

template

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# Variable Separable ODE

## Problem 1

Solve  $(1+x)dy - ydx = 0$

**Solution:** Dividing by  $(1+x)y$ , we get

$$\frac{dy}{y} = \frac{dx}{1+x}$$

from which it follows that

$$\begin{aligned}\int \frac{dy}{y} &= \int \frac{dx}{1+x} \\ \ln |y| &= \ln |1+x| + c \\ e^{\ln |y|} &= e^{\ln |1+x| + c} \\ y &= e^c(1+x)\end{aligned}$$

Relabeling the  $e^c$  as  $c$  then gives us

$$y = c(1+x)$$

**Answer:**  $y = c(1+x)$

## Problem 2

Solve  $\frac{dy}{dx} = -\frac{x}{y}$

**Solution:** Multiplying  $ydx$ , we get

$$ydy = -x dx$$

and integrating both sides gives us

$$\begin{aligned}\int ydy &= \int -x dx \\ \frac{y^2}{2} &= -\frac{x^2}{2} + c\end{aligned}$$

Solving for  $y$  gives us

$$y = \pm \sqrt{-x^2 + 2c}$$

**Answer:**  $y = \pm \sqrt{-x^2 + 2c}$

### Problem 3

Solve  $\frac{dy}{dx} = e^{3x+2y}$

**Solution:** Rewriting  $e^{3x+2y}$  as  $e^{3x}e^{2y}$ , we get

$$\frac{dy}{dx} = e^{3x}e^{2y}$$

Dividing by  $e^{2y}dx$  we get

$$\frac{dy}{e^{2y}} = e^{3x}dx$$

and integrating both sides gives us

$$\begin{aligned}\int \frac{dy}{e^{2y}} &= \int e^{3x}dx \\ -\frac{1}{2}e^{-2y} &= \frac{1}{3}e^{3x} + c \\ 3e^{-2y} &= -2e^{3x} + c\end{aligned}$$

**Answer:**  $3e^{-2y} = -2e^{3x} + c$

# Homogeneous ODE

## Problem 4

Solve  $2x^3ydx + (x^4 + y^4)dy = 0$

**Solution:** Each coefficient is a homogeneous function of degree four, so we can let  $x = vy$  and substitute to get

$$\begin{aligned}2(vy)^3y d(vy) + ((vy)^4 + y^4)dy &= 0 \\2v^3y^4(vdy + ydv) + (v^4y^4 + y^4)dy &= 0 \\2v^4y^4dy + 2v^3y^5dv + v^4y^4dy + y^4dy &= 0 \\3v^4y^4dy + 2v^3y^5dv + y^4dy &= 0 \\y^4(3v^4 + 1)dy + 2v^3y^5dv &= 0\end{aligned}$$

Now we can separate the variables and integrate to get

$$\begin{aligned}\int \frac{dy}{y} + \int \frac{2v^3}{3v^4 + 1} dv &= \int 0 \\ \ln |y| + \frac{1}{6} \ln |3v^4 + 1| &= c \\ 6 \ln |y| + \ln |3v^4 + 1| &= c \\ e^{6 \ln |y|} + e^{\ln |3v^4 + 1|} &= e^c \\ y^6 + (3v^4 + 1) &= c \\ y^6 + \frac{3x^4}{y^4} + 1 &= c \\ y^6 + \frac{3x^4}{y^4} &= c - 1\end{aligned}$$

Rewriting  $c - 1$  as  $c$ , we get

$$y^6 + \frac{3x^4}{y^4} = c$$

**Answer:**  $y^6 + \frac{3x^4}{y^4} = c$

$$3x^4y^2 + y^6 = c$$

**Problem 5**

Solve  $(x^2 + y^2)dx = (x^2 - xy)dy = 0$

**Problem 6**

Solve  $(x - y)dx + xdy = 0$