ROBÓTICA E AUTOMAÇÃO

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Questão 1

Letra a

$$\theta_{sd} = 0$$

$$p_{sd}^T = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$R_{sd} = \begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$T_{sd} = \begin{bmatrix} R_{sd} & p_{sd} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Letra b

$$\theta_{dc} = \pi$$

$$p_{dc}^T = \begin{bmatrix} 3 & 1 \end{bmatrix}$$

$$R_{dc} = \begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$T_{dc} = \begin{bmatrix} R_{dc} & p_{dc} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Letra c

$$\theta_{sd^{\backprime}} = \pi/2$$

$$p_{sd^{\backprime}}^T = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

$$R_{sd^{\backprime}} = \begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$T_{sd'} = \begin{bmatrix} R_{sd'} & p_{sd'} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Letra d

$$\theta_{sc'} = 3\pi/2$$

$$p_{sc'}^T = \begin{bmatrix} 1 & 5 \end{bmatrix}$$

$$R_{sc'} = \begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$T_{sd'} = \begin{bmatrix} R_{sc'} & p_{sc'} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Letra e

$$T_{sc'} = T_{sd'} T_{d'c'}$$

$$T_{d'c'} = T_{dc}$$

$$T_{sc'} = T_{sd'} T_{dc}$$

$$T_{sc'} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Questão 2

Letra a

$$w = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix}$$

$$\widehat{w} = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix}$$

Listing 1: Código no Matlab para calcular os autovalores de $\widehat{w}$.

```
1 w = [ sym('w1'); sym('w2'); sym('w3') ]
2
3 w_chapeu = [0 -w(3) w(2); w(3) 0 -w(1); -w(2) w(1) 0]
4
5 [v, d] = eig(w_chapeu)
```

$$v = \begin{bmatrix} \dfrac{w_1}{w_3} & \dfrac{-jw_2 - w_1w_3}{w_1^2 + w_2^2} & \dfrac{jw_2 + w_1w_3}{w_1^2 + w_2^2} \\ \dfrac{w_2}{w_3} & \dfrac{jw_1 - w_2w_3}{w_1^2 + w_2^2} & \dfrac{-jw_1 + w_2w_3}{w_1^2 + w_2^2} \\ 1 & 1 & 1 \end{bmatrix}$$

$$d = \lambda = \begin{bmatrix} 1 \\ \sqrt{-w_1^2 - w_2^2 - w_3^2} \\ -\sqrt{-w_1^2 - w_2^2 - w_3^2} \end{bmatrix}$$

$$\sqrt{-w_1^2 - w_2^2 - w_3^2} = \sqrt{-(w_1^2 + w_2^2 + w_3^2)} = \sqrt{-1} = j$$

$$\lambda = \begin{bmatrix} 1 \\ j \\ -j \end{bmatrix}$$

Para $\lambda = 1$:   Para $\lambda = j$:   Para $\lambda = -j$:

$$v = \begin{bmatrix} \dfrac{w_1}{w_3} \\ \dfrac{w_2}{w_3} \\ 1 \end{bmatrix} \qquad v = \begin{bmatrix} \dfrac{-jw_2 - w_1w_3}{w_1^2 + w_2^2} \\ \dfrac{jw_1 - w_2w_3}{w_1^2 + w_2^2} \\ 1 \end{bmatrix} \qquad v = \begin{bmatrix} \dfrac{jw_2 + w_1w_3}{w_1^2 + w_2^2} \\ \dfrac{-jw_1 + w_2w_3}{w_1^2 + w_2^2} \\ 1 \end{bmatrix}$$

Letra b

$$R = e^{\widehat{w}\theta}$$

Listing 2: Código no Matlab para calcular os autovalores de R.

```
1 w = [ sym('w1'); sym('w2'); sym('w3') ]
2
3 w_chapeu = [0 -w(3) w(2); w(3) 0 -w(1); -w(2) w(1) 0]
4
5 R = expm(w_chapeu * sym('theta'))
6
7 e = eig(R)
```

$$e = \lambda = \begin{bmatrix} 1 \\ e^{\theta\sqrt{-w_1^2-w_2^2-w_3^2}} \\ e^{-\theta\sqrt{-w_1^2-w_2^2-w_3^2}} \end{bmatrix}$$

$$\lambda = \begin{bmatrix} 1 \\ e^{\theta j} \\ e^{-\theta j} \end{bmatrix}$$

Questão 3

Listing 3: Código no Matlab para calcular a matriz de rotação R.

```
1  w = [ (3^(1/2))/3; -(3^(1/2))/3; (3^(1/2))/3 ]
2
3  w_chapeu = [0 -w(3) w(2); w(3) 0 -w(1); -w(2) w(1) 0]
4
5  theta = 4*pi/6
6
7  R = expm(w_chapeu * theta)
```

$$R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Questão 4

Questão 5

Questão 6

Questão 7

Questão 8

Questão 9

Questão 10