

## महाविद्या 6(A)

① एक त्रिभुज ABC में  $\angle B = 90^\circ$ ,  $AB = 5\text{cm}$ ,  $BC = 12\text{cm}$  हो तो उन  $\sin A$ ,  $\cos A$ ,  $\tan A$  के मान ज्ञात कीजिए।

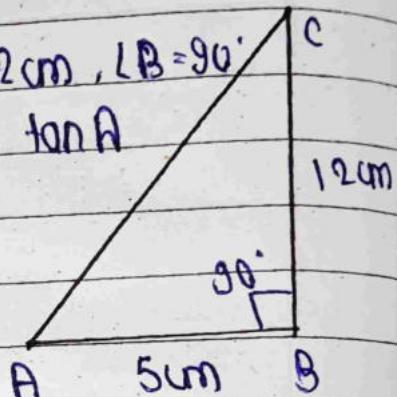
दिया है -

$\angle B = 90^\circ$ ,  $AB = 5\text{cm}$ ,  $BC = 12\text{cm}$ ,  $\angle B = 90^\circ$

सिर्फ जाना है -  $\sin A$ ,  $\cos A$ ,  $\tan A$

हम -  $\triangle ABC$  में  $\angle B = 90^\circ$

पाइथागोरस प्रमेय से



$$(AC)^2 = (AB)^2 + (BC)^2$$

$$AC^2 = 5^2 + 12^2$$

$$AC = \sqrt{25 + 144}$$

$$AC = \sqrt{169}$$

$$AC = 13\text{cm}$$

$$\sin A = \frac{\text{परिपथ}}{\text{आधार}} = \frac{12}{13}, \cos A = \frac{\text{आधार}}{\text{परिपथ}} = \frac{5}{13}$$

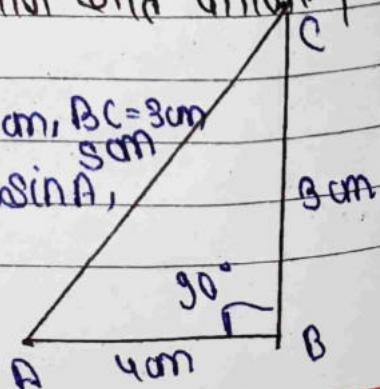
$$\tan A = \frac{\text{परिपथ}}{\text{आधार}} = \frac{12}{5}$$

②  $\triangle ABC$  में  $\angle B = 90^\circ$  अमानों ही ज्ञात  $\sin A$ ,  $\cos A$ ,  $\tan A$ ,  $\cot C$ ,  $\csc A$ ,  $\sec B$  तथा  $\csc C$  के मान ज्ञात कीजिए।

दिया है -

$AB = 4\text{cm}$ ,  $CA = 5\text{cm}$ ,  $BC = 3\text{cm}$   
कात जाना है -  $\cos C$ ,  $\cot C$ ,  $\sin A$ ,  $\csc A$

$\tan A$



$$\cdot \text{हल} - \sin A = \frac{\text{सम्बन्ध}}{\text{कर्तव्य}} = \frac{3}{5}$$

$$\tan A = \frac{\text{पर्याप्त}}{\text{छान्दोर}} = \frac{3}{4}$$

$$\cos C = \frac{\text{आधार}}{\text{पर्श}} = \frac{9}{5}$$

$$\cot C = \frac{AB}{BC} = 3$$

१. समष्टि ५

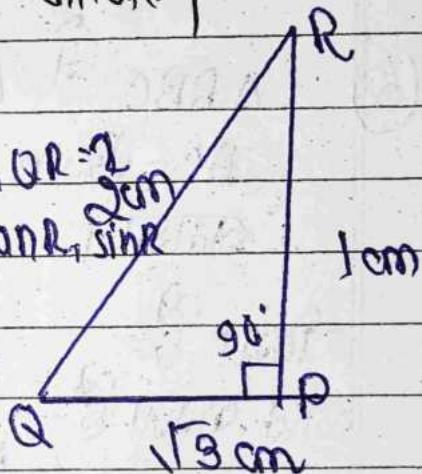
③  $\triangle PQR$  में  $LP$  अमितावी है, यदि  $QP = \sqrt{3}$ ,  $PR = 1$   
 तथा  $QR = \sqrt{2}$  हो तो  $\sin Q, \cos Q, \tan R,$   
 $\sin R$  के मान ज्ञात कीजिए।

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प्र० ४) यदि  $PQ = \sqrt{3}$ ,  $PR = 1$ ,  $QR = 2$   
 जात गया है  $\sin Q, \cos Q, \tan Q, \sin R$

४५-

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$$



$$\cos Q = \frac{31613}{3901} = \frac{\sqrt{3}}{2}$$

$$\tan R = \frac{\text{पर्याप्त}}{\text{आवश्यक}} = \sqrt{3}$$

$$\sin R = \frac{\text{मन्त्र}}{\text{जारी}} = \frac{\sqrt{3}}{2}$$

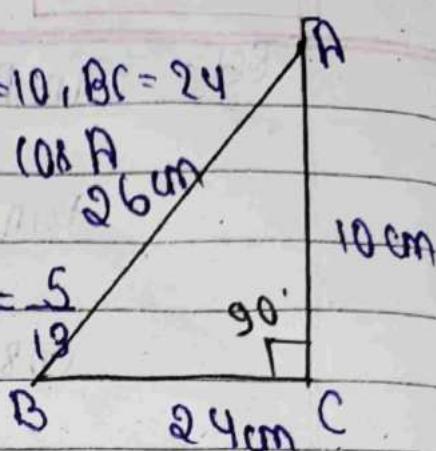
(4) अमर्जीत श्रियुज का ABC में BC अमर्जीत ही यादे  
 मुझे AB = 16, AC = 10, BC = 24 हो, तो  $\sin B$ ,  
 $\tan B$ ,  $\cos A$  की मात्र ज्ञात कीजिए।

दिया गया -

जात जरूरी -  $AB = 16, AC = 10, BC = 24$

$\sin B, \tan B, \cos A$

$26\text{cm}$



हल -  $\sin B = \frac{\text{पर्याप्त}}{\text{कर्ण}} = \frac{10}{26} = \frac{5}{13}$

$\tan B = \frac{\text{पर्याप्त}}{\text{आधार}} = \frac{10}{24} = \frac{5}{12}$

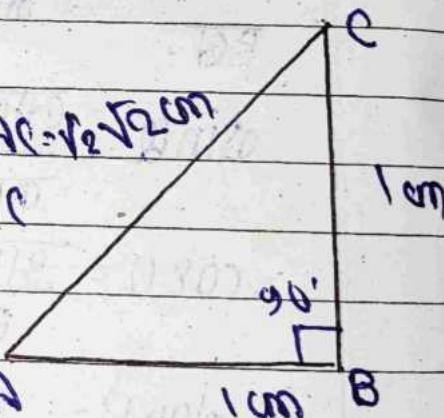
$\cos A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{24}{26} = \frac{12}{13}$

- (5)  $\triangle ABC$  में  $\angle B$  समकोण है। यदि  $AB = BC = 1\text{cm}$   
 $AC = \sqrt{2}\text{cm}$  हो तो  $\sin A, \tan C$  के मान  
 बताएं।

दिया गया -

जात जरूरी -  $AB = BC = 1, AC = \sqrt{2}\text{cm}$

$\sin A, \tan C$



$\sin A = \frac{\text{पर्याप्त}}{\text{कर्ण}} = \frac{1}{\sqrt{2}}$

$\tan C = \frac{\text{पर्याप्त}}{\text{आधार}} = \frac{1}{1} = 1$

- (6)  $\triangle PQR$  में  $\angle Q$  समकोण हो यदि  $PQ = 4\text{cm}, QR = 3\text{cm}$   
 $PR = 5\text{cm}$  हो तो  $\cos P, \tan P, \cos R$  के  
 मान जात लीजिए।

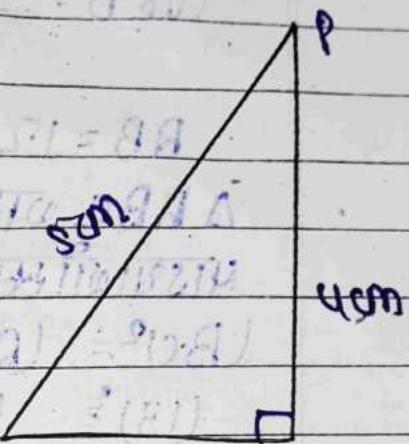
दिया है -  
बात जर्जा है -  $PQ = 4\text{cm}$ ,  $QR = 3\text{cm}$ ,  $PR = 5\text{cm}$   
(QD पर,  $\tan P$ ,  $\cos R$ )

हल -

$$\cos P = \frac{\text{आधार}}{\text{कर्ण}} = \frac{4}{5}$$

$$\tan P = \frac{\text{सम्मेलन}}{\text{आधार}} = \frac{3}{4}$$

$$\cos R = \frac{\text{आधार}}{\text{कर्ण}} = \frac{3}{5}$$



- ⑦ निम्नलिखित में से प्रत्येक में, तीज शिखों का मात्राय भवुपात में से कौन दिया है, अव्यक्ति अनुपातों को बात जीवा,

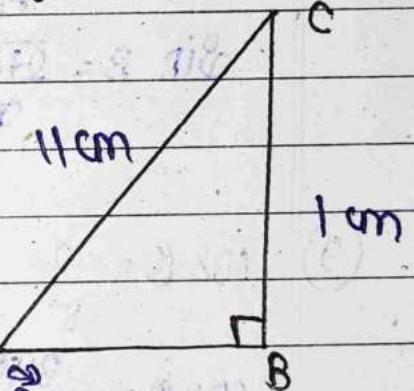
$$\sin A = \frac{1}{11}$$

$$\sin A = \frac{\text{सम्मेलन}}{\text{कर्ण}} = \frac{1}{11}$$

$$BC = 1, AC = 11$$

$\Delta ABC$  का मानों का विषय A है,

प्राप्तिगति विषय है -



$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(11)^2 = (AB)^2 + (1)^2$$

$$121 - 1 = (AB)^2$$

$$\sqrt{120} = AB$$

$$2\sqrt{30} = AB$$

$$\cos A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{2\sqrt{30}}{11}$$

$$\tan A = \frac{\text{सम्मेलन}}{\text{आधार}} = \frac{1}{2\sqrt{30}}$$

⑧

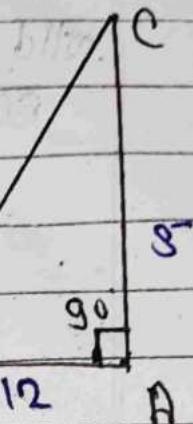
$$\cos B = \frac{12}{13}$$

$$\cos B = \frac{\text{आधार}}{\text{कर्ण}} = \frac{12}{13}$$

$$AB = 12, BC = 13$$

$\triangle ABC$  अमूलीता है।

पाठ्याग्रीसम प्रमेय में -



$$(BC)^2 = (AB)^2 + (AC)^2$$

$$(13)^2 = (12)^2 + (AC)^2$$

$$\sqrt{169 - 144} = AC$$

$$\sqrt{25} = AC$$

$$AC = 5$$

$$\sin B = \frac{\text{पर्याप्त}}{\text{कर्ण}} = \frac{5}{13}, \tan B = \frac{\text{पर्याप्त}}{\text{आधार}} = \frac{5}{12}$$

⑨

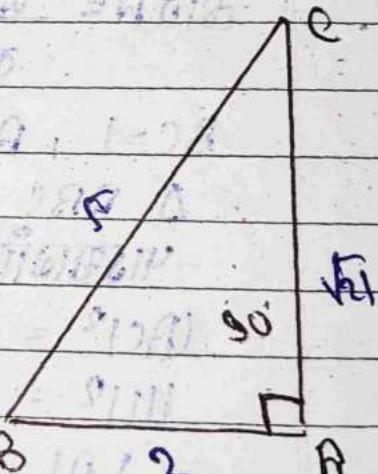
$$\cos O = \frac{2}{5}$$

$$\cos O = \frac{\text{आधार}}{\text{कर्ण}} = \frac{2}{5}$$

$$AB = 2, BC = 5$$

$\triangle ABC$  अमूलीता है।

पाठ्याग्रीसम प्रमेय में -



$$(BC)^2 = (AB)^2 + (AC)^2$$

$$(5)^2 = (2)^2 + (AC)^2$$

$$25 - 4 = AC^2$$

$$\sqrt{21} = AC$$

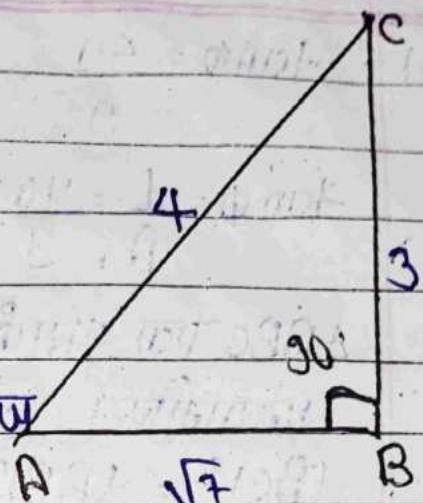
$$\sin O = \frac{\text{पर्याप्त}}{\text{कर्ण}} = \frac{\sqrt{21}}{5}, \tan O = \frac{\text{पर्याप्त}}{\text{आधार}} = \frac{\sqrt{21}}{2}$$

$$\textcircled{10} \quad \sin A = \frac{3}{4}$$

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{3}{4}$$

$$BC = 3, AC = 4$$

$\triangle ABC$  एक भमलीन त्रिभुज है।



पाठ्यागारस समेय क्ये -

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$16 = (AB)^2 + 9$$

$$AB = \sqrt{16 - 9}$$

$$AB = \sqrt{7}$$

$$\cos A = \frac{\text{आधार}}{\text{जर्म}} = \frac{\sqrt{7}}{4}, \tan A = \frac{\text{जर्म}}{\text{आधार}} = \frac{3}{\sqrt{7}}$$

$$\textcircled{11} \quad \tan C = \frac{1}{3}$$

$$\tan C = \frac{\text{जर्म}}{\text{आधार}} = \frac{1}{3} = \frac{AB}{BC}$$

$\triangle ABC$  एक भमलीन  $\triangle$  है।

पाठ्यागारस समेय क्ये -

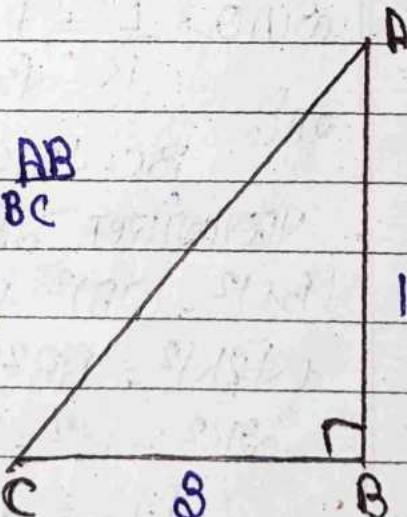
$$(AC)^2 = (AB)^2 + (BC)^2$$

$$AC^2 = (1)^2 + (3)^2$$

$$AC = \sqrt{1 + 9}$$

$$AC = \sqrt{10}$$

$$\sin C =$$



$$(12) \tan \phi = \frac{40}{9}$$

$$\tan \phi = \frac{L}{A} = \frac{40}{9} = \frac{AC}{AB}$$

$\triangle ABC$  एक व्यापकोन त्रिभुज है।

पाठ्यागारस प्रमेय से -

$$(BC)^2 = (AB)^2 + (AC)^2$$

$$BC^2 = (91)^2 + (40)^2$$

$$BC = \sqrt{81 + 1600}$$

$$BC = \sqrt{1681}$$

$$BC = 41$$

$$\sin \phi = \frac{L}{K} = \frac{40}{41}, \text{ ठोस } \phi = \text{आधार} = \frac{9}{41}$$

(13) यदि  $\sin \theta = \frac{1}{\sqrt{2}}$  तो शोध शिक्षण मित्रय अनुपात ज्ञात कीजिए।

$$\sin \theta = \frac{L}{K} = \frac{1}{\sqrt{2}}$$

$$\text{यदि } BC = \sqrt{2}K, AC = K$$

पाठ्यागारस प्रमेय से -

$$(BC)^2 = (AB)^2 + (AC)^2$$

$$(\sqrt{2}K)^2 = AB^2 + (1K)^2$$

$$2K^2 - K^2 = AB^2$$

$$AB = \sqrt{K^2}$$

$$AB = K$$

$$\text{ठोस } \theta = \frac{A}{L} = \frac{K}{\sqrt{2}K} = \frac{1}{\sqrt{2}}$$

$$\tan \theta = \frac{L}{A} = \frac{\sqrt{2}K}{K} = \sqrt{2}$$

$$\text{ठोस } \theta = \frac{L}{K} = \sqrt{2}K$$

$$\sec \theta = \frac{L}{A} = \frac{\sqrt{2}K}{K} = \sqrt{2}$$

$$\cot \theta = \frac{A}{L} = \frac{K}{\sqrt{2}K} = \frac{1}{\sqrt{2}}$$

(14) यदि  $\operatorname{sec} \theta = \sqrt{5}$  तो लीप्र पहुंच मिलीमिटीय अनुपात ज्ञात कीजिए।

$$\operatorname{sec} \theta = \frac{k}{l} = \sqrt{5}$$

यदि  $BC = \sqrt{5}k$ ,  $AB = lk$

पाठ्यालौस्य प्रमेय से -

$$(BC)^2 = (AB)^2 + (AC)^2$$

$$(\sqrt{5})^2 = (l)^2 + (AC)^2$$

$$5 - l^2 = AC^2$$

$$AC = \sqrt{4} = 2$$

$$\operatorname{sin} \theta = \frac{l}{k} = \frac{1}{\sqrt{5}}, \quad \operatorname{tan} \theta = \frac{l}{1} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$$\operatorname{cosec} \theta = \frac{A}{l} = \frac{1}{1} = 1, \quad \operatorname{cot} \theta = \frac{l}{A} = \frac{1}{1} = 1 \quad \operatorname{sec} \theta = \frac{k}{l} = \sqrt{5}$$

(15) यदि  $\operatorname{cosec} \theta = \frac{2}{\sqrt{3}}$ , तो लीप्र मिलीमिटीय अनुपात ज्ञात कीजिए।

$$\operatorname{cosec} \theta = \frac{k}{l} = \frac{2}{\sqrt{3}}$$

यदि  $BC = 2k$ ,  $AC = \sqrt{3}k$

पाठ्यालौस्य प्रमेय से

$$(BC)^2 = (AB)^2 + (AC)^2$$

$$(2)^2 = (AB)^2 + (\sqrt{3})^2$$

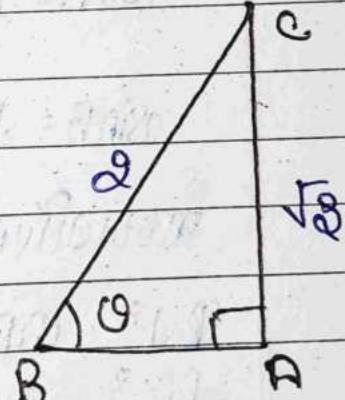
$$4 = AB^2 + 3$$

$$AB^2 = 4 - 3 = 1$$

$$AB = \sqrt{1} = 1$$

$$\operatorname{sin} \theta = \frac{\sqrt{3}}{2}, \quad \operatorname{cosec} \theta = \frac{1}{\frac{1}{2}} = 2, \quad \operatorname{tan} \theta = \sqrt{3}, \quad \operatorname{sec} \theta = 2$$

$$\operatorname{cot} \theta = \frac{1}{\sqrt{3}} \quad \underline{\text{AB}}$$



(16) यदि  $\cos A = \frac{1}{2}$ ,  $\sin B = \frac{\sqrt{3}}{2}$ , तो लिम्नाभिविष्ट

के मान का ग्राफ बनाओ।  $\sin A \cos B + \cos A \sin B$

$$\cos A = \frac{1}{2}$$

$$AB = 1, AC = \sqrt{2}$$

पाठ्यांशीरम् प्रमेय से -

$$(AC)^2 = (BC)^2 + (AB)^2$$

$$(\sqrt{2})^2 = (BC)^2 + (1)^2$$

$$\vartheta = BC^2 + 1$$

$$BC^2 = \vartheta - 1$$

$$BC = \sqrt{\vartheta - 1}$$

$$\sin A = \frac{1}{\sqrt{2}}$$

$$\cos A = \frac{1}{\sqrt{2}}$$

$$\sin B = \frac{\sqrt{3}}{2}$$

पाठ्यांशीरम् प्रमेय से -

$$(BC)^2 = (AB)^2 + (AC)^2$$

$$(\vartheta)^2 = AB^2 + (\sqrt{3})^2$$

$$\vartheta - 3 = AB^2$$

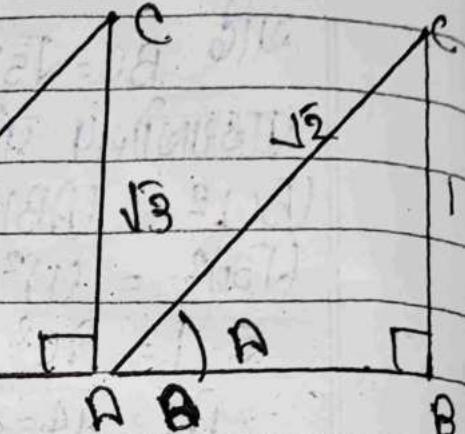
$$AB = \sqrt{1} = 1$$

$$\cos B = \frac{1}{\sqrt{2}}$$

(17) यदि  $\tan \theta = \frac{4}{3}$  तो लिम्न जीविष्ट कि

$$\sec \theta = ?$$

$$\csc \theta = ?$$



$$\Rightarrow \sin A \cos B + \cos A \sin B$$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\Rightarrow \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$\frac{1+\sqrt{3}}{2\sqrt{2}} \text{ Ans}$$

दिया है -  
मिल जाना -

$$\tan \theta = \frac{a}{b} \quad b \sec \theta \rightarrow 1$$

$\cos \sec \theta$

$$\tan \theta = \frac{c}{a} = \frac{a}{b} = \frac{BC}{AB}$$

$$BC = ak, AB = bk$$

$\triangle ABC$  में,

प्रमाणात्मक प्रमेय से -

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$AC^2 = (bk)^2 + (ak)^2$$

$$AC = \sqrt{b^2 k^2 + a^2 k^2}$$

$$AC = k\sqrt{a^2 + b^2}$$

$$\sec \theta = \frac{k\sqrt{a^2 + b^2}}{b} = \frac{\sqrt{a^2 + b^2}}{b}$$

$$\cos \theta = \frac{k\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{a^2 + b^2}}{a}$$

$$\text{L.H.S.} = \frac{b \sec \theta}{a \cos \theta} = \frac{k\sqrt{a^2 + b^2}}{a} = 1$$

Proved

(18)

यदि  $\cos A = \frac{4}{5}$  तो मिल कीजिए कि  $\tan A = ?$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cos A = \frac{4}{5} = \frac{AB}{AC}$$

$k$

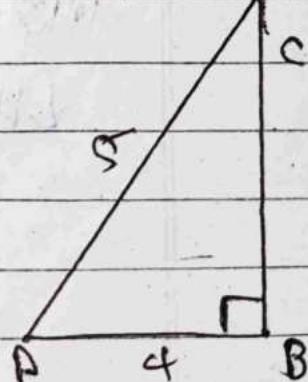
प्रमाणात्मक प्रमेय से -

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$5^2 = 4^2 + BC^2$$

$$BC^2 = 25 - 16$$

$$BC = \sqrt{9} = 3$$



$$\tan A = \frac{L}{A} = \frac{3}{4}, \quad \sin A = \frac{L}{K} = \frac{3}{5}, \quad \sec A = \frac{K}{A} = \frac{5}{4}$$

$$\frac{\tan A}{1 + \tan^2 A} = \frac{\sin A}{\sec A}$$

$$\frac{\frac{3}{4}}{1 + \left(\frac{3}{4}\right)^2} = \frac{\frac{3}{5}}{\frac{5}{4}}$$

$$\frac{\frac{3}{4}}{1 + \frac{9}{16}} = \frac{\frac{3}{5} \times \frac{4}{5}}{\frac{25}{16}}$$

$$\frac{\frac{3}{4} \times \frac{16}{16}}{16 + 9} = \frac{12}{25}$$

$$\frac{3}{4} \times \frac{16}{25} = \frac{12}{25}$$

$$12 = 12 \quad \text{Proved}$$

(19)  $\sin \theta = \frac{1}{2}$  तो सिंह गीजा की  ~~$\cos \theta - 4 \sin^3 \theta = 1$~~

दिया है-

$$\sin \theta = \frac{1}{2}$$

सिंह गीजा है-

$$\cos \theta - 4 \sin^3 \theta = 1$$

$$\text{L.H.S.} \Rightarrow 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3$$

$$\Rightarrow \frac{3}{2} - 4 \times \frac{1}{8}$$

$$\Rightarrow \frac{3}{2} - \frac{1}{2} = 1 \quad \text{Proved}$$

(Q) यदि  $\sin A = \frac{1}{\sqrt{3}}$  तो निम्नलिखित का मान ज्ञान कीजिए,

$$108A \cdot 108\operatorname{cosec} A + \tan A \cdot 8\operatorname{cosec} A$$

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{3}}$$

$$AC = 3, BC = 1$$

पाठ्यांगौर्य समीक्षा-

$$(AB)^2 = (AC)^2 + (BC)^2$$

$$(AB)^2 = AC^2 + BC^2$$

$$9 - 1 = AB^2$$

$$AB = \sqrt{8} \Rightarrow 2\sqrt{2}$$

$$108A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{8}{\sqrt{3}}$$

$$108\operatorname{cosec} A = \frac{\text{कर्ण}}{\text{आधार}} = \frac{8}{8}$$

$$\tan A = \frac{\text{आधार}}{\text{कर्ण}} = 1$$

$$\text{आधार} \quad 2\sqrt{2}$$

$$\operatorname{cosec} A = \frac{\text{कर्ण}}{\text{आधार}} = \frac{8}{2\sqrt{2}}$$

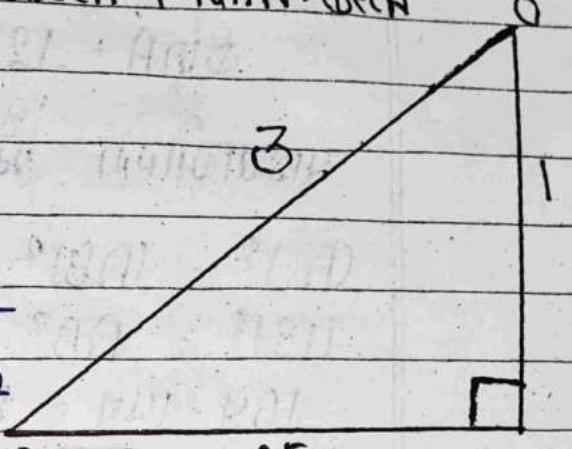
$$(108A \cdot 108\operatorname{cosec} A + \tan A \cdot 8\operatorname{cosec} A)$$

$$\frac{8\sqrt{3}}{3} \times 8 + 1 \times 8$$

$$8\sqrt{2} + 8$$

$$8\sqrt{2}$$

$$\frac{16\sqrt{2} + 8}{8} \quad \text{Ans}$$



(21) यदि  $18 \sin A = 12$  तो  $\sec A - \tan A$  का मान ज्ञात कीजिए।

$$\sin A = \frac{12}{18} = \frac{2}{3}$$

पाइथागोरस - त्रिभुज से

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(13)^2 = AB^2 + 12^2$$

$$169 - 144 = AB^2$$

$$AB = \sqrt{25} = 5$$

$$\sec A = \frac{13}{5} = 2.6$$

आधार 5

$$\tan A = \frac{12}{5} = 2.4$$

$\sec A - \tan A$

$$\frac{13}{5} - \frac{12}{5}$$

$$\frac{1}{5} \text{ उत्तर}$$

(22) यदि  $1080 = m$  तो  $\tan \theta - \cot \theta$  का मान ज्ञात कीजिए।

$$1080 = \frac{\text{आधार}}{n} = m$$

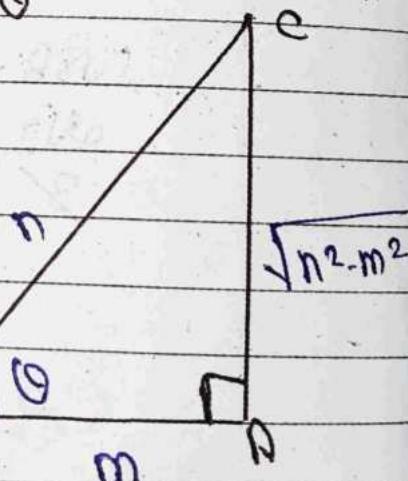
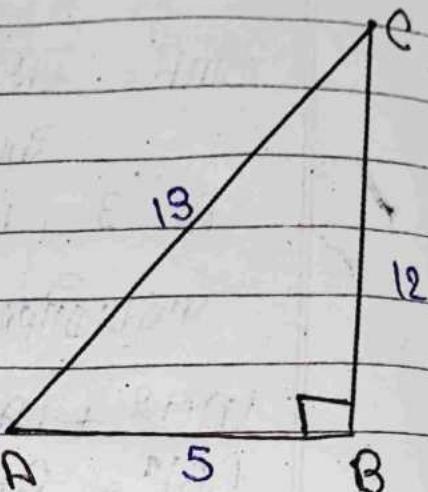
पाइथागोरस - त्रिभुज से

$$(BC)^2 = (AB)^2 + (AC)^2$$

$$n^2 = m^2 + AC^2$$

$$\sqrt{n^2 - m^2} = AC$$

$$AC = \sqrt{n^2 - m^2}$$



$$\tan \theta = \frac{\text{सम्ब}}{\text{आधार}} = \frac{\sqrt{n^2 - m^2}}{m}, \cot \theta = \frac{\text{आधार}}{\text{सम्ब}} = \frac{m}{\sqrt{n^2 - m^2}}$$

$$\Rightarrow \tan \theta - \cot \theta$$

$$\Rightarrow \frac{\sqrt{n^2 - m^2} - m}{m} = \frac{n^2 - m^2 - m^2}{m \sqrt{n^2 - m^2}} = \frac{n^2 - 2m^2}{m \sqrt{n^2 - m^2}}$$

(Q3) यदि  $\cos A = \frac{5}{13}$  तो  $\sin A - \cot A$  का मान ज्ञात कीजिए।

$$\cos A = \frac{5}{13} = \frac{\text{आधार}}{\text{मूल}}$$

$$AB = 5, AC = 13$$

पाठ्यालौकिक प्रमेय से

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(13)^2 = (5)^2 + BC^2$$

$$BC^2 = 169 - 25$$

$$BC = \sqrt{144}$$

$$BC = 12$$

$$\sin A = \frac{\text{पर्याप्त}}{\text{मूल}} = \frac{12}{13}, \cot A = \frac{AB}{BC} = \frac{5}{12}, \tan A = \frac{BC}{AB} = \frac{12}{5}$$

$$\Rightarrow \frac{\sin A - \cot A}{\cot A}$$

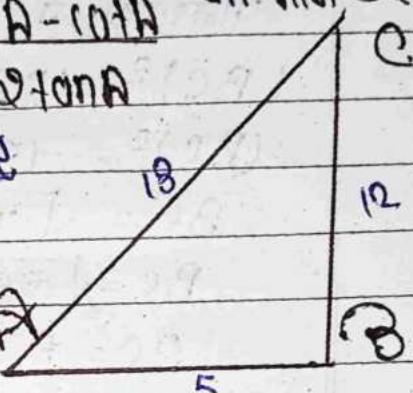
$$\frac{12}{13} - \frac{5}{12}$$

$$\Rightarrow \frac{2 \times 12}{5}$$

$$\Rightarrow \frac{144 - 65}{156} \Rightarrow \frac{79 \times 5}{156 \times 24} \Rightarrow \frac{395}{3744}$$

$$\frac{24}{5}$$

Ans



(64)

यादि

$$\tan \theta = -\frac{4}{3}$$

तो मिल वीथिए की

$$8 \sin \theta - 3 \cos \theta = -17$$

$$8 \sin \theta - 3 \cos \theta = 17$$

$$\tan \theta = \frac{4}{3} = \frac{\text{पर्याप्त}}{\text{अधिक}}$$

$$AB = 3, AC = 4$$

पार्श्वांकीय प्रमेय से -

$$(BC)^2 = (AB)^2 + (AC)^2$$

$$(BC)^2 = (3)^2 + (4)^2$$

$$BC = \sqrt{9+16}$$

$$BC = \sqrt{25}$$

$$BC = 5$$

$$8 \sin \theta = \frac{4}{5}, \quad 10 \cos \theta = \frac{3}{5}$$

$$L.H.S = 8 \sin \theta - 3 \cos \theta$$

$$8 \sin \theta + 3 \cos \theta$$

$$= \frac{8 \times 4}{5} - 3 \times \frac{3}{5}$$

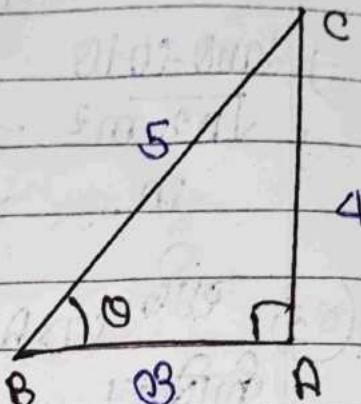
$$\frac{8 \times 4}{5} + 3 \times \frac{3}{5}$$

$$= \frac{8}{5} - \frac{9}{5}$$

$$\frac{8+9}{5}$$

$$= \frac{-1}{17} \Rightarrow \frac{-1 \times 5}{17} = \frac{-1}{17}$$

Proved



(Q5)

चाहि ती विक्ष कीजिए की

$$\tan \theta = \frac{7}{24}$$

 $= 17$  $\sin \theta - 0.80$  $\cos \theta - 0.60$ दिया है:-  
विक्ष करना है:-

$$\tan \theta = 24/7$$

$$\therefore \sin \theta - 0.80 = 17$$

$$\sin \theta + 0.80 \quad (B)$$

$$\tan \theta = \frac{\text{लम्ब}}{\text{आधार}} = \frac{24}{7}$$

$$AB = 7, BC = 24$$

पाठ्यालौरम् समीक्षा-

$$(BC)^2 = (AB)^2 + (AC)^2$$

$$(BC)^2 = 7^2 + 24^2$$

$$BC = \sqrt{49 + 576}$$

$$BC = \sqrt{625} = 25$$

$$\sin \theta = \frac{\text{लम्ब}}{\text{गंता}} = \frac{24}{25}, \cos \theta = \frac{\text{आधार}}{\text{गंता}} = \frac{7}{25}$$

$$L.H.S = \frac{\sin \theta - 0.80}{\sin \theta + 0.80}$$

$$= \frac{24 - 7}{25 + 25}$$

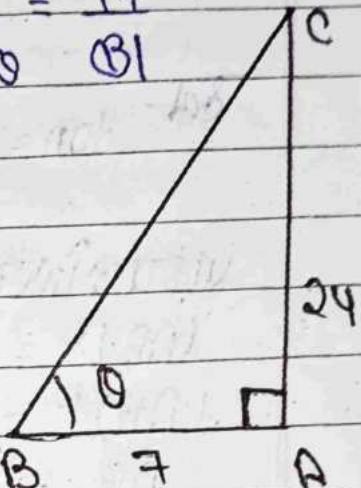
$$\frac{24 + 7}{25 + 25}$$

$$\Rightarrow \frac{17}{25} \Rightarrow \frac{17 \times 25}{25 + 25} = \frac{17}{50} \quad (B)$$

$$\frac{17}{50}$$

$$\frac{17}{50}$$

~~Approved~~



(Q6) यदि  $\tan A = \frac{1}{\sqrt{2}}$  तो विनाखिका का मान ज्ञात कीजिए :  $108A \cos B - \sin A \sin B$

दिया है :  $\tan A = 1$ ,  $\tan B = \sqrt{3}$

जात जाएगा है :  $108A \cos B - \sin A \sin B$

$$\text{हल} - \tan = 1 = \text{समान}$$

1 आधार

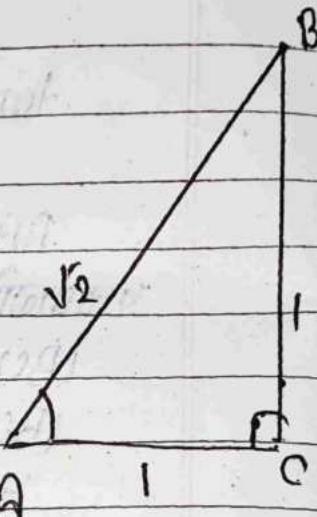
पाइथागोरस प्रमेय से -

$$(AB)^2 = (AC)^2 + (BC)^2 \quad \sqrt{2}$$

$$\therefore (AB)^2 = (1)^2 + (1)^2$$

$$AB = \sqrt{1+1}$$

$$AB = \sqrt{2}$$



$$\sin A = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}, \quad 108A = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\tan B = \frac{\sqrt{3}}{1} = \sqrt{3}$$

आधार

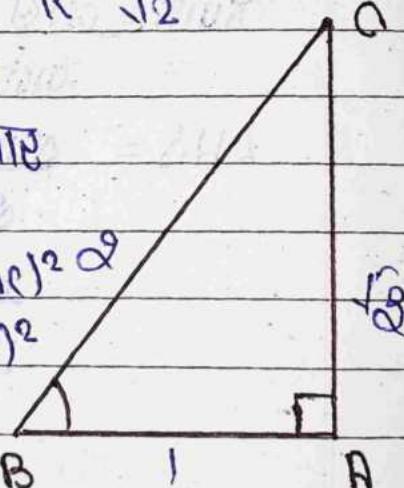
पाइथागोरस प्रमेय से -

$$(BC)^2 = (AB)^2 - (AC)^2$$

$$(BC)^2 = (1)^2 + (\sqrt{3})^2$$

$$BC = \sqrt{1+3}$$

$$BC = \sqrt{4} = 2$$



$$\sin B = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}, \quad \cos B = \frac{1}{2} = \frac{1}{2}$$

$$\cos A \cos B - \sin A \sin B$$

$$\frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \Rightarrow \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

Name \_\_\_\_\_

(Q7) यदि  $\tan \theta + \cot \theta = 2$  तो  $\tan^2 \theta + \cot^2 \theta$  का मान  
 बात कीजिए।

$$\tan \theta + \cot \theta = 2$$

दीनी पड़ी जा वर्ती लगती पर

$$(\tan \theta + \cot \theta)^2 = 4$$

$$\tan^2 \theta + \cot^2 \theta + 2 \times \tan \theta \times \cot \theta = 4$$

$$\left\{ \begin{array}{l} \tan \theta = \sin \theta, \cot \theta = \frac{\cos \theta}{\sin \theta} \\ \cot \theta = \frac{\cos \theta}{\sin \theta} \end{array} \right\}$$

$$\tan^2 \theta + \cot^2 \theta + 2 \times \frac{\sin \theta}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} = 4$$

$$\tan^2 \theta + \cot^2 \theta = 4 - 2$$

$$\tan^2 \theta + \cot^2 \theta = 2 \text{ Ans}$$

(Q8) यदि  $\sin A = 108A$  तो  $\tan^2 A - 28 \sec^2 A + 5$  का मान बात कीजिए।

दिया है -  $\sin A = 108A$

बात यहाँ है -  $\tan^2 A - 28 \sec^2 A + 5$

इस -  $\sin A = 108A$

लेनी पड़ी मैं  $108A$  से कागं छेके पर

$$\frac{\sin A}{108A} = \frac{108A}{108A}$$

$$\tan A = 1 = \text{मान}$$

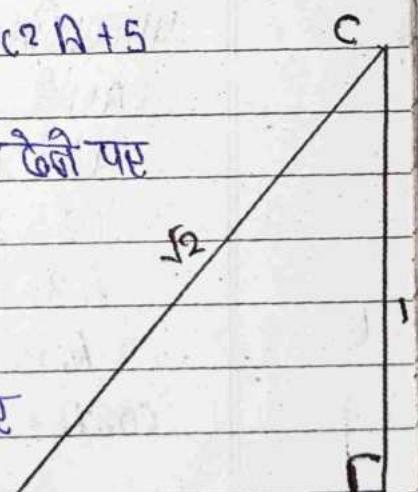
आहार

प्रश्नावाली से प्रमेय से -

$$(AC)^2 = (BC)^2 + (AB)^2$$

$$(AC)^2 = (1)^2 + (1)^2$$

$$AC = \sqrt{1+1} = \sqrt{2}$$



$$\sec A = \frac{\text{कर्ण}}{\text{आधार}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$9 + \tan^2 A - 2\sec^2 A + 5$$

$$2 \times 11^2 - 2 \times (\sqrt{2})^2 + 5$$

$$9 - 2 \times 2 + 5$$

$$9 - 4 + 5$$

$$7 - 4$$

Q Ans

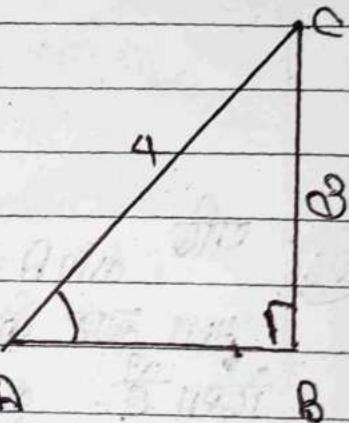
(79) यदि  $\sin A = \frac{3}{4}$ , तो  $\cos A$  और  $\tan A$  ज्ञात कीजिए।

दिया है -

$$\sin A = \frac{3}{4}$$

ज्ञात करना है :-  $\cos A$ ,  $\tan A$

$$\sin A = \frac{3}{4} = \frac{\text{पर्याप्त}}{\text{कर्ण}}$$



पाठ्यांकित समीकरण -

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(5)^2 = (4)^2 + (3)^2$$

$$16 = (AB)^2 + 9$$

$$AB = \sqrt{16 - 9}$$

$$AB = \sqrt{7}$$

$$\cos A = \frac{\text{आधार}}{\text{कर्ण}} = \frac{4}{5}$$

$$\tan A = \frac{\text{पर्याप्त}}{\text{आधार}} = \frac{3}{\sqrt{7}}$$

~~12/12~~

(Q80) यदि अवय समीक्षा मित्रीय अनुपात  $\cot\theta = \frac{13}{12}$  हो तो आवश्यकीयता कीजिए,

$$\cot\theta = \frac{\text{आधार}}{\text{लम्ब}} = \frac{13}{12}$$

$$AC = 13k, AB = 12k$$

पाठ्यांगोर्ध्वमेय रूप,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(13k)^2 = (12k)^2 + (BC)^2$$

$$169k^2 = 144k^2 + (BC)^2$$

$$169k^2 - 144k^2 = (BC)^2$$

$$BC = \sqrt{25k^2}$$

$$BC = 5k$$

$$\sin\theta = \frac{\text{लम्ब}}{\text{अवय}} = \frac{5k}{13k} = \frac{5}{13}$$

$$\tan\theta = \frac{\text{आधार}}{\text{लम्ब}} = \frac{12k}{5k} = \frac{12}{5}$$

$$\cot\theta = \frac{\text{लम्ब}}{\text{आधार}} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cos\theta = \frac{\text{लम्ब}}{\text{अवय}} = \frac{12k}{13k} = \frac{12}{13}$$

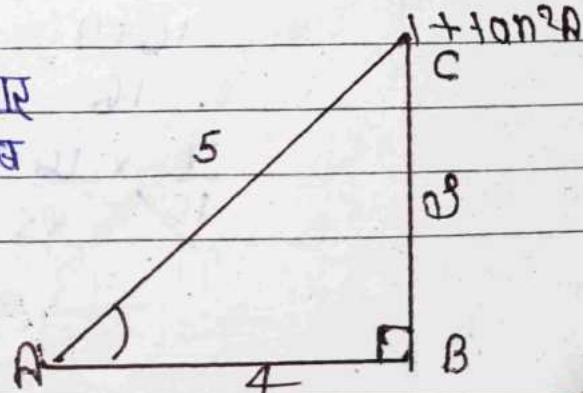
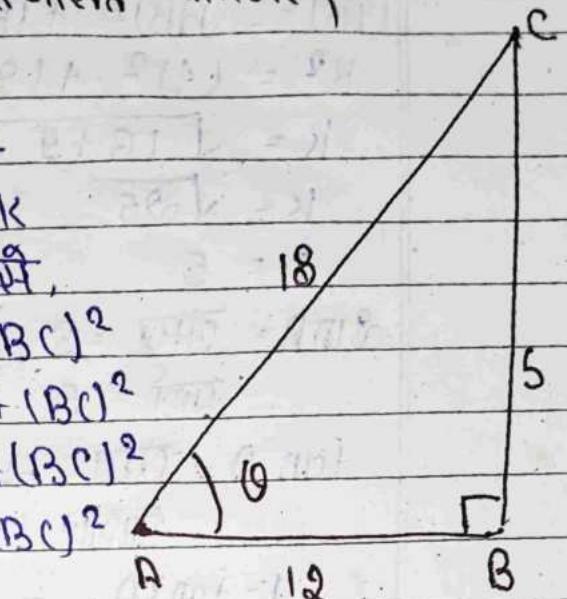
$$\sec\theta = \frac{\text{अवय}}{\text{लम्ब}} = \frac{13k}{5k} = \frac{13}{5}$$

(Q81) यदि  $\cot A = 4$  तो ज्ञात कीजिए कि  $1 + \tan^2 A$

$$= 108^2 A - \cot^2 A.$$

$$\cot A = 4 = \frac{\text{आधार}}{\text{लम्ब}}$$

$$BC = 3, AB = 4$$



पांडुचरीसम प्रमेय एवं -

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$k^2 = (4)^2 + (3)^2$$

$$k = \sqrt{16+9}$$

$$k = \sqrt{25}$$

$$k = 5$$

$$\sin A = \frac{\text{पर्याप्त}}{\text{कर्ता}} = \frac{3}{5}, \quad 108A = \frac{\text{आधार}}{\text{कर्ता}} = \frac{4}{5}$$

$$\tan A = \frac{\text{पर्याप्त}}{\text{आधार}} = \frac{3}{4}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{108^2 A}{\sin^2 A}$$

$$\frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{16 - 9}{25}$$

$$\frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{16-9}{25}$$

$$\frac{7}{16} \times \frac{16}{25} = \frac{7}{25}$$

$$\frac{7}{25} = \frac{7}{25}$$

*Proved*

(Q2)  $\triangle ABC$  में भूमि की तुलना में जियाका कीठा भूमि कीठा है,  $AB = 94\text{ cm}$  और  $BC = 7\text{ cm}$  है। इसकी विधि का मान ज्ञात कीजिए।

$$\text{i) } \sin A, \text{ ii) } \sin C$$

$$\text{i) } \sin A, \text{ ii) } \sin C$$

$\triangle ABC$  में  $AB$  भूमि है।

पाठ्यालोक समीक्षा की-

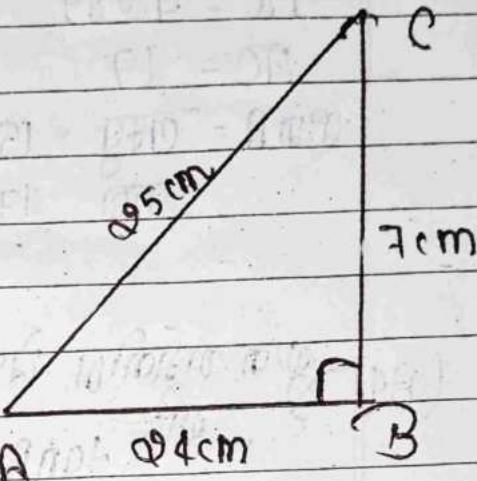
$$(AC)^2 = (AB)^2 + (BC)^2$$

$$k^2 = (94)^2 + (7)^2$$

$$k = \sqrt{576 + 49}$$

$$k = \sqrt{625}$$

$$k = 25$$



$$\text{i) } \sin A = \frac{\text{आधार}}{\text{तुलना}} = \frac{7}{25}, \quad \text{ii) } \sin A = \frac{\text{आधार}}{\text{तुलना}} = \frac{94}{25}$$

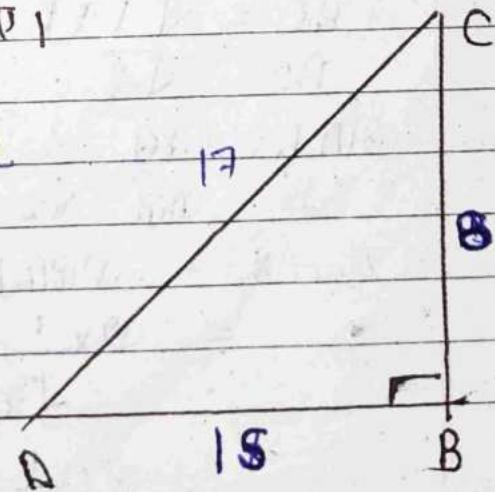
$$\text{ii) } \sin C = \frac{\text{तुलना}}{\text{आधार}} = \frac{94}{25}, \quad \text{iii) } \sin C = \frac{\text{तुलना}}{\text{आधार}} = \frac{7}{25}$$

(Q3) यदि  $15 \cot A = 8$  हो, तो  $\sin A$  और  $\sec A$  का मान ज्ञात कीजिए।

$$15 \cot A = 8$$

$$\cot A = \frac{8}{15} = \frac{\text{आधार}}{\text{तुलना}}$$

$$AB = 15, BC = 8$$



पाठ्यालोकन प्रमेय अ०-

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = 15^2 + 8^2$$

$$AC = \sqrt{225 + 64}$$

$$AC = \sqrt{289}$$

$$AC = 17$$

$$\sin A = \frac{\text{पर्याप्त}}{\text{कर्ता}} = \frac{15}{17}, \sec A = \frac{\text{अधिकारी}}{\text{आधार}} = \frac{17}{8}$$

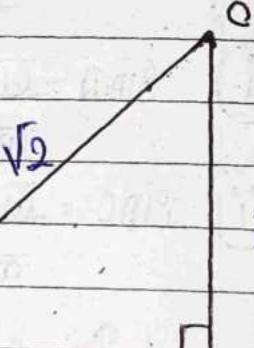
Ans

(84) एक व्यक्तिगत विभाजन में, विषयका कीठा B व्यक्तिगत  
है, यदि  $\tan A = 1$  तो व्यत्यापित कीजिए

$$\sin A \cos A = 1$$

$$\tan A = \frac{\text{पर्याप्त}}{\text{आधार}} = 1$$

$$AB = 1, BC = 1$$



पाठ्यालोकन प्रमेय अ०-

$$(AC)^2 = (BC)^2 + (AB)^2$$

$$(AC)^2 = (1)^2 + (1)^2$$

$$AC = \sqrt{1+1}$$

$$AC = \sqrt{2}$$

$$\sin A = \frac{\text{पर्याप्त}}{\text{कर्ता}} = \frac{1}{\sqrt{2}}, \cos A = \frac{\text{आधार}}{\text{कर्ता}} = \frac{1}{\sqrt{2}}$$

$$L.H.S = \sin A \cos A$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2}$$

Ans

(95)

$\triangle ABC$  में, भिन्न संकाय की तरफ  $B$  समाकोण है, तो  $\tan A = \frac{1}{\sqrt{3}}$  तो लिमाइमिप्रति के मान ज्ञात कीजिए।

$$(i) \sin A \cos C + \cos A \sin C \quad (ii) \cos A \cos C - \sin A \sin C$$

प्रश्न-

$$\tan A = 1 = \frac{\text{लम्ब}}{\text{धरा}}$$

 $\sqrt{3}$  धरा

$$AB = \sqrt{3}, BC = 1$$

पाठ्यालीकन प्रमेय की

$$(AC)^2 = (BC)^2 + (AB)^2$$

$$(AC)^2 = 1^2 + (\sqrt{3})^2$$

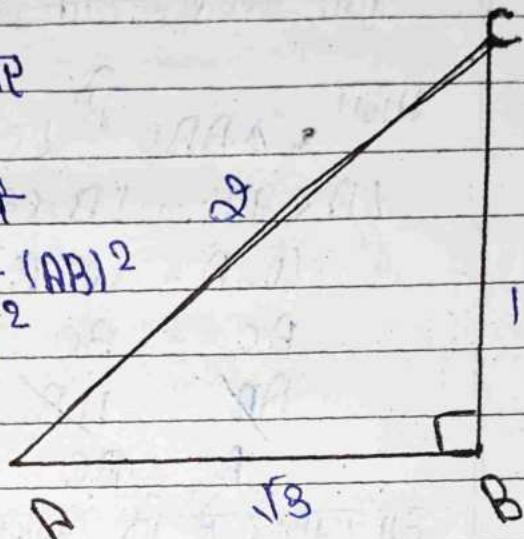
$$AC = \sqrt{1+3}$$

$$AC = \sqrt{4}$$

$$AC = 2$$

$$\sin A = \frac{\text{लम्ब}}{\text{कर्ति}} = \frac{1}{2}, \cos A = \frac{\text{धरा}}{\text{कर्ति}} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{लम्ब}}{\text{कर्ति}} = \frac{\sqrt{3}}{2}, \cos C = \frac{\text{धरा}}{\text{कर्ति}} = \frac{1}{2}$$



(i)

$$\sin A \cos C + \cos A \sin C$$

$$\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$\frac{1}{4} + \frac{3}{4}$$

~~Ans~~

(ii)

$$\cos A \cos C - \sin A \sin C$$

$$\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

~~Ans~~

(36)

यदि  $\angle A$  और  $\angle B$  व्युत्कील हैं, तो  
 $108A = 108B$ , तो इसका जी  $\angle A = \angle B$

$AC = BC$  [बाहर पूँछ के]

$AC = BC$  [बाहर की कोण के समान  
पूँछ बाहर होते हैं]  $\Rightarrow \angle A = \angle B$

माना  $\triangle ABC$  में  $\angle C = 90^\circ$

$\angle A < 90^\circ, \angle B < 90^\circ$

$$108A = 108B$$

$$\frac{AC}{AB} = \frac{BC}{AB}$$

$$AC = BC$$

इस बाणी है कि बाहर पूँछ के समान कोण बाहर होते हैं

$$\angle A = \angle B \quad \text{Proved}$$

(37)

$\triangle PQR$  में  $\angle Q$  घमकोण है  $\overline{PR+QR} = 25\text{cm}$  और  
 $PQ = 5\text{cm}$  तो  $108P, 108Q, 108R$  ज्ञात कीजिए।

$\triangle PQR$  में  $\angle Q$  घमकोण है।

पाठ्याग्रह समझ से -

$$(PR)^2 = (PQ)^2 + (QR)^2$$

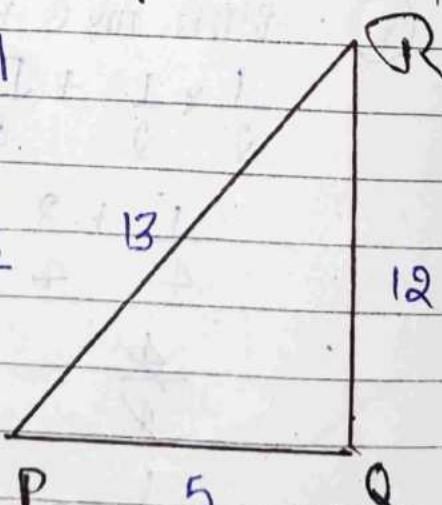
$$PR^2 = 5^2 + RQ^2$$

$$(PR)^2 - (RQ)^2 = 25$$

$$(PR+QR)(PR-QR) = 25$$

$$25(PR-QR) = 25$$

$$PR-QR = 1 \quad \text{--- (1)}$$



$$PR + QR = 25 \quad \text{--- (ii)}$$

मानी ① व ② से -

$$PR - QR = 1$$

$$\underline{PR + QR = 25}$$

$$2PR = 26$$

$$PR = \frac{26}{2}$$

$$PR = 13$$

PR का मान (ii) में

$$13 + QR = 25$$

$$QR = 25 - 13$$

$$QR = 12$$

$$\sin p = \frac{\text{पर्याप्त}}{\text{कर्ता}} = \frac{12}{13}$$

$$\cos p = \frac{\text{आधार}}{\text{कर्ता}} = \frac{5}{13}$$

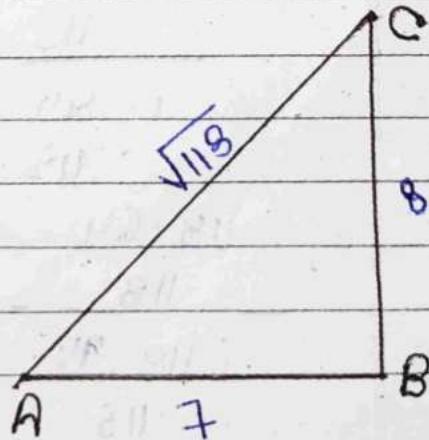
Answer

$$\tan p = \frac{\text{पर्याप्त}}{\text{आधार}} = \frac{12}{5}$$

(iii) यदि  $\cot \theta = \frac{7}{8}$  तो  $\frac{i}{i} \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$

(iv)  $\cot^2 \theta$  का मान

$$\cot \theta = \frac{\text{आधार}}{\text{पर्याप्त}} = \frac{7}{8}$$



पाठ्याग्रहण समीक्षा स्पै-

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = (7)^2 + (8)^2$$

$$K = \sqrt{49+64}$$

$$K = \sqrt{113}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{\sqrt{113}}, \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{7}{\sqrt{113}}$$

$$\textcircled{i} \quad \frac{(1+8\sin\theta)(1-8\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$\textcircled{n} \quad (1+\cos\theta) = \left(\frac{7}{8}\right)^2$$

$$\left(\frac{1+8}{\sqrt{113}}\right) \left(\frac{1-8}{\sqrt{113}}\right)$$

$$\left(\frac{1+7}{\sqrt{113}}\right) \left(\frac{1-7}{\sqrt{113}}\right)$$

Ans

$$11^2 - \left(\frac{8}{\sqrt{113}}\right)^2$$

$$11^2 - \left(\frac{7}{\sqrt{113}}\right)^2$$

$$\begin{aligned} & 1 - \frac{64}{113} \\ \therefore & \frac{113 - 64}{113} \\ & 113 - 64 \end{aligned}$$

$$\begin{aligned} \frac{113}{113-64} & \Rightarrow \frac{49}{113} \Rightarrow \frac{49 \times 113 - 49}{113 \cdot 64 - 64} \\ & \frac{64}{113} \end{aligned}$$

Ans

प्रश्नावली 8 (B)

(i)

$$\frac{\theta + \tan 30^\circ}{1 + \tan^2 30^\circ}$$

- (A)  $\sin 60^\circ$  (✓) (B)  $\cos 60^\circ$  (C)  $\tan 60^\circ$  (D)  $\sin 30^\circ$

हल-

$$\theta + \tan 30^\circ$$

$$1 + \tan^2 30^\circ$$

$$\Rightarrow \frac{\theta \times 1}{\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{\theta}{\sqrt{3}} + \frac{2 \times 3}{\sqrt{3} \times 2} = \frac{3}{2\sqrt{3}}$$

$$1 + \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1+1}{\sqrt{3}} = \frac{3+1}{3}$$

$$\Rightarrow \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3} = \frac{\sqrt{3}}{\frac{3}{2}} = \frac{2\sqrt{3}}{3} = \cos 60^\circ$$

Ans

(ii)

$$\theta - \tan 30^\circ$$

$$1 - \tan^2 30^\circ$$

- (A)  $\cos 60^\circ$  (B)  $\sin 60^\circ$  (C)  $\tan 60^\circ$  (D)  $\sin 30^\circ$

सूत्र-

$$\theta + \tan 30^\circ$$

$$1 - \tan^2 30^\circ$$

$$\Rightarrow \frac{\theta \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} = \frac{2\sqrt{3} \times 3}{2 \times 3} = \frac{3\sqrt{3}}{2}$$

$$\Rightarrow \frac{3\sqrt{3}}{2} = \tan 60^\circ$$

Ans

(iii)

$$1 - \tan^2 45^\circ$$

$$1 + \tan^2 45^\circ$$

- (A)  $\tan 90^\circ$  (B) 1 (C)  $8\sin 45^\circ$  (D) 0 (✓)

Ques:-

$$\frac{1 - (1)^2}{1 + (1)^2} \Rightarrow \frac{1 - 1}{1 + 1} \Rightarrow \frac{0}{2} = 0$$

Ans.

(iv)

$8\sin A = 8\sin A$  तब यात्य होता है, जबकि  $A$  वास्तव

- (A)  $0^\circ$  (✓) (B)  $30^\circ$  (C)  $45^\circ$  (D)  $60^\circ$

Sol:-

$$8\sin A \times 0^\circ = 8\sin 0^\circ$$

$$0 = 0$$

(v)

यदि  $\tan(A+B) = \sqrt{3}$  और  $\tan(A-B) = 1$ ,  
 $0^\circ < (A+B) < 90^\circ$ ,  $A > B$  तो  $A$  तथा  $\sqrt{3}$ '  
 $B$  का मान ज्ञात कीजिए।

$$\tan(A+B) = \sqrt{3}$$

$$\tan(A+B) = \tan 60^\circ$$

$$A+B = 60^\circ \quad \text{--- (i)}$$

$$\tan(A-B) = \frac{1}{\sqrt{3}}$$

$$\tan(A-B) = \tan 30^\circ$$

$$A-B = 30^\circ \quad \text{--- (ii)}$$

अमीर (i) व (ii) को समझे यह -

$$A+B = 60^\circ \quad \text{जाइन}$$

$$A - B = 30^\circ$$

$$A = 90^\circ$$

$$A = 45^\circ$$

A का मान ① में

$$45^\circ + B = 60^\circ$$

$$B = 60^\circ - 45^\circ$$

$$B = 15^\circ \quad \underline{\text{Ans}}$$

④ विकल्पित के मान जारी कीजिए-

$$\text{i) } \vartheta \tan^2 45^\circ + (\cos^2 30^\circ - \cos^2 60^\circ)$$

$$\Rightarrow \vartheta \times 1 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \vartheta \times 1 + \frac{3}{4} - \frac{3}{4}$$

$$\Rightarrow \vartheta \quad \underline{\text{Ans}}$$

$$\Rightarrow \sqrt{3} \times \frac{(\sqrt{6} - \sqrt{2})}{(\sqrt{6} + \sqrt{2}) (\sqrt{6} - \sqrt{2})}$$

ii)  $108.45^\circ$

$$\sec 30^\circ + (\cos \sec 30^\circ)$$

$$\Rightarrow \frac{1}{\sqrt{2}}$$

$$\frac{\vartheta + 2}{\sqrt{3}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}$$

$$\frac{\vartheta + \vartheta \sqrt{3}}{\sqrt{3}}$$

$$\sqrt{3}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{(\vartheta + 2\sqrt{3})}$$

$$\Rightarrow \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}}$$

$$\Rightarrow \sqrt{18 - \sqrt{6}}$$

$$\vartheta [(\sqrt{6})^2 - (\sqrt{2})^2]$$

$$\Rightarrow 3\sqrt{2} - \sqrt{6}$$

$$\vartheta (6 - 2)$$

$$\Rightarrow 3\sqrt{2} - \sqrt{6}$$

$$\vartheta \times 4$$

$$\Rightarrow \frac{3\sqrt{2} - \sqrt{6}}{8}$$

Answer

$$\text{III} \quad \frac{\cos 30^\circ + i \sin 45^\circ}{\cos 30^\circ + i \cos 60^\circ + i \sin 45^\circ}$$

$$= \frac{\frac{1}{2} + i - \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2} + i}$$

$$= \frac{\frac{1}{2} + i - \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2} + i}$$

$$= \frac{\frac{3}{2} - \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{3\sqrt{3} - 4}{2\sqrt{3}} \times \frac{2\sqrt{3}}{4 + 3\sqrt{3}}$$

$$= \frac{3\sqrt{3} - 4}{(4 + 3\sqrt{3})} \times \frac{(4 - 3\sqrt{3})}{(4 - 3\sqrt{3})}$$

$$= \frac{12\sqrt{3} - 9 \times 3 - 16 + 12\sqrt{3}}{(4)^2 - (3\sqrt{3})^2}$$

$$= \frac{24\sqrt{3} - 27 - 16}{16 - 27}$$

$$= 4\sqrt{3} - 43 - 11$$

$$= 4\sqrt{3} - 24\sqrt{3}$$

$$\cancel{\times 11} \quad \underline{\text{Ans}}$$

$$\text{IV} \quad 8 \sin 60^\circ (0830^\circ + 8 \sin 60^\circ)$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

Ans

$$\begin{aligned}
 & \textcircled{v} \quad 5 \cos^2 60^\circ + 4 \sin^2 30^\circ - 1 \cos^2 45^\circ \\
 & \quad \sin^2 30^\circ + \cos^2 30^\circ \\
 \Rightarrow & \quad 5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{1}{2}\right)^2 - 1 \times \left(\frac{1}{2}\right)^2 \\
 & \quad \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 \\
 \Rightarrow & \quad \frac{5 \times 1}{4} + \frac{4 \times 1}{4} - \frac{1}{4} \\
 & \quad \frac{1}{4} + \frac{3}{4} \\
 \Rightarrow & \quad \frac{5+16-1}{4} \\
 & \quad \cancel{4} \\
 \Rightarrow & \quad \frac{19+64-12}{12} \\
 \Rightarrow & \quad \frac{79-12}{12} \\
 \Rightarrow & \quad \frac{67}{12} \quad \underline{\text{Ans}}
 \end{aligned}$$

$$\begin{aligned}
 & \textcircled{5} \quad \text{सिल्ल कीप्रिया:-} \\
 & \cos^2 30^\circ + \sin^2 30^\circ = 1 \\
 \Rightarrow & \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1 \\
 \Rightarrow & \frac{3}{4} + \frac{1}{4} = 1 \\
 \Rightarrow & \frac{4}{4} = 1 \\
 & 1 = 1 \\
 & L.H.S = R.H.S
 \end{aligned}$$

$$\begin{aligned}
 & \textcircled{6} \quad \text{यदि } 0 = 45^\circ \text{ तो सिल्ल कीप्रिया } \sin 0 \cos 0 - \sin 20^\circ \\
 & \quad \cos 45^\circ \cos 45^\circ = \sin 2 45^\circ \\
 & \quad 0 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \sin 0 \times 45^\circ \\
 & \quad \cancel{0} \times \cancel{\frac{1}{\sqrt{2}}} \times \cancel{\frac{1}{\sqrt{2}}} = \sin 45^\circ \\
 & \quad \cancel{0} \times \cancel{\frac{1}{\sqrt{2}}} = \sin 45^\circ \\
 & \quad 1 = 1 \quad \underline{\text{Proved}}
 \end{aligned}$$

7 यदि  $\cos A = \frac{17}{15}$  तो मिहू कीजिए कि  $\sec A = ?$

$$\cos A = \frac{\text{लम्ब}}{\text{अधार}} = \frac{17}{15}$$

पाइथागोरस समीक्षा से -

$$(\text{कर्ण})^2 = (\text{आधा})^2 + (\text{लम्ब})^2$$

$$(17)^2 = A^2 + 15^2$$

$$289 = A^2 + 225$$

$$A^2 = 289 - 225$$

$$A = \sqrt{64}$$

$$A = 8$$

$$\sec A = \frac{\text{कर्ण}}{\text{आधा}} = \frac{17}{8}$$

आधार 8

Proved

8 जिम्न के मान बात कीजिए -

i)  $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

$$\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

0 Ans

ii)  $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

$$\frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$\frac{2\sqrt{3}}{4} \Rightarrow \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2}$$

Ans

9 जिम्नायिक के मान बात कीजिए ।

i)  $\tan 30^\circ \cdot \cot 60^\circ$

$$= \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3} \text{ Ans}$$

ii)  $\csc 90^\circ \cdot \sin 60^\circ \cdot \sin 45^\circ$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{4\sqrt{2}} = \frac{1}{4}\sqrt{\frac{3}{2}} \text{ Ans}$$

$\theta$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	0
$\cot \theta$	$\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\infty$
$\csc \theta$	$\infty$	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

## त्रिकोणमितीय वर्गसमीकरण (Trigonometrical Identities):-

जब या कठन ये आधिक दुष्प्रभावी समीकरणों की वर्गसमीकरण  
कहा जाता है तो वाकि एप्पी चरों के एप्पी मानों के सिए  
वर्गसमीकरण ही जाता है इसीलिए चरों के एप्पी मानों के  
भासीकरणों का लार्यों पक्ष और बायों पक्ष का  
मान बराबर हीता है।

$$(1) \quad \sin^2 \theta + \cos^2 \theta = 1$$

$$(2) \quad 1 + \tan^2 \theta = \sec^2 \theta$$

$$(3) \quad 1 + \cot^2 \theta = \csc^2 \theta$$

$$(4) \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$(5) \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$(6) \quad \cot \theta = \frac{1}{\tan \theta}$$

$$(7) \quad \tan \theta = \frac{1}{\cot \theta}$$

$$(8) \quad \sec \theta = \frac{1}{\cos \theta}$$

$$(9) \quad \csc \theta = \frac{1}{\sin \theta}$$

$$(10) \quad \sin^2 \theta = 1 - \cos^2 \theta$$

$$(11) \quad \cos^2 \theta = 1 - \sin^2 \theta$$

$$(12) \quad \tan^2 \theta = \sec^2 \theta - 1$$

$$(13) \quad \sec^2 \theta - \tan^2 \theta = 1$$

$$(14) \quad \cot^2 \theta = \csc^2 \theta - 1$$

$$(15) \quad \csc^2 \theta - \cot^2 \theta = 1$$

प्रश्नावली 8 (D)

① (i) गोणीयमितीय छविप्रती की ओर  $\cot A$  के पद में  $\sin A$ ,  $\sec A$  और  $\tan A$

$$(i) \sin A$$

$$\sin A = \frac{1}{\sec A}$$

$$= \frac{1}{0.8 \sec A}$$

$$[अवधिमिता] \quad 0.8 \sec^2 A - (\cot^2 A) = 1 \text{ है}$$

$$0.8 \sec^2 A = 1 + (\cot^2 A)$$

$$\sec^2 A = \sqrt{1 + (\cot^2 A)}$$

$$\sin A = \frac{1}{\sqrt{1 + (\cot^2 A)}} \quad \underline{\text{Ans}}$$

(ii)  $\sec A$

$$\sec A \text{ अवधिमिता} \quad \sec^2 A - \tan^2 A = 1 \text{ है} -$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\sec^2 A = 1 + \frac{1}{(\cot^2 A)}$$

$$\sec^2 A = (\cot^2 A + 1)$$

$$(\cot^2 A + 1)$$

$$\sec A = \sqrt{\frac{1 + (\cot^2 A)}{(\cot^2 A)}}$$

$$\sec A = \sqrt{1 + (\cot^2 A)}$$

$$1 + (\cot^2 A) \quad \underline{\text{Ans}}$$

(iii)  $\tan A$

$$\tan A = \frac{1}{\cot A} \quad \underline{\text{Ans}}$$

(ii) यदि अन्य सभी निम्नालिखीय प्रणाली की sec A  
के बर्णन में भिन्न हैं।

(i)  $\sin A$

$$\sin^2 A + \cos^2 A = 1$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\sin^2 A = 1 - 1$$

$$\sec^2 A$$

$$\sin^2 A = \frac{1}{\sec^2 A} - 1$$

$$\frac{1}{\sec^2 A}$$

$$\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

(iii)  $\cos A$

$$\cos^2 A = 1$$

$$\frac{1}{\sec^2 A}$$

(iv)  $\tan A$

$$\tan^2 A = \sec^2 A - 1$$

$$\tan A = \sqrt{\sec^2 A - 1}$$

(v)  $\cot A$

$$\cot^2 A = \frac{1}{\tan^2 A}$$

$$\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

(vi)  $\operatorname{cosec} A = \frac{1}{\sin A} \Rightarrow \frac{1}{\sqrt{\sec^2 A - 1}} \Rightarrow \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

$$\frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

(3)  $\sec^4 \theta - \tan^4 \theta = 1 + 2\tan^2 \theta$ .

$$L.H.S = (\sec^2 \theta)^2 - (\tan^2 \theta)^2$$

$$(\sec^2 \theta - \tan^2 \theta)(\sec^2 \theta + \tan^2 \theta) [a^2 - b^2]$$

$$1 \times (\sec^2 \theta + \tan^2 \theta) [\sec^2 \theta - \tan^2 \theta]$$

$$1 + \tan^2 \theta + \tan^2 \theta [1 + \tan^2 \theta = \sec^2 \theta]$$

$$1 + 2\tan^2 \theta = R.H.S$$

Proved

(4)  $1 + \cos 2\theta = (\cos \theta + \cot \theta)^2$

$$1 - \cos 2\theta$$

$$L.H.S = (1 + \cos 2\theta) \times \frac{1}{(1 - \cos 2\theta)}$$

$$(1 - \cos 2\theta) (1 + \cos 2\theta)$$

$$= (1 + \cos 2\theta)^2$$

$$(1)^2 - (\cos 2\theta)^2$$

$$= 1 + \cos^2 2\theta + 2\cos 2\theta \times 1$$

$$1 - \cos^2 2\theta$$

$$= 1 + \cos^2 2\theta + 2\cos 2\theta \times 1$$

$$\sin^2 2\theta$$

$$[1 - \cos^2 2\theta = \sin^2 2\theta]$$

$$= \frac{1}{\sin^2 2\theta} + \cos^2 2\theta + 2 \times \frac{1}{\sin 2\theta} \times \cos 2\theta$$

$$= \frac{1}{\sin^2 2\theta} + \frac{\cos^2 2\theta}{\sin^2 2\theta} + \frac{2 \times 1}{\sin 2\theta} \times \frac{\cos 2\theta}{\sin 2\theta}$$

$$= \sec^2 2\theta + \cot^2 2\theta + 2\sec 2\theta \cdot \cot 2\theta$$

$$= (\sec^2 2\theta + \cot^2 2\theta)^2 = R.H.S$$

Proved

(4)  $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \cos \theta) = 2$ .

$$L.H.S = \left( \frac{1 + \sin \theta + 1}{\cos \theta} \right) \left| \frac{1 + \cos \theta - 1}{\sin \theta} \right|$$

$$\Rightarrow \frac{(\cos\theta + 8\sin\theta + 1)}{\cos\theta} \cdot \frac{(\sin\theta + \cos\theta - 1)}{\sin\theta}$$

$$] \quad \sin\theta \cdot \cos\theta + 10\cos^2\theta - 10\cos\theta + 8\sin^2\theta + 8\sin\theta \cdot \cos\theta - 1 \\ \Rightarrow - \frac{\sin\theta \cdot (\cos\theta + 10\cos\theta - 1)}{\sin\theta \cdot \cos\theta}$$

$$\Rightarrow \frac{2\sin\theta \cdot \cos\theta + 10\cos^2\theta + 8\sin^2\theta - 1}{\sin\theta \cdot \cos\theta}$$

$$\Rightarrow \frac{2\sin\theta \cdot \cos\theta + 1 - 1}{\sin\theta \cdot \cos\theta} \quad [\sin^2\theta + \cos^2\theta = 1]$$

Proved

$$5) \quad \frac{\cos\theta}{1 + \sin\theta} = \frac{1 - 8\sin\theta}{\cos\theta} = \sec\theta - \tan\theta.$$

$$L.H.S = \frac{\cos\theta}{(1 + \sin\theta)} \times \frac{(1 - 8\sin\theta)}{(1 - 8\sin\theta)}$$

$$\Rightarrow \frac{\cos\theta (1 - 8\sin\theta)}{1 - 8\sin^2\theta}$$

$$\Rightarrow \frac{\cos\theta (1 - 8\sin\theta)}{\cos^2\theta} \quad [1 - 8\sin^2\theta = \cos^2\theta]$$

$$\Rightarrow \frac{1 - 8\sin\theta}{\cos\theta \cos\theta}$$

$$\Rightarrow \sec\theta - \tan\theta = R.H.S$$

Proved

$$6) \quad \frac{\tan\theta + 8\sin\theta}{\tan\theta - 8\sin\theta} = \frac{\sec\theta + 1}{\sec\theta - 1}$$

$$L.H.S = \frac{8\sin\theta + 8\sin\theta}{1080}$$

$$\frac{8\sin\theta}{1080} + \frac{8\sin\theta}{1080}$$

$$= \frac{\sin\theta}{1080} \left( \frac{1}{1080} + 1 \right)$$

$$\frac{\sin\theta}{1080} \left( 1 - 1 \right)$$

$$\Rightarrow \frac{\sec\theta + 1}{\sec\theta - 1} = R.H.S$$

Proved

$$(7) \quad (8\sec\theta - \tan\theta)^2 = \frac{1 - 8\sin\theta}{1 + 8\sin\theta}$$

$$L.H.S = 8\sec^2\theta + \tan^2\theta - 2\tan\theta \cdot \sec\theta$$

$$= \frac{1}{108^2\theta} + \frac{8\sin^2\theta}{108^2\theta} - \frac{2}{108\theta} \times \frac{8\sin\theta}{108\theta} \times 1$$

$$\Rightarrow \frac{1 + 8\sin^2\theta - 2\sin\theta}{108^2\theta}$$

$$= \frac{1 + 8\sin^2\theta - 2\sin\theta}{1 - 8\sin^2\theta} \quad [108^2\theta = 1 - 8\sin^2\theta]$$

$$= \frac{(1 - 8\sin\theta)^2}{(1)^2 - (8\sin\theta)^2}$$

$$= \frac{(1 - 8\sin\theta)^2}{(1 + 8\sin\theta)(1 - 8\sin\theta)}$$

$$\Rightarrow \frac{(1 - 8\sin\theta)^2}{(1 + 8\sin\theta)} = R.H.S$$

Proved

$$\textcircled{8} \quad (108A + 8\sec A)^2 + (\sin A + \cos \sec A)^2 = 7 + \tan^2 A +$$

(0t^2 A)

$$L.H.S = 108^2 A + 8\sec^2 A + 2(108A \cdot 8\sec A + 8\sin^2 A +$$

$$\cos \sec^2 A + 2\sin A \cdot \cos \sec A)$$

$$= ) 108^2 A + \tan^2 A + 1 + 2 \times \frac{1}{8\sec A} \times 8\sec A + 8\sin^2 A$$

$$+ 1 + 0t^2 A + 2 \times \frac{1}{8\sec A} \times 108\sec A$$

$$= ) 108^2 A + \tan^2 A + 1 + 2 + 8\sin^2 A + 1 + 0t^2 A + 2$$

$$= ) \tan^2 A + 0t^2 A + 6 + 8\sin^2 A + 108^2 A$$

$$= ) \tan^2 A + (0t^2 A + 6 + 1) [8\sin^2 A + 108^2 A = 1]$$

$$= ) 7 + \tan^2 A + 0t^2 A$$

~~Retained~~

$$\textcircled{9} \quad \frac{108A}{1 + \sin A} + \frac{1 + \sin A}{108A} = 0t8\sec A.$$

$$L.H.S \Rightarrow \frac{108^2 A + (1 + \sin A)^2}{(108A)(1 + \sin A)}$$

$$= ) \frac{108^2 A + 1 + 8\sin^2 A + 2 \times 8\sin A \times 1}{(108A)(1 + \sin A)}$$

$$= ) \frac{\sin^2 A + 108^2 A + 1 + 28\sin A}{(108A)(1 + \sin A)}$$

$$= ) \frac{1 + 1 + 28\sin A}{(108A)(1 + \sin A)} [8\sin^2 A + 108^2 A = 1]$$

$$= ) \frac{2 + 28\sin A}{(108A)(1 + \sin A)}$$

$$= ) \frac{2(1 + 14\sin A)}{(108A)(1 + \sin A)}$$

$$= ) \frac{2(1 + 14\sin A)}{(108A)(1 + \sin A)} \rightarrow \frac{2 \times 1}{108A} \rightarrow \frac{2}{108A}$$

~~Retained~~

$$\textcircled{16} \quad \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + 8\sec\theta \csc\theta.$$

$$\text{L.H.S.} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cos\theta}$$

$$= \frac{1}{\sin\theta \cos\theta}$$

$$= 1 - \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}$$

$$= 1 - \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta + \cos\theta}{\cos\theta \sin\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= 1 - \frac{\sin\theta \times \sin\theta}{\cos\theta (\sin\theta - \cos\theta)} + \frac{\cos\theta \times \cos\theta}{\sin\theta (\cos\theta - \sin\theta)}$$

$$= 1 - \frac{\sin^2\theta}{\cos\theta (\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta (\cos\theta - \sin\theta)}$$

$$= 1 - \frac{\sin^2\theta}{\cos\theta (\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta (\cos\theta - \sin\theta)}$$

$$= 1 - \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta (\sin\theta - \cos\theta)}$$

$$= 1 - \frac{1}{\sin\theta \cos\theta (\sin\theta - \cos\theta)}$$

$$= 1 - \frac{1}{\sin\theta \cos\theta (\sin\theta - \cos\theta)}$$

$$= 1 - \frac{\sin^2\theta - \cos^2\theta}{\sin\theta \cos\theta (\sin\theta - \cos\theta)}$$

$$= 1 - \frac{\sin^2\theta - \cos^2\theta}{\sin\theta \cos\theta (\sin\theta - \cos\theta)}$$

$$= 1 - \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{\sin\theta \cos\theta (\sin\theta - \cos\theta)}$$

$$= 1 - \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{\sin\theta \cos\theta (\sin\theta - \cos\theta)}$$

$$= 1 - \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{\sin\theta \cos\theta (\sin\theta - \cos\theta)}$$

$$\Rightarrow \frac{1}{108\theta \cdot \sin\theta} + \frac{8\sin\theta \cdot 108\theta}{108\theta \cdot \sin\theta}$$

$$= 1 \quad 8\sec\theta(108\theta + 1)$$

~~Proved~~

(ii)  $\left( \frac{-1+108\theta}{108\theta} \right) \left( \frac{-1-108\theta}{108\theta} \right) \cos\theta \sec^2\theta = 8\sec^2\theta.$

$$\text{L.H.S.} = \frac{(1+108\theta)(1-108\theta)}{108^2\theta} \times 108\sec^2\theta$$

$$= \frac{1-108^2\theta}{108^2\theta} \times 1$$

$$= \frac{\sin^2\theta \times 1}{108^2\theta} \quad [1 - \cos^2\theta = \sin^2\theta]$$

$$= \frac{1}{108^2\theta} = 1 \quad \text{Proved}$$

(19) (9)  $(8\sin\theta + 108\theta)^2 + (\sin\theta - 108\theta)^2 = 2.$

$$\text{L.H.S.} \rightarrow 8\sin^2\theta + 108^2\theta + 2 \cdot 8\sin\theta \cdot 108\theta + 8\sin^2\theta$$

$$+ 108^2\theta - 2 \cdot 8\sin\theta \cdot 108\theta$$

$$= 2 \cdot 8\sin^2\theta + 2 \cdot 108^2\theta$$

$$= 2(8\sin^2\theta + 108^2\theta)$$

$$= 2 \times 1$$

~~Proved~~

(b)  $(1-108\theta)(1+108\theta)(1+108^2\theta) = 1.$

$$\text{L.H.S.} = 1^2 - (108\theta)^2 \times 108\sec^2\theta$$

$$\Rightarrow 1 - 108^2\theta \times 108\sec^2\theta$$

$$= \frac{\sin^2\theta \times 1}{108^2\theta} \times 1$$

$$= 1 \quad \text{Proved}$$

$$(18) \quad (1 - \sin\theta)(1 + \sin\theta)(1 + \tan^2\theta) = 1.$$

$$\begin{aligned} L.H.S. &= (1^2 - 1\sin\theta)^2 \times \sec^2\theta \quad [(a+b)(a-b)=a^2-b^2] \\ &= 1 - \sin^2\theta \times \sec^2\theta \\ &= \cos^2\theta \times \sec^2\theta \quad [1 - \sin^2\theta = \cos^2\theta] \\ &= \frac{1}{\cos^2\theta} \times \sec^2\theta \\ &= 1 \quad \text{Proved} \end{aligned}$$

$$(14) \quad \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A}$$

$$\begin{aligned} L.H.S. &= (1 - \cos A) \times \frac{1 + \cos A}{\sin A} \\ &= \frac{(1)^2 - \cos^2 A}{\sin A} \quad [(a+b)(a-b)=a^2-b^2] \\ &= \frac{\sin^2 A}{\sin A (1 + \cos A)} \\ &= \frac{\sin^2 A}{1 + \cos A} \quad [1 - \cos^2 A = \sin^2 A] \\ &= \frac{\sin A}{1 + \cos A} \quad \text{Proved} \end{aligned}$$

$$(15) \quad (\cos\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$$

$$L.H.S. = \left( \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} \right)^2$$

$$= \frac{(1 - \cos\theta)^2}{\sin^2\theta}$$

$$\Rightarrow \frac{(1 - \cos\theta)(1 + \cos\theta)}{1 - \cos^2\theta}$$

$$= \frac{(1 - \cos\theta)(1 - \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)} = \frac{1 - \cos\theta}{1 + \cos\theta}$$

$$\text{Proved}$$

(16)

$$\frac{1-8\sin A}{1+\sin A} = \sec A - \tan A$$

$$\text{L.H.S.} \rightarrow \sqrt{\frac{(1-8\sin A) \times (1-8\sin A)}{(1+\sin A) \times (1-8\sin A)}}$$

$$= \sqrt{\frac{(1-8\sin A)^2}{1^2 - (8\sin A)^2}}$$

$$= \sqrt{\frac{(1-8\sin A)^2}{1-8\sin^2 A}}$$

$$= \sqrt{\frac{(1-8\sin A)^2}{\cos^2 A}} [1-8\sin^2 A = \cos^2 A]$$

$$= \frac{1-8\sin A}{\cos A} = \frac{1-\cos A}{\cos A} = 8\sec A - \tan A$$

*(Proved)*

(17)

$$\frac{1+8\sec A}{\sec A} = \sin^2 A$$

$$\sec A = \frac{1-\cos A}{\cos A}$$

$$\text{L.H.S.} \rightarrow \frac{1+1}{\cos A}$$

$$\rightarrow \frac{(\cos A + 1) \times (\cos A - 1)}{\cos A}$$

$$+ \quad + (\cos A - 1)$$

$$\rightarrow \cos^2 A - 1$$

$$\rightarrow (1 + \cos A) \times (1 - \cos A)$$

$$\rightarrow \frac{(\cos A + 1)}{\cos A}$$

$$\rightarrow \frac{1 - \cos^2 A}{\cos A}$$

$$\rightarrow \frac{1}{1 - \cos A}$$

$$\frac{1}{\cos A}$$

$$\rightarrow \sin^2 A [1 - \cos^2 A = \sin^2 A]$$

$$\rightarrow \frac{(\cos A + 1) \times (\cos A)}{(\cos A)^2}$$

(18)

परिस्थिति)  $108\cot^2 A = 1 + 10t^2 A$  की भावुकार्ये

$$108A - 8\sin A + 1 = 108\sec A + 10t A$$

$$\cot A + 8\sin A - 1$$

L.H.S.  $\Rightarrow$  अंग आए हुए के सत्यक पर्याप्ति में  $8\sin A$  की भावुकार्ये

$$\Rightarrow \frac{108A - 8\sin A + 1}{8\sin A} \\ \frac{108A + 8\sin A - 1}{8\sin A - 8\sin A}$$

$$\Rightarrow \frac{\cot A - 1 + 108\sec A}{\cot A + 1 - 108\sec A} \quad [108A - 10t A, 1 = 108]$$

$$\Rightarrow \frac{(\cot A + 108\sec A) - 1}{(\cot A - 108\sec A) + 1}$$

$$\Rightarrow \frac{(\cot A + 108\sec A) - (108\sec^2 A - 10t^2 A)}{10t A + 1 - 108\sec A}$$

$$\Rightarrow \frac{(\cot A + 108\sec A) - (\cot A + 10t A) 108\sec A - 10t A}{\cot A + 1 - 108\sec A}$$

$$\Rightarrow \frac{(\cot A + 108\sec A) (1 - 108\sec A) - (\cot A)}{1 - 108\sec A + (\cot A)}$$

$$\Rightarrow \cot A + \cot A$$

Proved

(19)

$$\sin\theta - \cos\theta \sin^2\theta = \tan\theta.$$

$$\cos\theta \cos^2\theta - \cos\theta$$

$$L.H.S \Rightarrow \frac{\sin\theta (1 - \cos\theta \sin^2\theta)}{\cos\theta (\cos^2\theta - 1)}$$

$$\frac{\sin\theta (1 - \cos\theta \sin^2\theta)}{\cos\theta (\cos^2\theta - 1)}$$

$$\Rightarrow \frac{\sin\theta (1 - \cos\theta \sin^2\theta)}{\cos\theta (1 - \cos^2\theta - 1)}$$

$$(1 - \cos^2\theta) [1 - \cos\theta \sin^2\theta]$$

$$\Rightarrow \frac{\sin\theta (1 - \cos\theta \sin^2\theta)}{\cos\theta (-\sin^2\theta - 1)}$$

$$(1 - \cos\theta \sin^2\theta)$$

$$\Rightarrow \frac{\sin\theta (1 - \cos\theta \sin^2\theta)}{(1 - \cos\theta \sin^2\theta)}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

~~Proved~~

(20)

$$(\sec A - \sin A), (\sec A - \cos A) = 1$$

$$-\tan A + \cot A$$

$$L.H.S \Rightarrow \left( \frac{1 - \sin A}{\sin A} \right) \left( \frac{1 - \cos A}{\cos A} \right)$$

$$\Rightarrow \left( \frac{1 - \sin 2A}{\sin A} \right) \left( \frac{1 - \cos 2A}{\cos A} \right)$$

$$\Rightarrow \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} \quad [1 - \sin^2 A = \cos^2 A] \\ \qquad \qquad \qquad [1 - \cos^2 A = \sin^2 A]$$

$$= \cos A \cdot \sin A$$

$$R.H.S \Rightarrow \frac{1}{\tan A + \cot A}$$

$$\Rightarrow \frac{1}{\sin A + \cos A}$$

$$\frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A}$$

$$\Rightarrow \frac{1}{\sin^2 A + \cos^2 A}$$

$$\frac{1}{\cos A \cdot \sin A}$$

$$\Rightarrow \frac{1}{\frac{1}{\cos A \cdot \sin A}} \quad [\sin^2 \theta + \cos^2 \theta = 1]$$

$$\Rightarrow 1 \times \cos A \cdot \sin A$$

$$\Rightarrow \cos A \cdot \sin A \quad [L.H.S = R.H.S]$$

Proved

$$(Q1) \quad \frac{1 + \tan^2 A}{1 + \cot^2 A} = \left( \frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A.$$

$$L.H.S = \frac{1 + \tan^2 A}{1 + \cot^2 A}$$

$$\Rightarrow \frac{1 + \tan^2 A}{1 + \frac{1}{\tan^2 A}} \quad \left[ \because \cot A = \frac{1}{\tan A} \right]$$

$$\Rightarrow \frac{1 + \tan^2 A}{\tan^2 A + 1}$$

$$= 1 + \tan^2 A \times \frac{\tan^2 A}{1 + \tan^2 A}$$

$$\Rightarrow \tan^2 A = \text{R.H.S} - \textcircled{1}$$

$$\text{L.H.S.} = \left( \frac{1 - \tan A}{1 + \tan A} \right)^2$$

$$= \left( \frac{1 - \tan A}{1 + \tan A} \right)^2$$

$$= \left( \frac{1 - \tan A}{\tan A - 1} \right)^2$$

$$= \left( \frac{1 - \tan A}{-(1 - \tan A)} \right)^2$$

$$= \left[ \frac{(1 - \tan A)}{1} \times \frac{-1}{(1 - \tan A)} \right]^2$$

$$= (- \tan A)^2 = \tan^2 A = \text{R.H.S.} = \textcircled{1}$$

अतः (1) व (11) समान

L.H.S. = R.H.S. (Proved)

(22)

$$\text{L.H.S.} = (\sec A + \tan A)(1 - \sin A) = 108A.$$

$$= \left( \frac{1 + \sin A}{\cos A} \right) (1 - \sin A)$$

$$= \left( \frac{1 + \sin A}{108A} \right) (1 - \sin A)$$

$$= \frac{1 - \sin A + \sin A - \sin^2 A}{108A}$$

$$= \frac{1 - \sin^2 A}{108A}$$

$$\Rightarrow \frac{\cos^2 A}{\sin A} \rightarrow \cos A = R.H.S$$

Proved

(Q9)

$$\frac{1 - \tan A}{\sec A + \tan A} = \frac{1 - \frac{1}{\cos A}}{\sec A + \frac{1}{\cos A}}$$

$$\frac{1 - \frac{1}{\cos A}}{1 - \frac{1}{\cos A}} = \frac{1 - \frac{1}{\cos A}}{1 + \frac{1}{\cos A}}$$

$$\frac{1 - \frac{1}{\cos A}}{1 - \frac{1}{\cos A}} = \frac{1 - \frac{1}{\cos A}}{1 + \frac{1}{\cos A}}$$

$$\frac{(1 - \frac{1}{\cos A}) - \frac{1}{\cos A}}{1 - \frac{1}{\cos A} + \frac{1}{\cos A}} = \frac{1 - \frac{1}{\cos A}}{1 + \frac{1}{\cos A}}$$

$$\frac{(1 - \frac{1}{\cos A}) - \frac{1}{\cos A}}{1 - \frac{1}{\cos A} + \frac{1}{\cos A}} = \frac{1 - \frac{1}{\cos A}}{1 + \frac{1}{\cos A}}$$

$$\frac{(1 - \frac{1}{\cos A}) - \frac{1}{\cos A}}{1 - \frac{1}{\cos A} + \frac{1}{\cos A}} = \frac{1 - \frac{1}{\cos A}}{1 + \frac{1}{\cos A}}$$

$$(1 - \frac{1}{\cos A}) = 1 + \frac{1}{\cos A} - (1 - \frac{1}{\cos A})$$

$$\frac{1 - \frac{1}{\cos A}}{(1 - \frac{1}{\cos A})} = \frac{1 + \frac{1}{\cos A} - (1 - \frac{1}{\cos A})}{(1 + \frac{1}{\cos A})}$$

$$\cancel{1 - \frac{1}{\cos A}} - \cancel{1 + \frac{1}{\cos A}} = \cancel{1 - \frac{1}{\cos A}} - \cancel{1 + \frac{1}{\cos A}}$$

$$\frac{\cancel{1 - \frac{1}{\cos A}}}{(1 - \frac{1}{\cos A})} = \frac{\cancel{1 + \frac{1}{\cos A}}}{(1 + \frac{1}{\cos A})}$$

$$\tan A = \frac{1}{\cos A}$$

$$L.H.S = \frac{1}{\cos A}$$

Proved

(Q4)

$$\frac{1}{108\operatorname{cosec} A - 108A} = \frac{1+108A}{8\sin A}$$

$$\text{L.H.S.} \Rightarrow \frac{1}{\frac{1}{8\sin A} - \frac{108A}{8\sin A}}$$

$$= 1 - \frac{1}{\frac{1-108A}{8\sin A}}$$

$$= 1 - \frac{8\sin A \times (1+108A)}{(1-108A)(1+108A)}$$

$$= 1 - \frac{8\sin A (1+108A)}{1-108^2 A}$$

$$\Rightarrow \frac{8\sin A}{8\sin^2 A} (1-108^2 A)$$

$$[1-108^2 A = \sin^2 A]$$

$$= 1 + \frac{108A}{\sin A}$$

$$\Rightarrow \frac{1+108A}{8\sin A}$$

Proved

(Q5)

$$\frac{1}{1-108A} + \frac{1}{1+108A} = \operatorname{cosec}^2 A$$

$$\text{L.H.S.} = \frac{1+108A + 1-108A}{(1-108A)(1+108A)}$$

$$= 1 - \frac{108}{1-108^2 A} = 1 - \frac{108}{\sin^2 A} = \operatorname{cosec}^2 A$$

Banned

## प्रश्नावली 8 (E)

अद्युतिकृतीय प्रश्न -  
यदि  $\sin \theta = \frac{3}{5}$  तो  $\tan \theta$  का मान है:-

①

- (i)  $\frac{3}{4}$  (v)  $\frac{4}{3}$  (ii)  $\frac{4}{5}$  (iii)  $\frac{3}{5}$  (iv)  $\frac{3}{5}$

हमें  $\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$

$\triangle ABC$  में पार्श्वान्तरीय प्रमेय है -

$$(\text{कर्ण})^2 = (\text{आधा})^2 + (\text{भूज})^2$$

$$(5)^2 = BC^2 + (3)^2$$

$$25 = BC^2 + 9$$

$$BC^2 = 25 - 9$$

$$BC = \sqrt{16} = 4$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

②

$\sin 15^\circ$  का मान है:-

108750

- (i) से कम (ii) 1 (v) (iii) से अधिक (iv) 10

हमें -

$$\Rightarrow \sin(90^\circ - 15^\circ)$$

108750

$$\Rightarrow \frac{10875^\circ}{10875^\circ} = 1$$

(Q3.)  $\sin 9^{\circ}$  का मान है:

$$(\text{i}) \sin 81^{\circ}$$

- (i) ये अस्तित्व (ii) 1 (iii) पर्याप्त गह (iv) 0.

$$\Rightarrow \frac{\sin(90^{\circ} - 81^{\circ})}{\sin 81^{\circ}} = \frac{\cos 81^{\circ}}{\sin 81^{\circ}} = 1$$

(4)  $\sin^2 \theta \cdot \sec^2 \theta$  का मान है

- (i) 10820 (ii) 10720 (iii) 10720 (iv) 10820