AMATH 582: HOME WORK 1

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Abstract. In this assignment, an acoustic pressure data is used to detect the target submarine under the sea. A 3d dataset with an half-hour increment over a 24 hours period is given. We are going to find the trajectory of the submarine through different methods including Time-averaging FFT, Noise-elimination, Gaussian filter.

1. Introduction and Overview

Our goal in this homework is to locate a submarine in the Puget Sound using noisy acoustic data. We do not know much about this submarine as it is a new technology that emits an unknown acoustic frequency that our need to detect. Unfortunately the submarine is moving so its location and path need to be determined. Broad spectrum recording of acoustics data obtained over 24 hours in half-hour increments is available to us. Below is a list of tasks to complete.

(1) Through averaging of the Fourier transform and visual inspection, determine the frequency signature (center frequency) generated by the submarine.
(2) Design and implement a Filter to extract this frequency signature in order to denoise the data anddetermine the path of the submarine. You can also plot the 3D path of the submarine if you like.
(3) Determine and plot the path of submarine during the 24 hour period. This information can be used to deploy a sub-tracking aircraft to keep an eye on your submarine in the future

2. Theoretical Background

2.1. Fourier Transform and FFT.

Given $f : \mathbb{R} \to \mathbb{R}$ we define its Fourier Transform

$$\mathcal{F}(f) \equiv \hat{f}(k) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx}\, dx$$

inverse FT for $g : \mathbb{R} \to \mathbb{C}$

$$\mathcal{F}^{-1}(g) \equiv \hat{g}(x) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(k) e^{ikx}\, dk$$

The fact that $\mathcal{F}^{-1}(\mathcal{F}(f)) = f$ is a consequence of the Fourier integral theorem. Fourier Transform gives us the ability to translate the function into frequency domain. It then comes in handy to compute different tasks through utilizing the Fourier properties such as Linearity, Shift/Translation, Differentiation, Integration, Convolution, etc,.

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The FFT is a modern strategy for scientists to compute the Fourier Transform of a function in a speedy way $O(NlogN)$. FFT is widely used for applications in engineering, music, science, and mathematics. In this assignment, the computation is based on 'fftn' function in Python Numpy's fft library, which is used to calculate Fourier Transform in 3D space. Most modern FFT algorithms include a shift in frequency coordinate, so a 'fftshift' function will be used to restore the shift.

2.2. **Noise-elimination.**

It is known that adding mean zero white noise to a signal is equivalent to adding mean zero white noise to its Fourier series coefficients. This fact enables one to reduce the noise by averaging measurements in the Fourier domain. The more measurements one has the better the effect of this averaging.

2.3. **Gaussian filter.**

The idea is applying a shifted Gaussian filter to the Fourier signal. The filtered Fourier signal eliminates the frequencies far away from its center frequency, and thus highlights the frequencies only near the center frequency. The Gaussian Filter in higher dimension is given by

$$F(\mathbf{k}) := e^{(-\tau||\mathbf{k}-\mathbf{k_0}||^2)}$$

with $\mathbf{k}$, $\mathbf{k_0}$ vectors.

3. Algorithm Implementation and Development

The procedure for solving the tasks are given below, separately.

(1) First calculate the fft of the original signal in 3D. Averaging by time-step of 1 to get rid of the mean 0 white noise. The frequency signature generated by submarine is then plotted in 3D frequency space.
(2) A Gaussian filter is designed in a way that picks the maximum absolute value of the frequency signature as its center. The Gaussian filter is applied to ffts of the original signal to de-noise the data. Regenerate the signal by ifft of the Fourier Transform. The path is determined by picking the max value of clean signal in each time-step.
(3) The path on x-y plane is plotted in the same manner as before, but projected on x-y plane. This will be helpful to track the submarine by deploying a sub-tracking aircraft.

4. Computational Results

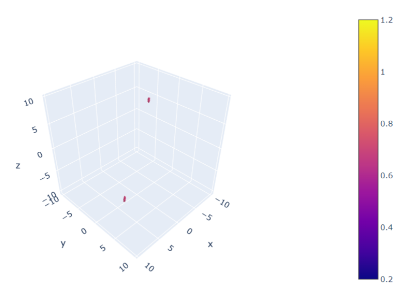
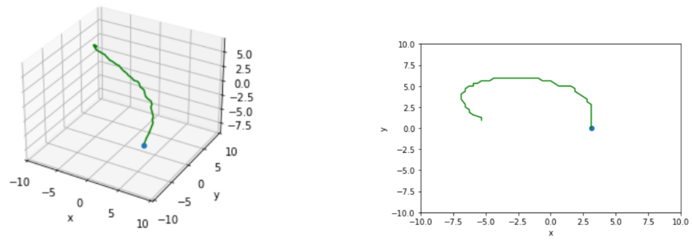


Figure 1. Frequency signature of the submarine in Fourier domain.

The solutions of the three tasks are shown here. The frequency signature is plotted by plotly's isosurface function. We can see there are two area colored in red, because the absolute value of FFT frequency is symmetric against the origin.1

The path of the submarine in 3-D and the path in x-y plane are shown below.2a



(A) path in 3D

(B) path in x-y plane

5. Summary and Conclusions

In order to locate the submarine, there are few steps most importantly to obtain the results. These steps include calculating the frequency signal using FFT, de-noising the data, finding the frequency signature, applying suitable Gaussian filter, and plotting the path of the submarine.

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