Feedback Control II

lecture 3

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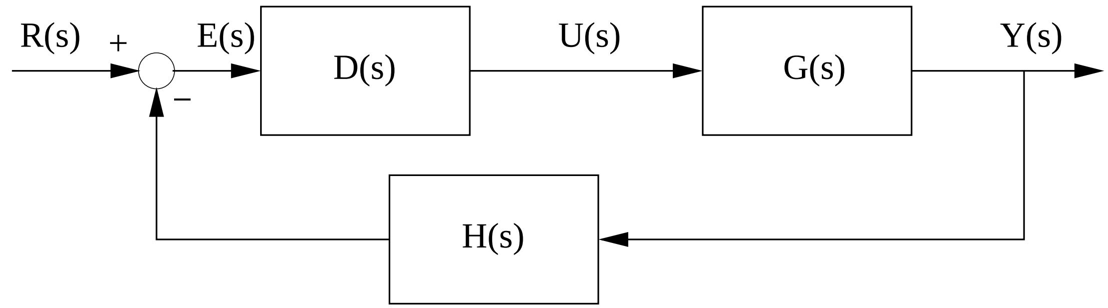
Standard set-up



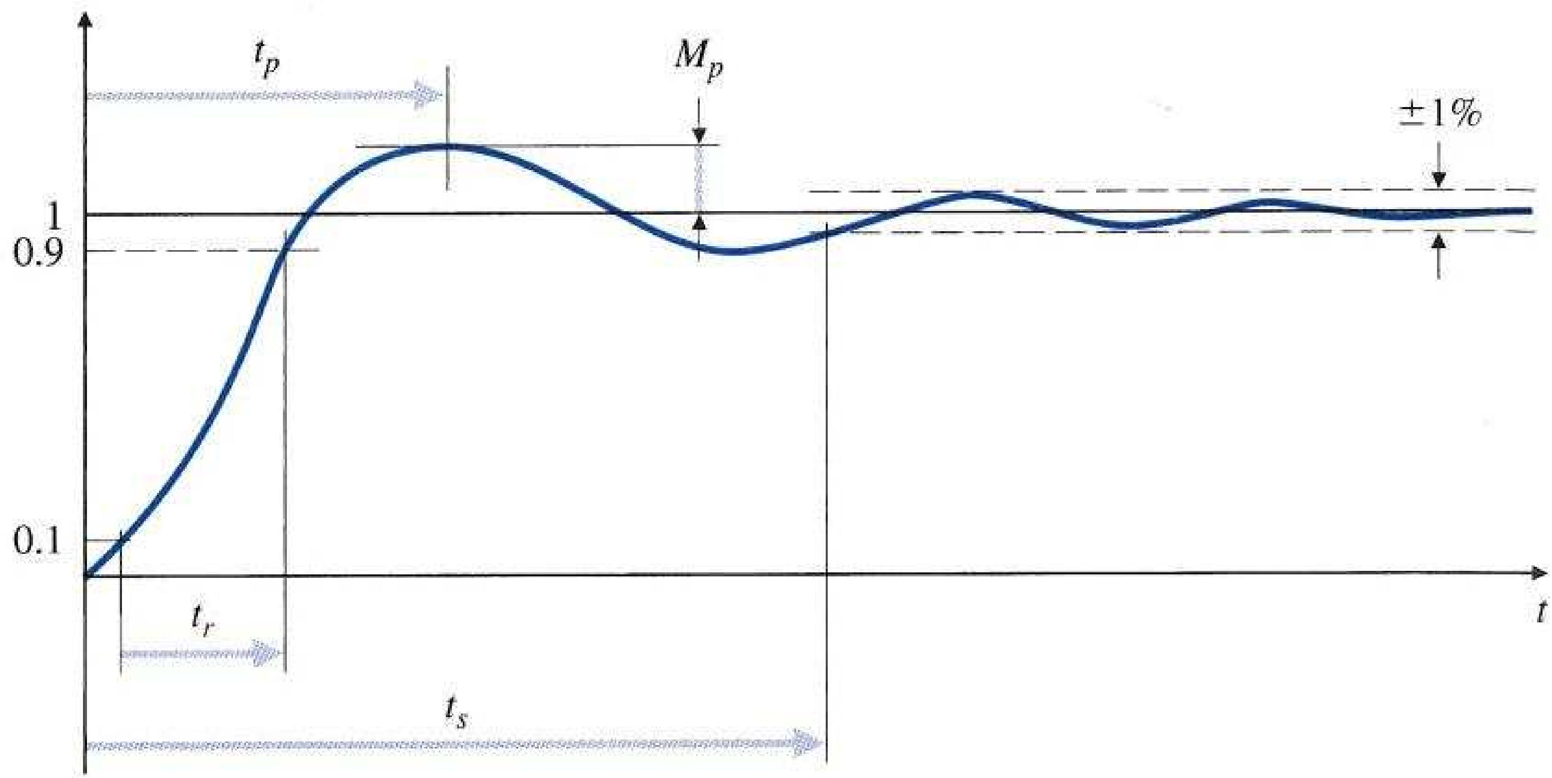
$$Closed \quad loop: \qquad T(s) = \frac{Y(s)}{R(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)H(s)}$$

$$Open \quad loop: \qquad L(s) = D(s)G(s)H(s)$$

$$Direct \quad term: \qquad D(s)G(s)$$

$$closed \quad loop = \frac{direct \quad term}{1 + open \quad loop}$$

Time domain specifications



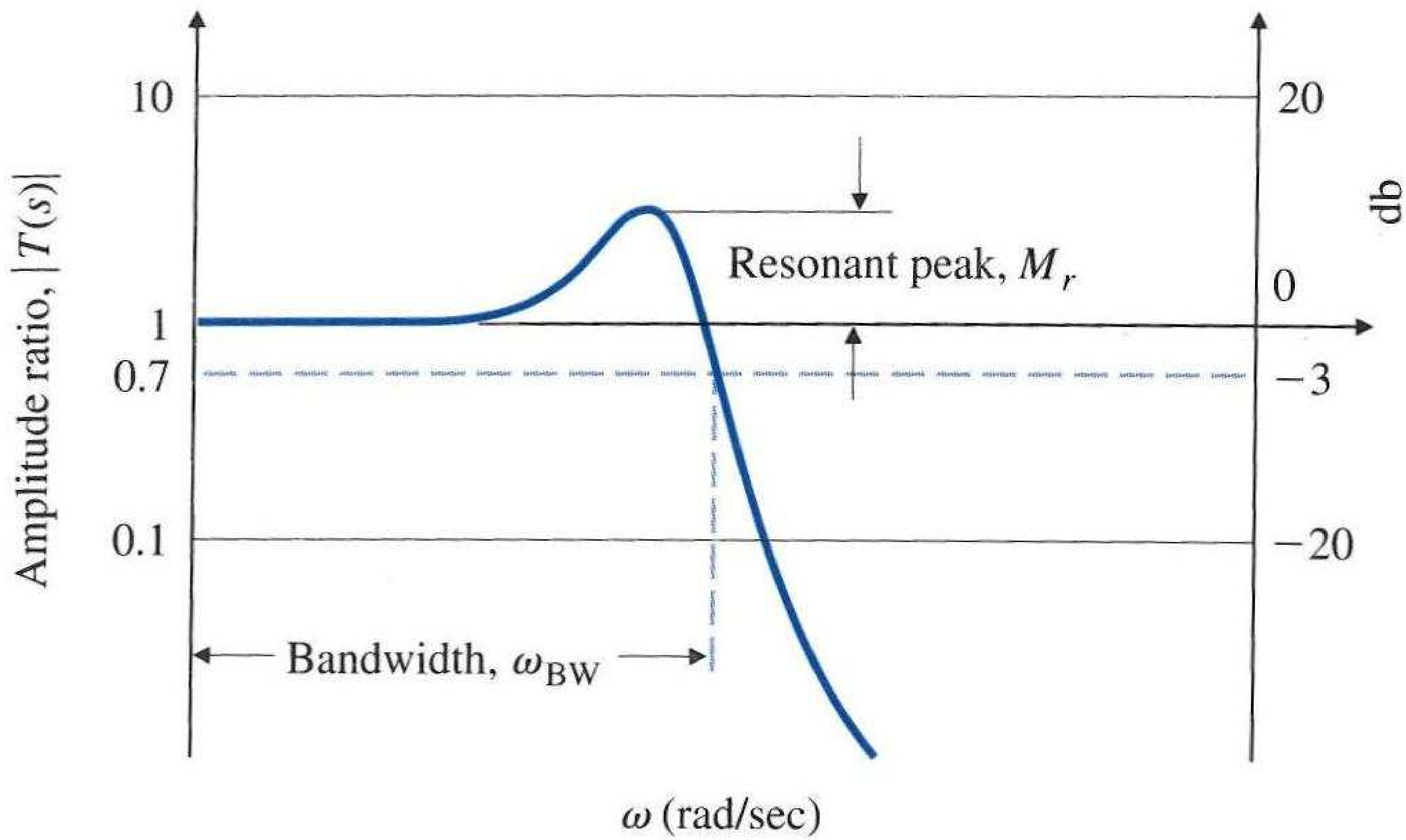
Rise time $t_r \sim$ stigetid

Settling time $t_s \sim$ indsvingningstid

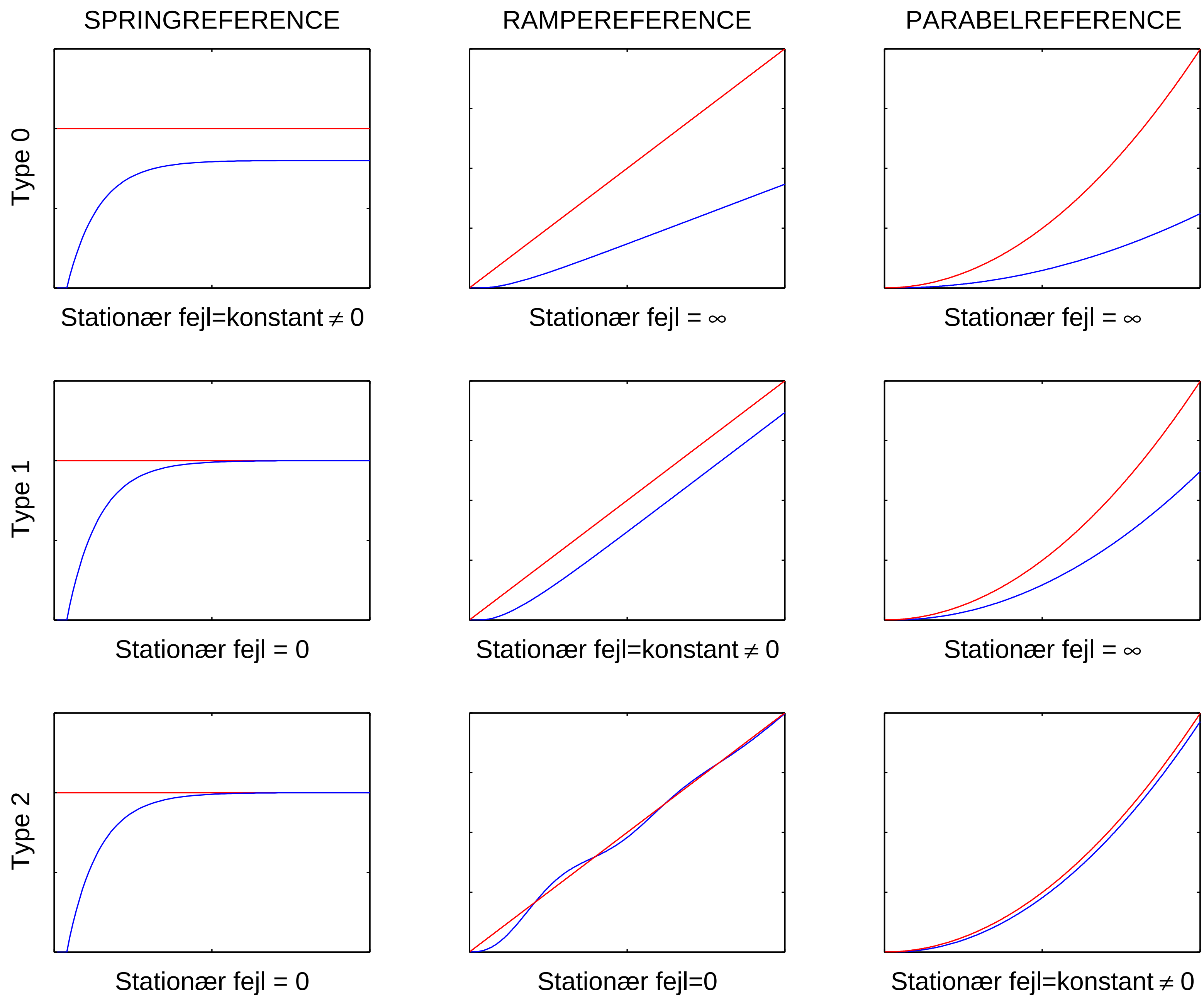
Overshoot $M_p \sim$ oversving

Frequency domain specifications

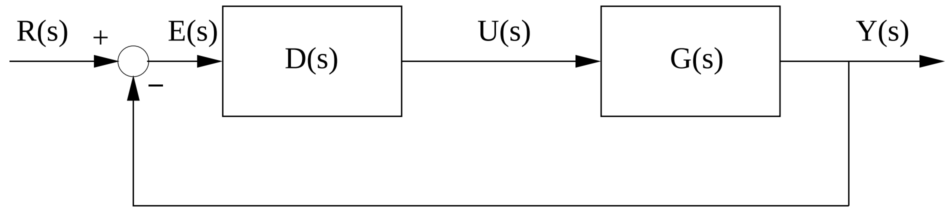
CLOSED LOOP



Steady state errors system type



Special Case: $H(s) = 1$

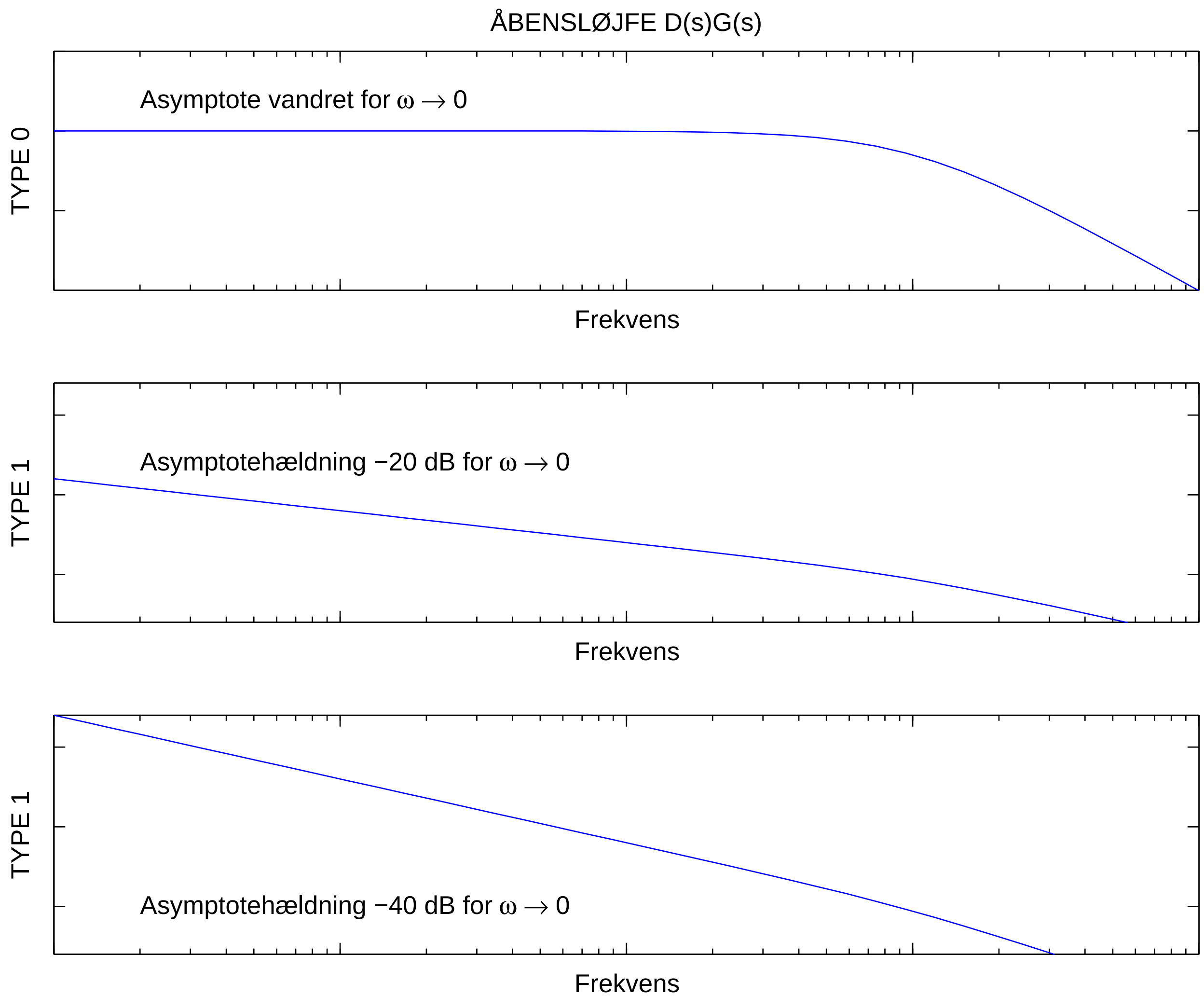


System type=number of poles in 0 in $D(s)G(s)$.

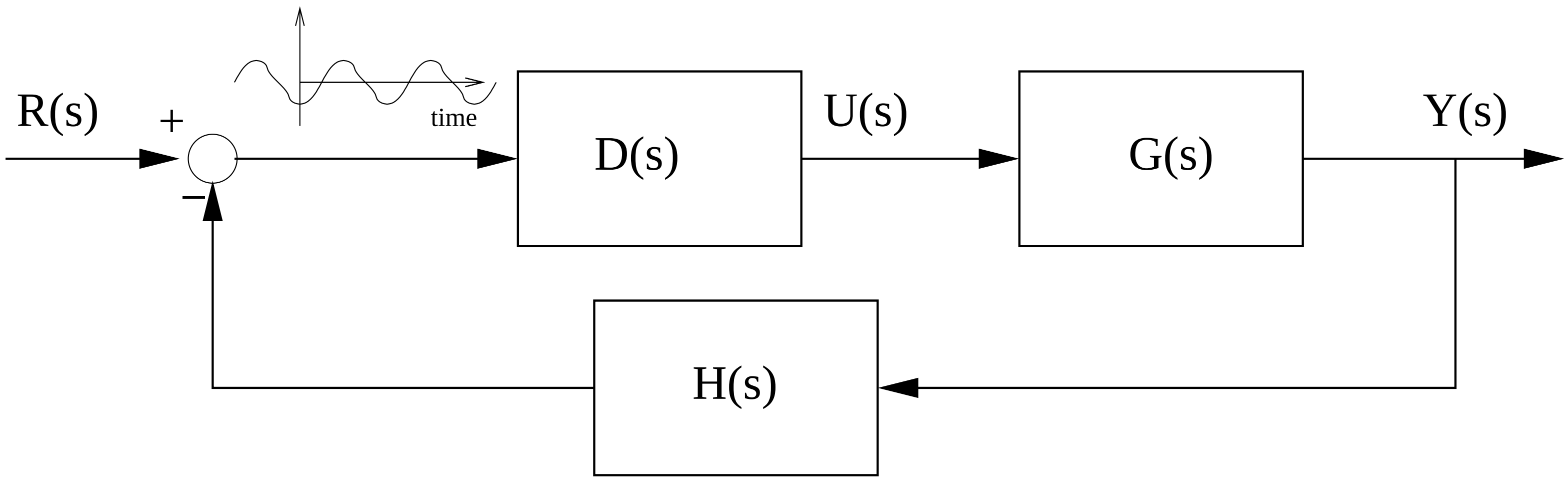
Example:
if $D(s)G(s) = \frac{10}{s(s+5)}$ a pole in 0 (an a pole in -5) system type = 1.
If $D(s)G(s) = \frac{10}{(s+10)(s+5)}$ poles in -10 and -5 system type = 0.

Notice that $D(s)G(s) = L(s)$ OPEN LOOP

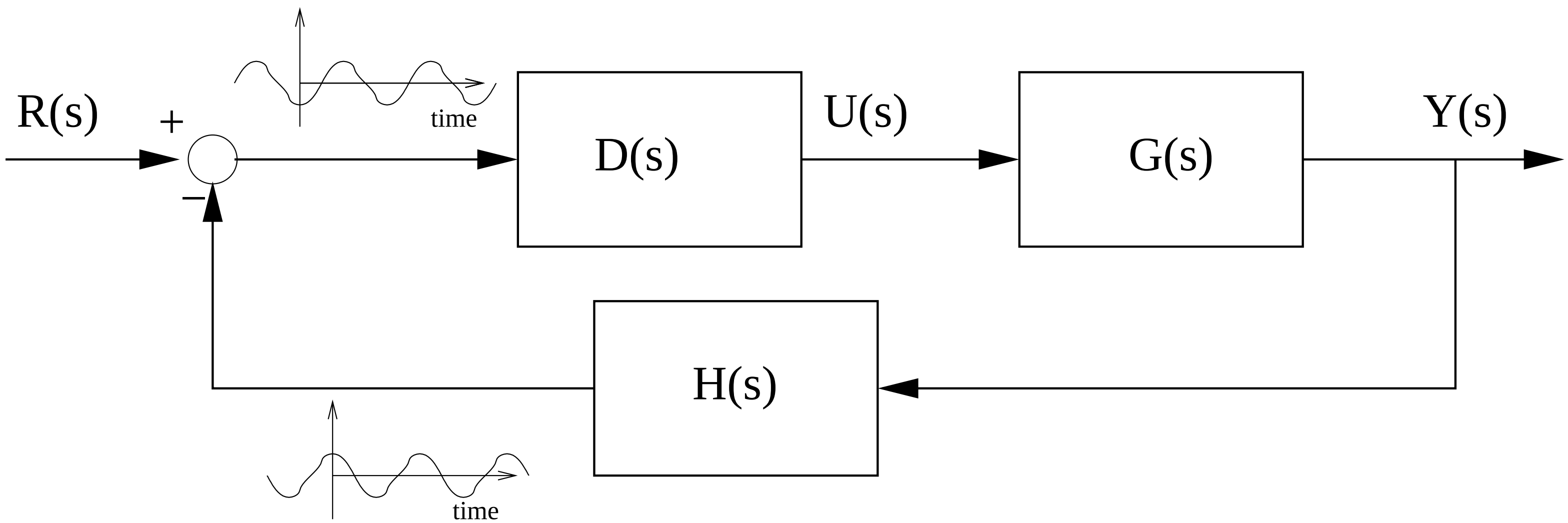
Special Case: $H(s) = 1$



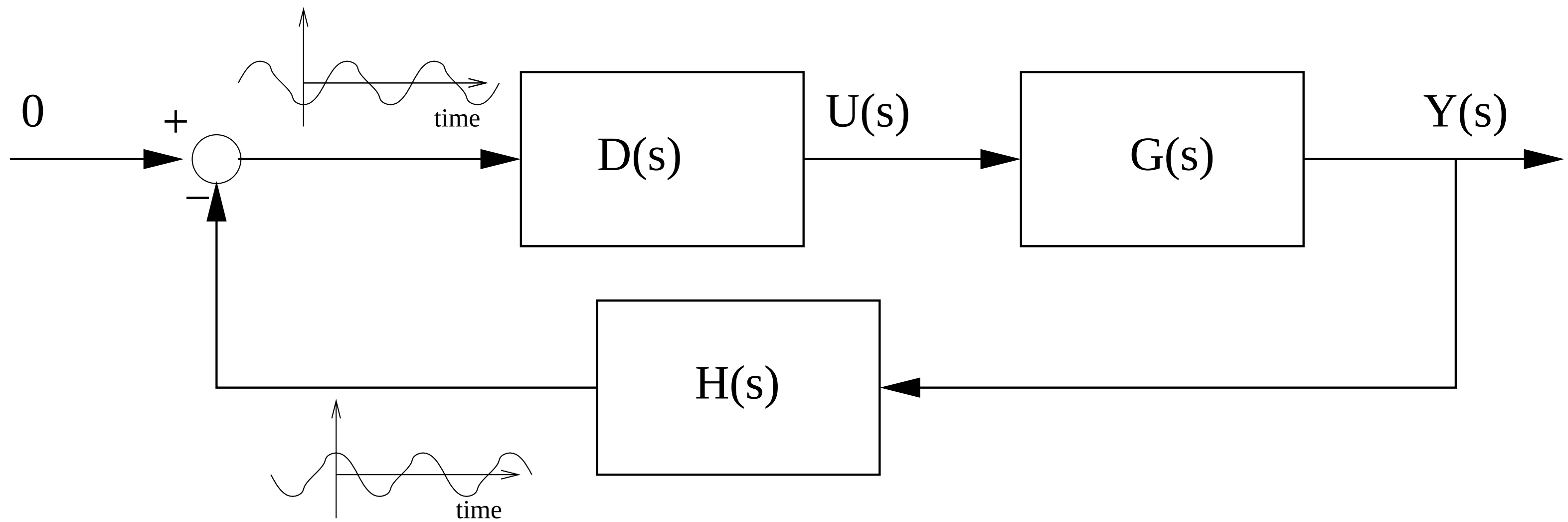
Frequency domain specifications



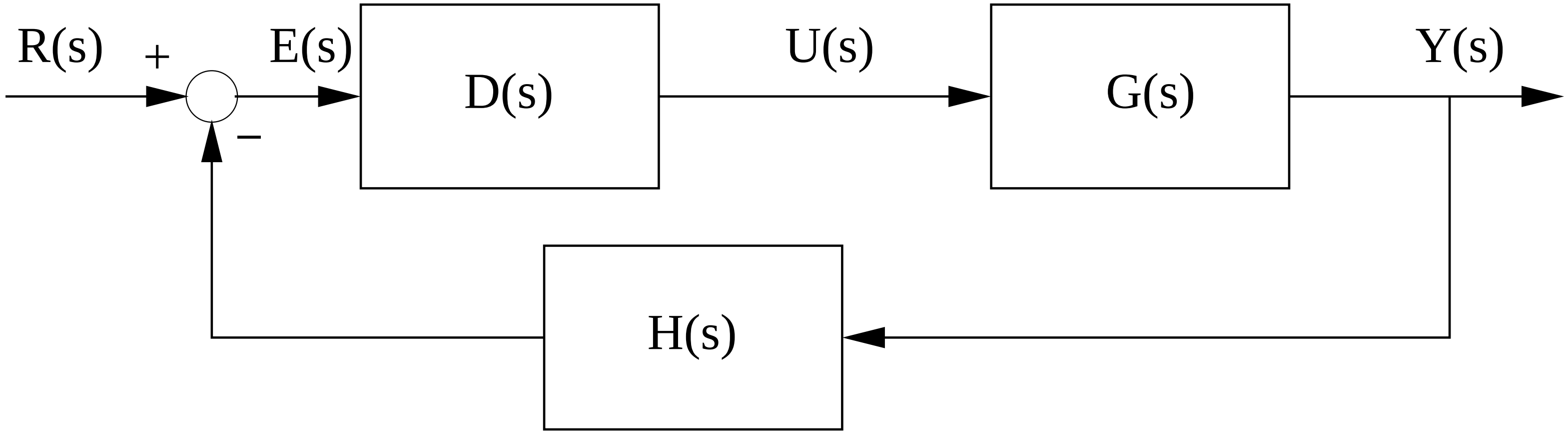
Frequency domain specifications



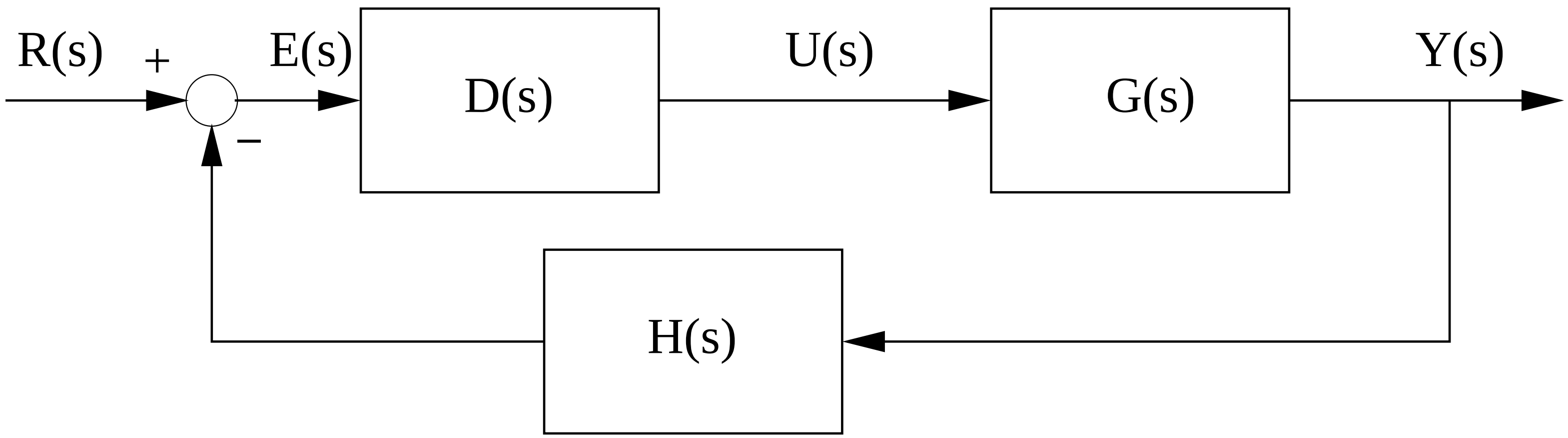
Frequency domain specifications



Frequency domain specifications



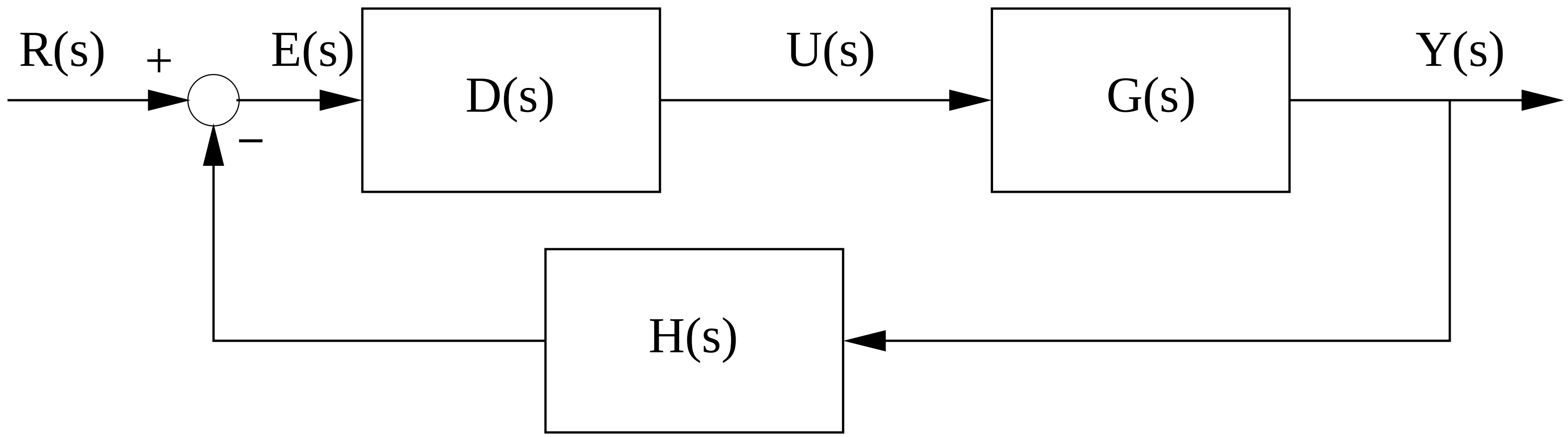
Frequency domain specifications



Oscillating:

$$D(s)G(s)H(s) = -1 = \ 1 \ \angle -180$$

Frequency domain specifications



Oscillating:

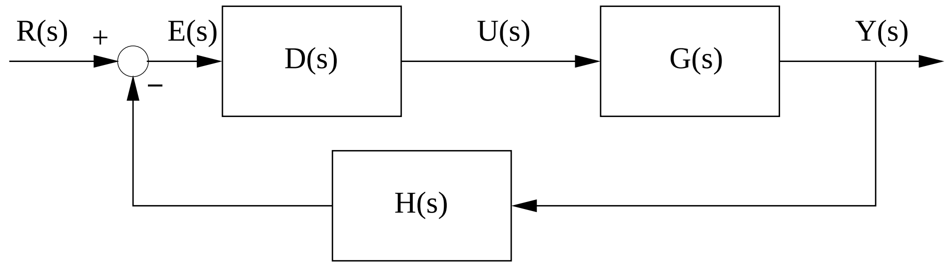
$$D(s)G(s)H(s) = -1 = \ 1 \ \angle -180$$

Unstable:

$$D(s)G(s)H(s) = \ > 1 \ \angle -180$$

If there is a frequency ω_1 where the phase, $\angle D(\omega_1)G(\omega_1)H(\omega_1)$, is -180 grader then the gain, $|D(\omega_1)G(\omega_1)H(\omega_1)|$, must be smaller than 1 (0 dB) for stability.

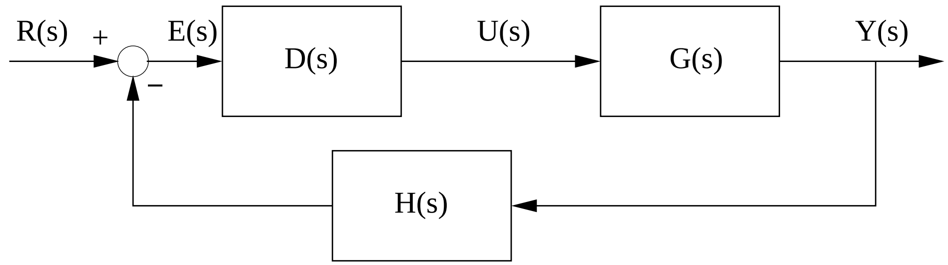
Frequency domain specifications



Oscillating:

$$D(s)G(s)H(s) = -1 = \ 1 \ \angle - 180$$

Frequency domain specifications



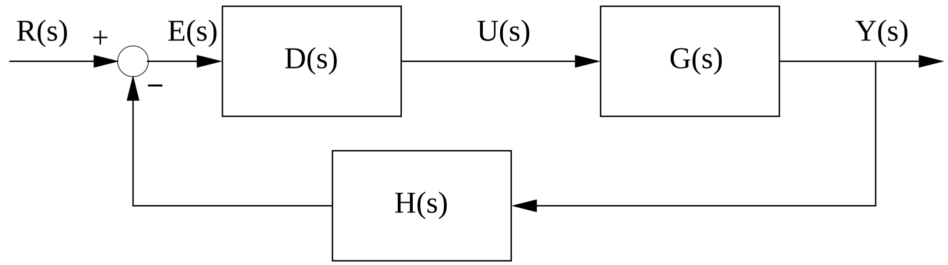
Oscillating:

$$D(s)G(s)H(s) = -1 = \; 1 \;\angle -180$$

Stability margins:

Gain margin (GM) is the number of dB missing before the gain is 0dB at the frequency where the phase is -180 degrees. Gain and phase is calculated on the open loop function!

Frequency domain specifications



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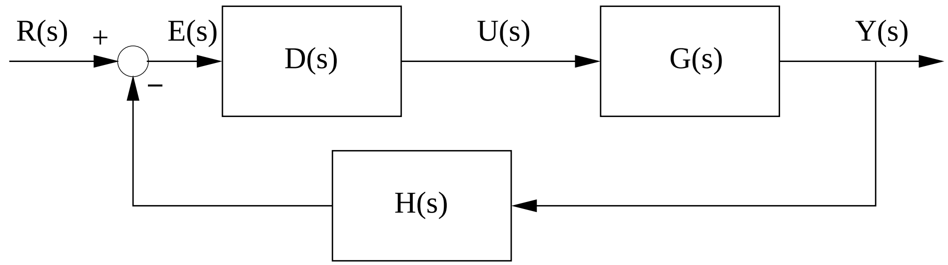
Phase margin, (PM) is the number of degrees missing before the phase is -180 at the frequency where the gain is 0 dB. Gain and phase is calculated on the open loop function!

Frequency domain specifications



Oscillating:

$$D(s)G(s)H(s) = -1 = \; 1 \; \angle - 180$$

Stability margins:

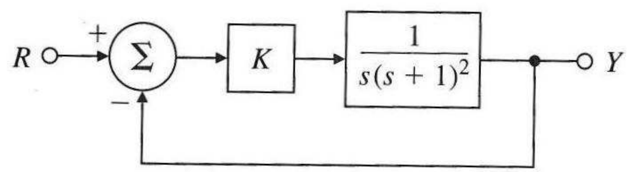
Gain margin (GM) is the number of dB missing before the gain is 0dB at the frequency where the phase is -180 degrees. Gain and phase is calculated on the open loop function!

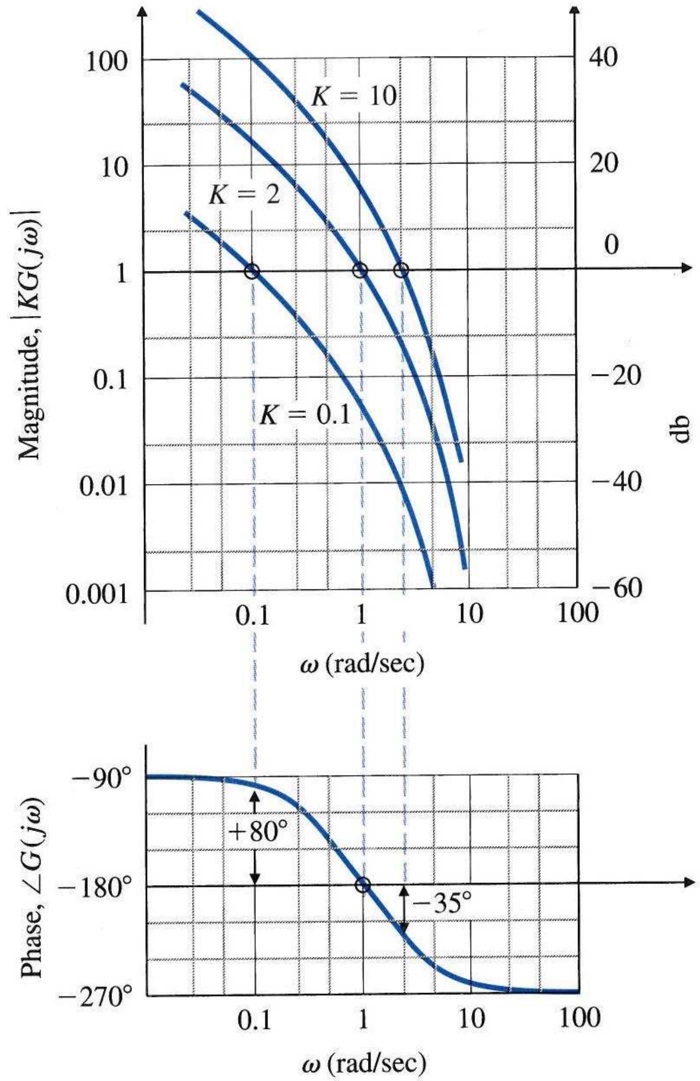
Phase margin, (PM) is the number of degrees missing before the phase is -180 at the frequency where the gain is 0 dB. Gain and phase is calculated on the open loop function!

Cross over frequency ω_c is the frequency where the gain is 0 dB.

Frequency domain specifications

Example





OPEN LOOP

- Gain margin
- Phase margin
- Cross over frequency

Controller specifications

Specifications:

Time domaine spec's	*Frequency domaine spec's*		*Type of spec.*
Closed loop:	Closed loop:	Open loop:	
Overshoot M_p	Resonant peak M_r	Phase margin PM	Stability
Rise time t_r	Bandwidth ω_{BW}	Crossover frequency ω_c	Dynamics
Settling time t_s			Dynamics
Peak time t_p	Resonant frequency ω_r		Dynamics
		Gain margin GM	Stability
Steady state error e_{ss}	Steady state error e_{ss}	Asymptote $\omega \to 0$	Steady state

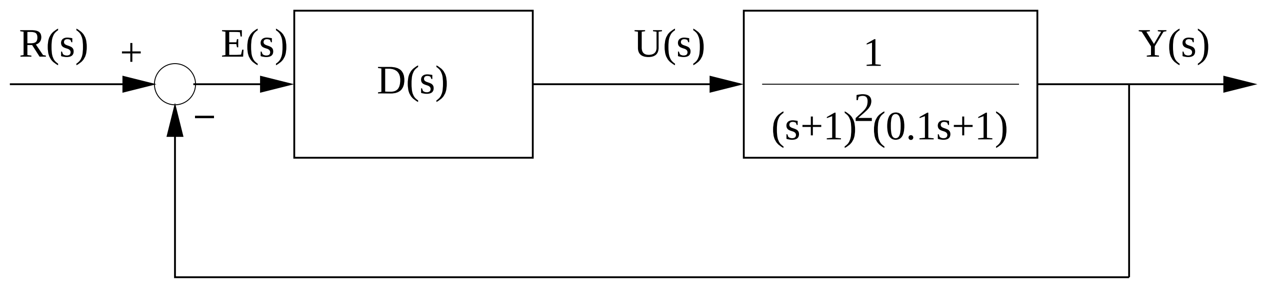
Controller specifications

For the standard 2. order cloosed loop system $T(s) = \frac{A}{s^2+2\zeta\omega_n s+\omega_n^2}$

Specifications	*Equations*
Overshoot,Resonant peak,Phase margin	$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$ $\quad M_r \approx \frac{1}{2sin\frac{PM}{2}}$ $\quad \zeta \approx \frac{PM}{100}$
Rise time,Bandwidth, Cross over frekvens	$t_r \approx \frac{1.8}{\omega_n}$ $\quad \omega_{BW} \approx 1.4 \cdot \omega_n$ $\quad \omega_c \approx 0.5 \cdot \omega_{BW}$
Settling time	$t_s = \frac{-ln(x)}{\zeta\omega_n}$ $\quad x = baand$
Peak time, Resonant frequency	$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$ $\quad \omega_r = \omega_n\sqrt{1-2\zeta^2}$

Frequency domain design

Design example

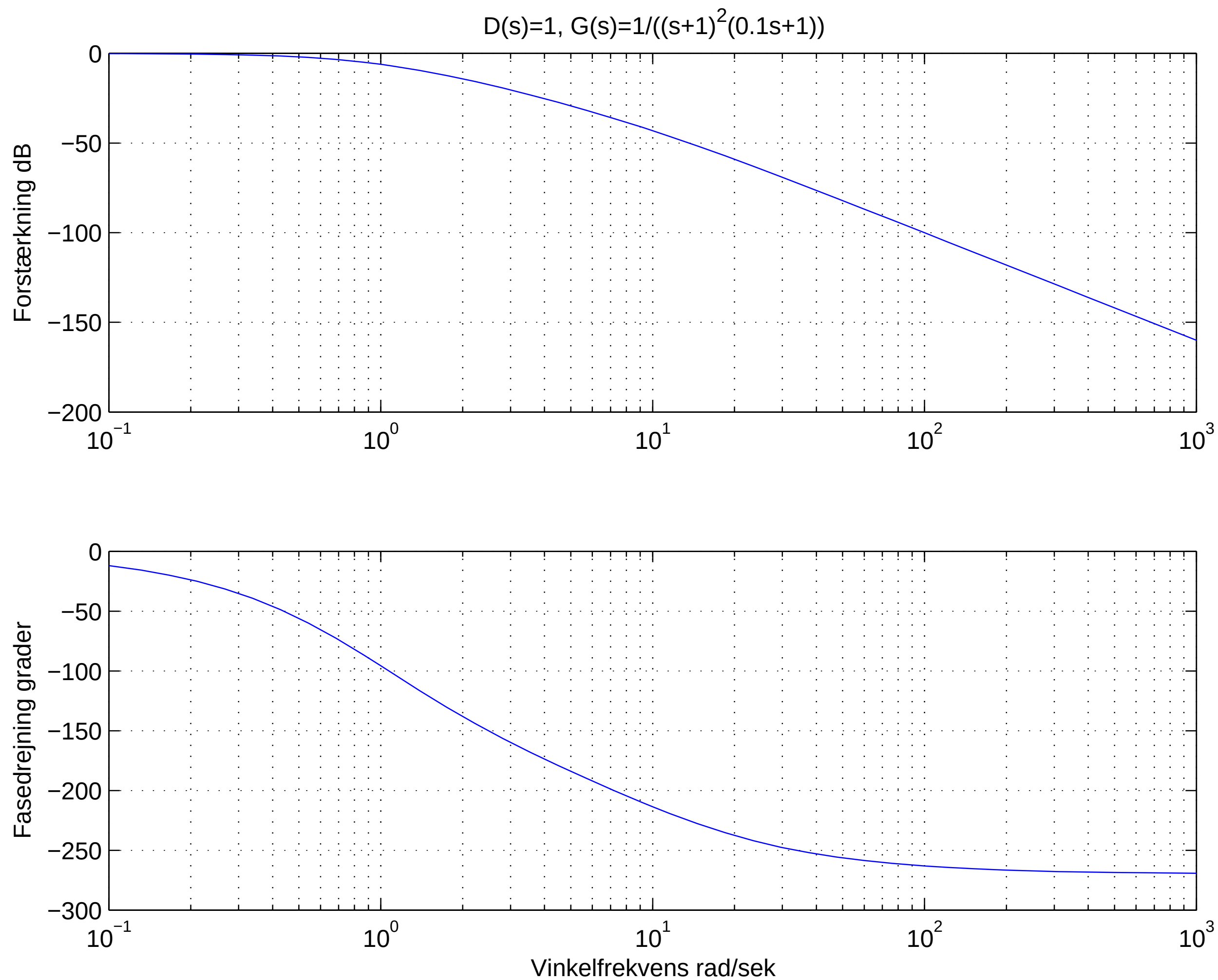


Specifications:

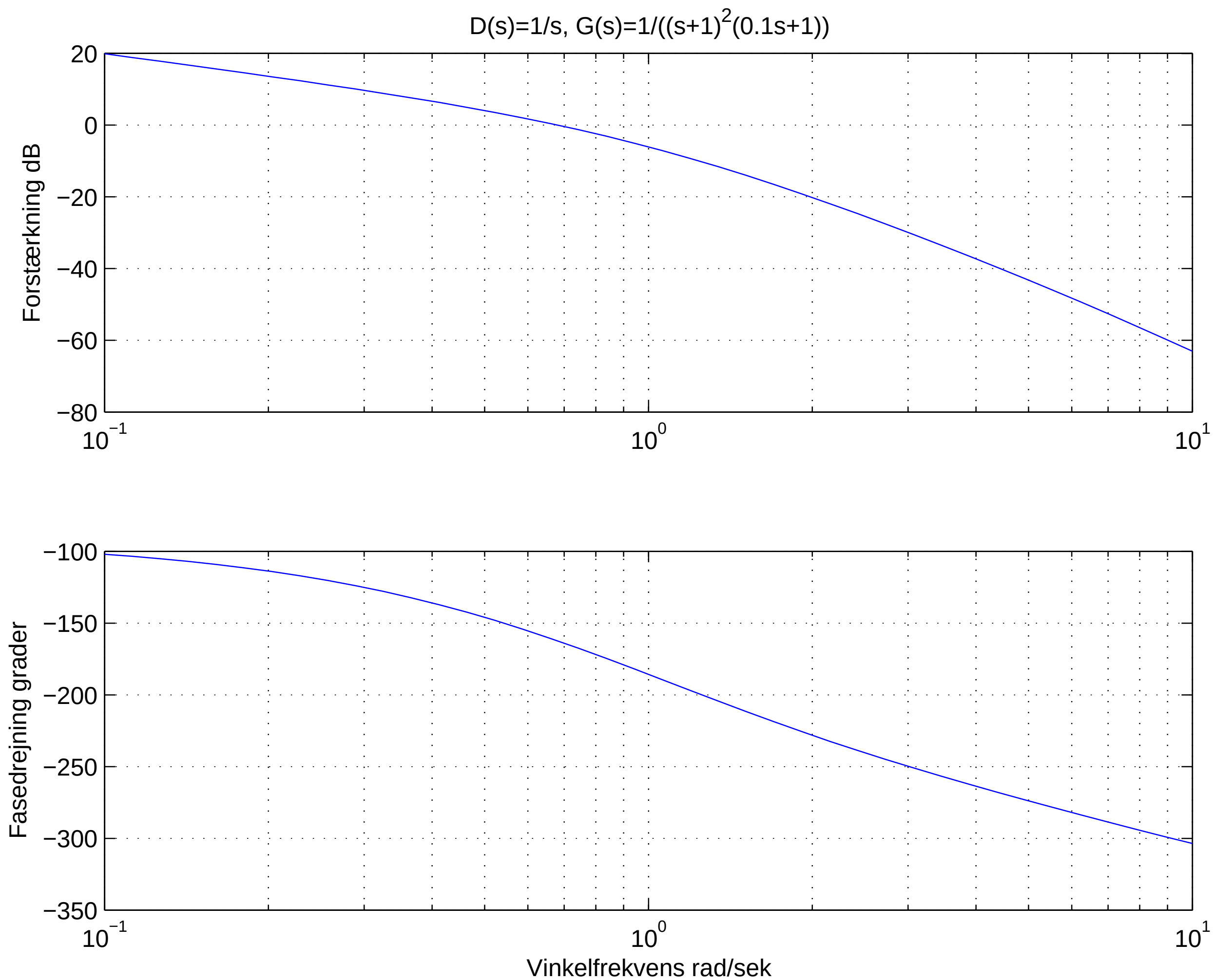
- Steady state error for step = 0
- Phase margin ≅ 45 degrees
- Cross over frequency > 0.5 rad/sek

Determine $D(s)$

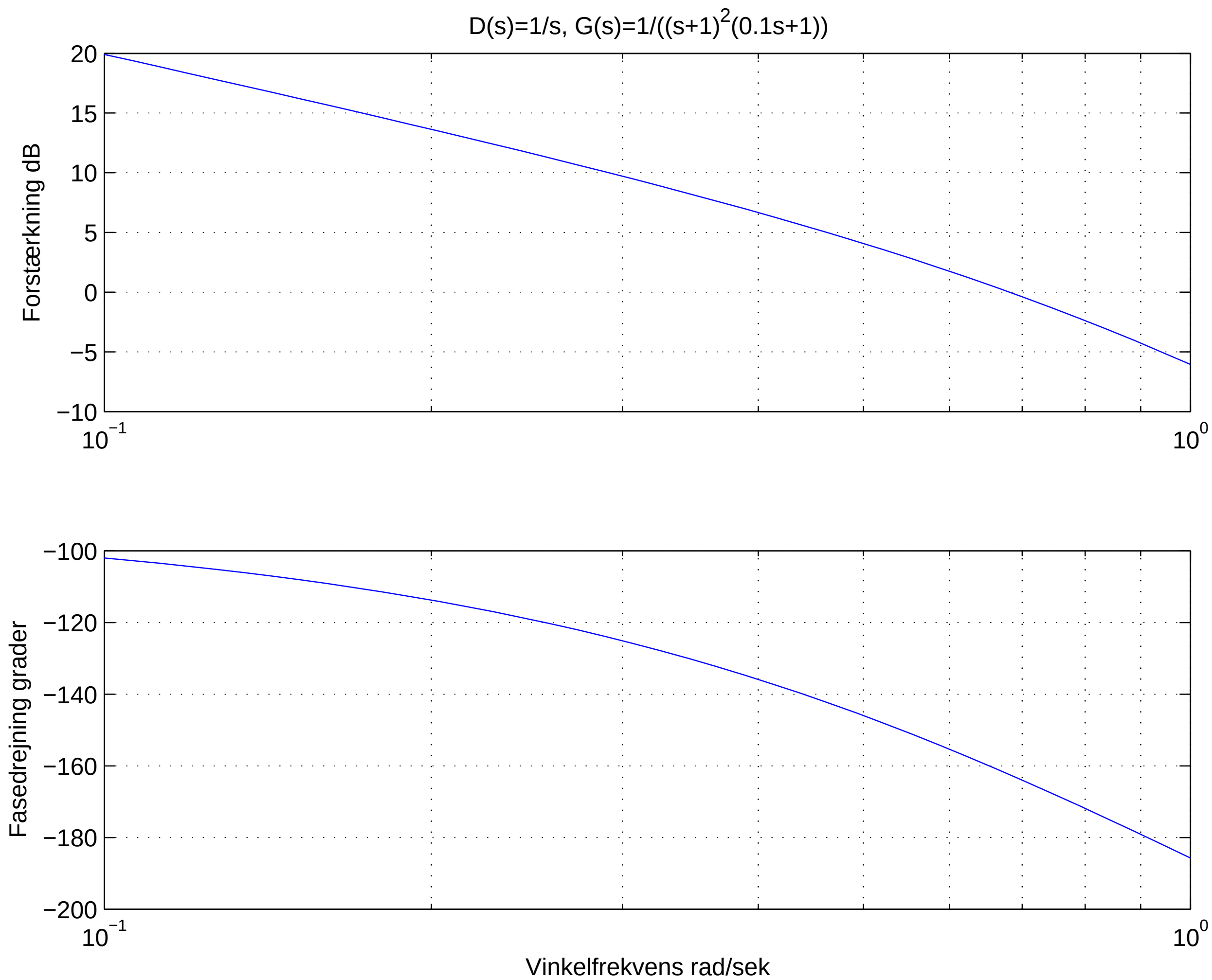
Frequency domain design



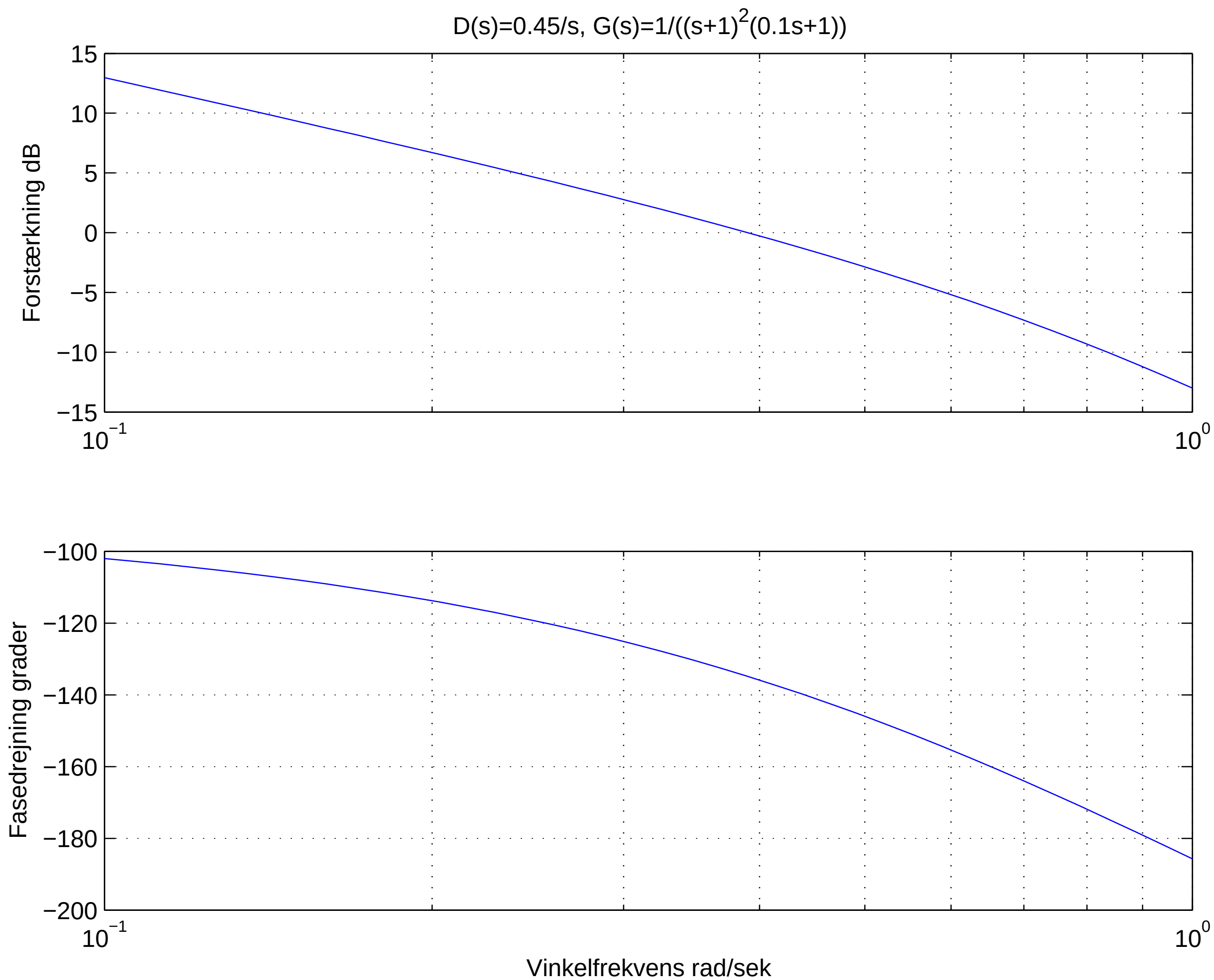
Frequency domain design



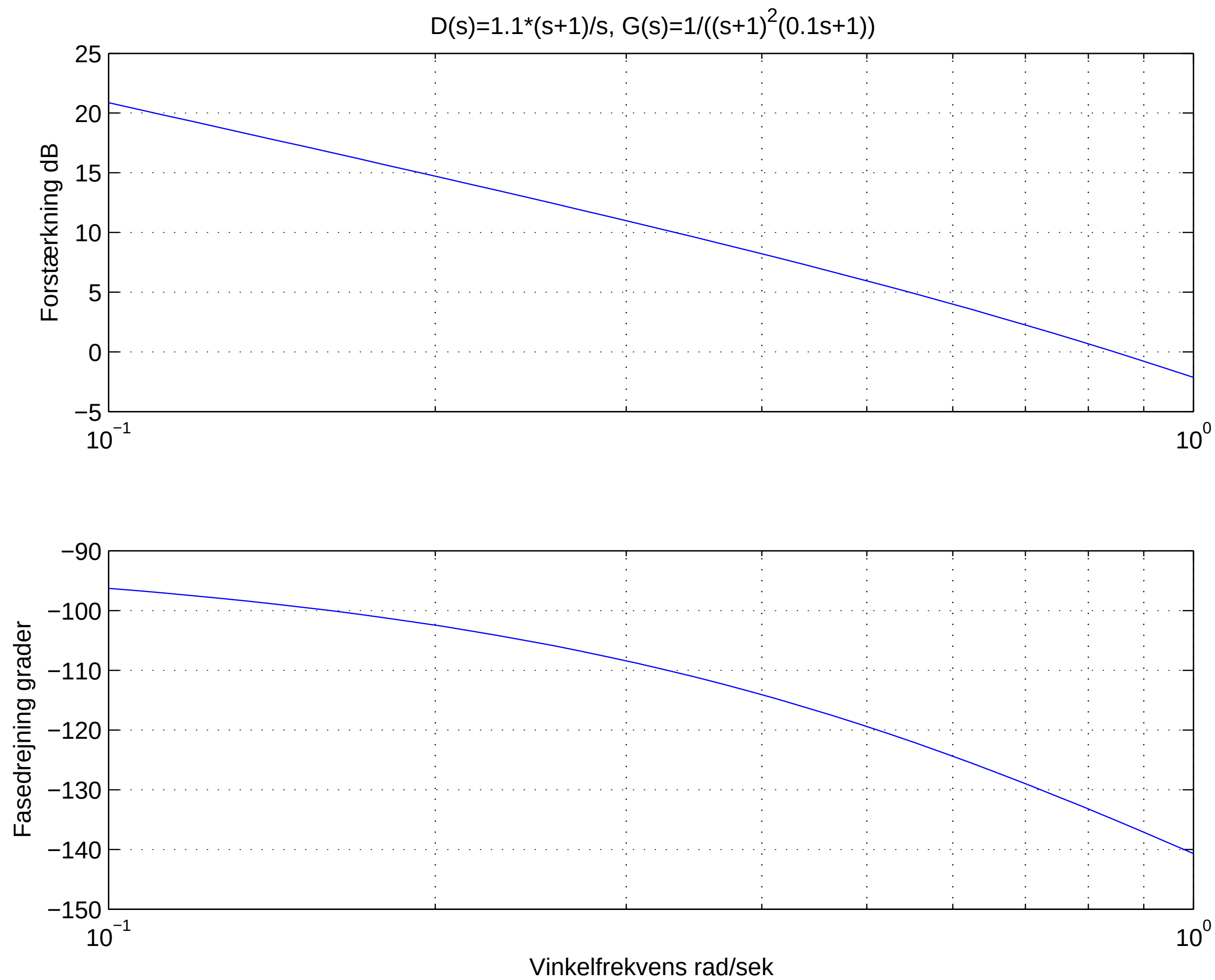
Frequency domain design



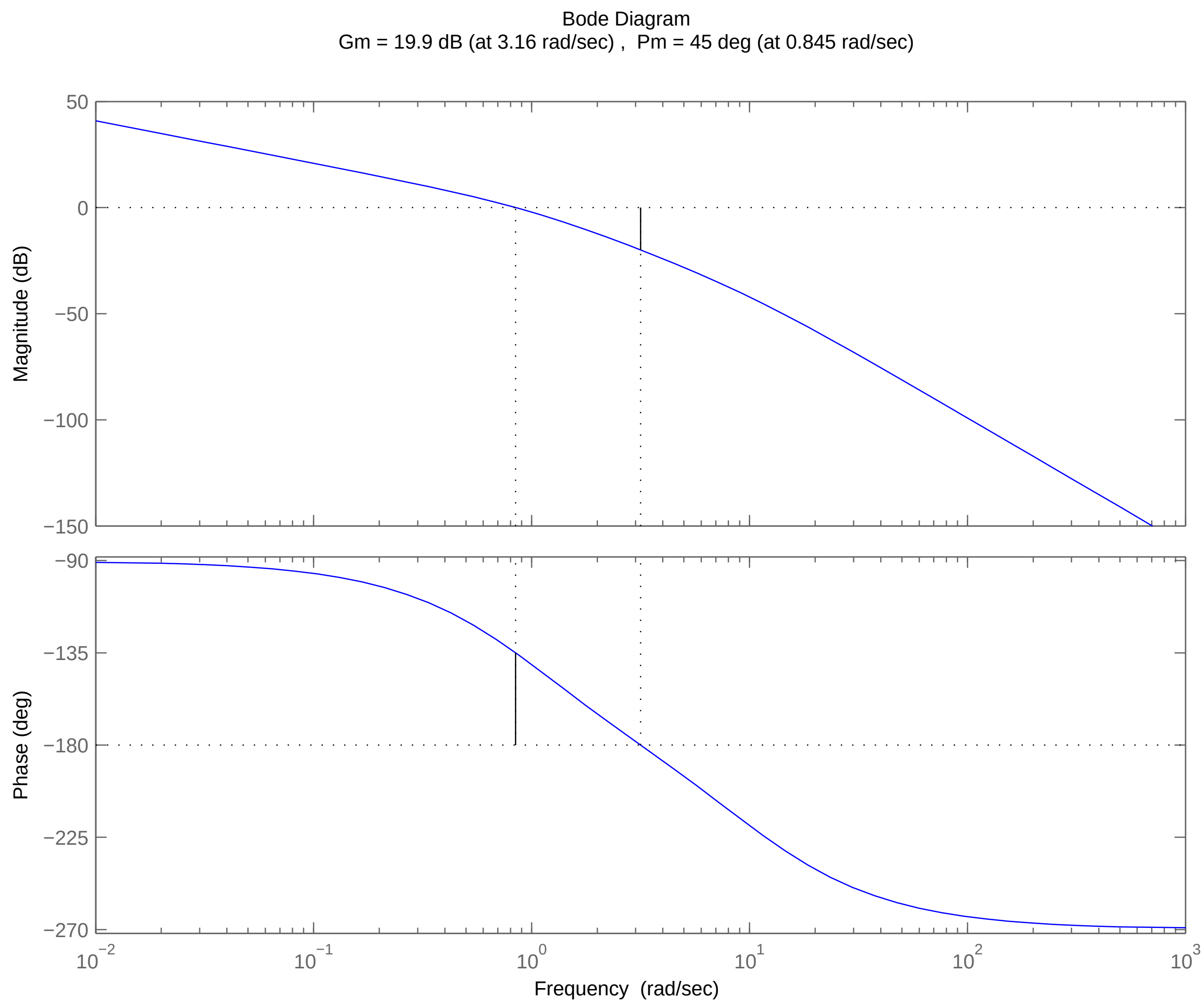
Frequency domain design



Frequency domain design



Frequency domain design



Computer implementation

Controller

$$D(s) = 1.11\frac{s+1}{s}$$

transform to computer form:

```
s=tf('s');
D=1.11*(s+1)/s;
G=1/((s+1)^2*(0.1*s+1));
T=feedback(D*G,-1); BW=bandwidth(T); % To determine samplingtime
Wsamplemin=20*BW; Tsmax=2*pi/Wsamplemin % We select 0.1 sek
Dd=c2d(D,0.1,'tustin') % to discrete filter form
[nomerator,denominator]=tfdata(Dd,'v'); % more digits
nomerator, denominator
```

$$u(k) = u(k-1) + 1.1655e(k) - 1.0545e(k-1)$$

Controller type

Controller

$$\begin{aligned} D(s) &= 1.11\frac{s+1}{s} = 1.11(1+\frac{1}{s}) \\ &= Proportional(1+Integral) = PI - controller \end{aligned}$$

A standard controller is:

$$\begin{aligned} D(s) &= K_p(1+\frac{1}{T_i \cdot s}+T_d \cdot s) \\ &= Proportional(1+Integral+Differential) \\ &= PID - controller \end{aligned}$$