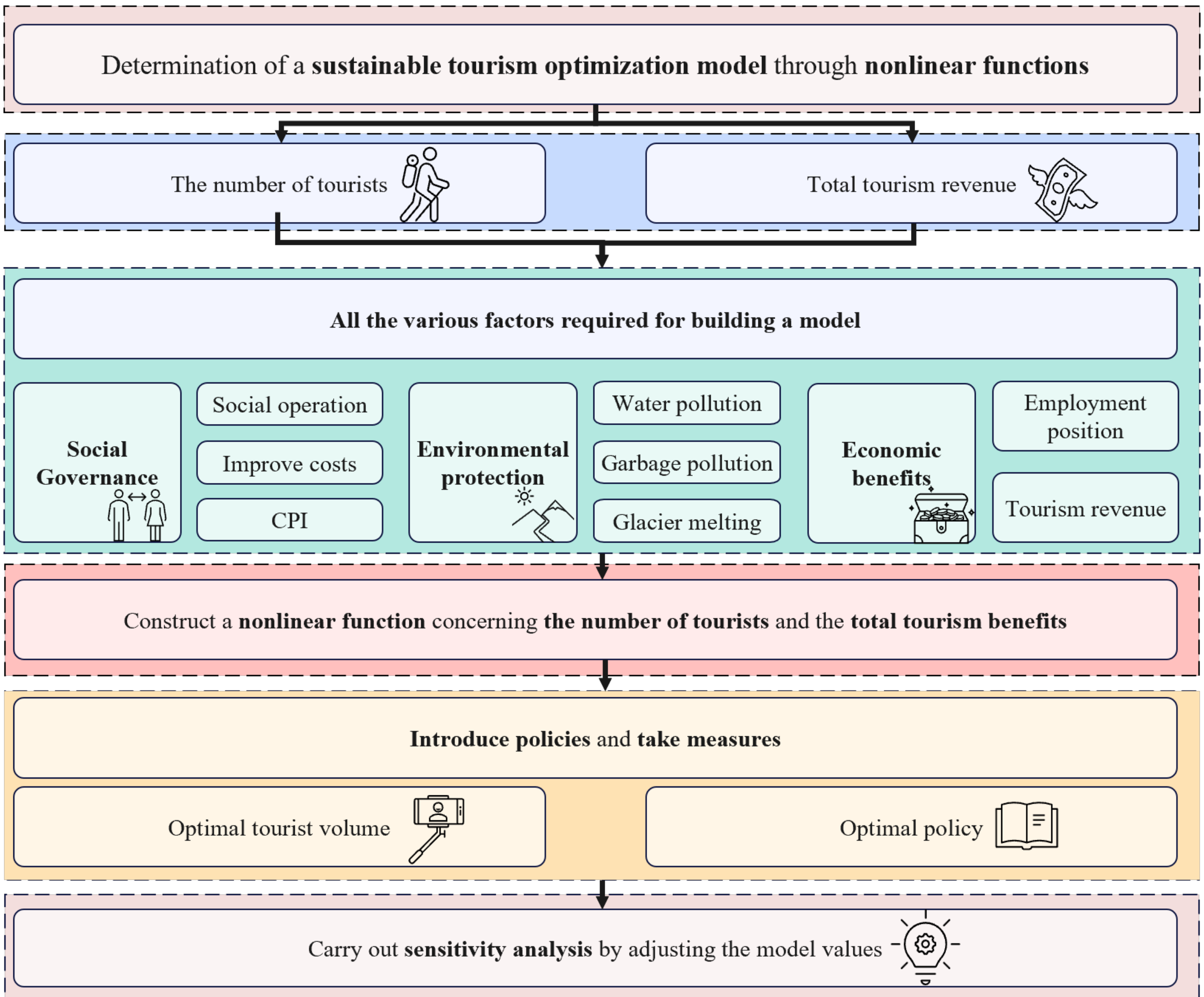


Figure 5: Flowchart for model construction

We transform the issue of sustainable tourism development into a functional relationship between tourism revenue and tourist numbers, where tourist numbers serve as the independent variable. Tourism revenue is influenced by economic **income** from tourism and **implicit costs** associated with tourism. Therefore, we first construct an overall nonlinear objective function, defining tourism revenue as the difference between tourism economic income and implicit costs, which are further categorized into **environmental protection costs** and **social governance costs**. The specific functional relationship is shown in Equation 1.

$$\max_{x} Z = B_r(x) - C_e(x) - C_s(x) \tag{1}$$

Where：

Z represents tourism revenue, xrepresents tourist numbers, $B_r(x)$, $C_e(x)$, and $C_s(x)$ represent the relationships between economic benefits, environmental protection costs, and social governance costs, respectively, and tourist numbers. We construct the nonlinear function Zand use nonlinear goal programming methods to maximize overall tourism revenue, thereby determining the optimal tourist population. Below is the detailed process for constructing models of economic benefits, environmental protection costs, and social governance costs in relation to tourist numbers.

5.1 Construction of a Model for Economic Revenue

Economic income is the primary driving force behind the tourism industry. It not only generates substantial employment opportunities in the labor market but also yields significant economic returns through associated commercial activities, contributing considerable tax revenue to national finances. Consequently, this study develops a comprehensive model of tourism industry economic income from three dimensions: **employment income**, **business income**, and **tax revenue** [8].

$$B_r(x) = B_e(x) + B_b(x) + B_t(x) \tag{2}$$

Where：

$B_e(x)$, $B_b(x)$ and $B_t(x)$ represent the relationships between employment income, business income, and tax revenue with the number of tourists x. The following sections detail the modeling for each of these three aspects.

The growth of the tourism sector has provided abundant employment opportunities and considerable income for residents. Initially, as tourist numbers increase, employment income grows rapidly. **However,** once job availability reaches saturation, the rate of income growth slows down, demonstrating a clear saturation effect. To capture this relationship, we employ a logarithmic function:

$$B_e(x) = \alpha_e \ln(\beta_e x + 1) \tag{3}$$

Where:

α_e is the scaling coefficient related to initial economic income, while β_e controls the relationship between tourism growth and tourism employment income.

Tourists typically have specific commercial demands that are closely tied to their individual economic conditions rather than the overall number of tourists or destination congestion. Therefore, the commercial demand per tourist remains relatively stable as tourist numbers increase, leading to a strong linear relationship. Based on this characteristic, we use a linear function to model the relationship between **business income** and **the number of tourists:**

$$B_b(x) = \alpha_b x \tag{4}$$

Where:

$B_b(x)$represents total commercial revenue, andα_b denotes the average commercial income per tourist.

High revenues from tourism activities generate substantial tax income for the country. This tax revenue can be linked to other incomes through tax rates. Therefore, we correlate tax revenue with other incomes using tax rates:

$$B_t(x) = t_e B_e(x) + t_b B_b(x) + T \tag{5}$$

Where:

t_e and t_b represent the tax rates on employment income and business income, respectively, while T denotes other taxes.

5.2 Model Construction for Environmental Protection Cost

Environmental protection efforts at tourist destinations are crucial for the sustainable development of tourism and the long-term health of **natural ecosystems**. The rapid increase in tourist numbers has led to a significant rise in domestic waste and sewage, necessitating substantial annual investments for treatment. Additionally, Juno City, home to **numerous glaciers**, faces increased carbon emissions from tourists, leading to higher temperatures and threatening the ecological environment and sustainable tourism. Therefore, this study explores the relationship between environmental protection costs and the number of tourists from three dimensions: **glacier protection costs, waste treatment costs**, and **sewage treatment costs,** constructing a corresponding model:

$$C_e(x) = G_c(x) + R_c(x) + W_c(x) \tag{6}$$

Where:

$G_c(x)$, $R_c(x)$ and $W_c(x)$represent the relationships between glacier protection costs, waste disposal costs, and sewage treatment costs with the number of tourists x.

Glaciers are a unique feature of Juno's tourism, attracting many visitors and playing a **vital** role in the local tourism industry. However, excessive tourism has severely degraded glaciers and triggered secondary disasters such as floods, causing significant economic losses. Figure 6 shows the melting of glaciers near Juneau from 1975 to 2023.Therefore, glaciers are considered an essential component of environmental protection costs. Tourist activities contribute to local temperature increases through carbon emissions, accelerating glacier melting. We quantify the relationship between tourist numbers and glacier melting by linking **tourist numbers to carbon emissions, carbon emissions to temperature, temperature to glacier melting, and glacier melting volume to governance costs** [8].

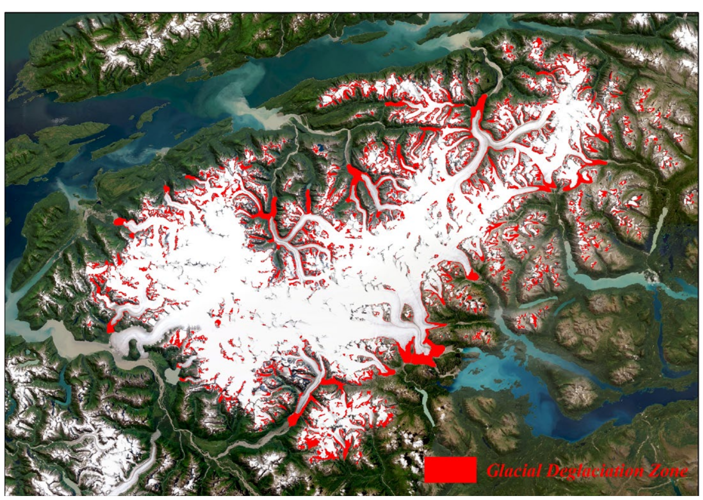


Figure 6: Visualization of melting glacier

Tourists indirectly affect glacier melting through carbon emissions. The amount of carbon emissions per tourist remains constant regardless of the increase in the number of tourists, implying a linear relationship between total carbon emissions and the number of tourists. The specific functional relationship is as follows:

$$C_{sum} = \bar{C}x \tag{7}$$

Where:

C_{sum} represents the total amount of carbon dioxide generated by tourists, and $\bar{C}$ denotes the average carbon emission per tourist

Carbon emissions influence glacier temperature via the greenhouse effect, thereby accelerating glacier melting. However, the relationship between **carbon emissions** and **temperature** is not linear; instead, as carbon emissions increase, the rate of temperature increase gradually slows down. This phenomenon can be explained physically: as temperature rises, it radiates heat to surrounding areas, causing the rate of temperature **increase** to slow down and eventually reach a balanced state. To more accurately describe the relationship between glacier temperature and carbon emissions, we use a logarithmic function for fitting, as shown below:

$$T = \mu_t \ln\left(\frac{C_0 + C_{sum}}{C_0}\right) + T_0 \tag{8}$$

Where:

T represents the temperature of glaciers with tourism activities, μ_t is the environmental sensitivity coefficient, C_0 is the base level of carbon dioxide emissions, and T_0 is the average temperature of glaciers without tourism activities.

Glacier melting is directly influenced by **glacier temperature**. In the absence of tourism activities, glaciers have a natural melting rate. Tourism activities cause glacier temperatures to rise, making the temperature higher than its natural melting point, thus promoting the melting process [8]. The melting function relationship between glaciers and temperature is as follows:

$$M = k_m(T - T_0)^{\alpha_m} + \beta_m \tag{9}$$

Where:

k_m represents the sensitivity coefficient of glaciers to temperature changes, T is the temperature of glaciers in the tourist destination, T_0 is the average temperature of glaciers without tourism activities, α_m is the acceleration index of ice and snow melting (usually greater than 1), indicating that the melting speed increases with rising temperature, and β_m is the background melting speed of glaciers without tourism activities.

To prevent the reduction of glacier attractiveness due to melting and reduce the possibility of secondary disasters caused by glacier melting, it is necessary to calculate the cost of glacier protection. The cost of glacier protection is a function of glacier melting as follows:

$$C_e(x) = \mu_p M \tag{10}$$

Where:

μ_p represents the cost required to maintain one cubic meter of glacier.

By solving the above equations simultaneously, we obtain the relationship between the glacier protection cost $C_e(x)$ and the number of tourists:

$$C_e(x) = \mu_p \left\{k_m \left[\mu_t \ln [\![\left(1 + \frac{\bar{C}x}{C_0}\right) \right]]\!]^{\alpha_m} + \beta_m\right\} \tag{11}$$

During tourism activities, tourists generate a large amount of sewage and garbage. Within a reasonable range, these pollutants can be naturally regulated and recycled back to nature, causing relatively small impacts on the ecological environment. However, once they exceed the limit of natural regulation capacity, these pollutants will cause irreversible damage to the ecological environment, requiring significant resources and costs to restore the ecosystem. Based on this assumption, we use an exponential function to fit and calculate the costs of garbage management and water pollution control to more accurately describe the costs associated with excessive pollution. The functions are as follows:

$$R_c(x) = \mu_r e^{k_r(x-x_r)} \tag{12}$$

$$W_c(x) = \mu_w e^{k_w(x-x_w)} \tag{13}$$

Where：

μ_r and μ_w represent the unit costs of garbage and wastewater treatment, respectively. k_r and k_w are adjustment coefficients controlling the growth rates of the adjustment curves to adapt to different types of ecosystems. x_r and x_w are the maximum capacities of the ecological environment for garbage and wastewater, respectively, which are the limits beyond which the maintenance cost for protecting the natural environment increases exponentially.

5.3 Model Construction of Social Maintenance Cost

The arrival of tourists not only consumes local social resources but also increases the costs associated with social governance and prices, leading to a decline in purchasing power and causing disruptions to the daily lives of residents, such as noise pollution. To mitigate these impacts, residents may need to implement community improvement measures to reduce the interference from issues like noise, thereby generating corresponding community improvement costs. Based on this, this section constructs the functional relationship between social governance costs and the number of tourists from three aspects: **community improvement costs, residents' living costs, and social operation costs**. The formula is as follows:

$$C_s(x) = C_i(x) + C_l(x) + C_o(x) \tag{14}$$

Among these, $C_i(x)$, $C_l(x)$, and $C_o(x)$ represent the relationships between community improvement costs, resident living costs, and social operation costs, respectively, and tourist numbers.

Although tourism brings significant economic benefits to residents, the large influx of tourists also leads to increased noise pollution, light pollution, and pressure on infrastructure, resulting in rising community improvement costs. However, there is a marginal effect between community improvement costs and tourist numbers; as tourist numbers increase, the rate of cost increase gradually slows down. Therefore, we use a power function to fit the community improvement costs to more accurately describe their relationship with tourist numbers. The functional relationship is as follows:

$$C_i(x) = \alpha_i x^{\beta_i} \tag{15}$$

where α_i is the growth coefficient controlling the rate of increase in community improvement costs, and β_i is the exponent of population size, typically less than 1, indicating a decreasing rate of cost increase.

As the number of tourists increases, local prices rise, reducing social purchasing power and leading to higher living costs for residents. However, the increase in living costs has a threshold; beyond this threshold, living costs tend to converge and reach a maximum value. Based on this, we introduce the Consumer Price Index **(CPI)** to measure the extent of price increases or decreases in purchasing power due to population growth, thereby calculating the living costs of residents [2]. The functional relationship is shown below:

$$\Delta CPI = CPI_m(1 - e^{-\alpha_c x}) \tag{16}$$

In this model, $\boldsymbol{\Delta CPI}$ represents the magnitude of the decrease in the Consumer Price Index (CPI), CPI_m denotes the maximum possible reduction in CPI, and ΔCPI ultimately converges to this value. The parameter α_c is used to adjust the growth rate of the function. Subsequently, the cost of living for residents is calculated based on their expenditure on identical daily necessities under reduced purchasing power [2].

$$C_l(x) = \frac{C_0}{(1 - \Delta CPI)} \tag{17}$$

Here, C_0 represents the cost of living for residents in the absence of tourism activities.

Regarding social operational costs, it is important to note that indigenous societies typically provide a certain level of resource redundancy for the native population. When tourist visits are low, visitors utilize these redundant resources, thereby generating certain social operational costs (such as transportation costs). However, as the number of tourists increases, the social surplus resources are rapidly depleted, leading to a sharp rise in social operational costs. Once the available resources are exceeded, the growth rate of social operational costs gradually stabilizes.

This phenomenon is exemplified by transportation costs: when the number of tourists is low, traffic flows smoothly; however, as the number approaches the traffic capacity limit, traffic nearly grinds to a halt, causing a significant increase in costs. As the number of tourists surpasses this limit, more visitors opt to walk, significantly slowing the increase in transportation costs. At this point, although traffic remains congested, newly added tourists often choose to walk, thereby alleviating the pressure on transportation costs. Overall, this process exhibits an S-shaped curve characteristic.

$$C_o(x) = \frac{\alpha_o}{1 + e^{-k_o(x - x_o)}} + \beta_o \tag{18}$$

Here, $C_o(x)$ represents the social operational cost, α_o denotes the upper limit of the social operational cost, k_o reflects the strength of the relationship between social operation and population, x_o is the maximum number of tourists that the society can accommodate while functioning normally, and β_o represents the basic cost of social operation.

5.4 Determination of Constraint Conditions

After constructing the nonlinear function, it is necessary to set reasonable constraints based on actual conditions to ensure that the optimal solution falls within the **feasible range**. These constraints should fully consider various limiting factors in reality, thereby ensuring that the optimal solution accurately reflects actual requirements. For this model, we only use tourist numbers as the independent variable and build the model from three aspects: economic benefits, environmental protection costs, and social governance costs. Therefore, by setting constraints on tourist numbers, appropriate environmental protection costs, and social governance costs, we limit the relevant parameters to suitable ranges to ensure that the final results align more closely with real-world conditions.

To determine the tourist numbers, we analyzed data from the past decade in Alaska (excluding 2020). The data show that tourist numbers primarily ranged between **0.9 million and 1.7 million**. To establish a reasonable range for tourist numbers, we calculated a **50%** fluctuation around the average tourist numbers over the past decade, as detailed in the following formula:

$$x_{\downarrow} < x < x_{\uparrow} \tag{19}$$

Here, $x_{\downarrow}$ and $x_{\uparrow}$ represent the lower and upper bounds of tourist numbers over the past decade, respectively. Based on this formula, the constraints on tourist numbers are as follows.

$$x \in [0.45m, 2.55m] \tag{20}$$

Regarding the constraints on environmental maintenance costs, during tourism activities, the number of tourists must not exceed the environmental carrying capacity. Exceeding this capacity can cause irreversible damage to the environment. Therefore, it is crucial to **reasonably limit** the number of tourists. To achieve this, we constrain natural environmental protection costs to control population levels by leveraging the relationship between environmental maintenance costs and population size, thereby preventing over-exploitation or destruction of natural resources. The specific constraint conditions are as follows:

$$E_{\downarrow} < C_e(x) < E_{\uparrow} \tag{21}$$

Here, $C_e(x)$ represents the environmental protection cost, $E_{\downarrow}$ represents the minimum environmental protection cost, and $E_{\uparrow}$ represents the maximum environmental protection cost. To determine the range of environmental protection costs, we estimate using wastewater treatment fees. The sewage discharge treatment fee is $[100m, 150m]$. Based on this parameter, we constrain the environmental protection cost to be within two to four times the sewage treatment fee, i.e., the constraint interval for environmental protection costs is $C_e(x) \in [200m, 600m]$.

Regarding the constraints on social governance costs, similar to environmental cost constraints, although the social environment has some self-regulating capacity, an excessive number of tourists can still disrupt the normal living order of local residents, increase social maintenance costs, and interfere with the daily lives of indigenous people. To maintain basic social governance and protect the rights of indigenous people as much as possible, it is necessary to constrain social maintenance costs to safeguard the fundamental interests of indigenous people.

$$S_{\downarrow} < C_s(x) < S_{\uparrow} \tag{22}$$

Here, $C_s(x)$ represents the social governance cost, $S_{\downarrow}$ and $S_{\uparrow}$ represent the minimum and maximum environmental protection costs, respectively. To determine **the range of social governance costs**, we use the public safety and public transportation expenses over the past decade for estimation. Given that public safety costs were abnormally high during the COVID-19 pandemic and are therefore excluded from consideration, we base our constraint estimation on the total sum of public safety and public transportation costs, which falls within the $[30m, 40m]$ interval. Based on this, we estimate the social governance cost range to be between 1.5 and 2 times the total sum, i.e., the constraint interval for social governance costs is $[45m, 80m]$.

5.5 Nonlinear Goal Programming

In Sections 5.1 to 5.4, we completed the construction of the tourism revenue and nonlinear objective function Z, as well as the determination of the associated constraints. The specific nonlinear functions and constraints are given in Equations (1), (19), (21), and (22). After constructing the nonlinear function and setting the constraints, we used the gradient descent method to find the optimal tourist number x to maximize the revenue Z.

First, we calculate the gradient of the objective function $Z(x)$ with respect to x, as shown in the following formula:

$$\nabla Z(x) = \nabla B_r(x) - \nabla C_e(x) - \nabla C_s(x) \tag{23}$$

Here, $\nabla B_r(x)$, $\nabla C_e(x)$, and $\nabla C_s(x)$represent the gradients of tourism revenue, environmental protection cost, and social governance cost, respectively.

Then, we specify the initial value x_0 and learning rate α, and use the gradient descent method to iteratively update x_0 according to the following rules:

$$x_{k+1} = proj\big(x_k + \alpha \nabla Z(x_k)\big) \tag{24}$$

Here, α is the learning rate, which controls the step size. We update x_k along the direction of the gradient ascent to ensure that x_{k+1} approaches the maximum value of the objective function more closely. During each update, if x_{k+1} satisfies the constraints, we directly compute $Z(x_{k+1})$ using the gradient descent method and check whether $Z(x_{k+1}) - Z(x_k)$ is less than the tolerance. If x_{k+1} does not satisfy the constraints, we project x_{k+1} into the feasible region defined by the constraints until $Z(x_{k+1}) - Z(x_k)$ is less than the tolerance. Finally, we stop the iteration and output x_{k+1} and $Z(x_{k+1})$ as the optimal tourist number and maximum tourism revenue.

5.6 Expenditure Plan for Additional Revenue

In formulating the revenue and expenditure plan, it is first necessary to clearly define the boundaries between **disposable income** and **non-disposable income**. Economic income primarily consists of employment income, business income, and tax revenue. Employment income and business income mainly reflect individual residents' earnings, which are difficult for the government to directly control through macroeconomic policies. In contrast, tax revenue is under government control and can be increased by adjusting tax rates. Therefore, we allocate a portion of the tax revenue as additional income to improve the conditions of tourist destinations and enhance their attractiveness.

Based on this approach, we use the additional income to transform natural and social environments, thereby increasing their capacity to accommodate tourists and enhancing their self-regulation capabilities. Specifically, we establish relationships between **tax revenue** and **natural maintenance costs** as well as social governance costs, incorporating the additional income into our model. The allocation is as follows: income is distributed to wastewater treatment x_w, waste disposal x_r, and social operation costs x_o. As additional income I is invested, the environmental carrying capacity for tourists gradually increases, shifting the relevant functions to the right. This strengthens the self-regulation capabilities of natural and social environments, thus achieving a virtuous cycle.

$$x_{w+1}(I) = x_w + \alpha_r I \eta_w \tag{25}$$

$$x_{r+1}(I) = x_r + \alpha_r I \eta_r \tag{26}$$

$$x_{o+1}(I) = x_O + \alpha_O I \eta_O \tag{27}$$

Here, $x_{w+1}(I)$, $x_{r+1}(I)$, and $x_{o+1}(I)$ represent the environmental carrying capacities for wastewater treatment, waste disposal, and social operation costs, respectively, after additional income is allocated. These are functions of the input I and increase as I increases. This indicates that as investment increases, the environment's capacity to accommodate tourists also increases; under the same number of tourists, the costs of environmental maintenance and social governance decrease. α_w, α_r, and α_o are the growth coefficients of environmental carrying capacity for the respective indicators, while η_w, η_r, and η_o are the conversion coefficients of investment into increased environmental carrying capacity.

5.7 Model Implementation and Results

This model quantifies the relationship between tourism revenue and tourist numbers by constructing a nonlinear objective function that incorporates economic income, environmental protection costs, and social maintenance costs. The function is constrained within a reasonable range. Ultimately, the gradient descent method is used to find the optimal balance point, which corresponds to the maximum revenue and optimal tourist number for the Juneau region. The specific implementation and results of the model are as follows:

First, parameter settings for the model. This model comprehensively models sustainable tourism in Juneau, with many parameters determined based on the actual conditions of Juneau. The specific methods for determining parameters are as follows:

1. Parameters are determined based on relevant data: For example, employment tax rate T_e and business tax rate T_b are determined by reviewing annual economic reports and using average or median values as references;

2. Parameters are determined based on relevant literature: For instance, the accelerated glacier melting rate α_m and background melting temperature β_m are determined by consulting authoritative literature;

3. Parameters are determined based on existing data: We set a reasonable range for the parameters and define an iterative interval. Each iteration result is compared with existing data, and the optimal parameters are obtained by minimizing residuals. By incorporating waste treatment and wastewater treatment cost data, and using parameters $k_r$and $k_w$to control the rate of image change, we compare each iteration result with existing data and ultimately select the parameter combination with the smallest residual;

4. If the above methods cannot resolve the issue, parameters are reasonably estimated using prior knowledge.

Parameter estimation results are detailed in the appendix 1.

We performed interpolation and densification of the tourist number data. To more accurately depict the relationship between tourist numbers and other factors, we collected ten years of tourist number data from Juneau, Alaska. However, ten years of data alone are insufficient to fully reveal the relationship between tourist numbers and other factors. Therefore, we used time interpolation to densify the data and added **random noise** (-0.05m, 0.05m) to the interpolated data to **enhance the model's robustness to noise**. Figure 7 shows the tourist number data after time interpolation.

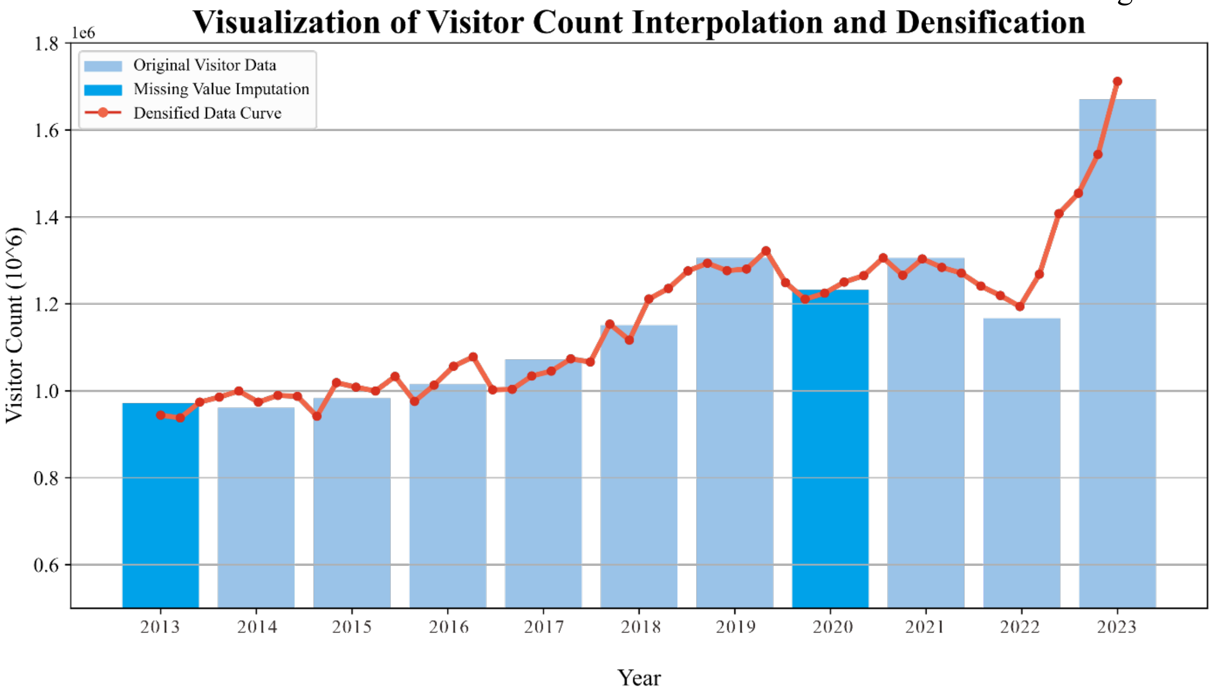


Figure 7: Data processing visualization

Secondly, we analyzed the relationship between economic income, environmental protection costs, and social maintenance costs with tourist numbers over the years. The specific changes are as follows:

As tourist numbers fluctuate, various revenues and costs also vary accordingly. In terms of economic income, commercial revenue accounts for more than 56% of total income and is most significantly influenced by tourist numbers. Regarding environmental protection costs, glacier protection represents the highest cost, accounting for 46% of total environmental protection costs. For social maintenance costs, when tourist numbers are **below 1.2 million**, community improvement costs are **lower** than residential living costs; however, when tourist numbers **exceed 1.2 million**, community improvement costs surpass residential living costs, and the gap gradually widens.

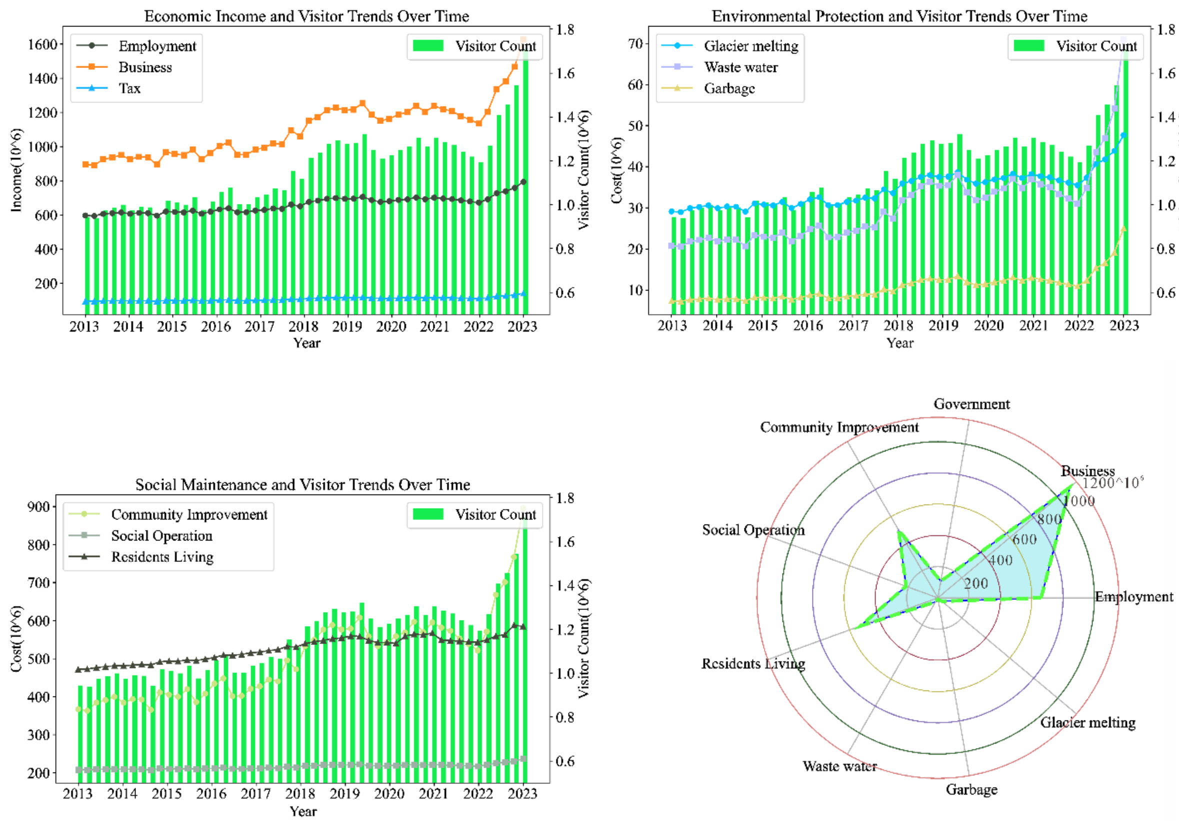


Figure 8: Time-varying graphs of various factors and tourist numbers

After constructing the nonlinear function and setting reasonable constraints, we employed the **gradient descent method** to solve for the optimal number of tourists and the corresponding maximum tourism revenue. To avoid falling into local optima, the experiment involved multiple runs with different initial values X, selecting the highest tourism revenue as the optimal result of the model. Additionally, by adjusting the tax rates in the tax expenditure model, we altered the investments in wastewater treatment, waste management, and social maintenance, thereby increasing environmental capacity and further optimizing tourism revenue.

We plan to allocate 5 million USD in tax revenue to this project, which will be deducted from the total income accordingly. Through model optimization, we found that investing part of the revenue in environmental governance and infrastructure development significantly increased tourism revenue, demonstrating that environmental investment can substantially enhance overall tourism earnings. The finally results show in Figure 8 that when the number of tourists reaches 1.08 million, the tourism revenue peaks at 356 million USD. Compared to the pre-optimization optimal tourist number of 0.86 million and revenue of 324 million USD, the optimization effect is very significant. Based on the above model, we can implement the following measures:

1. Implement a reservation system for visits to limit the number of tourists entering Juneau;

2. Encourage tourists to conserve water and reduce the use of single-use items;

3. Rely on residents to establish environmental protection organizations responsible for collecting scattered waste and promoting environmental awareness;

4. Increase government investment in wastewater treatment and waste management enterprises;

5. Install soundproof panels on walls in high-traffic areas to minimize the impact on residents;

6. Designate no-entry zones within scenic areas to reduce the impact of human activities on glacier melting.

6 Case Study

To validate the **applicability** and **versatility** of our model, we selected **Lhasa** in China and **Iceland** as new study areas. These two regions share certain similarities with Juneau, such as the presence of glacial landscapes; however, they also exhibit **significant differences**. Lhasa faces the issue of overdevelopment of tourism resources, while Iceland suffers from underdevelopment in this regard. We chose these two regions as case studies to test the model's versatility and applicability.

By conducting a comprehensive survey of the basic conditions in these two tourist destinations, we determined the values of various parameters in the model. For instance, since both areas belong to tundra ecosystems with low tolerance for waste and wastewater, and fragile ecological environments, we appropriately reduced the values of relevant parameters. After modifying and running the model, we obtained the optimal number of tourists and maximum revenue for both regions and compared them with the original data.

As shown in Figure 9, Lhasa's maximum tourism revenue reached 5216.5 million USD, with a tourist count of 23.4 million. Compared to the 2024, this represents an increase in income by 614.1 million USD despite a reduction of 14.2 million tourists. Iceland's maximum tourism revenue was 798.8 million USD, more than doubling the 2024, with only a 2.1 million increase in tourist numbers. Therefore, Lhasa can appropriately reduce the number of tourists entering Tibet and increase investment in local infrastructure and environmental protection; Iceland, on the other hand, can enhance the development of tourism resources to boost tourism revenue. This aligns with the actual situations of over-tourism in Lhasa and underdeveloped tourism resources in Iceland, indirectly proving the model's versatility.

Thus, for regions with underdeveloped tourism resources, we can adjust the model coefficients based on actual conditions to better predict the optimal number of tourists and maximum revenue.

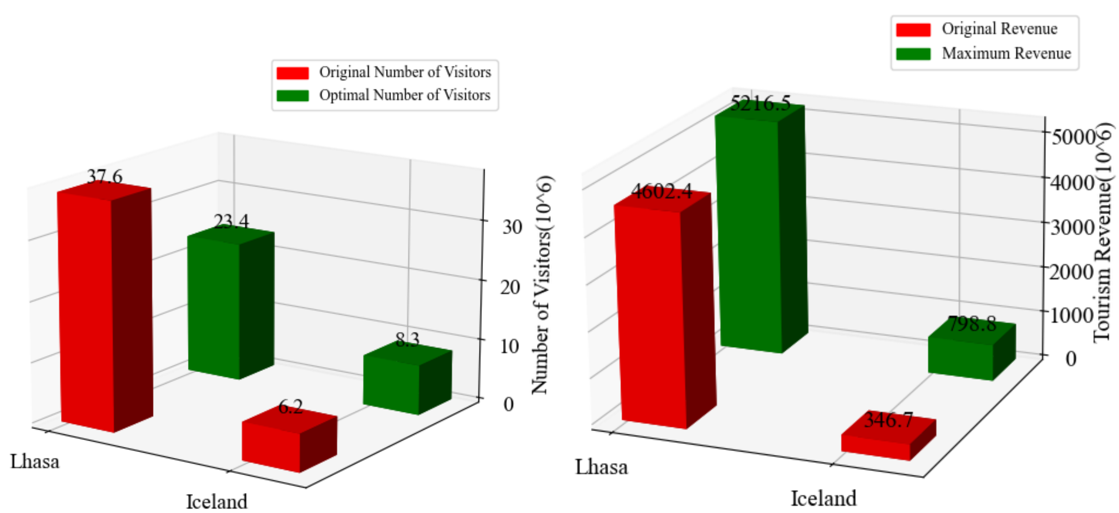


Figure 9: Schematic diagram of model verification effect

7 Sensitivity and Robustness Analysis

Sensitivity analysis of the model refers to the study of how changes in input parameters affect the output results when analyzing a mathematical model or system. By **varying input variables** or **parameters**, we can determine their impact on model behavior, thereby optimizing the model and gaining a better understanding of its dynamic characteristics. In the sensitivity analysis of the model, we use the optimized model for Juneau as a baseline and adjust the environmental carrying capacity growth coefficients α_w, α_r, and α_o in the additional expenditure plan to evaluate the impact of these parameters on the overall performance of the model.Specifically, we divide the range of α_w, α_r, and α_o from 0 to 0.2 into 10 equal intervals, with each interval receiving an investment of 1 million USD. We then observe changes in the optimal number of tourists and maximum revenue under different parameter settings. As shown in Figure 10, within the range of 0 to 0.2, the optimal number of tourists and maximum revenue both increase significantly as α_w, α_r, and α_o increase, approaching the model's predicted optimal values. Specifically, with a constant total investment, increases in α_w, α_r, and $\alpha_o$enhance wastewater and waste management capacities, allowing for more tourist-generated waste. Additionally, improvements in social infrastructure capacity and ecosystem self-regulation enable the attraction of more tourists and increased overall tourism revenue.

In the sensitivity analysis, the results near the optimal number of tourists (1.05 million) and optimal tourism revenue (356 million USD) remain stable as these coefficients change, without significant fluctuations. This further demonstrates the robustness and stability of the model, indicating its effectiveness in addressing challenges in sustainable tourism development.

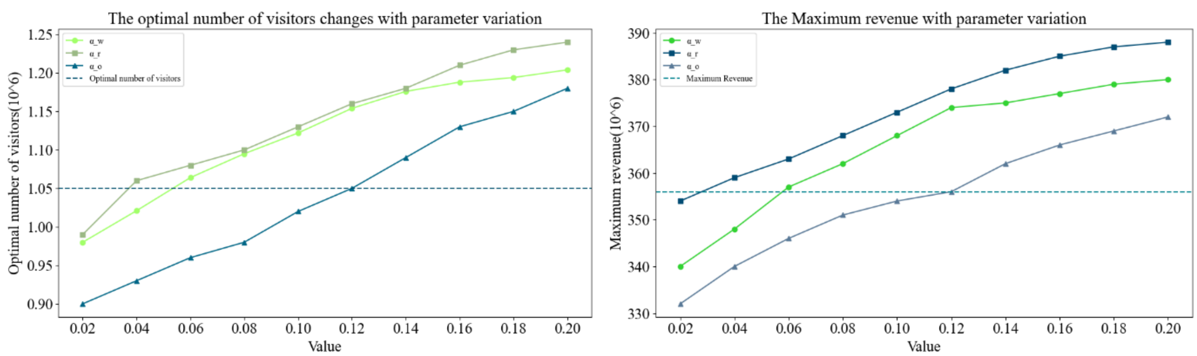


Figure 10: Sensitivity and robustness analysis results

8 Model Evaluation and Further Discussion

8.1 Strengths

1. The model comprehensively considers three dimensions: economic income, nature conservation, and social maintenance, involving multiple parameters. It provides a holistic analysis of sustainable tourism in Juneau from a global perspective.

2. By consolidating various complex indicators into the single variable of tourist numbers, the model achieves consistency and intuitiveness in its results.

3. The model's complex parameters have been validated against existing actual data, demonstrating strong alignment with real-world conditions. This allows for effective

recommendations addressing practical issues and accurate predictions of revenue and expenses.

4. The model does not solely focus on economic benefits but incorporates complex environmental variables, presenting the importance of the environment to human society through income and expenditure. It aims to highlight the feedback effect of the environment on the economy and society, which is a unique innovation of this model. We hope that governments, policymakers, and more individuals will recognize this significance.

5. Due to its involvement of multiple parameters, the model exhibits good adaptability and can be applied to different tourist regions.

8.2 Weaknesses

Due to the model's pursuit of high precision, future research will focus on achieving intelligent parameterization. The model does not account for the costs caused by secondary disasters resulting from glacier melting, which leads to an underestimation of the glacier protection costs. In the future, we will pay more attention to this cost factor.

8.3 Further Discussion

1. The model involves multiple parameters, so the optimization direction focuses on simplifying parameter settings.

2. The inclusiveness of residents in tourist destinations is an important consideration for government tourism policymaking. By conducting surveys and other methods, the inclusiveness of residents can be quantified and incorporated into the model to further enhance its completeness.

3. Due to the inclusion of numerous parameters, the model performs well across different regions and exhibits strong adaptability.

9 Conclusion

The rapid development of tourism not only brings substantial income to local residents but also has a profound impact on the local ecological environment and social structure. Therefore, achieving sustainable tourism development has become a core issue in current research. In this study, we focus on the phenomenon of overtourism in Juneau and develop a comprehensive tourism sustainability model that integrates economic, natural, and social dimensions. This model aims to predict optimal tourist numbers and **maximize tourism economic benefits** while **ensuring its broad applicability and robustness**. The specific conclusions are as follows:

1. We decompose the issue of sustainable tourism into three aspects: economic, natural, and social. By simplifying complex problems into the relationship between tourism revenue and tourist numbers, we construct a nonlinear objective function and use nonlinear programming methods to maximize tourism revenue. This process reduces problem complexity and enhances the operability of the model.

2. The model incorporates parameters such as additional expenses and environmental carrying capacity, integrating additional expenses to improve the natural environment and maintain social infrastructure. This not only enhances the self-regulation capabilities of natural and social systems but also reduces maintenance costs, thereby achieving higher economic efficiency. After optimization, we find that the optimal number of tourists for Juneau is 1.05 million, with maximum revenue reaching 356 million USD. This indicates that to achieve sustainable development, it is necessary to appropriately limit the number of tourists.

3. To verify the applicability and broad applicability of the model, we apply it to two different regions: Lhasa and Iceland, adjusting parameters according to local conditions. The results show that the model performs well in both regions, leading to reasonable recommendations: Lhasa should limit tourist numbers and increase investment in environmental protection, while Iceland should moderately expand tourism development. This further demonstrates the model's versatility and flexibility.

4. We conduct sensitivity analysis and robustness testing on the model by iterating the growth coefficient of environmental carrying capacity within a reasonable range and observing changes in optimal tourist numbers and maximum tourism revenue. Results indicate that although optimal tourist numbers and maximum revenue increase with the growth coefficient, overall changes remain within expected ranges, proving the model's good stability and robustness.

Based on the above conclusions, we recommend that Juneau first limit tourist numbers and invest more resources in environmental protection and social infrastructure to enhance its self-regulation capabilities and support sustainable tourism development. Specifically, during the summer when glaciers are prone to melting, a "limited tourism period" should be set in ecologically fragile glacial areas to protect the original natural environment and reduce secondary disasters caused by glacier melting.

References

[1] Tang, Y., Li, J. & Xing, C. Construction of an evaluation indicator system for the integrated development of sports, culture and tourism based on tourist perceptions. Sci Rep 15, 3330 (2025). https://doi.org/10.1038/s41598-025-87526-2

[2] Irawan, N. C., and Ei Hartoyo. "Environmental management and stakeholder roles in sustainable tourism development: a feasibility study." IOP Conference Series: Earth and Environmental Science. Vol. 1108. No. 1. IOP Publishing, 2022. https://doi.org/10.1088/1755-1315/1108/1/012068

[3] World Bank. Global Economic Prospects. Retrieved from https://www.worldbank.org

[4] Wake, B. "Tourism stepping up climate action." Nature Climate Change 1-1 (2024). https://doi.org/10.1038/s41558-024-02231-8

[5] Sun, Y., Faturay, F., Lenzen, M., et al. Drivers of global tourism carbon emissions. Nat Commun 15, 10384 (2024). https://doi.org/10.1038/s41467-024-54582-7

[6] Lenzen, M., Sun, Y., Faturay, F., et al. The carbon footprint of global tourism. Nature Clim Change 8, 522–528 (2018). https://doi.org/10.1038/s41558-018-0141-x

[7] Streimikiene, D, et al. "Sustainable tourism development and competitiveness: The systematic literature review." Sustainable development 29, 259-271 (2021). https://doi.org/10.1002/sd.2133

[8] Davies, B., McNabb, R., Bendle, J., et al. Accelerating glacier volume loss on Juneau Icefield driven by hypsometry and melt-accelerating feedbacks. Nat Commun 15, 5099 (2024). https://doi.org/10.1038/s41467-024-49269-y

Appendix 1

Reference values of some model parameters

Variable Name	**Value**	**Description**
α_e	3.9	Initial employment benefits
t_e	5%	Employment tax rate
t_b	5%	Business tax rate
$\bar{C}$	$41.18kg/a$	Per capita carbon emissions
μ_t	4.3	Climate sensitivity parameter
k_m	0.0383	The sensitivity coefficient of glaciers to temperature changes
α_m	3.0	The index of accelerated melting of ice and snow
μ_r	0.08	The cost of managing waste per unit volume
μ_w	0.147	The cost of treating a unit volume of sewage
k_r	1.6	The growth coefficient of garbage treatment costs
k_w	1.6	The growth coefficient of sewage treatment costs
x_r	1	The number of tourists corresponding to garbage saturation
x_w	1	The number of tourists corresponding to the saturation of sewage.
α_i	40000	Conversion parameter between community impact and cost

Memorandum

Connections Do Exist

To: Tourism Bureau of Juneau City
Subuject: Suggestions for Sustainable Development
Date: Sunday, January 26,2025

As the stewards of the Gastineau Channel, protectors of the Tongass National Forest, and guides for visiting tourists, we face a significant challenge: how to **implement effective measures** to maximize both **tourist satisfaction** and **the well-being of residents** in the context of overtourism. To address this issue, our research team has developed a **sustainable tourism model** that analyzes key factors from two perspectives: **explicit benefits** and **implicit costs**.

Explicit Benefits: The development of the tourism industry has significantly promoted social and economic progress, specifically through increased business revenue and job creation.

Implicit Costs: As overtourism intensifies, residents' dissatisfaction is rising, and environmental degradation is becoming more severe, leading to substantial increases in environmental protection and social governance costs. Notably, **glacier protection** incurs the highest cost and has the most profound impact on the ecosystem.

Through an in-depth analysis of these key factors, we have reached the following conclusions:

1.Implement a reservation system for visits to **limit the number of tourists** entering Juneau;
2. Encourage tourists to conserve water and reduce the use of single-use items;
4. Increase **government investment** in wastewater treatment and waste management enterprises;

We can adopt the following measures:

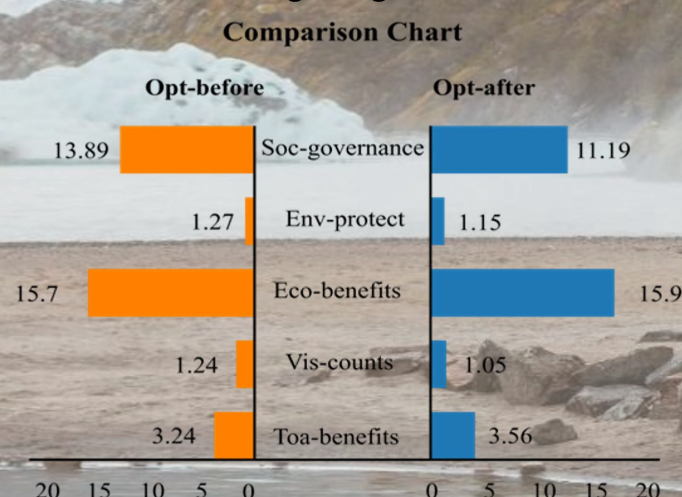
1. **Limit** the number of tourists visiting Juneau and implement a reservation system for visiting.

2. The government should **increase investment** in enterprises discharging pollutants and those dealing with garbage.

3. **Install sound insulation boards** on the walls of busy streets to reduce the impact on local residents.

4. The scenic area should draw a **red line** prohibiting foot traffic to reduce the impact of humans on the **melting of glaciers.**

The effect prediction after improvement is shown in the right figure:



Let's act immediately! Let's strive to create a more beautiful, happier and harmonious Juneau!

Team#2510151